Trabecular Topology: Computational Structural Design Inspired by Bone Remodeling

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Abstract
Bone remodeling is the process by which the internal morphology of bones in a healthy person or animal will adapt to the loads under which it is placed. This process makes bone stronger and performs better under daily loadings. It also gives a special topology to the trabecular bone. This thesis proposes a new computational structural design approach inspired by the trabecular bone topology and remodeling process and it can be applied to the 2D, 3D and building-scale structures. It reveals the importance of the connectivity in the structures and provides a innovative bio-inspired method for the future structural topology design.

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Chapter 1

Introduction

This thesis presents a new methodology to computationally design the topology of space frames inspired by the bone remodeling process. The first chapter identifies the need for new topology design methods and motivations for using bone as an inspiration of structural design. Then, it defines the research scope of this thesis and lists the outline for following chapters.

1.1 Motivation

With the advances of the building industry, many eclectic and unusually geometrically complex building designs have sprung up (Figure 1-1). Those irregular buildings which stand out from the other cookie-cutter buildings are incorporated with a great deal of irregularity and eccentricity. Consequently, they pose a difficult question to structural engineers about how to design innovative structural systems to these irregular buildings.

At the same time, large percentage of civil engineers still live in the orthogonal grid world. The orthogonal grid has many benefits. They are easy for design because of available design codes. They are easy for construction. However, orthogonal structure might not be the most efficient structures in terms of material distribution and structural efficiency, as yielded by the long history of structural optimization. Michell’s milestone study about least weight truss [Hemp and Chan, 1965] with stress constrains
Figure 1-1: Twisted, Diagrid and Tapered Irregular Buildings

shows the optimized structure is not an orthogonal structure with only horizontal and vertical elements, as shown in Figure 1-2.

In addition, due to the advances of new construction and manufacturing technology, such as additive manufacturing (also called 3D printing), the possibility of building complex geometry is increased and the potential cost is decreased. As a result, in the near future, the optimized non-orthogonal structure can be more cost-effective. The need for innovative structural topology design is brought up.

Bone has always been considered as an adaptive and strong material due to its micro-structure. There have been many studies about mechanical properties of bones in biomechanics and material science field. It is an interesting topic to use bone as an inspiration for structure design.

1.2 Bone-inspired Design

Bones serve as the skeleton of the human body. The main function of bones is to sustain both permanent and transient loads caused by the daily activity or special events of the human. It is obvious that bones grow with age. Actually, most bones
Figure 1-2: Michell’s Least Weight Truss

Figure 1-3: The Microstructure of Trabecular Bones
start to grow during the fetal period. They start from a soft, flexible cartilage and later ossify into a spongy mineral lattice. The lattice becomes harder when osteoblasts, specialized bone-forming cells, deposit more minerals. However, the above process is not enough to make bones functional. If you use a bone made from above process to lift heavy things, the bone may probably snap. But bones do not usually snap, because a more significant bone growth mechanism exists. Our body reinforces and builds bones wherever they are frequently used, which is called “Wolff’s Law”. In other words, if loads on a particular bone increase, the bone reshapes itself over time to become stronger to resist the loads. It is called bone remodeling process as well. “Wolff’s Law” actually makes the bone have a very special shape. For example, the femur, the longest and the strongest bone in the human body, has an angle of inclination in its head due to the gravity load of the human body. Most importantly, this special growth mechanism makes our bones can perform well under extreme loading conditions.

Because of the interesting properties of bone, it has been mimicked across different scales, as shown in Figure 1-4. Material scientists used bone’s micro-structure as an inspiration for material design; Designers used bone’s skeletal organic quality to decorate their furniture; Architects replaced the cylindrical columns with bone-like columns to create nature and dynamic appearance.

It is interesting as well to mimic bones in the structural engineering field. Just like
bones, the structural system of a high-rise building also serves as the load resisting system. It is subjected to both vertical loads and lateral loads. Vertical loads can come from gravity and movable objects in the building. Lateral loads are mainly caused by the wind and earthquake. Now, the similarity between bones and the structural system is very clear. So, if there is “Wolff’s Law” for bones, can it also be used to design and reinforce our structural system?

Developing a topology design algorithm for mimic the “Wolff’s Law” in bones is a new and interesting topic. Although many attempts have been made for structural optimization in the construction industry, studying nature’s optimization and applying it in construction is a new area and requires knowledge of structural engineering, material science, and biology. The potential outcome is a new optimization approach in the structural engineering field. It also has great potentiality for applications in 3D printing to generate optimal truss structures of a given volume.

1.3 Space Frames

Space frame is defined as “a structure system assembled of linear elements so arranged that forces are transferred in a three-dimensional manner” [Tsuboi et al., 1984]. Regular space frames in common practice are usually based on polyhedral packing, as shown in Figure 1-6.a. They are easy for space filling, modular construction and can be distorted globally for different aesthetic reasons (such as Figure 1-6.b).

