A FRAMEWORK FOR DETERMINING THE MECHANICAL PROPERTIES OF DISSIMILAR MATERIAL JOINTS

by

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ABSTRACT

An overview of the various techniques influencing the performance of dissimilar material joints is presented. Currently, there is no single analytical model capable of describing the elastic stresses arising from the interaction of two materials to a change in temperature. The applicability of continuum techniques is made difficult by the presence of singularities and the influence of geometry on mechanical loading. Numerical approaches are not immune to difficulties either. Due to the influence of singularities, peak stress metrics are very susceptible to the model formulation within finite element analysis. An alternative metric, based on strain energy, was shown to offer much higher precision while incorporating the effect of material properties, joint geometry and bonding temperature. In the past, many modeling efforts have been limited by the ability to predict and incorporate realistic mechanical properties which are valid over a wide range of temperature. Through a combination of deformation mechanism maps and dislocation mechanics, a material behavior model was developed which is sensitive to the dominant effects present in dissimilar material joints. The validity of this approach was evaluated by examining a number of case-studies and by mechanically testing copper : molybdenum dissimilar material joints fabricated by vacuum brazing.

Thesis Supervisor: Dr. Thomas W. Eagar
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<tr>
<td>$\alpha$</td>
<td>coefficient of thermal expansion</td>
</tr>
<tr>
<td>$\alpha_{\text{Taylor}}$</td>
<td>crystallographic geometric factor</td>
</tr>
<tr>
<td>$b$</td>
<td>Burgers vector</td>
</tr>
<tr>
<td>$\beta$</td>
<td>elastic axis in Dundurs space</td>
</tr>
<tr>
<td>$c$</td>
<td>dislocation line pinning constant</td>
</tr>
<tr>
<td>$C_n$</td>
<td>specific heat</td>
</tr>
<tr>
<td>$d$</td>
<td>density</td>
</tr>
<tr>
<td>$D_A$</td>
<td>core diffusion flux</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>strain</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear modulus</td>
</tr>
<tr>
<td>$h$</td>
<td>heat transfer coefficient</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Dundurs geometry dependent factor</td>
</tr>
<tr>
<td>$k$</td>
<td>spring constant</td>
</tr>
<tr>
<td>$k_{\text{thermal}}$</td>
<td>thermal conductivity</td>
</tr>
<tr>
<td>$F$</td>
<td>force</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>singularity index</td>
</tr>
<tr>
<td>$l$</td>
<td>differential length</td>
</tr>
<tr>
<td>$L$</td>
<td>thickness</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$\phi$</td>
<td>elastic axis in Dundurs space</td>
</tr>
<tr>
<td>$r$</td>
<td>radial distance</td>
</tr>
<tr>
<td>$\rho$</td>
<td>dislocation density</td>
</tr>
<tr>
<td>$R$</td>
<td>gas constant</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>stress</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
</tr>
<tr>
<td>$T_i$</td>
<td>initial temperature</td>
</tr>
<tr>
<td>$T_f$</td>
<td>temperature of quenching medium</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle</td>
</tr>
<tr>
<td>$\theta_{\text{edge}}$</td>
<td>Joint edge angle</td>
</tr>
<tr>
<td>$\theta_{\text{interface}}$</td>
<td>Joint interface angle</td>
</tr>
<tr>
<td>$U$</td>
<td>Strain energy</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>cell volume</td>
</tr>
<tr>
<td>$w$</td>
<td>width of a component</td>
</tr>
<tr>
<td>$x$</td>
<td>direction in cartesian coordinates</td>
</tr>
<tr>
<td>$z$</td>
<td>height in cylindrical coordinates</td>
</tr>
</tbody>
</table>
1. An Overview of Dissimilar Materials Joining

Abstract
Dissimilar material joints are becoming increasingly prevalent as the inherent limitations of conventional materials and their designs are reached. In this chapter, an overview of the literature establishes the techniques used to create these joints and the various chemical and mechanical concerns facing them. While many limitations due to chemical concerns have been solved, the mechanics of these systems are still poorly understood. A more thorough understanding of their properties and limitations will be created in subsequent chapters through the development of a tractable mechanical theory.

Introduction
New joining technologies are needed for advanced structural materials to realize their full potential. Unfortunately, the designs which utilize these advanced materials often suffer from many joining restrictions. As the use of materials becomes increasingly application-specific, the number of joints and the number of dissimilar material combinations increases. The combination of these factors creates a bottleneck which makes many material combinations unfeasible to assemble. A need to further understand the factors which limit the ability to join these materials is paramount for increasing their use and reliability.

When joining dissimilar materials, several important factors must be considered: chemical compatibility of materials, mechanical properties of the materials and overall joint design. Throughout the years, the issues associated with chemical compatibility have dominated much of the research and development work. As a result, many of the joining limitations that can be ascribed to these problems have been alleviated. Further advances in establishing reliable joining techniques requires a consideration of the mechanical behavior of materials and the effects of joint geometry. Both the paucity of material behavior data over wide temperature ranges and the complexity needed to accurately model mechanical
property evolution such as dynamic strain-hardening have stymied many modeling
attempts. By virtue of the detail required to describe a particular joining system, these
sophisticated analyses often obscure a more qualitative understanding of the processes at
hand.

It is the purpose of this thesis to expand the understanding of the factors which are critical
to the mechanical performance of dissimilar material joints. This initial chapter establishes
an overview of joining techniques and chemical concerns facing dissimilar material joints.
Chapter 2 examines the applicability of the various analytical techniques used to model
dissimilar material joints. Currently, there is no single analytical model capable of
describing the elastic stresses arising from the interaction of two materials to a change in
temperature. The applicability of continuum techniques is made difficult by the presence of
singularities and the influence of geometry on mechanical loading. Chapter 3 discusses the
many issues present in numerical modeling of dissimilar material joints. Frequently, a
metric such as peak stress is used to describe the strength or stress state within a joint.
Because of the influence of singularities, stress metrics are very susceptible to the setup of
the problem. An alternative metric, based on strain energy, was shown to be very precise.
Based on this result, an analytical method for predicting the elastic strain energy in a joint
which incorporates the effects of material properties, joint geometry and bonding
temperature is developed in Chapter 4. The ability to predict and incorporate realistic
mechanical properties has been a limitation present in many modeling efforts. Chapter 5
addresses this problem by developing a plastic mechanical model, sensitive to the dominant
effects present in dissimilar material joints which is valid over a wide range of
temperatures. Ultimately, the validity of this approach is evaluated by examining a number
of case-studies and by mechanically testing dissimilar material joints fabricated by vacuum
brazing—Chapter 6. A conclusion and a discussion of future work is presented in Chapter
7.
Joining Techniques

There are a number of techniques for joining materials, of which, bonding temperature is an effective distinguishing feature—Table 1.1 (Suganuma, 1990). Adhesive, cementicious, and mechanical joining are room temperature techniques, while joining occurs at an elevated temperature for the other methods. The entire assembly may be heated in the case of brazing, eutectic and diffusion bonding, or the heating may be localized, as in the case of fusion, induction and friction welding. Generally, procedures for joining dissimilar materials are limited to the elevated temperature processes. Of these, brazing and diffusion bonding are the most widely used (Santella, 1992).

Brazing requires a high melting interlayer material that wets both components of the joining system. The creation of the liquid phase reduces the degree of surface preparation required for solid-state bonding. Typically, the components are placed in a furnace and compressed under a slight load until the braze has flowed throughout the joint. Both diffusion bonding and brazing are often performed in vacuum or other protective atmospheres to enhance wetting and avoid oxidation of reactive components.

Diffusion bonding, which is a form of solid-state bonding, occurs at a temperature low enough that the components do not melt, yet high enough that diffusion is rapid. Joining

Table 1.1. Comparison of several joining method (Suganuma, 1990)

<table>
<thead>
<tr>
<th>Low temperature processes</th>
<th>Adhesive</th>
<th>Cement</th>
<th>Mechanical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strength (MPa)</td>
<td>20-80</td>
<td>&lt;10</td>
<td>10-50</td>
</tr>
<tr>
<td>Maximum Temperature (°C)</td>
<td>175</td>
<td></td>
<td>525</td>
</tr>
<tr>
<td>Cost</td>
<td>Low</td>
<td>Low</td>
<td>Low-medium</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High temperature processes</th>
<th>Brazing</th>
<th>Eutectic</th>
<th>Diffusion</th>
<th>Fusion</th>
<th>Friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strength (MPa)</td>
<td>100-500</td>
<td>200-400</td>
<td>100-1000</td>
<td>50-200</td>
<td>50-200</td>
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<tr>
<td>Maximum Temperature (°C)</td>
<td>525</td>
<td>925</td>
<td>1025</td>
<td>1025</td>
<td>225</td>
</tr>
<tr>
<td>Cost</td>
<td>Low-medium</td>
<td>Medium</td>
<td>Medium-high</td>
<td>Low-high</td>
<td>Low-medium</td>
</tr>
</tbody>
</table>
occurs as surfaces approach each other through a combination of diffusion and creep, which makes surface preparation extremely important to minimize porosity and reduce bonding time. Inserts of ductile metal foils are often used to accelerate the bonding process by enhancing deformation and reducing the joining temperature (Mizuhara & Huebel, 1986).

The eutectic system, which is sometimes referred to as active-brazing or transient liquid phase bonding, is intermediate between brazing and diffusion bonding. It combines the somewhat relaxed surface preparation techniques of brazing and the high temperature strength of diffusion joints. Because it requires a beneficial type of reactivity among the components to be joined, it is seldom used for ceramic : metal joints. The extent and rate of this reactivity are often difficult to predict, making other joining methods easier to achieve.

Techniques that employ localized heating, such as electron-beam or arc-welding are often not suitable for joining dissimilar materials. The high heat generated creates a localized molten region which fuses the surrounding materials together on cooldown. This requires an acceptable fracture toughness and thermal shock resistance which brittle materials, such as ceramics and intermetallics do not generally possess. Studies have shown that ductility in both the base metal and fusion zone are vital to producing high quality welds (Mizuhara, Huebel & Oyama, 1989).

**Chemical Concerns**
Reactivity is the primary chemical consideration when joining dissimilar materials. The lack of reactivity manifests itself in the form of non-wettability, i.e., when one material fails to adequately cover the surface of another. Generally, joint strength is proportional to coverage, making adequate wettability essential. On the other hand, excessive reactivity can lead to the formation of porosity or brittle intermetallics. For systems with widely disparate material properties, this fine balance can be difficult to achieve.
Structural ceramics are among the most stable compounds known and as a result they are very difficult to wet. This thermodynamic limitation can be overcome through the kinetics of chemical reactivity. If the interface material contains an element which forms a more stable oxide than the solid oxide on which the liquid metal is held, wetting or spreading of the liquid occurs (Loehman, 1989). For example, when bonding alumina and steel, active metals such as titanium or zirconium are often used in the form of vapor-deposited coatings or as a component within the braze alloy. Because titanium has a higher affinity for oxygen than aluminum, it effectively wets the ceramic while sharing enough metallic similarity with the steel to effectively join with it as well.

Chemical reactions between materials during joining have a significant influence on the strength of joints. Unfortunately, this is about the most definitive statement that can be made about the effect of reactivity. There are nearly as many sources in the literature stating reactivity improves strength as those which refute it (Kim & Kim, 1992; Ning, Okamoto, Mijamoto & Koreeda, 1991; Qin & Derby, 1992). In general, reactivity on the atomic scale improves bonding, while the formation of bulk amounts of porous ceramics or brittle intermetallics almost always proves deleterious.

These various issues can be more clearly understood by examining a molybdenum : silicon carbide braze joint using a copper interlayer with titanium as a metallization layer. As the amount of titanium increases by increasing the thickness of the sputtered metallization layer, the wettability of the ceramic increases. At a critical thickness, the wettability of the ceramic reaches a maximum. While the titanium reacts with the ceramic, it also diffuses into the copper, creating the brittle Ti₂Cu intermetallic. These competing reactions create a strength maximum for the overall joining system—Figure 1.1 (Naka, Tsuyoshi & Okamoto, 1990). The negative effects of reactivity are further illustrated by a diffusion bonded silicon nitride : silicon nitride joint using an aluminum interlayer. For a given temperature, a certain amount of time is required for adequate wettability between interlayer
and the ceramic. After an extended period of time, the formation of deleterious reaction products begins and joint strength is compromised—Figure 1.2 (Suganuma, Okamoto & Koizumi, 1987b).

The challenge is to develop an interlayer system of prescribed reactivity which effectively wets both materials while keeping the production of reaction products to a minimum. This is a rather complicated exercise as a dissimilar joining system with two separate components and an interlayer may form up to nine distinct phases. An example of this is shown in Table 1.2, where reaction products found in various metal : ceramic joints are listed (Okamoto, 1990). Given the complexity of this analysis, alloy systems have been designed using computational thermodynamics, which produce strong joints with relatively little dependence on bonding time (McDermid & Drew, 1991).

**Mechanical Concerns**
The ability to accommodate stresses due to thermal expansion mismatch between components is the primary mechanical problem facing dissimilar material joints. At the joining temperature, both components are in mechanical equilibrium. As the components cool from the joining temperature, the high thermal expansion material shrinks more than the low thermal expansion material. At room temperature, the low expansion material experiences a predominantly compressive stress, while the high expansion material is in tension—Figure 1.3. From the standpoint of fracture mechanics, this is fortunate, as low thermal expansion materials tend to be brittle—Table 1.3.
<table>
<thead>
<tr>
<th>Ceramic</th>
<th>Metal</th>
<th>Reaction Product</th>
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<tr>
<td>Al&lt;sub&gt;2&lt;/sub&gt;O&lt;sub&gt;3&lt;/sub&gt;</td>
<td>Ti</td>
<td>TiO, TiAl&lt;sub&gt;3&lt;/sub&gt;</td>
</tr>
<tr>
<td>Al&lt;sub&gt;2&lt;/sub&gt;O&lt;sub&gt;3&lt;/sub&gt;</td>
<td>Ti</td>
<td>Ti&lt;sub&gt;2&lt;/sub&gt;O&lt;sub&gt;3&lt;/sub&gt;, Ti-Al-O</td>
</tr>
<tr>
<td>Al&lt;sub&gt;2&lt;/sub&gt;O&lt;sub&gt;5&lt;/sub&gt;</td>
<td>Ti-Ni</td>
<td>Ti-Al-O</td>
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<td>Ti-Ni</td>
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<td>MnO, aMnO•bAl&lt;sub&gt;2&lt;/sub&gt;O&lt;sub&gt;3&lt;/sub&gt;•cSiO&lt;sub&gt;2&lt;/sub&gt;</td>
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<td>NbO&lt;sub&gt;y&lt;/sub&gt;</td>
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<tr>
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<td>NbO&lt;sub&gt;y&lt;/sub&gt;, Na, K</td>
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<td>Mo3Si, MoSi&lt;sub&gt;2&lt;/sub&gt;</td>
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<tr>
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<td>Ti</td>
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* contains SiO<sub>2</sub>
** contains free Si
Table 1.3. Thermal expansion and elastic modulus data for several materials

<table>
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<tr>
<th>Material</th>
<th>C.T.E. (x 10^-6 °C^-1)</th>
<th>Young’s Modulus (GPa)</th>
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<tr>
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</tr>
<tr>
<td>Si₃N₄</td>
<td>3</td>
<td>294</td>
</tr>
<tr>
<td>Tungsten</td>
<td>4</td>
<td>407</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>5</td>
<td>329</td>
</tr>
<tr>
<td>Al₂O₃</td>
<td>8</td>
<td>360</td>
</tr>
<tr>
<td>Titanium</td>
<td>10</td>
<td>117</td>
</tr>
<tr>
<td>Kovar</td>
<td>10</td>
<td>147</td>
</tr>
<tr>
<td>ZrO₂</td>
<td>10</td>
<td>131</td>
</tr>
<tr>
<td>Nickel</td>
<td>11</td>
<td>168</td>
</tr>
<tr>
<td>Iron</td>
<td>12</td>
<td>210</td>
</tr>
<tr>
<td>410 Stainless steel</td>
<td>14</td>
<td>210</td>
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<td>Copper</td>
<td>20</td>
<td>115</td>
</tr>
<tr>
<td>Aluminum</td>
<td>27</td>
<td>70</td>
</tr>
</tbody>
</table>

The contributions to residual stress can be separated into effects related to the component mechanical properties and the joint geometry. It is intuitive that materials with a great disparity in thermal expansion will be more difficult to join than materials that are identical. As this mismatch increases, the likelihood of failure increases accordingly (Crispin & Nicholas, 1982). High strength materials exacerbate the effect of thermal expansion mismatch because the mismatch energy is stored elastically, concentrating the residual stress along the interface of the joint.

Processing variables such as joining temperature and cooling rates are generally determined by the method used to join the components. Rapid cooling rates, associated with processes that use localized heat sources, can increase residual stress due to the presence of thermal gradients within components and the inherently greater deformation resistance of materials at high strain rates. Generally, residual stress scales with the deviation from bonding temperature, making processes using the lowest possible bonding temperature desirable.

Another important consideration is the geometry of the components that form the joint. For cylindrical butt joints measured with a strain gauge, the stresses were found to increase with increasing bonded area, making it difficult to form joints of large cross-sectional area (Suganuma, Okamoto & Koizumi, 1987a). In addition, rectangular components with
stress-risers created by their sharp corners were found to have higher stresses than similar cylindrical joints—Figure 1.4.

The use of a low flow stress material such as aluminum is often used to dissipate residual stress plastically (Xian & Si, 1992). As the joint begins to cool, the thermal strain energy that begins to develop is minimized by the deformation of the aluminum. This method is most effective when chemical reactions do not occur and the cooling rate is slow enough to allow for some degree of mechanical equilibration. This type of process cannot be used for high temperature applications due to the effects of creep.

Thermal expansion mismatch can also be controlled through the use of functionally gradient materials. By using interlayers with thermal expansions that gradually increment from the low expansion material to the high expansion material, each layer absorbs only a small portion of the total mismatch (Kawasaki & Watanabe, 1987). The residual stress can be minimized by increasing the number and thickness of the interlayers.

For high strength materials with a large thermal expansion mismatch, cracking may occur as the joint cools down from the joining temperature. Residual stress, concentrated at the interface is a driving force for crack propagation in dissimilar material joints. In practice, porosity or flaws at the interface are frequently the site of failure initiation (Bartlett & Evans, 1991). For elastic systems, such as a metal : ceramic joints, fractographs of room-temperature torsion specimens show that the initiation sites for cracking coincided with those for the calculated maximum stresses (He & Evans, 1991).

**Summary**

The challenges associated with dissimilar material joining can be classified as either chemical or mechanical in nature. Significant advances have been made in the field of chemical control. The initial wettability limitation has been overcome through the use of active metals, while a qualitative understanding of the formation of reaction products and
their influence on joint strength has begun. Finally, the ability to design an interlayer alloy of prescribed reactivity has been demonstrated.

References


**Figures**

![Graph showing strength of copper at interface of joint and overall strength of joint against thickness of Ti metallization layer](image)

**Figure 1.1**—*Joint strength as a function of titanium thickness (Naka, Tsuyoshi & Okamoto, 1990)*

![Graph showing strength against bonding time](image)

**Figure 1.2.** *Joint strength as a function of bonding time (Suganuma et al., 1987b)*
Figure 1.3. Schematic of a dissimilar material interface

Figure 1.4. Residual stress as a function of bonded area and geometry (Suganuma et al., 1987a)
Figure 1.5. Joint strength as a function of bonding temperature (Suganuma et al., 1987b)
2. An Examination of the Analytical Techniques used to Characterize Dissimilar Material Joints

Abstract
This chapter examines the types of analytical models applied to dissimilar materials joining and their general limitations. Often, these models are limited to strict geometries, elastic material behavior and are ultimately valid for very small temperature deviations. Currently, no analytical model is able to characterize the mechanical behavior of high aspect ratio, dissimilar material joints undergoing a change in temperature. The ability to predict stress at the interface of an adhesive joint was demonstrated through a correlation of analytical and numerical techniques. For all practical dissimilar material combinations, a stress singularity exists at the free edge of the joint, along the interface.

