On Relative Permeability: A New Approach to Two-Phase Fluid Flow in Porous Media

by

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B.Sc., The University of Texas at Austin (2013)

Submitted to the Department of Earth, Atmospheric and Planetary Sciences

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Abstract

Being valid for single-phase flow, Darcy’s law is adapted to two-phase flow through the standard approach of relative permeability, in which permeability, rather than being a unique property of the porous medium, becomes a joint property of the porous medium and each fluid phase. The goal of this study is to find a proper, alternate approach to relative permeability that can describe two-phase flow in porous media while maintaining sound physical concepts, specifically that of a unique permeability exclusive to the porous medium.

The suggested approach uses the concept of an average viscosity of the two-phase fluid mixture. Viscosity, the only fluid-characterizing term in Darcy’s law, should-at least partially-explain two-phase flow behavior by becoming the two-phase flow property that varies with the saturation ratio of the two fluid phases. Three common mathematical averages are tested as potential viscosity averages. Aspects of two-phase flow in pipes are then considered to see whether two-phase flow behavior in porous media can be attributed to the fluid mixture alone.

Total flow rate of the two-phase fluid mixture is modeled by using the fluid mixture average viscosity in Darcy’s law. Using two-phase flow data from Oak et al. (1990a, 1990b), the harmonic average weighted by the reduced fluid saturations represents the average viscosity of liquid-gas mixtures in steady-state flow in imbibition. Extracting flow rates of the individual phases from the total flow rate of the fluid mixture is the next, but crucial, step that determines whether the average viscosity approach can replace that of relative permeability in solving common reservoir engineering problems. Liquid-liquid flow in both drainage and imbibition, and liquid-gas flow in drainage are not represented by a simple viscosity average, which indicates the need for further study into more complex viscosity averages.

Thesis Supervisor: Frank Dale Morgan
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Chapter 1

Introduction

Henry Darcy, in his 1856 publication of “Les Fontaines Publiques de la Ville de Dijon”, established from experimental investigations a quantitative description of water flow through sand columns. Darcy’s contribution paved the way for the study of flow of homogeneous fluids through porous media (Muskat, 1937). The flow behavior of homogeneous fluids -single-phase fluids such as a single gas or liquid- is greatly understood. Two-phase fluid flow however, is fairly understood at the pore scale but not well represented at the macroscopic scale (Cosse, 1993). Laws describing such flows are based on experimental studies and are of great importance in applications such as petroleum reservoir engineering for quantifying the flow of oil and water or gas in a petroleum reservoir.

Lacking real, physical understanding of two-phase flow in porous media and needing a mathematical description of such flow to solve pressing day-to-day and industry-related problems, scientists resorted to the mathematically sufficient but conceptually misleading notion of relative permeability. In this approach, Darcy’s law which is valid for single-phase flow in porous media, can be used to quantify the flow rate of each phase of a two-phase fluid (for a specific saturation ratio of the two phases) by using the effective or relative permeability of the medium to each phase. In other words, for a given saturation ratio of the two phases, the two-phase flow is described by two separate Darcy’s equations: one for each phase. This gives the
erroneous consequence of coupling the medium's permeability to the fluid mixture instead of maintaining permeability as a characteristic of the medium. In his introduction to *Porous Media* (1992), Dullien states the fundamental problem that is the motivation behind this research, which is that a “true porous material should have a specific permeability, the value of which is uniquely determined by the pore geometry and is independent of the properties of the penetrating fluid”.

The suggested, alternative approach to two-phase flow is to describe the flow rate of the fluid mixture by a single Darcy’s equation and attribute the two-phase flow behavior to the fluid-related term in the equation, viscosity. The viscosity term however cannot be a simple, constant parameter anymore in this case, since it needs to explain the variation of two-phase flow as a function of the saturation ratios of the two fluid phases. The problem of quantifying the flow rate of each phase individually is then based on finding the relationship between the flow rates of the individual phases and the total fluid-mixture flow rate. This study is an attempt to reconcile the improper relative permeability representation of two-phase flow in porous media by finding a sound representation using the concept of the average viscosity of a two-phase fluid mixture.

An important limitation of this study must be pinpointed which concerns the simple nature of our suggested approach relative to the complex reality of flow in porous media. Attempting to describe two-phase flow in porous media using viscosity-centered models is a clear indication of the many aspects of fluid flow that are not being considered in this study. Fluid flow mechanisms other than flow due to viscous forces are not captured by an average viscosity model. Consequently, if other effects such as capillarity are strongly present in the experimental data, validating the new approach of an average viscosity becomes challenging because of interfering effects on the experimental results.

The rest of Chapter 1 clarifies some important concepts relating to two-phase flow, such as permeability, immiscibility and wettability. Chapter 2 explains the concept of relative permeability, the historical background behind its development starting
with Darcy's law for single-phase flow, the laboratory testing methods to measure relative permeability and how it is used to calculate the flow rates of the two fluid phases. Chapter 3 presents the relative permeability data used in this study and obtained by Oak (1990); Oak et al. (1990). Thus, the first three chapters include the necessary background information and the relative permeability data used in this study and collected by previous researchers. After having introduced the necessary concepts and terminology of relative permeability, Chapter 4 introduces the ideal, imaginary case of two-phase flow in which the complex two-phase flow behavior is absent and instead, each fluid phase flows as a single-phase flow satisfying Darcy’s law. Two flow scenarios are considered which represent the two extreme ends of two-phase flow. Chapter 5 describes the alternate approach of the average viscosity of a two-phase fluid mixture and tests the suggested average viscosity models on experimental data. Chapter 6 investigates average viscosity models from the literature developed for two-phase flow in pipes, with the goal of exploring similarities -if any- between two-phase flow in pipes and in porous media. Chapter 7 summarizes the conclusions from this study and lays out potential future work.

1.1 Background

1.1.1 Permeable, Porous Media

When discussing fluid flow in this study, we are concerned only with fluid flow (whether single-phase or two-phase flow) in permeable, porous media. A porous medium is any material that contains “relatively small spaces, so-called pores or voids, free of solids, imbedded in the solid or semisolid matrix” (Dullien, 1992). Such materials are common in nature and the industrial world and are widely used in various applications, which is why they are rigorously studied and continue to be of great interest. Under this definition of porous materials however, almost all solids and semisolids other than plastics, metals and some dense rocks satisfy this condition and are therefore deemed “porous”. Therefore, it is important to
understand what makes a permeable, porous material. For a porous medium to be permeable, “fluids must be able to penetrate through one face of a septum made of the material and emerge on the other side” (Dullien, 1992). This definition narrows down the pool of porous materials of interest, but it still includes virtually all solids and semisolids since their pore spaces permit the penetration of some fluid molecules through molecular diffusion. Thus, fluid penetration through viscous flow mechanism is an important condition for a material to be a true permeable, porous medium. Rocks are one type of porous, permeable media that are of significant interest because of their central role in two important fields: hydrology and petroleum engineering.

1.1.2 Immiscible fluids

The problem of two-phase flow investigated in this study is related to the flow of two immiscible fluids in porous media. Immiscible fluids are fluids that are present simultaneously in the pore spaces and separated from one another by interfaces (Dullien, 1992). When two immiscible fluids are present and meet at an interface, the concept of capillary pressure arises. In addition, when a solid surface exists with the two fluids such as the case of two fluid phases in the pore space of a porous medium, the concept of wettability comes into play.

1.1.2.1 Wettability

Wettability is the affinity of one fluid phase towards the solid surface of the porous medium in the presence of another fluid phase. Wettability of a fluid phase is described by its contact angle, which is the angle that the fluid phase interface makes with the solid surface. If the contact angle is less than 90 degrees, the fluid is said to be wetting the solid surface, while it is said to be nonwetting if the contact angle is greater than 90 degrees (Figure 1-1). In simple words, a wetting phase tends to spread over the solid surface while a nonwetting phase tends to “ball up” (Dullien, 1992). It is important to note that the concept of wettability of a particular fluid
phase is relative to another phase, i.e. a particular fluid can be wetting to the solid surface in one situation and nonwetting in another. For example, oil is usually the non-wetting phase to a rock surface when the other phase is water, whereas oil is the wetting phase to the same rock surface when air is the second phase. Two important terms related to wettability that will be used frequently in later sections are imbibition and drainage. Imbibition refers to the process of increasing the wetting phase saturation in a porous medium, thereby decreasing the saturation of the non-wetting phase. Drainage is the opposite process in which the nonwetting phase saturation is increasing while decreasing the wetting phase saturation.

Figure 1-1: Contact angle of water with a solid surface when water is the: (left) nonwetting phase (oil-wet system); (right) wetting phase (water-wet system). Taken from Glover’s online course notes.

1.1.2.2 Capillary Pressure

Another physical phenomenon that occurs when two immiscible fluid phases are present and meet at an interface is capillary pressure. Across the fluid-fluid interface, a pressure difference between the two phases exists and is called the capillary pressure, $p_c$:

$$p_c = p_{nw} - p_w$$

where the subscripts $w$ and $nw$ denote the wetting and nonwetting phases respectively. Usually, in a relative permeability test which will be explained in section 2.2, the capillary pressure is not measured and so the two fluid phases are assumed to be under the same hydraulic pressure. However, the capillary pressure
in a porous medium is a function of the saturation ratio of the two phases; it
decreases with increasing wetting phase saturation (Dullien, 1992). So, in the case
of non-uniform distribution of the two-phase saturation profile throughout a porous
medium, the capillary pressure varies locally from one location in the porous
medium to another.
Chapter 2

The Relative Permeability Approach

2.1 Historical Background

2.1.1 Single-Phase Flow in Porous Media

A homogeneous or single-phase fluid could be a single liquid or gas. A mixture of two completely miscible phases which remain miscible throughout the flow medium can also be regarded as a single-phase fluid (Muskat, 1937). A mixture of two fluid phases that results in an interface between the phases is not an example of a single-phase fluid but rather a two-phase fluid, such as a mixture of two immiscible liquids or a mixture of liquid and undissolved gas (Muskat, 1937). Henry Darcy, in his 1856 publication, presented the findings of his empirical investigations of water flow through vertical sand columns, which is an example of single-phase flow in porous media. Darcy’s experimental set-up comprised of homogeneous sand columns acting as water filters mounted vertically and supplied with water at the inlet face (Figure 2-1). Manometers at the inlet and outlet faces of the sand column measured the piezometric or hydraulic head, which is the sum of the pressure head at the column face and its elevation above a datum level. Darcy observed a linear relationship between the volume flow rate $Q$ of water through the sand column of a constant cross-sectional area $A$ and the hydraulic head difference between the two column faces. Darcy’s empirical expression came to
be known as Darcy’s law (Darcy, 1856), and can be written in its original form as:

\[ Q = KA\frac{\Delta h}{L} \]  

(2.1)

where \( L \) is the length of the sand column and \( \Delta h = h_1 - h_2 \) is the difference between the inlet and outlet hydraulic heads. The term \( \frac{\Delta h}{L} \) can be regarded as the hydraulic gradient as it is the hydraulic head difference across the column length. Since hydraulic head has units of length, the hydraulic gradient is dimensionless which makes the constant of proportionality \( K \) having units of velocity. \( K \) is termed the “coefficient of permeability” or “hydraulic conductivity” and is a characteristic of the sand column that describes the ease at which the fluid (water) is able to flow. Darcy’s law is a linear equation that describes unidirectional, slow, steady-state flow of water in the direction normal to the cross-sectional area of the porous medium. Note how Darcy’s law does not include a term characterizing the flowing fluid. This is because Darcy’s experiments were done using a single type of fluid, water.

