How to Integrate your Production and Logistics Strategy:

A New CLSP Formulation for a CPG Supply Chain

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Abstract

In manufacturing companies, production strategies prioritize maximizing line efficiency which favors large lot sizes and few setups. On the other hand, logistics strategies prioritize minimizing inventory costs which favors smaller lot sizes and more setups. This thesis provides a new mixed integer linear model formulation that optimizes lot sizes such that both manufacturing efficiency and inventory costs are considered simultaneously. The model solves a multi-machine capacitated lot sizing problem with novel extensions for multi-echelon inventory, transfer costs between inventory echelons, and a multi-echelon product setup hierarchy. The model includes extensions for setup-times and multiple non-identical machine capabilities. The multi-echelon inventory extension is applicable to firms that contract a third party logistics provider's warehouse to handle seasonal inventory. In this situation, the firm has two inventory holding cost structures and desires to optimize usage of the contracted warehouse. The multi-echelon setup extension is applicable to firms that manufacture products with similar characteristics such that they share a common machine setup cost at a category or aggregated level and a unique setup cost at an item or disaggregated level. When applied to benchmarking manufacturing data, the model demonstrates improved production plans that reduce inventory and setup costs by 30% in some scenarios. This thesis emphasizes how integrating production and logistics strategies can offer significant improvement to any firm's supply chain. In particular, firms with a multi-echelon inventory or setup cost structure can benefit from a model that accounts for these important cost drivers in planning its production.

Thesis Supervisor: Dr. Josué C. Velázquez-Martínez

Director, SCALE Latin America

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From Thomas; Special thanks to my lovely wife Julianna, without whom this would not have been possible.

From Ian; Thanks to my family and friends for their support and belief.

Contents

1 Introduction

Consumer packaged goods (CPG) companies typically operate on low margins. In 2015, the average profit margin for the food and beverage industry was 5.3% (Investopedia, 2015). Thus any supply chain cost reductions in these companies has a large impact on improving the profit margin of the company. At a 5% gross margin, supply chain cost savings of just 5% will double net profits (OByrne, 2016). For this reason, processes that can improve coordination across internal functions to reduce supply chain costs is a strong value proposition for companies that want to stay competitive in a low margin industry. Process improvements are particularly enticing because they require little investment; implementing policy changes only requires investment in personnel time since it focuses on improving utilization of existing assets.

Production Lot-sizing models are particularly useful to drive consistent and holistic processes as supply chain complexity increases. These models are able to assess multiple dimensions of cost trade-offs simultaneously and efficiently in way difficult for a manual process to emulate. The most basic model formulations balance the conflicting minimization of the costs to set up a machine to produce a product, and the minimization of inventory holding cost.

The smallest lot size for a product that satisfies demand is to only make the amount ordered. This results in the lowest amount of inventory, but more frequent setups and reduced machine efficiency. Increasing the lot sizes to anticipate future demand increases the amount of inventory held, but decreases the amount of setups and improves efficiency.

In light of the desire to improve cost performance of the production planning process, Niagara Bottling LLC (Niagara), a leading bottled water supplier in the United States, has partnered with MIT on this thesis to determine optimal lot sizes for its specific production layout and business needs. The goal is to formulate a model that optimizes production planning so that management can reap the benefits of paying less to produce the right products at the right time.

This thesis shows how applying a production lot-sizing model that incorporates the parameters unique to a firm can reveal the efficiency of existing policies, and provide insights on opportunities to improve.

1.1 Niagara Bottling LLC

The Niagara Bottling Company manufactures bottled water for its own brand and for private labels in numerous bottle and pack sizes. The company has numerous production plants across the nation. Each plant has multiple production lines that form, fill, pack, and palletize water bottles using a highly automated process. Each production line is not identical and can be differentiated by what items it can produce and how fast it can produce them.

Niagara faces a seasonal distribution of demand for bottled water as shown in Figure 1. This demand varies throughout the year with a typical peak demand season in the summer and a low season in the winter. During peak demand season, plant capacity is unable to meet demand. As a result, Niagara develops a production plan to build inventory during the low season in anticipation of the peak season. Niagara then stores the excess inventory in a third party warehouse (3PL) until demand is realized. The resulting network of plants, 3PL warehouses, and customers is illustrated in Figure 2.
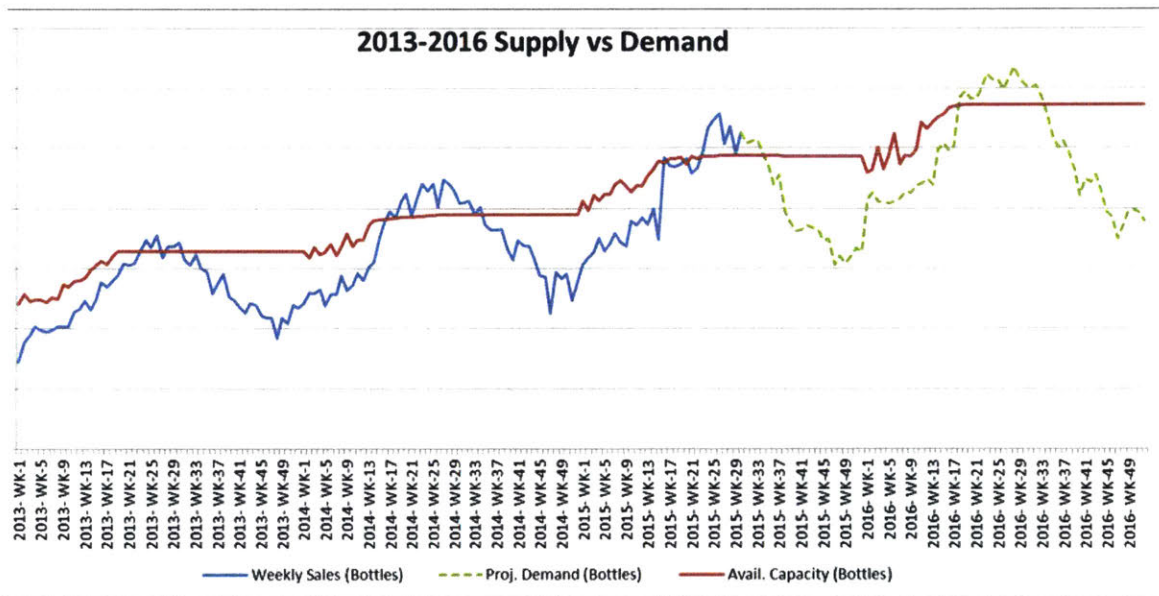

Figure 1: Relationship between Demand and Capacity throughout the year. In the middle of the year there is typically a peak season in which demand is greater than capacity.

Niagara has some limited inventory warehousing capacity at its plants. The plant warehousing cost structure is significantly lower than the cost structure for the 3PL warehouse. Additionally,

there are transfer costs associated with transporting and handling of inventory entering the 3PL warehouse. Niagara expects that because of this cost structure, inbound shipments to the 3PL warehouse should only occur during periods where inventory is being built for the peak season.

Instead, Niagara has observed that even during peak periods, inventory is being transferred to the 3PL warehouse as shown in Figure 3. This behavior is attributed to large lot sizes that maintain high utilization of the machines, but are incurring increased inventory holding costs. Hence this thesis focuses on optimizing Niagara's production planning process.
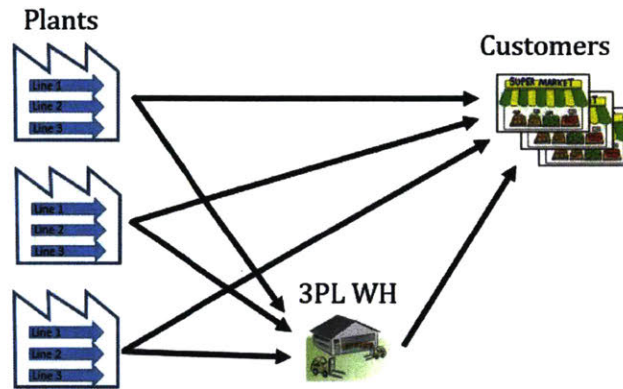

Figure 2: Illustration of Niagara's Logistics Network. Plants can send product directly to the customer or, when building inventory for future demand, to the 3PL Warehouse.
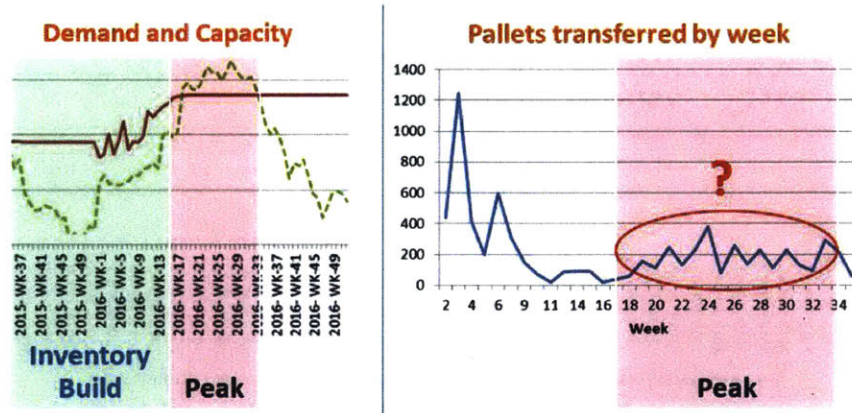

Figure 3: 3PL Inventory fluctuation during peak season.

