ANALYTICAL INVESTIGATION

OF SEAWORTHINESS STATISTICAL PARAMETERS

by

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- i -
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ABSTRACT

In order to determine the feasibility of evaluating seaworthiness performance along analytical lines, a systematic comparison is first made of experimentally measured and theoretically computed ship motions and phase angles. Tank data for a broad range of Series 60 models in regular waves are extracted from N.S.M.B. publications and correlated with model responses calculated by a digital computer program which is based on the Korvin-Kroukovsky linear theory of ship motions in conjunction with Grim's latest results on added mass and damping. Both directly ahead and astern seas are considered and emphasis is paid on the effects of variations of hull form parameters and weight distribution. It is concluded that with minor exceptions there is satisfactory correlation between the analytical and experimental results and recommendations towards further improvement are subsequently presented.

An attempt is next made to formulate kinematical seaworthiness on a rational basis. To this end, the importance of pitching, vertical acceleration, slamming, wetness and propeller emergence is discussed. It is then shown how the above ship responses and seaworthiness phenomena can be analyzed and hence assessed on the basis of simple statistical parameters and criteria. Following the statistical approach in the frequency domain, the previous computer program is modified and extended in order to examine these seaworthiness considerations. A 600 ft. Series 60 ship is chosen to illustrate the potential of the developed computer algorithm. Further recommendations are then given towards a systematic investigation aiming to establish the importance of seaworthiness in preliminary ship design.

Thesis Supervisor: Philip Mandel
Title: Professor of Naval Architecture
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Title</td>
<td>1</td>
</tr>
<tr>
<td>Abstract</td>
<td>11</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>iii</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>iv</td>
</tr>
<tr>
<td>List of Tables</td>
<td>vi</td>
</tr>
<tr>
<td>List of Figures</td>
<td>vii</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>ix</td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
</tbody>
</table>

## PART I

<table>
<thead>
<tr>
<th>Chapter I</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Linear Theory of Pitching and Heaving</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter II</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computational Procedure Considerations</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter III</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation Between Theory and Experiment and Discussion of Results</td>
<td>84</td>
</tr>
<tr>
<td>1. Previous Comparisons</td>
<td>84</td>
</tr>
<tr>
<td>2. Discussion of Results</td>
<td>85</td>
</tr>
<tr>
<td>3. Reasons for Discrepancies</td>
<td>89</td>
</tr>
<tr>
<td>4. Conclusions</td>
<td>94</td>
</tr>
<tr>
<td>5. Recommendations</td>
<td>94</td>
</tr>
</tbody>
</table>

## PART II

<table>
<thead>
<tr>
<th>Chapter IV</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>On Seaworthiness Considerations</td>
<td>96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter V</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Statistical Approach</td>
<td>104</td>
</tr>
<tr>
<td>1. The Excitation: Seaway</td>
<td>104</td>
</tr>
<tr>
<td>2. Linear System: Ship</td>
<td>108</td>
</tr>
<tr>
<td>3. The Responses: Pitch, Acceleration and Relative Motion and Velocity</td>
<td>110</td>
</tr>
</tbody>
</table>
CHAPTER VI
SHIP RANDOM RESPONSES AND SEAWORTHINESS
PHENOMENA

1. Pitching Amplitude
2. Vertical Acceleration
3. Slamming Occurrence
4. Wetness Occurrence
5. Propeller Immersion

CHAPTER VII
SAMPLE SEAWORTHINESS EVALUATION -
FURTHER RECOMMENDATIONS

BIBLIOGRAPHY

APPENDICES

A. MISCELLANEOUS PROBLEMS ARISING IN THE CORRELATION
   ATTEMPT

1. On Experimental Shortcomings With Regard to
   Directly Ahead and /or Astern Sea Cases
2. On Unified Motion Amplitude and Phase
   Definitions
3. Pitching and Heaving Periods of Series 60
   Models
4. Evaluation of Critical Wavelength to
   Shiplength Ratios

B. ANALYTICAL DETAILS OF THE LINEAR THEORY OF SHIP
   MOTIONS

C. THE FREQUENCY MAPPING PROBLEM FOR DIRECTLY AHEAD AND
   ASTERN SEAS

1. Directly Ahead Seas (χ = 180°)
2. Directly Astern Seas (χ = 0°)

D. ANALYSIS OF CERTAIN KINEMATIC RESPONSES IN REGULAR WAVES

1. Absolute Acceleration
2. Relative Motion and Velocity

E. DESCRIPTION OF COMPUTER PROGRAMS
# LIST OF TABLES

<table>
<thead>
<tr>
<th>I</th>
<th>SERIES 60 MODEL CHARACTERISTICS</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>LIST OF FIGURES OF (13) FROM WHICH EXPERIMENTAL DATA WERE OBTAINED FOR COMPARISON OF THEORY AND EXPERIMENT</td>
<td>15</td>
</tr>
<tr>
<td>III</td>
<td>SUMMARY OF SEAWORTHINESS EVALUATION FOR 606 FT. SHIP</td>
<td>139</td>
</tr>
<tr>
<td>AI</td>
<td>NON-DIMENSIONAL PITCHING AND HEAVING PERIODS OF MODELS FOR ZERO SPEED IN STILL WATER</td>
<td>A-10</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<p>| Page |
|---|---|
| 1 | THREE-DIMENSIONAL CONFIGURATION OF MODEL HULL PARAMETERS UNDER EXAMINATION | 12 |
| 2-9 | MODEL A - HEAVING AND PITCHING AMPLITUDES AND PHASE ANGLES IN DIRECTLY AHEAD AND ASTERN SEAS | 20-27 |
| 10-17 | MODEL B - HEAVING AND PITCHING AMPLITUDES AND PHASE ANGLES IN DIRECTLY AHEAD AND ASTERN SEAS | 28-35 |
| 18-25 | MODEL C - HEAVING AND PITCHING AMPLITUDES AND PHASE ANGLES IN DIRECTLY AHEAD AND ASTERN SEAS | 36-43 |
| 26-33 | MODEL D - HEAVING AND PITCHING AMPLITUDES AND PHASE ANGLES IN DIRECTLY AHEAD AND ASTERN SEAS | 44-51 |
| 34-41 | MODEL E - HEAVING AND PITCHING AMPLITUDES AND PHASE ANGLES IN DIRECTLY AHEAD AND ASTERN SEAS | 52-59 |
| 42-49 | MODEL F - HEAVING AND PITCHING AMPLITUDES AND PHASE ANGLES IN DIRECTLY AHEAD AND ASTERN SEAS | 60-67 |
| 50-57 | MODEL G - HEAVING AND PITCHING AMPLITUDES AND PHASE ANGLES IN DIRECTLY AHEAD AND ASTERN SEAS | 68-75 |
| 58-65 | MODEL C WITH VARYING RADIUS OF GYRATION - HEAVING AND PITCHING AMPLITUDES IN DIRECTLY AHEAD SEAS | 76-83 |
| 66 | ILLUSTRATING SLAMMING OCCURRENCE | 121 |
| 67 | ILLUSTRATING DEGREES OF WETNESS | 126 |
| 68 | ILLUSTRATING PROPELLER EMERGENCE AND SUBMERGENCE | 129 |
| 69 | ABSOLUTE AND MODIFIED NEUMANN SEA ENERGY DENSITY SPECTRA | 133 |
| 70 | RESPONSE AMPLITUDE OPERATOR AND SQUARED AMPLITUDE DENSITY SPECTRA OF PITCHING MOTION | 134 |
| 71 | RESPONSE AMPLITUDE OPERATOR AND SQUARED AMPLITUDE DENSITY SPECTRA OF VERTICAL ACCELERATION AT SLAMMING (WETNESS) STATION | 135 |</p>
<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
</tr>
<tr>
<td>73</td>
</tr>
<tr>
<td>74</td>
</tr>
<tr>
<td>A1</td>
</tr>
<tr>
<td>A2</td>
</tr>
<tr>
<td>C1</td>
</tr>
<tr>
<td>C2</td>
</tr>
<tr>
<td>C3</td>
</tr>
<tr>
<td>D1</td>
</tr>
<tr>
<td>D2</td>
</tr>
</tbody>
</table>
NOMENCLATURE

English Letters

\[ a \] = coefficient of equation of motion
\[ a \] = arbitrary value of random response
\[ A \] = coefficient of equation of motion
\[ A^* \] = area under amplitude density spectrum of generalized random variable
\[ A_r \] = area under amplitude density spectrum of generalized relative motion
\[ A_{r'} \] = area under amplitude density spectrum of generalized relative velocity
\[ A_{g'} \] = area under amplitude density spectrum of generalized absolute acceleration
\[ b \] = coefficient of equation of motion
\[ b \] = arbitrary value of random response
\[ B \] = breadth of ship or model
\[ B \] = coefficient of equation of motion
\[ B^* \] = station breadth of ship of model at designed waterline
\[ c \] = a constant
\[ c \] = coefficient of equation of motion
\[ c \] = gravity wave celerity
\[ C \] = coefficient of equation of motion
\[ C_B \] = block coefficient of ship or model
\[ C_P \] = prismatic coefficient of ship or model
\[ C_W \] = waterplane area coefficient
\[ d \] = coefficient of equation of motion
\[ D \] = coefficient of equation of motion

- ix -
\( D \) = propeller diameter

\( e \) = coefficient of equation of motion

\( E \) = coefficient of equation of motion

\( E \) = total energy of unit-width gravity wave

\( E \) = area under sea energy density spectrum

\( f \) = freeboard at any ship station

\( \tilde{f} \) = expected "frequency" of random fluctuation

\( \tilde{f}^* \) = number of slams in unit time

\( F \) = total vertical (heaving) force

\( F_0 \) = amplitude of time-varying heaving force

\( \tilde{F} \) = complex vertical (heaving) force

\( g \) = gravitational acceleration

\( g \) = coefficient of equation of motion

\( G \) = coefficient of equation of motion

\( h, h(t) \) = instantaneous wave elevation referred to absolute system

\( h(\xi) \) = instantaneous wave elevation referred to relative (moving) system

\( h_0 \) = amplitude of sinusoidal wave (half wave-height)

\( H \) = draft of ship or model

\( H (w_e) \) = complex frequency response of linear system

\( |H_{\Theta \Theta} (w_e)|^2 \) = response operator of pitching motion

\( |H_{\Phi \Phi} (w_e)|^2 \) = response operator of relative motion

\( |H_{\Phi \Phi} (w_e)|^2 \) = response operator of relative velocity

\( |H_{SS} (w_e)|^2 \) = response operator of absolute acceleration

\( J \) = longitudinal moment of inertia of ship or model

\( J (w_e) \) = Jacobian (determinant) of frequency transformation

\( k \) = maximum wave slope

\( k_\theta \) = non-dimensional longitudinal radius of gyration

- x -
\( k_z \) = vertical inertia coefficient
\( L \) = length of ship or model
\( \text{LCB} \) = longitudinal distance of centre of buoyancy from amidships
\( \text{LCG} \) = longitudinal distance of centre of gravity from amidships
\( m \) = mass of ship or model
\( m_k \) = kth moment of response amplitude density spectrum
\( M \) = total pitching moment
\( M_0 \) = amplitude of time-varying pitching moment
\( \hat{M} \) = complex pitching moment
\( N (\xi) \) = sectional damping coefficient
\( p \) = probability of occurrence
\( P_\xi (x) \) = probability density function of amplitude of random variable
\( P_{\xi, \alpha} (x, \hat{x}) \) = joint probability density function of amplitudes at
two random variables
\( r, r(\xi) \) = relative instantaneous motion of arbitrary ship point
\( \hat{r}, \hat{r}(\xi) \) = relative instantaneous velocity of arbitrary ship station
\( r_o \) = amplitude of relative motion
\( \hat{r}_o \) = amplitude of relative velocity
\( \hat{r}_c \) = critical value of random relative velocity
\( R (\alpha) \) = average time reliability
\( R_{\alpha \alpha} (\alpha) \) = reliability of acceleration assuming value less than \( \alpha \)
\( s \) = absolute vertical motion of arbitrary ship point
\( s_o \) = amplitude of absolute vertical motion
\( \dot{s} \) = absolute vertical velocity of arbitrary ship station
\( \dot{s}_o \) = amplitude of absolute vertical velocity
\( \ddot{s} \) = absolute vertical acceleration of arbitrary ship station
\( \ddot{s}_o \) = amplitude of absolute vertical acceleration
\( s^{(99)}_{1/10} \) = average of 1/10 highest amplitudes of vertical acceleration
experienced at the "slamming" station

- xi -
\( t \) = time

\( \bar{T} \) = expected "period" of random fluctuation

\( T_e \) = period of encounter

\( T_s \) = pitching or heaving natural period of ship or model

\( U \) = wind speed

\( V \) = speed of ship or model

\( x \) = coordinate of absolute (stationary) coordinate system

\( x \) = range variable of any random response amplitude

\( y \) = section depth of equal area semi-circle

\( \dot{z} \) = heaving motion of centre of gravity of ship or model

\( \dot{z} \) = heaving velocity of centre of gravity of ship or model

\( \ddot{z} \) = heaving acceleration of centre of gravity of ship or model

\( z_0 \) = amplitude of heaving motion

\( \bar{z} \) = complex heaving motion

**Greek Letters**

\( \alpha \) = acceleration level

\( \delta \) = theoretically computed heaving phase angle

\( \delta^* \) = heaving phase angle (as defined in report)

\( \delta \) = fraction of propeller diameter

\( \Delta \) = displacement of ship or model

\( \varepsilon \) = theoretically computed pitching phase angle

\( \varepsilon^* \) = pitching phase angle (as defined in report)

\( \varepsilon_{sh} \) = experimentally measured phase angle \((s = z \text{ for heave,}\n
s = \theta \text{ for pitch})

\( \varepsilon(w) \) = random phase angle

\( \varepsilon \) = "broadness" factor
\( \eta \) = fraction of ship or model draft
\( \varphi \) = pitching motion
\( \dot{\varphi} \) = pitching velocity
\( \ddot{\varphi} \) = pitching acceleration
\( \varphi_0 \) = amplitude of pitching motion
\( \varphi \) = complex pitching motion
\( \bar{\varphi}_{1/3} \) = average of 1/3 highest pitching amplitudes
\( \bar{\varphi}_{1/10} \) = average of 1/10 highest pitching amplitudes
\( \lambda \) = wavelength
\( \lambda_e \) = effective wavelength
\( \Lambda \) = tuning factor
\( \xi \) = coordinate of moving (ship) coordinate system
\( \bar{\xi} \) = average of amplitude of random response
\( \bar{\xi}_{1/3} \) = average of 1/3 highest amplitudes of random response
\( \bar{\xi}_{1/10} \) = average of 1/10 highest amplitudes of random response
\( \bar{\xi}_p \) = "most probable" amplitude of random response in one oscillation
\( \bar{\xi}_m \) = highest expected amplitude of random response occurring in N oscillations
\( \rho \) = water density
\( \rho(\xi) \) = phase angle of relative motion
\( \dot{\rho}(\xi) \) = phase angle of relative velocity
\( \sigma \) = phase angle of heaving force
\( \sigma_\xi \) = standard deviation of amplitude of random response
\( \tau \) = phase angle of pitching moment
\( \tau_z \) = non-dimensional natural heaving period of ship or model
\( \tau_\theta \) = non-dimensional natural pitching period of ship or model
\[ \phi(s) \quad \text{= phase angle of vertical acceleration} \]
\[ \xi(\omega) \quad \text{= absolute sea energy density spectrum} \]
\[ \xi_{ii}(\omega) \quad \text{= transformed (input) sea energy density spectrum} \]
\[ \xi_{\phi\phi}(\omega) \quad \text{= pitch amplitude density spectrum} \]
\[ \xi_{rr}(\omega) \quad \text{= relative motion amplitude density spectrum} \]
\[ \xi_{rr}(\omega) \quad \text{= relative velocity amplitude density spectrum} \]
\[ \xi_{ss}(\omega) \quad \text{= absolute acceleration amplitude density spectrum} \]
\[ \chi \quad \text{= heading angle} \]
\[ \chi_e \quad \text{= apparent heading angle} \]
\[ \hat{\psi} \quad \text{= theoretically computed phase angle of generalized ship motion} \]
\[ \hat{\psi}_w \quad \text{= generalized motion phase angle (as defined in report)} \]
\[ \hat{\psi}_0 \quad \text{= expected number of zero crossings of random fluctuation} \]
\[ \hat{\psi}_m \quad \text{= expected total number of maximum amplitudes in unit time} \]
\[ \omega \quad \text{= absolute wave frequency} \]
\[ \omega_e \quad \text{= frequency of encounter} \]
INTRODUCTION

Two main objectives determined the scope of the present investigation. The first is an evaluation of the accuracy of the linearized theory of pitching and heaving motions of a ship in regular waves performed by comparing theoretical calculations to experimental data for similar conditions. The second is an attempt to develop a uniform procedure for assessing seaworthiness performance in irregular seas, using only the analytical approach. Such an attempt is justified in view of the universality of the theoretical method, provided that the latter is validated.

The first part of the work consists of a reappraisal of the Korvin-Kroukovsky theoretical procedure for computing ship motions. Various considerations which arise in the correlation of the theoretical and experimental results are next discussed. Pitching and heaving motions of eleven Series 60 models differing in hull proportions and weight distribution and moving in directly ahead and astern seas are calculated by a recently developed computer program based on the aforementioned theory. Grim's latest results for added mass and damping are incorporated in the computer algorithm in preference to earlier, less accurate data. The theoretical results are subsequently compared with model data which are extracted from N.S.M.B. publications. For several reasons, the comparison is characterized by the absence of a norm so that the validation of the theory is based on how well the two sets of data match each other. From a large family of graphs it is concluded that the analytical procedure has attained a stage of practical application since with the exception
of a few discrepancies, the magnitude of the predictions as well as
the trends are similar to the ones obtained from experimental measurement.

In view of this agreement, an attempt to utilize the Korvin-
Kroukovsky method in evaluating seaworthiness performance in confused
seaways was undertaken. A limited but still useable formulation of
kinematical seaworthiness is first presented.

The underlying principles of the statistical approach in the
frequency domain are next reviewed and it is shown that quantitative
measures can be assigned to important ship responses and seaworthiness
phenomena. The computer program previously mentioned is modified and
extended in order to statistically define and assess random pitching and
vertical acceleration amplitudes as well as slamming, wetness and propeller
blade emergence. The whole analysis tentatively utilizes the concept of
a long-crested Neumann sea. The potential of the method and of the computer
algorithm are illustrated in the case of a Series 60, 600 ft. ship.

Analytical derivations and details which accompany any particular
chapter are included in the form of appendices, to which reference is
made whenever required. A detailed description of the structure and use
of the two computer programs is also given. References appear in
parentheses in the text and may be found listed in the Bibliography.
Equations, tables and figures shown in any given appendix are preceded
by a letter which corresponds to that appendix; those given in the main
text are simply numbered in successive order.
PART I

I. THE LINEAR THEORY OF PITCHING AND HEAVING

In the realm of a ten-year continuous and conscientious research activity in seakeeping problems, the profession has been confronted with a multitude of problems some of which have only partially been examined. In particular, the present status is characterized by an almost universally voiced need for an immediate trial application of our broad scientific knowledge. The present investigation was launched in order to assist in satisfying this need and as such, it is believed, represents the first attempt to utilize systematically an existing analytical tool. Since the latter may be identified as the "backbone" of the analysis at hand, the purpose of this chapter is to outline the basis and essential features of the Korvin-Kroukovsky theoretical procedure for computing heaving and pitching motions of a ship in regular waves.

The earliest and crudest version of the method was presented in (2), where the authors essentially amplified the studies of Kriloff, Weinblum, St. Denis and other pioneers in the field of ship oscillations. The first complete presentation of the procedure followed in 1955 (3) and was subsequently corrected and improved two years later (4). To this end, various discussers of (3) and in particular, Kaplan (5), were instrumental in pointing out certain mistakes of the 1955 exposition, while Pcy's analysis (6) motivated a more accurate definition of the velocity dependent terms in the equations of motion (see Appendix B, eq. B5). Finally, Jacobs (7) presented a more precise expression for the exciting force and hence extended the procedure to the analytical calculation of ship bending moments, as a result of which,
a unified computational approach was outlined in (8). The most recent reappraisal of the method appears in (9), whereas for a detailed study of the problem the interested reader is referred to (1).

The hydromechanical phase of the seakeeping problem employs the concepts of analytical dynamics of perfectly rigid bodies. In this respect, the ship is assumed to be free of elastic distortions in so far as spatial orientation is concerned. For its complete specification, the latter requires a six-fold set of differential equations corresponding to the system's six degrees of freedom; for our purposes, however, the analysis is considerably simplified by focusing attention in the longitudinal plane of symmetry only and by furthermore ignoring surging motion. The last point also implies neglect of propeller thrust fluctuations and hence the vessel, floating in an even keel condition, moves at constant speed. Rolling, swaying and yawing are excluded from the analysis by assuming that their associated effects are negligible. The seas, directly ahead or astern, are described by uniform, infinitely long-crested sinusoidal waves of small amplitude and the direction of the ship's motion is taken to be normal to the wave crests.

The mathematical model for the oscillating vessel, namely, the set of differential equations which describes the idealized motion in the vertical plane is next derived, on the basis of infinitesimal departures from the equilibrium position, by applying Newton's Second Law of Motion for pure translation and pure rotation about an invariant pitching axis, in the form,

\[ m \ddot{z}(w_e, t) = F(w_e, \lambda, \dot{z}, \ddot{z}, \ddot{\theta}, \dot{\theta}, t) \]

\[ J \ddot{\theta}(w_e, t) = M(w_e, \lambda, \ddot{z}, \dot{z}, \ddot{\theta}, \dot{\theta}, z, t) \]  

(1)

- 4 -
The exact total force and moment acting on a semi-submerged body moving in waves is a formidable hydrodynamic problem and, in order to render the method amenable to easy manipulation, the difficulty is circumvented by commencing with the "no-cross-flow" hypothesis, well known in aeronautical circles in connection with "slender-body" theory. Thus, the vessel is divided in elementary thin segments and, for analysis purposes, attention is directed to a typical control section. The latter is assumed initially to be typical of an infinitely long circular cylinder surrounded by a two-dimensional, irrotational flow pattern as caused by an inviscid and incompressible fluid. It should be emphasized that, in this manner, we do not allow for the interaction of adjacent sections, since we neglect longitudinal perturbation velocities. We see, therefore, that such an approach essentially replaces a three-dimensional problem by a summation of many two-dimensional ones, with the advantage that,

(a) One gains a clear, yet still partial, insight of the physical phenomenon

and

(b) The complete load distribution can be determined, once the motions have been computed, and hence, shearing forces and bending moments at any hull station can readily be determined by successive integrations.

A distinct feature of this theoretical approach is the fashion in which the forcing functions and coefficients of the equation of motion are determined. For this reason we shall now discuss in turn the manner in which the exciting, damping and restoring forces and moments are computed, independently modified and subsequently improved in order to approach reality as simply and correctly as possible.
By a judicious choice of the total velocity potential which incorporates linearly the effects of body presence, wave action as well as their interaction, the velocity field is linked, via Bernoulli's Theorem, to the pressure field around the control section. The inclusion of body-wave interaction represents a sufficiently accurate improvement over the past more elementary approaches, whereby we can forgo once and for all the now famous Froude-Kriloff hypothesis. In this manner, some of the dynamic aspects of our problem are easily established, for the elemental hydrodynamic force, whenever required, is simply obtained by integrating the pressure over the sectional shape.

The analysis of forces, as derived from potential theory considerations, is accomplished in two distinctive steps. At first, consideration is given to the "exciting" hydrodynamic force which is derived from the combined velocity potential due to (a) pure wave action and (b) its interaction with the present body, on the basis of high-frequency motion. The resulting expression is next generalized, in an independent fashion, in order to accommodate for the actually occurring low-frequency motion of a ship-shape-section conforming with F. M. Lewis' forms (4). A further minor correction is nowadays considered necessary in order to allow for energy dissipation and pressure variation with depth. This has been introduced by Jacobs (7) and constitutes a further improvement over the 1957 exposition (4).

In the present investigation the differential "exciting" force is defined in terms of the wave elevation and its first two time derivatives as shown in (7) and (8). The differential "exciting" moment about a stationary pitching axis at the ship's centre of
gravity is then obtained from the previous expression and two respective integrations over the ship length provide the forcing functions appearing on the RHS of the differential equations of motion (Eq. B1, Appendix B). Physically, they may be interpreted as the total time varying force and moment acting on a fully restrained two-dimensional ship system.

The differential hydrodynamic force and moment brought about by the ship's own motion are next derived by utilizing the velocity potential due solely to the body's presence in the two-dimensional flow regime. This force and moment, with the inclusion of suitable damping and restoring components to be discussed later, are occasioned by the section's oscillation in an originally calm water surface and yield, upon correction for low-frequency motion and integration over the ship length, terms which are transferred to the LHS of the equations. They result in polynomial form and are proportional to $\ddot{Z}$, $\dot{z}$, $\ddot{\beta}$, $\dot{\theta}$, $\delta$ and contribute, sometimes only partially, to the coefficients of the set (Eq. B1, Appendix B).

Five of these coefficients attain their final form (Eq. B5, Appendix B), once the remaining elemental forces and moments due to damping and restoring effects are independently included and integrated. The differential damping force per unit velocity of the control section is evaluated in a manner first proposed by St. Denis (10) on the basis of earlier work by Holstein and Havelock. The force depends on the encounter frequency and is also a function of the square of the ratio of the amplitude of the wave, created by the section, to the section's heaving amplitude. Finally, the elementary restoring force and moment are computed separately on the basis of purely physical arguments, as differential changes in displacement while the section's draft changes.
There are certain aspects of the theoretical tool described so far that allow the analyst to admit further elaboration in the whole procedure, if future, more accurate knowledge so permits.

For example, up until recently, in the absence of more suitable data, the correction for free-surface effects was, whenever required, applied in the form of a frequency dependent factor (4) originally derived by Ursell for an oscillating semi-circular cylinder. This can now be abandoned, however, for Grim's latest results allow a further refinement in the analysis by simultaneously treating frequency and section form modifications in one, more accurate step. The best resume of his ten-year work on added mass and damping for low-frequency oscillations of Lewis section shapes is given in (11). The data presented there were determined from a computer algorithm, similar to the one incorporated in the digital machine program used in this investigation for the same purposes and discussed in Appendix E.

The mathematical model described in this chapter pertains to a linear, time-invariant system. However, the analysis leads to a serious drawback in that most of the coefficients of the equations of motion result as functions of the encounter frequency. The essence of this disagreeable outcome will not be mentioned here; it is lucidly discussed by Cummins (12) who, while propounding a different model, referred to the Korvin-Kroukovsky theoretical procedure as a "shoe that doesn't really fit". Be that as it may, we cannot afford, on the other hand, to reject a useful device which functions fairly well and represents nature effectively albeit by force.
II. COMPUTATIONAL PROCEDURE CONSIDERATIONS

As mentioned earlier, the first objective of the present investigation was to assess in a systematic and comprehensive manner the potential of the linearized theory of ship motions in the form it exists at present. In this respect, we were particularly interested in examining the extent to which the theoretical tool accurately predicts the effects of variations in hull form delineation as well as hull weight distribution. For this purpose, use was made of the Netherlands Ship Model Basin series of experimental data on ship model behaviour in regular waves, as presented in (13) and (14).

At this juncture it seems mandatory to pose and recognize the underlying implications of our effort. Theory and experiment are, by necessity, the two avenues along which any engineering profession proceeds in order to establish, analyze and explain physical phenomena and ultimately provide useful information for applications. A perusal of our literature will immediately indicate that this is particularly true in the field of Naval Architecture. In this connection it will suffice then to state on the outset, that neither the theoretical or experimental approaches will be identified as a norm for our comparison purposes. This seems to be further validated by the fact that, at present, theory still rests on powerful and limiting assumptions while experiment is still linked with sources of systematic errors. In this work, therefore, the entire philosophy behind the correlation attempt will be interpreted as being one of "matching" the products of the analyst and experimenter, in the hope that further light will be shed on the shortcomings of both approaches.
Korvin-Kroukovsky in his comprehensive treatise (1), p. 210, concluded that the theoretical tool had now reached such a stage of practical perfection that the naval architect could make systematic use of it in computing system functions for seakeeping work. His statement was based on a relatively small family of experimental data and hand computations, and hence the question of whether his findings are valid or not is subject to critical evaluation in this investigation.

It must be appreciated that the labour involved in computations of ship motions by the linear theory method is tedious and time-consuming and for this reason recourse has been had to digital computer facilities. Haslum of the Department of Naval Architecture and Marine Engineering undertook the creation of a computer program, which, along with the collaboration of the author has been brought up into the working stage. A description of the program may be found in (15), although sufficient comments are also made in Appendix B of this work. The basic steps involved in the computation are essentially similar to the ones proposed in (6), whereby a judicious breakdown of the whole computation is allowed in suitable "packages" which can be easily modified or extended if this is deemed necessary. The program has been thoroughly examined in steps as well as on the whole in order to ensure the validity of the results reported herein. Thus, it may be stated safely that any departures or discrepancies are due to inherent imperfections of the theoretical procedure and/or tank measuring techniques and should not be considered as results of computational blunders.

The voluminous series of data published in (13), for Series 60 models of varying block coefficient, length to beam and length to draught ratios constitutes the first systematic seakeeping experimental
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The voluminous series of data published in (13), for Series 60 models of varying block coefficient, length to beam and length to draught ratios constitutes the first systematic seakeeping experimental
project to date and hence represented the main source for the comparison attempt. Attention has been focused to only a partial extract of the results reported there and in particular, consideration was given to non-dimensional pitching and heaving amplitudes together with their associated phase angles which correspond approximately to directly ahead and astern seas.

The reason for the above restriction is, of course, the essential limitations of the theoretical approach. It must also be noted that the word "approximately" has been used, since the experimental results of (13) pertained to actual heading angles of 10° and 170°. Further discussion on the latter point appears in Appendix A.

The correlation between theory and experiment, in so far as it is affected by geometrical variations, is accomplished over a 3-D range of model hull parameters, as shown schematically in Fig. 1. Although the original experimental work incorporated intermediate values in all three directions of Fig. 1, it is considered that the adopted variables adequately suffice for our purposes. Table I indicates the main particulars of the models chosen for the computation, while further information on hull form delineation, as required for the computer program, is given in Appendix E. In particular, the sectional area coefficients and load waterline breadths at various stations were obtained from Tables 4, 6 and 8 of the original paper on Series 60 models (16).

Throughout this investigation, it is assumed that all models were tested in an even keel condition and that, therefore, the longitudinal centres of gravity and buoyancy coincided. The examined model speed range covers four different Froude numbers of 0.10 (0.05) 0.20 in accord with the available results reported in (13). The wave
FIG. 1 THREE DIMENSIONAL CONFIGURATION OF MODEL HULL PARAMETERS UNDER EXAMINATION
TABLE I
Series 60 Model Characteristics

<table>
<thead>
<tr>
<th>MODEL</th>
<th>L Feet</th>
<th>B Feet</th>
<th>H Feet</th>
<th>L/B</th>
<th>L/H</th>
<th>B/H</th>
<th>C_B</th>
<th>C_P</th>
<th>C_W</th>
<th>LCB From % L</th>
<th>Pounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10.00</td>
<td>1.429</td>
<td>0.571</td>
<td>7.00</td>
<td>17.50</td>
<td>2.50</td>
<td>0.800</td>
<td>0.805</td>
<td>0.871</td>
<td>2.50F</td>
<td>407.14</td>
</tr>
<tr>
<td>B</td>
<td>10.00</td>
<td>1.429</td>
<td>0.571</td>
<td>7.00</td>
<td>17.50</td>
<td>2.50</td>
<td>0.700</td>
<td>0.710</td>
<td>0.787</td>
<td>0.55A</td>
<td>356.25</td>
</tr>
<tr>
<td>C</td>
<td>10.00</td>
<td>1.429</td>
<td>0.571</td>
<td>7.00</td>
<td>17.50</td>
<td>2.50</td>
<td>0.600</td>
<td>0.614</td>
<td>0.706</td>
<td>1.50A</td>
<td>305.36</td>
</tr>
<tr>
<td>D</td>
<td>10.00</td>
<td>1.429</td>
<td>0.417</td>
<td>7.00</td>
<td>24.00</td>
<td>3.43</td>
<td>0.700</td>
<td>0.710</td>
<td>0.787</td>
<td>0.55A</td>
<td>260.17</td>
</tr>
<tr>
<td>E</td>
<td>10.00</td>
<td>1.429</td>
<td>0.909</td>
<td>7.00</td>
<td>11.00</td>
<td>1.57</td>
<td>0.700</td>
<td>0.710</td>
<td>0.787</td>
<td>0.55A</td>
<td>567.13</td>
</tr>
<tr>
<td>F</td>
<td>10.00</td>
<td>1.176</td>
<td>0.571</td>
<td>8.50</td>
<td>17.50</td>
<td>2.06</td>
<td>0.700</td>
<td>0.710</td>
<td>0.787</td>
<td>0.55A</td>
<td>293.17</td>
</tr>
<tr>
<td>G</td>
<td>10.00</td>
<td>1.818</td>
<td>0.571</td>
<td>5.50</td>
<td>17.50</td>
<td>3.18</td>
<td>0.700</td>
<td>0.710</td>
<td>0.787</td>
<td>0.55A</td>
<td>453.22</td>
</tr>
</tbody>
</table>

*As calculated for fresh water based on above particulars.
All models have a radius of gyration $K_g = 0.24L = 2.4$ feet.
conditions are characterized by a constant wave height to skip length ratio equal to \( L/50 \) (\( h_o = 0.1 \text{ ft.} \)) and by five wavelength to shiplength ratios. The latter are slightly different from the ones given in (13) for reasons discussed in Appendix A.

