Load Bearing Interface Design for a Pan-Tilt Mechanism for Severe Marine Environments

by

Michael John Beautyman, Jr.

Submitted to the Department of Mechanical Engineering
in partial fulfillment of the requirements for the degrees of

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and

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See Appendix E - Drawings
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Abstract

The Naval Research Laboratory (NRL) requested the design of a two-axis gimbal device for the shipboard support of a sensor payload. Previous design efforts presented a low-mass two-axis (pan and tilt) machine. Vibration and shock testing induced failure in the interface between the payload and the tilt shaft, through which the control cabling connected to the sensors, taking the system out of service and creating a hazard for Sailors. This thesis proposes a tapered, hollowed shaft and flange interface connected by an interference fit that is preloaded and retained by a single hollowed bolt for ease of maintenance at sea.

This simplified design is a departure from existing rotary tapered interfaces, such as seen in machine tooling, and focuses on connecting massive payloads to their actuators when subjected to severe loading. This design is uniquely suited to withstand large bending moments and loading as demanded by military standards for shock. A custom rig was designed and constructed to subject reduced-scale designs to military standard environmental testing for shock in the laboratory. These test results were analyzed using moving average filtering to develop confidence intervals to validate the design mathematics. A full-scale prototype was manufactured and subjected to shock testing and analysis. The design exceeded all requirements and is ready for immediate integration into the gimbal. This research also revealed the potential for tapered interfaces to connect massive payloads to their actuators in industry.

Thesis Supervisor: Alexander H. Slocum
Title: Pappalardo Professor of Mechanical Engineering
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Chapter 1

Introduction

This design thesis was inspired by a system design problem put forth by the sponsor, the Naval Research Laboratory (NRL). In 2014, the NRL asked the Precision Engineering Research Laboratory (PERG) to "design, build, and test a two-axis pan tilt mechanism for shipboard use to support an NRL project" on which a "key feature is that the elevation mechanism’s axis of rotation will be based on a large diameter bearing such that the payload be placed at and project through the elevation mechanism." [Slocum, 2014] The resulting directional device machine is meant to focus in a specific direction along the azimuth and elevation as commanded by precision motor controllers. The machine must be design for exposure to green water (seawater which washes over the deck of a ship), wind loading, vibrations, and shock. The NRL is developing the payload, for which specifications are classified. All discussions of the payload in this thesis refer only to geometry and mass without detail of capability or purpose.

From 2015-2016, Nathan Mills brought the project through prototyping and initial testing. This testing revealed component failures that required new designs in order to meet NRL specifications. Critical among these design failures was the interface
joining the payloads to the horizontal shaft actuator (tilt shaft) that controls the rotation of the payloads to their zenith.

This thesis details the research, design, prototyping, and testing of an improved interface between the payloads and the tilt shaft, and its possible applicability beyond the mechanism specified by the NRL. This thesis also discusses the design of rigs to test the interface to the specified military standards within the laboratory. This thesis is unclassified.

The design process began with a review of literature on mechanical interfaces in robust systems, found in chapter 2. This ranged from patents for aviation components to discussions of high-speed tooling machinery. This research provided a baseline for the development of innovative interface designs and provided exposure to the existing methods for connecting payloads to shafts on rotating systems. It also ensured that no design effort would be duplicative or infringe on existing design patents.

From this effort came several candidate design solutions, detailed in section 3.1. These were analyzed for compliance to the NRL specifications as well as for simplicity, part count, maintainability, and weight. Section 3.2 discusses the design process.
and the development of mathematical models to facilitate rapid analysis of different designs, and section 3.5 reviews the manufacturing process for the reduced-scale model.

Sections 3.4 and 3.6 detail the design and manufacture of shock tests intended for a reduced-scale prototype in the laboratory and for a full-scale model. Due to the large mass of the actual payloads, different testing rigs were required for the reduced-scale and full-scale interfaces. This chapter also discusses the characterization of these rigs in order to control each experiment.

Chapter 4 presents the results and analysis of those shock tests, with observations about features of the test rigs and interfaces that influenced the data. This chapter also discusses the applicability and integration of this design to the NRL gimbal, and the potential applicability of this interface design for other systems connecting massive payloads to actuators. It concludes with a discussion of future work.

1.1 Requirements

1.1.1 Sponsor Requirements

Table 1.1 is a summary of the sponsor requirements pertinent to this research. Of particular importance to this thesis are the angular accelerations, payload size and mass, wind loading, wave loading, vibration, and shock requirements. The geometry of the payload was approximated from a mock-up model provided by the sponsor. This allowed for fairly accurate assumptions in calculating moments of inertia, distances, and other geometric quantities. The angular accelerations, wind and wave loading values, and the vibration and shock standards were used to calculate the maximum forces and moments on the machine.
Table 1.1: NRL requirements

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Threshold</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max angular speed ( [deg/s] )</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>Max angular speed ( [deg/s^2] )</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>Payload mass total ( [lbm] )</td>
<td>130</td>
<td>170</td>
</tr>
<tr>
<td>Payload dimensions each ( [in] )</td>
<td>9 x 7.5 x 6</td>
<td></td>
</tr>
<tr>
<td>Green water loading</td>
<td>6 ([psi])</td>
<td></td>
</tr>
<tr>
<td>Wind loading without damage</td>
<td>115 ([kts]) sustained, 120 ([kts]) gust</td>
<td></td>
</tr>
<tr>
<td>Vibration Resistance to MIL-STD-167-1A</td>
<td>4-15Hz: table amplitude 0.030in [in]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>16-25Hz: table amplitude 0.020in [in]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>26-33Hz: table amplitude 0.010in [in]</td>
<td></td>
</tr>
<tr>
<td>Shock Resistance</td>
<td>MIL-S-901D</td>
<td></td>
</tr>
<tr>
<td>Cable Bundle</td>
<td>20 ([mm]) diameter</td>
<td></td>
</tr>
</tbody>
</table>

1.1.2 Derived Requirements

My experience at sea as a Naval Officer informed several requirements, as did the lean principle of poka-yoke, or the reduction of chances for operator error. Simplicity was critical for assembly and disassembly on a moving vessel in a variety of conditions. Simplicity also informs maintainability - the ease with which Sailors could work on the interface while at sea. Ships spend the majority of their time far from their supply depots, and have limited cargo space for spare and replacement parts. For this reason, a reduced part count - another component of simplicity - was essential. All design efforts favored manufacturability where possible in order to reduce cost.

The interface design was developed from a mathematical model that predicted the ability of the interface to hold the payloads through all requirements. In order to validate this mathematical model, I designed and constructed a shock test in the laboratory. The full-scale model also required testing. Both of these tests had to impart large shock loads, meaning they were constructed to withstand those loads while providing a stable platform for measurement. Additionally, the accelerometer sensors used were bandwidth limited, meaning that they had a specific frequency
range over which they could operate and collect data. This meant designing the tests to control for the time interval of the shock test impulse to ensure the accelerometers would gather multiple data points across the peak acceleration. Chapter 3 discusses these test requirements in detail.

1.2 Characterizing the Problem

In the first iteration of the gimbal [Mills, 2016], the payloads connected to the tilt shaft via a circular 6-bolt pattern. These bolts were driven into the flat of the circular end face of the cylindrical shaft. During vibration and shock testing, this interface failed when the bolts sheared, causing the payloads to partially sever from the tilt shaft.

The existing geometry of the gimbal tilt housing constrained the general solutions for redesigning the tilt shaft. For example, the bearings on which the tilt shaft sit in the tilt housing defined the maximum shaft diameter and thus the maximum diameter of the tapered interface. These constraints are enumerated in section 3.7. The challenge was to redesign the shaft and interfacing components to withstand the required forces and moments while maintaining simplicity, manufacturability,
maintainability, and a low part count. In addition I avoided adding mass to the system and to reduced the number of parts required.

1.3 Executive Summary

The Naval Research Laboratory (NRL) requested the design of a two-axis gimbal device for the shipboard support of a sensor payload. Previous design efforts (Mills, 2016) presented a low-mass two-axis (pan and tilt) machine. Vibration and shock testing induced failure in the interface between the payload and the tilt shaft, through which the control cabling connected to the sensors, taking the system out of service and creating a hazard for Sailors. This thesis proposes a tapered, hollowed shaft and flange interface connected by an interference fit that is preloaded and retained by a single hollowed bolt for ease of maintenance at sea. This simplified design is a departure from existing rotary tapered interfaces, such as seen in machine tooling, and focuses on connecting massive payloads to their actuators when subjected to severe loading. This design is uniquely suited to withstand large bending moments and loading as demanded by military standards for shock.

The first iteration of the interface was a circular bolted pattern as pictured, which failed in shear during vibration and shock testing of the full gimbal. The gimbal was required to withstand green water loading, wind loading, select MIL-STD-167-1A vibrations, and MIL-S-901D shock. This interface, which moves the payloads, also needed to provide enough torque to do so without slipping circumferentially or axially. The shock requirement is the most demanding of these requirements, so I focused on that noting that the other requirements would be met as a result. I also derived several requirements from my own experience at sea and from the lean principle of poka-yoke, or the reduction of chances for operator error. Manufacturability, simplicity and
repeatability in assembly and disassembly, and a low part count were critical.

To meet these requirements I considered and rejected several potential solutions: a larger shaft and bolt pattern (bearing restricted), a helicopter-style rotary joint (too complex), a wind turbine joint (too many parts), and others. The optimal solution was a tapered interference fit, which facilitated repeated removal and replacement of the connecting flange while still providing a substantial retaining force and resisting shock-induced bending moments. Utilizing a commercially available flat head screw to provide the pre-load and retention reduced cost and part count while keeping the design familiar to the maintaining Sailor.

The design of the interface required examining existing tapers, such as those used for high-speed tooling, in automotive work, and in fastening ship propellers to their shafts. From these areas I obtained standards for tapers that could guide my design, though this use was unique in featuring multi-directional forces and moments and in seeking a reduced footprint. Solving by hand at first, I developed an equation for the ability of the taper to resist a bending moment due to shock. In the equation below, M represents that resisting moment for 90 degrees of the taper revolved around the x-axis, for a total resistance of four times that value.

The design required readily available shock testing, as the formal testing previously conducted was costly, time-intensive, and not local. I researched and rejected explosive testing (dangerous, difficult to setup, difficult to repeat) and simple drop testing (generally limited by bandwidth of available accelerometers) before settling on impact testing. Inspired by Charpy testing machines, I developed a mathematical spreadsheet that calculated the acceleration experienced by a mass on a fully characterized physical pendulum released from a given angle. I specified, designed, machined, and assembled a pendulum rig and components that contacted a large compression spring at the bottom of its arc, creating a known acceleration as a function of the spring constant.
and geometric and mass properties.

The pendulum rig and the predicted accelerations were validated using a test mass bolted to the pendulum arm, on which two accelerometers were attached. The pendulum rig was bolted in place and stiffened to ensure the accelerometers did not get poor data from movement of the system. Even so, the data collected was noisy from vibrations in the pendulum arm and the sensor mount. After adding a damper to the pendulum arm, I passed all results through a smoothing filter to account for the outlying data points. This filter used a moving mean. To determine the window over which the mean would be calculated, I plotted the sum of absolute differences between the smoothed and noisy data for different window sizes and identified the knee in the curve, noting that the maximum acceptable window was equivalent to the number of data points collected during the total impulse time. These results were then compared to the predicted values to develop confidence intervals for the test data.

After thus validating the pendulum, I had a reduced scale taper shaft and flange manufactured for testing. The pre-load was derived from the Propeller Installation Calculations for US Navy Ships [Shepstone, 2005], while the rest of the design incorporated other research and deviations from the standards surrounding tapers. The pre-load was designed to “fail” when subjected to a 20 g-force acceleration, meaning that it would come loose from the shaft. The flanges were pressed onto the shaft, and the bolts partly backed out to permit movement while securing the flanges from leaving the test rig entirely. As predicted, the model came loose at 20 g-force.

After validating the design code mathematics, I developed the full-scale model. This model was designed to meet the existing tilt shaft housing parameters, and thus was partially geometrically defined, such as in diameter where the bearings sit. Taking advantage of the additional resistance afforded by the hollowed bolt, and the ability
to increase the pre-load if desired, I reduced the engagement length of the interface to decrease the footprint of the gimbal. The full-scale tilt shaft was manufactured partially solid in the center to permit shock testing without collapse or other damage.

The full scale model was too massive for testing on the pendulum rig and required a drop test. For this, I developed a simply-supported beam that acted as a spring in series with the tilt shaft "beam," from which I could predict the acceleration when dropped from a given height. The intent of this test was to prove 60 g-force capability and then continue upwards until unable due to system failure or inability to test further. The tilt shaft and payload system was rigged to a shop crane using a tumble hitch, a stable quick-release knot used to support heavy loads without jamming. Neither the interface nor the simply supported beam failed during testing, and I was forced to conclude when I could no longer lift the shaft and payloads higher.

The data collected from the full-scale drop test was plotted and found to be significantly less noisy than the pendulum rig data. The impulse time contained approximately 26 data points, and the peaks of the 500 g-force accelerometer results comprised upwards of 17 points, indicating reliable maximum value data. For comparison, I still plotted a moving average of the data using the window size dictated by the sum of absolute differences plot.