Introduction
Analytical models define the behavior of a system based on a set of initial variables. Generally, they provide a quick solution for a given problem without the need for sophisticated numerical analysis. In the case of dissimilar material joining, these models require specific geometries, elastic material behavior and mathematics that are extremely difficult to evaluate from both an analytical and numerical perspective.

The analytical solution for a bimetallic strip is an example of a solution that is geometrically limited. It is used to analyze the mechanical behavior of two initially stress-free dissimilar materials undergoing a change in temperature. Once the width: height ratio becomes less than 10, beam theory is no longer accurate due to edge-effects (Kuo, 1989). The use of St. Venant’s theorem is required to satisfy the boundary conditions of the problem by ignoring the complex nature of the stresses at the free edge. Elaboration on this type of problem is contained within many texts on continuum mechanics (McClintock & Argon, 1965; Boresi & Chong, 1987).
The solution to the class of problems where the thickness of the component becomes significant in comparison to the width was first solved by Bogy (Bogy, 1968). Plane-strain and generalized plane stress dissimilar joints were modeled by prescribing arbitrary normal and shearing tractions on their free surfaces. A similar formulation was later used to predict stresses at the interface based on a more generalized far-field normal force (Bogy, 1975). The solution for the axisymmetric problem was subsequently solved by Agarwal (Agarwal, 1978). The mathematics required to analyze the stress within the joint using any of these techniques proves to be difficult beyond the point of being practical.

Conveniently, the solution to a characteristic equation involving the loading conditions and the elastic material properties provides an easy method for predicting the peak stress along the interface (Bogy, 1970). For most dissimilar material joints, this region of high stress is coincident with the location for failure initiation. Many additional analyses have considered the influence of more complex joint geometries through the use of numerical techniques (Lin & Hartmann, 1989; Munz & Yang, 1993; Chen & Nishitani, 1993).

In each of these analytical treatments, the stresses at the interface are calculated in terms of an externally applied load, for a system that is initially stress-free. This is applicable to adhesively bonded joints where residual stresses are very small because the deviation from bonding temperature is negligible. For joints formed at high temperature, this type of mathematical formulation is insufficient due to the presence of residual stress from thermal mismatch between materials. Currently, there are no "stand-alone" analytical models based on the interaction of the materials to a change in temperature. Further attempts to solve this type of problem in terms of material mechanical properties do so only with the assistance of numerical techniques such as finite element analysis (Munz, Fett & Yang, 1993; Munz & Yang, 1993).
Results and Discussion
In order to examine the applicability and limitations of the analytical process, it is necessary
to have an understanding of the key issues present in dissimilar material joints. The ability
to model these joints is often complicated by the presence of stress “singularities”. These
act to diminish the range over which the material behavior can be considered truly elastic.
Analytical analyses of adhesively bonded joints from prior researchers is used as a means
for verifying a finite element analysis model. Finally, similarities between mechanically
loaded (adhesively bonded) and thermally loaded (materials with a thermal expansion
mismatch) dissimilar material joints is discussed.

Definition of Singularity
Confusion often surrounds the term “singularity” as a popular connotation refers to it as an
infinite or undefined quantity. In fact, a singularity is little more than a region where a
function is no longer analytic or differentiable (Lapedes, 1978). For material systems of
practical significance, the magnitude of the singularity (or the singularity index) must be
bounded. If the stress in the singular region were infinite, the amount of the energy in the
system must also be infinite, which is non-physical. Furthermore, because all materials
plastically deform beyond a certain load, stresses cannot increase indefinitely. The
mathematical rationale behind the treatment of singularities in dissimilar material joining is
very similar to that of fracture mechanics, where a singularity exists at the crack-tip (Broek,

The singularity index is of practical importance because it allows for a measure of the peak
stress within the joint. Along the interface, near the edge of a dissimilar material joint, the
state of stress is dominated by the presence of a singularity of the form $x^{-\lambda}$, where $x$ is the
distance from the free-surface and $\lambda$ is the singularity index (Kuo & Chen, 1992). In the
“singular region”, the magnitude of the singularity provides a unique measure of the
intensity of the stress and strain (Suresh, 1991). An example of this is shown in Figure
2.1, where the relative magnitude of \( x^{-\lambda} \) is calculated for values representative of the singularity index for dissimilar material joints.

As the singularity index increases, so does the peak stress. With a negative singularity index, stresses decrease near the free surface of the joint. A singularity index of zero corresponds to stress that is independent of direction, which is expected for a joint formed from identical materials. A positive singularity index is the most detrimental, it means that stresses will rise rapidly at the free surface of the joint, where it is most vulnerable to cracking and subsequent failure (Terasaki, 1994).

**Constraints of Elasticity**

Another common limitation of stress analysis using analytical techniques is the incorporation of plastic behavior. Since many of the joining procedures for dissimilar materials involve elevated temperatures, it is useful to examine the range over which these materials are strictly elastic. A first order approximation can be performed by considering the stress on an isotropic material, constrained in all directions and subjected to a temperature change—Equation 2.1 (Roark, 1989).

\[
\sigma = \frac{E \alpha \Delta T}{1 - 2\nu} \quad (2.1)
\]

Where \( E \) is Young’s Modulus, \( \alpha \) is the coefficient of thermal expansion, \( \Delta T \) is the temperature change and \( \nu \) is Poisson’s ratio. By assuming that the Poisson’s ratio for a typical metal is 0.3 and creating a relationship between thermal expansion and modulus of typical structural materials, Equation 2.1 can be evaluated as a function of temperature and coefficient of thermal expansion—Figure 2.2. (This calculation and its approximations are detailed further in Appendix 2.1.) The materials are assumed to be stress-free at a temperature deviation of 0°C. Each contour line represents a 100 MPa increment in stress.
As this temperature deviation increases, the stress increases due to the constrained thermal expansion of the material.

For each material, the yield strength of its various alloy systems are shown. If fully constrained and subjected to a temperature change of 200°C, virtually all structural metals will begin to experience plastic flow. Unalloyed metals, which are commonly used as interlayers to reduce residual stress, will experience plastic flow with temperature changes of less than 50°C. For dissimilar joining systems with a positive singularity index, the onset of plasticity will occur at even smaller temperature changes, because the stresses at the interface will be significantly higher.

**Mechanically Loaded Systems**
A mechanically loaded, or adhesive joint, is formed between two materials with different mechanical properties which are joined under initially stress-free conditions. The stresses at the interface resulting from a typical tensile test can be determined using Bogy’s analysis—Appendix 2.2 (Bogy, 1975). The algebraic complexity of this calculation is greatly reduced by incorporating the material composite parameters first introduced by Dundurs (Dundurs, 1969)—Equations 2.2 and 2.3.

\[
\varphi = \frac{G_1 (\kappa_1 + 1) - (\kappa_2 + 1)}{G_2 (\kappa_1 + 1) + \kappa_2 + 1} \quad \text{(2.2)} \quad \text{and} \quad \beta = \frac{G_1 (\kappa_1 - 1) - (\kappa_2 - 1)}{G_2 (\kappa_1 + 1) + \kappa_2 + 1} \quad \text{(2.3)}
\]

Here, G represents the shear modulus for each component and kappa is a function of Poisson's ratio and the geometry of the joint—Equations 2.4 and 2.5.

\[
\kappa_{\text{plane stress}} = 3 - 4\nu \quad \text{(2.4)} \quad \text{and} \quad \kappa_{\text{plane strain}} = \frac{3 - \nu}{1 + \nu} \quad \text{(2.5)}
\]

The order of the singularity is found by the solution to the characteristic equation, Equation 2.6.
\[ \lambda^2 (\lambda^2 - 1) \phi^2 + 2 \lambda^2 \left[ \sin^2 (\psi) - \lambda^2 \right] \phi \beta + \left[ \sin^2 (\psi) - \lambda^2 \right] ^2 \beta^2 + \sin^2 (\psi) \cos^2 (\psi) = 0 \quad (2.6) \]

While the solution space for Equation 2.6 is valid for all possible values of Poisson's ratio (0 to 0.5) and ratios of elastic modulus (0 to infinity), a more practical subset should consider the following limitations: modulus ratios no greater than 10:1 and Poisson’s ratios between 0.25 and 0.4. The singularity index was evaluated numerically as a function of the Dundurs parameters for plane stress and plane strain loading conditions—Figures 2.4 and 2.5, respectively. (The details of this calculation are given in Appendix 2.3.)

In each of these figures, the light gray box represents the solution space for the more practical combination of engineering materials. The most notable feature of this constraint is the reduction of the region where a negative singularity index is predicted (shown by the shaded region). Such a condition would be particularly beneficial because the normal stresses are expected to decrease near the free surface of the joint. For identical materials, the singularity index is zero and the stress across the interface is uniform and equivalent to the applied far-field stress.

A positive singularity index is predicted for virtually all dissimilar material combinations of practical importance. This means that the peak stresses will occur near the free surface of the dissimilar material joint, along the interface. It is a commonly held rule that tensile failure initiates at the region of peak tensile stress while fatigue failure initiates at the surface of a material (Suresh, 1991). Whether the failure is due to tensile overload or fatigue, the predicted initiation location coincides with the free surface of the joint, along the interface. Because of this, it is expected that the singularity index can be used to predict relative joint strength.
The results of this analytical predictive method were compared to the stress profile predicted by the finite element analysis package Abaqus 5.4 on a DEC 5000/200 workstation. The geometry of the components and the loading conditions used for the simulation are shown in Figure 2.3. A line-load of 1 Pa was applied normal to interface along one end of the joint and the other end of the joint was constrained only in the y-direction. Finally, the mesh was designated x-symmetric along the centerline of the joint. Both plane stress and axisymmetric models were analyzed. The material properties used for the simulations and their analytically determined singularity indices are shown in Table 2.1. Simulations 1 through 6 correspond to systems which were solved using analytical methods by Bogy and Agarwal and were used as a means of verification for the finite element analysis results (Bogy, 1975; Agarwal, 1978). It is important to note that many material combinations used in these simulations are non-physical and were examined merely to explore the mathematical boundaries of the formulation.

The Mises stress along the interface for the plane stress model and axisymmetric models is shown in Figures 2.6 and 2.7, respectively. The numerical analysis did not converge for Simulations 1 and 2 because a finite stress causes an infinite expansion for a material of zero modulus. Beyond that, the results of these numerical analyses were qualitatively comparable to the prior analytical results. In each case, the shape of the various stress distributions was similar, however, the magnitudes varied due to difficulties caused by the inherent stress singularities. An in depth analysis of this variation is detailed in

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<th>Simulation #</th>
<th>$E_1$ (GPa)</th>
<th>$\nu_1$</th>
<th>$E_2$ (GPa)</th>
<th>$\nu_2$</th>
<th>$\lambda_{p\text{;stress}}$</th>
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<td>210</td>
<td>0</td>
<td>210</td>
<td>0.5</td>
<td>0</td>
<td>-0.053</td>
</tr>
<tr>
<td>9</td>
<td>210</td>
<td>0.3</td>
<td>70</td>
<td>0.3</td>
<td>0.060</td>
<td>0.080</td>
</tr>
</tbody>
</table>
Chapter 3.

For a given material combination and mechanical loading, the stresses predicted by the axisymmetric analysis were always greater than the corresponding plane stress model, which is corroborated by its higher singularity index. Most importantly, the singularity index effectively determined the magnitude of the peak Mises stress along the interface of each system. As a first order approximation, the solution to Equation 2.6 provides a quick estimation of the relative stresses for an adhesively bonded joint with a tensile load.

**Thermally Loaded Systems**

In contrast to mechanically loaded joints, thermally loaded joints are formed from two materials with a difference in thermal expansion, subjected to a change in temperature. One of the most frequent errors regarding the modeling of these systems is the use of plane-strain analysis. When a change in temperature occurs, the natural thermal expansion of each material is mathematically constrained in one direction, creating an artificial source of stress. In effect, this implies that the materials are joined to a material of zero thermal expansion and infinite modulus. Depending on the specifics of the modeling system, the stress in this direction may dominate the solution, influencing any composite function such as strain energy, Mises stress, or maximum principle stress.

The mechanics of these systems are very dependent on the geometry of the joint, making them difficult to model by conventional analytical techniques. Because of this, there is not a technique comparable to Bogy's analysis for estimating stresses within thermally loaded joints. Both the complex mathematical formalism and the amount of data required to accurately describe material behavior over a wide range of temperatures makes the development of such an analytical technique unlikely.

Finite element analysis was used to develop a metric capable of describing the relative stresses for a series of thermally loaded joints. An array of simulations identical to those
in Table 2.1 were modeled with the addition of a thermal expansion mismatch between materials to provide for a means of loading. The geometry of the thermally loaded joint was very similar to that used for the mechanically loaded joint—Figure 2.8. The centerline of the mesh was designated x-symmetric and one node along the intersection of the centerline and the interface was constrained in the y-direction to provide the sufficient boundary conditions. The temperature of a system which was initially stress-free and rigidly attached along the interface was changed by 1°C to determine the resulting stress profiles.

The results of the plane stress analysis and axisymmetric analysis are shown in Figures 2.9 and 2.10, respectively. Once again, Simulations 1 and 2 failed to converge due to the infinitesimal modulus of the second component. In general, the peak stress proved to be an unreliable indicator of the overall state of stress for the joint. Along the interface, near the edge of the joint, the stresses became erratic. Even after several attempts to further refine the mesh using more accurate elements and an increased mesh density, the peak stress failed to converge. These problems are covered in detail in the next chapter, Chapter 3.

While peak stress proved to be a poor metric, the strain energy of the system, “U”, as calculated by finite element analysis proved to be indicative of the overall state of stress—Table 2.2. If the erratic stress values at the edge of the interface are disregarded,

\[
\begin{array}{cccccccc}
\text{Simulation} \# & E_1 \ (\text{GPa}) & \alpha_1 & \nu_1 & E_2 \ (\text{GPa}) & \alpha_2 & \nu_2 & U (J) \\
1 & 210 & 10 & 0.5 & 0 & 25 & 0.5 & * & * \\
2 & 210 & 10 & 0.2 & 0 & 25 & 0.3 & * & * \\
3 & 210 & 10 & 0.5 & 23 & 25 & 0.3 & 0.132 & 0.530 \\
4 & 210 & 10 & 0.5 & 23 & 25 & 0 & 0.131 & 0.476 \\
5 & 210 & 10 & 0.5 & 70 & 25 & 0.5 & 0.301 & 1.250 \\
6 & 210 & 10 & 0.47 & 124 & 25 & 0 & 0.078 & 0.280 \\
7 & 210 & 10 & 0.3 & 210 & 25 & 0.3 & 0.567 & 1.830 \\
8 & 210 & 10 & 0 & 210 & 25 & 0.3 & 0.304 & 0.979 \\
9 & 210 & 10 & 0.3 & 70 & 25 & 0.3 & 0.304 & 0.979 \\
\end{array}
\]
the strain energy accurately depicts the relative state of stress within these joints. A
summation of the stress magnitude and the volume element over which it acts is used to
form the elastic energy. This effectively couples the state of stress to the geometry of the
joint. An analytical method to evaluate the strain energy of joints based on their
geometry and elastic mechanical properties is developed in Chapter 4.

Summary
The constraints of elasticity and geometry limit the applicability of many analytical
solutions to dissimilar material joining. A fully constrained metal, loaded by its own
thermal expansion, experiences plastic deformation at a temperature deviation of less
than 200°C. Because plasticity cannot be accounted for, this greatly limits the accuracy
of analytical techniques used to solve the residual stress state and should not be used for
systems that are joined at elevated temperatures.

As the width : height ratio of joints becomes less than 10 : 1, the stresses at the edge of
the interface becomes significant. The use of analytical models which require geometric
simplifications such as the bimetallic strip approach are no longer valid.

For adhesive joints, the use of Bogy's characteristic equation provides a means for
prediction of the stresses at the interface. Because this approach only considers the effect
of two materials, the amount of adhesive at the joint surfaces must be small. For all
practical dissimilar material combinations, stresses are predicted to peak along the
interface at the edge of the joint.

Many factors make the modeling of thermally loaded joints difficult. Techniques within
continuum mechanics fail because of the loading dependence on geometry. For
thermally loaded elastic joints, strain energy provides a metric capable of predicting the
overall state of stress.
References


Figures

Figure 2.1—Effect of singularity index on relative stress

Figure 2.2—Yield point of constrained structural materials
Figure 2.3—Schematic for mechanically loaded joints

Figure 2.4—Singularity index for mechanically loaded plane stress joints
Figure 2.5—Singularity index for mechanically loaded axisymmetric joints

Figure 2.6—Mises stress for mechanically loaded plane stress joints
Figure 2.7—Mises stress for mechanically loaded axisymmetric joints

Figure 2.8—FEA schematic for thermally loaded joints
Figure 2.9—Mises stress for thermally loaded plane stress joints

Figure 2.10—Mises stress for thermally loaded axisymmetric joints
Appendix 2.1—Plasticity due to constrained thermal expansion

This mechanical data has been taken from Smithells Metals Reference Book, the CRC Handbook, and the ASM Metals Handbook. X-values are room temperature linear expansion, while y-values are the room temperature modulus.

\[
\text{data} = \{[4.5, 407], [5.1, 329], [6.5, 279], [8.9, 117], [12, 210], [13.3, 210], [17, 120], [19.1, 83], [23.5, 50], [24, 70], [26, 45], [30, 105]\}
\]

\[
\text{pict} = \text{ListPlot}[[\text{data}, \text{Prolog} \rightarrow \text{PointSize}[.02]], \text{AxesLabel} \rightarrow \{"\text{C.T.E.} \ 10^{-6}\}, "\text{Modulus} \ \text{GPa}\}, \text{DisplayFunction} \rightarrow \text{Identity}];
\]

\[
\text{modulus} = \text{Fit}[[\text{data}, \{\text{cte} \cdot 50.1, \text{cte}\}, \text{cte}]
\]

\[
\text{Plot}[[\text{modulus}, \{\text{cte}, 3, 30\}, \text{DisplayFunction} \rightarrow \text{Identity}];
\]

\[
\text{Show}[[x, \text{pict}, \text{AxesLabel} \rightarrow \{"\text{C.T.E.} \ 10^{-6}\}, "\text{Modulus} \ \text{GPa}\}, \text{DisplayFunction} \rightarrow \text{Identity}];
\]

\[
\text{Display}[[\text{Function}]; \text{Prolog} \rightarrow \text{PointSize}[.02]];\]

\[
511.699 - 89.7777 \text{cte}^{0.5}
\]

Using the relation between the modulus and the linear coefficient of thermal expansion, the constrained stress for the materials is calculated. The 1000 term is here to accommodate modulus $10^9$ and coefficient of thermal expansion $10^{-6}$

\[
\text{stress} = \frac{(10^9 \ \text{modulus} \ 10^{-6} \ \text{cte} \ \text{deltaT})}{(1-2 \cdot 3)};
\]

\[
\text{ContourPlot}[[\text{stress}/10^6, \{\text{deltaT}, 0, 300\}, \{\text{cte}, 3, 27\},
\]

\[
\text{Contours} \rightarrow \text{Table}[100i, \{i, 1, 19\}],
\]

\[
\text{ContourShading} \rightarrow \text{False},
\]

\[
\text{FrameLabel} \rightarrow \{"\text{Temperature}\}, "\text{C.T.E.}\}];
\]

This yields the contour plot of $\Delta$Temperature [$^\circ$C] (x-axis) versus coefficient of thermal expansion [$10^{-6} 1/\text{C}$] (y-axis) showing Stress contours in 100 MPa increments. Stress increases from $\Delta$Temp=0 to $\Delta$Temp=300.
Appendix 2.2—An overview of Bogy's Analysis

The figure below is used as a guide for the plane stress solution of a bi-material strip with a plane normal interface in tension. The problem is reduced to a system of singular integral equations formed by the superposition of solutions for infinite strips and half planes. The order of the stress singularity at the intersection of the material interface and lateral boundary is then extracted from the integral equations. With this knowledge, the integral equations are solved numerically, yielding the bond tractions and displacements.