However, space frame with uniform-distributed material and topology might not be good for the structural efficiency. In addition, advanced architecture geometry enables the new ways of generating space frames and gives more ionic appearance for architectures. One example is the Beijing National Aquatics Centre(Figure 1-6.c). With the organic Weaire-Phelan structure as its outer wall, it got a unique nickname “Water Cube”. The Weaire-Phelan structure is a complex 3D geometry representing the foam of bubbles. With the randomness in the structure, it provides amazing and dynamic aesthetics. Thus, new potential space frame generation method will be
Figure 1-5: Stress Trajectories of Bones and its Similarities with Optimized Truss

Figure 1-6: Space Frames
interesting to explore.

1.4 Research Scope

This thesis presents a structural topology design algorithm for space frames inspired from the trabecular bone remodeling process. It is a rule-based algorithm with potential structural optimization. The thesis shows the application of the algorithm in two-dimensional design space, three-dimensional design space and building lateral loads resisting system.

1.5 Outline

The chapter 2 reviews existing research about bone remodeling, bone topologies and their relationship with architecture and engineering designs. The chapter 3 illustrates the proposed structural design method inspired from bone remodelling method. The chapter 4 shows the results of applying the proposed method to 2D, 3D and building-scale structures.
Chapter 2

Literature Review

The chapter 2 gives an overview about the previous biomechanics study of the topology of bones, lists examples of biomimicry in architecture and engineering and reviews existing space-frame generation method and rule-based topology optimization method.

2.1 Biomechanics and Bone Remodeling

People are always fascinated about the forms of nature. The exploration of bone micro-structures can be dated back to one hundred years ago, the work of Hermann Meyer. He discovered the “beautiful curving lines (in Figure 2-1) from the head to the tubular shaft of the bone” existed in human bones [Thompson et al., 1942]. The lines had a nice arrangement that each intercrossing was nearly as possible an orthogonal one. Carl Culmann, as the father of Graphic Statics, happened to see Meyer’s drawing of human’s trabecular bones and recognized that the arrangement of the micro-structures of trabecular bones was just like the diagram of the lines of stress in a loaded structure as shown in Figure 2-2, which demonstrated that nature was strengthening the bones precisely the manner and direction in which strength was needed.

Continuing their discovery, a more specific study with respect to bones was conducted by German anatomist and surgeon Julius Wolff. His Wolff’s Law states that
Figure 2-1: Head of the Human Femur in Section [Thompson et al., 1942]

Figure 2-2: Crane-Head and Femur [Thompson et al., 1942]
bones in a healthy person or animal will adapt to the loads under which it is placed. Stress trajectories of bones illustrate the existence of “Wolff’s Law”. In fact, our body reinforces and builds bones wherever they are frequently used. In other words, if loads on a bone change, the bone reshapes itself over time to become stronger and resist the loads. In the biological aspect of the case, bone as a highly plastic structure, its internal beam elements are constantly being formed and deformed, demolished and formed as new. “Wolff’s Law” can be identified as a kind of topology and shape optimization of structures in nature.

D’Arcy Thomson summarized all above work and provided his unique understanding of Wolff’s Law in his book On Growth and Form [Thompson et al., 1942]. As a pioneer of mathematical biology, Thompson is famous for his scientific explanation of morphogenesis, the process by which patterns and body structures are formed in plants and animals. D’Arcy Thomson stated that biologists of his day overemphasized evolution as the fundamental determinant of the form and structure of living organisms, and underemphasized the roles of physical laws and mechanics.

In his book On Growth and Form [Thompson et al., 1942], he used a chapter “On Form and Mechanical Efficiency” to analyze why the form of bones were efficient in terms of its mechanical properties. He pointed out bone is a very good material. Its tensile strength was as good as its crushing strength, which mean that it can be either used as a struct or tie. In comparison, cast iron and wrought iron both have a great discrepancy between their two strengths. He referred Wolff’s Law as a possible reason. He also illustrated that the stimulus for the growth of bones might be the strain in the microstructures and provided several possible ways for bones to reinforce itself, by increasing the number of trabeculae (beam elements in the structure), increasing the size of trabeculae or changing the chemical properties of each trabeculae.

Modeling “Wolff’s Law” has been popular in biomechanics for decades. Several computational methods to model it have already been developed. One famous mechanical approach is the “Stanford Model” [Jacobs et al., 1997], in which researchers considered bones as a continuous material and used an iterative method to update density $\rho$ of bones with respect to the stress distribution in the structure.
Recently, another group of researchers have used topology optimization to optimize the structure of trabecular bones [Boyle and Kim, 2011]. They constructed structures of a trabecular bone using a micro-FE model with 23.3 million finite elements and optimized material distribution based on elastic energy. Their results demonstrate a similar material distribution to that of real bones, which validates to some extent the existence of “Wolff’s Law” in the bone structures.

