Dynamic Pricing in Shared Mobility on Demand Service and its Social Impacts

by

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Abstract

In this thesis, we formulate and solve a profit maximization problem of shared mobility on demand service operations, and investigate the impact of such operations on performance of transportation system with a carefully designed case study. It is shown that our approach can generate much more profit than other basic strategies, though it has negative impacts on system performance, such as increasing congestion level and reducing capacity provided. We also consider possible regulation schemes on such profit-driven operations, and find that schemes related to the total reduced distance in system can achieve significant improvement. These findings indicate several research directions in future for better designing or regulating shared mobility on demand service from the system perspective.

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# Contents

1 Introduction 13

2 Literature Review 17

3 Problem Setting 21
   3.1 General Framework 21
   3.2 Traveler Behavior 23
   3.3 Operator’s Optimal Strategy 24
      3.3.1 Assortment Optimization 24
      3.3.2 Price optimization 26
      3.3.3 Relationship between Assortment and Price optimization 28
      3.3.4 Daily-level Optimization 29

4 Case Study 35
   4.1 General Framework 35
      4.1.1 Choice Model Specification 37
      4.1.2 Speed-density Model 38
      4.1.3 Public Policy: Tax Reduction 39
   4.2 Implementation 40
      4.2.1 Data Description and Processing 41
      4.2.2 Parameter Selection 43

5 Computation Results 49
   5.1 Operating Metrics and Strategies Comparison 49
5.2 System Performance ........................................ 54
  5.2.1 Low Original-cost Case ................................ 55
  5.2.2 High Original-cost Case ............................... 60

6 Conclusion ...................................................... 65
  6.1 Future Direction ............................................. 66
    6.1.1 Optimal Strategy on Private Side ....................... 66
    6.1.2 Optimal Strategy on Public Side ...................... 67
List of Figures

3-1 Request-level Decision Process Flow Chart .......................... 22
4-1 Simulation System Flow Chart ........................................... 35
4-2 Data Location .................................................................. 41
4-3 Demand Size Time Series ................................................. 42
5-1 Shared rates $p_{sh}$ across different settings ......................... 53
5-2 Congestion - Capacity Tradeoff among Strategies, $b_{f,o} = 2.5$ .... 58
5-3 Congestion - Capacity Tradeoff of PO under Tax Reduction, $b_{f,o} = 2.5$ 59
5-4 Congestion - Capacity Tradeoff among Strategies, $b_{f,o} = 5.0$ .... 62
5-5 Congestion - Capacity Tradeoff of PO under Tax Reduction, $b_{f,o} = 5.0$ 63
List of Tables

4.1 Summary of parameters ................................................. 47

5.1 Some possible strategies .................................................. 50

5.2 Operating statistics under fixed traffic time case with $r_{sh} = 0.6$, $N = 150$ 51

5.3 Average percentage difference in operating statistics, compared with PO, fixed traffic time cases .......................................................... 52

5.4 Capacity $M_{sh}$ and shared rate $\rho_{sh}$, fixed traffic time cases ........ 52

5.5 Average percentage difference in operating statistics, compared with PO, $b_{f,o} = 2.5$ .................................................. 55

5.6 Average percentage difference in system metrics, compared with background value, $b_{f,o} = 2.5$ .................................................. 56

5.7 Percentage difference in system metrics, compared with background value, $b_{f,o} = 2.5$, $\psi_C = High$, $\varphi = 0.4$ ............................. 57

5.8 Shared trips rate $\rho_{sh}$ and total reduced trip distances $\Delta_d (\times 10^4 km)$, $b_{f,o} = 2.5$, $\psi_C = High$, $\varphi = 0.4$ .............................................. 58

5.9 Average percentage difference in system metrics, compared with background value, under tax reduction, $b_{f,o} = 2.5$, $\varphi = 0.4$ ...... 60

5.10 Average percentage difference in operating statistics, compared with PO, $b_{f,o} = 5.0$ .................................................. 60

5.11 Average percentage difference in system metrics, compared with background value, $b_{f,o} = 5.0$ .................................................. 61
5.12 Percentage difference in system metrics, compared with background value, and total reduced trip distances $\Delta_d(\times 10^4 km)$, $b_{f,o} = 5.0$, $\psi_C = \text{High, $\varphi = 0.4$}$ 62

5.13 Average percentage difference in system metrics, compared with background value, under tax reduction, $b_{f,o} = 5.0$, $\varphi = 0.4$ 63
Chapter 1

Introduction

Recently, as several mobility matching platforms grow and gain influence, shared mobility and mobility on demand (MoD) service are becoming hot topics in the transportation community. Shared mobility refers to ride-sharing or car-sharing, which both try to share capital of transportation means to serve trips with lower cost, while MoD service refers to mobility service that operates based on customer requests or booking, and is becoming more practical and economic, as technology advances and the interaction between operator and users is more convenient. In this thesis, we will focus on the combination of ride-sharing and MoD service.

Intuitively, on one hand, MoD service provides a possibility to allocate travel capacity dynamically across space and time, and thus can reduce unnecessary usage of transportation infrastructure when demand drops, while increase throughput when demand surges. On the other hand, ride-sharing aggregates similar origin-destination demand, and reduces the need for supply in serving trips, so it can both reduce the number of vehicles on the road and provide more trips when necessary. Moreover, as they are making progress on different aspects of mobility service, it is not impossible to combine the two and exploit advantage from both sides. Thus, it seems that MoD service with the capability to provide ride-sharing service can be very promising in boosting social welfare.

Though recently there have been a great number of studies investigating the social impact of shared mobility or MoD service, and trying to support the intuition above,
there is a major problem: these studies tend to be too optimistic in ignoring the private interest of operators and solely focusing on specific social welfare related metrics, and the results can be quite unrealistic and biased. For example, even though we can optimize operation to provide much more capacity with shared mobility, such operation might not be economically reasonable for a private operator.

Trying to overcome such problem, and to have more accurate understandings of the social impact of emerging shared MoD services, in this thesis, we will think from the perspective of private side by modeling a profit-driven operator, and investigate its impact on the performance of transportation system with a carefully designed case study. We also consider some possible regulation schemes on such operations to gain insights on how policy should be developed under these settings.

To model the profit-driven property of the operator, we formulate a profit maximization problem to optimize operation decision on trip requests, such as providing different service options and adjusting prices. More specifically, in formulating the problem, we use discrete choice models to describe the traveler behavior, apply assortment and price optimization framework to model the request-level dynamics, and leverage insights from dynamic programming and reinforcement learning to capture temporal correlations among request-level decisions. We then exploit existing methods and techniques in these fields to design an efficient algorithm to solve the problem.

In the case study, we first compare the generated profit and other operating statistics of the strategy designed in this thesis with those of other basic ones, to validate our algorithm. Then we move to compare system performances and we indeed find that, in general, such profit-driven operator will not improve the system performance as well as the best case by the basic strategies. We also identify an important metric, the total reduced distance in the system, and find that policy related to this metric can improve the system performance under profit-driven operations. Finally, we also identify some important properties of these profit-driven operations. These findings indicate several future research directions, both for studies from the public perspective and from the private perspective.

The remainder of this thesis is organized as follows. Chapter 2 reviews recent
development and research effort in shared mobility, MoD service, especially under autonomous vehicle (AV) settings, in transportation research community. There is also a brief summary of the history of dynamic pricing and discrete choice modeling, and current development in assortment and price optimization under discrete choice models. Chapter 3 introduces our analyzing framework and the formulation of our major problem: daily-level profit maximization of shared MoD service with request-level control. Chapter 4 describes the settings for the case study, including both modelings of system dynamics and parameter selection. Chapter 5 summarizes results from the case study and discusses the implications. Chapter 6 presents our conclusions and provides several insights for future directions.
Chapter 2

Literature Review

Recently, there has been an increasing number of studies on the capacity of shared mobility, especially under settings with autonomous vehicle (AV). Fagnant et al. [18, 20] used simulation to investigate how shared autonomous vehicle (SAV), under certain operation rule, might improve transportation efficiency. Later, they [19] further included dynamic ride sharing in the simulation to show that SAV can also be profitable for the private operator. Alonso et al. [2] investigated how to do online dispatching to optimize service rate, and showed that such algorithm can improve service capacity by a factor of 3. Even though results from these studies might indicate shared mobility should be promoted, their studies are from public perspectives and do not consider the interest of the private operator. It is likely that a profit driven operator might voluntarily reduce its capacity to boost profit. Whether such operators will provide beneficial shared mobility to the society is still unknown.

At the same time, there is much more interest in optimizing operations of private mobility service operators. Other than discussing under the integrated settings of shared mobility, mobility on demand (MoD) and AV, they usually focus on a specific setting and solve a subproblem in general operation. One special setting that attracts much attention recently is the two-sided ride-sharing platform. Studies in this area [7, 8, 29, 41] tend to focus on optimal matching, and most likely to apply queueing theory in the modeling. There is also limited effort trying to solve other problems under this setting. Jung et al. [31] and Nourinejad and Roorda [40] developed algorithms to
solve dispatching problem; Fang et al. [21] investigated trade-off between private profit and public welfare with simple game theory approach; Chen and Sheldon [14] investigated the impact of dynamic pricing on supply; similarly Chen et al. [13] gathered data of Uber’s operation to learn how Uber’s strategy might alleviate the demand/supply imbalance, and they observe that surge prices have a strong negative impact on demand, while a weak positive impact on supply.

In terms of the one-sided market setting, most research interests are in the dispatching problem, which is closely related to traditional vehicle routing problem. One example is dial-a-ride problems (DARP). Recently there is a nice review by Cordeau and Laporte [15] summarizing attempts on this problem. For this problem, Hyytiä et al. [30] and Sayarshad and Chow [45] provided non-myopic solution methods that consider temporal correlations and improve operation performance in a longer time scale. There is also limited research on problems with other focuses. For example, to maximize profit, Atasoy et al. [4] formulated a very similar problem trying to address the difference between single passenger service and shared service, and incorporate traveler behavior, but focus on request-level assortment optimization method and cannot develop efficient strategy; to optimally rebalance fleet, Smith et al. [46] introduced a fluid model and develop mathematic programs upon it.

In some sense, our study is a hybrid of studies on one-sided market introduced above. More specifically, we mainly focus on profit maximization and traveler behavior as Atasoy et al. [4], but extend to consider price optimization and non-myopic optimization, which is the central topic in [30, 45]. As we are approaching our problem from a different perspective (parametric policy-based optimization), our study might also provide a different angle for their works.

The models we used in this study connect dynamic pricing approaches with discrete choice models to solve profit maximization problems with stochastic traveler behavior. In terms of dynamic pricing, there has been a long line of research in the setting of market design and revenue management. There is a nice review, though not very recently, on general dynamic pricing, by Elmaghraby and Keskinocak [17]. Theory of discrete choice analysis is also long established; Luce [36] and McFadden
[37] represent the first efforts in developing this theory, especially on the application of multinomial logit model (MNL). A very nice textbook on this subject is [9]. But the connection between these two fields begins to emerge only recently. In discussing possible modeling in airline revenue management, Talluri and Van Ryzin [48] first discussed the assortment optimization problem under MNL model, and sooner this topic attracts much attentions. Several other interesting studies on the property of assortment optimization under MNL model are [23, 35]. For interested readers, there is a review of assortment optimization under other discrete choice models by Kök and Fisher [32].

