On Public Debt and Exchange Rates

by

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Submitted to the Department of Economics
in partial fulfillment of the requirements for the degree of

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Abstract

This thesis provides theoretical tools relevant to policy makers in both public debt and exchange rate management. The first chapter studies the optimal composition and denomination of public debt. The last two analyze two relevant issues in determining exchange rate management. Chapter 2 looks at liquidity crises while the last chapter focuses in the behavior of the real exchange rate in developing countries. Both topics are fundamental blocks to understand exchange rate management.

In chapter 1 an optimal composition of public debt is derived for a government that minimizes tax distortions and faces uncertainty with respect to the outcome of current shocks in GDP, spending and inflation but, also, faces time consistency problems. In particular, optimal proportions of short, long nominal and foreign denominated debt are derived.

The model is tested using panel data on 12 OECD countries in the period 1957-1992. The results indicate, as the model predicted, that countries with innovations in inflation positively correlated with spending tend to have higher long nominal debt while pure inflation disturbances work in the opposite direction. Those with GDP and spending shocks strongly correlated with foreign inflation have a higher proportion of foreign denominated debt.

In addition, the determinants of time inconsistency incentives have a strong effect on the composition. As the ratio of debt over GDP increases countries reduce their proportion of long nominal debt.

In chapter 2 a model of external crises is developed focusing on the interaction between liquidity creation by financial intermediaries and foreign exchange collapses. The intermediaries role of transforming maturities is shown to result in larger movements of capital and a higher probability of crises. This resembles the observed cycle in capital flows: large inflows, crises and abrupt outflows. The model highlights how adverse productivity and international interest rate shocks can be magnified by the behavior of individual foreign investors linked together through their deposits in the intermediaries. An eventual collapse of the exchange rate can link investors' behavior even further. The basic model is then extended, quite naturally, to study the effects of capital flow contagion between countries.
Chapter 3 studies the real exchange rate appreciation process of developing countries. Also known as the Balassa-Samuelson effect, this process occurs when developing countries reduce their productivity gap with respect to developed countries. Its theoretical foundations are reviewed briefly in the paper. The presentation of the empirical evidence concentrates, first, on cross-section analysis of 138 countries and then explores some panel data estimates during the period 60-90. The results suggest that a 1% per capita growth is expected in average to generate a .15% real appreciation. Time series analysis on the real exchange rate of Korea, Taiwan, Singapore, Hong Kong and China confirms the cross-section results and convergence hypotheses are then analyzed. Finally, the implication of the Balassa-Samuelson effect to policy makers is explored with emphasis on the sustainability of exchange rate pegs.

Thesis Supervisor: Rüdiger Dornbusch
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This thesis is a product of my 4 years in the Ph.D. program at MIT. During this period I benefited from the interaction with several people. In the early stages, when I presented preliminary versions of the chapters of this thesis in the Money lunch and International Breakfast, many participants contributed with valuable comments and discussions. In particular, Daron Acemoglu, Andrew Bernard, Olivier Blanchard, Ricardo Caballero and John Heaton read or commented on chapters of this thesis.

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Chapter 1

On Public Debt Indexation and Denomination

1.1 Introduction

Debt management has always been an important practical question to policy makers around the world, who have to determine the allocation of public debt between long and short maturities and between foreign and domestic currency denominations. There is a wide dispersion of debt structures both across countries and across time for given countries\textsuperscript{1}. It is still a challenge to researchers in this field to provide insights on why those are the chosen allocations and how economic theory can provide tools to policy makers on their future decisions.

Debt management is defined as the decision to issue specific securities for a given level of debt as opposed to choosing the time path of total debt and taxes.

In several cases, the latter decision is well defined but the former gives an indeterminate answer. This is the case whenever there are an infinite number of ways of attaining the same path of total debt with a given set of securities. The objective of the chapter is to shed some light on the composition of debt when it is not indeterminate.

\textsuperscript{1}See graphs at the end of the chapter.
Of course, the two decisions are connected. In particular, the chapter analyses the effect of a higher total debt on the government's incentive to inflate away the nominal debt. This will lead to a different debt composition.

The irrelevance theorems of Stiglitz provide a good theoretical starting point. They basically amount to extend the Modigliani-Miller theorem of corporate finance to the case with Ricardian consumers. In this case, any change in the composition of public debt is irrelevant. With consumers that take into account future tax levies, demand for securities of different types adjust automatically to different supplies so that their prices do not change and there is no real effect on the economy. The second section gives a brief overview of the developments in this area.

The literature has generally focused on two sources of non-neutralities: distortionary taxes, and the potential time-inconsistency of the governments' optimal debt policy. In both cases, debt management will play a role. In the first case, debt management can minimize the distortions from non-lump sum taxes, while in the second case, debt management can be used as a commitment device.

Time-consistency introduces two opposing effects to consider in managing the debt. First, the government will issue a higher proportion of indexed debt or short-term nominal debt so as to make credible its pledge not to inflate away the debt. However, if the entire stock of debt were indexed, then fluctuations in real government spending would have to be financed in a larger extent by changes in taxes, which may be distortionary. In the model, optimal debt management involves trading off this second effect with the first one.

In section 3, a two period model highlights the main trade-off. First, the optimal composition of public debt is derived when there exist a commitment technology. In this case, the optimal debt structure will reflect only the hedging motives and will depend on the extent that the value of long nominal debt is negatively correlated with spending shocks. Then, the commitment assumption is relaxed and the optimal proportion of long nominal debt will decrease, reflecting the need to make more credible the pledge to not inflate away the debt.

\(^2\text{See Stiglitz[1983].}\)
The model is extended in several directions. First, GDP shocks are incorporated. These give an additional hedging role to long nominal debt, provided they are positively correlated with the real value of debt. Second, a foreign denominated asset is introduced, introducing a new instrument to satisfy both hedging and commitment objectives.

In section 4 a multiperiod model is presented in its full version, including both spending and GDP shocks and 3 assets. The main gain from extending the simple two period model is that it clarifies that the basic results still hold even when the government is allowed to smooth spending and GDP shocks through time.

Although the theoretical literature on the subject had some developments, its empirical counterpart is still relatively unexplored. This chapter contributes to the empirical literature on this area using data on public debt composition on 12 OECD countries from 1957-92, published by the respective central banks.

In section 6.2, the first stage of the empirical work is calculated. For each country, a time series of estimates of the relevant covariances between inflation, spending and GDP are estimated using coefficients obtained from a VAR. The output are 4 series of covariances (each point is a covariance estimate using different information set period) from 1968 to 1992.

In section 6.3, a panel data regression with 12 countries from 1968-92 is performed. Both fixed and random effects estimates are given. The objective is to find the determinants of the actual composition of public debt chosen on those countries. The data set on public debt composition is then used as independent variable and several predictions of the theoretical model are then tested. In particular, the focus is on the effect of the four relevant covariances, the level of debt and the variance of inflation on the chosen debt composition.
1.2 On the Irrelevance of Public Debt Management

It is well known that in a complete markets setting with no time consistency issues government debt policy is irrelevant. With distortionary taxes (no lump sum taxes available) Barro [1979] showed that there is an optimal debt policy which basically amounts to smoothing taxes through time. In this context, the debt composition (maturity, indexation or denomination) is still irrelevant. There are an infinite number of ways to generate the same path of total debt and taxes with a given set of securities.

Lucas and Stokey [1983] introduce time consistency problems in a real economy and show that a sufficiently rich maturity can implement the optimal debt policy. The time consistency problem arises because in each period governments have an incentive to change the current real interest rate by altering the path of the debt. In the absence of a commitment device, Lucas and Stokey show that a specific maturity structure can resolve the time consistency problem. However, they find that an optimal debt policy cannot be obtained in a monetary economy, where inflating the debt is a new time consistency problem.

Persson, Persson and Svensson [1987] extend the previous result to a monetary economy, introducing the possibility of issuing indexed securities. In this economy, the time consistency problem is resolved by the government maintaining a zero net position in nominal securities. In equilibrium, the government has a negative debt (or positive claims) in nominal terms with the private sector to offset the fact that consumers hold money. In this case, there is no temptation to inflate and the time consistent plan can be implemented.

However, the assumption of a fairly rich asset menu available to the government in a complete market setting or the existence of explicit contingent bonds is counterfactual. In practice, there is only nominal debt and indexed debt to the price level or foreign exchange. No debt contingent on GDP or government spending exist, for example. This is explained by moral hazard considerations on the side of the

\(^3\)Calvo and Guidotti[1990] and Bohn[1988]
government. However, there is *implicitly contingent* debt. Nominal debt promising nominal returns does not guarantee real returns or command over output, which is the relevant return for the consumer. Neither does foreign exchange debt.

An interesting question, then, is to ask what is the optimal composition of government debt given the availability of these three different assets?

In the following, the chapter focuses on the trade-off between time consistency problems versus tax-smoothing motives. The former generates a higher inflation than optimal which requires, in order to credibly commit to low inflation, a structure based on either indexed securities (to either foreign or domestic inflation) or on shorter maturities. Tax smoothing calls for issuing any security whose returns are negatively correlated with the tax needs of the government. This may be the case of both long nominal or foreign currency linked debt.

In the next section, an optimal composition of debt between long nominal, indexed and foreign denominated liabilities is derived in a two period example. The objective is to highlight the effects of both tax smoothing and time consistency considerations on the optimal composition, in the simplest framework available.

1.3 Two Period Examples

1.3.1 Two assets

In order to gain intuition, a simple two period example will be derived first. In this subsection, it is assumed that there are only two assets available to the government, indexed \(^4\) and nominal debt.

The objective of the government is to minimize distortions from taxes and inflation, both assumed to be quadratic. There are two sources of uncertainty: government spending and money demand are both assumed to be stochastic. The latter introduces uncertainty with respect to the rate of inflation.

---

\(^4\)In the empirical part of the chapter, this asset would be broadly interpreted as including all short term securities.
\[ \text{Min}_m E \left[ A \frac{r^2}{2} + \frac{\pi^2}{2} \right] \]  

(1.1)

In the first period, the government will choose the composition of the debt that it sells to the public and that will mature at the end of period two. Nominal bonds are sold at a nominal interest rate \( i \) while indexed bonds at a real rate of \( r \). There are no shocks or other financing decisions in the first period. In the second period, given the level of debt and its composition, the government decides about the optimal money growth. After that, the shocks are realized and taxes are levied to balance the budget.  

Consumers are in the background, they enter the model as risk neutral debt holders and will demand a nominal interest rate which includes a fixed real interest rate plus the expected rate of inflation.  

\[ 1 + i = (1 + \pi^e)(1 + r^*) \]  

(1.2)

The money market equation determines the equilibrium inflation rate:

\[ \pi = \dot{m} + \dot{v} \]  

(1.3)

where, \( \dot{m} \) is money supply growth created by the government and \( \dot{v} \) are velocity shocks that are assumed to be white noise.

In equilibrium, rational investors will anticipate the money supply growth decision of the government. In this model, this implies that investors anticipate the average inflation generated by the government.

\[ \pi^e = E[\pi] = \dot{m} \]  

(1.4)

The budget constraint of the government in period 2 is that taxes have to be equal

---

5 The budget has to balance since it is the last period. In the multiperiod model, taxes and money growth are decided simultaneously and the amount of debt financing adjusts for the shocks.

6 It is assumed that this real interest rate \( r^* \) is equivalent to the one demanded on indexed bonds. Even with risk neutrality, equation 1.2 is an approximation.
to spending plus the real value of debt. Linearizing around $\pi = \pi^e = 0$, the budget constraint can be written as:

$$\tau = \tilde{G} + B(1 + r^*)[1 - \theta(\pi - \pi^e)]$$

(1.5)

Where $\theta$ is the proportion of the debt in nominal terms.

**Commitment Solution**

First the commitment solution will be derived. It is assumed that government at 1 can credibly commit the decisions taken by government at 2. Therefore, in period 1 the government will choose both $\theta$ and the way government 2 will finance itself between taxes and money growth. Since investors will demand a higher nominal rate for any increase in announced money supply growth, from equation 1.4, the maximization problem of the government at 1 becomes:

$$\min_{\tilde{m}, \theta} \mathbb{E}[A \frac{\tau^2}{2} + \frac{\pi^2}{2}]$$

(1.6)

s.t.

$$\tau = \tilde{G} + B[1 - \theta(\pi - \pi^e)]$$

(1.7)

and,

$$\pi^e = \mathbb{E}[\pi] = \tilde{m}$$

(1.8)

where it is assumed, for simplicity, that $r^* = 0$.

The solution to this problem is:

$$\tilde{m}^* = 0$$

(1.9)

and,

$$\theta = \frac{\sigma_{ug}}{B\sigma_v^2}$$

(1.10)
where $\sigma$ is the standard deviation (or covariance term).

The optimal money supply growth is zero because higher nominal interest rates will increase proportionally to higher inflation rates and there will be no revenue benefits from announcing a higher path. At the same time, higher inflation rates imply higher distortion costs, so that governments will optimally commit to a zero inflation path.

The optimal proportion of the debt in nominal terms increases with the covariance of inflation with spending but diminishes with the variance of inflation. The intuition is that shocks to the other components of the budget should be optimally hedged. Governments should issue securities negatively correlated to other components of the budget.

Long nominal debt is a good hedging device for the government whenever shocks to spending (or any other component of the primary deficit) are positively correlated to inflation shocks and, therefore, negatively correlated to the debt value. This avoids having to raise taxes in bad states of the world.

On the other hand, pure inflation variance only introduces noise to the budget and induces the use of more indexed debt. This will be the case in countries that face large nominal shocks relative to real movements. The real value of the their debt would fluctuate without a corresponding change in other budgetary components. This induces the use of a high proportion of indexed debt. In the empirical section of the chapter, we will see that this is precisely the behavior of governments in high inflationary countries.

**Time Consistent Solution**

In this subsection, the assumption of commitment is relaxed.

If the government cannot commit its future behavior, it will face time consistency problems, arising from the fact that it controls partially the inflation rate. In period 2, it will be optimal to inflate some of the existing nominal debt. Ex-ante, rational

---

7For simplicity revenues from cash balance holdings are ignored.
investors will anticipate the future temptation to inflate, adjust expected inflation and demand a higher nominal interest rate. In this equilibrium the inflation rate will be higher than in the previous equilibrium where the government could commit itself.

It is interesting to calculate the effect of future incentives to inflate on the debt composition chosen in period 1.

Solving backwards, government in period two minimizes taxes and inflation costs, taken as given the level of debt $B$, its composition $\theta$ and the nominal interest rate $i$:

$$\min_{\tau, \pi} E\left[ A \frac{\tau^2}{2} + \frac{\pi^2}{2} \right]$$  \hspace{1cm} (1.11)

s.t.

$$\tau = \bar{G} + B[1 - \theta(\bar{m} + \bar{v} - \pi^e)]$$  \hspace{1cm} (1.12)

The first order condition yields:

$$m^* = AB\theta \Omega$$  \hspace{1cm} (1.13)

where

$$\Omega = (\bar{G} + B)$$  \hspace{1cm} (1.14)

where the first order condition 1.13 above uses equation 1.4. It says that the government at 2 equates the expected marginal costs of raising taxes and inflation. The equilibrium values of taxes and inflation, after shocks are realized are:

$$\bar{\tau} = \bar{G} + B(1 + \theta \bar{v})$$  \hspace{1cm} (1.15)

$$\pi = m^* + \bar{v}$$  \hspace{1cm} (1.16)

In comparison to the previous case, the absence of commitment increases the average rate of inflation, creating expected distortion costs. The incentive to inflate and the resulting equilibrium inflation depend on the proportion of nominal debt outstanding, $\theta$. Therefore, the government in period one would need to adjust its
optimal $\theta$ to reduce the incentive to inflate. Thus, it will deviate from the optimal proportion of nominal debt for hedging purposes. In other words, the absence of other commitment technologies forces the government to use indexation to satisfy two objectives, hedging and commitment, leaving it worse off.

The government in period one minimizes the loss function in period two choosing the appropriate indexation of debt. For this purpose, it is possible to explicitly derive the loss function that the government faces in period one (dropping irrelevant terms):

\[
Loss = A^2 B^2 \theta^2 \Omega^2 - 2\theta B A \sigma_{gy} + B^2 A \theta^2 \sigma_v^2
\]  
\[
(1.17)
\]

The first term in the right hand side of the equation is the price for not being able to commit to not inflate the nominal debt. It is the traditional inflationary bias of time consistent solutions. Indexing the debt is one of the commitment technologies. The government will now take into account this term when choosing its optimal proportion of long nominal debt. It is clear from here that setting $\theta = 0$ will completely avoid this type of cost. The second term is the hedging role of nominal debt. If inflation and government spending are positively correlated, tax rates will fluctuate less and governments can reduce distortions in the economy. The point is that it is optimal to have contingent debt that reduces the real value of debt when financing needs are higher. Nominal debt is implicitly contingent debt. The last term in the right works against issuing nominal debt. It is the variance of velocity shocks which affects inflation and changes the real value of nominal debt, which increases the variance of tax rates. Setting $\theta = 0$, completely indexing the debt, eliminates this last effect.

The government in period one minimizes its loss function and obtains the optimal $\theta^{op}$:

\[
\theta^{op} = \frac{\sigma_{gy}}{B(\Omega^2 A + \sigma_v^2)}
\]  
\[
(1.18)
\]

This optimal value has an additional term $\Omega^2 A$ when compared to the commitment case. The government now has to take into account the marginal cost, in terms of higher average inflation, of not indexing one extra unit of debt. This additional cost
will reduce the optimal proportion of long nominal debt.

The higher the term $\Omega^2 A$ the more tempted the government appears to want to inflate the debt. Thus, to credibly commit itself to not inflate, the government will use a higher proportion of indexed debt. This will reduce the amount of long nominal debt available for hedging purposes, deviating more from the commitment solution. It is interesting to note that the $\Omega^2 A$ term depends on both the level of total debt and of the average spending.

**Other Shocks**

The other relevant shocks to the budget are shocks to GDP. These determine the total revenue from taxes for a given tax rate. When GDP is negatively correlated to the inflation rate, as it will be when productivity shocks are dominant, then negative shocks to GDP will increase the marginal cost of taxes and at the same time increase inflation, which reduces the real value of debt. The negative correlation between higher marginal cost of taxes and real value of nominal debt provides a good hedge for the government.

Introducing productivity shocks in the analysis gives further role to nominal debt. Incorporating them in the previous example is straightforward. Assume $\tau$ is the tax rate and governments seek to smooth them and that lower case letters are the same variables expressed as percentage of GDP.

\[
\tilde{\tau} = \tilde{g} + b(1 - \theta\tilde{\pi})
\]  

(1.19)

A negative productivity shock reduces the tax base available to the government and implies that, for given expenditures, tax rates have to increase. In order to reduce fluctuations in tax rates, nominal debt can be used as a hedging mechanism, provided inflation and productivity shocks are negatively correlated. In bad states, when productivity shocks are negative, higher inflation will decrease the real payments on debt and dampen the need for a tax rate adjustment. Therefore, issuing nominal debt helps stabilize the tax rates.
Using the same steps as above, we obtain an optimal value for $\theta$ which takes into account the hedging role of nominal debt with respect to both government spending and the productivity shock.

$$
\theta^{op} = \frac{\sigma_{gu} - \sigma_{yv}}{B(A\Omega^2 + \sigma_v^2)} 
$$  

(1.20)

It is clear that the existence of a negative correlation between productivity shocks and inflation increases $\theta^{op}$. So nominal debt may have a hedging role, dampening both spending and productivity shocks.

### 1.3.2 3 Assets

In this section we will introduce the possibility of issuing debt denominated in foreign currency. The example is similar to the previous one but now the government in period one chooses both $\theta$ and $\theta^*$, the proportion in long nominal and foreign denominated debt.

In order to give an interesting role to the foreign denominated debt we allow for some correlation between foreign inflation and domestic variables: spending and GDP.

Assuming that PPP hold, the nominal exchange rate will fluctuate to compensate the difference between domestic and foreign inflation.

$$
(1 + \hat{c}) = \frac{(1 + \pi)}{(1 + \pi^*)} 
$$  

(1.21)

With perfect capital mobility between countries we obtain that real rates are equalized. Therefore, nominal interest rate at home and abroad can be written in a similar fashion $^{8}$

$$
1 + i = (1 + \pi^*)(1 + r^*) 
$$  

(1.22)

---

$^{8}$The same remarks made before concerning the approximation made at this point apply here.
\begin{align*}
1 + i^* &= (1 + \pi^e)(1 + r^*) \\
\intertext{Introducing these equations in the budget constraint, linearizing around } \pi = \pi^* = \pi^e = \pi^{*e} = 0, \text{ we obtain:} \\
\tilde{\tau} &= \tilde{G} + B(1 - \theta \tilde{\pi} - \theta^* \tilde{\pi}^*) \\
\intertext{Where } \tilde{\pi} \text{ and } \tilde{\pi}^* \text{ are innovations in inflation.} \\
\text{Again, solving the government's problem in period 2 first, the FOC conditions yield:} \\
m^* &= AB\theta \Omega \\
\intertext{and the resulting taxes are:} \\
\tilde{\tau} &= \tilde{G} + B[1 - \theta \tilde{\nu} - \theta \tilde{\nu}^*] \\
\intertext{Using this values in the expected loss function,} \\
\text{Loss} &= A^2 B^2 \theta^2 \Omega^2 - 2\theta B\sigma_{G\theta} - 2\theta^* B\sigma_{G\theta^*} + B^2 (\theta^2 \sigma_{\theta}^2 + \theta^2 \sigma_{\theta^*}^2 + 2\theta \theta^* \sigma_{\theta \theta^*}) \\
\intertext{Government one minimizes the loss function and obtains the optimal proportion of debt in nominal and foreign currency:} \\
\theta &= \frac{\sigma_{G\theta}^2 - \sigma_{G\theta^*} \sigma_{\theta^* \theta^*}}{B[(A^2 \Omega^2 + \sigma_{\theta}^2) \sigma_{\theta^*}^2 + \sigma_{\theta \theta^*}^2]} \\
\theta^* &= \frac{\sigma_{G\theta^*} - \sigma_{\theta^* \theta^*} \theta}{\sigma_{\theta^*}^2} \\
\intertext{The optimal proportions of nominal and foreign denominated debt increase when they have better hedging properties, i.e., when the covariance between spending and the relevant inflation increases. At the same time, pure inflation disturbances reduce}
the proportion issued of that liability.

