Modeling Of Polarimetric Scattering From Vegetation Using Radiative Transfer Theory

by

Angel Ramon Martinez

Submitted to the Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degrees of Bachelor of Science in Electrical Science and Engineering

and

Master of Engineering in Electrical Engineering and Computer Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May 1996

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May 28, 1996

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Chairman, Departmental Committee on Graduate Theses
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Abstract

Microwave Remote Sensing has become an invaluable tool in the study of Earth’s various terrains and biomasses. It has been used to probe sea ice, forests, and agricultural crops. Important in all these applications is the proper modeling of the terrain, ice, or vegetation being observed. These models serve as guides in the understanding the data collected by remote sensing platforms. In particular, this thesis focuses on an effort to increase the accuracy of current models of polarimetric scattering from vegetation. This is accomplished by introducing the use of coherent scattering terms in the calculation of the extinction matrix in the vector radiative transfer equation. It is found that coherent scattering increases the total scattering by 40% at low frequencies and only about 15% at higher frequencies. When considering the total loss due to absorption and scattering, (the case in which the imaginary and real parts of the dielectric constant are comparable) coherent scattering accounts for less than 1% at low frequencies and almost 10% at higher frequencies.

Thesis Supervisor: Jin Au Kong
Title: Professor of Electrical Engineering
Acknowledgments

First and foremost, I would like to thank Chih-Chien Hsu for his incredible patience, kindness and understanding. His help was invaluable in my work, from start to finish. I sincerely wish him the best of luck in his future endeavors.

Of course I am deeply indebted to Professor Kong who gave me the opportunity to work within the Remote Sensing Group. He is one of the best teachers I have ever had, and I could not have asked for a better thesis advisor.

Thanks to my wonderful wife Ochida. Without her constant love and support I would not have gotten as far as I have. You truly are the wind beneath my wings, sweetie.

Thanks to my parents and my sister, whose love and care helped me to rise and meet the challenges I encountered at MIT.

Thanks to all my friends in La Casa and NH4.

Finally, thank you God, for all the blessings You have given to me and for holding me up with Your strong hands when I needed them most.
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Chapter 1

Introduction

1.1 Introduction to Remote Sensing

Remote sensing has become an important aid in the study of earth’s resources. Previous methods of remote sensing of earth terrain have depended on optical or thermal systems. Obvious limitations for these systems included the impenetrability of clouds and daytime only use. Microwave remote sensing, although slightly affected by rain, is not bounded by such limitations and would thus provide an alternative to these techniques.

The advent of microwave polarimetric remote sensing has provided a powerful tool in the monitoring of Earth’s environment and terrain from orbiting satellites or high flying aircraft. One recent polarimetric system is the SIR-C/X-SAR (Synthetic Aperture Radar) System which has flown on the Space Shuttle. SAR systems will prove important for future observation of changes in ecosystems, such as determining exact relationships between the planet’s CO₂ supply and regenerating forests [1], and in giving important agricultural statistics [2]. The SIR-C/X-SAR provides the selection of multiple wavelengths as well as selectivity of the polarization of observation [3].

SAR images are intended to represent scattering matrix elements which in turn relate incident and scattered electromagnetic waves [4]. Full utilization of radar parameters ultimately leads to more detailed knowledge of geometric and electric surfaces under observation [5]. Specifically, microwave polarimetric sensing utilizes the
polarization within the return signal in analyzing the terrain under observation. This provides for better characterization of the land below in terms of foliage density, types of vegetation present, etc. than would be possible with backscatter cross sections alone [6].

1.2 Background

In order to make full and accurate use of remote sensing data, proper models of the terrain or vegetation in question must be developed. In the case of vegetation, these theoretical models must account for scattering from trunks, branches, and leaves.

There is a special challenge in the microwave remote sensing of vegetation. Its random structure, differing types of vegetation and variations in growth density produce a complicated return signal. In order to utilize the information from vegetation backscatter, models have been created to predict this electromagnetic scattering. Two main approaches are the continuous random medium approach [7] and the discrete scatterer approach [8].

For the continuous random medium model, the random medium is represented by a stochastic dielectric variation. For example, layers can be modeled with a composite permittivity of two terms; one representing an average and the other random permittivity fluctuations [9]. In the case of strong dielectric fluctuations within the medium, strong fluctuation theory has been developed [10].

