Individual and Systemic Risk Trade-offs Induced by Information Barriers in the Financial System

by

Georgia-Evangelia Katsargyri

Submitted to the Department of Electrical Engineering and Computer Science
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Barriers in the Financial System

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Abstract

Investment diversification is a risk management technique that allows to create balanced portfolios that achieve a certain rate of return on one’s investment, within a certain risk allowance. Despite the advantages it offers to investors, diversification has been strongly debated in the aftermath of the global financial crisis of 2007-2009, because it is believed to have potential adverse effects on systemic risk. In this thesis, we specifically investigate the adverse effects that limited information availability of investors, and the diversification choices they make due to that information, may have on the systemic risk of the financial system as a whole. Information availability here is seen as the level of awareness for each agent of the available options he can employ in order to diversify his portfolio in the given market, examined in terms of two so-called “information barriers”:

a) assets accessibility, representing private and public information offered to each investor about the available assets in the market,

b) agents diversifiability, representing the agent’s experience in processing this information in order to make better diversification decisions.

Building on an existing stylized financial system model, we enrich it by partitioning the assets and the investors according to their accessibility and diversifiability respectively.

Our contribution is threefold; we demonstrate a tradeoff between individual diversification activity and systemic risk induced by the two information barriers, we provide analytical characterization and numerical representation of the conditions under which diversification activity under limited information may amplify systemic risk and finally we observe and highlight a discrepancy that is created between actual and perceived risk for increasing level of information availability in the system.

Thesis Supervisor: Munther A. Dahleh
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To my parents, Vassilis and Vicky
I love traveling. I love adventurous journeys, where nothing goes according to the plan, where every day feels like a whole month passed, where a person I meet may become my friend for life, where my senses are at the edge. I enjoy these journeys from their beginning to their end, because they teach me new things, they flood me with unexpected feelings, they fill my moments with lifetimes.

My PhD was one such journey, with a big difference though. Instead of days that felt like months, there were months that felt like one day, the Groundhog Day... Every day was the same for a while, no significant changes, no interesting evolution, nothing unusual to remember. And this repetitive day needed patience to relive it, as well as the hope that at some point this tiny light will peek at the end of the tunnel, the one that everyone talks about. And as these days go by, without realizing it, you are building a tree, one that has a boring but sturdy trunk and multiple playful brunches. The brunches increase exponentially as you go, each one of them giving you many more of those, and you don’t know which one to follow... And all this feels overwhelming, but also invigorating, sometimes leading you to frustrating paths, other times leading you to little revelations and triumphs. Suddenly after a while, the brunches start meeting again and this light at the end of the tunnel fades in. And this is the point at which you realize that this tree created your own story, and all the Groundhog days built a strong trunk, and all the brunches had been worth exploring.

This is how I experienced my PhD story. This adventurous journey showed me how much I can endure and how much I can achieve. It made me travel through moments of numbing frustration and others of irrepressible enthusiasm and determination. It sculpted me to improve. This journey would have been very challenging if I had been alone. But I was extremely lucky to be surrounded by many special people, whose presence in my life was decisive and I would like to thank them all for that.

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Munther, I would like to thank you for the freedom that you gave me to pursue any direction I wanted and in any way I chose. Sometimes complete freedom can scare us, mainly when we don’t know how to use it. It scared me too at times... But now I appreciate how precious it was, because it helped me define what is important for me and become a more complete researcher and person. Thank you for your genuine support all these years, for having been there every time I needed to meet, even just for 5 minutes, for all the discussions about any kind of concern, research, life decisions, fears and dreams. Thank you for all the meetings that I came in frustrated and I left optimistic and motivated. Finally, thank you for teaching me that no challenge is a dead-end if I keep trying to solve it and for helping me find and trust my strengths.

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Chapter 1

Introduction

1.1 Diversification, a controversial tool

Diversification, as an effective risk management practice, has been recognized and consciously employed by humans for centuries. References of it can be found starting more than 21 centuries ago, in historical texts such as the Bible, the Talmud, or Shakespeare’s work, which all reflected peoples’ mentality at their corresponding era. The modern understanding of diversification dates back to the work of Harry Markowitz in the 1950s [38], where techniques of portfolio analysis are presented, aiming to determine the most suitable selection among a large number of securities - tradable financial assets - that best meets an investor’s needs.

Diversification is a risk management technique that allocates capital to a variety of assets in order to limit losses in the event of a fall in any single asset. It can be employed by anyone that makes any kind of investment. Already in the early AD years the Judaic law suggested a formula according to which everyone should split their assets into thirds: one third in business (buying and selling things), one third kept liquid (e.g. gold coins), and one third in land. Farmers, slightly in the past but more nowadays, perform agricultural diversification, in order to address risks related to changing consumer demands or government policy, extreme weather conditions, climate change, etc; they diversify by re-allocating some of a farm’s productive resources, such as land, capital and farm equipment to other farmers, different crops, or even non-farming activities such as restaurants and shops. In
the corporate world, diversification refers to the practice under which a firm extends into disparate fields, by entering a new industry or a market different from its core business, by developing a new product, etc. In finance, diversification is a portfolio strategy designed to reduce exposure to risk by combining a variety of investments, such as stocks, bonds and commodities, which are unlikely to all move in the same direction.

In the most simplistic way, diversification can be described by the proverb "Don’t put all your eggs in one basket". Dropping the basket will break all the eggs. Placing each egg in a different basket is more diversified. There is more risk of losing one egg, but less risk of losing all of them. In finance, an example of a non-diversified portfolio is to hold only one stock. This is risky: it is not unusual for a single stock to go down 50% in one year. It is less common for a portfolio of multiple stocks to go down that much, especially if they are selected at random and from a variety of industries. A portfolio of different kinds of investments will, on average, yield higher returns and pose a lower risk than any individual investment within the portfolio, because the positive performance of some investments will neutralize the negative performance of others. Volatility is limited by the fact that not all asset classes or industries move up and down in value at the same time or at the same rate.

Despite the aforementioned advantages it offers to investors, diversification in portfolio management has been strongly debated in the aftermath of the global financial crisis of 2007-2009. Individual risk, referring to the risk taken by an agent due to his own investment activity, and systemic risk, referring to the risk of the financial system as a whole due to the investment activities of all the agents, are both affected by agents’ unilateral efforts to diversify their investments. Sophisticated financial and hedging instruments, whose presence in the financial market has exploded the last few decades, facilitate portfolio diversification and individual risk management, but they have recently been criticized for potentially causing adverse effects on systemic risk, due to dangerously correlated investments [11].
1.2 Individual versus systemic risk and related literature

Individual and systemic risk management have been actively studied in the recent years.

1.2.1 Individual Risk

*Individual risk*, refers to the risk taken by an investor due to his own investment activity, and normally it can be substantially mitigated or eliminated from a portfolio by using adequate diversification. Individual risk management has been extensively studied in different contexts.

Several papers examine stock trading models, with some employing genetic network programming, such as [16, 35, 44, 52, 55], and other feedback-control methods, such as [6, 45, 36].

Extensive work focuses on portfolio management and hedging techniques, both in an intellectual ([50, 24, 47, 49, 53, 59, 46]) and a commercial context [58].

Finally, some researchers have investigated how factors such as demographics, experience level and personality may affect investors’ individual psychology and risk tolerance ([15, 26, 27, 39]).

By the variety of the aforementioned techniques and models, it is obvious that there is no generic diversification method that will meet the needs of every investor. Regardless of whether an investor is risky or conservative, there is a common driving factor for diversification; to create "balanced" portfolios, i.e. to design a balance of assets that creates a specific risk-return ratio, offering the opportunity to achieve a certain rate of return on one’s investment, in exchange for one’s willingness to accept a certain amount of risk. However, apart from this common framework, one’s personal time horizon, risk tolerance, investment goals, financial means and level of investment experience will play a large role in dictating the corresponding investment mix.
1.2.2 Systemic Risk

*Systemic risk* refers to the risk inherent to the financial system as a whole, due to individual activities of all the investors. There is no universal definition or formula that describes it, but it is generally perceived as a metric of the degree of negative consequences that can be caused to the larger body of the financial system due to individual investment actions. Depending on the chosen context or model, systemic risk may be chosen to represent the risk of big total loss in the entire financial system, the probability of multiple simultaneous breakdowns or losses common to multiple parts or entities of the system, or the possibility that a local event could spread like a domino effect and trigger severe instability or collapse of the system as a whole.

Financial systemic risk was mostly an unknown concept and sparsely addressed before the financial crisis of 2007-2009, so that only a few raised questions and previewed adverse consequences that its existence could potentially ignite. In his 1995 analysis [18], P. Davis seeks patterns common to several countries economies, in order to examine the relationship between the underlying financial structure in terms of borrowing and vulnerability to default, as well as to describe a widespread disorder in the financial sector which he calls systemic risk. [48, 4, 20] study propagation mechanisms of a local economic distress in a wide part of the financial system, with Allen and Gale though never mentioning the term "systemic risk" [4]. Finally, in an independent review in 2003, Kaufman and Scott discuss the alternative definitions and possible causes of systemic risk and attempt to evaluate existing bank regulations aiming to address it [33].

The Great Recession provides a prime example of systemic risk. Anyone who was invested in the market in 2008 saw the values of their investments change because of this market-wide economic event, regardless of what types of securities they held. The Great Recession affected many asset classes and many kinds of investors.

In the aftermath of the crisis, an explosion of discussions is noticed, both in the press ([9, 12, 1]) and in an academic context. The speech of Ben Bernanke, Chairman of the Fed in 2009, raised awareness of the systemic risk and its still vague connections with the global crisis, as well as stressed the imperative need to consciously reform the regulatory policies...
towards systemic risk mitigation [9]. Following the lead of Bernanke, many researchers, economists and policy makers focused their efforts on discovering the underlying systemic mechanisms that led to the subprime mortgage crisis, attempted to rigorously model systemic risk and its linkage to financial bubble formation ([13, 28, 29, 12, 19]) and looked into relevant regulatory policies ([3, 8, 40]).

