Essays in Informal Finance and Market Design Under Weak Institutions
by
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Abstract

The essays in this thesis span two important and related themes in development economics: understanding and relaxing constraints to small scale entrepreneurship and designing markets in environments with weak institutional enforcement. Methodologically, the essays marshal both theory and field experimentation to study these issues.

In joint work with Ernest Liu, Chapter 1 offers a new explanation for why microcredit and other forms of informal finance have so far failed to catalyze business growth among small scale entrepreneurs in the developing world, despite their high return to capital. We present a theory of informal lending that highlights two features of informal credit markets that cause them to operate inefficiently. First, borrowers and lenders bargain not only over division of surplus but also over contractual flexibility (the ease with which the borrower can invest to grow her business). Second, when the borrower’s business becomes sufficiently large she exits the informal lending relationship and enters the formal sector – an undesirable event for her informal lender. We show that in Stationary Markov Perfect Equilibrium these two features lead to a poverty trap and study its properties. The theory facilitates reinterpretation of a number of empirical facts about microcredit: business growth resulting from microfinance is low on average but high for businesses that are already relatively large, and microlenders have experienced low demand for credit. The theory features nuanced comparative statics which provide a testable prediction and for which we establish novel empirical support. Using the Townsend Thai data and plausibly exogenous variation to the level of competition Thai money lenders face, we show that as predicted by our theory, money lenders in high competition environments impose fewer contractual restrictions on their borrowers. We discuss robustness and policy implications.

In work with Reshmaan Hussam and Natalia Rigol, Chapter 2 explores a different facet of small-scale entrepreneurship. The impacts of cash grants and access to credit are known to vary widely, but progress on targeting these services to high-ability, reliable entrepreneurs is so far limited. We report on a field experiment in Maharashtra, India that assesses (1) whether community members have information about one another that can
be used to identify high-ability microentrepreneurs, (2) whether organic incentives for community members to misreport their information obscure its value, and (3) whether simple techniques from mechanism design can be used to realign incentives for truthful reporting. We asked 1,380 respondents to rank their entrepreneur peers on various metrics of business profitability and growth potential. We also randomly distributed cash grants of about $100 to measure their marginal return to capital.

We find that the information provided by community members is predictive of many key business and household characteristics including marginal return to capital. While on average the marginal return to capital is modest, preliminary estimates suggest that entrepreneurs given a community rank one standard deviation above the mean enjoy an 8.8% monthly marginal return to capital and those ranked two standard deviations above the mean enjoy a 13.9% monthly return. When respondents are told their reports influence the distribution of grants, we find a considerable degree of misreporting in favor of family members and close friends, which substantially diminishes the value of reports. Finally, we find that monetary incentives for accuracy, eliciting reports in public, and cross-reporting techniques motivated by implementation theory all significantly improve the accuracy of reports.

In Chapter 3 I highlight an under appreciated facet of centralized market design of critical importance to developing economies with weak contract enforcement: often market designers cannot force participants to join a centralized market. I present a theory in which centralizing a market is akin to designing a mechanism to which people may voluntarily sign away their decision rights and propose a new desideratum for mechanism and market design, termed \( \varepsilon \)-dominant individual rationality.

Loosely, \( \varepsilon \)-dominant individual rationality guarantees participation by assuring participants that each decentralized strategy is approximately dominated by a centralized strategy. I then provide two positive results about centralizing large markets. The first offers a novel justification for stable matching mechanisms and an insight to guide their design to achieve \( \varepsilon \)-dominant individual rationality. The second result demonstrates that in large games, any mechanism with the property that every player wants to use it conditional on sufficiently many others using it as well can be modified to satisfy \( \varepsilon \)-dominant individual rationality while preserving its behavior conditional on sufficient participation. The modification relies on a class of mechanisms we refer to as random threshold mechanisms and resembles insights from the differential privacy literature.

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Abhijit’s uncanny ability to see the model through what others might have perceived to be my incoherent ramblings has made its mark on all of the chapters in this thesis. Daron’s immediate access to every paper and model ever written by a (social?) scientist is legendary. But it is dwarfed by the ease with which he brings these resources to bear in understanding and advancing my work. Ben’s disciplined encouragement to put a field plan before my fieldwork has on many occasions saved my projects from failure. And his ability to identify when a project has failed has on equally many occasions saved me from wasting my time. Rob’s vision of theory in development economics is an inspiration. He was a source of constant support and encouragement, and at times, a healthy dose of macro. Scott’s enthusiasm for economics and for life is energizing. The third chapter of this thesis has been profoundly shaped by his guidance. And I am honored not only to be his student but also to call him a close friend.

I would be remiss not to acknowledge my collaborators and friends Ernest Liu, Vivek Bhattacharya, Natalia Rigol, Reshma Hussam, and Ran Shorrer. You all give credence to the aphorism that ‘You go to grad school to learn from your peers.’ Each of you has a distinct approach to and understanding of economics and each of you has imprinted a mini version of your voices into my subconscious. But it pales by comparison to the real thing. I hope we write many more papers together.

Finally, my family. Instead of undertaking the absurd task of articulating the extent of my gratitude for their support, I dedicate my thesis to my parents, my brother Aaron, and my wife-to-be Stephanie.
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Chapter 1

Keeping the Little Guy Down: A Debt Trap for Informal Lending

1.1 Introduction

Microcredit was long celebrated for its promise to lift the developing world out of poverty.\footnote{See e.g. the 2006 Nobel Peace Prize awarded to Muhammad Yunus and the Grameen Bank for the innovation and practice of microcredit.} Its proponents argued that, by offering a sustainable source of capital, microcredit would enable small scale entrepreneurs to leverage profitable investment opportunities and begin a path to a more prosperous future. These hopes were bolstered by a number of experimental studies that found that many microfirms in the developing world enjoy extremely high marginal returns to capital (on the order of five to ten percent per month; see De Mel, McKenzie, and Woodruff (2008), Fafchamps, McKenzie, and Woodruff (2014), and McKenzie and Woodruff (2008)). However, a variety of recent experimental and non-experimental evidence suggests that the impact of microcredit falls far short of previous expectations: on average, firms that randomly receive microcredit are no more profitable than those that do not (see Banerjee, Karlan, and Zinman (2015b) and Meager (2016) for a summary of the recent experimental evidence). A similar puzzle presents itself when considering other forms of informal finance available to these small firms: why haven’t informal financiers such as money lenders enabled these small scale entrepreneurs to leverage their high return to capital opportunities?

The proximate answer for why microcredit, and more broadly informal finance, has
so far failed to empower these entrepreneurs may lie in the various contractual features, other than the transfer of working capital, that are common in informal loans. Many of these features seem to stymie the borrower’s ability to invest her loan into high growth opportunities. One prominent example is that many microfinance institutions (MFIs) require that repayment begin immediately after the initial disbursement of funds and take place in frequent installments. The need to have cash on hand may restrict the borrower’s ability to undertake long term investments that serve to grow her business at the expense of short term output. Field, Pande, Papp, and Rigol (2013) describe a field experiment in which this restriction was relaxed for a random set of borrowers. Three years after the study took place, borrowers who received two-month grace periods had roughly 80% more business capital and enjoyed 41% higher profits than their counterparts who received standard contracts.

While money lenders are known to allow flexible repayment schedules, they may utilize other means to deter some forms of investment. For instance money lenders commonly require that borrowers work on their land (such as in tenancy arrangements) or that borrowers must forfeit their own land for the money lender to use for the duration of the loan (see e.g. Sainath (1996)). In the former case the lender ties up the borrower’s labor, preventing him from focusing on projects to expand his own productive capacity and in the latter case the lender ties up an asset that the borrower could otherwise put to productive use. A final contractual restriction common to both money lenders and microfinance is the use of guarantors (or joint liability, in the case of microfinance). Banerjee, Besley, and Guinnane (1994) theorized that guarantors might pressure borrowers to eschew profitable but potentially risky investments in favor of safer uses of the loan to ensure its repayment. And Fischer (2013) provides evidence from a lab in the field experiment that such pressures indeed exist.

The question then becomes why is the rigid enforcement of these contractual features commonplace? Often these restrictive features are attributed to ensuring the repayment of loans, since formal recourse is unavailable to many informal lenders. That they also restrict investment is largely seen as an unintended consequence. However, with the exception of immediate and frequent repayments, there is little conclusive evidence that

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2There may be disagreement about whether microfinance should be classified as a form of informal finance. In this paper, we refer to any form of lending as informal if relatively wealthy borrowers are likely to terminate the relationship in favor of more attractive sources of financing and if the lender has the capacity to influence the borrower’s project selection. We argue below that both of these criteria are satisfied by microfinance.
these contractual provisions actually serve to reduce default.\(^3\) And in Section 1.6 we suggest that even in the case of immediate and frequent repayments the story may not be clear.

We address both questions raised above by invoking an old explanation: informal lenders may benefit from keeping their borrowers in a debt trap, discouraging them from taking profitable investments to ensure they will continue to borrow for as long as possible (see e.g. Bhaduri (1973), and Bhaduri (1977)). However Braverman and Srinivasan (1981) and Braverman and Stiglitz (1982) argue profit maximizing lenders would not discourage such investments on the grounds that they should allow the efficient level of investment so long as they can extract the surplus. In this paper we argue that relatively wealthy borrowers may leave the informal lending relationship when they become eligible for cheaper, more formal sources of credit, providing a natural source of non-transferable utility. For money lenders it may be that their relatively wealthy borrowers become eligible for microloans, and for MFIs it may be that their borrowers become eligible for collateralized bank loans. In either case, relatively poor borrowers may not be able to commit to share the benefits of “formal sector” lending (should they ever reach it) with their informal lender, and thus informal lenders may not be able to extract the surplus of investment. This reopens the possibility that a profit maximizing lender would deliberately impose a debt trap.\(^4\)

Specifically, our model rests on three critical assumptions which are characteristic of much of the informal lending sector. First, borrowers who save and become sufficiently wealthy cease interaction with their informal lenders and enter the formal sector. While borrowers who engage in the formal lending sector enjoy its benefits, their informal lenders may regret losing customers. Second, borrowers and their informal lenders bargain not only over the division of surplus (i.e. the interest rate of the loan) but also over contractual restrictions which govern the ease with which the borrower can invest her loan to grow her business. Finally, neither the borrower nor the lender can commit to long term contracts. The borrower cannot commit to share the benefits of the formal sector (should she ever reach it) with her informal lender, and the lender cannot commit to provide favorable financing to the borrower in the future in exchange for the borrower’s

\(^3\)Gine and Karlan (2014) and Attanasio, Augsburg, De Haas, Fitzsimons, and Harmgart (2015) each provide experimental evidence that joint liability does not affect the likelihood of default. In contrast, using non-experimental variation, Carpena, Cole, Shapiro, and Zia (2013) finds that joint liability loans are more likely to be repaid.

\(^4\)Ray (1998) discusses another explanation for debt traps: borrowers who become sufficiently wealthy may default on their loans, causing lenders to discourage investments that help them grow their business.
cooperation in the short term.

For a stylized example, consider a fruit vendor. At low levels of wealth she operates a mobile cart. Each year she receives an endowment of cash and decides how to allocate it between two projects: a working capital project (e.g. buying fruits to sell throughout the week) and a fixed capital project (e.g. buying wood to expand from a mobile cart to a permanent stall from which to sell her fruits). If she invests in working capital, she begins to realize its returns immediately and consumes the proceeds from investment, as for the duration of the period she may not again have the minimum investment required to undertake the fixed capital project. In contrast, if she invests in the fixed capital project, she will forgo consumption for a period but have expanded her capacity for production in future periods. After several more business expansions she will gain access to the formal lending sector, and reap the corresponding benefits.

Each year she is also offered a loan from an MFI (her informal lender). Not only does the loan contract stipulate an initial cash transfer and an interest rate, but it also specifies whether she will be subjected to a variety of measures that make it difficult for her to undertake fixed capital investment and grow her business. For concreteness, assume it specifies whether the borrower must begin repayment immediately, or whether she can wait until the end of the period and repay in one lump installment. By controlling this additional feature, the lender may guide the borrower’s project selection. If the borrower can wait until the end of the year to repay her loan, she may be able to choose her project flexibly. In contrast, if the borrower must have cash on hand to repay her initial installments, she may need to invest in her working capital project at the beginning of the period. And if she has trouble saving cash and unutilized assets (for instance, because she feels pressured to share unutilized assets with family members) then by the time she has earned enough to be able to cover her initial installments she may no longer have the necessary cash to undertake her fixed capital project. That is, the moment at which the lender makes the initial cash transfer is special; this is the moment when the borrower has enough cash to undertake her fixed investment, and if she cannot undertake it immediately she may not be able to for the remainder of the period. Of course, accepting the contract is voluntary, so if the borrower does not find the initial cash transfer and interest rate sufficiently attractive to offset the additional contractual restrictions, she can reject it and allocate her own, smaller endowment flexibly among her projects.

We show that subject to a plausible contracting friction, an asymmetry arises between contracts that restrict the borrower’s ability to grow her business and those that do not.
If the lender is unable to set an interest rate which leaves the borrower with exactly the level of output she would have had in his absence, she will retain more utility from unrestrictive contracts in equilibrium. This asymmetry arises because she cannot commit to share the proceeds of business growth, and therefore values the investment of her residual income into fixed capital more highly than its investment into working capital. If this asymmetry is sufficiently large the borrower may get stuck in a debt trap; despite large welfare gains from growth, the lender imposes repeated contractual restrictions on the borrower and she remains in poverty (and borrowing from her informal lender) forever. We show such a debt trap occurs if and only if the additional surplus the borrower gains from unrestrictive contracts exceeds the additional social welfare generated from business growth.

Beyond establishing that firms offered access to credit often fail to reach their efficient size, the theory organizes a number of other well established empirical facts about microcredit. In our equilibrium, sufficiently wealthy borrowers always receive unrestrictive contracts. These are borrowers for whom business expansion is especially valuable due to their proximity to the formal sector, and thus a lender seeking to restrict their investment would need to compensate them with prohibitively low interest rates. This is consistent with many experimental estimates of the impact of microcredit which find that, while the marginal return to microcredit for the average firm is indistinguishable from zero, relatively richer business owners do exhibit high marginal returns to microcredit (see Angelucci, Karlan, and Zinman (2015), Augsburg, De Haas, Harmgart, and Meghir (2015), Banerjee, Duflo, Glennerster, and Kinnan (2015a), and Crepon, Devoto, Duflo, and Pariente (2015)).

A further empirical regularity noted in the above experiments is that demand for microcredit contracts is substantially lower than previously expected. This too emerges as a prediction of our theory. Because the lender transfers less surplus to borrowers via restrictive contracts than unrestrictive ones, borrowers who receive restrictive contracts may be nearer their indifference condition and their demand for credit may be low.

The model also sheds light on a number of nuanced comparative statics. Improving the attractiveness of the formal sector improves welfare of relatively richer borrowers because they anticipate eventually entering it. On the other hand, this improvement in the formal sector may harm the welfare of poorer borrowers and cause them to be trapped at even lower levels of wealth. Intuitively this is because of a “trickle down” effect whereby lenders anticipate that richer borrowers become more demanding, and restrict the invest-
ment of poor borrowers to ameliorate their increased bargaining power. This is especially striking given that fixing any lender behavior, an improvement in the formal sector unambiguously increases the borrower’s welfare. It is because of the lender’s endogenous response that this improvement harms the borrower.

Our comparative static on the borrower’s patience offers a counterpoint to a standard intuition that poverty traps are driven by impatience. In our model, the effect of increasing the borrower’s patience is ambiguous. Increasing the borrower’s patience increases her value of investment, and thus relatively richer borrowers who anticipate eventually entering the formal sector are made better off. However, similar to the comparative static on the attractiveness of the formal sector, increasing the borrower’s patience can make poor borrowers worse off. Anticipating that rich borrowers have improved bargaining power, the lender may restrict the investment of poor borrowers, tightening the debt trap.

Finally we provide two novel empirical facts in support of our theory. The first observation supports one of our key modeling assumptions. Relatively richer borrowers of a large Indian MFI terminate the informal borrowing relationship with higher frequency than their poorer counterparts. This creates potential for the MFI to desire to restrict the business growth of its borrowers to ensure a continued relationship.

Our second observation supports one of the model’s key testable predictions. We provide evidence of our comparative static on competition. During the 2002 Million Baht Program, the Thai government established village funds endowed with one million Baht in each of many villages. Importantly the size of these lending institutions was constant even across villages of varying population size, inducing plausibly exogenous variation in the per capita credit shock. Kaboski and Townsend (2012) leverage this variation to estimate the return to credit for the customers of these village funds. In contrast, we study the variation in contracts offered by money lenders, and argue that Thai money lenders resemble the informal lenders in our model. In particular we show that there is a steep decline in the likelihood a villager borrows from a money lender as a function of his household’s income. Thus we treat the Million Baht Program as exogenous variation in the level of competition faced by these informal money lenders.

We find that in villages with larger per capita credit shocks there is a decline in the incidence of restrictive contracts offered by money lenders, and argue that given the contractual patterns observed in these villages this is the unambiguous prediction of our theory. In essence borrowers in villages with more attractive village funds have better outside options, and the most efficient way for informal lenders to transfer surplus to their bor-
rowers is to relax contractual restrictions. Thus lenders who previously offered them restrictive contracts at relatively low interest rates now find it unprofitable to do so, and switch to loans which do not restrict the nature of investment. We invoke a number of placebo tests to argue that other theories are unlikely to explain the observed patterns.

A number of other papers offer explanations for the fact that credit markets operate inefficiently. Classic explanations include adverse selection (see e.g. Stiglitz and Weiss (1981)), moral hazard in project selection (Jensen and Meckling (1976)) and moral hazard in repayment (see e.g. Banerjee and Duflo (2010)). Bizer and DeMarzo (1992) suggest that credit markets may operate inefficiently when borrowers cannot commit to exclusive lending relationships and Green and Liu (2016) apply this logic in a development setting to argue that informal lenders may lend the least to borrowers with the most productive investment opportunities. While each of these theories offers an explanation for why credit may not allow firms to fully realize their growth potential, they struggle to match the other empirical regularities we note. Most notably, each of these theories predicts that firms will be credit constrained, and will demand as much credit as they are offered. In contrast our model offers an explanation for the empirical regularity that the demand for microcredit is low.

Finally, several papers offer theories that yield comparative statics of a similar flavor to our own. Petersen and Rajan (1995) argue that credit markets in which there is a high degree of competition for rich borrowers may feature more constrained lending to poor borrowers, as lenders in high competition environments are less able to reap the rewards of investment in poor borrowers. Jensen and Miller (2015) provide a theoretical model of a farmer choosing a level of education for his child. Highly educated children may opt to migrate to the city rather than assisting their parents with farm work, and therefore as the urban returns to education increase the parent may decrease the level of education he allows his child to reach. In both of these models, the comparative static unambiguously harms one of the parties. In contrast, in our model improving the attractiveness of the formal sector only harms poor borrowers by virtue of helping richer borrowers. We expand on this point in section 1.4.3.

The rest of the paper proceeds as follows. In Section 1.2 we describe the model. Section 1.3 characterizes the equilibrium of our game. Section 1.4 describes comparative statics. Section 1.5 discusses some extensions of the model where we relax some of our stylized assumptions. Section 1.6 documents our novel empirical facts. Section 1.7 concludes. All proofs are relegated to the appendix.
1.2 The Model

**Players, Actions, and Timing:** We study a dynamic game of complete information and perfectly observable actions. There are two players, a borrower (she) and a lender (he). Each period lasts length $dt$ and players discount the future at rate $\rho$. For analytical convenience we study the continuous time limit as $dt$ converges to 0. The borrower’s business is indexed with a state variable $w \in \{1, \ldots, n+1\}$ referred to as her business size.

At the beginning of each period the borrower has an endowment $E_w$, which may be augmented by a loan from her lender. She can invest her endowment into two projects: a working capital project $C$ and a fixed capital project $I$. The working capital project $C$ produces consumption goods which she uses to repay her lender and to eat, and the fixed capital project $I$ governs the rate at which her business size increases. The allocation of her endowment between these two projects may be influenced by contractual restrictions imposed by the lender. We defer detailed explanation of financial contracts, and transition between states to the discussion of timing below, after which we map the modeling assumptions to our earlier anecdote about a fruit vendor.

The timing within each period is as follows:

a) The lender offers a contract $\bar{\epsilon} = (R, a) \in C \equiv \mathbb{R}^+ \times \{I, C\}$, where $R$ represents the (contractable) repayment from the borrower to the lender, and $a$ represents the contractual restrictiveness.

b) The borrower chooses to accept or reject the contract. Formally she chooses a decision $d \in \{\text{Accept}, \text{Reject}\}$.

i. If she rejects the contract:

   i. She receives an endowment $E_w > 0$ to flexibly allocate between two projects.

   ii. She chooses an amount $c \leq E_w$ to invest into her *working capital project* $C$.

   A. We assume this project has linear return: $C(w, c) = q_w c$, for some $q_w > 1$ and the borrower consumes this output.

   iii. She chooses an amount $i = E_w - c$ to invest into her *fixed capital project* $I(w, i)$, the output of which is specified below.

   iv. The lender receives a flow payoff of 0
ii. If she accepts the contract

i. The lender transfers $T_w > 0$ working capital to the borrower, making her endowment $E_w + T_w$.\(^5\)

ii. If $a = C$, the borrower must invest everything in the working capital project. That is, she invests $i = 0$ in the fixed capital project and $c = E_w + T_w$ in the working capital project.

iii. If $a = I$, the borrower must invest $i = E_w + T_w - \frac{R}{q_w}$ in the fixed capital project and $c = \frac{R}{q_w}$ in the working capital project.

iv. The borrower repays $R$ to the lender who receives a flow payoff of $R - T_w$.

c) If the borrower invests $i$ into her fixed capital project $I$, her business size moves from state $w$ to $w + 1$ according to a Poisson process with arrival rate $\frac{i}{q_w}dt$ with $q_w > 0$ and remains constant otherwise.\(^6\)

d) If the game ever reaches state $n + 1$ both players cease acting.

i. The borrower receives a continuation payoff $U \equiv \frac{u}{\rho}$

ii. The lender receives a continuation payoff 0

e) Else the period concludes and after discounting the next one begins.

The timing above can be understood through the lens of the example in our introduction. The borrower is a fruit vendor, and at state $w$ she operates a mobile cart. At the beginning of the year she has a cash endowment $E_w$. If she rejects the lender’s contract then she flexibly allocates her endowment between her two projects: a working capital project $C$ which can be understood as purchasing fruits to sell during the week, and a fixed capital project $I$ which can be understood as buying raw materials to expand to a market stall from which she may have access to a broader market, improving her productivity. For every unit she invests in the working capital project, she produces $q_w > 1$ units of output. So $q_w$ may be thought of as the markup she enjoys from selling fruits, and $\phi_w$ may be thought of as the cost of fixed investment. The more she invests in fixed capital, the more likely she is to succeed in expanding her productive capacity by moving to state $w + 1$.

\(^5\)Note, we assume that $T_w$ is fixed, and therefore do not study the lender’s decision of loan size in this paper.
\(^6\)This assumption may be generalized to allow for any transition process in which the probability of transition from $w$ to any other state scales linearly with investment.
If instead she accepts the contract \(\langle R, a \rangle\), the lender transfers \(T_w\) working capital to the borrower and her endowment is \(E_w + T_w\). The borrower’s subsequent investment decision is determined exclusively by the contractual restriction \(a \in \{I, C\}\). If the contract specifies that the borrower should invest in the fixed capital project, that is \(a = I\), then she invests her entire endowment into fixed capital save for just enough which she invests into working capital to repay her debt. If instead \(a = C\) she invests her entire endowment into the working capital project. At the end of the period she repays her debt \(R\) to the lender, and they begin anew in the next period.

Though the stylized model above does not include an detailed description of the timing of output within a period, the contractual restriction \(a = C\) can be understood as the requirement of early and frequent repayments. If the lender demands that the borrower has cash on hand each day to repay a small fraction of her loan, she may not be able to initially invest in the long term, fixed capital project which may not return output for weeks. By the time she has generated enough income through her working capital project to ensure she can repay each installment, she may no longer have enough cash on hand to meet the minimum required investment in her fixed capital project, as would be the case if she has trouble saving cash from day to day (for instance because she faces pressure from her family to share underutilized assets). In contrast, a borrower uninhibited by a restrictive repayment plan \((a = I)\) may invest freely. Therefore we refer to a contract that specifies \(a = I\) as an unrestrictive contract and a contract that specifies \(a = C\) as a restrictive contract.

The borrower’s business expansion is represented by the discrete state space \(\{1, \ldots, n + 1\}\). Each state \(w\) represents a different business size (i.e. \(w = 1\) may be a mobile cart, \(w = 2\) a fixed stall, \(w = 3\) a small store and so on). State \(n + 1\) is a reduced form representation of the formal sector. The borrower enjoys (unmodeled) benefits of formal loans, and the lender receives a 0 continuation payoff having lost his customer. Notably, because the theorems below hold for any fixed investment cost and any number of states, it is straightforward to extend the model to accommodate a continuous state space. We discuss this further in Section 1.5.

**Parametric Assumptions:**

\(^7\)For technical convenience we assume that upon receiving an “unrestrictive” contract, the borrower must invest her endowment into the fixed capital project. Because we assume below that the borrower values investment in fixed capital more highly than she values investment in working capital, this assumption could be replaced by allowing her to invest flexibly upon receiving an unrestrictive contract with minimal consequence.
Arguably our most important parametric assumption is on the range of feasible repayment rates $R$.

**Assumption 1.1.** We assume that the feasible range of repayment rates satisfies $R \in [T_w, q_w T_w - h_w]$ with $h_w > 0$ for all $w$.

This assumption guarantees that if the borrower accepts the lender’s contract and sets aside $\frac{R}{q_w}$ of her endowment to invest in her working capital project for repayment, the residual endowment she can invest in either project is at least $E_w + \frac{h_w}{q_w}$ which necessarily exceeds the endowment $E_w$ she could have invested on her own. This can be motivated in a number of ways. Most straightforwardly, the borrower might be able to hide $h_w$ from her lender every period, and thus the repayment rate he sets is bounded above by the residual output resulting from the loan, $q_w T_w - h_w$. Alternatively one could assume that the borrower can renege on her debt in any period, in which case she must find a new lender at cost $v_w = q_w T_w - h_w$. Then the borrower would never repay a debt in excess of this cost. $^8$

The repayment ceiling is critical to many of our results below. Because the borrower cannot commit to share the proceeds from business expansion with her lender, she values investment in fixed capital more highly than she values investment in working capital. This in turn implies that the extra endowment $E_w + \frac{h_w}{q_w}$ the borrower retains induces an asymmetry between the utility she derives from restrictive and unrestrictive contracts. When this wedge is sufficiently large, the lender will impose inefficient contractual restrictions on the borrower, trapping her in poverty. $^9$

We next assume that both the borrower and lender are risk neutral.

**Assumption 1.2.** Both the borrower and lender enjoy a linear utility of consumption.

Assumption 1.2 implies that if either player receives a sequence of flow consumptions $\{u_t\}$, their lifetime utility is

$$\int_{0}^{\infty} e^{-\rho t} u_t dt$$

---

$^8$In the equilibrium of our model the borrower may not extract positive rents from the lending relationship. Thus, to take this microfoundation seriously, one can ensure that she always finds it profitable to find a new lender in the event of reneging on the first by assuming she receives an additional positive flow utility from interacting with any lender, that is unaffected by which loan she is offered. This can be motivated by an insurance benefit she receives from knowing her lender, that operates independently from the loans she receives every period.

$^9$For technical convenience we also require that the repayment level is bounded below, but that it must be larger than the principle transfer $T_w$ is unimportant.
Histories and Strategies: A history \( \tilde{h}_t \) is a sequence \( \{\tilde{c}_t, d_t, i_t, w_t\}_{t \leq t} \) of contracts, accept/reject decisions, investment allocations and business states at all periods prior to \( t \). We define \( \tilde{H}_t \) to be the set of histories up to time \( t \).

The lender's strategy is a sequence of (potentially mixed) contractual offers \( \tilde{c} = \{\tilde{c}(\tilde{h}_t)\}_{\tilde{h}_t \in \tilde{H}_t} \) where \( \tilde{c}(\tilde{h}_t) \in \Delta(C) \) is the probability weighting of contracts he offers the borrower following history \( \tilde{h}_t \). The borrower's strategy is a sequence of accept/reject decisions \( d = \{d(\tilde{h}_t, \tilde{c})\}_{\tilde{h}_t \in \tilde{H}_t, \tilde{c} \in C} \) and investment decisions in the event of rejection \( i = \{i(\tilde{h}_t, \tilde{c})\}_{\tilde{h}_t \in \tilde{H}_t, \tilde{c} \in C} \). Here \( d(\tilde{h}_t, \tilde{c}) \) denotes the probability the borrower accepts the contract \( \tilde{c} \) following history \( \tilde{h}_t \), and \( i(\tilde{h}_t, \tilde{c}) \) denotes the investment allocation the borrower undertakes following history \( \tilde{h}_t \) and rejecting contract \( \tilde{c} \).

Equilibrium: Our solution concept is the standard notion of Stationary Markov Perfect Equilibrium (henceforth equilibrium) which imposes that at every period agents are best responding to one another and that they only condition their strategies on payoff relevant state variables (in this case, business size). In particular, neither agent has the ability to commit to a long term contract.

Formally, a strategy profile \( (\tilde{c}, d, i) \) is an equilibrium if

a) \( \tilde{c}(\tilde{h}_t) \) is optimal for the lender at every \( \tilde{h}_t \) given the borrower's strategy \( (d, i) \).

b) \( d(\tilde{h}_t, \tilde{c}) \) and \( i(\tilde{h}_t, \tilde{c}) \) are optimal for the borrower at every \( \tilde{h}_t \) and for every contract \( \tilde{c} \) given the lender's strategy \( \tilde{c} \).

c) At any two histories \( \tilde{h}_t \) and \( \tilde{h}_t' \) for which \( w \) is the same, we have \( \tilde{c}(\tilde{h}_t) = \tilde{c}(\tilde{h}_t') \), \( d(\tilde{h}_t, \tilde{c}) = d(\tilde{h}_t', \tilde{c}) \), and \( i(\tilde{h}_t, \tilde{c}) = i(\tilde{h}_t', \tilde{c}) \).

By studying Stationary Markov Perfect Equilibria, we impose that the lender uses an impersonal strategy: any borrower with the same business size must be offered the same contract. This may be an especially plausible restriction in the context of large informal lenders such as microfinance institutions whose policy makers may be far removed from the recipients of their loans, rendering overly personalized contract offers infeasible.

1.3 Equilibrium Structure

We now describe the borrower and lender's equilibrium behavior and our main results about the structure of the equilibrium. Section 1.3.1 describes the borrower's autarky
problem and sets forth an assumption that guarantees the borrower will eventually reach the formal sector (state $n + 1$) in autarky. Section 1.3.2 describes the key incentives of the borrower and lender necessary to understand the structure of the equilibrium. Section 1.3.3 provides our main results: The equilibrium is unique, and under additional assumptions specified below the probability that the lender offers a restrictive contract is single peaked in the state. Thus, the lender’s poorest and richest clients may receive unrestricted contracts and grow faster than they would have in his absence. But borrowers with intermediate levels of wealth receive restrictive contracts every period and find themselves in a poverty trap. Notably, this poverty trap may exist even if the borrower would have reached the formal sector in autarky and even if the discounted utility from expanding to the formal sector is greater than the total surplus generated from investing the total endowment in working capital in every state. In Section 1.3.4 we argue that several well established empirical facts about microfinance can be contextualized through the lens of this equilibrium.

1.3.1 The Borrower’s Autarky Problem

First consider the borrower’s autarky problem. That is, the economic environment is as specified in Section 1.2, but the borrower is forced to reject the lender’s contract at all times (i.e. she must choose $d(\bar{h}_t, \tilde{c}) = 0$ for all histories $\bar{h}_t$ and contracts $\tilde{c}$).\(^{10}\)

Let $B^\text{aut}_w$ be the borrower’s continuation value in autarky in state $w$. This can be decomposed into a weighted average of her flow payoff in the time interval $[t, t + dt]$, and her expected continuation utility at time $t + dt$. We have

$$B^\text{aut}_w = \max_i q_w (E_w - i) dt + (1 - \rho dt) \left( \frac{i}{\phi_w} dt B^\text{aut}_{w+1} + \left( 1 - \frac{i}{\phi_w} dt \right) B^\text{aut}_w \right) \quad (1.1)$$

Fixing the optimal level of investment $i$ in state $w$, rearranging, and ignoring higher order terms we have

$$B^\text{aut}_w = \frac{q_w (E_w - i)}{\rho + \frac{i}{\phi_w}} + \frac{\frac{i}{\phi_w}}{\rho + \frac{i}{\phi_w}} B^\text{aut}_{w+1} \quad (1.2)$$

That is, the borrower’s autarky continuation value in state $w$ is a weighted sum of her flow consumption $q_w (E_w - i)$ and her continuation value upon increasing business

\(^{10}\)Alternatively, one can imagine that the borrower simply does not have access to a lender.
size, $B_{w+1}^{\text{aut}}$. Because equation 1.1 is linear in $i$ (and equation 1.2 is monotone in $i$), the borrower will choose an extremal level of investment. From here on we will use the notation $\kappa_w \equiv \frac{E_w}{\phi_w}$, which is the maximum speed the borrower can invest in fixed capital and grow in autarky. We have the following proposition about the borrower’s autarky behavior.

**Proposition 1.1.** The borrower invests her entire income in every state iff

$$\frac{q_w E_w}{\rho} \leq \left( \prod_{w'=w}^{n} \frac{\alpha_{w'}}{\rho} \right) \frac{u}{\rho} \text{ for all } w,$$

where $\alpha_w \equiv \frac{\kappa_w}{\rho + \kappa_w}$.

**Proof.** See appendix. \qed

The borrower’s autarky problem has an attractive structure. If she chooses to invest in fixed capital in state $w$ at every period then her continuation utility in state $w$ is $B_w^{\text{aut}} = \alpha_w B_{w+1}^{\text{aut}}$. That is, she spends a fraction $(1 - \alpha_w)$ of her expected, discounted lifetime in the current state, and a fraction $\alpha_w$ of her expected, discounted lifetime in all future states $w + 1$ and onwards. Likewise in state $w$ she anticipates spending a fraction $\prod_{w'=w}^{n} \alpha_{w'}$ of her expected, discounted lifetime in state $w + m$ and onwards if she invests in fixed capital at every period until reaching state $w + m$.

This property is closely related to the Poisson arrival of jumps. Letting $t$ denote the time of the jump and $\lambda$ be the arrival intensity, $v_1$ be the flow utility the borrower enjoys prior to a jump and $v_2$ the flow utility she enjoys post jump, the borrower’s utility is represented by:

$$E_t \left[ \int_0^t v_1 e^{-\rho s} ds + \int_t^\infty v_2 e^{-\rho s} ds \right] = \int_0^\infty \left[ \int_0^t v_1 e^{-\rho s} ds + \int_t^\infty v_2 e^{-\rho s} ds \right] e^{-\lambda t} dt = (1 - \alpha) \left( \frac{v_1}{\rho} \right) + \alpha \left( \frac{v_2}{\rho} \right)$$

where $\alpha \equiv \frac{\kappa}{\rho + \kappa}$. Thus the borrower’s utility from this process can be represented as the convex combination of her lifetime utility from staying in the initial state forever and her lifetime utility from staying in the post-jump state forever, where the weights on each are a function of the intensity of the arrival process. Having established that the borrower invests in fixed capital in all states in autarky if and only if $\frac{q_w E_w}{\rho} \leq \left( \prod_{w'=w}^{n} \frac{\alpha_{w'}}{\rho} \right) \frac{u}{\rho}$ for all $w$, 26
we make the following, stronger assumption and maintain it throughout the subsequent analysis.

**Assumption 1.3.** \( \frac{q_w E_w + h_w}{\rho} \leq \left( \prod_{w=0}^{n} \alpha_w \right) \frac{a}{\rho} \) for all \( w \).

Assumption 1.3 guarantees that the borrower would prefer to invest her income into fixed capital rather than invest it into working capital for any flow income stream weakly less than \( q_w E_w + h_w \). In addition to ruling out an uninteresting case in the analysis, Assumption 1.3 serves to highlight that the introduction of a lender may cause a poverty trap to emerge despite the autarkic borrower’s eventual entry into the formal sector. That is, business growth among borrowers with access to credit may be lower than growth among their counterparts without access to credit.

### 1.3.2 Relationship Value Functions

We now outline the borrower and lender’s relationship maximization problems and describe their value functions. Let \( B_w \) be the borrower’s equilibrium continuation utility at the beginning of a period in state \( w \), and let \( B_w ((R, a)) \) be her equilibrium continuation utility upon receiving the contract \( (R, a) \) in state \( w \). Further, define

\[
B_w^{\text{REJ}} = \max_i q_w (E_w - i) dt + (1 - \rho dt) \left( \frac{i}{\phi_w} d t B_{w+1}^w + \left( 1 - \frac{i}{\phi_w} dt \right) B_w^w \right)
\]

to be her equilibrium continuation utility upon rejecting a contract. These functions satisfy

\[
B_w ((R, C)) = \max \left\{ (q_w (E_w + T_w) - R) dt + (1 - \rho dt) B_w, B_w^{\text{REJ}} \right\}
\]

and

\[
B_w ((R, I)) = \max \left\{ (1 - \rho dt) \left( \frac{E_w + T_w - R}{\phi_w} dt B_{w+1}^w + \left( 1 - \frac{E_w + T_w - R}{\phi_w} dt \right) B_w^w \right), B_w^{\text{REJ}} \right\}
\]

where for both value functions above, the first expression in the brackets corresponds to the borrower’s continuation utility if she accepts the contract \( (R, a) \) and the second term corresponds to her continuation utility if she rejects the contract, she is left with her smaller endowment \( E_w \) and chooses her own allocation of investment.
The lender’s value function \( L_w \) in state \( w \) satisfies

\[
L_w = \max_{(R,a)} \left( R - T_w \right) dt + (1 - \rho dt) \left( L_w + \sum_{a=1}^{\infty} \frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} dt (L_{w+1} - L_w) \right)
\]

such that

\[
q_w (E_w + T_w) - R \geq \kappa_w (B_{w+1} - B_w) \quad \text{if} \quad a = C
\]

\[
T_w \leq R \leq q_w T_w - h_w
\]

Note that the lender’s maximization problem and constraints assume the lender never finds it optimal to offer the borrower a contract she will reject.\(^{11}\) The lender’s maximization problem also assumes that the borrower accepts any unrestrictive contract. This is the case so long as the borrower would invest her entire income in fixed capital if she were to reject the contract.\(^{12}\) The borrower accepts a restrictive contract \((R, C)\) if and only if her value of consuming what the lender offers is weakly higher than that of rejecting the contract and choosing her own allocation of investment, i.e.

\[
q_w (E_w + T_w) - R \geq \kappa_w (B_{w+1} - B_w).
\]

We refer to the above inequality as the borrower’s individual rationality constraint. In equilibrium the lender always offers one of three contracts:

- \((T_w, I)\) in states \( w \) where the lender’s value function satisfies \( L_{w+1} - L_w \geq \phi_w \) and thus he wants the borrower to expand as quickly as possible. In such states, the lender charges the lowest possible interest rate, \( T_w \).

- \((q_w T_w - h_w, I)\) in states where the lender prefers unrestrictive contracts but where \( L_{w+1} - L_w < \phi_w \) so that the lender’s preference for expansion is not so strong so as to drive the him to offer the borrower a higher than necessary flow payoff. In such

\(^{11}\) It is straightforward to show that in any Stationary Markov perfect equilibrium, either offering the borrower a restrictive contract with the highest acceptable repayment rate or offering her an unrestrictive contract with the highest feasible repayment rate will dominate offering the borrower a contract she would reject.

\(^{12}\) The borrower may invest in working capital in her outside option in equilibrium if she expects a high rate of investment in fixed capital from the lender. This case is dealt with in the appendix, but the analysis does not substantively differ from the above.
states the lender offers the highest possible interest rate, \( q_w T_w - h_w \).

- \( (q_w (E_w + T_w) - \kappa_w (B_{w+1} - B_w), C) \) in states where the lender prefers a restrictive contract, and therefore charges the highest acceptable interest rate.

**Expansion rents**

The lender’s maximization problem illuminates an important force in our model. If the lender offers the borrower a restrictive contract, he optimally offers her the most extractive repayment rate she finds acceptable, denoted by \( \hat{R}_w \). This repayment rate is determined by the borrower’s indifference condition between accepting the restrictive contract or investing in fixed capital at her autarkic rate. Receiving this contract at every period the borrower’s continuation utility would be

\[
B_w = (q_w (E_w + T_w) - \hat{R}_w) dt + (1 - \rho dt) B_w = (1 - \rho dt) (\kappa_w dt B_{w+1} + (1 - \kappa_w dt) B_w)
\]

Rearranging and ignoring higher order terms we have

\[
B_w = \frac{\kappa_w}{\rho + \kappa_w} B_{w+1} = \alpha_w B_{w+1}
\]

That is, if the lender offers the borrower the least generous acceptable restrictive contract the borrower’s continuation utility is exactly what it would be if she invested in fixed capital at her autarkic rate.

On the other hand, if the lender offers a maximally extractive unrestrictive contract in every period, the borrower’s continuation value will satisfy

\[
B_w = (1 - \rho dt) \left( \frac{E_w + \frac{h_w}{\phi_w}}{\phi_w} dt B_{w+1} + \left( 1 - \frac{E_w + \frac{h_w}{\phi_w}}{\phi_w} dt \right) B_w \right)
\]

Rearranging and ignoring higher order terms we have

\[
B_w = \frac{\gamma_w}{\rho + \gamma_w} B_{w+1} = \beta_w B_{w+1}
\]

where \( \gamma_w = \frac{E_w + \frac{h_w}{\phi_w}}{\phi_w} \) is the rate of expansion the borrower enjoys when she receives a maximally extractive unrestrictive contract, and \( \beta_w = \frac{\gamma_w}{\rho + \gamma_w} \) is the fraction of her discounted lifetime she expects to spend in state \( w + 1 \) and onwards if she invests in fixed capital at

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rate $\gamma_w$ in state $w$. Note that if the lender offers the borrower an unrestrictive contract, the borrower’s continuation utility is strictly higher than it would be in autarky, because she is allowed to invest strictly more into fixed capital than she would in autarky.

The difference between the borrower’s continuation value upon receiving an unrestrictive contract and upon receiving a restrictive one is $(\beta_w - \alpha_w) B_{w+1}$. We refer to this term as the expansion rent in state $w$. This asymmetry arises because of the ceiling on feasible repayment rates the lender may set. Recall, after transferring $T_w$ endowment to the borrower, the lender must set a repayment weakly less than $q_w T_w - h_w$ with $h_w > 0$. Thus upon accepting a loan and allocating $\frac{K}{q_w}$ to the working capital project for repayment, the borrower necessarily has a larger residual endowment to allocate to either project than she would have had on her own. Because she values investment in fixed capital more highly than she values investment in working capital, she values this extra income more highly when receiving unrestrictive contracts than she does when receiving restrictive ones.

As will be clear in the following sections, this expansion rent is critical for our main results. The lender may prohibit efficient growth by offering the borrower restrictive contracts, and in equilibrium will do so if and only if the expansion rent highlighted above exceeds the change in joint surplus resulting from expansion.

