Electricity Market Integration of Stochastic Renewable Resources: Efficiency and Risk Tradeoffs

by

Ian Michael Schneider

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Signature redacted

Author .........................................................

Institute for Data, Systems, and Society and the Department of Electrical Engineering and Computer Science

January 20, 2017

Signature redacted

Certified by ......................

Professor Munther A. Dahleh
Director, Institute for Data, Systems, and Society
Thesis Supervisor

Signature redacted

Certified by ......................

Mardavij Roozbehani
Principal Research Scientist, MIT
Thesis Supervisor

Signature redacted

Accepted by ......................

Professor Munther A. Dahleh
Director, Institute for Data, Systems, and Society

Signature redacted

Accepted by ......................

Professor Leslie A. Kolodziejski
Chair, Department Committee on Graduate Theses
Electricity generation from renewable sources is growing rapidly, but the variability and uncertainty of renewable resources like wind and solar energy can increase the costs of supplying reliable electricity. Competitive markets for wholesale electricity are widely used in the United States, but the regulatory details that govern their treatment of stochastic resources can have significant effects on efficiency and risk. This research analyzes how producers respond to market mechanisms intended to improve forecasting and long-term siting decisions. This thesis characterizes producer equilibrium strategies in competitive short term energy markets by examining the bidding behavior of energy market participants when energy imbalance payments are determined endogenously from market clearing conditions. The results show that the market-based pricing mechanism leads to better tradeoffs of system efficiency and risk compared to the case where penalties are exogenous, suggesting additional benefits of market-based penalty prices beyond those previously studied. This research also explores how long-term market investment equilibria are affected by current energy policies. It presents new analytical results showing how the Production Tax Credit (PTC) biases wind investment towards high-producing sites, but with higher overall levels of wind correlation, which can induce additional costs associated with reliability and system risk.

Thesis Supervisor: Professor Munther A. Dahleh
Title: Director, Institute for Data, Systems, and Society

Thesis Supervisor: Mardavij Roozbehani
Title: Principal Research Scientist, MIT
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Chapter 1

Introduction

The presence of renewable generation on the electric grid is rapidly expanding, due to environmental concerns and to increasing cost competitiveness with conventional power plants. For instance, wind energy accounted for 4.4% of electricity generation in the United States in 2014, an 8.3% increase over the previous year and a 92.1% increase over 2010 [1]. Wind is also expected to be the dominant source of new U.S. generation capacity in 2015 [1].

The uncertainty and intermittency of renewable generation can increase the overall cost of supplying reliable electricity. Competitive markets for wholesale electricity are utilized in many parts of the United States, but there are questions as to whether they can optimally manage the uncertainty and intermittency of renewable resources while ensuring system reliability. As such, there is interest in understanding how the special features of renewable energy generation impact the way operators develop their projects and sell wholesale energy. In particular, this thesis examines how policies impact market equilibria in electricity markets with renewable generation.

Policies that deal specifically with the characteristics of variable renewable energy can have major impacts on market equilibria in short- and long-term energy markets. For example, due to the stochasticity of wind availability, it may be important to have mechanisms to enforce the requirement that wind generators produce as much energy as offered in day-ahead markets. These types of policy mechanisms have been proposed, and are currently in use in many regional U.S. markets. Chapter 2 examines the effects of these policies on market equilibria, extending the current research by focusing in particular on markets where wind producers behave strategically and expect the same of other producers. Furthermore, due to the nascent status of the renewable energy industry, the costs of greenhouse gas emissions, and the political constraints surrounding carbon pricing, a federal Production Tax Credit (PTC) provides subsidies for wind energy per unit of energy production. Chapter 3 examines the long-term effects of this policy on equilibria in energy markets, through its impacts on optimal investment criteria for wind energy. The thesis presents optimal or equilibrium strategies by market players over both the short and long term planning horizons to help identify key tradeoffs in integrating the wind resource and to improve existing market design.
The externalities caused by the stochasticity of wind energy can be broadly classified into two groups, based on the duration of the planning period under consideration. First, the short-term uncertainty of the underlying wind resource requires additional costs for balancing and operating reserve requirements [2], compared to a system with traditional, controllable generation. This uncertainty may also increase risk of system failures. Second, the long-term variability in wind output implies that wind turbines can not be relied upon as firm capacity during hours of peak demand. This presents engineering challenges related to the assessment of reliability and may require improved methods for controlling or curtailing selected loads. It also presents economic challenges because the uncertainty of the wind-resource at peak hours can interfere with market mechanisms intended to ensure adequate capacity supply. This thesis does not attempt to specifically enumerate the costs associated with wind energy; rather, it develops metrics for system efficiency and risk under a particular equilibria outcome. These metrics are useful markers for observable and unobservable system costs associated with variable renewable energy under any particular set of energy market outcomes.

The research uses tools from economics, optimization, game theory, and mechanism design, and we hope the analytical work towards this thesis can help us extend the technical literature in some of these fields. Furthermore, the research is directly connected to policies that affect the shape of our energy landscape. Chapters 2 and 3 discuss the policy implications of proposed market designs and highlight conflicts with existing policy support for renewable electricity generation.

1.1 Bid Settlement Policies: Energy Market Effects

Compared to a system with controllable generation, the near-term uncertainty of wind resources imposes additional costs for balancing and reserve requirements [2]. Markets rules increasingly hold wind energy producers responsible for uncertainty in their output, in line with fair principles of cost-causation. However, these policies impact equilibrium bidding behavior, which can increase curtailment and system risk. Chapter 2 examines the effects of these rules on resulting energy market equilibria, and characterizes the outcomes in terms of system efficiency and risk.

Large wind producers are penalized for deviations from their day-ahead quantity bids in various ways in ERCOT, PJM, NYISO, ISO-NE [3], the UK, and Spain. Such policies have occasionally caused unexpected bidding behavior. In NYISO and ISO-NE, wind producers are required to settle the deviation between their day-ahead bid and real-time production at the real-time price [3]. This serves as an implicit penalty on forecast error, but with a positive or negative sign based on the difference between real-time and day-ahead energy prices. Some markets have not fully implemented the real-time settlement requirement: PJM exempts payments within a 5% deviation, and MISO and Alberta’s ESO do not require settlement at the real-time price [3].

Existing research focuses on the optimal bid strategies of wind producers under existing and theoretical market policies. In day ahead markets, wind producers place a quantity, price bid pair
for the sale of energy. The optimal price bid is well known to be the marginal cost of production in a perfectly competitive uniform-price electricity market, but research shows that profit maximizing producers will often bid higher than their marginal cost [25].

Wind producers also have the challenge of setting their bid quantity despite underlying forecast and price uncertainty. Academic papers have addressed this strategy in the case where the effective penalty price is uncertain and unaffected by the producers’ strategies [5] [6]. Results by Botterud et al. show that optimal bidding by wind power producers in day ahead markets is largely driven by the real-time price expectation [7].

In a model with separate penalties on positive and negative deviations, Bitar et al. show that the optimal bid is an explicit function of the price and penalty levels, based on the quantile function of the distribution of energy availability [8]. Dent et al. develop analytical results for the optimal forward strategy with an imbalance settlement requirement [9].

Previous work treats the deviation penalty level and/or the difference between day-ahead and real-time prices as exogenous, but price and penalty variables can be substantially affected by wind forecasts and bidding strategies [2] [10] [11]. Vilim and Botterud propose a model to capture the relationship between wind availability and market prices, and provide two bidding algorithms for maximizing profit in the case of high wind penetration [12]. Wind producer bidding strategies impact the distribution of errors between the bid quantity and the actual amount of wind energy produced, which in turn impacts the equilibrium bidding and curtailment strategies. This additional feature can have significant on resulting system efficiency and risk.

Similar to [12], we tackle a profit-maximizing problem for wind producers, but we use an analytically-focused model where the market impact of system errors is clear. We present new analytical results on optimal producer decisions, where player behavior affects realizations of the implicit penalty. Our model forces wind producers to settle at an endogenously-determined real-time price, as required in markets like NYISO and ISO-NE.

The results show that wind producers bid below their expected availability of production, even when they are not risk averse, because of the convexity of price sensitivity. They also show that the marginal-cost based penalty mechanisms lead to better tradeoffs between efficient use of wind power resources and system risk. Therefore, regulators and system operators should favor policies like the real-time settlement requirement to best tradeoff costs to system efficiency and risk.

1.2 Spot Market Pricing and Long Term Investment

The theory of spot market pricing is well-established, but open questions remain regarding long-run investments in electricity markets because of inflexible demand and because of market policies such as price caps. Capacity markets have attempted to fix the subsequent problems that result from suboptimal investment, but they end up returning many of the investment decisions to the central planner. Practical market design solutions are still debated in the present. Wind generation is
inexorably tied to the crucial questions regarding generation expansion for two reasons. First, it represents a major source of capacity expansion in recent years, and was the dominant source of new capacity in the U.S. in 2014. Second, it raises interesting questions regarding resource adequacy because it is variable and not controllable, and thus can not typically be relied upon to satisfy peak demand. Chapter 3 develops the marginal investment criteria for wind energy from the spot-market pricing theory, explains how it is altered by policies like the Production Tax Credit (PTC), and examines the effects of this policy on likely correlations in total wind energy output.

The theory behind spot pricing of electricity was developed in the early 1980's and applied to deregulated wholesale markets in the 1990's and 2000's. In 1982, Caramanis extended the spot pricing theory to show that under certain conditions spot pricing would lead to optimal investment decisions in generating capacity [16], and others have arrived at similar conclusions [17]. The basic model of spot pricing from Caramanis's 1982 work is used here to develop a simplified version of the profit incentive for investment in traditional and renewable generation [16].

However, early literature regarding electricity generation investments makes some assumptions that do not hold up in practice. As a result, wholesale energy markets have experienced a "missing money" problem when energy revenues are not sufficient to recoup generator investment costs. In a 2005 paper, Hogan attributes much of the missing money problem to the energy-market price cap, and proposes high price caps coupled with an operating reserve demand curve as a workable design that incentivizes more efficient investment in an energy-only market design [18]. In 2006, Joskow discussed the missing money problem in electricity markets and suggested policy reform to improve the efficiency of spot wholesale electricity markets [19]. Joskow argues that ultimately, the source of the "missing money" problem is that spot prices do not rise high enough during scarcity hours to cover the additional investment costs of the peaking plants.

Other proposed solutions for the revenue adequacy / "missing money" problems involve call options or mandated capacity markets for maintaining reliability and resource adequacy, [20] [21]. Joskow et al have written about design criteria for workable capacity markets as an alternative solution to the revenue adequacy problem [22] [23]. Cramton and Stoft review some of the essential problems in the resource adequacy problem, focusing in part on demand inflexibility, and evaluate the solutions discussed by several other authors [25].

Unfortunately, reliability capacity mechanisms in U.S. markets are typically flawed in practice. In particular, they often require the ISOs to make complicated regulatory decisions to govern investment. This leads to more centralized regulatory control of investment based on market questions that are difficult or impossible for policy makers to evaluate on suitable time scales. Furthermore, existing energy-only markets tend to contain low price caps and overstate the implied value of lost load (VOLL) through strict reliability standards, which contributes to the "missing money" problem and leads to an overemphasis on capacity markets as the solution, despite their inherent flaws.

Another portion of the literature focuses on the integration of wind energy in existing markets. In particular, researchers have noted that the system value of a wind investment depends heavily on
when it produces electricity. For instance, Joskow finds the levelized cost of energy (one standard method for comparing energy costs amongst different generator types) for wind and solar based on the prices at the times in which they are available [26] and argues that the time sensitive approach more reasonably estimates their value.

In a recent paper, Wolak presents research on mean and standard deviation trade offs in wind project siting [27]. Wolak describes the optimal frontier between average hourly output (or revenue) and the standard deviation of hourly wind/solar output (or revenue). He show that the actual mix in California is far from the efficient frontier, and he suggests that a theoretical adjustment of capacity siting could increase the expected value of capacity factor by 40% without increasing portfolio standard deviation (SD). Furthermore, measures of non-diversifiable energy and revenue risk are calculated using actual market portfolios and risk-adjusted optimal portfolios. Wolak discusses reliability externalities in the context of renewable energy, and places some blame for the sub-optimal allocation on support mechanisms for renewable energy. However, Wolak does not describe how support mechanisms would lead to the measured suboptimal renewable investment. This research uses decision theory to help explain why rational producers might invest in a sub-optimal way given current policy support mechanisms.

Others have discussed about the impact of wind energy and renewable support schemes on power prices, for instance Sáenz de Miera et. al [28] and Green and Vasilakos [29], but without focusing specifically on how support schemes impact the renewable investment decision.