In the case of a horizontal sand column, the inlet and outlet faces are at the same elevation. This simplifies the hydraulic head term \( h \) into a pressure head, since:

\[ h = h_p + z, \]  

(2.2)

where \( h_p \) is the pressure head and is related to the hydraulic pressure \( p \) through the specific weight of the fluid (the weight of a unit fluid volume: \( \rho g \)):

\[ h_p = \frac{p}{\rho g} \]  

(2.3)

Rewriting Darcy’s law for horizontal flow, we obtain (Bear, 1988):

\[ Q = K A \frac{\Delta h}{L} = K A \frac{\Delta p}{\rho g L} \]  

(2.4)

A generalized form of Darcy’s law was then developed to better describe the flow of
any fluid. The generalized form separates the hydraulic conductivity $K$ into two separate parameters: one pertaining to the porous medium and is called permeability, $k$, and another relating to the fluid which is viscosity, $\mu$. The generalized Darcy’s law becomes:

$$Q = -\frac{k}{\mu} A \frac{\Delta P}{L},$$

(2.5)

where the negative sign indicates that the fluid flow direction is in the direction of decreasing pressure. Comparing the generalized form to the original Darcy’s law, the relationship between the hydraulic conductivity $K$ and permeability $k$ is easily obtained:

$$K = \frac{kpg}{\mu},$$

(2.6)
Table 2.1: Various units of Darcy’s law used in petroleum engineering (Engler).

If the fluid viscosity $\mu$ is measured in centipoise (cP), volume flow rate $Q$ in $cm^3/s$, cross-sectional area $A$ in $cm^2$, and the pressure gradient $\Delta P/L$ in atmospheres per centimeter, the unit of permeability $k$ comes out to be $0.987 \times 10^{-12} m^2$ (or 0.987 $\mu m^2$). This unit of permeability is called a Darcy, which can be defined as the permeability of a medium in which 1 $cm^3/s$ of fluid with a viscosity of 1 cP flows through a cross-sectional area of 1 $cm^2$ under a pressure gradient of 1 atm/cm.

Table 2.1 lists the appropriate units for Darcy’s law in different unit systems.

### 2.1.2 Two-Phase Flow in Porous Media

**The Concept of Relative Permeability** In the case of two fluids flowing in a porous medium, Muskat (1949) is thought to be the first to apply the generalized Darcy’s law to each phase such that:

$$Q_1 = -\frac{k_1(S_1)}{\mu_1} A \frac{\Delta P_1}{L}$$
$$Q_2 = -\frac{k_2(S_2)}{\mu_2} A \frac{\Delta P_2}{L}$$

where $k_1$ and $k_2$ are no longer the medium’s permeability $k$, which is referred to in the literature as the *absolute* permeability when dealing with two-phase flow. The subscripted permeabilities $k_1$ and $k_2$ are called *effective* permeabilities of the medium to fluid #1 and fluid #2 respectively, and are non-linear functions of their corresponding fluid saturations. The concept of effective permeability is used to
represent the apparent reduction of the medium’s permeability in the case of two-phase flow. This reduction is indicated by the reduced volume flow rates of both fluids for a given pressure gradient compared to the case of single-phase flow. In other words, the simple, linear relationship between flow rate and pressure gradient in Darcy’s law is not sufficient to describe the complex behavior of two-phase flow. Effective permeability thus became the standard approach to adapt Darcy’s law to two-phase flow, and from there sprang the term *relative* permeability \( k_r \), which is simply the ratio of the \( i^{th} \) phase effective permeability, \( k_i \) to the absolute permeability of the medium, \( k \):

\[
k_{r,i} = \frac{k_i}{k} \quad i = 1, 2
\]  

(2.8)

where \( k_i \) and thus \( k_{r,i} \) are functions of the \( i^{th} \) fluid saturation \( S_i \).

Rewriting the two-phase flow equations using the definition of relative permeability, we obtain:

\[
Q_1 = -k_{r,1}(S_1) \frac{k \Delta P}{\mu_1 L}
\]

\[
Q_2 = -k_{r,2}(S_2) \frac{k \Delta P}{\mu_2 L}
\]

(2.9)

Relative permeability of each phase increases with the phase’s saturation, having a minimum value of zero at which the fluid phase is not flowing (giving an effective permeability of zero), and a maximum value of one at which the phase is the only flowing fluid and its effective permeability is equal to the absolute permeability. Figure 2-2 is an example of typical relative permeability curves of two-phase flow through a rock. The x-axis is usually the wetting phase saturation, which is water saturation in this case of water-oil flow. The y-axis is the value of relative permeability which ranges from 0 to 1 as mentioned above. The two curves in the figure are the relative permeabilities of water and oil, denoted by \( k_{rw} \) and \( k_{ro} \) respectively, and are often asymmetrical. As seen from the figure, as water saturation increases, relative permeability to water increases while relative permeability to oil decreases.

The range of water saturations over which the relative permeabilities are obtained is less than the full range of 0% to 100%. This is due to physical limitations in reaching the saturation levels beyond these two boundary values which are denoted
Figure 2-2: Typical relative permeability curves of two-phase flow in porous media (Cosse, 1993). The x-axis is usually the wetting phase saturation, which is water saturation in this example of water-oil flow. The increasing curve is the wetting phase relative permeability, $k_{ro}$, and the decreasing curve is the nonwetting phase relative permeability, $k_{rw}$.

by $S_{wi}$ and $S_{WM}$. $S_{wi}$ stands for irreducible water saturation and is the minimum water saturation after which water starts flowing. It is the fractional amount of water that is trapped in the medium that could not be extracted out of the medium when it was initially being saturated with oil. At this irreducible water saturation, the oil saturation is a maximum and is equal to $1 - S_{wi}$. The maximum water saturation on the other hand occurs when the oil saturation hits its minimum which is called the residual oil saturation, $S_{or}$, at which oil ceases to flow and water is the only flowing fluid. The maximum water saturation at this point is $1 - S_{or}$. The concept of a reduced saturation was therefore constructed in order to take these boundary
saturation levels into account and transform actual fluid saturations to some kind of "effective" saturation. The reduced water saturation is defined as:

\[ S_{w,\text{red}} = \frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}} \]  

(2.10)

so that at the irreducible water saturation, the reduced water saturation is zero and therefore no flow occurs, while at the maximum water saturation of \( 1 - S_{or} \), the reduced water saturation is 1 and the entire flow is that of the water phase. The concept of reduced saturation is similarly applied to the oil phase, such that:

\[ S_{o,\text{red}} = \frac{S_o - S_{or}}{1 - S_{wi} - S_{or}} \]  

(2.11)

Since the medium is assumed to be fully saturated with water and oil only,

\[ S_w + S_o = 1 \]  

(2.12)

The reduced oil saturation is simply:

\[ S_{o,\text{red}} = 1 - S_{w,\text{red}} \]  

(2.13)

This phenomenon of a minimum fluid phase saturation before which no outflow of the fluid is observed, can be explained through the concept of breakthrough pressure. The breakthrough pressure is the capillary pressure (pressure difference between the wetting and nonwetting phase, in this example between water and oil) that corresponds to the first appearance of a nonwetting phase on the outlet face of the sample (Dullien, 1992). This occurs when a "continuum of the nonwetting phase through a pore network" (Dullien, 1992) has just formed, allowing for outflow of the nonwetting phase. After this point, increasing capillary pressure results in more penetration of the nonwetting fluid (displacement of wetting fluid), until a point is reached at which increasing capillary pressure does not increase the nonwetting phase saturation. At this point, the wetting phase has reached its irreducible saturation and cannot be dis-
placed by further increase of the capillary pressure. It must be noted though that it has been experimentally shown that wetting phase saturation can be progressively decreased by increasing the externally applied capillary pressure and given long enough time for this to happen.

2.2 Relative Permeability Laboratory Measurements

Since the relative permeability concept was developed to represent the results of two-phase flow laboratory experiments, a good understanding of the relative permeability experimental procedure and results is necessary. There are two main categories of relative permeability laboratory tests: unsteady-state and steady-state tests.

In the unsteady-state method, the flow system does not reach equilibrium at any point during the experiment, and so the properties of the flow system continuously change with time. The fluid saturation at a given point in the medium will change with time, i.e. displacement of one fluid by another occurs. In this method, a cylindrical sample of a porous medium fully saturated with one fluid is injected with a second fluid. For example, let water be the initial fluid and oil be the injected fluid as shown in the schematic of Figure 2-3. During this process, water is being displaced out of the medium up to a certain limit, at which an immobile portion of the water phase is “stuck” in the medium. The medium is then mostly saturated with oil, and a small portion of the medium is filled with water making up the irreducible water saturation. This configuration of fluid saturations in the porous medium marks the starting point of the unsteady-state relative permeability test. Injection of water now begins, where initially, the produced fluid from the outlet face of the medium is comprised only of the oil phase, because the injected water front has not yet reached the outlet face. Once the fluid interface reaches the outlet face, the produced fluid is a combination of both water and oil. As time increases, the sample becomes more and more saturated with water and less saturated with oil, meaning that the injected fluid (water) is displacing the original fluid (oil) out of the sample. At specific, known saturation values
of the two fluids in the porous sample, the volume flow rates of the produced fluids are measured. The relative permeability of each phase can then be calculated using equation 2.9 by knowing the phase viscosity, the sample’s dimensions and “absolute” permeability and the pressure drop across the sample. The experimental test continues until only the injected fluid (water) is being produced from the outlet face, leaving behind an immobile, residual oil saturation in the medium. When the injected fluid is the wetting phase and the displaced fluid is the non-wetting phase, the process is termed imbibition, while the opposite process is called drainage (Cosse, 1993).

The steady-state test is simpler than the unsteady-state method, but is more time-consuming and thus expensive. The initial configuration of the fluid saturations in the medium is achieved in the same manner as in the unsteady-state test. Following the same example of a porous medium that is mostly oil-saturated with a small, irreducible water saturation, the steady-state test starts by injecting both fluids simultaneously into the sample under a specific ratio of the two fluids’ injection flow rates (Figure 2-4). The procedure is called steady-state, because at each ratio of injected flow rates, the two-phase flow system reaches a steady-state (given enough time) at which the pressure differential and ratio of produced fluids’ flow rates stabilize (Cosse, 1993). When steady-state is reached, the saturations of the two phases in the porous medium become constant across the length of the medium, and so there is no displacement of one fluid by the other (Bear, 1988). By varying the ratio of injected fluids’ flow rates over the entire range from 0 to 1, the entire range of fluid saturation ratios is covered. By measuring the produced fluid flow rates, relative permeability of each fluid can be calculated in the same manner as in the unsteady state method (Cosse, 1993).
Unsteady-State Relative Permeability Test

Figure 2-3: A schematic description of a relative permeability test using the unsteady-state method (Glover).
Steady-State Relative Permeability Test

Figure 2-4: A schematic description of a relative permeability test using the steady-state method (Glover).
2.3 Effects on Relative Permeability Measurements

From the earliest experiments of two-phase flow in porous media, it was apparent that the problem of two-phase flow is a complex phenomenon that depends on many different aspects. Predicting relative permeabilities of any two fluids comprising a two-phase fluid in a porous medium as a function of fluid saturation was becoming to look more challenging as more experimental work have shown the effects of different variables on the results of relative permeability tests. Some of the most prominent variables which have received great attention in order to understand the manner and magnitude of their effects on two-phase flow experiments are: fluid flow rate, wettability and interfacial tension, and saturation history or hysteresis.