1.1.1 Niagara Planning Process

Niagaras production planning process can be divided into three core groups. The groups are organized as follows along with their responsibilities:

- Demand Planning - Maintain, update, and publish strategic and tactical forecasts.

- Sales and Operations Planning (S&OP) - Create aggregate level production plans for the network and setting weekly inventory build targets for the summer peak season. They make strategic decisions on adding new plants and lines to the network.

- Supply Planning - Perform long-term network planning and tactical production planning

At the beginning of the year, using the strategic forecast from Demand Planning, the S&OP group creates a service plan for the year. This service plan is then revised, updated, and published every subsequent month to maintain tactical responsiveness.

The supply planning group then uses the service plan primarily as a guideline for projecting dynamic sourcing[1] and planning inventory build targets for the peak season. These strategies are integral to Niagara for maintaining high service levels since demand is seasonal with high variability.

In light of Niagara's production planning process, this thesis presents a model to determine optimal lot sizes that improves the tactical revision and updating of the service plan. The model will capture Niagara's production process, inventory build targets, and additional costs of 3PL warehousing and transfers. At a tactical level, the model will assist the supply planning group to plan network balancing and production scheduling on a weekly basis. The model will help Niagara remain responsive to demand while pursuing a cost effective inventory strategy.

1.2 Thesis Scope

Lot sizing models have been studied for over 100 years and there are many different models that capture a myriad of business features that exist in the corporate landscape. Thus it is critical to select a production lot-sizing model that matches the key cost structure of Niagara's operations.

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[1]shifting demand between regions to accommodate for capacity constraints

The literature review reviews existing literature on lot sizing problems and identifies a model that most closely match Niagara's business landscape.

The methodology section maps the existing model formulation parameters to Niagara's operations. Limitations of the existing formulations with respect to Niagara's business structure are highlighted. Thus this thesis presents novel extensions to the existing formulations where they fail to address key features of Niagara's operations.

The results section reviews the models performance against several benchmarking scenarios using historical company data. The model is compared against actual production data, the existing planning process at Niagara, and an additional manual planning heuristic using the Economic Order Quantity (Harris, 1913). The benchmarking analysis discussion highlights key insights, limitations, and cost saving opportunities. A sensitivity analysis explores the robustness of the model to changes in key parameter values.

The conclusion summarizes the key findings of the thesis and discusses further research opportunities to improve the model.

2 The Literature Review

This literature review is organized as follows. The first section explores lot sizing literature surveys which summarize the basic models and extensions applicable to lot sizing problems. The second section focuses on the two models most applicable to Niagara's business environment: the Economic Order Quantity (EOQ) and the Capacitated Lot Sizing Model (CLSP). The third section looks at which extensions and solution methods have been most researched in recent years. The fourth section considers literature that employs hybrid approaches that are computationally efficient to find exact solutions for lot sizing problems.

2.1 Economic Order Quantity EOQ

The Economic Order Quantity (EOQ) model has been a valuable management tool ever since it was first introduced by Harris in 1913 (Harris, 1913). The EOQ model calculates the total cost per item as a function of just three components: purchasing, production set-up, and inventory holding costs (Andriolo, Battini, Grubbström, Persona, & Sgarbossa, 2014). The optimal order quantity is derived such that the total cost per item is minimized.

In order to formulate this total cost equation, Harris makes several assumptions about the demand and production capacity for items being produced (Harris, 1913). Demand is assumed to be independent, continuous, and constant. The assumption of independent demand implies that the end-item is produced in one continuous process without requiring intermediate production of items (i.e. sub-assemblies) (Karimi, Fatemi Ghomi, & Wilson, 2003). Independent demand is also referred to as a single level problem (Bahl et al., 1987). Continuous and constant demand imply that the demand for the item is known and does not change (i.e. static and deterministic). Harris also assumes that resources to produce the item were available in abundance and capacity constraints are not considered. Unconstrained capacity problems such as this are also referred to as single item problems. When using this model for multiple items, studies simply solve the equation for each item independently. Including a capacity constraint introduces interdependency between items that requires more constraints and variables in models.

The first solution methodology for a multi-product capacitated EOQ model was using a Lagrangian algorithm (Whitin & Hadley, 1963). There are many studies on the robustness of the EOQ/EPQ model and how deviations from it only give small increases to average cost of a production policy (Dobson, 1988; Stadtler, 2007).

The robustness of the EOQ/EPQ model has led to many studies that explore variants and extensions of lot sizing. For example, Porteus (Porteus, 1985) uses the EOQ model to show that companies who invested in reducing setup costs could reduce costs. Other articles introduce uncertainty to EOQ parameters that are traditionally held constant (Weiss, 1982; Björk, 2011). More recent studies use the EOQ model for extensions such as multiple-product, single machine, space constraints, and multiple discrete delivery scenarios (Taleizadeh, Widyadana, Wee, & Biabanid, 2011; Taleizadeh, Cárdenas-Barrón, & Mohammadi, 2014).

In the case of Niagara, this study assumes that demand is independent and resources are constrained. The demand is independent because the total production from raw material to end product can be considered one continuous process. As such, the EOQ model is inadequate for Niagara's operations.

2.2 Single Level Constrained Resource Problems

Instead of the EOQ, Single Level Constrained Resource Problems are most appropriate for bottling operations such as Niagara's (Bahl et al., 1987). Bahl provides a framework for categorizing lot sizing problems, which is shown in Figure 4. Using this framework, the most appropriate classification for Niagara is the single level constrained resource category (SLCR).

There are many types of SLCRs, one of the models that allows for planning of multiple items is the Economic Lot Size and Scheduling Problem (ELSP). This problem maintains the assumptions of continuous constant demand, but introduces a constraint on the amount of items that can be produced at one time. An important feature of Niagara's business is the changing demand experienced during summer months. As such, further research on this set of problems is dismissed in favor of problems that allow for dynamic demand.

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                         Lot Sizing

                          Problems
                             /\
                            /  \
                           /    \
                          /      \
                         /        \
                        /          \
               Single Level          Multiple Level

          (Independent Demand)      (Dependent Demand)
                   /\                       /\
                  /  \                     /  \
                 /    \                   /    \
   Unconstrained Resources  Constrained Resources  Unconstrained Resources  Constrained Resources

          (SLUR)            (SLCR)            (MLUR)            (MLCR)
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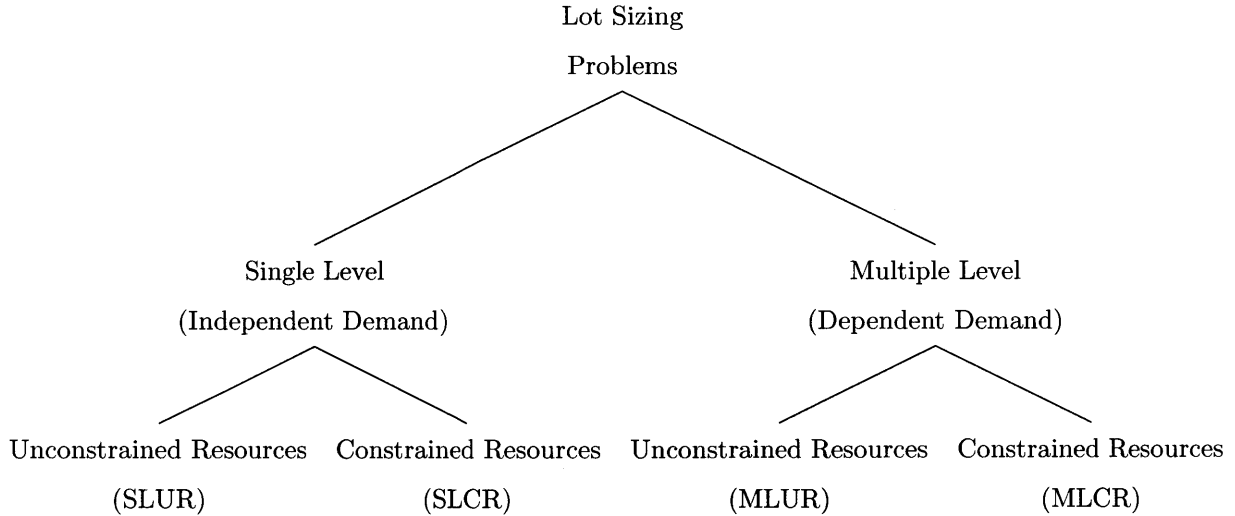
Figure 4: Lot Sizing Problem Formulation Categories. Adapted from (Bahl et al., 1987)

