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Singular optics and topological photonics

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For centuries, singularities in wave fields have been a mainstay of interest in multiple areas of physics, ranging from plasma physics and fluid dynamics to atmospheric physics to optics and photonics [1]. In optics, energy and momentum flow around singularities in the interference field can twist to form vortices, which carry angular momentum [2–4]. **Singular Optics** is the branch of physics that encompasses studies of structured light with localized and extended singularities, such as optical vortices in scalar optical fields and polarization singularities in vector fields [2,5,6]. It has been pioneered by the publication of a seminal paper of J F Nye and M V Berry ''Dislocations in wave trains'' in 1974 [7], and has recently celebrated its 40th anniversary.

Optical vortices—just like their analogs in tidal waves, superfluids, and superconductors—are intriguing phenomena with the rich physics, which offer fascinating applications in classical and quantum optics. For example, linear and angular momentum of photons in structured optical fields can be utilized to achieve non-contact optical trapping and manipulation of microscopic particles and biological molecules [8,9]. The inherent orthogonality of optical modes with different orbital angular momentum uniquely positions them for applications in polarization and wavelength division multiplexing to increase data capacity of both free-space and fiber-optic communications [10]. Diffraction-free and self-healing propagation of optical vortex fields offers further opportunities in this field [11]. Higher information content of structured light fields along with the possibilities for their quantum entanglement can also play a key role in the development of new protocols for quantum information processing [12–14]. Strong variations of optical fields around singularities makes structured light extremely sensitive to small changes in the medium and illumination conditions, which can be exploited for super-resolution imaging [15,16], on-chip optical switching [17–19], (bio)chemical sensing [20,21], spectroscopy [22], solar energy harvesting [23,24], and astronomical observations [25].

The key property of a vortex is the existence of a topological charge, which, in an optical vortex, takes the form of the winding number of an optical wavefront around the vortex core. Because this topological charge is discrete, or "quantized", it remains constant under small deformations or defects in the physical system; it can only be altered by a global change, such as mutual annihilation with an anti-vortex, another vortex of the opposite charge. Conservation of topological charges of optical vortices in structured light fields underlies many unusual optical phenomena, including existence of bound optical states in the radiation continuum [26] and topological transformations of optical metamaterials into a hyperbolic regime [27].

In recent years, researchers have found very different and more abstract types of singularities and topological indices, which can appear in optical systems, based on ideas originating in the physics of topological insulators [28–30]. In a periodic optical medium, such as photonic lattice or crystal, electromagnetic modes form band structures. The energy bands can themselves possess singularities in momentum space, such as Dirac cones in graphene [31] and photonic lattices [32,33], the latter intimately connected with vortices and angular momentum of light [34,35]. Within each band, the optical Bloch wavefunctions form a mathematical structure (a "fiber bundle") defined over the Brillouin zone, which possesses quantized topological charges. Just as a vortex cannot be eliminated by small deformations, a "topologically non-trivial" band possessing non-zero topological charge cannot be smoothly converted to a "topologically trivial" band.

The field of **Topological Photonics** aims to engineer devices to have topologically non-trivial photonic band structures, and to exploit their unusual and striking physical properties. In particular, 2D devices can exhibit topological edge states that circulate in a single direction along their boundaries, and are immune to back-scattering from defects or shape deformations. This was first demonstrated in 2009, using a magnetic photonic crystal operating at microwave frequencies [36]. Thereafter, topological edge states have been observed at the technologically crucial infrared-to-visible frequency range, using helical waveguide lattices and ring resonator lattices [37,38]. A wide variety of other platforms are now undergoing active investigation, including microwave circuits [39,40] and polaritonic resonator lattices [41,42].

This special issue aims to survey the state-of-the-art in Singular Optics and Topological Photonics and to chart new research directions. The issue is a contribution to the celebration of the United Nations observance of The International Year of Light and Light-based Technologies, IYL 2015. It is also dedicated to the two remarkable anniversaries celebrated in 2014, the 40th anniversary of the pioneering paper by J. F. Nye and M. V. Berry [7] and the 85th birthday of Prof. Marat Soskin, who has tirelessly spearheaded Singular Optics as an independent research field for the last 15 years [5].

This special issue contains 31 papers, which highlight the latest developments in this burgeoning research field and bridge fundamental concepts with emerging applications. The issue begins with an article by Barnett and co-authors, in which they lay down a comprehensive overview of the **nature of the spin and orbital parts of optical angular momentum** [43].

