Coupled Electromagnetic and Acoustic Wavefield Modeling in Poro-Elastic Media and its Applications in Geophysical Exploration

by

Matthijs W. Haarsen

Submitted to the Department of Earth, Atmospheric, and Planetary Sciences in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Geophysics

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 1995

© Massachusetts Institute of Technology 1995
All rights reserved

Signature of Author ............................................................
Department of Earth, Atmospheric, and Planetary Sciences
June 8, 1995

Certified by .................................................................
Professor M. Nafi Toksöz
Director, Earth Resources Laboratory
Thesis Advisor

Accepted by .................................................................
Professor Thomas H. Jordan
Department Head

AUG 29 1995  ARCHIVES
Coupled Electromagnetic and Acoustic Wavefield Modeling in Poro-Elastic Media and its Applications in Geophysical Exploration

by

Matthijs W. Haartsen

Submitted to the Department of Earth, Atmospheric, and Planetary Sciences on June, 1995, in partial fulfillment of the requirements for the degree of Doctor of Philosophy

Abstract

This thesis investigates the macroscopic dynamics of two-phase (fluid and solid) porous media possessing continuously distributed phases. A fluid saturated sedimentary rock is an example of such a material. When a mechanical disturbance propagates through it, a small amount of relative motion is induced between the fluid and solid phases. The driving force for this relative flow is a combination of pressure gradients set up by the peaks and troughs of a compressional wave and by grain accelerations. The relative flow and, therefore, induced streaming current is due to both compressional and shear waves. Mechanical waves, therefore, act as a current source for electromagnetic disturbances.

First, the governing equations that control the coupled electromagnetics and acoustics of porous media, including the transport equations through which all coupling occurs, are used to study electrical streaming currents through a homogeneous porous medium induced by seismic point sources. Then the coupled equations are solved in heterogeneous, i.e. layered porous media. Electroseismic boundary conditions at singular jumps in the macroscopic material properties are defined, and a macroscopic energy transfer consistent field-vector-formalism is derived and solved numerically in a layered medium. Different seismic point sources applied on either part of the two phase medium are derived in a poro-elastic medium. A method is presented to simulate the propagation of seismic and converted electromagnetic waves generated by a mechanical borehole source embedded in a layered poro-elastic medium. Electroseismic field experiments in shallow subsurface environments and in boreholes are analyzed using the developed electroseismic numerical algorithms.
The coupled transport equations, flux/force relations that relate the current and relative flow to potential and pressure gradients, are simplified to solve for the mechanically induced electrical streaming currents generated by seismc point sources in homogeneous porous media. The mechanical wave behavior is decoupled from the electromagnetic wave behavior to simplify the analysis of induced fluxes (relative fluid flow and current flow). The mechanically induced relative flow is determined by Green's function solution. The stationary-phase method is used to calculate streaming current radiation patterns for an explosive and vertical point source acting on the bulk phase and a pressure source acting on the fluid phase. The dynamic streaming current amplitude behavior with respect to porosity, permeability, and fluid chemistry is investigated at three different source center frequencies.

From the macroscopic coupled mechanical equations of motion, electromagnetic equations, and constitutive equations the acoustic-electromagnetic power balance in its global form in a porous medium is derived. The time rate at which the sources deliver mechanical and electromagnetic work to the coupled acoustic, electromagnetic disturbances is shown to equal the time rate of kinetic energy plus the time rate of deformation energy plus the time rate of coupled field energy plus the time rate of total stored electromagnetic energy in a volume plus the acoustic-electromagnetic powerflow through a surface bounding the volume.

Conditions are derived for uniqueness of solution of the coupled field equations. Particular attention is paid to continuity requirements at an interface where a jump in macroscopic medium parameters occurs. Electroseismic boundary conditions at singular jumps in the macroscopic material properties are determined. This defines a displacement-stress-EM wavefield vector whose components are continuous through a contrast. The governing electroseismic equations are transformed into a field-vector-formalism using a plane wave solution procedure. Wavefield eigenvectors and associated eigenvalues (wavefield slownesses) are derived for up and down going waves in porous media (i.e. fast compressional wave, slow compressional wave, rotational waves and electromagnetic waves). To guarantee a consistent macroscopic theory of energy transfer, conservation of electromagnetic and poro-elastic Poynting power upon crossing an interface is imposed by energy normalization of the up and down going vectors.

The macroscopic governing equations, transformed into a field vector formalism, are numerically solved using the very stable global matrix method. The effects of both pressure and shear seismic waves traversing a mechanical and/or electrical contrast are studied. When seismic waves traverse contrasts, dynamic current imbalances are induced which generate radiating electromagnetic disturbances. Amplitude versus offset and sensitivity to macroscopic medium properties of the seismic to electromagnetic signals are studied.

Relative flow and displacement Green's functions in dyadic form are derived. The final dyadic form is similar in form to the isotropic elastic dyadic Green's function. Source jump representations are derived for an explosive and vertical point source
using the developed Green’s tensors together with the deformation equations of the two phases. The explosive point source, which generates $P$ waves only, and the vertical point source, which generates both $P$ and $SV$ waves, are compared with respect to their conversion behavior into electromagnetic disturbances in layered porous medium.

The Biot-Rosenbaum model is extended by including the effect of a heterogeneous porous formation surrounding the borehole and by including the conversions of mechanical into electromagnetic waves at the mechanical and/or electrical contrasts in the poro-elastic formation. The method to solve the extended Biot-Rosenbaum problem is formulated as a boundary element technique. The singular properties of the Green’s functions are determined analytically using static Green’s functions to regularize the integrals. This is necessary to calculate the element’s self interaction.

Two real electroseismic datasets are interpreted using the developed electroseismic theory and numerical algorithms. Synthetics in Transpose Vertical ElectroSeismic Profiling (TVESP) geometry are generated to identify the electroseismic responses in borehole geometry from a soil-glacial till and glacial-till-bedrock interface. The synthetics in surface seismic geometry are calculated to extract converted electric field amplitudes versus dipole antenna offset which is compared against the real measured data result.

Thesis supervisor: M. Nafi Toksöz
Title: Professor of Geophysics
Acknowledgment

I would like to thank my advisor, Prof. Nafi Toksöz, for his advise and guidance throughout my study at MIT. I have always enjoyed our discussions of my own geophysical research, as well as general science, political and social issues. Part of his guidance has been the introduction to colleague scientists and the provision of a stimulating research environment with an always changing crew of visiting scientists and visiting professors some of whom have made, directly or indirectly, important contributions to this thesis. This environment has taught me the importance of communication and collaboration on all levels, especially with student colleagues. A frequent returning visiting professors was Michel Bouchon who taught me about numerical modeling of elastic wavefield propagation, scattering and the Boundary Integral Technique. I would also like to thank Prof. Ted Madden for sharing his ideas and intuition on complicated phenomena (his suggestions were often very useful and his intuition was often right).

This research started out with Steve Pride, who taught me about porous media and rock physics. I would like to thank him for the many inspiring discussions while he was at ERL and for his collaboration throughout this thesis work. I have to mention and thank Dr. Art Thompson, who drew new attention to electroseisics with his field experiments. He taught a very interesting seminar at ERL that broadened my scope of interests to include transport problems through porous rock, rock physics, and electroseisics. Art Thompson’s preliminary work and the research initiative and enthusiasm of Steve Pride in defining the coupled poro-elastic and electromagnetic wavefield problem are the props of this work.

I would like to thank my office mate, Chengbin Peng, for sharing his many interesting ideas. He introduced me to a numerical stable solution procedure to solve the coupled wavefield problem in layered poro-elastic media. What I know about borehole geophysics I owe him and Wenjie Dong. His presence stimulated me to work hard and I wished I could solve problems as efficient as he does. As already mentioned, I want to thank my friend, Wenjie Dong, for his help, contribution and stimulation, to solve the decoupled electroseismic wavefield problem by Green’s tensor solution.

When the theory and numerical model was completed I got invaluable help to measure the electroseismic conversions in ERL’s laboratory and in the field. I want to thank Dr. Zhenya Zhu for designing an electroseismic laboratory experiment with which he was the first to measure an electroseismic conversion at ultrasonic frequencies. I thank Oleg Mikhailov for making an electroseismic field experiment possible which resulted in successful measurements of electroseismic conversions. Impractical as we were, we got the necessary guidance from Prof. Ted Madden, Dr. Randy Mackie and Dr. Roger Turpening. Karl Butler at UBC, Vancouver, provided me with his collected electroseismic data. I want to thank him for that and the fruitful collaboration afterwards on the interpretation of the data.

I consider myself lucky to have met Matthias Imhoff, my office mate for four years,
whose positive criticism and morale boosts helped me to complete this thesis. I want
to single out his encouragement and help to transform me into a programmer of the
nineties. He taught me to code in C++ and assisted me in implementing all codes
on the nCUBE massive parallel computer. Ted Charette added some slickness to
my parallel programming and he and Joe Matarasse were a source of help in using
the always changing computer facilities. I want to thank Rick Gibson for always
being willing to read critically and improve manuscripts. I greatly benefitted from
his knowledge of wave propagation related issues. I began my studies at MIT/ERL
with, my later on, friend Rick Bennett. We developed over the years, many common
interests, mostly not related to geophysics, which have made the lunch hours and my
stay at MIT more enjoyable.

I would also like to thank Craig Schultz, Jie Zhang, and Bob Greaves for their ex-
pressed interest in this research and their general discussions. I have enjoyed working
with many other students who broadened my geophysical knowledge and I’d like to
acknowledge Dave Cist, Ningya Cheng, Rafi Katzman, Shirley Rieven, Weiqun Shi,
Francesca Sciré-Scappuzzo scientists, JungMo Lee and Sadi Kuleli, and staff Sara
Brydges, Naida Buckingham, Liz Henderson, and Sue Turbak.

This research was funded by the U.S. Department of Energy through contact number
DE-FG02-93ER14322.
The author held a SEG Foundation Scholarship award for two years was an OYO
fellow.
Contents

1 Introduction ......................................................... 14
  1.1 Background .................................................. 16
  1.2 Outline of the Thesis ......................................... 21

2 Dynamic Streaming Currents from Seismic Point Sources in Homogeneous Poro-Elastic Media. 28
  2.1 Introduction .................................................. 28
     2.1.1 Electrokinetics at the Pore Scale ...................... 28
     2.1.2 Macroscopic Transport Coefficients .................. 30
     2.1.3 Sensitivity of Transport Coefficients to Porosity and DC Permeability ..................... 34
     2.1.4 Macroscopic Coupled Electromagnetic and Poro-Elastic Field Equations ...................... 36
     2.1.5 Mechanically Induced Streaming Currents ............. 37
  2.2 Dynamic Streaming Currents Mechanically Induced by Point Forces ............. 40
     2.2.1 Stationary-Phase Solutions ................................ 42
  2.3 Dynamic Streaming Currents from Point Forces ................ 45
2.3.1 Streaming Current Behavior with respect to Frequency  48
2.3.2 Streaming Current Behavior with respect to Salinity  48
2.3.3 Streaming Current Behavior with respect to Permeability  49
2.3.4 Streaming Current Behavior with respect to Porosity  50

2.4 Discussion and Conclusions ............................... 51

3 Electroseismic Waves in Poro-Elastic Media 69

3.1 Introduction .............................................. 69
3.2 Energy Considerations in a Porous Medium ............... 70
3.3 Uniqueness Theorem ...................................... 73
3.4 Reciprocity Theorem in a Poro-Elastic Medium .......... 75
3.5 Eigenvalues and Eigenvectors of the System Matrices for Complex Fourier Parameters .................................. 76
3.6 Determination of the Different Wave Type Velocities. .... 78
  3.6.1 The Transverse Polarization ........................ 79
  3.6.2 The Longitudinal Polarization. ...................... 80
3.7 Phase Velocity and Specific Attenuation .................. 81
3.8 Displacement Normalized Eigenvectors and Eigenvalues of the System Matrix .................................. 82
3.9 Up and Down Going Wavefields and their Polarization .... 84
  3.9.1 The SV/TM Wavefield ................................ 85
  3.9.2 The $P_l/P_s$ Wavefields ............................. 86
  3.9.3 The SH/TE Wavefields ................................ 86
3.10 The ElectroSeismic Eigenvectors ........................................ 87
  3.10.1 The SV-TM Eigenvectors ......................................... 87
  3.10.2 The $P_f - P_s$ Eigenvectors .................................. 88
  3.10.3 The SH-TE Eigenvectors ....................................... 89
3.11 Eigenvector Normalization by Time-Averaged Poynting Power .... 89
  3.11.1 The SV and TM Vertical Power Flux ............................ 90
  3.11.2 The $P_f$ and $P_s$ Vertical Power Flux ...................... 90
  3.11.3 The SH TE Vertical Power Flux ............................... 91
3.12 Conclusions .......................................................... 91

4 Electroseismic Waves from Point Sources in Layered Media. ....... 93
  4.1 Introduction ......................................................... 93
  4.2 Macroscopic Coupled Electromagnetic and Poro-Elastic Field Equations 95
  4.3 Transform-Domain Electroseismic Wavefield Constituents in an Isotropic
      Poro-Elastic Layered Medium ................................... 98
    4.3.1 Transform-Domain Field Description ........................ 100
    4.3.2 Solution Procedure to Obtain Wavefield Constituents ...... 105
  4.4 The Global Matrix Method ........................................... 108
    4.4.1 Solution Procedure of the PSVTM and SHTE Problems ...... 109
    4.4.2 Radiation Conditions .......................................... 111
    4.4.3 Free surface Conditions ...................................... 111
  4.5 Point Forces in a Poro-Elastic Stratified Medium ................ 116
4.5.1 Mechanical Point Sources ........................................ 116
4.5.2 Electromagnetic Point Sources ............................... 120

4.6 Transformation Back to the Space Time Domain ............. 120

4.7 Numerical Electroseismogram Examples ....................... 123
  4.7.1 The Electroseismic Conversion at Mechanical Contrasts ... 124
  4.7.2 The Electroseismic Conversion at Electrical Contrasts ... 128
  4.7.3 Vertical ElectroSeismic Profiling (VESP) ................... 130

4.8 Conclusions .......................................................... 131

5 Electroseismic Waves in Layered Media using Dynamic Green’s Functions. 152

  5.1 Introduction ...................................................... 152

  5.2 Equations for Dynamic Poro-Elasticity ....................... 153
    5.2.1 Relative Flow Green’s Function with Force on Solid Phase . 155
    5.2.2 Relative Flow Green’s Function with Force on Fluid ...... 164

  5.3 Point Source Results in Poro-Elastic Media ................ 168
    5.3.1 Vertical Point Source ...................................... 168
    5.3.2 Explosive Point Source .................................... 168

  5.4 Mechanical and Electromagnetic Waves in Horizontally Layered Poro-Elastic Media ............................ 171
    5.4.1 Transformation Back to the Space Time Domain .......... 173

  5.5 Numerical Results ............................................... 174
    5.5.1 Electroseismsics in Surface Geometry .................... 175
5.5.2 Electroseismograms in VESP Geometry ................................ 178

5.6 Conclusions and Discussion .............................................. 179

6 Modeling of SeismoElectric effects from a Borehole Source in a Layered medium. ........................................... 197

6.1 Introduction ................................................................... 197

6.2 Strategy to Simulate Coupled Acoustic-Elastic and Electromagnetic Waves in Borehole Geometry ............................... 201

6.3 BEM Implementation ....................................................... 203

6.4 Boundary Condition Matrix ............................................. 204

6.5 Singularities when Source and Receiver Coincide .................. 209

6.6 Ring Source Results in Poro-Elastic Media ........................... 212

6.6.1 Vertical Ring Source ..................................................... 212

6.6.2 Radial Ring Source ..................................................... 214

6.6.3 Explosive Ring Source in Porous Medium ....................... 216

6.6.4 Explosive Ring Source in Fluid ..................................... 218

6.6.5 Explosive Volume Source in an Open Borehole ................. 219

6.7 Ring Sources in Horizontally Layered Poro-Elastic Media ....... 219

6.8 Mechanical and Electromagnetic Waves in Horizontally Layered Poro-Elastic Media ............................................. 222

6.9 Transformation Back to the Space Time Domain ................... 224

6.10 Numerical Results .......................................................... 225

6.10.1 Electroseismics in Surface Geometry ............................. 226
6.10.2 Electroseismograms in VESP Geometry ........................................... 228

6.11 Conclusions and Discussion ............................................................... 230

7  Real ElectroSeismic Data, Modeling and Analysis: The Haney Field
   Experiment  ............................................................................................... 248

  7.1  Introduction .......................................................................................... 248

  7.2  Electroseismic Data in Transpose Vertical ElectroSeismic Profiling (TVESP)
       Geometry ............................................................................................. 249

  7.3  Electroseismic Data in Surface Geometry .............................................. 250

  7.4  Electroseismic Modeling in Transpose Vertical ElectroSeismic Profiling
       Geometry ............................................................................................. 250

      7.4.1  Effect of the Soil-Till Interface ...................................................... 253

      7.4.2  Effect of the Bedrock Interface ..................................................... 255

      7.4.3  Real Data Comparison ................................................................. 255

  7.5  Electroseismic Modeling in Surface Geometry ...................................... 257

  7.6  Conclusions and Discussion .................................................................. 260

8  Summary and Conclusions ........................................................................ 277

  8.1  Future work ......................................................................................... 286

A  ElectroSeismic Field-Vector Formalism with Powerflow Normalized
   Eigenvectors .............................................................................................. 301

B  Derivation of Dynamic Displacement Green’s Functions ....................... 305

  B.1  Displacement Green’s Function with Force on Solid .......................... 305

  B.2  Displacement Green’s Function with Force on Fluid .......................... 310
B.3 Comments on the Derivations of the Green’s Functions .... 311
  B.3.1 Displacement Green’s Function with Force on Solid .... 311
  B.3.2 Displacement Green’s Function with Force on Fluid .... 314
  B.3.3 Relative Flow Green’s Function with Force on Fluid .... 316

C Dynamic and Static Green’s Functions in Poro-Elastic Media .... 319
  C.1 Displacement Green’s Function with Source on Frame .... 319
  C.2 Relative Flow Green’s Function with Source on Frame .... 320
  C.3 Displacement Green’s Function with Source on Fluid .... 322
  C.4 Relative Flow Green’s Function with Source on Fluid .... 322
  C.5 Static Green’s Functions in Poro-Elastic Media .... 323
    C.5.1 Static Displacement Green’s Function .... 323
    C.5.2 Static Relative Flow Green’s Function .... 324
Chapter 1

Introduction

In this thesis we are concerned with the macroscopic dynamics of two-phase (fluid and solid) porous media possessing continuously distributed phases. A packing of grains saturated by a fluid is an example of such a material. The term “macroscopic”, as used here, means that the wavelengths of the applied disturbances are much greater than the dimensions of the grains. When a macroscopic-mechanical disturbance propagates through such a material, a small amount of relative motion is induced between the fluid and solid phases. This relative flow will carry along the excess ions in the electric double layers near the grain surfaces. Thus, a mechanical wave can act as a current source for macroscopic-electromagnetic disturbances. Similarly, when an electromagnetic disturbance propagates, the electric field will act on the charge excesses of the double layers producing pressure gradients in the fluid and, in principle, macroscopic-mechanical disturbances. This thesis won’t contain numerical simulation of this so called osmosis effect.

This research involves an extensive discussion of the equations controlling electrokinetic and osmotic behavior, i.e. the governing coupled electromagnetic and acoustic-elastic equations describing electroseismic wave propagation through porous
media and solving numerically these coupled equations in homogeneous and heterogeneous porous formations. Dynamic electrokinetic phenomena, e.g. the electrical streaming current, induced by the compressional and shear waves from seismic point sources in homogeneous porous media are studied. The streaming current’s dependence on changing lithology and fluid chemistry is investigated.

Boundary conditions at singular jumps in the macroscopic material properties are determined. This defines a displacement-stress-EM wavefield vector whose components are continuous through the contrast. The macroscopic governing equations that control the coupled electromagnetics and acoustics of porous media are transformed into a field vector formalism using a plane wave solution procedure. Wavefield eigenvectors and eigenvalues (wavefield slownesses) are determined for the up and down-going waves in porous media (i.e. fast compressional wave, slow compressional wave, rotational waves, and electromagnetic waves). The main part of this thesis deals with electroseismic wave propagation through heterogeneous porous media, layered media. The electroseismic response is numerically modeled using this field vector formalism in horizontally layered poro-elastic media (shallow subsurface environments) or at depth in borehole geometry. At the contrasts in electrical and/or mechanical properties the traversing seismic waves induce a dynamic current imbalance across the interface that generates electromagnetic radiation, see figure 1-1. The amplitudes of the electromagnetic conversion components and their sensitivity with respect to lithological parameters are numerically studied. The converted electromagnetic disturbance is diffusive in nature. This implies a rapid decay in amplitude with traveled distance of the converted electromagnetic signal driven by a seismic source, and suggest the importance of the Vertical ElectroSeismic Profiling (VES P) geometry, where antennas are positioned close to the target of interest.

Field experiments are conducted in shallow subsurface environments and boreholes. The different electroseismic conversions are identified and understood using the newly developed numerical algorithms. Laboratory measurements at ultrasonic
frequencies are performed in cores and layered media to study the dynamic electrokinetic and electroseismic conversion phenomena. The data is primarily used to test the theoretical models.

Important additional lithological information is contained in electroseismic data, obtainable by using the electroseismic method, measuring both seismic and electric fields. The electroseismic method is believed to do better in (1) remotely characterizing subsurface lithologies and fluid types in shallow environments which are typically encountered in environmental and groundwater investigations, (2) detecting fluid chemistry contacts (oil-water type interfaces) and permeable/fractured zones surrounding a borehole at depth. This would require the development of an electric-acoustic logging tool.

1.1 Background

In 1936, Blau and Statham (1936) and Thompson (1936) conducted field experiments, where they applied a constant voltage to a pair of electrodes embedded in the ground and recorded the current modulation when seismic waves passed by the electrodes. They suggested that a change in ground resistivity due to elastic deformations caused by the seismic pulse resulted in modulation of the current (later called the "I-effect", (Ivanov, 1939)). In later experiments, Thompson (1939) showed that the current modulations were indeed due to the change of ground resistivity and not to a change in contact resistance between the electrodes and the ground caused by vibration.

Ivanov (1939) discovered an electroseismic effect of a different nature. He measured electric fields generated by seismic waves without applying any external voltage to the ground. He recorded an electrical pulse arriving at an antenna several milliseconds prior to the arrival of a seismic pulse. He called this effect the electroseismic effect of the second kind, or "E-effect". Ivanov realized that the presence of pore fluid
and pore fluid chemistry are essential in the electroseismic effect.

Most efforts to apply electroseismic coupling to hydrocarbon exploration were directed towards using antennas as electromagnetic geophones. Depending on the local soil conditions, Broding et al. (1963) concluded that antennas were not as sensitive as geophones.

Martner and Sparks (1959) reported a systematic study of electroseismic coupling using explosive sources placed at various depths. Their results were the first to show conversions of seismic to electromagnetic energy at depth, measured at the earth's surface with antennas. Thompson and Gist (1991, 1993) also made field measurements, clearly demonstrating that seismic waves can induce electromagnetic disturbances in saturated sediments in the earth. They recorded electrical fields generated when a seismic wave crosses an interface. The arrival time of the electrical signal provided seismic traveltimes to the interfaces of interest. Thompson and Gist correlated their electroseismic data with well logs and seismic reflection data to a depth of 300 m. Their data shows events that correlate with seismic reflectors (mechanical contrasts) and events that don’t correlate with seismic reflectors (possible electrical contrasts). Their observations are explained by the electrokinetic/osmotic coupling between seismic and electromagnetic signals discussed in this thesis.

There has not been much previous work in this direction. In 1944, Frenkel (1944) postulated equations that estimated the amount of relative fluid/solid motion induced by a seismic wave. He attempted to allow for flow-induced electric fields by employing, incorrectly, the Helmholtz-Smoluchowski equation. This equation assumes that the total electric current in the porous material, which is a sum of the mechanically driven streaming current (due to convection of double-layer ions) and an electrically-driven conduction current, is everywhere zero. Thus, in Frenkel's model, the generated electric field exists only where there is seismically-induced relative flow. Correctly, the total current should be present in Ampere's law and the full set of Maxwell's equations utilized. This is the more general result used here.
Neev and Yeatts (1989) also postulated a set of equations that attempt to model the interaction between mechanical waves and electric fields due to electrokinetics. They also do not allow for the full set of Maxwell's equations which leads them to the erroneous conclusion that mechanical shear waves do not generate electromagnetic disturbances. They only allow for electric fields generated by charge separation. A mechanical shear wave, however, generates a divergence-free (no induced-charge excess) streaming current that will act as a current source in Ampere's law thus demonstrating the need to use the complete set of Maxwell's equations. Both Neev and Yeatts and Frenkel completely ignore the frequency dependence of their proposed transport laws. The central problem with both of these cited works is that their equations are not derived from the underlying constituent properties.

In this thesis we use the macroscopic governing equations obtained by Pride (1994), who volume-averaged the continuum equations known to apply in the solid grains and fluid electrolyte. The final equations have the form of Maxwell's equations coupled to Biot's, (Biot, 1956) equations. The coupling arises naturally from just two postulates: 1) the solid grains have a uniform surface-charge density adsorbed to their surface, and 2) prior to the arrival of a disturbance, the net charge in a volume of the porous material is zero. Through the volume averaging procedure frequency dependent transport coefficients are obtained.

The electroseismic governing equations are valid only under the following assumptions, numerical model inputs must obey these assumptions and/or limitations. Only linear disturbances are considered (i.e., those that obey superposition). The fluid is assumed to be an ideal electrolyte thus restricting salt concentrations to less than one mole per liter. Both the solid grains and all the macroscopic-constitutive laws are assumed to be isotropic. All wave-induced diffusion effects (flux due to wave-induced ion-concentration gradients) are ignored. Two conditions must be fulfilled in order to make this assumption at the pore and grain scale: (1) the dielectric constant of the grains must be much less than that of the electrolyte (say a factor of ten or more),
and (2) the thickness of the double layer should be much less than the radii of curvature of the solid grains (Pride, 1994). The specific class of porous media that are of primary interest here are water-saturated sediments and sedimentary rocks. For these materials, both of these assumptions are valid. Lastly, no wave scattering from the individual grains of the porous material will be allowed for. This implies that wavelengths $\lambda$ are always much greater than grain sizes (e.g., $\lambda \geq 1$ mm). Thus, the largest applied frequency that can be considered (for mechanical waves) is on the order of $10^6$ Hz. This upper frequency limit, limits the maximum center frequency to be used in the ultrasonic laboratory data modeling.

Piezo-electric effects are not allowed for. This doesn’t limit the model if the elastic frame consists of crystalline minerals (e.g. centro symmetrical crystals) with zero piezo-electric coefficient. Common minerals that have non zero piezo-electric coefficients are quartz, sphalerite, nepheline and tourmaline, Russell and Barker (1991). Even for these minerals there is experimental evidence, Tuck et al. (1977) that random arrangements of crystallites cause cancellation of any large scale effect. The effective piezo-electric coefficient would be very small. Ghomsei et al. (1988) and Ghomsei and Templeton (1989) show that some coarse grained vein quartz samples that are coherent over tens of meters have a sizable piezo-electric coefficient. Since piezo-electricity is essentially a linear phenomenon, seismic input should elicit a piezo-electric response that contains only seismic frequencies. The electroseismic conversion measured in its near field also contains the seismic frequency of the source. Amplitudes and amplitude versus offset behavior seem to be similar and might cause confusion in data interpretation due to an ambiguous origin of the converted electromagnetic signals in certain piezo-electric media.

In addition to the streaming potential there are several mechanisms that can cause coupling between seismic and electromagnetic energy in porous rock containing a fluid in the pore space. For example, when a seismic pulse compresses a rock, causing a change in the pore volume and hence in the electrical conductivity. If there are cur-
rents flowing, such as magnetotelluric currents or currents artificially created (Wenner array), then modulation of the resistance will produce a modulation of the currents and associated voltages (Long and Rivers, 1975). Another coupling mechanism is due to self potentials in the earth, which are static voltage gradients that are electrochemical in origin. Volume changes induced by seismic pressure gradients modulate the separation of such potentials and can produce electromagnetic radiation.

Immiscible fluids in the pore space such as oil and water, have surface charges on their interfaces. The relative motion of the fluids and distortion of the interfaces produced by the seismic wave pulse creates local electric field charges and radiation. The model in this thesis has only one saturating fluid in the pore space and the above effect is not an issue.

The coupling between seismic and electromagnetic disturbances through charge transport induced by mechanical induced relative flow is probably the largest of the various other coupling mechanisms.

There are many other coupling mechanisms between seismic waves and other fields. An example is the seismomagnetic coupling between the earth's magnetic field and a seismic wave (Mikhailenko, 1991). These seismomagnetic effects arise after an explosion in an elastic media and are described by the elastic wave equation with added Lorentz force and Maxwell's equations with displacement velocity components \(E = \frac{\partial u}{\partial t} \times \frac{Ha}{\mu_0}\), with \(H_0\) the earth's magnetic field, \(O(10^{-4} T)\).

The influence of the earth magnetic field on the electroseismic conversions is completely negligible and therefore neglected in this thesis. The current generated by the Lorentz force is much smaller than that generated by convection of the free double layer ions.

La Cruz and Spanos (1989b, 1992) studied thermomechanical coupling during seismic wave propagation in a porous medium. They concluded that when thermomechanical coupling is an important attenuation mechanism in one of the media it is also observed to have a substantial effect on the mode conversions which occur at
the boundary (La Cruz and Spanos, 1992).

Although these other coupling mechanisms, thermal conduction, chemical reactions etc., are not studied in this thesis, it is possible to formulate theoretically the governing equations and define a transport equation system or Onsager matrix that relates the wave and/or diffusive fields to induced fluxes, Groot and Mazur (1984), which contain all coupling. The more involved equation system would have a similar structure as the electroseismic equation system. The solution procedure would in principle be similar to the approach discussed in this thesis.

1.2 Outline of the Thesis

This thesis is divided into eight chapters. Chapter 1 defines the subject of the thesis, reviews the background work concerning electroseismics and coupled wavefield phenomena in two phase poro-elastic formations, and outlines the studies covered by this thesis.

In chapter 2 mechanically induced streaming currents generated by seismic point sources in homogeneous isotropic porous media are studied. First the coupled transport equations in porous rock are discussed. These flux/force relations relate the current and the relative flow to potential and pressure gradients. The macroscopic electromagnetic and acoustic-elastic wave fields that drive the current and fluid flow can be obtained by solving the coupled poro-elastic (modified Biot's) and electromagnetic (Maxwell's) equations. In this study we decouple the mechanical wave behavior from the electromagnetic wave behavior to simplify the analysis of induced fluxes (relative fluid flow and current flow). The relative flow is determined by Green's function solution. The mechanically induced relative flow, decoupled from the electrically induced relative flow is used to determine the dynamic streaming current generated by mechanical point sources. The streaming current induced by the electrically induced
relative flow is shown to be second order in electrokinetic coupling coefficient and is therefore a second order effect and neglected.

Streaming current radiation patterns are calculated for an explosive source acting on the bulk, a pressure source acting on the fluid phase, and a vertical point force acting on the bulk using the stationary-phase method. The stationary-phase method allows a fast computation of the radiation patterns and no frequency assumption is required for this approach. The dynamic streaming current amplitude behavior with respect to porosity, permeability, and fluid chemistry is investigated at three different frequencies. Laboratory measurements are necessary to verify the obtained results. Experimental data analysed with the developed simple and rapid stationary-phase, theoretical streaming current amplitude predictions, as function of salinity or permeability, could provide lithological information of the homogeneous porous rock sample.

In chapter 3 a displacement-stress-EM wavefield vector is derived which is continuous through jumps in the macroscopic material properties. Wavefield eigenvectors and eigenvalues (wavefield slownesses) are derived for the up and downgoing waves in poro-elastic media (i.e. fast compressional wave, slow compressional wave, rotational waves, and electromagnetic waves). To guarantee a consistent macroscopic theory of energy transfer, conservation, and continuity of electromagnetic and poro-elastic Poynting power upon crossing, an interface is imposed. This results in a physical normalisation of the wavefield eigenvectors and also determines the wavefield vector components that are continuous across an interface.

In chapter 4 the macroscopic governing equations controlling the coupled electromagnetics and acoustics of porous media are numerically solved for the case of a layered poro-elastic medium. It is shown that these coupled equations decouple into two equation sets describing two uncoupled wavefield pictures. That is, the $PSVTM$ picture where the compressional and vertical polarized mechanical waves drive currents in the $PSV$ particle motion plane that couples to the electromagnetic wavefield components of the $TM$ mode. And the $SHTE$ picture where the horizontal polar-
ized rotational mechanical waves drive currents in the $SH$ particle motion plane that couples to the electromagnetic wavefield components of the $TE$ mode. The global matrix method is employed in computing electroseismograms in layered poro-elastic media in the $PSVTM$ picture.

The principal features of the converted electromagnetic signals are: (1) contacts all antennas at approximately the same time; (2) arrives at the antennas at half of the seismic travel time at normal incidence reflected $P$ waves; (3) changes sign on opposite sides of the shot.

The seismic pulse is shown to induce electric fields that travel with the compressional wavespeed and magnetic fields that travel with the rotational wavefield. The frequency content of the converted electromagnetic field has the same frequency content of the driving incident seismic pulse, as long as the propagation distances are much less than the electromagnetic skin depth.

Snapshots in time and converted electromagnetic amplitudes versus seismic point source - antenna offset are calculated for contrasts in mechanical and/or electrical medium property. Conversion happens there where the seismic wavefront passes a contrast in medium properties due to generated imbalances in current across the contrast. The $TM$ component amplitude radiation pattern away from the interface shows similarities with an effective electric dipole radiation pattern, or its dual an effective magnetic current loop radiation pattern centered right beneath the source at the contrast's depth. The $TM$ mode amplitudes decay rapidly with travelled distance and suggest the importance of a Vertical ElectroSeismic Profiling geometry to enhance recording of converted electromagnetic signals by positioning the antennas closer to the target (contrast) of interest.

In chapter 5 we first derive concise and numerically feasible relative flow and displacement Green's functions in dyadic form (these Green's functions are also used in chapter 2 to calculate the mechanically induced electrical streaming currents). We use an extended version of the Kupradze method which solves the wave equations and
force balance equations for a delta forcing function. The dynamic Green's function is expressed through three scalar quantities characterizing the propagation of \( SH \) and \( P_f - P_s - SV \) waves in a poroelastic isotropic medium. The final dyadic form is similar in form to the isotropic elastic dyadic Green's function. To solve simultaneously the macroscopic electromagnetic and acoustic-elastic wavefields in layered media we use the global matrix method. We derive source jump representations for an explosive and vertical point source using the relative flow and displacement Green's functions together with the deformation equations in the two phases. These mechanical sources generate fluid flow and electrical streaming currents in the porous medium. Both current and fluid flow are related by the transport equations through which all electromagnetic and mechanical coupling occurs.

We compare explosive and vertical point source behavior in two different geometries. The first example set is calculated in a surface seismic geometry and the second one in a Vertical ElectroSeismic Profiling (\( VESP \)) geometry. The conversion to electromagnetic waves from mechanical waves traversing boundaries is found to be mainly due to pressure gradients generated by \( P \) waves across the contrast. The conversion from rotational waves into electromagnetic waves is found to be much smaller.

In chapter 6 we present a method to simulate the propagation of seismic and converted electromagnetic waves generated by a mechanical borehole source embedded in a layered poro-elastic medium. The electroseismic conversions occur at both the borehole wall and layer boundaries. Most studies in electroseismic effects have been modeled and tested with seismic sources and detectors (geophones and antennas) at the surface. In this chapter we investigate the case of a seismic source in a borehole and receivers in the borehole, at the surface or embedded in the medium. The method is formulated as a boundary element technique where the poro-elastic displacement and relative flow Green's functions are calculated by the discrete wavenumber method. The singular properties of the Green's functions are determined analytically using static Green's functions to regularize the integrals, this is necessary to calculate the
element's self interaction. The borehole is cylindrical and its axis runs normal to the interfaces. The coupled electroseismic effects in the layered media are included by using the global matrix technique.

The developed method is an extension of the Biot-Rosenbaum (Rosenbaum, 1974) model, who applied the wavenumber integration technique to investigate the effect of formation permeability on Stoneley waves using Biot theory to model wave propagation effects of a homogeneous permeable formation surrounding a borehole. We extend the Biot-Rosenbaum model by including the effect of a heterogeneous permeable formation surrounding the borehole. The effect of formation permeable zones (or fractured zones) on Stoneley waves can now be investigated. The other modification is the inclusion of conversions of mechanical into electromagnetic waves at mechanical and/or electrical contrasts in the poro-elastic formation. The converted electromagnetic fields are sensitive to large permeability contrasts and fluid chemistry contrasts inside a reservoir. Using the electroseismic method downhole provides more information about permeability contrasts in the formation and additional lithological information (salinity of the fluids) (Parkhomenko and Gaskarov, 1971).

In chapter 7 we model two electroseismic data sets acquired at the Haney test site, near Vancouver, Canada. The objective in calculating a synthetic in Transpose Vertical ElectroSeismic Profiling (TVESP) geometry is to identify the electroseismic conversion responses from an organic rich soil-glacial till interface and a glacial till-bedrock interface. Three electroseismic conversions are identified in the dataset. Two $P$ wave to electromagnetic wavefield conversions are found to originate from the soil-till and till-bedrock contrast respectively. We call this electroseismic conversions of the first kind. The last electromagnetic conversion originates from a traveling $P$ wave along the interface. We call this electroseismic conversion a conversion of the second kind.

The objective in calculating a synthetic in surface seismic geometry is to obtain an electric field amplitude versus antenna offset curve of the converted electroseismic
signal of the first kind which we compare against the real data result. The model predicts the antenna offset and electric field amplitude at which the field is maximum. Two amplitude drop off regimes with offset can be identified. The second data set and simulation also shows a conversion originating from an interface wave moving with $SV$ wavespeed of the glacial till (an electroseismic conversion of the second kind).

Chapter 8 summarizes the important results and major conclusions of this thesis. Also future directions of electroseismic research, both at seismic and ultrasonic frequencies will be discussed. The mathematical details and summaries of derived equations used throughout the thesis are in the appendices.
The ElectroSeismic Method

Figure 1-1: The electroseismic method, where seismic motion in two phase media generates mechanical relative flow that induces an electrical streaming current. When seismic waves traverse a contrast in electrical and/or mechanical properties a dynamic current imbalance is induced across the interface, generating an electromagnetic disturbance.
Chapter 2

Dynamic Streaming Currents from Seismic Point Sources in Homogeneous Poro-Elastic Media.

2.1 Introduction

2.1.1 Electrokinetics at the Pore Scale

Electrokinetic phenomena are usually studied with respect to solid and liquid interfaces. When two phases of different chemical composition are in contact, an electric potential difference develops between the two interfaces. The anions from the electrolyte are adsorbed to the solid interface (i.e. a quartz matrix) leaving behind a net excess of cations distributed near the wall, producing an electrical double layer. There are several models of the electric double layer. One is the Gouy-Chapman type model (Bockris and Reddy, 1970; Dukhin and Derjaguin, 1974), a diffuse layer model where
the counter ions are attracted to the surface only by electrostatic forces. Another model of the solid/fluid interface which is currently favored is largely due to Stern (1924), who argued that the electrolyte ions and structured (hydrogen bonded) water molecules are not only electrostatically but also chemically adsorbed to the surface of the solid. This absorbed Helmholtz layer (Overbeek, 1952; Bockris and Reddy, 1970) contains immobile adsorbed ions and can also include ionized surface sites present on the grain surfaces, as shown in figure 2-1. Beyond this first layer of bound ions, there is a diffuse distribution of mobile ions whose position is determined by the Poison-Boltzman statistics. In the diffuse layer the ions are under the continued influence of the ordering chemical and dis-ordering thermal forces. The Stern model is therefore a composite of a Helmholtz layer and a Gouy-Chapman diffusion zone. The separation plane between the mobile and immobile charge is called the hydrodynamic slipping or shear plane. The potential at this shear plane is the $\zeta$-potential, which value is often used to characterize the double layer. The electric potential in neutral electrolyte (no excess charge) is defined to be zero. Modifications to the Stern model have been proposed. In particular, Graham (1947) suggested splitting the Helmholtz layer into an inner and outer region. The inner region is populated by dehydrated ions stuck to the bare solid with specific adsorption forces, while the outer region would contain the partly hydrated ions and structured (hydrogen bonded) water molecules which touch a hydrated solid rather than stick to a bare solid. There is experimental evidence (Derjaguin et al., 1987) suggesting that long range chemical forces from adsorption sites on the quartz surface may align water molecules for up to several hundred molecular distances away from the wall.

An important detail concerning the adsorbed layer is the fact that chemical adsorption of dissociated $OH^-$ and $H^+$ ions at the quartz surface is more dominant than absorption of the electrolyte anions in determining the potential (Gaudin and Fuerstenau, 1955; Fuerstenau and Modi, 1959). Interesting discussions that show the performance of the theoretical Stern model with respect to aqueous organic electrolyte
solutions can be found in Anderson (1959) and Fuerstenau and Modi (1959). The absorption mechanism involves now a combination of electrostatic attraction and long hydrocarbon chain association (Van der Waals attraction) which can be incorporated into the Stern framework of double layer description.

In this chapter we study the effects of dynamic relative motion between a solid and monovalent anorganic electrolytes. The Stern electrical double layer description is used to characterize the grain-electrolyte interfaces. The fluid is assumed to be an ideal electrolyte thus restricting salt concentrations to less than 1.0 mol/l. At this molarity the ion separation distance becomes such that electrostatic interaction energy becomes significant compared to thermal energy. There are numerous phenomena at the shear plane defined by its $\zeta$-potential. Examples of these 'electrokinetic phenomena' are electro-osmosis, electrophoresis and streaming potential (Bockris and Reddy, 1970). The $\zeta$-potential is fundamental in all these kinetic phenomena and will be discussed later on in this chapter.

### 2.1.2 Macroscopic Transport Coefficients

Transport equations to describe electrokinetic phenomena in a porous medium can be treated within the framework of non-equilibrium thermodynamics (Groot and Mazur, 1984). For linear processes the flux/force equations relating the conservation equations for fluid flow and current flow when there are electric potential and fluid pressure gradients are,

\[
\vec{J} = \sigma(\omega) \vec{E} + L(\omega) \left[ -\nabla P + i\omega \rho_j \vec{u}_s \right]
\]

\[
\tilde{w} = \frac{k(\omega)}{\eta} \left[ -\nabla P + i\omega \rho_j \vec{u}_s \right] + L(\omega) \vec{E}
\]

$\vec{J}$ is the electric current density, $\sigma(\omega)$ is the frequency dependent conductivity of the two phase medium, $k(\omega)$ the dynamic permeability, $L(\omega)$ the dynamic electrokinetic coupling coefficient and $\eta$ the fluid viscosity. The electric field strength $\vec{E}$ can be
expressed as the gradient of a potential, $\mathbf{E} = -\nabla \Phi$. The pressure $P$ in the pore fluid which has a density $\rho_f$, $\mathbf{u}_f$ is the displacement in the bulk material and $\mathbf{w}$ is the relative fluid-solid motion. The dots above the $\mathbf{u}$ and $\mathbf{w}$ vectors denote a derivative with respect to time. The first term in equation (Eq 2.1) is the conduction current contribution and the second term is the streaming current contribution to the total current. The forces which drive the relative flow are a combination of pressure gradients set up by a compressional wave, and by grain acceleration. The relative flow caused by grain acceleration can therefore be both due to compressional and shear waves. The first term in equation (Eq 2.2) represents Darcy's law. The second term in the same equation describes the amount of fluid flow caused by the electric field that drives the hydrated mobile excess charge in the diffuse boundary layer, an osmosis phenomenon.

The fact that the cross terms have the same coupling coefficients $L(\omega)$, a symmetry which is one of the fundamental results in the theory of irreversible processes is a statement of Onsager reciprocity (Onsager (1931a,b)). The coefficient equality implies the action of the electric field on the charge density to be reciprocal to the action of the fluid pressure on the charge. In other words, the mobility of the ion is independent of the charge moving through the fluid or the fluid moving around the charge.

In order to give explicit expressions for electrokinetic coefficient $L$ for porous media, the capillary model is often taken. The porous medium is assumed to be a composite of a bundle of capillaries. This approach, to obtain expressions for electrokinetic coupling coefficient is taken among others by Levine et al., (1975), Broz and Epstein (1976), Anderson and Koh (1977), Donath and Voigt (1986), and Manzanares et al. (1991). Another approach is to study the behavior of these coefficients numerically. Jin and Sharma (1991) developed a numerical model for electrochemical and electrokinetic coupling in inhomogeneous porous medium. Their network approach solves the coupled transport processes in inhomogeneous charged membranes/pores.
The numerical results provide insight into the relationships among pore structure, flow properties and coupling coefficients. They conclude that several obvious characteristics of real porous media are not present in capillary based network models. These differences between model and experiment are systematically discussed in Sharma (1988) and Kuo et al. (1988). Their streaming potential measurements conducted on mixtures of alumina and silicia yield results that differ significantly from that predicted by a simple weighted average capillary tube model. The streaming potential was found to be a function of the grain sizes of the individual components. This grain size effect on streaming potential in artificial spherical grain sands is also observed in data collected by Schriever and Bleil (1957).

Wurmschich and Morgan (1994) solved the 3-D static transport equations by a 3D finite difference scheme to investigate whether streaming potential measurements in boreholes and at the surface can be used to monitor subsurface flow and to detect subsurface flow patterns in oil reservoirs. Their model was limited by inadequate knowledge of the electrokinetic cross-coupling coefficients, which are important for determining streaming current/streaming potential magnitudes as a function of medium properties.

The coupling coefficients used in this paper, which are frequency dependent, are explicitly obtained by a volume average procedure of Maxwell's equations coupled to Biot's equations (Pride, 1994). They obtained low and high frequency limits of the transport coefficients and connected these functions with a smooth and simple postulated function analogous to Johnson et al.'s in their approach to define dynamic permeability.

Two possible functions that smoothly connect the low and high frequency regimes of the transport coefficients are;

$$\frac{k(\omega)}{k_0} = \left[1 - i \frac{\omega}{\omega_t} \frac{4}{m}\right]^{1/2} - i \frac{\omega}{\omega_t}^{-1}$$  (2.3)
\[
\frac{L(\omega)}{L_0} = \left[ 1 - i \frac{\omega}{\omega_t} \frac{m}{4\alpha_\infty} \left( 1 - 2\alpha_\infty \frac{d}{\Lambda} \right)^2 \left( 1 - i^{3/2} \frac{\omega \rho_f}{\eta} \right)^2 \right]^{-1/2}
\]  
(2.4)

The transition frequency \( \omega_t \) separates the low frequency viscous flow behavior from the high frequency inertial flow and is defined by,

\[
\omega_t = \frac{\phi}{\alpha_\infty k_0 \rho_f} \eta
\]  
(2.5)

and the dimensionless number \( m \) by,

\[
m = \frac{\phi \Lambda^2}{\alpha_\infty k_0}
\]  
(2.6)

This number consists only of geometry terms.

The static coupling coefficient \( L_0 \) is given by,

\[
L_0 = -\frac{\phi}{\alpha_\infty} \frac{\epsilon_0 \kappa_f \zeta}{\eta} \left[ 1 - 2\alpha_\infty \frac{d}{\Lambda} \right]
\]  
(2.7)

where \( \kappa_f \) is the relative fluid permittivity, \( \zeta \) is the zeta potential, \( \alpha_\infty \), the tortuosity (normally a number between 3 and 10) and \( \phi \) is the porosity. The \( \sqrt{\frac{\omega \rho_f}{\eta}} \) factor in equation (Eq 2.4) determines the viscous skin depth and \( \tilde{d} \leq \frac{3 \times 10^{-10}}{\sqrt{C}} \equiv \text{Debye length} \) (is a measure of double layer thickness) and \( C \) is the electrolyte concentration. The pore length parameter, \( \Lambda \), is the same as defined by Johnson, Koplik and Dashen, (Johnson et al., 1987).

An important factor affecting the electrokinetic coefficient is the \( \zeta \)-potential (see equations (Eq 2.4) and (Eq 2.7)). Theoretical determination of \( \zeta \) remains problematic due to the complexity of the adsorbed layer. The \( \zeta \)-potential employed in this paper is taken from experimental studies. A variety of researchers have determined the \( \zeta \)-potential as a function of electrolyte concentration from streaming potential data. The \( \zeta \)-potential we values are based on saturated NaCl, KCl quartz samples at \( T = 25^\circ C \) and \( \text{pH} = 7 \), determined by Gaudin and Fuerstenau (1955), Sidorva et al. (1975), and Hidalgo-Alvarez et al. (1985). Pride and Morgan (1991) applied regression analysis to their combined data and obtained the following expression for the \( \zeta \)-potential as
function of electrolyte concentration,\n\[
\zeta(V) = 0.008 + 0.026 \log_{10}(C). \tag{2.8}
\]

There have been few studies determining \(\zeta\)-potential for naturally occurring geological samples such as sandstones. Sharma et al., (1987) determined \(\zeta\)-potential from streaming potential data for samples of Berea sandstones. They obtained \(\zeta\)-potential values at \(pH = 7\) and \(T = 25^\circ C\) for \(10^{-4} - 10^{-3} M\) NaCl electrolyte which compare to the values obtained using equation (Eq 2.8) for a specific concentration. Morgan et al. (1989) and Ishido and Mizutani (1981) determined experimentally from streaming potential data \(\zeta\)-potentials as a function of temperature. Morgan found that \(\zeta\) was independent of temperature for crushed Westerly granite in \(10^{-2} M\) NaCl at \(pH = 5.5\). While Ishido found \(\zeta\) to change in magnitude by about -65 mV/\textdegree C for quartz in \(10^{-3} M\) \(\text{KNO}_3\) at \(pH = 6.1\). Streaming potential data collected in artificial spherical-grain sands (Schriever and Bleil, 1957) show that the \(\zeta\)-potential has a positive temperature coefficient whose approximate value is 0.039 \(\text{/}^\circ C\), and the streaming potential to have a positive temperature coefficient of 0.0535 \(\text{/}^\circ C\). More experimental work is needed to understand the \(\zeta\)-potential versus \(T\) relation. In our numerical calculations we used a constant 298 \(K\) temperature.

2.1.3 Sensitivity of Transport Coefficients to Porosity and DC Permeability

To investigate the transport coefficient behavior with respect to porosity and permeability changes we calculate the \(k_0/k(\omega)\) and \(L_0/L(\omega)\) ratios as function of frequency using equations (Eq 2.3) and (Eq 2.4). The real and imaginary parts of the \(k_0/k(\omega)\) and \(L_0/L(\omega)\) ratios versus porosity are shown in figures 2-4 and 2-5, the same ratios versus permeability are shown in figures 2-6 and 2-7.

There is a distinct frequency relaxation present in the \(k(\omega)\) and \(L(\omega)\) transport
coefficients. This relaxation frequency demarcates the transition from viscous flow to inertial flow. The \( k(\omega) \) relaxes at frequency \( \omega_t \), with \( \omega_t \) given in (Eq 2.5), while \( L(\omega) \) relaxes at a frequency \( \omega_t/m \), with \( m \) given in (Eq 2.6). The \( k(\omega) \) amplitude decreases as \( \omega \) when \( \omega > \omega_t \) while \( L(\omega) \) decreases as \( \omega^{1/2} \) when \( \omega > \frac{4\alpha^2 \omega_t}{m} \) (see \( Re[k_0/k(\omega)] \) and \( Re[L_0/L(\omega)] \) in figures 2-4, 2-5 and equations (Eq 2.3) and (Eq 2.4)).

The conductivity \( \sigma(\omega) \) which controls the conduction current relates current density to electric field strength and is defined by (Pride (1994),

\[
\sigma(\omega) = \phi \left[ \frac{\sigma_f}{\alpha_\infty} + \frac{2}{\Lambda} (C_{em} + C_{os}(\omega)) \right] \tag{2.9}
\]

With \( C_{em} \) and \( C_{os} \), the conductances due to electromigration and osmosis along the grain-pore fluid interfaces respectively,

\[
C_{em} = 4\tilde{d}q^26 \times 10^{26} C \left[ \cosh\left(\frac{2\zeta}{2kT}\right) - 1 \right] \tag{2.10}
\]

\[
C_{os}(\omega) = \frac{(\varepsilon_0\kappa_f\zeta)^2}{2\tilde{d}\eta} \left[ 1 - i^{3/2} \sqrt{\frac{\omega\rho_f}{\eta}} \right]^{-2} \tag{2.11}
\]

With \( q \), the elementary electron charge; \( C \), salinity; \( k \), Boltzmann’s constant; \( T \), temperature and \( \tilde{d} \), the Debye length which is a measure of the thickness of the diffuse double layer and defined as,

\[
\tilde{d} = \sqrt{\frac{q^2z^2n}{\varepsilon_0\kappa_f kT}} \tag{2.12}
\]

With \( z \) the ion valence and \( n \) the ionic concentration.

\( \sigma_f \) is the conductivity of the fluid and defined as,

\[
\sigma_f = (qz)^2bC6.022 \times 10^{26} \tag{2.13}
\]

With \( qzb \) the ionic mobility of charge and \( C6.022 \times 10^{26}qz \) the total amount of charge, \( 6.022 \times 10^{23} \) is Avogadro’s number.

In figures 2-8 and 2-9 the conductivity versus porosity and permeability are shown. The conductivity increases monotonically with increasing porosity. The conductivity
doesn't change linearly with porosity due to the porosity dependence in the $\Lambda[m]$, pore length parameter.

The conductivity decreases with increasing permeability. At a $10^{-16} \, m^2$ permeability, the $\Lambda$ parameter becomes comparable in value with the osmotic (Eq 2.11), and electromigration, (Eq 2.10), conductances. At a constant porosity and permeabilities less than $10^{-16} \, m^2$, the surface conductances become more important than the fluid phase conductivity. In figure 2-9 this behavior shows itself as a sharp increase in conductivity at the low permeability end of the figure. Physically this situation occurs when the pore space is filled with clays which don't decrease the porosity but greatly enhance the surface to pore volume ratio and therefore enhance the conduction along the pore surfaces.

2.1.4 Macroscopic Coupled Electromagnetic and Poro-Elastic Field Equations

The macroscopic fully coupled mechanical, electromagnetic equations and constitutive relations in volume averaged form describing the coupled field behavior in two phase porous medium are,

The electromagnetic field equations,

$$\nabla \times \vec{E} = i\omega \vec{B} \quad (2.14)$$

$$\nabla \times \vec{H} = -i\omega \vec{D} + \vec{J} \quad (2.15)$$

$$\vec{D} = \epsilon_0 \left[ \frac{\phi}{\alpha_{\infty}} (\kappa_f - \kappa_s) + \kappa_s \right] \vec{E} \quad (2.16)$$

$$\vec{B} = \mu_0 \vec{H} \quad (2.17)$$

The transport equations,

$$\vec{J} = \sigma(\omega)\vec{E} + L(\omega) \left[ -\nabla P + \omega^2 \rho_f \vec{z} \right] \quad (2.18)$$
\[ -i\omega \mathcal{W} = L(\omega)E + \frac{k(\omega)}{\eta} \left[ -\nabla P + \omega^2 \rho_f u_s \right] \]  

(2.19)

The mechanical field equations,

\[ \nabla \cdot \mathcal{I}_B = -\omega^2 [\rho_B u + \rho_f \mathcal{W}] \]  

(2.20)

\[ \mathcal{I}_B = [K_G \nabla \cdot u_s + C \nabla \cdot \mathcal{W}] l + G_{fr} \left[ \nabla u_s + \nabla u_s^T - \frac{2}{3} \nabla \cdot u_s l \right] \]  

(2.21)

\[ P = C \nabla \cdot u_s + M \nabla \cdot \mathcal{W} \]  

(2.22)

Equations (Eq 2.14) - (Eq 2.17) describe the electromagnetic wave fields and equations (Eq 2.20) - (Eq 2.22) the mechanical wavefields. All coupling is present in the transport equations (Eq 2.18) - (Eq 2.19).

The coefficients in the deformation equations are,

\[ K_G = H - \frac{4}{3} G = \frac{K_{fr} + \phi K_{fr} + (1 - \phi)K_s \Delta}{1 + \Delta} \]  

(2.23)

\[ C = \frac{K_f + K_s \Delta}{1 + \Delta} \]  

(2.24)

\[ M = \frac{1}{\phi} \frac{K_f}{1 + \Delta} \]  

(2.25)

Where the parameter \( \Delta \) is defined as,

\[ \Delta = \frac{K_f}{\phi K_s^2} [(1 - \phi)K_s - K_{fr}] \]  

(2.26)

The moduli \( K_{fr} \) and \( G_{fr} \) are the bulk and shear moduli of the framework of the grains, when the fluid is absent. The frame moduli may either be considered experimentally determined or may be obtained from approximate theoretical models for specific pore grain geometries. \( C \) and \( M \) are the incompressibilities used by Biot (1962a,b), they are complex and frequency dependent, allowing for losses in addition to those associated with relative flow.

### 2.1.5 Mechanically Induced Streaming Currents

The macroscopic current density in equation (Eq 2.18) has three distinct contributions, diffusion current density, conduction current density and streaming current density.
If frequencies are in the seismic frequency range of interest macroscopic diffusion currents can be neglected. Here we will concentrate on the dynamic mechanical streaming current contribution to the total macroscopic current. From the transport relations we deduce the streaming current induced by relative flow. Since there is an electrically and a mechanically induced relative flow, we also have electrically and mechanically induced streaming currents. Substituting equation (Eq 2.2) into equation (Eq 2.1), the total streaming current can be written as,

$$ J_s = -i\omega \frac{\eta}{k(\omega)} L(\omega)w - L(\omega)^2 E $$

(2.27)

We neglect the electrically induced part of the streaming current since it is second order in electrokinetic coupling coefficient $L[Cs/kg]$, also derived by Neev and Yeatts (1989). Defining an effective fluid density operator (Pride et al., 1993) that controls the magnitude of the wave induced relative flow $\rho_E[kg/m^3] = -\frac{\eta}{i\omega k(\omega)}$, the mechanical part of the streaming current can be written as,

$$ J_{sm} = -\omega^2 \rho_E L(\omega)w $$

(2.28)

The relative flow vector is obtained by Green's function solution, see appendix C. This relative fluid flow driven by pressure gradients and grain acceleration is determined from linearized force balances. In Pride et al. (1993) a geometric condition on non-linear flow in a homogeneous poro-elastic medium is obtained. The non-linear convection term of fluid flow is considered negligible if the pore radii are less than 100 $\mu m$ for water solution saturated media under a maximum allowed strain of $10^{-6}$. It is assumed that the above conditions are satisfied in the numerical modeling. Therefore the effect of turbulence upon electrokinetic phenomena (i.e. streaming potentials/currents) (Rutgers, 1957; Kurtz et al. (1976) ) can be ignored in this study.

Measurements to determine electrokinetic properties of capillary surfaces or porous media have been traditionally performed under static electric potential gradient or pressure gradient. Transient streaming potential modeling using Biot's consolidation work in the quasi static limit to obtain quasi static flow which drives a streaming
current through a porous medium was studied by Chandler (1981) and Chandler and Johnson (1981). The performed experimental studies of diffusive flow from transient pressure pulses in various porous structures, where streaming potentials were used to monitor fluid pressure, show that the Biot slow wave can be detected with the streaming potential. He proposed a streaming potential based logging tool that would indicate permeability variations.

Dynamic measurements of streaming potentials and streaming currents are discussed in Packard (1952), Groves and Sears (1975), Sears and Groves (1978), and Cerda and Non-Chom (1989). Their experiments are based on sinusoidally alternating fluid flow through capillaries and porous media to measure electrokinetic phenomena.

Direct measurements of streaming currents can be found in Rutgers et al. (1959) and Hurd and Hackerman (1955). They argue that investigations of electrokinetic phenomena traditionally have been restricted almost exclusively to measurements of the streaming potential because of the exceedingly low streaming currents. The advantage of measuring streaming currents is not to have to know the value of the surface conductance to determine the ζ-potential, which needs to be determined when streaming potential measurements are made. An additional disadvantage of streaming potential measurements is their sensitivity to temperature due to the temperature dependence of resistivity. Groves and Sears designed an experiment to measure alternating streaming currents resulting from monochromatic sinusoidally alternating fluid flow.

In this chapter we investigate numerically the effect of porosity, permeability and fluid chemistry on the dynamic streaming currents caused by point forces in porous media. The numerical results can be useful in designing an experiment to measure dynamic streaming currents generated by point sources/transducers in poro-elastic media. The theoretical streaming current prediction could possibly be used to invert for a medium parameter, like permeability, from dynamic streaming current data, provided the data is sensitive to the particular medium parameter.
2.2 Dynamic Streaming Currents Mechanically Induced by Point Forces

Using the relative flow Green’s function solution for a point source applied to the frame and fluid, equations (Eq C.7) and (Eq C.16) respectively, we obtain the relative flow vector generated by vertical point forces and explosive point forces (volume injection sources). Substituting this relative flow vector into the mechanical streaming current equation, (Eq 2.28) and using electrokinetic coefficient \( L(\omega) \) defined in equation (Eq 2.4), the dynamic streaming current components can be determined. We first calculate the streaming current components generated by a vertical point source applied on the frame. Then the streaming currents for an explosive point source applied on either the solid or fluid phase are given.

Vertical Point Force on Solid

The mechanical streaming current is calculated for a vertical point force (parallel to the symmetry axis) at the origin, \( \mathbf{F}(x) = \hat{z}\delta(x) \). Using Green’s theorem,

\[
\mathbf{w} = \int\mathbf{G}^w(\mathbf{x}) \cdot \mathbf{F}(x')dx' = g^w \hat{z} - \psi \nabla \frac{\partial}{\partial z} \Phi - \gamma \nabla \frac{\partial}{\partial z} \Phi^* \tag{2.29}
\]

With,

\[
\psi = \frac{1}{C} \left[ C^2 - HM + GC \frac{\rho_E}{\rho_f} \right] \tag{2.30}
\]

\[
\gamma = \frac{\omega^2 \rho_f \frac{HM}{C} + \omega^2 C \frac{\rho B \rho_E}{\rho_f}}{C} - H \omega^2 \rho_E - M \omega^2 \rho_B \tag{2.31}
\]

Where \( \mathbf{G}^w, g^w, \Phi, \Phi^* \) are defined in equations (Eq C.7), (Eq C.12), (Eq C.8) and (Eq C.10) respectively. Writing out in component form the following equations are obtained,

\[
J_r^t = \frac{1}{8\pi} \omega^2 \rho_E L(\omega) \int_{-\infty}^{\infty} \left[ l W_\beta H_1^{(2)}(lr) + m W_\alpha H_1^{(2)}(mr) \right]
\]
\[ J^*_z = \frac{i}{8\pi} \omega^2 \rho_E L(\omega) \int_{-\infty}^{\infty} k_z \left[ W^x_{\beta} H_0^{(2)}(m \rho) + W^x_{\alpha_1} H_0^{(2)}(m \rho) \right] e^{-ik_z z} d\rho \] (2.33)

Where the introduced variables, \( W^p_r \) and \( W^p_z \), with \( p = \beta \lor \alpha_1 \lor \alpha_2 \) are defined as,

\[ W^r_{\beta} = \psi B(ws) + \gamma B^*(ws), \quad W^z_{\beta} = \frac{-1}{G_{\rho_E}} \frac{1}{k_z^2} + \psi B(ws) + \gamma B^*(ws) \]
\[ W^r_{\alpha_1} = \psi A_1(ws) + \gamma A_1^*(ws), \quad W^z_{\alpha_1} = \psi A_1(ws) + \gamma A_1^*(ws) \]
\[ W^r_{\alpha_2} = \psi A_2(ws) + \gamma A_2^*(ws), \quad W^z_{\alpha_2} = \psi A_2(ws) + \gamma A_2^*(ws) \] (2.34)

With \( B(ws), A_q(ws) \) defined in equation (Eq C.9) and \( B^*(ws), A_q^*(ws) \) defined in equation (Eq C.9), with \( q = 1 \lor 2 \).

**Explosive Point Source on Solid and Fluid**

The relative flow vector for an explosive point source at the origin can be obtained by taking the divergence of the relative flow Green's function with respect to the source coordinates.

\[ \mathbf{w} = \nabla g^w - \psi \nabla \nabla^2 \Phi - \gamma \nabla \nabla^2 \Phi^* \] (2.35)

Where \( G^w, g^w, \Phi, \Phi^*, \psi, \gamma \) are defined in equations (Eq C.7), (Eq C.12), (Eq C.8), (Eq C.10), (Eq 2.30) and (Eq 2.31) respectively. The two relative flow components are,

\[ J^r = -\frac{i}{8\pi} \omega^2 \rho_E L(\omega) \int_{-\infty}^{\infty} k_z \left[ m W^r_{\alpha_1} H_1^{(2)}(m z) + n W^r_{\alpha_2} H_1^{(2)}(m z) \right] e^{-ik_z z} dz \] (2.36)
\[ J^z = \frac{1}{8\pi} \omega^2 \rho_E L(\omega) \int_{-\infty}^{\infty} k_z \left[ W^z_{\alpha_1} H_0^{(2)}(m z) + W^z_{\alpha_2} H_0^{(2)}(m z) \right] e^{-ik_z z} dz \] (2.37)

Where the introduced variables, \( W^p_r \) and \( W^p_z \), with \( p = \alpha_1 \lor \alpha_2 \) are defined as,

\[ W^x_{\alpha_1} = W^r_{\alpha_1} = (\psi A_1 + \gamma A_1^*) k^2_{\alpha_1} \]
\[ W^x_{\alpha_1} = W^z_{\alpha_1} = (\psi A_2 + \gamma A_2^*) k^2_{\alpha_1} \] (2.38)
With \( A_q(ws) \) defined in equation (Eq C.9) and \( A^*_q(ws) \) defined in equation (Eq C.9), with \( q = 1 \lor 2 \). In isotropic media only compressional waves are generated by an explosive point source.