This tapered interface successfully met the requirements for the gimbal and may be utilized in the prototype machine. It reduces the part count and the time required for maintenance and repair. It could be further reduced in size, but that would require an increase in pre-load and thus application torque. This design proves that tapers are a feasible method to connect massive loads to their actuators, and can use different geometry than established in automotive and marine industries. There is potential to use this technology with large actuators in industry. Future work on this project could include making a graphic user interface for the code such that a designer could
input some parameters and have a taper design iterated from the code.
Chapter 2

Literature Review

2.1 Introduction

The ubiquity of shafting interfaces in machine tooling and the marine industry has generated a substantial amount of analysis and research into their design and development. Although the literature covers a wide variety of analysis, this review will focus on three major themes that emerged throughout the research. These themes are: types of interfaces, interface mathematics, design, and dimensioning, and interface manufacturing. In addition, this literature review examines shock testing mechanical systems as required by this thesis. From this review, I developed potential interface designs and parameters to address the gimbal interface failure. While there were many resources relating interface design to torque, very little was available that explicitly discussed bending moment resistance. In fact, bending moments were listed as a tertiary function of tapered interfaces [Bosmanns and Tu, 2002].
2.2 Types of Interfaces

There are a variety of shafting interfaces used, mostly either bolted or tapered. Tapered interfaces range from the German HSK to the American CAT [Agapiou, 2005]. As the popularity of tapered tooling interfaces increased, designs such as 7/24 [Bosmanns and Tu, 2002] and KM [Lewis, 1999] were developed. Yet another article discussed the Japanese-designed BT shank and the attempts at standardizing taper designs geographically, as the author predicted the decline of steep-taper systems in the face of cheaper, quality standard tapers [Kocherovsky, 2000]. All of these tapered interfaces are designed for transmitting torque in rotating systems. While some of these systems build in protection through intentional slip, that is not a desired feature for the gimbal tilt shaft interface. These articles did not explore the bending moment resistance of these interface designs.

2.3 Interface Mathematics, Design, and Dimensioning

2.3.1 Mathematics

In addition to presenting the existing interfaces, the literature explored the mathematical models used to create resilient designs for specific purposes. While presenting a potential wind turbine joint design, one paper offered a way to assess the effects of the interference fit, such as the stress and displacement on the hub connecting the load to the cylindrical shaft [Kang et al., 2015]. A structural member can experience axial, torsion, and bending stresses and strains simultaneously. This combined loading can be determined using superposition [Vable, 2002]. The US Navy has sought solutions for securing propellers to tapered shafts. This can be solved using the required thrust and the geometry of the shaft and propeller hub. This provides the interface pressure
and the required push-on force [Shepstone, 2005]. For the purpose of the gimbal tilt shaft interface, this was invaluable information for designing the axial loading screw.

2.3.2 Design

Equally important to the design process as the equations surrounding the design were the lessons on critical features of shafting interfaces. Tapered interfaces are favorable to cylindrical because they are able to be re-tensioned, are detachable, do not significantly weaken the shaft, and are great for centering. Tapers ratios smaller than 1:10 in particular are detachable only with difficulty [Bosch, 2004], rendering them appropriate for a system that will be occasionally disassembled for maintenance. In addition to being optimal for joint centering, tapered interfaces can be most simply and effectively preloaded using a thread, such as the single large screw ultimately used in this design [Creitaru and Grigore, 2011]. Despite these advantages, the stiffest shaft tapers require end contact with the flange, for which the precision requirement is critical [Bossman and Tu, 2002].

2.3.3 Dimensioning and Components

While the marine industry has established a standard for long engagement lengths [DNV, 2015, LR, 1982], research shows that longer shaft lengths lead to more vibration [Bashir Asdaque and Behera, 2014]. Combined with the requirement to maintain a small footprint for the gimbal, this suggested that I explore decreasing the size of the taper engagement length. In machining, the tooling structure - the tapered connection - is the weakest link, underscoring the importance of designing a stiff joint [Agapiou, 2005]. Again the axial load from a threaded fastener rose to the top of possible ways to address this design need. Good taper designs are relatively minimal, generally
including three major couplings [Bossmanns and Tu, 2002].

2.4 Interface Manufacturing

It was noted that "improving the stress distribution of the hub is an effective approach to strengthen" a connection [Ling Xiong et al., 2013], suggesting that the design be manufactured to a precision finish and fit. Furthermore, "if precise tolerances are not achieved on the taper and face, both on the male and female taper forms, there can be a negative impact on the performance of the connection." [Hanna et al., 2002]. While press-fit connections can halve the endurance limit of the shaft material, a rolled interface increases endurance [Peterson and Wahl, ] . This dictated the manufacturing process to ensure and resilient and robust design.

2.5 Shock Testing

Shock tests are rapid events requiring extensive planning prior to execution. Some are more intensive than others, such as the multiple contact drop test designed to mimic the stresses of dropping small objects like mobile phones [Goyal and Buratynski, 2000]. Even the most simple drop test must be carefully setup. Accelerometers are best mounted with screws and must be chosen with due consideration to the bandwidth limitations of the sensor. Even dropping a 10 gram sensor from 1 m high can result in 30,000 g-forces [Endevco, 2016]. Explosive tests, while realistic, are dangerous, complex to setup, and risk limiting the test sample size to one [Schauer, 1962].
2.6 Conclusion

The literature review revealed best practices for the design of a torque-transmitting tapered interface, and established a tapered interface as the best option for the gimbal tilt shaft interface to the payloads. There was little information available on bending moments for these tapers, which were focused largely on small interfaces for tooling and not on connecting massive payloads to bi-directional actuators. Providing more information on bending moments for massive payload interfaces became a focal point for this thesis.
Chapter 3

Methods

As noted in Table 1.1, the tilt shaft and payload interface components must transmit torque for elevation positioning, provide axial retention against centripetal forces during azimuth positioning, vibration, or shock loading parallel to the tilt shaft, and resist the bending moment from loading the payloads orthogonally to the tilt shaft by water, wind, vibration, or shock. After observing the original interface fail during testing, Mills noted in his thesis:

The most destructive forces may be externally imposed due to wave impact or inertial due to shock. Both are expected to be exacerbated as payloads grow in size and weight. The machine . . . began to fail at the relatively benign shock level of 20g. [Mills, 2016]

Of those external forces, shock loading per MIL-S-901D was calculated to be the largest by an order of magnitude. Therefore the bending moment due to shock loading became the primary design concern, as resisting that moment would ensure the interface exceeded the other requirements. As noted in Chapter 2, most shaft to payload interface literature has been dedicated to torque, as in high-speed machining
and automotive design, suggesting that this design effort would be relatively unique.

In addition, this design would only be successful if simple to assemble and maintain on a ship at sea. Keeping the design intuitive was critical.

3.1 Analysis of Alternatives: Interface

The first step was to identify different means for connecting payloads to shafts. The literature review discussed in Chapter 2 identified or inspired a field of torque-transferring designs found in wind turbine construction, railroad cars, helicopter rotary wing attachments, and high-speed tooling for machinery. These ranged from cylindrical interference fits, to bolted features, to multi-part taper systems to establish the connecting friction forces.

Larger Shaft and Bolt Pattern The diameter of the shaft was defined by the bearings on which it rides in the tilt housing, holding it to a maximum 50 mm. A bolted feature mimicked that of the original design which failed shock testing, and was eliminated for predicted failure in shear stress.

Welded Flanges While this solution would offer a very strong connection between the payload flanges and the shaft, it would require a complete redesign of the much more complex tilt housing in order to split the housing and install the shaft. Installation from one end would not longer be possible.

Threaded Tilt Shaft A threaded shaft would allow the payload interface to be screwed on and off. However, such a design could easily be misaligned given the difficult of controlling the end position of a threaded fastener. This design would also induce a high stress concentration at the root of the internal thread on the payload
flange. This would require a removable alignment pin, which - along with the threads themselves - would present an entry point for salt water.

**Cylindrical Interference Fit** These are attached by either a press fit, requiring an arbor press or similar machinery, or a shrink fit, requiring an intensive heating operation. As this pan-tilt system is meant to be maintained - including possible assembly and disassembly - by Sailors on a ship at sea, such a design was either not possible or not practical. The repeatability of a cylindrical press fit was also a concern, as misalignment is easy and can cause galling on the surfaces of the mating components.

It became clear that a tapered interface would better meet the functional requirements. However, existing designs were overly complex and high part-count.

**Helicopter Rotary Joint** The literature review identified a patent for a coupling flange system for a hollow shaft connecting a helicopter main rotor and tail rotor. This design used five components that connected a cylinder to a flange via an expanding tapered element internal to the shaft cylinder. However, this requires numerous parts that would be complex to manufacture, a tedious assembly, and provided no means of repeatably aligning the two interfaces of the tilt shaft. Additionally the massive nut used to tighten the system would induce high stress concentrations, and the nut could not be backed off while maintaining pressure.

**Wind Turbine Joint** An article on transmitting torque for a wind turbine proposed using a two-piece tapered collar on a cylindrical shaft, tightened by a bolted flange.
It featured a tapered ring, a split tapered ring, and a flange that attached externally to the shaft cylinder. However, it was secured by a twelve bolt circular pattern, making it a laborious assembly and disassembly process even before connecting the payloads. It offered no features for repeatable alignment port and starboard.

**High-Speed Tooling** This space of machine design has long relied on tapers to securely and precisely hold tool heads, using technologies such as HSK, BT, CAT, and KM [Lewis, 1999]. However, these tools are often "pulled" into the taper from the opposite side, such as when a collet is installed in a milling machine. The symmetry of the tilt shaft design required that each interface be assembled independently of the other.

**Marine Joints** Like high-speed tooling, propeller interfaces to shafts are designed to transmit torque, albeit at lower speeds and much larger values. The marine
industry has long been using tapered interfaces, for which standards have been provided by several international classification societies such as DNV. These are used for rudder connections, propeller connections, and other interfaces in the ship control systems. These standards often rigidly define the geometry of the taper, and are designed for complex assembly and disassembly using numerous components.

![Diagram of Cone Coupling with Key](image)

Figure 3-4: Cone Coupling with Key [DNV, 2015]

### 3.1.1 Selection of Optimal Solution

The interface I was designing needed to be easily manipulated by maintainers ashore and at sea, meaning that alignment, assembly, and removal must be repeatable with basic hand-portable tools and a relatively low complexity/low part-count for when working in austere environments. A keyed taper allowed for a press fit that could be aligned by hand and pushed on. The small key way would ensure alignment between the two payloads. The press fit ensured that the interface would be capable of resisting the bending moment due to shock, of transmitting torque, and of resisting penetration.
during green water events. The taper made alignment for installation easy, sticking to poka-yoke principle and reducing the opportunity for the maintainer to induce galling of the material.

The next step was to design a simple tapered interface that could withstand the bending moment due to shock. The first iteration of this idea is seen in figure 3-5. Drawing on the best elements of the helicopter, wind turbine, and propeller designs, I developed an interface wherein the shaft and flange themselves are tapered. The axial force comes from a single large hollow screw that is threaded into the hollow tilt shaft. This permits the control cabling to connect to the payloads, allows a sailor to press on the flange with hand tools, and provides a pre-load to assist the interference fit through vibration and shock.

![Figure 3-5: Initial design sketch](image)

3.2 Development of the Interface Mathematical Model

3.2.1 Hand Calculations

The holding strength of a tapered interface is dependent upon the pressure exerted between the mating surfaces $P$, the coefficient of friction $\mu$, the engagement length $L$, the maximum radius of the engaged shaft taper $r_{\text{max}}$, and the engagement length $L$. The formula for the holding strength of a tapered interface is given by

$$
F = \frac{P \cdot L}{r_{\text{max}}}\mu
$$

where $F$ is the holding strength, $P$ is the pressure exerted, $L$ is the engagement length, $\mu$ is the coefficient of friction, and $r_{\text{max}}$ is the maximum radius of the engaged shaft taper.
and the taper angle $\alpha$ as defined in Figure 3-6.

![Diagram of shaft interface for hand calculations](image)

Figure 3-6: Diagram of shaft interface for hand calculations

Integrating over the engaged surface of the shaft taper revealed the governing equation for resisting the bending moment, seen in equation 3.1. The full derivation is included in appendix A.

$$M_{\text{resist}} = 4 \frac{P \mu}{3} L \sec \frac{\alpha}{2} (L^2 \tan^2 \frac{\alpha}{2} - 3Lr_{\text{max}} \tan \frac{\alpha}{2} + 3r_{\text{max}}^2)$$  \hspace{1cm} (3.1)

In order to determine the pressure in equation 3.1 I turned to the process used for propeller installation on US Navy ships. This process uses the larger of the required thrusts forward and astern and the dimensions of the taper in a quadratic equation to determine the interface pressure to prevent slip [Shepstone, 2005]. For this design, I determined that the maximum axial shock load - 60 g-force acting on the payload pass - was the required thrust. From this interface pressure and the contact area of the shaft and flange the push-on force was calculated. The next step was to consolidate these mathematical processes.

### 3.2.2 MATLAB Script

To capture all of the variables contributing to both the Shepstone math and the bending moment equation to facilitate iterating the design through different parameters and scales, I consolidated these calculations in a MATLAB script found
in appendix B. This code also assessed the material ability of all components - shaft, flange, and hollow screw - to ensure no component would fail during operation. This script drew heavily on threaded fastener equations from *Shigley's Mechanical Engineering Design 9th Edition* and Fastenal Engineering & Design Support's *Screw Thread Design*.

In order to validate the mathematics used to design the tapered interface to withstand 60 g-force, I needed to create a shock test that could be conducted locally and without the expense of commercial testing. The script concluded that the full-scale interface design would be able to withstand the shock requirements, as seen in appendix B. In order to validate this model, I iterated to a small version of the shaft and flange with a decreased push-on force and interface pressure. This reduced-scale model was designed to fail at far below the shock requirements. If the prototype performed as predicted, the same mathematics could be used for the larger model.