1.2.3 Individual versus Systemic Risk and Diversification Hazards

After the establishment of the concept of systemic risk in the financial and academic community, great interest started developing in the relationship between individual and systemic risk, that is in how systemic risk is affected by individual activities of investors, whose goal is to keep their risk exposure within their personal risk allowance, while maintaining a desirable return rate on their investment.

Lots of work has focused on financial contagion, which refers to the risks imposed by interlinkages and interdependencies in a financial system, where the failure of a single entity or cluster of entities can cause a cascading failure, which could potentially bankrupt the entire system. Back in 1998, Franklin Allen and Douglas Gale model a process through which shocks can ignite bank runs [4]. The failing of financial firms in 2008, which were either "Too Big To Fail" or "Too Interconnected to Fail", caused a spillover of the downturn in the larger economy. The domino effect has since been extensively investigated as a consequence of interconnections among investors due to interbank lending [2, 34, 41, 48], risk sharing [23, 7] and leverage [25, 54, 32, 14].

On the other hand, several researchers have looked into how individual diversification activity may have adverse effects on systemic risk due to dangerously correlated investments, that either increase the probability of multiple simultaneous breakdowns or losses in multiple parts or entities of the system ([57, 30, 10]), or destabilize asset prices by increasing their volatility ([43, 11, 17, 56]), two factors that both drive up the risk of a big total loss in the financial system as a whole.
1.2.4 Diversification under Limited Information

The history of speculative bubbles begins roughly with the advent of newspapers. One can assume that, although the record of these early newspapers is mostly lost, they regularly reported on the first bubble of any consequence, the Dutch tulipmania of the 1630s. Although the news media—newspapers, magazines, and broadcast media, along with their new outlets on the Internet, present themselves as detached observers of market events, they are themselves an integral part of these events. Significant market events generally occur only if there is similar thinking among large groups of people, and the news media are essential vehicles for the spread of ideas [51].


Robert Shiller, Nobel Prize-winning Yale University professor, commented in his "Irrational Exuberance" on how information spread by the media to large groups of people may be related to significant market events.

How does information availability really affect investors’ actions and in turn the entire financial system? So far, researchers have mostly focused on how information asymmetry among investors, information precision and reporting frequency affect the cost of capital ([22, 5, 31]). On the other hand, Morris and Shin investigated how enhanced dissemination of public information through the media and disclosures by market participants with high public visibility affect the social welfare, in a setting where agents have a coordination motive arising from a strategic complementarity in their actions [42]. According to their work, when the agents have no socially valuable private information, greater provision of public information always increases welfare, while, when agents also have access to independent sources of information, the welfare effect of increased public disclosures is ambiguous, especially when the private information is of high precision, compared to the public signal. They show that the agents "overreact" to the public signal while suppressing the information content of the private signal, so that the public signal noise is given more of an impact in the agents’ decisions than it deserves.

In this work we investigate how information availability of investors and the diversifica-
tion choices they make due to that information, and independently of the others, may have adverse effects on the systemic risk of the financial system as a whole. Information availability here is seen as the level of knowledge of each agent about the available options he can employ in order to diversify his portfolio in the given market.

1.3 Contribution

In this thesis we propose a novel model, based on an existing stylized model adopted from [14], in order to study how independent diversification activities of agents with limited information may affect systemic risk. The novelty of the model is in the introduction of two diversification barriers, that is assets accessibility and agents diversifiability, which constrain investment activity in the system. We enrich Caccioli’s model in [14] with these two realistic features, by partitioning assets according to their accessibility level and investors according to their diversifiability level. Both of these barriers are two different aspects of the investors’ information availability, with assets accessibility representing the investor’s awareness of the existing options they can employ to diversify, and agents diversifiability representing the investors’ experience in processing information that will allow him to make better diversification decisions and expand his investment activity. Information availability here thus includes both private and public information offered to each investor, as well as his experience in analysing this information in order to make better diversification decisions.

Our main contribution is the conclusion that the existence of these two information barriers causes a tradeoff between individual diversification and systemic risk, and the demonstration of conditions under which diversification activity under limited information magnifies systemic risk.

1.4 Structure of the thesis

The rest of the thesis is organized as follows:

- In Chapter 2 we introduce our model and formulate our problem.
• In Chapter 3 we attempt a theoretical approach to our problem, introducing some simplifying assumptions to our model.

• In Chapter 4 we develop a simulation model and we employ it to examine the sensitivity of our theoretical results to the simplifying assumptions.

• Finally, in Chapter 5 we present the conclusions and we summarize the contributions of the thesis.
Chapter 2

Model and Problem

In this chapter we introduce our model and formulate our problem, after presenting some concepts and quantities that will be used throughout the thesis, as well as we offer a motivating example that will give as some intuition about the cornerstone of this thesis.

2.1 Definitions

In this section we define some concepts that will be widely used throughout the thesis:

Asset: A resource with economic value that someone invests in, with the expectation that it will provide future benefit. The value of an asset is a random variable with an expected return and a risk that may negatively affect the wealth of the asset owner.

Agent: An entity that invests its capital in assets accessible to it for diversification, i.e. an investor. The goal of the agent is to choose the allocation of his capital on different assets in such a way that his expected return is maximized, while his investment risk remains below some tolerance level.

Financial System: A system that consists of a set of assets and a set of agents investing in those assets.

Financial Market: The set of the total of assets in the financial system.
**Risk:** The variance of a risky investment return.

**Idiosyncratic Risk:** Risk specific to an asset.

**Individual Risk:** Risk specific to an agent.

**Systemic Risk:** The risk inherent to the financial system as a whole, given the individual investments of all the agents on all the assets of the financial market.

### 2.2 Information Barriers

As already mentioned, in this work we investigate how information availability of investors and the diversification choices they make due to that information, independently of each other, may have adverse effects on the systemic risk of the financial system as a whole. Information availability here is seen as the level of awareness for each agent of the available options he can employ in order to diversify his portfolio in the given market. More specifically, this agent awareness is examined in terms of two aspects of information availability:

a) private and public information offered to each investor about the availability and accessibility of different assets in the market,

b) his experience in analysing this information in order to make educated diversification decisions.

In this work we introduce two so-called information barriers, that exactly represent the aforementioned aspects of information availability and which constrain investment activity in the system:

1) **Accessibility:** The information barrier that represents the investor’s awareness of the existing options he can employ to diversify.

2) **Diversifiability:** The information barrier that represents the investor’s experience or capability of processing information that will allow him to make better diversification decisions and expand his investment activity.
2.2.1 Assets Partition

Given the two information barriers, we divide the assets into two categories, according to their accessibility:

1) **Local**: Assets that are accessible only to agents that are naturally cognizant of them due to proximity or expertise.

These could include assets that are not accessible to every agent due to technical reasons, for example mortgage loans are not accessible to hedge funds but are accessible to commercial banks, as well as assets that are not attractive to every investor, for example agricultural products are not popular in the majority of investors, or a house in Miami would likely not be attractive to someone who lives in Los Angeles - both cities offer similar quality of life, having comparable weather and location characteristics, and are located inconveniently far from each other.

2) **Global**: Assets that are widely accessible in the market.

These could include assets that are literally accessible to any agent that is willing to expand into new investment areas without necessarily being specialized in them, e.g. stock market, as well as assets that are quite popular at a particular point in time due to word of mouth, promising public information, high return expectation, coordination incentives, etc, for instance Apple, Google and Tesla stocks in 2016, or mortgage-backed securities before the subprime mortgage crisis.

Apart from natural trends in the market, such as investors herding around up-and-coming industries - e.g. electric vehicles technology - the existence or emergence of global assets are often a result of enhanced dissemination of public information through the media and disclosures by market participants with high public visibility, or a result of particular financial instruments issued to make these assets more marketable to the investors. Such instruments may include options, collateralized debt obligations, mortgage-backed securities, hedging instruments, etc.
2.2.2 Agents Partition

Given the two information barriers, we divide the agents into two categories, according to their diversifiability:

1) **Small**: Agents that restrict their investment activity only to assets they are naturally cognizant of due to proximity, expertise, etc.

These include agents that are quite inexperienced and perform what Massa and Simonov call in [39] "familiarity-based investments", making decisions according to profession, geography, demographics, etc. They are not capable of performing efficient risk assessment and they prefer to restrict their diversification activity within assets and areas they are traditionally tied to, due to proximity, expertise, etc. Examples of such agents could be private investors, specialized banks, real estate agents, industry investors, households, etc.

2) **Institutional**: Agents that are willing to expand into new assets and investment areas without necessarily being specialized in them.

These include the most knowledgable and experienced investors, who have at their disposal the most efficient risk assessment tools and are able to make the most educated investments, being, as such, willing to use all the assets available to them in order to diversify as much as possible. Examples of such investors could be pension funds, mutual funds, money managers, insurance companies, investment banks, commercial trusts, endowment funds, hedge funds, etc.

The institutional agents represent the experienced investors that are willing to take new risks and enter new markets, while at the same time are acquainted with the options, methods and benefits of diversification. The small agents represent the amateur or specialized investors that choose to remain in their expertise field or comfort zone of their familiar assets.
2.3 Financial System Model

In this section we introduce our financial system model.

2.3.1 Notation

We consider a financial system with the following given characteristics:

- \( m \) assets, with expected return \( U_j \), idiosyncratic risk (or variance) \( S_j \) and covariance between two different assets \( j_1, j_2 \) equal to \( \text{cov}(j_1, j_2) \), for \( j, j_1, j_2 \in 1, \ldots, m \). Out of the \( m \) assets:
  - \( g \) are global and belong to the set \( G : |G| = g \)
  - \( m - g \) are local and belong to the set \( \overline{G} : |\overline{G}| = m - g \)

- \( n \) agents, with available capital \( k_i \) and risk tolerance \( t_i \), for \( i \in 1, \ldots, n \). Out of them:
  - \( b \) are institutional and belong to the set \( B : |B| = b \).
  - \( n - b \) are small and belong to the set \( \overline{B} : |\overline{B}| = n - b \).

- For each agent \( i \), let \( A_i \) be the set of all assets accessible to \( i \), with \( A_i \neq \emptyset \), \( \forall i \leq n \).

- For each agent-asset pair \((i, j)\), let \( a_{ij} \leq k_i \) be the capital allocation, i.e. the amount that \( i \) invests in \( j \).