With the discrete scatterer model, vegetation or some other structure of interest is modeled as a collection of random discrete scatterers. Previous modeling efforts often used spherical scatterers [11]. The more sophisticated scatterer models are needed to detail a vegetation structure's main trunk, branches and leaves. The vertical variance is accounted for by the use of distinct layers, each having it's own scattering structures. To solve for the scattering for this model, analytic wave theory coupled with Foldy's approximation have been utilized [12]. The distorted Born approximation [13] and radiative transfer theory [14] are also commonly used in such discrete scatterer models. The application of radiative transfer theory to this scattering problem is the focus of
this thesis.

Radiative transfer theory was used in the field of astrophysics by S. Chandrasekhar. The theory's objective is to determine the intensities created by an incident wave and the resulting scattering from the particles present in the space of interest. In its formulation, the intensity is calculated within the boundaries of planes. These planes arise from the stratification of an atmosphere, for example [15]. This of course lends itself well to the multilayered model used in remote sensing problems. A vector form of the radiative transfer theory, the vector radiative transfer equation, will be used for the vegetation scattering problem. The equation will be introduced in the next chapter and contains parameters vital to the modeling of the vegetation structures and the various scattering mechanisms present there.

1.3 Description Of The Thesis

The Vector Radiative Transfer Equation (VRTE) contains parameters which have been calculated to the first order. It is of interest, then, to understand what accuracy may be gained by adding higher order terms. Specifically, the scattering between elements in the vegetation model will now be used in the calculation of the extinction matrix, which describes the losses of the incident wave due to scattering and absorption.

Chapter 2 will delve more deeply into the physical and mathematical models used in solving for the scattering.

Chapter 3 will present results and conclusions.
Chapter 2

Modeling And Formulation

2.1 Vegetation Model

In the probing of forested areas, trees are the primary scattering structures. Proper modeling of trees is therefore imperative in understanding SAR measurements from forests. A tree's trunk, branches, and leaves all contribute to the electromagnetic backscatter. Measurements of radar backscatter have been conducted on some species of trees [16] and therefore serve as reference to computer and mathematical models.

In addition to the dielectric properties of vegetation, attention has been given to the creation of physical models. A generalized vegetation structure is modeled as layers containing different scatterer structures, as shown in Figure 2-1. Within

<table>
<thead>
<tr>
<th>Region</th>
<th>Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Crown Layer</td>
</tr>
<tr>
<td>2</td>
<td>Trunk Layer</td>
</tr>
<tr>
<td>3</td>
<td>Understory Layer</td>
</tr>
<tr>
<td>4</td>
<td>Ground</td>
</tr>
</tbody>
</table>

![Diagram of vegetation layers](image)

Figure 2-1: Regions
each layer, specific scattering architectures are described using the discrete scatterer model. The discrete scatterer model is appealing because it facilitates the matching of real vegetation parameters. Vegetation structures are composed of many different length scales, and each type has its own particular structure and form [17]. Including parameters such as trunk diameter, branch distribution, leaf size etc. to the model greatly increases its effectiveness.

For example, a plant or tree can be modeled as a main cylinder having attached to it independently and randomly distributed smaller cylinders (branches). The leaves themselves can be modeled as dielectric disks [8]. An example cylinder cluster used in modeling pine forests [18] is shown below:

![Cylinder Cluster Model For Pine Forest](image)

Figure 2-2: Cylinder Cluster Model For Pine Forest

modeling elements because their scattering functions are for the most part known and understood. In the case of the cylinder, the local scattering fields are computed in terms of Hankel and Bessel functions and then rotated to suit the necessary reference frame [19]. As mentioned before, leaves are often modeled as disks. The shape of the disks depends on the type of vegetation being studied. For soybean plants circular disks are used [2]. For pine forests, small cylinders represent coniferous needles as shown above in Figure 2-2. Deciduous trees have leaf shapes that are more elliptical
than circular, and have been used in backscattering experiments as well [20].

2.2 Formulation

In order to understand the information within the return signal of an SAR system it is necessary to create a $2 \times 2$ complex scattering matrix relating the incident wave with the scattered wave:

$$
\begin{bmatrix}
E_{h'} \\
E_{v'}
\end{bmatrix}
^{sc} = \frac{e^{i kr}}{\tau}
\begin{bmatrix}
f_{h'h} & f_{h'v} \\
f_{v'h} & f_{v'v}
\end{bmatrix}
\begin{bmatrix}
E_h \\
E_v
\end{bmatrix}
^{inc}
$$

(2.1)

where $f_{yx}$ quantifies the change of the incident polarization $x$ to scattered polarization $y$ caused by the scattering properties of the medium of interest. Analogous to this is the Stokes, or Mueller matrix which is a real $4 \times 4$ matrix [21] that also relates the incident polarizations with the scattered polarizations and is crucial in the development of scattering models. By measuring the magnitude and phase of the different polarizations present in the return signal, the construction of the Stokes matrix is possible [6].