2.3.1 Effect of Flow Rate

One of the major variables that was seen to affect relative permeability laboratory results was the rate of fluid flow used in the laboratory experiments. This obviously would pose a major concern, since the goal of many relative permeability tests is to be able to predict two-phase flow in oil reservoir conditions at different pressure gradients and thus, different flow rates. Many attributed the effect of the flow rate on relative permeability measurements to experimental errors, namely the capillary end effects which occur on the core boundaries (Osaba et al., 1951; Leverett and Lewis, 1941).

Capillary end effects arise because of the abrupt change in capillarity at the outlet end of the core, which causes the core to retain more of the wetting phase than the nonwetting phase due to capillary forces (Leverett and Lewis, 1941). This results in a higher saturation of the wetting phase at the outlet as compared to the rest of the core, resulting in a saturation gradient in the core. Such a saturation gradient causes a difference in capillary pressure between the two ends of the core which is equal to the difference between the pressure drops of the two phases. The actual pressure drop for the wetting phase would be less than that of the non-wetting phase, and so
if the former was assumed to be equal to the latter, then the wetting-phase relative permeability would be underestimated (Osaba et al., 1951). Several solutions to this problem have been proposed, one is to use high flow rates that would minimize the effect of capillary forces on the core boundaries. Another solution is to use the “Penn State Method” in which the test core is positioned (with capillary contact) between two samples of similar material such that the boundary effect is moved to the outer samples instead of the test core. Figure 2-5 from Osaba et al. (1951) shows the flow-rate dependence of relative permeability when the boundary effect is believed to be present (left), and the lack of such dependence when the boundary effect is believed to be eliminated through appropriate experimental set-up. Other researches however, disagree with the notion of the boundary effect being the source of relative permeability variation with flow rate. Maini et al. (1990) acknowledges that the boundary effect could be causing a small portion of the variation of relative permeability of water and oil with flow rate. However, the cause of the variation for the most part is attributed to changes in viscous fingering and fluid distribution in the core. Figure 2-6 shows the stark variation in relative permeability of water and oil with different flow rates. In general, relative permeability is seen to increase with increasing flow rate.

2.3.2 Effect of Interfacial Tension and Wettability

The effects of interfacial tension and wettability on relative permeability have been extensively studied, especially because of the important applications of changing wettability and interfacial tension in increasing oil recovery from petroleum reservoirs. Interfacial tension is the free energy at the interface between two immiscible fluids (Bear, 1988) and is equivalent to the amount of work needed to increase the area of this interface by a unit area. It has units of energy per unit area or force per unit length. The interfacial tension $\sigma_{12}$ between two immiscible fluids in contact with a solid is related to the contact angle $\theta$ between the solid and the denser fluid through Young’s equations:
Figure 2-5: Effect of varying flow rate (by varying pressure drop \( \Delta P \)) on results of relative permeability of oil in oil-gas flow is attributed to the "boundary effect" by Osaba et al. (1951).

Figure 2-6: Effect of varying flow rate on relative permeability of oil and water is attributed to changing fluid distribution in the core by Maini et al. (1990).
\[ \cos \theta = \frac{\sigma_{S1} - \sigma_{S2}}{\sigma_{S1}} \]

where \( \sigma_{S1} \) and \( \sigma_{S2} \) are the surface tensions between the solid surface and fluids 1 and 2 respectively. Remember that the contact angle is a direct measure of wettability of a fluid to a solid surface, and so the wettability of the two fluids to the solid surface depends on the interfacial tension between the fluids. Bardon and Longeron (1980) investigated the effect of varying oil and gas interfacial tension on relative permeability results. Interfacial tension was controlled by varying the equilibrium pressure and thus the composition of the liquid-vapor mixtures used in the experiments. The higher range of interfacial tension values (>1 mN/m) were directly measured, while lower interfacial tensions could only be calculated using a formula proposed by Weinaug and Katz (1943). The results from their unsteady-state relative permeability tests over the higher range of interfacial tensions showed increasing relative permeability of the wetting (oil) phase with decreasing interfacial tension (Figure 2-7). For the calculated, lower range of interfacial tension values, the increase of relative permeability with decreasing interfacial tension is much greater and happens for both phases.

Another study concerning the effect of wettability on relative permeability was done by Heaviside et al. (1987). The wettability of water to the core samples was controlled through different core cleaning methods. The goal was to test whether standard core cleaning techniques would produce relative permeability results representative of what actually happens in petroleum reservoirs. The answer is clear from Figure 2-8, in which different relative permeability curves correspond to core samples that have undergone different cleaning processes. The “restored” core sample is the core that is believed to have retained its original, intermediate wettability to water through proper cleaning techniques. It is seen to have the intermediate relative permeability values, whereas other cores that were treated to have different water wettabilities exhibit different relative permeability results.
Figure 2-7: Experimental results from Bardon and Longeron (1980) showing the effect of varying interfacial tension on relative permeability of oil ($k_{rL}$) and gas ($k_{rv}$) for interfacial tension values $>1$ mN/m (left) and $<1$ mN/m (right).

Figure 2-8: Experimental results from Heaviside et al. (1987) showing the effect of varying rock wettability through different core cleaning methods on water relative permeability.
2.3.3 Effect of Saturation History (Hysteresis)

Saturation history, or hysteresis, makes the complex two-phase flow problem even more problematic, because it affects relative permeability results from using the same fluid-pair and porous medium. Hysteresis refers to the effect in which the relative permeability of a fluid phase at a specific saturation depends on the path taken to reach this saturation (Osaba et al., 1951). In other words, the relative permeability curve for a fluid phase obtained by increasing its saturation is different from the relative permeability curve obtained in the opposite manner of decreasing saturation. This is seen in the difference between drainage and imbibition relative permeability curves (Figure 2-9), since the only difference between the two experiments is the direction of changing wetting phase saturation (refer to section 1.1.2.1). The difference between the results of drainage and imbibition is not quite understood but is attributed to the changing fluid distributions at the microscale level of pore spaces of the medium.

Figure 2-9: Typical relative permeability curves for the wetting (W) and nonwetting (NW) phases from drainage (D) and imbibition (I) experiments. (Efstathios et al., 2016).
2.3.4 Summary

Flow rate, surface tension and saturation history are three variables that are found to have prominent effects on relative permeability test results. Such variability caused by varying experimental conditions of the porous medium and the fluid phases imposes great challenges to testing new theories and modeling approaches. This is because in order to test the validity, or lack thereof, of a certain two-phase flow model, two main things must be known:

1. the region of experimental conditions at which the effect of such variables is minimized or negligible, such as the minimum flow rates (controlled by the imposed pressure drop) at which capillarity effects are insignificant and can thus be ignored.

2. the experimental conditions under which the experimental data being used for testing a proposed theory or model has been collected.

To determine what experimental conditions correspond to the data, appropriate measurements quantifying these conditions must be made, such as: contact angle measurements between the fluid phases and the porous medium surface for determining wettability conditions and interfacial tension measurements of the specific two fluid phases used in the experiment.

When using experimental data collected by other researchers, as is the case in this study, there is less control over ensuring experimental conditions suitable for the specific tested concept, unless one has the luxury of finding and accessing any dataset satisfying the desired criteria. Also, knowledge of the experimental conditions is limited by the reported experimental parameters and the exact experimental procedure. For the experimental data we have access to which is from Oak et al. (1990), comprehensive description of the rock core preparation, fluid composition, experimental set-up, laboratory conditions and raw measurements is provided. However, there is only so much details that the authors can provide. Therefore, any experimental details and parameters not considered by the authors
will be missing information that might be important in determining the validity of our proposed approach. Examples of such information for our specific data are whether the two fluid phases are mixed together before being injecting into the core, whether the flow rates are high enough for capillary effects to be unimportant, fluid interfacial tension measurements and core rock wettability measurements.
Chapter 3

Relative Permeability Data

The experimental data used in this study is from Oak (1990); Oak et al. (1990) and constitute two-phase flow experiments of water-oil, oil-gas, and water-gas systems. The specific fluids used to represent the different fluid phases are: brine with salinity of 55,000 ppm as the water phase, mineral oil as the oil phase and nitrogen gas as the gas phase. The relative permeability tests are steady-state, in which the two fluid phases are injected simultaneously into the core sample at constant rates. When the flow system reaches steady-state, that is, when the pressure differential across the core and the fluid saturations inside the core stabilize, the experimental measurements of pressure, fluid flow rates and fluid saturations are taken. The relative permeability for each phase is calculated at this specific steady-state using Darcy’s law for two-phase flow. The next stage of steady-state is then sought by changing the ratio of the two fluids’ injection rates in order to reach a new level of fluid saturations inside the core. Fluid saturations were measured using the X-ray absorption method, in which a different X-ray tracer is dissolved in each liquid phase (water and oil), the saturation of each liquid phase is calibrated to X-ray measurements of its corresponding tracer, and gas saturation in the case of a liquid-gas flow experiment is calculated by subtracting the measured liquid saturation from unity. By the end of each two-phase flow experiment, relative permeability data for each phase is obtained for the different saturations covered by the experiment.
The experimental apparatus was kept under constant room temperature (21.1 ± 0.6 degrees C) and 5.5 MPa gauge pressure. The experiments were conducted on three cylindrical cores of water-wet Berea sandstone that are 5.1 cm (2 inches) in diameter and 7.6 cm (3 inches) in length with different permeabilities: 200 mD (sample #6), 800 mD (sample #14) and 1000 mD (sample #13). Both drainage and imbibition experiments were carried out on each core sample (Table 3.2) with fluid viscosities shown in Table 3.1. Figures 3-1 to 3-3 are plots of relative permeability data versus wetting-phase saturation for each type of fluid-pair experiment (water-oil, oil-gas and water-gas), separated into drainage experiments (top plots) and imbibition experiments (bottom plots). Note that each plot contains data from multiple runs of the same experiment type, and in cases where different runs yielded different relative permeability results, the scatter of the data is significant.

Remember that drainage experiments start at the maximum saturation level of the wetting phase, which in the case of this dataset as seen from the top plots of the figures, is almost exactly at the 100% saturation level in all fluid-pair experiments. At this point, the wetting-phase curve (blue) is at its maximum and the nonwetting phase (red) is at its minimum. The drainage experiment then proceed by increasing the nonwetting-phase saturation thus reducing the wetting-phase saturation, with the effect of decreasing relative permeability of the wetting phase and increasing that of the nonwetting phase. This process continues until a minimum saturation level is reached, the irreducible saturation, which is seen to be around 25% in water-oil experiments, 35% in water-gas experiments and 30% in oil-gas experiments. The imbibition experiments are then carried out in the opposite direction, starting at the endpoint of the drainage experiment at which the wetting phase saturation is a minimum and the nonwetting-phase saturation is a maximum. The bottom plots of the figures are relative permeability data from imbibition tests. The wetting-phase saturation is increased while the nonwetting-phase saturation is reduced, until the maximum wetting-phase saturation is reached. Significant differences between drainage and imbibition tests for the same type of two-phase flow experiment are obvious from the figures. In imbibition tests of all fluid-pair
types, the wetting phase saturation reaches a maximum of 60-80% which is much lower than the 100% saturation level reached in most drainage tests. The shape of the imbibition relative permeability curves for the nonwetting phase is quite different from that of the drainage curves, with a sharper decline as a function of wetting-phase saturation. The shape of the wetting-phase relative permeability curve however looks similar between drainage and imbibition tests.