The two computational studies above both consider bone as a continuous material. However, some interesting features can also be seen if the bone is considered as a discrete lattice structure. A study about the relative abundances of nodes with different numbers of emanating branches [Reznikov et al., 2016] reveals that the connectivity of the micro-structures of bones is a main characteristic as shown in Figure 2-3.

### 2.2 Biomimicry in Architecture and Engineering

Biomimicry is one of the most popular approaches for architecture and engineering design. It requires not only to replicate the form of the nature, but to understand the rules behind the nature as well. Potential benefits people can get from biomimicry include innovative design, efficiency, sustainability and cost-saving.
2.2.1 Architecture Design

eVolo as an architecture and design magazine holds annual skyscraper competition every year, seeking innovative and creative future skyscraper design all over the world. Many selected designs are even beyond imagination but provide practical designers with potential creative solutions. One selected design called “Trabeculae: Re-imagining the Office Building” [Aiello, 2013] was designed by architects Dave Pigram, Iain Maxwell, Brad Rothenberg, and Ezio Blasetti. It mimics the micro-structure of trabecular bones (shown by Figure 2-4) to create a highly complex network and maximize the daylight, circulation, communication within the office building.

2.2.2 Engineering Design

Many nature materials are proved to be efficient and adaptable, and superior engineering. Learning from nature, some top structural engineering companies incorporate biomimicry into their structural system and create the most ionic and elegant efficient designs.

One famous example is Skidmore, Owings & Merrill LLP (SOM)’s competition
scheme for the China World Trade Center, Beijing [SOM, 2011]. It is inspired by the form of bamboo and it utilizes the structural properties of bamboo at the same time. In bamboo, long stems support for its foliage and response efficiently and effectively to lateral loads. Its unique geometric properties explained the reason as demonstrated in Figure 2-5. The nodes of the bamboos are not distributed evenly, as they get closer at the bottom and top and get far apart at the middle, which can be summarized by mathematical expressions. The reason is that the buckling failure can be prevented at the bottom of the stems with smaller spacing and more shear capacity can be provided at the top of the stem with smaller spacing, as shears are dominating than moment at the top of the stems. These concepts are applied to the structural systems of China World Trade Center Tower submission with smaller spacing of nodes at the bottom and top.

Biomimicry are also applied in the detailed connection design. SOM invents a Pin-Fuse Joint Frame (shown in 2-6) emulated from the pivotal movements of a human shoulder joint [SOM, 2011]. It remains fixed in the normal service condition and can be flexible during an extreme earthquake to dissipate the energy and prevent the potential damage. It can increase the durability and sustainability of the building.

2.3 Generation of Space Frames

As discussed in section 1.3, space frame generation in practice is based on polyhedral packings. One application is the tetrahedral meshing in computer graphics with Delaunay algorithm. It can be used for further solid material analysis such as finite element analysis. But it might not be a good algorithm for space truss generation. It does not consider potential loadings on the structure, which creating a stress field across the structure. The stress field requires changing the density of structures in different areas. In addition, the topology of the structure is not optimized. The connectivity of the truss is simple and repetitive. In comparison, in the real bones as discussed in section 2.1, there are three-branch, four-branch and five-branch nodes representing different connectivity. Complex topology generation method should be
Figure 2-5: China World Trade Center Competition Entry by SOM [SOM, 2011]
proposed. This section examines several existing innovative techniques to generate the space frames.

2.3.1 Stress Line Additive Manufacturing (SLAM) for 2.5-D Shells

SLAM [Tam et al., 2015] is a new proposed method for generating frame of the structure along the three-dimensional principal stress trajectories of 2.5-D structural surfaces. The stress lines represent the load path from the loaded point to the support point which potentially encodes the optimal topology of a structure (Figure 2-7). The proposed method can provide space frame for a 2.5 D structure. However, due to the difficulty of generating 3D principal stress trajectories, it is difficult to work with 3D structures.
Figure 2-7: Stress Line Additive Manufacturing (SLAM) for 2.5-D Shells [Tam et al., 2015]
2.3.2 Mechano-adaptive Space Frame Generation Based on Ellipsoid Packing

Another new proposed 3D space frame generation method [Felder et al., 2016] is inspired by the bone remodeling process. By generating ellipsoids aligned to the principle stress directions and updating the region of high stress with smaller ellipsoids and the region of anisotropic stress area with elongated ellipsoids (Figure 2-8). Then ellipsoid centres are connected by Delaunay tetrahedralization and the resultant structures are optimized for edges and relaxed by the dynamic relaxation method. However, in the paper, the detailed relationship between the size and shape of the ellipsoids with the stress field is unclear. In addition, only Delaunay tetrahedralization is used for exploring the topology, which might not be the best topology.