Figure 2.11—Superposition of strip and half-plane solutions
Appendix 2.3—Axisymmetric singularity calculation

These are the original simplifications as proposed by Dundurs in his 1969 review paper. Gamma is the modulus ratio of one material to the other. k is a factor that relates the Poisson's ratio to the loading condition.

\[
\begin{align*}
\alpha_l &= \frac{\text{gamma} \ (k1+1)-(k2+1)}{\text{gamma} \ (k1+1)+(k2+1)}; \\
\beta_l &= \frac{\text{gamma} \ (k1-1)-(k2-1)}{\text{gamma} \ (k1+1)+(k2+1)};
\end{align*}
\]

\[k1=(3-4v1); \quad k2=(3-4v2);\]

The mathematically possible alpha-beta space is found by varying each relation from its minimum to its maximum.

\[
\{\alpha_l,\beta_l\}/.\{\text{gamma} \rightarrow 9999, v1 \rightarrow 0, v2 \rightarrow 1/2\} \\
\{\alpha_l,\beta_l\}/.\{\text{gamma} \rightarrow 9999, v1 \rightarrow 1/2, v2 \rightarrow 0\} \\
\{\alpha_l,\beta_l\}/.\{\text{gamma} \rightarrow 0, v1 \rightarrow 1/2, v2 \rightarrow 0\} \\
\{\alpha_l,\beta_l\}/.\{\text{gamma} \rightarrow 0, v1 \rightarrow 0, v2 \rightarrow 1/2\}
\]

plot3=ListPlot[%, PlotJoined->True, AxesLabel->"alpha", "beta", DisplayFunction->Identity];

bound1=.25a +.25; 
bound2=.25a -.25;

\[
\begin{align*}
9997 &\quad 9999 \\
'9999' &\quad 19999 \\
9997 &\quad -(\frac{1}{10001}) \\
10001 &\quad -\frac{1}{10001}
\end{align*}
\]

(-1, -\frac{1}{2}) \\
(-1, 0)

A more realistic approach to the mechanical properties can assume that the modulus ratio of structural materials will be no greater than 10 to 1, or hence, its reciprocal. By the same token, Poisson's ratio doesn't vary much from .3, with .25<v<.4 being the norm.

\[
\{\alpha_l,\beta_l\}/.\{\text{gamma} \rightarrow 10, v1 \rightarrow .25, v2 \rightarrow .4\} \\
\{\alpha_l,\beta_l\}/.\{\text{gamma} \rightarrow 10, v1 \rightarrow .4, v2 \rightarrow .25\} \\
\{\alpha_l,\beta_l\}/.\{\text{gamma} \rightarrow 1, v1 \rightarrow .4, v2 \rightarrow .25\} \\
\{\alpha_l,\beta_l\}/.\{\text{gamma} \rightarrow 1, v1 \rightarrow .25, v2 \rightarrow .4\}
\]

plot4=ListPlot[%, PlotJoined->True, AxesLabel->"alpha", "beta", PlotStyle->GrayLevel[.5], DisplayFunction->Identity];

\((0.851852, 0.296296)\) \\
\((0.777778, 0.111111)\) \\
\((-0.851852, -0.296296)\) \\
\((-0.777778, -0.111111)\)

--Graphics--

This is from "The Plane Solution for Joined Dissimilar Elastic Semi-strips Under Tension" March 1975 by Bogy. g represents the order of the singularity while a and b represent Dundurs alpha and beta simplifications, respectively. The solutions for the singularity index are broken down into the positive (posval2) and negative (negval2) values.
\[ d = (\cos[g \pi/2]^2 - (1-g)^2) a b^2 + 2(1-g)^2 (\cos[g \pi/2]^2 \sin[g \pi/2])^2 \]

\[ g \text{zero} = \text{Plot}(.5a,(a,0,1),\text{DisplayFunction}->\text{Identity}); \]

\[ \text{posval1}=\text{Table}[\text{eqn=}\text{Solve}([d=0/.g->i,b]), \]
\[ \text{el}=\text{Evaluate}[\text{eqn}[[2]]]; \]
\[ \text{p1} := \text{If}[\text{Evaluate}[[1]>,\text{bound2}], \text{el}, \text{bound2}]; \]
\[ \text{e2}=\text{Evaluate}[\text{eqn}[[3]]]; \]
\[ \text{p2} := \text{If}[\text{Evaluate}[[2]<\text{bound1}], \text{e2}, \text{bound1}]; \]
\[ \text{Plot}([\text{p1}, \text{p2}]), \text{PlotRange}->\{-0.5,0.5\}, \]
\[ \text{DisplayFunction}->\text{Identity}, [i,0.05,0.35,0.05]]; \]

\[ \text{negval1}=\text{Table}[\text{eqn=}\text{Solve}([d=0/.g->i,b]), \]
\[ \text{el}=\text{Evaluate}[\text{eqn}[[2]]]; \]
\[ \text{p1} := \text{If}[\text{Evaluate}[[1]>,\text{bound2}], \text{el}, \text{bound2}]; \]
\[ \text{e2}=\text{Evaluate}[\text{eqn}[[3]]]; \]
\[ \text{p2} := \text{If}[\text{Evaluate}[[2]<\text{bound1}], \text{e2}, \text{bound1}]; \]
\[ \text{Plot}([\text{p1}, \text{p2}]), \text{PlotRange}->\{-0.5,0.5\}, \]
\[ \text{DisplayFunction}->\text{Identity}, [i,-0.02,-0.12,-0.02]]; \]

\[ \text{Show}([\text{posval1}, \text{negval1}, \text{plot3}, \text{plot4}, \text{gzero}]), \]
\[ \text{DisplayFunction}->\$\text{DisplayFunction}, \text{AxesLabel}-> \]
\[ \{"alpha","beta"\}]; \]

The solution to the singularity index can be found in a step-wise manner by finding the root of the characteristic equation for a known alpha and beta.

\[ d = (\cos[g \pi/2]^2 - (1-g)^2) a b^2 + 2(1-g)^2 (\cos[g \pi/2]^2 \sin[g \pi/2])^2 \]

\[ a1 = (\gamma (k1+1)-(k2+1) ) / (\gamma (k1+1)+(k2+1) ); \]
\[ b1 = (\gamma (k1-1)-(k2-1) ) / (\gamma (k1+1)+(k2+1) ); \]
\[ k1=3-4v; \]
\[ k2=3-4v; \]
\[ \gamma = 1; \]
\[ a=a1/.(v1->-0.5,v2->0) \]
\[ b=b1/.(v1->-0.5,v2->0) \]
\[ \text{sol}=\text{FindRoot}([d=0, (g,-1)]) \]
\[ -0.33333 \]
\[ -0.33333 \]
\[ (g -> -0.131501) \]
3. Finite Element Analysis Applied to Dissimilar Materials Joining

Abstract
A commercially available finite element analysis package was used to model a two material dissimilar joining system. By performing a sensitivity analysis using an elastic perfectly plastic material model, changes in mesh geometry and element type were shown to result in a high variance of data. When modeling a system which contains stress singularities such as dissimilar material joining, the use of a metric such as peak stress is unreliable. Results of an elastic model are governed by the mesh setup while the results for a plastic model are governed by the material response model for the weakest component.

Introduction
Finite element analysis offers an attractive alternative to problems that are difficult to solve by analytical methods. While there are analytical methods to determine the response of a homogenous system under the influence of a thermal gradient, the presence of two materials, each with unique thermal and mechanical properties, undergoing a change in temperature, presents a more difficult challenge. At the interface between the two materials, a mathematical singularity exists. The existence of a singularity can make both analytical analysis intractable and numerical techniques such as finite element analysis inaccurate (Levy & Papzian, 1991).

The complexity of the mathematics associated with analytical techniques generally preclude all but an “expert-user,” however, this is not the case with finite element analysis. The very ability to perform complex simulations with ease, may at the same time lead to a failure to fully understand the details of the modeling process. A thorough sensitivity analysis must be performed to examine the manner in which the problem is setup, the material model for each component and the presentation method for the data. Only then can
the influence of apparently external factors be determined and the results of the analysis verified.

**Problem Definition**
An ideal numerical model accurately answers the questions at hand without undo complexity. Mechanics simplifications take the physical 3-dimensionality of a system and decrease it to a more computationally manageable form. The most typical examples include: plane stress, where one dimension of the component is very small in relation to the others; plane strain, where one dimension of the component is very large in comparison to the others; and finally, axis-symmetry, where the component exhibits symmetry about one axis, such as a cylinder.

Geometric simplifications allow a fraction of the original problem to be modeled through the use of appropriate symmetry and boundary conditions. These simplifications must be carefully considered so that the problem of interest is modeled, not the influence of the constraints. Although many analyses have considered the effect of various complex loading conditions, only the most simple constraints will be used in this study (Kishimoto, Sugita, Notomi & Koizumi, 1993). Once these simplifications have been made, the setup of the problem is complete and the majority of the process becomes straightforward.

Mesh generation choices are no longer philosophically based so much as they are related to the desired numerical accuracy of the solution. The use of linear elements allows easy mesh generation at the expense of accuracy, while quadratic elements increase accuracy at the expense of computational speed. Successful finite element analysis involves an iterative process where an initial solution is obtained, detailing the most active regions of a problem. By increasing the mesh density in these regions, an accurate solution is attained while maintaining overall computational efficiency (Fagan, 1992).
**Mechanical Properties**

The mechanical model of the materials is a frequently overlooked portion of finite element analysis. Generally, this is due to the complexity required for the most simple analysis. Dissimilar joints are a particularly poignant example as a minimum of 9 variables are required to describe a purely elastic, two-component dissimilar material joint: elastic modulus, coefficient of thermal expansion, and Poisson's ratio for each component; width and height of the joint; and a change in temperature. Over the wide temperature ranges common in joining, many elastic properties which are generally considered temperature insensitive, may vary by more than 50% (Boyer & Gall, 1992), (Weast, 1985).

Various mechanisms influence the plastic response of materials during joining. At high temperature, recovery processes are rapid enough that they are able to offset loading due to thermal mismatch. As the temperature continues to decrease, recovery effects become increasingly sluggish, leading to strain-hardening and a rapid increase of residual stress. Material models that purport to incorporate plasticity without incorporating first-order effects such as strain rate sensitivity, hardening, softening, multi-stage plasticity and creep are of questionable accuracy. This chapter does not consider the material behavior beyond a simple elastic, perfectly plastic response. In Chapter 5, a methodology will be developed which addresses the dominant mechanisms of deformation observed in dissimilar material joining.

**Simplification of Data**

Developing a concise method for reporting data without losing valuable content is another inherent problem with finite element analysis. The files for a single simulation frequently contain more than 1 million data points and exceed 100 megabytes in size. Graphics are often used as a type of simplification, however, it is difficult to quantitatively compare the results of several simulations. An example of this is illustrated by elastic finite element analysis of the maximum principal residual stress at room temperature for a silicon nitride:
steel joint formed at 800°C—Figure 3.1 (Suganuma, Okamoto, Koizumi & Shimada, 1985). The stresses are dependent upon the location within the joint and tend to peak along the interface of the joint, near the free surface.

A way to overcome this data-overload is through the use of a single metric to describe the joining system. Ideally, the variable used as a metric should be insensitive to factors considered external to the problem of interest. Peak stress is commonly used for determining the relative mechanical strength of a dissimilar joining system. The link between these metrics and the finite element setup variables such as mesh density and element type is vital for establishing the validity of the simulation results. Unfortunately, these details are frequently omitted from many analyses.

**Experimental**
A two component dissimilar joint was analyzed with the finite element analysis package Abaqus 5.4 on a DEC 5000/200 workstation. A thorough sensitivity analysis, which considered the effects of mesh geometry and element type, was performed by varying each variable independently.

To more clearly focus on the effect of the finite element setup, the following assumptions were made: both materials are stress-free at the joining temperature and are rigidly attached along the interface; temperature equilibration throughout the system occurs instantaneously; and the mechanical properties of both materials are elastic, perfectly plastic with no temperature or stress dependence—Table 3.1.

A rectangular mesh was used to simulate two components, each with a height : width ratio of 4:1, joined along their width—Figure 3.2. Along the centerline of the mesh, each node

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus (GPa)</th>
<th>C.T.E. (1/°C)</th>
<th>Poisson’s ratio</th>
<th>Yield stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>70</td>
<td>$25 \times 10^{-6}$</td>
<td>0.3</td>
<td>50</td>
</tr>
<tr>
<td>Steel</td>
<td>210</td>
<td>$10 \times 10^{-6}$</td>
<td>0.3</td>
<td>500</td>
</tr>
</tbody>
</table>
was designated as x-symmetric, doubling the effectively modeled area. The effect of mesh
density was investigated by varying the number of elements along the x-axis. Additionally,
the elements were biased at successively closer intervals along the x-axis toward the free
surface to more effectively accommodate the active region of the problem. For each element
type, the effect of linear and quadratic geometry as well as reduced and full integration was
measured. These various combinations are referred to by simulation number and are
summarized in Appendix 3.1.

Both plane stress and axisymmetric geometries were examined in this study. In each case,
the width of the mesh was set to 0.5 m. The through thickness of the plane stress model
was set to $\pi/4$ m, so that the bonded area was equivalent to the axisymmetric model. A
schematic of the element and node numbers and a sample input file for Abaqus are given in
Appendices 3.2 and 3.3, respectively.

**Results and Discussion**

**Peak stress as a metric**

For both the plane stress and the axisymmetric models, a graph summarizing the peak
stresses for each component is given as a function of the simulation number. The results
for the plane stress analyses are shown in Figures 3.3 and 3.4, while the results for the
axisymmetric analysis are shown in Figures 3.5 and 3.6. A summary of this information is
given in Table 3.2.

The "steel" component was perfectly elastic for all but two of the axisymmetric
simulations — Figures 3.4 and 3.6. For an elastic material, the peak stresses are closely
related to the mesh setup. As the accuracy and density of the elements increased, a
consistently higher peak stress was predicted. For the most sensitive axisymmetric
meshes, Simulations #24 and #32, the peak stress was equal to the yield strength of the
material. The scatter for the peak Mises stress data was greater than 30%. Other stress
metrics, such as maximum principle stress and maximum 3rd invariant stress, exhibited similar scatter.

The peak Mises stress for the “aluminum” component appeared to be unaffected by variations in mesh setup—Figures 3.3 and 3.5. In each case, the temperature deviation was large enough to cause yielding for both the plane stress and axisymmetric models. Many finite element analysis packages, including Abaqus, use the Mises stress as a criterion for plasticity—once it surpasses the yield strength, the material plastically deforms. The use of an elastic, perfectly plastic model dictates that the peak Mises stress is equivalent to the predefined flow stress once yielding occurs, regardless of the amount of plastic strain. It is this overly simplified material model which makes the peak Mises stress appear to be a precise metric. In actuality, the flow stress of materials is related to both temperature and plastic strain.

Singularities are the primary cause of scatter when modeling dissimilar material joints using numerical methods such as finite element analysis. Their effect can be more easily visualized by examining the stresses predicted along the interface of a copper-molybdenum joint, using completely elastic properties, undergoing a 1°C temperature change—Figure 3.7. Along the interface, near the free edge of the joint, the stresses reach a maximum. The ability to accurately approach the inherent magnitude of the singularity increases with increases in mesh density and the use of more accurate elements. For an elastic material, the slope of the stress versus position curve is very steep in the singular region, making an accurate convergence of the solution difficult. Whenever the geometry

<table>
<thead>
<tr>
<th></th>
<th>Plane Stress</th>
<th></th>
<th>Axisymmetric</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Aluminum</td>
<td></td>
<td>Aluminum</td>
</tr>
<tr>
<td>Mises (MPa)</td>
<td>50.0 ± 0.0%</td>
<td>148 ± 15%</td>
<td>50.0 ± 0.0%</td>
<td>166 ± 32%</td>
</tr>
<tr>
<td>Max Principal (MPa)</td>
<td>56.5 ± 1.0%</td>
<td>74.4 ± 44.5%</td>
<td>143 ± 120%</td>
<td>96.9 ± 71%</td>
</tr>
<tr>
<td>Max 3rd Invariant (MPa)</td>
<td>50.0 ± 0.0%</td>
<td>75.1 ± 45.5%</td>
<td>28.3 ± 30.9%</td>
<td>139 ± 30.3%</td>
</tr>
<tr>
<td>Elastic Energy (J)</td>
<td>1,260 ± 3.7%</td>
<td>435 ± 3.3%</td>
<td>213 ± 4.8%</td>
<td>68.6 ± 3.4%</td>
</tr>
<tr>
<td>Plastic Energy (J)</td>
<td>23,800 ± 1.2%</td>
<td>0.0 ± 0.0%</td>
<td>6,230 ± 3.1%</td>
<td>0.02 ± 570%</td>
</tr>
</tbody>
</table>
of the mesh is changed, sensitivity analyses must be performed to determine whether apparent differences in peak stress are due to physical effects or a localized refinement of the mesh within the singular region.

**Strain energy as a metric**

While mesh setup influenced the magnitude of peak stresses to a large extent, it did not have a very substantial impact on either elastic energy or plastically dissipated energy. In most cases, the scatter for the elastic energy calculation varied by less than 4%—Table 3.2. Only two simulations predicted a small amount of plasticity in the axisymmetric steel component, leading to its extraordinarily high scatter. Unfortunately, the elastic energy is not a variable which is easily obtained from most finite element analysis packages. While Abaqus makes the strain energy available for the global model, there is not a “built-in” provision for calculating the energy based upon regions or components. Special code was written in order to process the data files and create this value—Appendix 3.4.

**Rationalization of Stresses**

While the magnitude of the peak stresses predicted by numerical analysis may be inaccurate, this type of analysis can be used to provide an estimation of the stress profiles that cannot be obtained by any other means. Techniques that employ strain gauges or use x-rays measure surface deformation and are limited to approximations of the full 3-dimensional stress states inherent in dissimilar material joints. Currently, neutron diffraction is the only technique capable of determining the in situ state of stress for 3-dimensional objects, however, this technique is quite expensive and its resolution is limited.

Through the use of continuum mechanics and simple logic, the trends of the stresses present in a cylindrical dissimilar material joint can be rationalized. This is most easily accomplished through the use of a cylindrical curvilinear system—Figure 3.8. The
conditions of mechanical equilibrium for cylindrical coordinates are given by the Equation 3.1 (McClintock & Argon, 1965).

\[
\frac{\partial \sigma_\theta}{\partial r} + \frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \sigma_\theta}{\partial z} + \frac{\sigma_\theta - \sigma_\theta}{r} = 0
\]
\[
\frac{\partial \sigma_\theta}{\partial r} + \frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \sigma_\theta}{\partial z} + 2 \frac{\sigma_\theta}{r} = 0 \tag{3.1}
\]
\[
\frac{\partial \sigma_\theta}{\partial r} + \frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \sigma_\theta}{\partial z} + \frac{\sigma_\theta}{r} = 0
\]

A cylindrical joint loaded by thermal expansion mismatch will retain its circular profile, which means that two of the shear components, \(\sigma_\theta\) and \(\sigma_\theta\) must vanish. In addition, any derivative in the theta direction is also zero. This leads to the following simplification of the equilibrium conditions—Equation 3.2.

\[
\frac{\partial \sigma_\theta}{\partial r} + \frac{\partial \sigma_\theta}{\partial z} + \frac{\sigma_\theta - \sigma_\theta}{r} = 0
\]
\[
\frac{\partial \sigma_\theta}{\partial r} + \frac{\partial \sigma_\theta}{\partial z} + \frac{\sigma_\theta}{r} = 0 \tag{3.2}
\]

The boundary conditions for this system are very simple. Because this system is loaded by the interaction of two materials to a change in temperature, there are no external tractions, which means that there are no normal forces at the surface of either component. In addition, the residual stress due to thermal expansion mismatch are expected to be the highest at the interface and gradually decrease with increasing distance. In order to minimize coordinate confusion by having the origin at the interface, the absolute value of \(z\) will be used in the following derivations. This leads to the following boundary conditions—Equation 3.3.

\[
\sigma_\theta \bigg|_{r \rightarrow \text{width}} = 0
\]
\[
\sigma_\theta \bigg|_{z \rightarrow \text{e}} = 0 \tag{3.3}
\]
Low expansion material
Upon cooling from the joining temperature, the low expansion material is smaller than its equilibrium size due to residual stress. This can be accounted for by a compressive stress is acting in the "rr" direction. As stated in the boundary conditions, this normal stress must vanish at the free surface. For the low expansion material, the bulk $\sigma_{rr}$ is negative and must increase to zero at the edge, making $\frac{\partial \sigma_{rr}}{\partial r} > 0$. From the boundary conditions, the stress must approach zero as the distance from the interface increases, making $\frac{\partial \sigma_{rz}}{\partial z} > 0$.