Price optimization is another model that represents such connection. As the problem is a continuous nonlinear optimization, most studies focus on investigating properties such as convexity, and rely on standard solver for computation. There have been a collection of works [16, 27, 47] discuss the structural property of pricing problem under MNL model. Recently, similar to the studies on assortment optimization, scholars are exploring more sophisticated choice model. For example, Gallego and Wang [24], and Li and Huh [34], tried to characterize properties of the problem under nested logit model (NL). For interested readers, there is a related review by Aydin and Ryan [6]. Finally, there are also some efforts to consider assortment optimization and price optimization at the same time [33], which provide us some insights on including both of them in this study.
Chapter 3

Problem Setting

3.1 General Framework

Assume there is a MoD service operator, who owns a fleet of vehicles and provides both on-demand single and shared trip service. Single service means the vehicle can only provide service for one trip at one time, while share service can provide service for trips sharing similar routing properties up to a certain capacity limit. The major decision process in the operation can be described as follows. When the operator receives a travel request from user $i$ at time $t$, it will search for vehicles that are feasible to provide service for each service type (e.g., empty vehicles for single service), and generate a feasible set of options $C_i$, where each option consists of trip information, such as service type, estimated travel time and distance, and financial information, such as fare and cost. The operator then generates the offer set $C'_i$ from the feasible set $C_i$ according to some control algorithms, including to change trip fare. On one hand, if the offer set $C'_i$ is not empty, the traveler $i$ will select one option $j \in C'_i$, or leave the service system and choose other alternatives, such as driving. This selection process is modeled by a discrete choice model which is, for simplicity, homogeneous for the whole population. If there is a selection $j$, the operator confirms the selection and updates status of its fleet. On the other hand, if the offer set is empty $C'_i = \emptyset$, the traveler will wait and request again in next time step. If this traveler cannot receive any offer within $T_d$ time steps, she will leave the service system and choose
other alternatives. A simple flow chart describing the process can be found below:

![Flow Chart](image)

**Figure 3-1: Request-level Decision Process Flow Chart**

And our major problem is to maximize the operator’s daily profit by selecting good offer generation algorithms, given such decision process. We further make several important assumptions in this work, mainly to focus on the request-level decisions, and simplify the difficulty in modeling and optimization of irrelevant settings and operations. We expected to relax these restrictions in future research.

1. We assume that there is only one operator of similar services, and there is no competition between this operator and other transportation service providers, i.e., they will not react to any change in this operator’s strategy.

2. We assume that several factors, including operating capital (e.g., fleet size), traveler’s perception of operating policy (e.g., knowledge of based fare structure), potential demand population, background system traffic flow and public policy, are fixed and will not be influenced by operator’s strategy, during the period of a day.

3. We assume that the operator has full knowledge of the demand in this day, both the underlying origin-destination distribution and the general traveler choice behavior.

4. Finally, we assume that other operation rules of this operator are simple. The
operator will process each request in order and will not do batch dispatch; it will not 
rebalance supply, and each empty vehicle is kept idle and stop at wherever they drop 
the last passengers off.

In following subsections, we will further elaborate the framework introduced above, 
and provide technical details on our modeling.

3.2 Traveler Behavior

In this subsection, we will describe how we model the traveler choice behavior. Assume 
for each trip request \( i \), we provide a set of options \( C_i = \{1, \cdots, n'\} \). For each option 
\( j \), there are several values in interest of the traveler: service type \( y_j \), standard fare 
\( f_j \), fare adjustment \( \delta_j \), and travel time (estimated arrival time - current time) \( T_j \). We 
assume there is a known utility function for each option \( U_j = U(y_j, f_j, \delta_j, T_j) \), which 
measures the utility of the option to the traveler. Then, the selection probability for 
option \( j \) is modeled by a multinomial logit model (MNL) [9, 36, 37]:

\[
p_j = \frac{\exp(U_j)}{\sum_{k=1}^{n'} \exp(U_k) + V_0}
\]

where \( V_0 \) is the preference for not selecting the service from the operator, and includes 
two parts: the alternative to taking original travel mode, such as driving or taking 
a taxi, and the alternative to cancel the trip. We can expand the expression of \( V_0 \) 
as \( \exp(U_o) + \exp(U_n) \), where \( U_o \) is the utility for the original travel mode, and \( U_n \) is 
the utility for trip cancellation. If we let \( V_j = \exp(U_j) \), the equation above can be 
simplified as

\[
p_j = \frac{V_j}{\sum_{k=1}^{n'} V_k + V_0}.
\]

Notice that this choice model characterize two effects at the same time: conversion from original trips to new services, and generation of new trips. Later in the 
case study, we can modify the parameter related to \( U_o \) and \( U_n \) to address each effect.

We should point out that, currently we assume this choice model to be homoge-
neous for the population. That is, all travelers behave in the same way and follow this model. This assumption might be too strong since people behave quite differently across the population. Thus a potential extension of this work is to consider more sophisticated models, such as mixed multinomial logit model (MMNL), which models several classes of travelers and their mixing effect.

3.3 Operator’s Optimal Strategy

In above, we assume that operator can control how to generate offers, and also knows how travelers will behave. Next, we will discuss how the operator, given information of a specific travel request, can make decision on offer set to optimize profit. We will introduce two models, assortment optimization and price optimization, which provide efficient ways to optimize the profit of this specific request under MNL model; then, by using dynamic programming formulation, we extend these request-level optimization methods for daily-level problem.

Before starting our discussion, let us recall that at the offer generation stage, the operator has access to a set of feasible options $C_i = \{1, \cdots, n\}$, and for each option $j$, there are service type $y_j$, standard fare $f_j$, fare adjustment $\delta_j$, cost $c_j$, and travel time $T_j$. The operator also knows the MNL choice model, the utility function $U_j = U(y_j, f_j, \delta_j, T_j)$, and exit preference $V_0$. Again, we use $V_j$ to represent $\exp(U_j)$. Finally, the operator tries to optimize the profit, which is defined by $R_j = f_j + \delta_j - c_j$.

3.3.1 Assortment Optimization

In this part, we consider the profit maximization problem with the decision variables to be some appropriate selections of assortment, or subsets of options to offer, while all other factors, including $y_j, f_j, \delta_j, T_j, c_j$, are set to be constant. That is, we want to provide a subset $C_i' \subseteq C_i$ to maximize the expected profit:

$$\max_{C_i' \subseteq C_i} \sum_{j \in C_i'} p_j(C_i') R_j = \max_{C_i' \subseteq C_i} \frac{\sum_{j \in C_i'} V_j R_j}{\sum_{j \in C_i'} V_j + V_0}$$
where $V_j$ and $R_j$, from the assumption, are constants. We can also use a $n$-dimensional binary vector $x$ to represent if element $j$ of $C_i$ is in $C'_i$; that is, $x_j = 1$ if and only if $j \in C'_i$. Then, the problem above can be simplified as

$$z = \max_{x \in \{0,1\}^n} \frac{\sum_{j=1}^n V_j R_j x_j}{\sum_{j=1}^n V_j x_j + V_0}$$

(3.1)

From the literature, we know that assortment optimization problem has some very nice properties under MNL model, and we can thus solve it efficiently [23, 35, 48]. Based on these works, we develop an easier solving algorithm for our setting:

**Proposition 1.** For optimization problem (3.1), the optimal solution $x^*$ and optimal value $z^*$ can be solved by the following iterative algorithm:

$$x^k = \arg \max_x \sum_{j=1}^n V_j (R_j - z^k) x_j$$

$$z^{k+1} = \frac{\sum_{j=1}^n V_j R_j x^k_j}{\sum_{j=1}^n V_j x^k_j + V_0}$$

with $z_0 < z^*$.

**Proof.** Let

$$f(z) = \max_{x \in \{0,1\}^n} \sum_{j=1}^n V_j (R_j - z) x_j$$

We first prove that to solve problem (3.1) is equivalent to solve equation $V_0 z = f(z)$, and then show the iterative algorithm can solve this equation.

Assume optimal solution and value of (3.1) are $x^*$ and $z^*$, respectively. We then have

$$V_0 z^* = \sum_{j=1}^n V_j (R_j - z^*) x_j^*$$

By the optimality of $x^*$ and $z^*$, we know that the right hand side is optimal (maximized) for every $x \in \{0,1\}^n$; if not, then we have some $x'$ such that $\sum_{j=1}^n V_j (R_j - z^*) x'_j > V_0 z^*$, which means we have

$$\frac{\sum_{j=1}^n V_j R_j x'_j}{\sum_{j=1}^n V_j x'_j + V_0} > z^*,$$
a contradiction. Thus, the above equation can be written as

\[ V_0z^* = \max_{z \in \{0,1\}^n} \sum_{j=1}^n V_j(R_j - z^*)x_j = f(z^*) \]

On the other hand, if we have \( V_0z^* = f(z^*) \), we know that for every \( x \in \{0,1\}^n \),

\[ \frac{\sum_{j=1}^n V_j R_j x_j}{\sum_{j=1}^n V_j x_j + V_0} \leq z^* \]

thus \( z^* \) is an upper bound on the right-hand side of (3.1). But there is also some \( z^* \) that reach equality, so we must have (3.1) holds for \( z^* \). That concludes the proof for the first part.

For the second part, we need to prove that the iterative algorithm given solves the equation \( V_0z = f(z) \). First of all, we notice that for a given \( z \), \( f(z) \) is easy to calculate: it is obvious that all \( j \) with \( R_j > z \) should be selected and all \( j \) with \( R_j < z \) should not be selected.

Then, we notice that \( f(z) \) is monotonically decreasing in \( R^+ \), while the left hand side \( V_0z \) is monotonically increasing. Given \( f(0) > 0 \), we can start from \( z^0 = 0 \) and apply Newton method to solve for optimal \( z \) and \( x \):

\[
\begin{align*}
x^k &= \arg \max_x \sum_{j=1}^n V_j(R_j - z^k)x_j \\
z^{k+1} &= z^k - \frac{f(z^k) - V_0z^k}{f'(z^k) - V_0} = \frac{\sum_{j=1}^n V_j R_j x_j^k}{\sum_{j=1}^n V_j x_j^k + V_0}
\end{align*}
\]

### 3.3.2 Price Optimization

In this part, we consider the profit maximization problem with decision variables being fare adjustment \( \delta_j \in R \). To make the problem tractable, we assume that the corresponding elasticity \( \partial U_j / \partial \delta_j = E_j(\delta_j) \) is continuous, monotonically decreasing, and satisfies regularity condition \( E_j(0) = 0 \). Then, the problem becomes

\[
z = \max_{\delta \in R^d} \sum_{j=1}^n p_j(\delta) R_j = \max_{\delta \in R^d} \frac{\sum_{j=1}^n V_j e^{E_j(\delta_j)}(f_j - c_j + \delta_j)}{\sum_{j=1}^n V_j e^{E_j(\delta_j)} + V_0} \tag{3.2}
\]
where \( V, f_j, c_j, E_j \) are known. Again, price optimization has some good structural properties under MNL model [3, 16, 27, 47], though the algorithms induced can be inefficient. Later, people find that, under certain conditions, this problem has nicer property [28] and efficient algorithms can be designed [1, 5], by using the same technique as that in assortment optimization to decouple the formulation. We compile these existing results and summarize them into following proposition:

**Proposition 2.** The optimization problem (3.2) is equivalent to equation

\[
V_0 z = \max_{\delta \in \mathbb{R}^n} \sum_{j=1}^{n} V_j e^{E_j(\delta_j)} (f_j - c_j - z + \delta_j) 
\]

(3.3)

**Proof.** Assume optimal solution and value of (3.2) are \( \delta^* \) and \( z^* \), respectively. We then have

\[
V_0 z^* = \sum_{j=1}^{n} V_j e^{E_j(\delta_j)} (f_j - c_j - z^* + \delta_j)
\]

By the optimality of \( \delta^* \) and \( z^* \), we know that the right hand side is optimal (maximized) for every \( \delta \in \mathbb{R}^n \); if not, then we have some \( \delta' \) such that \( \sum_{j=1}^{n} V_j e^{E_j(\delta'_j)} (f_j - c_j - z^* + \delta'_j) > V_0 z^* \), which means we have

\[
\frac{\sum_{j=1}^{n} V_j e^{E_j(\delta'_j)} (f_j - c_j + \delta'_j)}{\sum_{j=1}^{n} V_j e^{E_j(\delta'_j)} + V_0} > z^*,
\]

a contradiction. Thus, the above equation can be written as

\[
V_0 z^* = \max_{\delta \in \mathbb{R}^n} \sum_{j=1}^{n} V_j e^{E_j(\delta_j)} (f_j - c_j - z^* + \delta_j)
\]