The Wagner and Whitin algorithm (Wagner & Whitin, 1958) provides a method for considering dynamic demand. Extending this model to consider resource constraints produced a range of problems that is best classified by the duration of the time periods used. The large bucket problems use a longer time period in which multiple items can be set-up and produced. This period is typically a week with a planning horizon less than six months (Drexl & Kimms, 1997). A common large bucket formulation is the Capacitated Lot Sizing Problem (CLSP). This problem is known to be impractical to solve exactly for more than 10 machines, 20 items, and 3 periods (Drexl & Kimms, 1997). As a result, heuristic solutions methodologies dominate the literature (Karimi et al., 2003).

The small bucket formulations simplify the problem by using smaller periods of typically hours or shifts. The Discrete Lot Sizing Problem (DLSP) assumes that only a single item is produced in a period using the entire capacity. This means the lot size for the item produced in a period is always the same. The Continuous Setup Lot Sizing Problem (CSLP) and Proportional Lot Sizing Problem (PLSP) remove these restrictions to represent more realistic situations. CSLP allows for a continuous order size that can be less than the full capacity in a time period. The PLSP adds to this the ability to produce up to two items in a given period (Drexl & Kimms, 1997). Due to the short time periods used in small bucket problems, unmanageable problem sizes occur when considering a long time horizon.

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As mentioned above, the EOQ's robustness and low computation requirements make it easy to apply in most planning situations. However, it applies only when demand for a product is assumed constant over the year and each order is delivered in full immediately. Because this assumption is too restrictive to necessarily fit the Niagara problem, which must take into account seasonal demand patterns and capacity constraints, this study explored relevant literature on the CLSP formulation as well.

2.3 The Capacitated Lot Sizing Problem CLSP

The precursor to the CLSP is the Wagner-Whitin problem (Wagner & Whitin, 1958) which assumes a finite planning horizon, subperiods, varying demand, and no capacity constraints; this effectively makes it a single item problem. The CLSP, however, includes capacity constraints that extend the problem and address multiple items.

Numerous studies apply extensions to the basic CLSP model. The basic CLSP provides a fixed setup cost for each item; however, there are cases where the setup cost depends on how the machine was set up previously. For instance, the idea that a production configuration can still be used in the next period such that no setup costs or time is incurred is discussed by Almada-Lobo, et al (2007) (Almada-Lobo, Klabjan, Antónia carravilla, & Oliveira, 2007; Boctor, 2016). Gupta and Magnusson (2005) also describe a similar model that includes both the ability to carry-over setups and the sequence dependency of setups.

There are articles that explore manufacturing environments that have multiple machines or production lines that can make the same product. Furthermore, these machines may have varying production rates. For example, Bollapragada, Croce, and Ghirardi (Bollapragada, R., Croce, F. D., & Ghirardi, M. , 2011) describe a model for non-identical multiple machines. Other models describe the situation with multiple identically constrained machines where the production for an item can be split between multiple machines (Tempelmeier & Copil, 2016).

Another extension is parallel machines where products can be exclusively assigned to a machine or produced on several alternative machines (Tempelmeier & Copil, 2016; Marinelli, Nenni, & Sforza, 2007). There are articles that introduce a backlogging extension to better represent realistic production policies (Toledo, de Oliveira, & Morelato Frana, 2013; Karimi, Ghomi, & Wilson, 2006).

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There are studies presenting models where each setup cost and time depends on the configuration preceding it - Some articles also refer to this as sequence dependency (Tempelmeier & Copil, 2016; Almada-Lobo & James, 2010; Almada-Lobo et al., 2007). This extension touches on the lot scheduling problem that Niagara faces, but is not part of the scope of this paper.

All these extensions are attempts to better model real life production planning scenarios to provide valuable insights for planning managers. As a result, some studies incorporate more uncertainty in their models. For instance, Björk, Yao, and Huang use fuzzy logic to model how there is uncertainty in the exact number of products produced by machines in production runs (Björk, 2011; Yao, Huang, & Huang, 2007).

Unfortunately, and expectedly, the CLSP becomes more complicated the more extensions are added to it. Studies have shown the basic CLSP problem is NP Hard (Bitran, Haas, & Hax, 1981; Florian, Lenstra, & Rinnooy Kan, 1980; Maes, McCLain, & Van Wassenhove, 1991). There are studies that have attempted to solve the CLSP exactly (Eppen & Martin, 1987; Akbalik & Penz, 2009), but their methods have been limited to small capacities or a small number of items and machines. On the other hand, this thesis model is able to solve a variation of the CLSP for up to three Niagara facilities with large capacities, a larger number of items, and multiple machines.

2.4   Exact Solutions for the CLSP

The CLSP model is NP-hard, hence there are studies focused on developing heuristics that allow for feasible solutions that are close enough to optimality to be practical. Jans and Degraeve's survey (Jans & Degraeve, 2007) explores how metaheuristics are being applied to solve dynamic capacitated lot sizing models. One heuristic category covered by articles is Lagrangian transformations that decompose the NP hard problem into several easier to solve subproblems with good solution quality and short execution times (Alaoui-Selsouli, Mohafid, & Najid, 2012; Caserta & Rico, 2009). Other studies use Tabu search and obtain efficient results even for highly capacitated instances with high setup-to-holding-costs ratios (Almada-Lobo & James, 2010; Karimi et al., 2006).

However, when compared to other metaheuristics, genetic algorithms perform the best in terms of solution quality and speed than the results obtained by exact solution approaches and other heuristics (Guner Goren, Tunali, & Jans, 2010; Toledo et al., 2013). Jans and Degraeve

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contend that although there has not been any real breakthrough with genetic algorithms yet, the promising results in recent years indicate a likely increase in practical applications in the near future (Jans & Degraeve, 2007).

There are studies that focus on reformulating the CLSP problem to improve computational efficiency. For example, rewriting the model as a mixed-integer linear program has been proven effective in solving a variety of lot sizing problems (Belvaux & Wolsey, 2001). To solve a production planning and maintenance problem, Alaoui-Selsoulia, Mohafid, and Najid (Alaoui-Selsouli, Mohafid, & Najid, 2012) used a tightened' mixed-integer LP model together with a Lagrangian-based heuristic to achieve solutions very close to optimal [2].

Another approach was presented by Józefowska and Zimniak (Józefowska & Zimniak, 2008) that uses expert opinions from management to create model rules to tighten the solution space. The study then uses a genetic algorithm to find a set of potentially Pareto-optimal solutions.

As a contribution to the existing literature in lot-sizing problems, this thesis presents a new CLSP model formulation that captures optimal lot sizing behavior for Niagara's production and logistics functions. The model can be solved exactly by commercially available software. The proposed model also provides the unique extensions of a double-echelon inventory warehouse option, a transfer cost for utilizing a second echelon warehouse, adjusts productive capacity depending on setup decision variables, and solves a double-echelon setup hierarchy simultaneously.

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[2]all instances not solved to optimality have less than 0.97% deviation from the solver

3   Methodology

The model focuses on the needs of three facilities in a core region of Niagara's operations. These three facilities operate twelve production lines with unique production capabilities and efficiencies. Altogether, this region can produce over 90% of Niagara's items. In addition, the network includes a 3PL warehouse that Niagara uses for excess inventory. The demand in this region exceeds the capacity during peak seasons.

The combined capabilities of these facilities span the range of capability features in the rest of the Niagara network. Thus the thesis model is easily applied to other regions or facilities in Niagara's network, as shown in the results section.

This section examines the basic CLSP and discusses its inadequacies with respect to Niagara's supply chain and processes. These inadequacies prompt a new model formulation, the CLSP with Multi-Echelon Setup Costs and Inventory extensions, which is presented and discussed. This model formulation is capable of planning production for multiple items on a single machine.