- For each asset \( j \), let \( a_j \) be the total capital allocation, i.e. the total amount invested by all agents in asset \( j \), for \( j \in 1, \ldots, n \). By definition, \( a_j = \sum_{i=1}^{m} a_{ij} \).

2.3.2 Single Agent Optimization

As we discussed in Sec.1.1, an investor can reduce portfolio risk by diversifying, that is by holding a balanced combination of instruments that creates a specific risk-return ratio. Modern Portfolio Theory (MPT), or mean-variance analysis, is a mathematical framework for assembling such a portfolio in a way that the expected return is maximized for a given level of risk defined as variance [21, 37]. MPT was introduced by economist Harry
Markowitz in a 1952 essay [37], for which he was later awarded a Nobel Prize in economics. Its key insight is that an asset’s risk and return should not be assessed by themselves, but by how they both contribute to a portfolio’s overall risk and return.

MPT assumes that investors are risk averse, meaning that given two portfolios that offer the same expected return, investors will prefer the less risky one. Thus, a rational investor will not invest in a portfolio if a second portfolio exists with a more favorable risk-expected return profile, i.e. one will take on increased risk only if compensated by higher expected returns, while one who wants higher expected returns must accept more risk. In that sense, diversification may allow for the same portfolio expected return with reduced risk, or for a maximum expected return based on a given level of risk. In other words, it offers the opportunity to achieve a certain rate of return on one’s investment, in exchange for one’s willingness to accept a certain amount of risk, emphasizing that risk is an inherent part of higher reward.

Apart from the above common framework, the tradeoff between expected return and risk is evaluated differently by different investors, based on individual risk aversion characteristics, including risk tolerance, investment goals, financial means and level of investment experience.

Mathematical risk measurements are useful only to the degree that they reflect investors’ true concerns - there is no point minimizing a variable that nobody cares about in practice. MPT uses the mathematical concept of variance to quantify risk, which can be justified under the assumption of normally distributed returns. I should be noted that variance is a symmetric measure that counts abnormally high returns just as risky as abnormally low returns. Some would argue that, in reality, investors are only concerned about losses and do not care about the dispersion or tightness of above-average returns. However, due to the distribution symmetry, it is a good metric to evaluate the risk inherent to an investment.

In our model we employ the MPT framework, assuming that each agent $i$ solves the following optimization problem in order to choose the allocation of his capital on the different assets he invests in:
\[
\max_{a_{ij}} \sum_{j \in A_i} a_{ij} U_j
\]  

(2.1)

s.t.

\[
\sum_{j=1}^{m} a_{ij} \leq k_i
\]  

(2.1a)

\[
\sum_{j=1}^{m} a_{ij}^2 S_j + 2 \sum_{j=1}^{m} \sum_{r \neq j} a_{ij} a_{ir} \text{cov}(j, r) \leq t_i
\]  

(2.1b)

where the cost function in Eq. (2.1) represents the expected return of the agent and constraints (2.1a), (2.1b) represent the capital and risk tolerance constraints respectively, according to each agent’s characteristic, as described in Sec. 2.3.1.

### 2.3.3 Systemic Risk

Systemic risk, as seen earlier, is the risk of the financial system as a whole, given the individual investments \( a_{ij} \) of all the agents in all the assets of the financial market. As such, it is a function of the individual capital allocations \( a_{ij}, \forall i \in 1, \ldots, n, j \in 1, \ldots, m. \) In order to use consistent risk metrics at an individual and systemic level, we defined systemic risk as the variance of the entire financial system, given by the following formula:

\[
R(a_{ij}) = \text{var}\left(\sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij} U_j\right) \Rightarrow
\]

\[
R(a_{ij}) = \sum_{j=1}^{m} (\sum_{i=1}^{n} a_{ij})^2 S_j + 2 \sum_{j=1}^{m} \sum_{r \neq j} a_{ij} a_{ir} \text{cov}(j, r)
\]  

(2.2)

As we discussed in Sec. 1.2.2, systemic risk does not have an exact definition. In this work we use the variance of all the assets of the system as a systemic risk metric. Although, as we described in Sec. 2.3.2, variance penalizes for abnormally high returns just as much as abnormally low returns while, in reality, investors are only concerned about the latter,
considering normally distributed asset returns and their symmetry, variance does capture
the risk of very low returns of the system's total wealth, while it is also consistent with the
MPT risk metric.

**Systemic Risk versus Aggregate Risk:** Systemic risk should be distinguished from aggregate risk in the system. Systemic risk refers to the risk created by the interactions and interconnections of the agents that may lead to a collapse in the entire system, whereas aggregate risk represents the total of the risk perceived by the agents who are ignorant of the relationships among them and the consequences these may have on the system as a whole. Consequently, systemic risk represents the actual risk of the system and aggregate risk represents the perceived risk in the system. For the same aggregate (perceived) risk we may have system structures with higher systemic (actual) risk, due to dangerously correlated investments.

### 2.3.4 Graphical Representation

Based on the model described in [14], we represent our system using a bipartite graph, with agents and assets being the two disjoint sets of nodes, and capital allocations being the graph edges. Fig. 2-1 depicts one such financial system representation.

![Figure 2-1: Financial system graphical representation](image)

As we mentioned in Sec. 2.3.3, for the same aggregate risk we may have system structures with higher systemic risk, due to dangerously correlated investments. Fig. 2-2 gives the graphical representation of two systems with equal aggregate but different systemic risks.
Assuming assets of IID returns, the perceived risk by each agent in both cases is the idiosyncratic risk of the asset he invests in, so the aggregate perceived risk is equal to the sum of the perceived risks of all the agents. However, the systemic risk is lower in graph A, because the total system investment is perfectly diversified over all the available assets, while in graph B all agents invest their capital in one and only common asset, rendering their investments fully correlated.

![Diagram](image)

Figure 2-2: Financial systems with equal aggregate but different systemic risk

As discussed in Sec. 2.2, in this work we study how systemic risk may be adversely affected by the two information barriers, assets accessibility and agents diversifiability. In order to perform our analysis, we enrich the financial system graph with partitions of the nodes according to the two barriers, introducing thus small and institutional agents as well as local and global assets. Fig. 2-3 depicts one such financial system representation.

We notice that:

- Being able to make educated investments and diversify as much as possible by expanding into new assets, institutional agents opt to invest in all the accessible assets of the system, including all global ones, as well as any local assets they may be traditionally tied to.

- To the contrary, small agents restrict their investment activity by remaining in the comfort zone of only assets they are naturally cognizant of, which may however be
either global or local.

From a different point of view, we may use the degree of the nodes in order to divide them in the different categories. So, based on the degree:

1) Local are the assets with degree $= 1$ and global the ones with degree $> 1$.
2) Small are the agents with degree $= 1$ and institutional the ones with degree $> 1$.

### 2.4 Problem Formulation

In this work we investigate how information availability of investors and the diversification choices they make due to that information, independently of each other, may have adverse effects on the systemic risk of the financial system as a whole. As explained in Sec. 2.2, information availability is examined in terms of the two information barriers, i.e. accessibility and diversifiability, which represent the information offered to each investor about the available assets in the market, as well as his experience in analysing this information in order to make better diversification decisions. Consequently, our goal is to study how systemic risk performs with respect to these two information barriers.

We consider a financial system like the one described in Sec. 2.3 with the following characteristics:

- $B : |B| = b$, the set of institutional agents
- $G : |G| = g$, the set of global assets

- $A^{n \times m}$: matrix with elements $a_{ij}$, i.e. the capital allocation of agent $i$ to asset $j$. The non-negative values of the rows $\overline{A}_i$ indicate the set $A_i$ of all assets accessible to agent $i$.

Fig.2-4 depicts such a system.

![Figure 2-4: Financial system graphical representation.](image)

We define:

- $X \sim B, G, A$: the state of the system;

  the state of the system is a function of the set of institutional agents $B$, the set of global assets $G$ and all the capital allocations $A$ by all the agents to all the assets.

- $U \sim (\text{global} \leftrightarrow \text{local}) / (\text{small} \leftrightarrow \text{institutional})$ changes: the input of the system;

  the input of the system is a single change of status of one node in the system, i.e. the switch of an asset from local to global or vice versa, or the switch of an agent from small to institutional or vice versa.
\* \( R \sim X, U \): the output of the system;

systemic risk is the output of the system and it is a function of its state \( X \) as well as its input \( U \), i.e. a function of the status of all the nodes in the system and of any change of status that happened to one of the nodes.

Our goal is to analyze how systemic risk performs with respect to the two information barriers, that is how the output \( R \) of the system performs:

1. with respect to the given state \( X \) of the system, in terms of capital allocations from the agents to the assets, and

2. after the occurrence of an incremental change in the system, i.e. a change in the status of a single node, either an agent or an asset.

In other words we aim to study how incremental changes in information availability leading to changes in the graph structure of the financial system are reflected in systemic risk.

### 2.5 Motivating example

In this section we employ a motivating example in order to start building some intuition, about how systemic risk performs with respect to agents information availability in the system. As discussed in Sec. 2.2, information availability is seen as the level of information for each agent of the available options he can employ, both in terms of awareness and in terms of experience, in order to diversify his portfolio in the given market.

For our motivating example, we consider the extreme case of a financial system, where each agent may have either full or minimum information about the available assets in the market:

*Full information* refers to the agent having at his disposal information about all the assets of the system.
Minimum information refers to the agent having at his disposal information only about his single traditional asset.

Let each agent have full information with probability $p$ and in turn minimum information with probability $1 - p$, where $p$ is homogeneous across agents. For simplicity, we consider a financial system consisting of two agents and two assets, as the one depicted in Fig. 2-5. According to the definition of $p$, each one of the gray colored edges in the graph of Fig. 2-5 exists with probability $p$, while the black ones exist always, i.e. with probability 1.

![Figure 2-5: Full information availability with probability $p$.](image)

We assume that both agents and assets are homogeneous, in the sense that agents have the same capital and risk constraints, while assets have the same expected return and variance. We also assume IID assets returns. Given that each agent solves the optimization problem of Sec. 2.3.2, his decision will be to divide equally his capital among all his available assets, because he has no incentive to invest more or less in any of them given their homogeneity.