Table 3.1: Viscosities of fluids used in the two-phase fluid flow experiments carried out by Oak (1990); Oak et al. (1990) on different Berea sandstone core samples.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>#6</td>
<td>200</td>
<td>1.06</td>
<td>1.77</td>
<td>0.0187</td>
</tr>
<tr>
<td>#13</td>
<td>1000</td>
<td>1.05</td>
<td>1.43</td>
<td>0.0187</td>
</tr>
<tr>
<td>#14</td>
<td>800</td>
<td>1.05</td>
<td>1.39</td>
<td>0.0187</td>
</tr>
</tbody>
</table>

Table 3.2: Drainage (Dr.) and Imbibition (Imb.) Experiments of Water-Oil, Oil-Gas and Water-Gas Flow.

<table>
<thead>
<tr>
<th>Core Sample</th>
<th>Water-Oil Flow</th>
<th>Oil-Gas Flow</th>
<th>Water-Gas Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dr.</td>
<td>Imb.</td>
<td>Dr.</td>
</tr>
<tr>
<td>#6</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>#13</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>#14</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>

The next set of figures (3-4 to 3-6) regroup the same relative permeability data shown in the preceding figures based on the core sample used for the different two-phase experiments. It is very interesting to see how similar relative permeability results are (in both drainage and imbibition tests) from two-phase flow experiments on core sample #6 using different fluid pairs. Core samples #14 and #13 test results show more scatter, indicating varying relative permeability results when using different fluid pairs. However, a clear trend in the relative permeability curves is still seen from the experimental results on these two cores.
Two-Phase Flow Experiments Grouped by Fluid-Pair Type

1. Water-Oil Flow Experiments

WO Drainage Experiments on Different Core Samples

WO Imbibition Experiments on Different Core Samples

Figure 3-1: Relative permeability data from two-phase flow experiments of water and oil using 3 different core samples with different permeabilities. Different symbols mark different core samples while different colors distinguish the wetting water from the nonwetting oil. Top: Drainage experiment data. Bottom: Imbibition experiment data.
2. Oil-Gas Flow Experiments

Figure 3-2: Relative permeability data from two-phase flow experiments of oil and gas using 3 different core samples with different permeabilities. Different symbols mark different core samples while different colors distinguish the wetting oil from the nonwetting gas. Top: Drainage experiment data. Bottom: Imbibition experiment data.
3. Water-Gas Flow Experiments

Figure 3-3: Relative permeability data from two-phase flow experiments of water and gas using 3 different core samples with different permeabilities. Different symbols mark different core samples while different colors distinguish the wetting water from the nonwetting gas. Top: Drainage experiment data. Bottom: Imbibition experiment data.
Two-Phase Flow Experiments Grouped by Core Sample

1. Core Sample #6 ($k=200$ mD) Experiments

![Relative Permeability Data Graph](image)

**Figure 3-4:** Core sample #6 relative permeability data of wetting and nonwetting phases for each fluid-pair type: water-oil, oil-gas and water-gas. Different symbols mark different experiment types while different colors distinguish the wetting phase from the nonwetting phase. Top: Drainage test data. Bottom: Imbibition test data.
2. Core Sample #13 ($k=1000$ mD) Experiments

Figure 3-5: Core sample #13 relative permeability data of wetting and nonwetting phases for each fluid-pair type: water-oil, oil-gas and water-gas. Different symbols mark different experiment types while different colors distinguish the wetting phase from the nonwetting phase. Top: Drainage test data. Bottom: Imbibition test data.
3. Core Sample #14 (k=800 mD) Experiments

Figure 3-6: Core sample #14 relative permeability data of wetting and nonwetting phases for each fluid-pair type: water-oil, oil-gas and water-gas. Different symbols mark different experiment types while different colors distinguish the wetting phase from the nonwetting phase. Top: Drainage test data. Bottom: Imbibition test data.
Chapter 4

The Ideal Two-Phase Flow

Let us imagine the ideal case of two-phase flow in which the flow of each phase is that of a single-phase. In other words, each phase is unaware of and unaffected by the presence of the other phase, and so Darcy's law for single-phase flow applies to each phase separately. Two main cases of this hypothetical two-phase flow can be defined based on the configuration of the flowing phases in the medium: parallel-flow and series-flow. For each flow case, we consider the fluid mixture as a whole and describe its flow by applying Darcy's law to the fluid mixture. From this step, we define the concept of an "average viscosity" of the fluid mixture, which is equivalent to the fluid viscosity of a virtual, single fluid that would have the same flow rate as the two-phase mixture, under a given pressure drop across the length of the porous medium. We relate this average viscosity to the viscosities of the two fluid phases by establishing the pressure drop and flow rate relationships for each flow case.

4.1 Two-Phase Flow in Series

In the ideal case of two-phase flow in series, each fluid phase flows as a single-phase through a complete vertical section of the porous medium as shown in Figure 4-1. The two phases meet at a vertical interface away from the (left) inlet face of the medium, at a fraction distance of the total length of the medium $L$ equal to the saturation of fluid 1. The figure is a schematic snapshot of the two-phase flow at a
Figure 4-1: Sketch of the ideal two-phase flow in series in which each phase flows in a vertical section of the porous medium proportional to its saturation single moment in time, so that as time progresses, the fluid interface moves in the direction of flow towards the (right) outlet face of the medium. Here, we are depicting the case in which the medium is initially saturated with fluid 2, and fluid 1 is injected from the inlet face. The first relationship particular to this case of series flow is that the flow rates of the two phases are equal.

\[ Q_1 = Q_2 = Q_{\text{mix}} \]  

This must be the case, for if fluid 2 were to flow at a faster rate than fluid 1, the two phases must be separated by vacuum rather than a fluid interface, which is not possible. Similarly, fluid 1 cannot flow at a higher rate than fluid 2 since it will not have any room in the porous medium to flow into and thus its flow rate is limited by that of fluid 2. If we consider the two phases as a single “fluid mixture”, then the fluid mixture would have a flow rate \( Q_{\text{mix}} \) equal to the phases’ flow rates. The second relationship between the two phases and the fluid mixture is that the total pressure drop \( \Delta P \) across the length \( L \) of the porous medium equals the sum of the
individual phases' pressure drops.

\[ \Delta P = \Delta P_1 + \Delta P_2 \]  \hspace{1cm} (4.2)

where each fluid phase experiences a pressure drop over the length of the portion of the porous medium filled by the fluid. This fraction of this length relative to the entire length of the core is equal to the fluid saturation, i.e.

\[ L_i = S_i L \quad ; \quad i = 1, 2 \]  \hspace{1cm} (4.3)

where \( S_i \) is the fluid saturation of the \( i^{th} \) phase. Having the necessary relationships between the individual phases and the fluid mixture, we apply Darcy's law for single-phase flow on the flow of the fluid mixture:

\[ Q_{\text{mix}} = \frac{k}{\mu_{\text{avg}}} A \frac{\Delta P}{L} \]  \hspace{1cm} (4.4)

where the viscosity term here is the average viscosity \( \mu_{\text{avg}} \), which describes the fluid mixture rather than the individual phases. Using the pressure drop relationship in Equation 4.2, we can write:

\[ Q_{\text{mix}} = -\frac{k}{\mu_{\text{avg}}} A \frac{(\Delta P_1 + \Delta P_2)}{L} \]  \hspace{1cm} (4.5)

Since we assume that each phase flows as a single-phase in its portion of the porous medium, we can relate the pressure drop of each phase to its viscosity using Darcy's law while accounting for Equation 4.3, such that:

\[ Q_i = -\frac{k}{\mu_i} A \frac{\Delta P_i}{S_i L} \quad ; \quad i = 1, 2 \]  \hspace{1cm} (4.6)
Rearranging Equation 4.6 for fluids 1 and 2 and substituting the individual pressure drops in Equation 4.5, we obtain:

\[
Q_{mix} = -\frac{k}{\mu_{avg}} A \left[ \frac{Q_1 \mu_1 S_1}{kA} + \frac{Q_2 \mu_2 S_2}{kA} \right]
\]  

(4.7)

Cancelling the common terms, employing the equal relationship between the three flow rates (Equation 4.1) and rearranging, the average viscosity of the fluid mixture becomes:

\[
\mu_{avg} = S_1 \mu_1 + S_2 \mu_2
\]  

(4.8)

which is the arithmetic average of the two fluid viscosities weighted by their saturations. Comparing this ideal scenario to the manner in which relative permeability experiments of Oak et al. were conducted, we can obviously see that the series two-phase flow is not the ideal representation of these experiments. First, the series flow is the extreme representation of displacement flow, which is the nature of unsteady relative permeability tests whereas Oak et al.'s experiments are steady-state. It is an “extreme” representation because a vertical fluid interface does not exist in a real displacement experiment in porous media. If the interface were to be vertical and stay so throughout the experiment, then only one phase would be produced from the outlet face of the medium at any point in time: only the initial fluid would flow out until the interface reaches the outlet face, after which only the injected fluid starts flowing out. A more obvious difference between the ideal series flow and any two-phase flow experiment in porous media is that the flow rates for the two phases are not equal in a real experiment. Also, for the arithmetic average to represent the experiment average viscosity, the pressure drops of both phases over their saturated sections of the medium must be measured and not assumed to be equal. In relative permeability tests on the other hand, the pressure drop is measured across the whole medium and is assumed to be the same for both fluids.
4.2 Two-Phase Flow in Parallel

In the other case of ideal two-phase flow, the two fluid phases flow simultaneously through the cross-sectional area of the porous medium as shown in Figure 4-2. The partitioning of the cross-sectional area $A$ among the two phases depends on the volume fraction of the pore space occupied by each fluid, or simply their saturations. Basically, each phase flows through a fraction of the area $A$ that is equal to its saturation, i.e.

$$A_i = S_i . A$$

$$i = 1, 2$$

(4.9)

Unlike the two-phase flow in series, the pressure drop for the two phases is equal and is measured across the total length $L$ of the porous medium:

$$\Delta P_1 = \Delta P_2 = \Delta P$$

(4.10)

Figure 4-2: Sketch of the ideal two-phase flow in parallel in which each phase flows as a single-phase through a horizontal section of the porous medium proportional to its saturation.

Assuming each phase flows as a single-phase where Darcy's law applies, this results
in unequal flow rates of the two phases as long as they have different viscosities. We can thus write a separate Darcy's law for each phase as follows:

\[ Q_i = \frac{k}{\mu_i} (S_i A) \frac{\Delta P}{L} ; \quad i = 1, 2 \quad (4.11) \]

where for a fully saturated rock with only two phases: \( S_1 + S_2 = 1 \).