The relative flow vector caused by a volume injection source is,

\[
\mathbf{w} = \nabla \Upsilon^w_f
\]

(2.39)

With \( \Upsilon^w_f \) defined in (Eq C.16) The induced mechanical streaming current components are,

\[
J^r_z = -\frac{i}{8\pi} \omega^2 \rho E L(\omega) \int_{-\infty}^{\infty} \left[ m W^r_{\alpha_1} H^{(2)}_1(mr) + n W^r_{\alpha_2} H^{(2)}_1(nr) \right] e^{-ik_z z} dk_z
\]

(2.40)

\[
J^s_z = \frac{1}{4\pi} \omega^2 \rho E L(\omega) \int_{-\infty}^{\infty} k_z \left[ W^s_{\alpha_1} H^{(2)}_0(mr) + W^s_{\alpha_2} H^{(2)}_0(nr) \right] e^{-ik_z z} dk_z
\]

(2.41)

Where the introduced variables, \( W^r_p \) and \( W^s_p \), with \( p = \alpha_1 \lor \alpha_2 \) are defined as,

\[
W^r_{\alpha_1} = W^s_{\alpha_1} = \Lambda_1
\]

\[
W^r_{\alpha_2} = W^s_{\alpha_2} = \Lambda_2
\]

(2.42)

With \( \Lambda_1 \) and \( \Lambda_2 \) defined in equations (Eq C.17). And where the following horizontal wavenumbers are used, \( l = \left[ \frac{\omega^2}{\beta^2} - k_z^2 \right]^{1/2} \), \( m = \left[ \frac{\omega^2}{\alpha_1^2} - k_z^2 \right]^{1/2} \) and \( l = \left[ \frac{\omega^2}{\alpha_2^2} - k_z^2 \right]^{1/2} \).

\subsection{2.2.1 Stationary-Phase Solutions}

The method of stationary-phase yields an asymptotic solution to integrals of the form (Ben-Menahem and Singh, 1981; Bender and Orzag (1978), and Bleistein and Handelsman (1986)

\[
K(\lambda) = \int_{-\infty}^{\infty} g(\zeta)e^{i\lambda f(\zeta)}d\zeta
\]

(2.43)

In the above expression, \( \lambda \) is a large valued, positive constant. Because \( \lambda \) is large, the integrand oscillates rapidly and the contributions to the integral tend to cancel
except near the stationary value of \( f(\zeta) \), the value \( \zeta_0 \) where \( f'(\zeta_0) = 0 \) (the prime indicates differentiation with respect to \( \zeta \)). The leading order asymptotic behavior of the integral is then controlled by contributions from a small region around this stationary point. The desired asymptotic approximation to this equation is

\[
K(\lambda) \sim \left[ \frac{2\pi}{\lambda |f''(\zeta_0)|} \right]^{1/2} g(\zeta_0) e^{i\lambda f(\zeta_0)+(i\pi/4)\text{sgn}f''(\zeta_0)}
\tag{2.44}
\]

The solution will be inaccurate, and the method of stationary phase invalid, if the integrand is not regular (Bleistein and Handelsman, 1986).

An additional simplification can be made by applying the large-argument asymptotic forms for the Hankel functions (Abramowitz and Stegun, 1964),

\[
H_0^{(2)}(z) \sim \sqrt{\frac{2}{\pi z}} e^{-i(z-\pi/4)}
\]

\[
H_1^{(2)}(z) \sim \sqrt{\frac{2}{\pi z}} e^{-i(z-3\pi/4)}
\tag{2.45}
\]

Substituting equations, (Eq 2.45), into equations (Eq 2.32) and (Eq 2.33), and into equations (Eq 2.36) and (Eq 2.37) and into equations (Eq 2.40) and (Eq 2.41) shows that these far-field streaming current fields are of the form \( K(\lambda) \) suitable for application of the stationary-phase approach. The terms in each integral represent the contribution of the shear, fast and slow wave to the dynamic streaming current components. The stationary phase value \( k_0 \) is different for the three wave types. Considering the first term in the integral for the radial component in equation (Eq 2.32), the quantity analogous to \( i\lambda f(\zeta) \) in equation (Eq 2.43) is

\[-i[k_z z + lr] = iR [k_z \cos(\gamma) - l \sin(\gamma)]\]

\[
\tag{2.46}
\]

where \( z = -R \cos(\gamma) \) and \( r = R \sin(\gamma) \). Hence, \( \zeta = k_z \) and \( \lambda = R \). Setting

\[
\frac{df}{d\zeta} = \cos(\gamma) + \frac{k_z \sin(\gamma)}{(\omega^2 - k_z^2)^{1/2}} = 0
\]

\[
\tag{2.47}
\]

and solving for \( k_z \) gives the stationary wavenumber for the S-waves, \( k_0^S = (-\omega \cos(\gamma))/\beta \). Similarly, the analogous quantity for the compressional waves are \( k_0^{C1} = (-\omega \cos(\gamma))/\sigma_1 \).
and \( k_0^{\alpha_2} = (-\omega \cos(\gamma))/\alpha_2 \). Using these values for the wavenumbers \( k_0 \) and applying the stationary-phase asymptotic solution in equation (Eq 2.44) gives the following far-field dynamic streaming current fields.

**Vertical Point Force on Fluid**

\[
J_r^z = -\omega^2 \rho_E L(\omega) \frac{\sin(\gamma)}{4\pi R} \left[ \frac{\omega}{\beta} W^r_\beta(k_0^{\beta}) e^{-iR_{\alpha_1}} + \frac{\omega}{\alpha_1} W^r_{\alpha_1}(k_0^{\alpha_1}) e^{-iR_{\alpha_2}} \right] + \frac{\omega}{\alpha_2} W^r_{\alpha_2}(k_0^{\alpha_2}) e^{-iR_{\alpha_2}} 
\]

(2.48)

\[
J_z^z = \omega^2 \rho_E L(\omega) \frac{\cos(\gamma)}{4\pi R} \left[ \frac{\omega}{\beta} W^z_\beta(k_0^{\beta}) e^{-iR_{\alpha_1}} + \frac{\omega}{\alpha_1} W^z_{\alpha_1}(k_0^{\alpha_1}) e^{-iR_{\alpha_2}} \right] + \frac{\omega}{\alpha_2} W^z_{\alpha_2}(k_0^{\alpha_2}) e^{-iR_{\alpha_2}} 
\]

(2.49)

where \( W^r_p \) and \( W^z_p \), with \( p = \beta \lor \alpha_1 \lor \alpha_2 \) are defined in equations (Eq 2.34).

**Explosive Point Source on Solid and Fluid**

\[
J_r^z = \omega^2 \rho_E L(\omega) \frac{i\sin(\gamma)}{4\pi R} \left[ \frac{\omega}{\alpha_1} W^r_{\alpha_1}(k_0^{\alpha_1}) e^{-iR_{\alpha_1}} + \frac{\omega}{\alpha_2} W^r_{\alpha_2}(k_0^{\alpha_2}) e^{-iR_{\alpha_2}} \right] 
\]

(2.50)

\[
J_z^z = -\omega^2 \rho_E L(\omega) \frac{i\cos(\gamma)}{4\pi R} \left[ \frac{\omega}{\alpha_1} W^z_{\alpha_1}(k_0^{\alpha_1}) e^{-iR_{\alpha_1}} + \frac{\omega}{\alpha_2} W^z_{\alpha_2}(k_0^{\alpha_2}) e^{-iR_{\alpha_2}} \right] 
\]

(2.51)

where for the force applied on the solid \( W^r_p \) and \( W^z_p \), with \( p = \alpha_1 \lor \alpha_2 \) are defined in equations (Eq 2.38) and for the force applied on the fluid \( W^r_p \) and \( W^z_p \), with \( p = \alpha_1 \lor \alpha_2 \) are defined in equations (Eq 2.42). The "W" coefficients are now written with argument \( k_0 \) to emphasize that they are computed at the appropriate stationary-phase wavenumber. Since there is no low-frequency approximation, the solutions can be used to explore the changes in radiation pattern as the frequency of the
source increases. To compute the total radiation pattern of a given source, it is more convenient to reorganize the terms in equations (Eq 2.48), (Eq 2.49), (Eq 2.50) and (Eq 2.51) to express separately the $P_{fast}$, $P_{slow}$ and $S$-wave amplitudes. By simple arrangements the following radiation patterns are obtained,

Vertical point force,

$$M_\beta = \omega^2 \rho_EL(\omega)W_\beta^r(k_0^\beta)\frac{\omega^2 e^{-iR_\omega/\beta}}{4\pi R} \sin(\gamma)$$  \hspace{1cm} (2.52)

$$M_{\alpha_1} = \omega^2 \rho_EL(\omega)W_{\alpha_1}(k_0^{\alpha_1})\frac{\omega^2 e^{-iR_\omega/\alpha_1}}{4\pi R} \cos(\gamma)$$  \hspace{1cm} (2.53)

$$M_{\alpha_2} = \omega^2 \rho_EL(\omega)W_{\alpha_2}(k_0^{\alpha_2})\frac{\omega^2 e^{-iR_\omega/\alpha_2}}{4\pi R} \cos(\gamma)$$  \hspace{1cm} (2.54)

where $W_p^r$ and $W_p^z$, with $p = \beta \lor \alpha_1 \lor \alpha_2$ are defined in equations (Eq 2.34).

Explosive point source on solid and fluid,

$$M_{\alpha_1} = i\omega^2 \rho_EL(\omega)W_{\alpha_1}(k_0^{\alpha_1})\frac{\omega e^{-iR_\omega/\alpha_1}}{4\pi R}$$  \hspace{1cm} (2.55)

$$M_{\alpha_2} = i\omega^2 \rho_EL(\omega)W_{\alpha_2}(k_0^{\alpha_2})\frac{\omega e^{-iR_\omega/\alpha_2}}{4\pi R}$$  \hspace{1cm} (2.56)

where for the force applied on the solid $W_p^r$ and $W_p^z$, with $p = \alpha_1 \lor \alpha_2$ are defined in equations (Eq 2.38) and for the force applied on the fluid $W_p^r$ and $W_p^z$, with $p = \alpha_1 \lor \alpha_2$ are defined in equations (Eq 2.42).

### 2.3 Dynamic Streaming Currents from Point Forces

In this section the stationary-phase radiation pattern results and the dynamic streaming current sensitivities with respect to fluid salinity, permeability, and porosity are discussed. These streaming currents are induced by compressional and rotational waves generated by an unit strength vertical point force, and by compressional waves only due to an unit strength explosive point force acting on the solid frame and volume.
injection source acting on the fluid. The streaming current responses are calculated at three different frequencies, 20 Hz, 200 Hz and 2000 Hz. The compressional P waves generate relative flow set up by their peaks and troughs and induce streaming current by moving excess charges in the electric double layers near the grain surface (see figure 2-2). Both compressional and shear waves generate grain acceleration and therefore relative flow and streaming currents. The divergence free shear waves can't cause charge to separate but induce streaming currents sheets due to its relative fluid motion with respect to the actual moving solid grains (see figure 2-3). By symmetry considerations, the induced currents in the seismic pulse don't radiate electromagnetic waves away from the traveling pulse into the homogeneous poro-elastic medium. The effect of a point source in a layered poro-elastic medium is studied in chapters 4 and 5. In those chapters the conversion from mechanical waves into electromagnetic waves caused by mechanically and electrically induced streaming current inbalances across a mechanical and/or electrical boundary is theoretically investigated and numerically modeled.

In figure 2-10 the stationary-phase radiation patterns at 200 Hz are displayed for a vertical point force where equations (Eq 2.52) and (Eq 2.53) are used, an explosive point source on the solid and a volume injection source where equation (Eq 2.55) is used. The correctness of the stationary-phase solutions were checked against synthetic seismograms computed using the discrete-wavenumber method. This method evaluates the integral expressions by discretizing the integral over wavenumber using FFT methods to perform the frequency integrals.

The streaming current radiation pattern parameters are calculated at a permeability, $k = 10^{-12} m^2$, salinity, $C = 10^{-3} mol/l$ and a 20 % porosity. The unit force strength vertical point force induces the largest streaming current magnitudes compared to the other point sources, see figure 2-10. The rotational mechanical waves induce in this case approximately 10 times larger streaming currents compared to the streaming currents induced by the compressional waves in a medium with 20 percent
porosity.

In figure 2-11 the $P$ and $SV$ relative flow amplitudes, $W_\beta^r$ and $W_{\alpha_1}$, defined in equations (Eq 2.52) and (Eq 2.53) are shown. The increase in $W_\beta^r$ coefficient is largely due to the decrease in shear frame modulus with increasing porosity (a squishier material). The combined porosity effect on the medium and Biot’s moduli result in the more complicated $W_{\alpha_1}$ behavior.

At zero porosity, i.e. the elastic limit, the $W_\beta^r$ and $W_{\alpha_1}$ coefficients are of equal strength and normalized to one. At zero porosity the difference in induced streaming current amplitude is solely due to the difference in $P$ and $SV$ wavefield velocity, see equations (Eq 2.52) and (Eq 2.53). For non-zero porosities the $W_\beta^r$ and $W_{\alpha_1}$ amplitudes are different from each other, figure 2-11. The $P$ wave and $SV$ wave induced streaming current amplitudes are now not only determined by the wavefield velocities but also by the relative flow amplitudes. At a 20% porosity the combined effect of wave velocity and $W_\beta^r/W_{\alpha_1}$ coefficient difference results in a 10 times larger streaming current induced by the rotational waves than by the compressional waves. The unit strength explosive point source exciting the solid frame induces approximately $10^3$ times bigger streaming currents than a unit strength volume injection source.

In figures 2-12, 2-13, 2-14 and 2-15 the streaming current amplitude sensitivities with respect to fluid salinity, permeability, and porosity are shown. The maximum streaming current magnitudes from the stationary-phase radiation patterns are plotted against the studied medium parameter in the case of the vertical point force. The rows indicated by $a$, $b$ and $c$ in figures 2-12, 2-13 and 2-14 denote streaming currents induced by a vertical point force, and explosive point force acting on the solid frame and a volume injection source respectively. The columns indicate from left to right the frequency of the source, 20 $Hz$, 200 $Hz$ and 2000 $Hz$, respectively. In figure 2-15 the $S$ wave induced streaming currents are plotted against concentration, row $a$, permeability, row $b$ and porosity, row $c$. The columns denote again the vertical point source frequency, 20 $Hz$, 200 $Hz$ and 2000 $Hz$. 

47
2.3.1 Streaming Current Behavior with respect to Frequency

The relative flow as a function of electrolyte concentration \((10^{-5} - 1.0 \text{ mol/l})\) and porosity \((5\% - 45\%)\) changes per factor 10 in frequency (from 20 \text{ Hz} to 2000 \text{ Hz}) a factor 10 (vertical point force), 100 (explosive point source) and 1000 (volume injection source) in magnitude. The relative flow as a function of permeability \((10^{-14} - 10^{-9}m^2)\) doesn't change in magnitude for a vertical point force and a factor 10 in magnitude for both explosive point sources, per factor 10 in frequency.

The streaming current amplitudes as function of concentration and porosity changes approximately an additional factor 10 for all three point forces per factor 10 in frequency (total streaming current change is this factor 10 times the factor change in relative flow with frequency). While this factor is 100 for the streaming current amplitudes as a function of permeability.

The relative flow/streaming current increase as function of electrolyte concentration and porosity have a similar magnitude behavior, which differs from the relative flow/streaming current magnitude behavior as function of permeability, with frequency. The streaming current magnitude as function of permeability changes more rapid with frequency than the relative flow amplitudes (permeability equilibrates the relative flow) for all thee point forces. The opposite, a more rapid change in relative flow magnitude behavior with frequency is observed as function of concentration and porosity.

2.3.2 Streaming Current Behavior with respect to Salinity

In figures 2-12 and 2-15 the streaming current amplitude behavior with respect to changes in fluid ion concentration are shown for the different point forces. The streaming current decreases almost linearly with increasing molarity (logarithmic scale) for all three source types. This is consistent with the decrease of the double layer thick-
ness, predicted by double layer theory with increasing electrolyte concentration. The relative flow amplitudes on the other hand are almost constant over five orders of electrolyte concentration change. The electrokinetic coefficient amplitude has the same functional shape as the streaming current plots. There is a change in electrokinetic amplitude decrease at concentrations larger than 0.1 mol/l. The change in direction coefficient at 0.1 mol/l is caused by the change in electrokinetic coefficient (ζ-potential).

2.3.3 Streaming Current Behavior with respect to Permeability

In figures 2-13 and 2-15 the streaming current behavior with respect to changes in permeability are shown for the different sources. The streaming current behavior with respect to permeability is different for the three source types. The streaming current behavior induced by the $P$ wave for a vertical point force and by an explosion source exciting the solid frame, show a similar shape. The streaming current sheet behavior induced by the $S$ waves however shows a different shape at the 2000 Hz input source frequency. The streaming current shapes for a volume injection source at all three frequencies are different from the two other source types.

The relative flow amplitudes versus permeability show a sharp increase in relative flow amplitudes at $k = 10^{-11}m^2$ for all source types at 20 Hz. At 200 Hz this sharp increase in relative flow happens at $k = 10^{-12}m^2$. At this frequency another change in relative flow amplitude behavior is observed at $k = 10^{-10}m^2$ where a decrease in positive amplitude direction coefficient with respect to permeability sets in. The relative flow amplitude versus permeability has at 200 Hz taken a 'S' shaped curve. At 2000 Hz a similar 'S' curved relative flow shape is observed. The permeabilities where the relative flow amplitude direction coefficients change are at $k = 10^{-13}m^2$ and
$k = 10^{-11} m^2$ approximately for all three source types. The electrokinetic coefficient decreases rapidly with increasing permeability. The decreasing trend of the coupling coefficient and the increasing trend of the relative flow with increasing permeability tends to balance each other, resulting in a uniform streaming current response over 5 decades of permeability change for the vertical and explosive point source on the frame at 20 Hz. At the 200 Hz and 2000 Hz frequencies, the decrease in electrokinetic coefficient is stronger than the increase in relative flow amplitude, since the increase in the direction coefficient of the relative flow decreases for the high permeabilities. This causes a sudden drop in streaming current at $k = 10^{-10} m^2$ for the sources at 200 Hz and at $k = 10^{-11} m^2$ for the sources at 2000 Hz. The streaming currents induced by the $S$ waves show a different amplitude pattern at 2000 Hz than the $P$ wave induced streaming currents, corresponding to a different relative flow amplitude shape for the two wave types. The volume injection source has a maximum in streaming current at $k = 10^{-9} m^2$, $k = 10^{-10} m^2$ and $k = 10^{-12} m^2$ for a 20 Hz, 200 Hz and 2000 Hz source respectively.

The relative flow amplitudes have similar 'S' shaped curves as the vertical and explosion point source applied on the solid. The very low relative flow amplitude behavior for the low permeabilities and the very low electrokinetic coupling coefficient for the high permeabilities determine the streaming current curve shape as function of permeability.

### 2.3.4 Streaming Current Behavior with respect to Porosity

In figures 2-14 and 2-15 the streaming current behavior with respect to changes in porosity are shown for the different point sources. The $P$ and $S$ wavefield velocities together with the bulk densities for the different porosities are shown in table I. The streaming current trends with increasing porosity are very similar for all source types and all frequencies. An increase in streaming current direction coefficient is observed
at 15% porosity, corresponding to a change in relative flow behavior (squishier frame regime). The streaming current amplitudes increase relatively spoken less with increasing porosity at 2000 Hz than at 20 Hz and 200 Hz for all source types. The streaming currents induced by rotational waves generated by the vertical point force show different behavior than the streaming currents induced by compressional waves. Their direction coefficient at low porosities is different from the P wave induced streaming currents approaching zero porosity. The S wave induced streaming current response at 2000 Hz is different from the response at 20 and 200 Hz. The electrokinetic coupling coefficient increases linearly with increasing porosity for all frequencies. But the relative flow amplitudes, which increase with increasing porosity at 20 and 200 Hz, decrease with increasing porosity at 2000 Hz. This decreasing relative flow amplitude behavior combined with the more rapidly increase in electrokinetic coupling coefficient amplitudes with increasing porosity, results in a slightly increasing streaming current response with increasing porosity.

2.4 Discussion and Conclusions

The stationary-phase radiation approach allows a quick computation of radiation patterns, useful to investigate the amplitude variability with respect to different medium parameters. In addition no frequency restrictions are required for this approach, making the method useful to investigate the amplitude variability at different source frequencies. It is understood that the frequency constraints imposed by the theory prohibits the numerical radiation pattern study at the very high frequencies.

The relative flow/streaming current increases as function of electrolyte concentration and porosity have a similar magnitude behavior, which differs from the relative flow/streaming current magnitude behavior as a function of permeability, with frequency (20 Hz, 200 Hz, 2000 Hz). The streaming current magnitude as function
of permeability changes more rapid with frequency than the relative flow amplitude for all three point forces. While the opposite, a more rapid change in relative flow magnitude behavior with frequency, is observed as a function of concentration and porosity. The streaming current decreases with increasing electrolyte molarity for all three source types, consistent with the decrease of double layer thickness with increasing electrolyte concentration.

The streaming current increases monotonically with increasing porosity for all three source types.

The streaming current behavior with respect to permeability differs per source type and with frequency. A source applied on the fluid (volume injection source) has a different streaming current response versus permeability than a point force applied on the solid (explosion source applied on the solid frame, vertical point force).

The streaming current magnitude induced by $P$ waves generated by a vertical point force is approximately 10 times smaller than the streaming current sheet magnitudes generated by $S$ waves in the medium with 20% porosity. The relative change in streaming current sheet magnitudes versus porosity and concentration is less at high frequencies ($2000 \ Hz$) than at the low frequencies ($20 \ Hz$).

In future work, the numerical results can be used to design an experiment to measure dynamic streaming currents caused by point sources/transducers. The advantage of measuring streaming currents is not to have to know the value of the (surface) conductance, if one wants to determine the $\zeta$-potential. Since a dynamical measurement is made, electrode polarization might be less of a problem. An experiment analysed with simple and rapid stationary-phase, theoretical amplitude predictions (versus salinity, permeability, porosity) can be a useful tool to determine, for example, permeability from dynamic streaming current measurements.
TABLE I. Calculated medium parameters as function of porosity.

<table>
<thead>
<tr>
<th>Porosity (%)</th>
<th>P velocity (m/s)</th>
<th>S velocity (m/s)</th>
<th>Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3604.38</td>
<td>1808.28</td>
<td>2615</td>
</tr>
<tr>
<td>15</td>
<td>3282.60</td>
<td>1768.88</td>
<td>2445</td>
</tr>
<tr>
<td>20</td>
<td>3220.87</td>
<td>1746.69</td>
<td>2360</td>
</tr>
<tr>
<td>25</td>
<td>3182.67</td>
<td>1722.53</td>
<td>2275</td>
</tr>
<tr>
<td>35</td>
<td>3143.52</td>
<td>1667.08</td>
<td>2105</td>
</tr>
<tr>
<td>45</td>
<td>3131.68</td>
<td>1599.43</td>
<td>1935</td>
</tr>
</tbody>
</table>
Figure 2-1: The Quartz/electrolyte interface according to Stern. In the compact layer the electrolyte ions and structured water dipole molecules are electrostatically and chemically adsorbed to the surface of the solid. Beyond the compact layer of bound ions there is a diffuse distribution of mobile ions whose position is determined by Poisson-Boltzman statistics. The potential at the separation plane between mobile and immobile charge is called the $\zeta$-potential.
P waves: charge separation

However, no E-fields away from the pulse in homogeneous media.

Figure 2-2: Electric fields due to charge separation within a P wave seismic pulse. Streaming and conduction currents balance each other inside the P wave pulse. Therefore when compressional waves propagate through homogeneous medium, there is an electric field fixed to the wave with no extend outside the wave.
Figure 2-3: When divergence-free $S$ waves propagate through porous medium, current sheets are induced through grain accelerations. Magnetic fields with a polarization orthogonal to the seismic particle displacement are induced by these current sheets. Very small electric fields of the induction type are generated by this magnetic field. There is no current imbalance within the $S$ wave and therefore no radiating magnetic field outside the pulse.
Figure 2-4: Dependence of dc permeability to dynamic permeability ratio on porosity and frequency.
Figure 2-5: Dependence of dc electrokinetic coupling coefficient to dynamic electrokinetic coupling coefficient ratio on porosity and frequency.
Figure 2-6: Dependence of dc permeability to dynamic permeability ratio on permeability and frequency.
Figure 2-7: Dependence of dc electrokinetic coupling coefficient to dynamic electrokinetic coupling coefficient ratio on permeability and frequency.
Figure 2-8: conductivity versus porosity in a porous medium with $k = 10^{-12} m^2$ and $C = 10^{-3} mol/l$. The conductivity doesn’t change linearly with porosity due to the porosity dependence of the pore length parameter, $\Lambda [m]$. 
Figure 2-9: Conductivity versus permeability in a porous medium with $\phi = 20\%$ and $C = 10^{-3} \text{mol/l}$. At $k_0 < 10^{-16} \text{m}^2$ the pore length parameter $\Lambda[m]$ becomes comparable to the sum of the grain surface conductances. Physically this situation occurs when the pore space is filled with clays, which do not decrease the porosity but greatly enhances the surface to pore volume ratio and therefore enhance the conduction along the pore surfaces.
Figure 2-10: Streaming current radiation patterns for different unit strength seismic sources calculated at a 200 Hz center frequency. (a) vertical point force; (b) an explosive point force acting on the solid frame; (c) volume injection source.
Figure 2-11: Relative fluid-solid motion generated by $P$ and $S$ waves as function of porosity. The relative flow amplitude generated by the $P$-waves are affected by the porosity effect on the bulk and frame moduli of the solid and the compressibility of the saturating fluid. The increase in relative fluid-solid motion generated by $S$-waves with increasing porosity is due to the decrease of the shear frame modulus with increasing porosity.
Figure 2-12: Streaming current density versus salinity induced by $P$ waves generated by (a) vertical point force, (b) explosive point force acting on the frame and (c) volume injection source at a 20 Hz (left column), 200 Hz (center column) and 2000 Hz (right column) source center frequency.
Figure 2-13: Streaming current density versus permeability induced by $P$ waves generated by (a) vertical point force, (b) explosive point force acting on the frame and (c) volume injection source at a 20 Hz (left column), 200 Hz (center column) and 2000 Hz (right column) source center frequency.
Figure 2-14: Streaming current density versus porosity induced by $P$ waves generated by (a) vertical point force, (b) explosive point force acting on the frame and (c) volume injection source at a 20 Hz (left column), 200 Hz (center column) and 2000 Hz (right column) source center frequency.
Figure 2-15: Streaming current density sheets induced by $S$ waves generated by a vertical point force versus (a) concentration, (b) permeability and (c) porosity at a 20 Hz (left column), 200 Hz (center column) and 2000 Hz (right column) source center frequency.
Chapter 3

Electroseismic Waves in Poro-Elastic Media

3.1 Introduction

From the macroscopic coupled mechanical equations of motion, electromagnetic equations and constitutive equations, the acoustic-electromagnetic power balance in its global form in a porous medium is derived. The time rate at which the sources deliver mechanical and electromagnetic work to the coupled acoustic, electromagnetic disturbances is shown to equal the time rate of kinetic energy plus the time rate of deformation energy plus the time rate of coupled field energy plus the time rate of total stored electromagnetic energy in a volume plus the acoustic-electromagnetic powerflow through a surface bounding the volume.

Conditions are derived for uniqueness of solution of the coupled field equations. Particular attention is paid to continuity requirements at an interface where a jump in macroscopic medium parameters occurs. Two displacement-stress-EM wavefield
component vectors, characterizing the $PSVTM$ and $SHTE$ electroseismic wavefield pictures, that are continuous in horizontally layered isotropic porous media are derived.

A reciprocity relationship for the coupled electromagnetics and acoustics in a porous medium is derived. Representation integrals for mechanical and electromagnetic wavefields are obtained using the reciprocity theorem in its global form.

The equations of motion, Maxwell’s equations and the constitutive relations, rewritten into a first-order differential equation relate the derivatives with respect to depth of a displacement-stress-EM wavefield vector to a system matrix times this displacement-stress-EM vector. Using a plane wave ansatz, the wavefield slownesses are derived for the up and downgoing waves in poro-elastic media (i.e. fast compressional wave, slow compressional wave, rotational waves and electromagnetic waves). The wavefield eigenvectors are derived using the appropriate polarization vectors for all wave types, each belonging to an associated wavefield slowness (eigenvalue of the system matrix).

To guarantee a consistent macroscopic theory of energy transfer, conservation of electromagnetic and poro-elastic, time-cycle averaged, Poynting power is imposed upon crossing an interface. This results in a physical normalisation of the wavefield eigenvectors.

### 3.2 Energy Considerations in a Porous Medium

We investigate the exchange of acoustic and electromagnetic energy between a certain part of a porous medium and its surroundings. The part under consideration is located in a bounded volume $V$, closed in by a surface $S$. The unit vector along the normal to $S$ is denoted by $\hat{n}$. First we rewrite the macroscopic coupled mechanical, electromagnetic equations and constitutive relations, given in chapter 2, characteriz-
ing the coupled field behavior, in an appropriate form.

The electromagnetic field equations,

\[ \nabla \times \mathbf{E} + \mu \dot{\mathbf{H}} = \mathbf{M} \]  \hspace{1cm} (3.1)

\[ \nabla \times \mathbf{H} - \epsilon \dot{\mathbf{E}} - L \rho_E \ddot{\mathbf{w}} = -\mathbf{J} \]  \hspace{1cm} (3.2)

The mechanical field equations,

\[ -\rho_B \dddot{\mathbf{u}} - \rho_f \dddot{\mathbf{w}} + \nabla \cdot \mathbf{\tau} = \mathbf{f} \]  \hspace{1cm} (3.3)

\[ -\rho_f \dddot{\mathbf{u}} - \rho_E \dddot{\mathbf{w}} + \nabla S + L \rho_E \dot{\mathbf{E}} = \mathbf{F} \]  \hspace{1cm} (3.4)

The deformation relations,

\[ D = \varepsilon_0 \left( \frac{\phi}{\alpha_\infty} (\kappa_f - \kappa_s) + \kappa_s \right) \mathbf{E} \]  \hspace{1cm} (3.5)

\[ \mathbf{B} = \mu_0 \mathbf{H} \]  \hspace{1cm} (3.6)

\[ \mathbf{\tau} = \left[ (H - \frac{4}{3}G) \nabla \cdot \mathbf{u} + C \nabla \cdot \mathbf{w} \right] \mathbf{I} + G \left( \nabla \mathbf{u} + \nabla \mathbf{u}^T - \frac{2}{3} \nabla \cdot \mathbf{u} \mathbf{I} \right) \]  \hspace{1cm} (3.7)

\[ S = C \nabla \cdot \mathbf{u} + M \nabla \cdot \mathbf{w} \]  \hspace{1cm} (3.8)

With \( \cdot \) on top of the wavefield vectors denoting the derivatives with respect to time. \( S \) denotes the stress in the fluid. \( \mathbf{f} \) and \( \mathbf{F} \) are body forces per unit volume in the bulk and fluid phases respectively, \( \mathbf{J} \) and \( \mathbf{M} \) are the electric and magnetic current sources.

We first multiply (Eq 3.1) and (Eq 3.2) through from the left side with \( \mathbf{H} \) and \( \mathbf{E} \) respectively. We multiply equations (Eq 3.3) and (Eq 3.4) through from the right hand side with \( \dot{\mathbf{u}} \) and \( \dot{\mathbf{w}} \) respectively.

We will first manipulate the mechanical equations.

Using the obtained mechanical equations,

\[ \left( \nabla \cdot \mathbf{\tau} \right) \cdot \dot{\mathbf{u}} - \dot{\mathbf{f}} \cdot \dot{\mathbf{u}} = \rho_B \dddot{\mathbf{u}} + \rho_f \dddot{\mathbf{w}} \]  \hspace{1cm} (3.9)

\[ \nabla \cdot \mathbf{S} - \dot{\mathbf{F}} \cdot \dot{\mathbf{w}} = \rho_f \dddot{\mathbf{u}} + \rho_E \dddot{\mathbf{w}} - L \rho_E \dot{\mathbf{E}} \cdot \dot{\mathbf{w}} \]  \hspace{1cm} (3.10)
And identities,

\[
\nabla \cdot (\mathbf{I} \cdot \mathbf{u}) = \nabla \cdot (\mathbf{I} \cdot \mathbf{u}) - \mathbf{I} \cdot \nabla \mathbf{u} \tag{3.11}
\]

\[
\nabla \cdot (\mathbf{S}_L \cdot \mathbf{w}) = \nabla \cdot (\mathbf{S}_L \cdot \mathbf{w}) - \mathbf{S}_L \cdot \nabla \mathbf{w} \tag{3.12}
\]

Using the symmetry of the stress tensors, (Ben-Menahem and Singh, 1981), the following equations are obtained,

\[
\nabla \cdot (\mathbf{I} \cdot \mathbf{u}) - \mathbf{I} \cdot \mathbf{u} = \frac{1}{2} \frac{\partial}{\partial t} \mathbf{I} \cdot \mathbf{u} + \rho_B \frac{\partial \mathbf{u}}{\partial t} + \rho_f \frac{\partial \mathbf{u}}{\partial t} \tag{3.13}
\]

\[
\nabla \cdot (\mathbf{S}_L \cdot \mathbf{w}) - \mathbf{E} \cdot \mathbf{w} = \frac{1}{2} \frac{\partial}{\partial t} \mathbf{S}_L \cdot \mathbf{w} + \rho_f \frac{\partial \mathbf{w}}{\partial t} + \rho_E \frac{\partial \mathbf{w}}{\partial t} - \rho_E \mathbf{L} \cdot \mathbf{w} \tag{3.14}
\]

With \( \mathbf{E} = \frac{1}{2} (\nabla \mathbf{u} + \mathbf{u} \nabla) \) and \( \mathbf{W} = \frac{1}{2} (\nabla \mathbf{w} + \mathbf{w} \nabla) \). Subtracting equations (Eq 3.1) from (Eq 3.2) yields,

\[
\mathbf{H} \cdot \nabla \times \mathbf{E} - \mathbf{E} \cdot \nabla \times \mathbf{H} + \mu \mathbf{H} \cdot \dot{\mathbf{H}} + \epsilon \mathbf{E} \cdot \dot{\mathbf{E}} + \mathbf{L} \dot{\mathbf{E}} \cdot \mathbf{w} = \mathbf{M} + \mathbf{E} \cdot \mathbf{J} \tag{3.15}
\]

Using identity,

\[
\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot \nabla \times \mathbf{E} - \mathbf{E} \cdot \nabla \times \mathbf{H} \tag{3.16}
\]

We obtain,

\[
\nabla \cdot (\mathbf{E} \times \mathbf{H}) + \frac{\partial}{\partial t} \left[ \frac{1}{2} \mu \mathbf{H} \cdot \dot{\mathbf{H}} \right] + \frac{\partial}{\partial t} \left[ \frac{1}{2} \epsilon \mathbf{E} \cdot \dot{\mathbf{E}} \right] + \mathbf{L} \dot{\mathbf{E}} \cdot \mathbf{w} = \mathbf{M} + \mathbf{E} \cdot \mathbf{J} \tag{3.17}
\]

Adding equations (Eq 3.13), (Eq 3.14) and (Eq 3.17), integrating over a volume \( V \) and using Green's theorem, we obtain,

\[
\int_V \left[ \mathbf{I} \cdot \dot{\mathbf{u}} + \mathbf{E} \cdot \dot{\mathbf{w}} + \mathbf{H} \cdot \mathbf{M} + \mathbf{E} \cdot \mathbf{J} \right] \, dV = \\
- \frac{\partial}{\partial t} \left[ \int_V \frac{1}{2} (\rho_B \mathbf{u}^2 \mathbf{u}^2 + 2\rho_f \mathbf{u} \cdot \dot{\mathbf{w}}) \, dV \right] - \frac{\partial}{\partial t} \left[ \int_V \frac{1}{2} (\mathbf{I} \cdot \mathbf{E} + \mathbf{S}_L \cdot \mathbf{w}) \, dV \right] \\
+ \frac{\partial}{\partial t} \left[ \int_V \rho_E \mathbf{L} \mathbf{E} \cdot \dot{\mathbf{w}} \, dV \right] + \frac{\partial}{\partial t} \left[ \int_V \frac{1}{2} (\mu \mathbf{H} \cdot \dot{\mathbf{H}} + \epsilon \mathbf{E} \cdot \dot{\mathbf{E}}) \, dV \right] \\
+ \int_S \left[ \mathbf{I} \cdot \dot{\mathbf{u}} + \mathbf{S}_L \cdot \dot{\mathbf{w}} + \mathbf{E} \times \mathbf{H} \right] \, dS \tag{3.18}
\]

Equation (Eq 3.18) is the acoustic-electromagnetic power balance in its global form in a porous medium. The equation can be decomposed into the following contributions.
to the power balance.

\[ \dot{E}_{SRC} = \dot{E}_{KIN} + \dot{E}_{DEF} + \dot{E}_C + \dot{E}_{EM} + \hat{n} \cdot P \]  

(3.19)

The time rate at which the sources deliver mechanical and electromagnetic work to the coupled acoustic, electromagnetic disturbances equals the time rate of kinetic energy plus the time rate of deformation (potential) energy plus time rate of coupled field energy plus the time rate of total stored electromagnetic energy in a volume plus the combined acoustic-electromagnetic powerflow, \( P \), through boundary \( S \).

3.3 Uniqueness Theorem

The uniqueness theorem provides conditions under which the solution to the wave equation is unique. The conditions under which a solution to the wave equation is unique are the boundary conditions and radiation conditions. The nonuniqueness associated with the resonance solution for a lossless medium is not an issue here, since a lossy medium is considered (Chew, 1990).

Assume there are two solutions for a given set of sources denoted by \( E^1, H^1, u^1, w^1 \) and \( E^2, H^2, u^2, w^2 \). Let the differences be denoted as, \( \delta E, \delta H, \delta u \) and \( \delta w \). Since both field solutions satisfy equations (Eq 3.1), (Eq 3.2), (Eq 3.3) and (Eq 3.4), their differences satisfy the source free equations. By virtue of the positive definite character of the kinetic, strain and coupled field energies and the stored electric and magnetic energies for the difference fields requires the vanishing of the difference fields in region \( V \) for all time to guarantee uniqueness for specified conditions on the boundary \( S \), (Kong, 1990; Aki and Richards, 1980). The solution is unique for two bodies if the following surface integral is equal to zero, see also (Deresiewicz and Skalak, 1963),

\[ \int_{S_1} dS_1 \cdot \left[ (E^1 \times H^1 + \tau^1 \cdot \dot{u}^1 + S^1 \cdot \dot{w}^1) + \int_{S_2} dS_2 \cdot \left[ (E^2 \times H^2 + \tau^2 \cdot \dot{u}^2 + S^2 \cdot \dot{w}^2) \right] \right. \\
\left. + \int_{S_3} dS_3 \cdot \left( (E^1 \times H^1 + \tau^1 \cdot \dot{u}^1 + S^1 \cdot \dot{w}^1) - (E^2 \times H^2 + \tau^2 \cdot \dot{u}^2 + S^2 \cdot \dot{w}^2) \right) \right] \]  

(3.20)
This leads to the following requirements. At each point on the isolated boundary \((S_1\) and \(S_2\)) specification must be made of one term in each of the products.

\[
\left( \mathbf{t} \cdot \hat{n} \right) \cdot \mathbf{u}, \quad \left( \mathbf{S} \cdot \hat{n} \right) \cdot \mathbf{\dot{u}}, \quad \hat{n} \cdot \mathbf{E} \times \mathbf{H} \tag{3.21}
\]

At each point of the interface \(S_c\) we require the continuity across the interface of,

\[
\left[ \mathbf{E} \times \mathbf{H} + \mathbf{t} \cdot \mathbf{u} + \mathbf{S} \cdot \mathbf{\dot{u}} \right] \cdot \hat{n} \tag{3.22}
\]

Assume \(S_c\) to be a horizontal boundary with a \(\hat{n}_z\) normal to it. The condition in equation (Eq 3.22) is fulfilled by the continuity of the following wavefield components.

\[
\mathbf{B} = [u_x, u_y, u_z, w_z, \tau_{xz}, \tau_{yz}, \tau_{zz}, S, H_y, E_x, H_x, E_y]^T \tag{3.23}
\]

In isotropic poro-elastic media, the coupled electromagnetics and acoustic-elastic equations decouple into \(PSVTM\) and \(SHTE\) coupled electroseismic wavefield pictures. Thus we can define two displacement-stress-EM wavefield component vectors that are continuous in a horizontally layered medium.

\[
\mathbf{B}^{(PSVTM)} = [u_x, u_z, w_z, \tau_{xz}, \tau_{zz}, S, H_y, E_x]^T \tag{3.24}
\]

\[
\mathbf{B}^{(SHTE)} = [u_y, \tau_{yz}, H_x, E_y]^T \tag{3.25}
\]

For porous media in welded contact, the following boundary conditions can be defined.

- Continuity of normal and shear stresses in the bulk phase.
- Continuity of stress in the fluid phase.
- Continuity of normal and horizontal displacements in the bulk phase.
- Continuity of vertical relative flow, a statement of mass conservation across an interface.
- Continuity of tangential electric and magnetic fields.
3.4 Reciprocity Theorem in a Poro-Elastic Medium

We derive the reciprocity relationship for the coupled electromagnetic and acoustic equations in porous media. A reciprocity relation interrelates in a specific manner the field quantities of two non-identical physical states (denoted by superscripts A and B) that can occur in one and the same domain in space. Neither the source distribution of the mechanical and electromagnetic wavefields nor the media present in the two states need be the same (Fokkema and van de Berg, 1993).

Let the time harmonic fields $u^A$ and $u^A$ satisfy equations (Eq 3.3) and (Eq 3.4) and the time harmonic fields $E^A$ and $H^A$ equations (Eq 3.1) and (Eq 3.2). The stress tensor $\sigma^A$ and scalar $S^A$ are defined by equations (Eq 3.7) and (Eq 3.8). Similarly let $\mathbf{u}^B, \mathbf{u}^B, S^B, \mathbf{E}^B$ and $\mathbf{H}^B$ be the displacement, stress and electromagnetic field produced by body forces $\mathbf{f}^B, \mathbf{f}^B$ and electromagnetic sources $\mathbf{J}^B$ and $\mathbf{M}^B$. We obtain the following reciprocity theorem in its global form, using Gauss’ divergence theorem.

$$\int_S d\mathbf{S} \cdot \left[ \mathbf{E}^B \times \mathbf{H}^A - \mathbf{E}^A \times \mathbf{H}^B + \mathbf{E}^A \cdot \mathbf{H}^B \right] =$$

$$\int_V \left[ -i\omega \left[ \mu^A - \mu^B \right] \mathbf{H}^B \cdot \mathbf{H}^A + i\omega \left[ \epsilon^A - \epsilon^B \right] \mathbf{E}^B \cdot \mathbf{E}^A + \omega^2 \left[ \rho_f^A - \rho_f^B \right] (-i\omega) \mathbf{u}^A - \mathbf{u}^B + \omega^2 \left[ \rho_f^A - \rho_f^B \right] (-i\omega) \mathbf{u}^A \cdot \mathbf{u}^B + \omega^2 \left[ \rho_f^A - \rho_f^B \right] (-i\omega) \mathbf{u}^A \cdot \mathbf{u}^B - \omega^2 \left[ \rho_f^A - \rho_f^B \right] (-i\omega) \mathbf{u}^A \cdot \mathbf{u}^B + \omega^2 \left[ \rho_f^A - \rho_f^B \right] (-i\omega) \mathbf{u}^A \cdot \mathbf{u}^B -$$

$$\int_V \left[ \mathbf{H}^B \cdot \mathbf{M}^A + \mathbf{E}^B \cdot \mathbf{f}^A (-i\omega) \mathbf{u}^B + \mathbf{E}^A \cdot (-i\omega) \mathbf{u}^B \right] +$$

$$\left[ \mathbf{H}^A \cdot \mathbf{M}^B + \mathbf{E}^A \cdot \mathbf{f}^B \cdot (-i\omega) \mathbf{u}^A + \mathbf{E}^B \cdot (-i\omega) \mathbf{u}^A \right] \right] dV \quad (3.26)$$

When the two media are each other's adjoint then the interaction between the two states are only related by the source distributions (Kong, 1990; Fokkema and van de Berg, 1993).

$$\int_S d\mathbf{S} \cdot \left[ \mathbf{E}^B \times \mathbf{H}^A - \mathbf{E}^A \times \mathbf{H}^B +$$

75
\[
\begin{align*}
\mathbf{r}^B \cdot (-i \omega \mathbf{u}^A) - \mathbf{r}^A \cdot (-i \omega \mathbf{u}^B) + S^B I^A \cdot (-i \omega \mathbf{u}^A) - S^A I^B \cdot (-i \omega \mathbf{u}^B) & = \\
\int_V \left[ - \left[ H^B \cdot \mathbf{M}^A + E^B \cdot \mathbf{J}^A + F^B \cdot (-i \omega \mathbf{u}^A) + F^A \cdot (-i \omega \mathbf{u}^B) \right] + \\
\left[ H^A \cdot \mathbf{M}^B + E^A \cdot \mathbf{J}^B + F^A \cdot (-i \omega \mathbf{u}^A) + F^B \cdot (-i \omega \mathbf{u}^A) \right] \right] dV
\end{align*}
\] (3.27)

Consider the special case when \( \mathbf{u}^B, \mathbf{w}^B, \mathbf{E}^B, \mathbf{H}^B \) are the Green's tensors for a point source. Then,

\[
\begin{align*}
\mathbf{u}_i^B &= u_{i;k}^G(\mathbf{x}; \mathbf{x}'), \\
\mathbf{w}_i^B &= w_{i;k}^G(\mathbf{x}; \mathbf{x}'), \\
\mathbf{E}_i^B &= E_{i;k}^G(\mathbf{x}; \mathbf{x}'), \\
\mathbf{H}_i^B &= H_{i;k}^G(\mathbf{x}; \mathbf{x}').
\end{align*}
\] (3.28)

Where \( k \) denotes the direction of application of the point source. Substituting \( f_i(\mathbf{x}) = \delta_{ik} \delta(\mathbf{x} - \mathbf{x}'), F_i(\mathbf{x}) = \delta_{ik} \delta(\mathbf{x} - \mathbf{x}'), J_i(\mathbf{x}) = \delta_{ik} \delta(\mathbf{x} - \mathbf{x}'), \) or \( M_i(\mathbf{x}) = \delta_{ik} \delta(\mathbf{x} - \mathbf{x}') \) into equation (Eq 3.27) respectively and applying the sifting property of \( \delta_{ik} \) we obtain a representation integral for the \( \mathbf{u}_k(\mathbf{x}), \mathbf{w}_k(\mathbf{x}), \mathbf{E}_k(\mathbf{x}) \) and \( \mathbf{H}_k(\mathbf{x}) \) wavefields for \( \mathbf{x} \) in \( V \). It expresses the fields at any point in \( V \) in terms of the applied sources in \( V \) and field specifications on the boundary.

### 3.5 Eigenvalues and Eigenvectors of the System Matrices for Complex Fourier Parameters

The equations of motion, Maxwell's equations and the constitutive relations can be combined in such a way that only first-order depth derivatives of the displacement-stress-EM field components are needed. This means that plane waves can be studied in terms of the equation of state type (Aki and Richards, 1980),

\[
\frac{\partial \mathbf{B}}{\partial z} = \mathbf{A} \mathbf{B}
\] (3.30)

With the continuity vector \( \mathbf{B} \) defined by equations (Eq 3.24), (Eq 3.25) and \( \mathbf{A} \) defined in (Eq 4.36) and (Eq 4.37) for the \( \text{PSVTM} \) and \( \text{SHTE} \) coupled electroseismic wavefield pictures respectively. For the \( A_{NN} \) system matrices we can find \( N \) eigenvalues

76
and $N$ linearly independent eigenvectors. The most general solution $\mathbf{B}$ to equation (Eq 3.30) is some linear combination of the eigenvectors.

$$\mathbf{B} = \mathbf{D} \mathbf{W}$$

(3.31)

Where $\mathbf{W}$ is a vector of constants, weighting the columns of $\mathbf{D}$ that are to appear in the solution $\mathbf{B}$.

To determine analytic expressions of the eigenvalues and corresponding eigenvectors of the system matrices $\mathbf{A}$ (both PSVTM and SHTEN pictures) we follow a plane-wave solution procedure. First the linear equations of motion (linearity is well satisfied in homogeneous media under infinitesimal strain) describing the behavior of the compressional, rotational and electromagnetic fields in a fully saturated poro-elastic medium are obtained by applying the curl operator to equation (Eq 3.1) and substituting equation (Eq 3.2) into the obtained equation without magnetic and electric sources. To obtain the equations of motion in the composite and the fluid the deformation equations (Eq 3.7), (Eq 3.8) are substituted into equations (Eq 3.3), (Eq 3.4) respectively.

The following equations of motion are arrived at,

$$\nabla \times \nabla \times \mathbf{E} - \omega^2 \varepsilon \mu_0 \mathbf{E} + i\omega^3 \mu_0 \rho_E \mathbf{w} = 0$$

(3.32)

$$\left[ \left( H - \frac{4}{3} G \right) \nabla \cdot \left( \nabla \cdot \mathbf{u} \right) + C \nabla \cdot \left( \nabla \cdot \mathbf{u} \right) \right] +$$

$$G \left[ \nabla \cdot \nabla \mathbf{u} + \nabla \cdot \nabla \mathbf{u}^T - \frac{2}{3} \nabla \cdot \left( \nabla \cdot \mathbf{u} \right) \right] + \omega^2 \rho_B \mathbf{u} + \omega^2 \rho_f \mathbf{w} = 0$$

(3.33)

$$C \nabla \left( \nabla \cdot \mathbf{u} \right) + \omega^2 \rho_f \mathbf{u} + M \nabla \left( \nabla \cdot \mathbf{w} \right) + \omega^2 \rho_E \mathbf{w} - i\omega \rho_E \mathbf{E} = 0$$

(3.34)

With $\varepsilon = \varepsilon_0 \kappa(\omega) + \frac{i}{\omega} \left[ \sigma(\omega) - \frac{n^2(\omega)}{\kappa(\omega)} \right]$. We apply a Fourier transformation with respect to time and with respect to horizontal coordinates, to obtain the equations of motions in the transform domain. We replace the wave fields by plane-wave solutions, with their appropriate polarization, $\hat{u}$, $\hat{w}$ and $\hat{e}$ for the displacement, relative flow and electric field vectors respectively.

$$\mathbf{u} = U \exp(i k \cdot \mathbf{r}) \hat{u}$$

(3.35)
\[ w = W \exp(ik \cdot r) \hat{w} \]  \hspace{1cm} (3.36)

\[ E = E \exp(ik \cdot r) \hat{e} \]  \hspace{1cm} (3.37)

Where \( U, W \) and \( E \) are the plane wave amplitudes of the displacement, relative flow and electric field respectively which can be complex. The direction and magnitude of propagation, and phase velocity direction is given by \( k = \hat{k} \). The wave vector amplitude is expressed as \( \omega/\nu \), with \( \nu \) the appropriate wavefield velocity. The transform-domain equations of motion (note that the equations are divided through by \( \omega^2 \)) become,

\[ E \left[ -\frac{1}{v^2} ((\hat{k} \cdot \hat{e}) \hat{k} - \hat{e}) - \epsilon \mu_0 \hat{e} \right] + W \left[ i \omega \rho_E L \mu_0 \hat{w} \right] = 0 \]  \hspace{1cm} (3.38)

\[ U \left[ \rho_B \hat{u} - \frac{1}{v^2} ((H - G) \hat{k} (\hat{k} \cdot \hat{u}) + G \hat{u}) \right] + W \left[ \rho_f \hat{u} - \frac{1}{v^2} C \hat{k} (\hat{k} \cdot \hat{u}) \right] = 0 \]  \hspace{1cm} (3.39)

\[ E \left[ -i \frac{\rho_E L}{\omega} \hat{e} \right] + U \left[ \rho_f \hat{u} - \frac{1}{v^2} C \hat{k} (\hat{k} \cdot \hat{u}) \right] + W \left[ \rho_E \hat{u} - \frac{1}{v^2} M \hat{k} (\hat{k} \cdot \hat{u}) \right] = 0 \]  \hspace{1cm} (3.40)

### 3.6 Determination of the Different Wave Type Velocities.

To determine the velocities for the different wave types, the transverse and longitudinal polarizations are substituted into the equations of motion. The transverse polarization will lead to an equation set that determines the shear wave velocity and the \( EM \) wave velocity, while the longitudinal polarization leads to an equation set that determines the fast and slow compressional wave in the porous medium.
3.6.1 The Transverse Polarization

For the transverse polarization the displacement, relative flow and TM polarization are perpendicular to the wave vector, as given in,

\[ \hat{k} \cdot \hat{u} = \hat{k} \cdot \hat{w} = \hat{k} \cdot \hat{e} = 0 \]  \hspace{1cm} (3.41)

The different wave polarizations have all three unit magnitude. Substitution into the equations of motion, (Eq 3.38), (Eq 3.39) and (Eq 3.40) gives,

\[ \left[ \frac{1}{v^2} - \epsilon \mu_0 \right] E + [i \omega \rho \rho_0] W = 0 \]  \hspace{1cm} (3.42)

\[ G \left( \frac{1}{v^2} - \frac{\rho_B}{G} \right) U - \rho_f W = 0 \]  \hspace{1cm} (3.43)

\[ -i \rho \frac{L}{\omega} E + \rho_f U + \rho W = 0 \]  \hspace{1cm} (3.44)

The first two equations (Eq 3.42), (Eq 3.43) are used to express the electric and relative flow amplitudes in terms of the displacement amplitude. This gives,

\[ W = \frac{G}{\rho_f} \left[ \frac{1}{v^2} - \frac{\rho_B}{G} \right] U \]  \hspace{1cm} (3.45)

\[ E = -i \omega \rho \frac{G L \mu_0}{\rho_f} \left[ \frac{1/v^2 - \rho_B/G}{1/v^2 - \epsilon \mu_0} \right] U \]  \hspace{1cm} (3.46)

To determine the shear and TM wavefield velocity the determinant of the following equations, where equation (Eq 3.46) is rewritten in terms of U and W amplitudes using equations (Eq 3.42) and (Eq 3.44), need to be evaluated.

\[ G \left( \frac{1}{v^2} - \frac{\rho_B}{G} \right) U - \rho_f W = 0 \]  \hspace{1cm} (3.47)

\[ \rho_f U + \left( 1 - \frac{\rho E L^2 \mu_0}{[1/v^2 - \epsilon \mu_0]} \right) W = 0 \]  \hspace{1cm} (3.48)

Solving the determinant or characteristic equation of the above equations for the velocity \( v \), determines the up and down going transverse and EM wave field velocities,

\[ \frac{2}{v^2} = \frac{\rho_B - \rho_f^2/\rho_E}{G} + \epsilon \mu_0 + L^2 \rho_E \mu_0 \]

\pm \left[ \left( \frac{\rho_B - \rho_f^2/\rho_E}{G} - (\epsilon \mu_0 + L^2 \rho_E \mu_0) \right)^2 - 4 \left( \frac{\rho_f^2 L^2 \mu_0}{G} \right) \right]^{1/2} \]  \hspace{1cm} (3.49)
The + sign denotes the shear \((SV, SH)\) wavefield velocity and the - sign denotes the \(EM\) wavefield velocity. In the \(P - SV\) wavefield picture the \(EM\) wavefield velocity corresponds to the Transverse Magnetic \((TM)\) mode whereas in the \(SH - TE\) picture the \(EM\) velocity corresponds to the Transverse Electric \((TE)\) mode. Since an analytic expression for the two wavefield types is derived the eigenvectors, corresponding to the appropriate wave field eigenvector can now be determined for each horizontal slowness \(p\) (note \(k = \omega p\)).

\[
q_{SV,SH} = \sqrt{\frac{1}{v_{SV,SH}^2} - p^2}, \quad q_{TM,TE} = \sqrt{\frac{1}{v_{TM,TE}^2} - p^2}, \quad Im(q) > 0 \quad (3.50)
\]

### 3.6.2 The Longitudinal Polarization.

For the longitudinal polarization the displacement, relative flow and electric field polarizations are aligned to the wave vector, as expressed in,

\[
\hat{u} = \hat{w} = \hat{e} \quad (3.51)
\]

The different polarizations have all three unit magnitude. Substitution of the polarization vectors into the equations of motion (Eq 3.38), (Eq 3.39) and (Eq 3.40) yields,

\[
\begin{align*}
\quad i \omega \rho E \mu_0 L \mathbf{W} - \epsilon \mu_0 \mathbf{E} &= 0 \quad (3.52) \\
\left[ \rho_f - \frac{C}{v^2} \right] \mathbf{W} + \left[ \rho_B - \frac{H}{v^2} \right] \mathbf{U} &= 0 \quad (3.53) \\
\left[ \rho_E - \frac{M}{v^2} \right] \mathbf{W} + \left[ \rho_f - \frac{C}{v^2} \right] \mathbf{U} - \frac{i \rho E L}{\omega} \mathbf{E} &= 0 \quad (3.54)
\end{align*}
\]

The first two equations are used to express the electric and relative flow amplitudes in terms of the displacement amplitude. From equation (Eq 3.53) and (Eq 3.52) the following expressions are arrived at,

\[
\mathbf{W} = -\left[ \frac{\rho_B - \frac{H}{v^2}}{\rho_f - \frac{C}{v^2}} \right] \mathbf{U} \quad (3.55)
\]
\[
E = -\frac{i\omega \rho E L}{\epsilon} \left[ \frac{\rho_B - \frac{H}{v^2}}{\rho_f - \frac{C}{v^2}} \right] U \tag{3.56}
\]

To determine the fast and slow compressional (\(P_f, P_s\)) velocities, the determinant of the following equation set is set to zero (equation (Eq 3.56) is rewritten in terms of U and W amplitudes using equations (Eq 3.52) and (Eq 3.54)).

\[
\left[ \rho_B - \frac{H}{v^2} \right] U + \left[ \rho_f - \frac{C}{v^2} \right] W = 0 \tag{3.57}
\]

\[
\left[ \rho_B - \frac{C}{v^2} \right] U + \left[ \rho_E - \frac{M}{v^2} + i \frac{L^2 \rho_E}{\epsilon} \right] W = 0 \tag{3.58}
\]

Solving the determinant or characteristic equation of the above equations for velocity \(v\), the up and down going fast and slow wave velocities are obtained,

\[
\frac{2[H M - C^2]}{v^2} = \rho_B M + \rho_E \left[ 1 + \frac{L^2 \rho_E}{\epsilon} \right] H - 2\rho_f C
\]

\[
\pm \left( \left( \rho_B M + \rho_E \left[ 1 + \frac{L^2 \rho_E}{\epsilon} \right] H - 2\rho_f C \right)^2 - 4 \left( H M - C^2 \right) \left( \rho_E \rho_B \left[ 1 + \frac{L^2 \rho_E}{\epsilon} \right] - \rho_f^2 \right) \right)^{1/2} \tag{3.59}
\]

The + sign denotes the fast compressional (\(P_f\)) wavefield velocity and the - sign denotes the slow compressional (\(P_s\)) wavefield velocity. Since an analytic expression for the compressional wavefield type is derived the eigenvectors, corresponding to the appropriate wave field eigenvector can now be determined for each horizontal slowness \(p\) (note \(k = \omega p\)).

\[
q_{P_f} = \sqrt{\frac{1}{v_{P_f}^2} - p^2}, \quad q_{P_s} = \sqrt{\frac{1}{v^2} - p^2}, \quad \text{Im}(q) > 0 \tag{3.60}
\]

### 3.7 Phase Velocity and Specific Attenuation

The derived complex slownesses \(q_r\) with \(r = [P_f, P_s, SV, SH, TM, TE]\) are related to the phase velocities and attenuation in the material. The plane waves take the
following form

\[ \exp[i\omega q_r z] = \exp[-\omega \text{Im}(q_r)z] \cdot \exp[i\omega \text{Re}(q_r)z] \]  \hspace{1cm} (3.61)

The phase velocity and attenuation coefficient are defined as respectively

\[ v_r = \frac{1}{\text{Re}(q_r)}, \quad \alpha_r = \omega \text{Im}(q_r) \]  \hspace{1cm} (3.62)

The attenuation coefficient \( \alpha_r \) is related to the specific attenuation \( Q_r^{-1} \) as ((Toksoz and Johnson, 1981) chapter 1).

\[ Q_r^{-1} = \frac{2\alpha_r v_r}{\omega \left[ 1 - \frac{\alpha_r^2 v_r^2}{\omega^2} \right]} = \frac{2\text{Re}(q_r)}{\text{Re}(q_r) \left[ 1 - \frac{\text{Im}(q_r)^2}{\text{Re}(q_r)^2} \right]} \]  \hspace{1cm} (3.63)

It is common for both fast and shear waves, that \( \text{Im}(q_{P_j,SV,SH}) \ll \text{Re}(q_{P_j,SV,SH}) \). In this case equation (Eq 3.63) reduces to the familiar form \( Q_{P_j,SV,SH} = 2\frac{\text{Im}(q_{P_j,SV,SH})}{\text{Re}(q_{P_j,SV,SH})} \).

This is not the case for the slow waves and EM waves (slow waves and EM waves in the low frequency limit are diffusive).

In the numerical computation of the specific attenuation equation (Eq 3.63) is used for all wave types. The attenuation that is being accounted for in the above expressions is due to fractional heating associated with the relative flow, and that due to the expansion and compression of the viscous fluid phase (bulk-viscosity losses). Additional losses due to "squirt flow" mechanisms could be accounted for by defining complex, frequency dependent frame moduli (which are not used in the calculations).

### 3.8 Displacement Normalized Eigenvectors and Eigenvalues of the System Matrix

In this section the eigenvectors of the electroseismic system's matrix of an isotropic, stratified fully saturated porous medium will be determined. The porous medium system's matrix is given by equations (Eq 4.36) and (Eq 4.37) describing the wavefields.
in the \textit{PSVTM} and \textit{SHTF} picture respectively.

From the theory of matrices it is known that the eigenvalues follow from the
determinantal equation,

\[ \det(A_{M,N} - \Lambda_{M,N}) = 0 \]  \hspace{1cm} (3.64)

With \( \Lambda_{M,N} \) a diagonal eigenvalue matrix. The direct, but difficult approach can be
circumvented by using the plane wave solution approach which results in characteristic
equations which contain the wavefield velocities, see equations (Eq 3.49) and
(Eq 3.59). From the velocities in an unbounded medium the vertical slownesses,
which are the eigenvalues (equations (Eq 3.50) and (Eq 3.60)) of the system's matrix
can be determined.