It is interesting to note the effect of having liabilities in foreign currency. They introduce another instrument for hedging, provided the correlation between foreign inflation and other domestic budget components (in this case spending) are not zero. At the same time they are not subject to the incentive effects of the government, since it cannot create foreign inflation. ⁹

The trade-off with foreign denominated liabilities is that they may introduce more noise to the budget. If the foreign currency used to peg part of the debt is not very stable, in the sense of having a high variance of inflation, taxes will fluctuate more to compensate the movements in the real value of foreign denominated debt.

### 1.3.3 Foreign Exchange Risk Avoidance

It is straightforward to include real exchange rate risk in this simple set up. Assuming away Purchasing Power Parity the budget constraint of the government can be written in the following manner:

\[
\bar{r} = \bar{G} + B[1 - \theta\bar{\pi} - \theta^*\bar{q}] \tag{1.30}
\]

where,  \[
q = \pi - \hat{\pi} \tag{1.31}
\]

Now, the government has to take into account real exchange rate movements, or the difference between nominal devaluations and domestic inflation (this is exactly foreign inflation if PPP holds).

The optimal proportions are analogous to the previous ones:

---

⁹This is true in this model since we ruled out real exchange movements by assuming PPP. In a general case, it will be still true that governments cannot manipulate the real returns on foreign denominated debt provided that they do not affect systematically the real exchange rate.
\[
\theta = \frac{\sigma_{GQ}^2 - \sigma_{GQ}\sigma_{\pi q}}{B[(A^2\Omega^2 + \sigma_{\pi}^2)\sigma_q^2 + \sigma_{\pi q}^2]} \tag{1.32}
\]
\[
\theta^* = \frac{\sigma_{GQ}}{B\sigma_q^2} - \frac{\sigma_{\pi q}\theta}{\sigma_q^2} \tag{1.33}
\]

More real exchange rate risk, interpreted here as higher variance, induces less use of foreign denominated debt.

1.4 A Multiperiod Model

The multiperiod model has 5 elements.

1. Consumers hold three assets, real, nominal and foreign exchange debt. The real asset delivers \(1 + r\) units of income next period while the rest is denominated in nominal terms, either in domestic or foreign currency. Assuming risk neutrality, capital mobility and PPP, the returns on the assets will be given by:

\[
1 + i = (1 + \pi^e)(1 + r) \tag{1.34}
\]

\[
1 + i^* = (1 + \pi^{*e})(1 + r) \tag{1.35}
\]

Where \((1 + r)^{-1}\) is assumed equal to \(\beta\), the time preference discount rate.

The consumer maximizes utility intertemporally and while satisfying her intertemporal budget constraint. She pays taxes and faces inflation. Both are distortionary and are costly in terms of units of output. We assume that the output costs of higher taxes or inflation are represented by two convex functions \(h(\tau)\) and \(k(\pi)\) \(^{10}\). The problem of the consumer can be represented as:

\[
Max_{C_t} \quad U = E \sum_{j=0}^{\infty} \beta^j C_{t+j} \tag{1.36}
\]

\(^{10}\)This modeling of tax smoothing is based in the structure developed in Barro[1983], Bohn[1988] and Bohn[1990]
\[ B_{t+1} = Y_t[1 - \tau_t - h(\tau_t) - k(\pi_t)] - C_t \]
\[ + B_t[(1 - \theta_{t-1} - \theta^*_t)(1 + r) + \theta_{t-1} \frac{1 + i}{1 + \pi} + \theta^*_t \frac{1 + i^*}{1 + \pi^*}] \] (1.37)

Where \( B_t \) is the total real value of the consumers assets in period \( t \) and \( \theta_{t-1} \) and \( \theta^*_t \) are the proportions of total assets in nominal and foreign denominated bonds, respectively.

2. Government is benevolent and wants to maximize welfare but has only distortionary sources of funds available (taxes and inflation) to finance stochastic spending (can be related to productivity cycles) which includes payments on existing debt. The government has two instruments. The tax rate is set directly, while the inflation rate is only indirectly controlled through the money creation process, \( \hat{m} \). Shocks to money demand can change the equilibrium inflation rate after the government has set the money growth rate.

Its dynamic budget constraint can be written as:

\[ B_{t+1} = G_t - \pi_t Y_t + B_t(1 + r)[1 - \theta_{t-1} \hat{\pi}_t - \theta^*_t \hat{\pi}^*_t] \] (1.38)

Where \( \hat{\pi}_t \) and \( \hat{\pi}^*_t \) are the innovations in domestic and foreign inflation. \( Y, \ G \) and \( \pi's \) are random variables which are possibly correlated. The real payments on government debt are reduced by surprise inflation in both currencies, weighted by the proportion of debt in each one.

3. The equilibrium in the money market will determine the inflation rate.

\[ \hat{\pi} = \hat{m} + \hat{v} \] (1.39)

Where \( \hat{m} \) is the growth in money which will be determined by the government, \( \hat{v} \) is an exogenous shock which has mean zero and variance \( \sigma_v^2 \).

4. Equivalently, inflation in the foreign country is given by the money equilibrium equation, but the growth in money supply is exogenous for the domestic agents.
\[ \tilde{\pi}^* = \tilde{m}^* + \tilde{\nu}^* \] (1.40)

5. In addition, time consistency issues involved in dealing with nominal bonds will be taken into account. The government can inflate the nominal part of the debt through its choice of \( \tilde{m}_t \).

1.4.1 Solution Without Time Consistency Issues

In this section it is assumed that the government can commit to a desired path of inflation. Since, in equilibrium, rational consumers expect this path of inflation and demand an adequate nominal interest rate, the government cannot benefit from inflation revenue (we will abstract from pure money balances). Therefore, it will choose a zero inflation path, since there are positive inflation costs.

The optimal composition of debt will not be affected by the inflation decision because there are no time consistency issues. Therefore, in this section we can ignore the money supply decision and set the inflation cost \( k(\pi) \equiv 0 \). The government decision is to set a path of tax rates and to determine the optimal composition of debt.

Using 1.38 on 1.37 and substituting on consumers utility 1.36, the welfare maximizing government ends up minimizing tax distortions:

\[
\text{Max}_{x_{\theta_t, \theta_t^*}, \tau_t} \sum_{j=0}^{\infty} E \beta^j [Y_{t+j}(1 - h(\tau_{t+j}) - G_{t+j}] \quad (1.41)
\]

subject to dynamic budget constraint 1.38. \( h \) is increasing and convex.

The optimal tax policy in a world where tax distortions are convex is to smooth taxes\[ For a discussion if this means constant tax rate see Blanchard and Fischer[1989], chapter 11.3 \]
In this case, long nominal and foreign denominated debt have the role of "implicitly contingent debt".

It should be clarified that it is not equivalent to smooth taxes through time than through states of nature. The former implies that in response to a negative shock today taxes will have to be increased some time in the future while in the latter taxes would never need to be levied.

The solution to this problem is given by the following first order conditions:

\[ h'(\tau_t) = E[h'(\tau_{t+1})] \]  \hspace{1cm} (1.42)

\[ h'(\tau_t) = -E[h'(\tau_{t+1})\hat{\pi}_t] \]  \hspace{1cm} (1.43)

\[ h'(\tau_t) = -E[h'(\tau_{t+1})\pi^*] \]  \hspace{1cm} (1.44)

The first equation is traditional tax smoothing over time, debt is adjusted such that the tax rate is maintained constant. The last two conditions involve tax smoothing through states of nature. Optimality calls for setting the debt in nominal or foreign currency until the point where the covariance between the innovations in returns and the tax rate is zero. This avoids unnecessary fluctuations in the future tax rates. Using 1.42, and assuming quadratic costs, \( h(\tau) = \tau^2/2 \), the last two equations can be written as:

\[ E[\hat{\pi}_{t+1}\hat{\pi}_t] = Cov(\tau_{t+1}, \pi_t) = 0 \]  \hspace{1cm} (1.45)

\[ E[\hat{\pi}_{t+1}\pi^*] = Cov(\tau_{t+1}, \pi^*_t) = 0 \]  \hspace{1cm} (1.46)

The last two equations above give a tax smoothing role to nominal debt. By changing the debt composition the government can change the elasticity of debt with respect to domestic and foreign inflation and obtain the desired zero covariance conditions. Setting the covariance of inflation innovations with tax rates equal to zero minimizes the fluctuations in the tax rates and reduces the distortions in the economy. Therefore, by hedging the fluctuations of the government budget, the debt composition plays an important role.

In order to determine the optimal quantities \( \theta \) and \( \theta^* \) we solve for \( \hat{\tau}_t \), using the con-
dition that tax rates are set to remain constant on average. Using the intertemporal budget constraint of the government, the approximate solution is:

\[
\hat{\tau}_{t+1} = (1 - \phi) \exp(-\bar{y}) \left[ -b(\hat{\pi}_t + \hat{\pi}^*_{t-1}) + \sum_{i=0}^{\infty} \beta^i \hat{g}_{t+i+1} - \tau_t \sum_{i=0}^{\infty} \phi^i \hat{y}_{t+i+1} \right] \tag{1.47}
\]

Where \( \phi = \beta \exp(-\bar{y}) \) is a discount factor. All the variables in lower case are now expressed as relative to GDP. The equation just says that positive shocks in the present value of spending or unexpected increases in the value of debt will induce higher taxes but, because of smoothing, only by a proportion \((1 - \phi)\). Similarly, since revenues depend on output, positive shocks in current or future growth rates induce lower tax rates. Using equation 1.47 in the zero covariance condition above, one period ahead, we get the equations that determine \( \theta \) and \( \theta^* \):

\[
b \theta_t \text{Var} (\hat{\pi}) + b \theta_t^* \text{Cov} (\hat{\pi}, \hat{\pi}^*) - \text{Cov} (\hat{\pi}, PV (\hat{g})) + a \text{Cov} (\hat{\pi}, PV (\hat{y})) = 0 \tag{1.48}
\]

\[
b \theta_t^* \text{Var} (\hat{\pi}^*) + b \theta_t \text{Cov} (\hat{\pi}, \hat{\pi}^*) - \text{Cov} (\hat{\pi}^*, PV (\hat{g})) + a \text{Cov} (\hat{\pi}^*, PV (\hat{y})) = 0 \tag{1.49}
\]

Which gives:

\[
\theta_t = \frac{\sigma^2_{\pi} s_{t+1} - \sigma_{\pi \pi^*} S^*_{t+1}}{\Delta} \tag{1.50}
\]

\[
\theta_t^* = \frac{\sigma^2_{\pi^*} S^*_{t+1} - \sigma_{\pi \pi^*} S_{t+1}}{\Delta} \tag{1.51}
\]

Where \( s \) is the sum of the covariance of \( \pi \) with the present value of government spending and the weighted sum of future growth innovations (equivalently to \( s^* \)) and \( \Delta \) is the determinant of the covariance matrix of the inflation rates times \( b \).
As expected the higher the covariance with spending and the more negative the covariance with GDP growth, the better is the hedge and the higher is the proportion of nominal debt. Equivalently, if foreign inflation is correlated with any of the above variables then there is a hedging role for the foreign denominated debt. In section 6 we discuss the comparative statics of this result.

1.4.2 Including Time Consistency

Assume now that the government can use the money supply to generate higher inflation but faces inflation costs $k(\pi)$. In assuming a commitment technology, as in the previous section, the government at time $t$ would optimally choose the future path of inflation to be zero. This is so because, from $t + 1$ on, there are welfare costs but no benefits in terms of revenues, since consumers are rational and the Fisher equation holds (for each point increase in expected inflation there is an equivalent increase in nominal interest rates, such that the real rate remains constant).

The zero inflation path is not time consistent, however. Since there will be nominal debt expressed in domestic currency, the future government will be tempted to inflate the debt up to the point where the following condition holds:

$$h'(\pi_t)\theta_{t-1}B_t(1 + r) = E[k'(\pi_t)]$$

(1.52)

This equation equates the marginal cost of inflation with its marginal benefit associated with the reduction in distortionary taxes.

Therefore, when government at time zero decides about the optimal maturity of debt, it has to take into account the future temptation to inflate and its inflation costs.

Obviously, for there to exist a hedging role, the government in period $t$ must not observe the shocks at $t$ before deciding about its control variables, $m$ and $\tau$. Therefore, after the rate of growth of money and the tax rate are decided, shocks in productivity (or government spending) and velocity are realized, and the equilibrium inflation rate and product are determined.
The objective function of the government will be to use the debt composition to maximize hedging by reducing the covariance between innovations but taking into account the future inflation costs. Specifically, it has to take into account equation 1.52 which equates the marginal cost of inflation with its marginal benefit in terms of reduced tax rates. Since, for simplicity we are dealing with only one period bonds and, therefore, the indexation of the debt can only be used to commit next period government, the objective function of the government is to minimize the expected inflation and the variance in innovations next period. Assuming quadratic costs in inflation cost and dropping irrelevant terms, the problem of the government becomes:

$$\min_{\theta \in \Theta} \cdot 0.5[E[\pi_t]^2 + Var(\pi_{t+1})]$$  \hspace{1cm} (1.53)$$

The last term represents the costs associated with not hedging perfectly and the first term is the cost of higher inflation in average. Substituting for \( \pi_{t+1} \) and expected inflation from equations 1.47 and 1.52 respectively and deriving with respect to \( \theta \) and \( \theta^* \) yields:

$$b\Omega_{\pi\pi^*} \theta_t - \Omega_{PV(\theta)} + a\Omega_{PV(\theta)} + TC_{t+1} \theta_t = 0$$  \hspace{1cm} (1.54)$$

where \( \Theta \) is a column vector \( (\theta, \theta^*)' \), \( \Omega_{\pi\pi^*} \) is the covariance matrix of domestic and foreign inflation, \( \Omega_{PV(\theta)} \) is a vector of the covariances of inflation (domestic and foreign) with the PV of the government spending and \( \Omega_{PV(\theta)} \) is the equivalent covariance vector with the future growth innovations. Also,

$$TC_t = \begin{bmatrix} 0 & B^2(1 + r)^2 \tau_t \\ 0 & 0 \end{bmatrix}$$

The optimal proportion of debt in domestic nominal and foreign debt, \( \Theta \), is given by:
\[ \Theta_t = (b\Omega_{\pi\pi^*} + TC_{t+1})^{-1}S_{t+1} \]  

(1.55)

where:

\[ S = \Omega_{PV(y)} - a\Omega_{PV(\dot{y})} \]  

(1.56)

1.5 Comparative Statics and Conclusions

1.5.1 Changes in \( S \)

The optimal amount of nominal and foreign denominated debt for hedging purposes depends on the correlations between innovations in the real value of the debt and the other components of the budget. The focus here was on two obvious candidates: shocks to government spending and to income. The results show that the proportions of debt in each category depend on how well they are correlated with these shocks (which is summarized by the vector \( S \)). Since we should expect domestic inflation to be more correlated to these domestic shocks than foreign inflation, the results suggest why nominal, domestically denominated debt has been the preferred debt instrument in several countries (see graphs attached for OECD countries).

If innovations in foreign inflation are correlated to domestic shocks to the budget, there is a specific role to foreign inflation in hedging fluctuations in the tax rates. For example, if foreign inflation is positively correlated with domestic spending, total debt will lose its value exactly when it is more needed for budgetary reasons. Also, it is straightforward to relax the PPP assumption and to include shocks to the real exchange rate affecting the return on foreign denominated bonds. In this case, if the real exchange rate is positively correlated to GDP then there is an additional hedging property for foreign denominated debt.

This model predicts that closed economies, that have less correlation with the rest of the world, should rely more on domestic currency debt than open economies.
Incidentally, this is the pattern observed in several cases. US debt is completely denominated in dollars, while European countries have substantial foreign denominated debt. Among developing countries, Brazil has all its debt indexed to the its price level while Argentina ties its debt to the dollar.

1.5.2 Changes in $\Omega_{\pi,\tau}$

The results also show that holding constant $S$ and increasing the inflation variance will reduce the optimal amount of both nominal and foreign debt. Extra variation in returns, without corresponding changes in other components of the budget will only increase variance of tax rates, which is socially costly. This suggests that, for example, pure domestic monetary shocks will reduce the amount of nominal debt issued by the government and pure foreign monetary shocks reduce the proportion of foreign debt. The latter only suggests that pegging the debt to a currency that is not very stable will only import the foreign instability, a well-known result from models of stabilization. This may explain why countries with high fluctuations in their nominal variables have less incentive to issue nominal debt and may increase welfare by indexing the debt.

The model therefore suggest a "Poole"\textsuperscript{12} result to the public debt. The more monetary fluctuations relative to real shocks the higher should be the proportion of indexed or short debt.

1.5.3 Increases in $b$

Controlling for the hedging variables, the higher the debt the lower nominal debt a government should issue. This comes from the commitment role of both foreign and indexed debt. More debt induces more temptation to inflate and higher inflation in equilibrium. This induces more commitment which is the role of indexed and foreign debt. In the same way the higher the average taxes needed to finance the

\textsuperscript{12}Poole [1976] discusses interest rate versus money targeting focusing in the relative variances of shocks to the IS vs the LM
intertemporal budget constraint, say because of higher average spending, the higher the benefit of inflating and the lower the nominal debt is in equilibrium.

If hedging is relevant, foreign debt is a better commitment technology. Not only does it reduce the temptation to inflate but, if foreign shocks are correlated to domestic budget variables, it still performs the hedging role.

It is important to take into account the possibility of other commitment technologies, such as having an independent central bank. In this way, nominal debt can be issued without concern with credibility, since the fiscal authority has no power to inflate this debt. On the other side of this same argument, commitments through indexation or denomination may not be credible since pure repudiation of the debt is always a possibility of governments (although at a very high costs).

1.5.4 \( \theta \) nonnegative

We can restrict the values of \( \theta \) and \( \theta^* \) to be between 0 and 1, which means that there is no "negative debt" in any component of the debt, i.e., governments do not hold net positive claims with the private sector. In this case corner solutions may arise. If foreign inflation is correlated to the budget variable in the wrong way \( s^* < 0 \), and under no time consistency problems, the optimal solution will be \( \theta^* = 0 \) and \( \theta = 1 \) although you would want to hold negative debt in foreign denominated assets.

1.6 Empirical Analysis

The purpose of this section is to test the "hedging versus commitment roles" in determining the optimal composition of public debt. The analysis will focus on the joint behavior of the macroeconomic series that determine the budget constraint of the government (government spending, inflation and growth), the level of the debt and its composition for 12 OECD countries.
1.6.1 Data

We used quarterly series of government expenditures, GDP growth, money growth and inflation for 12 OECD countries; UK, Italy, Ireland, Belgium, Germany, France, Netherlands, Canada, Austria, Australia, Spain and US. Those are taken from the IFS and are from 1957 to 1992.

Debt composition was divided into short (less than one year or indexed), long and foreign denominated debt. The data was obtained from two sources: Missale[1994], where several series obtained directly from central banks are compiled for the period 1960-1992 and the OECD[1993] publication “Debt Management and Government Securities” where information on debt composition from 1980-1990 can be obtained.

The OECD countries were chosen as a sample because they offered the larger number of available observations.

1.6.2 First Stage: Obtaining the Covariance Series

We obtain 4 covariance series for each country: inflation with the present value of spending, inflation with the present value of growth, $\pi - \hat{e}$ with the present value of spending and $\pi - \hat{e}$ with the present value of growth.

The covariance structure of innovations in government spending (as percentage of GDP), growth and inflation was obtained for each country by performing a VAR of these variables and including money growth as extra information variable and using the information obtained from the calculated covariance matrix of the residuals. In order to obtain the covariance of innovations in inflation today with the future innovations in spending and growth, the impulse response coefficients were used. Therefore, the estimate of the covariance of inflation with the present value of both variables were obtained from the impulse response coefficients weighted by an increasing discount rate and the covariance matrix of residuals. The impulse response length used was of $t$ to $t+20$.