On the other hand, if we have (3.3) holds, we know that for every \( \delta \in \mathbb{R}^n \),

\[
\frac{\sum_{j=1}^{n} V_j e^{E_j(\delta'_j)} (f_j - c_j + \delta'_j)}{\sum_{j=1}^{n} V_j e^{E_j(\delta'_j)} + V_0} \leq z^*,
\]

thus \( z^* \) is an upper bound on the right-hand side of (3.2). But there is also some \( \delta^* \) that reach equality, so we must have (3.2) holds for \( z^* \). That concludes the proof.
With proposition 2, we have a special case that admit easy algorithm for optimal solution:

**Corollary 3.** For pricing problem with piecewise linear elasticity $E_j(x) = -e_{j1}x I(x < 0) - e_{j2}x I(x > 0)$ and $e_{j2} > e_{j1} > 0$ for all $j$, the optimal objective value $z^*$ can be solved by the following iterative algorithm:

$$
\delta_j^{k+1} = \begin{cases} 
z^k - f_j + c_j + 1/e_{j1} & z^k - f_j + c_j + 1/e_{j1} < 0 \\
z^k - f_j + c_j + 1/e_{j2} & z^k - f_j + c_j + 1/e_{j2} > 0 \\
0 & \text{otherwise}
\end{cases}
$$

$$
z^{k+1} = \frac{\sum_{j=1}^n V_j e^{f_j(\delta_j^{k+1})} (R_j + \delta_j^{k+1})}{\sum_{j=1}^n V_j e^{f_j(\delta_j^{k+1})} + V_0}
$$

**Proof.** The update on $z$ is simply the Newton method step, the same as that in proposition 1. To prove this corollary, we only need to prove that the update on $\delta$ solves the optimization problem on the right-hand side of (3.3). Since the problem is decoupled, for each option $j$, the optimal fare adjustment achieves

$$
\delta_j^{k+1} = \arg\max_{\delta_j \in R} e^{E_j(\delta_j)}(f_j - c_j - z^k + \delta_j)
$$

By taking derivative on both $\delta_j > 0$ and $\delta_j < 0$ parts, we know the optimal $\delta_j^{k+1}$ is given by

$$
\delta_j^{k+1} = \begin{cases} 
z - f_j + c_j + 1/e_{j1} & z - f_j + c_j + 1/e_{j1} < 0 \\
z - f_j + c_j + 1/e_{j2} & z - f_j + c_j + 1/e_{j2} > 0 \\
0 & \text{otherwise}
\end{cases}
$$

### 3.3.3 Relationship between Assortment and Price optimization

In this subsection, we deviate a little bit to discuss the relationship between the two models introduced above. Traditionally, there has been few discussion on this, since they are designed for different problems; though there are some studies that
try to model the pricing problem with assortment optimization [12], which provides a more efficient solution when the elasticity function is highly nonlinear. But in our problem setting, both are possible approaches, and a comparison between the two is necessary. In fact, it is not hard to show that, under MNL model, joint optimization in assortment and price is equivalent to solely price optimization; thus price optimization always provides better performance.

**Proposition 4.** Consider the following optimization problem (joint assortment and pricing)

\[
z = \max_{x \in \{0,1\}^n, \delta \in \mathbb{R}^n} \frac{\sum_{j=1}^{n} V_j e^{E_j(\delta_j)} (f_j - c_j + \delta_j) x_j}{\sum_{j=1}^{n} V_j e^{E_j(\delta_j)} x_j + V_0}
\]  

(3.4)

This problem is equivalent to pricing problem (3.2).

**Proof.** Assume the optimal value and solution for (3.3) is \(z^*\) and \((x^*, \delta^*)\), respectively. Assume for option \(j_0\), \(x^*_j = 0\). Then, for any pair \((x^*_j, \delta^*_j)\), we should have

\[
\begin{align*}
\sum_{j=1}^{n} V_j e^{E_j(\delta_j)} (f_j - c_j + \delta_j) x^*_j &= \sum_{j=1}^{n} V_j e^{E_j(\delta_j)} x^*_j + V_0 \\
&= \sum_{j=1}^{n} V_j e^{E_j(\delta_j)} (f_j - c_j + \delta_j) x^*_j + V_{j_0} e^{E_j(\delta_{j_0})} z^* x_{j_0} \\
&= \sum_{j=1}^{n} V_j e^{E_j(\delta_j)} x^*_j + V_{j_0} e^{E_j(\delta_{j_0})} x_{j_0} + V_0
\end{align*}
\]

And it is clear that if we let \(x_{j_0} = 1, \delta_{j_0} > z^* - f_{j_0} + c_{j_0}\), we will have

\[
z' = \frac{\sum_{j=1}^{n} V_j e^{E_j(\delta_j)} (f_j - c_j + \delta_j) x^*_j + V_{j_0} e^{E_j(\delta_{j_0})} (f_{j_0} - c_{j_0} + \delta_{j_0}) x_{j_0}}{\sum_{j=1}^{n} V_j e^{E_j(\delta_j)} x^*_j + V_{j_0} e^{E_j(\delta_{j_0})} x_{j_0} + V_0} \geq z^*
\]

Thus, in optimal solution \((x^*, \delta^*)\), all \(x^*_j\) should equal to 1. Then the problem reduces to price optimization.

### 3.3.4 Daily-level Optimization

The assortment optimization and price optimization frameworks introduced above only focus on profit at request level; this can be very myopic, for example, it prefers to provide single service which generates higher unit profit, even though it consumes more supply capacity. To mitigate such problems, we should consider approaches that address subsequent effects of the current decision, or, in literature, that are
“non-myopic”.

Now, let us consider a single decision step. Assume that currently the operator is given the state $X$, which is described by the current time $t$, and the location of its vehicles and their future plans, including where to pickup and dropoff. Then comes a random new request $w_1 \sim P_1(\cdot | X)$ with the corresponding feasible subset $C(w_1) = \{1, \ldots, n\}$. Consider possible control $u$ in set $U(C)$ (e.g. for assortment problem, $U(C) = \{0, 1\}^n$; for pricing problem, $U(C) = R^n$), and stochastic traveler choice behavior $w_2 \sim P_2(\cdot | u)$, we have the following Bellman equation [10], which characterizes the request-level optimal control considering impact on future:

$$V^*(X) = E_{w_1} \left\{ \max_{u \in U(C)} E_{w_2} \left\{ R(u, w_1, w_2) + V^*(f(X, u, w_1, w_2)) \right\} \right\}$$

while the optimal control is given by

$$u(X, w_1) = \arg \max_{u \in U(C)} E_{w_2} \left\{ R(u, w_1, w_2) + V^*(f(X, u, w_1, w_2)) \right\}$$

where $R$ is the profit function of a specific request, and $f$ is the state transition function.

Notice that the two optimization models in previous sections can be expressed in an abstract way as

$$u(X, w_1) = \arg \max_{u \in U(C)} E_{w_2} \{ R(u, w_1, w_2) \};$$

thus, to reach daily-level optimum, we only need to estimate future value function $V^*(f(X, u, w_1, w_2))$. We notice that this notion of future value is common in a variety of transportation management problem: for example, pricing problem under vehicle routing problem framework [22]; dial-a-ride problems (DARP) [30]; and joint dispatching and pricing problem [45]. But in general, the optimal function $V^*$ is hard to find, and these studies all develop problem-specific approaches. Sayarshad and Chow [45] provide an exhaustive summary of recent attempts in this line.

In this work, since $w_2$ is the realization of a selection, the request-level optimal
control problem can be further decompose into option level. That is, we want to consider function $V_j(X, Y_j(w_i), u)$ for option $j$ (where $Y_j(w_i) = (y_j, f_j, \delta_j, c_j, T_j)$ is all information contained in option $j$ for demand $w_i$) and $V_0(X)$ for not choosing any option, and optimize

$$u = \arg \max_{u \in U(C)} \left( \sum_{j=1}^{n} p_j(u)[R(Y_j, u) + V_j(X, Y_j, u)] + p_0(u)V_0(X) \right)$$

$$= \arg \max_{u \in U(C)} \left( \sum_{j=1}^{n} p_j(u)[R(Y_j, u) + V_j(X, Y_j, u)] + (1 - \sum_{j=1}^{n} p_j(u))V_0(X) \right)$$

$$= \arg \max_{u \in U(C)} \sum_{j=1}^{n} p_j(u)[R(Y_j, u) + V_j(X, Y_j, u) - V_0(X)]$$

where $p_j(u)$ is the selection probability for option $j$ under control $u$. Now, we can further define opportunity cost function $A_j$ to replace the value function $V_j$:

$$A_j(X, Y_j, u) = V_0(X) - V_j(X, Y_j, u)$$

If we have accurate information of $A_j$, we can solve the non-myopic optimal control problem easily, applying the same algorithms introduced in proposition 1 and corollary 3. But the estimation of $A_j$ is very difficult in general, as mentioned above. In this work, we consider a different approach. Since specific knowledge on $A_j$, no matter accurate or not, combining with request-level assortment/price optimization, provides us a specific strategy in making request-level decisions ("rollout" policy with respect to $A_j$ in DP/RL community), we can consider some parametric representations of $A_j(X, Y_j, u|\theta)$, and define corresponding rollout policy $\pi_\theta$. Then, we try to optimize the daily-level profit with respect to these parametric policies. Though such approach restricts the policy space, and thus might provide suboptimal policy, this is tractable and has very good performance in general.

Next, we discuss the details on how we design the parametric representation of $A_j$. Since this function can be interpreted as the expected cost of consuming some travel capacities for a certain period, this should only relate to the local information of the request. Thus, we can consider approximations $\hat{A}_j(\hat{X}, Y_j, u|\theta)$, where $\hat{X}$ is
a simple representation of local state of request, including following factors: local outflow intensity \( \lambda_{ot} \) at origin \( o \), local outflow intensity \( \lambda_{dt} \) at destination \( d \), local supply capacity at origin \( s_{ot} \), and local unit travel time at both origin and destination \( t_{ot}, t_{dt} \) (which is characterized by average travel speed in nearby links). That is, \( \hat{X} = (\lambda_{ot}, \lambda_{dt}, s_{ot}, t_{ot}, t_{dt}) \). We further consider \( \hat{A}_j \) to be dependent on service type, and linear in parameter \( \theta = (\theta_s, \theta_{sh}) \) in each case:

\[
\hat{A}_j(\hat{X}, Y_j, u|\theta) = \begin{cases} 
\theta_s^T \hat{X} & y_j = \text{single} \\
\theta_{sh}^T \hat{X} & y_j = \text{shared}
\end{cases}
\]

To summarize, in request level, we reduce to solve

\[
u = \arg \max_{u \in U(C)} \sum_{j=1}^{n} p_j(u)[R(Y_j, u) - \hat{A}_j(\hat{X}, Y_j, u|\theta)]
\] (3.5)

Given a specific \( \theta \), we can use simulation with the corresponding rollout policy \( \pi_\theta \) to evaluate the daily-level profit function \( V(\theta) \); then we can use any related optimization methods, such as those of stochastic optimization, to optimize \( \theta \).

From (3.5), we know that the request-level optimal control has the same form as assortment/price optimization, and their properties carry over. One important corollary is that, the proof of proposition 4 can also be applied in this setting. That means, in daily level, optimal price optimization is equivalent to optimal joint assortment and price optimization. Thus, we only need to focus on the optimal price optimization when comparing with myopic strategies. Nevertheless, we should notice that, in daily level, myopic price optimization will not always have better performance over myopic assortment optimization.