Now, considering the two phases as a single fluid mixture flowing through the entire cross-sectional area \( A \) of the medium at a flow rate \( Q_{\text{mix}} \), this flow rate must be the sum of the individual phases' flow rates:

\[ Q_{\text{mix}} = Q_1 + Q_2 \quad (4.12) \]

Now, using Equation 4.11 and applying Darcy's law for single-phase flow to the fluid mixture with the same \( \text{average} \) viscosity term as defined in the section on series flow, we rewrite Equation 4.12 as:

\[ \frac{k}{\mu_{\text{avg}}} A \frac{\Delta P}{L} = \frac{k}{\mu_1} S_1 A \frac{\Delta P}{L} + \frac{k}{\mu_2} S_2 A \frac{\Delta P}{L} \quad (4.13) \]

Canceling out the common terms, we obtain the viscosity averaging expression for the ideal case of two-phase flow in parallel, which is the harmonic average of the individual fluid viscosities weighted by their respective saturations:

\[ \frac{1}{\mu_{\text{avg}}} = \frac{S_1}{\mu_1} + \frac{S_2}{\mu_2} \quad (4.14) \]

Considering the properties of the ideal two-phase flow in parallel, it is obvious that this is an ideal representation of the steady-state two-phase flow experiments by Oak et al. The fluid configuration in the medium is the equilibrium configuration after enough time has been allowed to reach this steady-state for the given fluid injection rates. The flow rates of the two individual phases are expected to vary according to their viscosities. A single pressure drop is measured across the medium’s length and is assumed to be the same for both fluids. Thus, the harmonic
average of the fluid viscosities is expected to be the right averaging scheme in the case of a perfect two-phase flow in parallel.

On another note, comparing the flow rate equations for the individual phases in parallel flow (Equation 4.11) to the standard two-phase flow equations employing relative permeability (Equations 2.9), it turns out that in the ideal, parallel two-phase flow, the relative permeabilities of fluids 1 and 2 are simply equal to their saturations, i.e.:

\[ k_{r,i} = S_i \quad ; \quad i = 1, 2 \quad (4.15) \]

The plot of such ideal relative permeability curves would be two symmetric, diagonal lines with opposite slopes as shown in Figure 4-3. Also, the ideal relative permeability curves would be the same for any pair of fluid phases since the relative permeability here does not depend on any fluid property other than its saturation, whether the fluids are two liquids such as water and oil or a liquid and a gas such as water/oil and gas. Note also that the sum of the relative permeabilities would always equal to one which corresponds to an effective permeability equal to the medium’s absolute permeability. The plot of the ideal flow rate curves however for the two phases would be different for different two-phase systems, since the fluid viscosity goes into the calculation of flow rate.

### 4.2.1 Ideal Steady-State Water-Oil Flow

In the case of a water-oil flow, such as the flow tests from Oak et al. on core sample #6 with water and oil viscosities of 1.05 cP and 1.43 cP respectively, we calculate the ideal flow rates for a pressure difference of 1 atm across the core’s length similar to those shown in Figure 4-4. The flow rate curves are linear curves following Darcy’s law for single-phase flow. The total flow rate of the fluid mixture (black curve) is the sum of water and oil flow rates and so it too must be a linear curve. The total flow rate increases in the direction of increasing water saturation because of the lower water viscosity compared to oil and because of the inverse proportionality between flow rate and fluid viscosity.
For the case of water-oil flow with the specific viscosities mentioned earlier, the mixture average viscosity curve would be similar to the one shown in Figure 4-5. While the total flow rate increases linearly with water saturation, the mixture’s average viscosity decreases non-linearly following the harmonic-average function.

4.2.2 Ideal Steady-State Water-Gas Flow

Because of the significant difference in fluid viscosity between liquids and gases, a liquid-gas two phase flow is considerably different from liquid-liquid flow in certain aspects. As mentioned earlier, the ideal relative permeability curves would be the same for the liquid-gas case as those for the liquid-liquid case. The plot of ideal flow rate curves for water-gas flow is shown in Figure 4-6. The water flow rate is hardly seen to be increasing with water saturation because of the smaller range of water flow rate magnitudes as compared to that of gas. This is because for a given pressure gradient, the gas flow rate will be significantly higher than the water flow rate because of the much lower gas viscosity than water viscosity. While the average viscosity of the water-gas mixture is computed with the same harmonic average for

Figure 4-3: Relative permeability curves for the ideal scenario of two-phase flow in parallel where each phase flows as a single phase.
Figure 4-4: Water, oil and total flow rate curves for the ideal scenario of two-phase flow in-parallel where each phase flows as a pure single-phase flow.

Figure 4-5: Plot of the total flow rate as a function of water saturation alongside the plot of average mixture viscosity in the ideal scenario of water-oil flow in-parallel.
the water-oil mixture, the average viscosity curve of the former appears different from the water-oil case because of again, the stark difference between the water and gas viscosities (Figure 4-7).

4.2.3 Ideal Steady-State Oil-Gas Flow

The ideal two-phase flow of oil and gas is very similar to the flow of water and gas because of the liquid-gas nature of the fluid mixture. Plots of the ideal fluid flow rates and average viscosity are similar to those of water-gas flow and are seen in Figures 4-8 and 4-9.

4.2.4 Summary

The ideal two-phase flow has been defined as the flow in which each fluid phase flows as a single-phase, i.e. is unaffected by the flow of the other phase and perfectly satisfies Darcy’s law for single-phase flow. Two scenarios of such ideal, two-phase flow are described which constitute the two extreme cases of two-phase flow: parallel flow and series flow. Establishing the appropriate properties of the flow system for each case, the average viscosity expression corresponding to each case is derived. The appropriate viscosity averages are found to be the weighted harmonic average for the parallel flow case and the weighted arithmetic average for the series flow case, with the two fluid-phase saturations being the weighting factors for their corresponding fluid viscosities. Dealing with steady-state relative permeability test data from Oak et al., the parallel flow is considered the appropriate, ideal representation of the two-phase flow captured by the experimental data.

Comparing the “ideal” relative permeability curves to actual ones from two-phase flow experiments (see chapter 3), it is obvious that the real two-phase flow behavior is very different from the ideal one. Figure 4-10 overlays the ideal relative permeability curves of water-oil flow on the real water-oil relative permeability data from Oak et al. Note how the ideal curves are not plotted over the entire range of water saturation of 0-1, but rather between the irreducible water saturation $S_{w,irr}$
Figure 4-6: Water, gas and total flow rate curves for the ideal scenario of two-phase flow in-parallel where each phase flows as a pure single-phase flow.

Figure 4-7: Plot of the total flow rate as a function of water saturation alongside the plot of average mixture viscosity in the ideal scenario of water-oil flow in-parallel.
Figure 4-8: Oil, gas and total flow rate curves for the ideal scenario of two-phase flow in-parallel where each phase flows as a pure single-phase flow.

Figure 4-9: Plot of the total flow rate as a function of oil saturation alongside the plot of average mixture viscosity in the ideal scenario of oil-gas flow in-parallel.
Figure 4-10: An example of real versus ideal relative permeability curves for water-oil flow.

(0.30) and the maximum water saturation (here, it happens to be equal to 1 because of no residual oil saturation). This is because the ideal relative permeability curves are taken as the reduced saturations instead of the regular saturations, i.e. the weighting factors in the harmonic average are taken as the reduced saturations.

As seen in the figure, instead of the straight, diagonal lines, the real relative permeability curves are non-linear functions of saturation. In other words, flow rate is not a simple, linear function of the pressure differential across the porous medium. This just shows what has been known since the beginning of two-phase flow investigation, which is that the linear equation of Darcy’s law does not apply to each fluid phase part of a two-phase flow system. More specifically, the real relative permeability value is always less than the ideal one (data points lie below the ideal curves), which means that the real, two-phase flow suffers a deficiency in volume flow rates as compared to the real case of single-phase flow.
Chapter 5

The Average Viscosity Approach

5.1 Fluid Viscosity

Unlike solids which can withstand a shear stress by suffering strain or deformation, fluids continue to deform as long as they are subjected to shear stress. This continuous deformation is called flow, and viscosity is the property that describes the fluid’s resistance to this type of deformation or flow (Bear, 1988). In mathematical notation, viscosity $\mu$ is the constant of proportionality between the applied shear stress $\tau$ and the shear strain rate $\dot{\gamma}$, such that:

$$\tau = \mu \dot{\gamma}$$

(5.1)

where $\mu$ is called the dynamic viscosity with units of Pa.s in the M.K.S unit system and units of g.cm$^{-1}$ s$^{-1}$ or Poise in the C.G.S. unit system. Newtonian fluids are fluids whose dynamic viscosity is a constant (at a specific pressure and temperature) that is independent of the shear stress or strain rate. Kinematic viscosity $\nu$ is the ratio of the fluid’s dynamic viscosity to its density $\rho$, and has units of cm$^2$/s or Stoke in the C.G.S unit system.

$$\nu = \frac{\mu}{\rho}$$

(5.2)

As shown earlier in the case of one-phase fluid flow in porous media represented by
Darcy's law, viscosity is the only parameter characteristic of the flowing fluid. It is therefore sensible to investigate the viscosity of the fluid mixture in the case of two-phase flow in porous media, and test whether this concept of mixture viscosity can explain the two-phase flow behavior as a function of fluid saturation. We showed in the previous chapter that the harmonic average is expected to be the ideal representation of the steady-state two-phase flow data.

5.2 Mathematical Averages as Viscosity Averages

Knowing that results from actual relative permeability tests indicate fluid flow behavior quite far from the ideal steady-state flow, we still attempt to test common mathematical averages for our purpose of averaging two-phase viscosities. Modeled mathematical averages are compared to average viscosities calculated from the relative permeability data. Instead of the regular fluid saturations however, we use the reduced fluid saturations (see page 29) as weighting factors in order to eliminate the need for modeling the regions of fluid saturations with no flow data. The three mathematical averages tested are the arithmetic, harmonic, and geometric averages, which by taking the reduced saturations as weighting factors can be written as:

\[
\mu_{avg,A} = S_{1,red} \mu_1 + S_{2,red} \mu_2
\]

(Arithmetic Average) \hspace{1cm} (5.3)

\[
\frac{1}{\mu_{avg,H}} = \frac{S_{1,red}}{\mu_1} + \frac{S_{2,red}}{\mu_2}
\]

(Harmonic Average) \hspace{1cm} (5.4)

\[
\log_{10} \mu_{avg,G} = \log_{10} S_{1,red} \cdot \log_{10} \mu_1 + \log_{10} S_{2,red} \cdot \log_{10} \mu_2
\]

(Geometric Average) \hspace{1cm} (5.5)

where: \( S_{2,red} = 1 - S_{1,red} \).

We showed in Chapter 4 that the arithmetic average represents two-phase flow in-series while the harmonic average represents flow in-parallel. The geometric mean is tested because it is an intermediate averaging function that lies between the arithmetic and harmonic means.

As a reminder from Chapter 4, the average viscosity from the data is calculated by
applying Darcy's law on the fluid mixture and then rearranging the expression:

\[
Q_{tot} = \frac{k}{\mu_{avg}} A \frac{\Delta P}{L}
\]

\[
\mu_{avg} = \frac{k}{Q_{tot}} A \frac{\Delta P}{L}
\]  \tag{5.6}

Comparison between the mixture viscosity computed from the data and the mixture viscosity averages using the arithmetic, harmonic and geometric averages are shown in Figures 5-1 through 5-6. In the liquid-liquid case of water-oil flow, the computed mixture viscosity from either drainage or imbibition test data is alien to any mathematical average representation. In drainage experiments, the calculated average viscosity from the data peaks around the middle of the reduced saturation range, i.e. the range of saturations over which fluid flow is detected and relative permeability data are collected. In imbibition experiments, the calculated average viscosities from the data reach a similar peak at high water saturation near the maximum value.

