Measurement of Physician Hand & Patient Tissue Mechanical Impedance

by

David Mercado
B.S., University of Florida (2014)

Submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Author ..............................................................................................................................
Department of Mechanical Engineering
May 12, 2017

Certified by .................................................................
Neville Hogan
Sun Jae Professor of Mechanical Engineering
Thesis Supervisor

Certified by .................................................................
Brian Anthony
Principal Research Scientist, Dept. of Mechanical Engineering
Thesis Supervisor

Accepted by .................................................................
Rohan Abeyaratne
Chairman, Department Committee on Graduate Students
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This study investigates various methods by which a previously developed hand-operated actuated device is capable of measuring the mechanical impedance of both the compliant patient tissue in contact with its end effector and of the hand of the operator holding the device. The particular device being investigated is an actuated ultrasound probe originally designed to regulate the amount of force exerted by a sonographer to their patient during an ultrasound scan. We expect that quantifying the effective mechanical impedance of the operator hand will lead to improvements in the design and control of hand-operated devices. Improvements are being considered not only to improve the quality and reliability of the ultrasound scan, but to alleviate the chronic muscular and joint stress endured by sonographers over the course of their careers. An additional motivation behind this study is to augment the capabilities of physicians to diagnose medical conditions, such as breast cancer and liver cirrhosis, on the basis of tissue impedance without the need for an additional mechanism.

Several methods were developed for approximating mechanical impedance using the actuated ultrasound probe, based on the sensors available and models of the device dynamics. Due to sensor limitations, impedance measurement could only be effectively implemented for a single interface, instead of both concurrently. These methods involved immobilizing one end of the device. Experiments were conducted on artificial tissues in order to confirm that the methods developed were valid and reliable for measuring mechanical impedance of the body in contact with either the sonographer or patient end of the device.

Thesis Supervisor: Neville Hogan
Title: Sun Jae Professor of Mechanical Engineering

Thesis Supervisor: Brian Anthony
Title: Principal Research Scientist, Dept. of Mechanical Engineering
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Chapter 1

Introduction

1.1 Motivation

The purpose of this study was to develop techniques to measure the effective mechanical impedance of the arm of a sonographer while conducting an ultrasound exam using an existing force stabilization device. From personal experience and as confirmed across studies, we know that the human arm is not a perfectly rigid structure, even when providing nominally static support. An existing theory is that limb compliance plays a key role in enabling humans to interact with unknown surfaces and mechanical constraints, including the ability to operate manual tools in a stable manner despite an uncertain environment. (Hogan & Sternad, 2012) However, the mechanical impedance of the human arm also increases the dynamic complexity of contact interactions, making it more challenging to adequately model and quantify such behavior. For instance, a physician may decide to diagnose their patient by measuring the stiffness of an affected tissue using a hand-operated device. Since the arm of the physician is a compliant structure, it is necessary to take the arm impedance into consideration when calculating tissue stiffness from the device sensor readings. This study approaches this challenge by attempting to develop practical methods for measuring the physician arm and patient tissue impedances simultaneously during medical exams performed with a hand-operated device.

The primary reason to develop this measurement functionality is to be able to quantitatively assess and catalogue the variation in compliance of the human arm when applying increasing magnitudes of force or due to variations in arm configuration. One of the main concerns in the medical ultrasound field is the amount of fatigue undergone by sonographers. (Burnett, 2007; Evans, Roll, & Baker, 2009; Magnavita, Bevilacqua, Mirk, Fileni, & Castellino, 1999; Murphey & Milkowski, 2006; Murphy & Russo, 2000) Studying how sonographers modulate their impedance
while interacting with patient tissues could provide insight into some of the leading contributors to this fatigue and help us suggest modifications to standard practice. Furthermore, characterizing joint impedance modulation may result in new understanding of human interactive motion control, which could lead to the optimization of the design and motor control of manually operated devices for maximum performance and minimum discomfort.

As previously mentioned, the measurement functionalities developed in this study were intended to allow a sonographer to measure the mechanical impedance of a patient tissue while compensating for the impedance of their own arm. Previous studies have shown that the mechanical properties of body tissues and organs can serve as a reliable indicator of the onset and advancement of a variety of diseases and malignant abnormalities. (Cornu, Goubel, & Fardeau, 1998; Gefen et al., 2001; Klaesner et al., 2002; Rago et al., 2007; Wachter et al., 1996; Wellman, Howe, Dalton, & Kern, 1999; Ziol et al., 2005) Additionally, most current methods for measuring the rigidity of human parts involve procedures that can be performed externally and thus avoid the need for risky surgical procedures. (Arokoski et al., 2005; Zhai, Palmeri, Bouchard, & Nightingale, 2008) However, most of these methods are performed using devices that are not currently in use in standard medical diagnosis, or they involve organ-specialized computational analysis and additional physician training. (Arokoski et al., 2005; Zhai et al., 2008; Ziol et al., 2005)

The machine that is to host the impedance estimation techniques discussed above was originally designed to serve as an improvement on the probes used during ultrasound exams. Recently developed at the MIT Device Realization Laboratory, the actuated ultrasound probe, as it will be referred to in this document, is a motor-driven device that can stabilize the force applied by a sonographer to their patient to a desired constant value. (Gilbertson & Anthony, 2012; Gilbertson & Anthony, 2015) This is a device with a functionality that is useful during ultrasound exams. Therefore, developing an impedance measurement technique for the actuated probe would provide sonographers with an additional tool without requiring another physical system. Furthermore, the techniques proposed here are simple enough for the sonographer to learn in minutes and the methodology is universal enough such that it could be applied to a variety of tissues, muscles, and organs of interest.
1.2 Actuated Ultrasound Probe

The device being used for this study is the second prototype of an actuated ultrasound probe, a series of devices designed and built by Matthew Gilbertson at the MIT Device Realization Laboratory. (Gilbertson, 2014; Gilbertson & Anthony, 2012; Gilbertson & Anthony, 2015) The original design objective of the device was to maintain the force applied by a specialist to a patient during an ultrasound scan at a constant, desired value. The effort to help the sonographer maintain a constant force was initially begun by Gilbertson by using an ultrasound probe equipped with a force transducer to provide real-time visual feedback of the amount of force being applied. (Gilbertson & Anthony, 2013) During use, the sonographer is able to see how much force they are applying and has the opportunity to adjust their muscular effort to reach the desired pressure. The actuated probe takes this a step further - while the sonographer is still responsible for applying the base amount of force, the motorized device alleviates or intensifies the applied load by compensating for deviations from the desired value based on force sensory feedback.

The motivation behind constant force control lies in the sonogram images. The amount of pressure that is applied by an ultrasound probe to the patient distorts the geometry of the organs, veins, muscles, and other compliant structures underneath the contact area. Therefore, the geometry of the tissue being analyzed will be visually different on the resultant sonogram image depending on the amount of force applied by the sonographer. Previous studies have shown that this sonographer-applied force is highly variable across physicians, across trials performed by the same physician, and even over the course of a single trial, regardless of an explicit instruction to maintain a constant force. (Gilbertson & Anthony, 2013) Since the force applied by a sonographer is highly variable and the image geometry is dependent on the force applied by a sonographer, the geometry of the ultrasound images is highly variable as well. While most ultrasound images can provide physicians with a visual reference of the state of the analyzed tissue, two images taken during different trials cannot be compared confidently. By regulating the applied force, images from scans taken months apart from each other could be compared directly. This would provide sonographers with an additional diagnostic tool for assessing changes in the characteristics of the tissue over time.
Other devices have been previously developed with the objective of reducing variability in the input from the human. For instance, the Steady-Hand Robot was developed at John Hopkins University to reduce the amount of hand tremors during retinal microsurgery. (Mitchell et al., 2007) In this device, the externally mounted robot shares control with the human and tremor compensation is achieved in large part due to the additional lateral stiffness provide by the actuator. On the other hand, robots like the Da Vinci Robot from Intuitive Surgical and the Tokyo vitreoretinal surgical system use robots that are physically separated from the operator. (Broeders & Ruurda, 2001; Ueta et al., 2009) The human is the “master” that provides desired motions through a sensory-input controller and the machine “slave” performs the desired tasks, which are first processed and filtered to compensate for human variability. The actuated probe is like neither of these approaches, but is more similar to the Carnegie-Mellon University Micron, developed for microsurgery, and the Harvard voice-coil actuated beating heart surgical tool. (MacLachlan et al., 2012; Yuen et al., 2009; Yuen et al., 2010) In all three of these devices, the operator maintains control of the overall actions performed with the device, but an actuator installed within the tool compensates for the expected human error, such as hand tremor.

The actuated ultrasound probe can, in brief, be described as an ultrasound probe driven by a handheld linear actuator. At greater length, the device is composed of a support structure held by the physician; attached to this structure is a brushless rotary motor that provides torque to a linear actuator via a timing belt-gear system; a ball screw linear actuator then converts rotational motion to translation motion that can either move the attached ultrasound probe back and forth or increase and decrease the amount of force applied to the patient. The actuated probe serves as an intermediary device between the patient and sonographer that compensates for the variability of the input from the operator and ensures that the output received by the patient is at the desired level. In order to regulate the applied force, the device is equipped with sensors that measure the configuration, orientation, and position of its components, the amount of force observed at the probe, and whether the system is attempting to operate out of bounds.

Due to the ability of the device to modulate and collect data on position and force, it meets the requirements necessary to measure the mechanical impedance of an external structure. However, the number of external structures that the device can quantify distinctly and
simultaneously, along with the setup necessary to perform these estimates with sufficient accuracy, is not immediately obvious until the particular estimation methods for this particular device are developed, as will be done in Chapters 2 and 4. A critical decision was taken when considering which prototype should be used for examining impedance estimation capabilities. While Prototype 3 uses direct drive instead of a timing belt, its motor inertia is two orders of magnitude higher than the inertia of the Prototype 2 motor armature as well as the apparent screw inertia. (Gilbertson, 2014) The high inertia of Prototype 3 limited the motion amplitude when the device operated at high frequencies. Additionally, it is expected that the prototypes will display resonance at the natural frequency associated with the belt elasticity and the apparent inertia of the screw and motor. The lower motor inertia of Prototype 2 increases the frequency at which resonance occurs and allows a larger amplitude of motion when operating at high frequencies. This made this version of the device more suitable for this study.

1.3 Human Hand Impedance Modulation

As a handheld device, the actuated probe is supported by a human, specifically by a human arm. Similar to a rigid structure support, an arm support is responsible for providing the reaction forces necessary to maintain the device body relatively stationary despite external forces such as gravitational weight. Unlike a rigid structure support, an arm support accomplishes this by expending metabolic energy. In the case of the actuated probe, the majority of the desired pressure applied by the probe to the patient is provided by the effort of the operator arm. This is a basic functional characteristic that the actuated probe carries over from a standard ultrasound probe, and it is also the characteristic that most significantly contributes to the negative side effects associated with sonography. (Heyward, 1973)

Multiple surveys and studies have confirmed throughout the past decades that ultrasound technicians suffer from chronic muscular fatigue and musculoskeletal disorders over years of performing ultrasound scans on their patients. (Burnett, 2007; Evans, Roll, & Baker, 2009; Magnavita et al., 1999) In descending order, over 40% of respondents to one survey reported experiencing pain or discomfort in their shoulder, neck, wrist, upper back, and hand regions while
scanning patients. Pain or discomfort in the shoulder region is particularly common, with over 20% of respondents reporting severe shoulder pain while performing an ultrasound scan. (Evans et al., 2009)

Different operating strategies have been recommended to alleviate the pain and damage, such as adopting a standing posture during trials, reducing the amount of force applied to the patient, and using arm configurations that transfer the load through the skeleton instead of straining the joints. (Murphey & Milkowski, 2006; Murphy & Russo, 2000) However, these strategies do not completely resolve the problem and their long-term efficacy often takes years to confirm. An obvious solution to fatigue reduction is to minimize the amount of overall muscular effort required from the sonographer. If a reduction in applied force is an effective way to reduce muscular fatigue, the actuated probe and force-measurement probe could contribute by actively encouraging ultrasound technicians to maintain a low force during scanning.

Humans produce forces through muscular contraction. This actuation process is inherently nonlinear but can be linearized as a first-order Hill-type model. (Gasser & Hill, 1924) Previous studies on human and primate motion control have shown that a model of muscular control based on equilibrium point control is sufficient to predict unconstrained reaching motions. (Hogan 1984; Bizzi et al., 1984) If these models of muscular impedance modulation are assumed to extend to constrained motions, it is expected that as humans exert more force on an object, the effective rigidity of the arm increases as well. This is a hypothesis that could be confirmed by measuring the effective stiffness of the arm across a wide range of applied forces. It can also be predicted that if the dynamic interaction between the ultrasound probe and the tissue is unstable or oscillatory, the human arm will provide viscous damping to increase the stability of the interaction. If these predictions hold true in practice, then active and constant modulation of muscle impedance could be a root cause of muscular fatigue among sonographers.

Various models of this theory of interactive motion control have already been put forward, which argue that humans are able to interact with unknown constraints and external surfaces with variable properties by modulating the mechanical impedance of the arm rather than simply planning for a certain contact force or position trajectory. (Hogan & Sternad, 2012) This is a hypothesis that would benefit from further study; in particular, the manner in which impedance
is modulated and the factors that lead to modulation have not been fully characterized experimentally. The ability exhibited by humans to maintain stability despite interacting with unknown constraints is especially impressive when considering the difficulty with which robots are able to perform unplanned constrained interactions, despite having much greater processing capabilities. If impedance control within human arms were characterized, similar control techniques could then be applied to robotic arms. The interaction of the machines with unknown constraints may then provide evidence about the extent to which impedance control affects stability during contact tasks with high uncertainty.

An important consideration in determining how to improve the system is to examine the characteristics of the human-device interaction. For instance, it may be important to examine the dynamic behavior of the human arm and how it changes across different limb masses, arm configurations, and force production. If the frequency of resonance of the human arm is within the range of frequencies exhibited by the device during normal operation, it could inadvertently cause dynamic amplification of the sonographer arm joint motion. This could lead to a reduction in the control that the operator has over the device, it may introduce vibrations to the device itself and compromise any data collected, and it may increase the chances of muscular discomfort and fatigue.

1.4 Tissue Impedance as a Diagnostic Tool

Tissue impedance has already been shown in previous studies to indicate the progression of various medical conditions. Several of these correlations have become the basis of standard diagnostic practices in the medical field. However, there are opportunities in tissue impedance measurement as a diagnostic tool that have not been pursued either due to a lack of commonplace tools for performing the tests or due to a lack of data supporting that such correlations exist.

The most popular medical application of tissue impedance measurement as a diagnostic tool is the use of ultrasound elastography to measure the stiffness of the liver. The advancement of fibrosis of the liver due to the onset of Hepatitis C has been found to be strongly correlated with
the stiffness of the liver. (Ziol et al., 2005) In recent years this has been an exciting new development in the field because prior to the use of elastography, the standard method by which to diagnose the condition was to surgically extract a sample from the liver of the patient. Elastography eliminates the need for invasive procedures and it is a function that has been made standard across all contemporary ultrasound probes. This process does, however, involve a heavy amount of computation through software programs specialized at deciphering the stiffness of the liver while bypassing the impedance of the neighboring tissues, organs, and bones. This means that conducting elastography properly and reliably on other internal tissues similarly requires a computational algorithm designed specifically for identifying the properties of that organ.

Several scientific studies exist showing that static rigidity can be a reliable indicator for other medical conditions. For instance, for thousands of years it has been known that the stiffness of breast tissue is a prime indicator of the onset of breast cancer, leading to the common practice of self-inspection for lumps – tissue structures of noticeably different compressibility. In recent years, the differences in tissue stiffness between healthy breast tissue and tissue containing malignant growths have been quantified to confirm this common knowledge. (Wellman et al., 1999) Prior research has also shown the potential correlation between the onset and growth of thyroid nodules and tissue stiffness. (Rago et al., 2007) Stiffness has not only been shown to correlate with the onset of malignant growths, but in some cases also corresponds to changes in the characteristics of affected tissues. For instance, the stiffness of muscular tissue at the base of the foot was found to be indicative of the advancement of diabetes-related disabilities. (Klaesner et al., 2002)

Many of the tissue stiffness studies discussed have been performed using an indentation stiffness meter. (Arokoski et al., 2005; Wellman et al., 1999; Zhai et al., 2008) In essence, an independent machine was built that, when supported on a test table, presses a probe into the tissue of the test subjects. For all these indentation devices, the force is measured through a load cell and the length of indentation is either prescribed or measured through a position sensor such as a motor encoder or Hall effect sensor. The main drawback of these devices is that they only serve one purpose, which is to measure stiffness, and since most of them are ground-supported
and not handheld, the range of tissues that can be measured is limited by their geometry and location on the body.

While stiffness is currently being used as a diagnostic tool for multiple conditions, there are fewer instances identified of other tissue mechanical impedance properties, such as damping, correlating with a medical condition. The closest example found is of a study that observed how velocity-dependent damping at the knee joint varies with the advancement of fibromyalgia. (Gefen et al., 2001) The experiment performed involved releasing the leg of the subjects from a certain height and asking subjects to allow the leg to swing freely. The motion of the leg was measured in order to determine the logarithmic decrement of the oscillations. The study found that the motion damping significantly increased with the severity of the fibromyalgia across subjects. While not confirmed in the study, since fibromyalgia is a muscular condition, it is expected that the changes in joint damping were due to changes in the damping of the associated leg muscles. If this is the case, then it is possible that the methods proposed in this study could allow the actuated probe to detect the onset and advancement of fibromyalgia without asking patients to perform time-consuming and potentially painful tasks.

An additional diagnostic opportunity worth discussing is the correlation of muscular stiffness with the advancement of Duchenne muscular dystrophy (DMD). This is a direction that the MIT Device Realization Laboratory has already begun moving towards using the actuated ultrasound probe. (Koppaka, Wu, et al., 2014; Koppaka et al., 2014) It is therefore expected that the DMD project could significantly and directly benefit from the findings in this study.

If the mechanical impedance measurement techniques developed in this study prove valid, the actuated probe would be able to measure the mechanical impedance of all the tissues discussed in this section. This would expand the capacity of the device as a diagnostic tool beyond what is capable with simply ultrasound and elastography. Furthermore, because it is expected that the actuated probe will be able to measure tissue impedances in a general sense, rather than being limited to a set of tissues, the device has the potential to be used in scientific studies to determine any currently unknown correlations between medical conditions and the mechanical impedance of the associated tissues.
1.5 Objectives

The primary goal of the study presented here was to determine how the actuated probe can be equipped to perform mechanical impedance estimation. The following chapter discusses the device itself in more detail, along with a more rigorous definition of the mechanical impedance we are interested in. Based on the understanding of how the device interacts with the sonographer and the patient, an ideal sensory configuration is detailed in Chapter 3 that would be capable of estimating mechanical impedance of both the patient and the physician simultaneously without needing to model the dynamics of the device. Chapter 4 then lays out a more practical solution to this challenge using only the sensors already available on the actuated probe. However, this approach does require a model of the actuated probe dynamics.

Before developing a model of the device dynamics, the present sensors on the system were characterized in order to determine whether their performance would be sufficiently reliable for the purposes of this study. Based on the findings of the sensory analysis, the impedance measurement methods proposed in Chapter 4 are revised to reflect the existing limitations. Chapter 6 describes the necessary assumptions and experimental procedures required to confidently estimate the impedance of the sonographer arm and patient tissue using the device, taking into consideration the present sensor limitations.

Modeling of the device dynamics is presented in Chapter 7. This model relates the arm, tissue and device dynamics together and is therefore what enables us to determine the arm and tissue impedance characteristics from the sensor readings. Additionally, alternate models are proposed to better understand the device behavior and allow us to make the necessary and proper simplifying assumptions. The model of the device dynamics is validated experimentally by exciting the device at a discrete set of frequencies and analyzing the system response to the excitations. The manner in which the frequency response excitation is implemented on the actuated probe is outlined in Chapter 8 while the experimental process is discussed in detail in Chapter 9.

Finally, in order to validate whether the device is able to determine the mechanical impedance associated with the structures it interacts with, experiments were performed with the actuated probe to measure the mechanical impedance of Phantom artificial tissues. During
the first half of these trials, the “patient” end of the device made contact with the compliant artificial tissue while the “sonographer” end remained immobilized. Then, the experiments were repeated with the “sonographer” end in contact while the “patient” end remained immobilized. Because the properties of these tissues are known and time-invariant, the results from both procedures were expected to corroborate each other and be in accordance with the a priori knowledge of the tissues.

The continuation of this project was enabled by the results of these tests. Since the existing device was capable of estimating the artificial tissue impedance with a reasonable degree of accuracy and confidence, then the impedance measurement functionality can be extended to the actual procedural configuration with a human arm pressing the device upon a human tissue. This is the ultimate goal, for it will allow us to use the device clinically and in scientific applications to characterize the human body and address the motivations previously discussed.
Chapter 2

Setup for Impedance Measurement

2.1 Fundamental Device Operation

The device that will be used to carry out the impedance measurement trials presented in this study is the second iteration of the handheld actuated ultrasound probe, originally designed and built by Matthew Gilbertson at MIT. (Gilbertson, 2014; Gilbertson & Anthony, 2012; Gilbertson & Anthony, 2015) Figure 2-1 shows a side view of the device, with annotations indicating its primary components. The top right of the image shows a clamp mechanism. The clamp is what rigidly attaches the device to an ultrasound probe. This clamp is interchangeable in order to attach a variety of different ultrasound probes. Enough clearance is provided at the rear of the probe for the probe cable and the front face of the probe protrudes away from the device to make contact with the tissue of the patient. Because the ultrasound functionality is not necessary for impedance estimation, the mock plastic probe shown in the figure is used throughout this study.

The probe clamp sits on a support beam that attaches rigidly to the linear actuator carriage. During operation, the translation of this carriage directly causes the probe to move forwards and backwards, or to increase and decrease the amount of contact force. What transmits power to the carriage translation is the rotation of a threaded ball screw. The screw rotates along the internal female thread within the carriage, but cannot translate with the carriage. Linear rails along the inner walls of the device housing prevent the carriage from rotating, but allow it to translate relative to the screw. The rotation of the screw is itself powered by a brushless DC motor. In order to operate at a desired torque and speed range, power is transmitted from the motor shaft to the screw via a set of pinion gears connected by a timing belt. Friction between the sliding parts is minimized with lubricated ball bearings.
During operation, most of the device is covered with an outer plastic housing, shown in Figure 2-2, that keeps the moving mechanical parts internal and away from human contact. The sonographer grasps this device housing instead of grasping the probe directly. The flat end face of the probe is the only component that makes direct contact with the human tissue.

The amount of force applied on the patient by the probe is measured using a single-axis force transducer, shown in Figure 2-3. The force transducer attaches the probe clamp to the support beam in order to measure the force transferred between the two components. This force is nearly equivalent to the force exerted on the patient, particularly during quasi-static applications. As it is currently programmed, if the force reading is below or above the desired force of application, the motor will adjust the amount of torque to the linear actuator.

The device is also equipped with a motor encoder, shown in Figure 2-4, which keeps track of the rotation of the motor armature. During quasi-static operation, the motor rotation is converted to probe translation. This position reading is used to ensure that the carriage remains within the range of operation of the screw and to avoid collision with the end stops. The last sensor mounted on the device is a three-axis accelerometer, shown in Figure 2-5. The accelerometer reading is used to estimate the orientation of the device in space. The device orientation is used to compensate for the effects of gravity on the force transducer reading.
Figure 2-1: Side view of actuated ultrasound probe. The main functional components of the device are labeled and indicated.

Figure 2-2: Typical operation of an ultrasound scan using the actuated probe. An external housing attaches to the internal housing and prevents the sonographer from interacting with the mechanical parts. Image courtesy of M. Gilbertson.
Figure 2-3: Futek single axis force transducer attaches to the probe cantilever support on one end and the lower probe clamp on the other.

Figure 2-4: Optical quadrature encoder attaches to the Maxon brushless motor.

Figure 2-5: An accelerometer with three-axis measurement capability attaches to the external device housing.
2.2 Impedance Measurement

Before establishing how the actuated probe can be used to measure mechanical impedance, it is important to first discuss what that entails. Mechanical impedance is the physical quantity characterizing the dynamic relation between the force at a contact interface and the motion that interface undergoes. The inverse of impedance is admittance. In a linear static case, mechanical impedance reduces to stiffness, the ratio between the interface force and displacement. (Palazzolo et al., 2007) The use of impedance or admittance depends on the application, such as whether force or motion is treated as an input during analysis but the two functionally describe the same dynamics.

The mathematical definition of mechanical impedance $Z$ of a linear system is shown below. This can be expressed as the ratio between a function relating the contact force $f$ with its time derivatives and a function relating the interface displacement $x$ with its time derivatives. Alternatively, it can be expressed in the frequency domain.

$$Z = \frac{func(f, f', f'', \ldots; t)}{func(x, x', x'', \ldots; t)} = \frac{f(\omega)}{x(\omega)}$$

An interpretation of this equation is that for a linear system, it is sufficient to know a time history of motion and contact force in order to determine mechanical impedance. Alternatively, it is sufficient to have a history of the force-motion ratio across an appropriate frequency range to determine impedance. The typical manner of examining the mechanical impedance of a compliant structure would be to provide an external actuation to excite the system at the appropriate frequencies.

In this study, the mechanical impedance of primary interest is that of the sonographer hand during operation of the device. Following the definition above, the impedance of the sonographer hand is the dynamic relation between the motion and force exerted by the hand. In this application, the sonographer hand impedance is defined by the contact force at the interface between the hand and the device housing and the motion of this interface. If the sonographer hand was a rigid structure with infinite impedance, there would be no motion. In such case, for any motion of the hand-housing interface, the hand would provide an infinite amount of force in reaction. Conversely, if the hand provided a force acting on the housing that was independent of
the hand-housing motion it would indicate a structure with zero impedance, or infinite admittance. In the context of this application, characterizing the mechanical impedance between the sonographer hand and the actuated probe housing provides us with a quantitative description of the type of support the sonographer provides to the device during operation.

A mechanical impedance of secondary interest is that of the patient tissue that the ultrasound probe is making contact with. The dynamic relation between the force and motion at the probe-tissue interface is equivalent to the impedance of the tissue. The resistance that the tissue exhibits in response to probe actuation is a physical property inherent to the tissue. In some cases, the impedance of a tissue may be correlated with other properties, such as the presence of carcinogenic tissue. It may also be possible that the sonographer adjusts their grasp on the device or their muscular effort in response to their perception of the patient tissue impedance. Any such adjustment by the sonographer, conscious or otherwise, could result in changes in the effective hand impedance, providing a motivation to measure the tissue impedance if studying the hand impedance.

Since the actuated probe makes contact with a compliant human tissue while also being held by a compliant human arm, the operation of this device presents interactive dynamics that increase the complexity of the impedance characterization. Previous impedance studies have had the measurement device mounted rigidly to an inertial frame and focused on a single contact interface during a single trial. Since this study aims to focus on two contact interfaces using a device that is free to move in space, the main challenge is to incorporate the ability to distinguish the hand impedance and the tissue impedance from a single set of trials.

The analysis in this study is simplified by treating both impedances as unidirectional along the translational direction of the device. Additionally, it is assumed that both the sonographer and patient are ultimately supported by a structure that remains rigid relative to ground. For instance, it is assumed that the sonographer will not actively adapt their stance and will essentially maintain their thorax in place while the impedance measurement data is collected.

When characterizing impedance, the dynamic relation will be determined from the data. Depending on the order and linearity of the relation, certain terms may be referred to by their more common names. The stiffness of a structure, $k$, as mentioned previously can be defined in
this configuration as the ratio between the contact force and displacement. The structure’s damping, $b$, is the ratio between contact force and velocity. The inertial term, $m$, or the effective mass of the structure as seen by the interface, is the ratio between the force and acceleration. For many physical systems, including some of the ones examined in this study, force can be approximated as a linear relationship with motion and its derivatives with respect to time using only these terms.

$$f = kx + b\dot{x} + m\ddot{x}$$

In these cases, the stiffness, damping, and inertial terms can be defined as the following partial derivatives of this function.

$$k = \frac{\partial f}{\partial x} \quad b = \frac{\partial f}{\partial \dot{x}} \quad m = \frac{\partial f}{\partial \ddot{x}}$$

Since these are terms that are commonly assumed to be a constant ratio between force and a motion variable, they are often estimated individually. One common way to estimate stiffness is to compress an object with incremental amounts of force through multiple static tests. The stiffness is then equal to the ratio between the force applied and the resulting displacement. This method has been previously used to estimate stiffness with the actuated probe. Similarly, a weighing scale is commonly used to determine mass by observing the force produced by an object due to gravitational acceleration. However, other terms may also be observed, for example, arising from higher-order resonance and anti-resonance. In order to fully characterize the impedance of a structure that is highly dependent on configuration and task-dependent factors, static measures are not sufficient; it is necessary to collect dynamic data.

### 2.3 Computationally Minimal Impedance Measurement

#### 2.3.1 Minimal Sufficient Information

Mechanical impedance is the relationship between force and motion at an interface. Therefore, the simplest way to examine this relationship is by having immediate knowledge of the interface motion and its corresponding force. Any additional information about the dynamics
of the device is superfluous. The minimal information required to determine mechanical impedance are those two sets of data – force and motion at the interface.

In the case of this study, in order to measure the sonographer hand impedance it is necessary and sufficient to know the contact force between the actuated probe and the sonographer hand, \(f_s\), as well as the motion of that interface location. Since the housing end of the device is the rigid structure in contact with the hand, a measure of the motion of the device housing relative to an inertial frame of reference, \(x_h\), would be equivalent to tracking the motion of the contact interface. If the sonographer hand were to exhibit linear dynamics, its impedance would be defined by the relationship shown below between \(x_h\) and \(f_s\), taking their performance over time into consideration.

\[
Z_s = \frac{\text{func}(f_s, f_s', f_s''; \ldots; t)}{\text{func}(x_h, x_h', x_h''; \ldots; t)} = \frac{f_s(\omega)}{x_h(\omega)}
\]

Similarly, the patient tissue impedance can be estimated from a measure of the contact force between the ultrasound probe and the patient tissue, \(f_p\). Since the ultrasound probe is the rigid structure in contact with the hand, the ability to measure the motion of the probe end of the device relative to an inertial frame of reference, \(x_p\), would be equivalent to tracking the motion of the patient contact interface. If linearity is confirmed, the tissue impedance is defined by the relationship below.

\[
Z_p = \frac{\text{func}(f_p, f_p', f_p''; \ldots; t)}{\text{func}(x_p, x_p', x_p''; \ldots; t)} = \frac{f_p(\omega)}{x_p(\omega)}
\]

2.3.2 Practical Implementation

While it is simple to define the minimum theoretical measures needed for a straightforward impedance estimation, determining adequate sensors and their placement to gather these measures is not as simple. One way to gather the inertial motion data is to place an accelerometer on the housing and the carriage structures, as shown in Figure 2-6. In this setup, the device housing is rigidly attached to the grip, which is the portion of the housing end that the sonographer is in contact with. Similarly, the carriage attaches rigidly to the probe, the structure making direct contact with the patient tissue. Therefore, the motion data collected by these
accelerometers will be equivalent to the motion of the interface locations. Related quantities of interest, such as the positions and velocities, can in principle be integrated from the acceleration readings.

An alternative method to measure the inertial motion is to place visual markers on the housing and carriage. An external stationary camera would then track the positions of visual markers over the course of the trials. Part of the difficulty in this solution is placing the markers in locations that will remain in the line of sight of the camera at all times. Additionally, during an actual ultrasound exam the sonographer will need to change the orientation of the device to optimize the sonogram quality. Multiple cameras would be needed at that point in order to track the motion. Requiring hospitals to install motion tracking cameras ultimately goes beyond the original intent to only make use of an existing device.

Figure 2-6: Sensor configuration required for measuring mechanical impedance using minimal computation. Force transducers are placed at the patient and sonographer ends of the device. Accelerometers or a motion tracking system can be used to measure the inertial positions of the housing and carriage. The device dynamics relating the probe and housing ends of the device do not need to be known.
The best way to measure the contact forces is through the use of force transducers. Ideally, the force transducer would be placed at the contact interface, for instance between the probe and the patient tissue. However, this is not possible as it disturbs the contact interface and it may not capture the force accurately. The best practical solution is to place the force transducer on the rigid end of the device involved in contact, but further away from the contact point. The load cell would then serve as a junction between two masses composing the rigid device end. As shown in Figure 2-6, one of the load cells can be placed between the probe and the linear actuator carriage. The other load cell would be placed between the grip and device housing.