Next, several articles address the issues of **non-diffracting beam propagation, effects of turbulence, and beam stabilization**. In particular, Garcia-Gracia and Gutiérrez-Vega introduce a family of non-diffracting full Poincaré beams based on a superposition of non-diffracting Mathieu beams [44]. Makris and co-authors theoretically study superoscillatory superpositions of vectorial Bessel beams, which are diffraction-free and can support subwavelength features in their transverse electromagnetic fields, without the presence of evanescent waves [45]. Izdebskaya and colleagues study experimentally and numerically vortex stabilization in non-local media by using co-propagating spatial solitons [46]. Two papers discuss the effects of turbulence in the media on the angular momentum and quantum information content of the light beams. Aksenov *et al* numerically explore laser beams propagation through turbulent medium and show that the relative fluctuations of the orbital angular momentum (OAM) decrease with the increase of the initial topological charge of the beam, which offers low-noise method of optical communication based on optical vortices [47]. In turn, Goyal and co-authors experimentally study the secret key rate for quantum key distribution protocols in OAM for free space quantum communication in the presence of turbulence, and find that a quantum key can be securely distributed over distances comparable to those over which the entanglement survives [48]. Morgan *et al* investigate propagation of modulated multi-petal concentric optical vortex beams through turbid environments to explore the use of beams with orbital angular momentum in underwater free-space communications links [49].

The next two papers investigate the **free-space optical channels multiplexing** using twisted light beams. Willner and colleagues discuss design challenges and guidelines in using OAM multiplexing of multiple beams in free-space communications links [50]. Rodenburg *et al* experimentally demonstrate an interferometric protocol for multiplexing optical states of light, which enables preparation of either coherent superpositions or statistical mixtures of OAM states [51].

Interaction of structured light with matter has been studied by several groups in the context of forming hybrid plasmon-polariton or exciton-polariton vortex states and exciting atoms by twisted photons. Dzedolik and co-authors report on the formation of plasmon polariton vortex lattices on the metal surface formed the interference of the surface plasmon polaritons scattered by curvilinear boundaries [52]. Afanasev and colleagues calculate transition amplitudes and cross sections for excitation of hydrogen-like atoms by twisted photon states, and predict the transition rates into the high-OAM states to be comparable with the rates for electric dipole transitions if the atom is located near the phase singularity [53]. Schultz and colleagues employ a coherent two-photon Raman interaction to interface singular optical beams with a pseudo-spin-1/2 Bose-Einstein condensate, which serves as a q-plate for the atoms enabling conversion between atomic spin and orbital angular momentum states [54]. Rosanov *et al* investigate and compare dissipative vortex solitons and their complexes in wide-aperture lasers and in exciton-polariton lasers, and show that while the energy is stored both in the electromagnetic field and in the lasing medium, media with fast response can follow the topology of the field without inertia [55]. In turn, Yaparov and Taranenko report on the theoretical and experimental work on spatial solitons formed in optical bistable oscillators with laser or/and parametric gain, with the focus on the solitons dynamical properties [56].

Tailored light-matter interactions have also been shown to enable **shaping, switching and modulation of structured light**. Larocque et al demonstrate liquid crystal devices for tailoring the wavefront of optical beams through the Pancharatnam-Berry phase concept and generate an extensive range of shaped optical beams [57]. Silahli and colleagues investigate the differences in the structured light interactions with positive and negative-index materials, and predict azimuthal modulation instability of optical vortices with different topological charges in nonlinear negative-index materials [58]. Laudyn and co-authors study spatial solitons in chiral nematic liquid crystal (nematicons) and experimentally achieve all-optical and electro-optical soliton steering via interactions with structure defects [59]. Belyi *et al* report on the generation of wavefront phase dislocations of vortex Bessel light beams under acousto-optic diffraction in uniaxial crystals [60]. Bekshaev and colleagues investigate the diffraction of a circularly-symmetric optical vortex light beam on an opaque screen with a sharp edge, with the aim to use the fine structure details in the optical vortex trajectories to reveal the nature and spatial configuration of beams for metrological applications [61]. Grigoriev and co-authors analytically describe the spatial distribution of the polarization of a second harmonic of the beam with a singularity reflected from the surface of an isotropic gyrotropic medium [62]. Vasylkiv et al report on the experimental method for identification of the topological defects of optical indicatrix orientation, which appear in the glass samples with residual mechanical stresses, via generation of polarization optical singularities and optical vortices in the propagating optical beams [63].