The interest, then, is to correlate the magnitudes and phases of the return polarizations with physical properties within the scattering medium. Arriving at a mathematical representation of scattering within a random medium is an arduous undertaking. One must take into account various scattering and absorption mechanisms that occur in the scatterer filled volume of interest. An exact solution would entail the inclusion of high order terms describing multiple inter-particle scattering.

When an electromagnetic wave, whether it be visible light or microwave, enters a scattering medium losses to the incident wave occur. The attenuation is expressed as extinction and is the sum of absorption and scattering effects [22]. In order to quantify scattering effects, the density of the medium must be taken into account as well as the size and shape of the particles.

In a medium with high density the assumption of independent scattering is no longer feasible [14]. This independent scattering is referred to as incoherent scat-
tering and describes scattering from individual structures within scattering media. Coherent scattering, on the other hand, defines the inter-structural scattering that also takes place. As the density increases, the coherent intensity becomes comparable to the incoherent intensity translating into a greater effect in wave fluctuations by multiple scattering effects [23]. The radiative transfer theory, to be described later, is applicable to mediums with a particle density of less the 1% [24]. In forests, this particle density is only about .1% allowing the use of radiative transfer theory in calculating scattering. This low particle density also allows the inclusion of only incoherent intensities in calculations. However, it is the purpose of this thesis to understand how much accuracy is gained by including coherent effects.

Radiative transfer theory has produced a powerful formulation that represents the change in intensity fields through scattering and absorption within a medium. The equation of transfer can be expressed in the following manner: [15]

\[- \cos \theta \frac{dI(z, \theta, \phi)}{\kappa \rho \, dz} = I(z, \theta, \phi) - \zeta(z, \theta \phi) \tag{2.2}\]

where \(I\) is the intensity, \(\rho\) is the density of the material, and \(\zeta\) is the source function. The other parameter, \(\kappa\), is the extinction coefficient.

The vector radiative transfer equation has also served as a central mathematical formulation in the description of electromagnetic scattering from collections of distributed particles. It is a simply a vector representation of the radiative transfer theory already discussed. It is advantageous in that it economizes computational time and can allow for multiple scattering effects if solved numerically [25].

The vector radiative transfer equation is given as:

\[\cos \theta \frac{d\vec{I}(\theta, \phi, z)}{dz} = -\vec{\kappa}_e \cdot \vec{I}(\theta, \phi, z) + \int_{0}^{2\pi} d\phi' \int_{0}^{\pi} d\theta' \sin \theta' \vec{P}(\theta, \phi, \theta', \phi') \cdot \vec{I}(\theta', \phi', z) \tag{2.3}\]

where \(I\) is the intensity vector containing the Stokes parameters, and \(P\) is the phase matrix describing contributions from directions other than that of the incident direc-
tion. The matrix $P$ is given as:

$$
\overline{P}(\theta, \phi, \theta', \phi') = n_0 \times 
\begin{bmatrix}
\langle |f_{HH}|^2 \rangle & \langle |f_{VH}|^2 \rangle & Re(f_{HV}f_{HH}^*) & -Im(f_{HV}f_{HH}^*) \\
\langle |f_{HV}|^2 \rangle & \langle |f_{VV}|^2 \rangle & Re(f_{VV}f_{HH}^*) & -Im(f_{VV}f_{HH}^*) \\
2Re(f_{VH}f_{HH}^*) & 2Re(f_{VH}f_{HH}^*) & Re(f_{VV}f_{HH}^* + f_{VH}f_{HV}^*) & -Im(f_{VV}f_{HH}^* - f_{VH}f_{HV}^*) \\
2Im(f_{VH}f_{HH}^*) & 2Im(f_{VH}f_{HH}^*) & Im(f_{VV}f_{HH}^* + f_{VH}f_{HV}^*) & Re(f_{VV}f_{HH}^* - f_{VH}f_{HV}^*)
\end{bmatrix}
$$

(2.4)