2.3.3 Voronoi Mesh Generation

In a previous study about modulus strength reduction in bones [Kraynik and Gibson, 2000], professor Lorna Gibson, who set the foundation of understanding cellular solids including bones, and her students used Voronoi structure to represent the micro-structure of bones. Compared with Delaunay tetrahedralization, Voronoi structure is less connected. For a 2D Voronoi structure, each node only has three branches. For a 3D Voronoi structure, each node has four branches. As discussed
in 2.1, the most happened connectivity case in bones are 3 to 5. In comparison, 2D Delaunay triangulation will have 4-7 branches per node. As a result, Voronoi is a better representation for micro-structures of bones.

### 2.4 Structural Optimization

ESO (Evolutionary Structural Optimization) [Xie and Steven, 1993] / BESO (Bidirectional Evolutionary Structural Optimization) [Yang et al., 1998] are one of the most popular proposed rule-based topology optimization methods. They are based on the simple concept of slowly removing (or shifting) inefficient material so that the resulting shape of the structure evolves towards an optimum. ESO is only removing the material from least stressed part. BESO allows the adding material to the most stressed part. However, the work related to discrete space frame has not been fully explored. ESO existing studies are most based on the ground structure approach. Ground structure approach needs user to set up all possible elements in the frame structure, which is usually very difficult as there are too many possible conditions.

### 2.5 Summary of Existing Work and Research Question

In summary, the relationship between the material distribution of the trabecular bones and the loads has been discovered more than one hundred years ago. Most of people from biomechanics or medical fields are using continuous model to study it and there are only a few studies consider it as a discrete space structure. At the same time, in the architecture and engineering design field, biomimicry has becoming a hot topic and it has great potentiality for new innovations, sustainability and material-savings. Thus, it would be an interesting and promising topic to mimic the trabecular bones topology for new topology design in the architecture and engineering field. Several existing topology design methods include SLAM, Mechano-adaptive space frame generation use the loading conditions for the guidance of topology design.
However, none of them has detailed study of the connectivity of space frames. Existing rule-based topology optimization method ESO and BESO give potential solutions for space frames design but their topologies are restricted by the ground structure approach. Thus, a new and innovative computational topology generation method for space frames is needed.

This thesis will address following research questions:

1. How to generate a topology mimicking the micro-structures of trabecular bones?
2. How to update the topology according to the loads of the structure?
3. How to apply the method to 3D structures and building-scale structures?
Chapter 3

Methodology

In order to explore the bone-inspired topology design, this chapter introduces a rule-based topology optimization method that enables topology generation and modification based on the stress distribution. It first illustrates the flow of the algorithm and then a simple 2D cantilever beam example is used as a demonstration. Then the detailed algorithm design is presented in section 3.2. At last the implementation of the algorithm and evaluation method are presented.

3.1 Conceptual Overview of the Proposed Topology Design method

The proposed method is designed to update the topology of structures based on stress distribution of the structure.

1. The first step for the algorithm is to define the boundary conditions, loads and cross section of the space frames.

2. Then, Voronoi diagram is used to generate the base topology.

3. With the loads, finite element analysis is conducted to find the stress distribution of the structure.
Start

Define: BC, Loads, A

Generate Voronoi Diagram

Finite Element Analysis

Smaller than Maximum number of iterations

Yes

Force Density Method

Add one more element to release the most stressful element

No

Delete the least stressful element

End

Figure 3-1: Flowchart of the Proposed Topology Design Method
4. Based on the stress distribution, the least stressed element is deleted from the structure and an additional element is added next to the most stressed element.

5. The updated structure is re-analyzed using finite element analysis and back to step 4 until the maximum number of iterations are archived.

In addition, adding force density method to the algorithm is a proposed idea to improve the above method. However, the result is not very promising but it will be documented in the chapter 4.

**Simple Demonstration with a 2D Cantilever Beam**

First, a random 2D Voronoi diagram of a beam is generated, as shown in Figure 3-2. The 2D Voronoi diagram represents the micro-structures of beam. The beam is fixed on the left end and subjected to a point load on the top right corner. The stress distribution of the beam is analyzed with finite element analysis. For updating the structure, the least stressed element is deleted from the structure and an extra element is added at the node with the maximum stress, as marked in the orange circle. Afterwards, the beam is reanalyzed by finite element analysis and reinforced by the same method. The reinforcing process is repeated until the maximum number of iterations is reached.
3.2 Detailed Algorithm Design