It is of great insight value to compare how aggregate and systemic risk perform with respect to probability $p$. As discussed in Sec. 2.3.3, systemic risk refers to the risk created by the interactions and interconnections of the agents that may lead to a collapse in the entire system, whereas aggregate risk represents the total of the risk perceived by the agents who are ignorant of the relationships among them and the consequences these may have on the system as a whole. Diversification activity aims to decrease the individual risk of each agent and, thus, a higher value of $p$ should decrease the perceived risk of the system, i.e.
the aggregate risk. However it is not clear how $p$ would affect the systemic risk. If systemic risk does not decrease or, worse, increases with $p$, then the perceived risk, as a risk metric, omits a significant part of the risk of the system that is not noticed by the investors, but is existential and may trigger a systemic crisis much easier than what the investors believe.

Without loss of generality, we assume that $k_i = 1, \forall i = 1, 2$, where $k_i$ is the total capital invested by agent $i$ in the market. We denote by $S$ the return variance of each asset.

Figure 2-6: Full versus minimum information availability.

### 2.5.1 Systemic (Actual) Risk

According to the Law of Total Expectation, the expected systemic risk $\tilde{R}_s$, as a function of probability $p$, can be computed as follows:

$$
\tilde{R}_s(p) = P(M)E[\tilde{R}_s|M] + P(F)E[\tilde{R}_s|F] + P(H)E[\tilde{R}_s|H]
$$

where:

$M$: The event of both agents having minimum information,
$F$: The event of both agents having full information,
$H$: The event of one agent having minimum and the other one full information.

Thus,

$$\tilde{R}_s(p) = (1 - p)^2(1^2 + 1^2)S + p^2(1^2 + 1^2)S + 2p(1 - p)\left[\left(1 + \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right]S$$

$$= -S(p - 2)(p + 1)$$

### 2.5.2 Aggregate (Perceived) Risk

The expected aggregate risk $\tilde{R}_a$, as a function of probability $p$, is the sum of the risks perceived by each one of the two agents:

$$\tilde{R}_a(p) = 2\left[P(M_a)E[\tilde{R}_s|M_a] + P(F_a)E[\tilde{R}_s|F_a]\right]$$

where:

$M_a$: The event of an agent having minimum information,
$F_a$: The event of an agent having full information.

Thus,

$$\tilde{R}_a(p) = 2\left[(1 - p)S + p\left(\frac{1}{2^2} + \frac{1}{2^2}\right)S\right]$$

$$= -S(p - 2)S$$
2.5.3 Systemic versus Aggregate Risk

Fig. 2-7 depicts on a common graph the two expected risks, systemic and aggregate, as functions of the full information probability $p$. We notice that although the perceived risk decreases with increasing information, the actual risk is a concave function of it, so the distance between these two risks is increasing.

\[
\tilde{R}_s = -(p - 2)(p + 1) \\
\tilde{R}_a = -(p - 2)S
\]

Figure 2-7: Systemic risk versus aggregate risk.

In order to better understand the relationship between actual and perceived risk in the system we also compute the ratio of the two:

\[
\frac{\tilde{R}_s}{\tilde{R}_a} = \frac{-S(p - 2)(p + 1)}{-S(p - 2)} = p + 1
\]

We plot the ratio in Fig. 2-8. We notice that it is an increasing function of the full-information probability $p$. Intuitively, this means that the more information is given to the agents about their available diversification options, the bigger the difference between...
the perceived by the agents and the actual risk in the system is. The more knowledge the agents have, the more they can diversify and decrease their individual risk, without though driving the actual risk of the system to lower levels as well. In fact, while perceived risk decreases as more information is offered in the system, the actual systemic risk increases for \( p < \frac{1}{2} \). This discrepancy between the perceived and actual risk in the system is very important because it shows how ignorance of agents about the actual risk may lead to actions that are dangerous for the system as a whole.

2.6 Results Preview

In this thesis we study how independent diversification activities of agents with limited information may affect systemic risk. The main contributions of this work are stated next:

1. We show the existence of a tradeoff between individual diversification activity and systemic risk caused by the two information barriers.
2. We provide analytical characterization of phase transition thresholds for these barriers, which allow us to determine under which conditions diversification activity may amplify systemic risk.

3. We demonstrated a discrepancy between aggregate and systemic risk.

4. We develop and employ a simulation model to investigate the sensitivity of our risk performance results to the theoretical assumptions.
Chapter 3

Individual and Systemic Risk tradeoff induced by Information Barriers - An Initial Approach

As discussed in chapter 2, our goal in this work is to analyze how systemic risk performs with respect to the level of information that agents have at their disposal about the existing diversification options they can employ to build balanced portfolios. This level of information availability, represented by the two information barriers, accessibility and diversifiability, determines the capital allocations and the structure of the financial system graph, which in turn affects systemic risk.

In this chapter we attempt an initial approach of the problem by considering some assumptions that simplify the financial system and render the analysis easier to pursue.
3.1 Initial approach

3.1.1 Assumptions

We consider a financial system like the one described in Sec. 2.3, but adding the following assumptions:

**Assumption 1 (Symmetric System):** $m=n$

**Assumption 2 (Assets):** The assets are independent and identically distributed, i.e. $U_j = U$, $S_j = S$, $\forall j \in 1, \ldots, m$ and $\text{cov}(r, l) = 0$, $\forall r \neq l$.

**Assumption 3 (Agents):** The agents have identical risk and capital characteristics, i.e. $k_i = k$, $t_i = t$, $S \leq \frac{t}{k^2} |A_i|$, $\forall i \in 1, \ldots, n$. The inequality allows the agents to invest the total of their capital in the market.

**Assumption 4a (Local assets):** Each local asset receives investment from exactly one agent, i.e. if $j \in A_i$ for some $i \in 1, \ldots, n$, then $j \notin A_{i'}$, $\forall i' \neq i$, $\forall j \in \overline{G}$.

**Assumption 4b (Small agents):** No two small agents invest in the same asset i.e. $A_p \cap A_q = \emptyset$ if $p, q \in \overline{B}$.

The motivation behind the last assumption is aligned with our goal to demonstrate a trade-off caused purely by the diversification activity, due to the existence of global assets and institutional agents, and not by inherent investment correlations in the system.

3.1.2 Single Agent Optimization

According to the above assumptions, the *individual portfolio optimization* for each agent $i$ becomes:

$$\max_{a_{ij}} U \sum_{j \in A_i} a_{ij} \quad (3.1)$$
Solving this optimization problem, we are easily led to the following lemma:

**Lemma 1** *The optimal allocation of agent* \( i \) *is* \( a_{ij} = \frac{k}{|\mathcal{A}_i|} \) *for every asset* \( j \) *accessible to him, i.e. \( \forall j \in \mathcal{A}_i \).*

**Proof:** The Lagrangian of problem (3.1) is:

\[
f(a_{ij}, \lambda_i, \mu_i) = U \sum_{j \in \mathcal{A}_i} a_{ij} - \lambda_i \left( \sum_{j \in \mathcal{A}_i} a_{ij} - k \right) - \mu_i \left( \sum_{j \in \mathcal{A}_i} a_{ij}^2 - \frac{t}{S} \right)
\]

Taking the derivative with respect to \( a_{ij} \) equal to 0 we get:

\[
\frac{\partial f(a_{ij}, \lambda_i, \mu_i)}{\partial a_{ij}} = U - \lambda_i - 2\mu_i a_{ij} = 0 \Rightarrow a_{ij} = \frac{U - \lambda_i}{2\mu_i} = a_i, \forall j \in 1, \ldots, m
\]

Plugging \( a_i \) in constraint (3a) we compute the optimal allocations:

\[
\sum_{j \in \mathcal{A}_i} a_{ij} = a_i |\mathcal{A}_i| = k \Rightarrow a_i = \frac{k}{|\mathcal{A}_i|}
\]

which gives as the Lemma. Q.E.D.

### 3.1.3 Systemic Risk

According to the above assumptions, the *systemic risk* of the financial system becomes:

\[
R(a_{ij}) = S \sum_{j=1}^{n} \left( \sum_{i=1}^{n} a_{ij} \right)^2
\]
3.2 Examined Structures

This chapter studies how systemic risk performs with respect to information availability in a simplified system, like the one described in Sec. 3.1. More specifically, and as mentioned in Sec. 2.4, our goal is to study how incremental changes in information availability and in turn in the graph structure of the financial system are reflected in systemic risk. Towards this goal, two extreme and one more general type of graph structures are considered. The three structure types under investigation are presented next.

3.2.1 Extreme Structures

1) G-complete system

We define a system in which all assets are global, i.e. \( g = n \), as a G-complete system (Fig.3-1). In a G-complete system, all institutional agents invest in all the assets of the system.

![Figure 3-1: G-complete system: \( g = n \)](image)

Our goal is to study how systemic risk performs with respect to incremental changes in the number of institutional agents in the system (see Fig.3-2).
2) *B-complete system*

We define a system in which all agents are institutional, i.e. $b = n$, as a *B-complete system* (Fig.3-3). In a *B-complete* system, all global assets accept investments from all the agents of the system.

Our goal is to study how systemic risk performs with respect to incremental changes in the number of global assets in the system (see Fig.3-4).
3.2.2 Richer Structures

General system

Given the complex nature of graph structures, we will only focus on a subset of more general structures, which we generate according to the following steps (see Fig. 3-5):

**Step 1:** First, we create 1-1 connections between agents and assets, i.e. we connect each agent exclusively with exactly one asset, so no two assets can accept investment from the same agent and no two agents can invest in the same asset.

**Step 2:** Second, we introduce in the network the two information barriers, according to the
number of institutional agents and global assets that a given system has.

**Step 3:** Finally, we add the links that represent connections between all the institutional agents and all the global assets.

A typical structure built according to the above steps is shown in Fig. 3-6 and we will call it *general system*, for given $g$, $b$, and $n$.

As mentioned earlier, our goal is to study how systemic risk performs with respect to incremental changes either in the number of global assets or in the number of institutional agents in the system (see Fig.3-7).