For the liquid-gas case in drainage tests seen by the water-gas and oil-gas flow drainage experiments, the geometric average seems to represent the fluid mixture viscosity at lower liquid saturations up to 60-70% in most experiments which is around the middle of the reduced saturation range. At higher liquid saturations, the experimental fluid mixture viscosity diverges off the geometric average and sometimes even peaks at a high liquid-saturation value before falling off to reach the liquid viscosity at 100% liquid saturation. In imbibition tests however, the harmonic average fits the data almost exactly in all the experiments runs except for two (bottom water-gas and oil-gas imbibition experiments). However, there is still a huge gap between the calculated average viscosities between the data and the harmonic average (or any mathematical average) in many of the experiments near the maximum wetting phase saturation. If these large values are assumed to be erroneous and overestimated, this could be due to the measurements of pressure differential being larger than the true pressure values or to the flow rate measurements of one or both fluids being lower than their true values. This is
because the calculated average viscosity is directly proportional to the pressure differential and inversely proportional to the total flow rate (Equation 5.6).

Examining the flow rate measurements that correspond to the high jumps in the calculated average viscosity from the data of water-gas imbibition experiments for example, the flow rates are reported to be in the order of 0.1 cc/min for gas and in the order of 1 cc/min for water. Such low values especially for gas are subject to huge error factors.

Using the harmonic average viscosity to model the total flow rate of the fluid mixture in liquid-gas imbibition experiments, we obtain a satisfying fit between actual and modeled flow rates. Figure 5-7 shows the degree of fit for oil-gas and water-gas experiments.
1. a. Water-Oil Drainage Experiments

WO Drainage Experiments on Sample 6 - Average Viscosity

WO Drainage Experiments on Sample 13 - Average Viscosity
Figure 5-1: Fluid-mixture viscosity models using the three common mathematical averages compared to calculated fluid-mixture viscosities from water-oil flow data in drainage on different core samples.
1.b. Water-Oil Imbibition Experiments

WO Imbibition Experiments on Sample 6 - Average Viscosity

WO Imbibition Experiments on Sample 13 - Average Viscosity
Figure 5-2: Fluid-mixture viscosity models using the three common mathematical averages compared to calculated fluid-mixture viscosities from water-oil flow data in imbibition on different core samples.
2.a. Oil-Gas Drainage Experiments

![Graph showing Oil-Gas Drainage Experiments on Sample 6 - Average Viscosity](image1)

![Graph showing Oil-Gas Drainage Experiments on Sample 13 - Average Viscosity](image2)
Figure 5-3: Fluid-mixture viscosity models using the three common mathematical averages compared to calculated fluid-mixture viscosities from oil-gas flow data in drainage on different core samples.
2.b. Oil-Gas Imbibition Experiments

OG Imbibition Experiments on Sample 6 - Average Viscosity

OG Imbibition Experiments on Sample 13 - Average Viscosity
Figure 5-4: Fluid-mixture viscosity models using the three common mathematical averages compared to calculated fluid-mixture viscosities from oil-gas flow data in imbibition on different core samples.
3.a. Water-Gas Drainage Experiments

Figure 5-5: Fluid-mixture viscosity models using the three common mathematical averages compared to calculated fluid-mixture viscosities from water-gas flow data in drainage on different core samples.
3.b. Water-Gas Imbibition Experiments

Figure 5-6: Fluid-mixture viscosity models using the three common mathematical averages compared to calculated fluid-mixture viscosities from water-gas flow data in imbibition on different core samples.
Figure 5-7: Total flow rate of the liquid-gas fluid mixture modeled using the harmonic average viscosity, plotted against total flow rate calculated from two-phase flow data from Oak et al.
Chapter 6

Two-Phase Flow in Pipes

Behavior of two-phase fluid flow in non-porous media, such as pipes and conduits, is investigated to see if any correlations exist between the flow behaviors in the two different flow media. If any similarities are found, certain aspects of two-phase flow in porous media might be related to the nature of the fluid mixture itself regardless of the flow medium.

The first area of investigation is the method of averaging the viscosities of two fluids comprising a two-phase fluid mixture flowing in a pipe. The other investigated area is the Martinelli-Lockhart correlations between dimensionless parameters that relate single-phase pressure drop in pipes to two-phase pressure-drop in pipes for a given flow rate.

6.1 Average Viscosity Models for Two-Phase Flow in Pipes

Extensive research on the topic of average viscosity of two-phase fluid mixtures flowing in pipes has been carried out as early as the 1930's when Taylor (1932) derived a theoretical average viscosity of a liquid carrying spherical droplets of another liquid in suspension. Taylor's work was purely theoretical and was made by taking Einstein's equation for the viscosity of a fluid carrying solid spheres and
extending it to the case of liquid droplets. Later work on the average viscosity of a
two-phase fluid mixture focused on the case of a liquid phase carrying droplets or
bubbles of a gas phase, and used pipe-flow experiments to verify the proposed
average viscosity models. The main goal of these studies was to find an appropriate
average viscosity for the two-phase fluid flowing under a particular flow regime
(laminar vs. turbulent) and flow pattern (bubbly flow, slug flow, churn flow, annular
flow, etc...) that can predict the observed pressure loss over the length of the pipe.

In average viscosity models of two-phase flow in pipes, the fluid mixture is regarded
as whole and the average viscosity of the fluid mixture is a function of certain
parameters of the two-phase flow system that vary among different models. The two
fluid phases are first categorized into a continuous fluid phase called the carrier
phase, and a discontinuous phase made of suspended droplets or particles in the
carrier phase called the dispersed phase. Some of the most popular models that
have developed over decades of pipe-flow experimentation are:

\[
\frac{1}{\mu_{mix}} = \frac{\chi}{\mu_d} + \frac{(1 - \chi)}{\mu_c} \quad \text{(McAdams et al., 1942)} \tag{6.1}
\]

\[
\mu_{mix} = \chi \mu_d + (1 - \chi) \mu_c \quad \text{(Cicchitti et al., 1960)} \tag{6.2}
\]

\[
\nu_{mix} = \chi \nu_d + (1 - \chi) \nu_c \quad \text{(Duckler et al., 1964)} \tag{6.3}
\]

which is equivalent to: \( \mu_{mix} = \beta \mu_d + (1 - \beta) \mu_c \)

\[
\mu_{mix} = \beta \mu_d + (1 - \beta)(1 + 2.5\beta)\mu_c \quad \text{(Beattie and Whalley, 1982)} \tag{6.4}
\]

\[
\mu_{mix} = \frac{1 - \beta}{S_c} \quad \text{(Fourar and Bories, 1995)} \tag{6.5}
\]

where the “flow quality”, \( \chi \), is the kinematic mass concentration, or the ratio of the
mass flow rate of the dispersed phase to the mass rate of the fluid mixture:

\[
\chi = \frac{\dot{m}_d}{\dot{m}_{mix}} = \frac{Q_d \rho_d}{Q_{mix} \rho_{mix}} \tag{6.6}
\]

where \( \dot{m} \) is mass flow rate, \( Q \) is volume flow rate and \( \rho \) is fluid density. The fluid
mixture bulk density $\rho_{mix}$ is calculated as an arithmetic average of the phase densities weighted by their volume fractions $\alpha_c$ and $\alpha_d$:

$$\rho_m = \alpha_c \rho_c + \alpha_d \rho_d$$

where

$$\alpha_c = \frac{V_c}{V_{mix}} \quad ; \quad \alpha_d = \frac{V_d}{V_{mix}} = 1 - \alpha_c.$$

$\nu$ is the kinematic viscosity introduced earlier, which is equal to the ratio of the fluid dynamic viscosity to its density:

$$\nu = \frac{\mu}{\rho} \quad (6.7)$$

and $\beta$ is the dispersed phase volume flow rate fraction of the total fluid volume flow rate, i.e.:

$$\beta = \frac{Q_d}{Q_{mix}} \quad (6.8)$$

Similar to what has been done in Chapter 5 where calculated average viscosity of the two-phase fluid is compared to the mathematical average models, we test the above average viscosity models from two-phase pipe-flow experiments on the experimental data of two-phase flow in porous media. The volume fractions $\alpha_c$ and $\alpha_d$ needed to calculate the fluid mixture average density $\rho_m$, are taken as the reduced fluid saturations $S_{1,red}$ and $S_{2,red}$. The wetting phase is considered the continuous phase and the nonwetting phase is considered the dispersed phase. Figures 6-1 through 6-4 show the results of the comparison between the calculated and modeled average viscosity of the two-phase fluid mixture. The results for water-oil flow are shown for both drainage and imbibition experiments. However, only drainage experiment results are shown for liquid-gas flow (water-gas and oil-gas) because imbibition experiments were shown in section 5.2 to have their total flow rates modeled well using the harmonic average. The geometric average is included with the other models, since the geometric average has been seen to fit the liquid-gas flow data well in drainage experiments at low-to-intermediate wetting phase saturations.
From the figures of liquid-liquid flow (water-oil): Fluid mixture viscosities modeled using pipe-flow models are far off from the calculated average viscosity from the data for liquid-liquid flow in porous media. Two models show a significant peak in the mixture viscosity around the middle of the curve which lies at the middle of the reducible water saturation range. These models, namely the Fourar & Bories model and the Beattie & Whalley model, use weighting factors that are calculated from the flow rate measurements of the individual phases. Remember that $\beta$ is the ratio of the non-wetting phase volume flow rate to the total fluid volume flow rate. In other words, the mixture viscosity can only be modeled by knowing the volume flow rates. This is useful in the case where the pressure drop across the porous medium needs to be estimated from preset values of the phases volume flow rates. However, in the opposite case where the fluid volume flow rates are to be estimated for a given pressure drop, the mixture viscosity cannot be modeled without knowing the volume flow rates, which are the unknown values that are to be estimated in the first place.