Before we continue to the detail in optimizing \( \theta \), we should mention that elements of \( \hat{X} \) are in different scales; thus to improve optimization efficiency, we will do normalization for each element. That is, for each element, we divide it by the global average of that element. For example, for demand distribution \( \lambda_{ot} \) at location \( o \) and time \( t \), we normalize it by \( S \times T/M \) where \( S \) is the size of the region (in \( km^2 \)), \( T \) is
the total time steps (in mins), and $M$ is the total demand size during the targeted time period.

To optimize $\theta_j$, in this work we have tried several approaches, and we mostly focus on episodic update methods; that is, for a given $\theta_j$, we use episodic simulation to evaluate policy $\pi_\theta$, and then use the evaluation to update $\theta_j$. In computation we find that naive implementation of policy gradient method, such as REINFORCE [53], has a major drawback: it is very likely to fall into local optima. Since our representation of policy is of low dimension, we can consider (global) stochastic optimization approach. Specifically, we use Covariance Matrix Adaptation Evolution Strategy (CMA-ES) method [25, 26]. For the completeness of this thesis, we describe the process as below:

$$
\langle \theta \rangle_t^t = \frac{1}{\mu} \sum_{i \in \text{best } \mu} \theta_i^t
$$

$$
\theta_{t+1}^k = \langle \theta \rangle_t^t + \sigma_t \cdot B^t D^t z_{t+1}^k, \forall k \in 1, \cdots, \lambda
$$

$$
\langle z \rangle_{t+1} = \frac{1}{\mu} \sum_{i \in \text{best } \mu} z_{t+1}^i
$$

$$
p_{\sigma}^{t+1} = (1 - c_\sigma)p_{\sigma}^t + \sqrt{c_\sigma(2 - c_\sigma)} \cdot c_w \cdot B^t(z)^{t+1}_{\mu}
$$

$$
\sigma^{t+1} = \sigma^t \cdot \exp\left(\frac{1}{d_\sigma} \cdot \frac{||p_{\sigma}^{t+1}||_2 - \hat{x}_n}{\hat{x}_n}\right)
$$

$$
p_{C}^{t+1} = (1 - c_C)p_{C}^t + \sqrt{c_C(2 - c_C)} \cdot c_w \cdot B^t D^t(z)^{t+1}_{\mu}
$$

$$
Z_{t+1} = B^t D^t \left(\frac{1}{\mu} \sum_{i \in \text{best } \mu} z_{t+1}^i [z_{t+1}^i]^T \right) (B^t D^t)^T
$$

$$
C^{t+1} = (1 - c_{\text{cov}})C^t + c_{\text{cov}}(\alpha_{C} p_{C}^{t+1} [p_{C}^{t+1}]^T + (1 - \alpha_{C})Z^{t+1})
$$

in which $C^t = (B^t D^t)(B^t D^t)^T = B^T (D^t)^2 (B^t)^T$, $B^t$ is a unitary matrix and $D^t$ is a diagonal matrix. Given sample size $\lambda$ (normally set to be $10n$, where $n$ is the dimension of $\theta$), we can set the following parameters:

$$
\mu = \frac{\lambda}{4}, c_C = c_\sigma = \frac{4}{n + 4}, c_w = \sqrt{\mu}, d_\sigma = c_\sigma^{-1} + 1 = \frac{n + 8}{4}, \alpha_{\text{cov}} = \frac{1}{\mu},
$$

$$
c_{\text{cov}} = \alpha_{\text{cov}} \frac{2}{(n + \sqrt{2})^2} + (1 - \alpha_{\text{cov}}) \min(1, \frac{2\mu - 1}{(n + 2)^2 - \mu}), \hat{x}_n = \sqrt{n} \left(1 - \frac{1}{4n} + \frac{1}{21n^2}\right)
$$
Chapter 4

Case Study

In this section, we will design a case study to validate our algorithm, and to analyze its impact. We will first introduce the basic setting, along with several evaluation models; we then move to the actual data and determine parameters for a specific case.

4.1 General Framework

Since our model involves complex operation rule and high dimension of uncertainty, in this work, we develop a simple simulation system to assess the performance of our algorithm. Following is a figure describing the structure of this simulation system:

![Simulation System Flow Chart](image)

Figure 4-1: Simulation System Flow Chart

On the other hand, since we need to use simulation to train our algorithm, the
system cannot be too complicated. In this work, we make several simplifications, listed as follows:

(1) We consider square grid as the base map, where each link is 1km. With this simple network, we can simplify the computation requirement in searching for the route, this improves efficiency in simulation rollout. Later we can see that, this assumption is not too far from the background city we are considering.

(2) We discretize the time into minutes and simulate for one day at each time (1440 steps). With fixed discrete time, the computation steps become clear; and we don't need to design module to record system time specifically. Surely this will lead to some biases, especially in dynamic travel time setting; but the gain in computation efficiency is not negligible, so we stick to this setting.

(3) In dynamic travel time setting, we need to consider the impact of trips that stay in original travel mode. To simplify the computation, we assume all the potential request only consider non-sharable ground transportation service as the original travel mode; that is, the possible alternatives only include driving or taking a taxi. The limitation for not considering the influence of the public transit can be overcome in later research, as there is more data, and we can quantify both the traveler's behavior in such case and these services' impact on transportation network density.

To conduct this case study, there are several modules that require further details or external parameter as input:

(1) the choice model specification to compute traveler choice behavior;
(2) the fare and cost structure for the agent to control and update statistics;
(3) the operation rules, including searching radius for feasible vehicles, and detour limit;
(4) traffic density-time model to update system link travel time;
(5) the design of public regulations.

We will discuss each in subsequent sections.
4.1.1 Choice Model Specification

In the previous section, to focus on the optimization part, we only briefly introduce the choice model form, and have not discussed details of the utility function. In this subsection, we will cover this missing part.

For simplicity, we follow [4] and consider only the impact of service type/travel mode, travel time, and fare/cost. We also define the utility value with regard to the monetary unit, so we will be able to select realistic parameters for our model. Recall that for option \( j \), the travel time is \( T_j \), standard fare/cost is \( f_j \), and the fare adjustment is \( \delta_j \); the detailed equations are then given as below:

\[
U_j = \begin{cases} 
\mu(A SC_s - VOT \cdot T_j - f_j - E(\delta_j)) & y_j = \text{single} \\
\mu(A SC_{sh} - b_{v,sh} VOT \cdot T_j - f_j - E(\delta_j)) & y_j = \text{shared} 
\end{cases}
\]

\[
U_o = \mu(A SC_o - VOT \cdot T_o - b_{f,o} f_o)
\]

\[
U_n = \mu A SC_n
\]

where \( \mu \) is the scale factor; constant \( A SC \) characterize the utility of option that is not captured by the distance and the fare, e.g., the value of the trip, the comfort of the travel mode; factor \( VOT \) characterizes the average value of time of this population; \( E \) is the elasticity function on price adjustment \( \delta_j \), and in this work we define it to be a piecewise linear function

\[
E(x) = \begin{cases} 
e_1 x & x < 0 \\
e_2 x & x \geq 0 
\end{cases}
\]

with \( e_2 > e_1 > 0 \). This specification means that people are more sensitive towards price surge than price reduction.

There are two other parameters in the model that have physical concerns. Factor \( b_{v,o} \) characterize the relative time variability of shared service comparing with single service and original travel mode (e.g., driving, taxi); when given an estimated travel time, the one for the shared service is generally less reliable, and people normally
will consider this in their decision-making process. In this work, we select this factor according to the empirical ratio between the reliability of estimated travel time for shared and single service. That is, assume on average, the ratio between actual travel time and estimated travel time for shared service is $\eta_{sh}$, and the one for single service is $\eta_s$, we empirically select $b_{v,s}$ to be $\eta_{sh}/\eta_s$.

Another factor $b_{f,o}$ characterizes the average cost of original travel mode comparing with driving, and we use this factor to set different setting for the population: a lower $b_{f,o}$ represents a lower cost of staying in original travel mode, and the launch of the new service has less impact; while a higher $b_{f,o}$ represents a setting that original travel mode is costly and the new service can have greater impact.

We will defer the setting of actual parameters to next section, where we already introduce the data and realistic setting we are using.

### 4.1.2 Speed-density Model

When the market exposure rate of the service becomes large, the operation begins to have a significant impact on travel time, and thus not negligible in analyzing the system dynamics. In this work, to address this connection, we use a very basic triangular flow-density model [39] in settings where we need to calculate dynamic traffic time, i.e., dynamic traffic time settings. Specifically, we consider free flow density limit $k_m$, congestion density limit $k_c$, and free flow speed $v_m$, and consider the following flow-density relationship between flow $q$ and density $k$:

$$q = \begin{cases} 
    v_mk & k < k_m \\
    \frac{k-k}{k_c-k_m}v_mk_m & k_m < k < k_c
\end{cases}$$

we can then convert this to link travel time $T$, by the relation $q = vk = k/T$

$$T = \begin{cases} 
    1/v_m & k < k_m \\
    \frac{(k-c-k_m)k}{(k_c-k_m)v_m} & k_m < k < k_c
\end{cases}$$

38
We can set the parameters $v_m, k_m, k_c$ directly, without going to the actual context; currently, we assume $v_m = 60\text{km/h} = 1\text{km/min}$, $k_m = 1\text{veh/36m} = 28\text{veh/km}$ and $k_c = 1\text{veh/6m} = 167\text{veh/km}$, which seems to be reasonable in the context of city transportation network. Since the scale of $k$ does not matter (we can always scale the final travel time back), in implementation we let $k_m = 1$ and $k_c = 6$ to simplify computation.

The remaining thing to obtain an estimate of the travel time is how to combine the traffic density in the simulation with the background traffic density (which can be referred from the background travel time). We adopt a very simplified approach: to combine the two linearly. For time $t$ and link $l$, the estimated travel density $\hat{k}_{tl}$ is calculated as

$$\hat{k}_{tl} = (1 - \varphi)k_{tl} + \frac{1}{k_0}k'_{tl}$$

where $k_{tl}$ is the density derived from background travel time, $k'_{tl}$ is the density of vehicle/trips in the simulator, and $\varphi$ is the market exposure rate of the operator’s service (which quantify how much share of the population cannot be represented by the simulation process). Conversion factor $k_0$ is a parameter to make $k'_{tl}$ and $k_{tl}$ in the same scale, and in actual implementation, there is no consistent method to select it; in this work, we simply calibrate it to make the background travel time distribution consistent across different market exposure rate $\varphi$. $k_0$ for specific setting will be reported along with the result, in the next chapter.

4.1.3 Public Policy: Tax Reduction

One interesting extension of this work is how public sector manager can impose adaptive regulation that incentivizes the private operator to boost social welfare. As a preliminary, in this work, we can consider some fixed regulation schemes as settings, and see how such regulations can change the operator’s behavior. The results can then be compiled into later research.

Before we get to design the policy, we need first to identify several targets a public sector manager might care about. One thing we should certainly consider is capacity
provided, which most research in shared mobility is focusing on; and another very important factor is congestion level, since the operation is very likely to change the travel time distribution. In this work, we will solely focus on these two targets. We should notice that there is some tradeoff between them; when there is more capacity provided, there will be naturally more trips, thus lead to higher congestion level.