### 1.3.3 Results

We are now in a position to state our first result.

**Proposition 1.2.** An equilibrium exists and is generically unique.

**Proof.** See Appendix. 

The result follows by backward induction on the state. In any state $w$ the borrower’s accept/reject decision is pinned down by her state $w$ continuation value $B_w$ and her state $w + 1$ continuation value $B_{w+1}$. The primary subtlety arises from the fact that the borrower’s welfare in state $w$ is increasing in the probability the lender offers an unrestrictive contract in $w$. The more frequently the borrower anticipates unrestrictive contracts in $w$ the less demanding she will be of restrictive contracts. Formally, we define $\delta_w(p_w) \equiv p_w \gamma_w + (1 - p_w) \gamma_w$. It is straightforward to show that a borrower who expects a restrictive contract with probability $p_w$ in state $w$ will have a continuation utility
of $B_w (p_w) = \frac{\delta_w (p_w)}{\rho + \delta_w (p_w)} B_{w+1}$ which is decreasing in $p_w$. The lender determines the interest rate associated with restrictive contracts, $R_w (p_w)$, to solve

$$\kappa_w (B_{w+1} - B_w (p_w)) = q_w (E_w + T_w) - R_w (p_w)$$

from which it is immediate that $R_w (p_w)$ is decreasing in $p_w$. Thus it may be that when the borrower expects a restrictive contract with certainty the lender strictly prefers to offer an unrestricted contract, and when the borrower expects an unrestricted contract with certainty the lender strictly prefers to offer a restrictive contract. In such a case the unique equilibrium involves a strictly interior $p_w$ and the expansion rent is $(\beta_w - \frac{k_w}{\rho + \delta_w (p_w)}) B_{w+1}$.

A second subtlety is due to the possibility that in equilibrium the borrower invests her autarkic endowment in working capital after rejecting the lender’s contract in some state $w$. Despite Assumption 1.3, the borrower may invest her autarkic endowment in working capital in state $w$ if she expects to receive sufficiently attractive unrestricted contracts in state $w$, which causes $B_w$ to be near to $B_{w+1}$ and depresses the value of business expansion. We show that if there is an equilibrium in which the borrower invests her autarkic flow endowment in working capital in state $w$, this can only be due to the fact that the lender offers her an attractive unrestricted contract, which is feasible irrespective of the borrower’s autarkic action and therefore occurs across all equilibria. Thus after rejecting the lender’s contract in state $w$, she invests her autarkic flow endowment in working capital in any equilibrium.

**Equilibrium Contract Structure**

For the remainder of this section and the next we make the following parametric assumptions. We do so for simplicity and ease of exposition but argue in Section 1.5 that their complete relaxation does not change the qualitative lessons to be drawn from the model.

First we assume that the flow working capital output within the relationship is increasing and weakly concave in the state. Let $y_w \equiv q_w (E_w + T_w) - T_w$.

**Assumption 1.4.** $y_w > y_{w-1}$ for all $w$ and $y_w - y_{w-1} \geq y_{w+1} - y_w$ for all $w$.

Second we assume that the borrower’s autarky endowment $E_w$, the amount she can hide $h_w$, and the cost of investment in fixed capital $\phi_w$ are constant in $w$.

**Assumption 1.5.** $E_w = E_{w'} \equiv E$ for all $w, w'$, $h_w = h_{w'} \equiv h$ for all $w, w'$, and $\phi_w = \phi_{w'} \equiv \phi$ for all $w, w'$.  

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Note that Assumption 1.5 allows us to omit the subscripts on \( \kappa, \gamma, \alpha, \) and \( \beta. \)

Our next result regards the equilibrium organization of restrictive states and unrestricted states under the parametric assumptions above.

**Proposition 1.3.** In equilibrium, the probability the lender offers a restrictive contract \( p_w \) is single peaked in \( w. \)

**Proof.** See Appendix.

This result implies that in equilibrium the states can be partitioned into three regions of consecutive states: An initial region with only unrestricted contracts, an intermediate region in which both kinds of contracts are possible, and a final region in which only unrestricted contracts are offered. In the intermediate region, the probability a restrictive contract is offered is increasing (potentially reaching 1) and then decreasing. This is depicted in the figure below where white states denote unrestricted states, black states denote restrictive states and grey states denote mixing states.

![Diagram](image)

Borrowers who arrive at a state in which only restrictive contracts are offered never grow beyond it. The next natural question, therefore, is when do such states arise? We have the following result.

**Proposition 1.4.** In equilibrium, the probability the lender offers a restrictive contract \( p_w = 1 \) if and only if

\[
\beta \left( (L_{w+1} + B_{w+1}) - \frac{q_w(E + T_w) - T_w}{\rho} - \phi \right) \leq (\beta - \alpha) B_{w+1}
\]

The left hand side of the above inequality may be loosely understood as the social gain from investing in fixed capital at rate \( E + \frac{h}{q} \) rather than investing everything into working capital. If the borrower invests at rate \( E + \frac{h}{q}, \) then she and the lender expect to spend a fraction \( \beta \) of their lifetime in state \( w+1 \) and onwards. Once in \( w+1 \) they jointly enjoy continuation values of \( L_{w+1} + B_{w+1} \) but forgo the consumption they could have enjoyed in state \( w, \frac{q_w(E + T_w) - T_w}{\rho}, \) and the cost they incur from expansion is \( \beta \phi. \) In contrast, the right
hand side of the inequality is the borrower’s expansion rent: the additional surplus she commands from unrestricted contracts relative to restrictive ones. Thus if this expansion rent exceeds the social gain of business expansion, the lender will offer only restrictive contracts, pinning the borrower to the current state.

Note that because of the restrictions on feasible repayment rates, this is not a model of transferrable utility. Thus the left hand side of the above inequality should not literally be interpreted as a change in social welfare. Nevertheless we will sometimes abuse terminology and say that it is socially efficient to invest in fixed capital when the left hand side of the above inequality is positive.

We are now ready to discuss the intuition behind Proposition 1.3. When the borrower is near the formal sector, it is extremely costly to offer her a restrictive contract. For concreteness, consider a borrower in state $n$. A lender who offers this borrower a restrictive contract in every period needs to compensate her with $\frac{a^u}{\rho}$ consumption over the life of the relationship. For $u$ sufficiently high this is prohibitively costly. However, as the borrower becomes poorer it becomes cheaper to offer her a restrictive contract. Consider a borrower who is at state $w$ and who expects unrestricted contracts in all future states. A lender who offers this borrower a restrictive contract in every period needs to transfer her only $\frac{a^u \beta^{w-u} u}{\rho}$ consumption over the lifetime of the relationship. Thus as the borrower becomes poorer it becomes exponentially cheaper to offer her the restrictive contract.

In this intermediate region the expansion rent may become important. As discussed above, when the borrower’s expansion rent exceeds the social gain from business expansion, the lender offers only restrictive contracts, keeping her inefficiently small. Note that this poverty trap is created by the presence of the lender. Assumption 1.3 guarantees that in autarky the borrower would have grown to her efficient size.

Last, as the borrower becomes sufficiently poor her expansion rent $(\beta - \alpha) B_{w+1}$ decreases, as it is tied to her continuation value in the next state. Moreover, because of the concavity of the output from working capital investment, the joint surplus increase from expansion becomes increasingly large as the borrower becomes poorer. Thus sufficiently poor borrowers receive unrestricted contracts.

We close this section with a discussion of the source of this poverty trap. One crucial feature is that the minimum endowment residual of repayment the borrower enjoys when contracting with the lender, $E + \frac{h}{q_w}$, is strictly larger than the endowment she would have had on her own, $E$. We encode this fact in the following proposition.
Proposition 1.5. If $h = 0$, then $p_w = 0$ for any $w$ in which it is socially efficient to invest in fixed capital (i.e. whenever $\beta \left( (L_{w+1} + B_{w+1}) - \frac{q_w(E+T_w) - T_w}{\rho} - \phi \right) > 0$).

When the lender can choose interest rates flexibly enough such that the borrower can be left with exactly the same amount of income that she would have produced alone, he offers unrestricted contracts in any state in which the social gain from business expansion is positive. When the lender offers a restrictive contract, he gives the borrower just enough consumption to make her indifferent between accepting the contract and rejecting it and investing $E$ in fixed capital. But if the lender instead offers the borrower a maximally extractive unrestricted contract, the borrower remains indifferent, because the endowment she can invest into business expansion is exactly what she could have invested on her own. Since the total social surplus increases and the residual surplus accrues to the lender, he prefers unrestricted contracts.

While the poverty trap disappears when $h = 0$ it is important to note that the unique equilibrium still features inefficiently slow business expansion relative to the social optimum. A natural question then, is what contractual flexibility is required to reach the first best level of investment in fixed capital. It is straightforward to verify that equity contracts – contracts that allow the borrower to commit a fraction of her formal sector flow payoff to the lender in exchange for favorable unrestricted contracts – are sufficient to guarantee first best investment. However this is primarily a theoretical exercise, as the participants of informal financial markets rarely have the capacity to write equity contracts.

1.3.4 Connection to Empirical Evidence

At this point the model already starts to organize much of the empirical evidence on microcredit cited in our introduction. That firms fail to grow from being offered access to microcredit can be understood through the fact that in our model, firms who enter a state where the lender offers restrictive contracts (the black region in the figure above) never leave it, despite the fact that they would have continued to grow in autarky. That is, this is a model in which having access to an informal lender can reduce business growth.

While the microcredit studies listed above find low marginal returns to credit on average, a number of them find considerable heterogeneity in observed returns to credit. In particular they consistently find a long right tail in returns to credit – the largest businesses in areas that randomly received access to microcredit are substantially larger than the largest businesses in areas that did not. Our model sheds light on this heterogeneity.
to returns as well. Firms at very low and very high business sizes grow faster in the presence of a lender than in his absence (they grow at least at rate \(\gamma\) rather than \(\kappa\)). Whereas firms at intermediate business sizes may not grow at all in the presence of a lender.

In contrast to many other models with credit constrained borrowers, this model offers a novel explanation for the regular finding that demand for microcredit contracts is low. Borrowers in the restrictive region are pushed exactly to their individual rationality constraint – they are indifferent between taking loans and not. While the exact indifference of these borrowers may seem an artifact of the model, the intuition that the lender can push the borrower nearer to her outside option when preventing her from investing in business expansion seems robust. Thus these borrowers may be expected to waver on their decision to accept a loan. In the appendix we discuss an extension to the model in which the lender is incompletely informed about the borrower’s outside option. In equilibrium borrowers in the restrictive region sometimes reject his offer, whereas those in the unrestrictive region never do.

### 1.4 Comparative Statics

In this section we discuss how the equilibrium changes with respect to a number of comparative statics. Each of them emphasizes an important “trickle down” nature of our model. Namely, changes to the fundamentals of the contracting environment can have nuanced impacts on equilibrium contracts and welfare that vary depending on the borrower’s business size. We close this section with comparison to other theories that leverage similar comparative statics.

#### 1.4.1 Comparative Statics on the Borrower’s Continuation Utility \(u\) from Entering the Formal Sector.

Increasing the borrower’s continuation value \(u\) from entering the formal sector shifts the entire restrictive region leftward. The poverty trap is relaxed for rich borrowers but tightened for poor borrowers. The intuition behind this observation relies on the fact that, when his borrower is rich enough to be in the final unrestrictive region, increasing the attractiveness of the formal sector makes restrictive contracts more expensive for the lender.
because it increases the borrower's desire to expand. So the lender shifts towards unrestrictive contracts, relaxing the poverty trap for rich borrowers.

On the other hand, it is precisely this force that causes the lender to tighten the reins on poorer borrowers, increasing the likelihood he offers them restrictive contracts and trapping them at lower levels of wealth. Holding the lender's strategy fixed, increasing the attractiveness of the formal sector improves the borrower's bargaining power in all states. However, the richer the borrower is, the more her bargaining power improves because of her proximity to the formal sector. Thus the lender shifts towards restrictive contracts for poorer borrowers, to prevent them from reaching higher levels of wealth where they can exercise their additional bargaining power. This is encoded in the propositions below.

Let \( \bar{w} \equiv \text{argmax} \{ w : p_w = 1 \} \), and \( w \equiv \text{argmin} \{ w : p_w = 1 \} \).

**Proposition 1.6.** Increasing the attractiveness of the formal sector relaxes the poverty trap for relatively rich borrowers, but tightens it for poorer borrowers.

That is, \( \frac{d\bar{w}}{du} \leq 0 \) for \( w \geq \bar{w} \) with strict inequality for \( 0 < p_w < 1 \). \( \frac{dp_w}{du} \geq 0 \) for \( w < \bar{w} \) with strict inequality for \( 0 < p_w < 1 \).

*Proof. See appendix.*

The intuition for the above proposition is inextricably linked to the equilibrium effects on welfare, codified in the next proposition.

**Proposition 1.7.** Increasing \( u \) weakly decreases the lender's continuation value in all states, and strictly so for \( w \leq \bar{w} \). Increasing \( u \) strictly increases the borrower's continuation utility in all states \( w \geq w \), but can decrease it in states \( w < w \).

That is, \( \frac{dL}{du} \leq 0 \) for all \( w \) with strict inequality for \( w \leq \bar{w} \). \( \frac{dB}{du} > 0 \) for \( w \geq w \). For \( \rho > \frac{x}{x+\gamma} \), if \( p_{w-1} > 0 \) then \( \frac{dB_w}{du} < 0 \) for states \( w < w \).

*Proof. See appendix.*

Though some of the details are cumbersome, the intuition behind these results is instructive. The comparative static for states \( w > \bar{w} \) is most easily understood. Consider the largest state \( n \) at which the borrower remains in the informal sector. Increasing \( u \) makes it more expensive to offer the borrower a restrictive contract, because she finds investment in fixed capital more valuable. On the other hand, the borrower accepts any unrestrictive
contract due to her expansion rent. So the lender finds unrestrictive contracts relatively more attractive and shifts towards them if he previously chose an interior solution.

The borrower’s continuation utility increases for two reasons. She benefits from the increased prevalence of unrestrictive contracts, and conditional on entering the formal sector her utility increases. By assumption, the lender at least weakly prefers to offer unrestrictive contracts and since the utility he derives from doing so is unaffected, so is his equilibrium continuation value. This logic extends straightforwardly by backward induction to all states weakly larger than $\bar{w}$.

The story changes at or prior to $W$. By definition of $zP$, the lender offers a restrictive contract with certainty (i.e. $p_{\bar{w}} = 1$), and therefore transfers $aB_{\bar{w}+1}$ utility to the borrower over the lifetime of the relationship. Having already established that the borrower’s continuation utility in state $\bar{w} + 1$ increases with $u$, we can now see that the borrower’s utility also increases in state $\bar{w}$. However, now the increase in her utility results from a direct transfer from the lender, so his continuation utility in state $\bar{w}$ decreases. A similar conclusion is reached for all states $w \in \{w, \bar{w}\}$ by backward induction.

Finally, consider state $w - 1$, in which, by definition, the lender at least weakly prefers to offer an unrestrictive contract. Further, suppose that the preference is indeed weak, so that $p_{w-1} \in (0, 1)$ (i.e. the lender offers a restrictive contract with positive probability). First, note that since the borrower never grows beyond state $w$, increasing $u$ has no effect on social welfare in state $w - 1$. However, since the borrower’s equilibrium continuation utility in state $w$, $B_w$ increases, so does her expansion rent in state $w - 1$. Recall that her state $w - 1$ expansion rent $\left( \beta - \frac{x}{\rho + \delta(p_{w-1})} \right) B_w$ is a fraction of her continuation utility in state $w$. Because the borrower’s share of surplus from business expansion increases but the change in social surplus accruing from business expansion does not, the lender shifts towards restrictive contracts, slowing down the borrower’s growth.

Another way to understand this is the increase in the attractiveness of the formal sector trickles down and increases the borrower’s bargaining power in state $w$. Markov Perfection prevents her from committing not to exercise this additional bargaining power and, because state $w - 1$ borrowers are less affected, they become relatively more attractive and the lender shifts towards offering them restrictive contracts.

How this affects the borrower’s state $w - 1$ equilibrium continuation utility $B_{w-1}$ is in general ambiguous. That $B_w$ increases is a force towards increasing $B_{w-1}$. However, the rate at which she grows to state $w$ slows, which is a force towards reducing $B_{w-1}$.
For sufficiently impatient borrowers the latter force dominates, as impatience amplifies the difference between slow and fast rates of expansion, and \( B_{w-1} \) decreases in the attractiveness of the formal sector. The lender is made unambiguously worse off from the increase in \( u \), because he weakly prefers unrestrictive contracts and his state \( w \) continuation utility is decreasing in \( u \). Thus an increase in the attractiveness of the formal sector causes a Pareto disimprovement. Because the borrower cannot commit to forgo her improved bargaining power in state \( w \), the lender traps her in state \( w - 1 \) to both of their detriments. And the story continues in much the same way for all states prior to \( w - 1 \).

That increasing \( u \) can make the borrower worse off at some business state \( w \) is especially striking in light of the following consideration. Fix any (potentially non-equilibrium) lender behavior characterized by \( \{p_w\} \), such that in state \( w \) the lender offers a restrictive contract with probability \( p_w \) and an unrestrictive contract with probability \( 1 - p_w \). Then increasing \( u \) unambiguously makes the borrower strictly better off in all states. The restrictive contract in state \( w \) becomes more generous when \( B_{w+1} \) improves, and the borrower’s utility from receiving an unrestrictive contract in state \( w \) improves when \( B_{w+1} \) increases. This logic is codified in the following proposition.

**Proposition 1.8.** Fixing the lender’s behavior characterized by \( \{p_w\} \) defined above, increasing \( u \) strictly improves the borrower’s continuation utility in all states.

**Proof.** See appendix.

So fixing the lender’s behavior, regardless of what that behavior is, increasing \( u \) unambiguously improves the borrower’s continuation utility. It is because of an equilibrium adjustment to the lender’s behavior, namely that he shifts towards restrictive contracts, that impatient borrowers are made worse off at all business states \( w < w \).

### 1.4.2 Comparative Statics on the Borrower’s Level of Patience

A standard intuition about poverty traps is that they are driven by impatience. However in this model increasing patience has a very similar effect to increasing the attractiveness of the formal sector, and hence can tighten the poverty trap and make the borrower worse off at some levels of wealth. Let \( \rho^B \) be the borrower’s level of patience and \( \rho^L \) be the lender’s level of patience (and note that decreasing \( \rho^B \) is equivalent to increasing patience).
Proposition 1.9. Increasing the borrower's patience relaxes the poverty trap for relatively rich borrowers, but may tighten it for poorer borrowers.

That is, \( \frac{d p_w}{dp} \geq 0 \) for \( w > \bar{w} \) with strict inequality for \( p_w > 0 \). For \( w < \bar{w} \), the sign of \( \frac{d p_w}{dp} \) is ambiguous.

Proof. See appendix.

For rich borrowers above the highest pure restrictive state \((w > \bar{w})\), the comparative static on \( \rho^B \) works in exactly the same way as the comparative static on \( u \). Increasing the borrower’s patience increases how much she values business expansion. This causes her to be more demanding of restrictive contracts, but leaves the lender’s payoff from offering unrestrictive contracts unchanged. Thus, in all such states the lender shifts towards unrestrictive contracts, increasing the rate that these rich borrowers reach the formal sector.

For borrowers in pure restrictive states \((w \in \{w, \ldots, \bar{w}\})\), the comparative static on the borrower’s patience again works as it did for changes in the attractiveness of the formal sector. The borrower’s continuation utility in state \( w + 1 \), \( B_{w+1} \), increases so the amount of consumption she demands in return for contractual restrictions increases. This increases her welfare at the direct expense of the lender’s.

Finally, consider state \( w - 1 \). Recall the borrower’s expansion rent in this state is \( \left(\beta - \frac{x}{\rho^B + \delta(p_{W-1})}\right) B_W \). That her utility in state \( w \), \( B_W \), increases is a force towards increasing her expansion rent. However, as she becomes more patient, the difference she perceives between slow and fast rates of expansion is muted. That is \( \frac{d}{dp} \left(\beta - \frac{x}{\rho^B + \delta(p_{W-1})}\right) > 0 \), which is a force towards decreasing the expansion rent. Which of these two forces dominates is in general ambiguous, but we show in the appendix that these forces can resolve in favor of increasing the expansion rent. Thus, in contrast to standard models of poverty traps, increasing the borrower’s patience can make this poverty trap worse.

1.4.3 Comparison to Petersen and Rajan (1995) and to Jensen and Miller (2015)

There are a number of principal agent models that feature a comparative static similar to ours. Two such theories are those of Petersen and Rajan (1995) and Jensen and Miller
Petersen and Rajan study a credit market, and argue that if competition for rich borrowers becomes more fierce, lenders may invest less in their current borrowers. Interpreting this theory through the lens of our model, this is akin to reducing the attractiveness of the formal sector for the lender without changing the attractiveness of the formal sector for the borrower. Doing so reduces the lender’s payoff from unrestricted contracts and weakly increases the equilibrium probability of restrictive contracts in all states, making the borrower weakly worse off.

Jensen and Miller (2015) study Indian agricultural households in which parents decide a level of education for their children, and then children decide whether to stay at home and work on the farm or migrate to the city, leaving their parents behind. While education has positive returns in both locations, it has higher returns in the city. They show, both theoretically and empirically, that reducing the cost of migrating to the city causes parents to reduce educational investment in their children. Interpreting their insight in the language of this paper, theirs is an exercise in reducing the wealth level required to enter the formal sector in a model where the lender doesn’t need to respect the borrower’s individual rationality constraint. This is eminently sensible for a parent who chooses a child’s level of education, but may be less so for a lender offering a borrower a loan. Like Petersen and Rajan, the result of their comparative static is to make the borrower weakly worse off in all states.

In contrast to both theories, the welfare of rich borrowers in our model is unambiguously improved by improvements in the formal sector. The sources of improvement are twofold: borrowers who reach the formal sector receive higher utility, and borrowers enjoy more frequent unrestricted contracts because restrictive contracts become relatively more expensive for the lender. This improvement in the rich borrower’s welfare (and her inability to commit not to exercise her improved bargaining power) is absent in both theories highlighted above, yet is critical to our result that poor borrowers can be made worse off. The lender ameliorates this improvement in bargaining power by transitioning from away from unrestricted contracts, harming the borrower’s equilibrium welfare.

1.5 Extensions and Robustness

In this section we argue that the key intuitions highlighted above survive a number of extensions to the model. We begin by completely relaxing parametric Assumptions 1.4 and 1.5 and arguing that the qualitative lessons are unchanged.
We then extend the model to study direct competition. We show that if the incumbent lender has a sufficiently large lending advantage, the results are unchanged relative to the monopolist case above. If instead the incumbent has no advantage, the borrower necessarily reaches the formal sector in finite time in equilibrium. We further derive a sufficient condition for a monotone comparative static in the incumbent’s lending advantage which provides a testable prediction for the empirical exercise in Section 1.6.

In the appendix we explore several other extensions. First we allow for the lender to be incompletely informed about the borrower’s outside option and show that in equilibrium the borrower sometimes rejects the lender’s offer of a restrictive contract, providing an explanation for the low demand of microcredit. We then discuss an extension in which we allow the borrower to flexibly allocate a fraction of her income irrespective of contractual restrictions, and show that the lender may still restrict the rate at which the borrower grows relative to autarky.

1.5.1 Arbitrary production functions

In this section we relax Assumptions 1.4 and 1.5 and discuss how it affects our results. In particular we make no assumptions about $q_w, E_w, T_w, \phi_w$ or $h_w$ other than that $h_w > 0$ for all $w$ and Assumption 1.3 above, which guarantees that in autarky the borrower reaches the formal sector in finite time.

Structure of the Equilibrium

First, recall that Proposition 1.2 which states that the equilibrium is unique was shown without Assumptions 1.4 and 1.5 and thus continues to hold. In this section we discuss the structure of the unique equilibrium. A typical equilibrium is depicted below, with each circle representing a state and shaded circles representing states in which restrictive contracts are offered.

Even though in general we cannot say anything about the organization of restrictive and unrestrictive states, we argue that many of the empirical facts discussed in Section 1.4 can still be understood through the equilibrium above. In fact, with the exception
of heterogeneity in returns to credit, our explanation of the facts in that discussion only depended on the potential for each type of contract to coexist in a single equilibrium. As such we focus our attention for the remainder of this discussion on the prediction that wealthy borrowers will receive unrestrictive contracts and thus will enjoy high returns to credit.

To do so, we first outline how to transfer the insights in the above model to one with a countably infinite number of states. Given that our results do not depend on the number of states \( n \), or the cost of investment \( \phi_w \), this is a straightforward task. We define a sequence of games satisfying the above assumptions, each with successively more states.

Let \( \Gamma^1 \) be an arbitrary game satisfying the assumptions, with \( n \) business states.

For \( m > 1 \), let \( \Gamma^m \) be constructed in the following way:

- \( \Gamma^m \) has \( 2^{m-1}n \) business states, and let \( q_w^m, E_w^m, T_w^m, h_w^m, \) and \( \phi_w^m \) be the corresponding parameters for game \( \Gamma^m \).

- If \( w \) is even, set \( q_w^m = q_w^{m-1} / 2, E_w^m = E_w^{m-1} / 2, T_w^m = T_w^{m-1} / 2, \) and \( h_w^m = h_w^{m-1} / 2 \)

- If \( w \) is odd, set

\[
q_w^m \in \left[ \min \{q_{w-1}^m, q_{w+1}^m\}, \max \{q_{w-1}^m, q_{w+1}^m\} \right]
\]
\[
E_w^m \in \left[ \min \{E_{w-1}^m, E_{w+1}^m\}, \max \{E_{w-1}^m, E_{w+1}^m\} \right]
\]
\[
T_w^m \in \left[ \min \{T_{w-1}^m, T_{w+1}^m\}, \max \{T_{w-1}^m, T_{w+1}^m\} \right]
\]

and

\[
h_w^m \in \left[ \min \{h_{w-1}^m, h_{w+1}^m\}, \max \{h_{w-1}^m, h_{w+1}^m\} \right]
\]

- If \( w \) is even, set \( \phi_w^m = \phi_w^{m-1} / 2 \) and if \( w \) is odd, set \( \phi_w^m = \phi_w^{m-1} / 2 \)

Thus \( \Gamma^m \) has twice as many states at \( \Gamma^{m-1} \), and even states in \( \Gamma^m \) correspond to states in \( \Gamma^{m-1} \). The parameters in odd states take values intermediate to those in the surrounding states. Because the cost of investment in \( \Gamma^m \) is only half that in \( \Gamma^{m-1} \), a borrower investing in fixed capital at the same rate in either game would reach the formal sector in the same expected time. One way to understand \( \Gamma^m \) relative to \( \Gamma^{m-1} \) is that the borrower and lender appreciate more nuanced differences in the borrower's business size. Holding investment rate fixed, it takes the same amount of time to get from \( w \) to \( w + 2 \) in \( \Gamma^{m+1} \) as it does to get from \( w / 2 \) to \( w / 2 + 1 \) in \( \Gamma^m \), but along the way in \( \Gamma^m \) the borrower and lender realize an intermediate production function change. For \( m' > m \), we say \( \Gamma^{m'} \) is descended from \( \Gamma^m \) if
there is a sequence of games $\Gamma^m, \ldots, \Gamma^{m'}$ that can be derived in this manner. We have the following result.

**Proposition 1.10.** For any $\Gamma^m$, there is an $\bar{m}$ such that for all $m' > \bar{m}$, the equilibrium in any $\Gamma^{m'}$ descended from $\Gamma^m$ features a $\bar{w}$ such that for $w \geq \bar{w}$ the borrower reaches the formal sector in finite time starting from state $w$ if it is socially efficient to do so.

**Proof.** See appendix.

The above result says that for any game with sufficiently fine discrimination between states, all sufficiently wealthy borrowers receive unrestrictive contracts in equilibrium, and thus realize high returns to credit. The intuition is simple. Because entering the formal sector is efficient, the lender is unable to offer a sufficiently wealthy borrower (one who is sufficiently near to the formal sector) a restrictive contract she will accept. As the borrower and lender become arbitrarily discerning of different states, there will eventually be business states where the borrower is indeed sufficiently wealthy.

**Comparative statics**

As before, we can be fairly precise in describing how the equilibrium changes with respect to various fundamentals of the game. In this section we focus on the comparative static with respect to $u$.

Note that without loss of generality we can identify $m$ disjoint, contiguous sets of states $\{w_1, \ldots, w_1\}, \ldots, \{w_m, \ldots, w_m\}$ such that $w_m = \max \{w : p_w = 1\}$, $w_m = \max \{w : p_w = 1, p_{w-1} < 1\}$, and in general for $k \geq 1$, $w_k = \max \{w < w_{k+1} : p_w = 1\}$ $w_k = \max \{w \leq w_k : p_w = 1, p_{w-1} < 1\}$. An arbitrary set $\{w_k, \ldots, w_k\}$ is a contiguous set of states where restrictive contracts are offered with probability 1, and each pure restrictive state is contained in one of these sets.

We consider an impatient borrower and establish the following result.

**Proposition 1.11.** For impatient borrowers, the regions of contiguous restrictive states merge together as the formal sector becomes more attractive.

That is, for $\rho > \max_w \frac{x_0 \gamma_w}{x_0 \gamma_w + \gamma_w}, \frac{dp_w}{du} < 0$ for $w \in \{w_m + 1, n\}, \frac{dp_w}{du} > 0$ for $w \in \{w_{m-1} + 1, w_m - 1\}, \frac{dp_w}{du} < 0$ for $w \in \{w_{m-2} + 1, w_{m-1} - 1\}$ and so on.

**Proof.** See appendix.
Proposition 1.11 states that the highest region of pure restrictive states moves leftward, the second highest region moves rightward and so on. This is depicted in the following figure.

The intuition is as follows. For \((w_m \bar{w}_m)\), the analysis exactly follows that of Section 1.4.1, and hence it shifts leftward as \(u\) increases. But recall that for the impatient borrowers to the left of a restrictive state, the leftward shift lowers their utility. This is akin to lowering the utility of entering the formal sector, and hence for the next set of restrictive states \((w_{m-1} \bar{w}_{m-1})\) the analysis reverses and \(\bar{w}_{m-1}\) and \(w_{m-1}\) shift rightward. The rest follows by backward induction.

### 1.5.2 Direct Competition

Throughout the above analysis we have assumed the lender is a monopolist. In this section we introduce the possibility of a second lender who can make offers to the borrower. We label one lender the incumbent and one lender the entrant. The timing and technologies are the same as above, however now each period both lenders offer the borrower a contract. If the borrower accepts the incumbent’s contract everything proceeds as above. If the borrower accepts the entrant’s contract, everything proceeds as above except that the borrower incurs a non-pecuniary penalty of \(\psi dt > 0\). This penalty can be understood as a lending disadvantage the entrant suffers relative to the incumbent, perhaps because the incumbent is better equipped to screen or monitor its borrowers and thus borrowers interacting with the entrant undergo more costly screening processes.

We are now ready to state our first result.
Proposition 1.12. There exists a unique equilibrium in which the borrower accepts the incumbent's loan offer in all periods.

Proof. See Appendix.

The proof of the above result proceeds much in the same way as the proof of Proposition 1.2, except that rather than the borrower's outside option being that she can flexibly invest her endowment $E_w$, her outside option may now be to receive an attractive loan from the entrant and incur the non-pecuniary cost of $\psi dt$. Specifically, because the entrant never expects the borrower to accept his loan in the future, he is willing to offer the borrower any loan she would accept in the current period. Thus, in effect the borrower's outside option is whichever she prefers between flexibly allocating $E_w$, and flexibly allocating $E_w + T_w$ but incurring the non-pecuniary cost $\psi dt$.

Our next proposition aims to demonstrate that the intuitions derived under the monopolist case survive to the case with two lenders so long as $\psi$ is large enough. In contrast, as $\psi$ becomes sufficiently small, the equilibrium approaches the first best level of business expansion.

Proposition 1.13. There exists a $\bar{\psi} > 0$ such that for $\psi > \bar{\psi}$, the equilibrium probability $p_w$ that the incumbent lender offers the borrower a restrictive contract in state $w$ is the same as in the model with one lender.

So long as it is efficient to invest in business growth, there exists a $\bar{\psi} > 0$ such that for $\psi < \bar{\psi}$, the equilibrium probability $p_w$ that the incumbent lender offers the borrower a restrictive contract in state $w$ is 0. And as $\psi \to 0$, the equilibrium repayment rate $R$ charged by the incumbent lender converges to $T_w$ in each state $w$.

For a sufficiently strong lending advantage, the incumbent lender behaves as a monopolist. While this is intuitive, it may nevertheless be important to highlight that de facto monopoly power need not arise from being the only lender available. It may also arise from having low screening and monitoring costs relative to one's competitors. Proposition 1.13 also asserts that as the incumbent's lending advantage vanishes, he offers increasingly generous unrestricted contracts and the borrower's rate of business expansion approaches first best. This serves to highlight that in the limit of perfect competition, the inefficiencies highlighted in the model vanish.

Finally, we discuss comparative statics with respect to the incumbent's lending advantage. While the model is suitable to study the effect of a localized increase in competition
across many scenarios, we highlight only one, which motivates the empirical exercise to follow.

**Proposition 1.14.** If the equilibrium is characterized by a $\bar{w}$ such that $p_w = 1$ for $w \leq \bar{w}$, and $p_w \in (0, 1)$ for $w > \bar{w}$, then the comparative static with respect to the level of competition is unambiguous. As the incumbent’s lending advantage diminishes, the probability he offers restrictive contracts weakly declines in all states, and strictly so in states in which he had previously been offering a restrictive contract with positive probability. As competition increases, the incumbent is forced to transfer more surplus to his borrower, and the most efficient way to do so is to relax the contractual restrictions he imposes. In section 1.6.2, we provide empirical evidence for this testable prediction.$^{13}$

### 1.6 Novel Empirical Support

In this section we present two novel empirical observations in support of our theory. The first observation supports one of our key modeling assumptions; relatively richer borrowers are more likely to cease interaction with an MFI than are their poorer counterparts. Second we provide empirical evidence for a comparative static of our model. Leveraging exogenous variation in the level of competition facing money lenders in Thai villages, we show that competition causes a relaxation of the contractual restrictions they impose. We further argue that this is the unambiguous prediction of the model given the equilibrium contract structure we observe in the data. As discussed above, this comparative static prediction arises from the fact that money lenders in competitive markets face pressure to transfer additional surplus to their borrowers, and the primary method by which they do so is to ease contractual restrictions.

$^{13}$The comparative static in the empirical exercise may be more closely modeled by increasing the amount of money the entrant has to lend to the borrower. This would have exactly the same impact on the lending environment as reducing $\psi$. 

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1.6.1 Rich Borrowers Cease Interaction With an MFI

The data from this exercise are from Field et al. (2013). Their paper reports on an experiment conducted in partnership with Village Financial Services, a large Indian MFI. As discussed in the introduction, the authors randomly relaxed the contractual requirement that borrowers begin repayment immediately after loan disbursement; a random subset of borrowers received a two month grace period during which they did not need to meet any repayment obligation, and after which repayment took place in standard installments. The authors found that three years after the initial loan disbursement, borrowers in the treatment group reported weekly profits between Rs. 450 and Rs. 900 more than those in the control group. However they also report that borrowers in the treatment group default significantly more on average. While the probability that any amount of money remains in default at the end of the loan cycle more than quadruples (from about 2% in the control group to 9% in the treatment group), the increase in average amount in default is much more modest. On average borrowers in the treatment group default on an additional Rs. 150. Put another way, for a one time additional cost of Rs. 150, the grace period increased weekly business profits by Rs. 450 - Rs. 900. Why then, have MFIs (including Village Financial Services) maintained their strict repayment schedules that begin immediately after loan disbursement?

We propose that the answer lies in the pattern of default as a function of business size. The red line in Figure 1-1 below plots the expected amount in default a year after loan origination as a function of business profits three years after loan origination.1 As may be expected, there is a clear decreasing relationship between a borrower’s level of wealth and the amount of money she is expected to have in default.

However the blue line in Figure 1-1 plots the likelihood a borrower defaulted on any amount of money as a function of business profits three years after loan origination. In this case there is U-shaped relationship between profits and default; those with relatively low and relatively high business profits are the most likely to default. Table 1.1 presents this pattern in regression form. Specifically we regress

\[ Default_i = a + \beta_1 \ln prof_i + \beta_2 \ln prof sq_i + \gamma X_i + \epsilon_i \]

where \( Default_i \) is an indicator taking a value of 1 if borrower \( i \) has not completed repayment of her loan a year after origination (and substantially after the final tranche was

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14This is the publicly available measure of profits.
due), $lnprof$ is the log of borrower $i$'s business profits three years after loan origination, $lnprofitsq$ is the square of her profits three years after origination and $X_i$ is the vector of controls used in Field et al. (2013). As can be seen, $\beta_1$ is negative, $\beta_2$ is positive, and both are statistically significant once the vector of controls is included.

Taken together these findings paint a clear picture. Borrowers with large businesses are substantially less likely to finish repaying their loans but the amount of money they take in default is low. One interpretation of these findings is that these richer customers no longer found it worthwhile to attend repayment meetings, either because they found the biweekly meetings too time consuming or because they no longer saw the value in maintaining the option for further microloans. While the borrowers who do not completely repay their loans are only a subset of those who do not continue to borrow, the pattern is highly suggestive. The primary loss to the MFI may come from losing these customers rather than the default itself, and presents the possibility that the MFI may want to restrict the business growth of their customers.

### 1.6.2 Comparative Statics on Likelihood of Restrictive Contracts

Among the most distinctive features of our model is the comparative static of contractual restrictions with respect to the level of competition. Specifically, Proposition 1.14 makes an unambiguous, testable prediction and in this section we present evidence in its support in the context of informal borrowing in Thai villages.

To do so we leverage variation induced by the 2002 Million Baht Program, which endowed each village with a village fund with one million baht to lend. Importantly, the amount of money endowed to each village fund is invariant to the village size - thus smaller villages received more credit per capita than larger villages. Kaboski and Townsend (2012) leverage this variation to estimate the return to credit offered by village funds. In contrast, we focus on informal money lenders and their borrowers, and study how this change in competition affected the contracts they offered. As we show below, there is a steep decline in the likelihood a villager borrows from a money lender as a function of the villager’s wealth, and thus we think of informal money lenders and their borrowers as an empirical analogue of the informal sector in our model.

In what follows, we present an empirical definition of contractual restrictions, and show that borrowers subjected to these restrictions indeed anticipate lower income the following year. Further, we show there is a negative correlation between the imposition of
contractual restrictions and the interest rate, suggesting that money lenders may need to compensate borrowers whose investments they restrict. We next verify that the condition assumed by Proposition 1.14 is satisfied, and that as it predicts, money lenders in villages with larger increases in competition impose fewer contractual restrictions. Finally we invoke a variety of robustness and placebo tests to argue that this comparative static is not likely to be the result of other theories.

Data

The data used for this exercise were gathered as part of the ongoing Townsend Thai Project to track the financial lives of members of 64 Thai villages across 4 provinces: Chachoengsao, Lopburi, Sisaket, and Buriram. Specifically we utilize data from the household survey that tracked a representative sample of 15 households in each village on an annual basis. The dataset is extremely detailed; of particular relevance to this exercise it contains information about all loans received by study households (both formal and informal loans) including information on loan size, interest rates, collateral, consequence of default, and loan originator. The data also contain demographic information such as village size and composition, occupation, businesses operated, and a detailed breakdown of household income, and expectation about future income. For most of our regressions we utilize the unbalanced panel of loan level observations from survey rounds collected between 1997 (at the inception of the Townsend Thai Project) to 2007 (6 years after the Million Baht program was initiated).

Validity of the Natural Experiment

As described in Kaboski and Townsend (2012), two important elements make the Million Baht Program suitable for research. First, it was proposed during Prime Minister Thaksin Shinawatra’s election campaign following the dissolution of the Thai Parliament in November of 2000, and the program was then rapidly implemented between 2001 and 2002. So households were unlikely to anticipate the program in earlier years. Second the amount of credit endowed to each village was constant across villages of varying size, inducing variation in the per capita level of credit injected by the program. This motivates a continuous difference in differences strategy examining outcomes before and after the implementation of the program and across villages of varying sizes. That is, our principle
regressions of interest will be of the form

\[ \text{outcome}_{i,t} = \alpha + \beta_1 \text{in} \text{size}_v + \beta_2 \text{post}_{i,t} + \beta_3 \text{in} \text{size}_v \times \text{post}_{i,t} + \gamma X_{i,t} + \theta_t + \theta_h + \epsilon_{i,t} \]

where \( \text{in} \text{size}_v \) is the inverse size of the village, \( \text{post}_{i,t} \) is an indicator variable that is 1 if year \( t \) is after 2001, \( X_{i,t} \) is a vector of loan and household characteristics for the borrower of loan \( i \) in year \( t \), \( \theta_t \) is a year fixed effect, and \( \theta_h \) is a household fixed effect. All standard errors are clustered at the village level.

The validity of this regression rests on the orthogonality assumption

\[ \text{post}_{i,t}, \text{in} \text{size}_v \perp \epsilon_{i,t} | X_{i,t}, \theta_t, \theta_h \] (1.3)

Under the above assumption, \( \beta_3 \) captures the effect of treating small villages with a higher per capita credit shock than that of larger villages. To lend credibility to our identification assumption, we verify parallel trends in our pre-periods in the regressions to follow.

Further, in Table 1.2 we regress a number of village characteristics on inverse village size. We examine characteristics related to the village head, demographics, financial penetration, technology adoption, distance to a main road, and occupational distribution. The variables that have a significant relationship with inverse size are the number of agricultural cooperatives with a presence in the village, the education of the village head, and whether the village has common land that is shared among the villagers. In the regressions to follow we control for each of these characteristics as well as their interaction with \( \text{post}_{i,t} \). Finally, as in Kaboski and Townsend (2012), our main analysis restricts attention to villages with between 50 and 250 households, but we show it is robust to including all villages.

**Money Lenders are the Informal Sector**

We focus on borrowers in these villages who receive loans from money lenders. The two defining features of informal lenders in the model are that richer villagers are less likely to borrow from them, and that they have some capacity to influence the project selection of their borrowers. In this section we argue that rich villagers stop borrowing from money lenders. Figure 1-2 plots the likelihood a household borrows from a money lender as a function of its wealth. As can be seen, the poorest households are about three times as
likely to borrow from money lenders as are the richest households, creating the potential for money lenders to desire limiting the business growth of their clients.

**Contractual Restrictions**

Next we set forth two empirical definitions of contractual restrictiveness, one theoretically driven and one empirically driven. In line with our motivating examples our theoretical and primary measure of contractual restrictiveness is a binary indicator which takes a value of 1 if the borrower must forfeit productive capital for the duration of the loan or if the borrower must use a guarantor. In particular, in about 21% of cases in which land is used to collateralize a loan, the money lender is also given rights of use during the loan’s tenure. Because 60% of employed respondents report that farming is their primary means of income generation, and many more report it as a secondary means of income generation, we view this contractual feature as a restriction on the borrower’s ability to put a productive asset to use. Additionally, we code contracts which require a guarantor as restrictive, as guarantors may pressure borrowers to eschew profitable yet risky investments in favor of safer and less profitable ones to ensure repayment (see e.g. Banerjee et al. (1994) and Fischer (2013)).