The sign of the shear stress is most easily determined by considering the geometry of the joint with respect to the nomenclature for a negative shear strain—Figure 3.9. Both the top and bottom components have this geometry, so it stands to reason that $\sigma_{rz}$ is negative in both cases. Because of symmetry, there is no shear stress acting along the centerline of the joint, making $\frac{\partial \sigma_{rz}}{\partial r} < 0$. Along the length of the joint, $\sigma_{rz}$ increases from a negative value to zero, making $\frac{\partial \sigma_{rz}}{\partial z} > 0$.

From the simplified equilibrium equations, $\frac{\partial \sigma_{rz}}{\partial z}$ must be positive since both $\frac{\partial \sigma_{rr}}{\partial r}$ and $\sigma_{zz}$ are negative, which makes $\sigma_{zz} < 0$ at the interface.

The sign of $\sigma_{\theta\theta}$ can be understood by a geometric model—Figure 3.10. When cooled from the joining temperature, the component’s actual size is smaller than its equilibrium size, requiring a compressive hoop stress, $\sigma_{\theta\theta} < 0$. Finally, it can be assumed that this stress approaches zero with increasing distance from the interface. Not much more can be determined about this value because it is not contained within the equilibrium equations.
This exercise can be repeated for the high expansion material. In either case, the stresses predicted or rationalized by the analytical method agree qualitatively with those predicted by the numerical methods.

Summary
The most simple analysis performed in this study required a minimum of 9 variables to describe the elastic properties of the materials and the geometry of the joining system. Variables external to the joining problem, required for the mesh creation within finite element analysis have been shown to affect the resulting peak stress calculations significantly. Ideally, this should not have had any effect as the properties of the materials were not changed. For a system which contains a singularity, the stresses calculated for the elastic-regime of the materials are dominated by the mesh-setup. The magnitude of the peak stress was observed to increase with the mesh density and the use of higher precision elements. Because of this pronounced sensitivity, either a method for standardizing simulations or an alternative method of data comparison must be found.

References


Figures

Figure 3.1—Elastic finite element analysis of a silicon nitride: steel joint. Arrows represent direction of the maximum principal stress (Suganuma et al., 1985).
Figure 3.2—Finite element analysis mesh geometry

Figure 3.3—Peak stress for the aluminum section using plane stress elements
Figure 3.4—Peak stress for the steel section using plane stress elements

Figure 3.5—Peak stress for the aluminum section using axisymmetric elements
Figure 3.6—Peak stress for the steel section using axisymmetric elements

Figure 3.7—Singular stress field along the interface of a dissimilar material joint
Figure 3.8—Nomenclature for cylindrical coordinates

Figure 3.9—Nomenclature for negative shear stress

Figure 3.10—Schematic of compressive hoop stresses in the low expansion material
## Appendix 3.1—Description of numerical simulations

<table>
<thead>
<tr>
<th>Simulation #</th>
<th># of Elements Wide</th>
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<th>Element Type</th>
<th>Integration</th>
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Appendix 3.2—Mesh nomenclature

"Aluminum" component

"Steel" component

line of x-symmetry
Appendix 3.3—Abaqus “Input Deck” for Simulation #22

*HEADING
al_t0001.inp
** Mesh definitions for the bottom infinite mesh (Set #B4)
**
*NODE
00001, 0.000, 0.000
00041, 0.200, 0.000
10001, 0.000, 1.000
10041, 0.200, 1.000
**
*NGEN,NSET=NSB412
1, 1.1
*NGEN,NSET=NSB443
10001, 1.0041, 1
*NGEN,NSET=NSB414
1, 1.0001, 500
*NGEN,NSET=NSB423
41, 1.0041, 500
**
*NFILL
NSB412,NSB443,20.500
NSB414,NSB423,40.1
**
*ELEMENT,TYPE=CAX8R
1, 1.3, 1.0001, 2.500, 1002, 501
*ELGEN,ELSET=ESB4
1, 20, 2.1, 10, 1000, 100
**
** Mesh definitions for the bottom far-field mesh (Set #B3)
**
*NODE
20001, 0.000, 1.500
20041, 0.200, 1.600
**
*NGEN,NSET=NSB312
20001, 1.0041, 1
*NGEN,NSET=NSB343
20001, 1.0041, 1
*NGEN,NSET=NSB314
10001, 2.0001, 500
*NGEN,NSET=NSB323
10041, 2.0001, 500
**
*NFILL,BIAS=1.05
NSB312,NSB343,20.500
NSB314,NSB323,40.1
**
*ELEMENT,TYPE=CAX8R
1001, 1.0001, 110003, 110003, 110002, 110002, 30503, 30503
*ELGEN,ELSET=ESB3
1001, 20, 2.1, 10, 1000, 100
**
** Mesh definitions for the bottom coarse mesh (Set #B2)
**
*NODE
30001, 0.000, 1.800
30041, 0.200, 1.850
**
*NGEN,NSET=NSB212
20001, 1.0041, 1
*NGEN,NSET=NSB243
30001, 1.3041, 1
*NGEN,NSET=NSB214
20001, 1.3001, 500
*NGEN,NSET=NSB223
20041, 1.3001, 500
**
*NFILL,BIAS=1.05
NSB212,NSB243,20.500
*NFILL,BIAS=1.05,NSET=NSB2
NSB214,NSB223,40.1
**
*ELEMENT,TYPE=CAX8R
2001, 200001, 210003, 210003, 210002, 20503, 210002, 20501
*ELGEN,ELSET=ESB2
2001, 20, 2.1, 10, 1000, 100
**
** Mesh definitions for the bottom fine mesh (Set #B1)
**
*NODE
40001, 0.000, 2.000
40041, 0.200, 2.000
**
*NGEN,NSET=NSB112
30001, 30041, 1
*NGEN,NSET=NSB143
40001, 40041, 1
*NGEN,NSET=NSB114
30001, 30041, 500
*NGEN,NSET=NSB123
30041, 40041, 500
**
*NFILL,BIAS=1.05
NSB112,NSB143,20.500
*NFILL,BIAS=1.05,NSET=NSB1
NSB114,NSB123,40.1
**
*ELEMENT,TYPE=CAX8R
3001, 300001, 300003, 310003, 310001, 310002, 30503, 310002, 30501
*ELGEN,ELSET=ESB1
3001, 20, 2.1, 10, 1000, 100
**
** Create a global mesh set for the bottom meshes
**
*NSET,NSET=NSBALL
NSB1,NSB2,NSB3,NSB4
**
** ELSET,ELSET=ESBALL
ESB1,ESB2,ESB3,ESB4
**
** Material definitions for the bottom elements
**
*SOLID SECTION,ELSET=ESBALL
MATERIAL=BOTOM
ELASTIC
210E+12, 3
PLASTIC
500E-9
EXPANSION
100E-04
**
** Mesh definitions for the top fine mesh (Set #T1)
**
*NODE
50001, 0.000, 2.200
50041, 0.200, 2.150
**
*NGEN,NSET=NST112
40001, 40041, 1
*NGEN,NSET=NST143
50001, 50041, 1
*NGEN,NSET=NST114
40001, 50001, 500
*NGEN,NSET=NST123
40041, 50041, 500
**
*NFILL,BIAS=952
NST112,NST143,20.500
*NFILL,BIAS=1.05,NSET=NST1
NST114,NST123,40.1
**
*ELEMENT,TYPE=CAX8R
4001, 400001, 400003, 410003, 410001, 400002, 40503, 410002, 40501
*ELGEN,ELSET=EST1
4001, 20, 2.1, 10, 1000, 100
**
** Mesh definitions for the top coarse mesh (Set #T2)
**
*NODE
60001, 0.000, 2.500
60041, 0.200, 2.400
**
*NGEN,NSET=NST212
50001, 50041, 1
*NGEN,NSET=NST243
60001, 60041, 1
*NGEN,NSET=NST214
50001, 60001, 500
*NGEN,NSET=NST223
50041, 60041, 500
**
*NFILL,BIAS=952
NST212,NST243,20.500
*NFILL,BIAS=1.05,NSET=NST2

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**SN214,SN233,40.1**

**ELEMENT, TYPE=CAX8R
5001,50001,50002,5003,51001,51002,50503,51002,50501
*ELGEN,ELSET=EST1
5001,20,2,1,10,100,100**

** Mesh definitions for the top far-field mesh (Set #T3)**

**NODE
70001,0.000,3.000
70001,0.200,3.000**

*NGEN, NSET=NST312
60001,60004,1,1
*NGEN, NSET=NST343
70001,70004,1,1
*NGEN, NSET=NST314
60001,70001,500
*NGEN, NSET=NST323
60001,70004,1,500**

**NFILL, BIAS=952
NST312,NST343,20,500**

*NFILL, BIAS=1.05, NSET=NST3
NST314,NST323,40,1**

**ELEMENT, TYPE=CAX8R
6001,60001,60003,61001,60002,60503,61002,60501
*ELGEN, ELSET=EST3
6001,20,2,1,10,100,100**

** Mesh definitions for the top infinite mesh (Set #T3)**

**NODE
80001,0.000,4.000
80001,0.200,4.000**

*NGEN, NSET=NST412
70001,70004,1,1
*NGEN, NSET=NST443
80001,80004,1,1
*NGEN, NSET=NST414
70001,80001,500
*NGEN, NSET=NST423
70001,80004,1,500**

**NFILL
NST412,NST443,20,500**

*NFILL, BIAS=1.05, NSET=NST4
NST414,NST423,40,1**

**ELEMENT, TYPE=CAX8R
7001,70001,70003,71003,70002,70503,71002,70501
*ELGEN, ELSET=EST4
7001,20,2,1,10,100,100**

** Create a global mesh set for the top meshes**

**NSET, NSET=NSTALL

**NST1,NST2,NST3,NST4**

**ELSET, ELSET=ESTALL
EST1,EST2,EST3,EST4**

** Material definitions for the top elements**

**SOLID SECTION, ELSET=ESTALL, MATERIAL=ALUMINUM
*MATND, NAME=ALUMINUM
*ELASTIC
.700E+1, 3
*PLASTIC
.500E+8
*EXPANSION
.250E-04**

** Create a global node and element set**

**NSET, NSET=NSTALL
NSBALL,NSTALL
*ELSET, ELSET=ESTALL
ESBALL,ESTALL**

** Set Axis of symmetry along the left edge with a
** simple support along the midpoint of the joined elements**

**NSET, NSET=LEDG, GENERATE
1,80001,500
*BOUNDARY
LEDG, XSYMM
*BOUNDARY
40001,2**

** Create the the element set of interest**

**ELSET, ELSET=INT1
ESB1,EST1
*ELSET, ELSET=INT2
ESB2,EST2,INT1
*ELSET, ELSET=INT3
ESB3,EST3,INT2
*ELSET, ELSET=INT4
ESB4,EST4,INT3**

**

** DRAW
*INITIAL CONDITIONS, TYPE=TEMPERATURE
NSALL,1000.
*STEP, NL, GEOM, INC=100
*STATIC
*ENERGY FILE
*EL FILE
ELIN
ENER
SNV
*TEMPERATURE
NSALL,0.
*END STEP
Appendix 3.4—Program for obtaining energy data

PROGRAM PROCESS
C T ABULATE THE ELASTIC, PLASTIC, AND TOTAL ENERGY WITHIN THE VARIOUS ELEMENTS OF THE MESH
C SAV ES THE RESULTS AND CLEANS UP
C THE FOLLOWING VARIABLES ARE FOR COMMUNICATION WITH SUBROUTINES CREATED BY HKS
C IMP LICT REAL*: (A-A,1-M,2)
CHARACTER*(10) NAME
CHARACTER*(20) FABQUS
CHARACTER*(20) FOUNAME
CHARACTER*25 FNAME
CHARACTER*25 FHEADER
CHARACTER*25 FSTRING
CHARACTER*25 FFILENAME
DIMENSION A (1),RARRAY(1),JARRAY(1)

C DIMENSION EENERGY(10,100),ENERGY1(10,100),ENERGY2(10,100)
DIMENSION KENERGY(10,100),KENERGY1(10,100),KENERGY2(10,100)
DIMENSION MISES(10,100),KPRN(10,100),KPRF(10,100),KPRN(10,100)
DIMENSION INVARMAX(10,100),INVARMIN(10,100)
DIMENSION KINVARMAX(10,100),KINVARMIN(10,100)
DIMENSION ESTRDN(10,100),PSFSTRDN(10,100)
DIMENSION KESTRN(10,100),KPSFSTRDN(10,100)
DIMENSION ALLSH(100),ALLPD(100),ETOTAL(100),TIME(100)
DIMENSION KCOMP(10)

C FSTRING='0123456789'
C KCOMP IS AN INTEGER VARIABLE DETERMINING WHICH REGION OF THE MESH IS BEING READ BY JEL. THE FIRST ELEMENT IN REGION 1 IS KCOMP1. THE FIRST ELEMENT IN THE SECOND REGION IS KCOMP2.
C **** THESE VARIABLES MUST BE SET FOR EACH PARTICULAR MESH****
C THE 1ST ELEMENT IN MATERIAL 1 IS KCOMP1
C THE 2ND ELEMENT IN MATERIAL 2 IS KCOMP2
C **** THESE NUMBERS MUST BE SEQUENTIALLY ASCENDING****
C KCOMP(1)=1
C KCOMP(2)=2
C KCOMP(3)=999999
C DO 25 K=4,10
C KCOMP(K)=999999
C CONTINUE
C
C GRAB THE INPUT FILE NAME FROM THE KEYBOARD AND DISPLAY THE POSSIBLE CHOICES
C WRITE(6,*)
C CALL ULTLCMD (ts = ' fI'
C WRITE(6,*)
C WRITE(6,*)
C READ1,1(A1), FNAME
C WRITE(6,*)
C C PARSE FNAME FOR THE FIRST BLANK SPACE 'FNAME'
C C NOW, FNAME WITHOUT BLANKS IS 'FNAME(1:KNAME)' KNAME=INDEX( 'FNAME', ' ')+1
C C THIS IS WHERE GLOBAL LOOP BEGINS TO OPEN EACH FILE
C C FOUNAME=FNAME(1:KNAME)
C C CLEAR THE VARIABLES
C DO 66 KD1=1,10
C DO 67 KD2=1,100
C ALLLD(KD1)=0.
C ALLLPD(KD1)=0.
C ETOTAL(KD1)=0.
C TIME(KD1)=0.
C EENERGY(KD1,KD2)=0.
C ENERG Y1(KD1,KD2)=0.
C ENER GY2(KD1,KD2)=0.
C MISES(KD1,KD2)=0.
C KPRN(KD1,KD2)=0.
C KPRF(KD1,KD2)=0.
C INVARMAX(KD1,KD2)=0.
C INVARMIN(KD1,KD2)=0.
C ESTRDN(KD1,KD2)=0.
C PSFSTRDN(KD1,KD2)=0.
C KESTRN(KD1,KD2)=0.
C KPSFSTRDN(KD1,KD2)=0.
C
C 67 CONTINUE
C 66 CONTINUE
C FILE INITIALIZATION
C FABQUS=FOUTNAME(1:KNAME)
C KRN(1)
C KRN(1)=8
C KRN(2)=2
C KOUT=0
C CALL INTPF(FABQUS,KRN,KRN,KR N,KOUT)
C JUNIT=8
C CALL DBCRN(JUNIT)

C INITIALIZE THE SECTION, STEP, AND MAXSTEP VARIABLES
C KSECTION=1
C KSTEP=1
C KMAXSTEP=1
C
C LOOP ON ALL RECORDS IN RESULTS FILE
C DO 100 K=1,999999
C CALL DDBFIL(EARRAY,JRCD)
C IF(JRCD NE 100 TO 110)
C KEY=JARRAY(1,2)
C ** READ THE ELEMENT NUMBER
C IF(KEY,EQ.1) THEN
C JEL=JARRAY(1,3)
C DETERMINE BY USING THE ELEMENT NUMBER, WHICH SECTION OF THE MESH YOU ARE READING. LOOP THROUGH THE 10 USER-ASSIGNED VALUES FOR KCOMP
C KSECTION IS THE CURRENT SECTION
C KMAXSECTION IS THE LAST SECTION TO BE READ FROM THE FILE
C KSTEP IS THE CURRENT STEP OF THE SIMULATION
C KMAXSECTION IS THE FINAL STEP OF THE SIMULATION
C DO 150 K=1,9
C IF(JEL.GE.JCOMP(K)) AND (JEL.LT.JCOMP(K+4)) THEN
C KSECTION=K
C IF(KSECTION.GT.KMAXSECTION) THEN
C KMAXSECTION=KSECTION
C ELSE IF
C END IF
C 150 CONTINUE
C ** READ THE INVARIANT STRESS COMPONENTS
C MISES(KSECTION,KSTEP) IS THE MISES STRESS
C PRN1(KSECTION,KSTEP) IS THE HIGHEST PRINCIPAL STRESS
C PRN2(KSECTION,KSTEP) IS THE LOWEST PRINCIPAL STRESS
C ELSE IF(KEY,EQ.12) THEN
C IF ARRAY(3), GT MISES(KSECTION,KSTEP) THEN
C KSECTION=KSTEP=ARRAY(3)
C MISES(KSECTION,KSTEP)=JEL
C END IF
C IF ARRAY(6), LT PRN1(KSECTION,KSTEP) THEN
C KSECTION=KSTEP=ARRAY(6)
C PRN1(KSECTION,KSTEP)=JEL
C END IF
C IF ARRAY(8), GT PRN1(KSECTION,KSTEP) THEN
C KSECTION=KSTEP=ARRAY(8)
C PRN1(KSECTION,KSTEP)=JEL
C END IF
C IF ARRAY(9), GT INVARMAX(KSECTION,KSTEP) THEN
C INVARMAX(KSECTION,KSTEP)=ARRAY(9)
C KINVARMAX(KSECTION,KSTEP)=JEL
C END IF
C IF ARRAY(9), LT INVARMIN(KSECTION,KSTEP) THEN
C INVARMIN(KSECTION,KSTEP)=ARRAY(9)
C KINVARMIN(KSECTION,KSTEP)=JEL
C END IF
C ** READ THE STRAIN ENERGY DENSITY
ESTRден(KSECTION,KSTEP) IS THE HIGHEST ELASTIC STRAIN
ENERGY DENSITY
PSTRден(KSECTION,KSTEP) IS THE HIGHEST PLASTIC STRAIN
ENERGY DENSITY
ELSE IF (KEY.EQ.22) THEN
  IF (ARRAY(7).GT.ESTRден(KSECTION,KSTEP)) THEN
    ESTRден(KSECTION,KSTEP)=ARRAY(7)
  END IF
ENDIF

** READ THE ENERGY COMPONENTS FOR EACH ELEMENT OF
THE MESH
EENERGY(KSECTION,KSTEP)=EENERGY(KSECTION,KSTEP)+ARRAY(4)
PENERGY(KSECTION,KSTEP)=PENERGY(KSECTION,KSTEP)+ARRAY(5)
TENERGY(KSECTION,KSTEP)=TENERGY(KSECTION,KSTEP)+EENERGY(KSECTION,KSTEP)
ELSE IF (KEY.EQ.19) THEN
  EENERGY(KSECTION,KSTEP)=EENERGY(KSECTION,KSTEP)+ARRAY(4)
PENERGY(KSECTION,KSTEP)=PENERGY(KSECTION,KSTEP)+ARRAY(5)
TENERGY(KSECTION,KSTEP)=TENERGY(KSECTION,KSTEP)+EENERGY(KSECTION,KSTEP)
ENDIF