In the static case, the forces measured by the transducers are equivalent to the contact forces with the patient and sonographer. However, since the impedance trials will be dynamic, an analysis of the system is needed to compensate for the motion of the housing mass $m_{H1}$ between the force transducer measure $f_{FT,S}$ and the sonographer contact force $f_s$. A similar analysis is performed to compensate for the inertia of the probe mass $m_{P2}$ between the patient force $f_p$ and force transducer measure $f_{FT,P}$. The equations of motion below were derived from the free-body diagrams of the housing grip and probe, as shown in Figure 2-7. Based on the orientation of the figure, motion and contact forces are defined as positive towards the right direction. The measured load cell forces are defined as positive when the device is under compression.

$$m_{H1} \ddot{x}_h = f_s - f_{FT,S}$$  
$$m_{P2} \ddot{x}_p = f_{FT,P} + f_p$$

The contact forces can be estimated if the load cell readings and the accelerations of the contact interfaces are known. Since the accelerations $\ddot{x}_h$ and $\ddot{x}_p$ are being measured by the accelerometers, the contact forces can be determined from the algebraic sums below if the masses of the grip and probe are known. As noted earlier in the chapter, these are the signals needed to determine the mechanical impedances of interest.

$$f_s = f_{FT,S} + m_{H1} \ddot{x}_h$$  
$$f_p = -f_{FT,P} + m_{P2} \ddot{x}_p$$

Once the position data is collected and the contact forces are computed from the force transducer data, all the information is available for determining the mechanical impedances of the sonographer arm and patient tissue. Chapter 7 will discuss in greater detail the type of actuation used to excite the system across the appropriate range of frequencies to establish a sufficient characterization.
2.4 Model-Based Impedance Measurement

2.4.1 Model-Based Approach

In many cases, the sensors available on a device do not correspond directly with the force and motion variables necessary to determine mechanical impedance. For these cases the force and motion variables at the contact interfaces may be determined from the existing sensor data if a sufficient knowledge of the device dynamics is made available. The additional information about the system may be trivial to acquire. This was the case with the sensor configuration presented earlier in this chapter. The inertias of the probe and grip structures needed to be taken into account in order to determine the contact forces from the force transducer readings. The equations of motion used for those calculations were determined from two simple free-body diagrams that only account for a small fraction of the device dynamics.
In other situations, the additional information needed about the system dynamics may be extensive and additional computation and modeling may be required in order to relate the sensor readings to the contact force and motion variables. For instance, we have established that mechanical impedance $Z(\omega)$ is a function of force $f(\omega)$ and motion $x(\omega)$ at a contact interface. Yet it may be the case that measures of $f$ and $x$ are not attainable, but measures of other variables $Q_1(\omega), Q_2(\omega)$, etc. are. If $f$ and $x$ can be expressed in terms of the known $Q$ variables, then an estimate of mechanical impedance is attainable even when the sensors do not directly correspond with the force and motion states. The equations below show how mechanical impedance could be dependent on known variables $Q_n$ in lieu of the unknown variables $f$ and $x$, assuming a linear system.

$$\begin{align*}
f(\omega) &= \text{function}(Q_1, Q_2, \ldots; \omega) \\
x(\omega) &= \text{function}(Q_3, Q_4, \ldots; \omega) \\
Z(\omega) &= \frac{f(\omega)}{x(\omega)} = \frac{\text{function}(Q_1, Q_2, \ldots; \omega)}{\text{function}(Q_3, Q_4, \ldots; \omega)}
\end{align*}$$

The challenge to this approach is determining the dynamic relationships between the measured signals and the contact force and motion and implementing the necessary computations. Later chapters discuss the modeling and validation of these relationships at greater length.

### 2.4.2 Present Sensor Configuration

In the previous section, a sensor configuration was put forward that could be used to estimate impedance with the minimal amount of information required about the device dynamics. However, we now know that other sensor configurations may be sufficient for estimating mechanical impedance if the device dynamics are known. It is worthwhile then to examine whether the existing sensor configuration installed on the device provides sufficient information to determine the mechanical impedances of the sonographer hand and patient tissue simultaneously. There are three sensors presently installed on the actuated probe – a force transducer, a motor encoder, and an accelerometer. Their arrangement within the device is shown in the schematic in Figure 2-8.
2.4.3 Measured Signals

The quantities needed to define the hand and tissue mechanical impedances remain the same – the sonographer contact force \( f_s \), the motion of the housing \( x_h \), the patient tissue contact force \( f_p \), and the motion of the probe \( x_p \). As with the previous sensor configuration, there is a three-axis accelerometer attached to the housing of the device. When aligned along the direction of motion, the accelerometer measures the acceleration of the housing with respect to the inertial frame, \( \ddot{x}_h \). The acceleration measure \( \ddot{x}_h(t) \) can be converted into an expression of the housing motion in the frequency domain \( x_h(\omega) \). This motion reading is sufficient for tracking the contact interface between the housing and sonographer hand.

\[
\text{Accelerometer} \rightarrow \ddot{x}_h(t) \rightarrow x_h(\omega)
\]
Similarly, as with the previous sensor configuration, there is a force transducer placed between the probe and linear actuator carriage that gathers a force reading $f_{FT,p}(t)$, which can then be expressed in the frequency domain $f_{FT,p}(\omega)$.

$$\text{Force Transducer} \rightarrow f_{FT,p}(t) \rightarrow f_{FT,p}(\omega)$$

As discussed in Section 2.3, if the mass and acceleration of the probe are known, then the patient contact force can be determined. However, because there is no accelerometer installed on the probe, its acceleration $\ddot{x}_p$ is no longer directly accessible and the probe motion needs to be determined from the available sensors and the device dynamics.

Unlike the sensor configuration presented in the previous chapter, the actuated probe presently has an optical quadrature encoder attached to the device motor, which measures the rotation of the motor armature with respect to the device housing, $\theta_{MD}(t)$. This time signal can similarly be expressed as the motor armature motion in the frequency domain $\theta_{MD}(\omega)$. Although this is a motion measure, the motor rotation does not convert to the motion of either the probe or the housing by a simple proportional relationship, so further analysis is needed in order to determine how the encoder reading can be useful for estimating $x_p$ and $f_s$.

$$\text{Encoder} \rightarrow \theta_{MD}(t) \rightarrow \theta_{MD}(\omega)$$

It is also worth noting for the dynamic analysis that the torque input from the motor $\tau_m$ can be determined from the input command if the dynamics of the actuation controller are known. Future chapters will discuss these controllers at further length but at present it is assumed that this relationship is well-established. The frequency domain expression of the motor torque is denoted by $\tau_m(\omega)$.

$$\text{Motor Input} \rightarrow \tau_m(t) \rightarrow \tau_m(\omega)$$

### 2.4.4 High-Level Dynamics Analysis

In order to determine how states are related, we begin by re-examining Figure 2-8. The only external sources of energy acting on the actuated probe are the sonographer force, the patient force, and the motor torque. The motor torque acts specifically on the motor armature inertia and therefore has a direct effect on its motion $\theta_{MD}$. Additionally, the motor rotation is affected by the internal device dynamics, which are dependent on the relative motion between the motor,
carriage, and housing components. The dynamics governing the motor armature motion can then be expressed as a function of these variables.

\[ f(\theta_{MD}; t) = f(\tau_M, x_h, x_p; t) \]

The expression above also takes into consideration the time derivatives of the variables noted. As can be noted, the motor rotation \( \theta_{MD} \), housing motion \( x_h \), and torque input \( \tau_M \) are known or measured quantities. Therefore, if the dynamics governing the motor armature motion were known, or modeled with sufficient accuracy, then the probe end motion \( x_p \) could be determined from the available data.

\[ \tau_M, x_h, \theta_{MD} \rightarrow x_p \]

Similarly, the motion of the housing \( x_h \) is governed by the sonographer contact force and the internal device dynamics. This allows us to establish the following relationship.

\[ f(x_h; t) = f(f_s, \theta_{MD}, x_p; t) \]

As before, the only unknown is identified to be the sonographer force. Given a sufficiently accurate model of the housing dynamics, the sonographer contact force could be determined from the available experimental data.

\[ x_h, \theta_{MD}, x_p \rightarrow f_s \]

Lastly, now that the probe inertial position is known, the patient contact force can be determined from the force transducer reading following dynamic analysis performed in Section 2.3.2.

\[ x_p, f_{FT,p} \rightarrow f_p \]

The contact forces at each end of the device, along with the motion of those contact interfaces, are all determinable. Therefore, the mechanical impedances of the sonographer hand and patient tissue can be simultaneously determined from a single set of trials. Although the device dynamics have not been modeled thoroughly, this high-level analysis of the principal device components and the external forces they interact with provides us with the assurance that estimating mechanical impedance at both ends of the device is an attainable functionality using ideal versions of the existing sensors. A similar assurance when using the actual existing sensors, however, depends on the ability of the sensors to provide adequate performance when collecting data.
Chapter 3

Sensory Limitations

3.1 Sensor Calibration

3.1.1 Motor Encoder

The sensor that measures the motion of the motor on the device is a quadrature optical encoder. The code wheel on the encoder has two tracks that can individually measure position with a precision of 512 counts per rotation. However, the two tracks are offset from each other so that when their signals are read together they can measure rotation with a precision of 2048 counts per rotation and relay information on the direction of motion. (Maxon Motors, 2016) The accuracy of the device was confirmed by checking the reading output at various positions and through multiple revolutions, comparing against markers placed on the motor housing and shaft coupling. Figure 3-1 shows the markers used to visually confirm the calibration. The encoder count measured after each revolution was consistently a multiple of 2048 counts, with a maximum error of $\pm 3$ counts, or $\pm 0.53^\circ$. This error could be easily attributed to human error in the visual inspection, but even if it were due to sensor error the encoder static measures would still be deemed sufficiently accurate for this study. The software was updated to carry out the proper conversion from counts to radians.

3.1.2 Force Transducer

One of the key inputs to the Ultrasound Force Control Probe is the force reading from the Futek LSB200 load cell. This force transducer has a $\pm 45 \, N$ force-measuring capability along a single axis. (Futek Advanced Sensor Technology, 2016) This device has a rated output of $2 \, mV/V$ and an input of $\pm 5 \, V$, so it can measure forces at a resolution of $0.018 \, N$. While weighing only 9 g, the Futek part can withstand loads up to 445 N before failure. The load cell is placed between
the linear actuator cart and ultrasound probe clamp in order to measure the amount of force applied by the probe to the patient. (Gilbertson, 2010; Gilbertson, 2014)

Figure 3-1: A strip of tape was attached to the motor support structure with a marking that aligns with a similar marking on the motor coupling once per revolution. During calibration, these marking were used to visually inspect the encoder count after each revolution.

Figure 3-2: At left, the weight values of the calibration objects were confirmed using a precision scale. The calibration objects were then set on the probe contact surface in order to compare the force transducer readings with the expected weight of the objects. A vice was used to maintain the force transducer oriented vertically.
To ensure that the voltage reading from the force transducer was converted to the appropriate force equivalent, the force transducer was calibrated by applying known weight forces to it. To ensure that the force transducer measurement axis ran parallel with gravity, a spirit level was used to confirm that the probe end surface was parallel to the ground surface. A vice was used to maintain this orientation. A set of calibration objects was assembled with weights spanning the range of forces experienced during normal use. A precision scale, seen in Fig. 3-2, was used to confirm their nominal weight values. Individual and sets of calibration objects were placed on the probe end surface, as shown in Figure 3-2, to provide a thorough set of samples.

The steady state voltage readings for each applied weight were compiled and plotted to find the voltage-force relation. After adjusting the conversion and compensating for the constant bias, the static sensor error was approximated at 0.16% ± 0.08% based on 21 readings at various applied forces between 0 N and 6.4 N. Within the range tested, the force transducer static measures were deemed sufficiently accurate for this study. Further details on this calibration are found in Appendix A.

3.1.3 Accelerometer

The ADXL335 accelerometer presently on the actuated probe has a nominal operation range of ±35.3 m/s² and a nominal sensitivity of 3.3 cm/s². (Analog Devices, 2010) The three-axis accelerometer was calibrated by clamping it to an unperturbed surface with its y-axis positive direction aligned parallel to gravity. This orientation setup was similar to the force transducer calibration setup, shown in Figure 3-2. The sensor reading confirmed that the y-axis value was maximum at the clamped position. A sensor data sample of 5 seconds was recorded in the final configuration. The average of the data was calculated and noted as corresponding to an acceleration of 9.8 m/s². This process was repeated with the y-axis negative direction aligned with gravity; the resulting average value corresponds to an acceleration of −9.8 m/s². The conversion $k_{acc}$ from raw sensor reading to equivalent acceleration measure was performed as follows.
\[ k_{acc} = \frac{Raw_{+y} - Raw_{-y}}{Acc_{+y} - Acc_{-y}} = \frac{Raw_{+y} - Raw_{-y}}{2(9.8 \frac{m}{s^2})} \]

\[ \text{Acceleration} = k_{acc}(\text{Raw Sensor Value}) \]

This process was repeated for the x-axis and z-axis. This confirmed that the sensor was working as expected. Because of orthogonality, the y-axis accelerometer reading is expected to be zero when the x-axis or z-axis is aligned with gravity. The resulting error from zero was within the standard deviation of the sensor reading, 0.03 m/s^2. This confirmed that the prior calibration correctly converted to a zero acceleration reading and indicated that the accelerometer had a steady-state accuracy that is suited for this study.

### 3.2 Motion Sensory Noise Analysis

Before measuring mechanical impedance with the actuated ultrasound probe, it was necessary to quantify the confidence placed on the sensor readings. In addition to the steady state accuracy of the sensors, this included characterizing the noise associated with the sensor readings throughout the frequency range of interest. This analysis was first performed on the motion measurement instruments – the accelerometer and the motor encoder.

#### 3.2.1 Accelerometer Noise Performance

For the experimental noise characterization, the accelerometer was placed unperturbed on a static surface. Data of the static configuration was recorded for a period of over 40 seconds, as shown in Figure 3-3 in raw signal units. The signal was then adjusted for trending and mean-centered at zero. Figure 3-4 shows the de-trended signal in acceleration units, \( \frac{N}{kg} = \frac{m}{s^2} \).
Figure 3-3: Raw sensor reading of the accelerometer placed unperturbed on a static surface for over 40 seconds.

Figure 3-4: De-trended and mean-centered accelerometer reading, corresponding to the unperturbed static measure presented in Figure 3-3.
From the time series of the data, the noise associated with the signal can be quantified by its standard deviation of $0.0333 \text{ m/s}^2$. However, as can be seen from the plot, there appears to exist an underlying periodic component in the signal at a low frequency between 0.2 Hz and 0.3 Hz that is significantly contributing to the variance in the signal. This phenomenon was noted but not addressed in order to examine the signal noise at frequencies between 0.5 Hz and 50 Hz, the frequencies of interest in this study.

During the mechanical impedance measurement trials, the probe is actuated at a broad range of frequencies. Therefore, it is necessary to observe the frequency dependency of the noise associated with each sensor in order to evaluate the excitation frequency range through which its data can be confidently utilized for the impedance approximations. A discrete Fourier transform was performed on the square of the de-trended accelerometer noise in order to express the sensor variance in the frequency domain. The built-in MATLAB FFT algorithm was used to perform the discrete Fourier transform shown in Figure 3-5.

![Variance of Accelerometer Signal](image)

**Figure 3-5:** After the static accelerometer signal was de-trended and mean centered, the Fourier transform of subsequent error signal was computed to observe the signal variance as a function of frequency. Single sample population represented.
The power spectrum in Figure 3-5 indicates that the noise level of the acceleration reading remains relatively constant over the range of 0.5 Hz to 50 Hz. Over this range, the sensor noise can justifiably be approximated as white noise. However, this is only a single sample of the power spectrum of the sensor. As a result of the single sample, the variance of the estimate is large. (Bendat & Piersol, 1971) The analysis can be improved by splitting the original signal into separate time windows and evaluating the mean of the individual Fourier transforms. The 10 second windows, set as such to observe the noise above 0.1 Hz, have a 50% overlap to maintain each data sample effectively independent.

Figure 3-6: In order to increase the number of samples from which the signal noise is determined, the original accelerometer signal is divided into windows of 10 seconds to conserve data at frequencies above 0.1 Hz. Including windows with a 50% overlap increases the population size while retaining effective independence.
The power spectra from the seven windows were averaged into a single variance plot that more clearly indicates that the noise associated with the accelerometer reading is constant at approximately $10 \text{ m}^2/\text{s}^4$ from 1 Hz to 50 Hz. The variance is higher at lower frequencies and decreases substantially at higher frequencies.

This examination allowed us to describe the accelerometer noise performance mathematically as additive white noise. As a function, white noise is a random signal that has a constant power spectrum, adding equal amounts of energy to the signal at each frequency. The autocorrelation of a white noise process yields a constant variance across all frequencies. For the accelerometer, this assumption is approximately true within the frequency range of 1 Hz to 50 Hz, when the signal has an average variance of $\sigma_{wa}^2 = 11.01 \text{ m}^2/\text{s}^2$.

$$a_{\text{measured}} = a_{\text{true}} + w_a(t)$$

Where $w_a(t)$ has a variance of $\sigma_{wa}^2 = 11.01 \text{ m}^2/\text{s}^2$

![Variance of Accelerometer Signal](image)

Figure 3-7: After the static accelerometer signal was de-trended and mean centered, the Fourier transform of the subsequent error signal was computed to observe the signal variance as a function of frequency. Average of seven samples.
3.2.2 Motor Encoder Noise Performance

The noise associated with the motor encoder also needed to be assessed in order to evaluate the confidence that could be placed in position measurements taken with the device. A time sample of the encoder reading is shown in Figure 3-8, taken during a trial where the motor was slowly rotating prior to stopping. An unperturbed static measure could not be performed because the static frictional forces of the motor maintain the encoder reading constant and would therefore yield no information on the dynamic performance of the sensor.

![Time Sample of Non-Static Encoder Noise](image)

Figure 3-8: Raw sensor reading of the encoder as the motor rotates to a halt at the slowest velocity attainable. The static trials performed could not be used to assess the sensor variance at non-zero frequencies.

The encoder signal was de-trended and mean-centered at zero. The time windows were chosen as 50% overlaps of 20 second segments. As Figure 3-8 shows, there are portions in the data with a constant position reading through segments as long as 10 seconds. An analysis of one of these static sections would yield zero variance, which is unrepresentative of the dynamic
response. Setting the window size to 20 seconds avoids a discrete Fourier transform of a primarily static set of data. This also means that accelerometer noise can be analyzed for frequencies above 0.05 Hz instead of 0.1 Hz as was done for the encoder. This has little effect on the overall analysis since the primary frequency range of interest is between 0.5 Hz and 50 Hz.

Figure 3-10 shows the average power spectrum of the five window segments. In a similar manner as the accelerometer, the encoder has a constant level of variance within the frequency range from 1 Hz to about 50 Hz. As such, the encoder noise can be approximated mathematically as additive white noise in this region. Within the frequency range of 1 Hz to 50 Hz, the encoder signal has an average variance of \( \sigma_{w\theta}^2 = 315.51 \text{ } ct^2 = 0.0031 \text{ } rad^2 \), distributed equally among each frequency.

\[
\theta_{measured} = \theta_{true} + w_{\theta}(t)
\]

Where \( w_{\theta}(t) \) has a variance of \( \sigma_{w\theta}^2 = 315.51 \text{ } ct^2 = 0.0031 \text{ } rad^2 \)

Figure 3-9: In order to increase the samples from which the signal noise is determined, the original encoder signal is divided into windows of 20 seconds to conserve data at frequencies above 0.05 Hz and avoid exclusively static data. Including windows with a 50% overlap increases the population size while retaining effective independence.
Figure 3-10: After the quasi-static motor encoder signal was de-trended and mean centered, the Fourier transform of the subsequent error signal was computed to observe the signal variance as a function of frequency. Average of five samples.

3.2.3 Housing Motion Estimate

The inertial motion of the housing is measured by the accelerometer installed on the device. Since the motion of the probe is uniaxial, only one of the instrument’s axes is of interest when it is set parallel to the motion.

\[
\Delta x_h = \int \int a_{measured} \, dt = \int \int (a_{true} + w_a(t)) \, dt
\]

As mentioned earlier, \( w_a(t) \) has a variance of \( \sigma_{wa}^2 = 11.01 \, m^2/s^2 \), meaning that the measure of the acceleration of the housing end is expected to have a noise level with a standard deviation of \( \sigma_{wa} = 3.32 \, m/s \) constant through the frequency range of interest. This is not an insignificant amount; it is roughly equal to 0.34 g and it is 100 times greater than the standard deviation of the static accelerometer reading. To see how this noise affects the variance of the housing velocity and position measures \( w_a(t) \) needs to be integrated once and twice,
respectively. Integrals were calculated by dividing each variance value in Figure 3-7 by their respective $\omega^2$.

As Figure 3-11 shows, the housing velocity $\dot{x}_h$ estimate, when measured using the accelerometer, has an associated noise component that decreases with frequency. At 1 Hz, the noise has a variance of $2.9 \, m^2/s^2$, which drops to roughly $0.0004 \, m^2/s^2$ at 50 Hz. Since the noise varies with frequency, one way to examine its effect on the impedance measurement trials is to focus on the noise levels at the logarithmic middle of the frequency range of interest: 5 Hz. Preliminary data from one of the impedance measurement trials was used as a reference point. The trial chosen was a sinusoidal excitation at 5 Hz of an artificial Phantom tissue with a modulus of elasticity of 12.4 kPa; this was the median stiffness from the tissues available. Further details on these trials are covered in Chapter 8. For this contact interaction the velocity has an amplitude of approximately $47 \, mm/s$ at 5 Hz. At this frequency, the coefficient of variation associated with the housing end inertial velocity estimate, or the ratio of standard deviation and mean amplitude of the signal, is 1.0 or 100%.

Figure 3-11: Variance of the accelerometer signal when integrated to a velocity measure, as a function of frequency.
For the previously-described contact interaction, the device translation exhibits a 3 mm amplitude at 5 Hz. At this frequency, the position estimate from the accelerometer signal exhibits a variance of $10^{-5} \text{ m}^2$. This means that the inertial position, as estimated from the accelerometer measure, also has a coefficient of variation of 1.0, or 100%.

Figure 3-12: Variance of the accelerometer signal when integrated twice to a position measure, as a function of frequency.

3.2.4 Probe Motion Estimate

Since the motion of the probe is not measured directly, it is expressed as a weighted sum of known quantities. As discussed in Chapter 2, the inertial motion at the probe end of the device is equal to the sum of the inertial translation of the motor housing and the separation between the probe and the motor housing ends.

$$x_p = x_h + x_{p/h}$$

The relative motion between the two ends of the device $x_{p/h}$ is determined by the rotation of the motor. Establishing the dynamic relation between the motor armature and the probe end
is a main focus in the following chapters. For the purposes of determining whether the sensors are sufficiently precise in estimating motion, a simple model of the device will suffice. In this model, it is assumed that the timing belt between the motor and ball screw has infinite stiffness and that the ball screw experiences no backlash. Under this model, the relative translation between the probe and housing, the screw rotation \( \theta_S \), and the motor armature rotation \( \theta_M \) are proportional.

\[
\Delta x_{p/h} = n \theta_S = n \left( \frac{r_S}{r_m} \theta_M \right) = n \left( \frac{r_S}{r_m} \left[ \frac{1}{k_{enc}} \right] \right)
\]

The proportionality constants include \( n \) - the screw pitch relating screw revolution to probe translation, \( \frac{r_S}{r_m} \) - the gear ratio between the motor shaft and ball screw, and \( k_{enc} \) - the number of encoder counts per revolution.

\[
\Delta x_{p/h} = \frac{1}{rev} \theta_M = \frac{1}{2\pi \ rad} (\theta_{true} + w_\theta(t))
\]

The variance exhibited by the estimate of the relative position \( x_{p/h} \) is a direct result of the variance in the encoder white noise. Therefore, the variance associated with this estimate is \( \sigma^2_{xp/h} = 0.00008 \ mm^2 \) on average within the region of 1 Hz to 50 Hz. During the aforementioned contact interaction with the 12.4 kPa tissue at 5 Hz excitation, the device exhibits a 3 mm translation amplitude. This means that the relative position measure exhibits a coefficient of variation of 0.003, or 0.3%, due to encoder sensory noise.

The total uncertainty in the estimate of the inertial motion of the probe end of the device is a sum of the uncertainty associated with the inertial motion of the housing end and the relative position between the two ends. As covered earlier in this chapter, the measure of inertial position of the housing end \( x_h \) exhibits a coefficient of variation above 100% for the representative contact interaction examined. This means that the coefficient of variation for the estimate of the inertial motion of the probe end will also be above 100%.

### 3.2.5 Implications of Motion Sensory Noise

The amount of uncertainty associated with the existing accelerometer is unacceptably high for estimating position or velocity in the context of the intended impedance measurement. This means that the inertial motion of the device ends cannot be confidently measured using the
existing sensors on the actuated ultrasound probe. The ability to confidently estimate inertial motion remains a pre-requisite for simultaneous measurement of mechanical impedances due to contact interactions at both ends of the device. As an exception, relative motion might suffice if the sonographer and tissue impedances were non-zero over sufficiently different frequency ranges. However, these properties are unknown so this approach cannot be implemented. The replacement of the existing accelerometer with a sensor of greater dynamic precision is needed in order to carry out simultaneous impedance measurements.

On the other hand, the estimate of relative position between the probe end and housing end of the device using the motor encoder is expected to experience a relatively insignificant coefficient of variation of 0.3% due to the sensor. As the following chapter discusses, the ability to confidently measure this relative motion leaves the possibility open for measuring mechanical impedance under particular circumstances without the need of the accelerometer reading.

3.3 Force Sensory Noise Analysis

3.3.1 Force Transducer Noise Performance

Since mechanical impedance is the mapping between motion and force, the force sensor noise performance also needs to be characterized. The single force sensor on the device is the force transducer attached beneath the probe. In order to gather data on the load cell performance, the probe was held unperturbed in a static position for a period of over 12 seconds. The raw sensor signal measured during this time is shown in Figure 3-13. The data collected from this steady signal was then de-trended and mean-centered at zero, as is shown in Figure 3-14 in the corresponding force units. The data was split into independent segments of 4 seconds with a 50% overlap. The additional windows increased the sample population from which the sensor noise is determined while still retaining information above 0.25 Hz and independence from each other. The window size was chosen as 4 seconds in order to have at least 5 samples from the data as was done for the encoder.
Figure 3-13: Raw sensor signal of the force transducer maintained unperturbed in a nominally static configuration.

Figure 3-14: In order to increase the samples by which the signal noise is determined, the original load cell signal is divided into separate windows. The windows were kept to segments of 4 seconds, which conserves data at frequencies above 0.25 Hz. Including additional windows at a 50% overlap increases the population size while retaining independence from each other.
A Fourier transform of each of the window segments was performed by converting the noise data into the frequency domain using the MATLAB built-in FFT algorithm. The square of the resultant magnitude is equivalent to the signal variance as a function of the frequency. The processed data from the five window segments were averaged and displayed as the logarithmic plot in Figure 3-15. Within the frequency range of interest, 1 Hz to 50 Hz, the signal is generally constant and has an average variance of 0.25 $N^2$.

However, there appear to exist regions of amplified variance near 10 Hz, 20 Hz, and 30 Hz that raise suspicion about a constant variance assumption. In order to determine whether this is an intrinsic sensor phenomenon or due to the external perturbations particular to the trial analyzed, additional data from static, unperturbed force transducer calibration trials were examined. The total data amounted to 158 seconds. Because of the additional time, the window size is changed to 10 seconds to examine the data above 0.1 Hz. This yields 30 independent window segments. The averaged autocorrelation magnitude from these windows are displayed as the logarithmic plot in Figure 3-16.

Based on the plot in Figure 3-16, we can more confidently claim that the load cell exhibits a constant variance over the range of 1 Hz to 50 Hz. The average variance within this region is $\sigma_f^2 = 0.25 \, N^2$, the same amount calculated from the single trial. There still appears to be some heightened activity between 8 and 10 Hz, with the average variance in this region of $0.41 \, N^2$. However, this is much lower than the $1 \, N^2$ observed from Figure 5-16, indicating that this amplification was likely due to external vibrations unaccounted for during a portion of the trials. The force transducer can be justifiably approximated mathematically as introducing additive white noise $w_f(t)$ within the frequency range of 1 Hz to 50 Hz. The sensor has an average variance of $\sigma_f^2 = 0.25 \, N^2$, approximated to be distributed equally among each frequency.

$$F_{measured} = F_{true} + w_f(t)$$

Where $w_f(t)$ has a variance of $\sigma_f^2 = 0.25 \, N^2$
Figure 3-15: After the static force transducer signal was de-trended and mean centered, the Fourier transform of the subsequent error signal was computed to observe the signal variance as a function of frequency. Average of five sample populations.

Figure 3-16: After multiple static force transducer signals were de-trended and mean centered, the Fourier transform of the subsequent error signal was computed to observe the signal variance as a function of frequency. Average of 30 sample populations.
3.3.2 Patient Force Estimate

As would be expected, the contact force between the device probe end and the patient is measured primarily by the force transducer. However, even with the simple model assumed for the purposes of this chapter it is necessary to account for the contributions due to the inertia of the device components. The schematic in Figure 3-17 displays how the external forces, device component motion, and load cell sensor interact with each other. The equations of motion governing the patient force estimate are derived from the free-body diagram on the right side of the figure.

\[ m_{p1} \ddot{x}_p = F_{ft} - F_{patient} \]

\[ F_{patient} = F_{ft} - m_{p1} \ddot{x}_p = [F_{ft, true} + w_f(t)] - [m_{p1}(\ddot{x}_h + \ddot{x}_{p/h})] \]

Figure 3-17: Free-body diagram of the simple model of the actuated probe. The probe and housing ends of the device are attached but can move with respect to the other. The two external forces are due to the external contact with the sonographer hand and patient tissue. The force transducer separates the probe end and measures the load transferred across it.

We first examine the uncertainty contributions of the force transducer. The noise due to the sensor was previously approximated as white noise with a constant variance across all frequencies of interest. The corresponding standard deviation between 1 Hz and 50 Hz is \( \sigma_f = 0.50 \text{ N} \), on average. Using the 12.4 kPa artificial tissue excited at 5 Hz as the standard, the amplitude of the interaction force between the probe and the tissue is of 1.25 N. This means that
the patient force measure is expected to experience a coefficient of variation of 0.4 due to the force transducer sensory noise.

The uncertainty contributions due to the probe mass inertia estimates can now be examined. As discussed earlier in the chapter, the housing acceleration is measured with the accelerometer, which introduces a sensory white noise $w_a(t)$ with a constant standard deviation of $\sigma_a = 3.32 \, m/s^2$ across the frequencies of interest. The mass $m_{p1}$ is approximately equal to 68 g, based on estimates described in Chapter 7 and Appendix B. Therefore, the noise associated with the accelerometer on the force estimate is expected to be $\sigma = 0.23 \, N$, or 16.5% of the expected force measure. However, the acceleration experienced during this trial has an amplitude of $2.96 \, m/s^2$, meaning that the estimate of inertia using the accelerometer actually has a coefficient of variation of 1.1.

$$F_{patient} = F_{ft} - m_{p1} \left[ (a_{true} + w_a(t)) + \frac{d^2}{dt^2} \left( \frac{1 \, mm}{2\pi \, rad} \right) (\theta_{true} + w_\theta(t)) \right]$$

The translation of the probe relative to the acceleration of the housing is estimated with the motor encoder. The end-to-end relative acceleration would then be the second time derivative of the encoder signal. The motor encoder has an associated sensory noise, the white noise $w_\theta(t)$, which has an equivalent constant standard deviation of $\sigma_{xp/h} = 0.009 \, mm$ after the signal is converted to translation. However, this uncertainty will no longer be constant when the white noise is differentiated twice. The frequency spectrum can be derived by multiplying the standard deviation values by the frequency term $\omega$. The resulting end-to-end relative acceleration standard deviation frequency distribution is plotted in Figure 3-18. At 5 Hz, the standard deviation associated with the encoder acceleration estimate is $\sigma = 0.0089 \, m/s^2$. Using the acceleration amplitude of $2.96 \, m/s^2$ as the standard, the coefficient of variation associated with the encoder is 0.3%.
3.3.3 Hand Force Estimate

In a similar manner to the patient force, the hand force sensory noise analysis can be determined using the schematic in Figure 3-17. The equations of motion governing the patient force estimate are derived from the free-body diagram on the left side of the figure.

\[
m_{p2} \ddot{x}_p + m_H \ddot{x}_h = F_{\text{hand}} - F_{\text{ft}}
\]

\[
F_{\text{hand}} = F_{\text{ft}} + m_{p2} (\ddot{x}_h + \ddot{x}_{p/h}) + m_H \ddot{x}_h
\]

\[
= [F_{\text{ft,true}} + w_f(t)] + m_{p2} \left[ \frac{d^2}{dt^2} \left( \frac{1 \text{ mm}}{2\pi \text{ rad}} \right) (\theta_{\text{true}} + w_\theta(t)) \right] + (m_{p2} + m_H) [a_{\text{true}} + w_a(t)]
\]

As with the patient force estimate, all three sensors are involved in determining the hand force. We first examine the uncertainty contributions of the force transducer. The load cell white noise has a corresponding standard deviation between 1 Hz and 50 Hz of \( \sigma_f = 0.50 \, N \), on average. As with the patient force, the hand force measure has a coefficient of variation of 0.4 due to the force transducer sensory noise.
Based on estimates described in Chapter 7 and Appendix B, the masses $m_{p2}$ and $m_H$ are approximately equal to 68 g and 330 g, respectively. The housing inertial acceleration is measured with the accelerometer, which introduces a sensory white noise $w_a(t)$ with a constant standard deviation of $\sigma_a = 3.32 \text{ m/s}^2$ across the frequencies of interest. As mentioned before, this results in a coefficient of variation of 1.1 when estimating inertia at the 12.4 kPa tissue trial at 5 Hz.

