Misperceptions

by

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Abstract
This dissertation studies the causes and effects of misperceptions of the past history of the economy.

The first chapter shows that, under mild assumptions on the processes of information diffusion, it is possible to sustain an Animal Spirits equilibrium with the stylized features of the Business Cycle. Agents use their perceptions of the past as a coordination device, but the sunspot is endogenous to the economy, as it is its dynamic stochastic structure.

The second chapter shows empirically that in the real world perceptions matter much more than reality. We show that once announcements on the past evolution of the economy are introduced (announcements that might be untrue) the real past evolution loses all predictive power. What happens in the economy is determined by what people think today that happened in the past, not by what actually happened.

The third chapter shows that as long as there exists informational externalities, even if they affect only to the second moment of the variables, societies overconcentrate investment and underinvest in research. The amount of information on the past evolution of the economy becomes suboptimal. We will study the intimate relationship between information disclosure and agents' homogeneity (both variables being endogenous to the model)

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Introduction

Economic agents need to make predictions of future events in order to take the best possible action. To assume that they have rational expectations should mean that they make the best possible use of their available information in order to predict the future. In itself rational expectations should not imply that agents possess an intimate knowledge of the world, but that they make the best use of whatever it is that they know of the world.

Nevertheless rational expectations models have always assume that agents knew the history of the economy, the realization of all past economic events. In this dissertation I study the consequences of this not being the case, show that this is not the case, and explain why this could never be the case.

Noise from the past

In most rational expectations models the sources of all uncertainty that agents might face lie on the future. Agents know the whole history of the economy, and they make the best use of this information. The only reason why their predictions not always became true is the existence of unforeseeable and exogenous events that will take place in the future. The exogeneity of the shocks (that is the source of all fluctuations of the economy) is a modeling necessity, it is there by construction.

In the real world agents are uncertain not only of the evolution of future events, but also of the realization of economic variables in the past. People does not know exactly what happened. Agents have perceptions, not certainties. Individuals capture information due to their own experience, and interchange this information with other agents. Each agent know his own experience and has some information on the experiences of other agents, but this does not allows
them to know exactly what happened in the economy as a whole. When they take decisions they will use all their information, and so their perceptions are a meaningful economic variable in itself.

In this dissertation I will show that this has important implications in the collective behavior of an economy. It will allow us to interpret uncertainty, and economic fluctuations, as something endogenous to the economy. It also will allows us to introduce a meaningful notion of heterogeneity. In this context agents are different because they have different perceptions, how much this perceptions differ is endogenous to the evolution of the economy, and intimately related to the amount of available information.

**Structure of the dissertation**

The dissertation consists of three intimately related papers. Each of them could stand on their own and be self-explanatory, but they all deal with different aspects of the same topic: uncertainty on past evolution of the economy.

I have arranged them in a chronological order. I have done so because it reflects the way that I jumped from a problem to the next. I think that each chapter is the logical step that should follow the previous one, but I am probably biased. I have come to understand the issues following this road, but that does not mean that it is the best possible one. In this section I will explain what each of the papers talks about, and let the reader take the path of his choosing.

**Spirits but not so animal**

In chapter 1 individuals use their perception of the immediate past in order to predict the future and take the appropriate investment decision. Doing this animal spirits equilibrium is generated.

In essence the model is a coordination game, a sunspot model. The peculiarity is that the sunspot, the coordination device, is neither mysterious nor exogenous. The perception that agents have of the past acts as such coordination device.

Each agent collects information by observing the actions taken by a sample of individuals, so the perceptions are noisy (no agent is ever sure that what she sees is what really happened)
and may differ across agents. The amount of noise that agents face is in itself an endogenous variable, a product of the state of the economy.

The model generates a dynamic stochastic structure of aggregate investment with the stylized features of the business cycle.

Believing in lies

This chapter, joint work with Paul Schulstald, is an empirical study of the degree in which perceptions affect the evolution of the economy.

As an instrumental variable for perceptions we use the announcements that the government makes on GNP growth. This announcements are subject to a substantial degree of noise, and its accuracy improves with time. Several years after the first announcement, a revised number is published. This is the value that we consider to be the “truth”, and that is widely used in econometric regression.

We show that once announcements (perceptions) are taken into account, the true value of GNP growth has no predictive power in determining the future evolution of the economy. All the predictive power lies in the announcements, and not in the true level of growth. Actually we show that the variable that determines future growth is the unexpected part of the announcements. We also show that the effects of the announcements work by the way of affecting aggregate investment.

Shared knowledge

This chapter returns to the topics of the first one, but from another perspective. It shows that the amount of information available to the agents is endogenous to the economy, and that this implies that agents overconcentrate their investment in a few sectors and the society as a whole underinvests in experimentation.

Agents learn from the experiences of other agents, not only from their actions; they have unbiased perceptions of the truth. Nevertheless individuals affect other agents’ beliefs because their actions affect the second moment of the informational variables. In other words, the variance of the noise in the perception of the return of any given activity is an increasing function of the number of agents engaged in such an activity.
As long as the level of social interaction and the underlying precision of the observations are relatively large, we should expect the agents to behave in a very precise way. This behavior is unmodified for a huge range of informational parameters and it is characterized by excessive concentration of the investment in a few sectors.
Chapter 1

Spirits, but not so Animal

1.1 Introduction

Keynes considered generalized waves of optimism and pessimism among entrepreneurs a key determinant of aggregate investment. This is in line with a very old tradition of thought that considers business cycles driven, at least in part, by changes in the expectations on the return of investments, changes that are not originated by changes in the fundamentals of the economy. This paper falls in the same tradition. It generates an animal spirits equilibrium. The novelty on the paper is that it does not use exogenous coordination devices in order to generate a behavior in the endogenous variables that resembles that of business cycles.

I consider that the best way of explaining why this model is interesting is to first put it on the table, and then locate it in the context of the modern literature on the topic. Therefore, I must ask for a leap of faith from the readers and immerse them in a sketch of the model before they have decided if it is or it is not worth the effort.

The basic set up of the model is the following:

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I have been helped by the comments and suggestions of Daron Acemoglu, Abhijit Banerjee, Olivier Blanchard, John Hassler, Jesús Guillermo Llorente Álvarez, Julio Rotemberg, Paul Schulstad, Àngel Serrat i Tubert, Lones Smith, Robert Solow and everybody at the MIT Macro and Theory Workshops. Obviously all remaining errors are product of my ignorance and/or really bad luck. If someone is reading this with acceptable interest, I also need to thank the Divine Providence.
• Agents want to coordinate with each other. It is worth to invest only if most contemporaneous agents do so.

• Agents have private information on the state of the economy in previous periods. This take the form of a noisy signal on the level of investment the previous period. We will assume that they observe the actions taken by a sample of agents in the previous period.

• Agents have a common prior on the state of the economy at any given time. In a rational expectations equilibrium this belief must be the unconditional distribution of aggregate investment.

• In an animal spirits equilibrium agents will coordinate using the past as a coordination device. This is, they will invest only if given their private information and common prior on past performance, they believe that the past was a good time to invest. This strategy is a rational expectations equilibrium if simultaneously happens that

  – The time series structure that produces in aggregate investment is consistent with the common prior of the agents.

  – It is optimal for each agent to follow the strategy if everybody else is doing it.

We will see that there exists an animal spirits equilibrium where all the agents invest only if their private signal tells them that yesterday was a good time to invest. This signal makes them believe that the future will be a good time to invest, makes them optimistic. In this equilibrium aggregate investment will follow a stochastic time series structure with the same stylized features of business cycles. This is, it shows persistence (auto correlation) and its spectrum will assign high variances to the frequencies of the business cycle.

Imagine that the agents were able to see exactly what happened in the economy during the previous period. A possible equilibrium of the game would be the strategy “Invest only if yesterday most people invested”, which is the same that “Invest only if yesterday was a good time to invest”. In this equilibrium the past behavior of the economy is used as a coordination device. Imagine that at \( t = 0 \) the aggregate investment takes an exogenously given value \( y_0 \). At \( t = 1 \), and always thereafter, all the agents invest if \( y_0 \) was large, or nobody invest if it was
low. The economy achieves a stationary state in one period, and never moves away from it. If the economy started from scratch, with random values for $y_0$, a very large number of times, half of the realizations would finish with all the agents always investing and the other half with no agent ever doing it. In equilibrium there is strong hysteresis, but not fluctuations.

In our model fluctuations will appear because in equilibrium agents will face uncertainty on what was the state of the economy in previous periods, and so in the future. To understand this result it is useful to make precise the exact stochastic structure of the model:

In most rational expectations models uncertainty affects only the future realizations of the economy. The information set of the agents at time $t$ includes all the realizations of all the variables up to that point. Uncertainty arises because future realizations of this variables might deviate from its deterministic path due to exogenous shocks. This is not the case in the present model.

In our model the unique source of noise lies on the private signals, and they refer to the past performance of the economy. There is neither aggregate productivity shocks nor mysterious stochastic coordination devices. We will use exclusively noise in the perception that agents have on the past realizations of the variables and the dynamic stochastic structure of this noise will be endogenous to the economy. In a way the economy in itself will generate the noise and uncertainty that agents face.

In the real world we should expect agents to be unsure on a lot of things of the past. Agents that invested and were successful might have been so because they had good luck; they might not know for sure if it was a good idea to invest. Obviously their perception of the past must be close to the truth, but realistically it should be subject to some kind of noise. They may believe that was good to invest because they see that a lot of agents did it, but it could be that their sample had over selected people that took the wrong decision. Thus this signals are informative, but potentially may be misleading.

In the model we will need a degree of aggregate noise in this perceptions. We will need the number of agents that observe erroneous signals to be a random variable. We will discuss in further detail the significance of this hypothesis and how reasonable it is. So far it will be enough to say that this will happen if the number of agents is small, or if the idiosyncratic signals are correlated across agents.
This is the only source of aggregate noise that the economy will face. The main point of the paper is that the time series stochastic structure of this noise is determined in equilibrium; at each given moment in time its distribution function is determined by the performance of the economy. Imagine that almost everybody is investing, and so it is good to invest; the probability that an individual receives a signal saying that it is bad to invest must be very low. On the other hand assume that half of the agents are investing and half are not, it is as much likely for the sample to capture a majority of investors than it is to capture a majority agents that did not invest. The signal is much more noisy.

Therefore, the signals are more noisy when there is imperfect coordination than when there is close to perfect coordination. This fact will be important for understanding later claims.

How much this noise affects aggregate investment depends on the equilibrium that the economy takes.

There will be equilibria where this noise does not affect either the information that agents have or the actions that they will take. If all the agents follow the strategies “invest whatever your signal is” (or “never invest whatever your signal”), the best response is clearly to follow the same strategy as everybody else. Additionally, the signal will be uninformative, because you know ex-ante that yesterday everybody invested (or nobody did).

On the other hand there will be an animal spirits equilibrium where the signals will affect the information that agents have, and the course of their actions. In this equilibrium agents will update their priors using their noisy signal. Suppose that an agent receives a signal that says that most people invested during the previous period (so the past was a good time for investing). It is rational for him to expect that most other agents would also observe similar samples. Therefore if he expects others to follow the rule “Invest only if you see a sample where most people invest”, he should invest only if he sees a sample where most people invest. Thus the strategy is an equilibrium.

Business cycles follow a well determined pattern whose main characteristics are that presents persistence and that frequencies corresponding to events of a periodicity of several years have a big incidence on the total variance of the process.

Our equilibrium presents business cycles characteristics because when almost everybody
is investing the variance of the signals is very low; they will almost always be accurate, and the economy will remain around there for quite a long time. Sooner or later some individuals will make mistakes\(^1\) (because the signals are well defined random variables), in so doing they will move the economy away, towards the center. Now the signals are more noisy, and so the economy more volatile.

In other words, aggregate investment will stay a long time in its extreme values, where the variance of the signals is small, and a short one in the center, where this variance is large. The economy may jump from high to low levels of investment every so often; in this way there will be persistence and the share on the total variance of the series due to things that repeat itself every several periods will be high; that is, the spectrum will assign high variance to the frequencies of the business cycle. The aggregate investment will follow a business cycle structure.

Note that what is partly driving the business cycle is the existence of an informational externality.

In the model there is no ex-ante reason for the agents to coordinate using the past as a focal point. We will see that if there were other "exogenous" reasons for the agents to imitate the past, the animal spirits equilibrium would get reinforced. If past investment affects positively future returns, agents will have additional reasons to follow their signals, and so additional reasons for coordinating using the past.

This paper is obviously related to the literature on sunspots, multiple equilibrium and self-fulfilling prophecies. Azariadis (1981), Azariadis and Guesnerie (1985) and others have stressed the existence of equilibria with self-fulfilling expectations. In an overlapping generations context, an increase in the expected level of inflation produces changes in labor supply and the demand of money that will indeed result in a higher inflation. Woodford (1988) presents a model where small perturbations around a deterministic steady state are susceptible of producing stationary sunspots equilibria. He also manages to get persistence in the level of capital stock

\(^1\)Agents never make mistakes in the sense that they always adopt the optimal policy. When I say mistake I mean ex-post mistake; even though they have reacted optimally to their information, this information was not accurate, and so there was other action that offered a higher payoff.
out of iid sunspot noise, because near steady state the levels of capital are very close to each other. The flow of output does not presents persistence though.

In a more Keynesian tradition Diamond (1982), Diamond and Fudenberg (1989), Bryant (1983), and Shleifer (1986) present multiple equilibria models. Cooper and John (1988) show that the necessary and sufficient condition for obtaining multiple Pareto rankable equilibria in static models is the simultaneous presence of aggregate increased returns and strategic complementarities. In essence, these models are coordination games. Agents want to do whatever other agents are doing. They want to invest if everyone is investing; they do not want to if nobody is doing it. This literature explains some of the more important characteristics of booms and recessions, such as the existence of unemployment, interpreted as suboptimality; however, the lack of an explicit dynamic structure makes these models unable of explaining business cycles in itself.

Several authors, Howitt and McAfee (1992) most significantly, have added a more dynamic structure to these models by introducing an exogenous coordination device. This coordination device is a random variable that can take two values high or low with probabilities derived from a markovian transition matrix, its realizations being observed by all the agents. When it signals high agents know that everybody will interpret it as meaning "time to invest", therefore they invest because everybody is investing. When it signals low exactly the opposite happens, agents became pessimistic and do not invest. Animal spirits, waves of optimism and pessimism, are generated by coordination on this exogenous signaling process. The aggregates of the economy have the time series structure of the signals.

The main drawback of this approach is that the business cycle is exclusively derived from this exogenous coordination device. The dynamic structure of the aggregates of the economy is assumed ex-ante and not derived within the model. The aggregates have persistence because the transition matrix that drives the coordination process is such that this process has persistence. Thus at best the propagation mechanism in this models is exogenous to the economy.

On the other hand since the seminal work of Kydland and Prescott (1982) the real business cycle-type of models rests on the powerful concept of the propagation mechanism. These models transform a noise with no structure (productivity shocks typically), into random variables (the endogenous variables) that have a business cycle type of structure. In other words, the model is
a filter that transforms a flat spectrum into one that assigns high variances to the frequencies of the business cycle. The concept of a propagation mechanism is intellectually very attractive, it provides us with a powerful thinking tool. It allows us to explore comprehensibly the business cycle as the product of random shocks.

The model presented here works in the same tradition. An animal spirits equilibrium, that presents the stylized characteristics of a business cycle, is obtained without the help of exogenous coordinating variables and in a context of forward looking rational behavior. This is generated out of a noise without time series structure, noise that only affects the informational parameters of the economy. The propagation mechanism lays on the process of information diffusion.

This paper is also related to recent work on social learning. In Banerjee (1992) and Bikchandani, Hirshleifer and Welch (1992) it is shown how when people try to make decisions based partly on actions taken by other agents, it is possible to obtain herding. Agents imitate each other because "they may know better" and this has been exploited by the herding literature in order to explain such thing as fads and fashions. Kirman (1992) shows how processes of information diffusion and learning can generate aggregate behavior similar to the one that we traditionally assign to business cycles. His model is not an equilibrium one. Agents meet successively and interchange information, there is no equilibrium mechanism that generates their behavior.

The paper that I present is not a herding model, but captures some of the characteristics of the these learning models. It is a coordination game, but its dynamic structure is endogenous to the model. Individuals try to learn from the actions taken in the past by other individuals, how much information do they get from their samples depends on what actually happened in the past. The fact that their signals are informative is in itself an equilibrium result.

The rest of the paper is organized as follows. In section 2, I present a very simple example of the type of model that I have in mind. I solve numerically a toy model that contains all the points that this paper attempts to raise. Section 3 is the core of the paper. I will present and solve a simple formal model that makes sense of everything said so far. The most tedious and forgettable proofs are relegated to appendices at the end. Section 4 addresses some of the possible criticisms of the model. Section 5 summarizes the basic points and discusses possible
1.2 In a small, noisy world.

This section introduces a model that captures the intuition expressed above. In order to do so, we will focus on an economy with a finite number of agents. We will see under what circumstances it is possible to build endogenous animal spirits, and therefore a business cycle-type of structure for the time series of the aggregate variables. We will then try to understand the mechanisms that generate the business cycles in this context.

1.2.1 Set up of the model.

Utility function and idiosyncratic shocks.

In each period there are \( n \) agents. The generation born at \( t \) lives for only one period. Each individual is born with an endowment of one indivisible unit of the single good of this economy, they either invest it on market activity or use it in home production.

This is a world of short and risky lives; calling \( x_t^i \) the investment decision of individual \( i \) of generation \( t \), and \( \bar{y}_t^i \) the average investment of other agents in period \( t \), the utility that individual \( i \) will receive is:

\[
U_t^i = \bar{y}_t^i x_t^i - (v - s_t^i)x_t^i
\]  
(1.1)

Where \( v - s_t^i \) is a parameter that represents the opportunity cost of not investing. This opportunity cost depends on \( v \in (0, 1) \), a parameter, and an idiosyncratic shock \( (s_t^i) \) that can take three possible values:

- \( s_t^i = v \)
- \( s_t^i = -1 \)
- \( s_t^i = 0 \)

Agent \( i \) invests if and only if

\[
E(\bar{y}_t^i) \geq v - s_t^i
\]  
(1.2)

As \( x_t^i \) is either 0 or 1, \( \bar{y}_t \) belongs to the closed interval \( [0, 1] \), and so:
• If $s^i_t = v$ (positive shock) then, because $\bar{y}^i_t \geq 0$, the optimal decision is $x^i_t = 1$

• If $s^i_t = -1$ (negative shock) then, because $1 \geq \bar{y}^i_t$, the optimal decision is $x^i_t = 0$

• If $s^i_t = 0$ (no shock) then the agent will invest iff

$$E(\bar{y}^i_t) \geq v$$  \hspace{1cm} (1.3)

We will assume that the aggregate shock is always zero, so the shocks are distributed as an urn model without replacement. In each period exactly $\alpha$ agents will receive a positive shock, and exactly $\alpha$ a negative one. The distribution of the shocks across individuals will then be hypergeometric.

**Informational structure.**

All agents are born with a common prior on the unconditional distribution of the aggregate level of investment. That is, they know the probability that in any given period of time the aggregate has a value $\bar{y}_t \in \{0, \frac{1}{n}, \frac{2}{n}, ..., \frac{n-1}{n}, 1\}$. This distribution will be called $q$. In equilibrium, to be defined below, this distribution must be the true unconditional distribution of the aggregate investment. Additionally agents receive private information. They observe the actions taken at period $t - 1$ by a sample of size $m$ ($m < n$) of individuals. They do not know the value of $\bar{y}_{t-1}$, but they will update their prior ($q$) using this sample.

For the sake of simplicity we will assume that this sample consists of $m$ extractions with replacement from the pool of agents. So the sample of each individual will follow a binomial distribution with parameters $m$ and probability $\bar{y}_{t-1}$.

**Space of strategies**

The space of strategies defined below refers only to agents with $s^i = 0$, because, as it is clear from above, all other individuals have dominant strategies. A strategy is a map from the number of individuals in an agent’s sample that invested last period, $w^i_t$, to the set of actions $\{1, 0\}$. We will explore the strategies of the type “invest iff $w \geq \bar{w}$”, other strategies could possibly sustain equilibria, but are probably uninteresting. The strategy is summarized in the threshold $\bar{w}$. 