All other forms of collateral, and the absence of a collateral requirement, are coded as unrestrictive. Other forms of collateral include the deed to the borrower’s land (but allow the borrower to retain the rights of use for the duration of the loan), jewelry, the deed to their house, and the right to proceeds from future crop production. Importantly, none of these forms of collateral involve the transfer of productive capital from the borrower to the lender.

The defining feature of restrictive contracts in our model is that they restrict growth of the borrower’s business. Table 1.3 offers suggestive evidence that this may be the case in practice. Columns 1 and 2 present a regression of the borrower’s expected income in the following year on whether her money lender asks for a restrictive form of collateral. That is, we present regressions of the following form

\[
\log(y)_{i,t+1} = \alpha + \beta_{\text{restrictive},i,t} + X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}
\]

where \(\log(y)_{i,t+1}\) is the log of the borrower’s year \(t\) expectation of her year \(t + 1\) income, and all other variables are as defined above. In addition to household and year fixed effects, the controls \(X_{i,t}\) include the borrower’s income and her loan size. As can be seen
from the table, there is a negative and significant correlation between the borrower’s expected income and the imposition of restrictive contractual features. In the next two columns we present a regression of the form

$$\log(y)_{i,t+1} = \alpha + \sum \beta_{j,collateral_{i,t}} + X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}$$

where $collateral_{i,t}$ is 1 if the borrower of loan $i$ in year $t$ was asked for a collateral of type $collateral_j$. Several forms of collateral have large negative relationships with the borrower’s expected income. Consistent with the theoretically motivated definition of contractual restrictiveness, these are that the borrower forfeits her land, that the lender asks for multiple guarantors, and a catch-all category labeled “other.” Collateral in “other” are typically other forms of third party guarantees, such as using a third party’s assets to guarantee the loan. This motivates a data driven definition of contractual restrictiveness that takes a value of 1 if the collateral takes any of the preceding three forms and 0 otherwise. In Table 1.15 we show that our main regressions are robust to using this empirically driven notion of restrictiveness rather than our primary one.

Finally we verify that, as predicted by our theory, restrictive forms of collateral are correlated with a reduction in the interest rate. Specifically we regress

$$Monthly\text{Interest}_{i,t} = \alpha + \beta_{\text{restrictive}} + X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}$$

where $Monthly\text{Interest}_{i,t}$ is the monthly interest rate that the borrower is charged for the loan, and all other variables are as defined above. We present this regression in Table 1.4.

While one could imagine stories for the negative relationship between contractual restrictions and future expected income in which the arrow of causality points in the opposite direction (e.g. borrowers with low expectations about their future income are more tolerant of restrictive forms of collateral), it is a little tougher to do the same for the negative relationship between contractual restrictions and the interest rate. In particular, if contractual restrictions were a proxy for low quality borrowers, we would expect a positive relationship between the two variables. Thus, while acknowledging that the above relationships are merely correlational, we take them to be reassuringly consistent with the predictions of the model.
The Theory Predicts Increased Competition Corresponds to a Reduction in Restrictive Contracts

In general, the theory’s prediction about how money lenders should react to increased competition is ambiguous. On the one hand, competition among lenders increases the borrower’s bargaining power, which is a force increasing the frequency of unrestrictive contracts. On the other hand, if the lender anticipates a sufficiently large increase competition for relatively rich borrowers, the lender may increase the frequency of restrictive contracts for poorer borrowers. Which of these two forces dominates is in general sensitive to parametric assumptions.

Taking the theory literally, however, Proposition 1.14 asserts that if in equilibrium the lender offers all borrowers (above a certain wealth level) a restrictive contract with strictly interior probability, the comparative static is unambiguous. In such a case we should expect that the first force always dominates, and that after the program was introduced, villages with a larger increase in competition (i.e. smaller villages) should have a correspondingly larger decrease in the frequency of restrictive contracts. Figure 1-3 plots the likelihood a borrower is asked for a restrictive form of collateral as a function of his income and verifies that the condition of Proposition 1.14 is satisfied. Thus the theory makes an unambiguous prediction.\textsuperscript{15}

Comparative Static on the Level of Competition

We now turn to our main regression:

\[ restrictive_{i,t} = \alpha + \beta_1 \text{invsize}_v + \beta_2 \text{post}_{i,t} + \beta_3 \text{invsize}_v \ast \text{post}_{i,t} + \gamma X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t} \]

where all variables are as defined above. In addition to household and year fixed effects, the controls include the three village characteristics correlated with inverse size and their interactions with \text{post}_{i,t}, the borrower's income and loan size, and whether the household has a loan from the village fund. In our primary regressions we use data from 1997 to 2007, but also show in Table 1.14 that our results are robust to including only the three years before and after the program took effect. Note that our theoretical prediction is that

\textsuperscript{15}We do note that Proposition 1.14 requires the lender is following a mixed strategy between offering restrictive and unrestrictive contracts. While it is difficult to verify whether money lenders are indifferent between the two contracts in practice, it is reassuring that the true distribution of restrictive contracts is far from either corner case for all sufficiently wealthy borrowers.
\( \beta_3 < 0 \), as higher inverse village size corresponds to a larger credit shock, which should correspond to a greater relaxation in contractual restrictiveness.

Our main results are presented in Table 1.5. As can be seen, \( \beta_3 \) has the desired sign and large magnitude. For instance, \( \beta_3 \) is approximately \(-10.8\) in the OLS specification in column 1. The standard deviation of inverse village size is .006, so a one standard deviation increase in inverse village size corresponds to an approximately 6.5 percentage point decrease in the probability a restrictive form of collateral is demanded (off of a base of about 20 percentage points).

In Table 1.6, we verify parallel trends prior to 2002. Specifically, we regress

\[
\text{restrictive}_{i,t} = \alpha + \beta_1 \text{invsize}_v + \beta_2 \text{wave}_t + \beta_3 \text{invsize}_v \times \text{wave}_t + \gamma X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}
\]

and verify that we cannot reject \( \beta_3 = 0 \). We also regress

\[
\text{restrictive}_{i,t} = \alpha + \beta_1 \text{invsize}_v + \sum_{t=1997}^{2001} \beta_t \text{wave}_t + \sum_{t=1997}^{2001} \beta_t \text{invsize}_v \times \text{wave}_t + \gamma X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}
\]

and verify we cannot reject that \( \beta_t \) are jointly 0.

Finally we regress

\[
\text{restrictive}_{i,t} = \alpha + \beta_1 \text{invsize}_v + \sum_{t=1997}^{2007} \beta_t \text{wave}_t + \sum_{t=1997}^{2007} \beta_t \text{invsize}_v \times \text{wave}_t + \gamma X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}
\]

and plot the coefficients \( \beta_t \) in Figure 1-4. As can be seen, there does not seem to be a strong trend before or after 2002, but with the exception of 2005, all coefficients post 2002 are below 5 and all coefficients pre 2002 are above 5.

The Comparative Static is Not Driven by Borrower Selection

One threat to identification would be that the villagers who borrow from money lenders are selected differently in high and low competition environments. While our main regression would still capture the causal effect of competition on the incidence of restrictive contracts, it would be capturing an effect on the composition of borrowers rather than an effect on borrowers' bargaining power. Because we use an unbalanced panel (we use loan...
level observations), our inclusion of household fixed effects does not eliminate the threat of selection at the household level.

Nevertheless we argue selection is unlikely to be driving our results. First we examine whether fewer households borrow from money lenders in high competition environments. Specifically we regress

\[
\text{borrower}_{i,t} = \alpha + \beta_1 \text{invsize}_t + \beta_2 \text{post}_{i,t} + \beta_3 \text{invsize}_t \ast \text{post}_{i,t} + \gamma X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}
\]

where our unit of observation \(i, t\) is a household \(\times\) wave, our outcome \(\text{borrower}_{i,t}\) is an indicator taking a value of 1 if household \(i\) borrows from a money lender in wave \(t\) and 0 otherwise, and everything else is defined as above. These regressions are presented in Table 1.7. The estimates fluctuate in sign, are always small in economic terms, and are all far from statistically significant. For example, recalling that standard deviation on inverse village size is .006, the estimates in column 2 imply that a village with one standard deviation smaller size saw a .2% larger decline in the frequency with which villagers borrow from money lenders after the Million Baht Program.

Next we examine whether households borrow less money from money lenders in villages with higher competition. We regress

\[
\log(\text{amount})_{i,t} = \alpha + \beta_1 \text{invsize}_t + \beta_2 \text{post}_{i,t} + \beta_3 \text{invsize}_t \ast \text{post}_{i,t} + \gamma X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}
\]

where \(\log(\text{amount})_{i,t}\) is the log of the total amount borrowed from any money lender by household \(i\) in wave \(t\) and everything else is as defined above. The results are presented in Table 1.8. Again, the estimates fluctuate in sign, are always economically small, and far from statistically significant.

Finally Table 1.9 presents the results of a Heckman selection model of our primary specification. The selection equation assumes that the error terms are jointly normal and conditions on a number of village and borrower characteristics including \(\text{post}_{i,t}, \text{invsize}_t\) and \(\text{post}_{i,t} \times \text{invsize}_t\). Two observations are of note. First, the coefficient on interaction term \(\text{invsize}_t \ast \text{post}_{i,t}\) in the selection equation is far from significant. Second, the estimates in Table 1.9 are strikingly similar to the corresponding estimates in Table 1.5. Taken together these offer further reassurance that selection is not driving our results.
Ruling Out Alternative Theories

We now present a number of tests to rule out alternative theories. First we aim to rule out a general trend away from restrictive forms of collateral. Specifically we rerun our main regression, but rather than focusing on the population of borrowers interacting with informal money lenders, we focus on the borrowers who take loans from their neighbors. These lenders should have no desire to keep their borrowers in poverty, and thus we would expect no relationship between the variation induced by the Million Baht Program and the likelihood that these lenders ask for restrictive forms of collateral. The regressions are presented in Table 1.10. The estimate on the interaction term fluctuates in sign and magnitude and is never statistically significant.

Next, we aim to separate our theory from any other one that makes no distinction between different forms of collateral. Again focusing on the population of borrowers who take loans from money lenders we regress

\[ \text{unrestrictive}_{i,t} = \alpha + \beta_1 \text{invsize}_o + \beta_2 \text{post}_{i,t} + \beta_3 \text{invsize}_v \times \text{post}_{i,t} + \gamma X_{i,t} + \theta_v + \theta_t + \epsilon_{i,t} \]

where \( \text{unrestrictive}_{i,t} \) is an indicator taking a value of 1 if a money lender asks for any form of collateral that was not coded as restrictive and 0 otherwise. We present this regression in Table 11 from which it is apparent that there is a positive relationship between the interaction term and the likelihood a money lender asks for unrestrictive collateral. This is what one would expect if money lenders substitute away from restrictive forms of collateral toward unrestrictive forms of collateral in high competition environments. Thus any theory consistent with these results must draw a distinction between restrictive and unrestrictive forms of collateral, and must also predict the reduction in restrictive forms of collateral arising from increased competition from village funds.

Finally, we aim to rule out the possibility that the increased competition from village funds caused money lenders to screen their borrowers less carefully, and that this explains the shift away from restrictive forms of collateral in high competition environments. Though we do not observe screening efforts directly, we provide indirect evidence that this is not the case by examining how the interest rates charged by money lenders are influenced by competition. Specifically we regress

\[ \text{MonthlyInterest}_{i,t} = \alpha + \beta_1 \text{invsize}_o + \beta_2 \text{post}_{i,t} + \beta_3 \text{invsize}_v \times \text{post}_{i,t} + \gamma X_{i,t} + \theta_v + \theta_t + \epsilon_{i,t} \]
where all variables are as defined above. The results are presented in Table 1.12. Though the estimates are noisy, and diminish as we include finer levels of fixed effects, it appears that if anything money lenders charge higher interest rates in higher competition environments.\textsuperscript{16} Thus, a theory based on screening effort should predict that money lenders increased their screening efforts in high competition environments to compensate for the more adversely selected population of borrowers, and would thus predict a corresponding increase in restrictive contracts, counter to what we observe.

\section*{1.7 Discussion}

In this paper we explored a novel theory of informal lending, in which relatively rich borrowers cease lending from informal financiers and enter the formal sector. Each period the borrower and her informal lender bargain not only over the interest rate, but also over a contractual restriction that governs the borrower's ability to invest in business expansion. In contrast with earlier theories (e.g. Braverman and Srinivasan (1981) and Braverman and Stiglitz (1982)) we show that the borrower may indeed be caught in a debt trap when she cannot commit to share the benefits of entering the formal sector.

Our theory therefore reconciles the seemingly disparate facts that small scale entrepreneurs enjoy very high return to capital yet are unable to leverage microcredit and other forms of informal finance to realize those high returns. Moreover it offers an explanation for the robust findings that relatively wealthier business owners do enjoy high returns from microcredit and that on average the demand for microcredit is low.

The theory also offers nuanced predictions on comparative statics of the lending environment. Increasing the attractiveness of the formal sector improves the bargaining power of rich borrowers and hence increases their welfare and relaxes the poverty trap. However the same improvement may harm the welfare of poorer borrowers; anticipating that rich borrowers have improved bargaining power, the lender tightens contractual restrictions on poor borrowers to prevent them from reaching higher levels of wealth and exploiting their improved position. Similarly, and counter to standard intuitions, increasing the borrower's patience (and hence her value for business expansion) can make

\textsuperscript{16}While it may at first seem unintuitive that an increase in competition is accompanied by an increase in the interest rate charged by money lenders, it is entirely consistent with our theory. We have documented that as competition rises money lenders offer more unrestrictive contracts, which tend to carry higher interest rates than restrictive ones.
relatively poor borrowers worse off, and tighten the poverty trap. We finally showed that, while the model primarily studies a monopolist lender, similar effects can operate in contexts with imperfect competition, and derived intuitive comparative statics on the level of competition.

Then, studying money lenders in rural Thailand, we offered empirical evidence for the comparative static prediction on competition. Utilizing the Townsend Thai data and the plausibly exogenous shock to competition induced by the Million Baht Program, we found that Thai money lenders in environments with high competition reduced the frequency with which they imposed contractual restrictions more than money lenders in low competition environments. We argue that, because the same effects cannot be found for loans given by neighbors or for other forms of collateral, other theories are unlikely to explain this robust pattern.

In addition to the theories cited in the introduction, it is worth contrasting our theory with two other theories development economists seem to carry with them in the field. The first of these theories might sensibly be labeled as “blaming the borrower.” These are theories that allude to the argument that many borrowers are not natural entrepreneurs, and are primarily self employed for lack of being able to find steady wage work (see e.g. Schoar (2010)). While these theories enjoy some empirical support, they are at best a partial explanation of the problem as they are inconsistent with the large impacts of cash grants, cited in our introduction.

Second are the theories that assign blame to the lender for not having figured out the right lending contract. These theories are implicit in each of the experiments that evaluate local modifications to standard contracts (see e.g. Gine and Karlan (2014), Attanasio et al. (2015), and Carpena et al. (2013) on joint liability, Field et al. (2013) on repayment flexibility, and Feigemberg, Field, and Pande (2013) on meeting frequency). While many of these papers contribute substantially to our understanding of the ways in which microfinance operates, none have so far generated a lasting impact on the models that MFIs employ.

In contrast, ours is a theory that assumes that borrowers have the competence to grow their business, and that lenders are well aware of the constraints imposed on borrowers by the lending paradigm. Instead we focus on the rents that lenders enjoy from retaining customers, and the fact that customers who become sufficiently wealthy will find alternative forms of finance. Part of the value of this theory therefore may very well be its distance from the main lines of reasoning maintained by empirical researchers.
1.8 Appendix

1.8.1 Additional Extensions to the Model

The Borrower is Privately Informed About Her Outside Option

In this section we explore an extension in which the borrower maintains some private information about her outside option. In particular we augment the model such that the borrower’s autarkic endowment is privately known. If she rejects the lender’s contract she receives an endowment of $E_w + v_t$. Let $v_t \overset{iid}{\sim} G$ be a random variable privately known to the borrower, and redrawn each period in an iid manner from some distribution $G$. Further, assume that if the borrower accepts the lender’s contract, her endowment is still $E_w + T_w$. One way to understand this is that in the event that the borrower rejects the lender’s contract, a relative will offer her a gift of size $v_t$, which she can allocate flexibly between her projects. We make the following additional assumption on the range of $G$ to simplify the discussion.

Assumption 1.6. Let $G$ have bounded support with minimum $0$ and maximum $\bar{v}$ such that $\bar{v} < \frac{h_w}{q_w}$.

The above assumption guarantees that the borrower will accept any unrestrictive contract. However, if the lender offers the borrower a restrictive contract, he will now face a standard screening problem. Because he would like to extract the maximum acceptable amount of income, borrowers with unusually good outside options will reject his offer. This is encoded in the following proposition.

Proposition 1.15. The borrower never rejects an unrestrictive contract on the equilibrium path. However the borrower may reject restrictive contracts with positive probability.

Proof. See Appendix. \qed

This intuitive result offers an explanation for the low takeup of microcredit contracts referenced in the introduction. Lenders who offer restrictive contracts to borrowers aim to extract all of the additional income generated by the loan, but in doing so sometimes are too demanding and therefore fail to attract the borrower. In contrast, lenders who offer unrestrictive contracts necessarily leave the borrower with excess surplus, and therefore demand for these contracts is high.
The Borrower Flexibly Allocates a Fraction of Her Income

In this section we explore an extension to the model in which, even when subjected to contractual restrictions, the borrower maintains flexible control over a fraction of her income. In doing so we aim to show that our main result is qualitatively robust. Rather than finding that the borrower may remain in inefficiently small forever, we now find that having access to a lender may slow the borrower’s growth relative to her autarkic rate.

Formally, the model is as in Section 1.2 but after accepting a contract \((R, a)\), the borrower is free to invest a fraction \(s < 1\) of her endowment flexibly, irrespective of the contractual restriction \(a\) the lender imposes. Thus we have weakened the lender’s ability to influence the borrower’s project choice. We maintain all other assumptions from Sections 1.2 and 1.3, and make the following observation.

**Proposition 1.16.** For sufficiently small \(s\), the lender may impose the contractual restriction \(C\) on the equilibrium path. In such an equilibrium the borrower reaches the formal sector in finite time, but will grow more slowly than she would have in autarky.

*Proof.* See Appendix. \(\square\)

### 1.8.2 Omitted Proofs

**Proof of Proposition 1.1**

In state \(n\) the borrower chooses her investment allocation \(i\) to maximize

\[
B_n^{aut} = \max_{i \leq E_n} q_n (E_n - i) dt + e^{-\rho dt} \left( \left( 1 - e^{\frac{i}{\phi_n} dt} \right) B_{n+1}^{aut} + e^{\frac{i}{\phi_n} dt} B_{n+1}^{aut} \right)
\]

At the optimal level of \(i\), the borrower’s continuation utility can be rewritten as

\[
B_n^{aut} = \frac{q_n (E_n - i) dt + e^{-\rho dt} \left( \left( 1 - e^{\frac{i}{\phi_n} dt} \right) B_{n+1}^{aut} \right)}{1 - e^{\left( \frac{i}{\phi_n} + \rho \right) dt}}
\]

\[
\rightarrow \frac{q_n (E_n - i)}{\frac{i}{\phi_n} + \rho} + \frac{\frac{i}{\phi_n} + \rho B_{n+1}^{aut}}{\frac{i}{\phi_n} + \rho}
\]

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Because the problem is stationary, the borrower's maximization problem can equivalently be written as choosing $i$ to maximize her continuation utility above.

We now take the derivative of $B^\text{aut}_n$ with respect to $i$.

$$\frac{dB^\text{aut}_n}{di} = \frac{-q_n\phi_n - q_n\kappa_n}{\left(\frac{1}{\phi_n} + \rho\right)^2} + \frac{\rho}{\phi_n^2} B^\text{aut}_{n+1}$$

where recall $\kappa_w \equiv \frac{E_w}{\phi_w}$. The denominator is positive. The numerator is positive iff

$$-q_n\phi_n - q_n\kappa_n + \frac{\rho}{\phi_n} B^\text{aut}_{n+1} > 0$$

$$\iff \alpha_n B^\text{aut}_{n+1} > \frac{q_n E_n}{\rho}$$

We conclude that if $\frac{q_n E_n}{\rho} < \alpha_n B^\text{aut}_{n+1}$ then the borrower's value in state $n$ $B^\text{aut}_n$ is increasing in $i$ and otherwise it is decreasing. The result for earlier states follows similarly by backward induction. This completes our proof.

**Proof of Proposition 1.2**

The existence of an equilibrium follows standard arguments (See Maskin and Tirole (2001)). In this section we prove that generically the equilibrium is unique. We do so by backward induction on the state. In each state we first argue that if there exists an equilibrium in which the borrower invests her autarkic endowment in the working capital project conditional on rejecting the offered contract, then she does so in all equilibria and the lender gives her an unrestrictive contract with lower than necessary repayment rate.

We then argue that in any equilibrium where the borrower invests her autarkic endowment in the fixed capital project conditional on rejecting the offered contract (which, by the above statement, can only exist in the absence of an equilibrium in which the borrower invests her autarkic flow payoff in the working capital project), the lender's behavior is uniquely determined.

We first consider equilibrium behavior in state $n$.

**Lemma 1.1.** There is no equilibrium in which the borrower weakly prefers to invest her autarkic endowment in the working capital project in state $n$. 

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Proof. Suppose toward contradiction that upon rejecting the contract, the borrower weakly prefers investing her autarkic endowment in the working capital project. Then the borrower would accept any restrictive contract (which necessarily grants her more consumption than her outside option), and in state \( n \) the lender would clearly choose to give the borrower a maximally extractive restrictive contract in every period. The borrower’s equilibrium continuation value is \( B_n = \frac{h_n + q_n E_n}{\rho} \) which by Assumption 1.3 is less than \( \alpha_n \frac{u}{\rho} \). This contradicts that the borrower weakly prefers to consume her autarkic flow payoff.

We next establish that conditional on the borrower strictly preferring to invest her autarkic endowment in fixed capital in equilibrium, the lender’s behavior is uniquely determined.

**Lemma 1.2.** The probability the lender offers a restrictive contract in state \( n \), \( p_n \) is generically uniquely determined across any equilibrium in which the borrower strictly prefers to invest her autarkic endowment in the fixed capital project in state \( n \).

Proof. Assume the borrower strictly prefers to invest her autarkic endowment in the fixed capital project. Then if the lender offers the borrower any unrestrictive contract she accepts it, and the lender never benefits in state \( n \) from offering an excessively generous unrestrictive contract. Further, the lender never offers the borrower an excessively generous restrictive contract. So in equilibrium, the lender either offers the borrower the contract \( \langle q_n T_n - h_n, I \rangle \), or the contract \( \langle R(p), C \rangle \) for some \( R(p) \) that pushes the borrower to her outside option. Now conjecture that in equilibrium the lender offers the borrower a restrictive contract with probability \( p \).

Noting that the borrower receives the maximum of the utility from investing her outside option in fixed capital and from consuming \( q_n E_n + h_n \) upon receiving a restrictive contract, we have

\[
B_n(p) = p \left( \max \left\{ e^{-\rho dt} \left( 1 - e^{-\kappa dt} \right) \frac{u}{\rho} + e^{-\kappa dt} B_n(p) \right) \right) \\
+ (1 - p) \left( e^{-\rho dt} \left( 1 - e^{-\gamma dt} \right) \frac{u}{\rho} + e^{-\gamma dt} B_n(p) \right)
\]

\[
B_n(p) = \max \left\{ \frac{p}{1 - pe^{-(\rho + \kappa) dt}} \left( q_n E_n + h_n \right) dt + \frac{(1 - p) e^{-\rho dt} (1 - e^{-\gamma dt}) \frac{u}{\rho}}{1 - pe^{-\rho dt} - (1 - p) e^{-(\rho + \gamma) dt}} \right\}
\]

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It is straightforward to verify that $\frac{dB_n(p)}{dp} < 0$. This is intuitive as a restrictive contract pushes the borrower to her individual rationality constraint (if possible), whereas an investment contract does not.

The highest possible repayment rate $R(p)$ that can be required for a restrictive contract is pinned down by the borrower’s individual rationality constraint

\[
(q_n (E_n + T_n) - R(p)) \Delta t + e^{-\rho \Delta t} B_n(p) \geq e^{-\rho dt} \left( (1 - e^{-\kappa_n dt}) \frac{u}{\rho} + e^{-\kappa_n dt} B_n(p) \right)
\]

\[
q_n (E_n + T_n) - R(p) = \max \left\{ \frac{e^{-\rho dt}}{dt} \left( 1 - e^{-\kappa_n dt} \right) \left( \frac{u}{\rho} - B_n(p) \right), q_n E_n + h_n \right\}
\]

The maximal acceptable repayment rate is increasing in $B_n(p)$ - this is intuitive as the higher is the borrower’s continuation value in state $n$, the less she values investment. 17

Now, consider the lender’s decision of whether to offer an unrestrictive or restrictive contract. Fixing the borrower’s expectation that the lender offers a restrictive contract with probability $p$, in any period in which the lender offers an unrestrictive contract his utility is

\[
(q_n T_n - h_n - T_n) \Delta t + e^{-\rho dt} \left( 1 - e^{-\gamma_n dt} \right) L_n(p)
\]

If he offers a restrictive contract his utility is

\[
(R(p) - T_n) \Delta t + e^{-\rho dt} L_n(p)
\]

So he offers a restrictive contract if and only if the following incentive compatibility constraint holds

\[
(R(p) - T_n) \Delta t + e^{-\rho dt} L_n(p) \geq (q_n T_n - h_n - T_n) \Delta t + e^{-\rho dt} \left( 1 - e^{-\gamma_n dt} \right) L_n(p)
\]

\[
\iff
\]

\[
(q_n T_n - h_n - R(p)) \Delta t \leq e^{-(\rho + \gamma_n) dt} L_n(p)
\]

The left hand side is the additional consumption the lender must forgo to persuade the borrower to accept a restrictive contract, and the right hand side is the discounted expected loss the lender incurs from allowing the borrower to invest in fixed capital.

\[\text{Note that by Assumption 1.3, } q_n (E_n + T_n) - R(1) > q_n E_n + h_n \text{ for sufficiently small } dt, \text{ but in general } q_n (E_n + T_n) - R(p) \text{ may equal } q_n E_n + h_n \text{ for some } p < 1.\]
Note that the lender’s continuation utility $L_n(p)$ is weakly decreasing in $p$. This is so because the set of restrictive contracts the borrower will accept is decreasing in $p$, while the set of unrestrictive contracts is unchanged.

Thus the left hand side of the above inequality is weakly increasing in $p$, and the right hand side is weakly decreasing in $p$. Given the lender’s incentive compatibility constraint, we argue that generically there can only be one equilibrium level of $p$.

If at $p = 0$ (pure unrestrictive), the lender’s incentive compatibility constraint for unrestrictive contracts is strictly satisfied, i.e.

$$ (q_n T_n - h_n - R(0)) \, dt > e^{-(\rho + \gamma_n)dt} L_n(0) \quad (1.4) $$

then it will be strictly satisfied for all higher levels of $p$, contradicting that any $p > 0$ can be supported in equilibrium.

If at $p = 1$ (pure restrictive) the lender’s incentive compatibility constraint for restrictive contracts is strictly satisfied, i.e.

$$ (q_n T_n - h_n - R(1)) \, dt < e^{-(\rho + \gamma_n)dt} L_n(1) \quad (1.5) $$

then it will be strictly satisfied for all lower levels of $p$, contradicting that any $p < 1$ can be supported in equilibrium.

If neither of the above inequalities holds even weakly then by the intermediate value theorem there will be a $\bar{p}$ at which

$$ (q_n T_n - h_n - R(\bar{p})) \, dt = e^{-(\rho + \gamma_n)dt} L_n(\bar{p}) $$

Note that when the borrower believes she will receive a restrictive contract with probability $\bar{p}$, the amount of consumption she demands when given a restrictive contract, $q_n(E_n + T_n) - R(\bar{p})$, is strictly larger than than $q_nE_n + h_n$ (the minimum feasible consumption the lender can leave the borrower). To see this, note that by assumption

$$ (q_n T_n - h_n - R(0)) \, dt < e^{-(\rho + \gamma_n)dt} L_n(0) . $$

Now, supposing that $q_n(E_n + T_n) - R(\bar{p}) = q_nE_n + h_n$, we’d have $L_n(\bar{p}) = L_n(0)$ (because the borrower is willing to accept all feasible contracts in both cases), which would imply that $(q_n T_n - h_n - R(\bar{p})) \, dt < e^{-(\rho + \gamma_n)dt} L_n(\bar{p})$ and would contradict that the lender
is indifferent between restrictive and unrestrictive contracts. Therefore we know that
\[ q_n (E_n + T_n) - R(p) = q_nE_n + h_n \]
and thus \( \frac{dR(p)}{dp} < 0 \). At \( p > \bar{p} \) the lender will strictly
prefer investment loans and at \( p < \bar{p} \) the lender will strictly prefer consumption loans,
contradicting that any \( p \neq \bar{p} \) can be supported in equilibrium.\(^{18}\)

So far we have argued that in state \( n \) the borrower and lender’s behavior are uniquely
determined across all equilibria. We now proceed to complete the proof by backward
induction. Suppose in all states \( \bar{w} \geq w + 1 \) it has been shown that equilibrium behavior
is generically unique.

We split the proof for state \( w \) into the following two lemmas.

**Lemma 1.3.** *If there exists an equilibrium in which the borrower weakly prefers to invest her au-
tarkic endowment in the working capital project in state \( w \), she does so in all equilibria. Moreover,
the lender offers her the same (unrestrictive) contract across all equilibria.*

*Proof.* Assume that in equilibrium the borrower weakly prefers to invest her autarkic
endowment in working capital in the event of rejecting the lender’s contract in state \( w \).

Then the borrower would accept a maximally extractive restrictive contract. But if the
lender were to offer one in equilibrium, this would contradict that the borrower would
invest her autarkic endowment in working capital. To see this, note that Assumption 1.3
 guarantees
\[ \alpha_w B_{w+1} > \frac{q_w E_w + h_w}{\rho} \]
Thus the borrower would strictly prefer to invest her entire autarkic endowment \( E_w \) in
fixed capital than to accept the maximally extractive contract \( (q_w T_w - h_w, C) \) if she expects
that the lender will offer her that contract in all future periods in state \( w \).

So in equilibrium the lender must weakly prefer to offer the contract \( (q_w T_w - h_w, I) \)
to the contract \( (q_w T_w - h_w, C) \). Suppose in equilibrium the lender offers the borrower an
unrestrictive contract for which the borrower’s IR constraint binds. Then in every period
the borrower is indifferent between consuming \( q_w E_w \) and accepting the lender’s contract.
The borrower’s continuation utility is \( B_w = \frac{q_w E_w}{\rho} \), which, by Assumption 1.3, contradicts
her desire to invest her autarkic endowment in working capital.

The final possibility, which we expand on below, is that in equilibrium the lender offers
the borrower an unrestrictive contract \( (R, I) \) that is more generous than the borrower’s IR

\(^{18}\)Note that since both \( R(p) \) and \( L(p) \) can both be written in terms of exogenous parameters of the model,
it will hold generically that neither 1.4 nor 1.5 holds with equality.
constraint demands. But if the borrower’s IR constraint doesn’t bind when offered \( (R, I) \), then it must be that this is the lender’s unconstrained optimal contract and will continue to be feasible in any conjectured equilibrium in which the borrower strictly prefers to invest her autarkic endowment in fixed capital. Noting that, by assumption she weakly prefers to invest her autarkic flow income in working capital when offered the contract \( (R, I) \), this contradicts the existence of an alternative equilibrium.

Formally, if the borrower weakly prefers to invest her autarkic endowment in working capital, then the lender solves

\[
\max_R \left( R - T_w \right) dt + e^{-\rho dt} \left( \frac{1 - e^{-\frac{E_{w+T_w} + R}{\phi_w} dt}}{1 + \frac{E_{w+T_w} + R}{\phi_w} dt} \right) L_{w+1} - e^{-\frac{E_{w+T_w} - R}{\phi_w} dt} L_w
\]

s.t.

\[
e^{-\rho dt} \left( 1 - e^{-\frac{E_{w+T_w} + R}{\phi_w} dt} \right) B_{w+1} + e^{-\frac{E_{w+T_w} - R}{\phi_w} dt} B_w \geq q_w E_{w} dt + e^{-\rho dt} B_w
\]

\[T_w \leq R \leq q_w T_w - h_w\]

Note that, because the problem is stationary, we could equivalently solve for the contract that maximizes the lender’s continuation value in state \( w \). That is we can solve

\[
\max_R \left( R - T_w \right) dt + e^{-\rho dt} \left( 1 - e^{-\frac{E_{w+T_w} + R}{\phi_w} dt} \right) L_{w+1}
\]

s.t.

\[
e^{-\rho dt} \left( 1 - e^{-\frac{E_{w+T_w} + R}{\phi_w} dt} \right) B_{w+1} + e^{-\frac{E_{w+T_w} - R}{\phi_w} dt} B_w \geq q_w E_{w} dt + e^{-\rho dt} B_w
\]

\[T_w \leq R \leq q_w T_w - h_w\]

It is straightforward to show that the maximand is increasing if and only if

\[
L_{w+1} > \phi_w \frac{1 - e^{-\left( \frac{E_{w+T_w} + R}{\phi_w} \right) dt}}{q_w e^{-\rho dt}} - e^{-\frac{E_{w+T_w} - R}{\phi_w} dt} - e^{-\frac{E_{w+T_w} - R}{\phi_w} dt}
\]
and that 
\[ \frac{1-e^{-\left(\frac{E_{w}+T_{w}-R}{\Phi_{w}}\right) dt}}{e^{-\left(\frac{E_{w}+T_{w}-R}{\Phi_{w}}\right) dt}} \] is increasing and always less than 1 for \( R \leq q_{w}T_{w} - h_{w} \).

Thus the maximand is either monotonically decreasing, monotonically increasing, or increasing and then decreasing. If the borrower’s IR constraint doesn’t bind, the lender achieves a local maximum. And in any of these cases, the local maximum is also a global maximum in the range of repayments \( R \in [T_{w}, q_{w}T_{w} - h_{w}] \). This uniquely pins down the repayment rate in an equilibrium in which the borrower invests her autarkic endowment in working capital.

Further, if such an equilibrium exists, there could not also be an equilibrium in which the borrower strictly preferred to invest her autarkic endowment in fixed capital. In such an equilibrium the lender would be solving

\[
\max_{R} \left( R - T_{w} \right) dt + e^{-\rho_{d} t} \left( 1 - e^{-\frac{E_{w}+T_{w}-R}{\Phi_{w}} dt} \right) L_{w+1}
\]

s.t.

\[
e^{-\rho_{d} t} \left( 1 - e^{-\frac{E_{w}+T_{w}-R}{\Phi_{w}} dt} \right) B_{w+1} + e^{-\frac{E_{w}+T_{w}-R}{\Phi_{w}} dt} B_{w} \geq e^{-\rho_{d} t} \left( 1 - e^{-\frac{E_{w}}{\Phi_{w}} dt} \right) B_{w+1} + e^{-\frac{E_{w}}{\Phi_{w}} dt} B_{w}
\]

\( T_{w} \leq R \leq q_{w}T_{w} - h_{w} \)

where the borrower’s IR constraint clearly never binds. So in this case as well the lender achieves the global maximum within the range of repayments \( R \in [T_{w}, q_{w}T_{w} - h_{w}] \), contradicting that the borrower would invest her autarkic endowment in fixed capital in the event that she has rejected the lender’s contract. □

Thus if there exists an equilibrium where the borrower weakly prefers to invest her autarkic endowment in working capital in state \( w \), she does so in all equilibria. We complete the proof by noting that in any equilibrium in which the borrower strictly prefers invest her autarkic endowment in fixed capital (which by the Lemma 1.3 only occurs when there
is no equilibrium in which the borrower weakly prefers to invest her autarkic endowment in working capital, the lender’s behavior is uniquely pinned down.

**Lemma 1.4.** The probability the lender offers a restrictive contract in state \( w \), \( p_w \) is generically uniquely determined across any equilibrium in which the borrower strictly prefers to invest her autarkic endowment in the fixed capital project in state \( w \).

**Proof.** The proof proceeds exactly as in Lemma 1.2 and is thus omitted.

This completes the proof that the equilibrium is generically unique.

**Proof of Proposition 1.3**

We aim to show that in equilibrium the probability the lender offers the borrower a restrictive contract in state \( w \), \( p_w \), is single peaked in \( w \). We split the proof into two steps. First we show that conditional on the borrower investing her autarkic endowment in fixed capital in a given region of states \( w_k, \ldots, w_{k'} \) with \( k' > k \), the probability the lender offers a restrictive contact is single peaked in the state.

Second we show that if in equilibrium the lender offers the borrower an excessively generous unrestrictive contract in state \( w \), then he also does so for all states \( w' < w \), and hence \( p_w = 0 \) in all states \( w' < w \). Because we showed that in equilibrium the borrower only invests her outside option in working capital in states where she gets an excessively generous unrestrictive contract, this completes the argument.

**Lemma 1.5.** Suppose that in equilibrium the borrower invests her autarkic endowment in fixed capital in states \( \bar{w} - 1, \bar{w}, \) and \( \bar{w} + 1 \). Then if \( p_{\bar{w}} < p_{\bar{w}+1} \implies p_{\bar{w}-1} \leq p_{\bar{w}} \).

**Proof.** Assume that in equilibrium the borrower invests her outside option in fixed capital in states \( \bar{w} - 1, \bar{w}, \) and \( \bar{w} + 1 \). We begin by defining a function that implicitly determines the equilibrium probability \( p_{\bar{w}} \) that the lender offers the borrower a restrictive contract. To do so we first determine the borrower’s value in state \( \bar{w} \) as a function of the probability \( p \) she expects a restrictive contract. This allows us to determine the maximal repayment rate \( R(p) \) she is willing to accept for a restrictive contract given the probability \( p \) she expects the lender to offer a restrictive contract. Finally, \( R(p) \) allows us to determine the lender’s payoff from offering restrictive contracts, and by comparing this to his payoff from offering unrestrictive contracts we pin down the equilibrium probability \( p_{\bar{w}} \).
In state $\tilde{w}$, if in equilibrium the borrower receives a restrictive contract with probability $p_{\tilde{w}}$, her utility is

$$B_{\tilde{w}} (p_{\tilde{w}}) = e^{-\rho dt} \left( p_{\tilde{w}} \left( \left( 1 - e^{-\tau dt} \right) B_{\tilde{w}+1} + e^{-\tau dt} B_{\tilde{w}} \right) + (1 - p_{\tilde{w}}) \left( \left( 1 - e^{-\gamma dt} \right) B_{\tilde{w}+1} + e^{-\gamma dt} B_{\tilde{w}} \right) \right)$$

$$B_{\tilde{w}} (p_{\tilde{w}}) = \frac{p_{\tilde{w}} \left( e^{-\rho dt} - e^{-\left(\tau + \rho\gamma\right) dt} \right) B_{\tilde{w}+1} + (1 - p_{\tilde{w}}) \left( e^{-\rho dt} - e^{-\left(\gamma + \rho\tau\gamma\right) dt} \right) B_{\tilde{w}+1}}{1 - p_{\tilde{w}} e^{-\left(\tau + \rho\gamma\right) dt} - (1 - p_{\tilde{w}}) e^{-\left(\rho + \gamma\tau\gamma\right) dt}}$$

$$\rightarrow \frac{p_{\tilde{w}} \kappa B_{\tilde{w}+1} + (1 - p_{\tilde{w}}) \gamma B_{\tilde{w}+1}}{\rho + p_{\tilde{w}} \kappa + (1 - p_{\tilde{w}}) \gamma}$$

$$= \frac{\delta (p_{\tilde{w}})}{\rho + \delta (p_{\tilde{w}})} B_{\tilde{w}+1}$$

where $\delta (p_{\tilde{w}}) \equiv p_{\tilde{w}} \kappa + (1 - p_{\tilde{w}}) \gamma$.

Further recall that in equilibrium, the maximum repayment $R(p)$ that the borrower would accept is given by

$$(q_{\tilde{w}} (E + T_{\tilde{w}}) - R(p)) dt + e^{-\rho dt} B_{\tilde{w}} = \left( 1 - e^{-\tau dt} \right) B_{\tilde{w}+1} + e^{-\tau dt} B_{\tilde{w}}$$

$$R(p) dt = q_{\tilde{w}} (E + T_{\tilde{w}}) dt - e^{-\rho dt} \left( \left( 1 - e^{-\tau dt} \right) (B_{\tilde{w}+1} - B_{\tilde{w}}) \right)$$

Now, given the borrower’s equilibrium expectation $p_{\tilde{w}}$, we can calculate the lender’s payoff from offering a maximally extractive, acceptable restrictive or unrestrictive contract. Because the problem is stationary, we can determine which contract the lender prefers by comparing the lender’s lifetime utility if he were to offer only restrictive contracts or only unrestrictive contracts in state $\tilde{w}$. If the lender were to offer only restrictive contracts in state $\tilde{w}$ his utility would be

$$L_{\tilde{w}}^R (p_{\tilde{w}}) = (R (p_{\tilde{w}}) - T_{\tilde{w}}) dt + e^{-\rho dt} L_{\tilde{w}}^R (p_{\tilde{w}})$$

$$= (q_{\tilde{w}} (E + T_{\tilde{w}}) - T_{\tilde{w}}) dt - e^{-\rho dt} \left( \left( 1 - e^{-\tau dt} \right) (B_{\tilde{w}+1} - B_{\tilde{w}}) \right) + e^{-\rho dt} L_{\tilde{w}}^R (p_{\tilde{w}})$$

$$\rightarrow \frac{(q_{\tilde{w}} (E + T_{\tilde{w}}) - T_{\tilde{w}}) dt - e^{-\rho dt} \left( \left( 1 - e^{-\tau dt} \right) \left( \frac{\kappa}{\rho + \delta (p_{\tilde{w}})} B_{\tilde{w}+1} \right) \right)}{\rho}$$

On the other hand, if the lender were to offer only unrestrictive contracts in state $\tilde{w}$ his
utility would be

\[
L_{\bar{w}}^U(p_{\bar{w}}) = (q_{\bar{w}} T_{\bar{w}} - h - T_{\bar{w}}) dt + e^{-\rho dt} \left( 1 - e^{-\gamma dt} \right) L_{\bar{w}+1} + e^{-\gamma dt} L_{\bar{w}}^U(p_{\bar{w}})
\]

\[
= \frac{(q_{\bar{w}} T_{\bar{w}} - h - T_{\bar{w}}) dt + \left( e^{-\rho dt} - e^{-(\rho + \gamma) dt} \right) L_{\bar{w}+1}}{1 - e^{-(\rho + \gamma) dt}}
\]

\[
= (1 - \beta) \left( \frac{q_{\bar{w}} T_{\bar{w}} - h - T_{\bar{w}}}{\rho} \right) + \beta L_{\bar{w}+1}
\]

Next, consider the function \( g_{\bar{w}}(p) = L_{\bar{w}}^U(p) - L_{\bar{w}}^R(p) \). Note that \( L_{\bar{w}}^U(p) \) is independent of \( p \) and \( L_{\bar{w}}^R(p) \) is decreasing in \( p \), so \( g_{\bar{w}}(p) \) is increasing. If \( g_{\bar{w}}(1) < 0 \), then the unique equilibrium is for the lender to offer a restrictive contract with probability 1. If \( g_{\bar{w}}(0) > 0 \), then the unique equilibrium is for the lender to offer an unrestrictive contract with probability 1. Else, as shown in Proposition 1.2, generically there is a unique \( p_{\bar{w}} \in [0,1] \) such that \( g_{\bar{w}}(p_{\bar{w}}) = 0 \), and the unique equilibrium is for the lender to offer a restrictive contract with probability \( p_{\bar{w}} \).