Since in this work we are focusing on the stable operating stage, to have impacts on the operator, we should select some trip based policy. In this work, we will focus on **dynamic tax deduction**, which tries to incentivize the operator to provide ride-sharing capacity in situations that are highly congested or with high demand volume. To simplify the implementation, we consider piecewise linear function $TRD(\lambda_{ot}, \lambda_{dt})$ on outflow demand near origin and destination $\lambda$ at time $t$, and function $TRC(dT)$ on extra travel time $dT$, which is the difference between current estimated travel time $T$ and the one in free flow condition $T_0$:

\[
TRC(dT) = s_c \cdot (dT - \gamma_0 T_0) \cdot I(dT > \gamma_0 T_0)
\]

\[
TRD(\lambda_{ot}, \lambda_{dt}) = s_\lambda \cdot (\lambda_{ot} + \lambda_{dt} - \Lambda_0) \cdot I(\lambda_{ot} + \lambda_{dt} > \Lambda_0)
\]

where $I(\cdot)$ is the indicator function. In short, these functions are only activated when congestion or high demand exceeds the threshold. Again, we will determine those parameters, including thresholds $\gamma_0$ and $\Lambda_0$ and coefficient $s_c$ and $s_\lambda$, in next section, where we have an actual context.

### 4.2 Implementation

In this work, we set our simulation setting with respect to a middle size city Langfang in China. Following sections describe with details how we compile existing data as simulation inputs, and how we select parameters based on city’s economic statistics and regulation policies.
4.2.1 Data Description and Processing

Our dataset consists of real trajectory data of private trips in Langfang central region in a normal day. There are ~250,000 records in total, and each record is a tuple of: (series of location identified code, series of time stamps in seconds). One example is (53-54-07-34,125773-126469-126584-126698). A map of location identified code and the real map of the city are shown below:

![Map](image)

(a) Location identified code  (b) Corresponding map

Figure 4-2: Data Location

Given the scale of the city (central region ~ $7km \times 7km$), and the resolution of the network given, we will use a $10 \times 10$ grid in our simulation (recall each link has length $1km$). We can then map each original node into nodes in the grid, and transform the records into the new format (series of location node ID, series of time stamps in seconds).

Assuming these records are in the same trip distribution as the ground truth, we can generate demand distributions from them, and later use such distributions as demand generator in the simulation system.

The process of generating demand distribution can be described as follows: first, we reduce each trajectory record into trip origin-destination (OD) record, in the form
of tuple (start time in minutes, end time in minutes, origin node ID, destination node ID) (notice we also simplify time resolution from seconds to minutes). Then, to reduce storage requirement and simplify subsequent computation, we decompose OD pair into outflow distribution and inflow distribution, which are accumulated with the tuple (start time, origin node ID) and the tuple (end time, destination node ID), respectively. Finally, we smoothen these distributions using a uniform filter with radius to be 3km, and rescale them (with a single factor) to make the number of total trips per day smaller, so that we can reduce computation requirement, while not have too much variance in the results. Notice that this rescale process is directly connected with total demand size \( M \). After this processing, the resulted distribution will not be consistent with ground truth request distribution, but we should be fine if we just use it as input for simulation. The following figure shows the time series of demand (aggregated across different ODs), after rescaling with \( M \sim 11,000 \):

![Demand Size Time Series](image)

**Figure 4-3: Demand Size Time Series**

In the simulation process, we can use these distributions in the following way: in each time step (minute), we average the total number of inflow and outflow to obtain the number of OD requests we want to generate; and then for each request, we draw the origin and destination independently from the corresponding normalized
distributions.

We can also use these records to recover the background travel time distribution. Notice that, for this task, we do not need to assume that the observation is completely consistent with the ground truth. This time, we decompose each trajectory record into segments of link travel records (start time in seconds, end time in seconds, link ID). Then we can transform each record into link travel time record (link ID, trip time in minutes, travel time in minutes) by making the trip time element as the average of the start time and the end time. Finally, we obtain the link travel time distribution by averaging the travel time elements for same (link ID, trip time) tuple, and further extrapolating and smoothing with uniform filter across space and time. That is, if there is a missing tuple (link ID, trip time), we use the average value of nearby available records as the estimates; after filling all the missing tuple, we apply the uniform filter to further smooth the whole distribution.

4.2.2 Parameter Selection

First, we select the parameters for operations. In this work, we simplify the calculation of cost and fare. Cost is mainly related to distance \( d \):

\[
c = c_d \cdot d
\]

where parameter \( c_d \) is the unit distance cost. In this work, we set this cost to be consistent with fuel cost, with respect to the situation in Langfang. In China, fuel price is about $1 per liter (L) [52], and as current Chinese standard fuel efficiency is 7L/100km [50], this factor is approximately $0.07/km.

Both fare for single service \( f_s \) and fare for shared service \( f_{sh} \) are related to distance \( d \) and time \( t \):

\[
f_s = r_b + r_d \cdot d + r_t \cdot t
\]

\[
f_{sh} = (r_b + r_d \cdot d + r_t \cdot t)r_{sh}
\]

where \( r_b \) is the base fare for each trip, \( r_d \) is the unit distance fare, \( r_t \) is the unit time fare, and \( r_{sh} (< 1) \) is the uniform fare ratio between shared service and single service.
Again, we try to make the setting closer to that in Langfang, by referring to the taxi fare structure in the city [51]: currently, in Langfang, base fare per trip is $1, unit distance fare is $0.25/km, and unit waiting time fare $0.04/min. If we further assume on average 25% of total travel time is waiting time, we can use $0.01/min as an approximation for unit travel time fare.

For other operations rules, we have following setting: we define the max sharing capacity of parties to be 3, and the maximum waiting time for offer \( T_d = 5 \text{mins}. \) In finding feasible routes, for empty vehicles, we consider a feasible radius of 5km, and for vehicles available to share we consider a feasible radius of 2km. We also restrict that the maximum detour of vehicles available to share (the difference between distance/travel time by shared service and sum of those of separate routes) should not be over 2km and 5mins.

Next, let us look at the choice model specification. Recalled the choice model is in the form

\[
U_j = \begin{cases} 
\mu(ASC_s - VOT \cdot T_j - f_j - E(\delta_j)) & y_j = \text{single} \\
\mu(ASC_{sh} - b_{v,sh}VOT \cdot T_j - f_j - E(\delta_j)) & y_j = \text{shared} 
\end{cases}
\]

\[U_o = \mu(ASC_o - VOT \cdot T_o - b_{f,o}f_o)\]

\[U_n = \mu ASC_n\]

where \( E \) is a piecewise linear function

\[E(x) = \begin{cases} 
e_1 x & x < 0 \\
e_2 x & x \geq 0 
\end{cases}\]

with \( e_2 > e_1 > 0. \)

To select a reasonable VOT value, we retrieve several records on the income level in the city of Langfang, or nearby regions. From a report [49], in 2015, the average salary in the central region of Langfang is about $7,500/year, or about $20/day; another study [54] claims that, in 2011, the average hourly salary in Hebei province is about 2.55€. On the other hand, it is indicated [55] that the lowest daily income
rate at Langfang, at 2016, is about $2.5/hour. Combining these statistics, we think it is reasonable to set VOT as $1.8/hour, or $0.03/min.

For $b_{v,sh}$, we empirically set it to be $\eta_{sh}/\eta_{s}$, as discussed before; with simulation, we find this ratio on average is about 1.2.

For the fare elasticity component, we select $e_2 = 2$, $e_1 = 1$. This means, traveler regard price reduction as normal fare change, while price surge as an unexpected event and it should generate more disutility.

For other parameters, we select scale factor $\mu = 0.5$, following [4]; constant factors $ASC_s = 4.5$, $ASC_{sh} = 4$, $ASC_o = 5$, $ASC_n = 0$, based on our experience; and fare adjusting factor $b_{f,o}$ is subjected to change under different analyzing settings.

Finally, we will discuss the parameters of the tax reduction. Recall the trip-based tax reduction is in the following form

$$TRC = s_c \cdot (dT - \gamma_0 \bar{T}_0)^+$$
$$TRD = s_A \cdot (\lambda_t + \lambda_{d_t} - \Lambda_0)^+$$

For the congestion part, we select $s_c$ similar to the unit waiting time fare in taxi fare structure, which is $0.04/min$; since we want to restrict the effect of this tax reduction (in normal case waiting time should not be given any tax reduction), we should reduce this value by some factor. With trial and error, we find $0.025/min$ is a good selection, in that it would not become a large part of the operator’s profit but still has a significant impact ($\sim 5\%$). We also select $\gamma_0 = 1.5$, which means this tax reduction is only effective when the travel time is 2.5 times of the free flow condition, or less than 24km/h.

For the demand part, we try to select parameters based on the observation data. If we normalize all the outflow demand $\lambda_t$ by the overall average value (that is, the average of normalized demand equals 1), the 80%, 90%, 100% quantile of the normalized demand is about 2.4, 3.4, 11, respectively. To make sure that the tax reduction only targets a small fraction of situations, but still has some significance in those cases, we select the threshold for the demand to be 3.5, that is, slightly above
the 90\% quantile, and thus $\Lambda_0$ to be $2 \times 3.5 = 7$. Next we try to set the factor $s_\lambda$ to make this tax reduction a certain fraction of total fare. As the average trip in this simulation system has distance $5\text{km}$ and travel time $15\text{mins}$, average standard fare is approximately $1 + 0.25 \times 5 + 0.01 \times 15 = 2.4\$$. And from the data, the average of normalized demand that are over $3.5$ is about $4.5$, the average of normalized $\lambda_{ot} + \lambda_{dt}$ is about $2 \times 4.5 = 9$. if we want to have the tax reduction of high demand to be about $20\%$ of standard fare in average case, rate should be about $2.4\$ \times \frac{0.2}{(9 - 7)} = 0.24\$$. In this case, we select $s_\lambda$ to be $0.25\$. 

Finally, we use table 4.1 to summarize all parameters. TBD represents unknown parameters at this point, and will be selected in next chapter, in a case-specific manner.
### Simulation Setting

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Map size</td>
<td>$S = 10 \times 10 \text{ km}^2$</td>
</tr>
<tr>
<td>Total time steps</td>
<td>$T = 1440 \text{ mins}$</td>
</tr>
<tr>
<td>Maximum waiting time for offer</td>
<td>$T_d = 5 \text{ mins}$</td>
</tr>
<tr>
<td>Fleet size</td>
<td>$N = \text{TBD}$</td>
</tr>
<tr>
<td>Potential request size</td>
<td>$M = \text{TBD}$</td>
</tr>
<tr>
<td>Market exposure</td>
<td>$\varphi = \text{TBD}$</td>
</tr>
<tr>
<td>Background congestion level</td>
<td>$\psi_C = \text{TBD}$</td>
</tr>
</tbody>
</table>

### Fare and Cost Structure

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit distance cost</td>
<td>$c_d = 0.007$/km</td>
</tr>
<tr>
<td>Trip base fare</td>
<td>$r_b = 1$</td>
</tr>
<tr>
<td>Unit distance fare</td>
<td>$r_d = 0.25$/km</td>
</tr>
<tr>
<td>Unit time fare</td>
<td>$r_t = 0.01$/min</td>
</tr>
<tr>
<td>Shared service fare ratio</td>
<td>$r_{sh} = \text{TBD}$</td>
</tr>
</tbody>
</table>

### Choice Model Specification

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant factors for single service</td>
<td>$ASC_s = 4.5$</td>
</tr>
<tr>
<td>Constant factors for shared service</td>
<td>$ASC_{sh} = 4$</td>
</tr>
<tr>
<td>Constant factors for original mode</td>
<td>$ASC_o = 5$</td>
</tr>
<tr>
<td>Constant factors for trip cancellation</td>
<td>$ASC_n = 0$</td>
</tr>
<tr>
<td>Scale factor</td>
<td>$\mu = 0.5$</td>
</tr>
<tr>
<td>Elasticity for price reduction</td>
<td>$e_1 = 1$</td>
</tr>
<tr>
<td>Elasticity for price surge</td>
<td>$e_2 = 2$</td>
</tr>
<tr>
<td>Travel time variability</td>
<td>$b_{v,s} = 1.2$</td>
</tr>
<tr>
<td>Original mode cost</td>
<td>$b_{f,o} = \text{TBD}$</td>
</tr>
</tbody>
</table>

### Speed Density Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free flow speed</td>
<td>$v_m = 1\text{km/min}$</td>
</tr>
<tr>
<td>Free flow density limit</td>
<td>$k_m = 1$</td>
</tr>
<tr>
<td>Congestion density limit</td>
<td>$k_c = 6$</td>
</tr>
<tr>
<td>Density scale factor</td>
<td>$k_0 = \text{TBD}$</td>
</tr>
</tbody>
</table>

### Tax Reduction

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel time ratio threshold</td>
<td>$\gamma_0 = 1.5$</td>
</tr>
<tr>
<td>Demand threshold</td>
<td>$\Lambda_0 = 7$</td>
</tr>
<tr>
<td>TRC coefficient</td>
<td>$s_c = 0.025$/min</td>
</tr>
<tr>
<td>TRD coefficient</td>
<td>$s_{\lambda} = 0.25$</td>
</tr>
</tbody>
</table>

Table 4.1: Summary of parameters
Chapter 5

Computation Results

In this chapter, we will discuss results obtained from the case study. We will decompose the discussion in two parts: the first part focus on the operator side, and provide justification for the optimal pricing strategy developed; and the second part focus on the system side, trying to understand how the selected optimal pricing strategy might impact the performance of the transportation system, and how public policy, such as tax reduction, might change the impact. All these results are computed with our simulator/controller, which are based on C++, executed on Linux servers with 64GB, 8-core 2.0GHz CPU.