A second model formulation is presented that captures Niagara's capability to use multiple machines to perform dynamic production planning: the CLSP with Multi-Echelon Setup Costs and Inventory for Multiple Machines. This model allows Niagara to plan lot sizes for multiple machines with non-identical capabilities.

The models capture the behavior and not actual costs of Niagara's operations, while still providing realistic lot sizing results that can improve Niagara's production planning process.

3.1   The Basic CLSP

The basic CLSP serves as the starting point for the model developed for Niagara. The basic formulation as presented by Karimi (2003) is:

Indexes

$i = item$ $i = 1, 2, .., n$

$t = period$ $t = 1, 2, .., T$

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Nomenclature

T	number of periods in the planning horizon
X_{it}	production (lot size) of item i in period t
I_{it}	inventory of item i at the end of period t
	($I_{i1} = I_{iT} = 0$, without loss of generality)
Y_{it}	a binary variable that assumes value 1 if item i is produced in period t
	and 0, otherwise
R_t	available capacity in period t
d_{it}	demand for item i in period t
C_{it}	unit production cost of item i produced in period t
S_{it}	setup cost incurred if item i is produced in period t
$M_{it} = \sum_{k=1}^{T} d_{ik}$	upper bound on the production of item i in period t
a_i	unit resource consumption for item i
h_{it}	unit holding cost of item i at the end of period t

Minimize
$$Z = \sum_{i=1}^{n} \sum_{t=1}^{T} S_{it} \cdot Y_{it} + C_{it} \cdot X_{it} + h_{it} \cdot I_{it} \qquad (1)$$

Subject to
$$\sum_{i=1}^{n} a_i \cdot X_{it} \leq R_t \quad (t = 1, ..., T) \qquad (2)$$

$$X_{it} + I_{i,t-1} - I_{it} = d_{it} \qquad (i = 1, ..., n; \quad t = 1, ..., T) \qquad (3)$$

$$X_{it} \leq M_{it} Y_{it} \qquad (i = 1, ..., n; \quad t = 1, ..., T) \qquad (4)$$

$$Y_{it} \in 0, 1 \qquad (i = 1, ..., n; \quad t = 1, ..., T) \qquad (5)$$

$$X_{it} \leq 0 \qquad (i = 1, ..., n; \quad t = 1, ..., T) \qquad (6)$$

$$I_{it} \geq 0 \qquad (i = 1, ..., n; \quad t = 1, ..., T) \qquad (7)$$

The objective function in this basic formulation minimizes the sum of setup costs, inventory holding costs and production costs. The formulation provides a distinct production cost for each item in each time period.

In the case of Niagara, the difference in production costs between items does not impact the optimal solution. The minimum quantity of each item produced is determined by the demand constraint, while over-production is optimized by the inventory holding cost in the objective func-

tion. Therefore, production costs are not relevant for the objective function and are removed. The remaining variables of the objective function and constraints are addressed in the following sections.

3.1.1 Setup Costs

The setup cost is the cost incurred each time the machine is set up to run a product. These costs can come from a number of sources including direct labor to perform the setup, opportunity cost of lost production while the machine is down, or decreased utilization of a capital investment. For the case of Niagara, the most significant is the opportunity cost of lost production. As such, the setup cost is a function of the downtime associated with a setup, the production rate of the machine, and the lost revenue per bottle of the item.

As discussed in the literature review, Niagara's production consists of a continuous process from bottle extrusion through palletization of the final product. The final product for Niagara is a unitized pack of water bottles. The key differences between products are the bottle size, the number of bottles in a pack, the label, and type of water in each bottle. The combination of these features defines each individual item.
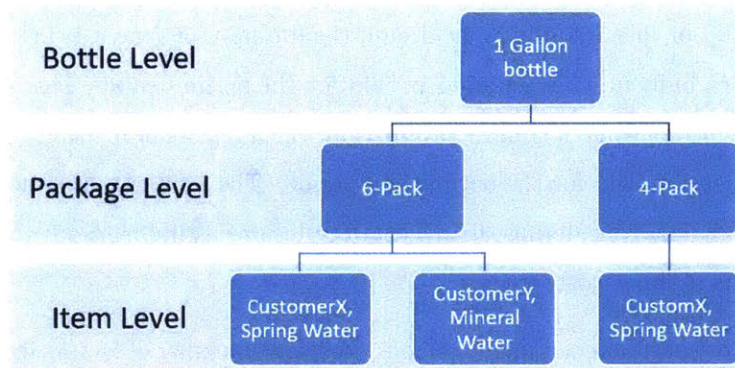
Figure 5: Hierarchical Nature of Product Setups

Figure 5 illustrates the hierarchical structure of any given item. Due to this continuous process, the entire production line stops to set up for each item being produced. The length of time required to set up for the next item depends on the activity, which depends on what item was being produced previously. For example, a bottle change takes longer than changing the packaging size.

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However, these two activities are executed in parallel, so the setup cost is associated with just the bottle change even if both bottle and package needed to be changed.

Unfortunately, the basic CLSP formulation only provides one setup cost per item per time period and cannot account for this complexity. Typically, accounting for this type of complex setup is handled by the small bucket problems such as the DLSP (Drexl & Kimms, 1997). However, these models use a time period too small to be practical for the tactical level of detail Niagara is seeking.

Additionally, in actual application, a setup cost is not necessarily incurred every time production occurs in a time period. In the event that the first item produced in a time period is the same as the last item produced in the previous time period, no additional setup is required. The basic CLSP model does not take this into account and overestimates the setup costs for these cases. However, capturing this behavior is more important on an operational level where sequencing of production is being planned. For the purposes of this tactical model, the overestimation does not impact the behavior of the model such that it is inconsistent with Niagara's process.

3.1.2 Holding Cost

The holding cost is a function of the cost of holding an item in inventory for a period of time, the quantity of that item being held, and the duration of time it is held. The model assumes that all inventory built in a time period is held for the entire period. This method leads to some overestimation since in some instances production which results in inventory occurs at the end of a period and thus is not held for the entire time period. The level of inaccuracy this introduces into the model is negligible considering the uncertainty in forecasting methods to produce the demand data, estimations of labor cost, etc.

Holding cost per item can take into consideration a number of factors including cost of storage space, insurance, labor to handle and manage inventory, and the cost of capital that is tied up by holding inventory (Taylor, 2010). Niagara determines holding cost as the physical cost of space and labor over an annual period. The annual holding cost is then divided by the average number of pallet footprints to determine a cost per pallet footprint per time period. The number of bottles that fit on a pallet and whether those pallets can be stacked are then used to convert footprint cost to a holding cost per bottle per time period. The calculation of holding cost for Niagara is

straight forward with the data available. However, the model only allows for one holding cost and one inventory position.

Holding Cost = Cost of space and Labor of a pallet footprint / number of bottles on a pallet [3]

As discussed in the introduction, Niagara in fact has two different inventory positions and cost structures for the plant and 3PL warehouses. This is a key feature for Niagara that is not addressed in the basic CLSP.

3.1.3 Capacity Constraint

The capacity constraint for the basic CLSP formulation is equation (2). Capacity for a production line is a function of the design production rate, the average production efficiency, and the uptime of the machine in a time period. Capacity is calculated using this function to determine the number of bottles a machine produces in a time period.

Capacity = Machine speed[4] (BPM) x Average efficiency (%) x Uptime in a period (minutes)

The model allows for specification of a different capacity for each time period. Niagara can adjust capacity between time periods to account for planned maintenance or changes in the number of shifts.

Because capacity is related to uptime of the machine, an important consideration is the lost capacity due to setup times. This loss of capacity can be significant for long setup times for the bottle level or when setups occur frequently. The capacity change from setups is significant to Niagara particularly during peak seasons when demand exceeds capacity. The basic CLSP does not account for setup times in its existing form.

3.1.4 Demand and Inventory Constraints

The demand constraint (3) establishes the relationship between demand, production, and inventory: Production and the change in inventory position must satisfy demand in a given period. When planning production, the demand is the forecasted sales in bottles for the time period. It can

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[3]double number of bottles per footprint if bottle/pack type is stackable

[4]The Bottles Per Minute that the machine was originally designed for

also include inventory build targets for the peak season, but this is better achieved through the use of ending inventory targets.

The basic CLSP specifies the starting and ending inventory as zero. In the case of Niagara, it's important to capture the inventory in the system at the beginning of the planning time horizon and the needed inventory at the end. Using this feature to capture inventory build targets allows the model to optimize based on Niagara's current inventory position and its inventory strategy at the end of the planning horizon.

