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recommended and analyzed for their error rate and decoding complexity.

Throughput Analysis for Code Division Multiple Access of the Spread Spectrum Channel

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Abstract—We consider the use of error correction codes of rate r on top of pseudonoise (PN) sequence coding for code division multiple access of the spread spectrum channel. The channel is found to have a maximum throughput of 0.72 and 0.36 based on the evaluation of channel capacity and cutoff rate, respectively. More generally, these two values are derived for given bandwidth expanding n/r versus n/N where n is the length of the PN sequence and N is the number of simultaneous users. It is found that to achieve the maximum throughput, n should be small. This implies that coding schemes with short PN sequences and low rate codes are superior in terms of throughput or antijam capability. The extreme case of $n=1$ corresponds to using a very low rate code with no PN sequence coding. Convolutional codes are recommended and analyzed for their error rate and decoding complexity.

I. INTRODUCTION

CODE division multiple access [1], [6] is attractive for mobile communication for the following reasons. First, a large number of users may subscribe to such a system while system performance (error rate) depends only on the number of simultaneous users. The channel degrades gracefully as the number of active users increases. Second, each user may transmit asynchronously and no scheduling of the channel is required. This uniformity makes entry of new subscribers easy. For such systems, the signal of the other active users becomes interference (having statistics that are approximately Gaussian) to a receiver. To reduce the effect of this interference, each user redundantly encodes its message and spreads the coded signal over a large spectral bandwidth. The obvious question then, is how many users (each with fixed data rate) may transmit simultaneously and reliably for a given spectral bandwidth? Is there a capacity analogous to the e^{-1} throughput of the slotted ALOHA system? Before answering this question, Section I describes the coding scheme and subsequently models the channel. A Gaussian approximation is used in the process. Section II then evaluates the total capacity and cutoff rate for the active users. For convolutional codes and sequential decoding, it is shown that the maximum throughput is $(\log_2 e)/4 = 0.36$.

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A similar result using a different model is derived by Viterbi and reported in [6]. Section III gives an analysis for the bit error probability and decoding complexity as a function of the constraint length and rate of the convolutional code, as well as the length of the PN sequence.

II. MODELING

Let N be the number of active users. Each user i , $1 \leq i \leq N$, is assigned an n bit PN sequence $\{c_{ij}\}_{j=1}^n$, $c_{ij} \in \{\pm 1\}$. A chip is defined as an interval of length d . The PN sequence carrier is the function

$$c_i(t) = \sum_{j=1}^n c_{ij}s(t-jd)$$

in which $s(t)$ equals 1 if $0 \leq t \leq d$, and 0 elsewhere. Each user encodes the binary (0 or 1) data stream $\{\dots u_{i,-1}u_{i,0}u_{i,1}\dots\}$ into the antipodal (1 or -1) code stream $\{\dots x_{i,-1}x_{i,0}x_{i,1}\dots\}$. The rate of encoding is r (≤ 1). A coding scheme using a convolutional code is shown in Fig. 1. The signal sent by each user¹ is

$$x_i(t) = \sqrt{P_i} \sum_{k=-\infty}^{\infty} x_{i,k}c_i(t-knd + D_i)$$

in which P_i is the power for the i th transmitter, and D_i is the delay for the i th user, which is randomly distributed in $[0, nd]$. The channel output is

$$y(t) = \sum_{i=1}^N x_i(t).$$

The i th receiver first acquires synchronization with the i th transmitter by estimating D_i , which we assume can be estimated accurately by some means. Demodulation is performed by match filtering and the values

¹Viterbi's derivation considers BPSK with a random phase $\cos \theta_i$ for each user. Consequently, the results for sum-capacity and sum-cutoff rates (1.44 and 0.72, respectively) are twice that of our model.