Light-matter interactions can also be tailored to achieve **localization of optical vortices and solitons** in linear or non-linear potential wells. Hoq and colleagues explore the existence, stability and dynamics of localized states (especially multi-site solitonic and vortex states) in anisotropic hexagonal and honeycomb lattices [64]. Knitter *et al* demonstrate formation of strongly confined optical resonances with high quality factors by combining a topological defect with a photonic crystal defect cavity, and achieve optically-pumped lasers with circulating near-field energy flow [65]. Dror and Malomed investigate self-trapping of topological modes with effective single- and double-well nonlinear potentials induced by spatial modulation of the local strength of the self-defocusing nonlinearity, and observe spontaneous formation of self-trapped modes, whose symmetry may be lower than that of the underlying modulation pattern [66].

Polarization pattern formation around singularities in optical fields have been studied in several papers, with the focus on the high-order singularities, whose properties have been less explored to date. In particular, Khajavi and Galvez investigate formation and transformations of first- and high-order disinclination patterns (including, lemon, star and monstar) via spatially-variable polarization of a light beam [67]. Vasnetsov and colleagues analyze the origin and morphological transformations of topological structures which appear in a vector space-variant field around points of circular polarization, and analytically derive the conditions of the existence a monstar-type point [68]. Otte et al experimentally study dynamical shaping of higher-order polarization singularities embedded in tailored vector beams, resulting in the generation of light fields associated with flowers, spider webs, or their hybrid networks, which offer applications in spatially extended optical tweezing and imaging [69].

Phase singularities in stochastic optical fields have been investigated to reveal the topological charge conservation and vortex screening effects. Roux derives the topological charge current for scalar stochastic optical fields and shows that it obeys a conservation equation, which enables computing the full topological charge current for any monochromatic, paraxial normally distributed stochastic optical field [70].

New **analytical methods and numerical modeling techniques** reveal novel aspects of structured and topological light propagation in free space, inside optical fibers, and through coherent optical networks. Ferrando and García-March present a novel procedure for solving the Schrödinger equation for multisingular vortex Gaussian beam, and demonstrate the power of the new technique on the problems of an initial Gaussian beam with two positive singularities and a negative one embedded in it as well as of a discrete-Gauss state [71]. Salguiero calculates different families of first-order two-component vector solitons existing in a solid-core photonic crystal fiber with the Kerr nonlinearity and analyzes the stability of the modes, reaching the conclusion that there exists a region with fully stable vortex modes [72]. Finally, Chong and Rechtsman propose a technique to realize a tachyonic band structure in a coherent network – such as an array of coupled ring resonators – by adding parity-time symmetric spatially balanced gain and loss to each node of the network, which opens the door to engineering optical delay lines of wide tunability with low loss [73]. Unlike the tight-binding models commonly used in condensed-matter and optical physics, a 'network model' does not describe a lattice in terms of a Hamiltonian and instead uses an evolution matrix to describe the propagation of waves through a network of directed links and nodes.

It is our hope that this collection of papers will help to trigger new ideas and to chart new research directions in this rapidly expanding field of structured and topological light by showcasing new advances in theory and experiment and highlighting new applications in free-space and on-chip optical communications, quantum cryptography, bio(chemical) sensing, metrology, and nano-manipulation.

[1] Berry M 2000 Making waves in physics *Nature* **403** 21

[2] Dennis M R, O'Holleran K and Padgett M J 2009 Chapter 5 Singular Optics: Optical Vortices and Polarization Singularities *Prog. Opt.* **53** 293–363

[3] Leach J, Dennis M R, Courtial J and Padgett M J 2005 Vortex knots in light *New J. Phys.* **7** 55

[4] Dennis M R, Kivshar Y S, Soskin M S and Swartzlander Jr G a 2009 Singular Optics: more ado about nothing *J. Opt. A Pure Appl. Opt.* **11** 90201

[5] Soskin M S and Vasnetsov M V 2001 Singular optics *Prog. Opt.* **42** 219–76

[6] Desyatnikov A S, Torner L and Kivshar Y S 2005 Optical vortices and vortex solitons *Progress in Optics, 47*