The research shows how wind investors optimally tradeoff between investments with higher overall production and those with marginally more valuable production, i.e. production that is more highly correlated with price. The research suggests that the PTC in particular biases wind investors to care marginally less about the value of a site's production. Furthermore, this thesis establishes links between these effects and covariance in overall wind production, arguing that fixed-price support mechanisms like the PTC lead to higher overall levels of wind correlation.
Chapter 2

Settlement Requirements and the Effect on Equilibrium Energy Bidding

This chapter examines the welfare implications of energy market equilibrium bidding behavior when energy imbalance payments are determined endogenously from market clearing conditions. Presently, in some markets, wind producers are required to settle their day-ahead electricity offers based on their actual production in real-time. This is usually justified by economic and cost-causation principles, but the policy has additional effects on curtailment and system risk.

Section 2.1 models the bidding and curtailment process as a two stage game played by wind producers with uncertain availability of energy production. Sections 2.3 and 2.2 analytically characterize the symmetric subgame perfect equilibrium (SPE) in the multiplayer game, focusing on the equilibrium curtailment and day-ahead bidding strategies, respectively. Section 2.4 provides general bounds on the SPE for more generic price sensitivity functions. Section 2.5 simulations to compare our results with the case where penalty prices are exogenous, as modeled in [8]. Section 2.6 analyzes price sensitivity data in Denmark and suggests how wind producers might bid in equilibrium in that market.

The results show that wind producers bid conservatively in response to convexity in the price sensitivity as a function of day-ahead forecast errors, even when they are not risk averse. They also show that the endogenous penalty mechanism induces better tradeoffs between efficient use of wind power resources and system risk. Therefore, regulators and system operators should favor penalty mechanisms that internalize wind producer behavior, like a real-time settlement requirement. For these sorts of penalty mechanisms, the bidding behavior of wind producers can be expected to induce less system costs than is suggested by previous results, because of the way that profit-maximizing producers consider the behavior of competitors while placing their own quantity bids.
2.1 The Penalty and Profit Model

2.1.1 Basic Model

Let \( w_i, i \in N = \{1, 2, ..., n\} \) represent the available energy production for the \( i \)th producer, measured in MWh of energy available during a specific period. Let \( q_i \) represent the a priori quantity bid placed by producer \( i \) in the day-ahead market for energy (MWh). The bids and available production correspond to the same period, for instance, the 15-minute or 1-hour blocks used by independent system operators (RTOs) to schedule resources; the analysis focuses on a single period.

Furthermore, consider \( w_i \) to be a realization of the random variable \( W_i \) of energy (wind) availability for producer \( i \). Assume that \( W_i \) are independently distributed and the realizations occur immediately before the curtailment period (second stage).

Define the bid-error \( x_i = w_i - q_i \) as difference between the realized energy availability and the day-ahead bid. Furthermore, define \( \hat{w}_i \), the final energy production, such that \( \hat{w}_i \leq w_i \) for all \( i \). In other words, if a player is able to produce \( w_i \) units of energy, they may curtail \( w_i - \hat{w}_i > 0 \) units of energy, for instance by adjusting the blade angle of some turbines. The variable \( \hat{w}_i \) represents their actual energy production in the settlement period. Analogous to the bid-error, define the generation error, \( \hat{x}_i = \hat{w}_i - q_i \).

Let \( p \) be the price established in the day-ahead market. Assume that \( p \) is exogenous, or, equivalently, the wind producers are price-takers in the day-ahead market. This assumption models the real-world situation where a large number of conventional generators participate in the day-ahead market, but infrequently make adjustments in the real-time market.

2.1.2 Real-Time Imbalance Prices

In a stylized model, consider a real-time price \( p_{rt} \) that differs from the day-ahead price \( p \) by a function of the system forecast error. Assume that the forecast error is simply the sum of the wind producer errors and the demand error \( \phi_D \). Note that the variable \( \phi_D \) is defined as the actual demand minus the demand forecast, both of which are negative values to reflect the fact that demand consumes electricity. Then, the real time price can be written as

\[
p_{rt} = p - g(\sum_j \hat{x}_j + \phi_D)
\]

with \( j = \{1, 2, ..., n\} \). Sections 2.3 and 2.2 focus on a simplified penalty function that is a linear function of the error, and Section 2.4 presents results for a more general form. Initially, therefore, assume \( g(x) = \mu x \), where \( \mu > 0 \) is constant. Then the \( i \)th producer’s profit function is given by:

\[
\pi_i = pq_i + p_{rt}\hat{x}_i = pq_i + p\hat{x}_i - \mu(\sum_j \hat{x}_j + \phi_D)\hat{x}_i
\]

In a system with high wind penetration, it is reasonable to assume that the differences between
day-ahead and real-time prices are largely driven by wind and demand forecast errors. The profit function in (3.1) makes this assumption explicit.

The model in (3.1) can also describe a real-world scenario where wind producers receive a nominal price $p$ for all of their energy production, less a penalty that is a function of total system error. If the applied deviation penalty price is set as the marginal costs for managing these deviations, then the resulting profit model is identical to that described in the previous section.

### 2.1.3 Game Model

This work models the decision process as a two-stage game where players seek to maximize the profit function given by (3.1). In the first stage, the day-ahead market, players simultaneously choose a bid $q_j$ for their energy production in a future period.

Just before the second stage, players realize their available production $w_i$ and thus, learn $x_i$ as well. In the second stage, players simultaneously choose a final output level $\hat{w}_i$. This is equivalent to deciding $\hat{x}_i$ since $q_i$ is now fixed. Player $i$ is curtailing its output if $\hat{x}_i < x_i$.

### 2.2 Stage Two: Optimal Curtailment Strategy

This section describes the optimal curtailment strategies for the second stage of the game described in Section II, under various assumptions on the information structure. First, we assume that players have full information regarding the production availability of all players. Then we relax this assumption and detail the equilibrium bidding strategy when players have no knowledge of the real-time production availability of other wind producers. The optimal stage one strategy is detailed in the following section, Section 2.3; stage two is detailed first because it informs the decision making process in stage one.

#### 2.2.1 Optimal Curtailment Strategy under Full Information

**Proposition 1.** Let $K \subseteq N$ be the group of producers that curtail $K = \{i | i \in N, \hat{x}_i < x_i\}$, and define $k = |K|$. Assume that the demand forecast is unbiased, i.e. $E[\phi_D] = 0$. Then the curtailting group $K$ is given by

$$i \in K \iff x_i > \frac{p - \mu \sum_{j \not\in K} x_j}{\mu(k + 1)}$$

(2.3)

Then, the $i$th player's optimal production in the curtailment stage of the two-stage competitive penalty game is

$$\hat{x}_i^* = \begin{cases} \frac{p - \mu \sum_{j \not\in K} x_j}{\mu(k + 1)} & \text{if } i \in K \\ x_i & \text{otherwise} \end{cases}$$

(2.4)
Proof. Each player seeks to find $\hat{x}_i^*$, the argmax of the following optimization problem:

$$\max_{\hat{x}_i \leq x_i} \mathbb{E}[\pi_i] = \mathbb{E}[pq_i + p\hat{x}_i - \mu(\hat{x}_i + \sum_{j \neq i} \hat{x}_j^* + \phi_D)\hat{x}_i]$$

(2.5)

Note that player $i$'s best response is a function of the generation errors of the other players. Furthermore, only the demand error $\phi_D$ is a random variable. We expand the objective function and take its derivative:

$$\frac{d\mathbb{E}[\pi_i]}{d\hat{x}_i} = p - 2\mu\hat{x}_i - \mu \sum_{j \neq i} \hat{x}_j^* - \mu \mathbb{E}[\phi_D]$$

(2.6)

Assume that $i$ satisfies the condition on the right hand side of (3.10). Then setting (3.13) equal to 0, and noting that $\mathbb{E}[\pi_i] = 0$ gives the first order condition for optimality for the curtailed output:

$$\mu \hat{x}_i = p - \mu \sum_j \hat{x}_j^*$$

(2.7)

Sum (3.14) over all $i \in K$, since each curtailing player sets $\hat{x}_i^*$ to satisfy the first order optimality condition:

$$\mu \sum_{i \in K} \hat{x}_i^* = k(p - \mu \sum_{j \in K} \hat{x}_j^*)$$

(2.8)

$$(k + 1)\mu \sum_{i \in K} \hat{x}_i^* = k(p - \mu \sum_{j \in K} \hat{x}_j^*)$$

(2.9)

Solving (2.9) in terms of $\sum_{i \in K} \hat{x}_i^*$ and substituting into (3.14) gives the following:

$$\mu \hat{x}_i^* = p - \frac{k(p - \mu \sum_{j \notin K} \hat{x}_j^*)}{k + 1} - \mu \sum_{j \notin K} \hat{x}_j^*$$

(2.10)

Simplifying the above, and taking $\hat{x}_j^* = x_j \forall j \notin K$, obtains the result for $i \in K$ detailed in (3.9).

The second order optimality condition is satisfied because second derivative of the objective function

$$\frac{d^2\mathbb{E}[\pi_i]}{d\hat{x}_i^2} = \frac{d^2}{d\hat{x}_i^2}(pq_i + p\hat{x}_i - \mu(\hat{x}_i + \sum_{j \neq i} \hat{x}_j + \mathbb{E}[\phi_D])\hat{x}_i)$$

(2.11)

$$= -2\mu < 0.$$
function with respect to its final output is positive at the original output,

$$\left. \frac{d\pi_i}{d\hat{x}_i} \right|_{\hat{x}_i} = -\mu x_i + \frac{p - \mu \sum_{j \notin K} \hat{x}_j}{k + 1}$$

(2.12)

$$p - \mu \sum_{j \notin K} x_j \geq -\mu x_i + \frac{p - \mu \sum_{j \notin K} x_j}{k + 1} \geq 0$$

because of the solution in (3.9) for the curtailing producers and the curtailment limits for the maximization problem (3.12). Producer $i$ would prefer to increase $\hat{x}_i$ beyond $x_i$, but the conditions of the maximization problem prevent this, so any producer that has value $x_i$ less than the right hand value in (3.10) will not curtail.

Therefore, the strategy set $(\hat{x}_1^*, \hat{x}_2^*, ..., \hat{x}_n^*)$ represents the unique symmetric Nash equilibrium when $\hat{x}_i^*$ is chosen $\forall i$ according to (3.9).

Remark. Determining the Set of Curtailing Players

The curtailing set $K$ does not need be determined recursively; it can be defined exactly through $n - k + 1$ operations on the ordered set of player errors. For instance, let $\bar{x}$ represent the ordered list of producer errors, $\bar{x} = \{\bar{x}_1, \bar{x}_2, ..., \bar{x}_N\}$ such that $\bar{x}_N \geq \bar{x}_{N-1} \geq ... \geq \bar{x}_2 \geq \bar{x}_1$. Then (3.9) implies

$$i \in K \iff x_i \geq x_j > \frac{p - \mu \phi_D - \mu \sum_{i=1}^{j-1} \bar{x}_i}{\mu(n - j + 2)}$$

(2.13)

for some $i, j \in N$. Starting from $\bar{x}_1$, simply calculate the right hand side of (3.11) until finding the value $x_j$ that exceeds the right-hand side. Any player with $x_i \geq x_j$ will curtail.

Remark. Upper Bound on Final Error

Not all positive errors will be curtailed, but there is a strict limit on the sum of the positive production errors. This is given by $\frac{m}{\mu(n+1)}$, or, $\frac{1}{\mu}$ as $n$ grows large.

First, observe the upper bound for the sum of non-curtailing producers, of which there are $n - k$. From (3.10),

$$\sum_{j \notin K} x_j \leq (n - k) \frac{p - \mu \sum_{j \notin K} x_j}{\mu(k + 1)}$$

$$\frac{k + 1 + n - k}{k + 1} \sum_{j \notin K} x_j \leq \frac{p(n - k)}{\mu(k + 1)}$$

(2.14)

$$\sum_{j \notin K} x_j \leq \frac{p(n - k)}{\mu(n + 1)}$$

Using (2.14) and the optimal behavior for curtailing producers, shown in (3.9), we calculate the
upper bound on the final curtailed values of all participants:

\[
\max_{x_1, x_2, \ldots, x_n} \sum_{j=1}^{n} x_j = \max_{x_1, x_2, \ldots, x_n} \sum_{j \notin K} x_j + \sum_{j \in K} x_j
\]

\[
= \max_{x_1, x_2, \ldots, x_n} \left( \sum_{j \notin K} x_j + k \cdot \frac{p - \mu \sum_{j \notin K} x_j}{\mu(k+1)} \right)
\]

\[
= \frac{kp}{\mu(k+1)} + \max_{x_1, x_2, \ldots, x_n} \left( \frac{\sum_{j \notin K} x_j}{(k+1)} \right)
\]

\[
\leq \frac{kp}{\mu(k+1)} + \frac{p(n-k)}{\mu(n+1)(k+1)}
\]

\[
= \frac{k-1}{k+1} \frac{pm}{\mu(n+1)}
\]

\[
\leq \frac{pm}{\mu(n+1)}
\]

when \( p \) and \( \mu \) are positive. Remember that \( k \in \mathbb{Z} \) with \( 0 \leq k \leq n \). The expression in the final line is upper-bounded by \( \frac{p}{\mu} \) as \( n \) approaches infinity.

