Essays on Industrial Organization

by

Hongkai Zhang

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at the

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Signature redacted

Signature of Author .................................................................
Department of Economics
15 May 2017

Certified by .................................................................
Glenn Ellison
Gregory K. Palm 1970 Professor of Economics
Thesis Supervisor

Signature redacted

Certified by .................................................................
Muhame Yildiz
Professor of Economics
Thesis Supervisor

Signature redacted

Accepted by .................................................................
Ricardo Caballero
Ford International Professor of Economics
Chairman, Department Committee on Graduate Students

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Abstract
This thesis consists of three chapters on information transmission and utilization in market-
places and their implications on firm competition and quality discovery.

The first chapter analyzes the dynamic effect of sponsored search advertising on quality
discovery on the Taobao.com retail platform. In a stylized model, a new product’s boosted
exposure from sponsored search ads could help the platform to infer how well the product
converts exposure to sales (the quality measure) and award top organic search ranks accordingly.
Hence, sellers with higher private quality signals would bid more aggressively for sponsored ads,
which accelerates the platform’s discovery of these high-quality products. An empirical analysis
echoes the stylized model and reveal a synergy between the platform’s PC and mobile interfaces.

The second chapter (co-authored with Sara Fisher Ellison and Christopher M Snyder) stud-
ies price dynamics for computer components sold on a price-comparison website. We estimate
a dynamic model of competition, backing out structural estimates of managerial frictions. The
estimated frictions are substantial, concentrated in the act of monitoring market conditions
rather than entering a new price. Coupled with supporting reduced-form statistical evidence,
our analysis provides a window into the process of managerial price setting and the microfounda-
tion of pricing inertia, issues of growing interest in industrial organization and macroeconomics.

The third chapter analyzes the revelation of hard information in a buyer-seller relationship.
The seller can choose whether and when to credibly reveal quality information and some buyers,
called prosumers, have greater taste for quality than others. This paper first analyzes an
alternating bargaining game allowing endogenous delay between communications in the fashion
of Admati and Perry(1987), and constructs an equilibrium with delayed revelation of quality.
The paper then analyzes an informed principal problem and found that under certain conditions,
both seller types choose the same truth-telling mechanism that maximizes the revenue of the
seller type with high quality. In both games, a high quality seller hides his quality information
before the buyer acts in order to extract surplus from prosumers.

Thesis Supervisor: Glenn Ellison
Title: Gregory K. Palm 1970 Professor of Economics

Thesis Supervisor: Muhamet Yildiz
Title: Professor of Economics
To My Love,

Yang Wang
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Chapter 1

Accelerated Quality Discovery through Sponsored Search Advertising in Online Marketplaces

1.1 Introduction

E-commerce platforms facilitate transactions between consumers and independent suppliers. Given the drastically reduced cost of listing and transacting, they often carry an enormous number of products. Consumers, typically limited by cognitive capacities or the form factor of the interface, can only investigate a few products carefully. Platforms could help the consumers to narrow their searches to high-quality products that they are likely to purchase, possibly through an informative search function that returns a ranked list of products. Both the platform and consumers would appreciate and benefit from information channels that improve the platform’s knowledge about product quality.

This paper examines a mechanism of quality discovery on an e-commerce platform, Taobao.com. To fulfill its informational role, when a consumer expresses her interest by searching for a key-

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word, the platform ranks the relevant products on the search result page. A prominent feature of the search result page on Taobao is that a list of sponsored products is displayed alongside the main, or organic, results and sellers can bid on per-click fees to be ranked in the sponsored list. The goal of this paper is to examine the effect of this feature on the platform's effectiveness as an information intermediary.

I look at the women's apparel categories where scores of small sellers retail numerous varieties of horizontally differentiated, nonbranded, seasonal goods, and consumers rely heavily on the keyword search. Although consumers can learn some basic information (product title, picture, price and monthly sales) with an effortless glance at the search result page, they incur search costs to investigate product detail pages. The information on those pages determines how likely a consumer viewing the product page will be convinced to make a purchase. This probability, called the conversion rate, is the primary quality measure of interest in this paper, so the word "quality" will refer to the conversion rate most of the time. The platform can learn very limited quality-relevant information from the picture-rich product pages, so it primarily relies on its data on observed consumer browsing and purchasing behaviors: the conversion rate of a product can be recognized by the platform only after sufficient consumer exposure. However, given the large number of new products, consumer exposure is scarce.

I set up a stylized model in Section 1.3 to illustrate that the sponsored ads system can help the platform allocate consumer exposure and discover high-quality products more efficiently by incorporating the sellers' private information in an incentive-compatible way. I consider a market with $N \geq 2$ new products each period with conversion rates drawn from a known distribution, and assume each product lasts for two periods. The platform has two slots to display to consumers. In any period, the platform sells the sponsored slot on the right to an entrant through a second price auction with simultaneous bids. For the organic slot on the left, the platform infers from last period data to identify and display the incumbent with the highest expected conversion rate.

A key element of my model is that the sellers have private signals about the quality of their products. An entrant's bid for the sponsored slot depends on how many extra sales the sponsored slot can bring, conditional on the entrant's private signal. In the current period, the entrant could receive additional sales from the exposure of the sponsored slot, and it expects this
benefit to increase with the private signal. Furthermore, the entrant has a dynamic incentive, as its sales from the sponsored exposure are observed by the platform to infer its expected conversion rate: an entrant with a higher private signal would be more optimistic about the sponsored exposure this period leading to the award of the organic slot in the next period. Section 1.3.2 shows entrants use bidding functions increasing in their private quality signals, and an entrant with a high private quality signal would bid much higher than the current-period benefit of the sponsored slot.

This monotonic bidding equilibrium for the sponsored slot has positive effects on the average qualities of both the sponsored slot and the organic slot. For the sponsored slot, because the entrant with the highest private signal wins the auction, the average quality of the sponsored slot increases in the accuracy of the private signals. Furthermore, because the sponsored exposure is directed to a new product with a high private signal, the platform’s experiment starts with a very promising candidate and is more likely to confirm a high-quality product for the organic slot in the next period.

To demonstrate these effects, Section 1.3.3 considers a benchmark environment, in which the platform randomly chooses a fixed number of entrants to display in the right-side slot without seller input, and use their observed sales to determine the product for the left-side slot in the next period. Section 1.3.4 compares the outcomes of the two environments and varies the private signal accuracy, the number of entrants and the number of consumers. The sponsored ads environment performs better when private signals are accurate and the number of entrants is high, because the total information of the entrants depends on these two factors. The performance of the benchmark environment depends heavily on the number of consumers, which caps the platform’s information power, and would not benefit from an increasing number of entrants as much as the sponsored ads environment.

To understand this quality discovery story from an empirical perspective in the setting of Taobao.com, I collect daily panel data on sponsored ads display, organic search ranks and 30-day sales for hundreds of products and thousands of keywords on Taobao.com by scraping the relevant data from Zhibi365.com\(^1\), an independent data service for sellers. The platform

\(^1\)Many existing papers on Taobao.com (for example, Chu and Manchanda (2016), Ju et al. (2013) and Fan et al. (2016)) use proprietary data obtained directly from the Taobao/Alibaba company to study suitable topics. While Taobao.com is open to coordinate with academic researchers, it often provides data at a certain aggregated
provides different organic and sponsored ranks on its PC website and mobile app, so I have recorded all four types of ranks. In Section 1.4, I develop an empirical specification that relates a product’s all types of search ranks under various keywords to consumer exposure and measures its relative conversion rate by matching exposure to sales.

A unique feature of Taobao.com perturbs the organic rankings with asynchronous weekly cycles and helps to identify the empirical model with exogenous variation. For historical reason, the platform nominally relists each product weekly at a time of week chosen by its seller. As the relisting time approaches, the product is considered to expire soon and given some priority in all relevant organic rankings. The priority disappears when the product is automatically relisted, and a new seven-day cycle starts. This feature, retained by Taobao.com as a way to disperse exposure across products, creates exogenous and frequent shuffling in organic ranks, and has a more significant role on the website than in the mobile app.

The estimation reveals some interesting contrasts of consumer behaviors on the mobile app and the desktop website. While consumer exposure is quite dispersed across organic ranks on the website, in the mobile app, consumer concentrates at the top ranks and diminishes very fast for lower ranks. This is consistent with the two interfaces’ difference in the physical factors: mobile users are limited in their abilities to scroll over the list of products and to jump back and forth between the list and individual products, so they rely on the organic ranking much more heavily.

A more striking comparison is that the organic rankings on the website have contributed only a quarter as many sales as their counterparts on the mobile app. This is partly due to the popularity of the mobile app exceeding the desktop website recently. However, this contrast is also consistent with a quality story: the organic rankings on the website shuffle significantly according to each product’s weekly cycle, contrasting to the much more stable organic rankings on the mobile app. Therefore, the mobile app offers more informative and reliable organic rankings so that the consumers expect the top organic products on the mobile app to have high quality, and are more willing to investigate those products, which leads to more sales.

level or from a small random sample, which is not suitable for the purpose of this work. The commercial data service, Zhibi365.com, provides a unique opportunity to utilize almost any publicly observable information on Taobao.com without the limitations imposed by Taobao.com for proprietary data. As a for-profit company serving thousands of sellers paying for subscriptions, Zhibi365.com is cost-effective for researchers due to the economy of scale. This work is the first to use this data source, to my knowledge.
In contrast to the case of organic rankings, the sponsored rankings on the website are much more important than those on the mobile app, most likely due to the ample space and easier browsing for the consumers. In fact, the sponsored rankings on the website contribute more sales than the organic rankings next to them: in this extreme case, because the sponsored rankings are determined through an informative auction and are much more stable, consumers are more willing to click the sponsored products than the organic results.

These pieces of evidence in Section 1.5 suggest the reality of the women's apparel category on Taobao.com is quite close to the stylized model in several aspects. First, since the dominant organic rankings are ones on the mobile app with consumer exposure concentrated at top ranks, they resemble the single top organic slot in the stylized model, which implies that new products have little organic exposure and the dynamic incentive to purchase sponsored ads is strong. Second, the high exposure received by the sponsored products on the website suggests the sponsored list can indeed of high quality and the consumers would respond to that. In contrast, the significant shuffling of desktop rankings resemble the random experiments in the benchmark environment, which degrades the quality of the ranking and suppresses consumer clicks.

My empirical analysis suggests a strong complementarity between the desktop interface and the mobile interface, which is of interest for platform strategies. The deep browsing pattern and the ample advertising capacity on the desktop interface enable quality-relevant information to emerge quickly. The mobile interface, given its strong influence on consumer behaviors, has made great use of this information from the desktop interface to improve efficiency. Essentially, this represents a positive externality generated by the consumers with lower search costs on the consumers with higher search costs. As most platforms today have multiple interfaces that generate heterogeneity in search costs, this complementarity is worth exploring so that platforms can improve the integration and differentiation of consumer experiences on multiple interfaces.

The estimation also yields a relative measure of the products' qualities and allows testing of the dynamic patterns postulated by the stylized model in Section 1.5.3. Products in the data set demonstrate aggressive initial advertising and enjoy high exposure from organic search rankings later.
This paper adds to the growing literature on sponsored search. Edelman and Ostrovsky (2007), Edelman et al. (2007) and Varian (2007) are the pioneering papers that examine the auction mechanisms and the bidding strategies on leading search engines. Athey and Nekipelov (2012) analyzes the equilibrium of the auction game with uncertain click-through rates. Chen and He (2011) and Athey and Ellison (2011) relate the advertisers' heterogeneous valuation of consumer clicks to their probability of converting clicks to transactions, and both show that sellers with the higher quality would bid more because they can achieve more transactions from sponsored exposure. Furthermore, Athey and Ellison (2011) endogenizes the number of consumer clicks received by each slot in the sponsored list, and captures that a sponsored ranking informative of quality would encourage more consumer clicks. This paper builds on these elements, and augments the sellers' bidding incentive by the intertemporal link between sponsored lists and organic lists, so that the dynamic patterns of sponsored ad advertising can be understood.

While the aforementioned papers treat the sponsored list in separation, a few papers explore the interaction of the organic list and the sponsored list. White (2013) considers both complementarity and substitution between the goals of the two lists: on the one hand, the higher quality of the organic list can attract more users and therefore allow higher revenue from the sponsored list; on the other hand, the high-quality merchants in the organic lists may compete with merchants interested in the sponsored slots and reduce their willingness-to-pay for the sponsored ads. Katona and Sarvary (2010) takes on a different substitution effect: free exposure in the organic list undermines the marginal benefit of sponsored exposure for the same advertiser.

This paper adds to above strand of research with two dynamic complementarities: First, the sellers' private information stimulates the competition for the sponsored slots, and the outcome of the sponsored slots helps to improve the quality of the organic ranking. Second, the higher quality of the organic list would attract more consumer exposure for top organic lots, so that the high-quality entrants have stronger incentives to bid for the sponsored slot in order to earn the top organic lots later.

This work also belongs to the literature on platforms and two-sided markets, as it considers a model of facilitated interactions between the two sides that enriches the generic match functions
modeled in the pioneering work (for example, Rochet and Tirole (2003) and Armstrong (2006)). The platform’s ability to extract information from early interactions and use it to improve future matchings is a new type of network effect, and its implications for welfare and competition should be explored in future work.

The empirical exercise in this paper connects to the literature on consumer search behaviors and the effectiveness of sponsored ads. Ellison and Ellison (2009) studies the effect of price rankings on consumer search and the sellers’ obfuscation strategies in response. Blake, Nosko and Tadelis (2016) analyzes consumer search patterns using a dataset from ebay and find that consumers on average conduct 36 searches over three days before making a purchase, which echoes the low conversion rates in my data and the delayed effects of exposure in my estimates. Through a large-scale field experiment on major search engines, Blake, Nosko and Tadelis (2015) shows the sponsored ads are informational instead of persuasive, which I assume in my model. Their work also highlights the endogeneity that experienced consumers searching for brand keywords would have purchased the relevant products regardless of their advertising. This endogeneity issue is also studied by Edelman (2013) and Lewis and Reiley (2014). By focusing on categories with non-branded, horizontally differentiated products and small sellers, I minimize this endogeneity problem.

The paper is organized as follows: Section 1.2 describes the empirical setting. Section 1.3 proposes a stylized model. Section 1.4 describes the data. Section 1.5 specifies and estimates an empirical model that relates ranks to sales. Section 1.6 concludes.

1.2 Empirical Setting

Taobao.com is a Chinese retailing platform that allows third-party sellers to list their products and sell to consumers. It was launched in 2003 by the parent company Alibaba, soon after Ebay’s entry in China to imitate its services. Because Alibaba was determined to support Taobao for market dominance, there have been no listing or transaction fees. This, among other reasons, drove Ebay and other major competitors out in a few years. Because most transactional services are free, Taobao only started profiting many years later, when advertising revenues took off, and soon dwarfed the revenue of Alibaba’s namesake wholesale platform. Taobao and its spin-
off, Tmall\textsuperscript{2}, contributed $5.9$ billion, or $77\%$ of total revenue, to the Alibaba Group in just the last quarter of 2016\textsuperscript{3}. Advertising services represented $70\%$ of this revenue, and grew by $47\%$ year-over-year. The dominance of advertising revenue distinguishes Taobao from other online retailing websites in terms of business models.

While Taobao carries so many categories suitable for online retailing, I focus on a subcategory of women’s apparel to highlight the information challenges on this platform. Women’s apparel is the largest top category on Taobao, representing more than $10\%$ of all transactions, and contains millions of listings from thousands of sellers. The vast majority of these listings are not affiliated with established brands, and they are known for good value for money at very affordable prices in the range of 20-150 RMB (3-20 USD)\textsuperscript{4}. In this very noisy market, consumers need to acquire information extensively prior to purchase. Also, the sheer numbers of sellers and varieties, coupled with seasonality and ever-changing fashion trends, challenge the platform to produce up-to-date recommendations to guide consumer browsing. These features separate the apparel category from categories with branded products and infrequent entry. For example, in the cellphone category, consumers often already know specifications of different models from media coverage and brand marketing, and it is relatively easy for the platform to recommend reputable sellers for a specific model based on sellers’ previous performance. I expect the dynamics in the cellphone category would be very different from the dynamics explored here.

In this section, I first describe the search function of Taobao.com on both desktop and mobile interfaces to give the context of analysis, and then discuss the factors affecting organic and sponsored rankings. A special feature of the platform with individual organic ranks affected by asynchronous weekly cycles is covered here and will then have an important role in the empirical section.

\textsuperscript{2}Tmall hosts major brands, their top distributors and some largest sellers migrated from Taobao. Search results on Taobao would include products from Tmall and the two websites share similar user experience and interface. Tmall charges 1-3\% fees for all transactions.


\textsuperscript{4}According to market data presented by an industry analyst: https://goo.gl/MlyUj1
1.2.1 Search on desktop and mobile interfaces

Taobao.com contains numerous products in almost every possible category listed by independent sellers. Given the vastness of the selection, consumers most often try to locate the products of interest through keyword search. After entering a keyword (or a combination of keywords) and hitting the search button, a search result page appears. An example, for the keyword "sweater", is given below:

![A desktop search result screenshot](image)

![Figure 1.1 A desktop search result screenshot](image)

On the left is the有机 search results, where the relevant products are ranked by Taobao.com’s proprietary algorithm. On the right is the sponsored search results, where a seller has to pay a per-click fee whenever a consumer clicks on its product. Each product in the lists has its picture and title shown, as well as its price, location, the reputation score of the seller and 30-day sales of the product. There are about 40-44 products in the organic list, occupying 4 columns and 10-11 rows. There are 17 products in the sponsored list. Twelve of them occupy the rightmost column, while 5 of them are located at the last row of the page. On a normal computer screen, about the first two rows of the organic results and the top 4 slots in the sponsored column are shown before the user scrolls down. One additional complication is that the first 3 slots in the

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Consumers can also reach some products by clicking through the curated promotions on the homepage, but this channel covers very few products. Consumers can also directly go to sellers’ virtual storefronts, but most small stores do not have many such consumers who are willing to start their shopping from the store.
organic results area are sometimes allocated to sponsored products. When this is the case, the sponsored products will be designated with a small label. Both the small label and the header of the sponsored list says "sellers' recommendation," and most consumers understand they are influenced by the sellers’ payment to the platform.

Parallel to search results on the website, consumers can also search within the mobile app:

![A mobile search result screenshot](image)

**Figure 1.2** A mobile search result screenshot

The mobile result is a single list of products presented in two columns that the consumer can scroll down to load more and more products. For every ten slots in the stream, there is one slot used to display a sponsored product. The mobile organic rankings are correlated with, but different from the organic rankings on the desktop, as the platform adopted different algorithms to adjust for the different consumer behavior patterns. Also, most sellers are allowed to submit different bids for the mobile sponsored slots.

From any search result page, consumers can click the product picture or title to reach a product page with more details about the product. Given the product’s subcategory, the platform mandates a set of possible attributes which are presented in a plain and modest way. Sellers use most space of the product page to feature even finer information, including product-specific attributes, additional pictures of the product and clarification of service terms. To ensure uniform rendering on all computers and mobile phones, most sellers choose to organize these additional details into a series of pictures, with all texts embedded using picture editing software. On a desktop, the product detail pictures usually span several screens. The picture-rich format of
the product page makes it hard for the platform to parse the information automatically.

1.2.2 Organic rankings

The ranking of the organic results aims to bring the most relevant, reliable and popular products to the front places. For relevance, the platform would check the product title, attributes allowed for the subcategory, and the seller's primary category. For reliability, the platform takes into account of seller reputation/service scores and product review scores. For popularity, the platform uses the product's sales in the past 30 days and the observed conversion rate, which is the ratio of the number of sales to the number of product page views.

Generating up-to-date and high-quality organic rankings is a challenging task. For a popular keyword, there could be hundreds of thousands of listings that qualify for the organic ranking. In
categories where products are mostly horizontally differentiated and non-branded, a significant proportion of products would appear very close in relevance and reliability measures. Hence, the popularity measure is the most differentiating element in the determination of the top ranks.

Among the factors in the popularity measure, the conversion rate is weighed heavily by the platform in its algorithm for the organic rankings for several reasons. First, the conversion rate is a key measurement that directly affects the volume of transactions. Second, it has a strong role in consumers' shopping experience. Consumers are more willing to click products for details when they often find the products investigated are worth purchasing, and would feel more satisfied overall. Finally, the conversion rate is more costly for sellers to manipulate in unfair ways. The challenge for using conversion rates is time pressure: the platform can only have a reliable observation of a product's conversion rate after the product receives enough clicks, and many products are only relevant in the medium term and become obsolete in the long term. Notably, this challenge is more or less also faced by other platforms with horizontally differentiated time sensitive products, such as Youtube for videos.

**Asynchronous weekly cycle.** A unique and very useful feature of the platform is a shuffling of organic ranking according to the listing time. While a product can exist on Taobao.com for as long as the seller wishes, the listing nominally expires every seven days and is automatically reinstated immediately afterwards. The product's sale records and review records are not interrupted by this nominal process. The only effect of this pseudo expiration is that products are given a greater advantage in the organic rankings as they are close to the expiration time, and this advantage disappears entirely at the beginning of the next seven-day cycle. On the desktop website, this force can boost a product's organic ranks drastically on the last day or the last hour of the cycle. On the mobile app, this force is weaker and the organic rankings do not shuffle as much. Below is an example of a product's desktop and mobile ranks under a given keyword. The desktop rank often improves significantly on the dates noted by horizontal
ticks, while the mobile rank shows some mild shocks on those dates.

![Graph showing organic ranks of desktop and mobile](image)

**Figure 1.4** An example of a product's organic ranks under a given keyword

The sellers can choose the expiration time by changing the starting time of their listings, and it is popular among sellers to choose an expiration time during evenings and weekends. The pseudo expiration time is not shown to consumers.

### 1.2.3 Sponsored rankings

The ranking of the sponsored results is determined through a generalized second price auction. The sellers submit bids for the pay-per-click charge, and the platform ranks the products by their bids multiplied by quality scores in the sponsored list. A product's quality score (QS) depends on its match to the keyword, the seller's historical effectiveness in sponsored ads promotion, the seller's service scores and the product's click-through-rate (CTR) under the keyword, which is calculated from its recent displays in the sponsored list. The product at the nth slot is actually charged a pay-per-click (PPC) equals to the bid of the product at the (n + 1)th slot adjusted...
by the ratio of their click-through rates plus 1 cent.

$$PPC^{(n)} = b^{(n+1)} \times QS^{(n+1)}/QS^{(n)} + 0.01$$

The sellers can see a normalized version of their QSs, on a scale of 1-10, where 10 means a product is among those with the highest QSs under the keyword. For experienced sellers, the first three components of the quality score are not very differentiating, and it is mainly the CTR that affects their bids and auction outcome. Even for the same seller, the CTRs of its products could vary and the seller could experiment with different products to find the one gives the highest QS. Sellers can also experiment with different titles and pictures for same products to improve its QS.

The evolution of the sponsored ads service, which is one of the platform’s main revenue channels, has been an iterative process with revisions aiming at improving the consumer experience. Besides the click-through-rate (CTR), the platform has also included in the quality score (QS) other factors that do not affect the advertising revenue but reflect the relevance of the product to the keyword searched, in order to deter sellers from hurting the consumer experience with irrelevant sponsored ads. Sellers are also encouraged to pay attention to the conversion rates of their sponsored products, and they are educated that the conversion rates they demonstrate following clicks on sponsored ads are recorded and can affect their organic ranks.

Taobao.com started its business without charging proportional fees from sales, and it has remained so. The platform today generates revenue mainly through promotional services including sponsored advertising. However, like many other platforms, Taobao.com’s business strategies have been prioritizing the total volume of transactions on the platform and the satisfaction of the consumers and the sellers in general over the platform’s own current period revenues. A primary concern of the platform today is to compete with other online retail websites for consumers and sellers so that it can maintain its dominant place in the Chinese online retail market. Taobao.com has also been creating externalities that are captured by other branches of its parent company, Alibaba. During its early days, Taobao.com was mainly for small sellers to market products that they purchased from wholesalers on Alibaba.com, and
Alibaba.com can make money from those wholesale transactions. This is still the practice for many sellers today. Also, Taobao.com is contributing consumer flow to Tmall.com, a sibling platform on which large sellers including manufacturers sell branded products and pay the platform 1%-3% transaction fee. Both externalities depend on the popularity of Taobao.com.

1.3 A stylized model

In this section, I set up a stylized market in which the platform has two slots to show. In the main analysis, one slot is for sponsored display. I argue that sponsored advertising in equilibrium encourages high-quality entrants to bid aggressively for the sponsored slot, which allows the platform to leverage on the seller’s private information to accelerate quality discovery. I then compare this outcome to a benchmark case where both slots are organic so that the platform determines the products displayed without seller input, and discuss what features in the setup would make a strong case for using one slot for sponsored advertising.

1.3.1 Setup

I consider a discrete time model with an infinite number of periods. Consider a simplified market with a continuum mass of $M$ consumers searching for a specific keyword. In this market, at the beginning of each period, $N$ new products enter and each product lasts 2 periods, so there are always $2N$ available products in total. Each product is solely characterized by its conversion rate, $\gamma_j$, which is the probability for a consumer having landed on the product page to purchase the product. On the search result page, the platform has two slots to display two products at any time.

![Diagram of the search result page](image)

Figure 1.5 Diagram of the search result page

Upon landing on the search result page, each consumer $i$ has to decide whether to click the
products’ links to read the product page for detailed information. Doing so incurs a search cost $c_i$, but if the consumer later decides to purchase the product, she will get a benefit $B$ and the seller will get a profit, normalized as 1. The consumer’s clicking and purchasing decisions are independent across products.

The consumers have heterogenous search costs uniformly distributed on $[0, 1]$. In equilibrium, the consumers have rational expectations about the conversion rates of the two products on the result page. Hence, a consumer with search cost $c_i$ would only click a product’s link on the left/right of the search result page if she believes the product on the left/right has an expected conversion rate of at least $c_i/B$. The two slots on the result page may receive different numbers of clicks from consumers if the products at those slots are chosen by different rules. I assume the consumers’ search costs are independent from their purchase behaviors conditional on clicking the product page.

The platform, sellers and consumers share a common prior that for each product, $\gamma_j$ is drawn from a gamma distribution with shape $\alpha_0$ and mean $\bar{\gamma}_0 = \alpha_0/\beta_0$. Before entry, each seller receives a private imperfect signal $q_j$ about its own conversion rate. The signal $q_j$ follows a Poisson distribution with mean $\gamma_j\pi_0$, so the seller has a gamma posterior for its conversion rate, with shape $\alpha_0 + q_j$ and mean $\bar{\gamma}_j^E = (\alpha_0 + q_j) / (\beta_0 + \pi_0)$, according to Bayes rule.

### 1.3.2 Platform with sponsored slot

In this subsection, I consider an environment with one sponsored slot on the result page. In particular, the platform assigns the right slot on the result page to an entrant through a second price auction with simultaneous bids, and allocates the left slot to the incumbent whom the platform believes to have the highest expected conversion rate among incumbents. The platform uses observed sales from incumbents’ last period to compute their expected conversion rates. Under this set of rules (denoted as $O|S$ as the platform has one organic slot and one sponsored slot), I solve for the equilibrium strategies of consumers and sellers.
In this environment, an entrant would bid for the sponsored slot with his posterior after observing his private signal. If the entrant loses the auction, it gets zero sales during the first period and the platform does not update belief about its conversion rate. If the entrant wins the auction, it pays the second highest bid, and receives consumer clicks $\Pi^s$ from the sponsored slot during its first period. Its sales $Q_j$ during this period, which follow a Poisson distribution with mean $\gamma_j \Pi^s$, are observed by the platform. The platform uses this observation to update its belief according to the Bayes rule, so its posterior about the incumbent's conversion rate $\gamma_j$ at the beginning of the next period is a gamma distribution with shape $\alpha_0 + Q_j$ and mean $\bar{\gamma}_j^p = (\alpha_0 + Q_j) / (\beta_0 + \Pi^s)$.

The platform places the incumbent of the highest expected conversion rate at the top organic place with random tie breaking. Hence, if the platform finds the incumbent who had the sponsored slot last period has an updated expected conversion rate greater than $\bar{\gamma}_0$, the expectation of its prior, this incumbent would be placed at organic slot; otherwise, a randomly chosen other incumbent gets the top organic slot. The single sponsored slot is allocated through a second price auction with simultaneous sealed bids.\footnote{It is assumed that the platform does not infer any information from the seller's bid per se, which is a realistic characterization of Taobao.com. The platform has not stated that the bid itself has any effects on the organic rankings, and the seller communities have been always working on tricks to pay as less as possible for a given number of clicks. Because of the significant heterogeneity among the sellers and their products, if the platform did relate the sellers' bids to their organic rankings in the future, it would be too much a burden for sellers to figure out their bidding strategies.}

An equilibrium of this market consists of three components: the consumer expectation of the conversion rate of the top organic slot $\bar{\gamma}^o$, the consumer expectation of the conversion rate
of the sponsored slot $\hat{\gamma}^s$, the bidding strategy of entrants. I consider an equilibrium where all entrants use the same bidding strategy, which can be denoted as $b^E \left( \hat{\gamma}_E^o \right)$. In an equilibrium, the consumers’ expectations are consistent with the long run average of the products at the respective slots, therefore, the numbers of clicks for the organic slot and the sponsored slot are $\Pi^o \left( \hat{\gamma}_o^o \right) = \hat{\gamma}_o^o BM$ and $\Pi^s \left( \hat{\gamma}_s^s \right) = \hat{\gamma}_s^s BM$, respectively.

The bidding strategy is an equilibrium strategy of the auction game, so it must satisfy the incentive compatibility constraint. In a second price auction, this generally means an entrant’s bid equals to its incremental benefit from the sponsored slot. The intuition discussed in the introduction section postulates that entrants with higher quality products would benefit more from the sponsored slot, because they not only can convert more exposures to sales in the current period, but also bear higher probability to earn the organic slot in the next period. This intuition is to be formalized into a proposition.

I define $W^s \left( \hat{\gamma}_s^e^1, \hat{\gamma}_s^s \right)$ as the probability perceived by entrant $e_1$ for him to be allocated the organic slot in the next period given: (1) entrant $e_1$ has posterior expected conversion rates $\hat{\gamma}_s^e_1$ after observing own private signals; (2) in the first period, entrant $e_1$ holds the sponsored slot; (3) the number of clicks of the top sponsored slot is $H' \left( W^o \right) = V^o BM$ and $H' \left( \hat{\gamma}_s^s \right) = \hat{\gamma}_s^s BM$, respectively.

The platform’s rules for slot allocation leads to the following lemma:

**Lemma 1.1** $W^s \left( \hat{\gamma}_s^e_1, \hat{\gamma}_s^s \right)$ increases in $\hat{\gamma}_s^e_1$. $W^s \left( \hat{\gamma}_0 + \frac{1}{\beta_0 + \pi_0}, \hat{\gamma}_s^s \right) > 0.5$.

**Sketch of Proof.** See Appendix 1 for the full proof and the exact formula for $W^s \left( \hat{\gamma}_s^e_1, \hat{\gamma}_s^s \right)$. The condition for entrant $e_1$ to earn the organic slot in the next period is that its sales from the sponsored exposure in this period, $Q^e_1$, is greater than the expected sales according to the platform’s prior. Conditional on $\hat{\gamma}_s^e_1$, $Q^e_1$ has a negative binomial distribution that increases in $\hat{\gamma}_s^e_1$ in the sense of first-order stochastic dominance, hence the probability for $Q^e_1$ to exceed a given threshold increases in $\hat{\gamma}_s^e_1$. When $\hat{\gamma}_s^e_1 = \hat{\gamma}_0 + \frac{1}{\beta_0 + \pi_0}$, according to Van de Ven and Weber (1993) (see the full proof for details), the median of the distribution of $Q^e_1$ is bounded from below by $\Pi^s \left( \hat{\gamma}_s^s \right) \hat{\gamma}_0$, so $Q^e_1$ is more likely to exceed $\Pi^s \left( \hat{\gamma}_s^s \right) \hat{\gamma}_0$ than not. The term $\frac{1}{\beta_0 + \pi_0}$ is needed because the negative binomial distribution is right-skewed. ■

Now I can show the following proposition:
Proposition 1.1 The entrant’s equilibrium bidding function $b^E(\cdot)$ is increasing.

**Sketch of Proof.** See Appendix 1 for the full proof. I consider an entrant $e_1$ with posterior expectation $\bar{\gamma}^{e_1}$. Denote his equilibrium bid as $b^{e_1}$. Take any alternative bid $b' < b^{e_1}$, the entrant’s expected payoff following bids $b^{e_1}$ would be higher than that with $b'$, given other players sticking to their equilibrium strategy. Holding the value of $b^{e_1}$ and $b'$ constant, I show this difference in payoffs would be higher for an entrant with higher posterior expectation $\bar{\gamma}^{e_1}$, which implies that for this entrant, $b'$ cannot be its equilibrium bid, so its equilibrium bid must be at least $b^{e_1}$.

Because $b^{e_1} > b'$, there is a positive probability that bid $b^{e_1}$ wins the auction while bid $b'$ cannot. The difference in payoffs is caused by this situation and has three parts. The first part, winning instead of losing the sponsored slot makes current period profits, which increases in the entrant’s posterior expected conversion rate proportionally. The second part, winning the sponsored slot in this period leads to the organic slot in the next period with probability $W^o(\bar{\gamma}^{e_1}, \bar{\gamma}^{s})$, which by Lemma 1.1 increases in $\bar{\gamma}^{e_1}$. Finally, the entrant also needs to pay for the sponsored slot, and the payment does not depend on $\bar{\gamma}^{e_1}$. Hence, the difference in payoffs following bids $b^{e_1}$ and $b'$ has all three components increase or hold constant in $\bar{\gamma}^{e_1}$, which completes the proof. 

Given the increasing bidding function, the sponsored slot always goes to the entrant with the highest private signal. Given the common prior, the entrants’ private signal follows a negative binomial distribution $NB(\alpha_0, \pi_0 + \beta_0)$. The $N$ private signals of all entrants form a sample of size $N$ of this distribution. Denote the maximum value in this sample as $\tilde{q}_0$. Then the expected quality of the sponsored slot in equilibrium is

$$\bar{\gamma}^s = E \left[ \frac{\tilde{q}_0 + \alpha_0}{\pi_0 + \beta_0} \right] = \frac{E[\tilde{q}_0] + \alpha_0}{\pi_0 + \beta_0}$$

Given $\bar{\gamma}^s$ and $\Pi^o(\bar{\gamma}^s)$, the stochastic procedure that generates the incumbent at the organic slot in equilibrium is fully specified, hence its expected conversion rate, $\bar{\gamma}^o$, is well defined in terms of the model parameters, although the exact formula is too complex to present here. For the purpose of presentation, I denote $\bar{\gamma}^o$ as a function:

$$\bar{\gamma}^o = \Gamma^o(M, B, N, \alpha_0, \beta_0, \pi_0)$$
In Section 1.3.4, both $\hat{\gamma}^o$ and $\hat{\gamma}^e$ will be numerically computed to compare against counterparts from a different environment.

I now use the local incentive compatibility (LIC) constraint to derive the last component of the equilibrium $b^E(\cdot)$. The entrant considers how marginally change his bid would affect his payoff: in equilibrium, an entrant should be indifferent about changing its bid marginally conditional on event $\Lambda(b)$. Because it is a second price auction, a small change in bid has no effect on any outcome unless the highest bid among the other entrants' bids, denoted as $b^E(\gamma^e)$, is the same as the entrant’s bid $b$. Denote this tipping event as $\Lambda(b)$.

Denote entrant $e_1$’s first period profit as $V^e_{1}$ and second period profit as $V^e_{2}$. Denote the winner of the sponsored slot at the first period as $w_1$. The local IC constraint gives that entrant $e_1$’s equilibrium bid $b^E(\gamma^e)$, which is also his payment for the sponsored slot at event $\Lambda(b^E(\gamma^e))$, should be just as high as the expected benefit of winning conditional on $\Lambda(b^E(\gamma^e))$, so that the entrant is indifferent about just winning or just losing the auction.

$$b^E(\gamma^e) = \frac{\mathbb{E}[V^e_{1} | w_1 = e_1, \Lambda(b^E(\gamma^e))] - \mathbb{E}[V^e_{1} | w_1 \neq e_1, \Lambda(b^E(\gamma^e))] + \mathbb{E}[V^e_{2} | w_1 = e_1, \Lambda(b^E(\gamma^e))] - \mathbb{E}[V^e_{2} | w_1 \neq e_1, \Lambda(b^E(\gamma^e))]}{(\text{LIC})}$$

Because $V^e_{1}$ is independent of $\Lambda(b^E(\gamma^e))$ given $w_1$, $\mathbb{E}[V^e_{1} | w_1 = e_1, \Lambda(b^E(\gamma^e))] = \Pi^s_{\gamma^e} \Pi^o_{\gamma^e}$ and $\mathbb{E}[V^e_{1} | w_1 \neq e_1, \Lambda(b^E(\gamma^e))] = 0$. Now I focus on the parts with $V^e_{2}$.

If entrant $e_1$ wins the sponsored slot, his probability of getting the organic slot in the next period is $W^s(\gamma^e, \hat{\gamma}^e)$. Conditional on winning, which means that his sales in the first period is above the threshold, he update his expected quality to $\hat{\gamma}^e_{1,w}(\gamma^e)$, which is a function of $\gamma^e$. Hence,

$$\mathbb{E}[V^e_{2} | w_1 = e_1, \Lambda(b^E(\gamma^e))] = W^s(\gamma^e, \hat{\gamma}^e) \Pi^o(\hat{\gamma}^o) \hat{\gamma}^e_{1,w}(\gamma^e)$$

If entrant $e_1$ just loses the sponsored slot (i.e. at event $\Lambda(b^E(\gamma^e))$) the winning entrant must have posterior expectation $\gamma^e_{1,n}$ the same as $\gamma^e$. Hence, the probability for entrant $e_1$ to get the top organic slot in the next period is $(1 - W^s(\gamma^e, \hat{\gamma}^e)) / (N - 1) = (1 - W^s(\gamma^e, \hat{\gamma}^e)) / (N - 1)$, so

$$\mathbb{E}[V^e_{2} | w_1 \neq e_1, \Lambda(b^E(\gamma^e))] = \frac{(1 - W^s(\gamma^e, \hat{\gamma}^e)) \Pi^o(\hat{\gamma}^o) \gamma^e}{N - 1}$$
Plug in the four terms into (LIC), I have the equilibrium bidding function:

\[ b^E (\bar{\gamma}^e_1) = \Pi^s (\bar{\gamma}^s) \bar{\gamma}^e_1 + W^s (\bar{\gamma}^e_1, \bar{\gamma}^s) \Pi^o (\bar{\gamma}^o) \bar{\gamma}^e_1, w (\bar{\gamma}^e_1) - \frac{(1 - W^s (\bar{\gamma}^e_1, \bar{\gamma}^s)) \Pi^o (\bar{\gamma}^o) \bar{\gamma}^e_1}{N - 1} \]

I summarize the solved equilibrium by the following proposition.

**Proposition 1.2** On a platform with rule set O|S, the unique symmetric equilibrium consists of three elements: the expected conversion rate of the sponsored slot \( \bar{\gamma}^s \), the expected conversion rate of the organic slot \( \bar{\gamma}^o \) and the bidding function of entrants \( b^E (\bar{\gamma}^e_1) \), and they are specified by (1), (2) and (3), respectively.

Now I discuss the bidding function with more details. The first component in \( b^E (\bar{\gamma}^e_1) \) is the static incentive from profits in the current period. The other two components are the dynamic incentives from profits in the next period. Regardless of the number of entrants \( N \), if \( W^s (\bar{\gamma}^e_1, \bar{\gamma}^s) > \frac{1}{2} \), the dynamic incentive is positive. Combining Lemma 1.1 and Proposition 1.2 gives the following corollary

**Corollary 1.1** An entrant with expected quality of at least \( \bar{\gamma}^o + \frac{1}{2(1 + \pi_0)} \) would always bid higher than the current-period benefit for the sponsored slot.

If \( N \) is small, an entrant would weigh the third term substantially. For an entrant with very low expected quality, he understands that if he could win the sponsored slot, it must be that the other entrants also have very low expected quality, so for the purpose of earning the organic slot in the next period, it is better to let another entrant demonstrate its low quality in the sponsored slot. Hence, the dynamic incentive could be negative for a low quality entrant and cause bid-shading.

When \( N \) is large, the third term is insignificant due to the fact that an entrant losing the auction has very small probability to be randomly assigned the organic slot in the next period. Then the entrant is only concerned with earning the top organic slot in the next round through winning the sponsored slot in this round, and the dynamic incentive is always positive.

In this environment, I have restricted the incumbents from bidding for the sponsored slot. If they were allowed to bid, for incumbents who did not had the sponsored slot in the previous
period and therefore have not updated their beliefs, their bids as incumbents would simply equal to their static incentives in the previous period, which corresponds to the first term in (3). If an incumbent have not earned the organic slot after a period in the sponsored slot, it should bid even lower due to the downward information updating. Hence, when N is large, the incumbents are most likely outbid by the top entrant, who has a very strong dynamic incentive on top of its static incentive. Therefore, the restriction that only entrants can bid is not very restrictive after all for large N.

1.3.3 Platform with two organic slots

In this subsection, I consider an alternative environment where the platform does not take any input from the seller and determine all the rankings with information it gains from observed sales. In particular, the platform commits to rank the incumbent with the highest posterior expected quality at the top organic slot on the left of the first page. The right slot on the first page is now also an organic slot at the platform's disposal. Assume that the platform constantly randomizes between an incumbent and n randomly chosen entrants within a period, and each entrant receives a $\delta \leq \frac{1}{n}$ proportion of the total exposure of the second organic slot, and the observed sales of the entrants are used by the platform to compute posterior expectations for these entrants in the same Bayesian fashion as in the previous subsection. Then for all entrants, the platform determines the entrants with the highest and the second highest quality (with random tie breaking), which will be placed in the top organic slot full-time and the second organic slot part-time, respectively. Denote this set of rules as $O|O(n, \delta)$. 

\[
\begin{array}{|c|c|}
\hline
\text{1st Organic} & \text{2nd Organic} \\
\hline
\Pi^{01} = \mathcal{Q}^{01}BM & \Pi^{02} = \mathcal{Q}^{02}BM \\
\hline
\end{array}
\]

Figure 1.7 Diagram of a platform with two organic slots
This benchmark setup aims to capture the essential trade-off for welfare comparison. With
the sponsored slot in the environment $O|S$, the platform loses control of a precious prominent
space, and delegates the assignment of this slot to an auction mechanism so as to use the
private information of the seller. The benchmark case gives that control back to the platform
so that more information gained from last periods can be used. Because now the platform needs
to place two incumbents at the first page, I also give the platform the ability to experiment
with even more entrants so as to discover more than one good products for the next period.
Also, whenever the platform has two good incumbents and need to divert some exposure from
them to experiment with entrants, it is always more efficient to first divert exposure from the
incumbent of lower quality. Hence, the assumption that the platform split the second organic
slot instead of the first is reasonable.