3.3 Analysis of Alternatives: Shock Testing

Researching shock testing led me to explosive testing, drop testing, and various forms of impact testing.

**Explosive Testing** Explosive testing is used for subjecting materials and systems to extreme environmental loading, with high heat, forces, and impulses. In particular, U.S. Navy ships are subject to shock testing after construction [Schauer, 1962]. Explosive testing presented concerns around access, safety, cost, complexity, and repeatability. Explosive testing facilities were not readily available and would add cost to the project. I considered conducting testing myself, but the risk and complexity
was prohibitive. Finally, explosive testing would risk the repeatability of the test if any components were to be damaged, jeopardizing the schedule and the tests.

**Drop Testing** Drop testing offered a more viable solution than explosive testing, using gravity as the means to accelerate the shaft and payload before bringing them to a sudden stop. Numerous methods have been explored in the mobile phone space [Goyal and Buratynski, 2000], but the masses and forces involved in testing this interface are much greater.

I first calculated the parameters of a test wherein I dropped the subsystem (connected shaft and payloads) onto a very stiff surface of known physical characteristics, in this case a steel plate. Using the natural frequency to calculate the impulse forces, time, acceleration, and deflection [Endevco, 2016], I soon recognized that the impulses seen in materials as stiff as the steel of the interface were too rapid. A successful test would require dropping the subsystem from impossibly low heights [Kausel, 2016]. Even if set up, the bandwidth of the available accelerometers would not capture the peak or not collect enough data points during the impulse to present valid data.

![Figure 3-7: Notes on accelerations of dropped objects, applied to the tilt shaft and payload [Kausel, 2016]](image)

In order to develop a more reasonable test, I had to extend the time over which the impact force acted. I could achieve this by attaching a more forgiving material
of known spring constant to the bottom of the payloads, and then dropping the subsystem onto a much stiffer material such as thick steel. The material would act as a spring, slowing the impulse and allowing for a higher drop. The process to characterize the test was as follows:

1. Identify a material of known modulus of elasticity, $E$

2. Calculate the spring constant $K$ from the modulus of elasticity and the area and thickness of the material, where

$$K = \frac{E \cdot A}{H}$$

(3.2)

3. Solve for the natural frequency,

$$\omega_n = \sqrt{\frac{K}{m}}$$

(3.3)

4. Solve for the initial velocity at impact,

$$u_0 = \frac{60 \cdot g}{-\omega_n}$$

(3.4)

5. Solve for the drop height using conservation of energy,

$$gh = \frac{1}{2} u_0^2$$

(3.5)

However, accelerometers are bandwidth limited, meaning the peak acceleration impulse could occur in between data points. While a laser interferometer or high-speed camera could capture such a peak, the equipment is costly and difficult to source,
and accelerometers are readily available in the laboratory. In order to fall within the bandwidth of those accelerometers, I considered various spring systems. Attaching a spring to each payload would create a measurable acceleration. The large mass of the subsystem dictated using heavy-duty coil springs, such as used in automobiles, or even leaf springs as seen in figure 3-8. While these spring systems did control the impulse time, they left unresolved the issue of maintaining a perfectly parallel drop such that both payload springs struck the contact plate simultaneously. If one spring contacted before the other, the accelerations experienced on each end would not match the objective of the test. The added time and complexity of manufacturing linear bearings and flexures to precisely control the drop test rendered this method untenable.

Impact Testing A pendulum can easily be characterized, allowing a researcher to know the kinetic and potential energies at any point along its arc, as well as its velocity. This presented an opportunity to develop a repeatable test wherein the shaft and payload system acted as the mass, which was then released from a known angular displacement to contact a spring of known constant, inducing the desired acceleration for the shock test on the interface. However, a simple pendulum on a wire or string does not offer torsional control of the mass, and an essential component of this test was that the shaft strike perpendicular to the spring, including a bending moment at the interface to the payload. Charpy impact tests, used to determine the amount of energy absorbed by a material during fracture, offer an example of a controlled and repeatable test using a physical pendulum to contact another object.

Inspired by Charpy testing rigs, I developed a mathematical design spreadsheet

Figure 3-8: A single stage multi leaf spring [Akar, 2017]
that calculated the acceleration experienced by a mass on a physical pendulum from a given angular displacement. The physical pendulum was a viable solution to create a repeatable shock test, and was selected for design and manufacture.

3.4 Development of the Reduced-Scale Shock Test

3.4.1 Requirements

The physical pendulum shock test rig needed to meet several derived requirements in order to facilitate local testing:

1. Physically small enough to be safely operated with the laboratory and easily manipulated and moved

2. Withstand the resultant forces and stresses of a low mass test interface released from up to 90 degrees angular displacement

3. A pendulum arm that prevented torsion of the test interface when released

4. A backstop to halt the pendulum arc and induce the required acceleration while maintaining alignment of the points of contact

5. Accommodate sensors, cabling, and interfaces required to capture the acceleration data
6. Controlled release of the test interface from a given angular displacement

7. Manufacturable on MIT campus with readily available materials and parts

3.4.2 Development

After considering a large pendulum rig for testing outdoors, it became clear than such an endeavor would challenge the manufacturing spaces available and would jeopardize the schedule as I was machining and assembling this rig. A smaller, local system was preferred. The optics table in the PERG laboratory was the ideal space to mount this system, as it provided a stable surface with mounting holes that could secure the pendulum rig from moving during testing. This space limited the pendulum arm to little more than 1.3 meters.

With the critical dimension of the pendulum arm determined I developed free body diagrams illustrating the forces experienced by the shock rig during testing from maximum angles, such as that seen in figure 3-10. These forces did not exceed

Figure 3-10: Free body diagram and sketch of shock pendulum concept

the specifications of the 80-20 T-slotted aluminum framing in the laboratory and ubiquitous in experimental structures. In order to minimize the moments on the
bolted joints of the structure, I minimized the width of the pendulum support structure, choosing to connect the uprights supporting the arm, and all other structural members, to a single large beam that would, in turn, be mounted to the optics table.

6061 aluminum square tubing was selected for the pendulum arm in order to maintain a low mass while ensuring alignment of the test interface and contact point by prohibiting twisting during travel. The tubing was treated as a simply supported beam, conservatively point loaded in the center at the time of impact. These calculations ensured that the pendulum arm would not fail or deform during maximum acceleration testing. The pendulum arm needed to be securely fastened to the pivot shaft. Circumferential clamps provide very good torque transmission, low stress concentration, and are easily milled [Slocum, 2008]. For this component attachment, a split-housing circumferential clamp ensured nearly continuous contact between the square tubing of the pendulum arm and the pivot shaft, which was sized to the inner dimension of the tubing. The pivot shaft itself was secured to minimize deflections utilizing Saint-Venant’s principle, which states that "several characteristic dimensions away from an effect, the effect is essentially dissipated." Maxwell’s reciprocity then suggests that an effect will dominate a system when applied over three to five characteristic dimensions of a system [Slocum, 2008]. The pivot shaft bearings with positioned accordingly, ensuring that the bearings would effectively resist moments applied to the shaft. I specified self-aligning bearings rated to withstand the calculated testing forces.

I considered attaching the spring to the swinging test subject but determined that machining it into the backstop at the bottom of the arc would simplify the manufacturing process and avoid potential inconsistency between test subjects, as only one mount would be required. The spring could have been in the forms discussed
previously as leaf springs, coils, or Belleville washer. Knowing that

$$I \frac{d^2 \theta}{dt^2} + \left( \frac{L}{2} m_{arm} g + L m_{test} g \right) \sin(\theta) = 0$$

(3.6)

and that the kinetic energy of a physical pendulum is

$$KE_{pp} = \frac{1}{2} I \left( \frac{d^2 \theta}{dt^2} \right)^2$$

(3.7)

and assuming no losses, I was able to determine the energy transferred into the spring. This provided the displacement of the spring, from which the force and then acceleration could be determined. The calculated displacements and the complexity of mounting a leaf spring or Belleville washer directed the use of a tempered steel jumbo compression spring.

Figure 3-11: Shock test compression spring and cap for tooling ball [McMaster-Carr, 2017]

Furthermore, each of these spring mechanisms presented a large contact area. The compression spring, with closed and ground flat ends, provided a means to mount a Hertzian contact point that would eliminate errors from minor misalignment. This was accomplished by machining a tapered cap that was mounted to the compression
spring, into which a steel tooling ball was inserted. Using a Hertzian contact design spreadsheet [Slocum, 2011], I determined that the contact at maximum acceleration would induce yielding in the aluminum of the pendulum arm. While repeated contact yielding would lead to a cold-formed spherical pocket upon repetition, I wanted to eliminate the possibility of unaccounted travel distance during spring compression. The test model mount required a bolt through the pendulum arm, so I located this bolt to be the contact point. The bolt head, also steel, would not deform from the Hertzian contact stresses.

In order to ensure accurate data on the acceleration of the test piece, the pendulum arm had to accommodate sensors aligned with the interface model and the point of contact. The mount also needed to be stiff in the direction of the acceleration to prevent the accelerometers from experiencing different accelerations than the test model. A simple bracket mount located the accelerometers directly behind the pendulum arm over the bolt used to both secure the test model and as the point of contact to the spring-mounted tooling ball as seen in figure 3-12.

This sensor platform was large enough to accommodate multiple sensors simultaneously for calibration and comparison while setting up the initial experiment to validate the testing model. From this sensor platform I ran the cabling up the pendulum arm to the pivot, securing the cables with enough slack to account for all angular displacements before connecting the cables to the sensor interface mounted on an upright.

The arm of the shock test rig needed to be released from a consistent angular displacement for each test, which required a release mechanism. Drop tests often incorporate electronic release systems. The shipping industry has informed the development of release hooks designed for massive systems, specified up to thousands of pounds.
[IDM, 2013]. These systems are expensive and require power connections to operate. Always seeking simplicity and a tie-in the nautical nature of this project, I decided to use a quick-release knot that can be untied under tension with a simple tug on the bitter end. The tumble hitch, seen in figure 3-13 is the most stable variation of the highwayman’s hitch, securely supporting large loads without jamming when released.

![Tumble Hitch Image](AnimatedKnots, 2016)

Figure 3-13: A tumble hitch [AnimatedKnots, 2016]

### 3.4.3 Manufacturing

All of the machined components of the shock testing rig were made of aluminum. The base was a 80-20 aluminum slotted beam measuring 9 cm by 9 cm, to which two 140 cm tall uprights were attached on either side with four brackets each. I drilled through the base beam and two pieces of 170 cm long aluminum angle stock to create triangular supportive brackets for the uprights, spaced using spare 80-20 slotted stock. The uprights were also connected by the 3/4 inch pivot shaft, which ran between a mounted ball bearing with cast iron housing on each upright. Those
bearings, as the brackets, were attached by end-feed fasteners in the T-slots. I milled a matching hole for the shaft in the top of the pendulum arm and a smaller \( \frac{1}{2} \) inch bolt hole above and orthogonal to that hole, then split the square tubing using a slot cutter on the mill to create a circumferential clamp. On the mass end of the pendulum arm I milled a \( \frac{3}{4} \) hole for a bolt to mount the test interface model, sensor platform, and act as the contact point. The sensor platform was an aluminum 90° angle stock into which I milled holes for the mounting bolt and each of the sensor connection bolts (10-32 and 4-40 screws requiring #7 and #30 drill bits for the 25 and 70 g-force accelerometers, respectively).

The coil spring needed a block into which it could be mounted and then attached to the double bracket securing it to the base beam. For this I faced a 2 in by 3.5 in by 2.5 in block of aluminum and then used a CNC mill to bore a 2.437 in diameter blind hole into which the spring could be mounted. I drilled and tapped the back of the block so that it could be attached to the double bracket using four 1/4-20 bolts. The final component was the tapered cap to hold the tooling ball and create the Hertzian contact point. I faced a 1.5 in long section of 2.5 in diameter aluminum bar stock, then used the lathe to create a boss matching the coil spring inner diameter at 1.687 in. The lathe taper function facilitated tapering to the 1 in diameter tooling ball face, into which I drilled a 1.25 in deep blind hole using a \( \frac{25}{64} \) drill bit to ensure a press fit. Once this was mounted, the machine was ready for testing with a control mass that would simulate the interface test model and validate the calculated relationship between mass, angular displacement of the pendulum arm, and acceleration upon contact with the spring.
3.5 Designing the Reduced-Scale Interface

3.5.1 Computer Modeling

The first iteration of the reduced-scale design was a shortened shaft and a cup-like flange designed to hold standard lifting weights on the end to permit testing at various payload masses. The center of the shaft was machined flat to sit flush on the pendulum arm, and the sides of that flat served to prevent twisting. In order to increase the mass of the flange/payload pieces and simplify manufacturing, I worked with Professor Slocum to design a more robust small shaft featuring chamfers instead of fillets wherever possible, and increased the flanges to hold Olympics weights.

![Figure 3-14: Exploded isometric view of the reduced-scale model](image)

3.5.2 Manufacturing

The model was sent out for quotes and the manufacturing contract awarded to Startsomething LLC. During the quote process, I received feedback on the drawings and requests for clarification on the design, such as the surface finish requirement and
any tolerances. This was an important learning point for the future, full-scale design, for which tolerances and finishes were critical.

Figure 3-15: Reduced scale model mounted to the pendulum rig

3.6 Development of the Full-Scale Shock Test

The full scale prototype of the interface design required a test using the payload models provided by the NRL, at a mass of 38.55 kg each. The laboratory shock rig was not designed to accommodate these forces, compelling me to develop a second shock test. Returning to the concept of a drop test, I examined the potential of utilizing the full scale model shaft and a simply-supported beam as springs in series to control the impulse.