The acceleration of the probe relative to the acceleration of the housing is estimated by deriving the motor encoder signal twice. As discussed in the previous section, the coefficient of variation associated with this measure is 0.3%.

### 3.3.4 Implications of Force Sensory Noise

For both the hand force and patient force estimates, the force transducer introduces a noise component with a standard deviation of $\sigma_f = 0.5 \text{ N}$, which translates to a 40% coefficient of variation in the examined case. This is higher than would be desired; a 20% total noise-to-mean ratio or lower would be preferable for all the measures of interest. However, the amount of noise associated with the load cell is still low enough to distinguish a waveform. Therefore, an estimate of the phase of a force sinusoid can be reasonably determined from the raw data. In order to determine the amplitude of such a sinusoid, additional filtering would be required to remove the noise at high and low frequencies and sharpen the signal exclusively about the frequency of excitation. So while improvements in the force sensor are recommended, the existing force transducer can be used for impedance estimation with additional signal processing.

The amount of uncertainty associated with the existing accelerometer can be best summarized by a 1.1 coefficient of variation when estimating inertia. The noise due to the sensor is higher than the mean value of the signal. This only serves to corroborate the previous conclusion that the existing accelerometer is unsuited for mechanical impedance estimation at this scale using the actuated probe.

When compensating for the inertia due to the motion between the probe and housing ends of the device, the motor encoder has an associated 0.3% coefficient of variation. This is a relatively insignificant amount. As a sensor, the existing motor encoder is expected to provide precise readings and is therefore fully suited for mechanical impedance estimation.
Chapter 4

Consecutive Impedance Measurement

4.1 Patient Impedance, Sonographer Immobilized

As concluded in the previous chapter, the accelerometer measure includes an amount of sensory noise that is greater than the expected signal mean and is therefore inadequate for this study. The accelerometer would have provided us with an estimate of the motion of the device housing with respect to the inertial frame. Due to the motor encoder, we still have an estimate of the motion of the housing relative to the probe end of the device. However, the mechanical impedances of the patient tissue and sonographer hand are defined by the inertial motions of the points of contact. Additional knowledge is needed about the location of the device in space in order to estimate the impedance at both contact interfaces simultaneously.

Despite the sensory limitations of the device at present, mechanical impedance of the two interfaces may still be determined separately using the actuated probe under particular circumstances. The first circumstance investigated is the case where the sonographer is replaced by a rigid clamp while the probe makes contact with the patient tissue. In this scenario, the housing end of the device is immobilized and its position with respect to the inertial frame is once again known since it remains constant throughout operation. A diagram of this configuration is shown in Figure 4-1, the force transducer and motor encoder installed on the device are the only sensors used to examine the device interaction with the patient contact force.

\[ x_h = \text{constant} \]

Since the variables \( x_h, \theta_{MD}, f_{FT,p}, \) and \( \tau_M \) are known, the patient tissue mechanical impedance can be determined in the same manner discussed in Section 2.4.
4.2 Sonographer Impedance, Patient Immobilized

We can now similarly examine the circumstance where the probe is immobilized and the sonographer grasps the device as if conducting an ultrasound scan. By rigidly clamping the device probe, the motion of the probe end is known, since its position with respect to the inertial frame remains constant throughout a trial. On the other hand, the motion of the housing end needs to be determined from the available sensors. The setup and sensor configuration is shown in Figure 4-2; the force transducer and motor encoder are the sensors used to examine the device interaction with the sonographer hand.

\[ x_p = \text{constant} \]

The variables \( x_p, \theta_{MD}, f_{FT,p}, \) and \( \tau_M \) are known through an impedance measurement trial performed with the immobilized probe. Since the probe remains static in this configuration, the...
force from the transducer $f_{FT,p}$ is equal to the contact force between the probe and the clamp $f_p$. The motion of the housing end $x_h$ can be determined from the equation of motion of the probe dynamics since it is the only unknown variable.

$$f(x_p; t) = f(f_p, x_h, \theta_{MD}; t)$$

$$f_p, x_p, \theta_{MD} \rightarrow x_h$$

Similarly, the contact force between the housing end of the device and the sonographer hand $f_s$ can be determined from the equation of motion of the housing end dynamics.

$$f(x_h; t) = f(f_s, \theta_{MD}, x_p; t)$$

$$x_h, x_p, \theta_{MD} \rightarrow f_s$$

The sonographer hand mechanical impedance can be determined from these two variables by mapping their relationship in the Laplace domain. The definition of impedance below only applies if the system is linear.

$$Z_s = \frac{\text{func}(f_s, f_s, \dot{f}_s, \ddot{f}_s, \ldots; t)}{\text{func}(x_h, \dot{x}_h, \ddot{x}_h, \ldots; t)} = \frac{f_s(\omega)}{x_h(\omega)}$$

Figure 4-2: Sensor configuration used for sonographer measurement with the patient end immobilized. A single force transducer is attached at the patient end of the device and an encoder measures the rotation of the motor. The probe clamp is immobilized to the inertial frame.
4.3 Limitations of Proposed Methods

By immobilizing one end at a time, as described in the previous sections, the mechanical impedances of the patient tissue and the sonographer hand can be determined separately. Determining the patient tissue impedance in this manner would be sufficient for the purposes of determining the progression of medical conditions. A human tissue can be approximated to have time-invariant impedance properties that do not vary due to the experimental procedures used. This includes the assumption that the patient does not contract any surrounding muscles as a conscious reaction to periodic force excitations applied on the tissue of interest. The obvious drawback of immobilizing the housing end of the device is that impedance of the sonographer hand is not determined since the sonographer is not involved in the procedure.

However, determining the hand impedance by immobilizing the probe end of the device is inadequate for examining the human control of this hand-operated device. The mechanical impedance observed at the human hand will be affected by factors such as arm configuration and muscular contraction. The effective inertia observed at the hand interface is at least partially due to the mass distribution of the upper and lower arm. Therefore, changes in the orientation of the sonographer arm will lead to differences in effective inertia. Having a physician hold an ultrasound probe that is partially immobilized prevents them from adjusting the device location and orientation to match typical clinical conditions. This results in an arm configuration that may not adequately represent clinical practice.

When muscles are contracted they produce an actuating force that is accompanied by an increase in tissue stiffness. When antagonist and agonist muscles co-contract, it leads to overall stiffening of the arm joint without necessarily producing a net force. (Hogan, 1984) The effective stiffness observed at the hand interface will be at least partially due to muscular contraction. Muscular contraction is controlled by the neural processes of the physician. The associated muscular contraction commands may depend on an assortment of variables which could include perceived stability, patient tissue impedance, arm configuration, and force applied. The concern is that a partially immobilized device provides guaranteed stability of the device angular orientation and does not provide a deformable tissue interaction on the end opposite the operator. Additionally, restricting the orientation of the device in space limits the allowable
configuration of the sonographer arm, specifically the wrist orientation. Experiments would be limited to observing the differences in arm impedance as a function of force exerted on the device in the arm configurations allowable by the immobilized device orientation. The results conducted from such tests may provide some insights into the effects of applied force on impedance but would ultimately fail to recreate the intended operation of the device.

4.4 Hand Impedance Measure, Patient Known

For an experimental measure of physician hand impedance to provide results that represent the clinical operation of the actuated ultrasound probe, the subject needs to hold a device that is free to move in space while simultaneously remaining in contact with a human tissue. The experimental setups discussed in Chapters 2 satisfy this condition, as does the circumstance presented in Figure 4-3. Trials conducted with this setup are performed with neither end clamped - the sonographer grasps the device by the housing end while the probe makes contact with a patient tissue. The main difference between this setup and the previously noted ones is that the patient tissue impedance is characterized beforehand. This additional knowledge about the system interaction is sufficient to determine the sonographer hand impedance without requiring an additional sensor.

This approach depends on the assumption that the mechanical impedance characteristics of the patient tissue are time-invariant. In this manner, the four known variables and properties are \( f_p/x_p, \theta_{MD}, f_{FT,p}, \) and \( \tau_M \). The characterization of the tissue impedance is useful in determining the inertial motion of the probe end \( x_p \). The following equation relates the force transducer reading to the contact force experienced at the interface of the probe with the tissue. The equation takes into consideration the inertia of the probe segment \( m_{P_2} \) between the sensor and the contact location.

\[
\begin{align*}
  f_p &= m_{P_2}x_p - f_{FT,p} \\
  \frac{f_p}{x_p}(\omega) &= m_{P_2}\omega^2 - \frac{f_{FT,p}(\omega)}{x_p(\omega)}
\end{align*}
\]
Figure 4-3: Sensor configuration used to measure the sonographer hand impedance when the device is in contact with a patient tissue of known impedance properties. A single force transducer is attached at the patient end of the device and an encoder measures the rotation of the motor.

Although there are originally two unknowns, the contact force $f_p$ and the acceleration $\dot{x}_p$, examining the equation and measured signals in the Laplace domain allows us to replace the unknown contact force $f_p(\omega)$ with the mechanical impedance of the patient tissue $Z_p(\omega) = f_p/x_p(\omega)$. Mechanical impedance is a property of the tissue, so the position-to-force behavior can be known even if the position and contact force are unknown. The motion of the probe $x_p(\omega)$ can then be determined from the relationship.

$$\frac{f_p}{x_p}(\omega), m_p, f_{FT,p}(\omega) \rightarrow x_p(\omega)$$

The motion of the device housing end $x_h(\omega)$ can be determined from the equation of motion defining the dynamics of the motor armature after converting to the frequency domain.

$$f(\theta_{MD}; t) = f(\tau_{MD}, x_h, x_p; t)$$

$$\tau_{MD}(\omega), x_p(\omega), \theta_{MD}(\omega) \rightarrow x_h(\omega)$$

In addition to the housing motion, the contact force between the device and the sonographer $f_s(\omega)$ is needed to determine the hand impedance. This amount can be determined from the
dynamics of the housing end after converting to the frequency domain. The definition of hand impedance below only applies if the system is linear.

\[ f(x_h; t) = f(f_s, \theta_{MD}, x_p; t) \]

\[ x_h(\omega), x_p(\omega), \theta_{MD}(\omega) \rightarrow f_s(\omega) \]

\[ Z_s(\omega) = \frac{f_s(\omega)}{x_h(\omega)} \]

The determination of the sonographer hand impedance could therefore be performed when there exists a prior characterization of the patient impedance. It is unnecessary to use the device accelerometer or immobilize part of the device using this method. The practical implementation of this approach would be as follows. Prior to the ultrasound scan performed by the sonographer, an estimate of the mechanical impedance of the tissue of interest is conducted following the process described in Section 4.2. For the tissue characterization, the housing end of the device would need to be clamped to a rigid surface in order to immobilize its motion while the probe end remains in contact with the tissue area in question. The challenge in this process is predicting and mimicking the probe orientation and skin placement location that the sonographer will use during the ultrasound scan.

After the tissue mechanical impedance is determined, the sonographer performs the ultrasound scan on the patient tissue. The sonographer hand impedance can be determined once the physician has reached the desired arm configuration, application force, and tissue surface contact. The process for characterizing impedance involves actuating the device through the appropriate frequency range and performing the necessary calculations from the data collected. The following chapters will model the device dynamics that the calculations are dependent on and discuss how the actuated excitation is performed with the device to gather the frequency response of the system.
Chapter 5

Dynamic Model of the System

5.1 Device Substructures

Impedance measurement is a functionality that can be carried out by any device that can impose a force on an external structure and measure the amount of deformation incurred. However, the computations necessary in order to extract the desired parameters from the sensor measures may depend heavily on a competent model of the device dynamics, as is the case for the experiments performed in this study. For this reason, this chapter aims to understand how the device functions, both on its own and when it is interacting with external bodies.

In order to model the device, it is important to recognize all the individual components that make up the machine, group components that are rigidly attached together, and relate how the different component groups interact with each other. To simplify the analysis, components that are rigidly attached are grouped together into a ‘substructure’ that in effect moves as a single inertial body and that will be referenced as such throughout this chapter.

The first substructure that can be identified is the housing. This is the main body of the device, the one that the sonographer holds on to, that supports and serves as a common reference for the remaining moving parts. The housing includes the supporting structure of the linear actuator as well as the supporting structure of the motor.

A second substructure can be identified by analyzing the bottom half of the device, shown on the left side of Fig. 5-1. Within the parts shown lies the motor structure, shown isolated on the right image of the figure. Its primary component is the motor armature, whose motion is defined by the rotation about its central axis. A coupling and pinion gear are attached to the motor armature via a shaft and bearing and can be included as part of the motor structure. Since the motor housing that holds the motor armature needs to remain clamped to the motor support, it
is included within the housing structure. The same applies to the pinion housing that supports and aligns the pinion rotation.

A third substructure is identified by inspecting the power transmission from the motor pinion to the screw pinion via the belt. The belt elasticity prevents the screw pinion from being rigidly coupled to the motor pinion, so the screw rotation can be treated as independent from the motor rotation. The screw pinion and its rigidly-attached screw constitute the screw structure. The belt has dynamics that will be examined later, but it is approximated to be massless so it is not considered an inertial body or separate substructure. Additionally, the structure supporting and aligning the screw, including the track and the track ends, is rigidly attached to the motor support and can be included in the housing structure. These components are shown in Figure 5-2.

The fourth and last substructure is defined by the motion of the probe and the cart of the linear actuator. The actuator cart simultaneously slides along the linear grooves of the track and the helical grooves of the lead screw. This is the component that converts rotational motion to linear motion. Since clamp and clamp support attach the probe rigidly to the linear actuator cart, they also make up the probe structure. Figure 5-3 shows a perspective view of the parts composing the probe structure.

![Motor Housing + Armature and Pinion Housing](image)

Figure 5-1: At left is a perspective view of the bottom half of the device with belt unattached; labels indicate functional components. The pinion housing, motor support, and motor housing belong to the device housing substructure. The remaining components, isolated at right, compose the motor substructure.
Figure 5-2: Perspective view of the ball screw substructure, composed of the screw pinion and screw. The supporting track and track ends belong to the device housing substructure.

Figure 5-3: Perspective view of the probe substructure, with labels indicating its components. The motion of this structure is equivalent to the translation of the probe and actuator cart.
5.2 Device Dynamics - Undamped, No Generalized Forces

5.2.1 Definition of Coordinates

A coordinate system is defined to determine the equations of motion for the system. Impedance can be defined in any coordinate frame, but since the device, sonographer arm, and patient tissue undeniably have inertial properties, coordinates should be established with respect to an inertial frame of reference. The static bench top where the device experiments were performed was used as the reference frame from which to measure the device motions and is referred to as “ground”. In order to simplify the model, it is assumed that the device and its components only translate and rotate along a single axis - the roll axis of the lead screw, which extends from the hand of the sonographer to the tissue of the patient. This assumes that the orientation and location of the axis of motion remains constant relative to the inertial frame. The implications of this assumption are that the sonographer does not reorient the device during the measurement and that the device does not slide along the surface of the patient tissue.

The equations of motion of the system were derived from the law of energy conservation as it applies to each device substructure, the component groupings defined in section 5.1. The translation and rotation coordinates associated with each body are outlined below and shown in Figure 5-4. All the coordinates are with respect to the inertial frame. The distance units used in this study are in centimeters (cm) and the rotation units are in radians (rad).

<table>
<thead>
<tr>
<th>Component</th>
<th>Translation</th>
<th>Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing</td>
<td>$x_h, \theta_h$</td>
<td>$x_p, \theta_p$</td>
</tr>
<tr>
<td>Motor Armature</td>
<td>$x_m, \theta_m$</td>
<td>$x_s, \theta_s$</td>
</tr>
<tr>
<td>Screw</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Additionally, the timing belt is modeled as a power transmission connection between the motor and the screw, with associated stiffness and damping properties. The motion of the belt can be expressed in terms of the motion of the connected bodies, but for convenience an intermediate coordinate $\Delta s$, the elongation of the belt, is used.

The nine coordinates outlined above can be reduced by examining the constraints on the components. For instance, the probe is only free to translate relative to the housing, therefore the rotation of probe and housing assumed to be common.

$$\theta_p = \theta_h$$
Furthermore, the motor and the screw are only free to rotate, their translations are constrained and equal to the device housing translations.

\[ x_s = x_m = x_h \]

The timing belt extends around two pinions, which are fixed to the ball screw and motor shafts. As shown in Figure 5-5, the belt is stretched by the rotations of the pinions relative to each other. Differing pinion radii result in an effective gear ratio. Pinion radii and rotations are defined as \( r_{ps}, \theta_{ps}, r_{pm}, \theta_{pm} \) at screw and motor.

\[ \Delta s = r_{pm} \theta_{pm} - r_{ps} \theta_{ps} \]

This conceptualization only holds true when the pinion axes of rotation maintain a constant separation and orientation. For this reason, the pinion rotations need to be expressed taking into account the motion of their common supporting structure. In this case, the pinion rotations used are relative to the rotation of the housing. Relative coordinates are expressed by summing the absolute coordinates − the coordinates with respect to the inertial frame of reference. The subscripts of the rotations \( \theta \) refer to the motions of the motor \( m \), screw \( s \), ground \( g \), and housing \( h \) while the slash denotes "with respect to".

\[ \theta_{pm} = \theta_{m/h} = \theta_{m/g} + \theta_{g/h} = \theta_m - \theta_h \]

\[ \theta_{ps} = \theta_{s/h} = \theta_{s/g} + \theta_{g/s} = \theta_s - \theta_h \]

\[ \Delta s = r_{pm} \theta_{pm} - r_{ps} \theta_{ps} = r_{pm} (\theta_m - \theta_h) - r_{ps} (\theta_s - \theta_h) \]

Finally, the rotation of the ball screw is coupled with the translation of the probe; it is this coupling that converts rotational effort to translational effort. Therefore, these two coordinates are not independent of each other, so one can be expressed in terms of the other. The conversion is given by the screw pitch, \( n \), measured in cm/rad. In a similar manner to the timing belt, the coordinate conversion needs to account for the motion of the common supporting structure. The screw rotation and probe translation are expressed with respect to the housing rotation and translation, then converted to absolute coordinates.

\[ -n \theta_{s/h} = x_{p/h} \rightarrow n(\theta_h - \theta_s) = x_p - x_h \]

\[ \theta_s = \frac{x_h}{n} - \frac{x_p}{n} + \theta_h \]
These five relations reveal that from our nine body-defined coordinates, only four independent coordinates are needed to describe the motion of the device. The following four coordinates can be therefore chosen as the generalized coordinates of the system.

\[ x_p, x_h, \theta_h, \theta_m \]

Figure 5-4: At left is a side view of the device defining the translational coordinates of the four substructures composing the device. At right is a perspective view of the back of the device defining the rotational coordinates of the substructures; \( \theta_h = \theta_p \) (not shown).

Figure 5-5: Graphical models of the belt dynamics, modeled as a spring-damper (left) that is stretched by a length, \( \Delta s \), due to the rotations of the two pinions, \( \theta_{pm} \) and \( \theta_{ps} \) (right).
5.2.2 Kinetic Energy

The kinetic energy, $T_h$, of the housing structure has two components. The first is due to the translation of the housing along $x_h$. The second is due the rotation of the housing about $\theta_h$, or roll about the rotational axis of the device. The parallel axis theorem is needed when determining the effective rotational inertia because the device axis of rotation does not necessarily coincide with the housing structure center of mass (COM). The actual axis of rotation of the device during normal use will vary in accordance with the external applied forces and torques. As an estimate this axis is approximated to run through the COM of the device along the direction $x_h$ and $x_p$ since this is the axis of rotation when there are no external forces. Below, the distance between the housing COM and the axis of rotation is given by $d_h$, $m_h$ is the total mass of the housing substructure, and $I_h$ is the roll moment of inertia at the housing substructure COM.

$$T_h = \frac{1}{2} m_h \dot{x}_h^2 + \frac{1}{2} (I_h + m_h d_h^2) \dot{\theta}_h^2$$

The kinetic energy of the probe substructure has similar components. Since the probe and housing rotations are common, $\theta_h$ is used in place of $\theta_p$. The distance between the probe COM and its axis of rotation is given by $d_p$, $m_p$ is the total mass of the probe substructure, and $I_p$ is the roll moment of inertia at the probe substructure COM.

$$T_p = \frac{1}{2} m_p \dot{x}_p^2 + \frac{1}{2} (I_p + m_p d_p^2) \dot{\theta}_h^2$$

The motor substructure has three kinetic energy components. The first is the translational kinetic energy along the axis of motion. Since the motor and housing translations are common, $x_h$ is used in place of $x_m$. The second is due to the rotation (roll) of the motor armature within the motor housing, defined by $\theta_m$. In this case the axis of rotation coincides with the COM of the motor. However, since the motor sits within the housing, the motor COM follows a circular motion whenever the housing rotates, resulting in a third term. This circular translation is tangent to the axial translation and is centered at the housing axis of rotation. Therefore, it can be defined by the rotation of the housing $\theta_h$ and the distance $d_m$ between the motor COM and the housing axis of rotation. Below, $m_m$ is the total mass of the motor substructure and $I_m$ is the roll moment of inertia of the motor substructure at its COM.

$$T_m = \frac{1}{2} m_m \dot{x}_h^2 + \frac{1}{2} I_m \dot{\theta}_m^2 + \frac{1}{2} m_m (d_m \dot{\theta}_h)^2$$
The kinetic energy of the screw substructure can be similarly determined. Below, $m_s$ is the total mass of the screw substructure, $I_s$ is the roll moment of inertia at the screw substructure COM, and $d_s$ is the distance from the screw COM to the housing axis of rotation. Since the screw and housing translations are common, $x_h$ is used in place of $x_s$.

Screw: $$T_s = \frac{1}{2} m_s \dot{x}_h^2 + \frac{1}{2} I_s \dot{\theta}_s^2 + \frac{1}{2} m_s (d_s \dot{\theta}_h)^2$$

The kinetic energy terms can be summed and grouped accordingly.

$$T = \frac{1}{2} (m_h + m_m + m_s) \dot{x}_h^2 + \frac{1}{2} m_p \dot{x}_p^2 + \frac{1}{2} (I_h + m_h d_h^2 + I_p + m_p d_p^2 + m_m d_m^2 + m_s d_s^2) \dot{\theta}_h^2$$

$$+ \frac{1}{2} I_m \dot{\theta}_m^2 + \frac{1}{2} I_s \dot{\theta}_s^2$$

The total kinetic energy equation can be further simplified by combining inertial terms.

$$m_H = m_h + m_m + m_p + m_s + m_p$$

$$I_H = I_h + m_h d_h^2 + I_p + m_p d_p^2 + m_m d_m^2 + m_p d_m^2 + m_s d_s^2 + m_p d_p^2$$

Total: $$T = \frac{1}{2} m_H \dot{x}_h^2 + \frac{1}{2} m_p \dot{x}_p^2 + \frac{1}{2} I_H \dot{\theta}_h^2 + \frac{1}{2} I_m \dot{\theta}_m^2 + \frac{1}{2} I_s \dot{\theta}_s^2$$

Substituting $\dot{\theta}_s$ by its kinematic equivalent:

$$T = \frac{1}{2} m_H \dot{x}_h^2 + \frac{1}{2} m_p \dot{x}_p^2 + \frac{1}{2} I_H \dot{\theta}_h^2 + \frac{1}{2} I_m \dot{\theta}_m^2 + \frac{1}{2} I_s \left( \frac{\dot{x}_h}{n} - \frac{\dot{x}_p}{n} + \dot{\theta}_h \right)^2$$

5.2.3 Potential Energy

We are assuming that the device only translates horizontally. The lack of vertical motion eliminates any change in potential energy due to gravity. Therefore, the potential energy within the system is solely stored in the torque transmission belt elasticity, $k_b$. The potential energy stored in the belt is expressed as follows.

$$V = \frac{1}{2} k_b \Delta s^2 = \frac{1}{2} k_b \left( r_{pm} (\theta_m - \theta_h) - r_{ps} (\theta_s - \theta_h) \right)^2$$

Substituting $\dot{\theta}_s$ by its kinematic equivalent:

$$V = \frac{1}{2} k_b \left( r_{pm} \theta_m - r_{pm} \theta_h - \frac{r_{ps}}{n} x_h + \frac{r_{ps}}{n} x_p \right)^2$$
5.2.4 Lagrangian Derivation of Equations of Motion

The dynamics of the system can be summarized by the Lagrangian, \( L \). In non-relativistic mechanics, this function is defined as a signed sum, \( L = T - V \), of the kinetic and potential energy. The system equations can be derived from the Lagrangian if the generalized forces \( \Xi_q \) on each generalized coordinate \( q \) are known. The generalized forces include external forces, damping, and other non-conservative forces. An initial assumption of an undamped system with no external forces allows us to focus on the derivation of the Lagrangian and assume there are no generalized forces. This simplification is made in order to clarify the system dynamics.

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \Xi_q = 0
\]

In the equation above, \( q \) is a generalized position coordinate and \( \dot{q} \) is its corresponding velocity coordinate. In this system, the kinetic energy, \( T \), is a function of only the velocities while the potential energy, \( V \), is a function of only the positions. Therefore, the equation can be further simplified.

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \frac{d}{dt} \left( \frac{\partial (T - V)}{\partial \dot{q}} \right) - \frac{\partial (T - V)}{\partial q} = 0
\]

\( T \neq f(q) \quad V \neq f(\dot{q}) \)

\[
\frac{d}{dt} \frac{\partial T}{\partial \dot{q}} + \frac{\partial V}{\partial q} = 0
\]

The partial derivatives of the potential and kinetic energy functions are summarized below.

The functions \( T \) and \( V \) are defined in the previous two sections.

\[
\frac{\partial T}{\partial \dot{x}_h} = m_H \ddot{x}_h + I_s \left( \frac{\dot{x}_h}{n^2} - \frac{\dot{x}_p}{n^2} + \frac{\dot{\theta}_h}{n} \right) \quad \frac{\partial T}{\partial \dot{x}_p} = m_p \ddot{x}_p + I_s \left( \frac{\dot{x}_p}{n^2} - \frac{\dot{x}_h}{n^2} + \frac{\dot{\theta}_h}{n} \right) \quad \frac{\partial T}{\partial \dot{\theta}_m} = I_2 \ddot{\theta}_m
\]

\[
\frac{\partial V}{\partial \dot{x}_h} = k_b \left( \frac{r_{ps}^2}{n^2} x_h - \frac{r_{ps}^2}{n^2} x_p + \frac{r_{pm} r_{ps}}{n} \theta_h - \frac{r_{pm} r_{ps}}{n} \theta_m \right) \quad \frac{\partial V}{\partial \dot{x}_p} = k_b \left( -\frac{r_{ps}^2}{n^2} x_h + \frac{r_{ps}^2}{n^2} x_p - \frac{r_{pm} r_{ps}}{n} \theta_h + \frac{r_{pm} r_{ps}}{n} \theta_m \right) \quad \frac{\partial V}{\partial \dot{\theta}_h} = k_b \left( \frac{r_{pm} r_{ps}}{n} x_h - \frac{r_{pm} r_{ps}}{n} x_p + \frac{r_{ps}^2}{n} \theta_h - \frac{r_{pm} r_{ps}}{n} \theta_m \right)
\]
\[ \frac{\partial V}{\partial \theta_m} = k_b \left( \frac{-r_{ps} r_{pm}}{n} x_h + \frac{r_{ps} r_{pm}}{n} x_p - r_{pm} \theta_h + r_{pm}^2 \theta_m \right) \]

The time derivatives are summarized below.

\[ \frac{d}{dt} \frac{\partial T}{\partial \dot{x}_h} = m_H \ddot{x}_h + I_s \left( \frac{\ddot{x}_p - \ddot{x}_h}{n^2} + \frac{\ddot{\theta}_h}{n} \right) \]

\[ \frac{d}{dt} \frac{\partial T}{\partial \dot{\theta}_h} = I_H \ddot{\theta}_h + I_s \left( \frac{\ddot{x}_h - \ddot{x}_p}{n} + \ddot{\theta}_h \right) \]

The terms can now be summed to yield the four equations of motion.

\[ \frac{d}{dt} \frac{\partial T}{\partial q} + \frac{\partial V}{\partial q} = 0; \quad q = x_h, x_p, \theta_h, \theta_m \]

1. \[ m_H \ddot{x}_h - \frac{I_s}{n^2} \left( \ddot{x}_p - \ddot{x}_h \right) + \frac{I_s}{n} \ddot{\theta}_h - \frac{k_b r_{ps}^2}{n^2} \left( x_p - x_h \right) - \frac{k_b r_{pm} r_{ps}}{n} \left( \theta_m - \theta_h \right) = 0 \]

2. \[ m_p \ddot{x}_p + \frac{I_s}{n^2} \left( \ddot{x}_p - \ddot{x}_h \right) - \frac{I_s}{n} \ddot{\theta}_h + \frac{k_b r_{ps}^2}{n^2} \left( x_p - x_h \right) + \frac{k_b r_{pm} r_{ps}}{n} \left( \theta_m - \theta_h \right) = 0 \]

3. \[ (I_H + I_s) \ddot{\theta}_h + \frac{I_s}{n} \left( \ddot{x}_h - \ddot{x}_p \right) - \frac{k_b r_{ps} r_{pm}}{n} \left( x_p - x_h \right) - k_b r_{pm}^2 \left( \theta_m - \theta_h \right) = 0 \]

4. \[ I_m \ddot{\theta}_m + \frac{k_b r_{ps} r_{pm}}{n} \left( x_p - x_h \right) + k_b r_{pm}^2 \left( \theta_m - \theta_h \right) = 0 \]

5.2.5 Change of Coordinates

Now that the equations of motion of the system are derived, it is no longer necessary to continue working in the coordinates of the inertial bodies. A new set of coordinates would preferably reduce the number of terms in the equations of motion by combining similar terms and revealing the underlying dynamic structure of the system. The number of position-dependent and acceleration-dependent terms can be reduced when two of the new coordinates are chosen as \[ x_D = x_p - x_h \] and \[ \theta_D = \theta_m - \theta_h \]. The new coordinate \[ x_D \] is the translational position of the probe with respect to the housing body. In a similar manner, \[ \theta_D \] is the rotation of the motor with respect to the rotation of the housing body.

A strategy for selecting the remaining coordinates is to identify ignorable coordinates - coordinates that the Lagrangian does not depend on, such as \[ q_{\text{ign}} \] below.

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_{\text{ign}}} = \frac{\partial L}{\partial q_{\text{ign}}} = 0 \]
Ignorable coordinates may still be used to describe the degrees of freedom of the inertial bodies but are not necessarily coupled to the rest of the system, and thus provide no additional insight into the system dynamics. As of now all the translational coordinates have been defined with respect to ground; however, the device as it is currently being analyzed is not attached to ground in any way. The translational motion of the device as a whole with respect to the inertial reference frame is unknown and has no effect on the remaining dynamics of the system. The translational position of the device center of mass \( x_{CM} \) is ignorable in this model and thus it is chosen as one of the new coordinates.

Similarly, the overall rotation of the device with respect to ground is also unknown yet is presumed to have no effect on the remaining dynamics of the system. The "center of mass" rotation, or the overall rotation of the device, is expected to be ignorable and thus it is chosen as one of the new coordinates. The new coordinates in terms of the previous coordinates are outlined below.

\[
\begin{align*}
x_{CM} &= \frac{m_p x_p + m_H x_h}{m_p + m_H} \\
x_D &= x_p - x_h
\end{align*}
\]

\[
\begin{align*}
\theta_{CM} &= \frac{l_H \theta_h + l_m \theta_m + I_s \theta_s}{l_H + l_m + I_s} \\
\theta_D &= \theta_m - \theta_h
\end{align*}
\]

The new coordinates were then substituted into the equations of motion (1) through (4). Linear combinations of these equations are shown below.