In Equation 2.3, $\overline{\kappa_e}$ is the extinction matrix describing loss through absorption and scattering. The characterization of $\overline{\kappa_e}$ with respect to the physical attributes of the scattering structure is of importance. This matrix contains magnitude and phase information necessary to relate the incident and scattered waves. The elements of the $\overline{\kappa_e}$ matrix are composed of scattering functions which are in turn defined by parameters such as the permittivity, size, orientation, and distribution of the scattering particles [14]. The extinction matrix is defined in the following manner:

$$
\overline{\kappa_e}(\theta, \phi) = \frac{2\pi n_0}{k} \begin{bmatrix}
2Im(f_{HH}) & 0 & Im(f_{HV}) & Re(f_{HV}) \\
0 & 2Im(f_{VV}) & Im(f_{VH}) & -Re(f_{VH}) \\
2Im(f_{VH}) & 2Im(f_{HH}) & Im(f_{VV} + f_{HH}) & Re(f_{VV} - f_{HH}) \\
-2Re(f_{VH}) & 2Re(f_{HH}) & -Re(f_{VV} - f_{HH}) & Im(f_{VV} + f_{HH})
\end{bmatrix}
$$

(2.5)

where $n_0$ is inverse volume and $k$ is the wave number. The $\overline{\kappa_e}$ matrix can be thought of as two separate matrices,

$$
\overline{\kappa_e} = \overline{\kappa_a} + \overline{\kappa_s}
$$

(2.6)

where $\overline{\kappa_a}$ symbolizes absorption loss and $\overline{\kappa_s}$ scattering loss.

In order to utilize the physical models and equations presented thus far, an effective formulation is needed to provide the scattering functions for the scattering portion of the extinction matrix. The optical theorem and its vector representation [26], relate the scattering function of an object with its cross section [27]. It is based
on the following equation:

\[ \sigma_e = \frac{4\pi}{k} \text{Im}\{f\} \]  \hspace{1cm} (2.7)

where \( f \) is of course the scattering function and

\[ \sigma_e = \sigma_a + \sigma_s \]  \hspace{1cm} (2.8)

is the cross-sectional area in terms of absorption and scattering of the scattering body. In using the optical theorem, the scattering amplitude function \( f \) must be found for the structure as a whole in order to accurately calculate the \( \sigma_s \) portion of total scattering loss. This can be accomplished when the structure in question is a simply-shaped body. However, the formulation for a structure representing trees and plants would be very complicated. It would require the use of finite difference methods to find an exact solution, needing a large amount of computing time.

Currently, the optical theorem is used to calculate scattering from each individual cylinder in the vegetation cluster model which is then summed to represent scattering from the entire structure. This has given reasonably accurate results when compared to field data. However, this method neglects coherent scattering that occurs between the elements which constitute the cluster model. The next step, therefore, is to derive the total scattering from a cluster in such a manner so as to include the coherent scattering effects. Mathematically, this inclusion may be understood by thinking of the extinction matrix as total loss due to a collection of structures, as opposed to the optical theorem which calculates loss for a single structure. In other words, the extinction matrix can be expressed as:

\[ \overline{\kappa_e} = n_o \cdot \left( \frac{4\pi}{k} \text{Im}\{f_{\text{effective}}\} \right) \]  \hspace{1cm} (2.9)

where \( f_{\text{effective}} \) is the forward scattering function of the entire structure.

The scattering portion of the extinction matrix, \( \overline{\kappa_s} \), will now be expressed as the
addition of two matrices representing incoherent and coherent scattering mechanisms.

\[
\overline{\kappa_e} = \overline{\kappa_a} + \overline{\kappa_{\text{sincoherent}}} + \overline{\kappa_{\text{coherent}}} \tag{2.10}
\]

The definition of these two pieces of the scattering loss may be derived from the expression describing the total power scattered from the structure in question. The scattered power from the cluster model is [6]:

\[
|S_{\text{total}}| = |S_0| + \sum_{j=1}^{N} |S_j|^2 + 2 \sum_{j=1}^{N} \text{Re}[S_0^* S_j e^{i(\theta_j - \theta_i^*)}] + \sum_{j=1}^{N} \sum_{i \neq j} S_i^* S_j e^{i(\theta_j - \theta_i^*)} \tag{2.11}
\]

where \(\theta\) is the phase factor created by differences in the incident and scattering vectors. The variable \(S\) can be described in terms of the scattering amplitude function and the incident \(E\) field in the following manner:

\[
|S_n|^2 = \frac{1}{r^2} |f_n|^2 |E|^2 \tag{2.12}
\]

where \(r\) is the distance between scatterer and observer. The first term in the scattered power expression, \(|S_0|^2\), is the power scattered from the main trunk while the second term represents individual scattering from branches. The last terms represent phase interference created by combined scattering elements as well as overall scattering [6].