This was also done for the series of excess inflation over depreciation (foreign inflation in the model) with domestic spending and growth.
For each year the procedure was repeated but with a different data set available to estimate the covariance matrix and the impulse response function. For each estimation, the last eleven years of data were used, such that every estimate in the series used 44 observations. In this way a time series of estimates of the 4 relevant covariances were obtained. The series start in 1967 in order to have at least 44 observations for each estimation (the last estimate is for 1992).

This was done for each of the 12 countries, using 2 lags of the variables growth, inflation, spending and exchange rate depreciation. However, some countries did not have data available for the whole sample and their series may begin at dates later than 1967.

The 4 covariance series for each country are graphed and presented in the figures at the end of the chapter. For the majority of the countries the correlation between innovations in inflation and spending or growth gives some role for the long nominal security as a hedging device: covariance inflation/PV of spending tend to be positive and inflation/growth negatively correlated.

Similarly, the covariances $\pi - \bar{e}$/spending tend to be positive although $\pi - \bar{e}$/growth is usually very close to zero.

### 1.6.3 Second Stage: Panel Data

The second stage uses the covariances obtained in stage 1 to estimate their influence in the choice of $\theta$ and $\theta^*$, in a panel data regression. The period of estimation is 1967-1992 for 12 countries in an unbalanced panel (some countries have fewer observations).

First the proportion of long nominal debt is regressed against the relevant covariances (domestic spending and growth with domestic and foreign inflation). Then we introduce the level of debt and the variance of inflation as additional regressors. Both fixed and random effects estimates are given. In some specifications a time trend is also introduced.

In order to control for other forms of commitment (besides the compositon of the debt), table 3 presents the regressions using an index of central bank independence
Finally, the proportion of foreign denominated debt is regressed against the same right hand side variables. Again both fixed and random effect estimates are given.

1.6.4 Results

The results of the regression are presented in tables 1-4. The expected signs are presented in the last row.

The covariances with spending fit the predictions of the model. The coefficient on the covariance of inflation/spending have the correct sign. It is significantly positive. It confirms the prediction of the model that the higher this covariance, the higher the proportion of long nominal debt. The same applies to the effect of the covariance \( \pi - \hat{\sigma} \) and spending. It is significantly negative. The higher the covariance of spending with the foreign variable the more useful is a foreign denominated security and the lower should be the proportion of long nominal debt.

The coefficients on the covariances of growth, however, give mixed results, some of them are not significant. Specifically, the covariance of inflation with growth shows no significant effect on the proportion of long nominal debt.

The level of debt seems to have a significant effect and in the direction predicted by the model. A higher level of debt reduces the long nominal proportion of debt. This supports the model since having a higher total debt affect the incentives to inflate and influences the decision of having long nominal debt.

The variance of inflation has a strong negative effect on the proportion of long nominal debt, as inflation uncertainty tend to reduce maturities. As predicted by the model, controlling for the effect of the covariances of inflation with spending and growth, higher variance of inflation should reduce the amount of long nominal debt.

In table 3, the effect of CBI on the proportion of long nominal debt is positive. If there is a commitment technology not to inflate, there is no need to use the composition of the debt for this matter, and we should observe a higher proportion of long

\[ ^{13} \text{The index is drawn from Cukierman (1992)} \]
nominal debt (as was shown in sections 3.1.1 and 4.1). The coefficients on Central Bank Independence (CBI) are positive and significant.

The results of the regressions using foreign denominated debt, in table 4, complement the results on tables 1-3. The covariances with spending have the correct signs while the covariances with growth are insignificant. The level of total debt as well as the variance of inflation increase the proportion of foreign denominated debt. In general, this table confirms the hypothesis that whenever a variable reduces the proportion of long nominal debt it tends to increase, to some extent, the proportion of foreign denominated debt. In other words, not all the reduction of long nominal debt is absorbed by more short term debt.
Table 1 - 12 OECD countries 1967-1992
Dependent Variable is the Proportion of Long Nominal Debt
(t-statistics in parentheses)

<table>
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<th></th>
<th>Cov($g^p, \pi$)</th>
<th>Cov($\hat{y}^p, \pi$)</th>
<th>Cov($g^p, \pi - \hat{e}$)</th>
<th>Cov($\hat{y}^p, \pi - \hat{e}$)</th>
<th>Trend</th>
<th>$\tilde{R}^2$</th>
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<td>.20</td>
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<td>FE</td>
<td>(3.92)</td>
<td>(0.68)</td>
<td>(-2.08)</td>
<td>(2.03)</td>
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<td></td>
</tr>
<tr>
<td>2</td>
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<td></td>
<td>.21</td>
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<tr>
<td>3</td>
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<td>(-3.5)</td>
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<tr>
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<td>(-2.31)</td>
<td>(2.06)</td>
<td></td>
<td>(-3.72)</td>
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</table>

The last row contains the expected signs predicted by the model.
Table 2 - 12 OECD countries 1967-1992
Dependent Variable is the Proportion of Long Nominal Debt
(t-statistics in parentheses)

<table>
<thead>
<tr>
<th></th>
<th>Cov($g^p, \pi$)</th>
<th>Cov($\hat{y}^p, \pi$)</th>
<th>Cov($g^p, \pi - \hat{\epsilon}$)</th>
<th>Cov($\hat{y}^p, \pi - \hat{\epsilon}$)</th>
<th>$D/Y$</th>
<th>$\sigma^2_\pi$</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.65</td>
<td>0.11</td>
<td>-0.56</td>
<td>0.37</td>
<td>-0.56</td>
<td>-0.10</td>
<td>(-0.58)</td>
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<tr>
<td></td>
<td>(1.92)</td>
<td>(0.22)</td>
<td>(-1.12)</td>
<td>(2.46)</td>
<td>(-6.13)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
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<td>0.10</td>
<td>-0.90</td>
<td>0.38</td>
<td>-0.66</td>
<td>-0.12</td>
<td>(-0.78)</td>
</tr>
<tr>
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<td>(2.13)</td>
<td>(0.14)</td>
<td>(-1.79)</td>
<td>(1.72)</td>
<td>(-6.03)</td>
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<td></td>
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<tr>
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<td>-0.43</td>
<td>-0.81</td>
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<td>-0.94</td>
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<td>(0.57)</td>
<td>(-6.03)</td>
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<td>(-4.63)</td>
</tr>
<tr>
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</tr>
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<td>(-1.03)</td>
<td>(0.98)</td>
<td>(-6.15)</td>
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<td>(-1.57)</td>
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<tr>
<td>6</td>
<td>2.87</td>
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<td>0.02</td>
<td>-0.47</td>
<td>-1.11</td>
<td>(-4.84)</td>
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<td>(0.07)</td>
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<td>(-2.02)</td>
</tr>
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<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The last row contains the expected signs predicted by the model.

$$\tilde{R}^2 = .36, \quad .38, \quad .23, \quad .27, \quad 0.46, \quad 0.52. \quad (1.57)$$
Table 3 - 12 OECD countries 1967-1992

Dependent Variable is the Proportion of Long Nominal Debt

(t-statistics in parentheses)

<table>
<thead>
<tr>
<th></th>
<th>$Cov(g^p, \pi)$</th>
<th>$Cov(\hat{g}^p, \pi)$</th>
<th>$Cov(g^p, \pi - \hat{e})$</th>
<th>$Cov(\hat{g}^p, \pi - \hat{\pi})$</th>
<th>D/Y</th>
<th>CBI</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 RE</td>
<td>1.67</td>
<td>0.10</td>
<td>-0.90</td>
<td>0.38</td>
<td>-0.66</td>
<td>-0.12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.13)</td>
<td>(0.14)</td>
<td>(-1.79)</td>
<td>(1.72)</td>
<td>(-6.03)</td>
<td>(-0.78)</td>
<td></td>
</tr>
<tr>
<td>2 RE</td>
<td>2.66</td>
<td>-0.05</td>
<td>-1.40</td>
<td>0.06</td>
<td>-0.93</td>
<td>0.91</td>
<td>-0.33</td>
</tr>
<tr>
<td></td>
<td>(2.47)</td>
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<td>(-2.4)</td>
<td>(2.15)</td>
<td>(-7.02)</td>
<td>(3.88)</td>
<td>(-1.78)</td>
</tr>
<tr>
<td></td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td></td>
</tr>
</tbody>
</table>

The last row contains the expected signs predicted by the model.

CBI is Central Bank Independence.
Table 4 - 9 OECD countries 1967-1992

Dependent Variable is the Proportion of Foreign Denominated Debt

(t-statistics in parentheses)

<table>
<thead>
<tr>
<th></th>
<th>$\text{Cov}(g^p, \pi)$</th>
<th>$\text{Cov}(\hat{g}^p, \pi)$</th>
<th>$\text{Cov}(g^p, \pi - \hat{e})$</th>
<th>$\text{Cov}(\hat{g}^p, \pi - \hat{e})$</th>
<th>$D/Y$</th>
<th>$\sigma^2_{\pi}$</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.21</td>
<td>-0.14</td>
<td>0.22</td>
<td>-0.08</td>
<td>.12</td>
<td>0.37</td>
<td>-0.01</td>
</tr>
<tr>
<td>FE</td>
<td>(-1.67)</td>
<td>(-.97)</td>
<td>(2.14)</td>
<td>(-1.01)</td>
<td>(2.98)</td>
<td>(2.37)</td>
<td>(-0.00)</td>
</tr>
<tr>
<td>2</td>
<td>-0.15</td>
<td>-0.27</td>
<td>0.28</td>
<td>-0.29</td>
<td>0.21</td>
<td>0.39</td>
<td>-0.04</td>
</tr>
<tr>
<td>RE</td>
<td>(-1.46)</td>
<td>(-1.66)</td>
<td>(2.28)</td>
<td>(-2.43)</td>
<td>(5.74)</td>
<td>(3.26)</td>
<td>(-0.52)</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

The last row contains the expected signs predicted by the model.

$$\bar{R}^2 = 0.23, \quad .27$$ (1.58)
1.7 Conclusions

This chapter contributes to the understanding of debt management and its optimal composition.

First, it considers the incentive problem faced by the government. It may be tempted to reduce the real return to nominal debt by increasing the inflation rate. In equilibrium, it ends up only with higher inflation rate and the same real payment. This induces the use of short and foreign denominated debt.

This presumption is tested using information on the level of debt in several countries. The higher the level of debt the more tempted to inflate a government becomes and the lower the proportion of long nominal debt we should see in the government portfolio. This is strongly confirmed by the data for the 12 OECD countries during the last 25 years.

Second, the chapter shows the relevance of long nominal and foreign denominated debt as hedging devices. The model highlights the fact that the composition is relevant even when governments maximize the utility of infinitely lived consumers who do not regard public debt as private wealth. It stresses the fact that governments would like to have state contingent liabilities and that long nominal or foreign denominated debt act as implicitly contingent debt. The hedging properties of those liabilities depend on the covariances between the inflation rate and the other components of the fiscal budget, namely, government spending and GDP. Inflation positively correlated with spending and negatively with GDP growth, increases the attractiveness of those liabilities. Pure fluctuations in inflation, however, just increase the variance of inflation and, therefore, tend to reduce the attractiveness of long nominal debt.

If these hedging properties are relevant, they must be reflected in the composition chosen by the OECD governments. The regression results, in general, confirm their relevance. Although the effect of the covariance inflation-growth is not significant, the covariance between inflation and spending has a strong significant effect on the debt composition chosen by governments. In addition, pure inflation variance clearly reduces the proportion of long nominal debt.
Foreign, Short, Indexed and Total Debt

UK

![Graph showing the proportions of foreign, short, indexed, and total debt over GDP from 1965 to 1990. The graph includes lines for each type of debt, with distinct markers for each category.](image)
Foreign, Short and Total Debt
BELGIUM

Short and Foreign Proportions


0 0.05 0.1 0.15 0.2 0.25 0.3

0 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2

Foreign  Short  Total Debt
Foreign, Short, Indexed and Total Debt
IRELAND

Short, Foreign and Indexed Proportions

Total Debt Over GDP

- Foreign
- Short
- Total Debt
- Indexed Debt
Foreign, Short and Total Debt
CANADA

Short and Foreign Proportions

D/Y

0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.05
0

Foreign  Short  Total Debt
Short and Total Debt
US

Short Proportion


Total Debt Over GDP

0.25 0.3 0.35 0.4 0.45 0.5 0.55 0.6 0.65

Short Proportion

Short

Total Debt
Ireland
Covariance Series of Inflation

PV of Expenditure
PV of Growth
Germany
Covariance Series of Inflation

- PV of Growth
- PV of Expenditure
Germany
Covariance Series of Foreign Inflation


-4 -3 -2 -1 0 1 2 3

PV of Growth  PV of Expenditure
Belgium
Covariance Series of Foreign Inflation

PV of Growth

PV of Expenditure
Finland
Covariance Series of Inflation

-2 -1 0 1 2 3 4

PV of Growth  PV of Expenditure
Finland
Covariance Series of Foreign Inflation
Australia
Covariance Series of Inflation

PV of Growth
PV of Expenditure
Australia
Covariance Series of Foreign Inflation

- PV of Growth
- PV of Expenditure
France
Covariance Series of Foreign Inflation

PV of Growth  PV of Expenditure
Spain
Covariance Series of Inflation

PV of Growth
PV of Expenditure
Spain
Covariance Series of Foreign Inflation

PV of Growth
PV of Expenditure
Italy
Covariance Series of Inflation

- Expenditure
- Domestic Growth
Italy
Covariance Series of Inflation

Foreign Expenditure  Foreign Growth
Austria
Covariance Series of Inflation

Expenditure
Growth
Austria
Covariance Series of Foreign Inflation

- Expenditure  - Growth
Netherlands
Covariance Series of Inflation

PV of Growth
PV of Expenditure
United Kingdom
Covariance Series of Inflation

PV of Expenditure

PV of Growth
United Kingdom
Covariance Series of Foreign Inflation
Chapter 2

Capital Flows and Crises: The Role of Liquidity

2.1 Introduction

The Mexican external crisis of December 1994 brought into question our basic understanding of these type of events. The collapse of the Peso was prompted by an initial devaluation and was characterized by a severe run against the foreign reserves caused by a sudden outflow of capital. The immediate preoccupation of the Mexican government (and several policy makers in the US) was to solve the very short run problem of rolling over the debt and avoiding the major step of announcing their default. The run against Mexican assets gives the impression that there is a strong component of a liquidity crisis involved, which is more similar to the models of the Bank Run literature than to the traditional models of balance of payment crises.¹

The established theoretical framework on balance of payment crises is based on the large literature on speculative attacks that followed the seminal article by Krugman (1979). The key starting point of this literature is that the government follows an inconsistent fiscal policy combined with a fixed exchange rate regime, that would eventually have to collapse. The major contribution, then, is to define exactly when

and how the collapse occurs.\(^2\)

Although it is still disputed whether there was a *hidden* fiscal deficit in the years preceding the Mexican crises of December of 1994, the normal measures of fiscal budget indicated that Mexico was running budget surpluses.\(^3\) But, even if there had been a *hidden* fiscal deficit or simply an increase in domestic credit, the run on the Mexican assets and the liquidity crisis that followed suggest that the observed “bank run” type of behavior has to be an essential part of the modelling of a Balance of Payment crises. The objective of this chapter is to introduce liquidity considerations into the theory of exchange rate collapses.

This chapter focuses on the interaction between liquidity, capital flows and exchange rate collapses in a structure inspired by the original bank run modelling of Diamond and Dybvig (1983). In this context, the chapter will show how these interactions imply strong amplification and propagation effects of a given initial shock and, therefore, can give a rationale for the observed large swings in capital flows that generate exchange collapses but that are not preceded by a proportional negative shock.

Liquidity considerations arise only in a world where there are intermediaries transforming maturities, offering liquid assets to their customers and, implicitly, allowing the possibility of runs on their assets. Thus, the introduction of intermediaries in the model is a synonym for liquidity creation and all its side effects.

The model below highlights the fact that there is an asymmetry between the time needed for investment to mature and the timing of investors. The latter are short sighted by necessity. They may need the money in the short run for their consumption or want to have liquid assets in order to have the flexibility to invest in other places in the short run. The intermediaries offer these assets to investors in order to attract them. On the other side they invest in production which needs time to mature (early interruptions are not profitable). In other words, they transform their illiquid assets

---

\(^2\)See, e.g., Agénor et al. (1992) and Dornbusch (1987).

\(^3\)As of September of 1994 the fiscal budget surplus - GDP ratio figures are as follows: 1.6\% in 1992, 1.0\% in 1993, and -0.5\% in 1994.
into liquid ones in order to attract capital. It is precisely this transformation that brings capital to the economy but it is also the one that introduces the possibility of runs. Ex-post, the good equilibrium is the one in which the intermediary offers liquid assets, there are no runs and (more) investment is realized. However, the possibility of runs and massive disruption does exist.

Intermediation, therefore, produces two main effects. On one hand, it can increase the capital inflows to the economy. By allowing more flexibility, offering more liquid assets, intermediaries improve the attractiveness of the economy in the eyes of the foreign investors. On the other hand, they may generate runs that induce large capital outflows, amplifying initial shocks that otherwise would not have generated crises.

The idea that higher capital inflows are related to increasing intermediation is a phenomena that has a strong counterpart in the real world. For instance, if we analyze the episodes of capital inflow surges studied in Schadler, et al. (1993), there is evidence that financial intermediation increased significantly during the time of the surges. Figure 2.1 presents real claims of the financial sector on the private sector during these episodes (Chile, Egypt, Mexico, Spain, and Thailand). The surge starts in quarter 0. It is clear from the figure that in all five countries financial intermediation increased during the surge. The composition of capital inflows is also interesting. Table 2-1 presents the figures for the countries in the study of Schadler et al. (1993). The main conclusion is that neither Foreign Direct Investment nor Portfolio Investment are the driving forces of these surges. Other capital—which is more associated with intermediation—explains the bulk of the inflows. This includes bonds, direct borrowing, and other short and long run fixed income instruments.

Intermediation, together with its creation of liquid assets, allows for the possibility of runs and crises but it does not generate crises by itself. Throughout the paper, we analyze three types of shocks: productivity, international interest rates, and exogenous need for liquidity by foreign investors. For each type of shock, there will be a cutoff point that determines a region where runs against the intermediary are the equilibrium outcome. This region is determined by the foreign investors, who decide whether to accelerate the timing of their withdrawals. With this region defined
Figure 2-1: Real Financial Claims on the Private Sector - Percentage of GDP

Table 2.1: Composition of Some Capital Inflow Surges

<table>
<thead>
<tr>
<th>First Year of Surge minus Previous Year, US$ mill.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Years of Surge</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Chile</td>
</tr>
<tr>
<td>Egypt</td>
</tr>
<tr>
<td>Mexico</td>
</tr>
<tr>
<td>Spain</td>
</tr>
<tr>
<td>Thailand</td>
</tr>
</tbody>
</table>

Source: IFS.
Note: The countries are those studied in Schadler et al. (1993). Colombia was left out because of lack of intermediation data.
we can explicitly determine the probability of crises. In this sense we depart from the standard “bank run” literature in which the outcome of the models are multiple self-fulfilling equilibria whose likelihood is not determined endogenously.

The interaction between exchange rate collapses and runs against the intermediaries is especially interesting. The effects work in both directions. The existence of runs against the intermediaries generates a sudden demand for reserves that may force a devaluation of the currency, independent of the fiscal policy followed by the government. On the other hand, an expected devaluation of the currency will change the return profile of the investment, increasing the benefits of early withdrawals, and, therefore, increasing the chances of a collapse.

The chapter is organized as follows. In section 2.2 we set up the simplest possible model with its basic components: foreign investors, intermediaries, technology and the central bank. As a useful benchmark, we initially solve the model for the capital flow pattern that would exist in the absence of intermediation. Then, we introduce intermediation, solve for the optimal early withdrawal policy, and identify the endogenous probability of runs. We show that this probability is strictly positive and does not decrease when intermediaries offer more liquidity. In section 2.2.2, we verify that runs effectively increase the capital outflows and in section 2.2.2 we propose that, under certain conditions, capital inflows may actually increase with intermediation. In section 2.3 we give a closed-form solution of the model using a Constant Relative Risk Aversion (CRRA) utility function and a Bernoulli distribution of the shocks. In several simulations, we show that capital inflows effectively increase with intermediation and we look at some comparative statics.

The relationship between runs on intermediaries and exchange rate collapses is explored in section 2.4. First, we verify that runs increase the probability of an exchange rate collapse. Then, we show that the possibility of a devaluation increases the region where runs against intermediaries are the unique equilibria. Finally, we analyze the interactions of two intermediaries with imperfectly correlated investment pools, showing that runs against an otherwise liquid intermediary can occur if there is a run against the other intermediary. This effect increases both the size and probability of
the collapse.

Once the main contributions of the chapter are completed, we explore some extensions. First, in section 2.5 we demonstrate how all the effects can still go through when the nature of the initial shock is changed. We explore the interesting case where the impulse is the international interest rate. Finally, in section 2.6 we show how the model can be used to explore contagion effects in capital flows. When there is a crisis in some part of the world and investors have more liquidity needs, the model shows how that a domestic crisis can easily result.