We now verify that \( p_{\bar{w}} \) is single peaked in \( w \) by considering the following five exhaustive cases:

1. \( 0 < p_{\bar{w}} < p_{\bar{w}+1} < 1 \)
2. \( 0 < p_{\bar{w}} < p_{\bar{w}+1} = 1 \)
3. \( 0 = p_{\bar{w}} < p_{\bar{w}+1} < 1 \)
4. \( 0 = p_{\bar{w}} < p_{\bar{w}+1} = 1 \)
5. \( 0 = p_{\bar{w}} = p_{\bar{w}+1} \) and \( g_{\bar{w}}(0) > g_{\bar{w}}(0) \)

We will provide the proof for the case where \( 0 < p_{\bar{w}} < p_{\bar{w}+1} < 1 \) and omit the others as they are all similar.
Because $p_{\bar{w}+1}$ is interior, we have

$$L_{\bar{w}+1} = L_{\bar{w}+1}^R(p_{\bar{w}+1}) = \frac{q_{\bar{w}+1}(E + T_{\bar{w}+1}) - T_{\bar{w}+1}}{\rho} - \frac{\kappa}{\rho + \delta(p_{\bar{w}+1})} B_{\bar{w}+2}$$

and

$$B_{\bar{w}+1} = \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}+1})} B_{\bar{w}+2}$$

Thus in state $w$ we have

$$g_{\bar{w}}(p_{\bar{w}}) = L_{\bar{w}}^U - L_{\bar{w}}^R(p_{\bar{w}})$$

$$= (1 - \beta) \frac{q_{\bar{w}}(E + T_{\bar{w}}) - T_{\bar{w}}}{\rho} + \beta \left( \frac{q_{\bar{w}+1}(E + T_{\bar{w}+1}) - T_{\bar{w}+1}}{\rho} - \frac{\kappa}{\rho + \delta(p_{\bar{w}+1})} B_{\bar{w}+2} \right)$$

$$- \left( \frac{q_{\bar{w}}(E + T_{\bar{w}}) - T_{\bar{w}}}{\rho} - \frac{\kappa}{\rho + \delta(p_{\bar{w}})} \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}+1})} B_{\bar{w}+2} \right)$$

$$= \beta (q_{\bar{w}+1}(E + T_{\bar{w}+1}) - T_{\bar{w}+1}) - (q_{\bar{w}}(E + T_{\bar{w}}) - T_{\bar{w}}) - (1 - \beta) \frac{h + q_{\bar{w}}E}{\rho} - \frac{\kappa}{\rho + \delta(p_{\bar{w}+1})} B_{\bar{w}+2} \left( \beta - \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}})} \right)$$

$$= 0$$

Similarly we have

$$g_{\bar{w}-1}(p) = L_{\bar{w}-1}^U - L_{\bar{w}-1}^R(p)$$

$$= \beta (q_{\bar{w}-1}(E + T_{\bar{w}-1}) - T_{\bar{w}-1}) - (q_{\bar{w}-1}(E + T_{\bar{w}-1}) - T_{\bar{w}-1}) - (1 - \beta) \frac{h + q_{\bar{w}-1}E}{\rho}$$

$$- \frac{\kappa}{\rho + \delta(p_{\bar{w}})} \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}+1})} B_{\bar{w}+2} \left( \beta - \frac{\delta(p_{\bar{w}})}{\rho + \delta(p_{\bar{w}})} \right)$$

$$= 0$$

Comparing the expression for $g_{\bar{w}-1}(p)$ to the that of $g_{\bar{w}}(p)$ we can see that the sum of first two terms is strictly larger in the expression for $g_{\bar{w}-1}(p)$ (by Assumption 1.4). That means that in order for $p$ to set $g_{\bar{w}-1}(p) = 0$ (if possible), we need that the third term in $g_{\bar{w}-1}(p)$ is strictly smaller than the third term in $g_{\bar{w}}(p)$. That is

$$- \frac{\kappa}{\rho + \delta(p_{\bar{w}})} \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}+1})} B_{\bar{w}+2} \left( \beta - \frac{\delta(p_{\bar{w}})}{\rho + \delta(p_{\bar{w}})} \right) < - \frac{\kappa}{\rho + \delta(p_{\bar{w}+1})} B_{\bar{w}+2} \left( \beta - \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}})} \right)$$

$$\iff \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}})} \left( \beta - \frac{\delta(p_{\bar{w}})}{\rho + \delta(p_{\bar{w}})} \right) > \left( \beta - \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}})} \right)$$
Recall that \( p_{\tilde{w}+1} > p_{\tilde{w}} \) by assumption. So \( \frac{\delta(p_{\tilde{w}+1})}{\rho + \delta(p_{\tilde{w}})} < 1 \). Thus

\[
\left( \beta - \frac{\delta(p_{\tilde{w}})}{\rho + \delta(p)} \right) > \left( \beta - \frac{\delta(p_{\tilde{w}+1})}{\rho + \delta(p_{\tilde{w}})} \right)
\]

\[
\Rightarrow \frac{\delta(p_{\tilde{w}})}{\rho + \delta(p)} > - \frac{\delta(p_{\tilde{w}+1})}{\rho + \delta(p_{\tilde{w}})}
\]

\[
\Rightarrow \frac{\delta(p_{\tilde{w}})}{\rho + \delta(p)} < \frac{\delta(p_{\tilde{w}+1})}{\rho + \delta(p_{\tilde{w}})}
\]

\[
\Rightarrow p < p_{\tilde{w}}
\]

On the other hand, if there is no \( p \geq 0 \) such that \( g_{\tilde{w}-1}(p) = 0 \), then \( g_{\tilde{w}-1}(0) > 0 \), and the unique equilibrium includes \( p_{\tilde{w}-1} = 0 \). This completes the argument for this case. As the remaining cases are similar they are omitted.

To complete the proof we now only need to address the possibility that \( p_{\tilde{w}} = 0 \) and that the lender offers the borrower an unrestrictive contract with smaller than necessary repayment in state \( \tilde{w} \) in equilibrium. We do so with the following lemma.

**Lemma 1.6.** Suppose \( p_{\tilde{w}} = 0 \) and the lender offers the borrower an unrestrictive contract with smaller than necessary repayment in state \( \tilde{w} \) in equilibrium. Then for all \( w < \tilde{w} \), \( p_w = 0 \) and the lender gives the borrower an unrestrictive contract with a smaller than necessary repayment.

**Proof.** Given that the lender offers an unrestrictive contract in state \( \tilde{w} \), he optimally sets the repayment rate to maximize

\[
L_{\tilde{w}}^U = \max_{T_{\tilde{w}} \leq R \leq \frac{q_{\tilde{w}} T_{\tilde{w}}}{\phi}} \left( R - T_{\tilde{w}} \right) + \frac{E + T_{\tilde{w}} \frac{q_{\tilde{w}}}{\phi} - \frac{q_{\tilde{w}}^2}{\phi}}{\rho + \frac{E + T_{\tilde{w}} \frac{q_{\tilde{w}}}{\phi}}{\phi}} L_{\tilde{w}+1}
\]

It is easily verified that the above objective function is monotonically decreasing (increasing) in \( R \) if and only if \( L_{\tilde{w}+1} \geq (\leq) \frac{q_{\tilde{w}} (E + T_{\tilde{w}}) - T_{\tilde{w}}}{\rho} + \phi \). That in state \( \tilde{w} \) the lender optimally offers a more generous than necessary repayment rate implies that \( L_{\tilde{w}+1} \geq \frac{q_{\tilde{w}} (E + T_{\tilde{w}}) - T_{\tilde{w}}}{\rho} + \phi \).

Now consider state \( \tilde{w} - 1 \), and suppose toward contradiction that the lender does not offer an excessively generous unrestrictive contract in \( \tilde{w} - 1 \). By the discussion above, this implies that

\[
L_{\tilde{w}} \leq \frac{q_{\tilde{w}-1} (E + T_{\tilde{w}-1}) - T_{\tilde{w}-1}}{\rho} + \phi
\]

(1.6)
Define \( w_R \) to be the lowest state larger than \( \bar{w} \) in which a restrictive contract is offered with positive probability (if no such state exists, the proposition holds trivially). We will show equation 1.6 implies that for all states \( w \in \{ \bar{w} - 1, \ldots w_R - 1 \} \) the lender offers the least generous unrestrictive contract, contradicting the premise of this lemma.

We know that \( L_{\bar{w}} \geq (1 - \beta) \left( \sum_{w'=\bar{w}}^{w_R-1} q_{w'} T_{w'} - h - T_{w'} \beta^{w'-\bar{w}} \right) + \beta^{w_R-\bar{w}} L_{w_R} \), as the lender would derive this utility if he allowed the borrower to invest in fixed capital at the slowest possible rate in each state \( w' > \bar{w} \) until \( w_R \) (by assumption he optimally allows the borrower to invest at a weakly higher rate).

Combining the above two inequalities we have that

\[
(1 - \beta) \left( \sum_{w'=\bar{w}}^{w_R-1} q_{w'} T_{w'} - h - T_{w'} \beta^{w'-\bar{w}} \right) + \beta^{w_R-\bar{w}} L_{w_R} \leq \frac{q_{\bar{w}} - 1}{\rho} \left( E + T_{\bar{w}} - 1 - T_{\bar{w}} - 1 \right) + \phi \tag{1.7}
\]

We now show that the above inequality together with concavity of \( \frac{q_{w'} (E + T_{w'}) - T_{w'}}{\rho} \) in \( w \) (Assumption 1.4) implies

\[
(1 - \beta) \left( \sum_{w'=\bar{w}+1}^{w_R} q_{w'} T_{w'} - h - T_{w'} \beta^{w'-\bar{w}} \right) + \beta^{w_R-(\bar{w}+1)} L_{w_R} \leq \frac{q_{\bar{w}} (E + T_{\bar{w}}) - T_{\bar{w}}}{\rho} + \phi \tag{1.8}
\]

The right hand side of equation 2.2 is \( \frac{q_{\bar{w}} (E + T_{\bar{w}}) - T_{\bar{w}}}{\rho} - \frac{q_{\bar{w}} - 1}{\rho} (E + T_{\bar{w}} - 1 - T_{\bar{w}} - 1) \) larger than that of 1.7. To evaluate the difference in left hand sides note
The first equality follows from rearrangement of terms, the second follows from substituting for \( L_{wR} \), the third follows from further rearrangement, the fourth inequality follows from deletion of a negative term, and the fifth inequality follows from the concavity of \( \psi w(E + Tw) - Tw \) in \( w \). Thus the difference in the left hand side of equations 2.2 and 1.7 is smaller than that of the right hand sides. So we conclude that equation 2.2 holds.

Proceeding by forward induction we have that

\[
(1 - \beta) \left( \sum_{w'=w+1}^{w_R} \frac{q(w'T_{w'} - h - Tw'\beta^{w'-(w+2)}}{\rho} \right) + \beta^{w_R-(w+1)} L_{wR} \leq qw(E + Tw) - Tw + \phi
\]

for all \( \bar{w} \) such that \( \bar{w} \leq w \leq w_R - 1 \). Note that at \( w = w_R - 1 \) the above equation implies that \( L_{wR} \leq \frac{qw_{R-1}(E + Tw_{R-1}) - Tw_{R-1}}{\rho} + \phi \) and hence at \( w_R - 1 \) the lender invests at the lowest possible rate. Therefore \( L_{wR-1} = (1 - \beta) \frac{qw_{R-1}(E + Tw_{R-1}) - Tw_{R-1}}{\rho} + \beta L_{wR} \). Proceeding backward the argument extends for all \( w \) such that \( \bar{w} \leq w \leq w_R - 1 \), so that \( L_w = (1 - \beta) \left( \sum_{w'=w}^{w_R} \frac{q(w'T_{w'} - h - Tw'\beta^{w'-w)}}{\rho} \right) + \beta^{w_R-w} L_{wR} \) for \( \bar{w} \leq w \leq w_R - 1 \) and hence the lender allows the borrower to invests at the lowest possible rate in all such states. This completes the contradiction.

Together the above two lemmas complete the proof.
Proof of Proposition 1.4

Recall from the proof of Proposition 1.2 that in equilibrium the lender offers a restrictive contract in state $w$ with probability 1 if and only if $L_R^w(1) \geq L_U^w$, where $L_R^w(p) \equiv \frac{q_w(E+T_w)-T_w}{\rho} - \frac{\kappa}{\rho + \delta(p)} B_{w+1}$ and $L_U^w \equiv (1 - \beta) \frac{q_w T_w - h - T_w}{\rho} + \beta L_{w+1}$. Now

$$L_R^w(1) \geq L_U^w \iff \frac{q_w (E + T_w) - T_w}{\rho} - \alpha B_{w+1} \geq (1 - \beta) \frac{q_w T_w - h - T_w}{\rho} + \beta L_{w+1} \iff -\alpha B_{w+1} > -(1 - \beta) \left( \frac{q_w E + h}{\rho} \right) + \beta \left( L_{w+1} - \frac{q_w (E + T_w) - T_w}{\rho} \right) \iff (\beta - \alpha) B_{w+1} > \beta \left( B_{w+1} + L_{w+1} - \frac{q_w (E + T_w) - T_w}{\rho} \phi \right)$$

which completes the proof.

Proof of Proposition 1.5

Suppose $h = 0$ and consider the lender’s behavior in state $n$. Fixing a probability $p_n$ that the borrower anticipates a restrictive contract in equilibrium, (and recalling that we can consider the lender’s continuation utility in state $n$ from a fixed action due to the stationarity of the problem), the lender’s utility from offering a restrictive contract is

$$L_R^n(p_n) = \frac{q_n (E + T_n) - T_n}{\rho} - \frac{\kappa}{\rho + \delta(p_n) \rho} \frac{u}{\rho} = \frac{q_n (E + T_n) - T_n}{\rho} - \alpha \frac{u}{\rho}$$

where the equality follows from the fact that $h = 0$. 

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On the other hand, his utility from offering an unrestricted contract is

\[ L_n^U(p_n) = \frac{q_nT_n - h - T_n}{\rho} (1 - \beta) = \frac{q_nT_n - h - T_n}{\rho} (1 - \alpha) \]

The lender prefers offering an unrestricted contract over a restrictive one if and only if

\[ \frac{q_nT_n - h - T_n}{\rho} (1 - \alpha) \geq \frac{q_n (E + T_n) - T_n}{\rho} - \alpha \frac{u}{\rho} \]

\[ (1 - \alpha) \frac{q_nT_n - h - T_n}{\rho} + \alpha \frac{u}{\rho} \geq \frac{q_n (E + T_n) - T_n}{\rho} \] (1.9)

The left hand side of inequality 1.9 is the sum of the borrower and lender’s welfares if the borrower invests in fixed capital at the slowest possible rate in the relationship, and the right hand side is the sum of their welfares from if the borrower invests in working capital. Thus if it is socially efficient to invest in fixed capital the lender strictly prefers unrestricted contracts, irrespective of the borrower’s expectation \( p_n \), and thus in equilibrium in state \( n \) the lender chooses unrestricted contracts with probability 1.

Moving backwards, the proof proceeds similarly.

**Proof of Propositions 1.6 and 1.7**

**Lemma 1.7.** For any state \( w > \bar{w} \), \( \frac{d p_w}{d w} \leq 0 \) with strict inequality if \( p_w > 0 \).

**Proof.** By definition, in states \( w > \bar{w} \), \( p_w < 1 \). Thus in equilibrium the lender at least weakly prefers offering the borrower an unrestricted contract. We can thus write the lender’s continuation utility in each such state as the utility he derives from offering an unrestricted contract at every period (fixing the borrower’s expectation at \( p_w \)).\(^{19}\) That is

\[ L_w = L_w^U \equiv (1 - \beta) \frac{q_wT_w - h - T_w}{\rho} + \beta L_{w+1} \]

\(^{19}\)Note that in full generality he may offer the borrower an unrestricted contract with positive transfer in state \( w \). If so his continuation utility is \( L_w^U = \left( 1 - \frac{q_w (E + T_w - R)}{q_w (E + T_w - R + \rho) + \rho} \right) \frac{R - T_w}{\rho} + \left( \frac{q_w (E + T_w - R)}{q_w (E + T_w - R + \rho) + \rho} \right) L_{w+1} \), but otherwise the argument goes through unchanged.
On the other hand, if the lender were to offer a minimally generous restrictive contract at every period (again, fixing the borrower’s expectation at \( p_w \)) he would receive a continuation utility of

\[
L_w = L^R_w(p_w) = \frac{q_w(E + T_w) - T_w}{\rho} - \frac{\kappa}{\rho + \delta(p_w)} B_{w+1}
\]

In state \( n \) \( L^U_n = (1 - \beta) \frac{q_n T_n - h - T_n}{\rho} \) which is not a function of \( u \). On the other hand \( L^R_n(p_n) = \frac{q_n(E + T_n) - T_n}{\rho} - \frac{\kappa}{\rho + \delta(p_n)} \frac{u}{\rho} \) is decreasing in \( u \). Hence if in state \( n \) \( L^U_n > L^R_n(0) \), then \( p_n = 0 \) and \( \frac{dp_n}{du} = 0 \). The lender’s continuation utility is \( L_n = (1 - \beta) \frac{q_n T_n - h - T_n}{\rho} \) and \( \frac{dL_n}{du} = 0 \). The borrower’s utility is \( B_n = \beta u \frac{1}{\rho} \) so \( \frac{dB_n}{du} > 0 \).

If \( L^I_n = L^C_n(p_n) \) for some \( p_n \in [0, 1] \) then \( p_n \) is the solution to

\[
g(p_n) = (1 - \beta) \frac{q_n T_n - h - T_n}{\rho} - \left( \frac{q_n(E + T_n) - T_n}{\rho} - \frac{\kappa}{\rho + \delta(p_n)} \right) u = 0
\]

Since \( \delta \) is a decreasing function, it is clear that \( \frac{dp_n}{du} < 0 \). But we still have \( L_n = (1 - \beta) \frac{q_n T_n - h - T_n}{\rho} \) so that \( \frac{dL_n}{du} = 0 \). \( B_n = \frac{\delta(p_n)}{\rho + \delta(p_n)} \frac{u}{\rho} \), so

\[
\frac{dB_n}{du} = \frac{\left( \frac{\delta(p_n)}{\rho + \delta(p_n)} \frac{dp_n}{du} \right)}{(\rho + \delta(p_n))^2} \frac{\rho u}{\rho + \delta(p_n)} > 0
\]

Proceeding backward to any state \( w > \bar{w} \), suppose \( \frac{dp_{w+1}}{du} > 0 \), \( \frac{dL_{w+1}}{du} = 0 \). Then the proof proceeds exactly as above. This completes the proof of the lemma.

We next consider the comparative static in states \( w \in \{w, \ldots, \bar{w}\} \).

**Lemma 1.8.** For \( w \in \{w, \ldots, \bar{w}\} \), generically \( \frac{dp_w}{du} = 0 \), \( \frac{dB_{w+1}}{du} > 0 \) and \( \frac{dL_{w}}{du} < 0 \).

**Proof.** By definition \( p_w = 1 \) for \( w \in \{w, \ldots, \bar{w}\} \). Generically this preference will be strict and thus \( \frac{dp_w}{du} = 0 \).

Recall that in Lemma 1.7 we established \( \frac{dB_{w+1}}{du} > 0 \). We also know that \( L_\bar{w} = L^R_{\bar{w}}(1) = \frac{q_\bar{w}(E + T_\bar{w}) - T_\bar{w}}{\rho} - \alpha B_{\bar{w}+1} \). Hence generically \( \frac{dL_w}{du} < 0 \). Further, \( B_\bar{w} = \alpha B_{\bar{w}+1} \) so \( \frac{dB_w}{du} = \alpha \frac{dB_{w+1}}{du} > 0 \).
For the remainder of the states \( w \in \{w, \ldots, \bar{w}\} \), the result follows from straightforward induction.

We now consider the comparative statics for \( w < w \) in the following three lemmas.

**Lemma 1.9.** Suppose \( p_{W-1} = 0 \). Then generically \( \frac{dp_w}{dw} = 0, \frac{dL_w}{dw} < 0, \text{ and } \frac{dB_w}{dw} > 0 \) for all \( w < w \).

**Proof.** If \( p_{W-1} = 0 \) and \( L^U_{W-1} > L^R_{W-1}(0) \), then the lender's preference for unrestrictive contracts is strict so \( \frac{dp_{W-1}}{dw} = 0 \). Further, Lemma 1.8 established that \( \frac{dL_W}{dw} < 0 \) and \( \frac{dB_W}{dw} > 0 \). Therefore, because \( L_{W-1} = (1 - \beta) \frac{q_{W-1}T_{W-1}h}{\rho} + \beta L_W \), we know \( \frac{dL_{W-1}}{dw} < 0 \). And \( B_{W-1} = \beta B_W \) so \( \frac{dB_{W-1}}{dw} > 0 \). Moving backwards proceeds by straightforward induction.

The remainder of the proof deals with the case for which \( p_{W-1} > 0 \). We split the analysis into two cases based on the players' level of patience.

**Lemma 1.10.** Suppose \( p_{W-1} > 0 \) and \( \rho > \frac{\kappa \gamma}{\kappa + \gamma} \). Then \( \frac{dp_w}{dw} > 0, \frac{dL_w}{dw} < 0, \text{ and } \frac{dB_w}{dw} < 0 \) for all \( w < w \).

**Proof.** Consider first state \( w - 1 \). We know \( p_{W-1} \) is the solution to \( g(p_{W-1}) = 0 \). So

\[
\beta \frac{(q_w(E + T_w) - T_w) - (q_{W-1}(E + T_{W-1}) - T_{W-1})}{\rho} - (1 - \beta) \frac{h + q_{W-1}E}{\rho} - \left( \beta - \frac{\delta(p_w)}{\rho + \delta(p_w)} \right) \frac{\kappa}{\rho + \delta(p_{W-1})} B_{W+1} = 0
\]

\[\iff\]

\[
\beta \frac{(q_w(E + T_w) - T_w) - (q_{W-1}(E + T_{W-1}) - T_{W-1})}{\rho} - (1 - \beta) \frac{h + q_{W-1}E}{\rho} = \left( \beta - \frac{\delta(p_w)}{\rho + \delta(p_w)} \right) \frac{\kappa}{\rho + \delta(p_{W-1})} B_{W+1}
\]

Note that the left hand side of the above equation is constant in \( u \).\(^{20}\) Thus

\(^{20}\)The right hand side of the above equation can be simplified by noting that \( p_W = 1 \), but we leave it in this more general form to economize on notation in the backward induction step.
\[
\frac{d}{du} \left( \beta \frac{\kappa}{\rho + \delta (p_{W})} - \frac{\kappa}{\rho + \delta (p_{W-1})} \frac{\delta (p_{W})}{\rho + \delta (p_{W})} \right) B_{W+1} = (9.10) \\
\Rightarrow \\
-\beta \frac{\kappa}{(\rho + \delta (p_{W}))^2} B_{W+1} + \frac{\kappa}{(\rho + \delta (p_{W-1}))^2} \frac{\delta (p_{W})}{\rho + \delta (p_{W})} B_{W+1} \\
- \frac{\kappa}{\rho + \delta (p_{W-1})} \left( \frac{\delta (p_{W})}{\rho + \delta (p_{W})} \right)^2 B_{W+1} \\
+ \left( \beta \frac{\kappa}{\rho + \delta (p_{W})} - \frac{\kappa}{\rho + \delta (p_{W-1})} \frac{\delta (p_{W})}{\rho + \delta (p_{W})} \right) \frac{dB_{W+1}}{du} = 0 \\
\Rightarrow \\
\frac{\kappa}{\rho + \delta (p_{W-1})} \frac{d p_{W-1}}{du} < 0 \\
\Rightarrow \\
\frac{d p_{W-1}}{du} > 0
\]

Where the second implication follows by removing positive terms from the right hand side and noting that \( p_{W} > p_{W-1} \) which implies that \( \frac{\delta (p_{W})}{\rho + \delta (p_{W})} \frac{\delta (p_{W})}{\rho + \delta (p_{W})} \leq \beta. \)

Next, note that
\[
L_{W-1} = L_{W-1}^U = (1 - \beta) \frac{q_{W-1} T_{W-1} - h}{\rho} + \beta L_{W} (1.11)
\]
so by Lemma 1.8 we know that \( \frac{d L_{W-1}}{du} < 0. \)

To find the sign of \( \frac{d B_{W-1}}{du} \) recall that \( \frac{\rho + \delta (p_{W-1})}{\delta (p_{W-1})} B_{W-1} = B_{W} = \frac{\delta (p_{W})}{\rho + \delta (p_{W})} B_{W+1}. \) Hence

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21Note that in full generality the lender may offer the borrower an investment loan with positive transfer in state \( w - 1. \) If so his continuation utility is \( L_{W-1}^U = \left( 1 - \frac{q_{W-1} (e + T_{W-1}) - R}{q_{W-1} (e + T_{W-1}) - R + \rho} \right) \frac{R - T_{W-1}}{\rho} + \left( \frac{q_{W-1} (e + T_{W-1}) - R}{q_{W-1} (e + T_{W-1}) - R + \rho} \right) L_{W}, \) and the interest rate becomes weakly higher but otherwise the argument to follow goes through unchanged.
we know that
\[
\frac{d}{du} \left( \beta \frac{\kappa}{\rho + \delta(p_{W-1})} + \frac{\delta(p_{W})}{\rho + \delta(p_{W})} \right) B_{W-1} = 0
\]
\[
\frac{d}{du} \left( \beta \frac{\kappa}{\delta(p_{W})} - \frac{\delta(p_{W-1})}{\rho + \delta(p_{W-1})} \right) B_{W-1} = 0
\]
\[
\frac{d}{du} \left( \beta \frac{\kappa}{\delta(p_{W})} - \frac{\delta(p_{W-1})}{\rho + \delta(p_{W-1})} \right) B_{W-1} = 0
\]
\[
- \left( \beta \rho - \kappa \right) \frac{d\delta(p_{W-1})}{dp_{W-1}} \frac{dp_{W-1}}{du} B_{W-1} + \frac{d\delta(p_{W-1})}{du} \left( \beta + \frac{\beta \rho - \kappa}{\delta(p_{W-1})} \right)
\]
where the second equality follows from noting that \( p_{W} = 1 \). Reducing we have
\[
\frac{dB_{W-1}}{du} = NEG(\beta \rho - \kappa) \quad (1.13)
\]
where \( NEG \) is a negative constant. The one subtle algebraic reduction in this final step is that
\[
\left( \beta + \frac{\beta \rho - \kappa}{\delta(p_{W-1})} \right) = \frac{\rho + \delta(p_{W-1})}{\delta(p_{W-1})} \left( \beta - \frac{\kappa}{\rho + \delta(p_{W-1})} \right) > 0.
\]
Since we have assumed \( \rho > \frac{\kappa \gamma}{\kappa + \gamma} \), which is equivalent to \( \rho \beta > \kappa \) we have \( \frac{dB_{W-1}}{du} < 0 \).

Moving backward to state \( W-2 \), suppose \( p_{W-2} > 0 \) (or \( p_{W-2} = 0 \), but \( L_{W-2}^{I} - L_{W-2}^{C}(0) = 0 \)). Then \( p_{W-2} \) is the solution to \( g_{W-2}(p_{W-2}) = 0 \). That is
\[
(1 - \beta) \frac{q_{W-2}T_{W-2} - h}{\rho} + \beta L_{W-1} - \left( \frac{q_{W-2}(E + T_{W-2})}{\rho} - \frac{\kappa}{\rho + \delta(p_{W-2})} B_{W-1} \right) = 0
\]
Differentiating both sides with respect to \( u \) we see
\[
\frac{dL_{W-1}}{du} = \left( \frac{dB_{W-1}}{du} \right) + \left( \frac{\kappa d\delta(p_{W-2})}{dp_{W-2}} \frac{dp_{W-2}}{du} \right) B_{W-1} + \left( \frac{\kappa dB_{W-1}}{du} \right) \rho + \delta(p_{W-2}) B_{W-1} = 0 \quad (1.14)
\]
We know that \( \frac{dL_{W-1}}{du} < 0 \) and \( \frac{dB_{W-1}}{du} < 0 \). Hence \( \frac{dp_{W-2}}{du} > 0 \). Further
\[
L_{W-2} = (1 - \beta) \frac{q_{W-2}T_{W-2} - h - T_{W-2}}{\rho} + \beta L_{W-1}
\]
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so \( \frac{dL_{W-2}}{du} < 0 \). And \( B_{W-2} = \frac{\delta (p_{W-2})}{\rho + \delta (p_{W-2})} B_{W-1} \) so

\[
\frac{dB_{W-2}}{du} = \frac{d\delta (p_{W-2})}{du} \frac{dp_{W-2}}{du} (p_{W-2}) B_{W-1} + \frac{\delta (p_{W-2})}{\rho + \delta (p_{W-2})} \frac{dB_{W-1}}{du} < 0 \quad (1.15)
\]

If instead we had \( p_{W-2} = 0 \) and \( L_{W-2}^L > L_{W-2}^R (0) \), then \( \frac{dp_{W-2}}{du} = 0 \). \( L_{W-2} = (1 - \beta) \frac{g_{W-2} T_{W-2}^h}{\rho} + \beta L_{W-1} \) so \( \frac{dL_{W-2}}{du} < 0 \). \( B_{W-2} = \beta B_{W-1} \) so \( \frac{dB_{W-2}}{du} < 0 \).

Because we used only that \( \frac{dB_{W-1}}{du} < 0 \) and \( \frac{dL_{W-1}}{du} < 0 \), moving backwards from state \( w-2 \) to state 0 is straightforward induction.

We now complete the proof of Propositions 1.6 and 1.7 by considering a patient borrower.

**Lemma 1.11.** Suppose \( p_{w-1} > 0 \) and \( \rho < \frac{\kappa T}{\kappa + \gamma} \). Then \( \frac{dp_{w}}{du} > 0 \) and \( \frac{dL_{w}}{du} < 0 \) for all \( w < w \).

**Proof.** In state \( w-1 \) everything follows as it did in Lemma 1.10 except that \( \frac{dB_{W-1}}{du} \), determined by equation (1.13) is positive. In state \( w-2 \), the considerations are similar. \( \frac{dp_{W-2}}{du} > 0 \) is determined by equation (1.10) (reducing all indices by 1) and \( \frac{dL_{W-2}}{du} < 0 \) is determined by (1.11) (reducing all indices by 1). However the sign of \( \frac{dB_{W-2}}{du} \), determined by (1.12) is now ambiguous.

Moving back to an arbitrary state \( w < w \) such that \( \frac{dB_{w+1}}{du} > 0 \), the considerations will be exactly the same as for \( w-2 \). In any state \( w < w \) for which \( \frac{dB_{w+1}}{du} < 0, \frac{dp_{w}}{du} > 0 \) is determined by equation (1.14), \( \frac{dL_{w}}{du} < 0 \) is determined by (1.11), and \( \frac{dB_{w}}{du} < 0 \) is determined by (1.15). This completes the proof.

Together Lemmas 1.7 through 1.11 complete the proof of Propositions 1.6 and 1.7.

**Proof of Proposition 1.8**

Fixing the lender’s behavior, the borrower’s continuation utility in state \( n \) is

\[
B_n (p_n) = \frac{p_n (1 - e^{-\gamma dt}) + (1 - p_n) (1 - e^{-\gamma dt})}{1 - p_n e^{-\gamma dt}} - \frac{1}{\rho} \int \left[ e^{-\gamma dt} - (1 - p_n) e^{-\gamma dt} \right] \frac{e^{-\rho dt} u}{\rho}
\]

81
which increases linearly in $u$. Moving backward, suppose $B_{w+1}$ is increasing in $u$. Then, noting that

$$B_w(p_w) = \frac{p_w (1 - e^{-\kappa dt}) + (1 - p_w) (1 - e^{-\gamma dt})}{1 - p_w e^{-(\rho + \kappa)dt} - (1 - p_w) e^{-(\rho + \gamma)dt}} e^{-\rho dt} B_{w+1}$$

increases linearly in $B_{w+1}$ completes the proof.

**Proof of Proposition 1.9**

The proof for states $w \geq w$ proceeds exactly as in Proposition 1.6 and is thus omitted. In this section we provide an example in which $\frac{dp_{w-1}}{d\rho^B} < 0$ so that making the borrower more patient can strengthen the poverty trap.

We prove this result with a three state model $w = \{1, 2, 3\}$ where the game ends if the borrower ever reaches state 3. We take

$$E = .15, q_1 = q_2 = q = 2, \phi = \frac{1}{2}, h = 100, T_1 = 600, T_2 = 1000, \frac{u}{\rho^B} = 2000 \text{ and } \rho^B = \rho^L = 1.$$ It is easily verified that Assumption 1.3 hold in states 1 and 2. That is,

$$u^2 \frac{u}{\rho^B} = \left(\frac{3}{1.3}\right)^2 2000 > qE + h > \frac{qE + h}{\rho^B} = 100.3$$

Now we verify that in state 2 the lender offers the borrower a restrictive contract with probability 1. If the borrower expects a restrictive contract with probability 1 then the lender gets the following continuation utility if he offers the borrower a restrictive contract in state 2.

$$L^R_2 (1) = \frac{q(E + T_2) - T_2}{\rho^L} - \alpha \frac{u}{\rho^B} = 1000.3 - \frac{3}{1.3} 2000 \approx 539$$

If instead the lender offers the borrower an unrestricted contract at every period in state 2, his continuation utility is

$$L^U_2 = \frac{qT_2 - h - T_2}{\rho^L} (1 - \beta) \approx 9$$

Because the lender finds it least appealing to offer a restrictive contract when the bor-
rower expects restrictive contracts with probability 1, we conclude that in the unique equilibrium the lender offers the borrower a restrictive contract with probability 1.

We next verify that in equilibrium, the lender mixes between restrictive and unrestrictive contracts in state 1.

First, consider the lender’s continuation utility in state 1 from offering the borrower a restrictive contract with probability 1 when she expects an restrictive contract with probability p.

\[
L_1^R(p) = \frac{q(E + T_1) - T_1}{\rho^L} - \max \left\{ \frac{qE + h}{\rho^L}, \frac{\kappa}{\rho^B + \delta(p)} \right\} B_2
\]

\[
\approx 600.3 - \max \left\{ 100.3, \frac{.3}{1 + .3p + 100(1 - p)} \right\} \left( \frac{.3}{1.3} \right) 2000
\]

Note that the repayment rate the lender must set is the larger of \(qT - h\) and what the lender must set so that the borrower achieves the utility she would have received from investing \(E\) in fixed capital.

If instead the lender were to offer an unrestrictive contract with probability 1, her state 1 continuation utility would be

\[
L_1^U = \frac{qT - h - T_1}{\rho^L} (1 - \beta) + \beta L_2 \approx \frac{500}{101} + \frac{100}{101} 539
\]

It is easily verified that \(L_1^R(0) > L_1^U > L_1^R(1)\) and hence the unique equilibrium in state 1 involves the lender mixing between restrictive and unrestrictive contracts. The probability \(p_1\) that the lender offers a restrictive contract is determined by the following equation.

\[
\frac{qT_1 - h - T_1}{\rho^L} (1 - \beta) + \beta L_2 = \frac{q(E + T_1) - T_1}{\rho^L} - \frac{\kappa}{\rho^B + \delta(p_1)} B_2
\]

\[
\Rightarrow \quad \frac{500}{101} + \frac{100}{101} \left( 1000.3 - \frac{.3}{1.3} 2000 \right) \approx 600.3 - \frac{.3}{1 + .3p_1 + 100(1 - p_1)} \left( \frac{.3}{1.3} 2000 \right)
\]

\[
\Rightarrow \quad p_1 \approx .99
\]
Now, recall the investment rent in state 1 is

\[
\left( \beta - \frac{\kappa}{\rho^B + \delta (p_1)} \right) B_2 \approx \left( \frac{100}{101} - \frac{.3}{1 + .3 p_1 + (1 - p_1) 100} \right) B_2
\]

We have

\[
\frac{d}{dp^B} \left( \beta - \frac{\kappa}{\rho^B + \delta (p)} \right) B_2 = \frac{d}{dp^B} \left( \beta - \frac{\kappa}{\rho^B + \delta (p)} \right) B_2 + \left( \beta - \frac{\kappa}{\rho^B + \delta (p)} \right) \frac{d}{dp^B} B_2
\]

Now,

\[
\frac{d}{dp^B} B_2 = \frac{d}{dp^B} \left( \beta - \frac{\kappa}{\rho^B + \delta (p)} \right) \frac{u}{\rho^B + .3 \rho^B}
\]

\[
\approx -\frac{.3}{(\rho^B + .3)^2} \frac{u}{\rho^B + .3} - \frac{.3}{\rho^B + .3 (\rho^B)^2}
\]

\[
\approx -\frac{.3}{(1.3)^2 2000} - \frac{.3}{1.3 2000}
\]

\[
\approx -816.56
\]

And,

\[
\frac{d}{dp^B} \left( \frac{100}{\rho^B + 100} - \frac{.3}{\rho^B + .3 p + (1 - p) 100} \right) \approx \left( \frac{100}{(\rho^B + 100)^2} + \frac{.3}{(\rho^B + .3 p + (1 - p) 100)^2} \right)
\]

\[
\approx .05
\]

So,

\[
\frac{d}{dp^B} \left[ \left( \beta - \frac{\kappa}{\rho^B + \delta} \right) B_2 \right] \approx -675.87 < 0
\]

Therefore making the borrower more patient increases the investment rent and reduces \( p_1 \).

**Proof of Proposition 1.10**

Fix a game \( \Gamma \) with \( n \) states, and a cost of fixed investment \( \{\phi_w\} \). Then for game \( \Gamma'' \) with \( m > 0 \), a borrower investing in fixed capital rate \( i \) in state \( 2^m n \) will derive a state \( 2^m n \)
continuation value of
\[ \frac{\frac{i}{\gamma_{2m_n}}}{\rho + \frac{i}{\gamma_{2m_n}} \rho} \]
which converges to \( \frac{u}{\rho} \) as \( m \) becomes large. If the borrower’s equilibrium expectation is that the lender will offer the restrictive contract with probability 1, then the lender’s continuation utility in state \( 2^m n \) from doing so is
\[ L_{2^m n}^R(1) = \frac{q_{2m_n} (E_{2^m n} + T_{2^m n}) - T_{2^m n}}{\rho} - \frac{\kappa_{2m_n}}{\rho + \kappa_{2m_n} \rho} \frac{u}{\rho} \]
which for sufficiently large \( m \) will be negative when it is socially efficient to invest.

On the other hand the lender’s continuation utility in state \( n \) if he offers an unrestrictive contract in every period is
\[ L_{2^m n}^U = \frac{q_{2m_n} T_{2^m n} - h - T_{2^m n}}{\rho} \left( 1 - \frac{\gamma_{2m_n}}{\rho + \gamma_{2m_n}} \right) \]
which is positive for all \( m > 0 \). Thus for sufficiently high \( m \), the lender will offer an unrestrictive contract with positive probability in state \( 2^m n \), completing the proof.

**Proof of Proposition 1.11**

For states \( w > \bar{w}_{m-1} \), the proof closely follows that of Proposition 1.6. Specifically, for states \( w > \bar{w}_m \), the proof follows that of Lemma 1.7. For states \( w \in \{ \bar{w}_m, \ldots, \bar{w}_{m-1} \} \) the proof follows that of Lemma 1.8. And for states \( w \in \{ \bar{w}_{m-1} + 1, \ldots, \bar{w}_m - 1 \} \) the proof follows that of Lemma 1.10.

For \( w \in \{ \bar{w}_{m-2} + 1, \bar{w}_{m-1} \} \) the logic of Proposition 1.6 is reversed, as \( \frac{dB_{\bar{w}_{m-1}}}{du} > 0 \). That is, in the state directly beyond the pure restrictive state \( \bar{w}_{m-1} \), the borrower’s continuation utility is decreasing in \( u \). In contrast, in the state directly beyond the pure restrictive state \( \bar{w}_m \), the borrower’s continuation utility is increasing in \( u \). So, the comparative static for \( w \in \{ \bar{w}_{m-2} + 1, \bar{w}_{m-1} \} \) comes directly from reversing the signs in Lemmas 1.8 and 1.10.

The proof proceeds similarly for all consumption regions \( \{ \bar{w}_{\bar{m}}, \ldots, \bar{w}_m \} \) for \( \bar{m} \leq m - 2 \).
Proof of Proposition 1.12

Lemma 1.12. In any equilibrium in which the borrower never accepts the entrant’s contract, the borrower must always be weakly better off than she would be from receiving the contract \( T_w, I \) or \( T_w, C \) from the entrant in state \( w \).

Proof. Consider any equilibrium in which the borrower never accepts the entrant’s contract. In such an equilibrium the entrant’s continuation value in any state is 0. Suppose toward contradiction that there is some state \( w \) in which with positive probability the incumbent offers the borrower a contract \( \tilde{c} \) that provides her with strictly less utility than she would receive from accepting the contract \( T_w, I \) or \( T_w, C \) from the entrant. Then there exists an \( \epsilon > 0 \) such that the borrower would strictly prefer the entrant’s contract \( T_w + \epsilon, I \) or \( T_w + \epsilon, C \) to the incumbent’s contract \( \tilde{c} \). Therefore by deviating to offer one of these contracts, the entrant could guarantee himself a positive payoff, contradicting the premise that the borrower never accepts the entrant’s contract in equilibrium. \( \square \)

Lemma 1.12 implies that in equilibrium the borrower’s outside option is the maximum of the utility she would receive from accepting the entrant’s contract \( T_w, I \) or \( T_w, C \), and the utility she would receive from allocating her autarkic endowment flexibly.

The remainder of the proof proceeds exactly as in Proposition 1.2, with this new individual rationality constraint in place of the borrower’s autarkic constraint, and is thus omitted.

Proof of Proposition 1.13

We first identify a \( \psi \) such that for \( \psi > \tilde{\psi} \), the equilibrium is the same as in the monopolist case. We determine \( \tilde{\psi} \) as follows. Let \( B_{w+1}^{\text{max}} \) be the maximal feasible (potentially out of equilibrium) state \( w + 1 \) continuation utility that the borrower can achieve. Further suppose that in state \( w \) the borrower anticipates the incumbent will never make a loan offer, inducing her to have maximal feasible value of investment in state \( w \). Then if she opts not to borrow from the entrant, her state \( w \) continuation utility is

\[
B_w = e^{-\rho dt} \left( (1 - e^{-\kappa dt}) B_{w+1}^{\text{max}} + e^{-\kappa dt} B_w \right)
\]
Let \( \tilde{\psi}_w \) satisfy
\[
\int_0^{\tilde{\psi}_w} \left( 1 - e^{-\frac{r_0 + T_0 - L}{\nu} dt} \right) B_{w+1}^{\max} + e^{-\frac{r_0 + T_0 - L}{\nu} dt} B_w \right) - \tilde{\psi}_w < e^{-\rho dt} \left( \left( 1 - e^{-\kappa_0 dt} \right) B_{w+1}^{\max} + e^{-\kappa_0 dt} B_w \right)
\]

ensuring that the borrower would prefer to invest her autarkic endowment rather than borrow from the entrant even when her value from investment is as high as is feasible in state \( w \). Now let \( \bar{\psi} = \max_{w} \tilde{\psi}_w \). Clearly in equilibrium, if the borrower ever rejects the incumbent’s contract then she will choose not to borrow from the entrant and her individual rationality constraint will be the same as in Proposition 1.2. Thus the equilibrium outcome will be the same as in the monopolist case.