3.2.1 Topology Design with Voronoi Diagram

Base topology generation is based on the Voronoi algorithm. It is inspired from a previous model of trabecular bone study [Kraynik and Gibson, 2000]. There are several benefits for using Voronoi algorithm. First it can generate random 2D and 3D space frames with the uniform distribution of material. Second, it is a good way to mimic the topology of the trabecular bones. Shown by the study of trabecular bone architecture [Reznikov et al., 2016], the connectivity of the micro-structures of bones has a specific character. Most nodes in the bones have three to five branches. The Voronoi algorithm can produce structures with on average three branches per node in 2D space and four branches in 3D space, which satisfies the topology of bones. However, in comparison, delaunay algorithm used by mechano-adaptive space frame generation method can give the connectivity from five to eight branches per node in 2D space, which is more than desired. In conclusion, Voronoi diagram is a better way to generate bone-like topology.
3.2.2 Adding or Removing Elements Based on Stress Distribution

The algorithm removes the elements based on the stress distribution. It is inspired from the growth mechanism of bones [Birmingham et al., 2016]. As shown in the scientific research, the stimuli of the bone growth are the strain in its micro-structure.

As stress is proportional to the strain for the same material in the elastic range, stress can be used as a guidance for element removing and adding. Other potential criteria are studied as well. Deflection is proved to be wrong. Taking a cantilever beam as an example, it is easy to see that the algorithm can keep adding elements to the cantilever area, which leads to the structure to be heavier and heavier at the tip. The defelction and elastic energy will get worse and worse.

3.2.3 Element Adding Methods

How to randomly add an element at the most stressed area sounds trivial but needs to be paid more attention to. This thesis sets up several rules for element adding.

Adding to the under-connected nodes

As discussed in the Voronoi diagram part, the proposed method is mimicking the topology of the trabecular bones. The connectivity of the micro-structure of trabecular bones are restricted. The maximum number of branches per node is no more than five. Thus, when adding elements to existing structures, the nodes that have more than five branches are not taken into considerations. Only if the structure has no more available under-connected nodes around for connecting, it will consider the nodes with more than 5 branches.

This proposed method makes sure that the adding elements part will not keep adding elements to the same area.
Select nodes based on topology

As shown in Figure 3-4, the most stressed element will first be identified by the finite element analysis. Then the algorithm will randomly choose one end of the most stressed element and check whether it is under-connected or not. If not, it will search for the second most stressed element for adding elements. If yes, it will base on the topology to add more elements. In Figure 3-4, the red node is assumed the under-connected end. It will first search for nodes connected with the selected under-connected node, which is marked as first-level connected nodes. Then the algorithm further searches nodes connected to first-level connected nodes but are not connected with the selected under-connected node, which is marked as second-level connected nodes. Then it randomly picks one node from second-level nodes to add an element between this node with the selected under-connected node.

3.2.4 Force Density Method

Force Density method is a method developed for the form-finding of a structure network [Schek, 1974]. It is based on the equilibrium of external loads and internal forces of each member on the structure. In the proposed topology design algorithm, the force density method is applied for smoothing structures.
3.3 Implementation

The proposed algorithm is implemented in the Rhino 3D with Grasshopper. Grasshopper is a graphical algorithmic design tool within Rhino 3D and it enables graphically programming structures and architectures. Karamba is the finite element analysis software in Grasshopper environment. It is incorporated into proposed algorithm for the structural analysis. The websites for above companies are shown as follow below,

Rhino 3D: https://www.rhino3d.com/
Grasshopper: http://www.grasshopper3d.com/
Karamba: http://www.karamba3d.com/

3.4 Evaluation

The structural performance is evaluated based on the elastic energy of the structures. The elastic energy $U$ is defined

$$U = \frac{1}{2} k * u^2 \text{ (kNm)}$$

where $k$ is the stiffness of each element and $u$ is the elongation of each element.

The elastic energy of the structure will be recorded during the computational process for evaluation of the structure performance. As the proposed method is a rule-based topology design method. It is not guaranteed that the elastic energy will be strictly decreasing during the computational process.
Chapter 4

Results

This chapter presents the results of applying proposed topology design algorithm to 2D, 3D and building-scale structures. Several potential improvements of the proposed method are also investigated.

4.1 2D Cantilever Beam

4.1.1 A Single Point Load

As presented in 3.1 Figure 3-2, the algorithm is first applied to a simple cantilever beam in the 2D design space. To study the effect of the width-depth ratio and its influence to results, three beams are set up with dimensions 3m*10m, 5m*10m and 7m*10m.