### 2.2 The Basic Model

International Investors are risk averse agents that maximize their expected utility of wealth, choosing their optimal portfolio allocation between a safe international asset and a risky foreign technology (*home* from the perspective of the receiving country).\(^4\) They solve

\[
\text{Max}_a \ E[U(\tilde{W})] 
\]

s.t.

\[
\tilde{W} = W_0(a\tilde{r} + (1 - a)r^*), \tag{2.2}
\]

where \(W_0\) is the initial endowment, henceforth set equal to 1. \(r^*\) and \(\tilde{r}\) are the gross returns on the safe international asset and the risky asset abroad, respectively.

Investors may have liquidity needs. They have a random probability of requiring the money. At time zero each investor does not know if he will need the money in the next period. We assume that the discount rate equals to 1.

Time is discrete and there are three periods. As in Diamond and Dybvig (1983), investors are divided between two types:

**Type 1 - Early Consumers** There is a proportion \(\theta\) of the population that needs the money in period one. Their utility function is \(U(W_1)\), where \(W_1\) is wealth.

---

\(^4\) Here we do not need a riskless international technology but only a safer one.
in period 1. These are the investors that will always interrupt the investment in period one.

**Type 2 - Late Consumers** They are in proportion $1 - \theta$ and their utility function is $U[W_2]$, where $W_2$ is wealth in period 2. These investors have the option to maintain their resources invested in the technology but may choose to withdraw in period 1 if this is more profitable.

Although each investor does not know what his type is in period 0, we will assume that the proportion of the population $\theta$ that have liquidity needs is fixed and known.\footnote{We normalize the total number of investors to be 1.} In section 2.6 we will relax this assumption and analyze the model when there is uncertainty with respect to the proportion of early consumers.

The return on the investment abroad is ultimately tied to a constant returns to scale technology. It is relatively irreversible, requiring some time to generate profits. The gross return on a unit invested in this technology is given by:

$$
\text{Return} = \begin{cases} 
\hat{R} & \text{if } t = 2 \\
q & \text{if } t = 1.
\end{cases}
$$

(2.3)

Here we assume that $q < r^*$. This captures the fact that investment is irreversible or illiquid. Illiquidity is defined as the cost to liquidate an asset in the short run. This cost is the difference between the return on the short run and the return per period of the technology in the long run. The technology generates $\hat{R}$ if it is not interrupted in period 1. This return has a publicly known distribution $\mathcal{G}(\hat{R})$. We assume its support has a lower bound $\underline{R} = q$.

The investors do not need to invest directly in the technology. They can use the services of the intermediaries, that compete à la Bertrand. The intermediaries role is to transform the illiquid technology into liquid assets, providing liquidity to potentially illiquid investors. Their liabilities may be composed of demand deposits (as in the case of the banks), other fixed income assets (investment banks or governments)
or simple quotas (as in mutual funds). Here we will simply assume that they offer the following contract to the investors:

$$
\tilde{r} = \begin{cases} 
\tilde{r}_2 & \text{in } t = 2 \\
r_1 & \text{in } t = 1.
\end{cases} \quad (2.4)
$$

The transformation of liquidity is done by investing the proceeds in the technology and offering the foreign investors a contract that pays a rate of return $r_1 \geq q$ in period 1. In this way, the intermediary will be effectively reducing the liquidity costs to the investors, which in case of necessity will obtain a better rate. Of course, this contract is feasible because the intermediaries, constrained by the technology, will pay a rate $r_2 \leq R$ in the second period. This reduction of the spread increases utility for sufficiently risk averse consumers.\(^6\)

The link between the rates in different periods is given by the resource constraint of the economy:\(^7\)

$$
\frac{r_1 \theta}{q} + \frac{r_2 (1 - \theta)}{R} = 1,
$$

so that the return promised in period two is given by:

$$
\tilde{r}_2 = \frac{\tilde{R} (1 - \frac{r_1 \theta}{q})}{1 - \theta}.
$$

(2.6)

It is immediately apparent from (2.6) that $r_1 \geq q$ implies $r_2 \leq R$.

The intermediaries compete à la Bertrand, offering investors better rates in order to attract capital and maximize profits. They end up with zero profits and offering a contract with interest rates that maximize investors utility.

The return in equation (2.6) is feasible if only early consumers withdraw in period one. However, the intermediary cannot distinguish between types and will have to honor the withdrawals of every investor. The return that it will effectively be able to

\(^6\)The model does not change in any substantial way if we allow the intermediaries to directly invest a portion of their portfolios in the international safe asset.

\(^7\)Initial wealth is one because individual endowments and the number of investors were both normalized to one.
offer will be:

\[ \tilde{r}_2 = \max \left\{ \frac{\tilde{R}(1 - \frac{r_1 f_1}{q})}{1 - \theta}, 0 \right\}, \quad (2.7) \]

where \( f_1 \) is the proportion of withdrawals in period 1 which cannot generate an outflow greater than what the technology is able to produce:

\[ r_1 f_1 \leq q. \quad (2.8) \]

The transformation of liquidity makes the intermediary vulnerable to runs. There is always the possibility that the expectation of a high number of withdrawals in period 1 (e.g. higher than the proportion of early consumers \( \theta \)) will drain the resources available to continue investing in the technology and the return promised to investors in period 2 may turn unprofitable. All the late consumers will have an incentive to withdraw early. This may generate a self-fulfilling run on the intermediary. Moreover, if the return promised in period one ends up being higher than the realized \( r_2 \) (under a normal proportion of withdrawals \( \theta \)), it will be optimal for everybody to withdraw in period one, and the run is the unique equilibrium outcome. In order to formally analyze the possibility of runs, the behavior of the intermediary under a run must be precisely defined.

We assume that in the case of a run the intermediary will distribute all its assets equally among the investors.\(^8\) Since the bank will have to interrupt all its investment in the technology to pay for the withdrawals, every investor will get \( q \). Thus, the final return profile is:

\[ \tilde{r} = \begin{cases} 
q & \text{in the case of run} \\
r_1 & \text{in } t = 1 \text{ if there is no run} \\
{\tilde{r}_2} & \text{"t = 2".} 
\end{cases} \]

The Central Bank fixes the nominal exchange rate \( e = \bar{e} \). In order to clearly depart from the exchange rate collapse literature, we will assume that the government is not

---

\(^8\)This can also be done as a "first come first serve basis", where the last investors in line do not get anything, as in Diamond and Dybvig(1983). See also section 2.6 where we follow this approach.
following an inconsistent policy: the treasury has a balanced budget and the central bank is not increasing domestic credit.

Also, we will initially assume that the authority has enough reserves RX to maintain the exchange rate fixed even in the event of capital outflows resulting from a liquidity crisis. Therefore, in this section, the returns to foreign investment can be thought of as denominated in the international currency (in order to simplify notation we will normalize the nominal exchange rate to be 1). The more interesting case where reserves are not sufficient to overcome a liquidity crises is analyzed in section 2.4.

Investment is carried out in period 0, the returns are known only in period 1, and realized in period 2. The timing of the model is given below where it is clear that all uncertainty is resolved in period 1:\footnote{We assume that there is no side-trading in the form of early consumers selling their "shares" of the intermediary to late consumers. In the model this is equivalent to assuming that the risk-free investment is not sufficient to finance these transactions. In the actual world we do not observe much of these transactions. A lack of an institutional arrangement and adverse selection considerations may explain this phenomenon.}

\begin{align*}
  t = 0 & \quad \begin{cases}
    \text{Banks specify } r_1 \text{ and } r_2, \\
    \text{Investors decide } a. \\
    \text{Investors learn their type,} \\
    \hat{R} \text{ is realized,}
  \end{cases} \\
  t = 1 & \quad \begin{cases}
    \text{Withdrawal decision made: possibility of runs,} \\
    \text{Central bank sustains or fails to sustain } \hat{e}.
  \end{cases} \\
  t = 2 & \quad \text{Patient investors get } r_2 \text{ if there was no run.}
\end{align*}

\subsection{Absence of Intermediation}

In the absence of intermediation the foreign investors still have the option to invest directly in the technology. The returns are given by the technology in (2.3) and the return on the safe asset $r^*$.

Since the proportion of early consumers is fixed at $\theta$, each investor knows the probability that he will need to withdraw in period 1. The maximization problem is
Max_a E[U[\tilde{W}]] = \theta U(aq + (1 - a)r^*) + (1 - \theta) \int_{\tilde{R}} U(aR + (1 - a)r^*)d\mathcal{G}(\tilde{R}), \quad (2.9)

where \( a \) is the amount (and proportion) of initial wealth invested in the technology.

Each investor has to worry only about his idiosyncratic shock (being a late or early consumer) and the macroeconomic shock \( \tilde{R} \). There is no need to worry about the possibility of exchange rate crises (which will generally affect the returns in the international currency) because we assume that the central bank has enough reserves \( R_x \) to sell to all the early consumers, after subtracting the current account deficit \( X \) from the total reserves \( RX \):

\[
R_x = RX - X \geq \theta qa^*,
\]

and, therefore, is able to sustain the fixed parity. Neither, is there the possibility of runs against domestic assets. There are no intermediaries to link the returns of the investors (here \( \tilde{R} \) and \( q \) do not depend on the behavior of the other investors), hence, the self-fulfilling run cannot exist.

The maximization in (2.9) implies an optimal amount invested in the country given by:

\[
a^*_{ni} = a^*_n(q, \tilde{R}, \theta, r^*), \quad (2.10)
\]

where the subscript \( ni \) stands for no intermediary. The flow of capital, in turn, will be given by:

\[
\begin{align*}
t & = 0 & a^*_{ni} \\
  t & = 1 & -\theta qa^*_{ni} \\
  t & = 2 & -(1 - \theta)Ra^*_{ni}.
\end{align*}
\]

### 2.2.2 Intermediation

Including the possibility of investment through intermediaries introduces two interesting features. First, the intermediary may offer a different return profile to the foreign
investor which may change his investment decisions. It will be particularly interesting when this new pattern increases the capital inflows to the country. Second, with intermediaries there is always the possibility of runs on their assets, provided they are transforming illiquid assets into liquid ones. This possibility has to be taken into consideration by the investor when choosing his portfolio allocation, since it affects the returns, as shown in (2.7).

Higher probability of runs

In order to precisely define the investors’ problem, we need to solve backwards and first obtain the probability of runs. The runs are defined when all the investors withdraw in period 1. Since early consumers are those who always withdraw in period 1, the runs will be determined only by late consumers, who may decide to withdraw early. These will choose to withdraw only if the payoff of waiting is lower than the payoff to immediate withdrawal. In terms of the model, the late consumers will accelerate their withdrawals if

\[ r_1 r^* \geq r_2, \]

which implies that there will be a cutoff in the realization of \( \hat{R} \), say \( \hat{R} \), such that for values smaller than \( \hat{R} \) a run is the unique equilibrium. The cutoff is determined by:

\[
r_1 = \frac{R(1 - \frac{r\theta}{q})}{1 - \theta} \rightarrow
\]

\[
\hat{R} = \frac{r_1 (1 - \theta)}{(1 - \frac{\theta}{q})}, \quad (2.11)
\]

where we have normalized \( r^* = 1. \)

The probability of a run will be given by \( \mathcal{G}(\hat{R}). \)

**Proposition 1** The probability of runs with intermediation is strictly positive. Also, when more liquidity is provided, the probability of runs does not decrease.\(^{11}\)

\(^{10}\)As explained below, we do not consider self-fulfilling runs here.

\(^{11}\)Liquidity provision was defined as setting \( r_1 > q \). More liquidity is increasing \( r_1 \), making it closer to \( \sqrt{R} \), which is the one-period-equivalent return of the technology.
The first part of the proposition is a straightforward consequence of the fact that intermediaries create liquidity which, using equation (2.11) implies that \( \hat{R} > q = R \), and therefore \( \mathcal{G}(\hat{R}) > 0 \). The second part is obtained by differentiating (2.11) with respect to \( r_1 \) and using the definition of liquidity provision by intermediaries \( (r_1 > q) \) we conclude that \( \frac{\partial \hat{R}}{\partial r_1} > 0 \). Given that \( \mathcal{G}'(\hat{R}) \geq 0 \) we establish that the probability of runs cannot decrease (and will most likely increase) with higher \( r_1 \).

In summary, for every \( R \leq \hat{R} \) the only possible equilibrium is a run. The probability of the equilibrium being a run does not decrease when the intermediary increases \( r_1 \), increasing the cutoff \( \hat{R} \).

In addition to the equilibria described above, there is always the possibility of a self-fulfilling run independent of the realization of \( \hat{R} \).\(^{12}\) If all the rest of the investors withdraw it is optimal for a specific investor to withdraw because the return in period 2 depends on the amount withdrawn in period 1 (see equation 2.8). There are two problems with this type of equilibrium. First, as in any sunspot equilibrium, there is not an endogenous probability of the occurrence of this event. A coordinating event is required and this has to be exogenously defined. Second, there are problems involved in defining rigorously the equilibrium concept because along the equilibrium path beliefs have to be correct.\(^{13}\) This means that without an exogenous coordinating event — which makes agents act in a particular way so that the initial beliefs turn out to be correct — the expected probability of a self-fulfilling run has to be zero (if it does not occur) or one (if it occurs). However, if this probability were one, agents would never invest in the first place since runs generate a return lower than the safe return \( r^* \). Thus, without a coordinating event the sunspot equilibrium has to have probability zero and the probability of a run will continue to be given by \( \mathcal{G}(\hat{R}) \).

\(^{12}\)Provided \( r_1 > q \), which is exactly the case when intermediaries create liquidity.

\(^{13}\)See, e.g., Postlewaite and Vives (1987) for more on the problems involved in specifying this as an equilibrium. See Fudenberg and Tirole (1991), pp. 99-100, for some problems that the requirement of correct beliefs along the equilibrium path may cause.
Investors' Problem, Runs and Capital Outflows

When agents invest through intermediaries, each foreign investor takes into account the probability of a run, $\mathcal{G}(\hat{R})$, and the return $q$ in this event. He now solves:

$$
Max_a E[U[\hat{W}]] = 
(1 - \mathcal{G}(\hat{R})) [\theta U(ar_1 + (1 - a)) + (1 - \theta) \int_{\hat{R}} U(a \hat{r}_2 + (1 - a)) d\mathcal{G}(\hat{R})] 
+ \mathcal{G}(\hat{R}) U(a q + (1 - a)),
$$

(2.12)

which gives an optimal investment policy with an intermediary:

$$
a^*_i = a^*_i(r_1, q, \theta, \Omega),
$$

(2.13)

where $\Omega$ includes all the parameters in the distribution. The flow of capital in this case will be given by:

$$
t = 0 \quad a^*_i \\
t = 1 \begin{cases} 
-\theta r_1 a^*_i & \text{with probability } (1 - \mathcal{G}(\hat{R})) \\
-qa^*_i & \text{with probability } \mathcal{G}(\hat{R}) 
\end{cases} \\
t = 2 \begin{cases} 
-(1 - \theta) \hat{r}_2 a^*_i & \text{with probability } (1 - \mathcal{G}(\hat{R})) \\
0 & \text{with probability } \mathcal{G}(\hat{R}) 
\end{cases}
$$

**Proposition 2** There are proportionally more capital outflows in period 1 with intermediation and, particularly, in the event of runs, i.e., $\theta q < \theta r_1 < q$.

The second inequality says that capital outflow in period one is higher with runs. This comes from the fact that the intermediary cannot contract to pay to investors in $t=1$ more than the technology allows (i.e. $r_1 \theta < q$; see equation (2.8)). The first inequality is a straightforward consequence of the fact that intermediaries create liquidity $r_1 > q$.

The increased capital outflows means that with a run against the intermediaries there will be a higher demand on the central banks foreign reserves. We assume
in this section that the central bank has enough reserves, after paying net imports payments, to pay for the capital outflows (i.e., \( R x = R X + X \geq \theta r_1 a^*_i \)).

**Intermediaries Competition and Capital Inflows**

The intermediaries, knowing the investors' function \( a^*_i = a^*_i(r_1, \hat{R}, \theta, \Omega) \), will choose the rate \( r_1 \) to attract more investment and maximize profits. Bertrand competition among intermediaries will lead to zero profits and an \( r_1 \) that maximizes investors utility:

\[
\begin{align*}
Max_{r_1} E[U[\hat{W}]] &= (1 - G(\hat{R}))[\theta U(a^*_i r_1 + (1 - a^*_i)) + (1 - \theta) \int_{\hat{R}}^{\hat{R}} U(a^*_i \hat{r}_2 + (1 - a^*_i))dG(\hat{R})] \nonumber \\
&+ G(\hat{R})U(a^*_i q + (1 - a^*_i)) \tag{2.14}
\end{align*}
\]

subject to equation (2.13).

This gives us an equilibrium \( r_1^* \):

\[
r_1^* = r_1^*(q, \Omega, \theta). \tag{2.15}
\]

Plugging this equilibrium \( r_1^* \) back in the investment function (2.13) we get the equilibrium capital inflows with intermediaries.

**Proposition 3** There exist utility functions and distribution functions such that capital inflows in period 0 increase with intermediation.

In the next section we work out a closed-form solution where \( a^*_ni \leq a^*_i \) (constant relative risk aversion utility function and Bernoulli distribution). Even though investors rationally expect crises in bad states of nature, the benefits from the liquidity provision by intermediaries will more than compensate that effect and will induce them to invest a higher proportion of their portfolio in the economy.
2.3 A Closed-Form Solution: CRRA Utility and Bernoulli Distribution

In order to solve this problem explicitly we will assume a specific distribution for \( G(\tilde{R}) \). In particular we assume:

\[
\tilde{R} = \begin{cases} 
\tilde{R} & \text{with probability } \alpha \\
q & \text{"} \quad 1 - \alpha
\end{cases}
\]

We also assume a constant relative risk aversion utility function (CRRA).

The maximization for the case where \( \hat{R} < \tilde{R} \) becomes:

\[
\max_{a_i, r_1} \quad \alpha \frac{(aq + 1 - a)^{1-\gamma}}{1 - \gamma} + \\
\quad \quad \quad (1 - \alpha)\left[\theta \frac{(ar_1 + 1 - a)^{1-\gamma}}{1 - \gamma} + (1 - \theta) \frac{\tilde{R}(1 - \gamma) a (r_1^{th} - 1)^{1-\gamma}}{1 - \gamma}\right],
\]

where \( \gamma \) is the coefficient of risk-aversion.

The FOCs for this case are given by:

\[
\frac{(r_1 - 1)\theta(1 - \alpha)}{(a(r_1 - 1) + 1)^\gamma} - \frac{\alpha(1 - q)}{(1 - a + aq)^\gamma} + \frac{(1 - \alpha)(1 - \theta)(r_2^{th} - 1)}{(1 - a - ar_2^{th})^\gamma} = 0 \tag{2.16}
\]

and

\[
\frac{\theta(1 - \alpha)a}{(a(r_1 - 1) + 1)^\gamma} + \frac{(1 - \alpha)(1 - \theta)a\tilde{R}q}{(1 - a - ar_2^{th})^\gamma} = 0, \tag{2.17}
\]

where \( r_2^{th} \) is given by equation (2.6) applied to \( \tilde{R} \).

The closed-form solutions for the capital inflow \( a_i^* \) and liquidity provision \( r_1^* \) are given in the appendix. The solution for the capital inflow without intermediary, \( a_{ui}^* \), is presented there as well. Although it is possible to compute partial derivatives from the closed-form solutions, for simplicity we present here some simulations using a concrete numerical example. Figure 2-2 presents the optimal capital inflows with and without intermediaries, and the optimal liquidity provision for different parameter values. The baseline case has the following parameter values: \( \tilde{R} = 1.7, q = 0.8, \).
\( \alpha = 0.6, \theta = 0.2, \gamma = 2. \) These parameters imply the following results: \( a_i^* = 0.942, \)
\( r_i^* = 1.054, \) and \( a_{ni}^* = 0.753. \) That is, intermediation results in liquidity provision
—even in excess of the risk-free rate—, an increase in capital inflows, and an increase
in the probability of collapse —which changes from zero to \( 1 - \alpha. \)

Figure 2-2 show that for parameter values where the intermediaries provide liquid-
ity, that is \( r_i^* > q = .8, \) capital inflows under intermediation are systematically higher.
In principle, there are two opposite effects determining the amount of investment when
there is intermediation. On one hand, by providing liquidity, intermediaries make in-
vestment in the country more attractive to potentially illiquid investors. On the other
hand, the provision of liquidity by intermediaries allows for the possibility of runs and
makes rational investors more cautious with regard to investing in the country. In
the example shown here, the liquidity effect dominates the risk of being forced to early
withdraw (in the case of a run). Notice that for some parameter values the amount of
inflows with intermediaries is the same as without intermediation (i.e., \( a_i^* = a_{ni}^* \)). At
these points \( r_i^* = q, \) and the return and probability of the different states that the
investor faces are identical, regardless of the presence of intermediation. Interestingly,
for parameters values at which the intermediaries (optimally) offer illiquid contracts,
i.e. \( r_i^* < q = .8, \) there are fewer capital inflows.