**Remark. Upper Bound on Curtailment**

The inequality (2.14) suggests a lower bound on the curtailment level and a corresponding upper bound on the amount that any individual player curtails, in the absence of demand error. From the optimal strategy in (3.9) and from (2.14), we have \( \forall i \in K \), for any realizations of the production availability \((x_1, x_2, \ldots, x_n)\),

\[
\hat{x}_i^* \geq \frac{p}{\mu(n+1)}
\]

Therefore, no profit-maximizing player will curtail more than the amount \( x_i - \frac{p}{\mu(n+1)} \), and no player with a bid-error less than or equal to the right hand side of (2.16) will ever curtail. More generally, an individual player will never curtail their final output to a value lower than their initial bid \( q_i \).

### 2.2.2 Relaxing Information Assumptions: No Shared Information

The preceding analysis is based on the strong assumption that each player has full information regarding the available production of each producer. While the ISO might collect and disseminate information for the wind production ex-post, the realized production capacity will most likely not be known ex-ante.

**Proposition 2.** Consider the case where each producer has no knowledge of the wind production availability nor the demand error of other producers, \( x_{-i} = \{x_j| j \neq i\} \). Then the optimal curtailment strategy for player \( i \) is to choose:
\[ \hat{x}_i^* = \min \left\{ \frac{p - \mu E[\sum_{j \neq i} \tilde{x}_j^*]}{2\mu}, x_i \right\} \] (2.17)

Proof. When player \( i \) has a given error, \( x_i' \), the expected value of the sum of the other players' post-curtailment errors is given by \( E[\sum_{j \in K} x_j | x_i = x_i'] \). Then, starting from (3.12), with \( E[D_\phi] = 0 \), player \( i \)'s expected profit function is given by

\[ E[p_i] = p q_i + p \hat{x}_i - \mu (\hat{x}_i + E[\sum_{j \neq i} \tilde{x}_j^* | x_i = x_i']) \hat{x}_i \] (2.18)

However, at any given round, the bids \( q_i \) are already set for all players, and thus the bid-errors \( x_i \) are independent by assumption. Furthermore, in the model with no shared information, players must make their curtailment decisions without knowledge of the realized bid-errors of other players, and thus their optimal generation errors \( \hat{x}_j^* \) are just a function of their own errors \( x_j \) and some previously defined curtailment limit.

Thus, \( E[\sum_{j \neq i} \hat{x}_j^* | x_i = x_i'] = E[\sum_{j \neq i} \tilde{x}_j^*] \), and player \( i \) creates a curtailment rule according to the first order optimality condition for their best response in expectation, as follows:

\[ 2\mu \hat{x}_i^* = p - \mu E[\sum_{j \neq i} \tilde{x}_j^*] \] (2.19)

Then by algebraic manipulation we achieve the profit-maximizing final output value. Player \( i \) chooses to curtail to this value when possible, and does not curtail otherwise, as expressed in (2.17).

\[ \square \]

2.3 Stage One: Optimal Bidding Strategy

In the first round of the two stage game, each player \( i \) places a quantity bid \( q_i \). Players seek to place a bid that optimizes the expected value of their profit based on the realizations of energy availability in the second round and curtailment strategies described in the previous section.

In the game with full information, it is difficult to analytically characterize the expected profits because of the dependency between player \( i \)'s production availability and the post-curtailment production of other players. However, the game of no shared information is readily analyzable because of the independence of post-curtailment generation errors, as described in section IIID. We will focus on this case in order to elucidate important features of the optimal first round bids.
2.3.1 Optimal First Round Bids with No Shared Information in the Second Round

**Proposition 3.** The unique symmetric subgame perfect equilibrium is defined by stage one bid strategies \((q^*_1, q^*_2, \ldots, q^*_n)\), where each player \(i\) chooses \(q^*_i\) such that \(E[\hat{x}^*_i] = 0 \forall i\), and by stage two curtailment strategies \((x^*_1, x^*_2, \ldots, x^*_n)\), with \(x^*_i\) given by (2.17).

Proof. Each player’s post-curtailment errors in the second round, \(\hat{x}^*_i\) will conform to the optimal strategy given in (2.17). To simplify the notation, define the sum of errors due to demand and all players besides \(i\) as \(s_{-i} = \sum_{j \neq i} \hat{x}^*_j + \phi_D\) and \(S_{-i}\) the corresponding random variable of that sum. Then define the curtailment level

\[
z_i = \frac{p - \mu E[S_{-i}]}{2\mu} \tag{2.20}
\]

Then (2.17) is equivalent \(\hat{x}^*_i = \min\{z_i, x_i\}\). The player seeks to maximize the expected value of its profit:

\[
\text{maximize } E[\pi_i] = E[pq_i + px^*_i + \mu(x^*_i + \sum_{j \neq i} \hat{x}^*_j)\hat{x}^*_i] \tag{2.21}
\]

In the remainder of the section, for notational simplicity, we fix \(i \in N\) arbitrarily and drop the subscript \(i\) from all variables \(\pi_i, w_i, x_i, q_i,\) and \(z_i\), that refer to these values for some player \(i\).

Define random variable \(W\) which has realizations \(w\). The expectation in (2.21) is naturally taken over the uncertainty in the player’s wind availability \(W\); the PDF and CDF of \(W\) are given by \(f_w(w)\) and \(F_w(w)\). We expand the expected value and take the derivative with respect to \(q\), using the Leibniz rule for differentiation under the integral sign:

\[
\frac{dE[\pi]}{dq} = p(1 - F_w(z + q)) + 2\mu \int_{-\infty}^{z+q} (w - q)f_w(w)dw + \mu E[S_{-i}]F_w(z + q) \tag{2.22}
\]

where \(E[S_{-i}]\) and \(z\) are constants because of the independence of the post-curtailment errors, and where the demand error term vanishes due to its independence and because \(E[\phi_D] = 0\).

The first order condition for profit maximization is that \(q^*\) is chosen such that (2.22) is equal to 0. We confirm that \(E[\pi]\) is concave in \(q\), by applying the Leibniz differentiation rule to the above:

\[
\frac{d^2E[\pi]}{dq^2} = -2\mu F_w(z + q) \leq 0 \tag{2.23}
\]

This confirms that \(q^*\) is the unique maximizer of the expected profit. By definition, we have the post curtailment generation error:

\[
E[\hat{x}^*] = \int_{-\infty}^{z+q} (w - q)f_w(w)dw + \int_{z+q}^{\infty} z f_w(w)dw \tag{2.24}
\]
It follows from (2.22) and (2.24) that
\[ \frac{dE[x]}{dq} = p(1 - F_w(z + q)) + 2\mu E[\hat{x}^*] - 2\mu z(1 - F_w(z + q)) + \mu E[S_{-i}]F_w(z + q) \] (2.25)

By setting the right hand side of (2.25) equal to zero and using (2.29), the definition of \( S_{-i} \), and the linearity of expectation with \( E[\phi_D] = 0 \) we have another interpretation of the first order condition:
\[ E[\hat{x}_j^*] = -\frac{\sum \{E[\hat{x}_j^*] \}}{2\mu} \] (2.26)

This holds generally for any player \( i \), so in equilibrium each player will choose \( q \) such that (2.26) is true. By summing over all \( i \), for any \( n \in Z^+, \mu > 0 \), clearly (2.26) requires that
\[ E[\hat{x}_i^*] = 0 \quad \forall i. \] (2.27)

Thus, the equilibrium bid strategies are \((q_1^*, q_2^*, ..., q_n^*)\) chosen to satisfy the first order condition in (2.27).

**Remark.** In light of this result, we can simplify the curtailment limit in (2.29) and the associated optimal strategy given in (2.17). Player \( i \)'s overall strategy is as follows:

1. Choose the curtailment limit \( z_i = \frac{p}{2\mu} \)
2. For a given distribution of wind speeds, \( f_{w_i}(w_i) \), choose a first round bid \( q_i \) s.t. \( E[\hat{x}_i^*] = 0 \), as defined in (2.24).
3. Curtail according to the optimal strategy \( \hat{x}_i^* = \min\{z_i, x_i\} \) with \( x_i \) realized in the second round.

**Remark.** There is a unique \( q_i \) which satisfies the conditions in 2) in the previous Remark. For the given, fixed strategy, with \( z_i > 0 \), then the expected value of the final post-curtailment error \( E[\hat{x}_i^*] \) is decreasing in \( q_i \). From (2.24), we can take the derivative with respect to \( q \),
\[ \frac{\partial E[\hat{x}]_i}{\partial q} = \int_{-\infty}^{z+q} -f_w(w)dw = -F_w(z + q) \] (2.28)

Since \(-F_w(z + q) \in [0,1]\forall q\), then the expected value of the post-curtailment error is continuous and non-increasing on the real line. Furthermore, \( \lim_{q \to \infty} E[\hat{x}_i^*] = z \) and \( E[\hat{x}_i^*] \to -\infty \) as \( q \to \infty \), so there must exist a unique \( q \) such that \( E[\hat{x}_i^*] = 0 \). This justifies the operation in step 2) of the previous remark, and shows that the appropriate \( q_i \) can be uniquely found.

**Remark.** It is also possible to rewrite the statement of the proof in terms of the expected value of the final wind-output, post curtailment. Let \( \bar{w}_i = q_i + \hat{x}_i^* \). Then the sub game perfect equilibrium result from this section is equivalent to the strategy where \( \forall i \ q_i^* = E[\bar{w}_i] \).
2.4 Conditions on the Game Theoretic Equilibrium with a General Penalty Model

The previous sections assumed that some linear function mapped the real-time wind availability error to an expected price difference between real-time and day-ahead prices, i.e. \( p_{rt} = p - g(\sum \hat{x}_j + \phi_D) \) with \( g(x) = \mu x \) and \( \mu > 0 \). Now, consider the case where the price sensitivity may be more significant for negative errors, i.e. where \( g : \mathbb{R} \rightarrow \mathbb{R} \) is continuously differentiable, increasing, and concave. Then \( \forall x \in \mathbb{R} \, g'(x) > 0 \). Furthermore, assume \( g(0) = 0 \); this represents the fact that no price difference is expected when real-time conditions are unchanged from the day-ahead forward market.

For this generalized function, the observed price sensitivity is at least as high for negative bid errors as it is when there is more energy availability than suggested by the day-ahead clearing. This variation in price sensitivity is evident in power markets because of the supply curve shape and because of practical constraints on generator operation. In particular, because of start-up costs, the convexity of the supply curve in the region of typical demand, and because of short-term start-up constraints that make it especially hard to bring extra energy in real-time, prices can spike when there is an availability shortage.

2.4.1 Curtailment Strategy for a General Penalty Model

For the general price sensitivity function described in this section, it is not possible to explicitly characterize the Nash equilibria of the curtailment stage game. This is due to the fact that the uniqueness of any optimal curtailment action \( \hat{x}_i^* \) is not guaranteed because for some fixed \( E_{S-i} \in \mathbb{R} \), the second order conditions of the profit maximization problem do not hold. However, it is possible to bound the set of Nash equilibria. In particular, in any equilibria strategy set, no player will curtail to a final production value lower than their initial day-ahead bid.

**Proposition 4.** Assume the real-time price is given by \( p_{rt} = p - g(\sum \hat{x}_j + \phi_D) \) with \( g : \mathbb{R} \rightarrow \mathbb{R} \) continuously differentiable, increasing, and concave, and with \( g(0) = 0 \). Then a profit-maximizing wind producer will never curtail below their original bid, i.e. \( \forall i \in N, \hat{x}_i^* < x_i \rightarrow \hat{x}_i^* \geq 0 \), as long as the real-time price remains positive.

**Proof.** Player \( i \) seeks to find the optimal post-curtailment output \( \hat{x}_i^* \) that solves the following optimization problem:

\[
\max_{\hat{x}_i \leq x_i} \mathbb{E}_{S-i}[\pi_i]
\]  

(2.29)

where \( \pi_i = pq_i + p\hat{x}_i - g(\hat{x}_i + s_{-i})\hat{x}_i \). Then we take the derivative of the objective function to find points that satisfy the first-order conditions,
\[
\frac{d}{d\hat{x}_i} \mathbb{E}_{S_{-i}, \phi_D}[\pi_i] = \mathbb{E}_{S_{-i}} \left[ \frac{d\pi_i}{d\hat{x}_i} \right] 
\]  
\hspace{1cm} (2.30)

Assume that the random variable \( S_{-i} \) has finite support; there are positive and negative limits to the total sum of wind and demand errors. This and the continuous differentiability of \( g(\cdot) \) implies that the absolute value of the derivative is upper bounded by some constant for all \( \hat{x}_i \) and \( s_i \). This justifies the equality through the bounded convergence theorem. Then the FOC for \( \hat{x}_i \) follows:

\[
0 = \mathbb{E}_{S_{-i}}[p - g'(x_i^* + S_{-i})x_i^* - g(x_i^* + S_{-i})] 
\]  
\hspace{1cm} (2.31)

Furthermore, the derivative of the objective function is strictly positive evaluated at all \( \hat{x}_i < 0 \). Fix \( \hat{x}_i < 0 \). Then

\[
\mathbb{E}_{S_{-i}}[p - g'(\hat{x}_i + S_{-i})\hat{x}_i - g(\hat{x}_i + S_{-i})] 
> \mathbb{E}_{a_{-i}}[p - g(\hat{x}_i + S_{-i})] 
\geq 0 
\]  
\hspace{1cm} (2.32)

where the first inequality is due to \( g'(\cdot) > 0 \) and \( \hat{x}_i < 0 \), and because of the linearity of expectation, and the second inequality is because of the assumption that the real-time price is assumed to be greater than or equal to 0. Therefore, \( \hat{x}_i^* < x_i \leq 0 \) can not be a solution to the optimization problem in 2.31, because it always achieves a strictly worse value of the optimization function than \( x_i \).