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If \( \bar{w} = 0 \), then the strategy is "always invest" regardless of what you see. If \( \bar{w} = m + 1 \), then the strategy is "never invest". If \( 0 < \bar{w} \leq m \), then we are allowing for herding. If a strategy of this type is an equilibrium, then individuals will herd. The more agents are observed investing the more likely it is to invest. We will concentrate in pure strategies.

**Equilibrium concept**

A symmetric equilibrium is a strategy \( \bar{w} \), and a prior distribution on the aggregate \( q \); such that:

1. If all other individuals are playing with the strategy \( \bar{w} \), for me it is optimal to play the same strategy, given \( q \).

2. The prior \( q \) is the unconditional distribution of the aggregate output generated by the strategy \( \bar{w} \).

The first condition insures optimality, while the second implies rational expectations. In essence we are looking for Perfect Bayesian Equilibria.

In this model an agent would not care about what past agents did unless this can give him some information about what contemporary players will do. She cares about \( E(\bar{y}_i) \), and so individual \( i \) will try to estimate the probability that other individual \( j \) will invest. She will estimate \( \Pr(x_i^j = 1 \mid w_i^j, q) \), where \( w_i^j \) is the number of individuals that he observed in the last period.

We have an equilibrium if the strategy \( \bar{w} \) produces the unconditional distribution \( q \) and

\[
\Pr(x_i^j = 1 \mid w_i^j, q) \geq \nu \iff w_i^j \geq \bar{w}
\]  \hspace{1cm} (1.4)

That is, observing a sample where the number of agents investing is smaller than the threshold makes the expected profit of investment negative; while observing an amount larger or equal than it, makes it positive.

**1.2.2 Solution to the model.**

First we derive the process for aggregate output generated by a strategy \( \bar{w} \). Then we will use simulations to see what set of strategies can generate an equilibrium. We will see then in what
circumstances it is plausible to find "animal spirits".

Transition matrix and unconditional distribution of \( y_t \).

The number of agents that individual \( i \) observes investing as driven by a binomial distribution with parameters \( m \) and \( y_{t-1} \), so the probability of observing \( w_{it} = w \) is

\[
\Pr(w_{it} = w \mid y_{t-1}) = \binom{m}{w} (y_{t-1})^w (1 - y_{t-1})^{m-w},
\]

(1.5)

and the probability of observing \( w_{it} < \bar{w} \)

\[
\Pr(w_{it} < \bar{w} \mid y_{t-1}) = \sum_{w=0}^{\bar{w}-1} \Pr(w_{it} = w \mid y_{t-1}).
\]

(1.6)

In appendix A it is proven that if all the agents follow the strategy \( \bar{w} \) the Markov process that drives \( y_t \) is described by a transition matrix \( Q(\bar{w}) \) defined exclusively by \( \bar{w} \). \( Q(\bar{w}) \) is a \((n - 2\alpha + 1) \times (n - 2\alpha + 1)\) dimensional matrix, whose element \( i, j \) is

\[
\begin{pmatrix}
(n - 2\alpha) & (1 - \Pr(w_{it} < \bar{w} \mid y_{t-1}))^{y-\alpha} \\
(y_t - \alpha) & (\Pr(w_{it} < \bar{w} \mid y_{t-1}))^{n-2\alpha-(y-\alpha)}
\end{pmatrix}
\]

(1.7)

and denotes the probability that given that \( y_{t-1} = \frac{\alpha-1+i}{n} \), then \( y_t = \frac{\alpha-1+i}{n} \).

This Markov process has an unique ergodic set (there are no zeros in the stochastic matrix for suitable values of \( \bar{w} \)), this ensures the existence of an unconditional distribution for the process, denoted by \( q(\bar{w}) \).

Updating the prior.

When agent \( i \) observes \( w_{it} = w \), he will update his prior using Bayes rule:

\[
q(y_{t-1} = y \mid w_{it} = w) = \frac{\Pr(w_{it} = w \mid y_t = y) \times q(y_t = y)}{\sum_{y=\alpha}^{\frac{n-\alpha}{n}} \Pr(w_{it} = w \mid y_t = y) \times q(y_t = y)}
\]

(1.8)

Then the probability that he assigns to the event that other agents without a shock invest
is:

\[
\Pr(x_i^t = 1 \mid w_i^t) = \Pr(w_i^t \geq \bar{w} \mid w_i^t) = \sum_{y=a}^{n-\alpha} \Pr(w_i^t \geq \bar{w} \mid y) \times q(y \mid w_i^t)
\]  

(1.9)

The number of agents that will invest is \(\alpha\) (agents with positive shocks), plus the number of agents that have no shock and invest. This is a random variable that from \(i\)'s point of view follows a binomial distribution with parameters \(n - 2\alpha - 1\), and probability \(\Pr(x_i^t = 1 \mid w_i^t)\). So given his information:

\[
E(y_i^t \mid w_i^t) = \frac{\alpha + (n - 2\alpha - 1) \times \Pr(w_i^t \geq \bar{w} \mid w_i^t)}{n - 1}
\]  

(1.10)

They will invest iff

\[
E(y_i^t \mid w_i^t) \geq v \quad \Leftrightarrow \quad \Pr(w_i^t \geq \bar{w} \mid w_i^t) \geq \frac{v(n - 1) - \alpha}{n - 1 - 2\alpha}
\]  

(1.11)

If \(v = \frac{1}{2}\), then they will invest iff

\[
\Pr(w_i^t \geq \bar{w} \mid w_i^t) \geq \frac{1}{2}
\]  

(1.12)

In order for \(\bar{w}\) to be an equilibrium, if \(v = \frac{1}{2}\), it must be true that:

\[
\forall w_i^t \in \{0, 1, ..., \bar{w} - 1\} \rightarrow \Pr(w_i^t \geq \bar{w} \mid w_i^t) < \frac{1}{2}
\]  

(1.13)

and

\[
\forall w_i^t \in \{\bar{w}, \bar{w} + 1, ..., m\} \rightarrow \Pr(w_i^t \geq \bar{w} \mid w_i^t) \geq \frac{1}{2}
\]  

(1.14)

### 1.2.3 Equilibria

When \(v\) is around \(\frac{1}{2}\) there are three equilibria for this game, the obvious “never invest” and “always invest” (in our nomenclature \(\bar{w} = 0\) and \(\bar{w} = m+1\) respectively) and an unique herding equilibrium, that is the interesting one because it is susceptible of generating “animal spirits”.

The animal spirits equilibrium corresponds with the strategy “invest if half or more of your sample has invested”. This is so because this strategy generates an unconditional distribution of the aggregate that is perfectly symmetric; i.e., ex-ante the economy could be in the “good” or
in the “bad” state with equal probability. If knowing this you see more than half of your sample investing, an agents posterior of being in the “good” state will be higher than the posterior of being in the “bad” one. You think then that a lot of individuals are observing samples similar to yours, with more than half of the agents investing, this will induce them to invest, and so induces you to invest. It is the belief that people are optimistic that makes you optimistic. It should then come as no surprise that in all the simulations done with \( \bar{w} = \frac{n}{2} + 1 \) (when this is an integer) an equilibrium is found.

There is no other equilibrium herding strategy because a small asymmetry in the strategies produces a large asymmetry in the unconditional distribution. If the strategy is biased towards investment (i.e., \( \bar{w} \) smaller than \( \frac{n}{2} \)) the prior is that it is extremely probable that almost everybody with no shock is investing. Even if an agent does not see a single individual investing, this is not enough to make you believe that most of the people are seeing less than \( \bar{w} \) investors, so it is not optimal for you to follow the strategy, so it is not an equilibrium.

This can be seen using simulations. Most of the simulations presented are expressed in four graphs. The one in the NW corner represents the distribution probabilities of \( y_t \) conditional on \( y_{t-1} \) being \( \alpha, \frac{n}{2} \) and \( n - \alpha \) respectively. The one in the NE corner represents the unconditional distribution of the aggregate. The one in the SW corner is a simulation obtained from the transition matrix of the process. And the one in the SE corner indicates if the strategy is an equilibrium; if it is so, the value of the function plotted (Pr(\( x_t^j = 1 \mid w_t^j \)) as a function of \( w_t^j \)) has to be bigger than 0.5 for \( w \geq \bar{w} \) and smaller for the rest of the integer values.

In figure 1-1 the strategy is not symmetric, the SE corner indicates that they are not an equilibrium. It can be sustained as an equilibrium only if the value of \( v \) is very high because the strategy is biased towards investment.

The equilibrium herding strategy is an equilibrium for a continuum of values of \( v \) for which

\[
\Pr(x_t^j = 1 \mid \bar{w} - 1) \leq \frac{v(n-1) - \alpha}{n-1 - 2\alpha} < \Pr(x_t^j = 1 \mid \bar{w})
\]

For a continuum of values of \( v \) around \( \frac{1}{2} \) the strategy is an equilibrium, because the symmetric unconditional distribution implies that observing more than \( \frac{n}{2} \) investors produces a posterior larger than \( \frac{1}{2} \); and observing less than that, of less than \( \frac{1}{2} \).
Figure 1-1:
1.2.4 Idiosyncratic risk, individual uncertainty, and animal spirits

The essence of the herding equilibrium implies a notion of persistence. When people imitate their ancestors' actions the ancestors' outcome is likely to be repeated, yet volatility appears as a consequence of the noisy structure of the model. Agents are subject to a high degree of uncertainty, and this allows for misrepresentations that with the time can turn out to be true. Agents can have misconceptions about the past because the sample that they get is relatively small. The existence of idiosyncratic risk (the shocks) generates the possibility that, even if everybody without shocks is producing, a relatively large number of individuals see low levels of investment, and thus becomes pessimistic, and thus decides not to invest.

When a group of agents believe wrongly that the previous period was pretty bad they assume that most people believe what they do, and so believing that others are pessimistic, they themselves become pessimistic. The past acts as a sunspot.

The point is that after they stop investing there are more than $\alpha$ agents not investing and it is easier that the process is repeated; it becomes easier that a bigger set of individuals decide that bad times are ahead. In taking a decision individuals also affect the beliefs of the people observing them, and so their actions. Therefore movements from the good to the bad state (and vice versa) become an event of positive measure.

In summary, volatility arises because:

- Individuals suffer from high levels of uncertainty and can make mistakes. The level of individual uncertainty increases with the idiosyncratic risk and decreases with the sample size.

- The number of individuals that make mistakes is in itself a random variable (a consequence of the world being "small")

There is some aggregate noise, but ex-ante it has no structure. It is the model in itself that gives a business cycles type of stochastic structure to the aggregate noise.

When the economy is "not too noisy" the unconditional probability distribution is "double-bell-shaped" with the probability of getting an average value being almost zero (see figure 1-2).
Figure 1-2:

This indicates that when the economy is in a state, good or bad, it is almost impossible to leave it. Ex-ante the economy is as likely to be in the good as in the bad state, but once it arrives to one of these states, it gets locked-in. It is very unlikely that an agent makes a mistake, and even if she does it is most likely that she will not induce other agents to make mistakes, because their sample is significantly large.

If the economy were played again and again from scratch, half of the time it would be in the good state forever, and half of the time in the bad state forever. The strategy is the same, but the outcomes are radically different in one circumstance or the other. This is in line with the traditional results of the herding literature; small perturbations at the beginning of the process generate strong consequences forever, the process presents strong hysteresis.

When the level of uncertainty that individuals confront increases (either because there is more idiosyncratic risk, or a smaller sample) this stops being true. The unconditional distribution is still double-bell-shaped, but now the probability of getting average values is away from zero (see figure 1-3).
In this case the economy is literally "jumping" from one state to the other every so often. When it is in one state it will tend to remain there for quite a long time, but some times a cascade will be generated and the state could change. It is the relatively high level of individual uncertainty what allows a relatively high number of mistakes to be produced every so often, and so volatility is generated. When the economy is in an extreme the individuals have a low but significant level of uncertainty, most of the time they won't make mistakes, but every so often it will happen. The closer the economy is to the center, the more likely mistakes are; the signal becomes less informative in average values because there is more or less the same number of agents investing and not investing and the probabilities that the sample tells you one thing or the other are roughly equal. The conditional distribution is flatter towards the center, the economy is more volatile and movements towards an extreme are more likely.

If we increase the level of uncertainty even further the unconditional distribution becomes single-bell-shaped (see figure 1-4), the economy is most of the time at an average level of investment. But we still have persistence, when \( y_t \) is below average, it will most likely be below average also at \( t + 1 \). When the economy is below (above) average the conditional distribution has a maximum in below (above) average values.

1.3 When the world is large; a general model.

There are two problems with the previous model as \( n \) becomes large, first that the sample size becomes a set of measure zero. This could possibly be taken care of. Anyway the world is not infinite, and for any finite value of \( n \) the sample is enough to sustain the herding equilibrium for people with symmetric priors and risk neutral preferences. More important is the other implication, that aggregate uncertainty becomes zero. The number of individuals making mistakes stops being a random variable and so the process becomes a difference equation with two stationary points, everybody investing and nobody investing.

It is a necessary condition for this kind of equilibrium that the number of agents that make mistakes be a random variable. Durlauf (1990, 1991) and Jovanovich (1987) have created models where out of an infinite number of agents and individual risk, is possible to generate aggregate noise. The essence of doing this is to escape from the law of the large numbers either
Figure 1-4:
by making the response to the individual shocks correlated across individuals or by having some aggregate noise in higher moments.

For the sake of simplicity I will assume that the perception that agents have of the past, a random variable for each agent, is correlated across individuals. In this way the law of the large numbers does not hold. This noise affects only to informational parameters of the economy. There is no necessity of real shocks because their purpose in the previous section was to generate some informational noise that now will be assumed. Additionally this noise, as in the previous section, will have no ex-ante structure, and the model in itself will generate the business cycles structure desired. The basic intuitions of the previous section will hold.

1.3.1 Set up

Now we will not need idiosyncratic technological shocks. Each of the risk neutral agents will invest if and only if they expect half or more of the other agents to invest\(^2\). Given that now the weight of an agent on the aggregate will be zero, they will invest if and only if

\[
E_t^i(y_t) \geq \frac{1}{2},
\]

where the indexes \(i\) and \(t\) represent the information of individual \(i\) at period \(t\).

There is a continuum of \(2\pi\) agents uniformly distributed in a circumference of perimeter \(2\pi\). In each period each agent makes an investment decision of 0 or 1, so:

\[
y_t = \int_0^{2\pi} \frac{x_t(i)}{2\pi} di \quad \Rightarrow \quad y_t \in [0, 1] \tag{1.16}
\]

**Informational structure**

Each agent will have at each moment two pieces of information:

- A prior on the distribution of the aggregate that is common across all agents that we will call \(q(y)\).

---

\(^2\)This is not an important assumption, the threshold for investment could be whatever value in \((0, 1)\) and the model would work fine, but it would complicate things unnecessarily.
• A privately observed signal \( s_t(i) \) with information about the level of activity in the previous period. This replaces the sampling process of the previous section. For the sake of simplicity we will assume that this signal takes only two values: 1 (or high, or “optimistic”) or 0 (or low, or “pessimistic”).

Each agent \( i \) receives a signal as information of the state of the world yesterday. The signal indicates, with noise, if yesterday had been a good time to invest; the better that yesterday was for investing, the more likely that the signal will say the truth. This signal (updating the prior) summarizes the perception that agent \( i \) has of the state of the world.

Each agent knows that if the aggregate had a value \( y_{t-1} \) yesterday, she has a probability \( p(y_{t-1}) \) of observing a high signal, and \( 1 - p(y_{t-1}) \) of observing a low one. The function \( p(y) \) is common knowledge and is required to have the following properties:

1. \( p(y) = 1 - p(1 - y) \)
2. \( p(0) > 0 \)
3. \( p(y) \) is not decreasing in all points of \([0,1]\)

The first requirement implies symmetry. The probability of receiving a high signal when the aggregate is at a 99% of its potential is the same that the one of receiving a low one when the aggregate is at a 1% of its potential. It implies that \( p(\frac{1}{2}) = \frac{1}{2} \).

The second insures that always there will be people making mistakes.

The third implies that the higher the aggregate level is, the more likely it is to receive a high signal. Together with the first condition, it implies that the variance of the signals achieves a maximum at \( y = \frac{1}{2} \), it is not decreasing for values of \( y \) in \([0, \frac{1}{2}] \) and symmetric around \( \frac{1}{2} \).

**Perceptions of reality**

I like to think of the model as a set of producers of intermediate goods that sell to a common final good manufacturer. Each producer lives in his small market, his small island. In this circumstance it is likely that the information that arrives to a given producer is closely related to the one that arrives to a producer of a similar good. They are in relatively close markets, a shock in a given market is reflected in the surrounding ones, but not (or little) in faraway
markets. In essence, the information of a producer will be correlated with the one of individuals close to him.

In our simple model the only relevant notion of proximity is the informational one. The correlation between \( s_t(i) \) and \( s_t(j) \) will be a given and known function of the distance between \( i \) and \( j \). So given that for all the individuals

\[
\text{Var}(s_t(i)) = p(y) \cdot (1 - p(y))
\]  

(1.17)

Then:

\[
E [(s_t(i) - p(y)) (s_t(j) - p(y))] = p(y) \cdot (1 - p(y)) \cdot \rho(i, j)
\]  

(1.18)

Where \( \rho(i, j) \) is a function such that:

1. \( \rho(i, i) = 1 \ \forall i \)
2. \( \rho(i, j) \geq 0 \ \forall i, j \)
3. \( \rho(i, j) = \rho(j, i) \ \forall i, j \)
4. \( \forall i, j : \rho(i, j) = \rho(i, j + 2\pi) \)
5. \( \forall i \) there exist a set of positive measure \( \Omega(i) \) such that \( \forall j \in \Omega(i), \rho(i, j) > 0 \)
6. \( \forall i \) there exist a set of positive measure \( \Phi(i) \) such that \( \forall j \in \Phi(i), \rho(i, j) < 1 \)

The first condition implies that each individual is perfectly correlated with herself. The second that nobody expects to have a perception of reality completely opposed (negatively correlated) to the one of other individuals. Everybody lives in the same world, if I am wrong this does not imply that others are right, and if I am right I should not expect others to be systematically wrong.

The third and fourth conditions imply symmetry. Once location is taken into account, everybody is ex ante identical to everybody else.

The fifth condition is the crucial one. By insuring that a relatively large number of individuals move together it will produce aggregate noise. This will allow us to fight back the law of the large numbers.
The sixth insures a degree of heterogeneity; without it a common aggregate shock would exist.

The average correlation is then:

\[ R = \int_0^{2\pi} \frac{\rho(i,j)}{2\pi} dj \]

(1.19)

and clearly \( R \in (0, 1) \) and is independent of \( i \).

I think of \( \rho(i,j) \) as a sinusoidal function on \( j \) that achieves a maximum value of 1 at \( j = i \) and a minimum of 0 at \( j = i \pm \pi \), but much more general formulations are allowed.

Given this setup the conditional probabilities\(^3\) that agents assign of other agents investing are:

- If \( i \) receives a signal \( s_t(i) = 1 \), the conditional probability of \( j \) receiving also a signal with value 1 is:

\[ \Pr(s_t(j) = 1 \mid s_t(i) = 1) = 1 - (1 - p(y))(1 - \rho(i,j)) \]

(1.20)

- If \( i \) receives a signal \( s_t(i) = 0 \), the conditional probability of \( j \) receiving also a signal with value 1 is:

\[ \Pr(s_t(j) = 1 \mid s_t(i) = 0) = p(y) \ (1 - \rho(i,j)) \]

(1.21)

**Space of strategies**

A strategy is a mapping from the set of private information to the set of possible actions, a function from \( \{0,1\} \) to \( \{0,1\} \). As before we will study only stationary pure strategies, meaning pure strategies that are not indexed by time\(^4\). There are only four possible strategies of this type:

1. Never invest: \( x(0) = 0; \ x(1) = 0 \).

2. Always invest: \( x(0) = 1; \ x(1) = 1 \).

3. Invest if you get a high signal: \( x(0) = 0; \ x(1) = 1 \).

---

\(^3\)See appendix B

\(^4\)This will exclude strategies of the type “when \( t<100 \) invest only if \( s=1 \), when \( t=100 \) invest whatever you see, and when \( t>100 \) never invest”.
4. Invest if you get a low signal: \( x(0) = 1; x(1) = 0. \)

**Equilibrium concept**

In order to insure rational expectations we will require perfect Bayesian consistency to the relevant equilibrium concept. We will study only symmetric equilibria, in a strong sense. This is, equilibria where all the agents play the same strategy all the time. One could imagine these strategies as behavior rules that pass from parents to children.