The assumption about competition among intermediaries means that the liquidity
provided in equilibrium is the optimal one from the investors’ point of view. However,
the optimal level of intermediation from the recipient country’s point of view —which
takes into account the trade-off between the size of capital inflows and the probability
of crisis— is not necessarily the same. If a country prefers to have a low crisis
risk rather than larger capital inflows, capital movement controls, Tobin taxes, and
intermediation controls might be desirable. The experience of Chile during the last
two years provides a good example of such policies (see, e.g., Corbo and Hernandez, 1995).

\[ ^{14} \text{In general, there is a third effect. By changing the wealth of investors, intermediation can}
\text{potentially change 'investors' risk-aversion and, consequently, the amount invested. In our example}
\text{we have left out this effect by fixing the relative risk-aversion.} \]
Figure 2-2: Capital Inflows Simulations
Figure 2-2 allows us to analyze some of the comparative statics involved in the problem. As expected, a higher good-state return, \( \bar{R} \), and a higher early-liquidation return (bad-state return), \( q \), increase the inflows both with and without intermediation. More important, however, is the fact that both the difference between the two inflows and the provision of liquidity increase. A higher probability of a higher return (that is a higher \( \alpha \)) has similar effects. Finally, if a higher proportion of Late-Consumer type of investors (that is a lower \( \theta \)) is expected, there are more inflows and intermediation in equilibrium. However, the difference between inflows with and without intermediation is not monotonic. Initially, it increases when \( \theta \) decreases, but then it decreases. What happens is that at very low values of \( \theta \) the problem with and without intermediation become identical, with no agents taking advantage of the liquidity that intermediaries provide.

### 2.4 Exchange Rate Collapses

The model presented so far has analyzed the effect of financial intermediation on both capital inflows and outflows. This section extends the model in order to investigate the interactions between runs against intermediaries and balance of payments collapses in economies with a fixed exchange rate.

The introduction of an upper bound to the stock of reserves in our previous model both amplifies and propagates the runs against the intermediaries. First, there is the effect of runs on the sustainability of the exchange rate. Relaxing our previous assumption of sufficiently high level of reserves, runs can generate abnormal capital outflows that may force a devaluation. This will be the case if the Central Bank is not able to finance the sudden outflow, in the short run, borrowing immediately against future reserves.\(^{15}\) Thus, outflows generated by runs against intermediaries—even against a small number—will put pressure on the exchange rate and will propagate

\(^{15}\)This is the typical assumption in the Balance of Payments Collapse literature. This will typically be the case if the required future fiscal policy is not credible or if there is risk of strategic repudiation. In this model, the assumption implies that there are no immediate public compensatory flows of capital.
the effects of a negative shock to the rest of the economy. Second, given that forced devaluations are now possible and that portfolio returns depend on them, investors have to recalculate their optimal allocation and the optimal time to withdraw. The anticipation of a devaluation produces strong incentives for a run against the Central Bank. As in the case of intermediaries offering bank-type deposits, the position in the line of the central bank matters because a devaluation produces a capital loss to those at the end of the line. Therefore, even if the investors’ portfolios include “liquid” intermediaries or direct investment, these agents may have incentives for early liquidation because the returns measured in the international currency are affected by the eventual devaluation. Typically, there will be runs in more states of nature. This is the amplification effect that exchange collapses have on intermediaries’ crises.

There is an alternative link between intermediation and Balance of Payments. If intermediaries have a fiscal-backed deposit insurance system, runs against intermediaries will produce an extra burden on the fiscal sector. This extra burden, in turn, will both bring forward a Balance of Payments crises and make it more likely. This link is investigated in Calvo (1995).

In what follows below we will concentrate on the direct amplification and propagation effects between exchange collapses and intermediaries that were described above. The effect of runs against the intermediaries on the sustainability of the exchange rate is investigated first in section 2.4.2. Then, the feedback of exchange collapses on runs are analyzed in sections 2.4.3 and 2.4.4.

### 2.4.1 The Economy Under Fixed Exchange Rate

Before introducing the possibility of devaluations, we need to be more specific with respect to the units in which the projects and the final returns to the investor are measured. The projects are investment opportunities in the non-tradable goods sector, with returns measured and paid in the local currency. Therefore, a devaluation of the currency reduces the return on the foreign investment.

There are $N$ intermediaries that compete à la Bertrand, each one with a pool of projects which gives an aggregate return $\bar{R}_i$. We assume that these returns are not
perfectly correlated, and, for simplicity, that have the same c.d.f. \( G(\cdot) \).\(^{16}\)

The rest of the economy is represented by a sequence of current account deficits \( X_t \) which are exogenous to the model. We assume the current account surplus in period 2 is high enough to finance the highest possible capital outflow in period 2, which, in turn, is given by the maximum possible realization of \( \hat{R} \).\(^{17}\)

There are two key assumptions about central bank behavior. First, under a fixed exchange rate regime, it will try to maintain the exchange rate fixed whenever it is possible. In period one, the authority would like to keep the exchange rate fixed at the level it started in period 0.\(^{18}\) In the event of a devaluation in period 1, given the assumption of a current account surplus in period 2, the Central Bank will fix the exchange rate at the new level. Second, we assume that the central bank follows the following rule-of-thumb in the case of being forced to devalue. As long as the amount of net reserves \( Rx \) (reserves \( RX \) net of current account deficit) is bigger than the demand for reserves (or capital outflows) the exchange rate is kept fixed. If the demand for reserves is higher than the net reserve stock, reserves are exchanged at the fixed exchange rate until they hit a predetermined-specified level \( Rx_{\text{min}} \). At that level the remaining reserves are publicly auctioned so as to clear the market.

With these assumptions, for a given stock of net reserves in period 1, \( Rx = RX + X \), and a given demand for reserves in period 1, \( F/c \), where \( F \) is capital outflows measured in local currency, the exchange rates will take the following values

\(^{16}\)One intermediary would dominate the existence of many intermediaries if administration costs and sector-specific knowledge were not important. We assume here that they are important, meaning that more than one intermediary is optimal. At the same time, these costs make full diversification suboptimal.

\(^{17}\)This assumption precludes exotic cases in which future returns and capital repatriation are so high, that there is a Balance of Payments crisis in period 2.

\(^{18}\)Normalized to be equal to 1.
at the end of each period:

\[
\begin{align*}
e_0 &= 1 \\
e_1 &= \begin{cases} 
1 & \text{if } F \leq Rx \\
1 + \frac{F - Rx}{Rx_{\min}} & \text{otherwise}
\end{cases} \\
e_2 &= e_1.
\end{align*}
\]

In period one, if there are not enough reserves, the exchange rate will increase so that the demand for reserves will match the remaining supply.

Investors, in turn, will face the following exchange rates in period 1:

\[
e_1 = \begin{cases} 
1 & \text{if } F \leq Rx \\
1 & \text{with prob. } \alpha & \text{if } F > Rx \\
1 + \frac{F - Rx}{Rx_{\min}} & \text{with prob. } 1 - \alpha & \text{if } F > Rx,
\end{cases}
\]

where \( \alpha = \frac{(Rx - Rx_{\min})}{F} \). Of course, the smaller \( Rx_{\min} \), the higher the devaluation.

### 2.4.2 The Effect of Intermediation Runs on the Exchange Rate

A run against a financial intermediary has a simple direct effect on the exchange rate determination. Given an amount of reserves and a current account deficit level, these runs increase both the probability of a Balance of Payments crisis, and, if there is a collapse, the size of the devaluation. The non-linearities produced by the intermediation process make small real shocks in project returns translate into Balance of Payment crises.

In terms of the model, and in the simple case of one intermediary, outflows of capital increase by \( \Delta = a^*_i(q - \theta r_1) \) when there is a run, where \( a^*\theta r_1 \) is the "normal" capital outflow. If we assume that there is no Balance of Payment crisis under the "normal" capital outflow, the extra outflow translates into a Balance of Payment crisis if \( \Delta > Rx - a^*\theta r_1 > 0 \). That is, if the Central Bank does not have enough reserves
to sustain the extra capital outflow that results from the run on the intermediary. Moreover, if there is a devaluation, the new exchange rate level will be given by $1 + (a_i^* q - R x) / R x_{\text{min}}$.

Given our assumption that under a “normal” capital outflow there is no exchange collapse, we can extract the probability of collapses from the likelihood of runs against the intermediaries. If we denote by $R^c$ the early withdrawal policy cutoff for $\tilde{R}$, the probability of a crisis will be simply given by $\mathcal{G}(R^c)$.

**Proposition 4** Under a fixed exchange rate regime, the probability of devaluation increases when there is intermediation and the risk of runs.

Under our assumptions, where we normalized the probability of exchange rate collapse to zero if there are no runs against the intermediary, the proposition will be true when $\mathcal{G}(R^c) > 0$. Following the same reasoning as in proposition 1, this proves to be indeed correct.

### 2.4.3 The Effect of Exchange Collapses on Runs: 1 Intermediary

In this section we will show that an expected devaluation will increase the probability of a run against the intermediary (holding constant the feedback from runs on intermediaries to devaluations, shown to exist in the previous section).

 Investors who are able to keep the investment until period 2 will evaluate whether it is convenient to withdraw in period 1. As in the simple model, there will be a cutoff $R^c$, such that if the project return is higher than $R^c$ it is optimal not to withdraw. The cutoff level in this case will depend on the reserve level of the central bank, the current account deficit, and the reserve level at which the authority auctions the remaining reserves.\(^{20}\) In particular, given the amount invested in period 0, $a_i^*$, the

---

\(^{19}\)As shown below, in section (2.4.3) it is not always the case that this is the same cutoff as before, $\tilde{R}$.

\(^{20}\)If we allow for a sunspot equilibrium it is possible to have a full collapse of the intermediary independently of the amount of reserves.
cutoff which defines optimal early withdrawal is uniquely defined by:

\[
R^c = \begin{cases} 
\hat{R} & \text{if } a^*r_1\theta \leq Rx \\
R' & \text{otherwise}
\end{cases}
\]

where \(\hat{R} = r_1 (1 - \theta) / \left(1 - \frac{r_1\theta}{q}\right)\) is our previous cutoff. If reserves are not enough to finance "normal outflows", we can show that the expected devaluation changes the cutoff to \(R'\), which is defined by the implicit equation:

\[
U \left[\frac{a^*\hat{r}_2}{e_2} + 1 - a^*\right] = \\
\alpha U \left[a^*r_1 + 1 - a^*\right] + (1 - \alpha) U \left[\frac{a^*r_1}{e_2} + 1 - a^*\right],
\]

(2.18)

where \(\hat{r}_2 = R' \left(1 - \frac{r_1\theta}{q}\right) / (1 - \theta)\), and where \(\alpha\) is as defined above, with \(F = a^*r_1\theta\).

If \(a^*r_1\theta \leq Rx\), then there is no devaluation if late consumers do not run and the returns are the same as in the simple model. If \(a^*r_1\theta > Rx\), then there is devaluation with probability 1, and there exist a unique \(R'\) such that late consumers are indifferent between early and late withdrawal, taking into account the effect of a devaluation (with \(F = a^*r_1\theta\)). \(R'\) exists and is unique because, given \(F\) the RHS of equation (2.18) is constant and the LHS is monotonic and continuous in \(R'\) (assuming a well behaved utility function: continuous, with \(U'() > 0\) and \(U''() < 0\)).

It is worth noticing that the cutoff \(\hat{R}\) is the same as before, in the case in which there were sufficient reserves to finance any capital outflow. The main result, however, is summarized in the following proposition.

**Proposition 5** If devaluations are expected, runs against the intermediary are more likely.

Proving this proposition amounts to showing that \(G(\hat{R}) < G(R')\), or, equivalently,

\[
\frac{r_1 (1 - \theta)}{1 - \frac{r_1\theta}{q}} < R'.
\]

The inequality can be verified by noticing that if \(a^*r_1\theta > Rx\), then \(1 < e_2\), regardless of the existence of a run against the intermediary. Therefore, the LHS of equation...
(2.18), which is equal to a convex combination of two terms, has to be bigger than $U [a^* r_1 / e_2 + 1 - a^*]$, the smallest of the two terms of the combination. Comparing the arguments of the two functions and using the fact that $U''(.) > 0$, yields the result.

Given this proposition and the previous one in section 2.4.2, runs against intermediaries and exchange rate collapses have a reinforcing effect on each other. This will be investigated in the next section where we do not keep the probability of devaluation constant.

2.4.4 Early Withdrawal Decision: 2 Intermediaries

An interesting interaction between a fixed exchange rate regime and the intermediation process occurs when there is more than one intermediary. In this case, we can show the total effect of having intermediation on both exchange rate crises and the probability of runs, taking into account their mutual feedback (shown to exist in the last two sections).

Potentially, the return on the investment in all intermediaries matters for the decision of early withdrawal from a particular intermediary. The return of other intermediaries matters because the exchange rate affects the final return and the size of an eventual devaluation is a function of the total amount withdrawn in period 1. In general, the early withdrawal solution will be characterized by multiple Nash-equilibria.

Restricting our attention to symmetric solutions in the case of two intermediaries (indexed by $i$ and $j$) we now characterize the Nash-equilibrium strategies. Depending on the amount of reserves in period 1, three different cases can be isolated. In the first one the amount of reserves in period 1 is sufficient to cover the outflows generated by the runs against one or both intermediaries in addition to the "normal" capital outflow (that is the non-run outflow). In this case the decision rule is the same as in the simple case: withdraw in period 1 if and only if $\hat{R} < \tilde{R}$, with $\hat{R}$ defined as above (notice that the strategy in this case is independent of the return of the other intermediary).

In the second case, where reserves are enough to cover the "normal" outflow of cap-
ital, but not sufficient to additionally finance the outflow of a run in one intermediary, the equilibrium strategies can depend on the portfolio returns of both intermediaries. In particular, assuming that $2a^*r_1\theta \leq Rx < a^*q + a^*r_1\theta$, and that $Rx_{\min}$ is sufficiently high (but less that $Rx$), the optimal strategies are characterized as follows:21

There are two cutoff values for $\tilde{R}_i$, $R_{hi}^c$ and $R_{li}^c$, such that for $\tilde{R}_i < R_{hi}^c$ early withdrawal is optimal, and for $R_{hi}^c \leq \tilde{R}_i$ late withdrawal is optimal, regardless of $\tilde{R}_j$. For $R_{li}^c \leq \tilde{R}_i < R_{hi}^c$, the withdrawal decision depends on the realization of the return of the other intermediary $\tilde{R}_j$. If $\tilde{R}_j < R_{li}^c$, then early withdrawal is optimal, and if $R_{hi}^c \leq \tilde{R}_j$ late withdrawal is optimal. If both returns are between the two cutoff values there exist three Nash-equilibria: two pure strategy equilibria (both investors withdraw or both choose to wait) and a mixed strategy one (early withdrawal with probability $\lambda_i$, which in turn depends on the realization of the returns). Moreover, given $a^*$—the amount invested through each intermediary— the cutoff $R_{hi}^c$ is determined by the implicit equation (2.18), with $F = a^*q + a^*r_1\theta$.

Given the central bank policy, the lower bound cutoff $R_{li}^c$ is given by $\tilde{R}$. Returns below $\tilde{R}$ will trigger early withdrawal regardless of the exchange rate, and therefore regardless of $\tilde{R}_j$. This is so because a devaluation will never turn (relatively) less attractive an early withdrawal (given the possibility of getting $e = 1$). The upper bound $R_{hi}^c$ defines the region where higher returns will induce late withdrawal even if there is a devaluation. This cutoff is defined at the highest level of the exchange rate in the absence of a run against $i$, which occurs when there is a run against $j$. Given that particular exchange rate level, the assumptions about $Rx_{\min}$, and a well behaved utility function, it is always possible to find an $R'$ that solves equation (2.18). Let $R_{hi}^c$ be equal to this $R'$. Since the LHS is increasing in $R'$ returns higher that $R_{hi}^c$ make late withdrawal strictly preferred. When $R_{li}^c \leq \tilde{R}_i < R_{hi}^c$, early withdrawal is optimal if and only if there is a devaluation and hence the importance of the realization of $\tilde{R}_j$.

---

21If $Rx_{\min}$ is not high enough, it is not possible to insure that $R'$ is increasing in $F$, and the proposed solution does not need to hold. To show that $R'$ is increasing in $F$ totally differentiate equation (2.18).
In the third case, where reserves are not enough even to cover the “normal” outflow (so that a devaluation occurs with probability 1), the equilibrium strategies will also depend on the returns of both intermediaries because runs will affect the size of the devaluation. In this case we have $R_x < 2a^*r_1\theta$ and again there are two cutoff values for $\tilde{R}_i$, $R_{ii}^c$ and $R_{ii}^l$, which determine the optimal withdrawal policy. If $R_{x_{\text{min}}}$ is sufficiently high, these cutoffs are determined by the implicit equation (2.18), with $F = a^*q + a^*r_1\theta$ and $F = 2a^*r_1\theta$, respectively. For $R_{ii} < R_{ii}^c$ and $R_{ii}^c \leq R_{ii}$ early and late withdrawal are optimal respectively, regardless of $R_j$. For $R_{ii}^c \leq R_{ii} < R_{ii}^e$, the optimal strategy depends on $R_j$ as in the second case.

**Proposition 6** With an eventual unsustainable fixed exchange rate and two or more intermediaries, both the probability of runs against intermediaries and the probability of a Balance of Payments crisis increase (vis-à-vis the case of a sustainable fixed exchange rate or one intermediary).

Following similar steps as in the case of one intermediary it is straightforward to show that $\tilde{R} \leq R_{ii}^c < R_{ii}^e$, which gives the result.\textsuperscript{22}

### 2.5 International Interest Rates

There is a lively debate in the literature about the role of external factors in determining capital flows to (or from) LDCs. There is some evidence that movements in the international interest rate are an important determinant of the direction of capital flows to (or from) LDCs.\textsuperscript{23} However, it is fairly difficult to justify how rather modest changes in the US interest rates can determine the magnitude of these impressive capital inflow and outflow surges. This is certainly the case of a crisis, when the magnitudes of the capital outflows are much larger than the ones predicted by fundamentals.

The structure developed in the previous sections is suitable to show how relatively small shocks may generate large swings in capital flows and, in the case of insufficient

\textsuperscript{22}Again, we need to assume here that $R_{x_{\text{min}}}$ is high enough so that $R'$ is increasing in $F$.\textsuperscript{23} See, e.g., Calvo, Leiderman and Reinhart (1993).
reserves, even an exchange rate crisis. Although the focus up to this point has been the role of internal (or country specific) factor shocks, exemplified by productivity shocks, it is straightforward to extend the model in order to include external factors as the initial impulse.

An initial increase of US interest rates, for example, may prompt more than the normal withdrawals if late consumers have the incentive to withdraw early to take advantage of better opportunities abroad. If this is reinforced by the contract offered by intermediaries, basically offering liquidity and reducing the cost of withdrawal at short notice, the incentive is even higher and a surge of capital outflows may occur. Capital inflows can also be explained if the intermediation process becomes endogenous. For instance, a small inflow prompted initially by a drop in the international interest rate can produce a surge if there are thick market externalities in the process of intermediation, which, in turn facilitate the liquidity provision process.

Using the same methodology as in the case of internal factors, there will a cutoff $r^*$, such that for second-period interest rates higher than $r^*$ all late consumers will have an incentive to withdraw early.\textsuperscript{24} 25 The probability of crises will be given by $G(r^*)$ which will be strictly positive and non decreasing in $r_1$. The runs against the intermediaries will generate a larger outflow and, in the absence of enough international reserves, this may trigger a devaluation. The more liquidity creation by intermediaries, the smaller will be the cutoff and, therefore, higher realizations of the international interest rate will be able to generate a run.

An important consideration is that because it is an external shock, the international interest rate simultaneously affects all intermediaries (and countries) and, hence, could help explain the generalized effect that movements in the US interest rate produce in capital flows across countries. Moreover, if this was the source of instability, cross-country insurance schemes would not work.

\textsuperscript{24}See Hellwig (1994) for a similar model based on the Diamond and Dybvig approach to analyze the interest rate risk. The focus of that paper is quite different from this one; it aims to analyze the optimality of deposit contracts when the interest rate is stochastic.

\textsuperscript{25}The cutoff in this case is $r^* = \frac{R(1 - \frac{\epsilon^2}{2})}{r_1(1 - \beta)}$
2.6 Contagion

One of the puzzles of capital flows to less developed countries has been their excess correlation. The comovement of capital flows is still present after controlling for the effect of fundamentals (different sets of them have been tried) and cannot be easily explained away by simple theories.\(^\text{26}\)

This section contributes to the understanding of this phenomena stressing that the correlation between capital outflows from LDCs can be reinforced by the non-linearities caused by runs against intermediaries. If there is a shock that increases the liquidity needs of foreign investors, then the existence of intermediation may generate a crisis elsewhere. This suggest that shocks are not only magnified within a country but can also be propagated to other countries in a way that simple regressions of capital flows on fundamentals may not capture.

In this section, we will relax the assumption of known proportions of early investors. There will be an exogenous shock to \(\theta\), which means that the proportion of early consumers that will need the money in period one is unknown. This is a shortcut for a particular external shock that affects the liquidity needs of the investors and generate endogenous crises. In particular we will have in mind the "contagion effect," the effect of crises in one country and the resulting withdrawal needs, affecting the outcome of another country (withdrawals from Latin American investment funds prompted by lack of confidence following the Mexican 1994 crisis provides an example). The probability distribution of \(\theta\), \(\mathcal{F}(\cdot)\) is publicly known.