With the aid of the known eigenvalues, explicit expressions for the accompanying
eigenvectors can be derived. The eigenvectors and eigenvalues relate to the system
matrix through,

\[ D_{M,I}^{-1}A_{I,N}D_{J,N} - \Lambda_{M,N} = 0 \]  \hspace{1cm} (3.65)

Where \( D_{M,N} \) is an 8 by 8 (\textit{PSVTM} picture) or 4 by 4 (\textit{SHTF} picture) eigencolumn
matrix. To solve the 8 by 8 or 4 by 4 systems, (Eq 3.64) and (Eq 3.65) is a straight
forward but rather unfeasible task. This direct approach can be circumvented by
calculating the polarization vectors of the up and down going wavefield. The ob-
tained polarization vector identities are expressed as functions of the free horizontal
slowness. We determine the eigenvectors for the up and down going \( P_f \), \( P_s \) wave
fields and \( SV \), \( TM \) and \( SH \), \( TE \) wave fields by using the polarization vectors, the
\textit{PSVTM} and \textit{SHTF} wavefield amplitudes, (equations (Eq 3.45), (Eq 3.46), (Eq 3.52)
and (Eq 3.53)) and displacement, relative flow and electric field solutions (equations
(Eq 3.35), (Eq 3.36) and (Eq 3.37)).
3.9 Up and Down Going Wavefields and their Polarization

An up and down going wavefield vector $\hat{k} = k\hat{k}$ is defined for the PSVTM system.

$$\hat{k}^\pm = [pv, 0, \pm qv]$$  \hspace{1cm} (3.66)

Where $p$ denotes the horizontal slowness and $q$ the vertical slowness. The + sign corresponds to an up going wave vector and the - sign corresponds to a down going wave vector. The velocity $v$ determines what wave type is dealt with. The up and down going wavefield eigenvector component amplitudes are multiplied with their physical polarization. The following vector identities expressed as functions of the free horizontal slowness variable are,

$$\hat{\hat{y}} \times \hat{k}^\pm = [\pm qv, 0, -pv]^T$$  \hspace{1cm} (3.67)

$$\begin{bmatrix} \hat{\hat{y}} \times \hat{k}^\pm \end{bmatrix} \hat{k}^\pm = \begin{bmatrix} \pm qpv^2 & 0 & q^2v^2 \\ 0 & 0 & 0 \\ -p^2v^2 & 0 & \mp pv^2 \end{bmatrix}$$  \hspace{1cm} (3.68)

$$\hat{k}^\pm \begin{bmatrix} \hat{\hat{y}} \times \hat{k}^\pm \end{bmatrix} = \begin{bmatrix} \pm qpv^2 & 0 & p^2v^2 \\ 0 & 0 & 0 \\ -q^2v^2 & 0 & \mp qv^2 \end{bmatrix}$$  \hspace{1cm} (3.69)

$$\begin{bmatrix} \hat{\hat{y}} \times \hat{k}^\pm \end{bmatrix} \hat{k}^\pm + \hat{k}^\pm \begin{bmatrix} \hat{\hat{y}} \times \hat{k}^\pm \end{bmatrix} = \begin{bmatrix} \pm 2qpv^2 & 0 & (q^2 - p^2)v^2 \\ 0 & 0 & 0 \\ (q^2 - p^2)v^2 & 0 & \mp 2qpv^2 \end{bmatrix}$$  \hspace{1cm} (3.70)

$$\hat{k}^\pm \hat{k}^\pm = \begin{bmatrix} p^2v^2 & 0 & \mp pv^2 \\ 0 & 0 & 0 \\ \pm pv^2 & 0 & q^2v^2 \end{bmatrix}$$  \hspace{1cm} (3.71)
\[
\hat{y} \hat{k}^\pm + \hat{k}^\pm \hat{y} = \begin{bmatrix}
0 & pv & 0 \\
pv & 0 & \pm qv \\
0 & 0 & \mp qv
\end{bmatrix}
\] (3.72)

### 3.9.1 The SV/TM Wavefield.

Their polarization vectors are,

\[
\hat{u} := \hat{\epsilon} = \hat{y} \times \hat{k}^\pm
\] (3.73)

Substituting equation (Eq 3.73) into equations (Eq 3.35) and (Eq 3.45), the displacement and relative flow field vectors are obtained. Substituting these equations into the bulk deformation equation and deformation equation of the fluid the stress fields in the bulk and fluid phases are obtained. The electric field vector is obtained by substituting (Eq 3.75) into (Eq 3.46) and the magnetic field vector is obtained using equation,

\[
H = \frac{1}{i \omega \mu_0} \nabla \times \mathbf{E} = \frac{1}{i \omega \mu_0} \mathbf{E} \left[ \hat{k} \times \hat{e} \right] \exp(i \hat{k} \cdot \mathbf{r})
\] (3.74)

The following mechanical (SV) and electromagnetic (TM) wavefield vector components are determined,

\[
u = U \left[ \hat{y} \times \hat{k}^\pm \right] \exp(i \hat{k} \cdot \mathbf{r})
\] (3.75)

\[
w = U \frac{G}{\rho_f} \left[ \frac{1}{v^2} - \frac{\rho_f}{G} \right] \left[ \hat{y} \times \hat{k}^\pm \right] \exp(i \hat{k} \cdot \mathbf{r})
\] (3.76)

\[
\mathbf{E} = \frac{-i \omega L G \mu_0 \rho E}{\rho_f} \left[ \frac{1}{v^2} - \frac{\rho_f}{G} \right] \left[ \hat{y} \times \hat{k}^\pm \right] \exp(i \hat{k} \cdot \mathbf{r})
\] (3.77)

\[
\mathbf{H} = \frac{-i \omega L G \rho E}{\nu \rho_f} \left[ \frac{1}{v^2} - \frac{\rho_f}{G} \right] \hat{y} \exp(i \hat{k} \cdot \mathbf{r})
\] (3.78)
3.9.2 The $P_f/P_s$ Wavefields

Their polarization vectors are,

$$\hat{\mathbf{u}} = \hat{\mathbf{e}} = \hat{\mathbf{k}}^\pm$$  \hspace{1cm} (3.81)

The displacement and relative flow vectors are obtained by substituting the polarization equation (Eq 3.81) into equations (Eq 3.35) and (Eq 3.55) respectively. The stresses in the bulk and fluid phases are obtained by substituting the displacement and relative flow vectors into the appropriate deformation equations. The electric field is obtained by substituting (Eq 3.82) into (Eq 3.56). The curl of the electric field, see equation (Eq 3.74), results in a zero magnetic field. The following mechanical ($P_f/P_s$) and electric fields of the coulombic attraction type are determined,

$$\mathbf{u} = U \hat{\mathbf{k}} e^{(i\mathbf{k} \cdot \mathbf{r})}$$  \hspace{1cm} (3.82)

$$\mathbf{w} = U \left[ \frac{\rho_B - H}{\rho_f - C \nu^2} \right] \hat{\mathbf{k}}^\pm e^{(i\mathbf{k} \cdot \mathbf{r})}$$  \hspace{1cm} (3.83)

$$\mathbf{\tau} = \frac{U i \omega}{v} \left[ \left[ H - 2G \right] - \left[ \frac{\rho_B - H}{\rho_f - C \nu^2} \right] C \right] \mathbf{I} + 2G \hat{\mathbf{k}}^\pm \hat{\mathbf{k}}^\pm e^{(i\mathbf{k} \cdot \mathbf{r})}$$  \hspace{1cm} (3.84)

$$\mathbf{S} = \frac{U i \omega}{v} \left[ C - \left[ \frac{\rho_B - H}{\rho_f - C \nu^2} \right] M \right] e^{(i\mathbf{k} \cdot \mathbf{r})}$$  \hspace{1cm} (3.85)

$$\mathbf{E} = \frac{U i \omega L \rho_E}{\epsilon} \left[ \frac{\rho_B - H}{\rho_f - C \nu^2} \right] M \hat{\mathbf{k}}^\pm e^{(i\mathbf{k} \cdot \mathbf{r})}$$  \hspace{1cm} (3.86)

$$\mathbf{H} = 0$$  \hspace{1cm} (3.87)

3.9.3 The SH/TE Wavefields.

Their polarization vectors are,

$$\hat{\mathbf{u}} = \hat{\mathbf{e}} = \hat{\mathbf{y}}$$  \hspace{1cm} (3.88)
The displacement and relative flow vectors are obtained by substituting (Eq 3.88) into (Eq 3.35) and (Eq 3.45). The stresses in both phases are obtained by substituting these vectors into the deformation equations. The electric field vector is obtained by substituting (Eq 3.89) into (Eq 3.46). The magnetic field vector is obtained by using Amperes law. The following mechanical (SH) and electromagnetic (TE) wavefield components are obtained,

\[
\begin{align*}
\mathbf{u} &= U \hat{y} e^{(ik \tau)} \\ 
\mathbf{w} &= U \frac{G}{\rho_f} \left[ \frac{1}{v^2} - \frac{\rho_B}{G} \right] \hat{k} e^{(ik \tau)} \\ 
\mathbf{\tau} &= U iG \frac{\omega}{v} \left[ \hat{k} \hat{y} \hat{k} \hat{y}^{\pm} \right] e^{(ik \tau)} \\ 
S &= 0 \\
\mathbf{E} &= U \frac{-i \omega \rho_B G \mu_0}{\rho_f} \left[ \frac{1}{v^2} - \frac{e_B}{G} \right] \hat{y} e^{(ik \tau)} \\ 
\mathbf{H} &= U \frac{-i \omega \rho_B G L}{\rho_f v} \left[ \frac{1}{v^2} - \frac{e_B}{G} \right] \left[ \hat{k} \hat{y} \right] e^{(ik \tau)} 
\end{align*}
\]

\[\text{(3.89)}\]

\[\text{(3.90)}\]

\[\text{(3.91)}\]

\[\text{(3.92)}\]

\[\text{(3.93)}\]

\[\text{(3.94)}\]

### 3.10 The ElectroSeismic Eigenvectors

Substituting equations (Eq 3.68) - (Eq 3.72) into the derived $SV/TM$, $P_f/P_s$ and $SH/TE$ wavefields yield the final eigenvectors for up and down going mechanical and electromagnetic waves in poro-elastic media.

#### 3.10.1 The SV-TM Eigenvectors

\[
\begin{align*}
\mathbf{u}_x &= \pm q v \\
\mathbf{u}_z &= -p v
\end{align*}
\]

\[\text{(3.95)}\]

\[\text{(3.96)}\]
\[ w_z = -\frac{pvG}{\rho_f} \left[ \frac{1}{v^2} - \frac{\rho_B}{G} \right] \]  
(3.97)

\[ \tau_{zx} = i\omega v \left[ q^2 - p^2 \right] G \]  
(3.98)

\[ \tau_{zz} = \mp 2i\omega qpvG \]  
(3.99)

\[ S = 0 \]  
(3.100)

\[ H_y = \frac{-i\omega LG \rho_E}{v\rho_f} \left[ \frac{1}{v^2} - \frac{\rho_B}{G} \right] \]  
(3.101)

\[ E_x = \mp i\omega qvLG \mu \frac{\rho_E}{\rho_f} \left[ \frac{1}{v^2} - \frac{\rho_B}{G} \right] \]  
(3.102)

The previous derived SV/TM velocities (Eq 3.49), \( v \), determine if the eigenvector describes a SV or TM wavefield. The upper signs in the eigenvectors are the down going wavefields while the lower signs are the up going wavefields. If there are no sign options than the up and down going wavefield components are the same.

### 3.10.2 The \( P_f - P_s \) Eigenvectors

\[ u_x = pv \]  
(3.103)

\[ u_z = \pm qv \]  
(3.104)

\[ w_x = \mp \left[ \frac{\rho_B - \frac{H}{\sqrt{3}}}{\rho_f - \frac{C}{\sqrt{3}}} \right] qv \]  
(3.105)

\[ \tau_{zx} = \pm 2i\omega qpvG \]  
(3.106)

\[ \tau_{zz} = \frac{i\omega}{v} \left[ H - 2Gp^2v^2 - \left[ \frac{\rho_B - \frac{H}{\sqrt{3}}}{\rho_f - \frac{C}{\sqrt{3}}} \right] M \right] \]  
(3.107)

\[ S = \frac{i\omega}{v} \left[ C - \left[ \frac{\rho_B - \frac{H}{\sqrt{3}}}{\rho_f - \frac{C}{\sqrt{3}}} \right] M \right] \]  
(3.108)

\[ H_y = 0 \]  
(3.109)

\[ E_x = -\frac{i\omega \rho_E Lpv}{\epsilon} \left[ \frac{\rho_B - \frac{H}{\sqrt{3}}}{\rho_f - \frac{C}{\sqrt{3}}} \right] \]  
(3.110)
The previous determined $P_f/P_s$ velocities, (Eq 3.59), $v$, determine if the eigenvector is a fast or slow wavefield eigenvector.

### 3.10.3 The SH-TE Eigenvectors

\[
\begin{align*}
  u_y &= 1 \quad (3.111) \\
  \tau_{yz} &= \pm i\omega qG \quad (3.112) \\
  H_x &= \mp i\omega LGq\rho E \left[ \frac{1}{v_s^2} - \frac{\rho_p}{G} \right] \left[ \frac{1}{v_s^2} - \epsilon\mu \right] \quad (3.113) \\
  E_y &= -i\omega LG\mu \rho E \left[ \frac{1}{v_s^2} - \frac{\rho_p}{G} \right] \frac{1}{v_s^2} - \epsilon\mu \quad (3.114)
\end{align*}
\]

The previous determined $SH/TE$ wavefield velocities (Eq 3.49), $v$, determine if the eigenvector is a mechanical $SH$ or electromagnetic $TE$ wavefield eigenvector.

### 3.11 Eigenvector Normalization by Time-Averaged Poynting Power

The eigenvectors as derived in the previous section are normalized by unit displacement. In this section the eigenvector normalization by powerflow through a horizontal plane is outlined. This normalization scheme guarantees continuity of the combined poro-elastic Poynting vector (Ben-Menahem and Singh, 1981), and electromagnetic Poynting vector (Stratton, 1941), in the direction perpendicular to the wavefront upon crossing a horizontal plane. This yields a consistent macroscopic theory of energy transfer.

The combined electromagnetic and mechanical, time-averaged (averaged over one
cycle in time) Poynting vector crossing a horizontal boundary (the normal to the plane is taken to be the $z$ direction) in the \textit{PSVTM} and \textit{SHTE} picture, using equation (Eq 3.22) yields,

$$\langle S_z^{(PSVTM)} \rangle = \frac{1}{4} (E_x H_y^* + E_y^* H_x - i \omega [(u_z \tau_{zz}^* - u_z^* \tau_{zz}) + (u_z^* \tau_{zz} - u_z \tau_{zz}) + (w_z S^* - w_z^* S)])$$

$$\langle S_z^{(SHTE)} \rangle = \frac{1}{4} (E_y H_x^* + E_x^* H_y - i \omega [(u_y \tau_{yx}^* - u_y^* \tau_{yx})])$$

Where $*$ denotes the complex conjugate and the following identities have been used.

$$\langle S^{(mech)} \rangle = \frac{1}{2\pi} \int_0^{2\pi} (Rez_1)(Rez_2) d\omega t = \frac{1}{2} Re[Z_1 Z_2^*]$$

$$\langle S^{(EM)} \rangle = \frac{1}{2\pi} \int_0^{2\pi} E \times H d\omega t = \frac{1}{2} Re[E \times H^*]$$

With $z_{1/2}(x, t) = Z_{1/2}(x)e^{-i\omega t}$, $H(x, t) = H(x)e^{-i\omega t}$ and $E(x, t) = E(x)e^{-i\omega t}$. Substituting the derived eigenvectors into equations (Eq 3.115) and (Eq 3.116) gives the normalization factor for each wave type.

### 3.11.1 The SV and TM Vertical Power Flux

$$\langle S_z \rangle = \frac{\omega^2 G}{4} \left[ \left( \frac{q v}{v^*} + \frac{q^* v}{v} \right) \left( \frac{G \mu}{\rho_i^2} \rho E \rho_E^* LL^* \tau_T^* + 1 \right) \right]$$

### 3.11.2 The $P_f$ and $P_\gamma$ Vertical Power Flux

$$\langle S_z \rangle = \frac{\omega^2}{4} \left[ \left( \frac{q v}{v^*} + \frac{q^* v}{v} \right) (H + (\gamma L + \gamma L^*) C + \gamma L \gamma L^* M) \right]$$
3.11.3 The SH TE Vertical Power Flux

\[
\langle S_t \rangle = \frac{\omega^2}{4} \left[ (q^* + q) \left( \frac{\rho_E \rho_E^*}{\rho_f^2} LL^* \mu \gamma_T \gamma_T^* G + 1 \right) \right]
\]  
(3.121)

Where \( \gamma_T \) and \( \gamma_L \) are defined as,

\[
\gamma_T = -\left[ \frac{\frac{1}{v_T^2} - \frac{\rho_B}{G}}{\frac{1}{v_T^2} - \epsilon \mu} \right]
\]  
(3.122)

\[
\gamma_L = -\left[ \frac{\rho_B - H}{\rho_f^2} \right]
\]  
(3.123)

To obtain the normalization with respect to time-averaged Poynting power perpendicular to the plane wave front, we dot equations (Eq 3.119), (Eq 3.120) or (Eq 3.121) into the real part of the wave direction vector, which is \( Re[pv, 0, qv] \). The velocity \( v \) determines which eigenvector wave type is meant, \( * \) denotes the complex conjugate and \( q \) is the vertical slowness and \( p \) is the horizontal slowness of a wave type.

3.12 Conclusions

From the macroscopic coupled equations, an acoustic-electromagnetic power balance equation in its global form in a porous medium is derived. The reciprocity relationship and representation intergrals are determined for the coupled electromagnetic and acoustic equations in a porous medium. The continuity requirements at an interface where a jump in macroscopic medium parameter occurs are determined using the uniqueness theorem. Eigenvalues (wavefield slownesses) and their eigenvectors are derived for each wavetype (fast compressional, slow compressional, rotational and electromagnetic waves) using a plane wave solution procedure, which decomposes the system matrix into eigenrow, eigencolumn and diagonal eigenvalue matrices. Conservation of electromagnetic and poro-elastic Poynting power results in
a normalisation that guarantees energy conservation upon crossing an interface. A field-vector-formalism is hereby established which is based on the macroscopic governing equations that control the coupled electromagnetics and acoustics of porous media and can be numerically solved to obtain reflection and transmission coefficients at jumps in macroscopic medium properties.
Chapter 4

Electroseismic Waves from Point Sources in Layered Media.

4.1 Introduction

When seismic waves propagate through a fluid saturated sedimentary material a small amount of relative fluid-solid motion is induced (the motion of the pore fluid to the solid matrix defines relative flow). The driving force for the relative flow is a combination of pressure gradients set up by the peaks and troughs of a compressional wave and by grain accelerations. The relative flow caused by grain accelerations can therefore be due to either compressional or shear waves.

A fluid electrolyte in contact with a solid surface chemically adsorbes the anions from the electrolyte to the solid wall leaving behind a net excess of ions distributed near the wall. This region is known as an electric double layer (Bockris and Reddy, 1970). The diffuse distribution of mobile ions, with a higher concentration ions in the region close to the adsorbed layer and more and more diffuse towards the neutral
electrolyte are free to move when the fluid moves.

The seismic wave motion which generates the relative flow also induces a ‘streaming’ electrical current due to flow of double-layer ions. This induced streaming current acts as a current source in Maxwell’s equations. Therefore, when seismic waves travel through fluid saturated sedimentary materials current systems are setup in the material inducing non-radiating fields. When the seismic waves hit a contrast in electrical and/or mechanical properties the current systems on both sides form a complex dynamic current system across the interface generating electromagnetic waves.

In this chapter, the macroscopic equations controlling such behavior will be numerically solved for the case of a layered medium. A global matrix method (Chin et al., 1984); Mal, 1988a,b) is employed that solves simultaneously for all the macroscopic electromagnetic and poro-elastic wavefield properties. The equations solved are the coupled poro-elastic (modified Biot’s equations) and electromagnetic (Maxwell’s) equations. Biot’s equations are modified with an induced body force on Biot’s equation of relative flow (where the electromagnetic field induces a mechanical force, an osmosis phenomenon). Ampère’s law in Maxwell’s equations has an external current source (where the relative flow has induced an electrical streaming current, an electrokinetic phenomenon). The coupling is given by the transport equations, these flux/force relations relate the current and the relative flow to potential and pressure gradients. The EM field converted from seismic waves traversing boundaries are associated with a change in the following properties: changes in elastic properties (e.g. porosity, bulk and frame moduli), changes in fluid flow permeability, changes in fluid-chemistry that affect the amount of double layer ions free to move in the double layer (e.g., bulk free-ion concentration, pH).

A thick, relative to the mechanical wavelengths, permeable sand layer example will be presented to show the effect of a mechanical contrast on the mechanical and electromagnetic wavefield components. A fresh water/brine contrast model is used to show the effect of an electrical contrast on the mechanical and electromagnetic wave-
field components. Both models are also used in the calculation of converted magnetic and electric field amplitudes versus seismic source-antenna offset at different distances from the interface.

Snapshots in time are calculated to follow the conversion evolution of mechanical waves into electromagnetic waves. The rapid decay in amplitude with traveled distance of the converted electromagnetic signal driven by the seismic source with a seismic center frequency suggests the Vertical ElectroSeismic Profiling geometry, where antennas are positioned close to the target of interest. The last two numerical examples are calculations in a VESP geometry.

4.2 Macroscopic Coupled Electromagnetic and Poro-Elastic Field Equations

Assuming an $e^{-i\omega t}$ dependence, the macroscopic fully coupled mechanical, electromagnetic equations and constitutive relations in volume averaged form describing the coupled field behavior in two phase porous medium are (Pride, 1994),

The electromagnetic field equations,

\[ \nabla \times E = i\omega B \]  
\[ \nabla \times H = -i\omega D + J + C \]  
\[ D = \varepsilon_0 \left[ \frac{\phi}{\alpha_\infty} (\kappa_f - \kappa_s) + \kappa_s \right] E \]  
\[ B = \mu_0 H \]

The transport equations,

\[ J = \sigma(\omega)E + L(\omega) \left[ -\nabla P + \omega^2 \rho_f u_s + f \right] \]  
\[ -i\omega \mathbf{w} = L(\omega)E \cdot \frac{k(\omega)}{\eta} \left[ -\nabla P + \omega^2 \rho_f u_s + f \right] \]
The mechanical field equations,

\[
\nabla \cdot \tau_B = -\omega^2 [\rho_B \mathbf{u} + \rho_f \mathbf{w}] + \mathbf{F}
\]

\[
\tau_B = [K_G \nabla \cdot \mathbf{u}_s + C \nabla \cdot \mathbf{w}] \mathbf{I} \cdot G_{fr} \left[ \nabla \mathbf{u}_s + \nabla \mathbf{u}_s^T - \frac{2}{3} \nabla \cdot \mathbf{u}_s \mathbf{I} \right]
\]

\[
-P = C \nabla \cdot \mathbf{u}_s + M \nabla \cdot \mathbf{w}
\]

With \( C \) an external electric source and, \( \mathbf{F} \) and \( \mathbf{I} \) bodyforces acting on the bulk and fluid phase of the two phase porous medium respectively. The coefficients in the flux equations relating the equations of mass, here fluid flow and current flow when there are potential and pressure gradients, are the frequency dependent conductivity of the two phase medium, \( \sigma(\omega) \), the dynamic permeability, \( k(\omega) \) and the fluid viscosity, \( \eta \). The first term in equation (Eq 4.5) is the conduction current contribution and the second term is the streaming contribution to the total current. The forces which drive the relative flow are a combination of pressure gradients set up by the peaks and troughs of a compressional wave and by grain acceleration. The relative flow caused by grain acceleration can therefore be both due to compressional and shear waves. The fact that the cross terms have the same coupling coefficient \( L(\omega) \) is a statement of Onsager reciprocity. The coefficient equality implicates the action of the electric field on the charge density to be reciprocal to the action of the fluid pressure on the charge. Or the mobility of the ion is independent of the charge moving through the fluid or the fluid moving around the charge.

The frequency dependent coefficients are obtained in Pride (1994), where they are explicitly obtained by a volume averaging procedure of Maxwell's equations coupled to Biot's equations. Two possible functions that smoothly connect the low and high frequency regimes of the transport coefficients are,

\[
\frac{k(\omega)}{k_0} = \left[ 1 - i \frac{\omega}{\omega_t} \frac{4}{m} \right]^{1/2} - i \frac{\omega}{\omega_t}
\]

\[
\frac{L(\omega)}{L_0} = \left[ i - i \frac{\omega}{\omega_t} \frac{m}{4 \alpha_\infty} \left( 1 - 2 \alpha_\infty \frac{d}{\Lambda} \right)^2 \left( 1 - i^{3/2} \frac{\omega \rho_f}{\eta} \right)^2 \right]^{-1/2}
\]

96
The transition frequency $\omega_t$ separates the low frequency viscous flow behavior from the high frequency inertial flow and is defined as,

$$\omega_t = \frac{\phi}{\alpha_\infty k_0 \rho_f} \eta \frac{\eta}{\alpha_\infty k_0} (4.12)$$

The dimensionless number $m$ is defined as,

$$m \equiv \frac{\phi \Lambda^2}{\alpha_\infty k_0} (4.13)$$

This number consists only of geometry terms. The low frequency coupling coefficients $L_0$ is,

$$L_0 = \frac{\phi \varepsilon_0 \kappa_f \zeta}{\alpha_\infty \eta} \left[ 1 - 2\alpha_\infty \frac{\tilde{d}}{\Lambda} \right] (4.14)$$

With $\kappa_f$, the relative fluid permittivity, $\zeta$, the zeta potential, and $\alpha_\infty$, the tortuosity (normally a number between 3 and 10). The $\sqrt{\frac{\varepsilon_0 \kappa_f \zeta}{\eta}}$ factor in equation (Eq 4.11) determines the viscous skin depth and $\tilde{d} \leq \frac{3 \times 10^{-10}}{\sqrt{C}} \equiv$ Debye length (which is a measure of the electric double layer thickness) and with $C$ the salinity of the pore fluid. The $\Lambda$ parameter is the same as defined by Johnson et al, (1987). An important factor affecting the electrokinetic coefficient is the $\zeta$-potential. The $\zeta$-potential employed in the numerical calculations is taken from experimental studies. These $\zeta$-potential values are based on saturated NaCl, KCl quartz samples at $T = 25^\circ C$ and $pH = 7$, determined by Gaudin and Fuerstenau (1955), Sidorova et al. (1975), and Hidalgo-Alvarez et al. (1985). Regression analysis applied on their combined data sets resulted in the following $\zeta$-potential as function of electrolyte concentration.

$$\zeta(V) = 0.008 + 0.026log_{10}(C) (4.15)$$

The coefficients in the deformation equations are,

$$K_G = H - \frac{4}{3} G = \frac{K_{fr} + \phi K_{fr} + (1 - \phi) K_s \zeta}{1 + \Delta} (4.16)$$

$$C = \frac{K_f + K_s \Delta}{1 + \Delta} (4.17)$$

$$M = \frac{1}{\phi} \frac{K_f}{1 + \Delta} (4.18)$$
Where the parameter $\Delta$ is defined as,

$$\Delta = \frac{K_f}{\phi K_s^2} [(1 - \phi)K_s - K_{fr}]$$  \hspace{1cm} (4.19)

The moduli $K_{fr}$ and $G_{fr}$ are the bulk and shear moduli of the framework of the grains, when the fluid is absent. The frame moduli may either be considered experimentally determined or may be obtained from approximate theoretical models for specific pore grain geometries. $C$ and $M$ are the incompressibilities used by Biot (1962b) and Pride et al. (1992); they are complex and frequency dependent, allowing for losses in addition to those associated with relative flow.

4.3 Transform-Domain Electroseismic Wavefield Constituents in an Isotropic Poro-Elastic Layered Medium

In this section the compressional $(P_f, P_s)$, rotational $(SV, SH)$ and electromagnetic fields $(TM, TE)$ in layered isotropic fully saturated poro-elastic media are investigated. Only an explosive source will be numerically modeled, it will be shown however how to allow for other sources as well. The seismic wave motion which causes relative flow induces a 'streaming' electrical current due to the excess cation motion in the electric double layer. The induced streaming current acts as a current source in Amperes current law, equations (Eq 4.2) and (Eq 4.5). Therefore when seismic waves travel through fluid saturated porous media current systems are set up. When compressional waves travel through homogeneous porous material, the pressure gradients set up by the peaks and troughs cause charge to separate, see figure 4-1. This induces within the seismic pulse a system of electric fields that travels with the compressional wave speed. The streaming current is exactly balanced by the conduction

98
current within the $P$ wave pulse and therefore there is no net current imbalance and the seismic pulse cannot act as an electromagnetic source and radiate electromagnetic waves away from the pulse.

When rotational waves travel through homogeneous porous material, the grain accelerations which are equivoluminal, set up current sheets, inducing electric fields with the same polarization as the particle displacements, see figure 4-2. The current sheets induce within the seismic pulse a magnetic field system that travels with the rotational mechanical wavespeed. By symmetry of the induced current sheets within the seismic pulse, the seismic pulse does not radiate electromagnetic fields.

Compressional and vertical polarized rotational mechanical waves can only generate currents in the $PSV$ particle motion plane in homogeneous isotropic poro-elastic media. This couples to the electromagnetic wavefield components of the $TM$ mode. We will call this from now on the $PSVTM$ coupled electroseismic picture. The horizontal polarized rotational mechanical waves can only generate current in the $SH$ particle motion plane in homogeneous isotropic poro-elastic media. This couples to the electromagnetic wavefield components of the $TE$ mode. We will call this from now on the $SHTE$ coupled electroseismic picture.

But when seismic waves hit a contrast in electrical and/or mechanical properties the current systems within the seismic pulse on both sides become unbalanced and form a complex dynamic current system across the interface generating electromagnetic waves. These $EM$ fields converted from seismic waves traversing boundaries are associated with a change in the following properties: changes in elastic properties (e.g. porosity), changes in fluid flow permeability, and changes in fluid-chemistry that affect the amount of cations free to move in the double layer (e.g. bulk free-ion concentration, pH).
4.3.1 Transform-Domain Field Description

There are assumed to be $ND$ layers. Each layer is characterized by its (frequency dependent) elastic, electromagnetic, hydraulic and electrical transport properties. The fundamental input properties used in the numerical modeling, which determine all wavefield velocities, the electrical conductivity and the electrokinetic coupling coefficients, are: porosity, dc permeability, bulk modulus of the solid and fluid phase, the frame's bulk and shear modulus, the densities of the solid and fluid phase, the viscosity of the fluid phase, the temperature of the bulk material, the tortuosity, the salinity of the fluid and the relative permittivities of the solid and fluid phases.

First the transform-domain field equations describing the wave behavior in each layer need to be determined. Next those field components that are discontinuous across an interface between two media with different poro-elastic and/or electromagnetic properties are eliminated. The resulting differential equations are written in the form of a first-order ordinary matrix differential equation (Aki and Richards, 1980). Since the geometry has azimuthal symmetry it is natural to use cylindrical coordinates $(r, \phi, z)$ with $z$ being depth. First the following definitions involving the horizontal components and horizontal derivatives are employed (e.g., Hudson, 1969a; Kennett, 1983),

\[
    u_Y = \frac{1}{r} \left[ \frac{\partial}{\partial r} (ru_r) + \frac{\partial}{\partial \phi} u_\phi \right] \\
    u_H = \frac{1}{r} \left[ \frac{\partial}{\partial r} (ru_\phi) - \frac{\partial}{\partial \phi} u_r \right] \\
    \tau_{r\phi} = \frac{1}{r} \left[ \frac{\partial}{\partial r} (r \tau_{r\phi}) + \frac{\partial}{\partial \phi} \tau_{\phi\phi} \right] \\
    \tau_{r\phi} = \frac{1}{r} \left[ \frac{\partial}{\partial r} (r \tau_{\phi\phi}) - \frac{\partial}{\partial \phi} \tau_{r\phi} \right]
\]

\[
    < E, H >_Y = \frac{1}{r} \left[ \frac{\partial}{\partial r} (r < E, H >_r) + \frac{\partial}{\partial \phi} < E, H >_\phi \right] \\
    < E, H >_H = \frac{1}{r} \left[ \frac{\partial}{\partial r} (r < E, H >_\phi) - \frac{\partial}{\partial \phi} < E, H >_r \right]
\]
\[ \nabla_1^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \] (4.26)

to rewrite the governing macroscopic coupled electromagnetic and poro-elastic field equations. It is advantageous to employ the invariance properties of the configuration. The equations are first Fourier transformed to the temporal frequency, where linear time invariance of the system is assumed (note \( \frac{\partial}{\partial t} \rightarrow -i\omega \)). To take advantage of the shift invariance of the configuration in the horizontal plane, a Finite Fourier Hankel transform is performed over the horizontal coordinates \( r \) and \( \phi \) thus providing a cylindrical wave decomposition.

This transform pair is defined as,

\[ \text{HF}[\psi(r, \phi)] = \hat{\psi}(k, n) = \frac{1}{2\pi} \int_0^\infty \int_0^{2\pi} drJ_n(kr) d\phi \ e^{-i\phi} \psi(r, \phi) \] (4.27)

\[ \text{HF}^{-1}[\hat{\psi}(k, n)] = \psi(r, \phi) = \int_0^\infty dk \ \sum_{n=-N}^N J_n(kr) e^{i\phi} \hat{\psi}(k, n) \] (4.28)

Note the property \( \text{HF}[\nabla_1^2 \psi] = -k^2 \hat{\psi} \) (Watson, 1966). The magnitude \( N \) in the summation is determined by azimuthal symmetry of the point source. For spherically symmetric sources there is no azimuthal dependence, then \( N = 0 \). If the source can be described using an arbitrarily directed force vector, then \( N \leq 1 \), while if it can be described using a second order moment tensor, then \( N \leq 2 \) (Kennett, 1983).

The first-order linear matrix differential equation satisfied by \( B_{j(m)} \) is,

\[ \frac{\partial}{\partial z} B_{j(m)} = A_{i,j}^{(m)} B_{i(m)} + S_{i(m)} \text{ for } z_{m-1} < z < z_m, \ m = 2, ..., ND - 1 \] (4.29)

\[ \frac{\partial}{\partial z} B_{i(1)} = A_{i,j}^{(1)} B_{j(1)} \text{ for } -\infty < z < z_1 \] (4.30)

\[ \frac{\partial}{\partial z} B_{i(ND)} = A_{i,j}^{(ND)} B_{j(ND)} \text{ for } z_{ND-1} < z < \infty \] (4.31)

With \( S_j \) a stress-displacement-EM source jump representation across a source level. Where the field vector \( B_j \) represents the transform-domain displacement-stress-EM field components in the subdomain \( D_m \) and is defined for the two electroseismic wavefield pictures as, see chapter 3,

\[ B_{j(m;PSVTM)} = [\hat{u}_1, \hat{u}_z, \hat{w}_z, \hat{\tau}_{1z}, \hat{S}, \hat{H}_2, \hat{E}_1]^T \] (4.32)

101
\[ B_j^{(m,SHTE)} = [\hat{u}_2, \hat{\tau}_2, \hat{H}_1, \hat{E}_2]^T \]  

(4.33)

These wavefield vector components are just the ones that are continuous across an interface. The field vector components are arrived at by requiring the continuity of the electromagnetic and poro-elastic Poynting vector in the direction perpendicular to the wavefront upon crossing an interface. This also guarantees a consistent macroscopic theory of energy transfer, see chapter 3. The necessary boundary conditions are sufficient for the uniqueness of the solution of the macroscopic field equations as far as this uniqueness is based on energy considerations. Since there are four different up and down going traveling wavetypes in each poro-elastic layer (\(P_{fast}, P_{slow}, SV\) and \(TM\)) in the PSVTM picture, they impose eight boundary conditions that require continuity of eight wavefield components at both sides of the interface. The \(SHTE\) has two up and down traveling wavetypes and therefore four boundary conditions are required. The explicit boundary conditions that determine uniquely the displacement-stress-EM field components, (Eq 4.32) and (Eq 4.33), are discussed in chapter 3.

For isotropic media the first-order ordinary differential equations decompose into two sets that describe the PSVTM picture, the compressional \(P_{fast}\) and \(P_{slow}\), vertical polarized shear and \(TM\) polarized EM wavefield behavior, and the \(SHTE\) picture, the horizontal polarized shear and \(TE\) polarized EM wavefield behavior. The first picture describes the changes in \(z\) of the field quantities \(B_j^{(PSVTM)} = [\hat{u}_1, \hat{u}_z, \hat{w}_z, \hat{\tau}_1, \hat{\tau}_{zz}, \hat{T}_p, \hat{H}_2, \hat{E}_1]^T\). The second picture describes the changes in \(z\) of the field quantities \(B_j^{(SHTE)} = [\hat{u}_2, \hat{\tau}_2, \hat{H}_1, \hat{E}_2]^T\).

The horizontal components are defined as follows,

\[ \hat{u}_1 = \frac{\hat{u}_V}{ik}, \quad \hat{\tau}_1 = \frac{\hat{\tau}_{Vz}}{ik}, \quad \hat{H}_2 = \frac{\hat{H}_H}{ik}, \quad \hat{E}_1 = \frac{\hat{E}_V}{ik} \]  \hspace{1cm} (4.34)

\[ \hat{u}_2 = \frac{\hat{u}_H}{ik}, \quad \hat{\tau}_2 = \frac{\hat{\tau}_{Hz}}{ik}, \quad \hat{H}_1 = \frac{\hat{H}_V}{ik}, \quad \hat{E}_2 = \frac{\hat{E}_H}{ik} \]  \hspace{1cm} (4.35)

If a line source parallel with the stratification is considered instead of a pointsource, then a plane wave decomposition is employed. Working in the \((x, y, z)\) cartesian
coordinates, assuming the line source to be parallel with the y axis and taking a Fourier transform of the equations of motion with respect to x results in exactly the same equations as are obtained when the following identification of the horizontal components in the displacement-stress-EM vectors are made: \( \hat{u}_1 = \hat{u}_x, \hat{u}_2 = \hat{u}_y, \hat{t}_1 = \hat{t}_{xx}, \hat{t}_2 = \hat{t}_{yz}, \hat{H}_H = \hat{H}_y, \hat{H}_V = \hat{H}_x, \hat{E}_H = \hat{E}_y, \hat{E}_V = \hat{E}_x \). Thus using this mapping, it follows that the solution of \( \frac{\partial}{\partial z} B_j = A_{IJ} B_J \) applies directly to the line source problem as well. Additionally, the cylindrical wave reflection/transmission coefficients to be obtained are necessarily identical to the plane wave coefficients.

\( A_{ij}^{(PSVTM)} \) is an 8 by 8 system matrix, and \( S_j^{(PSVTM)} \) is an 8 by 1 source vector, while \( A_{ij}^{(SHTE)} \) is an 4 by 4 system matrix, and \( S_j^{(SHTE)} \) an 4 by 1 source vector. The system matrix entries depend on frequency and frequency dependent poro-elastic, electric and magnetic medium properties and on the horizontal slowness \( p \) (note \( k = \omega p \)). The first-order linear differential matrix equations describing the \( PSVTM \) case in the transform domain can be written as,

\[
\frac{\partial}{\partial z} B_j^{(PSVTM)} = \begin{bmatrix}
\Delta^{(Biot;PSVTM)} & \Delta^{(osmosis;PSVTM)} \\
\Delta^{(electrokinetic;PSVTM)} & \Delta^{(Maxwell;PSVTM)}
\end{bmatrix} B_j^{(PSVTM)} \tag{4.36}
\]

The first order differential matrix equation describing the \( SHTE \) case in the transform domain is,

\[
\frac{\partial}{\partial z} B_j^{(SHTE)} = \begin{bmatrix}
0 & \frac{1}{\sigma} & 0 & 0 \\
-\omega^2 (\beta - p^2 G) & 0 & 0 & 0 \\
\omega^2 L \rho_f & 0 & 0 & -i \omega \epsilon \left(1 - \frac{p^2}{\mu}\right) \\
0 & 0 & -i \omega \mu & 0
\end{bmatrix} B_j^{(SHTE)} \tag{4.37}
\]

Where the 6 by 6 submatrix, \( PSVTM \) picture, and 2 by 2 submatrix, \( SHTE \) picture,

\[
A_{ij}^{(Biot;PSVTM)} = \begin{bmatrix}
-\omega p a_{11} & a_{12} \\
-\omega^2 a_{21} & -\omega p a_{11}^T
\end{bmatrix}, \quad A_{ij}^{(Biot;SHTE)} = \begin{bmatrix}
0 & \frac{1}{\sigma} \\
-\omega^2 (\beta - p^2 G) & 0
\end{bmatrix} \tag{4.38}
\]
relate the mechanical stress-displacement wavefields with depth.

Where the 2 by 2 submatrix, $PSVTM$ picture, and 2 by 2 submatrix, $SHTE$ picture,

$$A_{ij}^{(Maxwell;PSVTM)} = a_{33}, \quad A_{ij}^{(Maxwell;SHTE)} = \begin{bmatrix} 0 & -i\omega \left(1 - \frac{\nu^2}{\mu^2}\right) \\ -i\omega \mu & 0 \end{bmatrix} \quad (4.39)$$

relate the electric and magnetic wavefield components with depth.

Where the 6 by 2 submatrix, $PSVTM$ picture, and 2 by 2 submatrix, $SHTE$ picture,

$$A_{ij}^{(electrokinetic;PSVTM)} = \begin{bmatrix} -\omega^2 a_{31} & -i\omega p a_{32} \\ \omega^2 L \rho_f & 0 \end{bmatrix} \quad (4.40)$$

$$A_{ij}^{(electrokinetic;SHTE)} = \begin{bmatrix} \omega^2 L \rho_f & 0 \\ 0 & 0 \end{bmatrix}$$

relate the electric and magnetic wavefield components, converted from the mechanical wavefields, with depth.

And where the 2 by 6 submatrix, $PSVTM$ picture, and 2 by 2 submatrix, $SHTE$ picture,

$$A_{ij}^{(osmosis;PSVTM)} = \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}, \quad A_{ij}^{(osmosis;SHTE)} = 0 \quad (4.41)$$

relate the mechanical wavefield components, converted from electric and magnetic wavefields, with depth.

with the submatrices given by,

$$a_{11} = \begin{bmatrix} 0 & 1 & 0 \\ \alpha & 0 & 0 \\ -\beta & 0 & 0 \end{bmatrix}, \quad a_{12} = \begin{bmatrix} \frac{1}{G} & 0 & 0 \\ 0 & \frac{M}{\Delta} & -\frac{C}{\Delta} \\ 0 & -\frac{C}{\Delta} & \frac{H}{\Delta} - \frac{p^2}{\rho E} \end{bmatrix}, \quad a_{13} = 0 \quad (4.42)$$

$$a_{21} = \begin{bmatrix} B - 2p^2G(1+\alpha) & 0 & 0 \\ 0 & \rho_B & \rho_f \\ 0 & \rho_f & \frac{\epsilon}{\rho E \epsilon - \rho E \epsilon^2} \end{bmatrix}, \quad a_{23} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (4.43)$$

$$a_{31} = \begin{bmatrix} L \rho_f & 0 & 0 \\ 0 & 0 & \frac{\rho E \rho_f}{\epsilon - \rho E \epsilon^2} \end{bmatrix}, \quad a_{32} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.44)$$

104
\[
\begin{bmatrix}
0 & i\omega \epsilon \\
\bar{i}\omega \mu \left(1 - \frac{\rho^2}{\mu (r - \rho_E L)}\right) & 0
\end{bmatrix}
\]

The following coefficients have been used in the matrices above.
\[
\alpha = 1 - \frac{2GM}{\Delta} \quad \beta = \frac{\rho_I}{\rho_E} - \frac{2GC}{\Delta}
\]
\[
\Delta = HM - C^2 \quad B = \rho_B - \frac{\rho_J^2}{\rho_E}
\]
\[
\epsilon = \epsilon_0 \kappa(\omega) + \frac{\sigma(\omega)}{\omega}
\]

(4.45)

(4.46)

(4.47)

4.3.2 Solution Procedure to Obtain Wavefield Constituents

To obtain the wavefields in each layer, the continuity conditions at each boundary are used.
\[
\lim_{z \rightarrow z_m} B_J^{(m+1)} - \lim_{z \rightarrow z_m} B_J^{(m)} = 0 \quad m \neq s
\]

(4.48)

At the source level \(z = z_s\), where the localized source is situated, \(B_J\) jumps by a finite amount. At the source level,
\[
\lim_{z \rightarrow z_s} B_J^{(s+1)} - \lim_{z \rightarrow z_s} B_J^{(s)} = S_J
\]

(4.49)

With \(S_J\) the source vector. A linear transformation on each of the \(ND\) field vectors \(B_J^{(m)}\) is carried out. Through it, a field-vector-formalism is obtained in which a decomposition of \(B_J^{(m)}\) into up and downgoing fields is manifest. Let \(W_N^{(m;PSVTM)}\) be the 8 by 1 vector and \(W_N^{(m;SHTE)}\) the 4 by 1 vector that is related to \(B_J^{(m)}\) by the linear transformation,
\[
B_J^{(m)} = D_{J,N}^{(m)} W_N^{(m)}
\]

(4.50)

Where the composition matrix \(D_{J,N}^{(m)}\) is subject to a convenient choice. From the theory of matrices \(D_{J,N}^{(m)}\) is taken the eigencolumn matrix of the system matrix \(A_{I,J}^{(m)}\)
\[
D_{J,N}^{(m;PSVTM)} = \begin{bmatrix}
b_J^{(m;-P_I)}, b_J^{(m;-P_S)}, b_J^{(m;-SV)}, b_J^{(m;-TM)}, \\
b_J^{(m;+P_I)}, b_J^{(m;+P_S)}, b_J^{(m;+SV)}, b_J^{(m;+TM)}
\end{bmatrix}
\]

(4.51)

105
\[ D^{(m;SHTE)}_{J,N} = \begin{bmatrix} b^{(m;-SH)}_J, b^{(m;-TE)}_J, b^{(m;+SH)}_J, b^{(m;+TE)}_J \end{bmatrix} \] (4.52)

Where \( b^{(m;\pm r)}_J \), \( r \in P_{\text{fast}}, P_{\text{slow}}, SV, SH, TM, TE \) denote the eigenvectors of the system matrices \( A^{(m)}_{i,j} \). The eigenvectors are derived in chapter 3. The \( b^{(m;\pm r)}_J \) eigenvectors and their normalizations used in this chapter are given in appendix A.

For later convenience the elements of \( W^{(m)}_N \) are denoted \( W^{(m;\pm)}_r \), with \(+\) corresponding to upgoing and \(-\) corresponding to downgoing waves,

\[ W^{(m;PSVTM)}_N = \begin{bmatrix} W^{(m;-)}_{P_I}, W^{(m;-)}_{P_s}, W^{(m;-)}_{SV}, W^{(m;-)}_{TM} \\ W^{(m;+)}_{P_I}, W^{(m;+)}_{P_s}, W^{(m;+)}_{SV}, W^{(m;+)}_{TM} \end{bmatrix}^T \]

\[ W^{(m;SHTE)}_N = \begin{bmatrix} W^{(m;-)}_{SH}, W^{(m;-)}_{TE}, W^{(m;+)}_{SH}, W^{(m;+)}_{TE} \end{bmatrix}^T \] (4.53) (4.54)

Substituting equation (Eq 4.50) into equation (Eq 4.29) and premultiplication by \( D^{(m)-1}_{M,I} \) yields the desired matrix differential equation for \( W^{(m)}_N \),

\[ \frac{\partial}{\partial z} W^{(m)}_M = \Lambda^{(m)}_{M,N} W^{(m)}_N \] (4.55)

Where \( \Lambda^{(m)}_{M,N} \) is the diagonal matrix of the eigenvalues \( q^{(m;\pm r)} \) of system matrix \( A^{(m)}_{i,j} \), and the diagonal matrix \( \Lambda^{(m)}_{M,N} \) is defined as,

\[ \Lambda^{(m;PSVTM)}_{M,N} = \text{diag} \left[ q^{(m;-P_I)}, q^{(m;-P_s)}, q^{(m;-SV)}, q^{(m;-TM)}, q^{(m;+P_I)}, q^{(m;+P_s)}, q^{(m;+SV)}, q^{(m;+TM)} \right] \] (4.56)

\[ \Lambda^{(m;SHTE)}_{M,N} = \text{diag} \left[ q^{(m;-SH)}, q^{(m;-TE)}, q^{(m;+SH)}, q^{(m;+TE)} \right] \] (4.57)

For each subdomain \( D_m \) the system of first-order linear differential equations is now mutually uncoupled. The field vector \( W^{(m)}_N(z) \) of the subdomain \( D_m \) is coupled to the field vector \( W^{(m+1)}_N(z) \) of the subdomain \( D_{m+1} \) through the boundary conditions, equation (Eq 4.48) at the interface \( z = z_m \) between \( D_m \) and \( D_{m+1} \).

\[ \lim_{z \downarrow z_m} D^{(m+1)}_{J,N} W^{(m+1)}_N(z) - \lim_{z \uparrow z_m} D^{(m)}_{J,N} W^{(m)}_N(z) = 0 \quad m \neq 0 \] (4.58)

While at the source level \( z = z_s \),

\[ \lim_{z \downarrow z_s} D^{(s+1)}_{J,N} W^{(s+1)}_N(z) - \lim_{z \uparrow z_s} D^{(s)}_{J,N} W^{(s)}_N(z) = S_J \] (4.59)
Since there are \( ND - 1 \) interfaces, equations (Eq 4.58) and (Eq 4.59) represent \( 8 \times (ND - 1) \), \( PSVTM \) picture, or \( 4 \times (ND - 1) \), \( SHTE \) picture, linear algebraic equations of which the eight or four that follow from equation (Eq 4.59) are inhomogeneous. Next, for each subdomain \( D_m \), the field vector \( W^{(m)}_N(z) \) is written in terms of the linear independent functions \( \Gamma^{(m;\pm r)}(z) \) that express the decomposition into up and downgoing fields in \( D_m \).

\[
\Gamma^{(m;PSVTM)}_{M,K} = \text{diag} \left[ \Gamma^{(m;-P_f)}, \Gamma^{(m;-P_s)}, \Gamma^{(m;-SV)}, \Gamma^{(m;-TM)}, \Gamma^{(m;+P_f)}, \Gamma^{(m;+P_s)}, \Gamma^{(m;+SV)}, \Gamma^{(m;+TM)} \right]
\]

(4.60)

\[
\Gamma^{(m;SHTE)}_{M,K} = \text{diag} \left[ \Gamma^{(m;-SH)}, \Gamma^{(m;-TE)}, \Gamma^{(m;+SH)}, \Gamma^{(m;+TE)} \right]
\]

(4.61)

Where \( \Gamma^{(m)}_{M,K}(z) \) are solutions of differential equations (Eq 4.29).

\[
\frac{\partial}{\partial z} \Gamma^{(m)}_{M,K} = \Lambda^{(m)}_{M,N} \Gamma^{(m)}_{N,K}
\]

(4.62)

The diagonal elements \( \Gamma^{(m;\pm r)}(z) \) of \( \Gamma^{(m)}_{M,K}(z) \) are given by,

\[
\Gamma^{(m;-r)}(z) = \text{exp}[i\omega q^{(m;-r)}(z - z_m)] \quad \text{for} \quad z_{m-1} < z < z_m
\]

(4.63)

\[
\Gamma^{(m;+r)}(z) = \text{exp}[-i\omega q^{(m;+r)}(z - z_{m-1})] \quad \text{for} \quad z_{m-1} < z < z_m
\]

(4.64)

The source vector \( S_I \) contains a concentrated source and has the form,

\[
S_I = \hat{\phi}(\omega) F_I \delta(z - z_s)
\]

(4.65)

Where \( \hat{\phi}(\omega) \) is the Fourier transform of the source signature \( \phi(t) \) and \( F_I \) is the 8 by 1 or 4 by 1 vector that depends on the nature of the source. Combining equations (Eq 4.55), (Eq 4.58) - (Eq 4.60) and (Eq 4.65) the following expression for \( W^{(m)}_N(z) \) is arrived at,

\[
W^{(m)}_N(z) = \hat{\phi}(\omega) \Gamma^{(m)}_{N,M}(z) \tilde{W}^{(m)}_M
\]

(4.66)

The coefficient \( \tilde{W}^{(m)}_M \) expresses the action of the source as well as the influence of the poro-elastic and electromagnetic properties of each layer. They follow from the
boundary conditions at each of the \( ND - 1 \) interfaces, equations (Eq 4.58) and (Eq 4.59). The transform-domain field vector \( B_j^{(m)} \) of the subdomain \( D_m \) is related to \( \tilde{W}_M^{(m)} \) via the linear transformation,

\[
B_j^{(m)}(k, \omega, z) = \hat{\phi}(\omega) D_{J,N} \Gamma_{N,M}^{(m)}(z) \tilde{W}_M^{(m)}
\]

(4.67)

The righthand side of equation (Eq 4.67) can be recognized as the superposition of eight \( \text{PSVTM} \) picture, four \( \text{SHTE} \) picture terms, each corresponding to a transform-domain field component. In the \( \text{PSVTM} \) picture four are upward and four are downward traveling field constituents in subdomain \( D_m \) while in the \( \text{SHTE} \) picture two upward and two downward field constituents are propagating in the same subdomain. In accordance with equations (Eq 4.53) and (Eq 4.54) \( \tilde{W}_N^{(m)} \) is written as,

\[
\tilde{W}_N^{(m;\text{PSVTM})} = \left[ \tilde{W}_P^{(m;+)} \begin{array}{c} \tilde{W}_P^{(m;-)} \\ \tilde{W}_S^{(m;+)} \\ \tilde{W}_S^{(m;-)} \\ \tilde{W}_{TM}^{(m;+)} \\ \tilde{W}_{TM}^{(m;-)} \end{array} \right] \quad (4.68)
\]

\[
\tilde{W}_N^{(m;\text{SHTE})} = \left[ \tilde{W}_S^{(m;+)} \begin{array}{c} \tilde{W}_S^{(m;-)} \\ \tilde{W}_H^{(m;+)} \\ \tilde{W}_H^{(m;-)} \\ \tilde{W}_{TE}^{(m;+)} \\ \tilde{W}_{TE}^{(m;-)} \end{array} \right] \quad (4.69)
\]

The radiation conditions on \( B_j^{(1)}(z) \) and \( B_j^{(ND)}(z) \) as \( |z| \to \infty \) result in,

\[
\tilde{W}_r^{(1;\pm)} \equiv 0 \quad \tilde{W}_r^{(ND;\pm)} \equiv 0 \quad (4.70)
\]

This leaves a total of \( 8 \times (ND-1) \) unknown coefficients \( \tilde{W}_r^{(m;\pm)} \) \( \text{PSVTM} \) picture or \( 4 \times (ND-1) \) unknown coefficients in the \( \text{SHTE} \) picture, being just equal to the number of linear algebraic equations (Eq 4.58) and (Eq 4.59).

### 4.4 The Global Matrix Method

The global matrix method is used to solve simultaneously the unknown \( \tilde{W}_r^{(m;\pm)} \) coefficients in all layers. The formulation was developed by Chin et al. (1984). The
global matrix formulation has several important features: (1) Multiple sources can be treated, the produced wavefields are simply superposed, (2) The up and down going wavefield coefficients are simultaneously solved for in all layers, therefore the wavefield is known to all receivers at all places in the medium, (3) Time stability problems do not occur because only decaying exponents are needed. The Thomson-Haskell propagator matrix approach (Schmidt and Tango, 1986) doesn’t have these advantageous features.

### 4.4.1 Solution Procedure of the PSVTM and SHTE Problems

In the PSVTM picture the eigencolumn matrix, for each subdomain \( m \), of the system matrix \( A_{I,J}^{(m;PSVTM)} \) is defined as

\[
D_{J,N}^{(m;PSVTM)} = \begin{bmatrix}
\Phi(m;P_{f},-P_{s}), & \Phi(m;-SV,-TM), & \Phi(m;+P_{f},+P_{s}), & \Phi(m;+SV,+TM)
\end{bmatrix} \quad (4.71)
\]

\[
= \begin{bmatrix}
M_{P,Q}^{(m,PSVTM; \pm)}, & M_{P,Q}^{(m,PSVTM; \pm)} \\
N_{P,Q}^{(m,PSVTM; \pm)}, & N_{P,Q}^{(m,PSVTM; \pm)}
\end{bmatrix} \quad (4.72)
\]

With \( M_{P,Q}^{(m,PSVTM; \pm)} \) and \( N_{P,Q}^{(m,PSVTM; \pm)} \) 4 × 4 matrices.

In the SHTE picture the eigencolumn matrix for each subdomain \( m \), of the system matrix \( A_{I,J}^{(m;SHTE)} \) is defined as

\[
D_{J,N}^{(m;SHTE)} = \begin{bmatrix}
\Phi(m;-SH,-TE), & \Phi(m;+SH,+TE)
\end{bmatrix} = \begin{bmatrix}
M_{P,Q}^{(m,SHTE; \pm)}, & M_{P,Q}^{(m,SHTE; \pm)} \\
N_{P,Q}^{(m,SHTE; \pm)}, & N_{P,Q}^{(m,SHTE; \pm)}
\end{bmatrix} \quad (4.73)
\]

With \( M_{P,Q}^{(m,SHTE; \pm)} \) and \( N_{P,Q}^{(m,SHTE; \pm)} \) 2 × 2 matrices.

The submatrices \( \Phi \) contain the eigenvectors \( b_{j}^{(m;\pm r)} \), displacement/powerflow normalized, with \( \pm r \in P_{f}, P_{s}, SV, TM \) the up and downgoing wavefield types (see appendix
A). The propagator matrix is defined as, see equations (Eq 4.63) and (Eq 4.64),

\[
\Gamma_{M,N}^{(m)} = \exp[\Lambda_{M,N}^{(m)}(z_m - z_{m-1})] = \begin{bmatrix}
E_{P,Q}^{(m)} & 0 \\
0 & E_{P,Q}^{(m)-1}
\end{bmatrix}
\]  \hspace{1cm} (4.74)

With the \( E_{P,Q}^{(m)} \) matrices defined to be,

\[
E_{P,Q}^{(m;PSVTM)} = \text{diag} \left( \exp[i\omega q^{(m;P_f)}(m;P_s) \Delta z], \exp[i\omega q^{(m;SV)} \Delta z], \exp[i\omega q^{(m;TM)} \Delta z] \right)
\]  \hspace{1cm} (4.75)

\[
E_{P,Q}^{(m;SHTE)} = \text{diag} \left( \exp[i\omega q^{(m;SH)} \Delta z], \exp[i\omega q^{(m;TE)} \Delta z] \right)
\]  \hspace{1cm} (4.76)

With \( q^{(m;P_f,P_s)} = \sqrt{\frac{1}{\nu_{P_f,P_s}^2} - p^2} \), \( q^{(m;SV,SH)} = \sqrt{\frac{1}{\nu_{SV,SH}^2} - p^2} \), \( q^{(m;TM,TE)} = \sqrt{\frac{1}{\nu_{TM,TE}^2} - p^2} \)

the vertical slowness for the compressional fast wave mode, the compressional slow
wave mode, the vertical polarized shear wave mode, the horizontally polarized shear
wave mode, the transverse magnetic \( EM \) mode and the transverse electric \( EM \)
mode. The choice of branch cuts is defined according to the computational methods used to perform the inverse Fourier and Hankel transforms; \( Im(\omega q^{(m;P_f,P_s)}) > 0 \),
\( Im(\omega q^{(m;SV,SH)}) > 0 \), \( Im(\omega q^{(m;TM,TE)}) > 0 \).

For \( m \neq s \), see equation (Eq 4.58) and applying the linear transformation (Eq 4.67),
the interface continuity condition can be written as,

\[
\begin{bmatrix}
M_{P,Q}^{(m;-)} & M_{P,Q}^{(m;+)} \\
N_{P,Q}^{(m;-)} & N_{P,Q}^{(m;+)}
\end{bmatrix}
\begin{bmatrix}
E_{P,Q}^{(m)} & 0 \\
0 & E_{P,Q}^{(m)-1}
\end{bmatrix}
\begin{bmatrix}
p^{(m;-)} \\
p^{(m;+)}
\end{bmatrix} = 0
\]  \hspace{1cm} (4.77)

\[
\tilde{W}_{M}^{(m;PSVTM)} = \begin{bmatrix}
p^{(m;PSVTM,-)} \\
p^{(m;PSVTM,+)}
\end{bmatrix}, \quad \tilde{W}_{M}^{(m;SHTE)} = \begin{bmatrix}
p^{(m;SHTE,-)} \\
p^{(m;SHTE,+)}
\end{bmatrix}
\]  \hspace{1cm} (4.78)
4.4.2 Radiation Conditions

The radiation condition for halfspaces requires the conditions of no incoming waves from $|z| \to \infty$, implying $\mathcal{E}^{(1; -)} = 0$ and $\mathcal{E}^{(ND; +)} = 0$. The interface conditions at $z = z_i$ become

$$M_{P,Q}^{(1; +)} E_{P,Q}^{(1; -)} E^{(1; +)} - M_{P,Q}^{(2; -)} E^{(2; -)} - M_{P,Q}^{(2; +)} E^{(2; +)} = 0$$
$$N_{P,Q}^{(1; +)} E_{P,Q}^{(1; -)} E^{(1; +)} - N_{P,Q}^{(2; -)} E^{(2; -)} - N_{P,Q}^{(2; +)} E^{(2; +)} = 0$$

At $z = z_N$ the interface conditions are,

$$M_{P,Q}^{(ND-1; -)} E_{P,Q}^{(ND-1; -)} E^{(ND-1; -)} + M_{P,Q}^{(ND-1; +)} E_{P,Q}^{(ND-1; -)} E^{(ND-1; +)} - M_{P,Q}^{(ND; -)} E^{(ND; -)} = 0$$
$$N_{P,Q}^{(ND-1; -)} E_{P,Q}^{(ND-1; -)} E^{(ND-1; -)} + N_{P,Q}^{(ND-1; +)} E_{P,Q}^{(ND-1; -)} E^{(ND-1; +)} - N_{P,Q}^{(ND; -)} E^{(ND; -)} = 0$$

4.4.3 Free surface Conditions

To include a free surface, the free surface conditions for the mechanical wavefields and the ordinary electromagnetic boundary conditions at the porous medium-air interface need to be complied with. The mechanical normal and tangential stresses and pressure in the two phase medium have to vanish at the free surface, while the tangential electric and magnetic fields obey the porous medium air-boundary conditions. When we neglect the osmosis effect at the free surface, the up and downgoing fast, slow, and shear wave amplitudes can be related through the vanishing of the stress and fluid traction boundary conditions. The up and downgoing electric and magnetic wavefields at the free surface are related through the ordinary electromagnetic boundary conditions. All downgoing wave type amplitudes are now related to the upgoing wave amplitudes and satisfy all free surface conditions. The reflected wavefield from the free surface are phase delayed to the interface below and included in the phase
matching across this boundary.

In this analysis the following matrices are used: The $3 \times 3$ matrix, $S_{33}^{(1;\mp)}$, which contains the mechanical eigenvector components that relate the mechanical wavefield amplitudes to the normal and shear bulk stresses and pressure in the pore fluid; the $2 \times 3$ matrix, $U_{23}^{(1;\mp)}$, which contains the mechanical eigenvector components that relate the mechanical wavefield amplitudes to the tangential magnetical and electrical wavefields; the $2 \times 2$ matrix, $V_{22}^{(1)}$, which contains the electromagnetic eigenvector components that relate the EM wavefield amplitudes to the tangential magnetical and electrical wavefields in the porous medium; and the $2 \times 1$ matrix, $W_{21}^{(0;\pm)}$, which contains the electromagnetic eigenvector components that relate the EM wavefield amplitudes to the tangential magnetical and electrical wavefields in the upper half-space (air). Where the superscript 1 denotes the first layer, 0 the halfspace above the free surface, and $\mp$ denotes eigenvector components relating to the down or up going wavefields. Using the eigencolumn components listed in appendix A, the following matrices are constructed, necessary to express the reflected wavefield amplitudes from the free surface in terms of upgoing wave amplitudes.

\[
S_{33}^{(1;\mp)} = \begin{bmatrix}
\pm 2i\omega pq v_{p_f} G \\
\pm 2i\omega pq v_{p_s} G \\
\i\omega v_{p_f} \left[ C - \frac{\rho_B - \frac{\mu}{v_{p_f}^2}}{\rho_f - \frac{\mu}{v_{p_f}^2}} M \right] \\
\i\omega v_{p_s} \left[ C - \frac{\rho_B - \frac{\mu}{v_{p_s}^2}}{\rho_f - \frac{\mu}{v_{p_s}^2}} M \right] \\
0
\end{bmatrix}
\]

\[
U_{23}^{(1;\mp)} = \begin{bmatrix}
0 \\
0 \\
-\i\omega \rho_{LE} \frac{\rho_B - \frac{H}{v_{p_f}^2}}{\rho_f - \frac{\mu}{v_{p_f}^2}} \\
-\i\omega \rho_{LE} \frac{\rho_B - \frac{H}{v_{p_s}^2}}{\rho_f - \frac{\mu}{v_{p_s}^2}} \\
\mp \i\omega \sigma v_{sv} L \frac{\rho_B - \frac{H}{v_{p_f}^2}}{\rho_f - \frac{\mu}{v_{p_f}^2}} \\
\mp \i\omega \sigma v_{sv} L \frac{\rho_B - \frac{H}{v_{p_s}^2}}{\rho_f - \frac{\mu}{v_{p_s}^2}} \\
\end{bmatrix}
\]

112
\[
V_{22}^{(1)} = \begin{bmatrix}
-i\omega LGG_{\mu} & -\frac{\nu_{lm}}{v_{lm} p_{f}} & -\frac{\nu_{lm}}{v_{lm} p_{f}} \\
-i\omega LGG_{\mu} & -\frac{\nu_{lm}}{v_{lm} p_{f}} & -\frac{\nu_{lm}}{v_{lm} p_{f}} \\
-i\nu_{lm} LGG_{\mu} & -\nu_{lm} LGG_{\mu} & -\nu_{lm} LGG_{\mu}
\end{bmatrix}
\]

\[
W_{21}^{(0;+)} = \begin{bmatrix}
-q v_{lm} \\
\sqrt{\frac{\rho}{\mu}}
\end{bmatrix}
\]

(4.79)

The up and downgoing mechanical wavefield amplitudes in the first layer are the fast wave amplitudes, \(p_{1}^{(1;\pm)}\), the shear wave amplitudes, \(p_{2}^{(1;\pm)}\), and the slow wave amplitudes, \(p_{3}^{(1;\pm)}\), which can be related through the vanishing stress and fluid traction boundary conditions at the free surface.

\[
p_{1-3}^{(1;\pm)} = -\left[S_{33}^{(1;\pm)}\right]^{-1} S_{33}^{(1;\pm)} p_{1-3}^{(1;\pm)}
\]

The electromagnetic boundary conditions are the usual continuity of the magnetic, \(H_\phi\), and electric, \(E_r\), components of the electromagnetic TM mode. The radiation in the space above the free surface requires the condition of no incoming EM waves from \(z \to -\infty\), implying \(p_{4}^{(0;-)} = 0\), downgoing TM wave amplitude is zero. The EM boundary conditions at the free surface are expressed as follows,

\[
U_{23}^{(1;-)} p_{1-3}^{(1;-)} + U_{23}^{(1;+)} p_{1-3}^{(1;+)} + V_{22}^{(1)} \begin{bmatrix} p_{4}^{(1;-)} \\ p_{4}^{(1;+)} \end{bmatrix} = W_{21}^{(0;+)}
\]

(4.80)

Using equation (Eq 4.80), the following relation between the up and downgoing EM wave amplitudes propagating in the first layer can be obtained.

\[
\begin{bmatrix} p_{4}^{(1;-)} \\ p_{4}^{(1;+)} \end{bmatrix} = \left[V_{22}^{(1)}\right]^{-1} \left(W_{21}^{(0;-)}\right)^{-1} \left(U_{23}^{(1;-)} S_{33}^{(1;-)} p_{1-3}^{(1;+)} + U_{23}^{(1;+)} p_{1-3}^{(1;+)}\right)
\]

(4.81)

Eliminating the \(p_{4}^{(0;+)}\) amplitude, the downgoing TM amplitude in the first layer can be expressed in terms of the upgoing TM and mechanical wavefield amplitudes.

\[
p_{4}^{(1;-)} = \frac{1}{\text{det}(V_{22}^{(1)})} \left[\text{det}(V_{22}^{(1)}) p_{4}^{(1;+)} + \Xi(2,1) p_{1}^{(1;+)} + \Xi(2,2) p_{1}^{(1;+)} + \Xi(2,3) p_{3}^{(1;+)}\right]
\]

113
\[ - \Xi(1, 1)p_1^{(1;+)} - \Xi(1, 2)p_2^{(1;+)} - \Xi(1, 3)p_3^{(1;+)} \] 

(4.82)

With,

\[ \Theta = \frac{V_{22}^{(1)}(2, 2)W_{21}^{(0;\cdot)}(1) - V_{22}^{(1)}(1, 2)W_{21}^{(0;\cdot)}(2)}{-V_{12}^{(1)}(2, 1)W_{21}^{(0;\cdot)}(1) + V_{22}^{(1)}(1, 1)W_{21}^{(0;\cdot)}(2)} \] 

(4.83)

\[ \Xi_{23} = -U_{23}^{(1;\cdot)} \left[ S_{33}^{(1;\cdot)} \right]^{-1} S_{33}^{(1;\cdot)} p_{1-3}^{(1;+)} + U_{23}^{(1;\cdot)} p_{1-3}^{(1;+)} \] 

(4.84)

Equations (Eq 4.80) and (Eq 4.82) relate the upgoing wave amplitudes to the downgoing wave amplitudes satisfying the free surface conditions. The downgoing wave fields reflected from the free surface are phase delayed to the next interface and included in the global boundary condition matrix. The obtained global matrix is identical in shape to the matrix obtained for a radiating upper halfspace. Inverting the global matrix yields only the upgoing wave amplitudes in the first layer. The downgoing wave fields are calculated using equations (Eq 4.80) and (Eq 4.82).