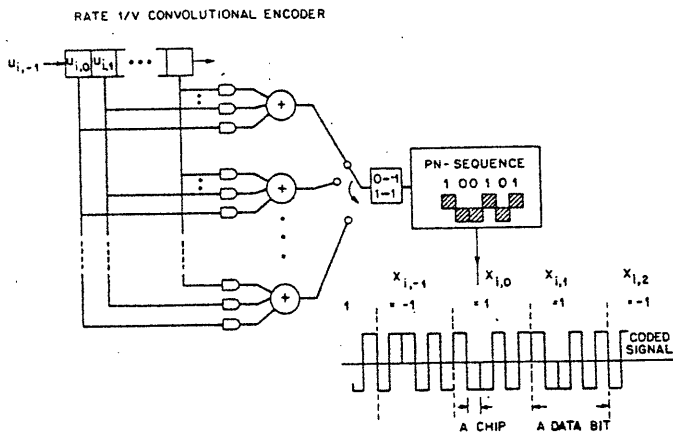


Fig. 1. Coding scheme for the spread spectrum channel.

$$y_{i,k} = \left(nd^2 \sum_{\substack{j=1 \\ j \neq i}}^N P_j \right)^{-1/2} \int_{-D_i + kn d}^{-D_i + (k+1)nd} y(t) \cdot c_i(t - kn d + D_i) dt$$

are obtained. The $y_{i,k}$'s are subsequently quantized. We shall assume either hard quantization or no quantization. The channel, as viewed by the i th user, can be characterized by $P(y_{i,k}/x_{i,k})$. Assuming that the PN sequences are generated randomly with equiprobable use of 1 and -1,² it follows readily that the random variable $y_{i,k}$ has mean

$$E(y_{i,k}/x_{i,k} = m) = m \left(nP_i / \sum_{\substack{j=1 \\ j \neq i}}^N P_j \right)^{1/2}$$

and variance

$$E((y_{i,k} - \bar{y}_{i,k})^2/x_{i,k} = m) = 1$$

for $m = 1$ or $= -1$. Since $y_{i,k}$ is the sum of a large number of random variables, the statistics of $y_{i,k}$ are approximately Gaussian by the law of large numbers. Hence, by defining

$$N_{o,i}/2 = \sum_{\substack{j=1 \\ j \neq i}}^N P_j d$$

which has the interpretation of equivalent one-sided noise spectral density [5], and dropping the subscript k ³ we have the approximation

²There is the question of whether specifically designed PN sequences (such as the Gold code [2]) may achieve a higher capacity than PN sequences that are generated randomly. The answer is no, due to the asynchronism of the transmitters. The proof of this statement follows from a coding theorem for the asynchronous multiple access channel [3].

³The $y_{i,k}$'s may not be independent for different k 's. This dependence, however, is weak and diminishes as the number of users N becomes large. Exploiting this dependence for achieving lower error probability or higher throughput requires a more sophisticated decoding algorithm. We restrict ourselves to simple decoder structures. The issue of channel capacity with restrictions on the structures of the decoder is treated in [3].

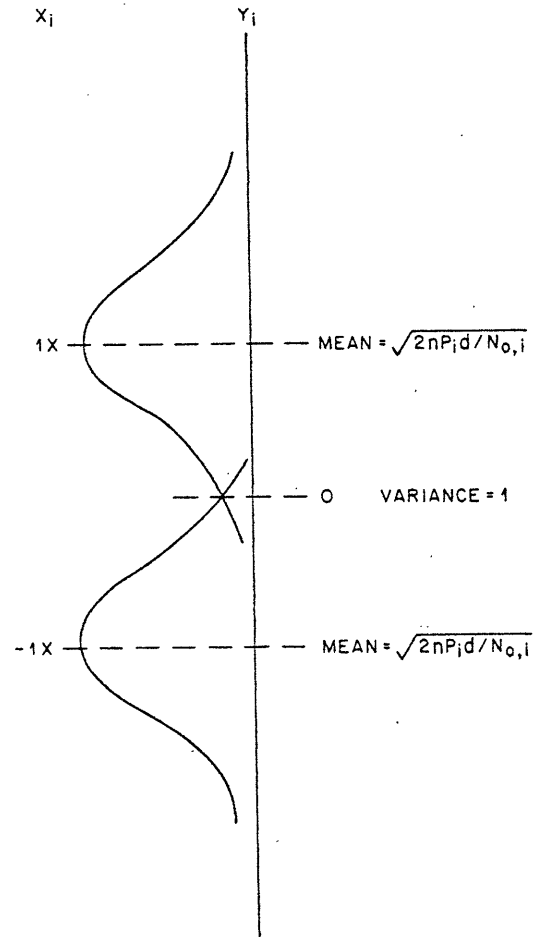


Fig. 2. The binary input Gaussian output channel.