We now prove the existence of a \( \bar{\psi} > 0 \) such that for \( \psi < \bar{\psi} \) the incumbent offers an unrestrictive contract with probability 1 in every period. This is so because by Lemma 1.12, for \( \psi = 0 \) the borrower’s outside option is to take whichever she prefers of the maximally generous restrictive and unrestrictive contracts from the entrant at no additional cost. So long as it is efficient to invest in business expansion, the borrower will always prefer the maximally unrestrictive contract. By continuity, there exists a \( \bar{\psi} > 0 \) such that the same will be true for any \( \psi < \bar{\psi} \). So for \( \psi < \bar{\psi} \) the incumbent must offer unrestrictive contracts in equilibrium. And by the same logic, as \( \psi \to 0 \), the incumbent’s equilibrium contract offer must converge to the maximally generous unrestrictive contract. This completes the proof.

**Proof of Proposition 1.14**

**Lemma 1.13.** If business growth is efficient and \( p_n \in (0,1) \), \( \frac{dp_n}{d\psi} \geq 0 \), \( \frac{dB_n}{d\psi} \leq 0 \), and \( \frac{dL_n}{d\psi} = 0 \).

**Proof.** Suppose that in state \( n \) the equilibrium probability the incumbent lender offers a restrictive contract is \( p_n \in (0,1) \). If \( \psi \) is sufficiently high that the borrower’s outside option is to invest her own autarkic endowment rather than borrow from the entrant, then \( \frac{dp_n}{d\psi} = 0 \) and the conclusion of the lemma is satisfied. Else the borrower’s outside option is to borrow from the entrant and incur the non pecuniary cost of \( \psi dt \).

Now suppose that in equilibrium the borrower derives more utility from the maximally extractive unrestrictive contract than she does from the maximally extractive but acceptable restrictive contract (and hence her expansion rent is positive). Then \( p_n \) will be determined as it was in Lemma 1.7, respecting the borrower’s modified individual
rationality constraint. Decreasing $\psi$ has the same impact as increasing $U$, it improves the borrower's outside option so that she becomes more demanding of restrictive contracts. As in Lemma 1.7 this causes an equilibrium shift towards unrestrictive contracts, so $\frac{dP_n}{d\psi_n} > 0$. The lender remains indifferent between the two types of contracts so his continuation utility $L_n$ is unaffected.

Last suppose that in equilibrium the borrower derives the same utility from the maximally extractive unrestrictive contract as she does from the maximally extractive but acceptable restrictive contract (and hence her expansion rent is zero). Then by the logic is Proposition 1.5, it must be that $p_n = 0$, contradicting the premise of the lemma.

We complete the proof of the proposition by backward induction.

**Lemma 1.14.** Consider state $w$ for which $p_w \in (0, 1)$ for and for which $\frac{dL_{w+1}}{d\psi} = 0$ and $\frac{dB_{w+1}}{d\psi} < 0$. Then $\frac{dP_w}{d\psi} \geq 0$.

**Proof.** The proof for this lemma proceeds in the same way as the proof for Lemma 1.13, however when considering how demanding the borrower is of restrictive contracts, her outside option now improves both due to the increase in competition in the current state and to her improved continuation value in state $w + 1$. Otherwise the proof is the same and is thus omitted.

Together Lemmas 1.13 and 1.14 complete the proof of the result.

**Proof of Proposition 1.15**

**Lemma 1.15.** The borrower never rejects an unrestrictive contract on the equilibrium path.

**Proof.** Assumption 1.6 guarantees that if the borrower receives an unrestrictive contract, she necessarily invests more in the fixed capital project than she could have in autarky. Thus by the logic in Lemma 1.3, the borrower would only ever reject the maximally extractive unrestrictive contract if in equilibrium she gets a more generous unrestrictive contract with certainty.

**Lemma 1.16.** The borrower may reject a restrictive contract on the equilibrium path.

**Proof.** To prove this lemma we need only find an example in which the borrower rejects a restrictive contract with positive probability. To do so we modify the example from the
proof of Proposition 1.9. Specifically consider the one state example in which we take $E = .15$, $q = 2$, $\phi = \frac{1}{2}$, $h = 100$, $T = 1000$, $\frac{u}{\rho^{\alpha}} = 2000$ and $\rho^B = \rho^L = 1$. We define the distribution $G$ such that $v = 0$ with probability $1 - \epsilon$, and $v = 45$ with probability $\epsilon$.

We verified in the proof of Proposition 1.9 that this example satisfies Assumption 1.3 and that in equilibrium the lender offers the borrower a restrictive contract with probability 1. Clearly for sufficiently small $\epsilon$, the lender would prefer to offer the least generous restrictive contract that borrowers of type $v = 0$ would accept. The loss the lender suffers from being rejected with probability $\epsilon$ is vanishing. In contrast, if the lender offers a contract that both types of borrowers would accept, he incurs a first order loss in order to compensate the high type borrower for the $v = 45$ additional forgone investment. \qed

**Proof of Proposition 1.16**

Define $\beta_s \equiv \frac{\gamma - \frac{q}{\rho + \gamma}}{\frac{q}{\rho + \gamma} - \frac{\phi}{\rho}}$. Now suppose in state $w$ the borrower anticipates a restrictive contract with probability 1. It is straightforward to show that the borrower’s expansion rent when she can invest a fraction of her endowment $s$ flexibly no matter the contractual restriction is $(\beta_s - \alpha)B_{w+1}$. Further as in Proposition 1.4, in equilibrium the borrower receives a restrictive contract with certainty in state $w$ if and only if

$$(\beta_s - \alpha)B_{w+1} \geq \beta \left( B_{w+1} + L_{w+1} - q_w (E + T_w) - T_w - \phi \right).$$

For $s < \frac{E_w}{T_{w+1} + T_w}$, the borrower will grow more slowly in equilibrium than in autarky in any state in which the above condition is satisfied.

**1.8.3 Tables and Figures**
Table 1.1: Default is U-Shaped in Income in Field et. al. Data

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<th>Doesn't Complete Repayment</th>
<th>Doesn't Complete Repayment</th>
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<td>-0.167*</td>
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<td>(0.0945)</td>
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<td>Log Proits Sq.</td>
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Notes: Robust standard errors in parentheses. All columns are borrower level OLS regressions. The outcome variable is whether the borrower has completed repayment a year after disbursal. Controls are those included in Field et. al. (2013) and include borrower education, household size, religion, literacy, marital status, age, household shocks, business ownership at baseline, financial control, home ownership, and whether the household has a drain. Log Profits are measured three years after loan disbursal - this is the publicly available measure.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Table 1.2: Village Covariates Correlated With Inverse Size

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<th>(1)</th>
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<td>Village Head Age</td>
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<td>-532.3**</td>
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<tr>
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<td>(8.377)</td>
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<th>(14)</th>
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<td>(21.76)</td>
<td>(17.26)</td>
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<td>(0.249)</td>
<td>(0.199)</td>
</tr>
<tr>
<td>N</td>
<td>55</td>
<td>54</td>
<td>55</td>
<td>55</td>
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</table>

Notes: Robust standard errors in parentheses. All columns are village level OLS regressions.

* p < 0.10, ** p < 0.05, *** p < 0.01
### Table 1.3: Restrictive Loans are Correlated with Lower Income Expectation

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<td>Log Expected Income</td>
<td>Log Expected Income</td>
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<td>-0.193***</td>
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<td>0.609***</td>
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<tr>
<td>Log Loan Size</td>
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<td></td>
<td>(0.0187)</td>
<td>(0.0177)</td>
<td>(0.0180)</td>
<td>(0.0160)</td>
</tr>
<tr>
<td>Land - borrower use</td>
<td>-0.0401</td>
<td>-0.103</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0674)</td>
<td>(0.0809)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Land - lender use</td>
<td>-0.275**</td>
<td>-0.345***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.132)</td>
<td>(0.120)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Future Crop</td>
<td>0.0644</td>
<td>-0.0893</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.182)</td>
<td>(0.127)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td>-0.238</td>
<td>-0.199**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.163)</td>
<td>(0.0935)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single Guarantor</td>
<td>0.0705</td>
<td>-0.0919</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0609)</td>
<td>(0.0651)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mult. Guarantor</td>
<td>-0.182**</td>
<td>-0.191**</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.0753)</td>
<td>(0.0743)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>1999</td>
<td>1783</td>
<td>1999</td>
<td>1783</td>
</tr>
<tr>
<td>Wave FEs</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Village FEs</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Household FEs</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is the log of expected income in the next year. Land - borrower use is an indicator taking the value 1 if land is used as collateral but the borrower uses it. Land - lender use is an indicator taking the value of 1 if the borrower forfeits his land to the lender. Income and loan size are trimmed at the 99th percentile.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Table 1.4: Restrictive Loans Are Correlated with Lower Interest Rate

<table>
<thead>
<tr>
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<th>(1) Monthly Interest</th>
<th>(2) Monthly Interest</th>
<th>(3) Monthly Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Restrictive</td>
<td>-0.0328***</td>
<td>-0.0326***</td>
<td>-0.0168</td>
</tr>
<tr>
<td></td>
<td>(0.00910)</td>
<td>(0.0101)</td>
<td>(0.0125)</td>
</tr>
<tr>
<td>Log Income</td>
<td>-0.00423</td>
<td>0.000321</td>
<td>0.000263</td>
</tr>
<tr>
<td></td>
<td>(0.00384)</td>
<td>(0.00515)</td>
<td>(0.00737)</td>
</tr>
<tr>
<td>N</td>
<td>1101</td>
<td>1068</td>
<td>878</td>
</tr>
<tr>
<td>Wave FEs</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Village FEs</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Household FEs</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable the monthly interest rate. We do not control for loan size as it is used to construct interest rates. Income and interest rate are trimmed at the 99th percentile.

* p < 0.10, ** p < 0.05, *** p < 0.01
Table 1.5: Main Comparative Static on Village Fund Intensity

<table>
<thead>
<tr>
<th></th>
<th>(1) Restrictive</th>
<th>(2) Restrictive</th>
<th>(3) Restrictive</th>
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</thead>
<tbody>
<tr>
<td>Inv. Size</td>
<td>8.893*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.126)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post</td>
<td>0.222***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0815)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post*Inv. Size</td>
<td>-10.87**</td>
<td>-13.27**</td>
<td>-21.65***</td>
</tr>
<tr>
<td></td>
<td>(6.364)</td>
<td>(5.351)</td>
<td>(5.572)</td>
</tr>
<tr>
<td>Log Loan Size</td>
<td>0.0613***</td>
<td>0.0421***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0142)</td>
<td>(0.0140)</td>
<td></td>
</tr>
<tr>
<td>Log Income</td>
<td>-0.00516</td>
<td>-0.00320</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0126)</td>
<td>(0.00957)</td>
<td></td>
</tr>
<tr>
<td>Village Fund</td>
<td>0.0770</td>
<td>0.130***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0581)</td>
<td>(0.0296)</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>1710</td>
<td>1710</td>
<td>1620</td>
</tr>
<tr>
<td>Wave FEs</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Village FEs</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Household FEs</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is whether the money lender demands a restrictive form of collateral. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* p < 0.10, ** p < 0.05, *** p < 0.01
Table 1.6: Parallel Trends Prior to 2002

<table>
<thead>
<tr>
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<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Restrictive</td>
<td>Restrictive</td>
<td>Restrictive</td>
</tr>
<tr>
<td>Wave</td>
<td>0.00787</td>
<td>0.0115</td>
<td>-0.0183</td>
</tr>
<tr>
<td></td>
<td>(0.0321)</td>
<td>(0.0298)</td>
<td>(0.0183)</td>
</tr>
<tr>
<td>Inv. Size</td>
<td>-625.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5916.8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wave*Inv. Size</td>
<td>0.317</td>
<td>-0.121</td>
<td>2.841</td>
</tr>
<tr>
<td></td>
<td>(2.960)</td>
<td>(3.086)</td>
<td>(1.709)</td>
</tr>
<tr>
<td>Log Loan Size</td>
<td>0.0453***</td>
<td>0.0146</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0159)</td>
<td>(0.0157)</td>
<td></td>
</tr>
<tr>
<td>Log Income</td>
<td>-0.0282*</td>
<td>-0.0233</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0150)</td>
<td>(0.0179)</td>
<td></td>
</tr>
<tr>
<td>Village Fund</td>
<td>-0.00676</td>
<td>0.107</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0411)</td>
<td>(0.0734)</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>916</td>
<td>914</td>
<td>826</td>
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<tr>
<td>Household FEs</td>
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<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Village FEs</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Controls</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is whether the money lender demands a restrictive form of collateral. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* p < 0.10, ** p < 0.05, *** p < 0.01
Table 1.7: Whether a Household Borrows from Money Lenders Regressed on post*invsize

<table>
<thead>
<tr>
<th></th>
<th>(1) Borrow From Money Lender</th>
<th>(2) Borrow From Money Lender</th>
<th>(3) Borrow From Money Lender</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inv. Size</td>
<td>3.421</td>
<td>-0.109***</td>
<td>-0.101</td>
</tr>
<tr>
<td></td>
<td>(6.093)</td>
<td>(0.0378)</td>
<td>(0.0729)</td>
</tr>
<tr>
<td>Post</td>
<td>-0.109***</td>
<td>-0.101</td>
<td>-0.0637</td>
</tr>
<tr>
<td></td>
<td>(0.0378)</td>
<td>(0.0729)</td>
<td>(0.0788)</td>
</tr>
<tr>
<td>Post*Inv. Size</td>
<td>1.810</td>
<td>-0.739</td>
<td>-0.528</td>
</tr>
<tr>
<td></td>
<td>(3.427)</td>
<td>(3.471)</td>
<td>(3.798)</td>
</tr>
<tr>
<td>Log Income</td>
<td>-0.0127</td>
<td>-0.00263</td>
<td>-0.00263</td>
</tr>
<tr>
<td></td>
<td>(0.00954)</td>
<td>(0.00848)</td>
<td></td>
</tr>
<tr>
<td>Village Fund</td>
<td>0.0238</td>
<td>-0.0251</td>
<td>-0.0251</td>
</tr>
<tr>
<td></td>
<td>(0.0156)</td>
<td>(0.0196)</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>6782</td>
<td>6782</td>
<td>6715</td>
</tr>
<tr>
<td>Wave FEs</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Village FEs</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Household FEs</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the village level in parentheses. All columns are villager x time level OLS regressions. The outcome variable is whether the villager borrows from a money lender in a given survey wave. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* p < 0.10, ** p < 0.05, *** p < 0.01
Table 1.8: Total Household Borrowing from Money Lenders Regressed on post*invsize

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Log Total Borrowing From Money Lenders</td>
<td>Log Total Borrowing From Money Lenders</td>
<td>Log Total Borrowing From Money Lenders</td>
</tr>
<tr>
<td>Inv. Size</td>
<td>39.29</td>
<td>(60.64)</td>
<td>39.29</td>
</tr>
<tr>
<td>Post</td>
<td>-0.964**</td>
<td>(0.368)</td>
<td>-0.920</td>
</tr>
<tr>
<td>Post*Inv. Size</td>
<td>13.60</td>
<td>(33.72)</td>
<td>-11.82</td>
</tr>
<tr>
<td>Log Income</td>
<td>-0.0505</td>
<td>(0.0956)</td>
<td>0.00934</td>
</tr>
<tr>
<td>Village Fund</td>
<td>0.308*</td>
<td>(0.159)</td>
<td>-0.216</td>
</tr>
<tr>
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<td>6782</td>
<td>6782</td>
<td>6715</td>
</tr>
<tr>
<td>Wave FEs</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Village FEs</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Household FEs</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
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<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the village level in parentheses. All columns are villager x time level OLS regressions. The outcome variable is the log of how much the villager borrows from a money lender in a given survey wave. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Table 1.9: Heckman Selection Model

<table>
<thead>
<tr>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
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<tr>
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<td>Restrictive</td>
<td>Restrictive</td>
<td>Restrictive</td>
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<tr>
<td>Inv. Size</td>
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<td></td>
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<tr>
<td></td>
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<td>(5.206)</td>
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</tr>
<tr>
<td>Post</td>
<td>0.231***</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.0896)</td>
<td></td>
</tr>
<tr>
<td>Post*Inv. Size</td>
<td>-10.96*</td>
<td>-13.10**</td>
<td>-37.79</td>
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<tr>
<td></td>
<td></td>
<td>(6.355)</td>
<td>(5.353)</td>
</tr>
<tr>
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<td>0.0613***</td>
<td>0.0414</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0141)</td>
<td>(0.0587)</td>
</tr>
<tr>
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<td>(0.0136)</td>
<td>(1.661)</td>
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<td>(0.214)</td>
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<td>0.872</td>
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<td>(16.59)</td>
<td>(16.60)</td>
<td>(4.937)</td>
</tr>
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<td>Post</td>
<td>-0.244</td>
<td>-0.247</td>
<td>-0.247**</td>
</tr>
<tr>
<td></td>
<td>(0.172)</td>
<td>(0.171)</td>
<td>(0.120)</td>
</tr>
<tr>
<td>Post*Inv. Size</td>
<td>-5.092</td>
<td>-5.015</td>
<td>-5.018</td>
</tr>
<tr>
<td></td>
<td>(9.189)</td>
<td>(9.184)</td>
<td>(6.610)</td>
</tr>
<tr>
<td>Log Income</td>
<td>-0.134***</td>
<td>-0.135***</td>
<td>-0.135***</td>
</tr>
<tr>
<td></td>
<td>(0.0310)</td>
<td>(0.0310)</td>
<td>(0.0136)</td>
</tr>
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<td>24916</td>
<td>24916</td>
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<tr>
<td>Wave FE(s)</td>
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<td>Yes</td>
</tr>
<tr>
<td>Village FE(s)</td>
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<td>No</td>
</tr>
<tr>
<td>Household FE(s)</td>
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<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
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<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level Heckman Selection models under the assumption that errors are jointly normally distributed. The outcome variable is whether the money lender demands a restrictive form of collateral. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Controls in the selection equation include all non fixed effect regressors except for loan size in main specification and whether the household's primary means of income generation is farm or nonfarm work. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)
Table 1.10: Placebo Test: Main Regression with Population of Borrowers From Neighbors Rather Than Money Lenders

<table>
<thead>
<tr>
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<th>(1) Restrictive</th>
<th>(2) Restrictive</th>
<th>(3) Restrictive</th>
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</thead>
<tbody>
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<td></td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>Post</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.147)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post*Inv. Size</td>
<td>7.824</td>
<td>18.80</td>
<td>-6.507</td>
</tr>
<tr>
<td></td>
<td>(12.56)</td>
<td>(11.48)</td>
<td>(12.28)</td>
</tr>
<tr>
<td>Log Loan Size</td>
<td>0.0971***</td>
<td>0.0890***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0207)</td>
<td>(0.0228)</td>
<td></td>
</tr>
<tr>
<td>Log Income</td>
<td>-0.0467*</td>
<td>-0.0151</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0268)</td>
<td>(0.0230)</td>
<td></td>
</tr>
<tr>
<td>Village Fund</td>
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<td>-0.0869</td>
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</tr>
<tr>
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<td>(0.0693)</td>
<td>(0.0677)</td>
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<td>745</td>
<td>649</td>
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<td>Yes</td>
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<tr>
<td>Village FEs</td>
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<td>Yes</td>
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</tr>
<tr>
<td>Household FEs</td>
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<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
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<td>Yes</td>
<td>Yes</td>
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</tbody>
</table>

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is whether the neighbor demands a restrictive form of collateral. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* p < 0.10, ** p < 0.05, *** p < 0.01
<table>
<thead>
<tr>
<th></th>
<th>(1) Unrestrictive</th>
<th>(2) Unrestrictive</th>
<th>(3) Unrestrictive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inv. Size</td>
<td>1.503</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6.379)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post</td>
<td>-0.0971</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0725)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post*Inv. Size</td>
<td>4.165</td>
<td>12.03**</td>
<td>14.34***</td>
</tr>
<tr>
<td></td>
<td>(5.664)</td>
<td>(5.869)</td>
<td>(5.366)</td>
</tr>
<tr>
<td>Log Loan Size</td>
<td>0.0878***</td>
<td>0.0665***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0147)</td>
<td>(0.0155)</td>
<td></td>
</tr>
<tr>
<td>Log Income</td>
<td>-0.0331**</td>
<td>-0.00913</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0144)</td>
<td>(0.0135)</td>
<td></td>
</tr>
<tr>
<td>Village Fund</td>
<td>-0.128***</td>
<td>-0.0177</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0435)</td>
<td>(0.0421)</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>1710</td>
<td>1710</td>
<td>1620</td>
</tr>
<tr>
<td>Wave FEs</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Village FEs</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Household FEs</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is whether the money lender demands an unrestricted form of collateral. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Table 1.12: Main Regression with Interest Rates as Outcome Variable

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Monthly Interest</td>
<td>Monthly Interest</td>
<td>Monthly Interest</td>
</tr>
<tr>
<td>Inv. Size</td>
<td>-0.547</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.186)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post</td>
<td>-0.0456**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0195)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post*Inv. Size</td>
<td>1.959</td>
<td>0.512</td>
<td>-0.197</td>
</tr>
<tr>
<td></td>
<td>(1.751)</td>
<td>(2.104)</td>
<td>(2.056)</td>
</tr>
<tr>
<td>Log Income</td>
<td></td>
<td>-0.00366</td>
<td>-0.00254</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00455)</td>
<td>(0.00697)</td>
</tr>
<tr>
<td>Village Fund</td>
<td></td>
<td>-0.0362*</td>
<td>-0.0395***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0208)</td>
<td>(0.0140)</td>
</tr>
<tr>
<td>N</td>
<td>860</td>
<td>859</td>
<td>757</td>
</tr>
<tr>
<td>Wave FEs</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Village FEs</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Household FEs</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is the monthly interest rate. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post, but not loan size as it is used to construct the outcome variable. Income and interest rate are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Table 1.13: Robustness Check: Main Comparative Static Without Restriction on Village Size

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Restrictive</td>
<td>Restrictive</td>
<td>Restrictive</td>
</tr>
<tr>
<td>Inv. Size</td>
<td>6.259**</td>
<td>(3.089)</td>
<td></td>
</tr>
<tr>
<td>Post</td>
<td>0.205***</td>
<td>(0.0570)</td>
<td></td>
</tr>
<tr>
<td>Post*Inv. Size</td>
<td>-9.173**</td>
<td>(4.119)</td>
<td>-9.837</td>
</tr>
<tr>
<td>Log Loan Size</td>
<td>0.0628***</td>
<td>(0.0137)</td>
<td>0.0408***</td>
</tr>
<tr>
<td>Log Income</td>
<td>-0.00170</td>
<td>(0.0122)</td>
<td>-0.00207</td>
</tr>
<tr>
<td>Village Fund</td>
<td>0.0821</td>
<td>(0.0558)</td>
<td>0.125***</td>
</tr>
</tbody>
</table>

N      | 1826   | 1824   | 1721   |
Wave FEs | No     | Yes    | Yes    |
Village FEs | No    | Yes    | No     |
Household FEs | No   | Yes    | Yes    |
Controls | No     | Yes    | Yes    |

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is whether the money lender demands a restrictive form of collateral. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile.

* p < 0.10, ** p < 0.05, *** p < 0.01
Table 1.14: Robustness Check: Main Comparative Static Using Data Only From 1999-2004

<table>
<thead>
<tr>
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<th>(1) Restrictive</th>
<th>(2) Restrictive</th>
<th>(3) Restrictive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inv. Size</td>
<td>8.374</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.995)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post</td>
<td>0.196**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0842)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post*Inv. Size</td>
<td>-10.35</td>
<td>-7.474</td>
<td>-11.51**</td>
</tr>
<tr>
<td></td>
<td>(6.554)</td>
<td>(4.812)</td>
<td>(4.723)</td>
</tr>
<tr>
<td>Log Loan Size</td>
<td>0.0609***</td>
<td>0.0543***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0160)</td>
<td>(0.0176)</td>
<td></td>
</tr>
<tr>
<td>Log Income</td>
<td>-0.00407</td>
<td>-0.00658</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0201)</td>
<td>(0.0130)</td>
<td></td>
</tr>
<tr>
<td>Village Fund</td>
<td>0.0635</td>
<td>0.0888***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0646)</td>
<td>(0.0228)</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>1189</td>
<td>1188</td>
<td>1124</td>
</tr>
<tr>
<td>Wave FEs</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Village FEs</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Household FEs</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is whether the money lender demands a restrictive form of collateral. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households and only waves collected between 1999 and 2004 are included.

* p < 0.10, ** p < 0.05, *** p < 0.01
Table 1.15: Robustness Check: Main Comparative Static with Data Driven Definition of Restrictiveness

<table>
<thead>
<tr>
<th></th>
<th>(1) Restrictive</th>
<th>(2) Restrictive</th>
<th>(3) Restrictive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inv. Size</td>
<td>0.0578 (5.149)</td>
<td>0.224 (1.37)</td>
<td>0.320 (1.38)</td>
</tr>
<tr>
<td>Post</td>
<td>0.186** (0.0811)</td>
<td>0.224 (0.137)</td>
<td>0.320 (0.138)</td>
</tr>
<tr>
<td>Post*Inv. Size</td>
<td>-8.709 (6.490)</td>
<td>-7.608 (5.609)</td>
<td>-15.55*** (5.553)</td>
</tr>
<tr>
<td>Log Loan Size</td>
<td>0.0444*** (0.0119)</td>
<td>0.0148 (0.0115)</td>
<td></td>
</tr>
<tr>
<td>Log Income</td>
<td>-0.00607 (0.0112)</td>
<td>-0.00491 (0.00949)</td>
<td></td>
</tr>
<tr>
<td>Village Fund</td>
<td>0.0597* (0.0326)</td>
<td>0.0842*** (0.0290)</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>1710</td>
<td>1710</td>
<td>1620</td>
</tr>
<tr>
<td>Wave FEs</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Village FEs</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Household FEs</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is whether the money lender demands a restrictive form of collateral, using the data driven definition of restrictive. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Figure 1-1: Patterns of Default by Wealth Level

Figure 1-2: Probability Household Borrows from Money Lender by Wealth Level
Figure 1-3: Probability Money Lender Uses Restrictive Collateral by Wealth Level

Figure 1-4: Coefficients on $invsize_t \times year_t$
Chapter 2

Targeting High Ability Entrepreneurs Using Community Information: Mechanism Design In The Field

2.1 Introduction

Identifying high-ability microentrepreneurs is essential to generating sustainable economic growth among the poor. Empirical studies from Sri Lanka, Ghana, and Mexico in which microenterprises are randomly assigned to receive small cash grants find average marginal returns to capital between 4.5% and 25% per month (De Mel et al. (2008), Fafchamps et al. (2014), McKenzie and Woodruff (2008)). But these estimates of average marginal returns mask substantial heterogeneity amongst firms. For instance, while the average marginal return to capital among male business owners in Sri Lanka was 4.5% per month, the quantile treatment effect ranges from 0% - 32.5% per month. However, the authors are unable to predict marginal returns to capital on the basis of the observable information they collected. Thus the problem of identifying high-growth potential microentrepreneurs remains of first order importance for institutions interested in providing capital to microfirms.

In the absence of asymmetric information, finding and empowering potential high-growth entrepreneurs would be easy. The presence of screening costs, however, poses a serious barrier to providing finance (Stiglitz and Weiss (1981)). Though credit registries for small scale borrowers are emerging in many parts of the developing world, they are
still very much in their infancy, and asymmetric information continues to plague the informal lending industry in these areas. As such, there is a serious need for tools that can overcome hidden information problems and accurately target capital to firms that can use it well. Low-cost, reliable tools to discern high-growth potential firms may allow financial institutions to extend larger or more flexible loan contracts to these entrepreneurs, fostering economic development while maintaining profitability (Field et al. (2013)). Additionally, as evidence accumulates about the substantial positive impacts of unconditional transfers, there is rising interest among governments and non-profits in disbursing grants to poor households (e.g. Haushofer and Shapiro (2016); Blattman, Fiala, and Martinez (2014); De Mel et al. (2008); Fafchamps et al. (2014); McKenzie and Woodruff (2008)). Enabling targeting on the basis of micro-entrepreneurs’ ability might increase the viability and development impact of these initiatives.

What progress has been made thus far in targeting high-growth entrepreneurs? Existing research focuses on developing metrics for assessment of entrepreneurs’ individual characteristics. The earliest literature uses family characteristics and personal background; De Mel et al. (2008) use tests of cognitive ability and personality which evaluate focus, competitiveness, etc.; Klinger, Khwaja, and del Caprio (2013) develop a psychometric test that analyses ethics and character, intelligence, attitudes and beliefs, and business skills. Yet, as De Mel et al. (2008) recognize, most measures of entrepreneurship have been developed using high-income country entrepreneurs who may face vastly different constraints from their developing country counterparts. Such tests might thus be a poor metric of what makes a successful entrepreneur in the developing world.

We report on experimental evidence that suggests community information can be used to target high growth entrepreneurs. We pursue this case along two dimensions. First, we document that community members have valuable information regarding the entrepreneurial characteristics of their peers. Second, we establish that, while the natural inclination of community members is to distort their reports in favor of their family and friends, an array of simple techniques from mechanism design theory are effective in re-aligning community incentives towards truthfulness, substantially increasing the value of reports.

Specifically, we asked community members to rank their peers on various metrics of business growth potential (such as profits and marginal return to capital), borrower reliability (probability of a late payment or default), and other key firm characteristics. We then randomly allocated grants to some of these firms to induce growth. At the time of
eliciting rankings we varied whether respondents were told their reports would influence the likelihood that their peers would receive grants. This induced an organic incentive to shift reports in favor of family and friends. We then cross randomized various mechanisms to encourage truth-telling: paying respondents for the accuracy of reports using peer prediction mechanisms, eliciting reports in public or private, and using cross-reporting techniques to infer which peers each community member was most likely to favor.

We find that respondents have valuable information about one another and that peer reports contain residual information that is not predicted by observable household and business-level characteristics. Using baseline data, we find respondents are able to predict one another’s household income, value of household assets, business profits, work hours, medical expenses, and score on a digit span memory test. On average, a 1 percentile increase in the average rank is associated with a 0.23 percentile increase in the outcome variable. Other than work hours, respondents continue to predict these outcomes even after controlling for a plethora of easily verifiable and also harder-to-observe demographic, household, and business characteristics. Thus ranks contain valuable residual information about community members beyond that which a loan officer may be able to observe.

More surprisingly, community members were also able to predict the impact of cash grants. Using follow up data on business profits for those who randomly did and did not receive our grants, we find that community members are well able to predict which of their peers enjoy high marginal return to capital. Though on average the impact of our grant on business profits was modest, those who were ranked one standard deviation above the mean enjoyed an 8% monthly marginal return to capital, and those who were ranked two standard deviations above the mean enjoyed a 14% monthly marginal return to capital. The value of community rankings is not diminished by the inclusion of a variety of respondent and business characteristics.

However, we also find that community members distort their reports when they are told that their information will influence the distribution of grants. The correlation between community reports and true outcomes is on average 40% lower, implying a significant amount of manipulation in reports and severely limiting their usefulness. We see manipulations directly when examining how peers rank themselves, their family members, and people who are identified by group members as a respondent’s close friends. Although respondents manipulate their reports, we find that our interventions can sig-
nificantly improve their accuracy. Giving respondents monetary incentives when they report in private increases the accuracy of reports by 75%. Asking respondents to report in public increases the accuracy of reports by 89%. As with manipulations, we also document direct evidence that incentives and public reports make it less likely that respondents favor themselves, their family members, or their close friends in the group.

Our primary motivation is to identify entrepreneurs with the highest growth potential (as measured by marginal returns), since allocating grants to these individuals may be efficient. However institutions targeting the poor may have a variety of welfare priorities. We demonstrate that community information can be used to allocate resources using other types of selection criteria such as wealth, health burden, and average business profitability. By additionally asking community members how they would prefer to allocate the grants, we also estimate how communities themselves trade-off efficiency and redistribution.

This project builds on work done by Rigol and Roth (2016). In a lab in the field experiment, we tested the effectiveness of two different types of monetary incentive schemes to elicit truthful reporting: one that relies on ex-post verification of outcomes and one peer prediction incentive scheme that relies on a correlation in community reports to pay respondents based only on their own reports and the contemporaneous reports of their peers. The lab experiment took place in a controlled setting in which farmers were presented with a simple and transparent trade off - to lie or to tell the truth - when asked to report on their farmer peers in the same village. Reassuringly, we find that the two rules are equally effective in eliciting truthful responses. However, relative to a payment rule that relies on ex-post verification of outcomes, peer prediction schemes are much easier to implement for a variety of reasons we discuss below. So, in the present experiment we paid our respondents using the Robust Bayesian Truth Serum of Witkowski and Parkes (2012), a peer prediction scheme identified to be particularly effective in our lab experiment. To our knowledge, this is the first large-scale field experiment to use a peer prediction mechanism to incentivize respondents in a developing country setting and we are among the first to use peer elicitation outside the lab.1

Our work adds to the literature that studies the use of community reports for targeting. Alatas, Banerjee, Hanna, Olken, and Tobias (2012) explore the value of community information in targeting the poor in Indonesia. The authors ask villagers to select the village’s poorest members who are then eligible for a limited number of government transfers.

1The notable exception is John, Lowenstein, and Prelec (2012) who use BTS to measure the prevalence of questionable research practices among academic psychologists.
They find that communities have information about the consumption of their members over and above the information contained in the standard proxy means tests. Relative to theirs, our project furthers the understanding of community targeting along several directions. First, we quantify the value of community information over a much broader set of characteristics. Alatas et al. (2012) measured community knowledge of wealth, which may be easily observable to community members and hence an easy test case. So our ability to target high growth potential entrepreneurs along arguably harder to observe metrics is an encouraging step. Second, we identify that incentives to report truthfully are of first order importance in the elicitation of community information. While Alatas et al. (2012) do examine whether elite capture poses a problem for community reporting, we take a substantially closer look by manipulating respondents' incentives to misreport their information and identifying the likely beneficiaries of each respondent's manipulations. We conclude that there is a substantially larger amount of misreporting than Alatas et al. (2012) are able to identify. Finally we identify a variety of mechanisms effective in encouraging truth-telling, substantially improving the informativeness of community reports.

Closely related is Bryan, Karlan, and Zinman (2015), who randomly give existing borrowers incentives for referring a new borrower and for the repayment outcomes of their referral. They find strong peer enforcement effects but no evidence of peer selection effects in excess of the screening that the bank already does. Their interpretation is that the original clients do not have useful information regarding the individuals they refer beyond the information available to the bank. In contrast to our study, theirs is conducted in South Africa, which has a well-functioning credit bureau. In settings without a well-functioning credit bureau, one might expect community information to be of considerably more value.

The rest of the paper proceeds as follows. Section 2.2 describes our experimental setting and design, Section 2.3 describes the data, Section 2.4 discusses our key specifications, Section 2.5 discusses our results, and Section 2.6 concludes. All tables are relegated to the appendix.
2.2 Description of the Experiment

2.2.1 Sampling and Study Population

Our intervention took place in Amravati, a city of about 550,000 persons in the state of Maharashtra, India. Within Amravati, we selected nine neighborhoods that each had an abundance of microenterpreneurs. In September 2015, we conducted a complete door-to-door census of these neighborhoods, which encompassed 5,573 households. Each person in the household who was engaged in self-employment activities responded to our census. Every respondent reported the total value of their enterprise’s durable assets and inventory (excluding the value of land and buildings), their total number of permanent employees, and their business sector. We identified a sample of 1,576 households that had at least one enterprise with (1) USD 1,000 or less in total working and durable capital and (2) no paid, permanent employees. We excluded farmers and self-employed service persons, such as domestic helpers and teachers. Our selection follows the same criteria as recent “cash-drop” experiments (see e.g. De Mel et al. (2008)).

In October 2015, we recruited these households to participate in our study. At the time of recruitment, households were informed that we would be conducting a project to study entrepreneurship and business growth and that some households would be randomly selected to receive a USD 100 grant. Ultimately, 1,380 households agreed to participate in the study. We conducted baseline surveys of these households in December 2015 - April 2016. These 1,380 households were then organized into groups of five based on geographic proximity. We created a total of 277 groups across all neighborhoods. After all respondents in a neighborhood had completed the baseline survey, the groups in that neighborhood were invited to participate in an activity at the community hall, where the lottery would be conducted. At the hall, respondents completed the ranking activity (described in section 2.2.2) and, upon its completion, a public lottery was held and the winners of the USD 100 grant were announced.

2 The neighborhoods are: Belpura, Vilash Nagar, Mahajan Pura, Akoli, New Saturna, Old Saturna, Wadali, and Pathan Chawk.
3 If there were multiple business owners in the household, we selected households that had less than USD 2000 in combined business capital.
4 We organized respondents into groups that would minimize the geographic distance between study households. The total number of respondents per neighborhood was not always a multiple of 5, so some groups had 4 or 6 clients.
2.2.2 Experimental Design

The aim of this study is to investigate whether knowledge that community members hold about one another can be useful for making decisions regarding the allocation of business grants. We ask a number of related questions. First, do community members have accurate knowledge regarding their entrepreneur peer's business growth potential? To address this question, we instructed community members rank their peers across several personal and enterprise characteristics that are predictive of business growth (such as profits and marginal returns to capital) and borrower reliability (probability of a late payment or default). We compare community reports to business and household outcomes elicited in a baseline household survey to determine the predictiveness of these measures. To assess whether respondents can predict one another’s marginal return to capital, we utilize follow up data on business profits for those who did and did not randomly receive our grants (discussed below). Importantly, the baseline household survey was conducted before respondents knew they would be ranking their peers. In addition, we assess whether respondents' reports provide useful information beyond what could be gathered through measurement of observable household and business characteristics or psychometric tests.

Community knowledge—even if accurate— is only useful for allocative decision-making if those collecting the community information can be reasonably confident that they will gather truthful reports. But eliciting reliable information from community members is not a straightforward task. If respondents know that information they provide about their peers will be used for targeting resources, they may distort their reports to favor persons with whom they have stronger social ties. Rigol and Roth (2016) find that, in a low stakes lab experiment, farmers in a village setting distort their reports in favor of family and close friends. In this paper, we measure the level of distortion that occurs when stakes are high: half of our study participants, whom we refer to as being in the revealed treatment, were told that their reports would influence the probability that their peers would receive cash grants of USD 100 (grants are randomly allocated to a third of our sample). This is a shock approximately equivalent to one and a half months of business profits for the average business in our sample. The control group, whose rankings do not influence grant allocation, provide an estimate of the quality of information embedded in communities in the absence of incentives to distort responses. Comparing the informativeness of the two groups' reports establishes whether community members strategically misreport.

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5The treatment name, revealed, refers to community reports being revealed to a principle who would make use of them.
information to affect allocative decisions (i.e. the possibility of receiving a grant).

If distortions can be corrected, their possibility does not eliminate the utility of community reports. We experimentally test whether two simple techniques from mechanism design theory can effectively correct distortions: (1) asking respondents to report in public or in private and (2) offering monetary incentives for the accuracy of reports. Screening mechanisms that are common in practice and that rely on community information (such as joint liability and participatory rural appraisals) require public reporting. If respondents fear negative social repercussions of truth-telling, mechanisms that involve public reporting may discourage reliable reporting. On the other hand, if respondents fear that misreports will cause them to be perceived as liars by their peers, public reports may lead to more truthful elicitation. To test whether reporting in public would increase or decrease the accuracy of reports, our respondents were randomly assigned to make reports in public (ranks were visible to the entire group) and or private (group members did not see ranking reports). The difference in the accuracy of reports between these two groups reveals whether privacy is an important criterion for elicitation of peer information.

In order to test whether monetary incentives impact accuracy of reports, we randomly assigned respondents to either receive no monetary incentive or to receive an incentive delivered via the Robust Bayesian Truth Serum of Witkowski and Parkes (2012). We discuss the details of the payment rule and our rationale for choosing it in the following section.

In addition to these treatments, we included two non randomized features of the elicitation exercise. First, we asked all respondents to report about their peers using two different methods: Ranking the five members of the group relative to one another, and separately placing the five members of the group in quintiles relative to the entire neighborhood. The relative ranking may be a coarse measure of the group’s knowledge but has the advantage that it induces a zero-sum game in the sense that elevating the status of any member necessarily lowers the status of another, which may deter misreporting. On the other hand, quintiles may contain more information but are also easily manipulable; everyone can be placed into the highest position. The last elicitation mechanism we evaluate is closely related to the cross reporting techniques which play a prominent role in mechanism design and implementation theory. We asked each respondent to identify who each other respondent was closest to (and hence most likely to lie about) within the group. We also asked each respondent to identify who in the group would give the most accurate ranking.

In summary, our respondents were cross-randomized (at the group level) to give their
ranking reports under the following three treatment conditions, for a total of eight treatment cells: Revealed vs Not Revealed (R0 vs R1), Public vs Private (P0 vs P1), and Incentives vs No Incentives (I0 vs I1). Since important predictors of our outcomes of interest such as occupation, religion, caste, and other socio-demographic characteristics are frequently correlated with the area in which respondents live, we stratified our treatments by geographic clusters. Since there are eight treatment groups, we created geographic clusters of eight proximate groups. We then randomized our 8 experimental interventions amongst the eight groups in each cluster. We also randomly selected one-third of our sample to receive USD 100 grants. Selection of grant winners was done via public lottery and is explained in detail in Section 2.2.4.

2.2.3 Peer Prediction and the Robust Bayesian Truth Serum

Delivering monetary incentives is a somewhat thorny endeavor. A natural inclination might be to pay respondents based on the closeness of their reports with ex-post, objectively measured outcomes. However this approach has several drawbacks. First, it may be costly or even impossible to measure ex-post outcomes; indeed, this is one of the primary motivations of relying on community reports. Second, in cases where it is possible to measure ex-post outcomes via surveys, respondents who know their reports will determine the incentive payments of their peers may lie - a particularly undesirable outcome in a research setting where survey reports are a primary measure of ground truth. Thus we were in need of an alternative method.

As stated above, we delivered monetary incentives via the Robust Bayesian Truth Serum (RBTS) of Witkowski and Parkes (2012). RBTS is part of a class of mechanisms known as peer prediction mechanisms. The major innovation in these payment rules is that they eschew reliance on ex-post verifiable information to calculate payments. The tradeoff is that these are very complicated payment rules, which rely on comparison between reports from the respondent about the question of interest (which we refer to as first order beliefs) and community expectations about the distribution of responses to the question of interest (which we refer to as second order beliefs). Roughly speaking, the payment rule rewards respondents who choose responses that are under predicted by the community more than those who choose responses that are over predicted by the community. Details

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6 As some neighborhoods had a number of groups indivisible by 8, some clusters have fewer than eight groups. For these clusters, we randomly selected groups to receive one of the eight treatments.