Table II indicates the figures of (13), from which values of pitching and heaving non-dimensional amplitudes and their phases were obtained, for every model examined. For a limited number of cases, these data were checked with similar ones reported in later publications of the N.S.M.B. Tank and were found to be in very good agreement. The above statement refers in particular to models B and C in ahead seas (\( \chi = 170^\circ \)). For the former, Reference (17) provides dimensionless pitch and heave amplitudes and phases in tabular form (Tables V-VIII) for three Froude numbers (0.15, 0.20, 0.25), while for the latter, Figs. 16 and 17 of (14) report heave and pitch amplitudes only, (\( k_e = 0.24 \) model). Referring finally to the computational approach, Appendix E includes a typical output of the machine calculation which serves to indicate the manner in which the theoretical results were obtained in every case.

The following sign conventions and phase definitions are adopted in this report although the present status of seakeeping research is characterized by complete non-uniformity in data presentation among theoreticians on the one hand and experimental establishments on the other.

a) Heave is positive upwards and pitch is positive when the bow is up.

b) Amplitudes of motions are considered positive for all directly ahead and astern wave lengths.

c) Phase angles are defined as lags when referred to the maximum wave elevation at the midship section of the model; furthermore, their range is restricted to 0-180\(^\circ\) only.
<table>
<thead>
<tr>
<th>Model</th>
<th>Heaving Motion</th>
<th>Pitching Motion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-dimensional Amplitude*</td>
<td>Phase Angle (lag)*</td>
</tr>
<tr>
<td></td>
<td>$z_0/h_0$</td>
<td>$\delta$</td>
</tr>
<tr>
<td>A</td>
<td>15(a) - 15(b)</td>
<td>16(a)-16(b)</td>
</tr>
<tr>
<td>B</td>
<td>24(a) - 24(b)</td>
<td>25(a)-25(b)</td>
</tr>
<tr>
<td>C</td>
<td>33(a) - 33(b)</td>
<td>34(a)-34(b)</td>
</tr>
</tbody>
</table>

*To be suitably interpreted, as discussed in Appendix A
As a result of the adopted definitions, a short analysis had to be made in order to bring the theoretical and experimental results on a common basis for comparison purposes. Appendix A includes an amplification of the previous comments and a description of the manner in which the two sources of data were matched.

The results of the first phase of the correlation attempt are reported in the form of graphs of non-dimensional motion amplitudes versus wave length to ship length ratio, Figs. 2-57. Heave is divided by the wave amplitude $h_0$ and pitch, in radian measure, by the maximum wave slope $\frac{2\pi h_0}{\lambda}$. Superimposed on the same graphs are given the phase lag variations, as defined earlier. Full and dotted lines indicate theoretically computed motion amplitudes and phase angles respectively. Experimentally determined motion amplitudes are represented by circles whereas experimental phase angles are shown by full spots. For every model depicted in Fig. 1, there correspond eight sets of such graphs for the four Froude numbers in ahead and astern seas respectively.

A prime requisite of a particular presentation is that of reflecting results in a comprehensive and digestible form while at the same time retaining and bringing out all salient features. Although we have endeavored to do so, it is felt that the adopted variables for graph presentation may still be vulnerable to criticism. It is realized, for instance, that an abscissa of model speed or Froude number might have been more appropriate, especially so because the effect of frequency of encounter would be depicted. Since, however, the results of (13) and (14) were purposely restricted in the linear
range it would not have been possible, within the available speed range, to reveal peaks in amplitude and phase variations. A wavelength to shiplength abscissa, however, did accommodate these important effects. Again it may be pertinent to restate that our aim was to establish a degree of correlation, without trying to analyze trends and assess the significance of important variables.

In order to incorporate in the final picture the consequences of synchronism, the critical wavelength to shiplength ratio corresponding to resonant conditions is indicated by an arrow in every graph. The determination of the latter involved some special thought in view of the well-known non-uniqueness and ambiguities which arise, especially in astern seas (23) when one tries to assess it in terms of a given speed of advance and natural response period. The relevant steps taken in this connection are discussed in Appendix A.

The second and minor phase of the comparison endeavor was solely concerned with the effect of longitudinal inertia (weight) distribution on ship motions. The experimental data required in this case were obtained from Figs. 16 and 17 of (14). In this latter work, Model C of Fig. 1 was ballasted in four additional ways so as to yield non-dimensional radii of gyration of \( k_\theta = 0.21, 0.225, 0.255 \) and 0.270. The previous discussion in the main body of this chapter with regard to presentation, etc., applies also in this phase of the investigation, with the following exceptions due to insufficient model data:

(a) Only pitch and heave amplitudes were compared,
(b) Attention was fixed to directly ahead seas only (\( \chi = 180^\circ \)),
and
(c) The results are given for three Froude numbers of 0.15, 0.20, 0.25.
Since the case of \( k_o = 0.24 \) has already been examined previously, Figs. 58-65 of this report deal with the remaining four only.

Text continues on page 84
### Keys to Figures 2-65

<table>
<thead>
<tr>
<th></th>
<th>Theoretical</th>
<th>Experimental</th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motion Non-dimensional Amplitude</td>
<td></td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Motion Phase Angle (Lag)</td>
<td>- - - - -</td>
<td></td>
<td>- - - - -</td>
<td>-</td>
</tr>
</tbody>
</table>
FIG. 2  MODEL A IN DIRECTLY AHEAD SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO WAVEHEIGHT RATIO
FIG. 3 MODEL A IN DIRECTLY AHEAD SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIPLENGTH RATIO
- 21 -
FIG. 5  MODEL A IN DIRECTLY AHEAD SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 6  MODEL A IN DIRECTLY ASTERN SEAS

HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIPLENGTH RATIO
FIG. 7  MODEL A IN DIRECTLY ASTERN SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIPLENGTH RATIO

- 25 -
FIG. 6  MODEL A IN DIRECTLY ASTERN SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIPLength RATIO

- 26 -
FIG. 9  MODEL A IN DIRECTLY ASTERN SEAS
FITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE AMPL.
VERSUS WAVELENGTH TO SHEPHERD HATIO

- 27 -
FIG. 10 MODEL B IN DIRECTLY AHEAD SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 11 MODEL B IN DIRECTLY AHEAD SEAR
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 12 MODEL B IN DIRECTLY AHEAD SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIPLENGTH RATIO

- 30 -
FIG. 13 MODEL B IN DIRECTLY AHEAD SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 15 MODEL R IN DIRECTLY ASTERN SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIPLENGTH RATIO
FIG. 16  MODEL D IN DIRECTLY ASTERN SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIPLENGTH RATIO
FIG. 18  MODEL C IN DIRECTLY AHEAD SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIPLENGTH RATIO
FIG. 20  MODEL C IN DIRECTLY AHEAD SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO

- 36 -
FIG. 21  MODEL C IN DIRECTLY AHEAD SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIPLENGTH RATIO

- 39 -
FIG. 22  MODEL C IN DIRECTLY ASTERN SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 23  MODEL C IN DIRECTLY ASTERN SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 24  MODEL C IN DIRECTLY ASTERN SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 25 MODEL C IN DIRECTLY EASTERN SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 26 MODEL D IN DIRECTLY AHEAD SEAS

HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 27  MODEL D IN DIRECTLY AHEAD SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO

- 45 -
Fig. 28 Model D in directly ahead seas
Pitching non-dimensional amplitude and phase angle
versus wavelength to ship length ratio

- 46 -
FIG. 29. MODEL B IN DIRECTLY AHEAD SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
Fig. 30 Model D in Directly Astern Seas

Harving Non-Dimensional Amplitude and Phase Angle
versus Waveheight to Shiplength Ratio

- 48 -
FIG. 31  MODEL D IN DIRECTLY ASTERN SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 32  MODEL D IN DIRECTLY ASTERN SEAS

PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 33 MODEL D IN DIRECTLY ASTERN SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO

- 51 -
FIG. 34  MODEL E IN DIRECTLY AHEAD SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 35 MODEL E IN DIRECTLY AHEAD SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIPLENGTH RATIO

- 53 -
FIG. 36 MODEL E IN DIRECTLY AHEAD SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 38 MODEL B IN DIRECTLY ASTERN SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 39 MODEL E IN DIRECTLY AFT SEAS
HEAVING, NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 40 MODEL E IN DIRECTLY ASTERN SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE VERSUS WAVELENGTH TO SHIPLENGTH RATIO
FIG. 41 MODEL E IN DIRECTLY ASTERN SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO

- 59 -
FIG. 42  MODEL F IN DIRECTLY AHEAD SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 43 Model F in Directly Ahead Seas
Heaving Non-Dimensional Amplitude and Phase Angle
Versus Wavelength to Ship Length Ratio
FIG. 44  MODEL F IN DIRECTLY AHEAD SEAS

PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 45. MODEL F IN DIRECTLY AHEAD SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 46 MODEL F IN DIRECTLY ASTERN SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIPLENGTH RATIO

- 64 -
FIG. 47  MODEL F IN DIRECTLY ASTERN SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 48 MODEL F IN DIRECTLY ASTERN SEAS

PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 49  MODEL F IN DIRECTLY ASTERN SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIPLength RATIO

- 67 -
FIG. 50. MODEL G IN DIRECTLY AHEAD SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 51 MODEL Q IN DIRECTLY AHEAD SEAS

HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIPLENGTH RATIO
FIG. 52  MODEL G IN DIRECTLY AHEAD SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIPLENGTH RATIO
FIG. 53. MODEL G IN DIRECTLY AHEAD SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO

- 71 -
**FIG. 54: MODEL G IN DIRECTLY ASTERN SEAS**

**HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLEVERSUS WAVELENGTH TO SHIPLENGTH RATIO**
FIG. 55  MODEL G IN DIRECTLY ASTERN SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIPLENGTH RATIO
Fig. 56  Model G in directly astern seas
PITCHING NON-DIMENSIONAL AMPLITUDE AND PHASE ANGLE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 57 MODEL G IN DIRECTLY ASTERN SEAS
Pitching non-dimensional amplitude and phase angle versus wavelength to shiplength ratio
FIG. 58 MODEL C (λ = 0.210) IN DIRECTLY AHEAD SEAS
HEAVING NON-DIMENSIONAL AMPLITUDE
VERSUS WAVELENGTH TO SHIPLENGTH RATIO

- 76 -
FIG. 59  MODEL C (k_b=0.225) IN DIRECTLY AHEAD SEAS

HEAVING NON-DIMENSIONAL AMPLITUDE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 60 MODEL C (kₐ=0.255) IN DIRECTLY AHEAD SEAS

HEAVING NON-DIMENSIONAL AMPLITUDE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO

- 78 -
FIG. 61   MODEL C ($k_g = 0.270$) IN DIRECTLY AHEAD SEAS

HEAVING NON-DIMENSIONAL AMPLITUDE
VERSUS WAVELENGTH TO SHIPLENGTH RATIO

- 79 -
FIG. 62. MODEL C (k_8=0.210) IN DIRECTLY AHEAD SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE
VERSUS WAVELENGTH TO SHIPLength RATIO
FIG. 63 MODEL C (k = 0.225) IN DIRECTLY AHEAD SEAS
PITCHING NON-DIMENSIONAL AMPLITUDE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 64  MODEL C \((k_g=0.255)\) IN DIRECTLY AHEAD SEAS

PITCHING NON-DIMENSIONAL AMPLITUDE
VERSUS WAVELENGTH TO SHIP LENGTH RATIO
FIG. 65  MODEL C (k_a=0.270) IN DIRECTLY AHEAD SEAS

PITCHING NON-DIMENSIONAL AMPLITUDE
VERSUS WAVELENGTH TO SHIPLength RATIO

- 83 -
III. CORRELATION BETWEEN THEORY AND EXPERIMENT
AND DISCUSSION OF RESULTS

1. Previous Comparisons

Before discussing in detail the results shown in Figs. 2-65, which include both directly ahead and astern seas, a brief review of earlier comparisons of similar nature, but made only for directly ahead seas, is advisable. Of particular importance will be the indication of whether the present computer calculations, utilizing Grim's latest results for added mass and damping, differ in any appreciable manner from earlier computations.

If we restrict ourselves to the latest formulation of the Korvin-Kroukovsky theoretical procedure, then (3) and (7) appear to be the only references in which pertinent data are presented. In particular the first, (3), illustrated the application of the analytical tool to eight widely different models and in a sense established the applicability of the method. Of the models used in (3), No. 1445 is sufficiently similar to Model C (Figs. 18-25) of the present investigation, although not exactly alike in length and proportions. Three years later, Jacobs, (7), compared the theoretical and experimental motions of a T-2 tanker model, whose lines may be considered comparable to those of Series 60 forms and which was also included in (3) as model No. 1444. Both (3) and (7) used Ursell's factor and Prohaska's curves in computing added mass and Grim's data for damping which at the time were less accurate. Also, (7) utilized slightly more precise expressions for the excitation terms than (3) did. In both references, the experimental results used in the comparison were those of 5 ft. models tested at Davidson Laboratory. Finally, Gerritsma's correlations reported in (24) and (25) dealt with Series 60
models which approximately correspond to Models A, B and C (Figs. 2-25) of the present report. For our purposes, however, the latter work is of less significance because all coefficients of the equations of motion were first determined experimentally and then substituted in the set (Bl) to compute motions and phase angles.

The agreement between theory and experiment achieved in (3) and (7) was described as very satisfactory. This conclusion was based on graphs whose ordinate scales were large and which did not easily indicate percentage departures of theory from experiment. This was particularly true in the case of phase angles. Theoretical pitching motion amplitudes were found to be more accurate than heaving amplitudes. In the latter case serious overestimations as compared to experimental data were found to occur in the vicinity of syncronism. Gerritsma subsequently indicated that this was due to underestimation of heave damping (24), (26). Analytically computed phase angles did not exhibit the same standard of correlation as motions did and were found to be very susceptible to computational errors.

2. Discussion of Results

The current comparison with NSMB models (13) shown in Figs. 2-57, indicates that, in directly ahead seas, reasonably good correlation is achieved for heave amplitude. An exception is found in the case of Model E (Figs. 34 and 35) where theory fails to reveal the correct trends and grossly exceeds measured values. This may be due to numerical errors which probably arise for low B/H ratios in the subroutine of the computer program which calculates damping and added mass according to Grim's theory. Apart from this discrepancy and for practically all wavelengths, maximum deviations between theoretical and experimental results for the remaining models are about 7% and hence agreement can be termed satisfactory.
However, for the wavelengths corresponding to resonance, theory overestimates experimental heaving amplitudes by about 15-20%, which is in accord with previous findings (3), (7), (25). As indicated earlier, this is due to the underestimation of heave damping as computed by the analytic approach.

Agreement in directly ahead seas is much better for pitch than for heave although the trends are not the same for different models. For models A, B, C and D (Figs. 2-33) at all speeds, theoretical results are below experimental data and the effect is more pronounced as the wavelength is increased. In the case of Model E (Figs. 36-37) large discrepancies are not observable as with heaving motions. In this instance, theory is about 10-15% higher than experiment especially at resonance. Excellent agreement is found in the case of model F (Figs. 44-45) where it may be observed that maximum deviations are less than 3%.

With respect to phase angles in directly ahead seas, it will be seen that the theoretical predictions are usually higher than experiment and this is true for both pitch and heave. Since phase angles are more susceptible to both computational and experimental errors, agreement should perhaps be interpreted as satisfactory whenever deviations are less than about 15-20%. Discrepancies usually occur at wavelengths equal to model length. Apparent disagreement is also observable at very short wavelengths, but this is mainly due to the manner in which experimental data have been plotted. Specifically, the fact that motion amplitudes are allowed to assume negative as well as positive values for presentation purposes, often causes the experimental phase angles to be reflected about the 0° axis, and hence to have the opposite sign from that analytically predicted. Pitching phase angles as computed by theory are much closer to the experimental data than heaving phase angles.
This is particularly obvious in the case of models F and G (Figs. 44-45 and 52-53). Theoretical phase angles for the heaving motion are about 25-30% higher than corresponding measured ones.

In the case of directly astern seas, heaving motion is underestimated by theory and the deviations increase with wavelength and are in some cases as high as 40%. For most models, agreement of pitching motion amplitudes in directly astern seas is excellent, although in the case of Models A and B (Figs. 8-9 and 16-17) theory is about 30% less than experiment. No comparisons have been made of phase angles in astern seas, but general indications are that theory reveals the expected trends.

The second part of the current correlation which is concerned with the effect of weight distribution is illustrated in Figs. 58-65. For small radii of gyration, theoretical heaving motion amplitudes are about 10% less than the experimental ones, but the situation is reversed and slightly worsened as \( k_\theta \) is increased. Agreement in pitching motion appears to be similar, and the tendency here is for the experimental data to be 15-20% higher, particularly at high wavelengths.

All figures for ahead seas indicate that for wavelengths less than about half the length of the model, both pitching and heaving motions are negligible whilst for wavelengths higher than about twice the model length, heaving becomes equal to the wave amplitude and pitching corresponds to the maximum wave slope. The range \( 0.9L < \lambda < 1.5L \) excites the models the most. On the contrary, astern seas do not seem to induce appreciably large responses in heave or pitch and amplitudes tend to increase in a linear fashion with wavelength. These findings are in accord with those of earlier investigations.

The arrows shown in Figs. 2-65 which correspond to the wavelengths for resonant conditions are usually situated near the points of
maximum response. In the case of pitch, the indications are much more accurate due to the fact that the actual pitching periods of the models have been used in the computation. Even though the heaving periods for the models have been estimated by an approximate formula, the dotted arrows tend to be located at the proper positions; in many cases, however, the maximum response occurs at much lower wavelengths.

Theory predicts trends with variations in main hull parameters and weight distribution which are similar to the ones noted by the authors of (13) and (14) on the basis of their experimental results. Since the effects of variations in the main hull parameters depicted in Fig. 1, appear to be negligible for both pitching and heaving motions in the case of directly astern seas, the discussion will be mainly concerned with ahead seas. Block coefficient appears to be somewhat important only. In particular, both motions tend to diminish slightly with increasing block coefficient, but at high speed resonance conditions, reductions of the order of 25% for both pitch and heave are achieved between $C_B = 0.60$ and $C_B = 0.80$. On the contrary, an examination of the results given for models G, B, F indicates that the length to breadth ratio has no effect whatsoever. The most important influence on both motions is offered by the length to draught ratio variations. For high wavelengths, distinct increases in heave are observed whenever the draught is increased, whereas the effect on pitch is slightly less pronounced. However, in the case of low wavelengths, both motion amplitudes decrease as the draught is increased. Finally, the importance of adopting a small radius of gyration is demonstrated in Figs. 58-65 where it may be seen that both heave and pitch motions increase considerably as $k_g$ is increased, particularly in the vicinity of synchronism.
3. Reasons for Discrepancies

In the introduction of Chapter II it was noted that both the current methods of experimental measurement and the current status of theoretical knowledge are replete with sources of errors. The following discussion gives some indication of the inadequacies of both approaches, particularly as they are related to each other, and hence provides partial explanation for those differences between theory and experiment which did arise in the present investigation.

On the experimental side, the following sources of discrepancies can be identified:

a) The models used by Wossers et al. (13) were free to move in all six degrees of freedom. Even though their effects are generally considered to be negligible in directly ahead or astern seas, rolling, surging, yawing and swaying are expected to influence to some degree the finally recorded amplitudes of pitch and heave. The theory used in this investigation does not take into account any of these cross-coupling effects; it treats motion in the longitudinal plane of symmetry only and furthermore it presumes the absence of surge.

b) These comments bear directly on our approach of interpreting the $\chi = 170^\circ$ and $\chi = 10^\circ$ of (13) as pertaining, after a slight modification, to directly ahead and astern seas respectively. However, as discussed in Appendix A, there is evidence to suggest that this reason may not be decisive and hence deviations arising from this approach may be within the experimental accuracy of the reported data.
c) All models tested at N.S.M.B. were equipped with bilge keels and were furthermore self-propelled. Bilge keels provide some heave damping which would tend to decrease measured motion amplitudes. Thrust fluctuations on the other hand may have some effect on the measurement of experimental phase angles.

d) Wave height measurement appears to be one of the most important sources of error in model basin work. This item is currently receiving serious attention as there is a growing evidence to suggest that recorded wave amplitudes may be considerably different from what they were intended to be. The presence of the model obviously modifies the wave pattern and this effect is also not allowed for in the theoretical approach.

e) Wall effects, especially in the case of low speeds and small wavelengths, may alter the picture materially and yield erroneous answers.

f) Owing to the surging motion of the models, determination of the exact phase angle is a matter of considerable difficulty. This has been noted in (13).

g) The manner in which data are finally presented may be quite important in revealing or obscuring the actual experimental conditions. One such example has been mentioned earlier with regard to phase angle presentation. For motions in short wavelengths the convention of allowing the amplitude to take negative values leads to unreasonable discrepancies.
The voluminous data presented by Vossers et al have been shown in (13) to be approximately similar to previously reported data, when due allowance is first made for the differences in model size and proportions, measuring techniques and environmental conditions. This seems to justify the previous findings of Gerritsma (27) and Abkowitz (28).

However, one tends to accept with some reluctance the trends of various curves especially when the degree of scatter is not at all demonstrated. Vossers et al assert that their experimental accuracy is within 5%. Comparisons of data made at random between their own publications indicated that, at some instances, deviations of up to 7-10%, especially for phase angles, can occur so that one should interpret their results with due caution. After all, to quote Vasta (73): "No-one believes experimental data except the man who takes it".

On the theoretical side, the following items affect the applicability of the analytical approach:

a) Linearized theory is applicable to a ship or model whose section lines are vertical at the waterline. As may be seen in (3), the excellent agreement in the case of the destroyer model and the complete failure of the theoretical approach in the case of the yacht are ample proof of the importance of "wall-sidedness" at the waterline. Series 60 aft section lines do not comply with the above prerequisite and hence this may be one of the reasons why the current comparison does not achieve the standard of the destroyer model correlation (3).

b) The large inclinations of the section lines mentioned above largely affect the computation of added mass and damping,
since both are derived for Lewis section forms. For small motions these effects will, of course, be negligible but the same is not true at resonance, as has been demonstrated herein in the case of heaving motion.

c) Throughout a whole cycle of motion the Korvin-Kroukovsky procedure assumes that the coefficients of the equations of motion are constant. For small excitation amplitudes, this may be sufficiently correct, but large discrepancies can be attributed to this very reason in the case of particularly high responses at resonance. In this instance, the assumptions of constant hull submergence, constant waterline width and invariability of the mean section draught are certainly not valid.

d) Motion damping is assumed to be independent of forward speed, proportional to the first power of the vertical velocity only and any possible effects of viscosity are neglected. No account is taken of the actual distribution of damping or its dependence on speed and frequency. The recent results of (29) have shown that heaving damping coefficients of the forward sections of a Series 60 model increase with speed whilst the corresponding coefficients aft decrease and can even attain negative values. It is obvious that such an effect will influence considerably the actual values of the coefficients of the equations of motion which are dependent on the distribution of damping along the model length. Phase angle determination which has been shown to be predominantly dependent on these coefficients will be expected to be in error also.
Fortunately, total damping is not radically affected by speed or frequency and it has been shown (29) that Grim's work compares favourably in this respect with experimentally measured values. Thus, the coefficients which include the integrated values of damping should be reasonably accurate.

e) The work of Gerritsma and Beukelman, (29), also dealt with the effects of speed and frequency on the distribution of added mass. It is there shown that a strong shift forward of the distribution of added mass can occur with increasing frequency while at low frequencies several sections show negative values of added mass. Speed effects in this case appear to be less important. Since a large number of coefficients of the equation of motion depend on the distribution of the added mass, their values may be in error whenever the frequency effects are significant. Coefficient a which is dependent on the total mass will not be materially affected, however, since Grimm's total added mass again appears to agree with the integrated experimental value of (29).

f) The restoring coefficients will be correct only as long as the motions are small and the section lines fulfill the requirement of "wall-sidedness" at the waterline.

g) Although Havelock, Korvin-Kroukovsky, and Vossers have shown (1), that three-dimensional effects are not very important for normal ship forms, serious discrepancies may be due to "end" effects when large section areas are present at the aft end of the ship or model.
4. Conclusions

In spite of the aforementioned errors in both the experimental and theoretical prediction of ship motions, Figs. 2-65 show that the pitching and heaving amplitudes as well as phase angles for normal ship forms are reasonably well predicted by the Korvin-Kroukovsky theoretical procedure. Although agreement is slightly worse than what earlier references have shown and although it is still far from ideal, an attempt towards assessing kinematical seaworthiness in ahead irregular seas on the basis of linear theory was still judged to be worthwhile. Such a task was therefore undertaken in the second part of the present work.

5. Recommendations

The results of the first part of the current investigation suggest several important extensions and future research projects:

a) Since no correlation of theoretical and experimental motion phase angles for the case of directly astern seas is shown in this work, it is suggested that an attempt be made to this end. If the model data of (13) are used for this correlation, the phase angles will have to be obtained by extrapolation from the limited graphs appearing there and hence the findings will have to be interpreted with caution.

b) References (13) and (14) provide experimental amplitudes of relative bow motion whilst (14) also presents similar data for bow and stern accelerations. The expressions given in Appendix D permit an instructive comparison of the above results with similar ones computed analytically. It is suggested that the theoretically derived amplitudes be compared to:
(1) the experimental ones presented in (13) and (14) and, to

(2) the amplitudes calculated in a similar way as in Appendix D, but using the experimental results for pitch and heave motions and phase angles.

Such an investigation will be of particular importance in establishing the validity of the assumptions made in computing motions, velocities and accelerations of various points along the ship hull and will also show whether reasonable estimates can be made of these important responses for seaworthiness considerations.

c) The manner in which the results of the present correlation have been reported reveal the effects of only a very limited number of seakeeping variables. A useful extension would be their interpretation on a different coordinate system, e.g., pitching motion versus volumetric ratio. It is expected that different forms of presentation will depict the importance of other, perhaps hitherto hidden variables.

d) The potential of the available computer program should be utilized in assessing the effects on motions of other hull parameters such as the waterplane area coefficient and longitudinal position of the center of buoyancy.

e) A most instructive comparison for the case of zero model speed, appears to be one between the Korvin-Kroukovsky strip theory and the more complete three-dimensional approach proposed by Grim in (11) and (30). Since (30) has already
dealt with exactly similar models as the ones used in this work, such an investigation is urgently recommended in order to reveal the importance of three-dimensional effects.

f) A systematic comparison of theoretically computed coefficients of the equations of motion with similar ones derived by experiment should be commenced; in this context Gerritsma's work, (24) and (25), will be extremely valuable.

g) Similarly, a systematic comparison should be made of theoretical and experimental heaving forces and pitching moments. The work of Golovato (31), Watson (32) and Gersten (33) will be of particular assistance in this respect.

The above suggestions can readily be accomplished on the basis of present knowledge. Further analysis is needed, however, for the main objective of improving and amplifying the existing approach. In this respect the following items are important:

a) On the basis of the work done under f) and g) cited above, more precise definitions of the equations of motion and of the excitation terms should be sought.

b) Since it is known that the most important errors are introduced by the inability to accurately predict damping, especially at resonance conditions, the extent to which Grim's procedure for computing damping and added mass is valid should be re-examined. Finally, Porter's valuable work (34) should also be incorporated in the computer program and individually assessed and the same should be done, if possible, with Tasaí's results (35). Also for wide
V-section forms the work of Kaplan and Jacobs (36) becomes of importance, while the whole research process under this item should pay due regard to the findings and recommendations of (37).
PART II

IV. ON SEAWORTHINESS CONSIDERATIONS

The powerful concepts of the theory of random processes, as recognized by St. Denis and Pierson in their pioneering paper (38), have alleviated the perplexities of the behaviour of sea-borne vehicles under realistic sea conditions and have furthermore provided a sound basis for analysis purposes. A next logical step consists of incorporating the products of the statistical operation in preliminary design procedures.

In this connection, however, the need still exists for further clarification of the important factors, before the naval architect can visualize a suitably defined goal and proceed to systematize his design approach. One obviously desirable goal, that of higher sustained ship speed, brought about the realization that a rational form of design procedure should include seaworthiness considerations. Lewis' work (39), (40), (41), has shown that it is by no means limitations in power that cause speed loss and undesirable delays, but inability to cope effectively with nature's elements. Rough water effects such as structural damage, shipping of green seas, severe vibrations, loss of controllability and maneuverability and sustained human discomfort are not only objectionable per se, but also force upon the ship operator reductions in power below the capacity of his power plant.

Before 1953, seakeeping research was identified by painstaking attempts on the part of few investigators who, although handicapped by the absence of the statistical device, utilized their insight and
experience from multiple voyages in delineating and classifying aspects of ship behaviour in confused seaways.

In particular, Kent played a vital role in this connection and his numerous papers* (42), (43), (44) discuss most of the pertinent seakeeping criteria, some of which can now be handled in mathematical form.

The effect of seaworthiness on preliminary design considerations is different for the two broad classes of vessels which traverse the oceans, namely, military and merchant ships. For the military ships, shipping of water which endangers personnel activities and excessive accelerations or large stern motions which may forbid aircraft recovery are obviously important factors. Once it can be shown that a particular scientific suggestion could even moderately improve the foregoing parameters and thus augment the military effectiveness of naval ships, the adoption of the suggestion would be a matter of course.

With the private shipowner, however, there is a well-known conservatism in accepting novel ideas. One is sure to expect that unless it has been proven beyond a shadow of a doubt that, say, a reduced block coefficient will, in the long run, prove economically beneficial despite the reduced capacity, such a measure will never be adopted. Stated otherwise, the analyst's task, in this case, consists of conclusively demonstrating the economic attractiveness of his proposition.

The plethora of seaworthiness parameters which can be put forward for both military and commercial ships indicate one very important

*Ref. (1) lists 21 papers under Kent's name.
point. Seaworthiness cannot be assessed or defined in terms of a single number, property or event. By necessity, one must employ many such numbers since, as will be seen later, each may be of importance in a particular case. Thus, in order not to forfeit generality, we shall view seaworthiness as a property of ships which is described by a group of important responses and phenomena. These can be either ignored or incorporated when dealing with a particular type of vessel, however, the designer should have available a complete list of them from which he can select those important to his problem.

The definition of seaworthiness which we shall contemplate will pay sole emphasis to the preliminary design of displacement type vessels. In this way we shall hope that the ship master will be forced to reduce speed or change heading at much higher sea states and that the destructive effects of the sea fury will only commence in exceptionally severe weather. Finally, to further crystallize our main aim, we shall not be concerned with mechanical devices, such as antipitching fins, which could meliorate the ship behaviour problem.

Since we shall proceed in this work along analytical lines, which are at present quite restrictive, our scope will also be restrictive. Thus, we shall treat irregular seas which come directly ahead from the bow only. Such a restriction fortunately excludes few generalities since, broadly speaking, head seas represent the worst possible conditions for displacement vessels. It must, however, be pointed that, the effects of beam seas are not negligible and should be included, if possible, in future work.
Safety and kinematic performance, the latter to be shortly discussed, will not be decoupled, but will on the contrary be viewed as being strongly interrelated. Safety will be assured if optimum performance is achieved, although the converse is not true. This strong interrelationship between safety and performance is not in accord with earlier views on the problem (45); in fact, seaworthiness and seakindliness were two distinctive terms used to separately describe these two aspects of the problem.