We will show that intermediation will generate an amplification effect on the capital outflows. The realized proportion of early consumers determines the "normal" capital outflow which can then be compared to the outflow in the case of crisis. For sufficiently high \(\tilde{\theta}\), crises will occur and withdrawals will exceed the normal proportion \(\tilde{\theta}\).

In terms of methodology, in order to show the equivalence to our previous treatment, in this section we will explore the case where the intermediary, facing a crisis,

\(^{26}\)This has been documented in Valdés (1995). See also Calvo and Reinhart (1995).
pays in a "first come, first serve basis." Under this scheme the last investors to withdraw do not get anything if the contracted $r_1$ is higher than the liquidation return, i.e., if there is liquidity provision.\footnote{This follows Diamond and Dybvig (1983)}

2.6.1 No Intermediaries

With stochastic $\theta$ the investor solves

$$\max_{a_{ni}} [U(\bar{W})] = \int_0^1 \{ \bar{\theta}U(aq + (1-a)) + (1-\bar{\theta})U(aR + (1-a))\} dF(\bar{\theta}).$$

Again, for the utility function with constant relative risk aversion (CRRA) with parameter $\gamma$, the optimal $a^{*}_{ni}$ will be given by:

$$a^{*}_{ni} = \frac{(1-\Psi)}{(1-q) + \Psi(R-1)},$$

where

$$\Psi \equiv \left[ \frac{E[\theta](1-q)}{(1-E[\theta])(R-1)} \right]^{\frac{1}{\gamma}}.$$

2.6.2 With Intermediaries

As in section 2.2, the cutoff for runs is found by equating the payoffs of withdrawing in period 1 versus period 2, at the realized $\bar{\theta}$:

$$r_1 = \frac{R(1-r_1^{\bar{\theta}})}{1-\bar{\theta}} \rightarrow$$

$$\bar{\theta} = \frac{R - r_1}{r_1\left(\frac{R}{q} - 1\right)} \quad (2.22)$$

Differing from section 2.2, we will assume that the intermediary will pay $r_1$ until it has no money. After that point, everybody in line will receive a zero return.
Assuming that there is a uniform probability of being in the position $s_j$ of the line, we can determine the probability $p$ of receiving $r_1$ in the case of a run:

$$p = \frac{q}{r_1}$$

The maximization problem for the investor now depends on both $\hat{\theta}$ and $p$:

$$\begin{align*}
\text{Max}_{a_1} & \quad \left[ 1 - \mathcal{F}(\hat{\theta}) \right] \left[ pU(ar_1 + (1 - a)) + (1 - p)U(1 - a) \right] + \\
& \quad \mathcal{F}^{-1}(\hat{\theta}) \int_{0}^{\hat{\theta}} \left\{ \hat{\theta}U(ar_1 + 1 - a) + (1 - \hat{\theta})U(a \frac{R(1 - \frac{r_1}{q})}{(1 - \hat{\theta})} + 1 - a) \right\} \frac{d\mathcal{F}}{\mathcal{F}(\hat{\theta})}.
\end{align*}$$

This will generate an optimal amount of investment through the intermediary:

$$a_i^* = a_i^* \{ R, q, r_1, \Omega \}.$$  \hspace{1cm} (2.23)

In the case of a run, the excess capital outflow (the amplification effect produced by intermediation) is equal to:

$$1 - \hat{\theta},$$

which occurs with probability $\mathcal{F}(\hat{\theta})$.

### 2.6.3 Uniform Distribution

This subsection explores the concrete case in which $\theta$ is distributed uniformly between 0 and $\theta_{\text{max}} \leq 1$.

Competition among intermediaries implies that the solution of this problem is equivalent to the solution of the following program:

$$\begin{align*}
\text{Max}_{a_1, r_1} & \quad \left[ 1 - \frac{\hat{\theta}}{\theta_{\text{max}}} \right] \left[ pU(ar_1 + (1 - a)) + (1 - p)U(1 - a) \right] + \\
& \quad \int_{0}^{\hat{\theta}} \left\{ \hat{\theta}U(ar_1 + 1 - a) + (1 - \hat{\theta})U(a \frac{R(1 - \frac{r_1}{q})}{(1 - \hat{\theta})} + 1 - a) \right\} \frac{d\theta}{\theta_{\text{max}}}.
\end{align*}$$

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The rather complicated FOCs are omitted. Notice, however, that the choice of $r_1$ will affect $\hat{\lambda}$, and, therefore, the probability of a crisis. A concrete example where the intermediary increases capital flows, utility, and volatility is given by the following set of parameters. The parameters were chosen to show the trade-off between the size of capital inflows and the probability of crisis.

If $\theta^{max} = 0.6$, $R = 5.6$, $q = 0.4$, and $\gamma = 3$, the investment volume without intermediation, $a_{ni}^*$, is 0.26. With intermediation the results are: $a_{ni}^* = 0.28$ (more investment), $r_1^* = 0.86$ (liquidity effect), and $\hat{\lambda} = 0.43$ (more volatility). The implied run probability is 0.28.

2.7 Conclusion

Exchange rate crisis sometimes occur in a disproportional manner. The resulting capital flows and price movements happen with a force above and beyond any observable initial impulse, generated by an external or internal event. In addition, some crises seem to have a strong component of a run on liquid assets, where a large proportion of the investors (if not all of them) try to cash in their investments ahead of the rest and transfer them abroad. The magnitude and size of the devaluation that follows suggest that this behavior is important and that it is worthwhile to attempt to introduce them into our standard exchange rate collapse models.

In this chapter we stressed the role of run behavior on exchange rate crises and capital flows. We showed that intermediaries, by offering assets that pay a better return in the case of early withdrawal, allow the possibility of runs and magnify the outflows of capital (in particular, in bad states of nature) relative to the no intermediation case.

Also, we showed that if credit is funneled through liquidity creating intermediaries, internal or external adverse shocks may generate runs and large exchange rate devaluations that otherwise would not have occurred. The devaluation, then, propagates the shocks to the rest of the economy. Therefore, it is the fragile financial situation of the intermediaries that allows the propagation and amplification of a given initial
shock and produces strong capital movements and exchange rate overreaction.

Interestingly, we find the effect working in the other direction, as well. The expectation of an exchange rate collapse exacerbates the financial fragility of the intermediaries by reducing the return of the investments in the event of runs, measured in foreign currency units. Therefore, the mutual interaction between financial fragility and exchange collapses can multiply and amplify an initial adverse shock and resemble the magnitude of the crises that are sometimes observed in reality.

The financial fragility of intermediaries raises two valid questions. First, is there a competitive structure that generates this fragile situation? In the model of the chapter, the existence of relatively illiquid investments and investors that have strong liquidity needs, combined with Bertrand competition between intermediaries, produces a situation where the main role of the intermediaries is to create liquidity. The financial fragility situation is embedded in this role.

Second, with rational investors, does the financial fragility still allow us to reproduce the observed surges in capital flows that precede the crisis? Under reasonable assumptions about the utility function and the distribution function of the shocks, we were able to simulate several cases where capital inflow increases with intermediation, even though rational investors anticipate the possibility of runs. The liquidity provision services provided by intermediaries more than compensate for the risk of runs.

The focus on the financial fragility of liquidity creating intermediaries may help explain the different nature of some exchange rate collapses. In Latin America or other recently stabilized countries, where intermediaries are still readily available to offer liquid assets (as a consequence of the previous inflationary environment), external crises take the full proportion, with a bank run phenomenon as a major part of the collapses. In other countries, with less creation of liquid assets, exchange rate crises are costly events, but do not reproduce the bank run effects.

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28See the graph and description in the introduction.
Appendix: Closed-Form Solutions

This appendix presents the closed-form solutions of the optimal levels of investment and liquidity provision for the case of a CRRA utility function and a Bernoulli distribution. The maximization problem and the corresponding FOCs were presented in section 2.3. To find \( a_i^* \) and \( r_1^* \) explicitly we solve equation (2.16) for \( a \) (simplifying terms using equation (2.17)), solve equation (2.17) for \( a \), and equate. The final solutions are given by:

\[
a_i^* = \frac{\Phi_2 \left\{ \theta \bar{R} \Phi_1 + q (1 - \theta) \right\} - \Phi_1 \left\{ \theta \bar{R} + q (1 - \theta) \right\}}{(1 - q) \Phi_2 \left\{ \theta \bar{R} \Phi_1 + q (1 - \theta) \right\} - \Phi_1 \left\{ (\theta - q) \bar{R} + q (1 - \theta) \right\}}
\]

and

\[
r_1^* = \frac{q \left[ \Phi_1 \left\{ \bar{R} - (1 - \theta) \right\} + (1 - \theta) \right]}{\frac{\theta \bar{R} \Phi_1 + q (1 - \theta)}{\Phi_2 \left\{ \theta \bar{R} \Phi_1 + q (1 - \theta) \right\} - \Phi_1 \left\{ (\theta - q) \bar{R} + q (1 - \theta) \right\}}} \times \frac{(1 - q) \Phi_2 \left\{ \theta \bar{R} \Phi_1 + q (1 - \theta) \right\} - \Phi_1 \left\{ (\theta - q) \bar{R} + q (1 - \theta) \right\}}{\left\{ \theta \bar{R} \Phi_1 + q (1 - \theta) \right\} (1 - \Phi_2) \left\{ \theta \bar{R} \Phi_1 + q (1 - \theta) \right\}},
\]

where

\[
\Phi_1 \equiv \left( \frac{q}{\bar{R}} \right)^{\frac{1}{\gamma}}
\]

and

\[
\Phi_2 \equiv \left( \frac{\alpha \left\{ \bar{R} (q - \theta) - q (1 - \theta) \right\}}{\bar{R} (1 - \alpha) (1 - q)} \right)^{\frac{1}{\gamma}}.
\]

Note that for the problem to be well defined we need to restrict the parameter values such that \( \bar{R} (q - \theta) - q (1 - \theta) \geq 0 \).

For the case of no intermediation, the optimal investment level is given by:

\[
a_{ni}^* = \frac{1 - \Phi_3}{1 - q + \Phi_3 (\bar{R} - 1)},
\]
where

$$\Phi_3 \equiv \left( \frac{(1 - q) \{ \theta + (1 - \theta) (1 - \alpha) \}}{(1 - \theta) \alpha (R - 1)} \right)^{\frac{1}{2}}.$$
Chapter 3

The Long Run Appreciation Process

3.1 Introduction

The determination of the equilibrium real exchange rate has been for long time a constant theme in the economic literature, both theoretically and empirically, partially reflecting its relevance to policy makers. The correct assessment of its fundamental value is an important consideration for policy makers, who often need to determine a nominal value for their exchange rate peg, set the correct band in a target zone regime, or simply determine if their currency is over or undervalued for fiscal and monetary policy.

For developing countries this assessment is even more complicated. Long lasting current account deficits do not necessarily imply that there is some deep misalignment in the exchange rate. They are consistent with the early stage of the development process of developing countries, that are expected to generate future receipts that will pay for this prolonged borrowing. In this context, it is even harder to extract any information from the balance of payments to assess if the exchange rate is overvalued.

This chapter analyses the real exchange rate of developing countries, focusing on its value during fast growth periods. In particular, we will be focusing on the Balassa-Samuelson effect: the long run real exchange rate appreciation process of developing
countries. In contrast to Purchasing Power Parity Theory (PPP), which predict that the law of one price will end up prevailing, the Balassa-Samuelson effect predicts a sliding real exchange rate. This is a clear departure from PPP and provides a different benchmark to predict the equilibrium real exchange rate. Once the natural appreciation tendency is accepted, a prolonged period of growth and appreciation should not be considered a disequilibrium phenomena, that soon will exert mean reverting pressures towards its equilibrium value.

The Balassa-Samuelson theory implies that this appreciation tendency is a consequence of the relatively higher price of non tradable goods in richer countries. Large productivity differences in the tradable sector between developed and developing countries deliver a high relative wage that is transmitted, through the labor market, into high relative wages in the non tradable sector. The resulting relatively higher price of non tradables in developed countries imply that real exchange rates of developing countries seem undervalued, when compared to the PPP benchmark. Therefore, as the developing countries grow and reduce their productivity differential in the tradable sector, without a similar improvement in their non tradable sector, their exchange rate should tend to appreciate.

The chapter starts, in section 3.2, with a theoretical derivation of the Balassa-Samuelson effect on an intertemporal context. With open capital markets and factor mobility, higher relative growth in tradables delivers naturally the appreciation process.

In the chapter the labor productivity growth is proxied by growth in GDP per capita. We assume that the relative productivity of tradables is growing faster than in nontradables in the growth process of developing countries. In the absence of direct data on the relative productivity growth of non tradables, the real exchange rate measure is regressed against GDP per capita.

Initially, in section 3.3.2 we analyze a cross section of 138 countries, from the Penn World Tables\(^1\), where the law of one price does not seem to hold and a systematic relationship between per capita GDP and real exchange rate is observed. Then several

\(^{1}\text{Summer and Heston (1991)}\)
sub samples of the data are analyzed, in particular a subdivision in 4 quartiles by income per capita, where the negative relationship does not hold within each subdivision, suggesting that the Balassa-Samuelson effect hold mainly for large differences in income.

The time series information of the data is captured in section 3.3.3 where I analyze the appreciation dynamics of the countries in the sample during the period 1960-1990. In section 3.3.3 panel data regressions were performed and reinforce the cross-section results.

In order to focus on fast growing episodes of developing countries, five Asian economies were selected as case studies, namely Taiwan, Singapore, Hong Kong, Korea and China, and analyzed in section 3.3.4 and their appreciation rates decomposed.

Finally, the implication of the Balassa-Samuelson effect to exchange rate policy in developing countries is explored in section 3.5.1, where the sustainability of long lasting exchange pegs is reassessed given the natural real appreciation tendency of growing economies.

### 3.2 Theory and Balassa-Samuelson Effect

In this section\(^2\) we will be looking at an economy with factor mobility and open capital markets. There are two goods in this economy: Tradable and Non tradable. We assume that the terms of trade are given so that we can treat tradable goods as a composite good. We define the real exchange rate as the relative price of tradables, \(e\).

The representative consumer maximizes utility intertemporally:

\[
\max_{\{C_t, C_{N_t}\}} U = \int_0^\infty \exp(-\beta t) \frac{(C_T^\alpha C_N^{1-\alpha})^{1-\gamma}}{1-\gamma},
\]

where \(\beta\) is the discount factor and \(\frac{1}{\gamma}\) is the intertemporal elasticity of substitution. The subscripts \(T\) and \(N\) refer to tradable and non tradable sector, respectively.

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\(^2\)This section is based on Rogoff(1991) and Dornbusch(1983).
Her budget constraint is:

\[ \dot{B}_t = r^* B_t + e_t C_{Tt} + C_{Nt} - Y_t, \]  

(3.2)

where \( Y \) is total income measured in terms of tradables and \( B \) is the amount of foreign debt.

For simplicity and to avoid this country to accumulate or decumulate forever, we will assume that \( \beta = r^* \).

Production of both tradables and non tradables is given by Cobb Douglas production functions:

\[ Y_T = A_T(K_T)^{\theta_T} \]  

(3.3)

\[ Y_N = A_N(K_N)^{\theta_N}, \]  

(3.4)

where \( K \) is the capital – labor ratio in respective sector and \( A \) is the productivity parameter. \( Y \) is also expressed in per capita terms.

The consumption of non tradables cannot be smoothed internationally and therefore must be equal to the domestic production in every period.

\[ Y_N = C_N. \]  

(3.5)

In contrast tradables can be exchanged internationally using the capital markets. Assuming a non ponzi condition (such that it will not be possible to accumulate debt for ever) we get:

\[ \int_0^\infty C_{Tt} \exp(-r^* t) = \int_0^\infty Y_{Tt} \exp(-r^* t) - B_0. \]  

(3.6)

The condition above guarantees that the current account deficits or surplus are sustainable in the long run. Initial deficits will be paid by future surplus or, at least, will not be allowed to accumulate in an explosive debt.

In every period we must have that the relative price of non tradables in terms of
tradables is equal to the ratio of marginal utilities:

\[ e = \frac{\alpha C_N}{(1 - \alpha)C_T} = \frac{\alpha Y}{(1 - \alpha)C_T}, \]  

(3.7)

therefore the evolution of the composition of tradables and non-tradables will depend on the time path of the relative price of non-tradables. Also, the intertemporal allocation of consumption will depend on the time trend of the relative price of non-tradables. If the price of non-tradables is expected to fall, consumers will optimally transfer consumption to the future, since they can always adjust their consumption to take advantage of a future lower price of their basket. Defining the real exchange rate as \( e = \frac{1}{p} \), the relevant real interest rate of this economy is the international interest rate \( r^* \) plus the rate of depreciation of the exchange rate. The depreciation rate and the growth rate of consumption will be given by:

\[ \frac{\dot{e}}{e} = \frac{\dot{C}_N}{C_N} - \frac{\dot{C}_T}{C_T}, \]  

(3.8)

and

\[ \frac{\dot{C}}{C} = (1 - \alpha)\frac{1 - \gamma}{\gamma} \left\{ \frac{\dot{e}}{e} \right\}, \]  

(3.9)

where \( C \) is the overall consumption level given by \( C_T + \frac{C_N}{e} \).

The last result is a consequence of the fact that we assumed that the rate of time preferences is equal to the foreign interest rate and, also, from the fact that the relevant interest rate for the domestic consumers is the sum of the foreign interest rate plus the depreciation rate, \( r^d = r^* + \frac{\dot{e}}{e} \).

The consumption path of tradables and non-tradables is given by:

\[ \frac{\dot{C}_T}{C_T} = (1 - \gamma)\frac{1 - \alpha}{\gamma} \left\{ \frac{\dot{e}}{e} \right\}, \]  

(3.10)

\[ \frac{\dot{C}_N}{C_N} = \frac{\alpha(\gamma - 1) + 1}{\gamma} \left\{ \frac{\dot{e}}{e} \right\}. \]  

(3.11)

For intertemporal elasticities of substitution greater than 1, which is the value of the intratemporal elasticity of substitution, the consumption of tradables increases
with the depreciation process. If the substitution of tradables for non tradables is greater than the intertemporal effects then the result above is reversed. The consumption of non tradables will always decrease with the appreciation process.

The most interesting fact of this open economy with factor mobility is that the evolution of the real exchange rate, which we have seen determines the composition and time path of consumption, is determined only by the supply side of the economy. Its path is given by the differential growth in productivity between the tradable and non tradable sectors. This is easily seen from the profit maximization of firms in both sectors.

Given international capital mobility, the marginal product of capital in both sectors will be given by the foreign interest rate and, therefore, will define the appropriate capital/labor ratio in the tradable sector. The wage in this sector will be given by this capital/labor ratio,

\[
\begin{align*}
    r^* &= A_T \theta_T K_T^{\theta_T-1} \rightarrow K_T^*, \\
    w &= A_T (1 - \theta_T) K_T^{\star \theta_T} \rightarrow w^*.
\end{align*}
\]

(3.12)

If factors are perfectly mobile, as we are assuming in this section, then wages and real interest rate in the non tradable sector must be the same. This fact will determine simultaneously the equilibrium real exchange rate and the capital/labor ratio in this sector:

\[
\begin{align*}
    w^* &= \frac{A_N (\theta_N \gamma - 1)}{e} K_N^{\theta_N} \\
    r^* &= \frac{A_N (\theta_N \gamma - 1)}{e} K_N^{\theta_N-1} \\
    \end{align*}
\] \rightarrow \{K_N^*, e^*\}

(3.13)

Therefore, given foreign interest rate and the productivity parameters in both sectors we can determine the factor demand decision and the equilibrium real exchange rate (the relative price of tradables). This will give us that the real appreciation path is given by the difference in productivity growth in tradables versus non tradable
sector. Differentiating equations 3.12-3.13 we have:

\[
\frac{\dot{e}}{e} = \dot{A}_N - \left(\frac{1 - \theta_N}{1 - \theta_T}\right) \dot{A}_T,
\]

(3.14)

where hats are rates of growth.

Here we have our main result. In the case that the growth in productivity is higher in the tradable sector than in the non tradable, then we obtain the famous Balassa-Samuelson finding that the real exchange rate tends to appreciate in growing economies. In the empirical section we will verify this proposition both in cross section and time series analysis.

Also, since the rate of growth of consumption and the evolution of the composition of consumption depend on the appreciation rate we have the following results: total consumption of non tradable is decreasing and consumption of tradables is increasing if the intertemporal elasticity of substitution is not very big (see equations 3.10 and 3.11).

Given an assumption about \( \gamma \), the intertemporal elasticity of substitution, we can trace the behavior of the trade balance. For high enough elasticities, with \( \gamma \) lower or equal to 1, the consumption of tradables is decreasing, or at most is constant, while the production is increasing since more resources are allocated to this sector. Therefore we have an initial trade deficit which becomes a trade surplus as production of tradables increase. On the other hand, if the intertemporal elasticity of substitution is too low and consumption of tradables is increasing with the appreciation then the trade balance will depend on the rate of productivity growth.

\[
\{A_N, A_T\}_0^\infty \rightarrow \{\epsilon\}_0^\infty \rightarrow \{CA\}_0^\infty,
\]

(3.15)

where \( CA \) stands for current accounts.
3.3 Empirical Evidence

This section analyses the exchange rate dynamics in a growth environment. Estimates for 138 countries of real exchange rate and GDP per capita in Purchasing Power Parity are obtained from the Penn World Table that is related to Heston & Summers (1991).