Given a strategy and the process of information diffusion, a certain probability distribution on \( y_t \) will be generated for each possible level of \( y_{t-1} \): \( F(y \mid y_{t-1}) = \Pr(y_t < y \mid y_{t-1}) \), with density function \( f(y \mid y_{t-1}) \).

This will generate an unconditional distribution \( Q(y) \), whose density \( q(y) \) will be such that:

\[
q(y) = \int_0^1 f(y \mid z)q(z)dz
\]

(1.22)

This unconditional distribution will be the prior that all agents have each moment on the distribution of \( y_{t-1} \).

When an individual receives a signal, she will update her prior using it:

\[
q(y_{t-1} \mid s_t(i) = 1) = \frac{p(y_{t-1}) \cdot q(y_{t-1})}{\int_0^1 p(y) \cdot q(y) \cdot dy}
\]

(1.23)

\[
q(y_{t-1} \mid s_t(i) = 0) = \frac{(1 - p(y_{t-1})) \cdot q(y_{t-1})}{\int_0^1 (1 - p(y)) \cdot q(y) \cdot dy}
\]

(1.24)

The strategy is an equilibrium if and only if using this updated prior and the knowledge of the way that other people's information relates to their own:

\[
\forall s_t(i) \quad x(s_t(i)) = 1 \quad \text{iff} \quad E[y_t \mid s_t(i)] \geq \frac{1}{2}
\]

(1.25)

**1.3.2 Animal spirits equilibrium.**

In order for the strategy “invest if you observe high, do not invest if you observe low” to be an equilibrium, it must be true that when an agent observes \( s_t(i) = 1 \), she expects that most
people are going to invest today, and when she observes \( s_t(i) = 0 \), she expects that most people will not invest today, even though the signal in itself only carries information about the past, the signal depends only on the level of the aggregate yesterday. So the strategy is an equilibrium only if

\[
E(y_t \mid s_t(i) = 1) \geq \frac{1}{2} \geq E(y_t \mid s_t(i) = 0)
\]  
\[
(1.26)
\]

The strategy is an equilibrium if the observation of \( s_t(i) = 1 \) makes agents believe that most people will observe the same, and so most people will be optimistic about the future. Again, the belief in other people optimism makes you optimistic.

The expected value of the contemporaneous aggregate level is:

\[
E(y_t \mid s_t(i)) = \int_0^{2\pi} E(x_t(j) \mid s_t(i)) \frac{dj}{2\pi}
\]  
\[
(1.27)
\]

and if everybody follows the strategy:

\[
E(y_t \mid s_t(i)) = \int_0^{2\pi} E(s_t(j) \mid s_t(i)) \frac{dj}{2\pi}
\]  
\[
(1.28)
\]

Now, given equations (1.20), (1.21), (1.23) and (1.24):

\[
E(s_t(j) \mid s_t(i) = 1) = \int_0^1 (1 - (1 - p(y))(1 - \rho(i,j))) \cdot q(y \mid s_t(i) = 1) \cdot dy
\]

\[
= 1 - (1 - \rho(i,j)) \frac{\int_0^1 p(y)(1-p(y))q(y)dy}{\int_0^1 p(y)q(y)dy}
\]  
\[
(1.29)
\]

and

\[
E(s_t(j) \mid s_t(i) = 0) = (1 - \rho(i,j)) \frac{\int_0^1 p(y)(1-p(y))q(y)dy}{\int_0^1 (1-p(y))q(y)dy}
\]  
\[
(1.30)
\]

So, when \( s_t(i) = 0 \):

\[
E(y_t \mid s_t(i) = 0) = (1 - R)\frac{\int_0^1 p(y)(1-p(y))q(y)dy}{\int_0^1 (1-p(y))q(y)dy}
\]  
\[
(1.31)
\]

and when \( s_t(i) = 1 \):

\[
E(y_t \mid s_t(i) = 1) = 1 - (1 - R)\frac{\int_0^1 p(y)(1-p(y))q(y)dy}{\int_0^1 p(y)q(y)dy}
\]  
\[
(1.32)
\]
The unconditional distribution is symmetric\(^5\): \( q(y) = q(1-y) \). In addition, the unconditional expected probability of receiving high or low signals is \( \frac{1}{2} \)\(^6\).

Thus the strategy is an equilibrium (so (1.26) hold) if and only if:

\[
(1 - R) \cdot 2 \cdot \int_0^1 p(y) (1 - p(y)) q(y)dy \leq \frac{1}{2}
\]  
\[ (1.33) \]

We know that the variance of the signals is a number between \( p(0) (1 - p(0)) \) and \( \frac{1}{4} \). \( \int_0^1 p(y) (1 - p(y)) q(y)dy \) is the expected unconditional variance of the signals, that is also a number between \( p(0) (1 - p(0)) \) and \( \frac{1}{4} \), because it is a weighted average of the variances. So:

\[
\int_0^1 p(y) (1 - p(y)) q(y)dy \leq \frac{1}{4}
\]  
\[ (1.34) \]

With which we are almost done, because \((1 - R) \in (0, 1)\), so

\[
(1 - R) \cdot 2 \cdot \int_0^1 p(y) (1 - p(y)) q(y)dy < 2 \cdot \int_0^1 p(y) (1 - p(y)) q(y)dy \leq \frac{1}{2}
\]  
\[ (1.35) \]

Which proves that the strategy “invest if you think that yesterday was a good time for investing, do not invest if you do not think so” can be sustained as an equilibrium.

1.3.3 Dynamics

Even though it is very difficult to get a closed form for \( F(y \mid y_{t-1}) \), we will be able to get a good amount of information on it by looking at the first two moments of this distribution.

Conditional mean and variance.

Aggregate output will follow a Markov process. If at \( t - 1 \) the aggregate was \( y_{t-1} \), then the expectation\(^7\) at \( t \):

\[
E(y_t \mid y_{t-1}) = p(y_{t-1})
\]  
\[ (1.36) \]

---

\(^5\)See appendix C.
\(^6\)See appendix D.
\(^7\)See appendix E.
And its variance:

\[ \sigma^2[y_t \mid y_{t-1}] = p(y_{t-1}) \cdot (1 - p(y_{t-1})) \cdot R \]  

(1.37)

As is clear, it is the fact that the signals are correlated across individuals what allows for aggregate noise. But the statement should be more general, as long as the law of the large numbers does not apply we will obtain aggregate fluctuations. We have used non independent random variables, but other schemes are plausible.

**Persistence volatility and mean reversion.**

We already know that \( E[y_t \mid y_{t-1}] = p(y_{t-1}) \). This means that the aggregate level of investment will *tend* to increase when \( p(y_{t-1}) \) is above the 45 degree line, and to decrease when is below.

If \( p(y) \) is like the one drawn in figure 1-5 (recall that \( p(y) \) is the process of information diffusion that we consider exogenous), there will be two points \( y^* \) and \( 1-y^* \), that we will call "attraction points", such that if the variance of the aggregate were 0 \( (R = 0) \) would be the only stable long run solutions of the differential equation that would drive the process of the economy: \( y_t = p(y_{t-1}) \). This economy would have a non-stable equilibria at \( y = \frac{1}{2} \). For all initial values smaller than \( \frac{1}{2} \) the economy would converge at \( y^* \), and for all initial points bigger than \( \frac{1}{2} \) to \( 1-y^* \). The conditions that we have imposed on \( p(y) \) insure that there exists at least one accumulation point (that would be \( \frac{1}{2} \) if it were unique).

The interesting thing is that the economy will *tend* to move towards the closer attraction point, but *need not do so*. The economy will tend to move towards its expected value, but there exists a positive probability that it moves in the opposite direction. Here is where the effects of the informational externality are visible.

Imagine, in figure 1-5, that the economy is at or near the low attraction point \( y^* \). At this point the variance of the aggregate is positive, *and increasing*. Even though we would expect the economy to remain around \( y^* \) (which it will do most of the time), it can happen that a relatively large number of individuals observe a high signal moving the aggregate towards the right, say to a point between \( y^* \) and \( \frac{1}{2} \). At this new point the variance is *higher* than before, the conditional distribution is flatter. That implies that new movements towards the right are more likely than before; even if we still would expect the economy to move towards \( y^* \), we will do it with less confidence. It is easier that a relatively large number of individuals observe a
Figure 1-5:
high signal.

This is the effect of the informational externality. When the economy is far away from the center, the probability of individuals making mistakes is relatively low because the unconditional variance of the signals is very low. So it is not very likely for the economy to move away, the aggregate level is persistent. But if it moves towards the center, and sooner or later it will, the probability that individuals receive the wrong signal increases substantially. Aggregate uncertainty increases accordingly. When a small group of individuals makes a mistake increases the volatility of the signal, and makes more likely for others individuals to make mistakes. Prediction is more difficult, and it is possible that we cross \( \frac{1}{2} \). Once in high levels of output we will tend to stay there, but again every so often we will move to the left, and the process will go on, with long periods staying around \( y^* \) or \( 1-y^* \), but with jumps every so often between high and low states, in between the jumps a kind-of-mean reverting process will take place; things tend to move toward the attractor, but this is only a "local mean", not the mean of the process (clearly \( 1/2 \)). In other words, the unconditional distribution will have maxima in \( y^* \) and \( 1-y^* \), and a local minimum at \( \frac{1}{2} \). A good deal of the total variance of the time series of the output will come from things that repeat itself every so often (the time between jumps will have an expected duration), one of the characteristics of the business cycles.

Even if the only attraction point is \( \frac{1}{2} \) (figure 1-6), we will have some degree of persistence. At the attraction point the variance achieves a maximum, so the aggregate is particularly unstable. Movements towards the extremes will go back to the center (mean reversion), but relatively slowly, because the distribution in the extremes is more concentrated around its conditional expectation.

1.4 Comments on multiplicity of equilibria.

The animal spirits equilibrium is not unique. Actually all four strategies can be sustained as an equilibrium\(^8\). I do not know up to what point we should be worried about it. After all in coordination games with a dynamic setting is inevitable the existence of multiple equilibria. In a sunspot equilibrium is as much an equilibrium to invest when the sun is covered with spots as

---

\(^8\)The strategies "always invest" and "never invest" produce priors that assign probability one to \( y = 1 \) and \( y = 0 \) respectively. The "fourth" strategy, invest iff \( s_t(i) = 0 \), is shown in appendix F to be an equilibrium.
Figure 1-6:
to invest when it is spotless, as long as everybody agrees to do so. There are, though, several good reasons to make the case for the animal spirits equilibrium in the present model.

The basic intuition of the animal spirits equilibrium is that if the reduced form of the world has persistence, you will like to follow your perception of the immediate past, because it is likely to be close to what will happen in the future. In the model as has been stated the only ex-ante source of persistence is the fact that we centered our study on stationary strategies. We will see below that given this the only equilibrium robust to the presence of uninformed agents is the animal spirits one.

We would like the equilibrium to be robust to the presence of "uninformed" agents; to be such that if an agent starts playing, being the world in an equilibrium, she would play the equilibrium strategy. In our model if an agent has to play knowing that the world is in an equilibrium, but not which one, she will play the animal spirits strategy.

The intuition is that the signal does not only carry information about what physically happened in the past, it also carries information about which was the equilibrium in place yesterday, and given that we concentrate in stationary strategies, on which one will be the equilibrium tomorrow. We will see that for a Martian who falls on earth with complete ignorance of what strategies agents are playing, her best bet is to follow what the signal tells.

More formally, let \( U(k, s, x) \) be the utility that an individual playing \( x \) expects to get if she observes \( s \) and everybody else is playing strategy \( k \). There are \( K \) equilibrium strategies of the form \( X_k(s) \), the \( k\)th of them giving an expected payoff of \( U(k, s, x) \) when \( s \) is observed and an action \( x \) is taken\(^9\). The equilibrium strategy \( l \) is robust to the presence of uninformed agents if and only if:

\[
\forall s \quad X_l(s) = \text{Arg max}_x \left( \sum_{k=1}^{K} \frac{1}{K} U(k, s, x) \right)
\]

So, if you are uninformed about which is the equilibrium strategy that everybody else is

\(^9\)Because \( k \) is an equilibrium, \( U(k, s, X_k(s)) \geq U(k, s, x) \).
playing, and thus you assign equal priors to all equilibrium strategies; when you see a signal \( s \), you will follow the action indicated by the robust strategy. It is as if you were playing that equilibrium strategy.

In appendix G it is shown that if we assign to the agents a payoff function such that they get \( y_t - \frac{1}{2} \) if they invest and 0 if they do not, the animal spirits equilibrium is the only one robust in the sense expressed above.

The point is that if you assign an equal prior to all the possible equilibria and then you see \( s_t(i) = 1 \), your posterior that the equilibrium is "never invest" is very low, so you have incentives to invest, and vice versa when you see \( s_t(i) = 0 \). The best bet is to always follow the animal spirits strategy, to always follow the past.

If there were other sources of persistence, as it should be expected in the real world, the animal spirits equilibrium would get reinforced. The more reasons an agent has to believe that tomorrow will be similar to yesterday, the more reasons she has to follow her perception of the past. If there are externalities in capital, past investment affecting positively present returns, the higher you believe that past investment was, the more you would like to invest independently of what other agents do today.

In the economy that we have studied the only source of persistence was the derived from the animal spirits equilibrium. It is only natural to expect that persistence is also derived for other, more physical, causes. For instance, the return on investment may depend on aggregate capital, and not aggregate investment. Introducing exogenous sources of persistence "reinforces" the animal spirits equilibrium.

Assume that the return to present investment is \( y_{t-1} + y_t - 1 \), so that there is an intertemporal externality due to past investment as in Acemoglu(1993). In this case if an agent has flat priors on the strategies that other agents may take, and so flat priors on both \( y_{t-1} \) and \( y_t \), she will follow the animal spirits strategy. Note that now we do not require the world to be in an equilibrium. It is not that there are dominant strategies, because in an equilibrium how much we trust the signal depends on the equilibrium strategy, but an individual very uncertain on other people’s strategies will play the animal spirits strategies.
1.5 Conclusions

We have seen that to follow one's perception of the past can be sustained as an equilibrium, and that given some particular schemes of distribution of the information, that seem to be pretty plausible, this equilibrium would reproduce some of the stylized facts of the business cycles.

The past is used as a coordination mechanism, the equilibrium is a sunspot, with the particularity that the transition matrix is endogenous, and it is such that produces the kind of stochastic processes that we were looking for.

One could question why individuals do not coordinate in other, possibly exogenous, variables. Why is the (not perfectly observable) past chosen instead of other variables? Why is the past chosen as a focal point?

On this I have a double line of defense. On one hand people can learn to know, learn to believe in sunspots. As in Howitt and McAfee (1992) the equilibrium could be such that people can learn to believe in it, if the rest of the agents are coordinating in the same way. Probably an even better line of defense is the most obvious one: in all coordination games agents will look for a focal point; a focal point must be in one way or another observed by all individuals, and there is nothing more observable than the past. Everybody has a feeling about what happens with the economy, and everybody is likely to be right, so it is difficult to think of a more plausible focal point that the perception that agents have of the past.

There is even more; if the past affects directly to present returns, agents will have an additional, and direct, incentive to herd and doing so to coordinate with the past. We have build the model without stocks, but the existence of state variables would by itself generate persistence. The more persistence there is, the more reasons agents have to do something close to what they did in previous periods, and so the more natural it becomes to coordinate using the past as reference.

In the real world the externalities are local, a shoe manufacturer in Boston does not care that much about the potato harvest in Idaho. The coordination process will be local too, this has been explored by Durlauf in a series of papers. My model tries to capture this with a simple set up, to be a simplification. That is why the perception of the past is assumed to be
correlated, but not perfectly, across agents and why I do not believe that the assumption of the unobservability of past aggregate investment is too restrictive.

Agents may know the true value of aggregate investment, but not the statistic that they care about (an average of investment weighted by the distance to the agent), to learn it would be way too expensive. They have nevertheless a perception of the state of the economy; each one thinks that was good or bad to invest in the previous period, and most likely they are right.

The machinery developed in this paper could prove itself useful, but in order to make it truly realistic it would be necessary to specify models where the externality is local, and so individuals want to coordinate only with the people around (the aggregate rates could be meaningless for them), and where individuals are subject to uncertainty about other people's reaction functions (probably generated by stochastic growth). The most difficult problem that this context would require to solve is that individuals want to predict not only a number, but a distribution, whose functional form does not need to be closed.

This model does not pretend to say that there are no real sources of business cycles, but makes the point that it is possible to conceptualize them without exogenous productivity shocks. Local productivity shocks could well be at the source of the informational noise. The model would be identical and would create cycles out of white noise shocks.

Models are generally simplifications of reality and this model is more so than most of them, but tries to make the point that the distribution of information across the economy and the willingness of agents to coordinate their investments could in itself generate a behavior of the aggregate with a business cycle structure. The point of the model is to show that it is possible to generate animal spirits equilibrium using only informational noise on the state of the economy, a noise that in itself depends on the performance of the economy. Surely this is not the only source of recessions and booms, but its contribution to the process could be important and should be determined empirically.
Chapter 2

Believing in Lies: The Effect of GNP Announcements on Fluctuations of GNP Growth

2.1 Introduction

In rational expectations models, agents forecast the future based upon their knowledge of the structural form of the economy and their perception of the history of the economy. This chapter is focused on the empirical study of how these perceptions are created and the effect of these perceptions upon the future path of the economy.

In economic models, agents are typically assumed to be fully and correctly informed about the entire past history of the economy. Agents are assumed to know precisely the realizations of all past variables necessary for the prediction of future realizations. The only uncertainty that agents face is due to the existence of unforeseeable exogenous shocks that will occur in the future. The real world, however, is considerably more complex. Uncertainty does not affect only future variables, there is also uncertainty about the true level of past economic activity (such as the true level of real Gross National Product last quarter). Agents do not really know

This paper is joint work with Paul Schulstald. We have been the comments and advice of Daron Acemoglu, Abhijit Banerjee, Olivier Blanchard, Ricardo Caballero and the participants in the MIT Macro Workshop. Obviously we are solely responsible for all remaining errors.

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the complete history of the economy (not even the history which is relevant for prediction of future aggregate activity), they have perceptions about the past. From their observations they have probability distributions on the realizations of the economic variables in the past. Thus, to forecast the future behavior of the economy, it is necessary to know both the actual past performance of the economy and the past performance of the economy as perceived by agents today. As agents take actions, at least in part, based upon these perceptions, the future path of the economy will be influenced by these perceptions. The perception that agents have today about the economy’s past performance is itself an important economic variable.

In this chapter, we investigate and establish the importance of agent perceptions about past activity as a determinant of present and future activity. Our primary contribution is to document the influence of announcements of past growth rates of U.S. real GNP growth upon future real GNP growth. We find a striking result: in a regression explaining the growth rate of real GNP for quarter $t$, the true growth rate of the economy during quarter $t - 1$ does not matter if the growth rate at $t - 1$ which agents in the economy perceived during quarter $t$ is also included in the regression. We consider this to be very interesting and important evidence that agents’ beliefs about economic conditions are an important influence upon the aggregate economy. We also show, through examination of the components of real GNP, that the influence of agent beliefs works through aggregate investment.

Our work is new and addresses a subject not previously explored in the literature. The closest paper in spirit to ours is Oh and Waldman (1990). They study the effects of announcements in the leading economic indicators upon future economic activity. Their primary result is that predictions of future growth do in fact influence the realization of what is predicted. Thus, if agents receive information which states that aggregate activity is likely to be high in the future (even if this information is based on incorrect data), the information has a positive effects on movements of future output. Our work is also related to Acemoglu (1993) who models the investment accelerator exploiting agent uncertainty about the level of past aggregate investment.

The rest of this chapter is organized as follows. In Section 2.2, we describe our empirical approach. Section 2.3 presents our main results. In Section 2.4 we examine whether our results are evidence of ‘animal spirits’. Section 2.5 concludes.