5.1 Operating Metrics and Strategies Comparison

In this section, to focus on the operator side and to compare different strategies, we will concentrate on cases with following simplifications: there is only generation effect, that is, we let $b_{f,o} = \infty$ to remove alternative $U_o$; the market exposure rate is low and the impact on the traffic time is negligible, and we avoid the computation with speed-density model and consider fixed traffic time; and we consider only the cases with no public policy intervention (i.e., tax reduction). These simplifications help us to better understand properties of different strategies and therefore we should be able to identify several most interesting ones for later discussion. They also greatly simplify computation complexities (simulation of one day costs about 1 seconds) and
we can investigate more settings.

In this case, we select potential request population \( M \) as \( \sim 11,000 \), since simulation shows that such size is enough to reduce the variance of results in simulation. We then choose the following two factors as input and analyze how their impacts on the results: shared service fare ratio \( r_{sh} \in \{0.4, 0.6, 0.8\} \), and fleet size \( N \in \{50, 100, 150, 200\} \). We select this set of \( N \) as we observe that for \( N \) to be greater than 150, the increase in the number of passengers with greater \( N \) becomes very small. And we will discuss a collection of strategies: for basic strategies, we have the one to provide all feasible options if possible (S+Sh), to only provide feasible single options (S), to only provide feasible shared options (Sh). For myopic decision strategies, we only look at myopic pricing strategy (PM). For optimal strategies, we focus on optimal assortment strategy (AO) and optimal pricing strategy (PO). We notice that there are more hybrid strategies available, such as to only provide shared options but with dynamic pricing (Sh+PM). But proposition 4 in Chapter 4 indicates that, optimal pricing strategy is equivalent to optimal joint pricing and assortment strategy, thus better than all other variants in terms of profit; thus we simply include PM and AO as compared baselines, and ignore all other possibilities, even though they might have some other superior performances in terms of other metrics, e.g., congestion alleviation effect. The following table summarizes other potential variants, for future reference:

<table>
<thead>
<tr>
<th>All options</th>
<th>Single service only</th>
<th>Shared service only</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Control</td>
<td>S+Sh</td>
<td>S</td>
</tr>
<tr>
<td>Request-level Assortment</td>
<td>AM</td>
<td>S+AM</td>
</tr>
<tr>
<td>Request-level Pricing</td>
<td>PM</td>
<td>S+PM</td>
</tr>
<tr>
<td>Daily-level Assortment</td>
<td>AO</td>
<td>S+AO</td>
</tr>
<tr>
<td>Daily-level Pricing</td>
<td>PO</td>
<td>S+PO</td>
</tr>
</tbody>
</table>

Table 5.1: Some possible strategies

In analyzing the result, we will focus on a collection of metrics. First of all, we will look at profit, revenue, and cost of the operations. We will also look at the number of passengers serviced, and the travel time and distance of the whole fleet. Finally, to assess the service quality, we will look at the average waiting time across serviced
passengers, both for offer and pickup. Waiting time for offer refers to the gap between the time of the first request, and the time of receiving first offer set; while waiting time for pickup refers to the gap between the time of trip confirmation and the time of actual pickup.

Now, we look at results under a specific setting, $r_{sh} = 0.6$ and $N = 150$, to gain an impression of how different strategies can be. These results are summarized by table 5.2. In the following tables, we will use AWT to refer to average waiting time. In this sample case, we can see that PO indeed has a higher profit comparing with other strategies, mainly because the pricing algorithm can target the population with high willingness to pay when there is an abundant supply.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>S+Sh</th>
<th>S</th>
<th>Sh</th>
<th>AO</th>
<th>PM</th>
<th>PO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit ($)</td>
<td>15601.7</td>
<td>13034.9</td>
<td>14713.1</td>
<td>16522.1</td>
<td>15602.7</td>
<td>17860.6</td>
</tr>
<tr>
<td>Revenue ($)</td>
<td>18785.2</td>
<td>16169.1</td>
<td>17629.3</td>
<td>19721.5</td>
<td>18775.4</td>
<td>21025.9</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>3183.52</td>
<td>16169.1</td>
<td>2916.26</td>
<td>3199.41</td>
<td>3172.74</td>
<td>3165.27</td>
</tr>
<tr>
<td>Serviced Passengers</td>
<td>7768</td>
<td>6045</td>
<td>7760</td>
<td>7677</td>
<td>7794</td>
<td>7616</td>
</tr>
<tr>
<td>Travel Distance (km)</td>
<td>45479</td>
<td>44774</td>
<td>41661</td>
<td>45706</td>
<td>45325</td>
<td>45218</td>
</tr>
<tr>
<td>Travel Time (h)</td>
<td>2036</td>
<td>1939</td>
<td>1925</td>
<td>2028</td>
<td>2027</td>
<td>2006</td>
</tr>
<tr>
<td>AWT, Offer (s)</td>
<td>0.90</td>
<td>2.75</td>
<td>0.35</td>
<td>1.00</td>
<td>0.82</td>
<td>0.73</td>
</tr>
<tr>
<td>AWT, Pickup (s)</td>
<td>7.20</td>
<td>8.18</td>
<td>6.71</td>
<td>7.16</td>
<td>7.12</td>
<td>7.08</td>
</tr>
</tbody>
</table>

Table 5.2: Operating statistics under fixed traffic time case with $r_{sh} = 0.6$, $N = 150$

To obtain clearer comparisons under general settings, we summarize the average percentage difference for selected strategies, compared with optimal pricing strategy (PO), across different settings of $N$ and $r_{sh}$, with table 5.3. The reason we use average percentage difference, rather than the average value, is that values vary greatly across settings, especially those with different fleet size $N$, and we cannot directly compare the average.

From this table, we can see that optimal pricing strategy has a much higher profit than the others, but at the cost of providing much fewer trips. It is also interesting to find that, for PO, even though the total travel time of fleet is smaller, the total travel distance is greater; this means that PO tries to avoid trips that pass through congested area. In terms of service quality, this table shows that, for PO, the average waiting time for pickup is comparable to others, and in many cases, much better;
Table 5.3: Average percentage difference in operating statistics, compared with PO, fixed traffic time cases

while the average waiting time for offer is generally much smaller, which means that PO tends to restrict capacity for traveler with higher willingness to pay, rather than to provide every possible capacity, as by other basic strategies.

Next, we can look at the capacity provided by the operator, and focus on three strategies: PO, S+Sh, and Sh. We select the latter two strategies as the baseline since they provide more capacity than PO, as indicated above. The following table 5.4 summarizes the total passenger serviced $M_s + M_{sh}$, shared trips serviced $M_{sh}$, and ratio between shared trips and total trips $\rho_{sh} = M_{sh} / (M_s + M_{sh})$, across different setting for each strategy.

Table 5.4: Capacity $M_{sh}$ and shared rate $\rho_{sh}$, fixed traffic time cases

From table 5.4, we have several observations. First, the shared service fare ratio $r_{sh}$
has different impact on basic strategies and optimal pricing PO: for basic strategies, as the shared service fare goes higher, the total utility of the offer set decreases, and traveler are less likely to make trips; but at the same time, operator has more incentive to provide shared service and release extra capacity, so with PO, the capacity $N_s + M_{sh}$ actually goes up.

Second, when supply goes up, there is more capacity in the system, and the chance of having empty vehicles around the request point increases; thus the shared rate $\rho_{sh}$ generally decreases as supply increases. Also, shared service become less attractive since capacity worths less, so for PO, the change in shared rate $\rho_{sh}$ becomes less sensitive to shared service fare ratio $r_{sh}$.

These observations are summarized in figure 5-1:

![Figure 5-1: Shared rates $\rho_{sh}$ across different settings](image)

From this section, we can conclude that, supply size $N$ and shared service fare ratio $r_{sh}$ have a strong impact on how much capacity the operator can generate; and when this operator is profit-driven, the results could be significantly different from others, such as those basic ones. Generally, a profit-driven operator will provide less capacity than the operator with no control, as it will focus on exploiting trips that generate more profit; but in some circumstances, especially when supply is limited and fare for shared service is not too low, this profit-driven operator will voluntarily provide more capacity and close the gap.
5.2 System Performance

Recall that in system level, we want to focus on global capacity and congestion level. Next, we will define the metrics for subsequent analysis.

On global capacity, we will focus on the ratio $\rho$ between the number of trips realized (including single service, shared service, and original travel modes), and the number of total requests:

$$\rho = \frac{M_s + M_{sh} + M_o}{M_s + M_{sh} + M_o + M_n}$$

where $M_s$, $M_{sh}$, $M_o$, and $M_n$ are the number of shared trips, single trips, original trips, and trip cancellations from the target population, respectively.

On congestion level, we use a simple metric: average system link travel time

$$T_{ave} = \frac{\sum_t \sum_{e \in E} k_{e,t} T_{e,t}}{\sum_t \sum_{e \in E} k_{e,t}}$$

where $T_{e,t}$ and $k_{e,t}$ are the link travel time and density for link $e$ at time $t$, respectively; $E$ is the set of links in the system.

Now we discuss some scenario settings for tests in this section. To capture the variability in travel time, in this section we consider dynamic traffic time settings, along with some significant market exposure $\varphi$. Specifically, we will consider $\varphi \in \{10\%, 20\%, 40\%\}$.

We also consider a varying background congestion level $\psi_C \in \{\text{Low}, \text{Medium}, \text{High}\}$, where each level is derived by multiplying the background travel density of each link by certain factor. Specifically, a medium $\psi_C$ corresponds to the original case (factor = 1.0), a low $\psi_C$ corresponds to the case with factor = 0.8, and a high $\psi_C$ corresponds to the case with factor = 1.2.

Since the variance in the simulation is much greater with dynamic traffic time setting, we use a larger demand size to obtain more accurate estimation of targeted metrics. We consider $M \sim 110,000$ requests, which is 10 times larger than that in fixed traffic time settings.
Finally, we consider fleet size \( N \in \{500, 750, 1000, 1250\} \), and shared service fare ratio \( r_{sh} \in \{0.4, 0.6, 0.8\} \). Notice that the ratio \( N/M \) is finer than that in fixed traffic time cases, as this provides more information and we can have a better understanding of the dynamics under dynamic traffic time settings.

In this section, we again focus on three strategies: PO, S+Sh, and Sh. We select PO as it represents the optimal strategy targeting operating profit, while the two basic strategies, S+Sh and Sh, are selected since they have best system performances in fixed traffic time case.