Since now the sellers have no actions to take, the equilibrium of this environment only
involves consumers' rational browsing behaviors. Assuming consumers cannot distinguish the
platform's randomization, they would perceive the expected quality of the second organic slot as
the weighted average of the $n+1$ products. Hence, the equilibrium consists of the expected con-
version rates of the first and second organic slots, $\tilde{\gamma}^{o1}$ and $\tilde{\gamma}^{o2}$, respectively. In this equilibrium,
the first and second organic slots receive exposure $\Pi^{o1} (\tilde{\gamma}^{o1}) = \tilde{\gamma}^{o1} BM$ and $\Pi^{o2} (\tilde{\gamma}^{o2}) = \tilde{\gamma}^{o2} BM$.

Given $\Pi^{o2} (\tilde{\gamma}^{o2})$, an entrant displayed will receive exposure $\delta\Pi^{o2}$, and its number of sales, $Q_j$, according to the gamma prior of the conversion rates, follows a negative binomial distribution
$\text{NB}(n_j, \frac{\Pi^{o2} (\tilde{\gamma}^{o2})}{\Pi^{o2} (\tilde{\gamma}^{o2}) + \beta_0})$ and its expected conversion rate will be updated to $\frac{Q_j + \alpha_0}{\Pi^{o2} (\tilde{\gamma}^{o2}) + \beta_0}$. Hence,
the sample of the updated expected conversion rates of all $N$ entrants consists of $n$ draws
from this distribution transformed from a negative binomial distribution, and $N - n$ fixed
elements equals to $\tilde{\gamma}^0$. Denote the expectations of the first and second highest elements of this
sample, defined in terms of $\tilde{\gamma}^{o2}$ and fundamental parameters, as $\tilde{\gamma}^{(N)} (\tilde{\gamma}^{o2}, \alpha_0, \beta_0, B, M)$ and $\tilde{\gamma}^{(N-1)} (\tilde{\gamma}^{o2}, \alpha_0, \beta_0, B, M)$.

The following proposition characterizes the equilibrium with the above notation:

**Proposition 1.3** On a platform with rule set $O|O (n, \delta)$, in equilibrium, the expected conver-
sion rate of the second organic slot, $\tilde{\gamma}^{o2}$, is the fixed point of the following transformation:

$$\tilde{\gamma}^{o2} = (1 - \delta n) \tilde{\gamma}^{(N-1)} (\tilde{\gamma}^{o2}, \alpha_0, \beta_0, B, M) + \delta n \tilde{\gamma}^0$$
and the expected conversion rate of the first organic slot, \( \gamma_{01} \), is

\[
\gamma_{01} = \gamma^{(N)}(\gamma_{02}, \alpha_0, \beta_0, B, M)
\]

In the next subsection, I will numerically compute \( \gamma_{02} \) and \( \gamma_{01} \).

1.3.4 Welfare comparison

In this subsection, I compare the equilibria of the two environments from a welfare perspective. The total welfare is proportional to the total number of transactions. In the first environment, \( O|S \), the total number of transaction is

\[
\Psi_{O|S} = \pi^o \gamma^o + \pi^s \gamma^s = \left( (\gamma^o)^2 + (\gamma^s)^2 \right) BM.
\]

Similarly, in the second environment, \( O|O_\delta \), the total number of transaction is

\[
\Psi_{O|O_\delta} = \pi^{o1} \gamma^{o1} + \pi^{o2} \gamma^{o2} = \left( (\gamma^{o1})^2 + (\gamma^{o2})^2 \right) BM
\]

Hence, in the rest of this subsection, I will focus on comparing \( \gamma^o \) to \( \gamma^{o1} \) and \( \gamma^s \) to \( \gamma^{o2} \).

First, I focus on the first environment. The analysis of the bidding functions shows that the sellers' private information are incorporated in the ranking mechanism to improve accelerate quality discovery, so the qualities of the organic and the sponsored slots depends on the amount of the entrants' private information, which is measured by \( \pi_0 \).

Below is a graph depicting the equilibrium quality levels, \( \gamma^o \) and \( \gamma^s \), of the organic slot and the sponsored slot in the first environment \( O|S \), as the power of the private information, \( \pi_0 \),
varies. Other parameters are fixed at $N = 8, \alpha_0 = 0.1, \beta_0 = 10, B = 20, M = 1000$.

The graph shows that both $\hat{\gamma}^o$ and $\hat{\gamma}^s$ increases in $\pi_0$. The improvement in $\hat{\gamma}^s$ is purely due to better private information of the entrants. The improvement in $\hat{\gamma}^o$ can be attributed to two factors: First, the higher is $\pi_0$, the more likely that it is the actually better entrant is allocated at the sponsored slot. The sponsored exposure thus can better help the platform to recognize the high quality. Second, because equilibrium exposure of the sponsored slot $\Pi^s$ is linear function of $\hat{\gamma}^s$, which increases in $\pi_0$, more consumers are willing to click the sponsored product, therefore the platform just receive better information regarding the quality of that product.

Second, I focus on the second environment with two organic slots. I plot the expected quality of the two slots against $n\delta$ for $n = 2, 4, 8$ in Figure 1.9 and Figure 1.10. Other parameters are fixed at the same values, i.e. $N = 8, \alpha_0 = 0.1, \beta_0 = 10, B = 20, M = 1000$. Across the range of $n\delta$, the quality of the first organic slot increases in $n\delta$ in Figure 1.9 since more exposure is
used to infer the qualities of the entrants.

Figure 1.9 Equilibrium quality of the first organic slot in the environment with only organic slots

The quality of the second organic slot is non-monotonic in \( n \delta \) in Figure 1.10 due to the trade-off between two usages of the exposure of the second organic slot: obtaining information for entrants sacrifices the chance to utilize those obtained information. In fact, at the current set of parameters, because exposure is so scarce, the second best incumbent inferred from data is not so much better than the average\(^8\), so even the peaks of these curves are very close to the

\(^8\)For \( n = 8 \), if too little exposure is allocated to entrants such that the expected sales according to the prior is close to zero, the second best incumbent would likely have zero sales and therefore have an expected quality that is even lower than the mean of the prior, which explains the downward bend of the solid curve for \( n = 8 \).
prior mean, $\tilde{\gamma}^0 = 0.01$.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Equilibrium quality of the second organic slot in the environment with only organic slots}
\end{figure}

Hence, in the interest of total number of transactions $\Psi_{O|O(n, \delta)}$, it is best to allocate all second slot exposure for informational purpose by choosing $n\delta = 1$. Also, Figure 1.10 suggests it is beneficial to experiment with all entrants. These observations will not hold true when the number of products and the number of consumers are both very large. In that case information is no longer scarce, so the platform can easily identify a second best incumbent of very high quality, and the marginal informational benefit from more exposure for entrants is small, which warrants experimenting with a subset of entrants using partial exposure.

Finally, I compare the two environments. The expected quality levels in Figure 1.8, 1.9 and 1.10 show that, even at the optimal set of $n$ and $\delta$, the environment $O|O(n, \delta)$ is dominated by the environment $O|S$. The second organic slot in the second environment cannot match the informativeness of the sellers' private signal, so using the second slot for random experiments leads to very low expected quality in comparison to that of the auction winner.

The informativeness of the private signals is comparable to an amount of exposure of $\pi_0 N$, while the exposure received by the second organic slot in the second environment is slightly above $\tilde{\gamma}^0 BM$. The first environment would have an advantage over the second if $\pi_0 N$ is large or if $\tilde{\gamma}^0 BM$ is small.
The table below gives more comparisons for the expected qualities as the number of products \( N \) and the number of consumers \( M \) vary. For all entries, I fix \( \alpha_0 = 0.1, \beta_0 = 10, B = 20 \). For the entries for the environment \( O|S \), I fix \( \pi_0 = 30 \); for all entries for the environment \( O|O(n, \delta) \), I choose \( n = N \) and \( \delta = 1/N \). To save space, the numbers are in percentage.

| \( N \) = 4 | \( O|S \) | \( \gamma^0 : 3.01 \) | \( \gamma^s : 2.93 \) | \( O|O(n, \delta) \) | \( \gamma^0 : 3.11 \) | \( \gamma^0^2 : 1.00 \) | \( M = 500 \) | \( O|S \) | \( \gamma^0 : 4.96 \) | \( \gamma^s : 4.92 \) | \( O|O(n, \delta) \) | \( \gamma^0 : 4.14 \) | \( \gamma^0^2 : 1.00 \) |
| \( O|S \) | \( \gamma^0 : 4.97 \) | \( \gamma^s : 4.93 \) |
| \( O|O(n, \delta) \) | \( \gamma^0 : 4.79 \) | \( \gamma^0^2 : 1.00 \) |
| \( N = 8 \) | \( O|S \) | \( \gamma^0 : 6.50 \) | \( \gamma^s : 6.49 \) | \( O|O(n, \delta) \) | \( \gamma^0 : 5.85 \) | \( \gamma^0^2 : 1.00 \) | \( M = 1000 \) | \( O|S \) | \( \gamma^0 : 4.97 \) | \( \gamma^s : 4.93 \) | \( O|O(n, \delta) \) | \( \gamma^0 : 4.79 \) | \( \gamma^0^2 : 1.00 \) |
| \( O|O(n, \delta) \) | \( \gamma^0 : 5.22 \) | \( \gamma^0^2 : 1.00 \) |
| \( N = 12 \) | \( O|S \) | \( \gamma^0 : 6.50 \) | \( \gamma^s : 6.49 \) | \( O|O(n, \delta) \) | \( \gamma^0 : 5.22 \) | \( \gamma^0^2 : 1.00 \) | \( M = 2000 \) | \( O|S \) | \( \gamma^0 : 4.96 \) | \( \gamma^s : 4.92 \) | \( O|O(n, \delta) \) | \( \gamma^0 : 5.22 \) | \( \gamma^0^2 : 1.00 \) |

The left panel of the table shows that the environment \( O|S \) benefits more from increasing numbers of entrants. In this environment, more entrants means more draws of private signals, and the best private signal improves with the size of the pool. For the environment \( O|O(n, \delta) \), since the exposure that can be allocated to the entrants is limited, splitting the exposure to more entrants reduce the accuracy of the platform’s information on each entrant. Hence, at higher \( N = 12 \), the environment \( O|S \) outperforms the other by a large margin, while at lower \( N = 4 \), the environment \( O|S \) underperforms.

The right panel of the table shows that the performance of the environment \( O|O(n, \delta) \) depends on the number of consumers, which directly affects how much exposure the platform can use to test the qualities of the entrants. The environment \( O|S \) does not react much to the number of consumers in the range of comparison, since the exposure of the sponsored slot is only used to determine whether that particular product is of good quality, which can be accomplished quite accurately with modest exposure.

1.4 Data

My data is obtained from a third party data service, Zhibi365.com, which uses a cluster of servers to record public information on Taobao.com on a daily basis. Every day, Zhibi365 conducts searches on Taobao.com and its mobile app using a list of 500,000 keywords provided by Taobao.com, and records all the information on the search result pages (top 6 pages for the
PC website and top 100 products for the mobile app), including the organic and sponsored lists of products, and for each product, its title, price and sales in the past 30 days as displayed in the search results. The information collected by Zhibi365.com is sorted out by product ID and seller ID, so that users of Zhibi365.com can query all the relevant ranks and other daily records of a given product or all the products of a given seller. Zhibi365.com also allows to query the ranks by sales within subcategories defined by Taobao.com.

Given Taobao.com's vast landscape and my research goal, I decided to draw data from the women's apparel category, which features large number of nonbranded products with frequently new product introductions, so that timely quality information is important to consumers and challenging for the platform. The seasonality guarantees a large number of new products in the early fall season.

Because Zhibi365.com organizes its data and queries based on seller ID and product ID, I first established a pool of products by recording the top 500 best selling products in the women's sweater category on Dec 03 2015, on which day the total sales of women's sweater category peaked. Then I tracked the sellers of these 500 products, and for each seller, I downloaded all the data relevant for this seller from Zhibi365.com, including the products it sold over time and the ranks of those products as discussed earlier. I kept the products that had appeared for more than 60 days in the ranks data, and ended up with 1500 products. From the user interface of Zhibi365.com, I collected daily data series covering seven months for these products.

The platform, Taobao.com, officially releases cross-sectional summary data on the 500,000 keywords. The list is intended for the sellers to find keywords to purchase sponsored ads for, so for each keyword, Taobao.com gives impression index (number of searches conducted for the keyword), click index (average number of clicks of a sponsored product), average click-through rate of sponsored product, average conversion rate of sponsored product, competitiveness for sponsored slots, and average per-click fee for sponsored ad. All of the rates are average values that aim to provide a relative idea about the effectiveness of the keyword in order to facilitate sellers purchasing sponsored ads. Zhibi365.com merges this data with its keyword search data and show them together, so I have collected this data from Zhibi365.com as well.

The descriptive statistics are presented in Table 1.2. The first two variables, Price and 30DaySales, are observed once per day for each product. The mean of prices (about $10)
confirms that most products are not of premium brands and therefore the effect of marketing outside of the platform is minimal. Most products do not change price over time. Some observations of prices are missing, but I will not use price information due to the great variation among the products. Also, On average these products have very substantial sale numbers, so the platform should care about conversion rates and also has reasonable data to infer conversion rates.

I scraped four types of ranks from Zhibi365.com, including both organic and sponsored ranks on the PC website and the mobile app interfaces. The four types of ranks have different supports due to truncation in data collection done by Zhibi365.com. Observed ranks appear to be uniformly distributed in the data, even after controlling for popularity of keywords, which in fact suggests significant heterogeneity among products’ rankings: the products ranking in top slots in popular keywords are receiving much more exposure than the products that not even show up in the observed range for those keywords.

Keyword-level cross-section variables are released by Taobao.com. While the impression index just measures how many times the keyword is searched, the click index and the average click-through rates and conversion rates are measured only for the sponsored products on the PC website. Taobao.com does not disclose the relevant length of the time interval used in computing the impression index, and the best guess is a week. The three variables specific to the sponsored products indicate that click-through rates are quite low and that the conversion rates are even lower.

Table 1.2 Variables and Descriptive Statistics
<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Product-level time-series variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Price</em></td>
<td>Price in CNY</td>
<td>62.4</td>
<td>47.2</td>
<td>9.9</td>
<td>518</td>
<td>43,279</td>
</tr>
<tr>
<td><em>30DaySales</em></td>
<td>Quantity sold in past 720 hours</td>
<td>758</td>
<td>2,195</td>
<td>0</td>
<td>39,109</td>
<td>71,556</td>
</tr>
<tr>
<td><strong>Product-keyword-level time-series variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>rank</em></td>
<td>Organic search rank on PC</td>
<td>97.6</td>
<td>57.5</td>
<td>1</td>
<td>243</td>
<td>812,711</td>
</tr>
<tr>
<td><em>rank</em></td>
<td>Sponsored search rank on PC</td>
<td>51.76</td>
<td>33.9</td>
<td>1</td>
<td>102</td>
<td>$1 \times 10^6$</td>
</tr>
<tr>
<td><em>rank</em></td>
<td>Organic search rank in mobile app</td>
<td>40.7</td>
<td>26.5</td>
<td>1</td>
<td>119</td>
<td>777,450</td>
</tr>
<tr>
<td><em>rank</em></td>
<td>Sponsored search rank in mobile app</td>
<td>36.3</td>
<td>26.6</td>
<td>1</td>
<td>111</td>
<td>234,547</td>
</tr>
<tr>
<td><strong>Keyword-level cross-section variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Impression-</em></td>
<td>Relative measure of displays during an interval of an undisclosed length</td>
<td>$40,077 \times 7 \times 10^5$</td>
<td>$0 \times 8 \times 10^7$</td>
<td></td>
<td></td>
<td>44,117</td>
</tr>
<tr>
<td><em>ClickIndex</em></td>
<td>Relative measure of clicks to sponsored products during the same interval</td>
<td>525</td>
<td>3,381</td>
<td>0</td>
<td>$2 \times 10^6$</td>
<td>44,117</td>
</tr>
<tr>
<td><em>AveCTR</em></td>
<td>Average percentage click-through rate</td>
<td>2.52</td>
<td>2.29</td>
<td>0</td>
<td>100</td>
<td>44,117</td>
</tr>
<tr>
<td><em>AveCR</em></td>
<td>Average percentage conversion rate</td>
<td>0.73</td>
<td>3.56</td>
<td>0</td>
<td>100</td>
<td>44,117</td>
</tr>
</tbody>
</table>

Two features of the data are of important empirical relevance. First, as mentioned in the empirical setting section, the platform has a mechanism to automatically vary each product's ranks in the organic results by a 7 day cycle starting at a time specified by the seller. In particular, a product is gradually moved up in the organic ranks as time moves towards the end of the 7 day cycle, which marks a nominal expiration time of the listing. At the expiration time, the listing is immediately and automatically reinstated, and a new cycle starts as the product is moved down in the organic ranks abruptly. This exogenous rank change helps to identify the effect of organic ranks on sales.

Second, one limitation of the available quantity data is that it consists of daily records of *30DaySales*, but the records are not necessarily taken at the same time on each day. In the empirical analysis I will use a flexible specification that allows imperfectly recovered daily sales to be properly related to the observed daily ranks.

### 1.5 Effects of sponsored and organic ranks on sales

In this section I connect search ranks to the sales they are responsible for. The unique 7-day asynchronous shuffling feature of the platform creates high-frequency, broad-support and, most
importantly, exogenous variation in organic ranks, which plays a crucial role in the identification of the effect of the organic ranks. Because the platform gives the 7-day timer different influences on the PC website and the mobile app interface, the effects of the two types of organic ranks can be reliably separated. The sharp day-to-day changes also allow studying the delay between consumers seeing search results and making purchase. For the sponsored ranks, the exogenous variation is primarily supplied by the frequent involuntary rank changes due to other sellers' bids.

Depending on the nature (organic or sponsored) and the interface (desktop or mobile) of the rank, the four types of ranks could have different effects on sales that reflect interesting patterns of consumer behavior. The estimation in this part is to be interpreted in tandem with the stylized model in Section 1.3. First, inspired by the stylized model, the empirical model in Section 1.5.1 specifies a linear relation between exposure and sales, with the coefficient representing a relative measure of the product's quality, which will allow relating quality to rank dynamics in Section 1.5.3. Second, the estimated exposure measure by ranks can testify whether the stylized model, simplified with just two slots, is conceptually reflective of the actual market on Taobao.com. Section 1.5.2 discusses the relevant evidence supporting the setup of the stylized model.

1.5.1 Specifying effects of ranks on sales

Inspired by the conversion rate model, I explore a specification identifying the following relationship:

\[
Sales_{jt} = \gamma_j E_{jt} + \mu_{jt} + \varepsilon_{jt}
\]

where \(Sales_{jt}\) is the sale happens on date \(t\), \(E_{jt}\) is the product's exposure through searching results, \(\mu_{jt}\) is the product's expected sales from other unobserved channels, and \(\varepsilon_{jt}\) is an error term reflecting randomness and variation in the sales process.

The parameters to be estimated are \(\{\gamma_j\}\), parameters generating \(\mu_{jt}\) and parameters generating \(E_{jt}\) from the product's ranks. Now I specify the parameters leading to \(E_{jt}\) and \(\mu_{jt}\).

For each of a product's four ranks (\(\{p(c), m(obile)\}\) by \(\{organic), s(ponsored)\}\), I estimate a rank function to measure how much exposure decreases as the rank number increases. The
rank functions have similar form

\[ g^{p,o}(\text{rank}^{p,o}_{j,k,t}) = \left[ \frac{(\alpha^{p,o} + \text{rank}^{p,o}_{j,k,t})}{(\alpha^{p,o} + 1)} \right]^{-\beta^{p,o}} \]

\[ g^{p,s}(\text{rank}^{p,s}_{j,k,t}) = \delta^{p,s} \left[ \frac{(\alpha^{p,s} + \text{rank}^{p,s}_{j,k,t})}{(\alpha^{p,s} + 1)} \right]^{-\beta^{p,s}} \]

\[ g^{m,o}(\text{rank}^{m,o}_{j,k,t}) = \delta^{m,o} \left[ \frac{(\alpha^{m} + \text{rank}^{m,o}_{j,k,t})}{(\alpha^{m} + 1)} \right]^{-\beta^{m,o}} \]

\[ g^{m,s}(\text{rank}^{m,s}_{j,k,t}) = \delta^{m,s} \left[ \frac{(\alpha^{m} + \text{rank}^{m,s}_{j,k,t})}{(\alpha^{m} + 1)} \right]^{-\beta^{m,s}} \]

The exposure at the top PC website organic slot is normalized to 1, and \( \delta^{p,s}, \delta^{m,o} \) and \( \delta^{m,s} \) measures the exposure of top slots in other types of ranks. The logic of imposing some parameters to be common across rank functions is to respect the way webpage/smartphone app displays the search results. For the PC results, because the organic and sponsored products are displayed distinctively in two separated areas, they could potentially have very different rank functions. However, because they are grouped into pages, so for large rank numbers, exposures of these two types of results should decay in a proportional way. Hence, I impose that the two function shares their \( \beta^{p} \), but allows them to have different \( \alpha^{p,o} \) and \( \alpha^{p,s} \), so that for small ranks, the two function could have quite different shape. For the mobile results, because the sponsored product is blended into the flow of products, it is natural to use a single multiplicative factor to account for the effect of the sponsored products and keep the shape of the functions (determined by \( \alpha^{m} \) and \( \beta^{m} \)) the same.

To aggregate the exposure from multiple keywords, I use the relative average number of clicks, \( \text{ClickIndex}_{k} \), to weigh the thousands of keywords. So the exposure from the same type of ranks can be aggregated over keywords, for example:

\[ E^{p,o}_{j,t} = \sum_{k} \text{ClickIndex}_{k} g^{p,o}(\text{rank}^{p,o}_{j,k,t}) \]

I then aggregate for PC and mobile interfaces separately

\[ E_{j,t}^{p} = E_{j,t}^{p,o} + E_{j,t}^{p,s} \]

\[ E_{j,t}^{m} = E_{j,t}^{m,o} + E_{j,t}^{m,s} \]
Because consumers could delay purchase after viewing the product page, the sales in date \( t \) may be related to exposures in previous days, and this pattern of delay could be different for consumers searching through PC and those searching through mobile. I aggregate the total effective exposure that is responsible for sales between \( t \) and \( t - 1 \) as

\[
E_{j,t}^{\text{all}} = \frac{m_{\bar{p}}}{\tau_{\bar{p}} - 1} E_{j,t-1}^{\bar{p}} + \frac{m_{\bar{p}}}{\tau_{\bar{p}} - 1} E_{j,t-2}^{\bar{p}} + \frac{m_{\bar{p}}}{\tau_{\bar{p}} - 1} E_{j,t-3}^{\bar{p}} + E_{j,t+1}^{m_{\bar{p}}} + \frac{m_{\bar{p}}}{\tau_{\bar{p}} - 1} E_{j,t-1}^{m_{\bar{p}}} + \frac{m_{\bar{p}}}{\tau_{\bar{p}} - 1} E_{j,t-2}^{m_{\bar{p}}} + \frac{m_{\bar{p}}}{\tau_{\bar{p}} - 1} E_{j,t-3}^{m_{\bar{p}}}
\]

Ideally, if the data collecting actions are exactly aligned and every consumer who would purchase do so without delay, I would expect \( \tau_{\bar{p}} = \tau_{\bar{p}}^{-1} = 1 \) while \( \tau_{\bar{p}}^{-2} = \tau_{\bar{p}}^{-3} = \tau_{\bar{p}}^{m_{\bar{p}}} = \tau_{\bar{p}}^{m_{\bar{p}}} = 0 \). However, due to consumers delay in purchase action, ranks from earlier days can potentially play a role here. This specification is also robust to misalignment in data collection time.

I also notice the possible effect of 30DaySales to consumer demand. First, it is possible that consumers are more likely to click/purchase if 30DaySales is higher, especially when the product is in the initial stage and 30DaySales is relatively small. Hence, both \( E_{j,t}^{\text{all}} \) and the constant term will be adjusted by \( 30\text{DaySales}^\kappa \) where \( \kappa \) is expected to be in \((0,1)\). Second, because the product I am looking at are seasonal, so while my story is good at explaining how the product get advertised and discovered, I rely on 30DaySales to model the product’s obsolescence: after the day at which 30DaySales_{j,t} peaks (peakDay), I shrink the effect of exposure and the constant term exponentially. Hence, after these two adjustment, I have

\[
\mu_{j,t} = 30\text{DaySales}_{j,t}^\kappa e^{-\chi \max\{t-\text{peakDay}, 0\}}
\]

\[
E_{j,t} = E_{j,t}^{\text{all}} \mu_{j,t}
\]

Next, I deal with the left-hand-side variable, the daily sales. The sales data available is the sum of sales made in the past 30 days, recorded daily. To transform this data into daily sales, I take the difference of consecutive observations. Ideally, if 30DaySales is collected every 24 hours, the difference between two consecutive observations of 30DaySales should just be the difference between the sales of two days that are 30 days apart.

\[
30\text{DaySales}_{j,t} - 30\text{DaySales}_{j,t-1} = Sales_{j,t} - Sales_{j,t-30}
\]
However, due to the randomness in data collection time, $30\text{DaySales}_{j,t} - 30\text{DaySales}_{j,t-1}$ could represent the difference between sales during two intervals that have equal lengths of $\varphi_{j,t}$ days, where $\varphi_{j,t}$ is a random number in $(0,2)$ with $E[\varphi_{j,t}] = 1$. Hence,

$$30\text{DaySales}_{j,t} - 30\text{DaySales}_{j,t-1} = \varphi_{j,t} (Sales_{j,t} - Sales_{j,t-30})$$

Bring in the RHS variables, I have

$$30\text{DaySales}_{j,t} - 30\text{DaySales}_{j,t-1} = \varphi_{j,t} (Sales_{j,t} - Sales_{j,t-30}) = \varphi_{j,t} \gamma_j (E_{j,t} - E_{j,t-30}) + \varphi_{j,t} (\mu_{j,t} - \mu_{j,t-30})$$

$$= \gamma_j (E_{j,t} - E_{j,t-30}) + (\mu_{j,t} - \mu_{j,t-30}) + (\varphi_{j,t} - 1) \gamma_j (E_{j,t} - E_{j,t-30}) + (\varphi_{j,t} - 1) (\mu_{j,t} - \mu_{j,t-30}) + \varepsilon_{j,t}$$

Because $E[\varphi_{j,t}] = 1$ and $\varphi_{j,t}$ is independent from $E$ and $\mu$, $(\varphi_{j,t} - 1) \gamma_j (E_{j,t} - E_{j,t-30}) + (\varphi_{j,t} - 1) (\mu_{j,t} - \mu_{j,t-30})$ can be counted as part of the error term in a least square regression.

Now, the empirical model is completely specified and it can be estimated by a nonlinear least square regression of the differences in consecutive $30\text{DaySales}$ on 30-day differences in exposure $E_{j,t}$ and extra sales $\mu_{j,t}$. Because $(\varphi_{j,t} - 1) \gamma_j (E_{j,t} - E_{j,t-30}) + (\varphi_{j,t} - 1) (\mu_{j,t} - \mu_{j,t-30})$ is treated as part of the error term, for observations with greater $(E_{j,t} - E_{j,t-30})$ and $(\mu_{j,t} - \mu_{j,t-30})$, the error term has higher variance, which creates a heteroskedasticity problem. I apply the feasible least square method, that is, I first estimate the model without weighting, and then weigh each observation by the inverse of the squared residuals from the first round of estimation and run estimation again.

1.5.2 Results

The final estimates are presented below. Confidence intervals are computed by bootstrap with resampling at the product level. Unfortunately, due to the limited number of products and the heterogeneity across products, these confidence intervals are quite wide. For current results, I will focus on point estimates and discuss their implications about the consumers’ usage of the platform, in connection to dynamic story told in the stylized model.
<table>
<thead>
<tr>
<th>PC exposure parameters</th>
<th>Mobile exposure parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Organic Date Weight</strong></td>
<td><strong>Organic Date Weight</strong></td>
</tr>
<tr>
<td>$\tau^P_{1}$</td>
<td>$\delta^{m,o}$</td>
</tr>
<tr>
<td>$0.60$</td>
<td>$25.5$</td>
</tr>
<tr>
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<td>$[0.0$</td>
</tr>
<tr>
<td>$1.41]$</td>
<td>$251.0]$</td>
</tr>
<tr>
<td>$\alpha^P$</td>
<td>$\alpha^m$</td>
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<tr>
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<td>$7.76$</td>
</tr>
<tr>
<td>$[5.07$</td>
<td>$[7.07$</td>
</tr>
<tr>
<td>$6.99]$</td>
<td>$12.16$</td>
</tr>
<tr>
<td>$\beta^P$</td>
<td>$\beta^m$</td>
</tr>
<tr>
<td>$0.39$</td>
<td>$0.95$</td>
</tr>
<tr>
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<td>$[0.04$</td>
</tr>
<tr>
<td>$0.84]$</td>
<td>$2.46$</td>
</tr>
<tr>
<td>Sponsored</td>
<td>Sponsored</td>
</tr>
<tr>
<td>$\delta^{p,s}$</td>
<td>$\delta^{m,s}$</td>
</tr>
<tr>
<td>$306.8$</td>
<td>$1013.3$</td>
</tr>
<tr>
<td>$[14.2$</td>
<td>$[19.7$</td>
</tr>
<tr>
<td>$2895]$</td>
<td>$5038.9$</td>
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<tr>
<td>$\alpha^{p,s}$</td>
<td>$\kappa$</td>
</tr>
<tr>
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<td>$0.428$</td>
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<td>$7.22]$</td>
<td>$0.513$</td>
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<tr>
<td>Sponsored</td>
<td>$\chi$</td>
</tr>
<tr>
<td>$30DaySales Adjustment$</td>
<td>$-0.041$</td>
</tr>
<tr>
<td></td>
<td>$[-0.054$</td>
</tr>
<tr>
<td></td>
<td>$-0.029]$</td>
</tr>
</tbody>
</table>

The rank functions: PC website vs. mobile

The shape of the rank functions are best understood through graphs. The graph below is the percentage distribution of exposures over the top 100 organic ranks for the two interfaces. The parameters behind this graph are $\alpha^{p,o}, \beta^p, \alpha^m$ and $\beta^m$.
There is a stark contrast between organic results on PC and on Mobile, reflecting the difference between $\beta^p$ and $\beta^m$. The exposure is quite dispersed across organic ranks on the PC website, while in the mobile app, exposure diminishes very fast and most exposure is contributed by top ranks. This is consistent with the difference in consumer behaviors on the two terminals. Desktop consumers, with the aid of the monitor and mouse, can easily scroll through many products, and open multiple tabs to examine in detail. Mobile consumers who have to scroll up and down on small screen can hardly go too far down the list. Also on mobile the consumer has to examine the products sequentially, so patience may wear out fast. This comparison, based on the points estimates, are quite intuitive and encouraging, with the caveat that the confidence intervals for $\beta^p$ and $\beta^m$ are quite wide and overlap with each other, perhaps due to excessive heterogeneity across products in a data set of limited size.

Relative importance of organic ranks: PC website vs. mobile

The estimate of $\delta^{m,o}$ is quite large. After adjusting for the mobile organic ranking’s faster decline in exposure, it suggests that for a given keyword, the total exposure from the top 100 organic ranks in the mobile app is four times as high as the total exposure from the top 250 organic ranks on the PC website. This comparison is again based on the point estimates, and the confidence interval of $\delta^{m,o}$ has a lower end close to zero, so the following interpretation should be taken with a grain of salt.

One important reason behind the importance of the mobile interface is the boost of mobile app usage. Because more people have mobile phones and mobile shopping can utilize fractured time, the platform now indeed has more transactions on mobile than on PC. However, usage ratio alone seems not enough to explain the huge difference, which may actually has to do with how the platform arrange the organic results on PC website and the mobile app.

On the PC side, the platform does not rank the products very strictly by their quality, potentially because consumers can look through large number of products easily. One piece of evidence is that on the PC website, the nominal once-per-week expiration time has a much stronger effect on products’ organic ranks than on the mobile app. Since the nature of this mechanism is to intentionally shuffle most products’ ranking so that every product gets some exposure, it certainly reduces the general quality of the organic ranking on the PC website. By
way of comparison, on the mobile side, since consumers have limited attention span over the ranks, the platform is very careful about the ranks, and the ranks, despite having 7-day cycles as well, is much more stable.

Another piece of evidence is that, despite Zhibi365.com collects data on about top two hundred slots on the PC website, twice as many as on the mobile app, products in my sample get similar number of records on the PC website and on the mobile app. Since my sample consist of potentially more serious sellers, it is suggested that the PC website rankings are more dispersed across products and sellers, and are therefore of lower expected quality.

If the mobile organic search ranking is more reliable and informative, consumers should be more willing to click the results on the mobile app, which is indeed suggested by the estimates. This is consistent with the equilibrium assumption in the stylized model that consumers’ willingness-to-view increases with the expected quality of the products presented at a given slot.

**Relative importance of sponsored ranks: PC website vs. mobile**

There are much fewer number of sponsored slots in the mobile app than on the website. Also, the difference between $\beta^m$ and $\beta^p$ suggest that exposure diminishes much faster with ranks in the mobile app. Hence, although the estimated scaling coefficient $\delta^{m,s}$ for the mobile sponsored ranks is about three times as high as $\delta^{p,s}$ for the PC website sponsored ranks, the estimates suggest that for a given keyword, the total sponsored exposure of the top 100 sponsored ranks is about six times as high as that of the sponsored ranks among the top 100 ranks in the mobile app. The desktop users, despite being outnumbered by mobile users, are contributing more clicks to sponsored products, which is consistent with their greater browsing capacity. Because the platform can use the data from the PC website sponsored slots to learn about the products’ quality and improve the mobile organic rankings, the desktop users have generated a positive externality on the mobile users. This interpretation, however, is also subject to the caveat of wide confidence intervals of $\delta^{m,s}$ and $\delta^{p,s}$.

**Relative importance of organic and sponsored ranks**

Given the very large estimate of the scaling coefficient for the sponsored ranks on the PC website, $\delta^{p,s}$, the estimates suggest the exposure related to the organic results on the PC
website is dwarfed by that of the sponsored results. Given the wide confidence intervals of $\delta^{p,s}$, the true comparison could be less significant. The idea of sponsored results receiving more exposure is somewhat consistent with the consumers’ rational expectations of the qualities of the products in the two lists. While the organic results are shuffling, because the sponsored results are generated by auction, it is likely the top places are hold by aggressive advisors, so it is very possible that the average quality of sponsored products is higher than that of the organic ones, so the consumers could pay even more attention to the sponsored products.

Both $\delta^{p,s}$ and $\delta^{m,s}$ are very large even relative to $\delta^{m,o}$, while this could be an artifact of estimation noises implied by the confidence intervals, it is also possible that the sponsored exposure estimates are biased upwards due to an endogeneity issue. Because sponsored advertising is not the only paid promotional service on Taobao.com, when sellers purchase sponsored ads, they may simultaneously invest in other types of promotional service that boost sales. If that’s the case, the effects of other promotional services could be mistaken as part of the effect of the sponsored ranks and cause bias.

**Purchase delay and the effect of 30DaySales**

Other parameters appear to be intuitive. The $\tau$ parameters models consumers delaying purchase, and their estimates, albeit noisy, shows exposure today can convert to sales three days later, which echoes the finding of Blake, Nosko and Tadelis (2016) that consumers on ebay on average conduct searches over a period of three days for each purchase.

Finally, the first 30DaySales parameter $\kappa$ confirms that 30DaySales have a very concave effect on consumer purchase. When 30DaySales is small, consumers are skeptical and a boost in 30DaySales is very important; When the product is already well established, the consumers do not respond to 30DaySales much. The second 30DaySales parameter $\chi$ describes a reasonable fade-out stage possibly due to seasonality.

**Relation to the stylized model**

Several aspects of the estimates echo the theoretical model. First, the organic exposure across the platform is highly concentrated to the top ranks in the mobile app, highlighting that the emergence of the mobile interface brings the market closer to the narrative of the limited number
of slots in the stylized model. The concentration means there is very limited exposure left for other organic ranks. Given the large number of products, exposure for new product is very scarce. According to the welfare comparison in Section 1.3.4, these features give edge to the environment with sponsored slots relative to the environment without those.

Second, the seven-day shuffling in the organic ranks, especially that on the PC website, is similar to the environment without sponsored slot described in Section 1.3.3. The estimates find exposure received by shuffled PC website organic rankings is dwarfed by that of the more stable mobile organic rankings; therefore, consistent with the simulation in Section 1.3.4, this shuffling seems to discourage consumer clicks on the PC website by lowering the information value in the rankings. The very quick shuffling on the PC website gives each product just a small amount of organic exposure, while for products that purchase sponsored ads, they receive far more exposure from sponsored slots than from their shuffled organic slots. These results supports that the sponsored slot mechanism has a much prominent role in new product discovery that the "random experiment" allowed by the seven-day cycles.

1.5.3 Dynamic Patterns

The empirical estimates provide a way to aggregate a product's exposure of the same type across keyword-rank pairs. In the graph below, I translate a product's appearance in keyword searches into its aggregate organic and sponsored exposure using the estimated empirical model. Note that each exposure type is normalized by its average for this product. The curves demonstrate the dynamics postulated in the stylized model: The product first purchased sponsored exposure (the dashed curve) aggressively. Meanwhile, its organic exposure (the solid curve) increases gradually. When the organic exposure stabilizes at a high level, the product stops purchasing sponsored exposure. While this is the pattern of a particular product, many other products
share the same dynamic features.

![Graph showing Dynamic exposure patterns of a product]

Figure 1.12 Dynamic exposure patterns of a product

Potentially due to the heterogeneity across products, the endogeneity problem for the sponsored exposure and the limited data size, the relative conversion rates estimated from the empirical model demonstrate very large variance, making it difficult to relate the dynamic patterns of organic and sponsored exposure to the products' conversion rates in a statistical framework.

1.6 Conclusion

In this paper, I have analyzed the sponsored ads scheme on a retail platform as an informational channel that incorporates the sellers' private quality signals to accelerate the discovery of high-quality entrants. The theoretical model finds this channel to be prominent when the private quality signals are strong, the number of new products is high and consumer exposure is relatively scarce. Using data on publicly observable search ranks and 30-day sales, I have dissected the relation between various types of search ranks and the sales they are responsible for, which echoes the theoretical analysis' characterization of the informational landscape.

Using data on both the PC website and the mobile app interfaces, my empirical analysis identifies striking differences and an interesting synergy between the two. For a given keyword,
the major organic ranks on the PC website only contribute a quarter as many sales in total as the major organic ranks in the mobile app. In contrast, the major sponsored ranks on the PC website contributes six times as many sales in total as the major sponsored slots in the mobile app. The ranks functions suggest that consumers using the PC website are less affected by the rankings and click through a broad set of products, generating quality-relevant data through purchase decisions. The mobile consumers are found to rely heavily on the rankings and to only pay attention to the top results, so they represent the prize for top products that have demonstrated their qualities. This prize in return prompts sellers to compete for the sponsored exposure, which is primarily from the PC website.

Online platforms have revolutionized the supply side of many markets and enabled small or even individual suppliers to provide products or services at affordable transaction costs. On the demand side, online platforms connect to consumers more and more through convenient yet limited interfaces: the mobile app is a prominent example, and other rising venues with limited interactions include smart TVs, virtual assistants and connected devices taking voice commands. The clash of the vast selection of goods and the limited information processing ability of the users highlights the informational role of the platforms, which is to provide high-quality recommendations. The tech industry has already advanced very far in schemes collecting and analyzing information from the demand side, such as user reviews and personalized recommendations based on user histories. The example of sponsored advertising on Taobao.com, from a different perspective, suggests that encouraging the supply side to provide information in an incentive compatible way could be an equally important frontier to explore.
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1.7 Appendix

**Proof of Lemma 1.1.** For the first part, given \( \hat{\gamma}^e \), entrant \( e_1 \) would earn the top organic slot in the next period if and only if the platform’s posterior expectation of \( e_1 \)'s conversion rate, \( \bar{\gamma}^P_{e_1} = (Q^{e_1} + \alpha_0)/(\Pi^e(\hat{\gamma}^e) + \beta_0) \), is greater than that of \( e_2 \), which is just the common prior \( \bar{\gamma}_0 = \alpha_0/\beta_0 \). Hence, the threshold for \( Q^{e_1} \) such that entrant \( e_1 \) earn the top organic slot in the next period is \( Q(\hat{\gamma}^e) = \Pi^e(\hat{\gamma}^e) \bar{\gamma}_0 \).

Intuitively, the higher is \( \hat{\gamma}^e \), entrant \( e_1 \) would be more optimistic about \( Q^{e_1} \), and believe there is a higher probability for \( Q^{e_1} \) to exceed \( Q(\hat{\gamma}^e) \). Thus the first part should be true. To be rigorous, \( W^e(\hat{\gamma}^{e_1}, \hat{\gamma}^e) \) is well-defined according to the information updating process. Since the exposure at the sponsored slot is \( \Pi^e(\hat{\gamma}^e) \), the sales number follows a Poisson distribution with \( \lambda = \Pi^e(\hat{\gamma}^e) \gamma^{e_1} \). Because the entrant’s belief for \( \gamma^{e_1} \) is a gamma distribution with shape parameter \( (\beta_0 + \pi_0) \hat{\gamma}^{e_1} \) and scale parameter \( (\beta_0 + \pi_0) \), his belief for the sales number \( Q^{e_1} \) is a negative binomial distribution

\[
Q^{e_1} \sim NB \left( (\beta_0 + \pi_0) \hat{\gamma}^{e_1}, \frac{\Pi^e(\hat{\gamma}^e)}{\Pi^e(\hat{\gamma}^e) + (\beta_0 + \pi_0)} \right)
\]

Note that this negative binomial distribution can be related to the number of successes until \( (\beta_0 + \pi_0) \hat{\gamma}^{e_1} \) failures happens in an infinite sequence of independent Bernoulli trials with success probability \( \frac{\Pi^e(\hat{\gamma}^e)}{\Pi^e(\hat{\gamma}^e) + (\beta_0 + \pi_0)} \). Hence, when \( (\beta_0 + \pi_0) \hat{\gamma}^{e_1} \) is higher, for any realization of the Bernoulli sequence, the realization of \( Q^{e_1} \) is higher, hence the probability for \( Q^{e_1} \) to exceed a given threshold is higher.

Van de Ven and Weber (1993) proves that the median of a negative binomial distribution \( NB(r, p) \) is bounded from below by \( [(r - 1)p/(1 - p)] + 1 \). Hence, if \( \hat{\gamma}^{e_1} = \hat{\gamma}_0 + \frac{1}{\beta_0 + \pi_0} \), the median of the distribution of \( Q^{e_1} \) is bounded from below by \( Q(\hat{\gamma}^e) \), which gives the second part of the lemma.