The main goals are to see if there is any systematic relationship between GDP per capita and real exchange rate in a cross section analysis and, also, to verify the proposition that for individual countries, specially those that have grown substantially in the last decades, there is a tendency to have a real exchange rate appreciation during their growth process. This will be done in some detail, analyzing subgroups of countries divided by their income per capita. Also, several fast growing countries, particularly the east Asian economies, will be analyzed thoroughly, concentrating on their appreciation process.

3.3.1 Definitions, Data and Procedures

Real Exchange Rate Definition

In this section we define the real exchange rate as the ratio of nominal exchange rate and the estimate of the purchasing power parity:

\[ RER = \frac{E}{PPP} \]

Where E is the nominal exchange rate expressed as national currency per unit of US dollar and PPP=P/P* is the purchasing power parity estimate (or, national currency units necessary to buy one unit of US dollar worth of the country’s GDP).

This definition is related to the real exchange rate defined as the relative price of nontradables. Since prices of tradables are equalized internationally, the above definition can be rewritten as the relative price of nontradables.

The real exchange rate estimate can be obtained directly from the "Price Level" estimate in Heston & Summers. The "Price Level", which is the column 13 in PWT5, is just \( \frac{PPP}{E} \), the inverse of our definition of real exchange rate.

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Estimates of GDP

Heston & Summers data also provide estimates of GDP in PPP. These are obtained using the four International Comparisons Project's (ICP) benchmarks: 70, 75, 80 and 85. The ICP divides GDP in about 150 categories (including 110 Consumption, 35 investment and 5 government items), grouping individual items into one of these categories. For each category, country's item prices are expressed as ratios of the corresponding item prices of a numeraire country, the United States, and then averaged to take into account for missing products in some countries. The country average in each category is denominated in national currency relative to the US dollars: \( \frac{P_i}{P_{us}} \). In addition to this itemized prices, the countries provide expenditure data in their own currency for each of the 150 categories. To obtain the expenditure in US category price, we divide the expenditures by the price ratios:

\[
\frac{P_i Q_i}{P_i P_{us}} = P_{us} Q_i,
\]

for every expenditure j.

These individual expenditures expressed in dollars must be aggregated to obtain the overall GDP in PPP. This aggregation must be done in a way to preserve the comparability of the GDP's across countries. Therefore an average of the relative prices in all countries are used to aggregate categories in a global GDP estimate. These average relative price aggregation gives rise to GDP's measured in "International Dollars" or PPP estimation. The level of prices are normalized such that the US GDP is the same in international dollars as in American dollars.

It's worth to note that services, that are considered "comparison resistant", are also included in these estimates of GDP.

The final product of the ICP's are these GDP estimates in PPP terms for the benchmark year. Using this data, Summer & Heston extrapolate the series to obtain estimates of GDP for the rest of the period, 1960-1988. They also extend the results to countries which did not participate in the original project. In their extrapolation, they estimate four measures of GDP:
- **RGDP**

  Real GDP. Column 6 in H& S. Based in the ICP results for 1985, they create a "real" series using the growth rates of the constant-price national account GDP series. This estimates are expressed in 1985 international prices.

- **CGDP**

  GDP per Capita in current international prices. In order to obtain the CGDP for years other than 1985 they need to obtain PPP estimates for all the years. This is done applying the implicit GDP price index (from the constant-price and current price national account) in the 1985 PPP estimates to get other years estimates. These new estimates, along with the current-price GDP's expressed in the countries own currency are used to get the GDP's in current international dollars.

- **RGDPCH**

  Same as RGDP but uses chain index to capture the change in relative prices along the years, instead of maintaining 1985 international prices as a basis of comparison.

- **RGDPTT**

  Same as RGDP but including the effect of changes in the terms of trade in the value of the countries output. The net foreign balance is valued in current prices while the domestic absorption part is the same as in RGDP.

In this study, we used the CGDP series in our cross section regressions and plots. This series is more appropriate when comparing countries GDP in a specific year since it uses the more recent relative prices instead of the 1985 benchmark prices. For our purposes, the advantage of using CGDP instead of RGDP was not existent or relevant since we ended up comparing countries GDP only in 1985 and 1988.

When looking at the behavior of specific countries along the whole period, the series used were either the RGDP or RGDPCH, which are comparable across time.
(in contrast to CGDP which uses different average relative prices to calculate PPP estimates each year). The results did not depend on which series was used, although the absolute values of the two GDP series differ substantially, specially for years far away from the 1985 benchmark.

The Summer and Heston data extended the results of the ICP to more countries. For example, in the 1985 benchmark there were only 57 countries surveyed but the Summer and Heston data offers information on 138 countries. This is done, either extrapolating results from other surveys, when the country participated in at least one of those, or using estimates based on surveys conducted by the United Nations as part of a post allowance program designed to equalize salaries of high-ranking executives assigned to different countries. The PPP's for these countries are more problematic since they are based on the baskets of high-income non nationals living in big cities rather than a broader measure appropriate to a GDP measure. In order to mitigate this problem, they found a structural relationship for surveyed countries between their PPP's and their post allowance PPP's. This relationship was used to obtain the PPP's of non-surveyed countries from their post allowance ones.

This study uses benchmark as well as non benchmark countries in the cross section regressions and, also, in the individual country analyses. Therefore, the quality of the data varies substantially across our chosen countries (see table 3.1 below, for more details).

We also used the GDP's measured in US dollars, converted using the nominal exchange rate. From the PWT5 table of H& S this is easily obtained multiplying column 9, CGDP, by the "price level", PPP/E, on column 13.

3.3.2 Cross Section Regressions and Results

The first regression inputs all 138 countries using their GDP in PPP term, CGDP, relative to the US (US=1) , and their real exchange rate, c/PPP. The results are shown in figure 3-1 and table 3.1. The solid line is the predicted values from the regression of the log of the real exchange rates on the log of the relative GDP per capita. The coefficient value is highly negative (-0.29) and highly significant (t-stat
<table>
<thead>
<tr>
<th>Description</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>138 Countries in PPP</td>
<td>-0.28</td>
<td>-10.34</td>
<td>0.04</td>
</tr>
<tr>
<td>100 &quot; &quot; 1990</td>
<td>-0.36</td>
<td>-8.54</td>
<td>0.42</td>
</tr>
<tr>
<td>138 Countries in U$\S$ GDP</td>
<td>-0.26</td>
<td>-17.08</td>
<td>0.68</td>
</tr>
<tr>
<td>57 original countries in PPP</td>
<td>-0.32</td>
<td>-8.13</td>
<td>0.54</td>
</tr>
<tr>
<td>1st Quartile in U$\S$ GDP</td>
<td>-0.14</td>
<td>-0.68</td>
<td>0.02</td>
</tr>
<tr>
<td>2nd Quartile in U$\S$ GDP</td>
<td>0.05</td>
<td>0.17</td>
<td>-0.03</td>
</tr>
<tr>
<td>3rd Quartile in U$\S$ GDP</td>
<td>-0.13</td>
<td>-0.62</td>
<td>-0.02</td>
</tr>
<tr>
<td>4th Quartile in U$\S$ GDP</td>
<td>-0.42</td>
<td>-4.81</td>
<td>0.40</td>
</tr>
<tr>
<td>Panel 115 countries 60-88</td>
<td>-0.11</td>
<td>-7.13</td>
<td>0.016</td>
</tr>
<tr>
<td>&quot; &quot; &quot; Random Effects</td>
<td>-0.16</td>
<td>-15.22</td>
<td>0.065</td>
</tr>
</tbody>
</table>

-10.33).

It is apparent from the regression and graph that there is a nonlinear negative relationship between real exchange rate and GDP per capita. The more richer is the country the closer its real exchange rate is to one, which is the value predicted by the strong version of the PPP theory. Moreover, this relationship is not linear, the real exchange is decreasing at higher rates between low and middle income than between middle and high income countries. This result suggests a "convergence" property for real exchange rates: the poorer is the country the more depreciated is its real exchange rate. Notice that this property depends on the convergence in GDP per capita which is not a settled issue in the literature.

Using the new version of the PWT table (mark 5.5) which updates the series to 1990, we replicated the regression and the results are shown for hundred countries in figure 3-2.

It is interesting to note that China is a complete outlier in the figure. Its real exchange rate of 8.38 is way above the other countries, even if only comparing between the poor countries. If the "convergence" property is correct this implies a higher rate of growth or, maybe, a higher rate of appreciation. The other outliers are Mozambique (9.33) and Laos (12.77).

As mentioned in the previous section, the inclusion of all countries in the cross
Figure 3-1: Real Exchange Rate for All 138 Countries in 1985

Estimates of GDP in Purchasing Power Terms Relative to US

Real Exchange Rate = e/PPP

GDP per Capita (US = 1 is U$16779)
Figure 3-2: Rel Exchange Rate for 100 Countries in 1990

Cross Section: Real Exchange Rate vs GDP per Capita in PPP terms

Real Exchange Rate = e/PPP

GDP/pop in PPP over US GDP/pop (US = 1 is US$21,571)
section regression implies dealing with less reliable data due to nonbenchmarking countries. In order to test the robustness of the above results to these errors, we run a regression including solely the 57 countries surveyed in 1985. The results did not change substantially (see figure 3-3).

The regressions of the real exchange rate against GDP per capita in US dollars leads to similar results and conclusions as the one using GDP in PPP terms. We can recover the estimate of GDP in U$ from the GDP in PPP dividing by the real exchange rate:

\[ GDP_{U\$} = \frac{CGDP}{e/PPP} \]

Figure 3-4 graphs this relationship between e/PPP and GDP in U$. It is very similar to figure 3-1, but seems more concentrated. The fit is even better (Rbar squared of .68) as can be seen from the regression results in table 3.1. This suggest that if the main interest relies only on the predictability of the above negative relationship, using GDP in US dollars is more appropriate.

An interesting exercise is to look at subgroups of the 138 countries shown in figure 3-4. This will show how robust is the relationship between real exchange rate and GDP per capita. In other words, we want to see if the general negative relationship obtained in figures 3-1 and 3-4 for large differences in income is also obtained when we concentrate in countries with similar levels of income.

Figures 3-5-3-8 and table 3.1 show the results when we subdivided the sample in 4 quartiles. It is apparent that the negative correlation does not follow when we are looking at close levels of incomes. It seems that the phenomenon exists only between big differences in income. The highest quartile, however, still delivers a negative correlation.

### 3.3.3 Panel Data results

We estimated the model for 115 countries between 1960-88. We estimated both the "fixed effect" and the "random effect" estimators. The negative coefficients are significant in both equations with t statistics smaller than -7 and -15.
Figure 3-3: Real Exchange Rate for the Original 57 Countries in 1985 ICP Survey

Cross Section: Real Exchange Rate vs GDP per Capita

Real Exchange Rate = PPP/e

GDP per capita in PPP / US GDP per capita in PPP

Observed
Predicted
Figure 3-4: Cross Section: Real Exchange Rate vs GDP in U$S

Real Exchange Rate for all 138 Countries in 1985
Figure 3-5: Real Exchange Rate for First Quartile in 1985

Real Exchange Rate vs GDP per Capita

Real Exchange Rate = \text{PPP}/e

GDP per capita in PPP / US GDP per capita in PPP

O Observed
--- Predicted
Figure 3-6: Real Exchange Rate for Second Quartile in 1985

Cross Section: Real Exchange Rate vs GDP per Capita

![Graph showing the relationship between Real Exchange Rate and GDP per Capita in PPP, with observed and predicted data points.](image)
Figure 3-7: Real Exchange Rate for Third Quartile in 1985

Cross Section: Real Exchange Rate vs GDP per Capita

Real Exchange Rate = \( \text{PPP} / e \)

GDP per capita in PPP / US GDP per capita in PPP

Legend:
- ○ Observed
- - Predicted
Figure 3-8: Real Exchange Rate for Fourth Quartile in 1985

Cross Section: Real Exchange Rate vs GDP per Capita

Real Exchange Rate = PPP/e

GDP per capita in PPP / US GDP per capita in PPP

0 Observed
----- Predicted
The results are presented in table 3.1.

**Cross Section Dynamics**

Up to this point, we established some strong relationships between GDP per capita and the real exchange rate. These were level relationships stating that the higher the country's GDP per capita the lower its real exchange rate.

The next logical step is to inquire about growth rates, focusing in the real appreciation process that "catching up" countries experience.

As a starting point, we looked at the 30 year rates of growth of GDP per capita and real exchange rate of all the 115 countries in our sample, in the period 1960-1989. As in the previous sections GDP measures are in PPP terms from the Heston & Summers data set and real exchange rate are defined as nominal exchange rates (NC/U$) over PPP estimates. Therefore, positive growth rates of the real exchange are depreciation rates. We calculated the means, standard deviation and correlation of the growth rates. The results are presented below.

<table>
<thead>
<tr>
<th>Growth and Depreciation Rates 60-88, 115 Countries Included</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Means</td>
</tr>
<tr>
<td>Medians</td>
</tr>
<tr>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Correlation</td>
</tr>
</tbody>
</table>

Since we have included all the countries and not only the "catching up" countries, i.e., not only those that have successfully reduced their gap in GDP per capita when compared to the United States (or any other benchmark country), we don't necessarily expect average appreciation rates, i.e., negative depreciation rates in the table. As a matter of fact, the results confirm an average positive depreciation rate for the countries as a whole. Although we get a negative correlation between the two series, the positive average depreciation and the large standard deviation of both the depreciation rate and the GDP growth cast doubts on continuing our analysis based on the whole sample of countries.
The natural following step, and the most interesting from our point of view, is to look at countries that have reduced their GDP gap with respect to the western industrialized countries, growing at higher rates during the period 1960-1989. In particular, given our previous result that the negative relationship between real exchange rate and GDP per capita is more likely to occur when dealing with large differences in income, we should concentrate our attention on countries which were able to increase substantially their GDP per capita through very high growth rates during this period. Therefore, we sorted the 115 countries in our sample by growth rates and calculated the statistics for the highest "n" countries, where n ranged between 11 to 29.

All the countries included in the sequential subgroups have growth rates that range from 530% to 150%, i.e., countries that increased their GDP per capita by more than one and a half times in 30 years. In comparison the US GDP per capita increased by 83% in the same period.

The results for the highest 11 countries are presented in table 3.2. As expected, higher average growth rates are associated with higher average appreciation rates (negative values in the table). Also, the correlation of both series is fairly negative for all the sub groups. However, the standard deviation of the exchange rate depreciation process is extremely high. For the highest 11 countries the average depreciation rate is approximately 9% while the standard deviation is 34%. This means that we have a great dispersion in the way countries exchange rate behave in the "catching up" process; while in average we get a depreciation rate we still have several countries which appreciate while increasing their GDP per capita. The medians were included to give a better picture of the process since they are less influenced by outliers. The results do not seem to depend on outliers.

In the above results we have relied in 30 year rates, that is the behavior of the real exchange rate during the whole period of our analysis. It is of interest to look at the behavior of shorter horizons, like 10 year rates, in order to capture the dynamics of the exchange rate during the fastest periods of growing economies. This will not only concentrate our attention to even higher growth rates but also give us a picture of how the countries dynamics differ in short and strong pushes.
Table 3.2: Growth and Depreciation Rates, highest 11 countries included

<table>
<thead>
<tr>
<th>30-year growth rates, 60-89</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>Means</td>
</tr>
<tr>
<td>Medians</td>
</tr>
<tr>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Correlation</td>
</tr>
</tbody>
</table>

Table 3.3: Growth and Depreciation Rates, highest 8 countries included, 60-89

<table>
<thead>
<tr>
<th>10-year growth rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>Means</td>
</tr>
<tr>
<td>Medians</td>
</tr>
<tr>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Correlation</td>
</tr>
</tbody>
</table>

In the analysis below, we started calculating all the 10 year growth rates of the 115 countries between 1960 and 1989. This gives us, for each country, 19 growth rates, from which we chose the higher one. Having one growth rate for each country, we sorted the countries by growth rates and again calculated the means, standard deviation and correlation of the series.

The results are presented in table 3.3. There are very high average growth and appreciation rates but also very big standard deviations. The average appreciation rates are comparatively much higher than the 30 year rates. Also, the correlation is more negative than the ones seen before.\(^3\)

---

[^3]: Due to an outlier, the average appreciation rate when including more than 10 countries becomes a depreciation rate. The medians, which are not so influenced by extreme observations continue to be negative.
Table 3.4: Average Growth in PPP Terms and Quality of Data

<table>
<thead>
<tr>
<th></th>
<th>ICP’s</th>
<th>Average Growth 73-80</th>
<th>Average Growth 80-88</th>
</tr>
</thead>
<tbody>
<tr>
<td>KOREA</td>
<td>70, 75, 80,85</td>
<td>5.2</td>
<td>6.9</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>80, 85</td>
<td>5.9</td>
<td>6.0</td>
</tr>
<tr>
<td>Singapore</td>
<td>None</td>
<td>6.1</td>
<td>4.9</td>
</tr>
<tr>
<td>Taiwan</td>
<td>None</td>
<td>6.3</td>
<td>5.3</td>
</tr>
</tbody>
</table>

### 3.3.4 Country studies: Results and Dynamics

#### The Selection of Countries

The selection of countries was made based on a twofold criterion. We looked for countries that had both, past high growth rates and reliable data. As the table below turns clear, in several cases we were not successful in obtaining this two qualities for the same country. Some of the countries did not participate in any ICP survey.

As commented in section 3.3.1, for the country studies below, we used both the RGDP and RGDPCH series in the Heston and Summers data.

#### Korea

The period covered in our regressions is from 1960-1989. During this period South Korea had very high average growth rate per capita (growth measured in GDP in PPP terms): 6.7, 6.9 and 5.2 for the periods 1960-1973, 1973-1980 and 1980-1988 respectively. The quality of the data is pretty good since Korea participated in all ICP’s surveys, therefore receiving a B grade from Summers & Heston.

The results for Korea confirm the cross section findings. There is a significant negative nonlinear relationship between Korean GDP per capita and its real exchange rate. This is an important result since it shows that the above negative relationship is not only a result of a cross section analysis but effectively extends to the behavior of individual countries along their growth process. Figure 3-9 graph the predicted and observed points for Korea’s real exchange rate.

It is interesting to note that Korean figures suggest an "overshooting" of the real exchange rate. During the 60's and, specially, along the 70's, Korean high rates
Figure 3-9: Real Exchange Rate and Per Capita Income for Korea 1960-88

Real Exchange Rate = e/PPP

Korean GDP per capita in PPP / US GDP per capita in PPP

Legend:
- Observed
- Predicted
of growth brought along side a severe appreciation of the real exchange rate. This process led the real exchange rate to reach a low 1.4 value which was not sustainable in the short run since for the rest of the period, basically during the eighties, the economy experienced a period of both growth and depreciation of the real exchange rate. In 1988 the real exchange rate was back to its low 1.4, appreciating even further in 1989.

In order to gain insight on how the Korean appreciation process evolved we graphed both the nominal exchange rate and the PPP estimate in the same figure 3-10.

During most of the period, from 1968-1986, the nominal exchange rate depreciated vigorously, specially in the 80’s. Until 1980 the real exchange rate appreciated continuously although the nominal exchange rate kept depreciating. This was possible through sufficiently higher inflation at home than abroad, as seen by the falling $\frac{\mu_x}{p}$ curve. Higher depreciation rates in the first half of the 80’s, however, were able to induce real depreciations generating a brief interruption in the long run appreciation process. It seems that the depreciation of the real exchange rate is induced by the nominal movements of the exchange rate, instead of being induced by higher inflation abroad than at home.

**Taiwan, Singapore and Hong Kong**

Figures 3-11-3-13 show the results of the regressions for Taiwan, Singapore and Hong Kong.

Although the data for these countries is less reliable (see table above), the results for the three countries are pretty consistent with each other. There is a significant long run real exchange appreciation while experiencing high growth rates in all countries. Nevertheless, all of them experienced periods with both GDP growth and real exchange depreciation. This is equivalent to our previous finding for the behavior of the Korean real exchange rate during the first half of the 80’s.

This last point alerts to the pitfalls involved in taking too seriously the strong negative relation between real exchange rate and GDP per capita and expecting, in
Figure 3-10: Decomposition of Korean Appreciation, 1965-1990
the short or medium run, a systematic real appreciation during the growth process.

China

The methodology used by Heston and Summers to derive the GDP estimates for China differs from other countries. It uses 1975 estimates of I.B. Kravis in his "An Approximation of the Relative Real per Capita GDP of the People's Republic of China" an extrapolate the results to the base year of 1985. With this base year estimate the rest of the series is obtained through the same procedure in described in section II.

China's figure 3-14 looks at the behavior of China's real exchange rate in the last 30 years and reveals a completely different pattern than in the other Asian economies studied. Even though China's GDP per capita grew steadily during this period, its real exchange rate depreciated in real terms (this process was driven mainly by nominal depreciations). From the perspective of our previous results this is a puzzle.