(1.1) = (1) + (2)

\[
(m_H + m_p) \ddot{x}_{CM} = 0
\]

(2.1) = (3) + (4)

\[
(l_H + l_m + I_s) \ddot{\theta}_{CM} - \frac{l_s}{n} \ddot{x}_D = 0
\]

(3.1) = \( m_h (2) - m_p (1) \)

\[
\frac{m_h m_p}{2} \dddot{x}_D + (m_H + m_p) \left( \frac{l_s}{n^2} \dddot{x}_D - \frac{l_s}{n} \dddot{\theta}_{CM} + \frac{l_s l_m}{n (l_H + l_m + I_s)} \dddot{\theta}_D + \frac{k_{ps} r_{ps}^2}{n^2} x_D + \frac{k_{pm} r_{pm} r_{ps}}{n} \theta_D \right) = 0
\]

(4.1) = \( (l_H + l_s) (4) - l_m (3) \)

\[
l_m (l_H + l_s) \dddot{\theta}_D + \frac{l_s l_m}{n} \dddot{x}_D + (l_H + l_m + l_s) \left( \frac{k_{ps} r_{ps} r_{pm}}{n} x_D + k_{p} r_{pm} r_{ps} \theta_D \right) = 0
\]
5.2.6 State-Space Representation

Now we would like to represent the dynamics equations in matrix form such that we can expose the structure of the system dynamics. The equations of motion are rearranged in order to isolate the acceleration variables on one side of the equation. This can be done by writing the equations in matrix form, where \([q]\) is a vertical vector of the generalized position coordinates, \([\dot{q}]\) is a vector of the corresponding accelerations, \([K]\) is the stiffness matrix containing the coefficients of the position terms, and \([M]\) is the mass matrix containing the acceleration term coefficients.

\[
[M][\ddot{q}] = [K][q] \quad \Rightarrow \quad [\ddot{q}] = [M]^{-1}[K][q]
\]

The resulting system equations are presented below:

\[
\ddot{x}_D = \frac{k_b}{\left(\frac{I_s}{n^2}(1 + \frac{m_H}{m_p}) + m_H(1 + \frac{I_s}{I_H})\right) \left(1 + \frac{I_s}{m_p} \left(1 - \frac{r_{pm}r_{ps}}{r_{ps}}\right)\right)} \left(-\frac{r_{ps}^2}{n^2}x_D - \frac{r_{pm}r_{ps}}{n} \theta_D\right)
\]

\[
\ddot{\theta}_D = \frac{k_b}{\left[\frac{I_s(1 + \frac{m_H}{m_p}) + m_Hn^2(1 + \frac{I_s}{I_H})}{m_Hn^2(I_H + I_m + I_s) + \left(1 + \frac{m_H}{m_p}\right) \left(\frac{I_s}{I_m} + \frac{I_s}{I_H} \left(1 - \frac{r_{ps}}{r_{pm}}\right)\right)}\right]} \left(-r_{pm}^2\theta_D - \frac{r_{ps}r_{pm}}{n}x_D\right)
\]

\[
\ddot{x}_{CM} = 0 \quad \ddot{\theta}_{CM} = 0
\]

For simplicity, we substitute the inertial terms in the denominator of each equation with the representative values, \(M_x\) and \(I_\theta\). These parameters are named as such because \(M_x\) is in units of kilograms while \(I_\theta\) is in units of \(kg \cdot m^2\). The equations were then rearranged using the state-space matrix, shown below, following the form \([\dot{\rho}] = [A][\rho]\) where \(\rho\) is a state.

\[
\begin{align*}
\dot{x}_{CM} &= 0 \\
\dot{\theta}_{CM} &= 0 \\
\ddot{x}_D &= \frac{k_b}{M_x} \left(-\frac{r_{ps}^2}{n^2}x_D - \frac{r_{pm}r_{ps}}{n} \theta_D\right) \\
\ddot{\theta}_D &= \frac{k_b}{I_\theta} \left(-r_{pm}^2\theta_D - \frac{r_{ps}r_{pm}}{n}x_D\right)
\end{align*}
\]
Although they do not appear in the system equations, the velocity variables need to be included in the state-space representation to explicitly state that the acceleration variables are the second time derivatives of the position variables. The dotted lines are included for visual organization. Vertically, the blocks are separated to indicate whether the resulting equations are for a velocity or acceleration term. Horizontally, the blocks are separated by whether the associated variable being multiplied is a position or velocity term.

The most immediate observation is that the matrix is singular – there are two empty rows and columns. This confirms that for an ungrounded model of the system, the COM coordinates are ignorable and can be discarded in further analysis.

The second observation is that two acceleration variables are dependent on the two corresponding position variables. Just as importantly, the relationship of each acceleration variable with its corresponding position variable is negative. This is indicative of two inertial bodies coupled together via an elastic element with a stiffness that restores the centers of mass of the two bodies back to equilibrium. However, since the accelerations of the bodies are not dependent on their velocities, noted by the empty bottom right block, the system is undamped and will exhibit oscillatory behavior. The behavior of the system will be inspected in greater detail in the next section by analyzing its modes and corresponding frequencies. For the modes of the system to be more intuitive, the coordinates are changed to the rotations that stretch and compress the belt - the pinion rotations $\theta_{pm}$ and $\theta_{ps}$. In the equations below, $n$ is the pitch ratio, expressed in cm/rad.

Motor Rotation: $\theta_{pm} = \theta_{D}$
Screw Rotation: \[ \theta_{ps} = \theta_s - \theta_h = \frac{x_h}{n} - \frac{x_p}{n} = -\frac{x_D}{n} \]

The ‘four-state’ system matrix with the new coordinates and the ignorable coordinates removed is presented below.

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} \theta_{ps} \\ \dot{\theta}_{ps} \\ \theta_{pm} \\ \dot{\theta}_{pm} \end{bmatrix} &= \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & -k_b r_{ps}^2 / M_x n^2 & k_b r_{ps} r_{pm} / M_x n^2 & 0 \\
0 & k_b r_{ps} r_{pm} / l_\theta & -k_b r_{pm}^2 / l_\theta & 0
\end{bmatrix} \begin{bmatrix} \theta_{ps} \\ \dot{\theta}_{ps} \\ \theta_{pm} \\ \dot{\theta}_{pm} \end{bmatrix}
\end{align*}
\]

### 5.3 Estimates of System Parameters

#### 5.3.1 Part Documentation

As seen throughout the previous section, the dynamic performance of the device is dependent on a number of parameters, including inertia, stiffness, and power transmission. Therefore, it was necessary to catalog what the numerical values of each of these parameters is so that further analysis could be performed. For more intricate components, such as the motor and the linear actuator, manufacturer documentation was referenced to confirm measurements and provide estimates when measurements could not be taken.

The motor used in this iteration of the device is a slender, current-controlled Maxon brushless motor, part number 232241. The nominal voltage is 24 V and the maximum continuous current it can be driven at is 2.71 A. The rotational inertia of the rotor is \(1.27 \, g \cdot cm^2\) and the rotational inertia of the motor housing can be approximated from the external dimensions. (Maxon Motors, 2013)

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The rotational work produced by the motor is converted to linear work by the linear actuator, an NSK Monocarrier, part number MCM02005P02K. The carriage has a nominal stroke of 5.0 cm and the ball screw has a lead of 2.0 mm per rotation. The rotational inertia of the screw is \(0.93 \, g \cdot cm^2\), a value that was later confirmed using a disassembled Monocarrier. (NSK Motion & Control, 2015)
5.3.2 Power Transmission Constants

Power transmission in the device occurs between the motor and ball screw, and between the ball screw and the linear carriage. The torque produced by the motor is transmitted to the ball screw through a timing belt, which extends between a pinion mounted to the motor shaft and a pinion fastened to the ball screw. The torque transmitted and the rotation of the ball screw relative to the motor is dependent on the ratio between the pinion radii. This ratio was previously catalogued to be a 2:1 reduction and the pinion radii were confirmed through caliper measurements. (Gilbertson, 2014)

\[ r_{p,m} = 0.45 \text{ cm} \quad r_{p,s} = 0.90 \text{ cm} \]

The power transmission from rotation to linear motion occurs at the contact of the ball screw threads and the internal threads of the linear carriage. This conversion is governed by the ratio between rotational and linear motion, denoted by the ball screw lead. The lead value is provided by the manufacturer and was confirmed through a caliper measurement of the total length of the ball screw. For our purposes, the lead is expressed as:

\[ n = 0.2 \frac{\text{cm}}{\text{rev}} = \frac{0.1 \text{ cm}}{\pi \text{ rad}} \]

5.3.3 Inertial Parameters

The relevant inertial parameters in the system can be partitioned into linear and rotational inertia. Linear inertias were measured by disassembling the device and weighing each component separately using a precision scale and converting to mass. The masses of all the parts composing each substructure were summed together and catalogued in Table 5.1.

The rotational inertias of the individual parts were calculated using the weight measurements from the precision scales and relevant dimensions from caliper measurements. The rotational inertia of the motor armature is the nominal value provided by the manufacturer. The rotational inertias composing each lumped inertia term used in the model equations were summed accordingly and catalogued in Table 5.2. Further explanation on the calculations performed and full tables of the individual inertial and dimensional measurements are presented in Appendix B.
Table 5.1: Linear inertia parameters used in the device model and system equations.

<table>
<thead>
<tr>
<th>Term</th>
<th>Inertia (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_H)</td>
<td>330</td>
</tr>
<tr>
<td>(m_p)</td>
<td>136</td>
</tr>
<tr>
<td>(M_x)</td>
<td>2900</td>
</tr>
</tbody>
</table>

Table 5.2: Rotational inertia parameters used in the device model and system equations.

<table>
<thead>
<tr>
<th>Term</th>
<th>Inertia ((g \cdot cm^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_H)</td>
<td>1570</td>
</tr>
<tr>
<td>(I_m)</td>
<td>4.59</td>
</tr>
<tr>
<td>(I_s)</td>
<td>2.84</td>
</tr>
<tr>
<td>(I_\theta)</td>
<td>4.60</td>
</tr>
<tr>
<td>(I_{SS})</td>
<td>2.94</td>
</tr>
</tbody>
</table>

5.3.4 Apparent Motor Inertia

Excitation of the device is performed through the control of the brushless Maxon actuator. An important consideration when implementing impedance characterization using frequency response is the bandwidth of the excitation. A larger apparent inertia experienced by the actuator makes it more difficult to reverse directions quickly and thus limits the range of frequencies that can be analyzed with the device. The apparent inertia observed by the motor can be estimated by assuming an infinitely stiff transmission belt and no device housing rotation. The equations of motion derived in Section 5.2 can be used to make this estimate, using \(\theta_m\) as the coordinate and ignoring external and dissipative forces.

\[
I_m \ddot{\theta}_m = f_b r_{pm} + \tau_m \quad \quad I_{SS} \ddot{\theta}_s = -f_b r_{ps}
\]

\[
\theta_s = \frac{r_{pm}}{r_{ps}} \theta_m
\]

\[
I_m \ddot{\theta}_m + I_{SS} \frac{r_{pm}^2}{r_{ps}^2} \dddot{\theta}_m = I_{m,app} \dddot{\theta}_m = \tau_m
\]
Multiple design prototypes of the actuated probe were originally developed. For the prototype used in this investigation, the apparent motor inertia is $I_{m, app} = 5.33 \, g \cdot cm^2$. One of the other prototypes used a motor capable of higher torques, but at the cost of higher motor inertia. The rotational inertia of the alternate motor is $I_{m, alt} = 92.5 \, g \cdot cm^2$. (Gilbertson, 2014)

In this design, the motor rotation is equal to the screw rotation and motor directly drives the screw.

$$I_{m, alt} \ddot{\theta}_m + I_{SS} \ddot{\phi}_m = I_{m, alt, app} \ddot{\theta}_m = \tau_m$$

For this alternative prototype, the apparent motor inertia is $I_{m, alt, app} = 95.4 \, g \cdot cm^2$. The apparent inertia is 18x greater for the alternative prototype, leading to a significant reduction in frequency bandwidth. Due to its lower motor armature and gear reduction, the slender Maxon motor prototype was chosen as the candidate to investigate the device capabilities as a mechanical impedance measurement tool.

5.3.5 Estimate of Belt Stiffness

There is a single elastic element modeled within the device – the compliance of the timing belt responsible for power transmission. The resonant frequency of the device depends on this value and thus a numerical estimate of the belt stiffness is needed to make a confident prediction of when to anticipate dynamic amplification. To estimate this belt stiffness, the elongation of the belt was measured in response to increments in static loading.

The experimental setup, shown in Figure 5-6, involved immobilizing the motor pinion with a vice clamped to the workbench. The screw pinion was then suspended from the motor pinion by placing the gear teeth within the timing belt grooves. This replicates the configuration in which the belt is tensioned when the device is assembled. In order to create tension in the belt, weights are hung from the motor pinion using the string shown in the figure. Since the motor pinion is exclusively supported by the timing belt, the weight increments correspond to the amount of tension supported by the belt.

The elasticity of the belt was determined by measuring the vertical deflection of the motor pinion as the weight was incremented. The tip of a dial gauge was placed directly below the center of the pinion to measure this motion with a precision of 0.0005 inches. The measurement
device was similarly immobilized to the workbench using a clamped vice to provide a common reference coordinate. After multiple trials, it was determined that the elasticity of the belt is approximately 8500 N/m. Calculations performed to arrive at this value as well as the data collected from this trial are included in Appendix C.

Figure 5-6: Experimental setup of the belt stiffness estimation trial. The top pinion and the measurement device were immobilized with respect to the workbench. The bottom pinion is pulled down by incremental sets of weights and supported exclusively by the belt elastic force that links the two pinions.

5.4 Eigenvalues & Modes

The eigenvalues, $\lambda$, of the system matrix determine the natural frequencies of the system. This lets us know when to expect resonance in the system and at what range of frequencies the motion of the components will be in-phase or out-of-phase with each other. Similarly, the modes of the system, $V$, let us know the relative amount that each coordinate will change at its
respective natural frequency. All motions exhibited by the system can be written as linear sums of the behavior denoted by these modes. The eigenvalue problem can be solved by converting the state-space representation from the time domain to the Laplace domain.

\[
\dot{\rho} = A\rho \quad \rightarrow \quad s[\rho(s)] = A[\rho(s)] \quad \rightarrow \quad \lambda [V] = A[V]
\]

A derivation of the eigenvalue equation can be produced by inspecting the transformation from time to frequency domain. Each variable, \( q \), can be expressed as an amplitude, \( \Phi \), multiplied by a complex exponential defined by both frequency, \( \omega \), and time, \( t \). This structure allows us to represent time derivatives easily.

\[
\rho = \Phi e^{j\omega t} \quad \dot{\rho} = j\omega \Phi e^{j\omega t}
\]

In general, it is not necessary for the exponential to be strictly complex, but it is necessarily so for a passive physical system with no dissipation, as is the case for the present system model. The change in representation can just as easily be applied to matrices, including the state-space matrix arrangement. Since the exponential term is shared among all terms, it can be removed from the resulting equation.

\[
[\rho] = [\Phi] e^{j\omega t} \quad [\dot{\rho}] = j\omega [\Phi] e^{j\omega t}
\]

\[
[\dot{\rho}] = [A][q] \rightarrow j\omega [\Phi] e^{j\omega t} = [A][\Phi] e^{j\omega t}
\]

\[
j\omega [\Phi] = [A][\Phi]
\]

The equation above is a general equation that applies across all frequencies. In order to find the fundamental modes of the system, the general \( j\omega \) is replaced with a particular value, \( \lambda \), known as an eigenvalue. For each eigenvalue, there exists a set of amplitudes \( \Phi \) that is constant – this is the corresponding eigenvector \( V \). The number of solutions to the eigenvalue problem depends on the size of the \( A \) matrix and whether it is non-singular.

\[
\lambda [V] = [A][V]
\]

The resulting eigenvalues and eigenvectors for the undamped, unforced system outlined in Section 5.2.6 are shown below. The numerical values found in the Section 5.3 were substituted in place for the symbolic terms. Each eigenvector is a mode of the system; the values in the vector are normalized quantities describing how each coordinate behaves relative to the others under that mode. The magnitude of an eigenvalue is the natural frequency at which its corresponding mode dominates the system. The values presented correspond to the ‘four-state’ system matrix.
The ignorable COM coordinates would have resulted in $\lambda = 0$, further indicating that the COM positions and velocities are irrelevant to the dynamics of the ungrounded system.

$$\lambda_{1,2} = \pm j \ 1644 \ \frac{rad}{s} = \omega_n \ \rightarrow \ f_n = 262 \ Hz$$

$$\lambda_{3,4} = 0$$

$$V_{1,2} = \begin{bmatrix} \dot{\theta}_{ps} \\ \dot{\theta}_{pm} \\ \theta_{ps} \\ \theta_{pm} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -0.319 \end{bmatrix} \quad V_{3,4} = \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ 1 \end{bmatrix}$$

The way to interpret the eigenvalues and eigenvectors in terms of the actual device behavior is first to note that there are only two distinct modes in the system, since two of the eigenvalues are actually just complex conjugates of each other. Mode $V_{3,4}$ is indicative of the device operating as designed – the screw rotation follows the motor rotation without delay, with the rotational speed amplified by the gear ratio 2:1. This mode does not display oscillatory behavior, as confirmed by the corresponding zero frequency.

Mode $V_{1,2}$ exhibits more interesting dynamics – the screw and motor rotate in opposite directions. In this mode, the belt is repeatedly stretched and compressed, with the screw and motor oscillating out of phase. The ratio between the two motions is dependent on the amount of inertia attached to each coordinate, as this determines how quickly each coordinate can accelerate and decelerate. The non-zero eigenvalue is the frequency at which pure oscillation occurs, in radians per second; this is the natural frequency of the system. Based on this model, the stiffness of the belt is expected to result in a natural frequency of oscillation $\omega_n = 262 \ Hz$. For motions at all other frequencies, the device will operate as a sum of the two modes, displaying some coordinated rotation and some belt compression.

Another way to solve for the eigenvalues and modes is to express the undamped system in a second-order state-space form relating position directly to acceleration.

$$[M][\ddot{q}] = [K][q]$$

$$[\ddot{q}] = [M]^{-1}[K][q] = [A][q]$$

$$s^2[q(s)] = [A][q(s)] \ \rightarrow \ \lambda^2[V] = [A][V]$$
The COM coordinates are not included since they are decoupled from the rest of the system. By expressing the system equations in a second-order form, the integrators are removed as well, so the state matrix reduces to a 2x2 matrix. The following state matrix is composed of the same coefficients expressed in Section 5.2.6, but substitutes the symbolic variables with the actual system values, as outlined in the Section 5.3.

\[
\frac{d^2}{dt^2} \begin{bmatrix}
\theta_{ps} \\
\theta_{pm}
\end{bmatrix} = 10^6 \times \begin{bmatrix}
-2.332 & 1.166 \\
0.744 & -0.372
\end{bmatrix} \begin{bmatrix}
\theta_{ps} \\
\theta_{pm}
\end{bmatrix}
\]

The eigenvalues and eigenvectors found using this method are equivalent to the ones found previously. The only differences are that the duplicate modes are eliminated and velocity is no longer displayed as part of the mode.

\[
\lambda_1^2 = -2\,704\,000 \frac{rad^2}{s^2} \rightarrow \lambda_1 = \pm j 1644 \frac{rad}{s} = \omega_n \rightarrow f_n = 262 Hz \\
\lambda_2 = 0
\]

\[
V_1 = \begin{bmatrix}
\theta_{ps} \\
\theta_{pm}
\end{bmatrix} = \begin{bmatrix}
1 \\
-0.319
\end{bmatrix} \quad V_2 = \begin{bmatrix}
\theta_{ps} \\
\theta_{pm}
\end{bmatrix} = \begin{bmatrix}
0.5 \\
1
\end{bmatrix}
\]

5.5 Intermediate and Alternate Models

Additional models and assumptions about the device dynamics are evaluated in order to more fully comprehend and predict what the behavior of the device is during use. These alternate models are also useful in ensuring that the current model is sufficiently thorough yet simple enough to avoid unnecessary complexities.

5.5.1 Assumption: Screw and Motor Inertias Dominant

The current model can be simplified further by assuming that the only coordinates of importance are the rotations of the screw and the motor. This assumes that the other possible motions and coordinates, including the rotation of the housing and the center of mass translation, are either negligible or irrelevant to the dynamics of interest. One way to implement this is by clamping the housing unit to an inertial frame of reference through a rigid support. This
assumption breaks down when realizing that during an actual trial, the support to the housing is the sonographer arm; humans cannot provide rigid support, and quantifying this arm stiffness is one of the main motivations of this project. However, making these assumptions allows us to validate the dominant modes of the current model and evaluate the contributions of housing rotation and rigid body translation.

The equations of motion are derived by revisiting the belt dynamics and performing a force and torque analysis of the free-body diagram in Figure 5-7. Translational inertias are considered negligible since translational motion is ignorable.

\[
\begin{align*}
\Delta s &= r_{pm} \theta_{pm} - r_{ps} \theta_{ps} \\
\tau_{ps} &= r_{ps} f_{SD} = r_{ps} (k_b \Delta s) \\
I_s \ddot{\theta}_{ps} &= \tau_{ps} = k_b r_{ps} r_{pm} \theta_{pm} - k_b r_{ps}^2 \theta_{ps} \\
I_m \ddot{\theta}_{pm} &= \tau_{pm} = k_b r_{pm} r_{ps} \theta_{SD} - k_b r_{pm}^2 \theta_{pm}
\end{align*}
\]

The resulting equations of motion can then be rewritten as acceleration equations and arranged in a 2x2 second-order state-space matrix using the parameters from Section 5.3.
\[
\frac{d^2}{dt^2} \begin{bmatrix} \theta_{ps} \\ \theta_{pm} \end{bmatrix} = 10^6 \begin{bmatrix} -2.41 & 1.205 \\ 0.746 & -0.373 \end{bmatrix} \begin{bmatrix} \theta_{ps} \\ \theta_{pm} \end{bmatrix}
\]

The modes and natural frequencies of this system can then be found by solving the eigenvalue problem outlined in Section 5.4.

\[
\lambda_1^2 = -2.783 \, 000 \, \frac{rad^2}{s^2} \quad \rightarrow \quad \lambda_1 = \pm j 1668 \, \frac{rad}{s} = \omega_n \quad \rightarrow \quad f_n = 265.5 \, Hz
\]

\[
V_1 = \begin{bmatrix} \theta_{ps} \\ \theta_{pm} \end{bmatrix} = \begin{bmatrix} 1 \\ -0.309 \end{bmatrix}
\]

\[
\lambda_2 = 0
\]

\[
V_2 = \begin{bmatrix} \theta_{ps} \\ \theta_{pm} \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}
\]

The values found from this simplified system are similar to the values found from the undamped, ungrounded full system. The natural frequency computed here is 265.5 Hz, 1.3% higher than the natural frequency of the full system. The associated oscillatory mode is only different by 3.1%, when comparing the smaller values after normalizing. Inspecting the equations, it is evident that the only differences between the two systems are the effective screw and housing inertias. The inertial contributions due to the probe translation does not place as significant a burden on the power transmission as the screw rotation. Additionally, the housing rotation seems to have a negligible effect on the effective housing inertia. This confirms that the belt stiffness, motor inertia, and screw inertia are the dominant parameters in the oscillatory mode of the device. The second mode, representing non-oscillatory motion, is exactly the same as before since it only depends on gear ratio between the pinions.

### 5.5.2 Assumption: Zero Belt Stiffness; Motor Torque Included

We now assume the belt stiffness to be zero, such that the screw is not driven by the motor. Additionally, motor torque production \( \tau_m \) is included as an external generalized force. In order to determine which components the torque is acting on, we examine the virtual work performed by the torque. For each coordinate \( q \), the external forces and torques are "turned on" and the remaining three generalized velocities are set to zero. The forces and torques are allowed to perform an infinitesimal amount of work \( \delta W_q \) by moving the affected inertial objects across some
infinitesimal distance $\delta x$ or rotation $\delta \theta$. This virtual work is equal to $\Xi_q \delta q$, or the virtual work done by the total externalized general force $\Xi_q$ on the coordinate $q$. This process allows us to determine which forces and torques affect each coordinate, in what manner, and what the total sum is.

\[
\begin{align*}
\Xi_{\theta h} \delta \theta_h &= -\tau_m \delta \theta_h = \delta W_{\theta h} \\
\Xi_m \delta \theta_m &= \tau_m \delta \theta_m = \delta W_m \\
\Xi_{xh} \delta x_h &= 0 = \delta W_{xh} \\
\Xi_{xp} \delta x_p &= 0 = \delta W_{xp}
\end{align*}
\]

The system dynamics given these external generalized forces are outlined below.

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \Xi_q
\]

\[
m_H \ddot{x}_h + I_s \left(\frac{\dot{\theta}_h + \dot{x}_h - \dot{x}_p}{n^2} + \frac{\dot{x}_p}{n^2}\right) = 0 \\
m_p \ddot{x}_p - I_s \frac{\dot{\theta}_h}{n} = 0
\]

\[
l_H \ddot{\theta}_h + I_s \left(\frac{\ddot{x}_h - \ddot{x}_p}{n^2} + \frac{\dot{x}_p}{n^2}\right) = -\tau_m \\
l_m \dot{\theta}_m = \tau_m
\]

The equations above indicate that the motor armature and device housing rotations are driven by the motor torque, but in opposite directions. Further analysis of the equations reveals that the screw rotations are strongly coupled to the housing rotation.

\[
\left(\frac{\ddot{x}_h}{n} - \frac{\ddot{x}_p}{n} + \frac{\dot{x}_p}{n^2}\right) = \dot{\theta}_s \quad \rightarrow \quad l_H \ddot{\theta}_h + l_s \ddot{\theta}_s = -\tau_m
\]

So while the motor torque does not drive the screw directly, the rotational acceleration of the housing requires that the screw accelerate as well. This is because the rotation of housing includes the rotation of the probe, a structure that the screw is kinematically constrained to. It is this same kinematic constraint that results in the linear accelerations of the probe and housing due to screw acceleration.

\[
m_p \ddot{x}_p = -m_H \ddot{x}_h = \frac{l_s}{n} \ddot{\theta}_s
\]

The probe and housing move in opposite directions to prevent the acceleration of the device center of mass. The culmination of this exercise is to show that even when the belt is removed from the device, the torque from the motor can drive the linear actuator since the motions of the system substructures are coupled to one another. This conclusion is only valid, however,
under the assumption that the device housing is free to rotate; if the rotation of the housing were
fixed, the screw and probe would be decoupled from the motor entirely.

5.5.3 Assumption: Infinite Belt Stiffness; Motor Torque Included

It is now assumed that the belt compliance is zero, such that the screw is driven directly by
the motor. This means that the belt is infinitely stiff and its elongation is always zero.

\[ k_b = \infty \rightarrow \frac{1}{k_b} = 0 \]

\[ \Delta s = r_{pm}(\theta_m - \theta_h) - r_{ps}(\theta_s - \theta_h) = 0 \rightarrow \theta_s = \frac{r_{pm}}{r_{ps}} \theta_m + \left(1 - \frac{r_{pm}}{r_{ps}}\right) \theta_h \]

The energy terms of the system need to be reexamined following this new relationship.
Potential energy is zero because there are no longer any potential energy storage elements. The
absence of a spring also eliminates a degree of freedom, so the position of the probe must now
be expressed in terms of the other coordinates.

\[ V = 0 \]

\[ T = \frac{1}{2} m_H \ddot{x}_h^2 + \frac{1}{2} m_p \ddot{x}_p^2 + \frac{1}{2} I_H \dot{\theta}_h^2 + \frac{1}{2} I_m \dot{\theta}_m^2 + \frac{1}{2} I_s \dot{\theta}_s^2 \]

\[ T = \frac{1}{2} \left[ m_H \ddot{x}_h^2 + m_p \left(n \frac{r_{pm}}{r_{ps}} (\dot{\theta}_h - \dot{\theta}_m) + \ddot{x}_h\right)^2 + I_H \dot{\theta}_h^2 + I_m \dot{\theta}_m^2 + I_s \left(\frac{r_{pm}}{r_{ps}} \dot{\theta}_m + \dot{\theta}_h - \frac{r_{pm}}{r_{ps}} \dot{\theta}_h\right)^2 \right] \]

The equations of motion are derived from the Lagrangian, with the generalized forces
remaining unchanged from the previous exercise.

\[ \frac{\partial L}{\partial q} = 0 \]

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = m_H \ddot{x}_h + m_p \left(n \frac{r_{pm}}{r_{ps}} (\dot{\theta}_h - \dot{\theta}_m) + \ddot{x}_h\right) = \Xi_{xh} \]

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_h} = \left(I_H + I_s \left(1 - \frac{r_{pm}}{r_{ps}}\right)^2\right) \ddot{\theta}_h + I_s \left(\frac{r_{pm}}{r_{ps}} - \frac{r_{ps}^2}{r_{ps}^2}\right) \ddot{\theta}_m + m_p \left(n^2 \frac{r_{pm}^2}{r_{ps}^2} \ddot{\theta}_h - n^2 \frac{r_{pm}^2}{r_{ps}^2} \ddot{\theta}_m + n \frac{r_{pm}}{r_{ps}} \ddot{x}_h\right) = \Xi_{\theta h} \]
\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_m} = \left( I_m + I_s \frac{r_{pm}^2}{r_{ps}^2} \right) \ddot{\theta}_m + I_s \left( \frac{r_{pm}}{r_{ps}} - \frac{r_{pm}}{r_{ps}} \right) \ddot{\theta}_h - m_p \left( n^2 \frac{r_{pm}^2}{r_{ps}^2} \ddot{\theta}_h - n^2 \frac{r_{pm}^2}{r_{ps}^2} \ddot{\theta}_m + n \frac{r_{pm}}{r_{ps}} x_h \right) = \Xi_{\theta m}
\]
\[
\Xi_{xh} = 0, \quad \Xi_{\theta h} = -\tau_m, \quad \Xi_{\theta m} = \tau_m
\]

Solving for the system of equations in order to decouple the coordinates, where \( \Lambda \) is a chain of inertial terms common to all equations:
\[
\frac{\Lambda}{m_p n \frac{r_{pm}}{r_{ps}^2} \left( I_m + I_H + I_s \left( \frac{r_{pm}}{r_{ps}} + w \right)^2 \right)} x_h = 0
\]
\[
\frac{\Lambda}{\left( I_m + I_s \left( \frac{r_{pm}}{r_{ps}} + \frac{r_{pm}^2}{r_{ps}^2} \right) \right) (1 + m_h)} \ddot{\theta}_h = -\tau_m
\]
\[
\frac{\Lambda}{\left( I_H + I_s \left( \frac{r_{pm}}{r_{ps}} + \frac{r_{pm}^2}{r_{ps}^2} \right) \right) (1 + m_h)} \ddot{\theta}_m = \tau_m
\]

An observation that can be derived from these equations is a comparison between the rotational inertias that the motor torque acts on. Constant \( \alpha \) is common to both equations.

Equivalent Motor Housing: \( \alpha \left( I_H + I_s \left( w \frac{r_{pm}}{r_{ps}} + \frac{r_{pm}^2}{r_{ps}^2} \right) \right) \ddot{\theta}_h = -\tau_m \)

Equivalent Motor Armature: \( \alpha \left( I_m + I_s \left( w \frac{r_{pm}}{r_{ps}} + \frac{r_{pm}^2}{r_{ps}^2} \right) \right) \ddot{\theta}_m = \tau_m \)

The housing inertia \( I_H \) is the largest of the three. This is expected since its components have the largest geometries and \( I_H \). Since \( I_H > I_m \), it is expected for the housing rotation to accelerate less than the motor rotation for a given torque.

If the inertias of the screw and motor armature are much smaller than those of the rest of the system, such that \( I_H \gg I_m \) and \( I_H \gg I_s \) then the rotation of the housing is expected to be insignificant compared to that of the motor. This is especially true when the gear ratio is very high, \( r_{ps} \gg r_{pm} \).
5.6 System Inputs: External Forces & Dissipation

Relations have been established for the undamped, unforced, ungrounded device behavior. However, this is unrepresentative of the device dynamics during intended use so it is necessary to include damping, external forces, and grounded contact interfaces. These terms can be included as system inputs, resulting in equations that provide applicable relations between the states, actuator inputs, and sensor outputs.