Furthermore, an exact expression of \( W^e(\hat{\gamma}^{e_1}, \hat{\gamma}^e) \) can be derived. Denote the CDF of \( Q^{e_1} \) as \( F_{Q^{e_1}} \), then

\[
W^e(\hat{\gamma}^{e_1}, \hat{\gamma}^e) = 1 - F_{Q^{e_1}} \left( (\Pi^e(\hat{\gamma}^e) + \pi_0) \hat{\gamma}_0 - q_0 \right)
\]

Because the CDF of a negative binomial distribution \( NB(r, p) \) can be defined by the regularized
incomplete beta function $1 - I_p (k + 1, r)$,

$$W^* (\tilde{\gamma}^{e_1}, \tilde{\gamma}^s) = \int_{\tilde{\gamma}^{e_1} \leq \gamma \leq \tilde{\gamma}^s} \left( (\Pi^* (\tilde{\gamma}^s) + \pi_0) \tilde{\gamma}_0 - q_0 + 1, (\beta_0 + \pi_0) \tilde{\gamma}^{e_1} \right)$$

Proof of Proposition 1.1. I consider an entrant $e_1$ with posterior expectation $\tilde{\gamma}^{e_1}$, after observing his private signal. Denote his equilibrium bid as $b^{e_1} = b^E (\tilde{\gamma}^{e_1})$. Denote $U (b, \tilde{\gamma}^{e_1})$ as the entrant’s payoff for bidding $b$, given the entrant’s expected quality $\tilde{\gamma}^{e_1}$ and other players’ playing equilibrium strategy. Because the equilibrium bid $b^{e_1}$ maximizes $U (b, \tilde{\gamma}^{e_1})$, for any $b' < b^{e_1}$, $U (b^{e_1}, \tilde{\gamma}^{e_1}) - U (b', \tilde{\gamma}^{e_1}) \geq 0$. I argue that $U (b^{e_1}, \tilde{\gamma}^{e_1}) - U (b', \tilde{\gamma}^{e_1})$ increases in $\tilde{\gamma}^{e_1}$, which would implies that for small $d\gamma > 0$, $U (b^{e_1}, \tilde{\gamma}^{e_1} + d\gamma) - U (b', \tilde{\gamma}^{e_1} + d\gamma) > U (b^{e_1}, \tilde{\gamma}^{e_1}) - U (b', \tilde{\gamma}^{e_1}) > 0$, so $b^E (\tilde{\gamma}^{e_1} + d\gamma)$ is at least as high as $b^E (\tilde{\gamma}^{e_1})$.

The two bids, $b^{e_1}$ and $b'$, only generate a difference in payoff when $b^{e_1}$ wins the auction while $b'$ not. Denote $e_h$ as the entrant other than $e_1$ with the highest posterior expectation. It suffices to compare payoffs when entrant when $b^E (\tilde{\gamma}^{e_h})$ lies right between $b'$ and $b^{e_1}$.

$U (b^{e_1}, \tilde{\gamma}^{e_1}) - U (b', \tilde{\gamma}^{e_1})$ contains three components. The first component is the difference in profit in the first period, which is $\Pi^s \tilde{\gamma}^{e_1}$. The second component, is related to the change in the profit in the next period. Now focus on this case. If entrant $e_1$ bids $b^{e_1}$, he wins the sponsored slot and the probability for him to get the top organic slot in the next period is $W^* (\tilde{\gamma}^{e_1}, \tilde{\gamma}^s)$. Because earning the top organic slot means his sales in this period is higher than the threshold, so his posterior expectation conditional on earning the top organic slot, denoted as $\tilde{\gamma}^{e_1, w}$, is higher than $\tilde{\gamma}^{e_1}$. If entrant $e_1$ bids $b'$, he loses the sponsored slot to entrant $e_h$, then entrant $e_1$ will earn the top organic slot in the next period with probability $(1 - W^* (\tilde{\gamma}^{e_h}, \tilde{\gamma}^s)) / (N - 1)$. Hence, the difference in the total profit from both periods can be expressed as

$$\int_{b' < b^E (\tilde{\gamma}^{e_h}) < b^{e_1}} \left[ \Pi^s \tilde{\gamma}^{e_1} + W^* (\tilde{\gamma}^{e_1}, \tilde{\gamma}^s) \Pi^o \tilde{\gamma}^{e_1, w} \frac{(1 - W^* (\tilde{\gamma}^{e_h}, \tilde{\gamma}^s)) \Pi^o \tilde{\gamma}^{e_1}}{N - 1} \right] f (\tilde{\gamma}^{e_h}) d\tilde{\gamma}^{e_h}$$

$$= \int_{b' < b^E (\tilde{\gamma}^{e_h}) < b^{e_1}} \left[ W^* (\tilde{\gamma}^{e_1}, \tilde{\gamma}^s) \Pi^o \tilde{\gamma}^{e_1, w} + \left( \Pi^s - \frac{(1 - W^* (\tilde{\gamma}^{e_h}, \tilde{\gamma}^s)) \Pi^o \tilde{\gamma}^{e_1}}{N - 1} \right) \tilde{\gamma}^{e_1} \right] f (\tilde{\gamma}^{e_h}) d\tilde{\gamma}^{e_h}$$

Because $W^* (\tilde{\gamma}^{e_1}, \tilde{\gamma}^s)$ and $\tilde{\gamma}^{e_1, w}$ increases in $\tilde{\gamma}^{e_1}$, the first term increases in $\tilde{\gamma}^{e_1}$ rapidly. If $N$ is greater than $\Pi^o / \Pi^s + 1$, the second term also increases in $\tilde{\gamma}^{e_1}$, and this difference in total
profits would increase in $\xi^{e_1}$.

The last element of $U(b^{e_1}, \xi^{e_1}) - U(b', \xi^{e_1})$ is the difference in auction payment. Since this is a second price auction, this difference only has to do with bids and does not change with $\xi^{e_1}$. Hence, overall, $U(b^{e_1}, \xi^{e_1}) - U(b', \xi^{e_1})$ has all three components increase or hold constant in $\xi^{e_1}$. ■
Chapter 2

Costs of Managerial Attention and Activity as a Source of Sticky Prices: Structural Estimates from an Online Market

2.1 Introduction

Managerial frictions can be broadly construed as a cost associated with the manager's overlooked but essential input into the operation of a firm. Textbook industrial-organization models leave little room for managerial frictions. Consider, for example, the ubiquitous pricing decision. Firms in most industries have some discretion over their output price, which may require dynamic adjustment to changing supply and demand conditions. When does the manager make these price changes? By how much? Textbook industrial-organization models would say price is changed precisely when and by the amount dictated by strategic considerations based on all available information. Other economic subdisciplines have paid more attention to the frictions

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in this pricing process. Macroeconomists have developed a class of models in which price inertia plays a key role in the monetary transmission mechanism, leading to a large theoretical and empirical macro literature studying a range of frictions such as menu costs, the costs of monitoring rivals’ prices, managerial inattention, and the implicit contracts that firms have with their customers to maintain prices. Organizational economists have long been interested in the internal processes that firms use to overcome the complexities associated with price setting in practice, dating back to Simon’s (1962) claim that “Price setting involves an enormous burden of information gathering and computation that precludes the use of any but simple rules of thumb as guiding principles,” and to Cyert and March’s (1963) classic book documenting the rule of thumb used by a department store to price its merchandise.

In this paper, we help bridge the gap between these literatures by extending methods used in industrial organization to estimate dynamic models to provide some of the first structural estimates of the sort of managerial frictions posited by macro and organizational economists. We study the pricing decisions of a set of rival firms selling computer components in an online marketplace, Pricewatch, which ranks the lowest-price firms in the most prominent positions, generating the highest sales. Price changes were relatively frequent in this market (the average spell between price changes lasting about a week) yet far from continuous. Initial reduced-form analysis coupled with information from manager interviews provide evidence that managerial frictions played a role in this price inertia, motivating us to propose a structural model to quantify these frictions. Our fine-grained data (hourly price observations for scores of firms over a year) combined with fluctuating market conditions and the jockeying for price rank generate a rich sample of price changes, an ideal environment to estimate a dynamic model of competitive price adjustment. We use the model to back out structural estimates of managerial frictions, separately identifying a manager’s cost of monitoring the market from the menu cost he or she faces when entering a new price. These structural estimates provide insight into the microfoundations of the observed price inertia, a perennial concern of macro and organizational economists, but also of interest to industrial-organization economists concerned with price dynamics.

We estimate our dynamic, structural model following the two-step method of Bajari, Benkard, and Levin (2007) (BBL) for estimating structural parameters in the profit function. The first
step involves estimating reduced-form strategies constituting a Markov-perfect equilibrium. The second step involves finding structural parameters in the profit function that rationalize the estimated strategies such that no player can improve his payoff by deviating to some other strategy. To allow for feasible estimation in our complex setting and to accommodate the possibility of boundedly rational managers—a natural possibility in our setting in which managerial frictions are the focus—we make several modifications to BBL’s procedure. The reduced-form policy functions estimated in the first step are not required to constitute a Markov-perfect equilibrium based on the universe of state variables but rather are taken to be rule-of-thumb strategies based on a subset of salient ones. In the second stage, we search for structural parameters rationalizing the estimated rule of thumb as undominated by any deviation in the class of admissible rules of thumb. Rather than requiring that the firm’s strategy to be optimal in every state, we impose the weaker requirement that the firm chooses a rule of thumb ex ante maximizing its expected present discounted value of the flow of surplus over the play of the game, where the expectation is taken over the distribution of salient states experienced by the firm. The computational burden that the full state space would impose is daunting. Just considering permutations of the scores of sample firms across price ranks, putting aside many other payoff-relevant variables (costs, margins, distances between firms in price space, etc.), there are billions times more such permutations in a single period than grains of sand on earth. Even if estimation were feasible for the econometrician, it may be unrealistic to assume that managers could compute the best responses each instant conditional on such a complex state space, especially in our setting in which the simple act of monitoring rivals’ prices will present a substantial managerial friction, to say nothing of performing wildly complicated calculations. Our approach—as one of the first attempts to accommodate boundedly rational players in a dynamic structural model—presents its own challenges, to be successful requiring the specification of a parsimonious policy functions providing a good fit to observed play. We will thus need to pay considerable attention to the specification of the functional form and covariates included in the policy function and to the measurement of the policy function’s goodness of fit. To improve the fit of the policy functions, we modify the treatment of firm heterogeneity in BBL’s framework. Instead of estimating the mean, variance, and other parameters characterizing the continuous distribution of structural parameters across firms, we postulate discrete types of firms, use machine-learning techniques
to partition firms into the types ex ante, and estimate separate structural parameters for each type. They key benefit of assuming several discrete types is that we can improve the flexibility of the policy-function specification by allowing its coefficients to vary freely across types. For parsimony, we end up postulating three types; if we instead try the structural estimation with a single policy function, its fit and the structural estimates are quite poor.

Consistent with recent work microfounding price stickiness, including contributions to the macro theory literature by Alvarez, Lippi, and Paciello (2011) and Bonomo et al. (2015), we build two separate managerial frictions into the structural model, (a) a cost of monitoring market conditions to determine the appropriate timing and size of a price change and (b) a menu cost, i.e., the physical cost of recording a new price. Though we do not observe monitoring, we are still able to identify the cost of this latent action through the exclusion restriction that certain payoff-relevant variables are only available to the manager after monitoring. We find substantial monitoring costs—around $68 per episode for the most prominent firm type—but virtually no menu costs. The division of managerial frictions matters for firm profit. In the counterfactual exercise in which these costs are reversed, with costless monitoring and a $68 menu cost, the firm’s net profit rises substantially because the firm is able to tailor its price policy more closely to market conditions. In another counterfactual exercise, we document substantial profit gains from an increasingly automated price-setting policy (which firms went on to implement after our sample period). Our structural estimates confirm the findings of Alvarez, Lippi, and Paciello (2015), who use survey data to calibrate their previously cited dual-cost model, finding that the monitoring cost is three times the menu cost of changing price.

A limit of our approach is that our conclusions are derived from a small and specialized market, not a broader set of markets representative of the U.S. economy. But even then, we would argue that more general lessons can be drawn from our findings here, and the detail of the results more than outweighs the disadvantage of specificity. For instance, measuring price stickiness in an online marketplace is a useful complement to much of the existing empirical evidence, which so far has concentrated on brick-and-mortar stores.¹ Also, we would argue

¹Exceptions include Arbatskaya and Baye (2004), Chakrabarti and Scholnick (2005), Lünneman and Wintr (2006), Gorodnichenko and Weber (2015), and the “Billion Prices Project” (Cavallo and Rigobon 2012) in progress.
that simply understanding the price-setting mechanisms in one market very well, as we can with our unusually detailed data, can help us predict which factors are likely to be important in many other markets. Finally, structural estimation would be difficult without the fine-grained, market-specific data provided by our setting. Our structural approach enables us to simulate counterfactual managerial costs for a firm, including higher costs that the manager of a brick-and-mortar store might face.

The paper is structured as follows. Section 2.2 reviews related literatures. Section 2.3 provides background on the Pricewatch market. Section 2.4 describes the data. Section 2.5 highlights several important reduced-form results from the data. Besides motivating the setup of the structural model, these reduced-form results contribute independent insights into price-changing behavior, which have the appeal of being non-technical and fairly assumption-free. The rest of the paper is devoted to specification and estimation of the structural model. Section 2.6 specifies the three components of our structural model: the value, profit, and policy functions. Section 2.7 presents estimates of the policy function and assesses the goodness of its fit. Section 2.8 discusses identification of the structural parameters, presents the structural estimates, and provides a sensitivity analysis. Section 2.9 conducts counterfactual simulations showing how a firm’s pricing behavior changes when managerial costs are varied from the estimated levels. The last section concludes. An appendix provides technical details on the structural estimation.

2.2 Literature Review

In this section, we provide more detail on where our paper fits in the literatures of several subfields. Methodologically, our paper lies in the industrial-organization literature, in particular the literature that structurally estimates dynamic games, including seminal papers by Aguirregabiria and Mira (2007); Pakes, Ostrovsky, and Berry (2007); Pakes, et al.(2015); and BBL cited in the introduction. Our estimation strategy is based on BBL, modified to allow boundedly rational managers to choose a rule-of-thumb policy. Our paper studies the same Pricewatch market and uses the same data as Ellison and Ellison (2009a, 2009b). Those papers focus on the demand side, estimating demand elasticities with respect to price, rank, avoided sales tax,
and geographic proximity between buyer and seller. We shift the focus to the supply side, analyzing pricing dynamics, allowing for the strategic interaction among firms, and structurally estimating managerial frictions. We will take the demand estimates from Ellison and Ellison (2009a) as an input into our analysis.

Our paper touches on several other strands of the industrial-organization literature. The field has a distinguished history of interest in price rigidity, dating at least back to Gardiner Means's testimony to Congress testimony about inflexibility of prices during the Depression (Means 1935, documented in Terkel 1970). Stigler and Kindahl (1970) and Carlton (1986) bypass the Consumer Price Index (CPI) computed by the Bureau of Labor Statistics (BLS) and indices computed by other administrative agencies, drilling into the underlying transactions data to establish a body of stylized facts about price rigidity.

Price rigidity is but one issue under the broader heading of dynamic pricing. Another issue that has received attention among empirical industrial-organization economists is documenting the occurrence of Edgeworth cycles (Edgeworth 1925, formalized by Maskin and Tirole 1988), in which firms gradually undercut each other until they reach the zero-profit level where they stay until one relents, resulting in a dramatic price rise. Edelman and Ostrovsky (2010) and Zhang (2010) document Edgeworth cycles in sponsored-search and online-advertising auctions, respectively.² Our setting resembles these Internet auctions in that firms continuously “bid” for favorable rank positions by cutting their margins—indeed winning the rank-1 position often requires the firm to have a negative margin of price over cost. Our reduced-form analysis will document cycles in ranks, but in reverse of the Edgeworth pattern: a given firm’s rank gradually rises as others undercut as they adjust their prices for secular declines in cost, punctuated by sharp drops in the given firm’s rank when the manager attends to pricing, readjusting to a target rank.


²A number of papers attempt to characterize Edgeworth cycles and other interesting pricing dynamics in gasoline markets, including Castanias and Johnson (1993); Eckert and West (2004); Noel (2007a, 2007b, 2008); Hosken, McMillan, and Taylor (2008); Atkinson (2009); Lewis (2009); Wang (2009); and Doyle, MuchLegger, and Samphantharak (2010).
Closest to our setting is Baye, Morgan, and Scholten (2004), who document price dispersion in an price-comparison website using rich data collected with a web-scraping technology. Although we also document price dispersion in our reduced-form analysis, this is not one of our core results, which are structural estimates of managerial frictions. However, our core results can be viewed as contributing to the price-dispersion literature. Our finding that substantial managerial frictions contribute to price stickiness provides a novel explanation for equilibrium price dispersion. Most explanations rely on costly price search à la Stahl (1989), but managerial frictions could also generate a distribution of prices for homogeneous goods in equilibrium. That is, our structural estimates raise the possibility that supply-side rather than demand-side frictions drive price dispersion.

Our paper is related to several subfields outside of industrial organization. Substantively, our analysis is related to a large literature in macroeconomics on sticky prices, including notable papers by Blinder et al. (1998); Bils and Klenow (2004); Nakamura and Steinsson (2008); and Eichenbaum, Jaimovich, and Rebelo (2011). As summarized in Klenow and Malin's (2011) handbook chapter, this literature uses a range of different data sources from manager surveys to supermarket scanner data to price data used by the BLS to compute the CPI. These are typically reduced-form studies documenting facts about frequency and size of price changes as well as identifying sectors with the most price rigidity (nondurable goods, processed goods, goods with noncyclical demand, and goods sold in more concentrated markets). Comparing our methodology to this literature's, we also document descriptive facts in our from an initial reduced-form analysis, but our main goal is structural estimation of managerial frictions, which this literature does not undertake. Comparing our data to this literature’s, each has certain advantages. The macro studies tend to use comprehensive datasets of retail prices, representative of the economy, whereas we look at a single product. The tradeoff is that their datasets have less detail for each product. For example, BLS price data is collected monthly, so price movements within a month are not recorded. We have hourly data, and we do, in fact, observe many price changes within a month. We observe prices for most of the rivals operating in the market, allowing us to estimate firms’ reactions to their rivals’ price movements. The BLS does not attempt to sample all the substitutes in a product space. The nature of our market and the length of our time series results in our recording dozens of price changes for each firm, allowing
us to estimate a detailed policy function for price changes at the firm level. We also have unusually detailed auxiliary data on wholesale costs and quantities, integral to the structural estimation.³

Ours is closest to a suite of papers from macro as well as empirical trade that back out managerial frictions in price setting by, in effect, measuring the imperfect pass through of input-price shocks (generated by exchange-rate or commodity-price movements, depending on the study) to product prices. Two pioneering studies estimate models of price setting by a single agent with costly adjustment, Slade (1998) in the market for crackers sold at grocery stores, Davis and Hamilton (2004) in the market for wholesale gasoline. More recently, Nakamura and Zerom (2010) and Goldberg and Hellerstein (2013) move beyond single-agent models to oligopolies, using a static oligopoly model including menu costs. Nakamura and Zerom (2010) find a menu cost of $7,000 in their calibration exercise, far greater than any managerial friction we estimate, though their figure is still a small percentage (less than 0.25%) of revenue. Goldberg and Hellerstein (2013) estimate a menu cost of $60 to $230, or 1% to 3% of revenue in their structural estimation of the static oligopoly model, in the range of our estimated monitoring cost, considerably higher than the negligible menu cost we found in our online setting. Similar to these previous papers, we use variation in input prices, in our case the wholesale price of memory chips, to provide crucial exogenous variation behind our estimation of managerial frictions. We diverge from these seminal papers in two ways, by incorporating the dynamics of firm interactions and by distinguishing two types of managerial costs, a crucial distinction, given that we find menu costs to be dominated by a different form of managerial friction, monitoring costs.

³A recent addition to the macro literature, a working paper by Gorodnichenko and Weber (2015), studies pricing in an online platform serving many retailers, generating data of similar richness as ours. Like most of this literature, the authors do not estimate parameters governing firm interactions or managerial decisions, so, although their empirical setting is similar to ours, their goals and methods are quite different. A study of the market for used cars in Germany in Artinger and Gigerenzer's (2012) working paper spans several literatures, macro as well as organizational economics. In the spirit of the macro literature, in particular Blinder et al. (1998), the first part of their paper presents descriptive evidence from extensive manager surveys. The second part of their paper gathers online price data from hundreds of car dealers to test Simon's (1955) model of aspirational pricing. While we share their goal of understanding what drives firms to change online prices, the direction of our analysis is quite different, estimating a model of price changing by firms and structural parameters capturing managerial frictions.
2.3 Empirical Setting

Our empirical setting is the online marketplace for computer components mediated by Pricewatch, previously studied by Ellison and Ellison (2009a, 2009b); see those papers for additional details on the empirical setting. During the 2000-01 period during which our data were collected, the Pricewatch marketplace was composed of a large number of small, undifferentiated e-retailers selling memory upgrades, CPUs, and other computer parts. These retailers tended to run bare-boned operations with spartan offices, little or no advertising, rudimentary websites, and no venture capital. A large fraction of their customers came through Pricewatch. Instead of click-through fees, retailers paid Pricewatch a monthly fee to list products. Customers could use Pricewatch to locate a product in one of two ways, either typing a product description into a search box or running through a multi-layered menu to select one of a number of predefined product categories. For example, clicking on “System Memory” and then on “PC100 128MB SDRAM DIMM” would return a list of products in that category sorted from cheapest to most expensive in a format with twelve listings per page. The full list for a pre-defined category could span dozens of pages with listings from scores of retailers. Figure 2.1 contains the first page of a typical list for the memory module in our study, downloaded during our sample period.

The Pricewatch ranking exhibited substantial churn from day to day and even from hour to hour. Figure 2.2 illustrates the movement in prices (top row of panels) and ranks (bottom row of panels) for three representative retailers of PC100 128MB memory modules in our sample during a representative month. The first and third retailers changed price six times during the month, the second retailer twice. Even the second firm’s slower rate of price change is quite rapid compared to findings in the macro literature from U.S. Bureau of Labor Statistics data that the median length of spells between price changes of 4.3 months (Bils and Klenow, 2004). While frequent, retailers’ price changes were still far from continuous. When firms were not changing prices, their ranks continued to move, bumped up or down by rival price changes, leading a firm’s rank to fluctuate much more than its price.

Based on information from a detailed interview of a manager of one of the retailers participating on Pricewatch, we can identify several possible reasons for this churn in the rankings. First, wholesale prices could be quite volatile for some products. Retailers would receive wholesale-price quotes via daily emails, and they often fluctuated from day to day. Short-
term fluctuations could be up or down, but as these are electronic components, the long-term trend was downward. The daily price quotes were relevant to the manager’s operations as they typically carried little or no inventory, ordering enough to cover just the sales since the previous day’s order. A second reason for turnover in the rankings was that managers did not continuously monitor each individual product’s rank on the Pricewatch website, making the instantaneous changes needed to maintain a particular rank. Retailers typically offered scores of products in different categories on Pricewatch; it would be impractical for a manager to continuously monitor all of them even he or she attended to no other managerial tasks. Certain high-volume products, such as the specific memory modules we study, could merit more attention, but this might involve checking at most one or two times a day. During our study period, managers had to enter price changes manually into Pricewatch’s database, as automated price setting was not introduced until 2002. Each adjustment would thus involve a fixed cost in both determining and entering the appropriate price.

A retailer’s rank on Pricewatch was a key determinant of its sales and profits. The first panel of Figure 2.3, derived from demand estimates from Ellison and Ellison (2009a), shows how a retailer’s daily sales vary with rank. The bulk of sales go to the two or three lowest-priced retailers in this market, but positive sales still accrue to many additional retailers on the list. For later reference in our calculation of firm profits, we will label the curve $Q(Rank_t)$. A significant source of profit in this market came from the “upselling” strategy documented by Ellison and Ellison (2009a), by which a firm attracts potential customers with low prices for the “base” memory module, but then tries to induce them to upgrade to a more expensive one. The second panel of Figure 2.3 again uses results from Ellison and Ellison (2009a) to back out an estimate of the hourly profit from this upselling strategy as a function of the firm’s rank. The shape results from two opposing forces: at higher ranks the firm has fewer potential customers, but the customers it does attract are advantageously selected be more likely to upgrade. For later reference in our calculation of profits, which will incorporate returns from the upselling strategy, we will label the curve $U(Rank_t)$.

Returning to Figure 2.2, it illustrates another feature of the Pricewatch market that will play a large role in our analysis: heterogeneity in retailers’ pricing strategies. We have already seen that retailers varied in the frequency of price change. They also appear to have targeted
different segments of the ranking, with the first firm maintaining low ranks, the second allowing itself to drift into middle ranks, and the third content with high ranks. To accommodate this heterogeneity in the later estimation, we will allow the parameters to vary freely across different types of firms. In fact, looking ahead to Section 2.5.1, the three firms in Figure 2.2 are representatives of the three types into which we will ultimately classify our sample using machine-learning methods. This analytical treatment allows us to be agnostic about the source of the firms' strategy heterogeneity, whether differences in firm costs, such as the cost incurred by a manager to monitor market conditions or to compute and enter a price change, or differences in a firm's ability to convert customers attracted by its rank position into sales of the base product or upgraded product (i.e., differences in the $Q(Rank_{it})$ or $U(Rank_{it})$ function.

To summarize the main take-aways from this brief overview of the market, competition among retailers for Pricewatch rank, a key competitive variable, led to frequent although far from continuous price changes. Our interview information suggested that this pricing inertia was due in part to managerial frictions, motivating further reduced-form work providing clearer documentation of managerial inertia and motivating structural estimation of a model of managerial costs involving in monitoring the market and changing price. Visual evidence of substantial heterogeneity in firms' pricing strategies will motivate our accommodating this heterogeneity in the estimation in a flexible way.

2.4 Data

Our data come from Ellison and Ellison (2009a). The authors scraped information on the first two pages of listings (12 listings per page for a total of the 24 lowest-price listings) from the Pricewatch website for several computer components. We focus on the category of 128MB PC100 memory modules because it is the most active and highest volume of the categories collected and because of the homogeneity of the products in this category. The authors recorded the name of the firm (as well as other information about the firm such as its location), the name of the specific product, and its price. By linking names of firms and products over time, we are able to trace pricing strategies of individual firms for individual products (taking the conservative approach of assuming that a change in the product's name indicates a change in
product offering). The authors scraped this continuously updated information every hour from May 2000 to May 2001 (with a few interruptions).

Ellison and Ellison (2009a, 2009b) supplemented the Pricewatch data with proprietary data from a retailer who sold through Pricewatch. This retailer provided information on its quantity sold and wholesale acquisition cost. We take this cost to be common across retailers justified by the fact that the typical retailer carried little inventory, did not have long-term contracts with suppliers, and had access to a similar set of wholesalers.

A large number of firms made brief appearances on the Pricewatch lists. Since we are interested in the dynamics of firms' pricing patterns, we study firms that were present for at least 1,000 hours during the year (approximately one-eighth of our sample period) and changed price while staying on the list at least once. For a small number of firms who had multiple products on the first two pages of Pricewatch simultaneously during some periods, we excluded their observations during those periods. We were left with 43 firms appearing at some point during the year, at most 24 present on the first two pages of Pricewatch at any particular moment. Although the excluded retailers do not constitute observations, we do use them to compute relevant state variables for rivals including rank, density of neighboring firms, etc.

Based on these data, we created a number of variables to describe factors that might be important to firms' decisions about timing and magnitude of price changes. Figure 2.2 demonstrated the importance of the rank of that firm on the Pricewatch list for firm outcomes. We also included margin, length of time since its last price change, number of times a firm has been "bumped" (i.e., had its rank changed involuntarily) since its last price change, and so forth. Table 2.1 provides a description of these variables and summary statistics.

Most of the variables can be understood from the definitions in the table, but a few require additional explanation. Placement measures where a firm is between the next lower- and next higher-priced firms in price space. For example, if three consecutive firms were charging $85, $86, and $88, the value for Placement for the middle firm would be \(0.33 = (86 - 85)/(88 - 85)\). Density is a measure of the crowding of firms in the price space around a particular firm. It is defined as the difference between the price of the next higher-priced firm minus the price of the firm three spaces below divided by 4. For example, if five consecutive firms charged $84, $84, $85, $86, and $88, the value for Density for the firm charging $86 would be \(1 = (88 - 84)/4\).
QuantityBump reflects relative changes in a retailer's order flow caused by being bumped from its rank. It is calculated using the $Q(Rank_t)$ function in Figure 2.2, in particular proportional to $\ln(Q(Rank_t)/Q(Rank_{t'}))$, where $t$ is the current period and $t'$ is the period in which the retailer last changed price. CostTrend and CostVol are computed by regressing the previous two weeks of costs on a time trend and using the estimated coefficient as a measure of the trend and the square root of the estimated error variance as a measure of the volatility. The definitions of the remaining variables are self-explanatory.

Turning to the descriptive statistics, we see that the average price for a memory module in our sample was $69, with a considerable standard deviation of 35.1. Most of this variation is over time, with prices typically above $100 at the beginning of the period and down in the $20s by the end, mirroring a large decline in the wholesale cost of these modules. The mean spell between price changes was 117.55 hours—about five days—similar to what we saw for the first and third of the representative firms in Figure 2.2. Wholesale cost showed a strong downward trend over our sample period, falling an average $0.19 per day, but was quite volatile.

2.5 Reduced-Form Evidence

Before diving into the structural model, we pause in this section to report several results from a reduced-form analysis of the data. Besides motivating the setup of the structural model, the results contribute independent insights into price-changing behavior, which have the appeal of being non-technical and fairly assumption-free.

2.5.1 Firm Heterogeneity

To accommodate the strategy heterogeneity illustrated by the representative firms in Figure 2.2 in our subsequent empirical analysis, we will adopt a compromise between combining all firms in one sample and performing separate firm-by-firm analysis. Separate firm-by-firm analysis sacrifices power, perhaps unnecessarily; results in a profusion of parameters that are hard to digest; and, most importantly, selects a non-random set of firms, eliminating almost all of the less active firms due to too few observations. To balance these three concerns with a desire to allow heterogeneity in our model, we will classify firms into a small number of strategic types.
and allowed the model parameters to differ freely across the types.

To partition the firms into types, we employed a popular machine-learning technique called "cluster analysis"; see Romesburg (2004) for a textbook treatment. To be clear, the cluster analysis referred to here is not the same as "clustering the standard errors," which is the familiar way of adjusting standard errors for correlation among related observations (which as indicated in the table notes we do throughout, clustering by firm). The first step is to select a set of dimensions along which the firms could be differentiated. We chose seven variables that we judged were instruments under the firms' control rather than outcomes depending on external factors.\(^4\) Next, the variables are standardized so that each has a standard deviation of 1, preventing the variable with the largest variance from dominating the assignment. Starting with every firm its own cluster, the algorithm proceeds by identifying which clusters are most similar, measured by the sum of Euclidean distances between all firms in the two clusters, and iteratively combines them.\(^5\) We iterated until three clusters were left, which we thought achieved a balance between spanning most of the important firm heterogeneity and still obtaining enough of each type enough to analyze empirically with some confidence. We will allow estimates of most structural parameters to freely vary across the three clusters, henceforth referred to as types \(\tau = 1, 2, 3\).

Table 2.2 provides variable means by each of the three firm types. The 22 firms of type 1 generally occupy the lower price ranks and change prices relatively frequently. The eight firms of type 2 occupy the middle price ranks and tend to change prices infrequently. The 13 firms of type 3 generally charge the highest prices but are more active in changing prices than type 2. This characterization echoes what we saw in Figure 2.2 for the three firms representing each type. The means for Margin show that type 1 firms earn the lowest margins (at least captured by this measure), followed by type 2, followed by type 3. The means of Rank show the same

\(^4\)The variables include the firm's target Rank and Placement, the firm's mean values of NumBump, SinceChange, and FirstPage, and the firm's variance of SinceChange. We also included the fraction of time the firm was present in our sample. Firm \(i\)'s target value of a given variable is the mean of that variable computed for the subset of periods that immediately follow a price change by \(i\).

\(^5\)The method of starting with each item in a separate cluster and combining them until the target is reached is called the agglomeration method. The use of Euclidean distance (the sum of squared differences in the standardized variables) to measure difference and the criterion of combining clusters that have the smallest sum of squared differences is called Ward's method. These are the standard options in Stata 14's cluster command, which we used to perform the cluster analysis.
pattern, with type 1 occupying the lowest ranks, followed by type 2, followed by type 3. The means of SinceChange show that type 1 and type 3 change price more than twice as often as type 2.

2.5.2 Distribution of Price Changes

Figure 2.4 provides more refined evidence on the distribution of the size and spells of price changes by firm type. Panel A provides histograms of the size of price changes. The bar for zero change has been omitted for readability since firms did not change price in the vast majority of hours. Note that the height of the other bars are unconditional masses, i.e., not conditioned on a price change. The distributions are roughly unimodal for the three types, the mass falling off for larger magnitude price changes. The mode for all types is at a price reduction of $1. While there is mass for price increases as well as reductions, more mass is on the reduction side, consistent with the downward trend in wholesale costs throughout our sample. A characteristic feature of type 2 firms is that they change price only infrequently, which shows up in the histograms as relatively little overall mass in the histogram compared to the other types. The ratio of tail mass to mode mass is higher for type 2 firms than others, suggesting that their infrequent price changes tend to be larger in magnitude than other types'.

Panel A can be related to the theoretical and empirical literatures from macroeconomics on price stickiness. Our histograms are roughly unimodal, inconsistent with theoretical models such as Golosov and Lucas (2007) that, under the assumption of cheap monitoring and expensive price changing, predict bimodal distributions, with most of the mass on large price changes in the tails than on small price changes in the center of the distribution. We will actually provide a microfoundation for the unimodal price distributions shown here when we estimate managerial costs. In particular, we estimate a high cost of monitoring and low cost of price changing. The empirical macro literature has found unimodal distributions in other settings such as Figure 2 in Midrigan (2011) and Figure II in Klenow and Kryvtsov (2008). The magnitudes of our firms' price changes are smaller than in Klenow and Kryvtsov (2008), who report over 35% of price changes exceeding 10% of the initial price in absolute value, while only 7% our our price changes are this large.

Panel B plots a kernel density estimate of the distribution of spells between price changes.
The distribution looks similar for types 1 and 3. These frequent price changers necessarily have shorter spells. Most of the mass is concentrated in spells shorter than 200 hours. The mass for type 2 firms spreads out more uniformly over longer spells, even as high as 600 hours.

The distribution of spells that we report has a shape similar to those previously documented, but with a compressed scale. For instance, the spell distributions for our type 1 and type 3 firms look quite similar to Figure IV in Klenow and Kryvtsov (2008) and Figure VIII in Nakamura and Steinsson (2008), but instead of a time scale stretching to 12 or 18 months respectively, ours covers just one month. The difference could be due in part to the nature of the data: much of the previous literature used monthly price data, missing more frequent price changes that our hourly data would pick up. However, the analysis of survey data in Blinder et al. (1998) finds that a large majority of managers change the prices of their most important products no more than twice in a year. The managers of our firms are simply more active than those in these other studies, perhaps due to volatile wholesale costs or low managerial costs.

Panel C plots the absolute value of the price change as a nonparametric function of spell using Cleveland’s (1979) locally weighted regression smoothing (LOWESS). The graph for type 1 firms shows an upward slope, with larger changes made after longer spells. The type 2 graph is also initially upward sloped. There is a downturn for longer spells, but this non-monotonicity is estimated from too few observations to be conclusive. The type 3 graph shows a flat relationship between size and spell. The graph for type 2 is higher than the other types’, indicating that the magnitude of type 2’s price changes tend to be larger even conditional on spell. Although there is not a single pattern emerging here that can be used to make a clean comparison, our results at least are not at odds with what the previous literature has found for the relationship between spell length and size of price change, such as Figure VIII in Klenow and Kryvtsov (2008).

2.5.3 Managerial Inertia

We have seen that price changes, while frequent, were far from continuous, on the order of once per week rather than each hour. This pricing inertia could be due in principle to many factors. The integer constraint on prices could combine with only slowly moving market forces to produce the infrequent price changes. Perhaps firms are reluctant to engender a rival response
to a price cut. Figure 2.5 provides evidence that the inertia is due at least in part to costs of managerial activity.

The figure plots the residual probability of a price change (after partialling out other covariates as controls) during each hour measured in Eastern Time, estimated separately for retailers operating on the East and West Coasts. Retailers supply a national market via Pricewatch, so if there were no managerial costs, presumably the timing of activity would be similar on the two coasts, responding to the same national demand factors. In fact the probability functions peak at different points, at 11 a.m. for East Coast and 8 p.m. for West Coast retailers.

Interestingly, the peak for the West Coast retailers is not simply shifted by the three-hour difference between Eastern Time and the local time for West Coast retailers, as might be expected if the retailers on the two coasts served separate markets but faced the same pattern of managerial costs during the day. Instead, the peak on the West Coast is shifted by ten hours, to 8 p.m. Eastern Time, which is 5 p.m. in their local time. One explanation, supported by the interview subject, is that the market has already been operating for a few hours by the time the West Coast managers arrive at work, so they are well-advised to set prices in the evening before they leave. The evening might also be a less busy time for them since orders might have started falling off at least from customers in the East. In contrast, the East Coast manager would arrive to a very slow order flow at 8 a.m. Eastern Time and have the leisure to adjust prices at that point before orders picked up for the day. In the absence of managerial costs, it is difficult to rationalize why retailers on the two coasts would pick the opposite ends of the day to do most of their price changes.

2.6 Structural Model

This section presents the model that will be the basis for our structural estimation. We begin in the next subsection by discussing the key object in dynamic structural estimation, the firm's value function. The subsections after that discuss the components that go into computation of the value function.
2.6.1 Value Function

Firm $i$ participates in the market each period from the current one, $t$, until it exits at time $T_i$. Its objective function is the present discounted value of the stream of profits given by the value function

$$V_i(s_t; \sigma, \theta) = E \left[ \sum_{k=t}^{T_i} \delta^k \pi_{ik} \right],$$

where $s_t$ represents the state of the game in period $t$, $\sigma$ is the vector over $i$ of firms’ strategies $\sigma_i$, $\delta$ is the discount factor,

$$\pi_{it} = \pi(s_t, \sigma(s_t), \theta)$$

is the static profit function, reflecting the payoff from one period (one hour in our empirical setting) of play, and $\theta$ is a vector of parameters. The expectation is taken over the distribution of all possible game play and evolution of private shocks starting from $s_t$.

The focus of this study is on obtaining structural estimates of $\theta$, which will include measures of managerial costs, upselling profits, and other variables of central interest. In essence, we will compare firm $i$’s value function when it plays equilibrium strategy $\sigma_i$ to that when it plays some deviation $\tilde{\sigma}_i$, maintaining rivals’ play as specified by $\sigma$. The estimated $\theta$ will be those values minimizing violations of the dominance of $\sigma_i$ over $\tilde{\sigma}_i$. As in BBL, we are not required to solve for the equilibrium. Instead, we can observe the equilibrium from the data. Still, we need to compute the value function in and out of equilibrium, which in turn requires three components: (a) specification of the profit function $\pi_{it}$; (b) an estimate of firm $i$’s policy function, $\tilde{\sigma}_i(s_t)$, which, following BBL, we will use in place of the equilibrium strategy $\sigma_i(s_t)$; and (c) treatment of the expectations operator. The remainder of this section will be devoted to specifying the profit function and policy function. Expectations will be computed by averaging the present discounted value of the profit stream from many simulated runs of the market, as described in more detail in Section 2.8 on the structural estimation.

For tractability and to better suit our empirical setting, our model will depart in several ways from the standard BBL framework. We touch on the departures here but discuss them in more detail below in the relevant sections. First, either because of limited information, cognition, or memory, the manager is assumed to only be able to consider a restricted set $\hat{S}$ of the overall state space. For example, rather than keeping track of the whole price history.
for each rival, manager i may view his current rank as an adequate summary statistic of the combined effect of these actions. Second, the manager is assumed to follow a rule-of-thumb policy that is a function of the restricted state space \( \hat{S} \). He does optimize, not over the price each moment, but over the long-run choice of this policy, maximizing the present discounted value of profits over the set of admissible policies \( PF \). Given the overwhelming complexity of the period-by-period optimization problem involving the full state space, it is unlikely that managers were fully neoclassical—all the more in our setting in which we have qualitatively documented substantial managerial frictions, which we aim to quantify. Our approach has the advantage of being able to accommodate behavioral managers. The disadvantage of model is that it will be misspecified if managers are more neoclassical or behavioral in a different way than we allow. It will thus be essential for us to specify a class of policy functions that can closely match observed firm behavior. We will devote careful attention to this specification and to gauging its resulting goodness of fit.

2.6.2 Profit Function

Our detailed specification of the profit function \( \pi_{it} \) draws on our rich information about the business strategies of the firms operating in this market and the costs they face, as well as institutional details and estimates from Ellison and Ellison (2009a):

\[
\pi_{it} = \text{Base}_{it} + \text{Upsell}_{it} - \mu_t \text{Monitor}_{it} - \chi_t \text{Change}_{it},
\]

where

\[
\text{Base}_{it} = Q(\text{Rank}_{it})(\text{Price}_{it} - \text{Cost}_i) \quad (2.4)
\]

\[
\text{Upsell}_{it} = U(\text{Rank}_{it}). \quad (2.5)
\]

The first term \( \text{Base}_{it} \) accounts for the profits from the sale of the base version of the memory modules, equaling the quantity sold times the margin per unit. The quantity sold is the function \( Q(\text{Rank}_{it}) \) from Figure 2.3A. The second term \( \text{Upsell}_{it} \) accounts for the upselling strategy discussed in Section 2.3, whereby the firm attracts potential customers with low prices for the base product but then induces some of them to upgrade to more expensive versions of the
memory module. It is given by the function $U(Rank_{it})$ from Figure 2.3B.\(^6\)

The final two terms in (2.3) reflect the two types of managerial costs that we seek to estimate, both of which vary by firm type $\tau$. The coefficient $\mu_\tau$ on the indicator $Monitor_{it}$ for whether firm $i$ monitors in period $t$ will provide an estimate of the cost of monitoring. The last term involves $Change_{it}$, which is our label for the indicator $1\{\Delta_{it} \neq 0\}$ for whether firm $i$ changes price in period $t$, where we define $\Delta_{it} = Price_{it} - Price_{i,t-1}$. The coefficient $\chi_\tau$ on this indicator will provide an estimate of the cost of price change over and above any monitoring cost. We do not observe monitoring, but, just as we will generate simulated values for $Rank_{it}$, $Price_{it}$, and other variables when we compute expectations of the value function, we can simulate $Monitor_{it}$ using the model of monitoring behavior embedded in the policy function discussed in the next subsection.

The specification of managerial costs $\mu_\tau$ and $\chi_\tau$ as fixed parameters represents a departure from BBL’s framework, which specifies a distribution over structural parameters with a mean and variance to be estimated. Our approach of estimating a single fixed parameter eases the computational burden, important given we allow the parameters to differ across the three types $\tau$. We also have several conceptual reasons for our approach. The policy function specified in the next subsection already incorporates arrival of opportunities for managerial actions based on both stochastic as well as deterministic factors. Our estimates of the structural parameters can be thought of as the marginal cost of taking the relevant managerial action in opportune periods dictated by the policy function. Implicit in this formulation is the presumption that managerial actions are not taken in other periods in part because of high, perhaps prohibitive, draws for random managerial costs. To explain the relative rarity of price changes observed on a hourly basis, an estimated distribution of managerial costs would have to be skewed toward large values compared to our estimate for opportune periods. Estimating the distribution of managerial costs would require specifying the functional form for the distribution and the hour-to-hour correlation structure, which could be quite complex. Our policy function captures the time-series properties of opportunities for managerial action simply, by including covariates such as $SinceChange$, $Night$, and $Weekend$, among others.