READ THE ENERGY WRITTEN BY ABAQUS FOR THE ENTIRE
MESH
THE FOLLOWING ARE WRITTEN BY ABAQUS AND READ FOR
CONFIRMATION
ALLSEL(KSTEP) IS THE ELASTIC ENERGY FOR THE ENTIRE MESH
ALLPL(KSTEP) IS THE PLASTIC ENERGY FOR THE ENTIRE MESH
ELSE IF (KEY.EQ.199) THEN
  ETOTAL(KSTEP)=ARRAY(14)
  ALLSEL(KSTEP)=ARRAY(4)
  ALLPL(KSTEP)=ARRAY(6)
ENDIF

GET THE INCREMENT NUMBER OF THE SIMULATION
ELSE IF (KEY.EQ.2000) THEN
  TIME(KSTEP)=ARRAY(1)
ENDIF

IF (KSTEP.GT.KMAXSTEP) THEN
  KMAXSTEP=KSTEP
ENDIF

ONE STEP HAS BEEN MADE THROUGH THE RECORD
END IF

CONTINUE
CONTINUE
CONTINUE
WRITE(6,*) K1

WRITE(6,*) KMAXSTEP
WRITE THE DATA TO A FILE IN THE FOLLOWING ORDER
1. $Name$ in the file
2. + suffix .sum
DATE
KMAXSECTION,KMAXSTEP
SECTION x
STEP x
ELASTIC ENERGY
PLASTIC ENERGY
TOTAL ENERGY
MISO, ELEMENT #
PRINE, ELEMENT #
PRINJ, ELEMENT #
3RD INVAR MAX. ELEMENT #
3RD INVARIANT MIN. ELEMENT #
ELASTIC STRAIN DENSITY, ELEMENT #
TIME
REPEAT STEP (TO KSTEPMAX)
REPEAT MATERIAL (TO KSECTIONMAX)
OPEN(UNIT=99,FILE=OUTNAME(1:4)FNAME(5::),STATUS=UNKNOWN)
WRITE(99,*) KMAXSECTION,KMAXSTEP
BEGIN LOOP ON SECTION
DO 760 K=1,KMAXSECTION
BEGIN LOOP ON STEP
DO 770 K2=1,KMAXSTEP
WRITE(99,250) EENERGY(K1,K2)
WRITE(99,250) PENERGY(K1,K2)
WRITE(99,250) ATotal(K1,K2)
WRITE(99,840) MISO(K1,K2), KMISO(K1,K2)
WRITE(99,840) PRIN(K1,K2), KPRIN(K1,K2)
WRITE(99,840) PRINJ(K1,K2), KPRINJ(K1,K2)
USEVARMAX(K1,K2), KDEVARMAX(K1,K2)
USEVARMIN(K1,K2), KDEVARMIN(K1,K2)
WRITE(99,840) ESTRден(K1,K2), KESTRден(K1,K2)
WRITE(99,850) TIME(K2)
STOP
770 CONTINUE
760 CONTINUE
CLOSE(UNIT=99)
CLOSE(UNIT=81)
VARIOUS FORMATING STATEMENTS
840 FORMAT(12.3,,16)
850 FORMAT(12.3)
980 FORMAT(A)
STOP
END
Appendix 3.5—Elastic stress summary for a Cu : Mo joint
The following graphs represent the stresses present in a copper : molybdenum joint with completely elastic properties, cooled from 820°C to 20°C. Both the coefficient of thermal expansion and the modulus are temperature dependent. A plastic analog of this analysis is presented in Appendix 5.2.

Nomenclature for the contour and line graphs

<table>
<thead>
<tr>
<th>Abaqus notation</th>
<th>Notation from Figure 3.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>S11</td>
<td>$\sigma_{rr}$ (Pa)</td>
</tr>
<tr>
<td>S12</td>
<td>$\sigma_{zz}$ (Pa)</td>
</tr>
<tr>
<td>S22</td>
<td>$\sigma_{zz}$ (Pa)</td>
</tr>
<tr>
<td>S33</td>
<td>$\sigma_{\theta\theta}$ (Pa)</td>
</tr>
<tr>
<td>SP1</td>
<td>minimum principal stress (Pa)</td>
</tr>
<tr>
<td>SP3</td>
<td>maximum principal stress (Pa)</td>
</tr>
<tr>
<td>MISES</td>
<td>Von Mises stress (Pa)</td>
</tr>
<tr>
<td>INV3</td>
<td>3rd invariant stress (Pa)</td>
</tr>
<tr>
<td>SENER</td>
<td>elastic strain energy ($J m^{-3}$)</td>
</tr>
</tbody>
</table>

Trace location information for the line graphs

***A stress Above the interface in the high cte material
***B stress Below the interface in the low cte material
***C stress along the Centerline of the joint
***E stress along the Edge of the joint
4. Modeling Dissimilar Material Joints through an Energy Approach

Abstract
Finite element analysis was used to develop an understanding of the variables which most prominently affect the residual stress within a dissimilar material joint. The results from an elastic analysis have been used to create an analytical method of determining the mechanical strain energy in terms of the material mechanical properties and the joint geometry. From a thermodynamic perspective, it can be shown that large dissimilar material joints are inherently weaker than smaller ones.

Introduction
The ability to model the state of stress in a dissimilar joint can be used to gauge the relative strength of various joints. Current analytical techniques are unable to incorporate the plastic response of materials and are unable to account for all but the most simple joint geometries. Numerical techniques, such as finite element analysis, are able to solve these problems in a convenient manner, however, sensitivity analyses have shown that the use of peak stress as a metric is unreliable for systems which contain singularities.

The use of elastic strain energy as a metric has been shown to be indicative of the loading within thermally loaded joints. Strain energy effectively incorporates the effects of joint geometry and it is relatively insensitive to the effect of mesh setup variables within finite element analysis. By performing an array of finite element simulations, an analytical method of determining strain energy can be developed which is sensitive to both material mechanical properties and joint geometry.

Continuum derivation
A simple analysis which considers the state of stress at the interface between two dissimilar materials forms the basis for this derivation. Consider some material "A" with a thermal
expansion is greater than material "B", joined at high temperature and then cooled to room temperature. The deformation of these components can be separated into a component due to the natural thermal expansion of the materials plus a component due to the residual stress at the interface. Using Hooke's law \( \sigma = \varepsilon \cdot E \) and thermal expansion \( \varepsilon = \alpha \cdot \Delta T \), the comparative strain for either component can be determined as a function of the imposed stress at the interface. Assuming that a rigid bond exists between materials, the strain between components is equal, leading to Equation 4.1.

\[
\alpha_A(T_{Bond} - T) + \frac{\sigma}{E_A} = \alpha_B(T_{Bond} - T) - \frac{\sigma}{E_B} \quad (4.1)
\]

This can be solved in terms of the mathematically imposed interfacial stress to form Equation 4.2.

\[
\sigma = \frac{E_A E_B}{E_A + E_B} (T_{Bond} - T)(\alpha_B - \alpha_A) \quad (4.2)
\]

For a purely elastic body, the strain energy in either component can be determined in terms of the material properties and the deviation from the joining temperature.

\[
U_A = \frac{1}{2} \left( \frac{E_A E_B^2}{(E_A + E_B)^2} \right) (T_{Bond} - T)^2 (\alpha_B - \alpha_A)^2 \quad (4.3) \text{ and}
\]

\[
U_B = \frac{1}{2} \left( \frac{E_A^2 E_B}{(E_A + E_B)^2} \right) (T_{Bond} - T)^2 (\alpha_B - \alpha_A)^2 \quad (4.4)
\]

Thus, the strain energy of the total system is the sum of Equations 4.3 and 4.4.

\[
U_T = \frac{1}{2} \left( \frac{E_A E_B}{E_A + E_B} \right) (T_{Bond} - T)^2 (\alpha_B - \alpha_A)^2 \quad (4.5)
\]

The use of continuum mechanics based on a stress / strain approach makes it difficult to incorporate the effects of joint geometry. In addition, this derivation assumes completely
elastic behavior. Disregarding these obvious limitations, the basic formalism of this strain energy approach will be assessed using finite element analysis.

**Numerical Modeling**

Finite element analysis was performed using Abaqus 5.4 on a DEC 5000/200 workstation with the following assumptions: both materials are stress-free at the joining temperature and are rigidly attached along the interface; temperature equilibration throughout the system occurs instantaneously; and the mechanical properties of both materials are perfectly elastic with no temperature or stress dependence. The mechanical properties of the system were based on an aluminum : steel joint with a total temperature change of 1000°C—Table 4.1.

The geometry nomenclature for the meshes used in the analyses is shown in Figure 4.1. The mesh is symmetric along the y-axis and pinned at the interface. Each component is formed from reduced integration quadratic elements biased toward the joint edge and interface. The standard geometry for the components was 0.5 m wide and 2.0 m high. The through thickness of the plane stress analysis was set to \( \pi/4 \) m thick, so that the bonded area was identical to the axisymmetric model.

**Results and Discussion**

The effect of each variable on the strain energy of the joint was determined by changing one variable at a time while holding the others constant. The results of these analyses are separated into material effects and geometric effects of the joint. The strain energy for each

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus (GPa)</th>
<th>( \alpha ) (1/°C)</th>
<th>Poisson's ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>70</td>
<td>( 25 \times 10^6 )</td>
<td>0.3</td>
</tr>
<tr>
<td>Steel</td>
<td>210</td>
<td>( 10 \times 10^6 )</td>
<td>0.3</td>
</tr>
</tbody>
</table>

*Table 4.1 — Mechanical properties used for finite element analysis*
series of simulations is presented as a function of the independent variable.

**Material Effects**

**Modulus**
The effect of modulus was determined by keeping the modulus of the "steel" component constant, while varying the modulus of the other component from 10 GPa to 310 Gpa in 20 Gpa increments—Figure 4.2. Each vertical series of symbols represents the results of a single simulation. The energy for each component was determined along with the total energy of the combined system. The line connecting these point was created using Equation 4.5. In each case, the curve fit was very accurate. It is interesting to note that the material with the lowest modulus actually "contains" the most energy of the joint. For both the plane stress and axisymmetric models, the functionality of the curve fit was the same, only the multiplying constant varied.

**Thermal expansion**
The effect of thermal expansion mismatch was determined by varying the linear coefficient of thermal expansion of the "aluminum" component from $3 \times 10^{-6} \text{ C}^{-1}$ to $27 \times 10^{-6} \text{ C}^{-1}$ while keeping the other variables constant—Figure 4.3. Again, the functionality of the calculated strain energy was as predicted by the continuum derivation, varying with the square of the thermal expansion difference for each component. The amount of strain energy within the components is not influenced by their respective thermal expansions.

**Joining temperature**
The dependence of strain energy on deviation from joining temperature was modeled at 100°C intervals from 1000°C to 0°C—Figure 4.4. As the components cooled, the strain energy was proportional to the square of the temperature deviation. A summary of the material effects and the fitted equations are given in Table 4.2
Table 4.2—The effect of mechanical properties on strain energy

<table>
<thead>
<tr>
<th></th>
<th>Plane Stress</th>
<th>Axisymmetric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus (GPa)</td>
<td>$8,570 \frac{E_A E_B}{E_A + E_B}$</td>
<td>$17,660 \frac{E_A E_B}{E_A + E_B}$</td>
</tr>
<tr>
<td>Thermal expansion ($10^{-6}$ 1/°C)</td>
<td>$2,140 (\alpha_B - \alpha_A)^2$</td>
<td>$4,370 (\alpha_B - \alpha_A)^2$</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>$0.47 (T_o - T)^2$</td>
<td>$0.99 (T_o - T)^2$</td>
</tr>
</tbody>
</table>

**Geometric Effects**

The dependence of the strain energy on joint geometry was determined by varying the following variables: width, aspect ratio, edge angle and interface angle. This nomenclature is referred to in Figure 4.1.

**Width**

The effect of joint width on strain energy was determined by keeping the height : width ratio of the joint constant while varying the width from 0.1 m to 2.6 m. There was a distinct difference in the way that the plane stress and axisymmetric models reacted to a change in width. For the plane stress model, strain energy was proportional to the square of the width while the energy of the axisymmetric model was proportional to the cube of the radius—Figure 4.5.

This effect can be understood by re-examining the initial argument from the analytical derivation. For the plane stress model, the through-thickness of the joint is infinitesimal by definition, in effect, making the interface between the components one-dimensional. For a given change in temperature, the unconstrained offset between components at the interface increases as the size of the joint increases. Consequently, the restoration force at the interface is much larger as well. The strain energy caused by this interaction is dependent upon the mechanical properties of the components and the size of the interface. For the plane stress geometry, this can be modeled like a one dimensional spring.
\[ F = k x \text{ and } U_{\text{plane stress}} = \int F \cdot dx = \frac{1}{2} kx^2 \quad (4.6) \]

Here, the constant “k” represents the mechanical properties of the materials and “x” represents the width of the interface. A similar analogy can be used for the case of the axisymmetric model, except that its interface is circular. The strain energy created by expanding a disc radially can be determined in a similar fashion—Equation 4.7.

\[ F = k r \text{ and } U_{\text{axisymmetric}} = \int \int F r \, dr \, d\theta = 2\pi k \int r^2 \, dr = \frac{2\pi}{3} kr^3 \quad (4.7) \]

The energy predicted by Equations 4.6 and 4.7 is due to residual stress, which will tend to reduce the strength of a joint. A bounding function can be created by considering the positive energy which joins the components and is proportional to the area of the interface. For the plane stress model, it is proportional to the width of the interface while the axisymmetric model is proportional to the square of the radius. With regards to interface size, the strain energy will always increase at a greater rate than the bonding energy, which means that the formation of a large joints are energetically unfavorable.

**Aspect Ratio**

By holding the width of the joint constant and varying the height of both components, the effect of joint aspect ratio (height : width) on strain energy was determined—Figure 4.6. As the aspect ratio of the joint increases, so does the strain energy. At a height : width ratio of approximately 2:1, the strain energy of the joint asymptotically approaches a maximum. Beyond this point, the strain energy of the system does not increase appreciably. This type of behavior is common in diffusion and heat transfer problems and can be modeled conveniently by using the “error function”—Equation 4.8.

\[ \text{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\gamma^2} \, d\gamma \quad (4.8) \]
Because an analytical method does not exist for this type of system a rigid proof of this relationship is virtually impossible, however, it is not difficult to understand the roots of this behavior. The stress at the interface is generated by two effects: volume interaction of two dissimilar materials and an increase in stiffness of the joint structure with increasing thickness. Strain energy is created as each successive layer of material is influenced by the perturbation at the interface. As the thickness of the joint increases, the volume of interacting material increases, which leads to an increase in the strain energy. Concurrently, the stiffness of the total structure increases with thickness, increasing the stresses at the interface. A practical manifestation of this phenomena is that joints formed from components of low height: width ratios will be easier to form than thicker components.

**Interface angle**
The effect of the interface angle was measured by varying it while keeping the overall width of the joint constant. The minimum energy for the total joint occurred at an interface angle of 0°, however, the minima for the individual components occurred at slightly different angles—Figure 4.7. At the angular extremes, the strain energy was dominated by the effect of an increased interfacial area. Because the width of the joint was kept constant, the interfacial area increased with the width / cosine of the interface angle.

**Edge angle**
As the edge angle was varied, the energy of the joint remained relatively constant (Figure 4.8), however, it affected the peak stress values significantly—Figure 4.9. In general, the peak stress increased with an increase in the edge angle which corresponds to an increase in mesh density in the material of higher modulus. This is a fundamental limitation of the finite element process applied to dissimilar material joining. Whether these geometric changes actually lead to higher stresses or are an artifact of the modeling system is very difficult to determine.
Table 4.3—The effect of joint geometry on strain energy

<table>
<thead>
<tr>
<th></th>
<th>Plane Stress</th>
<th>Axisymmetric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width (m)</td>
<td>$1.60 \times 10^6 , \text{width}^2$</td>
<td>$7.11 \times 10^6 , \text{width}^3$</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>$482,000 , \text{Erf} \left( \frac{2.25 , \text{height}}{\text{width}} \right)$</td>
<td>$983,000 , \text{Erf} \left( \frac{2.98 , \text{height}}{\text{width}} \right)$</td>
</tr>
<tr>
<td>Edge Angle (°)</td>
<td>$460,000 + 534 , \theta_{\text{edge}} + 195 , \theta_{\text{edge}}^2$</td>
<td>$933,000 + 2930 , \theta_{\text{edge}} + 421 , \theta_{\text{edge}}^2$</td>
</tr>
<tr>
<td>Interface Angle (°)</td>
<td>$1.81 \times 10^6 - 1.35 \times 10^6 , \cos \theta_{\text{interface}}$</td>
<td>$3.87 \times 10^6 - 2.94 \times 10^6 , \cos \theta_{\text{interface}}$</td>
</tr>
</tbody>
</table>

Summary

For each simulation, the results of the numerical analysis agreed with the mathematical form predicted by the simple analytical model. The functionality of the “materials effects” were the same for the plane stress and axisymmetric analyses. Once the materials for a dissimilar material joint are selected, the only remaining design element is the geometry of the joint. Finite element analysis was used to examine the strain energy created by dissimilar material butt joints undergoing a change in temperature. These results can be summarized for plane stress and axisymmetric models in Equations 4.9 and 4.10, respectively.

$$U_{\text{total}} = 0.32 \frac{1}{2} \frac{E_A E_B}{E_A + E_B} (\alpha_A - \alpha_B)^2 (T_{\text{Bond}} - T)^2 (\text{width})^2 \text{Erf} \left( \frac{\text{height}}{\text{width}} \right) \quad (4.9)$$

$$U_{\text{total}} = 0.33 \frac{2\pi}{3} \frac{E_A E_B}{E_A + E_B} (\alpha_A - \alpha_B)^2 (T_{\text{Bond}} - T)^2 (\text{width})^3 \text{Erf} \left( \frac{\text{height}}{\text{width}} \right) \quad (4.10)$$

Because the deleterious strain energy always scales at a higher order than the bonding energy that holds the joint together, small joints are expected to be more durable than large joints of similar materials and aspect ratio.
Figures

Figure 4.1—Nomenclature for joint geometry

Figure 4.2—Strain energy as a function of modulus
Figure 4.3—Strain energy as a function of coefficient of thermal expansion

Figure 4.4—Strain energy as a function of temperature
Figure 4.5—Strain energy as a function of joint width

Figure 4.6—Strain energy as a function of joint aspect ratio
Figure 4.7—Strain energy as a function of joint interface angle

Figure 4.8—Strain energy as a function of joint edge angle
Figure 4.9—Peak Mises stress as a function of joint edge angle
5. Plastic Materials Response over a Wide Range of Temperature

Abstract
A powerful yet tractable method for modeling the mechanical behavior of materials over a wide range of temperature was created. By developing a characteristic strain rate for the dissimilar joining system being studied, the use of first-order dislocation mechanics and deformation mechanism maps are used to create a yield surface as a function of temperature and plastic strain. Iterations between finite element analysis and the external mechanical model are performed to ensure that the mechanical properties calculated by the external material model concur with the strain rate predicted by the finite element model.

Introduction
When forming a dissimilar material joint, the temperatures encountered may range from room temperature to the melting point of the material. Over this temperature range, many factors contribute to the mechanical behavior of the system. Mechanical loading can take place due to thermal gradients within a material or a difference in thermal expansion between materials. To predict the residual stress for these types of systems, an attempt must be made to model the dominant effects as accurately as possible.