Therefore, \( \forall x < 0, x \) can not satisfy the first order condition in (2.31). Define the set \( B = \{ x \in [0, \infty) | x \text{satisfies FOC} \} \). From the previous statement, it is clear that any point that satisfies the FOC must be in \( B \). Then, since the FOC is a necessary condition for unconstrained optimization, any solution \( \hat{x}_i^* \in B \cup \{ x_i \} \). Any player \( i \) will not curtail below the minimum of their wind availability \( w_i = x_i + q_i \) and their bid \( q_i \), i.e. the point at which \( \hat{x}_i^* = 0 \). The final production \( \hat{x}_i^* \geq \min(0, x_i) \).

Thus \( \forall i \in N, \hat{x}_i^* < x_i \rightarrow \hat{x}_i^* \geq 0 \). \( \square \)

### 2.4.2 Day-Ahead Bidding Strategy for a General Penalty Model

Consider the general model for the deviation penalty function, where producers' second-round strategies achieve the equilibrium conditions detailed in the previous subsection. Then the first stage Nash equilibrium for the producer bid strategies suggests that producers bid, on average, no more than the expected value of their-post curtailment production. When the function \( g(\cdot) \) is concave and non-linear, the average bid is strictly less than the expected value of their-post curtailment production.

**Proposition 5.** Consider the payoff function given in subsection 2.4 with \( g(\cdot) \) is increasing, concave, and continuously differentiable, and with \( g(0) = 0 \). The average optimal bidding strategy has the upper bound \( \sum_i q_i^* \leq \sum_i \mathbb{E}[w_i] \).
Proof. Fix $z_i$ as above but for the more general case; $z_i$ is the argument that maximizes the following unconstrained optimization problem:

$$
\max_{\hat{z}_i} \pi_i = pq_i + p\hat{z}_i - g(\hat{z}_i + \sum_{j \neq i} \hat{x}_j + \phi_D)\hat{z}_i
$$  

(2.33)

Then $\forall i, x_i^* = \min\{x_i, z_i\}$. When finding the optimal bidding strategy $q_i^*$, treat $z_i$ as fixed, because it is chosen in the second round but with no new information after the first round, so player $i$ already knows its optimal curtailment strategy for the second round during the first round game. Since $z_i$ is the result of an unconstrained optimization problem, it must always be the case that the FOC is satisfied. Following from 2.31,

$$
p = E_{S_{-i}}[g'(z_i + S_{-i})z_i + g(z_i + S_{-i})]
$$  

(2.34)

Then $q_i^*$ solves the the optimization problem 2.21, replacing $\mu$ with $g(\cdot)$. Since $q_i^*$ is chosen over the unconstrained real line, it must satisfy the first order condition:

$$
0 = \frac{dE[\pi_i]}{dq_i} \bigg|_{q_i^*} = E_{S_{-i}}[g(1 - F_{W_i}(z_i + q_i^*)) + \int_{-\infty}^{z_i+q_i^*} g(w_i - q_i^* + S_{-i})f_{W_i}(w_i)dw_i + \int_{-\infty}^{z_i+q_i^*} g'(w_i - q_i^* + S_{-i})(w_i - q_i^*)f_{W_i}(w_i)dw_i]
$$  

(2.35)

The first equality is given by the definition of expectation and by Leibniz differentiation. The definition of $w_i$ gives that $E[h(w_i, x)] = \int_{-\infty}^{z_i+q_i^*} h(w_i, x)f_{W_i}(w_i)dw_i + \int_{z_i+q_i^*}^{\infty} h(z_i, x)f_{W_i}(w_i)dw_i = \int_{-\infty}^{z_i+q_i^*} h(w_i, x)f_{W_i}(w_i)dw_i + (1 - F_{W_i}(z_i + q_i^*))h(z_i + q_i^*, x)$. The final line of (2.35) applies this equality and the result in (2.34) to cancel terms.

However, by applying Jensen’s inequality, the final line of (2.35) is upper-bounded by

$$
g(E_{S_{-i},\bar{w}_i}[\bar{w}_i - q_i^* + S_{-i}]) + E_{S_{-i},\bar{w}_i}[g'(\bar{w}_i - q_i^* + S_{-i})(\bar{w}_i - q_i^*)]
$$  

(2.36)

which as an upper bound must therefore be greater than or equal to 0 for any $q_i^*$ that satisfies the first order condition.

Assume towards a contradiction that $\sum_i q_i^* > \sum_i E[\bar{w}_i]$. Then, $\forall i$ is must be the case that $E_{S_{-i}} < 0$. To see this, fix $E_{S_{-i}} = \beta \geq 0$. Then, from (2.36), $E[\bar{w}_i] - q_i^* \geq -\beta$, because otherwise both terms of (2.36) are assured to be negative, since $g(\cdot)$ is increasing, with $g(0) = 0$. But then $E[\bar{w}_i] - q_i^* + S_{-i} \geq -\beta + \beta = 0$, and $E[\bar{w}_i] - q_i^* + S_{-i} = \sum_j E[\bar{w}_j] - \sum_j q_j < 0$, so there is a contradiction.

Therefore, $\forall i E_{S_{-i}} < 0$. But if $E_{S_{-i}} < 0$ then it must be that $q_i^* \leq Ew_i$. Proof: Fix $E_{S_{-i}} < 0$
and $q_i^* > E[w_i]$. Then the left hand term of (2.36) is negative because $E[\hat{w}_i - q_i^* + S_{-i}] < 0$ by linearity of expectation, and $\forall x < 0, g(x) < 0$ since $g(\cdot)$ increasing with $g(0) = 0$. Furthermore, the right hand term of 2.36 is also strictly less than zero because $g'(\cdot)$ is strictly positive and because $E[q_i^*] > E[w_i]$. Then, $q_i^*$ can not be an optimal solution to the bidding problem for player $i$ because $\frac{dE[q_i]}{dn_i}|q_i^* < 0$, so the first order condition is not satisfied by $q_i^*$. This contradiction, and the result that $\forall E[S_{-i} < 0$, implies that $\forall q_i^* \leq E[w_i]$, which contradicts the original assumption when summed over $i$. Therefore, it must be that $\sum_i q_i^* \leq \sum_i E[w_i]$, completing the proof.

Remark. The strategy set $\forall q_i^* = E[\hat{w}_i]$ is a subgame perfect equilibrium only in the case where $g(\cdot)$ is linear. Subsection 2.3.1 showed that this strategy set is a SPE in the case where $g(\cdot)$ is linear. Furthermore, $q_i^* < E[\hat{w}_i]$ for concave, nonlinear $g(\cdot)$, where the proof follows from the above but with equation (2.36) as a strict upper bound on the first order condition.

2.5 Computational Results: Penalty Comparison

This section presents computational results for optimal bid and curtailment behaviors. In general, the developed algorithms use the exact results as described in sections III and IV. However, for calculating the equilibrium bids in the case of full wind information, we find the equilibrium bid strategies through a convergence process on players’ beliefs regarding $E[z]$ and $E[S_{-i}]$.

In addition, for comparison to the results of the games modeled here with endogenous penalties, we simulate players’ best responses to exogenous penalties. This simulation adapts the model in [8] to align the penalty structure with that studied here. The model is detailed here for convenience. The individual player’s profit is given by:

$$\pi_i = pq_i + p\hat{x}_i - \lambda_+(-\hat{x}_i) - \lambda_+(-\hat{x}_i)^+$$

(2.37)
We consider cases in which $\lambda_+ > p$, which is equivalent to a positive penalty on over-production. Then player $i$ will curtail all positive deviations to zero, i.e. $x_i^* = \min(x_i, 0)$ and optimal $q_i^*$ is chosen, based on [8], such that:

$$q_i^* = F_{\lambda_+}^{-1} \left( \frac{p}{\lambda_+ + p} \right)$$

with $F_{\lambda_+}^{-1}(\cdot)$, the quantile function of $F_{\lambda_+}(\cdot)$ described above.

The tested system includes $n = 20$ players, with i.i.d. production availability modeled as a normal distribution with a mean of 15 MWh and standard deviation equal to 1.78 MWh, an approximation of real-world uncertainty based on results for system-wide forecasting errors in ERCOT [14]. The simulation ignores the effect of the demand error, by setting $\phi_D = 0$ to avoid adding excess noise in the results.

For particular values of $p$, $\mu$, and $\lambda_-$, differences in the bidding and curtailment strategies drive very different outcomes in terms of the bids and final energy production. Figure 2-1 shows a frequency histogram of the generation-bid outcomes $\sum_i \hat{x}_i^*$, normalized by the sum of the optimal bids in each case, with parameters $p = 80$, $\mu = 100$, $\lambda_- = 100$. The resulting optimal bids $q^*$ are $\{13.99, 13.86, 14.75\}$ for the three cases presented.

![Risk vs. Efficiency for Wind Deviation Penalties](image)

Figure 2-2: Risk vs. efficiency for three model situations, as the price/penalty ratio varies from very low (bottom left) to very high (top right).

Comparing endogenous and exogenous penalties at a particular penalty level is not appropriate because of the different penalty structures. Therefore, we compare profits, efficiency, and risk across the different penalty structures as $\frac{p}{\mu}$ and $\frac{p}{\lambda_-}$ vary on $(0, \infty)$. The system conditions or system operator preferences determine the price/penalty ratio to achieve performance at some point along the frontier.
We measure efficiency $\eta$ as the average percentage of available wind energy utilized and system risk $\beta$ as the mean squared generation-bid error, normalized by the squared bid value:

$$
\eta = \mathbb{E}\left[ \frac{\sum \hat{w}_i}{\sum w_i} \right] \quad \beta = \mathbb{E}\left[ \frac{\sum (\hat{x}_i^*)^2}{\sum q_i^2} \right]
$$

(2.39)

Note that low efficiency (low $\eta$) is equivalent to high curtailment, on average. High values of $\$ indicate the presence of large system-wide deviations, which may present failure risks to the electricity system or result in especially high balancing costs.

In Figure 2-2, we see how the log of the system risk and the efficiency vary over the price/penalty range. As we expect, for all models, curtailment decreases and risk of large deviations increases as the price/penalty ratio increases.

At all levels of efficiency, the exogenous price model suggests higher risk of large system deviations through greater mean square error deviations. The endogenous penalty model features improved performance, even when players curtail without observing the actual deviations of the other players. The difference is stark at high efficiency. The exogenous case suggests much higher levels of system risk, about 10 times the average squared error vs. the endogenous case with no shared information, at a reasonable target efficiency of 95%.

![Producer Profit under over Price-Penalty Range under Various Penalty Models](image)

Figure 2-3: Producer profit for a single, one hour settlement period over a range of price/penalty ratios

Finally, we examine how producer profit is affected by the price/penalty ratio, which we measure by holding the energy price constant and varying the penalty. As evident in Figure 2-3, producers’ profits increase in expectation as the price/penalty ratio increases. Expected profits are maximized
in the full wind-error information case. Thus, players have incentives to cooperate to share information in repeated games, even if a profit-maximizing player might choose to lie in a one-shot game.

Figure 2-4: Estimated Polynomial and Bilinear Regressions Price Sensitivity in Denmark Electricity Markets

### 2.6 Computational Results: Deviation Effects on Real-Time Prices

This section estimates parameterized functions $g(\cdot)$ for the Denmark NordPool electricity markets and provides examples of how wind producers would optimally bid in response to those best-fit functions. NordPool runs as a day-ahead market, with 'Regulating Power' sold in near real-time markets to settle deviations versus the day-ahead offers. Regulating power is priced separately in different directions, termed 'up regulation' and 'down regulation', with at most one of these purchased in non-zero quantity in each period based on the overall error versus day-ahead bids.

Therefore, the real-time price is analogous to the sum of the binding aspects of each of these prices; in any period $t$, $p_{rt} = p_u \mathbb{1}_{\{V_u > 0\}} + p_d \mathbb{1}_{\{V_d < 0\}}$. Here, $p_u$ and $V_u$ are the price and volume of up-regulation purchased (i.e. when the day-ahead markets are short real time demand), and $p_d$ and $V_d$ are the price and volume of down-regulation purchased. The sum of up- and down-regulation, as defined in the NordPool market, is thus the negative of the sum of bid-quantity errors defined in this paper, i.e. $\phi_D + \sum_{i \in N} x_i^* = -(V_u + V_d)$. The NordPool market data for two Denmark Market Areas, termed 'DK1' and 'DK2,' from 2014 and 2015 [15], was combined and adjusted to match the sign and notation of the model presented in this paper.