Input

Table 4.1: Basic Assumptions for 2D Cantilever Beams with a Single Point Load

<table>
<thead>
<tr>
<th>Dimensions (Depth*Length)</th>
<th>3m * 10m, 5m* 10m, 7m * 10m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary Conditions</td>
<td>Fixed at the Left Side of the Structure</td>
</tr>
<tr>
<td>Loads</td>
<td>One point load at the tip of the structure 20 kN</td>
</tr>
<tr>
<td>Cross Section</td>
<td>Internal Element Diameter - 1 cm in Diameter</td>
</tr>
<tr>
<td></td>
<td>Boundary Element Diameter - 0.1 cm in Diameter</td>
</tr>
</tbody>
</table>
### Results

The result of the algorithm is summarized in Figure 4-1. It illustrates how the structures changes during the application of the algorithm, the images of the structures at iteration 0, 250 and 500 are shown with their elastic energies. Figure 4-2 records the change of the elastic energy in each iterations.

### Discussion

As show in Figure 4-1, after applying the proposed method, the material distribution of the structures is changed. It changes from uniform distribution to truss-like distribution. In fact, the results reveal that the shape of the structures after applied the algorithm follows the stress trajectories of the structure. If a cantilever beam is only subjected to a single load, the stress trajectory is shown in Figure 4-3. The shapes of the structures at iteration 500 are similar to the half set of the stress trajectories.

How the results relate to the micro-structure of bones? In this case study, only a point load is applied to the structure. As a result, the stress trajectories are relatively simple. For a bone like structure, loads are often more complex. Human usually have

---

**Figure 4-1: Results for 2D Cantilever Beams with One Single Load**

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>3m * 10m</td>
<td>Iteration 0</td>
</tr>
<tr>
<td>5m * 10m</td>
<td>Elastic Energy: 74.79 kNm</td>
</tr>
<tr>
<td>7m * 10m</td>
<td>Elastic Energy: 49.01 kNm</td>
</tr>
<tr>
<td></td>
<td>Elastic Energy: 43.92 kNm</td>
</tr>
</tbody>
</table>

---

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Figure 4-2: Elastic Energy vs Iterations for 2D Cantilever Beams with a Single Load distributed and cyclic loads on the trabecular bones. Thus, the micro-structures of bones are more complex. At present, the result is not comparable to the structures of these real bones. However, it is highly possible that a similar rule-based growth algorithm exists in the human body, because the micro-structures of bones are also similar to stress trajectories. Further study needs to be done to prove that hypothesis.

In addition, the elastic energy of the structure decreases during “rule-base” optimization. As shown by Figure 4-2, the elastic energy decreases during optimization. To be noticed, proposed rule-based topology algorithm does not have the control of elastic energies. However, after the algorithm is applied, elastic energy decreases, which means the structure is getting more efficient. The result reveals that proposed rule-based topology design method is successful in terms of improving structural performance.

### 4.1.2 Two Point Loads

Instead of applying a single point load at the tip of the structure, two point loads are simultaneously applied to the 2D cantilever beams.
Input

Table 4.2: Basic Assumptions for 2D Cantilever Beams with Two Point Loads

<table>
<thead>
<tr>
<th>Dimensions (Depth*Length)</th>
<th>3m * 10m, 5m* 10m, 7m * 10m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary Conditions</td>
<td>Fixed at the Left Side of the Structure</td>
</tr>
<tr>
<td>Loads</td>
<td>Two point loads at top &amp; bottom of the tip : Each 10 kN</td>
</tr>
<tr>
<td>Cross Section</td>
<td>Internal Element Diameter - 1 cm in Diameter</td>
</tr>
<tr>
<td></td>
<td>Boundary Element Diameter - 0.1 cm in Diameter</td>
</tr>
</tbody>
</table>

Results

The result of the algorithm is summarized in Figure 4-5. Figure 4-6 records the change of the elastic energy in each iterations.

Discussion

Compared with the single-load case, the two-load case shows different material distribution, as shown in Figure 4-7. For 3m*10m, the material distribution shape is
Elastic Energy:

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Iteration 0</th>
<th>Iteration 250</th>
<th>Iteration 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>3m * 10m</td>
<td>82.67 kNm</td>
<td>45.56 kNm</td>
<td>13.97 kNm</td>
</tr>
<tr>
<td>5m * 10m</td>
<td>56.29 kNm</td>
<td>47.54 kNm</td>
<td>23.19 kNm</td>
</tr>
<tr>
<td>7m * 10m</td>
<td>53.06 kNm</td>
<td>39.69 kNm</td>
<td>21.19 kNm</td>
</tr>
</tbody>
</table>

Figure 4-5: Results for 2D Cantilever Beams with Two Loads

Elastic Energy vs Iterations

Figure 4-6: Elastic Energy vs Iterations for 2D Cantilever Beams with Two Loads
concave, similar to the single load cases. However, for 7m*10m case, material distribution shape is convex, which follows completely different set of stress trajectories. For 5m*10m case, the material distribution is uniform across the height.