\(^6\)We normalize the profit function for a firm that moves off of the first two Pricewatch pages to $4.70, our projection of a firm’s hourly profit at rank 25 based on Ellison and Ellison’s (2009a) parametric estimates for ranks 1–24. The structural estimates are robust to the choice of this normalization.
2.6.3 Policy Function

The next piece needed for structural estimation is the policy functions, or models of firms’ strategies. An estimate of this model provides the $\hat{\sigma}_i(s_t)$ that will be substituted for equilibrium strategies $\sigma_i(s_t)$ to compute the value function (2.1) in the simulations. The model reflects the reality that price changes in this market are infrequent relative to our data frequency. To a first order, the best prediction of the firm’s price next hour is its current price. We will thus focus on modeling the timing and size of price change episodes, with the firm maintaining the price outside of these episodes. Consistent with our belief that firms are not engaging in complicated calculations of optimal policy based on hundreds of state variables each hour, we want the model to be simple, streamlined, and reflective of just the variables that firms are likely to be able to monitor and process. Also, we want the model to be a predictive empirical description of what firms actually do.

We thus model price changes as coming from a two-step process. The manager knows some components of the market state vector at all times, information he receives essentially “for free.” Based on these state variables, the manager’s first step is to decide whether to attend to the market to gather information needed for a pricing decision. We will call this behavior “monitoring” and denote the decision to do so with the indicator function $\text{Monitor}_{it}$. In our setting, we think of monitoring as visiting the Pricewatch website and internalizing the relevant information, involving an opportunity cost of cognition and time. Through monitoring, the manager gains additional information on state variables, including current rank and the distribution of competitors’ prices, and computes the new desired price. If the new desired price is different from the current price and the costs of changing it are justified, then he enters it in the Pricewatch form, and changes it on his own website, again involving costs in terms of cognition and time.

The two-step process can account for periods of excess inertia, during which the manager keeps price constant even though market conditions would warrant a price change. Inertia can come from three sources. First, the manager may not be aware of the changed market conditions because he did not monitor. Second, the benefit from making a desired price change, especially a small price change, may not justify the managerial cost of entering it. Third, if the desired price change is smaller than a whole dollar unit in which Pricewatch prices are denominated,
price may stay constant. The two-step process is also consistent with anecdotal evidence from our interview subject and broader survey evidence (see Blinder et al., 1998) that managers often monitor market conditions including rival prices without changing their own price.

We specify the manager’s latent desire to monitor, $\text{Monitor}^*_{it}$, as

$$
\text{Monitor}^*_{it} = X_{it}\alpha + \epsilon_{it},
$$

(2.6)

where $X_{it}$ is a vector of explanatory variables, $\alpha$ is a vector of coefficients to be estimated, which are allowed to differ across firm types $\tau$, and $\epsilon_{it}$ is an error term. If $\text{Monitor}^*_{it} \geq 0$, then the firm monitors; i.e., $\text{Monitor}_{it} = 1$. Otherwise, if $\text{Monitor}^*_{it} < 0$, then the firm does not monitor; i.e., $\text{Monitor}_{it} = 0$.

In essence, equation (2.6) embodies the manager’s forecast of the costs and benefits of monitoring, so the explanatory variables can only include state variables known by the manager before monitoring. In addition, the variables must be important shifters of either the cost or benefit of monitoring. We specify the following parsimonious list:

$$
X_{it} = \left( \text{Night}_t, \text{Weekend}_t, \text{CostVol}_t, \text{CostTrend}_t^+, |\text{CostTrend}_t^-|, \text{QuantityBump}_t^+, |\text{QuantityBump}_t^-|, \ln \text{SinceChange}_it, (\ln \text{SinceChange}_it)^2 \right)
$$

(2.7)

We assume the manager is automatically aware of the time and day. The variables $\text{Night}_t$ and $\text{Weekend}_t$ are included to reflect that the cost of monitoring varies in a predictable way over a week: the cost of monitoring at 2am on Sunday morning might be high if that’s when a manager typically sleeps, and the benefit might be low because few sales would be made around that time anyhow. The manager is also assumed to be aware of the day’s wholesale cost—recall that he receives emails every day from the wholesalers—and can glean volatility and trends from the pattern of costs over the past couple of weeks. Presumably the gains to monitoring are greater the more conditions including costs are fluctuating. We include $\text{CostVol}_t$ to capture the magnitude of recent unpredictable fluctuations and $\text{CostTrend}_t$ to capture recent trends. Both rapidly rising and rapidly falling costs would lead the manager to monitor more. To
allow asymmetry\(^7\) in the concern for rising or falling costs, a rising cost trend, \(CostTrend_t^+ = CostTrend_t \times 1\{CostTrend_t > 0\}\), enters (2.6) separately from a falling cost trend, \(CostTrend_t^- = |CostTrend_t| \times 1\{CostTrend_t < 0\}\). The latter variable appears in absolute value so that we would anticipate the two variables’ coefficients to have the same sign if not magnitude.

We assume that the manager is roughly aware of changes in his order flow resulting from being bumped in the ranks and will be more likely to monitor if there has been a large change, whether an increase or decrease. Thus (2.6) includes \(QuantityBump_t\). Recall that this variable is computed by translating the current rank and rank at the previous price change into a quantity change using the function in Figure 2.3A. This predicted change in order flow is a proxy for the quantity signal the manager observes. The proxy diverges from the signal because the firm’s actual sales depend on random market fluctuations on top of any predictable effect of a rank change. The proxy also diverges from the signal because the manager may only be vaguely aware of actual sales in any given hour. Again, to allow for asymmetries, increases in order flow, \(QuantityBump_t^+\), enter separately from decreases, \(QuantityBump_t^-\).

The last set of variables, functions of \(SinceChange_{it}\), enter in a flexible, nonlinear way, allowing for various patterns of managerial attention, including monitoring the market at regular intervals as well as periods of intense monitoring, in which several price changes may follow in succession, followed by periods of inattention during which price stays constant independent of market conditions. Including this variable can help us tease out time dependence from state dependence in price monitoring behavior.

Conditional on monitoring, the manager may decide to change price based on the information acquired. Let \(\Delta_{it}^* = Price_{it} - Price_{it-1}\) denote the size of the price change the manager would desire if price were a continuous variable and the menu cost \(\chi_r\) of changing price were ignored just for period \(t\). Assume this latent variable is given by

\[
\Delta_{it}^* = \begin{cases} 
Z_{it} \beta_r + u_{it} & \text{if } Monitor_{it} = 1, \\
0 & \text{if } Monitor_{it} = 0,
\end{cases}
\]  

\(2.8\)

\(^7\)Some evidence suggests that prices could be stickier in one direction versus the other. For instance, Borenstein, Cameron, and Gilbert (1997) identify an asymmetry in the response of wholesale gasoline prices to cost increases versus decreases.
where $Z_{it}$ is a vector of explanatory variables, $\beta_\tau$ is a vector of coefficients to be estimated, which again are allowed to differ across firm types $\tau$, and $u_{it}$ is an error term. The actual price change $\Delta_{it} = Price_{it} - Price_{i,t-1}$ may diverge from the latent price change $\Delta^*_t$ for two reasons. First, rather than being continuous, prices were denominated in whole dollars on Pricewatch. Second, because changing price is not in fact costless, the manager must weigh the benefits of changing price if cost were no object reflected by $\Delta^*_t$ against the menu cost $\chi_\tau$. Assuming the manager’s willingness to pay to change price is an increasing function of the size of the desired price change, we can relate the observed price change $\Delta_{it}$ to the actual $\Delta^*_t$ by specifying cut points along the real line

$$
\cdots < C_{\tau}^{-5} < C_{\tau}^{-4} < C_{\tau}^{-3} < C_{\tau}^{-2} < C_{\tau}^{-1} < C_{\tau}^{0} < C_{\tau}^{1} < C_{\tau}^{2} < C_{\tau}^{3} < C_{\tau}^{4} < C_{\tau}^{5} < \cdots \tag{2.9}
$$

Then

$$
\Delta_{it} = \begin{cases} 
  k & \text{if } \Delta^*_t \in (C_{\tau}^k, C_{\tau}^{k+1}), \\
  -k & \text{if } \Delta^*_t \in (C_{\tau}^{-(k+1)}, C_{\tau}^{-k}).
\end{cases} \tag{2.10}
$$

Thus, for example, an observed price increase of $1$ corresponds to a latent price change satisfying $\Delta^*_t \in (C_{\tau}^{1}, C_{\tau}^{2})$. If the manager monitored but did not change price, then $\Delta^*_t \in (C_{\tau}^{-1}, C_{\tau}^{1})$. To deal with the fact that many elements of the manager’s decision to change price—the function mapping the size of his desired price change into his willingness to pay to change price, the distribution of the error term $u_{it}$, of course the menu cost $\chi_\tau$, which is yet to be estimated—we adopt a flexible specification for the $C_{\tau}^k$, allowing them to be free parameters estimated in a similar way as the cut points in an ordered probit, and allowing them to differ across firm types $\tau$.

The explanatory variables can include state variables the manager learns as a result of monitoring in addition to those known before. We specify the following parsimonious set:

$$
Z_{it} = \left( \text{CostTrend}_{it}, \text{CostChange}_{it}, \text{Margin}_{it}, \text{NumBump}_{it}, \right. \\
\left. \text{Density} \times \text{NumBump}, \text{Placement}_{it}, \text{Rank}_{it}, \text{RankOne}_{it} \right). \tag{2.11}
$$

The higher are forecasted costs, $\text{CostTrend}_{it}$, and the more costs have risen since firm $i$ last
changed its price, $\text{CostChange}_{it}$, the higher the firm's desired price. $\text{Margin}_{it}$ may also factor into price changes, a firm with low margins being more likely to increase price and one whose margins are already high less likely to further increase price and more likely to lower.

The remaining variables in $Z_{it}$ are the sort of state variables revealed by monitoring. After visiting the Pricewatch website, the firm learns its current rank and thus the number of ranks it was bumped since the last price change. The firm can use this information to return itself to its desired rank, so decreasing price if it was bumped up in the ranks (gauged by a positive $\text{NumBump}_{it}$) and increasing price if it was bumped down. A low-ranked firm may have less of an incentive to cut price to increase it sales; this effect may be particularly strong for a firm occupying rank 1: further price reductions will only result in a small increase in sales because most of the demand elasticity is with respect to rank, which the firm cannot improve beyond 1. We, therefore, include an indicator, $\text{RankOne}_{it}$. We include $\text{Density}_{it}$ because the presence of a thicket of close competitors may affect pricing incentives. For example, in a dense price space, firms may have less incentive to increase price because this will result in a more severe rank change. Specifically, we include $\text{Density}_{it}$ interacted with $\text{NumBump}_{it}$ because density will likely matter more for firms that have a need to change price as proxied by a bump from the previous preferred ranking. A firm's $\text{Placement}_{it}$ between its nearest rivals will also affect its desired price in potentially complex ways.

The list of explanatory variables in $Z_{it}$ deliberately excludes some of the variables that appear in the monitoring equation. For example, $\text{Night}_t$ is included in the monitoring equation to reflect the fact that checking the Pricewatch website at 2 a.m. would typically be more costly than during the workday. However $\text{Night}_t$ should have little effect on the desired price change conditional on monitoring because, conditional on already being on the website, changing price is no more difficult at 2 a.m. than 10 a.m. The same logic applies to $\text{Weekend}_t$. While the time since price was last changed may affect the desire to monitor—one possibility is that with more time for market conditions to change, more information is to be gained—conditional on the information gained through monitoring such as $\text{Rank}_{it}$ and $\text{NumBump}_{it}$, $\text{SinceChange}_{it}$ has no obvious role in price setting and thus is excluded from $Z_{it}$.
2.7 Estimation of the Policy Function

2.7.1 Identification

Though the dependent variable $\text{Monitor}_{it}$ in the monitoring equation (2.6) is not itself observable, the coefficients $\alpha_t$ can still be estimated because the dependent variable $\Delta_{it}$ in the price-change equation (2.10) is observable, and $\text{Monitor}_{it}$ enters that equation through (2.8). This begs the question of how determinants of monitoring are identified separately from determinants of price changing when both ultimately combined to produce a price change. Because $\text{Monitor}_{it}$ is an indicator while $\Delta_{it}$ takes on a larger range of integer values, the two can be identified purely through functional-form restrictions, for example, assuming restricting variables to enter both linearly. We obtain stronger identification from the inclusion of variables such as $\text{SinceChange}_{it}$, $\text{Night}_{it}$, and $\text{Weekend}_{it}$ in the monitoring equation excluded from the price-change equation based on the argument that, conditional on monitoring, they should have not incremental effect on the latent desire to change price. Denote this set of variables $X_{it} \setminus Z_{it}$.

To better understand how the parameters in the monitoring equation are identified, consider sample periods $t$ in which values of the variables $Z_{it}$ in the price-change equation strongly predict a price change. If price changes materialize in one subset of these periods and not another, the model looks to see whether variables in $X_{it} \setminus Z_{it}$ can systematically explain the difference. Values of $X_{it} \setminus Z_{it}$ correlated with a strongly predicted price change materializing as actual price changes will receive weight as increasing the probability of monitoring. By similar logic, variables included in $Z_{it} \setminus X_{it}$ can help identify determinants of the desire to change price separately from monitoring.

While it is easy to justify the inclusion of the explanatory variables in the respective functions, it may be harder to assert that this strategy model is sufficient to describe firms’ behavior. Firms could have executed much more complex strategies, including nonlinear functions of the included variables, interactions of them, and additional variables such as the characteristics of neighboring firms in the Pricewatch ranking. More generally, one might worry whether a reduced-form policy function could ever capture the intricacies involved in full-blown expected value maximization each period. We argue that our approach still has some appeal in our empirical setting. First, our approach is computationally much less burdensome than a fully
rational model. Second, it is reasonable to suppose managers used fairly simple rules of thumb to make the high-frequency decisions to monitor and change price rather than continually evaluating expected value functions based on scores of state variables each period. Such calculations would overwhelm the direct cost of checking websites and calculating and typing in new prices involved in the managerial actions. Third, as discussed in Section 2.7.4, we have encouraging results from a goodness of fit test to determine the match between simulations from our estimated strategies and observations in the data. Finally, we applied a machine-learning method (gradient-boosted machine) to generate a policy function without imposing functional-form restrictions on the included variables. Goodness of fit was not significantly improved by the machine-learning method relative to our reported policy function.

2.7.2 Estimation Details

The policy function consists of equations (2.6)–(2.11). We estimate these equations jointly using maximum likelihood taking the errors \( e_{it} \) and \( u_{it} \) to be independent standard normal random variables. Without the monitoring stage, the price-change stage of the model would be equivalent to an ordered probit, where various intervals would correspond to various discrete price changes. With the monitoring stage, the likelihood of observations in which firm \( i \) does not change price at time \( t \) is

\[
L(\Delta_{it} = 0) = 1 - \Phi(X_{it} \alpha_{\tau}) + \Phi(X_{it} \alpha_{\tau}) \left[ \Phi(C_{\tau}^{-1} - Z_{it} \beta_{\tau}) - \Phi(C_{\tau}^{-1} - Z_{it} \beta_{\tau}) \right],
\]

(2.12)

where \( \Phi \) is the standard normal distribution function. The likelihood of, for example, a \( k \) dollar price increase is

\[
L(\Delta_{it} = k) = \Phi(X_{it} \alpha_{\tau}) \left[ \Phi(C_{\tau}^{k+1} - Z_{it} \beta_{\tau}) - \Phi(C_{\tau}^{k} - Z_{it} \beta_{\tau}) \right].
\]

(2.13)

Adding the monitoring stage scales up the probability of no price change and scales down the probability of any given sized price change.

The literature refers to this type of model as a zero-inflated ordered probit. Harris and Zhao
(2007) show in Monte Carlo experiments that the maximum likelihood estimator of this model performs well in finite samples. We group the few price reductions of $5 or more together and similarly group the few price increases of $5 or more so that we only need to estimate thresholds down to $C_\tau^{-5}$ and up to $C_\tau^5$.

### 2.7.3 Results

Table 2.3 presents estimates of $\alpha_\tau$, $\beta_\tau$, and $C_\tau^k$ by firm type $\tau$. Consider the results for type 1 firms first, as these represent the majority of firms, comprising the biggest sample. *Night* and *Weekend* show up as important determinants of whether a firm monitors. This result is not surprising but is a telling indicator of the importance of managerial attention. Managers evidently take evenings and weekends off and do not bother monitoring the market then. Managers are also more likely to monitor if wholesale cost is volatile, if there has either been a sharp trend in wholesale cost or a sharp change in order flow recently, or if there has been a long spell since the last price change. These estimates are consistent with intuition and are quite precisely estimated for type 1 firms. Conditional on monitoring, a price increase for a type 1 firm is associated with a wholesale cost increase, a firm being bumped down, rank being low, rank being 1, and there being few firms close by in price space. Again, these results are consistent with intuition as well as our conversations with one of the firm managers.

A likelihood-ratio test rejects homogeneity of the policy function across types at better than the 1% level. While there are quantitative differences among the results for the firm types, there are generally not large qualitative differences, especially for the $\alpha_\tau$ coefficients. Our framework is flexible enough to accommodate the difference in activeness between type 2 firms and the rest. In the monitoring equation, the coefficients for $\ln \text{SinceChange}$ and $(\ln \text{SinceChange})^2$ of type 2 firms constitute a quadratic function of $\ln \text{SinceChange}$ that is decreasing when $\text{SinceChange} < 586$. So a type 2 firm would only monitor when it has accumulated enough change in QuantityBump or it has been a very long time since the last price change. Conditional on monitoring, a type 2 firm requires a more significant latent size of price change to trigger actual price changes because the size of the interval of no price change, $C_\tau^1 - C_\tau^{-1}$, is larger for $\tau = 2$ than other types.
2.7.4 Goodness of Fit

We will use a series of figures to assess the goodness of fit of the estimated policy function \( \hat{\sigma} \), in particular whether it fits well enough to be suitable for the later structural estimation. As discussed in the next section, after substituting the specified profit function as well as estimates of the structural parameters, firm \( i \)'s value function \( \hat{V}_i(\hat{\sigma}, \theta) \) reduces to a linear function of just a few aggregates, in particular, the total hours firm \( i \) spends at each rank and the total times it changes its price. Hence we will evaluate goodness of fit by comparing aggregates simulated from the policy function to their values in the actual data.

To provide some technical details on how the simulations were run, we start with a given firm type and consider all the observations at the \( it \) level for that type. For each \( it \), we construct 20 simulated forward histories lasting 720 hours. The simulations use the actual market data for state variables where possible (e.g., for cost histories), simulating firm behavior by substituting current state variables as well as random draws for error terms \( e_{it} \) and \( u_{it} \) into equations (2.6) and (2.8) of the policy function. Averaging the aggregate over all the simulations for all \( it \) observations of that firm type produces a value that can be compared to the average in the actual data of 720-hour forward histories starting from each candidate observation. To be precise, the aggregates are discounted rather than simple sums, using the same annual discount factor of 0.95 used in the value functions in the structural estimation. Over the 720-hour horizon, discounting has a negligible effect on the results.

Panel A of Figure 2.6 compares simulated time spent at each rank (solid black curves) to actual (dashed grey curves) for the three firm types. The actual curves have quite different levels and shapes across firm types, yet the simulated curves are able to fit each quite well. Panel B compares the simulated number of price changes from the estimated policy function (the black squares) to the averages in the actual data (the grey circles). Again, the fit is quite close, with the markers essentially overlapping and moving together across the types of firm: moderate for type 1, low for type 2, and high for type 3. The close fit is not surprising given the policy function is a reduced form that was estimated in part to maximize the likelihood of the observed frequency of price changes. However, the close fit was not guaranteed. The maximum likelihood estimation targeted size as well as number of price changes; we see that the joint estimation does not harm the fit for number alone. More importantly, the policy functions
were estimated to fit individual behavior; letting their interaction on the market play out over a length of time could generate feedback causing behavior to diverge from actual outcomes. We see in Panel A that this is not the case. Taken together, the results from Figure 2.6 suggest that the estimated policy function will provide good estimates of the sums that are the essential inputs into the value functions in the structural estimation.

Figure 2.7 provides a more refined assessment of goodness of fit. The policy function should not only be able to fit the forward history on average across states but also fit the forward history in any state \( s \) in which the firm finds itself. The figure compares simulated to actual profit conditional on various states including various initial ranks in Panel A and margins in Panel B. (To save space, we just show the results for type 1 firms; the fit for types 2 and 3 is similar.) We focus on profit in this figure as opposed to time spent at each rank in the previous figure because profit is a convenient summary statistic for the distribution of times spent at each rank, allowing us to reduce the dimensionality of the graph. While we do not yet have all the components of profit \( \pi_{it} \) from equation (2.3)—the managerial-cost components are of course yet to be structurally estimated—we can estimate the first two components \( Base_{it} \) and \( Upsell_{it} \) using equations (2.4) and (2.5). Because monetary profits are not observed even in the actual data, they have to be estimated in both cases—using simulated prices in the former case and actual prices in the latter. Because the figure displays the results separately for each initial state \( s \), each average is now taken only over initial observations at the \( it \) level that qualify for state \( s \).

Panel A compares the simulated to actual monetary profit going forward for 720 hours conditional on rank in the initial period. The solid black curve for profit based on simulated prices closely matches the dashed grey one for profit based on actual prices over the whole range of the horizontal axis. Of course the closeness of the graphs could be a symptom, not of good fit, but of the stability of the environment, with initial rank correlating highly with profits at least over an horizon as short as 720 hours. To investigate this possibility, we have added a curve (lighter with dot markers) representing the naïve forecast that the firm earns the same profit in each of the 720 hours as it does in the first. This naïve forecast ends up overestimating profit conditional on top ranks, because it has not taken into account the other firms’ reaction to firm \( i \)’s top rank, which is to undercut firm \( i \). For similar reasons it ends up underestimating profit.
conditional on bottom ranks. Our estimated policy function fits dramatically better for both high and low ranks, suggesting that our policy function likely captures this dynamic correctly.

Panel B similarly compares simulated, actual, and naïve estimates of monetary profit conditional on a type 1 firm's margin (price minus wholesale cost in levels) in the initial state \( s \). Again our estimated policy simulation fares well, while the naïve prediction underestimates profit conditional on negative margins and overestimates profit conditional on positive margins because does not properly incorporate the firm’s future price adjustments and the cascade of rival responses. This comparison suggest that our policy function has likely captured this dynamic correctly.

2.8 Estimation of Structural Parameters

2.8.1 Identification

For tractability and to better suit our empirical setting, our model will depart in several ways from the standard BBL framework. We touch on the departures here but discuss them in more detail below in the relevant sections. First, either because of limited information, cognition, or memory, the manager is assumed to only be able to consider a restricted set \( \hat{S} \) of the overall state space. For example, rather than keeping track of the whole price history for each rival, manager \( i \) may view his current rank as an adequate summary statistic of the combined effect of these actions. Second, the manager is assumed to follow a rule-of-thumb policy that is a function of the restricted state space \( \hat{S} \). He does optimize, not over the price each moment, but over the long-run choice of this policy, maximizing the present discounted value of profits over the set of admissible policies \( PF \). Given the overwhelming complexity of the period-by-period optimization problem involving the full state space, it is unlikely that managers were fully neoclassical—all the more in our setting in which we have qualitatively documented substantial managerial frictions, which we aim to quantify. Our approach has the advantage of being able to accommodate behavioral managers. The disadvantage of model is that it will be misspecified if managers are more neoclassical or behavioral in a different way than we allow. It will thus be essential for us to specify a class of policy functions that can closely match observed firm behavior. We will devote careful attention to this specification and to gauging its resulting
goodness of fit.

We follow the broad outlines of the BBL approach to estimate our structural parameters with a few modifications. While BBL assume the econometrician observes the game play of a Markov perfect Nash equilibrium, we assume a weaker solution concept such that the observed game play is a Nash equilibrium in which each player chooses a rule-of-thumb policy function based on a limited set of state variables to maximize the present discounted value of profits. The equilibrium policy function is that estimated in the previous section. We then calculate the simulated counterpart of the value function in equation (2.1), which involves approximating the distribution of the states considered by the firm. Assuming firms have consistent beliefs about game play—the premise of the BBL method—then a natural approximation of the distribution of states in firms' consideration sets is the empirical distribution observed in the data, denoted $\hat{S}$. This approximation is also consistent with our policy-function estimation, which matches firms' behavior to the same distribution of states.

In the final step, we search for structural parameters $\theta$—which here is the vector of managerial costs $\mu_\tau$ and $\chi_\tau$—that rationalize the estimated policy function as the equilibrium one, i.e., the one among the class of admissible policy functions maximizing the value function. Let $PF(\alpha_\tau, \beta_\tau, C^k_\tau)$ denote the admissible class of policy functions, comprised by substituting alternative values for the estimated parameters in the monitoring and price-changing equations. Our identification condition is that no deviation in $PF(\alpha_\tau, \beta_\tau, C^k_\tau)$ can dominate the estimated policy function $\hat{\sigma}_i$. Formally,

$$E_{s \in \hat{S}}[V_i(s; \hat{\sigma}_i, \theta)] \geq E_{s \in \hat{S}}[V_i(s; \bar{\sigma}_i, \bar{\sigma}_{-i}, \theta)] \quad \text{for all } \bar{\sigma}_i \in PF(\alpha_\tau, \beta_\tau, C^k_\tau).$$  \hspace{1cm} (2.14)

We estimate $\theta$ by finding values that satisfy (2.14) for a large number of deviations $\bar{\sigma}_i$. The details of our specific estimator and our choice of deviations are provided in the appendix.

The intuition behind how $\theta$ is identified is straightforward. Our estimate of the equilibrium policy function $\hat{\sigma}_i$ will entail some rates of monitoring and price changing. Holding the benefits of these actions constant, their rates should be inversely related to their costs, $\mu_\tau$ and $\chi_\tau$ respectively. For example, a high equilibrium monitoring rate entailed by $\hat{\sigma}_i$ must imply a low $\mu_\tau$. If instead $\mu_\tau$ were extremely high, the set of deviations $PF(\alpha_\tau, \beta_\tau, C^k_\tau)$ is rich enough
that one could be found involving a lower rate of monitoring, reducing the number of times
\( \mu_r \) is subtracted from the profit stream, which for a given benefit of monitoring would increase
the value function. Similarly, a high equilibrium rate of price change (conditional on the rate
of monitoring) can only be consistent with a low \( \chi_r \). What may be difficult in practice is
distinguishing in the data an equilibrium with frequent monitoring but inert price changing from
one with infrequent monitoring but hair-trigger price changing. But that is a difficulty with
the identification of the policy function—discussed already in Section 2.7.1—not the structural
parameters. A well-identified policy function will deliver distinct rates of monitoring and price
changing that can be used to identify \( \mu_r \) and \( \chi_r \). If the policy function is not well identified,
then different bootstrapped samples will lead to large swings in the rates of monitoring and
price changing, leading to large bootstrapped standard errors on the structural parameters.
Thus the standard errors on the structural parameters will serve as a natural check on the
identification of the policy function.

To identify the structural parameters, it is crucial that the richness of the set of deviations
\( PF(\alpha_\tau, \beta_\tau, C_\tau^k) \) be exploited. Not all deviations give useful information. For example, a de-
viation generating negative margins most of the time would be dominated by the equilibrium
strategy for any managerial-cost parameters. As discussed further in the next subsection, we
will sample from distributions of deviations that have a realistic chance of being profitable to
maximize the power of identification assumption (2.14).

The restriction of deviations to \( \hat{\sigma}_t(s_t) \in PF(\alpha_\tau, \beta_\tau, C_\tau^k) \) is where our procedure accommodates behavioral managers, who pursue rule-of-thumb strategies short of fully rational strategies. A fully rational strategy would have to dominate all conceivable deviations including, for example, a one-time price increase of $1 in any given hour, a deviation which is not in \( PF(\alpha_\tau, \beta_\tau, C_\tau^k) \). Given the difficulty of solving for the fully rational strategies in our setting, we are reluctant to examine deviations that only "work" (i.e., generate inequalities in the correct direction) if firms are fully rational. The restricted set of deviations we consider reflects the idea that firms experiment among a simpler class of pricing rules to discover the most profitable of them. By restricting the set of deviations, our identification assumption is weaker than the standard assumption in BBL, allowing for estimation of structural parameters that is robust to certain forms of behavioral pricing. The only potential pitfall for structural identification
would be if the deviation set were not rich enough to span possible combinations of monitoring and price-changing rates. Our model avoids this pitfall, however: fixing all the other parameters, varying the constant terms in \( \alpha_r \) and \( \beta_r \) over \((-\infty, \infty)^2\) can produce any combination of monitoring and price-changing rates in \((0,1)^2\). The true drawback to the restriction \( \delta_i \in PF(\alpha_r, \beta_r, C^k) \) lies in the possibility of misspecifying equilibrium strategies, hence the importance of establishing good fit of the policy function in Section 2.7.4.

Additional technical details on the estimation of the structural parameters have been relegated to the appendix, allowing us to turn directly to the results.

### 2.8.2 Results

Table 2.4 presents the estimates of the structural parameters from the profit equation (2.3). Recall that we rely on estimates of \( Base_{it} \) and \( Upsell_{it} \) from previous research, leaving managerial costs \( \mu_r \) and \( \chi_r \) as the only structural parameters to be estimated. We measure \( \mu_r \) and \( \chi_r \) in dollars.

Type 1 firms are estimated to have a substantial monitoring cost of around $72, significantly different from 0 at better than the 5% level. While at first glance this estimate may seem high, further consideration suggests it is plausible. The monitoring cost covers a series of managerial activities including reviewing recent sales and inputs, acquiring competition status, and integrating information from several sources. If these activities occupy about an hour of the manager’s time, the monitoring cost should be comparable to the manager’s hourly pay.

The estimated cost of changing price for a type 1 firm is in fact negative, \(-6.3\), although insignificantly different from 0. Using the 95% confidence interval, we can reject that costs of changing price exceed $4.1 at better than the 5% level. The estimates suggest that managers find it costly to continually attend to the market but once they do attend, the menu costs of electronically updating the price are fairly trivial. This estimate confirms the notion that the advent of e-commerce can practically eliminate physical menu costs. The stark contrast between the two types of managerial costs are in align with a series of recent macro papers. For example, Alvarez, Lippi and Paciello (2015) finds the cost of reviewing information is three times as large as the cost of changing price, and Zbaracki, Ritson, Levy, Dutta and Bergen (2004) finds the managerial cost behind the decision of price change is about six times as large.
as the physical cost of the actual price change.

Turning to type 2 firms, the estimate of the monitoring cost at $48 is a bit lower though qualitatively similar to that for type 1. The 42.4 estimate for the cost of changing price is substantially higher than that for type 1’s. However, the remarkably wide confidence interval $[-177.2, 145.8]$ around the estimate indicates that the cost estimate is essentially uninformative. The small number (eight) of type 2 firms and the infrequency of their price changes combine to generate few price-change observations that go into identifying the policy function. This shows up in standard errors for the policy-function parameters in Table 2.3 that are around five times higher for type 2 than type 1 firms, leading to the wide confidence interval seen here. The clustering procedure that divided firms up into types allowed us to keep heterogeneous type 2 firms from contaminating the estimates for other types but did not provide enough data to allow credible estimation of that type’s managerial costs.

The estimates for type 3 firms are almost identical to those for type 1. They also have a substantial monitoring cost of around $68 and negative but insignificant cost of changing price of $-7.4$. One might have thought the differences between type 1 and type 3 firms stem from differences in managerial capacity. Perhaps type 1 managers occupy the low ranks because they can cheaply monitor and respond to this active segment of the market. In fact, while type 1 and type 3 firms occupy different ranks, they do not differ much in their price-changing behavior, as Figure 2.4 showed. This similarity naturally translates into similar managerial costs. One possible story is that similar managers are indifferent among various rank positions because high margins and upselling profits (per consumer) at high ranks balance low sales. The indifferent managers are content then to spread and fill out the rank space.

Our preferred estimates given in the bottom of Table 2.4 re-estimate the structural parameters after imposing a non-negativity constraint on costs. This is a compelling theoretical restriction because it is hard to imagine monitoring or inputting prices providing a utility boost. Perhaps more importantly, the assumption plays an essential role in separately identifying the components of managerial costs in certain cases, in particular when the policy function happens to generate a price change for almost all monitoring episodes. To understand this essential role, consider the limiting case in which the manager changes price every time he monitors. Then $\mu_r = 100$ and $\chi_r = 0$ would produce the same net profit as $\mu_r = 1,100$ and $\chi_r = -1,000$, as
indeed would every linear combination of $\mu_r$ and $\chi_r$ on the line including these points. The structural parameters would be unstable and huge positive values for $\mu_r$ and huge negative values for $\chi_r$ could result. Imposing the constraint $\mu_r, \chi_r \geq 0$ eliminates this instability and selects plausible values of the structural parameters.\footnote{Although our estimated policy functions happen to generate substantially more monitoring episodes than price changes, for some bootstrapped samples, the estimated policy functions generate similar numbers of monitors and price changes, leading to the problem of wildly positive values of $\mu_r$ and negative values of $\chi_r$. To address this problem in the unconstrained estimation, for a valid bootstrap draw, we required the number of monitors to exceed the number of price changes by at least 5%. The percentage of valid bootstraps is listed in Table 2.4. A side benefit of imposing the non-negativity constraint is that it eliminates the problem of invalid bootstraps, as indicated by 100% of the bootstrapped sample being valid, as stated at the bottom of the table.} The non-negativity constraint binds for type 1 and 3’s cost of changing price. That estimate becomes precisely 0, while the cost of monitoring falls slightly for them and its confidence interval tightens. The estimates for type 2 remain unchanged although the 95% confidence interval around that type’s monitoring cost now includes 0 and ranges up to 210.9, reinforcing the impression that the structural estimates for that type are simply uninformative.

2.8.3 Sensitivity

This section gauges the sensitivity of the results to several specification choices made in constructing the profit and value functions. Regarding monetary profits, we adopted a particularly simple specification in equation (2.3), the markup of price over unit cost times quantity, measuring unit cost as the wholesale acquisition cost of a memory module using our daily data. In practice, firms may have had other unit costs. Credit-card purchases involve a transaction fee, typically 2.25%. There is also a small chance of loss or breakage of the wholesale product. Offsetting these extra costs, firms gained extra revenue through the approximately $3 difference between actual shipping costs and the $9.99 they commonly charged the consumers. Our presumption is that these extra costs and revenues largely cancel each other out, leaving equation (2.3) as an accurate representation of monetary profits. Table 2.5 shows how the structural parameters change with alternative specifications of monetary profits.

The first row for both the cost of monitoring and price change repeats the baseline estimates from the previous table for comparison. The structural costs presented have all been estimated imposing the non-negativity constraint. Following the baseline estimates, the next
row recomputes the structural estimates after adding $2 to unit cost. In general, higher unit costs lead the estimates of managerial costs to fall. Intuitively, the higher are unit costs, the lower are unit profits; lower managerial costs are required to justify the observed frequency of managerial activity. Qualitatively, though, the results do not change much, as the monitoring cost still dominates the cost of price changing for type 1 and type 3 firms. Indeed, the cost of price changing continues to hit the zero lower bound for type 1 and type 3 firms but it remains positive for type 2 firms.

Our specification of monetary profits used the function in Figure 2.3B for upselling profits. The next row for each managerial cost re-estimates the structural parameter cutting the assumed upselling profits in half for that type. Again, as expected, this leads to lower estimates for managerial costs. For example, monitoring costs for type 1 firms falls from $68.4 to $26.0.

While this is a large quantitative change, still the qualitative picture remains that for type 1 and type 3 firms, the monitoring cost is substantial while the cost of price changing is negligible. Notably, after this reduction in upselling profits, type 2 firms start to look much like the others. The cost of price changing hits the zero lower bound for them, too. Therefore, we cannot be sure whether the inactivity of type 2 firms is explained by a high cost of changing price or a lack of add-on profits. Some anecdotal evidence for the latter interpretation comes from examining firms' product webpages. While it is quite common for type 1 and type 3 firms to have complex webpages emphasizing available upgrades, most type 2 firms have very simple webpage designs offering little information on premium options.

The last row re-estimates the structural parameters changing the profit specified for firms that disappear from our sample of the first two Pricewatch pages because they rise to rank 25 or higher. As mentioned in footnote 6, this was set to $4.70, but here we cut it in half. This causes very little change in the estimated managerial cost for type 1 firms because they spend very little time at high ranks. For types-2 and 3, estimated monitoring costs rise to offset the higher benefit from adjusting price to avoid falling off the list. For all three types, the qualitative results are insensitive to this change in specification.
2.9 Counterfactuals

To assess the importance of managerial costs for the firm's pricing behavior, in this section we present counterfactual exercises in which we shock a single firm $i$'s managerial costs and see how its pricing behavior and profit change, leaving other firms the same as before. In effect, this section performs the inverse exercise from the structural estimation in the previous section. In the structural estimation, we estimated a policy function from actual pricing behavior and used this to infer firms' managerial costs. Here, we posit a vector of managerial costs for $i$ and search over policy functions for the one maximizing its simulated profits. For each candidate policy function, we want to simulate $i$'s new pricing behavior and profit assuming that all other firms continue with their originally estimated managerial costs, equivalent to assuming that their pricing behavior is given by the originally estimated policy functions. This is exactly the simulation we did for the 1,800 deviations during the structural estimation, so we can treat these deviations as candidate policy functions, and use the recorded simulations to recalculate simulated profits with the new managerial costs for these deviations, and pick the one with the highest profits as the firm's optimal policy at the new managerial costs. In fact, for this purpose, we did a much finer search by simulating for additional several thousands candidate policy functions that are drawn sequentially from the proximity of the existing promising ones. To save space we will focus on the case in which $i$ is a type 1 firm. The results are shown in Figure 2.8.

Panel A shows the monthly number of prices changes for the firm as the costs of monitoring and price change vary from 0 to 100 dollars. With a large yet finite pool of high quality candidate policy functions, we are able to identify 31 policy functions as optimal for some given vectors of managerial costs. The surface is therefore marked by 31 plateaus, which indicate regions of costs for which we identified the same strategy as optimal. Even as costs of monitoring and price changing go to 0, the frequency of price change does not grow without bound. In other words, in the complete absence of frictions associated with price change, it is not optimal for a firm to continuously tweak its price. There are three reasons for this. First, other firms have retained their positive costs, so they do not respond continuously. This results in stretches when state variables do not change during which $i$'s optimal strategy is to keep price constant. Second, changing price, especially downwards, may trigger other firms' reaction and intensity
future competition, so there is a dynamic incentive to wait for a while between price changes. Third, prices in this market are posted in whole dollar amounts, so even if a firm continuously monitored its optimal continuous price, it still would not want to change the price until the optimal price exceeded the threshold necessary to move the price a whole dollar up or down. When $i$ has no costs of monitoring or price changing, it ends up changing price around once a day.

Panel B explores the same counterfactual exercise but now focuses on a different outcome variable: $i$'s monthly profits. These are the net profits from equation (2.3), which subtract off the new managerial costs with which we are shocking firm $i$, accumulated over the month. While the surface in the previous panel had discrete jumps reflecting the discrete changes to firm $i$'s pricing policy, the continuous changes in $i$'s costs smooth out its profit function in this panel. We will highlight several key insights that can be drawn from the graph. The graph shows that the division between the two types of managerial costs matters for profit. At the estimated costs of $68.4$ for monitoring and $0$ for price changing, the firm's net profit is $6,070$. Holding the total managerial cost of a joint episode of monitoring and price changing constant at $68.4$ but shifting more of the joint cost from monitoring to price changing, the firm's monthly profits monotonically increase from $6,070$ to $6,496$. In principle, simple arithmetic could explain this gain: holding constant the firm's monitoring and price-changing episodes, it gains from shifting more of the joint cost onto price changing because it monitors multiple times for each price change, so managerial costs would be lower with free monitoring and $68.4$ price changing rather than vice versa. In fact the gain here is due to a more subtle change in the firm's strategy. When monitoring is expensive, the firm ends up forgoing some prime opportunities to change price and changing price is some other less than prime circumstances knowing that it would be too expensive to delay and keep tabs on the market. As monitoring becomes cheaper, the firm can keep almost continual tabs on the market and change price in exactly the right states.

At the estimated costs of $68.4$ for monitoring and $0$ for price changing, the firm's optimal strategy leads it to monitor an average of 10.7 times per month, generating managerial costs of $68.4 \times 10.7 = 731.9$. If the costs are shifted to $0$ for monitoring and $68.4$ for price changing, the firm's optimal policy now leads it to continually monitor and change price 12.1 times a month, generating managerial costs of $68.4 \times 12.1 = 827.6$. Aggregate managerial costs are thus
higher with more weight shifted from monitoring to price changing. So it is not an arithmetic reduction in cost that leads to the counterfactual profit increase. Rather this increase in profit comes from improvements in the firm's pricing policy, possible when the managerial activity with additional option value—monitoring—becomes cheaper even as the managerial activity without option value—price changing—becomes more expensive.

Another key insight from Panel B, which can be drawn from considering the height of the surface, is the potentially large gain to adopting technologies that decrease managerial costs, potentially thousands of dollars a month just for this one product. In particular, based on numbers in this graph, the monthly net profits would increase by over $1,500 if the managerial costs decreased from the estimated levels to zero. One would underestimate the profits improvement assuming the firm simply saves the managerial cost, which is about a half of $1,500, without showing a policy response. Re-optimizing the policy function to a suitably active one contributes the other half of the profit improvement. The gain would presumably be multiplied if the technology could be used to reduce managerial costs for the scores of other products the retailers marketed on Pricewatch. In fact the retailers did move to automated pricing soon after the time period of our data, consistent with our estimates of potentially large gains from doing so.

2.10 Conclusion

In this paper, we studied firms' price-changing behavior in an online market for computer components. Special features of this market made it particularly suitable for study: firms were ranked according to price with lower-price firms receiving more prominent listings and the bulk of the sales on the market; this ranking system, coupled with rapidly changing market conditions, gave firms an incentive to change price frequently as they jockeyed for position. The abundance of price-changing episodes over the year of high-frequency (hourly) observations offers an opportunity to precisely estimate a structural model of price-changing behavior.

We used some initial reduced-form evidence to direct the structural modeling. While firms were free to change price continuously, as frequent as the price changes were, they were far from continuous. Despite competing in a nationally integrated market (at least to some extent),
managers on the East and West Coast changed prices at times during the day suiting their shifted schedules, and managers in all locations rarely changed prices on weekends. We took this as reduced-form evidence that the costs of managerial activity provided a source of pricing inertia, motivating a structural model allowing us to separately estimate both the managerial cost of monitoring the market and the managerial cost of entering the new price (a pure menu cost). We were also careful to incorporate firm heterogeneity in the structural model, based on visual examination of price and rank paths for some representative firms showing systematic differences in their strategies, with some firms changing price at least weekly while others only once or twice a month and with some firms aggressively targeting prominent ranks while others higher ranks with lower sales. We incorporated firm heterogeneity in the structural estimates by using a machine-learning method to cluster sample firms method into three types and then estimating policy functions and structural parameters allowed to freely vary across types.

Given the emphasis on frictions in managerial behavior, and the prohibitive complexity of the state space in our setting, we built a dynamic model of a boundedly rational manager. The manager changes price according to a rule of thumb based on a subset of state variables attended to rather than making the price change that would be optimal given the full state space each instant. The key step for our approach to work is to estimate a rule of thumb—in the language of dynamic structural estimation, a policy function for price changes—that accurately captures managerial behavior. We do this by allowing for a sufficiently rich set of carefully selected state variables, further enriched by allowing for different policy functions across heterogeneous firm types, and then determining that the policy function performs well across a suite of goodness-of-fit exercises. Following BBL, we estimate the structural parameters—here, the managerial costs of monitoring and changing price—as those rationalizing the estimated policy function as being more profitable than deviations. The new feature added to accommodate bounded rationality is that deviations are restricted to the class of admissible rules of thumb rather than any arbitrary price change.