**Figure 3-6:** General Financial System

**Figure 3-7:** General Financial System: incremental changes in the number of a) the institutional agents and b) the global assets.


3.3 Systemic Risk Performance

3.3.1 Graph Partition

In order to compute the systemic risk as defined in Eq. 3.2, we first need to calculate the capital allocations $a_{ij}$ from every agent $i$ to every asset $j$. In order to do that, we partition the characteristics of the graph according to the two information barriers, accessibility and diversifiability. More specifically, we consider the following partitions:

**Assets Partition**

We partition the assets based on their adjacent agents:
1) $G_B = \{ j : j \in G, i \in B \forall i \text{ s.t. } a_{ij} \geq 0 \}$: these assets receive investments from institutional agents only.
2) $G_{\bar{B}} = \{ j : j \in G, i \in \bar{B} \text{ for } i \text{ s.t. } j \in A_i \}$: these assets receive investments from all institutional agents and from one small agent.
3) $\bar{G}_B = \{ j : j \in G, i \in B \text{ for } i \text{s.t. } j \in A_i \}$: the only investor of these assets is an institutional agent.
4) $\bar{G}_{\bar{B}} = \{ j : j \in G, i \in \bar{B} \text{ for } i \text{s.t. } j \in A_i \}$: the only investor of these assets is a small agent.

**Assets Partition**

We partition the agents based on their adjacent assets.
1) $B_G = \{ i : i \in B, j \in G \forall j \text{ s.t. } a_{ij} \geq 0 \}$: these agents invest in the global assets only.
2) $B_{\bar{G}} = \{ i : i \in B, j \in \bar{G} \text{ for } j \in A_i \}$: these agents invest in all global assets and in one local asset.
3) $\bar{B}_G = \{ i : i \in \bar{B}, j \in G \text{ for } j \in A_i \}$: the only investment of these agents is in one global asset.
4) $\bar{B}_{\bar{G}} = \{ i : i \in \bar{B}, j \in \bar{G} \text{ for } j \in A_i \}$: the only investment of these agents is in one local asset.
Edges Partition

We partition the edges based on their adjacent nodes (see Fig. 3-8).

1) Type $\overline{B} = \{(i, j) : i \in \overline{B}\}$: these edges contain a small agent.

2) Type $B_{\overline{G}} = \{(i, j) : i \in B_{\overline{G}}\}$: these edges connect an institutional agent with a local asset.

3) Type $B_G = \{(i, j) : i \in B_G\}$: these edges connect an institutional agent with a global asset.

![Figure 3-8: Edge types](image)

Using Lemma 1 and the above edge partition, we can now calculate the capital allocations $a_{ij}$ from every agent $i$ to every asset $j$:

1) Type $\overline{B}$ edges $\rightarrow a_{ij} = a_{\overline{B}} = k$

2) Type $B_{\overline{G}}$ edges $\rightarrow a_{ij} = a_{B_{\overline{G}}} = \frac{k}{g+1}$

3) Type $B_G$ edges $\rightarrow a_{ij} = a_{B_G} = \frac{k}{g}$

Let $T_{As}$ represent the type of asset and $T_{Ag}$ the type of agent according to the above partitions. Then, the systemic risk can be written as follows:

$$R(a_{ij}) = S \sum_{T_{As}} \sum_{j \in T_{As}} \left( \sum_{T_{Ag}} \sum_{i \in T_{Ag}} a_{ij} \right)^2$$

(3.3)
3.3.2 Theorems

In this section we present the systemic risk performance with respect to incremental changes in the status of nodes in the system - agents and assets. As defined in Sec. 2.3, and according to the assumptions of Sec. 3.1.1, every financial system we examine has the following characteristics:

- \( m \) assets, with expected return \( U \), idiosyncratic risk \( S \) and covariance between two different assets \( j_1, j_2 \) equal to 0, for \( j, j_1, j_2 \in 1, \ldots, m \). Out of the \( m \) assets:
  - \( g \) are global and belong to the set \( G : |G| = g \)
  - \( m - g \) are local and belong to the set \( G^c : |G^c| = m - g \)

- \( n \) agents, with available capital \( k \) and risk tolerance \( t \), for \( i \in 1, \ldots, n \). Out of them:
  - \( b \) are institutional and belong to the set \( B : |B| = b \).
  - \( n - b \) are small and belong to the set \( B^c : |B^c| = n - b \).

- For each agent-asset pair \((i, j)\), \( a_{ij} \) is the capital allocation from agent \( i \) to asset \( j \).

As computed in Sec. 3.3.1, the capital allocations \( a_{ij} \) for the different types of edges are given by:

1) \( a_{ij} = a_B = k \)

2) \( a_{ij} = a_{B^c} = \frac{k}{g+1} \)

3) \( a_{ij} = a_{B^c} = \frac{k}{g} \)

In the rest of this section we carry out a theoretical analysis that examines how systemic risk evolves with respect to incremental changes in the number \( b \) of institutional agents and the number \( g \) of global assets that are present in a financial system. In this analysis, the systems we examine have the structures described in Sec. 3.2, while \( b \) and \( g \) represent the two information barriers, accessibility and diversifiability respectively.
**G-complete system**

A *G-complete* system is characterized by \( g = n \), i.e. all institutional agents invest in all the assets of the system.

**Theorem 1** For a *G-complete* system, \( R \) is a concave function of \( b \). Specifically, \( R \) is increasing with \( b \), \( \forall b \leq \frac{n}{2} \) and decreasing with \( b \), \( \forall b \geq \frac{n}{2} \).

**Proof:** As shown in Fig. 3-9, a *G-complete* system only includes edges of types \( \overline{B} \) and \( B_G \).

![Diagram of G-complete system](image)

**Figure 3-9:** *G-complete* system: incremental changes in the number of the institutional agents.

Based on this observation, we compute the systemic risk \( R \) as a function of the institutional agents \( b \):

\[
R(a_{ij}) = S \sum_{T_{As}} \sum_{J \in T_{As}} (\sum_{T_{Ag}} \sum_{i \in T_{Ag}} a_{ij})^2
\]

\[
= S \left[ \sum_{j \in G_B} (\sum_{T_{Ag}} \sum_{i \in T_{Ag}} a_{ij})^2 + \sum_{j \in G_{\overline{B}}} (\sum_{T_{Ag}} \sum_{i \in T_{Ag}} a_{ij})^2 \right]
\]

\[
= S \left[ \sum_{j \in G_B} (\sum_{i \in B_G} a_{BG})^2 + \sum_{j \in G_{\overline{B}}} (\sum_{i \in B_G} a_{BG})^2 + \sum_{i \in B_{\overline{G}}} a_{BG}^2 \right]
\]

\[
= S \left[ \sum_{j \in G_B} (\sum_{i \in B_G} \frac{k}{n})^2 + \sum_{j \in G_{\overline{B}}} (\sum_{i \in B_G} \frac{k}{n} + k)^2 \right]
\]

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\[ R(b) = S k^2 \left[ \sum_{j \in G_B} \left( \frac{b}{n} \right)^2 + \sum_{j \in G_B^2} \left( \frac{b}{n} + 1 \right)^2 \right] = S k^2 \left[ b \left( \frac{b}{n} \right)^2 + (n - b) \left( \frac{b}{n} + 1 \right)^2 \right] = S k^2 \left( \frac{-b^2}{n} + b + n \right) = R(b). \]

The derivative of \( R \) with respect to \( b \) is equal to:

\[
\frac{\partial R(b)}{\partial b} = S k^2 \left( \frac{-2b}{n} + 1 \right)
\]

From Eq.3.4 we notice that \( b < \frac{n}{2} \Rightarrow \frac{\partial R(b)}{\partial b} > 0 \) and \( b > \frac{n}{2} \Rightarrow \frac{\partial R(b)}{\partial b} \leq 0 \).

Q.E.D.

**B-complete system**

A B-complete system is characterized by \( b = n \), i.e. all global assets accept investments from all the agents of the system.

**Theorem 2** For a B-complete system, \( R \) is decreasing with \( g, \forall g > 0 \).

*Proof:* As shown in Fig. 3-10, a B-complete system only includes edges of types \( B_G \) and \( B_G^2 \).

![Diagram of B-complete system](image)

Figure 3-10: B-complete system: incremental changes in the number of the global assets.
Based on this observation, and following steps similar to the previous section's, we compute the systemic risk $R$ as a function of the global assets $g$.

$$R(a_{ij}) = S \sum_{T_{as}} \sum_{j \in T_{as}} (\sum_{T_{ag}} \sum_{i \in T_{ag}} a_{ij})^2 =$$

$$= S \left[ \sum_{j \in G_B} \sum_{i \in B_G} a_{BG} + \sum_{j \in \overline{G_B}} \sum_{i \in B_{\overline{G}}} a_{BG} \right]^2 + \sum_{j \in G_B} \sum_{i \in B_G} a_{BG}^2$$

$$= S \left[ \sum_{j \in G_B} \sum_{i \in B_G} \frac{k}{g} + \sum_{j \in \overline{G_B}} \sum_{i \in B_{\overline{G}}} \frac{k}{g+1} \right]^2 + \sum_{j \in G_B} \sum_{i \in B_G} \frac{k}{g+1}^2$$

$$= Sk^2 \left[ \sum_{j \in G_B} (1 + \frac{n-g}{g+1})^2 + \sum_{j \in \overline{G_B}} \frac{1}{(g+1)^2} \right] =$$

$$= Sk^2 \left[ \sum_{j \in G_B} \frac{n+1}{g+1} + \sum_{j \in \overline{G_B}} \frac{1}{(g+1)^2} \right] =$$

$$= Sk^2 \left[ g(n+1)^2 + \frac{n-g}{(g+1)^2} \right] =$$

$$= Sk^2 n \frac{(n+2)g + 1}{(g+1)^2} = R(g)$$

The derivative of $R$ with respect to $g$ is equal to:

$$\frac{\partial R(g)}{\partial g} = Sk^2 n \left[ \frac{n+2}{(g+1)^2} - 2 \frac{(n+2)g + 1}{(g+1)^3} \right] \quad (3.5)$$

From Eq. (3.5) we notice that $\frac{\partial R(g)}{\partial g} \geq 0$ for $g \leq \frac{n}{n+2} < 1$, so $\frac{\partial R(g)}{\partial g} \leq 0$, $\forall g \geq 1$, i.e. $\forall g > 0$ given that $g \in \mathbb{N}$. Hence, $R(g)$ is decreasing with $g$, for all $g > 0$.