[7] Nye J F and Berry M V 1974 Dislocations in wave trains *Proc. R. Soc. London A. Math. Phys. Sci.* **336** 165–90

[8] Taylor M A, Waleed M, Stilgoe A B, Rubinsztein-Dunlop H and Bowen W P 2015 Enhanced optical trapping via structured scattering *Nat. Photonics* **9** 669–73

[9] Dholakia K and Čižmár T 2011 Shaping the future of manipulation *Nat. Photonics* **5** 335–42

[10] Bozinovic N, Yue Y, Ren Y, Tur M, Kristensen P, Huang H, Willner A E and Ramachandran S 2013 Terabit-scale orbital angular momentum mode division multiplexing in fibers. *Science* **340** 1545–8

[11] Mazilu M, Stevenson D J, Gunn-Moore F and Dholakia K 2009 Light beats the spread: "non-diffracting" beams *Laser Photon. Rev.* **4** 529–47

[12] Molina-Terriza G, Torres J P and Torner L 2007 Twisted photons *Nat. Phys.* **3** 305–10

[13] Peacock A C, Steel M J, Hensen B, Reimer C, Migdall A, Polyakov S, Fan J, Bienfang J, Clark A S, Grassani D, Silverstone J W, Franson J D, Humphreys P C, Xiong C, Carolan J and Harris N C 2016 QUANTUM OPTICS. The time is right for multiphoton entangled states. *Science* **351** 1152–3

[14] Mair A, Vaziri A, Weihs G and Zeilinger A 2001 Entanglement of the orbital angular momentum states of photons *Nature* **412** 313–6

[15] Maurer P C, Maze J R, Stanwix P L, Jiang L, Gorshkov A V., Zibrov A A, Harke B, Hodges J S, Zibrov A S, Yacoby A, Twitchen D, Hell S W, Walsworth R L and Lukin M D 2010 Far-field optical imaging and manipulation of individual spins with nanoscale resolution *Nat. Phys.* **6** 912–8

[16] D'Aguanno G, Mattiucci N, Bloemer M and Desyatnikov A 2008 Optical vortices during a superresolution process in a metamaterial *Phys. Rev. A* **77** 43825

[17] Boriskina S V. and Reinhard B M 2012 Molding the flow of light on the nanoscale: from vortex nanogears to phase-operated plasmonic machinery *Nanoscale* **4** 76

[18] Boriskina S V. and Reinhard B M 2011 Adaptive on-chip control of nano-optical fields with optoplasmonic vortex nanogates *Opt. Express* **19** 22305

[19] Kim H, Park J, Cho S-W, Lee S-Y, Kang M and Lee B 2010 Synthesis and dynamic switching of surface plasmon vortices with plasmonic vortex lens *Nano Lett.* **10** 529–36

[20] Maeda E, Lee Y, Kobayashi Y, Taino A, Koizumi M, Fujikawa S and Delaunay J-J 2012 Sensitivity to refractive index of high-aspect-ratio nanofins with optical vortex. *Nanotechnology* **23** 505502

[21] Svedendahl M, Verre R and Käll M 2014 Refractometric biosensing based on optical phase flips in sparse and short-range-ordered nanoplasmonic layers *Light Sci. Appl.* **3** e220

[22] Ahn W, Boriskina S V, Hong Y and Reinhard B M 2012 Electromagnetic field enhancement and spectrum shaping through plasmonically integrated optical vortices *Nano Lett.* **12** 219–27

[23] Kuang P, Eyderman S, Hsieh M-L, Post A, John S and Lin S-Y 2016 Achieving an accurate surface profile of a photonic crystal for near-unity solar absorption in a super thin-film

architecture *ACS Nano* acsnano.6b01875

[24] Boriskina S V., Ghasemi H and Chen G 2013 Plasmonic materials for energy: From physics to applications *Mater. Today* **16** 375–86

[25] Lavery M P J, Speirits F C, Barnett S M and Padgett M J 2013 Detection of a spinning object using light's orbital angular momentum. *Science* **341** 537–40

[26] Zhen B, Hsu C W, Lu L, Stone A D and Soljačić M 2014 Topological nature of optical bound states in the continuum *Phys. Rev. Lett.* **113** 257401

[27] Tong J, Mercedes A, Chen G and Boriskina S V. 2014 Local field topology behind light localization and metamaterial topological transitions *Singular and Chiral Nanoplasmonics* ed S V. Boriskina and N I Zheludev (Pan Stanford) pp 259–84