For the types of firms for which we have reliable estimates (types 1 and 3), we estimate a cost of monitoring of roughly $60 and essentially no cost of price changing. (The estimates for type 2 firms are considerably noisy because there are relatively few of these firms and they changed price infrequently, leaving few price-changing episodes to use to estimate the model.)
While these managerial-cost estimates can be moved around by introducing or removing factors in the profit function, the qualitative results remain. Managerial costs primarily arise in the monitoring stage; the further cost of inputting price changes is fairly trivial, suggestive of technological features of this e-commerce market.

This paper fills several gaps in the economics literature. First, we extend methods in the literature on dynamic structural estimation to accommodate behavioral agents who may behave according to rules of thumb based on a subset of state variables. This is an important extension in our setting, where our central focus is on limits to managerial capacity, but may be realistic in other settings as well. Second, we contribute to the empirical macroeconomics literature on retail price stickiness. Like that literature, we suggest that the costs of managerial attention and activity may be important. Our novel contribution is a carefully specified dynamic model of pricing behavior that can generate actual structural estimates of these costs. Finally, we contribute to the behavioral-economics literature by, first, providing a framework for structural estimation in the presence of behavioral agents and, second, by providing numerical estimates of the cost of managerial activity, providing an insight into the psychological barriers of actions such as changing prices that are typically regarded as automatic in neoclassical models.
References


Table 2.1: Variable Definitions and Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Firm-level variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>Measure of density in price space of firms with nearby ranks</td>
<td>0.60</td>
<td>0.40</td>
<td>0</td>
<td>3</td>
<td>111.276</td>
</tr>
<tr>
<td>Margin</td>
<td>Percentage markup over wholesale cost, $100(Price - Cost)/Cost</td>
<td>1.01</td>
<td>5.65</td>
<td>-20.50</td>
<td>20.38</td>
<td>111.276</td>
</tr>
<tr>
<td>NumBump</td>
<td>Net number of ranks bumped since last price change</td>
<td>1.30</td>
<td>3.40</td>
<td>-22</td>
<td>21</td>
<td>111.276</td>
</tr>
<tr>
<td>Placement</td>
<td>Placement between adjacent firms in price space</td>
<td>0.58</td>
<td>0.42</td>
<td>0</td>
<td>1</td>
<td>111.276</td>
</tr>
<tr>
<td>Price</td>
<td>Current listed price in dollars</td>
<td>70.1</td>
<td>34.6</td>
<td>25.0</td>
<td>131.0</td>
<td>111.276</td>
</tr>
<tr>
<td>QuantityBump</td>
<td>Relative change in hourly sales resulting from rank bump</td>
<td>-0.16</td>
<td>0.36</td>
<td>-2.08</td>
<td>2.48</td>
<td>111.276</td>
</tr>
<tr>
<td>Rank</td>
<td>Rank of listing in price-sorted order</td>
<td>10.75</td>
<td>6.77</td>
<td>1</td>
<td>24</td>
<td>111.276</td>
</tr>
<tr>
<td>RankOne</td>
<td>Indicates whether firm is at rank 1</td>
<td>0.06</td>
<td>0.24</td>
<td>0</td>
<td>1</td>
<td>111.276</td>
</tr>
<tr>
<td>SinceChange</td>
<td>Hours since firm last changed price</td>
<td>117.55</td>
<td>146.45</td>
<td>1</td>
<td>1,113</td>
<td>111.276</td>
</tr>
<tr>
<td><strong>Market-level variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td>Wholesale cost</td>
<td>66.40</td>
<td>36.85</td>
<td>23</td>
<td>129</td>
<td>7,740</td>
</tr>
<tr>
<td>CostTrend</td>
<td>Trend in Cost over previous two weeks</td>
<td>-0.19</td>
<td>0.71</td>
<td>-2.06</td>
<td>1.53</td>
<td>7,740</td>
</tr>
<tr>
<td>CostVol</td>
<td>Volatility of Cost over previous two weeks</td>
<td>1.64</td>
<td>1.08</td>
<td>0.00</td>
<td>4.36</td>
<td>7,740</td>
</tr>
<tr>
<td>Night</td>
<td>Indicates hour from midnight to 8 a.m. EST</td>
<td>0.33</td>
<td>0.47</td>
<td>0</td>
<td>1</td>
<td>7,740</td>
</tr>
<tr>
<td>Weekend</td>
<td>Indicates Saturday or Sunday</td>
<td>0.29</td>
<td>0.45</td>
<td>0</td>
<td>1</td>
<td>7,740</td>
</tr>
</tbody>
</table>

Notes: Firm-level variables vary over indexes $i$ and $t$. Market-level variables vary over $t$. For these variables, descriptive statistics are for the time series of one observation per period.
### Table 2.2: Variable Means by Firm Type

<table>
<thead>
<tr>
<th>Variable</th>
<th>Combined types (43 firms)</th>
<th>Type 1 (22 firms)</th>
<th>Type 2 (8 firms)</th>
<th>Type 3 (13 firms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>0.60</td>
<td>0.63</td>
<td>0.52</td>
<td>0.56</td>
</tr>
<tr>
<td>Margin</td>
<td>1.01</td>
<td>-0.72</td>
<td>1.24</td>
<td>6.20</td>
</tr>
<tr>
<td>NumBump</td>
<td>1.30</td>
<td>1.00</td>
<td>2.12</td>
<td>1.64</td>
</tr>
<tr>
<td>Placement</td>
<td>0.58</td>
<td>0.58</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>Price</td>
<td>70.06</td>
<td>68.09</td>
<td>85.42</td>
<td>64.90</td>
</tr>
<tr>
<td>QuantityBump</td>
<td>-0.16</td>
<td>-0.17</td>
<td>-0.18</td>
<td>-0.11</td>
</tr>
<tr>
<td>Rank</td>
<td>10.75</td>
<td>7.78</td>
<td>13.37</td>
<td>18.06</td>
</tr>
<tr>
<td>RankOne</td>
<td>0.06</td>
<td>0.09</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>SinceChange</td>
<td>117.55</td>
<td>99.88</td>
<td>255.08</td>
<td>93.33</td>
</tr>
<tr>
<td>Observations</td>
<td>111,276</td>
<td>71,460</td>
<td>16,904</td>
<td>22,912</td>
</tr>
</tbody>
</table>

**Notes:** Shown are means only of variables varying over indexes $i$ and $t$. Column for combined firms repeats information from Table 2.1 for comparison.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Type 1 firms</th>
<th></th>
<th>Type 2 firms</th>
<th></th>
<th>Type 3 firms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Std. err.</td>
<td>Coefficient</td>
<td>Std. err.</td>
<td>Coefficient</td>
<td>Std. err.</td>
</tr>
<tr>
<td><strong>Monitoring estimates α</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-2.25***</td>
<td>(0.10)</td>
<td>-1.71***</td>
<td>(0.54)</td>
<td>-2.83***</td>
<td>(0.25)</td>
</tr>
<tr>
<td>Night</td>
<td>-0.68***</td>
<td>(0.05)</td>
<td>-0.84***</td>
<td>(0.23)</td>
<td>-1.30***</td>
<td>(0.21)</td>
</tr>
<tr>
<td>Weekend</td>
<td>-0.47***</td>
<td>(0.04)</td>
<td>-0.61***</td>
<td>(0.20)</td>
<td>-0.71***</td>
<td>(0.11)</td>
</tr>
<tr>
<td>CostVol</td>
<td>0.04***</td>
<td>(0.01)</td>
<td>0.04</td>
<td>(0.06)</td>
<td>0.05*</td>
<td>(0.03)</td>
</tr>
<tr>
<td>CostTrend⁺</td>
<td>0.13***</td>
<td>(0.04)</td>
<td>0.33***</td>
<td>(0.15)</td>
<td>0.18</td>
<td>(0.12)</td>
</tr>
<tr>
<td></td>
<td>CostTrend⁻</td>
<td>0.09***</td>
<td>(0.03)</td>
<td>0.02</td>
<td>(0.21)</td>
<td>0.24***</td>
</tr>
<tr>
<td>QuantityBump⁺</td>
<td>0.39***</td>
<td>(0.08)</td>
<td>0.69***</td>
<td>(0.26)</td>
<td>0.13</td>
<td>(0.35)</td>
</tr>
<tr>
<td>QuantityBump⁻</td>
<td>0.36***</td>
<td>(0.05)</td>
<td>0.15</td>
<td>(0.26)</td>
<td>0.20</td>
<td>(0.19)</td>
</tr>
<tr>
<td>ln SinceChange</td>
<td>0.25***</td>
<td>(0.05)</td>
<td>-0.23</td>
<td>(0.17)</td>
<td>0.47***</td>
<td>(0.12)</td>
</tr>
<tr>
<td>(ln SinceChange)²</td>
<td>-0.05***</td>
<td>(0.01)</td>
<td>0.02</td>
<td>(0.02)</td>
<td>-0.07***</td>
<td>(0.02)</td>
</tr>
<tr>
<td><strong>Price change estimates β</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CostTrend</td>
<td>0.11***</td>
<td>(0.05)</td>
<td>0.25</td>
<td>(0.22)</td>
<td>-0.31***</td>
<td>(0.13)</td>
</tr>
<tr>
<td>CostChange</td>
<td>0.06***</td>
<td>(0.01)</td>
<td>0.06</td>
<td>(0.04)</td>
<td>0.09***</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Margin</td>
<td>-0.02**</td>
<td>(0.01)</td>
<td>0.04</td>
<td>(0.04)</td>
<td>-0.11***</td>
<td>(0.03)</td>
</tr>
<tr>
<td>NumBump</td>
<td>-0.05***</td>
<td>(0.02)</td>
<td>0.07</td>
<td>(0.05)</td>
<td>-0.05</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Density x NumBump</td>
<td>-0.10***</td>
<td>(0.04)</td>
<td>-0.28***</td>
<td>(0.08)</td>
<td>-0.06</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Placement</td>
<td>0.30***</td>
<td>(0.08)</td>
<td>0.19</td>
<td>(0.32)</td>
<td>0.07</td>
<td>(0.17)</td>
</tr>
<tr>
<td>Rank</td>
<td>-0.04***</td>
<td>(0.01)</td>
<td>-0.09**</td>
<td>(0.04)</td>
<td>-0.05**</td>
<td>(0.02)</td>
</tr>
<tr>
<td>RankOne</td>
<td>0.62***</td>
<td>(0.11)</td>
<td>a</td>
<td>a</td>
<td>a</td>
<td>a</td>
</tr>
<tr>
<td>Cutoff C⁻¹</td>
<td>-0.15</td>
<td>(0.14)</td>
<td>-1.80</td>
<td>(0.48)</td>
<td>-1.62</td>
<td>(0.37)</td>
</tr>
<tr>
<td>Cutoff C¹</td>
<td>0.76***</td>
<td>(0.14)</td>
<td>0.55</td>
<td>(0.87)</td>
<td>-0.44</td>
<td>(0.43)</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-6,174.2</td>
<td></td>
<td>-383.0</td>
<td></td>
<td>-1,337.9</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>71,460</td>
<td></td>
<td>16,904</td>
<td></td>
<td>22,912</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Coefficients from maximum likelihood estimation of equations (2.6) and (2.10) separately for each firm type. Heteroskedasticity-robust standard errors clustered by firm reported in parentheses. Because most of the observations with RankOne = 1 are in group 1, equations estimated for groups 2 and 3 constrain RankOne coefficient to be the same as estimated for group 1, 0.62. Model includes cutoffs Ck for k ∈ {-5, -4, -3, -2, -1, 1, 2, 3, 4, 5}; for space considerations we only report C⁻¹ and C¹. Statistically significant in a two-tailed test at the *10% level, **5% level, ***1% level.
Table 2.4: Structural Estimates of Cost Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type 1 firms</th>
<th></th>
<th>Type 2 firms</th>
<th></th>
<th>Type 3 firms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>95% c.i.</td>
<td>Estimate</td>
<td>95% c.i.</td>
<td>Estimate</td>
<td>95% c.i.</td>
</tr>
<tr>
<td>Without non-negativity constraint</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost of monitoring, ( \mu_r )</td>
<td>72.1***</td>
<td>[52.8, 104.2]</td>
<td>48.0***</td>
<td>[11.6, 283.2]</td>
<td>67.7***</td>
<td>[37.4, 133.0]</td>
</tr>
<tr>
<td>Cost of changing price, ( \chi_r )</td>
<td>-6.3</td>
<td>[-40.5, 4.1]</td>
<td>42.4</td>
<td>[-177.2, 145.8]</td>
<td>-7.4</td>
<td>[-67.4, 22.2]</td>
</tr>
<tr>
<td>Valid bootstraps</td>
<td>100%</td>
<td></td>
<td>92%</td>
<td></td>
<td>93%</td>
<td></td>
</tr>
<tr>
<td>Imposing non-negativity constraint</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost of monitoring, ( \mu_r )</td>
<td>68.4***</td>
<td>[50.1, 87.2]</td>
<td>48.0*</td>
<td>[0.0, 207.8]</td>
<td>63.0***</td>
<td>[37.5, 103.8]</td>
</tr>
<tr>
<td>Cost of changing price, ( \chi_r )</td>
<td>0.0</td>
<td>[0.0, 4.1]</td>
<td>42.4</td>
<td>[0.0, 145.8]</td>
<td>0.0</td>
<td>[0.0, 22.2]</td>
</tr>
<tr>
<td>Valid bootstraps</td>
<td>100%</td>
<td></td>
<td>100%</td>
<td></td>
<td>100%</td>
<td></td>
</tr>
</tbody>
</table>

Notes: 95% confidence intervals computed via bootstrapping using 200 runs. Bootstrap is taken to be valid if estimated policy function generates at least 5% more monitoring episodes than price changes. Unless a non-negativity constraint is imposed, structural parameters cannot be independently identified and estimates become unstable as the number of price changes converges to the number of monitoring episodes. Statistically significantly different from 0 in a two-tailed test at the *10% level, **5% level, ***1% level.
### Table 2.5: Sensitivity of the Structural Parameters to Adjustments in the Profit Function

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type 1 firms</th>
<th>Type 2 firms</th>
<th>Type 3 firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>95% c.i.</td>
<td>Estimate</td>
</tr>
<tr>
<td>Cost of monitoring, $\mu_r$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline estimates</td>
<td>68.4***</td>
<td>[50.1, 87.2]</td>
<td>48.0*</td>
</tr>
<tr>
<td>Adding $2$ to unit cost</td>
<td>48.1***</td>
<td>[33.9, 61.6]</td>
<td>41.7</td>
</tr>
<tr>
<td>Cutting upselling profits in half</td>
<td>26.0***</td>
<td>[15.3, 38.2]</td>
<td>22.7</td>
</tr>
<tr>
<td>Cutting rank 25+ profits in half</td>
<td>71.8***</td>
<td>[53.1, 91.4]</td>
<td>71.7***</td>
</tr>
<tr>
<td>Cost of changing price, $\chi_r$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline estimates</td>
<td>0.0</td>
<td>[0.0, 4.1]</td>
<td>42.4</td>
</tr>
<tr>
<td>Adding $2$ to unit cost</td>
<td>0.0</td>
<td>[0.0, 2.3]</td>
<td>10.1</td>
</tr>
<tr>
<td>Cutting upselling profits in half</td>
<td>0.0</td>
<td>[0.0, 0.6]</td>
<td>0.0</td>
</tr>
<tr>
<td>Cutting rank 25+ profits in half</td>
<td>0.0</td>
<td>[0.0, 3.1]</td>
<td>74.0</td>
</tr>
</tbody>
</table>

Notes: Baseline estimates are those from Table 2.4 imposing the non-negativity constraint. All robustness exercises also impose this constraint. Statistically significantly different from 0 in a two-tailed test at the *10% level, **5% level, ***1% level.
Figure 2.1: Example Pricewatch webpage

<table>
<thead>
<tr>
<th>BRAND</th>
<th>PRODUCT</th>
<th>DESCRIPTION</th>
<th>PRICE</th>
<th>NET</th>
<th>DATE DR.</th>
<th>DEALER PHONE</th>
<th>CST PARTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generic</td>
<td>PRICE FOR ONLINE ORDERS ONLY - 128MB PC100 SRAM DIMM 16x64 Gold leads</td>
<td>* LIMIT ONE - Easy installation in stock</td>
<td>$9.69 INSURED</td>
<td>10/12/00 12:40:35 AM CST</td>
<td>Computer Craft Inc. 800-487-4910</td>
<td>FL-MEM-128-DIMM-GT</td>
<td></td>
</tr>
<tr>
<td>Generic</td>
<td>ONLINE ORDERS ONLY - 128MB SRAM PC100 16x64 168pin</td>
<td>* LIMIT ONE</td>
<td>$69 INSURED</td>
<td>10/11/00 1:59:56 PM CST</td>
<td>Connect Computers 800-277-6287</td>
<td>CA-P9960021</td>
<td></td>
</tr>
<tr>
<td>Generic</td>
<td>PRICE FOR ONLINE ORDER - 128MB PC100 SRAM DIMM</td>
<td>* LIMIT ONE - In stock</td>
<td>$70</td>
<td>10/11/00 2:11:00 PM CST</td>
<td>1st Choice Memory 449-888-3010</td>
<td>CA-P9960021</td>
<td></td>
</tr>
<tr>
<td>Generic</td>
<td>PRICE FOR ONLINE ORDER - 128MB PC100 SRAM DIMM 16x64 - LIMIT ONE</td>
<td>* LIMIT ONE - In stock - with Lifetime Warranty</td>
<td>$72</td>
<td>10/10/00 11:30:39 AM CST</td>
<td>Memtop.com 877-918-5676</td>
<td>CA-P9960021</td>
<td></td>
</tr>
<tr>
<td>Generic</td>
<td>IN STOCK - 128MB PC100 3.3VOL Unbuffered SRAM Gold Lead 168 Pin, 7/8ns - with Lifetime warranty</td>
<td>* LIMIT ONE Not compatible with 512 Machine</td>
<td>$74</td>
<td>10/11/00 1:54:41 AM CST</td>
<td>PortaTech 800-487-1527</td>
<td>CA-P9960021</td>
<td></td>
</tr>
<tr>
<td>Generic</td>
<td>PRICE FOR ONLINE ORDERS ONLY - 128MB True PC100 SRAM DIMM - Gold leads - LIMIT ONE</td>
<td>* LIMIT ONE</td>
<td>$74</td>
<td>10/9/00 6:53:25 PM CST</td>
<td>1st Choice Memory 449-888-3010</td>
<td>CA-P9960021</td>
<td></td>
</tr>
<tr>
<td>Housebrand</td>
<td>128MB PC100 3.3v SRAM DIMM 16x64 - LIMIT ONE</td>
<td>* LIMIT ONE</td>
<td>$74</td>
<td>10/11/00 1:55:36 AM CST</td>
<td>1st Choice Memory 449-888-3010</td>
<td>CA-P9960021</td>
<td></td>
</tr>
<tr>
<td>Generic</td>
<td>128MB PC100 16x64 PC100 168pin, 7/8ns - with Lifetime warranty</td>
<td>* LIMIT ONE</td>
<td>$74</td>
<td>10/11/00 1:55:36 AM CST</td>
<td>1st Choice Memory 449-888-3010</td>
<td>CA-P9960021</td>
<td></td>
</tr>
<tr>
<td>Generic</td>
<td>128MB 168pin TRUE PC100 SRAM OBM 16x64</td>
<td>DIMM16x64 168pin - In stock - with Lifetime Warranty</td>
<td>$75</td>
<td>11/01/00 2:37:00 PM CST</td>
<td>Sunset Marketing, Inc. 800-350-9530</td>
<td>MD-P9960021</td>
<td></td>
</tr>
<tr>
<td>Generic</td>
<td>128MB PC100 16x64 PC100 168pin</td>
<td>* LIMIT ONE</td>
<td>$77</td>
<td>10/12/00 6:37:45 AM CST</td>
<td>PC-Coast 800-877-9442</td>
<td>IL-P9960021</td>
<td></td>
</tr>
<tr>
<td>Generic</td>
<td>IN STOCK, PC100, 128MB, 168pins DIMM NonECC - with Lifetime warranty</td>
<td>* LIMIT 5</td>
<td>$77</td>
<td>10/9/00 5:11:10 PM CST</td>
<td>Augustus Technology, Inc. 877-406-5381</td>
<td>CA-P9960021</td>
<td></td>
</tr>
<tr>
<td>Generic</td>
<td>128MB PC100 BNS 16x64 168pin SRAM - one year warranty</td>
<td>* LIMIT ONE</td>
<td>$78</td>
<td>10/11/00 5:16:36 PM CST</td>
<td>Computer Super Sale 800-305-4930</td>
<td>IL-P9960021</td>
<td></td>
</tr>
<tr>
<td>Generic</td>
<td>PRICE FOR ONLINE ORDERS ONLY - PC100 128MB Nonbuffered, NonECC 16x64 SRAM DIMM 3.3V - with Lifetime warranty</td>
<td>* LIMIT ONE - with Lifetime warranty</td>
<td>$78</td>
<td>10/13/00 6:20:59 PM CST</td>
<td>Jazz Technology USA, LLC 888-485-8827</td>
<td>CA-P9960021</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Page downloaded October 12, 2000.
Figure 2.2: Price and rank series for representative firms of each type

Notes: Series start later for type-2 and type-3 firms because they entered during the month.
Figure 2.3: Effect of rank on various retailer outcomes

Notes: Panel A is derived from Ellison and Ellison’s (2009a) demand estimates, based on their regression of the natural log of quantity on linear rank. Exponentiating yields the equation graphed, $Q(Rank_{it}) = 4.56(1 + Rank_{it})^{-1.29}$. Panel B is derived from Ellison and Ellison’s (2009a) estimates of upselling profit. They estimate that a retailer sells an additional $0.97(1 + Rank_{it})^{-0.77}$ units of a medium-quality product with an average markup of $15.69$ and an additional $0.49(1 +$
Figure 2.4: Distribution of size and spell of price changes by firm type

Panel A: Histograms showing distribution of size of price changes

Panel B: Kernel density estimates of distribution of price spells

Panel C: LOWESS estimates of magnitude of price change as function of spell

Notes: For readability, histograms in panel A omit bin for zero price change. When this bar is included, the proportions for each firm type sum to 1. Horizontal axis truncated at ±10 price changes. Densities in panel B estimated using Epanechnikov kernel. Cleveland’s (1979) locally weighted regression smoothing (LOWESS) estimated in panel C. A single outlying observation with a spell of over 500 dropped in the graph for type-3 firms. Panel B and C estimated in Stata 14 using default bandwidths.
Figure 2.5: Price-changing activity during the day for retailers on different coasts

Notes: Graphs show residual probability of price change each hour estimated after partialing out other covariates. So that the probability functions integrate to 1, they have been converted into conditional probabilities, i.e., conditional on a price change occurring during the day. Probabilities computed from estimates from a maximum likelihood model similar to that reported in Table 3 but with Night indicator replaced by suite of indicators for Eastern Time hour and interactions between this suite and an indicator for whether the supplier is located on the West Coast. Model estimated on subsample of East and West Coast suppliers only.
Figure 2.6: Goodness of fit of estimated policy function by firm type

Panel A: Fit to time spent at each rank

- **Type-1 firms**
- **Type-2 firms**
- **Type-3 firms**

- Simulations using estimated policy function
- Sample

Panel B: Fit to number of price changes

- **Type-1 firms**
- **Type-2 firms**
- **Type-3 firms**

- Simulations using estimated policy function
- Sample

Notes: Variables on vertical axis are discounted using same 0.95 annual factor used to compute value functions in the structural estimation. Over the short 720-hour horizon, discounting has a negligible effect on the graphs.
Figure 2.7: Goodness of fit of estimated policy function across various initial states

Panel A: State = initial rank

Panel B: State = initial margin

Notes: To save space, graphs show results only for type-1 firms. Monetary profit involves just the first two components of $\pi_t, Base_{it} + Upsell_{it}$, not the managerial costs structurally estimated later. Monetary profit for the actual sample is also estimated but estimated based on firms' actual prices as opposed to the simulations, which use prices from the estimated policy function. Naïve forecast assumes firm earns same profit in each of the 720 hours as in the first. Profits discounted using same 0.95 annual factor used to compute value functions in the structural estimation. Over the short 720-hour horizon, discounting has a negligible effect on the graphs.
Figure 2.8: Counterfactual scenarios with different managerial costs

Panel A: Number of price changes

Panel B: Profit net of managerial costs

Notes: Counterfactuals show firm's response to a shock to its managerial costs. Dots show estimated managerial costs. To save space, graphs show results only for type-1 firms. Net profit subtracts managerial costs from π. Profits discounted using same 0.95 annual factor used to compute value functions in the structural estimation. Over the short 720-hour horizon, discounting has a negligible effect on the graphs.
2.11 Appendix

This appendix provides additional technical details on the estimation of the structural parameters omitted from the text.

To transform our identification condition (2.14) into an estimator of $\theta$ requires empirical analogs to the expectations over value functions appearing there, which we will compute via simulation. Our empirical analogue to the first expectation, $E_{s \in \hat{S}}[V_i(s; \hat{\sigma}, \theta)]$, is

$$\hat{E}V_i(\hat{\sigma}, \theta) = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} \min\{T, 720\} \sum_{t_{mn}=0}^{T} \delta^{t_{mn}} \pi(s_{t_{mn}}, \hat{\sigma}(s_{t_{mn}}), \theta).$$

There are three summations in (2.15). The first sum simulates the expectation over initial states represented by the $E_{s \in \hat{S}}$ operator. We do this by taking $M$ draws from the set of state vectors observed in the data, $\hat{S}$, and averaging the result (hence the division by $M$). The second sum simulates the expectation implicit in the value function $V_i$; this expectation is over the distribution of all possible histories of game play and private shocks starting from the given initial state. We compute this expectation by simulating $N$ histories for each of the $M$ initial states and averaging the resulting value functions (hence the division by $N$). The third sum adds up the profit stream implicit in the value function.

The other expectation, $E_{s \in \hat{S}}[V_i(s; \hat{\sigma}_i, \hat{\sigma}_{-i}, \theta)]$ from the identification condition (2.14) is similarly transformed into its empirical analogue $\hat{E}V_i(s; \hat{\sigma}_i, \hat{\sigma}_{-i}, \theta)$. Instead of all firms behaving according to the estimated policy function $\hat{\sigma}$, firm $i$ deviates to the policy function $\hat{\sigma}_i \in PF(\alpha, \beta, C_k)$ in the simulation, resulting in profit $\pi(s_{t_{mn}}, \hat{\sigma}_i(s_{t_{mn}}), \hat{\sigma}_{-i}(s_{t_{mn}}), \theta)$ as the new summand in (2.15).

The upper limit in the third sum is modified from the value function as originally appears in (2.1). Instead of calculating the value function over an indeterminate number of periods ending with firm $i$'s exit at time $T_i$, we just add up the profit during the first month—720 hours to be precise. We do this to reduce the accumulation of simulation errors as the period becomes longer. Given the nature of deviations we are considering, with the firm deviating to a whole new policy function for the entire game, profits are stationary in all simulations. Hence the average per-period profit over 720 hours is an unbiased estimate of the average over the
whole game. This is not true of some excluded deviations, for example, a one-time increase in price of $1; such a deviation could generate a complicated impulse response, which would lead the average profit over a truncated period to diverge from that over the full game. We set the discount factor to an annual value of 0.95, though discounting turns out to be fairly inconsequential over the month horizon we are considering.

Calculating $\overline{EV}_i$ is further expedited following BBL’s insight that when the profit function is linear in the structural parameters, this linearity is inherited by $\overline{EV}_i$ because it is essentially an average over these linear profits. Thus, for example, we can write $\overline{EV}_i(s; \hat{\sigma}, \theta)$ as the dot product of two vectors

$$\overline{EV}_i(s; \hat{\sigma}, \theta) = \overline{EV}_i(s; \hat{\sigma}) \cdot \overrightarrow{\theta}_\tau. \quad (2.16)$$

Here $\overline{EV}_i(s; \hat{\sigma})$ is a vector with four components, corresponding to the four terms in the profit function in (2.3). The first component is the sum of the stream of firm $i$’s base profits in a simulation, $\sum_{t_{mn}=0}^{T_{720}} \text{Base}_{it_{mn}}$, averaged over the $MN$ simulations, where firms behave according to their estimated policy functions in each simulation. Similarly, the remaining three components are the averages over the $MN$ simulations of, respectively, the total upselling profit over a simulation, $\sum_{t_{mn}=0}^{T_{720}} \text{Upsell}_{it_{mn}}$; the number of times firm $i$ monitored market conditions during a simulation, $\sum_{t_{mn}=0}^{T_{720}} \text{Monitor}_{it_{mn}}$; and the number of times firm $i$ changed price during a simulation, $\sum_{t_{mn}=0}^{T_{720}} \mathbb{1}\{\Delta_{it_{mn}} \neq 0\}$. The second vector in (2.16) includes the structural parameters for a firm of type $\tau$: i.e., $\overrightarrow{\theta}_\tau = (1, v_\tau, \mu_\tau, \chi_\tau)$. The linearity in (2.16) means that the summary statistics $\overline{EV}_i(s; \hat{\sigma})$ are all that need to be saved from the $MN$ simulations to later compute $\overline{EV}_i(s; \hat{\sigma}, \theta)$ for any given $\theta$ because one just needs to take the dot product of the summary statistics and subvectors of the given $\theta$. Hence the simulation and estimation steps can essentially be conducted independently.

The other expectation estimate can be expressed similarly as

$$\overline{EV}_i(s; \hat{\sigma}, \hat{\sigma}_{-i}, \theta) = \overline{EV}_i(s; \hat{\sigma}, \hat{\sigma}_{-i}) \cdot \overrightarrow{\theta}_\tau, \quad (2.17)$$

in this case conducting the simulations with firm $i$ using its deviation strategy $\hat{\sigma}_i$ in the simulation while other firms use their estimated policy functions.

With these estimates of the expectations in identification condition (2.14) in hand, we can
to estimation of the structural parameters \( \theta \). Following BBL, define \( Q \) to be the change in the value function caused by a deviation in strategy:

\[
Q(\hat{\sigma}, \hat{\sigma}_i, \theta) = \left\lfloor \hat{E}V_i(s; \hat{\sigma}, \theta) - \hat{E}V_i(s; \hat{\sigma}_i, \hat{\sigma}_{-i}, \theta) \right\rfloor 
\]

\[
= \left( \hat{E}V_i(s; \hat{\sigma}) - \hat{E}V_i(s; \hat{\sigma}_i, \hat{\sigma}_{-i}) \right) \cdot \hat{\theta}_i, 
\]

(2.18)

where (2.19) follows from (2.16) and (2.17). The force of identification assumption (2.14) here is that \( Q \) will be non-negative for sufficiently accurate estimates of the expectations and for \( \theta \) sufficiently close to the true structural parameters. We will estimate \( \theta \) by assessing a penalty for violations of non-negativity and choosing the value that minimizes the sum of squared penalties over the deviations considered:

\[
\hat{\theta} = \arg\min_{\{\hat{\theta}_i|\tau=1,2,3\}} \left\{ \sum_{\{\hat{\sigma}_i|\tau=1,2,3\}} [\min\{Q(\hat{\sigma}, \hat{\sigma}_i, \theta), 0\}]^2 \right\}. 
\]

(2.20)

For each firm type, we considered a set of 2,000 deviations \( \sigma_i \) from the set of policy functions \( PF(\alpha, \beta, C_k^i) \). Each deviation was generated by adding three perturbations to the type's estimated policy function. The first perturbation involves pure random noise: we multiplied the coefficients \( \hat{\alpha}_\tau, \hat{\beta}_\tau \) by log-uniform noise terms and shifted the cut points \( \hat{C}_k^i \) by uniform noise terms. The second perturbation involves adding correlated noise terms to the constant term in \( \hat{\alpha}_\tau \) and the cut points \( \hat{C}_k^i \), such that the deviations amount to an experiment with changing the frequency of monitoring and frequency of price change conditional on monitoring in opposite directions. The third perturbation is to add correlated noise terms to the cut points \( \hat{C}_k^i \), such that the deviations amount to experiments with trade off between small and large price changes. We chose these perturbations to reflect actual tradeoffs that firms might face in their decision-making. For each deviation \( \hat{\sigma}_i \), we calculate the value function statistics \( \hat{E}V_i(s; \hat{\sigma}_i, \hat{\sigma}_{-i}) \) with \( \hat{M} = 10,000 \) random draws of initial states with replacement and \( N = 1 \) simulation for each initial state. For the estimated policy, we calculate the value function statistics \( \hat{E}V_i(s; \hat{\sigma}) \) with \( M = 1,000,000 \) random draws of initial states with replacement and \( N = 1 \) simulation for each initial state to attain greater accuracy.

Because we have chosen a large number of deviations and a large number \( MN \) of simulations
for each deviation, the second-stage estimate, $\hat{\theta}$, is quite accurate conditional on the first-stage estimate $\hat{\sigma}$. The main source of randomness in $\hat{\theta}$ therefore is the potential error in the first stage. To account for this main source of error, we create 200 bootstrapped samples and repeat the whole estimation procedure. For each bootstrapped sample $n = 1, \ldots, 200$, we derive a new estimate, $\hat{\sigma}^{(n)}$, of the policy function. For each $n$, we carry out the second-stage using $\hat{\sigma}^{(n)}$ in the place of $\hat{\sigma}$ in (2.18), (2.19) and (2.20) and generate new sets of deviations by perturbing $\hat{\sigma}^{(n)}$. The resulting structural estimates $\hat{\theta}^{(n)}$, to minimize the sum of squared penalties represented by the new $Q$ function. Confidence intervals can be constructed from quantiles of the set $\{\hat{\theta}^{(n)} | n = 1, \ldots, 200\}$. We could have used bootstrapping to compute standard errors for the policy-function parameters $\alpha, \beta, \text{ and } C^F_t$ reported in Table 2.3. It turns out that these bootstrapped standard errors are quite close to the standard errors from the maximum-likelihood procedure we chose to report. To simplify the estimation, we did not incorporate uncertainty in $Q(Rank_t)$ and $U(Rank_t)$ in our standard errors, but we note that those expressions were fairly precisely estimated in Ellison and Ellison (2009a).
Chapter 3

Keeping Good Quality as a Surprise

3.1 Introduction

In markets where products have quality variation, it is crucial to understand how sellers reveal their quality information. Seminal unraveling results from Milgrom and Roberts (1986) and Grossman and Hart (1980) state that when the seller has verifiable and private quality information, unraveling will start from the top quality so that every quality type, except the worst one, will have to reveal and verify their quality immediately, because concealing quality is to be punished by the buyer’s skeptical belief. It is argued that voluntary revelation happens before the buyer makes informed decisions based on the true quality of the product.

However, in many markets where products or services have complex quality attributes, often some rare and premium attributes are only appreciated by a minority of buyers. When a seller supplying a particular attribute meets with a buyer demanding it, if their rare match is not common knowledge, the seller is not only at the risk of being mistaken as lacking the attribute, as in the unraveling argument, but is also vulnerable to the buyer’s tactic to pretend as not interested in that attribute. This new consideration could affect the seller’s revelation strategy. In this paper I formalize these strategic interactions and emphasize the general intuition that temporary concealing of quality information could enable the seller to charge a higher price in

\[\text{\footnotesize{I would like to thank Glenn Ellison and Muhamet Yildiz for invaluable advice and support. This paper benefited from discussions with Mihai Manea, Harry Di Pei, Juuso Toikka and Alexander Wolitzky. I also thank participants in MIT Theory lunch for their very helpful comments and suggestions.}}\]
a good match.

I consider a seller endowed with a good, for example, a house, trying to sell to a buyer that he is randomly matched to. To simplify the exposition, I focus on a single attribute conveniently referred to as quality, which can be either high or low. The quality is fixed and the seller cannot change it. In the example of selling a house, the attribute could be a home automation system, which may or may not have been installed by the owner/seller. Only a small minority of the buyers are tech-savvy and therefore sensitive to this quality, who are referred to as Prosumers, while the rest of the buyers are indifferent about this quality, who are referred to as Consumers. The buyer’s type is private\(^1\). If the good is of high quality, the seller wants to transfer the good at a higher price whenever the buyer appreciates the quality, that is, the seller of a "smart" house wants to sell at a premium if the buyer is tech-savvy.

For the seller to achieve this goal, two conditions immediately emerge. The first condition, of course, is that the seller must be able to verify the quality to the buyer, as otherwise a seller with a low quality good can always perfectly pretend high quality and there is no ground for the buyer to pay more.

Having verifiable quality is not enough though. The second condition is that quality information must remain private unless the seller chooses to verify and reveal it. To see its importance, suppose verification is mandatory and public or can be costlessly done by the buyer such that the high quality of a seller becomes common knowledge at the very beginning, then a strategic quality-sensitive buyer (Prosumer) has an incentive to pretend as a quality-indifferent Consumer. Depending on the protocol of the engagement and the scarcity of Prosumers, the seller has to either use some costly screening/signaling to distinguish the buyer’s types, or forgo the quality premium and treat the buyer as she claims.

If the seller has access to quality verification and has control of its revelation, he could potentially reveal quality only after a Prosumer has signaled her type, in order to charge a price reflecting the Prosumer’s true value. In a bargaining environment, a buyer may use delays to signal her type. In particular, I consider the protocol proposed in Admati and Perry

\(^1\)This information structure applies to very common situations in which one party has some information but this party do not know its significance to the counterparty. If we interpret the seller as a start-up company and the buyer as a private equity firm who may buy a share of the company, any verifiable information the company has related its profitability fits into this structure, because the private equity firm may have better knowledge about what characteristics can transform into future success.
Starting with the seller, each party makes alternating offers, and can freely choose a delay before responding to the other party’s offer, during which the other party cannot do anything. For the seller, he can choose whether to reveal quality whenever he makes an offer.

Under this bargaining protocol, I propose that the following equilibrium outcome exists and may even be unique under certain parameter ranges: the seller makes the first offer without revelation, such that the buyer would only accept if she does not value quality. A Prosumer who value quality would make a counter offer after a delay. The delay would signal the buyer’s type, so the seller would then reveal the quality upon receiving the delayed counter offer, and as all private information gets revealed they trade at Rubinstein price. More details about this equilibrium and the proofs of its existence and uniqueness are covered in Section 3.2 and 3.

I would like to highlight the intuition behind the seller’s non-revelation decision at time zero. If he reveals the quality without knowing the buyer’s type, the revelation would create a large difference in valuation between the type of the buyer who values quality and the type who does not. Such difference augments the importance of the buyer’s private information and a prosumer buyer has a strong incentive to pretend to care little about quality. According to Admati and Perry (1987), if the prosumer type is rare, a revealed high quality seller cannot afford distinguishing buyer types with a long delay and would sell as if the buyer is of the Consumer type, in which case a Prosumer takes the full benefit of high quality as her information rent. In contrast, if the seller tries to separate the buyer’s types before the revelation of quality, because the difference in expected valuation is small, only a short delay is needed, and after separation, the seller can charge the Prosumer a proper price with full confidence. In other words, by inducing the buyer to signal her type before knowing the quality, the seller minimizes the power of the buyer’s private information.

The alternating bargaining protocol with delays encompasses natural contexts for real-world strategic interactions. A delay, which discounts future payoffs with some factor, could model any process that may cause the bargaining to break up with some probability. In a highly liquid housing market, for example, if the two parties decide to meet again on the next day, either side may strike a deal with someone else before the scheduled meeting. In a business negotiation, either delegate may be withdrawn by their superior prematurely if they decide to exchange offers in the next meeting. While being realistic, a bargaining model is inherently restrictive.
in terms of communication methods and does not give the seller full control of communication, leaving about half of the surplus to the buyer even with complete information.

To explore the seller’s best possible revenue without the aforementioned restrictions, I further analyze the same information and value configuration with the informed principal framework formulated by Myerson (1983). In this framework, the seller is the principal who knows his own type and announce a mechanism that maps the parties’ reports about their types to a trading probability and a monetary transfer. The buyer could update her belief based on the announced mechanism, and if she agrees to participate, both parties make reports to the mechanism which then implements the outcome.

I show that under certain parameter ranges, the seller must announce the same mechanism regardless of quality, and not reveal quality before the buyer reporting to the mechanism. In equilibrium, a high quality seller benefits from concealing quality information, and receives an expected revenue very close to the expected value of the high quality good, while a low quality seller receives an expected revenue as if his low quality is public. The good is transacted with certainty unless the seller has low quality and the buyer is quality sensitive. The detailed analysis is covered in Section 3.4.

The predictions under both frameworks are in contrast to the seminal unraveling results from Milgrom and Roberts (1986) and Grossman and Hart (1980). In these works, the seller also has verifiable and private quality types, but the buyer types do not differ in how much to value quality. Because the seller type with the best quality among any concealing types could always benefit from revelation, the unraveling starts from the top quality. Also in equilibrium, the receiver would hold a skeptical belief whenever some information is concealed, such that the concealing sender is punished by the low payoff following the skeptical belief. It is predicted that all types of the seller would voluntarily reveal the type before the buyer takes action so that the action chosen by the buyer reflect the quality premium, which implies a policy that mandates the disclosure of quality information is not necessary. Hagenbach, Koessler and Perez-Richet (2014) generalizes the concepts such as the order of types and skeptical beliefs, and characterizes the minimum requirements on evidence structure and payoffs that give rise to full revelation.

In my setup, both types of the seller choose not to reveal before the buyer’s first move and
there is no unraveling. This is because the seller will have the chance to reveal later, and as the proof will show, even the maximum punishment imposed by the buyer’s skeptical belief is quite limited and can be outweighed by the aforementioned concern to minimize the power of the buyer’s private information. Also, as the seller now benefits from its type being private, it is no longer true that mandating disclosure of seller’s quality has no effect, and whether the seller has control of its quality information would affect both the price and quantity (timing) of the trade. In fact, now a policy that mandates the disclosure of quality information would preclude the seller from charging more for higher quality, so the policy implication depends on the level of socially optimal compensation for high quality, which must consider the development cost of the high quality good or service incurred prior to the seller-buyer match.

The delayed revelation of verifiable information is also predicted by the developing literature on optimal information disclosure, for example, Bergemann and Pesendorfer (2014) analyzes the optimal sequential release of information in an auction with ascending price. A common feature of this literature is that while the seller can determine the distribution of the verifiable signals and when the buyer can access the signals, the seller himself does not observe the signal, hence the revelation itself creates private information for the buyer. My model is different from this literature: the seller here knows which type he would reveal, and the buyer knows in advance how her initial private information interacts with the seller’s revelation.

The informed principal analysis applies the solution concepts from Myerson (1983) and shares some common features with Koessler and Skreta (2014): first, the seller has zero cost and thus only cares about revenue; second, the value of the good to the buyer depends on both players’ type. The key difference is that in this paper the seller’s type is verifiable when he makes report to the mechanism, while it is unverifiable in Koessler and Skreta (2014), in which the IC constraints of the seller dictate that in any feasible mechanisms, all types of the seller have the same interim revenue. But with verifiable seller types, according to Koessler and Perez-Richet (2014), which provides a general framework for implementability with verifiable reports, the seller’s IC constraints can be dropped, leading to very different predictions.