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2.2 GNP Announcements and Revisions

To measure the perceptions that agents have about the past is not straightforward. In economics, more attention has been devoted to studying agent's expectations about the future economy as opposed to their perceptions about how the economy performed in the past. Fortunately the manner in which economic variables are measured, and how their magnitudes are publicized, provides us with an adequate instrumental variable for agents beliefs.

We will exploit the fact that governments make announcements, which become common knowledge, about the past performance of the economy. These announcements are noisy measurements of the true realizations of economic variables. The accuracy of the measurements depends upon the amount of information available to the statistical agency at any point in time. Mankiw and Shapiro (1986) show that the noise in the revisions is due to the appearance of unforeseeable new information. Clearly, the longer the period of time the statistical agency has to formulate an estimate of an economic variable, the more accurate will be that measurement. Some years after, the statistical agency makes a final estimate of activity for that time period. We denote this final estimate to be the 'truth'. It is this number that economists use when doing econometrics and that is widely available in digital format. The crucial point is that the first announcement and the final estimate (the truth) differ substantially. The announcements are composed of the truth plus a white noise error term. We will assume that the latest announcement reflects the state of opinion about the realizations of the economic variables at any given moment of time, an appropriate assumption given that the announcements receive much attention in the media after they are announced. In this chapter, we will study the usefulness of both announcements and truth in predicting future rates of growth of the economy and future growth rates of the components of real GNP.

Our work focuses upon announcements of real GNP in the United States. As Mankiw and Shapiro write, "GNP is probably the most closely watched economic series. Almost all observers - economists, policy makers, and the press - consider it the primary measure of the health of the macroeconomy. Estimates of GNP, therefore, receive much attention". As is well documented, there is considerable noise in the measurement of past GNP. The first 'advance'

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1We focus upon GNP rather than GDP because until 1991 the U.S. government used GNP as its primary measurement of aggregate activity.
estimate,\(^2\) based upon preliminary and incomplete source data, is released soon after the end of a quarter. As additional data becomes available, 'preliminary' and 'final' estimates are released thirty and sixty days after that first announcement. Each month, the Commerce Department publishes its most recent estimates of real GNP growth for the previous six quarters.\(^3\) The final comprehensive benchmark revisions are published after five years. These latest estimates are based on new and revised source data and also are influenced by updated seasonal factors, shifts in the base period and by definitional changes. In general, the initial estimate, based upon the roughest data, is subject to the most noise. We consider the final comprehensive estimate of quarter GNP as the 'true' estimate of quarterly GNP.\(^4\) Our sample period of our announcements is from January 1967 to July 1991. The first date was selected because before January 1967 the first estimate of monthly real GNP was announced two months after the end of a quarter, and we wanted a measure of perceptions consistent across time. The last date was chosen because in October 1991, the Commerce Department ceased publishing initial estimates of real GNP and switched to a GDP based national accounting system. Additionally it is necessary to have a long time lag between the last announcement in our sample and the present in order for the last observations of the true rate of growth in our sample to be sufficiently updated. In Table 2.1, we present descriptive statistics of the size of the measurement error in the first advance estimate of real GNP growth and the true rate of growth of real GNP. In Appendix H we discuss the sources and collection of the data.

### 2.3 Results for U.S. Real GNP

One of the well known macroeconomic facts is that past rates of growth of real GNP have predictive power for future real GNP growth. The time series of real GNP growth exhibits persistence. A high rate of growth last quarter is typically associated with a high rate of

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\(^2\)We do not make use of the "flash" estimate. This estimate was published fifteen days before the end of the quarter during the final years of the 1970's and the first years of the 1980's. It was originally intended for exclusive use by policy makers, and was not intended to be made public, due to the large amount of noise that it possessed. The flash estimate began to receive public attention inducing its discontinuation.

\(^3\)In principle each of these numbers could influence future activity.

\(^4\)In practice, we work with growth rates and not levels so we need not worry about periodic revisions to the national accounts which, by changing the base year of prices, changes systematically all past measurements of GNP.
Table 2.1: Descriptive Statistics

<table>
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<th>$\bar{m}_{t-1}$</th>
<th>$g_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-.000693</td>
<td>.00670</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>.00545</td>
<td>.00963</td>
</tr>
<tr>
<td>Maximum</td>
<td>.0135</td>
<td>.0295</td>
</tr>
<tr>
<td>Minimum</td>
<td>-.0185</td>
<td>-.0267</td>
</tr>
<tr>
<td>Time Period</td>
<td>1967.1–91.3</td>
<td>1960.1–91.3</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>100</td>
<td>126</td>
</tr>
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growth next quarter. This basic truth of the business cycle can be and has been explained by the existence of physical state variables (such as investment or capital stock) which induce correlation in the first derivative of real GNP over time.

In this section, we show that once perceptions are taken into account this basic stylized fact is untrue. Once government announcements are controlled for, the true past rate of real GNP growth no longer has predictive power for future real GNP. Instead it is the rate of real GNP growth that agents observed (i.e. government announcements) and believed about previous real GNP growth that has predictive power. Then in Section 2.3.1, we demonstrate that perceptions influence real GNP through aggregate investment.

To examine the effect of government announcements upon real GNP growth presupposes an econometric model which explains real GNP growth. We take a reduced form approach and model growth as an ARMA process. Then the effects of adding announcements to the model are studied.

Our final estimation\(^5\) of the ARMA model on the time series of the (true) growth rate of real GNP\(^6\)

\[
g_t = \alpha_0 + \alpha_1 g_{t-1} + \epsilon_t
\]  

(2.1)

where \(g_t\) is the true growth rate of real GNP, \(\alpha_0\) and \(\alpha_1\) are parameters to be estimated and \(\epsilon_t\) is an error term. The results of the estimation of equation (2.1) are in the first column of Table 2.2. As would be expected, the coefficient on past growth is significant and greater than zero.

To examine the importance of the announcements of real GNP we add the first announcement of real GNP growth for period \(t - 1\), denoted \(\hat{g}_{t-1}\), announced during period \(t\):

\[
g_t = \alpha_0 + \alpha_1 g_{t-1} + \alpha_2 \hat{g}_{t-1} + \epsilon_t.
\]  

(2.2)

We concentrate upon the first announcement of real GNP growth for period \(t - 1\) that is announced one month into period \(t\).\(^7\) The fact that \(g_{t-1}\) and \(\hat{g}_{t-1}\) are highly correlated might

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\(^5\)We report only the final results for the ARMA processes that we studied. These are the simplest regressions that generated white noise error terms.

\(^6\)Note that the \(g_t\) is \(ln(y_t/y_{t-1})\), where \(y_t\) is real GNP during quarter \(t\). Thus the average level of \(g_t\) over the period 1960 - 1991 is .0061 implying an annual growth rate of 2.4% over this period.

\(^7\)The addition of other announcements does not change the results.
lead to multicollinearity problems. Alternatively, we can estimate the following equivalent equation:

\[ g_t = \alpha_0 + \alpha_1 g_{t-1} + \alpha_2 \hat{m}_{t-1} + \epsilon_t \]  

(2.3)

where \( \hat{m}_{t-1} \equiv \hat{g}_{t-1} - g_{t-1} \) is the mistake made in the announcement \( \hat{g}_{t-1} \).\(^8\)

The results of the estimation of equations (2.2) and (2.3), reported in the second and third columns of Table 2.2, are quite striking. When real GNP announcements are added to the regression, the true rate of growth during quarter \( t - 1 \) loses all of its predictive power. Only the initial estimate of last quarter’s rate of growth has predictive power. This is important evidence that perceptions of agents are significant economic variables in their own right - indeed in our regression the true level of the variable has no statistical significance.\(^9\)

Our final regression, from which \( g_{t-1} \) is omitted, is reported in the final column of Table 2.2.

Another way of see our point is the following. Imagine that an economist had been assigned with task of predicting real GNP growth for the current quarter. The economist knows that the published measurements of the past performance are noisy. Assume further that our economist is truly fortunate; he is fully informed about the true rate of growth during the previous quarters, information available only to him. Our results indicate that in this circumstance the economist would not use the additional information to forecast future growth, for forecasting purposes he can do no better than use the announcements.

Thus persistence of real GNP growth is induced by the fact that announcements are highly correlated with the truth but is not produced by the truth by itself. It is the commonly held belief that the past was good what produces a good future. This is true independent of whether the immediate past was a period of high growth. There are no physical state variables producing autocorrelation between past and future, rather it is perceptions which yield autocorrelation.

\(^8\)We also tested the series \( \hat{m}_{t-1} \) using GARCH to see whether the conditional variance of the announcements changed over time. We found that it did not.

\(^9\)Both the announcements and the “truth” are deseasonalized series. We do not think that this affects our results.

When the “true” series is deseasonalized, the values of GNP used are that of the revised (“true”) series, not the announcements. In the event that there exists autocorrelation in the deseasonalized series due to the method of seasonal correction and not due to economic factors, that autocorrelation should appear in equation (2.2) independent of the inclusion of the announcements.
Table 2.2: Regression Results

<table>
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<th>Parameter</th>
<th>Equation 1</th>
<th>Equation 2</th>
<th>Equation 3</th>
<th>Equation 4</th>
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</thead>
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<td>.00372*</td>
<td>.00374*</td>
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<td>(.00108)</td>
<td>(.00108)</td>
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<td>$g_{t-1}$</td>
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<td>.0176</td>
<td>.380*</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>(.0847)</td>
<td>(.149)</td>
<td>(.0994)</td>
<td>—</td>
</tr>
<tr>
<td>$\hat{g}_{t-1}$</td>
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<td>.363*</td>
<td>—</td>
<td>.376*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.148)</td>
<td>—</td>
<td>(.0928)</td>
</tr>
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<td>$\hat{m}_{t-1}$</td>
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<td>—</td>
<td>.363*</td>
<td>—</td>
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<td></td>
</tr>
<tr>
<td>Time Period</td>
<td>1960.3–91.4</td>
<td>1967.1–91.4</td>
<td>1967.1–91.4</td>
<td>1967.1–91.4</td>
</tr>
<tr>
<td>Sample Size</td>
<td>126</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>.105</td>
<td>.126</td>
<td>.126</td>
<td>.135</td>
</tr>
<tr>
<td>Durbin-Watson</td>
<td>2.10</td>
<td>1.99</td>
<td>1.99</td>
<td>1.98</td>
</tr>
<tr>
<td>Box-Pierce Q on residuals</td>
<td>13.19</td>
<td>9.38</td>
<td>9.38</td>
<td>9.24</td>
</tr>
</tbody>
</table>

Standard deviations in parentheses. Starred coefficients are significant at the 5% confidence level.
The picture that this presents is that of a very volatile economy whose connections across time are much weaker than previously thought.

We find more evidence for our hypothesis by conducting an additional exercise. Recall that not all the information contained in an announcement is new information, some part of a given announcement could have been forecasted from previous announcements (after all, real GNP is serially correlated). We will now show that the variable which has predictive power for real GNP growth is the component of the announcement which is genuinely new information.

Before the ‘advance’ estimate of real GNP growth during \( t - 1 \) agents could forecast its value based upon the announcements made the previous quarter on the GNP growth during \( t - 2 \). Define \( s^t_{t-1} \), the unexpected part of the announcement made at \( t \) about growth at \( t - 1 \), as the error term in the following regression equation:

\[
\hat{g}^t_{t-1} = \beta_0 + \beta_1 \hat{g}^{t-1}_{t-2} + s^t_{t-1}
\]  

(2.4)

where \( \beta_0 + \beta_1 \hat{g}^{t-1}_{t-2} \) is the best linear prediction, at time \( t - 1 \), of the announcement at time \( t \) of the rate of growth of the economy at \( t - 1 \). As should be expected, \( s^t_{t-1} \) is white noise; its descriptive statistics are found in Table 2.3.

The results of regressing the true rate of growth on its past values and past values of the ‘surprise’, i.e.:

\[
g_t = \alpha_0 + \alpha_1 g_{t-1} + \alpha_2 s^t_{t-1} + \epsilon_t
\]  

(2.5)

are in Table 2.4. Again, the true past rate of growth is not statistically different from zero once the unexpected announcement is introduced.

In the second column of Table 2.3 we report the results of the regression of the unexpected component of the announcement on past values of itself and true growth, none of them explain, at all, this part of the announcement. This insures that the result is not due to the used for collecting data. Our final equation, again omitting the insignificant variable \( g_{t-1} \), is presented in the last column.
Table 2.3: Descriptive Statistics for $s_{t-1}$

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>0.00867</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0233</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.0284</td>
</tr>
<tr>
<td>Box-Pierce Q statistic</td>
<td>16.49</td>
</tr>
</tbody>
</table>

2.3.1 Through Which Components Do Perceptions Matter?

The regressions results presented above demonstrate that agents’ perceptions matter as economic variables in their own right. In this subsection, we investigate through which component of real GNP do perceptions matter. Our intuition suggests that our results of the previous section will be stronger if it is found that perceptions matter through investment, a forward looking variable and an especially volatile component of GNP.

To explore the effect of measurement mistakes upon the components of real GNP, we estimate ARMA models to explain the growth rates of the major components of real GNP: consumption, investment and government spending.\(^{10}\) For each of the components $i$ ($i = \{1, 2, 3\}$) we estimate the following regression, analogous to equation (2.3):

$$c_t^i = \gamma_0^i + \gamma_1^i c_{t-1}^i + \gamma_2^i \tilde{m}_{t-1} + \epsilon_t^i \tag{2.6}$$

where $c_t^i$ is the growth rate of component $i$ at time $t$, $\{\gamma_0^i, \gamma_1^i, \gamma_2^i\}$ are parameters to be estimated,

\(^{10}\)We do not use net exports because its growth rate is not a well defined variable.
Table 2.4: Regression Results using Unexpected Component of Announcements

<table>
<thead>
<tr>
<th>Endogenous variable</th>
<th>$g_t$</th>
<th>$s_{t+1}^t$</th>
<th>$g_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>.00533*</td>
<td>-.000838</td>
<td>.00610*</td>
</tr>
<tr>
<td></td>
<td>(.00121)</td>
<td>(.00117)</td>
<td>(.000909)</td>
</tr>
<tr>
<td>$g_{t-1}$</td>
<td>.124</td>
<td>.149</td>
<td>——</td>
</tr>
<tr>
<td></td>
<td>(.130)</td>
<td>(.125)</td>
<td></td>
</tr>
<tr>
<td>$s_{t-1}^t$</td>
<td>.288*</td>
<td>-.115</td>
<td>.381*</td>
</tr>
<tr>
<td></td>
<td>(.144)</td>
<td>(.138)</td>
<td>(.105)</td>
</tr>
<tr>
<td>Time Period</td>
<td>1967.2–91.4</td>
<td>1967.3–91.4</td>
<td>1967.2–91.4</td>
</tr>
<tr>
<td>Sample Size</td>
<td>99</td>
<td>98</td>
<td>99</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>.109</td>
<td>.00870</td>
<td>.110</td>
</tr>
<tr>
<td>Durbin-Watson</td>
<td>2.10</td>
<td>2.01</td>
<td>1.96</td>
</tr>
<tr>
<td>Box-Pierce Q</td>
<td>11.79</td>
<td>16.88</td>
<td>11.88</td>
</tr>
</tbody>
</table>

Standard deviations in parentheses. Starred coefficients are significant at the 5% confidence level.
\( \hat{\mu}_{t-1} \) is (as before) the mistake in the announcement of real GNP growth, and \( \epsilon_i^t \) is a white noise error term. The results of the estimation of (2.6) for aggregate consumption \((i = 1)\), investment \((i = 2)\) and government expenditure \((i = 3)\) are in Table 2.5.

Amongst decision-makers in the economy, our intuition indicates that investors are the most likely candidates. The results are in accord with this basic intuition. Investment, the most volatile component of GNP, is most influenced by measurement mistakes. From our results of the estimation of the investment equation (column 2), note that the point estimate of the influence of the perceptions variable is much higher than in the equation using the growth rate of real GNP. This is as expected, an aggregate shock that works only through investment should have a much larger influence upon the growth rate of aggregate investment than upon the growth rate of the economy as a whole. Note also that the perceptions variable does not help explain, in a statistical sense, the growth rates of aggregate consumption and government expenditure suggesting that these variable are much less sensitive to perceptions than is investment.

### 2.4 Animal Spirits?

One would be tempted to interpret our results as support for the belief that such non-economic (or at least difficult to quantify) factors such as ‘animal spirits’, are important determinants of economic fluctuations. This is may be so, but our results do not by themselves demonstrate the existence of animal spirits.

Announcements on past GNP growth could affect future GNP growth in basically two ways:

1. Knowing that real growth is serially correlated, agents may use the past level of growth in order to forecast current or future growth. Their knowledge of the past is limited, but they can use the announcement in order to estimate what happened and from there forecast what will happen in the future.

2. Agents might use the announcements independent of their informational content about past economic activity. They might use the announcement as something different than an estimation of the past.

Anything that falls in this category could loosely be interpreted as ‘animal spirits’, but there are several formal models that predict that they would be used independently of
Table 2.5: Regression Results for Components of Real GNP

<table>
<thead>
<tr>
<th></th>
<th>$c_t=$consumption</th>
<th>$c_t=$investment</th>
<th>$c_t=$government spending</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>.00538*</td>
<td>.00341</td>
<td>.00286*</td>
</tr>
<tr>
<td></td>
<td>(.00100)</td>
<td>(.00487)</td>
<td>(.00100)</td>
</tr>
<tr>
<td>$c_{t-1}$</td>
<td>.254*</td>
<td>.269*</td>
<td>.219*</td>
</tr>
<tr>
<td></td>
<td>(.0986)</td>
<td>(.0999)</td>
<td>(.101)</td>
</tr>
<tr>
<td>$\bar{m}_{t-1}$</td>
<td>.00616</td>
<td>1.55*</td>
<td>.103</td>
</tr>
<tr>
<td></td>
<td>(.110)</td>
<td>(.781)</td>
<td>(.146)</td>
</tr>
<tr>
<td>Time Period</td>
<td>1967.1–91.4</td>
<td>1967.1–91.4</td>
<td>1967.1–91.4</td>
</tr>
<tr>
<td>Sample Size</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>.0448</td>
<td>.0676</td>
<td>.0270</td>
</tr>
<tr>
<td>Durbin-Watson</td>
<td>2.07</td>
<td>1.92</td>
<td>2.00</td>
</tr>
<tr>
<td>Box-Pierce Q</td>
<td>32.86</td>
<td>26.43</td>
<td>24.98</td>
</tr>
</tbody>
</table>

Standard deviations in parentheses. Starred coefficients are significant at the 5% confidence level.
their value as estimators of the past. In particular agents could use the announcements as an extraneous coordination device, a sunspot.

The fact that announcements are used a lot, does not imply that they are used too much. In order to show the existence of “animal spirits” we need to demonstrate that announcements are used for more than simply knowledge of what happened. In this section we will try to test exactly that. Our results will be found to be inconclusive. We are unable to say with certainty if announcements are used for more than estimating the past. Nevertheless we consider it to be an interesting exercise.

We examine a simple model in which agents should make most use of the announcement in order to estimate the past. In the context of this model we then test to see if agents are using the announcements for more than mere estimation. If, in this model, agents were to use the announcement “too much” (i.e., for anything different than an estimation of past growth), then we believe this would be evidence that “animal spirits” would indeed exist.

Assume that the only thing that affects an agent’s decision is the GNP growth of the economy during the last period. This is of course a crude assumption however it ensures that agents have maximum incentive to estimate past real GNP growth using all the information available to them. Whatever the “true” model of the world is, in it the agents would make less use of the announcement on aggregate activity than in our model. If with this model as the alternative hypothesis we cannot reject the null hypothesis that agents use the announcement for more than for estimation of the past, then with the “true” model of the world we could not reject it neither.

If we assume that agents know only the announcement (and thus possess no private information), our previous result comes as no surprise. They only care about the aggregate past and their only estimate of it is the announcement, so their actions today (and consequently aggregate output today) will depend only on the announcement. However, we pose for ourselves a more difficult problem.