We will assume a linear system, an assumption which has been repeatedly shown to hold good for destroyer forms despite the rather obvious non-linearities which physically characterize it. The sea wave elevation possesses a stochastic time history and hence any generalized response of the ship must also be of the same nature. Furthermore, in order to obtain meaningful answers we make use of the concepts of ergodicity and simultaneously assume that the probability density functions of all resulting random variables are of the Gaussian type. The next chapter provides an amplification of the previous statements while reviewing the potential application of the statistical theory. In the past, several attempts have been made to understand the ship motion problem on the basis of average conditions. Such an approach, essentially based on the sinusoidal seaway concept, is inadequate and yields erroneous results.

We next borrow the most useful tool from the theory of random processes. This is the complex frequency response which provides us with a unique description of the random response in the frequency domain. The square of the absolute value of this quantity has been referred to as the "response amplitude operator" (48) and implies a linear transformation
whereby the excitation's squared amplitude density spectrum yields the corresponding description of any response.

The parameters which we have decided to employ are simple statistical descriptions of the kinematic responses of the ship or combinations of these responses. The statistical descriptions need not necessarily be the same for every response and therefore the most meaningful definitions have been chosen in a particular case. Taken all together, however, these parameters define the kinematic seaworthiness properties of a ship. Independent considerations, stemming from experience, analysis or experiment may then impose acceptable magnitudes to these statistical parameters.

On the basis of past knowledge and needs we have examined several parameters, as shown in Chapter VI, which describe:

1) Pitching Motion Amplitude
2) Absolute Acceleration Amplitude at any Station of the Vessel
3) Slamming Occurrence
4) Wetness Occurrence
5) Propeller Immersion

The adopted parameters are restrictive in many senses and deal with but few of the many aspects of a vessel's behaviour in rough seas. This is the penalty for the formidable complexity which faces the analyst and narrows his knowledge. Increased resistance and power, speed loss, vibration and strength considerations have been regarded as lying outside the scope of the present work.

By making use of the analytical tool, the preceding seaworthiness parameters for a particular type of vessel may be related in the design stage to the following ship characteristics:
(a) Hull shape and dimensions, below as well as above the load waterline with particular emphasis on the forebody section.

(b) Internal weight distribution.

In order to assess their influence, it seems advisable to decouple these two characteristics and start by examining the first while keeping the second constant. For its unique specification, the geometry of the hull requires a complete lines plan. However, such information is not available in the preliminary stages of the design and hence we must resort to hull form parameters such as ratios of dimensions, block and prismatic coefficients.

The most suitable weight distribution can next be evaluated by systematically varying the longitudinal radius of gyration, once the geometrical variations have been tentatively secured to small "bands". In the latter case, however, the freedom for drastic departures is very limited and the nature of cargo together with strength considerations confine the designer in his evaluation.
V. THE STATISTICAL APPROACH

1. The Excitation: Seaway

The ever-changing, chaotic and unpredictable pattern of seaway elevations defies any deterministic description. A typical wave record is a continuous random time series with the basic property that analytical specification over all time is impossible. As a result, the oceanographer and naval architect resort to the powerful techniques of Generalized Harmonic Analysis and Probability Theory. Any wave record can then be considered as a typical realization of a large family or ensemble of such records.

The process of defining this ensemble in a meaningful manner requires the assumptions of (a) stationarity and (b) equivalence of temporal and ensemble averages. Both of these assumptions are embraced by the concept of ergodicity, which is frequently employed in many other engineering fields, but must be retested for our case. This has been done on numerous occasions and it has been shown that the seaway generating mechanism can be regarded as invariant once the wind conditions remain sensibly constant, and also that the normal density law reasonably characterizes the distribution of random elevations over the whole time axis.

Wiener's fundamental work (46) has rigorously shown that any random process, such as a wave time history, can be defined by either its,

(a) autocorrelation function in the time domain,

or

(b) squared amplitude density spectrum in the frequency domain.
In our analysis, we follow the approach originally propounded in (38) and choose the second which is more general and flexible and which, by the Wiener Theorem (46), (47) is the Fourier Transform of the first.

The relationship between wave elevation and total energy for a unit-width gravity wave is, from Lamb (48),

\[ E = \rho g \int_{0}^{\lambda} h^2(t) \, dx \quad (2) \]

Since the squared amplitude density spectrum of a particular sea condition involves the square of the wave elevation, it represents, with the exception of a constant factor, the average energy distribution in a given frequency (or wave length) band. This band centered about a discrete frequency, when multiplied by the average spectrum height at the same frequency, yields the energy contribution of the particular seaway component present in the spectrum configuration and specified by that frequency (wavelength).

In essence, therefore, we imagine the sea to be made up of an infinite number of sinusoidal gravity waves, whose phases are randomly distributed at a particular time instant; the phases can assume with equal probability any value between 0°-360°. The mathematical equivalent of the above assumption, is in the form of a Lebesgue integral given by:

\[ h(t) = \int_{0}^{\infty} \cos \left[ wt + \epsilon(w) \right] \sqrt{S^2(w)} \, dw \quad (3) \]

The efforts of the oceanographers over the past decade have provided both:

(a) analytic spectrum definitions as functions of wind speed, which represent idealized sea conditions,
and actually recorded and numerically defined spectra for a particular wind speed at a particular location in the ocean.

The first spectra are obviously more flexible for analytical investigations but yield qualitative information only, whilst the second lose their quantitative significance by being rare in existence and restrictive in the general case.

For his purposes, the ship designer requires knowledge of the seaway time and space average conditions along the route of the proposed ship. Since this has not as yet been accomplished, we have chosen the widely used Neumann formulation (1), (49), but have paid full attention to the implications of this choice. This uni-directional sea energy density spectrum forms the starting point of our probabilistic methodology and is given by, (1),

$$g(w) = \frac{2c}{w_0^2} \frac{\pi^3}{6} \exp \left[ - \frac{2s^2}{U^2 w_0^2} \right]$$  \hspace{1cm} (4)

where, \( c = 8.27 \times 10^{-4} \text{ ft}^2 \text{sec}^{-5} \)

In the above form and for a limited wind speed range, the spectrum defines the distribution of energy in an idealized, fully developed, long-crested sea as a function of the wave component frequencies. It is recognized, of course, that there is nothing sacred about this particular expression and that any other analytical formulation could equally well serve our purposes.

It must be borne in mind now, that the excitation definition given by eq. (4) is related to an absolute or fixed coordinate system with its origin at any point in the ocean. In the most general case, however, the ship system is in motion and may thus be described best
by a non-stationary coordinate system with its origin at the centre of
gravity of the ship. For this reason, the forcing function must be
transformed and referred to the same coordinate axes. Stated otherwise,
we are interested in a description of the seaway as it applies to the
ship moving at a given speed and heading angle.

The associated problem of frequency transformation which
emerges in this connection has been analyzed in detail in (36);
further discussion, with particular emphasis on directly ahead and
astern seas, appears in Appendix C of this work. It is shown that the
relationship between the actual "input" energy spectrum and the absolute
one defined by eq. (4) is:

$$\xi_{ii}(\omega_{e}) = \xi(\omega) J(\omega_{e})$$  \hspace{1cm} (5)

It may be seen from above that the important variable is
now the frequency of wave encounter which in turn is a function of
ship speed and heading angle. It is shown in Appendix C that for
directly ahead seas, with which we shall be solely concerned, the
frequency of encounter and Jacobian of the frequency transformation are
related to the absolute wave frequency by:

$$\omega_{e} = \omega (1 + \frac{\omega V}{g})$$  \hspace{1cm} (6)

and

$$J(\omega_{e}) = \left[ 1 + \frac{4 \omega_{e} V}{g} \right]^{-1/2}$$  \hspace{1cm} (7)

and that, therefore, the final form of the sea spectrum generated for
arbitrary wind and ship speeds by the computer program developed in this
work, becomes:

$$\xi_{ii}(\omega_{e}) = \frac{2 c g^{2} 2^{3/2} \omega_{e}^{3/2}}{6 \lambda (1 + \frac{\omega_{e} V}{g})} \exp\left[ - \frac{2 g^{2}}{U^{2} \omega_{e}^{2}} \right]$$  \hspace{1cm} (3)

where the absolute wave frequency is obtained from the inverse transformation:
\[ \omega = - \frac{1 - \sqrt{1 + \frac{4 \omega_c V}{2g}}}{\frac{2V}{g}} \] (9)

2. **Linear System: Ship**

So far in our analysis, consideration was given to the definition of the random process which excites the ship system. We now endeavor to examine the dynamic properties of the vessel as related to a definite deterministic excitation. This is in accord with our basic assumptions of linearity and time invariance which allow us to define the system uniquely by its generalized response to either a:

(a) unit impulse excitation in the time domain,

or (b) unit amplitude sinusoidal excitation in the frequency domain.

In accord with the remarks made in the first section of this chapter we choose to proceed in the latter domain. This is dictated by simplicity and clearer visualization of the ensuing phenomena and affords special ease in both the analytic and experimental attacks to the problem.

Applying next the principle of linear superposition which states that the response of a system to an arbitrary excitation is the sum of the responses of the system to each of the excitation's components, we arrive at the fundamental relationship of the statistical approach, which is treated in the next section of this chapter. Insofar as motions of normal ship forms in head seas is concerned, the work of Dalzell (50), (51) and Lewis and Numata (21), has established the validity of the basic assumption with which seakeeping research was launched ten years ago. It is now generally
accepted that, for engineering purposes, the ship essentially behaves in a linear fashion for a surprisingly wide range of ship speeds, heading angles and wave severity.

The frequency response function of a linear system is in general complex and, as such, provides information with regard to both amplitude and phase. The fundamental relationship of the statistical approach, however, makes use of the square of the absolute magnitude of this function only. Hence, in the process of transforming linearly the input density spectrum to yield the output density spectrum, it is obvious that any phase information is necessarily lost. In our literature, the term "response amplitude operator" has been frequently employed to denote this transforming function, although sometimes certain authors deviate and use this term to indicate slightly different forms of this operator.

For this investigation, we define and examine the following kinematic responses:

(a) pitching motion of the ship,

(b) absolute acceleration of any point along the length of the hull,

(c) relative motion and velocity of any point along the length of the hull with respect to the adjacent travelling wave.

Later in this work, the latter two responses are referred to specific fore and aft locations along the ship's hull in order to assess the kinematical conditions which pertain to slamming and wetness occurrence, acceleration-induced human discomfort and satisfactory propeller performance.
The various responses examined in this work are discussed in Chapter VI and analyzed in Appendix D. Their corresponding "response amplitude operators" are digitally computed by forcing the ship system with a unit amplitude wave excitation of progressively increasing frequency of encounter and each time squaring the resulting real amplitudes.

3. The Responses - Pitch, Acceleration, Relative Motion and Velocity

If a linear system is forced by a random input function then, the response function will necessarily be random also. It follows that the description of the response in the frequency domain may again be in the form of a spectrum. For linear, time invariant systems the relationship between excitation and response spectra is given by, (47), (52):

$$\hat{\eta}_{\infty}(\omega_e) = |\mathbb{H}(\omega_e)|^2 \hat{s}_{\infty}(\omega_e)$$

(10)

The simplicity of the above equation is remarkable as compared with the equivalent convolution integral in the time domain since it only involves a simple multiplication. The response spectrum's distinct importance, however, lies in the fact that by employing the hypothesis of ergodicity the total area under the configuration is found to be equal to the mean square value of the random amplitude of the output function. This follows easily from the Wiener Theorem (46), (47):

$$\Phi(\tau) = \int^{\infty}_{-\infty} \hat{s}_{\infty}(\omega_e) \cos \omega_e \tau \, d\omega_e$$

(11)

Since sea density spectra are represented over the semi-infinite interval \((0, \infty)\) the lower limit of the above integral can, without loss of generality, be substituted by zero.

For zero time shift, the equation for the autocorrelation function results in:
\( \varphi_{\infty}(0) = \int_{0}^{\infty} \delta_{\infty}(w_e) \, dw_e = A^* \)  \( (12) \)

The equality of time and ensemble averages finally permits us to establish that:

\[ \varphi_{\infty}(0) = A^* = \int_{-\infty}^{\infty} x^2 \, p_e(x) \, dx = \sigma^2 \]

\( (13) \)

From an analysis of many seakeeping records with relatively "narrow" density spectra it has been confirmed that, amplitudes chosen at random from a particular time history tend to conform to a Gaussian (normal) distribution law. Rice's fundamental work (53) anticipated the above findings, for he indicated that the peak to peak excursions of a similar random process should obey the Rayleigh distribution. This basically equivalent statement has been shown to hold good in the ship case by Jasper's analysis of full-scale data (54).

Whichever way one prefers to interpret his statistical results the essential point which now emerges is that by using the response density spectrum and the additional assumption of zero mean value, the probability density function of the random response amplitude can be uniquely established.

Thus, the Gaussian and Rayleigh probability functions are given, in terms of the area under the response spectrum, as:

\[ p_e(x) = \frac{1}{\sqrt{2\pi A^*}} \exp\left[-\frac{x^2}{2 A^*}\right] \quad \text{for } -\infty < x < \infty \]

\( (14) \)

and

\[ p_{2e}(x) = \frac{2x}{A^*} \exp\left[-\frac{x^2}{A^*}\right] \quad \text{for } x \geq 0 \]

\( (15) \)

Since this investigation deals mainly with the amplitudes of the various random variables our statistical inferences will be made through equation (14). The probabilities of occurrence of important "events" are then obtained from the following fundamental relationships:
\[ p \left[ a < x < b \right] = \int_{a}^{b} p_{x}(x) \, dx \]
\[ p \left[ x < a \right] = \int_{-\infty}^{a} p_{x}(x) \, dx \quad (16) \]
\[ p \left[ x > a \right] = \int_{a}^{\infty} p_{x}(x) \, dx \]

Summarizing now the available statistical parameters which may be employed to describe the random amplitude seaworthiness responses, we have from (1), (72):

(a) **Standard Deviation of Amplitude**,  
\[ \sigma_{x} = (A^*)^{1/2} \quad (17) \]

(b) **Average (Expected) Amplitude**,  
\[ \bar{x} = 0.866 \ (A^*)^{1/2} \quad (18) \]

(c) **Average of One-third Highest Amplitudes**,  
(Also termed Significant or Characteristic Amplitude),  
\[ \bar{x}_{1/3} = 1.415 \ (A^*)^{1/2} \quad (19) \]

(d) **Average of One-tenth Highest Amplitude**,  
\[ \bar{x}_{1/10} = 1.800 \ (A^*)^{1/2} \quad (20) \]

(e) **"Most Probable" Amplitude in One Oscillation**,  
\[ \bar{x}_{p} = 0.707 \ (A^*)^{1/2} \quad (21) \]

(f) **Highest Expected Amplitude Occurring in N Oscillations**  
\[ \bar{x}_{m} = (\log N \ A^*)^{1/2} \quad (22) \]

The above expressions are strictly applicable for narrow band processes only. In later work, Cartwright and Longuet-Higgins ( ) have shown that if the response spectrum extends over an appreciably large range of encounter frequencies, the parameters cited above should be multiplied by a "broadness factor" defined as:

- 112 -
\[
e = \left[ \frac{m_0 m_4 - m_2^2}{m_0 m_4} \right]^{1/2} \tag{23}
\]

where,

\[
m_k = \int_0^\infty s_{oo}(\omega_e) \omega_e^k d\omega_e \quad (k = 0, 1, 2, \ldots) \tag{24}
\]

As may be seen, \( e \) depends on the area and the first two even moments of the spectrum and in a sense measures the relative "width" of the spectrum.

The moments of the response spectrum can provide us with additional statistical information, such as:

\( g \) **Expected Number of Zero Crossings in Unit Time**

\[
\hat{t}_0 = \frac{1}{\pi} \left( \frac{m_2}{m_0} \right)^{1/2} \tag{25}
\]

\( h \) **Expected Total Number of Maximum Amplitudes in Unit Time**

\[
\hat{t}_m = \frac{1}{\pi} \left( \frac{m_4}{m_2} \right)^{1/2} \tag{26}
\]

\( i \) **Expected(Average)"Period" of Random Fluctuation**

\[
\bar{T} = 2\pi \left( \frac{m_0}{m_2} \right)^{1/2} \tag{27}
\]

or alternatively,

\( j \) **Expected(Average)"Frequency" of Random Fluctuation**

\[
\bar{f} = \frac{1}{2\pi} \left( \frac{m_2}{m_0} \right)^{1/2} \tag{28}
\]

An important extension of Rice's analysis for the case of a narrow band random process and its time derivative has been made by Tick (56). Specifically, he derived the expected number of occurrences in unit time of an "event" described by a combination of three Gaussian random processes. For our description of slamming, we shall neglect one of the random variables, in which case, his expression simplifies to:

- 113 -
$$\tilde{F}^* = \frac{1}{2\pi} \left[ \frac{1}{\tilde{x}^2} \right]^{1/2} \exp \left[ -\frac{1}{2} \left( \frac{\tilde{a}^2}{\tilde{x}^2} + \frac{\tilde{b}^2}{\tilde{x}^2} \right) \right]$$ \hspace{1cm} (29)

Finally, if we assume that $\xi$ and $\tilde{\xi}$ are uncorrelated, or otherwise independent, we can also establish the joint probability density function which characterizes these two random variables in the form:

$$P_{\xi \tilde{\xi}} (x, \tilde{\xi}) = \frac{1}{2\pi} \left( \xi^2 \tilde{\xi}^2 \right)^{-1/2} \exp \left[ -\frac{1}{2} \left( \frac{\xi^2}{\tilde{\xi}^2} + \frac{\xi^2}{\tilde{\xi}^2} \right) \right]$$ \hspace{1cm} (30)

In his treatise, Korvin-Kroukovsky (p. 214 of (1)) pointed out the need for a statistical measure which would estimate "severity" quantitatively and which would at the same time be physically meaningful and amenable to mathematical manipulation. Such a concept is offered by the average time reliability with which we can expect that a certain response (or responses) will not exceed a predetermined limiting value. The average time reliability is defined as:

$$R (\alpha) = 1 - p (\xi > \alpha)$$ \hspace{1cm} (31)

where $p(\xi > \alpha)$ is the probability that the random response $\xi$ will exceed an undesirable magnitude $\alpha$. The significance of such a measure in the case of slamming, wetness and acceleration is apparent. It can also be used for other statistically described ship responses such as bending moments and stresses, in which case, one may even be able to dispense with safety factors.

Using such a criterion, the ship designer is provided with a concrete "number", which he can optimize according to his specifications. Also, such a measure indicates implicitly the proportion of time that one must be prepared to allow for the occurrence of a deliriuous sea effect. Finally, the most significant feature of the proposed statistical inference is that it is not limited to narrow band responses, since it depends only on the assumption of a zero mean Gaussian process.
VI. SHIP RANDOM RESPONSES AND SEAWORTHINESS PHENOMENA

Having briefly outlined the fundamental concepts of the statistical approach, we now examine the manner in which the various kinematic responses of the ship combine to form important seaworthiness phenomena. Statistical measures for quantitatively determining the severity of the responses (and in turn the phenomena) will then be presented. Finally, in some cases, criteria or standards for acceptable seaworthiness performance, based in part on full-scale experience and log-data analysis, will be indicated for particular classes of ships.

1. Pitching Amplitude

Broadly speaking, motions per se are not vitally important for most commercial vessels; an exception may be found in the case of fishing vessels (57) and ferryboats. Due to their relatively small length, these types of ships tend to suffer excessive pitching motions, with the result that, in the first case, the personnel activities are hindered especially at the stern, while in the second, severe discomfort may be felt by the passengers who are not permanently accommodated over the usually short period of voyage.

For combatant vessels, and in particular, aircraft carriers (58), the problem may be different. In this instance, the requirements for launching and recovering aircraft on the flight deck suggest the adoption of an upper limit on the bow or stern motions. For this reason, the design of aircraft carriers should incorporate considerations of pitching motion.

Furthermore, pitching does contribute drastically to the total vertical motion of points remote from the vessel's centre of gravity and,
consequently, attenuates the occurrence of various phenomena such as slamming, shipping of green seas and emergence of propeller. It also causes distinct increases in resistance and hence speed loss, and is directly responsible for large bow and/or stern accelerations (39), (42), (59).

In the present investigation, the response operator for the pitching motion is obtained directly from the solution, equation (B 12), of the set of differential equations, equations (B1), by simply squaring the resulting amplitude, \( \Theta^2 \). It is herein denoted by:

\[
|H_{\Theta}(\omega_e)|^2 = (\Theta^2)
\]

(32)

where \( \omega_e \) is the frequency of encounter with a unit-amplitude sinusoidal wave. The corresponding response density spectrum is designated as \( \Phi_{\Theta}(\omega_e) \).

The "significant" amplitude and the average of 1/10 highest amplitudes of pitch, indicated respectively by:

\[
\bar{\Theta}_{1/3} = 1.415 (A_\Theta)^{1/2}
\]

(33)

and

\[
\bar{\Theta}_{1/10} = 1.800 (A_\Theta)^{1/2}
\]

(34)

appear to be the best statistical measures in this instance. It is not possible to set forth criteria for acceptable pitching amplitudes at this time.

2. Vertical Acceleration

In severe weather, the combined pitching and heaving motions of a ship create objectionable amplitudes of vertical acceleration especially at the ends. These result in human discomfort or even injury and also tend to shift cargo in remote holds, dislocate objects and affect the proper functioning of delicate equipment (39), (42). Preliminary calculations
(22), (60), have indicated that motion-induced acceleration when compared to gravitational acceleration can attain undesirable magnitudes.

The annoying effects of acceleration appear to assume the same degree of importance for both commercial and naval ships. Although at present, modern cargo liner design ensures concentration of accommodation amidships and hence alleviates the situation with regard to crew comfort, proper thought must be given with passenger ships to acceleration limits throughout the whole length of the vessel in the early stages of the design. Finally, with naval vessels we are not so much interested in comfort, but rather with the mental fatigue and human intolerance as well as hampering of military activities, such as aircraft recovery, to particularly high levels of acceleration.

It is not the purpose of this investigation to attempt an analysis of the basic causes of human discomfort and motion sickness or ascertain their limits. There exists, however, an urgent need for a methodical survey and correlation of the results of a large number of scattered reports, e.g. (61)-(67), so as to extract important information for design purposes.

The frequency of a given sinusoidal oscillation appears to be the governing factor in establishing thresholds of perception, unpleasantness, discomfort and intolerance (64), (65). For a random amplitude ship response, however, the concept of frequency is meaningless and thus one may have to consider the frequency of the maximum spectrum ordinate or a certain bandwidth about it as the dominating frequency (or band). For the ship motion frequency range, there is evidence to suggest that the rate of acceleration, usually referred to as jerk or jolt, is the stimulus of human reaction (64), while the actual amplitude of acceleration becomes
meaningful for slightly higher frequencies. If this is so, then clearly the rate of change of acceleration should also enter the list of important seakeeping responses. Such a criterion will even more be substantiated by the fact that the occurrence of slamming is associated with a very rapid change of acceleration (1).

In this work, for reasons of simplicity, we have examined the random amplitude of the vertical acceleration experienced at Station No. 1 (0.1 L aft of F.P.). This station has no special significance as far as this particular response is concerned; it was selected only because in later work it is utilized for the assessment of wetness and slamming phenomena.

The expression for the amplitude of the absolute vertical acceleration in regular seas for any ship station is derived in Section I of Appendix D, equation (D9). The response operator for the acceleration is denoted by:

\[ |H^{zz}(\omega_e)\|^2 = \left[ \frac{\bar{z}}{\omega_e} (G_l) \right]^2 \]  \hspace{1cm} (35)

where \( G_l \) = Distance from centre of gravity, \( G \), to Station 1 and \( \omega_e \) has the same meaning as in equation (32).

Since magnitude as well as duration of a particular acceleration level are of importance in this instance, it is suggested that one should consider two statistical measures for acceleration, namely, the average of \( 1/10 \) highest amplitudes and the "average time reliability" with which we can expect that, for a particular sea condition, the adopted upper limit will never be exceeded. These parameters have been designated respectively by:

\[ \overline{z}(G_l)_{1/10} = 1.800 \left( A_g \right)^{1/2} \]  \hspace{1cm} (36)
and

$$R_e (\alpha) = 1 - p \left[ \bar{\alpha} > \alpha \right]$$

(37)

where $\alpha$ is the appropriate acceleration level.

3. Slamming Occurrence

One of the most deleterious effects of the sea on a vessel moving at high speed is caused by the impact of the re-entering bow on the water surface. Slamming is characterized by a series of irregular blows which impose high pressure loads on the ship's plating and is accompanied by high-frequency transient vibration (43), (68).

It is obvious that such a phenomenon can weaken or even destroy the forward hull structure. In the latter event, combatant vessels fail to fulfill their operational requirements while both naval and merchant ships do suffer increased repair bills. Even in the absence of damage, slamming causes increased resistance, objectionable vibration and motion sickness. Furthermore, the mere possibility of the occurrence of slamming forces the ship operator to slow down which, for merchant ships, means reduction of profits.

The exact mechanism of this highly non-linear phenomenon requires the examination of the effects of ship speed, keel-to-wave attitude and phasing of wave motions with respect to the moving vessel. Within our present knowledge, a complete analysis cannot be advanced and we shall therefore admit a simplified solution to the problem.

The following factors are generally considered to be conducive to the occurrence of slamming (56), (69):

1) Forefoot* emergence

*In this work, the "forefoot" is defined as the intersection of the vertical at Station 1 and the horizontal keel line.
2) Large relative vertical velocity between forefoot and wave,
3) Small angle in the longitudinal plane between keel and wave profile.

Because the general expression involving the last factor cannot be practically applied, the last factor will be ignored and the first two will be considered on the basis of linear theory. For this purpose we require the response amplitude operators of the motion and velocity of an arbitrary hull point relative to the adjacent wave profile. These were obtained from general expressions developed in Section 2 of Appendix D in the case of regular waves.

Following the simplified analysis of Tick (56), slamming will herein be assumed to occur whenever,

a) The instantaneous motion relative to the wave has exceeded the value \(-H\), i.e., \(r (\dot{c}) < -H\)

b) The velocity of the slamming station relative to the wave has exceeded a critical value, \(\dot{r}_c\), i.e., \(\dot{r} (\dot{c}) > \dot{r}_c\)

where \(\dot{r}_c > 0\).

The kinematical conditions stated above are conveniently illustrated in Fig. 66. It will be seen from the latter sketch that a more accurate first requirement would be:

\[ r (\dot{c}) < -H \cos \theta \]

where \(\theta\) is the instantaneous random pitch angle. Since knowledge of the latter quantity is never envisaged, we are forced to employ the linearizing assumption:

\[ \cos \theta = 1 \]

Quoting from Tick (56): "The condition \(\dot{r} (\dot{c}) > \dot{r}_c > 0\) implies that the keel was previously out of the water, and the relative velocity
FIG. 66 ILLUSTRATING SLAMMING OCCURRENCE
between keel and sea surface exceeds a critical value $\hat{r}_c$." Thus, finally, the expected number of slams in unit time is given by (56):

$$\bar{\bar{\bar{r}}} = \frac{1}{2\pi} \left[ \frac{p_2}{p_1} \right]^{1/2} \exp \left[ -\frac{1}{2} \left( \frac{H^2}{p_1} + \frac{\hat{r}_c^2}{p_2} \right) \right]$$

(38)

where

$$p_1 = \left[ r \sqrt{\text{CFL}} \right]^2$$

(39)

$$p_2 = \left[ \hat{r} \sqrt{\text{CFL}} \right]^2$$

(40)

are the mean square values of the random relative motion and velocity of the appropriate point. In accord with our statistical approach, the latter quantities are obtained as the areas under the response spectra:

$$s_{\bar{r}_T}(w_e) = |H_{\bar{r}_T}(w_e)|^2 \ s_{\bar{r}_T}(w_e)$$

(41)

and

$$s_{\bar{r}_r}(w_e) = |H_{\bar{r}_r}(w_e)|^2 \ s_{\bar{r}_r}(w_e)$$

(42)

In a foot-pound-second system, equation (38) gives the expected number of slams in one second. For full scale purposes, it is advisable to multiply the above expression by 3,600 and hence obtain the expected number of slams per hour.

A realistic prediction of slamming occurrence requires knowledge of $\hat{r}_c$, a quantity which is obviously difficult to ascertain. From an analysis of experimental results of a 5 ft. model, Tick (56) obtained the approximate value of 2.2 f.p.s. In the absence of more accurate information, full-scale predictions in this investigation were based on:

$$\hat{r}_c = 2.2 \sqrt{\frac{L}{5}} \ \text{ft. sec.}^{-1}$$

Since the distribution of all random amplitudes of the various ship responses have been considered to be Gaussian, an additional statistical
property may be established. To this end, it is assumed that the random amplitude of the relative motion and its time derivative are linearly independent. For Gaussian processes this implies that they are also statistically independent. Hence, the probability of the slamming phenomenon occurring can be obtained as the product of the probabilities with which the random relative motion and velocity assume certain values.

Since,

\[ p[r < -H] = \frac{1}{\sqrt{2\pi A_r}} \int_{-\infty}^{-H} \exp\left[-\frac{x^2}{2A_r}\right] \, dx \]  \quad (43)

and

\[ p[\dot{r} > \dot{r}_c] = \frac{1}{\sqrt{2\pi A_{\dot{r}}}} \int_{\dot{r}_c}^{\infty} \exp\left[-\frac{x^2}{2A_{\dot{r}}}\right] \, dx \]  \quad (44)

it follows that the probability of occurrence of slamming can be estimated from the product of equations (43) and (44). Also, the average time reliability with which we can expect slamming not to take place is given by:

\[ 1 - p[r < -H] \cdot p[\dot{r} > \dot{r}_c] \]  \quad (45)

It is obvious that the inclusion of a particular velocity level as a prerequisite of slamming has the effect of reducing the probability of the occurrence phenomenon and similar comments can be made with regard to a corresponding reduction of the expected number of slams in unit time as given by equation (38). By the same token, the neglect of the keel to wave condition noted earlier, will tend to exaggerate slightly the statistical predictions.

From a design point of view, the criteria and statistical measures of slamming developed in the preceding provide a means of assessing the adequacy of draft and shape of forward body in minimizing slamming. Although the estimation of slamming occurrence is hindered by the fact that the linear theory is inaccurate whenever the hull is out of the water, it is expected that predictions of the right order of magnitude can be achieved by the indicated approach (56).
4. Wetness Occurrence

Shipping of large quantities of water over the bow of a ship is one of the most distinct effects of heavy seas. As with slamming, this phenomenon occurs under special combinations of weather severity, ship speed and heading and phasing of the wave elevation and the forebody's motions. Water over the stern, usually referred to as pooping, occurs more rarely in practice (42) and for this reason it is not treated in this investigation.

The oncoming green seas augment a ship's resistance, force reduction in sustained speed, and can damage or displace exposed fittings, cargo hatches and other structures and equipment on the forward deck. At the same time, work on exposed decks becomes dangerous for personnel. For naval ships, refueling and efficient use of armament and equipment are hampered. Similar remarks can be made in connection with small commercial vessels such as trawlers and cross-channel ships. In the case of passenger ships, even moderate shipment of water in combination with the prevailing wind can create a very uncomfortable and annoying environment for the passengers.

A complete analysis of the kinematical conditions which cause the occurrence of this undesirable phenomenon is again complex. By necessity our examination will concentrate on the motion of a typical deck point with respect to the adjacent wave profile. The theoretical approach is more accurate in this case, as compared with slamming, since wetness occurrence always preassumes the submergence of the ship hull.

Although water may be shipped over the bow from many directions, we shall treat only head seas. We shall furthermore neglect the phasing
of irregular bow and wave motions and also assume that the stochastic sea
history possesses maxima which are widely separated from each other. It
appears (70) that shipping of water usually occurs at a point somewhat
aft of the forward perpendicular and hence we shall assume that Station No. 1
(0.1 L from F.P.) is the "wetness" station, as well as the "slamming" station.

Following similar lines as in (70), the following wetness criteria
or degrees of severity can be recognized:

a) **Dry Condition** - In this case the wave elevation is always
below the level of the weather deck at side. The relative
motion of a deck point lying on the vertical at Station 9
will illustrate this case mathematically through the inequality:

\[ r (C1) \leq f \]  \hspace{1cm} (46)

where \( f \) is the freeboard to the weather deck at Station 1.

b) **Wet Condition** - Whenever the wave formation reaches slightly
higher than the weather deck, moderate quantities of water
will be considered to cover the forward part of the ship.
Analytically, we restrict the relative motion of the deck
point at Station 1 to satisfy:

\[ 5/4 f \geq r (C1) \geq f \]  \hspace{1cm} (47)

c) **Very Wet Condition** - Shipping of green seas and severe bow
submergence will be assumed to occur whenever:

\[ r (C1) \geq 5/4 f \]  \hspace{1cm} (48)

and this will be considered as the most adverse wetness
condition.