3.1.5   Remaining Constraints

Equation 4 ensures that if a machine is not setup for a product, than that machine cannot produce that product. The remaining constraints (5), (6), and (7) specify that setups are binary, and that inventory and production must be positive.

3.2   CLSP with Multi-Echelon Setup Costs and Inventory Extensions

Addressing the shortcomings of the basic CLSP requires use of existing extensions discussed in the literature as well as novel extensions developed by the authors. The revised CLSP formulation with extensions is as follows:

Indexes

$i = item$ $\qquad i = 1, 2, .., n$

$t = period$ $\qquad t = 1, 2, .., T$

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Nomenclature

T number of periods in the planning horizon

X_{it} production (lot size) of item i in period t

I_{it} inventory in bottles of item i at the end of period t

W_{it} 3PL inventory in bottles of item i at the end of period t

Is_i starting inventory in bottles of item i

Ws_i starting 3PL inventory in bottles of item i

Ie_i ending inventory in bottles of item i

We_i ending 3PL inventory in bottles of item i

Y_{it} a binary variable that assumes value 1 if item i is produced in period t and 0, otherwise

Z_{jt} a binary variable that assumes value 1 if an item from category j is produced in period t and 0, otherwise

R_{1t} available production capacity in bottles in period t

R_{2t} available inventory capacity in bottles at plant warehouse in period t

d_{it} demand for item i in period t

S_{1i} basic level setup cost incurred if item i is produced

S_{2i} higher level setup cost incurred if item i is produced

K_{1it} basic level setup time in bottles for item i in period t

K_{2it} higher level setup time in bottles for item i in period t

h_{1i} holding and handling cost per bottle of item i in plant warehouse

h_{2i} holding and handling cost per bottle of item i in 3PL warehouse

h_3 average fixed cost of transferring inventory of item i to 3PL warehouse in each time period

H_{it} a binary variable that assumes value 1 if item i is transferred to 3PL in period t and 0, otherwise

n number of items

N_j number of items in category J

M arbitrary large number

$$\text{Minimize} \quad \sum_{t=1}^{T} \left[\sum_{i=1}^{n} [(h_{1i} \cdot I_{it}) + (h_{2i} \cdot W_{it}) + (h_3 \cdot H_{it}) + (S_{2i} \cdot Y_{it})] + \sum_{j=1}^{J} (S_{1j} \cdot Z_{jt}) \right] \qquad (8)$$

$$\text{Subject to} \quad \sum_{i=1}^{n} X_{it} + \sum_{i=1}^{n} (K_{1i} \cdot Y_{it}) + \sum_{j=1}^{J} (K_{2j} \cdot Z_{jt}) \leq R_{1t} \quad \forall t \qquad (9)$$

$$d_{i,t} = X_{i,t} + (I_{i,t-1} + W_{i,t-1}) - (I_{it} + W_{it}) \quad \forall i,t \qquad (10)$$

$$\sum_{i=1}^{n} I_{it} \leq R_{2t} \quad \forall t \qquad (11)$$

$$\sum_{i=1}^{n} [W_{it} - W_{i,t-1}] \leq M \cdot H_{it} \quad \forall t \qquad (12)$$

$$\sum_{Nj-1}^{Nj} Y_{jt} \leq M \cdot Z_{jt} \quad \forall j,t \qquad (13)$$

$$\sum_{i=1}^{n} X_{i,t} \leq M \cdot Y_{it} \quad \forall t \qquad (14)$$

$$X_{it} \geq 0 \quad \forall i,t \qquad (15)$$

$$Y_{it} \in 0,1 \quad \forall i,t \qquad (16)$$

$$Z_{jt} \in 0,1 \quad \forall j,t \qquad (17)$$

$$H_{it} \in 0,1 \quad \forall i,t \qquad (18)$$

$$I_{it} \geq 0 \quad \forall i,t \qquad (19)$$

$$W_{it} \geq 0 \quad \forall i,t \qquad (20)$$

3.2.1 Multi-Echelon Setups

The available methods in the literature for addressing Niagara's complex setup structure focus on solving the sequence order for all items being produced in a time period (Tempelmeier & Copil, 2016; Almada-Lobo & James, 2010; Almada-Lobo et al., 2007). This greatly adds to the complexity of the model and computational time required. In addition, it provides a level of detail not needed for Niagara's planning process.

Instead, a solution is identified that allows for application of a second setup cost in the event a higher level setup occurs. For example, as shown in Figure 6, if all items set up in a time period are

of the same packaging size, an item level setup cost is incurred for each item setup and a packaging level setup cost is incurred once. Alternatively, as seen in Figure 7, if all items produced in a time period are either of two packaging sizes, each item setup incurs an item level setup and two packaging level setups are incurred. Each item and package size can have a unique setup cost. This formulation encourages a machine to switch between package sizes less frequently, which accurately reflects Niagara's production process.
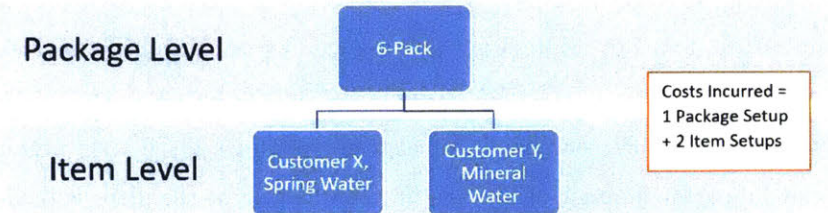

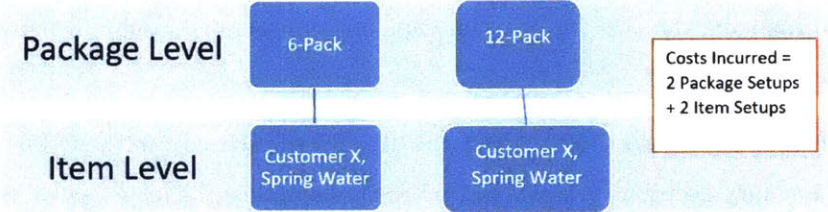Figure 6: Incurred costs for different items with same packaging size

Figure 7: Incurred costs for different items with different packaging size

To capture this multi-echelon behavior, an additional decision variable Z_{jt} is added to the objective function (8) and constraints (9), (13), and (17), which increases the complexity. In this model, the size of the matrix is only the number of different categories at the higher level and the number of time periods. In contrast, a sequence order formulation would require a decision variable matrix for all combinations of every item in every time period, thus making the problem much larger.

This new model formulation extension addresses two echelons at one time. In Niagara's case, the model runs either the bottle level and the packaging level, or the packaging level and the item level. A third echelon can be added, but is not included in the scope of this thesis due to time restrictions. A double-echelon system fully addresses the majority of Niagara's production lines, which can only produce one bottle size. If a machine only produces one bottle size, it can only have

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setup costs for a package level and an item level.

A few production lines can produce products with all three levels. For these lines, the model can still be used to plan production for bottle size and category at the same time which still addresses Niagara's immediate business needs.

3.2.2 Multi-Echelon Inventory

The literature does not address the specific case of using multiple inventory positions and holding costs. The addition of decision variable W_{it} in the formulation[5] represents the amount of inventory held at the 3PL warehouse for a given item in a given time period. In addition, a constraint that limits the amount of bottles that can be held at the plant is included[6]. This forces the use of the 3PL warehouse when plant warehouse is at capacity. The use of the 3PL warehouse, when the plant is not at capacity, is controlled by the higher holding cost of the 3PL in the objective function. Minimizing the cost for the same amount of inventory results in minimizing the use of the 3PL warehouse.

The literature also does not address the use of a transfer cost in the CLSP formulation. The addition of a binary decision variable, H_{it}, represents any time the inventory or an item in the 3PL warehouse increases from the previous period[7]. This variable is multiplied by a fixed transfer cost that accounts for the typical transportation and handling cost incurred during a transfer event[8]. The fixed transfer cost uses an average number of truckloads sent from the plant to the 3PL during the specified time period in the model. For example, the fixed transfer cost may assume fifty truckloads are sent from the plant to the 3PL in a week during a peak season. The number of truckloads used should be based on historical data for the particular plant and season being modeled.

Using a variable cost per bottle would increase the accuracy of the model. However, the assumptions and complexity this adds to the model was not within the scope of this thesis. Although less accurate, this approach does create the desired behavior in the model: Excessive transfers into the 3PL warehouse are avoided in favor of larger less frequent transfers.