We assume that agents also possess some private information. It is realistic to assume that agents know, or at least have accurate knowledge of, the level of activity in their sector of the economy. We continue to assume that agents care only about the past level of aggregate activity. The next logical step is to assume that sectorial activity and aggregate activity are
correlated. To summarize agents have two pieces of information:

1. They know what happened in their sector, so an agent in sector \( i \) \( (i = \{1, 2, ..., K\}) \) knows the rate of growth in sector \( i \) during \( t - 1 \): \( x^i_{t-1} \). Additionally, we posit that:

\[
  x^i_{t-1} = g_{t-1} + \epsilon^i_{t-1},
\]

where \( \epsilon^i_{t-1} \sim N(0, \sigma^2_\epsilon) \).

2. In addition, all agents receive an announcement of the economy's growth rate last quarter:

\[
  \hat{g}_{t-1} = g_{t-1} + \hat{m}_{t-1},
\]

where we assume \( \hat{m}_{t-1} \sim N(0, \sigma^2_m) \).

Agent \( i \) with information set \( I^i_t = [\hat{g}_{t-1}^i, x^i_{t-1}] \) faces a signal extraction problem in order to calculate her expectation of last period's real GNP growth. Application of Bayes' rule yields:

\[
  E[g_{t-1}|I^i_t] = (1 - \zeta)\hat{g}_{t-1}^i + \zeta x^i_{t-1}
\]

where

\[
  \zeta = \frac{\sigma^2_m}{\sigma^2_\epsilon + \sigma^2_m}.
\]

We assume that agents make their decision based on their beliefs about the level of \( g_{t-1} \) at time \( t \):

\[
  x^i_t = E[F(g_{t-1})].
\]

Assuming that \( F \) is linear, (2.11) becomes:

\[
  x^i_t = v + \gamma \left( (1 - \zeta) \hat{g}_{t-1}^i + \zeta x^i_{t-1} \right)
\]

Summation across agents yields equation (2.2):

\[
  g_t = \alpha_0 + \alpha_1 g_{t-1} + \alpha_2 \hat{g}_{t-1}^i
\]
with
\[ \alpha_0 = \nu, \]
\[ \alpha_1 = \gamma \zeta, \]
\[ \alpha_2 = \gamma (1 - \zeta). \]  

(2.14)

If announcements are only used to predict the past level of aggregate growth then it should be true that
\[ \frac{\alpha_2}{\alpha_1} = \frac{(1 - \zeta)}{\zeta} = \frac{\sigma^2_\xi}{\sigma^2_m}. \]  

(2.15)

Alternatively agents use the announcement for more than mere estimation of the past. (For example, the past announcement is an extraneous variable upon which agents coordinate.) In this case, the following equation holds:
\[ x^t_i = \nu + \gamma E[g_{t-1}|I^t_i] + \delta g^t_{t-1}. \]  

(2.16)

from which aggregate GNP growth would again follow equation (2.2), with
\[ \alpha_0 = \nu, \]
\[ \alpha_1 = \gamma \zeta, \]
\[ \alpha_2 = \gamma (1 - \zeta) + \delta. \]  

(2.17)

Assuming \( \delta > 0, \)
\[ \frac{\alpha_2}{\alpha_1} = \frac{(1 - \zeta)}{\zeta} + \frac{\delta}{\gamma \zeta} > \frac{\sigma^2_\xi}{\sigma^2_m}. \]  

(2.18)

Thus a test of whether agents in the economy use announcements today for more than mere knowledge of what happened last quarter is a test of whether \( \frac{\alpha_2}{\alpha_1} > \frac{\sigma^2_\xi}{\sigma^2_m}. \)

From Table 2.2 we obtain our estimates for \( \alpha_1 \) and \( \alpha_2. \) From the descriptive statistics reported in Table 2.1, \( \sigma^2_m \) is estimated to be \( 2.90 \times 10^{-5}. \)

We can calculate \( \sigma^2_\xi \) using data from the Productivity Database of Wayne B. Gray. This dataset is a panel of 450 four digit SIC sectors containing extensive annual data from 1958 to 1989, including estimates of value added and a price deflator for each sector. Taking \( \sigma^2_\xi \) to be the average of the variances each year, we obtain an estimate of \( \sigma^2_\xi \) to be \( \sigma^2_\xi = .0195. \)

As is apparent, the idiosyncratic sectorial noise is orders of magnitude larger than the
measurement noise. This makes us reject the null hypothesis that 'animal spirits' exists versus the alternative that the model, as previously stated, is true. We reject the null hypothesis, but the alternative hypothesis would, undoubtedly, also be rejected by the data. Our test is admittedly very simple, it does not proof or disprove the existence of "animal spirits". It is included here is only because we do not want our results to be misinterpreted as showing something that they actually do not.

2.5 Conclusion

In this chapter, we document the importance of agents' perceptions about past economic variables. For real GNP growth in the U.S., we show that the perceptions that economic agents have about what happened in the past is a much more important determinant of current growth than are the true levels of growth in previous quarters. Our primary result is that when both the true rate of growth during quarter \( t - 1 \) and the (first) rate of growth at \( t - 1 \) that the government announced at \( t \) are included in a linear model explaining real GNP growth during quarter \( t \), the true rate of growth at \( t - 1 \) does not matter. The beliefs of agents determine the future path of the economy much more so than past actions. We then showed that perceptions influence the aggregate economy through their influence upon aggregate investment. Our result questions the sources of the business cycle. If, as we argue, perceptions play an important role in aggregate fluctuations, then the role of other possible explanatory factors is necessarily smaller.
Chapter 3

Shared Knowledge

3.1 Introduction

Economic agents have to make predictions. They have to allocate scarce resources among different possible uses, so they would like to know the payoffs of these uses.

In making predictions they will use two different sets of information:

1. Their knowledge of the structure of the economy. In particular of its links across time, the way in which what happened in the past determines what will happen in the future.

2. Their knowledge of what actually happened in the past. As long as temporal links do exist, the knowledge of the past evolution of the economy is a crucial piece of information.

In this paper I am going to argue that the amount of available information on the past evolution of the economy is not an exogenous variable, but an endogenous one. We will see that the information disclosed to the agents at any moment of time is determined by the heterogeneity of their beliefs, a variable that will be modeled as endogenous to the economy. This will allow us to make strong predictions on the collective behavior of the agents. The following example intends to clarify what this paper is about.

\textsuperscript{6}The origin of this paper lies in conversations that I had with Daron Acemooglu. His suggestions have been instrumental in the birth of this paper. I can not be sufficiently grateful for it.

I was also helped by comments made by Andrés Almazán, Abijit Banerjee, Olivier Blanchard and the participants in the MIT Theory workshop.

Obviously I am solely responsible for all remaining errors.

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The endogenous accuracy of the perceptions of the past.

In 1972 few, if any, companies were making research in energy-efficient bulbs. Everybody had a prior on what return such an investment would pay, and the commonly shared view considered it quite a foolish investment.

Nowadays all the major firms in the sector are producing new and every time more efficient lamps. This is so even if the price of energy is not that much different, in real terms, today than what it was before the oil crisis. This is an indication that 25 years ago investment in energy efficient products would have been as wise as it is today. Nevertheless, the recession of the seventies was necessary in order for the companies to learn it.

The reason is that the priors that everybody had were not being tested. This kept the priors unchanged, which prevented them to be tested, et cetera...

The investment in research on low consumption bulbs had a very good rate of return, but this particular piece of information lied outside the information set of the agents. The rate of return that such an investment would have had during 1971 was, in 1972, part of the past evolution of the economy, but the information that the agents had on it was very inaccurate.

The interesting point is that this information was very inaccurate due to endogenous reasons. We can speculate, for a moment, that if a group of crazy outliers made the investment in 1971, assuming this investment was successful, and this success was publicized, then everybody would have tested their priors. In those circumstances most agents would have placed investments in such technologies before the '73 shock hit; but, you need the outliers, their success and its publicity.

At this stage it is probably clear that we are going to talk a lot about informational externalities. The hypothetical outliers, by engaging in an apparently foolish investment, would have produced a big positive externality on the whole of the economy. Had they invested, and had their actions been observed, they would have greatly enriched the information available to everybody else.

So in this context, agents produce informational externalities, but the structure of these externalities differs from most papers related to the topic. Note that I am assuming that agents not only observe the actions that other people took, but also the payoffs that these actions generated. In this respect the paper departs from the body of literature on the topic.
Externalities on the second moments of informational variables

We are going to assume that at any given moment of time, everybody gets an unbiased signal on the value of each of the relevant variables. The actions that agents take affect the variance of these signals, their accuracy, but not their expected value. The summation of all the signals will always reveal the truth.

What I am saying is that in 1972 every bulb manufacturer was receiving a signal on the value of investment in energy efficient technology, and that the expected value of each of these signals said the truth. Nevertheless the accuracy of these signals was very low, and so they scarcely amounted for information.

Imagine that there actually was a lonely outlier and that the perceived rates of return on an investment in efficient bulbs is known for everybody to be very volatile. This is because there is a lot of idiosyncratic noise, or simply because nobody can ever be 100% certain that what he perceives that happens to others is what really happened to them. Well, our outlier made the investment and it was a success. Even in these circumstances the bulb manufacturers would not have tested their priors substantially. They would have attributed the outlier's perceived success to just a lot of luck, or simply to inaccurate information. The high volatility of the perceived return with respect to the well established prior would have made the value of the first practically nil.

On the other hand, the priors would have been tested if instead of a single outlier there would have been a group of them. Lots of successful outliers cannot be lucky, they must be right.

The presence of externalities that affect the second moments of the informational variables is sufficient to generate herds of people not investing in energy-efficient technology. It is not necessary to assume that agents do not observe the outcomes of the actions taken by others; a noisy perception of the outcomes is enough to generate a very interesting collective behavior.

Typically informational externalities are modeled as affecting directly the first moment of the random variables that define an agent’s view of the world. That is, when an agent takes an action, he changes the beliefs that other agents have by affecting the expected value that they assign to the different random variables which they face. The most clear example of this is the literature on herding (Banerjee[3] and Bikchandani et al.[4] for instance).

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There agents observe the actions taken by other agents, but they do not observe their outcome. Some agents have private information (a noisy signal) on which is the best action to take, otherwise they start with flat priors on them. If you observe that most agents are taking a given action, not knowing how happy they are with their decisions, you would think that it is very likely that they are right even if you have a signal that says otherwise. You would then ‘herd’, do as they did, and by doing so you would be inducing more and more people to join the herd.

The key of this ‘they might know better’ type of herding is that agents do receive biased information from other agents, because they are unable to observe the outcome of other agents’ actions. The summation of the signals that they receive from other people would not reveal the truth. If they were capable of knowing the satisfaction that other agents get from their actions, the whole construction would fall and herding would disappear. There would be fast, optimal, learning.

The key assumption is the unobservability of other people’s outcomes, because the biased information that this induces. Not being an innocuous assumption, we should examine whether it is far away from reality, a task which I will not attempt to do. Nevertheless, it seems to me that, at least in what respects to macroeconomics, the value of such assumption is dubious. After all, firms make their books public, and any interested investor knows which firms are having abnormally high benefits and which ones are incurring in shameful losses.

In this paper I present a model in which agents do observe the outcomes, but with some noise. The information that they interchange will be unbiased. In spite of it, their collective behavior could rightly be categorized as ‘herding’.

**Links across time**

A few pages ago I made reference to two sets of information that the agents use: information on the structure of the economy and information on its past behavior. So far I have talked only about the second, as it is what the model intends to explain; but the first is not less important and some of the assumptions that I make on it are not completely innocuous.

We will assume that the return that different actions pay is independent of the activity of the agents, completely exogenous. This intends to bring both simplicity and clarity: simplicity
because otherwise the model would be orders of magnitude more complex, and clarity because we are interested in the study informational effects, not physical ones. In our context the actions that agents take affect the payoffs that they will get in the future because they affect their view of the world, not the world in itself.

Additionally we will build on the hypothesis that agents know the structure of the economy. They will not need to expend time and resources in learning how what actually happened in the past affects the events that will happen in the future. All their effort will be concentrated in learning what happened in the past. I may agree that this is quite a courageous assumption, but it greatly simplifies the model, allowing me to write the paper, and helps to focus it in its intended object of study: the endogeneity of perceptions.

There is a last assumption that is worth mention here which refers to the particular form of the links across time that agents will face. I will assume that the rates of return in the different sectors are random variables, the realizations of well defined stationary stochastic processes with a positive autocorrelation. Additionally, and this is not so important, this processes will be assumed to be independent among them.

The past rate of return in any sector will have predictive power on what will happen with this rate of return in the future, because the autocorrelation is different from zero. Additionally, the fact that the processes are supposed to be stationary, mean reverting, implies that agents will always expect the rates of return to move towards their long run average. After a period in which the disclosure of information on this variable was minimal the priors will converge to the long run mean of the variable. This turns out to be crucial for the conclusions that I will present. Even if the model itself allows for the presence of non stationary processes, its qualitative results would differ. To understand its solution in such a case would require further work.

**Heterogeneity across agents**

The amount of information disclosed on the return of a sector depends on the number of agents that are investing in it. If all the agents of the economy share the same opinions, they will all invest in the same sector because they will all think that is the one promising the highest payoff. The direct consequence of it is that no new information gets disclosed on any of the
other sectors. In our bulb manufacturing example we started by saying that all the agents had, in 1972, a *commonly* shared prior.

There is an intimate relationship between heterogeneity and information disclosure. If everybody invests in the same sector, the information disclosed on that sector will be very accurate. In other words, everybody will observe approximately the true value of the variable, and so all the agents will share very similar views of the world. In the same manner, if no information is disclosed, the priors of all the agents will converge to the long run mean of the variable, producing very homogeneous beliefs.

If beliefs across agents are heterogeneous, the aggregate investment will be diversified across the different sectors; consequently, there will be information disclosure in all the sectors. If they are very homogeneous, the amount of information disclosed will be high in one sector and minimal in the rest.

This dynamic relationship between information disclosure and homogeneity of beliefs is in the core of the explanation of the economy's behavior.

In the next section I present the model that we will use as the thinking tool in the rest of the paper. After that, the following three sections will pave the road towards the solution of the model. Section 3.3 studies how the perceptions differ across agents, the evolution of the heterogeneity of the perceptions; section 3.4 focuses on how the distribution of investment will be determined, and section 3.5 studies the dynamics of the model.

Section 3.6 will finally solve the model using numerical simulations. It may seem surprising now, but in section 3.7 we will use the model to help us understand things as diverse as why Japan's MITI is so successful, why the mileage of the cars was so low in 1972, and why it took so long for the Soviet Union to break down.

We will conclude by summarizing the findings and establishing the conclusions that they imply.
3.2 Set up of the Model

There is a continuum of risk neutral agents in the interval $(0, 1)$. Each agent has an unit of investment good available at each period. They have to invest it in one of $K$ investment opportunities. At time $t$ sector $k$ ($k = \{1, 2, \ldots, K\}$) pays an exogenous amount $R_t^k$ per unit of investment. We do not need to deal with intertemporal decision problems, because there is no accumulation (the good is perishable) and the informational structure rules out experimentation (see below).

The rates of return of each sector follow independent $AR(1)$ processes, all of them with the same autocorrelation and variance of the perturbations:

$$R_t^k = \rho R_{t-1}^k + e_t^k \quad \text{where} \quad e_t^k \sim N(0, \sigma) \quad (3.1)$$

We will call $x_t^k$ to the amount of aggregate investment in sector $k$ at time $t$.

3.2.1 Structure of the information; Externalities.

Before playing at time $t$, the information set of an individual consists of the perception that he has on the performance of each sector at all previous times. He has been collecting information since the beginning of time. This information consists in what he believes that each sector paid at each previous time, his perceptions coming in the form of signals. With all this information they establish priors on what is going to be the return on each sector at $t$.

The information that an agent receives (the signals) is independent of what he chooses to do. This rules out experimentation. Agents have no incentive to sacrifice some income (not maximizing expected income) in order to get information. By assuming this we will substantially simplify the model and make it tractable.

Perceptions of the past.

The perception that an individual has of the past does not depend exclusively on his own experience. Individuals observe how happy other individuals are with the actions they took, and communicate with each other. In all economies information on past events flows from one individual to another. The perceptions that any agent has on the state of the economy depends
not only on what he observes, but also in what others observe and how this information is transmitted to him. In other words the new information that an agent receives in a given period of time depends on:

1. The information that he receives because he invests in some place and gets a return from it.

2. The information that other agents receive because they too are investing in some sector.

3. The way that they interchange this information.

To model realistically all the micro characteristics of a process of information diffusion is a task well beyond the reach of this paper. Nevertheless, I will try to capture what I believe are the relevant characteristics of such a process by using a, hopefully reasonable, reduced form. First I will list the characteristics of the information diffusion process that I would like to capture. Later I will propose a reduced form that includes them.

1. An agent’s perception must have idiosyncratic characteristics. That is, it must be different for different agents. This is so because the rate of return that each agent observes will have idiosyncratic components (people are subject to idiosyncratic shocks in the rates of return). Additionally, the information that any individual gets from others will also depend on who he is because, for instance, who are the agents that exchange information with a given agent depends on his location.

2. I will assume that the perception that agents have is always unbiased, so that if they were able to pool their information they will get the truth out of it. This is not necessarily the case in the real world, but by doing this I am making things harder to the model. I am taking away the most obvious effect of informational externalities. We will see that there is herding even in this case.

Additionally, as long as we believe that agents interchange information on the outcomes of their actions this seems to be the natural assumption; we are ruling out the ‘they-might-know-better’ effect, and doing so we do not have ‘informational cascades’.
3. One would think that the precision in the perception that individuals have on what happened during the previous period depends on the number of individuals investing in any given sector. If nobody is investing in a sector, agents are going to receive no new information on the rate of return in that sector during that period. On the contrary, if all the agents are investing in a sector we should assume that the level of precision in the agents' perceptions on what happens in that sector is relatively high, because there is a big pool of information before they start interchanging it.

4. It is difficult to know how much the aggregate investment level in a sector affects the precision of the observations.

If an agent gets his information by sampling from the pool of agents, we would have a linear relationship, so that an increase of a one percent in the number of agents investing in \( k \) increases the precision of the perceptions of the sector \( k \)'s return by a one percent.

But that is not the only possible world. In a 'yellow journalism' society, the bigger the aggregate investment in a given sector, the bigger the attention that this sector receives from the news media, and so the more information on this sector that is reported, increasing exponentially the precision of the information that everybody gets (there is a bigger pool of information, and it is reported much more extensively). An increase of a one per cent in the aggregate investment produces an increase in the precision of more than a one per cent.

Alternatively, the media could be making 'structural' research, focusing on the processes themselves, and not relaying that much in the experiences of the agents. In such a case, an increase of a one percent in the number of agents that engage in a certain activity would increase the level of precision in less than a one percent.

The moral is that on this respect we should be quite open. Not knowing how the real world is, we should not burn our bridges by doing a too restrictive modelization.

In order to capture all the points expressed above I will assume that after playing at any time \( t \) each agent receives \( K \) signals, one for each sector. Each signal is the summation of
the true rate of return in the corresponding sector plus some noise that is independent across sectors, individuals and time.

Call this signal $S_t^k(i)$, then:

$$S_t^k(i) = R_t^k(i) + \frac{1}{\sqrt{P_t^k}} \epsilon_t^k(i)$$  (3.2)

Where $\epsilon_t^k(i)$ comes from a standard normal independent across sectors, individuals and time. $\frac{1}{P_t^k}$ is the variance of the noise; $P_t^k$ the precision of the signals.

The elasticity of the precision of the signals with respect to the aggregate investment in a sector is an exogenous constant, that depends on the information diffusion process of the society; in the ‘sampling’ case would be one, in a ‘yellow journalism’ society it would be bigger than one, and in the ‘structural information’ case smaller than one:

$$P_t^k = A \left( x_t^k \right)^p$$  (3.3)

A high $p$ represents a high level of social interaction, a situation where there are big informational externalities. In such a society an agent’s decision to invest in $k$ and not in $j$, has important consequences on the accuracy of the information that all the agents get on both sectors.

If the elasticity of the precision with respect to the aggregate investment were zero ($p = 0$) there would be no informational externality. The information would be independent of the evolution of the economy, and we would have an economy with no social interaction.

All values of $p$ between 0 and infinity are admissible, the higher the value the bigger the level of social interaction in the economy.