There are two hypothesis to explain this striking result. First is the poor reliability of the data. The estimates for China may be very inefficient leading to wrong results. The cross section results using Heston and Summer data seem to confirm this hypothesis since China is a complete outlier in the cross section regression (see figure 3-1). Summers and Heston's estimate of China's national price level gives an exchange rate of 8.38, the third highest number in the sample of 100 countries. The fitted curve exchange rate for a GDP per capita of about 5% of US GDP per capita is 2.9. This estimate is based on extrapolation of previous studies and may not be completely accurate. A more accurate estimate of China's price level comes from Ren Ruoen and Chen Kai (1993) who gives an estimate for the real exchange rate and GDP per capita in 1986 in 3.29 and US$ 1044 respectively. We updated to 1990 using the growth in our RGDP (constant price) and price level series (which uses the GDP deflators). The 1990 estimates ended up in 3.74 and the GDP per capita at about 7% of US GDP per capita, respectively, which positions China closer to the fitted curve but still above the predicted equilibrium value of 2.18. However, even using Ren Ruoen's estimate, which puts China closer to the cross section fit curve,
Figure 3-11: Real Exchange Rate and Per Capita Income for Singapore, 1960-85

Real Exchange Rate = e/PPP

Observed
Predicted

Singapore GDP per capita in PPP / US GDP per capita in PPP
Figure 3-12: Real Exchange Rate and Per Capita Income for Taiwan, 1950-88
Figure 3-13: Real Exchange Rate and Per Capita Income for Hong Kong, 1960-88
Figure 3-14: Real Exchange Rate and Per Capita Income for China

Chinese GDP per capita in PPP / US GDP per Capita in PPP

Real Exchange Rate = e/PPP
the dynamics are still the same. Our estimates for the real exchange rate in other years are an extrapolation of the 1986 estimate using the GDP deflators and nominal exchange rates in the same way as Heston and Summers. Therefore, our dynamics replicate their results: high GDP growth rates with real exchange rate depreciations.

The other hypothesis is that there is a strong deliberate intervention by the Chinese policy makers to depreciate the exchange rate in real terms in order to continue obtaining foreign aid from the international institutions. Figure 3-15 sheds light on this hypothesis. It graphs both the Chinese nominal and real exchange depreciation. The two curves seem to follow each other closely suggesting that nominal exchange rate depreciations are driving the real process even in periods of higher inflation at home than abroad. This corroborates to the results from the Korean case, where the only real depreciating period seem to be driven by nominal exchange rate depreciation.

It is worth to note that the depreciation rates, nominal and real, increase substantially since the 1980’s.

In the late 80’s, using the official exchange rate to calculate the real exchange rate leads to two consecutive years, 87-89, of appreciation, in the opposite direction of the long run process. This conclusion, however, is reversed if we take into account that since 1987 a new swap exchange market exists and use these values in the Heston and Sumner series. This results in a continuing real depreciation for China since the 1960, losing the small appreciation period.
Figure 3-15: Decomposition of Chinese Depreciation, 1968-1990
Table 3.6: Growth and Appreciation Rates for Korea 65-89

<table>
<thead>
<tr>
<th>Annual Rates</th>
<th>GDP</th>
<th>Exchange Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Means</td>
<td>13.98</td>
<td>-2.61</td>
</tr>
<tr>
<td>Medians</td>
<td>14.39</td>
<td>-4.56</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.77</td>
<td>6.93</td>
</tr>
<tr>
<td>Correlation</td>
<td></td>
<td>0.37</td>
</tr>
</tbody>
</table>

Table 3.7: Growth and Appreciation Rates for Taiwan 70-90

<table>
<thead>
<tr>
<th>Annual Rates</th>
<th>GDP</th>
<th>Exchange Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Means</td>
<td>12.82</td>
<td>-2.59</td>
</tr>
<tr>
<td>Medians</td>
<td>13.44</td>
<td>-.53</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.12</td>
<td>7.57</td>
</tr>
<tr>
<td>Correlation</td>
<td></td>
<td>0.60</td>
</tr>
</tbody>
</table>

3.3.5 Convergence Analysis

Up to this point, we established some strong relationships between GDP per capita and the real exchange rate. These were level relationships stating that the higher the country's GDP per capita the lower its real exchange rate.

The next logical step is to inquire about growth rates, focusing in the real appreciation process that "catching up" countries experience.

As a starting point, we looked at the appreciation processes of the same countries studied in the previous report: Singapore, Taiwan, Hong Kong and Korea. These are countries that experienced high growth rates in the period 1960-1990. At the same time, as we have already seen, their exchange rate appreciated in real terms. The tables 3.6-3.9 below look more carefully at this joint process:

The four countries studied follow similar patterns. In the first place, they all experienced very high annual average rates of GDP per capita growth, reason why they were chosen to be included in our case studies in the first place. Their GDP growth processes seem fairly stable, at least based in their small standard deviations.
Table 3.8: Growth and Appreciation Rates for Singapore 70-90

<table>
<thead>
<tr>
<th>Annual Rates</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GDP</td>
</tr>
<tr>
<td>Means</td>
<td>12.56</td>
</tr>
<tr>
<td>Medians</td>
<td>13.87</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>5.28</td>
</tr>
<tr>
<td>Correlation</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.9: Growth and Appreciation Rates for Hong Kong 70-90

<table>
<thead>
<tr>
<th>Annual Rates</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GDP</td>
</tr>
<tr>
<td>Means</td>
<td>12.50</td>
</tr>
<tr>
<td>Medians</td>
<td>12.07</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>5.21</td>
</tr>
<tr>
<td>Correlation</td>
<td></td>
</tr>
</tbody>
</table>

More interesting is their appreciation processes. In all the cases we have average annual appreciation rates of at least 0.75% and reaching 2.61% in the case of Korea. The correlation of real exchange depreciations and GDP growth rates is always negative and significant. This gives a built in tendency to expect a appreciation tendency during "catching up", which have an effect of reducing the equilibrium domestic interest rates in the long run. Lower interest rates are related with a boom in the asset market and in the land prices.

The appreciation rates, however, have large standard deviations. This confirms our previous results that the long run trend in the real exchange rate is not smooth and small real depreciation periods are often the case. This will probably have effect on the asset pricing and interest rates, at least in the short run.

3.4 Summary of results obtained

- Cross Section
- Using 100 countries in 1990, we obtain significant negative correlations between real exchange rate (e/PPP) and GDP estimated in PPP terms. Regressions fit well and we get some "Convergence" results.

- Equivalently, there is a significant negative correlation between real exchange rate using U$ GDP estimates. Fitted curve predict better than the one in (i).

- Subdividing the sample in smaller groups ranked by GDP per capita, we lose the strong negative correlations within each group. Therefore, our previous result seems to depend on large differences in income.

• Country Studies

  - Figures for Korea, Singapore, Taiwan and Hong Kong show clear tendency to appreciate in real terms while experiencing high sustained growth.

  - Looking the figures more carefully reveals that during the long run appreciation process there is always a small brief real depreciation period after several years of appreciation. The long run process, however, continues to evolve.

  - Decomposition of Real Appreciation: Appreciation occurs even in periods of great nominal exchange depreciation. Inflation at home greater than abroad maintains the appreciation process. In the brief periods of real depreciation, the nominal exchange rate seems to drive the process.

  - China is a complete outlier in the Cross Section study using Heston & Summers data set. Its real exchange rate of 8.4 is way above fitted curve. Using Ren Ruosen estimate of 1986 per capita GDP of u$1044 and PPP of 0.8709, China's position in the cross section is closer to the fitted curve. Using either estimate, the dynamic behavior of China's real exchange rate is completely different from the other growing countries. It experienced sustained real depreciation from 1960 to 1990.

• Panel Data Results
Results either using fixed effects or random effects confirm the negative coefficients on the regression of c/PPP on GDP per Capita.

3.5 Appreciation and Stabilization Plans

This section stresses the relevance of our previous finding that the equilibrium Real Exchange Rate is negatively correlated with GDP per capita,⁴ to developing countries. In particular to stabilizing countries.

Different growth rates in productivity between tradable and non tradable induces appreciations which imply that developing countries in their growth process should expect a more appreciated equilibrium real exchange rate as opposed to going back to the previous level, as implied by Purchasing Power Parity theories.

On the other hand, exchange rate based stabilization plans end up appreciating their currencies and seem to maintain their currencies overvalued for some time. For instance, the stabilizations in Latin America: Mexico (1987), Argentina (1991), Chile (1982) and Brazil (1994). Also, stabilizations in Eastern Europe resorted to fixing the nominal exchange rate in order to control inflation. This plans leave the policy makers in the following dilemma. First, simply devaluing will sometimes imply loosing the nominal anchor and the inflation control. On the other hand, doing nothing seems to be inconsistent in the long run, since the real exchange rate is overvalued (with the corresponding current account imbalances) and will end up either in a balance of payments crisis or in a deep recession as a consequence of the lack of competitiveness.

Therefore, developing countries end up in a curious situation. In the short run they are trapped with an appreciated exchange rate that can undermine their efforts to stabilize and ultimately grow, while in the long run this same exchange rate will not be considered overvalued, as productivity increases and the equilibrium real exchange rate will approach the actual exchange rate.

This section stresses the transition period where future gains in productivity and the corresponding real exchange rate appreciation cannot be capitalized in immediate

⁴The latter is a proxy for productivity in the non tradable sector.
gains in the stabilization process. Therefore, only if the developing country manages to overcome the transition period it could benefit from a more appreciated exchange rate.

This section will argue that the existence of a long run tendency to appreciate will mitigate this dilemma reducing the equilibrium real exchange rate while countries still maintain their nominal exchange rate fixed (or in a crawling peg) and overvalued. In contrast, if Purchasing Power Parity was the rule then there was no way out of this dilemma: the amount of the current currency appreciation will have to be exactly compensated with future depreciations. The extent in which this effect is strong enough to offset the need to a nominal devaluation depends on the specific conditions in the stabilized country, in particular, the extent that their productivity gap is reduced. The more you are able to gain in productivity during the transition period, the higher is your prospects of succeeding. However, since usually in the transition period there is a trade off between growth and inflation, the optimal amount of inflation that the government allows will determine the extent of the productivity growth and, therefore, the equilibrium real exchange rate appreciation.

The model below will argue that the sustainability of the reform will depend on both the extent of the initial overvaluation and the growth prospects of the country. The higher the growth prospects, the more patient will be the external investors and more willingly they will be to finance a longer transition period. Similarly, an extreme case of initial overvaluation will both lead to more current account deficits and, afterwards, to recession which will undermine growth prospects. Low growth prospects and very appreciated exchange rate will lead to the failure of the plan.

If there is a trade off between inflation and growth then it is argued that a possible optimal solution will be to allow some degree of inflation to generate growth and a long run appreciation process that will ease current pressures in the balance of payments.
3.5.1 Exchange Rate Based Stabilization Plans and their Sustainability

In the previous section we saw that an intertemporal model with differential growth in tradables/non tradables would deliver an optimized path for the real exchange rate and for the current account. We will assume that these are our equilibrium values in the sense that not only the current account path is consistent with the intertemporal budget constraint but also the real exchange rate is determined uniquely by supply factors.

If the productivity of tradables grows at a faster rate than the productivity of non tradables we have that developing countries will have a tendency to appreciate as they grow. What is the importance of this effect in stabilization policies that use the exchange rate as a nominal anchor?

The stabilization dilemma can be formalized in a model where inflation exhibits inertia and accelerates whenever the rate of wage inflation or the rate of depreciation exceed the inflation rate. Wages are indexed to the inflation rate. In this set up a reduction of the rate of depreciation (for simplicity set to zero) implies that inflation will not disappear immediately and the consequence is an appreciation of the real exchange rate.

With an overvalued exchange rate the government faces the dilemma of maintaining the nominal exchange rate fixed and facing the risks of both capital flight and recession (brought by less external demand) or depreciating and loosing the major goal of stabilizing inflation at low levels. Therefore, in some cases the overvaluation is kept for years

How sustainable is this situation? What is the role of long run appreciating expectations in this situation?

To illustrate this dilemma we will sketch a simple model which will capture the basic dilemma.

There are two ways to approach the sustainability issue. The main difference is in the way the overvaluation ends. One is to consider that appreciated real exchange
rate will end up driving the country to a deep recession, given the lack of competitiveness, and after a while will generate deflation and, therefore, a real depreciation. The other approach is to admit that capital flows are rather autonomous and not necessarily work out to adjust the internal disequilibriums of the stabilizing country. As a consequence of deteriorating expectations with respect to the sustainability of the plan, at a certain point in time capital stops entering the country, and even flights out, and the stabilizing country have no other option than to devalue its currency.

This is the approach adopted in the formalization below.

Features of the model:

a) Capital Inflows sustain overvaluation. Actual Exchange Rate, however, is overvalued. Capital still enters although at expense of a probability of default.

b) The finance of the current account deficits is done by foreign individual investors who do not internalize their influence on the aggregate finance situation of the country. They take individual decisions given their individual return on their project and a global default risk of the country.

b) The overvaluation period may end with capital outflows which forces depreciation and destroys the stabilization plan. A successful plan is one with no capital outflows and no need to a drastic depreciation.

c) The expected time of the retreat of capitals is given by the prospects of future capital account balances, which are determined by expectations of future overvaluation and productivity growth.

d) The expected time of the retreat of capital will be a proxy to the chances of success of the stabilization plan.

e) If investors take into account what the other investors are doing then there is a coordination issue in financing a stabilizing country. Your decision to invest in the country depends in other investors decisions. Multiple equilibria. If everybody decides to repatriate capital (or simply not to invest) then it is optimal to an individual investor to do it. Alternatively, if everybody believes in the country then it may be optimal to the individual investor to refinance this country.

f) If an overvalued exchange rate affects your growth prospects then there is a
policy issue to consider. More inflation fighting may undermine your chances of success by reducing future growth prospects.

Assuming the equilibrium real exchange rates are determined by the model in the previous section and are not influenced by the current path of real exchange rates. In other words, we are assuming that the differential productivity growth is not influenced by the actual real exchange rate. Therefore in every point in time we have defined the equilibrium real exchange rate in the absence of government intervention:

The government uses the nominal exchange rate as a stabilizing instrument which leads to a real exchange rate appreciation. This is a consequence of the fact that inflation does not fall immediately to zero at the moment of stabilizing. Indexation leads to a residual inflation after the shock which ends up appreciating the currency in real terms. For our purposes it is enough to assume that after the stabilization:

$$e^*_t \geq e_t.$$  \hspace{1cm} (3.16)

The current account balance will know be associated with the new real exchange rate and will deviate from the optimal path. Therefore we can define the "excess" CA deficit as a function of the real exchange rate discrepancy:

$$\Delta CA_t = F (e^*_t - e_t)$$  \hspace{1cm} (3.17)

Suppose, hypothetically, that productivity growth path is independent of the actual real exchange rate given by the government efforts to reduce inflation. Then we should expect that the equilibrium real exchange rate would approximate the actual rate.

However, the two paths for the actual and equilibrium exchange rates are not independent. They are correlated, which makes the issue more difficult to policy makers and more interesting. Otherwise, we could simply recommend a do nothing option to the policy dilemma.

If the overvaluation leads to a path of current account deficits that seems to be inconsistent with the intertemporal budget constraint, the question to ask is how long
the appreciation rate will be sustainable?

Suppose that there are \( n \) investors that individually are small enough to influence the economy's path of current account but in the aggregate determine the extent that this country can use the international capital market to smooth consumption and, in this case, to sustain a stabilization plan.

Suppose each one of them make the decision looking at the future paths of finance needed given the actual exchange rate policy. The probability of receiving the amount invested is assumed given by the sum of future current account balances over the current debt stock:

\[
\Pi = \max \left\{ \min\{1, \frac{\sum_{t_0}^{\infty} (1 + r^*)^{-(i-t_0)} E(CA_i/I_{t_0})}{B_{t_0}} \}, 0 \right\}.
\]  

This says that the probability of default is the ratio described above and it is bounded between one and zero. Current accounts in excess of the debt stock don't make the probability greater than one and negative values of this sum leads to expect a total loss, which is equal zero.

In particular, it is important to note that:

\[
\Pi_0 = \mathcal{G}(E[\{a_{Nt}, a_{Tt}\}_{0}^{\infty}, \{e_t\}_{0}^{\infty}])
\]

The probability of default at the beginning of the plan will depend on the growth prospects and the expected path for the exchange rate set by the government. Nothing here is said about the independence of both factors. They may be correlated and have strong interactions.

We can think of \( \Pi \) also as a proportion of the amount invested that will be paid by the borrower country to this individual investor. It is rational to lend to the stabilizing country even when \( \Pi \) is less than 1, provided that their expected utility from the investment is positive. Assume for simplicity that agents are risk neutral or that the revenues in each state are in "utils". Then the investment decision will be
given by the rate of gross excess returns $r$ in investing in this country and in $\Pi$:

$$E[U] = \Pi f(1 + r) \geq 0$$

(3.20)

The cut off point is then given by:

$$\Pi = \frac{1}{1 + r}$$

(3.21)

Each period the investors readjust the probability of a country default given by the shocks in the determinants of the current account. In our set up this is given by the difference between the exchange rate set by the government and the equilibrium one, given by our previous model. Therefore, the important shocks to the current account are the actual real exchange rate shocks and to productivity, which we have seen determine the equilibrium real exchange rate. We could assume some time processes for both and derive the time process for CA (instead, we could simply use a random walk).

$$\{e_t, a_{N_t}, a_{T_t}\}_t^\infty \rightarrow \{CA\}_t^\infty \rightarrow \{\Pi\}_t^\infty$$

(3.22)

Suppose the investors are identical both in their relative risk and their information sets. Then their cut-off points are the same and the aggregate withdrawal of foreign capital is abrupt. The dependent economy will not be able to borrow abroad to finance its stabilization plan after this cut off point. The economy will be forced to depreciate to balance its current account. If indexation and stickiness are still present in the economy, this will lead to more inflation and, in some cases, destroy the stabilization plan.

For a given process of the current account we can calculate the expected time to hit the cut off point. The expected time of the capital outflows is an important determinant of the chances of success of the plan. It gives the likelihood of a determinant path to never hit the cut off point. The expected time will depend on the moments of the process, which are given by the moments of the processes for the real exchange rate and the productivity differential. More interesting, it will also depend on the
degree of appreciation at the beginning of the stabilization plan.

\[ E[T^*] = \mathcal{H}(e - e^*, \Pi_0), \tag{3.23} \]

where \( H_1 \leq 0 \) and \( H_2 \leq 0 \).

Therefore, what this shows is that it is important to not begin the stabilization plan with a too appreciated exchange rate since the success of the plan, which depends on the time available for reforms, will crucially depend on this initial condition. A too appreciated exchange rate may also reduce the growth prospects at the beginning of the plan may be interesting even to overshoot your initial exchange rate (although at expense of other important costs).

The expected time is relevant if the plan is less than fully credible. Then the amount of time available to produce a positive shock is important. The success of the plan may depend on the availability of time to boost productivity as a consequence of stabilization payoffs (less inflation, less uncertainty and more growth) which are sometimes combined with more openness to foreign trade. Thus, reforms may take time and since they are not perfectly credible at the beginning of the plan, the expected time is relevant to the success of the plan. If productivity manages to increase substantially after the plan, even if not immediately after the stabilization, then the chances to maintain the exchange rate at its level without resorting to a real depreciation increases substantially.

### 3.6 Conclusion

Both theory and evidence suggest that the real exchange rates and GDP per capita move in opposite directions in the long run. Theory implies that this relationship is a consequence of the relatively higher price of non tradable goods in richer countries. Large productivity differences in the tradable sector between developed and developing countries deliver a high relative wage which is transmitted, through the labor market, into high relative wages in the non tradable sector. Therefore, as the
developing countries grow and reduce their productivity differential in the tradable sector, without a similar improvement in their non tradable sector, their exchange rate should tend to appreciate.

Empirically, a cross section analysis of 100 countries in 1990 show that in average for every 1% increase in GDP per capita the real exchange rate appreciates by 0.36%. This relationship between real exchange rates and GDP per capita is non-linear and is more easily observed when looking at large differences in income per capita. Small movements in GDP per capita will not generate observable differences in the real exchange rate since short run deviations from the long run equilibrium will dominate. For example, concentrating only in countries in the 2nd quartile of the distribution of GDP per capita does not generate any significant pattern for the real exchange rates. Therefore, when looking at specific countries through time we chose to concentrate on 4 out of the 5 fastest growing countries: Taiwan, Korea, Singapore and Hong Kong. Their GDP per capita growth rates, between 1973-1988 were 5.8%, 6%, 5.5% and 6%, respectively.

The figures for these countries reveal that in every case the real exchange rate appreciated during "catching up" periods: Taiwan(-32%), Korea(-42%), Singapore(-30%) and Hong Kong(-21%). These results corroborate the cross section conclusions above, giving a time dimension to the relationship between real exchange rate and GDP per capita.

However, this process of appreciation is not monotonic. The long run appreciation process is usually interrupted by short depreciation periods but the general tendency to appreciate ends up prevailing. A real appreciation of a currency may occur through either a nominal appreciation of the exchange rate or by higher inflation at home than abroad, or both. In each of the case studies above, appreciation occurs even in periods of nominal depreciation, through higher inflation at home. In contrast, in the brief periods of real depreciation, nominal depreciation seem to drive the process.
Bibliography