5.6.1 Sonographer Hand Interface

The primary external contact of interest is the interface between the actuated probe housing and the sonographer hand. Measuring the impedance observed at the sonographer hand is the main goal of the device enhancement and as such needs to be included as a system input. The sonographer hand contact \( \hat{c}_s \) can initially be modeled as a force \( f_s \) from the hand of the sonographer acting on the device housing along the \( x_h \) axis, and a torque \( \tau_s \) that keeps the device from rotation about the \( \theta_h \) axis. The contact force and torque with the sonographer hand are composed of a nominal quasi-static amount generated to maintain the device in the desired position and a variable amount determined by the hand mechanical impedance. The directions of \( f_s \) and \( \tau_s \) are noted by the unit vectors \( \hat{e}_{x_h} \) and \( \hat{e}_{\theta_h} \) in the equation below.

\[
\hat{c}_s = f_s \hat{e}_{x_h} + \tau_s \hat{e}_{\theta_h}
\]

5.6.2 Patient Tissue Interface

The second external contact is the interface between the probe and patient. The impedance of the patient tissue will affect the dynamics of the device and therefore affect the interaction between the device and sonographer hand. Additionally, patient tissue impedance measurement is a secondary objective of the device enhancement. The patient tissue interface, \( \hat{c}_p \), can be modeled as a force from the patient tissue \( f_p \) acting on the probe along the \(-x_p\) axis. This contact force is similarly composed of a nominal quasi-static amount and a variable amount determined by the hand mechanical impedance. The direction of \( f_p \) is noted by the unit vector \( \hat{e}_{x_p} \).

\[
\hat{c}_p = -f_p \hat{e}_{x_p}
\]
5.6.3 Motor Torque

The brushless motor is the sole controlled energy producing element in the device. This is the element that is used to dictate the frequency, amplitude, and operating point of the device behavior. The motor torque $\tau_m$ can only act on the motor armature and the motor housing along the coordinate $\theta_{pm} = \theta_m - \theta_h$. The directions of motion that $\tau_m$ act along are noted by the unit vector $\hat{e}_{\theta_m}$ and $\hat{e}_{\theta_h}$ in the equation below.

$$\hat{\tau}_m = \tau_m(\hat{e}_{\theta_m} - \hat{e}_{\theta_h})$$

5.6.4 Belt Damping

An important energy dissipation element is the damping component of the timing belt. Although the belt damper does not significantly affect the device dynamics during low-frequency operation, once the device motion nears its natural frequency, the belt damping is an important factor in limiting dynamic amplification due to resonance. Moreover, including belt damping improves the general accuracy and completeness of the model. Fig. 5-5 is referenced to define the force due to the belt damping.

$$f_{d,b} = b_b \cdot \Delta s = b_b \cdot \frac{d}{dt}(r_{pm}\theta_{pm} - r_{ps}\theta_{ps})$$

$$f_{d,b} = b_r r_{pm}\dot{\theta}_{pm} - b_b r_{ps}\dot{\theta}_{ps}$$

It is important to note that the belt damping force can only act on the motor armature and the ball screw by exerting a torque, as shown in Figure 5-7. This requires using the motor and screw pinion radius as the lever arms that are perpendicular to the force direction and the respective axes of rotation. The directions of motion that $\tau_{d,b}$ act along are noted by the unit vector $\hat{e}_{\theta_m}$ and $\hat{e}_{\theta_s}$ in the equation below.

$$\hat{\tau}_{d,b} = (f_{d,b} \cdot r_{ps})\hat{e}_{\theta_s} - (f_{d,b} \cdot r_{pm})\hat{e}_{\theta_m}$$

5.6.5 Velocity-Dependent Sliding Friction

Another set of dissipative elements is the friction between components sliding or moving past each other. Some of this “sliding friction” may be constant during motion, as would be the case
for Coulomb damping. To simplify the dynamic model, constant sliding friction is ignored since it introduces nonlinear components. Although this decision was maintained, it would have been revised if the system did display direction-dependent nonlinearities in the collected data.

The type of sliding friction that can be included in the model without difficulty is velocity-dependent damping. One example of this is the velocity-dependent motor damping $b_m$, proportional to the rotational speed between the motor armature and the motor housing. This type of friction also exists between the rotation of the screw and the housing, and between the translation of the probe over the housing. Since the screw rotation and the relative translations are kinematically constrained, these velocity-dependent frictions can be lumped into one parameter $b_s$ proportional to the screw rotational speed relative to the housing.

$$
\tau_{f,m} = -b_m \theta_{pm} \\
\tau_{f,s} = -b_s \theta_{ps} \\
\dot{\tau}_f = \tau_{f,s} \theta_{ps} + \tau_{f,m} \theta_{pm}
$$

### 5.6.6 Static Housing Rotation Assumption

An important assumption that will be made throughout the rest of the study is that the device housing does not rotate and can thus be fully ignored. There are a few reasons both out of necessity and from practical and model observations to support this assumption.

The inertia of the housing is large enough in comparison to the inertia of the screw and motor armature that housing rotations, even during operation without external forces, are negligible in comparison to the screw and motor armature rotations. This was the main conclusion from analysis of the alternate model in section 5.3.3. Since it provides negligible additional insight into the system dynamics, including $\theta_h$ will not significantly affect the estimates of the sonographer arm and patient tissue mechanical impedances.

When a sonographer is holding the device, the torque applied by their hand to the device is always working to maintain the housing from rotating. This would be indicative of a torsional mechanical impedance component of the sonographer arm. However, the housing rotation cannot be measured with the current sensors available, and the motion is sufficiently negligible.
that it could be difficult for motion tracking or an accelerometer to provide signals distinguishable from noise.

It is unfeasible and unnecessary to measure or estimate $\theta_h$, so the coordinate is ignored to simplify the model and facilitate the manipulation of the system equations.

5.6.7 Model Update with External Forces

To implement the static housing rotation assumption, the system inputs and coordinates are updated to reflect a static $\theta_h$ and that any torques acting on the device housing will be absorbed by the semi-rigid support structure of the sonographer arm.

\[
\theta_{ps} = \theta_s - \theta_h = \theta_s \\
\theta_{pm} = \theta_m - \theta_h = \theta_m \\
\theta_s = \frac{x_h}{n} - \frac{x_p}{n} + \theta_h = \frac{x_h}{n} - \frac{x_p}{n}
\]

Sonographer Contact Force: \( \dot{c}_s = f_s \dot{e}_{xh} + \tau_s \dot{e}_{\theta h} = f_s \dot{e}_{xh} \)

Patient Contact Force: \( \dot{c}_p = -f_p \dot{e}_{xp} \)

Motor Torque: \( \dot{t}_m = \tau_m \dot{e}_{\theta m} \)

Belt Damping: \( \dot{f}_{d,b} = f_{d,b} (r_{ps} \dot{e}_{\theta s} - r_{pm} \dot{e}_{\theta m}) \)

Sliding Friction: \( \dot{t}_f = \tau_{f,s} \dot{e}_{\theta s} + \tau_{f,m} \dot{e}_{\theta m} \)

The external and dissipation forces can be summed as the vector, \( \vec{F} \).

\[
\vec{F} = f_s \dot{e}_{xh} - f_p \dot{e}_{xp} + \tau_m \dot{e}_{\theta m} + f_{d,b} \left( \frac{r_{ps}}{n} (\dot{e}_{xh} - \dot{e}_{xp}) - r_{pm} \dot{e}_{\theta m} \right) + \frac{\tau_{f,s}}{n} (\dot{e}_{xh} - \dot{e}_{xp}) + \tau_{f,m} \dot{e}_{\theta m}
\]

As mentioned in 5.3.1, the system inputs can be included in the model equations by treating them as generalized forces, \( \Xi_q \), the summation of the external forces acting on the inertial bodies dominated by the coordinate \( q \).

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \Xi_q
\]

The derivation of the generalized forces for the three remaining coordinates is shown below.

\[
\delta W_{xh} = \vec{F} \cdot \delta x_h = \left( f_s + \frac{f_{d,b}r_{ps}}{n} + \frac{\tau_{f,s}}{n} \right) \delta x_h = \Xi_{xh} \delta x_h
\]

\[
\delta W_{xp} = \vec{F} \cdot \delta x_p = \left( -f_p - \frac{f_{d,b}r_{ps}}{n} - \frac{\tau_{f,s}}{n} \right) \delta x_p = \Xi_{xp} \delta x_p
\]
\[ \delta W_{\theta_m} = \hat{F} : \delta \theta_m = (\tau_m - f_{d,b} r_{ps} + \tau_{f,m}) \delta \theta_m = \Xi \delta \theta_m \]

The generalized forces for each coordinate can then be set equal to the inertial and stiffness terms from the Lagrange derivation shown in Section 5.2.4, updated to reflect the housing rotation assumption. The resulting equations can then be arranged in matrix form in order to isolate the acceleration terms and rearrange them into COM and relative coordinates.

\[
[M][\ddot{q}] = [K][q] + [\Xi]\]
\[
[\ddot{q}] = [M]^{-1}[K][q] + [M]^{-1}[\Xi]\]

\[ \dot{x}_{CM} = \left(\frac{1}{m_p + m_H}\right)(f_s - f_p) \]
\[ \dot{\theta}_s = -\frac{k_b r_{ps}}{l_{ss}} (r_{ps} \theta_s - r_{pm} \theta_m) + \frac{r_{ps}}{l_{ss} f_{d,b}} + \frac{1}{l_{ss}} \tau_{f,s} + \frac{m_p n}{l_{ss} (m_p + m_H)} f_s + \frac{m_H n}{l_{ss} (m_p + m_H)} f_p \]
\[ \dot{\theta}_m = \frac{k_b r_{pm}}{l_{m}} (r_{ps} \theta_s - r_{pm} \theta_m) - \frac{r_{pm}}{l_{m} f_{d,b}} + \frac{1}{l_{m}} \tau_{f,m} + \frac{1}{l_{m}} \tau_{f,m} \]
\[ l_{ss} = \frac{m_p m_H n^2 + l_s m_p + l_s m_H}{m_p + m_H} \]

The system equations can now be organized into state-space matrix representation. The form is presented below, with states \( \rho \), inputs \( u \), state matrix \( A \), and input matrix \( B \). The terms \( f_{d,b} \), \( \tau_{f,s} \), and \( \tau_{f,m} \) are replaced by their equivalents.

This model shows that the motor torque only acts directly on the acceleration of the motor. Additionally, it shows that the sonographer force and the patient force act on the screw acceleration, causing the probe to move relative to the housing. Prior to the inclusion of system inputs, the COM translation coordinate \( x_{CM} \) was treated as ignorable. This coordinate becomes useful, however, in providing an additional relationship between the sonographer and patient contact forces. By representing the device as a single object, Figure 5-8 shows the free-body diagram that corresponds with \( \dot{x}_{CM} \) equation.

\[ \frac{d}{dt} [\rho] = [A][\rho] + [B][u] \]
\[
\begin{aligned}
\frac{d}{dt} \begin{bmatrix}
    \dot{x}_{CM} \\
    \dot{\theta}_s \\
    \dot{\theta}_m \\
\end{bmatrix} &=
\begin{bmatrix}
    0 & 0 & 0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 1 & 0 \\
    0 & -\frac{k_br^2_{ps}}{l_{SS}} & \frac{k_br_{ps}r_{pm}}{l_{SS}} & 0 & 0 & 0 & 0 \\
    0 & -\frac{k_br_{pm}^2}{l_{SS}} & \frac{k_br_{pm}^2}{l_{SS}} & 0 & 0 & 0 & 0 \\
    \frac{k_br_{ps}r_{pm}}{I_m} & \frac{k_br_{ps}r_{pm}}{I_m} & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
    x_{CM} \\
    \theta_s \\
    \theta_m \\
\end{bmatrix} \\
+ \begin{bmatrix}
    0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    1 & -1 & 0 & 0 & 0 & 0 & 0 \\
    \frac{m_p + m_H}{m_H n} & \frac{m_p + m_H}{m_H n} & 0 & 0 & 0 & 0 & 0 \\
    \frac{1}{l_{SS}(m_p + m_h)} & \frac{1}{l_{SS}(m_p + m_h)} & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
    f_s \\
    f_p \\
    \tau_m \\
\end{bmatrix}
\end{aligned}
\]

Figure 5-8: Free-body diagram of the device treated as a single solid object. The device has a combined mass of \((m_c + m_h)\) that is accelerated by the contact forces \(f_s\) and \(f_p\).

5.7 System Outputs

5.7.1 Measured Signals

The device sensors were introduced and calibrated in Chapter 3. Including them in the model as the system output will allow us to determine whether the states of the system are observable.
The motor encoder measures the position of the motor armature relative to its housing, a quantity already represented as $\theta_m$ in our model.

$$\theta_{enc} = \theta_m$$

The force transducer measures a sum of the force of contact with the patient $f_p$ and the inertia of the probe mass between the sensor and the contact interface.

$$f_{ft} = m_{p1} \ddot{x}_p + f_p = m_{p1} \ddot{x}_{CM} - \frac{m_{p1} m_H n}{m_p + m_H} \dot{\theta}_s + f_p$$

The sensor equations can now be represented in state-space matrix representation. The form is presented below, with states $\rho$, inputs $u$, outputs $y$, and output matrices $C$ and $D$.

$$[y] = [C][\rho] + [D][u]$$

$$\begin{bmatrix} \theta_{enc} \\ f_{ft} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 38.31 & -19.15 & 0 \end{bmatrix} \begin{bmatrix} x_{CM} \\ \dot{\theta}_s \\ \dot{\theta}_m \\ \ddot{x}_{CM} \end{bmatrix} + \begin{bmatrix} 0 \\ (55.97) b_s + (0.00453) b_b \\ (-0.00227) b_b \end{bmatrix}$$

$$+ \begin{bmatrix} f_s \\ f_p \\ \tau_m \end{bmatrix}$$

5.7.2 Observability

A system is considered fully observable when the time history of all the model variables can be reconstructed based on the given set of system outputs. To determine whether all the states in the model are observable from the sensor outputs, we inspect the rank of the observability matrix $Obs$. $C$ is the state output matrix, $A$ is the state matrix, and $n$ is the number of states in the system.

$$Obs = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

When the device is ungrounded, as modeled using the 6-state $A$ matrix, the sensor output matrix results in an observability matrix with a rank of 4. This indicates that the motion of the device, as defined by the variables included in the model, cannot be fully determined from the
encoder and force transducer data. As mentioned previously, the COM translation is defined by the external contact forces, not the internal dynamics of the device, and neither sensor measures motion relative to the inertial frame of reference.

5.8 Sonographer End Immobilized

Since the states of the device are not fully observable during intended use of sonographer and patient contact, the two particular configurations outlined in Chapter 4 are reexamined. The first is the imposed circumstance where the device housing, the sonographer end, is immobilized to the inertial frame. This guarantees that the housing translation and rotation are constant and that the only non-ignorable coordinates are the probe translation and motor rotation. This also means that the screw rotation is directly proportional exclusively to the probe translation.

\[ x_h = \text{const} \quad \theta_h = \text{const} \]
\[ x_p, \theta_m \quad \theta_s = -\frac{x_p}{n} \]

The equations of motion as derived from the Lagrangian are revised to eliminate the equations derived for coordinates \( x_h \) and \( \theta_h \). Additionally, all terms proportional to the immobilized states are removed from the remaining equations, presented below.

\[
\begin{align*}
(1) \quad m_p \ddot{x}_p + \frac{l_s}{n^2} \dddot{x}_p + \frac{k_b r_{ps}^2}{n^2} x_p + \frac{k_b r_{pm} r_{ps}}{n} \theta_m &= -f_p - \left( \frac{b_b r_{ps}^2 + b_s}{n^2} \right) \dot{x}_p - \frac{b_b r_{ps} r_{pm}}{n} \dot{\theta}_m \\
(2) \quad l_m \ddot{\theta}_m + \frac{k_b r_{ps} r_{pm}}{n} \dot{x}_p + k_b r_{pm}^2 \theta_m &= \tau_m - \frac{b_b r_{ps} r_{pm}}{n} \dot{x}_p - (b_b r_{pm}^2 + b_m) \dot{\theta}_m
\end{align*}
\]

The equations are then arranged into state space form, substituting the parameters with their numerical values as found in Section 5.3. The outputs of the system are the encoder and force transducer sensor readings, as defined in the previous section. The coordinate \( \theta_s \) is used instead of \( x_p \) in order to avoid numerical errors due to the disparity in magnitude between the translational and rotational units.

The observability matrix for this set of outputs and device configuration is of full rank. This means that the sensors presently on the device are sufficient to reconstruct the model variables when the sonographer end of the device is immobilized.
In Chapter 7, the device will be actuated across a series of frequencies in order to confirm whether this model is sufficiently accurate. Some of the experiments conducted involve immobilizing the device housing using a vise and actuating the probe without a contact impedance at the patient end. This is equivalent to configuration modeled in this section when \( f_p \) is set to zero. One way to compare the model with the experimental data is to examine the frequency response function between the measured sensor data and the torque input to the device. This can be compared to the transfer functions predicted by the model.

The transfer function relating output \( Y(s) \) and input \( U(s) \) in a system is a function in the Laplace domain computed from the state space matrices of the system dynamics model, as shown below. The input matrix \( B \) should include the contributions of the input \( U \) and the output matrices \( C \) and \( D \) should only include the contributions of the system dynamics that result in the output \( Y \). The term \( \text{eye}_n \) refers to an identity matrix of size \( n \times n \).

\[
Y(s) = [C(s \ast \text{eye}_n - A)^{-1}B + D]U(s)
\]

The transfer functions relating the encoder and force transducer outputs to the motor input are summarized below. Since damping terms are unknown and may be negligible, they are noted by the terms \( b_j \).

\[
\frac{\theta_m(s)}{\tau_m(s)} = 2.179 \times 10^6 \frac{s^2 + b_1s + 2\ 299\ 000}{s^4 + b_2s^3 + (2\ 671\ 000 + b_3)s^2 + b_4s}
\]

\[
\frac{f_{ft}(s)}{\tau_m(s)} = -5.818 \times 10^7 \frac{s^2 + b_6s^3}{s^4 + b_2s^3 + (2\ 671\ 000 + b_3)s^2 + b_4s}
\]
Something to notice about \( f_{ft}/\tau_m \) is that there is a pole-zero cancellation at the origin. Additionally, when the damping terms \( b_4 \) and \( b_6 \) are set to zero, there is a double pole-zero cancellation at the origin. This typically indicates that the system is uncontrollable, unobservable, or both. From a physical perspective, if the only output of the system is the force transducer, then it is not possible to estimate both the screw and motor rotation. The combination of the force transducer and motor encoder is necessary for that, as was discussed in Chapter 4. An observability matrix was constructed using \( f_{ft} \) as the only output and it has a rank of 2, confirming that the system is not fully observable.

In terms of controllability, the motor torque is the only input to the system and due to its direct connection to the motor, it is expected that \( \tau_m \) can control \( \theta_m \). It is less clear whether the motor can additionally control \( \theta_s \), since the screw rotation is strongly coupled to the motor rotation. A controllability matrix is assembled to inspect further, as defined below.

\[
Cont = [B \ AB \ \cdots \ A^{n-1}\!B]
\]

Although the resulting controllability matrix for \( f_{ft}/\tau_m \) is of full rank, a singular value decomposition of the matrix reveals that it is ill-conditioned, with two of the values 6 orders of magnitude greater than the other two. Since the controllability matrix is the same for \( \theta_m/\tau_m \), the same remarks apply.

An observability matrix was constructed for \( \theta_m/\tau_m \), which is of full rank. This is not unexpected since this equation does not display a double pole-zero cancellation at the origin. However, a singular value decomposition showed that this matrix is ill-conditioned, with one of the values 8 orders of magnitude greater than two of the remaining ones.

### 5.9 Patient End Immobilized

The second particular configuration outlined in Chapter 4 is the imposed circumstance where the ultrasound probe, the patient end of the device, is immobilized to the inertial frame. This guarantees that the probe translation and housing rotation are zero and that the only non-ignorable coordinates are the housing translation and motor rotation. This also means that the screw rotation is directly proportional to the housing translation.
\[ x_p = \text{const} \quad \theta_h = \text{const} \]
\[ x_h, \theta_m \quad \theta_s = \frac{x_h}{n} \]

The equations of motion as derived from the Lagrangian are revised to eliminate the equations derived for coordinates \( x_p \) and \( \theta_h \). All terms proportional to the immobilized states are removed from the remaining equations, presented below.

(1) \[ m_H \ddot{x}_h + \frac{l_s}{n^2} \ddot{x}_h + k_b \frac{r^2_{ps}}{n^2} x_h - \frac{k_b r_{pm} r_{ps}}{n} \theta_m = f_s - \frac{\left(b_b r^2_{ps} + b_s\right)}{n} \dot{x}_h + \frac{b_b r_{ps} r_{pm}}{n} \dot{\theta}_m \]

(2) \[ I_m \ddot{\theta}_m - \frac{k_b r_{ps} r_{pm}}{n} x_h + k_b r^2_{pm} \theta_m = \tau_m + \frac{b_b r_{ps} r_{pm}}{n} \dot{x}_h - \left(b_b r^2_{pm} + b_m\right) \dot{\theta}_m \]

The equations are then arranged into state space form, substituting the parameters with their numerical values as found in Section 5.3. The coordinate \( \theta_s \) is used instead of \( x_h \).

\[
\frac{d}{dt} \begin{bmatrix} \theta_s \\ \theta_m \\ \dot{\theta}_s \\ \dot{\theta}_m \end{bmatrix} = 10^6 \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2.156 & 1.078 & -2.552 b_b - 3.15 b_m & 1.276 b_b \\ 0.746 & -0.373 & 0.882 b_b & -0.441 b_b - 2.179 b_m \end{bmatrix} \begin{bmatrix} \theta_s \\ \theta_m \\ \dot{\theta}_s \\ \dot{\theta}_m \end{bmatrix} + 10^4 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 217.9 & 0 \\ 0 & 0 & 0 & 226.5 \end{bmatrix} \begin{bmatrix} f_s \\ f_p \\ \tau_m \end{bmatrix}
\]

\[
\begin{bmatrix} \theta_{\text{enc}} \\ f_{ft} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 226.5 & -113.2 & 268 b_s + 330.9 b_b & -134 b_b \end{bmatrix} \begin{bmatrix} \theta_s \\ \theta_m \\ \dot{\theta}_s \\ \dot{\theta}_m \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_s \\ f_p \\ \tau_m \end{bmatrix}
\]

The outputs of the system are the encoder and force transducer sensor readings. The force transducer sensor reading is used to measure the hand force and housing inertia. The derivation of this equation was presented in Chapter 3.

\[ f_{ft} = f_s - m_H \ddot{x}_h \]

The observability matrix for this set of outputs and device configuration is of full rank. This means that the sensors presently on the device are sufficient to reconstruct the model variables when the patient end of the device is immobilized.

Some of the device characterization experiments discussed in Chapter 7 involved immobilizing the ultrasound probe using a vise and actuating the device housing without a
contact impedance at the sonographer end. This is equivalent to the configuration presented here when $f_s$ is set to zero. As described in the previous section, the developed model can be compared to the experimental data to validate the equations of motion. Specifically, the frequency response functions between the measured sensor data and the torque input to the device were compared against the transfer functions predicted by this model.

The transfer functions relating the encoder and force transducer outputs to the motor input are summarized below. Since damping terms are unknown and may be negligible, they are noted by the terms $b_j$.

$$\frac{\theta_m(s)}{\tau_m(s)} = 2.179 \times 10^6 \frac{s^2 + b_1 s + 2 \, 156 \, 000}{s^4 + b_2 s^3 + (2 \, 529 \, 000 + b_3) s^2 + b_4 s}$$

$$\frac{f_{ft}(s)}{\tau_m(s)} = -2.467 \times 10^8 \frac{s^2 + b_6 s^3}{s^4 + b_2 s^3 + (2 \, 529 \, 000 + b_3) s^2 + b_4 s}$$

The equations found above are of the same form as those presented in Section 5.8, with different coefficient values but at the same magnitude. Therefore, all the previous comments about observability, controllability, and pole-zero cancellation apply as well.

The models developed in this chapter can be revised or developed further if the predicted behavior does not match the observed behavior from the device characterization experiments. Furthermore, there is also additional information that can be gathered from the dynamic model that was not covered within this chapter. For instance, since the immobilized systems are fully observable, it is possible to implement an observer to reproduce all the states in the system. Observers are useful if it is necessary estimate state parameters in real time or as an alternative to determine them during data processing, but were not necessary in this study. With a model of the device in place, the parameters necessary to estimate hand and tissue impedance can be determined from the output sensor data. But before doing so it is necessary to validate the model experimentally.
Chapter 6

Frequency Response Excitation

6.1 Frequency Response Excitation Methods

As has been stated throughout the previous chapters, the mechanical impedance at a contact interface is defined by the contact force $f$ and the corresponding contact motion $x$. For a linear system, the mechanical impedance is equal to the ratio of the two variables, when expressed as a function of frequency.

$$Z(\omega) = \frac{f(\omega)}{x(\omega)}$$

Since impedance is a function of $\omega$ and not necessarily a single value or a simple relationship, it is useful to represent it graphically. Because this mathematical expression may be complex-valued, it is convenient to express impedance with plots of magnitude and phase vs. frequency. There are visually apparent elements in these plots, such as asymptotic behavior, which can serve as indicators of the characteristics of the impedance dynamics, such as the order of the system, resonant frequencies, and levels of dynamic amplification.

$$Mag(Z(\omega)) = |Z(\omega)| = \sqrt{\text{Im}(Z(\omega))^2 + \text{Re}(Z(\omega))^2}$$

$$Phase(Z(\omega)) = \angle Z(\omega) = \tan^{-1}\left(\frac{\text{Im}(Z(\omega))}{\text{Re}(Z(\omega))}\right)$$

It is important to note that the impedance frequency response function $Z(\omega)$ is a mathematical representation of a physical behavior that occurs in a real-valued world, and as such it is vital to understand how the behavior correlates with the model. Given a perfect mathematical model of impedance, the magnitude $|Z(\omega)|$ is equal to $G(\omega)$, the ratio between the contact force and contact position magnitudes at the frequency $\omega$, as determined from experimental data. The phase $\angle Z(\omega)$ predicted by the model similarly corresponds to the phase
\( \phi(\omega) \), the time delay between the force and position data normalized by the period \( T \) of the frequency \( \omega \).

\[
G(\omega) = \frac{|F(\omega)|}{|X(\omega)|}
\]

\[
\phi(\omega) = -\frac{t_{lag}(\omega)}{T(\omega)} \times 360^\circ
\]

If the impedance frequency response function is unknown or requires validation, the impedance gain and phase values \( G(\omega) \) and \( \phi(\omega) \) can be determined experimentally by performing a frequency response analysis. The mathematical model can then be developed or adjusted to fit the experimental behavior. As the name suggests, a frequency response analysis requires exciting the system across a particular set of frequencies and measuring how the system behaves in response to this input. Multiple methods for frequency excitation exist, but the ones of interest to us use discrete sinusoidal inputs, swept sinusoidal inputs, and stochastic inputs.

### 6.1.1 Discrete Sinusoidal Excitation

The simplest frequency response analysis is based on discrete sinusoidal excitation. In this method, a sinusoidal input of constant amplitude actuates the system at a single frequency \( \omega \). If the system is linear, then, when steady state is reached (i.e. after all transients have decayed to zero) all system variables will exhibit sinusoidal behavior of constant amplitude at the frequency of the input excitation. Since the frequency is the same for all variables, the gain at \( \omega_{\text{excitation}} \) can be computed after determining the magnitudes of the contact position and force. The phase at \( \omega_{\text{excitation}} \) is determined from the time delay between the two variables. The amplitudes of the measured sinusoidal time signals are used to determine the gain and the time delay is based on when the signals cross their mean values. Figure 6-1 shows how these values can be determined from the experimentally collected signals.

\[
G(\omega) = \frac{|F(\omega)|}{|X(\omega)|} = \frac{f_{\text{amp}}(\omega)}{x_{\text{amp}}(\omega)} = \frac{\max(f(t)) - \mean(f(t))}{\max(x(t)) - \mean(x(t))}
\]

\[
t_{\text{lag}}(\omega) = t_{f,\text{mean}}(\omega) - t_{x,\text{mean}}(\omega)
\]
During discrete sinusoidal excitation, each test provides a single sample point on the plots of gain and phase vs. frequency. In order to have a comprehensive characterization of the mechanical impedance, the system needed to be excited across a sufficiently broad range of frequencies, with a sufficiently small separation between tested frequencies to yield a desirable resolution. For the trials performed in this study, the frequencies tested were between 0.1 Hz and 50 Hz. This is the range in which we hypothesized the tissue dynamics could be observed. The standard used in this study is to sample at least 3 logarithmically spaced frequencies per decade. Higher resolutions may be necessary at frequencies where the dynamic behavior is rapidly changing; further tests are conducted to provide this additional resolution.

It is important to remember that this approach inherently assumes a linear system – each variable follows a sinusoidal behavior if the input is sinusoidal. In order to confirm the linearity of the system, a sinusoid can be fitted to the experimental data using least squared error optimization. The sinusoidal fits should match the input frequency of excitation. The mean residual error of the fit can be compared to the amplitude of the sinusoid in order to determine how well the data approximates sinusoidal behavior. If the residual-to-mean ratio approaches zero, linearity is an acceptable assumption.
A downside of exciting the system at a series of discrete frequencies is the time required to characterize the frequency response of the mechanical impedance. The characterization resolution is limited by the number of trials conducted, and each trial length is a function of the inverse of the frequency and the number of cycles collected. For the application that this study focuses on, one of the objectives is to measure the mechanical impedance of the sonographer while they are performing an ultrasound scan. This is a task that is accomplished within a matter of seconds; if the posture and applied effort is prolonged for too long, the effects of fatigue could set in and the sonographer’s impedance characteristics may change over the course of the task. As such, the characterization trials should also be completed within a matter of seconds. This will not be attainable by exciting the system at discrete frequencies if they extend below 1 Hz.

6.1.3 Stochastic Excitation

The other form of system excitation being considered is stochastic, where the input to the system is a white noise process. As discussed in Chapter 3, white noise is a random signal that maintains a constant power spectral density. In an ideal white noise signal, there is an equal amount of power associated with each frequency. As a consequence, a single white noise input would actuate the device at all frequencies. Additionally, the signal is unique and non-repeating. This is exemplified by its autocorrelation function, which yields an infinite mean square value at time delay \( \tau = 0 \). If a white noise signal were to be filtered, for instance due to the device system dynamics, the cross-correlation function between the filtered and original signal would result in a finite distribution across time delays due to the phase shift and gain between the two signals at different frequencies. (Bendat & Piersol, 2011, p. 134)

In relevance to this study, if both the input and the outputs of the system were measured accurately and reliably, the gain and phase between the two variables could be determined for all frequencies from a single stochastic input. Because the signal inherently excites the system at every frequency simultaneously, the length of the commanded input is determined by the longest period, or the lowest frequency, of interest. This significantly reduces the amount time required to conduct an impedance characterization trial while also providing more information about the frequency response of the system.
In implementation, a theoretical white noise signal can neither be created nor measured. Most notably, the highest frequency that can be included in the signal is limited by the sampling rate of the actuating and measurement devices. Additionally, the lowest frequency is defined by the duration of the signal. A time-sampled white noise signal cannot provide the infinite amount of frequency information that a continuous signal theoretically could, but provides significantly more frequency information than discrete sinusoidal excitation using the same device.

Determining the phase and gain between a stochastic input signal and the resulting outputs is more computationally intensive than the sinusoidal excitation. For sinusoidal excitation, the gain and phase are determined by estimating the amplitudes and time delays of two sinusoidal waves. For stochastic excitation, the gain and phase can be determined by computing the correlations between random data vectors and performing Fourier transforms to convert from the time domain to frequency domain. These computations require higher computational power and processing time and can be performed after the conclusion of the trial.

It is important to remember that this approach inherently assumes a linear time-invariant (LTI) system. In order to confirm the linearity of the system, the stochastic input can be selected as a white noise signal with a Gaussian distribution of amplitudes. If the input follows a Gaussian distribution and the system is linear, the output must also follow a Gaussian distribution. (Bendat & Piersol, 2011, p. 164) This can be proven by computing the probability density for the output variable amplitudes and comparing it to the probability density of a Gaussian distribution of the same mean and standard deviation as the output. The mean error between the two should approach zero when the system is linear.

Another thing to consider about a stochastic input is that since the input is adding equal amounts of energy across all frequencies of excitation, it adds only a small amount of energy to the system at any particular frequency. In contrast, a sinusoidal signal adds a large amount of energy to a single frequency. In this matter, a stochastic signal is preferable, particularly if the system exhibits resonance at one of the frequencies tested. If too much energy is added into the system at the frequency of resonance, the system may behave outside its limits of linearity, and the output may reflect the effects of nonlinearities instead of the LTI system.
It is expected that a stochastic excitation input would considerably reduce the impedance characterization data collection time by an order of magnitude compared to sinusoidal excitation. Since the signal is primarily determined by the lowest frequency of interest, the device would be able to characterize impedance down to 0.1 Hz within a duration of 50 seconds. This is within the time limits imposed by the intended testing conditions, making stochastic excitation the preferred candidate from the methods examined.

6.2 Implementation of Frequency Analysis via Device

To perform the impedance measurement experiments, a signal needs to be commanded to the motor input and the sensors need to reliably collect data on the system outputs. The manner in which both of these tasks are performed depends on the architecture of the control and data acquisition electronics used. In this study, three electronics architectures are considered, each defined by its control electronics – data acquisition board, motion card, and direct amplifier control architectures. As the following sections explain, each setup is best suited for a particular type of excitation input – stochastic, discrete sinusoidal using sensory feedback, and feedforward discrete sinusoidal.