Using the scattered power expression as a guide, the incoherent and coherent portions can now be expressed in the following manner:

\[
\overline{\kappa_{\text{sincoherent}}} = n_o \cdot \int_{\Omega'} \left\{ |f_0|^2 + \sum_{n=1}^{N} |f_n|^2 e^{i\phi_n} \right\} \tag{2.13}
\]

\[
\overline{\kappa_{\text{coherent}}} = \frac{1}{V} \cdot \int_{\Omega'} \left\{ 2N \text{Re}[\langle f_m \rangle \langle f_n^* \rangle e^{-i\phi_m}] + \hat{N} (i\hat{N} - 1) \langle f_m \rangle \langle f_n^* \rangle <e^{i\phi_m} \rangle <e^{-i\phi_n} \rangle \right\} \tag{2.14}
\]

In the both equations, the scattering functions are those of a simple cylinder calculated by the optical theorem. The term \(f_0\) is the scattering from the main trunk, and \(f_n\) and \(f_m\) are scattering from secondary structures such as branches and needles. The angled brackets denote ensemble averaging which is used to take into account variations in
physical dimensions and angles. The phase $\phi$ represents the phase delays between the structures in the cluster.

In the hopes of introducing greater accuracy, the coherent terms will now be included in the formulation of the $\overline{\kappa_e}$ matrix. This accuracy will not incur a high price in complexity as would be the case with the use of the optical theorem since the added terms are comprised of already known scattering functions. An already developed simulation program will be modified to reflect this new formulation in the radiative transfer equation.

### 2.3 Solution Method

Once the extinct and phase matrices are constructed, a solution for $\overline{I}(\theta, \phi, z)$ may be found. The solving of the radiative transfer equation is an iterative process usually taken to the first or second order. Using boundary conditions defined by adjoining layers, the intensity vectors are found within each layer of the medium. The solution process will now be demonstrated for the two layer case [28, 29].

![Two Layer Medium Diagram](Image)

Figure 2-3: Two Layer Medium
Within Region 1, the radiative transfer equations are expressed as:

\[
\cos \theta \frac{d\vec{I}(\theta, \phi, z)}{dz} = -\vec{\kappa}_e(\theta, \phi) \cdot \vec{I}(\theta, \phi, z) + \vec{S}(\theta, \phi, z)
\] (2.15)

\[
-\cos \theta \frac{d\vec{I}(\pi - \theta, \phi, z)}{dz} = -\vec{\kappa}_e(\pi - \theta, \phi) \cdot \vec{I}(\pi - \theta, \phi, z) + \vec{W}(\theta, \phi, z)
\] (2.16)

where \( \vec{I}(\theta, \phi, z) \) is the upward going Stokes vector and \( \vec{I}(\pi - \theta, \phi, z) \) represents the downward going Stokes vector. The terms \( \vec{W}(\theta, \phi, z) \) and \( \vec{S}(\theta, \phi, z) \) are source terms which include scattering intensities emanating from directions other than that of propagation.

The boundary conditions will now be considered. At the boundary \( z = 0 \),

\[
\vec{I}(\pi - \theta, \phi, z = 0) = \vec{T}_0 \delta (\cos \theta - \cos \theta_0) \delta (\phi - \phi_0)
\] (2.17)

while further down at \( z = -d \),

\[
\vec{I}(\theta, \phi, z = -d) = \vec{R}(\theta) \vec{I}(\pi - \theta, \phi, z = -d)
\] (2.18)

where \( \vec{R}(\theta) \) is a matrix composed of Fresnel reflection coefficients describing reflection at the boundary between Regions 1 and 2 for each polarization. It should be noted that the two boundary conditions are defined for \( 0 \leq \theta \leq \frac{\pi}{2} \).

Utilizing the boundary conditions along with the original radiative transfer equations, a set of integral equations are produced. In order to determine the first order solution, the source terms \( \vec{S} \) and \( \vec{W} \) are set to zero, leaving an equation of the form of

\[
\frac{d\vec{I}}{ds} = -\vec{\kappa}_e \vec{I}
\] (2.19)

which can be solved with eigenvalue methods.