The preceding simplified criteria illustrated in Fig. 67 do
not take account of the wave formation which is induced by the bow's
motion and which may be superimposed on the oncoming sea wave. Our
WETNESS STATION (No. 1)

DRY CONDITION
\[ r(G1) \leq f \]

WET CONDITION
\[ 1.25f > r(G1) \geq f \]

VERY WET CONDITION
\[ r(G1) \geq 1.25f \]

FIG. 67 ILLUSTRATING DEGREES OF WETNESS
elementary treatment also assumes that whenever the deck point at Station No. 1 is submerged, all points forward of it are also submerged while points aft of it are above the water surface.

The three degrees of wetness elaborated above, could alternatively be defined by examining jointly the relative motion of two points longitudinally separated on the forward deck and then establish similar criteria. Such an approach would probably be more realistic in delineating levels of wetness severity because it would ensure the submergence or emergence of a significant length of the forward deck.

The statistical measure which will be most useful is the average time reliability with which the wetness point on the deck is not submerged. Different reliabilities can, of course, be obtained for the three wetness criteria mentioned earlier. Since the amplitude operator of this particular ship response is derived in exactly the same manner as for slamming, further discussion of this point is not necessary.

From the design point of view, wetness criteria can provide a basis for ascertaining the adequacy of freeboard and sheer forward. Since the specification of freeboard for merchant ships is presently based on crude damage survivability criteria and not on seaworthiness considerations it will be interesting to see how currently specified merchant ship freeboards match the suggested wetness criteria. Finally, it should be recognized that for tankers and bulk carriers, green water over the deck poses for fewer problems than for other kinds of merchant ships, hence freeboard assessment for the former types of ship need not conform to any wetness criteria.

5. Propeller Immersion

The last and perhaps the least important effect of the sea with which we shall be concerned, is related to the undesirable emergence of
the propeller blades which occasionally occurs in severe weather conditions. As far as the author is aware, none of the seakeeping analyses to date have dealt with this event. This is probably due to the fact that with steam turbines and fast diesel engines, propeller "racing" is not the hazard that it was with steam reciprocating engines.

However, blade emergence does cause thrust breakdown, noise and vibration generation as well as augmented pitting and corrosion of the propeller. As Kempf has noted as far back as 1937 (71), it is essential for optimum operation to ensure that the blade tips are at all times well submerged, not just submerged beneath the water.

The previous utilization of the relative motion of an arbitrary point as an important kinematic response in describing slamming and wetness occurrence can be simply extended to the problem of propeller emergence. For our purposes, the computation of the relative motion of the aftfoot, the latter defined as the intersection of the A.P. (Station10) and the horizontal keel line, will provide a sufficiently accurate measure of the propeller's actual motion.

If we assume that in general, the propeller centerline is at a distance \( \overline{TH} \) \((0 < \eta < 1)\) below the designed waterline, as shown in Fig. 68, then one may proceed to establish a reasonable limit to the motion of the blade tips which are originally at a depth of \( \overline{TH} - D/2 \), where \( D \) is the propeller diameter. Thus, for motion in confused seas, we shall expect the blade tip(s) to be just submerged whenever:

\[
  r(\log) = - (\overline{TH} - D/2) = D/2 - \overline{TH} \tag{49}
\]

provided that the ship was originally in an even keel condition. \( \log \) is the (negative) distance from the centre of gravity, \( G \), to Station10 (A.P.)
BLADE TIP(S) SUBMERGED
WHEN \( r(10G) \geq -[\eta H - \frac{D}{2} (1 + 28)] \)

FIG. 68 ILLUSTRATING PROPELLER EMERGENCE
AND SUBMERGENCE
A minimum tip submergence may most conveniently be expressed in terms of the propeller diameter, say, \( \delta D \), in which case, one can ensure optimum propeller performance if the simplified condition:

\[
\begin{align*}
r(\omega G) &> - \left[ \frac{\pi H}{D} - \frac{D}{2} \left( 1 + 2 \delta \right) \right] \\
& (50)
\end{align*}
\]

is at all times satisfied.

Again, the most appropriate statistical measure will be the average time reliability with which we can expect the above inequality to hold throughout a given voyage. Just how significant such a criterion is remains to be seen; it is appreciated, however, that the requirements of propeller performance probably cannot be absolutely decisive in the selection of hull design parameters. One may also expect that if the slamming and wetness criteria are met, adequate blade tip submergence will also be achieved. Whether this is true or not will be examined in the subsequent sections.
VII. SAMPLE SEAWORTHINESS EVALUATION--FURTHER RECOMMENDATIONS

The main objective of the previous three chapters has been the establishment of a unified procedure whereby the assessment of seaworthiness performance can eventually become a matter of course in the preliminary stage of ship design. Since the computations involved in such an evaluation are extremely laborious and time consuming, the computer program utilized in the first part of this investigation was subsequently modified and extended so as to provide a relatively fast means of determining the kinematical performance of any ship moving at a particular speed in an ahead, irregular seaway. A complete description of the computer algorithm is given in Appendix E. Briefly, its scope at present involves the computation of the amplitude operators and spectra of the responses discussed in Appendix D so as to provide statistical measures of:

(a) the random responses, and

(b) certain seaworthiness phenomena, which are determined by these responses, as described in Chapter VI.

A typical example of what may be accomplished along the lines discussed herein is provided by the examination of a Series 60 ship, 600-feet long, with lines conforming to those of model B of Fig. 1, moving at a Froude number of 0.20 in two different sea states. This choice is not based on the good agreement of theory and experiment for this particular ship form as described in Part I of the present work, but is rather dictated by the fact that refs. (22) and (60) have already investigated this specific case.
The main characteristics of the chosen ship are shown below; further hull form particulars appear in Appendix E:

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>600.000 ft.</td>
</tr>
<tr>
<td>B</td>
<td>85.714 ft.</td>
</tr>
<tr>
<td>H</td>
<td>34.285 ft.</td>
</tr>
<tr>
<td>Δ</td>
<td>35,265 tons</td>
</tr>
<tr>
<td>C_B</td>
<td>0.700</td>
</tr>
<tr>
<td>C_P</td>
<td>0.710</td>
</tr>
<tr>
<td>C_W</td>
<td>0.787</td>
</tr>
<tr>
<td>L/B</td>
<td>7.00</td>
</tr>
<tr>
<td>L/H</td>
<td>17.50</td>
</tr>
<tr>
<td>B/H</td>
<td>2.50</td>
</tr>
<tr>
<td>LCB(LCG)</td>
<td>3.30 ft. aft of X</td>
</tr>
<tr>
<td>Radius of gyration</td>
<td>144.00 ft.</td>
</tr>
</tbody>
</table>

The results of the machine calculations (see typical output listing in Appendix E) are illustrated in Figs. 69-74 and summarized in Table III. In order to compare the kinematical performance of the ship in fully-developed seas of varying severity, two Neumann sea spectra for wind speeds of 25 and 30 knots were used. These correspond roughly to sea states 6 and 7. The manner chosen for their presentation in Fig. 69, indicates the effect of the transformation from the absolute to the encounter frequency domain. The modified spectra were calculated with the aid of equation (8), while the absolute spectra were separately

Text continues on page 140
FIG. 60  ABSOLUTE AND MODIFIED NEUMANN SEA ENERGY DENSITY SPECTRA
FIG. 70. RESPONSE AMPLITUDE OPERATOR AND SQUARE AMPLITUDE DENSITY SPECTRA OF ELONGATED MOTION
FIG. 72 RESPONSE AMPLITUDE OPERATOR AND SQUARED AMPLITUDE
DENSITY SPECTRA OF RELATIVE MOTION OF SLAMMING
(WINDNESS) STATION
FIG. 2: RESPONSE AMPLITUDE OPERATOR AND SQUARED AMPLITUDE DENSITY SPECTRA OF RELATIVE VELOCITY OF SLAMMING STATION.
FIG. 74  RESPONSE AMPLITUDE OPERATOR AND SQUARED AMPLITUDE
DENSITY SPECTRA OF RELATIVE MOTION OF PROPELLER STATION

-139 -
### TABLE III
Summary of Seaworthiness Evaluation for 600' Ship
(Ship Speed = 16.5 knots)

<table>
<thead>
<tr>
<th>Wind Speed (Knots)</th>
<th>25 (Sea State 6)</th>
<th>30 (Sea State 7)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.71°</td>
<td>3.36°</td>
</tr>
<tr>
<td></td>
<td>2.17°</td>
<td>4.28°</td>
</tr>
<tr>
<td></td>
<td>6.89 ft/sec²</td>
<td>12.80 ft/sec²</td>
</tr>
<tr>
<td></td>
<td>(0.214 g)</td>
<td>(0.398 g)</td>
</tr>
<tr>
<td></td>
<td>49.4%</td>
<td>41.3%</td>
</tr>
<tr>
<td>Slamming Occurrence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slams per hour</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>100%</td>
<td>98.9%</td>
</tr>
<tr>
<td>Wetness Occurrence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reliability of achieving</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) Dry Condition</td>
<td>99.8%</td>
<td>96.2%</td>
</tr>
<tr>
<td>(b) Wet Condition</td>
<td>0.2%</td>
<td>2.3%</td>
</tr>
<tr>
<td>(c) Very Wet Condition</td>
<td>0.0%</td>
<td>1.5%</td>
</tr>
<tr>
<td>Propeller Immersion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reliability of ensuring adequate propeller immersion</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>84.4%</td>
<td>72.9%</td>
</tr>
<tr>
<td>Wind Speed (Knots)</td>
<td>25 (Sea State 6)</td>
<td>30 (Sea State 7)</td>
</tr>
<tr>
<td>-------------------</td>
<td>------------------</td>
<td>------------------</td>
</tr>
<tr>
<td></td>
<td>1.71°</td>
<td>3.36°</td>
</tr>
<tr>
<td></td>
<td>2.17°</td>
<td>4.28°</td>
</tr>
</tbody>
</table>

1. **Pitching Motion**

"Significant" Amplitude Average of 1/10 Highest Amplitudes

2. **Vertical Acceleration**

Average of 1/10 Highest Amplitudes Reliability of Vertical Acceleration not exceeding 0.3g, \( R_{gg} \) (0.3g)

3. **Slamming Occurrence**

Slams per hour Reliability of not sustaining Slamming

4. **Wetness Occurrence**

Reliability of achieving (a) Dry Condition (b) Wet Condition (c) Very Wet Condition

5. **Propeller Immersion**

Reliability of ensuring adequate propeller immersion
computed in accord with equation \((4)\) by an algorithm not described herein. In every case, the ordinates of the absolute sea spectra were computed at wave frequencies which were derived through the use of equation \((9)\) for values of \(\omega_e = 0.0(0.1) 2.4\) rad/sec and a ship speed of 27.8 ft/sec (16.5 knots).

For each of the five examined responses, Figs. 70-74 show plotted values of the amplitude operator and squared-amplitude density spectra for the two aforementioned sea conditions. The importance of resonance is observable in all of the amplitude operator curves, where the peaks occur in the immediate vicinity of the ship's natural frequency of pitching. This might have been anticipated, since all examined responses which deal with the behaviour of points remote from the center of gravity are primarily influenced by the angular rotation of the body.

It is interesting to note that the response amplitude operators for the relative motion at the slamming and propeller stations attain a value of unity at high frequencies of wave encounter*. This assumption made in the investigations reported in (22) and (60) is now fully justified by the formal analytic approach. By comparing Figs. 72 and 74 it may also be seen that the relative motion at the forward end is about twice as high as the corresponding one at the stern, a result which agrees with previous experience and expectations.

---

*At high frequencies the ship fails to respond to the waves at all, so that the amplitude of the relative motion of a point on the ship with respect to the wave profile equals the wave amplitude.
Generally speaking, the trends of all response amplitude operator curves are in accord with those of earlier works. An interesting and novel situation arises in Fig. 73, where it may be seen that the relative velocity at Station No. 1 increases with increasing frequency of encounter, beyond the dominant frequency range.

This result may be easily explained by examining the definition of the relative velocity, equation (D20); for large encounter frequencies, the ship motions being essentially zero, the response amplitude operator essentially behaves as a plot of $\omega_e^2$ versus $\omega_e$. Finally, all squared-amplitude density spectra demonstrate the effects of increasing sea severity and indicate the relative behaviour of the selected ship in these idealized seaways.

Although not shown in this report, the results of complementary computations for the relative motion and vertical acceleration response operators at the forward perpendicular, checked favorably with the ones obtained in (22) and (60), where the experimental motion data of (13) was used rather than the theoretically computed motions of this report. This check validated the present computer calculations and generally indicated that statistical results obtained from the completely analytical approach do not differ significantly from the results obtained by using model data. Since only one ship at a given speed has been examined, one must refrain from generalizations; it appears however that, by and large, the present status of theory and experiment yield approximately the same statistical answers.
Table III is an illustrative summary of the statistical measures of the random ship responses and seaworthiness phenomena which were computed for the selected ship operating at one speed in two different sea conditions. Most of the statistical results were directly obtained from the machine output, as shown in Appendix E, whereas the probabilities of various phenomena occurring were determined from standard tabulations of the normal curve, using the equation:

\[
\Pr \{a < \xi < b\} = \frac{1}{\sqrt{2\pi}} \int_{a}^{b} \frac{1}{\sigma_{\xi}} \exp \left(-\frac{x^2}{2}\right) \, dx
\]  

(51)

where \(\xi\) represents a non-standard normal variable of zero mean and standard deviation \(\sigma_{\xi}\). The latter quantity is always calculated by the computer program for any random ship response and appears in the machine output.

To complete and evaluate the data contained in Table III necessitates knowledge of certain particulars of the ship form, etc., which, for our purposes, were assumed to be as follows:

1. Maximum permissible acceleration level, \(a = 0.3 \, g\).
   This seems to be a reasonable value as discussed, for example, in (66).

2. Freeboard at Station No. 1, \(f = 26 \, \text{ft}\).
(3) Propeller diameter, $D = 22$ ft. For this type of ship, $(13)$ shows that the center line of the propeller is approximately situated at a depth of 60% of the draft, so that $\eta = 0.6$. Adequate propeller immersion was arbitrarily selected as being 20% of the diameter, so that $\delta = 0.20$.

(4) In the absence of more accurate knowledge, the critical relative velocity between ship and wave for slamming occurrence was estimated to be $24$ ft/sec, by simply scaling the value quoted by Tick (56).

Referring to Table III, the results for the pitching motion and acceleration appear to be of the same order of magnitude as previous investigations and full-scale observations have shown them to be. The same cannot be said about slamming occurrence however, for the crude assumption about the relative velocity appears to yield very low estimates indeed. As it is, no one as yet has measured the critical velocity which, obviously, must depend on a host of other variables. The only reassurance in the slamming prediction is offered by the wetness indications which imply that the chosen ship at a Froude number of 0.20 does not appear to be drastically affected by sea states as high as No. 7. Contrary to our expectations in Chapter VI, Table III indicates that adequate propeller immersion is not automatically ensured if slamming and wetness criteria are met. It must be appreciated however, that a sustained blade tip
immersion at 4.4 ft. below water surface is probably very stringent. Relaxation of that requirement could possibly result in simultaneous satisfaction of slamming, wetness, and propeller immersion criteria, provided of course that realistic values are used throughout.

In concluding the second phase of the present investigation, it may be stated that a trial application of presently available knowledge has been made in effort to predict systematically the trends of realistic ship performance. We have seen how seaworthiness considerations can be approximately evaluated but have not yet proven how important such considerations are or can be. This is indeed the next logical step.

With the hope that the results of this work will assist in pursuing further research towards the ultimate goal of designing "efficient" ships on a rational basis, the following recommendations are made:

(1) The computer program described in Appendix E at present computes only a few of a large number of important ship responses and hence extension of its capabilities appears necessary. The machine time required for a seaworthiness evaluation, similar in scope to the one presented in this Chapter, is about 1.76 minutes. As computer times go, this is a very long time interval which necessarily hinders a systematic examination of all pertinent variables. Time
optimization is therefore specifically recommended in order to economically allow an investigation of much wider scope than the present.

(2) As discussed in Appendix C, the frequency mapping problem must be reexamined and if necessary formulated on a different basis. If this is impossible, then other means must be devised in order to efficiently obtain statistical results for the case of directly astern seas, which, so far has not received any attention.

(3) The Neumann formulation for the sea spectrum was used in this work and the results are therefore valid only as long as the latter is representative of actual sea conditions. It is therefore important to prepare subroutines which would provide different excitation definitions in the computation cycle and which will be either based on analytic formulations or actually measured sea spectra. The definitions of Darbyshire, Roll-Fischer (1), Vossers (14), and Pierson* for the sea spectrum will be of significance in this connection.

*As yet unreleased.
In Chapter III it was pointed out that Porter's work (34) on added mass and damping of two-dimensional forms should be incorporated in the computer program which calculated ship motions. Having done so, it would be instructive to pursue the statistical predictions and see if any distinct differences arise when using the latter data. In this connection it is also of importance to note that Grim's results for added mass and damping are presented in (11) on the basis of a frequency parameter (in our notation \( \frac{\omega_0^2 B^*}{2g} \)) which extends up to 1.4. If this if the upper limit for which Grim's data are correctly computed by the subroutine incorporated in our program, then a simple calculation shows that for the ship examined in this chapter, the results are valid only up to a frequency of encounter of about 1.15 rad/sec. As may be seen in Figs. 70-74, the computation of response amplitude operators necessarily involved frequencies much higher than that. Since no obvious computational blunders have appeared, it is presumed that Grim's subroutine functioned correctly at those higher frequencies or that the errors, if any, were suppressed by the magnitudes of other variables. Since Porter has recently reported
calculations for high frequency motion
\[ \omega_s B^* \]
\[ \frac{2}{2g} = 13.0 \), an examination of the two
sources of data is obviously imperative.

(5) It was suggested in Chapter IV that rate of
change of acceleration may be of some importance
in connection with human response. If this is
proven so, then clearly a suitable extension of
the seaworthiness considerations to include such
a variable is highly recommended.

(6) The computer program should be modified and so
generalized so as to permit the evaluation of
acceleration (or rate of acceleration) profiles
along the whole length of a given ship. This
would probably be of significance in the design
of passenger ships.

(7) The criteria of wetness and slamming discussed in
Chapter VI must be further reexamined and suitably
interpreted on the basis of full-scale or model
behaviour trends, before meaningful estimates can
be assured. In particular, emphasis should be
paid on the measurement or prediction of the act-
tual relative velocities which occur at the instant
of slamming.
(8) All statistical measures discussed in this work are based on the narrow-spectrum assumption. It would be a relatively simple matter to examine just how well this assumption is fulfilled by every random response, if the broadness factor, equation (23), is also computed at the end of the statistical analysis. This item should be considered after the machine time involved in the computations is well minimized.

(9) Since the computer program is capable of computing bending moments in regular waves, it will be of importance to attempt a correlation of analytically predicted and experimentally measured values. The N.S.M.B. has already published pertinent data for the same models as the ones used in Part I of this work and therefore such an evaluation would be relatively straightforward.

(10) If the work under item (9) indicates that the correlation is sufficiently reasonable then, clearly, the next step should be a statistical prediction of bending moments along the same lines as followed herein and using an improved version of the computer algorithm.

The inherent capabilities of a completely analytical approach to the seaworthiness problem are obvious. It goes without saying that the potential of a generalized computer program which is entirely based on such a concept will finally afford a rigorous evaluation of all seakeeping variables which characterize our problem.
BIBLIOGRAPHY


BB-1


BB-2


(34) Porter, W., "Pressure Distributions, Added-Mass, and Damping Coefficients for Cylinders Oscillating in a Free Surface", University of California, Institute of Engineering Research, Series No. 82, Issue No. 16, July 1960.


(36) Kaplan, P., and Jacob, W., "Two-Dimensional Damping Coefficients from Thin Ship Theory", and "Theoretical Motions of Two Yacht Models in Regular Head Seas on the Basis of Damping Coefficients Derived from Wide V-Forms", Davidson Laboratory Notes No. 586 and 593, June 1960.


BB-5
APPENDIX A
MISCELLANEOUS PROBLEMS ARISING IN THE CORRELATION ATTEMPT

1. On Experimental Shortcomings With Regard to Directly Ahead and/or Astern Sea Cases.

Early investigations (15) with the experimental facilities of the N.S.M.B. Seakeeping Tank (19) have shown that significant "wall effects" result when directly ahead and/or astern seas are simulated. These are caused by the reflection of model generated waves and materially affect measured motions. To eliminate such discrepancies, the snake-type wave generators of the above laboratory have since been used to produce wave systems which propagate at limiting angles of 170° and 10° to the model's direction of motion and therefore, the experimental results reported in (13) and (14) require modification for our purposes.

In order to bridge the "10° gap" to either 180° or 0° headings and hence correlate theory and experiment on a rational basis, one can proceed in two alternative ways:

(a) By extrapolation of the results given for various heading angles,

or

(b) By pursuing the suggestions of Weinblum and St. Denis (20).

The first approach is illustrated in Fig. A1, which is essentially a reproduction of Fig. 10 of (13). By plotting values of any model response and its associated phase angle on a basis of heading angle and by noting the symmetrical properties of the resulting graphs about the ordinates erected at 0° (360°) and 180°, one can easily extrapolate for the latter abscissae. In our case, this procedure requires crossplots from 24 graphs for each of the five reported wavelength to shiplength ratios.

A-1
FIG. A1 EXTRAPOLATION OF MODEL DATA FOR $\chi = 0^\circ$ AND $\chi = 180^\circ$
For the sake of time and labor, the second approach appeared more advantageous and was finally adopted. By viewing in an elementary manner a model's geometric attitude relative to an oblique wave system, the authors of (20) proposed the utilization of directly ahead and/or astern wave results in estimating motions and phases in oblique seas.

Although herein we are interested in the reverse problem, the principle remains the same. If a model moves in a regular wave system of actual length, \( \lambda \), which is propagating at an angle, \( \chi \), to the model's direction of motion, then one can imagine the model progressing in a fictitious wave system of effective wavelength \( \lambda_e = \lambda / \cos \chi \). Provided that the wave height is taken to be the same for both systems and that the model speed is properly modified in order to ensure the same frequency of encounter, it may be seen that, for small heading angles in any case, the approach appears rather plausible. Fig. A2 serves to illustrate the above argument, which has been shown by Lewis and Numata (21) to hold good for heading angle deviations of up to 30\(^\circ\), when examined on a Series 60 model. As discussed in the latter reference, one can anticipate discrepancies by employing the results of this simplified interpretation of what is otherwise an extremely complicated hydrodynamic problem. In particular, the so-called Smith effect is different since the pressure pattern in the wave depends on the actual wavelength, the effect of forward speed per se is neglected and finally the wave profiles on both starboard and port sides are not symmetrical as would otherwise be in a directly ahead or astern sea. Although the above effects may be of significance in the general case, it must be noted that in the present investigation the heading angle variation is only 10\(^\circ\), in which case one can safely assume that any discrepancies are probably comparable to those arising from experimental errors.
\[
\lambda_e = \lambda / \cos \chi
\]

<table>
<thead>
<tr>
<th>ITEM</th>
<th>ACTUAL WAVE SYSTEM</th>
<th>FICTITIOUS WAVE SYSTEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEIGHT</td>
<td>( h_0 )</td>
<td>( h_0 )</td>
</tr>
<tr>
<td>LENGTH</td>
<td>( \lambda )</td>
<td>( \lambda_e = \lambda / \cos \chi )</td>
</tr>
</tbody>
</table>

\[
F_{\text{AHEAD}} = F_{\text{EXPT}} - F(\chi) \left( \lambda / L \right)^{1/2}
\]

\[
F_{\text{ASTERN}} = F_{\text{EXPT}} + F(\chi) \left( \lambda / L \right)^{1/2}
\]

where \( F(\chi) = \left( 2\pi \right)^{-1/2} \left[ \cos \left( 180 - \chi \right) \right]^{-1/2} - \cos \left( 180 - \chi \right)^{-1} \)

**FIG. A2 RELATIONS BETWEEN ACTUAL AND FICTITIOUS WAVE SYSTEMS**
It has been shown in (22), where the same procedure was utilized, that the modifications required for the present analysis are not significant. In particular, the Froude number for ahead and astern seas may be taken to be the same as given in the experimental data of (13) and (14) for \( \chi = 10^\circ \) and \( \chi = 170^\circ \) respectively, since the correction turns out to be negligible even for the largest wave length to ship length ratio \( (\lambda / L = 1.8) \). The wave lengths, however, are to be increased by 1 1/2\% when simulating directly ahead and astern seas.

In summary, the computer input data relating to model speed and wave geometry, which are necessary for the correlation attempt with the N.S.M.B. experimental results, are given below:

\[
\begin{align*}
\text{Wave Amplitude}, & \quad h_o = 0.1 \text{ ft.} \\
\text{Wave Length}, & \quad \lambda = 6.0960 (3.0480) 18.2880 \text{ ft.} \\
\text{Model Speed}, & \quad V = 1.7945 (0.8972) 4.4861 \text{ ft. sec}^{-1}
\end{align*}
\]

Since the length of all models is \( L = 10.00 \) ft., the wave lengths, given above, correspond to the five wave-length to ship-length ratios,

\[
0.6096, 0.9144, 1.2192, 1.5240, 1.8288
\]

whereas, the speed range covers the four Froude numbers, for which results are available, namely,

\[
0.10, 0.15, 0.20, 0.25.
\]

2. On Unified Motion Amplitude and Phase Angle Definitions

This section deals with several problems which arise in connection with the different conventions and definitions adopted by,

(a) the personnel of the N.S.M.B. when publishing their model data (13), (14), (17),

(b) the authors of (8), on the basis of which the computer program has been developed,
and the author of the present work, on the basis of which the computer output results were interpreted and presented.

With regard to motion definitions, Vossers et al (13) consider heave positive upwards and pitch positive when the bow is displaced down from the designed waterline. Any model response is then defined to be:

\[ s(t) = s = s_o \cos (\omega_e t + \phi_h) \]  
(A1)

where, \( s = z \) or \( \theta \)

and \( s_o = z_o \) or \( \theta_o \)

The phase angle, \( \phi_h \) is so defined that the maximum value (amplitude) of the motion is leading the maximum wave elevation at the model's midship section. The wave motion referred to amidships is then,

\[ h(t) = h = h_o \cos (\omega_e t) \]  
(A2)

and is considered positive upwards. With regard to data presentation, Vossers et al also allow the amplitudes to assume negative values for reasons discussed in (13), so that, in the latter case,

\[ (-s_o) \cos (\omega_e t + 180^\circ + \phi_h) = s_o \cos (\omega_e t + \phi_h) \]  
(A3)

Turning now to the analytical side, Jacobs et al (8) define any model response as,

\[ s(t) = s = s_o \cos (\omega_e t - \psi) \]  
(A4)

where, \( s = z \) or \( \theta \)

\( s_o = z_o \) or \( \theta_o \)

\( \psi = \delta \) or \( \epsilon \)

with heave positive upwards and pitch positive when the bow is up. The phase angle, \( \psi \), in this case, is so defined that the maximum value of the motion lags the maximum positive wave slope at the model's centre of
gravity, i.e., in this case the wave motion, as referred to the C.G. is given by,

$$h = h_0 \sin (\omega e t)$$  \hspace{1cm} (A5)$$

and considered positive when the wave elevation is above the undisturbed waterline.

In order to compare the two different notations on a uniform basis, it was decided to proceed as follows. With regard to wave elevation, it seems convenient to adopt the same convention employed by both analysts and experimenters. Next, all motion amplitudes are considered positive throughout the whole cycle and are plotted as such (see Figs. 2-65). The computed amplitudes, which are obtained from the machine output and are always positive, are then directly comparable to the absolute values of the experimental motion amplitudes of (13) and (14).

It appears more convenient to refer all phase angles to the midship section, since, in a ship's lifetime the longitudinal centre of gravity never remains fixed at one point and also, from hydrodynamic considerations the centre of geometry is always justified as a unique reference point. For direct comparison, it also appears preferable to relate the maximum of the model motion to the maximum of the wave motion at the midship section. Thus, we define any model response in accord with equation (A4) and the wave elevation in accord with equation (A2); in this manner, the model motion always lags the wave motion.

Finally, for presentation purposes, we restrict the phase angle variation to lie within the range $0^\circ$-$180^\circ$ and hence negative lags, wherever shown in Figs. 2-57, may be considered as leads.
With the above definitions, it follows that both experimental and theoretical phase angles require modification. Focusing our attention to the first category, it may be seen that separate corrections apply for pitch and heave phase angles. Thus, the former must be increased by $180^\circ$ in order to allow for the positive direction of motion. Then, both must be modified depending upon whether the corresponding motion amplitude is given as positive or negative. If negative, equation (A3) requires that the angle must be decreased by $180^\circ$ whereas, if positive, no such measure is, of course, required. Finally, according to the adopted convention, a reversal in sign must be made in order to bring all phase angles on a common basis for comparison.

Before proceeding further, it must be noted that, since the pitch and heave phase angles reported in (13) are given for $\chi = 50^\circ$ ($40^\circ$) $170^\circ$, the necessary results for directly astern ($\chi = 10^\circ$) must be approximated. The validity of extrapolated values at this instance was considered doubtful and, for this reason, the correlation between theoretical and experimental phase angles was effected for directly ahead seas only.

The theoretically computed phase angles, on the other hand, are reported herein for both directly ahead and astern seas. Prior to presentation and in accord with the agreed definitions, the relevant information obtained from the computer output listings was modified in two distinct steps. The first, a time transformation, allowed for the change in temporal origin, while the second, a space transformation, referred the time origin from the centre of gravity to the midship section. A short analysis indicates that for a given LCG, reckoned positive if forward of amidships, the "modified" phase angle for directly ahead seas is given by,

\[ \text{A-8} \]
\[ \psi^* = \psi + \left[ \frac{\log}{\lambda} - 0.25 \right] 360^\circ \] \tag{A6}

while, for directly astern seas which "overtake" the model (see section 4 of this Appendix), by,

\[ \psi^* = \psi - \left[ \frac{\log}{\lambda} - 0.25 \right] 360^\circ \] \tag{A7}

where,

\[ \psi^* = \delta^* \text{ or } \epsilon^* \]

is the phase angle corresponding to our generalized motion definition

\[ s = s_0 \cos (\omega t - \psi^*) \]

and

\[ \psi = \delta \text{ or } \epsilon \]

is the phase angle obtained from the computer outputs.

3. Pitching and Heaving Periods of Series 60 Models

In order to compute the critical wave length to ship length ratio which corresponds to resonance conditions, one requires the model's natural periods (frequencies) for heaving and pitching motions. As far as the author is aware, for the Series 60 models under examination, only pitching periods in still water and zero forward speed have been published by the N.S.M.B. The pertinent values for the first phase of the comparison attempt are tabulated in Table AI, as given in the discussion of (13). With regard to the second phase, they were obtained from Table II of (14).