6.2.1 Implementation of Stochastic Excitation

One of the biggest impediments to the implementation of stochastic system identification is the amount of computational power required. Portable and older systems tend to suffer from limitations on processing capabilities that make them incapable of performing the necessary linear algebra on large vectors of data. However, because the actuated probe is tethered to a bench top computer, this is not a concern. The input signal can be created before the trial using a random number generator to produce a white noise signal with a Gaussian amplitude distribution. The software available on the computer is well-suited for the post-processing and graphical display of the collected data. Additionally, the sensor readings can be read and stored at a rate of 200 Hz directly by the benchtop computer. This sampling rate is limited by the processing speed of the computer due to the additional processes that it runs simultaneously.
The sampling rate could be improved by using a data acquisition board, but since the system being characterized is mechanical with a non-negligible amount of inertia, the relevant dynamics are expected to occur below 50 Hz, which is within the available sampling rate.

The preferred way to implement the stochastic system input is by commanding a white noise signal directly to the actuator. The amplifier that presently delivers power to the motor can receive commands from a software provided by the manufacturer or from an independent motion card. The amplifier software allows the user to send direct commands to the amplifier but only in the form of a sinusoidal voltage input or a constant voltage. The commands that the motion card can send to the amplifier are either based on position or force feedback from the device. A white noise signal can be sent to the motion card in the form of the desired force or position value, but then the input command to the motor would be dependent on sensory feedback control. This is not only unnecessary for stochastic excitation, but it adds further complexity and uncertainty to the system dynamics and it introduces noise from the sensor performance in addition to the desired white noise signal. Therefore, the electronics presently in place are not able to perform a stochastic characterization of the system in the preferred manner.

Stochastic characterization will still be necessary when performing trials with sonographers and patients. To best address this concern, the actuated probe electronics would need to be reconfigured so that a data acquisition board manages the communication between the user interface computer and the motor amplifier. A stochastic input signal could be stored on the data acquisition board prior to the start of the trial; the board would then be able to output the white noise voltage command directly to the amplifier and simultaneously collect sensor data independent of and at a much higher sampling rate than the user interface computer. A diagram explaining the transfer of data using a data acquisition board is shown in Figure 6-2.

Before conducting trials in conjunction with an ultrasound scan, there are a series of experiments that should be conducted in order to confirm that the device is able to measure mechanical impedance accurately and reliably within the frequency range appropriate for human arm and tissue impedance. For the purposes of determining the device capabilities, trial length will not be a constraint. As such, initial impedance characterization experiments, including the ones discussed in this study, are performed using discrete sinusoidal excitation. Part of the appeal
of using discrete sinusoidal excitation is the computational simplicity. This means that the system identification results are less dependent on hardware processing limitations. Furthermore, one of the secondary future applications of this device is the ability to use it as an educational tool in courses on system identification and frequency response. Discrete sinusoidal excitation is pedagogically simpler and more intuitive so establishing this method would be advantageous in this context.

![Diagram of control architecture](image)

Figure 6-2: Data acquisition board control architecture explained graphically. The arrows denote the signal transmissions between the components, which are organized by function. This is the electronics setup that is required to perform stochastic excitation in the preferred manner.

6.2.2 Motion Card Control Architecture

At present, the motor input can be controlled directly through the amplifier software or through an independent motion card. Motion card control architecture is defined by the use of a National Instruments PCI-7538 motion card to implement a PID controller. The motion card serves as a separate processor and data acquisition board – it receives top-level commands from the operator computer, collects raw data from the sensors, carries out the required compensator
computations, and communicates commands with the motor amplifier. The motion card electronics are designed specifically for implementing PID control - it reads the sensor data and accordingly adjusts the current sent to the motor at a rate of 16 kHz. (National Instruments, 2008) By comparison, PID control performed on the operator computer can run at 200 Hz due to the additional processes running simultaneously and the processor limitations.

Figure 6-3: Motion card control architecture explained graphically. The arrows denote the signal transmissions between the components, which are organized by function.

Figure 6-3 shows the signal transmissions between all the electronics involved in the motion card control architecture. As stated before, the NI motion card receives position and force feedback from the encoder and load cell sensors. It then compares the data with the desired value, either a force or a position, set by the user through the PC. After performing the PID calculations, using parameters set by the user prior to the trial, the motion card adjusts the voltage signal sent to the amplifier. This voltage is proportional to the current input desired for the motor. The amplifier, a Copley Controls Accelnet, then performs the required commutation and pulse width modulation to ensure that the desired current is sent to the motor. (Copley
Controls, 2016) Force and position values are not stored by the motion card, but the data is relayed to the PC via the motion card.

The computer serving as a user interface uses LabVIEW to write the controller PID gains and the desired force or position value to the motion card. The LabVIEW interface also collects and writes the data from the motion card. The specific steps taken during a typical experimental session through the LabVIEW graphical user interface are outlined below.

1. Initialize device motors and sensors.
2. Set the controller type and the PID parameters.
3. Set the values for the desired sinusoid, including amplitude, frequency, and mean offset.
4. Turn on the controller. Since the desired value is constantly computed and updated to the motion card, this task is set at the highest processing priority. The update loop runs at 200 Hz, the same frequency as the sampling rate.
5. Collect position and force data from the motion card. This loop runs at 200 Hz, the highest sampling rate attainable due to processing limitations. Following the Nyquist-Shannon sampling theorem, sinusoidal signals below 100 Hz can be re-constructed.

6.2.3 Position Control Sinusoidal Excitation

One of the two controllers implemented using the motion card architecture is PID position control. The user initially designates a desired position $\theta_{desired}$ for the motor armature. The controller then compares the position reading from the motor encoder, $\theta_{actual}$, with the desired value to determine the error, $e_{\theta}$. The current that the motor is operating at is then adjusted based on the current error (proportional), how long the error has endured (integral), and how fast the error is changing (derivative). These parameters give the controller its name and the constants that determine the level of controller effort $k_p, k_i, k_d$ are set by the user, along with the integration limit, $I_L$, the window that the error is integrated over. The associated equations are presented below and the visual representation is shown in Figure 6-4.

$$e_{\theta} = \theta_{desired} - \theta_{actual}$$

$$i(t_1) = k_p \left( \frac{d}{dt} \theta(t_0) + k_p e_{\theta}(t_0) + k_i \int_{t_0-I_L}^{t_0} e_{\theta} dt \right)$$
Figure 6-4: The position control feedback loop represented graphically. The plant represents the motor dynamics involved in converting input current to output rotational position, including motor inertia and reaction torque due to the mechanical impedance of the device under test.

The amplitude of $\theta_{desired}$ was set to correspond to a probe translational amplitude of 1 to 2 cm at low frequencies. The PID gains were then set to minimize the effects of nonlinearities, such as static friction, and hardware malfunction, such as motor input signal saturation. These values varied by trial and are presented as part of the experimental procedures in the later chapters. For a linear time-invariant system, a sinusoidal input will result in a steady-state sinusoidal output of the same frequency. Following the assumption that the device is an LTI system, the PID gains were adjusted to values that resulted in sinusoidal position and force outputs that matched the frequency of the sinusoidal input. A real-time plot of the actual position was used to manually adjust the gains. In an attempt to maintain consistency, the PID gains remained unchanged across all trials with the same position input amplitude. Tests with additional amplitudes of $\theta_{desired}$ were conducted to check for an amplitude dependency in the frequency response.

Since the motion card does not have a function generator, $\theta_{desired}$ needed to be constantly updated by the user computer to produce the sinusoidal inputs. The sampling frequency of the computer limited the input to below 200 Hz. The lowest frequency tested was 0.1 Hz to limit the test trial time for a given input to below 2 minutes. At least 10 cycles were collected during each of the trials. Successive individual measurement trials were conducted from 0.1 Hz to 50 Hz in semi-logarithmic increments to gather sufficient data within the spectrum.
6.2.4 Force Control Sinusoidal Excitation

PID force control was the second controller implemented with the motion card architecture. Similar to the PID position control, the user designates a desired force transducer measure \( f_{\text{desired}} \) that they would like the probe to experience. The controller then compares the force reading from the load cell \( f_{\text{actual}} \) with the desired value to determine the error \( e_f \). The operating current of the motor is then adjusted proportional to the integral, derivative, and proportional error. The gains \( k_p, k_I, k_D \) are set by the user, along with the integration limit, \( I_L \). The controller equations are presented below along with the visual representation of the force feedback loop shown in Figure 6-5.

\[
e_f = f_{\text{desired}} - f_{\text{actual}}
\]

\[
i(t_1) = k_D \left( \frac{d}{dt} e_f(t_0) + k_P e_f(t_0) + k_I \int_{t_0}^{t_0-1} e_f dt \right)
\]

Figure 6-5: The force control feedback loop represented graphically. The plant represents the device dynamics involved in converting input current to output force from the probe.

Trials were similarly performed in semi-logarithmic increments starting from 0.1 Hz. The amplitude of \( f_{\text{desired}} \) was set to correspond to a load between 1 and 5 N since this is the typical force exerted during an ultrasound scan. Load amplitude was varied to check for load dependency in the system dynamics. The PID gains for the force control trials were set in the same manner described for position control.

The most noticeable difference among this set of trials was that the highest frequency achievable was 10 Hz. This limit was due to a breakdown in communication and control of the
motor, characterized by maximum output. This phenomenon was possibly due to saturation of the motor command. From initial trials it was observed that as the frequency of operation increased, $f_{\text{actual}}$ could no longer track $f_{\text{desired}}$ as closely and the force amplitude decreased. The increase in $e_f$ led to an increase in the command sent to the motor to compensate, which contributed to the saturation. Additionally, it was noted in Chapter 5 that the force sensor has a constant noise with a standard deviation of $\sigma_f = 0.5\ N$. When the amplitude of $f_{\text{actual}}$ decreases, the controller feedback becomes more susceptible to the load cell uncertainty. The following chapter discusses the filtering necessary to reduce the noise and gather data from the force sensor reliably. This filtering cannot be implemented on the feedback to the force controller, leading to unreliable sensor data under certain conditions. A sensor feedback that is no longer reflective of the actual state of the device results in instability and a breakdown in control effort.

### 6.2.5 Direct Amplifier Control Architecture

Direct amplifier control architecture is defined by the use of the Copley Control Accelnet amplifier to directly control the current signal sent to the motor. The amplifier is capable of running as a function generator, so upon setting the desired frequency and amplitude of the input sine wave, the amplifier independently ensures that the input current signal meets these criteria. As shown in Figure 6-6, the NI motion card is still used in this architecture, but only to relay the sensor information to the user interface computer, not for control purposes.

The computer serving as a user interface uses CME2, a software from Copley Controls, to communicate with the Copley amplifier directly. A LabVIEW interface is used to initialize the motors and collect the data from the NI motion card. The specific steps taken during a typical current control experimental session are outlined below.

1. Initialize device motors and sensors via LabVIEW.
2. Set up the function generator within CME2.
3. Set the values for the desired output sinusoid including amplitude, and frequency within the CME2 function generator.
4. Turn on the amplifier function generator. Current control within the amplifier runs independently of the user interface computer at 15 kHz. (Copley Controls, 2016)

5. Collect position and force data from the motion card via the LabVIEW software. The amplifier software does not have access to the force transducer readings and so it cannot presently be used for full data collection. Because LabVIEW is not sending values to the motion card during current control, the data collection loop runs at the highest priority at a frequency of 200 Hz.

![Diagram of control architecture]

Figure 6-6: Direct control architecture explained graphically. The arrows denote the signal transmissions between the components, which are organized by function.

### 6.2.6 Current Control Sinusoidal Excitation

Current control is performed by bypassing the NI motion card and having direct access to the amplifier. Unlike the NI motion card, the amplifier can be programmed to independently produce a sinusoidal signal so it is not limited by the processing speed of the user interface computer. Current can be inputted at two orders of magnitude faster than through motion card control, but the signals were conservatively kept to a maximum frequency of 200 Hz. The limiting factor in
this method is then the frequency of the data collection software, which can confidently
reconstruct sinusoidal signals below 100 Hz when following the Nyquist-Shannon sampling
theorem.

Trials were performed by exciting the device at discrete sinusoidal frequencies, starting at 50
Hz and successively reducing the frequency in semi-logarithmic decrements. The current
amplitude was kept at the allowable maximum of 3 A for all trials. At about 4 Hz the probe motion
was no longer symmetric, as it would drift further in the direction of least resistance – away from
the tissue impedance. After a few cycles, this would lead to the probe losing contact with the
tissue. This problem could be resolved by applying a bias to the current input to the motor, but
this feature was not available on the amplifier function generator.

With the implementation of three different controllers for inputting a discrete sinusoidal
input, there are enough options available to compare which method is most reliable, or whether
a combination of controllers is preferable due to their limitations at high or low frequencies.
Additionally, performing the same experiment using different controllers can be used to
corroborate the results and provide confidence in the methods used. The following chapters
present the discrete sinusoid excitation trials performed using these controllers in order to,
among other things, confirm that the implementation described in this chapter is capable of
performing mechanical impedance characterization.
Chapter 7

Model Validation & Characterization

7.1 Experimental Characterization of Device

While most of the parameters in the system could be found by measuring the properties of the individual components of the disassembled device with external tools, some could only be estimated by assembling the actuated probe and measuring the behavior using its internal measurement devices. This was particularly necessary when estimating the effective inertia, stiffness, and dissipation in the system.

Figure 7-1: Device setup during the experimental characterization of the actuated probe. At left, the housing end of the device is clamped to the workbench while the probe end is free to move in space. At right, the probe end of the device is clamped to the workbench while the housing end of the device is free to move.
7.1.1 Experimental Setup

Before the actuated probe is placed in interaction with deformable structures, such as a human hand or tissue, it is possible to first observe the unloaded system behavior. Since a model of the unloaded system was previously developed in Chapter 5, the predicted dynamics can be compared against the behavior exhibited by the device without interaction with an external impedance. For these experiments, the probe was rigidly attached on one end to a clamp while the opposite end is allowed to move freely. Figure 7-1 shows the device with the housing end clamped (left) and the probe end clamped (right). To confirm the rigidity of the clamp support, tests were conducted with the accelerometer placed on the probe end of the device and then repeated with the accelerometer placed on the housing end.

7.1.2 Procedures

For each of the configurations, the system was consecutively excited by sinusoidal inputs via the motor. For half of the trials, the excitations occurred by inputting a sinusoidal current directly to the motor using the amplifier software. These trials initiated at a 50 Hz excitation using a 3 A amplitude, the maximum allowable by the amplifier. Data was collected at 5 Hz decrements until 15 Hz. Below this frequency, data was collected at 12 Hz and below in decrements of 2 Hz. In this lower range of frequencies, the current amplitude was reduced as necessary in order to prevent the moving end of the device from reaching the travel limits of the ball screw. For most sets of experiments, the lowest frequency attainable using current control was 4 or 5 Hz. At this frequency, the current amplitude had to be reduced to 2.2 A and the device exhibited irregular periods of lack of motion, possibly due to inability to overcome static friction.

For the other half of the trials, a desired sinusoidal trajectory was inputted to the position feedback controller using the motion card architecture. Experiments were started at 0.1 Hz and repeated in semi-logarithmic increments: 0.2, 0.5, 1, 2, 3, 4, 5, 6, 8, 10, 12, 15 Hz. The maximum frequency that could be achieved using the position feedback controller was 20 Hz. At higher frequencies, the motor would suddenly vibrate vigorously shortly after beginning the trial. The current control trials were conducted at this frequency without a problem so this is not due to a limitation of the motor hardware. This erratic behavior is likely due to the implementation of the
feedback controller. The particular reason was not determined but it is possible that the controller effort exceeds the saturation limits of the current input to the motor.

To ensure that the system dynamics remained constant through each set of experiments, the PID gains were maintained constant for all trials. The gain $k_P$ proportional to the error in encoder position was adjusted at the 0.1 Hz trial to achieve the desired amplitude without exhibiting oscillatory overshoot. The integral and derivative gains $k_I$ and $k_D$ were not needed and were set to zero. The motor current $i_m$ was controlled by the error between the actual and desired encoder positions $\theta$ and $\theta_{des}$, $k_P$, and an unknown gain $c_m$ due to hardware implementation.

$$i_m = c_m [k_P(\theta_{des} - \theta)]; \quad k_P = 10$$

The desired position amplitude was set to 10000 encoder counts from 0.1 Hz to 8 Hz; this corresponds with a linear motion with an amplitude of 0.98 cm. For all position trials, the desired amplitude had to be reduced to 5000 counts at 10 Hz in order to avoid the vigorous motor vibrations, and to 2000 counts for some of the 20 Hz trials.

### 7.1.3 Clamp Rigidity

To confirm that the clamp used to immobilize the device to the reference frame was rigid, the accelerometer placement was alternated between the probe end and housing end of the device for all trials. Even though the accuracy of the accelerometer was deemed questionable for the quantitative computations of this study, the accelerometer reading is sufficiently reliable to indicate the presence of motion. As expected, the level of activity from the accelerometer was comparable to the static noise levels when it was placed on the immobilized portion of the device but considerably higher when the accelerometer was placed on the moving end of the device.

To further analyze this observation, the accelerometer reading was de-trended and centered at zero for the four experimental setups when the excitation frequency was 20 Hz. Figure 7-2 shows the Fourier transform of the square of the data sets plotted as a function of frequency. When the accelerometer is installed on the free end of the device, there is a spike in variance that is not observed when the accelerometer is attached to the clamped end. This observation confirms that the vice prevents the clamped end from exhibiting motion and that the free end follows the sinusoidal oscillation at the frequency of excitation, as explained below.
Figure 7-2: Power spectral density of the accelerometer reading at trials with 20 Hz excitation, for each setup configuration. At top left, the probe end is clamped and the accelerometer is on the free end. At top right, accelerometer is mounted on the clamped probe end. At bottom left, accelerometer placed on clamped housing end. At bottom right, the housing end is clamped, with accelerometer placed on the free probe end.

When a Fourier transform is performed on an ideal sinusoid, the power spectral density manifests itself as a delta function at the frequency of oscillation. (Bendat & Piersol, 1971) In Figure 7-2, there is a maximum value at approximately 20 Hz – the frequency of excitation – but the peak appears to decay at the surrounding frequencies instead of dropping sharply. This is due to the finite length of the signal collected as well as the discrete Fourier transform algorithm. A
DFT needs to be of a specific length $n$ in order for the algorithm to converge. (Strang, 2007) If the signal length is not equal to one of these $n$ values, the algorithm appends additional zero-valued points to the signal. The resulting plot is a Fourier transform of a sine function and a rectangular function, which results in a sum of an impulse function and a sinc function, resulting in the gradual dropoff.

### 7.1.4 Time Data Analysis

One way to examine the system dynamics is to relate the measures collected for each of the discrete frequency tests. As discussed in the previous chapter, by sampling the device behavior across a range of frequencies, a frequency response plot can be compiled that shows the relationship between two variables. These can be formed from the ratio of the signal amplitudes and the phase shift between them for each sampled frequency. Because the system excitation is sinusoidal, a sinusoidal behavior is expected from all variables if the system were linear and time invariant. A least squares fit algorithm, the MATLAB built-in “fit” function, was applied to the time series data in order to approximate the signals as sinusoidal waves. The residual between the fit and the data can provide insight into the validity of the linearity assumption as well as the effects of the sensor noise.

Initially, a fit was assumed using the equation shown below. The amplitude $A$ is initially approximated as the maximum value within the zero-centered signal and the phase shift $C$ is initially approximated as 0. The frequency of oscillation is set to the nominal frequency of excitation. A sample fit is shown in Figure 7-3.

$$\theta = A \sin(\omega_{\text{excitation}} 2\pi t + C)$$

For this fitting method, the residual between the fit and the data was not constant throughout data set and the fit consistently underestimated the signal amplitude. It is possible that the fit and the data don’t follow the same frequency, causing them to become in and out of phase. The least squares fit was revised to allow the frequency to vary as a fitted parameter. The initial value for $B$ is the excitation frequency $\omega_{\text{excitation}}$ that was previously assumed. A sample fit is shown in Figure 7-4.

$$\theta = A \sin(B 2\pi t + C)$$
Figure 7-3: At left is the sinusoidal fit to the measured encoder data. The amplitude and phase are fitted to the data while the frequency is assumed. At right is the residual between the fit and the data.

Figure 7-4: At left is a portion of the sinusoidal fit to the measured encoder data. The amplitude, phase, and frequency are fitted to the data using initial estimates. At right is the residual between the fit and the data.

Compared to the fit in Figure 7-3, the fit in Figure 7-4 follows the data more closely and consistently, as is confirmed by a smaller and more uniform residual. This observation was made
across multiple trials and as such it was concluded that although a nominal frequency is known, accuracy is improved by assuming small deviations can occur from this nominal value.

It is possible that the fitted values of the signal – phase, frequency, amplitude – are not time-invariant as would be expected. It may be worthwhile examining smaller segments of the data and averaging the results from the separate windows. The variance of these values across windows will be useful in providing a measure of the time-variance of the signal. Figure 7-5 shows the residual of the fits when the data from the previous figures is divided into windows of 10 cycles which are analyzed separately. Residual values are lower than those shown in Figure 7-4.

![Residual of Sinusoidal Fit to Encoder, p40](image)

Figure 7-5: Residuals between the sinusoidal fits and the encoder data points. The collected data was segmented into windows consisting of 10 cycles which were then analyzed independently.

When the sinusoidal variables are fitted across independent windows of 10 cycles, the values average to $A = 32.14 \pm 0.10 \text{ rad}$, $B = 1.991 \pm 0.005 \text{ Hz}$, $C = -1.41 \pm 0.28 \text{ rad}$ for the data presented in Figure 7-5. The low standard deviations of these fitted values are characteristic of a system that can be approximated as time-invariant. The average residual for the fits in this set of windows is 1.53 radians, which is 4.8% of the amplitude. The error between the fit and the data is low enough to confidently assume that the data is sinusoidal; and since the excitation is sinusoidal as well, this implies the dominance of linear dynamics in the system. Further variations on the curve fitting processing were attempted, including overlapping the windows by 50% in
order to double the sample size. However, there were no further noticeable improvements in the fit residual or the variance of the fitted variables.

### 7.1.5 Additional Processing for Load Cell Data

The sinusoidal fitting process was relatively simple for the encoder data, but that was not the case for the load cell. The sensory analysis performed in Chapter 3 revealed that the load cell exhibits noise levels of approximately 30% of the amounts measured in this study. To determine whether additional filtering is necessary to extract useful information from this sensor, the window-segmented sinusoidal fitting process discussed in the previous section was applied to the load cell data from the same trial. The sinusoid equation used in the load cell fit, shown below, uses the same variables as the encoder fit. Results from this process are presented in Figure 7-6.

\[
F = A \times \sin(B \times 2\pi \times t + C)
\]

![Sinusoidal Fit to Load Cell, p40](image)

![Residual of Sinusoidal Fit to Load Cell, p40](image)

Figure 7-6: At left is a portion of the sinusoidal fit to the measured load cell data. The amplitude, phase, and frequency are fitted to the data using initial estimates. The data is analyzed in independent windows consisting of 10 cycles. At right are the residuals between the fit and the data.

The load cell data itself looks noisy and a sinusoidal pattern is not visually apparent. The standard deviation of the fit, 0.26 N, is greater than the fitted amplitude of the signal. When averaged across the windows, the fitted parameters are \( A = 0.19 \pm 0.01 \) N, \( B = 1.993 \pm 0.009 \) Hz, \( C = 1.13 \pm 0.74 \) rad. Based on their standard deviations, the values of \( A \) and \( B \) appear to be consistent across the window segments, suggesting that a sinusoidal signal at the
frequency of excitation was measured by the sensor despite the noise. Inspecting the power spectral density of the measured signal, shown in Figure 7-7, provides greater insight on how to address the noise.

![Variance of Load Cell](image)

Figure 7-7: Discrete Fourier transform of the squared de-trended load cell signal.

Upon inspection, Figure 7-8 reveals a “spike” in activity at the frequency of excitation with mirrored decaying exponentials indicative of sinusoidal behavior at that frequency. There is also increased activity at higher frequencies around 5 Hz and 10 Hz. Since the only frequency of interest is the frequency of excitation, this higher frequency activity was treated as the source of the noise removed using a filter in order to improve the fit.

A first order bandpass filter was used to create a low cutoff at half the frequency of excitation and a high cutoff frequency that is twice the excitation frequency. The filter was applied to the load cell data using the MATLAB built-in Butterworth filter function. A transfer function of the bandpass filter is expressed below and Figure 7-8 shows the power spectral density of the load cell signal after applying the filter.

\[
\frac{f_{\text{load, filt}}}{f_{\text{load}}} (\omega) = \frac{\omega}{\left(\omega + \frac{\omega_{\text{excitation}}}{2}\right)\left(\omega + 2\omega_{\text{excitation}}\right)}
\]
Figure 7-8: Discrete Fourier transform of the squared de-trended load cell signal, after application of a bandpass filter. Cutoff frequencies were $\omega_{\text{low}} = 1 \text{ Hz}$, $\omega_{\text{high}} = 4 \text{ Hz}$.

Figure 7-9: At left is a portion of the sinusoidal fit to the measured load cell data, after being filtered to remove noise. The amplitude, phase, and frequency are fitted to the data using initial estimates. The data is analyzed in independent windows consisting of 10 cycles. At right are the residuals between the fit and the data.
A sinusoidal fit is applied to the filtered load cell data in order to evaluate the changes in the fitted parameters. Figure 7-9 shows a plot of the fit of the filtered load cell data and of the residual between the two.

Upon inspection of Figure 7-9, a sinusoidal pattern is now visually apparent and the amount of noise has been reduced. The standard deviation of the fit was reduced to 0.06 N, about 25% of the unfiltered standard deviation and about 30% of the calculated signal amplitude. As for the fitted variables, when averaged across windows they are $A = 0.191 \pm 0.006 \, N$, $B = 1.997 \pm 0.006 \, Hz$, $C = 1.25 \pm 0.60 \, rad$. These amounts and their standard deviations are not markedly different from the results found using the unfiltered data. Performing additional comparisons on other sets of data between filtered and unfiltered load cell measures yielded similar findings. However, the fitting algorithm had a failure rate of approximately 10% when applied to the unfiltered load cell data, but less than 1% when applied to the filtered load cell data. This was a sufficient enough improvement to warrant the continued used of a bandpass filter.

7.1.6 Summary of Time Data Analysis

For all the unloaded discrete sinusoidal excitation trials each set of data is split into windows of 10 cycles. A sinusoidal equation was fit that minimizes the mean square error between the fit and the data values. A 50% window overlap was used in order to maximize the number of independent data sets available. If for a given trial there were less than 10 windows in total, the number of cycles per window was decreased until 10 windows were created from the trial. For each discrete frequency within a set of trials, the fitted coefficients $A, B, C$ were averaged across the data set windows. For the load cell data, the bandpass filter described in the previous section was used prior to the sinusoid fitting.

The coefficient $C$ which is indicative of the phase of the sinusoid is highly sensitive to changes in the frequency of oscillation coefficient $B$. For this reason, when finding the phase between two signals it is important that the frequency $B$ of both signals are identical. For the trials discussed here, when computing the fit for the load cell data the frequency coefficient $B$ is set as an invariant amount $B_{enc}$, the frequency coefficient from the encoder fit. This ensures that the phase between the encoder and load cell is determined appropriately.
7.2 Frequency Response Analysis

7.2.1 Current Input, Encoder Output

Once the amplitude and phase of each of the variables measured or commanded during each trial is known, the ratio of amplitudes – the gain – and the phase between two variables can be examined as a function of frequency. The first relationship of interest is between the input current from the amplifier and the output position reading from the encoder. The current is only known for trials performed using the amplifier function generator. This limits the data available to the range of 5 – 50 Hz. A plot of the gain between the current input and motor encoder position is shown in Figure 7-10. Since the data collection system does not have direct access to the current input signal, the phase cannot be accurately estimated.

![Frequency Response](image)

Figure 7-10: Gain plot showing the amplitude ratio between the motor current input and the motor encoder measure as a function of frequency.

To analyze the plot, we first establish a model where the motor torque $\tau_m$ is proportional by an unknown factor $c_m$ to the motor input current $i_m$. From Figure 7-10, there appears to exist an asymptote with a gain decrease of two orders of magnitude per frequency decade. In this region,
the motor current and torque are proportional to the motor acceleration; this indicates that an inertial term \( I_{m,\text{app}} \), the apparent motor inertia, is dominant.

Unfortunately, not much is known about the dynamics at lower frequencies but since an elastic element \( k_{m,\text{app}} \) could be dominant in this region, due to the belt elasticity for instance, it is included within the assumption of a second-order system. Similarly, at frequencies below those tested, a velocity-dependent damping term \( b_{m,\text{app}} \) may be dominant. This behavior is not observed in the plot but it is possible that non-zero apparent motor damping exists since the system exhibits dissipative behavior. The motor dynamics may be modeled as shown below. Later sections will revisit whether the elastic and dissipative element are non-zero or whether they could be eliminated.

\[
i_m = \frac{\tau_m}{c_m} = \frac{1}{c_m} (I_{m,\text{app}} \ddot{\theta}_m + b_{m,\text{app}} \dot{\theta}_m + k_{m,\text{app}} \theta_m)
\]

This motor dynamics equation can be rewritten as a frequency response function.

\[
\frac{\theta_m}{i_m}(\omega) = \frac{c_m}{I_{m,\text{app}} \omega^2 + b_{m,\text{app}} \omega + k_{m,\text{app}}}
\]

The magnitude of the transfer function \( |\theta_m/i_m| \) can then be fitted to the gain between the measured variables \( \theta_m \) and \( i_m \).

\[
|\frac{\theta_m}{i_m}|(\omega) = \frac{c_m}{\sqrt{(-I_{m,\text{app}} \omega^2 + k_{m,\text{app}})^2 - (b_{m,\text{app}} \omega)^2}} \approx \text{Gain} \left( \frac{\theta_m}{i_m} \right)(\omega)
\]

Since the inertial term is dominant at higher frequencies, the magnitude equation can be simplified so that the inertial term \( \frac{c_m}{I_m} \) is approximately proportional to the gain times frequency squared in this range.

\[
\frac{c_m}{I_{m,\text{app}}} \approx \omega^2 \ast \text{Gain} \left( \frac{\theta_m}{i_m} \right)(\omega); \quad \omega > 10 \text{ Hz}
\]

This inertial term is approximated by computing the mean of \( \omega^2 \ast \text{Gain} \) for all values above 10 Hz, the region dominated by the inertial asymptote. \( I_{m,\text{app}} \) has units of \( kg \cdot m^2 \) while \( c_m \) has units of \( N \cdot m/Amps \) since it is the conversion between torque and current.

\[
\frac{I_{m,\text{app}}}{c_m} \approx 5 \times 10^{-5} A \cdot s^2
\]
In Sections 5.8 and 5.9, transfer functions were developed relating the motor rotation $\theta_m$ to the motor torque $\tau_m$. Since $\tau_m$ is proportional to the motor current $i_m$, the equation developed from the model in Chapter 5 can be compared against the plots presented in this section. As mentioned above, the plot only provides clear evidence of an inertia-dominated system, which corresponds to two poles in the denominator. The equations presented at the end of Chapter 5 indicate the presence of a pair of zeros and four poles. However, it was also mentioned that the controllability and observability matrices for that system were ill-conditioned. One possibility is that two poles in the system are dominant while the pair of zeros and a pair of poles are not significant to the relevant dynamics. If this were the case, then the model equations may tentatively agree with the data shown here, although this conclusion depends on several factors that are not yet clear, such as the values of $b_{m,app}$ and $k_{m,app}$.

7.2.2 Current Input, Load Cell Output

A frequency response plot is similarly assembled relating the current input to the load cell measured output. From the plot in Figure 7-11, the gain between input current and force measured appears to be approximately constant above 10 Hz. Below 10 Hz, the magnitude appears to increase by an order of magnitude for every decade in frequency. However, there are not enough data points to draw the conclusion. Based on this observation, the load cell signal can be approximately modeled as proportional to the current, and therefore proportional to torque.

$$f_t = c_f i_m = c_f c_m \tau_m$$

If a direct and lossless transfer of power is assumed, the ball screw would convert the torque from the motor to force at the probe end. This assumption would result in a load cell reading that is proportional to the motor torque and current input, which is what the frequency response data demonstrates. The proportionality constant can be approximated by computing the mean gain from the data plotted in Figure 7-11.

$$c_f \approx 0.2 \frac{N}{A}$$
Figure 7-11: Gain plot showing the amplitude ratio between the motor current input and the load cell measure as a function of frequency.

At the end of Chapter 5, an equation was developed from the model relating $f_{ft}$ to $\tau_m$ which can be directly compared against the data in Figure 7-11. The model equation predicted at least one pole-zero cancellation at the origin, tentatively two depending the damping parameters. The remaining two poles are expected to be located at approximately 260 Hz. This pole location is an order of magnitude higher than the frequencies tested. Therefore, assuming an accurate model, the equation $f_{ft}/\tau_m$ can be approximated as a constant within the frequency range of interest for this study. This appears to be consistent with the data presented in the figure, particularly above 10 Hz. It remains unclear if the data below 10 Hz is indicative of low frequency dynamics.