The parameter $A$ represents the level of ‘underlying precision’. The maximum possible level of precision, attainable when all the agents are investing in the same sector.

Given that the amount of investment in any sector is not bigger than one, the precision is always decreasing in $p$. This implies that higher levels of social interaction have a direct effect in lowering the precision. In order to compensate for this effect, when we make comparative
statics we will have to move not only \( p \), but also the level of ‘underlying precision’\(^1\).

### 3.2.2 Generating a prior about the future.

Each agent builds his belief about the future based on the information available to him: his perception of the history of the economy. This is, the collection of all the signals that he has received.

The signals are normally distributed, as a consequence his priors will also be normally distributed. In order to see this let’s go back to the first time he plays.

At the origin of time he had no information other than the knowledge of the stochastic process driving the returns (equation 3.1), so his prior on the return in any sector would be normally distributed with mean zero and variance \( \frac{\sigma}{1-p^2} \). After receiving a signal on what happened at \( t = 1 \), he would update his prior on the realization of the returns at \( t = 1 \), and after that, and based in his knowledge of equation 3.1, he will establish a prior on the value of the returns at \( t = 2 \).

We will now see that if the prior on the realization of the returns at any time \( t \) that an agent has before playing at \( t \) is normally distributed, then the prior on the returns at \( t + 1 \) after playing at \( t \) will also be normally distributed. This implies that all the priors are going to be normally distributed at all times, because at \( t = 1 \) they already were so.

Assume then that at \( t \), before investing, individual \( i \) believes that \( R^k_t \) is distributed from a normal with a mean \( \mu^k_t(i) \) and a variance \( V^k_t \):

\[
R^k_t(\cdot) \sim N(\mu^k_t(i), V^k_t) \quad (3.4)
\]

Independently of what he chooses to do, after playing he gets an unbiased signal on the rate of return in each sector with a (known) precision \( P^k_t \).

\(^1\)The effect of the informational externality is on the change in the precision derived by a change in the number of agents investing: \( \frac{\partial P^k_t}{\partial N^k_t} \).

The effect of a change in the level of social interaction \( \frac{\partial P^k_t}{\partial p^k_t} \) is an exercise of comparative statics, not an externality.
When the prior is updated (using Bayes law) the posterior on $R_t^k$ is also a normal distribution

$$R_t^{k+}(i) \sim N\left(\theta_t^k \mu_t^k(i) + \left(1 - \theta_t^k\right) S_t^k(i), \theta_t^k V_t^k\right)$$ \hspace{1cm} (3.5)$$

where:

$$\theta_t^k = \frac{1}{1 + P_t^k V_t^k}$$ \hspace{1cm} (3.6)$$
is the weight that agents assign to the prior when updating their belief. If $\theta_t^k$ is close to one (because the signal is very noisy or the prior very accurate) the agents put all the weight on the priors. If it is close to zero, they put all the weight on the signal.

Now let’s go back to equation 3.1; from the point of view of $i$ before playing at $t+1$ $R_t^k$ is normally distributed, and he knows that $e_t^k$ is so too. Thus given his information he will perceive $R_{t+1}^k$ as a normally distributed random variable,

$$R_{t+1}^k(i) \sim N\left(\mu_{t+1}^k(i), V_{t+1}^k\right)$$ \hspace{1cm} (3.7)$$

with mean:

$$\mu_{t+1}^k(i) = \rho \left(\theta_t^k \mu_t^k(i) + \left(1 - \theta_t^k\right) S_t^k(i)\right)$$ \hspace{1cm} (3.8)$$

and variance:

$$V_{t+1}^k = \rho^2 \frac{V_t^k}{1 + P_t^k V_t^k} = \rho^2 \theta_t^k V_t^k + \sigma$$ \hspace{1cm} (3.9)$$

The priors change across agents because they depend on the whole history of signals that the agents receive, and these signals are different for different agents. They have different perceptions of the past, and this induces different beliefs about the future.

On the other hand the variance of the beliefs depends only in the past variance of the signals, something that the agents know and that is common for all of them. $P_t^k$ and $V_t^k$ are common to all the agents. The priors differ only in their means.

### 3.3 Distribution of beliefs across agents

We have seen how the beliefs are generated, and that they may differ across agents; now we will see how much do they differ. We will see that the expected return is always distributed as
a normal, and we will determine how this distribution does evolve. The strategy is equal that in the previous section, first we show that at the origin of time the expected return is normally distributed across agents; then that if at any moment of time the distribution is a normal, it will always be so afterwards. Doing this we will also identify the stochastic differential equations that drive the beliefs of the agents.

Let’s go back again to the first time that they played. They had common flat priors, and then they updated them using their signals, as a consequence the prior that an individual $i$ has on the value of the return in sector $k$ at $t = 2$ is a normal with mean:

$$
\mu_2^k(i) = \rho \left( 1 - \theta^k_t \right) S_{1}^k(i)
$$

(3.10)

Across agents $S_{1}^k(i)$ is a normal with mean $R_1^k$ (what actually happened) and variance $\frac{1}{P_t^k}$. So, across agents $\mu_2^k(i)$ will also be a normal, with mean

$$
\rho \left( 1 - \theta^k_t \right) R_1^k
$$

(3.11)

and variance:

$$
\rho^2 \left( 1 - \theta^k_t \right)^2 \frac{1}{P_t^k}
$$

(3.12)

Now let’s assume then that at some time $t$ (that is, immediately before playing at $t$) the expected return was normally distributed across agents:

$$
\mu_t^k(i) \sim N \left( \tilde{\mu}_t^k, M_t^k \right)
$$

(3.13)

Where $\tilde{\mu}_t^k$ is the average expectation on $R_t^k$, the ‘consensus’ expected return, and $M_t^k$ measures the degree of heterogeneity in the beliefs of $R_t^k$, the dispersion in these beliefs.

The expected return at $t + 1$ for agent $i$ is

$$
\mu_{t+1}^k(i) = \rho \left( \theta^k_t \mu_t^k(i) + \left( 1 - \theta^k_t \right) S_t^k(i) \right)
$$

(3.14)

Given that $\mu_t^k(i)$ and $S_t^k(i)$ are independently distributed normals (across agents), then
$\mu_{t+1}^{k}(i)$ will also be a normal,

$$
\mu_{t+1}^{k}(i) \sim N(\bar{\mu}_{t+1}^{k}, M_{t+1}^{k})
$$

where its mean is

$$
\bar{\mu}_{t+1}^{k} = \rho^{k} \left( \theta_{t}^{k} \bar{\mu}_{t}^{k} + (1 - \theta_{t}^{k}) R_{t}^{k} \right) \tag{3.15}
$$

and its variance

$$
M_{t+1}^{k} = \rho^{2} \theta_{t}^{k} \left( \theta_{t}^{k} M_{t}^{k} + (1 - \theta_{t}^{k}) V_{t}^{k} \right) \tag{3.16}
$$

Thus across agents the expected return is always distributed as a normal, because we know that this was already the case at time $t = 2$.

Equations 3.15 and 3.16 are probably the most important for understanding the solution to the model, so later we will return to then for a more careful study, but so far let’s remark some of their implications.

When the signals are much more accurate than the priors so that the agents weigh the signal much more than the prior ($\theta_{t}^{k}$ is close to zero), the ‘consensus’ belief tracks the truth very well, ($\bar{\mu}_{t+1}^{k} \simeq \rho R_{t}^{k} = E(R_{t+1}^{k} \mid R_{t}^{k})$) and the dispersion of the beliefs is minimal, because everybody trusts what it ‘sees’, and everybody ‘sees’ more or less the same signal.

If the opposite happens (the priors are much more accurate than the signals) everybody moves its expected value towards zero (the long run mean), and so the range of beliefs decreases. Consequently both the average belief and the dispersion move towards zero independently of the realizations of $R_{t}^{k}$.

### 3.4 Aggregate Investment

Each individual has zero weight, so they are unable to change the variance of the signals by investing in one sector or another. This makes the individual decision problem quite trivial. They are risk neutral and they have no incentive to experiment, so they will invest everything in the sector from where they expect to get the highest return.
So the aggregate investment in sector \( k \) (\( x^k_t \)) will be the number of agents for whom:

\[
\mu^k_t(i) > \mu^j_t(i) \quad ; \forall j \neq k
\]

(3.17)

Given the distribution of the beliefs across agents and the fact that there is a continuum of agents, \( x^k_t \) will be exactly the probability that the previous condition holds.

\( \mu^k_t(i) \) depends on the history of signals that \( i \) received and the true rate of return in sector \( k \) at each different moment, but all this is independent across sectors, and the signals are also independent across agents. Thus \( \mu^k_t(i) \) and \( \hat{\mu}_t(i) \) are independent random variables, both of them normally distributed.

The probability that 3.17 holds (\( x^k_t \)) is then the cumulative distribution function of a Standard multivariate normal with \( K - 1 \) variables\(^2\). Calling the variables \( j = \{1, 2, ..., k - 1, k + 1, ..., K\} \), the integration limit of variable \( j \) is

\[
\frac{\tilde{\mu}^k_t - \tilde{\mu}^j_t}{\sqrt{M^k_t + M^j_t}}
\]

(3.18)

and the correlation between variables \( j \) and \( h \):

\[
\frac{M^k_t}{\sqrt{M^k_t + M^j_t}} \sqrt{M^j_t + M^h_t}
\]

(3.19)

If all the agents share very similar beliefs with respect to all the sectors (so that \( M^k_t \) is close to zero for all \( k \)), then everybody does the same thing; everybody invests in the sector that offers the highest expected payoff\(^3\). This is so unless their beliefs are identical across sectors \( (\tilde{\mu}^k_t = \tilde{\mu}_t \quad \forall k) \), in which case the aggregate investment will also be identical across sectors.

On the other hand, if the beliefs are not homogeneous across agents, the distribution of investment will follow an extremely non-linear function of their averages and dispersions, allowing for diversification.

\(^2\)See appendix J

\(^3\)In such a case the denominator of 3.18 is always zero, and given that it is sufficient to have a single integration limit close to minus infinity to have a probability close to zero; then the probability will be almost zero in all the sectors where the numerator is not always positive, and in the sector where it is always positive, it will be one.
3.5 Dynamics

The state of nature is defined by the real aggregate shocks on the return of the sectors. These rates of return are the only exogenous variable. The noise in the signals is whipped out by the law of the large numbers, its only purpose being to generate heterogeneity in the beliefs of the agents and, by doing so, the possibility of diversification of the investment. It has an effect through its variance, because it affects the weight that agents put on their signals, and in doing so, the dispersion of the beliefs.

The dynamic structure of the model is the following:

Given the averages and dispersion of the beliefs referring to the returns at $t$, the investment in each sector is determined:

$$x_t^k = \Phi(\{\tilde{\mu}_t^j, M_t^j\} \forall j)$$

(3.20)

This level of investment induces the precision with which each agent will observe the returns at $t$:

$$P_t^k = A \left( x_t^k \right)^p$$

(3.21)

The precision of the signals and the variance of the priors at $t$ determines the share of the prior in the update for each sector, how much do they trust their priors:

$$\theta_t^k = \frac{1}{1 + P_t^k V_t^k}$$

(3.22)

With it we can calculate the variance of the priors the following period:

$$V_{t+1}^k = \rho^2 \theta_t^k V_t^k + \sigma$$

(3.23)

The only exogenous variable is the rate of return at $t$, and this enters exclusively through the average belief at $t+1$

$$\tilde{\mu}_{t+1}^k = \rho \left( \theta_t^k \tilde{\mu}_t^k + (1 - \theta_t^k) R_t^k \right)$$

(3.24)

Finally, the variance of the priors and the past value of the dispersion will generate the
Figure 3-1:

dispersion of beliefs at t+1:

\[ M_{t+1}^k = \rho^2 \theta_t^k \left( \theta_t^k M_t^k + \left( 1 - \theta_t^k \right) V_t^k \right) \] (3.25)

The function \( \Phi \) has no closed form (there is no closed form for the CDF of a multivariate normal); consequently, it is not possible to find an analytical solution to the model. So we have to run simulations and see how the results change when the level of ‘social interaction’ \( p \) and the ‘underlying precision’ \( A \) change. Before doing so it is convenient to have a careful look at the previous equations and make an exercise that will help us understand the results.

Let’s take equations 3.22, 3.23 and 3.25; additionally let’s assume that the precision of the observations is fixed. All this accounts for a very non-linear system of differential equations. The solutions of \( M \) as a function of the level of precision (assuming \( \rho = 0.75 \) and \( \sigma = 1 \)) appear in figure 3-1.

There it is clear that when the precision of the observations is either very small or very
large, the dispersion of the beliefs becomes zero rather fast. We have already seen the intuition for this.

- If the precision of the observations is quite good, all the weight of the posteriors falls on the signals. The very fact that the precision is high implies that all the signals will be quite similar, their value very close to what actually happened. Thus agents will have very similar posteriors.

- If the precision is very low, they will put all the weight on the priors, but by doing so the range of beliefs gets smaller; the heterogeneity decreases, eventually being zero.

So, the agents have very homogeneous beliefs if in each sector the precision is either very high or very low. As we saw, this implies that the investment will be very concentrated in one sector.

The degree of precision in the signals of any sector $k$ depends on the underlying precision $\lambda$, the level of social interaction $p$, and the aggregate investment in the sector. Even if the investment is very low (but positive) the precision level can be quite high, provided that either $\lambda$ is very big and/or $p$ very small. Conversely, even if almost everybody is investing in a sector the precision might be very small.

Now let’s imagine 3 polar cases:

1. $\lambda$ is quite big and $p$ quite small, so that even if the number of agents investing in a sector is very small, the precision is quite big.

   In this case the dispersion of the beliefs will always be almost zero in all the sectors. This implies that almost everybody will invest in the sector that in the immediate past produced the highest payoff.

   There will always be a few outliers that make ‘mistakes’. They are very useful for the society because they generate signals with quite a high level of precision indicating the evolution of the non-optimal sectors. Thus almost all the members of the society are continuously keeping track of the evolution of the payoffs in all the sectors, and the aggregate investment is always close to one in the sector with the highest expected return.
2. \(A\) is quite small and \(p\) quite big, so that even if almost everybody were investing in a sector the precision in the signals of that sector would be very small.

This means that they cannot see anything, they will never update their signals significantly. They start with a common prior (zero, the long run distribution of the returns), and they stay with it forever. The dispersion of the beliefs is almost zero, but the investment is distributed evenly across sectors because the expected return is zero in all of them.

3. \(A\) and \(p\) are such that if the number of agents investing is very small, the precision is quite small (at the left of the peak in figure 3-1); and if the investment is close to one, quite big (at the right of the peak in figure 3-1).

Imagine that almost everybody is investing in the right place, the dispersion of beliefs will be nil in all the sectors, so they will keep concentrating the investment in one sector. They are able of keeping a good track of what happens in that sector, but they are unable to get information from the other sectors. The beliefs in all the other sectors will converge to zero (the long run mean) rather fast.

Almost everybody will invest in the chosen sector as long as it is paying above the long run average, even if other sectors are paying more. There is an obvious overconcentration of the investment in one of the sectors, excluding the possibility of learning. Only when the rate of return of the ‘herd’ is below the long run average will they diversify their investment and begin learning what is the best thing that they can do.

Assume that when they diversify they invest \(\frac{1}{K}\) in each sector. They will learn fast if doing so the precision of the signals is quite big, and again overconcentrate their investment in one of the sectors. On the other hand if when they diversify the precision in each sector is not so high, say that is in the ‘peak’ of figure 3-1, the learning process will be much slower, the recession longer.
3.6 Solution

As I said before, the solution of the model comes from the hand of simulations. All the simulations presented are done with parameters $\rho = .75$ and $\sigma = 1$, but changing these parameters does not change the qualitative results at all. Due to computational reasons all the simulations are done with $K = 4$, but as it will be clear this will provide us with enough information to adequately discuss how the solutions would be if there were more sectors. The simulations are the product of averaging the results for 25 different histories of 1000 observations each.

Each figure shows the result for an endogenous variable as a function of the logs of $A$ and $p$. This allows for comparisons when there are wide changes in the informational parameters. In figure 3-2 the graphs in the top are the 3-dimensional representation, while the figures in the bottom are the contour map of the respective surfaces. The columns represent the average production, the average Herfindal Index and the average dispersion of beliefs respectively.

To say that the model is non-linear is an understatement, a glimpse to the simulations is
proof enough of it. Things change suddenly for small changes in the parameters and remain steady for very large ranges of them. In order to understand the model we have to take a careful look at the figures and think about them having in mind the arguments given in the previous section.

To a first approximation we can divide the space defined by \( A \) and \( p \) in three regions that correspond with the 3 polar cases that we already saw. Inside these regions changes in the informational parameters do not induce significant changes in the expected return.

Now is when it comes in handy to make some exercises of comparative statics.

**Increasing the level of social interaction**

The first polar case exposed in the previous section corresponds to the region where the average payoff is maximum (see the first column of figure 3-2). In this area the precision of the signals is relatively high even if almost nobody is investing in a sector. This induces \( \omega^H \), homogeneous beliefs (see the second column), and thus big concentration of the investments (the third).
Figure 3-3 clarifies why this is so. There we represent the solution for \( M \) in the system of differential equations that we saw in the previous section, but instead of being a function of the precision (as in figure 3-1), here is a function of its components \( (A \) and \( p) \) when the investment is very close to zero\(^4\). It is clear that this first region corresponds with the area of the space \( A \times p \) where the beliefs are very homogeneous even if a minimal number of agents is investing in the sector. The variance of the priors is always relatively high, because of the stochastic structure of the returns (see equation 3.23, the variance of the priors is always bigger than \( \sigma \)), its precision always much smaller than the precision of the signals (even if an extremely small number of individuals is investing), so in doing the update almost all the weight is given to the signals, to the new information. In this area the agents keep very good track of the ranking of the sectors from best to worse. There is always a very small number of agents making ‘mistakes’, because the returns are continuously changing and to learn takes time. If the rates of return were constant, eventually everybody would be investing in the ‘right’ stuff. The noisy structure of the model induces some people to take the wrong decisions, but doing so they generate a big amount of information that the society, as a whole, uses.

If the level of social interaction were higher, the precision would decrease, both when almost everybody is investing and when almost nobody is doing it; but the decrease will be bigger when \( x \) is close to zero:

\[
\frac{\partial A x^p}{\partial p} = \left( \frac{x}{y} \right)^p \log \left( \frac{x}{y} \right) > 0 \iff x > y
\]

So, going back to figure 3-1, before increasing \( p \) the precision is always far away and to the right of the peak. As we go increasing \( p \) and the precision decreases, we move towards the peak faster for small values of \( x \). There are no substantial changes as long as the precision is at the right of the peak for small values of \( x \), because the signals are always much more accurate than the priors. The investment keeps concentrated in the right stuff.

Eventually the precision when the number of investors is very small arrives to the peak of figure 3-1 (or figure 3-3). Here, if the investment were very concentrated in one sector the heterogeneity of the beliefs in the other sectors would be quite high. However, this implies that

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\(^4\)Actually ‘very close to zero’ in this context means the machine epsilon, the smallest number bigger than zero that the computer recognizes as different from zero. This is a really small number.
the investment is not going to be very concentrated in any sector to start with (the denominators of the integration limits in equation 3.20 will not be close to zero). So in this range of parameters we should expect smaller concentration levels, more heterogeneity in the beliefs and a decrease in average output, and that is exactly what happens.

The 'peak' in figure 3-3 corresponds with the increase in average heterogeneity (see the second column of figure 3-2) and the 'valley' in the average concentration index (see third column). It also corresponds with the big fall in average output from the 'high' to the 'middle' plateaus. This can be seen more clearly in the contour 'maps', we are talking about the clearly defined lower 'arm' in all of them.

In this small region most of the agents 'do the right thing', but there is a substantial number of agents investing in sectors that did not do well in the immediate past. Again, if there was no change in the returns, they would eventually learn and concentrate their investment, but the optimal action changes continuously, and this allows for heterogeneity in the beliefs.

If \( p \) increases the average output falls into the 'middle' plateau. In this region when almost nobody invests the precision of the observations is at the left of the peak in figure 3-1 (above it in figure 3-3), while it is still at the right when almost everybody does it. If the investments were very diversified among the sectors the precision of the signals would be at the right of the peak. This can be seen in figure 3-4, which represents the solution of the already familiar system of differential equations that defines \( M \), this time for an investment level of \( \frac{1}{4} \).