### 5.2.1 Low Original-cost Case

In this section, we discuss the result with original cost factor \( b_{f,o} = 2.5 \). This means the cost for original mode is low, and the impact of the newly launch service is not significant. Under this setting, we select density adjustment factor \( k_0 = 0.95 \).

<table>
<thead>
<tr>
<th>Strategy</th>
<th>S+Sh</th>
<th>Sh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>-11.18%</td>
<td>-26.10%</td>
</tr>
<tr>
<td>Revenue</td>
<td>-7.99%</td>
<td>-22.61%</td>
</tr>
<tr>
<td>Cost</td>
<td>7.8%</td>
<td>-5.61%</td>
</tr>
<tr>
<td>Serviced Passenger</td>
<td>22.09%</td>
<td>15.40%</td>
</tr>
<tr>
<td>Travel Distance</td>
<td>7.80%</td>
<td>-5.61%</td>
</tr>
<tr>
<td>Travel Time</td>
<td>12.40%</td>
<td>-2.54%</td>
</tr>
<tr>
<td>AWT, Pickup</td>
<td>13.65%</td>
<td>10.70%</td>
</tr>
</tbody>
</table>

Table 5.5: Average percentage difference in operating statistics, compared with PO, \( b_{f,o} = 2.5 \)

First, we still compare some basic operating metrics across the selected strategies. Table 5.5 summarize the average percentage difference for selected strategies, compared with optimal pricing strategy (PO), across different settings. Again, it is clear that the optimal pricing strategy is much better than the two others in terms of profit and pickup time, but much worse in terms of the number of passengers serviced. This again shows that PO has some kind of trip selection criteria, and will reduce capacity for higher profit.

Next, we will look at the system metrics. Table 5.6 summarizes the average
<table>
<thead>
<tr>
<th>$\psi_C$</th>
<th>$\varphi$</th>
<th>S+Sh</th>
<th>Sh</th>
<th>PO</th>
<th>S+Sh</th>
<th>Sh</th>
<th>PO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>0.1</td>
<td>0.91%</td>
<td>0.56%</td>
<td>0.87%</td>
<td>9.42%</td>
<td>8.70%</td>
<td>7.32%</td>
</tr>
<tr>
<td>Medium</td>
<td>0.1</td>
<td>0.92%</td>
<td>0.51%</td>
<td>0.95%</td>
<td>9.01%</td>
<td>8.38%</td>
<td>7.14%</td>
</tr>
<tr>
<td>High</td>
<td>0.1</td>
<td>1.41%</td>
<td>0.8%</td>
<td>1.22%</td>
<td>8.26%</td>
<td>7.82%</td>
<td>7.14%</td>
</tr>
<tr>
<td>Low</td>
<td>0.2</td>
<td>1.63%</td>
<td>0.95%</td>
<td>1.87%</td>
<td>9.43%</td>
<td>8.50%</td>
<td>7.79%</td>
</tr>
<tr>
<td>Medium</td>
<td>0.2</td>
<td>2.84%</td>
<td>0.97%</td>
<td>2.08%</td>
<td>8.58%</td>
<td>8.18%</td>
<td>7.55%</td>
</tr>
<tr>
<td>High</td>
<td>0.2</td>
<td>3.36%</td>
<td>1.99%</td>
<td>3.14%</td>
<td>7.93%</td>
<td>7.60%</td>
<td>6.94%</td>
</tr>
<tr>
<td>Low</td>
<td>0.4</td>
<td>4.86%</td>
<td>-0.35%</td>
<td>5.71%</td>
<td>8.74%</td>
<td>8.25%</td>
<td>7.62%</td>
</tr>
<tr>
<td>Medium</td>
<td>0.4</td>
<td>12.35%</td>
<td>4.45%</td>
<td>14.41%</td>
<td>7.96%</td>
<td>7.85%</td>
<td>6.87%</td>
</tr>
<tr>
<td>High</td>
<td>0.4</td>
<td>23.52%</td>
<td>10.22%</td>
<td>19.58%</td>
<td>6.42%</td>
<td>7.11%</td>
<td>5.37%</td>
</tr>
</tbody>
</table>

Table 5.6: Average percentage difference in system metrics, compared with background value, $b_{f,o} = 2.5$

percentage difference across different settings for each strategy, compared with the following background value:

<table>
<thead>
<tr>
<th>$\psi_C$</th>
<th>$T_{ave}(mins)$</th>
<th>$\varphi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>1.666</td>
<td>0.844</td>
</tr>
<tr>
<td>Medium</td>
<td>2.272</td>
<td>0.832</td>
</tr>
<tr>
<td>High</td>
<td>3.063</td>
<td>0.815</td>
</tr>
</tbody>
</table>

There are several findings from table 5.6. First, the global capacity with PO is consistently lower than those with other two strategies, while strategy Sh has lowest congestion level. Second, when the market exposure $\varphi$ is not very high ($\varphi \in \{0.1, 0.2\}$), the difference from strategies are negligible: we cannot observe significant difference in $T_{ave}$, while 1% difference in $\varphi$ will only lead to less than $\max\{\rho\} \cdot \max\{\rho\} \cdot 1% = 0.2 \times 0.844 \times 0.01 = 0.17\%$ of total potential requests. So PO is not significantly worse than two other basic strategies in terms of system performance. Finally, when the market exposure rate $\varphi$ goes to 0.4, the system dynamics become much different. As congestion level increase, the congestion become much more severe, with a much higher $T_{ave}$, while the increment in capacity $\rho$ drop rapidly. In this situation, strategy Sh is much more robust towards congestion, as expected; but we also surprisingly find that PO is more robust in congestion against S+Sh, while the gap in $\rho$ is not widened.

To investigate what actually happen, we take further analysis on case with $\psi_C =$
High and market exposure $\varphi = 0.4$, and summarize the results in table 5.7. We can see that, PO is better than S+Sh only when the supply is very high ($N = 1250$).

<table>
<thead>
<tr>
<th>$N$</th>
<th>$r_{sh}$</th>
<th>$S+Sh$</th>
<th>Sh</th>
<th>PO</th>
<th>$\rho_{S+Sh}$</th>
<th>Sh</th>
<th>PO</th>
</tr>
</thead>
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<td>-2.45%</td>
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<td>7.37%</td>
<td>4.15%</td>
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<td>4.64%</td>
<td>-1.69%</td>
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<td>5.96%</td>
<td>6.57%</td>
<td>4.89%</td>
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<tr>
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<td>7.96%</td>
<td>1.37%</td>
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<td>5.07%</td>
<td>5.60%</td>
<td>5.61%</td>
</tr>
<tr>
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<td>6.17%</td>
<td>-0.42%</td>
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<td>8.23%</td>
<td>8.89%</td>
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</tr>
<tr>
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<td>13.09%</td>
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<td>19.93%</td>
<td>10.75%</td>
<td>14.87%</td>
<td>5.55%</td>
<td>5.84%</td>
<td>5.45%</td>
</tr>
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<td>18.42%</td>
<td>7.08%</td>
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<td>9.27%</td>
<td>6.17%</td>
</tr>
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<td>14.14%</td>
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<td>7.41%</td>
<td>6.02%</td>
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<td>19.05%</td>
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<td>5.66%</td>
<td>5.10%</td>
</tr>
<tr>
<td>1250</td>
<td>0.4</td>
<td>35.87%</td>
<td>17.43%</td>
<td>26.91%</td>
<td>8.03%</td>
<td>9.03%</td>
<td>4.63%</td>
</tr>
<tr>
<td>1250</td>
<td>0.6</td>
<td>47.80%</td>
<td>23.41%</td>
<td>33.28%</td>
<td>5.98%</td>
<td>7.06%</td>
<td>4.90%</td>
</tr>
<tr>
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<td>60.85%</td>
<td>29.69%</td>
<td>19.26%</td>
<td>3.97%</td>
<td>5.17%</td>
<td>6.25%</td>
</tr>
</tbody>
</table>

Table 5.7: Percentage difference in system metrics, compared with background value, $b_f, o = 2.5$, $\psi_C = \text{High}$, $\varphi = 0.4$

Further observation on each cell in table 5.7 shows that, as there is more supply, there is generally more traffic $\rho$ in the system, thus more severe congestion. This matches with our intuition, and is consistent across different strategies. But there is one very interesting difference between the basic strategies and PO: when the shared service fare ratio $r_{sh}$ increases, basic strategies tend to have more severe congestion, while PO does not have this property. We point out that for basic strategies, this trend is universal across different settings, but most significant in high demand, high congestion situation. One possible explanation is that, when $r_{sh}$ increases, shared service becomes less favorable, and shared trip rate $\rho_{sh}$ drops; this then leads to more traffic density in the system, thus more severe congestion. We might then speculate that the shared trip rate $\rho_{sh}$ is also the reason for PO's better performance over basic strategies, but it is not the case.

As we can see in the table 5.8, the total shared rate, defined to be the ratio between shared service trips and total trips $\rho_{sh} = \frac{M_{sh}}{M_s + M_{sh} + M_o}$, of PO is never much greater than those of the other two strategies, but sometimes it has better performance in alleviating congestion. On the other hand, we recognize another factor
<table>
<thead>
<tr>
<th>$N$</th>
<th>$r_{sh}$</th>
<th>$\rho_{sh}$</th>
<th>$\Delta_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.4</td>
<td>0.36</td>
<td>10.63</td>
</tr>
<tr>
<td>500</td>
<td>0.6</td>
<td>0.31</td>
<td>7.78</td>
</tr>
<tr>
<td>500</td>
<td>0.8</td>
<td>0.25</td>
<td>5.07</td>
</tr>
<tr>
<td>750</td>
<td>0.4</td>
<td>0.40</td>
<td>11.12</td>
</tr>
<tr>
<td>750</td>
<td>0.6</td>
<td>0.34</td>
<td>7.55</td>
</tr>
<tr>
<td>750</td>
<td>0.8</td>
<td>0.28</td>
<td>3.80</td>
</tr>
<tr>
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<td>0.4</td>
<td>0.50</td>
<td>10.28</td>
</tr>
<tr>
<td>1000</td>
<td>0.6</td>
<td>0.35</td>
<td>6.70</td>
</tr>
<tr>
<td>1000</td>
<td>0.8</td>
<td>0.34</td>
<td>3.94</td>
</tr>
<tr>
<td>1250</td>
<td>0.4</td>
<td>0.51</td>
<td>9.38</td>
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<td>0.6</td>
<td>0.35</td>
<td>6.04</td>
</tr>
<tr>
<td>1250</td>
<td>0.8</td>
<td>0.28</td>
<td>3.06</td>
</tr>
</tbody>
</table>

Table 5.8: Shared trips rate $\rho_{sh}$ and total reduced trip distances $\Delta_d(\times 10^4 km)$, $b_{f,o} = 2.5$, $\psi_C = \text{High}$, $\varphi = 0.4$

that has a significant impact: total reduced trip distances $\Delta_d$. $\Delta_d$ is defined to be the difference between the total accumulated trip distance from those who take the operator's service, or the required trip distance of to fulfill demand, and the total distance traveled by the operator's fleet. That is, $\Delta_d$ describes how many kilometers is saved with the operator's service. From table 5.8, we can see that when PO outperforms the two others in congestion level, either it has a much higher distance reduction $\Delta_d$, or a much lower capacity $\rho$, or both.

Figure 5-2: Congestion - Capacity Tradeoff among Strategies, $b_{f,o} = 2.5$
Thus, we might speculate that, the distance reduction $\Delta_d$ has an impact on the congestion-capacity relationship. This can actually be supported by figure 5-2, in which each circle represent the tuple $(T_{ave}, \rho, \Delta_d)$ for a specific setting and strategy. More specifically, $(T_{ave}, \rho)$ is mapped to the location of the center, while $\Delta_d$ is proportional to the radius of the circle. We can see that, larger $\Delta_d$ can actually push the $(T_{ave}, \rho)$ curve to the upper left, thus making positive contributions to the system. Then we might have a guess on why PO is more robust towards high congested situation: in such situation, it voluntarily incentivizes travelers with long distance trips to share rides, and then increase $\Delta_d$, while basic strategies cannot incorporate such dynamic decisions.