Since no information is available for the natural heaving periods, these were approximated from an expression given by Mandel (23) in the form,

\[ \tau_z = 2\pi \left[ \frac{H}{L} \frac{C_B}{C_W} (1 + k_z) \right]^{1/2} \] \tag{A9}

where \( k_z \) is a vertical inertia coefficient, strictly applicable to ellipsoids, and obtainable from Fig. 8 of (20), where it is plotted.
### TABLE A1

**NON-DIMENSIONAL PITCHING AND HEAVING PERIODS OF MODELS**

**FOR ZERO SPEED IN STILL WATER**

<table>
<thead>
<tr>
<th>Model</th>
<th>$\tau_0$</th>
<th>$C_B$</th>
<th>$C_w$</th>
<th>$C_B/C_w$</th>
<th>$H/L$</th>
<th>$L/B$</th>
<th>$B/2H$</th>
<th>$k_z$</th>
<th>$\tau_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.866</td>
<td>0.800</td>
<td>0.871</td>
<td>0.918</td>
<td>0.0571</td>
<td>7.00</td>
<td>1.25</td>
<td>1.16</td>
<td>2.12</td>
</tr>
<tr>
<td>B</td>
<td>1.848</td>
<td>0.700</td>
<td>0.787</td>
<td>0.889</td>
<td>0.0571</td>
<td>7.00</td>
<td>1.25</td>
<td>1.16</td>
<td>2.08</td>
</tr>
<tr>
<td>C</td>
<td>1.848</td>
<td>0.600</td>
<td>0.706</td>
<td>0.850</td>
<td>0.0571</td>
<td>7.00</td>
<td>1.25</td>
<td>1.16</td>
<td>2.03</td>
</tr>
<tr>
<td>D</td>
<td>1.758</td>
<td>0.700</td>
<td>0.787</td>
<td>0.889</td>
<td>0.0417</td>
<td>7.00</td>
<td>1.72</td>
<td>1.60</td>
<td>1.95</td>
</tr>
<tr>
<td>E</td>
<td>2.153</td>
<td>0.700</td>
<td>0.787</td>
<td>0.889</td>
<td>0.0909</td>
<td>7.00</td>
<td>0.78</td>
<td>0.83*</td>
<td>2.42</td>
</tr>
<tr>
<td>F</td>
<td>1.722</td>
<td>0.700</td>
<td>0.787</td>
<td>0.889</td>
<td>0.0571</td>
<td>8.50</td>
<td>1.03</td>
<td>1.00</td>
<td>2.02</td>
</tr>
<tr>
<td>G</td>
<td>1.955</td>
<td>0.700</td>
<td>0.787</td>
<td>0.889</td>
<td>0.0571</td>
<td>5.50</td>
<td>1.59</td>
<td>1.67</td>
<td>2.32</td>
</tr>
</tbody>
</table>

*Extrapolated from Fig. 8 of (20).*
as a function of L/B and B/2H. The calculations for $\tau_z$ pertaining to
the first phase of the correlation investigation are included in Table
AI, whereas, the experimental and computed ones for the second phase are
summarized below:

<table>
<thead>
<tr>
<th>Model $k_\theta$</th>
<th>$\tau_\theta$</th>
<th>$\tau_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.210</td>
<td>1.79</td>
<td>2.03</td>
</tr>
<tr>
<td>0.225</td>
<td>1.87</td>
<td>2.03</td>
</tr>
<tr>
<td>0.255</td>
<td>1.99</td>
<td>2.03</td>
</tr>
<tr>
<td>0.270</td>
<td>2.06</td>
<td>2.03</td>
</tr>
</tbody>
</table>


The period of wave encounter for a given ship speed $V$ and
heading angle $\chi$ is given in terms of the wave length as:

$$ T_e = \frac{\lambda}{\left(\frac{g_0}{2\pi}\right)^{1/2} - V \cos \chi} \quad \text{(A10)} $$

It is desirable to non-dimensionalize the above expression
and furthermore introduce the tuning factor,

$$ \Lambda = T_s / T_e $$

Thus, using

$$ V = F (gL)^{1/2} \quad \lambda = L (\lambda/L) $$

and

$$ T_s = T_s (gL)^{1/2} $$
equation (A10) takes the form,

$$ T_s = \frac{\Lambda \lambda / L}{\left(\frac{1}{2\pi}\right)^{1/2} (\lambda/L)^{1/2} - F \cos \chi} \quad \text{(A11)} $$

Solving for $\lambda/L$, we get,

$$ \lambda/L = \frac{1}{8\pi} \left(\frac{\tau_s}{\Lambda}\right)^2 \left[1 \pm \left(1 - \frac{8\Lambda \pi F \cos \chi}{\tau_s}\right)^{1/2}\right]^2 \quad \text{(A12)} $$

This expression has now to be considered separately for
directly ahead and astern seas. The analysis, similar to the one pre-

A-11
sented in (23), assumes that the ship speed and hence Froude number, is always positive. Resonance conditions are, as usual, typified by a tuning factor whose absolute magnitude is unity.

4-1. Directly Ahead Seas \((\chi = 180^0, \Lambda = 1)\)

In this case, equation (A12) simplifies to:

\[
\frac{\lambda}{L} = \frac{\tau_s^2}{8\pi} \left[ 1 + \left(1 + \frac{\delta \pi F}{\tau_s} \right)^{1/2} \frac{2}{2} \right]
\]  

(A13)

Physically speaking, in the case of directly ahead seas we can anticipate only one critical wavelength to ship length ratio which will cause the ship "system" to oscillate in a resonant fashion. The mathematical model, however, typified by equation (A13), yields two such ratios, one of which is obtained by squaring a negative number. A little experimentation with the above equation shows that, when the negative sign in front of the radical is retained, the resulting dimensionless wavelengths are unrealistic. For this reason, in this investigation only the positive sign has been considered and the critical wavelength-to-shiplength ratios so obtained are the ones depicted in Figs. 2-65.

4-2. Directly Astern Seas \((\chi = 0^0)\)

This case must be considered in two separate steps (23), according to whether the wave celerity is less than or greater than the ship speed.

(a) Following Seas \((\Lambda = -1, c < V)\)

The resulting expression for the latter subdivision is similar to Equation (A13) and hence the statements made previously in connection with directly ahead seas apply in this case also. Since, however, the maximum model speed and minimum wavelength investigated in (13) and (14) were \(V = 4.4861\) \text{ft. sec}^{-1} (\(F = 0.25\)) and \(\lambda = 6.096\) \text{ft.} (\(\lambda/L = 0.6\) for \(\chi = 10^0\))

A-12
respectively, the inequality \( c < V \) is never satisfied for our purposes, since the wave celerity always assumes a minimum value of \( c = 5.59 \text{ ft. sec}^{-1} \).

(b) **Overtaking Seas** (\( \Lambda = 1, c > V \))

In this case, equation (A12) simplifies to:

\[
\frac{\Lambda}{L} = \frac{r^2}{c} \left[ 1 + \left( 1 - \frac{8\pi F}{\tau_s} \right)^{1/2} \right]^2
\]  

(A14)

Three possible answers may now arise, depending upon whether \( 8\pi F/\tau_s \) is less, equal or greater than unity. In the first case, we get two real values, in the second, we get one and in the third, we obtain two complex and hence unrealizable wavelength to shiplength ratios.

An inspection of the experimental speed range of (13) and (14) and of the non-dimensional natural periods for pitching and heaving, listed in Table A1 and at the end of Section 3 of this Appendix, indicates that, for all models in the present investigation, the condition:

\( 8\pi F/\tau_s > 1 \)

is always satisfied. Due to this circumstance, it follows that in accord with the prevailing experimental conditions, no identification of possible critical wavelength-to-shiplength ratios is possible within the range 0.6096-1.8288.
APPENDIX B

ANALYTICAL DETAILS OF THE LINEAR THEORY OF SHIP MOTIONS

In order to achieve completeness and simultaneously provide easy reference, the fundamental analytical expressions of the Korvin-Kroukovsky method, as utilized in the computer program, are summarized in this Appendix. Following the nomenclature and definitions adopted in (8), the coupled set of linear differential equations describing the two-degree-of-freedom ship system, takes the form,

\[
\begin{align*}
\bar{a}\ddot{Z} + b\dot{Z} + cZ + d\ddot{\theta} + e\dot{\theta} + g\theta &= \bar{F} \exp(i\omega_e t) \\
A\ddot{\theta} + B\dot{\theta} + C\theta + D\ddot{Z} + E\dot{Z} + GZ &= \bar{M} \exp(i\omega_e t)
\end{align*}
\]  

(B1)

The above equations result from equilibrium considerations of the hydrodynamic forces and moments called into play by the ship's oscillations in the plane of symmetry, when meeting head or astern regular waves. For this reason the analysis ignores steady, continuously acting forces due to buoyancy, gravity and suction pressures. Following the principles of classical dynamics, these forces and moments are obtained by applying Newton's Second Law of Motion to both transatory and rotational displacements of the body's centre of gravity.

The wave induced excitation force and moment may be thought of as being imposed on a fully restrained ship and appear on the RHS of the set (B1). They have the useful property that they are functions of the wave elevation and its two first time derivatives, thereby allowing ease in algebraic manipulation (7). They are defined in complex notation as,

\[
\begin{align*}
\bar{F} \exp(i\omega_e t) &= F_0 \exp(-i\sigma) \exp(i\omega_e t) = F_0 \exp\left[i\left(\omega_e t - \sigma\right)\right] \\
\bar{M} \exp(i\omega_e t) &= M_0 \exp(-i\tau) \exp(i\omega_e t) = M_0 \exp\left[i\left(\omega_e t - \tau\right)\right]
\end{align*}
\]  

(B2)

B-1
The differential exciting force acting on a control section distant $\xi$ from the origin of the moving coordinate system (ship's C.G.), has been given in (8) in the simplified form,

$$\frac{dF}{dx} = \frac{dF_1}{dx} \cos \omega_t t + \frac{dF_2}{dx} \sin \omega_t t$$

$$= \left[ \left( \xi_1 \sin \frac{2\pi \xi}{\lambda} + \xi_2 \frac{2\pi \hbar c_w}{\lambda} \cos \frac{2\pi \xi}{\lambda} \right) \exp \left( - \frac{2\pi \xi}{\lambda} \right) \right] \cos \omega_t t$$

$$+ \left[ \left( \xi_1 \cos \frac{2\pi \xi}{\lambda} - \xi_2 \frac{2\pi \hbar c_w}{\lambda} \sin \frac{2\pi \xi}{\lambda} \right) \exp \left( - \frac{2\pi \xi}{\lambda} \right) \right] \sin \omega_t t$$

(B3)

where,

$$\xi_1 = \frac{4\pi^2 \hbar c_w^2}{\lambda^2} (\rho S k_{2 k_4})$$

and,

$$\xi_2 = N (\xi) - V \frac{d \rho S k_{2 k_4}}{d \xi}$$

while the differential exciting moment of this force about the C.G. is given by $\frac{dF}{dx} d\xi$. Integration of the above two quantities over the ship length results in the values of the total time-varying exciting force and moment, which are considered as the real parts of the expressions (B2). Thus,

$$F = F_1 \cos \omega_t t + F_2 \sin \omega_t t$$

$$M = M_1 \cos \omega_t t + M_2 \sin \omega_t t$$

$$= \sqrt{F_1^2 + F_2^2} \cos \left[ \omega_t t - \arctan \frac{F_2}{F_1} \right] \quad \text{and} \quad = \sqrt{M_1^2 + M_2^2} \cos \left[ \omega_t t - \arctan \frac{M_2}{M_1} \right]$$

$$= F_0 \cos (\omega_t t - \sigma)$$

$$= M_0 \cos (\omega_t t - \tau)$$

(B4)

The analysis of the forces and moments which correspond to the ship's free oscillations in calm water yields terms which appear on the LHS of the set (B1) and are proportional to the instantaneous heaving and pitching displacements, velocities and accelerations. All twelve coefficients of the above terms are independent of the speed per se and
the body's space orientation, but depend on the frequency of encounter, with the exception of \( c \) and \( G \). The final expressions for the coefficients of the equation of motion used in the machine computation are listed below:

\[
\begin{align*}
a &= m + \rho \int (Sk_2k_4) \, d\xi \\
b &= \int N (\xi) \, d\xi \\
c &= \rho g \int B^* \xi \, d\xi \\
d &= \rho \int (Sk_2k_4) \xi \, d\xi \\
e &= \int N (\xi) \xi \, d\xi - 2\nu \rho \int (Sk_2k_4) \xi \, d\xi - V \rho \int d (Sk_2k_4)/d\xi \xi \, d\xi \\
g &= \rho g \int B^* \xi \, d\xi - V \int N (\xi) \, d\xi \\
A &= J + \rho \int (Sk_2k_4) \xi^2 \, d\xi \\
B &= \int N (\xi) \xi^2 \, d\xi - 2\nu \rho \int (Sk_2k_4) \xi \, d\xi - V \rho \int d (Sk_2k_4)/d\xi \xi^2 \, d\xi \\
C &= \rho g \int B^* \xi^2 \, d\xi - V \int N (\xi) \xi \, d\xi + \nu^2 \rho \int d (Sk_2k_4)/d\xi \xi \, d\xi \\
D &= \rho \int (Sk_2k_4) \xi^2 \, d\xi \\
E &= \int N (\xi) \xi \, d\xi - V \rho \int d (Sk_2k_4)/d\xi \xi \, d\xi \\
G &= \rho g \int B^* \xi \, d\xi
\end{align*}
\]  

(B5)

Assuming now that sufficient time has elapsed for any transient disturbances to be damped out, we seek particular solutions of the non-homogeneous set (B1), which correspond to the steady-state responses of the system. Since the forcing functions are sinusoidal and the system is linear and time-invariant, we expect that any response will also be sinusoidal of the same frequency as the excitation and with generally different amplitude and phase. We therefore assume solutions of the form,

\[
\begin{align*}
z(t) &= z = \tilde{z} \exp (i\omega_e t) \\
\text{and} \quad \Phi(t) &= \Phi = \tilde{\Phi} \exp (i\omega_e t)
\end{align*}
\]  

(B6)

with the arbitrary definitions,

\[
\begin{align*}
\tilde{z} &= z_0 \exp (-i\xi) \\
\text{and} \quad \tilde{\Phi} &= \Phi_0 \exp (-i\xi)
\end{align*}
\]  

(B7)

B-3
Upon substitution in the original equations (B1), a conscientious algebraic manipulation yields the following expressions for the "complex" heaving and pitching amplitudes,

\[
\bar{Z} = \frac{\bar{F}S - \bar{M}Q}{FS - QR} \quad \text{and} \quad \bar{\Theta} = \frac{\bar{M}P - \bar{F}R}{FS - QR} \quad (B8)
\]

where,

\[
\bar{F} = F_0 (\cos \sigma - i \sin \sigma) \\
\bar{M} = M_0 (\cos \tau - i \sin \tau) \\
P = (c - a w_e^2) + i b w_e \\
S = (C - A w_e^2) + i B w_e \\
Q = (g - d w_e^2) + i e w_e \\
R = (G - D w_e^2) + i E w_e \quad (B9)
\]

It follows from (B8) and (B9) that the "complex" amplitudes may be expressed as,

\[
\bar{Z} = z_1 - i z_2 = \sqrt{z_1^2 + z_2^2} \exp \left[-i \arctan \frac{z_2}{z_1} \right] \quad (B10)
\]

and

\[
\bar{\Theta} = \theta_1 - i \theta_2 = \sqrt{\theta_1^2 + \theta_2^2} \exp \left[-i \arctan \frac{\theta_2}{\theta_1} \right]
\]

Considering the real parts of (B6), we finally obtain,

\[
z = \text{Re} \bar{Z} \exp (i w_et) \\
= \text{Re} \sqrt{z_1^2 + z_2^2} \exp \left[i (w_et - \arctan \frac{z_2}{z_1}) \right] \\
= z_0 \cos (w_et - \delta) \quad (B11)
\]

and

\[
\Theta = \text{Re} \bar{\Theta} \exp (i w_et) \\
= \text{Re} \sqrt{\theta_1^2 + \theta_2^2} \exp \left[i (w_et - \arctan \frac{\theta_2}{\theta_1}) \right] \\
= \theta_0 \cos (w_et - \epsilon) \quad (B12)
\]
APPENDIX C

THE FREQUENCY MAPPING PROBLEM FOR DIRECTLY AHEAD AND ASTERN SEAS

The concise application of the theory of random processes to the ship system case is, in some respects, hindered by complications which arise from the use of the frequency of wave encounter as a fundamental variable. Since the whole motion analysis problem is treated in the frequency domain, it is important to understand the reasons why such difficulties arise.

Two main causes can be identified which necessitate special treatment and which, in the case of astern seas, can even render certain problems unsolvable:

1) The seaway is most conveniently defined with respect to a stationary reference point. As a result, any typical idealized sea energy density spectrum is originally formulated as a function of the absolute wave frequency. It is recognized, however, that this is not the final form of the "input" spectrum because, in the most general case, the system is non-stationary and, furthermore, "directional". Since meaningful ship responses can only be obtained by referring them to the ship's coordinate system, it is necessary to linearly transform the absolute input spectrum in order to take account of the vessel's speed and direction of motion. As will be shown, it is on account of this transformation that the mathematical model fails to provide us with a unique answer.

2) In contrast to electromagnetic or acoustic waves whose velocity is independent of frequency, deep sea gravity waves are characterized by a velocity which is a function of the length of the wave.
The authors of (38) provided a concise first treatment to the problem. However, the requirements of establishing an efficient algorithm for digital computer application necessitated further elaboration of their results in order to treat the cases of both directly ahead and astern seas. In the process of this elaboration, certain weaknesses were revealed in their particular line of analysis.

Excluding "oblique" seas, the most general expression which relates frequency of encounter and absolute wave frequency for a given and always positive ship speed is:

$$\omega_e = \omega + \omega \frac{V}{g}$$

$$= \omega (1 + \omega \frac{V}{g})$$

where the + sign accounts for directly ahead seas and the - sign for directly astern seas. It may be seen that an alternative definition could allow the ship speed to be either positive or negative depending upon whether the wave celerity is in the opposite or same direction, so that, for both directly ahead and astern seas,

$$\omega_e = \omega (1 + \omega \frac{V}{g})$$

Either of these two alternate definitions admits the possibility of obtaining a negative frequency of encounter. Physically speaking, measured frequencies (periods) are always positive. Hence, a negative frequency indicates motion in an opposite direction to that previously assumed.

One could bypass the apparent awkwardness of these formulations and ensure a positive frequency of encounter by a definition of the form:

$$\omega_e = \omega |1 + \omega \frac{V}{g}|$$

(C3)
Such an expression, however, is not amenable to further analysis
along the lines which we must now pursue. We commence by examining
directly ahead and astern seas separately and by focusing attention to
the transformation from \( \omega \) to \( \omega_e \).

1. **Directly Ahead Seas (\( \chi = 180^\circ \))**

(a) Wave Frequency Independent Variable, \( \omega_e = f(\omega) \)

In this case equation (C1) specializes to:

\[
\omega_e = \omega (1 + \frac{\omega V}{g}) \tag{C4}
\]

where \( V \) is considered always positive. For a given absolute wave
frequency or, alternatively, wave length, equation (C4) shows that the
frequency of encounter \( \omega_e \) is a unique function of the independent
variable \( \omega \). Fig. C1 is a plot of equation (C3), for \( V = g \) ft. sec\(^{-1}\)
and illustrates the one-to-one relationship between \( \omega_e \) and \( \omega \).

The plane bounded by and extending at infinity to the right
of the vertical axis may be conveniently referred to as the \( \omega - 180^\circ \)
plane and represents the functional relationship \( \omega_e = f(\omega) \) for part of
the Primitive Region I, of the St. Denis and Pierson paper (38).

(b) Encounter Frequency Independent Variable, \( \omega = f'(\omega_e) \).

It is the author’s belief that a flexible computer algorithm
capable of handling the seaworthiness problem in its entirety, requires
that the frequency of encounter be the independent variable. If the
frequency of wave encounter is read in as input data or, even better,
uniformly generated in equally spaced intervals, then at later stages
approximate integration can be simply effected and important statistical
information thereby deduced.

Assuming then that the entire computation is started with a
particular frequency of encounter, there arises the following important
FIG. C1 RELATIONSHIP BETWEEN FREQUENCY OF ENCOUNTER AND ABSOLUTE WAVE FREQUENCY FOR DIRECTLY AHEAD SEAS ($\chi = 180^\circ$)

\[ V = 32.2 \text{ ft/sec} \]

\[ \omega_e = \omega [1 + \omega \frac{V}{g}] \]
question: For a given ship speed and heading angle, which absolute wave frequency (wavelength) caused that particular frequency of encounter? The answer to this question must be known since one must compute the energy spectrum (see equation (8)) and also certain absolute frequency dependent terms in the Korvin-Kroukovsky analytical procedure (see Appendix B).

In the case of directly ahead seas (χ = 180°) the solution of equation (C4), with w as the unknown variable is:

\[
\omega = -\frac{\frac{1}{2} + \sqrt{1 + \frac{4 \omega e V}{2V}}}{\frac{V}{2}}
\]  

(C5)

At first sight, the double sign in front of the radical may rouse suspicion as to the uniqueness of the solution; the requirement, however, that a positive frequency of encounter could only have resulted from a positive wave frequency, demands that the positive sign be neglected and, hence:

\[
\omega = -\frac{1 - \sqrt{1 + \frac{4 \omega e V}{2V}}}{\frac{V}{2}}
\]  

(C6)

Thus, as might have been anticipated, in the case of directly ahead seas, it is always possible to ascertain which wavelength causes a particular frequency of encounter. Fig. C2 is a plot of equation (C6) for \( V = g \) ft. sec\(^{-1} \) and exhibits the unique inverse relationship \( \omega = f'(\omega_e) \).

The modification of the sea energy spectrum, \( \mathbf{E}(\omega) \), from the absolute to the encounter frequency domain, is then achieved by noting that the input average energy must be preserved (38). Mathematically:

\[
E = \int_{0}^{\infty} \mathbf{E}(\omega) \, d\omega = \int_{0}^{\infty} \mathbf{E}_{11}(\omega_e) \, d\omega_e
\]  

(C7)
\[ \omega = -\frac{g}{2V} \left[ 1 - \sqrt{1 + \frac{4\omega_e V}{g}} \right] \]

\( V = 32.2 \text{ ft/sec} \)

FIG. C2 RELATIONSHIP BETWEEN ABSOLUTE WAVE FREQUENCY AND FREQUENCY OF ENCOUNTER FOR DIRECTLY AHEAD SEAS \( (\chi_e = 180^\circ) \)

c-6
so that,

$$\Phi_{ii}(\omega_e) = \Phi(\omega) \frac{d\omega}{d\omega_e} \quad (C8)$$

From equation (C6), it follows that:

$$\frac{d\omega}{d\omega_e} = \left[ 1 + \frac{4}{g} \frac{\omega_e}{V} \right]^{-1/2} \quad (C9)$$

In the most general case, where the heading angle, \( \chi \), is explicitly involved as a variable, one is required to determine \( \frac{d\omega}{d\omega_e} \) and \( \frac{d\chi}{d\chi_e} \) from a set of equations:

\[
\begin{align*}
\omega_e &= \Omega(\omega, \chi) \\
\chi_e &= \chi(\chi)
\end{align*}
\quad (C10)
\]

This preassumes that it is possible to obtain unique functional relationships for \( \omega \) and \( \chi \) in terms of \( \omega_e \) and \( \chi_e \). If this is not so, as in our case, then by differentiating the above set with respect to \( \omega \) and \( \chi \), a solution of the inverse derivatives \( \frac{d\omega}{d\omega_e} \) and \( \frac{d\chi}{d\chi_e} \) is ensured if and only if the determinant of the coefficients of the pertinent set is non-zero. This determinant is referred to as the Jacobian of the transformation, which in the case of directly ahead seas may be shown (38) to be:

\[
J(\omega_e) = \frac{d\omega}{d\omega_e} \quad (C11)
\]

In view of the above relationship, equation (C8) may be restated in the more familiar form:

\[
\Phi_{ii}(\omega_e) = J(\omega_e) \Phi(\omega) \quad (C12)
\]

2. **Directly A stern Seas \((\chi = 0^0)\)**

(a) Wave Frequency Independent Variable, \( \omega_e = f(\omega) \)

In this case, equation (C1) specializes to:
\[ \omega_e = \omega \left[ 1 - \frac{\omega \mathbf{V}}{g} \right] \]  \hspace{1cm} (C13)

where, again, \( V \) is considered always positive. A close examination of the latter equation reveals that \( \omega_e \) can now attain negative values whenever \( \frac{\omega \mathbf{V}}{g} > 1 \). In order to circumvent the awkwardness of this result, the authors of (38) proposed a new definition in this region so as to ensure positive frequencies of encounter. Thus, we may redefine equation (C13) in terms of:

\[
\omega_e = \begin{cases} 
\omega (1 - \frac{\omega \mathbf{V}}{g}) & \text{for } 0 \leq \omega \leq g/V \\
-\omega (1 - \frac{\omega \mathbf{V}}{g}) & \text{for } g/V < \omega \leq \infty 
\end{cases} \hspace{1cm} (C14)
\]

It may now be seen that, for \( 0 \leq \omega \leq g/V \) the frequency of encounter increases with increasing absolute frequency, until it reaches a maximum of \( 1/4 \frac{g}{V} \) at \( \omega = \frac{g}{2V} \). It then decreases with increasing \( \omega \) and becomes identically zero at \( \omega = \frac{g}{V} \). Since \( c = \frac{g}{\omega} \), a physical interpretation of the above results is afforded in terms of the relative magnitudes of the ship and wave speeds. Thus, for \( 0 \leq \omega \leq g/V \) the wave "overtakes" the ship while for \( \frac{g}{V} < \omega \leq \infty \) it "follows" it. At \( \omega = \frac{g}{V} \) no identification of relative motion between wave system and the ship can be made. Fig. C3 summarizes conveniently the discussion in this section and indicates plots of the set (C14).

(b) Encounter Frequency Independent Variable, \( \omega = f'(\omega_e) \)

Without further analysis, it may be seen from the above figure that it is not possible within \( 0 \leq \omega_e < \frac{1}{4} \frac{g}{V} \) to identify a unique wave frequency (wavelength) which causes a particular frequency of encounter. For this reason, it is not possible to exhibit, in the case of directly astern seas, a plot similar to Fig. (C2). On account of the above
FIG. C3 RELATIONSHIP BETWEEN FREQUENCY OF ENCOUNTER AND ABSOLUTE WAVE FREQUENCY FOR DIRECTLY ASTERN SEAS (χ = 0°)
ambiguity, the authors of (38) have found it expedient to divide the \( \omega \) axis in the three primitive regions shown in Fig. C3, and outline a procedure which would identify the inverse relationship \( \omega = f'(\omega_e) \).

In any event, the essential problem of ascertaining which \( \omega \) causes a given \( \omega_e \), in the case of directly astern seas remains unsolvable, even if the expedient of (38) is employed. Unfortunately, it has not been possible as yet to devise a different line of analysis which would resolve this undesirable result and provide a sound basis for analyzing directly astern seas.

Another main reason why no attempt has been finally made to incorporate this case in our statistical investigation is because of the dubious results this particular mathematical model yields with regard to the transformed energy spectrum of the sea. Specifically, it is shown in (38) that for \( \omega_e = \frac{E}{NV} \) the Jacobian of the transformation vanishes so that, at that point, the energy spectrum attains an infinite value. The exact definition of a density spectrum suggests in this case the existence of a unique sinusoidal wave component which adds, quite distinctly, to the remaining components of the spectrum configuration. This is difficult to interpret and furthermore, tends to shake the foundation on which our fundamental assumption rests, namely, that a purely random process, such as a wave record, is made up of an infinite number of seaway components whose phases attain values in a random fashion.

In summary, it is the author's belief that there exists an urgent need for further examining the implications of this particular problem, modifying the basis on which the whole concept of frequency mapping rests and thus providing a uniform, meaningful and general basis for applying the theory of random processes to the seaworthiness problem.
APPENDIX D

ANALYSIS OF CERTAIN KINEMATIC RESPONSES IN REGULAR WAVES

In this Appendix analytic expressions are developed for the absolute acceleration and relative motion and velocity of any point along the length of the ship (or model) (the term "absolute" is used to designate acceleration with respect to the earth, and the term "relative", responses with respect to the moving wave profile). For these responses to be meaningful, it is necessary to refer them to a moving coordinate system whose origin is at the ship's centre of gravity.

In our derivation we make the following simplifying assumptions:

(a) The ship (or model) is assumed to be a perfectly rigid body moving with constant speed in regular waves coming directly ahead from the bow.

(b) Consistent with the linear theory of pitching and heaving motions, the body's departures from the equilibrium position are considered to be infinitesimally small.

(c) In accord with the conventions adopted in Appendix A, the heaving and wave motions are considered positive upwards while the pitching motion is positive when the bow is up.

(d) The motion, velocity and acceleration are those of a point on an axis parallel to the base line of the ship and passing through the center of gravity which is assumed to be originally at the undisturbed water line.
1. Absolute Acceleration

We begin by considering the instantaneous vertical motion of a point, P, distant, $\xi$ from the centre of gravity. The hull coordinate, $\xi$, is positive forward. From Fig. D1, it may be seen that the total departure of $R_s(\xi, t)$, from the undisturbed position of equilibrium (no wave), is due to a vertical translation of the C.G. (heave) and a rotation about the C.G. (pitch). Hence, for a given frequency of wave encounter, $\omega_e$, the motion of P, being sinusoidal at a frequency equal to the encounter frequency and positive upwards, is given by:

$$s(\xi, t) = s(\xi) = z + \xi \Theta \sin \Theta$$

(D1)

where,

$$z = z_o \cos (\omega_e t - \delta)$$

(D2)

$$\Theta = \Theta_o \cos (\omega_e t - \epsilon)$$

(D3)

The phase angles $\delta$ and $\epsilon$ are herein defined in the same manner as in (8). For small pitching angles, $\Theta$, equation (D1) may be "linearized" to:

$$s(\xi) = z + \xi \Theta$$

(D4)

In order to derive the instantaneous acceleration of P which is induced from the ship's oscillations, equation (D4) must be differentiated twice with respect to time. Thus:

$$\ddot{s}(\xi) = \ddot{z} + \xi \dddot{\Theta}$$

(D5)

and since, from equations (D2) and (D3),

$$\ddot{z} = -z_o \omega^2_e \cos (\omega_e t - \delta)$$

(D6)

$$\dddot{\Theta} = -\Theta_o \omega^2_e \cos (\omega_e t - \epsilon)$$

(D7)

it follows, by substitution in (D5), that:
FIG. D1 ABSOLUTE MOTION OF ARBITRARY HULL POINT IN REGULAR WAVES
\[ \ddot{z}(\xi) = \ddot{z}_0(\xi) \cos \left[ \omega_e t - \phi(\xi) \right] \quad (D8) \]

where,
\[
\ddot{z}_0(\xi) = \left[ \left( - z_0 \cos \delta - \xi \theta_0 \cos \epsilon \right)^2 + \right. \\
\left. \left( - z_0 \sin \delta - \xi \theta_0 \sin \epsilon \right)^2 \right]^{1/2} \omega_e^2 \quad (D9)
\]

and
\[
\phi(\xi) = \arctan \left[ \frac{z_0 \sin \delta + \xi \theta_0 \sin \epsilon}{z_0 \cos \delta + \xi \theta_0 \cos \epsilon} \right] \quad (D10)
\]

The amplitude operator of the "absolute acceleration" response of the ship-system is then exemplified by:
\[
\left[ \ddot{z}_0(\xi) \right]^2
\]

provided that it has been evaluated for the specific case of motion in a unit-amplitude sinusoidal wave. Although not explicitly shown above, the acceleration response amplitude operator, so derived, corresponds to the particular frequency of encounter which defines the wave motion. Equation (D9), specifically derived for the point, P, is obviously valid for any other point above or below P and originally lying on the same vertical line perpendicular to the undisturbed water line.

2. Relative Motion and Velocity

For the purposes of defining the kinematical conditions which will permit us to examine slamming, wetness and adequate propeller immersion, we need to consider the motion of a point, P, relative to the wave profile. The nomenclature which we shall employ is similar to the one used in (8). Relative to a fixed origin in space, the moving surface profile is described, at any time, t, by:
\[
h = h_0 \sin \frac{2\pi}{\lambda} (x - ct) \quad (D11)
\]
The above equation represents the instantaneous elevation at a point \( x \) of a wave moving with celerity, \( c \), to the left of the positive space axis. For a ship moving with speed, \( V \), in the direction of the positive \( x \) axis, as shown in Fig. D2, the wave system expressed by (D11) corresponds to directly ahead seas \( (\chi = 180^\circ) \).