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[5]Equations (8), (10), (12), (20)
[6]Equation (11)
[7]Equation (12)
[8]Equation (8)

The demand and inventory constraints[9] are updated to reflect that demand can be satisfied by either inventory in the 3PL warehouse or the plant.

3.2.3 Setup Times

The use of setup times is discussed in the multi-machine model presented by Bollapragada, Della Croce, and Ghirardi (Bollapragada et al., 2011). This concept is extended to work with the multi-echelon setups discussed in section 3.2.1. The setup time for each level is converted into the number of bottles less the amount the machine can produce in a time period. The setup times multiplied by the number of setups that occur in each time period is subtracted from the capacity of the machine in that period[10].

3.2.4 Demand and Capacity

The model provides for a single demand and capacity value per item, per time period. This formulation works well if one machine produces all of the particular item that is being planned. However, if the item is produced by multiple machines and demand cannot be assigned to a particular machine, then the model cannot differentiate how much is to be produced on each machine. If the machines have identical cost structures, how the production is split between the machines is trivial. However, Niagara uses multiple machines within a plant with varying production rates and setup times. As such, a further model extension is needed to address the use of multiple-machines.

3.3 CLSP for Multi-Echelon Inventory, Multi-Echelon Setups, and Multiple Machines

As previously noted, Bollapragada et al., (2011) provides a formulation for multiple non-identical machines. This model is adopted and synthesized with the single machine extensions described above. The resulting model formulation (P) is as follows:

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[9]Equations (9), (10), (11)

[10]Equation (9)

Indexes

$i = item$ $i = 1, 2, .., n$

$t = period$ $t = 1, 2, .., T$

$j = machine$ $j = 1, 2, .., J$

$l = category$ $l = 1, 2, .., L$

Nomenclature

T	number of periods in the planning horizon
X_{ijt}	production (lot size) of item i on machine j in period t
I_{it}	inventory in bottles of item i at the end of period t
W_{it}	3PL inventory in bottles of item i at the end of period t
Is_i	starting inventory in bottles of item i
Ws_i	starting 3PL inventory in bottles of item i
Ie_i	ending inventory in bottles of item i
We_i	ending 3PL inventory in bottles of item i
Y_{ijt}	a binary variable that assumes value 1 if item i is produced on machine j in period t and 0, otherwise
Z_{ljt}	a binary variable that assumes value 1 if category l is produced on machine j in period t and 0, otherwise
C_{ij}	a binary variable that assumes value 1 if item i can be produced on machine j and 0, otherwise
R_{1t}	available production capacity in bottles in period t
R_{2t}	available inventory capacity in bottles at plant warehouse in period t
d_{it}	demand for item i in period t
S_{1ij}	basic level setup cost incurred if item i is produced on machine j
S_{2lj}	higher level setup cost incurred if category l is produced on machine j
K_{1ij}	first level setup time in bottles for item i on machine j
K_{2lj}	second level setup time in bottles for category l in on machine j
h_{1i}	holding and handling cost per bottle of item i in plant warehouse
h_{2i}	holding and handling cost per bottle of item i in 3PL warehouse
h_3	average fixed cost of transferring inventory of item i to 3PL warehouse in each time period
H_{it}	a binary variable that assumes value 1 if item i is transferred to 3PL in period t and 0, otherwise
n	number of items
N_j	number of items in category J
M	arbitrary large number

$$\text{Minimize} \quad \sum_{t=1}^{T} \Big[\sum_{i=1}^{n} [(h_{1i} \cdot I_{it}) + (h_{2i} \cdot W_{it}) + (h_3 \cdot H_{it})] + \sum_{j=1}^{J} (S_{1ij} \cdot Y_{ijt}) \Big] + \sum_{l=1}^{L} \sum_{j=1}^{J} [S_{2lj} \cdot Z_{ljt}]]$$

(21)

$$\text{Subject to} \quad \sum_{i=1}^{n} X_{ijt} + \sum_{i=1}^{n} (K_{1ij} \cdot Y_{ijt}) + \sum_{j=1}^{J} (K_{2lj} \cdot Z_{ljt}) \le R_{1jt} \quad \forall t$$

(22)

$$d_{it} = \sum_{j=1}^{J} X_{ijt} + (I_{i,t-1} + W_{i,t-1}) - (I_{it} + W_{it}) \quad \forall i, t$$

(23)

$$\sum_{i=1}^{n} I_{it} \le R_{2t} \quad \forall t$$

(24)

$$\sum_{i=1}^{n} [W_{it} - W_{i,t-1}] \le M \cdot H_{it} \quad \forall t$$

(25)

$$\sum_{Nj-1}^{Nj} Y_{ijt} \le M \cdot Z_{ljt} \quad \forall l, j, t$$

(26)

$$\sum_{i=1}^{n} X_{ijt} \le M \cdot Y_{ijt} \quad \forall t$$

(27)

$$\sum_{i=1}^{n} X_{ijt} \le M \cdot C_{ij} \quad \forall i, j, t$$

(28)

$$X_{ijt} \ge 0 \quad \forall i, j, t$$

(29)

$$Y_{ijt} \in 0, 1 \quad \forall i, j, t$$

(30)

$$Z_{ljt} \in 0, 1 \quad \forall l, j, t$$

(31)

$$H_{it} \in 0, 1 \quad \forall i, t$$

(32)

$$C_{ij} \in 0, 1 \quad \forall i, j$$

(33)

$$I_{it} \ge 0 \quad \forall i, t$$

(34)

$$W_{it} \ge 0 \quad \forall i, t$$

(35)

3.3.1 Machine Index

The addition of the index j for machines allows capacity and setups to be specified for each machine independently. Demand is still specified on a per item per time period basis, thus the sum of production for an item across all machines is used along with inventory to meet demand[11].

3.3.2 Capability Constraint

In addition to unique capacity and setup cost structures across machines, there can also be unique capabilities with respect to the items a machine can produce. The literature review did not reveal an extension for addressing this feature of Niagara's business. In order to address this, the model includes a binary variable C_{ij} that specifies if an item is produced on a machine. If the item is not produced on the machine, then production for that item on that machine must be zero[12].

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[11]Equation (23)
[12]Equation (28)

4 Results

The proposed model formulation (P) is capable of planning production for the representative region in Niagara's network. The results demonstrate model performance relative to actual production data from 2015 during the peak season.

The data quality for the representative region creates some issues for cost performance comparison between the model and the actual data. The plants in this region produce for demand in other regions[13], however the demand data is only for the region. There is not indication from the actual production data which production satisfies the regional demand as opposed to demand from other regions. Therefore, although the representative region scenario demonstrates the full functionality of the model, additional scenarios and methods are used for further cost performance benchmarking.

The first additional benchmarking method uses actual production data from a single plant in another region where isolated production and demand data is available[14]. The data is further modified such that the production data meets the same constraints as the model. For example, production and inventory must satisfy demand in every time period and production in a time period cannot exceed rated capacity. These modifications allow a more direct comparison between the model performance relative to actual production data.

The second additional method focuses on benchmarking the model against the production planning process as it exists today at Niagara. This method involves giving the same model inputs to a Niagara production planner who used their current processes and knowledge to create a production plan.

As part of the second benchmarking method, a further comparison is made with a simple heuristic that determines lot size based on the EOQ model.

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[13]dynamic sourcing

[14]production is matched with demand inputs

4.1 Representative Region Planning Results

The representative region consists of three plants and one 3PL warehouse. The plants have a total of twelve production lines, which are capable of producing fourteen different bottle and package size categories. Table 1 provides an overview of which lines are capable of producing which products.

Bottle Size	Package Size	C1	C2	C3	C4	P1	P2	P4	P5	P6	P7	R1	R2
A	24P	0	0	0	0	1	1	1	1	0	1	1	1
	32P	0	0	0	0	1	0	1	1	0	1	1	1
	35P	0	0	0	0	0	0	1	1	0	1	1	1
	40P	0	0	1	1	0	0	1	1	0	1	1	1
B	15P	0	1	0	0	0	0	0	0	0	0	0	0
	20P	0	1	0	0	0	0	0	0	0	0	0	0
C	15P	0	1	0	0	0	0	0	0	0	0	0	0
D	24P	0	0	0	0	0	0	0	0	1	0	0	0
	12P	0	0	0	0	0	0	0	0	1	0	0	0
E	24P	1	0	0	0	0	0	0	0	0	0	0	0
	20P	1	0	0	0	0	0	0	0	0	0	0	0
	35P	1	0	0	0	0	0	0	0	0	0	0	0
	70P	1	0	0	0	0	0	0	0	0	0	0	0
	80P	1	0	0	0	0	0	0	0	0	0	0	0

Table 1: Representative Region Production Line Capabilities

The planning period selected is eight weeks and occurs at the end of a peak period[15]. The data and analysis during this period have the following features:

- No inventory building is occurring yet for future peak seasons.