To include a source, the source jump conditions are transferred to the bottom interface of the source layer. Collecting all the interface condition equations in a matrix, in the \textit{PSVT M} picture a \((ND - 1) \times 8 \times (ND - 1) \times 8\) dimensional and in the \textit{SHT E} picture a \((ND - 1) \times 4 \times (ND - 1) \times 4\) dimensional "global" matrix is constructed. Inverting this matrix equation the amplitudes for the down and up going \textit{PSVT M} or \textit{SHT E} wave amplitudes are obtained.

The system that has to be solved for is,

\[
\begin{bmatrix}
M_{(1;+)}^{(1;\cdot)} E_{(1;\cdot)}^{(1;\cdot)} & -M_{(2;\cdot)}^{(2;\cdot)} & -M_{(2;\cdot)}^{(2;+)} \\
N_{(1;+)}^{(1;\cdot)} E_{(1;\cdot)}^{(1;\cdot)} & -N_{(2;\cdot)}^{(2;\cdot)} & -N_{(2;\cdot)}^{(2;+)} \\
M_{(s;\cdot)}^{(s;\cdot)} E_{(s;\cdot)} & M_{(s;\cdot)}^{(s;\cdot+)} E_{(s;\cdot)}^{(s;\cdot-1)} & -M_{(s+1;\cdot)}^{(s+1;\cdot)} & -M_{(s+1;\cdot)}^{(s+1;+)} \\
N_{(s;\cdot)}^{(s;\cdot)} E_{(s;\cdot)} & N_{(s;\cdot)}^{(s;\cdot+)} E_{(s;\cdot)}^{(s;\cdot-1)} & -N_{(s+1;\cdot)}^{(s+1;\cdot)} & -N_{(s+1;\cdot)}^{(s+1;+)} \\
M_{(nd+1;\cdot)}^{(nd;\cdot)} E_{(nd;\cdot)}^{(nd-1)} & M_{(nd+1;\cdot)}^{(nd+1;\cdot)} E_{(nd;\cdot)}^{(nd-1)-1} & -M_{(nd;\cdot)}^{(nd;\cdot)} & 0 \\
N_{(nd+1;\cdot)}^{(nd;\cdot)} E_{(nd;\cdot)}^{(nd-1)} & N_{(nd+1;\cdot)}^{(nd+1;\cdot)} E_{(nd;\cdot)}^{(nd-1)-1} & -N_{(nd;\cdot)}^{(nd;\cdot)} & 0 \\
\end{bmatrix}
\begin{bmatrix}
\vec{W} \\
\vec{S}^- \\
\vec{S}^+ \\
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
\end{bmatrix}
\]
Chin et al. (1984) show that the Thomson–Haskell method (Thomson, 1950; Haskell, 1953) can be viewed as treating the coefficient matrix of the linear matrix of the linear system in a block diagonal form and applying a forward marching algorithm for its solution. Chin’s formulation is a shooting method for a two point boundary value problem. Gaussian elimination with partial pivoting yields stable solutions. To avoid time stability problems (exploding exponentials) the $\tilde{W}_M$ vector is multiplied with a diagonal phase matrix $F_{IJ}$ that cancels the potentially unstable upgoing wave phase factors.

\[
F_{IJ} = \text{diag} \left[ E_{P,Q}^{(1)}, I_{P,Q}, E_{P,Q}^{(2)}, ..., E_{P,Q}^{(ND)}, I_{P,Q} \right]
\]

Where $I_{P,Q}$ is the identity matrix. If we define $\tilde{W}_M = F_{MN} X_N$, $X_N$ can be shown to satisfy,

\[
\begin{bmatrix}
M^{(1;+)} & -M^{(2;-)} & -M^{(2;+)} E^{(2)} \\
N^{(1;+)} & -N^{(2;-)} & -N_{P,Q}^{(2;+)} E^{(2)} \\
M^{(s;-)} E^{(s)} & M^{(s;+)} & -M^{(s+1;-)} & -M^{(s+1;+)} E^{(s+1)} \\
N^{(s;-)} E^{(s)} & N^{(s;+)} & -N^{(s+1;-)} & -N^{(s+1;+)} E^{(s+1)} \\
M^{(nd-1;-)} E^{(nd-1)} & M^{(nd;+)} & -M^{(nd;-)} & 0 \\
N^{(nd-1;-)} E^{(nd-1)} & N^{(nd-1;+)} & -N^{(nd;-)} & 0
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
S^- \\
S^+
\end{bmatrix}
\]

All terms that contained $\exp[-i\omega q^{(m;P_f,P_2)} \Delta z]$, $\exp[-i\omega q^{(m;S_S,S_H)} \Delta z]$, $\exp[-i\omega q^{(m;T,M,TE)} \Delta z]$ are eliminated. These exponentials cause numerical instability when the wavefield goes evanescent (Dunkin, 1965; Kennett, 1983; Schmidt and Tango, 1986).
4.5 Point Forces in a Poro-Elastic Stratified Medium

To model a general source, the moment tensor representing a body force equivalent in a poro-elastic medium has been used. Since the effect of the source is an internal process within a volume $V$, its total momentum and total angular momentum must be conserved (Aki and Richards, 1980). Therefore the total force and momentum about any fixed point must be zero (i.e. $\int_V h(\tau)dV(\tau) = 0$ and $\int_V h(\tau) \times (\tau - \tau_s)dV(\tau) = 0$, with $h$ a body force and $\tau_s$ a fixed point in space). The above conditions imply the following body force equivalent, $h_j = M_{jk} \frac{\partial}{\partial \tau_k} \delta(\tau - \tau_s) \wedge M_{jk} = M_{kj}$.

The following equation has to be evaluated when a cylindrical coordinate system is used,

$$h = M \cdot \nabla \delta(\tau - \tau_s) = \begin{bmatrix} m_{rr} & m_{r\phi} & m_{rz} \\ m_{\phi r} & m_{\phi \phi} & m_{\phi z} \\ m_{z r} & m_{z \phi} & m_{zz} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r} \frac{\partial}{\partial \phi} \\ \frac{\partial}{\partial z} \end{bmatrix} \frac{\delta(r)}{r} \delta(\phi) \delta(z - z_s) \tag{4.86}$$

4.5.1 Mechanical Point Sources

First an explosion source, vertical line source and vertical dipole source uniformly acting on the solid frame and pore fluid are derived. Due to symmetry, these sources have no angular dependence. Then a couple source acting in the $z$ direction with a spacing between the forces in the $r$ direction, having angular dependence, is outlined. Sources uniformly acting on the two phase medium are given by,

$$H = h = m \nabla \delta(\tau - \tau_s) = m \nabla \left[ \frac{\delta(r)}{r} \delta(\phi) \delta(z - z_s) \right] \tag{4.87}$$

with $H$ a source acting on the bulk material and $h$ a source acting on the pore fluid phase of the porous medium.

In component form the explosion source is given by,

$$h_r = m \frac{\partial}{\partial r} \left[ \frac{\delta(r)}{r} \right] \delta(\phi) \delta(z - z_s) \tag{4.88}$$
\[ h_\phi = m \frac{\delta(r)}{r^2} \frac{\partial}{\partial \phi} [\delta(\phi)] \delta(z - z_s) \]  
\[ h_z = m \frac{\delta(r)}{r} \delta(\phi) \frac{\partial}{\partial z} [\delta(z - z_s)] \]  

The horizontal component of the explosion source using definition (Eq 4.20) reads,

\[ h_V = \frac{1}{r} \left[ \frac{\partial}{\partial r} (rh_r) + \frac{\partial}{\partial \phi} h_\phi \right] = m \delta(z - z_s) \nabla_1^2 \left[ \frac{\delta(r) \delta(\phi)}{r} \right] \]  

The horizontal and vertical component of the explosion source after Hankel transformation and mapping on a plane wave description reads,

\[ \hat{h}_1 = \frac{\hat{h}_V}{i\omega p} = ikm \delta(z - z_s) \]  
\[ \hat{h}_z = m \frac{\partial}{\partial z} \delta(z - z_s) \]

The vertical line source components are given by (Eq 4.92) and \( \hat{h}_z = 0 \) and the vertical dipole force is given by \( \hat{h}_1 = 0 \) and (Eq 4.93).

A vertical couple source in the \( z \) direction with a spacing in between the forces in the \( r \) direction is given by,

\[ h_r = h_\phi = 0, \quad h_z = m \frac{\partial}{\partial r} \left[ \frac{\delta(r)}{r} \right] \delta(\phi) \frac{\partial}{\partial z} \delta(z - z_s) \]

The vertical component of the vertical couple source after Hankel transformation, (Sneddon, 1951), and mapping on a plane wave description read,

\[ \hat{h}_1 = 0, \quad \hat{h}_z = \pm \frac{\partial}{\partial z} \delta(z - z_s) \]

The effect of a general mechanical point source in a stratified porous medium is accommodated by specifying a jump in the displacement stress vector \( B_J \), across a horizontal plane containing the source (Hudson, 1969a; Kennett, 1983).

To obtain this jump vector, the governing poro-elastic constitutive equations, (Eq 4.8) and (Eq 4.9), and the Fourier transformed equations of motion with a point source uniformly acting on the solid frame and pore fluid in a poro-elastic medium,

\[ -\omega^2 [\rho_B \ddot{u} + \rho_f \ddot{u}] = \nabla \cdot \tau + \dot{H} \]  
\[ -\omega^2 [\rho_f \ddot{u} + \rho_E \ddot{w}] = -\nabla P + \ddot{h} \]  

117
are Hankel transformed and manipulated into a set of equations that have all derivatives with respect to \( z \) on the left hand side, and a right hand side identical to the mechanical submatrix of the electroseismic system matrix (Eq 4.38). The first set describes the change in \( z \) of the field quantities \([\hat{u}_1, \hat{u}_z, \hat{w}_z, \hat{\tau}_1, \hat{\tau}_{zz}, \hat{S}]\), where the mapping given in equations (Eq 4.34), (Eq 4.35) is applied to describe the \( P_f - P_s - SV \) case,

\[
\frac{\partial}{\partial z} \begin{bmatrix} \hat{u}_1 \\ \hat{u}_z \\ \hat{w}_z \\ \hat{\tau}_1 \\ \hat{\tau}_{zz} \\ \hat{S} \end{bmatrix} = A_{IJ}^{(Biot;PSVTM)} \begin{bmatrix} \hat{u}_1 \\ \hat{u}_z \\ \hat{w}_z \\ \hat{\tau}_1 \\ \hat{\tau}_{zz} \\ \hat{S} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \hat{H}_1 - \frac{\rho_L}{\rho_E} \hat{h}_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \hat{H}_z \\ 0 \end{bmatrix}
\]

(4.98)

Where \( A_{IJ}^{(Biot;PSVTM)} \) is defined in equation (Eq 4.38).

To include some form of source in the stratification we must solve the following inhomogeneous equation,

\[
\frac{\partial}{\partial z} B(z) - A(p, z) B(z) = F(z)
\]

(4.99)

First we multiply equation (Eq 4.99) through with a propagator matrix \( P^{-1}(z, z_s) \) from the left hand side to obtain an integration factor.

\[
P^{-1}(z, z_s) \left[ \frac{\partial}{\partial z} B(z) \right] - \left[ \frac{\partial}{\partial z} P^{-1}(z, z_s) \right] B(z) = P(z_s, z) F(z)
\]

(4.100)

where we have used the relationship \( P(z_s, z) = P^{-1}(z, z_s) \).

Then we integrate with respect to \( z \) and multiply with \( P(z_s, z) \), we obtain,

\[
B(z) = P(z_s, z_s) B(z_s) + \int_{z_s}^{z} d\zeta P(z, \zeta) F(\zeta)
\]

(4.101)

A point source can be manipulated into some dipolar contribution, (Kennett, 1983). This description is given by,

\[
F_I(z_s) = F_I^{(1)} \delta(z - z_s) + F_I^{(2)} \frac{\partial}{\partial z} \delta(z - z_s)
\]

(4.102)
with $F_I^{(1)}$ and $F_I^{(2)}$ the dipolar contributions. Substituting equations (Eq 4.102) into equation (Eq 4.101), we obtain,

$$
\int_{z_s}^z d\zeta \, \mathcal{P}(z, \zeta) \left[ F_I^{(1)}(z - z_s) + F_I^{(2)} \frac{\partial}{\partial z} \delta(z - z_s) \right] =
H(z - z_s) \mathcal{P}(z, z_s) \left[ \mathcal{P}(z, z_s) F_I^{(1)}(z - z_s) - \frac{\partial}{\partial \zeta} \mathcal{P}(z, \zeta) \right]_{\zeta = z_s} E_I^{(2)}
$$

(4.103)

with $H(z)$ the Heaviside step function.

Using the relationships, $\mathcal{P}(z, \zeta) = \mathcal{P}^{-1}(z, z_s)$ and $\frac{\partial}{\partial \zeta} \mathcal{P}^{-1}(z, z_s) = -\mathcal{P}^{-1}(z, z_s) A(p, z)$, we obtain a displacement-stress jump vector condition across a source plane.

$$
S_I(z_s) = \hat{\phi}(\omega) \left[ B_I(z_s^+) - B_I(z_s^-) \right] = \hat{\phi}(\omega) \left[ F_I^{(1)} + A_{pI}^{(Biot, PSVTM)}(p, z_s) F_I^{(2)} \right]
$$

(4.104)

with $\hat{\phi}(\omega)$, the Fourier transform of the source signature $\phi(t)$.

The final displacement-stress jump representations of some mechanical point sources in a poro-elastic isotropic medium are,

Explosive point source:

$$
S_{I}^{(P_P, PSV)}(z_s) = \hat{\phi}(\omega) \left[ 0, \frac{m}{\Delta} (M - C), \frac{m}{\Delta} \left( H - C - \frac{\Delta p^2}{\rho_E} \right), i\omega pm \left( 1 - \frac{\rho_I}{\rho_E} + \beta - \alpha \right), 0, 0 \right]^T
$$

(4.105)

Vertical line source:

$$
S_{I}^{(P_P, PSV)}(z_s) = \hat{\phi}(\omega) \left[ 0, 0, 0, i\omega pm \left( 1 - \frac{\rho_I}{\rho_E} \right), 0, 0 \right]^T
$$

(4.106)

Vertical couple source:

$$
S_{I}^{(P_P, PSV)}(z_s) = \hat{\phi}(\omega) \left[ 0, \pm\omega pm \frac{(M - C)}{\Delta}, \pm\omega p \frac{m}{\Delta} \left( H - C - \frac{\Delta p^2}{\rho_E} \right), \pm i\omega^2 p^2 m (\beta - \alpha), 0, 0 \right]^T
$$

(4.107)

Vertical dipole source:

$$
S_{I}^{(P_P, PSV)}(z_s) = \hat{\phi}(\omega) \left[ 0, \frac{m}{\Delta} (M - C), \frac{m}{\Delta} \left( H - C - \frac{\Delta p^2}{\rho_E} \right), i\omega pm \left( 1 - \frac{\rho_I}{\rho_E} \right), 0, 0 \right]^T
$$

(4.108)

\[119\]
4.5.2 Electromagnetic Point Sources

The effect of an electrical point source in a stratified porous medium is accommodated by specifying a jump in the electromagnetic field component vector $[\hat{H}_2, \hat{E}_1]^T$ across a horizontal plane containing the source. To obtain this jump vector, Maxwell’s equations with an external current source $\mathcal{C}$ in Fourier transform domain are Hankel transformed and manipulated into a set of equations that have all derivatives with respect to $z$ on one side.

\[
\frac{\partial}{\partial z} \begin{bmatrix} \hat{H}_2 \\ \hat{E}_1 \end{bmatrix} = \begin{bmatrix} 0 & i\omega \epsilon \\ i\omega \mu (1 - \frac{\sigma}{\mu \epsilon}) & 0 \end{bmatrix} \begin{bmatrix} \hat{H}_2 \\ \hat{E}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\rho \hat{C}_z}{\epsilon} \end{bmatrix} + \begin{bmatrix} -\hat{C}_1 \\ 0 \end{bmatrix} \tag{4.109}
\]

Using $\hat{C}_1 = 0$ and $\hat{C}_z = m \frac{\partial}{\partial z} \delta(z - z_s)$ and equation (Eq 4.104) with $A_{ij}^{(\text{Biot;PSVT})}$ replaced by $A_{ij}^{(\text{Maxwell;PSVT})}$ the jump representation of a vertical electric dipole source is obtained.

\[
S_{ij}^{(TM)}(z_s) = \hat{\phi}(\omega) [i\omega \rho m, 0]^T \tag{4.110}
\]

4.6 Transformation Back to the Space Time Domain

In this section the back transformation to the space-time domain of $B_j(k, \omega, z)$, equation (Eq 4.67) is performed. An inverse Fourier transform is applied to go back to the space-time domain. An inverse Hankel transform is applied to obtain the 3D spatial dependence of the displacements, stresses, electric and magnetic fields. The horizontal components of the displacements, stresses, electric and magnetic fields require additional integration over $r$ and $\phi$ to obtain, $u_r, u_\phi, \tau_{rz}, \tau_{rz}, H_r, H_\phi$ and $E_r, E_\phi$ due to the definitions of $u_V, u_H, \tau_{Vz}, \tau_{Hz}, H_V, H_H$ and $E_V, E_H$.

\[
u_z(\omega, r, \phi, z) = \int_0^\infty k \, dk \sum_{n=-N}^N [J_n(kr)\tilde{u}_z(\omega, k, n, z)] e^{i\omega \rho m} \tag{4.111}
\]
Identical relations exist for \( w_z, \tau_{zz} \) and \( S = -P \). The horizontal components may be recovered using the following equations, \( \text{(Kennett, 1983)} \).

\[
\begin{align*}
\mathbf{u}_r(\omega, r, \phi, z) &= \int_0^\infty k \, dk \sum_{n=-N}^N \left[ \frac{n}{kr} J_n(kr) \tilde{u}_2(\omega, k, n, z) ight. \\
&\quad \left. - ij'_n(kr) \tilde{u}_1(\omega, k, n, z) \right] e^{in\phi} \\
\mathbf{u}_\phi(\omega, r, \phi, z) &= \int_0^\infty k \, dk \sum_{n=-N}^N \left[ \frac{n}{kr} J_n(kr) \tilde{u}_1(\omega, k, n, z) ight. \\
&\quad \left. + ij'_n(kr) \tilde{u}_2(\omega, k, n, z) \right] e^{in\phi}
\end{align*}
\] (4.112)

Identical relations exist for \( \tau_{rz}, \tau_{xz} \) in terms of \( \tilde{\tau}_1 \) and \( \tilde{\tau}_2 \), for \( E_r, E_\phi \) in terms of \( \tilde{E}_1 \) and \( \tilde{E}_2 \) and \( H_r, H_\phi \) in terms of \( \tilde{H}_1 \) and \( \tilde{H}_2 \). The above representations may be regarded as a superposition of cylindrical waves whose order dictates the nature of their azimuthal modulation. At each frequency and angular order the radial contribution is obtained by superposing all horizontal wavenumbers \( k \). This corresponds to including all propagating waves at the level \( z \) within the stratification, from vertically to purely horizontal traveling waves, including the evanescent waves. At any particular distance \( r \) the relative contributions of the wavenumbers are imposed by the radial phase functions \( J_n(kr) \).

The integrals, \( \text{(Eq 4.111)} \), \( \text{(Eq 4.112)} \), \( \text{(Eq 4.113)} \) are evaluated by the discrete summation over many wavenumbers or the so called Discrete Wavenumber Method (Bouchon and Aki, 1977).

The discretization of the radial wavenumber \( k \) in cylindrical coordinates introduces periodicity into the source distribution. The original single source problem changes after discretization into periodic concentric sources around the original source. The periodicity of these sources, or the distance between two adjacent circular sources, \( L \), is related to the discretization interval of the wavenumber, \( \Delta k \), by the sampling relation,

\[
L = \frac{2\pi}{\Delta k}
\] 
(4.114)
L and therefore $\Delta k$ is determined by assuming a receiver located at $x_r = (r_0, z_0)$ and a source at $x_s = (0, z_s)$ on the symmetry axes of the medium configuration. Given the time window to record radiated waves from 0 to $t_{max}$, 'pseudo' waves radiated from the periodic sources are not allowed to enter this time window. This requirement is,

$$\sqrt{(L - r_0)^2 + (z_0 - z_s)^2} > v_{fastest} t_{max} \quad (4.115)$$

or

$$L > r_0 + \sqrt{v_{fastest}^2 t_{max}^2 - (z_0 - z_s)^2} \quad (4.116)$$

The sampling equation becomes now,

$$\Delta k < \frac{2\pi}{r_0 + \sqrt{v_{fastest}^2 t_{max}^2 - (z_0 - z_s)^2}} \quad (4.117)$$

The above equation is the criterion for choosing the sample rate for the discrete wavenumber summation.

To perform the summation, the singularities of the integrands are moved further from the real $k$ axes. This is done by adding a small imaginary part to the real frequency (analytic continuation into the complex plane), i.e. $\omega = \omega_R + i\omega_I$ with $\omega_I > 0$. If $e^{i\omega t}$ was used in the time to frequency transformation there would be a minus sign for the imaginary part of the frequency. The effect of complex frequency moves the singularities into the first and third quadrant of the complex $k$-plane. The use of complex frequency has the effect of smoothing the spectrum and enhancing the first motions relative to later arrivals. This effective attenuation is used to minimize the influence of the neighboring fictitious sources introduced by discrete $k$. The effect of the imaginary part of the frequency can be removed from the final time domain solution by inverse complex Fourier transform with the complex frequency with the same imaginary part used in the argument of the exponential function in the Fourier transform. The magnitude of the imaginary part is usually chosen to be,

$$\omega_I = \frac{\pi}{t_{max}} \quad (4.118)$$
Larger $\omega_I$ increases the attenuation for later arrivals, but also magnifies the numerical noise for late times. If $\omega_I$ is chosen too small, the attenuation may not be large enough to damp out the arrivals from the fictitious sources.

### 4.7 Numerical Electroseismogram Examples

Four different numerical models are calculated in the $PSVTM$ wavefield picture. The first example is a 100 m thick porous sand layer sandwiched in between two identical halfspaces that are less porous than the layer. The fluid chemistry is the same in all three layers. The source/receiver medium configuration is depicted in figure 4-3. This example is meant to study electroseismic conversions as a result of a change in mechanical properties. Mechanical displacement seismograms and electroseismograms ($TM$ mode components) are calculated and displayed in figure 4-4. The converted magnetic and electric field amplitude behavior as function of antenna position are determined at three different distances from the mechanical contrast, see figure 4-5.

In figures 4-6 and 4-7, four time slice based snapshots are shown of the mechanical wavefront traversing an interface and of the conversion of the mechanical wavefield into the electromagnetic $TM$ mode components.

The second example studies the effect of a change in fluid salinity, which only affects the medium's electrical properties, on the conversion to electromagnetic waves. The source/receiver medium configuration is depicted in figure 4-8. The mechanical displacement seismograms and electroseismograms ($TM$ components) are calculated and displayed in figure 4-9. The converted magnetic and electric field amplitude behavior as function of antenna position at three different distances from the electrical contrast are shown in figure 4-10. The four snapshots, figures 4-11 and 4-12 in time are calculated to trace in time the converted $TM$ wavefield pattern generated by a
mechanical wavefront traversing the electrical contrast.

The last two examples show the electroseismic effect in a Vertical Electroseismic Profiling (VESP) setting. The seismic source is located in the upper halfspace and the recording geophone/antennas are positioned vertically crossing two mechanical contrasts in the first VESP example (see figure 4-13) and an additional electrical contrast in the second VESP example (see figure 4-15).

The modeled mechanical source in all examples is a mechanical explosion source. To model true amplitudes, the diagonal elements in the seismic moment matrix, equation (Eq 4.86), need to be replaced by a realistic value. Using the relation \( m = \frac{4\pi a^3}{3} K_G \Delta \Theta \), with \( a \) the radius of the nonlinear deformation zone around the source, \( K_G \) a coefficient from the deformation equation given in equation (Eq 4.16), \( \Delta \Theta \) the fractional change in volume and \( E = \frac{4\pi a^3}{3} K_G (\Delta \Theta)^3 = \epsilon [4.7 \times 10^6 J/Kg] C \), with \( E \) the total energy, \( C \) the source weight in kilograms, and \( \epsilon \) an efficiency factor expressing the fraction energy converted into sound, the following expression for \( m \) is obtained.

\[
m = \sqrt[3]{\frac{4\pi a^3}{3} K_G [4.7 \times 10^6 J/Kg] C \epsilon}
\]

(4.119)

In the numerical calculation \( m = 4.4 \times 10^6 J \) is used, corresponding to \( C = 1 Kg \), \( a = 1 m \), \( K_G = 10^9 Pa \) and \( \epsilon = 10^{-3} \).

4.7.1 The Electroseismic Conversion at Mechanical Contrasts

An explosion source is positioned 100 \( m \) above the first mechanical contrast in the upper halfspace. Fifteen receivers are positioned symmetrically in a straight horizontal line at both sides of the source 95 \( m \) above the interface. The receiver spacing is 10 \( m \). The medium parameters of the two halfspaces and the 100 \( m \) thick sand layer are given in table I. Based on these medium properties velocities are calculated for the
mechanical fast wave, the slow wave, the shear wave, and the electromagnetic $TM$ wave. The complex velocities and calculated bulk conductivities are given in table II. A slow wave and $TM$ wave velocity range is given since these two wave phenomena are diffusive.

When an explosion source is set off, only $P$ waves are generated in an isotropic poro-elastic medium. Numerically the $8 \times 1$ displacement-stress-EM wavefield component vector is calculated at each geophone/antenna position. In figure 4-4 only the mechanical displacement seismograms ($u_z$ and $u_r$ components) and the electroseismograms ($TM$ mode components) are shown. All plots are seismogram scaled. The amplitudes of the first 40 msec in the $E_r$ and $H_\phi$ electroseismograms are multiplied by a factor 50 to enhance the electroseismic converted field generated at the first interface.

At the mechanical contrast the $P$ wave reflects as a $P$ wave, a converted $SV$ wave, and a converted $TM$ wave. The direct wave is omitted from all seismograms. The mechanical seismograms show the $PP$ reflection and $PSV$ conversion generated at the top and bottom of the thick sand layer. The converted $TM$ wavefield components show up at all antennas at approximately half the two way $P$ wave travel time for normal incidence reflection. Since the $TM$ wavefield velocity is at least two orders of magnitude faster than the $P$ wavefield (see table II), the travel time spent by the $TM$ waves in traveling upward to the antennas is negligible and the total two way traveltime appears to coincide with the oneway traveltime of the incoming $P$ wave at normal incidence reflection. The hyperbolas arriving at a later time in the $E_r$ and $H_\phi$ electroseismograms have the same travel time as the $P$ reflection (compare the mechanical seismogram with the $E_r$ electroseismogram in figure 4-4 and the same traveltime as the $PSV$ conversion (compare mechanical seismogram with the $H_\phi$ electroseismogram) respectively. In the first case, the compressional waves traveling through homogeneous porous medium cause pressure gradients which cause a charge to separate. This induces, within the seismic pulse, a system of electric
fields that travel with the compressional wave speed. As explained before, the seismic pulse doesn’t radiate electromagnetic fields away from the pulse. Therefore, when the reflected P wave pulse passes the antennas, an electric field is registered inside the P wave pulse. In the second case, the vertical polarized rotational mechanical waves traveling through homogeneous porous medium cause grain accelerations, setting up current sheets. This induces within the seismic pulse magnetic fields that travel with rotational mechanical wave speed. Therefore, when the converted SV wave pulse passes the antennas, a magnetic field is recorded inside the SV wave pulse. The electric and magnetic field strengths in the seismic pulse are in this case larger than the converted TM wavefield components strengths. The \( E_r \) hyperbolas arriving after the electroseismic conversion are associated with the electric fields inside the P wave pulse traveling with fast wave velocity, while the \( H_\phi \) hyperbolas arriving after the electroseismic conversion are associated with the magnetic fields inside the SV wave pulse traveling with S wavespeed velocity. The TM conversions from the bottom mechanical contrast have amplitudes too small to be identified in the electroseismogram with the scaling used.

In figure 4-5 the converted electric and magnetic absolute field amplitudes versus antenna offsets are shown at 20, 50 and 95 m from the interface. The field amplitude at each antenna position is determined by calculating a root mean square amplitude inside an estimated pulse window in the time domain. The converted TM wavefield, driven by the seismic pulse frequency, is diffusive; the real and imaginary parts of the TM velocity are almost equal, and, therefore, the largest signals are measured by antennas closest to the interface. The frequency content of the electromagnetic field is the same as the frequency content of the incident seismic wave. Since we are in the \( \frac{\sigma}{\omega\varepsilon} >> 1 \), conducting medium regime we can find a skindepth \( \delta = \frac{2}{\omega\mu_0\sigma} \), with \( \mu_0 = 4\pi \times 10^{-7} \text{ henry/m} \) the permeability of the medium (Kong, 1990). If the propagation distance is much less than the skindepth, the near field of the radiating interface, then the frequency contents in the mechanical and electromagnetic fields
are the same. The increase in amplitude with increasing source-antenna offset and a later decay in amplitude with offset, show similarities with amplitudes that would be recorded if the interface was replaced by a seismically induced electric dipole right under the source (the amplitude drop off results with antenna offset show that the effective source is not a simple dipole but rather a more complicated multipole).

To investigate the $TM$ wavefield conversions in more detail, snapshots in time are calculated around the first mechanical contrast. In figures 4-6 and 4-7 four successive time snapshots are displayed, showing the mechanical $u_z$ wavefield component, the top figure, the electromagnetic $H_\phi$ component, middle figure and the electromagnetic $E_\tau$ component, bottom figure at 26.6 msec, 28.9 msec, 30.5 msec and 32.0 msec after the shot respectively. The wavefields are determined at 120 by 60 receiver/antenna positions. The receiver/antenna spacings are 1 m in the horizontal direction and 3 m in the vertical direction. At $t = 26.6$ msec the Ricker wavelet, (Hosken, 1988) front has reached the interface at -90 m, see left column in figure 4-6. The $t = 26.6$ msec snapshot displaying the magnetic field component of the $TM$ mode shows an amplitude radiation pattern of an equivalent magnetic current loop, with a position centered under the seismic source at distance 60 m, with field lines pointing at one side into the paper and at the other side out of the paper. The magnetic field diffuses quickly away from the interface. The $t = 26.6$ msec snapshot displaying the electric field component of the $TM$ mode shows an amplitude radiation pattern of an electric dipole, positioned right under the seismic source at a distance 60 m. The largest electric fields are associated with the field inside the $P$ wave pulse. At later times, when a larger part of the seismic pulse has traversed the interface, the magnetic current loop diameter increases and the $TM$ wavespeed differences above and below the interface become visible. The current system imbalances across the interface change in direction in accordance with the pulse polarity, and the magnetic and electric field polarities flip accordingly. The last two snapshots show complex patterns of mechanical field convergences into electromagnetic fields. More than one
magnetic current loops appear with alternating field line directions and the inner loop diameter increases. Also more electric dipoles with opposite "dipole type moments" appear there where the wavefront passes. The combined $E_r$ and $H_\phi$ amplitude pattern away from the interface has a predominantly effective electrical dipole character, or its dual, an effective magnetic current loop character away from the interface. In both cases the amplitudes decay fast with traveled distance.

4.7.2 The Electroseisninc Conversion at Electrical Contrasts

The explosion source is again positioned $100$ $m$ above an electrical contrast in the upper halfspace. Fifteen receivers are positioned symmetrically in a straight horizontal line at both sides of the source $95$ $m$ above the interface to record the reflection from the contrast and $95$ $m$ below the interface to record the transmissions through the contrast. The receiver spacing is $10$ $m$. The medium input parameters describing the two halfspaces are given in table I. The calculated medium velocities and bulk conductivities based on the medium parameters listed in table I are shown in table III.

In figure 4-9 the mechanical displacement seismograms ($u_z$ and $u_r$ components) and the electroseismograms ($TM$ mode components) are shown. All four plots are seismogram scaled. At the electrical contrast the $P$ wave reflects only as converted $TM$ waves. Since there is no mechanical contrast, the $P$ wave doesn’t reflect or convert into shear waves. The mechanical $u_r$ and $u_z$ seismograms show the transmitted wave fields through the electrical contrast at the geophones $95$ $m$ below the interface which are essentially the direct $P$ wave field from the explosion source in the upper halfspace. The $TM$ wavefield components are recorded at antennas at approximately one way $P$ wave traveltime from source to interface. Since the direct wave in the upper halfspace is omitted, the $E_r$ wavefield seismogram doesn’t show induced electric
fields within the seismic pulse that travel with compressional wave speed.

In figure 4-10 the converted electric and magnetic absolute field amplitudes versus antenna offsets are shown at 20, 50 and 95 m from the interface. An increase in amplitude with increasing source-antenna offset up to a global maximum is observed. When the source-antenna offset is further increased, the amplitudes decrease monotonically. The amplitude pattern shows similarities with amplitudes that would be recorded if the interface had been replaced by a seismically induced electric dipole or magnetic loop centered right under the seismic source. If figures 4-5 and 4-10 are compared, the field amplitude curves have similar shape. The amplitudes, however, are a factor 10 bigger for the converted TM mode generated at the electrical contrast when compared to the converted TM mode amplitudes converted at the electrical contrast.

In figures 4-11 and 4-12 four successive snapshots in time are displayed, showing the mechanical $u_z$ wavefield component, top figure, the electromagnetic $H_\phi$ component, middle figure and the electromagnetic $E_r$ component, bottom figure at 26.6 msec, 28.9 msec, 30.5 msec and 32.0 msec after the shot, respectively. The number of receivers/antennas and their spacings in horizontal and vertical directions are identical to the mechanical contrast case. The top halfspace has a salinity of 1.0 mol/l and the lower halfspace has a salinity of 0.001 mol/l. This results in a larger conductivity and lower electromagnetic wavefield velocity in the upper halfspace than in the lower halfspace (see table III). Therefore, the electric fields within the incoming seismic $P$ wave pulse are small in amplitude when compared to the electric fields within the mechanical $P$ wave in the lower halfspace. This amplitude behavior can be clearly observed in the electromagnetic $E_r$ component snapshots. Tracing the transient TM component wavefield pattern fronts in figures 4-11 and 4-12, the TM wavefield velocity difference in both halfspaces is evident. The complex convergence patterns from mechanical wavefield into electromagnetic TM fields are similar to the patterns observed around the mechanical contrasts (compare figures 4-6 and 4-7 with
4-11 and 4-12). The resultant $E_r$ and $H_\phi$ amplitude pattern away from the interface has again a predominant effective electrical dipole character or, its dual, an effective magnetic current loop character, centered under the source at the contrast's depth. The amplitudes of the $TM$ mode transients decay rapidly with traveled distance.

### 4.7.3 Vertical ElectroSeismic Profiling (VESP)

The converted $TM$ mode amplitude behavior with distance, discussed when the surface electroseismic calculations were discussed, suggests that the VESP geometry setting is an important one. Since with this technique the antennas are positioned close to the target of interest, larger converted electromagnetic signals can be recorded before they become too attenuated with distance. With the VESP technique the electroseismic method can be applied to targets, and electrical and/or mechanical contrasts, at greater depths.

The first VESP example is a three layer model with the first interface at 300 m depth and the second interface at 400 m depth. The first receiver/antenna is positioned at a horizontal offset of 50 m and a depth of 155 m. The receiver/antenna depth spacing is 10 m. The medium description with its most important electroseismic medium parameters is shown in figure 4-13. In figure 4-14 the numerically calculated VESP's are shown. The top plot is the mechanical displacement response, and the middle and bottom plots show the $E_r$ and $H_\phi$ $TM$ mode components, respectively. The top VESP in figure 4-14 shows the $P$ wave reflections and converted $SV$ wave reflection at top and bottom interface of the "sand reservoir". The direct $P$ wave in the top halfspace is omitted. The other two vertical electroseismic profiles show the converted $TM$ mode components at the two contrasts. The converted $TM$ wavefield components show up at all antennas at virtually the same time. The high electrical conductivity in the middle layer attenuates the electric field amplitudes
completely. The later arrivals in the $E_r$ VESP are electric fields within the seismic pulse that travel with the compressional wave speed. The $H_\phi$ component VESP shows the fast decay of the converted electroseismic fields with distance. The converted $H_\phi$ component of the $TM$ mode is much larger in amplitude than the induced magnetic fields inside the $SV$ wave pulse. The main reason is the close antenna position to the target of interest.

The last numerical example has a new electrical contrast added to the previous model, see figure 4-15. The 100 m thick "reservoir sand" is now divided into two sands with identical mechanical properties saturated with two fluids of different salinity. In figure 4-16 the calculated VESP's are shown. The mechanical displacement response is identical to the previous calculation without the electrical contrast. But the $TM$ mode component VESP's are modified with an extra electroseismic conversion at the electrical contrast.

### 4.8 Conclusions

A global matrix method is described which solves the macroscopic equations controlling the coupled electromagnetics and acoustics of porous media numerically in layered poro-elastic media driven by arbitrary seismic point sources. The coupled equations decouple into a $PSVTM$ and a $SHTE$ picture. The induced current motion plane determines to which electromagnetic mode the mechanical waves are coupled. Seismic motion, which generates relative flow, induces a 'streaming' electrical current due to the flow of double-layer ions. The driving force for the relative flow is a combination of pressure gradients set up by the peaks and troughs of a compressional wave and by grain accelerations. The relative flow and, therefore, current can be due to either compressional and shear waves, although the two cases are of significantly different nature.
Compressional waves traveling through homogeneous porous medium cause pressure gradients which cause charge to separate. This induces within the seismic pulse a system of electric fields that travel with the compressional wavespeed. Rotational waves traveling through a homogeneous porous medium cause grain accelerations and set up current sheets. This induces within the seismic pulse magnetic fields that travel with the rotational wavespeed. Therefore, when the seismic pulse passes an antenna, an electric field is recorded inside the \( P \) wave pulse and a magnetic field is recorded inside the \( S \) wave pulse. The seismic pulse doesn’t radiate electromagnetic waves away from the pulse.

Radiating electromagnetic wavefields are converted from seismic waves, however, when contrasts in mechanical and/or electrical properties are traversed. The principal features of the converted electromagnetic signals are: (1) contacts all antennas at approximately the same time; (2) arrives at the antennas at half of the seismic travel time at normal incidence reflected \( P \) waves; and (3) changes sign on opposite sides of the shot. The frequency content of the converted electromagnetic field has the same frequency content of the driving incident seismic pulse, as long as the propagation distances are much less than the electromagnetic skin depth.

Root mean square converted electromagnetic amplitudes versus seismic point source-antenna offset are calculated for a mechanical porosity contrast and an electrical, salinity contrast at different depths. The amplitude curves are similar in shape, first a strong increase in amplitude to a global maximum is observed with increasing antenna offset and next a monotonic decrease in amplitude with increasing antenna offset. The amplitudes decrease rapidly with traveled distance.

Four snapshots in time show the wavefield there where the seismic wavefront passes the interface creating current inbalances across the interface. Equivalent sources can be identified with the conversion to electromagnetic waves. A magnetic current loop can be identified with the \( H_\phi \) component snapshots and electric current dipoles can be identified with the \( E_r \) snapshots. The total conversion to electromagnetic waves
can be either represented by an effective current dipole or an effective magnetic loop since they are each others dual. The $TM$ component amplitude versus offset curves recorded at some distance from the interface show similarities with the wavefield that would be obtained if the interface was replaced by an equivalent electric dipole or magnetic current loop positioned right beneath the seismic source at the contrast's depth. The amplitude versus offset results and the electroseismograms confirm such an effective conversion right beneath the source.

The VESP modeling shows the rapid decay of the converted electroseismic signals with distance. The antennas close to the target of interest show larger amplitudes in the converted signal than electromagnetic signals inside the seismic pulse. With increasing distance from the contrast the electromagnetic signal inside the seismic pulse totally dominates in amplitude the converted fields in the electroseismograms with increasing distance from contrast to antenna.
TABLE I. The mechanical contrast and the electrical contrast medium properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Top and bottom halfspaces</th>
<th>Sand layer</th>
<th>Fresh water halfspace</th>
<th>Brine halfspace</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi[%]$</td>
<td>15</td>
<td>30</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>$k_0[\text{m}^2]$</td>
<td>$10^{-12}$</td>
<td>$10^{-12}$</td>
<td>$10^{-12}$</td>
<td>$10^{-12}$</td>
</tr>
<tr>
<td>$k_s[\text{Pa}]$</td>
<td>$3.6 \times 10^{10}$</td>
<td>$3.6 \times 10^{10}$</td>
<td>$3.6 \times 10^{10}$</td>
<td>$3.6 \times 10^{10}$</td>
</tr>
<tr>
<td>$k_f[\text{Pa}]$</td>
<td>$2.2 \times 10^{9}$</td>
<td>$2.2 \times 10^{9}$</td>
<td>$2.2 \times 10^{9}$</td>
<td>$2.2 \times 10^{9}$</td>
</tr>
<tr>
<td>$k_{fr}[\text{Pa}]$</td>
<td>$9.9 \times 10^{9}$</td>
<td>$9.9 \times 10^{9}$</td>
<td>$9.9 \times 10^{9}$</td>
<td>$9.9 \times 10^{9}$</td>
</tr>
<tr>
<td>$g_{fr}[\text{Pa}]$</td>
<td>$9.0 \times 10^{9}$</td>
<td>$9.0 \times 10^{9}$</td>
<td>$9.0 \times 10^{9}$</td>
<td>$9.0 \times 10^{9}$</td>
</tr>
<tr>
<td>$\eta[\text{Pas}]$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$1.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\rho_s[\text{Kg/m}^3]$</td>
<td>$2.7 \times 10^{3}$</td>
<td>$2.7 \times 10^{3}$</td>
<td>$2.7 \times 10^{3}$</td>
<td>$2.7 \times 10^{3}$</td>
</tr>
<tr>
<td>$\rho_f[\text{Kg/m}^3]$</td>
<td>$1.0 \times 10^{3}$</td>
<td>$1.0 \times 10^{3}$</td>
<td>$1.0 \times 10^{3}$</td>
<td>$1.0 \times 10^{3}$</td>
</tr>
<tr>
<td>$C[\text{mol/l}]$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>1.0</td>
<td>$1.0 \times 10^{-3}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$T[K]$</td>
<td>298</td>
<td>298</td>
<td>298</td>
<td>298</td>
</tr>
<tr>
<td>$\kappa_f$</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>$\kappa_s$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$\alpha_\infty$</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

134
TABLE II. Calculated wavefield velocities and bulk conductivities used in the mechanical contrast model. The diffusive slow wave and $EM$ wavefield velocities are listed at zero and source center frequency. The real and imaginary parts of the fast and shear waves are listed.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Upper and lower halfspace</th>
<th>Sand reservoir layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{pf}[m/s]$</td>
<td>$3282.62, -0.1037$</td>
<td>$3158.56, -0.7419$</td>
</tr>
<tr>
<td>$v_{ps}[m/s]$</td>
<td>$21.2/146.2, -13.3/-119.1$</td>
<td>$(16.7/111.6, -10.5/-100.3)$</td>
</tr>
<tr>
<td>$v_s[m/s]$</td>
<td>$1769.14, -1.39$</td>
<td>$(1669.25, -1.53)$</td>
</tr>
<tr>
<td>$v_{TM}[m/s]$</td>
<td>$(318890/2.04 \times 10^6, -201223/-2.02 \times 10^6)$</td>
<td>$(7159/45665, -4517/-45326)$</td>
</tr>
<tr>
<td>$\sigma[S/m]$</td>
<td>$0.000388$</td>
<td>$0.77$</td>
</tr>
</tbody>
</table>

TABLE III. Calculated wavefield velocities and bulk conductivities used in the electrical contrast model. The diffusive slow wave and $EM$ wavefield velocities are listed at zero and source center frequency. The real and imaginary parts of the fast and shear waves are listed.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Top fresh water saturated halfspace</th>
<th>Bottom brine saturated halfspace</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{pf}[m/s]$</td>
<td>$(3282.62, -0.1037)$</td>
<td>$(3282.62, -0.1037)$</td>
</tr>
<tr>
<td>$v_{ps}[m/s]$</td>
<td>$(21.2/146.2, -13.3/-119.1)$</td>
<td>$(21.2/146.1, -13.3/-119.1)$</td>
</tr>
<tr>
<td>$v_s[m/s]$</td>
<td>$(1769.14, -1.39)$</td>
<td>$(1769.14, -1.39)$</td>
</tr>
<tr>
<td>$v_{TM}[m/s]$</td>
<td>$(318890/2.04 \times 10^6, -201223/-2.02 \times 10^6)$</td>
<td>$(10124/64580, -6389/-64100)$</td>
</tr>
<tr>
<td>$\sigma[S/m]$</td>
<td>$0.000388$</td>
<td>$0.385$</td>
</tr>
</tbody>
</table>
Figure 4-1: Electric fields due to charge separation within a $P$ wave seismic pulse. Streaming and conduction currents balance each other inside the $P$ wave pulse. Therefore when compressional waves propagate through homogeneous medium, there is an electric field fixed to the wave with no extend outside the wave.
Figure 4-2: When divergence-free $S$ waves propagate through porous medium, current sheets are induced through grain accelerations. Magnetic fields with a polarization orthogonal to the seismic particle displacement are induced by these current sheets. Very small electric fields of the induction type are generated by this magnetic field. There is no current imbalance within the $S$ wave and therefore no radiating magnetic field outside the pulse.
"Thick" Permeable Sand Layer

fc = 200 Hz

5 m

100 m

Fluid Chemistry = 0.001 mol/l
porosity = 30%
permeability = 1 D

"Sand"

Figure 4-3: The thick permeable sand medium configuration.
Figure 4-4: The mechanical displacement component seismograms and the $TM$ mode component electroseismograms calculated for the thick permeable sand medium configuration.
Figure 4-5: Converted electric and magnetic root mean square wavefield amplitudes versus antenna offsets calculated at 20, 50 and 95 m from a mechanical contrast generated by an explosive point source.
Figure 4-6: Time slice snapshots calculated at 120 by 60 geophone/receiver positions around a mechanical contrast at $t = 26.6\, \text{msec}$ and $t = 28.9\, \text{msec}$ after explosion. Top snapshot is $u_z$ component of mechanical displacement wavefield, middle snapshot is $H_\phi$ component of electromagnetic $TM$ mode, bottom snapshot is $E_r$ component of electromagnetic TM mode.
Figure 4-7: Time slice snapshots calculated at 120 by 60 geophone/receiver positions around a mechanical contrast at $t = 30.5$ msec and $t = 32.0$ msec after explosion. Top snapshot is $u_z$ component of mechanical displacement wavefield, middle snapshot is $H_\phi$ component of electromagnetic $TM$ mode, bottom snapshot is $E_r$ component of electromagnetic TM mode.
Fresh Water / Brine

Figure 4-8: The fresh water - brine medium configuration.
Figure 4-9: The mechanical displacement component seismograms and the $TM$ mode component electroseismograms calculated for the fresh water - brine medium configuration.
Figure 4-10: Converted electric and magnetic root mean square wavefield amplitudes versus antenna offsets calculated at 20, 50 and 95 m from an electrical contrast generated by an explosive point source.
Figure 4-11: Time slice snapshots calculated at 120 by 60 geophone/receiver positions around an electrical contrast at \( t = 26.6 \) msec and \( t = 28.9 \) msec after explosion. Top snapshot is \( u_z \) component of mechanical displacement wavefield, middle snapshot is \( H_\phi \) component of electromagnetic \( TM \) mode, bottom snapshot is \( E_r \) component of electromagnetic TM mode.
Figure 4-12: Time slice snapshots calculated at 120 by 60 geophone/receiver positions around an electrical contrast at $t = 30.5 \text{ msec}$ and $t = 32.0 \text{ msec}$ after explosion. Top snapshot is $u_z$ component of mechanical displacement wavefield, middle snapshot is $H_{\phi}$ component of electromagnetic $TM$ mode, bottom snapshot is $E_r$ component of electromagnetic TM mode.
Figure 4-13: Vertical ElectroSeismic Profiling medium configuration.
Figure 4-14: The mechanical displacement component seismogram and the $TM$ mode component electroseismograms calculated for a Vertical ElectroSeismic Profiling medium configuration.
Vertical ElectroSeismic Profiling (VESD)

Figure 4-15: Vertical ElectroSeismic Profiling medium configuration with additional electrical contrast.
Figure 4-16: The mechanical displacement component seismogram and the $TM$ mode component electroseismograms calculated for a Vertical ElectroSeismic Profiling medium configuration with additional electrical contrast inside the sand reservoir.
Chapter 5

Electroseismic Waves in Layered Media using Dynamic Green’s Functions.

5.1 Introduction

Green’s functions are the response of the medium to point excitations. To seek these fundamental solutions, a force density corresponding to a harmonic point force which acts on the solid matrix is added to the force balance on the bulk material. Similarly, a harmonic volume injection source which corresponds to a point pressure source is added to the force balance on the fluid phase in the pore space. The solution approach is similar for both point source excitations on the bulk and fluid phase respectively.

Dynamic Green’s functions in porous saturated media have been studied by different authors. Burridge and Vargas (1979) give a transient solution in the far field. They obtain their solution by using the Laplace transform method. The Kernel of
their inverse transform is not given in terms of simple functions. Norris (1985) introduces point forces in the fluid and gets its solution by the Fourier transform method. Bonnet (1987) provides a harmonic solution in analogy with thermoelasticity. Boutin et al. apply the theory of homogenization for periodic structures (Auriault, 1980; Auriault et al., 1985) to obtain a description for the harmonic behavior of periodic saturated porous media. Using their homogenization results, they obtain a Green's matrix description of the dynamic behavior in porous saturated media. In this chapter, as opposed to other poro-elastic Green's function papers, we derive expressions for the dynamic relative flow Green's functions by solving the volume averaged force balance and wave equations with a general source distribution acting on the solid matrix and a pressure source which acts on the fluid phase, using a modified version of the Kupradze (1979) method. In the appendices the dynamic displacement Green's functions are derived for completeness.

Using the relative flow and displacement Green's functions together with the deformation equations in both phases, we derive source jump representations for an explosive and vertical point source acting on the frame. We then calculate electroseismograms in layered media driven by mechanical sources using the global matrix technique. In the numerical examples we compare the conversions from $P$ and $SV$ mechanical waves into electromagnetical waves in two different medium geometries.

5.2 Equations for Dynamic Poro-Elasticity

The dynamics of porous media was first developed by Biot (1956, 1962a,b) using a phenomenological approach. More recently the equations of motion and constitutive relations for the linear dynamics of a two phase fluid/solid, isotropic, porous material have been derived by a direct volume averaging of the equations of motion and constitutive relations known to apply in each phase (Pride et al., 1992). In this chapter
we use these volume averaged coupled equations, coupled since the continuity of the stress requires the fluid motion to be coupled to the solid motion along the pore walls. By allowing for both average fluid displacements and average solid displacements in the equations of motion, the possibility of average relative fluid/solid motion is allowed for.

The force balance equations on fluid and bulk phase are,

\[-i\omega w = \frac{k(\omega)}{\eta} \left[ -\nabla P + \omega^2 \rho_f y_s \right]
\]

\[\nabla \cdot \tau = -\omega^2 [\rho_B u + \rho_f w] - E
\]

(5.1)  (5.2)

The fluid and bulk deformation equations are,

\[-P = C\nabla \cdot y_s + M\nabla \cdot w + S
\]

\[\tau_B = [K_G \nabla \cdot y_s + C \nabla \cdot w]I + G_{fr} \left[ \nabla y_s + \nabla y_s^T - \frac{2}{3} \nabla \cdot y_s I \right]
\]

(5.3)  (5.4)

The point body force on the elastic frame is denoted by $E$ and $S$ is a pressure source which acts on the fluid in the pore space. The bulk density is defined as the sum of the phase volume weighted fluid and solid densities, $\rho_B = \phi \rho_f + (1 - \phi) \rho_s$. $k(\omega)$ and $\eta$ in equation (Eq 5.1) are the dynamic permeability and fluid viscosity respectively. $y_s$, $\tau_B$ and $w$ are the solid displacement, bulk stress and relative flow vector respectively. The relative flow is defined as $w = \phi (y_f - y_s)$, i.e. the relative displacement difference in the two phases.

The coefficients in the deformation equations are,

\[K_G = H - \frac{4}{3} G = \frac{K_{fr} + \phi K_{fr} + (1 - \phi) K_s \Delta}{1 + \Delta}
\]

\[C = \frac{K_f + K_s \Delta}{1 + \Delta}
\]

\[M = \frac{1}{\phi} \frac{K_f}{1 + \Delta}
\]

(5.5)  (5.6)  (5.7)

Where the parameter $\Delta$ is defined as,

\[\Delta = \frac{K_f}{\phi K_s^2} [(1 - \phi) K_s - K_{fr}]
\]

(5.8)
The moduli $K_f$ and $G_f$ are the bulk and shear moduli of the framework of the grains, when the fluid is absent. The frame moduli may either be considered experimentally determined or may be obtained from approximate theoretical models for specific pore grain geometries. $C$ and $M$ are the incompressibilities used by Biot (1962b) and Pride et al. (1992), they are complex and frequency dependent, allowing for losses in addition to those associated with relative flow.

5.2.1 Relative Flow Green’s Function with Force on Solid Phase

To solve for the dynamic Green’s functions, the deformation equation (Eq 5.4) is substituted into the bulk force balance equation (Eq 5.2). Secondly the pressure equation (Eq 5.3) is substituted into the modified Darcy type equation (Eq 5.1). The mechanical field behavior is completely decoupled from the electromagnetic field behavior in solving the mechanical Green’s functions. The electric field which acts on the charges in the double layer, an osmosis effect acting as a drag force, is neglected (see chapter 4).

The resulting frequency domain wave equations in the bulk and fluid phase read in grouped $x$ and $y$, transverse components and $z$, vertical component form.

\[ G \left( \nabla_t^2 u_t + G \frac{\partial^2}{\partial z^2} u_t + (H - G) \nabla_t \nabla_t \cdot u_t + (H - G) \frac{\partial}{\partial z} \nabla_t u_z \right) + \omega^2 \rho_B u_t + \omega^2 \rho_f w_t = -F_t \]  \[5.9\]

\[ C \left[ \nabla_t \nabla_t \cdot w_t + \frac{\partial}{\partial z} \nabla_t w_z \right] + \omega^2 \rho_B u_z + \omega^2 \rho_f w_z = -F_z \]  \[5.10\]

\[ C \left[ \nabla_t \nabla_t \cdot u_t + \frac{\partial}{\partial z} \nabla_t u_z \right] + M \left[ \nabla_t \nabla_t \cdot w_t + \frac{\partial}{\partial z} \nabla_t w_z \right] + \omega^2 \rho_f u_t + \omega^2 \rho_E w_t = -f_t \]  \[5.11\]
\[
C \left[ \frac{\partial}{\partial z} \nabla_t \cdot u_t + \frac{\partial^2}{\partial z^2} u_z \right] + M \left[ \frac{\partial}{\partial z} \nabla_t \cdot w_t + \frac{\partial^2}{\partial z^2} w_z \right] + \omega^2 \rho_f u_z + \omega^2 \rho_E w_z = -f_z \tag{5.12}
\]

Where the transverse displacement, transverse relative flow and transverse point force are defined as respectively, \( u_t = u_x \hat{x} + u_y \hat{y}, \) \( w_t = w_x \hat{x} + w_y \hat{y} \) and \( F_t = F_x \hat{x} + F_y \hat{y}, \) \( f_t = f_x \hat{x} + f_y \hat{y} \) \((f_x = \frac{\partial}{\partial x} S, f_y = \frac{\partial}{\partial y} S, f_z = \frac{\partial}{\partial z} S),\)

\( S \) is a pressure source.

To solve for the relative flow scalar shear Green’s function equation (Eq 5.11) is rewritten into the form,

\[
u_t = \frac{-1}{\omega^2 \rho_f} \left( \omega^2 \rho_E w_t + C \left[ \nabla_t \nabla_t \cdot u_t + \frac{\partial}{\partial z} \nabla_t u_z \right] + M \left[ \nabla_t \nabla_t \cdot w_t + \frac{\partial}{\partial z} \nabla_t w_z \right] \right) \tag{5.13}
\]

The transverse displacement vectors in equation (Eq 5.9) are substituted by equation (Eq 5.13). This new equation can be solved by taking the transverse curl, defined as \( \nabla_t \times [\nabla - \frac{\partial}{\partial z} \hat{z}] \times \) on both sides. The gradient terms disappear because \( \nabla_t \times \nabla_t w_t = 0. \) The following equation is obtained,

\[
- G \frac{\rho_E}{\rho_f} \left[ \nabla_t \nabla_t \times w_t + \frac{\partial^2}{\partial z^2} \nabla_t \times w_t + \frac{\omega^2}{G} \left[ \rho_B - \frac{\rho_f}{\rho_E} \right] \nabla_t \times w_t \right] = -\nabla_t \times F_t \tag{5.14}
\]

The solution of this equation in terms of the transverse curl of \( w_t, \) using Green’s superposition theorem is,

\[
\nabla_t \times w_t = \int_V g^w(x, x') \nabla_t' \times F_t' \, dx'
\tag{5.15}
\]

Where \( x = (x, y, z) \) is the receiver location and \( x' = (x', y', z') \) is the source location. \( g^w(x, x') \) is the Green’s function of the scalar wave equation,

\[
\nabla_t^2 g^w + \frac{\partial^2}{\partial z^2} g^w + \frac{\omega^2}{G} \left[ \rho_B - \frac{\rho_f}{\rho_E} \right] g^w = \frac{1}{G \rho_E} \delta(x - x')
\tag{5.16}
\]

If \( e^{-i\omega t} \) time dependence is assumed for the wavefield, the 3D Green’s function \( g^w \) is,

\[
g^w(x, x') = -\frac{\rho_f}{\rho_E} \frac{e^{i k_0 R}}{4\pi G R}
\tag{5.17}
\]

Where \( R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \) is the distance from the source to the receiver and \( k_0 = \omega/\sqrt{G/(\rho_B - \rho_f/\rho_E)} \) is the wave number.

156
To solve for the $P_f - P_s - SV$ scalar Green's functions, the transverse divergence of equations (Eq 5.9) and (Eq 5.10), and the $z$ derivative of equations (Eq 5.11) and (Eq 5.12) are taken. These equations can be solved using a modified version of the Kupradze technique outlined for an isotropic elastic medium. To simplify the analysis the coupled equations are written in a matrix format.

\[
\begin{bmatrix}
  R_z & (H - G)\nabla_i^2 & L_z^C & C\nabla_i^2 \\
  (H - G)\frac{\partial^2}{\partial z^2} & R_t & C\frac{\partial^2}{\partial z^2} & L_t^C \\
  L_z^M & C\nabla_i^2 & L_z^M & M\nabla_i^2 \\
  C\frac{\partial^2}{\partial z^2} & L_t^C & M\frac{\partial^2}{\partial z^2} & L_t^M \\
\end{bmatrix}
\begin{bmatrix}
  \nabla_i \cdot u_t \\
  \frac{\partial}{\partial z} u_z \\
  \nabla_i \cdot w_t \\
  \frac{\partial}{\partial z} w_z \\
\end{bmatrix} =
\begin{bmatrix}
  \nabla_i \cdot F_t \\
  \frac{\partial}{\partial z} F_z \\
\end{bmatrix}
\]  
(5.18)

Where the variables $R_z, R_t, L_z^C, L_t^M, L_z^M, L_t^M$ are defined as

\[
L_z^C = C\nabla_i^2 + \omega^2 \rho_f, \quad L_t^C = C\frac{\partial^2}{\partial z^2} + \omega^2 \rho_f
\]  
(5.19)

\[
L_z^M = M\nabla_i^2 + \omega^2 \rho_E, \quad L_t^M = M\frac{\partial^2}{\partial z^2} + \omega^2 \rho_E
\]  
(5.20)

\[
R_z = H\nabla_i^2 + G\frac{\partial^2}{\partial z^2} + \omega^2 \rho_B, \quad R_t = G\nabla_i^2 + H\frac{\partial^2}{\partial z^2} + \omega^2 \rho_B
\]  
(5.21)

There exists now the option to rewrite the 4 by 4 differential operator matrix into a 2 by 2 differential operator matrix operating on the displacement components or the relative flow components. The system describing the displacements in the medium is solved in appendix B. This relative flow system has the following form,

\[
\begin{bmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{bmatrix}
\begin{bmatrix}
  \nabla_i \cdot w_t \\
  \frac{\partial}{\partial z} w_z \\
\end{bmatrix} =
\begin{bmatrix}
  \nabla_i \cdot F_t \\
  \frac{\partial}{\partial z} F_z \\
\end{bmatrix}
\]  
(5.22)

Where the variables $P_{11}, P_{12}, P_{21}, P_{22}$ are defined as,

\[
P_{11} = L_z^C + \theta \left[ R_z \left( L_z^C L_z^M - C M \frac{\partial^2}{\partial z^2} \nabla_i^2 \right) \right] + (H - G)\nabla_i^2 \left( -C L_z^M \frac{\partial^2}{\partial z^2} + M L_z^C \frac{\partial^2}{\partial z^2} \right)
\]  
(5.23)

\[
P_{12} = C\nabla_i^2 + \theta \left[ R_z \left( M L_z^C \nabla_i^2 - C L_t^M \nabla_i^2 \right) \right] + (H - G)\nabla_i^2 \left( -C M \nabla_i^2 \frac{\partial^2}{\partial z^2} + L_z^C L_t^M \right)
\]  
(5.24)

157
\[ P_{21} = C \frac{\partial^2}{\partial z^2} + \theta \left[ (H - G) \frac{\partial^2}{\partial z^2} \left( L_i^C L_i^M - CM \frac{\partial^2}{\partial z^2} \nabla_i^2 \right) \right] + R_t \left( -CL_i^M \frac{\partial^2}{\partial z^2} + ML_i^C \frac{\partial^2}{\partial z^2} \right) \]  
(5.25)

\[ P_{22} = L_i^C + \theta \left[ (H - G) \frac{\partial^2}{\partial z^2} \left( ML_i^C \nabla_i^2 - CL_i^M \nabla_i^2 \right) \right] + R_t \left( -CM \nabla_i^2 \frac{\partial^2}{\partial z^2} + L_i^C L_i^M \right) \]  
(5.26)

Where,
\[ \theta = \frac{-1}{L_i^C L_i^C - C^2 \nabla_i^2 \frac{\partial^2}{\partial z^2}} \]  
(5.27)

In the modified Kupradze method, the unknowns of the system are expressed in terms of the adjoint of the system matrix, which is the transpose of its cofactor matrix. The product of a matrix with its adjoint is the identity matrix scaled by its determinant.