7 See Prelec (2004) for a seminal contribution to this literature.
about the implementation of RBTS and the intuition behind its incentive compatibility are relegated to the appendix.

Whether these payment rules are as effective as simpler payment rules for respondents with low numeracy is an empirical question addressed in Rigol and Roth (2016). In a population similar to that of the current study, Rigol and Roth (2016) test the difference between paying incentives via a simple rule based on ex-post accuracy and via a peer prediction payment rule. The first rule was easy for surveyors to explain and for respondents to understand; respondents were paid based on the closeness of their report and the ex-post realized outcomes. On the other hand, as in our current study, surveyors did not attempt to explain the peer prediction method. They merely elicited first and second order beliefs and asserted to respondents that they would maximize their incentive payments by telling the truth. Rigol and Roth (2016) find that the additional accuracy induced by the simple ex-post incentive is statistically and economically indistinguishable from that induced by the peer prediction method for eliciting information about borrower reliability and entrepreneurial ability.

That upon first exposure, respondents treat the two payments schemes equivalently is reassuring; because in our experiment each respondent was only exposed to our payment scheme once, we feared no loss from relying on the peer prediction scheme. However, if RBTS does not function as theoretically predicted, over a longer period respondents may experiment and discover that truth telling is not their optimal strategy. As such, Rigol and Roth (2016) take the analysis a step further by estimating the higher order beliefs of respondents in the sample. Not only do respondents report as accurately when facing the complicated incentive scheme, but we provided evidence that truth-telling is the strategy that maximizes their subjective expected payments from RBTS. Thus, even over periods of repeated exposure, respondents should continue to tell the truth.\(^8\) Details of this exercise are replicated in the appendix of this paper.

\(^8\)Note, this is the standard to which we hold commonly used mechanisms even in the developed world. For example, medical students are placed in their first residency via the deferred acceptance algorithm. They largely do not know how it functions but they are assured that truthfully stating their preferences is their best strategy. And because we have theoretical guarantees that stating their true preferences is indeed their best option, the profession feels confident in giving them this advice.
2.2.4 Ranking Questions and Implementation Method

We asked respondents to rank their peers on a series of dimensions. We collected information about the following criteria: highest level of education attained, marginal returns of the peers’ business if she were to receive an Rs.6000 grant, household average monthly income over the past year, projected monthly profits of the peers’ business if she were to be given an Rs.6000 grant, total value of household assets, number of hours that their peers work, total household medical expenses in the previous 6 months, loan repayment trouble over the past year, and digit span memory test. For marginal returns, income, profits, and assets, we asked respondents to rank their peers relative to one another as well as to place them in quintiles of the community distribution. For the remainder of the questions, respondents were asked to report only relative ranks. We also asked a subset of groups to report who they thought deserved to receive the grant. We did not provide any criteria for this ranking and asked respondents to choose what they thought were important criteria.

To minimize respondent fatigue, each respondent answered only a subset of these questions. All members of the same group were asked the same ranking questions. In Figure 2-1 below, we lay out the question randomization structure. Because incentivized groups also had to report second order beliefs in addition to ranks, they were only be asked to answer a total of 7 questions, while non-incentivized groups answered 10. The order of the first 3 questions was always the same and groups were cross-randomized between P0/P1 and 10/I1. The first question was always about education and we primarily intended it to be a practice round. We chose to elicit education quintiles as it allowed us to explain the quintile rankings early. The next two questions were always about marginal return quintiles and relative rankings. In the relevant groups we elicited marginal return information in public and with incentives but we never used marginal return information to affect the distribution of grants because we did not want reports in this dimension to be adulterated by strategic behavior. For questions 4-7, we randomly picked two of three questions: income, assets, and profits. These were cross-randomized with all 3 of our treatments and we elicited both relative rankings and quintiles. Lastly, we randomized questions 8-10 with the public and private treatments only so as to minimize the amount of time respondents spent doing the rankings exercise. Notice that because income, assets, and profits were also in the rotation for Q8-Q10, we have more data on relative rather than quintile rankings for these questions.

After all baseline surveys were completed in a particular neighborhood, groups were
invited to a large community hall to conduct the ranking exercise. One group was invited to conduct the exercise at a time. As soon as a respondent arrived in the hall, he or she was seated behind a privacy screen along with a surveyor. The screen was placed both to reassure the respondents in the privacy treatment that their responses would never be visible to others in the group, but also to avoid potential coordination. Respondents were given name cards with the names of all of the peers they would be ranking. To explain complicated concepts and to minimize variation across surveyors in implementation of the treatments, we created animated videos to guide respondents through the exercise. In the videos we explained the definition of a quintile and how to do a quintile ranking, and the definitions of marginal returns to capital, profits, income, and assets.

For groups in the Public treatment, although respondents gave their ranks behind their privacy screens, they were asked to move with their rankings to the center of the hall at the completion of each ranking. While the pretext of the move to the center was that the lead surveyor had to record everyone’s answers, the purpose was actually that peers could clearly observe each others’ rankings. In the privacy treatment, respondents never interacted with other people in the group until all rankings were completed.

For those who received the incentives treatment, the videos explained that incentives would be paid for truthfulness of the responses. Respondents were told that people who reported what they truly believed were more likely to receive higher incentive payments than those who did not report what they truly believed. Since RBTS incentive payments also required respondents to report their second order beliefs, the videos were used to explain what second order beliefs were. For each of her peers, each respondent was given 20 orange coins and was asked to place the coins in proportion to how she thought others would rank her peer. Payments were calculated and distributed in private by the surveyor at the end of each ranking question. Groups that did not receive incentive treatments were not asked to report second order beliefs and were not paid for their reports.

At arrival, respondents were told that at the end of the exercise, a lottery would be conducted to choose the grant winners. Each person was given 20 lottery tickets and was told that at the end, all people present in the room would put their lottery tickets inside a basket and the winner would be selected by picking out lottery tickets. For groups in the revealed treatment, after completion of the marginal returns relative rankings, the video explained that for the next 4 rankings they would be able to help determine the lottery winner. Respondents were told the person that was ranked the highest by the

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*Surveyors report that respondents always looked at their peers' rankings.*

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group for each round would receive extra lottery tickets. Since we wanted, as much as possible, to keep the probability of selecting the winner balanced across relative ranks, only 1 extra lottery ticket was awarded for winning a round. Respondents, however, did not know how many extra lottery tickets we were awarding each round until all of the ranking exercises were over. At that point the winners were given their extra tickets and the lottery was conducted in the presence of all respondents.

**Answered by all Respondents**

**Answered by Respondents in 10**

<table>
<thead>
<tr>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Q6</th>
<th>Q7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>Asset</td>
<td>Profits</td>
<td>Quintile</td>
<td>Income</td>
<td>Asset</td>
<td>Profits</td>
</tr>
<tr>
<td>Education Quintile</td>
<td>MR Quintile</td>
<td>MR Relative</td>
<td>Income</td>
<td>Asset</td>
<td>Profits</td>
<td>Quintile</td>
</tr>
</tbody>
</table>

**Treatments Randomized:**
- P-Public vs. Private
- I-Incentives vs No Incentives
- R-Revealed vs. Not Revealed

**Figure 2-1: Question Randomization**

### 2.3 Data and Sample Characteristics

The data for the experiment come from respondent surveys. Baseline surveys were conducted between December 2015 and April 2016 in the privacy of the respondent’s home. Respondents were asked detailed information about the economic activities of the household. Specifically, households were asked demographic, occupation, work days and hours, and income information about all household members. They were asked to report about health expenditures and well-being as well as outstanding loans and loan repayment issues. Each business owner was asked to report about the revenues, costs, and profits, inventories, business assets, as well as other aspects of their own businesses. Importantly, all of this was done before respondents knew anything about the ranking exercise. Therefore we have detailed business module of all household businesses.
Business owners were asked to complete a psychometric questions section as well as the digit span memory test. A new method that has been utilized to identify credit-worthy entrepreneurs, developed by the Entrepreneurial Finance Lab, largely relies on psychometric questions. Although the precise test used by EFL in commercial institutions is proprietary, it relies on psychometric assessments similar to those used in De Mel et al. (2008), who find that psychometric tests predict whether people will become entrepreneurs versus wage workers in Sri Lanka. We therefore pose the same questions to our respondents, found in the appendix of this paper, as those utilized in their study. Respondents answered each question on a scale of one to five indicating whether they strongly disagreed, disagreed, do not agree or disagree, agreed, or strongly agreed with the statement. The questions are organized according to categories developed by industrial psychologists: polychronicity measures the willingness to juggle multiple tasks at the same time (Bluedorn, Kalliath, Strube, and Martin (1999)); impulsiveness is a measure of the speed at which a person makes decisions and savings attitudes (Barratt Impulsiveness Scale); tenacity measures a person’s ability to overcome difficult circumstances (Baume and Locke (2004)); achievement is a measure of satisfaction in accomplishing a task well (McClelland (1985)); and locus of control measures a person’s willingness to put themselves in situations outside of their control (Rotter (1966)).

Households were asked to report on a household assets list in which we asked whether the household owned a particular type of asset, how many pieces, and the current resale value of those assets. Our surveyors were trained to verify that the household actually owned the assets about which they reported.

Baseline data is utilized to validate whether respondents can predict information about their peers. We emphasize that this data was collected before any respondent in a particular neighborhood was aware that they would be asked to report about the household or business activities of their neighborhoods. Once we have collected follow up data, we will test whether respondents’ ranks predict future outcomes such as the observed marginal returns to the grant.

Given the selection criteria for the sample described in Section 2.2.1, it is no surprise that our sample is composed of relatively poor microentrepreneurs. Our sample households report earning approximately Rs.9000 per month in total earnings from all income-generating activities, or USD 5 per day, and have approximately $7000 in total household wealth. As is common in developing countries, poor households diversify across different types of income-earning activities: in 50% of our sample households there is at
least one fixed salary or daily wage worker and approximately a fifth of our sample has more than one household business. Medical expenses contribute to a very large portion of household expenses: on average respondents report spending nearly 30% of their monthly earnings on health-related expenditures.

Approximately 60% of the business owners in our sample are male. On average, the firm owners are 40 years old and have about 7 years of formal education. 30% of them work in manufacturing, another 30% in retail, and another 30% work in services with the remainder being spread across construction and livestock rearing. They work 45 hours during an average week and earn about Rs.4500 in profits per month from the businesses. When asked to project the increase in monthly profits that they expected if they were to receive the USD 100 grant, they report expecting to increase their monthly profits by an average of Rs.2000 (USD 35).

Peer predictions data comes from the group ranking exercises. Business owners in our sample are well informed about the peers in their group. We find that in less than 1% of the cases a respondent reported not recognizing a peer in her group. On average, they visited one another 22 times over the previous 30 days. 23% report discussing private family or business issues with the peer. To establish a baseline level of knowledge that peers in the group had about one another, we asked them to report some observable and easily verifiable information about one another. We asked respondents whether their peers owned a motorcycle (50% of our sample has at least one motorcycle in the home). 83% of respondents correctly identified when a peer did not own a motorcycle and 70% correctly identified when a peer owned a motorcycle. This number rises to 80% if we include other motorized vehicles in the criteria. We also asked whether there were children living in their peers’ homes. As with motorcycle ownership, approximately 80% were able to correctly identify when there were and there were no children in their peers’ homes.

### 2.4 Empirical Strategy

We present the regression specifications to answer the questions posed in Section 2.2.2. To address how well peers can predict one another’s baseline outcomes we use the following model.

\[
\text{Outcome}_{ikjc} = \alpha_0 + \alpha_1 \text{Rank}_{ikjc} + \gamma_c + \epsilon_{ikjc}
\]  

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where \( i \) indexes the person being ranked (rankee), \( j \) indexes his group, and \( k \) indexes the person doing the ranking (respondent). \( \text{Outcome}_{ikjc} \) is the percentile at which person \( i \)'s outcome value lies in the distribution of that outcome for the sample. \( \text{Rank}_{ikjc} \) is the percentile at which the rank that person \( k \) provides for person \( i \) in group \( j \) lies in the distribution of ranks for the sample. \( \gamma_c \) is a fixed effect for each cluster of groups that form the basis for our stratification. Standard errors are clustered at the group level. In all Tables presented in the paper we additionally include a surveyor and month of baseline survey fixed effect.

We measure outcomes and ranks in percentiles to maintain comparability across outcomes and reporting methods (ranks vs quintiles), and to ease interpretation. A 1 percentile increase in the rank is, therefore, associated with an \( \alpha_1 \) percentile increase in the outcome variable distribution.

To estimate the residual information that ranks provide over and above observables, we will estimate regression 2.1 with the addition of \( X_{ijkc} \), a vector of person \( i \)'s baseline characteristics that may impact the outcome value. If ranks have residual predictive power, we expect \( \alpha_1 \) to remain positive and significant. We will also estimate a related version of this model in which, rather than using each \( i-k \) pair as the unit of observation, we will average across all of the reports that peers in group \( j \) gave about person \( i \).

\[
\text{Rank}_{ijc} = \frac{1}{n} \sum_{k=1}^{n} \text{Rank}_{ikjc}
\]

\[
\text{Outcome}_{ijc} = \alpha_0 + \alpha_1 \text{Rank}_{ijc} + \gamma_c + \epsilon_{ijkc}
\]

To address whether respondents can predict one another’s marginal return to capital we estimate the following model.

\[
\text{Profit}_{ijkct} = \alpha_0 + \alpha_1 \text{Winner}_{it} + \alpha_2 \text{Rank}_{ijkc} + \alpha_3 \text{Winner}_{it} \times \text{Rank}_{ijkc} + \alpha_4 X_{ijkc} + \alpha_5 \text{Winner}_{it} \times X_{ijkc} + \gamma_c + \gamma_t + \epsilon_{ijkct}
\]

where \( \text{Profit}_{ijkct} \) is the profit for person \( i \) being ranked by person \( k \) in group \( j \), cluster \( c \) and time \( t \), \( \text{Winner}_{it} \) is an indicator for whether person \( i \) has (randomly) received a grant at time \( t \), \( \gamma_t \) is a wave fixed effect and all other variables are as defined above. \( \alpha_2 \) captures the average increase in marginal return to capital associated with being one rank higher in the community prediction. In some specifications we control for borrower and business characteristics \( X_{ijkc} \) as well as its interaction with \( \text{Winner}_{it} \) to measure the additional information conveyed in community reports.
To answer question (2), whether respondents manipulate their rankings when they have a strategic incentive to do so, we test how the accuracy of their responses changes when they are assigned to the revealed treatment (they are told that their ranks would affect the probability that their peers win the lottery). Specifically, we estimate

\[
\text{Outcome}_{ikjc} = \beta_0 + \beta_1 \text{Rank}_{ikjc} + \beta_2 \text{Rank}_{ikjc} \times \text{Revealed}_{jc} + \beta_3 \text{Revealed}_{jc} + \gamma_c + \epsilon_{ikjc} \quad (2.2)
\]

where \text{Revealed}_{jc} is a dummy for whether group \( j \) was assigned to the revealed treatment. \( \beta_1 \) indicates the accuracy of reports in the group that was told that was told nothing about reports being used to inform the distribution of grants and so respondents had no explicit incentive to misreport. \( \beta_2 \) is then the differential effect of being assigned to the revealed group. If respondents manipulate rankings, we expect \( \beta_2 \) to be negative - the correlation between reports and true outcomes should be lower for this group.

A more direct way to test whether respondents are lying is to check whether they manipulate the reports that they give about peers in the group that they share a close relationship with (measured by self and cross reports). Specifically we estimate,

\[
\text{Rank}_{ikjc} = \pi_0 + \pi_1 \text{Member}_{ijc} + \pi_2 \text{Member}_{ijc} \times \text{Revealed}_{jc} + \pi_3 \text{Revealed}_{jc} + \gamma_c + \epsilon_{ikjc}
\]

\( \text{Member}_{ijc} \) is a dummy that will signify one of three types of relationship between the rankee and the respondent: they are family members, they are close peers (as reported by other group members), or the it is the rank that the respondent gives herself. If \( \pi_2 > 0 \), then respondents up rank these individuals irrespective of their true characteristics.

Lastly, to answer question (3) - whether monetary incentives or reporting in public/private improve the accuracy of reports - we estimate the following model

\[
\text{Outcome}_{ikjc} = \theta_0 + \theta_1 \text{Rank}_{ikjc} + \theta_2 \text{Rank}_{ikjc} \times \text{Public}_{jc} + \theta_3 \text{Rank}_{ikjc} \times \text{Incentives}_{jc} + \epsilon_{ikjc} \quad (2.3)
\]
\[ \theta_4 \text{Rank}_{ikjc} \times \text{Public}_{jc} \times \text{Incentives}_{jc} + \theta_5 \text{Public}_{jc} \times \text{Incentives}_{jc} + \theta_6 \text{Public}_{jc} + \theta_7 \text{Incentives}_{jc} + \gamma_c + \epsilon_{ijc} \]

*Public* is a dummy indicating whether the group is assigned to report rankings in public (vs private). *Incentives* is a dummy for whether the group is assigned to receive personal incentives for its reports. \( \theta_1 \) indicates the accuracy of reports when respondents report in private and receive no incentives. \( \theta_2 \) is the differential effect of reporting in public without incentives. \( \theta_3 \) is the differential effect of reporting in private and receiving incentives, so if incentives improve rankings, we expect this to be positive. Lastly \( \theta_4 \) is the differential effect of reporting in public and receiving incentives relative to either treatment on its own.

### 2.4.1 Randomization Check

In online appendix Table 1, we present the randomization check of baseline characteristics by treatment. To check for balance we estimate the model

\[
\text{Characteristic}_{ijc} = \tau_0 + \tau_1 \text{Treatment}_{jc} + \gamma_c + \epsilon_{ijc}
\]

where \( \text{Treatment}_{jc} \) is a dummy for whether the group was assigned to the \textit{Revealed}_{jc} treatment (columns 1 and 2), the \textit{Incentives}_{jc} treatment (columns 3 and 4), and the \textit{Public}_{jc} treatment (columns 5 and 6). The odd columns show the average of each characteristic for the control group in each block. So column 1 shows the means of characteristics for groups that were assigned to \textit{Not Revealed}. The even columns show \( \tau_1 \) for each treatment (the difference between treatment and control characteristics). The characteristics in rows 1-6 are at the household level. The remainder pertain to the business owner that was selected to participate in the study. As would be expected by chance, there is some imbalance across a few characteristics between the various treatments. Groups assigned to \textit{Revealed} report lower yearly income and slightly fewer years of education, and the incentivized group has somewhat lower baseline assets than the unincentivized group and lower projected monthly profits if they were to receive the grant. There is no significant difference in the means of characteristics in the \textit{Public} treatment. We also present the results of a joint test of statistical significance, and cannot reject that all groups are drawn from the same population.
2.5 Results

2.5.1 How Much Do Respondents Know About Baseline Characteristics?

The most basic way to quantify how much respondents know about community members is to estimate the correlation between the ranks given to a respondent by her peers and a respondents' true outcome value for that ranking (regression model 2.1). In Table 2.1, we pool across all treatments and report the results at the ranker-rankee pair level of observation. The outcome variables, denoted in the column headings, were elicited from clients (rankees) during the baseline survey, which was conducted before respondents had any knowledge of the rankings portion of the study and before any ranking exercises had been conducted in the community. They correspond exactly to the outcome that peers were asked to rank on during the ranking exercises. Self-reported marginal returns are the outcome values in columns 1 and 2.\footnote{While self reported marginal returns to capital may seem unreliable, in previous work done by the authors, we find that self-reported marginal returns are predictive of true marginal returns in cash drop studies. And as stated above, once our followups are completed we will be able to measure and evaluate community predictiveness of true marginal returns to capital.} In columns 3 and 4, the outcome is average monthly household income over the past year. In columns 5 and 6, the outcome value is the clients' predicted monthly profits if they were to receive a $100 grant.\footnote{We use this outcome as it corresponds to the ranking question posed to respondents. In online appendix Table 1 we show that the results remain nearly identical if we use average yearly profits as the outcome variable.} In columns 7 and 8, the outcome is the total value of household assets. In column 9, we report a households total medical expenses in the past six months. In column 10, the outcome variable is the average number of hours the client works per week and in column 11 we report the total number of digits the client remembered during a digit span memory test. Following De Mel et al. (2008), we trim variables in columns 1-10 at the 99.5% level.\footnote{Digit span is a commonly used test for working memory. Respondents are shown flashcards with an increasing number of digits and asked to recall the numbers from memory. The surveyor records the total number of digits that the respondent correctly repeated back.}

Both the outcome variables and \textit{Rank} have been converted to percentiles of the distribution of observed outcome values. So a 1 percentile increase in the rank is associated with an $a_1$ percentile increase in the outcome variable distribution. For all outcome measures, the reports are highly predictive. Peers are informed about about relatively observable aspects of their neighbors' lives, such as household assets, but also much less easily observable characteristics including working memory and work ethic and ones that are...
notoriously hard to predict such as business profits and marginal returns to capital.

The level of accuracy of the prediction varies depending on the question. Respondents are most accurately able to predict peers' household assets: a 1 percentile increase in the relative rank of assets is associated with a 0.19 percentile increase in the distribution of household assets in our sample population. Unsurprisingly, the most difficult outcome to predict accurately is marginal returns. We find that a 1 percentile increase in the marginal returns relative rank is associated with a 0.078 percentile increase in the self-reported marginal return to capital. We asked respondents to rank their peers relative to others in the group and also relative to the community by reporting the quintile of the outcome distribution that they believe the peer to be in. In theory, quintile rankings could be more useful as they could contain more information about the true position of the peer vis-a-vis other similar microentrepreneurs. In both the even and odd columns 1-8 of Table 2.1 the outcome variable is the same, but what changes is the method of reporting. In the odd columns, the regressor is the percentile in the relative rank distribution and in the even columns the regressor is the percentile of the quintile rank distribution. By comparing the odd and even columns, we find that relative and quintile rankings are equally informative.

Respondents may have idiosyncratic preferences for misreporting about certain peers in their group and may otherwise make idiosyncratic errors. One way to reduce the influence of the errors is by averaging across all reports given about a particular group member. We do so and present these results in Table 2.2, where the unit of observation is the rankee. We observe that the average reports are significantly more predictive of all outcome variables. While a 1 percentile increase in the profits rank leads to a 0.14 percentile increase in the profits distribution in Table 2.1, it is associated with a 0.25 percentile increase in Table 2.2.

While community reports contain valuable information about community members, are they informative beyond what would be captured by observables? To test whether there is residual information, we add household and business-level controls to the regressions in Table 2.2 and observe whether the Rank variable continues to be statistically and economically significant. We present the most conservative version of this exercise in Table 2.3. In addition to adding easily observable household and demographic characteristics that would be verifiable by a principal - such as gender of the main business owner, education, age, household size, household composition - we also add self-reported information about the household's primary business. Specifically, we control for the number
of household businesses, the value of business assets, business revenues in the previous 30 days, and average yearly profits. The control variables tend to predict the outcome variables in the directions we would expect. Average yearly profits are positively predictive of nearly all outcome measures (except digit span). Due to correlation with average yearly profits, revenues is nearly always insignificant except in predicting profits in columns 5 and 6 (average yearly profits is omitted as a control in this regression). Larger households, as well as households with more business owners, have higher household income as well as more assets. Despite controlling for a plethora of household and business characteristics, ranks continue to significantly predict outcomes. The one exception is in column 10: covariates appear to contain fully overlapping information with the work hours ranks. In online appendix Table 3, we have replicated Table 2.3, but removed hard to observe business characteristics: total business capital, revenues, and average yearly profits.

In Table 2.3 and online appendix Table 3, we see that marginal returns ranks continue to be informative even after controlling for a set of household and business characteristics. Another set of characteristics that have been identified as predictive of credit worthiness and entrepreneurial aptitude are psychological questions that identify characteristics such as tenacity, polychronicity, and optimism (see Klinger, Khwaja, and Carpio 2013). We therefore test how well psychometric questions perform at predicting self-reported marginal returns and how they compare to community rankings in Table 2.4. The regressors are labeled according to the psychological trait for which they are meant to proxy (the specific wording of the statement is found in the Appendix). In column 1, we regress self-reported marginal returns percentile on all of the psychometric questions that we collected. The traits that are strongly predictive of marginal returns fall into two categories: optimism and achievement. Optimism negatively predicts marginal returns: business owners who are more likely to agree with the statements “In times of uncertainty I expect the best” and “I’m always optimistic about the future” and those who are more likely to disagree with “If something can go wrong with me, it will” have lower self-reported marginal returns. People who agree with the statement “Part of my enjoyment in doing things is improving my past performance” tend to have higher marginal returns. Both the direction of prediction of growth and the adjusted $R^2$ on this regression (0.05) is in line with the results found by De Mel et al. (2008).

In column 2 of the table we replicate the result of column 1 of Table 2.2. We see that the adjusted $R^2$ on this regression is equivalent to the psychometric regression, but the predictive power of community ranks is twice as large as any of the psychometric questions.
In addition to asking respondents to rank one another on marginal returns, we also asked respondents to rank their peers on their ability to grow a business. The specific wording of the question asked was

"Consider a world in which everyone in your group – no matter how rich they are right now, or how poor they are, or how small or big their business or their current loan is – is starting at exactly zero, at exactly the same place as everyone else. Now we give each of you Rs.20,000, and we ask you to each, individually, start any type of business you want. Who do you think will grow their business the most and make the most profits from that business? What we are trying to understand is: who is most talented in growing a business?"

While this question has no natural analogue in the data, much like the psychometric questions, it was meant to proxy for entrepreneurial ability using concepts and wording that were more familiar to the respondent. The question was asked to only a subset of respondents as explained in Section 2.2.4, but in column 3 we see that despite the small sample size, the question is strongly predictive of marginal returns. A 1 percentile increase this ranking increases marginal returns by 0.2 percentile. The adjusted $R^2$ also increases to 0.15. Do psychometrics and community ranks provide overlapping information? To test this, in columns 4 and 5, we include both in a regression to predict self reported marginal returns. We find that ranks continue to be predictive even after controlling for psychometrics and given that the coefficient on marginal returns remains nearly identical to columns 2 and 3, it appears that the information is not overlapping.

2.5.2 Can Respondents Predict One Another’s True Marginal Return To Capital?

We next ask whether respondents can predict one another’s true marginal return to capital, measured by comparing profits of those who did and did not receive grants, collected during followup surveys. We are in the midst of collecting follow up data, so the results in this section should be viewed as indicative. In general we find that, while some of our estimates are noisy, community information appears to be quite valuable in identifying the entrepreneurs with high marginal return to capital. Tables 2.10 - 2.13 make the case.

In Table 2.10 we examine the relationship of a variety of measures of grant expenditure (of those who received our grant) with the average community ranking of marginal return to capital. We find that community members ranked highly on average spend
more of their own money augmenting the grant to buy larger business assets, spend a higher fraction of the grant on business assets, and allocate less of the grant to household expenditures and savings.

Table 2.11 presents our main result: community rankings are highly predictive of marginal return to capital. As outlined in Section 2.4, our primary specification is

$$\text{Profit}_{ikjt} = \alpha_0 + \alpha_1 \text{Winner}_{it} + \alpha_2 \text{Rank}_{ijc} + \alpha_3 \text{Winner}_{it} \times \text{Rank}_{ijc} + \alpha_4 X_{ijc} + \alpha_5 \text{Winner}_{it} \times X_{ijc} + \gamma_c + \gamma_t + \epsilon_{ikjt}$$

where \( \text{Profit}_{ikjt} \) measures either business profits or household income of person \( i \) in period \( t \), depending on the specification, \( \text{Winner}_{it} \) is an indicator for whether person \( i \) received a grant at or before period \( t \), and \( \text{Rank}_{ijc} \) is the average rank assigned to person \( i \) by the members of group \( j \) in cluster \( c \). \( \alpha_3 \) measures the average additional marginal return to capital an entrepreneur enjoys for every additional rank he is assigned by his group. Across the various specifications in Table 2.11, \( \alpha_3 \) is large and positive, with varying levels of statistical significance. The standard deviation on the average ranking of marginal return to capital is .83. Therefore, while the estimates in column 2 imply that the average marginal return to capital in the whole population was about 3.7% per month, the average entrepreneur ranked one standard deviation above the mean enjoyed an 8.8% monthly marginal return to capital, and an entrepreneur ranked two standard deviations above the mean enjoyed a 13.9% marginal return to capital. Thus community information appears to be extremely valuable in targeting grants.

Tables 2.12 and 2.13 examine how much the value of community reports diminishes when controlling for other business and household characteristics, as well as their interaction with \( \text{Winner}_{it} \). The regressions in Table 2.12 control for the entrepreneur's gender and the type of business she is in, as these are easily observable characteristics that tend to be predictive of marginal return to capital. Nevertheless we find that community information is almost orthogonal to these characteristics; the estimates in Table 2.12 are strikingly similar to those obtained without controls. In addition to gender and business category, Table 2.13 presents estimates that also control for baseline profits. If anything, controlling for baseline profits seems to enhance the value of community information, as the estimates in Table 2.13 are substantially larger than the preceding ones. Thus it seems that community information is valuable in identifying entrepreneurs with high marginal return to capital even if the implementing organization has access to a large variety of
observable demographic and business information.

2.5.3 Do Peers Distort their Responses?

Aside from quantifying how much information neighbors have about one another, the second major goal of this project is to quantify whether and how much peers misreport in high stakes settings. Half of our sample was informed that they could affect the probability that their peers (or themselves) would win the $100 grant (the other half was not given any information about how their rankings would be used). One test of whether respondents behave strategically is to compare the accuracy of reports in groups in which ranks affected the grant allocation (Revealed) and groups in which ranks had no effect on grant distribution.

As explained in Section 2.2.4, we implemented this treatment only for the income, profits, and assets ranking questions. Therefore in Table 2.5, we only show these three outcome variables in columns 1 to 3. As in previous tables, the treatment and outcome variables are standardized in percentile terms. To increase power, in columns 1-3 we pool across quintile and relative ranks and in column 4 we pool across the previous 3 columns. The Rank variable captures the accuracy of the report in the control group (Not Revealed). The Rank * Revealed coefficient tells us whether the rankings are differentially informative when respondents are told their ranks will be used to help determine grant allocation. First, we note that in the control group, predictions are all significant at the 1% level and range between 0.15 percentile increase in income and 0.23 percentile increase in assets. We cannot reject that respondents do not distort their ranks when reporting about income, although the standard error on the coefficient is large. On the other hand, for both profits and assets, the coefficient on Rank * Revealed is large, negative, and significant. This implies that responses are significantly less accurate when respondents have an incentive to behave strategically. We should note that this was not ex-ante obvious: the Revealed treatment may have had a positive effect since revealing ranks may have caused respondents to focus and take the exercise more seriously. Looking at the pooled estimate in column 4, we see that, on average, responses become 40% less accurate in the Revealed group.

We asked respondents to rank their peers in the group relative to one another and also relative to the community via quintiles. Quintile rankings have the advantage of potentially being more informative for a principal that is interested in understanding who are,
for example, the best entrepreneurs in the community, not just in the group. However, quintile rankings may also be easier to manipulate. Relative ranks have a zero-sum feature that ranking someone higher necessarily harms another person in the group. For quintiles, however, group members could simply say that all of their peers are the best in the community. We test both of these hypotheses - that quintile ranks are more informative in the absence of distortionary incentives and that they are more manipulable in their presence - in Table 2.6. We show the results separately for relative ranks and for quintile ranks. Column 7 pools columns 1, 3, and 5 (relative rankings) while column 8 pools across 2, 4, and 6 (quintile rankings). In the control group, quintile ranks are never more (or less) informative than relative ranks: the coefficient on \( \text{Rank} \) in the odd and even columns are nearly numerically identical for all questions. The Revealed treatment does not have an impact on the accuracy of quintile or relative ranks for household income. We do, however, observe a differential impact of the treatment in the quintile and relative ranks of profits and assets. As predicted, the coefficient on \( \text{Rank} \times \text{Revealed} \) is more negative in the quintile as compared to the relative regressions. The difference is significant in the pooled versions of the regressions: while the Revealed treatment reduces the accuracy of relative ranks by 20%, it reduces the accuracy of the quintile ranks by 40%. Thus relative ranks may be a more robust way to elicit community information.

Respondents manipulate their reports when they have an incentive to do so. We now turn to predicting the beneficiaries of each respondent’s manipulations. In Table 2.7 we directly check how respondents ranked different types of peers in the group. The outcome variable in this set of regressions is the rank that respondent \( i \) gives to her peer \( j \). Our regressors include dummies for whether respondents are ranking themselves or their family members. We also asked group members to cross-report who they thought was each group members’ closest peer in the group. In other words, we asked respondent \( i \) to report who she thought was person \( j \)’s closest friend in the group. We created a binary variable Closest Peer which takes a value 1 if two or more people in the group reported that person \( i \) is group member \( j \)’s closest peer. The coefficients Family, Self, Closest Peer indicate the average percentile difference in the rank given to each of these groups over other peers in the control group (Not Revealed). Even in the absence of incentives to misreport, respondents rank themselves 6 percentage points higher (significant at the 1% level). They rank their family members 2 percentage points higher, although this is not significant, and they rank their peers 4 percentage points higher (significant at the 1% level). The peers result remains significant even if we exclude family members from the Closest Peer group, which implies that neighbors are well informed about the inter-
personal relationships of their group members beyond what may be easily observable by an outsider (familial connections). The fact that respondents rank these groups of people higher in the control group, however, does not necessarily imply that they are behaving strategically: it may be that people rank those that they know better more highly simply because they are better informed about these people and feel more confident about these reports. To test whether they lie to favor themselves or those who they are close to we compare the control group to the Revealed group. We see that they are much more likely to rank themselves highly. The average percentile rank increases by 200% in the Revealed group. Although noisy, they also appear to be ranking their family members more highly. The only group that does not seem to benefit are Closest Peer. In the next section, we will give further evidence of reports manipulations.

2.5.4 Can Mechanism Design Tools Correct Incentives to Misreport?

In the previous section we provided evidence that respondents distort their reports to favor themselves and their family members when they have a strategic incentive to do so. These distortions have a substantial impact on the accuracy of reports, particularly when respondents have an incentive to misreport such as when their information is used to allocate a desired good. Can we use mechanism design tools to generate incentives for truthful reporting? We test two tools: incentive payments for the accuracy of reports and reporting in public versus private. In Table 2.8, we provide evidence that reports become more accurate when respondents in either the Public, Incentives, or Public and Incentives treatments. The coefficient on Rank indicates the accuracy of reports in groups in which respondents do not receive incentives and report in private. Reports are informative and, in the pooled outcomes regression in column 4, a 1 percentile increase in the rank is associated with a 0.1 percentile increase in the outcome variable.

Providing groups with incentives, when group members report in private, increases the accuracy of predictions for both income and profits over the private groups that receive no incentives by between 86% and 179%. We cannot reject that the incentives have no effect on assets ranks, but the incentive boost remains large and significant in the pooled specification. It is ex-ante ambiguous whether the public treatment should lead to more or less accurate predictions: on the one hand group members, the fact that group members can observe a respondents’ ranks may lead to policing and therefore coordination on the truth. On the other hand, respondents may find themselves implicitly pressured to up-rank their friends in the group. While noisy in columns 1 and 2, we see that
the public treatment increases the accuracy of reports. In the case of assets, the public treatment has a large and significant effect. When we pool across all variables in column 4, the public treatment without incentives and the incentives treatment in private have what appears like an equivalently-sized effect.

We break down the ranks of self, family, and close peer ranks by public and incentive treatments in Table 2.9. The patterns are consistent with the results observed in Table 2.8. In the private treatment without incentives, respondents up rank themselves, their family members, and their close peers by 16.5 percentage points, 9.7 percentage points, and 5.6 percentage points respectively. Consistent with Table 2.8, offering monetary incentives or eliciting reports in public both mitigate this tendency substantially.

2.6 Conclusion

We find that community members have valuable residual information about their peers that can be useful in targeting. Not only can community members identify one another's business characteristics, but they can also predict which of their peers have high return to capital. By distributing cash grants and measuring profits over time we find that we can reliably identify entrepreneurs with profitable investment opportunities. In particular, while the average impact of our grant on business profits was modest, community information can reliably identify entrepreneurs with upwards of 14% monthly marginal return to capital.

While community information appears to be quite valuable for targeting, we also find that its accuracy is sensitivity to the conditions under which it is elicited. In particular, we identify a natural tendency for respondents to favor their friends and family members. Moreover this tendency is amplified when respondents are told that their reports influence the distribution of grants. However we also find that a variety of techniques motivated by mechanism design theory are effective in realigning incentives for truthfulness. In particular, small monetary payments for accuracy, eliciting reports in public, rather than in private, and cross reporting techniques to identify which community members are likely to favor one another all substantially improved the accuracy of reports. We therefore hope that the techniques identified in this paper may prove useful in improving the targeting of financial services.
## APPENDIX

### Tables

#### Table 2.1: What Respondents Know: Individual Regressions

<table>
<thead>
<tr>
<th>Rank</th>
<th>Marginal Return</th>
<th>Marginal Return</th>
<th>Income</th>
<th>Income</th>
<th>Profits</th>
<th>Profits</th>
<th>Assets</th>
<th>Assets</th>
<th>Medical Expenses</th>
<th>Work Hours</th>
<th>Digitspan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Relative</td>
<td>Quintile</td>
<td>Relative</td>
<td>Quintile</td>
<td>Relative</td>
<td>Quintile</td>
<td>Relative</td>
<td>Quintile</td>
<td>Relative</td>
<td>Relative</td>
<td>Relative</td>
</tr>
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<td>0.0766***</td>
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<td>0.141***</td>
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<td>2324</td>
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Robust standard errors clustered at group level in parentheses. The model includes cluster, surveyor, and date of survey fixed effects. The outcome variable is the percentile of the outcome in the column header. The regressor is the percentile of the rank given to a respondent by each random, computed by question.

#### Table 2.2: What Respondents Know: Average Regressions

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<tr>
<th>Rank</th>
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<th>Marginal Return</th>
<th>Income</th>
<th>Income</th>
<th>Profits</th>
<th>Profits</th>
<th>Assets</th>
<th>Assets</th>
<th>Medical Expenses</th>
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</table>

Robust standard errors clustered at group level in parentheses. The model includes cluster, surveyor, and date of survey fixed effects. The outcome variable is the percentile of the outcome in the column header. The regressor is the percentile of the average rank given to a respondent, computed by question.
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</table>

Robust standard errors clustered at group level in parentheses. The model includes cluster, survey, and date of survey fixed effects. The outcome variable is the percentile of the outcome in the outcome header. The regressor is the percentile of the average rank given to a respondent, computed by question. Due to space constraints, the coefficients on sector controls are not shown.
## Table 2.4: The Predictiveness of Psychometrics and Ranks

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<th>Rank</th>
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<th>(2) Marginal Return</th>
<th>(3) Marginal Return</th>
<th>(4) Marginal Return</th>
<th>(5) Marginal Return</th>
</tr>
</thead>
<tbody>
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<td>0.007 (0.026)</td>
<td>0.011 (0.025)</td>
<td>0.011 (0.027)</td>
<td>-0.069** (0.027)</td>
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<tr>
<td>Impulsiveness 2</td>
<td>0.013 (0.041)</td>
<td>0.011 (0.040)</td>
<td>-0.115* (0.040)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Impulsiveness 3</td>
<td>0.009 (0.034)</td>
<td>0.011 (0.032)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimism 1</td>
<td>-0.069** (0.028)</td>
<td>-0.067** (0.028)</td>
<td>-0.069 (0.026)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimism 2</td>
<td>0.067** (0.030)</td>
<td>0.065** (0.030)</td>
<td>0.067 (0.030)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimism 3</td>
<td>-0.037 (0.034)</td>
<td>-0.032 (0.034)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimism 4</td>
<td>-0.077** (0.032)</td>
<td>-0.072** (0.032)</td>
<td>-0.077 (0.032)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenacity 1</td>
<td>0.013 (0.032)</td>
<td>0.015 (0.032)</td>
<td>0.013 (0.032)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tenacity 2</td>
<td>0.034 (0.029)</td>
<td>0.033 (0.030)</td>
<td>0.034 (0.030)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polychronicity 1</td>
<td>0.036 (0.039)</td>
<td>0.040 (0.039)</td>
<td>0.036 (0.039)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polychronicity 2</td>
<td>-0.054 (0.035)</td>
<td>-0.055 (0.034)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polychronicity 3</td>
<td>-0.032 (0.030)</td>
<td>-0.037 (0.030)</td>
<td>-0.032 (0.030)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Locus of Control 11</td>
<td>-0.019 (0.028)</td>
<td>-0.012 (0.026)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Locus of Control 11</td>
<td>0.051 (0.033)</td>
<td>0.043 (0.033)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Achievement 1</td>
<td>0.035 (0.032)</td>
<td>0.033 (0.032)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Achievement 2</td>
<td>0.070** (0.032)</td>
<td>0.064** (0.031)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Organization 1</td>
<td>-0.011 (0.027)</td>
<td>-0.012 (0.027)</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>


N: 128 128 128 128 128

* p < 0.05  ** p < 0.01  *** p < 0.001
Table 2.5: Do Respondents Distort Their Responses? Individual Regressions

<table>
<thead>
<tr>
<th></th>
<th>(1) Income</th>
<th>(2) Profits</th>
<th>(3) Assets</th>
<th>(4) Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank*Revealed</td>
<td>0.010</td>
<td>-0.077**</td>
<td>-0.141***</td>
<td>-0.068**</td>
</tr>
<tr>
<td></td>
<td>(0.043)</td>
<td>(0.035)</td>
<td>(0.046)</td>
<td>(0.028)</td>
</tr>
<tr>
<td>Rank</td>
<td>0.149***</td>
<td>0.153***</td>
<td>0.228***</td>
<td>0.178***</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.025)</td>
<td>(0.034)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>Revealed</td>
<td>-0.018</td>
<td>0.045</td>
<td>0.119***</td>
<td>0.042*</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.033)</td>
<td>(0.039)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>Mean of the</td>
<td>9325.46</td>
<td>7022.62</td>
<td>472153.54</td>
<td>.</td>
</tr>
<tr>
<td>Dependent Variable</td>
<td>[7435.95]</td>
<td>[5285.32]</td>
<td>[677786.22]</td>
<td>[.]</td>
</tr>
<tr>
<td>N</td>
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<td>6040</td>
<td>5128</td>
<td>16957</td>
</tr>
<tr>
<td>Controls</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Robust standard errors clustered at group level in parentheses. The model includes cluster, surveyor, and date of survey fixed effects. The outcome variable is the percentile of the outcome in the column header. The regressor is the percentile of the rank given to a respondent by each rankee, computed by question.
Table 2.6: Do Respondents Lie in Relative and Quintile Responses?