Two separate functions were considered for price sensitivity of the day-ahead minus real-time
Figure 2-5: Risk vs. Efficiency for SPE Strategies for Denmark Wind Producers over Day-Ahead Price Range

price divergence, a polynomial function and a bi-linear function. Define $y = p - p_{rt} = (p - p_u)I_{\{V_u > 0\}} + (p - p_d)I_{\{V_d < 0\}}$ and $x = -(V_u + V_d)$. The polynomial and bilinear functions, their least-squares best-fit parameter values, and the corresponding coefficients of determination are described below, and graphed along with the underlying data in Figure 2-4.

### Polynomial Best-Fit

$$f_p(x) = c_2 x^2 + c_1 x + c_0$$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_2$</td>
<td>$-0.0003$</td>
</tr>
<tr>
<td>$c_1$</td>
<td>$0.8082$</td>
</tr>
<tr>
<td>$c_0$</td>
<td>$37.5955$</td>
</tr>
</tbody>
</table>

$r^2 = 38.70$

### Bilinear Best-Fit

$$f_b(x) = (ax + a_0)I_{\{x < 0\}} + ((a - b)x + (a_0 - b_0))I_{\{x > 0\}}$$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$0.4929$</td>
</tr>
<tr>
<td>$a_0$</td>
<td>$-37.2221$</td>
</tr>
<tr>
<td>$b$</td>
<td>$0.3181$</td>
</tr>
<tr>
<td>$b_0$</td>
<td>$-172.1548$</td>
</tr>
</tbody>
</table>

$r^2 = 48.95$

An F-test can be used to compare each of these regression fits to the best fit linear regression, for instance of the form considered in Sections 2.2 and 2.3 of this paper. The simpler linear model is nested within both the polynomial or bilinear model. Applying the standard F-test for nested models allows for rejection of the null hypotheses that the enhanced models (polynomial and bilinear, respectively), provides no additional information about the data $y$, with significance at the 0.01
level. This suggests the importance of the more general exposition in Section 2.4 and the use of the polynomial and bilinear models here.

However, the least-square best-fit parameters for two models both include non-zero intercepts, contrary to our original assumptions. This suggests that there may be remaining arbitrage opportunities in electricity markets, or that there may be discontinuity between generator price offers in day-ahead versus real-time markets. Further research should focus on understanding this apparent discrepancy.

A computational iterative learning method was developed to find a symmetric subgame perfect equilibrium for the bidding and curtailment games when wind producers have equivalent assumptions about the best-fit sensitivity functions for price divergence as a result of real-time output errors. Assume that wind forecast errors are normal and i.i.d, (note: this does not require that wind availability is independent across producers), and that consumption forecast error is also normal and independent from production errors.

Figure 2-5 presents market outcomes results for both measured price sensitivity functions for two different producer concentration scenarios. These scenarios represent high and low levels, respectively, of market concentration. In the 'high concentration' scenario, five wind producers are responsible for 90% of the variance in real-time settlement volumes. In the 'low concentration' scenario, twenty wind producers are responsible for just 50% of the variance in real-time settlement volumes. In each, demand bid errors are responsible for the remaining error variance.

Figure 2-5 shows the risk and efficiency bid and curtailment equilibrium strategies for wind producers in each market concentration scenario. Risk and efficiency are defined as in Section 2.5, equations (2.39) and (??). The left and right graphs chart the equilibrium risk versus efficiency when producers view the price sensitivity as best estimated by the polynomial and bilinear best-fit functions detailed in this section, respectively. The bidding and curtailment strategies, and accompanying risk and efficiency, are calculated for day-ahead prices at the 2.5%,10%,20%,30%,40%,50%,60%,70%,80%,90%, and 97.5% quantile levels.

At very low day-ahead prices, wind producers in equilibrium curtail when their realized availability is as little as one-half standard deviation greater than the mean availability; this helps prevent the real-time price from going very low or even negative. As the day-ahead price rises, the curtailment limit rises, so the total amount of curtailment decreases because both the frequency and size of curtailment events decrease. For prices above the 30% quantile, curtailment events are very rare across market concentration types and price sensitivity assumptions. Lower concentration markets actually experience greater or equal mean squared bid-generation error versus high concentration markets, but the difference is only evident at low prices. At medium to high prices, producers curtail very little and so the bid-generation squared errors are nearly fixed.
2.7 Conclusion

This paper presented an abstract model for endogenously priced imbalance settlements in electricity markets. Analytical solutions for the optimal curtailment and bid strategies were presented for the case when price sensitivity can be approximated by a locally linear function. The model suggests that market-based penalty prices achieve much better trade-offs between efficiency and system risk than exogenous penalty models, because producers expect that their competitors will react in a similar way to real-time price movement, decreasing its overall effect.

Results were also presented for a more general class of estimated price sensitivity functions. In particular, when the price sensitivity function is increasing, concave, and crosses through the origin, producers do not curtail below their original bid amounts, and they bid weakly less than their mean availability. This analysis provides a general model for profit maximization for wind producers who have assumed or measured the real-time price sensitivity as a function of day-ahead bid errors, and who are able to estimate the mean observed difference between aggregate wind production and wind producer bids in the system. The two stage process of estimating a functional form for price sensitivity and applying the results in Section 2.4 should appeal to merchant wind producers as a natural way to maximize profits.

Parameterized models were presented and estimated for the price sensitivity function, using two years worth of demand, wind, and price data in the NordPool Denmark Markets. Further work could help improve sensitivity function estimation or explore how network constraints impact price sensitivity. Computational results were presented to detail the market risk and efficiency from symmetric subgame perfect equilibrium strategies over the range of day-ahead prices. This suggests that wind producers are very unlikely to curtail except in cases where day-ahead prices are low. In future work, we hope to tackle related questions in capacity and other ancillary service markets in order to understand how price and resource stochasticity impact optimal bidding behaviors of renewable generators and storage units in these types of markets.
Chapter 3

Wind Subsidies and the Effect on Investment and Long-Run Equilibria

This chapter examines the wind investment decision and reviews the impact of policy interventions such as price caps, capacity markets, and the PTC on wind investment in the broader system context. It also connects the effects of the PTC on individual investment criteria to overall effects in the long-run equilibrium of generation investment.

Section 3.1 derives a simple representation of the investment decision in wind generation. Section 3.2 shows how this extends to trade-offs in capacity factor and price correlation, describing how the PTC might affect investment siting decisions. Section 3.3 links the effects of the PTC to a system-wide perspective, showing how fixed support systems can lead to higher variance of system output. Section 3.4 analyzes medium and long-run impacts of wind development with various levels of correlation. Section 3.5 addresses policy impacts and concludes.

The research shows how the stochasticity of the wind resource implies a trade-off between sites with higher capacity factors and higher covariance with prices at the efficient frontier. The analysis suggests that the Production Tax Credit (PTC), along with some capacity mechanisms, bias wind investment towards high-producing sites but with lower covariance of their variable output with market prices. Furthermore, since wind production depresses prices, this bias is linked to covariance between wind sites. Fixed-price support mechanisms like the PTC lead to market equilibriums with higher levels of wind correlation.

3.1 Optimal Investments in Intermittent Generation

The value of any new investment in a electricity market is theoretically equal to the value that investment can earn in the spot market for electricity. For a theoretical electricity spot market, the optimal investment criteria is of the form first characterized in [16] for spot pricing. For a traditional
generator \( j \), considering a marginal investment \( k_j \), the optimal investment criteria is given by

\[
\frac{\partial I}{\partial k_j} = E \left[ \sum_{t=0}^{T} \left( \frac{\partial Y_{j,max}(t)}{\partial k_j} + \frac{\partial Y_{j,min}(t)}{\partial k_j} \right) + \frac{\partial C_j}{\partial k_j} \right]
\]

(3.1)

where \( \frac{\partial I}{\partial k_j} \) is the marginal capital cost of new investment, \( t \) refers to a single settlement period in the market, and \( T \) refers to the investment lifetime. Furthermore, \( \frac{\partial Y_{j,min}(t)}{\partial k_j} \) and \( \frac{\partial Y_{j,max}(t)}{\partial k_j} \) refer to the marginal change in the minimum and maximum generation capacity constraints of the generator, and \( -\mu_{j,min}(t) \) and \( -\mu_{j,max}(t) \) are the optimal values of the Lagrange multipliers for the generator min and max availability constraints in the optimal pricing problem. The final term, \( \frac{\partial C_j}{\partial k_j} \) refers to the change in variable costs due to investment, for instance through an improvement in heat-rate or reduced variable maintenance costs.

This exposition focuses only on questions of generation expansion. Therefore, assume \( \frac{\partial Y_{j,min}(t)}{\partial k_j} = 0 \) and \( \frac{\partial C_j}{\partial k_j} = 0 \). Then, the marginal benefits of additional investment are simply the benefits from pushing out the maximum production constraint in the spot market optimization problem. Using complementary slackness, Caramanis shows that in periods \( t \) when the maximum production constraint is binding for generator \( j \),

\[
\mu_{j,max}(t) = \pi_j(t) - \frac{\partial C_j}{\partial Y_j(t)} > 0
\]

(3.2)

where \( \frac{\partial C_j}{\partial Y_j(t)} \) is generator \( j \)'s marginal cost (MC) of production and \( \pi_j(t) \) is the market price seen by generator \( j \) in period \( t \). In the competitive market, the clearing price is the same for all generators \( \pi_j(t) = \pi(t) \ \forall j, t \).

Finally, assume that if the maximum production constraint for generator \( j \) is binding in period \( t \), it would still be binding despite a marginal constraint improvement due to investment \( k_j \). This allows for a reinterpretation of a simplified 3.1 and 3.2 as follows:

\[
\frac{\partial I}{\partial k_j} = E \left[ \sum_{t=0}^{T} \left( \frac{\partial Y_{j,max}(t)}{\partial k_j} \max\{0, \pi(t) - \frac{\partial C_j}{\partial Y_j(t)} \} \right) \right]
\]

(3.3)

Equation 3.3 states a simple relationship regarding the marginal investment decisions of generators. In equilibrium, the marginal cost of a capacity investment decision should be the marginal benefit of that investment in expectation, where the total marginal benefit is the sum of the marginal benefits over all periods. In the case of efficient economic dispatch, than in any period \( t \), the marginal benefit of the additional capacity is the amount of that capacity that is available for production, times the difference between the spot price and the generator's marginal cost, when that value is greater than 0. If the generator's marginal cost is greater than or equal to the spot price in any period \( t \), the marginal benefit of extra investment in that period is 0.

Traditional generators usually operate with high reliability, so the value \( \frac{\partial Y_{j,max}(t)}{\partial k_j} \) may be close
to 1 for all periods \( t \). Moreover, the operator of a mid-range or peaking plant can carefully plan upgrades and reliability maintenance for the shoulder season when average demand is low, so that as much capacity as possible is available in all periods when the maximum output constraint is binding. Thus, while the expected value of \( \frac{\partial Y_{j,\text{max}}(t)}{\partial k_j} \) and its variation in \( t \) are important to the investor, they may not have a major differentiating impact between traditional generation investments.

On the other hand, the availability of additional generation capacity due to the investment is very important for valuing the investment in intermittent generation, like wind. In the case of wind and solar generation, the marginal cost is approximately 0, since there are no fuel costs. Correspondingly, if \( k_j \) represents investment in renewable generation, then \( \frac{\partial C_j}{\partial Y_j(t)} = 0 \) \( \forall t \). To align the notation with existing work on wind generation, let

\[
\frac{\partial Y_{j,\text{max}}(t)}{\partial k_j} = f_k(t) \frac{\partial W_j}{\partial k_j}
\]

where \( f_k(t) \) is the capacity availability of the new investment in time \( t \) and \( \frac{\partial W_j}{\partial k_j} \) is the incremental capacity addition due to investment \( k_j \). To simplify the analysis, normalize \( \frac{\partial W_j}{\partial k_j} = 1 \) so all investments have the same cost per unit of capacity. Note that the capacity availability \( f_k(t) \) is a random variable whose probability distribution varies in \( t \), as is \( \pi(t) \). Therefore, the capacity factor of the new wind investment is \( \mathbb{E}\left[ T \sum_{t=0}^{T} f_k(t) \pi(t) \right] \). Then, the spot pricing model leads to the intuitive investment value for a wind project:

\[
\frac{\partial I}{\partial k_j} = \mathbb{E}\left[ T \sum_{t=0}^{T} \left( f_k(t) \pi(t) \right) \right]
\]

It is possible to rewrite 3.5 in terms of the marginal distributions of the random variables \( f_k \) and \( \pi \), which have marginal distributions equally weighting all possible future values of \( t \). For example, pick \( f_k \) so that the distribution of \( f_k \pi \) conditioned on \( t = t' \) is equal to the distribution of \( f_k(t') \pi(t') \) for all \( t' \in \{1, \ldots, T\} \), and likewise for each of the marginals. Define a random variable \( S \sim U\{1, 2, \ldots, T\} \). Then,

\[
\frac{\partial I}{\partial k_j} = \mathbb{E}\left[ T \sum_{t=0}^{T} \left( f_k(t) \pi(t) \right) \right] = T \mathbb{E}\left[ \sum_{t=0}^{T} f_k(t) \frac{1}{T} \right] = T \mathbb{E}_S \left[ \mathbb{E}_{f_k \pi} [f_k \pi | S] \right] = T \mathbb{E}_{f_k \pi} [f_k \pi]
\]

The equation in 3.6, the marginal investment condition for wind generation, can be interpreted as follows: the marginal value for each unit of investment in wind generation equals the investment horizon times the joint expectation of the product of the random variables for the price of electricity.
and the capacity availability of the new generation investment.