7.2.3 Encoder – Load Cell Relationship

The frequency response magnitude and phase plots relating the encoder position reading to the load cell force reading are shown in Figure 7-12. Because the encoder and load cell readings
are always being measured synchronously, the magnitude and phase plots can be reproduced from all the data collected, regardless of control configuration.

The first thing to notice is that the data points from the current control trials align with the data points from the position control trials within the overlapping frequency range. If these points were noticeably misaligned or displayed contradicting trends it would be an indication that the relationship between the motor motion and the load cell reading is being affected by the controller configuration used.

The next step is to identify the order of the relationship by detecting asymptotes in the phase and magnitude data. For all of the current-controlled data and most of the position-controlled data, the magnitude shows a steady decrement of 2 orders of magnitude for every decade in frequency. Within part of this same range, between 2 and 10 Hz, the phase appears to approach \(-\pi = -180^\circ\).

Another set of asymptotes exists at frequencies below 1 Hz. The values indicate a constant magnitude and a phase that approaches 0°. In this region the motor position is proportional to the load cell reading. In a physical interpretation, this relationship can be interpreted as the dominance of an elastic element. The presence of these two regions is a defining characteristic of a second-order mass-spring-damper system.

Above 10 Hz, the phase appears to depart away from \(-180^\circ\). One possible reason for this is due to a time delay between the two measures due to the difference in sensor dynamics. This may also be evidence of a high frequency zero. The slope of the magnitude plot does not show any significant change at higher frequencies so it cannot confirm this explanation. It is expected that at frequencies higher than those tested, the effects of this phase departure will become more apparent. Due to this, greater attention is placed on the second order dynamics.

These trends are observed for all sets of experiments. However, the asymptotes and natural frequency of the system vary depending on whether the housing or the probe of the device is immobilized. In order to extract the device parameters from the frequency response plot, the following second order equation was fitted to the data.

\[
\frac{\theta_m}{f_f}(\omega) = \frac{1}{m\omega^2 + b\omega + k}
\]
Figure 7-12: Frequency response plots showing the frequency relationship between the motor current input and the load cell measure. At top is the gain, the ratio between the amplitudes of the variable signals. At bottom is the phase between the two signals. The black line is a model fit of the data found by approximating asymptotic values. The green line is a computational least squared error fit the magnitude data. The red line is a computational non-linear least squared error fit of the magnitude and gain.
Based on when the phase crosses $-90^\circ$, the natural frequency of the system shown in Figure 7-12 is at approximately 2 Hz. This frequency is determined by the real component of the roots of the transfer function denominator.

$$\omega_n = \sqrt{\frac{k}{m}}$$

The first fit performed is an asymptote fit where the parameters are individually estimated based on magnitude values at the different asymptotic regions. The $k$ term is dominant at lower frequencies where the asymptote is constant in the position-force plot. The average of the data points in this region is approximately equal to $1/k$.

$$\frac{1}{k} \approx \text{Gain}\left(\frac{\theta_m}{f_{ft}}\right)(\omega); \quad \omega < 1 \text{ Hz}$$

Similarly, the $m$ term is dominant at higher frequencies where the asymptote is constant in the acceleration-force plot. After integrating the data twice, by multiplying the gain by the square of the frequency, the average of the data points in this asymptotic region is approximately equal to $1/m$.

$$\frac{1}{m} \approx \omega^2 \ast \text{Gain}\left(\frac{\theta_m}{f_{ft}}\right)(\omega); \quad \omega > 3 \text{ Hz}$$

When the phase plot is approximately equal to $-90^\circ$, at about the natural frequency, the damping term is dominant. After finding the first derivative of the data, by multiplying the gain by the frequency, the maximum value is approximated as $1/b$.

$$\frac{1}{b} \approx \max\left[\omega \ast \text{Gain}\left(\frac{\theta_m}{f_{ft}}\right)(\omega)\right]$$

In the plots on the previous page, the resulting asymptote fit is plotted as a black line. Although the phase plot is not used directly in the approximation of the parameters, the analytic fit appears to fit the phase data as well.

The remaining fits were performed computationally by fitting the transfer function to the data using various algorithms. The green line indicates a least squared error fit of only the magnitude data. The equation used for the fit reflects this.

$$\left|\frac{\theta_m}{f_{ft}}(\omega)\right| = \frac{1}{\sqrt{(-m\omega^2 + k)^2 - (b\omega)^2}}$$
A final fit is performed using the MATLAB function tfest, which uses a nonlinear least-squares weighted algorithm to fit the transfer function to the complex-valued data. The resulting fit, shown in red, is the only fit employed that uses both the magnitude and phase plots to determine an analytic solution.

At the end of Chapter 5, relationships were derived from the system model $\theta_m/\tau_m$ and $f_{ft}/\tau_m$. The equation relating the outputs $\theta_m$ and $\tau_m$ can be found by combining these two relations. It is important to note that the four poles in $\theta_m/\tau_m$ and $f_{ft}/\tau_m$ are the same and will therefore cancel out.

$$\frac{\theta_m}{f_{ft}}(s) = 0.0375 \frac{s^2 + b_1 s + 2299000}{s^2 + b_6 s^3}$$

Assuming that $b_6$ is negligible, the equation above predicts that the relationship between the motor encoder position and the force transducer measure is dominated by an inertial term. The two zeros would not affect the dynamics until around 260 Hz, which is beyond the frequency range of interest. Figure 7-12 shows that an inertial term is dominant between 2 Hz and 50 Hz. Below 1 Hz, though, the relationship between the two variables is proportional, evident of an elastic element. At $0.1 \, N/m$ this stiffness is approximately five orders of magnitude lower than the estimated belt stiffness. The source of this low stiffness element is unclear, but it is apparent that it was not included in the system model and thus there is a mismatch with the behavior exhibited in this section.

7.2.4 Position Controller Input, Encoder Output

For some of the trials, the PID position controller was used to input the sinusoidal excitation. Since only the proportional gain is used, the control equation can be written as follows. The constant $c_{PID}$ refers to a constant to account for any proportionality or conversion between the control equation and the commanded current $i_m$.

$$i_m = c_{PID}[k_p(\theta_{des} - \theta_m)]$$

The current $i_m$ can be replaced by the equivalent motor dynamics found in the previous section so the control equation is only in terms of the motor position.
\[
\frac{I_{m,\text{app}}}{c_m} \ddot{\theta}_m + \frac{b_{m,\text{app}}}{c_m} \dot{\theta}_m + \frac{k_{m,\text{app}}}{c_m} \theta_m = c_{\text{PID}}[k_p(\theta_{\text{des}} - \theta_m)]
\]

When this equation is expressed as a function of frequency, the transfer function between the encoder position reading and the desired position can be expressed as follows.

\[
\frac{\theta_m}{\theta_{\text{des}}} (\omega) = \frac{k_p c_{\text{PID}}}{\frac{I_{m,\text{app}}}{c_m} \omega^2 + \frac{b_{m,\text{app}}}{c_m} \omega + \frac{k_{m,\text{app}}}{c_m} + k_p c_{\text{PID}}}
\]

This predicted relationship can be compared with the collected encoder data and outputted control command. Figure 7-13 shows the gain between the desired encoder position and the actual position.

From the figure, two asymptotes are apparent. Below 2 Hz, as the frequency \( \omega \) approaches 0, the gain remains constant at approximately unity. This indicates that the actual position is closely following the desired position. This behavior is predicted by the transfer function as \( \omega \) approaches 0. Above 5 Hz, as the frequency \( \omega \) grows larger, the gain decreases at a rate of approximately 40 decibels for every frequency decade. This behavior is also predicted by the transfer function; as \( \omega \) approaches infinity, the inertia term becomes dominant, causing the magnitude to decrease at a rate of 40 dB per frequency decade.

Based on when the phase crosses \(-90^\circ\), the natural frequency of the system shown in Figure 7-13 is between 3 and 4 Hz. This frequency is determined by the real component of the roots of the transfer function denominator.

\[
\omega_n = \sqrt{\frac{k_p c_{\text{PID}} c_m + k_{m,\text{app}}}{I_{m,\text{app}}}}
\]

Since the \( \theta_m/\theta_{\text{des}} \) frequency response plot seems to match the behavior predicted by the \( \theta_m/\theta_{\text{des}} \) transfer function, the equation is fitted to the data in order to determine the value of \( c_{\text{PID}} \) and \( b_m/c_m \).
Figure 7-13: Frequency response plots showing the frequency relationship between the commanded desired encoder position and the actual encoder reading. At top is the gain, the ratio between the amplitudes of the variable signals. At bottom is the phase between the two signals.
An additional observation can be made by inspecting the phase plot in Figure 7-13. As predicted by the second order model, the phase approaches 0° as the frequency approaches zero and then drops to approximately −180° after the natural frequency. However, the phase continues to drop below −180° above 5 Hz at an increasing rate. Since there is no evidence of additional poles in the gain plot, this phase drop appears to be due to a time delay between the controller input and the sensor output. This is a common occurrence in physical implementations of feedback controllers. A pure time delay can be included in the system model with the additional term $e^{-j\tau\omega}$ in the denominator.

\[
\frac{\theta_m}{\theta_{des}}(\omega) = \frac{k_p c_{PID}}{\frac{I_{m,app}}{c_m} \omega^2 + \frac{b_{m,app}}{c_m} \omega + \frac{k_{m,app}}{c_m} + k_p c_{PID} + e^{-j\tau\omega}}
\]

The magnitude of this time delay is 1 and has not effect on the gain plot of the function. The additional phase change due to the time delay is equal to $-\tau\omega$.

\[
\angle e^{-j\tau\omega} = \tan^{-1} \left( \frac{-\sin(\tau\omega)}{\cos(\tau\omega)} \right) = -\tau\omega
\]

The value of the time delay $\tau$ can be estimated from the departure of the phase from −180° at the higher frequencies between 8 Hz and 20 Hz. The time delay is estimated as $\tau = 23 \pm 4 ms$. This level of time delay results in an additional 180° drop in phase before 20 Hz, which would be severely limiting during feedback control.

However, to understand why the stability of the system is not affected by this time delay, it is important to revisit the motion card control architecture discussed in Section 6.2.2. The desired position is outputted and recorded by the user interface computer. A significant amount of the delay is found in the transfer of the desired signal to the NI motion card. Additionally, the measured signals have to be transferred from the NI motion card to the user interface computer before they are recorded, resulting in additional delay between the two recorded signals. Since the motion feedback control is handled by the NI motion card, the feedback loop is unaffected by these transmission delays. In order to estimate the true feedback delay, the desired motion signal used by the motion card during feedback needs to be returned to the user interface computer along with the measured signals. In this manner, both recorded signals are affected by the same amount of transmission delay.
7.2.5 Position Controller Input, Load Cell Output

For the PID position controller trials, the current $i_m$ can be replaced by the equivalent motor dynamics found in Section 7.2.2.

$$f_t/c_f = c_{PID}[k_p(\theta_{des} - \theta_m)]$$

When this equation is expressed as a function of frequency, the transfer function between the force transducer reading and the desired position can be expressed as follows.

$$f_t/\theta_{des}(\omega) = c_f c_{PID} k_p - c_f c_{PID} k_p \frac{\theta_m}{\theta_{des}}$$

$$f_t/\theta_{des}(\omega) = \frac{c_f c_{PID} k_p [I_{m,app} \omega^2 + b_{m,app} \omega + k_{m,app}]}{I_{m,app} \omega^2 + b_{m,app} \omega + k_{m,app} + c_{m} k_p c_{PID}}$$

This transfer function predicts the presence of a pair of poles with the same natural frequency as the poles of $\theta_{enc}/\theta_{des}$. It also predicts the presence of a pair of zeros, whose natural frequency is defined below. This frequency is lower than the natural frequency of the poles.

$$\omega_n = \sqrt{\frac{k_{m,app}}{I_{m,app}}}$$

Figure 7-14 shows the gain and phase between the desired encoder position and the actual position. The gain plot shows an asymptote of constant magnitude at frequencies below 1 Hz, followed by an increase of two orders of magnitude per frequency decade, and finally leading to a period of constant magnitude between 4 Hz and 8 Hz. This suggests the presence of a pair of lower frequency zeros and a pair of higher frequency poles. The activity above 10 Hz signifies that there are additional physical or controller dynamics not included in the model. At low frequencies, the phase approaches $40^\circ$ asymptotically, which is followed by an increase that reaches $140^\circ$ before the phase drops back to $40^\circ$ between 3 Hz and 4 Hz. This type of drop is likely due to a pair of poles, the location of which is determined by the roots of the transfer function denominator. The denominator of the equation presented here, $f_t/\theta_{des}(\omega)$, has the same roots as $\theta_m/\theta_{des}(\omega)$, presented in the previous section. Both frequency response plots display a phase drop between 3 Hz and 4 Hz. This corroborates the presence of a pair of poles at this location. Additionally, the rise in phase between 0.5 Hz and 3 Hz could be due to a pair of
zeros at approximately 1 Hz. However, the model predicts a low frequency asymptote at $0^\circ$ and for the phase to approximate $180^\circ$ before the phase drop at 3 Hz. Coupled with the observations of the gain plot, the fit of the model to the data is tentative but it is not as strong as the other analytic fits presented in this chapter.

The frequency drop at higher frequencies can be modeled as a time delay as was done in Section 7.2.4. Using the data between 8 Hz and 20 Hz, the time delay is approximated as $\tau = 22 \pm 5 \text{ ms}$. This is not significantly different than the delay observed between the desired position and the measured motion. As discussed earlier, most of this delay is likely due to the transmission delays between the computer used to record the data and the electronics implementing the feedback controller.

A notable observation from Figures 7-13 and 7-14 is that at high frequencies, the phase continues to become more negative instead of approaching a constant asymptote. This is not a behavior predicted by the transfer functions, but it is a behavior typically associated with time delay between the system input and output.

Since the relationships between the outputs motor encoder and force transducer and the input desired position are known, the relationship between the two outputs can be determined and compared to the transfer function derived from Figure 7-12.

\[
\frac{\theta_m(\omega)}{\theta_{des}} = \frac{\theta_m}{\theta_{des}} = \frac{c_m}{c_f J_m, app \omega^2 + b_m, app \omega + k_m, app} = \frac{1}{m \omega^2 + b \omega + k}
\]

A notable observation from this calculation is the cancellation of poles due to the proportional position controller input. Based on this calculation, the relationship between the force transducer and motor encoder is dictated by the apparent mass-spring-damper as observed from the motor. In essence, the zeros of $f_{ft}/\theta_{des}$ are equal to the poles of $\theta_m/f_{ft}$.

\[
m = \frac{c_f}{c_m} J_m, app \quad b = \frac{c_f}{c_m} b_m, app \quad k = \frac{c_f}{c_m} k_m, app
\]
Figure 7-14: Frequency response plots showing the frequency relationship between the commanded desired encoder position and the load sensor reading. At top is the gain, the ratio between the amplitudes of the variable signals. At bottom is the phase between the two signals.
Chapter 8

Phantom Tissue Experiments

8.1 Experimental Setup

Several experiments were conducted to confirm and evaluate the ability of the actuated probe to measure mechanical impedance. Before conducting trials with human subjects, initial validation trials were conducted on Phantom tissues, artificial tissues stored in cylindrical containers designed to mimic the properties of human tissues, shown in Fig. 8-1. (Computerized Imaging Reference Systems, 2013) These phantom trials were useful in streamlining the experimental procedures and adjusting the control parameters to maintain the force exerted by the device within a range that would be suitable for human trials. The trials also provided insight into how to best incorporate the control methods discussed in Chapter 6 during the impedance estimation trials. Additionally, the time-invariant properties of the Phantom tissues, such as the nominal Young’s Modulus, were critical to corroborate how well the attempted methods estimated impedance.

Figure 8-1: Phantom artificial tissue used during trials. Composed of the trademarked material, Zerdine, they are designed to imitate the mechanical properties of human tissues and organs.
8.1.1 Phantom Patient Trials

For half the trials, the probe end of the device is used to measure the impedance of three Phantom tissues of varying rigidities. This is intended to replicate the device contact with the patient tissue. For these trials, the motor housing of the device was immobilized, as shown in Fig. 8-2, using a vice clamped to a workbench. A spirit level was used to ensure that the translational motions of the device were horizontal and not affected by gravity. During the setup, the probe was maintained at the center of its motion range.

The phantom tissues used during the trials were composed of the patented material Zerdine stored in a cylindrical container with an exposed face. (Computerized Imaging Reference Systems, 2013) The three phantom tissues used were distinguished by their nominal Modulus of Elasticity: 4.0 kPa, 12.4 kPa, and 50.2 kPa. The cylinders were clamped to the workbench, the common inertial frame, with the exposed tissue surface positioned flush with the probe surface, as shown in Fig. 8-3. Rounded supports, shown in Fig. 8-4, were used to fix the location of the cylinders and prevent them from slipping.

Figure 8-2: During the Phantom patient trials, the device housing is clamped to the workbench inertial frame with a vice at the motor support.
Figure 8-3: Full experimental setup during Phantom patient trials. Phantom tissue is clamped to the workbench using rounded supports, device housing is immobilized with a vice that is clamped to the workbench, and the probe makes contact with the Phantom exposed face.

Figure 8-4: Rounded supports, 3D-printed from ABS plastic, used to clamp the Phantom tissues.
8.1.2 Phantom Sonographer Trials

The Phantom sonographer experiments were similar to the Phantom patient trials. The distinguishing feature of these experiments was that they were conducted using the sonographer end of the device in contact with the external tissue. This was intended to replicate the device interaction with the sonographer hand. A replica of the mock probe, shown in Fig. 8-5, was designed based on the mock probe end face geometry to ensure that the Phantom sonographer trials and the Phantom patient trials contact dynamics were as similar as possible. The replica was 3D-printed out of ABS and attached to the housing of the device facing in the opposite direction of the probe. As can be seen in Fig. 8-6, the replica is placed adjacent the belt and motor pinion, clamped between the pinion housing and motor housing support. Additional supports were included to distribute the applied loads and sufficient clearance was maintained between the replica and the moving components of the device.

When designing and attaching the replica of the probe, it was noted that the device was designed to be compact. As such, the tolerances and clearances between parts are typically less than a millimeter. This became a limitation when modifying the device, including the attachment of the replica and adjusting the belt stiffness. Small adjustments during assembly of the device resulted in moving components making contact, creating unnecessary friction in the system. This challenge could be overcome with heightened attention to precision.

One of the objectives of these tests was to confirm that the same impedances could be measured accurately regardless of the end of the device making contact with the tissue. This is important because the housing end is the one the sonographer is holding, so if we want to be able to measure their arm impedance during ultrasound exams, we need to be confident that impedances can be measured with this end. Replicating the contact interaction with the tissue allowed us to directly compare the impedance characterization results between the Phantom sonographer and Phantom patient trials.

For trials where a phantom tissue was measured using the housing end of the device, the probe was immobilized using a vice clamped to a workbench, as shown in Fig. 8-7. Due to its size and irregular shape, the probe was removed and the probe clamp served as the vice contact surface. Removing this component is not expected to affect the dynamics or measurements for
These trials. A spirit level was similarly used to ensure that the translational motions of the device were horizontal and not affected by gravity.

The setup of the Phantom tissues for these trials followed the same process described in the previous section. The Phantom tissues used were the same for both sets of experiments.

Figure 8-5: A replica of the mock probe, 3D-printed from ABS plastic, was assembled onto the motor structure and motor pinion housing to provide the housing end with a surface that the Phantom tissue could make contact with. The design was based on the geometry of the mock probe to ensure that the contact area used in patient and sonographer trials was equal.

Figure 8-6: Probe replica attachment to the housing end of the device. This additional part is rigidly attached to the motor support and pinion support without disturbing the motion of the moving components of the device.
8.2 Procedures

During the Phantom tissue characterization trials, the free end of the device was repeatedly actuated by the brushless motor using discrete sinusoidal excitation, a process previously discussed in Chapter 6. Similar to the unloaded experiments presented in Chapter 7, the measured sensor signals and the input excitation were used to determine the dynamic properties of the system. The mechanical impedance of the tissue in contact with the device can then be isolated from the measured dynamics. The manner in which this is performed was discussed at a conceptual level in Chapter 4, and later in Chapter 5 a model of the system was proposed in order to determine the specific dynamic relations guiding this system.
To finish the experimental setup for each set of experiments, the probe was actuated towards the Phantom tissue until making contact with the exposed surface. The encoder reading at this location was then set to zero in order to establish a useful position reference. The probe was then actuated against the tissue until reaching a distance of 1 cm. All sinusoidal excitation trials were centered about this position, unless the position amplitude of the probe motion required a depth greater than 1 cm. This centering process helped ensure continuous contact. The device probe needed to remain in contact with the Phantom throughout the duration of the trial in order to accurately capture the interactive dynamics between the tissue and the machine.

The discrete sinusoidal frequency excitation of the device was performed using three different controllers, whose hardware and software implementation is discussed at greater length in Chapter 6. During current control, measurement trials were conducted one frequency at a time, starting at 50 Hz and decrementing in a semi-logarithmic manner to capture sufficient data within the frequency range of interest. The lowest frequency tested was determined by when the probe would lose contact with the Phantom tissue. In these situations, the probe would move further away from the tissue with each cycle, exacerbated by the longer periods of motion. This could be countered with a bias force, but this option was not available through the amplifier control software. The frequencies tested for most current control trials were 6, 8, 10, 12, 15, 20, 25, 30, 35, 40, 45, and 50 Hz. The current sinusoidal input had an amplitude of 3 A for all trials.

During force and position control, the measurement trials were started at 0.1 Hz and incremented in a semi-logarithmic manner. PID gains were adjusted at the 0.1 Hz trial whenever the Phantom tissue or desired amplitude was changed. The proportional gain was adjusted until the feedback sensor reading amplitude matched the desired amplitude within 5%. Derivative and integral gains were initially adjusted to observe the effects on the tracking performance, but this type of feedback was ultimately not used nor was necessary for these trials. Typical contact forces exerted by a sonographer on patient vary between 5 N and 15 N. This was used as a reference guide when setting the bias contact force at 10 N. The desired force amplitudes ranged between 1 N and 5 N in order to remain within this range. Similarly, the desired position amplitude was set taking into consideration the corresponding force readings observed at 0.1 Hz. Gains and desired amplitudes are summarized in Table 8.1.
The highest frequency tested using position and force control was determined by when the motor would become uncontrollable. As mentioned in the previous chapter, this occurrence was typified by violent vibrations or high speed rotation in a single direction, possibly due to saturation of control effort. The tested frequencies for most position control experiments were 0.1, 0.2, 0.5, 1, 2, 3, 5, 10, and 15 Hz. The tested frequencies for most force control experiments were 0.1, 0.2, 0.5, 1, 2, 3, 5, and 10 Hz.

Table 8.1: Controller parameters used for each set of experiments. The gain proportional to the feedback error was set for each Phantom tissue at the 0.1 Hz trial to minimize tracking error. Two sets of trials were performed for each Phantom configuration using different desired amplitudes.

<table>
<thead>
<tr>
<th>Proportional Gain, $K_p$</th>
<th>Desired Amplitude</th>
<th>Proportional Gain, $K_p$</th>
<th>Desired Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0 kPa</td>
<td>50</td>
<td>5000 cts $\approx$ 2.4 mm</td>
<td>----</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10000 cts $\approx$ 4.9 mm</td>
<td>----</td>
</tr>
<tr>
<td>12.4 kPa</td>
<td>100</td>
<td>3500 cts $\approx$ 1.7 mm</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6000 cts $\approx$ 2.9 mm</td>
<td></td>
</tr>
<tr>
<td>50.2 kPa</td>
<td>50</td>
<td>2000 cts $\approx$ 1.0 mm</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3500 cts $\approx$ 1.7 mm</td>
<td>2.5 N, 5 N</td>
</tr>
</tbody>
</table>

### 8.3 Results

#### 8.3.1 Phantom Sonographer Trials

The data collected from the trials was processed using the sinusoidal fitting methods performed on the unloaded trial data discussed in Chapter 7. The amplitude and phase for each measured and commanded variable was determined from the least squares sinusoidal fits. One of the first observations made was that the force control trials yielded noisier data, which resulted in higher standard deviation during the sinusoidal fitting and higher rates of fitting failure. For the trials performed on the 12.4 kPa Phantom tissue, the force control position data consistently had a standard deviation 10 times greater than standard deviation of the position
data collected using position control. Additionally, the force controller trials had a more restricted frequency range of operation than the position controller trials. For the 4.0 kPa tissues, trials could not be effectively performed at any frequency using force control due to poor error tracking.

Overall, force control did not provide any additional insight or value that was not already provided by position control and confidence in the data collected was lower. This is not unexpected since the sensor used for force feedback exhibits much higher noise levels than the sensor used for position feedback, as was shown in Chapter 3. As such, the magnitude and phase values determined from the force controller trials were not used to analyze the Phantom tissue dynamics.

To determine the Phantom tissue dynamics, the data from the current control and position control trials were combined in order to have a continuous set of samples from 0.1 Hz to 50 Hz. The gain and phase between the encoder and force transducer data, $\theta_{enc}/f_{ft}$, was computed for each trial. Using the models of the partially immobilized system derived in Chapter 5, the motor performance relations determined in Chapter 7, and the commanded control input, the gain and phase between the housing motion and the force transducer, $x_h/f_{ft}$, was determined. Because the range of tested frequencies were below the resonance frequency of the belt and because of simple controller dynamics, this conversion was roughly equivalent to a constant gain determined primarily by the gear ratios and the unit conversions.

$$\frac{x_h}{f_{ft}} = \frac{\theta_{enc}}{f_{ft}} \cdot \frac{x_h}{\theta_{enc}} \approx \left( n \frac{\tau_{ps}}{\tau_{pm}} \frac{2\pi \text{ rad}}{2048 \text{ cts}} \right) \frac{\theta_{enc}}{f_{ft}}$$

The difference between load cell measure $f_{ft}$ and the contact force $f_s$ with the sonographer tissue is simply a matter of additional inertia, as shown below. In this situation, it was easier to analyze the mechanical admittance of $x_h/f_{ft}$ and determine the impedance characteristics of the tissue, $Z_s = f_s/x_h$, after finding analytic solutions that fit the data. Figures 8-8, 8-9, and 8-10 show the frequency response plots, magnitude and phase, for the Phantom sonographer trials on the 4.0 kPa, 12.4 kPa, and 50.2 kPa Phantom tissues. Experimental data points are distinguished by the type of controller used.

$$f_{ft} = f_s + m_h \ddot{x}_h \quad \frac{f_{ft}}{x_h} - m_h \omega^2 = \frac{f_s}{x_h} = Z_s$$
Figure 8-8: Frequency response plot for data taken using the sonographer end of the device on the Phantom artificial tissue with a 4.0 kPa modulus of elasticity. This includes the gain and phase between the position and force signals. Experimental data points are distinguished by the controller used in its collection. Transfer function fits are distinguished by line types.
Figure 8-9: Frequency response plot for data taken using the sonographer end of the device on the Phantom artificial tissue with a 12.4 kPa modulus of elasticity. This includes the gain and phase between the position and force signals. Experimental data points are distinguished by the controller used in its collection. Transfer function fits are distinguished by line types.
Figure 8-10: Frequency response plot for data taken using the sonographer end of the device on the Phantom artificial tissue with a 50.2 kPa modulus of elasticity. This includes the gain and phase between the position and force signals. Experimental data points are distinguished by the controller used in its collection. Transfer function fits are distinguished by line types.
An important observation from the frequency response gain data was that for all plots, there is a low frequency asymptote where the gain \( x_h/f_{ft} \) is constant. At low frequencies, the phase plot displays a constant phase that approaches \( 0^\circ \). This indicates that in this region, the dynamics are dominated by a stiffness element. Additionally, the gain plots show a high frequency asymptote with a decrement of two orders of magnitude per frequency decade. At high frequencies, the phase values across all trials approach \(-180^\circ\). This is consistent with an inertia-dominated system.

Between these two asymptote regions, there is a brief but continuous transition observed for all gain plots and most phase plots. Furthermore, the gain plot shows a dynamic amplification peak in the transitional frequency regions, which is typically indicative of an underdamped pair of poles. This set of observations is consistent with a second-order system. As such the mechanical admittance data in the figures can be modeled as a mass-spring-damper, with equivalent inertia \( m_s + m_h \), damping \( b_s \), and stiffness \( k_s \) terms. This relationship is expressed below in the frequency domain.

\[
Y_{s+h}(\omega) = \frac{x_h}{f_{ft}}(\omega) = \frac{1}{(m_s + m_h)\omega^2 + b_s\omega + k_s}
\]

To determine the inertial, damping, and stiffness characteristics dictating the tissue mechanical impedance, three different analytic solutions were fitted to the data. The asymptote fit was a solution where the \( m_s + m_h, k_s, \) and \( b_s \) terms were estimated based on an average of the gain data at low frequencies, an average of the second derivative of the gain data at high frequencies, and the maximum value of the first derivative of the gain data. The magnitude fit was a least squares computational solution using only the magnitude data and the complex fit was a weighted non-linear least squares solution using both magnitude and phase data. These fitting methods were discussed in greater detail in Chapter 7. The three analytic fitted models were plotted with the data in Figures 8-8, 8-9, and 8-10.

As mentioned earlier, the impedance characteristics of the Phantom tissues \( Z_s \) can be determined by subtracting the inertial effects of the moving housing end from the impedance \( Z_{s+h} \). Based on the device characterization performed in Chapter 7, the mass of the housing carriage is approximated as \( m_h = 271 \, g \). The mechanical impedance of the Phantom tissues can be summarized by the relationship below.
\[ Z_s(\omega) = \frac{f_s}{x_h}(\omega) = m_s\omega^2 + b_s\omega + k_s \]

Tables 8.2, 8.3, and 8.4 summarize the fitted terms for the 4.0 kPa, 12.4 kPa, and 50.2 kPa Phantom tissues. These values are included for each of the transfer functions fitted to the data. The tables additionally catalogue the natural frequency \( f_{n,s} \) and the damping ratio \( \zeta_s \). These parameters can be computed from the fitted \( m_s, b_s, k_s \) values using the following equations and serve as a complimentary representation of the tissue dynamics.

\[
\begin{align*}
  f_{n,s} &= \frac{1}{2\pi} \sqrt{\frac{k_s}{m_s}} \\
  \zeta_s &= \frac{b_s}{2\sqrt{k_s m_s}}
\end{align*}
\]

The natural frequency and damping ratio of the admittance plotted in the figures is also included in the tables, computed by replacing \( m_s \) with \( m_s + m_h \) in the equations above.

Table 8.2:  Impedance characteristics for the Phantom tissue with Young’s modulus, \( E_{nom} = 4.0 \text{ kPa} \), measured with the special probe in sonographer configuration.

<table>
<thead>
<tr>
<th></th>
<th>Asymptote Fit</th>
<th>Magnitude Fit</th>
<th>Complex Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stiffness, ( k_s ) (N/m)</strong></td>
<td>256</td>
<td>300</td>
<td>263</td>
</tr>
<tr>
<td><strong>Mass, ( m_{s+h} ) (g)</strong></td>
<td>296</td>
<td>317</td>
<td>720</td>
</tr>
<tr>
<td><strong>Damping, ( b_s ) (N·s/m)</strong></td>
<td>5.4</td>
<td>7.6</td>
<td>9.8</td>
</tr>
<tr>
<td><strong>Natural Freq., ( f_{n,s+h} ) (Hz)</strong></td>
<td>4.7</td>
<td>4.9</td>
<td>3.0</td>
</tr>
<tr>
<td><strong>Damping Ratio, ( \zeta_{s+h} )</strong></td>
<td>0.31</td>
<td>0.39</td>
<td>0.36</td>
</tr>
<tr>
<td><strong>Tissue Mass, ( m_s ) (g)</strong></td>
<td>25</td>
<td>46</td>
<td>449</td>
</tr>
<tr>
<td><strong>Tissue Nat. Freq., ( f_{n,s} ) (Hz)</strong></td>
<td>16.0</td>
<td>12.9</td>
<td>4.1</td>
</tr>
<tr>
<td><strong>Tissue Damp. Ratio, ( \zeta_s )</strong></td>
<td>1.06</td>
<td>1.02</td>
<td>0.49</td>
</tr>
</tbody>
</table>
Table 8.3: Impedance characteristics for the Phantom tissue with Young’s modulus, $E_{nom} = 12.4 \, kPa$, measured with the special probe in sonographer configuration.