Through the linear transformation,

\[
x = \xi + Vt
\]  

(D12)
equation (D11) can now be referred to the ship coordinate system with origin at the C.G., and thus becomes:

\[
h (\xi) = h_o \sin \frac{2\pi \xi}{\lambda} \left[ \xi + (V + c) t \right]
\]  

(D13)

The motion of any point, \( P \), relative to the wave elevation can then be conveniently described at any instant by:

\[
r (\xi, t) = h (\xi, t) - s (\xi, t)
\]  

(D14)
or

\[
r (\xi) = h (\xi) - s (\xi)
\]  

(D15)
where \( h (\xi) \) and \( s (\xi) \) are given by equations (D13) and (D14). Substituting in (D15) and simplifying we finally obtain:

\[
r (\xi) = r_o (\xi) \cos \left[ \omega t - \rho (\xi) \right]
\]  

(D16)
where,

\[
r_o (\xi) = \left[ h_o \sin \frac{2\pi \xi}{\lambda} - z_o \cos \delta - \xi \phi_o \cos \epsilon \right]^2 + \left[ h_o \cos \frac{2\pi \xi}{\lambda} - z_o \sin \delta - \xi \phi_o \sin \epsilon \right]^{1/2}
\]  

(D17)
and

\[
\rho (\xi) = \arctan \left[ \frac{h_o \cos \frac{2\pi \xi}{\lambda} - z_o \sin \delta - \xi \phi_o \sin \epsilon}{h_o \sin \frac{2\pi \xi}{\lambda} - z_o \cos \delta - \xi \phi_o \cos \epsilon} \right]
\]  

(D18)
The relative velocity between the point, \( P \), and the wave can then be obtained by differentiating (D16) once with respect to time and may be shown to be:
FIG. D2 RELATIVE MOTION OF ARBITRARY HULL POINT IN REGULAR WAVES
\[ z(\tau) = y_0(\tau) \cos \left[ \frac{2\pi}{\lambda} \left( x - z(\tau) \right) \right] \] (3.255)

where

\[ y_0(\tau) = \eta_0 \left( \frac{1 + \cos \frac{2\pi}{\lambda} \left( x - z(\tau) \right) \sin \eta_0 \sin \theta}{\lambda} \right)^{1/2} \] (3.256)

and

\[ \frac{z(\tau)}{\eta_0} = \operatorname{arctan} \left[ \frac{2\pi}{\lambda} \left( \frac{x - \zeta_0 \sin \frac{2\pi}{\lambda} \left( x - z(\tau) \right) \sin \theta}{\lambda} \right) \right] \] (3.257)

From the above treatment, it follows that at any instant, if

\[ r(\tau) < c \] the point \( P \) is emerged from the water,
\[ r(\tau) = c \] the point \( P \) and the water profile coincide,
\[ r(\tau) > c \] the point \( P \) is immersed under the water.

By the same token, inequalities of the form:

\[ r(\tau) < \ell \text{ and } r(\tau) > \ell \]
\[ r(\tau) < -\ell \text{ and } r(\tau) > -\ell \]

approximately describe the condition for which points at distances \( \ell \), above
and \( -\ell \) below \( P \), are emerged or immersed at any instant. As in Section 1
of this Appendix, the amplitude operators for the relative motion and
velocity responses of the ship system are given by:

\[ z_0(\xi) \quad \text{and} \quad \left| z_0(\xi) \right|^2 \]

whenever the excitation input to the system is caused by a unit-amplitude
sinusoidal wave.
## APPENDIX E

### DESCRIPTION OF COMPUTER PROGRAMS

**Table of Contents**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nomenclature of Computer Programs</td>
<td>E-2</td>
</tr>
<tr>
<td>Introduction</td>
<td>E-8</td>
</tr>
<tr>
<td>PROGRAM A</td>
<td>E-10</td>
</tr>
<tr>
<td>Listing of MAIN program</td>
<td>E-12</td>
</tr>
<tr>
<td>Typical Input Data Listing (Option 1)</td>
<td>E-15</td>
</tr>
<tr>
<td>Typical Output Data Listing (Option 1)</td>
<td>E-16</td>
</tr>
<tr>
<td>Input Data for Computations of Part I</td>
<td>E-17</td>
</tr>
<tr>
<td>PROGRAM B</td>
<td>E-21</td>
</tr>
<tr>
<td>MAIN Program</td>
<td>E-21</td>
</tr>
<tr>
<td>Order of Program and Input Data (Fig. El)</td>
<td>E-23</td>
</tr>
<tr>
<td>Flow Diagram of MAIN Program (Figs. E2a,b,c)</td>
<td>E-24</td>
</tr>
<tr>
<td>Ship or Model Representation for Computation Purposes (Fig.E3)</td>
<td>E-27</td>
</tr>
<tr>
<td>Subroutine SPECTR</td>
<td>E-28</td>
</tr>
<tr>
<td>Subroutine ADMAB</td>
<td>E-28</td>
</tr>
<tr>
<td>Subroutine COEFF</td>
<td>E-29</td>
</tr>
<tr>
<td>Subroutine EXCITE</td>
<td>E-29</td>
</tr>
<tr>
<td>Subroutine MOTION</td>
<td>E-30</td>
</tr>
<tr>
<td>Subroutine PARAM</td>
<td>E-33</td>
</tr>
<tr>
<td>Subroutine BENDSH</td>
<td>E-34</td>
</tr>
<tr>
<td>Subroutine STATIS</td>
<td>E-34</td>
</tr>
<tr>
<td>Subroutine SIMPS</td>
<td>E-35</td>
</tr>
<tr>
<td>Description of Input Data</td>
<td>E-36</td>
</tr>
<tr>
<td>Typical Input Data Listing (Seaworthiness Evaluation)</td>
<td>E-41</td>
</tr>
<tr>
<td>Description of Output Data</td>
<td>E-42</td>
</tr>
<tr>
<td>Typical Output Data Listing (Seaworthiness Evaluation)</td>
<td>E-44</td>
</tr>
<tr>
<td>Listing of Program</td>
<td>E-45</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>ABAR(I)</td>
<td>see subroutine ADMAB</td>
</tr>
<tr>
<td>ADDA(I)</td>
<td>see subroutine COEFF</td>
</tr>
<tr>
<td>ALPHA</td>
<td>phase lag of shearing force</td>
</tr>
<tr>
<td>AVERB</td>
<td>average relative motion of slamming station</td>
</tr>
<tr>
<td>AVERDB</td>
<td>average relative velocity of slamming station</td>
</tr>
<tr>
<td>AVERP</td>
<td>average relative velocity of propeller station</td>
</tr>
<tr>
<td>AVESDD</td>
<td>average absolute acceleration at slamming station</td>
</tr>
<tr>
<td>AVETHE</td>
<td>average pitch amplitude</td>
</tr>
<tr>
<td>AVHRB</td>
<td>average of one-tenth highest relative motions of slamming station</td>
</tr>
<tr>
<td>AVHRDB</td>
<td>average of one-tenth highest relative velocities of slamming station</td>
</tr>
<tr>
<td>AVHP</td>
<td>average of one-tenth highest relative motions of propeller station</td>
</tr>
<tr>
<td>AVHSDD</td>
<td>average of one-tenth highest absolute accelerations at slamming station</td>
</tr>
<tr>
<td>AVHTHE</td>
<td>average of one-tenth highest pitch amplitudes</td>
</tr>
<tr>
<td>BEEB(I)</td>
<td>see subroutine COEFF</td>
</tr>
<tr>
<td>BETA</td>
<td>phase lag of bending moment</td>
</tr>
<tr>
<td>BMIMAG</td>
<td>sine term of bending moment</td>
</tr>
<tr>
<td>BMNULL</td>
<td>bending moment amplitude</td>
</tr>
<tr>
<td>BMREAL</td>
<td>cosine term of bending moment</td>
</tr>
<tr>
<td>BFL</td>
<td>ship (model) length between perpendiculairs</td>
</tr>
<tr>
<td>BSTAR(I)</td>
<td>full beam (breadth) of ship (model) station</td>
</tr>
<tr>
<td>CGGC(I)</td>
<td>see subroutine COEFF</td>
</tr>
<tr>
<td>CONST</td>
<td>$0.001654g^2\pi^3$</td>
</tr>
</tbody>
</table>
CTFST see subroutine BENDSH
CW wave celerity
CXFST cosine term of exciting force
DEENFR increment of encounter frequency
DELTA phase lag of heaving motion
DELV increment of ship or model speed
DELW increment of wind speed
DELWL increment of wavelength
DISPL ship (model) displacement
DIX(I) $\frac{d(Sk_2k_4)}{d\xi}$
DMASS(I) station mass
DRAFT(I) station depth (draft)
DWEIGH(I) station weight
EMNULL pitching moment amplitude
ENFRMA maximum frequency of encounter
ENFRMI minimum frequency of encounter
ENOX(I) station damping coefficient
EPSIL phase lag of pitching motion
FNULL heaving force amplitude
FRRODE Froude number
GAMMA specific weight of water
GRAV gravitational acceleration
GSQUAR $\xi^2$

E-3
H  interval of Simpson's integration
HELP1--HELP5  square roots of RSDSDD, RSTRB, RSTRDB, RSTRP, RSTTHE
I  program variable
INCRS  see subroutine BENDSH
KRIT  see subroutine BENDSH
M  no. of ship or model stations
MAXKRI  see subroutine BENDSH
MINKRI  see subroutine BENDSH
N  highest station number
NN  program control variable and number of seaoworthiness computations
OMEGA  absolute wave frequency
OMEGAE  frequency of encounter
PI  \( \pi = 3.1415926 \)
QUANT(I)  \( \frac{S_{k_2}k_4}{2} \)
RAORB(I)  amplitude operator of slamming station relative motion
RAORDB(I)  amplitude operator of slamming station relative velocity
RAORP(I)  amplitude operator of propeller station relative motion
RAOSDD(I)  amplitude operator of slamming station absolute acceleration
RAOTHE(I)  amplitude operator of pitching motion
RIZA  \( 1.0 + 4\omega_e V/g \)
RO  density of water
RSDRB(I)  amplitude density spectrum of slamming station relative motion
RSDRDB(I)  amplitude density spectrum of slamming station relative velocity
RSDRP(I)  amplitude density spectrum of propeller station relative motion
RSDSDD(I) amplitude density spectrum of slamming station absolute acceleration
RSDTHE(I) amplitude density spectrum of pitching motion
RSTRB    mean square relative motion of slamming station
RSTRDB   mean square relative velocity of slamming station
RSTRP    mean square relative motion of propeller station
RSTSDD   mean square absolute acceleration at slamming station
RSTTHE   mean square pitching motion
SECOE(I)  section area coefficient
SHIMAG    sine term of shearing force
SHNULL    shearing force amplitude
SHREAL    cosine term of shearing force
SIGMA     phase lag of heaving force
SIGRBD    significant relative motion of slamming station
SIGRDB    significant relative velocity of slamming station
SIGRP     significant relative motion of propeller station
SIGSDD    significant absolute acceleration of propeller station
SIGTHE    significant pitching motion
SKLAM     see subroutine EXCITE and BENDSH
SPECMI(I)  see subroutine BENDSH

E-5
SXFST  sine term of heaving force
SYMPS  area under any curve (see subroutine SIMPS)
TAU    phase lag of pitching moment
TI(I)   see subroutine MOTION
TIMAG  sine term of pitching motion
TMASS  total mass of ship or model
TNULL  amplitude of pitching motion
TR(I)   see subroutine MOTION
TREAL  cosine term of pitching motion
UR(I)   see subroutine MOTION
UI(I)   see subroutine MOTION
V      ship or model speed
VMAX   maximum speed of ship or model
VMIN   minimum speed of ship or model
VOITH  \( \omega^2 \)
VOMEGA(I) frequency of encounter (used for printing output)
VP     ship or model speed in knots (used for printing output)
W      wind speed
WA     wave amplitude
WAVEN  wave number
WL     wavelength
WMAX   maximum wind speed
WMIN   minimum wind speed
\( \Xi(i) \)  longitudinal coordinate of moving (ship) system
\( Y \)  see subroutine SIMPS
\( Y_{\text{NERT}} \)  weight moment of inertia of ship or model
\( Z_{\text{IMAG}} \)  sine term of heaving motion
\( Z_{\text{NULL}} \)  amplitude of heaving motion
\( Z_{\text{REAL}} \)  cosine term of heaving motion
Introduction

Two computer programs have been used for the calculations presented in this report. The first, herein designated as Program A, was originally written by K. Haslum and was subsequently debugged and completed with the author's collaboration. This program was extensively used in the computations of the heaving and pitching motion amplitudes and phase angles, the results of which have been shown in Figs. 2-65.

The second program, herein designated as Program B, utilizes certain subroutines of Program A and represents an extension of the latter which permits computation of seaworthiness responses and their statistical measures along the lines discussed in Chapters IV, V, and VI. The following points should therefore be noted in connection with the two computer algorithms:

(a) Their main programs differ in conception and structure.
(b) Subroutines ADMAB, COEFF, EXCITE, MOTION, and SIMPS are similar in both programs and have been used for the same purposes. Minor differences occur only in their DIMENSION and COMMON statements. Also, subroutine SIMPS has been made slightly more general in Program B.
(c) In order to achieve the objectives of the second phase of the present investigation, three new subroutines have been incorporated in Program B. These are SPECTR, PARAM, and STATIS.
(d) With a different input option, as discussed in (15), Program A has the ability to compute shearing forces and bending moments at any station of a ship or model moving in regular waves. Since this was not our purpose however, subroutine BENDSH, which essentially performs the above task, was always bypassed in the main computation cycle through the inclusion of a proper control card. Subroutine BENDSH has been retained, but not used in Program B, in case future work along statistical lines may require the examination of random bending moments.

Since Program A has been extensively described in (15), only a brief resume and listing of the main program is presented in this Appendix. Also included are typical input-output listings and the pertinent hull-form data for the models examined in Part I of the present work. Program B is described in detail in the subsequent pages of this appendix and, for reasons stated earlier, the discussion includes the salient points of Program A. The variables listed in the nomenclature are those of both programs.
PROGRAM A

This program is a computer algorithm of the step-by-step procedure outlined in (8) for the calculation of ship (model) motions and bending moments on the basis of the Korvin-Kroukovsky linear theory of pitching and heaving. Since a detailed description of the program is already available in ref. (15) and since most of the subroutines of this program are discussed under Program B* in subsequent sections, only a brief description of its MAIN program and input-output is given.

The purpose of the MAIN program which is listed at the end of the section is to perform certain minor calculations, control the computation cycle from subroutine to subroutine and handle all input and output data. The input information required for the computation is almost exactly similar to that of Program B. We shall hence refrain from details in this section and refer the reader to the description on input data for Program B. A typical input listing is included in the Appendix followed by a complete list of the hull-form data of the Series 60 models A through G used in the correlation attempt of Part I. A typical output listing is also included.

The computer output capabilities of Program A come under three options, of which the present work has made sole use of option

*With the exception of subroutine BENDSH which is not used in the present work.
No. 1. This is related to the computation of motions only and assumes knowledge of the ship's radius of gyration.

The main variable for Program A is the wavelength and the basic computation cycle repeats the fundamental loop, first for different wavelengths and then returns if the ship speed is changed. For every complete calculation the program prints out the wavelength (WL), ship speed (V), and frequency of encounter (OMEGA). The units are consistent and correspond to those chosen by the first READ statement (see Program B). It then prints out all twelve coefficients of the equation of motion, and the exciting force and moment amplitudes and phase angles as well as their cosine and sine term components.

The heave and pitch amplitudes, cosine and sine terms as well as phases are given in dimensional and non-dimensional forms. Heave amplitude is non-dimensionalized by the wave amplitude and pitch amplitude by the maximum wave slope. Phases are given in radians when motion amplitudes are dimensional and in degrees where the motion amplitudes are non-dimensional. A typical output listing is included at the end of this brief description of Program A.
"MAIN" PROGRAM LISTING OF PROGRAM A

DIMENSION Y(21), TR(6), TI(6), ADDA(4), BEEB(4), CGGC(4), UR(6), UI(6),
1D MASS(21), QUANT(21), SKLAM(21), BSTAR(21), CXFST(21), SXFST(21),
2CTFST(21), STFST(21), XI(21), DIX(21), ENOXI(21), DRAFT(21), DWEIGH(21),
3SEO(E1421), Aabar(21),
COMMON Y, J, SYMP, DXI, ADDA, BEEB, CGGC, ZREAL, ZIMAG, TREAL, TIMAG, ZNULL,
1TNULL, DELTA, EPSIL, TR, TI, V, D MASS, QUANT, OMEGA, SKLAM, KRI T, RO, GRAV,
2BSTAR, C XFST, SXFST, ALPHA, SHNULL, XI, BET, BMNULL, GAMMA, DIX, M, WA, WAVEN
3, CW, E NOXI, SIGMA, TAU, FNUL, EMNUL, DRAFT, DWEIGH, SECOE, TMASS, N, UR, UI,
4ABAR, PI, SHRREAL, SHIMAG, BMREAL, BMIMAG, YNERT
900 FORMAT (10, 4F10.4)
901 FORMAT (3F10.4)
902 FORMAT (F10.4)
903 FORMAT (3F20.3)
904 FORMAT (4F20.3)
905 FORMAT (2F20.4)
906 FORMAT (7F10.4)
907 FORMAT (3I10)
922 FORMAT (120)
C

SHIP AND WATER CHARACTERISTICS
PI = 3.1415926
1001 READ 900, N, BPL, GAMMA, GRAV, DISPL
PRINT 900, N, BPL, GAMMA, GRAV, DISPL
RO = GAMMA / GRAV
FN = N
DXI = BPL / FN
M = N + 1
1002 READ 901, (BSTAR(I), SECOE(I), DRAFT(I), I = 1, M)
PRINT 901, (BSTAR(I), SECOE(I), DRAFT(I), I = 1, M)
J = M
TMASS = DISPL / GRAV
1003 READ 905, RADGYR, XI(1)
PRINT 905, RADGYR, XI(1)
IF (RADGYR) 2, 2, 5
2 DO 3 I = 1, M
1004 READ 902, DWEIGH(I)
PRINT 902, DWEIGH(I)
3 DM ASS(I) = DWEIGH(I) / GRAV
HOMENT = 0.0
DO 4 I = 1, M
L = I - 1
FL = L
4 HOMENT = HOMENT + DWEIGH(I) * FL
XI(1) = DXI * HOMENT / DISPL
5 DO 6 I = 2, M
L = I - 1
FL = L
6 XI(I) = XI(I) - DXI * FL
IF (RADGYR) 7, 7, 9
7 YNERT = 0.0
DO 8 I = 1, M
8 YNERT = YNERT + DWEIGH(I) * (XI(I) * XI(I) + (DXI * DXI / 12.)
GO TO 1005
9 YNERT = DISPL * RADGYR * RADGYR
1005 READ 907, MINKR, MAXKRI, INCR
PRINT 907, MINKR, MAXKRI, INCR
1006 READ 906, WA, SWL, BWL, DELWL, VMIN, VMAX, DELV
PRINT 906, WA, SWL, BWL, DELWL, VMIN, VMAX, DELV
V = VMIN
11 WL = SWL
12 IF (MINKR) 14, 14, 13
13 KRI T = MINKR

E-12
14 WAVEN=2.*PI/WL
   CW=SORTF(GRAV/WAVEN)
   OMEGAE=WAVEN*ABSF(CW+V)
   CALL ADMAA
   CALL COEFF
909 FORMAT(61H)
   WL
   V
   0
1MEGAE)
   PRINT 909
   PRINT 903, WL,V,OMEGAE
913 FORMAT(58H)
   ADDA
   BEEB
   CG
   1GC)
   PRINT 910
   PRINT 903, (ADDA(I),BEEB(I),CGGC(I),I=1,4)
   CALL EXCITE
911 FORMAT(78H)
   EXCITING-FORCE
   COSINE-TERM
   SINE
   1-TERM
   PHASE)
   PRINT 911
   FTWO=-TI(2)
   PRINT 904, FNULL,TR(2),FTWO,SIGMA
912 FORMAT(78H)
   EXCITING-MOMENT
   COSINE-TERM
   SINE
   1-TERM
   PHASE)
   PRINT 912
   EMWO=TI(5)
   PRINT 904, EMNULL,TR(5),EMWO,TAU
   CALL MOTION
913 FORMAT(78H)
   HEAVE
   COSINE-TERM
   SINE
   1-TERM
   PHASE)
   PRINT 913
   PRINT 904, ZNULL,ZREAL,ZIMAG,DELTA
914 FORMAT(78H)
   PITCH
   COSINE-TERM
   SINE
   1-TERM
   PHASE)
   PRINT 914
   PRINT 904, TNULL,TREAL,TIMAG,EPSIL
   L=(N/2)+1
   DELTA=DELTA*57.295779
   EPSIL=EPSIL*57.295779
   TNULL=TNULL/(WA*WAVEN)
   ZNULL=ZNULL/WA
   IF(MINKRI)16,16,15
15 CALL BENDSH
921 FORMAT(21H)
   KRIT)
   PRINT 921
   PRINT 922, KRIT
915 FORMAT(78H)
   DYN.SHEAR-FORCE
   COSINE-TERM
   SINE
   1-TERM
   PHASE)
   PRINT 915
   PRINT 904, SHNULL,SHREAL,SHIMAG,ALPHA
916 FORMAT(78H)
   DYNSBEND.MOMENT
   COSINE-TERM
   SINE
   1-TERM
   PHASE)
   PRINT 916
   PRINT 904, BMNULL,BMREAL,BMIMAG,BETA
   SHNULL=SHNULL*1BPL/(GAMMA*(BSTAR(L)*WA)**2)
   BMNULL=BMNULL/(GAMMA*WA*BSTAR(L)*BPL*BPL)
   ALPHA=ALPHA*57.295779
   BETA=BETA*57.295779
917 FORMAT(47H)
   THE SAME QUANTITIES IN DIMENSIONLESS FORM
16 PRINT 917
   DWL=WL/BPL
   FROUDE=V/SORTF(GRAV*BPL)

E-13
918 FORMAT(60H WL/BPL)
   LEGAE)
   PRINT 918
   PRINT 903, DWL,FROUDE,OMEGAE
919 FORMAT(78H HEAVE)
   PHASE)
   PRINT 919
   PRINT 904, ZNULL,DELTA,TNULL,EPSIL
   IF(MINKRI) 19,19,17
920 FORMAT(78H DYN.SHEAR-FORCE)
   PHASE)
   PRINT 920
   PRINT 904, SHNULL,ALPHA,BMNULL,BETA
   IF(KRIT-MAXKRI) 18,19,19
17 IF(WL=BWL) 20,21,21
18 KRIT=KRIT+INCRS
   GO TO 15
19 IF(WL-BWL) 20,21,21
20 WL=WL+DELWL
   GO TO 12
21 IF(ABSF(V)-ABSF(VMAX)) 22,25,25
22 V=V+DELV
   GO TO 11
25 CONTINUE
   GO TO 1001
END

TOTAL 145

E-14
TYPICAL INPUT DATA LISTING (OPTION 1)

*  DATA  20  10.0  62.4  32.2  407.14
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   1.345  0.851  0.571
   1.216  0.758  0.571
   1.017  0.627  0.571
   0.719  0.419  0.571
  0.229  0.618  0.057
          2.4       4.75