3.2 Capacity Factor and Price Correlation Trade offs in Wind Siting Decisions

The marginal investment results for an intermittent generator in the previous section are well understood, but the form suggests interesting effects on wind generation siting decisions that are not frequently discussed. In particular, there is a natural trade-off at the margin between capacity factor and covariance between energy production and electricity price, as suggested by equation 3.5, which is expanded here using the definition of covariance:

\[
\frac{\partial I}{\partial k_j} = T \mathbb{E}[f_k \pi] = T (\mathbb{E}[f_k] \mathbb{E}[\pi] + \text{Cov}(f_k, \pi))
\] (3.7)

Next, consider marginal investments \( k_j \) and \( k_j' \), with \( \frac{\partial I}{\partial k_j} = \frac{\partial I}{\partial k_j'} \). For simplicity, assume the two investments have the same time-horizon and involve the same marginal capacity addition. Then

\[
\mathbb{E}[f_k] \mathbb{E}[\pi] + \text{Cov}(f_k, \pi) = \mathbb{E}[f_{k'}] \mathbb{E}[\pi] + \text{Cov}(f_{k'}, \pi) = \varphi
\] (3.8)

for some constant \( \varphi \). By rewriting as a difference of two equal terms, observe that:

\[
\mathbb{E}[\pi] + \frac{\text{Cov}(f_k, \pi) - \text{Cov}(f_{k'}, \pi)}{\mathbb{E}[f_k] - \mathbb{E}[f_{k'}]} = 0
\] (3.9)

By considering the marginal differences between the investments and taking the limit as these differences become increasingly small, we can characterize the slope of the efficient frontier:

\[
\frac{\partial \text{Cov}(f_k, \pi)}{\partial \mathbb{E}[f_k]} = -\mathbb{E}[\pi]
\] (3.10)

This suggests that an incremental decrease in the capacity factor of a potential new investment would need to be coupled with an incremental increase of \( \mathbb{E}[\pi] \) units of covariance to remain on the efficient frontier.

The wind investment data from Wolak’s 2015 paper [27] allows for additional insight by plotting the efficient frontier of capacity factor and correlation with prices based on real-world investment data. Wolak provides metrics for plant output and price coefficient of variation and for correlation between energy output and price, but does not include the plant capacity factors in the results. Using covariance for each plant’s output with price and the average wholesale price in California, Figure 3-1 displays the required capacity factor any given plant would need to achieve in order to be on the efficient frontier, in accordance with equation 3.10 for the ‘No PTC’ case. For comparison purposes, a single arbitrary plant is fixed as a "swing" plant with a set capacity factor \( \mathbb{E}[f_k] = 0.30 \). The analysis assumes that all sites experienced equivalent average prices and had equivalent marginal
investment costs per unit of capacity. Note that some of the plants would be inefficient investments in the real-world, and would be below the efficient frontier. However, by using real-world data for the covariance (from [27]) to simulate the efficient frontier, the data gives a sense of the level of variation in capacity factor (a range of around 6%) one could expect from real-world plants that represent efficient investment decisions.

3.2.1 The Effect of the PTC on Investment Behavior

The Production Tax Credit (PTC) has been used since 1992 to stimulate investment in some renewable energy technologies, including wind. It was most recently renewed in December 2015 and extended through 2019. As of 2016, the PTC for wind is $0.023/kWh, but it will begin to decline in 2017.

The PTC, a fixed-rate subsidy for wind energy, affects marginal decisions along the investment frontier, resulting in a different characterization of the frontier versus the no-subsidy case described by 3.10. Starting from 3.8, for all investments $k_j, k'_j$ along the investment frontier

$$E[f_k]E[\pi] + E[f_k]\phi_{PTC} + \text{Cov}(f_k, \pi) = E[f_{k'}]E[\pi] + E[f_{k'}]\phi_{PTC} + \text{Cov}(f_{k'}, \pi) = \varphi$$  (3.11)

for some constant $\varphi$, with a fixed PTC $\phi_{PTC}$. Now, following the same derivation in 3.10, observe:
When the production tax credit is considered, the efficient frontier exhibits a much steeper slope in trading off the relationship to high prices and the capacity factor of a potential site. This serves to bias investment towards sites with proportionally higher expected capacity factors, but with proportionally lower covariance of output and price.

It is important to consider how the PTC affects this bias, but it is hard to estimate from available data. As a preliminary test, we again use the data provided by Wolack to examine the effect of the PTC in biasing the efficient investment frontier. The same swing plant is fixed; assume that it is located along the efficient frontier with a fixed $E[f_k] = 0.30$. The average electricity price is assumed to be the average traded price at the Northern California NP-15 hub over all of 2015, so $E[\pi] = 0.03705$, based on data from the EIA. Figure 3-1 provides a comparison of the efficient frontier with and without the PTC. Clearly, the PTC biases investments with higher capacity value at the expense of an improvement in covariance with price. When a fixed feed-in subsidy (the PTC) is provided to wind generators, investments will be made that are well below the true efficient frontier, at the expense of investments with higher value in the efficient market. Notably, if the subsidy was provided as a multiplier of the wholesale price of electricity in each period in which the generator was producing, the subsidy would no longer bias investment towards high capacity factors at the expense of improved production-price covariance.

In the aforementioned paper, Wolak states that the fixed-price Feed-in-Tariffs (FIT) that currently finance most wind projects could be the cause of suboptimal investment [27]. While Feed-in-Tariffs are only present in some states, they represent the most extreme form of the above analysis, one in which the effective average market price seen by the assets $E[\pi] = 0$ and the fixed tariff $\phi$ is the FIT rate. For states where utilities are required to offer a fixed-rate FIT, like California and Maine, the resulting investment frontier would be shown as a horizontal line on the graph in Figure 3-1. In that extreme case, the capacity factor dominates the investment decision.

### 3.2.2 The Effect of Capacity Markets on Investment Behavior

In some markets, wind assets are allowed to participate in forward capacity markets, based on a capacity value that is determined by the regulator. For instance, in ISO-NE, the capacity value is separated by the summer and winter seasons and calculated based on the 5 year rolling average of medium net generation in those hours in the relevant season (e.g. 1-6 pm in summer months June-September). In NYISO, new onshore resources receive a 10% default capacity credit for summer capacity and a 30% default capacity credit for winter capacity, while in PJM, new wind projects received a 13% default capacity credit [31], prior to changes in the PJM Capacity Market in 2015.

The capacity allowance can be factored in to the above equations similarly to $\phi_{PTC}$, as an additional rate based on the administratively determined capacity value and the capacity auction.
clearing price. Depending on the form of the capacity allowance, this could actually serve to bias the investment frontier in any direction. For instance, the capacity value mechanisms in NYISO [31] provide a fixed credit based on capacity, which does not take into account the actual performance of the resource at all. This is inefficient, because it would bias outcomes towards investments with relatively lower marginal costs for capacity expansion but lower average value of the energy produced and/or lower average capacity factor. PJM’s new Capacity Performance Model, approved by FERC in June 2015 [32], is a step in the right direction because it has more stringent penalties for non-performance, which, if implemented correctly could lead to correct valuations of the time-sensitive nature of the wind capacity. However, some have argued that the associated penalties still do not sufficiently penalize non-compliance, so this improved capacity metric still most likely undervalues covariance between capacity availability and price versus the efficient market.

3.2.3 The Effect of Price Caps

The same analysis also suggests that price caps in energy-only markets might have an impact the marginal wind investments as analyzed in the previous subsections, but ultimately neither the magnitude nor the direction of the impact can be explained by the methods used here. We hypothesized that a price cap would uniformly reduce the expected benefits from a site with marginally better covariance between production and energy prices, and therefore bias investments away from the efficient frontier and towards increasing emphasis on capacity factor, as was the case with the PTC. However, because of the linear nature of covariance, it is easy to come up with examples (if improbable) where this is not the case. For instance, consider two marginal investments \( k_j \) and \( k_j', k_j \) along the investment frontier. Assume that they have to equal mean capacity factors, but that investment \( k_j \) is ultimately more attractive because

\[
\text{Cov}(f_k, \pi) > \text{Cov}(f_{k'}, \pi).
\]

Furthermore, assume that this improvement in covariance is driven by the fact that

\[
\mathbb{E}[f_k | \pi = \bar{\pi}] > \mathbb{E}[f_{k'} | \pi = \bar{\pi}] \forall \bar{\pi} \in [\mathbb{E}[\pi], \bar{\pi}]
\]

and that the improvement holds true even though investment \( k_j \) is less attractive in periods of really high prices (i.e. high demand), i.e. even though

\[
\mathbb{E}[f_k | \pi = \bar{\pi}] < \mathbb{E}[f_{k'} | \pi = \bar{\pi}] \forall \bar{\pi} \in [\bar{\pi}, \infty)
\]

Next, consider a parallel market but with a price cap at \( \bar{\pi} \). It’s easy to see that the gap in value between these investments is actually increased as a result of the price cap. Thus, a price cap, in some cases, can actually bias marginal investment in the direction of investments that were already more attractive. That said, the way that this works is still by undervaluing the impact of the wind generation in periods that are constrained by the price cap, so a price cap still ultimately provides an inefficient incentive that can affect the wind investment decision.

In general, a price cap is inefficient in that it prevents available energy from realizing a price
that is actually commensurate with demand. However, even though we expect that a price cap might generally bias investment towards projects that have lower covariance with demand, this is not always the case. The effect of a price cap on individual investments depends on the details of the joint distribution of the site's wind availability and of demand, and is subject to a high-degree of uncertainty.

3.2.4 Policy Improvement: Proportional PTC

As explained previously, the current form of the Production Tax Credit (PTC) for wind producers clearly biases investment. This section argues that a different subsidy, provided as a multiplier of the wholesale price for electricity, would eliminate the bias present in the current form of the PTC. Here, we expand and provide evidence for that assertion.

Consider the expanded investment criterion in equation 3.7, but where the price received for electricity in a single time period is no longer \( \pi \) but rather \( \lambda \pi \), for some \( \lambda > 1 \), where the exact value of \( \lambda \) can be set by policy-makers. For instance, \( \lambda \) could be set to achieve modeled cost equivalence with the existing PTC level \( \phi_{PTC} \). Under this alternative subsidy model, the investment value is given by the following:

\[
\frac{\partial I}{\partial k_j} = T \mathbb{E}[f_k \lambda \pi] = T(\mathbb{E}[f_k] \mathbb{E}[\lambda \pi] + \text{Cov}(f_k, \lambda \pi))
\] (3.15)

As before, it is possible to characterize the marginal investments along the investment frontier, for investments with the same time-horizon and involving the same marginal capacity addition:

\[
\mathbb{E}[f_k] \mathbb{E}[\lambda \pi] + \text{Cov}(f_k, \lambda \pi) = \mathbb{E}[f_{k'}] \mathbb{E}[\lambda \pi] + \text{Cov}(f_{k'}, \lambda \pi) = \varphi
\] (3.16)

for some constant \( \varphi \). Rewriting 3.16 as a difference gives that:

\[
\lambda \mathbb{E}[\pi](\mathbb{E}[f_k] - \mathbb{E}[f_{k'}]) + \lambda \text{Cov}(f_k, \pi) - \lambda \text{Cov}(f_{k'}, \pi) = 0.
\] (3.17)

The properties of expectation and covariance allow us to move the constant \( \lambda \) outside of those terms. Dividing both terms by the difference in capacity factors, gives

\[
\mathbb{E}[\pi] + \frac{\lambda(\text{Cov}(f_k, \pi) - \text{Cov}(f_{k'}, \pi))}{\lambda(\mathbb{E}[f_k] - \mathbb{E}[f_{k'}])} = 0.
\] (3.18)

where the \( \lambda \) terms cancel. Then, taking the limits as marginal differences become increasingly small, we characterize the investment frontier according to

\[
\frac{\partial \text{Cov}(f_k, \pi)}{\partial \mathbb{E}[f_k]} = -\mathbb{E}[\pi]
\] (3.19)

which is equivalent to the original condition in 3.10. Therefore, a subsidy distributed proportionally
to the real-time price of electricity eliminates the investment bias towards higher capacity factor sites, pushing investment towards sites the produce more valuable electricity.