<table>
<thead>
<tr>
<th></th>
<th>(1) Income Relative</th>
<th>(2) Income Quintile</th>
<th>(3) Profits Relative</th>
<th>(4) Profits Quintile</th>
<th>(5) Assets Relative</th>
<th>(6) Assets Quintile</th>
<th>(7) Relative Questions Quintile</th>
<th>(8) Quintile Questions Quintile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank**</td>
<td>-0.011</td>
<td>-0.016</td>
<td>-0.059</td>
<td>-0.130**</td>
<td>-0.111</td>
<td>-0.225***</td>
<td>-0.050</td>
<td>-0.112**</td>
</tr>
<tr>
<td></td>
<td>(0.070)</td>
<td>(0.074)</td>
<td>(0.066)</td>
<td>(0.073)</td>
<td>(0.068)</td>
<td>(0.072)</td>
<td>(0.039)</td>
<td>(0.047)</td>
</tr>
<tr>
<td>Rank***</td>
<td>0.246**</td>
<td>0.252***</td>
<td>0.277***</td>
<td>0.252***</td>
<td>0.310***</td>
<td>0.322***</td>
<td>0.261***</td>
<td>0.274***</td>
</tr>
<tr>
<td></td>
<td>(0.047)</td>
<td>(0.056)</td>
<td>(0.048)</td>
<td>(0.049)</td>
<td>(0.046)</td>
<td>(0.050)</td>
<td>(0.024)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>Revealed</td>
<td>-0.008</td>
<td>-0.003</td>
<td>0.024</td>
<td>0.064</td>
<td>0.080*</td>
<td>0.135***</td>
<td>0.005</td>
<td>0.057*</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.047)</td>
<td>(0.041)</td>
<td>(0.045)</td>
<td>(0.044)</td>
<td>(0.045)</td>
<td>(0.024)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>Mean of the</td>
<td>9202.27</td>
<td>9244.57</td>
<td>6949.58</td>
<td>6857.15</td>
<td>46651.65</td>
<td>458235.98</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent</td>
<td>7420.04</td>
<td>7496.99</td>
<td>5170.26</td>
<td>5196.07</td>
<td>642915.39</td>
<td>649158.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>772</td>
<td>672</td>
<td>834</td>
<td>774</td>
<td>774</td>
<td>659</td>
<td>3007</td>
<td>2105</td>
</tr>
<tr>
<td>Controls</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Robust standard errors clustered at group level in parentheses. The model includes cluster, surveyor, and date of survey fixed effects. The outcome variable is the percentile of the outcome in the column header. The regressor is the percentile of the average rank given to a respondent, computed by question.
Table 2.7: Do Respondents Manipulate Rankings?

<table>
<thead>
<tr>
<th></th>
<th>(1) Rank</th>
<th>(2) Rank</th>
<th>(3) Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family Member*Revealed</td>
<td>0.0307</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0228)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Family Member</td>
<td>0.0193</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0172)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closest Peer*Revealed</td>
<td></td>
<td>-0.0207</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0185)</td>
<td></td>
</tr>
<tr>
<td>Closest Peer</td>
<td></td>
<td>0.0444***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0128)</td>
<td></td>
</tr>
<tr>
<td>Self*Revealed</td>
<td></td>
<td>0.0909***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0234)</td>
<td></td>
</tr>
<tr>
<td>Self</td>
<td></td>
<td>0.0565***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0139)</td>
<td></td>
</tr>
<tr>
<td>Revealed</td>
<td>-0.0212***</td>
<td>0.00601</td>
<td>-0.0162**</td>
</tr>
<tr>
<td></td>
<td>(0.00780)</td>
<td>(0.00588)</td>
<td>(0.00627)</td>
</tr>
</tbody>
</table>

Mean of the Dependent Variable 0.60 0.60 0.60  
[0.29] [0.29] [0.29]

N 12005 17015 17015

Controls Yes Yes Yes

Robust standard errors clustered at group level in parentheses. The model includes cluster fixed effects. The outcome variable are all of the ranks provided for all ranking questions asked. The regression includes a control for each question and for how confident the respondent felt about the ranking. Column (1) contains fewer observations due to missing family relationship information.
Table 2.8: The Impact of Incentive Payments and Public Reporting on Accuracy of Individual Reports

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Income</td>
<td>Profits</td>
<td>Assets</td>
<td>Total</td>
</tr>
<tr>
<td>Rank</td>
<td>0.113***</td>
<td>0.061*</td>
<td>0.112***</td>
<td>0.100***</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.036)</td>
<td>(0.037)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Rank*Incentives</td>
<td>0.098*</td>
<td>0.109**</td>
<td>0.009</td>
<td>0.075**</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.052)</td>
<td>(0.063)</td>
<td>(0.038)</td>
</tr>
<tr>
<td>Rank*Public</td>
<td>0.063</td>
<td>0.072</td>
<td>0.142**</td>
<td>0.089**</td>
</tr>
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<td></td>
<td>(0.059)</td>
<td>(0.050)</td>
<td>(0.061)</td>
<td>(0.041)</td>
</tr>
<tr>
<td>Rank*Public_Incentives</td>
<td>-0.158*</td>
<td>-0.148**</td>
<td>-0.101</td>
<td>-0.146**</td>
</tr>
<tr>
<td></td>
<td>(0.084)</td>
<td>(0.073)</td>
<td>(0.095)</td>
<td>(0.058)</td>
</tr>
<tr>
<td>Public_Incentives</td>
<td>0.155**</td>
<td>0.146**</td>
<td>0.034</td>
<td>0.118**</td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
<td>(0.068)</td>
<td>(0.082)</td>
<td>(0.050)</td>
</tr>
<tr>
<td>Public</td>
<td>-0.052</td>
<td>-0.055</td>
<td>-0.083</td>
<td>-0.056</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td>(0.048)</td>
<td>(0.052)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>Incentives</td>
<td>-0.125**</td>
<td>-0.113**</td>
<td>-0.042</td>
<td>-0.096***</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
<td>(0.047)</td>
<td>(0.057)</td>
<td>(0.034)</td>
</tr>
</tbody>
</table>

Mean of the

| Dependent Variable        | [7435.95]    | [5285.32]    | [677786.22]  | .             |

N                         | 5789         | 6040         | 5128         | 16957         |

Controls                   | Yes          | Yes          | Yes          | Yes           |

Robust standard errors clustered at group level in parentheses. The model includes cluster, surveyor, and date of survey fixed effects. The outcome variable is the percentile of the outcome in the column header. The regressor is the percentile of the average rank given to a respondent, computed by question.
Table 2.9: The Impact of Incentive Payments and Public Reporting on Accuracy of Individual Reports

<table>
<thead>
<tr>
<th></th>
<th>(1) Rank</th>
<th>(2) Rank</th>
<th>(3) Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family Member</td>
<td>0.097***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Family*Incentives</td>
<td>-0.057*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Family*Public</td>
<td>-0.066**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Family<em>Incentives</em>Public</td>
<td>0.056</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.042)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closest Peer</td>
<td>0.056***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closest Peer*Incentives</td>
<td>-0.054**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closest Peer*Public</td>
<td>-0.033</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closest Peer<em>Incentives</em>Public</td>
<td>0.090**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self</td>
<td></td>
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</tr>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.023)</td>
<td></td>
</tr>
<tr>
<td>Self*Public</td>
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<td>-0.025</td>
<td></td>
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<td>(0.021)</td>
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</tr>
<tr>
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<td></td>
<td></td>
<td>(0.034)</td>
<td></td>
</tr>
<tr>
<td>Incentives*Public</td>
<td>-0.016</td>
<td>-0.022**</td>
<td>-0.015</td>
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<td>(0.010)</td>
<td>(0.011)</td>
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<td>(0.009)</td>
<td>(0.008)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Public</td>
<td>0.007</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.006)</td>
<td>(0.007)</td>
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<td>Mean of the Dependent Variable</td>
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<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>[0.29]</td>
<td>[0.29]</td>
<td>[0.29]</td>
</tr>
<tr>
<td>N</td>
<td>21099</td>
<td>29844</td>
<td>29844</td>
</tr>
<tr>
<td>Controls</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Robust standard errors clustered at group level in parentheses. The model includes cluster, surveyor, and date of survey fixed effects. The outcome variable is the per-
Table 2.10: Grant Expenditures

<table>
<thead>
<tr>
<th></th>
<th>(1) Rs. Added to Grant Amount</th>
<th>(2) Business Expenditures</th>
<th>(3) Household Expenditures</th>
<th>(4) Amt of Grant Saved</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Rank</td>
<td>351.586</td>
<td>683.875***</td>
<td>-391.044***</td>
<td>-292.831***</td>
</tr>
<tr>
<td></td>
<td>(265.463)</td>
<td>(127.386)</td>
<td>(97.376)</td>
<td>(101.430)</td>
</tr>
<tr>
<td>Outcome Variable Mean</td>
<td>847</td>
<td>4554</td>
<td>753</td>
<td>693</td>
</tr>
<tr>
<td>N</td>
<td>444</td>
<td>444</td>
<td>444</td>
<td>444</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the Group level in parentheses. All regressions include survey month and surveyor fixed effects. * p ≤ 0.10, ** p ≤ 0.05, *** p ≤ 0.10.

Table 2.11: Returns No Controls

<table>
<thead>
<tr>
<th></th>
<th>(1) Profits</th>
<th>(2) Trim Profits</th>
<th>(3) Log Profits</th>
<th>(4) Income</th>
<th>(5) Trim Income</th>
<th>(6) Log Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winner*Average Rank</td>
<td>351.410</td>
<td>366.373</td>
<td>0.319**</td>
<td>702.195*</td>
<td>676.852*</td>
<td>0.118</td>
</tr>
<tr>
<td></td>
<td>(297.358)</td>
<td>(295.644)</td>
<td>(0.148)</td>
<td>(389.402)</td>
<td>(388.813)</td>
<td>(0.097)</td>
</tr>
<tr>
<td>Winner</td>
<td>-1050.338</td>
<td>-882.108</td>
<td>-0.848*</td>
<td>-1794.383</td>
<td>-1719.862</td>
<td>-0.326</td>
</tr>
<tr>
<td></td>
<td>(830.810)</td>
<td>(817.938)</td>
<td>(0.483)</td>
<td>(1142.007)</td>
<td>(1141.339)</td>
<td>(0.295)</td>
</tr>
<tr>
<td>Outcome Variable Mean</td>
<td>4695</td>
<td>4599</td>
<td>7</td>
<td>8323</td>
<td>8316</td>
<td>9</td>
</tr>
<tr>
<td>N</td>
<td>5340</td>
<td>5314</td>
<td>5340</td>
<td>5339</td>
<td>5313</td>
<td>5340</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the Group level in parentheses. All regressions include survey month and surveyor fixed effects. * p ≤ 0.10, ** p ≤ 0.05, *** p ≤ 0.10.
### Table 2.12: Returns with Observable Controls

<table>
<thead>
<tr>
<th></th>
<th>(1) Profits</th>
<th>(2) Trim Profits</th>
<th>(3) Log Profits</th>
<th>(4) Income</th>
<th>(5) Trim Income</th>
<th>(6) Log Income</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Winner</strong> Average Rank</td>
<td>395.168</td>
<td>402.798</td>
<td>0.335**</td>
<td>721.168*</td>
<td>690.924*</td>
<td>0.111</td>
</tr>
<tr>
<td></td>
<td>(302.056)</td>
<td>(301.194)</td>
<td>(0.151)</td>
<td>(403.256)</td>
<td>(402.447)</td>
<td>(0.099)</td>
</tr>
<tr>
<td><strong>Winner</strong></td>
<td>-2826.914*</td>
<td>-2697.521</td>
<td>-1.900</td>
<td>-1556.390</td>
<td>-1536.941</td>
<td>-0.330</td>
</tr>
<tr>
<td></td>
<td>(1684.708)</td>
<td>(1687.015)</td>
<td>(1.337)</td>
<td>(2545.332)</td>
<td>(2548.325)</td>
<td>(0.520)</td>
</tr>
<tr>
<td><strong>Outcome Variable Mean</strong></td>
<td>4695</td>
<td>4599</td>
<td>7</td>
<td>8323</td>
<td>8316</td>
<td>9</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>5336</td>
<td>5310</td>
<td>5336</td>
<td>5335</td>
<td>5309</td>
<td>5336</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the Group level in parentheses. All regressions include survey month and surveyor fixed effects. *p < 0.10, **p < 0.05, ***p < 0.10.

### Table 2.13: Returns with Observable and Business Controls

<table>
<thead>
<tr>
<th></th>
<th>(1) Profits</th>
<th>(2) Trim Profits</th>
<th>(3) Log Profits</th>
<th>(4) Income</th>
<th>(5) Trim Income</th>
<th>(6) Log Income</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Winner</strong> Average Rank</td>
<td>862.316***</td>
<td>867.135***</td>
<td>0.467***</td>
<td>1047.927**</td>
<td>1018.101**</td>
<td>0.181*</td>
</tr>
<tr>
<td></td>
<td>(262.483)</td>
<td>(260.014)</td>
<td>(0.150)</td>
<td>(405.568)</td>
<td>(405.491)</td>
<td>(0.102)</td>
</tr>
<tr>
<td><strong>Winner</strong></td>
<td>-1384.817</td>
<td>-1242.767</td>
<td>-1.577</td>
<td>-506.894</td>
<td>-468.060</td>
<td>-0.146</td>
</tr>
<tr>
<td></td>
<td>(1343.045)</td>
<td>(1335.504)</td>
<td>(1.309)</td>
<td>(2413.272)</td>
<td>(2417.933)</td>
<td>(0.488)</td>
</tr>
<tr>
<td><strong>Outcome Variable Mean</strong></td>
<td>4695</td>
<td>4599</td>
<td>7</td>
<td>8323</td>
<td>8316</td>
<td>9</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>5336</td>
<td>5310</td>
<td>5336</td>
<td>5335</td>
<td>5309</td>
<td>5336</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the Group level in parentheses. All regressions include survey month and surveyor fixed effects. *p < 0.10, **p < 0.05, ***p < 0.10.
Details for Robust Bayesian Truth Serum

This discussion is based on Rigol and Roth (2016)

Theory and Intuition

In this appendix section we discuss the details of the Robust Bayesian Truth Serum, an intuition for the underlying incentive properties, and our implementation of the payment rule in the field. The following discussion of the model is based on Witkowski and Parkes (2012).

Suppose there is a binary state of the world \( t \in \{h, l\} \) (high, low) representing the entrepreneurial quality of a community member. Agents get a binary signal which is informative of the state of the world. That is each agent receives a signal \( s \in \{h, l\} \) which may represent what they observe about their peer (e.g. they appear responsible, smart etc). Suppose further that all agents share a common prior about the state of the world such that they all agree on the prior probability of a high state, and they all agree on the distribution of signals conditional on the state. Let \( P_h = P(s_j = h | s_i = h) \) be the probability an agent assigns to one of his peers receiving a high signal conditional on himself receiving a high signal, and analogously let \( P_l = P(s_j = h | s_i = l) \). We say the common prior is admissible if \( P_h > P_l \), which in English implies that the probability that one’s peer receives a high signal is higher if the agent himself receives a high signal. Many natural distributions satisfy this weak requirement.

In order to define the RBTS we must first define the quadratic scoring rule. Let

\[
R_q(y, \omega) = \begin{cases} 
2y - y^2 & \text{if } \omega = 1 \\
1 - y^2 & \text{if } \omega = 0 
\end{cases}
\]

Imagine an agent trying to predict whether some true state \( \omega \) is 1 or 0. The quadratic scoring rule has the property that his expected score is maximized by reporting his true belief about the probability the state \( \omega \) is 1 (see e.g. Selten, 1998).

The RBTS is implemented as follows. Every agent states their first order belief (their signal), in a report \( x_i \in \{0, 1\} \) (imagine \( x_i = 1 \) corresponding to \( s_i = h \)). Further they report their second order belief \( y_i \in [0, 1] \) (this is the fraction of the population they believe will report a high signal, \( x_k = 1 \)). For each agent \( i \), assign them a peer agent \( j \),
and a reference agent \( k \), and calculate

\[
y'_i = \begin{cases} 
y_j + \delta & \text{if } x_i = 1 \\
y_j - \delta & \text{if } x_i = 0 
\end{cases}
\]

for arbitrary \( \delta \). The RBTS payment for agent \( i \) is

\[
u_i = R_q (y'_i, x_k) + R_q (y_i, x_k)
\]

The main theorem of Witkowski and Parkes (2012) is that under the assumption of an admissible prior and risk neutral agents, there is a Bayes’ Nash Equilibrium in which all agents report their first and second order beliefs truthfully.

The intuition behind the payment rule is fairly straightforward. The payment rule has two components. The second component incentivizes the agent to be truthful about his second order beliefs. That is, the agent is paid via the quadratic scoring rule to predict what some reference agent \( k \) will announce as his signal. And by the discussion above, agent \( i \) maximizes his expected payment from this component of the scoring rule by truthfully announcing his belief \( y_i \) about the likelihood agent \( k \) will announce a high signal. In simpler terms, the payment rule rewards agent \( i \) for choosing a second order belief as close as possible to the truth (the realized distribution of first order beliefs).

The first component of the payment rule incentivizes the agent to be truthful about his first order beliefs. The term \( y'_i \) takes an arbitrary person \( j \)’s second order belief \( y_j \) and either raises or lowers it depending on \( i \)’s report \( x_i \). RBTS pays agent \( i \) \( R_q (y'_i, x_k) \), and so \( i \) wants \( y'_i \) to be as near as possible to the true distribution of responses in the population. The admissibility assumption guarantees that if person \( j \) were to know that person \( i \)’s signal were high, then person \( j \) would increase his assessment as to the number of people in the group who received high signals. Likewise, if \( j \) were to learn that \( i \)’s signal were low, \( j \) would lower his assessment about the number of people in the group who received high signals. In effect the mechanism raises or lowers \( j \)’s assessment based on \( i \)’s report, and then pays \( i \) based on the closeness of this modified report to the truth. Thus \( i \) can do no better than to tell the truth.
Practical Implementation

We used this payment rule in the field to incentivize rank order responses about members of each group. The model and payment rule, however, were designed for binary responses. Thus while responses contain a rank ordering of 5 people, we treat each ranking as a composite response to 25 yes/no questions of the form “Is person $i$ the highest ranking individual in the group?” “Is he the second highest?” and so on. We elicited second order beliefs of the form “How many people will say person $i$ is the highest ranking individual in the group?” “How many will say he is the second highest?” and so on. From there we directly applied the payment rule, calibrated so that the expected difference between payments arising from truthful and deceptive answers was large. Note that the accuracy of responses across various questions in a single ranking were correlated, but under the assumption of risk neutrality (which is maintained throughout the peer prediction literature and may be empirically reasonable with respect to moderate sums of money), these correlations are irrelevant.

Incentive Compatibility Exercise for RBTS

Throughout the experiment we told respondents that they would maximize their personal payoffs if they reported truthfully. While RBTS is truthful under certain reasonable assumptions about how beliefs are formed, its incentive compatibility under the empirical distribution of beliefs in practice remains an open question. We therefore evaluate whether respondents are maximizing their subjective expected utility by telling the truth.

Due to the coarseness of our elicited measures of belief, we cannot verify directly whether or not the respondents’ priors are admissible. However we can determine the distribution of payoffs respondents can expect to receive under alternative responses to see whether they succeeded in maximizing their subjective expected utility. Respondents’ payments depend on the distribution of first order beliefs (i.e. the empirical distribution of responses about the question of interest) and on the distribution of second order beliefs. Therefore, to determine whether truth telling is incentive compatible, we must understand what the respondent believes are the distributions of first and second order beliefs in the population. We obtain the former for free; respondents’ beliefs about the distribution of first order beliefs are their second order beliefs, and we elicited these in our survey. We did not, however, elicit their beliefs about the distribution of second order beliefs: their third order beliefs. We must therefore construct those. The intuition behind
the construction is presented in the following three steps:

a) We assume that respondents hold a common prior. If so, we can back out their third order beliefs from (a) the distribution of second order beliefs conditional on each first order belief and (b) their belief about how probable each first order belief is. The latter corresponds to her second order beliefs. 13

b) We approximate the distribution of second order beliefs in the population conditional on any given first order belief with the (sparse) empirical distributions we observe.

c) Given second and third order beliefs, we can calculate a respondent’s subjective expected utility from reporting the truth (her stated first order belief) and from any other report. 14 Specifically, we assume that the report the respondent has given is her true belief and calculate her payment. Holding constant her own second order belief and the first and second order beliefs of her peers, we then calculate her payments for every other possible report she could have given.

The results from this exercise are presented in Figure 2-2 below. Column 1 of the figure depicts the percentage of instances in which telling the truth gives the largest payment, column 2 depicts the percentage of instances that telling the truth results in the second largest payment, etc. Taking the graph at face value, telling the truth maximizes the respondent’s subjective expected utility in about 35% of instances and it minimizes her subjective expected utility in only about 10% of instances. An ideal graph would place all of its weight in the first column.

13 If agents have common priors then conditional on the signal they receive, they would update to have the same posterior belief. We stress here that we elicited ranks and not signals. Therefore two agents who report the same rank do not necessarily have the same posterior as the rank is a coarse measure of a signal.

14 Notice that we only utilize incentivized data since it is only for these data that we collected second order beliefs.
The observed departure from this ideal may be due to the failure of our assumptions required by RBTS holding in practice, or by our noisy approximation of third order beliefs. To evaluate this, we perform a simulation in which we generate data that perfectly abides by all of the assumptions required for the incentive compatibility of RBTS. We generate groups of artificial agents, each of whom holds a common prior and receives a signal about the skill level of their peers. Agents update their priors based on these signals and these form the basis of their second and third order beliefs, each of which we can compute.

Because the data is generated to be perfectly consistent with the assumptions of RBTS, the agent always maximizes his expected utility by telling the truth. Next we compress our simulation data to correspond exactly to the data we collected from our respondents: just first and second order beliefs about group rank. This allows us to have two data sets (collected and simulated) that contain identical level of detail. We then generate the same graph as we did for our collected data and present it in Figure 2-3.
The graph produced with the collected and with the simulated data are strikingly similar. We therefore conclude that our noisy approximation of third order beliefs could be to blame for the observed weights in columns 2 through 5, and argue that our test yields the strongest evidence in favor of the incentive compatibility that could be achieved via this method. Therefore, telling respondents that they will maximize their expected payments by reporting truthfully may indeed be good advice.
Entrepreneurial Psychology

Impulsiveness:

- I plan tasks carefully.
- I make up my mind quickly
- I save regularly.

Optimism:

- In uncertain times I usually expect the best.
- If something can go wrong for me, it will.
- I’m always optimistic about my future.
- Generally speaking, most people in this community are honest and can be trusted

Locus of Control

- A person can get rich by taking risks.
- I only try things that I am sure of.

Tenacity

- I can think of many times when I persisted with work when others quit
- I continue to work on hard projects even when others oppose me.

Polychronicity:

- I like to juggle several activities at the same time
- I would rather complete an entire project every day than complete parts of several projects.
- I believe it is best to complete one task before beginning another.

Achievement

- Part of my enjoyment in doing things is improving my past performance
- If given the chance, I would make a good leader of people.
Organized person:

- My family and friends would say I am a very organized person
3.1 Introduction

A defining feature of centralized markets across many contexts is that they are voluntary. That is, centralized markets operate in a broader economic environment, and often market participants cannot be forced to use a centralized market rather than the decentralized institutions it was meant to displace. Thus well-designed centralized markets need not only ensure desirable outcomes conditional on garnering sufficient participation, but they must also guarantee the safety of participation relative to the available decentralized options. In this paper we propose one such guarantee and study markets for which it is satisfied.

Section 3.2 outlines our formal approach to the study of centralized market design. We analyze a general model of mechanism design in the presence of a pre-existing game. The pre-existing game specifies the players, feasible actions, and payoffs. A designer may introduce a mechanism (alternatively referred to as a centralized market), to which players may sign away their decision rights (i.e. participate in the centralized market). Players who do so select a message to send to the mechanism, which then acts on their behalf in a pre-specified manner as a function of the whole set of messages it receives. The mechanism is voluntary in the sense that players may act decentrally by choosing one of their original actions, and the mechanism cannot condition the actions of centralized participants on those of the players who act decentrally.
This may be a sensible approach for studying many market design applications. For instance, hospitals and residents who use the centralized clearinghouse known as the National Residency Matching Program are committed to follow through with their assigned match. However there is no legal barrier preventing members of either side of the market from declining to use the mechanism and finding a match outside of the clearinghouse. This was becoming increasingly common in the early 1990s before the clearinghouse was redesigned to better accommodate couples seeking two jobs.

Outside of market design many other centralized markets fall within this framework. Centralized financial markets such as the New York Stock Exchange elicit supply and demand curves from traders and have the authority to implement complex market clearing trades on their behalf. However traders have and frequently exercise the ability to execute trades outside of the centralized market, perhaps to the detriment of other users. Crowd funding platforms like Kickstarter and online marketplaces like Groupon offer to centralize transactions (financing new projects and purchasing goods and services) such that users can offer to participate conditional on sufficient participation by others. But users retain the option to transact outside of these centralized platforms.

Within this framework we study a variety of desiderata which may ensure that players will use the centralized mechanism. In Section 3.3 we identify a desideratum we term \( \varepsilon \)-dominant individual rationality (\( \varepsilon \)-dominant IR). A mechanism is \( \varepsilon \)-dominant individually rational if, given any decentralized strategy any player may consider, there is an alternative centralized strategy that achieves at most \( \varepsilon \) less utility relative to the decentralized strategy no matter what strategy profile other players follow, and performs strictly better for strategy profiles in which sufficiently many players participate in the centralized market. Notably, this definition does not specify that there is a centralized strategy that an agent can follow that performs almost as well as all decentralized strategies regardless of the strategies others follow. In fact, we show that this latter criterion may be unachievable in many settings of interest. While \( \varepsilon \)-dominant individual rationality leaves open the possibility that for any given centralized strategy a player may follow, she may ex-post regret not having followed another decentralized strategy, it still offers a compelling justification for players to use the centralized market. For any decentralized strategy a player is considering following \textit{ex-ante}, a centralized action exists that \( \varepsilon \)-dominates it. And for \( \varepsilon = 0 \), the definition implies that each decentralized strategy is dominated by a
centralized one.

We establish a number of positive results about $\epsilon$-dominant individual rationality. Namely, we show it can be achieved in two applications of interest, and that the pursuit of such mechanisms illuminates several desirable design features.

In Section 3.4 we study a large two-sided matching market and show that with simple modification, any stable matching mechanism can be $\epsilon$-dominant IR (with $\epsilon$ becoming arbitrarily small for increasingly large markets). The result arises from the fact that in matching markets with small cores, if proposer $i$ could achieve a match with receiver $j$ by proposing decentrally, he could also achieve a match with receiver $j$ with high probability by using the centralized market and listing $j$ first. Additionally the centralized procedure protects proposer $i$ from wasting offers on those who would reject them, offering a clear benefit relative to the decentralized market. Thus, so long as all receivers participate in the centralized market, every decentralized strategy a proposer might consider is $\epsilon$-dominated by a corresponding centralized strategy. This guarantee arises from the combination of the facts that the market has a small core and that the centralized mechanism finds a stable match. So in addition to offering an insight that guides the design of stable matching markets, the analysis constitutes a non-cooperative justification of stability, which is traditionally thought of as a cooperative solution concept. Methodologically, the analysis relies heavily on techniques closely related to those of Immorlica and Mahdian (2005) and Kojima and Pathak (2009).

In Section 3.5 we show that in arbitrary large games, any mechanism with the property that every player wants to use it conditional on sufficiently many other players using it, can be modified to be $\epsilon$-dominant IR while preserving the behavior of the mechanism conditional on a sufficiently large fraction of players participating (again, such that $\epsilon$ can become arbitrarily small for increasingly large markets). The intuition for the result relies on a threshold property of the modified mechanism, such that the mechanism elicits a default strategy from every participant and plays the default strategy unless a sufficiently large number of players choose to participate. Conditional on sufficiently many players participating, the modified mechanism behaves exactly as the original mechanism would have. However, this modification on its own may not make the mechanism $\epsilon$-dominant IR for small $\epsilon$ because players may anticipate strategy profiles in which their decision to enter the centralized market is pivotal in determining whether other players use their default action or the action specified by the mechanism. We solve this using a technique inspired by the differential privacy literature; we add a random perturbation to
the threshold, so that given any strategy profile the probability that any agent is pivotal is vanishingly small in large markets.

Important to note in both applications, while the $\epsilon$ regret allowed by $\epsilon$-dominant IR can be made arbitrarily small in large markets, the benefit each agent receives from using the mechanism in any strategy profile with sufficiently many others also participating does not vanish with the size of the market. Thus, in the large market limit, every decentralized strategy is weakly dominated by a centralized strategy offered by the mechanism.

Our paper fits into several literatures. Our model closely resembles that of Kearns, Pai, Rogers, Roth, and Ullman (2015), Myerson (1991), Kalai, Kalai, Lehrer, and Samet (2010), Forges (2013) and especially Monderer and Tennenholtz (2009) and Ashlagi, Monderer, and Tennenholtz (2007). These papers study the set of equilibria implementable via some mechanism in a one shot game and by and large obtain permissive answers. All of these ask questions about the existence of an equilibrium outcome, while we focus on equilibrium uniqueness and related criteria.

There has been a recent interest in applied models in which players can choose not to participate in particular mechanisms. Sönmez (1999) and Kesten (2012), among others, study a form of manipulation in school choice problems in which schools and students prearrange their matches prior to use of a centralized mechanism. Ekmekci and Yenmez (2014) study a complete information model of school choice in which schools may opt out of the central matching mechanism. They show that the Deferred Acceptance mechanism has no equilibrium in undominated strategies that attracts all schools to participate and propose modifications. We reach different results using large market models, with incomplete information about preferences, where the concerns they focus on are minimal. In contrast to these papers we study a general framework. And rather than asking whether there exists an equilibrium in which market participants use the centralized market, we ask when we can design markets that guarantee participation.

### 3.2 The Model

**The Underlying Game:** Let $\Gamma := (N, \{A_i, \Theta_i, u_i\}_{i \in N}, q)$ be a normal form game of (potentially) incomplete information, where $N = \{1, \ldots, N\}$ is the finite set of players, and for every $i \in N$, $\Theta_i$ is a set of types, $A_i$ is a finite set of actions, and $u_i : \Theta \times A \to \mathbb{R}$ is a Bernoulli utility function, where $\Theta = \prod_{i \in N} \Theta_i$ and $A = \prod_{i \in N} A_i$. Last, $q$ is a probability distri-
bution over $\Theta$ from which each player derives their conditional beliefs using Bayes law. Mixed strategy action profiles are defined in the usual way and we assume that agents are expected utility maximizers. We write $X_S = \prod_{i \in S} X_i$, and for a vector $x$ we write $x_i$ to refer to its $i$'th component, $x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N)$ and $x_S = \prod_{i \in S} x_i$.

**Mechanisms:** A mechanism is a tuple $M = (R, (\mu_S)_{S \subseteq N})$, where $R = \prod_{i \in N} R_i$ is a finite message space and for every subset $S \subseteq N$, $\mu_S$ is a mapping from messages in $R_S$ to distributions over action profiles in $A_S$.

The key novelty is that players are not required (but have the option) to sign away their decision rights to the mechanism, which is akin to participating in the mechanism in the standard analysis. Those that sign away their decision rights specify some message $r_i \in R_i$. For each possible set $S$ of players that sign away their decision rights to the mechanism, $\mu_S : R_S \rightarrow \Delta(A_S)$ is a mapping from instructions that the players in $S$ send the mechanism to a mixed strategy profile on behalf of those players. Note that, because using the mechanism is voluntary, a different outcome function $\mu_S$ must be specified for each possible coalition $S$ that might sign away their decision rights.

A note about the functionality of mechanisms is in order. A mechanism is a form of coalitional commitment and coordination. Players who sign away their decision rights to the mechanism are committed to following through with the mechanism’s recommendation, and the mechanism can be used to coordinate players actions when the state is incompletely known by some players. Finally, our notation implies that mechanisms can implement correlated distributions over action profiles.

**The Induced Game:** A mechanism $M$ applied to $\Gamma$ induces a simultaneous move game $\Gamma^M := \langle N, \{A_i \cup R_i, \Theta, \bar{u}_i\}_{i \in N}, q \rangle$. Here, $E \left[ \bar{u}_i(r_S, a_{N \setminus S}, \theta) \mid \theta_i \right] = E \left[ u_i(\mu_S(r_S), a_{N \setminus S}, \theta) \mid \theta_i \right]$ and all other objects are defined in the natural way.

In words the induced game is as follows. Players simultaneously choose whether or not to relinquish their decision rights to the mechanism. A player $i$ who signs away his or her decision rights also sends an instruction $r_i$ from the set $R_i$. Instructions are mapped in a pre-specified manner into actions on behalf of the participating players as determined by the relevant outcome function $\mu_S$, corresponding to the coalition $S$ that signed away their decision rights. All players in $N \setminus S$ who opt to keep their decision rights choose

---

4The solution concept we use in the Section 3.3 is trembling hand equilibrium which is defined over finite action spaces. The assumption of finite action and message spaces can be relaxed with a suitable generalization of trembling hand equilibrium to games with infinitely many actions (see Simon and Stinchcombe (1995)).
their own actions. Then outcomes are realized. Note that the mechanism can condition the actions of participating players on the messages of other participating players, but cannot condition the actions of participating players on the actions of those who choose to retain their decision rights and act on their own behalf.

3.3 \( \epsilon \)-Dominant Individual Rationality

We next offer a new desideratum for mechanisms in this framework. Let \( \Sigma \) be a subset of all strategies \( S := \Pi_{i \in N} (\sigma_i : \Theta_i \to \Delta (A_i \cup R_i)) \).

**Definition 3.1.** We say a mechanism is \( \epsilon \)-dominant individually rational with respect to \( \Sigma \subseteq S \) if for all \( i \in N \), for all \( \theta_i \in \Theta_i \), and for all \( a_i \in A_i \), there is a centralized strategy \( r_{ai} \) such that:

a) (\( \epsilon \)-Regret): \( E[u(r_{ai}, \sigma_{-i}, \theta_i)] > E[u(a_i, \sigma_{-i}, \theta_i)] - \epsilon \) for all \( \sigma_{-i} \in \Sigma_{-i} \)

b) (\( k \)-Attraction): There exists a \( k < 1 \) and \( \Delta > 0 \) such that \( E[u(r_{ai}, \sigma_{-i}, \theta_i)] > E[u(a_i, \sigma_{-i}, \theta_i)] + \Delta \) when \( \sigma_{-i} \in \Sigma_{-i} \) and \( \frac{\# \{j \neq i \in N: \sigma_j(\theta_j) \in \Delta (R_i) \forall \theta_j \in \Theta_j \}}{n} > k \).

A mechanism is \( \epsilon \)-dominant individually rational with respect to \( \Sigma \) if, for every player \( i \) and every decentralized action \( a_i \), there is a centralized action \( r_{ai} \) such that as long as all other players choose strategies in \( \Sigma \), \( r_{ai} \) guarantees player \( i \) an expected payoff at least within \( \epsilon \) of \( a_i \). Furthermore, if sufficiently many players use centralized actions with certainty, then \( r_{ai} \) guarantees \( i \) a strictly better expected payoff than \( a_i \).

A few remarks are in order. We define a mechanism to be \( \epsilon \)-dominant IR with respect to a particular subset of strategies \( \Sigma \) because in some applications, agents need not worry about strategies that are deemed infeasible. In our two-sided matching application below, for example, we will restrict attention to anonymous and undominated strategies, which may be sensible in a setting where proposers cannot coordinate their offers based on the names of receivers.

It is also important to note that we define \( \epsilon \)-dominant IR with respect to strategies that other players may follow, rather than the actions they take. \( \epsilon \)-dominant IR allows for the possibility that a player \( i \) will regret participating in the centralized market after discovering the types of players he is interacting with and the actions they follow, but rules out the possibility of (more than \( \epsilon \)) regret before discovering these realizations but

\[ \epsilon \text{-regret} = \sum_{i} \epsilon \text{-regret}_i \]

\[ \text{for } i \in N, \theta_i \in \Theta_i, \text{ and } a_i \in A_i \]

\[ \text{and } \sigma_{-i} \in \Sigma_{-i} \]

\[ \text{with } \epsilon \text{-regret}_i \]
after discovering the strategies of his opponents. This form of *ex-post* dominance is closely related to the insights in Azevedo and Budish (2015).

The guarantee of $\epsilon$-dominant IR is that for every decentralized action $a_i$ in $A_i$, player $i$ has a centralized action $r_{a_i}$ that performs at most $\epsilon$ worse regardless of what strategies other players follow. Importantly, however, it does not guarantee that there is a single centralized strategy $r_i$ that performs at least within $\epsilon$ of the payoff of every decentralized strategy regardless of what others do. We formalize this stronger notion in the next definition.

**Definition 3.2.** We say a mechanism is $\epsilon$-strongly dominant individually rational with respect to $\Sigma \subseteq S$ if for all $i \in N$, for all $\theta_i \in \Theta_i$, there is a centralized strategy $r_i$ such that

a) $\mathbb{E}u(r_i, \sigma_{-i}, \theta_i) > \mathbb{E}u(a_i, \sigma_{-i}, \theta_i) - \epsilon$ for all $\sigma_{-i} \in \Sigma_{-i}$, and for all $a_i \in A_i$.

b) There exists a $k < 1$ and $\Delta > 0$ such that $\mathbb{E}u(r_i, \sigma_{-i}, \theta_i) > \mathbb{E}u(a_i, \sigma_{-i}, \theta_i) + \Delta$ for all $a_i \in A_i$ when $\sigma_{-i} \in \Sigma_{-i}$ and $\frac{\#\{j \in N: \sigma_j(\theta_j) \in \Delta(R_j) \forall \theta_j \in \Theta_j\}}{n} > k$.

Note any mechanism which is $\epsilon$-strongly dominant IR is also $\epsilon$-dominant IR. An $\epsilon$-strongly dominant individually rational mechanism further assures participants that by choosing their centralized strategy $r_i$, they won’t regret not having chosen any decentralized strategy, regardless of the strategies other players follow. In contrast, a mechanism that is merely $\epsilon$-dominant individually rational allows for the possibility that after having chosen a centralized strategy and observing the strategies that other players are following, player $i$ may regret not having chosen another decentralized strategy. The guarantee made by an $\epsilon$-dominant IR mechanism is that a player $i$ considering any particular decentralized strategy $a_i$ can safely switch to the centralized strategy $r_{a_i}$ without experiencing more than $\epsilon$ regret. While the guarantee made by an $\epsilon$-strongly dominant IR mechanism is more appealing, in Section 3.5 we show that in many situations it is unattainable for sufficiently small $\epsilon$, regardless of the size of the market.

### 3.4 An Application to Two Sided Matching Markets

In this section we discuss an application to large two sided matching markets. We aim to capture several insights that may guide matching market design. First, standard implementations of centralized markets may be prone to adverse selection issues. Second, these concerns can be alleviated using simple techniques already observed in practice, which make the mechanism $\epsilon$-dominant individually rational. Lastly, while it is natural
to attribute the success of stable matching markets to the fact that they implement stable matches, we will argue that they have another desirable feature that may contribute to their success. Stability is paramount in assuring $e$-dominant individual rationality; it guarantees that any potential match a proposer could acquire decentrally will very likely also be attainable through the centralized match.

### 3.4.1 A Decentralized Matching Market

We begin with a model of a decentralized, one to one matching market adapted from the large market model of Immorlica and Mahdian (2005) and Kojima and Pathak (2009) to a non-cooperative environment.\(^6\)

**Players and Actions:** We let the set of players $N = F \cup W$ be divided into disjoint sets of $n$ firms (proposers) and $n$ workers (receivers).\(^7\) A firm’s action corresponds to making an offer to a worker. With slight abuse of notation we write $A_f = W \cup \{f\}$, where the choice of $a_f = f$ corresponds to not making any worker an offer. A worker’s action corresponds to an acceptance policy, which is a function from the set of any subset of firms who may have chosen him to a selection among them. That is $A_w = \{G_w : 2^F \to F \cup \{w\}\}$.

A matching of a subset of workers and firms $v : \tilde{W} \cup \tilde{F} \to \tilde{W} \cup \tilde{F}$ is a function from a subset of workers $\tilde{W} \subseteq W$ and firms $\tilde{F} \subseteq F$ to itself such that (i) $v(w) = w$ if $v(w) \not\in \tilde{F}$ for all workers $w$, (ii) $v(f) = f$ if $v(f) \not\in \tilde{W}$ for all firms $f$ and (iii) $v(w) = f$ if and only if $v(f) = w$. A strategy profile results in a matching $v$ where a worker $w$ and firm $f$ are matched if $a_f = w$ and $a_w(F') = f$ where $F' \equiv \{f' : a_{f'} = w\}$ is the set of firms who selected worker $w$.

**Preferences and Information Structure:** Each worker is given a popularity weight $p_w > 0$ about which he or she is privately informed. Each firm has a random preference list of size $K < n$ drawn in the following manner.

- Step 1: Draw a worker $w$ at random from all workers with weights $p_w$. This is $f$’s first choice worker.
- Step k: If firm $f$ has $K$ workers on his list, terminate the algorithm. Else, draw a

\(^6\)Our analysis relies heavily on there being a small core, but much less on other details of the environment. As such it seems likely that the intuitions developed in this section would extend to other matching models with small cores such as e.g. Ashlagi, Kanoria, and Leshno (2016).

\(^7\)The balanced number of agents on each side is of no material importance, and is done only to economize on notation c.f. Ashlagi et al. (2016) and Coles, Gonczarowski, and Shorrer (2016).
worker \( w \) at random from all workers with weights \( p_w \). If \( w \) has been selected in a previous step, discard him and repeat this step. If \( w \) has never been selected in a previous step, this is \( f \)'s \( k \)'th choice.

Workers are endowed with a preference list in a similar fashion, with the only difference being that they have full preference lists (of length \( n \)).

Cardinal preferences for all firms (workers) are given by a function from the rank of their partner to \((0,1)\), \( u_f : \{1, \ldots, k\} \rightarrow (0,1) \) \( u_w : \{1, \ldots, n\} \rightarrow (0,1) \). For all firms (workers) we assume \( u_f(w) = \frac{(n-\text{rank of } w)}{n} \) \( u_w(f) = \frac{(n-\text{rank of } f)}{n} \), and normalize the utility of being unmatched, and the utility of being matched to someone who was not ranked, to 0.

As stated above, we assume that \( p_i \) is agent \( i \)'s private information. However we assume that the distributions \( D_W \) and \( D_F \) from which \( p_w \) and \( p_f \) are drawn are common knowledge. We assume \( D_W \) and \( D_F \) are symmetric, such that given any permutation \( \pi(\cdot) \) of firms or workers, \( D \circ \pi(\cdot) = D(\cdot) \). Thus players understand that their preferences may be correlated, but do not know which firms and workers are popular before preferences are drawn.

Finally, similar to Kojima and Pathak (2009), we make the following assumption on the support of \( D_W \) and \( D_F \).

**Assumption 3.1 (Market Thickness).** There is some constant \( C < \infty \) such that for every realization of \( D_W \) \( D_F \) and every worker \( w \) and \( w' \) (firm \( f \) and \( f' \)), \( \frac{p_w}{p_{w'}} < C \left( \frac{p_f}{p_{f'}} < C \right) \).

The above assumption requires that the most popular worker (firm) cannot be arbitrarily more popular than the least popular worker (firm). We make this assumption so that, as Kojima and Pathak show, not only do most workers and firms have a unique stable match, but that without knowledge of others' preferences, all firms and workers have an arbitrarily high probability of having a unique stable match as the market becomes large.