Performing the modified Kupradze method yields,
\[ \begin{bmatrix} \nabla_i \cdot w_t \\ \frac{\partial}{\partial z} w_z \end{bmatrix} = \int_{V} \begin{bmatrix} P_{22} & -P_{12} \\ -P_{21} & P_{11} \end{bmatrix} \Phi(x, x') \begin{bmatrix} \nabla_i \cdot F_i' \\ \frac{\partial}{\partial z} F_z' \end{bmatrix} dx' \]  
(5.28)

Substituting equation (Eq 5.28) into equation (Eq 5.22) yields,
\[ [P_{11}P_{22} - P_{12}P_{21}] \Phi(x, x') = \delta(x, x') \]  
(5.29)

Substituting equations (Eq 5.19), (Eq 5.20) and (Eq 5.21) into equations (Eq 5.23), (Eq 5.24), (Eq 5.25) and (Eq 5.26) and carrying out the algebra the following expressions are obtained,

\[ P_{11} = K_t + \frac{\gamma}{\xi} \nabla_i^2, \quad P_{12} = \left[ \psi + \frac{\gamma}{\xi} \right] \nabla_i^2 \]  
(5.30)

\[ P_{21} = \left[ \psi + \frac{\gamma}{\xi} \right] \frac{\partial^2}{\partial z^2}, \quad P_{22} = K_t + \frac{\gamma}{\xi} \frac{\partial^2}{\partial z^2} \]  
(5.31)

Where the introduced variables, \( \gamma, \psi, \xi \) and \( K_t, K_z \) are defined to be,
\[ \gamma = \frac{HM}{C} \omega^2 \rho_f + C \omega^2 \frac{\rho_B \rho_E}{\rho_f} - H \omega^2 \rho_E - M \omega^2 \rho_B \]  
(5.32)
\[ \psi = \frac{1}{C} \left[ C^2 - HM + G \frac{\rho E}{\rho_f} \right] \]  
\[ \xi = C \nabla_t^2 + C \frac{\partial^2}{\partial z^2} + \omega^2 \rho_f \]  
\[ K_t = -G \frac{\rho E}{\rho_f} \nabla_t^2 - \frac{1}{C} \left[ HM - C^2 \right] \frac{\partial^2}{\partial z^2} + \omega^2 \left[ \rho_f - \frac{\rho E \rho_B}{\rho_f} \right] \]  
\[ K_z = -\frac{1}{C} \left[ HM - C^2 \right] \nabla_t^2 - G \frac{\rho E}{\rho_f} \frac{\partial^2}{\partial z^2} + \omega^2 \left[ \rho_f - \frac{\rho E \rho_B}{\rho_f} \right] \]  

The scalar equation (Eq. 5.29) can be solved using the Fourier transform method which yields,

\[ \Phi(k_x, k_y, k_z, \omega) = \frac{-1}{\left( K_t - \frac{\omega^2}{\xi} k^2 \right) \left( L_z - \frac{\omega^2}{\xi} k_z^2 \right) - \left( \psi + \frac{\omega^2}{\xi} \right) k^2 k_z^2} \]  

Where \( k^2 = k_x^2 + k_y^2 \) is the transverse or horizontal wavenumber. The inverse Fourier transform is applied to return to spatial coordinates. The analysis is performed in cylindrical coordinates, therefore the cartesian coordinates \((x - x', y - y', z - z')\) have to be changed into cylindrical coordinates \((D, \Theta_D, z)\) and the cartesian wavenumbers \((k_x, k_y, k_z)\) into the cylindrical wavenumbers \((k, \Theta_k, k_z)\). Integration over \( \Theta_k \) results in a zeroth order Bessel function, \( 2\pi J_0(kD) = \int_0^{2\pi} d\Theta k e^{i k D \cos(\Theta_k - \Theta_D)} \). Performing the inverse Fourier transform, equation (Eq. 5.37) becomes,

\[ \Phi(x, x', \omega) = \frac{1}{(2\pi)^2 G(HM - C^2)\rho E/\rho_f} \int_0^\infty dk \int_{-\infty}^{\infty} dk_z \left[ k J_0(kD) \frac{(\omega^2 \rho_f - k^2 C - k_z^2 C)}{(k_z^2 - \nu_{\alpha 1}^2)(k_z^2 - \nu_{\alpha 2}^2)(k_z^2 - \nu_{\beta}^2)} e^{ik(z - z')} \right] \]  

With \( D = \sqrt{(x - x')^2 + (z - z')^2} \), and \( \nu_{\alpha 1}^2 = k_{\alpha 1}^2 - k^2, \nu_{\alpha 2}^2 = k_{\alpha 2}^2 - k^2, \nu_{\beta}^2 = k_{\beta}^2 - k^2 \) representing the roots of the denominator in equation (Eq. 5.38). The roots of the denominator of equation (Eq. 5.38) corresponding to the fast P (\( P_f \)), slow P (\( P_s \)) and SV waves are respectively,

\[ \nu_{\alpha 1}^2 = \frac{\omega^2 [\rho_B M + \rho_E H - 2 \rho_f C]}{2 [HM - C^2]} + \frac{\sqrt{\omega^4 [\rho_B M + \rho_E H - 2 \rho_f C]^2 - 4 [HM - C^2] \omega^4 [\rho_B \rho_E - \rho_f^2]}}{2 [HM - C^2]} - k^2 \]  

159
\[ \nu_{\alpha 2} = \frac{\omega^2 [\rho_B M + \rho_E H - 2\rho_f C]}{2 [HM - C^2]} - \sqrt{\frac{\omega^4 [\rho_B M + \rho_E H - 2\rho_f C]^2 - 4 [HM - C^2] \omega^4 [\rho_B \rho_E - \rho_f^2]}{2 [HM - C^2]}} - k^2 (5.40) \]

\[ \nu_\beta^2 = \frac{\omega^2}{G} \left[ \frac{\rho_B - \rho_f^2}{\rho_E} \right] - k^2 \quad (5.41) \]

To perform the \( k_z \) integral Cauchy's theorem is applied to calculate the contributions of the six poles at \( k_z = \pm \nu_{\alpha 1}, k_z = \pm \nu_{\alpha 2} \) and \( k_z = \pm \nu_\beta \). The poles lie on the real \( k_z \) axis rendering the integral undefined. By adding a small complex part to the frequency \( \omega \), the poles are shifted into the complex plane, resulting in a well-defined integral. If \( \text{Im} [\nu_{\alpha 1}] > 0, \text{Im} [\nu_{\alpha 2}] > 0 \) and \( \text{Im} [\nu_\beta] > 0 \) is assumed for \( z - z' > 0 \) the contour is closed in the upper half of the complex \( k_z \) plane. Similarly, for \( z - z' < 0 \), the integral is equal to the pole contribution at \(-\nu_{\alpha 1}, -\nu_{\alpha 2} \) and \(-\nu_\beta \). Summing the contributions of all poles yields,

\[ \Phi(x, x', \omega) = \frac{1}{4\pi G (HM - C^2) \rho_E / \rho_f} \left[ \int_0^\infty \frac{i}{(\nu_{\alpha 1}^2 - \nu_\beta^2) (\nu_{\alpha 1}^2 - \nu_{\alpha 2}^2)} k J_0(kD) \frac{(\omega^2 \rho_f - k_{\alpha 1}^2 C)}{\nu_{\alpha 1}} e^{i\nu_{\alpha 1}|z - z'|} dk \right. \]

\[ + \int_0^\infty \frac{i}{(\nu_{\alpha 2}^2 - \nu_\beta^2) (\nu_{\alpha 2}^2 - \nu_{\alpha 1}^2)} k J_0(kD) \frac{(\omega^2 \rho_f - k_{\alpha 2}^2 C)}{\nu_{\alpha 2}} e^{i\nu_{\alpha 2}|z - z'|} dk \]

\[ + \int_0^\infty \frac{i}{(\nu_\beta^2 - \nu_{\alpha 1}^2) (\nu_\beta^2 - \nu_{\alpha 2}^2)} k J_0(kD) \frac{(\omega^2 \rho_f - k_\beta^2 C)}{\nu_\beta} e^{i\nu_\beta|z - z'|} dk \] \quad (5.42)

When \( g^w \) and \( \Phi \) are determined, the transverse curl, the transverse divergence and the \( z \) derivative of the relative flow vector can be determined.

\[ \nabla_t \times w_t = \int_V \nabla_t g^w(x, x') \times F'_t dx' \quad (5.43) \]

\[ \nabla_t \cdot w_t = \int_V \nabla_t \left[ K_t + \frac{\gamma}{\xi} \frac{\partial^2}{\partial z^2} \right] \Phi(x, x') \cdot F'_t dx' \]

\[ - \int_V \left[ \psi + \frac{\gamma}{\xi} \right] \nabla_t^2 \frac{\partial}{\partial z} \Phi(x, x') \hat{z} \cdot F'_t dx' \quad (5.44) \]

160
\[
\frac{\partial}{\partial z} w_z = \int_V \frac{\partial}{\partial z} \left[ K_z + \frac{\gamma}{\xi} \nabla^2 \right] \Phi(x, x') \hat{z} \cdot F' dx' \\
- \int_V \left[ \psi + \frac{\gamma}{\xi} \right] \frac{\partial^2}{\partial z^2} \nabla_t \Phi(x, x') \cdot F' dx'
\]

(5.45)

To recover the total relative flow vector equations (Eq 5.43), (Eq 5.44) and (Eq 5.45) are rewritten using some vector identities and applying the integration by parts technique.

Equation (Eq 5.43) is rewritten using vector identity \( g \nabla'_t \times f' = \nabla'_t \times \left[ g f' \right] - \nabla'_t g \times f' \), and relation \( \nabla'_t g = - \nabla_t g \).

\[
\nabla_t \times w_t = \int_V \left[ \nabla'_t \times \left[ g(x, x') F'_t \right] + \nabla_t g(x, x') \times F'_t \right] dx'
\]
\[
= \int_V \nabla_t g(x, x') \times F'_t dx'
\]

(5.46)

The first integral in equation (Eq 5.46) is zero because this integral can be expressed as an integral evaluated at the boundary surface of the source volume which vanishes because \( F_t \) is a body force and is not supported on the boundary surface.

The first integral in equation (Eq 5.44) is rewritten using vector identity \( \phi \nabla_t \cdot f = \nabla_t \cdot [\phi f] - \nabla_t \phi \cdot f \) and applying Green’s theorem to it. The second integral in equation (Eq 5.44) is integrated by parts. The induced surface integrals vanish again because the transverse body force \( F_t \) is not supported on a boundary. Relation \( \nabla_t g = - \nabla_t g \) is used in both integral evaluations. The total relative flow vector is recovered using vector identity \( \nabla^2_t w_t = \nabla_t \nabla_t \cdot w_t - \nabla_t \times \nabla_t \times w_t \).

\[
w = w_t + w_z \hat{z}
\]

(5.47)

\[
= \frac{1}{\nabla^2_t} \left[ \nabla_t \nabla_t \cdot w_t - \nabla_t \times \nabla_t \times w_t \right] + \hat{z} \int \frac{\partial}{\partial z} w_z dz
\]

(5.48)

\[
= \int_V \left[ g^w \hat{z} + \hat{z} \hat{z} \left[ K_z + \frac{\gamma}{\xi} \frac{\partial^2}{\partial z^2} \right] \Phi - g \right] - \left[ \psi + \frac{\gamma}{\xi} \right] \frac{\partial}{\partial z} \left[ \nabla_t \hat{z} + \hat{z} \nabla_t \right] \Phi \\
+ \frac{\nabla_t \nabla_t}{\nabla^2_t} \left[ \left[ K_t + \frac{\gamma}{\xi} \nabla^2_t \right] \Phi - g^w \right] \cdot F' dx'
\]

(5.49)

Where the following identities have been used,

\[
\nabla_t \times \left[ \nabla_t g \times F'_t \right] = \nabla_t \nabla_t g \cdot F'_t - \nabla^2_t g F'_t
\]

(5.50)
\[ \mathbf{I} = \mathbf{I}_t + \dot{\mathbf{z}} \hat{z} \]  
\[ F_t = \mathbf{I}_t \cdot \mathbf{F} \]  
\[ \frac{1}{\nabla^2_t} \nabla^2 \mathbf{I}_t = 1 \]  
\[ (5.51) \]  
\[ (5.52) \]  
\[ (5.53) \]

Equation (Eq 5.47) states that the relative flow can now in principle be determined for any source by convolving the source with a certain function and integrating it over the source volume. This is exactly the statement of Green's superposition theorem. This certain function (the integrand of equation (Eq 5.47)) is then the Green's function for the wave equation.

The dynamic Green's function (tensor), denoted by \( \mathbf{G} \), expressed in dyadic form is given by,

\[ \mathbf{G} = g^{w} \mathbf{I} + \dot{\mathbf{z}} \hat{z} \left( \left[ K_t + \frac{\gamma}{\xi} \nabla \mathbf{I}_t \right] \Phi - g^{w} \right) - \left[ \psi + \frac{\gamma}{\xi} \frac{\partial^2}{\partial z^2} \left[ \nabla \mathbf{I}_t \dot{z} + \dot{z} \nabla \mathbf{I}_t \right] \Phi \right] + \frac{\nabla \nabla \mathbf{I}_t}{\nabla^2 \mathbf{I}_t} \left( \left[ K_t + \frac{\gamma}{\xi} \frac{\partial^2}{\partial z^2} \right] \Phi - g^{w} \right) \]  
\[ (5.54) \]

Where \( g^{w}, \psi, \gamma, \xi, \Phi, K_t, K_t \) are defined in equations (Eq 5.17), (Eq 5.42), (Eq 5.32), (Eq 5.33), (Eq 5.34), (Eq 5.35) and (Eq 5.36) respectively.

Using the Sommerfeld representation of a point source, \( \Phi \) can be simplified into the following form,

\[ \Phi = B(\text{ws})g_{\beta} + A_1(\text{ws})g_{\alpha_1} + A_2(\text{ws})g_{\alpha_2} \]  
\[ = B(\text{ws})e^{ik_{\beta}R \frac{4\pi R}{4\pi R}} + A_1(\text{ws})e^{ik_{\alpha_1}R \frac{4\pi R}{4\pi R}} + A_2(\text{ws})e^{ik_{\alpha_2}R \frac{4\pi R}{4\pi R}} \]  
\[ (5.55) \]

With the introduced variables \( B(\text{ws}), A_1(\text{ws}), A_2(\text{ws}) \), where \( (\text{ws}) \) denotes the scalar Green’s function amplitudes belonging to the relative flow Green's function with source on the solid frame, defined as,

\[ B(\text{ws}) = \frac{1}{G[H\mathcal{M} - C^2] e^{ik_{\beta}R} \frac{4\pi R}{4\pi R}} \frac{-k_{\beta}^2 C + \omega^2 \rho_f}{k_{\beta}^2 (k_{\beta}^2 - k_{\alpha_1}^2)(k_{\beta}^2 - k_{\alpha_2}^2)} \]  
\[ A_1(\text{ws}) = \frac{1}{G[H\mathcal{M} - C^2] e^{ik_{\alpha_1}R} \frac{4\pi R}{4\pi R}} \frac{-k_{\alpha_1}^2 C + \omega^2 \rho_f}{k_{\alpha_1}^2 (k_{\alpha_1}^2 - k_{\beta}^2)(k_{\alpha_1}^2 - k_{\alpha_2}^2)} \]  
\[ A_2(\text{ws}) = \frac{1}{G[H\mathcal{M} - C^2] e^{ik_{\alpha_2}R} \frac{4\pi R}{4\pi R}} \frac{-k_{\alpha_2}^2 C + \omega^2 \rho_f}{k_{\alpha_2}^2 (k_{\alpha_2}^2 - k_{\beta}^2)(k_{\alpha_2}^2 - k_{\alpha_1}^2)} \]  
\[ (5.56) \]
When equations (Eq 5.17), (Eq 5.55), (Eq 5.32), (Eq 5.33), (Eq 5.34), (Eq 5.35) and (Eq 5.36) are substituted into equation (Eq 5.54) the following expression is obtained,

\[
\mathcal{G} = g^w \mathbb{1} + \left[ \frac{1}{C}(HM - C^2) - G_{\rho E} \rho_f \right] \nabla \nabla \Phi \\
- \left[ \frac{HM}{C} C \omega^2 \rho_f + C C \omega^2 \rho E \rho_f - H \omega^2 \rho E - M \omega^2 \rho B \right] \nabla \nabla \Phi^* \\
+ \left[ \hat{\varepsilon} \hat{\varepsilon} + \frac{\nabla_t \nabla_i}{\nabla_i^2} \right] \left[ -\frac{1}{C} (HM - C^2) \right] \nabla^2 \Phi \\
+ \frac{\left( \frac{HM}{C} C \omega^2 \rho_f + C C \omega^2 \rho E \rho_f - H \omega^2 \rho E - M \omega^2 \rho B \right)}{\left( C \nabla_i^2 + C \frac{\rho^2}{\rho_x^2} + \omega^2 \rho_f \right)} \nabla^2 \Phi \\
+ \omega^2 \left( \rho_f - \frac{\rho E \rho B}{\rho_f} \right) \Phi - g^w \tag{5.57}
\]

Where \( \Phi^* \) is defined as,

\[
\Phi^* = B^*(w_s) g_\beta + A^*_1(w_s) g_{\alpha_1} + A^*_2(w_s) g_{\alpha_2} \tag{5.58}
\]

\[
B^*(w_s) = \frac{B(w_s)}{\omega^2 \rho_f - C k_{\beta}^2}, \quad A_1^*(w_s) = \frac{A_1(w_s)}{\omega^2 \rho_f - C k_{\alpha_1}^2}, \quad A_2^*(w_s) = \frac{A_2(w_s)}{\omega^2 \rho_f - C k_{\alpha_2}^2} \tag{5.59}
\]

Substituting \( g^w \), equation (Eq 5.17) and \( \Phi \), equation (Eq 5.55) into equation (Eq 5.57) and using equations (Eq B.25), (Eq 5.66) and (Eq 5.59) yields the dynamic relative flow Green’s function (see appendix B for the algebraic details),

\[
\mathcal{G}^w = g^w \mathbb{1} + \left[ \frac{1}{C}(HM - C^2) - G_{\rho E} \rho_f \right] \nabla \nabla \Phi \\
- \left[ \frac{HM}{C} C \omega^2 \rho_f + C C \omega^2 \rho E \rho_f - H \omega^2 \rho E - M \omega^2 \rho B \right] \nabla \nabla \Phi^* \tag{5.60}
\]

Where the superscript \( w \) denotes the Greens’s function to be a relative flow Green’s function and the subscript \( s \) denotes the point force applied to the solid matrix.

The dynamic relative flow Green’s function rewritten in compact form yields,

\[
\mathcal{G}^w = g^w \mathbb{1} + \nabla \nabla \gamma^w \\
\gamma^w = \sum_{i \neq l \neq m \neq \beta, \alpha_1, \alpha_2} \frac{\psi (k_i^2 M - \omega^2 \rho E) - \gamma}{G (HM - C^2) \frac{\rho E}{\rho_f} (k_i^2 - k_l^2) (k_i^2 - k_m^2) g_i} \tag{5.61}
\]

163
5.2.2 Relative Flow Green's Function with Force on Fluid

To solve for this Green's function we write the coupled equations in the same matrix format as equation (Eq 5.18) with the driving force now on the fluid equations instead of the bulk equations. The first step in the solution procedure is to rewrite this 4 by 4 differential operator matrix into a 2 by 2 system where the differential operator matrix operates on the relative flow components and where the right hand side source vector contains the force components on the fluid phase in the pore matrix. The system has now the following form,

\[
\begin{bmatrix}
F_{11} & F_{12} \\
F_{21} & F_{22}
\end{bmatrix}
\begin{bmatrix}
\nabla_t \cdot w_t \\
\frac{\partial}{\partial z} w_z
\end{bmatrix}
= 
\begin{bmatrix}
\nabla_t \cdot f_t \\
\frac{\partial}{\partial z} f_z
\end{bmatrix}
\] (5.62)

Where the variables \(F_{11}, F_{12}, F_{21}, F_{22}\) are defined as,

\[
F_{11} = L_z^M + \Theta \left[ L_z^C \left( L_z^C R_t - C(H - G) \nabla_t^2 \frac{\partial^2}{\partial z^2} \right) 
+ C \frac{\partial^2}{\partial z^2} \left( -(H - G)L_z^C \nabla_t^2 + CR_z \nabla_t^2 \right) \right]
\] (5.63)

\[
F_{12} = M \nabla_t^2 + \Theta \left[ C \nabla_t^2 \left( L_z^C R_t - C(H - G) \nabla_t^2 \frac{\partial^2}{\partial z^2} \right) 
+ L_z^C \left( -(H - G)L_z^C \nabla_t^2 + CR_z \nabla_t^2 \right) \right]
\] (5.64)

\[
F_{21} = M \frac{\partial^2}{\partial z^2} + \Theta \left[ L_z^C \left( CR_t \frac{\partial^2}{\partial z^2} - L_z^C (H - G) \frac{\partial^2}{\partial z^2} \right) 
+ C \frac{\partial^2}{\partial z^2} \left( -C(H - G) \nabla_t^2 \frac{\partial^2}{\partial z^2} + L_t^C R_z \right) \right]
\] (5.65)

\[
F_{22} = L_z^M + \Theta \left[ C \nabla_t^2 \left( CR_t - L_z^C (H - G) \frac{\partial^2}{\partial z^2} \right) 
+ L_t^C \left( -C(H - G) \nabla_t^2 \frac{\partial^2}{\partial z^2} + L_t^C R_z \right) \right]
\] (5.66)

where,

\[
\Theta = \frac{-1}{R_t R_z - (H - G)^2 \nabla_t^2 \frac{\partial^2}{\partial z^2}} \] (5.67)
Performing the modified Kupradze method yields,
\[
\begin{bmatrix}
\nabla_t \cdot \mathbf{w}_t
\end{bmatrix} = \int_V \begin{bmatrix}
F_{22} & -F_{12}
\end{bmatrix} \begin{bmatrix}
\Phi(x, x')
\end{bmatrix} \begin{bmatrix}
\nabla_t' \cdot \mathbf{f}_t'
\end{bmatrix} \, dx'
\]

Substituting equation (Eq 5.68) into equation (Eq 5.62) yields,
\[
[F_{11}F_{22} - F_{12}F_{21}] \Phi(x, x') = -\delta(x, x')
\]

Substituting equations (Eq 5.19), (Eq 5.20) and (Eq 5.21) into (Eq 5.63), (Eq 5.64), (Eq 5.65) and (Eq 5.66) and carrying out the algebra, the following expressions are obtained.

\[
F_{11} = L_z + \frac{\mu_z}{\delta}, \quad F_{12} = \left[ M - \frac{C^2}{H} \right] \nabla_t^2 + \frac{\chi_t}{\delta}
\]

\[
F_{21} = \left[ M - \frac{C^2}{H} \right] \frac{\partial^2}{\partial x^2} + \frac{\chi_x}{\delta}, \quad F_{22} = L_t + \frac{\mu_t}{\delta}
\]

Where the introduced variables $L_t, L_z, \mu_t, \mu_z, \chi_t, \chi_z, \delta$ are defined to be,

\[
L_t = \left[ M - \frac{C^2}{H} \right] \frac{\partial^2}{\partial x^2} + \omega^2 \rho_E
\]

\[
L_z = \left[ M - \frac{C^2}{H} \right] \nabla_t^2 + \omega^2 \rho_E
\]

\[
\mu_t = \gamma \frac{\partial^4}{\partial z^4} + \gamma \nabla_t^2 \frac{\partial^2}{\partial z^2} + \xi \left[ \frac{\partial^2}{\partial z^2} - H \omega^4 \rho_f \nabla_t^2 - \omega^6 \rho_f \rho_B \right]
\]

\[
\mu_z = \gamma \nabla_t^4 + \gamma \nabla_t^2 \frac{\partial^2}{\partial z^2} + \xi \nabla_t^2 - H \omega^4 \rho_f^2 \frac{\partial^2}{\partial z^2} - \omega^6 \rho_f \rho_B
\]

\[
\chi_t = \gamma \nabla_t^4 + \gamma \nabla_t^2 \frac{\partial^2}{\partial z^2} + \left[ \xi + H \omega^4 \rho_f^2 \right] \nabla_t^2
\]

\[
\chi_z = \gamma \frac{\partial^4}{\partial z^4} + \gamma \left[ \frac{\partial^2}{\partial z^2} + \left[ \xi + H \omega^4 \rho_f^2 \right] \frac{\partial^2}{\partial z^2} \right]
\]

\[
\delta = \left[ H \nabla_t^2 + H \frac{\partial^2}{\partial z^2} + \omega^2 \rho_B \right] \left[ G \nabla_t^2 + G \frac{\partial^2}{\partial z^2} + \omega^2 \rho_B \right]
\]

\[
\gamma = \frac{GC^2}{H} - \omega^2 \rho_B - 2GC \omega^2 \rho_f
\]

\[
\xi = \frac{C^2}{H} \omega^4 \rho_B^2 - 2C \omega^4 \rho_f \rho_B - G \omega^4 \rho_f^2
\]

165
The scalar equation (Eq 5.69) can be solved using the Fourier transform method. After Fourier transformation the \( k_z \) integral is performed using Cauchy’s theorem to calculate the contribution of the six poles. Summing the contributions of all poles yields,

\[
\Phi (x, x', \omega) = \frac{1}{4\pi G \omega^2 \rho_E (HM - C^2)} \times \left[ \int_0^\infty \frac{ik J_0(kD)}{(\nu_{a1}^2 - \nu_{\beta}^2)} \frac{(\omega^2 \rho_B - k_{a1}^2 H)(\omega^2 \rho_B - k_{a1}^2 G)}{\nu_{a1}} e^{i\nu_{a1}|z - z'|} dk + \int_0^\infty \frac{ik J_0(kD)}{(\nu_{a2}^2 - \nu_{\beta}^2)} \frac{(\omega^2 \rho_B - k_{a2}^2 H)(\omega^2 \rho_B - k_{a2}^2 G)}{\nu_{a2}} e^{i\nu_{a2}|z - z'|} dk + \int_0^\infty \frac{ik J_0(kD)}{(\nu_{\beta}^2 - \nu_{a1}^2)} \frac{(\omega^2 \rho_B - k_{\beta}^2 H)(\omega^2 \rho_B - k_{\beta}^2 G)}{\nu_{\beta}} e^{i\nu_{\beta}|z - z'|} dk \right] \quad (5.81)
\]

The total relative flow vector can be recovered from \( \nabla_t \cdot w_t \) and \( \frac{\partial}{\partial z} w_z \) (note that \( \nabla_t \times w_t = 0 \), because \( \nabla_t \times f_t = \nabla_t \times \nabla_t S = 0 \)). The following expression for the relative flow vector is obtained.

\[
w = \int_V \left[ \dot{z} \dot{\hat{z}} \left( L_z + \frac{\mu_z}{\delta} \right) \Phi - \left( \left( M - \frac{C^2}{H} \right) + \frac{\chi^*}{\delta} \right) \frac{\partial}{\partial z} \nabla_t \dot{\hat{z}} \Phi + \left[ \left( M - \frac{C^2}{H} \right) + \frac{\chi^*}{\delta} \right] \frac{\partial}{\partial z} \dot{\hat{z}} \nabla_t \Phi \right] + \frac{\nabla_t \nabla_t}{\nabla_t^2} \left( L_t + \frac{\mu_t}{\delta} \right) \Phi \cdot f \, dx' \quad (5.82)
\]

With,

\[
\chi^* = \gamma \nabla_t^2 + \gamma \frac{\partial^2}{\partial z^2} + \left[ \xi + H \omega^4 \sigma^2 \right] \quad (5.83)
\]

The integrand of equation (Eq 5.82), call it \( G \), is a Green’s function for the fictitious source, \( f \), on the fluid phase (with \( f = \nabla S \), and \( S \) a pressure source). To obtain the relative flow Green’s function due to a pressure source, the divergence of the integrand with respect to the source coordinates in equation (Eq 5.82) has to be taken (use relation \( \nabla' = -\nabla \)). The algebraic manipulations are discussed in the appendix B.

\[
G_{\sigma}^w = \nabla \cdot G
\]

166
\[ = \frac{1}{\ell^2} \left[ H \left( \omega^4 \rho_f^2 - \omega^4 \rho_E \rho_B \right) \nabla \nabla^2 \Phi + \left( -\omega^6 \rho_E \rho_B^2 + \omega^6 \rho_f^2 \rho_B \right) \nabla \Phi \right. \\
+ \left. \left( -\omega^2 \rho_E G \left[ H \nabla^4 + \nabla^2 \omega \rho_B \right] \right) \nabla \Phi \right] \]

(5.84)

With,

\[ \Phi = B(wf)g_{\beta} + A_1(wf)g_{\alpha_1} + A_2(wf)g_{\alpha_2} \]

(5.85)

With the introduced variables \( B(wf), A_1(wf), A_2(wf) \), where \( (wf) \) denotes scalar Green’s function amplitudes belonging to the relative flow Green’s function with a source applied on the fluid phase, defined as,

\[ B(wf) = \frac{1}{G \omega^2 \rho_E [HM - C^2]} \frac{\left( \omega^2 \rho_B - k_{\beta}^2 H \right) \left( \omega^2 \rho_B - k_{\beta}^2 G \right)}{\left( k_{\beta}^2 - k_{\alpha_1}^2 \right) \left( k_{\beta}^2 - k_{\alpha_2}^2 \right)} \]

(5.86)

\[ A_1(wf) = \frac{1}{G \omega^2 \rho_E [HM - C^2]} \frac{\left( \omega^2 \rho_B - k_{\alpha_1}^2 H \right) \left( \omega^2 \rho_B - k_{\alpha_1}^2 G \right)}{\left( k_{\alpha_1}^2 - k_{\beta}^2 \right) \left( k_{\alpha_1}^2 - k_{\alpha_2}^2 \right)} \]

(5.87)

\[ A_2(wf) = \frac{1}{G \omega^2 \rho_E [HM - C^2]} \frac{\left( \omega^2 \rho_B - k_{\alpha_2}^2 H \right) \left( \omega^2 \rho_B - k_{\alpha_2}^2 G \right)}{\left( k_{\alpha_2}^2 - k_{\beta}^2 \right) \left( k_{\alpha_2}^2 - k_{\alpha_1}^2 \right)} \]

(5.88)

Using the amplitudes \( B(wf), A_1(wf), A_2(wf) \) and the scalar wave equations, \( \nabla^2 g_i + k_i^2 g_i = -\delta(x - x') \) with, \( i = \beta \vee \alpha_1 \vee \alpha_2 \), equation (Eq 5.84) can be rewritten into its final relative flow Green’s function form,

\[ G_{wf}^w = \nabla \Upsilon_{wf}^w, \quad \Upsilon_{wf}^w = \Lambda_1 g_{\alpha_1} + \Lambda_2 g_{\alpha_2} \]

(5.89)

Where,

\[ \Lambda_1 = \left[ H G \omega^2 \rho_E \left( k_{\alpha_1}^2 k_{\beta}^2 - k_{\alpha_1}^4 \right) + \omega^4 \rho_E \rho_B G \left( k_{\alpha_1}^2 - k_{\beta}^2 \right) \right] A_1^*(wf) \]

\[ \Lambda_2 = \left[ H G \omega^2 \rho_E \left( k_{\alpha_2}^2 k_{\beta}^2 - k_{\alpha_2}^4 \right) + \omega^4 \rho_E \rho_B G \left( k_{\alpha_2}^2 - k_{\beta}^2 \right) \right] A_2^*(wf) \]

\[ A_1^*(wf) = \frac{A_1(wf)}{\left( \omega^2 \rho_B - k_{\alpha_1}^2 H \right) \left( \omega^2 \rho_B - k_{\alpha_1}^2 G \right)} \]

(5.90)

\[ A_2^*(wf) = \frac{A_2(wf)}{\left( \omega^2 \rho_B - k_{\alpha_2}^2 H \right) \left( \omega^2 \rho_B - k_{\alpha_2}^2 G \right)} \]
5.3 Point Source Results in Poro-Elastic Media

A basic application of Green's functions is the calculation of the displacement and relative flow components generated by point sources. In this section the stress displacement fields generated by vertical and explosive point sources are calculated. Both sources are applied on the solid frame of the porous medium.

5.3.1 Vertical Point Source

The displacements are calculated for a vertical point force (parallel to the symmetry axis of the cylindrical coordinate frame) at the origin, \( F(x) = \hat{z}\delta(x) \).

Using Green's theorem we obtain,

\[
\textbf{u} = \int_V \textbf{G}_z^u \cdot F(x')dx' = g^u \hat{z} - \Psi \nabla \frac{\partial \phi}{\partial z} - \Gamma \frac{\partial}{\partial z} \phi^* \tag{5.91}
\]

With \( \textbf{G}_z^u \), \( g^u \), \( \Gamma \), \( \Psi \), \( \phi \) and \( \phi^* \) defined in (Eq B.29), (Eq B.5), (Eq B.16), (Eq B.17), (Eq B.22) and (Eq B.27).

Similarly the relative flow vector generated by a vertical point force is given by,

\[
\textbf{w} = \int_V \textbf{G}_z^w \cdot F(x')dx' = g^w \hat{z} - \phi \nabla \frac{\partial \phi}{\partial z} - \gamma \nabla \frac{\partial \Phi^*}{\partial z} \tag{5.92}
\]

With \( \textbf{G}_z^w \), \( g^w \), \( \gamma \), \( \psi \), \( \Phi \) and \( \Phi^* \) defined in equations (Eq 5.60), (Eq 5.17), (Eq 5.32), (Eq 5.33), (Eq 5.55), (Eq 5.58).

5.3.2 Explosive Point Source

The displacement vector for an explosive point source at the origin is obtained by taking the divergence of the displacement Green's function with respect to the source
coordinates. We obtain,

\[ \mathbf{u} = \nabla g^u - \Psi \nabla \nabla^2 \phi - \Gamma \nabla \nabla^2 \phi^* \]  
\( \text{(5.93)} \)

With \( g^u, \Gamma, \Psi, \phi, \phi^* \) defined in equations (Eq B.5), (Eq B.16), (Eq B.17), (Eq B.22), (Eq B.27).

Similarly the relative flow vector generated by an explosive point source is given by,

\[ \mathbf{w} = \nabla g^w - \psi \nabla \nabla^2 \Phi - \gamma \nabla \nabla^2 \Phi^* \]  
\( \text{(5.94)} \)

With \( g^w, \gamma, \psi, \Psi, \Phi^* \) defined in equations (Eq 5.17), (Eq 5.32), (Eq 5.33), (Eq 5.55), (Eq 5.58). Substituting the displacement and relative flow vectors into the deformation equations, (Eq 5.4) and (Eq 5.3) the bulk stress components and pore fluid pressure are obtained. From the displacement, relative flow, stress and pressure expressions for point sources, the received displacements, relative flows, stresses and pressures for a given plane wave at the receiver away from the point source in the layered porous formation can be manipulated into the following form.

\[ u_1(r, z) = \int_0^\infty \hat{u}_1(k, z) k J_0(kr) dk \]
\[ u_2(r, z) = \int_0^\infty \hat{u}_2(k, z) k J_0(kr) dk \]
\[ w_z(r, z) = \int_0^\infty \hat{w}_z(k, z) k J_0(kr) dk \]
\[ \tau_1(r, z) = \int_0^\infty \hat{\tau}_1(k, z) k J_0(kr) dk \]
\[ \tau_{zz}(r, z) = \int_0^\infty \hat{\tau}_{zz}(k, z) k J_0(kr) dk \]
\[ S(r, z) = \int_0^\infty \hat{T}_p(k, z) k J_0(kr) dk \]  
\( \text{(5.95)} \)

With \( \hat{u}_1 = \frac{\hat{u}_V}{\sqrt{k}} \) and \( u_V = \frac{1}{r} \left[ \frac{\partial}{\partial r}(ru_r) + \frac{\partial}{\partial \phi}(u_\phi) \right], \hat{\tau}_1 = \frac{\hat{\tau}_V}{\sqrt{k}} \) and \( \tau_{Vz} = \frac{1}{r} \left[ \frac{\partial}{\partial r}(r\tau_{rz}) + \frac{\partial}{\partial \phi}(\tau_{\phi z}) \right]. \)

The displacement stress wavefield vector for a vertical point source is given by,

\[ \begin{bmatrix} u_1, & u_2, & w_z, & \tau_1, & \tau_{zz}, & S \end{bmatrix}_{vps}^T = \sum_{p=\beta,\alpha_1,\alpha_2} \left( \begin{bmatrix} \frac{\text{sgn}(z-z')}{4\pi} \frac{1}{ik} U_p^r, & \frac{i}{4\pi} U_p^z, & \frac{i}{4\pi} W_p^z, & -i \frac{G}{4\pi} S_p^r, & \frac{\text{sgn}(z-z')}{4\pi} S_p^2, & \frac{\text{sgn}(z-z')}{4\pi} S_p^3 \end{bmatrix} \right) \times e^{-in_p|z-z'|} \]  
\( \text{(5.96)} \)
Where the introduced variables, $W_{eta}', W_{\alpha_1}', W_{\alpha_2}', U_{\beta}', U_{\alpha_1}', U_{\alpha_2}', W_{\beta}^z, W_{\alpha_1}^z, W_{\alpha_2}^z, U_{\beta}^z, U_{\alpha_1}^z, U_{\alpha_2}^z$ are defined as,

\[
W_{\beta}' = \left[ \Pi' B^*(ws) - \Delta' B(ws) \right] k^2, \quad U_{\beta}' = \left[ \Xi' B^*(us) - \Gamma' B(us) \right] k^2 \quad (5.97)
\]

\[
W_{\alpha_1}' = \left[ \Pi' A_{\alpha_1}'(ws) - \Delta' A_{\alpha_1}(ws) \right] k^2, \quad U_{\alpha_1}' = \left[ \Xi' A_{\alpha_1}'(us) - \Gamma' A_{\alpha_1}(us) \right] k^2 \quad (5.98)
\]

\[
W_{\alpha_2}' = \left[ \Pi' A_{\alpha_2}'(ws) - \Delta' A_{\alpha_2}(ws) \right] k^2, \quad U_{\alpha_2}' = \left[ \Xi' A_{\alpha_2}'(us) - \Gamma' A_{\alpha_2}(us) \right] k^2 \quad (5.99)
\]

\[
W_{\beta}^z = -\frac{\rho f}{\rho E G\nu_\beta} - \Delta' B(ws)\nu_\beta + \Pi' B^*(ws)\nu_\beta, \quad U_{\beta}^z = \frac{1}{G\nu_\beta} - \Gamma' B(ws)\nu_\beta + \Xi' B^*(ws)\nu_\beta \quad (5.100)
\]

\[
W_{\alpha_1}^z = -\Delta' A_{\alpha_1}(ws)\nu_{\alpha_1} + \Pi' A_{\alpha_1}'(ws)\nu_{\alpha_1}, \quad U_{\alpha_1}^z = -\Gamma' A_{\alpha_1}(us)\nu_{\alpha_1} + \Xi' A_{\alpha_1}'(us)\nu_{\alpha_1} \quad (5.101)
\]

\[
W_{\alpha_2}^z = -\Delta' A_{\alpha_2}(ws)\nu_{\alpha_2} + \Pi' A_{\alpha_2}'(ws)\nu_{\alpha_2}, \quad U_{\alpha_2}^z = -\Gamma' A_{\alpha_2}(us)\nu_{\alpha_2} + \Xi' A_{\alpha_2}'(us)\nu_{\alpha_2} \quad (5.102)
\]

With variables $A_q(ws), A_q^*(ws), B(ws), B^*(ws)$ defined in equations (Eq 5.56) and (Eq 5.59) and variables $A_q(us), A_q^*(us), B(us), B^*(us)$ defined in equations (Eq B.23) and (Eq B.28) with $q = 1 \lor 2$ With,

\[
\Gamma' = G - \frac{1}{M} \left[ HM - C^2 \right] \quad (5.103)
\]

\[
\Xi' = -2\omega^2 \rho f C + M\omega^2 \rho f^2 \rho E + C^2 \omega^2 \rho E \quad (5.104)
\]

\[
\Delta' = \frac{1}{C} \left[ HM - C^2 \right] - G\rho E \rho f \quad (5.105)
\]

\[
\Pi' = \frac{HM}{C} \omega^2 \rho f + C\omega^2 \frac{\rho B \rho E}{\rho f} - H\omega^2 \rho E - M\omega^2 \rho B \quad (5.106)
\]

Variables $S_1^p, S_2^p, S_3^p$ with $p = \beta \lor \alpha_1 \lor \alpha_2$ are defined as,

\[
S_1^\beta = 2k^2 \left[ U_{\beta}' - \frac{1}{G\nu_\beta} \right], \quad S_1^{\alpha_1} = 2k^2 U_{\alpha_1}'^z, \quad S_1^{\alpha_2} = 2k^2 U_{\alpha_2}'^z
\]

\[
S_2^p = (H - 2G) U_p' + C \left( W_{p}' + \nu_p W_{p}' \right) + H\nu_p U_p^z
\]

\[
S_3^p = C \left( U_p' + \nu_p U_p^z \right) + M \left( W_{p}' + \nu_p W_{p}' \right) \quad (5.107)
\]

The displacement stress wavefield vector for an explosive point source is given by,

\[
\begin{bmatrix}
  u_1, \ u_2, \ w_z, \ \tau_1, \ \tau_{zz}, \ S
\end{bmatrix}_{\exp}^T = \sum_{p=\alpha_1,\alpha_2} \left( \begin{array}{c}
-\frac{i}{4\pi} k^2 U_p' + \frac{\text{sgn}(z-z')}{4\pi} W_{p}', \ \frac{\text{sgn}(z-z')}{4\pi} W_{p}'^z, \ \frac{-\text{sgn}(z-z')}{4\pi} \frac{i k}{4\pi} S_1^p, \ \frac{-i}{4\pi} S_2^p, \ \frac{-i}{4\pi} S_3^p \n\end{array} \right) 
\times e^{-\nu_p\sqrt{|z-z'|}} \quad (5.108)
\]

170
With variables, $W_{\alpha_1}^r, W_{\alpha_2}^r, U_{\alpha_1}^r, U_{\alpha_2}^r, W_{\alpha_1}^z, W_{\alpha_2}^z, U_{\alpha_1}^z, U_{\alpha_2}^z$ defined as,

\[
W_{\alpha_1}^r = \left( -\Delta' A_1(ws) + \Pi' A_1^*(ws) \right) \frac{k_{\alpha_1}^2}{\nu_{\alpha_1}}, \quad U_{\alpha_1}^r = \left( -\Gamma' A_1(us) + \Xi' A_1^*(us) \right) \frac{k_{\alpha_1}^2}{\nu_{\alpha_1}} \\
W_{\alpha_2}^r = \left( -\Delta' A_2(ws) + \Pi' A_2^*(ws) \right) \frac{k_{\alpha_2}^2}{\nu_{\alpha_2}}, \quad U_{\alpha_2}^r = \left( -\Gamma' A_2(us) + \Xi' A_2^*(us) \right) \frac{k_{\alpha_2}^2}{\nu_{\alpha_2}} \\
W_{\alpha_1}^z = \left( -\Delta' A_1(ws) + \Pi' A_1^*(ws) \right) k_{\alpha_1}^2, \quad U_{\alpha_1}^z = \left( -\Gamma' A_1(us) + \Xi' A_1^*(us) \right) k_{\alpha_1}^2 \\
W_{\alpha_2}^z = \left( -\Delta' A_2(ws) + \Pi' A_2^*(ws) \right) k_{\alpha_2}^2, \quad U_{\alpha_2}^z = \left( -\Gamma' A_2(us) + \Xi' A_2^*(us) \right) k_{\alpha_2}^2 
\]

With variables $A_q(ws), A_q^*(ws)$ defined in equations (Eq 5.56) and (Eq 5.59) and variables $A_q(us), A_q^*(us)$ defined in equations (Eq B.23) and (Eq B.28) with $q = 1 \lor 2$.

Variables $S_1^p, S_2^p, S_3^p$ with $p = \alpha_1 \lor \alpha_2$ are defined as,

\[
S_1^p = Gk^2 \left( U_p^z + \nu_p U_p^r \right) \\
S_2^p = (H - 2G) k^2 U_p^r + C \left( k^2 W_p^r + \nu_p W_p^z \right) + H \nu_p U_p^z \\
S_3^p = C \left( k^2 U_p^r + \nu_p U_p^z \right) + M \left( k^2 W_p^r + \nu_p W_p^z \right) 
\]

(5.109)

### 5.4 Mechanical and Electromagnetic Waves in Horizontally Layered Poro-Elastic Media

We first transform the macroscopic governing equations that control the coupled electromagnetics and acoustics of porous media, see chapter 3, into a field-vector-formalism that can be solved numerically. The global matrix method (Chin et al., 1984) is employed to solve simultaneously the macroscopic electromagnetic and acoustic-elastic wavefield amplitudes which contribute to current and fluid flow. The current and fluid flow are related by the transport equations through which all electromagnetic and mechanical coupling occurs, i.e. currents generated by the acoustic-elastic wavefields, an electrokinetic phenomenon and fluid flow generated by the electromagnetic field, an osmosis phenomenon. In the global matrix method we propagate
a wavefield vector which contains the mechanical stress/displacement and electromagnetic wavefield components that are continuous through the horizontal boundaries in the layered porous medium, see chapter 4.

For isotropic media the first-order ordinary differential equations, the equations of state, relating the wavefields at all depths to the medium properties at depth, decompose into two equation sets. The first set describes the $PSV TM$ picture, which is the situation where the compressional and vertical polarized mechanical waves drive currents in the $PSV$ particle motion plane that couple to the electromagnetic wavefield components of the $TM$ mode. The second set describes the $SHT E$ picture where the horizontal polarized rotational waves drive currents in the $SH$ particle motion plane that couples to the electromagnetic wavefield components of the $TE$ mode. In this chapter we compute electroseismograms generated by two mechanical point sources in the $PSV TM$ polarization picture only.

The effect of a general mechanical point source in a stratified porous medium is accommodated by specifying a jump in the displacement-stress vector across a horizontal plane (see Hudson, 1969a,b; Kennett, 1983. The wavefield components in the displacement-stress vector calculated for a vertical point source and explosive point source, equations (Eq 5.96) and (Eq 5.108), respectively, are the wavefields that are continuous at the horizontal boundaries in the stratified medium for the $PSV TM$ picture. We define the total displacement-stress-EM wavefield vector as follows,

\[ B = [u_1, u_2, w_z, \tau_1, \tau_{zz}, S, H_2, E_1]^T \] (5.111)

At \( z = z' \), with \( z' \) the source depth only the \( u_1, \tau_{zz} \) and \( S \) wavefields jump in the vertical point source representation across a dummy interface, where a dummy interface denotes a boundary across which none of the medium properties change. The final jump representation across a source plane for a vertical point source, using equations (Eq 5.96) and (Eq 5.111) reads,

\[ \mathcal{S}(z') = \left[ \lim_{z \to z'} B(z) - \lim_{z \to z'} \mathcal{I}(z) \right] \]
\[
= \sum_{p=\beta,\alpha_1,\alpha_2} \left[ \frac{1}{2\pi} \frac{1}{ik} U_r^p, 0, 0, 0, \frac{1}{2\pi} S_2^p, \frac{1}{2\pi} S_3^p, 0, 0 \right]^T \tag{5.112}
\]

At \( z = z' \), with \( z' \) the source depth only the \( u_z, \tau_1 \) and \( \tau_{zz} \) wavefields jump in the explosive point source representation across a dummy interface. The final jump representation across a source plane for an explosive point source, using equations (Eq 5.108) and (Eq 5.111) reads,

\[
\mathbf{S}(z') = \left[ \lim_{z \rightarrow z'} B(z) - \lim_{z \rightarrow z} B(z) \right]
\]

\[
= \sum_{p=\alpha_1, \alpha_2} \left[ 0, \frac{1}{2\pi} U_r^z, \frac{1}{2\pi} W_r^z, \frac{1}{2\pi} \frac{1}{ik} S_1^p, 0, 0, 0, 0 \right]^T \tag{5.113}
\]

Note that the last two electromagnetic wavefield components in equations (Eq 5.112) and (Eq 5.113) are zero since no electromagnetic source is applied.

### 5.4.1 Transformation Back to the Space Time Domain

An inverse Hankel transform is applied to obtain the 3D spatial dependence of the displacements, stresses, and electric and magnetic fields. The horizontal components of the displacements, stresses, electric and magnetic fields require additional integration over \( r \) and \( \phi \) to obtain, \( u_r, u_\phi, \tau_{\phi z}, \tau_{rz}, H_r, H_\phi \) and \( E_r, E_\phi \) due to the definitions of \( u_V, u_H, \tau_{Vz}, \tau_{Hz}, H_V, H_H \) and \( E_V, E_H \).

\[
u_z(\omega, r, \phi, z) = \int_0^\infty k \, dk \sum_{n=-N}^N \left[ J_n(\omega) \hat{u}_z(\omega, k, n, z) \right] e^{in\phi} \tag{5.114}
\]

Identical relations exist for \( w_z, \tau_{zz} \) and \( S \). The horizontal components may be recovered using the following relations (Kennett, 1983).

\[
u_r(\omega, r, \phi, z) = \int_0^\infty k \, dk \sum_{n=-N}^N \left[ \frac{n}{kr} J_n(\omega) \hat{u}_2(\omega, k, n, z) \right] - \, iJ'_n(\omega) \hat{u}_1(\omega, k, n, z) e^{in\phi} \tag{5.115}
\]

\[
u_\phi(\omega, r, \phi, z) = \int_0^\infty k \, dk \sum_{n=-N}^N \left[ \frac{n}{kr} J_n(\omega) \hat{u}_1(\omega, k, n, z) \right] + \, iJ'_n(\omega) \hat{u}_2(\omega, k, n, z) e^{in\phi} \tag{5.116}
\]
Identical relations exist for $\tau_x$, $\tau_{\phi}z$ in terms of $\hat{r}_1$ and $\hat{r}_2$, for $E_r, E_\phi$ in terms of $\hat{E}_1$ and $\hat{E}_2$ and $H_r, H_\phi$ in terms of $\hat{H}_1$ and $\hat{H}_2$. The above representations may be regarded as a superposition of cylindrical waves whose order dictates the nature of their azimuthal modulation. At each frequency and angular order the radial contribution is obtained by superposing all horizontal wavenumbers $k$. This corresponds to including all propagating waves at the level $z$ within the stratification, from vertically to purely horizontal traveling waves, including the evanescent waves. At any particular distance $r$ the relative contributions of the wavenumbers are imposed by the radial phase functions $J_n(kr)$.

5.5 Numerical Results

Three different numerical models are calculated in the PSVTM wavefield picture. The first example is a surface electroseismic model example, see figure 5-1. The electroseismic response of two different point sources, an explosive and vertical point source are modeled. In figures 5-2 and 5-3 the mechanical and electroseismograms ($TM$ mode components) are displayed which include the direct waves of the explosive and vertical point source respectively. In figures 5-4 and 5-6 the reflection results are shown, i.e. the direct wavefields are not included.

The last two examples show the electroseismic effect in a Vertical ElectroSeismic Profiling (VES) setting. The seismic source is located in the upper halfspace and the recording geophone/antennas are positioned vertically crossing two mechanical/electrical contrasts in the first $VES$ example (see figure 5-7) and an additional fluid chemistry contrast in the second $VES$ example (see figure 5-12). In figures 5-8 and 5-9 the calculated results are shown for an explosive and vertical point source respectively, including the direct wavefields. In figures 5-10 and 5-11 the results are shown without the direct wavefields. Figures 5-13 and 5-14 show the additional elec-
troseismic response of the included fluid chemistry contact within the reservoir sand, generated by an explosive and vertical point source.

5.5.1 Electroseismics in Surface Geometry

The point source is positioned 100 m above the contrast in the upper halfspace. Fifteen antennas/geophones are positioned symmetrically in a straight horizontal line at both sides of the source 95 m above the interface. The receiver spacing is 10 m. The medium properties of the two halfspaces and the calculated wave velocities and bulk conductivities, calculated at 100 Hz source center frequency, are given in tables I, II, III.

When an explosion source is set off only P waves are generated in an isotropic poro-elastic medium. In the figures the mechanical displacement seismograms (u_z and u_r components) and the electroseismograms (TM mode components) are shown. All plots are seismogram scaled. At the mechanical contrast the P wave reflects as a P wave, converts into a SV wave and converts into an electromagnetic TM disturbance. The converted TM wavefield components show up at all antennas at approximately half the two way P wave traveltime for normal incidence reflection. Since the TM wavefield velocity is at least 2 orders of magnitude faster than the P wavefield, the traveltime spent by the TM waves traveling upward to the antennas is negligible and the total two way traveltime appears to coincide with the oneway traveltime of the incoming P wave at normal incidence reflection.

The hyperbolas arriving at a later time in the E_r and H_φ electroseismograms appear at the same traveltime as the PP reflection and PSV conversion respectively. In the first case the compressional waves traveling through homogeneous porous medium cause pressure gradients which cause the charge to separate (Coulombic charge attraction type mechanism). The induced conduction and streaming currents inside the pulse cancel each other and therefore the seismic pulse cannot act as an radiating
antenna. But the seismic pulse does carry a system of electric fields that travel with the compressional wavespeed. Therefore, when the reflected wavefield \( P \) wave pulses pass the antennas, an electric field is registered inside the \( P \) wave pulse. The antennas appear to act as geophones.

In the second case the vertical polarized rotational waves traveling through homogeneous porous medium cause grain accelerations which induce streaming current sheets inside the seismic pulse. These current sheets induce within the seismic pulse magnetic fields that travel with the \( SV \) wave speed. Therefore, when the converted \( SV \) wavelet passes an antenna a magnetic field is recorded inside the \( SV \) wave pulse.

The converted \( TM \) wavefields, driven with the mechanical frequencies, are diffuse. The frequency content of the electromagnetic field is approximately the same as the frequency content of the input mechanical source. Since the propagation distance of the converted electromagnetic disturbance is much less than the electromagnetic skindepth, the converted \( EM \) signals do not lose much of their frequency content and a seismic pulse behavior is maintained.

In figures 5-2, 5-3 and 5-4, 5-6 we study the effect of a different point source. In figures 5-4 and 5-6, the vertical point source synthetics are shown. A vertical point source generates \( P \) as well as \( SV \) waves as opposed to the explosive point source which produces \( P \) waves only in an isotropic medium. The vertical point source results show the additional \( SV \) reflection in the mechanical seismograms. The \( SV \) induced magnetic fields show up in the \( H_{\phi} \) electroseismograms. A noticeable difference between the explosive and vertical point source results is the direct \( EM \) wavefields generated by the vertical point force. These fields show up at approximately zero traveltime in the electroseismograms in figures 5-3 and 5-6. The electroseismograms are slightly shifted in time to avoid signal wrapping in time, which will be introduced by the non-causal Ricker wavelet (Hosken, 1988) convolution. Physically, a vertical point source induces a vertical streaming current imbalance across a hypothetical source plane, due to the mechanically created vertical relative flow. The electro-
magnetic fields associated with this mechanically induced vertical streaming current implies a jump in electromagnetic fields to accommodate for the zero $H_\phi$ and $E_r$ field specification in the vertical jump representation across a source plane, see equation (Eq 5.112).

In figure 5-5 the expected radiation pattern for a vertical point source above a contrast is outlined. The cartoon shows that the largest signal, denoted by the length of the arrows comes from the direct mechanically induced electromagnetic wavefields. This amplitude behavior can be identified in the electroseismograms in figure 5-3. Note the quick amplitude decay of the electromagnetic signal with traveled distance. The amplitude behavior of the reflected electromagnetic signal is outlined in figure 5-5 using the method of images, for a hypothetical perfect reflector case. The horizontal component of the electromagnetic field is zero right at the source position and increases with horizontal distance. This amplitude behavior can be identified in the electroseismograms in figure 5-6. The direct electromagnetic and reflected electromagnetic fields arrive at the same time at the antennas for the used time scale. The direct wavefield dominates in amplitude the reflected fields, compare figure 5-6 with figure 5-3 and the size of the arrows in the cartoon in figure 5-5. The electroseismic conversion of an incoming $SV$ wave generated by the vertical point source, is insignificant when compared to converted electromagnetic signals generated by pressure gradients induced by $P$ waves. At the used electroseismogram scaling the $SV$ to $EM$ conversion is not visible in figure 5-6.

The direct wavefields and direct $P$ wave induced electric fields and $SV$ wave induced magnetic field amplitudes dominate the seismograms, see figures 5-2 and 5-3. At the plotted scales the converted electromagnetic signals are not discernible in the figures. Signal processing needs to be applied to remove the direct arrivals so as to be able to observe the conversions. Since the velocities of the signals are very different in size, $f - k$ filter analysis would separate the converted from the mechanically induced electromagnetic fields. The magnetic field component electroseismogram in figure 5-2
doesn’t show direct fields since an explosive point source doesn’t produce \(SV\) waves.

### 5.5.2 Electroseismograms in VESP Geometry

In a \(VESP\) geometry the antennas are positioned close to the target of interest and therefore larger converted electromagnetic signals can be recorded before they become too attenuated with traveled distance. With the \(VESP\) technique the electroseismic method can be applied to targets, electrical and/or mechanical contrasts, at greater depths.

The first \(VESP\) example is a sandwiched reservoir sand in between two low permeability halfspaces with a smaller porosity. The first receiver/antenna is positioned at a horizontal offset of 20 \(m\) and depth of 40 \(m\). The receiver/antenna spacing is 5 \(m\). The model description with some of the key electroseismic medium parameters is shown in figure 5-7. The complete set of medium properties and the at source center frequency calculated wavefield velocities and bulk conductivities are given in tables I, II and III. In figures 5-8 and 5-9 the numerically calculated \(VESP\)'s are shown for an explosive source and a vertical point source including the direct wavefields. In figures 5-10 and 5-11 the same calculations are shown without the direct wavefields. The top plot shows the mechanical displacement response, the center and bottom plots show the \(E_r\) and \(H_\phi\) \(TM\) mode components respectively. The converted \(TM\) wavefield components show up at all receiving antennas at approximately the same time. These events can be recognized as the straight vertical lines in the figures. The large electrical conductivity, high salinity, in the sand reservoir layer attenuates the electric field amplitudes almost completely. All later arrivals in the \(E_r\) \(VESP\) are the electric fields within the seismic pulse that travel with compressional wavespeed. The later arrivals in the \(H_\phi\) \(VESP\) are the magnetic fields within the seismic pulse that travel with the rotational wavespeed. The main difference between an explosive point source and vertical point source \(VESP\) are the events associated with the \(SV\) waves.
generated by the vertical point source. The vertical point source shows at zero time the electromagnetic fields generated by the source. The electromagnetic conversions due to $P$ wave induced pressure gradients across an interface are much larger than the converted $TM$ waves induced by the $SV$ waves. In the $E_r$ electroseismogram in figure 5-11 and 5-14 a $SV$ conversion can be distinguished. When we compare the $H_\phi$ component electroseismograms in figures 5-8 and 5-9 we see that the largest $H_\phi$ fields are the direct $SV$ wave induced $H_\phi$ fields, see figure 5-9. The converted $H_\phi$ fields recorded close to the interface are larger in amplitude than the $SV$ wave induced $H_\phi$ fields, these $SV$ waves are the $P$ wave converted events, see figure 5-8.

The last numerical example has an additional electrical contrast added to the previous model, see figure 5-12. The 100 m thick "reservoir sand" is divided into two sands with identical mechanical properties saturated with two fluids of different salinity. In figures 5-13 and 5-14 the calculated $VESPs$ are shown for an explosive and vertical point source respectively. The mechanical response is identical to the previous calculations without the electrical fluid contrast, see figures 5-10 and 5-11. But the $TM$ mode component $VESPs$'s show an additional electroseismic conversion generated at the electrical contrast.

5.6 Conclusions and Discussion

Numerically feasible relative flow and displacement Green's functions in dyadic form are derived in an isotropic poro-elastic medium. A field vector formalism is used to solve numerically the macroscopic equations that control the coupled electromagnetics and acoustics of porous media. To solve numerically the macroscopic electromagnetic and acoustic wavefields in layered media a global matrix solution procedure is employed.

In this chapter two mechanical source jump representations for an explosive and
vertical point source acting on the elastic frame are derived. The wavefield components in the source jump representation are obtained by relative flow and displacement Green's function solution together with the deformation equations in the two phases. The mechanical vertical point source is shown to generate radiating electromagnetic fields. The vertical point force induces a step in vertical electrical streaming current which imposes a jump in electromagnetic fields to accommodate for the zero $H_\phi$ and $E_r$ field components in the vertical jump representation across the source plane. The direct and reflected electromagnetic wavefield amplitudes versus offset suggest an associated electrical dipole behavior for the vertical point force.

The conversion to electromagnetic waves from mechanical waves traversing boundaries is found to be mainly due to pressure gradients generated by the $P$ waves across the contrast. The conversion from rotational waves into electromagnetic waves is found to be much smaller.

The $VES$P modeling shows the rapid decay of the converted electromagnetic signals with distance. The antennas close to the target of interest show larger amplitudes in the converted signal than the induced electromagnetic signals inside the seismic pulse. An important application of the electroseismic method in a borehole geometry is the detection of fluid chemistry contrasts inside a reservoir. While the mechanical waves are not sensitive to the fluid saturating phase, an electromagnetic conversion is generated at these fluid (i.e. oil/water) contrasts.
TABLE I. The medium properties defining the layers used in the numerical calculations.

<table>
<thead>
<tr>
<th>Property</th>
<th>Top and bottom halfspaces</th>
<th>Salinity I layer</th>
<th>Salinity II layer</th>
<th>Salinity III halfspace</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi[%]$</td>
<td>15</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>$k_0[m^2]$</td>
<td>$10^{-14}$</td>
<td>$10^{-12}$</td>
<td>$10^{-12}$</td>
<td>$10^{-12}$</td>
</tr>
<tr>
<td>$k_s[Pa]$</td>
<td>$3.0 \times 10^{10}$</td>
<td>$1.0 \times 10^{10}$</td>
<td>$1.0 \times 10^{10}$</td>
<td>$1.0 \times 10^{10}$</td>
</tr>
<tr>
<td>$k_f[Pa]$</td>
<td>$2.2 \times 10^9$</td>
<td>$2.2 \times 10^9$</td>
<td>$2.2 \times 10^9$</td>
<td>$2.2 \times 10^9$</td>
</tr>
<tr>
<td>$k_{fr}[Pa]$</td>
<td>$8.0 \times 10^9$</td>
<td>$3.0 \times 10^9$</td>
<td>$3.0 \times 10^6$</td>
<td>$3.0 \times 10^9$</td>
</tr>
<tr>
<td>$g_{fr}[Pa]$</td>
<td>$9.0 \times 10^9$</td>
<td>$8.5 \times 10^9$</td>
<td>$8.5 \times 10^9$</td>
<td>$8.5 \times 10^9$</td>
</tr>
<tr>
<td>$\eta[Pas]$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$1.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\rho_s[Kg/m^3]$</td>
<td>$2.7 \times 10^3$</td>
<td>$2.7 \times 10^3$</td>
<td>$2.7 \times 10^3$</td>
<td>$2.7 \times 10^3$</td>
</tr>
<tr>
<td>$\rho_f[Kg/m^3]$</td>
<td>$1.0 \times 10^3$</td>
<td>$1.0 \times 10^3$</td>
<td>$1.0 \times 10^3$</td>
<td>$1.0 \times 10^3$</td>
</tr>
<tr>
<td>$C[mol/l]$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$1.0 \times 10^{-2}$</td>
<td>$1.0 \times 10^{-1}$</td>
<td>$1.0$</td>
</tr>
<tr>
<td>$T[K]$</td>
<td>298</td>
<td>298</td>
<td>298</td>
<td>298</td>
</tr>
<tr>
<td>$\kappa_f$</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>$\kappa_s$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$\kappa_\infty$</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
TABLE II a. The calculated (at source center frequency) wavefield velocities and bulk conductivities in the surface seismic and VESP geometry models. Both real and imaginary parts of the calculated wavefield velocities are listed.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Upper and lower halfspace</th>
<th>Salinity I layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>fast wave velocity $[m/s]$</td>
<td>(3154.97, -0.00011)</td>
<td>(2502.21, -0.0524)</td>
</tr>
<tr>
<td>slow wave velocity $[m/s]$</td>
<td>(5.52, -5.28)</td>
<td>(40.54, -38.17)</td>
</tr>
<tr>
<td>shear wave $[m/s]$</td>
<td>(1768.85, -0.00248)</td>
<td>(1648.32, -0.258)</td>
</tr>
<tr>
<td>TM wave velocity $[m/s]$</td>
<td>(827466.0, -792166.0)</td>
<td>(192248.0, -184061.0)</td>
</tr>
<tr>
<td>conductivity ($\sigma[S/m]$)</td>
<td>0.000416</td>
<td>0.00771</td>
</tr>
</tbody>
</table>

TABLE II b. The calculated (at source center frequency) wavefield velocities and bulk conductivities in the surface seismic and VESP geometry models. Both real and imaginary parts of the calculated wavefield velocities are listed.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Salinity II layer</th>
<th>Salinity III halfspace</th>
</tr>
</thead>
<tbody>
<tr>
<td>fast wave velocity $[m/s]$</td>
<td>(2502.21, -0.50)</td>
<td>(2502.21, -0.0524)</td>
</tr>
<tr>
<td>slow wave velocity $[m/s]$</td>
<td>(39.63, -37.26)</td>
<td>(40.54, -38.17)</td>
</tr>
<tr>
<td>shear wave $[m/s]$</td>
<td>(1648.32, -0.24)</td>
<td>(1648.32, -0.258)</td>
</tr>
<tr>
<td>TM wave velocity $[m/s]$</td>
<td>(18815.4, -17977.0)</td>
<td>(60834.4, -18244.3)</td>
</tr>
<tr>
<td>conductivity ($\sigma[S/m]$)</td>
<td>0.077</td>
<td>0.77</td>
</tr>
</tbody>
</table>
Surface ElectroSeismic Geometry

Figure 5-1: An explosive and vertical point source 100 m above an interface in a surface Electroseismic geometry setting.
Figure 5-2: The mechanical displacement component seismograms and the TM mode component electroseismograms, including direct waves, calculated for an explosive point source above an one interface model.
Figure 5-3: The mechanical displacement component seismograms and the $TM$ mode component electroseismograms, including direct waves, calculated for a vertical point source above an one interface model.
Figure 5-4: The mechanical displacement component seismograms and the $TM$ mode component electroseismograms, without direct waves calculated for an explosive point source above an $\epsilon \tau e$ interface model.
Figure 5-5: Radial electric field radiation patterns (direct and reflected) for a vertical electric streaming current source induced by a vertical point source above a contrast. The length of the vectors are a measure of the field amplitudes.
Figure 5-6: The mechanical displacement component seismograms and the $TM$ mode component electroseismograms, without direct waves, calculated for a vertical point source above an one interface model.
Figure 5-7: An explosive and vertical point source in a Vertical ElectroSeismic Profiling geometry (reservoir sand saturated with one fluid type).
Figure 5-8: The mechanical displacement component seismogram and the $TM$ mode component electroseismograms calculated for an explosive point source including direct waves in a Vertical ElectroSeismic Profiling geometry.
Figure 5-9: The mechanical displacement component seismogram and the $TM$ mode component electroseismograms calculated for a vertical point source including direct waves in a Vertical ElectroSeismic Profiling geometry.
Figure 5-10: The mechanical displacement component scismogram and the $TM$ mode component electroseismograms calculated for an explosive point source without direct waves in a Vertical ElectroSeismic Profiling geometry.
Figure 5-11: The mechanical displacement component seismogram and the TM mode component electroseismograms calculated for a vertical point source without direct waves in a Vertical ElectroSeismic Profiling geometry.
Vertical ElectroSeismic Profiling (VESP)

Figure 5-12: An explosive and vertical point source in a Vertical ElectroSeismic Profiling geometry (reservoir sand saturated with two fluid types).
Figure 5-13: The mechanical displacement component seismogram and the $TM$ mode component electroseismograms calculated for an explosive point source without direct waves in a Vertical ElectroSeismic Profiling geometry including a additional electrical contrast in the sand reservoir.
Figure 5-14: The mechanical displacement component seismogram and the $TM$ mode component electroseismograms calculated for a vertical point source without direct waves in a Vertical ElectroSeismic Profiling geometry including an additional electrical contrast in the sand reservoir.
Chapter 6

Modeling of SeismoElectric effects from a Borehole Source in a Layered medium.