3.6.1 Requirements

In order to facilitate local testing the drop test needed:

1. to be physically small enough to be moved into and safely operated within a campus space.
2. to withstand the resultant forces and stresses of a nearly 80 kg subsystem experiencing 60 g-forces.

3. to accommodate sensors, cabling, and interfaces required to capture the acceleration data.

4. a controlled release of the test model.

5. to be manufactured on MIT campus with readily available materials and parts.

3.6.2 Development

A testing solution to ensure the precise alignment concern was to drop the shaft and payload subsystem onto a simply supported cylindrical beam such that the shaft being tested contacted the supported beam. The simply supported beam acted as a spring where

\[ K = \frac{48EI}{L^3} \]  

(3.8)

and the tilt shaft behaved as a beam cantilevered from the point of contact, which is a spring where

\[ K = \frac{3EI}{(\frac{L}{2})^3}. \]  

(3.9)

Treating these as springs acting in series, I solved for an equivalent spring constant where

\[ \frac{1}{K_{eq}} = \frac{1}{K_{ss}} + \frac{1}{K_{shaft}} \]  

(3.10)

and determine the height from which to drop the shaft and payloads. This method permitted large spring constants that made it a viable candidate for testing the full scale interface from a drop height that did not exceed the length of the accelerometer cables.
Both the simply supported beam and the shaft containing the developed interface needed to withstand the stress of the impact without plastic deformation or failure. The maximum flexural stress in a beam of symmetric cross-section can be calculated using the equation

$$\sigma_{max} = \frac{Mc}{I} \quad (3.11)$$

where $M$ is the applied moment, $I$ is the moment of inertia of the cross-sectional area, and $c$ is the maximum distance from the neutral axis - in this case, the radius. The maximum shear stress in a beam of circular cross-section is given by

$$\tau_{max} = \frac{4V}{3A} \quad (3.12)$$

where $V$ is the shear force and $A$ is the cross-sectional area of the beam. These stresses were calculated for the interface shaft and compared to the yield and shear yield stresses for 1018 steel. In order to ensure the strength of the test shaft, I determined to keep it solid through testing, after which it could be hollowed for the required control cabling and installed in the prototype pan-tilt machine. The stresses for the simply supported spring beam were calculated and compared similarly in a custom design spreadsheet, using the Von Mises stress to ensure the material would not yield during testing. The Hertzian contact stresses experienced by the two shafts in contact were calculated using the Hertzian contact design spreadsheet to confirm that the shafts would not deform. Finally, I calculated the stress at the pin of the simple support to verify that it would not shear, nor would it cause failure of the beam at the joint.

The sensors now had to be attached to the payloads. This was achieved by tapping the payloads for 10-32 and 4-40 screws to mount the 25 and 70 g-force accelerometers,
respectively. The cabling ran clear of the drop area. As with the pendulum shock
rig, a tumble hitch provided a controlled release. Because two cylindrical shafts were
colliding at a Hertzian contact point, the acceleration experienced by the payloads
was not affected if the tilt shaft was not parallel to the ground at contact.

3.6.3 Manufacturing

The beam was turned from steel round bar stock. Two upright steel supports were
welded to a base plate, on which a third support was placed orthogonal to the first
two. The beam was pinned between the parallel supports and allowed to rest on the
orthogonal support as seen in figure 3-16. This simply supported structure was then
placed beneath a shop crane that was used to hold the shaft and payload subsystem.
The interface was raised to the appropriate height for the test.

3.7 Designing the Full-Scale Interface

3.7.1 Computer Modeling

This model was designed to meet the existing tilt shaft housing parameters, and
thus was partially geometrically defined, such as in diameter where the bearings sit.
Taking advantage of the additional resistance afforded by the hollowed bolt, and the ability to increase the pre-load if desired, I reduced the engagement length of the interface to decrease the footprint of the gimbal. To compensate for this reduction in engagement length, I reduced the taper angle as well. As discussed, the model for testing was not hollow throughout to prevent damage when point loaded at the midpoint of the beam, something that will not happen in the gimbal. The drop test model also did not include any external features, such as the elevation hard stop or the cable entry. Figure 3-18 shows the version as ready for use in the gimbal.
3.7.2 Manufacturing

The model was manufactured by the MIT Central Machine Shop over the course of a week. The shaft taper and flange taper were completed to a 16 surface finish and tested with engineer's blue to ensure complete contact throughout.
Chapter 4

Results

4.1 Pendulum Rig Validation Results

I tested the pendulum rig using a known mass in order to characterize the system for conducting future tests. I conducted five swings at ten degree intervals between ten and seventy degrees. The first data collected was noisy from the vibrations of the pendulum arm, so I added material as a damper with two sided tape as seen in figure 4-1.

I used a moving average to filter the data. A moving average depends upon the window size used. I determined the maximum window size from the impulse time of the shock test, selecting as many points at 10 kHz sampling rate would fit into that period. In order to determine the window size choice, I used the code in appendix C to plot the sum of absolute differences between the filtered and raw data over every window size up to the predetermined maximum. Where
the curve steadied was the window size used. An example of this, performed on the
tests at twenty degrees, is seen in figure 4-2.

![Figure 4-2: Determining window size using SAD at 20 degrees](image)

After filtering the data, I was able to determine the peak acceleration for each
swing at each angle. These, in turn, were analyzed in a normal distribution to
determine the average and the 95% confidence interval of the results. Finally, the
averages were plotted over each angle with a best-fit curve as seen in figure 4-3.

The pendulum rig was successful in delivering the required smaller shocks necessary
to test the reduced-scale model. Even with the added damping and bolting the system
in place there was substantial vibration and noise. A future design should explore
a more stiff pendulum arm and increased lateral support for the pivot point of the
pendulum.
4.2 Reduced-Scale Shock Test Results

After thus validating the pendulum, I tested the reduced-scale model. The push-on force was calculated such that the interface would "fail" when subjected to a 20 g-forces acceleration, meaning that the flange would come loose from the shaft. The flanges were pressed onto the shaft, and the bolts backed out partly to permit movement while securing the flanges from leaving the test rig entirely.

During the first tests, the model did not "fail" as predicted. I returned first to the push-on force calculations, iterating them to a very low push on force (and thus very low required torque to apply – too low, in fact, for the wrenches available). I used a force gauge to push on the flanges with ever lower forces, finally inducing failure on a swing. Returning to the MATLAB code, the source of the error became obvious immediately. The distance used to calculate the bending moment of the payload at impact had not been scaled down to match the test shaft and flange dimensions, and thus was predicted to be much larger than actually experienced during testing.

I corrected the MATLAB code moment arm inputs, and proofed the code for other errors. Running the code again predicted failure at 20 g-forces using the same
push-on force originally desired. I secured the flanges, marked the intersection of the flange ends and the shaft, and then backed the screws out several turns so that any loosening would be readily apparent but would prevent the flanges from departing the shaft entirely. This test proved successful, suggesting that the math was correct and that a larger scale design was feasible.

![Acceleration plot from 20 degree test](image)

Figure 4-4: Acceleration plot from 20 degree test

### 4.3 Full-Scale Shock Test Results

The full scale drop test used both a 70 and a 500 g-force accelerometer. As the results rapidly exceeded 70 g-forces, the data presented here is only that from the 500 g-force sensor. Additionally, the 70 g-force sensor cable severed during the third test, rendering it unusable for the last two tests.

The data collected from the full-scale drop test was significantly less noisy than the pendulum rig data. The impulse time contained approximately 26 data points, and the peaks of the 500 g-force accelerometer results comprised upwards of 17 points, indicating reliable maximum value data. For comparison, I still plotted a moving
average of the data using the window size dictated by comparing the sum of absolute differences to the window size. This analysis can be found in appendix D.

4.4 Conclusion

This tapered interface successfully met the requirements for the gimbal and may be utilized in the prototype machine. It reduces the part count and the time required for maintenance and repair. It could be further reduced in size, but that would require an increase in pre-load and thus application torque. This design proves that tapers are a feasible method to connect massive loads to their actuators, and can use different geometry than established in automotive and marine industries. There is potential to use this technology with large actuators in industry. Future work on this project could include making a graphic user interface for the code such that a designer could input some parameters and have a taper design iterated from the code.
Figure 4-6: Test 2

Figure 4-7: Test 3
Figure 4-8: Test 4
Appendix A

Hand Calculations

I calculated the taper retaining moment and that of a cylinder with only 30 degrees of contact on either side to see how the two compared. Those calculations are shown in figures A-1 and A-2, respectively.
Figure A-1: Taper Calculations
Figure A-2: Cylinder Calculations for 30 Degrees Contact
Appendix B

Interface Mathematical Model

% This code determines the interface's ability to withstand all
% loading and force requirements, as well validating that the shaft,
% flange, and screw will not fail materially in the process.

clf  
close all  
clear all  
clc

% Physical Constraints

g=9.81;           %gravity [m/s^2]
E=200E9; %Young's Modulus of AISI 4340 Stainless Steel [Pa]
nu=0.29; %Poisson's Ratio of AISI 4340 Stainless Steel
rho=7850; %Density of AISI 4340 Stainless Steel [kg/m^3]
sigma_yield=972E6; %Tensile Yield Strength of AISI 4340 SS [Pa]

mu=0.213650328; %measured coefficient of friction for flange and shaft [Pa]

%shear yield stress of screw per Shigley Eqn 5-21 [Pa]
screw_shear=0.577*screw_tensile;

%shock requirement [m/s^2]
shock=60*g;

%max angular velocity [rad/s]
omega_req_max=convangvel(100, 'deg/s', 'rad/s');

%max angular acceleration [rad/s^2]
alpha_req_max=convangacc(100, 'deg/s^2', 'rad/s^2');

d_tilt_inner=0.020; %clearance for control cabling [m]
r_tilt_inner=d_tilt_inner/2; %clearance for control cabling [m]
% Tilt Shaft and Payload Dimensions

% PAYLOAD
m_payload=77.1107/2; %mass of each payload [kg]
LCG_payload=0.2; %distance to center of gravity of payload [m]

% TILT SHAFT
d_tilt=0.05; %large end diameter of tilt shaft to fit bearing [m]
r_tilt=d_tilt/2; %large end radius of tilt shaft [m]

% TILT FLANGE AND HOUSING
t_r=0.013; %retainer thickness [m]
t_b=0.016; %tilt shaft bearing thickness [m]
L_offset=0.002; %prevents flange from exceeding taper [m]

t_flange=0.015; %flange vertical thickness [m]
d_flange_outer=0.05988; %outer diameter of flange hub on taper [m]
r_flange_outer=d_flange_outer/2; %outer radius of flange hub on taper [m]
chamfer_flange=0.002; %chamfer [m]

% FLANGE SCREW
pitch diameter of 1 1/4-12 3A screw [m]
d_m_bolt=convlength(1.1959, 'in', 'm');

minor diameter of 1 1/4-12 3A screw [m]
d_minor_bolt=convlength(1.1508, 'in', 'm');
r_minor_bolt=d_minor_bolt/2; %minor radius of 1 1/4-12 3A screw [m]
lead=convlength(1/12, 'in', 'm'); %inverse of threads per inch [m]
thread_angle=60; %UNC thread angle [degrees]
%flange screw average collar diameter [m]
d_c=convlength((2.438+1.25)/2, 'in', 'm');
screw_length=convlength(3, 'in', 'm'); %screw length [m]
screw_thread_length=convlength(3-.683, 'in', 'm'); %threaded length [m]
screw_head_height=convlength(0.683, 'in', 'm'); %head height of screw [m]
screw_head_diameter=convlength(2.438,'in','m'); %diameter of screw head [m]

% Shepstone Taper Input Variables

M_a=m_payload*shock; %thrust of axial shock on payload [N]
t=16; %inverse of shaft taper
L_interface=.038; %length of interface between flange and shaft [m]

%distance from tilt shaft bearing to payload center of gravity [m]
L_moment=(.5*t_b) + t_r + L_offset + L_interface + t_flange + LCG_payload;

c_tilt_min=1/t; %shaft taper (diameter to length)

taper_angle=atand(1/(2*t));

d_a=d_tilt-(L_interface*c_tilt_min); %small end diameter tilt shaft [m]
\begin{verbatim}
111 d_m = d_tilt - (L_interface / (2 * t)); % mean shaft diameter [m]
112
113 r_tilt_taper_max = (d_tilt / 2) - chamfer_flange * tand(taper_angle);
114 r_tilt_taper_min = d_a / 2;
115
116 % Shepstone Quadratic Equation Constants [non-dimensional]
117 a = (mu^2) * cosd(taper_angle)^2 - sind(taper_angle)^2;
118 b = -(mu^2 + 1) * 2 * M_a * cosd(taper_angle) * sind(taper_angle);
119 c_tilt_min = (mu * M_a * sind(taper_angle))^2 - (M_a * cos(taper_angle))^2;
120
121 % required radial force to prevent slip for axial thrust [N]
122 M_i = (-b + sqrt(b^2 - 4 * a * c_tilt_min)) / (2 * a);
123
124 area = pi * L_interface * d_m; % contact area of flange and shaft [m^2]
125
126 p = M_i / area; % required average interface pressure [Pa]
127
128 % required push on force [N]
129 F_d = M_i * (sind(taper_angle) + mu * cosd(taper_angle)) / ...
130     (cosd(taper_angle) - mu * sind(taper_angle));
131
132
133
134
135 % Calculate Screw Torque with Shigley Equation 8-25a
136
137 Torque = (F_d * d_m_bolt / 2) * ((lead + pi * mu * d_m_bolt * secd(thread_angle)) / ...
138     / (pi * d_m_bolt - mu * lead * secd(thread_angle))) + (F_d * mu * d_c / 2); % [N*m]
139
140
71
\end{verbatim}
% Check that screw and internal threads will not yield

n_t=thread_length/lead;  %number of threads engaged on screw (all)