  0.1  6.096  18.2880  3.048  1.7945  4.4861  0.8972

TOTAL:  26*

E-15
TYPICAL OUTPUT DATA LISTING (OPTION 1)

```
* XEQ

ENTRY POINTS TO SUBROUTINES REQUESTED FROM LIBRARY,
    SETUP  (Cshm)  (RTN)   (SPHM)  (FIL)   SQRT   SIN
    ATAN  exp1   exp12

.15 MINUTES ELAPSED SINCE START OF JOB

EXECUTION
    20  10.0000  62.4000  32.2000  407.1400

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    1.4120  0.9200  0.5710
    1.3450  0.8510  0.5710
    1.2160  0.7580  0.5710
    1.0170  0.6270  0.5710
    0.7190  0.4190  0.5710
    0.2290  0.6180  0.0570
    2.4000  4.7500

    -1    1    1

                        1.7945  4.4861  0.8972
    0.1000  6.0960  18.2880  3.0480
                    V
    WL  6.096
    ADDA  1.794
          24.213
          19.029
          -39.160
          -1.360
          2.360
          5225.951
    EXCITING-FORCE  175.430
                      SINE-TERM
    130.518
    EXCITING-MOMENT  6.285
                      SINE-TERM
    21.260
    HEAVE  -19.789
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           0.004
           0.008
    PITCH  COSINE-TERM
           SINE-TERM
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          -1.012

THE SAME QUANTITIES IN DIMENSIONLESS FORM
    WL/BPL  Froude
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    HEAVE  PHASE
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          72.520
          0.794
          779.262
    WL  9.144
    ADDA  BEEB
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    PITCH  SINE-TERM
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E-16
### INPUT DATA FOR COMPUTATIONS OF PART I

Particulars of Models A, B, C, D, E, F, G as used

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**TOTAL** 49

E-20
PROGRAM B

As stated in the introduction to this Appendix, Program B represents an extension of Program A which permits analytical prediction of ship performance in ahead irregular seas. The computer algorithm is divided in the MAIN program and nine subroutines thereby allowing efficient machine computation in minimum time and future extensions and/or modifications.

MAIN Program

The purpose of the MAIN program is essentially threefold:

(a) To allow a convenient and systematic handling of all input and machine output data. The required input information for the computations and the output data options of the program are discussed in detail in later sections.

(b) To compute the values of essential variables which are common in certain subroutines and/or in repeated calculation cycles, and

(c) to control computation loops and transfer information from one subroutine to another.

Figure E1 is a schematic of the computer program's composition, computation sequence and order of input data. Since most subroutines involve pure algebraic computations, only a flow diagram of the MAIN program is presented in this description. Thus, Fig. E2 illustrates the structure of the MAIN program, the calling sequence of various subroutines and the logical operations which are involved in the calculation process.
The ship or model is represented by a discrete combination of masses as shown in Fig. E3 which is equivalent to the ship's representation in the "strip" theory of Korvin-Kroukovsky. The MAIN program at first reads in the required input information and control cards which decide upon whether a complete seaworthiness evaluation is desired or whether the corresponding heaving and pitching motions need only be computed for checking purposes. The prime difference between the MAIN programs of Program A and B is that the former performs a cycle of computations by specifying wavelengths whereas the latter utilizes the frequency of encounter as the fundamental variable of every computation cycle. For this reason the MAIN program first computes the actual wavelength and absolute wave frequency along the lines discussed in Appendix C and then commences the whole computation cycle which is made up of three basic loops. The outermost loop involves the speed of the examined ship or model, the intermediate one the system's excitation and the innermost one the frequency of encounter. These concepts are conveniently illustrated in Fig. E2. In this way, computations are first performed for a certain number of frequencies of encounter and then repeated for different wind speeds each time the ship speed being the basic variable which alters the system's state. When all the specified computations are performed, the MAIN program prints out all the necessary data in a manner discussed later. At the end of this section, a complete listing of the entire program is presented together with typical input and output listings.
FIG. E1 ORDER OF PROGRAM AND INPUT DATA
FIG. E2a FLOW DIAGRAM OF MAIN PROGRAM

START

READ SHIP DATA

RO=\text{GAMMA}/\text{GRAV}

DXI=\text{LBP}/\text{N}

TM\text{ASS}=\text{DISPL}/\text{GRAV}

\text{IS RADGYR GIVEN?}

\text{YES}

\text{COMPUTE } X(I) \text{ FOR } I=1, M

\text{YNERT}=\text{DISPL} \cdot \text{RADGYR} \cdot \text{RADGYR}

\text{READ ALL PARTICULARS}

\text{IS NN GIVEN?}

\text{NO}

\text{YES}

\text{READ WIND DATA}

\text{PRINT ALL DATA}

\text{IS NN GIVEN?}

\text{NO}

\text{YES}

\text{PRINT WIND DATA}

\text{V}=\text{VMIN}

\text{FROUDE}=V/\sqrt{\text{GRAV} \cdot \text{BFL}}

\text{COMPUTE } YNERT

\text{COMPUTE } X(I) \text{ FOR } I=1, M
FIG. E2b  FLOW DIAGRAM OF MAIN PROGRAM (cont.)
FIG. E3 SHIP OR MODEL REPRESENTATION FOR COMPUTATION PURPOSES
Subroutine SPECTR

The sole function of this subroutine is to generate the ordinates of the transformed sea energy spectrum. The analytical expression used for this purpose is that of Neumann, as exemplified by equation (4) in the main text. If the ship speed is zero, the subroutine calculates the absolute spectrum ordinate for any given frequency of encounter (in this case equal to the absolute wave frequency). If the ship speed is non-zero and positive (corresponding to ahead seas), the subroutine yields the transformed spectrum ordinate in accord with equation (8). In the latter case, the absolute frequency is first determined in the main program from the inverse transformation represented by equation (9). The spectrum ordinates are designated by SPECM (I) and correspond to the frequency of encounter for which the whole seaworthiness computation cycle is performed at any instant.

Subroutine ADMAB

This subroutine, obtained from the staff of the Davidson Laboratory of the Stevens Institute of Technology, calculates according to Grim's theory (11),

(a) the added mass per unit length at each station, which is designated as QUANT (I), and,

(b) the ratio of the amplitude of the emitted wave to the amplitude of heave at each station, which is designated as ABAR (I).

Although individually checked for normal Lewis ship sections, the range of the numerical applicability of the subroutine has not been thoroughly examined.
Subroutine COEFF

This subroutine evaluates numerically the expressions (B5) for the coefficients of the equations of motion. The correspondence between coefficients and program variables is shown below:

\[
\begin{bmatrix}
a & b & c \\
d & e & g \\
D & E & G \\
A & B & C \\
\end{bmatrix}
= 
\begin{bmatrix}
\text{ADDA} (1) & \text{BEEB} (1) & \text{CGGC} (1) \\
\text{ADDA} (2) & \text{BEEB} (2) & \text{CGGC} (2) \\
\text{ADDA} (3) & \text{BEEB} (3) & \text{CGGC} (3) \\
\text{ADDA} (4) & \text{BEEB} (4) & \text{CGGC} (4) \\
\end{bmatrix}
\]

The damping coefficient at each station, denoted by ENOX (I) is obtained from:

\[
N(\xi) = \frac{pg^2(\bar{A})^2}{\omega_e^3}
\]

where \(\bar{A}\) is the quantity designated in subroutine ADMAB as \(\text{ABAR}(I)\) and the remaining symbols have their usual meaning. The numerical integrations are performed by subroutine SIMPS with an integration interval \(H = DXL\).

Subroutine EXCITE

Subroutine EXCITE computes the excitation force and moment as discussed in Appendix B. The elemental excitation force at a given ship station is computed from equation (B 3) by splitting the above expression in two parts. The first designated \(\text{CXPST}(I)\) is in phase with \(\cos \omega_t\)
and is given by the first bracketed term of equation (B 3). The second
designates SXFST (I) is inphase with sin \omega e t and is given by the second
bracketed term of equation (B 3). The quantities \phi_1 and \phi_2 of equation
(B 3) are correspondingly referred to as FKLAM and SKLAM (I). SKLAM (I)
is conveniently subscripted for subsequent utilization in subroutine
BENDSH if the computation of bending moments is also required. The total
excitation force and moment are finally obtained from equation (B 4) by
integrating over the ship length the appropriate elemental values using
subroutine SIMPS with H = DXI.

Subroutine MOTION

Whenever the main program calls this subroutine, the excitation
terms and the coefficients of the equations of motion (B1) are already
available and hence the heaving and pitching motion amplitudes and phase
angles can readily be determined. Subroutine MOTION performs this task
by first expressing the set (B9) in the following form:

\[ \begin{align*}
\ddot{F} &= F_r + iF_i \\
\ddot{M} &= M_r + iM_i \\
P &= P_r + iP_i \\
Q &= Q_r + iQ_i \\
R &= R_r + iR_i \\
S &= S_r + iS_i
\end{align*} \]
The various products in the set (B 8) are then evaluated as shown below:

\[
\begin{align*}
\overline{MQ} &= (M_r Q_r - M_i Q_i) + i(M_r Q_i + M_i Q_r) \\
\overline{FS} &= (F_r S_r - F_i S_i) + i(F_r S_i + F_i S_r) \\
\overline{QR} &= (Q_r R_r - Q_i R_i) + i(Q_r R_i + Q_i R_r) \\
\overline{PS} &= (P_r S_r - P_i S_i) + i(P_r S_i + P_i S_r) \\
\overline{FR} &= (F_r R_r - F_i R_i) + i(F_r R_i + F_i R_r) \\
MP &= (M_r P_r - M_i P_i) + i(M_r P_i + M_i P_r)
\end{align*}
\]

so that (B 8) can finally be computed from

\[
\begin{align*}
\tilde{z} &= Z\text{REAL} - iZ\text{IMAG} \\
\tilde{\sigma} &= T\text{REAL} - iT\text{IMAG}
\end{align*}
\]

where

\[
\begin{align*}
Z\text{REAL} &= \frac{AC + BD}{C^2 + D^2} \\
Z\text{IMAG} &= \frac{AD - BC}{C^2 + D^2} \\
T\text{REAL} &= \frac{BC + FD}{C^2 + D^2} \\
T\text{IMAG} &= \frac{ED - FC}{C^2 + D^2}
\end{align*}
\]

and

\[
\begin{align*}
A &= M_r Q_r - M_i Q_i - F_r S_r + F_i S_i \\
B &= M_r Q_i + M_i Q_r - F_r S_i - F_i S_r \\
C &= Q_r R_r - Q_i R_i - P_r S_r + P_i S_i \\
D &= Q_r R_i + Q_i R_r - P_r S_i - P_i S_r \\
E &= F_r R_r - F_i R_i - M_r P_r + M_i P_i \\
F &= F_r R_i + F_i R_r - M_r P_i - M_i P_r
\end{align*}
\]

E-31
The amplitudes and phase angles of heaving and pitching motions are finally computed in accord with equations (B11) and (B12), i.e.,

\[ Z_o = \frac{\sqrt{(Z_{\text{REAL}})^2 + (Z_{\text{IMAG}})^2}}{Z_{\text{REAL}}} \]

\[ \phi_o = \frac{\sqrt{(T_{\text{REAL}})^2 + (T_{\text{IMAG}})^2}}{T_{\text{REAL}}} \]

and

\[ \delta = \arctan \left\{ -\frac{Z_{\text{IMAG}}}{Z_{\text{REAL}}} \right\} \]

\[ \epsilon = \arctan \left\{ -\frac{T_{\text{IMAG}}}{T_{\text{REAL}}} \right\} \]

The correspondence between program variables and the quantities referred to above is as follows:

\[
\begin{bmatrix}
P_r & P_i \\
F_r & F_i \\
Q_r & Q_i \\
R_r & R_i \\
M_r & M_i \\
S_r & S_i
\end{bmatrix}
\begin{bmatrix}
TR(1) & TI(1) \\
TR(2) & TI(2) \\
TR(3) & TI(3) \\
TR(4) & TI(4) \\
TR(5) & TI(5) \\
TR(6) & TI(6)
\end{bmatrix}
\]
and

\[
\begin{bmatrix}
E & F \\
C & D \\
A & B
\end{bmatrix}
= 
\begin{bmatrix}
UR(3) & UI(3) \\
UR(4) & UI(4) \\
UR(5) & UI(5)
\end{bmatrix}
\]

It should be noted that TR(2), TI(2), TR(5) and TI(5) are furnished by subroutine EXCITE.

Subroutine PARAM

In its present restricted form subroutine PARAM computes the amplitude operators of the following responses in the order indicated:

a) vertical acceleration at Station No. 1
b) relative motion at Station No. 1
c) relative velocity at Station No. 1
d) relative motion at Station No. 10
e) pitching motion

With the exception of item e) which is already available, the necessary computations make use of the equations developed in Appendix D. In particular:

a) is determined from equation (D9)
b) and d) are determined from equation (D17)
and c) is determined from equation (D20)
Subroutine BENDSH

For a complete description of this subroutine the interested reader is referred to (15) and to the more complete analysis of (7) and (8) which present the basic analytical treatment involved in the computation of ship bending moments in regular waves. Since this subroutine is at present not used by the computer program under description, we shall refrain from further details.

Subroutine STATIS

By the time this subroutine is encountered in the main computation cycle, the computer program has already calculated the sea spectrum ordinates and the amplitude operators of the responses discussed in Chapter VI. These quantities correspond to the specified frequencies of encounter for given wind speed and ship speeds respectively. The function of this subroutine is then to calculate the ordinates of the squared amplitude density spectra of the above response at the same frequencies of encounter and finally to integrate these curves numerically in order to obtain the necessary statistical information. Integration is performed by subroutine SIMPS with an integration interval of $H = \text{DELENFR}$. It should be noted that for $\omega_e = 0$, this subroutine sets to zero all ordinates of the various statistical measures of the random responses which are functions of the areas under the response density spectra as discussed in Chapter V.
Subroutine SIMPS

This subroutine is an integrator and calculates numerically various integrals and areas under curves by using Simpson's rule. At any stage of the computation cycle, the number of ordinates furnished to this subroutine must be odd. Thus any integral is evaluated in accord with:

\[ \int_{a}^{b} f(x) \, dx = \frac{H}{3} \left[ Y_1 + 4Y_2 + 2Y_3 + \ldots + 4Y_N + Y_{N+1} \right] \]

where \( Y(I) \) are the integrand ordinates and \( I \) rises from 1 to \( J \). \( Y(I), H, \) and \( J \) must be preset appropriately to ensure that \( J \) is an odd number.
Description of Input Data

The MAIN program includes seven READ statements, numbered 1001(1) 1007, whose purpose is to (a) transmit the basic input information required for the computations and (b) incorporate the control cards which decide on the path of the computation and subsequently the form in which the output information is to be given. Two input options are possible which yield correspondingly different output information. The first option is associated with the computation of seaworthiness performance whereas the second furnishes, for checking purposes, the same output as option No. 1 or Program A.

Unless specifically stated in the following, numerical data must be of the floating point type. Any consistent system of units may be used, the decision being made by the user for the input variables of the first READ statement (No. 1001). At the end of the FORTRAN or binary deck and after the *DATA card, the basic information for the computations should be given in the following order, one card per READ statement, unless otherwise stated:

(a) **READ Statement 1001**

1) $N$ = number of stations the ship is divided into.

This number is also equal to the highest station number when the latter runs from 0 at the F.P. through $N$ at the A.P. $N$ must be an **integer** and also, due to the structure of subroutine SIMPS, it must be **even**. Unless


the DIMENSION statements are increased, 
N should always be less than 20.

2) BPL = length between perpendiculards of ship 
or model.

3) GAMMA = specific weight of water

4) GRAV = gravitational acceleration

5) DISPL = displacement of ship or model

(b) **READ Statement 1002**

This statement requires a number of cards equal to $N + 1$
giving, for each station starting from the F.P., the following 
information per card:

1) BSTAR(I) = full beam (breadth) at that station
2) SECEO(I) = sectional area coefficient at that 
   station divided by the area of the 
   circumscribing rectangle at that sta-
   tion.

3) DRAFT(I) = actual depth (draft) of section at 
   that station.

c) **READ Statement 1003**

This card must always be included since it is a control card 
which dictates the computer to bypass subroutine BENDSH. The follow-
ing information is assumed known and must therefore be given:

1) RADGYR = radius of gyration of ship or model
2) XI(1) = distance of center of gravity of ship 
or model from the F.P.
d) **READ Statement 1004**

This statement is never encountered in the computations described herein; therefore no input card should be included for this statement.

e) **READ Statement 1005**

The input card to this statement must always be included since the printed information serves as a controller to the program. Ordinarily, MINKRI, MAXKRI and INCRES are input variables for subroutine BENDSH which however is never used in the present computations. For this reason, the following numerical values must be given so as to instruct the computer to bypass BENDSH and avoid related calculations:

\[
\begin{align*}
\text{MINKRI} & = -1 \\
\text{MAXKRI} & = 1 \\
\text{INCRES} & = 1
\end{align*}
\]

NN is the number of successive computations in the innermost loop of the MAIN program i.e. the number of frequencies of encounter (other than \( \omega_e = 0.0 \)) for which the ship motions, sea energy spectrum, response amplitude operator and response spectrum ordinates are to be evaluated. If motions are only desired for checking purposes, then NN must not be given. In this case the computer output of Program B is identical to that of Program A's Option No. 1 described earlier. If seaworthiness computations are only required then NN must be given and, owing to the program's structure, must be even (See also READ Statement 1006). It is of importance to note that the values of all variables appearing on this card must be integer numbers.

E-38
f) **READ Statement 1006**

The following information should be included on the input card for this statement:

1) **WA** = wave amplitude (half the wave height).
   For seaworthiness computations the value of WA must be unity.

2) **ENFRMT** = Smallest frequency of encounter, other than zero, for which the computations must be performed.

3) **ENFRMA** = largest frequency of encounter for which the computations must be performed. See note below.

4) **DEENFR** = increment of frequency of encounter.

**Note**
As may be seen from Fig. E3, the values of ENFRMA and DEENFR decide the number of successive computations in the innermost main loop of the program. This number must be equal to **NN** and must be even. Also, since the computation for \( \omega_e = 0 \) is automatically performed by subroutine STATIS, it follows that the following relationship must always be satisfied:

\[
NN = \frac{ENFRMA}{DEENFR}
\]
5) \text{VMIN} = \text{smallest ship or model speed for which the computations must be performed.}

6) \text{VMAX} = \text{largest ship or model speed for which the computation must be performed.}

7) \text{DELV} = \text{increment of ship or model speed}

g) \text{READ Statement 1007}

The input card for this statement must only be inserted in the data whenever the computations are to be made for seaworthiness considerations. If only the motions are required then, no card must be included. In the former case the following information must be printed on the card:

1) \text{WMIN} = \text{smallest wind speed (in knots) for which the computations are to be made.}

2) \text{WMAX} = \text{maximum wind speed (in knots) for which the computations are to be made.}

3) \text{DElw} = \text{increment of wind speed (in knots).}
### TYPICAL INPUT DATA LISTING

(Seaworthiness Evaluation)

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TOTAL 50
Description of Output Data

Program B was so written so as to allow the user the possibility of extracting two types of output information. With the inclusion of a proper control card (see READ statement 1005), he can either obtain the complete seaworthiness results or the intermediate computation of the ship (model) motions which are subsequently used in the statistical analysis. To this end, the output routines and formats of Program A have been left unchanged in the MAIN program and hence in this case the output listing is exactly similar to the one corresponding to input option No. 1 of Program A, as described earlier in this Appendix.

The results of the seaworthiness evaluation on the other hand are given in the output listing in the following order. At first, the ship length is printed out and then the ship speed and Froude number under examination. Depending upon the number of wind speeds involved in the calculations at one ship speed, the computer prints out the following information for each sea condition:

(1) the frequency of encounter, OMEGAE (I)

and the corresponding

(2) modified sea energy spectrum ordinate, SPECM (I)

(3) squared amplitude of vertical acceleration at the slamming station, RAOSDD (I)

(4) squared amplitude of the relative motion at the slamming station, RAORB (I)

(5) squared amplitude of the relative velocity of the slamming station, RAORDB (I)
(6) squared amplitude of the relative motion at
the propeller station, RAORP (I)

(7) squared amplitude of the pitching motion,
RAOTHE (I)

These are arranged for convenience in columns as shown in the
typical output listing at the end of this section. Since all the above
responses have been computed for a unit-amplitude wave excitation, their
squared amplitudes correspond to the discrete ordinates of the "response
operator" curves. The density spectra ordinates of the above responses
are then printed out in a similar fashion. For easy reference the
encounter frequency and sea spectrum ordinates are reprinted while the
spectrum ordinates of the various responses are listed in the same
column as the corresponding amplitude operator ordinates.

Finally the relevant statistical information obtainable from
these curves is printed out in the Hollerith characters in the same
order as items (3) through (7) listed above. For each response, the
area and the square root of the area are printed as well as the average,
significant and average of one-tenth highest amplitudes. The output
described above is repeated for any wind speed within a given ship speed
of an examined ship.
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**TYPICAL OUTPUT DATA LISTING (Seaworthiness Evaluation)**

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SLAMMING STATION MEAN SQUARED ACCELERATION = 33.069 FT/SEC/SEC.

SQUARE ROOT OF ABOVE QUANTITY = 5.751

SLAMMING STATION AVERAGE ACCELERATION = 4.980 FT/SEC/SEC.

SLAMMING STATION SIGNIFICANT ACCELERATION = 8.137 FT/SEC/SEC.

SLAMMING STATION ONE TENTH AVERAGE ACCELERATION = 10.351 FT/SEC/SEC.

SLAMMING STATION MEAN SQUARED RELATIVE MOTION = 149.327 FEET.

SQUARE ROOT OF ABOVE QUANTITY = 12.220

SLAMMING STATION AVERAGE RELATIVE MOTION = 10.582 FEET.

SLAMMING STATION SIGNIFICANT RELATIVE MOTION = 17.291 FEET.

SLAMMING STATION ONE TENTH AVERAGE RELATIVE MOTION = 21.996 FEET.

SLAMMING STATION MEAN SQUARED RELATIVE VELOCITY = 135.722 FEET/SEC.

SQUARE ROOT OF ABOVE QUANTITY = 11.650

SLAMMING STATION AVERAGE RELATIVE VELOCITY = 10.089 FEET/SEC.

SLAMMING STATION SIGNIFICANT RELATIVE VELOCITY = 16.485 FEET/SEC.

SLAMMING STATION ONE TENTH AVERAGE RELATIVE VELOCITY = 20.970 FEET/SEC.

PROPELLER STATION MEAN SQUARED RELATIVE MOTION = 49.790 FEET.

SQUARE ROOT OF ABOVE QUANTITY = 7.056

PROPELLER STATION AVERAGE RELATIVE MOTION = 6.111 FEET.

PROPELLER STATION SIGNIFICANT RELATIVE MOTION = 9.985 FEET.

PROPELLER STATION ONE TENTH AVERAGE RELATIVE MOTION = 12.701 FEET.

PITCH MEAN SQUARED AMPLITUDE = 3.531 DEG.

SQUARE ROOT OF ABOVE QUANTITY = 1.879

PITCH AVERAGE AMPLITUDE = 1.627 DEG.

PITCH SIGNIFICANT AMPLITUDE = 2.659 DEG.

PITCH ONE TENTH AVERAGE AMPLITUDE = 3.382 DEG.
THIS PROGRAM COMPUTES SEAWORTHINESS STATISTICAL PARAMETERS FOR A SHIP MOVING IN AN AHEAD IRREGULAR NEUMANN SEAWAY, BY EMPLOYING THE KORVIN-KROUKOVSKY LINEAR THEORY OF HEAVING AND PITCHING MOTIONS IN REGULAR WAVES ALTERNATIVELY, BY THE INCLUSION OF A CONTROL CARD, IT COMPUTES ONLY SHIP MOTIONS AND, IF REQUIRED, THE SHEARING FORCE AND BENDING MOMENT AT ANY HULL STATION.

MAIN PROGRAM

DIMENSION Y(81), TR(6), TI(6), ADDA(4), BEEB(4), CGGC(4), UR(6), UI(6),
1DMASS(21), QUANT(21), SKLAM(21), BSTAR(21), CXFST(21), SXFST(21),
2CTFST(21), STFST(21), XI(21), DIX(21), ENOXI(21), DRAFT(21), DWEIGH(21),
3SECOE(21), ABAR(21), SPECM(81), VOMEGA(81), RAOSSDD(81), RAORB(81),
4RAORDB(81), RAORP(81), RAOTHE(81), RSDSDD(81), RSDRDB(81),
5RSDRP(81), RSDTHE(81), ZREAL, ZIMAG, TREAL, TIMAG, ZNULL, 1TNULL, DELTA, EPSIL, TR, TI, V1, DPMASS, QUANT, OMEGAE, SKLAM, KRIT, RO, GRAV,
2BSTAR, CXFST, SXFST, ALPHA, SHNULL, XI, BETA, BMNULL, GAMMA, DIX, M, WA, WAVE
3, CW, ENOXI, SIGMA, TAU, FNNULL, EMNULL, DRAFT, DWEIGH, SECOE, TMAC, N, JR, UI,
4ABAR, PI1, SHREAL, SHIMAG, BMREAL, BMIMAG, YNERT, VOITH, RIZA, OMEGA, MM, NN, H
5, KK, W, VOMEGA, SPECM, RAOSSDD, RAORB, RAORDB, RAORP, RAOTHE, RSDRDB, RSDSDD,
6RSDRPB, RSDRP, RSDTHE, RSDSDD, RSTRP, RSTRDB, RSTTHE, HELP1, HELP2,
7HELP3, HELP4, HELP5, AVESDD, AVERB, AVERDB, AVERP, AVETHE, SIGSDD, SIGSRB,
8SIGRDB, SIGRP, SIGTHE, AVHSDD, AVHRB, AVHRDP, AVHRP, AVHTHE, CONST, GSQUR,
9DEENFR
PI = 3.1415926

READ IN SHIP CHARACTERISTICS

1001 READ 900, N, BPL, GAMMA, GRAV, DISPL