$$P(y_i/x_i = m) = \exp\left(-\left(y_i - m\sqrt{2nP_i d / N_{o,i}}\right)^2 / 2\right) / \sqrt{2\pi}$$

The channel (actually the channel plus quantizer) can be modeled by the channel transition diagram in Fig. 2 and Fig. 3 for the cases of no quantization and hard quantization, respectively. Fig. 2 is essentially a binary input Gaussian output channel, whereas Fig. 3 is a binary symmetric channel (BSC) with crossover probability $p = Q(\sqrt{2nP_i d / N_{o,i}})$ in which

$$Q(z) = \int_{-\infty}^{-z} e^{-x^2/2} dx / \sqrt{2\pi}$$

Consider the case of no coding on top of the PN sequence (i.e., $x_{i,k} = 1$ if $u_{i,k} = 1$ and $x_{i,k} = -1$ if $u_{i,k} = 0$). The channel is therefore the BSC of Fig. 3. Assuming equal power for all users, the bit error probability is upper bounded by

$$Q(\sqrt{2nP_i d / N_{o,i}}) = Q(\sqrt{sn/(N-1)}) \cong Q(S^{-1/2})$$

in which S is the total throughput (N/n bits/chip). This upper bound is shown in Fig. 4. It is readily seen that the throughput has to be reduced substantially for tolerable error probability. An immediate conclusion is that coding should be used on top of PN sequence coding to achieve a lower bit error probability.

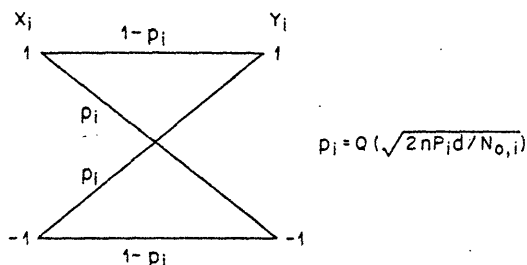


Fig. 3. The binary symmetric channel.

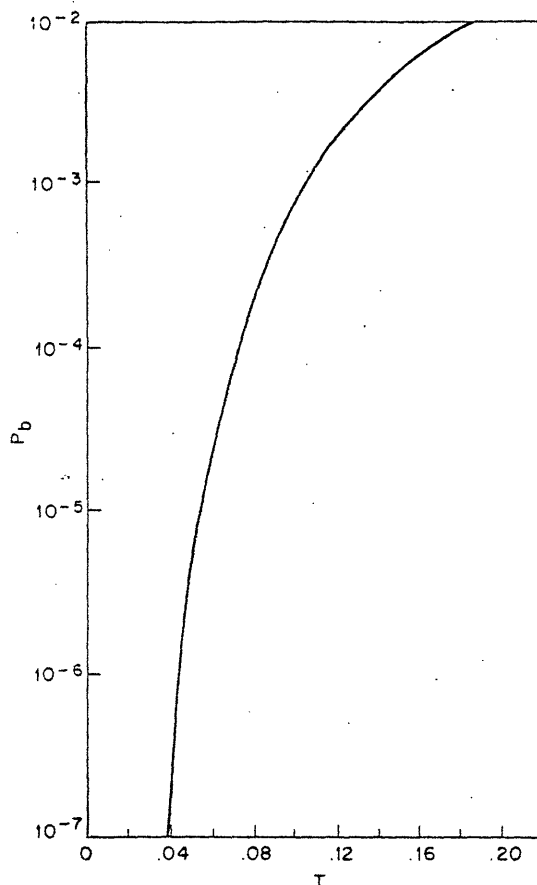


Fig. 4. Error probability versus throughput with no error correction code.