The mechanical behavior of materials depends upon many more variables than are readily measurable by experimental techniques. Factors such as prior loading history, strain rate dependence and the influence of temperature make the properties increasingly difficult if not impossible to model accurately. Because of this, order of magnitude assumptions must be adopted into models that are not completely verifiable—comments Professor Daniel Drucker:
"There is, in fact, no feasible way at all to perform more than a small subset of the full range of tests that are needed in the plastic regime to determine just the broad outline of the complete set of incremental relations between stress and strain for any material of comparable strength. Consequently it is necessity, not just convenience, that forces idealization of plastic behavior in the absence of experimental information." (Drucker, 1988)

There are numerous methods which the literature presents as viable methods of modeling such complex behavior. In general, they are either too theoretical, where none of the material data is available to evaluate the model; or they are too empirical, where the information is derived from curve fitting, yielding very little information applicable to other systems.

The work of Frost and Ashby is a notable exception to either of these stereotypes (Frost & Ashby, 1982). A series of competing deformation processes are combined to form a "deformation-mechanism map". The use of variables and concepts familiar to engineers aids in the creation of a detailed model without the use of overly complex mathematics. Experimental data coupled with mathematical generalizations is able to predict material behavior in the form of yield strength (interchangeably referred to as "deformation resistance" or "flow stress") as a function of temperature and strain rate.

An example of a deformation map for copper with an average grain size of 0.1 mm is shown in Figure 5.1. This approach considers five distinct mechanisms of deformation: (1) material overload, which is based on the ideal strength of the material; (2) plasticity by thermally activated dislocation glide, which is limited by lattice resistance or obstacles such as precipitates or second phase dispersions; (3) deformation by twinning; (4) creep, which can be limited by glide or diffusion processes; and finally (5) diffusional flow, where lattice, grain boundary or dislocation core diffusion may be the limiting factor.

This approach works well to establish a steady-state baseline for material behavior—at a given strain rate, the yield stress can be determined as a function of the temperature. The
effect of kinetic processes such as strain hardening and recrystallization are not considered. When modeling dissimilar material joints, the considerably more complex task of incorporating the effects of plastic strain and temperature must be done simultaneously.

Results and Discussion
The modeling formalism presented in this section is based upon a copper : molybdenum braze joint, formed at 850°C in a vacuum tube furnace. While the results are very system specific, the methodology behind this approach is quite general and may be applied to almost any joining system. All of the material behavior calculations were done using the mathematical analysis package, Mathematica, and are detailed in Appendix 5.1. The finite element analysis was performed on a Dec 5000/200 using Abaqus 5.4.

Thermal Gradients
Whenever possible, it is desirable to reduce the complexity of the system being modeled. For a dissimilar material joint undergoing a change in temperature, the effect of thermal gradients can be effectively eliminated by considering the relative cooling rate of the components within the furnace setup. This can be mathematically verified through the use of the Biot number, a dimensionless number that is an indicator of the external to internal resistance of heat flow—Equation 5.1.

\[ Bi = \frac{h L}{k_{thermal}} \] (5.1)

The heat transfer coefficient at the surface of the material is represented by \( h \) (\( W \text{ cm}^{-1} \text{ K}^{-1} \)), \( L \) is the semi-thickness of the material (cm) and \( k_{thermal} \) is the thermal conductivity of the material (\( W \text{ cm}^{-1} \text{ s}^{-1} \text{ K}^{-1} \)). When the Biot number is less than 0.1, the heat flow out of the material is the rate controlling process, making it a fair assumption that the material is at a uniform temperature from the external surface to the interior (Carslaw & Jaeger, 1959). This condition, referred to as Newtonian cooling, can be used to inductively determine the
validity of the assumption. For an infinitely long cylinder, the Newtonian cooling rate is given by Equation 5.2.

\[
\frac{T - T_f}{T_i - T_f} = \exp\left(\frac{-2ht}{rdC_p}\right)
\] (5.2)

Where \(T\) is the temperature (°C), \(T_f\) is the temperature of the quenching medium (°C), \(T_i\) is the initial temperature (°C), \(t\) is the time (s), \(r\) is the radius of the cylinder (cm), \(\rho\) is the density (g cm\(^{-3}\)) and \(C_p\) is the specific heat of the material (J g\(^{-1}\) °C\(^{-1}\)) (Geiger & Poirier, 1973).

The empirical cooling rate was obtained by heating the components to 850°C in a vacuum furnace and measuring the time versus temperature data with a thermocouple—Figure 5.2. Such a method measures the effects of specimen geometry and various cooling mechanisms directly, without the need to make assumptions. For components of similar geometry, molybdenum will determine the thermal equilibration rate due to its lower thermal conductivity—Table 5.1.

This data can be used to determine the validity of the Newtonian cooling approximation by solving for the heat transfer coefficient, \(h\) from Equation 5.2 and subsequently determining the value of the Biot number—Figure 5.3. Immediately after the power to the furnace is shut off, the heat transfer coefficient is very high, then it decreases gradually. Even when the peak value of the heat transfer coefficient is used, the Biot number is still far less than 0.001, making the assumption of thermal equilibrium valid.

<table>
<thead>
<tr>
<th>Property</th>
<th>Copper</th>
<th>Molybdenum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Heat (J g(^{-1}) K(^{-1}))</td>
<td>0.385</td>
<td>0.245</td>
</tr>
<tr>
<td>Density (g cm(^{-3}))</td>
<td>8.9</td>
<td>10.2</td>
</tr>
<tr>
<td>Thermal Conductivity (W cm(^{-1}) K(^{-1}))</td>
<td>3.9</td>
<td>1.4</td>
</tr>
<tr>
<td>Thermal Diffusivity (cm(^{2}) K(^{-1}))</td>
<td>1.14</td>
<td>0.56</td>
</tr>
</tbody>
</table>
**Thermal mismatch strain rate**

The strain rate within a dissimilar material joint is dependent upon many factors including the geometry of the joint, the location within the joint, the material combination and the cooling rate of the furnace. Using analytical methods, it is impossible to take all of these factors into consideration simultaneously. However, a first-order approximation can be made which considers only the thermal expansion mismatch of components, disregarding the effects of mechanical interaction.

The coefficient of thermal expansion at a specific temperature can be calculated using the linear thermal expansion data for each material—Equation 5.3. This data is available for a wide range of materials, from room temperature to their melting point (University Press, 1970-79). Often, an approximation is made which determines the average coefficient of thermal expansion over a range of temperature—Equation 5.4.

\[
\alpha = \frac{1}{L} \frac{dl}{dT} \quad (5.3) \quad \text{and} \quad \overline{\alpha} = \frac{1}{L} \frac{1}{T_f - T_i} \int_{T_i}^{T_f} \frac{dl}{dT} \quad (5.4)
\]

For most metals, the coefficient of thermal expansion increases significantly from room temperature to the melting point of the material. Accordingly, the average coefficient of thermal expansion should accurately represent the desired temperature regime.

Using the basic formula for thermal expansion, the differential strain between each material can be approximated using Equation 5.5. The strain rate is given by the derivative of this equation with respect to time. Because temperature is the natural variable for mechanical properties, the chain rule is used to make it more convenient—Equation 5.6.

\[
\varepsilon = \Delta \alpha \cdot T \quad (5.5)
\]

\[
\frac{d\varepsilon}{dt} = \frac{d\Delta \alpha}{dt} \cdot T + \Delta \alpha \frac{dT}{dt} = \left( \frac{d\Delta \alpha}{dT} \cdot \frac{dT}{dt} \right) T + \Delta \alpha \frac{dT}{dt} \quad (5.6)
\]
Given the cooling rate shown in Figure 5.2, the characteristic strain rate for a copper-molybdenum joint based strictly on the thermal expansion mismatch is calculated in Figure 5.4. For a given strain rate, the yield strength for each material can be calculated using the method of Frost & Ashby. A comparison of the mechanical properties based on the cooling rate of the vacuum furnace and the limit of the Newtonian cooling is shown in Figure 5.5. For any given material and joint geometry, there exists an infinite number of possible cooling rates. The Newtonian cooling rate is specific for a given material/geometry combination and establishes the fastest cooling rate which maintains thermal equilibrium within the material. At moderate temperatures, the mechanical behavior varies significantly with the strain rate. As the deformation rate increases, yield strength increases as kinetically dependent means of plasticity such as creep come less prominent.

**Addition of Plastic Strain**

Once the steady-state temperature response of the material has been determined, the effect of plastic strain must be added. Although no precise data exists for the hardening of materials over wide temperature ranges, an engineering approximation can be made by using first order dislocation mechanics. It is assumed that the only dislocation source is plastic deformation and the only dislocation sink is annihilation due to the effects of time and temperature.

A first order examination of the increase in dislocation density with respect to the average distance traveled is covered in most metallurgical textbooks (Dieter, 1986). This derivation can be calibrated to empirical data by adding a term, “C”, which considers the average distance a dislocation line travels before becoming either pinned or annihilated—Equation 5.7 (Argon, expected 1996).

\[
\frac{dp^+}{d\varepsilon} = \frac{\sqrt{p}}{bc} \quad (5.7)
\]
Prior investigators have used numerical simulations to determine that "C" is a constant on the order of 15. At low temperatures, where recovery effects are extremely slow, C dominates the response of the Equation 5.7 and can be used as a calibration factor. For this study, a value of 40 for C was found to provide a good match with experimental stress-strain data for copper (Paydar, Tong & Akay, 1994).

Once the effect of dislocation production has been determined, the more complicated effect of annihilation must be considered. The relative rate of dislocation annihilation within the cell and cell walls of a polycrystalline material can be characterized as a function of lattice diffusion and dislocation core diffusion (Prinz & Argon, 1984). It has subsequently been found that the dislocation density of the cell walls is at least 3 orders of magnitude larger than the cell interiors, which dominates the material response (Argon & Haasen, 1993). Experimental work on a copper system similar to that used in this study has shown that the dislocation core term sufficiently describes the behavior at the low strain rates encountered in dissimilar material joining, which leads to Equation 5.8 (Wlassich, 1995).

\[
\frac{dp^-}{dt} = \frac{-2\sqrt{2}}{\pi(1-v)} D_c \frac{G\Omega}{RT} \rho^3 \quad (5.8)
\]

The mechanical response of the materials is determined by solving the partial differential equation formed by Equations 5.7 & 5.8 as a function of time, temperature and strain. The complexity of this calculation is drastically reduced by considering interrelations between the independent variables: cooling rate establishes a relationship between time and temperature while the peak strain of a characteristic element can be determined as a function of temperature. This ultimately creates a relationship between strain rate and temperature, reducing the competition of dislocation production and annihilation to an ordinary differential equation — Equation 5.9.

\[
\frac{d\rho}{d\varepsilon} = \frac{dp^+}{d\varepsilon} + \frac{dp^-}{dt} \frac{dt}{d\varepsilon} \quad (5.9)
\]
Dislocation density can only be directly measured using a transmission electron microscope which makes the manipulation of Equation 5.9 rather unwieldy. Yield strength is an effective macroscopic measure of the dislocation density and the relation between the two is established by Equation 5.10 (Li, 1963).

\[ \sigma_{\text{yield}} = \alpha G b \sqrt{\rho} \quad (5.10) \]

Alpha is a numerical constant which depends on the crystal structure of the material. For face centered cubic metals, such as copper, it has a value of approximately 0.3. G is the shear modulus, b is the Burgers vector, and rho is the dislocation density. The use of Equation 5.10 and a further use of the chain rule allows for the solution of the yield strength as a function of material constants, plastic strain, strain rate and temperature—Equation 5.11.

\[ \frac{d\sigma}{d\varepsilon} = \frac{d\rho}{d\varepsilon} \frac{d\sigma}{d\rho} = \frac{1}{2\alpha_{\text{Taylor}} b^2 c G} - \frac{\sqrt{2}}{\pi(1 - v)} D_c A_c \left( \frac{\Omega}{RT} \right) \frac{\sigma^5}{\alpha^6 b^n G^5} \frac{dt}{d\varepsilon} \quad (5.11) \]

The initial conditions needed to solve Equation 5.11 are provided by the deformation mechanism maps. At each temperature, the competing effects of dislocation annihilation and production can be evaluated numerically to obtain a yield stress / plastic strain / temperature map—Figure 5.6. At zero strain, the yield stress is predicted by the deformation maps using the temperature dependent strain rate. At high temperature, the effect of annihilation overcomes the rate of dislocation production as evident by the decrease in hardening. Within this yield surface, the characteristic element traverses a path from the lower right to the upper left in a hemispherical manner. At the joining temperature, neither of the materials have plastically yielded. As the temperature decreases, the characteristic strain continues to grow in a monotonic fashion.
Feedback from FEA

The assumption that mechanical loading does not influence the mechanical properties of materials can be evaluated by comparing the strain rate predicted by Equation 5.6 with the strain rate predicted by finite element analysis. For the copper : molybdenum model system, the strain rate predicted by the numerical analysis was nearly an order of magnitude greater than that predicted by the thermal expansion mismatch alone. It can be concluded that the mechanical effects significantly influence the loading of dissimilar material joints and a method for accommodating this effect must be made.

Because the strain rate is a quantity that cannot be predicted using analytical techniques, it must solved for implicitly—Figure 5.7. The first two steps in this procedure, “Selection of materials” and “Measurement of process variables”, have already been covered. The next two steps, “Development of mechanical properties” and “Evaluation within finite element analysis” are coupled processes which must be solved for in an iterative fashion. Within the external material model, an input strain rate is used to determine the mechanical properties for each plastic component as a function of temperature and plastic strain. Next, the finite element model uses this material behavior to predict a characteristic strain rate as an output. If the mechanical behavior of the input material is “too weak”, excessive deformation will occur, producing a strain rate that is “too high”. Conversely, if the input material behavior is “too strong”, the predicted strain rate will be “too low”. This is a self-convergent process—after several iterations, the input strain rate matches the output strain rate and the simulation is complete—Figure 5.8.

This methodology offers a bounded solution that is quite computationally efficient. By using the maximum strain characteristic to a particular material system, cooling rate and joint geometry, the time required for a fully plastic finite element simulation is reduced to several hours. Calculation of the mechanical properties via a personal computer takes
approximately 30 minutes per iteration. While this procedure may sound obtuse, it is quite tractable, given the complexity of the modeling task.

**Correlation to elastic results**
The strain energy results from the analytical method developed in Chapter 4 are compared to the fully plastic numerical model using the methodology developed in this chapter. For this particular system, the strain energy predicted by an elastic analysis is at least one order of magnitude greater than that predicted by the plastic analysis—Figure 5.9. This leads to a rather interesting analogy. A number of investigators have shown that up to 10 percent of the work needed to plastically deform a metal remains as elastically stored energy (Reed-Hill & Abbaschian, 1992). Even when considering the many processes that occur when joining dissimilar materials, this same type of relationship is observed. The input work is not created by external tractions, rather, the elastic potential energy from the interaction of the two materials. For the fully plastic simulation, the total elastic energy of the joint was approximately 2-3% of that predicted by an elastic analysis. This is merely an order of magnitude approximation which depends on both the material properties and processing parameters of the joint. Systems with limited ductility such as high strength metal: ceramic joints are expected to have significantly higher strain energies.

A simulation using the cooling rate for a sample at its Newtonian limit shows the effect of cooling rate. The amount of stored elastic energy was 33% higher for the sample cooled at the limit of thermal equilibration—Figure 5.10. Due to the slow cooling rate, creep mechanisms allow much of the mismatch energy between components to be dissipated plastically. Significant levels of elastic strain do not increase appreciably until 400°C, which corresponds to another general material properties rule. Generally, creep processes do not become significant until the temperature increases beyond one-half of the absolute melting point of a material. For copper this corresponds to 678 K or 405°C. Below this
temperature, the creep kinetics are so sluggish that no practical cooling rate is slow enough to avoid the increase in residual stress.

While this plastic analysis is certainly more system specific than the generalized elastic analyses, the same trends with respect to joint geometry are observed. A fully plastic analysis where the radius of an axisymmetric butt joint was varied had functionality identical to the purely elastic analysis. Both the plastically dissipated energy and the stored elastic energy were proportional to the radius of the component cubed—Figure 5.11. This means that the plastic zone increases disproportionately with an increase in joint size. Regardless of the material behavior formulation used, it reiterates that large joints are thermodynamically unfavorable. For this particular system, the amount of energy is very small and would most likely be more noticeable in a system where elastic energies were higher such as a ceramic : metal joint.

Appendix 5.2 is presented as the plastic analog of the stress summary given in Appendix 3.5. The effect of the singularity is most pronounced when examining the elastic strain energy and plastic strain energy contour graphs. In both cases, the majority of the energy is located along the interface near the free surface of the joint. While the equilibrium stresses are the highest here, this is compounded by the effects of strain hardening. For this reason alone, it is recommended that the outer surface of the joint be machined away to avoid future propagation of a fatigue crack.

**Summary**
The processes which occur in a dissimilar joint as it is cooled from the joining temperature are very difficult to model, however, a bounded approach using strain rate as an implicit variable makes the analysis tractable. By considering the relative rate for each of these processes coupled with the initial conditions provided by the deformation mechanism maps, a model which is sensitive to strain rate, plastic strain and temperature can be created for almost any material. For the model copper : molybdenum system, the effect of thermal
gradients were eliminated by comparing the calculated heat transfer coefficient from a
Newtonian cooling approximation to the time versus temperature history of the vacuum
furnace. The use of a first order phenomenological approach which incorporates variables
such as diffusion rate, grain size, and furnace cooling rate, was used to estimate the yield
strength as a function of temperature and plastic strain. Feedback from the finite element
analysis was used in an iterative manner to refine the material properties of the joint. This
mechanical data behavior can be easily incorporated into finite element analysis, allowing a
high level of detail while maintaining overall computational efficiency.

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Figures

![Graph showing deformation map for copper](image)

*Figure 5.1—Deformation map for copper*
Figure 5.2—Vacuum furnace cooling data

Figure 5.3—Heat transfer coefficient calculated from furnace data
Figure 5.4—Characteristic strain rate as a function of temperature

Figure 5.5—Yield strength of copper as a function of cooling rate
Figure 5.6—Yield strength as a function of temperature and plastic strain

Figure 5.7—Schematic of analysis process
Figure 5.8—Convergence of material model and numerical simulation

Figure 5.9—Comparison of energy contributions as a function of temperature
Figure 5.10—Strain energy as a function of cooling rate

Figure 5.11—Strain energy as a function of component radius
Appendix 5.1—Plastic material model

Deformation Map Calculations

This approach is identical to that used by Frost and Ashby. The use of basic material parameters such as diffusion coefficients and activation energies are used to create a first order approximation of the yield stress / temperature surface as a function of strain rate.