In theory, the variable \( \lambda \) could be chosen to that the total expected profit for producers would be equal to that under the current PTC regime, i.e.

\[
TE[\lambda f_k \pi] = TE[f_k \pi + \phi_{PTC}]
\]

(3.20)

for the average or marginal producer. Then, the expected costs of the program are equivalent in either case:

\[
TE[(\lambda - 1)f_k \pi] = TE[\phi_{PTC}]
\]

(3.21)

However, the utility function of total profits is not necessarily linear for any potential investor. When considering the natural concavity of producer utility or additional aversion to risk, it becomes clear that increased incentive costs are required to maintain the same level of investment. Consider a specific potential investor \( m \) who weights profits by an increasing, concave utility function \( u_m(\cdot) \), with \( u(0) = 0 \). Assume that under the PTC regime, the expected utility for this investor exactly equals their reservation price, i.e. they are investing at the margin. Therefore, for the producer to move ahead with an equivalent investment under the proportional price regime, their expected utility must equal or exceed that under the PTC:

\[
TE[u_m(\lambda f_m \pi)] \geq TE[u_m(f_m \pi + \phi_{PTC})]
\]

(3.22)

**Proposition 6.** Consider a marginal producer with strictly concave and non-decreasing utility of profit, with \( u(0) = 0 \). In order for this producer to invest under the proportional PTC, i.e. for the condition in 3.22 to hold, the expected incentive / subsidy to that investor in the proportional incentive regime must exceed that under the PTC:

\[
TE[(\lambda - 1)f_m \pi] > TE[\phi_{PTC}f_m]
\]

(3.23)

**Proof.** By rearranging the marginal investment requirement in 3.22, using the linearity of expectation, observe that:

\[
0 \leq TE[u_m(\lambda f_m \pi) - u_m(f_m \pi + \phi_{PTC}f_m)]
\]

(3.24)

\[
< TE[u_m(\lambda f_m \pi - (f_m \pi + \phi_{PTC}f_m))]
\]

\[
\leq Tu_m(E[\lambda f_m \pi - (f_m \pi + \phi_{PTC}f_m)])
\]

The second inequality is due to the strict concavity of \( u_m(\cdot) \), since \( u(a + b) \leq u(a) + u(b) \) implies that \( u(c) \leq u(a) + u(c - a) \) and therefore \( u(c) - u(c - a) \leq u(a) \), as above. It holds strictly if \( u_m(\cdot) \) is strictly concave. The third inequality is true by Jensen's inequality.

Since \( u_m(0) = 0 \) and \( u_m(\cdot) \) non-decreasing, \( u(x) > 0 \to x > 0 \). From the inequalities in 3.24, it
is therefore true that

\[ \mathbb{E}\left[ \lambda f_m \pi - (f_m \pi + \phi_{PTC} f_m) \right] > 0 \quad (3.25) \]

By rearranging terms and using the linearity of expectation, it is clear that this is equivalent to 3.23 above.

In practice, wind investors may exhibit concave utility curves or general aversion to risk. For this more general class of producers with concave utility curves, it is more expensive on average to subsidize the marginal investment (and thus, to subsidize the same amount of total investment, given equal opportunities) under a price-based proportional incentive versus the traditional PTC. There are two main benefits, however, to the proportional incentive. First, the proportional incentive regime leads to increased average value for wind energy production, as argued in this section. Second the proportional incentive regime leads to lower variance of total wind output, as Section 3.3 will argue. A higher total output variance increases potential grid security costs, which are hard to price in electricity markets, and it can increase the difficulty of decarbonizing the electricity sector. The additional costs of the proportional incentive must be seen as a trade-off and evaluated versus the benefits described here.

Besides costs, there are some additional considerations required of any subsidy comparison. One goal of subsidies, like the Production Tax Credit, is to provide support to a nascent industry. The nominal goal of either incentive type, either increasing wind generation or increasing total wind energy value created, is a reasonable framing of that intent.

On the other hand, subsidies for wind energy are also used to help reduce greenhouse gas emissions in the electricity sector. For this goal, the existing PTC might seem a more efficient fit, since it provides an incentive to wind energy based on total kWh of energy produced (and thus roughly based on kWh of fossil fuels provided). In fact, if a PTC encourages wind producers to bid below their marginal cost of production, for instance to bid \( \frac{\partial C}{\partial y_j(t)} - \phi_{PTC} \), then the producers will essentially only receive the value of the credit (vs. the counter-factual where there is no credit and wind producers bid their MC) in the cases where wind is displacing some fossil fuel energy.

However, the traditional PTC also serves as an inefficient incentive for greenhouse gas reduction because it does not take into account variation in emissions from different sources. In a system where marginal production is frequently a single type of generator, but with varying levels of efficiency and therefore cost (for instance, California or New York with natural gas), the proportional PTC might actually be more reflective of the level of marginal emissions reductions.
3.3 Output-Price Covariance and Variance of Total System Wind Output

The fixed-rate subsidy analyzed in this section is clearly inefficient in the sense that they bias wind investment away from the socially optimal portfolio. These regulatory failures may help explain the under performance of existing wind assets versus an efficient portfolio, as evidenced by Wolak for wind and solar generators in California [27].

Besides covariance with price, variance of total system output is an important metric for the efficiency of any group of wind investments. High variance of total system output is equivalent to higher intermittency [27], which is associated with higher price swings, lower system stability, and greater challenges for integrating renewable resources. From the perspective of a single producer \( k \), define residual wind output as the random variable sum of capacity availability of all other producers \( S_{-k} \equiv \sum_{i \neq k} f_i \). This section develops theoretical results to link the covariance of a particular producer with price \( \text{Cov}(f_k, \pi) \) with covariance of a particular producer with remaining system production \( \text{Cov}(f_k, S_{-k}) \) and total system wind output variance \( \text{Var}(f_k + S_{-k}) \) for any \( k \). This allows us to extend the previous results to predict that policies like the PTC will lead to increased system wind variance as compared to investments in the unsubsidized or proportionally subsidized case.

The key fact that connects these results is that wholesale energy market prices are decreasing in wind output. Energy production from wind generators has 0 or near-0 marginal costs, and thus it nearly always clears in any auctions when it is available. Therefore, wind production reduces thermal generation demand and lowers the clearing price for electricity. Since the supply curve is increasing, then price is necessarily a decreasing function of wind output. The magnitude of this effect depends on electricity demand during the current period - if the demand curve shifts during periods of already low demand, the supply curve may be relatively flat in the base-load region and the price suppression may be small. However, the direction is clear - in any given period, increased wind production decreases electricity clearing prices. As such, it is useful to model price as a decreasing function of available wind generation, i.e. \( \pi = g(S_{-k}) \) with \( g(\cdot) \) strictly decreasing. The price earned by potential new investment \( k \) at any given time is strictly decreasing in the output of all other wind producers.

Therefore, for any given wind investment, covariance with energy price is directly influenced by the covariance between its output and that of other wind generators. The following proposition makes this clear:

**Proposition 7.** Consider two wind power sites whose availability is characterized by random variables \( f_k \) and \( f_k' \), satisfying \( E[f_k] = E[f_k'] \). Assume that the conditional expectation functions of \( f_k \) and \( f_k' \) with respect to the residual wind outputs \( S_{-k} \) are single crossing. Assume further that price is a strictly decreasing function of residual output \( S_{-k} \). Then, a higher output covariance with respect to price is equivalent to a lower output covariance with respect to residual wind output.
That is,

\[ \text{Cov}(f_k, \pi) < \text{Cov}(f'_k, \pi) \iff \text{Cov}(f_k, S_{-k}) > \text{Cov}(f'_k, S_{-k}) \]  \tag{3.26} 

**Proof.** Fix random variables \( f_k \) and \( f'_k \) such that \( \mathbb{E}[f_k] = \mathbb{E}[f'_k] \) and that conditional expectations \( \mathbb{E}[f_k|S_{-k}] \) and \( \mathbb{E}[f'_k|S_{-k}] \) are single crossing. That is, \( \exists y \) such that either

\[
\forall x \geq y \quad \mathbb{E}[f_k|S_{-k} = x] \geq \mathbb{E}[f'_k|S_{-k} = x] \quad \text{and} \quad \forall x \leq y \quad \mathbb{E}[f_k|S_{-k} = x] \leq \mathbb{E}[f'_k|S_{-k} = x]
\]

or

\[
\forall x \geq y \quad \mathbb{E}[f_k|S_{-k} = x] \leq \mathbb{E}[f'_k|S_{-k} = x] \quad \text{and} \quad \forall x \leq y \quad \mathbb{E}[f_k|S_{-k} = x] \geq \mathbb{E}[f'_k|S_{-k} = x]
\]  \tag{3.27} 

Note that no assumption is made about the direction of the single crossing property. Furthermore, the random variables \( f_k \) and \( f'_k \) might have very different joint probability distributions with the existing wind availability \( S_{-k} \). This single crossing property is satisfied by many types of joint distributions. For instance, if both \( (f_k, S_{-k}) \) and \( (f'_k, S_{-k}) \) are bivariate normal, then the functions \( \mathbb{E}[f_k|S_{-k} = x] \) and \( \mathbb{E}[f'_k|S_{-k} = x] \) are linear and are single crossing at the point \( y = \mathbb{E}[S_{-k}] \).

First, the proof will establish that the first condition of 3.26 implies the second. The proof of the converse follows the same logic. Assume \( \text{Cov}(f_k, \pi) < \text{Cov}(f'_k, \pi) \). Then

\[
0 > \text{Cov}(f_k, \pi) - \text{Cov}(f'_k, \pi) = \mathbb{E}[f_k \pi] - \mathbb{E}[f_k \pi] + \mathbb{E}[f'_k \pi] = \mathbb{E}[(f_k - f'_k)g(S_{-k})]
\]  \tag{3.28} 

where the final equality is due to linearity of expectation, \( \mathbb{E}[f_k] = \mathbb{E}[f'_k] \), and by rewriting price as a decreasing function of the random variable \( S_{-k} \).

Due to the single crossing property, assume \( 3s_0 \) that is the crossing-point separator that satisfies the conditions in 3.27. Assume that the random variable \( S_{-k} \) has support on some range that includes \( s_0 \) in its interior, i.e. there is a set \( A = \{ x \in \mathbb{R} \mid 0 < \mathbb{P}(S_{-k} \leq x) < 1 \} \) and \( s_0 \in \text{int}(A) \).

Note that since \( \mathbb{E}[f_k] = \mathbb{E}[f'_k] \) and \( g(s_0) \) is just a constant, \( \mathbb{E}[(f_k - f'_k)g(s_0)] = 0 \). Subtracting this from both sides of the equation 3.29 implies that

\[
0 > \mathbb{E}[(f_k - f'_k)(g(S_{-k}) - g(s_0))] = \mathbb{E}[(\mathbb{E}[f_k|S_{-k}] - \mathbb{E}[f'_k|S_{-k}])g(S_{-k}) - g(s_0))]
\]  \tag{3.29} 

where the equality is due to the law of total expectation. But this requires that the single crossing property is governed by the first line of 3.27. If it were not, then the second line must hold true and \( \forall S_{-k} = s < s_0, \mathbb{E}[f_k|S_{-k}] \geq \mathbb{E}[f'_k|S_{-k}] \) by the single crossing property and \( g(S_{-k}) > g(s_0) \) since \( g(\cdot) \) decreasing. Similarly, \( \forall S_{-k} = s \geq s_0, \mathbb{E}[f_k|S_{-k}] \leq \mathbb{E}[f'_k|S_{-k}] \) by the single crossing property and \( g(S_{-k}) \leq g(s_0) \). But combined, these statements imply that \( (\mathbb{E}[f_k|S_{-k}] - \mathbb{E}[f'_k|S_{-k}])g(s_{-k}) - (g(s_0)) \geq 0 \) almost surely. This implies that \( \mathbb{E}[(\mathbb{E}[f_k|S_{-k}] - \mathbb{E}[f'_k|S_{-k}])g(S_{-k}) - g(s_0)] \geq 0 \), which
is a contradiction with (3.29). Therefore, the second line of 3.27 can not be true, so the first must be. But then, \((E[f_k|S_{-k}] - E[f'_k|S_{-k}])(S_{-k} - s_0) > 0\) almost surely, which implies:

\[
0 < E[(E[f_k|S_{-k}] - E[f'_k|S_{-k}])(S_{-k} - s_0)] = E[(f_k - f'_k)(S_{-k} - s_0)] = E[(f_kS_{-k}) - E[f'_kS_{-k}]]
\]

(3.30)

\[
= E[(f_kS_{-k}) + E[f_kE[S_{-k}] - E[(f'_kS_{-k}) - E[f'_kE[S_{-k}] = Cov(f_k, S_{-k}) - Cov(f'_k, S_{-k})]
\]

The first and third equality use the fact that \(E[f_k] = E[f'_k]\), and the final equality uses the definition of covariance. Therefore, the proof and the final line of (3.30) shows that \(Cov(f_k, \pi) < Cov(f'_k, \pi)\) implies \(Cov(f_k, S_{-k}) > Cov(f'_k, S_{-k})\). The proof of the converse follows in the same way, but replacing the inequalities with their converse. This establishes the equivalence of the two statements in 3.26.

\[\square\]

Remark. While Proposition 2 proven in the case where the conditional expectations are single crossing, the single crossing property is not a requirement for the result in any specific case. Roughly, if the conditional expectations of the outputs of two potential sites with respect to total system wind output are ‘nearly’ single crossing, then the result will hold. For instance, for a specific pair of sites, if the conditional expectations are about equal over some range, or have a second ‘crossing’ conditioned on some set with sufficiently small measure, then for these sites it will remain true that the ranking of each of their covariances with price will be equivalent to the ranking of their covariance with system output.