III. CAPACITY AND CUTOFF RATES

The capacity of a channel is the upper bound on the rate of reliable communication. The cutoff rate is the value of the random coding exponent function with $\rho = 1$ [4]. When sequential decoding is used, the cutoff rate is the maximum rate of communication over the channel with a bounded average number of computations for decoding an information bit. For the BSC of Fig. 3, the cutoff rate and capacity (in bits/code symbol, i.e., x_i) are derived in [4], giving

$$R_{o,i} = 1 - \log_2 \left(1 + \sqrt{4p_i(1-p_i)} \right)$$

$$C_i = 1 - H(p_i)$$

in which

$$p_i = Q\left(\sqrt{2n P_i d / N_{o,i}}\right)$$

and H is the binary entropy function. For the binary input Gaussian output channel, we have [4]

$$R_{o,i} = 1 - \log_2 \left(1 + e^{-n P_i d / N_{o,i}} \right)$$

$$C_i = \log 2\pi e - \int_{-\infty}^{\infty} P^i(y) \log_2 P^i(y) dy$$

in which

$$P^i(y) = (P_{-1}^i(y) + P_1^i(y))/2$$

and

$$P_m^i(y) = P(Y_i = y_i / x_i = m)$$

$$= \exp\left(-\left[y_i - m\sqrt{2n P_i d / N_{o,i}}\right]^2 / 2\right) / \sqrt{2\pi}$$

We shall assume henceforth that the transmitting power is equal for all users. Define $\beta = n/N$, the ratio of the length of the PN sequence to the total number of active users. Summing over the N users and normalizing by n , the number of chips in a code symbol x_i , the sum of the capacity and cutoff rate (in bits/chip) for the BSC is

$$R_o^T(\beta) = \frac{N}{n} R_{o,i} = \beta^{-1} \left(1 - \log_2 \left(1 + \sqrt{4p(1-p)} \right) \right)$$

$$C^T(\beta) = \frac{N}{n} C_i = \beta^{-1} (1 - H(p))$$

in which

$$p = Q\left(\sqrt{n/(N-1)}\right) \approx Q(\beta^{1/2}) \quad \text{for large } N.$$

For the binary input Gaussian channel [4],

$$R_o^T(\beta) = \beta^{-1} \left(1 - \log_2 \left(1 + e^{-\beta/2} \right) \right)$$

$$C^T(\beta) = \beta^{-1} \left(-\log_2 2\pi e - \int_{-\infty}^{\infty} P(y) \log_2 P(y) dy \right)$$

in which

$$P(y) = (P_1(y) + P_{-1}(y))/2$$

and

$$P_m(y) = \exp\left(-\left[y - m\beta^{1/2}\right]^2 / 2\right) / \sqrt{2\pi}$$

These four functions of β are shown in Fig. 5. All four functions are observed to be monotonically decreasing in β . It can be readily shown that all four converge to the function $1/\beta$ for large values of β . Asymptotic evaluation of these functions for vanishing values of β gives

$$R_o^T = \frac{1}{2\pi} \log_2 e = 0.2296$$

$$C^T = \frac{1}{\pi} \log_2 e = 0.4592$$

for the BSC, and

$$R_o^T = \frac{1}{4} \log_2 e = 0.3607$$

$$C^T = \frac{1}{2} \log_2 e = 0.7213$$

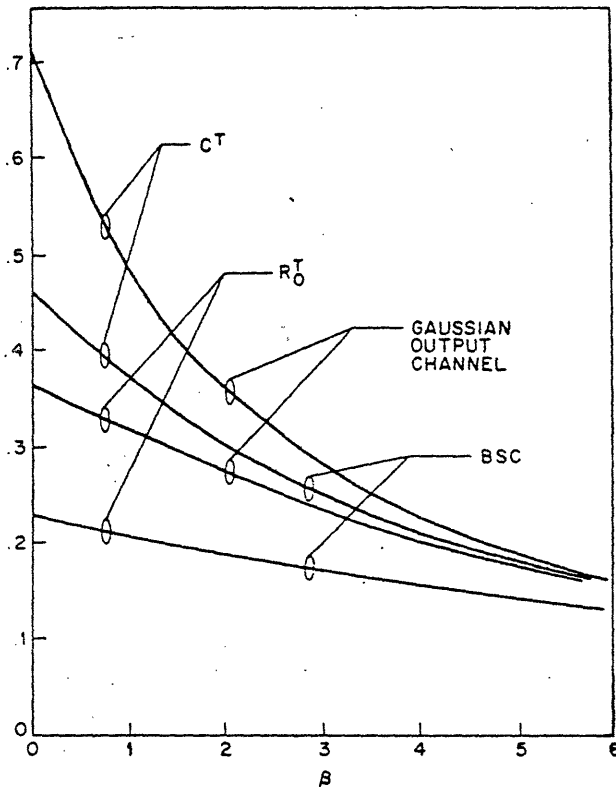


Fig. 5. Capacity and cutoff rates.