Imagine that the informational parameters fall inside this region and people have flat priors on what is the best thing to do. They will diversify their investment. Doing so they will get very accurate signals, and in the next period they would concentrate their investment in the more promising sector. From then on, and until things change, they will get very accurate information about what is happening with that sector, but they will be getting no practical information on the evolution of the other sectors. The beliefs get very homogeneous, because all the weight is given to the signals in the 'chosen' sector (and the signals have low variance) and to the prior in the other sectors (and so the priors converge to zero because the payoffs are mean reverting). Homogeneity reinforces the situation, because the investment gets concentrated, and this produces homogeneous beliefs, et cetera.

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The individuals will diversify their investment only when the return of the sector that they observe falls below the long run average. Then they will learn fast, and again concentrate their signals, so that the situation remains the same, perhaps with the investment concentrated in a different sector, but extremely concentrated nevertheless. So the average Herfindal Index (second column of figure 3-2) is very high, while the average dispersion of beliefs is very low (third column).

The average output (first column of figure 3-2) in this region is substantially lower than in the first ‘plateau’ because the society only keeps track of what happens in one sector. Any of the other sectors could get a series of positive shocks and start paying above the one that carries almost all the investment. Actually this will happen most of the time, but the number of people that observes it is too small for the society as a whole to notice.

If the world were not changing continuously, the information that these outliers generate would be used sooner or later. Little by little more people would move to the sector with the highest payoff. A stationary stochastic world prevents this from happening because:

1. Mean reversion implies that the priors are continuously moved towards the long run mean, and the priors have a huge weight if the precision of the observations is low. The two effects work in the same direction, inducing the agents to believe that the sector is close to its unconditional expected value and minimizing heterogeneity.

2. The amount of information that the outliers generate is extremely low, so the learning process would be very slow in any case. The probability of the event ‘the sector where everybody invest goes back to its mean before agents learn that another sector is paying more’ is close to one.

It is in this region where we can talk properly of herding. Agents ‘follow the herd’ because it is paying above the long run average, so they are not doing too badly, and they are unable to see what happens in the rest of the world. This is a conformist society where investment in research is nil. Experimentation could produce higher returns for the economy as a hole, but nobody has the incentives to sacrifice expected income in order for everybody to learn.
Imagine that the level of social interaction increases even further, so that when agents diversify their investment the precision of the observations is small, generating substantially heterogeneous beliefs (i.e., when there is diversification the precision of the observations is in the peak of figure 3-1). We are now in the area that produces the peak in figure 3-4, and that corresponds with the ‘upper arm’ in all the contour maps.

Here when the agents concentrate their investment the heterogeneity would be quite small, and so far they would be behaving as in the previous region. But sooner or later the sector will start paying below the long run average, inducing diversification of the investment. Now the learning process will be much slower than it was in the previous region, and the beliefs will be heterogeneous for a longer period.

This accounts for the sharp falls in average production and concentration, and the increase in heterogeneity observable in this area.

The smaller the precision at the diversification stage, the slower the learning process. If we keep increasing \( p \) the learning process becomes all too slow.

If at the diversification stage the precision of the signals is very low, the weight in the update falls overwhelmingly on the priors. The agents do not receive new information, and very soon all of them will use as a prior the long run distribution of the rates of return. In this area, beyond the ‘upper arm’, agents *never* learn. The beliefs are homogeneous, but the investment is equally distributed among all the sectors because they expect all of them to produce the same return (the numerator of the integration limits is zero).

**Increasing the underlying precision of the observations.**

One can think of two effects due to an increase in \( A \).

1. The standard: people have better information, so they will do better.

2. The perverse: everybody has better information, so more people will do the right thing; but doing so the society may be overconcentrating their investment and not learning about the evolution of most sectors.

The most graphic way of seeing the second effect is in figure 3-2.
There, when $A$ is very small the amount of information that agents receive is nil; the beliefs are very homogeneous because all the weight is put on the priors. If we increase $A$ so that the precision level falls inside the ‘peak’ of figure 3-1, people begin to learn. To learn means to make use of the signals, and given that they differ across individuals, there is an increase in the degree of heterogeneity.

If $A$ increases even further, the signals get more precise, and as a consequence the view of the world that agents have gets more homogeneous. So we observe a sharp fall in $M$.

For relatively small values of $p$ the level of precision is mainly determined by $A$:

$$\lim_{p \to 0} A x^p = A$$

In this region a decrease in heterogeneity, associated with an increase in $A$, does not have negative consequences (the first effect dominates). Both if almost nobody invests, or if everybody does it, the increase in precision of the observations is roughly the same. The second effect is very small because the informational externalities are almost non-existent.

On the other hand if the level of social interaction is relatively large, the negative effects of overconcentration are apparent. Say that $p$ and $A$ are such that we are on top of the ‘arm rest’ in the first column of figure 3-2. There diversification of the investment produces a relatively high level of heterogeneity in the beliefs (see figure 3-4). If $A$ increases there will be an increase in the concentration of the investment. Additionally the increase in the information generated in the ‘minority’ sectors is not as big as the increase generated in the one where most people are investing:

$$\frac{\partial}{\partial A} A x^p = x^p$$

There is a net loss in information, because the number of agents in ‘minority’ sectors decreases when homogeneity increases. Herds ‘begin’ to be formed, eventually the economy falls in the second ‘plateau’ where further increases in $A$ have an effect only if they make the precision in the minority sectors to fall in the ‘peak’ of figure 3-3, because there the homogeneity of the beliefs increases and ‘bad herds’ are broken relatively fast.
3.7 Some Examples

As in most macroeconomics papers that end up being mostly theoretical, one of the most difficult moments appears when the model has to be taken from the theoretical Valhalla and brought down to the crude world of reality.

Evidence of the kind of effects on which I have commented is, at most, scant. Nevertheless this should not be too discouraging. On one hand such a simple model does not intend to be a precise description of a very complex reality. On the other there is enough evidence how to allow me to make some points, both normative and positive.

The model gives support to dirigiste industrial policies, such as the ones profusely employed by Japan’s MITI. These policies, as much praised by their supporters as criticized by their detractors, are based in the belief that firms, if left to themselves, will not generate an optimal allocation of resources among the different possible investments. The government gives incentives to the existence of firms in a very wide range of sectors, even if most companies would opt otherwise for not being in them. The resulting economy is quite flexible to get in different fields as they open; it discovers new fields of action with more flexibility than it would if it were directed exclusively by market forces. From a strictly orthodox point of view these policies should be rapidly discarded, but few people will dare to doubt the success story that is post war Japan.

Another, more direct, piece of evidence is provided by the oil crisis of the seventies, and the posterior surge in energy-efficient products. Previously to the crisis it would have seemed bizarre to place big investments in order to decrease the appetite for energy of most products. During the crisis it became imperative. In the car industry, for instance, most firms found themselves with an obsolete product portfolio⁵. A new range of energy efficient products was developed. New fields of research were born, ranging from recycling to mere efficiency improvement. After the crisis the price of energy returned to pre-1973 levels, but the new industry did not disappear. If any, it expanded. Nowadays in the Ruhr region there are more workers in ‘green’ industries

⁵Some remarkable exceptions come to mind, in particular the Japanese car industry.
than in coal and steel activities. There was probably a market for all of these new activities previously to the recession, but few people noticed it, and the ones who did were not perceived by the rest of the world; a big shock was necessary for most of us to see it.

In a somewhat more esoteric way the model also explains why it took so long for the Soviet Union to disappear.

Even if any moderately neutral observer could have perceived in the mid sixties that the Soviet way of doing things was vastly more inefficient than the Western one, a surprisingly large number of people did not. Actually, the people that really mattered did not. Surprisingly as it may seem the Soviet intelligentsia believed in what it preached (see Hosking[16]), even a good number of outside observers did not see the system as inherently inferior\textsuperscript{6}. People tend to disregard an opponent’s achievements, that is in the human character; so one should not be surprised that the Soviet intelligentsia did not compare their system with the Western one. More surprising is the fact that they did not pay due attention to the Hungarian experience of economic liberalization.

The explanation that the model gives is that, compared with before the revolution, the Soviet Union was not faring so badly. The standard of living was improving substantially, and that was so in most countries that adopted their system. They had no incentives to experiment with new policies. Only when the system fell by its own weight -be the shock called Gorvachov, or Afghanistan, or whatever- did the Hungarian experience take a new light. An experiment that had been successful for quite a long time was rediscovered as the ‘new’ promising path to be taken.

The system fell from within, all the new establishment (the people that took power after the revolution of 1990) was part of the former Soviet intelligentsia. This are people that even well into the eighties still professed an almost blind trust in the old orthodoxy. It was necessary a big crash for them to see that the system was plainly inefficient, something that would have been obvious if the Hungarian experience could have been perceived before.

Even after these events, the range of disagreement on what is the best path to take is quite

surprising, both among different countries of the former Soviet Block and within most of them. One is tempted to interpret this as the heterogeneity in beliefs, and consequent diversification, that the model predicts after a sharp fall.

3.8 Conclusions

The main moral of this paper is that the presence of informational externalities may lead to overconcentration of the investment activity in a few sectors, even if the externalities affect only the second moment of the information that agents receive.

We have seen that for a wide range of informational parameters the behavior of the agents, and so their payoff, does not change with changes in these parameters, and that can be rightly categorized as 'herding'. In this environment herding is not a consequence of people thinking that other agents 'might-know-better'. Everybody does the same thing because different courses of action are not being explored, nobody has incentives to explore the unbeaten path. In our context herding takes the form of overconcentration of the investment, something that can be interpreted as underinvestment in research.

This overconcentration of the investment appears when the level of underlying precision of the information that the individuals receive ($A$) is relatively high and at the same time there exists a substantial amount of social interaction (high $p$). The high level of $A$ allows the signals to be quite accurate when most people are investing in one sector, thus inducing a high degree of homogeneity in the perception that the members of the society have of that sector. Additionally, the high $p$ implies that, in order to be heard, the number of agents investing in a sector has to be big. If only a few outliers invest in some sector, the beliefs of everybody on the evolution of that sector become more homogeneous because all the weight of the updating is given to the priors, and the processes that drive the rates of return are mean reversing. Thus one effect feeds the other and we end up with a very homogeneous society: everybody sees what happens in one sector and assumes that all the others are near their long run average. Not until the sector in which they are investing starts paying below the long run average do they diversify their investment, and really learn what happens in the rest of the world.

That the results are not linear comes as no surprise, that they are so much non linear it is
probably more so. This is due to the big nonlinearity in the heterogeneity of the priors. When the perceptions are either bad or good, relatively to the priors, the agents will tend to share very similar views on the evolution of the return. When they are bad because agents expect the sector to go back to its long run average. When they are good because all the agents observe the same thing, and so they all share basically the same information.

It is only for a relatively small range of values in the precision of the signals that heterogeneity is generated; only when the precision of the signals is roughly of the same magnitude as the precision of the priors. In any case the evolution of both variables is endogenous to the economy; what happens in one sector affects the view that individuals have of other sectors, at least by default, by quitting experimentation.

We have also seen that across-the-board improvements in the accuracy of the information that agents receive may have quite perverse consequences. More information for everybody is not always good if we share that information.

In a world of short-sighted people, individuals are going to be constantly insecure of what is the best thing to do, and their views on it are necessarily going to differ; as a consequence, new paths are continuously going to be explored, and new roads found. In a world where agents posses almost 20 x 20 vision, they are going to be overconfident on what is the best course to take, nobody is going to explore off the beaten paths, and the society will be defenseless when exogenous shocks hit the chosen road.

There are only a few remarks left to make. I cannot deny the criticism that I have been using the mechanism of information diffusion as a black box. Undoubtedly the mechanism should be endogenous to the economy, and infinitely more complex than I have assumed. However reduced forms should be acceptable if they accurately represent a complex reality in a simplified way. By allowing for a very flexible functional form, and by not assuming any range of parameters, I have insured that whatever reality is, it is captured by some point in the space defined by $A$ and $p$.

Most of the paper is a long exercise of comparative statics. It determines the effects of changes in the informational parameters. Nevertheless, whatever the values of the parameters,
in the real world they are most likely to be constant.

As long as we believe that both $A$ and $p$ are relatively large, we should expect agents to behave in a very precise way. This behavior is unmodified for a huge range of parameters and it is characterized by excessive concentration of the investment in a few sectors. Thus, it results in underinvestment in experimentation for the society as a whole.
Appendix A

Transition Matrix

All the agents follow a given strategy \( \bar{w} \). At \( t-1 \) the aggregate was \( y_{t-1} \), then at \( t \) it will take the value \( y_{t} = \frac{\alpha}{n} \) if and only if all the agents without shock see \( w_{i}^{t} < \bar{w} \).

There are \( n - 2\alpha \) individuals without a shock, so the probability of all of them observing a number of investors smaller than the threshold is

\[
\Pr(y_{t} = \frac{\alpha}{n} \mid y_{t-1}) = \binom{n - 2\alpha}{0} \left(1 - \Pr(w_{i}^{t} < \bar{w} \mid y_{t-1})\right)^{0} \left(\Pr(w_{i}^{t} < \bar{w} \mid y_{t-1})\right)^{n - 2\alpha} \quad (A.1)
\]

Using the same arguments the probability that only one agent without shock observes investment above the threshold is

\[
\Pr(y_{t} = \frac{\alpha + 1}{n} \mid y_{t-1}) = \binom{n - 2\alpha}{1} \left(1 - \Pr(w_{i}^{t} < \bar{w} \mid y_{t-1})\right)^{1} \left(\Pr(w_{i}^{t} < \bar{w} \mid y_{t-1})\right)^{n - 2\alpha - 1} \quad (A.2)
\]

And in general \( \forall y \in \{\alpha, \alpha + 1, ..., n - \alpha \} \):

\[
\Pr(y_{t} = y \mid y_{t-1}) = \binom{n - 2\alpha}{y - \alpha} \left(1 - \Pr(w_{i}^{t} < \bar{w} \mid y_{t-1})\right)^{y - \alpha} \left(\Pr(w_{i}^{t} < \bar{w} \mid y_{t-1})\right)^{n - 2\alpha - (y - \alpha)} \quad (A.3)
\]
Appendix B

Conditional probabilities

The covariance of the signals is:

\[ E [(s_t(i) - p(y)) (s_t(j) - p(y))] = \]
\[ \Pr [s(i) = 1, s(j) = 1] (1 - p(y))^2 + \Pr [s(i) = 1, s(j) = 0] (1 - p(y)) (-p(y)) \]
\[ + \Pr [s(i) = 0, s(j) = 1] (1 - p(y)) (-p(y)) + \Pr [s(i) = 0, s(j) = 0] (p(y))^2 \]  \hspace{1cm} (B.1)

Assuming further symmetry:

\[ \Pr [s(i) = 1, s(j) = 0] = \Pr [s(i) = 0, s(j) = 1] \]  \hspace{1cm} (B.2)

Then:

\[ E [(s_t(i) - p(y)) (s_t(j) - p(y))] = \Pr [s(i) = 1, s(j) = 1] (1 - p(y))^2 \]
\[ -2 \Pr [s(i) = 0, s(j) = 1] (1 - p(y)) p(y) + \Pr [s(i) = 0, s(j) = 0] (p(y))^2 \]  \hspace{1cm} (B.3)

Given that:

\[ \Pr [s(i) = 1, s(j) = 1] + \Pr [s(i) = 1, s(j) = 0] = p(y) \]  \hspace{1cm} (B.4)

and

\[ \Pr [s(i) = 0, s(j) = 0] + \Pr [s(i) = 0, s(j) = 1] = 1 - p(y) \]  \hspace{1cm} (B.5)

Then, using 1.18

\[ p(y) \cdot (1 - p(y)) \cdot \rho(i,j) = p(y) \cdot (1 - p(y)) - \Pr [s(i) = 1, s(j) = 0] \]  \hspace{1cm} (B.6)
and so, the probability that individuals $i, j$ observe different signals is:

$$\Pr [s(i) = 1, s(j) = 0] = p(y) \cdot (1 - p(y)) \cdot (1 - \rho(i, j))$$  \hspace{1cm} (B.7)

The probability that both observe 1 is:

$$\Pr [s(i) = 1, s(j) = 1] = p(y) \cdot [1 - (1 - p(y)) \cdot (1 - \rho(i, j))]$$  \hspace{1cm} (B.8)

And the probability that they observe both 0 is:

$$\Pr [s(i) = 0, s(j) = 0] = (1 - p(y)) \cdot [1 - p(y) \cdot (1 - \rho(i, j))]$$  \hspace{1cm} (B.9)

For each value of $y$ then, the conditional probability of $j$ to observe 1, given that $i$ has observed one:

$$\Pr [s(j) = 1 \mid s(i) = 1] = \frac{\Pr [s(i) = 1, s(j) = 1]}{\Pr [s(i) = 1]}$$  \hspace{1cm} (B.10)

$$= (1 - (1 - p(y)) \cdot (1 - \rho(i, j)))$$

And the probability that $j$ observes 1 given that $i$ observed 0:

$$\Pr [s(j) = 1 \mid s(i) = 0] = \frac{\Pr [s(i) = 0, s(j) = 1]}{\Pr [s(i) = 0]} = p(y) \cdot (1 - \rho(i, j))$$  \hspace{1cm} (B.11)
Appendix C

Symmetry of the unconditional distribution

Now let’s look at the unconditional distribution. Given that our strategy is perfectly symmetric and that the process of information distribution is also perfectly symmetric, it must be the case that the conditional distributions must be symmetric, that is:

\[ F(y \mid y_{t-1}) = 1 - F(1 - y \mid 1 - y_{t-1}) \]  \hspace{1cm} (C.1)

and differentiating with respect to \( y \):

\[ f(y \mid y_{t-1}) = f(1 - y \mid 1 - y_{t-1}) \]  \hspace{1cm} (C.2)

Now the unconditional distribution:

\[ q(y) = \int_0^1 f(y \mid x)q(x)\,dx = \int_0^1 f(y \mid x) \left( \int_0^1 f(x \mid z)q(z)\,dz \right)\,dx \]  \hspace{1cm} (C.3)

Now let’s define a succession of infinite variables \( \{x_1, x_2, x_3, \ldots\} \), then always that \( q(y) \) exists, must be the case that:

\[ q(y) = \int_0^1 \int_0^1 \int_0^1 \ldots f(y \mid x_1)f(x_1 \mid x_2)f(x_2 \mid x_3)\ldots dx_1 dx_2 dx_3 \ldots \]  \hspace{1cm} (C.4)
And:

$$q(y) = \int_0^1 \int_0^1 \int_0^1 ... f(1-y | 1-x_1)f(1-x_1 | 1-x_2)f(1-x_2 | 1-x_3)... dx_1 dx_2 dx_3... \quad (C.5)$$

Now, defining $z_n = 1 - x_n$,

$$q(y) = \int_0^1 \int_0^0 \int_0^0 ... f(1 - y | z_1)f(z_1 | z_2)f(z_2 | z_3)...(-dz_1)(-dz_2)(-dz_3)... = q(1 - y) \quad (C.6)$$

So the unconditional distribution is symmetric.
Appendix D

Expected unconditional probabilities.

We know that

\[ p(y) = (1 - p(1 - y)) \]  \hspace{1cm} (D.1)

and

\[ q(y) = q(1 - y) \]  \hspace{1cm} (D.2)

In such a case

\[ \int_0^1 p(y)q(y)dy = 1 - \int_0^1 p(1 - y)q(y)dy = 1 - \int_0^1 p(1 - y)q(1 - y)dy \]  \hspace{1cm} (D.3)

Defining \( z = 1 - y \):

\[ \int_0^1 p(y)q(y)dy = 1 - \int_0^1 p(z)q(z)dz \]  \hspace{1cm} (D.4)

\[ \int_0^1 p(y)q(y)dy = \frac{1}{2} \]  \hspace{1cm} (D.5)

So the unconditional expected probability of receiving a high signal is \( \frac{1}{2} \). The same argument shows that the unconditional expected probability of receiving a low one is also \( \frac{1}{2} \).
Appendix E

Moments of the conditional distribution.