From the figures of liquid-gas (water-gas / oil-gas) flow in drainage: The Beattie & Whalley model is the most representative model in general of the data of two-phase flow in porous media. However, it still carries the same problem of requiring flow rate information in order to estimate the average mixture viscosity. At wetting phase saturations ranging from the irreducible saturation to about 70% (which is about the middle of the reduced saturation range), the geometric average viscosity is the closest fit to the data among all the models.
1.a. Water-Oil Drainage Experiments

sample 6 WO Drainage Experiments: Pipe-Flow Mixture Viscosity Models

sample 13 WO Drainage Experiments: Pipe-Flow Mixture Viscosity Models

- data
- McAdams
- Cicchilli
- Duckler
- Beatle
- Fourar & Bories
- geometric volume average
Figure 6-1: Two-phase pipe-flow average viscosity models compared to calculated average viscosity from drainage experimental data of water-oil flow in porous media.
1.b. Water-Oil Imbibition Experiments

Sample 6 WO Imbibition Experiments: Pipe-Flow Mixture Viscosity Models

Sample 13 WO Imbibition Experiments: Pipe-Flow Mixture Viscosity Models

- data
- McAdams
- Cicchitti
- Duckler
- Beattie
- Fourar & Bories
- harmonic volume average
Figure 6-2: Two-phase pipe-flow average viscosity models compared to calculated average viscosity from imbibition experimental data of water-oil flow in porous media.
2. Oil-Gas Drainage Experiments

Sample 6 OG Drainage Experiments: Pipe-Flow Mixture Viscosity Models

Sample 13 OG Drainage Experiments: Pipe-Flow Mixture Viscosity Models

Legend:
- data
- McAdams
- Broditch
- Duckler
- Beattie
- Fourar & Bories
- geometric volume average

Wetting phase saturation, \( S_w \)
Figure 6-3: Two-phase pipe-flow average viscosity models compared to calculated average viscosity from drainage experimental data of oil-gas flow in porous media.
Figure 6-4: Two-phase pipe-flow average viscosity models compared to calculated average viscosity from drainage experimental data of water-gas flow in porous media.
6.2 Lockhart-Martinelli Correlations

Lockhart and Martinelli (1949) developed dimensionless correlations between the pressure drop for each phase under two-phase flow in pipes and the pressure drop for each phase under single-phase flow. It has been observed that the fluid pressure drop across the pipe is significantly different than the pressure drop observed in single-phase flow for the same volume flow rate. Their work focused on the flow of liquid and gas, and used air and different liquids (benzene, kerosene, water and different oils) in their experiments. Their main idea is that correlations between the two-phase flow case and the single-phase flow cases should be between the dimensionless parameters: Martinelli Parameter $X$, liquid friction factor $\phi_l$ and gas friction factor $\phi_g$ which are defined as follows:

1. **The gas friction factor, $\phi_g$:**

   \[
   \phi_g^2 = \frac{(\frac{\Delta P}{\Delta L})_{2-ph,g}}{(\frac{\Delta P}{\Delta L})_{1-ph,g}} \tag{6.9}
   \]

   which is the ratio of the pressure drop for the gas phase flowing at volume flow rate $Q_g$ under two-phase flow, to the virtual pressure drop for the gas phase if it was flowing at the same flow rate $Q_g$ but under single-phase flow.

2. **The liquid friction factor, $\phi_l$:**

   \[
   \phi_l^2 = \frac{(\frac{\Delta P}{\Delta L})_{2-ph,l}}{(\frac{\Delta P}{\Delta L})_{1-ph,l}} \tag{6.10}
   \]

   which is the ratio of two-phase pressure drop to the virtual single-phase pressure drop for the same flow rate of the liquid phase.

3. **The Martinelli parameter, $X$:**

   \[
   X = \frac{\phi_g}{\phi_l} = \frac{(\frac{\Delta P}{\Delta L})_{2-ph,g}/(\frac{\Delta P}{\Delta L})_{1-ph,g}}{(\frac{\Delta P}{\Delta L})_{2-ph,l}/(\frac{\Delta P}{\Delta L})_{1-ph,l}} \tag{6.11}
   \]
Since under two-phase flow, the pressure drop for the two phases \( \frac{\Delta P}{\Delta L} \)_{2-ph,g} and \( \frac{\Delta P}{\Delta L} \)_{2-ph,l} are equal, then:

\[
X = \frac{\frac{\Delta P}{\Delta L} \text{l-ph,l}}{\frac{\Delta P}{\Delta L} \text{l-ph,g}}
\]

which is the liquid-to-gas ratio of the single-phase pressure drops. Using Darcy's law to relate the flow rate of each phase to its single-phase pressure drop, the Martinelli parameter can be written as:

\[
X = \frac{\frac{Q_l}{\mu_l}}{\frac{Q_g}{\mu_g}}
\]

The correlations are to be developed between the phases friction factors, \( \phi_g \) and \( \phi_l \), and the Martinelli parameter \( X \). Such correlations have been investigated by Lockhart and Martinelli as well as other researchers (Chisholm, 1967; Fourar and Bories, 1995) for different combinations of the flow regimes of the two phases, namely: laminar-laminar, laminar-turbulent, turbulent-laminar and turbulent-turbulent. Chisholm (1967) derived an analytical expression for the correlations that can apply to any flow regime combination by changing a constant value, \( C \), in the expression below for the liquid friction factor:

\[
\phi_l^2 = 1 + \frac{C}{X} + \frac{1}{X^2}
\]

And so the gas friction factor by definition of the Martinelli parameter is equal to:

\[
\phi_g^2 = \phi_l^2 X = X^2 + CX + 1
\]

In the case of a laminar-laminar flow which is the case in two-phase flow in porous media, the value of the constant \( C \) is equal to 5. We test to see if the correlations found by Chisholm apply on flow rate and pressure data gathered from 2-phase flow experiments in porous rock. We follow the below steps in calculating the necessary
1. In a two-phase flow experiment, measure the volume flow rates for each phase, \( Q_g \) and \( Q_l \), and the corresponding pressure drops \( \frac{\Delta P}{\Delta L} \) for two-phase flow and \( \frac{\Delta P}{\Delta L} \) for single-phase flow.

2. For each measured flow rate \( Q \) of each phase from the two-phase flow experiment, calculate the single-phase pressure drop \( \frac{\Delta P}{\Delta L} \), which is the pressure drop that would be measured if the phase were to flow as a single phase at this flow rate \( Q \).

3. For each phase, take the ratio of the two-phase pressure drop to the single-phase pressure drop for the given range of flow rates \( Q \) measured from the two-phase experiment. This ratio is the function \( \phi \) for each phase.

4. Take the ratio \( X = \frac{\frac{\Delta P}{\Delta L} \text{ liquid}}{\frac{\Delta P}{\Delta L} \text{ gas}} \), which is the liquid-to-gas ratio of the computed single phase pressure drops.

5. Plot \( \phi_g \) and \( \phi_l \) as functions of the Martinelli parameter, \( X \).

Figures 6-5 through 6-8 show the results of comparing the computed Lockhart-Martinelli parameters for the data of two-phase flow in porous media, to the model developed for two-phase flow in pipes by Chisholm. Data from imbibition and drainage experiments are shown for water-oil flow, but only drainage results are shown for liquid-gas experiments (water-gas and oil-gas). The models developed for two-phase flow in pipes resemble the data from drainage two-phase flow in porous media in most cases. This shows that two-phase flow in non-porous media suffers a similar effect to that in porous media. The effect in the Lockhart-Martinelli correlations is represented by the increased pressure-drop in two-phase flow as compared to single-phase flow for a given flow rate. This statement is analogous to the effect observed in two-phase flow experiments in porous media, which is that the flow rates of the individual phases in two-phase flow suffer a deficit compared to single-phase flow rates for a given differential pressure.
1. a. Water-Oil Drainage Experiments

WO Drainage sample 6: Lockhart-Martineilli correlation

\[ \phi_i = \frac{d \phi_i}{d \phi_i_{\text{exp}}} \]

Martinelli parameter \( X = \frac{\phi_i}{\phi_i_{\text{exp}}} \)

WO Drainage sample 13: Lockhart-Martineilli correlation

\[ \phi_i = \frac{d \phi_i}{d \phi_i_{\text{exp}}} \]

Martinelli parameter \( X = \frac{\phi_i}{\phi_i_{\text{exp}}} \)
Figure 6-5: Computed wetting and nonwetting friction factors $\phi$ versus the Martinelli Parameter $X$ from data of two-phase flow in porous media for water and oil in drainage. The data is compared to the model developed from two-phase flow in pipes by Chisholm (1967).
1.b. Water-Oil Imbibition Experiments

WO Imbibition Sample 6: Lockhart-Martinelli Correlations

$\phi = \frac{\mu_p \phi_h \phi_o}{\mu} \left( \frac{\mu_p}{\mu_o} \right)
$

WO Imbibition Sample 13: Lockhart-Martinelli Correlations

$\phi = \frac{\mu_p \phi_h \phi_o}{\mu} \left( \frac{\mu_p}{\mu_o} \right)
$

Martinelli parameter $X = \frac{\phi_h \phi_o \mu_p}{\phi \mu}$
Figure 6-6: Computed wetting and nonwetting friction factors $\phi$ versus the Martinelli Parameter $X$ from data of two-phase flow in porous media for water and oil in imbibition. The data is compared to the model developed from two-phase flow in pipes by Chisholm (1967).
2. Oil-Gas Drainage Experiments

**OG Drainage sample 6: Lockhart-Martinelli correlation**

\[ \phi = \frac{\text{oil mass flow rate}}{\text{gas mass flow rate}} \]

\[ \phi = \left( \frac{1}{\phi_{\text{wetting}}} - 1 \right) \left( \frac{1}{\phi_{\text{non-wetting}}} - 1 \right)^{0.5} \]

\[ \phi_{\text{wetting}} = \left( 1 + 5/X + 1/X^2 \right)^{0.5} \]

\[ \phi_{\text{non-wetting}} = \left( 1 + 5X + X^2 \right)^{0.5} \]

**OG Drainage sample 13: Lockhart-Martinelli correlation**

\[ \phi = \frac{\text{oil mass flow rate}}{\text{gas mass flow rate}} \]

\[ \phi = \left( \frac{1}{\phi_{\text{wetting}}} - 1 \right) \left( \frac{1}{\phi_{\text{non-wetting}}} - 1 \right)^{0.5} \]

\[ \phi_{\text{wetting}} = \left( 1 + 5/X + 1/X^2 \right)^{0.5} \]

\[ \phi_{\text{non-wetting}} = \left( 1 + 5X + X^2 \right)^{0.5} \]
OG Drainage sample 14: Lockhart-Martinelli correlation

Figure 6-7: Computed wetting and nonwetting friction factors $\phi$ versus the Martinelli Parameter $X$ from data of two-phase flow in porous media for oil and gas in drainage. The data is compared to the model developed from two-phase flow in pipes by Chisholm (1967).
3. Water-Gas Drainage Experiments

Figure 6-8: Computed wetting and nonwetting friction factors $\phi$ versus the Martinelli Parameter $X$ from data of two-phase flow in porous media for water and gas in drainage. The data is compared to the model developed from two-phase flow in pipes by Chisholm (1967).
6.3 Summary

Average viscosity models from two-phase fluid flow in pipes were tested for two-phase flow in porous media using the experimental data from Oak et al. Liquid-liquid flow still fails to be represented even remotely by any average viscosity model. Effective average viscosity at the middle of the reduced fluid saturation range reaches extremely high values, up to 4 times as much as the oil viscosity in most cases. Two models from the pipe-flow literature (Beattie & Whalley, Fourar & Bories) display a similar feature of increased average viscosity at intermediate, reduced wetting saturations. The modeled average viscosity peak however is significantly lower than the actual peak seen in the data. Also, the problem with such models is the need for flow rate information in order to model the average viscosity. This is useful in the case of predicting the pressure drop for given fluid flow rates, but is unproductive in the case of predicting flow rates for a given pressure drop.

Average viscosity models from pipe-flow literature fairly model liquid-gas mixtures. The Beattie & Whalley model manages to represent this type flow at intermediate reduced saturations. At low, reduced wetting saturations, the geometric average fits the data more than any other model. At high, reduced wetting saturations, the effective average viscosity from the data portrays a similar peak effect seen in the liquid-liquid case, where effective average viscosities are higher than either fluid phase viscosity.

Such exaggerated, effective average viscosities seen in the liquid-liquid and liquid-gas cases at certain saturation regions are computed from the data, which correspond to extremely low total fluid flow rates. If such low flow rate values are caused by processes unrelated to viscous flow in the porous medium, then the data would reflect inaccurate average viscosity values. This is because average viscosities computed from the data are obtained by using Darcy's law to relate total flow rate to the mixture viscosity only without taking into account other factors that can be
responsible for the low fluid flow in porous media.