- Any transfer of inventory into the 3PL warehouse during this period is due to larger production lot sizes to avoid setup costs.

- This scenario isolates inventory management during a period when 3PL warehouse storage is not strictly needed.

- The demand and production for the planned case and the actual data are shown in Figure 8. The values have been normalized to the typical production capacity, such that 100% is the production capacity for all lines in the model.

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[15]when inventory positions can be assumed to be zero

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- Initially the beginning inventory and nominal capacity are not sufficient to satisfy demand which makes the model infeasible. Thus capacity and beginning inventory are adjusted to satisfy constraints for model feasibility. The actual production data suggests that the plant was not able to meet demand in the first week, thus creating backorders. These backorders were addressed by increasing output in the next period. Since this model does not allow for backorders, the increase in capacity was applied in the first week to provide a set of feasible inputs.

The model size consists of 3,504 decision variables. The software, Gurobi, solves 9.8 million iterations before the set time limit of 30 minutes is reached. The resulting solution is estimated by the software to be 2.9% from the optimal solution.
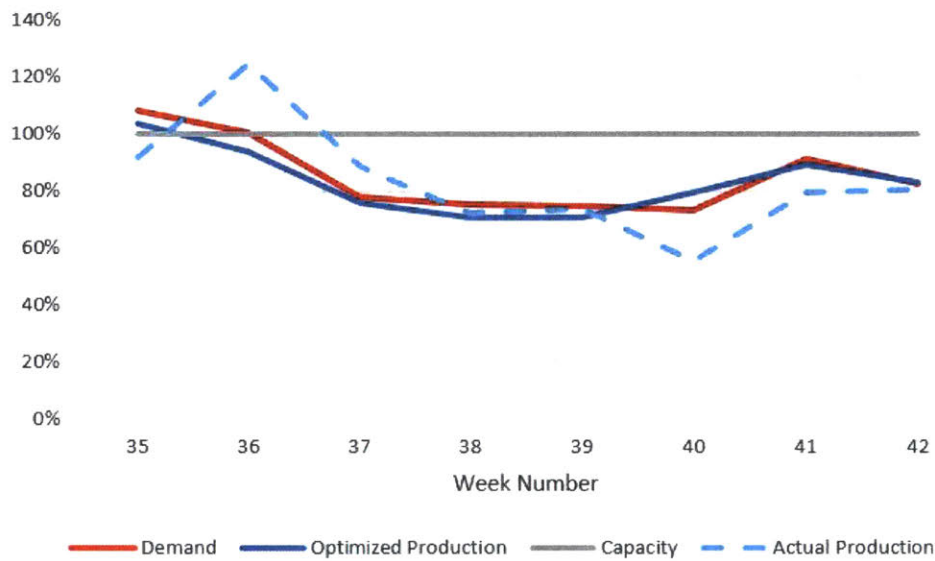

Figure 8: Representative Region demand versus actual production data and the optimized model

The results show that the model closely matches demand, whereas the actual production data shows an inventory build at the beginning followed by inventory burn in the later periods. This implies a better solution is achieved by making to order[16] as opposed to making to stock[17].

Table 2 summarizes the results from the optimized model as compared to actual data. The

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[16] satisfying demand as it materializes
[17] building inventory to meet future demand in the same time horizon

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data is normalized with respect to the total cost calculated for the actual production and inventory data such that the total cost for actual data is $1,000. The actual production data provided the number of setups and the amount of inventory held. These values were then multiplied by the same setup, holding, and transfer costs as used in the model.

Table 2: Cost difference between actual data and optimized model results

		Actual	Optimized
Setup Costs	Package Size	$ 686	$ 428
	Bottle Size	$ 182	$ 181
Inventory Costs	Plant	$ 58	$ 61
	3BA	$ 47	$ 3
	Transfers	$ 29	$ 2
Total		$ 1,000	$ 675

The optimization model shows a 33% reduction in costs as compared to the actual production data. The majority of these savings come from a reduction in setups. This is a remarkable outcome because the model was able to reduce setups without significantly increasing inventory holding costs. However, these results include the value of perfect foresight which the company did not have at the time it made production decisions. In addition, the actual amount produced exceeded demand by 17% which suggests that some production satisfied demand outside the region. Another factor that contributes to production in excess of demand is that some production is planned to maintain safety stock which is not included in the model. However, the results still show several cost saving opportunities for Niagara to pursue.

The modeled versus actual inventory positions are shown in Figure 9. The data is normalized with respect to the total plant inventory capacity. Only 5% of the plant inventory capacity is initially used based on the actual inventory data. The optimization model immediately moves the 3PL inventory into the plants to minimize holding costs. The actual data suggests that 3PL inventory was used to satisfy demand, but was not moved into the plant. In reality some cost would be associated with these transfers to the plant that is not captured in the model. These results show that Niagara can explore improving plant warehouse utilization as a cost saving strategy.
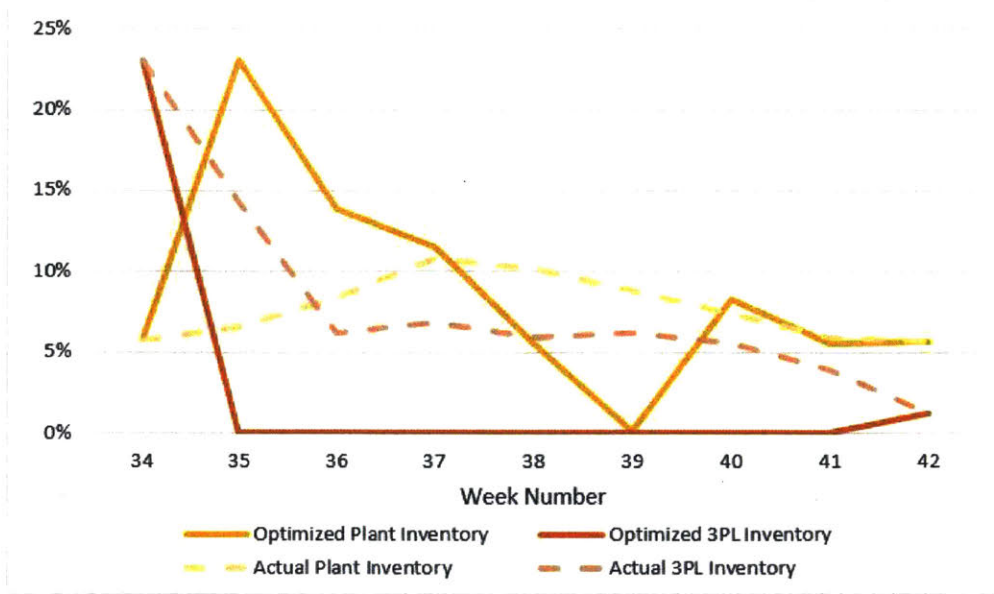
Figure 9: Inventory Positions for Plant and 3PL warehouses comparing actual data to optimized model

The calculation of transfer cost when comparing against actual data requires some further explanation. As described in section 3.2.2, the model input for transfer cost is based on the average transfer size observed in the actual inventory data. In this case, the actual inventory records showed an average transfer event involved two truckloads. The model outputs show that the average transfer event requires six truckloads. Therefore, the four transfer events for the optimized results are updated to use the transfer cost associated with six truckloads, while the actual results use the two truckload transfer event cost.

The results also highlight a difference in the number of transfer events from the plant to the 3PL warehouse. The optimized model minimizes transfer costs to the 3PL by never increasing 3PL inventory except to meet ending inventory requirements. This results in only four transfer events in the optimized model as compared to 47 transfer events in the actual inventory data. This suggests that Niagara can explore reducing unnecessary transfer events to the 3PL warehouse as a key cost saving strategy. These results show that the model can develop a production plan that reduces transfer events and setups without impacting inventory storage costs.

Given these valuable insights, other benchmarking methods are explored to corroborate the results.

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4.2 Single Plant Production Data Benchmarks

Due to the overproduction issue in the representative region, a better data set is selected for benchmarking cost performance of the model against actual production data. In this case, only a single plant that produces a single bottle size with four package sizes is selected. The time horizon is eight weeks during which demand is near capacity.

However, the actual production data still showed some instances where demand exceeded production which resulted in infeasible negative inventory positions. Thus, for the purposes of benchmarking, the production data is altered to ensure positive inventory throughout the selected time horizon. The initial and ending inventory from actual production data is used for the model inputs.