The paper is organized as follows. Section 3.2 formally describes the model and the equilibrium concept under the bargaining protocol. Section 3.3 proves the existence of an equilibrium with delayed revelation of seller’s verifiable information. Section 3.4 employs a chain of argu-
ments to show that under stricter conditions, the outcome in Section 3.3 is the unique outcome of the bargaining game. Section 3.5 carries out the analysis in an informed principal framework. Section 3.6 concludes.

3.2 Model

In this section, I lay out the model fundamentals, the sequential equilibrium solution concept and review some results from Admati & Perry (1987).

A seller sells an indivisible good to a buyer. The Seller has two possible types \{High, Low\} which can be interpreted as the quality of the good. To abstract away from the complexity of the usual interdependent value bargaining problem, I assume both types have zero cost, so the goal of the seller is to maximize expected revenue.

The Buyer has two preference types \{Consumer, Prosumer\}. The value for the buyer is \( V(t_s, t_B) \), a function of both parties’ types. Type Consumer is indifferent about quality, \( V(H, C) = V(L, C) = \alpha \). Type Prosumer is picky about quality, so \( V(H, P) = \beta^H > V(L, P) = \beta^L \). In addition, I focus on values \( \beta^H > \alpha > \beta^L \) in most parts of the paper. This configuration can naturally arise for goods or services with complex attributes: buyers who are well-informed about the importance of a feature, or prefer a feature strongly, would be disappointed by products lacking the feature, while the others are quite indifferent.

Before anything happens, each player has an initial belief about the other party’s type. The initial beliefs are common knowledge. Denote the buyer’s initial belief as \( \gamma_0 \) and the seller’s initial belief as \( \pi_0 \). Throughout the paper, it is assumed that the Prosumer type has a lower expected value for the good given the skeptical prior belief,

\[
E[V(t_s, P) | \gamma_0] = \gamma_0 \beta^H + (1 - \gamma_0) \beta^L < E[V(t_s, C) | \gamma_0]
\]

The key informational assumption of this paper is that the seller can only reveal the true type, therefore the revelation is credible and permanent. Alternatively, it can be interpreted as if there is a signal known to the seller taking two values, \( \{H, L\} \), and the seller type is defined by the value of his signal. The seller can reveal the signal and the signal is verifiable by the buyer (or a third party that can punish false report).
3.2.1 Bargaining protocol

The bargaining protocol used here was proposed in Admati & Perry (1987) and later adopted by, for example, Cramton (1992):

- The seller makes the first offer at \( t = 0 \).
- After each offer, the responding party can either counter offer or accept after a chosen time \( \tau \).
  - \( \tau \) is at least \( \Delta \).
  - The party who proposed the offer cannot amend it before the other party responds.

Payoffs are discounted by \( e^{-rt} \) where \( t \) is the time of acceptance. Denote \( \delta = e^{-\tau\Delta} \) so that the minimum time between actions induces a discount of \( \delta \). Denote the payoff for a buyer with value \( V \) paying \( p \) at \( t \) as \( U(V, p, t) \), so \( U(V, p, t) = (V - p) e^{-rt} \).

The key feature of this protocol is that the party receiving an offer can choose a delay before responding to it and nothing can happen during the delay. This feature enables parties with private and unverifiable information to signal themselves in Admati & Perry (1987) and Cramton (1992). It will serve a similar purpose here, although the way it works will be moderated by the seller's revelation strategy.

Communication delay causes discount in utility is not only due to preference for earlier consumption, but more importantly also related to exogenous termination of the negotiation during the delay. For example, when the buyer states that she will take a few days to consider the seller’s offer and other options, what really matters is the possibility that the buyer decides to pursue other options and give up the bargaining with this seller, and the discount in utility cause by this consideration time is far more significant than that simply delaying the deal by a few days. Because in real world negotiations, the two parties could be reasonably concerned with various exogenous termination events, they have much freedom to choose the discount or the effective length of a delay without spending long time on waiting.
3.2.2 Information revelation

While the seller can reveal his type credibly, this revelation is a form of communication comparable to sending a price offer, and potentially more complex. Hence, I impose a restriction that the seller can only reveal his type when he makes a price offer. The commitment to not revealing the type while waiting for a response will be equally important in supporting the equilibrium as the commitment to not amending the price offer while waiting for a response.

More specifically, the seller information revelation is modeled as a decision made before the decision of price offer. When it is the seller’s turn to move (at $t = 0$ or after receiving a counter offer) and his type has not been revealed, the seller

1. first decides whether to reveal his type if an offer is to be made,
2. then decides whether to accept (if responding to an offer) or make an offer,
3. then decides when to accept or what offer to make and when,
4. carries out the decisions.

The precise sequence of decisions is summarized in Figure 3.1.

![Figure 3.1](image)

3.2.3 Equilibrium Concept

A history is a sequence of offers exchanged and the latest revelation state, denoted by $H^N = (p^n, t^n)_{n=1}^N$, where $(p^n, t^n)$ is the $n$th price offer and the time that offer was made, and $\hat{r}_s = H, L, \emptyset$ corresponds to the three cases in which High, Low or no type of the seller has been revealed. The active player at $H^N$ is the player that is to respond to $(p^n, t^n)$. A strategy
for a player specifies, at each possible history where the player is active, the player’s response (counter offer or accept, and for the seller only, reveal or not) and the time of response (delay). Each type of a player may play different strategies. Given history $\mathcal{H}^N$, the belief of the active player is about the counter party’s type.

I explore sequential equilibria which consist of strategies of each type and beliefs of the active player at every possible history. The seller’s belief is $\pi (H^N) = \Pr (t_B = C|\mathcal{H}^N)$ and the buyer’s belief is $\gamma (H^N) = \Pr (t_S = H|\mathcal{H}^N)$. The common prior is that $\pi (\emptyset) = \pi_0$ and $\gamma (\emptyset) = \gamma_0$.

To avoid some trivial equilibria, I impose the assumption below following Admati and Perry (1987).

**Assumption 3.1 (A1)** If a player can obtain the same payoff by making fewer offers, then he makes fewer offers.

I also impose a version of intuitive criterion, which refines off-equilibrium beliefs, in the spirit that after a deviation that is very unreasonable for one type but not for the other type, the belief should put zero weight on the former type. The rigorous definition is discussed in Appendix A.1.1, here I give a more verbal version:

**Assumption 3.2 (A2)** A type b player $i$ can convince the other player $j$ that she is indeed type b through a deviation such that type $b$ is the only type of player $i$ that could be not worse off in some equilibrium following the deviation and player $j$’s being convinced.

Two simple observations about the model:

- In equilibrium, if the equilibrium response is acceptance, it must happens without delay, i.e. the active player accepts exactly $\Delta$ after the offer is received.

- If in equilibrium, $\pi (H^N) = 0$, i.e. if the seller is convinced that the buyer is the Prosumer type, then a seller of type $t_s$ will accept an offer of $\hat{p}$ if and only if $\hat{p} \geq \frac{\delta t_s}{1+\delta}$, and will otherwise reveal his type and counter offer with $\frac{\delta t_s}{1+\delta}$.

---

$^2$In a sequential equilibrium, beliefs are independent of the holder’s type, because all types have common prior and beliefs must be updated through the Bayesian rule, either based on the same on-equilibrium history or the same off-equilibrium trembling.
3.2.4 Post revelation equilibria: Admati & Perry (1987)

Before diving into the full equilibrium of this bargaining model, I first review some very useful results from Admati & Perry (1987), hereafter AP. Consider the moment at which the seller has just decided to reveal his type $t_S$ and to make an offer instead of accept. The seller should make the offer without delay because conditional on revelation, delay has no effect on the buyer’s belief. Suppose the seller’s belief, based on public history so far, is that $\Pr(\text{Consumer}) = \pi$. The game starting from the seller’s price offer chosen at this point is equivalent to the bargaining game in AP, so I will denote this game as $APE(\pi, t_S)$.

AP considers a bargaining game with the same protocol, but only the buyer has private information. Based on the revealed type of the seller, the buyer knows her value to be higher or lower depending on her type. Denote the higher value as $h$ and lower value as $l$, which equal to $\beta^H$ and $\alpha$, or $\alpha$ and $\beta^L$ depending on the seller’s revealed type. According to AP, there are at least two types of equilibria. In a separating equilibrium, the seller offer $\frac{h}{1+\delta}$ at $t = 0$; the high type buyer accepts and the low type counter offers $\frac{h}{1+\delta}$ after a delay at $t = \Gamma$, which convinces the seller of her true type and is accepted by the seller$^3$. The delay should just prevent the high type buyer from imitation ($U(h, \frac{h}{1+\delta}, 0) = U(h, \frac{h}{1+\delta}, \Gamma)$), so it increases in $h/l$. In a pooling equilibrium, the seller offer $\frac{l}{1+\delta}$ at $t = 0$ and both types accept, so a high type buyer is treated as a low type buyer.

If the buyer is more likely to have the low value, the seller will incur significant cost if he tries to induce a separating equilibrium that delays the transaction with the low value type buyer, so he will have to make the pooling offer. AP identifies sharp conditions for pooling to be the unique outcome. This is the intuition against the seller revealing at the very beginning: if the seller reveals High type, and the common prior for Prosumer type is less than $\frac{\alpha}{2\beta^H}$, the theorems in AP suggest that the seller will have to treat the buyer as a Consumer and give up the additional value of a Prosumer for sure. The equilibrium I construct in the next section will improve the seller’s payoff with delayed revelation.

$^3$An intuitive result shown in AP is that, if after a history, the seller has a degenerate belief ($\Pr(\text{Consumer}) = 0$ or 1) consistent with the buyer’s true type, the game has a unique equilibrium outcome afterwards, in which they trade without delay at the equilibrium price of a familiar Rubinstein bargaining model with discount factor $\delta$. Meanwhile, if the seller’s degenerate belief is opposite to the truth, the buyer is allowed to flip the seller’s belief with a delay in some equilibrium paths.
3.3 An equilibrium with delayed revelation

In this section I show that there exists a set of conditions on the parameters $\alpha, \beta^H, \beta^L, \pi_0, \gamma_0$ and $\delta$ that imply that the game has a sequential equilibrium in which the seller reveals his type only after the buyer signals her type through a delay. The previous section noted that if Consumer is more common, separating buyer types would be too expensive if High quality is revealed up-front. In contrast, separating buyer types prior to revelation could potentially be less costly for two reasons: first, because the Consumer type has the higher expected value before revelation, this more common type can transact before the Prosumer type; second, while a delay will still reduce the payoff of the seller from a Prosumer type, the delay could be quite short if the Prosumer and Consumer types have similar expected value without knowing quality for sure. If the seller could separate the Prosumer with an acceptable cost by concealing his type temporarily, he would get a share of the surplus due to his high quality. More precisely, I show the existence of an equilibrium in which the high type seller sells to a Prosumer at a price reflecting the high value after a delay. To simplify the notation, I will give this outcome a convenient name.

**Definition 3.1** An assessment is said to be Separating before Revelation with Delay $\tau$ if it has the following outcome:

- Both seller types offer $p^0 = \frac{\alpha}{1+\delta}$ at $t = 0$ without revelation.
- The Consumer type accepts without delay
- The Prosumer type counteroffers $p^P = \frac{\delta \beta^L}{1+\delta}$ after a delay $\tau$
  - The Low type seller accepts this offer
  - The High type seller counteroffers $\frac{\beta^H}{1+\delta}$ and reveals $H$. The offer is accepted by the Prosumer type buyer

If a sequential equilibrium is Separating before Revelation with Delay $\tau$, upon receiving the equilibrium offer $p^P$ after delay $\tau$, the seller holds belief that $\Pr(\text{Prosumer}) = 1$, therefore decides to reveal his type. This outcome is depicted in the Figure 3.2.
This outcomes provides the equilibrium path. To fill the supporting off-path play and beliefs, first note \( \tau \) has to be large enough to make a Consumer prefer accepting \( p^0 \) to counter-offering as the Prosumer does, and has to be small enough to make the Prosumer prefer counter-offering after \( \tau \) to accepting \( p^0 \). Intuitively, such \( \tau \) could exist because the Consumer type loses more from delay due to the higher expected value \( \alpha \). In addition to this, I have to impose some additional restrictions on the parameters \( \alpha, \beta^H, \beta^L, \pi_0, \gamma_0 \) and \( \delta \) and specifies the off-equilibrium paths such that each type of the seller indeed finds revealing at the beginning is no better than playing this equilibrium. I also have to assign proper beliefs after out-of-equilibrium behaviors such that those behaviors are not profitable for the deviator. The goal of this section is to discuss a full sequential equilibrium that is Separating before Revelation with Delay \( \tau \) and introduce the following theorem:

**Theorem 3.1** There exists functions \( \gamma^* (\alpha, \beta^H, \beta^L) \), \( \pi^* (\alpha, \beta^H, \beta^L, \delta) \), \( \beta^* (\alpha, \beta^L, \delta, \gamma_0) \) and \( \sqrt{\gamma^*(\alpha, \beta^H, \beta^L, \delta, \gamma_0)} \) such that if the following conditions for \( \alpha, \beta^L, \beta^H, \delta, \pi_0, \gamma_0 \) are satisfied:

1. **high quality is sufficiently rare:**
   \[
   \gamma_0 < \gamma^* (\alpha, \beta^H, \beta^L)
   \]

2. **The Prosumer type is sufficiently rare:**
   \[
   \pi_0 > \pi^* (\alpha, \beta^H, \beta^L, \delta)
   \]
3. The Prosumer type’s willingness to pay for quality is sufficiently large:

\[ \beta^H > \beta^* (\alpha, \beta^L, \delta, \gamma_0) \]

then there exist a sequential equilibrium consistent with A1 and A2 that is Separating before Revelation with Delay \( \tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0) \).

The first two conditions highlight the informational challenge that the supply and demand for good quality are very rare and therefore a high surplus match is hard for either parties to recognize, so without a proper delay mechanism, transactions would never reflect the added value of the high quality. The third condition confirms that, because delaying the transaction with Prosumer discounts realized surplus from a great match, this surplus have to be large enough for the two parties to coordinate on the delay. The full proof for this theorem is included in Appendix A.2. I will describe the important features of this equilibrium in the rest of this section, and give the arguments that determine \( \gamma^* (\alpha, \beta^H, \beta^L) \), \( \pi^* (\alpha, \beta^H, \beta^L, \delta) \), \( \beta^* (\alpha, \beta^L, \delta, \gamma_0) \) and \( \tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0) \).

### 3.3.1 Deviations and their deterrence

I first discuss the four most straightforward deviations in a Separating before Revelation with Delay \( \tau \) sequential equilibrium and gather conditions for no-deviation. Figure 3.3 summarizes the deviations discussed below.
1. A Consumer does not want to pretend to be a Prosumer by making the latter’s equilibrium offer, \( p^P \) after delay \( \tau \). This leads to the condition for \( \tau = \tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0) \)

If a Consumer carries out this deviation, the seller will update his belief to the opposite of the truth. A low quality seller would accept the offer \( p^P \), while a high quality seller will make the counter offer \( \frac{\delta^H}{1+\delta} \). If a deviating Consumer accept this counter offer, she has essentially get the same treatment as a Prosumer, which translates to lower expected payment for the good. However, the deviating Consumer can even improve upon this already lower payment by rejecting this offer and use a delay to convince the seller of her true type. In this way she will only need to pay \( \frac{\delta_0}{1+\delta} \). This off equilibrium outcome is depicted in Figure 3.3 below as Dev A.

Because \( \tau \) is a signal used by a Prosumer to distinguish herself from a Consumer, the
length of $\tau$ needs to be just long enough to make a Consumer indifferent between the equilibrium outcome (accepting the initial offer $\frac{\alpha}{1+\delta}$ without delay) and outcome of Dev A. The specific expression of $\tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0)$ is given in Appendix A.2.1.1. Because the expected payment in Dev A (a bit lower than the Prosumer’s equilibrium payment) could be quite close to $\frac{\alpha}{1+\delta}$, the delay could be quite short. This contrasts the delay following revelation, which would be used to separate two types of buyers with very different values $\alpha$ and $\beta^H$ or $\beta^L$. The heterogeneity between values are greatly reduced due to the concealing of the finer quality information.

2. A Prosumer does not want to pretend to be a Consumer by accepting $p^0$ at $t = 0$. This leads to the condition $\gamma_0 \leq \gamma^* (\alpha, \beta^H, \beta^L)$.

If high quality was common enough, a Prosumer would rather take the bet at the beginning to purchase the unknown quality good at a lower price, and to avoid the high likelihood of paying high price for high quality after a delay. Hence, high quality must be rare enough. The specific expression of $\gamma^* (\alpha, \beta^H, \beta^L)$ is given in Appendix A.2.1.2. Because this deviation ends the game, it is not depicted in Figure 3.3.

3. A type $H$ seller does not want to reveal his type at the very beginning. This leads to the condition $\beta^H \geq \beta^* (\alpha, \beta^L, \delta, \gamma_0)$ and a lower bound for $\pi_0$.

The most salient feature of the equilibrium outcome is that even the $H$ high type seller does not reveal at $t = 0$. This would only makes sense if the revenue for each type in this equilibrium is higher than the payoff if the type is known to be $H$. To compare the revenue, on the one hand, in a Separating before Revelation with Delay $\tau$ equilibrium, for a $H$ high type seller, the trading price is $\frac{\alpha}{1+\delta}$ with a Consumer or $\frac{\beta^H}{1+\delta}$ with Prosumer after a delay $\tau$. On the other hand, if a seller of type $H$ revealed his type at the very beginning, according to AP, if $\pi_0$ is large enough, there exist a sequential equilibrium following revelation, in which the seller simply charges $\frac{\alpha}{1+\delta}$ from both types of the buyer without delay. This outcome is depicted in Figure 3.3 as Dev B. Comparing the equilibrium outcome to the Dev B outcome for the seller, his revenue from a Consumer is the same, but in the separating equilibrium, he receives a higher revenue from a Prosumer after a delay $\tau$. Given the delay determined by $\tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0)$, the type $H$ seller should
prefer not to reveal at the beginning if the prize, $\frac{3H}{1+3}$, is large enough. The specific expression of $\beta^* (\alpha, \beta^L, \delta, \gamma_0)$ is given in Appendix A.2.1.3.

4. A type $L$ seller does not want to reveal his type at the very beginning. This leads to another lower bound for $\pi_0$.

If a Low seller reveals his type at the very beginning, the Consumer type buyer would have a higher value and the continuation game becomes $APE (\pi_0, L)$. I require that $\pi_0$ is high enough such that a separating equilibrium as described by AP exists after revelation. The exact condition is given in the appendix. In this separating equilibrium depicted in Figure 3.3 as Dev C, the seller gets the same payments from the two types of the buyer as in the Separating before Revelation with Delay $\tau$ sequential equilibrium, except that the delay before the transaction with a Prosumer becomes longer, as proved in Appendix A.2.1.4. Hence, this deviation cannot be profitable.

The four deviations discussed above all lead to unambiguous belief updating: the deviating type either takes the equilibrium action of the other type, or takes an action that drastically removes uncertainty. The outcomes following these deviations are either straightforward or are predicted right away by AP. To complete the construction of the Separating before Revelation with Delay $\tau$ equilibrium, I now switch focus to the beliefs updating rules and equilibrium outcome following more subtle deviations, such as making offers that no type would make in equilibrium. For example, what should the seller think if the buyer makes a counter offer higher than $p^O$ before $\tau$? what should the buyer think if the seller, without revealing type, makes an initial offer below $p^O$? Note I will only discuss such deviations before the seller reveals type, because after revelation, the off-equilibrium beliefs and actions are already covered in Admati & Perry and are parts of the separating equilibrium and the pooling equilibrium as they describe.

5. Seller makes off-equilibrium offer without revelation. This leads to another lower bound for $\pi_0$.

I propose the following simple belief updating rule: whenever the seller deviates by making a different offer $p'$ and/or at different time without revelation, the buyer would put probability 1 on seller being the Low type. It is shown in Appendix A.2.2.1 that if Consumer
is very likely ($\pi_0$ is large enough, exact condition given in appendix), the seller would always try to sell to the Consumer at the Rubinstein price first, and Prosumer will only come back after a delay that is longer than $\tau$. Hence, this deviation cannot be profitable for the seller.

6. Buyer makes off equilibrium offer before revelation.

If following the equilibrium offer $p^0$ at $t = 0$, the buyer deviates with an off-equilibrium counter offer $p' \geq \frac{\delta \beta L}{1+\delta}$ at time $\tau'$ (offers less than $\frac{\delta \beta L}{1+\delta}$ will never be accepted and therefore is dominated), the seller would update his belief such that the off-equilibrium counter offer does not make the buyer better off. To achieve this, in the equilibrium I propose, the seller would compare two hypothetical payoffs: the payoff for the Consumer type buyer to accept the offer $p^0$ without delay and the payoff for the Consumer buyer if the seller is convinced that she is a Prosumer after her deviation ($p', \tau'$) and reveal his type. If the former is greater or equal to the latter, the seller would put probability 1 on the buyer being a Prosumer; otherwise, the seller would put probability 1 on buyer being a Consumer.

This belief updating rule thus ensures the Consumer type buyer has no incentive to deviate with an off-equilibrium counter offer: if the deviation convinces the seller that she is a Prosumer, her expected payoff would be worse; if the deviation convinces the seller that she is a Consumer, she will pay no less than $p^0$ and gets the good no faster. For the Prosumer type, a graphical proof is provided in Appendix A.2.2.2, showing that this belief updating rule also deters a Prosumer from deviation.

So far I have described the equilibrium outcome following all possible one-step deviations from the equilibrium strategy, but I have not yet covered the outcome following a second deviations and beyond. Fortunately, no additional conditions are needed to deter those deviations. The appendix has a rigorous account of the whole assessment. Appendix A.2.3 also checks whether the off-equilibrium belief updating in my equilibrium is consistent with A2.
3.3.2 Summary

So far I have constructed a Separating before Revelation with Delay $\tau^*$ equilibrium and discussed how deviations can be deterred at suitable parameters. Now I summarize the results in the context of Theorem 3.1.

$\tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0)$, $\gamma^* (\alpha, \beta^H, \beta^L)$ and $\beta^* (\alpha, \beta^L, \delta, \gamma_0)$ in Theorem 3.1 are determined by considerations regarding the deviation types 1, 2 and 3. Analysis for deviation types 3, 4 and 5 generates three lower bounds for $\pi^0$, and the maximum of these bound gives $\pi^* (\alpha, \beta^H, \beta^L, \delta)$. The exact expressions are provided in Appendix A.2.4.

The conditions of Theorem 3.1 are hence simply the collection of all the conditions in this section, and these conditions guarantee all deviations from the equilibrium path can be deterred properly. A complete proof is given in the appendix, but all the ingredients in this theorem are already discussed.

Take the limit: $\delta \rightarrow 1$

As $\delta \rightarrow 1$, or equivalently, $\Delta \rightarrow 0$, there is no exogenous limitation on communication intensity. However, the delay predicted by Theorem 3.1 is almost unaffected, which confirms that the delay in equilibrium is caused by the information friction.

I take the limits of the functions in Theorem 3.1 as $\delta \rightarrow 1$ and obtain the following:

**Theorem 3.2** Let $\gamma^* (\alpha, \beta^H, \beta^L)$, $\pi^* (\alpha, \beta^H, \beta^L, \delta)$, $\beta^* (\alpha, \beta^L, \delta, \gamma_0)$ and $\tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0)$ be the functions derived for Theorem 3.1. If the following conditions for $\alpha, \beta^L, \beta^H, \pi_0, \gamma_0$ are satisfied:

1. high quality is rare:
   \[
   \gamma_0 < \gamma^* (\alpha, \beta^H, \beta^L)
   \]

2. Prosumer is rare:
   \[
   \pi_0 > \lim_{\delta \rightarrow 1} \pi^* (\alpha, \beta^H, \beta^L, \delta)
   \]

3. $\beta^H$ is high enough:
   \[
   \beta^H > \lim_{\delta \rightarrow 1} \beta^* (\alpha, \beta^L, \delta, \gamma_0)
   \]
then there exist \( \delta < 1 \) such that for any \( \delta > \delta \), there exist a sequential equilibrium consistent with \( A1 \) and \( A2 \) that is Separating before Revelation with Delay \( \tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0) \), and \( \lim_{\delta \rightarrow 1} \tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0) > 0 \).

In fact, at the limit, some functions have simpler expressions. The delay at the limit is

\[
\lim_{\delta \rightarrow 1} \tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0) = \frac{1}{r} \log \left[ \frac{\gamma_0}{2 - \frac{\alpha}{\beta^H}} + (1 - \gamma_0) \left(2 - \frac{\beta^L}{\alpha}\right) \right] > 0
\]

\( \pi^* \) at the limit is

\[
\lim_{\delta \rightarrow 1} \pi^* (\alpha, \beta^H, \beta^L, \delta) = \max \left\{ 1 - \frac{\alpha}{\beta^H}, \frac{\beta^L}{2\alpha}, \frac{\beta^H}{2\alpha + \beta^L} \right\}
\]

**Numerical Example**

Because the conditions 1 and 3 in Theorem 3.1 and 3.2 are two inequalities about \( \gamma_0 \) and \( \beta^H \), a natural concern is that whether they indeed define a nonempty set of parameters. To show the conditions 1 and 3 for \( \gamma_0 \) and \( \beta^H \) are coherent, I consider the \( \delta \rightarrow 1 \) limit, fix \( \alpha = 7, \beta^L = 5 \) and draw the graph of \( \gamma^* (\alpha, \beta^H, \beta^L) \) and \( \beta^* (\alpha, \beta^L, \gamma_0) \) on the \( \gamma_0 - \beta^H \) plane in Figure 3.4. The shaded area is the set of \( \gamma_0 \) and \( \beta^H \) that satisfy condition 1 and 3 in Theorem 3.2.
Meanwhile, given $\alpha$ and $\beta^L$, condition 2 in Theorem 3.2 translates to

$$\pi_0 > \max \left\{ \frac{\beta^H - 7}{\beta^H}, \frac{5}{14}, \frac{\beta^H}{9 + \beta^H} \right\}$$

$$= \begin{cases} 
1 - \frac{9}{9 + \beta^H} & \text{if } \beta^H \leq 31.5 \\
1 - \frac{7}{\beta^H} & \text{if } \beta^H > 31.5 
\end{cases}$$

Hence, if $\beta^H = 9$, the ex ante probability of the Prosumer type is bounded by 0.5. This numerical example shows that the conditions of Theorem 3.2 are not overly restrictive, and there is a wide range of parameters that can give arise to Separating before Revelation with Delay $\tau^*$ $(\alpha, \beta^H, \beta^L, \gamma_0)$.

Comparing this range of parameters required by Theorem 3.2 to extreme beliefs implies that the High quality seller’s payoff increases in the relative rareness of his quality. If High quality is not rare at all so that $\gamma_0 = 1$ but demand for High quality is rare so that $\pi_0$ is close to 1, AP predicts that a pooling equilibrium could be the unique outcome and the seller will never get a portion of the surplus due to a good match. If High quality is very rare so that $\gamma_0$ is close
to zero but demand for High quality is abundant so that $\pi_0$ is close to zero, a High quality seller could easily reveal his type at the beginning and capture a fair portion of the Prosumer type's additional value at the high price of $\frac{\beta^H}{1+\gamma}$. If both supply and demand for High quality are rare, as characterized by Theorem 3.1 and 3.2, a High quality seller can still capture a fair portion of the Prosumer type's additional value at the high price of $\frac{\beta^H}{1+H}$, albeit after a delay. This comparative statics resemble the outcome in a competitive market with price takers, but is achieved through a bargaining protocol.

### 3.4 Uniqueness of the equilibrium with delayed revelation

The previous section describes a Separating before Revelation with Delay $\tau^*$ equilibrium that exists under a range of parameters, which implies that the usual unravelling argument in the verified information revelation literature cannot guarantee the revelation of a good signal at the very beginning. In this section I show that if Prosumer is rare enough, this equilibrium is in fact the only equilibrium, which implies that there is no way for the buyer to use pessimistic beliefs to induce revelation at the beginning.

**Theorem 3.3** Let $\gamma^* (\alpha, \beta^H, \beta^L)$, $\tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0)$ and $\tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0)$ be the functions derived for Theorem 3.1. There exists a function $\pi^{**} (\alpha, \beta^H, \beta^L, \delta)$ such that if the following conditions for $\alpha, \beta^L, \beta^H, \delta, \pi_0, \gamma_0$ are satisfied:

1. **high quality is rare:**
   \[ \gamma_0 < \gamma^* (\alpha, \beta^H, \beta^L) \]

2. **Prosumer is rare:**
   \[ \pi_0 > \pi^{**} (\alpha, \beta^H, \beta^L, \delta) \]

3. **Prosumer and Quality match is strong:**
   \[ \beta^H > \beta^* (\alpha, \beta^L, \delta, 0) \]

then the unique sequential equilibrium outcome consistent with A1 and A2 is Separating before Revelation with Delay $\tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0)$.
The complete proof is in Appendix A.3, and I describe the blueprint here. First I will show that under certain conditions, revelation cannot happen with the seller’s initial offer. This follows from the following line of argument:

1. At higher π₀ (note π** (α, β⁺, β⁻, δ) > π⁺ (α, β⁺, β⁻, δ)), the equilibrium outcome after the seller deciding to reveal, described in the previous section, is unique according to AP. So for H high type seller, the payoff following revelation at the beginning is quite bad.

2. At higher β⁺ (β⁺ (α, β⁺, δ, 0) > β⁺ (α, β⁺, δ, γ₀)), if the seller makes a non-revealing offer no greater than \( \frac{α}{1+δ} \) and has belief π₀ satisfy the previous point, a Consumer type buyer would always accept the offer regardless of her belief. This is not surprising because if either type of the seller is revealed, a Consumer is to pay \( \frac{α}{1+δ} \). Some careful analysis gives that, when β⁺ is high enough, there is no way a Consumer can get the seller to accept anything lower than \( \frac{α}{1+δ} \).

3. Given that in equilibrium a Consumer would accept an offer \( \frac{α}{1+δ} \) made by a seller with initial belief, I further show that in equilibrium, the Prosumer must separate herself from a Consumer with offer \( p^P \) and delay \( r^* (p₀, γ₀, δ, α, β⁺, β⁻) \).

4. Combine 2 - 3, a seller can guarantee the separation of the buyer types if he makes a non-revealing initial offer, which gives higher payoff than the unique outcome following revelation, pinned down in 1.

Second, I pin down the seller’s initial offer given no revelation. The core argument is that because a Consumer type can signal herself credibly as permitted by the intuitive criterion A2, the seller cannot get her to accept any offer higher than \( \frac{α}{1+δ} \). Hence the best for both types of the seller is to make initial offer equal to \( \frac{α}{1+δ} \).

3.4.1 Take the limit: \( δ \to 1 \)

As \( δ \to 1 \), or equivalently, \( Δ \to 0 \), Theorem 3.3 becomes

**Theorem 3.4** Let \( γ^* (α, β⁺, β⁻) \), \( β^* (α, β⁺, β⁻, δ, γ₀) \) and \( r^* (α, β⁺, β⁻, δ, γ₀) \) be the functions derived for Theorem 3.1 and \( π^{**} (α, β⁺, β⁻, δ, γ₀) \) be the function derived for Theorem 3.3. If the following conditions for \( α, β⁺, β⁻, π₀ \) and \( γ₀ \) are satisfied:

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1. **high quality is rare:**

\[ \gamma_0 < \gamma^* (\alpha, \beta^H, \beta^L) \]

2. **Prosumer is rare:**

\[ \pi_0 > \lim_{\delta \to 1} \pi^{**} (\alpha, \beta^H, \beta^L, \delta) \]

3. **Prosumer and Quality match is strong:**

\[ \beta^H > \lim_{\delta \to 1} \beta^* (\alpha, \beta^L, \delta, 0) \]

then there exist \( \delta < 1 \) such that for any \( \delta > \delta \), the unique sequential equilibrium outcome consistent with A1 and A2 is Separating before Revelation with Delay \( \tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0) \), and \( \lim_{\delta \to 1} \tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0) > 0 \).

\( \lim_{\delta \to 1} \tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0) \) is the same limit given after Theorem 3.3, so Theorem 3.4 suggests that although there is no exogenous limitation on communication intensity, the delay could still be inevitable as the Prosumer type buyer uses delay to signal her type.

At the limit, the expressions for \( \pi^{**} \) and \( \beta^* \) are quite simple, so I present them below

\[ \lim_{\delta \to 1} \pi^{**} (\alpha, \beta^H, \beta^L, \delta) = \max \left\{ 1 - \frac{\alpha}{2\beta^H}, \frac{\beta^L}{2\alpha}, \frac{\beta^H}{\beta^H + 2\alpha - \beta^L} \right\} \geq \pi^* (\alpha, \beta^H, \beta^L) \]

\[ \lim_{\delta \to 1} \beta^* (\alpha, \beta^L, \delta, 0) = 2\alpha - \beta^L \]

**Numerical Example**

To show the conditions for \( \gamma_0 \) and \( \beta^H \) are coherent, I fix \( \alpha = 7, \beta^L = 5 \) and draw the graph of \( \gamma^* (\alpha, \beta^H, \beta^L) \) and \( \beta^* (\alpha, \beta^L) \) on the \( \gamma_0 - \beta^H \) plane in Figure 3.5, taking the limit \( \delta \to 1 \). Now the shaded area, formed by \( \gamma_0^*, \beta^H = 2\alpha - \beta^L = 9 \) and the \( \beta^H \) axis, represents the set of \( \gamma_0 \) and \( \beta^H \) that satisfy condition 1 and 3 in Theorem 3.4.
In comparison with Figure 3.4, only a small area in Figure 3.4 is deemed as not able to support a delay before revelation as an inevitable outcome in Figure 3.5.

Meanwhile, given $\alpha, \beta^L$ and $\beta^H > 9$, condition 2 in Theorem 3.4 translates to

$$\tau_0 > \max \left\{ \frac{2\beta^H - 7}{2\beta^H}, 14 \frac{\beta^H}{9 + \beta^H} \right\}$$

$$= 1 - \frac{7}{2\beta^H}$$

Hence, the ex ante probability of the Prosumer type is bounded by $\frac{7}{2\beta^H}$.

### 3.5 Mechanism Design with an Informed Principal

The alternating bargaining protocol described above is a realistic yet tractable way to model the interaction between the two parties with the aforementioned information structure, but it is not the only way. Some components in the alternating bargaining protocol are essential for the intuition behind the delayed revelation and the quality-sensitive transaction price. First,
the protocol gives that it is costly to separate buyer types with different values, and the cost (delay) increases with the value heterogeneity. Second, the protocol provides some commitment power for the seller: because seller cannot communicate to the buyer during the delay chosen by the Prosumer buyer, the seller commits to sell at most $e^{-rr^p} < 1$ unit of the good if the buyer reports her type as a Prosumer.

But some restrictions inherent to the alternating bargaining protocol are not necessary for the intuitions to carry about. For example, this protocol specifies that the two parties have equal bargaining power under complete information, but one can easily come up with an alternative model where the seller has all the bargaining power, for example, the seller is a monopolist selling a durable good\(^4\). More importantly, under this bargaining protocol, each type of the seller has to trade with a Prosumer type at almost the same time, yet one could imagine the Prosumer can signal more effectively if she can choose to delay trading longer when the quality is low.

To distinguish the intrinsic informational tension from the effects of these restrictions, in this section I pursue a more general mechanism design problem formulated by Myerson (1983) so that the seller has full control of the communication protocol and thus all the bargaining power. Now the seller, who knows his own type, directly announce a mechanism and commit to it. The buyer then updates her belief based on the announced mechanism, and if she agrees to participate, both parties make report to the mechanism which then implements the outcome. Because the buyer can make false report while the seller cannot, the mechanism is truthfully implementable if and only the buyer's incentive compatible constraint and the participation constraint hold.

I focus on mediated mechanisms, that is, the seller can announce a mechanism that take reports from the two parties simultaneously, as if there is a mediator, and implements an

\(^4\)For example, consider a monopolist (a real estate developer) selling durable goods (apartments). A reasonable pricing strategy is to offer introductory discounted prices at the beginning to accommodate Consumers who are ready to buy an apartment quickly. Then the monopolist can market the remaining apartments to Prosumers who are looking for some special features at higher prices. Because the special features are not common, the Prosumers are skeptical before the monopolist explicitly market on them. Potentially, the Consumers would have an incentive to wait if they also think there is a high probability that the monopolist does not have the special feature and will have to cut price to sell to the Prosumers. The monopolist has to commit to not start marketing to Prosumers too soon or offer limited choices to the Prosumers to prevent the Consumers from waiting. This could be achieved by offering price guarantee to the early buyers of the good for a limited time.

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outcome as a function of the reports. I find there is a unique sequential equilibrium in terms of interim payoffs of this extensive form game. In this equilibrium, both types of the seller announce the same mechanism, both parties make truthful reports to the mechanism, and the high quality seller gets an expected revenue that is even slightly higher than the expected value of the high quality good.

3.5.1 Model

Consider the same fundamentals as the bargaining game: A seller sells an indivisible good to a buyer. The Seller has two possible types \{High, Low\}. Both types have zero cost, so the goal of the seller is to maximize expected revenue. The Buyer has two preference types \{Consumer, Prosumer\}. The value for the buyer is \(V(t_S, t_B)\), a function of both parties' types. A Consumer is indifferent about quality, \(V(H, C) = V(L, C) = \alpha\). A Prosumer is picky about quality, so \(V(H, P) = \beta^H > V(L, P) = \beta^L\). Before anything happens, each player has an initial belief about the other party's type. The initial beliefs are common knowledge. Denote the buyer’s initial belief as \(\gamma_0 = \Pr(H)\) and the seller’s initial belief as \(\pi_0 = \Pr(C)\).

Assume the following for \(\alpha, \beta^H, \beta^L, \pi_0\) and \(\gamma_0\):

1. \(\beta^H > \alpha > \beta^L\), and the Prosumer type has a lower expected value for the good given the skeptical prior belief, i.e.

\[E[V(t_s, P) \mid \gamma_0] = \gamma_0 \beta^H + (1 - \gamma_0) \beta^L \leq E[V(t_s, C) \mid \gamma_0]\]

2. \(\pi_0 \alpha > \beta^L\) so if Low quality is common knowledge, the seller optimizes revenue by setting a fixed price at \(\alpha\) and only sells to the Consumer.

3. \(\alpha > (1 - \pi_0) \beta^H\) so if High quality is common knowledge, the seller optimizes revenue by setting a fixed price at \(\alpha\) and sells to both types of the buyer.

Mediated Mechanism

A mediated mechanism \(\varphi : T_S \times T_B \rightarrow [0, 1] \times R_+\) maps the seller's verifiable report \(t_S \in T_S = \{H, L\}\) and the buyer's unverifiable report of her type \(t_B \in T_B = \{C, P\}\) to a transaction.
where $q$ is the probability that the good is transferred to the buyer and $p$ is the transfer of nonnegative money from the buyer to the seller. A revelation principle gives it is without loss of generality to restrict the seller’s report to \{H, L\} and the buyer’s to \{C, P\}. I use $\varphi_q$ and $\varphi_p$ to represent the two component of $\varphi$. I denote $u_i(\varphi|t_i)$ as the expected payoff of type $t_i$ of player $i$ if both players truthfully report their types and trade is carried out according to $\varphi$.

### 3.5.2 Feasible Mechanism

I say that a mechanism $\varphi$ is feasible if it is an equilibrium that both players are willing to participate and truthfully report their types. With verifiable seller types, the IC constraints for seller disappears. Also given $\varphi_p \geq 0$, the IR constraints for seller is trivially satisfied. Hence, only the buyer’s IC and IR constraints matter. I denote $u_B(\varphi|t_B')$ as the payoff for a type $t_B$ buyer to misreport her type as $t'$, assuming the seller reports truthfully.

**Definition 3.2** A mechanism is feasible if the IC constraints for the buyer

\[
\begin{align*}
    u_B(\varphi|P) - u_B(\varphi|C, P) & \geq 0 \quad (3.1) \\
    u_B(\varphi|C) - u_B(\varphi|P, C) & \geq 0 \quad (3.2)
\end{align*}
\]

---

5 Implicitly, either party is allowed to report nothing or any other messages. The justification behind limiting the reports to types is the revelation principle. The revelation principle for verifiable information has been studied in several works (e.g. Bull and Watson (2007)), which in general state that if every type can report the collection of all available messages, then the evidence structure satisfies the normality condition and the revelation principle holds.

6 With unverifiable information, a feasible mechanism must satisfies (interim) incentive compatibility and individual rationality constraints for all players. Now with the seller having verifiable information, I can apply the Theorem 1 from Koessler, and Perez-Richet (2014) to show that the IC constraint for the seller is no longer needed to ensure an equilibrium (intuitively, because the seller only cares about revenue in this case and each type of the seller only has one deviation available, which is to report nothing (or anything unverifiable), such deviation can be easily deterred by treating any deviating report as a report of the seller type with lower expected revenue).

7 With unverifiable information, a feasible mechanism must satisfies (interim) incentive compatibility and individual rationality constraints for all players. Now with the seller having verifiable information, I can apply the Theorem 1 from Koessler, and Perez-Richet (2014) to show that the IC constraint for the seller is no longer needed to ensure an equilibrium (intuitively, because the seller only cares about revenue in this case and each type of the seller only has one deviation available, which is to report nothing (or anything unverifiable), such deviation can be easily deterred by treating any deviating report as a report of the seller type with lower expected revenue).

\[
\begin{align*}
    u_B(\varphi|C, P) &= \left[\gamma_0 \beta^H (1 - \gamma_0) \beta^L \right] \left[\varphi_q(H, C) \right] - E \left[\varphi_p(t_S, t_B) | t_B = C \right] \\
    u_B(\varphi|P, C) &= \left[\gamma_0 \alpha (1 - \gamma_0) \alpha \right] \left[\varphi_q(L, P) \right] - E \left[\varphi_p(t_S, t_B) | t_B = P \right]
\end{align*}
\]
and the IR constraints for the buyer

\[
\begin{align*}
    u_B(\varphi|P) & \geq 0 \quad (3.3) \\
    u_B(\varphi|C) & \geq 0 \quad (3.4)
\end{align*}
\]

hold.

I denote the set of feasible mechanisms as \( \mathcal{F} \), which will be derived in the next subsection.