%minimum major diameter of internal threads [m]
d_internal_thread=convlength(1.25, 'in', 'm');

%cross-sectional area through which internal thread shear occurs calculated
%from Fastenal Engineering and Design Support "Screw Thread Design"
%document using INCH units to match the equation. 12 is the threads per
%inch, 3 is the engagement length, 1.2386 is the minimum major diameter of
%external threads from efunda, 1.2019 is the maximum pitch diameter of the
%internal threads [in^2]
area_internal_threads=pi*12*3*1.2386*((1/(2*12))+0.57735*(1.2386-1.2019));

%convert cross-sectional area of internal threads to metric [m^2]
area_internal_threads=area_internal_threads*convlength(1, 'in', 'm')^2;

%axial stress in screw per Shigley Eqn 8-8 [Pa]
axial_stress_screw=F_d/(pi*(r_minor_bolt^2-r_tilt_inner^2));

%bending stress at root of thread per Shigley Eqn 8-11 [Pa]
bending_stress_screw=(6*F_d)/(pi*d_minor_bolt*n_t*lead);

%transverse shear stress at center of thread root per Shigley Eqn 8-12 [Pa]
shear_screw_thread=(3*F_d)/(pi*d_minor_bolt*n_t*lead);

%shear stress at internal thread per Fastenal Eng & Design Support [Pa]
shear_internal=F_d/area_internal_threads;
if axial_stress_screw>screw_tensile || bending_stress_screw>screw_tensile...
    || shear_screw_thread>screw_shear || shear_internal>screw_shear
    display('Screw will not provide pre-load.')
else display('Screw will provide pre-load.')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Check that taper will transmit torque
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

I\text{x}\text{x}=484265.17; \quad \% \text{from SolidWorks mass properties [lbm*mm}^2\text{]} 
I\text{x}\text{x}=I\text{x}\text{x}\cdot\text{convmass}(1, \text{'}\text{lbm}', \text{'}\text{kg}')\cdot(1/1000)^2; \quad \% \text{convert to metric [kg*m}^2\text{]} 

\text{Torque}\_\text{tilt}=I\text{x}\text{x}\cdot\alpha\_\text{req}\_\text{max}; \quad \% \text{max torque required [N*m]}

if \text{M}_i\cdot\text{d}_m<\text{Torque}\_\text{tilt}
    display('Taper will not transmit actuating torque.')
else display('Taper will transmit actuating torque.')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Check that centripetal force will not unseat payload
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

\% \text{centripetal force at payload LCG [N]}
F\_\text{centripetal}=m\_\text{payload}\cdot\text{L}_\text{moment}\cdot(\omega\_\text{req}\_\text{max}^2);

if F\_\text{centripetal}>M\_a
% Check that shock bending moment will not unseat payload

% taper retaining moment per integration in design notebook p.20 [N*m]

M_shock=m_payload*L_moment*shock;  %bending moment due to shock [N*m]

M_taper_hold=4*((p*mu)/3)*L_interface*secd(taper_angle)*(L_interface^2*...
   (tand(taper_angle))^2-3*L_interface*r_tilt_taper_max*...
   tand(taper_angle)+3*r_tilt_taper_max^2);

M_total_hold=abs(M_taper_hold)+(screw_tensile*(pi*(r_minor_bolt^2-...
   r_tilt_inner^2))*d_c);

if M_shock>M_total_hold
    display('Taper will not hold shock moment.')
else
    display('Taper will hold shock moment.')
end

% Check that large diameter of taper will not yield

% Centroidal Moment of Inertia for Hollow Shaft [m^4]
\[ I_x = \left( \frac{\pi}{64} \right) (r_{\text{tilt taper max}}^4 - r_{\text{tilt inner}}^4) \]

\[ M_{\text{stress}} = m_{\text{payload}} \times g \times L_{\text{moment}} \quad \text{%bending moment [N}\cdot\text{m]} \]

\[ c_{\text{d tilt}} = r_{\text{tilt taper max}} \quad \text{%distance to neutral axis [m]} \]

\[ \text{circumstress}_{\text{d tilt}} = -\frac{(r_{\text{tilt taper max}}^2 - r_{\text{tilt inner}}^2) \times p}{(r_{\text{tilt taper max}}^2 - r_{\text{tilt inner}}^2)} \quad \text{%circumfrential stress [Pa]} \]

\[ \text{radialstress}_{\text{d tilt}} = -p \quad \text{%radial stress [Pa]} \]

\[ \text{bendstress}_{\text{d tilt}} = M_{\text{stress}} \times c_{\text{d tilt}} / I_x \quad \text{%bending stress [Pa]} \]

\[ \text{if bendstress}_{\text{d tilt}} > \sigma_{\text{yield}} \quad \text{|| circumstress}_{\text{d tilt}} > \sigma_{\text{yield}} \quad \text{||...} \]

\[ \text{radialstress}_{\text{d tilt}} > \sigma_{\text{yield}} \]

\[ \text{display('Tilt shaft will yield at large end of taper.')} \]

\[ \text{else display('Tilt shaft will not yield at large end of taper.'}) \]

\[ \text{end} \]

\[ \text{c_{\text{d a}} = r_{\text{tilt taper min}} \quad \text{%distance to neutral axis [m]} \]

\[ \text{circumstress}_{\text{d a}} = -\frac{(r_{\text{tilt taper min}}^2 - r_{\text{tilt inner}}^2) \times p}{(r_{\text{tilt taper min}}^2 - r_{\text{tilt inner}}^2)} \quad \text{%circumfrential stress [Pa]} \]

\[ \text{radialstress}_{\text{d a}} = -p \quad \text{%radial stress [Pa]} \]

\[ \text{bendstress}_{\text{d a}} = M_{\text{stress}} \times c_{\text{d a}} / I_x \quad \text{%bending stress [Pa]} \]

\[ \text{if bendstress}_{\text{d a}} > \sigma_{\text{yield}} \quad \text{|| circumstress}_{\text{d a}} > \sigma_{\text{yield}} \quad \text{||...} \]

\[ \text{radialstress}_{\text{d a}} > \sigma_{\text{yield}} \]

\[ \text{display('Tilt shaft will yield at small end of taper.'}) \]
else display('Tilt shaft will not yield at small end of taper.\n\nend

% Check that open hub of flange will not yield

% Tilt taper thickness at large end of taper [m]
t_flange_hub_min=r_flange_outer-r_tilt_taper_max;

% Inner flange radius at large diameter taper [m]
r_flange_inner_max=r_tilt_taper_max;
r_flange_outer_max=r_flange_outer;  % Outer flange radius (constant) [m]

I_x_flange=(pi/64)*(r_flange_outer^4-r_flange_inner_max^4);  % [m^4]
c_flange=r_flange_outer_max;  % Distance to neutral axis [m]

% Circumferential stress [Pa]
circum_stress_r_flange_max=((r_flange_outer_max^2+r_flange_inner_max^2)*p)/(...
    (r_flange_outer_max^2-r_flange_inner_max^2));

radial_stress_r_flange_max=-p;  % Radial stress [Pa]
bend_stress_r_flange_max=M_stress*c_flange/I_x_flange;  % Bending stress [Pa]

if bend_stress_r_flange_max>sigma_yield || circum_stress_r_flange_max...
    >sigma_yield || radial_stress_r_flange_max>sigma_yield
    display('Flange will yield at open end of hub.\nelse display('Flange will not yield at open end of hub.\n
end

76
% Check that closed hub of flange will not yield

% flange thickness at small end of taper [m]
t_flange_hub_max=r_flange_outer-r_tilt_taper_min;

% inner flange radius at large diameter taper [m]
r_flange_inner_min=r_tilt_taper_min;

I_x_flange_min=(pi/64)*(r_flange_outer^4-r_flange_inner_min^4);  % [m^4]

% circumferential stress [Pa]
circum_stress_r_flange_min=((r_flange_outer_max^2+r_flange_inner_min^2)*p)/...
            (r_flange_outer_max^2-r_flange_inner_min^2);

radial_stress_r_flange_min=-p;  % radial stress [Pa]

% bending stress [Pa]
bend_stress_r_flange_min=M_stress*c_flange/I_x_flange_min;

if bend_stress_r_flange_min>sigma_yield || circum_stress_r_flange_min...
    >sigma_yield || radial_stress_r_flange_min>sigma_yield
    display('Flange will yield at closed end of hub.')
else display('Flange will not yield at closed end of hub.')
end

% End of Script
Appendix C

Shock Testing Analysis Mathematical Code

1 % This script analyzes the shock rig confidence tests with damper
2 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
3
4 clf
5 clear all
6 close all
7 clc
8 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
9
10 % 10 Degrees
11 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
12
13 % Create matrices for time and accelerometer
Test10 = csvread('10Degrees.csv',1,0);
Test10_time = Test10([1:end],[1]);
Test10_1 = Test10([1:end],[3]);
Test10_2 = Test10([1:end],[6]);
Test10_3 = Test10([1:end],[9]);
Test10_4 = Test10([1:end],[12]);
Test10_5 = Test10([1:end],[15]);

% Determine window size for moving average by plotting the sum of absolute
% differences for different window sizes and finding the knee in the curve,
% where the window is smallest and the curve seems to flatten. Maximum
% window size is calculated from a 10 kHz sample rate over the duration of
% the impulse.
windowSizes = 1 : 1 : 89;
for k = 1 : length(windowSizes);
    smoothed1 = movmean(Test10_1, windowSizes(k));
    sad1(k) = sum(abs(smoothed1 - Test10_1));
    smoothed2 = movmean(Test10_2, windowSizes(k));
    sad2(k) = sum(abs(smoothed2 - Test10_2));
    smoothed3 = movmean(Test10_3, windowSizes(k));
    sad3(k) = sum(abs(smoothed3 - Test10_3));
    smoothed4 = movmean(Test10_4, windowSizes(k));
    sad4(k) = sum(abs(smoothed4 - Test10_4));
    smoothed5 = movmean(Test10_5, windowSizes(k));
    sad5(k) = sum(abs(smoothed5 - Test10_5));
end

figure('Name','10 Degrees - SAD')
subplot(5,1,1);
plot(windowSizes, sad1, 'b*-', 'LineWidth', 2);
title('SAD 10-1')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,2);
plot(windowSizes, sad2, 'b*-', 'LineWidth', 2);
title('SAD 10-2')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,3);
plot(windowSizes, sad3, 'b*-', 'LineWidth', 2);
title('SAD 10-3')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,4);
plot(windowSizes, sad4, 'b*-', 'LineWidth', 2);
title('SAD 10-4')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,5);
plot(windowSizes, sad5, 'b*-', 'LineWidth', 2);
title('SAD 10-5')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');

% Moving Average Filter
Test10_1smooth = smoothdata(Test10_1,'movmean',44);
Test10_2smooth = smoothdata(Test10_2,'movmean',44);
Test10_3smooth = smoothdata(Test10_3,'movmean',44);
Test10_4smooth = smoothdata(Test10_4,'movmean',44);
Test10_5smooth = smoothdata(Test10_5,'movmean',44);

% Find maximum accelerations of each run, then average
Max10_1 = max(abs(max(Test10_1)),abs(min(Test10_1)));
Max10_1smooth = max(abs(max(Test10_1smooth)),abs(min(Test10_1smooth)));
Max10_2 = max(abs(max(Test10_2)),abs(min(Test10_2)));
Max10_2smooth = max(abs(max(Test10_2smooth)),abs(min(Test10_2smooth)));
Max10_3 = max(abs(max(Test10_3)),abs(min(Test10_3)));
Max10_3smooth = max(abs(max(Test10_3smooth)),abs(min(Test10_3smooth)));
Max10_4 = max(abs(max(Test10_4)),abs(min(Test10_4)));
Max10_4smooth = max(abs(max(Test10_4smooth)),abs(min(Test10_4smooth)));
Max10_5 = max(abs(max(Test10_5)),abs(min(Test10_5)));
Max10_5smooth = max(abs(max(Test10_5smooth)),abs(min(Test10_5smooth)));

M_10 = [Max10_1 Max10_2 Max10_3 Max10_4 Max10_5];
M_10 = mean(M_10);

G_1 = [Max10_1smooth; Max10_2smooth; Max10_3smooth; Max10_4smooth; ... Max10_5smooth];
G_10 = mean(G_1);

% Determine 95% confidence interval of data
pd10 = fitdist(G_1,'Normal');
ci10 = paramci(pd10);

% Plot raw and smooth data
figure('Name','10 Degrees')
plot(Test10_time,Test10_3,'-','LineWidth',1)
hold on
plot(Test10_time,Test10_3smooth,'-','LineWidth',1)
hold on
xlim([-0.01 0.09])
title('Acceleration v. Time for 10 Degrees at 10 kHz')
xlabel('Time [s]')
ylabel('Acceleration [G-forces]')
legend('Data','Smoothed Data')
str = {'Max Measured Acceleration = M_10,' ...
'Mean Moving Average Acceleration = G_10'};
annotation('textbox',[.5 .7 .1 .1], 'String', str, 'FitBoxToText','on');