[28] Peano V, Brendel C, Schmidt M and Marquardt F 2015 Topological phases of sound and light *Phys. Rev. X* **5** 31011

[29] Lu L, Joannopoulos J D and Soljačić M 2014 Topological photonics *Nat. Photonics* **8** 821–9

[30] Chong Y 2013 Optical devices: Photonic insulators with a twist *Nature* **496** 173–4

[31] Novoselov K S, Geim A K, Morozov S V., Jiang D, Katsnelson M I, Grigorieva I V., Dubonos S V. and Firsov A A 2005 Two-dimensional gas of massless Dirac fermions in graphene *Nature* **438** 197–200

[32] Leykam D and Desyatnikov A S 2016 Conical intersections for light and matter waves *Adv. Phys. X* **1** 101–13

[33] Boriskina S V. 2015 Quasicrystals: Making invisible materials *Nat. Photonics* **9** 422–4

[34] Song D, Paltoglou V, Liu S, Zhu Y, Gallardo D, Tang L, Xu J, Ablowitz M, Efremidis N K and Chen Z 2015 Unveiling pseudospin and angular momentum in photonic graphene *Nat. Commun.* **6** 6272

[35] Diebel F, Leykam D, Kroesen S, Denz C and Desyatnikov A S 2016 Conical diffraction and composite Lieb bosons in photonic lattices *Phys. Rev. Lett.* **116** 183902

[36] Wang Z, Chong Y, Joannopoulos J D and Soljačić M 2009 Observation of unidirectional backscattering-immune topological electromagnetic states *Nature* **461** 772–5

[37] Rechtsman M C, Zeuner J M, Plotnik Y, Lumer Y, Podolsky D, Dreisow F, Nolte S, Segev M and Szameit A 2013 Photonic Floquet topological insulators *Nature* **496** 196–200

[38] Khanikaev A B, Hossein Mousavi S, Tse W-K, Kargarian M, MacDonald A H and Shvets G 2012 Photonic topological insulators *Nat. Mater.* **12** 233–9

[39] Chen W-J, Jiang S-J, Chen X-D, Zhu B, Zhou L, Dong J-W and Chan C T 2014 Experimental realization of photonic topological insulator in a uniaxial metacrystal waveguide *Nat. Commun.* **5** 5782

[40] Hu W, Pillay J C, Wu K, Pasek M, Shum P P and Chong Y D 2015 Measurement of a topological edge invariant in a microwave network *Phys. Rev. X* **5** 11012

[41] Karzig T, Bardyn C-E, Lindner N H and Refael G 2015 Topological Polaritons *Phys. Rev. X* **5** 31001

[42] Bardyn C-E, Karzig T, Refael G and Liew T C H 2015 Topological polaritons and excitons in garden-variety systems *Phys. Rev. B* **91** 161413

[43] Barnett S M, Allen L, Cameron R P, Gilson C R, Padgett M J, Speirits F C and Yao A M 2016 On the natures of the spin and orbital parts of optical angular momentum *J. Opt.* **18** 64004

[44] Garcia-Gracia H and Gutiérrez-Vega J C 2016 Polarization singularities in nondiffracting Mathieu–Poincaré beams *J. Opt.* **18** 14006

[45] Makris K G, Papazoglou D G and Tzortzakis S 2016 Invariant superoscillatory electromagnetic fields in 3D-space *J. Opt.*

[46] Izdebskaya Y, Krolikowski W, Smyth N F and Assanto G 2016 Vortex stabilization by means of spatial solitons in nonlocal media *J. Opt.* **18** 54006

[47] Aksenov V P, Kolosov V V, Filimonov G A and Pogutsa C E 2016 Orbital angular momentum of a laser beam in a turbulent medium: preservation of the average value and variance of fluctuations *J. Opt.* **18** 54013

[48] Goyal S K, Ibrahim A H, Roux F S, Konrad T and Forbes A 2016 The effect of turbulence on entanglement-based free-space quantum key distribution with photonic orbital angular momentum *J. Opt.* **18** 64002

[49] Morgan K S, Miller J K, Cochenour B M, Li W, Li Y, Watkins R J and Johnson E G 2016 Free space propagation of concentric vortices through underwater turbid environments *J. Opt.* **18** 104004