Cu Material data

\[
\begin{align*}
\omega &= 1.18 \times 10^{-29}; \\
b &= 2.56 \times 10^{-10}; \\
T_n &= 1356; \\
\mu_0 &= 4.21 \times 10^4 \times 10^6; \\
\mu_\text{var wrt} &= -0.54; \\
D_{ov} &= 2.0 \times 10^{-5}; \\
Q_v &= 197 \times 10^3; \\
D_{ob} &= 5.0 \times 10^{-15} / d; \\
Q_b &= 104 \times 10^3; \\
D_{oc} &= 1.0 \times 10^{-24} / ac; \\
Q_c &= 117 \times 10^3; \\
\eta &= 4.8; \\
A_{\text{val}} &= 7.4 \times 10^5; \\
\alpha' &= 794; \\
\tau_\alpha &= 6.3 \times 10^{-3} \mu_0; \\
\gamma_{\text{m}} &= 10^6; \\
\delta_{\text{F}} &= 0.5 \mu_0 b^3; \\
\text{grainsize} &= 100. \times 10^{-6}; \\
k &= 1.381 \times 10^{-23}; \\
R &= 8.314; \\
ac &= 0.5 (ac \ D_{oc}) / (d \ D_{ob}); \\
d &= \sqrt{ac / 2}; \\
\mu_\text{m} &= \mu_0 (1 + (T-300) / T_n \ \mu_\text{var wrt}); \\
D_v &= D_{ov} \ \exp[-Q_v / (R T)]; \\
D_b &= D_{ob} \ \exp[-Q_b / (R T)]; \\
D_c &= D_{oc} \ \exp[-Q_c / (R T)]; \\
A &= \sqrt{3} \times (n+1) \ \text{Aval};
\end{align*}
\]

1. Elastic Collapse
   for fcc alpha=0.06 for bcc alpha=0.1

\[
\gamma_{\text{m}} = \text{If} (\sigma < 0.06 \ \mu_0, 10^{-50}, 10^{10}) / N;
\]

2. Plasticity Limited by Discrete Obstacles

\[
\gamma_{\text{m}} = \gamma_{\text{m}} \ \exp[-\delta_{\text{F}} / (k T) \ (1-\sigma / \tau_\alpha)] / N;
\]

3. Plasticity limited by lattice resistance

\[
\begin{align*}
\gamma_{\text{map}} &= 1. \times 10^{-11}; \\
\delta_{\text{F}} &= 0.1 \ \mu_0 b^3; \\
\tau_\alpha &= 1. \times 10^{-2} \ \mu_0; \\
\gamma_{\text{m}} &= \text{If} (\sigma / \mu_0 > 10^{-3}, \text{Abs} (\gamma_{\text{map}} (\sigma / \mu_0)^2 \ \exp[-\delta_{\text{F}} / (k T) \ (1-(\sigma / \tau_\alpha)^{3/4})] / N, 10^{-50});
\end{align*}
\]

4. Power law creep by glide alone

\[
A_2 = A; \\
\text{Deff} = D_v (1+10ac / (b^2) \ (\sigma / \mu_0)^2 \ D_{oc} / D_{v}); \\
\gamma_{\text{m}} = A_2 \ \text{Deff} \ \mu_0 b / (k T) \ (\sigma / \mu_0)^n / N;
\]

5. Harper-Dorn creep (used for Al and Pb only due to lack of data)

\[
A_\text{h} = 0; \\
(\gamma_{\text{m}} = A_\text{h} \ D_v \ \mu_0 b / (k T) \ (\sigma / \mu_0) / N \ (*) \\
\gamma_{\text{m}} = 1. \times 10^{-50};
\]

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6. Power Law breakdown

\[ A2' = A2/(\alpha' \cdot n); \]
\[ n' = n; \]
\[ \text{gamma6} = A2' \cdot \text{Deff} \cdot \mu \cdot b/(k \cdot T) \cdot (\text{Sinh}[\alpha' \cdot \text{sigma}/\mu])^{n'}/N; \]

7. Diffusional Flow

\[ \text{Ddiff} = Dv(1 + \pi \cdot d/\text{grainsize} \cdot \text{Db/Dv}); \]
\[ \text{gamma7} = 42 \cdot \text{sigma} \cdot \text{omega}/(k \cdot T \cdot \text{grainsize}^2) \cdot \text{Ddiff}/N; \]

- The final addition

\[ \text{gammanet} = \text{Evaluate}[(\text{gamma1} + \text{Max}[\text{Min}[\text{gamma2}, \text{gamma3}], \text{gamma4}, \text{gamma6}] + \text{Max}[\text{gamma5}, \text{gamma7}])/N; \]

- Deformation Maps

\[
\text{data1} = \text{Table}[\{\text{Ti}, \text{Tm}; \text{sigma} = 10^\{j \cdot \mu; \text{Log}[10, \text{gammanet}], (j, -6, -1, 1),
\{i, 0.001, 1.001, 0.020}\};
\text{sigma} = .;
\text{T} = .;
\text{ListPlot3D}[\text{data1}, \text{PlotRange} -> (-10, 3), \text{ViewPoint} -> (0.666, -3.028, 1.356),
\text{AxesLabel} -> \{"T", "\text{Stress}"}, \text{FrameTicks} -> \text{None}];
\]

\[
\text{ListContourPlot}[\text{data1}, \text{ContourShading} -> \text{False},
\text{Contours} -> \text{Table}[\text{conval}, \{\text{conval}, -10, 3, 1\}],
\text{FrameLabel} -> \{"T", "\text{Log Stress}"}, \text{FrameTicks} -> \text{None}];
\]
Temperature versus Time Data

The thermal history of the furnace is read as temperature versus time data.

\[
\text{tempdata}=\text{ReadList}[^\text{\"FUCK\Bryan\Bry\\'s Ph.D. Thesis:\ Chapters:5. Plastic Response:850C.data\", Number,RecordLists->True};
\]

\[
\begin{align*}
\text{mint}&=\text{Min}[\text{Transpose}[\text{tempdata}][[1]]]; \\
\text{maxt}&=\text{Max}[\text{Transpose}[\text{tempdata}][[1]]]; \\
\text{mintT}&=\text{Min}[\text{Transpose}[\text{tempdata}][[2]]]; \\
\text{maxtT}&=\text{Max}[\text{Transpose}[\text{tempdata}][[2]]];
\end{align*}
\]

\[
\text{eqn1}=\text{Interpolation}[\text{tempdata}, \text{InterpolationOrder}->4]; \\
\text{Plot}[\text{eqn1}[t],(t,\text{mint},\text{maxt}),\text{AxesLabel}->\{\"Time s\",\"Temp K\"\}, \\
\text{PlotRange}->\{\text{mintT},\text{maxtT}\}];
\]

\[
\text{eqn2}=\text{Interpolation}[\text{Transpose}[\text{Transpose}[\text{tempdata}][[2]]], \\
\text{Transpose}[\text{tempdata}][[1]]]]; \\
\]

□ Change in Temperature as a function of Temperature

\[
\text{temp}=(T-293); \\
\text{D[eqn1[t],t]}; \\
\text{data}=\text{Delete}[\text{Table}[\{\text{eqn1}[t],\%,(t,\text{mint},\text{maxt},(\text{maxt}-\text{mint})/100)]\},1]; \\
\text{p1=ListPlot[data,DisplayFunction->Identity];} \\
\text{eqn3= Fit[data, \{Temp^2,1\},T];} \\
\text{p2=Plot[eqn3,\{T,\text{mint},\text{maxt}\},AxesLabel}->\{\"Temp K\",\"dT / dt\"\},DisplayFunction->Identity];} \\
\text{Show[\{p1,p2\},DisplayFunction}->\$DisplayFunction,PlotRange}->\{-2,0\}, \\
\text{AxesLabel}->\{\"Temp K\",\"dT / dt\"\}];
\]

\[
\begin{align*}
\text{dT} / \text{dt} & \quad \text{600} \quad \text{800} \quad \text{1000} \\
& \quad \text{Temp K}
\end{align*}
\]

□ Evaluation of Heat Transfer Coefficient

The assumption of Newtonian cooling can be verified by determining the heat transfer coefficient at each stage of the joining process coupled with the joint geometry and material properties.
Maximum Strain Rate for Newtonian Cooling

To provide an upper bound for this joining problem where stresses due to thermal gradients do not influence the problem (the limit of Newtonian cooling), a constant Biot number of 0.1 will be used to determine the value of the heat transfer coefficient and ultimately, hypothetical time versus temperature data.

Development of a Characteristic Strain Rate

Using the thermal expansion data, the characteristic strain rate for these two materials can be found using the time versus temperature data.

Linear Expansion for Materials

\[ \text{mo} = 4.697 \times 10^{-6} (T-293) + 9.756 \times 10^{-10} (T-293)^2 + 9.403 \times 10^{-14} (T-293)^3; \]
\[ \text{cu} = 1.685 \times 10^{-5} (T-293) + 2.702 \times 10^{-9} (T-293)^2 + 1.149 \times 10^{-12} (T-293)^3; \]
\[ \text{ctemo} = \text{D}[\text{mo}, T]; \]
\[ \text{ctecu} = \text{D}[\text{cu}, T]; \]
Strain Rate as a function of Temperature

\[ e_{\text{eqn6}} = (-T \cdot (\text{ctemo-ctecu}) \cdot T) + (\text{ctemo-ctecu}) \cdot e_{\text{eqn3}}); \]
\[ \text{Plot}\{e_{\text{eqn6}}, \{T, \text{minT}, \text{maxT}\}, \text{AxesLabel} \rightarrow \{"Temp K", "Strain rate 1/s"}\}; \]

Strain rate 1/s

Strain Rate Feedback from FEA

In this section, the yield surface is iteratively refined using the prior data from the finite element analysis data from the previous simulation. Generally, 4 or 5 simulations are required for the input yield surface to converge with the output strain rate.

\[ \text{strdata=Prepend[strdata,\{1993,0\}]}; \]
\[ \text{minT=Min[Transpose[strdata][[1]]]}; \]
\[ \text{maxT=Max[Transpose[strdata][[1]]]}; \]
\[ \text{ListPlot[strdata, AxesLabel} \rightarrow \{"Temp K", "Strain"\}, \text{PlotJoined} \rightarrow \text{True}]}; \]
■ Yield Strength

The yield strength as a function of strain rate is determined by evaluating the strain rate as a function of log strain rate and log stress. At a particular temperature, an interpolation is used to create a function of log stress and log strain rate. The yield stress, sigma, is then found by inserting the appropriate value of the strain rate from the previous strain rate/temperature relationship back into the interpolation function.

```math
\text{data} = \text{Table}[
\text{search} = \text{Table}[\text{sigma} = 10^j \mu; \{\log(10, \text{gamma} + \text{sigma}), j\}, \{j, -5, -2, .005\}];
\text{sigma} = 10^\text{Interpolation}[\text{search}] \{\log(10, \text{eqn15}) \mu; (T, \text{sigma}), (T, \text{minT}, \text{maxT}, 20)\};
```
eqn7 = Interpolation[data];
Plot[eqn7[T]/10^6, {T, minT, maxT}, AxesLabel -> {"Temp K", "Yield MPa"}, PlotRange -> {0, 200}];

### Incorporation of Strain-Hardening

Once the yield strength as a function of temperature has been determined, the work of Argon is used to incorporate the effect of dislocation production as a function of strain and dislocation annihilation as a function of time. The resulting differential equation is solved at each temperature as a function of strain.

\[
yield = Table[
\]
\[
alpha = 0.25;
poisson = 0.343;
c = 5;
\]
\[
rhoadd = \frac{1}{(b \cdot c)} \cdot p^{(1/2)};
A2 = 2 \cdot \sqrt{2} \cdot \mu \cdot \omega \cdot a \cdot c \cdot \frac{1}{\pi} \cdot (1 - \text{poisson}) \cdot k \cdot v \cdot c \cdot b^4 \cdot Tm; 
\]
\[
rhooneg = -A2 \cdot v \cdot c \cdot b^4 \cdot Tm/T \cdot \text{Exp}[-\frac{Qc}{(RT)}] \cdot p^3 \cdot \frac{1}{eqn15};
\]
\[
dsigma = 0.5 \cdot \alpha \cdot \mu \cdot b \cdot p^{(-1/2)};
\]
\[
rho = (s[e]/(alpha \cdot mu \cdot b))^{2};
\]
\[
total = (rhoadd * dsigma rho) / p \rightarrow rho;
\]
\[
{T, HDSolve[\{s'[\text{e}] == total, s[0] == eqn7[T], s[\text{e}], (\text{e}, 0, 1.)[[1, 1, 2]]\},
{T, 293, 273 + 820, 20]};
\]
\[
(yield [[1, 2]] / . e -> 1) / 10^6
\]
288.161

### Stress / Strain / Temperature Surface

The following graphs are used to determine the validity of the simulation. The value for "C", the distance traveled before a dislocation line is immobilized is adjusted to empirical stress/strain data. The contour plot shows the stress/strain/temperature yield surface that is used within finite element analysis.
Saving the Results

The results are saved to a text-file which is subsequently incorporated into the input deck for Abaqus.

```
data = Table[If[ (yield[[j, 2]]/10^6) < 290, yield[[j, 2]]/10^6, 290], e, yield[[j, 1]]-273], {j, Length[Transpose[yield][[2]]]}, {e, 0, Max[Transpose[strdata][[2]]], .01}];

```
Appendix 5.2—Plastic stress summary for a Cu : Mo joint

The following graphs represent the stresses present in a copper : molybdenum joint for a fully plastic analysis, cooled from 820°C to 20°C. Both the coefficient of thermal expansion and the modulus are temperature dependent.

Nomenclature for the contour and line graphs

<table>
<thead>
<tr>
<th>Abaqus notation</th>
<th>Notation from Figure 3.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>S11</td>
<td>$\sigma_n$ (Pa)</td>
</tr>
<tr>
<td>S12</td>
<td>$\sigma_m$ (Pa)</td>
</tr>
<tr>
<td>S22</td>
<td>$\sigma_z$ (Pa)</td>
</tr>
<tr>
<td>S33</td>
<td>$\sigma_{\theta\theta}$ (Pa)</td>
</tr>
<tr>
<td>SP1</td>
<td>minimum principal stress (Pa)</td>
</tr>
<tr>
<td>SP3</td>
<td>maximum principal stress (Pa)</td>
</tr>
<tr>
<td>MISES</td>
<td>Von mises stress (Pa)</td>
</tr>
<tr>
<td>INV3</td>
<td>3rd invariant stress (Pa)</td>
</tr>
<tr>
<td>SENER</td>
<td>elastic strain energy (J m$^{-3}$)</td>
</tr>
<tr>
<td>PENER</td>
<td>dissipated plastic strain energy (J m$^{-3}$)</td>
</tr>
<tr>
<td>PEEQ</td>
<td>equivalent plastic strain</td>
</tr>
</tbody>
</table>

Trace location information for the line graphs

***A stress Above the interface in the high cte material
***B stress Below the interface in the low cte material
***C stress along the Centerline of the joint
***E stress along the Edge of the joint
Chapter 6—Experimental Verification

Abstract
The mechanical theory presented in the earlier chapters is verified in this chapter by a series of case studies and experimental analysis. All of the trends predicted by the modeling formalism are corroborated by the experimental results. This is particularly unique with respect to the geometrical aspects of the joints, for which a mechanical theory has not existed previously. Strict quantitative correlation is unrealistic for systems of this complexity as the uniqueness of each particular environment and the influence of unforeseen variables tend to dominate the mechanics.

Introduction
Many issues related to dissimilar material joining have been addressed in this thesis. The use of experimental evidence gathered by other researchers is used as a proof of the methodology presented in this thesis. Their work will be used to lend support to the general approach of strain energy as a metric as well as the various influences of material properties and joint geometry. Additional experiments related to the influence of joint geometry and strength were required to provide a more comprehensive analysis.

Case Studies
Strain energy
In the microelectronics industry, mechanical models for dissimilar materials are used to predict the overall reliability for a series of components. Because of the inherently complex geometries and range of materials that must be modeled, finite element analysis is often used. Here, the limitations of peak stress analysis are just beginning to be examined. Using a sensitivity analysis similar to the one in Chapter 3, the peak stresses and strains in a solder joint for a surface-mount resistor were modeled (Paydar, Tong & Akay, 1994). By varying the mesh element density and element type without changing the mechanics of
the actual joining problem, peak stresses were shown to vary by greater than 25% while the peak strain varied by nearly 50%. The accuracy of a peak stress approach is further compromised when it is extended to predict the fatigue life of a joint through a Coffin-Manson type relation.

Conversely, the use of strain energy has been very effective when applied to thermal fatigue modeling of leadless chip carriers. The mechanical behavior of solder joints subjected to several thermal fatigue cycles was numerically modeled using a critical accumulated strain energy failure criterion which incorporated a detailed material model for solder (Pan, 1994). The strain energy determined by finite element analysis closely corresponded with empirical fatigue data for a wide variety of thermal loading conditions—Figure 6.1.

**Mechanical properties**
The effect of thermal expansion mismatch on joint strength has been studied by diffusion bonding metals of various coefficients of thermal expansion to alumina (Crispin & Nicholas, 1982). As the thermal expansion mismatch between the alumina and the metals increased, the strength of the joint decreased—Figure 6.2. Substantial differences in both the types and growth rates of intermetallics were observed, however, the mismatch of the joints appears to be the single largest factor for determining the strength of the joints.

Another series of experiments was performed at higher temperature concerning the joining of silicon nitride (Kang, Dunn, Selverian & Kim, 1989). Two high strength, low thermal expansion metals (commercially pure molybdenum and Incoloy 909) were brazed to silicon nitride. The shear strength of these joints were found to be inversely proportional to the relative strain energy of the material couple.

**Cooling Rate**
Two trends dominate the effect of joint cooling rate on strength: at slow cooling rates, the amount of deleterious phases may increase, while a rapid cooling rate can lead to thermal
stresses within a material and a minimization of various deformation processes that lead to stress relief. This effect has been measured quantitatively by Hatakeyama et. al. (Hatakeyama, Suganuma & Okamoto, 1986). Alumina was bonded to stainless steel using a combination of titanium and molybdenum interlayers. As the cooling rate from the joining temperature increased, the strength of the joints decreased—Figure 6.3. Again, intermetallic growth was larger for the slower cooling rates, however, this did not appear to have a significant impact on the strength of the joints.

Joint geometry

Contact Area
The geometry of many joining systems is significantly more complex than the examples studied in this thesis. Not only are the dissimilar material joints in service subjected to loading due to thermal expansion mismatch, they may also be subjected to other forms of external mechanical loading. The leads for an electronic package are subjected to stresses from both the thermal expansion mismatch between the solder and the printed circuit board and the thermal expansion mismatch between the package and the circuit board. By studying the influence of these effects from the perspective of fatigue, it was found that the local interaction between the solder and the lead material is crucial for determining the fatigue life of the whole assembly (Gupta & Barker, 1994). For all of the various lead configurations examined, the "local" interaction of the lead and solder proved to be several orders of magnitude more important than the "global" package/board interaction. By increasing the compliance of the leads, a more reliable assembly was created by minimizing stresses.

Another type of electronic assembly called a pin grid array is very sensitive to issues associated with geometry and thermal fatigue. These high performance packages may contain several hundred metal pins, which are press fitted into a ceramic substrate. When this package is joined to the circuit board at high temperature, the deflection at the edge of the ceramic is greatest, putting the pins at the furthestmost edges under the greatest load.
The reliability of these components as a whole is inversely proportional to the module size—ceramic modules greater than 50 mm in size present a serious reliability concern (Tummala, 1989).

**Thickness**
Another important geometric consideration of dissimilar material joints is the thickness (or aspect ratio) of the components that are to be joined. This effect has been studied by creating diffusion bonded silicon nitride : stainless steel joints of various thicknesses (Suganuma, Okamoto, Koizumi & Shimada, 1985). Shear testing showed that most failures occurred within the ceramic material and that the joint strength decreases drastically as the thickness of the silicon nitride increased—Figure 6.4. Similar phenomena have been observed by Naka et al. (Naka, Kubo & Okamoto, 1989)

**Experimental**
Many of the case studies presented in this chapter have used ceramics component in their joining structure. From a materials perspective, ceramics are often the focus of joining research because their structural and electrical properties are quite different from the more conventional materials they replace. From a practical perspective, it easy to assess their joining ability because they have very little damage tolerance, which often obviates the need for extensive mechanical testing.

An objective of this research was to develop a general method for modeling the mechanical properties of structural materials. Many dissimilar material joints will experience deformation that extends beyond the regime of purely elastic behavior due to the manner of joining and the nature of application. The material model developed in Chapter 5 was tested using joining systems with varying levels of plasticity.

The model dissimilar joining system used in this study was comprised of copper and molybdenum. A molybdenum alloy called TZM (0.5% Ti—0.08% Zr—0.04% C) was used as the high strength, low thermal expansion component for all dissimilar material
joints created in this study. For the low strength, high thermal expansion material, both an oxygen free high conductivity copper (OFHC) and a commercially available particle dispersion strengthened copper alloy (GlidCop AL-50, (SCM Metal Products, 1994)) were used. While the thermal expansion and elastic modulus for these copper alloys are virtually identical, their plastic responses are very different—Table 6.1 and Figure 6.5.

Samples approximately 2 inches in length were cut from rod stock. One end was polished with 320 grit abrasive paper while mounted in a jig to ensure that the mating surfaces for each sample were as flat as possible. Following machining, they were thoroughly cleaned with soap and water, then ultrasonically cleaned in trichloroethylene. The samples were then placed into a joining fixture along with one of two different braze foil inserts—Table 6.2. This assembly was inserted into the cool-zone of a vacuum tube furnace and pumped to $10^{-5}$ torr while preheating took place. Once the furnace reached its designated temperature, the joining fixture was moved into the hot zone of the furnace. After a specified period of time, the assembly was moved away from the hot zone, allowing it to cool slowly in vacuum.