%%%%%%%%%%%%%%%%%%%%%%%%%%
% 20 Degrees
%%%%%%%%%%%%%%%%%%%%%%%%%%

% Create matrices for time and accelerometer
Test20 = csvread('20Degrees.csv',1,0);
Test20_time = Test10([1:end], [1]);
Test20_1 = Test20([1:end], [3]);
Test20_2 = Test20([1:end], [6]);
Test20_3 = Test20([1:end], [9]);
Test20_4 = Test20([1:end], [12]);
Test20_5 = Test20([1:end], [15]);

% Determine window size for moving average by plotting the sum of absolute
% differences for different window sizes and finding the knee in the curve,
% where the window is smallest and the curve seems to flatten. Maximum
% window size is calculated from a 10 kHz sample rate over the duration of
% the impulse.
windowSizes = 1 : 1 : 89;
for k = 1 : length(windowSizes);
    smoothed1 = movmean(Test20_1, windowSizes(k));
    sad1(k) = sum(abs(smoothed1 - Test20_1));
    smoothed2 = movmean(Test20_2, windowSizes(k));
    sad2(k) = sum(abs(smoothed2 - Test20_2));
    smoothed3 = movmean(Test20_3, windowSizes(k));
    sad3(k) = sum(abs(smoothed3 - Test20_3));
    smoothed4 = movmean(Test20_4, windowSizes(k));
    sad4(k) = sum(abs(smoothed4 - Test20_4));
    smoothed5 = movmean(Test20_5, windowSizes(k));
    sad5(k) = sum(abs(smoothed5 - Test20_5));
end

figure('Name','20 Degrees - SAD')
subplot(5,1,1);
plot(windowSizes, sad1, 'b*-', 'LineWidth', 2);
title('SAD 20-1')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,2);
plot(windowSizes, sad2, 'b*-', 'LineWidth', 2);
title('SAD 20-2')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,3);
plot(windowSizes, sad3, 'b*-', 'LineWidth', 2);
title('SAD 20-3')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,4);
plot(windowSizes, sad4, 'b*-', 'LineWidth', 2);
title('SAD 20-4')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,5);
plot(windowSizes, sad5, 'b*-', 'LineWidth', 2);
title('SAD 20-5')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');

% Moving Average Filter
Test20_1smooth = smoothdata(Test20_1, 'movmean', 44);
Test20_2smooth = smoothdata(Test20_2, 'movmean', 44);
Test20_3smooth = smoothdata(Test20_3, 'movmean', 44);
Test20_4smooth = smoothdata(Test20_4, 'movmean', 44);
Test20_5smooth = smoothdata(Test20_5, 'movmean', 44);

% Find maximum accelerations of each run, then average
Max20_1 = max(abs(max(Test20_1)), abs(min(Test20_1)));
Max20_1smooth = max(abs(max(Test20_1smooth)), abs(min(Test20_1smooth)));
Max20_2 = max(abs(max(Test20_2)), abs(min(Test20_2)));
Max20_2smooth = max(abs(max(Test20_2smooth)), abs(min(Test20_2smooth)));
Max20_3 = max(abs(max(Test20_3)), abs(min(Test20_3)));
Max20_3smooth = max(abs(max(Test20_3smooth)), abs(min(Test20_3smooth)));
Max20_4 = max(abs(max(Test20_4)), abs(min(Test20_4)));
Max20_4smooth = max(abs(max(Test20_4smooth)), abs(min(Test20_4smooth)));

85
Max20_5 = max(abs(max(Test20_5)), abs(min(Test20_5))); 
Max20_5smooth = max(abs(max(Test20_5smooth)), abs(min(Test20_5smooth))); 
M_20 = [Max20_1 Max20_2 Max20_3 Max20_4 Max20_5]; 
M_20 = mean(M_20); 
G_2 = [Max20_1smooth; Max20_2smooth; Max20_3smooth; Max20_4smooth;...
Max20_5smooth]; 
G_2 = mean(G_2); 

% Determine 95% confidence interval of data 
pd20 = fitdist(G_2, 'Normal'); 
ci20 = paramci(pd20); 

% Plot raw and smooth data 
figure('Name', '20 Degrees') 
plot(Test20_time, Test20_5, '-','LineWidth',1) 
hold on 
plot(Test20_time, Test20_5smooth, '-','LineWidth',1) 
hold on 
xlim([-0.01 0.09]) 
title('Acceleration v. Time for 20 Degrees at 10 kHz') 
xlabel('Time [s]') 
ylabel('Acceleration [G-forces]') 
legend('Raw Data', 'Smoothed Data') 
str = {'Max Measured Acceleration = ' M_20,...
'Mean Moving Average Acceleration = ' G_2}; 
anotation('textbox', [.5 .7 .1 .1], 'String', str, 'FitBoxToText', 'on');
% 30 Degrees

% Create matrices for time and accelerometer
Test30 = csvread('30Degrees.csv',1,0);
Test30_time = Test30([1:end], [1]);
Test30_1 = Test30([1:end], [3]);
Test30_2 = Test30([1:end], [6]);
Test30_3 = Test30([1:end], [9]);
Test30_4 = Test30([1:end], [12]);
Test30_5 = Test30([1:end], [15]);

% Determine window size for moving average by plotting the sum of absolute
% differences for different window sizes and finding the knee in the curve,
% where the window is smallest and the curve seems to flatten. Maximum
% window size is calculated from a 10 kHz sample rate over the duration of
% the impulse.
windowSizes = 1 : 1 : 89;
for k = 1 : length(windowSizes);
    smoothed1 = movmean(Test30_1, windowSizes(k));
sad1(k) = sum(abs(smoothed1 - Test30_1));
    smoothed2 = movmean(Test30_2, windowSizes(k));
sad2(k) = sum(abs(smoothed2 - Test30_2));
    smoothed3 = movmean(Test30_3, windowSizes(k));
sad3(k) = sum(abs(smoothed3 - Test30_3));
    smoothed4 = movmean(Test30_4, windowSizes(k));
sad4(k) = sum(abs(smoothed4 - Test30_4));
    smoothed5 = movmean(Test30_5, windowSizes(k));
sad5(k) = sum(abs(smoothed5 - Test30_5));
end
```matlab
figure('Name','30 Degrees - SAD')
subplot(5,1,1);
plot(windowSizes, sad1, 'b*-', 'LineWidth', 2);
title('SAD 30-1')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,2);
plot(windowSizes, sad2, 'b*-', 'LineWidth', 2);
title('SAD 30-2')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,3);
plot(windowSizes, sad3, 'b*-', 'LineWidth', 2);
title('SAD 30-3')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,4);
plot(windowSizes, sad4, 'b*-', 'LineWidth', 2);
title('SAD 30-4')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,5);
plot(windowSizes, sad5, 'b*-', 'LineWidth', 2);
title('SAD 30-5')
grid on;
xlabel('Window Size');
```
ylabel('Sum of Absolute Differences');

% Moving Average Filter
Test30_1smooth = smoothdata(Test30_1,'movmean',52);
Test30_2smooth = smoothdata(Test30_2,'movmean',52);
Test30_3smooth = smoothdata(Test30_3,'movmean',52);
Test30_4smooth = smoothdata(Test30_4,'movmean',52);
Test30_5smooth = smoothdata(Test30_5,'movmean',52);

% Find maximum accelerations of each run, then average
Max30_1 = max(abs(max(Test30_1)),abs(min(Test30_1)));  
Max30_1smooth = max(abs(max(Test30_1smooth)),abs(min(Test30_1smooth))); 
Max30_2 = max(abs(max(Test30_2)),abs(min(Test30_2)));    
Max30_2smooth = max(abs(max(Test30_2smooth)),abs(min(Test30_2smooth))); 
Max30_3 = max(abs(max(Test30_3)),abs(min(Test30_3)));      
Max30_3smooth = max(abs(max(Test30_3smooth)),abs(min(Test30_3smooth))); 
Max30_4 = max(abs(max(Test30_4)),abs(min(Test30_4)));    
Max30_4smooth = max(abs(max(Test30_4smooth)),abs(min(Test30_4smooth))); 
Max30_5 = max(abs(max(Test30_5)),abs(min(Test30_5)));      
Max30_5smooth = max(abs(max(Test30_5smooth)),abs(min(Test30_5smooth))); 

M_30 = [Max30_1 Max30_2 Max30_3 Max30_4 Max30_5];    
M_30 = mean(M_30);

G_3 = [Max30_1smooth; Max30_2smooth; Max30_3smooth; Max30_4smooth; Max30_5smooth];
G_30 = mean(G_3);

% Determine 95% confidence interval of data
pd30 = fitdist(G_3,'Normal');
ci30 = paramci(pd30);
% Plot raw and smooth data

figure('Name','30 Degrees')
plot(Test30_time,Test30_3,'-','LineWidth',1)
hold on
plot(Test30_time,Test30_3smooth,'-','LineWidth',1)
hold on
xlim([-0.01 0.09])

title('Acceleration v. Time for 30 Degrees at 10 kHz')
xlabel('Time [s]')
ylabel('Acceleration [G-forces]')
legend('Raw Data','Smoothed Data')
str = {'Max Measured Acceleration = ' M_30,...
      'Mean Moving Average Acceleration = ' G_30};
annotation('textbox',[.5 .7 .1 .1],'String',str,'FitBoxToText','on');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 40 Degrees
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Create matrices for time and accelerometer
Test40 = csvread('40Degrees.csv',1,0);
Test40_time = Test40([1:end],1);
Test40_1 = Test40([1:end],3);
Test40_2 = Test40([1:end],6);
Test40_3 = Test40([1:end],9);
Test40_4 = Test40([1:end],12);
Test40_5 = Test40([1:end],15);
% Determine window size for moving average by plotting the sum of absolute
% differences for different window sizes and finding the knee in the curve,
% where the window is smallest and the curve seems to flatten. Maximum
% window size is calculated from a 10 kHz sample rate over the duration of
% the impulse.

windowSizes = 1 : 1 : 89;
for k = 1 : length(windowSizes);
    smoothed1 = movmean(Test40_1, windowSizes(k));
    sad1(k) = sum(abs(smoothed1 - Test40_1));
    smoothed2 = movmean(Test40_2, windowSizes(k));
    sad2(k) = sum(abs(smoothed2 - Test40_2));
    smoothed3 = movmean(Test40_3, windowSizes(k));
    sad3(k) = sum(abs(smoothed3 - Test40_3));
    smoothed4 = movmean(Test40_4, windowSizes(k));
    sad4(k) = sum(abs(smoothed4 - Test40_4));
    smoothed5 = movmean(Test40_5, windowSizes(k));
    sad5(k) = sum(abs(smoothed5 - Test40_5));
end

figure('Name','40 Degrees - SAD')
subplot(5,1,1);
plot(windowSizes, sad1, 'b*-', 'LineWidth', 2);
title('SAD 40-1')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,2);
plot(windowSizes, sad2, 'b*-', 'LineWidth', 2);
title('SAD 40-2')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,3);
plot(windowSizes, sad3, 'b*-', 'LineWidth', 2);
title('SAD 40-3')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,4);
plot(windowSizes, sad4, 'b*-', 'LineWidth', 2);
title('SAD 40-4')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,5);
plot(windowSizes, sad5, 'b*-', 'LineWidth', 2);
title('SAD 40-5')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');

% Moving Average Filter
Test40_1smooth = smoothdata(Test40_1,'movmean',52);
Test40_2smooth = smoothdata(Test40_2,'movmean',52);
Test40_3smooth = smoothdata(Test40_3,'movmean',52);
Test40_4smooth = smoothdata(Test40_4,'movmean',52);
Test40_5smooth = smoothdata(Test40_5,'movmean',52);

% Find maximum accelerations of each run, then average
Max40_1 = max(abs(max(Test40_1)),abs(min(Test40_1)));
Max40_1smooth = max(abs(max(Test40_1smooth)),abs(min(Test40_1smooth)));
Max40_2 = max(abs(max(Test40_2)),abs(min(Test40_2)));

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Max40_2smooth = max(abs(max(Test40_2smooth)), abs(min(Test40_2smooth)));
Max40_3 = max(abs(max(Test40_3)), abs(min(Test40_3)));
Max40_3smooth = max(abs(max(Test40_3smooth)), abs(min(Test40_3smooth)));
Max40_4 = max(abs(max(Test40_4)), abs(min(Test40_4)));
Max40_4smooth = max(abs(max(Test40_4smooth)), abs(min(Test40_4smooth)));
Max40_5 = max(abs(max(Test40_5)), abs(min(Test40_5)));
Max40_5smooth = max(abs(max(Test40_5smooth)), abs(min(Test40_5smooth)));

M_40 = [Max40_1 Max40_2 Max40_3 Max40_4 Max40_5];
M_40 = mean(M_40);

G_4 = [Max40_1smooth; Max40_2smooth; Max40_3smooth; Max40_4smooth;...
       Max40_5smooth];
G_40 = mean(G_4);

% Determine 95% confidence interval of data
pd40 = fitdist(G_4,'Normal');
ci40 = paramci(pd40);

% Plot raw and smooth data
figure('Name','40 Degrees')
plot(Test40_time,Test40_3,'-','LineWidth',1)
hold on
plot(Test40_time,Test40_3smooth,'-','LineWidth',1)
hold on
xlim([-0.01 0.09])
title('Acceleration v. Time for 40 Degrees at 10 kHz')
xlabel('Time [s]')
ylabel('Acceleration [G-forces]')
legend('Raw Data','Smoothed Data')
str = {'Max Measured Acceleration = ' M_40,...
'Mean Moving Average Acceleration = ' G_40);
annotation('textbox', [.5 .7 .1 .1], 'String', str, 'FitBoxToText', 'on');