The reason can be explained from deep beam theory. In the two-load cases, the loads at the tip are symmetric. As a result, symmetric material distribution is expected which is shown in 5m * 10m case, but not other two cases. If fix end material distribution is investigated, it is clear that both 3m * 10 m and 7m * 10m cases have an uniform material distribution. The boundary of the final structure covers 80 percent of its original depth. But for the 7m*10m beam, the boundary only remains at the top half. As review by a previous study about fixed end stress distribution of deep beam [Ahmed et al., 1998], the shear stress and bending stress at the top of the fixed end is slightly larger than at the bottom of the fixed end. Deep beam is the beam with span depth ratio more than 2.0. When the span depth ratio is larger than 2.0, the assumption of beam bending theory cannot be used. The reason is that in the deep beam the assumption “Plane section remains plane after bending” is no longer valid. The strain distribution is not linear. As a result, the stress will not distribute uniformly. That’s why there will be more material distributed at the top half of the fixed end for 7m*10m case. With more material at the top for the fixed end, the structure will follow different sets of stress trajectories.

In addition, the elastic energy decreases when the number of iteration increases which proves the designed algorithm improves the structural performance.
4.1.3 Sensitivity Analysis with Connectivity

In the methodology, the adding of new elements will be limited to the under-connected nodes, which are defined as has no more than five branches connecting with that node. To test whether this assumption is a good assumption, a sensitivity test is conducted. For comparison, the definition of the under-connected nodes is changed to no more than six or seven branches per node. 200 iterations are conducted and the elastic energy is recorded.

Input

Table 4.3: Basic Assumptions for Sensitivity Test with Connectivity

<table>
<thead>
<tr>
<th>Dimensions (Depth*Length)</th>
<th>3m * 10m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary Conditions</td>
<td>Fixed at the Left Side of the Structure</td>
</tr>
<tr>
<td>Loads</td>
<td>One point load at the top of the tip : 10 kN</td>
</tr>
<tr>
<td>Cross Section</td>
<td>Internal Element Diameter - 1 cm in Diameter</td>
</tr>
<tr>
<td></td>
<td>Boundary Element Diameter - 0.1 cm in Diameter</td>
</tr>
</tbody>
</table>

Results

The result of the sensitivity analysis is summarized in Figure 4-8. Figure 4-9 records the change of the elastic energy in each iteration.

Discussion

As shown in Figure 4-8, the restriction of the connectivity makes the structure have less dense region. Because the more stressed area is reinforced with more elements, the nodes are quickly being over-connected. If the maximum number of nodes are restricted, then the element adding process can be spread out to other area. Most importantly, this process not only controls the material distribution but also influences the elastic energy in the structure. As shown in Figure 4-9, only in the five branches case the elastic energy decreases during application of the algorithm. It shows that the connectivity is one of the most important character of the structure and it can
**Definition of Under-Connected Nodes**

Figure 4-8: Sensitivity Analysis with Connectivity

Figure 4-9: Elastic Energy vs Iterations for Sensitivity Analysis
largely affect the structure efficiency. It also proves that the assumption about limiting connectivity is a valid and important assumption.

4.2 3D Cantilever Beam

The proposed topology design algorithm is applied to 3D Cantilever Beams for improving structural performance as well, as shown in Figure 4-10. This section presents the results for 3D cantilever beam design and optimization.

Input

Table 4.4: Basic Assumptions for 3D Cantilever Beams

<table>
<thead>
<tr>
<th>Dimensions (Depth<em>Width</em>Length)</th>
<th>3m<em>3m</em>10m 3m<em>5m</em>10m 3m<em>7m</em>10m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary Conditions</td>
<td>Fixed at the Left Side of the Structure</td>
</tr>
<tr>
<td>Loads</td>
<td>Point Loads at Nodes of the Tip: 2kN per Node</td>
</tr>
<tr>
<td>Cross Section</td>
<td>Internal Element Diameter - 1 cm in Diameter</td>
</tr>
<tr>
<td></td>
<td>Boundary Element Diameter - 0.1 cm in Diameter</td>
</tr>
</tbody>
</table>
Result

The result of the 3D cantilever beams is summerized in Figure 4-11. Figure 4-12 records the change of the elastic energy in each iterations. To be noticed, all boundaries of the structures are not shown in Figure 4-11.

Discussion

As shown in Figure 4-12, the performance of the structure is improved for the 3m * 3m * 10m case and 3m * 7m * 10m case. However, for the 3m * 5m * 10m case the structural efficiency starts to decrease after 100 iterations. It shows that the algorithm cannot guarantee the better performance for all cases. Especially, there is no control over the stopping criteria. Further study is needed for control the stopping criteria and improving the proposed method.
4.3 Lateral Loads Resistance System of Tall Buildings

The algorithm can be used for building-scale structure in the lateral loads resisting system design. In this section, proposed topology design method is applied to a tall building with 150 meters height.