6.1 Introduction

Many rocks in the upper crust have porosity and are characterized as porous, permeable media. The determination of fluid-flow properties as well as fluid properties of sedimentary rocks is of great importance in groundwater hydrology studies, environmental problems and the evaluation of hydraulic conductivity and fluid contrasts inside reservoirs in the petroleum industry. Full waveform acoustic logging offers an effective tool for characterizing permeable sand-shale sequences found in sedimentary formations. The stratigraphic permeable traps are important in the exploration and production of hydrocarbons.

The theoretical foundations of the study of acoustic wave propagation in a cylin-
drical borehole was laid by Biot (1952), who presented a derivation of the period equation for borehole guided waves and their dispersion characteristics. The current technique to model borehole acoustic wave propagation through heterogeneous formation is the finite difference method (Stephen et al., 1985; Zhu and McMechan, 1991; Hassanzadeh, 1991; Cheng, 1994). The numerical difficulty of using explicit time marching \( FD \), to model wave phenomena through porous media is the dispersive nature of Biot’s (1956) slow wave, and the coupling of this wave with the fast and shear waves. The wavenumber intergration technique has been applied to calculate wave propagation in homogeneous porous formations. Rosenbaum (1974) investigated the effect of formation permeability, using Biot theory, at high frequencies, combined with open pore boundary conditions proposed by Deresiewicz and Skalak (1963) to model wave propagation effects of a homogeneous permeable formation surrounding a borehole. Schmitt et al. (1988) investigated the case of a borehole surrounded by multiple radially layered isotropic poro-elastic layers and modified later his theory to the more complicated case of a borehole surrounded by a radially transversely isotropic layered poro-elastic medium (Schmitt, 1989).

In this chapter we generalize the Biot-Rosenbaum model with two modifications. We apply a \( BEM \) modeling technique to simulate wave propagation inside the borehole and outside the borehole in a heterogeneous poro-elastic formation. At the mechanical and/or electrical boundaries in the formation, the conversions from mechanical into electromagnetical waves are included. Before we outline the method to simulate the propagation of seismic and converted electromagnetic waves generated by a mechanical borehole source embedded in a layered poro-elastic medium, we want to discuss the importance of measuring the acoustic fields and electromagnetic fields inside the borehole with respect to their dependence on permeability and fluid chemistry.

The fundamental mode of a guided wave traveling in a fluid column, the Stoneley wave, is an interface wave, and therefore sensitive to formation properties, such as
density, moduli and, most importantly, permeability. Field experiments clearly indicate the effect of formation permeability on borehole Stoneley waves (Williams et al., 1984). Theoretical models were developed to study the correlation between permeability and Stoneley wave propagation. White (1983) and Hsui and Toksöz (1986) developed low frequency models of Stoneley wave propagation. Rosenbaum (1974) investigated the effects of permeability on high frequency borehole acoustic waves. Burns and Cheng (1986) went a step further and inverted for in-situ permeability from Stoneley wave velocity and attenuation using the Biot-Rosenbaum model. Winkler et al. (1989) performed laboratory model experiments on borehole Stoneley wave propagation to evaluate the applicability of the Biot theory to logging in porous formations. The theory was found to be in agreement with the experiments.

Others have tried to obtain fracture information from Stoneley wave measurements. Hornby et al. (1989) and Tang (1990) model both attenuation and reflection of Stoneley waves from a single plane fracture. They conclude that it takes a rather large fracture aperture, of the order of centimeters, to get a significant Stoneley wave attenuation. Paillet et al. (1989) suggest that in-situ fractures may consist of an array of flow passages or fracture layers instead of a single fluid layer. This suggests to model fractures as a permeable zone in the formation. Key parameters used to characterize the permeable zone are thickness of the zone, permeability, porosity and tortuosity. Since the last three parameters are typical parameters to describe a porous model we can use Biot-Rosenbaum theory (Tang and Cheng, 1993) to model Stoneley wave characteristics in the permeable zone. Tang et al. (1991) developed a simplified model for Stoneley wave propagation in permeable formation, which is consistent with the Biot-Rosenbaum theory in the presence of a hard formation.

The approach we present in this chapter relies on the important conclusion that fractured zones can be characterized using Biot's theory. The model has no real frequency constraints, as long as we are in the seismic/acoustic logging regime and is applicable to heterogeneous porous formations. Winkler et al. (1989) predict in their
theoretical calculations that in a $k = 10^{-14} m^2$ rock, the effect of permeability, is to decrease the Stoneley velocity by less than 1 percent which is less than the uncertainty in Stoneley velocity measurements. Therefore, they conclude that a $k = 10^{-14} m^2$ permeability is probably a lower limit to what can be expected to be measured in the field using Stoneley velocities.

Additional information about very low permeability zones might be obtained using the converted electric fields at high permeability contrasts. The amplitude of converted electromagnetic field is sensitive to large permeability contrasts of the permeable sand impermeable shale type. The permeability of the formation can play an important role in the determination of the formation’s electrical conductivity. The electrical conductivity increases monotonically with increasing porosity since the amount of bulk electrolyte increases linearly with porosity, see chapter 2. This increase in conductivity is not linear with porosity due to the porosity dependence of the pore length parameter, which influences the conductivity enhancement due to surface conductances. The electrical conductivity decreases with increasing permeability. At very small permeabilities the surface conductances become important and dominate in value the conductivity contribution of the bulk conductivity in the fluid phase, see chapter 2. Physically this situation occurs when the pore space is filled with clay, reducing the permeability of the formation, but not affecting the porosity, thus greatly enhancing the conductances along the pore surfaces, see chapter 2. When mechanical waves traverse such a permeability contrast the induced electric fields at both sides of the interface experience a different electrical conductivity and cause a dynamic current imbalance which causes electromagnetic radiation.

Electroseismic conversions are also sensitive to fluid chemistry contrasts inside a homogeneous reservoir. The Stoneley waves in the borehole won’t be affected by the different fluids, provided that densities and viscosities of the two fluids are approximately the same. The body waves in the surrounding poro-elastic formation are also not affected since the different saturating fluids cause only a negligible mechanical
6.2 Strategy to Simulate Coupled Acoustic-Elastic and Electromagnetic Waves in Borehole Geometry

The BEM modeling technique (general discussion on different types of Boundary Element Modeling implementations (Brebia and Dominguez, 1989)) is formulated to solve a vertical waveguide (borehole) which penetrates a horizontal flat layered poroelastic medium. We follow closely the boundary element based modeling technique proposed by Bouchon (1993). He used the method to solve the wavefield problem in an infinite open borehole and in the surrounding isotropic layered elastic formation. Dong (1993) added two extensions to Bouchon's modeling approach, he generalized the formation to include transversely isotropic layers and he included casing in his formulation.

In our formulation at the layer boundaries, mechanical waves are partly converted into electromagnetic fields, satisfying all mechanical and electromagnetical boundary conditions. The conversion to electromagnetic fields induced by the mechanical tube waves is also included. But these electromagnetic fields don't satisfy the EM boundary conditions at the borehole wall.

We first describe the BEM implementation. Since 4 boundary conditions have to be complied with at the borehole wall, 4 fictitious ring sources need to be introduced. The $P$ wave radiation in the borehole is uniquely described by a fluid volume injection ring source. The $P_f - SV - P_s$ wave radiation in the poro-elastic formation are determined by the vertical, radial ring sources applied to the frame and by an explosive ring source on the fluid phase respectively. The field vector components
for each ring source are obtained by integrating the appropriate Green's functions directly over a ring. The poro-elastic Green's functions are given in appendix C and derived in chapter 5. The final fields are obtained by surface integrating the dynamic Green's functions. The integrals are first reduced to horizontal wavenumber integrals and solved by discrete wavenumber summation (Bouchon and Aki, 1977). BEM modeling requires the calculation of a square boundary condition matrix containing the wavenumber interaction results between all elements. To calculate the element's self interaction, the singular properties of the Green's functions need to be determined. The stress field for the dynamic Green's functions with a force on the solid phase is singular, while the displacement for the dynamic Green's functions with a force on the fluid is singular. The contribution of the stress and displacement singularities, when source and receiver coincide, are determined analytically by integration of the static Green's functions over a half spherical surface.

The last part of this chapter discusses the inclusion of horizontal layers coupled to the borehole using the BEM modeling technique. At the horizontal interfaces, the electroseismic equations, see chapter 2, added with boundary conditions are solved numerically. Biot's equations with an induced body force on Biot's equation of relative flow, and Maxwell's equations with an induced current source, where all the mechanical to electromagnetic coupling is present in the transport equations, are solved simultaneously. A mechanical poro-elastic displacement/stress and coupled electromagnetic wavefield component vector as a function of depth can be determined, which equals the so called system matrix, containing all medium properties and electrokinetic and osmotic coupling, times the same displacement/stress/EM wavefield component vector. The eigenvalues (slownesses) and corresponding up and down going fast, slow, SV and TM polarized wavefield eigenvectors of the coupled electromagnetic and acoustic-elastic system matrix are derived in chapters 3 and 4.

We use the global matrix method (Chin et al., 1984) to solve simultaneously the macroscopic electromagnetic and acoustic-elastic wavefields in layered media. We
derive source jump representations for the introduced ring sources using the relative flow and displacement Green's functions together with the deformation equations in both phases. At the borehole wall, only the mechanical waves are matched. The effect of the borehole on the electromagnetic fields is assumed to be insignificant since the electromagnetic wavelengths are for typical circumstances (100 Hz seismic center frequency in the numerical modeling) on the order of a kilometer, while the borehole radius has typically a diameter of 20 cm.

In this chapter we will discuss the numerical results of the derived vertical, radial and volume injection ring sources in layered porous media. These calculation routines constitute the kernel of the BEM algorithm. The sum of the weighted ring sources determines the wavefields radiated into the formation. The weighting of each source is determined by solving a linear system of equations that incorporate the mechanical boundary conditions on the borehole wall. The calculation of all element interactions for all wavenumbers for all frequencies is numerically feasible on a massive parallel computer (the BEM algorithm is implemented on the nCUBE) but its long runtime doesn't allow practical wave propagation study. Therefore we have chosen to present the numerical comparison of the three ring sources in poro-elastic layered media. The electromagnetic radiation induced by the mechanical vertical and radial ring sources is discussed. The conversion behavior to electromagnetic disturbances generated by $P$ and $SV$ waves is studied for the three different ring sources.

### 6.3 BEM Implementation

To solve the mechanical boundary conditions at the borehole wall, we represent the borehole wall as a distribution of secondary sources. The borehole is discretized into small cylindrical elements and each element is represented by four sources: a volume source generating wavefields diffracted into the fluid, $V_i^f$, a vertical, $F_i^v$, and radial,
$F_i^r$, ring source acting on the elastic frame and a volume injection ring source, $F_i^v$, acting on the pore fluid. The last three sources give rise to the poro-elastic wavefields radiated into the formation. The density of the fictitious sources' wavefields are assumed to be constant on each element. To calculate an element's self interaction, the singular properties of the Green's functions need to be determined. The stress field for the dynamic Green's functions with a force on the elastic frame is singular, while the displacement/relative flow for the dynamic Green's functions with a force on the pore fluid is singular. The contribution of the stress and displacement/relative flow singularities are determined analytically by integration of the static Green's functions over a half spherical surface when source and receiver coincide. The strength of each source is obtained by solving a linear system of equations that incorporate the mechanical boundary conditions at the borehole wall.

6.4 Boundary Condition Matrix

Within each element the fluid - porous boundary conditions have to be satisfied (Deresiewicz and Skalak, 1963; Lovera, 1987; La Cruz and Spanos, 1989a).

- Normal displacement in the fluid matches the net normal displacement in the poro-elastic medium.

- The stress in the fluid matches the bulk normal stress in the poro-elastic medium.

- The stress in the fluid matches the stress in the fluid phase in the poro-elastic medium.

- The poro-elastic tangential bulk stress vanishes.

\[ [u_r + w_r]_{r=r_0^+} - u_r|_{r=r_0^-} = 0 \]
\[
\begin{align*}
\tau_{rr}|_{r=r_0^+} - \tau_{rr}|_{r=r_0^-} &= 0 \\
S|_{r=r_0^+} - \tau_{rr}|_{r=r_0^-} &= 0 \\
\tau_{rz}|_{r=r_0^+} &= 0
\end{align*}
\] (6.1)

The boundary conditions are satisfied at the center of each element. The displacements, stresses and pressure at the center of each element are due to the four fictitious sources on all elements. The indirect formulation is used to calculate the displacement at the j-th element due to a source at the i-th element. At the j-th element, the boundary conditions become,

\[
\begin{align*}
\sum_{i=1}^{N_e} A_{ji}^V V_i^l + \sum_{i=1}^{N_e} A_{ji}^v F_i^v + \sum_{i=1}^{N_e} A_{ji}^r F_i^r + \sum_{i=1}^{N_e} A_{ji}^e F_i^e &= E_j^v \\
\sum_{i=1}^{N_e} B_{ji}^V V_i^l + \sum_{i=1}^{N_e} B_{ji}^v F_i^v + \sum_{i=1}^{N_e} B_{ji}^r F_i^r + \sum_{i=1}^{N_e} B_{ji}^e F_i^e &= E_j^{rr} \\
\sum_{i=1}^{N_e} C_{ji}^V V_i^l + \sum_{i=1}^{N_e} C_{ji}^v F_i^v + \sum_{i=1}^{N_e} C_{ji}^r F_i^r + \sum_{i=1}^{N_e} C_{ji}^e F_i^e &= E_j^S \\
\sum_{i=1}^{N_e} D_{ji}^V V_i^l + \sum_{i=1}^{N_e} D_{ji}^v F_i^v + \sum_{i=1}^{N_e} D_{ji}^r F_i^r + \sum_{i=1}^{N_e} D_{ji}^e F_i^e &= E_j^S
\end{align*}
\] (6.2)

Where \(A_{ji}^V, A_{ji}^v\) and \(A_{ji}^r\) represent displacements at the j-th element due to the volume, vertical, radial and explosive ring sources of unit strength at the i-th element respectively. They are surface integrals of the appropriate Green's functions over the surfaces of the borehole. The \(B\) and \(C\)'s are the radial stresses and the stresses in the fluid phase of the poro-elastic formation at the j-th element due to sources at the i-th element. The \(D\)'s are the tangential stresses at the j-th element due to sources at the i-th element. They are the surface integrals of the appropriate stress Green's functions. The \(E\)'s are the source fields excited at the j-th element.

The number of elements depends on the time window and the fastest wave speed. the element heights \(dz\) satisfy,

\[
dz \leq \frac{\min(c, \beta, \alpha)}{3f}
\] (6.3)

205
The number of elements, $N_e$ is given by,

$$N_e = \frac{t_{\text{max}} \times \max(c, \beta, \alpha)}{dz} = \frac{3f \times t_{\text{max}} \times \max(c, \beta, \alpha)}{\min(c, \beta, \alpha)} \quad (6.4)$$

With $j$ ranging from 1 to $N_e$, $4 \times N_e$ equations are obtained that need to be solved for $4 \times N_e$ unknowns. The fields inside and outside the borehole can be calculated knowing these fictitious sources. The surface integration of the Green's functions is accomplished in two steps. The integrals are first transformed into wavenumber integrals, where the Sommerfeld integral representation for the function $\frac{e^{-ikR}}{4\pi R}$ is used. The wavenumber integration is then evaluated by the discrete wavenumber method. The scalar Green's functions $g_i$ with $i = \beta \lor \alpha_1 \lor \alpha_2 \lor f$, are expressed in a horizontal wavenumber integral using,

$$g_i = \frac{-i}{4\pi} \int_0^\infty \frac{k}{\nu_i} J_0(kD)e^{-i\nu_i|z-z'|}dk \quad (6.5)$$

Where $\nu_i = \sqrt{k_i^2 - k^2}$ and $D = \sqrt{r^2 + r_0^2 - 2rr_0 \cos(\phi - \phi_0)}$. The spherical wavefront is now represented by infinitely many cylindrical wavefronts. Using the addition theorem for the zeroth order Bessel function (Watson, 1966)

$$J_0(kD) = \sum_{m=0}^{\infty} \epsilon_m J_m(kr)J_m(kr_0) \cos m(\phi - \phi_0), \quad (6.6)$$

one obtains

$$g_i = \frac{-i}{4\pi} \sum_{m=0}^{\infty} \epsilon_m \cos m(\phi - \phi_0) \int_0^\infty \frac{k}{\nu_i} J_m(kr)J_m(kr_0)e^{-i\nu_i|z-z'|}dk \quad (6.7)$$

Where $\epsilon_m = 2 - \delta_{m0}$, and $J_m$ is the $m$-th order Bessel function of the first kind. The integrand of the above wavenumber integral simplifies when the surface integration $r_0d\phi_0dz$ is performed, due to the orthogonality of the sine and cosine functions over the integration range 0 to 2 $\pi$. The final result of this integration represents the response of a circular ring source. The $dz$ part of the surface integral can be carried out analytically through the integration of the ring source results over $z'$. Since the integral over element height depends on horizontal wavenumbers, its evaluation
must precede the wavenumber integration. The integration over the source element coordinate $z'$ is of two types, an integral of a complex exponential and an integral of a sign function multiplied by a complex exponential function. Both integral types can be performed analytically.

\[
\int_{z_l}^{z_h} e^{-iv(z-z')}dz' = \begin{cases} 
\frac{1}{i\nu} \left( e^{-iv(z-z_h)} - e^{-iv(z-z_l)} \right) & \text{for } z > z_h; \\
\frac{1}{i\nu} \left( e^{-iv(z_h-z)} - e^{-iv(z_l-z)} \right) & \text{for } z < z_l; \\
\frac{1}{i\nu} \left( e^{-iv(z_h-z)} + e^{-iv(z-z_l)} - 2 \right) & \text{for } z_l < z < z_h. 
\end{cases} \tag{6.8}
\]

and,

\[
\int_{z_l}^{z_h} \operatorname{sgn}(z - z')e^{-iv(z-z')}dz' = \begin{cases} 
\frac{1}{i\nu} \left( e^{-iv(z-z_h)} - e^{-iv(z-z_l)} \right) & \text{for } z > z_h; \\
\frac{1}{i\nu} \left( e^{-iv(z_h-z)} - e^{-iv(z_l-z)} \right) & \text{for } z < z_l; \\
\frac{1}{i\nu} \left( e^{-iv(z_h-z)} - e^{-iv(z-z_l)} \right) & z_l < z < z_h. 
\end{cases} \tag{6.9}
\]

The element surface integration of the Green's function reduces to a horizontal wavenumber integral. The field integrals are evaluated by the discrete summation, the so called Discrete Wavenumber Method (Bouchon and Aki, 1977). The discretization of the radial wavenumber $k$ in cylindrical coordinates introduces periodicity into the source distribution. The original single source problem changes after discretization into periodic concentric sources around the original source. The periodicity of these sources or the distance between two adjacent circular sources, $L$, is related to the discretization interval of the wavenumber, $\Delta k$, by the sampling relation,

\[
L = \frac{2\pi}{\Delta k} \tag{6.10}
\]

$L$ and therefore $\Delta k$ is determined by assuming a receiver located at $x_r = (r_0, z_0)$ and a source at $x_s = (0, z_s)$ on the symmetry axes of the medium configuration. Given the time window to record radiated waves from 0 to $t_{max}$, 'pseudo' waves radiated from the periodic sources are not allowed to enter this time window. This requirement is,

\[
\sqrt{(L - r_0)^2 + (z_0 - z_s)^2} > v_{\text{fastest}}t_{\text{max}} \tag{6.11}
\]
or

\[ L > r_0 + \sqrt{v_{fastest}^2 t_{max}^2 - (z_0 - z_s)^2} \]  \hspace{1cm} (6.12)

The sampling equation becomes now,

\[ \Delta k < \frac{2\pi}{r_0 + \sqrt{v_{fastest}^2 t_{max}^2 - (z_0 - z_s)^2}} \]  \hspace{1cm} (6.13)

The above equation is the criterion for choosing the sample rate for the discrete wavenumber summation.

To perform the summation, the singularities of the integrands are moved further from the real \( k \) axes. This is done by adding a small imaginary part to the real frequency (analytic continuation into the complex plane), i.e. \( \omega = \omega_R + i\omega_I \) with \( \omega_I > 0 \). If \( e^{i\omega t} \) was used in the time to frequency transformation there would be a minus sign for the imaginary part of the frequency. The effect of complex frequency moves the singularities into the first and third quadrant of the complex \( k \)-plane. The use of complex frequency has the effect of smoothing the spectrum and enhancing the first motions relative to later arrivals. This effective attenuation is used to minimize the influence of the neighboring fictitious sources introduced by discrete \( k \). The effect of the imaginary part of the frequency can be removed from the final time domain solution by inverse complex Fourier transform with complex frequency, using the same imaginary part in the argument of the exponential function in the Fourier transform. The magnitude of the imaginary part is usually chosen to be,

\[ \omega_I = \frac{\pi}{t_{max}} \]  \hspace{1cm} (6.14)

Larger \( \omega_I \) increases the attenuation for later arrivals, but also magnifies the numerical noise for late times. If \( \omega_I \) is chosen too small, attenuation may not be large enough to damp out the arrivals from the fictitious sources.
6.5 Singularities when Source and Receiver Coincide

In boundary integral or boundary element techniques, integration of the Green’s function over boundary surfaces is necessary. When receiver and source position coincide, the Green’s function becomes singular. The displacement and relative flow Green’s function have first order singularities which are removable when integrated over the surface. The stress Green’s function has a second order singularity and the surface integral is not defined. Here the principle value of the improper integral has to be calculated. The contribution of this second order singularity to the integral can be evaluated analytically. When the receiver and the source approach each other, the surface integral of the dynamic Green’s functions can be regularized using the static Green’s function (Kupradze, 1963). The improper integral of the dynamic Green’s function is reduced to the integration of the static Green’s functions over a half spherical surface around the source/receiver point. First the displacement and stress field components are calculated using the static Green’s functions, equations (Eq C.23) and (Eq C.29) for a vertical point force.

\[ u_x = \frac{[HM - C^2 - GM]}{8\pi G [HM - C^2]} \left[ \frac{(z - z')(x - x')}{R^3} \right] \]  
\[ (6.15) \]

\[ u_y = \frac{[HM - C^2 - GM]}{8\pi G [HM - C^2]} \left[ \frac{(z - z')(y - y')}{R^3} \right] \]  
\[ (6.16) \]

\[ u_z = \frac{1}{4\pi GR} - \frac{[HM - C^2 - GM]}{8\pi G [HM - C^2]} \left[ \frac{R^2 - (z - z')}{R^3} \right] \]  
\[ (6.17) \]

Notice that the displacements have a first order singularity when \( x \to x' \). The stress in a poro-elastic medium along the \( \hat{z} \) direction on a surface with the normal in the \( \hat{x} \) direction is,

\[ \tau_{xz} = G \left[ \frac{\partial}{\partial x} u_z + \frac{\partial}{\partial z} u_x \right] \]
\[ = \frac{x - x'}{4\pi GR^3} - \frac{[HM - C^2 - GM]}{8\pi G [HM - C^2]} \left[ \frac{2R^3(x - x') - 3R(x - x')(z - z')^2}{R^6} \right] \]  
\[ (6.18) \]
The stress has a second order singularity when the source and receiver position coincide at \( R = 0 \). The stress integrated over a surface results in a finite value. The magnitude of this value can be obtained by integrating \( \tau_{xz} \) over all possible \( dydz \) surrounding the source. This integration can be replaced by integration over a half sphere around the source point. This surface is defined by \( x - x' = R \sin(\theta) \cos(\phi) \), \( y - y' = R \sin(\theta) \sin(\phi) \), \( z - z' = R \cos(\theta) \). Then the surface mapping between \( dydz \) and the differential spherical surface becomes, \( dydz \sin(\theta) \cos(\phi) = R^2 \sin(\theta) \) \( d\theta d\phi \). Integrating the stress, \( \tau_{xz} \) yields,

\[
\int \tau_{xz} dydz = \left[ \frac{-1}{4\pi} + \frac{HM - C^2 - GM}{4\pi (HM - C^2)} \right] \int_{\pi/2}^{\pi} d\phi \int_0^\pi \sin(\theta) d\theta \\
- \frac{3(HM - C^2 - GM)}{4\pi (HM - C^2)} \int_{\pi/2}^{\pi} d\phi \int_0^\pi \sin(\theta) \cos^2(\theta) d\theta = -\frac{1}{2}
\] (6.19)

For a point force in the \( \hat{x} \) direction, the displacement and relative flow components are,

\[
u_x = \frac{1}{4\pi GR} - \frac{HM - C^2 - GM}{8\pi G (HM - C^2)} \left[ \frac{R^2 - (x - x')}{R^3} \right]
\] (6.20)

\[
u_y = \frac{HM - C^2 - GM}{8\pi G (HM - C^2)} \left[ \frac{(x - x')(y - y')}{R^3} \right]
\] (6.21)

\[
u_z = \frac{HM - C^2 - GM}{8\pi G (HM - C^2)} \left[ \frac{(x - x')(z - z')}{R^3} \right]
\] (6.22)

\[
w_x = \frac{-1}{4\pi GR} + \frac{HM - C^2 - GC}{8\pi G (HM - C^2)} \left[ \frac{R^2 - (x - x')}{R^3} \right]
\] (6.23)

\[
w_y = -\frac{HM - C^2 - GC}{8\pi G (HM - C^2)} \left[ \frac{(x - x')(y - y')}{R^3} \right]
\] (6.24)

\[
w_z = -\frac{HM - C^2 - GC}{8\pi G (HM - C^2)} \left[ \frac{(x - x')(z - z')}{R^3} \right]
\] (6.25)

The normal stress is given by,

\[
\tau_{xz} = H \frac{\partial}{\partial x} u_x + (H - 2G) \frac{\partial}{\partial y} u_y + (H - 2G) \frac{\partial}{\partial z} u_z + C \left( \frac{\partial}{\partial x} w_x + \frac{\partial}{\partial y} w_y + \frac{\partial}{\partial z} w_z \right)
\] (6.26)

210
Integrating the stress, $\tau_{xx}$ yields,

$$
\int \tau_{xx} dydz = \left[ -\frac{1}{4\pi G} + \frac{3H(HM - C^2 - GM)}{8\pi G(HM - C^2)} + \frac{2(H - 2G)(HM - C^2 - GM)}{8\pi G(HM - C^2)} \right] \int_{\pi/2}^{\pi} d\phi \int_0^\pi \sin(\theta) d\theta \\
+ \left[ \frac{C}{4\pi G} - \frac{5C(HM - C^2 - GC)}{8\pi G(HM - C^2)} \right] \int_{\pi/2}^{\pi} d\phi \int_0^\pi \sin(\theta) d\theta \\
+ \left[ \frac{-3H(H - 2G)(HM - C^2 - GM)}{8\pi G(HM - C^2)} + \frac{3C(HM - C^2 - GC)}{8\pi G(HM - C^2)} \right] \\
\times \int_{-\pi/2}^{\pi/2} \int_0^\pi \left[ \sin^3(\theta) \sin^2(\phi) + \sin^2(\theta) \cos(\theta) \right] d\phi d\theta \\
+ \left[ \frac{3H(HM - C^2 - GM)}{8\pi G(HM - C^2)} + \frac{3C(HM - C^2 - GC)}{8\pi G(HM - C^2)} \right] \\
\times \int_{-\pi/2}^{\pi/2} \int_0^\pi \sin^3(\theta) \cos^2(\phi) d\phi d\theta = \frac{1}{2}
$$

(6.27)

The point source potential and pressure, describing the acoustic behavior in the fluid phase of the poro-elastic solid, have first order singularities when source and receiver coincide. These singularities however are removable, integration of potential and pressure over an infinitesimal surface around $x = x'$ approaches zero when the surface shrinks to a point. The point displacement has a second order singularity, since the gradient of the Green’s function is taken. Integration of the displacement over a spherical surface centered at $x = x'$ approaches a constant as the radius of the sphere shrinks to zero.

Using equations (Eq C.16) and (Eq C.17), the $x$ component of the relative flow caused by a pressure source can be obtained. Integrating the relative flow, $w_x$, over all possible $dydz$ surrounding the source yields,

$$
\int w_x dydz = \lim_{R \to 0} \left[ \int_{-\pi/2}^{\pi/2} d\phi \int_0^\pi \left( \Lambda_1(ik_{a1}R - 1) \frac{ik_{a1}R}{4\pi} \right) \Lambda_2(ik_{a2}R - 1) \frac{ik_{a2}R}{4\pi} \right] d\theta \\
+ \Lambda_2(ik_{a2}R - 1) \frac{ik_{a2}R}{4\pi} \sin(\theta) \right] d\theta \\
= \frac{1}{2} \left[ \Lambda_1 + \Lambda_2 \right] = \frac{1}{2} \frac{H}{HM - C^2}
$$

(6.29)

The $x$ component of the bulk displacement caused by a pressure source is obtained by taking the divergence of equation (Eq C.29). Integrating the displacement, $u_x$,
over all possible $dydz$ surrounding the source yields,

$$\int u_x dydz = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi \int_0^\pi \left( \frac{C}{4\pi [HM - C^2]} \sin(\theta) \right) d\theta = -\frac{1}{2} \frac{C}{HM - C^2}$$  \hspace{1cm} (6.30)

To obtain the jump in the fluid displacement due to a \textbf{volume injection} source, the displacement potential $\Phi$ [dim $m^2$] will be introduced. The fluid displacement is related to the displacement potential as follows,

$$u^{(f)} = \nabla \Phi = C\underline{u} + M\underline{w}$$  \hspace{1cm} (6.31)

$$\int u_x^{(f)} dydz = C \int u_x dydz + M \int w_x dydz = \frac{1}{2}$$  \hspace{1cm} (6.32)

\section*{6.6 Ring Source Results in Poro-Elastic Media}

In this section the displacements, relative flows and stresses and pressures of the different ring sources are obtained by integrating the appropriate Green's functions directly over a ring.

\subsection*{6.6.1 Vertical Ring Source}

For a vertical ring source located at $(r_0, z')$, $F^v(x) = \delta(z - z')\delta(r - r_0)$. The integration over the angle $\phi_0$ is non zero only for $m = 0$, due to the orthogonality of the set \{1, $\cos \phi$, $\cos 2\phi$,\ldots\}. Substituting equation (Eq C.2) into displacement Green's function (Eq C.1) and using equation (Eq 6.7) to rewrite the scalar Green's functions the displacements at point $(r, z)$ due to a ring of unit vertical forces at $x_0 = (r_0, z')$ are obtained. When the relative flow Green's function in equation (Eq C.7) is used instead, the relative flows at point $(r, z)$ due to a ring of unit vertical forces at $x_0 = (r_0, z')$ are obtained.

$$u_r = -\text{sgn}(z - z') \frac{r_0}{2} \int_0^\infty J_1(kr)J_0(kr_0)$$
\[
\times \left[ U_{\beta e^{-i\omega t}} z - z' \right] + U_{a_1 e^{-i\omega t}} z - z' \right] + U_{a_2 e^{-i\omega t}} z - z' \right] dk
\]

\[ u_z = -i \frac{r_0}{2} \int_0^\infty k J_0(kr) J_0(kr_0) \]

\[
\times \left[ U_{\beta e^{-i\omega t}} z - z' \right] + U_{a_1 e^{-i\omega t}} z - z' \right] + U_{a_2 e^{-i\omega t}} z - z' \right] dk
\]

\[ w_r = -\text{sgn}(z-z') \frac{r_0}{2} \int_0^\infty J_1(kr) J_0(kr_0) \]

\[
\times \left[ W_{\beta e^{-i\omega t}} z - z' \right] + W_{a_1 e^{-i\omega t}} z - z' \right] + W_{a_2 e^{-i\omega t}} z - z' \right] dk
\]

\[ w_z = -i \frac{r_0}{2} \int_0^\infty k J_0(kr) J_0(kr_0) \]

\[
\times \left[ W_{\beta e^{-i\omega t}} z - z' \right] + W_{a_1 e^{-i\omega t}} z - z' \right] + W_{a_2 e^{-i\omega t}} z - z' \right] dk
\]

Substituting equations (Eq 6.33) - (Eq 6.36) into the stress equation in cylindrical coordinates and incorporating the static contribution to \( \tau_{rz} \), when source and receiver position coincide, the stresses are obtained at \( x = (r, z) \).

\[ \tau_{rr} = -\text{sgn}(z-z') \frac{r_0}{2} \int_0^\infty J_0(kr_0) I_1 dk \]

\[ I_1 = \sum_{p=\beta,a_1,a_2} \left[ (Hk^p + CkW^p + (H-2G)\nu_p kU^p + C\nu_p kW^p) J_0(kr) \right] - 2Gk^p J_1(kr) e^{-i\omega t} \]

\[ \tau_{rz} = -\frac{1}{2} \delta(z-z') + iG \frac{r_0}{2} \int_0^\infty k J_1(kr) J_0(kr_0) I_2 dk \]

\[ I_2 = 2k \left[ U^z_{\beta} - \frac{1}{G \nu_p} e^{-i\omega t} z - z' \right] + 2kU^z_{a_1} e^{-i\omega t} z - z' \right] + 2kU^z_{a_2} e^{-i\omega t} z - z' \right] \]

\[ \tau_{zz} = -\text{sgn}(z-z') \frac{r_0}{2} \int_0^\infty J_0(kr) J_0(kr_0) I_3 dk \]

\[ I_3 = \sum_{p=\beta,a_1,a_2} \left[ \left( (H-2G)k^p + CkW^p + H\nu_p kU^p + C\nu_p kW^p \right) \right] e^{-i\omega t} z - z' \]

\[ S = -\text{sgn}(z-z') \frac{r_0}{2} \int_0^\infty J_0(kr) J_0(kr_0) I_4 dk \]

\[ I_4 = \sum_{p=\beta,a_1,a_2} \left[ (Ck^p + \nu_p kU^p + MkW^p) \right] e^{-i\omega t} z - z' \]

Where the introduced variables, \( W^p_{\beta}, W^p_{a_1}, W^p_{a_2}, U^p_{\beta}, U^p_{a_1}, U^p_{a_2}, W^z_{a_1}, W^z_{a_2}, U^z_{a_1}, U^z_{a_2} \) are defined as,

\[ W^p_{\beta} = [\Pi B^*(ws) - \Delta B(ws)] k^2, \quad U^p_{\beta} = [\Xi B^*(ws) - \Gamma B(ws)] k^2 \]

213
\[ W_{\alpha_1}^r = [\Pi A_1^*(ws) - \Delta A_1(ws)] k^2, \quad U_{\alpha_1}^r = [\Xi A_1^*(us) - \Gamma A_1(us)] k^2 \quad (6.42) \]
\[ W_{\alpha_2}^r = [\Pi A_2^*(ws) - \Delta A_2(ws)] k^2, \quad U_{\alpha_2}^r = [\Xi A_2^*(us) - \Gamma A_2(us)] k^2 \quad (6.43) \]
\[ W_{\beta}^z = -\frac{\rho_f}{\rho E} \frac{1}{G \nu_\beta} - \Delta B(ws) \nu_\beta + \Pi B^*(w_2) \nu_\beta, \quad U_{\beta}^z = \frac{1}{G \nu_\beta} - \Gamma B(ws) \nu_\beta + \Xi B^*(w_2) \quad (6.44) \]
\[ W_{\alpha_1}^z = -\Delta A_1(ws) \nu_{\alpha_1} + \Pi A_1^*(ws) \nu_{\alpha_1}, \quad U_{\alpha_1}^z = -\Gamma A_1(ws) \nu_{\alpha_1} + \Xi A_1^*(ws) \nu_{\alpha_1} \quad (6.45) \]
\[ W_{\alpha_2}^z = -\Delta A_2(ws) \nu_{\alpha_2} + \Pi A_2^*(ws) \nu_{\alpha_2}, \quad U_{\alpha_2}^z = -\Gamma A_2(ws) \nu_{\alpha_2} + \Xi A_2^*(ws) \nu_{\alpha_2} \quad (6.46) \]

Where variables \( A_q(ws), A_q^*(ws), B_q(ws), B_q^*(ws) \) are defined in equations (Eq C.9) and (Eq C.11) and variables \( A_q(us), A_q^*(us), B_q(us), B_q^*(us) \) are defined in equations (Eq C.3) and (Eq C.5), with \( q = 1 \lor 2 \).

With,
\[ \Gamma = G - \frac{1}{M} [HM - C^2] \quad (6.47) \]
\[ \Xi = -2 \omega^2 \rho_f C + M \omega^2 \frac{\rho_f^2}{\rho E} + \frac{C^2}{M \omega^2 \rho_E} \quad (6.48) \]
\[ \Delta = \frac{1}{C} [HM - C^2] - G \frac{\rho E}{\rho_f} \quad (6.49) \]
\[ \Pi = \frac{HM}{C} \omega^2 \rho_f + C \omega^2 \frac{\rho B \rho E}{\rho_f} - H \omega^2 \rho_E - M \omega^2 \rho_B \quad (6.50) \]

### 6.6.2 Radial Ring Source

For a radial ring source \( F^r(x) = \left[ \hat{\phi} \cos(\phi - \phi_0) - \hat{\phi} \sin(\phi - \phi_0) \right] \delta(r - r_0) \delta(z - z'). \)

Using the displacement Green's function, (Eq C.1), and the scalar Green's functions, equations (Eq C.2) and (Eq C.6), which are rewritten using equation (Eq 6.7) and the orthogonality property in the integration over angle \( \phi_0 \), the displacements at \((r, z)\) due to a ring of unit radial forces at \( x' = (r_0, z') \) are obtained. When the relative flow Green's function, equation (Eq C.7), and the scalar Green's functions, equations (Eq C.8) and (Eq C.12), are used instead the relative flow vector components \((r, z)\)
due to a ring of unit radial forces at \( x' = (r_0, z') \) are obtained.

\[
\begin{align*}
  u_r &= i \frac{r_0}{2} \int_0^\infty J_1(kr)J_1(kr_0) \\
  &\quad \times \left[ U_\beta e^{-i\nu_p |z-z'|} + U_\alpha e^{-i\nu_\alpha_1 |z-z'|} + U_\alpha e^{-i\nu_\alpha_2 |z-z'|} \right] dk \quad (6.51) \\
  u_z &= \text{sgn}(z-z') \frac{r_0}{2} \int_0^\infty k J_0(kr)J_1(kr_0) \\
  &\quad \times \left[ U_\beta e^{-i\nu_p |z-z'|} + U_\alpha e^{-i\nu_\alpha_1 |z-z'|} + U_\alpha e^{-i\nu_\alpha_2 |z-z'|} \right] dk \quad (6.52) \\
  w_r &= i \frac{r_0}{2} \int_0^\infty J_1(kr)J_1(kr_0) \\
  &\quad \times \left[ W_\beta e^{-i\nu_p |z-z'|} + W_\alpha e^{-i\nu_\alpha_1 |z-z'|} + W_\alpha e^{-i\nu_\alpha_2 |z-z'|} \right] dk \quad (6.53) \\
  w_z &= \text{sgn}(z-z') \frac{r_0}{2} \int_0^\infty k J_0(kr)J_1(kr_0) \\
  &\quad \times \left[ W_\beta e^{-i\nu_p |z-z'|} + W_\alpha e^{-i\nu_\alpha_1 |z-z'|} + W_\alpha e^{-i\nu_\alpha_2 |z-z'|} \right] dk \quad (6.54)
\end{align*}
\]

Substituting equations (Eq 6.51) - (Eq 6.54) into the stress equation in cylindrical coordinates and incorporating the static contribution to \( \tau_{rr} \), when source and receiver position coincide, the stresses are obtained at \( x = (r, z) \). When equations (Eq 6.51) - (Eq 6.54) are substituted into the pressure equation in cylindrical coordinates, the fluid traction is obtained at \( x = (r, z) \).

\[
\begin{align*}
  \tau_{rr} &= -\frac{1}{2} \delta(z-z') + i \frac{r_0}{2} \int_0^\infty J_1(kr_0)I_1 dk \\
  I_1 &= \sum_{p=\beta,\alpha_1,\alpha_2} \left[ (HkU_p^r + CkW_p^r - (H-2G)k\nu_p U_p^z - Ck\nu_p W_p^z)J_0(kr) \\
  &\quad - 2G U_p^r \frac{J_1(kr)}{r} e^{-i\nu_p |z-z'|} \right] \\
  \tau_{rz} &= \text{sgn}(z-z') \frac{r_0}{2} \int_0^\infty J_1(kr)J_1(kr_0)I_2 dk \\
  I_2 &= G \left[ (\nu_\beta U_\beta - k^2 U_\beta) e^{-i\nu_\beta |z-z'|} \right. \\
  &\quad + (\nu_{\alpha_1} U_{\alpha_1} - k^2 U_{\alpha_1}) e^{-i\nu_{\alpha_1} |z-z'|} \\
  &\quad + (\nu_{\alpha_2} U_{\alpha_2} - k^2 U_{\alpha_2}) e^{-i\nu_{\alpha_2} |z-z'|} \right] \\
  \tau_{zz} &= i \frac{r_0}{2} \int_0^\infty J_0(kr)J_1(kr_0)I_3 dk \\
  I_3 &= \sum_{p=\beta,\alpha_1,\alpha_2} \left[ (H-2G)kU_p^r + CkW_p^r - Hk\nu_p U_p^z - Ck\nu_p W_p^z \right] e^{-i\nu_p |z-z'|} \\
  &\quad \left. e^{-i\nu_p |z-z'|} \right] (6.55)
\end{align*}
\]
\[ S = \frac{\tau_0}{2} \int_0^\infty J_0(kr)J_1(kr_0)I_4 dk \]
\[ I_4 = \sum_{i=\beta,\alpha_1,\alpha_2} \left[ C(kU_{p}^r - \nu_p kU_{p}^r) + M(kW_{p}^r - \nu_p kW_{p}^z) \right] e^{-iwp|z-z'|} \] (6.58)

Where the introduced variables, \( W_{\beta}^r, W_{\alpha_1}^r, W_{\alpha_2}^r, U_{\beta}^r, U_{\alpha_1}^r, U_{\alpha_2}^r, W_{\beta}^z, W_{\alpha_1}^z, W_{\alpha_2}^z, U_{\beta}^z, U_{\alpha_1}^z, U_{\alpha_2}^z \) are defined as,

\[ W_{\beta}^r = \frac{\rho_f}{\rho_E G \nu_\beta} k + B(ws)\Delta \frac{k^3}{\nu_\beta} - B^r(ws)\Pi \frac{k^3}{\nu_\beta}, U_{\beta}^r = -\frac{k}{\nu_\beta G} + \Gamma B(us)\frac{k^3}{\nu_\beta} - \Xi B^r(us)\frac{k^3}{\nu_\beta} \] (6.59)

\[ W_{\alpha_1}^r = A_1(ws)\Delta \frac{k^3}{\nu_{\alpha_1}} - A_1^r(ws)\Pi \frac{k^3}{\nu_{\alpha_1}}, U_{\alpha_1}^r = \Gamma A_1(us)\frac{k^3}{\nu_{\alpha_1}} - \Xi A_1^r(us)\frac{k^3}{\nu_{\alpha_1}} \] (6.60)

\[ W_{\alpha_2}^r = A_2(ws)\Delta \frac{k^3}{\nu_{\alpha_2}} - A_2^r(ws)\Pi \frac{k^3}{\nu_{\alpha_2}}, U_{\alpha_2}^r = \Gamma A_2(us)\frac{k^3}{\nu_{\alpha_2}} - \Xi A_2^r(us)\frac{k^3}{\nu_{\alpha_2}} \] (6.61)

\[ W_{\beta}^z = [\Pi B^r(ws) - \Delta B(ws)] k, \quad U_{\beta}^z = [\Xi B^r(us) - \Gamma B(us)] k \] (6.62)

\[ W_{\alpha_1}^z = [\Pi A_1^r(ws) - \Delta A_1(ws)] k, \quad U_{\alpha_1}^z = [\Xi A_1^r(us) - \Gamma A_1(us)] k \] (6.63)

\[ W_{\alpha_2}^z = [\Pi A_2^r(ws) - \Delta A_2(ws)] k, \quad U_{\alpha_2}^z = [\Xi A_2^r(us) - \Gamma A_2(us)] k \] (6.64)

Where variables \( A_q(ws), A_q^r(ws), B_q(ws), B_q^r(ws) \) are defined in equations (Eq C.9) and (Eq C.11) and variables \( A_q(us), A_q^r(us), B_q(us), B_q^r(us) \) are defined in equations (Eq C.3) and (Eq C.5), with \( q = 1 \vee 2 \).

With variables, \( \Gamma, \Xi, \Delta, \Pi \) defined in equations (Eq 6.47).

### 6.6.3 Explosive Ring Source in Porous Medium

For an explosive ring source located at \((r_0, z')\), \( F^r(x) = V\delta(z-z')\delta(r-r_0) \). Using the displacement Green’s function, (Eq C.13), and the scalar Green’s function, equation (Eq C.14), which are rewritten using equation (Eq 6.7) and the orthogonality property in the integration over angle \( \phi_0 \), the displacements at \((r, z)\) due to a ring of explosive point forces at \( x' = (r_0, z') \) are obtained. When the relative flow Green’s function, equation (Eq C.16), and the scalar Green’s function, equation (Eq C.17), are used
instead the relative flow vector components \((r, z)\) due to a ring of explosive point forces at \(z' = (r_0, z')\) are obtained.

\[
\begin{align*}
  u_r &= -\frac{1}{2 H M - C^2} \delta(z - z') \\
  &+ i \frac{r_0}{2} \int_0^\infty k^2 J_1(kr) J_0(kr_0) \left[ U_{a_1}^e e^{-i\omega_1|z-z'|} + U_{a_2}^e e^{-i\omega_2|z-z'|} \right] dk \\
  u_z &= -\text{sgn}(z-z') \frac{r_0}{2} \int_0^\infty k J_0(kr) J_0(kr_0) \\
  &\times \left[ \nu_{a_1} U_{a_1}^e e^{-i\omega_1|z-z'|} + \nu_{a_2} U_{a_2}^e e^{-i\omega_2|z-z'|} \right] dk \\
  w_r &= \frac{1}{2 H M - C^2} \delta(z - z') \\
  &+ i \frac{r_0}{2} \int_0^\infty J_1(kr) J_0(kr_0) \left[ \Lambda_1 k^2 \nu_{a_1} e^{-i\omega_1|z-z'|} + \Lambda_2 k^2 \nu_{a_2} e^{-i\omega_2|z-z'|} \right] dk \\
  w_z &= -\text{sgn}(z-z') \frac{r_0}{2} \int_0^\infty k J_0(kr) J_0(kr_0) \left[ \Lambda_1 e^{-i\omega_1|z-z'|} + \Lambda_2 e^{-i\omega_2|z-z'|} \right] dk
\end{align*}
\] (6.65-6.68)

Substituting equations (Eq 6.65) - (Eq 6.68) into the stress equation in cylindrical coordinates yields the stresses at \(x = (r, z)\). When equations (Eq 6.51) - (Eq 6.68) are substituted into the pressure equation in cylindrical coordinates, the fluid traction is obtained at \(x = (r, z)\).

\[
\begin{align*}
  \tau_{rr} &= i \frac{r_0}{2} \int_0^\infty J_0(kr_0) J_1 dk \\
  I_1 &= \sum_{p=a_1, a_2} \left[ \left( H k^2 U_p^e + (H - 2G) \nu_p^2 k U_p^e + C \Lambda_p \left( \frac{k^3}{\nu_p} + k \nu_p \right) \right) J_0(kr) \\
  &- 2G U_p^e k^2 J_1(kr) \right] e^{-i\nu_p|z-z'|} \\
  \tau_{rz} &= \text{sgn}(z-z') r_0 \int_0^\infty k J_0(kr) J_0(kr_0) I_2 dk \\
  I_2 &= \sum_{p=a_1, a_2} G \nu_p k U_p^e e^{-i\omega_1|z-z'|} \\
  \tau_{zz} &= i \frac{r_0}{2} \int_0^\infty k J_0(kr) J_0(kr_0) I_3 dk \\
  I_3 &= \left[ H \nu_{a_1}^2 U_{a_1}^e + (H - 2G) k^2 U_{a_1}^e + C \Lambda_1 \left( \frac{k^2}{\nu_{a_1}} + \nu_{a_1} \right) \right] e^{-i\omega_1|z-z'|} \\
  &+ \left[ H \nu_{a_2}^2 U_{a_2}^e + (H - 2G) k^2 U_{a_2}^e + C \Lambda_2 \left( \frac{k^2}{\nu_{a_2}} + \nu_{a_2} \right) \right] e^{-i\omega_2|z-z'|} \\
  S &= i \frac{r_0}{2} \int_0^\infty J_0(kr) J_0(kr_0) I_4 dk
\end{align*}
\] (6.69-6.71)

217
\[ I_4 = \left[ C \left( k^2 + \nu_{\alpha_1}^2 \right) kU_{\alpha_1}^c + MA_1 \left( \frac{k^3}{\nu_{\alpha_1}} + k\nu_{\alpha_1} \right) \right] e^{-i\omega_1 |z-z'|} + \left[ C \left( k^2 + \nu_{\alpha_2}^2 \right) kU_{\alpha_2}^c + MA_2 \left( \frac{k^3}{\nu_{\alpha_2}} + k\nu_{\alpha_2} \right) \right] e^{-i\omega_2 |z-z'|} \quad (6.72) \]

Where the introduced variables \( U_{\alpha_1}^c, U_{\alpha_2}^c \) are defined as,

\[ U_{\alpha_1}^c = - \left[ \gamma + \Omega \right] \frac{k^2}{\nu_{\alpha_1}} (A_1(wf) + A_1^*(wf)) \quad (6.73) \]
\[ U_{\alpha_2}^c = - \left[ \gamma + \Omega \right] \frac{k^2}{\nu_{\alpha_2}} (A_2(wf) + A_2^*(wf)) \quad (6.74) \]

Where variables \( A_q(wf), A_q^*(wf) \) are defined in equations (Eq C.18) and (Eq C.17), with \( q = 1 \lor 2 \).

With,

\[ \gamma = \frac{1}{C} \left[ C^2 - HM + GC\frac{\rho E}{\rho_f} \right] \quad (6.75) \]
\[ \Omega = \frac{HM}{C} \omega^2 \rho_f + C\omega^2 \frac{\rho B \rho E}{\rho_f} - H\omega^2 \rho_E - M\omega^2 \rho_B \quad (6.76) \]

### 6.6.4 Explosive Ring Source in Fluid

To determine an explosive ring source in the borehole fluid, the displacement potential is obtained by integrating \( g(x, x') = e^{-ik\rho|x-x'|}/(4\pi|x-x'|) \) with respect to angle \( \phi \),

\[ \phi = \frac{ir_0}{2} \int_0^{\infty} \frac{k}{\nu_f} J_0(kr)J_0(kr_0)e^{-i\omega t |z-z'|} dk \quad (6.77) \]

The fluid displacement components are obtained by taking the derivative with respect of \( r \) and \( z \) of equation (Eq 6.77).

\[ u_r = -\frac{1}{2} \delta(z - z') + \frac{ir_0}{2} \int_0^{\infty} \frac{k^2}{\nu_f} J_1(kr)J_0(kr_0)e^{-i\omega t |z-z'|} dk \quad (6.78) \]
\[ u_z = -\text{sgn}(z - z') \frac{r_0}{2} \int_0^{\infty} kJ_0(kr)J_0(kr_0)e^{-i\omega t |z-z'|} dk \quad (6.79) \]

The stresses in the fluid, which are the negations of the fluid pressure, are,

\[ \tau_{rr} = \rho\omega^2 \frac{ir_0}{2} \int_0^{\infty} \frac{k}{\nu_f} J_0(kr)J_0(kr_0)e^{-i\omega t |z-z'|} dk \quad (6.80) \]
\[ \tau_{zz} = \tau_{rr} \]  
\[ \tau_{rz} = 0 \] (6.81) (6.82)

### 6.6.5 Explosive Volume Source in an Open Borehole

The driving initial data for an explosive source at the center of a fluid filled borehole at the boundary wall are,

\[
\begin{align*}
\tau_r &= \frac{V_s}{4\pi} i \int_0^\infty \frac{k^2}{\nu_f} J_1(kr_0)e^{-i\nu_f|z-z_s|} dk \\
\tau_{rr} &= \frac{\rho\omega^2 V_s}{4\pi} i \int_0^\infty \frac{k}{\nu_f} J_0(kr_0)e^{-i\nu_f|z-z_s|} dk \\
\tau_{rz} &= 0
\end{align*}
\] (6.83) (6.84) (6.85)

Where the vertical position of the source and the element position are denoted by \( z_s \) and \( z \) respectively. The strength of the volume source \( V_s \) [dim \( m^3 \)] is taken 1600 \( cm^3 \).

### 6.7 Ring Sources in Horizontally Layered Poro-Elastic Media

To include the ring sources into layered media, we first manipulate the wavefield components that will be continuous across the horizontal boundaries into the following form:

\[
\begin{align*}
\hat{u}_1(r, z) &= \int_0^\infty \hat{u}_1(k, z)k J_0(kr) dk \\
\hat{u}_2(r, z) &= \int_0^\infty \hat{u}_2(k, z)k J_0(kr) dk \\
\hat{w}_1(r, z) &= \int_0^\infty \hat{w}_1(k, z)k J_0(kr) dk \\
\hat{\tau}_1(r, z) &= \int_0^\infty \hat{\tau}_1(k, z)k J_0(kr) dk
\end{align*}
\]
\[
\tau_{zz}(r, z) = \int_{0}^{\infty} \hat{\tau}_{zz}(k, z)kJ_{0}(kr)dk \\
S(\tau, z) = \int_{0}^{\infty} \hat{S}(k, z)kJ_{0}(kr)dk
\]  
(6.86)

With \( \hat{u}_1 = \frac{\hat{u}_V}{ik} \) and \( u_V = \frac{1}{r} \left[ \frac{\partial}{\partial r}(r u_r) + \frac{\partial}{\partial \phi}(u_\phi) \right], \hat{\tau}_1 = \frac{\tau_{zz}}{ik} \) and \( \tau_{Vz} = \frac{1}{r} \left[ \frac{\partial}{\partial r}(r \tau_{zz}) + \frac{\partial}{\partial \phi}(\tau_{\phi z}) \right] \).

The displacement stress wavefield vector for a vertical ring source is given by,

\[
\begin{bmatrix}
  u_1, \ u_z, \ w_z, \ \tau_1, \ \tau_{zz}, \ S
\end{bmatrix}_v^T = \sum_{p=\beta,\alpha_1,\alpha_2} \left( \begin{bmatrix}
  \frac{1}{ik} \left( -\text{sgn}(z-z') U_p^r \right), -iU_p^z, -iW_p^z, \frac{1}{ik} \left( iGS_1^p \right), \\
  -\text{sgn}(z-z') S_{2}^p, -\text{sgn}(z-z') S_{3}^p \end{bmatrix} \frac{r_0}{2} J_0(kr_0) e^{-i\nu_p|z-z'|} \right)^T
\]
(6.87)

With variables, \( W_{\beta}, W_{\alpha_1}, W_{\alpha_2}, U_{\beta}, U_{\alpha_1}, U_{\alpha_2}, W_{\beta}, W_{\alpha_1}, W_{\alpha_2}, U_{\beta}, U_{\alpha_1}, U_{\alpha_2} \) defined in (Eq 6.41) and (Eq 6.44). Variables \( S_1^p, S_2^p, S_3^p \) with \( p = \beta \lor \alpha_1 \lor \alpha_2 \) are defined as,

\[
\begin{align*}
S_1^\beta &= 2k^2 \left( U_p^\beta - \frac{1}{G\nu_p} \right), \quad S_1^{\alpha_1} = 2k^2 U_{\alpha_1}^z, \quad S_1^{\alpha_2} = 2k^2 U_{\alpha_2}^z \\
S_2^p &= (H - 2G)U_p^p + CW_p^r + H\nu_p U_p^z + C\nu_p W_p^z \\
S_3^p &= C \left( U_p^r + \nu_p U_p^z \right) + M \left( W_p^r + \nu_p W_p^z \right)
\end{align*}
\]
(6.88)

The displacement stress wavefield vector for a radial ring source is given by,

\[
\begin{bmatrix}
  u_1, \ u_z, \ w_z, \ \tau_1, \ \tau_{zz}, \ S
\end{bmatrix}_r^T = \sum_{i=\beta,\alpha_1,\alpha_2} \left( \begin{bmatrix}
  \frac{1}{ik} \left( iU_p^r \right), \text{sgn}(z-z') U_p^z, \text{sgn}(z-z') W_p^z, \frac{1}{ik} \left( \text{sgn}(z-z') GS_1^p \right), \\
  iS_2^p, iS_3^p \end{bmatrix} \frac{r_0}{2} J_1(kr_0) e^{-i\nu_p|z-z'|} \right)^T
\]
(6.89)

With variables, \( W_{\beta}, W_{\alpha_1}, W_{\alpha_2}, U_{\beta}, U_{\alpha_1}, U_{\alpha_2}, W_{\beta}, W_{\alpha_1}, W_{\alpha_2}, U_{\beta}, U_{\alpha_1}, U_{\alpha_2} \) defined in (Eq 6.59) and (Eq 6.62). Variables \( S_1^p, S_2^p, S_3^p \) with \( p = \beta \lor \alpha_1 \lor \alpha_2 \) are defined as,

\[
\begin{align*}
S_1^p &= U_p^r \nu_p - k^2 U_p^z \\
S_2^p &= (H - 2G) U_p^r + CW_p^r - H\nu_p U_p^z - C\nu_p W_p^z \\
S_3^p &= C \left( U_p^r - \nu_p U_p^z \right) + M \left( W_p^r - \nu_p W_p^z \right)
\end{align*}
\]
(6.90)
The displacement stress wavefield vector for an explosive ring source is given by,

\[
\begin{bmatrix}
u_1, \ u_z, \ w_z, \ \tau_1, \ \tau_{zz}, \ S \nend{bmatrix}^T = \sum_{p=\alpha_1, \alpha_2}
\left(\frac{1}{ik} \left(ik^2 U_p^e\right), -\text{sgn}(z - z')\nu_p U_p^e, -\text{sgn}(z - z')\Lambda_p, \frac{1}{ik} \text{sgn}(z - z') S_p^p, \right.
\left.
\left(\frac{1}{2i} S_2^p \tau_0 J_0(\alpha_p r) e^{-i \nu_p |z - z'|}\right)^T
\right.
\]

(6.91)

With variables, \(U_{\alpha_1}^e, U_{\alpha_2}^e\) defined in equations (Eq 6.75). Variables \(S_1^p, S_2^p, S_3^p\) with \(p = \alpha_1 \lor \alpha_2\) are defined as,

\[
S_{1}^{p} = 2Gk^2 \nu_i U_i^e
\]

(6.92)

\[
S_{2}^{\alpha_1} = H \nu_{\alpha_1}^2 / k U_{\alpha_1}^e + (H - 2G)i U_{\alpha_1}^e + C \Lambda_1 \left(\frac{k^2}{\nu_{\alpha_1}} + \nu_{\alpha_1}\right)
\]

(6.93)

\[
S_{2}^{\alpha_2} = H \nu_{\alpha_2}^2 / k U_{\alpha_2}^e + (H - 2G)k U_{\alpha_2}^e + C \Lambda_2 \left(\frac{k^2}{\nu_{\alpha_2}} + \nu_{\alpha_2}\right)
\]

(6.94)

\[
S_{3}^{\alpha_1} = C \left(k^2 + \nu_{\alpha_1}^2\right) U_{\alpha_1}^e + M \Lambda_1 \left(\frac{k^2}{\nu_{\alpha_1}} + \nu_{\alpha_1}\right)
\]

(6.95)

\[
S_{3}^{\alpha_2} = C \left(k^2 + \nu_{\alpha_2}^2\right) U_{\alpha_2}^e + M \Lambda_2 \left(\frac{k^2}{\nu_{\alpha_2}} + \nu_{\alpha_2}\right)
\]

(6.96)

To match the boundary conditions at the borehole wall the \(w_r\) and \(\tau_{rr}\) field components are needed, see equation (Eq 6.1). The radial relative flow component is obtained from the Fourier transformed deformation equation in the fluid, expressed in cylindrical coordinates.

\[
\dot{S}(k, z) = C \left[\ddot{u}_1(k, z) + \frac{\partial}{\partial z} \dot{u}_z(k, z)\right] + M \left[\ddot{w}_1(k, z) + \frac{\partial}{\partial z} \dot{w}_z(k, z)\right]
\]

(6.97)

Therefore,

\[
\dot{w}_1(k, z) = -\frac{\partial}{\partial z} \dot{w}_z(k, z) + \frac{1}{M} \left[\dot{S}(k, z) - C \left(\ddot{u}_1(k, z) + \frac{\partial}{\partial z} \dot{u}_z(k, z)\right)\right]
\]

(6.98)

Its radial component can be recovered using equation (Eq 6.105).

The radial stress component is obtained by using the deformation equation for the bulk in cylindrical coordinates.

\[
\tau_{rr}(r, z) = H \frac{\partial}{\partial z} u_r(r, z) + [H - 2G]u_1(r, z) + C \left[w_1(r, z) + \frac{\partial}{\partial z} w_z(r, z)\right]
\]

(6.99)
With \( u_1(r, z) \), \( w_2(r, z) \) obtained using equation (Eq 6.86) and, \( w_1(r, z) \) is obtained using (Eq 6.55) and (Eq 6.86) and, \( u_r(r, z) \) is obtained using (Eq 6.86) and (Eq 6.105).

### 6.8 Mechanical and Electromagnetic Waves in Horizontally Layered Poro-Elastic Media

We first transform the macroscopic governing equations that control the coupled electromagnetics and acoustics of porous media, into a field-vector-formalism that can be solved numerically. The global matrix method (Chin et al., 1984) is employed to solve simultaneously the macroscopic electromagnetic and acoustic-elastic wavefield amplitudes which contribute to current and fluid flow. The current and fluid flow are related by the transport equations through which all electromagnetic and mechanical coupling occurs, i.e. currents generated by the acoustic-elastic wavefields, an electrokinetic phenomenon and fluid flow generated by the electromagnetic field, an osmosis phenomenon. In the global matrix method we propagate a wavefield vector which contains the mechanical stress/displacement and electromagnetic wavefield components that are continuous through the horizontal boundaries in the layered porous medium (Haartsen and Pride, 1995).

In isotropic media the first order ordinary differential equations, the equations of state, relating the wavefields at all depths to the medium properties at depth decompose into two equation sets. The first set describes the \( PSVTM \) picture, which is the situation where the compressional and vertical polarized mechanical waves drive currents in the \( PSV \) particle motion plane that couple to the electromagnetic wavefield components of the \( TM \) mode. The second set describes the \( SHTE \) picture where the horizontal polarized rotational waves drive currents in the \( SH \) particle motion plane that couples to the electromagnetic wavefield components of the \( TE \) mode.
In this chapter we compute electroseismograms generated by three mechanical ring sources in the \textit{PSVTM} polarization picture only. The effect of a general mechanical ring source in a stratified porous medium is accommodated by specifying a jump in the displacement-stress vector across a horizontal plane (see Hudson, 1969a,b; Kennett, 1983). The wavefield components in the displacement-stress vector calculated for a vertical, radial and explosive ring source, equations (Eq 6.87), (Eq 6.89) and (Eq 6.91) respectively, are the wavefields that are continuous at the horizontal boundaries in the stratified medium for the \textit{PSVTM} picture, see chapter 3. We define the total displacement-stress-EM wavefield vector as follows,

\[ \mathcal{B} = [u_1, u_2, w_z, \tau_1, \tau_{zz}, S, H_2, E_1]^T \]  

(6.100)

At \( z = z' \), with \( z' \) the source depth, only the \( u_1, \tau_{zz} \) and \( S \) wavefields jump in the vertical ring source representation across a dummy interface. Where a dummy interface denotes a boundary across which none of the medium properties change. The final jump representation across a source plane for a vertical ring source, using equations (Eq 6.87) and (Eq 6.100) reads,

\[ \mathcal{S}(z') = \left[ \lim_{z \uparrow z'} \mathcal{B}(z) - \lim_{z \downarrow z'} \mathcal{B}(z) \right] \]

\[ = \sum_{p=\beta, \alpha_1, \alpha_2} \left[ -\frac{r_0}{ik^2} U_p J_1(\kappa r_0), 0, 0, 0, -r_0 S_2^p J_0(k r_0), \right. \]

\[ \left. -r_0 S_3^p J_0(k r_0), 0, 0 \right]^T \]  

(6.101)

At \( z = z' \), with \( z' \) the source depth, only the \( u_1, \tau_1 \) and \( \tau_{zz} \) wavefields jump in the radial ring source representation across a dummy interface. The final jump representation across a source plane for a radial ring source, using equations (Eq 6.89) and (Eq 6.100) reads,

\[ \mathcal{S}(z') = \left[ \lim_{z \uparrow z'} \mathcal{B}(z) - \lim_{z \downarrow z'} \mathcal{B}(z) \right] \]

\[ = \sum_{p=\beta, \alpha_1, \alpha_2} \left[ 0, r_0 U_p^z J_1(k r_0), r_0 W_p^z J_1(k r_0), r_0 \frac{1}{ik} G S_2^p J_1(k r_0), \right. \]

\[ \left. 0, 0, 0, 0 \right]^T \]  

(6.102)

223
At \( z = z' \), with \( z' \) the source depth, only the \( u_z, \tau_1 \) and \( \tau_{zz} \) wavefields jump in the explosive ring source representation across a dummy interface. The final jump representation across a source plane for an explosive ring source, using equations (Eq 6.91) and (Eq 6.100) reads,

\[
S(z') = \left[ \lim_{z \to z'} B(z) - \lim_{z \to z'} B(z) \right] \\
= \sum_{p=\beta,\alpha_1,\alpha_2} \left[ 0, -r_0 v_p U_p^e J_0(kr_0), -r_0 A_p J_0(kr_0), \frac{r_0}{ik} S_p^e J_0(kr_0), 0, 0, 0, 0 \right]^T 
\]

(6.103)

Note that the last two electromagnetic wavefield components in equations (Eq 6.101), (Eq 6.102) and (Eq 6.103) are zero since no electromagnetic source is applied.