Based on these results, the covariance between price and a single plant’s output can be thought of as a variable partially determined by two separate factors, the covariance between that plant’s output and the load demand, and the covariance between that plant’s output and the output of other wind generators. Policies that undervalue covariance with price will lead to investments at the margin that have lower covariance with price. The statement in (3.26) implies that these policies will also shift investments at the margin so they have higher variance with the rest of the wind portfolio. Furthermore, increased variance of individuals with the remaining portfolio is equivalent to increased variance of the entire portfolio. Consider a system with \(N\) wind sites. Then \(\text{Var}(\sum_{k \in N} f_k) = \sum_{k \in N} \text{Var}(f_k) + \sum_{k, j \in N, k \neq j} \text{Cov}(f_k, f_j) = \sum_{k \in N} \text{Var}(f_k) + \sum_{k \in N} \text{Cov}(f_k, S_{-k})\).

Thus subsidies that reduce the marginal impact of price covariance on the investment decision bias investment decisions towards a system with higher portfolio variance.

High variance of portfolio wind generation outputs results in lower value of the available wind production and lower prices paid to generators, but it also increases reliability concerns. This can manifest in two ways. First, it increases the need for fast-ramping capacity, which could impose additional costs on the system or necessitate the creation of new ancillary service markets. Second,
the reliability problem could increase resource adequacy concerns because the wind portfolio has incrementally lower "firm" (i.e. with high probability) availability as the standard deviation increases. A well-functioning energy-only market, such as the one described by Hogan [18] with an operating reserve demand curve, should help to counteract that and should provide incentives for efficient wind investment, in line with the frontier described in 3.10. However, policies like the PTC that affect the efficient frontier could increase externalities related to system reliability and increase tension regarding the inefficiencies of existing capacity mechanisms.
3.4 Impacts of Wind Generation on System Prices and Long-Run Market Equilibrium

The previous section showed that increased wind variance would exert downward pressure on electricity prices at precisely the times when production is likely to be high at individual generators. This section models that result to elucidate the effects of wind generation on market prices and long-run market equilibriums. This section will also show how the difference in correlation amongst wind investments can have an impact on the suppression of market prices.

This analysis focuses on a simplified market model with three types of traditional generators, a "baseload" plant that models a coal plant, a "midrange" plant that models a combined cycle plant, and a "peaker" plant that models a combustion turbine. The fixed and variable costs for the plants are estimated based on levelized cost of energy (LCOE) analysis from the EIA, except we assume a 50% increase in fuel costs for the natural gas plants. At current prices, natural gas plants dominate versus the ‘baseload’ plant, so this is intended to show the effects of wind on a market where there is more competition between new investment in competing generation technologies. The model is intended to provide a general analysis of the trade off between fixed and variable prices. The EIA parameter values are just used to ground the cost curves with some basis in reality; the specific values are not essential to our analysis.

Current market conditions and the load duration curve are based on data from ERCOT in 2014, with a peak demand of 66.5 GW. It is assumed that wind energy is always dispatched when available, with a marginal cost of 0, so it is considered as negative load. We find the load duration curve for the net load, which is the demand minus wind generation in any given hour.

We make that counter-factual assumption that the market was in long-run equilibrium in 2014, with the given mix of wind and traditional generation, in order to examine the effects of an expansion of wind. Generator are modeled as having a linear total cost curve, with the x-axis as hours of operation, with the y-intercept based on the total fixed costs and the slope based on the marginal costs of that particular generator, as shown in Figure 3-2. Clearly, in an efficient long-run equilibrium, the optimal mix of plants should be such that the plant type operating for t hours of the year should be the plant type that minimizes total fixed costs for that number of hours of operation t, i.e. if the total cost of a plant type j, that is operated for t hours in a year, is given by $C_j(t)$, than any plant operating for t hours must be of type $\arg\min_j\{C_j(t)\}$ in the theoretical equilibrium.

Figure 3-2 shows that the long-run equilibrium implies that each type of generator is operating in the range of hours in which that generator type has the lowest total cost. For example, 16.9 GW of peaker capacity is installed, operating up to 1014 hours, while any plant that operates more than 6864 hours is a baseload plant, in line with the long-run equilibrium that minimizes total cost, and 27 GW of baseload plants are installed.

Next, imagine that wind investment triples the annual production of wind energy, with no change
in demand or in the array of available options for traditional generation. Consider two extreme cases, where the new wind is perfectly correlated to the existing resource, and where the new wind is independent of the existing wind resource. As argued in the previous sections, the system should expect higher portfolio variance amongst new investments under a policy that undervalues the effect of covariance with price, which is modeled in an extreme case here where new generation is perfectly correlated with existing generation.

3.4.1 Effect on System Prices

The bottom graph of Figure 3-4 displays the effect on the net-load duration curve for the correlated and uncorrelated wind investment cases. The correlated wind investment results in a steeper net-load duration curve, so the peak demand hours have higher demand versus the uncorrelated investment case, while the lowest demand hours have lower demand versus the uncorrelated investment case.

To calculate the short-run effect on system prices, we use the load duration curves in Figure 3-4, but apply the existing equilibrium resource mix to see which resource type sets the price in various hours, with the capacities for the existing resource mix shown in Figure 3-2. Figure 3-3 displays the results for the base-case and for highly correlated and uncorrelated wind expansion, the number of hours in a year that each generator type sets the price.

As expected, wind expansion serves to reduce the price of electricity, which is clear because resources with lower marginal cost (baseload) clear the market more frequently, while resources with higher marginal cost (peaker) clear the market less frequently. This helps validate the previous
assumptions that wind serves to suppress market prices. The empirical estimates of correlation in
our model also show that the more highly correlated wind output has lower covariance with price,
as predicted by the model in Section 3.3. Using the market-clearing generators in Figure 3-3, and
assuming the market clears at their marginal cost, \( \text{Corr}(f_{hc}, \pi_{hc}) = -0.6056 \) while \( \text{Corr}(f_{uc}, \pi_{uc}) = -0.4073 \) (and similarly for covariance), where “hc” and “uc” represent the wind production and
corresponding prices in the highly correlated and uncorrelated investment cases. As expected, a
higher correlation amongst the new wind investment leads to a more negative correlation of the
output with the new prices in each period.

It is also interesting to compare the market prices in the case of highly correlated wind investment
and uncorrelated wind investment, as seen in Figure 3-3. Notably, in the case of highly correlated
wind investment we see less price suppression from the wind resource, so the same amount of wind
capacity has less price benefits for consumers of electricity.

From this analysis, two important themes emerge. First, the analysis provides additional evi-
dence to highlight the relationship between the covariance of price and wind availability and the
total portfolio variance. If wind output is highly correlated in the investment portfolio, then each
individual plant’s output is likely to be more negatively correlated with price, and the value of
its energy will be lower. Therefore, as suggested, inefficient investment mechanisms, such as those
introduced in the previous section, are likely to result in a wind portfolio mix that has an ineffi-
ciently high variance. Furthermore, the correlation amongst wind outputs affects price suppression.
Correlated wind outputs weaken the price reduction effects of wind in the medium-term. Thus,
inefficient investment mechanisms decrease the price benefits of new wind investment. While the
inefficiency is apparent from a price perspective, it also has reliability impacts that may impart
additional costs.

3.4.2 Effect on Long-Run Equilibrium

While wind energy places downward pressure on energy market prices in the short and medium-
run, a more careful analysis is required to understand the effect of wind energy on the long-run
equilibrium market prices. In particular, the long-run distribution of system prices in a theoretical
market equilibrium should be based only on the total cost curves of the traditional generators with
firm capacity [19] [28].

As explained by Sáenz et al., the implication of this is that wind-producers will drive prices
down in the short-run, and it will become clear in the medium-run that there is an over investment
in baseload plants compared to the new long-run efficient outcome. In time, however, investment
decisions will actually shift capacity towards more peaker plants and should actually return prices
towards the levels that existed in the absence of wind [28].

The reason for this is clear: in the long-run equilibrium, any plant operating for \( t \in [0, 8760] \)
hours should be the plant that minimizes the total cost curve at that particular level of operation.
For instance, if the total cost of a plant type \( j \) is given by \( C_j \), then the long-run equilibrium
Figure 3-4: Potential future net load curves under different wind investments, and calculation example of future capacity of different resources under different wind investment scenarios.

cost curve is given by $C(t) = \min_j \{C_j(t)\}$, and any plant operating for $t$ hours must be of type $\arg \min \{C_j(t)\}$ in the theoretical equilibrium.

This leads to the interesting result that wind should not substantially affect prices in a long-run equilibrium, because the technology with the lowest cost for any range of $t$ will run exactly that many hours in a year, over the very long-run when market entry and exit are taken into account. Therefore, Figure 3-4 can be used to analyze the potential capacity of different generation types under an expansion of wind in ERCOT, based on the two load duration curves for correlated and uncorrelated future wind resources. The results are shown in Figure 3-5.

As evident in Figure 3-5, the highly correlated equilibrium leads an increase in the capacity served by midrange plants, but actually decreases the capacity served by both baseload and peaker plants. This is explained by the fact that in the correlated wind investment scenario, the slope of the load duration curve is generally steeper in the middle of the graph, and less steep at the ends,
because correlated wind resources lead to more hours with a high or low level of demand. As a result, the midrange plants actually meet an increased portion of total capacity in equilibrium.

This result goes against conventional wisdom, which is that more highly correlated wind results in increased market appetite for peaker plants and less so for baseload plants. In both cases of significant wind investment the model suggests long-run suppression of base-load capacity, as expected. However, compared to the equilibrium in the uncorrelated case, more midrange capacity and less peaker capacity is present in the equilibrium for highly correlated wind investment.

In both cases of expanded wind investment, wind covers an increasing amount of demand in all hours. In the strictest interpretation of reliability, grid operators might find it necessary to have capacity available for all of that potential demand, in case no wind is available during hours of peak demand in a future year. The size of this capacity, technically necessary to cover demand in the absence of wind but unused in the current year, represented by the yellow bar in Figure 3-5. Alternatively, it provides a practical way of measuring the expected benefits wind provides as a capacity source.

As one might expect, in the uncorrelated case, the wind meets a greater level of capacity demand. However, it is difficult to compare the uncertainty in this value across different wind investment cases. Future research could use wind data across any given electricity system to estimate how variance in total portfolio output affects the tails of the distribution.

If operators accept that wind has a lower firm value as resource capacity than its contribution towards net demand in any given year, this suggests that there will be some traditional generator units demanded by the market or by regulation that actually do not operate at all in an average
year. In theory, peak scarcity pricing could work the same way for these units as other peakers, except that scarcity rents would rise even higher to reflect the lower number of operating hours in expectation in a given year.

3.5 Conclusion

This chapter examines optimal investment decisions in wind capacity, based on a simplification of analytical research on spot market pricing theory. It examined the investment frontier and considered tradeoffs between marginally better capacity factors and marginally better covariance of output and prices.

The analysis shows that decisions along this investment frontier are biased fixed price subsidies like the production tax credit (PTC), resulting in wind investment that is marginally less likely to be available in periods of high prices and/or demand. Further results show that lower covariance of a wind site’s output with price is equivalent to higher covariance with the remainder of system wind output. Therefore, subsidies like the PTC lead to investment decisions at the margin that inefficiently increase the total variance of the wind portfolio. The effects of portfolio variance on long-run market equilibriums were presented. Contrary to the typical assumption, more highly correlated wind investments actually support less peaker capacity in the long-run equilibrium versus a portfolio of wind investments with less correlation in their outputs. Future work could examine other costs or externalities of high wind portfolio variance, or estimate the magnitude of the effects described here in a real electricity system. In addition, many of the costs associated with high variability of renewable resources are not currently assigned to generators that contribute to that variability. In future research, we hope to focus on mechanism design for capacity markets that allows variable generation to receive fair remuneration for their contributions towards capacity while also efficiently taking into account reductions in value due to variability and the joint distributions of that variability with stochastic generation and demand.
Bibliography