Lastly, we say a matching \( \nu \) is stable if (i) every worker \( w \) prefers \( \nu(w) \) to being unmatched and similarly for firms, and (ii) there is no pair \( (w, f) \) such that \( w \) prefers \( f \) to \( \nu(w) \) and \( f \) prefers \( w \) to \( \nu(f) \). If there is such a pair, we say that \( (w, f) \) blocks the matching \( \nu \).

**Strategies:** In the analysis that follows we restrict attention to anonymous, undominated strategies. Anonymity implies that agents only consider strategies that condition their offer or acceptance decisions on their own preferences. We maintain anonymity so
that there are potential gains from coordinating the market through a mechanism, and to
ensure that in all strategy profiles each agent is arbitrarily likely to have a unique stable
match given stated preferences as the market becomes large.

3.4.2 Stable Matching Mechanisms

We now proceed to describe a *Stable Matching Mechanism* - a mechanism that captures
the main features of many centralized matching markets - and the game it induces. The
message space $R_i$ for a worker $w$ or a firm $f$ is the set of all strict preference orderings
over the other side of the market (of size $K$ for firms and $n$ for workers).\footnote{This can be relaxed to requiring that firms list a constant fraction $\lambda$ of all workers and workers list any number of firms up to $K$.} For any set of
delegators $S = S_W \cup S_F \subseteq W \cup F$, $\mu_S(r_{S_W}, r_{S_F})$ determines a stable matching among the
participants in $S$, $v_S$, makes offers on behalf of each firm to the worker assigned to them,
and accepts these offers on behalf of the corresponding firms.\footnote{That is the mechanism selects a strategy $G_o : 2^F \to F \cup w$ for each participating worker $w$ that accepts his stable match $f$ whenever it is among the set of offers he receives.} Thus any firm can make
a decentralized offer or it can opt to enter the centralized market and participate in the
stable matching mechanism. Similarly any worker can stay out of the market and accept
any offer he receives, or he can enter the market and commit to accepting a centralized
offer he receives.

In this setting, the mechanism has the potential to solve the coordination problems
present in the decentralized market. Firms who use the mechanism are guaranteed to
make non overlapping offers, and so every firm stands to benefit from coordination. For-

mally, we have the following observation:

**Observation 3.1.** In a market of any size $n > 2$, and for any stable matching mechanism $M$,
there exists a $k < 1$ such that $M$ satisfies $k$ - attraction.

As such one may hope that for sufficiently large markets, all stable matching mecha-
nisms are $\epsilon$ - dominant individually rational with respect to anonymous, undominated
strategies for arbitrarily small $\epsilon$. Our first result is that this is not the case.

**Proposition 3.1.** There exists an $\epsilon > 0$ such that for any matching market, no stable match-
ing mechanism is $\epsilon$ - dominant individually rational with respect to anonymous, undominated
strategies.

The proof is relegated to the appendix. While many strategy profiles may be used to
demonstrate this failure of $\epsilon$ - dominant IR, we focus on two instructive strategy profiles.
in particular. One resembles an adverse selection problem. Popular firms and workers fear that only unpopular counterparts will enter the centralized market. Thus by entering the centralized market themselves, they are doomed to be matched to a low ranking counterpart with high probability. In contrast, had they stayed out of the centralized market they would have had a high probability to be matched with a counterpart they prefer. The second strategy profile an agent may worry about highlights that concerns about market thickness may be self-fulfilling. An agent conjecturing he would be alone in the centralized market would do strictly better to operate in the decentralized market, ensuring that there is a strict equilibrium in which no one participates in the centralized market.

That centralized markets can suffer from adverse selection and lack of thickness is unsurprising. What may be surprising is that a simple adjustment to the centralizing mechanism can eliminate these concerns and guarantee participation.

3.4.3 The Modified Stable Matching Mechanism

This section highlights two simple ideas. First, by allowing workers who join the centralized market to accept offers from either part of the market, rather than forcing them to accept offers from firms who also joined the centralized market, entering becomes a dominant strategy for the workers. Second, conditional on all workers using the centralized market and the fact that the mechanism implements a stable outcome given stated preferences, all firms may be enticed to use the centralized market as well. This is due to the fact that in a large market where most participants have at most one stable partner, the mechanism guarantees that with high probability they can be matched centrally to any worker they could have achieved decentrally by entering the centralized market and listing that worker first. A somewhat subtle logic underlies this second observation and understanding it is the goal of this section.

We now introduce the modified stable matching mechanism. The message space $R_j$ is the same as above; workers and firms who sign away their decision rights to the mechanism communicate their preference list. The mechanism calculates a stable match among the players who signed away their decision rights and makes offers on behalf of firms to their assigned stable match. However, the mechanism does not force workers to accept their assigned stable partner. Instead, workers are allowed to accept their most preferred offer
regardless of whether the firm who made that offer was part of the centralized match.\(^\text{10}\)

We are now ready to state our main result.

**Theorem 3.1.** For every \(\varepsilon > 0\) there exists an \(\bar{n}\) such that for any matching market of size \(n > \bar{n}\), the modified stable matching mechanism is \(\varepsilon\)-dominant IR with respect to anonymous, undominated strategies.

**Proof.** See appendix. \(\Box\)

The logic behind this result follows a cost-benefit analysis. As we argued above, allowing workers to accept their best offer from any firm makes it a dominant strategy for them to enter; workers who enter don’t change the behavior of firms in the decentralized market and make themselves eligible to receive offers from firms in the centralized market. Hence for workers, the benefit to entering is the possibility of garnering an additional offer and there is no cost.

For firms, we perform the cost benefit analysis on a particular pair of strategies. We compare \(f\)'s payoff from the decentralized strategy of making an offer to worker \(w\), with its payoff from the centralized strategy of signing away its decision rights and submitting its true preference ordering but with \(w\) is listed first. The benefit of the centralized strategy is clear. If \(w\) would reject \(f\)'s decentralized offer, there is some chance that it is because \(w\) received a more preferred centralized offer, in which case the mechanism will have \(f\) make an offer to someone else instead. Thus, the mechanism satisfies \(k\)-attraction for firms.

Relative to its decentralized strategy, the cost of \(f\)'s centralized strategy is that \(w\) may have accepted it had it proposed decentrally, but the mechanism will match \(f\) with someone it disregards with \(w\) (or no one at all). A first sign that this cost is small comes directly from the definition of stability. For an arbitrary strategy profile, let \(\bar{F}\) be the set of firms other than \(f\) who sign away their decision rights to the mechanism. If \(w\) would have accepted \(f\)'s decentralized offer, it means that he prefers \(f\) to whatever centralized match \(\bar{f} \in \bar{F}\) he received. Thus \(w\) and \(f\) block any tentative matching that pairs \(w\) with \(\bar{f}\) and \(f\) with someone it likes less than \(w\). The primary wrinkle in this line of logic is that by joining the centralized match, \(f\) may displace another firm that would trigger a chain of displacements that would finally cause \(w\) to be matched to some new firm \(\bar{f}\) that he prefers to both \(\bar{f}\) and \(f\).

\(^{10}\)That is, the mechanism chooses a strategy \(G : 2^F \to F\) on behalf of each worker \(w\) such that the firm \(f\) selected is the one that worker \(w\) said he most preferred from the subset that made him offers.
The bulk of the proof is devoted to showing that cost imposed on $f$ of this wrinkle is small. The proof relies crucially on the small core result of Immorlica and Mahdian (2005) that as $n$ becomes large most firms and workers have at most one stable partner. However, our Theorem 3.1 does not follow immediately from their result. From a result similar to Immorlica and Mahdian's we know that in the centralized market that includes $W \cup \tilde{F}$, worker $w$ has at most one stable partner $\tilde{f}$ with high probability. We also know that in the market that includes $W \cup \tilde{F} \cup f$, $w$ has at most one stable partner with high probability. Their results do not directly imply, however, that $w$ is matched with $f$ with high probability in the latter market.\(^{11}\) We show this to be the case, with the following argument.

a) First we rely on the small core result similar to that of Immorlica and Mahdian (2005) that an arbitrary worker $\tilde{w}$ has at most one stable partner with high probability in the market that includes $W \cup \tilde{F}$.\(^{12}\)

b) Next we invoke a result of Demange, Gale, and Sotomayor (1987) that workers who have at most one stable match cannot manipulate the firm optimal stable matching mechanism.

c) We now consider the firm optimal stable match in the centralized market that includes $W \cup \tilde{F} \cup f$, which can be implemented via the firm proposing deferred acceptance algorithm. The output of the algorithm is insensitive to the order in which proposals are made, so we run the algorithm such that all firms in $\tilde{F}$ continue to make offers until all of them are either matched or have exhausted their list of acceptable matches before $f$ makes any offers. At this point, the tentative matching is the firm optimal stable matching in the market that includes only $W \cup \tilde{F}$. Next $f$ proposes to workers in order of its preference list until one of them prefers it to his tentative match. By assumption, worker $w$, the first worker who firm $f$ proposes to, will hold its offer.

d) If upon further proposals in the algorithm $w$ receives an offer from some firm $\tilde{f}$ whom he prefers to $f$, we argue that he also could have profitably manipulated the firm optimal stable matching mechanism in the market that included only $W \cup \tilde{F}$ - by truncating his preference list right above his previous tentative match. By step \(^{11}\)We further remark that their results do not apply to our setting as ours is a setting of strategic behavior, and as such we need to verify that the set of stable matchings is small with high probability given reported preferences arising from any anonymous and undominated strategy profile.
\(^{12}\)Note this is not implied by their results because the set of firms participating in the market may be small and thus existing large market results do not immediately apply.
2, this means that he had more than one stable partner in the market that includes only $W \cup \bar{F}$, which we know occurs with arbitrarily small probability.

e) Thus, with arbitrarily high probability, $F$ will be matched with $w$ in the firm optimal stable matching in the market that includes $W \cup \bar{F} \cup f$. We now reapply the small core result from step 1 to this extended market to show that with high probability $w$ and $f$ are paired in all stable matchings.

The intuition seems not to rely on specific details of the market. By allowing workers to accept offers from anyone regardless of their participation, it becomes a dominant strategy for them to enter. And in markets with small cores, stable matching mechanisms make it safe for firms to enter the market as well because, with high probability, entry won’t cause a firm to lose anyone she may have attained via a decentralized offer.

In the appendix we show that this intuition does not hold vacuously. In other commonly used but unstable mechanisms such as the Boston Mechanism or the Rank Sum Mechanism, firms may be matched to partners they disprefer to those they could have attained decentrally. Intuitively, these two mechanisms privilege workers who list the firm highly regardless of the firm’s preferences.

One last note is in order about the policy suggestion of allowing workers to accept offers from anyone in the market regardless of participation in the centralized mechanism. This almost exactly mirrors a policy suggestion made by Niederle, Proctor, and Roth (2006) when considering how to centralize the job market for new gastroenterologists. In particular they proposed doctors should not be bound to offers they receive from any hospital weakly prior to the date of the centralized match, regardless of the form in which the offer was delivered. They understood that this would make it a dominant strategy for doctors to use the centralized mechanism. The analysis above illuminates the role of stable matching in encouraging the hospitals to follow suit.

### 3.5 Threshold Mechanisms in Large Market Games

In this section we argue that in large games, if a mechanism satisfies $k$-attraction, then an intuitive modification allows it to satisfy $\epsilon$-dominant individual rationality, while preserving the mechanism’s behavior in the event that sufficiently many players use it. That is, given a mechanism for which it is in every player’s best interest to use it conditional on sufficiently many others using it as well, we present a modification that is $\epsilon$-dominant
IR, where $\epsilon$ goes to zero as the market becomes large. We also show that the same cannot be said for $\epsilon$-strong dominant individual rationality.

The idea highlighted in this section relies on what we term threshold mechanisms. A threshold mechanism is one that acts on players' behalves if and only if sufficiently many sign away their decision rights. Else it implements pre-specified default actions on each player's behalf. Mechanisms like this arise in many natural environments. At its inception, Groupon allowed sellers to offer discounts, but to condition them on sufficient demand. Similarly, Kickstarter allows users to volunteer financing for entrepreneurs if sufficiently many others do so, and otherwise returns their money as well. For an example in the market design literature, the NRMP has a clause that the centralized match will be cancelled in the event that fewer than 70% of all applicants use it. We will show that, with minor modification, this class of mechanisms is quite powerful.

Participation in a threshold mechanism entails a potential tradeoff. On the one hand, entry allows agents to condition their behavior on the behavior of others. On the other hand, agents may fear that by entering themselves, they are pivotal in triggering other participants thresholds and are thus affecting the behavior of the other participants in undesirable ways. We show that a modified threshold mechanism, which adds a small amount of noise to the threshold, eliminates this tradeoff. By participating in the mechanism agents can condition their actions on their peers' actions while the worry that they are pivotal vanishes.\(^\text{13}\)

Formally, we are interested in arbitrary games with utility functions bounded between 0 and 1. In addition to a standard report, a threshold mechanism $M = (R, \{\mu_S\}_{S \subseteq N})$ elicits a default action $a_i$. That is, $R_i = \hat{R}_i \times A_i$. Let $S$ be the set of players who sign away their decision rights, and for each $i \in S$ let $r_i = (\hat{r}_i, a_i)$. Let $x = |S|$ be the number of players who sign away their decision rights. Then

$$
\mu_S(r_S)_i = \begin{cases} 
\hat{\mu}_S(r_S)_i & \text{if } x \geq \bar{x} \\
 a_i & \text{if } x < \bar{x}
\end{cases}
$$

That is, if $x \geq \bar{x}$ then the mechanism plays an action on behalf of each player $i \in S$ that is a function of the messages that all players in $S$ submitted to the mechanism. Crucially,

\(^{13}\text{Such randomness is not typical in formal rules of real-life mechanisms, but could be thought of as additional randomness that our models do not capture (e.g. would a seller on Groupon actually purchase 3 coupons himself if the cut-off is almost achieved?)}

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if the threshold $\bar{x}$ is not met, then no player conditions his or her action on any other player's message.

A noisy threshold mechanism has the same message space, but also draws a random number $y \sim F$ for an arbitrary distribution $F$. It then generates a new "effective threshold" $\bar{x} = \bar{x} + y$. And then it applies the same $\mu_S$ from above to the effective threshold $\bar{x}$.

We are now ready to state our next result.

**Theorem 3.2.** For all $\epsilon > 0$ and $k < 1$ there is an $n$ such that for any $n > \bar{n}$ and any mechanism $M = (R, \{\mu_S\}_{S \subseteq N})$ which satisfies $k$ - Attraction when applied to a game $\Gamma$ with $|N| = n$, there is a noisy threshold mechanism $\bar{M} = (R, \{\bar{\mu}_S\}_{S \subseteq N})$ which is $\epsilon$ - dominant IR and satisfies $\bar{\mu}_N = \mu_N$.

**Proof sketch.** The logic of this result is as follows. We assume that $M$ has the property that for sufficiently large $k$, each player $i$ strictly prefers to play a centralized strategy if at least a fraction $k$ of other players use centralized strategies. Let $\bar{R} = (R \times A)$. We take $F$ to be the uniform distribution with support on $[-\sqrt{n}, 0]$ and $n$ to be large enough that $\frac{\sqrt{n}}{n} < 1 - k$, and the threshold to be any $\bar{x} \in (k + \frac{\sqrt{n}}{n}, 1)$

For any centralized strategy, the probability that a particular player $i$ triggers the threshold is at most $\frac{1}{\sqrt{n}}$. Now consider any decentralized strategy $a_i$ and the centralized strategy $\bar{r}_{a_i} = (r_{a_i}, a_i)$. We once again perform a cost-benefit analysis.

- For strategy profiles in which sufficiently few people sign away their decision rights such that the probability that $i$'s threshold is met is 0, the two strategies are payoff equivalent.

- For strategy profiles in which sufficiently many people sign away their decision rights such that $i$'s threshold is met with probability 1 regardless of $i$'s participation, $i$ strictly prefers the centralized strategy by assumption.

- For strategy profiles in which there is positive probability that $i$'s participation is pivotal in triggering at least one other player's threshold, $i$'s loss in this case is bounded below by $\frac{1}{\sqrt{n}}$.

Thus the cost of the centralized strategy relative to the decentralized strategy is at most $\frac{1}{\sqrt{n}}$ which vanishes as $n$ becomes large. Note that an ordinary threshold mechanism would not have this property. For certain strategy profiles, agent $i$ would be pivotal with probability 1 and thus his cost from participation would not be vanishing.
This result resembles insights from several literatures. Levine and Pesendorfer (1995) note that large market models where any agent can only observe aggregate actions with noise resemble continuum models in the limit, while large market models where agents can perfectly observe aggregate actions do not. There as well, the intuition comes from the fact that agents can be pivotal actors in the model without noise.

Also closely related is the literature on differential privacy, which explores the different ways in which adding noise to the output of algorithms makes it safe for participants to truthfully report inputs. For instance, Hsu, Huang, Roth, and Wu (2016) show that by calculating general equilibrium prices and then adding a small random perturbation in a large economy, it becomes an approximately dominant strategy for agents to report their true demand functions. And in a model closely related to this one, Kearns et al. (2015) show that in several classes of large market games in which equilibria can sensibly be perturbed, players can be induced to report their private information to a mechanism who then advises them to take actions that correspond to a slightly perturbed ex-post equilibrium. Their result differs from ours in that the perturbation in our model makes it safe to enter the centralized market regardless of what others do, while the perturbation in their model makes participation an approximate equilibrium without guaranteeing their desired equilibrium will be chosen.

We close this section with a negative result about $\epsilon$-strong dominant individual rationality. While the above result pertains to all large games, random matching games are a natural sub class to study large market limits. Formally, for our final result we are interested in the following class of random matching games. Let $\Gamma := (N, A = \{A_i\}_{i \in N}, u = \{u_i\}_{i \in N})$ be a normal form game of complete information, with players $N$, actions $A$, and utilities $u$ as defined in Section 3.2. Let $\Gamma^n := (n \times N, A, u)$ be a game with $n$ copies of each player in $\Gamma$, and where payoffs to each player are determined by choosing $n$ uniformly random subsets of players such that each subset has a copy of each player and applying the utility functions $u_i$ to the actions of the players in each subset. We are now ready to state our final result.

**Theorem 3.3.** Consider any game $\Gamma$ which does not have an equilibrium in dominant strategies. Then there exists an $\epsilon > 0$ such that for all $n$, no mechanism applied to $\Gamma^n$ is $\epsilon$-strong dominant individually rational.

**Proof.** See appendix.
The proof of the above result considers strategy profiles where player $i$ chooses the
centralized strategy $r_i$ that supposedly $\epsilon$-dominates his decentralized strategies, and
the other players all choose decentralized strategies. In such a case, the maximal regret
is clearly bounded from below by the worst case regret player $i$ suffers in the purely
decentralized game, and this is non vanishing in large markets for any game $\Gamma$ where
some player $i$ does not have a dominant strategy.

3.6 Discussion

We have laid out a model of voluntary centralized market design that respects the con-
straint that participants may subvert the centralized market and use their pre-existing
decentralized actions. Within this general framework we set forth a new desideratum
which may guarantee participation and which we argue offers guidance for market de-
sign. To satisfy the notion of $\epsilon$-dominant individual rationality a market must provide
each player with the guarantee that for each of their decentralized strategies there is a cor-
responding centralized strategy which will perform almost as well (within $\epsilon$) no matter
what other players do. Moreover, if sufficiently many others use the centralized market,
the centralized strategy must perform strictly better than its corresponding decentralized
strategy. While such a guarantee does not promise that players will not regret following a
particular centralized strategy, it does ensure that every decentralized strategy is $\epsilon$-dom-
inated by a centralized strategy. Thus for sufficiently small $\epsilon$, one may reasonably expect
that all players will follow centralized strategies.

We then analyzed two applications of the general framework to understand the details
of particular centralized markets. First we analyzed large two sided labor (matching)
markets and demonstrated that issues of adverse selection may plague standard imple-
mentations of centralized stable matching markets. However the analysis also demon-
strates that a minor modification to stable matching mechanisms - allowing the receiving
end of the market to accepts any offers they receive regardless of whether the proposer
was also part of the centralized market - allows the market to satisfy $\epsilon$-dominant indi-
vidual rationality, where $\epsilon$ vanishes in the size of the market. In the modified market it
is a strictly dominant strategy for receivers to enter the centralized market. And condi-
tional on all receivers using the centralized market, we established that every decentral-
ized strategy a proposer might follow is $\epsilon$-dominated by a corresponding centralized
strategy. This guarantee arises from the combination of the facts that the market's core
becomes small as it becomes large, and that the centralized market mechanism computes a stable matching. Thus our analysis also constitutes a non-cooperative justification of stability, which is traditionally considered a cooperative solution concept. Further, the modification we identify closely resembles the suggestion made by Niederle et al. (2006) when they considered how to centralize the market they considered. We believe that $\epsilon$-dominant individual rationality offers the conceptual framework necessary to fully understand the strength of their suggestion.

Last, we analyzed a general class of large games and showed that threshold mechanisms - mechanisms that take pre-specified actions on behalf of players only if they get sufficiently many participants - can be quite powerful. Namely, any mechanism with the critical mass property that every agent wants to use the mechanism so long as all others use it as well, can be modified to be $\epsilon$-dominant IR by converting it into a threshold mechanism. And again, as the market becomes large $\epsilon$ converges to zero. The principle subtlety arises from the possibility that agents worry they are pivotal in triggering the mechanism's threshold. To circumvent this concern we added a small perturbation to the mechanisms' threshold, which guarantees that no matter the strategy profile agents follow, no agent is pivotal with high probability. Threshold mechanisms like the ones we use are commonly observed in the real world and our analysis may shed some light on their practicality.

**Appendix**

**Omitted Proofs For Section 3.4**

Throughout this appendix, we fix $u$, $K$, and $C$, and allow the rest of the characteristics of the market to vary.

**Proposition (3.1).** There exists $\epsilon > 0$ such that no stable matching mechanism is $\epsilon$-dominant IR with respect to anonymous undominated strategies for a market of any size.

Our main concern is about adverse selection. However, for the sake of this proof we construct an equilibrium where the concern stems from the lack of participation (thin market). We do this for three reasons. First, the construction is simple. Second, we construct an equilibrium, so it is a traditional prediction for the outcome of the game. And third, in the equilibrium we construct, entry is safe for no one: unilateral deviation to entry is associated with non-negligible losses for all players. We will use the following claims:
Claim 3.1. Say $\sigma^*$ is an equilibrium profile of the decentralized game. Then it corresponds to an equilibrium in decentralized strategies of the game augmented by the standard deferred acceptance mechanism.

**Proof.** If no other participant is using the mechanism, by delegating to the mechanism the agent will ensure herself the lowest possible utility. A deviation to another decentralized strategy was also possible in the original (non-augmented) game.

Claim 3.2. In the decentralized game, choosing truthfully is a dominant strategy for workers.

**Proof.** Since workers do not affect the offers they receive, their best course of action is to take the best offer they receive.

**Fact 3.1.** There exists $M \in \mathbb{R}_+$ such that conditional on the identities of any subset of workers other than $i$ on a firm's preference lists, the probability that worker $i$ appears in a particular position on the list lies in \((1 - \frac{M}{n}) p_i, (1 + \frac{M}{n}) p_i\).

**Proof.** Follows immediately from writing Bayes' formula and noting that the expression is an average over expressions of the form \((1 + O(1/n))^{2K} p_i\).

**Remark.** We will use Fact 3.1 to make several approximations in what follows.

**Claim 3.3.** There exists $\delta > 0$ such that any equilibrium in anonymous undominated strategies of the decentralized game assures a payoff greater than $\delta$ to workers and firms of all types.

**Proof.** Assume a firm makes an offer to its first ranked worker. Making an offer to a worker that is not included in its preferences is dominated. Thus the probability that no firm other than this firm makes an offer to this worker is greater than (approximately) \((1 - \frac{KC}{n})^{n-1}\) which is positive and converges to $\exp(-KC) > 0$ as the market grows large, assuring a positive lower bound on workers utility (here we use Claim 3.2 that assures the offer is accepted).

For workers the probability receiving an offer from firm $i$ is greater than (approximately) $\frac{1}{Cn+1}$ (an approximation of the probability that the worker is ranked in the place on the firm's list that leads to an offer). Since offers are conditionally independent (conditional on $p_i$s) the conditional (and unconditional) probability to receive an offer from some player is at least (approximately) $1 - (1 - \frac{1}{Cn+1})^n$ which is positive and converges to $1 - \exp(-1/C) > 0$. The independence of firms and workers preferences takes care of the rest.
Proof. (of the proposition) The existence of an anonymous undominated equilibrium of the decentralized game follows from standard arguments. From Claim 3.3 it yields all players utility of at least $\delta > 0$. From Claim 3.1 this equilibrium corresponds to an equilibrium of the mediated game. Furthermore, delegating to the mechanism is assured to yield a payoff of zero if all others use the equilibrium strategies.

We are now ready to prove our main result of the section.

**Theorem (3.3).** For all $\epsilon > 0$ there exists $\bar{n}$ such that for all $n > \bar{n}$ the modified stable matching mechanism is $\epsilon$-dominant IR with respect to $\Sigma$.

**Claim 3.4.** For workers, any decentral strategy is dominated by reporting their true preferences to the mechanism.

Proof. By Claim 3.2 and the transitivity of the dominance relation, it is sufficient to show that reporting truthfully to the mechanism dominates choosing truthfully from all available offers. This holds, because by delegating to the mechanism the worker will continue to receive all offers it would have had he opted out, and potentially another offer. Since the choice would be according to actual preferences (due to truthful reporting), this means that the expended choice set must lead to weakly greater utility. There are clearly some profiles of strategies where the weak inequality will not bind (e.g., when all other participants delegate and are truthful).

**Claim 3.5.** Submitting an empty list is a dominated strategy for firms.

Proof. Submitting an empty list assures firms the lowest possible utility. Making a decentral offer to their most preferred worker dominates this strategy.

**Lemma 3.1.** For all $\epsilon > 0$ there exists $\bar{n}$ such that for all $n > \bar{n}$ the modified stable matching mechanism is $\epsilon$-dominant IR with respect to $\Sigma$ for workers.

Proof. $\epsilon$-safety of truthful reporting follows from Claim 3.4, where we showed that it dominates any decentral action. We now show that truthful reporting leads to a $\Delta > 0$ increase in utility relative to any decentral strategy when a fraction $k$ of agents participate in the match.

Assume that a fraction $\alpha$ of firms are using the mechanism. Then $\left(1 - \frac{KC}{n}\right)^{(1-\alpha)n}$ is an approximate lower bound on the probability that none of the non-delegators finds a particular worker acceptable. Hence, for large $n$, the probability that some non-delegator
finds the worker acceptable is approximately upper-bounded by \(1 - \exp \left((1 - \alpha)KC\right)\).\(^{14}\) Thus, since making a decentral offer to an unacceptable worker is dominated (thus not in \(\Sigma\)), the expected utility from not delegating is bounded above by \(\left[1 - \exp \left((1 - \alpha)KC\right)\right] \cdot u_w(1)\). For sufficiently large \(\alpha < 1\) this expression is arbitrarily close to 0.

On the other hand, the probability that one of the delegators ranks the worker first is approximately bounded below by \(\left[1 - \left(1 - \frac{1}{Cn}\right)^{an}\right]\) (an approximate lower bound of the probability of being on the particular place on the actual preferences list that translates to the first rank on submitted ROL). In this case, any stable matching would have the worker matched with this firm, or a more preferred one. Using the independence of worker and firm preferences, this gives a large-market approximate lower bound on utility of \(M := \left[1 - \exp \left(-\frac{a}{c}\right)\right] E[u_w(x)]\) where \(x\) is a random variable distributed uniformly on \((0, 1)\) if \(u\) is continuous. Without continuity, one can use monotonicity to derive a different bound.

Setting \(k = \frac{1}{2} + \frac{a^*}{2}\) where \(a^*\) assures that the upper bound is close to 0, \(\Delta = \frac{1}{2}M\), and \(n'\) such that all approximations are sufficiently tight, completes the proof. \(\square\)

**Definition.** Given a firm \(f\) and a worker \(w\), firm \(f\)'s centralized strategy \(w_{\text{truth}}\) ranks \(w\) first, and then the other \(K - 1\) acceptable workers in the order of true preferences if \(w\) is acceptable. Otherwise, it truthfully reports \(f\)'s preferences.

**Lemma 3.2.** Fix preferences and a pure strategy profile in \(\Sigma\) of all other players but \(f\). If \(w_{\text{truth}}\) yields \(f\) a lower payoff than a decentral offer to \(w\), then \(w\) has two stable partners with respect to reported preferences of delegators when \(f\) doesn't delegate, or when \(f\) delegates and plays \(w_{\text{truth}}\).

**Proof.** By Claim 3.4, all workers delegate to the mechanism. Additionally, if \(w\) is unacceptable, this is obvious because a decentral offer to \(w\) assures a non-positive payoff, whereas \(w_{\text{truth}}\) assures a non-negative payoff. Otherwise, \(w\) is acceptable, and we consider three cases. First, if \(f\)'s decentral offer would have been rejected. In this case, \(f\) cannot lose (and could potentially gain) by playing \(w_{\text{truth}}\). Second, if \(f\) would be matched with \(w\) if he plays \(w_{\text{truth}}\). In this case, again, he would lose nothing from playing \(w_{\text{truth}}\) relative to making a decentral offer to \(w\). \(f\) could only lose in the third case, where its decentral offer to \(w\) would be accepted, but he would not be matched with \(w\) if he plays \(w_{\text{truth}}\).

Subcase 1: \(f, w\) are a stable pair in the market that includes \(f\) reporting \(w_{\text{truth}}\).

Subcase 2: \(f, w\) are not a stable pair in the market that includes \(f\) reporting \(w_{\text{truth}}\).

\(^{14}\)Convergence is uniform over values of \(a\).
In Subcase 1, \( w \) has two stable mates (his assigned match and \( f \)). For Subcase 2, we use the notation \( v(\emptyset), v(w), v(w_{\text{truth}}) \) to denote the matchings resulting from \( f \) reporting the ROL in parenthesis. Note that \( v(\emptyset)[w] \), the mate assigned to \( w \) had \( f \) not participated, was ranked lower than \( f \) according to \( w \)'s reported preferences (otherwise, we are not in case 3). Since \( w \) is ranked first by \( f \) in \( w_{\text{truth}}, v(w_{\text{truth}})[w] \), \( w \)'s assigned mate when \( f \) plays \( w_{\text{truth}} \), must be weakly more preferred than \( f \) by stability. From Demange et al. (1987) we know that \( v(w)[f] = f \). That is, had \( f \) submitted an ROL ranking \( w \) only, \( f \) would have remained unmatched (as they are not a stable pair under \( w_{\text{truth}} \), and \( w \) is more preferred by \( f \) than his assigned mate, according to reported preferences). But since \( v(w) \) is stable with respect to reported preferences, \( w \)'s mate must be more desirable than \( f \) according to \( w \)'s reported preferences. This means that \( w \) has two stable partners in the market where \( f \) doesn't participate \( (v(\emptyset)[w] \neq v(w)[w]) \), because \( v(w) \) is also stable when \( f \) does not participate (there are fewer restrictions to satisfy, and \( f \) is not matched to any worker).

We next claim that in a large market (large \( n \)), for any (anonymous undominated) pure-strategy profile played by others, any \( p_i \) profile (that satisfies \( \frac{p_i}{p_j} < C \)), and any profile of worker preferences, the probability that \( w \) has two stable partners with respect to reported preferences of delegators when \( f \) doesn't delegate, or when \( f \) delegates and plays \( w_{\text{truth}} \) is minuscule. The upshot is that the same applies conditional on the (less precise) information that \( f \) holds. To this end, we now fix a profile of \( p_i \)'s, of worker preferences, and a profile of pure strategies. Since we only care about the centralized match result, strategies in \( \Sigma \) could be summarized by permutations for workers, and permutations and a cutoff \( k_f \leq K \) for firms.

Following Knuth (1997) we use the "principle of deferred decisions." To this end, we will use the following random algorithms to find upper bounds for the probability of the event that \( w \) has more than one stable partner.

**Algorithm 3.1.** Step 1: Set \( i = 2 \), and let \( v \) be the empty matching, and \( \succ_f \) be an empty ROL for each firm \( f \).

Step 2: If \( f_i \) is matched to a worker according to \( v \), or if \( \succ_{f_i} \) is of size \( K_{f_i} \) (the cutoff \( f_i \)'s strategy specifies), set \( i = i + 1 \). If \( i = n + 1 \), stop.

Else, draw a worker \( w' \) to be ranked last (for now) according to \( \succ_{f_i} \). Use Bayes’ rule to find the probability distribution, given the strategy of \( f_i \) (the mapping from preferences to submitted lists), and knowledge of \( p_i \)s. Draw according to this probability, and independently of any previous step conditional on this probability. If \( w' \) prefers \( f_i \) to his match according to \( v \), “divorce” \( w' \) from his current match and set \( v(w') = f_i \) and \( v(f_i) = w' \). set \( i = 2 \) and repeat.
Algorithm 3.1 runs the firm proposing deferred acceptance, while drawing the necessary parts of firm preferences in each stage. The distribution of resulting matches is equal to the distribution of matches if we first draw preferences and only then run Deferred Acceptance on reported ROLs (as determined by the pure strategies we fixed). While our algorithm is different than that of Kojima and Pathak (2009), the rationale is identical to the one they provide (this approach was pioneered by Knuth (1997)).

**Algorithm 3.2.** Input: \(w\).

1. **Step 1:** Run Algorithm 3.1. Set \(i = 2\).
2. **Step 2:** If \(w\) is unmatched at the end of Step 1 stop. Else, divorce \(w\) from \(v(w)\).
3. **Step 3:** If \(f_i\) is matched to a worker according to \(v\), or if \(\succ_f\) is of size \(K_{f_i}\) (the cutoff \(f_i\)'s strategy specifies), set \(i = i + 1\). If \(i = n + 1\) stop.

   Else, draw a worker \(w'\) to be ranked last (for now) according to \(\succ_f\). Use Bayes' rule to find the probability distribution conditional on the previous steps, given the strategy (the mapping from preferences to submitted lists), and knowledge of \(p_i\). Draw according to this probability, and independently of any previous step conditional on this probability. If \(w' \neq w\) prefers \(f_i\) to his match according to \(v\), "divorce" him from that firm and set \(v(w') = f_i\) and \(v(f_i) = w\). If \(w' = w\) return "1" and terminate. Set \(i = 2\). Repeat.

Algorithm 3.2 is akin to the Stochastic Rejection Chains Algorithm of Kojima and Pathak (2009). It returns 1 if a rejection chain that begins with \(w\) returns to \(w\). This is a necessary (but not sufficient) condition for \(w\) to have multiple stable partners.

**Algorithm 3.3.** Input: \(l\)

1. **Step 1:** Draw \(f_1\)'s \(l\)-th preferred worker using Bayes' law, and denote it by \(w\).
2. **Step 2:** Run algorithm 3.2 with the input \(w\).

**Algorithm 3.4.** input: \(l\)

1. **Step 1:** Run Algorithm 3.1. Draw \(f_1\)'s \(l\)-th preferred worker using Bayes' law, and set \(w\) to be this worker. If \(w\) prefers \(f_1\) to his match according to \(v\), "divorce" \(w\) from his current match and set \(v(w) = f_1\) and \(v(f_1) = w\). Set \(i = 1\).
2. **Step 2:** If \(f_i\) is matched to a worker according to \(v\), or if \(\succ_f\) is of size \(K_{f_i}\) (the cutoff \(f_i\)'s strategy specifies), set \(i = i + 1\) and repeat. If \(i = n + 1\), set \(i = 1\) and move to the next step.

   Else, draw a worker \(w'\) to be ranked last (for now) according to \(\succ_f\). If \(i = 1\), draw the best ranked mate that was not previously drawn (simulating the strategy "\(l_{truth}\)"). Use Bayes'
rule to find the probability distribution, given the strategy of \( f_i \) (the mapping from preferences to submitted lists), and knowledge of \( p_i \)'s. Draw according to this probability, and independently of any previous step conditional on this probability. If \( w' \) prefers \( f_i \) to his match according to \( v \), "divorce" \( w' \) from his current match and set \( v(w') = f_i \) and \( v(f_i) = w' \). Set \( i = 1 \) and repeat.

**Step 3:** Divorce \( w \) from \( v(w) \).

**Step 4:** If \( f_i \) is matched to a worker according to \( v \), or if \( f_i \) is of size \( K_{f_i} \) (the cutoff \( f_i \)'s strategy specifies), set \( i = i + 1 \). If \( i = n + 1 \) stop.

Else, draw a worker \( w' \) to be ranked last (for now) according to \( v \). If \( i = 1 \), draw the best ranked mate that was not previously drawn (simulating the strategy "ltruth"). Use Bayes' rule to find the probability distribution conditional on the previous steps, given the strategy (the mapping from preferences to submitted lists), and knowledge of \( p_i \)'s. Draw according to this probability, and independently of any previous step conditional on this probability. If \( w' \neq w \) prefers \( f_i \) to his match according to \( v \), "divorce" him from that firm and set \( v(w') = f_i \) and \( v(f_i) = w \). If \( w' = w \) return "1" and terminate. Set \( i = 1 \). Repeat.

**Claim 3.6.** On the run of Algorithms 3.3 and 3.4, whenever preferences are drawn for firms, worker \( i \) is drawn with probability \( q = \left(1 - \frac{M}{n}\right)p_i\left(1 + \frac{M}{n}\right)p_i\).

**Proof.** Follows from Fact 3.1.

**Lemma 3.3.** For any \( l \in \{1, ..., K\} \), the probability that the \( l \)-th worker on \( f_1 \)'s preferences list has two stable partners when \( f_1 \) is opts out, or when it plays \( l_{\text{truth}} \) is smaller than the sum of the probability that Algorithm 3.3 returns 1 and the probability that Algorithm 3.4 returns 1.

**Proof.** Fix \( l \). The probability that Algorithm 3.3 returns 1 with the input \( l \) is greater than the probability that \( f_1 \)'s \( l \)-th ranked worker has two stable mates if \( f_1 \) opts out of the mechanism (see Kojima and Pathak (2009)). Additionally, the probability that Algorithm 3.4 returns 1 with the input \( l \) is greater than the probability that \( f_1 \)'s \( l \)-th ranked worker has two stable mates if \( f_1 \) plays \( l_{\text{truth}} \) (for the same reasons). Finally, for any two events \( A \) and \( B \), it always holds that \( Pr\{A \cup B\} \leq Pr\{A\} + Pr\{B\} \).

**Lemma 3.4.** For any \( l \in \{1, ..., K\} \), the probability that Algorithm 3.3 with input \( l \) returns "1" is \( o\left(\frac{1}{n}\right) \).

**Proof.** From Fact 3.1 and the fact that the number of preferences draws is bounded by \( K \times n \), if we draw all preferences according to the strategies played by the participants, the distribution of ROLs will have a bounded likelihood ratio relative to the distribution.
where each firm reports truthfully the first $K_f$ workers on its list (with bounds of the order $\exp(2MK)$ and $\exp(-2MK)$ when $n$ is large). Thus, the result follows from the proof of Kojima and Pathak (2009), as with probability $1 - o(\frac{1}{n})$ the $l$-th ranked worker is not among the workers with more than one stable match under “truthful preferences,” and $\text{Const} \times o(\frac{1}{n}) = o(\frac{1}{n})$.

**Lemma 3.5.** For any $I \in \{1, \ldots, K\}$, the probability that Algorithm 3.4 with input $I$ returns “1” is $o(\frac{1}{n})$.

**Proof.** (sketch) As in the previous Lemma, we can assume that all reports are truthful. Here, too, the proof of Kojima and Pathak (2009) applies (even though the statement is different). The main idea is that after Step 2 there are many workers that are at least as popular as the first worker on $f_1$’s ROL, and many of them are unmatched. Thus, at any later stage these workers are more likely to receive an offer relative to $f_1$ (when preferences are drawn truthfully).

**Lemma 3.6.** For all $\epsilon > 0$ there exists $\tilde{n}$ such that for all $n > \tilde{n}$ the modified stable matching mechanism is $\epsilon$-dominant IR with respect to $\Sigma$ for firms.

**Proof.** We will compare the strategy $I$, of making a decentral offer to the firm’s $l$-th ranked worker, to the strategy $I_{truth}$. Safety follows from Lemmas 3.3-3.5. For $k$-attractiveness note that the probability that the $l$-th ranked firm will appear first on another firm’s ROL, when at least fraction $\alpha$ of firms participate in the mechanism and submit non-empty ROLs (empty ROLs are not in $\Sigma$ by Claim 3.5) is greater than approximately $(1 - \frac{C}{n})^n \rightarrow \exp(-C\alpha) > 0$. Conditional on this event, the probability that the second ranked firm appears on no other firm’s true preference list (thus ROL, since ranking an unacceptable firm is dominated, and $\Sigma$ excludes dominated strategies) $1 - (1 - \frac{1}{\alpha})^n \rightarrow 1 - \exp(-\frac{C}{n}) > 0$. Furthermore, since worker preferences are independent, if another firm ranked the same worker first the probability that the worker prefers this firm is at least $\frac{1}{C+1}$. Thus, as long as $\alpha$ is bounded away from 0, there are non-diminishing gains from coordination with other firms.

**Proof.** (of the theorem) Follows from Lemma 3.1 and Lemma 3.6.
Proof of Theorem 3.5

Proof. Let \( i \in N \) with type \( \theta_i \) be an arbitrary player in game \( \Gamma \) who does not have a dominant strategy and let

\[
\epsilon_{a_i} \equiv \max_{a_j} \max_{a_{-i}} [\mathbb{EU}(a_j, a_{-i}, \theta_i) - \mathbb{EU}(a_i, a_{-i}, \theta_i)].
\]

\( \epsilon_{a_i} \) can be thought of as the maximal regret that player \( i \) can feel when following strategy \( a_i \). Note that by the fact that player \( i \) does not have a dominant strategy, \( \epsilon_{a_i} \) is positive for all \( a_i \). Now let \( \bar{\epsilon} \equiv \min_{a_i} \epsilon_{a_i}. \)

We will show that for all \( n \geq 1 \), no mechanism \( M \) applied to \( \Gamma^n \) is \( \epsilon \)-strongly dominant individually rational for \( \epsilon < \bar{\epsilon}. \) Suppose to the contrary that \( M \) was \( \epsilon \)-strongly dominant IR, and let \( r_i \) be the strategy that \( \epsilon \)-dominates all decentralized strategies for player \( i \). Now let

\[
\epsilon_{r_i} \equiv \max_{a_j} \max_{a_{-i}} [\mathbb{EU}(a_j, a_{-i}, \theta_i) - \mathbb{EU}(r_i, a_{-i}, \theta_i)].
\]

Note that \( \epsilon_{r_i} \) is the maximal regret that player \( i \) can feel when he plays action \( r_i \) and all other players follow decentralized strategies (and hence is less than the maximal regret he can feel when all other players are unrestricted in the strategies they play). By definition, \( \epsilon_{r_i} = \epsilon_{a_i} \) for some \( a_i \) and hence \( \epsilon_{r_i} \geq \bar{\epsilon} > \epsilon. \) Thus \( r_i \) does not satisfy \( \epsilon \)-regret, which contradicts the assumption that \( M \) is \( \epsilon \)-strongly dominant individually rational.

\( \square \)
Bibliography