### 6.9 Transformation Back to the Space Time Domain

An inverse Hankel transform is applied to obtain the 3D spatial dependence of the displacements, stresses, electric and magnetic fields. The horizontal components of the displacements, stresses, electric and magnetic fields require additional integration over \( \tau \) and \( \phi \) to obtain, \( u_r, u_\phi, \tau_{r\phi}, \tau_{rz}, H_r, H_\phi \) and \( E_r, E_\phi \) due to the definitions of \( u_V, u_H, \tau_{Vz}, \tau_{Hz}, H_V, H_H \) and \( E_V, E_H \).

\[
u_z(\omega, r, \phi, z) = \int_0^\infty k \, dk \, \sum_{n=-N}^{N} [J_n(kr) \hat{u}_z(\omega, k, n, z)] e^{i n \phi} 
\]

(6.104)

Identical relations exist for \( w_z, \tau_{zz} \) and \( S \). The horizontal components may be recovered using the following relations (Kennett, 1983).

\[
u_r(\omega, r, \phi, z) = \int_0^\infty k \, dk \, \sum_{n=-N}^{N} \left[ \frac{n}{kr} J_n(kr) \hat{u}_z(\omega, k, n, z) \right. \\
- \left. i J'_n(kr) \hat{u}_1(\omega, k, n, z) \right] e^{i n \phi} 
\]

(6.105)
\[ u_\phi(\omega, r, \phi, z) = \int_0^\infty k \, dk \sum_{n=-N}^N \left[ \frac{n}{kr} J_n(kr) \hat{\epsilon}_1(\omega, k, n, z) \right. \\
+ \left. iJ'_n(kr) \hat{\epsilon}_2(\omega, k, n, z) \right] e^{i n \phi} \] (6.106)

Identical relations exist for \( \tau_{rz}, \tau_{\phi z} \) in terms of \( \hat{\tau}_1 \) and \( \hat{\tau}_2 \), for \( E_r, E_\phi \) in terms of \( \hat{E}_1 \) and \( \hat{E}_2 \) and \( H_r, H_\phi \) in terms of \( \hat{H}_1 \) and \( \hat{H}_2 \). The above representations may be regarded as a superposition of cylindrical waves whose order dictates the nature of their azimuthal modulation. At each frequency and angular order the radial contribution is obtained by superposing all horizontal wavenumbers \( k \). This corresponds to including all propagating waves at the level \( z \) within the stratification, from vertically to purely horizontal traveling waves including the evanescent waves. At any particular distance \( r \) the relative contributions of the wavenumbers are imposed by the radial phase functions \( J_n(kr) \).

### 6.10 Numerical Results

In this section we will discuss the vertical, radial and volume injection ring source results in layered media. The final \( BEM \) algorithm calls these vertical, radial and volume injection ring sources in layered medium functions for each source/receiver element combination, including the self interacting elements which are analytically included. The wavefield components, calculated at the receiving elements, that match the boundary conditions across the borehole wall, are calculated by discrete wavenumber integration and each element interaction fills an entry in the square boundary condition matrix. The wavenumber integral becomes computationally more and more expensive with decreasing vertical wavefield propagation, i.e. the interaction between neighboring elements are computationally more expensive than the interaction between further separated elements. Even with all the interacting element symmetries exploited, the \( BEM \) solution in layered media is prohibitively computationally ex-
pensive. Although numerically feasible on a massive parallel computer, the BEM algorithm is implemented on the nCUBE, the code didn't have a practical runtime performance.

Therefore, we discuss in this section only the secondary ring source results for three models in the $PSVTM$ wavefield picture. The first example is a simple surface electroseismic model example, see figure 6-1. The electroseismic response of the three different ring sources are modeled. In figures 6-2, 6-4 and 6-6 the mechanical and electroseismograms ($TM$ mode components) are depicted for a volume injection ring source, a vertical, and a radial ring source. Only the reflection/conversion results are shown, i.e. the direct wavefields are not included. The last two examples show the electroseismic effect in a Vertical ElectroSeismic Profiling ($VESP$) setting. The seismic ring source is located in the upper halfspace and the recording geophone/antenna are positioned vertically crossing two mechanical/electrical contrasts in the first $VESP$ example (see figure 6-7) and an additional fluid chemistry contrast in the second $VESP$ example (see figure 6-11). In figures 6-8, 6-9 and 6-10 the calculated results are shown for the volume injection ring source, the vertical ring source and the radial ring source respectively. Figures 6-12, 6-13 and 6-14 show the additional electroseismic response of the included fluid chemistry contrast within the sand reservoir generated by the three ring sources.

6.10.1 Electroseismics in Surface Geometry

The ring source is positioned 100 m above the contrast and has a 100 Hz center frequency. Fifteen antennas/geophones are positioned symmetrically in a straight horizontal line at both sides of the source, 95 m above the interface. The receiver spacing is 10 m. The medium properties of the two halfspaces and the calculated wave velocities and bulk conductivities, calculated at source center frequency, are given in tables I, II and III. All plots in figures 6-2, 6-4 and 6-6 are seismogram scaled. When
a volume injection ring source is set off only $P$ waves are generated in an isotropic poro-elastic medium while the vertical and radial ring sources generate $P$ as well as $SV$ waves. The mechanical displacement seismograms ($u_z$ and $u_r$ components) in figures 6-4 and 6-6 show clearly the difference in amplitude variability versus offset to be expected for vertical and radial ring sources. For example, the $u_z$ component in the $PP$ reflection is maximum at zero offset reflection for a vertical ring source, while the same wavefield component is minimum at zero offset reflection for a radial ring source.

The converted $TM$ wavefield components show up at all antennas at approximately half the twoway $P$ wave traveltime for normal incidence reflection. The hyperbolas arriving at later time in the $E_r$ and $H_\phi$ electroseismograms appear at the same traveltime as the $PP$ reflection, $PSV$ conversion and $SVSV$ reflection, $SVP$ conversion for the vertical and radial ring sources only. The electric fields traveling with compressional wavespeed through a homogeneous porous medium are caused by charge separation generated by pressure gradients inside the seismic pulse. The magnetic fields traveling with rotational wave speed through homogeneous porous medium are induced by current sheets generated by the divergence free grain accelerations.

The vertical and radial ring sources generate radiating $EM$ wavefields. These fields show up in the electroseismograms, figures 6-4 and 6-6, at approximately zero traveltime. The electroseismograms are slightly shifted in time to avoid signal wrapping in time, which will be introduced by the convolution of the impulse response with a non-causal Ricker wavelet (Hosken, 1988). Physically, a vertical and radial ring source induces a vertical and radial streaming current, respectively, at a hypothetical source plane, due to a mechanically driven vertical and radial relative flow. The electromagnetic fields associated with these mechanically induced streaming currents at the source plane need to be balanced by electromagnetic fields that make the $E_r$ and $H_\phi$ field component entries in the vertical and radial jump representations.
across the source plane vanish, see equations (Eq 6.101) and (Eq 6.102), respectively. Due to symmetry properties, the volume injection ring source doesn't radiate $EM$ fields away from the mechanical source. In figures 6-3 and 6-5 the expected radiation pattern for a vertical and radial ring source above a contrast is outlined. The amplitude behavior of the reflected electromagnetic signal is depicted for a hypothetical perfect reflector case, using the method of images. The horizontal component of the reflected electromagnetic field is zero right at the center of the ring sources and increases with horizontal distance. This amplitude behavior can be identified in the electroseismograms in figures 6-4 and 6-6 and the cartoons in figures 6-3 and 6-5. The length of the arrows in figures 6-3 and 6-5 are an indication of the the absolute field amplitudes. The reflected $EM$ wavefields generated by the radial ring source are much smaller than the fields generated by a vertical ring source.

The electroseismic conversion of an incoming $SV$ wave generated by the vertical and radial ring sources are insignificant to the pressure gradient, $P$ wave induced converted electromagnetic signals. The $SV$ to $EM$ conversion is not discernible at the used electroseismogram scaling in figures 6-4 and 6-6. The importance of a dynamic vertical pressure gradient imbalance in the conversion to electromagnetic signals is evident when we compare the volume injection and vertical ring source results with the radial ring source result. The $P$ wave conversion into electromagnetic signal is orders of magnitude smaller in the radial ring source synthetics, since its $P$ wave radiation pattern points mainly into the horizontal direction parallel to the contrast, as opposed to the volume injection and vertical ring source radiation pattern that point their $P$ wave energy mainly towards the contrast.

6.10.2 Electroseismograms in VESP Geometry

In the $VESP$ geometry the antennas are positioned close to the target of interest and therefore larger converted electromagnetic signals can be recorded before they become
too attenuated with traveled distance. With the $VESP$ technique the electroseismic method can be extended to targets, electrical and/or mechanical contrasts, at greater depths. The first $VESP$ example is a sandwiched reservoir sand in between two low permeability halfspaces with smaller porosity. The first receiver/antenna is positioned at 20 m horizontal offset and a depth of 40 m. The receiver/antenna spacing is 5 m. The model is depicted in figure 6-7. The complete set of medium properties and the at source center frequency calculated wavefield velocities and bulk conductivities are given in tables I, II and III. In figures 6-8, 6-9 and 6-10 the numerically calculated $VESP$'s are shown for a volume injection ring source and, a vertical, radial ring source. The top plot shows the mechanical displacement response, the center and bottom plots show the $E_r$ and $H_\phi$ TM medium components respectively.

The converted TM wavefield components show up at all receiving antennas at approximately the same time. These events can be recognized as the straight vertical lines in the figures. All later arrivals in the $E_r VESP$ are the electric fields within the seismic pulse that travel with the compressional wavespeed. The later arrivals in the $H_\phi VESP$ are the magnetic fields within the seismic pulse that travel with the rotational wavespeed. When we compare the electroseismograms generated by a vertical and radial ring source, we notice the very strong $SV$ and $SV$ induced signals in the radial ring source seismogram and electroseismograms. The vertical ring source seismogram shows strong $P$ wave conversions into TM signal. The conversions from $SV$ into $TM$ signal is very weak for the shear waves generated by the vertical ring source. The radial ring source on the other hand shows strong $SV$ to electromagnetic wave conversions. The $SV$ wave converted signals are in this case larger in amplitude than the $P$ wave converted signals.

The last numerical example has an additional electrical contrast added to the previous model, see figure 6-11. The 100 m thick "reservoir sand" is divided into two sands with identical mechanical properties saturated with two fluids of different salinity. In figures 6-12, 6-13 and 6-14 the calculated $VESP$'s for a volume injection
ring source, a vertical and radial ring source are shown respectively. The mechanical response is identical to the previous calculations without the electrical fluid contrast, see figures 6-8, 6-9 and 6-10. But the $TM$ mode component $VESP$'s show an additional electroseismic conversion generated at the electrical contrast.

6.11 Conclusions and Discussion

The Biot-Rosenbaum model is extended by including the effect of a heterogeneous formation surrounding the borehole and by including the conversions of mechanical into electromagnetic waves at the mechanical and/or electrical contrasts in the poro-elastic formation. The method is formulated as a boundary element technique where the poro-elastic mechanical wavefield components to be matched across the borehole wall are calculated by the discrete wavenumber method, integrating the dynamic poro-elastic Green's function wavefield solutions. The coupled electromagnetic and acoustic wavefields in layered media are included by employing the global matrix technique.

The singular properties of the stress field for the dynamic Green's functions with a force applied on the solid phase and the singular property of the displacement for the dynamic Green's function with a force applied on the fluid phase, when source and receiving element coincide are determined analytically by integration of the derived static Green's functions over a half spherical surface.

Four secondary ring sources are introduced—a volume injection ring source, generating wavefields diffracted into the fluid, a vertical and radial ring source acting on the elastic frame, and a volume injection ring source acting on the pore fluid. The last three sources give rise to the wavefields radiated into the poro-elastic formation.

The developed $BEM$ algorithm needs to be optimized to decrease its runtime on the nCUBE to allow waveform modeling of seismic and converted electromagnetic
waves in a borehole embedded in a layered porous medium. The Stoneley waves and converted electromagnetic signals both contain information about permeability. The converted electromagnetic signals are also sensitive to fluid chemistry contrasts (i.e. oil-water contact) in a homogeneous reservoir. The study of these effects on the full waveforms awaits the optimization of the developed algorithm.

In this chapter three mechanical source jump representations for the three ring sources in the poro-elastic formation are derived. These jump conditions are included in the global matrix technique and constitute the kernel program functions of the BEM algorithm. The numerical results show that the vertical and radial ring sources generate radiating electromagnetic fields. The induced step in electrical streaming current imposes a jump in electromagnetic fields to accommodate for the predefined zero magnetic and electric field components (since no electromagnetic source is applied) in the vertical and radial ring source jump representations across the source plane.

The conversion to electromagnetic disturbances is found to be mainly due to pressure gradients generated by the $P$ waves across the contrasts for the vertical and volume injection ring sources. Even though rotational waves, generated by the vertical ring source, induce larger streaming currents, they don’t necessarily create larger current imbalances across the interface and therefore larger converted electromagnetic disturbances, whereas in the radial ring source case, the $SV$ wave induced current imbalances across the contrasts account for the main conversions to electromagnetic disturbances. This illustrates the magnitude dependence of the electroseismic conversions from $SV$ waves, on the $SV$ radiation pattern orientation with respect to the interface.
### TABLE I. The medium properties in the layers used in the numerical calculations.

<table>
<thead>
<tr>
<th>Property</th>
<th>Top and bottom halfspaces</th>
<th>Salinity I layer</th>
<th>Salinity II layer</th>
<th>Salinity III halfspace</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi[%]$</td>
<td>15</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>$k_0[m^2]$</td>
<td>$10^{-14}$</td>
<td>$10^{-12}$</td>
<td>$10^{-12}$</td>
<td>$10^{-12}$</td>
</tr>
<tr>
<td>$k_s[Pa]$</td>
<td>$3.0 \times 10^{10}$</td>
<td>$1.0 \times 10^{10}$</td>
<td>$1.0 \times 10^{10}$</td>
<td>$1.0 \times 10^{10}$</td>
</tr>
<tr>
<td>$k_f[Pa]$</td>
<td>$2.2 \times 10^9$</td>
<td>$2.2 \times 10^9$</td>
<td>$2.2 \times 10^9$</td>
<td>$2.2 \times 10^9$</td>
</tr>
<tr>
<td>$k_{fr}[Pa]$</td>
<td>$8.0 \times 10^9$</td>
<td>$3.0 \times 10^9$</td>
<td>$3.0 \times 10^9$</td>
<td>$3.0 \times 10^9$</td>
</tr>
<tr>
<td>$g_f[Pa]$</td>
<td>$9.0 \times 10^9$</td>
<td>$8.5 \times 10^9$</td>
<td>$8.5 \times 10^9$</td>
<td>$8.5 \times 10^9$</td>
</tr>
<tr>
<td>$\eta[Pa\text{s}]$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$1.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\rho_s[Kg/m^3]$</td>
<td>$2.7 \times 10^3$</td>
<td>$2.7 \times 10^3$</td>
<td>$2.7 \times 10^3$</td>
<td>$2.7 \times 10^3$</td>
</tr>
<tr>
<td>$\rho_f[Kg/m^3]$</td>
<td>$1.0 \times 10^3$</td>
<td>$1.0 \times 10^3$</td>
<td>$1.0 \times 10^3$</td>
<td>$1.0 \times 10^3$</td>
</tr>
<tr>
<td>$C[mol/l]$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$1.0 \times 10^{-2}$</td>
<td>$1.0 \times 10^{-1}$</td>
<td>$1.0$</td>
</tr>
<tr>
<td>$T[K]$</td>
<td>298</td>
<td>298</td>
<td>298</td>
<td>298</td>
</tr>
<tr>
<td>$\kappa_f$</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>$\kappa_s$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$\alpha_{\infty}$</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
TABLE II a. The calculated (at source center frequency) wavefield velocities and bulk conductivities in the surface seismic and VESP geometry models. Both the real and imaginary parts of the calculated wavefield velocities are listed.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Upper and lower halfspace</th>
<th>Salinity I layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>fast wave velocity [$m/s$]</td>
<td>(3154.97, -0.00011)</td>
<td>(2502.21, -0.0524)</td>
</tr>
<tr>
<td>slow wave velocity [$m/s$]</td>
<td>(5.52, -5.28)</td>
<td>(40.54, -38.17)</td>
</tr>
<tr>
<td>shear wave [$m/s$]</td>
<td>(1768.85, -0.00248)</td>
<td>(1648.32, -0.258)</td>
</tr>
<tr>
<td>TM wave velocity [$m/s$]</td>
<td>(827466.0/-792166.0)</td>
<td>(192248.0/-184061.0)</td>
</tr>
<tr>
<td>conductivity ($\sigma[S/m]$)</td>
<td>0.000416</td>
<td>0.00771</td>
</tr>
</tbody>
</table>

TABLE II b. The calculated (at center frequency) wavefield velocities and bulk conductivities in the surface seismic and VESP geometry models. Both the real and imaginary parts of the calculated wavefield velocities are listed.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Salinity II layer</th>
<th>Salinity III halfspace</th>
</tr>
</thead>
<tbody>
<tr>
<td>fast wave velocity [$m/s$]</td>
<td>(2502.21, -0.5)</td>
<td>(2502.21, -0.0524)</td>
</tr>
<tr>
<td>slow wave velocity [$m/s$]</td>
<td>(39.63, -37.26)</td>
<td>(40.54, -38.17)</td>
</tr>
<tr>
<td>shear wave [$m/s$]</td>
<td>(1648.32, -0.24)</td>
<td>(1648.32, -0.258)</td>
</tr>
<tr>
<td>TM wave velocity [$m/s$]</td>
<td>(18815.4, -179.77.0)</td>
<td>(60834.4, -58244.3)</td>
</tr>
<tr>
<td>conductivity ($\sigma[S/m]$)</td>
<td>0.077</td>
<td>0.77</td>
</tr>
</tbody>
</table>
Surface ElectroSeismic Geometry

Figure 6-1: An explosive, vertical and radial ring source, 100 m above an interface in a surface Electroseismic geometry setting.
Figure 6-2: The mechanical displacement component seismograms and the $TM$ mode component electroseismograms, without direct waves, calculated for an explosive ring source above an one interface model.
Vertical Ring Source above Contrast

Figure 6-3: Radial electric field radiation patterns (direct and reflected fields) for vertical streaming currents induced by a vertical ring source above a contrast.
Figure 6-4: The mechanical displacement component seismograms and the $TM$ mode component electroseismograms, without direct waves, calculated for a vertical ring source above an one interface model.
Figure 6-5: Radial electric field radiation patterns (direct and reflected fields) for radial streaming currents induced by a radial ring source above a contrast.
Figure 6-6: The mechanical displacement component seismograms and the TM mode component electroseismograms, without direct waves, calculated for a radial ring source above an one interface model.
Figure 6-7: An explosive, vertical and radial ring source in a Vertical ElectroSeismic Profiling geometry (reservoir sand saturated with one fluid type).
Figure 6-8: The mechanical displacement component seismogram and the $TM$ mode component electroseismograms calculated for an explosive ring source without direct waves in a Vertical ElectroSeismic Profiling geometry.
Figure 6-9: The mechanical displacement component seismogram and the $TM$ mode component electroseismograms calculated for a vertical ring source without direct waves in a Vertical ElectroSeismic Profiling geometry.
Figure 6-10: The mechanical displacement component seismogram and the $TM$ mode component electroseismograms calculated for a radial ring source without direct waves in a Vertical ElectroSeismic Profiling geometry.
Vertical ElectroSeismic Profiling (VESD)

Figure 6-11: An explosive, vertical and radial point source in a Vertical ElectroSeismic Profiling geometry (reservoir sand saturated with two fluid types).
Figure 6-12: The mechanical displacement component seismogram and the $TM$ mode component electroseismograms calculated for an explosive ring source without direct waves in a Vertical ElectroSeismic Profiling geometry with an additional electrical contrast in a sand reservoir.
Figure 6-13: The mechanical displacement component seismogram and the TM mode component electroseismograms calculated for a vertical ring source without direct waves in a Vertical ElectroSeismic Profiling geometry with an additional electrical contrast in a sand reservoir.
Figure 6-14: The mechanical displacement component seismogram and the $TM$ mode component electroseismograms calculated for a radial ring source without direct waves in a Vertical ElectroSeismic Profiling geometry with an additional electrical contrast in a sand reservoir.
Chapter 7

Real ElectroSeismic Data, Modeling and Analysis: The Haney Field Experiment

7.1 Introduction

At the Haney test site, near Vancouver, Canada, a clear electroseismic conversion is measured at interfaces between road fill, glacial till and bedrock. The site is located on an unimproved dirt road that runs along the side of a steep slope in the UBC Research Forest. The knowledge of the subsurface is summarized in figure 7-1 and table I. Based on two boreholes, drilled 4.4 m apart near either edge of the road encountered the interface with the glacial till at about 3 m depth. The deeper borehole, drilled to a depth of about 11 m, penetrated the glacial till without encountering any obvious change in lithology. Seismic refraction studies indicated that the bedrock begins at a depth of 15 m.
The road fill is very permeable and has the properties of an organic rich soil. It consists of wood pieces and a minor amount of gravel in a matrix of decayed organic matter, silt and sand sized particles. The cores taken from the glacial till are competent and this "non" permeable glacial till is composed mainly of silt and clay sized particles with some gravel and cobbles. Some key material properties for the road fill and glacial till are given in table I. The \( P \) wave velocities are reasonably accurate and were obtained from seismic studies. The porosities and permeabilities are educated guesses based on the inspection of the borehole cores and the literature. The values in table I are characteristic of moderately permeable clean sands and moderately permeable glacial tills (Freeze and Cherry, 1979). The electrical resistivity estimates are based on two Wenner array soundings, see table I.

### 7.2 Electroseismic Data in Transpose Vertical ElectroSeismic Profiling (TVESP) Geometry

In figure 7-2 the electroseismic response is shown of blasting caps in a borehole recorded at the surface at the Haney test site. Figure 7-1 shows the experimental layout used to acquire the data. The shot was located at 5.5 \( m \) depth (about 2.5 \( m \) below the organic soil-glacial till boundary). The electroseismic responses were measured by sixteen 2 meter dipoles at the surface 2 to 32 \( m \) north of the borehole. The electroseismic data was collected on a standard 8 channel system. Several shots needed to be detonated at 5.5 \( m \) depth in order to record all of the traces. Powerline noise has been removed from the electrical traces by applying sinusoid subtraction in the least square sense, and by subtracting the remote dipole record from each of the other dipoles (see Butler and Russell, 1993). The electroseismic data is displayed in trace normalized form.
7.3 Electroseismic Data in Surface Geometry

Figure 7-8 shows the electroseismic response of sledgehammer impacts at the Haney test site. Figure 7-7 shows the experimental layout used to acquire the data. The electroseismic data responses are recorded at offsets of 2 to 26 m from the shot point. 2 m dipole antennas were used. The electroseismic traces are average stacks of 20 to 80 sledgehammer blows. The dipole data at 2-10 m offset are stacked a 20 times, while the 12-26 m dipole data are stacked a 80 times. Again the powerline harmonics have been removed from the electroseismic data by sinusoid subtraction, and for the more distant dipoles by the subtraction of a remote dipole record. The electroseismic data in figure 7-8 is displayed in normalized form, no AGC has been applied.

7.4 Electroseismic Modeling in Transpose Vertical ElectroSeismic Profiling Geometry

The objective of this modeling is to identify the origin of the electroseismic arrivals in figure 7-2. Before we start to do so, a few words about the theoretical model and its input medium properties. An assumption underlying the forward input model is the medium's composition of a poro-elastic frame saturated with a compressible fluid. The top organic soil violates the assumption of consolidation, i.e. grains in contact constructing an elastic frame.

Geli et al. (1987) discuss seismic propagation through a surface layer with the properties of loose sediment and the implications of a highly permeable and unconsolidated sediment on the transition frequency which discriminates two frequency domains—a low frequency viscous flow behavior domain and a high frequency inertial flow domain. Saturated unconsolidated materials at seismic frequencies can show
inertial flow behavior, but the combination of viscosity, fluid density and permeability (see table II) in our three layer model at the used center frequency implies viscous flow dominated physics in all layers.

The medium properties listed in table II are obtained by matching the calculated velocities, table III, to the seismically measured velocities, table I. The salinity is determined by matching the bulk conductivities obtained with Wenner array soundings to the calculated conductivity. We have used the literature to reduce the non-uniqueness in the determination of the rock moduli set, that defines the measured velocities. The saturating phase in the top soil is a gas, while the till and bedrock are saturated with the much less compressible fluids. The densities, viscosities and bulk moduli of gasses, water and brines, experimentally determined under varying parameters of state are discussed in Liebermann (1948), Hertzfeld and Litovitz (1959), and Sette (1961). The listed permeabilities are characteristic for moderately permeable sands and moderately permeable glacial tills (see Freeze and Cherry, 1979). The permeability of the top soil may be underestimated.

The porosity, bulk moduli of the saturating fluids, the bulk modulus of the solid and the bulk modulus of the frame formed by the grains, characterize a water-mineral-gas system and its compressional and shear velocities. A lot of experimental data and models exist that list the physical rock property measurements as a function of another free rock property (porosity, for example) or predict rock properties theoretically developed (Berryman, 1992), or empirically obtained functions for most of the above mentioned properties. The frame moduli is an exception and seems the hardest to get. The accumulation of measurements, however, of the elastic constants of minerals, and the compressional and shear wave velocities and densities of rocks and sediments permits the calculation of the system's bulk moduli, and the values of the bulk moduli of the saturating phase and minerals for enough rocks and sediments can be used to define approximate relationships between frame moduli and porosities (see, for example, Hamilton, 1971). Another approach to obtain a value for the frame
moduli is to invert the compressional and shear velocities. Where the forward model to predict the velocities are based on poroelasticity, contact theory and constitutive parameters derived from carefully controlled, experimental testing (Murphy et al., 1993).

The obtained bulk and frame moduli estimates of the solid in the three layer model are verified against data collected by Hamilton (1971) and Gregory (1976), and models and quality quartz sandstone data in Murphy et al. 1993). Gregory shows bulk modulus variability versus porosity bins (\(\phi < 10\%\), \(\phi = 10 - 25\%\) and \(\phi > 25\%)\) of consolidated sedimentary rocks under moderate confining pressure and effects of fluid saturation on the bulk modulus versus the above mentioned porosity bins. The listed bulk moduli in table II are consistent with his data. The order of magnitude lower bulk modulus in the top soil implies, according to Gregory's data, a 100% gas saturation. Hamilton shows the shear frame modulus versus mean grain-size and porosity of sediments. His obtained frame moduli in coarse unconsolidated sediment sands with a 30% porosity corresponds to the listed frame shear modulus in the top soil. The low porosity frame moduli in the glacial till and bedrock correspond to measurements discussed in Gregory (1976). In Murhpy et al. (1993), shear and bulk frame moduli versus porosity plots are shown for a clean quartz sand. We used this data as an upper bound on the used frame moduli in our model.

The calculated velocities in table III show the very small compressional velocity of the soil. Indeed, in laboratory experiments (Domenico, 1976; 1977), it is demonstrated that the compressional velocity drops significantly with the presence of gas in the pore fluid. He also showed the influence on velocity of how uniformly the gas and liquid mixture is distributed in the pores. Since we have no estimate of the saturation level in the top soil, we have assumed the top soil to be saturated with a gas and the grain pore boundaries to be wetted with a saline fluid. According to Gregory (1976), gas causes a substantial reduction in the elastic moduli, see table II, and this effect is most noticable at shallow depths and at high porosity. A compressional and
shear wave velocity ratio of less than two indicates either a consolidated material (glacial till case) or the presence of gas in an unconsolidated material (top soil case). The compressional and shear velocity values and ratio correspond to measurements discussed in Simmons and Brace (1965) of bedrock under low confining pressure.

To understand the various electroseismic arrivals in figure 7-2, we simplify the input model to discriminate the wavefields originating from the two, soil-till and till-bedrock, interfaces. We first model the soil-till interface at 3 m depth. The mechanical displacement component results are shown in figure 7-5 and are labeled "\( u_r \) and \( u_z \) through top layer". The electroseismic response, the \( TM \) mode components are shown in figure 7-6 and are labeled "\( E_x \) and \( H_y \) through top layer". We then model the effect of the bedrock interface at 15 m depth. The mechanical seismograms are shown in figure 7-5 and are labeled "\( u_r \) and \( u_z \) from bedrock". The electroseismic response are shown in figure 7-6 and labeled "\( E_x \) and \( H_y \) from bedrock". The modeled source is an explosive point source with a 300 Hz center frequency. All plots are convolved with a Ricker wavelet (Hosken, 1988) and trace scaled. The seismograms and electroseismograms are 20 msec phase delayed. This is done to facilitate the comparison with the real data, where the first 20 msec are an indication of the noise level.

### 7.4.1 Effect of the Soil-Till Interface

The following events can be distinguished in the mechanical displacement component seismograms in figure 7-5. The fastest event is the \( P \) wave generated by the explosive point source that travels with the \( P \) wave velocity of the glacial till along the till-soil boundary and the organic rich soil layer. The slower parallel event is this same \( P \) wave interface wave that converts to \( SV \) and travels as a \( SV \) wave through the top soil to the receivers. Note the amplitude difference of the two events in the \( u_r \) and \( u_z \) component seismograms. As we expect, the \( u_z \) component shows big \( P \) wavefield
amplitudes whereas the $u_r$ amplitudes are largest in the $SV$ wavefields. The remaining slower event in the $u_r$ component seismogram is the $PSV$ conversion generated at the till-soil boundary. The $u_z$ component seismogram shows the $PP$ transmission. These events arrive at late time far offset, since then most of the traveled distance is through the slow top soil.

Figure 7-6 displays the electroseismic response of a $P$ wavefront from an explosive point source traversing a till-soil interface. Both $E_x$ and $H_y$ component seismograms in figure 7-6 show the electroseismic conversion at the $P$ wave traveltime distance to the till-soil boundary at normal incidence. The events emerging at the onset of the electroseismic conversion show another electroseismic conversion generated by a $P$ wave that travels along the till-soil interface with the glacial till compressional velocity. While this mechanical disturbance travels along the interface, a moving current imbalance is set up across the interface that causes electromagnetic radiation. The nature of this conversion mechanism is clearly different from the $P$ bodywave conversion into electromagnetic waves. It is unclear why it appears that the moving current imbalance along the interface causes only an electromagnetic response at one antenna position. The question why interacting electromagnetic wavefields, generated by a moving current imbalance source, give rise to a constructive electromagnetic response, seemingly originating from one point along the interface needs further study.

All other events in the electroseismograms in figures 7-6 are bodywave induced fields. The wavefields traveling with compressional wavespeed through homogeneous porous medium are caused by charge separation generated by pressure gradients inside the seismic pulse. The magnetic fields traveling with rotational wavespeed through the homogeneous porous medium are induced by current sheets generated by divergence-free grain accelerations (Haartsen and Pride, 1995). The dynamic current systems inside the bodywaves are balanced, and therefore the $P$ and $SV$ wavefield pulses don’t act as radiating antennas. When the bodywaves pass the antennas, the antennas register inside the $P$ wave pulse an electric field and inside the $SV$ wave pulse a
magnetic field, thus the antennas appear to act as geophones.

The two $P$ wave induced electric fields in the $E_x$ component electroseismograms are the electric fields induced by the $P$ wave that travels along the till-soil interface and as a $P$ through the top soil, and the $PP$ transmission through the till-soil interface. The $SV$ wave induced magnetic field in the $H_y$ component electroseismogram is the magnetic field induced by the conversion to a $SV$ wave from the $P$ till-soil interface wave. The magnetic field associated with the $P$ to $SV$ conversion doesn’t show at the used scaling in figure 7-6.

### 7.4.2 Effect of the Bedrock Interface

The interpretation of the bedrock effect is simpler than the till-soil interface effects. The mechanical component seismograms show the direct $P$ wave, the $P$ reflection and the $P$ to $SV$ conversion from the bedrock at the geophones. The electroseismograms in figure 7-6 show a weak electromagnetic conversion from the till-bedrock interface. The $E_x$ component of the electromagnetic conversion is hardly discernible at the plotted scale, since the electroseismogram is dominated by the strong direct $P$ wave induced electric field. The other events in the $E_x$ electroseismogram are the $P$ wave induced electric fields in the direct $P$ wave and the $P$ wave reflection from the bedrock. The $H_y$ component electroseismogram shows the $SV$ wave induced field inside the $P$ to $SV$ conversion from the bedrock. Since an explosive point source doesn’t generate $SV$ waves, we don’t have a magnetic field associated with a direct wavefield.

### 7.4.3 Real Data Comparison

Figure 7-2 shows the real data acquired at 2 $m$ spaced electrodes, and figures 7-3 and 7-4 show the $E_x$ and $H_y$ ($TM$ mode component) modeling results. When we compare
the $H_y$ component electroseismogram in figure 7-4 with the real data in figure 7-2, we can observe two electroseismic conversions. The first originates from the till-soil interface and the second from the till-bedrock interface. In the $E_z$ component electroseismogram (figure 7-3), we also distinguish the onset of these two electroseismic conversions. But the strong induced electric fields in the bodywaves, due to the very squishy top soil (low frame moduli), makes it difficult for an appropriate plotting scale to show all events properly.

The main differences between the modeled results and the data are the absence of coherent body wave induced electromagnetic fields and the frequency content of the signals. The absence of body wave induced fields we contribute to the physics of mechanical wave propagation through unconsolidated material. This is also probably the main reason of the lower frequency content in the real data, with the higher frequencies more attenuated by the unconsolidated soil.

We explain the event emerging at the onset of the electroseismic conversion as an electroseismic conversion generated by the $P$ wave that travels along the till-soil interface with glacial till compressional velocity. This interface wave generates a moving current imbalance along the till-soil interface that converts into electromagnetic radiation.

It is unfortunate that due to the top soil properties no tracking body wave induced electric fields can be identified. The results from a few other electroseismic data studies (Ivanov, 1939; Martner and Sparks, 1959; Thompson and Gist, 1991, 1993), and a currently ongoing field data acquisition effort at ERL/MIT, show that strong electromagnetic signals are induced by mechanical body waves. Thompson's experiment required an extensive data processing effort to remove the body wave induced signals before the converted arrivals could be distinguished. The body wave induced electromagnetic signal is also observable in the laboratory measurements on homogeneous (Zhu et al., 1994) and, layered porous rock samples (Haartsen et al., 1995). The Haney data set, which as a result of the unconsolidated top soil masks
the induced fields in the body waves, shows without extensive signal processing three clear electroseismic conversions.

The electroseismic conversion generated by the moving \( P \) wave along the till-soil interface is a newly observed electroseismic phenomenon that can be numerically modeled. The ERL/Hamilton dataset also shows electroseismic conversions of the propagating interface wave conversion type.

### 7.5 Electroseismic Modeling in Surface Geometry

The objective of this modeling is to compare the amplitude in a converted electric field, generated at an organic rich soil - glacial till interface at 3 \( m \) depth versus antenna offset, see figure 7-8, against the numerically modeled results in figures 7-9 and 7-10. The mechanical displacement, \( u_r \) and \( u_z \) component, seismograms are depicted in figure 7-11. The final electric field versus antenna position results, data versus synthetics comparison, are shown in figure 7-12.

The medium properties used in the numerical calculations are listed in table II. The wavefield velocities and bulk electrical conductivities are calculated using these rock properties in each layer and match the seismically and electrically obtained parameters in table I.

The electroseismic data acquired at antennas, 2–10 \( m \) offset (stacked a 20 times) are recorded with 2 \( m \) dipole antennas with their centers spaced one meter, while the far offset traces 12–26 \( m \) (stacked a 80 times) are recorded with dipole antennas with their centers spaced two meters. The odd trace numbers in between 11 - 25 \( m \) display the source trigger signatures. The traces are 20 \( msec \) time delayed and trace scaled.

The simulated data in figures 7-9 and 7-10 are calculated using an explosive point source with a 100 \( Hz \) center frequency. A more appropriate point source to simulate a sledgehammer impact would be a vertical point source. The effect of the till-bedrock
interface is not modeled since the data doesn't suggest that enough energy has propagated to the 15 m deep interface to convert into an observable signal. As explained before, the unconsolidated top soil doesn't propagate a tracking body wave induced electric field. Although unfortunate for comparison with seismics, the advantage of the top soil's masking body wave behavior enables us to simplify the modeling to just the calculation of the electroseismic conversion response of the soil-till interface in a model without a free surface. The data doesn't show an electric signal induced by the normally strong direct P waves and surface waves (ground roll). Based on the observations, the free surface effects are taken to be of no importance in this rather unique dataset. Modeling the electroseismic response of the interface in a full space model, reduces the number of events in the electroseismogram and simplifies the electric field amplitudes versus offset analysis, since the amplitudes can now be obtained without the help of signal processing.

In figures 7-9 and 7-10 the $E_x$ and $H_y$ TM mode components of the forward modeling are shown. In figure 7-11 the mechanical displacement components are shown. In this figure the reflected P and SV conversion are shown. The $E_x$ component electroseismogram shows the induced electric fields within the P wave traveling with the compressional wavespeed. The $H_y$ component electroseismogram shows the induced magnetic field within the converted SV wave traveling with the rotational wavespeed. The discontinuity in amplitude of the body wave induced fields in the electroseismograms in figures 7-9, 7-10 and 7-11 is due to a smaller scaling factor for times bigger than 70 msec. This is done to emphasize the non-importance of the bodywave induced electromagnetic fields in the real data. The required small frame moduli in the top soil, necessary to model the velocities in the top soil, induce large relative flow and induce strong electromagnetic fields within the seismic pulses. We face here a limitation of the model that cannot account for the physics of unconsolidated material.

When we compare the data (figure 7-8) against the $E_x$ component seismogram
(figure 7-9), we distinguish two types of electromagnetic conversion. The first one is the electromagnetic conversion of the $P$ wave traversing through the soil-till interface. Its arrival time corresponds to the coil $P$ wave traveltime distance to the soil-till boundary at normal incidence. The electromagnetic conversion shows up at all antennas at approximately the same traveltime since the electromagnetic wavespeed is orders of magnitude faster than the mechanical wavespeed (see table III).

The second type electromagnetic conversion is a conversion generated by a head wave traveling along the soil-till interface. While this mechanical disturbance travels along the interface, a moving current imbalance is set up across the interface that causes electromagnetic radiation. The modeling and data show a conversion associated with a refraction that travels with the $SV$ wavespeed of the glacial till along the interface. We have to be careful in our interpretation of the $E_z$ electroseismogram, figure 7-9 where the bodywave refraction and conversion associated with this refraction both show up in the electroseismogram. It is evident and explained why this bodywave refraction doesn't appear in the real data in figure 7-8.

In figure 7-12 we show the electric field amplitude versus offset of the electroseismic conversion of the first kind read from the real data and $E_z$ electroseismogram. The modeling predicts the antenna offset at which the electric field is maximum. The antenna offset with maximum electric field equals the depth to the interface at which the conversion occurred. The maximum electric field amplitude in the synthetics is forced to match the maximum data amplitude by changing the source strength of the explosive source. The obtained source strength corresponds to an effective 250 g dynamite source.

The amplitude drop off around the maximum is underestimated by the model. The model predicts zero amplitude in a converted electric signal at zero offset and a steep drop off in amplitude for the offsets less than 2 m will be modeled. Two amplitude drop off regimes are determined in both curves. The real data amplitudes decrease almost linearly in the 3 to 5 m offset range and taper towards zero electric
field amplitude in the 5 to 12 m offset range. The synthetic data amplitudes decrease almost linearly in the 3 to 8 m offset range and taper towards zero electric field amplitude in the 8 to 15 m antenna offset range. The 5–12 m offset range and 8-15 m offset range are similar in curvature and field amplitude. The main difference between the two curves is the 3 m shift to the farther offsets in the synthetics where the linear amplitude drop off stops and changes into a polynomial drop off of an order higher than one. We contribute this difference to the source radiation pattern difference.

An explosive point source was modeled while a sledgehammer impact is better represented by a vertical point source. An unclear point is the effect of the unconsolidated - consolidated contact boundary conditions. The induced dynamic current imbalance might change significantly the electric field amplitude curve shape and cannot be properly modeled with the theoretical model based on two consolidated materials in welded contact.

7.6 Conclusions and Discussion

We interpreted two datasets acquired at the Haney test site, near Vancouver, Canada. The first electroseismic dataset showed the electroseismic response of blasting caps in a borehole at 5.5 m depth (about 2.5 m below the organic soil-glacial till boundary) recorded at the surface by sixteen 2 meter dipoles. The second data set showed the electroseismic response of sledgehammer blows recorded with 2 m dipole antennas at offsets of 2 to 26 m from the shot at the surface.

The objective of the modeling of the first dataset in TVESP geometry is to identify the origin of the electroseismic arrivals. Three distinct electroseismic conversions of two different conversion mechanisms can be identified in the data. The first electroseismic conversion is the conversion of an upgoing P wavefront from the explosive point source into electromagnetic waves, when traversing the till-soil interface. It's
arrival time equals the $P$ wave travel time from source to till-soil interface. The second parallel event is the electroseismic conversion of the downgoing $P$ wave into electromagnetic waves, when traversing the till-bedrock interface. It's arrival time at the dipole antennas equals the $P$ wave traveltime from source to till-bedrock interface. Both conversions are of the same conversion mechanism type. The mechanical waves induce a dynamic current imbalance across an interface, which causes the conversion into electromagnetic waves.

The third electroseismic conversion is generated by a $P$ wave that travels along the till-soil interface with the glacial till compressional velocity. While this mechanical disturbance travels along the interface, a moving current imbalance is set up across the interface that causes electromagnetic radiation. The physical picture of the question why interacting converted electromagnetic wavefields generated along the interface give rise to a constructive electromagnetic response, seemingly originating from one point along the radiating interface, needs further study. The conversion mechanism is clearly different from the bodywave conversion into electromagnetic waves when traversing a contrast.

The physics of the unconsolidated top soil prohibits the propagation of bodywaves which induce electromagnetic fields that are in consolidated media recorded at the antennas within the seismic pulse. These usually strong signals are modeled since the forward input model is based on the assumption that all layers are constituted of a poro-elastic frame saturated with a compressible fluid. The top organic soil violates this assumption and cannot properly be modeled. The advantage of this rather unusual dataset is that the data shows the weak electroseismic conversions generated at the two contrasts only. In consolidated media signal processing would have to be applied to remove the strong body wave induced signals before the usually weak electroseismic conversions can be distinguished.

The electroseismic modeling in surface geometry is compared against the sledgehammer responses recorded at the surface. The main objective of this comparison
was to look at how well the model predicts the electric field amplitude versus the
electric dipole offset of the electroseismic conversion of the first kind. The modeling
predicts the antenna offset at which the conversion occurred. The maximum electric
field, approximately 1.1 mV/m, is matched by the model using an effective 250 g
dynamite explosion source. The amplitude drop off is underestimated by the model,
which we contribute to the different radiation pattern of a vertical impact source. The
amplitude drop off of two offset regimes are determined in both real and synthetic
electric field versus offset curves. The change in amplitude drop off regime is 3 m
shifted towards the farther offsets in the synthetics. We contribute this again to the
vertical impact source radiation pattern and to a possible different converted electric
field pattern at an unconsolidated-consolidated interface.
TABLE I. Material properties obtained from seismic and electrical measurements at the Haney test site.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Top Soil</th>
<th>Glacial Till</th>
</tr>
</thead>
<tbody>
<tr>
<td>fast wave velocity [m/s]</td>
<td>280.0</td>
<td>2200.0</td>
</tr>
<tr>
<td>conductivity ((\sigma[S/m]))</td>
<td>0.00042</td>
<td>0.00022</td>
</tr>
</tbody>
</table>

TABLE II. The mechanical and electrical medium properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Top Soil</th>
<th>Glacial Till</th>
<th>Bedrock</th>
</tr>
</thead>
<tbody>
<tr>
<td>porosities ((\phi[%]))</td>
<td>30</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>dc permeability ((k[m^2]))</td>
<td>(10^{-11})</td>
<td>(10^{-16})</td>
<td>(10^{-16})</td>
</tr>
<tr>
<td>bulk modulus solid ((k_s[Pa]))</td>
<td>(2.0 \times 10^9)</td>
<td>(1.0 \times 10^{10})</td>
<td>(3.6 \times 10^{10})</td>
</tr>
<tr>
<td>bulk modulus fluid ((k_f[Pa]))</td>
<td>(1.0 \times 10^5)</td>
<td>(2.2 \times 10^9)</td>
<td>(2.2 \times 10^9)</td>
</tr>
<tr>
<td>frame bulk modulus ((k_{fr}[Pa]))</td>
<td>(8.0 \times 10^7)</td>
<td>(1.0 \times 10^9)</td>
<td>(9.0 \times 10^9)</td>
</tr>
<tr>
<td>frame shear modulus ((g_{fr}[Pa]))</td>
<td>(8.0 \times 10^7)</td>
<td>(1.0 \times 10^9)</td>
<td>(2.0 \times 10^{10})</td>
</tr>
<tr>
<td>fluid viscosity ((\eta[Pa s]))</td>
<td>(1.0 \times 10^{-4})</td>
<td>(1.0 \times 10^{-3})</td>
<td>(1.0 \times 10^{-3})</td>
</tr>
<tr>
<td>density solid ((\rho_s[Kg/m^3]))</td>
<td>(2.2 \times 10^3)</td>
<td>(2.7 \times 10^3)</td>
<td>(3.0 \times 10^3)</td>
</tr>
<tr>
<td>density fluid ((\rho_f[Kg/m^3]))</td>
<td>1.29</td>
<td>1.0 \times 10^3</td>
<td>1.0 \times 10^3</td>
</tr>
<tr>
<td>salinity ((C[mol/l]))</td>
<td>(5.0 \times 10^{-4})</td>
<td>(1.3 \times 10^{-3})</td>
<td>(1.0 \times 10^{-3})</td>
</tr>
<tr>
<td>temperature ((T[K]))</td>
<td>298</td>
<td>298</td>
<td>298</td>
</tr>
<tr>
<td>permittivity ((\kappa_f))</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>permittivity ((\kappa_s))</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>tortuosity ((\alpha_\infty))</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
TABLE III. Calculated wavefield velocities and bulk conductivities in each layer at source center frequency.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Top Soil</th>
<th>Glacial Till</th>
<th>Bedrock</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_p [m/s]$</td>
<td>301.0</td>
<td>2383.0</td>
<td>4064.0</td>
</tr>
<tr>
<td>$v_p [m^3]$</td>
<td>(1.61, -1.58)</td>
<td>(2.45, -2.42)</td>
<td>(4.5, -4.39)</td>
</tr>
<tr>
<td>$v_s [m/s]$</td>
<td>185.0</td>
<td>1347.0</td>
<td>2559.0</td>
</tr>
<tr>
<td>$v_{TM} [m/s]$</td>
<td>(1.57 $10^6$, -1.54 $10^6$)</td>
<td>(2.39 $10^6$, -2.63 $10^6$)</td>
<td>(2.69 $10^6$, -2.63 $10^6$)</td>
</tr>
<tr>
<td>$\sigma [S/m]$</td>
<td>0.00042</td>
<td>0.00022</td>
<td>0.00018</td>
</tr>
</tbody>
</table>
Figure 7-1: Blasting cap data acquired at the Haney test site in TVESP geometry.
Figure 7-2: Real electroseismic data collected at the Haney test site in TVESP geometry.
Figure 7-3: Modeled electric field component electroseismogram in TVESP geometry.
Figure 7-4: Modeled magnetic field component electroseismogram in TVESP geometry.
Figure 7-5: The mechanical displacement component response from the top organic soil-glacial till interface and the glacial till-bedrock interface.
Figure 7-6: The $T\!M$ mode component response from the top organic soil-glacial till interface and the glacial till-bedrock interface.
Unconsolidated Soil - Glacial Till Contrast at Haney

\[ f_c = 100 \text{ Hz} \]

\[ \text{salinity} = 0.0013 \text{ mol/l} \]
\[ \text{porosity} = 5\% \]
\[ \text{permeability} = 0.0001 \text{ D} \]
\[ \text{conductivity} = 0.00022 \]

\[ V_p = 2383 \text{ m/s} \]
\[ V_s = 1347 \text{ m/s} \]

"Glacial Till"

Figure 7-7: Real electroseismic data collected acquired at the Haney test site in surface geometry.
Figure 7-8: Real electroseismic data acquired at the Haney test site in surface geometry.
Figure 7-9: Modeled electric field component electroseismogram in surface geometry.
Figure 7-10: Modeled magnetic field component electroseismogram in surface geometry.
Figure 7-11: The mechanical displacement component response and $TM$ mode component response from the top organic soil/glacial till interface.
Figure 7-12: Comparison of measured converted electric field amplitudes against modeled converted electric field amplitudes versus dipole antenna offset.
Chapter 8

Summary and Conclusions

This thesis investigates the macroscopic dynamics of two-phase (fluid and solid) porous media, possessing continuously distributed phases. A fluid saturated sedimentary rock is an example of such a material. When a mechanical disturbance propagates through it, a small amount of relative motion is induced between the fluid and solid phases. The driving force for this relative flow is a combination of pressure gradients set up by the peaks and troughs of a compressional wave and by grain accelerations. The relative flow and therefore induced streaming current is due to both compressional and shear waves. Mechanical waves therefore act as a current source for electromagnetic disturbances.

First, the governing equations that control the coupled electromagnetics and acoustics of porous media, including the transport equations through which all coupling occurs, are used to study electrical streaming currents through homogeneous porous medium induced by seismic point sources. Then the coupled equations are solved in heterogeneous, i.e. layered porous media. Electroseismic boundary conditions at singular jumps in the macroscopic material properties are defined and a macroscopic energy transfer consistent field-vector-formalism is derived and solved.
numerically in a layered medium. Different seismic point sources applied on either part of the two phase medium are derived in a poro-elastic medium. A method is presented to simulate the propagation of seismic and converted electromagnetic waves generated by a mechanical borehole source embedded in a layered poro-elastic medium. Electroseismic field experiments in shallow subsurface environments and in boreholes are analyzed using the developed electroseismic numerical algorithms. Ultrasonic laboratory measurements in artificial porous layered media are made to test the theoretical models in the frequency regime where inertial flow determines the $k(\omega)$ and $L(\omega)$ transport coefficients.

The coupled transport equations, flux/force relations that relate the current and relative flow to potential and pressure gradients, are simplified to solve for the mechanically induced electrical streaming currents generated by seismic point sources in homogeneous porous media. The mechanical wave behavior is decoupled from the electromagnetic wave behavior to simplify the analysis of induced fluxes (relative fluid flow and current flow). The mechanically induced relative flow is determined by Green's function solution. The stationary-phase method is used to calculate streaming current radiation patterns for an explosive and vertical point source acting on the bulk phase and a pressure source acting on the fluid phase. The dynamic streaming current amplitude behavior with respect to porosity, permeability and fluid chemistry is investigated at three different source center frequencies.

From the macroscopic coupled mechanical equations of motion, electromagnetic equations and constitutive equations, the acoustic-electromagnetic power balance in its global form in a porous medium is derived. The time rate at which the sources deliver mechanical and electromagnetic work to the coupled acoustic, electromagnetic disturbances is shown to equal the time rate of kinetic energy plus the time rate of deformation energy plus the time rate of coupled field energy plus the time rate of total stored electromagnetic energy in a volume plus the acoustic-electromagnetic powerflow through a surface bounding the volume.
Conditions are derived for uniqueness of solution of the coupled field equations. Particular attention is paid to continuity requirements at an interface where a jump in macroscopic medium parameters occurs. Electroseismic boundary conditions at singular jumps in the macroscopic material properties are determined. This defines a displacement-stress-EM wavefield vector whose components are continuous through a contrast. The governing electroseismic equations are transformed into a field-vector-formalism using a plane wave solution procedure. Wavefield eigenvectors and associated eigenvalues (wavefield slownesses) are derived for up and down going waves in porous media (i.e. fast compressional wave, slow compressional wave, rotational waves, and electromagnetic waves). To guarantee a consistent macroscopic theory of energy transfer, conservation of electromagnetic and poro-elastic Poynting power upon crossing an interface is imposed by energy normalisation of the up and down going vectors. The macroscopic governing equations, transformed into a field vector formalism are numerically solved using the very stable global matrix method. The effects of both pressure and shear seismic waves traversing a mechanical and/or electrical contrast are studied. At contrasts, when seismic waves cross, dynamic current imbalances are induced which generate radiating electromagnetic disturbances. Amplitude versus offset and sensitivity to macroscopic medium properties of the seismic to electromagnetic signals are studied.

Relative flow and displacement Green’s functions in dyadic form are derived. The final dyadic form is similar in form to the isotropic elastic dyadic Green’s function. Source jump representations are derived for an explosive and vertical point source using the developed Green’s tensors together with the deformation equations of the two phases. The explosive point source, which generates P waves only, and the vertical point source, which generates both P and SV waves, are compared with respect to their conversion behavior into electromagnetic disturbances in layered porous medium.

The Biot-Rosenbaum model is extended by including the effect of a heteroge-
neous porous formation surrounding the borehole and by including the conversions of mechanical into electromagnetic waves at the mechanical and/or electrical contrasts in the poro-elastic formation. The method to solve the extended Biot-Rosenbaum problem is formulated as a boundary element technique. The singular properties of the Green's functions are determined analytically using static Green's functions to regularize the integrals. This is necessary to calculate the element's self interaction.

Two real electroseismic datasets are interpreted using the developed electroseismic theory and numerical algorithms. Synthetics in Transpose Vertical ElectroSeismic Profiling (TVESP) geometry are generated to identify the electroseismic responses in borehole geometry from a soil-glacial till and glacial-till-bedrock interface. The synthetics in surface seismic geometry are calculated to extract converted electric field amplitudes versus dipole antenna offset which is compared against the real measured data result.

Through these extensive investigations and modeling we have obtained the following conclusions,

**Electrokinetic Streaming Currents in Homogeneous Porous Medium**

The stationary phase method allows a quick computation of streaming current radiation patterns, useful to investigate the amplitude variability with respect to medium properties. No frequency restrictions are required making the method useful to investigate the amplitude variability at different source center frequencies (provided that the center wavelength is greater than the grain sizes). The mechanically induced relative flow is decoupled from the electrical induced relative flow to determine the dynamic streaming currents generated by seismic point sources. The streaming current induced by electrically induced relative flow is shown to be second order in electrokinetic coupling coefficient and is therefore a second order effect and neglected. The streaming current increases monotonically with increasing porosity for all three source types (volume injection source, explosive, and vertical point source applied on
the frame). The streaming current decreases with increasing electrolyte molarity for all three source types, consistent with the decrease of electrical double layer thickness with increasing salinity.

The streaming current behavior with respect to permeability differs per source type and with source center frequency. The volume injection source, force applied on the fluid phase, has a different streaming current response versus permeability than the explosive and vertical point source, with forces applied on the solid phase.

The streaming current magnitude induced by the $P$ waves generated by a vertical point source is smaller than the streaming current sheet magnitudes generated by $S$ waves. The amplitude factor between $P$ and $S$ wave induced streaming currents is porosity dependent. The $P$ wave induced streaming current is a factor 10 smaller than the $S$ wave induced streaming current in a medium with a 20 percent porosity. The relative change in streaming current sheet magnitudes versus porosity and salinity is less at high frequencies than at low frequencies. A large induced mechanical streaming current does not imply a large conversion upon traversing a contrast in macroscopic medium properties.

The streaming current magnitude as function of permeability changes more rapid with frequency than the relative flow amplitude for all three point sources, while the opposite, a more rapid change in relative flow magnitude behavior with frequency, is observed as function of salinity and porosity.

**Electroseismic Waves in Heterogeneous Porous Medium**

The coupled electromagnetics and acoustics of isotropic poro-elastic media decouple into $PSVTM$ and $SHTE$ electroseismic wavefield pictures. The induced current motion plane determines to which electromagnetic polarization mode the mechanical waves are coupled. Compressional waves traveling through a homogeneous porous medium cause pressure gradients which cause charge to separate. This induces within the seismic pulse a system of electric fields that travel with compressional wavespeed.
Rotational waves traveling through a homogeneous porous medium cause grain accelerations which set up current sheets. This induces within the seismic pulse mainly magnetic fields that travel with the rotational wavespeed. Therefore, when the seismic pulse passes an antenna, an electric field is recorded inside the $P$ wave pulse and a magnetic field is recorded inside the $S$ wave pulse. The seismic pulse doesn't radiate electromagnetic waves away from the pulse.

Radiating electromagnetic wavefields are converted from seismic waves, however, when contrasts in mechanical and/or electrical properties are traversed. The principal features of the converted electromagnetic signals are: (1) contacts all antennas at approximately the same time; (2) arrives at the antennas at half of the seismic twoway travel time from source to interface; and (3) changes sign on opposite sides of the shot. The frequency content of the converted electromagnetic field has the same frequency content of the driving incident seismic pulse, as long as the propagation distances are much less than the electromagnetic skin depth.

Two mechanical source jump representations for an explosive and vertical point source acting on the elastic frame are derived using the developed poro-elastic Green's tensors. The different point sources are included in the global field vector formalism which is solved numerically. The mechanical vertical point source is shown to generate radiating electromagnetic wavefields. The vertical point source is believed to induce a step in the vertical streaming current which imposes a jump in the electromagnetic fields to accommodate for the predefined zero magnetic and electric field components (no electromagnetic source is applied) in the vertical jump source representation across a source plane. The direct and reflected electromagnetic wavefield amplitudes versus offset suggest an associated electric dipole behavior for a vertical point force. The conversion to electromagnetic waves from mechanical waves traversing boundaries is found to be mainly due to pressure gradients generated by the $P$ waves across the contrast. Even though rotational waves, generated by the vertical point source, induce larger streaming currents, they do not necessarily create larger
streaming current imbalances across an interface. The radiation pattern orientation with respect to the interface determines the magnitude of the electroseismic conversion from $S$ waves.

**Electroseismic Effects From a Borehole Source Embedded in a Layered Medium**

The *VES*P modeling results show rapid decay of the converted electromagnetic signals with traveled distance. The antennas close to the target of interest show larger amplitudes in converted signals than the induced electromagnetic signals inside the seismic pulse. This suggested the possible application of electroseismsics in borehole geometry. Therefore, a *BEM* modeling method is developed to simulate the propagation of seismic and converted electromagnetic waves generated by a mechanical borehole source embedded in a layered medium. Since 4 boundary conditions have to be complied with at the borehole wall, 4 fictitious sources need to be introduced. The *P* wave radiation inside the borehole is uniquely described by a fluid volume injection source. The $P_f - SV - P_s$ wave radiation in the poro-elastic formation are determined by the vertical, radial ring sources applied on the frame and an explosive ring source applied on the fluid phase. The sum of the weighted ring sources determine the wavefields inside the borehole and the outside formation. The weighting of the ring sources is determined by solving a linear system of equations that satisfy the mechanical boundary conditions on the borehole wall. The total calculation on the nCUBE wasn't fast enough to study full waveform borehole solutions. Therefore, the electroseismic radiation of the three ring sources (kernel functions of the *BEM* algorithm) in poro-elastic media are compared. The vertical and radial ring sources generate radiating electromagnetic waves away from the mechanical source. The conversion to electromagnetic disturbances is found to be mainly due to pressure gradients generated by the *P* waves across the contrasts for the vertical and explosive ring source. Whereas, in the radial ring source case, the *SV* wave induced current
imbances across the contrast account for the main conversions to electromagnetic disturbances.

**Real Electroseismic Data**

The response of blasting caps in a borehole recorded at the surface is interpreted using the developed algorithms. Three distinct electroseismic conversions of two different conversion mechanisms are identified in this dataset and predicted in the modeling. Two electroseismic conversions are conversions from a $P$ wave pulse from the explosive point source into electromagnetic signals, when traversing the two different interfaces positioned above and below the source. The third electroseismic conversion is generated by a $P$ wave that travels along an interface separating two media with large velocity contrast. While the mechanical disturbance travels along the interface, a moving current imbalance is generated across the interface, causing electromagnetic radiation.

The sledgehammer responses recorded at the surface with dipole antennas are compared against electroseismic synthetics in surface geometry. The modeling predicts the antenna offset at which the maximum converted electric field amplitude occurred. The amplitude drop off rate is underestimated by the model, which is contributed mainly to the different radiation pattern of a vertical impact source which was modeled as an explosive point source.

**Expected Converted Electromagnetic Field Amplitudes**

The converted electromagnetic wavefield components caused by a 1 $kg$ dynamite source in stiff sandstones, characterized by large frame and bulk moduli ($3000m/s < v_p < 4000m/s$), are expected to generate a measurable electric field amplitude in the $\mu V/m$ range and a magnetic field amplitude in the $nA/m$ range when the traveled distance from contrast to dipole antenna is larger than 20 $m$. However, in loose, squishy sandstones, characterized by smaller frame moduli ($300m/s < v_p < 1000m/s$), a 0.25 $kg$ dynamite source is expected to generate a measurable electric field amplitude in
the $mV/m$ range and magnetic field amplitudes in the $\mu V/m$ range when the traveled distance from contrast to dipole antenna is smaller than 20 m.

**Sensitivity of Converted Electromagnetic Signal on Lithology**

A jump in medium properties across an interface will affect the dynamic current imbalance across this interface and generate a radiating converted electromagnetic field. A porosity contrast contributes to a current imbalance through a difference in conductivity with porosity, $\sigma(\omega)$, a difference in electrokinetic coupling coefficient with porosity, $L(\omega)$, and a difference in the amount of induced relative flow with porosity, $-\nabla P + \omega^2 \rho_f u_s$, which, substituted into the transport equation relating current flow to electrical potential and pressure gradients, results in this current imbalance. A salinity contrast contributes to the current imbalance through the salinity effect on conductivity and electrokinetic coefficient (function of $\zeta$—potential which is dependent on salinity). The frame moduli, which determine the squishiness of the elastic frame, controls the amount of induced relative flow and, therefore, the induced electrical streaming current on both sides of the contrast in frame moduli.

A permeability contrast (of the permeable sand-impermeable shale type) contributes to the current imbalance through a difference in conductivity, since the pore length parameter, $\Lambda$ depends on permeability. The permeability effect on induced relative flow is second order, its effect is only an equilibration of the pressure gradient due to fluid transport. The electrical conductivity increases with decreasing permeability. At very small permeabilities the surface conductances become important and dominate in value the conductivity contribution of the bulk conductivity in the fluid phase. This situation occurs when the pore space is filled with clay, reducing the permeability of the formation, not affecting the porosity, but greatly enhancing the conductances along the pore surfaces. When mechanical waves traverse such a permeability contrast, the induced electric fields at both sides of the interface experience a different electrical conductivity and create a dynamic current imbalance which causes electromagnetic radiation.
8.1 Future work

The coupled electromagnetic and acoustic wavefield propagation in a poro-elastic medium is an involved and challenging problem since electromagnetic wave theory, poro-elastic wave and medium theory, and electrical/hydraulic transport in two phase media need to be addressed. Each individual topic is an on-going research topic in itself and, therefore, there is enough room to improve, modify, simplify/approximate or generalize to more complicated media, the electroseismic problem. Solving the coupled forward wavefield problem has shown the influence of lithological (transport) parameters on the wavefield components. A systematic study needs to be performed on the sensitivity of macroscopic medium properties (permeability and salinity) on the converted electromagnetic wavefield components.

Frechet derivatives with respect to fluid salinity and permeability are derived and await implementation on the nCUBE. I developed an inversion scheme which would invert for lithological parameters in layered poro-elastic media. A "joint" inversion, using both seismic and electromagnetic wavefield information, would be performed by minimizing the reference data and the modeled data using a L2 norm (analogous to an inversion scheme discussed in Zhao (1994)).

The developed BEM code needs to be optimized, which is possible, to decrease its runtime on the nCUBE. This would then allow the full waveform modeling of seismic and converted electromagnetic waves in a borehole embedded in a layered poro-elastic medium. Stoneley waves and converted electromagnetic signals contain information about permeability. The converted electromagnetic signals would provide information about fluid chemistry contrasts in a homogeneous reservoir. The development of an acoustic-electric logging tool would be required to acquire data to test the theoretical model and to guide further logging developments.