Finally, we want to see if the tax reduction has any effect. With discussion above, we should believe that the tax reduction on congestion ($TRC$) should be more effective, since it is correlated to the travel time (which is then related to travel distance), while the tax reduction on demand size ($TRD$) is correlated with the number of trips. Figure 5-3 confirms our guess, and shows that tax reduction can really improve the optimal pricing strategy.

![Figure 5-3: Congestion - Capacity Tradeoff of PO under Tax Reduction, $b_{f,o} = 2.5$](image)

To close this subsection, we calculate the average percentage difference in system metrics, for PO with tax reduction under high market exposure rate $\varphi = 0.4$; the results are summarized in table 5.9. Even though they show some improvement, they
Table 5.9: Average percentage difference in system metrics, compared with background value, under tax reduction, $b_{f,o} = 2.5$, $\varphi = 0.4$

are still not comparable with strategy Sh.

5.2.2 High Original-cost Case

In this section, we discuss the result with original cost factor $b_{f,o} = 5.0$. This means the cost for original mode is relatively high, and the impact of the newly launch service will be more significant. Under this setting, we select density adjustment factor $k_0 = 0.8$.

Table 5.10: Average percentage difference in operating statistics, compared with PO, $b_{f,o} = 5.0$

First, we still compare some basic operating metrics across different strategies. Table 5.10 summarizes the average percentage difference for selected strategies, compared with optimal pricing strategy (PO), across different settings. Again, it is clear that the optimal pricing strategy is much better than the two others in terms of profit and pickup time, but much worse in terms of the number of passengers serviced.
Next, we will look at the system metrics. Table 5.11 summarizes the average percentage difference across different settings for each strategy, compared with the following background value:

<table>
<thead>
<tr>
<th>$\psi_C$</th>
<th>$\varphi$</th>
<th>$T_{ave}$ (mins)</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>0.1</td>
<td>1.81%</td>
<td>1.38%</td>
</tr>
<tr>
<td>Medium</td>
<td>0.1</td>
<td>1.98%</td>
<td>1.50%</td>
</tr>
<tr>
<td>High</td>
<td>0.1</td>
<td>2.79%</td>
<td>2.17%</td>
</tr>
<tr>
<td>Low</td>
<td>0.2</td>
<td>4.13%</td>
<td>3.03%</td>
</tr>
<tr>
<td>Medium</td>
<td>0.2</td>
<td>4.80%</td>
<td>3.55%</td>
</tr>
<tr>
<td>High</td>
<td>0.2</td>
<td>7.52%</td>
<td>5.63%</td>
</tr>
<tr>
<td>Low</td>
<td>0.4</td>
<td>14.23%</td>
<td>9.40%</td>
</tr>
<tr>
<td>Medium</td>
<td>0.4</td>
<td>28.58%</td>
<td>17.07%</td>
</tr>
<tr>
<td>High</td>
<td>0.4</td>
<td>47.96%</td>
<td>31.90%</td>
</tr>
</tbody>
</table>

Table 5.11: Average percentage difference in system metrics, compared with background value, $b_{f,o} = 5.0$

Again, the global capacity with PO is consistently lower than those with other two strategies, while strategy Sh has lowest congestion level. Even though the congestion become more severe compared with low original cost case, when the market exposure $\varphi$ is not very high ($\varphi \in \{0.1, 0.2\}$), the difference among strategies are still insignificant: difference in $T_{ave}$ is about 1%, while difference in $\rho$ is no more than 3%, which will only lead to less than $\max\{\varphi\} \cdot \max\{\rho\} \cdot 3\% = 0.2 \times 0.745 \times 0.03 = 0.48\%$ of total potential requests. So again, PO is not significantly worse than two other basic strategies in terms of system performance, and we only need to focus on high market exposure rate ($\varphi = 0.4$) situation, which shows more interesting dynamics.

Again, we focus on case with $\psi_C = \text{High}$ and market exposure rate $\varphi = 0.4$; the system metrics under each settings are summarized in table 5.12. As expected, the tradeoff between congestion and capacity prevails here, and $\Delta_d$ can move the tradeoff curves, even though the effect is not very consistent (look at cases such as
\[(N, p_s) = (1250, 0.4) \text{ or } (750, 0.8)\].

<table>
<thead>
<tr>
<th>(N)</th>
<th>(r_{sh})</th>
<th>(T_{ave})</th>
<th>(\rho)</th>
<th>(\Delta_d)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>S+Sh</td>
<td>Sh</td>
<td>S+Sh</td>
</tr>
<tr>
<td>500</td>
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<td>14.18%</td>
<td>8.96%</td>
<td>25.55%</td>
</tr>
<tr>
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<tr>
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<td>18.22%</td>
<td>11.73%</td>
<td>16.77%</td>
</tr>
<tr>
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<td>22.10%</td>
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</tr>
<tr>
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<td>98.60%</td>
<td>50.59%</td>
<td>46.20%</td>
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</table>

Table 5.12: Percentage difference in system metrics, compared with background value, and total reduced trip distances \(\Delta_d\times10^4 km\), \(b_{f,\alpha} = 5.0\), \(\psi_C = \text{High}\), \(\varphi = 0.4\).

If we plot the congestion-capacity tradeoff figure 5-4 as in the previous subsection, we can find that PO has a much smaller varying region than the other two; this can make it more robust in the extreme cases. Such robustness might due to the optimal nature that tries to control its capacity expansion and exploit the more profitable part in the population.

![Figure 5-4: Congestion - Capacity Tradeoff among Strategies, \(b_{f,\alpha} = 5.0\)](image)

Finally, we will analyze the effect of tax reduction. This time, with figure 5-5, we
find the effect to be very vague, even though relatively tax reduction with congestion (\(TRC\)) is better than tax reduction with demand (\(TRD\)). We suspect that under these extreme cases, a simple public policy will not make much difference, and a delicate control framework is needed.

![Figure 5-5: Congestion - Capacity Tradeoff of PO under Tax Reduction, \(b_{f,o} = 5.0\)](image.png)

To close this subsection, we calculate the average percentage difference in system metrics, for PO with tax reduction under high market exposure rate \(\varphi = 0.4\); the results are summarized in table 5.13. Again, even though there is some improvement, such strategies are still not comparable with strategy Sh.

<table>
<thead>
<tr>
<th>(\psi_C)</th>
<th>Sh</th>
<th>PO</th>
<th>(T_{ave})</th>
<th>PO+(TRC)</th>
<th>PO+(TRD)</th>
</tr>
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<tbody>
<tr>
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<td></td>
</tr>
<tr>
<td>Medium</td>
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<td>27.55%</td>
<td>25.10%</td>
<td>24.29%</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>31.90%</td>
<td>34.16%</td>
<td>31.12%</td>
<td>33.94%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\psi_C)</th>
<th>Sh</th>
<th>PO</th>
<th>(\rho)</th>
<th>PO+(TRC)</th>
<th>PO+(TRD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>18.34%</td>
<td>15.93%</td>
<td>16.19%</td>
<td>15.94%</td>
<td></td>
</tr>
<tr>
<td>Medium</td>
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<td>13.80%</td>
<td>14.22%</td>
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<td></td>
</tr>
<tr>
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<td>12.82%</td>
<td>10.72%</td>
<td>11.27%</td>
<td>10.60%</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.13: Average percentage difference in system metrics, compared with background value, under tax reduction, \(b_{f,o} = 5.0, \varphi = 0.4\)
Chapter 6

Conclusion

In this thesis, to investigate the behavior of profit-driven shared MoD service operator, we formulate a daily-level profit maximization problem with dynamic decisions on trip requests. We then design an efficient algorithm to solve for daily-level optimal strategy, and investigate some of its mathematical properties. Next, with a carefully designed case study, we show that the optimal pricing strategy (PO), which incorporates the opportunity cost of capacity consumed, has superior performance in profit compared with other strategies. But we also show that, this strategy can lead to inefficiency of the system, mainly because it has stricter criteria for providing capacity, especially shared ones.

Nevertheless, detailed analysis on the results from the case study provides us some insights on how we could regulate such profit-driven operators and improve system performance. We identify an important metric, the total reduced distance $\Delta_d$, which is central to improve the congestion-capacity tradeoff. We observe that, the tax reduction with regard to congestion $TRC$, which provides incentives for shared trips with long travel distance under congestion situation and thus is closely related to $\Delta_d$, has certain effects on improving the system performance of optimal pricing strategy. This can provide some references for future regulation design.

Finally, we notice that the capacity control by the optimal pricing strategy provides some robustness in controlling congestion towards extreme cases, e.g., high market exposure, high supply volume, and severe congestion. This advantage should
be an objective to pursue, besides nominal objectives such as expected congestion level, even for operator driven by social welfare.

6.1 Future Direction

6.1.1 Optimal Strategy on Private Side

In this work, we have been using a low-dimensional representation $\hat{A}$ of the optimal opportunity cost function $A^*$ to reduce computation complexity. But since we used episodic update approach, the computation requirement is still large: for each scenario in the dynamic travel time case, it takes $\sim 5$ hours for CMA-ES method to train for 20 steps. Thus, it is necessary to consider value-based approach and incorporate them to design stepwise update methods. Recently, we find that simplified linear programming formulation can provide us some good approximations for the value function, and we can then use function approximation to further simplify computation complexity. We also observe that, since the state $X$ contains spatial-temporal information, convolution neural network can be a powerful tool here. Thus, deep reinforcement learning methods, such as A3C [38], seem to be worth trying.

Another possible method to improve training speed, and even to improve policy stability, is to consider homogeneous policy across different scenarios. Intuitively, a good representation of $\hat{A}$ should be effective regardless of environment settings. If we can find such representation by proper feature selection, we might be able to train the parameter with randomly generated scenarios, and the result should be much more robust towards changes in settings than the current results.

In the problem formulation, we assume that the operator has full knowledge of the demand. There are two directions for extensions on this assumption: that the operator does not know the traveler behavior, or the operator does not know the future demand distribution. One possibility is still based on existing behavior model, though the actual parameters need to be estimated in an online manner. We notice that there are already several works on assortment optimization that address such
concerns: [11, 42, 44]. Another possibility is model-free approaches, where a large class of reinforcement learning methods can fit in. Finally, we can also consider robust approaches [43], which can be a good alternative for initial guess, if we have limited data as well as some form of knowledge of models, but also have a great level of uncertainties.

Another extension, at a higher level, is to combine the pricing problem with dispatching problem, which is a hot topic in recent research. We notice that in our current optimal pricing strategy, there is some effect in demand management, which is closely related to good dispatching algorithm. We believe that the non-myopic formulation can provide some connection between the two problems, and under this reinforcement framework, the integrated problem can be formulated and solved.

Finally, we should notice that, all discussion above are still in the sense of short-term operation, since we assume the environment to be fixed. In reality, analysis and modeling of long-term behavior dynamics are unavoidable, and this could be an exciting field for future research. Again, model-free reinforcement learning approaches could be very promising, and we look forward to more applications of these methods in transportation research area.

6.1.2 Optimal Strategy on Public Side

As we mentioned earlier, given the profit-driven operator, we can investigate how the government might design regulation strategy to increase social welfare. From the conclusion in this study, we can first focus on static strategies that incentivize the private operator to pursue higher $\Delta_d$. This can also be the objective for operator driven by social welfare.

We can also consider dynamic regulation strategies, where control is provided at trip level. In this case, we also need to formulate a dynamic decision problem for the government. And as optimal strategies of both parties rely on the others, there will be a game, similar to the competition among operators. This should be a very interesting topic for us to investigate in the future.
Bibliography