Q.E.D.
General system

A more general system structure, generated according to Sec. 3.2.2, is examined here. Before analyzing the general system case, we present the following Lemma that will be needed for the proof of the general case theorem.

**Lemma 2** $X$ is a random variable with $E[X] = \frac{bg}{n}$ and $E[X^2] = \frac{bg}{n} - \frac{(n-1)b^2g^2}{n^3}$.

*Proof:* Let $X = |B_G|$ the number of agents that only invest in global assets. Given Assumption 3, it is obvious that $|G_B| = |B_G| = X$.

For a general system of known $n$, $g$ and $b$, each realization of the exact allocations of institutional agents and global assets give a different $X$. We consider realizations according to a probability distribution, such that each asset is equally likely to be global with probability $p_g = \frac{g}{n}$, each agent is equally likely to be institutional with probability $p_b = \frac{b}{n}$ and the occurrence of global assets is independent of the occurrence of institutional agents.

Let also $I_i$ indicator variables such that:

$$I_i = \begin{cases} 1 & \text{if } i \in B_G \\ 0 & \text{otherwise} \end{cases}$$

Taking into account that the occurrence of global assets is independent of the occurrence of institutional agents, we compute:

$$p_t = p(I_i = 1) = p(i \in B)p(j \in G, \forall j \in A_i(G,B)) =$$

$$= p(i \in B)(1 - p(j \in G)) = p_b(1 - (1 - p_g)) = p_g p_b$$

Thus

$$p_t = \frac{bg}{n^2}.$$ 

Also

$$E[I_i] = \frac{bg}{n^2} \text{ and } E[I_i^2] = \frac{bg}{n^2}. $$

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But according to the definition of $X$

$$X = \sum_{i=1}^{n} I_i$$

from which we get:

$$E[X] = E[\sum_{i=1}^{n} I_i] = \sum_{i=1}^{n} E[I_i] = \sum_{i=1}^{n} \frac{bg}{n^2} = \frac{bg}{n} \quad (3.6)$$

Also

$$X^2 = (\sum_{i=1}^{n} I_i)^2 = \sum_{i=1}^{n} (I_i)^2 + 2 \sum_{i=1}^{n} \sum_{j \neq i} I_iI_j$$

$$E[X^2] = E[\sum_{i=1}^{n} (I_i)^2 + 2 \sum_{i=1}^{n} \sum_{j \neq i} I_iI_j] =$$

$$E[X^2] = \sum_{i=1}^{n} E[(I_i)^2] + 2 \sum_{i=1}^{n} \sum_{j \neq i} E[I_iI_j] =$$

$$= nE[I_i] + 2 \frac{n(n-1)}{2} E[I_i]E[I_j] \Rightarrow$$

$$\Rightarrow E[X^2] = \frac{bg}{n} + \frac{(n-1)b^2g^2}{n^3} \quad (3.7)$$

Q.E.D.

Now we can proceed by presenting our main theorem:

**Theorem 3** For a general system the following statements hold:

a) $\exists g^*(n)$ s.t. $R$ is increasing with $b$, $\forall g \leq g^*(n)$ and a concave function of $b$, $\forall g \geq g^*(n)$.

Also, $g^* \rightarrow \frac{n}{2}$ for $n \rightarrow \infty$.

b) $\exists b^*(n, g)$ s.t. $R$ is increasing with $b$, $\forall b \leq b^*(n, g)$ and decreasing with $b$, $\forall b \geq b^*(n, g)$, if $g \geq g^*(n)$.
Proof: Following steps similar to the previous sections', we compute the systemic risk \( R \).

Figure 3-11: General Financial System: incremental changes in the number of the institutional agents (red) and the number of global assets (green).

Omitting the details we get:

\[
R(b, g) = Sk^2 \left\{ \frac{(1 + g)^2 n^4 + bn^2[-1 - g + g^2(2 + n)]}{n^3(g + 1)^2} + \right.
\]
\[
+ \left. \frac{b^2 g[1 - n + n^3 - 2g(n - 1 + n^2)]}{n^3(g + 1)^2} \right\} \tag{3.8}
\]

The partial derivative of \( R \) with respect to \( b \) is equal to:

\[
\frac{\partial R}{\partial b} = Sk^2 \left[ 2bg \left( \frac{n^3 - 2g(n^2 + n - 1) - n + 1}{n^3(g + 1)^2} \right) + \right.
\]
\[
+ \left. \frac{n^2 (g^2(n + 2) - g - 1)}{n^3(g + 1)^2} \right]
\]

Our goal is to investigate how the above derivative behaves with respect to \( b \).

- \( R \) increasing with \( b \): \( \frac{\partial R}{\partial b} \geq 0 \)

\[
\frac{\partial R}{\partial b} \geq 0 \Rightarrow \frac{2bg}{n^2} \left( n^3 - 2g(n^2 + n - 1) - n + 1 \right) \geq -g^2(n + 2) + g + 1
\]
Assuming $n \to \infty$ the limit of the above expression is:

$$2bg(n-2g) \geq -g^2(n+2) + g + 1$$

- If $g \leq \frac{n}{2}$ \Rightarrow $\frac{\partial R}{\partial b} \geq 0 \Rightarrow b \geq 0 \geq -\frac{g^2(n+2) - g - 1}{2g(n-2g)}$. So, $R$ is increasing for all $b$, for $g \leq \frac{n}{2}$.

- If $g \geq \frac{n}{2}$ \Rightarrow $\frac{\partial R}{\partial b} \geq 0$ \Rightarrow $b \leq -\frac{g^2(n+2) - g - 1}{2g(n-2g)}$. So, $R$ is increasing for $b \leq -\frac{g^2(n+2) - g - 1}{2g(n-2g)}$ and $g \geq \frac{n}{2}$.

$R$ decreasing with $b$: $\frac{\partial R}{\partial b} \leq 0$

$$\frac{\partial R}{\partial b} \leq 0 \Rightarrow \frac{2bg}{n^2}(n^3 - 2g(n^2 - n + 1) + n - 1) \leq -g^2(n+2) + g + 1$$

Assuming $n \to \infty$ the limit of the above expression is:

$$2bg(n-2g) \leq -g^2(n+2) + g + 1$$

- If $g \leq \frac{n}{2}$ \Rightarrow $\frac{\partial R}{\partial b} \leq 0 \Rightarrow b \leq -\frac{g^2(n+2) - g - 1}{2g(n-2g)} \leq 0$

But by definition $b \geq 0$, so $\frac{\partial R}{\partial b} \leq 0$ cannot hold.

- If $g \geq \frac{n}{2}$ \Rightarrow $\frac{\partial R}{\partial b} \leq 0 \Rightarrow b \geq \frac{g^2(n+2) - g - 1}{2g(2g-n)}$

So, $R$ is decreasing for $b \geq \frac{g^2(n+2) - g - 1}{2g(2g-n)}$ and $g \geq \frac{n}{2}$.

To summarize:

$$g^*(n) = \frac{n^3 + n - 1}{2(n^2 - n + 1)} \to \frac{n}{2} \quad (3.9)$$

$$b^*(n, g) = \frac{g^2(n+2) - g - 1}{2g(2g-n)} \quad (3.10)$$

Q.E.D.
3.4 Interpretation

The above results are summarized in the graph illustrated in Fig. 3-12. The graph depicts the performance of systemic risk with respect to the two information barriers, *accessibility* and *diversifiability*, which are represented by $b$ (the number of institutional agents in the system) and $g$ (the number of global assets in the system) respectively. On the graph, $n$ represents the total number of agents and the total number of assets in the system.

![Graph of Systemic Risk Performance](image)

Let us highlight on the graph the results of our analysis for the different kinds of structures.

**G-complete system**

According to *Theorem 1*, in a G-complete system $R$ is a concave function of $b$, achieving its maximum at $b^* = \frac{n}{2}$.

The rounded-point red line on the graph of Fig. 3-13 depicts Theorem 1.
Figure 3-13: Systemic risk performance - Theorem 1

**B-complete system**

According to *Theorem 2*, in a *B-complete system* $R$ is a decreasing function of $g$. The square-point blue line on the graph of Fig.3-14 depicts Theorem 2. For increasing number of global assets in the financial system, the curves of the systemic risk with respect to $b$ drop.

**General system**

The graph of Fig. 3-15 describes the performance of systemic risk with respect to the two information barriers in a *general system*. Specifically it depicts the more general Theorem 3, while the rounded-point and square-point lines highlight the limits of Theorems 1 and 2 respectively.
Figure 3-14: Systemic risk performance - Theorem 2

Figure 3-15: Systemic risk performance - Theorem 3
From the graph we see that the presence of the two information barriers in the financial system causes a tradeoff between individual diversification and systemic risk. This tradeoff can be summarized as follows:

- **Benchmark**: A fully diversified financial system, where every agent invests in every asset, is optimal with respect to both systemic and individual risk.

- **Aligned individual diversification activity and systemic risk**: When the majority of agents diversify their portfolio over the majority of assets, individual diversification activity is aligned with systemic risk.

- **Trade-off between individual diversification and systemic risk**: When a minority of agents invest in the majority of assets, or many agents invest in a minority of assets, then individual diversification activity and systemic risk are in conflict and a tradeoff emerges.

As shown in Fig. 3-16, on the blue highlighted region an increase in the level of information in the system mitigates systemic risk, which means that in this region specifically individual diversification activity and systemic risk are aligned. This is not true though for the rest of the regions, where increased information magnifies systemic risk.

The above analysis provides us with a tool that helps identify when increasing levels of information availability in the system may lead to systemic risk amplification. The computed thresholds \( g^*(n) \) and \( b^*(n, g) \) of the two information barriers dictate which are the regions of the systemic risk performance graph and the levels of information availability for which the above adverse phenomenon may occur.
Figure 3-16: Systemic risk performance - The blue area represents where increase of information in the system is good.
Chapter 4

Individual and Systemic Risk tradeoff induced by Information Barriers - A General Approach

In chapter 3 we considered some assumptions that simplified our financial system model in order to make our analysis more tractable. The question that raises naturally is how these assumptions limit our results generalization. In the current chapter we relax some of the above assumptions and we revisit our goal, which is to investigate how systemic risk is affected by the information availability of agents.