The conditional expectation:

\[
E(y_t \mid y_{t-1}) = E \left[ \int_0^{2\pi} \frac{z_t(j)}{2\pi} dj \mid y_{t-1} \right] \\
= \int_0^{2\pi} E[z_t(j) \mid y_{t-1}] dj \\
= \int_0^{2\pi} E[y_t(j) \mid y_{t-1}] dj = p(y_{t-1})
\]  
(E.1)

And its variance:

\[
\sigma^2 [y_t \mid y_{t-1}] = E \left[ (y_t - p(y_{t-1}))^2 \mid y_{t-1} \right] \\
= E \left[ \left( \int_0^{2\pi} \frac{s_t(i)}{2\pi} - p(y_{t-1}) \right) dj \right] \mid y_{t-1} \\
= \int_0^{2\pi} E \left[ \left( \int_0^{2\pi} \frac{s_t(i)}{2\pi} - p(y_{t-1}) \right) dj \right] \mid y_{t-1} \\
= \int_0^{2\pi} p(y_{t-1}) \left( 1 - p(y_{t-1}) \right) \int_0^{2\pi} p(i,j) dj di \\
= p(y_{t-1}) \cdot (1 - p(y_{t-1})) \cdot R
\]  
(E.2)
Appendix F

The fourth strategy is an equilibrium.

F.1 Symmetry of the Unconditional Distribution with the fourth strategy.

Given a level of output at $t - 1$ agents are as likely to receive the wrong signals under strategy 3 or under strategy 4 (this is so because the process of information distribution is exogenous and does not depend on the strategy). Calling $f_4(\cdot)$ to the conditional probability under the "jumping" strategy:

$$f(x \mid y) = f_4(1 - x \mid y)$$

This is so because $f(x \mid y)$ is the probability that in the animal spirits equilibrium a $x$ percent of the agents receive the signal $s_t(i) = 1$, if the equilibrium is the fourth one, when a $x$ percent of the agents receive this signal, a $1 - x$ percent invest.

In this circumstances:

$$f_4(1 - x \mid 1 - y) = f(x \mid 1 - y) = f(1 - x \mid y) = f_4(x \mid y) \quad (F.1)$$
and it follows from above that $q_4$ is symmetric and

$$
\int_0^1 p(y)q_4(y)dy = \int_0^1 (1 - p(y))q_4(y)dy = \frac{1}{2}
$$

**(F.2)**

### F.2 Equilibrium

If everybody is following the fourth strategy $x_t(j) = 1 - s_t(j)$. Thus:

$$
E(y_t \mid s_t(i)) = \int_0^{2\pi} \frac{E(x_t(j) \mid s_t(i))dy}{2\pi} = \int_0^{2\pi} \frac{E(Pr(s_t(j) = 0 \mid s_t(i)))}{2\pi}
$$

**(F.3)**

When $s_t(i) = 1$:

$$
Pr(s_t(j) = 0 \mid s_t(i) = 1) = (1 - p(y))(1 - \rho(i, j))
$$

**(F.4)**

And when $s_t(i) = 0$:

$$
Pr(s_t(j) = 0 \mid s_t(i) = 0) = 1 - p(y)(1 - \rho(i, j))
$$

**(F.5)**

So:

$$
E(y_t \mid s_t(i) = 1) = (1 - R)\int_0^{1} \frac{p(y)(1 - p(y))q_4(y)dy}{\int_0^{1} p(y)q_4(y)dy}
$$

$$
= (1 - R) \cdot 2 \cdot \int_0^{1} p(y)(1 - p(y))q_4(y)dy
$$

**(F.6)**

and

$$
E(y_t \mid s_t(i) = 0) = 1 - (1 - R)\int_0^{1} \frac{p(y)(1 - p(y))q_4(y)dy}{\int_0^{1} (1 - p(y))q_4(y)dy}
$$

$$
= 1 - (1 - R) \cdot 2 \cdot \int_0^{1} p(y)(1 - p(y))q_4(y)dy
$$

**(F.7)**

As we saw $0 < \int_0^{1} p(y)(1 - p(y))q_4(y)dy < \frac{1}{4}$ (this is true for whatever density function $q$), which implies:

$$
E(y_t \mid s_t(i) = 1) < \frac{1}{2}
$$

**(F.8)**

and

$$
E(y_t \mid s_t(i) = 0) > \frac{1}{2}
$$

**(F.9)**

So the strategy is an equilibrium.
Appendix G

Robustness of the Animal Spirits equilibrium to the presence of uninformed agents.

We have four equilibria:

- $e_1$: always invest.
- $e_2$: never invest.
- $e_3$: animal spirits.
- $e_4$: invest iff $s_t(i) = 0$.

All of them generate equilibria, but only the Animal Spirits one is robust to the presence of uninformed agents; this is, in the hypothetical case that there appears a Martian who does not know in which equilibrium the economy is, after observing her signal she would play the Animal Spirits strategy. We can consider that everybody is a Martian that knows that everybody else is also a Martian, then they will know that everybody is playing the Animal Spirits strategy, and so the best that they can do is to play it too.

The point is that if you assign a equal prior to all the possible equilibria and then you see $s_t(i) = 1$, your posterior that the equilibrium is “never invest” is very low, so you have
incentives to invest, and vice versa when you see \( s_t(i) = 0 \). The best bet is to always follow the animal spirits strategy, to always follow the past.

### G.1 Priors and posteriors.

If an agent knows that the world is at an equilibrium, but is completely ignorant about which on it is, we should consider that she would assign flat priors across the equilibria.

Calling \( \epsilon \) to the equilibrium in place (that from the point of view of the Martian is a random variable),

\[
Pr(\epsilon = e_k) = \frac{1}{4} \quad \forall i
\]

Before playing she receives a signal about what happened yesterday, and she knows that the equilibrium is atemporal. Whatever the equilibrium was yesterday, today we will have the same one.

So she will update her prior on which the equilibrium is by using the signal:

\[
Pr(\epsilon = e_k \mid s_t(i)) = \frac{Pr(s_t(i) \mid \epsilon = e_k)}{\sum_{k=1}^{4} Pr(s_t(i) \mid \epsilon = e_k)}
\]

The probabilities of receiving one or the other signal given the equilibrium are:

\[
\begin{align*}
Pr(s_t(i) = 1 \mid \epsilon = e_1) &= (1 - p_0) \quad Pr(s_t(i) = 0 \mid \epsilon = e_1) = p_0 \\
Pr(s_t(i) = 1 \mid \epsilon = e_2) &= p_0 \quad Pr(s_t(i) = 0 \mid \epsilon = e_2) = (1 - p_0) \\
Pr(s_t(i) = 1 \mid \epsilon = e_3) &= \frac{1}{2} \quad Pr(s_t(i) = 0 \mid \epsilon = e_3) = \frac{1}{2} \\
Pr(s_t(i) = 1 \mid \epsilon = e_4) &= \frac{1}{2} \quad Pr(s_t(i) = 0 \mid \epsilon = e_4) = \frac{1}{2}
\end{align*}
\] (G.1)

So:

\[
\begin{align*}
Pr(\epsilon = e_1 \mid s_t(i) = 1) &= \frac{1 - p_0}{2} \quad Pr(\epsilon = e_1 \mid s_t(i) = 0) = \frac{p_0}{2} \\
Pr(\epsilon = e_2 \mid s_t(i) = 1) &= \frac{p_0}{2} \quad Pr(\epsilon = e_2 \mid s_t(i) = 0) = \frac{1 - p_0}{2} \\
Pr(\epsilon = e_3 \mid s_t(i) = 1) &= \frac{1}{4} \quad Pr(\epsilon = e_3 \mid s_t(i) = 0) = \frac{1}{4} \\
Pr(\epsilon = e_4 \mid s_t(i) = 1) &= \frac{1}{4} \quad Pr(\epsilon = e_4 \mid s_t(i) = 0) = \frac{1}{4}
\end{align*}
\] (G.2)
G.2 Payoffs.

The utility that the Martian would expect to get, conditional to the equilibrium in place and the signal that she receives:

\[
EU\left(s_t(i) = 1, \varepsilon = e_1\right) = \frac{1}{2} \quad EU\left(s_t(i) = 0, \varepsilon = e_1\right) = \frac{1}{2}
\]
\[
EU\left(s_t(i) = 1, \varepsilon = e_2\right) = -\frac{1}{2} \quad EU\left(s_t(i) = 0, \varepsilon = e_2\right) = -\frac{1}{2}
\]
\[
EU\left(s_t(i) = 1, \varepsilon = e_3\right) = \frac{1}{2} - A_3 \quad EU\left(s_t(i) = 0, \varepsilon = e_3\right) = A_3 - \frac{1}{2}
\]
\[
EU\left(s_t(i) = 1, \varepsilon = e_4\right) = A_4 - \frac{1}{2} \quad EU\left(s_t(i) = 0, \varepsilon = e_4\right) = \frac{1}{2} - A_4
\]

Where

\[
A_3 = 2 \cdot (1 - R) \cdot \int_0^1 p(y) (1 - p(y)) q(y) dy < \frac{1}{2}
\] (G.4)

and

\[
A_4 = 2 \cdot (1 - R) \cdot \int_0^1 p(y) (1 - p(y)) q_4(y) dy < \frac{1}{2}
\] (G.5)

When the Martian observes \(s_t(i) = 1\) if she invest, she expects to get:

\[
U(x_t(i) = 1 \mid s_t(i) = 1) = \frac{1 - p_0}{2} \cdot \frac{1}{2} + p_0 \left(-\frac{1}{2}\right) + \frac{1}{4} \left(\frac{1}{2} - A_3\right) + \frac{1}{4} \left(A_4 - \frac{1}{2}\right)
\] (G.6)

And if she observes \(s_t(i) = 0\):

\[
U(x_t(i) = 1 \mid s_t(i) = 0) = p_0 \cdot \frac{1}{2} + \frac{1 - p_0}{2} \left(-\frac{1}{2}\right) + \frac{1}{4} \left(A_3 - \frac{1}{2}\right) + \frac{1}{4} \left(\frac{1}{2} - A_4\right)
\] (G.7)

The Martian would invest whatever she sees (strategy 1) only if both G.6 and G.7 are non-negative. She will never invest (strategy 2) only if both G.6 and G.7 are non-positive. She will follow the Animal Spirits strategy if and only if G.6 is non-negative and G.7 is non-positive. She will follow the fourth strategy only if G.6 is non-positive and G.7 is non-negative.
G.3 “Martian Proof” equilibrium.

If our Martian invest after observing \( s_t(i) = 1 \), she gets:

\[
U(x_t(i) = 1 | s_t(i) = 1) = \frac{1-p_0}{2} + \frac{p_0}{2} \left( -\frac{1}{2} \right) + \frac{1}{4} \left( \frac{1}{2} - A_3 \right) + \frac{1}{4} \left( A_4 - \frac{1}{2} \right) = \frac{1}{2} \left[ \frac{1}{2} - p_0 + (1 - R) \left( \int_0^1 p(y) (1 - p(y)) \, q_4(y) \, dy - \int_0^1 p(y) (1 - p(y)) \, q_4(y) \, dy \right) \right] 
\]

and after observing \( s_t(i) = 0 \),

\[
U(x_t(i) = 1 | s_t(i) = 0) = \frac{p_0}{2} \left( -\frac{1}{2} \right) + \frac{1}{4} \left( A_3 - \frac{1}{2} \right) + \frac{1}{4} \left( \frac{1}{2} - A_4 \right) = \frac{1}{2} \left[ p_0 - \frac{1}{2} + (1 - R) \left( \int_0^1 p(y) (1 - p(y)) \, q_4(y) \, dy - \int_0^1 p(y) (1 - p(y)) \, q_4(y) \, dy \right) \right] = -U(x_t(i) = 1 | s_t(i) = 1) 
\]

So the necessary and sufficient condition for the animal spirits strategy to be a robust equilibrium is:

\[
\frac{1}{2} - p_0 + (1 - R) \left( \int_0^1 p(y) (1 - p(y)) \, q_4(y) \, dy - \int_0^1 p(y) (1 - p(y)) \, q_4(y) \, dy \right) > 0 \quad (G.10)
\]

The variance of the signals is bounded by \( p_0 (1 - p_0) \) and \( \frac{1}{4} \), and so is a weighted average of all this possible variances. From this it follows that

\[
\int_0^1 p(y) (1 - p(y)) \, q_4(y) \, dy - \int_0^1 p(y) (1 - p(y)) \, q_4(y) \, dy \geq p_0 (1 - p_0) - \frac{1}{4} \quad (G.11)
\]

Thus, given that \( p_0 < \frac{1}{2} \) (and so \( p_0 (1 - p_0) - \frac{1}{4} < 0 \)):

\[
\frac{1}{2} - p_0 + (1 - R) \left( \int_0^1 p(y) (1 - p(y)) \, q_4(y) \, dy - \int_0^1 p(y) (1 - p(y)) \, q_4(y) \, dy \right) \geq \frac{1}{2} - p_0 + (1 - R) \left( p_0 (1 - p_0) - \frac{1}{4} \right) = \frac{1}{2} - p_0 + p_0 (1 - p_0) - \frac{1}{4} - R \left( p_0 (1 - p_0) - \frac{1}{4} \right) > \frac{1}{4} - (p_0)^2 > 0
\]

Which proves that the Animal Spirits equilibrium is the only one that is robust to the presence of uninformed agents.
Appendix H

Data Appendix

In this appendix we explain the sources of our data used in Chapter 4.

The time series for the true level of real GNP are taken from the United States National Income and Product Accounts. Our data on the initial estimates of current dollar GNP were recorded from the Survey of Current Business. For the time period January 1967 to October 1991, we recorded the monthly estimates of the previous five quarters of current dollar GNP for which estimates are available. Our regression results reported above concentrate upon the first estimate of real GNP growth for quarter \( t - 1 \) published during quarter \( t \). This information is typically published (and widely disseminated) one month after the end of quarter \( t - 1 \). Our data on the true levels of real GNP and its breakdown into components are also from the NIPA accounts.

As noted in the text, our sectoral data is from the Productivity Database. This dataset has extensive annual data at the four-digit level. From this source, we extract series on current dollar value added in each sector and deflate these series to construct real value added in each sector. The sector growth rates are then constructed by forming the differences of the natural logarithms.
Appendix I

Industrial Production

In this appendix, we report results of a regression explaining the growth rate of monthly industrial production (denoted \( ip_t \)). The regression equation we estimate is

\[
ip_t = \alpha + \sum_{j=1}^{11} \delta_j \ S_j + \sum_{j=1}^{12} \phi_j \ \hat{i}_{t-j}^t + \sum_{j=1}^{12} \rho_j \ ip_{t-j} + \epsilon_t \quad (I.1)
\]

where \( S_j \ (j = 1, ..., 11) \) are seasonal dummy variables, \( \hat{i}_{t-j}^t \) is the rate of growth of industrial production for month \( t - j \) announced during month \( t \), \( \{\delta_1, ..., \delta_{11}, \phi_1, ..., \phi_{12}, \rho_1, ..., \rho_{12}\} \) are parameters to be estimated and \( \epsilon_t \) is an error term.\(^1\) The results of the estimation of equation (I.1) are reported in Table I.1.\(^2\) The seasonal noise makes the interpretation substantially more difficult than previous regressions. The estimate of the rate of growth of monthly production two months and four months previous are both significant and of the appropriate sign. The coefficients \( \phi_{10} \) and \( \phi_{11} \) (associated with announcements of monthly growth eleven and twelve months previous) are also significant. Thus this regression does not display the same pattern of significant coefficients as with seasonally adjusted real GNP reported in the text. The \( R^2 \) of the regression is much higher than before but that is because a substantial portion of the variance of \( ip_t \) is due to seasonal variation explained (in a statistical sense) by the seasonal dummy variables.

\(^1\)Note during month \( t \) the government makes its first announcement of the rate of growth during month \( t - 2 \).
\(^2\)To save space we report only those coefficients that are significant at the 5% level of confidence.
Table I.1: Industrial Production Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$-0.0364^*$</td>
</tr>
<tr>
<td></td>
<td>(0.00755)</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.170*</td>
</tr>
<tr>
<td></td>
<td>(0.0751)</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>0.104*</td>
</tr>
<tr>
<td></td>
<td>(0.0515)</td>
</tr>
<tr>
<td>$\phi_{10}$</td>
<td>0.144*</td>
</tr>
<tr>
<td></td>
<td>(0.0708)</td>
</tr>
<tr>
<td>$\phi_{11}$</td>
<td>$-0.287^*$</td>
</tr>
<tr>
<td></td>
<td>(0.0709)</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.166*</td>
</tr>
<tr>
<td></td>
<td>(0.0562)</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>$-0.123^*$</td>
</tr>
<tr>
<td></td>
<td>(0.0667)</td>
</tr>
<tr>
<td>$\rho_{12}$</td>
<td>0.466*</td>
</tr>
<tr>
<td></td>
<td>(0.0690)</td>
</tr>
<tr>
<td>Time Period</td>
<td>1964.1-91.1</td>
</tr>
<tr>
<td>Sample Size</td>
<td>324</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.849</td>
</tr>
<tr>
<td>Durban-Watson</td>
<td>1.96</td>
</tr>
</tbody>
</table>

Standard deviations in parentheses. Starred coefficients are significant at the 5% confidence level.
Appendix J

Aggregate Investment

Agents will invest in the sector that has the highest expected return.

Aggregate investment in sector \( k \) is the number of agents for whom:

\[
\mu^k_i(i) - \mu^j_i(i) > 0 \quad \forall j \neq k
\]  

(J.1)

It is clear that

\[
x^k_i = \Pr \left\{ \mu^k_i(i) - \mu^j_i(i) > 0 \quad \forall j \neq k \right\} = \\
\Pr \left\{ \frac{(\mu^k_i(i) - \mu^j_i(i) - (\bar{\mu}^k_i - \bar{\mu}^j_i)}{\sqrt{M^k_i + M^j_i}} > \frac{(\bar{\mu}^k_i - \bar{\mu}^j_i)}{\sqrt{M^k_i + M^j_i}} \right\} = \\
\Pr \left\{ \frac{(\mu^j_i(i) - \bar{\mu}^j_i) - (\mu^k_i(i) - \bar{\mu}^k_i)}{\sqrt{M^k_i + M^j_i}} < \frac{(\bar{\mu}^k_i - \bar{\mu}^j_i)}{\sqrt{M^k_i + M^j_i}} \right\}
\]

(J.2)

Both, \( \mu^k_i(i) \) and \( \mu^j_i(i) \) are independent random variables, both of them normally distributed:

\[
\mu^k_i(i) \sim N \left( \bar{\mu}^k_i, M^k_i \right) \\
\mu^j_i(i) \sim N \left( \bar{\mu}^j_i, M^j_i \right)
\]

(J.3)

where

\[
\text{covariance} \left( \mu^k_i(i), \mu^j_i(i) \right) = 0
\]

so:

\[
\mu^k_i(i) - \mu^j_i(i) \sim N \left( \bar{\mu}^k_i - \bar{\mu}^j_i, M^k_i + M^j_i \right)
\]

(J.4)

Define \( \nu^j \) as:
\[
\nu^j = \frac{(\mu^k_i - \bar{\mu}^j_i) - (\mu^h_i - \bar{\mu}^j_i)}{\sqrt{M^k_i + M^j_i}}
\]

Then:

\[
\nu^j \sim N(0, 1)
\]  
(J.5)

and, \(\forall j, h \neq k:\)

\[
E(\nu^j \cdot \nu^h) = \frac{M_i^k}{\sqrt{M^k_i + M^j_i} \sqrt{M^k_i + M^h_i}}
\]  
(J.6)

Define now the vector \(v:\)

\[
v = \{\nu^1, \nu^2, ..., \nu^{k-1}, \nu^{k+1}, ..., \nu^K\}
\]  
(J.7)

and the vector \(l:\)

\[
l = \left\{ \frac{\mu^j_i - \bar{\mu}^j_i}{\sqrt{M^k_i + M^j_i}} \right\}, \quad j = \{1, 2, ..., k - 1, k + 1, ..., K\}
\]  
(J.8)

Then:

\[
x^k_i = \Pr \{v < l\}
\]  
(J.9)

The joint CDF of \(v\) is a normal a standard multivariate normal with a matrix of variances defined by the correlations in equation J.6.
Bibliography


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