At ERL/MIT we began collecting electroseismic field data at two different sites with a 24-channel, 36-bit field data acquisition system. We are experimenting with
different recording geometries to obtain the optimal quality data. We can very effectively remove powerline noise and tellurics using the signals from two perpendicular dipoles located far from the source. In the data collected so far, we have observed mechanical wave induced electromagnetic signals and both types of electroseismic conversions at a shallow boundary with large velocity contrast. We will continue these studies to obtain better resolution data at greater depths. Electroseismic filtering and other signal processing techniques need to be developed. This data processing scheme developed in PROMAX™ is geared to enhance the electroseismic conversions and will facilitate the interpretation of the electroseismic data. The processing scheme needs to be tested first on synthetic data to quantify its performance before it is applied to real data.

The numerical results discussed in chapter 2 can be used to design an experiment in ERL's ultrasonic laboratory to measure dynamic streaming currents caused by point sources/transducers in porous rock. The advantage of measuring streaming currents is not to have to know the surface conductances to determine the $\zeta$-potential. An experiment analysed with the developed stationary-phase streaming current amplitude prediction versus salinity or permeability can be used to invert for permeability for instance from dynamic streaming current measurements.

An ultrasonic (center frequency transducer is 100 k$Hz$) experiment has been carried out to measure the electroseismic response in a layered artificial porous medium sample. Electroseismic conversions from both $P$ and $SV$ waves traversing a contrast have been measured and modeled at these high frequencies (Haartsen et al., 1995). This disproves the importance of a conversion of the diffusive Biot slow wave pressure gradient into an electric field, since we are in the inertial flow regime at these frequencies, where the slow wave just becomes another propagatory mode. The transport coefficients $k(\omega)$ and $L(\omega)$ are now in a different frequency relaxation regime and its effect on the electroseismic conversions and body wave induced electromagnetic fields need further study.
The reciprocal effect, where an electromagnetic disturbance propagates and the electric field will act on the charge excesses in the electrical double layer producing pressure gradients in the fluid and, in principal, macroscopic mechanical disturbances, have not been studied in this thesis. This osmosis phenomenon can readily be numerically studied using the developed theory and numerical algorithms and can possibly be measured in a carefully designed laboratory experiment.
References


Burridge, R. and C. Vargas, The fundamental solution in dynamic poroelasticity, 


Chin, R., G. Hedstrom, and L. Thigpen, Matrix methods in synthetic seismograms, 


Paillet, F., C. Cheng, and X. Tang, Theoretical models relating acoustic tube wave attenuation to fracture permeability - reconciling model results with field data,


Appendix A

ElectroSeismic Field-Vector Formalism with Powerflow Normalized Eigenvectors

A linear transformation of the vector $B^{(PSVTM)}$ is performed, through it a field-vector-formalism is obtained in which a decomposition of $B^{(PSVTM)}$ into up and downgoing fields is manifest. Let $W^{(PSVTM)}$ be the 8 by 1 vector that is related to $B^{(PSVTM)}$ by the linear transformation,

$$B^{(PSVTM)} = D^{(PSVTM)} W^{(PSVTM)}$$  \hspace{1cm} (A.1)

The $D^{(PSVTM)}$ is the eigencolumn matrix of system matrix $A^{(PSVTM)}$ \((\frac{\partial}{\partial z} B = AB)\) and defined as,

$$D^{(PSVTM)} = \begin{bmatrix} b_j^{(m;P_f)}, b_j^{(m;-P_f)}, b_j^{(m;-P_s)}, b_j^{(m;-SV)}, b_j^{(m;-TM)}, b_j^{(m;+P_f)}, b_j^{(m;+P_s)}, b_j^{(m;+SV)}, b_j^{(m;+TM)} \end{bmatrix}$$  \hspace{1cm} (A.2)

301
The superscript denote downgoing eigenvectors and the + superscript denote upgoing wavefield eigenvectors. The wavefield eigenvector $\mathbf{B}^{(PSVTM)}$ is derived to be,

$$
\mathbf{B}^{(PSVTM)} = [u_x, u_z, w_z, \tau_{xz}, \tau_{zx}, S, H_y, E_z]^T
$$

(A.3)

The up and downgoing compressional eigenvectors, bulk displacement normalized, are given in the following matrix,

$$
\begin{bmatrix}
\begin{array}{cc}
pv_{pf} & pv_{ps} \\
\pm qv_{pf} & \pm qv_{ps} \\
\mp \frac{\rho_B - \frac{H}{v_{pf}^2}}{\rho_f - \frac{C}{v_{pf}^2}} qv_{pf} & \mp \frac{\rho_B - \frac{H}{v_{ps}^2}}{\rho_f - \frac{C}{v_{ps}^2}} qv_{ps} \\
\pm 2i\omega pqv_{pf}G & \pm 2i\omega pqv_{ps}G
\end{array}
\end{bmatrix}
\begin{bmatrix}
b^{(m;P_f)}_j \\
b^{(m;P_s)}_j
\end{bmatrix}
\end{equation}

(A.4)

With the compressional velocities determined by,

$$
2\left[\frac{HM - C^2}{v^2}\right] = \rho_B M + \rho_E \left[1 + \frac{L^2 \rho_E}{\epsilon}\right] H - 2 \rho_f C
$$

$$
\pm \left[\left(\rho_B M + \rho_E \left[1 + \frac{L^2 \rho_E}{\epsilon}\right] H - 2 \rho_f C\right)^2
- 4 \left(HM - C^2\right) \left(\rho_E \rho_B \left[1 + \frac{L^2 \rho_E}{\epsilon}\right] - \rho_f^2\right)\right]^{1/2}
$$

(A.5)

The + sign denotes the fast compressional $(P_f)$ wavefield velocity, $v_{pf}$, and the - sign denotes the slow compressional $(P_s)$ wavefield velocity, $v_{ps}$.
The up and downgoing shear and TM wavefield eigenvectors, bulk displacement normalized, are given in the following matrix,

$$
\begin{pmatrix}
\pm qv_{sv} & \pm qv_{tm} \\
-pv_{sv} & -pv_{tm} \\
-\frac{pv_{sv}G}{\rho_f} \left[ \frac{1}{v_{sv}^2} - \frac{\rho E}{G} \right] & -\frac{pv_{tm}G}{\rho_f} \left[ \frac{1}{v_{tm}^2} - \frac{\rho E}{G} \right] \\
izv_{sv} \left[ q^2 - p^2 \right] G & izv_{tm} \left[ q^2 - p^2 \right] G \\
\mp 2i\omega qpv_{sv}G & \mp 2i\omega qpv_{tm}G \\
0 & 0 \\
-\frac{izv_{sv}LG_{\mu_E}}{\rho_f} \left[ \frac{1}{v_{sv}^2} - \epsilon \mu \right] & -\frac{izv_{tm}LG_{\mu_E}}{\rho_f} \left[ \frac{1}{v_{tm}^2} - \epsilon \mu \right] \\
\mp izv_{sv}LG_{\mu} \left[ \frac{1}{v_{sv}^2} - \epsilon \mu \right] & \mp izv_{tm}LG_{\mu} \left[ \frac{1}{v_{tm}^2} - \epsilon \mu \right]
\end{pmatrix}
$$

(A.6)

With the shear and TM wavefield velocities determined by,

$$
\frac{2}{v^2} = \frac{\rho_B - \rho_f^2/\rho_E}{G} + \epsilon \mu_0 + L^2 \rho_E \mu_0
$$

$$
\pm \left[ \frac{\rho_B - \rho_f^2/\rho_E}{G} - \left( \epsilon \mu_0 + L^2 \rho_E \mu_0 \right) \right]^2 - 4\frac{\rho_f^2 L^2 \mu_0 G}{G} \right]^{1/2}
$$

(A.7)

The + sign denotes the shear (SV, SH) wavefield velocity and the - sign denotes the EM wavefield velocity. If a normalization by vertical power flow is preferred the following normalization factors for each wave type need to be used.

**The SV and TM vertical power flux**

$$
\langle S_x \rangle = \frac{\omega^2}{4} G \left[ \left( \frac{q v}{v^*} + \frac{q^* v}{v} \right) \left( \frac{G \mu}{\rho_f^2} \rho E \rho E^* L L^* \gamma_T \gamma_T^* + 1 \right) \right]
$$

(A.8)

**The Pf and P, vertical power flux**

$$
\langle S_x \rangle = \frac{\omega^2}{4} \left[ \left( \frac{q v}{v^*} + \frac{q^* v}{v} \right) \left( H + (\gamma_L + \gamma_L^*) C + \gamma_L \gamma_L^* M \right) \right]
$$

(A.9)

Where $\gamma_T$ and $\gamma_L$ are defined as,

$$
\gamma_T = -\left[ \frac{1}{v_{sv}^2} - \frac{\rho E}{G} \right]
$$

(A.10)
\[ \gamma_L = -\frac{\rho_B - \frac{H}{\sqrt{\gamma}}}{\mu_f - \frac{C}{\nu^2}} \]  \hspace{1cm} (A.11)

To obtain the normalization with respect to time-averaged Poynting power perpendicular to the plane wave front, we dot equations (Eq A.8) or (Eq A.9) into the real part of the wave direction vector, which is \( \text{Re}[pv, 0, qv] \). The velocity \( v \) determines which eigenvector wave type is meant, \( * \) denotes the complex conjugate and \( q \) is the vertical slowness and \( p \) is the horizontal slowness of a wave type.
Appendix B

Derivation of Dynamic Displacement Green’s Functions

B.1 Displacement Green’s Function with Force on Solid

To solve for the displacement scalar shear Green’s function equation (Eq 5.11) is rewritten into the form

\[ w_t = \frac{-1}{\omega^2 \rho_E} \left( \omega^2 \rho_f u_t + C \left( \nabla_t \cdot u_t + \frac{\partial}{\partial z} \nabla_t u_z \right) + M \left[ \nabla_t \cdot w_t + \frac{\partial}{\partial z} \nabla_t w_z \right] \right) \quad (B.1) \]

The transverse relative flow vectors in equation (Eq 5.9) are substituted by equation (Eq B.1). This new equation can be solved by taking the transverse curl, defined as

\[ \nabla_t \times = \left[ \nabla - \frac{\partial}{\partial z} \hat{z} \right] \times \] on both sides. The gradient terms disappear because \( \nabla_t \times \nabla_t u_t = 0 \). The following equation is obtained,

\[ G \nabla_t \nabla_t \cdot u_t + G \frac{\partial^2}{\partial z^2} \nabla_t \times u_t + \omega^2 \left( \rho_B - \frac{\rho_f^2}{\rho_E} \right) \nabla_t \times u_t = -\nabla_t \times F_t \quad (B.2) \]

305
The solution of this equation in terms of the transverse curl of \( u_t \), using Green’s superposition theorem is,
\[
\nabla_t \times u_t = \int_{\mathcal{V}} g^u(x, x') \nabla'_t \times F'_t \, dx'
\]  
(B.3)

Where \( x = (x, y, z) \) is the receiver location and \( x' = (x', y', z') \) is the source location. \( g^u(x, x') \) is the Green’s function of the scalar wave equation,
\[
\nabla^2_t g^u + \frac{\partial^2}{\partial z^2} g^u + \frac{\omega^2 \left( \rho_B - \rho_E^2 \right)}{G} g^u = \frac{-1}{G} \delta(x - x')
\]  
(B.4)

If \( e^{-i\omega t} \) time dependence is assumed for the wavefield, the 3D Green’s function \( g \) is,
\[
g^u(x, x') = \frac{e^{ik_0 R}}{4\pi GR}
\]  
(B.5)

Where \( R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \) is the distance from the source to the receiver and \( k_0 = \omega / \sqrt{G/(\rho_B - \rho_E^2)} \) is the wave number.

To solve for the \( P_f - P_s - SV \) scalar Green’s functions we have to rewrite the 4 by 4 differential operator matrix, equation (Eq 5.18), into a 2 by 2 differential operator matrix operating on the displacement components. This system has the following form,
\[
\begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix}
\begin{bmatrix}
\nabla_t \cdot u_t \\
\frac{\partial}{\partial z} u_z
\end{bmatrix}
= \begin{bmatrix}
\nabla_t \cdot F_t \\
\frac{\partial}{\partial z} F_z
\end{bmatrix}
\]  
(B.6)

Where the variables \( K_{11}, K_{12}, K_{21}, K_{22} \) are defined as,
\[
K_{11} = R_z + \Theta \left[ L_z^C \left( L_t^M L_z^C - M C \nabla_t^2 \frac{\partial^2}{\partial z^2} \right) 
+ C \nabla_t^2 \left( -M \frac{\partial^2}{\partial z^2} L_z^C + C L_z^M \frac{\partial^2}{\partial z^2} \right) \right]
\]  
(B.7)

\[
K_{12} = (H - G) \nabla_t^2 + \Theta \left[ L_z^C \left( C L_t^M \nabla_t^2 - M \nabla_t^2 L_t^C \right) 
+ C \nabla_t^2 \left( -M C \frac{\partial^2}{\partial z^2} \nabla_t^2 + L_t^C L_z^M \right) \right]
\]  
(B.8)
\[ K_{21} = (H - G) \frac{\partial^2}{\partial z^2} + \Theta \left[ C \frac{\partial^2}{\partial z^2} \left( L_t^M L_z^C - M C \nabla_i^2 \frac{\partial^2}{\partial z^2} \right) \
+ L_t^C \left( -M \frac{\partial^2}{\partial z^2} L_z^C + C L_t^M \frac{\partial^2}{\partial z^2} \right) \right] \tag{B.9} \]

\[ K_{22} = R_t + \Theta \left[ C \frac{\partial^2}{\partial z^2} \left( C L_t^M \nabla_i^2 - M L_t^C \nabla_i^2 \right) \right. \n\left. + L_t^C \left( -M C \frac{\partial^2}{\partial z^2} \nabla_i^2 + L_t^C L_z^M \right) \right] \tag{B.10} \]

where,

\[ \Theta = \frac{-1}{L_t^2 L_t^M - M^2 \nabla_i^2 \frac{\partial^2}{\partial z^2}} \tag{B.11} \]

In the modified Kupradze method, the unknowns of the system are expressed in terms of the adjoint of the system matrix, which is the transpose of its cofactor matrix. The product of a matrix with its adjoint is the identity matrix scaled by its determinant. Performing the modified Kupradze method yields,

\[ \left[ \begin{array}{c} \nabla_t \cdot u_t \\ \frac{\partial}{\partial z} u_z \end{array} \right] = \int_V \left[ \begin{array}{cc} K_{22} & -K_{12} \\ -K_{21} & K_{11} \end{array} \right] \phi(x, x') \left[ \begin{array}{c} \nabla_t' \cdot F_t' \\ \frac{\partial}{\partial z'} F_z' \end{array} \right] dx' \tag{B.12} \]

Substituting equation (Eq B.12) into equation (Eq B.6) yields,

\[ [K_{11}K_{22} - K_{12}K_{21}] \phi(x, x') = -\delta(x, x') \tag{B.13} \]

Substituting equations (Eq 5.19), (Eq 5.20) and (Eq 5.21) into equations (Eq B.7), (Eq B.8), (Eq B.9) and (Eq B.10) and carrying out the algebra the following expressions are obtained,

\[ K_{11} = L_z + \frac{\Gamma}{\Xi} \nabla_t^2, \quad K_{12} = \left[ \Psi + \frac{\Gamma}{\Xi} \right] \nabla_t^2 \tag{B.14} \]

\[ K_{21} = \left[ \Psi + \frac{\Gamma}{\Xi} \right] \frac{\partial^2}{\partial z^2}, \quad K_{22} = L_t + \frac{\Gamma}{\Xi} \frac{\partial^2}{\partial z^2} \tag{B.15} \]

Where the introduced variables, \( \Gamma, \Psi, \Xi \) and \( L_t, L_z \) are defined to be,

\[ \Gamma = -2\omega^2 \rho_f C + M \omega^2 \frac{\rho_f^2}{\rho_E} + \frac{C^2}{M} \omega^2 \rho_E \tag{B.16} \]

307
\[ \Psi = \frac{1}{M} \left[ HM - C^2 - GM \right] \]  

\[ \Xi = M \nabla_t^2 + M \frac{\partial^2}{\partial z^2} + \omega^2 \rho_E \]  

\[ L_t = G \nabla_t^2 + \frac{1}{M} \left[ HM - C^2 \right] \frac{\partial^2}{\partial z^2} + \omega^2 \left[ \rho_B - \frac{\rho_I}{\rho_E} \right] \]  

\[ L_z = \frac{1}{M} \left[ HM - C^2 \right] \nabla_t^2 + G \frac{\partial^2}{\partial z^2} + \omega^2 \left[ \rho_B - \frac{\rho_I}{\rho_E} \right] \]  

The scalar equation (Eq B.13) can be solved using the Fourier transform method. Next we perform the \( k_z \) integral using Cauchy’s theorem. Carrying out similar steps as in the relative flow Green’s function derivation, the following Green’s tensor expressed in dyadic form is obtained,

\[
\begin{align*}
G &= g^u I + \hat{z} \hat{z} \left( \left[ L_z + \frac{\Gamma}{Z} \nabla_t^2 \right] \phi - g^u \right) \quad - \left[ \Psi + \frac{\Gamma}{Z} \right] \frac{\partial}{\partial z} \left[ \nabla_t \hat{z} + \hat{z} \nabla_t \right] \phi \\
&\quad + \frac{\nabla_t \nabla_t}{\nabla_t^2} \left( \left[ L_t + \frac{\Gamma}{Z} \frac{\partial^2}{\partial z^2} \right] \phi - g^u \right)
\end{align*}
\]  

Where \( g^u, \phi, \Gamma, \Xi, \Psi, L_z, L_t \) are defined in equations (Eq B.5), (Eq B.22), (Eq B.16), (Eq B.17), (Eq B.18), (Eq B.19) and (Eq B.20) respectively.

Using the Sommerfeld representation of a point source, \( \phi \) can be simplified into the following form.

\[ \phi(\mathbf{u}) = B(\mathbf{u}) g_\beta + A_1(\mathbf{u}) g_{1\alpha_1} + A_2(\mathbf{u}) g_{1\alpha_2} \]

\[ = B(\mathbf{u}) \frac{e^{ik_\beta R}}{4\pi R} + A_1(\mathbf{u}) \frac{e^{ik_{1\alpha_1} R}}{4\pi R} + A_2(\mathbf{u}) \frac{e^{ik_{1\alpha_2} R}}{4\pi R} \]  

With the introduced variables \( B(\mathbf{u}), A_1(\mathbf{u}), A_2(\mathbf{u}) \), where \( (\mathbf{u}) \) denotes scalar Green’s function amplitudes belonging to the displacement Green’s function with a source applied on the solid frame, defined as,

\[ B(\mathbf{u}) = \frac{1}{G \left[ HM - C^2 \right]} \frac{-k_\beta^2 M + \omega^2 \rho_E}{\left( k_\beta^2 - k_{1\alpha_1}^2 \right) \left( k_\beta^2 - k_{1\alpha_2}^2 \right)} \]  

\[ A_1(\mathbf{u}) = \frac{1}{G \left[ HM - C^2 \right]} \frac{-k_{1\alpha_1}^2 M + \omega^2 \rho_E}{\left( k_{1\alpha_1}^2 - k_\beta^2 \right) \left( k_{1\alpha_1}^2 - k_{1\alpha_2}^2 \right)} \]  

\[ A_2(\mathbf{u}) = \frac{1}{G \left[ HM - C^2 \right]} \frac{-k_{1\alpha_2}^2 M + \omega^2 \rho_E}{\left( k_{1\alpha_2}^2 - k_\beta^2 \right) \left( k_{1\alpha_2}^2 - k_{1\alpha_1}^2 \right)} \]  

308
and \( g_\beta, g_{\alpha_1}, g_{\alpha_2} \) are the scalar Green’s functions of the following scalar wave equations.

\[
\nabla^2 g_i + k_i^2 g_i = -\delta(x - x') \quad \text{with,} \quad i = \beta \vee \alpha_1 \vee \alpha_2
\]  

(B.25)

When equations (Eq B.5), (Eq B.22), (Eq B.16), (Eq B.17), (Eq B.18), (Eq B.19) and (Eq B.20) are substituted into equation (Eq B.21) the following expression is obtained,

\[
G = g^u I + \left[ G - \frac{1}{M} (HM - C^2) \right] \nabla \nabla \phi - \left[ -2 \omega^2 \rho_f C + M \omega^2 \frac{\rho_1^2}{\rho_E} + \frac{C^2}{M \omega^2 \rho_E} \right] \nabla \nabla \phi^* \\
+ \left[ \hat{z} \frac{\nabla \nabla \phi}{\nabla \nabla t} \right] \frac{1}{M} (HM - C^2) \nabla^2 \phi + \frac{-2 \omega^2 \rho_f C + M \omega^2 \frac{\rho_1^2}{\rho_E} + \frac{C^2}{M \omega^2 \rho_E}}{M \nabla^2 t + M \frac{\rho_1^2}{\omega^2 \rho_E} + \omega^2 \rho_E} \nabla^2 \phi \\
+ \omega^2 \left( \rho_B - \frac{\rho_1^2}{\rho_E} \right) \phi - g^u
\]  

(B.26)

Where \( \phi^* \) is defined as,

\[
\phi^* = B^* (us) g_\beta + A^*_1 (us) g_{\alpha_1} + A^*_2 (us) g_{\alpha_2}
\]  

(B.27)

With the variables \( B^* (us), A^*_1 (us), A^*_2 (us) \) (variables \( B (us), A_1 (us), A_2 (us) \) divided by \( \Xi \) ) defined as,

\[
B^* (us) = \frac{B (us)}{\omega^2 \rho_E - M k_\beta^2}, \quad A^*_1 (us) = \frac{A_1 (us)}{\omega^2 \rho_E - M k_{\alpha_1}^2}, \quad A^*_2 (us) = \frac{A_2 (us)}{\omega^2 \rho_E - M k_{\alpha_2}^2}
\]  

(B.28)

Substituting \( g^u \), equation (Eq B.5) and \( \phi \), equation (Eq B.22) into equation (Eq B.26) and using equations (Eq B.25), (Eq B.23) and (Eq B.28) yields the dynamic displacement Green’s function,

\[
G^u_s = g^u I + \left[ G - \frac{1}{M} (HM - C^2) \right] \nabla \nabla \phi - \left[ -2 \omega^2 \rho_f C + M \omega^2 \frac{\rho_1^2}{\rho_E} + \frac{C^2}{M \omega^2 \rho_E} \right] \nabla \nabla \phi^*
\]  

(B.29)

Where the superscript \( u \) denotes the Green’s function to be a displacement Green’s function and subscript \( s \) denotes a point force applied to the solid matrix. The
dynamic displacement Green's function rewritten in compact form yields,

\[ G_s^u = g_s^u \frac{\partial}{\partial t} + \nabla \nabla T_s^u \]

\[ T_s^u = \sum_{i \neq \beta \neq \alpha} \frac{\Psi (k^2 M - \omega^2 \rho_E) - \Gamma}{G (H M - C^2) (k_i^2 - k_f^2) (k_i^2 - k_m^2)} g_i \]  \hspace{1cm} (B.30)

**B.2 Displacement Green's Function with Force on Fluid**

The derivation of the displacement Greens's function due to a fictitious source \( f \) on the fluid phase (with \( f = \nabla S \) and \( S \) a pressure source) is identical to the derivation of the relative flow Green's function due to a force on the solid matrix. To obtain the displacement Green's function due to a pressure source the divergence with respect to the source coordinates of equation (Eq 5.60) has to be taken (and use relation \( \nabla' = -\nabla \)). After some algebraic manipulations we obtain,

\[ G_f^u = \nabla \cdot G_s^u \]

\[ = \frac{1}{C} \left[ C^2 - H M + G C \frac{\rho_E}{\rho_f} \right] \nabla \Pi \]

\[ + \left[ \frac{H M}{C} \omega^2 \rho_f + C \omega^2 \frac{\rho_B \rho_E}{\rho_f} - H \omega^2 \rho_E - M \omega^2 \rho_B \right] \nabla \Pi^* \]  \hspace{1cm} (B.31)

Where \( \Pi \) and \( \Pi^* \) are defined as,

\[ \Pi = - \left[ A_1(ws) k_{\alpha_1}^2 \frac{e^{ik_{\alpha_1} R}}{4\pi R} + A_2(ws) k_{\alpha_2}^2 \frac{e^{ik_{\alpha_2} R}}{4\pi R} \right] \]  \hspace{1cm} (B.32)

\[ \Pi^* = - \left[ A_1^*(ws) k_{\alpha_1}^2 \frac{e^{ik_{\alpha_1} R}}{4\pi R} + A_2^*(ws) k_{\alpha_2}^2 \frac{e^{ik_{\alpha_2} R}}{4\pi R} \right] \]  \hspace{1cm} (B.33)

The variables \( A_1(ws), A_2(ws) \) and \( A_1^*(ws), A_2^*(ws) \) are defined in equations (Eq 5.55) and (Eq 5.58) respectively.

The displacement Green's function due to a pressure source rewritten in compact
form yields,
\[
\begin{align*}
\mathcal{G}_j^u &= \nabla \Upsilon_j^u \\
\Upsilon_j^u &= \sum_{i \neq j \neq m \in \alpha_1, \alpha_2} \frac{\psi \left( k_i^2 M - \omega^2 \rho_E \right) - \gamma}{G \left( H M - C^2 \right) \rho_f^E \left( k_i^2 - k_j^2 \right) \left( k_i^2 - k_m^2 \right)} k_i^2 g_i
\end{align*}
\quad (B.34)
\]

### B.3 Comments on the Derivations of the Green's Functions

#### B.3.1 Displacement Green's Function with Force on Solid

The derivation of the displacement Green's function with a source applied on the solid is started with equation (Eq B.21).

\[
\begin{align*}
\mathcal{G} &= g^u L \\
&+ \frac{\ddot{z}}{\ddot{z}} \bigg( \frac{1}{M} (H M - C^2) \nabla^2 \phi - \frac{1}{M} (H M - C^2) \frac{\partial^2}{\partial z^2} \phi \bigg) \\
&+ \omega^2 \left( \rho_B - \frac{\rho_f^2}{\rho_E} \right) \phi + \left( \frac{\Gamma}{\Xi} \nabla^2 \phi - \frac{\Gamma}{\Xi} \frac{\partial^2}{\partial z^2} \phi \right) - g^u
\end{align*}
\quad (B.35)
\]

With \( \Gamma, \Xi, \phi \) and \( g^u \) defined by equations (Eq B.16), (Eq B.18), (Eq B.22) and (Eq B.5) respectively. Equation (Eq B.35) can be rewritten into the form,

\[
\begin{align*}
\mathcal{G} &= g^u L \\
&+ \left( G - \frac{1}{M} (H M - C^2) - \frac{\Gamma}{\Xi} \right) \left[ \frac{\ddot{z}}{\ddot{z}} \frac{\partial^2}{\partial z^2} \phi + \frac{\partial}{\partial z} \left( \nabla_t \ddot{z} + \frac{\partial}{\partial z} \nabla_t \phi \right) + \nabla_t \nabla_t \phi \right]
\end{align*}
\]

11
\[ + \left( \frac{1}{M} (HM - C^2) \nabla^2 \phi + \omega^2 \left( \rho_B - \frac{\rho_I}{\rho_E} \right) \phi + \frac{\Gamma}{2 \Omega} \nabla^2 \phi - g^u \right) \nabla_i \nabla^i \]  

Evaluating the term preceding operator \[ \nabla_i \nabla^i \] yields,

\[ \frac{1}{M} (HM - C^2) \nabla^2 \phi + \omega^2 \left( \rho_B - \frac{\rho_I}{\rho_E} \right) \phi + \frac{\Gamma}{2 \Omega} \nabla^2 \phi - g^u = \]

\[ \frac{1}{M \nabla^2 + \omega^2 \rho_E} \left[ (HM - C^2) B(\omega s) \nabla^4 + \left( H \omega^2 \rho_E + M \omega^2 \rho_B - 2 \omega^2 \rho_f C \right) \nabla^2 + \omega^4 \left( \rho_{E \rho B} - \rho_I^2 \right) B - \frac{1}{G} \left( M \nabla^2 + \omega^2 \rho_E \right) \right] g_\beta + \]

\[ \frac{1}{M \nabla^2 + \omega^2 \rho_E} \left[ (HM - C^2) \nabla^4 + \left( H \omega^2 \rho_E + M \omega^2 \rho_B - 2 \omega^2 \rho_f C \right) \nabla^2 + \omega^4 \left( \rho_{E \rho B} - \rho_I^2 \right) \right] (A_1(\omega s) g_{\alpha_1} + A_2(\omega s) g_{\alpha_2}) \]

With \( B(\omega s), A_1(\omega s) \) and \( A_2(\omega s) \) defined by equation (Eq B.23). Substituting \( B(\omega s) \) into the term, defined by \( P_\beta \), preceding the \( \beta \) scalar Greens function and using the following identity,

\[ \left( k_\beta^2 - k_{\alpha_1}^2 \right) \left( k_\beta^2 - k_{\alpha_2}^2 \right) = \frac{\omega^4 \left( \rho_B - \frac{\rho_I^2}{\rho_E} \right)^2}{G^2} - \frac{\omega^2}{G} \left( \rho_B - \frac{\rho_I^2}{\rho_E} \right) \frac{\omega^2 \left( \rho_B M + \rho_E H - 2 \rho_f C \right)}{HM - C^2} + \frac{\omega^4 \left( \rho_{E \rho B} - \rho_I^2 \right)}{HM - C^2} \]

yields,

\[ P_\beta g_\beta = \frac{1}{\left( k_\beta^2 - k_{\alpha_1}^2 \right) \left( k_\beta^2 - k_{\alpha_2}^2 \right)} \frac{1}{G} \left[ \nabla^4 + \frac{\omega^2 \left( \rho_B M + \rho_E H - 2 \rho_f C \right)}{HM - C^2} \nabla^2 \right] \]

\[ + \frac{\omega^2 \left( \rho_B M + \rho_E H - 2 \rho_f C \right)}{HM - C^2} \frac{\omega^2}{G} \left( \rho_B - \frac{\rho_I^2}{\rho_E} \right) - \frac{\omega^4 \left( \rho_B - \frac{\rho_I^2}{\rho_E} \right)}{G^2} \right] g_\beta \]

Dividing equation (Eq B.39) through with \( \nabla^2 g_\beta + \frac{\omega^2}{G} \left( \rho_B - \frac{\rho_I^2}{\rho_E} \right) g_\beta \) results in,

\[ P_\beta g_\beta = \frac{1}{\left( k_\beta^2 - k_{\alpha_1}^2 \right) \left( k_\beta^2 - k_{\alpha_2}^2 \right)} \]

312
\[
\times \left[ \frac{1}{G} \nabla^2 + \frac{\omega^2 (\rho_B M + \rho_E H - 2 \rho_f C)}{G(HM - C^2)} - \frac{\omega^2}{G} \left( \rho_B - \frac{\rho_f^2}{\rho_E} \right) \right] \\
\times \left[ \nabla^2 g_\beta + \frac{\omega^2}{G} \left( \rho_B - \frac{\rho_f^2}{\rho_E} \right) g_\beta \right]
\]

(Eq. B.40)

Evaluating the \( \alpha \) scalar Greens functions part of equation (Eq. B.37) using the following identity,

\[
(HM - C^2) \nabla^4 + \omega^2 (\rho_B M + \rho_E H - 2 \rho_f C) \nabla^2 + \omega^4 (\rho_E \rho_B - \rho_f^2) \\
= \left( \nabla^2 + k_{\alpha_1}^2 \right) \left( \nabla^2 + k_{\alpha_2}^2 \right)
\]

(Eq. B.41)

yields,

\[
\left[ (HM - C^2) \nabla^4 + \omega^2 (\rho_B M + \rho_E H - 2 \rho_f C) \nabla^2 + \omega^4 (\rho_E \rho_B - \rho_f^2) \right] \\
\left[ A_1^*(us) g_{\alpha_1} + A_2^*(us) g_{\alpha_2} \right] / (HM - C^2) = \\
\left[ A_1^*(us) \left( \nabla^2 + k_{\alpha_1}^2 \right) \delta(x - x') + A_2^*(us) \left( \nabla^2 + k_{\alpha_2}^2 \right) \right] \delta(x - x')
\]

(Eq. B.42)

With \( A_1^*(us) \) and \( A_2^*(us) \) defined by equation (Eq. B.28). Where \( \nabla^2 + k_i^2 g_i = \delta(x - x') \), \( i = \alpha_1 \lor \alpha_2 \) has been applied. The term preceding operator \( [\ddot{z} \ddot{z} + \nabla \nabla^*] \) in equation (Eq. B.36) has now become,

\[
\frac{1}{G} \left[ \frac{1}{(k_{\alpha_1}^2 - k_{\beta}^2) (k_{\alpha_1}^2 - k_{\alpha_2}^2)} + \frac{1}{(k_{\alpha_2}^2 - k_{\beta}^2) (k_{\alpha_2}^2 - k_{\alpha_1}^2)} \right] \nabla^2 \delta(x - x') \\
+ \frac{1}{G} \left[ \frac{k_{\alpha_2}^2}{(k_{\alpha_1}^2 - k_{\beta}^2) (k_{\alpha_1}^2 - k_{\alpha_2}^2)} + \frac{k_{\alpha_1}^2}{(k_{\alpha_2}^2 - k_{\beta}^2) (k_{\alpha_2}^2 - k_{\alpha_1}^2)} \right] \delta(x - x') \\
+ \frac{1}{G} \left( \frac{k_{\alpha_2}^2}{(k_{\alpha_1}^2 - k_{\beta}^2) (k_{\alpha_1}^2 - k_{\alpha_2}^2)} \right) \frac{\omega^2 (H \rho_E + M \rho_B - 2 \rho_f C)}{HM - C^2} \\
- \frac{\omega^2}{G} \left( \rho_B - \frac{\rho_f^2}{\rho_E} \right) \delta(x - x') = 0
\]

(Eq. B.43)

Where the following identities have been used,

\[
\frac{1}{G} \left[ \frac{1}{(k_{\alpha_1}^2 - k_{\beta}^2) (k_{\alpha_1}^2 - k_{\alpha_2}^2)} + \frac{1}{(k_{\alpha_2}^2 - k_{\beta}^2) (k_{\alpha_2}^2 - k_{\alpha_1}^2)} \right] \left[ (k_{\alpha_2}^2 - k_{\beta}^2) \\
- (k_{\alpha_1}^2 - k_{\beta}^2) + (k_{\alpha_1}^2 - k_{\alpha_2}^2) \right] = 0
\]

(Eq. B.44)
\[
\frac{1}{G} \left( k_{\alpha_2}^2 - k_{\beta}^2 \right) \left( k_{\alpha_1}^2 - k_{\alpha_2}^2 \right) \left( k_{\alpha_1}^2 - k_{\beta}^2 \right) \left( k_{\alpha_2}^2 - k_{\beta}^2 \right) \left[ k_{\alpha_2}^2 \left( k_{\alpha_2}^2 - k_{\beta}^2 \right) - k_{\alpha_1}^2 \left( k_{\alpha_1}^2 - k_{\beta}^2 \right) \right] \\
+ \left( k_{\alpha_1}^2 - k_{\alpha_2}^2 \right) \left( \frac{\omega^2 (H \rho_E + M \rho_B - 2 \rho_f C)}{HM - C^2} - \frac{\omega^2 (\rho_B - \rho_f^2)}{G (\rho_E)} \right) \right] = 0 \quad (B.45)
\]

The equals zero in equation (Eq B.45) becomes transparent if the following identities are recognized.

\[
k_{\beta}^2 = \frac{\omega^2}{G} \left( \frac{\rho_B - \rho_f^2}{\rho_E} \right), \quad k_{\alpha_1}^2 + k_{\alpha_2}^2 = \frac{\omega^2 (H \rho_E + M \rho_B - 2 \rho_f C)}{HM - C^2} \quad (B.46)
\]

Since the \( \ddot{\mathbf{z}} + \frac{\nabla \Phi}{\rho E} \) term in equation (Eq B.36) is proven to equal zero, the remainder of that equation is recognized to be the dynamic displacement Green's function, equation (Eq B.29). Similar steps need to be taken in the derivation of the relative flow Green's function with a source on the solid.

### B.3.2 Displacement Green's Function with Force on Fluid

Taking the divergence of the relative flow Green's function with source on solid, \( G^w_f \) with respect to the primed source coordinates, using the relation \( \nabla' = \nabla \), and defining \( G^w_f = \nabla \cdot G^w_s \) yields,

\[
G^w_f = \frac{\rho_f}{\rho_E G} \nabla g_B - \left[ \frac{1}{C} (HM - C^2) - G^E \frac{\rho_B}{\rho_f} \right] \nabla \nabla^2 \Phi \\
+ \left[ \frac{HM}{C} \omega^2 \rho_f + C \omega^2 \frac{\rho_B \rho_E}{\rho_f} - H \omega^2 \rho_E - M \omega^2 \rho_B \right] \nabla \nabla^2 \Phi^* \\
= \frac{\rho_f}{\rho_E G} \nabla g_B - \left[ \frac{1}{C} (HM - C^2) - G^E \frac{\rho_B}{\rho_f} \right] \nabla \left( B(ws) \left[ -k_{\beta}^2 g_B - \delta(x - x') \right] \right) \\
+ A_1(ws) \left[ -k_{\alpha_1}^2 g_{\alpha_1} - \delta(x - x') \right] \\
+ A_2(ws) \left[ -k_{\alpha_2}^2 g_{\alpha_2} - \delta(x - x') \right] \\
+ \left[ \frac{HM}{C} \omega^2 \rho_f + C \omega^2 \frac{\rho_B \rho_E}{\rho_f} - H \omega^2 \rho_E - M \omega^2 \rho_B \right] \nabla \left( B^*(ws) \left[ -k_{\beta}^2 g_B - \delta(x - x') \right] \right) \\
+ A_1^*(ws) \left[ -k_{\alpha_1}^2 g_{\alpha_1} - \delta(x - x') \right] + A_2^*(ws) \left[ -k_{\alpha_2}^2 g_{\alpha_2} - \delta(x - x') \right] \quad (B.47)
\]

314
Where $\nabla^2 + k_i^2 g_i = -\delta(x - x')$, $i = \alpha_1 \vee \alpha_2 \vee \beta$ has been applied. With $\Phi, \Phi^*$ defined by equations (Eq 5.55) and (Eq 5.58) respectively and variables $B(ws), A_1(ws), A_2(ws)$ defined by equation (Eq 5.56), and variables $B^*(ws), A_1^*(ws), A_2^*(ws)$ by equation (Eq 5.59). Using equation (Eq B.38) and $B(ws), B^*(ws)$ and the relation, $k_\beta^2 = \frac{\omega^2}{G} \left( \rho_B - k_\alpha^2 \right)$, it can be shown after some algebra that,

$$
\left[ \frac{\rho_f}{\rho_E G} + \frac{1}{C} (HM - C^2) - \frac{G \rho_E}{\rho_f} \right] B(ws) k_\beta^2 \nabla g_\beta = 0 \quad (B.48)
$$

The Dirac delta function part in equation (Eq B.47)

$$
\left[ \frac{1}{C} (HM - C^2) - \frac{G \rho_E}{\rho_f} \right] [B(ws) + A_1(ws) + A_2(ws)] \\
- \left[ \frac{HM}{C} \omega^2 \rho_f + C \omega^2 \rho_B \rho_E \rho_f - H \omega^2 \rho_E - M \omega^2 \rho_B \right] \times [B^*(ws) + A_1^*(ws) + A_2^*(ws)] \nabla \delta(x - x') = 0 \quad (B.49)
$$

is shown to equal zero, since,

$$
B^*(ws) + A_1^*(ws) + A_2^*(ws) = \frac{1}{G(HM - C^2) \rho_E} \rho_f \\
\times \left[ \frac{k_\alpha^2 - k_\alpha^2}{(k_\beta^2 - k_\alpha^2)} - \frac{k_\alpha^2 - k_\alpha^2}{(k_\beta^2 - k_\alpha^2)} \right] = 0
$$

$$
B(ws) + A_1(ws) + A_2(ws) = \frac{1}{G(HM - C^2) \rho_E} \rho_f \\
\times \left[ \frac{\omega^2 \rho_f \left( k_\alpha^2 - k_\alpha^2 \right) - \left( k_\beta^2 - k_\alpha^2 \right) \left( k_\beta^2 - k_\alpha^2 \right)}{(k_\beta^2 - k_\alpha^2) (k_\beta^2 - k_\alpha^2) (k_\beta^2 - k_\alpha^2)} \right] \times C \left( k_\beta^2 (k_\alpha^2 - k_\alpha^2) - k_\alpha^2 (k_\beta^2 - k_\alpha^2) + k_\alpha^2 \left( k_\beta^2 - k_\alpha^2 \right) \right) \nabla \delta(x - x') = 0 \quad (B.50)
$$

The final displacement Green's function (identical to equation (Eq B.31)) with source on fluid becomes,

$$
G_f^u = \left[ \frac{1}{C} (HM - C^2) - \frac{G \rho_E}{\rho_f} \right] \nabla \Pi + \left[ \frac{HM}{C} \omega^2 \rho_f + C \omega^2 \rho_B \rho_E \rho_f - H \omega^2 \rho_E - M \omega^2 \rho_B \right] \nabla \Pi^*
$$

315
\[ \Pi = - \left[ A_1(ws)k_{\alpha_1}^2g_{\alpha_1} + A_2(ws)k_{\alpha_2}^2g_{\alpha_2} \right] \]
\[ \Pi^* = - \left[ A_1^*(ws)k_{\alpha_1}^2g_{\alpha_1} + A_2^*(ws)k_{\alpha_2}^2g_{\alpha_2} \right] \] (B.51)

### B.3.3 Relative Flow Green’s Function with Force on Fluid

The derivation of the relative flow Green’s function with a force applied on the fluid starts with the integrand of equation (Eq 5.82).

\[
\begin{align*}
\mathcal{G} &= \ddot{z} \left[ \left( M - \frac{C^2}{H} \right) \nabla^2 - \left( M - \frac{C^2}{H} \right) \frac{\partial^2}{\partial z^2} \right] + \omega^2 \rho_E \\
&+ \left( \frac{\gamma \nabla^2_i + \gamma \frac{\partial^2}{\partial z^2} + \xi}{\delta} \right) \nabla^2 - \left( \frac{\gamma \nabla^2_i + \gamma \frac{\partial^2}{\partial z^2} + \xi}{\delta} \right) \frac{\partial^2}{\partial z^2} \\
&+ \left[ \left( M - \frac{C^2}{H} \right) + \frac{\gamma \nabla^2_i + \gamma \frac{\partial^2}{\partial z^2} + \xi + H \omega \rho_f^2}{\delta} \right] \frac{\delta}{\partial z} \left( \nabla_i \ddot{z} + \ddot{z} \nabla_i \right) \Phi \\
&+ \frac{\nabla_i \nabla_t}{\nabla^2_i} \left[ \left( M - \frac{C^2}{H} \right) \nabla^2 - \left( M - \frac{C^2}{H} \right) \nabla^2_i \right] + \omega^2 \rho_E \\
&+ \left( \frac{\gamma \nabla^2_i + \gamma \frac{\partial^2}{\partial z^2} + \xi}{\delta} \right) \nabla^2 - \left( \frac{\gamma \nabla^2_i + \gamma \frac{\partial^2}{\partial z^2} + \xi}{\delta} \right) \nabla^2_i \\
&+ \left[ \frac{\omega H \rho_f^2 \nabla^2_i - \omega^6 \rho_f^2 \rho_B}{\delta} \right] \Phi
\end{align*}
\] (B.52)

With \( \gamma, \xi, \delta \) and \( \Phi \) defined in equations (Eq 5.79), (Eq 5.80), (Eq 5.78) and (Eq 5.85).

Equation (Eq B.52) can be written into the form,

\[
\begin{align*}
\mathcal{G} &= - \left( M - \frac{C^2}{H} \right) + \frac{\gamma \nabla^2_i + \xi + H \omega \rho_f^2}{\delta} \\
&\times \left[ \ddot{z} \frac{\partial^2}{\partial z^2} \Phi + \frac{\delta}{\partial z} \left( \nabla_i \ddot{z} + \ddot{z} \nabla_i \right) \Phi + \nabla_i \nabla_t \Phi \right] \\
&+ \left( M - \frac{C^2}{H} \right) \nabla^2 \Phi + \frac{\gamma \nabla^2_i + \xi}{\delta} \nabla^2 \Phi + \left( \omega^2 \rho_E - \frac{\omega^6 \rho_f^2 \rho_B}{\delta} \right) \Phi \\
&\times \left[ \ddot{z} + \frac{\nabla_t \nabla_t}{\nabla^2_i} \right]
\end{align*}
\] (B.53)
Taking the divergence of equation (Eq B.53) with respect to the primed source coordinates and using the relation \( \nabla' = -\nabla \) yields,

\[
\nabla \cdot \mathbf{G} = \left( M - \frac{C^2}{H} \right) \nabla \nabla^2 \Phi + \frac{\gamma H^2}{\delta} \nabla \nabla^2 \Phi + \frac{H \omega^4 \rho_i^2}{\delta} \nabla \nabla^2 \Phi \\
- \left( \frac{1}{\partial z} + \nabla_t \right) \left[ \left( M - \frac{C^2}{H} \right) \nabla^2 \Phi + \frac{\gamma H^2}{\delta} \nabla \nabla^2 \Phi + \left( \omega^2 \rho_E - \frac{\omega^6 \rho_i^2 \rho_B}{\delta} \right) \Phi \right] \\
= \frac{1}{\delta} \left[ H \left( \omega^4 \rho_i^2 - \omega^4 \rho_E \rho_B \right) \nabla \nabla^2 \Phi + \left( -\omega^6 \rho_E \rho_B^2 + \omega^6 \rho_i^2 \rho_B \right) \nabla \Phi \right] \\
- \omega^2 \rho_E \left( H \nabla^4 + \nabla^2 \omega^2 \rho_B \right) \nabla \Phi
\]  

(B.54)

Using the identity, \( k_i^2 = \frac{\omega^2}{G} \left( \rho_B - \frac{\rho_i^2}{\rho_E} \right) \), and defining \( \mathbf{G}_f^w = \nabla \cdot \mathbf{G} \), equation (Eq B.54) can be rewritten into the form,

\[
\mathbf{G}_f^w = -HG \omega^2 \rho_E \left[ k_i^2 \nabla \nabla^2 + \nabla^2 \nabla \nabla^2 \right] \left( A_1^*(w_f)g_{a_1} + A_2^*(w_f)g_{a_2} + B^*(w_f)g_{\beta} \right) \\
- \omega^4 \rho_E \rho_B G \left[ k_i^2 \nabla + \nabla^2 \nabla \right] \left( A_1^*(w_f)g_{a_1} + A_2^*(w_f)g_{a_2} + B^*(w_f)g_{\beta} \right) \\
= -HG \omega^2 \rho_E \left[ k_i^2 \nabla \nabla^2 \left( A_1^*(w_f)g_{a_1} + A_2^*(w_f)g_{a_2} \right) \\
+ \nabla \nabla^2 \left( A_1^*(w_f) \left[ -k_i^2 - \delta(x - x') \right] + A_2^*(w_f) \left[ -k_i^2 - \delta(x - x') \right] \right) \right] \\
- \omega^4 \rho_E \rho_B G \left[ k_i^2 \nabla \left( A_1^*(w_f)g_{a_1} + A_2^*(w_f)g_{a_2} \right) \\
+ \nabla \left( A_1^*(w_f) \left[ -k_i^2 - \delta(x - x') \right] + A_2^*(w_f) \left[ -k_i^2 - \delta(x - x') \right] \right) \right] \\
+ HG \omega^2 \rho_E B^*(w_f) \nabla \nabla^2 \delta(x - x') + \omega^4 \rho_E \rho_B G B^*(w_f) \nabla \delta(x - x') \)  

(B.55)

With \( A_1^*(w_f), A_2^*(w_f) \) and \( B^*(w_f) \) defined by equation (Eq 5.90). Where \( \nabla^2 + k_i^2 g_i = -\delta(x - x'), i = \alpha_1 \lor \alpha_2 \) has been applied. First the the Dirac delta function part in equation (Eq B.55),

\[
[H G \omega^2 \rho_E + \omega^4 \rho_E \rho_B G] \\
\times (\nabla \nabla^2 + \nabla) \left[ A_1^*(w_f) + A_2^*(w_f) + B^*(w_f) \right] \delta(x - x') = 0
\]  

(B.56)

is shown to equal zero, since,

\[
A_1^*(w_f) + A_2^*(w_f) + B^*(w_f) = \frac{1}{G \omega^2 \rho_E \left( HM - C^2 \right)} \\
\times \left[ \left( k_i^2 - k_i^2 \right) - \left( k_i^2 - k_i^2 \right) + \left( k_i^2 - k_i^2 \right) \right] \\
\left( k_i^2 - k_i^2 \right) \left( k_i^2 - k_i^2 \right) \left( k_i^2 - k_i^2 \right)
\]  

(5.37)
Using the scalar wave equations $\nabla^2 + k_i^2 g_i = -\delta(x - x'), i = \alpha_1 \lor \alpha_2$ for the second time in equation (Eq B.55) the following equation is obtained.

$$
G_f^w = H G \omega^2 \rho_E \left( k_{\alpha_1}^2 - k_{\beta}^2 \right) \nabla \left[ A_1^*(w_f) \left( k_{\alpha_1}^2 g_{\alpha_1} - \delta(x - x') \right) \right] \\
+ H G \omega^2 \rho_E \left( k_{\alpha_2}^2 - k_{\beta}^2 \right) \nabla \left[ A_2^*(w_f) \left( k_{\alpha_2}^2 g_{\alpha_2} - \delta(x - x') \right) \right] \\
+ \omega^4 \rho_E \rho_B G \left[ \left( k_{\alpha_1}^2 - k_{\beta}^2 \right) \nabla A_1^*(w_f) g_{\alpha_1} + \left( k_{\alpha_2}^2 - k_{\beta}^2 \right) \nabla A_2^*(w_f) g_{\alpha_2} \right]
$$

(B.58)

The Dirac delta function part in equation (Eq B.58),

$$HG \omega^2 \rho_E \left[ \left( k_{\beta}^2 - k_{\alpha_1}^2 \right) A_1^*(w_f) + \left( k_{\beta}^2 - k_{\alpha_2}^2 \right) A_2^*(w_f) \right] \nabla \delta(x - x') = 0
$$

(B.59)
is zero, since,

$$
\frac{HG \omega^2 \rho_E}{G \omega^2 \rho_E (HM - C^2)} \left[ \frac{\left( k_{\beta}^2 - k_{\alpha_1}^2 \right) \left( k_{\beta}^2 - k_{\alpha_2}^2 \right) - \left( k_{\beta}^2 - k_{\alpha_2}^2 \right) \left( k_{\alpha_1}^2 - k_{\beta}^2 \right)}{\left( k_{\alpha_1}^2 - k_{\beta}^2 \right) \left( k_{\alpha_2}^2 - k_{\beta}^2 \right)} \right] = 0
$$

(B.60)

The final relative flow Green's function (identical to equation (Eq 5.89) and (Eq 5.90)) with source on fluid becomes,

$$
G_f^w = \left[ HG \omega^2 \rho_E \left( k_{\alpha_1}^2 k_{\beta}^2 - k_{\alpha_1}^4 \right) + \omega^4 \rho_E \rho_B G \left( k_{\alpha_1}^2 - k_{\beta}^2 \right) \right] A_1^*(w_f) \nabla g_{\alpha_1} \\
+ \left[ HG \omega^2 \rho_E \left( k_{\alpha_2}^2 k_{\beta}^2 - k_{\alpha_2}^4 \right) + \omega^4 \rho_E \rho_B G \left( k_{\alpha_2}^2 - k_{\beta}^2 \right) \right] A_2^*(w_f) \nabla g_{\alpha_2}
$$

(B.61)
Appendix C

Dynamic and Static Green’s Functions in Poro-Elastic Media

C.1 Displacement Green’s Function with Source on Frame

To evaluate the element surface integrals, the following poro-elastic Green’s functions are used. The detailed derivation of the dynamic Green’s functions is outlined in chapter 5.

\[
\mathbf{G}^u = g^u \mathbf{L} + \left[ G - \frac{1}{M} (HM - C^2) \right] \nabla \nabla \phi - \left[ -2\omega^2 \rho_f C + M\omega^2 \frac{\rho_f^2}{\rho_E} + \frac{C^2}{M} \omega^2 \rho_E \right] \nabla \nabla \phi^* \quad (C.1)
\]

Where the superscript \( u \) denotes the Green’s function to be a displacement Green’s function and subscript \( s \) denotes a point force applied to the solid matrix. With,

\[
\phi = B(us) \frac{e^{ik_0R}}{4\pi R} + A_1(us) \frac{e^{ik_{1R}}}{4\pi R} + A_2(us) \frac{e^{ik_{2R}}}{4\pi R} \quad (C.2)
\]
With the introduced variables \( B(us), A_1(us), A_2(us) \), where \( (us) \) denotes scalar Green’s function amplitudes belonging to the displacement Green’s function with a source applied on the solid frame defined as,

\[
B(us) = \frac{1}{G[HM - C^2]} \left( -k_\beta^2 M + \omega^2 \rho_E \right) \left( k_\alpha^2 - k_\alpha^2 \right) \left( k_\beta^2 - k_\beta^2 \right) \\
A_1(us) = \frac{1}{G[HM - C^2]} \left( -k_\alpha^2 M + \omega^2 \rho_E \right) \left( k_\alpha^2 - k_\alpha^2 \right) \left( k_\beta^2 - k_\beta^2 \right) \\
A_2(us) = \frac{1}{G[HM - C^2]} \left( -k_\alpha^2 M + \omega^2 \rho_E \right) \left( k_\alpha^2 - k_\alpha^2 \right) \left( k_\beta^2 - k_\beta^2 \right) \\
\text{(C.3)}
\]

Where \( \phi^* \) is defined as,

\[
\phi^* = B^*(us)g_\beta + A_1^*(us)g_\alpha_1 + A_2^*(us)g_\alpha_2 \quad \text{(C.4)}
\]

With the variables \( B^*(us), A_1^*(us), A_2^*(us) \) defined as,

\[
B^*(us) = \frac{B(us)}{\omega^2 \rho_E - Mk_\beta^2}, \quad A_1^*(us) = \frac{A_1(us)}{\omega^2 \rho_E - Mk_\alpha^2}, \quad A_2^*(us) = \frac{A_2(us)}{\omega^2 \rho_E - Mk_\alpha^2} \quad \text{(C.5)}
\]

With the displacement scalar shear Green’s function,

\[
g^\mu(x,x') = \frac{e^{ik_0 R}}{4\pi GR} \quad \text{(C.6)}
\]

Where \( R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \) is the distance from the source to the receiver and \( k_0 = \omega/\sqrt{\rho_B/\rho_E} \) is the wave number.

### C.2 Relative Flow Green’s Function with Source on Frame

\[
G^w_{\phi^*} = g^\mu L + \left[ \frac{1}{C}(HM - C^2) - G^\rho_E \rho_f \right] \nabla \nabla \Phi \\
- \left[ \frac{HM}{C} \omega^2 \rho_f + C4\omega\rho_B \rho_E \rho_f - H\omega^2 \rho_E - M\omega^2 \rho_B \right] \nabla \nabla \phi^* \quad \text{(C.7)}
\]
Where the superscript \( w \) denotes the Greens's function to be a relative flow Green's function and the subscript \( s \) denotes the point force applied to the solid matrix. With,

\[
\Phi = B(\omega s)\frac{e^{ijk_R}}{4\pi R} + A_1(\omega s)\frac{e^{ik_0^s R}}{4\pi R} + A_2(\omega s)\frac{e^{ik_0^s R}}{4\pi R} \tag{C.8}
\]

With the introduced variables \( B(\omega s), A_1(\omega s), A_2(\omega s) \), where \( (\omega s) \) denotes scalar Green's function amplitudes belonging to the relative flow Green's function with a source applied on the solid frame defined as,

\[
B(\omega s) = \frac{1}{G[H\mu - C^2] e_F \rho_f \left( k_\beta^2 - k_{\alpha_1}^2 \right) \left( k_\beta^2 - k_{\alpha_2}^2 \right)} \frac{-k_\beta^2 C + \omega^2 \rho_f}{-k_{\alpha_1}^2 C + \omega^2 \rho_f} \tag{C.9}
\]

\[
A_1(\omega s) = \frac{1}{G[H\mu - C^2] e_F \rho_f \left( k_{\alpha_1}^2 - k_\beta^2 \right) \left( k_{\alpha_1}^2 - k_{\alpha_2}^2 \right)} \frac{-k_{\alpha_1}^2 C + \omega^2 \rho_f}{-k_{\alpha_2}^2 C + \omega^2 \rho_f} \tag{C.9}
\]

\[
A_2(\omega s) = \frac{1}{G[H\mu - C^2] e_F \rho_f \left( k_{\alpha_2}^2 - k_\beta^2 \right) \left( k_{\alpha_2}^2 - k_{\alpha_1}^2 \right)} \frac{-k_{\alpha_2}^2 C + \omega^2 \rho_f}{-k_{\alpha_1}^2 C + \omega^2 \rho_f} \tag{C.9}
\]

Where \( \Phi^* \) is defined as,

\[
\Phi^* = B^*(\omega s)g_\beta + A_1^*(\omega s)g_{\alpha_1} + A_2^*(\omega s)g_{\alpha_2} \tag{C.10}
\]

With the variables \( B^*(\omega s), A_1^*(\omega s), A_2^*(\omega s) \) defined as,

\[
B^*(\omega s) = \frac{B(\omega s)}{\omega^2 \rho_f - Ck_\beta^2}, \quad A_1^*(\omega s) = \frac{A_1(\omega s)}{\omega^2 \rho_f - Ck_{\alpha_1}^2}, \quad A_2^*(\omega s) = \frac{A_2(\omega s)}{\omega^2 \rho_f - Ck_{\alpha_2}^2} \tag{C.11}
\]

The relative flow scalar shear Green’s function is given by,

\[
g^w(\mathbf{x}, \mathbf{x}') = -\frac{\rho_f}{\rho E} \frac{e^{ik_0^s R}}{4\pi GR} \tag{C.12}
\]

Where \( R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \) is the distance from the source to the receiver and \( k_0 = \omega/\sqrt{G/(\rho_B - \rho_E)} \) is the wave number.
C.3 Displacement Green’s Function with Source on Fluid

\[ G^w_f = \nabla \cdot G^w_s = \frac{1}{C} \left[ C^2 - H M + G C^{\rho_E \rho_f} \right] \nabla \Pi + \left[ \frac{H M}{C} \omega^2 \rho_f + C \omega^2 \frac{\rho_B \rho_E}{\rho_f} - H \omega^2 \rho_E - M \omega^2 \rho_B \right] \nabla \Pi^* \]  \hspace{1cm} (C.13)

Where \( \Pi \) and \( \Pi^* \) are defined as,

\[ \Pi = -A_1(ws)k_{\alpha_1}^2 \frac{e^{ik_{\alpha_1} R}}{4\pi R} - A_2(ws)k_{\alpha_2}^2 \frac{e^{ik_{\alpha_2} R}}{4\pi R} \]  \hspace{1cm} (C.14)

\[ \Pi^* = -A_1^*(ws)k_{\alpha_1}^2 \frac{e^{ik_{\alpha_1} R}}{4\pi R} - A_2^*(ws)k_{\alpha_2}^2 \frac{e^{ik_{\alpha_2} R}}{4\pi R} \]  \hspace{1cm} (C.15)

The variables \( A_1(ws), A_2(ws) \) and \( A_1^*(ws), A_2^*(ws) \) are defined in equations (Eq C.8) and (Eq C.10) respectively.

C.4 Relative Flow Green’s Function with Source on Fluid

\[ G^w_f = \nabla \Pi^*, \quad \Pi^* = \Lambda_1 g_{\alpha_1} + \Lambda_2 g_{\alpha_2} \]  \hspace{1cm} (C.16)

Where,

\[ \Lambda_1 = \left[ H G \omega^2 \rho_E \left( k_{\alpha_1}^2 k_{\beta}^2 - k_{\alpha}^2 \right) + \omega^4 \rho_E \rho_B G \left( k_{\alpha_1}^2 - k_{\beta}^2 \right) \right] A_1^*(wf) \]

\[ \Lambda_2 = \left[ H G \omega^2 \rho_E \left( k_{\alpha_2}^2 k_{\beta}^2 - k_{\alpha}^2 \right) + \omega^4 \rho_E \rho_B G \left( k_{\alpha_2}^2 - k_{\beta}^2 \right) \right] A_2^*(wf) \]

\[ A_1^*(wf) = \frac{A_1(wf)}{\left( \omega^2 \rho_B - k_{\alpha_1}^2 H \right) \left( \omega^2 \rho_B - k_{\alpha_1}^2 G \right)} \]

\[ A_2^*(wf) = \frac{A_2(wf)}{\left( \omega^2 \rho_B - k_{\alpha_1}^2 H \right) \left( \omega^2 \rho_B - k_{\alpha_1}^2 G \right)} \]  \hspace{1cm} (C.17)
With the introduced variables $A_1(wf), A_2(wf)$, where $(wf)$ denotes scalar Green's function amplitudes belonging to the relative flow Green's function with a source applied on the fluid phase defined as,

$$A_1(wf) = \frac{1}{G \omega^2 \rho_E [HM - C^2]} \frac{(\omega^2 \rho_B - k_{a_1}^2 H) (\omega^2 \rho_B - k_{a_1}^2 G)}{(k_{a_1}^2 - k_{a_1}^2) (k_{a_1}^2 - k_{a_2}^2)}$$

$$A_2(wf) = \frac{1}{G \omega^2 \rho_E [HM - C^2]} \frac{(\omega^2 \rho_B - k_{a_2}^2 H) (\omega^2 \rho_B - k_{a_2}^2 G)}{(k_{a_2}^2 - k_{a_2}^2) (k_{a_2}^2 - k_{a_1}^2)}$$

(C.18)

### C.5 Static Green’s Functions in Poro-Elastic Media

#### C.5.1 Static Displacement Green’s Function

To determine the singular behavior of the dynamic Green’s function when a field point approaches the source point, the static case must be considered. This regularizes the surface integrals of the dynamic Green’s function, Kupradze (1963). When $\omega$ is set to zero, a static Green’s function is obtained. To obtain the $g^u$ and $\phi$ equations in the static limit we have to be solve,

$$G \nabla_i^2 g^u + G \frac{\partial^2}{\partial z^2} g^u = -\delta (x - x')$$

(C.19)

$$\frac{G}{M} [HM - C^2] \left[ \nabla_i^2 + \frac{\partial^2}{\partial z^2} \right]^2 \phi = -\delta (x - x')$$

(C.20)

The solution of equation (Eq C.19) is

$$g^u = \frac{1}{4\pi G R} \frac{1}{R}$$

(C.21)

The solution of equation (Eq C.20), using Poisson’s equation $\nabla^2 \frac{1}{4\pi R} = -\delta (x - x')$ and the identity $\nabla^2 R = \frac{2}{R}$ is,

$$\phi = \frac{M}{8\pi G [HM - C^2]} R$$

(C.22)
With $R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$. Substitution of $\phi$ and $g$ into equation (Eq C.1) in the static limit yields,

$$G = \frac{1}{4\pi G} \frac{1}{R^4} - \frac{1}{8\pi} \left[ \frac{HM - C^2 - GM}{HM - C^2} \right] \nabla \nabla R$$  \hspace{1cm} (C.23)

Taking the isotropic elastic limit, physically meaning the collapse of the pore space ($\phi \to 0$), of equation (Eq C.23) gives,

$$G = \frac{1}{4\pi \mu} \frac{1}{R^4} - \frac{\lambda + \mu}{8\pi \mu(\lambda + 2\mu)} \nabla \nabla R$$  \hspace{1cm} (C.24)

Which is the isotropic elastic static Green’s function, Love (1944).

### C.5.2 Static Relative Flow Green’s Function

To obtain the $g^w$ and $\Phi$ equations in the static limit we have to solve,

$$- G \frac{\rho_E}{\rho_f} \nabla_i^2 g^w - G \frac{\rho_E}{\rho_f} \frac{\partial^2}{\partial z^2} g^w = -\delta (x - x')$$  \hspace{1cm} (C.25)

$$G \frac{\rho_E}{C \rho_f} \left[ HM - C^2 \right] \left[ \nabla_i^2 + \frac{\partial^2}{\partial z^2} \right] \Phi = -\delta (x - x')$$  \hspace{1cm} (C.26)

The solution of equation (Eq C.25) is

$$g^w = -\frac{1}{4\pi G} \frac{1}{R_{\rho_f}^{ex}}$$  \hspace{1cm} (C.27)

The solution of equation (Eq C.26), using Poisson’s equation $\nabla^2 \frac{1}{4\pi R} = -\delta (x - x')$ and the identity $\nabla^2 R = \frac{2}{R}$ is,

$$\Phi = \frac{C \rho_f}{8\pi G \rho_E [HM - C^2]} R$$  \hspace{1cm} (C.28)

With $R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$. Substitution of $\phi$ and $g$ into equation (Eq C.7) in the static limit yields,

$$G = -\frac{1}{4\pi G} \frac{1}{R^4} + \frac{1}{4} \left[ \frac{HM - C^2 - G C_{\rho_f}^{ex}}{8\pi C \rho_f [HM - C^2]} \right] \nabla \nabla R$$  \hspace{1cm} (C.29)
In the static limit, $\omega \to 0$, the effective fluid density reduces to the fluid density in the pore space, $\rho_E \to \rho_f$. 