Input

To simply the design, it is assumed that all the faces have same topology design. To make sure the faces share the same boundry connected points, one face is flipped three times to form four faces as the lateral load resisting system. The base topology of the structure is shown in Figure 4-13.

Table 4.5: Basic Assumptions for building-scale Structures

<table>
<thead>
<tr>
<th>Dimensions (Width<em>Width</em>Height)</th>
<th>40m * 40m * 150m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary Conditions</td>
<td>Fixed at the Fundation of the Structure</td>
</tr>
<tr>
<td>Loads</td>
<td>Wind Loads at Top : 10,000 kN in Total</td>
</tr>
<tr>
<td>Cross Section</td>
<td>Column Sections - 1m in Diameter</td>
</tr>
<tr>
<td></td>
<td>Bracing Sections - 0.3m in Diameter</td>
</tr>
</tbody>
</table>
The result of the building-scale structures is summerized in Figure 4-14. Figure 4-15 records the change of the elastic energy in each iterations. To be noticed, the direction of the wind load is in one single direction. In the proposed method, the element removing and adding is based on the stress distribution. However, the stress distribution is different for four faces. As a result, the face for analyzing stress distribution should be the face parallel to the wind load direction. Otherwise, the material adding and removing is not based on the forces in the main lateral load resisting system. In addition, there are some noise points in the elastic energy curve. They are removed from the curve. The reason that noise points happen because the structure might have local instability during certain iterations. However, with adding additional elements in the next iteration the instability can be resolved. To make the results more readable, the noise points are removed.

The total drift of the building at the first iteration is 5.31m. After applying the algorithm for 260 iterations, it has the lateral drift value 0.50m.
Figure 4-14: Results for Building-Scale Structures

Figure 4-15: Elastic Energy vs Iterations for Building-Scale Structures
Discussion

The material distribution of the final structures is very interesting. Only considering one face, it has the zig-zag shape. If all four faces are considered, it is a spatial cross-bracing structure. Compared with 2D cantilever beam, it is not singly concave or convex any more. The reason is that the lateral load resisting system has two parallel faces to resist lateral loads. But they have different topology. Their topology is anti-symmetrical to each other. As a result, the material distribution needs to fulfill requirements of both faces. Thus, the topology is different from previous cases but more interesting.

Moreover, the lateral drift reduces to 10 percent of the original drift, which is a very good structural performance improvement.

4.4 Proposed Improvement of the Algorithm

The author has proposed several improvement methods for this bone-inspired topology design method. However, the results are not promising at present. For the purpose of the future research, they are documented here as references.

4.4.1 Multi-loads Switching Method

In many existing optimization methods, only one single loading case is considered. Especially for the optimization of the lateral loads, the optimized structure is asymmetrical. The most common solution for this problem is to impose symmetry on to the structure. However, it loses the potentiality of other interesting structural forms. Multi-loads switching method is proposed here for designing of multi-loads condition. When the proposed topology design method is applied, different load cases are switched between different iterations. It is possible to generate a good design satisfying the performance requirements of all load cases.
4.4.2 Force Density Method

The designed topology of structures after many iterations of the proposed algorithm has dense and sparse zones. To make it more clear and elegant, the force density method is added to the algorithm to smooth the structures. However, the author encounters difficulties to design a good force density input for the force density method. Further study is needed to add relaxation method or smoothing method to the proposed algorithm.
Chapter 5

Conclusion

This chapter summarizes the future research work for the proposed structural topology design method and concludes with the potential impact of this research.

5.1 Main Contributions

1. This thesis reviews the existing space frame generation methods and identifies the needs for an innovative space frame topology design method. It also reminds the importance of biomimicry in architecture and engineering.

2. It develops an innovative structural topology design method inspired from trabecular bones, which can be applied to 2D structures, 3D structures and building-scale structures.

3. It discovers and proves the importance of the connectivity in the structures. The change of the connectivity in the structures can totally change the performance of the structures.

5.2 Future Work

In the future, detailed parameters should be studied in the proposed trabecular topology inspired structural design method. Some questions to addressed are as
follows.

1. How does the Voronoi density affect results?

2. How to decide the number of iterations for running the algorithm?

3. How to improve proposed multi-loads switching method?

4. How to improve proposed force density method?

5.3 Potential impact

The proposed computational structural topology design method has no control over elastic energy. However, it achieves the optimization goal of the 2D, 3D and building-scale structures and improves the structural performance for most cases, although there are also some limitations. It reveals the relationship between the connectivity of structures and structural performance, which is first discovered in trabecular bones. The proposed method provides a new understanding of nature optimization in trabecular bones in the field of biomechanics. It demonstrates that the natural processes can be used as an inspiration for structural design and optimization.
Bibliography