<table>
<thead>
<tr>
<th></th>
<th>Asymptote Fit</th>
<th>Magnitude Fit</th>
<th>Complex Fit</th>
</tr>
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<tr>
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<td>752</td>
<td>613</td>
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<tr>
<td><strong>Mass, $m_{s+h}$ (g)</strong></td>
<td>281</td>
<td>449</td>
<td>358</td>
</tr>
<tr>
<td><strong>Damping, $b_s$ (N·s/m)</strong></td>
<td>5.9</td>
<td>9.3</td>
<td>3.8</td>
</tr>
<tr>
<td><strong>Natural Freq., $f_{n,s+h}$ (Hz)</strong></td>
<td>7.4</td>
<td>6.5</td>
<td>6.6</td>
</tr>
<tr>
<td><strong>Damping Ratio, $\zeta_{s+h}$</strong></td>
<td>0.23</td>
<td>0.25</td>
<td>0.13</td>
</tr>
<tr>
<td><strong>Tissue Mass, $m_s$ (g)</strong></td>
<td>10</td>
<td>178</td>
<td>87</td>
</tr>
<tr>
<td><strong>Tissue Nat. Freq., $f_{n,s}$ (Hz)</strong></td>
<td>38.7</td>
<td>10.3</td>
<td>13.4</td>
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<tr>
<td><strong>Tissue Damp. Ratio, $\zeta_s$</strong></td>
<td>1.19</td>
<td>0.40</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Table 8.4: Impedance characteristics for the Phantom tissue with Young’s modulus, $E_{nom} = 50.2 \, kPa$, measured with the special probe in sonographer configuration.

<table>
<thead>
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<th>Asymptote Fit</th>
<th>Magnitude Fit</th>
<th>Complex Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stiffness, $k_s$ (N/m)</strong></td>
<td>1630</td>
<td>1980</td>
<td>1760</td>
</tr>
<tr>
<td><strong>Mass, $m_{s+h}$ (g)</strong></td>
<td>289</td>
<td>368</td>
<td>424</td>
</tr>
<tr>
<td><strong>Damping, $b_s$ (N·s/m)</strong></td>
<td>4.1</td>
<td>2.0</td>
<td>7.9</td>
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<tr>
<td><strong>Natural Freq., $f_{n,s+h}$ (Hz)</strong></td>
<td>12.0</td>
<td>11.4</td>
<td>10.3</td>
</tr>
<tr>
<td><strong>Damping Ratio, $\zeta_{s+h}$</strong></td>
<td>0.09</td>
<td>0.04</td>
<td>0.14</td>
</tr>
<tr>
<td><strong>Tissue Mass, $m_s$ (g)</strong></td>
<td>18</td>
<td>130</td>
<td>153</td>
</tr>
<tr>
<td><strong>Tissue Nat. Freq., $f_{n,s}$ (Hz)</strong></td>
<td>48.0</td>
<td>19.7</td>
<td>17.1</td>
</tr>
<tr>
<td><strong>Tissue Damp. Ratio, $\zeta_s$</strong></td>
<td>0.38</td>
<td>0.06</td>
<td>0.24</td>
</tr>
</tbody>
</table>
8.3.2 Phantom Patient Trials

The analysis of the data from the Phantom patient trials was similar to the analysis performed on the data from the Phantom sonographer trials. A notable adjustment is that the difference between load cell measure $f_{ft}$ and the contact force $f_p$ with the patient tissue is now determined by the inertia of the probe end, $m_{p1} = 68 \text{ g}$. The equations dictating the relationship between $f_{ft}$, $f_p$, probe end inertia $m_{p1} \ddot{x}_p$, and patient tissue impedance $Z_p$ is summarized below.

$$f_{ft} = f_p + m_{p1} \ddot{x}_p$$
$$\frac{f_{ft}}{x_p} - m_{p1} \omega^2 = \frac{f_p}{x_p} = Z_p$$

Figures 8-11, 8-12, and 8-13 show the frequency response plots, magnitude and phase, for the Phantom patient trials on the 4.0 kPa, 12.4 kPa, and 50.2 kPa Phantom tissues. Experimental data points are distinguished by the type of controller used. Data collected using the force controller was not used to determine the frequency response function.

Just as with the Phantom sonographer trials, the mechanical impedance of each tissue and probe mass can be modeled as a physical second-order system, with equivalent mass $m_{p1} + m_{pz}$, damping $b_{pz}$, and spring stiffness $k_{pz}$ terms. Tables 8.5, 8.6, and 8.7 summarize these characteristics for the 4.0 kPa, 12.4 kPa, and 50.2 kPa Phantom tissues. These values are included for each of the transfer functions fitted to the data. The tables additionally catalogue the natural frequency $f_{n,s}$ and the damping ratio $\zeta_s$ of the tissues. These parameters were computed in the same manner described in the previous section.
Figure 8-11: Frequency response plot for data taken using the patient end of the device on the Phantom artificial tissue with a 4.0 kPa modulus of elasticity. This includes the gain and phase between the position and force signals. Experimental data points are distinguished by the controller used in its collection. Transfer function fits are distinguished by line types.
Figure 8-12: Frequency response plot for data taken using the patient end of the device on the Phantom artificial tissue with a 12.4 kPa modulus of elasticity. This includes the gain and phase between the position and force signals. Experimental data points are distinguished by the controller used in its collection. Transfer function fits are distinguished by line types.
Figure 8-13: Frequency response plot for data taken using the patient end of the device on the Phantom artificial tissue with a 50.2 kPa modulus of elasticity. This includes the gain and phase between the position and force signals. Experimental data points are distinguished by the controller used in its collection. Transfer function fits are distinguished by line types.
Table 8.5: Impedance characteristics for the Phantom tissue with Young’s modulus, $E_{nom} = 4.0 \, kPa$, measured with the probe in patient configuration.

<table>
<thead>
<tr>
<th></th>
<th>Asymptote Fit</th>
<th>Magnitude Fit</th>
<th>Complex Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stiffness, $k_{pz} (N/m)$</strong></td>
<td>109</td>
<td>162</td>
<td>139</td>
</tr>
<tr>
<td><strong>Mass, $m_{pz+p1} (g)$</strong></td>
<td>87.5</td>
<td>72.1</td>
<td>127</td>
</tr>
<tr>
<td><strong>Damping, $b_{pz} (N\cdot s/m)$</strong></td>
<td>1.3</td>
<td>2.6</td>
<td>2.3</td>
</tr>
<tr>
<td><strong>Natural Freq., $f_{n.pz+p1} (Hz)$</strong></td>
<td>5.6</td>
<td>7.5</td>
<td>5.3</td>
</tr>
<tr>
<td><strong>Damping Ratio, $\zeta_{pz+p1}$</strong></td>
<td>0.21</td>
<td>0.39</td>
<td>0.27</td>
</tr>
<tr>
<td><strong>Tissue Mass, $m_{pz} (g)$</strong></td>
<td>19.5</td>
<td>4.1</td>
<td>59</td>
</tr>
<tr>
<td><strong>Tissue Nat. Freq., $f_{n.pz} (Hz)$</strong></td>
<td>11.9</td>
<td>31.6</td>
<td>7.7</td>
</tr>
<tr>
<td><strong>Tissue Damp. Ratio, $\zeta_{pz}$</strong></td>
<td>0.46</td>
<td>1.62</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Table 8.6: Impedance characteristics for the Phantom tissue with Young’s modulus, $E_{nom} = 12.4 \, kPa$, measured with the probe in patient configuration.

<table>
<thead>
<tr>
<th></th>
<th>Asymptote Fit</th>
<th>Magnitude Fit</th>
<th>Complex Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stiffness, $k_{pz} (N/m)$</strong></td>
<td>438</td>
<td>486</td>
<td>516</td>
</tr>
<tr>
<td><strong>Mass, $m_{pz+p1} (g)$</strong></td>
<td>81</td>
<td>98</td>
<td>204</td>
</tr>
<tr>
<td><strong>Damping, $b_{pz} (N\cdot s/m)$</strong></td>
<td>4.9</td>
<td>7.3</td>
<td>3.8</td>
</tr>
<tr>
<td><strong>Natural Freq., $f_{n.pz+p1} (Hz)$</strong></td>
<td>11.7</td>
<td>11.2</td>
<td>8.0</td>
</tr>
<tr>
<td><strong>Damping Ratio, $\zeta_{pz+p1}$</strong></td>
<td>0.42</td>
<td>0.53</td>
<td>0.19</td>
</tr>
<tr>
<td><strong>Tissue Mass, $m_{pz} (g)$</strong></td>
<td>13</td>
<td>30</td>
<td>136</td>
</tr>
<tr>
<td><strong>Tissue Nat. Freq., $f_{n.pz} (Hz)$</strong></td>
<td>28.9</td>
<td>20.1</td>
<td>9.8</td>
</tr>
<tr>
<td><strong>Tissue Damp. Ratio, $\zeta_{pz}$</strong></td>
<td>1.03</td>
<td>0.96</td>
<td>0.23</td>
</tr>
</tbody>
</table>
Table 8.7: Impedance characteristics for the Phantom tissue with Young’s modulus, $E_{nom} = 50.2 \text{ kPa}$, measured with the probe in patient configuration.

<table>
<thead>
<tr>
<th></th>
<th>Asymptote Fit</th>
<th>Magnitude Fit</th>
<th>Complex Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stiffness, $k_{pz} (N/m)$</strong></td>
<td>1520</td>
<td>1780</td>
<td>1450</td>
</tr>
<tr>
<td><strong>Mass, $m_{pz+p1} (g)$</strong></td>
<td>90</td>
<td>131</td>
<td>69</td>
</tr>
<tr>
<td><strong>Damping, $b_{pz} (N\cdot s/m)$</strong></td>
<td>3.6</td>
<td>6.7</td>
<td>5.4</td>
</tr>
<tr>
<td><strong>Natural Freq., $f_{n,pz+p1} (Hz)$</strong></td>
<td>20.7</td>
<td>18.6</td>
<td>24.4</td>
</tr>
<tr>
<td><strong>Damping Ratio, $\zeta_{pz+p1}$</strong></td>
<td>0.15</td>
<td>0.22</td>
<td>0.29</td>
</tr>
<tr>
<td><strong>Tissue Mass, $m_{pz} (g)$</strong></td>
<td>22</td>
<td>63</td>
<td>1</td>
</tr>
<tr>
<td><strong>Tissue Nat. Freq., $f_{n,pz} (Hz)$</strong></td>
<td>41.5</td>
<td>26.7</td>
<td>191</td>
</tr>
<tr>
<td><strong>Tissue Damp. Ratio, $\zeta_{pz}$</strong></td>
<td>0.31</td>
<td>0.31</td>
<td>0.27</td>
</tr>
</tbody>
</table>

8.4 Analysis

As mentioned previously the frequency response data for all sets of trials reveal a distinctive second-order relationship between probe position and contact force. Low frequency and high frequency asymptotes are visible in all phase and gain figures. Dynamic amplification in the gain is present for all sets of trials at the transition between the asymptotes. For most phase plots, there is a brief but continuous transition from $0^\circ$ to $-180^\circ$. The most noticeable exceptions are in Figures 8-9 and 8-13, which lack transitional phase data points. As an added note, the phase at higher frequencies appears to approach $-160^\circ$ for all plots instead of $-180^\circ$. The load cell – encoder relationship, as was observed in Section 7.2.3, similarly had a phase of $-160^\circ$ instead of $-180^\circ$ above 10 Hz so this discrepancy may not be due to the tissue dynamics.

There is a notable overlap in the data collected using different controllers – both position control and current control yield approximately the same gain and phase values at common frequencies. If the data did not agree when using two distinct methods, it would put into question the validity of the implementation of the discrete sinusoidal excitation
The stiffness element dominant at lower frequencies was the simplest to extract from the data since it only required finding the average of the values within the asymptote. The data was generally consistent in this region – the standard deviation of the asymptote fit estimate of $k_p$ and $k_s$ was 11% or lower for all trials. The only exception was the 4.0 kPa Phantom sonographer trial, summarized in Table 8.2, which exhibits a slightly higher standard deviation of 20%. Upon inspection of Figure 8-8, there appears to be two slightly distinct asymptotes – the trials with a lower position amplitude yielded a high stiffness. The cause behind this particular case is unknown since the experimental setup remained the same between trials with the same Phantom tissue.

The tissue stiffness measurements were consistently higher by approximately 150 N/m when measured using the sonographer end of the device. For the 4.0 kPa tissue, this meant that the estimates were different by a factor of 2.3, but for the 50.2 kPa tissue the estimate was only 8% higher. This artifact may be due to unforeseen differences in experimental variables. Although the mock probe replica used for sonographer trials was designed to match the mock probe used for the patient trials, there may be a geometric or material property difference between the two that results in the difference.

Additionally, some experimental factors were not controlled as precisely as others. The most relevant example is the placement location of the probe on the tissue. For each trial the probe was placed in contact with the center of the tissue. This location was approximated and no measurement tools were used to check for accuracy. For all patient end trials, a visual marker was used to ensure that the device was clamped at the same vertical position. The same was done for all sonographer end trials. However, since the markers were different for the two configurations, the vertical position differed by approximately 5 mm. This may have been sufficient to account for the difference in measured stiffness.

The modulus of elasticity, or Young’s modulus $E_p$, is the nominal characteristic used to distinguish the different Phantom tissues tested. As is shown below, $E_p$ is proportional to the stiffness of the tissue $k_{phantom}$. In this study, the probe area $A_{probe}$ remains constant for all trials and the effective length of the Phantom tissue $l_{phantom}$ is assumed to be approximately constant for all Phantom tissues.
While comparing measured tissue stiffness to its nominal modulus of elasticity is not a precise confirmation, it is a valuable reality check that confirms the reliability of the estimates. The ratio between the modulus of elasticity between the three tissues is 1:3.1:12.5. For the patient end trials, the measured stiffness values had a ratio of 1:4:13.9 and for the sonographer end trials the ratio was 1:2.4:6.5. However, if the stiffness values from the sonographer trials were adjusted to compensate for the additional 150 N/m discussed earlier, the ratio between the stiffness values would have a ratio of 1:4.3:14.4. The source of this discrepancy remains to be identified, but if it is an experimental artifact then the measured stiffness values would appear to increase at a rate comparable to the increase in tissue modulus of elasticity. It was previously noted that this observation would provide evidence of an accurate stiffness approximation.

The combined masses $m_{pz} + m_{p1}$ and $m_s + m_h$ were estimated from the plotted admittance values at the high frequency asymptotes. For the probe end measurements, estimates of $m_{pz+p1}$ were consistent at $86 \pm 5 \, g$. When subtracting the inertial effects of the probe, the estimates were more modestly estimated at $18 \pm 5 \, g$. Since the mass of the probe is greater than the mass of the tissue, the precision of the tissue mass is reduced due to the higher relative variability.

For the sonographer end measurements, estimates of $m_{s+h}$ were also generally consistent at $289 \pm 8 \, g$. This higher estimate is expected since the mass of the housing structure, $m_h$, is four times larger than the effective probe mass, $m_{p1}$. Additionally, a higher magnitude is typically associated with a higher standard deviation. When subtracting the inertial effects of the housing, the tissue mass was estimated at $18 \pm 8 \, g$. In this set of measures, the standard deviation is equal to half the tissue mass being estimated. This makes the estimate of the tissue mass $m_s$ susceptible to errors in the approximation of the effective housing end inertia $m_h$.

One of the main takeaways from this analysis is that a smaller effective probe inertia results in a higher precision and accuracy in the estimate of the tissue inertia. This essentially means that the housing end of the device is unable to accurately estimate the impedance of structures of low inertia. Experiments need to be performed with a human operator in order to determine...
whether the typical sonographer hand impedance has a sufficiently high inertia to be accurately estimated using the device housing end. Intuitively, it is expected for the hand inertia to be sufficiently high to distinguish from the housing inertia with an acceptable precision. From a basic understanding of joint dynamics, it is expected for the mass of the sonographer arm to contribute significantly to the effective inertia observed at the hand, making it considerably greater than the mass of the artificial tissue measured in this study.

Phantom tissues with higher stiffness displayed a higher natural frequency. This is consistent with the equation for natural frequency in a second-order system, $\omega_n = \sqrt{\frac{k}{m}}$. The natural frequencies of all the plotted data were below 20 Hz and were correlated with tissue stiffness. This confirms that the observed dynamic amplification is not due to the belt dynamics; the predicted belt resonant frequency is an order of magnitude higher than the measured frequencies and is not highly sensitive to changes in the contact impedance. This additionally means that the power transmission between the motor and ball screw actuator can be approximated as proportional within the tested frequency range.

An additional observation is that the natural frequencies observed from the plots were consistently lower for the sonographer trials than they were for the patient trials. This is expected because the effective probe inertia between the tissue and the force transducer was four times higher for sonographer trials than for patient trials. This should lead to a difference in measured natural frequency by a factor of 0.5, assuming low tissue inertia. The data showed a difference in measured natural frequency by a factor of 0.66, on average. However, this value was also affected by the discrepancy in measured stiffness.

The most difficult parameter to extract from the data is the damping value due to the sparse amount of data at the resonance frequency. Although an estimate of the damping is performed for each set of trials, the confidence in this estimate is low. Sufficient data is available for each set of trials, though, to assert that all plotted dynamic systems are underdamped due to the presence of dynamic amplification in the gain and brief transition in phase at the natural frequency. No conclusive statements can be made about the damping ratio of the tissue impedance but most estimates included in the tables suggest most Phantom tissues are underdamped.
Chapter 9

Conclusions

9.1 Accomplishment of Objectives

The main objective of this study was to examine whether a previously developed medical device could be repurposed to measure the mechanical impedance of the physician operator hand and the patient tissue it comes into contact with. This was not fully realized within the content of this document. The presented work identified the hardware limitations preventing the device from accomplishing this main objective but developed the mathematical models, device characterization, and software implementation necessary to measure mechanical impedance of an external structure. To demonstrate the present device capabilities, trials were conducted to measure the mechanical impedance of a single external structure using the ultrasound probe device.

9.2 Continuing Improvements

Mechanical impedance measurement of a single tissue has been shown to be viable in the work presented in this study. The next step is to measure the mechanical impedance of the patient tissue and the sonographer hand simultaneously, as this continues to be the main objective of this study. As mentioned previously, the primary hurdle to accomplishing this is the sensory limitation. Most critically, the accelerometer installed on the device needs to be replaced with a sensor capable of tracking the inertial motion of the device while maintaining sufficiently low noise levels within the frequency range of operation.

The experimental trials already performed have provided insight on additional modifications that could lead to improvements in the results collected from the mechanical impedance measurement trials. For instance, one of the most significant limitations is the sparse amount of
data available within the frequency range of dynamic amplification. One way to address this concern is to collect more data at the frequencies below and above resonance. However, conducting more trials requires more time that will not be available with human subjects. The process of measuring mechanical impedance using discrete frequency excitation can be accomplished in less than an hour, but preferably it should be accomplished in under a minute.

The trial time can be substantially reduced by using stochastic excitation instead of discrete sinusoidal excitation. This process also yields significantly more frequency response data since in theory it excites the system at all frequencies within a specified range. In order to implement the transition to stochastic excitation, a microcontroller will be used to directly send commands to the motor amplifier. This will replace the motion card PID controller and amplifier function generator used in the trials presented earlier. Additionally, a data acquisition board will be used to store the sensor signals and input motor current at a higher sampling rate than what is currently in use. Consistent and precise control and acquisition electronics operating at a high speed are necessary to successfully conduct stochastic system identification.

With regards to experimental procedures, improvements can be made in controlling the variables that the impedance measurement may be sensitive to. In particular, additional precision is needed in the placement location of the probe on the measured tissue. A shift of approximately 5 mm in placement of the mock probe may have accounted for some of the differences observed in the measured tissue impedances across trials. This will become more critical in future studies with human tissues and human operators due to the additional variables introduced into the setup and increased difficulty in controlling the precision of the experimental setup.

9.3 Future Work

9.3.1 Characterization of Impedance in Human Physiology

Upon accomplishing the functionality of measuring the mechanical impedance of the patient tissue or and human hand simultaneously, the ultrasound device can be used as a tool for research on human physiology. In particular, the probe can be used to measure the mechanical
impedance of the hand of the sonographer during operation of an ultrasound scan. One of the research applications is to study how the effective mechanical impedance as observed from the hand varies with muscular effort, arm configuration, and direction of applied load. This could provide further insight into how the current techniques employed by sonographers during ultrasound scans may be detrimental to their health. This could be useful in determining how to reduce the probability and intensity of chronic fatigue and joint pain.

Furthermore, understanding hand impedance modulation could shed light on the motion control strategies employed by humans. Knowledge of this could be used to improve the design and control of hand-operated devices. Additionally, the motion control strategies could be implemented on robotic arms to replicate the human ability to achieve stability despite limited knowledge of the interaction dynamics.

The device can also be used to measure the mechanical impedance of specific tissues among healthy subjects and among patients suffering from a medical condition that is believed to affect the body part in question. Any significant correlations found from these studies could then be explored further and potentially be established as an additional clinical diagnostic measure. Initial work is already in progress to use the mechanical impedance measurement capabilities of the device to further investigate changes in muscular properties in patients with Duchenne muscular dystrophy.

9.3.2 Educational Modules

One of the related goals of this project is to develop educational modules based on the theoretical and experimental exercises performed in this study. Despite the in-depth modeling and analysis, most of what is presented here could be replicated by an engineering graduate student if the simplifying assumptions are initially provided. The educational modules will be created for use in graduate-level control engineering classes that specifically focus on system modeling and the characterization of systems through frequency response. The module could be designed with the intent of either exploring discrete sinusoidal or stochastic excitation.
9.3.3 Improvements on Ultrasound Probe Design

The completion of this study has shed light into several of the improvements that can be made to the device in future iterations. One of the main things that needs to be improved is the clearance allowed between moving parts. This specifically refers to the clearances between the motor housing and the coupling, between the pinion housing and the pinion/belt/coupling, and between the external ergonomic structure and the internal wiring and moving components. The changes proposed would increase the dimensions of the overall device by less than a centimeter but would minimize the current difficulty in assembly and the margin of error during human operation.

Additionally, the system should be inspected to determine unexpected sources of compliance, such as the cantilever beam formed by the hollowed probe support. During the trials, the device would at times exhibit unwanted vertical vibrations, which introduces additional unmodeled dynamics and increases the variability in performance. Increasing the rigidity of the system could reduce these vibrations and improve the precision of the measures.

Another consideration is changing the gear ratio. At very high stiffness, the deformation of the tissue is small, so when a gear ratio is close to unity the motor does not need to turn much to achieve the motion. A higher gear ratio, on the other hand, would allow the motor a larger range of motion to operate. This increases the precision of the encoder reading as well as the resolution of the actuator.

The device could be capable of measuring mechanical impedances in a simpler manner if different or additional sensors were installed on the device. This is not a necessary change but it is a consideration in the future if it is not desirable to model the dynamics of the device or if a device will be developed with the particular purpose of measuring mechanical impedance.
Appendix A

Force Transducer Calibration

As described in section 3.1.2, the force transducer was calibrated by applying known weights to the probe end. The following table and plot show the readings from the force transducer prior to calibration as well as the expected force readings based on the mass of the objects used. The linear fit indicates that after accounting for the zero offset the reading underestimates the actual force by 3%. The linear regression perfectly fits the data.

Table A.1: Expected and actual force readings for the test objects before adjusting the calibration of the force transducer.

<table>
<thead>
<tr>
<th>Mass (g)</th>
<th>Expected Weight (N)</th>
<th>Load Cell Reading (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.51</td>
</tr>
<tr>
<td>50.0</td>
<td>0.49</td>
<td>0.99</td>
</tr>
<tr>
<td>100.0</td>
<td>0.98</td>
<td>1.46</td>
</tr>
<tr>
<td>150.0</td>
<td>1.47</td>
<td>1.93</td>
</tr>
<tr>
<td>234.7</td>
<td>2.30</td>
<td>2.76</td>
</tr>
<tr>
<td>334.7</td>
<td>3.28</td>
<td>3.73</td>
</tr>
<tr>
<td>500.0</td>
<td>4.91</td>
<td>5.29</td>
</tr>
<tr>
<td>677.4</td>
<td>6.65</td>
<td>7.00</td>
</tr>
<tr>
<td>862.1</td>
<td>8.46</td>
<td>8.76</td>
</tr>
<tr>
<td>1012.1</td>
<td>9.93</td>
<td>10.20</td>
</tr>
</tbody>
</table>
Figure A-2: Linear regression of the relationship between applied weight and measured force, after adjusting the calibration of the force transducer.

The following table and plot show the readings from the force transducer after adjusting the calibration as well as the expected force readings based on the mass of the objects used. The linear fit indicates that after accounting for the zero offset the reading overestimates the actual force by 0.4%, which is as close as precise as the calibration allows. The linear regression perfectly fits the data.

Table A.2: Expected and actual force readings for the test objects after adjusting the calibration of the force transducer.

<table>
<thead>
<tr>
<th>Mass (g)</th>
<th>Expected Weight (N)</th>
<th>Load Cell Reading (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.36</td>
</tr>
<tr>
<td>50.0</td>
<td>0.49</td>
<td>0.85</td>
</tr>
<tr>
<td>100.0</td>
<td>0.98</td>
<td>1.35</td>
</tr>
<tr>
<td>150.0</td>
<td>1.47</td>
<td>1.84</td>
</tr>
<tr>
<td>500.0</td>
<td>4.91</td>
<td>5.29</td>
</tr>
<tr>
<td>550.0</td>
<td>5.40</td>
<td>5.78</td>
</tr>
<tr>
<td>600.0</td>
<td>5.89</td>
<td>6.27</td>
</tr>
<tr>
<td>650.0</td>
<td>6.38</td>
<td>6.76</td>
</tr>
</tbody>
</table>
Figure A-2: Linear regression of the relationship between applied weight and measured force, after adjusting the calibration of the force transducer.
Appendix B

Device Inertia Measurements

As discussed in section 5.3.3, the inertial parameters used in the model equations are calculated from mass and dimensional measurements of the individual device parts. The linear inertia parameters were computed using the following summations of selected individual masses.

\[
m_p = \sum_{i} m_i = 136 \text{ grams}
\]

\[J \ni \text{Probe, Upper Probe Clamp, Lower Probe Clamp, Ball Bearings, Cart}\]

\[
m_H = \sum_{i} m_i = 330 \text{ grams}
\]

\[K \ni \text{Ball Screw, Screw Pinion, Motor Pinion, Pinion Housing, Pinion Bearing, Motor Coupling, Motor Housing Support, Motor, Linear Actuator Track, Track End 1 & 2, Limit Switches, Belt}\]

The rotational inertia parameters were computed using the following summations of selected rotational inertia terms. \(I_{Z,i}\) denotes the rotational inertia of part \(i\) about its center of mass along the \(Z\)-direction, the previously defined singular coordinate of motion of the device. If the center of mass of part \(i\) does not lie along the axis of rotation of the inertial parameter, an additional term \(m_i d_i^2\) is needed to satisfy the parallel axis theorem.

\[
I_m = \sum_{i} I_{Z,i} = 4.59 \text{ g} \cdot \text{cm}^2
\]

\[M \ni \text{Motor Pinion, Pinion Bearing, Motor Coupling, Motor Armature, Belt}\]

\[
I_s = \sum_{i} I_{Z,i} = 2.84 \text{ g} \cdot \text{cm}^2
\]

\[N \ni \text{Ball Screw, Screw Pinion}\]
\[ I_H = \sum_{i}^{L} (I_{Z,i} + m_i d_i^2) + \sum_{i}^{M} m_i d_i^2 + \sum_{i}^{N} m_i d_i^2 = 1570 \text{ g} \cdot \text{cm}^2 \]

\( L \) \( \equiv \) Probe, Upper Probe Clamp, Lower Probe Clamp, Ball Bearings, Cart, Pinion Housing, Motor Housing Support, Motor Housing, Linear Actuator Track, Track End 1 & 2, Limit Switches

The rotational inertias of the individual parts were computed using the following equations, based on the geometry of the part.

For cylindrical objects:
\[
I_{Z,i} = \frac{1}{2} m_i r_i^2
\]

For rectangular objects:
\[
I_{Z,i} = \frac{1}{12} m_i (h_i^2 + w_i^2)
\]

For point masses:
\[
I_{Z,i} = m_i d_i^2
\]

The value \( d_i \) is the distance between the part \( i \) and the relevant axis of rotation. In the cases where a part had to be divided into separate geometries, \( d_i \) is the distance between the subcomponent \( i \) center of mass (COM) and the overall COM of the part. When solving for the housing inertia \( I_H \), the value \( d_i \) is the distance between the part \( i \) and axis of rotation of the housing. The rotational axis of \( I_H \) is approximated to coincide with the device COM since this term is roughly equivalent to the overall device rotational inertia. The process for determining \( d_i \) included measuring \( x_i \), the distance between the subcomponent COM and a reference surface on the device, for all relevant geometries. The location of the free-body axis of rotation with respect to this reference surface, \( x_{rot} \), is equal to the equivalent COM.

\[
x_{rot} = \frac{\sum m_i x_i}{\sum m_i} \equiv \text{all relevant components } i
\]

\[
d_i = x_i - x_{rot}
\]

The table in the following page summarizes the numerical values used in the calculations discussed, organized by part. Subcomponents are included when the part was not of simple geometry and required a sum of multiple rotational inertia terms. The last column refers to the distance \( d_i \) between the COM of the part \( i \) and the overall device COM.
Table B.1: Inertial parameters associated with each part and subcomponent of the actuated ultrasound probe. Relevant geometric dimensions needed to compute moments of inertia are included in the table.

<table>
<thead>
<tr>
<th>Part</th>
<th>Mass (g)</th>
<th>Effective radius (cm)</th>
<th>Z-Rot Inertia (g cm²)</th>
<th>Offset from COM (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probe (may vary)</td>
<td>45</td>
<td>76.09</td>
<td>2.50</td>
<td></td>
</tr>
<tr>
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<td>Quantity</td>
<td>h</td>
<td>w</td>
<td>d</td>
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<td>w = 1.70</td>
<td>1.53</td>
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<td>w = 1.30</td>
<td>4.03</td>
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</table>
<pre><code>                           |          | d = 1.10|         |     |             |
</code></pre>
<p>| Motor                       | 76       | 0.80    | 23.04   | 2.20|             |
| Motor Housing               |          |         | 21.77   |     |             |
| Motor Armature              |          |         | 1.27    |     |             |
|                            |          |         | 92.5 (Version 3) |     |             |
| Linear Actuator Track      | 107      | 106.4   | 0.55    |     |             |
| Bottom                      | 64.2     | h = 0.30| w = 2.80| 42.43|             |
| Sides                       | 42.8     | h = 0.40| w = 0.70| 2.32 |             |
|          | d = 1.20|         |     |             |
| Track End 1                 | 29       | 34.55   | 0.20    |     |             |
| Full Track End 1            | 34       | h = 2.15| w = 2.80| 35.31|             |
| Bearing Hole                | -5       | 0.65    | -0.76   |     |             |
| Track End 2                 | 17       | h = 1.40| w = 2.80| 0.20 |             |
| Limit Switches              | 12       | h = 0.30| w = 1.30| 0.20 |             |
| Belt                        | 1        | 0.45    | 0.20    | 2.20 |             |</p>
Appendix C

Belt Stiffness Estimation

As discussed in Section 5.3.5, the stiffness of the timing belt was determined by applying a load to create tension in the belt and measuring the resulting deflection of the suspended pinion. Five separate sets of data were collected with different combinations of tensioning weights to increase the sample size and minimize potential errors in the approximation. Table C.1 summarizes the data from the five data sets, including suspended mass and corresponding deflection.

Table C.1: Pinion deflection sustained in response to the additional suspended mass. This deflection is due to the elasticity in the timing belt. Five separate sets of data were collected.

<table>
<thead>
<tr>
<th>Suspended Mass (grams)</th>
<th>Pinion Deflection (0.001 inch)</th>
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<tr>
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<td>200</td>
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<tr>
<td>250</td>
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<td>300</td>
<td>2.5</td>
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<td>350</td>
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<tr>
<td>500</td>
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<td>550</td>
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<td>800</td>
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<tr>
<td>850</td>
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</table>
Figure C-1 shows a plot of the data presented in Table C.1. A least squares linear regression was fitted to each set of data in order to determine the relationship between belt tension and static elongation, or its stiffness. The average of the five data sets estimates the belt compliance at $4.66 \times 10^{-6} \text{in/g}$. Converting the suspended mass to the corresponding gravitational force and converting to metric units results in a belt stiffness estimate of $8500 \text{N/m}$. A noteworthy observation is that when smaller masses were suspended from the belt, the fitted slope is higher in value. This corresponds to a lower stiffness value. This is likely because when the weight applied is low, the belt is not fully wrapped around the pinions and maintains some of its undeformed curvature. When the belt is fully tightened, the only element of stiffness is along the length of the straightened belt. The belt was fully tightened during all trials in this study to achieve this higher stiffness and to avoid belt slip over the gears.

Figure C-1: Measured deflection of the suspended pinion as a function of the suspended mass. Five separate sets of data were collected, distinguished by marker color. Least squares linear regressions were fitted to each set of data to determine the linear proportionality between the two variables. Since the suspended mass is proportional to the tension experienced by the belt, the slope of the linear regression to the data was used to determine the belt stiffness.
References