### 3.5.3 Seller Efficient Mechanism

In this subsection I characterize the set of mechanisms that maximizes the seller’s expected revenue in some sense. Before knowing the types of either players, denote the ex ante expected payment as \( \Pi \), so

\[
\Pi = \pi_0 E [\varphi_p(t_S, t_B) | t_B = C] + (1 - \pi_0) E [\varphi_p(t_S, t_B) | t_B = P]
\]

\[
= \gamma_0 u_S(\varphi|H) + (1 - \gamma_0) u_S(\varphi|L) \quad (3.5)
\]

**Definition 3.3** A feasible mechanism \( \varphi^* \) is seller efficient if it maximizes ex ante expected payment \( \Pi \), i.e. it solves

\[
\max_{\varphi \in \mathcal{F}} \gamma_0 u_S(\varphi|H) + (1 - \gamma_0) u_S(\varphi|L)
\]

In Appendix A.5, I solve this maximization problem, and construct feasible mechanisms that achieve the maximum efficiency. Essentially, in a seller efficient mechanism, the good is always transferred unless a Low type seller is matched with a High type seller, when the trading probability is \( \varphi_q^*(L, P) = \frac{\gamma_0(\beta^H - \alpha)}{(1 - \gamma_0)(\alpha - \beta^L)} < 1 \) such that a Consumer is just indifferent about misreporting or not. This is because that reducing \( \varphi_q^*(L, P) \) is the most efficient way to satisfy the IC constraint of a Consumer. The IC constraint for a Prosumer is not binding. The IR constraints are binding, so the seller receives the full expected value of the transferred good. The maximum revenue is hence

\[
\Pi^* = \max_{\varphi \in \mathcal{F}} \Pi = \pi_0 \alpha + (1 - \pi_0) \cdot (1 - \gamma_0) \cdot \frac{\gamma_0 (\beta^H - \alpha)}{(1 - \gamma_0) (\alpha - \beta^L)} \quad (3.6)
\]
It is shown in the appendix that for any pair of nonnegative interim revenues of the seller $(u_S(\varphi^*|H), u_S(\varphi^*|L))$ such that $\gamma_0 u_S(\varphi^*|H) + (1 - \gamma_0) u_S(\varphi^*|L) \leq \Pi^*$, there exists a feasible mechanism $\varphi^*$. To summarize,

**Theorem 3.5** A feasible mechanism $\varphi^*$ is seller efficient if and only if

$$\gamma_0 u_S(\varphi^*|H) + (1 - \gamma_0) u_S(\varphi^*|L) = \Pi^*$$

The set of interim revenues $(u_S(\varphi|H), u_S(\varphi|L))$ of feasible mechanisms is

$$\{(u_S(\varphi|H), u_S(\varphi|L)) | \gamma_0 u_S(\varphi^*|H) + (1 - \gamma_0) u_S(\varphi^*|L) \leq \Pi^*, u_S(\varphi|H) \geq 0, u_S(\varphi|L) \geq 0\}$$

The set of expected revenues conditional on the seller’s types of the seller efficient mechanisms is therefore represented by the downward sloping line in Figure 3.6. The triangle formed by this line and the two axes represents the set of feasible mechanisms.

Note each seller efficient mechanism maximizes the average interim revenue of the two seller types with weights $(\gamma_0, 1 - \gamma_0)$, because

$$\max_{\varphi \in \mathcal{F}} \gamma_0 u_S(\varphi|H) + (1 - \gamma_0) u_S(\varphi|L) = \Pi^*$$
Before diving into the informed principal problem, two benchmark cases are worth considering. First, if the seller has private types that is unverifiable, as in Koessler and Skreta (2014), a feasible mechanism has to also satisfy the IC constraints of the seller, that is, 

\[ u_S(\varphi|H) = u_S(\varphi|L), \]

or the seller would misreport his type. So the dotted line in Figure 3.6, connecting (0,0) to \((\Pi^*,\Pi^*)\), represents the set of feasible mechanisms with unverifiable seller types, which is a subset of the set of feasible mechanisms with verifiable seller types. I call a mechanism a seller optimal mechanism with unverifiable types, if 

\[ u_S(\varphi|H) = u_S(\varphi|L) = \Pi^*. \]

Second, if the seller's type is common knowledge before the players make reports, a mechanism is feasible if and only if the IC and IR constraints hold conditional on each type of the seller. These constraints are more restrictive than the constraints with private seller types. The optimal mechanism problem under these constraints has a simple conclusion: conditional on the seller's type is common knowledge, each seller type's optimal mechanism is equivalent to posting a fixed price. Because it is assumed that \(1 - \pi_0 < \frac{\alpha}{\beta_H}\) and \(\pi_0 < \frac{\beta_L}{\alpha}\), the High type seller should fix the price at \(\alpha\) and both buyer types get the good with probability 1; meanwhile the Low type seller should fix the price at \(\alpha\) as well and the Consumer type gets the good with probability 1 while the Prosumer type never gets the good. I call this mechanism the full-information optimal mechanism \(\tilde{\varphi}\) and the expected revenues in this optimal mechanism the full-information optimal revenue, denoted as \(\tilde{u}^H = \alpha\) for the High type and \(\tilde{u}^L = \pi_0\alpha\) for the Low type, which are illustrated by the dashed line in Figure 3.6.

Because constraints with public seller types are more restrictive, full-information optimal revenue is in the interior of set of seller's feasible payoffs in Figure 3.6. \(\Pi^* - (\gamma_0\alpha + (1 - \gamma_0)\pi_0\alpha)\) is the ex ante benefit for the seller to have private types. All seller efficient mechanisms attains this benefit. The informed principal problem to be addressed next is precisely asking how the two seller types will split this benefit if the seller is choosing the mechanism.

### 3.5.4 The informed principal problem

I have derived the seller efficient mechanisms so far and showed all such mechanisms can help the seller to achieve the same ex ante benefit in comparison with the full-information optimal revenue.
mechanism, but it remains a question which seller efficient mechanism will arise naturally, especially when the seller has more control of the interaction.

Consider the informed principal problem as formulated by Myerson (1983): First, the seller, who privately knows his type, announce a mechanism. Then the buyer can decide whether to participate. If she does not participate, both players get zero payoff. If she participates, each player makes a report to the mediator and the outcome according to the mechanism is implemented. The key consideration is that the buyer can infer about the seller’s type from the mechanism he announced. Myerson (1983) proposes multiple solution concepts to characterize the reasonable outcome of the informed principal problem. While Myerson (1983) assumes nonverifiable types, verifiable types can be treated as nonverifiable types plus a large penalty for misreporting in the outcome of the mechanism, so the analysis tools can be adapted here.

The first result to be adapted is the inscrutability principle, which says there is no loss of generality in assuming that all types of the principal (seller) would choose the same mechanism in equilibrium. The justification is that, if there exists an equilibrium that the two types of the seller would announce different mechanisms \( \varphi_H \) and \( \varphi_L \), then it is also an equilibrium that both types announcing some \( \varphi \) such that \( \varphi(H, \cdot) = \varphi_H(H, \cdot) \) and \( \varphi(L, \cdot) = \varphi_H(L, \cdot) \), because the communication allowed by the mechanism choice is incorporated as part of the mechanism, and the truthtelling incentives are preserved.

Myerson (1983) defines a mechanism that is consistent with a sequential equilibrium of this mechanism selection game as an expectational equilibrium. In the mechanism selection stage, the seller can deviate to choose any generalized mechanism that allows arbitrary sets of reports and arbitrary sets of recommended outcome. Observing the deviation, the buyer updates her belief to \( \mu \). Then the players must play a Bayesian Nash equilibrium of the generalized mechanism given \( \mu \), which specifies a reporting strategy of the players (may include rejection, and not necessarily truthtelling) and gives an equilibrium outcome \((\tilde{q}, \tilde{p})\). An expectational mechanism is defined below:

**Definition 3.4** A mechanism \( \varphi \) is an expectational mechanism if it is feasible and for every generalized mechanism \( \tilde{\varphi} \), there exists a buyer belief \( \mu \) such that there exist a BNE of \( \tilde{\varphi} \) given \( \mu \) such that every seller type weakly prefers \( \varphi \) to this BNE outcome.
Apparently, the definitions of the expectational mechanism only restricts the seller’s interim payoff $u_S(\varphi|\pi_S)$, so if a feasible mechanism $\varphi$ is an expectational mechanism, any other feasible mechanism $\varphi'$ with the same interim payoff for the seller is also a core(expectational) mechanism.

Koessler and Skreta (2014) solves a parallel informed principal problem with unverifiable seller types. So their results naturally serve as a benchmark, with which a comparison can highlight the distinctive nature of verifiability.

**Expectational Equilibrium**

With verifiable seller types, I show in Appendix A.6 there is a unique outcome in terms of the interim payoffs:

**Theorem 3.6** A feasible mechanism is an expectational equilibrium if and only if it is seller efficient and the Low type seller gets exactly his full-information optimal interim revenue. That is,

\[ u_S(\varphi|L) = \pi_0 \alpha \]
\[ u_S(\varphi|H) = \frac{1}{\gamma_0} \left( \Pi^* - (1 - \gamma_0) u_S(\varphi|L) \right) \]

In comparison with the full-information optimal mechanism, the above theorem gives a very sharp prediction: In an expectational equilibrium, the High type seller enjoy the full pie of the seller’s ex ante benefit of having private instead of public types, while the Low type seller is indifferent about whether quality information can be hidden. The equilibrium mechanism is a seller-efficient mechanism, and the good is transferred with positive probability between a Low type seller and a Prosumer, but the surplus generated from this transfer is allocated to the High type seller, because the Low type seller still only receive revenue equal to the surplus generated from transferring his good to a Consumer as if his type is public.

The stark difference in the effects of private types on the two seller types contrasts that in the bargaining equilibrium, where the High type is able to fare much better with private types, and the Low type seller also benefits to a less extent from having private types. Note that in the bargaining protocol, each type of the seller has to trade with a Prosumer type at almost the same time, but this restriction is removed in the mechanism design problem, and
each type of seller can have different trading probability with a Prosumer. This comparison therefore suggests the Low type seller’s benefit in the bargaining equilibrium is a consequence of this restriction imposed by the protocol.

If the seller types are unverifiable, Koessler and Skreta (2014) shows that the seller optimal mechanisms are expectational equilibria. In this setting, the seller optimal mechanisms with unverifiable types give interim revenues \((\Pi^*, \Pi^*)\), and this is the best possible outcome for either types of the seller. Without verifiability, such an expectational equilibrium is still seller efficient and achieve the maximum benefit of private information. However, as the two seller types must have the same interim revenue, the Low type seller, with a lower full-information optimal revenue, enjoys more of the benefit of private information than the High type. In fact, in this particular situation, the High type’s interim revenue, \(\Pi^*\), is lower than his full-information optimal revenue \(\alpha\), so the High type is hurt by the combination of privacy and nonverifiability, and the Low type is enjoying a benefit due to the ability of pretending as the High type.

The comparisons above imply the seller’s willingness-to-pay conditional on his type for verification and the ability to hide the verification outcome from the buyer before she reveals her type. Without verifiability, a High type seller expects to get at most \(\Pi^*\). If verification mandates disclosure, a High type seller would receive his full-information optimal revenue \(\alpha\), which improves upon \(\Pi^*\). With voluntary verification and disclosure, a High type seller could expect the unique expectational equilibrium/sequential equilibrium payoff of \(\frac{1}{\gamma_0} (\Pi^* - (1 - \gamma_0) \pi_0 \alpha)\), which improves upon \(\alpha\). Meanwhile, a Low type seller would hope verifiability does not exist so he can pretend to have better quality, and he is indifferent about disclosure policy given verifiability.

### 3.6 Conclusion

This paper analyzes the revelation of hard information when two parties transact a good. The buyer’s valuation of the good depends on both the buyer’s type (taste), and the seller’s type (quality). The seller can choose whether and when to credibly reveal quality information during communications. This paper analyzes an alternating bargaining game allowing endogenously
chosen delay between communications in the fashion of Admati and Perry (1987), and shows there exists an equilibrium involving delayed revelation of quality information as a way to extract surplus from the buyer type with a greater taste for quality. Under some parameters, this is the unique outcome.

Broadly speaking, the uncertainties in quality and taste affect the outcome of the bargaining in a way that is consistent with the relative abundance of supply and demand in a competitive market. The additional payoff attained by a high quality seller from a prosumer would be the highest if demands of high quality, represented by the ex ante likelihood of the buyer being a prosumer, are very common. This is the case captured by the unraveling prediction of Grossman and Hart (1980) and Milgrom and Roberts (1986), which state that the seller should fully reveal all hard information and immediately receive higher payments for higher qualities. On the other extreme, a high quality seller may not get any additional payoff for his quality, if his high quality is revealed up-front so that supply of quality is abundant, and the buyer’s likelihood of demanding the high quality is low so that demand for quality is low. This is a case captured by Admati and Perry (1987). This paper focuses on the intermediate case, where both supply and demand are scarce, and the outcome is also somehow intermediate as well, as a high quality seller does receive a portion of the added value, albeit after a delay.

The alternating bargaining protocol is a realistic yet tractable way to model the interaction between the two parties with the aforementioned information structure, but it is not the only way. To distinguish the intrinsic informational tension from some restrictions of the bargaining protocol, I also considered a more general mechanism design problem formulated by Myerson (1983) so that the seller has full control of the communication protocol and thus all the bargaining power. If the seller’s type is public, a High quality seller cannot profitably discriminate a Prosumer with higher price, so his optimal mechanism is to charge the Consumer’s valuation of the good. If the seller’s type is private, it is found that under some specification of the parameters, the unique equilibrium outcome is that the seller, regardless of quality, chooses a truth-telling mechanism that improves the revenue of the High quality seller upon the public type case and leave the Low quality seller’s revenue unchanged. Therefore, a High quality seller appreciates the privacy of the seller’s type, while a Low type quality is indifferent.

The effects of seller’s quality being private on social welfare can be derived for the bargaining
setup and the mechanism design setup. In the bargaining equilibrium, a Prosumer would receive the good after a delay, no matter the quality of the good. Had the quality been public, under the parameters of interest, a Prosumer would receive the high quality good without delay, this alone could improve the total surplus to a higher level than the equilibrium with private quality types. In the mechanism design problem, regardless of the seller's type being public or private, the good is transferred with certainty unless a Low quality seller is matched with a Prosumer. With private seller types, the good is transferred from a Low quality seller to a Prosumer with a positive probability, while with public seller types, the good would not be transferred at all for this match. Hence, the total surplus is higher in the expectational equilibrium with private seller types. The contrasting welfare implications of the two models suggest the welfare implication is dependent on the protocol and the allocation of bargaining power. Indeed, if the buyer has all the bargaining power and dictates that the good is always transferred at a very low price, the total surplus would always be maximized regardless of the information structure.

A more relevant welfare question, although not explicitly specified in my models, is about the incentive for the High quality seller to obtain high quality at the first place. In both the bargaining model or the mechanism design model, the common conclusion is that the High quality seller would always prefer to hide his type temporarily to improve his payoff. Also, in both models the difference between expected payoffs of the two seller types would be much smaller and independent of $\beta^H$ if the seller's type is public, because a Prosumer can always receive the treatment of a Consumer when the quality is high. Therefore, with private seller types, a seller would have much stronger incentive to obtain high quality, and in the long run, more high quality goods would be developed to meet the Prosumer's taste and social welfare would benefit.
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Appendix A

Formal proofs for Chapter 3

A.1 Model Details

A.1.1 Definition of Intuitive Criterion

First, I formalize the notion that a deviation is reasonable:

Definition A.1 For an equilibrium \( \mathcal{E} \), given a history \( H^N \), a deviation \( (p^{N+1}, t^{N+1}) \) without revelation by player \( i \) is "reasonable for type \( b \) who claims type \( \bar{b} \)" if the following is true:

1. player \( i \)'s type has not been revealed by \( t^{N+1} \).

2. After the deviation \( (p^{N+1}, t^{N+1}) \), there exist some equilibrium of the continuation game such that

   (a) This equilibrium is consistent with player \( j \) updates belief to \( \Pr(i \text{ is type } \bar{b}) = 1 \) after the deviation (this may or may not be the belief updating specified in \( \mathcal{E} \) and the equilibrium of the continuation game need not to be consistent with \( \mathcal{E} \))

   (b) a type \( b \) player is not worse off in this equilibrium in comparison with the equilibrium payoff according to \( \mathcal{E} \) after \( H^N \).

Building on this definition, intuitively, a type \( b \) player \( i \) can convince the other player through a deviation that is only reasonable for type \( b \) who claims type \( b \) :
Assumption A.1 (A2) For any equilibrium $E$, given any history $H^N$, if a deviation $(p^{N+1}, t^{N+1})$ by player $i$ satisfies the following:

1. $(p^{N+1}, t^{N+1})$ is reasonable for type $b$ who claims type $b$ and is not reasonable for type $b'$ who claims type $b$.

2. Switching $b$ and $b'$ in 1. makes the statement false.

Then in equilibrium $E$, player $j$ must update belief to $Pr(i \text{ is type } b) = 1$ after player $i$’s deviation $(p^{N+1}, t^{N+1})$.

Condition 2 in A2 simply rules out the case in which a type $b'$ player $i$ can use a similar intuitive argument. This assumption is stronger than Cho (1987).

A.1.2 Post revelation equilibria details: Admati & Perry (1987)

AP consider a bargaining game with the same protocol, but only the buyer has private information: the buyer knows her value to be $h$ or $l$. The bargaining starts with the seller making the first offer and the seller’s initial belief is common knowledge. Translating to this subgame, $h$ and $l$ would be $\alpha$ and $\beta^L$ or $\beta^H$ and $\alpha$ depending on the seller’s type and the seller’s initial belief is $Pr(\text{Consumer}) = \pi$. I review the relevant results from AP below. Conditional on the seller’s type being commonly known, my definition of sequential equilibrium, A1 and A2 coincide with the definitions and assumptions used in AP, so I can directly apply these results into the construction of equilibrium in my model.

According to AP, there are at least two types of equilibria. In a separating equilibrium, the seller offer $\frac{h}{1+\delta}$ at $t = 0$; the high type buyer accepts and the low type counter offers $\frac{\delta l}{1+\delta}$ after a delay at $t = \Gamma$, which is accepted by the seller. The delay should just prevent the high type buyer from imitation:

$$U \left( h, \frac{h}{1+\delta}, 0 \right) = U \left( h, \frac{\delta l}{1+\delta}, \Gamma \right)$$

so it increases in $\frac{h}{l}$ and $e^{-r\Gamma} = \left( \frac{1}{\delta} + \left( 1 - \frac{1}{l} \right) \right)^{-1}$. In a pooling equilibrium, the seller offer $\frac{l}{1+\delta}$ at $t = 0$ and both types accept.

The equivalence of A2 is discussed in the appendix.
AP also identify sharp range for π such that a unique equilibrium obtains, which is consistent with the intuition that separating only happens when probability of high type buyer is high enough.

[AP Proposition 4.1, 4.2] If $\Pr(V = h) > \frac{2h}{h+\delta(h-l)-\delta^2}$, the separating equilibrium exists; furthermore, if $\Pr(V = h) > \frac{h}{h+\delta(h-l)-\delta^2}$ the separating equilibrium is unique.

[AP Proposition 5.1, 5.2] If $\Pr(V = h) \leq \frac{h}{h+\delta(h-l)-\delta^2}$, the pooling equilibrium exists; furthermore, if $\Pr(V = h) < \frac{h}{h+\delta(h-l)-\delta^2}$ the pooling equilibrium is unique.

A.2 Proof of Theorem 3.1

A.2.1 Major Deviations

I first study the consequences of the four major deviations in my set-up. Three of them lead to immediate revelation of the seller's type, so the outcomes are straightforward applications of AP, reviewed earlier. Whenever AP allow multiple equilibria, I explicitly choose one as the equilibrium path following the deviation.

**Consumer does not imitate Prosumer**

In any Separating before Revelation with Delay τ equilibrium, regardless of the buyer's type, if the buyer has made the equilibrium offer $p^P$ of a Prosumer after delay τ, the seller must be convinced of the buyer being a Prosumer. A type $L$ seller would accept this offer, while a type $H$ seller would decide to revealed his type being $H$. This bring us to the subgame of $APE(O, H)$.

A natural yet inaccurate prediction would be that the game proceeds as if the buyer is a Prosumer and it is common knowledge, so they will trade at $\frac{2H}{1+\delta}$ without delay regardless of the buyer's true type. However, an application of the intuitive criterion would allow a Consumer to use a delay to signal her true type and the outcome is depicted in Figure 3.2 as Dev A.

**Lemma A.1** In $APE(O, H)$, there exist a unique sequential equilibrium, in which the seller makes offer $\frac{2H}{1+\delta}$ without delay, which a Prosumer accepts, and which a Consumer rejects and
counter-offers \( \frac{\delta \alpha}{1 + \delta} \) after a delay

\[
\tau^H = \frac{1}{r} \log \left( \frac{1}{\delta} + 1 - \frac{\alpha}{\beta^H} \right)
\]

which the seller accepts.

**Proof.** This is a straightforward application of AP Proposition 4.2. The full equilibrium is described in AP.

To simplify notation later, define \( \bar{p}^C \) such that type C is different between paying \( \bar{p}^C \) immediately and paying \( \frac{\delta \alpha}{1 + \delta} \) after a delay \( \tau^H + \Delta \):

**Definition A.2** \( \bar{p}^C \) is defined by

\[
U(\alpha, \bar{p}^C, 0) = U(\alpha, \frac{\delta \alpha}{1 + \delta}, \frac{1}{r} \log \left( \frac{1}{\delta} + 1 - \frac{\alpha}{\beta^H} \right) + \Delta)
\]

Now let us summarize what would happen if a Consumer deviates by imitating a Prosumer’s equilibrium offer \( p^P \) after a delay \( \tau \) (which is shown as Dev A in the diagram): because the seller is convinced \( \pi = 0 \), a Low quality seller would accept \( p^P \) while a High quality seller would make the Consumer feel as if she pays \( \bar{p}^C \). The expected payoff for the Consumer to carry out this deviation equal to \( U(\alpha, \gamma_0 \bar{p}^C + (1 - \gamma_0) p^P, \tau + \Delta) \). Note, \( \bar{p}^C \) is greater than \( \frac{\delta \alpha}{1 + \delta} \) but lower than \( \frac{\beta^H}{1 + \delta} \), so \( \gamma_0 \bar{p}^C + (1 - \gamma_0) p^P \) is smaller than \( \frac{\alpha}{1 + \delta} \). Therefore a sufficient delay, \( \tau \), is necessary to make sure a Consumer would rather accept \( \frac{\alpha}{1 + \delta} \) without delay.

**Lemma A.2** In a Separating before Revelation with Delay \( \tau \) assessment, while responding to \( \bar{p} \), a Consumer cannot be better off by making the counter offer as a Prosumer does if and only if

\[
e^{-r\tau} \leq \frac{\delta \alpha}{1 + \delta} / (\alpha - \gamma_0 \bar{p}^C - (1 - \gamma_0) p^P)
\]

**Proof.** The Consumer is not better off if

\[
U(\alpha, \gamma_0 \bar{p}^C + (1 - \gamma_0) p^P, \tau) \leq U(\alpha, \bar{p}, 0)
\]

and some algebra transform it to the condition in the lemma.
To construct the Separating before Revelation with Delay $\tau$ equilibrium in this section, I pick $\tau$ to be the value that satisfies the Lemma A.2 at equality, and denoted this value as the function $\tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0)$ in Theorem 3.1. With some algebra,

**Definition A.3** $\tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0)$ is the value of of $\tau$ that satisfies Lemma A.2 at equality, i.e.

$$\tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0) = -\frac{1}{r} \log \left( \frac{\delta_0}{(\alpha - \gamma_0 \beta^C) (1 + \delta) - (1 - \gamma_0) \delta \beta^L} \right)$$

(A.1)

It is worth noting that $\tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0)$ decreases in $\gamma_0$. From now on I will concentrate on a Separating before Revelation with Delay $\tau^*$ equilibrium. As will be shown in the proof of Theorem 3.3, this $\tau^*$ is actually the only choice that is consistent with the intuitive criterion.

Prosumer does not accept $p^0$

A Prosumer can always choose to simply accept $p^0$ without delay. It is straightforward to compare this payoff to her payoff according to the definition of Separating before Revelation with Delay $\tau^*$. It is rational for a Prosumer to decide to counter offer rather than to accept if and only if

$$\gamma_0 U \left( \beta^H, \frac{\beta^H}{1 + \delta}, \tau^* + \Delta \right) + (1 - \gamma_0) U \left( \beta^L, \frac{\delta \beta^L}{1 + \delta}, \tau^* + 2\Delta \right)$$

$$\geq U \left( \gamma_0 \beta^H + (1 - \gamma_0) \beta^L, p^0, \Delta \right)$$

(A.2)

Because the LHS increases in $\gamma_0$ not as fast as the RHS, the inequality transforms to a upper bound for $\gamma_0$. In fact, this is the only condition for $\gamma_0$ in this section, so the upper bound is in fact the function $\gamma^* (\alpha, \beta^H, \beta^L)$ in Theorem 3.1. Plug in $\tau^*$, after some algebra, $\gamma^*$ is defined by

**Definition A.4** $\gamma^* (\alpha, \beta^H, \beta^L)$ is the maximum $\gamma$ satisfying (A.2), or determined by

$$\frac{p_0}{1 - \gamma^*} + \frac{\gamma^* (p^P - p^C) p_0 - (\alpha p^P - E[\beta|\gamma^*] p^C)}{1 - \gamma^*} = \frac{\delta \beta^L}{\alpha - E[\beta|\gamma^*]}$$

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where \( \hat{p}_P \) is defined as
\[
U(\beta_H, \hat{p}_P, 0) = U\left(\beta_H, \frac{\beta_H}{1 + \delta}, \Delta\right)
\]

**Lemma A.3** In a Separating before Revelation with Delay \( \tau^* \) assessment, after receiving \( p_0 \), it is rational for a Prosumer to decide to counter offer rather than to accept if and only if

\[
\gamma_0 < \gamma^*(\alpha, \beta^H, \beta^L)
\]

In Admati & Perry (1987) or Cramton (1992), it is not a concern that the type with lower value would mimic the type with higher value, because the low value type should be more patient. However, here no-deviation for the low type, Prosumer, is a concern. Note the delay \( \tau^* \) equates \( U(\alpha, \gamma_0 \hat{p}_C + (1 - \gamma_0) p^P, \tau^*) \) to \( U(\alpha, p_0, 0) \), where \( \gamma_0 \hat{p}_C + (1 - \gamma_0) p^P \) is the equivalent price paid by a Consumer, and is less than the equivalent price paid by a Prosumer, as the latter pays equivalently \( \gamma_0 \beta^H + (1 - \gamma_0) p^P \) in expectation. So a Prosumer gets less reduction in payment after the delay, and \( \tau^* \) might be too long for her. The more precise analysis will be given in the proof of the unique theorem.

**Type \( H \) seller does not reveal at the beginning**

If a type \( H \) seller deviates by deciding to reveal his type at the very beginning of the game, we are at \( APE(\pi_0, H) \). I want the outcome following this deviation to be bad for the seller so as to deter revelation, so I find conditions that gives rise to a pooling equilibrium, of which the outcome is depicted in Figure 2 as Dev B:

**Lemma A.4** In \( APE(\pi_0, H) \), if the seller’s initial belief is sufficiently pessimistic so that

\[
Pr(t_B = P) = 1 - \pi_0 < \frac{\alpha}{\beta^H}
\]

then there exist a sequential equilibrium in which the seller makes offer \( \frac{\alpha}{1 + \delta} \) without delay and a buyer of either type accepts.
Proof. This is a straightforward application of AP Proposition 5.1. The full equilibrium is described in AP. ■

So under the condition of this lemma, if a type \( H \) seller reveals type at \( t = 0 \), I can construct the off-equilibrium path to be such that he sell at \( \frac{\alpha}{1+\delta} \) to both types and have to give up all the surplus between \( \alpha \) and \( \beta^H \) to type \( P \) buyer. Looking at this from the Prosumer buyer's perspective, she would love to learn the product is of high quality, but once this is shown, a sophisticated Prosumer would try to pretend she doesn’t care, which is a tactic used frequently in real life by buyers when seller has revealed so many good things about the product at the very beginning.

The condition for a type \( H \) seller to conceal his type at the very beginning of the game hence needs to compare his payoff in the target equilibrium and his payoff above, which is depicted as Dev B in the diagram. The comparison leads to the following condition

\[
\beta^H \geq \frac{\alpha}{\delta e^{-r^*}} \quad (A.5)
\]

Lemma A.5 If (A.4) holds and in a Separating before Revelation with Delay \( \tau^* \) assessment the outcome of \( APE(\pi_0, H) \) is the pooling equilibrium as in Lemma A.4, it is rational for a type \( H \) seller to decide to conceal his type at \( t = 0 \) if and only if

\[
\beta^H \geq \frac{\alpha}{\delta e^{-r^*}}
\]

Proof. No matter the seller reveals at \( t = 0 \) or not, according to the Separating before Revelation with Delay \( \tau \) definition and the pooling equilibrium of \( APE(\pi_0, H) \), the seller sells to the Consumer type at \( \frac{\alpha}{1+\delta} \) without delay. So the comparison boils down to the discounted revenue from a Prosumer. The seller should conceal his type if and only if selling to type \( P \) at \( \frac{\beta^H}{1+\delta} \) after a delay of \( \tau + 1 \) is better than \( \frac{\alpha}{1+\delta} \) without delay, which is

\[
\frac{\alpha}{1+\delta} < \frac{\beta^H}{1+\delta} e^{-r(\tau^*+1)}
\]

Plug in \( \tau^* \) to get (A.5). ■

In fact this is the only condition for \( \beta^H \) in this section, which gives the definition of \( \beta^*(\alpha, \beta^H, \beta^L, \delta, \gamma_0) \) in Theorem 3.1:
Definition A.5 \( \beta^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0) \) is the minimum \( \beta^H \) satisfying Lemma A.5, i.e.

\[
\beta^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0) = \frac{\alpha}{1 - e^{-\tau^*}}.
\]

Type \( L \) seller does not reveal at the beginning

If a type \( L \) seller deviates by deciding to reveal his type at the very beginning of the game, we are at \( APE(\pi_0, L) \). A Consumer type buyer now has the higher value \( \alpha \). Because my analysis focuses on the case the Consumer type is more common, a separating equilibrium is reasonable here. To simplify the analysis and avoid trivial discussions, I find a condition such that the separating equilibrium, depicted in Figure 2 as Dev C, indeed exists:

Lemma A.6 In \( APE(\pi_0, L) \), if the seller’s initial belief is sufficiently optimistic so that

\[
\Pr(t_B = C) = \pi_0 \geq \frac{\beta^L \alpha + \delta (\alpha - \beta^L)}{1 + \delta} - \frac{\delta \beta^L}{1 + \delta} \quad (A.6)
\]

Then there exist a sequential equilibrium in which the seller makes offer \( \frac{\alpha}{1 + \delta} \) without delay, which a Consumer accepts, and which a Prosumer rejects and counter offers \( \frac{\delta \beta^L}{1 + \delta} \) after a delay

\[
\tau^L = -\frac{1}{r} \log \frac{\alpha}{(1 + \delta) \alpha - \delta \beta^L}
\]

which the seller accepts.

Proof. This is a straightforward application of AP Proposition 4.1. The full equilibrium is described in AP.

So under the condition of this lemma, if a type \( L \) seller reveals his type at \( t = 0 \), I can construct the off-equilibrium path to be the above separating outcome, depicted as Dev C.

Comparing a Low type seller’s payoff in a Separating before Revelation with Delay \( \tau^* \) assessment and in \( APE(\pi_0, L) \) leads to the following condition:

Lemma A.7 If (A.6) holds and in a Separating before Revelation with Delay \( \tau^* \) assessment the outcome of \( APE(\pi_0, L) \) is the separating equilibrium as in Lemma A.6, it is rational for a
Proof. Note in a Separating before Revelation with Delay \( \tau \) assessment, for each buyer type, the type \( L \) seller gets the same payment as in \( APE(\pi_0, L) \), the only difference is the length of delay before a type \( P \) seller pays. So if the delay can be shortened by concealing the information, i.e. \( \tau^* < \tau^L \), the low type seller would love to conceal.

Note, mathematically, \( \tau = -\frac{1}{r} \log \left( \frac{\delta_0}{(1+\delta)(1-(1-\gamma_0)\beta\delta^L)} \right) \) decreases in \( \gamma_0 \) and equals \( \tau^L = -\frac{1}{r} \log \left( \frac{\delta_0}{(1+\delta)(1-(1-\gamma_0)\beta\delta^L)} \right) \) when \( \gamma_0 = 0 \). Hence, \( \tau^* < \tau^L \) is always satisfied, and Low type seller has no incentive to reveal.

The diagram below summarizes three of the deviations discussed above.

Figure 2

A.2.2 Off-equilibrium offers
The previous subsection covered several salient and obvious deviations whose outcome are predicted right away by AP. However, the players are free to play actions with ambiguous implication. For example, what should the seller think if the buyer makes a counter offer higher than \( p^D \) before \( \tau \)? what should the buyer think if the seller, without revealing type, makes an initial offer below \( p^0 \)? These are the questions to be covered in this part. Note I need only discuss such deviations before the seller reveals type, because the off-equilibrium beliefs and actions are already covered in Admati & Perry and are part of the separating equilibrium and the pooling equilibrium as they describe.

**Seller makes off equilibrium offer without revelation**

In the Separating before Revelation with Delay \( \tau \) equilibrium I propose, whenever the seller deviates by making a different offer \( p' \) and/or at different time without revelation, the buyer would put probability 1 on seller being the Low type. There are two possible deviations from the equilibrium path.

First, if seller makes an off equilibrium initial offer \( p' \) without revelation, the buyer updates her belief to \( \Pr (\text{Low}) = 1 \) and respond to this offer as in the separating equilibrium of \( APE(\pi_0, L) \). In particular, the buyer should react as if she is in the following subgame: the buyer's value is either \( \alpha \) or \( \beta^L \); the seller's belief is \( \Pr (\alpha) = \pi_0 \); the seller has made an initial offer \( p' \).

On the one hand, if the seller's true type is Low, he would just play the continuation game as in the same separating equilibrium of \( APE(\pi_0, L) \). Because we already know in such separating equilibrium of \( APE(\pi_0, L) \), the Low type seller's optimal action is to make initial offer \( p = \frac{\alpha}{1+\delta} \), so any other \( p' \) cannot give higher payoff than \( p' = \frac{\alpha}{1+\delta} \), which is just as good as deciding to reveal Low type. Hence, for a Low type seller's initial offer, I only need to consider deviations with revelation as in Lemma A.7, and no new conditions are needed to prevent deviations with an off-equilibrium offer without revelation.

On the other hand, if seller is High type, he can reveal the true type in later communications. If \( p' \leq \frac{\alpha}{1+\delta} \), the Consumer type will accept and the Prosumer type will come back after a delay \( -\frac{1}{\tau} \log \frac{\alpha-p'}{\alpha-p^H} + \Delta \), when the seller can reveal his type and guarantee a revenue of \( \frac{p^H}{1+\delta} \). So higher deviating \( p' \) would allow higher revenue from a Consumer type but also delay the time when
Prosumer comes back. This trade-off depends on π₀, the seller’s belief Pr(Consumer). The following condition guarantees the seller’s payoff increases in p' as long as p' \leq \frac{\alpha}{1+\delta}.

\[
\pi_0 \geq \frac{\delta \beta^H}{\delta \beta^H + \alpha (1+\delta) - \delta \beta^L} \tag{A.7}
\]

Also, if p' > \frac{\alpha}{1+\delta}, the Consumer type will counter offer immediately with p'' = \frac{\delta \alpha}{1+\delta} and the Prosumer type will come back after a delay as if p' = \frac{\alpha}{1+\delta}, so p' > \frac{\alpha}{1+\delta} is worse than p' = \frac{\alpha}{1+\delta}. Therefore, given (A.7), the seller’s best deviation without revelation is to offer p' = \frac{\alpha}{1+\delta} - \varepsilon at t = 0 and the delay before he gets revenue from a Prosumer is simply \( \pi^L + \Delta \). The outcome following this deviation differs from the seller’s outcome in a Separating before Revelation with Delay \( \tau^* \) assessment only in the delay before the the seller gets revenue from a Prosumer. Because we have shown \( \pi^{P^*} \leq \pi^L \) for Lemma A.7, this deviation cannot improve seller’s payoff.

The following Lemma summarizes the analysis about the seller’s off equilibrium initial offer:

**Lemma A.8** If in a Separating before Revelation with Delay \( \tau^* \) assessment, in addition to the conditions of Lemma A.5 and Lemma A.7, (A.7) holds and the buyer’s strategy following an off-equilibrium initial offer coincides with his strategy in the separating equilibrium as in Lemma A.6, it is rational for both types of the seller to make the initial offer \( p^0 = \frac{\alpha}{1+\delta} \) without revelation.

**Proof.** Conditions of Lemma A.5 and Lemma A.7 guarantee that the payoff from the Separating before Revelation with Delay \( \tau^* \) outcome is no worse than revelation at the beginning for both types. The above discussion further establishes that given how the buyer reacts to the off-equilibrium initial offer, (a) a Low type seller’s payoff from an off equilibrium initial offer is no better than revelation at the beginning; (b) given (A.7), a High type seller’s payoff from an off equilibrium initial offer is no better than his payoff from the Separating before Revelation with Delay \( \tau^* \) outcome. So for both types, the payoff from the Separating before Revelation with Delay \( \tau^* \) outcome is the best under the aforementioned conditions. ■

Second, if the seller deviates in response to the buyer’s counter offer \( p^P \) at \( \tau \), the buyer should respond to the offer \( p' \) as in the pooling equilibrium of APE(0,L). No matter the type
of the seller, the expected outcome for the seller is no better than revealing his type and sell at \( \frac{\beta^L}{1+\delta} \), so no new conditions in this case are needed to prevent deviations with an off-equilibrium offer without revelation.

**Buyer makes off equilibrium offer before revelation**

If following the equilibrium offer \( p^0 \) at \( t = 0 \), the buyer deviates with an off-equilibrium counter offer \( p' \geq \frac{\delta \beta^L}{1+\delta} \) at time \( \tau' \) (offers less than \( \frac{\delta \beta^L}{1+\delta} \) will never be accepted and are dominated), the seller should update his belief such that the counter offer does not make the buyer better off. To achieve this, in the Separating before Revelation with Delay \( \tau \) equilibrium I propose, the seller would put probability 1 on buyer being a Prosumer if

\[
U (\alpha, p^0, \Delta) \geq (1 - \gamma_0) U (\alpha, p', \tau' + \Delta) + \gamma_0 U (\alpha, \tilde{p}^C, \tau' + \Delta)
\]  

(A.8)

and put probability 1 on buyer being a Consumer otherwise.

The LHS of the inequality is the payoff for the Consumer type buyer to accept the offer \( p^0 \) without delay; the RHS of the inequality is the payoff for the Consumer buyer if the seller is convinced that she is a Prosumer after her deviation \( (p', \tau') \) and reveal his type. This belief updating rule thus ensures the Consumer type buyer has no incentive to deviate with an off-equilibrium counter offer: if the deviation convinces the seller that she is a Prosumer, her expected payoff would be worse; if the deviation convinces the seller that she is a Consumer, she will pay no less than \( p^0 \) and gets the good no faster.

**A graphical representation of buyer’s incentives**

To show this off-equilibrium belief prevents a Prosumer from deviation, I employ a graphical analysis. In the following diagram, I first draw the set of \( (p, \tau) \) that can just convince the seller that buyer is type \( P \), as defined by (A.8) at equality. Call this curve \( IDC \) (Indifference curve for a Consumer). \( (p^P, \tau^*) \) is then the intersection of \( IDC \) and \( p = \frac{\delta \beta^L}{1+\delta} \)

Second, I draw the set of \( (p', \tau') \) that satisfy the following equation

\[
U (E[\beta | \gamma_0], p^0, \Delta) = (1 - \gamma_0) U (\beta^L, p', \tau' + \Delta) + \gamma_0 U (\beta^H, \tilde{p}^P, \tau' + \Delta)
\]
The LHS is the payoff for the Prosumer type buyer to accept the offer $p^0$ without delay; the RHS is the payoff for the Prosumer buyer if the seller is convinced that she is a Prosumer after a counter offer $(p, \tau)$. Call this curve $IDP$ (Indifference curve for a Prosumer).

$IDP$ starts lower than $IDC$ (because $\hat{p}_C < \hat{p}_P$) and is also flatter than $IDC$ (because $\alpha > E[\beta | \gamma_0]$). Under the condition of Lemma A.3, the two curves intersect above $p^p = \frac{\delta \beta}{1 + \delta}$ because the Prosumer prefers $(p^p, \tau^*)$ to accepting $p^0$, i.e. $(p^p, \tau^*)$ is below $IDP$.

Now I show that $(p^P, \tau^*)$ can be made the Prosumer's best move. First, it is the best move among the counter offers above or on $IDC$, i.e. the best move such that the seller is convinced that the buyer is a Prosumer. Second, for counter offers below $IDC$, the seller will regard the buyer as a Consumer. I can now specify the assessment following this as a new Separating before Revelation with Delay $\tau$ assessment, starting by the seller counter offers $\frac{\alpha}{1 + \delta}$ without revelation. Now, unless the Prosumer can do something to change the seller's belief, she has to pay $\frac{\alpha}{1 + \delta}$ or $\frac{\delta \alpha}{1 + \delta}$, both are even worse than accepting $p^0$ at the very beginning. The Prosumer's best action for this counter offer is actually to counter offer $p^P$ after another delay of $\tau$, which is also worse than doing this at the first place. The appendix offers a more rigorous proof.

Figure 3
Other deviations

So far I have described the equilibrium outcome following all possible one-step deviations from the equilibrium strategy, but I have not yet covered the outcome following a second deviations and beyond. Fortunately, no additional conditions are needed to deter those deviations. The appendix has a rigorous account of the whole assessment.

A.2.3 Intuitive criterion

Note Admati & Perry used a different version of intuitive criterion, but in the one-sided incomplete information setting, our versions are equivalent. So anything happens after revelation is consistent with A2.

I need to check whether the off-equilibrium belief updating in my equilibrium is consistent with A2.

• Buyer updates belief to $\gamma = 0$ after seller’s deviation.

This would conflict A2 if some seller's deviation would benefit the $High$ type and hurt the $Low$ type if the buyer is convinced of $High$ type. However, the $Low$ type cannot be hurt more than $High$ type in this way.

• Seller updates belief to $\pi = 0$ or 1 after buyer’s deviation.

1. A2 would not mandate updating to $\pi = 1$, because if the seller is convinced that the buyer is a Consumer type after a deviation, the buyer will be charged at least $p_0$, which is no better than the equilibrium payoff.

2. When seller updates belief to $\pi = 1$ after a deviation in my equilibrium, a Consumer type would benefit if the seller instead updates his belief to $\pi = 0$, so A2 in this case cannot mandate updating to $\pi = 0$.