The first order solution for both the upward going and downward going intensities are [14]

\[
\vec{I}(\pi - \theta, \phi, z) = \vec{E}(\pi - \theta, \phi) \vec{D}(\beta(\pi - \theta, \phi)z \sec \theta) \vec{E}^{-1}(\pi - \theta, \phi) \vec{I}_0
\]
\[ \times \delta (\cos \theta - \cos \theta_0) \delta (\phi - \phi_0) \]
\[ + \int_0^z dz' \left\{ \overline{E}(\pi - \theta, \phi) \overline{D}(\beta(\pi - \theta, \phi)(z - z') \sec \theta) \cdot \overline{E}^{-1}(\pi - \theta, \phi) W(\theta, \phi, z') \right\} \]
(2.20)

\[ \overline{I}(\theta, \phi, z) = \overline{E}(\theta, \phi) \overline{D}(-\beta(\theta, \phi) \sec \theta(z + d)) \overline{E}^{-1}(\theta, \phi) \overline{R}(\theta) \]
\[ \cdot \overline{E}(\pi - \theta, \phi) \overline{D}(-\beta(\pi - \theta, \phi) d \sec \theta) \overline{E}^{-1}(\pi - \theta, \phi) \overline{I}_0 \]
\[ \times \delta (\cos \theta - \cos \theta_0) \delta (\phi - \phi_0) \]
\[ + \overline{E}(\theta, \phi) \overline{D}(-\beta(\theta, \phi) \sec \theta(z + d)) \int_0^d dz' \left\{ \overline{E}^{-1}(\theta, \phi) \right\} \]
\[ \cdot \overline{R}(\theta) \overline{E}(\pi - \theta, \phi) \overline{D}(-\beta(\pi - \theta, \phi) d \sec \theta)(z' + d) \]
\[ \cdot \overline{E}^{-1}(\pi - \theta, \phi) W(\theta, \phi, z') \right\} \]
\[ + \int_0^z dz' \overline{E}(\theta, \phi) \overline{D}(\beta(\theta, \phi) \sec \theta(z' - z)) \]
\[ \cdot \overline{E}^{-1}(\theta, \phi) S(\theta, \phi, z') \]
(2.21)

where the 4x4 matrix \( \overline{D} \) contains the diagonal elements \( \exp (\beta_i(\theta, \phi) z \sec \theta) \). The \( \beta_i \) values are eigenvalues and the \( \overline{E} \) are the corresponding eigenmatrices.

This solution was found with the use of the method of variation of parameters. An iterative solution is also described in [14]. The steps detailed above can be easily extrapolated for the \( n \) layer case, and, more specifically, the four layers used in vegetation models.
Chapter 3

Results And Conclusions

3.1 Data

A FORTRAN program was developed to simulate polarimetric backscattering in vegetation and was used heavily in collecting data for this thesis. The parameters given to the program included frequency, incident angle, size and density of trees, and average physical measurements for the trunk, branches, and leaves. The following table is representative of the program's input parameters.
<table>
<thead>
<tr>
<th>Frequency</th>
<th>$1 \times 10^9 Hz$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incident Angle</td>
<td>$45^\circ$</td>
</tr>
<tr>
<td>Dielectric Constant Of Trunk</td>
<td>$28.6 + i12$</td>
</tr>
<tr>
<td>Dielectric Constant Of Secondary Branch</td>
<td>$28.6 + i12$</td>
</tr>
<tr>
<td>Dielectric Constant Of Leaves</td>
<td>$28.6 + i12$</td>
</tr>
<tr>
<td>Radius Of Trunk</td>
<td>$8.70 \times 10^{-2}m$</td>
</tr>
<tr>
<td>Length Of Trunk</td>
<td>$5.85 m$</td>
</tr>
<tr>
<td>Radius Of Secondary Branch</td>
<td>$4.69 \times 10^{-2}m$</td>
</tr>
<tr>
<td>Length Of Secondary Branch</td>
<td>$0.47 m$</td>
</tr>
<tr>
<td>Radius Of Leaves</td>
<td>$1.2 \times 10^{-3}m$</td>
</tr>
<tr>
<td>Length Of Leaves</td>
<td>$0.15 m$</td>
</tr>
<tr>
<td>Density Of Trees</td>
<td>$4.95 \times 10^{-2}/m^2$</td>
</tr>
<tr>
<td>Density Of Secondary Branches</td>
<td>$7.00/m$</td>
</tr>
<tr>
<td>Density Of Leaves</td>
<td>$450/m$</td>
</tr>
<tr>
<td>Angle Of Secondary Branches</td>
<td>$70^\circ$</td>
</tr>
<tr>
<td>Angle Of Leaves</td>
<td>$70^\circ$</td>
</tr>
</tbody>
</table>