[50] Willner A E, Xie G, Li L, Ren Y, Yan Y, Ahmed N, Zhao Z, Wang Z, Liu C and Willner A J 2016 Design challenges and guidelines for free-space optical communication links using orbital-angular-momentum multiplexing of multiple beams *J. Opt.* **18** 74014

[51] Rodenburg B, Magaña-Loaiza O S, Mirhosseini M, Taherirostami P, Chen C and Boyd R W 2016 Multiplexing free-space channels using twisted light *J. Opt.* **18** 54015

[52] Dzedolik I V, Lapayeva S and Pereskokov V 2016 Vortex lattice of surface plasmon polaritons *J. Opt.* **18** 74007

[53] Afanasev A, Carlson C E and Mukherjee A 2016 High-multipole excitations of hydrogen-like atoms by twisted photons near a phase singularity *J. Opt.* **18** 74013

[54] Schultz J T, Hansen A, Murphree J D, Jayaseelan M and Bigelow N P 2016 Creating full-Bloch Bose–Einstein condensates with Raman q -plates *J. Opt.* **18** 64009

[55] Rosanov N N and Fedorov S V 2016 Topology of energy fluxes in vortex dissipative soliton structures *J. Opt.* **18** 74005

[56] Yaparov V V and Taranenko V B 2016 Topological solitons in optical oscillators *J. Opt.* **18** 74017

[57] Larocque H, Gagnon-Bischoff J, Bouchard F, Fickler R, Upham J, Boyd R W and Karimi E 2016 Arbitrary optical wavefront shaping via spin-to-orbit coupling *J. Opt.*

[58] Silahli S Z, Walasik W and Litchinitser N M 2016 Modulation instability of structured-light beams in negative-index metamaterials *J. Opt.* **18** 54010

[59] Laudyn U A, Kwasny M, Sala F A and Karpierz M A 2016 All-optical and electro-optical switches based on the interaction with disclination lines in chiral nematic liquid crystals *J. Opt.* **18** 54011

[60] Belyi V N, Khilo P A, Kazak N S and Khilo N A 2016 Transformation of phase dislocations under acousto-optic interaction of optical and acoustical Bessel beams *J. Opt.* **18** 74002

[61] Bekshaev A, Chernykh A, Khoroshun A and Mikhaylovskaya L 2016 Localization and migration of phase singularities in the edge-diffracted optical-vortex beams *J. Opt.* **18** 24011

[62] Grigoriev K S, Makarov V A and Perezhogin I A 2016 Formation of the lines of circular polarization in a second harmonic beam generated from the surface of an isotropic medium with nonlocal nonlinear response in the case of normal incidence *J. Opt.* **18** 14004

[63] Vasylkiv Y, Skab I and Vlokh R 2016 Identification of the topological defects of optical indicatrix orientation in CaB4O7 glasses *J. Opt.* **18** 84006

[64] Hoq Q E, Kevrekidis P G and Bishop A R 2016 Discrete solitons and vortices in anisotropic hexagonal and honeycomb lattices *J. Opt.* **18** 24008

[65] Knitter S, Liew S F, Xiong W, Guy M I, Solomon G S and Cao H 2016 Topological defect lasers *J. Opt.* **18** 14005

[66] Dror N and Malomed B A 2016 Solitons and vortices in nonlinear potential wells *J. Opt.* **18** 14003

[67] Khajavi B and Galvez E J 2016 High-order disclinations in space-variant polarization *J. Opt.* **18** 84003

[68] Vasnetsov M V, Soskin M S, Pas'ko V A and Vasil'ev V I 2016 A Monstar portrait in the interior *J. Opt.* **18** 34003

[69] Otte E, Alpmann C, Denz C, L A D, T D K and Č and Woerdemann M, Alpmann C E M and D C 2016 Higher-order polarization singularitites in tailored vector beams *J. Opt.* **18** 74012

[70] Roux F S 2016 Topological charge conservation in stochastic optical fields *J. Opt.* **18** 54005

[71] Ferrando A and García-March M A 2016 Analytical solution for multi-singular vortex Gaussian beams: the mathematical theory of scattering modes *J. Opt.* **18** 64006

[72] Salgueiro J R 2016 Vector–vortex solitons in nonlinear photonic crystal fibers *J. Opt.* **18** 74004

[73] Chong Y D and Rechtsman M C 2016 Tachyonic dispersion in coherent networks *J. Opt.* **18** 14001