for the binary input Gaussian channel. (It is well known that hard quantization reduces the capacity by a factor of $\pi/2$ [4].) It is noteworthy that small values of β correspond to very noisy channels in Figs. 2 and 3, when the cutoff rate can be shown to be half the capacity. In fact, both R_o^T and C^T for diminishingly small β can be derived alternatively using a very noisy channel model [4]. The asymptotic result for large β suggests that using long PN sequences decreases the sum of the capacity and cutoff rate. The lesson is that we should use very low rate encoders and short PN sequences so that β can be made small. The smallest value of β is achieved for $n=1$ which corresponds to using a very low rate code with no PN sequence coding.

IV. ERROR PROBABILITY AND DECODING COMPLEXITY

This section studies the upper bound on error probability as a function of complexity. For constraint length k and rate $1/v$ convolutional codes, the ensemble average of expected number of bit errors for an error event starting at a given time [4] is upper bounded by

$$\begin{aligned}
 P_b &< 2^{-k v R_{o,i}} / [1 - 2^{-(v R_{o,i} - 1)}]^2 \\
 &= 2^{-k v \beta R_o^T(\beta)} / [1 - 2^{-(v \beta R_o^T(\beta) - 1)}]^2 \\
 &= [(1 + e^{-1/2 S v}) / 2]^{v k} / [1 - 2((1 + e^{-1/2 S v}) / 2)^v]^2.
 \end{aligned}
 \tag{1}$$

The last equality follows from

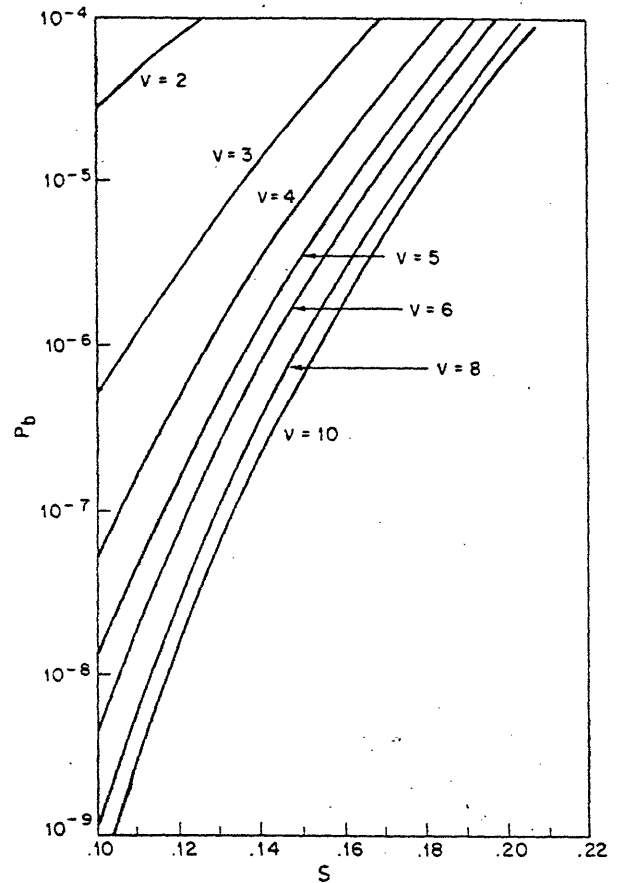


Fig. 6. Upper bound on P_b versus throughput for $k=10$.