The program solves the vector radiative equation and produces, among other results, the backscattering coefficients. These backscattering coefficients, $\sigma_{yx}$, are the normalized radar cross-section which relate incident wave polarization $x$ to scattered wave polarization $y$ and is expressed in dB. This measurement encompasses the whole area of interest, which in our case would be a particular section of forest.

The main purpose of this thesis is to understand the effects of the inclusion of coherent terms in the calculation of the extinction matrix. The first set of simulation results provide a measure of the relative value of $\bar{\kappa}_{\text{coherent}}$ compared to the incoherent scattering as frequency and density were varied. The simulations were run using a needle-less model in order to have a simpler view of the scattering due to the trunk, primary, and secondary branches alone.

The effect of density on coherent scattering is straightforward; the greater the number of structures within a given volume, the more multiple scattering will take
place. The following graph shows this for the case in which the density of the secondary branch in the pine tree model was varied from .77 to 7 branches/meter and the frequency was held constant at .1 GHz.

![Graph showing percent of incoherent scattering loss vs. density of secondary branches (l/m)].

Figure 3-1: Coherent Scattering Compared To Incoherent Scattering Over Density

The effect of frequency on coherent scattering is important when the resulting wavelength is comparable to the scattering structures. As the incident wave strikes upon a structure, components within the structure re-radiate their own fields in many directions. Consider the phase factor for the incident wave, \( \exp(ikr) \) or \( \exp(ir\frac{2\pi}{\lambda}) \) where \( r \) is the structure length. If the wavelength is much larger than the structure length, this phase factor becomes negligible, meaning that the re-radiated field is more coherent, or focused, in a specific direction. Thus, at low frequencies (larger wavelengths), the inter-structural scattering between components of lengths comparable to the wavelength is stronger and more detectable. Conversely, at high frequencies
(smaller wavelengths) the phase factor is not negligible thus the re-radiated fields are randomly directed and average themselves out. This phenomena can be observed in Figure 3-2.

![Graph showing percent of incoherent scattering loss over frequency.]

Figure 3-2: Coherent Scattering Compared To Incoherent Scattering Over Frequency

It is interesting to note that the coherent values are not insignificant in comparison to the incoherent values at low frequencies. This is of course due to the fact that the incident wavelength is comparable to the length of the secondary branches. As the frequency increases, the coherent scattering shows a downward trend. However there maybe fluctuation as seen in the slow rise after .8 GHz. This is due to the random phase factor of the scattered wave, which can exhibit oscillatory behavior because of scattering contributions from different angles.
In order to understand what percentage of the total loss was due to coherent scattering, a measurement was made of

\[
\frac{\kappa_{\text{coherent}}}{\kappa_{\text{incoherent}} + \kappa_a}
\]  

(3.1)

again as frequency and density were varied.

Figure 3-3: Coherent Scattering Compared To Total Loss Over Density
Although the coherent scattering was only a small percentage of the total loss, the same change over density is observed.

![Graph showing percent of total loss over frequency](image)

Figure 3-4: Coherent Scattering Compared To Total Loss Over Frequency

Figure 3-4 shows that as the frequency increased, the coherent scattering became a larger portion of the total loss. This is due to the fact that the absorption loss stays roughly constant over frequency, but the scattering loss rises. Although the coherent scattering loss decreases as a percentage of the incoherent scattering loss over frequency, it still accounts for a significant amount of the total loss. It is expected that the backscattering coefficients will follow the same trend.
3.2 Conclusions

A few observations may be drawn from this preliminary data. First, increasing density of scatterer structures leads to an increase in coherent scattering. Second, at low frequencies (.1 GHz) coherent scattering is appreciable when compared to incoherent scattering and adds approximately 40% more to the total scattering loss. This contribution drops to 15% as the frequency rises past 1 GHz. Furthermore, when the medium has an appreciable imaginary part in the dielectric constant, the coherent scattering loss is less than 1% at low frequencies but accounts for almost 10% of total loss at frequencies past 1.5 GHz.
References