% 50 Degrees

% Create matrices for time and accelerometer
Test50 = csvread('50Degrees.csv', 1, 0);
Test50_time = Test50([1:end], [1]);
Test50_1 = Test50([1:end], [3]);
Test50_2 = Test50([1:end], [6]);
Test50_3 = Test50([1:end], [9]);
Test50_4 = Test50([1:end], [12]);
Test50_5 = Test50([1:end], [15]);

% Determine window size for moving average by plotting the sum of absolute
% differences for different window sizes and finding the knee in the curve,
% where the window is smallest and the curve seems to flatten. Maximum
% window size is calculated from a 10 kHz sample rate over the duration of
% the impulse.
windowSizes = 1:1:89;
for k = 1: length(windowSizes);
    smoothed1 = movmean(Test50_1, windowSizes(k));
    sad1(k) = sum(abs(smoothed1 - Test50_1));
    smoothed2 = movmean(Test50_2, windowSizes(k));
    sad2(k) = sum(abs(smoothed2 - Test50_2));
    smoothed3 = movmean(Test50_3, windowSizes(k));
    sad3(k) = sum(abs(smoothed3 - Test50_3));
end
smoothed4 = movmean(Test50_4, windowSizes(k));
sad4(k) = sum(abs(smoothed4 - Test50_4));

smoothed5 = movmean(Test50_5, windowSizes(k));
sad5(k) = sum(abs(smoothed5 - Test50_5));

end

figure('Name','50 Degrees - SAD')
subplot(5,1,1);
plot(windowSizes, sad1, 'b+-', 'LineWidth', 2);
title('SAD 50-1')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,2);
plot(windowSizes, sad2, 'b+-', 'LineWidth', 2);
title('SAD 50-2')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,3);
plot(windowSizes, sad3, 'b+-', 'LineWidth', 2);
title('SAD 50-3')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,4);
plot(windowSizes, sad4, 'b+-', 'LineWidth', 2);
title('SAD 50-4')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
% Moving Average Filter
Test50_lsmooth = smoothdata(Test50_1,'movmean',52);
Test50_2smooth = smoothdata(Test50_2,'movmean',52);
Test50_3smooth = smoothdata(Test50_3,'movmean',52);
Test50_4smooth = smoothdata(Test50_4,'movmean',52);
Test50_5smooth = smoothdata(Test50_5,'movmean',52);

% Find maximum accelerations of each run, then average
Max50_1 = max(abs(max(Test50_1)),abs(min(Test50_1)));
Max50_1smooth = max(abs(max(Test50_lsmooth)),abs(min(Test50_lsmooth)));
Max50_2 = max(abs(max(Test50_2)),abs(min(Test50_2)));
Max50_2smooth = max(abs(max(Test50_2smooth)),abs(min(Test50_2smooth)));
Max50_3 = max(abs(max(Test50_3)),abs(min(Test50_3)));
Max50_3smooth = max(abs(max(Test50_3smooth)),abs(min(Test50_3smooth)));
Max50_4 = max(abs(max(Test50_4)),abs(min(Test50_4)));
Max50_4smooth = max(abs(max(Test50_4smooth)),abs(min(Test50_4smooth)));
Max50_5 = max(abs(max(Test50_5)),abs(min(Test50_5)));
Max50_5smooth = max(abs(max(Test50_5smooth)),abs(min(Test50_5smooth)));

M_50 = [Max50_1 Max50_2 Max50_3 Max50_4 Max50_5];
M_50 = mean(M_50);

G_5 = [Max50_lsmooth; Max50_2smooth; Max50_3smooth; Max50_4smooth;...
      Max50_5smooth];
G_50 = mean(G_5);

% Determine 95% confidence interval of data
pd50 = fitdist(G_5,'Normal');
ci50 = paramci(pd50);

% Plot raw and smooth data
figure('Name','50 Degrees')
plot(Test50_time,Test50_3,'-','LineWidth',l)
hold on
plot(Test50_time,Test50_3smooth,'-','LineWidth',1)
hold on
xlim([-0.01 0.09])
title('Acceleration v. Time for 50 Degrees at 10 kHz')
xlabel('Time [s]')
ylabel('Acceleration [G-forces]')
legend('Raw Data','Smoothed Data')
str = {'Max Measured Acceleration = ' M_50,...
      'Mean Moving Average Acceleration = ' G_50};
annotation('textbox',[.5 .7 .1 .1],'String',str,'FitBoxToText','on');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 60 Degrees
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Create matrices for time and accelerometer
Test60 = csvread('60Degrees.csv',1,0);
Test60_time = Test60([1:end],[1]);
Test60_1 = Test60([1:end],[3]);
Test6O_2 = Test6O([1:end],[6]);
Test6O_3 = Test6O([1:end],[9]);
Test6O_4 = Test6O([1:end],[12]);
Test6O_5 = Test6O([1:end],[15]);

% Determine window size for moving average by plotting the sum of absolute differences for different window sizes and finding the knee in the curve, where the window is smallest and the curve seems to flatten. Maximum window size is calculated from a 10 kHz sample rate over the duration of the impulse.
windowSizes = 1 : 1 : 89;
for k = 1 : length(windowSizes);
    smoothed1 = movmean(Test6O_1, windowSizes(k));
    sad1(k) = sum(abs(smoothed1 - Test6O_1));
    smoothed2 = movmean(Test6O_2, windowSizes(k));
    sad2(k) = sum(abs(smoothed2 - Test6O_2));
    smoothed3 = movmean(Test6O_3, windowSizes(k));
    sad3(k) = sum(abs(smoothed3 - Test6O_3));
    smoothed4 = movmean(Test6O_4, windowSizes(k));
    sad4(k) = sum(abs(smoothed4 - Test6O_4));
    smoothed5 = movmean(Test6O_5, windowSizes(k));
    sad5(k) = sum(abs(smoothed5 - Test6O_5));
end

figure('Name','60 Degrees - SAD')
subplot(5,1,1);
plot(windowSizes, sad1, 'b-*', 'LineWidth', 2);
title('SAD 60-1')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,2);
plot(windowSizes, sad2, 'b*-', 'LineWidth', 2);
title('SAD 60-2')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');

subplot(5,1,3);
plot(windowSizes, sad3, 'b*-', 'LineWidth', 2);
title('SAD 60-3')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');

subplot(5,1,4);
plot(windowSizes, sad4, 'b*-', 'LineWidth', 2);
title('SAD 60-4')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');

subplot(5,1,5);
plot(windowSizes, sad5, 'b*-', 'LineWidth', 2);
title('SAD 60-5')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');

% Moving Average Filter
Test60_1smooth = smoothdata(Test60_1, 'movmean', 52);
Test60_2smooth = smoothdata(Test60_2, 'movmean', 52);
Test60_3smooth = smoothdata(Test60_3, 'movmean', 52);
Test60_4smooth = smoothdata(Test60_4, 'movmean', 52);
Test60_5smooth = smoothdata(Test60_5, 'movmean', 52);
% Find maximum accelerations of each run, then average
Max60_1 = max(abs(max(Test60_1)),abs(min(Test60_1)));
Max60_1smooth = max(abs(max(Test60_1smooth)),abs(min(Test60_1smooth)));
Max60_2 = max(abs(max(Test60_2)),abs(min(Test60_2)));
Max60_2smooth = max(abs(max(Test60_2smooth)),abs(min(Test60_2smooth)));
Max60_3 = max(abs(max(Test60_3)),abs(min(Test60_3)));
Max60_3smooth = max(abs(max(Test60_3smooth)),abs(min(Test60_3smooth)));
Max60_4 = max(abs(max(Test60_4)),abs(min(Test60_4)));
Max60_4smooth = max(abs(max(Test60_4smooth)),abs(min(Test60_4smooth)));
Max60_5 = max(abs(max(Test60_5)),abs(min(Test60_5)));
Max60_5smooth = max(abs(max(Test60_5smooth)),abs(min(Test60_5smooth)));

M_60 = [Max60_1 Max60_2 Max60_3 Max60_4 Max60_5];
M_60 = mean(M_60);

G_6 = [Max60_1smooth; Max60_2smooth; Max60_3smooth; Max60_4smooth;...
      Max60_5smooth];
G_60 = mean(G_6);

% Determine 95% confidence interval of data
pd60 = fitdist(G_6,'Normal');
ci60 = paramci(pd60);

% Plot raw and smooth data
figure('Name','60 Degrees')
plot(Test60_time,Test60_3,'-','LineWidth',1)
hold on
plot(Test60_time,Test60_3smooth,'-','LineWidth',1)
hold on
xlim([-0.01 0.09])
title('Acceleration v. Time for 60 Degrees at 10 kHz')
xlabel('Time [s]')
ylabel('Acceleration [G-forces]')
legend('Raw Data','Smoothed Data')
str = {'Max Measured Acceleration = ' M_60,...
       'Mean Moving Average Acceleration = ' G_60};
annotation('textbox',[.5 .7 .1 .1],'String',str,'FitBoxToText','on');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 70 Degrees
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Create matrices for time and accelerometer
Test70 = csvread('70Degrees.csv',1,0);
Test70_time = Test70([1:end],[1]);
Test70_1 = Test70([1:end],[3]);
Test70_2 = Test70([1:end],[6]);
Test70_3 = Test70([1:end],[9]);
Test70_4 = Test70([1:end],[12]);
Test70_5 = Test70([1:end],[15]);

% Determine window size for moving average by plotting the sum of absolute
% differences for different window sizes and finding the knee in the curve,
% where the window is smallest and the curve seems to flatten. Maximum
% window size is calculated from a 10 kHz sample rate over the duration of
% the impulse.
windowSizes = 1 : 1 : 89;
for k = 1 : length(windowSizes);
    smoothed1 = movmean(Test70_1, windowSizes(k));
sad1(k) = sum(abs(smoothed1 - Test70_1));
smoothed2 = movmean(Test70_2, windowSizes(k));
sad2(k) = sum(abs(smoothed2 - Test70_2));
smoothed3 = movmean(Test70_3, windowSizes(k));
sad3(k) = sum(abs(smoothed3 - Test70_3));
smoothed4 = movmean(Test70_4, windowSizes(k));
sad4(k) = sum(abs(smoothed4 - Test70_4));
smoothed5 = movmean(Test70_5, windowSizes(k));
sad5(k) = sum(abs(smoothed5 - Test70_5));
end

figure('Name','70 Degrees - SAD')
subplot(5,1,1);
plot(windowSizes, sad1, 'b*-', 'LineWidth', 2);
title('SAD 70-1')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,2);
plot(windowSizes, sad2, 'b*-', 'LineWidth', 2);
title('SAD 70-2')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,3);
plot(windowSizes, sad3, 'b*-', 'LineWidth', 2);
title('SAD 70-3')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
subplot(5,1,4);
plot(windowSizes, sad4, 'b-*', 'LineWidth', 2);

```matlab
title('SAD 70-4')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');
```

subplot(5,1,5);

plot(windowSizes, sad5, 'b-*', 'LineWidth', 2);

title('SAD 70-5')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');

% Moving Average Filter

```matlab
Test70_1smooth = smoothdata(Test70_1, 'movmean', 52);
Test70_2smooth = smoothdata(Test70_2, 'movmean', 52);
Test70_3smooth = smoothdata(Test70_3, 'movmean', 52);
Test70_4smooth = smoothdata(Test70_4, 'movmean', 52);
Test70_5smooth = smoothdata(Test70_5, 'movmean', 52);
```

% Find maximum accelerations of each run, then average

```matlab
Max70_1 = max(abs(max(Test70_1)), abs(min(Test70_1)));
Max70_1smooth = max(abs(max(Test70_1smooth)), abs(min(Test70_1smooth)));
Max70_2 = max(abs(max(Test70_2)), abs(min(Test70_2)));
Max70_2smooth = max(abs(max(Test70_2smooth)), abs(min(Test70_2smooth)));
Max70_3 = max(abs(max(Test70_3)), abs(min(Test70_3)));
Max70_3smooth = max(abs(max(Test70_3smooth)), abs(min(Test70_3smooth)));
Max70_4 = max(abs(max(Test70_4)), abs(min(Test70_4)));  
Max70_4smooth = max(abs(max(Test70_4smooth)), abs(min(Test70_4smooth)));  
Max70_5 = max(abs(max(Test70_5)), abs(min(Test70_5)));  
Max70_5smooth = max(abs(max(Test70_5smooth)), abs(min(Test70_5smooth)));  
```
\[ M_{70} = [\text{Max70}_1 \text{ Max70}_2 \text{ Max70}_3 \text{ Max70}_4 \text{ Max70}_5]; \]
\[ M_{70} = \text{mean}(M_{70}); \]
\[ G_7 = [\text{Max70}_1\text{smooth}; \text{Max70}_2\text{smooth}; \text{Max70}_3\text{smooth}; \text{Max70}_4\text{smooth};... \text{Max70}_5\text{smooth}]; \]
\[ G_{70} = \text{mean}(G_7); \]

% Determine 95\% confidence interval of data
\[ \text{pd70} = \text{fitdist}(G_7, 'Normal'); \]
\[ \text{ci70} = \text{paramci}(\text{pd70}); \]