Lockhart-Martinelli correlations were investigated next, which describe through dimensionless parameters the increased pressure drop in the case of two-phase flow in pipes as compared to the case of single-phase flow, for the same value of volume flow rate. Different models of such correlations have been developed by many researchers, and Chischolm’s generalized model is taken here as the reference Lockhart-Martinelli correlation. Chischolm’s model fits many of the liquid-liquid flow experiments in porous media in drainage only, and most of the liquid-gas flow experiments in drainage. The similarity between this model from two-phase flow in pipes and results from two-phase flow in porous media shows that the two types of flow experience the same effect of increased pressure drop in the two-phase case compared to the single-phase case.
Chapter 7

Conclusions

7.1 Summary

An alternate approach to two-phase flow in porous media has been investigated. The approach is based on the concept of an average viscosity of a two-phase fluid mixture flowing in a porous medium. By formulating the idealized two-phase flow, we found that the arithmetic and harmonic averages weighted by fluid saturations constitute the theoretical bounds to the average viscosity of a two-phase fluid mixture. These two averages as well as the geometric average, which is an intermediate average between the two, were tested on data from Oak et al. (1990) of two-phase flow experiments on three Berea sandstone cores. The testing was done by comparing the average viscosity models to the effective average viscosity computed from the experimental data by applying Darcy’s law to the fluid mixture (see page 69). The results show strong similarities among the different core samples on which the two-phase flow experiments were conducted, but show striking variation between (1) liquid-liquid flow and liquid-gas flow, and between (2) drainage and imbibition.

Liquid-liquid vs. Liquid-gas Flow While the three mathematical average viscosities form the bounds to most of the data in liquid-gas flow, liquid-liquid flow data show an entirely different trend in the effective average viscosity (Figure 7-1).
The data lies far from all three averages, peaking at values much larger (up to $4\times$) than either fluid-phase viscosity.

Figure 7-1: An example of the average viscosity for liquid-liquid flow (left) versus liquid-gas flow (right), showing the different average viscosity trends between the two cases.

**Drainage vs. Imbibition**  Results from two-phase flow carried out with the same fluid pair on the same core sample showed significant variation between drainage and imbibition experiments. An example of such variation is the difference between results in liquid-gas flow (Figure 7-2). The harmonic average weighted by the reduced fluid saturations represents the average viscosity of liquid-gas flow in imbibition experiments only, while results from drainage experiments show a different average viscosity that is not represented by the arithmetic, harmonic nor geometric average.

**Two-phase Flow in Pipes**  Testing average viscosity models from the literature for two-phase flow in pipes, it is apparent that models that come close to representing the average viscosity of two-phase flow in porous media must include information about the individual phases' flow rates. Beattie & Whalley's model which is the most representative model among the other models for liquid-liquid and liquid-gas flow, uses the parameter $\beta$ which is ratio of the flow rate of one phase to the total flow rate of the fluid mixture. Figure 7-3 is an example for liquid-liquid (left) and liquid-gas (right) flow showing Beattie Whalley's model (purple) plotted
Figure 7-2: An example of the different average viscosity trends for liquid-gas flow between drainage (left) and imbibition (right). Against other pipe-flow models and the effective average viscosity computed from the data for two-phase flow in porous media.

Figure 7-3: Beattie & Whalley’s model for the average viscosity of two-phase fluid mixtures flowing in pipes is the closest model to the effective average viscosity of two-phase flow in porous media.

### 7.2 Relative Permeability vs. Average Viscosity

While the relative permeability approach is not based on accurate physical concepts, it has the advantage of predicting the flow rate of each phase at any saturation level by simply using the corresponding relative permeability value in Darcy’s law. However, obtaining the appropriate value is not trivial, for it needs to be measured from a two-phase flow experiment involving an intricate experimental set-up. Also,
for accurate flow rate predictions for a given porous medium such as a specific oil reservoir rock, the relative permeability test needs to be carried out on a core sample from the same reservoir. If the type of the relative permeability test is steady-state, obtaining the relative permeability values over the range of fluid saturations requires a long period of time that can last for weeks.

In the case of liquid-gas flow in imbibition, the required information for modeling the average viscosity of the two-phase fluid mixture is the fluid viscosities, the irreducible liquid (wetting) saturation and the residual gas (nonwetting) saturation. The latter two quantities are needed to compute the reduced saturations corresponding to the regular saturations. The total flow rate of the two phases can then be modeled by applying Darcy's law to the fluid mixture as a whole and substituting the harmonic average viscosity for the equation's viscosity term.

Table 7.1 compares the relative permeability approach to the average viscosity approach in terms of the necessary input quantities, intermediate calculations (if any), application of Darcy's law and output quantities. The average viscosity approach is still not fully developed and needs extensive laboratory investigation. Extracting the individual phases flow rates from the total flow rates is the crucial step that needs to be explored, and if proven successful, will make the average viscosity approach an alternative approach to relative permeability for two-phase flow problems in porous media.

### 7.3 Limitations to the Average Viscosity Approach

It has been shown from the dataset on the three Berea sandstone cores that the harmonic average viscosity applies only to liquid-gas flow in imbibition. For the other two cases of two-phase flow involving two immiscible liquids or liquid-gas in drainage, the average viscosity model using any of the three common mathematical averages does not represent the data-inferred average viscosity. For these cases where a simple mathematical average fails to represent the actual average viscosity,
Table 7.1: Relative Permeability vs. Average Viscosity Approach

<table>
<thead>
<tr>
<th>Input</th>
<th>Relative Permeability</th>
<th>Average Viscosity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1. “Absolute” permeability of the porous medium, ( k )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. Porous medium cross-sectional area ( A ), length ( L )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Pressure drop ( \Delta P ) across the medium’s length</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. Fluid viscosities: ( \mu_w, \mu_{nw} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Relative Permeabilities: ( k_w, k_{nw} )</td>
<td>3. Irreducible wetting-phase &amp; residual nonwetting-phase saturations: ( S_{nw,r}, S_{nw,r} )</td>
</tr>
<tr>
<td>Intermediate step</td>
<td>-</td>
<td>1. calculate reduced saturations ( S_{w,red}, S_{nw,red} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. calculate fluid mixture average viscosity ( \mu_{avg} )</td>
</tr>
<tr>
<td>Darcy’s law</td>
<td>( Q_i = \frac{k_i}{\mu_i} A \frac{\Delta P}{L} ; i=w,nw )</td>
<td>( Q_{tot} = \frac{k_{w}}{\mu_{avg}} A \frac{\Delta P}{L} )</td>
</tr>
<tr>
<td>Output</td>
<td>1. Individual flow rates: ( Q_w, Q_{nw} )</td>
<td>1. Total flow rate, ( Q_{tot} )</td>
</tr>
</tbody>
</table>

a model from the literature of two-phase flow in pipes that incorporates flow rate information comes closer to representing the data. This is a probable indicator that fluid viscosities and fluid saturations do not suffice as the only parameters in the appropriate viscosity average of a two-phase fluid mixture. The actual averaging functions might need to be a non-linear function of fluid saturations, pressure drop, flow rates and/or other parameters not captured by in Darcy’s law. Since Darcy’s law describes fluid flow due to viscous forces, other effects related to flow in porous media that would be absent in pipes, such as wettability and capillarity, might be playing a role in the macroscale behavior of two-phase flow in porous media and thus should be accounted for in the average viscosity model.

Seeing the varying results between drainage and imbibition experiments, liquid-liquid and liquid-gas flow and in some cases between different runs of the same exact experiment, the major challenge facing a research study similar to this is clearly related to guaranteeing high-quality experimental data. In order to test a
new concept such as that of an average viscosity of a two-phase fluid mixture in porous media, that can be modeled using known information about the flow system, the two-phase flow data must be made sure to be appropriate, precise and accurate enough to be applicable for testing purposes. For our specific case of an average viscosity modeled with only fluid saturations and viscosities, the experimental data must correspond to two-phase flow in which viscous forces are the dominating forces. Other phenomena related to fluid flow in porous media must be negligible in the specific flow experiments on which the new theory is being tested. For example, capillarity effects can be assumed to be absent by choosing the appropriate pressure drops that would yield high enough flow rates for the specific experimental fluids, thus situating the flow experiment in the region of low capillary effects. Another source of errors that must be taken care of in two-phase flow experiments is flow rate measurements. In many cases, flow rates reach low values that are susceptible to large instrumental errors. This in turn affects relative permeability results and does not ensure suitable testing of the average viscosity concept.

Realizing the difficulty in collecting experimental data from one experiment of two-phase flow in porous media, let alone multiple experiments with varying fluid pairs, porous media samples, and saturation paths (drainage vs. imbibition), the second major challenge concerns the amount of precise and accurate data. Validating the average viscosity concept requires collecting enough data from a representative sample size of two-phase flow experiments. Collecting enough experimental data under controlled and known experimental conditions using the steady-state method for example requires several months at the least and can span a few years depending on the number of experiment, flow rates, and permeability of the porous media. The two challenges related to experimental data thus outline the future work needed to carry on this study.
7.4 Future Work

The average viscosity approach needs experimental testing by collecting new data from appropriate, two-phase flow experiments. Suitable flow rates (or pressure drops) must be chosen such that viscous forces are the dominating forces in the two-phase flow system. If this study is to be continued, the main set of two-phase flow experiments would compare the results from flowing different fluid pairs with different viscosity ratios, since the appropriate average viscosity must apply to different fluid viscosity pairs in the same manner.

Changing the fluid pairs however is tricky, because the fluid properties in relation to each other (such as interfacial tension) and in relation to the porous medium (wettability) must be held constant, in order to study the effect of viscosity only on the two-phase flow. Comparing results of effective average viscosity in liquid-liquid (water-oil) flow to liquid-gas (water/oil - gas) flow for example, it is apparent that the two types of fluid pairs behave very differently. However, it is unknown what properties of the flow system (other than fluid viscosity) have changed drastically by changing the specific fluid pairs from one to the other, which have caused the major change in the calculated effective viscosity.

The experimental procedure of future two-phase tests must also be fixed for the different fluid-pair experiments, mainly the type of flow experiment (drainage vs. imbibition) and the range of fluid flow rates used. By measuring the pressure drop across the porous medium and the produced fluids’ volume flow rates, the effective average viscosity is computed by applying Darcy’s law for single-phase flow to the two-phase fluid mixture. The simple mathematical averages should be tested against the average viscosities computed from the experimental results as a first step. If these models are still unsatisfying even under the appropriate experimental conditions and fixed system properties, non-linear models may be the plausible candidates for an average viscosity.

From relative permeability test data, changing the two-phase flow experiment from
drainage to imbibition is seen to cause a significant change to the flow behavior. For example, the harmonic viscosity average applies to the imbibition case but not the drainage case for liquid-gas flow. The categorization of a two-phase flow experiment into drainage or imbibition is by definition related to the difference between the wettabilities of two phases to the medium: drainage is the process of increasing nonwetting phase saturation while imbibition is the process of increasing the wetting phase. Therefore, investigating the effect of varying the degree of wettability on the difference in results between drainage and imbibition can answer some questions regarding the hysteresis effect on two-phase flow and the applicability of the average viscosity method.


H. Darcy. Les fontaines publiques de la ville de dijon [the public fountains of the city of dijon], 1856.


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