4.1 Relaxed Assumptions

In our initial analysis we assumed identical agents and assets characteristics, as well as a tight capital constraint for all the agents. In order to generalize our results, we completely relax the above assumptions, maintaining however uncorrelated assets returns.

We consider a financial system like the one described in Sec. 2.3, but we update the assumptions of Sec. 3.1.1 as follows:
Assumption 1 (Symmetric System) - remains: m=n

Assumption 2 (Assets) - updated: The assets are uncorrelated i.e. \( \text{cov}(r, l) = 0, \forall r \neq l. \)

Assumption 3 (Agents) - relaxed

Assumption 4a (Local assets) - remains: Each local asset receives investment from exactly one agent, i.e. if \( j \in A_i \) for some \( i \in 1, \ldots, n \), then \( j \notin A_{i'}, \forall i' \neq i, \forall j \in \overline{G}. \)

Assumption 4b (Small agents) - remains: No two small agents invest in the same asset i.e. \( A_p \cap A_q = \emptyset \) if \( p, q \in \overline{B}. \)

As already mentioned before, both assumptions 2 and 4 are aligned with our goal to demonstrate a tradeoff caused purely by the diversification activity, due to the existence of global assets and institutional agents, and not by inherent investment correlations in the system.

### 4.2 Challenges

Relaxing these assumptions, the single agent optimization becomes:

\[
\max_{a_{ij}} \sum_{j \in A_i} a_{ij} U_j \quad (4.1)
\]

s.t.

\[
\sum_{j=1}^{m} a_{ij} \leq k_i \quad (1a)
\]

\[
\sum_{j=1}^{m} a_{ij}^2 S_j \leq t_i \quad (1b)
\]

On the other hand, systemic risk becomes:

\[
R(a_{ij}) = \sum_{j=1}^{n} (\sum_{i=1}^{n} a_{ij})^2 S_j \quad (4.2)
\]

In order to perform a similar analysis and compute the information barriers thresholds as in
Ch. 3, we would need to compute the capital allocations in the system. However, given the uneven asset returns and agents characteristics, Lemma 1 does not hold any more and the computation of $a_{ij}$'s becomes less trivial. Let us illustrate the challenge of this computation in figures 4-1, 4-2 and 4-3.

Figure 4-1: Single agent optimization: 1st case

Figure 4-2: Single agent optimization: 2nd case
We notice from the figures that $a_{ij}$ are set-valued functions, so it is not possible to simply plug them into the systemic risk formula and derive an analytic expression to describe it. Consequently, we cannot perform an analysis similar to chapter 3. Instead, we employ a simulated model in order to draw conclusions about the general case of the financial system. We present the model in Sec. 4.3.

4.3 Simulated Model

In this section we present the simulated financial system model that we employed, in order to draw conclusions about the performance of systemic risk as a function of information availability in the general case of the financial system, i.e. in the case where our initial simplifying assumptions are relaxed.

In order to build our model we implement the following functions:

- We randomly generate agents and assets characteristics - i.e. capital and risk tolerance for the agents, expected return and variance for the assets - driven out of uniform distributions.
• For given assets and agents characteristics and for every possible pair \((b, g)\), we generate multiple realizations of the network structure, where \(b\) the number of institutional agents and \(g\) the number of global assets in the system. For example, Fig. 4-4 illustrates two different structure realizations for \(b = g = 3\).

![Figure 4-4: Different structures for \(b = g = 3\)](image)

• For each structure realization, we perform multiple single agent optimizations, implementing the one described in Sec. 2.3.2, in order to compute optimal capital allocations of every agent. We then compute the systemic risk of the structure given those optimal capital allocations.

• We compute the average systemic risk over all the generated structures, for every pair \((b, g)\).

In order to ensure the validity of our model we performed a sanity check. We imposed to our model the assumptions of chapter 3, we computed the expected systemic risk according to the analytical form that was derived in Sec. 3.3.2 and we plotted both experimental and analytical average risk for comparison.

The results of the sanity check are shown in Fig. 4-5. In A we revise the results of our theoretical analysis in chapter 3 (same as Fig. 3-12). B depicts in a common graph the expected systemic risk, plotted according to formula 3.8, and the average systemic risk, computed by our model over all possible structure realizations for every pair \((b, g)\). We notice that the results of the simulated model are very close to the theoretical results of chapter 3, so we confirmed that our simulation model can represent reliably the financial systems that we are examining.
Figure 4-5: Sanity check
4.4 Results Sensitivity to Assumptions

In this section we present the results of our simulated model. In order to be consistent with the previous discussion, here again we present our results by plotting the average systemic risk with respect to the number \( b \) of institutional agents (diversifiability) and the number \( g \) of global assets (accessibility). For clarity, it is important to point out the following details about the graphs that will be presented in the next section:

- The rounded-point red line depicts Theorem 1 and represents a system where all the assets are global (\( G\)-complete system).

- The square-point blue line depicts Theorem 2 and represents a system where all the agents are institutional (\( B\)-complete system).

  Note: The blue square points have different sizes depending on \( g \), i.e. their size increases for increasing \( g \).

- The single point that is illustrated as a red circle in a blue square represents a system where all agents invest in all assets (fully diversified system).

In the next section, we compare the results of our simulations with the analytical results of chapter 3, evaluate the restrictions imposed by the assumptions of sec. 3.1.1, and explain their differences, similarities and causes.

4.4.1 Assumptions Relaxation

In this section, we relax the assumptions one by one and we compare the results of our simulations with the analytical results, for which all the assumptions were taken into account.

- **Tightness relaxation of capital constraint:** Here we modify Assumption 3 and specifically we drop the inequality \( S \leq \frac{1}{k^2}|A_i| \), which ensured that the agents invest the total of their capital in the market.
Result: The output of our model does not change, as we see in Fig. 4-6. Similarly to our theoretical analysis, where the capital constraint was chosen to be tight, here as well we notice that:

- **G-complete** system: In such a system, $R$ is again a concave function of $b$. When the majority of agents are institutional and diversify their portfolio, individual diversification activity improves systemic risk, while when the majority of agents are small, increased diversification activity magnifies the risk of the system.

- **B-complete** system: In such a system, $R$ is again decreasing with $g$. The more assets are offered to the agents for diversification, the more the risk of the system is mitigated.

Figure 4-6: Risk performance for: *Tightness relaxation of capital constraint.*
- **General** system: When the majority of agents are institutional and the majority of assets are global (high G - high B region), then relaxing the diversifiability barrier leads to the reduction of systemic risk. On the other hand, when a small fraction of agents are institutional, or a small subset of assets is global, then relaxing the diversifiability barrier increases systemic risk. Thus a tradeoff emerges between agents’ diversifiability and systemic risk. In such a system, the curves of $R$ with respect to $b$ for a given $g$ are concave if $g > g^*$ and increasing if $g < g^*$, where $g^*$ the number of global assets that maximizes the risk in the system.

- **Fully diversified** system: A fully diversified financial system, where every agent invests in every asset, is again optimal with respect to both systemic and individual risk.

- **Inhomogeneous agents:** Here we modify differently Assumption 3, by now considering diverse risk and capital characteristics among agents, which means that $k_i = k$ and $t_i = t$ don’t hold any more.

Result: The output of our model does change, as we see in Fig. 4-7. Differently from our theoretical analysis where the agents were homogeneous, here we notice that:

- **G-complete** system: In such a system, $R$ is not necessarily a concave function of $b$. Instead we notice a noisier curve, moving around a concave trend.

- **B-complete** system: The results here don’t change.

- **General** system: Similarly with a **G-complete** system, here as well the curves of $R$ with respect to $b$ for a given $g$ are not concave or increasing any more, but rather noisy curves moving around a concave trend for $g > g^*$ and around an increasing trend if $g < g^*$, $g^*$ being the number of global assets that maximizes the risk in the system.

- **Fully diversified** system: The results here don’t change.
• **Inhomogeneous assets**: Here we modify *Assumption 2*, by considering diverse statistical characteristics among assets, which means that $U_j = U$ and $S_j = S$ don’t necessarily hold any more.

**Result**: The output of our model changes, as we see in Fig. 4-8. Differently from our theoretical analysis where the assets were homogeneous, here we notice that:

- **G-complete system**: In such a system, $R$ is not necessarily a concave function of $b$. Instead we notice a noisier curve, moving around a concave trend.

- **B-complete system**: In such a system, $R$ is not necessarily a decreasing function of $g$.

- **General system**: Similarly with a *G-complete* system, here as well the curves
of $R$ with respect to $b$ for a given $g$ are not concave or increasing any more, but rather noisy curves moving around either a concave trend or an increasing trend.

- **Fully diversified system**: A fully diversified financial system, where every agent invests in every asset, is not necessarily optimal with respect to systemic risk.

- **All of the above**: Here we incorporate into our model all of the above modifications simultaneously.

**Result**: Combining all the changes, we get results similar to Fig. 4-9, where we notice that:

- **G-complete system**: In such a system, $R$ is not necessarily a concave function of $b$. Instead we notice a noisier curve, moving around a concave trend.
Figure 4-9: Risk performance for: All assumption relaxations incorporated.

- **B-complete** system: In such a system, $R$ is not necessarily a decreasing function of $g$.

- **General** system: Similarly with a $G$-complete system, here as well the curves of $R$ with respect to $b$ for a given $g$ are not concave or increasing any more, but rather noisy curves moving around either a concave trend or an increasing trend.

- **Fully diversified** system: A fully diversified financial system, where every agent invests in every asset, is not necessarily optimal with respect to systemic risk.

### 4.4.2 Interpretation

In Fig. 4-10 we summarize all the above results. Although the assumptions relaxation does change our results, we still notice a tradeoff between individual diversification activity
and systemic risk. The tradeoff is not as clean as when all the assumptions of chapter 3 hold. Instead, we notice noisy curves of the systemic risk with respect to $b$, moving around concave or increasing trends.