$$R_o^T(\beta) = \beta^{-1} (1 - \log_2(1 + e^{-\beta/2}))$$

derived in Section II for the binary input Gaussian channel, and the total throughput for the N users is

$$S = (N/n) \cdot (1/v) = 1/(v\beta).$$

This upper bound on P_b versus S , the degree of congestion of the channel, is shown in Fig. 6 for $k=10$ and v from 2 to 10. The result shows that using a large v gives far superior error probability for given k . Consequently, using a small β also improves the antijam capability of a user. By setting to zero the denominator of the upper bound on P_b , we have

$$S = -1 / (2v \ln [2(1/2)^{1/v} - 1]).$$

This value of S versus v is given in Fig. 7 together with the corresponding value of $\beta (= 1/(Sv))$. It is observed that a large v (consequently, a small β) increases the value of S . A value of v around 5 would give a value of S close to the maximum R_o^T .

From the bound on P_b in (1) the error probability is decreasing at a rate that is exponential in $nvkR_o^T(\beta)/N$. It is noteworthy that nv is the number of chips for the output on a transition of the trellis. Consider the use of Viterbi decoding, for which decoding complexity is measured by k (the trellis therefore has 2^{k-1} states), for fixed nv and N . Consider changing the value of β . (Since $\beta = n/N$ and nv is fixed, a smaller β would imply a smaller n and a larger

v	β	MAX S (BITS/CHIP)
2	1.765	.284
3	1.066	.313
4	0.766	.326
5	0.599	.334
6	0.493	.339
7	0.418	.342
8	0.363	.344

Fig. 7. Maximum throughput for rate $1/v$ code.

v .) Due to the fact that $R_o^T(\beta)$ is maximized by small values of β , the complexity k is minimized for small values of β at a fixed error probability.

Using small β , however, may require a decoder complexity larger than that reflected by the value of k due to two reasons. First, decoding can be performed in units of code symbols since circuits on a chip are available for demodulating the entire PN sequence carrier (which is considered as a code symbol in the channel transition diagram). A small β would require a large v , the number of code symbols on each transition of the trellis. Second, the use of small β requires finer quantization for decoding since the channel (Fig. 2) becomes more noisy. The merging of the four functions in Fig. 5 in fact shows that rough quantization may be used with little effect on R_o^T or C^T for large β .

Since user interference is severe in such a system, using a long constraint length code is crucial for low error probability. Unfortunately, Viterbi decoding is too complex to implement for constraint lengths beyond 10. We suggest searching for good convolutional codes to decrease error probability and increase total throughput. (Our previous bound on P_b is a random coding bound for an ensemble of codes chosen at random.) A directory of good encoders would be built up from which each subscriber would choose an encoder. The maximum size of the directory, which determines the number of subscribers for the system, depends on the density of good codes times the volume of the code space. We believe that searching for good convolutional encoders [4] is a better and more tractable way to improve code orthogonality than trying to design PN sequences with good autocorrelation and low cross correlation, which is difficult due to code asynchronism.

A long PN sequence may also make synchronization at the receiver easier to achieve. It should be noted that two synchronizations have to be performed, namely the synchronization of the PN sequence carrier and the synchronization of the convolutional encoder, up to the code symbols. For the case of $n=1$, there is no need for PN

sequence synchronization. In practice, a synchronization signal can be sent on top of the coded signal for easy code synchronization.

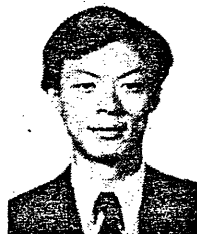
Priorities among the users can be set up by the authorization of transmitting power levels for the users. Using a higher transmitting power, a user may choose to increase its data rate or decrease its error rate. The analysis in this paper can be carried further to show that the capacity for each user is directly proportional to its transmitting power. Fading, due to distance and poor weather conditions, can also cause variations of signal power and result in dramatically deteriorated system performance. The system, in order to operate reliably in most conditions, would have to include a power margin that reduces the total throughput of the system. Also, power monitoring for fairness is difficult in practice.

V. CONCLUSION

It has been suggested [5] that CDMA for the spread spectrum channel should operate at a throughput of about 10 percent for reasonable error performance. The analysis in this paper gives a maximum throughput of 36 percent based on the cutoff rate. In practice, the throughput would have to be reduced for a smaller decoding complexity achieving a tolerable bit error rate. We have shown that a throughput of 20 percent is feasible in practice.

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