A.2.4 Summary

So far I have constructed a Separating before Revelation with Delay $\tau^*$ equilibrium with most of the details and showed that most natural deviations cannot improve the payoff of the deviator. Now I summarize the results in the context of Theorem 3.1.
\[ \tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0), \tau^* (\alpha, \beta^H, \beta^L), \gamma^* (\alpha, \beta^H, \beta^L) \text{ and } \beta^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0) \text{ in Theorem 3.1 are given in Definition A.3, A.4 \text{ and } A.5.} \]

\[ \pi^* (\alpha, \beta^H, \beta^L, \delta) \text{ summarizes the rest of the conditions in this section, namely, (A.4) in Lemma A.4, (A.6) in Lemma A.6 and (A.7) in Lemma A.8, all of which provide lower bound for } \pi_0. \]

**Definition A.6** \[ \tau^* (\alpha, \beta^H, \beta^L, \delta) = \max \left\{ 1 - \frac{\alpha \beta^L}{\alpha + \delta (\alpha - \beta^L) - \alpha \delta^2}, \frac{\delta \beta^H}{\delta \beta^H + \alpha (1 + \delta) - \delta \beta^L} \right\} \]

**Take the limit: } \delta \to 1 \]

As \( \delta \to 1 \), or equivalently, \( \Delta \to 0 \), there is no exogenous limitation on communication intensity. However, the delay predicted by Theorem 3.1 is almost unaffected. Taking the limit,

\[ \tau^* (\alpha, \beta^H, \beta^L, \gamma_0) = \lim_{\delta \to 1} \tau^* = -\frac{1}{r} \log \frac{\alpha}{(2\alpha - \gamma_0 \left(2\alpha - \alpha \frac{\beta^H}{2\beta^H - \alpha}\right)) - (1 - \gamma_0) \beta^L} \]

The third condition of Theorem 3.1 becomes

\[ \beta^H > \left(2\alpha - \gamma_0 \left(2\alpha - \alpha \frac{\beta^H}{2\beta^H - \alpha}\right)\right) - (1 - \gamma_0) \beta^L \]

\[ \Leftrightarrow (\beta^H - \alpha) \frac{2\beta^H - (1 - \gamma_0) \alpha}{2\beta^H - \alpha} > (1 - \gamma_0) (\alpha - \beta^L) \]

Because the LHS increases in \( \beta^H \), this condition implies a lower bound on \( \beta^H \). Denote the lower bound as \( \beta^* (\alpha, \beta^L, \gamma_0) \) such that

\[ (\beta^* - \alpha) \frac{2\beta^* - (1 - \gamma_0) \alpha}{2\beta^* - \alpha} = (1 - \gamma_0) (\alpha - \beta^L) \]

**A.3 Proof of Theorem 3.3**

**A.3.1 Unique equilibrium after revelation**

I simply applies AP Proposition 5.2 and 4.2 to get the sharper version of Lemma A.4 and Lemma A.6, such that for some \( \pi^{**} (\alpha, \beta^H, \beta^L, \delta) \), the outcomes in Lemma A.4 and Lemma A.6 are the unique outcomes if \( \pi_0 > \pi^{**} \).
Lemma A.9 In $APE(\pi_0, H)$, if the seller’s initial belief is pessimistic enough such that
\[ \Pr(t_B = P) = 1 - \pi_0 < \frac{\alpha}{\beta^H} \left( \frac{\beta^H + \delta(\beta^H - \alpha)}{\beta^H + \delta(\beta^H - \alpha)} \right) - \alpha \delta^2 \] \tag{A.9}
then in the unique sequential equilibrium the seller makes offer $\frac{\alpha}{1+\delta}$ without delay and buyer of either type accepts.

Lemma A.10 In $APE(\pi_0, L)$, if the seller’s initial belief is optimistic enough such that
\[ \Pr(t_B = C) = \pi_0 \geq \frac{\beta^L}{\alpha} \] \tag{A.10}
Then in the unique sequential equilibrium, the seller makes offer $\frac{\alpha}{1+\delta}$ without delay, which Consumer accepts, and Prosumer counter offers $\frac{\delta^L}{1+\delta}$ after a delay
\[ \tau^L = -\frac{1}{r} \log \frac{\alpha}{(1 + \delta) \alpha - \delta \beta^L} \]
which the seller accepts.

From these two lemmas, I define the higher threshold function $\pi^{**}$ as
\begin{equation}
\pi^{**}(\alpha, \beta^H, \beta^L, \delta) = \max \left\{ 1 - \frac{\alpha}{\beta^H} \frac{\beta^H + \delta(\beta^H - \alpha)}{\beta^H + \delta(\beta^H - \alpha)} \frac{\delta}{\alpha}, \frac{\delta \beta^H}{\delta + \alpha(1+\delta) - \delta \beta^L} \right\}
\end{equation}

A.3.2 Characterization of the separating path

Now, let’s consider the subgame where S has just made an offer without revealing $\alpha$. I first analyze the strategy of type P buyer, taking the type C buyer’s strategy as given. The goal of this subsection is to establish that given type C buyer accepting an offer without revelation, the type P buyer must signal his type with a delay in a unique manner that is consistent with the equilibrium described in Theorem 3.1. For the seller, this is saying that the delay before a Prosumer makes a counter offer following the seller’s initial non-revealing offer is somewhat short, so both types of the seller are ok with the delay.

To start, suppose in equilibrium, following some offer $p_0$ made by the seller, the buyer updates belief to some $\gamma$ and the Consumer type is to accept this offer. I want to find out
the Prosumer’s possible reaction to this offer. In Section 3.3, I showed that if \( p_0 = \frac{\alpha}{1+\delta} \), it is consistent with an equilibrium that a type P buyer counter offer \( p^P \) after a delay \( \tau \). Here I analyze a general \( p_0 \), and find conditions such that a Prosumer can distinguish herself with a similar delay and pin down the unique separating action that could arise in an equilibrium.

I follow a similar line of reasoning as in Section 3.3.4.2. Because in equilibrium, type \( C \) would accept \( p_0 \), so if in equilibrium type \( P \) does not accept \( p_0 \), the seller would update his belief to \( \pi = 0 \) upon receiving the Prosumer’s counter offer. Therefore, if in equilibrium, \( B \) makes a counter offer \( \hat{p} \), \( S \) would accept the counter offer if and only if \( \hat{p} \geq \frac{\delta \beta^S}{1+\delta} \), and otherwise reveal his type and counter offer with \( \frac{\delta \beta^S}{1+\delta} \). Hence, in equilibrium, a type \( P \) buyer would choose some \( \hat{p} \geq \frac{\delta \alpha^L}{1+\delta} \), because otherwise the seller would reject for sure and counter offer with higher prices, and the Prosumer can do better by offering \( \frac{\delta \alpha^L}{1+\delta} \).

Define \( P^P (t, p_0, \gamma) \) such that a Prosumer with belief \( \gamma \) is indifferent between accepting \( p_0 \) now or counter offering \( P^P (t, p_0, \gamma) \) after an delay \( \tau \). Draw \( P^P \) against \( \tau \) and call it IDP. Similarly define \( P^C (t, p_0, \gamma) \) such that a Consumer with belief \( \gamma \) is indifferent between accepting \( p_0 \) now or counter offer \( P^P (t, p_0, \gamma) \) after an delay \( \tau \), assuming the seller react to the counter offer as if it is the equilibrium offer made by type \( P \). Recall this means a High quality seller will reveal type and the Consumer has to use delay to signal her type, which is equivalent to paying \( \hat{p}^C \) without delay. Draw \( P^C \) against \( \tau \) and call it IDC. The equations for \( P^P (t, p_0, \gamma) \) and \( P^C (t, p_0, \gamma) \) are

\[
\begin{align*}
((1 - \gamma) (\beta^L - P^P (t, p_0, \gamma)) + \gamma (\beta^H - \hat{p}^P)) e^{-\tau \gamma} &= \mathbb{E} [\beta | \gamma] - p_0 \\
((1 - \gamma) (\alpha - P^C (t, p_0, \gamma)) + \gamma (\alpha - \hat{p}^C)) e^{-\tau \gamma} &= \alpha - p_0
\end{align*}
\]

some algebra brings it to

\[
\begin{align*}
P^P (\tau, p_0, \gamma) &= \frac{1}{1 - \gamma} (\mathbb{E} [\beta | \gamma] (1 - e^{-\tau \gamma}) + (p_0 e^{-\tau \gamma} - \gamma \hat{p}^H)) \\
P^C (\tau, p_0, \gamma) &= \frac{1}{1 - \gamma} (\alpha (1 - e^{-\tau \gamma}) + (p_0 e^{-\tau \gamma} - \gamma \hat{p}^C))
\end{align*}
\]

The two curves are drawn in the diagram below.

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Note the following features:

1. \( P^P(\Delta, p_0, \gamma) < P^C(\Delta, p_0, \gamma) \). Because if B counter offer \( \hat{p} \) immediately and S believes B is Prosumer, both types of B pay P for low quality good, but the Consumer type pays \( \hat{p}^C \) for high quality while the Prosumer type pays \( \hat{p}^H > \hat{p}^C \), so the Consumer type would equate paying \( P_0 \) for sure to counter offering a higher \( \hat{p} \).

2. \( \frac{\partial p^C(\tau, p_0, \gamma)}{\partial \tau} < \frac{\partial p^P(\tau, p_0, \gamma)}{\partial \tau} < 0 \). Because \( \alpha > E[\beta|\gamma] \), the Consumer type has higher expected value and is less patient.

Therefore, the two curves may intersect somewhere. Assuming they do,

\[
p^P(\tau, p_0, \gamma) = p^C(\tau, p_0, \gamma) = p^*
\]

\[
e^{\tau + \tau^*} = 1 + \frac{\gamma (\hat{p}^H - \hat{p}^C)}{\alpha - E[\beta]}
\]

\[
p^* = \frac{1}{1 - \gamma} \left( \left( 1 + \frac{\gamma (\hat{p}^H - \hat{p}^C)}{\alpha - E[\beta]} \right) p_0 - \left( \frac{\alpha \hat{p}^H - E[\beta] \hat{p}^C}{\alpha - E[\beta]} \right) \gamma \right)
\]
The intersection is meaningful if only $p^* \geq \frac{\delta \beta^L}{1+\delta}$, in which case the two curves and $p = \frac{\delta \beta^L}{1+\delta}$ can form the shaded area. I focus on this case and prove the following Lemma. If $p^* < \frac{\delta \beta^L}{1+\delta}$, there will be no separating equilibrium path following $p^C$.

Before proving the lemma, I first somehow simplify the condition $p^* \geq \frac{\delta \beta^L}{1+\delta}$. With some algebra, it can be shown that if $\alpha > E[\beta|\gamma]$ and $\bar{p}^C > p_0 > \frac{E[\beta^L]}{1+\delta}$, $p^*$ would decrease in $\gamma$, hence

$$p^* \geq \frac{\delta \beta^L}{1+\delta} \iff \gamma \leq \gamma^{**} (p_0, \alpha, \beta^H, \beta^L) \tag{A.11}$$

and $\gamma^{**}$ is determined by

$$p^* = \frac{\delta \beta^L}{1+\delta}$$

$\gamma^{**}$ is independent of $\pi$ because $p^*$ is independent of $\pi$. Apparently, by definition,

$$\gamma^{**} \left( \frac{\alpha}{1+\delta}, \alpha, \beta^H, \beta^L \right) = \gamma^* (\alpha, \beta^H, \beta^L)$$

Now I apply the intuitive criterion to pin down the unique equilibrium strategy of a Prosumer in the following lemma.

**Lemma A.11** Consider a history $H^N$ without revelation of $t_S$ and the buyer is active with updated belief $\gamma$. Suppose in equilibrium a Consumer type is to buy at $p_0$ at $t^C$. If the following conditions are true,

- The Consumer type has higher expected value $\alpha > E[\beta|\gamma]$.
- $p_0$ is neither too large nor too small $\bar{p}^C > p_0 > \frac{E[\beta^L]}{1+\delta}$.
- high quality is rare $\gamma \leq \gamma^* (\alpha, \beta^L, \beta^C, p_0)$,

then type $P$ must counter offer $p^P = \frac{\delta \beta^L}{1+\delta}$ at

$$t^P = t^C + \left( -\frac{1}{r} \log \frac{\alpha - p_0}{\alpha - \gamma \bar{p}^C - (1-\gamma) \frac{\beta^L}{1+\delta}} \right) - \Delta$$

**Proof.** First I assume in equilibrium type $P$ does not accept offer $p_0$. As discussed above, type $P$ must counter offer some $\hat{p}$ such that $\frac{\delta \beta^L}{1+\delta} < \hat{p} < p_0 < \frac{\delta \beta^H}{1+\delta}$, and $S$ of low quality would
accept and $S$ of high quality would reveal $\alpha = \alpha^H$ and counter offer immediately with $\frac{\alpha}{1+c}$. Now I figure out what exact counter offer $\hat{p}$ and time of counter offer could arise in an equilibrium. I argue $(p^P, t^P)$ is the only counter offer that can arise in an equilibrium.

The equilibrium counter offer made by type $P$ must be in the shaded area, for otherwise either type $C$ would like to mimic type $P$ or type $P$ would rather accept $p_0$. Suppose the equilibrium counter offer is not $(p^P, t^P)$ but some other $\hat{p}'$ and $\hat{t}'$ in the shaded area. Then there must exist some deviation $(p^P + \epsilon, t^P + \epsilon)$ with very small $\epsilon$ such that: A. If following the deviation, the seller is convinced that the buyer is a Prosumer, then such deviation would be good for type $P$ but bad for type $C$; B. if following the deviation, the seller is convinced that the buyer is a Consumer, then such deviation would still be bad for type $C$. Hence, Assumption 3.2 gives that the seller must be convinced of the buyer being a Prosumer by a deviation to $(p^P + \epsilon, t^P + \epsilon)$, so a type $P$ buyer can improve payoff with this deviation, which implies that $\hat{p}'$ and $\hat{t}'$ cannot be the equilibrium counter offer. Only $(p^P, t^P)$ can survive this refinement, and both types of the buyer has no incentive to deviate.

Second, I show that in equilibrium type $P$ must not accept offer $p_0$. Suppose that’s the equilibrium, then a Prosumer can similarly signal his type with a deviation to some $(p^P + \epsilon, t^P + \epsilon)$ and improves payoff, so type $P$ accepting $p_0$ cannot arise in an equilibrium.

Hence, $(p^P, t^P)$ is the unique equilibrium action of a Prosumer in response to the seller’s offer $p_0$, given the Consumer’s acceptance. ■

Corollary A.1 the delay $\left( -\frac{1}{r} \log \frac{\alpha - p_0}{\alpha - \gamma p_C - (1-\gamma) \frac{p C}{1+c}} \right)$ decreases in $\gamma$.

Corollary A.2 the threshold $\gamma^* (p_0, \alpha, \beta^L, \beta^L)$ increases in $p_0$.

A.3.3 Consumer type must accept offer no less than $p^0$

The above lemma states that the type $P$ buyer has the ability to distinguish herself through delay. The next two lemma discuss the strategy of type $C$ buyer. The core message of these two lemmas, is that in the battle between the Consumer and the seller without revelation, $p = \frac{\alpha}{1+c}$ is a middle point that no party can get better deal.
Lemma A.12 Suppose the seller just made an offer $P_0 \leq \frac{\alpha}{1+\delta}$ without revelation and the seller's belief is $\pi_0$ that satisfies Lemma A.9 and Lemma A.10. If

$$\beta^H \delta^2 + \delta \beta^L > \alpha (1 + \delta) \quad (A.12)$$

then a type C buyer must accept the offer in equilibrium.

To prove this lemma, a key point is to make sure the Consumer cannot get a better deal with the following strategy: both types of the buyer pools with counter offering some $\hat{P} < \frac{\alpha}{1+\delta}$ such that the Low type seller accepts this offer and the High type seller reveals type and continue as in the pooling equilibrium of $APE (\pi_0, H)$. This strategy is not trivially impossible because the Prosumer has been given incentive to pool on the counter offer $\hat{P}$: although such offer makes her buy Low quality product at a higher price, she can conceal her type to get a better price on High quality product. Hence, if $Pr(High)$ is high enough, the Prosumer should be happy to pool. However, a high $Pr(High)$ also means a Low type seller can potentially ask for a higher price. This line of argument leads to the following lemma:

Lemma A.13 Let $p$ be the lowest offer that the buyer would ever make in equilibrium under the following restrictions:

1. The seller's type has not been revealed yet.

2. The seller has belief $\pi_0$ that satisfies Lemma A.9 and A.10, and the buyer has any belief.

3. The two types of the buyer pool on this offer.

4. The Low type seller accepts this offer.

If

$$\beta^H \delta^2 + \delta \beta^L > \alpha (1 + \delta)$$

then $p \geq \frac{\delta_0}{1+\delta}$.

Proof. Let me first give some intuition for this proof. To get the Low type seller with belief $\pi_0$ to accept this pooling offer $p$, this offer would be significantly higher than $\frac{\beta^L}{1+\delta}$, so the
Prosumer is really overpaying the Low type seller by making this pooling offer. To make it up, it must be that the Prosumer buyer looks forward to be able to buy from the High type seller at $\frac{\alpha}{1+\delta}$ without being recognized as a Prosumer. However, the condition $\beta^H \delta^2 + \delta \beta^L > \alpha (1 + \delta)$ (which says that even if the buyer threat with $\gamma = 0$, a High type buyer still find it is better to force the buyer to signal her type through delay) guarantees that it is impossible to prevent the High type from trying to distinguish the two types of the buyers.

To start the formal proof, suppose following some offer by the seller and no revelation, the buyer updates her belief to $\gamma$ and makes an offer $p$ according to an equilibrium strategy. Suppose $p < \frac{\delta \alpha}{1+\delta}$. I will show either the seller would not accept it or the Prosumer can deviate from $p$ for better payoff.

Because the two buyer types pool on this offer, upon receiving it, the seller still holds belief $\pi_0$. Now consider what would happen if the seller counter offers $p' = \frac{\pi_0}{1+\delta} + (1 - \gamma) \frac{p}{\delta}$ without revelation. $\gamma$ is very small yet positive. There are several possible equilibrium outcomes:

1. If both types of the buyer accept this offer, then the Low type seller would not accept the buyer’s early offer $p$, because $\delta p' > p$.

2. If the Consumer type accepts and the Prosumer type counter offers, or if the two buyer types makes different counter offers, then the High type seller can use the counter offer $p'$ to separate the buyer’s types. Under condition,

$$\beta^H \delta^2 + \delta \beta^L > \alpha (1 + \delta)$$

it can be shown no matter what belief the buyer has upon receiving $p'$, the High type seller would get higher payoff than $\frac{\alpha}{1+\delta}$. Hence, in the equilibrium outcome following the buyer’s offer $p$, the High type seller will receive higher payoff than $\frac{\alpha}{1+\delta}$ for sure, which requires the separation of the buyer types. Hence, a Prosumer, in the equilibrium outcome following the buyer’s offer $p$, would eventually get the good after revelation if the seller is High type. This just eliminate the Prosumer’s incentive to pool on $p$. With some algebra, it can be shown that the Prosumer can gets a payoff higher than making the offer $p$ by using a delay to signal her type.
3. If the both types pool on a counter offer, note the lowest they can make is $p$ and the lowest price they need to pay the high type is $\frac{\alpha}{1+\delta}$. Hence, if the buyer updates belief to any $\gamma > \gamma$, pooling on a counter offer is not as good as just accepting $p'$. On the other hand, if the buyer updates belief to some $\gamma < \gamma$, the Prosumer would not want to pay high price to the Low type in exchange for low price to the High type, so still it cannot be an equilibrium that both types pool on a counter offer.

4. If the Prosumer type accepts and the Consumer type makes a counter offer, then a Low type seller will later charge the Consumer type $\frac{\alpha}{1+\delta}$, which is even higher than $p'$, so the Consumer type should accept $p'$ instead. Hence, this cannot be an equilibrium outcome.

So no matter what's the equilibrium outcome following the seller's counter offer $p'$, either the Low type would not accept $p$ or the Prosumer can deviate from $p$ for better payoff. It must be that $p \geq \frac{\alpha}{1+\delta}$. 

Now I can apply Lemma A.13 to prove Lemma A.12.

**Proof.** Lemma A.12 is in some sense no surprise, because if the buyer updates her belief to either of the extremes, a Consumer must expect to pay $\frac{\alpha}{1+\delta}$ according to Lemma A.9 and A.10. This lemma claims that even for intermediate beliefs, a Consumer cannot expect to pay any less. I will show that in any equilibrium path following the seller's offer, a type $C$ buyer cannot purchase at an equivalent price lower than $P_0$.

First, assume in equilibrium, the type $C$ buyer can buy with positive probability at some price $\hat{p} < \frac{\alpha}{1+\delta}$ at some $\hat{t}$ before revelation, and the type $P$ buyer has zero probability of getting this outcome. If $\hat{p}$ was offered by the buyer, then this offer would reveal the buyer's type to be $C$, but the seller knowing the buyer is of type $C$ would not accept any offer below $\frac{\alpha}{1+\delta}$, so $\hat{p}$ cannot be accepted. If $\hat{p}$ was offered by the seller, note a type $H$ seller would never offer price lower than $\frac{\alpha}{1+\delta}$, as he can simply reveal his type to guarantee this payoff. Therefore $\hat{p}$ must be offered by the seller of type $L$. Then this offer itself is equivalent to revelation, which will be discussed in the third point.

Second, assume in equilibrium, both the type $C$ and type $P$ buyer can buy with positive probability at some price $\hat{p} < \frac{\alpha}{1+\delta}$ at some $\hat{t}$ before revelation, such that type $C$ prefers this equilibrium to accepting $P_0$. If $\hat{p}$ was offered by the seller, the offer would signal the seller being
Low type, which will be discussed in the third point. For now, let’s focus on the case where two types of the buyer pool in making offer \( \hat{p} \). Again, \textit{High} type seller would not accept this offer, so it must be that a \textit{Low} type seller would accept this offer. However, this is exactly forbidden by Lemma A.13.

Third, assume in equilibrium, the type \( C \) buyer can buy with positive probability at some price \( \hat{p} < \frac{\delta}{1+\delta} \) at some \( \hat{t} \) after revelation. Choose the equilibrium with the smallest \( \hat{t} \). It must be that the revelation was type \( L \). Suppose at the initial belief \( \gamma \), if seller type is revealed to be \( L \), the equilibrium is separate so type \( L \) seller first sell to type \( C \) at \( \frac{\delta}{1+\delta} \). Then to get \( \hat{p} < \frac{\delta}{1+\delta} \), it must be that \( \gamma \) was somehow updated to be even lower. That is, there is an equilibrium event that happened before revelation such that type \( C \) was indifference between buying at that event and wait until revelation. Hence, the payoff for type \( C \) in this equilibrium is equal to the payoff of buying at that event before revelation, then as shown in the first two point, such payoff cannot be better than accepting \( P_0 \).

A.3.4 No revelation with the initial offer

The lemmas proven so far in this section leads to the inevitability of concealing of the seller’s type at the initial offer.

**Proposition A.1** Under conditions (A.9) of Lemma A.9, (A.10) of Lemma A.10, (A.12) of Lemma A.12 and \( \gamma_0 < \gamma^* (\alpha, \beta^H, \beta^L) \), in equilibrium, the \textit{High} type seller must not reveal his type with the initial offer.

**Proof.** Let’s assume for a while that there exist an equilibrium in which a \textit{High} type seller reveal his type with the initial offer with positive probability. Consider an alternative move of the \textit{High} type seller by making \( p_0 = \frac{\alpha}{1+\delta} \) without revealing. Lemma A.12 pins down that following this offer, a \textit{Consumer} type buyer must accept this offer. And the reaction of a \textit{Prosumer} type buyer would depend on the buyer’s updated belief \( \gamma \) upon receiving \( p_0 \).

If \( \gamma < \gamma_0 \), then according to Lemma A.11, a \textit{Prosumer} must separate from a \textit{Consumer} with a delay as defined in Lemma A.11. Because the delay decreases in \( \gamma \), seller’s payoff is the lowest when \( \gamma = 0 \) given \( \gamma < \gamma_0 \). Some algebra shows that the (A.12) of Lemma A.12 guarantees that even at \( \gamma = 0 \), a \textit{High} type seller still gets higher payoff from making offer \( p_0 \) than from...
revealing his type.

If \( \gamma \geq \gamma_0 \), then separation is not possible in equilibrium (need some extra part in Lemma A.11), and the Prosumer must pool with the Consumer and accept \( p_0 \). This means a Low type seller can guarantee a revenue of \( p_0 \) if he makes a non-revealing initial offer \( p_0 = \frac{\alpha}{1+3} \), which is higher than his payoff from revelation. Hence in this equilibrium where a High reveal his type with the initial offer with positive probability, a Low type seller would not reveal type with the initial offer. Furthermore, if the Low type seller makes his equilibrium initial offer, the buyer must update belief to some \( \gamma < \gamma_0 \), and he must get at least \( \frac{\alpha}{1+3} \) after discounting. This is impossible unless the Low type seller makes an offer \( \frac{\alpha}{1+3} \), but this would reject \( \gamma \geq \gamma_0 \). [Need some elaboration here]

So to summarize, if there existed some equilibrium in which a High type seller reveal his type with the initial offer with positive probability, the High type seller could have at least deviated with a non-revealing initial offer \( p_0 = \frac{\alpha}{1+3} \) to improve his payoff. Hence, such equilibrium does not exist.

A.3.5 Pin down the seller's initial offer

Given nonrevelation of the seller, there are two possibilities. One is that both types of the seller pool on the same initial non-revealing offer, which is what happened in the equilibrium I proposed in last section, where both types pool on \( p^0 \). Here I will show that given pooling, this initial offer is actually the unique possibility. The other is that the two types offer differently such that upon receiving the offer, the buyer has one of the extreme beliefs about the seller's type. In this case, the outcome would be equivalent to revealing seller's type, hence it is not as good as pooling on \( p^0 \), as compared in the previous section.

Lemma A.14 Suppose in equilibrium the seller makes a pooling initial offer \( p_0 > \frac{\alpha}{1+3} \) without revelation and \( \gamma_0 < \gamma^* (\alpha, \beta^H, \beta^L) \). Then in equilibrium a Consumer would counter offer \( \frac{\alpha}{1+3} \) without delay and the seller would accept, also a Prosumer would counter offer \( p^P \) after a delay \( \tau^* \).

Proof. First I argue that in equilibrium, the Consumer cannot accept this offer. Suppose she does in equilibrium, then according to Lemma A.11, in equilibrium a Prosumer would
separate herself through a delay and gain strictly higher payoff than accepting \( P_0 \). Now consider a deviation of counter offering with \( \alpha - \frac{\alpha - P_0}{\delta} - \varepsilon \) without delay, where \( \varepsilon \) is positive and small. If the seller is convinced that the buyer is a Consumer, then the seller must accept this offer because a Consumer’s value is \( \alpha \) regardless of the seller’s type. This outcome makes a Consumer better off (because \( \varepsilon > 0 \)) but a Prosumer worse off (because the Prosumer in equilibrium would get a payoff strictly higher than accepting \( P_0 \). Check the math here more careful). On the other hand, if the seller is convinced that the buyer is a Prosumer, the Low type seller would accept and the High type seller would counter offer \( \frac{\beta_H}{1+\delta} \). This is bad for both types of the buyer. Hence, Assumption 3.2 gives that this deviation must convince the seller of buyer being a Consumer, and the Consumer therefore can improve her payoff by this deviation.

Second, given that a Consumer is to counter offer, the counter offer may or may not separate her from a Prosumer. If it doesn’t, that is, in equilibrium the Consumer and the Prosumer pools on the same counter offer, then according to Lemma A.13, the offer must be at least \( \frac{\delta a}{1+\delta} \) or no seller would accept. Anyhow some careful analysis would show that the Prosumer type would then rather deviate by a delayed offer than pool with the Consumer. So the counter offer, in response to \( P_0 \), must be separating, then the Consumer in equilibrium must counter offer \( \frac{\delta a}{1+\delta} \) or she is just waiting time for nothing. That offer will be accepted, and given that, the Prosumer must counter offer with a delay as predicted. ■

Now I can pin down the unique initial offer

**Lemma A.15** Under the conditions of Theorem 3.5, in equilibrium, the seller must make the initial offer \( p^0 = \frac{\alpha}{1+\delta} \), without revelation.

**Proof.** If the two types of the seller pool, the previous lemma gives that \( P_0 > \frac{\alpha}{1+\delta} \) is not as good as \( \frac{\alpha}{1+\delta} \). Also, \( P_0 < \frac{\alpha}{1+\delta} \) is not as good as \( \frac{\alpha}{1+\delta} \) too, because a Consumer would accept and the Prosumer may or may not separate, in either case, the save on delay before the Prosumer trades cannot compensate the loss on revenue from the Consumer type.

Second, if they separate, the buyer has one of the extreme beliefs about the seller’s type. In this case, the outcome would be equivalent to revealing seller’s type, hence it is not as good as pooling on \( p^0 \), as compared in the previous section. ■
A.3.6 Summary

Lemma A.15 (initial offer by the seller must be \( p^0 = \frac{\alpha}{1+\delta} \)), Lemma A.12 (a Consumer must accept this offer) and Lemma A.11 (A Prosumer must separate with a delay \( \tau^* \)) together give that Separating before Revelation with Delay \( \tau^* (\alpha, \beta^H, \beta^L, \delta, \gamma_0) \) is the unique equilibrium outcome given the conditions stated in Theorem 3.3.

A.4 Notations for the mechanism design model

I denote \( u_i (\varphi|t_i) \) as the expected payoff of type \( t_i \) of player \( i \) if both players truthfully report their types and trade is carried out according to \( \varphi \). In particular,

\[
\begin{align*}
   u_S (\varphi|H) &= \begin{bmatrix} \pi_0 & 1 - \pi_0 \end{bmatrix} \begin{bmatrix} \varphi_p (H, C) \\ \varphi_p (H, P) \end{bmatrix} \\
   u_S (\varphi|L) &= \begin{bmatrix} \pi_0 & 1 - \pi_0 \end{bmatrix} \begin{bmatrix} \varphi_p (L, C) \\ \varphi_p (L, P) \end{bmatrix} \\
   u_B (\varphi|P) &= \begin{bmatrix} \gamma_0 & 1 - \gamma_0 \end{bmatrix} \begin{bmatrix} \beta^H \varphi_q (H, P) \\ \beta^L \varphi_q (L, P) \end{bmatrix} - \begin{bmatrix} \varphi_p (H, P) \\ \varphi_p (L, P) \end{bmatrix} \\
   &= \begin{bmatrix} \gamma_0 \beta^H (1 - \gamma_0) \beta^L \end{bmatrix} \begin{bmatrix} \varphi_q (H, P) \\ \varphi_q (L, P) \end{bmatrix} - E_{ts} [\varphi_p (ts, P)] \\
   u_B (\varphi|C) &= \begin{bmatrix} \gamma_0 & 1 - \gamma_0 \end{bmatrix} \begin{bmatrix} \alpha \varphi_q (H, C) \\ \alpha \varphi_q (L, C) \end{bmatrix} - \begin{bmatrix} \varphi_p (H, C) \\ \varphi_p (L, C) \end{bmatrix} \\
   &= \begin{bmatrix} \gamma_0 \alpha (1 - \gamma_0) \alpha \end{bmatrix} \begin{bmatrix} \varphi_q (H, C) \\ \varphi_q (L, C) \end{bmatrix} - E_{ts} [\varphi_p (ts, C)]
\end{align*}
\]

I denote \( u_B (\varphi|t'_B, t_B) \) as the payoff for a type \( t_B \) buyer to misreport her type as \( t'_B \), assuming the seller reports truthfully, so
A mechanism is feasible if the IC constraints for the buyer hold.

\begin{align}
    u_B(\varphi|C,P) &= \left[ \gamma_0 \beta^H (1 - \gamma_0) \beta^L \right] \begin{bmatrix} \varphi_q(H,C) \\ \varphi_q(L,C) \end{bmatrix} - E \left[ \varphi_p(t_S, t_B) | t_B = C \right] \\
    u_B(\varphi|P,C) &= \left[ \gamma_0 \alpha (1 - \gamma_0) \alpha \right] \begin{bmatrix} \varphi_q(H,P) \\ \varphi_q(L,P) \end{bmatrix} - E \left[ \varphi_p(t_S, t_B) | t_B = P \right]
\end{align}

**Definition A.8** A mechanism is feasible if the IC constraints for the buyer

\begin{align}
    u_B(\varphi|P) - u_B(\varphi|C,P) &\geq 0 \quad (A.13) \\
    u_B(\varphi|C) - u_B(\varphi|P,C) &\geq 0 \quad (A.14)
\end{align}

and the IR constraints for the buyer

\begin{align}
    u_B(\varphi|P) &\geq 0 \quad (A.15) \\
    u_B(\varphi|C) &\geq 0 \quad (A.16)
\end{align}

hold.

I denote the set of feasible mechanisms as $\mathcal{F}$.

### A.5 Proof of Theorem 3.5

The maximization problem’s Lagrangian is

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\[ L = \pi_0 E_{t_s} [\varphi_p (t_S, C)] + (1 - \pi_0) E_{t_s} [\varphi_p (t_S, P)] \]

\[ + \lambda_1 \left( \begin{bmatrix} \varphi_q (H, P) \\ \varphi_q (L, P) \end{bmatrix} - E_{t_s} [\varphi_p (t_s, P)] \right) \]

(\text{IC for Prosumer})

\[ - \left[ \begin{bmatrix} \varphi_q (H, C) \\ \varphi_q (L, C) \end{bmatrix} + E_{t_s} [\varphi_p (t_s, C)] \right] \]

\[ + \lambda_2 \left( \begin{bmatrix} \varphi_q (H, C) \\ \varphi_q (L, C) \end{bmatrix} - E_{t_s} [\varphi_p (t_s, C)] \right) \]

(\text{IC for Consumer})

\[ + \lambda_3 \left( \begin{bmatrix} \varphi_q (H, P) \\ \varphi_q (L, P) \end{bmatrix} - E_{t_s} [\varphi_p (t_s, P)] \right) \]

(IR for Prosumer)

\[ + \lambda_4 \left( \begin{bmatrix} \varphi_q (H, C) \\ \varphi_q (L, C) \end{bmatrix} - E_{t_s} [\varphi_p (t_s, C)] \right) \]

(IR for Consumer)

where \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 \) are the multipliers for the IC and IR constraints for the buyer. In fact, in addition to these, \( \varphi_q \) are also constrained to be in \([0, 1]\). To save space, I have not written these constraints explicitly. I will show that \( \lambda_1 = 0 \) and \( \lambda_2, \lambda_3, \lambda_4 > 0 \), i.e. IC constraint for Prosumer is not binding, which will give \( \varphi_q (L, C) = \varphi_q (H, C) = \varphi_q (H, P) = 1 \). Then I will compute the maximum using the three binding constraints.

1. Because \( \partial L / \partial E_{t_s} [\varphi_p (t_s, C)] = \pi_0 + \lambda_1 - \lambda_2 - \lambda_4 = 0 \), \( \lambda_2 + \lambda_4 > 0 \). Also, Because
   \[ \partial L / \partial \varphi_q (L, C) = -\lambda_1 (1 - \gamma_0) \beta^L + \lambda_2 (1 - \gamma_0) \alpha + \lambda_4 (1 - \gamma_0) \alpha > (1 - \gamma_0) \beta^L (-\lambda_1 + \lambda_2 + \lambda_4) = (1 - \gamma_0) \beta^L \pi_0 > 0 \]. Hence, \( \varphi_q (L, C) = 1 \).

2. Because \( \partial L / \partial E_{t_s} [\varphi_p (t_s, P)] = 1 - \pi_0 + \lambda_2 - \lambda_1 - \lambda_3 = 0 \), \( \lambda_1 + \lambda_3 > 0 \). Also, Because
   \[ \partial L / \partial \varphi_q (H, P) = \lambda_1 \gamma_0 \beta^H - \lambda_2 \gamma_0 \alpha + \lambda_3 \gamma_0 \beta^H > \gamma_0 \beta^H (\lambda_1 - \lambda_2 + \lambda_3) = \gamma_0 \beta^H (1 - \pi_0) > 0 \]. Hence, \( \varphi_q (H, P) = 1 \)

3. Because \( \partial L / \partial E_{t_s} [\varphi_p (t_s, C)] + \partial L / \partial E_{t_s} [\varphi_p (t_s, P)] = 1 - \lambda_3 - \lambda_4, \lambda_3 + \lambda_4 = 1 > 0 \)

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4. Given $\varphi_q(L,C) = 1, \alpha > \gamma_0 \beta^H + (1 - \gamma_0) \beta^L$, it can be verified that

$$
\begin{bmatrix}
\gamma_0 \alpha \\ 1 - \gamma_0 \alpha
\end{bmatrix}
\begin{bmatrix}
\varphi_q(H,C) \\ \varphi_q(L,C)
\end{bmatrix}
> \begin{bmatrix}
\gamma_0 \beta^H \\ (1 - \gamma_0) \beta^L
\end{bmatrix}
\begin{bmatrix}
\varphi_q(H,C) \\ \varphi_q(L,C)
\end{bmatrix}
$$

Hence, IR for Consumer and IC for Prosumer cannot be both binding, i.e. $\lambda_1 \lambda_4 = 0$.

5. If $\lambda_4 > 0$, then $\lambda_1 = 0$. If $\lambda_4 = 0$, then $\lambda_3 = 1$, and $\pi_0 + \lambda_1 - \lambda_2 = 0$. I show

$$
\frac{\partial L}{\partial \varphi_q(L,P)} = (1 - \gamma_0) (\lambda_1 \beta^L - \lambda_2 \alpha + \lambda_3 \beta^L)
< (1 - \gamma_0) (\lambda_1 \beta^L - \lambda_2 \alpha + \beta^L)
< (1 - \gamma_0) (\lambda_1 \alpha - \lambda_2 \alpha + \pi_0 \alpha) \text{ (use condition } \beta^L < \pi_0 \alpha)
= 0
$$

Hence, $\varphi_q(L,P) = 0$. Then

$$
\begin{bmatrix}
\gamma_0 \beta^H \\ 1 - \gamma_0 \beta^L
\end{bmatrix}
\begin{bmatrix}
\varphi_q(H,P) \\ \varphi_q(L,P)
\end{bmatrix}
> \begin{bmatrix}
\gamma_0 \alpha \\ 1 - \gamma_0 \alpha
\end{bmatrix}
\begin{bmatrix}
\varphi_q(H,P) \\ \varphi_q(L,P)
\end{bmatrix},
$$

which, together with A.17, implies that the two IC constraints cannot be both binding, i.e. $\lambda_1 \lambda_2 = 0$. Because $\lambda_2 + \lambda_4 > 0, \lambda_2 > 0$, hence, $\lambda_1 = 0$.

Hence, it must be that $\lambda_1 = 0$.

6. Because $\lambda_1 = 0$, $\partial L/\partial \varphi_q(L,C) = (\lambda_2 + \lambda_4) \gamma_0 \alpha > 0$. So $\varphi_q(L,C) = 1$

7. Because $\lambda_1 = 0, \lambda_2 + \lambda_4 = \pi_0$. Because $\lambda_3 + \lambda_4 = 1, \lambda_4 > \lambda_2$. Hence, IR for Consumer is binding.

8. Assume $\lambda_2 = 0$, then only the IR constraints, it must be that $\varphi_q = 1, E_{ts} [\varphi_p(tS,P)] = \gamma_0 \beta^H + (1 - \gamma_0) \beta^L$ and $E_{ts} [\varphi_p(tS,C)] = \alpha$, which implies that $E_{ts} [\varphi_p(tS,C)] > E_{ts} [\varphi_p(tS,P)]$. However, This will violates IC for Consumer. Hence, it must be that $\lambda_2 > 0$.

9. Given $\lambda_3, \lambda_4, \lambda_2 > 0$ and $\lambda_1 = 0$, note that $\partial L/\partial \varphi_q(L,C) > 0$, so $\varphi_q(L,C) = 1$. Then I
solve for $E_{tS} [\varphi_p (tS, P)]$, $E_{tS} [\varphi_r (tS, C)]$ and $\varphi_q (L, P)$ from the three binding constraints and get

$$\varphi_q^* (L, P) = \frac{\gamma_0 (\beta^H - \alpha)}{(1 - \gamma_0) (\alpha - \beta^L)}$$  \hspace{1cm} \text{(A.18)}$$

$$E_{tS}^* [\varphi_p (tS, C)] = \alpha. E_{tS}^* [\varphi_r (tS, P)] = \frac{\gamma_0 \alpha (\beta^H - \beta^L)}{\alpha - \beta^L}$$ \hspace{1cm} \text{(A.19)}$$

10. Hence, the maximum of $\pi_0 E_{tS} [\varphi_p (tS, C)] + (1 - \pi_0) E_{tS} [\varphi_r (tS, P)]$ is computed to be $\Pi^*$. 

A.6 Proof of Theorem 3.6

Proof. I use the figure below to pin down the unique interim revenues of the seller.

![Figure 9](image-url)

Figure 9

First, for any feasible mechanisms,

$$\gamma_0 u_S (\varphi|H) + (1 - \gamma_0) u_S (\varphi|L) \leq \Pi^*$$

this inequality is represented by the solid feasible frontier line. This line and the two axes form a triangle representing the interim revenues of all feasible mechanisms.

Second, an expectational equilibrium must give each seller type at least his full-information
optimal interim revenue, because for the full-information optimal mechanism \( \varphi \), at any belief of the buyer, the buyer would report truthfully and each type of the seller will get the full-information optimal revenue in any equilibrium. Hence,

\[
\begin{align*}
    u_S(\varphi|H) &\geq \alpha \\
    u_S(\varphi|L) &\geq \pi_0 \alpha
\end{align*}
\]

as illustrated by two dashed lines. These conditions pin down the possible expectational equilibria into the shaded triangle area.

Third, consider the mechanism \( \varphi^* \) with \( \varphi^*_q(\cdot, \cdot) \equiv 1, \varphi^*_p(\cdot, \cdot) \equiv V(\cdot, \cdot) \). Note the seller would report truthfully in any equilibrium of \( \varphi^* \), because it is always true that \( \varphi^*_p(H, t_B) \geq \varphi^*_p(L, t_B) \). Also, because the seller can always guarantee zero payoff by a truthful report, she would not never reject the mechanism. Hence, in any equilibrium of \( \varphi^* \), the interim revenue of the two seller types, denoted as \( (u^H, u^L) \), must be a convex combination of \( (\varphi^*_p(H, C), \varphi^*_p(L, C)) = (\alpha, \alpha) \) and \( (\varphi^*_p(H, P), \varphi^*_p(L, P)) = (\beta^H, \beta^L) \). This set of represented by the dotted segment in the figure.

If a mechanism \( \varphi \) in the shaded triangle is in the lower-left side of the dotted segment, then in any equilibrium of \( \varphi^* \), at least one type of the seller would be getting more in \( \varphi^* \) than in \( \varphi \), and \( \varphi \) cannot be an an expectational equilibrium. In fact, the right corner of the shaded triangle, which represents the interim revenue predicted by this theorem, is exactly on the dotted segment, because

\[
(u_S(\varphi|H), u_S(\varphi|L)) = \left( \frac{1}{\gamma_0} (\Pi^* - (1 - \gamma_0) u_S(\varphi|L)), \pi_0 \alpha \right) \]

\[
= \frac{\pi_0 \alpha - \beta^L}{\alpha - \beta^L} (\alpha, \alpha) + \left( 1 - \frac{\pi_0 \alpha - \beta^L}{\alpha - \beta^L} \right) (\beta^H, \beta^L)
\]

so the left corner of the shaded triangle is the only possible interim revenues of an expectational equilibrium.\(^2\)

\(^2\)This interim revenue coincide with the following equilibrium of \( \varphi^* \): the buyer holds belief \( \Pr(\text{High}) = \frac{\alpha - \beta^L}{\alpha - \beta^L} \), so the expected payment for reporting Prosumer is \( \alpha \) and both buyer types are indifferent. So it is an equilibrium that both buyer types randomizes such that the total probability that the seller reports Consumer is \( \frac{\pi_0 \alpha - \beta^L}{\alpha - \beta^L} \).
Myerson (1983) has shown that there always exists at least one expectational equilibrium (or, because the definition of the expectational equilibrium is identical to that of the sequential equilibrium for the informed principal extensive form game, and there always exist a sequential equilibrium), so my proof can ends here, claiming that because the left corner of the shaded triangle is the only possible interim revenues of an expectational equilibrium, it must also be sufficient to guarantee any feasible mechanism with this interim revenue to be an expectational equilibrium. 

3In fact, it can be shown that this interim revenue is a neutral equilibrium, the strongest solution concept in Myerson (1983), by showing that if a new enforceable outcome with verification and the following utilities becomes available

\[
\begin{array}{ccc}
    C & P & P \\
    H & \alpha, 0 & \frac{\alpha - \beta L}{\beta - \alpha} \alpha, 0 \\
    L & \alpha, 0 & 0, 0
\end{array}
\]

then a mechanism always choosing this outcome is a strong solution. This would be a plain application of the method of Myerson (1983)