The fact that the assumptions relaxation creates a noisy tradeoff, where concavity and monotonicity are not as clean, is due to the existence of inhomogeneous agents and assets in the system:

**Inhomogeneous Assets**
Systemic risk seems to decrease when we give to the agents more identical global assets, because they are able to diversify more, however this is not the case when these assets have different characteristics. In such a case, making an asset global is not necessarily a good thing if, for example, it has very high idiosyncratic risk. The agents will take advantage of
it and will rebalance their portfolio in a way optimal for themselves, but the systemic risk may increase, because part of the overall capital gets transferred to riskier assets.

**Inhomogeneous Agents**

Similarly to the case of inhomogeneous assets, inhomogeneous agents also contribute to a noisy tradeoff. It is not obvious that making an agent institutional is good or bad for the system. Depending, for example, on how rich that agent is compared to his peers, him diversifying his portfolio over new assets may be good if the result is a more even distribution of the system's wealth, but it may as well increase systemic risk, if the agent adds lots of capital to an already overloaded asset. Hence, agents with different characteristics and relations among them can have very different effects on systemic risk, which ruins the clean concavities and monotonicities of the systemic risk performance.

Our simulation model shows for a general system that the tradeoff between individual diversification activity and systemic risk remains, but we cannot any more compute specific thresholds. We can only notice the trends, according to which it is not always the case that diversification is good for the system. Although our theoretical model in chapter 3 was a bit simplistic, it was a great tool to prove the existence of this tradeoff and build some intuition about the mechanisms and the conditions that create it.

Going back to our initial goal, the analyzed tradeoff represents the relationship between systemic risk and the level of information availability in the system. As explained earlier, diversifiability and accessibility represent two aspects of this information availability.

In order to intuitively explain this tradeoff, we can think that:

a) **global assets** represent *investment diversification* in the system, given that they provide every agent with the potential to diversify his individual portfolio, while

b) **institutional agents** represent *investment correlations* in the system, given that all these agents gather around the global subset of the market and may cause systemically unbalanced investments.

When co-existing, the interaction of these two conflicting phenomena can cause either positive or negative effects to the systemic risk. In a realistic framework, where the system is normally far from *G-complete* or *B-complete*, this trade-off is not to be ignored.
This points to a handful of important problems to investigate further, involving when and which global assets should be triggered in a market given some knowledge about the percentage of particularly active investors in the system, or how changes in investing trends, due to speculation or fear, or information spread to large groups of people, may affect the balance of the systemic risk and lead to significant market events.

4.5 Systemic versus Aggregate Risk

As explained in Sec. 1.2.2, systemic risk refers to the risk inherent to the financial system as a whole, due to individual activities of all the investors. It is generally perceived as a metric of the degree of negative consequences that can be caused to the larger body of the financial system due to individual investment actions. The magnification of systemic risk can be very dangerous for the sustainability of the financial system, so it is important to understand what are the mechanisms that drive this magnification. In this work we show under what conditions diversification activity may magnify systemic risk. However this magnification would not be of any interest if it were driven exclusively by the size of the
system. It is important to show that systemic risk increases faster than the number of agents in the system and it does not scale just due to the presence of more capital in the risky market. In order to show that, we employ the following simple example:

Consider a financial system with $n$ agents and $n$ assets. Without loss of generality, we consider that the capital available to each agent is $k=1$ and that his risk tolerance allows him to invest all of it in the market (similarly to Assumption 3 in Chapter 3). We also consider that the assets are homogeneous with $U$, $S$ being their expected return and variance respectively. Let's assume that initially each agent only knows of one asset. We then assume that, e.g. public information is revealed about one of these assets and makes it popular to all the agents (see Fig.4-12). Then $n-1$ agents split their capital between their old asset and the new global one, while the 1 agent that already had access to the new global asset keeps all of his capital on that one.

![Figure 4-12: One new asset offered to all agents](image-url)
The aggregate risk in the system is equal to the total of all individual risks of all agents:

$$R_a = \sum_{i=1}^{n} R_i = (n - 1) \left( \frac{1}{2} \right)^2 S + \left( \frac{1}{2} \right)^2 S + S = \frac{n - 1}{2} S + S$$

$$R_a = \frac{n + 1}{2} S$$

The systemic risk is:

$$R_s = S \sum_{j=1}^{n} \left( \sum_{i=1}^{n} a_{ij} \right)^2 = (n - 1) \frac{S}{4} + \left( 1 + \frac{n - 1}{2} \right)^2 S$$

$$R_s = \frac{n(n + 3)}{4} S$$

In general a bigger system, where more agents invest their capital in the risky financial market, is expected to be overall a riskier system. This can be also noticed by the fact that the aggregate risk of the system scales linearly with the size of the system, i.e. with the number of the agents in the system:

$$R_a = \frac{n + 1}{2} S = f(n)$$

However, this is not the effect that we want to capture with the systemic risk. The latter is supposed to capture the risk added to the system due to agents individual investment actions and not due to their number. Therefore examining scalings with respect to the number of agents we see that systemic risk increases quadratically with respect to $n$:

$$R_s = \frac{n(n + 3)}{4} S = g(n^2)$$

Furthermore we notice that the systemic risk increases linearly in comparison to the aggregate risk, with respect to $n$, as can be seen by computing the rate of the systemic over the aggregate risk:
\[
\frac{R_s}{R_a} = \frac{\frac{n(n+3)}{4} S}{\frac{n+1}{2} S} = \frac{n(n+3)}{2(n+1)} = h(n) \tag{4.3}
\]

The above example presents one case where the ratio \( \frac{R_s}{R_a} \) increases linearly with the size of the system. Trying different changes in the system, we can get results in the whole spectrum between constant and linear ratio of systemic and aggregate risk.

With this brief analysis we confirm that systemic risk does capture a hidden danger in the system that gets magnified just because of the actions and interconnections of the agents and not because of their number. This way we validate that our risk metric measures systemic risk and not just aggregate risk. Our metric confirms that it is something about a group of individual investments that increases the risk substantially and not just the size of the system.

As discussed in Sec. 2.3.3, systemic risk refers to the risk created by the interactions and interconnections of the agents that may lead to a collapse in the entire system, whereas aggregate risk represents the total of the risk perceived by the agents who are ignorant of the relationships among them and the consequences they may have on the system as a whole.

Revisiting the motivating example of Sec.2.5, the ratio of the systemic and aggregate risk is an increasing linear function of the full-information probability \( p \). This means that the more information is given to the agents about their available diversification options, the bigger the difference between the perceived by the agents and the actual risk in the system is.

A similar intuition holds for our main problem as well, as demonstrated by Eq. 4.3. Fig.4-13 compares the systemic to aggregate risk ratio for the motivating example and for our simulated model. We notice that in both cases the ratio is increasing with information availability, represented by full-information probability in A and with diversifiability and accessibility in B. The more knowledge the agents have, the more they can diversify and decrease their individual risk, without though driving the actual risk of the system to lower levels as well. In fact, while perceived risk decreases as more information is offered in the system (see Fig.4-14), the actual systemic risk increases. This discrepancy between the perceived and actual risk in the system is very important because it shows again how
ignorance of agents about the actual risk may lead them to actions that are dangerous for the system as a whole.

Figure 4-13: Systemic to aggregate risk ratio with respect to information: A) Motivating example, B) Information barriers model.
Figure 4-14: Systemic versus aggregate risk with respect to information: A) Motivating example, B) Information barriers model.
Chapter 5

Conclusions

In this thesis we study how independent diversification activities of agents with limited information may affect systemic risk. Towards this goal we propose for the financial system a novel model, based on an existing stylized model adopted from [14]. The novelty of the model is that we introduce in it two information barriers, assets accessibility and agents diversifiability. Both of these barriers are two different aspects of the investors' information availability, with assets accessibility representing the investors' awareness of the existing options they can employ to diversify, and agents diversifiability representing the investors' experience in processing information that will allow them to make better diversification decisions and expand their investment activity. We enrich Caccioli's model with these two realistic features, by partitioning assets according to their accessibility level and investors according to their diversifiability level.

Our main contribution is the conclusion that the existence of these two information barriers causes a tradeoff between information availability and systemic risk, and the demonstration of conditions under which increased information in the system magnifies systemic risk. According to our main conclusion, limited information availability affects agents actions, but it is not necessarily true that the more information they are given the better it is for the system. Information barriers not only induce this tradeoff between individual diversification activity and systemic risk, but also a discrepancy between aggregate (perceived) and systemic (actual) risk, which renders agents ignorant of the risk they may be adding to the
system and allows them to potentially lead the system to a financial crisis.

Summarizing, the contributions of this work are stated next:

1. We showed the existence of a tradeoff between individual diversification activity and systemic risk caused by the two information barriers.

2. We provided analytical characterization of phase transition thresholds for these barriers, which allow us to determine under which conditions diversification activity may amplify systemic risk.

3. We demonstrated a discrepancy between aggregate and systemic risk.

4. We developed and employed a simulation model to investigate the sensitivity of our risk performance results to the theoretical assumptions.

The financial crisis of 2007-2009 is largely attributed to the creation and promotion of a plethora of sophisticated financial instruments, that made their underlying assets accessible to agents willing to expand into new investment areas. They were created mostly by a few big agents (e.g. banks), represented a small subset of the market assets (e.g. house related assets, mortgages, etc) and were then promoted and sold to investors that wanted to diversify their portfolios and were strongly influenced by the trends and the information spread during those days. Although some assumptions in this work create a simplistic framework, our theoretical and numerical analysis demonstrate that our model may offer a good tool to understand potential causes of that crisis. For example, during speculation times, more and more investors were willing to try new investment options (become institutional) and herded around the same instruments (global assets). This phenomenon of increasing concentration of agents on a subset of assets may have led to higher systemic risk and greater losses when the bubble exploded.

The most ambitious future extension of this work is to turn this model into an actionable tool, that will be useful both at an individual level, that would let the investor know how much risk he is adding to the system, as well as at a systemic level, for a social planner
whose goal is to come up with techniques that mitigate systemic risk whenever the information availability in the system changes.
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