% Plot raw and smooth data
\[ \text{figure('Name','70 Degrees')} \]
\[ \text{plot(}\text{Test70\_time,Test70\_3,'-}', 'LineWidth', 1) \]
\[ \text{hold on} \]
\[ \text{plot(}\text{Test70\_time,Test70\_3smooth,'-}', 'LineWidth', 1) \]
\[ \text{hold on} \]
\[ \text{xlim([-0.01} \text{ 0.09])} \]
\[ \text{title('Acceleration v. Time for 70 Degrees at 10 kHz')} \]
\[ \text{xlabel('Time [s]')} \]
\[ \text{ylabel('Acceleration [G-forces]')} \]
\[ \text{legend('Raw Data','Smoothed Data')} \]
\[ \text{str = {'}Max Measured Acceleration = ' M_{70},...\}
\[ \text{'Mean Moving Average Acceleration = ' G_{70});} \]
\[ \text{annotation('textbox', [.5 .7 .1 .1], 'String', str, 'FitBoxToText', 'on');} \]

% Plot the acceleration versus angle
\[ \text{Angle} = [10; 20; 30; 40; 50; 60; 70]; \]
Acc = [G_10; G_20; G_30; G_40; G_50; G_60; G_70];
myfit = fit(Angle,Acc,'poly2');

figure('Name','Acceleration v. Angle')
plot(Angle,Acc,'+')
hold on
plot(myfit)
hold on

title('Acceleration v. Time for 70 Degrees at 10 kHz')
xlabel('Angle [deg]')
ylabel('Acceleration [G-forces]')
legend('Location','northwest','Mean Maximum Accelerations')
Appendix D

Full Scale Testing Analysis

Mathematical Code

% This script plots the results of the full scale drop test

clf
clear all
close all
clc

% Run 1 Results with 70G and 500G Sensors

% Create matrices for time, 70G data, and 500G data
Run1 = csvread('Runl.csv',1,0);

Run1_time = Run1([1:end], [1]);

Run1_time = Run1_time - Run1_time(1,1);

Run1_70 = Run1([1:end], [2]);

Run1_500 = Run1([1:end], [3]);

% Determine window size for moving average by plotting the sum of absolute differences for different window sizes and finding the knee in the curve, where the window is smallest and the curve seems to flatten. Maximum window size is calculated from a 10 kHz sample rate over the duration of the impulse.

windowSizes = 1 : 1 : 26;

for k = 1 : length(windowSizes);
    smoothed70 = movmean(Run1_70, windowSizes(k));
    sad70(k) = sum(abs(smoothed70 - Run1_70));
    smoothed500 = movmean(Run1_500, windowSizes(k));
    sad500(k) = sum(abs(smoothed500 - Run1_500));
end

figure('Name','Run 1 - SAD')
%subplot(2,1,1);
%plot(windowSizes, sad70, 'b*-','LineWidth', 2);
%title('SAD Run 1 70G')
%grid on;
%xlabel('Window Size');
%ylabel('Sum of Absolute Differences');
%subplot(2,1,2);
plot(windowSizes, sad500, 'b*-','LineWidth', 2);
title('SAD Run 1 500G')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');

% Moving Average Filter
Run1_70smooth = smoothdata(Run1_70,'movmean',4);
Run1_500smooth = smoothdata(Run1_500,'movmean',7);

% Find maximum accelerations captured by each sensor
Max_70_1 = max(abs(max(Run1_70)),abs(min(Run1_70)));
Max_500_1 = max(abs(max(Run1_500)),abs(min(Run1_500)));
Max_70_1_smooth = max(abs(max(Run1_70smooth)),abs(min(Run1_70smooth)));
Max_500_1_smooth = max(abs(max(Run1_500smooth)),abs(min(Run1_500smooth)));

% Plot raw and smooth data on single figure each
figure('Name','Run 1 - 70G Sensor')
plot(Run1_time,Run1_70,'-o','LineWidth',1,'color','k')
hold on
plot(Run1_time,Run1_70smooth,'-d','LineWidth',1,'color','r')
hold on

figure('Name','Run 1 - 500G')
plot(Run1_time,Run1_500,'-o','LineWidth',1,'color','k')
hold on
plot(Run1_time,Run1_500smooth,'-d','LineWidth',1,'color','r')
hold on

title('Acceleration v. Time for Full Scale Drop Test from 0.04445 m')
xlabel('Time [s]')
ylabel('Acceleration [G-forces]')
legend('500G Sensor Data','500G Moving Average')
str = {'Max Measured Acceleration 500G=' Max_500_1,...
    'Max Moving Average Acceleration 500G=' Max_500_1_smooth};
annotation('textbox',[.2 .75 .1 .1], 'String',str,'FitBoxToText','on');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Run 2 Results with 70G and 500G Sensors
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Create matrices for time, 70G data, and 500G data
Run2 = csvread('Run2.csv',1,0);
Run2_time = Run2([1:end],[1]);
Run2_time = Run2_time - Run2_time(1,1);
Run2_70 = Run2([1:end],[2]);
Run2_500 = Run2([1:end],[3]);

% Determine window size for moving average by plotting the sum of absolute
% differences for different window sizes and finding the knee in the curve,
% where the window is smallest and the curve seems to flatten. Maximum
% window size is calculated from a 10 kHz sample rate over the duration of
% the impulse.
windowSizes = 1 : 1 : 26;
for k = 1 : length(windowSizes);
    smoothed70 = movmean(Run2_70, windowSizes(k));
    sad70(k) = sum(abs(smoothed70 - Run2_70));

smoothed500 = movmean(Run2_500, windowSizes(k));
sad500(k) = sum(abs(smoothed500 - Run2_500));
end

figure('Name','Run 2 - SAD')
% subplot(2,1,1);
% plot(windowSizes, sad70, 'b*-', 'LineWidth', 2);
% title('SAD Run 2 70G')
% grid on;
% xlabel('Window Size');
% ylabel('Sum of Absolute Differences');
% subplot(2,1,2);
plot(windowSizes, sad500, 'b*-', 'LineWidth', 2);
title('SAD Run 2 500G')
gird on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');

% Moving Average Filter
Run2_70smooth = smoothdata(Run2_70, 'movmean', 4);
Run2_500smooth = smoothdata(Run2_500, 'movmean', 7);

% Find maximum accelerations captured by each sensor
Max_70_2 = max(abs(max(Run2_70)), abs(min(Run2_70)));
Max_500_2 = max(abs(max(Run2_500)), abs(min(Run2_500)));
Max_70_2_smooth = max(abs(max(Run2_70smooth)), abs(min(Run2_70smooth)));
Max_500_2_smooth = max(abs(max(Run2_500smooth)), abs(min(Run2_500smooth)));

% Plot raw and smooth data on single figure each
% figure('Name','Run 2 - 70G')
% plot(Run2_time,Run2_70,'-o','LineWidth',1,'color','k')
% hold on
% plot(Run2_time,Run2_70smooth,'-d','LineWidth',1,'color','r')
% hold on
% title('Acceleration v. Time for Full Scale Drop Test from 0.2 m')
% xlabel('Time [s]')
% ylabel('Acceleration [G-forces]')
% legend('70G Sensor Data','70G Moving Average')
% str = {'Max Measured Acceleration 70G= ' Max_70_2,'Max Moving Average Acceleration 70G= ' Max_70_2_smooth};
% annotation('textbox',[.2 .75 .1 .1],'String',str,'FitBoxToText','on');

figure('Name','Run 2 - 500G')
plot(Run2_time,Run2_500,'-o','LineWidth',1,'color','k')
hold on
plot(Run2_time,Run2_500smooth,'-d','LineWidth',1,'color','r')
hold on
title('Acceleration v. Time for Full Scale Drop Test from 0.2032 m')
xlabel('Time [s]')
ylabel('Acceleration [G-forces]')
legend('500G Sensor Data','500G Moving Average')
str = {'Max Measured Acceleration 500G= ' Max_500_2,'Max Moving Average Acceleration 500G= ' Max_500_2_smooth};
annotation('textbox',[.2 .75 .1 .1],'String',str,'FitBoxToText','on');
% Create matrices for time and 500G data
Run3 = csvread('Run3.csv',1,0);
Run3_time = Run3([1:end],[1]);
Run3_time = Run3_time - Run3_time(1,1);
Run3_500 = Run3([1:end],[3]);

% Determine window size for moving average by plotting the sum of absolute
% differences for different window sizes and finding the knee in the curve,
% where the window is smallest and the curve seems to flatten. Maximum
% window size is calculated from a 10 kHz sample rate over the duration of
% the impulse.
windowSizes = 1 : 1 : 26;
for k = 1 : length(windowSizes);
    smoothed500 = movmean(Run3_500, windowSizes(k));
    sad500(k) = sum(abs(smoothed500 - Run3_500));
end

figure('Name','Run 3 - SAD')
plot(windowSizes, sad500, 'b*-', 'LineWidth', 2);
title('SAD Run 3 500G')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');

% Moving Average Filter
Run3_500smooth = smoothdata(Run3_500,'movmean');

% Find maximum accelerations captured by each sensor
Max_500_3 = max(abs(max(Run3_500)),abs(min(Run3_500)));
Max_500_3_smooth = max(abs(max(Run3_500smooth)),abs(min(Run3_500smooth)));

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% Plot raw and smooth data on single figure
figure('Name','Run 3 - 500G')
plot(Run3_time,Run3_500,'-+','LineWidth',1,'color','k')
hold on
plot(Run3_time,Run3_500smooth, '-d','LineWidth',1,'color','r')
hold on

title('Acceleration v. Time for Full Scale Drop Test from 1.524 m')
xlabel('Time [s]')
ylabel('Acceleration [G-forces]')
legend('500G Sensor Data','500G Moving Average')
str = {'Max Measured Acceleration 500G=' Max_500_3,...
'Max Moving Average Acceleration 500G=' Max_500_3_smooth};
annotation('textbox',[.2 .75 .1 .1],'String',str,'FitBoxToText','on');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Run 4 Results with 500G Sensor (70G Cable Severed During Run 3)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Create matrices for time and 500G data
Run4 = csvread('Run4.csv',1,0);
Run4_time = Run4([1:end],[1]);
Run4_time = Run4_time - Run4_time(1,1);
Run4_500 = Run4([1:end],[2]);

% Determine window size for moving average by plotting the sum of absolute
% differences for different window sizes and finding the knee in the curve,
% where the window is smallest and the curve seems to flatten. Maximum
% window size is calculated from a 10 kHz sample rate over the duration of
% the impulse.
windowSizes = 1 : 1 : 26;
for k = 1 : length(windowSizes);
    smoothed500 = movmean(Run4_500, windowSizes(k));
    sad500(k) = sum(abs(smoothed500 - Run4_500));
end

figure('Name','Run 4 - SAD')
plot(windowSizes, sad500, 'b*-', 'LineWidth', 2);
title('SAD Run 4 500G')
grid on;
xlabel('Window Size');
ylabel('Sum of Absolute Differences');

% Moving Average Filter
Run4_500smooth = smoothdata(Run4_500, 'movmean');

% Find maximum accelerations captured by each sensor
Max_500_4 = max(abs(max(Run4_500)), abs(min(Run4_500)));
Max_500_4_smooth = max(abs(max(Run4_500smooth)), abs(min(Run4_500smooth)));

% Plot raw and smooth data on single figure
figure('Name','Run 4 - 500G')
plot(Run4_time,Run4_500,'-+','LineWidth',1,'color','k')
hold on
plot(Run4_time,Run4_500smooth,'-d','LineWidth',1,'color','r')
hold on

title('Acceleration v. Time for Full Scale Drop Test from 0.8128 m')
xlabel('Time [s]')
ylabel('Acceleration [G-forces]')
legend('500G Sensor Data','500G Moving Average')
str = {'Max Measured Acceleration 500G=' Max_500_4,...
'Max Moving Average Acceleration 500G=' Max_500_4_smooth};

annotation('textbox', [.2 0.75 .1 .1], 'String', str, 'FitBoxToText', 'on');
Appendix E

Drawings
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MILLIMETERS
SURFACE FINISH: 16
TOLERANCES:
LINEAR:
ANGULAR:
FINISH:
DEBUR AND BREAK SHARP EDGES
DO NOT SCALE DRAWING

<table>
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<tr>
<th>NAME</th>
<th>SIGNATURE</th>
<th>DATE</th>
<th>MATERIAL:</th>
<th>DWG NO.</th>
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</thead>
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<td>Michael Beautyman</td>
<td>2017.03.21</td>
<td>1018 Cold Rolled Steel</td>
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<td>CHK'D</td>
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<td>Q.A</td>
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</tbody>
</table>

SOLIDWORKS Educational Product. For Instructional Use Only

Shock Test Shaft
SOLIDWORKS Educational Product. For Instructional Use Only

Test Flange

UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MILLIMETERS
SURFACE FINISH: 16
TOLERANCES:
LINEAR:
ANGULAR:

NAME SIGNATURE DATE
DRAWN Michael Beautyman 2017.03.21

TITLE:

MATERIAL:
1018 Cold Rolled Steel

WEIGHT:

SCALE: 1:2

Sheet 1 of 1
NOTE: Please manufacture 2 of this part.

Dimensions and Tolerances:
- Linear Tolerance: +0.004
- Angular Tolerance: 0.020

Surface Finish: 16 inside tape

Material: 1018 Steel

Weight: 8.4 THRU ALL

Rev.

Name: 
Signature: 
Date: 

Title: Beautyman Tilt Flange

Drawn: 
Checked: 
Approved: 
Manufactured:

Revision: A

Sheet 1 of 1
18-8 Shoulder Screw 1/2 diameter x 3" long shoulder, 3/8-16 thread size

Simply Supported Beam

UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
SURFACE FINISH:
TOLERANCES:
LINEAR:
ANGULAR:

DO NOT SCALE DRAWING

REVISION

NAME SIGNATURE DATE TITLE:

MATERIAL:

WEIGHT:

SCALE 1:5

SHEET 1 OF 1
Bibliography