M = N + 1
1002 READ 901, (BSTAR(I), SECOE(I), DRAFT(I), I = 1, M)
1003 READ 905, RAGYR, XI(I)
RO = GAMMA / GRAV
GSKUR = GRAV * GRAV
CONST = 0.001654 * GSKUR * PI * PI
FN = N
DXI = BPL / FN
TMSS = DISPL / GRAV
J = M
H = DXI

CALCULATION OF STATION COORDINATES AND MOMENT OF INERTIA

IF (RAGYR) 2, 2, 5
THE NEXT 10 INSTRUCTIONS ARE NOT USED IN THE PRESENT WORK

2 DO 3 I = 1, M
1004 READ 902, DWEIGH(I)
PRINT 902, DWEIGH(I)
3 DMass(I) = DWEIGH(I) / GRAV
HOMENT = 0.0
DO 4 I = 1, M
L = I - 1
FL = L
4 HOMENT = HOMENT + DWEIGH(I) * FL
E-47
XI(I)=DXI*HOMENT/DISPL

C

5 DO 6 I=2,M
L=I-1
FL=L
6 XI(I)=XI(I)-DXI*FL
IF (RADGYR) 7,7,9
C

THE NEXT 5 INSTRUCTIONS ARE NOT USED IN THE PRESENT WORK

7 YNERT=0,0
DO 8 I=1,M
8 YNERT=YNERT+DWEIGH(I)*XI(I)*XI(I)+(DXI*DXI/12.)
GO TO 1005
9 YNERT=DISPL*RADGYR*RADGYR
C

READ IN NN,WAVE AMPLITUDE AND ENCOUNTER FREQUENCY,SHIP SPEED,
WIND SPEED AND B.M. STATION RANGES

C

1005 READ 907, MINKRI,MAXKRI,INCRES,NN
1006 READ 906, WA,ENFRMI,ENFRMA,DEENFR,VMIN,VMAX,DELV
IF (NN) 1007,10,1007
1007 READ 901, WMIN,WMAX,DELW
C

PRINTING INSTRUCTIONS

C

10 PRINT 900, N,BPL,GAMMA,GRAV,DISPL
PRINT 901,(BSTAR(I),SECOE(I),DRAFT(I),I=1,M)
PRINT 905, RADGYR,XI(I)
PRINT 907, MINKRI,MAXKRI,INCRES,NN
PRINT 906, WA,ENFRMI,ENFRMA,DEENFR,VMIN,VMAX,DELV
IF (NN) 11,12,11
11 PRINT 901, WMIN,WMAX,DELW
C

V=VMIN
12 FROUDE=V/SQRTF(GRAV*BPL)
IF (NN) 14,16,14
C

PRINTING INSTRUCTIONS

C

14 PRINT 923, BPL
VP=V/1.689
PRINT 924, VP,FROUDE
C

W=VMIN
15 KK=2
16 OMEGAE=ENFRMI
17 IF (V) 19,18,20
18 OMEGA=OMEGAE
GO TO 21
19 CONTINUE
20 RIZA=1.0+((4.0*OMEGAE*V)/GRAV)
OMEGA=(1.0-SQRTF(RIZA))/((-2.0*V)/GRAV)
21 VOITH=OMEGA*OMEGA
WAVEN=VOITH/GRAV
CW=SQRTF(GRAV/WAVEN)
WL=(2.0*PI*GRAV)/VOITH
IF (NN) 22,23,22

E-48
EXECUTION OF MAIN COMPUTATION CYCLE

22 CALL SPECTR
23 CALL ADMAB
 CALL COEFF
   IF (NN) 25, 24, 25

PRINTING INSTRUCTIONS

24 PRINT 909
 PRINT 903, WL, V, OMEGA E
 PRINT 910
 PRINT 903, (ADDA(I), BEEB(I), CGGC(I), I=1,4)

25 CALL EXCITE
   IF (NN) 27, 26, 27

PRINTING INSTRUCTIONS

26 PRINT 911
   Ftwo=TI(2)
 PRINT 904, FNULL, TR(2), Ftwo, SIGMA
 PRINT 912
 EMTWO=-TI(5)
 PRINT 904, EMNULL, TR(5), EMTWO, TAU

27 CALL MOTION
   IF (NN) 28, 29, 28
28 CALL PARAM
   GO TO 33

PRINTING INSTRUCTIONS

29 PRINT 913
 PRINT 904, ZNULL, ZREAL, ZIMAG, DELTA
 PRINT 914
 PRINT 904, TNULL, TREAL, TIMAG, EPSIL

30 L=(N/2)+1
 DELTA=DELTA*.57*295779
 EPSIL=EPSIL*.57*295779
 TNULL=TNULL/(WA*WAVEN)
 ZNULL=ZNULL/WA
 DWL=WL/BPL

PRINTING INSTRUCTIONS

PRINT 917
 PRINT 918
 PRINT 903, DWL, FROUDE, OMEGA E
 PRINT 919
 PRINT 904, ZNULL, DELTA, TNULL, EPSIL
 IF (MINKRI) 33, 33, 30

THE NEXT 18 INSTRUCTIONS ARE NOT USED IN THE PRESENT WORK

30 KRIT=MINKRI
31 CALL BENDSH
PRINT 921
PRINT 922, KRIT
PRINT 915
PRINT 904, SHNULL, SHREAL, SHIMAG, ALPHA
PRINT 916
PRINT 904, BMNULL, BMREAL, BMIMAG, BETA
   L=N/2
SHNULL=SHNULL*BPL/(GAMMA*(BSTAR(L)*WA)**2)
BMNULL=BMNULL/(GAMMA*WA*BSTAR(L)*BPL*BPL)
ALPHA=ALPHA+57.295779
BETA=BETA+57.295779

C
PRINT 920.
PRINT 904, SHNULL, ALPHA, BMNULL, BETA

C
IF (KRIT=MAXKRI) 32,33,32
32 KRIT=KRIT+INCRE
   GO TO 31

C
IF (OMEGAE=ENFRMA) 34,35,35
34 VOMEGA(KK)=OMEGAE
   OMEGAE=OMEGAE+DEENFR
   KK=KK+1
   GO TO 17
35 IF (NN) 36,38,36
36 CALL STATIS

C

PRINTING INSTRUCTIONS

PRINT 925, W
PRINT 926
PRINT 927, (VOMEGA(KK), SPECM(KK), RAOSDD(KK), RAOORB(KK), RAOORDB(KK), R1ORP(KK), RAOORH(KK), KK=1, MM)
PRINT 928
PRINT 927, (VOMEGA(KK), SPECM(KK), RSDSDD(KK), RSDRB(KK), RSDRDB(KK), R1SDP(KK), RSDORH(KK), KK=1, MM)
PRINT 929, RSTSDD
PRINT 930, HELP1
PRINT 931, AVESDD
PRINT 932, SIGSDD
PRINT 933, AVHSDD
PRINT 934, RSTRB
PRINT 930, HELP2
PRINT 935, AVERB
PRINT 936, SIGRB
PRINT 937, AVHRB
PRINT 938, RSTRDB
PRINT 930, HELP3
PRINT 939, AVERDB
PRINT 940, SIGRDB
PRINT 941, AVHRDB
PRINT 942, RSTRP
PRINT 930, HELP4
PRINT 943, AVERP
PRINT 944, SIGRP
PRINT 945, AVHRP
PRINT 946, RSTTBE
PRINT 930, HELP5
PRINT 947, AVETHE
PRINT 948,SIGTHE
PRINT 949,AVHTHE
C
IF (W-WMAX) 37,38,37
37 W=W+DELW
GO TO 15
38 IF(ABS(F(V)-ABS(F(VMAX))) 39,40,40
39 V=V+DELV
GO TO 13
40 CONTINUE
GO TO 1001
C
INPUT-OUTPUT FORMAT SPECIFICATIONS
C
900 FORMAT(110,4F10.4)
901 FORMAT(3F10.4)
902 FORMAT(F10.4)
903 FORMAT(3F20.3)
904 FORMAT(4F20.3)
905 FORMAT(2F20.4)
906 FORMAT(7F10.4)
907 FORMAT(4110)
909 FORMAT(61H)
1MEGA(E)
910 FORMAT(58H)
1GC)
911 FORMAT(78H)
WL
ADD
912 FORMAT(78H)
V
BEEB
913 FORMAT(78H)
O
CG
914 FORMAT(78H)
1-TERM
915 FORMAT(78H)
EXCITING-FORCE
916 FORMAT(78H)
EXCITING-MOMENT
917 FORMAT(47H)
PHASE)
918 FORMAT(60H)
THE SAME QUANTITIES IN DIMENSIONLESS FORM)
919 FORMAT(78H)
OM
920 FORMAT(78H)
FROUDE
921 FORMAT(21H)
1EGAE)
922 FORMAT(120)
1ITCH
923 FORMAT (15H1SHIP LENGTH = F7.2X*6H FEET*)
924 FORMAT (1H1,51HTHE FOLLOWING CALCULATIONS ARE FOR A SHIP SPEED, V=
925 FORMAT (13H1WIND SPEED =F10.2X*7H KNOTS*)
926 FORMAT (1H0,7H OMEGAE,7X,5HSPECM,6X,6HRAOSDD,6X,5HRAORB,6X,6HRAORD
927 FORMAT (7(F8.3X))
928 FORMAT (1H0,7H OMEGAE,7X,5HSPECM,6X,6HRSDD,6X,5HRSRB,6X,6HRSRD
929 FORMAT (45HOSLAMMING STATION MEAN SQUARED ACCELERATION =F8.3,12H F
930 FORMAT (32H SQUARE ROOT OF ABOVE QUANTITY =F6.3)
E-51
931 FORMAT (4OH SLAMMING STATION AVERAGE ACCELERATION =F8.3,12H FT/SEC 1/SEC)  
932 FORMAT (44H SLAMMING STATION SIGNIFICANT ACCELERATION =F8.3,12H FT 1/SEC/SEC)  
933 FORMAT (50H SLAMMING STATION ONE TENTH AVERAGE ACCELERATION =F8.3,  
112H FT/SEC/SEC)  
934 FORMAT (48HOSLAMMING STATION MEAN SQUARED RELATIVE MOTION =F8.3,6H  
1 FEET)  
935 FORMAT (43H SLAMMING STATION AVERAGE RELATIVE MOTION =F8.3,6H FEET  
1)  
936 FORMAT (47H SLAMMING STATION SIGNIFICANT RELATIVE MOTION =F8.3,6H  
1FEET)  
937 FORMAT (53H SLAMMING STATION ONE TENTH AVERAGE RELATIVE MOTION =F8  
1.3,6H FEET)  
938 FORMAT (50HOSLAMMING STATION MEAN SQUARED RELATIVE VELOCITY =F8.3,  
110H FEET/SEC)  
939 FORMAT (45H SLAMMING STATION AVERAGE RELATIVE VELOCITY =F8.3,10H F  
1EET/SEC)  
940 FORMAT (49H SLAMMING STATION SIGNIFICANT RELATIVE VELOCITY =F8.3,1  
10H FEET/SEC)  
941 FORMAT (55H SLAMMING STATION ONE TENTH AVERAGE RELATIVE VELOCITY =  
1F8.3,10H FEET/SEC)  
942 FORMAT (49HOPROPELLER STATION MEAN SQUARED RELATIVE MOTION =F8.3,6  
1H FEET)  
943 FORMAT (44H PROPELLER STATION AVERAGE RELATIVE MOTION =F8.3,6H FEET  
1)  
944 FORMAT (48H PROPELLER STATION SIGNIFICANT RELATIVE MOTION =F8.3,6H  
1 FEET)  
945 FORMAT (54H PROPELLER STATION ONE TENTH AVERAGE RELATIVE MOTION =F  
18.3,6H FEET)  
946 FORMAT (31HOPITCH MEAN SQUARED AMPLITUDE =F6.3,5H DEG)  
947 FORMAT (26H PITCH AVERAGE AMPLITUDE =F8.3,5H DEG)  
948 FORMAT (30H PITCH SIGNIFICANT AMPLITUDE =F8.3,5H DEG)  
949 FORMAT (36H PITCH ONE TENTH AVERAGE AMPLITUDE =F8.3,5H DEG)  
END  

TOTAL 332*
LIST8

SUBROUTINE SPECTR
DIMENSION Y(81),TR(6),TI(6),ADDA(4),BEEB(4),CGGC(4),UR(6),UI(6),
1DMASS(21),QUANT(21),SKLAM(21),BSTAR(21),CXFST(21),SXFSF(21),
2CFTFST(21),STFST(21),XI(21),DIX(21),ENOXI(21),DRAFT(21),DWEIGH(21),
3SECOE(21),ABAR(21),SPEC(81),VOMEGA(81),ROASDD(81),RAORB(81),
4RAORDB(81),RAORP(81),ROAOY(81),RSDSDD(81),RSDRD(81),RSDRDB(81),
5RSDRP(81),RSTDRE(81),COMON(21),SYMS,DXI,ADDA,BEEB,CGGC,ZREAL,ZIMAG,ZREAL,TIMAG,ZNULL,
1TNULL,DDELTA,EPST,TR,TV,DMASS,QUAN,OMEGA,SKLAM,KRIT,RO,GRAV,
2BSTAR,CXFST,SXFSF,ALPHA,SHNULL,XXI,BETA,BAHNULL,OMEGA,D,WA,WAVEN
3,CW,ENOXI,SIGMA,TAU,FNULL,EHNULL,DRAFT,DWEIGH,SECOE,TMASS,N,UR,UI,
4ABAR,P1,SHREAL,SHREAL,SHREAL,SHREAL,SHREAL,SHREAL,SHREAL,SHREAL,SHREAL,
5VOMEGA,ROASDD,ROASDD,ROASDD,ROASDD,ROASDD,ROASDD,ROASDD,ROASDD,
6RSDRD,RSDRD,RSDRD,RSDRD,RSDRD,RSDRD,RSDRD,RSDRD,RSDRD,
7HELP3,HELP3,HELP3,HELP3,HELP3,HELP3,HELP3,HELP3,HELP3,
8SIGRAD,SIGRAD,SIGRAD,SIGRAD,SIGRAD,SIGRAD,SIGRAD,SIGRAD,SIGRAD,
9AVHSDD,AVHRD,AVHRD,AVHRD,AVHRD,AVHRD,AVHRD,AVHRD,AVHRD,
10AVHTHE,AVHTHE,AVHTHE,AVHTHE,AVHTHE,AVHTHE,AVHTHE,AVHTHE,AVHTHE,
11CONST,GSQUAR
U=W*1.689
POWER=((-2.0*GSQUAR)/(VOITH*U))
IF (V) 200,200,201
200 SPEC(KK)=(CONST*EXPF(Power))/((OMEGA)**6)
GO TO 290
201 DIVIDE=SQRTF(RIZA)
SPEC(KK)=(CONST*EXPF(Power))/((OMEGA)**6)*DIVIDE
290 RETURN
END
* LIST 8

SUBROUTINE ADMAB

DIMENSION Y(81), TR(6), TI(6), ADDA(4), BEEB(4), CGGC(4), UR(6), UI(6),
1 MPA(21), QUANT(21), SCLAM(21), BSTAR(21), CXSTF(21), SXSTF(21),
2 CTSTF(21), STSF(21), XI(21), DIX(21), ENOXI(21), DRAFT(21), DWEIGH(21),
3 SECOE(21), ABAH(21), SPECM(81), VOMEGA(81), RAOSDD(81), RAORB(81),
4 RAORDB(81), RAORP(81), RAOIN(81), RDSDD(81), RSDRB(81), RSDRB(81),
5 RSDRP(81), RSDTHE(81)

DIMENSION SY(10), SZ(10), SSB(10), SPB(10), SDB(10), SSA(10), SPA(10),
0 DIMENSION SLW(10), SDA(10), EPA(5*6), EQA(5*6), EPB(5), EPC(5), EPX(5),
0 DIMENSION EQX(5), EQY(5)

COMMON YJ, SYMP, DXI, ADDA, BEEB, CGGC, ZREAL, ZIMAG, TREAL, TIMG, ZNULL,
0 TNULL, DELTA, EPSIL, TR, TI, VDMAS, QUANT, OMEGAE, SKLAM, KRT, RO, GRAV,
2 BSTAR, CXSTF, SXSTF, ALPHA, SHNULL, XI, BETA, BMNULL, GAMMA, DIX, M, WA, WAVE,
3, CW, ENOXI, SIGMA, TAU, FNULL, EMNULL, DRAFT, DWEIGH, SECOE, TMASS, N, UR, UI,
4, ABAH, PI, SHREAL, SHIMAG, BMREAL, BMIMAG, YMERT, VOITH, RIZA, OMEGA, MM, NN, H,
5, KK, LW, VOMEGA, SPECM, RAOSDD, RAORB, RAORDB, RAORP, RAOIN, RDSDD,
6 RSDRB, RSDRP, RSDTHE, RSDRD, RSDRD, RSRD, RSRD, RSTH, HELP1, HELP2,
7, HELP3, HELP4, HELPS, AVESDD, AVERB, AVERDB, AVERP, AVETHE, SIGSDD, SIGDB,
8 SIGRDB, SIGRP, SIGTHE, AVHSSD, AVHRB, AVHRDD, AVHRP, AVHTHE, CONST, GQUAR,

DO 7499 I = 1, SN

IF (SFRPA) 7001, 7001, 7002

7001 QUANT(I) = 0.0
0 ABAH(I) = 0.0
0 GO TO 7499

7002 SBBB = SECOE(I)
0 SBB = BSTAR(I)
0 SBH = BSTAR(I)/(2*DRAFT(I))
0 SFRPB = SFRPA

7003 SAN = 3.14159 + (SBBB * 4.0 - 3.14159) * SBH / (SBH + 1.0) ** 2
0 SWA = 5.5165 - 1.57078 * SAN
0 SAZ = (2.35619 + SQRTF(SWA)) / SAN
0 SA = (SBH + 1.0) * SAZN / (SBH + 1.0)
0 SB = SAZN - 1.0
0 SW = SFRPA / (1.0 + SA + SB)
0

8003 SY = SFRPA
0 SBB = 3.14159 * SINF(SY)
0 SSA = SINF(SY) * LOGF(1.781 * SY) - 1.57078 * COSF(SY) - SY
0 SSO = SSA + 0.30556 * SY ** 3 - 0.01903 * SY ** 5
0 SFPI = 0.0
0 SFQI = 0.0
0 SQ = -0.05236
0 SWF = 0.0
0 SLWM = 0.0
0 DO 8004 LS = 1, 10
0 SLS = L5
0 SSL = LSL * 0.15708
0 SNL = SINF(SSL)
0 SN3L = SINF(3.0 * SSL)
0 SY(LS) = SW**((1.0 + SA) * COSF(SSL) + SB * COSF(3.0 * SSL))
0 SLS = SW**((1.0 - SA) * SNL - SB * SN3L)
0 SEZ = 3.14159 / EXPF(SZ(LS))
0 SSB(LS) = SEZ * SINF(SY(LS))
0 SPB(LS) = SEZ * COSF(SY(LS))
0 SDB(LS) = SBBL(LS) - SBBQ(1.0 - SLS) / (10.0)
0 SYZ = SY(LS) * SY(LS)
0 SYZ = SYZ * SY(LS)
0 SYZ = SY(LS) * (SZ(LS) - S2Z(LS))
0 SYZ = SY(LS) * S2Z(LS)
0 SYZ = SY(LS) * SZ(LS)

E-54
SLOG = 0.31831*LOG(F(1.781*SQRT(SY2+SZ2)))
STAN = 0.50*0.31831*ATANF(SZ(LS)/SY(LS))
SSAT(LS) = SSB(LS)*SLOG-SPB(LS)*STAN-SY(LS)*(1.0+0.91667*SZ2)
SSA(LS) = SSA(LS)+SY3*(0.30556+0.01903*(1.0+0.30556+0.01903*(SY2-SZ2)))
SPA(LS) = SPA(LS)+SZ3*(0.30556+0.01903*(SZ2-10.0*SY2))
SPA(LS) = SPA(LS)+SY2*(0.30556+0.08681*SZ(LS)+0.01903*(SZ2-10.0*SY2))
SPA(LS) = SPA(LS)+SYZ*(0.89514*SY3)-0.75*SZ2
SPA(LS) = SPA(LS)+SY2*(0.75-0.08681*SY2+0.52083*SZ2)
SDD(LS) = SSA(LS)-SSA0*(1.0-LS/10.0)
SQ = -SQ
SM = (1.0+SA)*SNSL+3.0*SB*SN3SL)*(1.5708+SQ)
SFQ1 = SFQ1+SPB(LS)*SFM
SPF1 = SPF1+SPA(LS)*SFM
SWF = SWF+2FM/EXPF(SZ(LS)*SFRPB/SFRPA)
SLWN = SFM*SEZ/(16.28318+40.0*SQ)
SLW(LS) = SLWM+SLWN
SLWM = SLW(LS)+SLWN

8004 CONTINUE
DO 8010 LS=1,9
SL = LS
8010 SLW(LS) = SLW(10)*SLS/10.0-LSLW(LS)
SFQ1 = SFQ1-0.50*SPB(10)*SFM
SPF1 = SPF1-0.50*SPA(10)*SFM
SWF = SWF-0.50*SPB(10)*SFM
EPA(1,1) = SSA0
EQA(1,1) = SSB0
EPC(1) = SLW(10)
SQ = 0.05236
DO 8005 KS=2,5
EPA(KS,1) = 0.0
EQA(KS,1) = 0.0
EPC(KS) = 0.0
SK = (KS-1)*2
SQ = 0.05236
DO 8005 MS=1,9
SQ = -SQ
SM = MS
SMSN = 1.27324*(0.15708+SQ)*SINF(SK*SM*0.15708)
EPA(KS,1) = EPA(KS,1)+SDD(1)*SMSIN
EQA(KS,1) = EQA(KS,1)+SDB(1)*SMSIN
EPC(KS) = EPC(KS)+SLW(1)*SMSIN

8005 CONTINUE
SAA = SAA+SAA+3.0*SB
SASA = SAA+SAA+3.0*SA+SB
EPA(1,2) = -SW
EPA(2,2) = -1.0-0.21221*SW
EPA(3,2) = -SA-0.02122*SW
EPA(4,2) = -SAA-0.00606*SW
EPA(5,2) = -SAAA-0.00253*SW
EPA(1,3) = -0.33333*SW
EPA(2,3) = -0.38197*SW
EPA(3,3) = -1.0-0.13642*SW
EPA(4,3) = -SA-0.02358*SW
EPA(5,3) = -SAA-0.00868*SW
EPA(1,4) = 0.20*SW
EPA(2,4) = 0.15158*SW
EPA(3,4) = 0.17684*SW
EPA(4,4) = 1.0-0.09646*SW

E-55
EPA(5,4) = -SA - 0.02040 * SW
EPA(1,5) = -0.1429 * SW
EPA(2,5) = 0.09903 * SW
EPA(3,5) = 0.06752 * SW
EPA(4,5) = 0.11427 * SW
EPA(5,5) = -1.0 - 0.07428 * SW
EPB(1) = 1.0 + SA + SB
EPB(2) = 0.63662 * (0.33333 * (1.0 + SA) - 1.80 * SB)
EPB(3) = 0.31831 * (0.0667 + 0.0667 * SA + 1.28571 * SB)
EPB(4) = 0.63662 * (0.00952 + 0.00952 * SA + 0.11111 * SB)
EPB(5) = 0.31831 * (0.00793 + 0.00793 * SA + 0.008182 * SB)
DO 8006 KS = 1, 5
DO 8006 LS = 2, 5

8006 EQA(KS, LS) = 0.0
NEQ = 5

9903 IEPB = 7070
NEP = NEQ + 1
DO 9933 IEO = 1, NEQ
DO 9948 LEQ = 1, NEQ

9948 EQA(LEQ, NEP) = 0.0
EPA(IEQ, NEP) = 0.0
IEQY = 1

IF (EPA(IEQ, 1) > 9934, 9931, 9934)

9931 IF (EQA(IEQ, 1) > 9934, 9910, 9934)

9934 EQP = EPA(IEQ, 1) * EPA(IEQ, 1) + EQA(IEQ, 1) * EQA(IEQ, 1)
EPT1 = EPA(IEQ, 1)
EQT1 = EQA(IEQ, 1)
DO 9935 JEQ = 1, NEP
EPT = (EPA(IEQ, JEQ) * EPT1 + EQA(IEQ, JEQ) * EQT1) / EQP
EQT = (EQA(IEQ, JEQ) * EPT1 - EPA(IEQ, JEQ) * EQT1) / EQP
EPA(IEQ, JEQ) = EPT

9935 EQA(IEQ, JEQ) = EQT
IEQX = 0
IEQY = 2
IF (IEQ-NEQ) > 9937, 9938, 9910

9938 MEQX = IEQ - 1
MEQY = 1
GO TO 9939

9937 MEQY = IEQ + 1
MEQX = NEQ

9939 DO 9940 LEQ = MEQY, MEQX
IEQX = 1, IEQX + 1
EQP = EPA(LEQ, 1)
EQP = EQA(LEQ, 1)
DO 9940 JEQ = 1, NEP
EPT = EPA(IEQ, JEQ) * EQP - EQA(IEQ, JEQ) * EPQ
EQT = EPA(IEQ, JEQ) * EPQ + EQA(IEQ, JEQ) * EQP
EPA(LEQ, JEQ) = EPA(LEQ, JEQ) - EPT

9940 EQA(LEQ, JEQ) = EQA(LEQ, JEQ) - EQT
IEQY = 3
IF (IEQ-1) > 9910, 9944, 9945

9945 IF ((NEQ-1) - IEQX > 9910, 9944, 9938

9944 DO 9946 LEQ = 1, NEQ
DO 9946 JEQ = 1, NEQ
NEQ = JEQ + 1
EPA(LEQ, JEQ) = EPA(LEQ, NEQ)

9946 EQA(LEQ, JEQ) = EQA(LEQ, NEQ)
9933 CONTINUE

E-56
DO 9953 IEQ=1,NEQ
   EPX(IEQ)=0.0
   EQX(IEQ)=0.0
   EPY(IEQ)=0.0
   EQY(IEQ)=0.0
DO 9953 JEQ=1,NEQ
   EPX(IEQ)=EPX(IEQ)+EPA(IEQ,JEQ)*EPB(JEQ)
   EQX(IEQ)=EQX(IEQ)+EQA(IEQ,JEQ)*EPB(JEQ)
   EPY(IEQ)=EPY(IEQ)+EPA(IEQ,JEQ)*EPC(JEQ)
   EQY(IEQ)=EQY(IEQ)+EQA(IEQ,JEQ)*EPC(JEQ)
9953 CONTINUE
GO TO 8009
9906 FORMAT (81H THIS SUBROUTINE ADMAB IS NOT ABLE TO FIND THE ADDED MAS
     IS AND DAMPING COEFFICIENT)
9910 PRINT 9936
GO TO 7499
8009 SF10=9.0*SB*(0.2-0.14286*SA-0.03704*SAA-0.01818*SAAA)
   SF10=SF10-((1.0+SA)*(0.3333+0.06667*SA+0.02837*SAA+0.01587*SAAA))
   SF1=SF10-SW*(1.0+SA)*78540
   SF20=-(1.0+SA)*(0.06667+0.02837*SA+0.01587*SAA)
   SF20=SF20-9.0*SB*(0.14286+0.03704*SA+0.01818*SAAA)
   SF2=SF20-SW*0.78540*SB
   SF3=-(1.0+SA)*(0.02837+0.01587*SA)-9.0*SB*(0.03704+0.01818*SA)
   SF4=-(1.0+SA)*0.01587-9.0*SB*0.01818
   SPF=EPX(1)*SFP1-EQX(1)*SFQ1+EPX(2)*SF1+EPX(3)*SF2+EPX(4)*SF3
   SPF=SPF+EPX(5)*SF4
   SC=SPF/(0.7854*0.01818+SAR)**2
   SAR=3.14159*SW*SQRF(EPX(1)**2+EQX(1)**2)
   QUANT(I)=SC*(PI*(BSTAR(I)**2)*RO/B])**2
9999 ABAR(I)=SAR
7499 CONTINUE
RETURN
END
LIST8

SUBROUTINE COEFF

* Dimensions and COMMON statements

COMMON Y, SYMPS, DXI, ADDA, BEEB, CGGC, ZREAL, ZIMAG, TREAL, TIMAG, ZNULL, 
  1, TNULL, DELTA, EPSIL, TR, TI, V, DMASS, QUANT, OMEGAE, SKLAM, KRT, RO, GRAY, 
  2, BSTAR, CXFST, SXFST, ALPHA, SHNULL, XI, BETA, BMNULL, GAMMA, DIX, M, WA, WAVEN 
  3, CW, ENOXI, SIGMA, TAU, FNUL, EMNULL, DRAFT, DWEIGH, SECOE, TMASS, N, URU, 
  4, ABAR, PI, SHREAL, SHIMAG, BMREAL, BMIMAG, YNERT, VOITH, RIZA, OMEGA, MM, NN, H 
  5, KK, W, VOMEKA, SPECM, RAOSSD, RAOB, RAOQDB, RAOQP, RAOQTH, RSDDR, RSDSD, 
  6, RRSRDDB, RSRDP, RSDTAE, RSTDD, RSTRP, RSTRD, RSTRB, RSTDB, RSTQ, HELP1, HELP2, 
  7, HELP3, HELP4, HELP5, AVESS, AVERB, AVERDB, AVERP, AVETHE, SIGSDD, SIGRB, 
  8, SIGRB, SIGRP, SIGTHE, AVHSSD, AVHRB, AVHRDP, AVHTP, AVHTHE, CONST, GSQUAR 

J = M

H = DXI

SMALL A = ADDA(1)
DO 10 I = 1, M

10 Y(I) = QUANT(I)
CALL SIMPS
ADMS = SYMPS
ADDA(1) = ADMS + TMSS
C
CAPITAL A = ADDA(4)
DO 21 I = 1, M

21 Y(I) = QUANT(I) * (XI(I) ** 2)
CALL SIMPS
ADDA(4) = SYMPS + YNERT/GRAY
C
SMALL D = ADDA(3) = CAPITAL D = ADDA(2)
DO 30 I = 1, M

30 Y(I) = QUANT(I) * XI(I)
CALL SIMPS
ADDA(2) = SYMPS
ADDA(3) = ADDA(2)
C
SMALL B = BEEB(1)
DO 40 I = 1, M

ENOXI(I) = (GAMMA * GRAY * (ABAR(I) ** 2)) / (OMEGAE ** 3)

40 Y(I) = ENOXI(I)
CALL SIMPS
BEEB(1) = SYMPS
C
CAPITAL B = BEEB(4)
DO 50 I = 1, M

50 Y(I) = ENOXI(I) * (XI(I) ** 2)
CALL SIMPS
BEEB(4) = SYMPS
DIX(I) = QUANT(2) / (2 * DXI)
DIX(M) = QUANT(N) / (2 * DXI)
DO 55 I = 2, N

55 DIX(I) = (QUANT(I-1) - QUANT(I+1)) / (2 * DXI)
DO 56 I = 1, M

56 Y(I) = DIX(I) * XI(I) * XI(I)
CALL SIMPS
BEEB(4) = BEEB(4) - ABSF(V1 * (2 * ADDA(2) * SYMPS)
C
CAPITAL E = BEEB(3)
DO 60 I = 1, M

60 Y(I) = DIX(I) * XI(I)
CALL SIMPS
ETHRE = SYMPS * (ABSF(V))
DO 63 I = 1, M

E-58
63  \( Y(I) = \text{ENOXI}(I) \times X(I) \) 
CALL SIMPS
EONE = SYMPS
ETWO = 2.0 * (ABS(\text{V})) * ADMS
BEEB(3) = EONE - ETHRE
SMALL E = BEEB(2)
BEEB(2) = BEEB(3) - ETWO
SMALL C = CGGC(1)
DO 70 I = 1, M
70  \( Y(I) = \text{BSTAR}(I) \) 
CALL SIMPS
CGGC(1) = GAMMA * SYMPS
CAPITAL G = CGGC(3)
DO 80 I = 1, M
80  \( Y(I) = \text{BSTAR}(I) \times X(I) \) 
CALL SIMPS
CGGC(3) = SYMPS * GAMMA
CAPITAL C = CGGC(4)
DO 71 I = 1, M
71  \( Y(I) = \text{BSTAR}(I) \times (X(I) \times X(I)) \) 
CALL SIMPS
CGGC(4) = GAMMA * SYMPS - (ABS(\text{V})) * BEEB(3)
SMALL G = CGGC(2)
CGGC(2) = CGGC(3) - (ABS(\text{V})) * BEEB(1)
RETURN
END
LIST8

SUBROUTINE EXCITE
DIMENSION Y(I),TR(6),TI(6),ADDA(4),BEEB(4),CCGC(4),UR(6),UI(6),
1DMASS(21),QUANT(21),SKLAM(21),BSTAR(21),CXSFT(I),SXFST(21),
2CFST(I),STFT(21),XI(I),DIX(21),ENOXI(21),DRAFT(21),DWEIGH(21),
3SECO(21),ABAR(21),SPECM(81),OMEGA(81),RAOSDD(81),RAORB(81),
4RAORDB(81),RAORP(81),RAOTHE(81),RSDSDD(81),RSDRB(81),RSDRDB(81),
5RSDRP(81),RSDTHE(81)

COMMON Y,J,SYMPS,DXI,ADDA,BEEB,CCGC,REAL,ZREAL,IMAG,TEMAG,ZNUL,
1TNULL,DELTA,EPSIL,TR,TV,DMASS,QUANT,OMEGA,SKLAM,KRIT,RO,GRAV,
2BSTAR,CXSFT,SXFST,ALPHA,SHNULL,XI,BETA,SMNULL,GAMMA,DIX,M,WA,WAVE,
3,CW,ENOXI,SIGMA,TAU,FNULL,EMNULL,DRAFT,DWEIGH,SECO,TMAS,NU,UR,UI,
4ABAR,PI,SHREAL,SHIMAG,EN REAL,SMIMAG,YNERT,VOITH,RIZA,OMEGA,MM,NN,
5,KK,W,VOMEGA,SCPM,RAOSDD,RAORB,RAORDB,RAORP,RAOTHE,RSDRDB,RSDSDD,
6RSDRB,RSDRP,RSDTHE,RRSTDD,RRSTRP,RRSTRDB,RRSTBP,FACT,HELPR1,HELP2,
7HELP3,HELP4,HELP5,AVESDD,AVEX,AVRED,AVRED,AVERP,AVETHE,AVSDD,AVSR,
8SIGSRB,SRP,SRP,SIGTHE,AVHSDD,AVHRB,AVHRD,AVHRP,AVHTHE,CONST,GSPR,
J=M
H=DXI
DO 90 I=1,M
FKLAM=(GAMMA*BSTAR(I)-((WAVE*C)**2)*QUANT(I))*WA
SKLAM(I)=ENOXI(I)-(DIX(I)*ABSF(V))
CXSFST(I)=(FKLAM*SINF(WAVE*X(I)))+(WAVE*C*WA)*SKLAM(I)
1*COSF(WAVE*X(I)(I))+(EXP(-WAVE*DRAFT(I)*SECO(1))
SXFST(I)=(FKLAM*COSF(WAVE*X(I)))-(WAVE*C*WA)*SKLAM(I)
1*SINF(WAVE*X(I)(I))+(EXP(-WAVE*DRAFT(I)*SECO(1))
C FONE
90 Y(I)=CXSFST(I)
CALL SIMPS
TR(2)=SYMPS
C -FTWO
DO 92 I=1,M
92 Y(I)=SXSFST(I)
CALL SIMPS
TI(2)=SYMPS
C MONE
DO 93 I=1,M
93 Y(I)=CXSFST(I)*XI(I)
CALL SIMPS
TR(5)=SYMPS
C -MTWO
DO 94 I=1,M
94 Y(I)=SXSFST(I)*XI(I)
CALL SIMPS
TI(5)=SYMPS
SIGMA=ATANF(-TI(2)/TR(2))
TAU=ATANF(-TI(5)/TR(5))
FNULL=SORTF(TR(2)**2+TI(2)**2)
EMNULL=SORTF(TI(5)**2+TR(5)**2)
RETURN
END

TOTAL 51*
* LISTB

SUBROUTINE MOTION

DIMENSION Y (81), TR(6), TI(6), ADDA(4), BEEB(4), CGGC(4), UR(6), UI(6),
1DABS(21), QUANT(21), SKLAM(21), BSTAR(21), CXFST(21), SXFST(21),
2CTFST(21), STFST(21), XI(21), DIX(21), ENOXI(21), DRAFT(21), DWEIGH(21),
3SECOE(21), ABAR(21), SPECM(81), VOMEGA(81), RAOSDD(81), RAORB(81),
4RAORDB(81), RAORP(81), RAOTHE(81), RSDSDD(81), RSRDB(81), RSRDDB(81),
5RSRDP(81), RSDTHE(81).

COMMON Y,J, SYMPS, DXI, ADDA, BEEB, CGGC, ZREAL, ZIMAG, TREAL, TIMAG, ZNULL,
1TNULL, DELTA, EPSIL, TR, TI, V, DMASS, QUANT, OMEGAE, SKLAM, KRT, RO, GRAV,
2BSTAR, CXFST, SXFST, ALPHA, SHNULL, XI, BETA, BMNULL, GAMMA, DIX, M, WA, WAVEN
3, C, W, ENOXI, SIGMA, TAU, FNULL, EMNULL, DRAFT, DWEIGH, SECOE, TMASS, N, UR, UI,
4ABAR, PI, SHREAL, SHIMAG, BMREAL, BMIMAG, YNERT, VOITH, RIZA, OMEGA, MM, NN, H
5, K, W, OMEGAE, SPECM, RAOSDD, RAORB, RAORDB, RAORP, RAOTHE, RSRDB, RSRDDB,
6RSRDP, RSDTHE, RSTSDD, RSTRP, RSRDB, RSTRB, RSTHE, HELP1, HELP2,
7HELP3, HELP4, HELP5, AVESDD, AVERB, AVERDB, AVERP, AVETHE, SIGSDD, SIRGB,
8SIGRDB, SIGRP, SIGTHE, AVHSDD, AVHRB, AVHRDB, AVHRP, AVHTHE, CONST, GSQUR.

DO 105 J=1, 4
GO TO (100, 101, 102, 103, J)

100 I=1
GO TO 104

101 I=3
GO TO 104

102 I=4
GO TO 104

103 I=6

104 TR(I)=CGGC(J)-ADDA(J)*(OMEGAE**2)

105 TI(I)=BEEB(J)*OMEGAE

DO 110 I=2, 3
DO 110 K=1, 2
IF(K*I-4) 108, 110, 108

108 IPK=I+K

UR(IPK)=TR(I)*TR(K+3)-TI(I)*TI(K+3)-TR(K)*TR(I+3)+TI(K)*TI(I+3)

UI(IPK)=TR(I)*TI(K+3)+TI(I)*TR(K+3)-TR(K)*TI(I+3)-TI(K)*TR(I+3)

110 CONTINUE

DO 111 I=1, 6
UR(I)=UR(I)/100000000000.

111 UI(I)=UI(I)/100000000000.

DENUM=UR(4)**2+UI(4)**2

ZREAL=(UR(5)*UR(4)+UI(5)*UI(4))/DENUM

ZIMAG=(UR(5)*UI(4)-UI(5)*UR(4))/DENUM

TREAL=(UR(3)*UR(4)+UI(3)*UI(4))/DENUM

TIMAG=(UR(3)*UI(4)-UI(3)*UR(4))/DENUM

ZNULL=SQRF(ZREAL**2+ZIMAG**2)

TNNULL=SQRF(TREAL**2+TIMAG**2)

DELTA=ATANF(ZIMAG/ZREAL)

EPSIL=ATANF(TIMAG/TREAL)

RETURN

END
* LIST8

SUBROUTINE BENDSH

DIMENSION Y(81), TR(6), TI(6), ADDA(4), BEEB(4), CGGC(4), UR(6), UI(6),
    DMONTH(21), QNUT(21), CKLAM(21), BSTAR(21), CFXST(21), SXFST(21),
    2CFST(21), STFST(21), XI(21), DIX(21), ENOXI(21), DRAFT(21), DWEIGH(21),
    3SCEOE(21), A2BAR(21), SPECTM(81), VOMEGA(81), RAOSDD(81), RAORB(81),
    4RAOORB(81), 8RAOORB(81), RAOORPB(81), RAOTHE(81), R5DSDD(81), R5DSRR(81),
    5RSDD(81), RSDTHE(81)

COMMON Y, J, SYMPS, DXI, ADDA, BEEB, CGGC, ZREAL, ZIMAG, TREAL, TIMAG, ZNULL,
    1TNULL, DELTA, EPSIL, TR, TI, V, DMONTH, QNUT, CKLAM, BSTAR, RO, GRAV,
    2BSTAR, CFXST, SXFST, ALPH, SHNULL, XI, ETA, BMNULL, GAMMA, DIX, H/W, A/V, W/AVEN
3, C/K, ENOXI, SIGMA, TAU, FMNULL, EMNULL, DRAFT, DWEIGH, SECOE, TMASS, N, UR, UI,
4ABAR, PI, SIGMA, TAU, FMNULL, EMNULL, DRAFT, DWEIGH, SECOE, TMASS, N, UR, UI

COMMON Y, J, SYMPS, DXI, ADDA, BEEB, CGGC, ZREAL, ZIMAG, TREAL, TIMAG, ZNULL,
    1TNULL, DELTA, EPSIL, TR, TI, V, DMONTH, QNUT, CKLAM, BSTAR, RO, GRAV,
    2BSTAR, CFXST, SXFST, ALPH, SHNULL, XI, ETA, BMNULL, GAMMA, DIX, H/W, A/V, W/AVEN
3, C/K, ENOXI, SIGMA, TAU, FMNULL, EMNULL, DRAFT, DWEIGH, SECOE, TMASS, N, UR, UI,
4ABAR, PI, SIGMA, TAU, FMNULL, EMNULL, DRAFT, DWEIGH, SECOE, TMASS, N, UR, UI

COMMON Y, J, SYMPS, DXI, ADDA, BEEB, CGGC, ZREAL, ZIMAG, TREAL, TIMAG, ZNULL,
    1TNULL, DELTA, EPSIL, TR, TI, V, DMONTH, QNUT, CKLAM, BSTAR, RO, GRAV,
    2BSTAR, CFXST, SXFST, ALPH, SHNULL, XI, ETA, BMNULL, GAMMA, DIX, H/W, A/V, W/AVEN
3, C/K, ENOXI, SIGMA, TAU, FMNULL, EMNULL, DRAFT, DWEIGH, SECOE, TMASS, N, UR, UI,
3, CH, ENQX, SIGMA, TAU, FNULL, EMNULL, DRAFT, DWEIGHT, SECOE, TMASS, N, U1,
4, AABAR, P, SHREAL, SHIMAG, BMREAL, BMIMAG, YNERT
SUM = Y(1) + Y(J)
KAPL = J - 1
DO 151 I = 2, KAPL
   X = (-1.)***I
   IF (X) 151, 150, 150
150   Y(J) = 2.*Y(J)
151   SUM = SUM + 2.*Y(J)
symp = (DXI/3.)*SUM
RETURN
END
LIST8
SUBROUTINE STATS
DIMENSION Y(81),TR(6),TI(6),ADDA(4),BEEB(4),CGGC(4),UR(6),UI(6),
1DMASS(21),QUANT(21),SKLAM(21),BSTAR(21),CXFST(21),SXFST(21),
2CFST(21),STFST(21),X1(21),DIX(21),ENOXI(21),DRAFT(21),DWEIGH(21),
3SECOE(21),ABAR(21),SPECM(81),VOMEGA(81),RAOSDD(81),RAORB(81),
4RAORDB(81),RAORP(81),RAOTHE(81),RSDSDD(81),RSDRB(81),RSDRDB(81),
5RSDRP(81),RSDTHER(81)
COMMON Y,J,SYMPS,DXI,ADDA,BEEB,CGGC,ZREAL,ZIMAG,TREAL,TIMAG,ZNULL,
ITNULL,DELTA,EPSIL,TR,TV,DMASS,QUANT,OMEGA,E,SKLAM,KRIT,RO,GRAY,
2BSTAR,CXFST,SXFST,ALPHA,SHNULL,X1,BETA,BMNULL,GRAMMA,DIX,M,A,WA,
VAVEN,CE,ENOXI,SIGMA,TAU,FNULL,EMNULL,DRAFT,DWEIGH,SECOS,MASS,N,UR,UI,
4ABAR,PI,SHREAL,SHIMAG,BMREAL,BMIMAG,YNERT,VOITH,RIZA,OMEGA,MM,NN,H,
5KK,W,VOMEGA,SPECM,RAOSDD,RAORB,RAORDB,RAORP,RAOTHE,RSDDB,RSDSDD,
6RSDRB,RSDRP,RSDTHER,RSTSDD,RSTRP,RSTRDB,RSTRB,RSTTHE,HELP1,HELP2,
7HELP3,HELP4,HELP5,AVESDD,AVERB,AVERDB,AVERP,AVETHE,SIGSDD,SIGRB,
8SIGGRD,SIGPR,SIGTHE,AVHSDD,AVHRB,AVHRDB,AVHRPB,AVHTHE,CONST,GSPAR,
9DEEENFR
VOMEGA(1)=0.0
SPECM(1)=0.0
RSDSDD(1)=0.0
RSDRB(1)=0.0
RSDRDB(1)=0.0
RSDRP(1)=0.0
RSDTHER(1)=0.0
MM=NN+1
DO 700 KK=2,MM
RSDSDD(KK)=RAOSDD(KK)*SPECM(KK)
RSDRB(KK)=RAORB(KK)*SPECM(KK)
RSDRDB(KK)=RAORDB(KK)*SPECM(KK)
RSDTHER(KK)=RAOTHE(KK)*SPECM(KK)
700 RSDRP(KK)=RAORP(KK)*SPECM(KK)
H=DEEENFR
J=MM
DO 701 I=1,MM
701 Y(I)=RSDSDD(I)
CALL SIMPS
RSTSDD=SYMPS
HELP1=SQRTF(RSTSDD)
AVESDD=0.866*HELP1
SIGSDD=1.415*HELP1
AVHSDD=1.800*HELP1
DO 702 I=1,MM
702 Y(I)=RSDRB(I)
CALL SIMPS
RSTRB=SYMPS
HELP2=SQRTF(RSTRB)
AVERB=0.866*HELP2
SIGRB=1.415*HELP2
AVHRB=1.800*HELP2
DO 703 I=1,MM
703 Y(I)=RSDRDB(I)
CALL SIMPS
RSTRDB=SYMPS
HELP3=SQRTF(RSTRDB)
AVERDB=0.866*HELP3
SIGRDB=1.415*HELP3
AVHRDB=1.800*HELP3
DO 704 I=1,MM
704 Y(I)=RSDRP(I)
CALL SIMPS
E-65
RSTRP=SYMPS
HELP4=SQRTF(RSTRP)
AVERP=0.866*HELP4
SIGRP=1.415*HELP4
AVHRP=1.800*HELP4
DO 705 I=1,MM
705 Y(I)=RSDTHE(I)
CALL SIMPS
RSTTNE=SYMPS
HELP5=SQRTF(RSTTNE)
AVETHE=0.866*HELP5
SIGTHE=1.415*HELP5
AVTTHE=1.800*HELP5
RETURN
END
* LIST8
SUBROUTINE SIMPS
DIMENSION Y(81), TR(6), TI(6), ADDA(4), BEEB(4), CGGC(4), UR(6), UI(6),
1DMASS(21), QUANT(21), SKLAM(21), BSTAR(21), CXFST(21), SXFST(21),
2CTFST(21), STFST(21), XI(21), DIX(21), ENOXI(21), DRAFT(21), DWEIGHT(21),
3SECOE(21), ABAR(21), SPECM(81), VOMEGA(81), RAOSDD(81), RAORB(81),
4RAORDB(81), RAORP(81), RAOTHE(81), RSDSDD(81), RSDRB(81), RSDRDB(81),
5RSDRP(81), RSDTHE(81),
COMMON Y, J, SYMPS, DXI, ADDA, BEEB, CGGC, ZREAL, ZIMAG, TREAL, TIMAG, ZNULL,
1TNUL, DELTA, EPSIL, TR, TI, V, DMASS, QUANT, OMEGA, E, CR, K, I, RO, GR
2BSTAR, CXFST, SXFST, ALPHA, SHNULL, XI, BETA, BMNULL, GAMMA, DIX, MW, WA, WAV
3, CW, ENOXI, SIGMA, TAU, FNULL, EMNULL, DRAFT, DWEIGHT, SECOE, TMASS, N, UR, UI,
4ABAR, PI, SHREAL, SHIMAG, BMREAL, BMIMAG, YNERT, VOITH, RIA, OMEGA, MM, NN, H
5, KK, W, VOMEGA, SPECM, RAOSDD, RAORB, RAORDB, RAOORP, RAOORP, RAOORP, RSDSDD,
6RSDRDB, RSDRP, RSDTHE, RSDSDD, RSTRP, RSTRDB, RSTRB, RSTHE, HELP3, HELP4,
7HELP5, HELP5, AVESDD, AVERB, AVERDB, AVERP, AVEPBET, SIGSDD, SIGRD
8SIGRD, SIGS, SIGT, AVHSDD, AVHRB, AVHRDB, AVHRDP, AVHTHE, CONST, GSQUAR
SUM=Y(1)+Y(J)
KARL=J-1
DO 801 J=2,KARL
X=(-1.0)*J
IF (X) 801 801,800,800
800 Y(J)=2.0*Y(J)
801 SUM=SUM+2.0*Y(J)
SYMPS=(H/3.0)*SUM
RETURN
END