Following joining, a minimal amount of machining was required to form a concentric, linear joint without any visible cracks or voids at the surface. The interfacial region was further polished with 600 grit abrasive cloth and placed in a desiccator prior to mechanical

<table>
<thead>
<tr>
<th>Table 6.1—Properties of materials used in this study</th>
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<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Modulus (GPa)</td>
</tr>
<tr>
<td>C.T.E. $(\times 10^{-6} , ^\circ \text{C}^{-1})$</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
</tr>
<tr>
<td>Yield strength (MPa)</td>
</tr>
<tr>
<td>Melting point ($^\circ \text{C}$)</td>
</tr>
<tr>
<td>Density (g cm$^{-3}$)</td>
</tr>
<tr>
<td>Thermal conductivity (W cm$^{-1}$ K$^{-1}$)</td>
</tr>
</tbody>
</table>

Page 122
Table 6.2—Braze alloys examined in this study

<table>
<thead>
<tr>
<th>Composition</th>
<th>Solidus</th>
<th>Liquidus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ticusil</td>
<td>830</td>
<td>850</td>
</tr>
<tr>
<td>69 Ag</td>
<td></td>
<td></td>
</tr>
<tr>
<td>27 Cu</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Ti</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nicoro</td>
<td>1000</td>
<td>1030</td>
</tr>
<tr>
<td>62 Cu</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35 Au</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Ni</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

testing. The tensile testing method proposed by the ASTM requires that two truncated cones of one material be bonded to a disc of the second material. Because the loading within these joints is extremely geometry dependent, the efficacy of such a test is questionable. It does not represent the geometry of a typical dissimilar material joint, nor is it easy to fabricate. As a consequence, many researchers have found it necessary to develop testing methods of their own (Suganuma, 1990).

For this study, simple tensile specimens were prepared. The geometry was chosen so that the gauge length for each material was at least 1.5 times the diameter of the joint. Mechanical testing was performed using an Instron tensile test machine with compression grips. All samples were tested at a uniform speed of 0.1 inch min⁻¹ and the load cell data was measured using a strip chart recorder.

Results and Discussion

Braze alloy

Nicoro
Joints produced using the Nicoro braze alloy had high strength for all materials except the GlidCop—Table 6.3. A temperature / time schedule of 1100°C for 18 minutes produced control OFHC : OFHC joints with strength identical to the parent material. Failure was purely ductile and occurred near the joint with a great deal of extension. Because slippage of the grips occurred as the load increased, accurate strain data were not available. For the TZM : TZM control joints, the strength of the joint was so high that the test was stopped
Table 6.3—Mechanical test data for Nicoro joints

<table>
<thead>
<tr>
<th>System</th>
<th>Strength (MPa)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFHC : OFHC</td>
<td>218</td>
<td>very good ductility, sample yielded at joint</td>
</tr>
<tr>
<td>TZM : TZM</td>
<td>&gt; 570</td>
<td>strength exceeded testing capabilities of grips</td>
</tr>
<tr>
<td>OFHC : TZM</td>
<td>222</td>
<td>sample yielded away from joint in copper</td>
</tr>
<tr>
<td>GlidCop : GlidCop</td>
<td>57.5</td>
<td>very poor wetting of braze</td>
</tr>
</tbody>
</table>

due to deformation caused by the compressive grips. The OFHC : TZM dissimilar material joint also had superior mechanical properties. In each case, failure occurred due to necking within the copper, away from the joint.

Joints formed from the as-received GlidCop AL-60 had very poor strength. In general, joining difficulties associated with this material have been ascribed to agglomeration of the Al₂O₃ reinforcing phase at the interface, sacrificing both strength and ductility (Santella, 1992). These problems are most often encountered when using silver based brazes, however, even the gold based Nicoro braze alloy did not produce satisfactory results.

**Ticusil**

In general, the lower melting point of the Ticusil makes it preferable to the Nicoro, however, considerable difficulty is had when forming joints using silver based brazes and GlidCop (Samal, 1992). Joints produced using the as-received Ticusil had a blistered appearance near the interface and came apart during handling. By plating the GlidCop with a 1.5 mil layer of copper (applied according to Procedure-A of Appendix 1 in (Samal, 1992)) substantially better results were obtained.

A temperature / time schedule of 925°C for 18 minutes produced control OFHC : OFHC joints with the identical strength of the parent material. The TZM : TZM control joints, produced strengths in excess of 500 MPa. Again, the OFHC : TZM dissimilar material interface had greater strength than the OFHC parent material. The mode of failure was identical to the Nicoro specimens, with the necking occurring away from the joint.
Table 6.4—Mechanical test data for Ticusil joints

<table>
<thead>
<tr>
<th>System</th>
<th>Strength (MPa)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFHC : OFHC</td>
<td>227</td>
<td>very good ductility, sample yielded at joint</td>
</tr>
<tr>
<td>TZM : TZM</td>
<td>&gt; 500</td>
<td>test stopped</td>
</tr>
<tr>
<td>OFHC : TZM</td>
<td>227</td>
<td>sample yielded away from interface in copper</td>
</tr>
<tr>
<td>GlidCop : GlidCop</td>
<td>379</td>
<td>very little elongation</td>
</tr>
<tr>
<td>GlidCop : TZM</td>
<td>195</td>
<td>fracture propagated into molybdenum</td>
</tr>
</tbody>
</table>

Effect of radius

Copper plated GlidCop in three different diameters was vacuum brazed to TZM with the Ticusil brazing alloy. The length of time was optimized for each diameter due to the difference in thermal mass. If the amount of time was too short, incomplete brazing occurred, leading to poor wettability and strength. Conversely, if the time at temperature was too long, there was the possibility that diffusion could occur through the copper plating leading to the previously mentioned agglomeration problems. For the 3/8", 1/2" and 5/8" diameter samples, 14 minutes, 18 minutes and 25 minutes with a 925°C preheat were used, respectively.

As the radius of the interface increased, the average stress required to break the joint decreased—Table 6.5. When a dissimilar material joint is tensile tested, there are two separate types of loading that must be considered: the tensile loading described in Chapter 2 and loading due to thermal mismatch described in Chapters 4 and 5. The stresses due to tensile loading rely solely on the mechanical properties of the materials and are not geometry dependent. Numerically, strain energy due to thermal expansion mismatch has

Table 6.5—Radius and tensile strength of GlidCop: TZM joints

<table>
<thead>
<tr>
<th>Initial radius (inch)</th>
<th>Strength (MPa)</th>
<th>Time at 925°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.375</td>
<td>213</td>
<td>14 minutes</td>
</tr>
<tr>
<td>0.500</td>
<td>195</td>
<td>18 minutes</td>
</tr>
<tr>
<td>0.625</td>
<td>182</td>
<td>25 minutes</td>
</tr>
</tbody>
</table>
been shown to increase as the size of the joint increases. While this effect has been observed for thermal fatigue, this is the first demonstration of its extension to tensile testing.

**Effect of joint edge angle**

As mentioned previously, butt joints formed from the OFHC and TZM failed due to necking within the bulk copper away from the interface. Mohr’s circle is a convenient method of visualizing the mechanics behind this effect—Figure 6.6 (Gere, 1984).

According to the Tresca criterion for yielding, the magnitude of the shear stress is responsible for plastic deformation, which is a function of the maximum principal stresses—Equation 6.1.

\[
\sigma = \left( \frac{\sigma_{\text{applied}} - \sigma_{\text{residual}}}{2} \right)
\]

For a material in simple tension, the shear stress is influenced only by the magnitude of the applied tensile stress. Upon cooling from the joining temperature, the copper develops a tensile residual stress along the direction of the interface. While the applied tensile stress has not changed, the presence of this residual stress acts to decrease the magnitude of the shear stress of the material near the interface. For systems where the strength of the joint is greater than that of the parent material, this predicts that yielding will occur away from the interface, in a region where the residual stresses are lower.

If the edge angle of these joints is changed, the situation becomes more complicated. The residual stresses are still expected to decrease with increasing distance away from the interface, however, deformation cannot occur as readily due to the increased cross-section of the copper. Several OFHC : TZM joints using the Ticusil braze alloy were prepared using a 925°C preheat for 18 minutes. The joints were machined to various taper angles and tensile tested. The initial diameter of the interface between the copper and molybdenum was used for all subsequent stress calculations.
The strength of the joint increased steadily with joint edge angle until approximately $5^\circ$, where the failure mode shifted from necking within the bulk copper to fracture along the interface—Figure 6.7. For the samples where necking was the mode of failure, the strength values are quite predictable. As the taper angle increased, the deformation zone became closer to the interface, ultimately causing a brittle fracture which propagated into the molybdenum—Figure 6.8. In comparison to plastic deformation, fracture is heavily influenced by factors such as misaligned grips during tensile testing or surface imperfections left after machining.

These joints were modeled numerically using the methodology developed in Chapter 5. The predicted strength was obtained by applying a steadily increasing tensile load to an axisymmetric mesh complete with residual stress from thermal loading. At a certain point, the load became sufficiently high so that the simulation became unstable due to necking. A correlation of the measured and predicted strengths is shown in Figure 6.9 and given in Table 6.6. The predicted geometry of the joints at yield is shown in Figure 6.10.

It is important to note that the measured strength is based upon the ultimate load and the initial dimensions of the joint—no consideration of reduced cross-section due to necking was made. While this effect is considered for the numerical model, no criteria for failure at the interface was specified. Consequently, the values for the measured stress are lower than expected, while the predicted strength is higher than expected when fracture of the interface dominates the behavior of the joint. Considering these caveats, both the predicted yield load and geometry correspond closely with the measured values.
Table 6.6—Effect of joint edge angle

<table>
<thead>
<tr>
<th>Edge angle (°)</th>
<th>Measured strength (MPa)</th>
<th>Predicted strength (MPa)</th>
<th>Mode of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>227</td>
<td>253</td>
<td>necking in bulk copper</td>
</tr>
<tr>
<td>1.25</td>
<td>240</td>
<td>268</td>
<td>necking in bulk copper</td>
</tr>
<tr>
<td>2.5</td>
<td>243</td>
<td>280</td>
<td>necking in bulk copper</td>
</tr>
<tr>
<td>3.75</td>
<td>250</td>
<td>292</td>
<td>necking in bulk copper</td>
</tr>
<tr>
<td>5.0</td>
<td>232</td>
<td>302</td>
<td>fracture along interface</td>
</tr>
<tr>
<td>7.5</td>
<td>264</td>
<td>319</td>
<td>fracture along interface</td>
</tr>
<tr>
<td>10.0</td>
<td>241</td>
<td>332</td>
<td>fracture along interface</td>
</tr>
</tbody>
</table>

Summary
A number of case studies have been shown which demonstrate that strain energy is an effective metric for describing and predicting the mechanical properties of dissimilar material joints. Experimentally, the correlation between tensile strength and geometry was established for brazed butt joints formed between copper and molybdenum. By increasing the cross-section of the copper relative to the molybdenum, tapered joints resulted in increased load bearing capability while maintaining failure tolerance. The methodology developed in this thesis provides a valuable first approximation which allows the trends of these joints to be established without requiring an in-depth analysis.

References


Figures

**Figure 6.1**—Fatigue life as a function of strain energy (Pan, 1994)

**Figure 6.2**—Joint strength as a function of thermal expansion (Crispin & Nicholas, 1982)
Figure 6.3—Joint strength as a function of cooling rate (Hatakeyama, Suganuma & Okamoto, 1986)

Figure 6.4—Joint strength as a function of component thickness (Suganuma, Okamoto, Koizumi & Shimada, 1985)
Figure 6.5—Yield strength of copper alloys for 1 hour at temperature (SCM Metal Products, 1994)

Figure 6.6—The effect of constrained loading on joint strength
Figure 6.7—Photograph of tensile tested OFHC : TZM joints

Figure 6.8—Photomicrograph of OFHC : TZM brittle fracture
Figure 6.9—Predicted and measured OFHC : TZM strength versus joint edge angle
Figure 6.10—Predicted deformation geometry versus joint edge angle
7. Conclusion and Future Work

Conclusion

- Analytical models cannot accurately predict the state of stress for dissimilar material joints
- Generalizations can be formed about these systems using predictive methods
  - The relative strength of adhesive joints relies mainly on the elastic material properties
  - Geometry determines the properties of a thermally loaded dissimilar material joint
  - Plasticity reduces the strain energy of dissimilar joining by more than an order of magnitude
  - Patterned interfaces may offer an alternative to gradient interlayers for stress reduction
- Real life applications are frequently a combination of the mechanically and thermally loaded conditions

The modeling of dissimilar material joining is a rather young field. Within the last two decades, an analytical method was developed for describing the stresses present in an adhesively bonded joint undergoing tensile forces. Approximately 10 years after that, the first numerical simulations were performed for joints with a coefficient of thermal expansion mismatch undergoing a change in temperature. Presently, the field is entering its next phase by incorporating more realistic material properties and utilizing fully plastic numerical analysis.

Analytical models cannot accurately predict the state of stress for dissimilar material joints because it is very difficult to accommodate plastic behavior to even a rudimentary level. For any dissimilar material combination of practical interest, a stress singularity exists along the interface near the free surface of the joint. Because of this, any appreciable load guarantees that some degree of inelastic deformation will occur, significantly affecting the state of stress. For a dissimilar material joint undergoing a change in temperature, plasticity occurs with a temperature change of approximately 50°C. This, coupled with the complexity created of a high aspect ratio and a lack of external tractions virtually guarantees that a rigid analytical solution will never be found.
Numerical analyses are not immune to difficulties when modeling these systems either. Peak stress analyses suffer from convergence problems due to stress singularities. A sensitivity analysis where only the mesh setup parameters were varied showed that the peak stress values may vary by more than 30%—Chapter 3. For thermally loaded joints, strain energy is a more appropriate metric. It incorporates the effects of the material properties and the joint geometry while remaining insensitive to the problem formulation within finite element analysis.

A realistic model must be able to accurately incorporate the mechanical properties of each material, the joint geometry and the process variables. Each of these considerations has an influence on the others, which ultimately determines the mechanical properties of a joint.

After the effects of plastic deformation and creep are considered, less than 5% of the strain energy predicted by a purely elastic approach remains. While this approach was able to describe the behavior of a copper : molybdenum butt joint very accurately, it requires the use of a workstation and finite element analysis.

Generalizations based on purely elastic behavior are able to predict many trends without the need for intensive computation. In Chapter 4, a set of “design rules” was created based on the elastic strain energy of a two component, thermally loaded dissimilar joint. The strain energy for a plane stress and an axisymmetric model can be separated into effects related to the materials and the joint geometry—Equation 7.1 and 7.2.

\[
U_{\text{plane stress}} \propto \frac{E_A E_B}{E_A + E_B} \left( \alpha_A - \alpha_B \right)^2 \left( T_{\text{Bond}} - T \right)^2 \left( \text{width} \right)^2 \text{Erf} \left( \frac{\text{height}}{\text{width}} \right) \]  

(7.1)

\[
U_{\text{axisymmetric}} \propto \frac{E_A E_B}{E_A + E_B} \left( \alpha_A - \alpha_B \right)^2 \left( T_{\text{Bond}} - T \right)^2 \left( \text{radius} \right)^3 \text{Erf} \left( \frac{\text{height}}{\text{radius}} \right) \]  

(7.2)

For both the plane stress and axisymmetric models, the functionality of the variables related to the material properties are identical. By minimizing the joining temperature and using
materials that have closely matched coefficients of thermal expansion, the strength and reliability of these joints are expected to increase.

For both plane stress and axisymmetric models, the strain energy of the joint increased with increasing aspect ratio. At a height : width ratio of approximately 2 : 1, the strain energy of the joint asymptotically approaches a maximum. Beyond this point, the strain energy of the system does not increase appreciably. With regard to interface size, the strain energy will always increase at a greater rate than the bonding energy, which means that the formation of large joints are energetically unfavorable.

Because of the complexities inherent in joint design, it would be ideal to always perform the most comprehensive analysis possible. While it is impossible to create a concise mechanical theory which incorporates all the factors inherent in the design of dissimilar material joints, the methodology developed in this thesis provides a valuable first approximation. Now that the mechanics of a particular joining process can be considered concurrently with the chemical compatibility issues, the ability to model and successfully implement these joints will be significantly improved.

**Future Work**

*Mechanically anisotropic interlayers*

For typical dissimilar material joints, much of the strength of the interface is used to constrain the transverse expansion of the components. This constraint creates residual stresses so high that some components may fail or fracture may occur along the interface. By creating a mechanically anisotropic interlayer between the two components, it is expected that this transverse thermal expansion mismatch can be accommodated while affording a high strength joint.

Recent studies have shown that crack growth between dissimilar materials can be disrupted through the use of a "patterned interface" (Oh, Rodel, Cannon & Ritchie, 1987), (Cannon, Dalgleish, Dauskardt, McNaney & Ritchie, 1992). Rather than transmitting the load force
directly to the growing crack front, a patterned interface continuously blunts the crack-tip, forcing repropagation—Figure 7.1. The fracture toughness for samples of glass bonded with a patterned copper interlayer was nearly an order of magnitude greater than those using a standard, continuum interface.

Studies performed on shaft-hub connections between dissimilar materials have produced similar results (von Esebeck, 1994). In each case, an aluminum oxide hub was brazed onto a stainless steel shaft and mechanically tested in torsion. By varying the geometry of the aluminum oxide, joints with the patterned interfaces had load carrying abilities that were in some cases twice as strong as conventional designs—Figure 7.2.

Another approach to managing the thermal expansion mismatch between components has been the use of mechanically anisotropic buffer layers referred to as “strain-isolators.” These materials resemble steel wool, but are made of high temperature materials such as Inconel. When forming large area joints of dissimilar materials, the compliance of this buffer layer accommodates the thermal expansion mismatch. Because this material is essentially isotropic, the same transverse compliance that allows for thermal expansion mismatch hinders its normal strength.

A new design is proposed which maintains the transverse compliance while maintaining high normal strength. Rather than using an isotropic material such as the strain-isolators, this interface would be anisotropic, with an ordered array of fibers normal to the interface—Figure 7.3. Fracture along the interface is an inherent problem with a continuum bonding approach. Using the modeling approach developed in this thesis, it can be shown that the strain energy of a patterned interface is much lower than a continuum approach.

**Mechanical rationale**

An elastic analysis for a plane stress model can be performed using the design rules created in Chapter 4. Consider a continuum interface of width “x” and a patterned interface of the same total width, separated into “n” equally sized, smaller “sub-bonds”. From the design rules, the
energy of each bonded region is proportional to the square of its width. This yields a comparison between the patterned interface and the continuum interface—Equations 7.3 and 7.4.

\[ U_{\text{patterned}} \propto \sum \left( \frac{x}{n} \right)^2 = \frac{x^2}{n} \]  \hspace{1cm} (7.3) \hspace{1cm} \text{while} \hspace{1cm} \frac{x^2}{n} \hspace{1cm} (7.4)

For the patterned interface, the strain energy is reduced by a factor of “n” when compared to the continuum approach. Obviously, there are limitations to this simple analysis—with an infinite number of sub-bonds, it predicts zero strain energy. Clearly such a joint would be impractical to assemble. The impingement of stress fields for neighboring bonds was simulated using finite element analysis for a completely elastic copper : molybdenum butt joint. As the separation distance between each of the sub-bonds increases, the total strain energy of the system decreases—Figure 7.4. At an offset of 100%, which corresponds to 50% coverage by area, the strain energy for the patterned interface system decreases by 8%.

References
Figures

Figure 7.1. Schematic of crack bridging by bulged metal-film segments (Cannon, Dalgleish, Dauskardt, Oh & Ritchie, 1991).

Figure 7.2. Torsional strength for two types of brazed dissimilar material hub-shaft connections (von Esebeck & Beitz, 1994).
Figure 7.3. Schematic of discrete bonding approach.

Figure 7.4. Effect of bond spacing on total strain energy of patterned interfaces.
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