Similar to the representative region data, some periods see actual capacity exceed the nominal values. For the sake of accurate benchmarking, the setup times are set to zero and capacities are increased to match production data when in excess of nominal capacity.

Actual production data has some limitations for benchmarking because the model is planning with hindsight whereas the production plan was implemented with more limited data about the future demand. In addition, operational production planning is done on an item level whereas this benchmark is performed on a bottle-package category level. The result is an overestimation of cost savings because the model can make more optimal decisions when ignoring demand at an item level. Even so, when the model plans production on an item level it still shows a 16% savings as shown in Table 3.

Table 3: Cost Differences at a bottle-package level when solving at an Item level

		Actual		Category Optimized		SKU Optimized	
		Cost	% of Total	Cost	Cost Dif.	Cost	Cost Dif.
Setup Costs	Machine 1	$ 33	3%	$ 52	59%	$ 47	44%
	Machine 2	$ 98	10%	$ 33	-67%	$ 40	-59%
	Machine 3	$ 69	7%	$ 32	-54%	$ 47	-32%
	SubTotal	$ 200	20%	$ 104	-48%	$ 135	-33%
Inventory Costs	Plant Holding	$ 97	10%	$ 94	-4%	$ 96	-2%
	3PL Holding	$ 475	47%	$ 469	-1%	$ 463	-3%
	Transfer Costs	$ 227	23%	$ 76	-67%	$ 151	-33%
	SubTotal	$ 800	80%	$ 638	-20%	$ 710	-11%
Total		$ 1,000	100%	$ 742	-26%	$ 845	-16%

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Table 3 compares the total cost of the item level optimized model, the bottle-package category level, and actual production costs. In order to make the comparison, the item level setup costs were removed from the item level model and actual production cost analysis. These results show that the model can help Niagara achieve cost savings regardless if optimized on the category or item levels when compared to historical data.

An additional method is used to overcome these limitations and improve the cost performance benchmarking: benchmarking against the production planning process currently in use at Niagara.

4.3   Manual Production Planning Benchmark

In this benchmarking method, the same inputs for the optimization model are also given to a Niagara planner to manually create a production plan using current best practices.

In addition, a simple manual heuristic is used to manually plan the production data based on the EOQ formula developed by Harris (Harris, 1913), which was also used as a benchmark. The EOQ method uses either the average demand or the calculated EOQ for each item over the time horizon. If the order frequency of the EOQ is greater than the time period of the model, one week in this case, then the average demand is ordered every period. Otherwise, the EOQ is planned for the item using the specified order frequency. These values are then adjusted to satisfy demand and capacity constraints.

The results show a 41% improvement of the optimized model over the current planning practice at Niagara. A breakdown of the cost differences between these benchmarks are shown in Table 4. Figure 10 and Figure 11 show that the majority of the cost is at the 3PL warehouse because of the inventory build requirements over the planning horizon and a low level of slack capacity during the planning horizon. These results show little difference between the total amount produced and inventory placement between benchmarks.

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Table 4: Benchmarking Cost Comparisons

|		Optimized		Actual		EOQ Rule		Manual Planned	
		Cost	% of Total	Cost	Cost Dif.	Cost	Cost Dif.	Cost	Cost Dif.
Setup Costs	Machine 1	$ 52	5%	$ 44	-15%	$ 44	-15%	$ 44	-15%
	Machine 2	$ 44	4%	$ 133	200%	$ 49	11%	$ 93	111%
	Machine 3	$ 43	4%	$ 93	115%	$ 113	161%	$ 44	2%
	SubTotal	$ 140	14%	$ 270	94%	$ 206	48%	$ 182	30%
Inventory Costs	Plant Holding	$ 126	13%	$ 131	4%	$ 123	-3%	$ 127	1%
	3PL Holding	$ 632	63%	$ 640	1%	$ 680	8%	$ 724	15%
	Transfer Costs	$ 102	10%	$ 306	200%	$ 136	33%	$ 374	267%
	SubTotal	$ 860	86%	$ 1,078	25%	$ 939	9%	$ 1,226	42%
Total		$ 1,000	100%	$ 1,348	35%	$ 1,146	15%	$ 1,407	41%
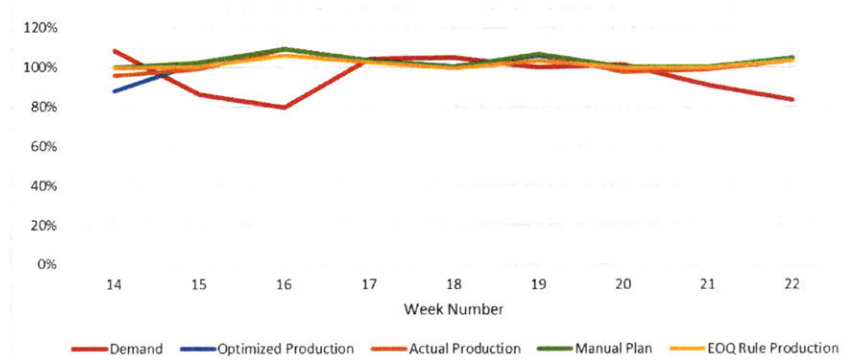
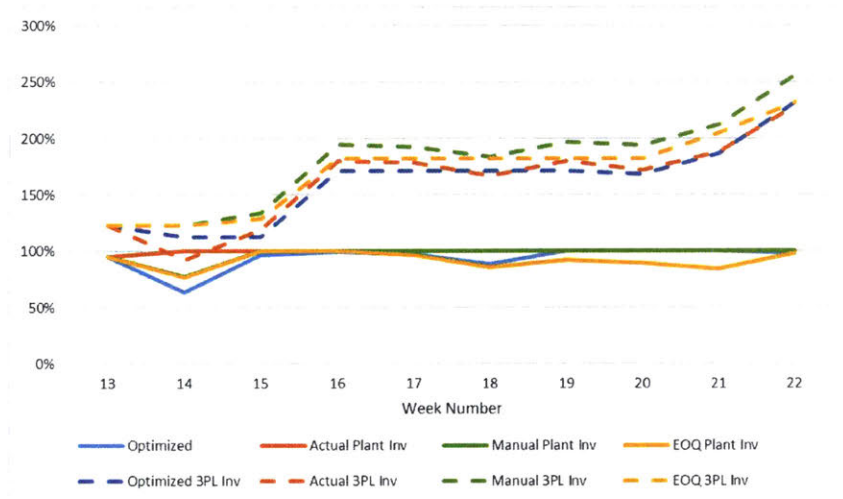Figure 10: Benchmarking Production Comparisons

Figure 11: Benchmarking Inventory Comparisons

The largest difference between the optimized models and the benchmarks is the minimization of transfer costs and the more distributed use of all machines at the plant. The typical planning

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process does not explicitly take into account transfer costs to the 3PL warehouse, which adds a dimension to the planning problem that is hard to optimize manually.

The difference in setup costs is more striking and less intuitive. The model is able to produce the same amount of product while building less inventory to satisfy demand using 30% fewer setups as compared to the manual method. The manual method assigns one or two fast moving products to one or two of the lines and then uses the third line for flexibility to produce slower moving items and demand variations. In contrast, the model coordinates the flexibility of all three lines producing all items on all lines to reduce overall setups.

4.4 Sensitivity Analysis

As previously discussed, the intention of the model is not to accurately portray supply chain costs, but to inform on optimal behavior. The relationship between holding and setup costs is key to differentiating optimal behavior. High relative setup costs encourages building items to stock. The production lot sizes are typically in excess of demand and held in inventory until the balance of demand materializes. High relative holding costs encourages building to order. Since producing every item every period as demand materializes increase the number of setups, the relative holding cost needs to be higher to justify building to order.

To test the sensitivity of the model to the relationship between these costs, the representative region data is more appropriate than the plant benchmarking data. This is because demand is below capacity and there are no inventory build requirements in the representative region scenario. As a result, the choice to build inventory is solely a factor of setup cost avoidance. The sensitivity analysis focuses on a fast moving product that accounts for 50% of the total supply chain costs for the representative region scenario.

The sensitivity scenarios varies holding cost and setup costs by a factor of two and four. Table 5 compares the total inventory held and number of setups for each scenario. The results show the expected change in behavior: decreasing setups as setup costs increase; increasing setups as holding cost decreases.

	Base	2xSetup	4xSetup	2xHolding	4xHolding	Actual
Setups (Qty)	68	65	64	71	72	107
Plant Inventory (M bottles)	20.12	39.32	55.06	10.47	8.61	39.92
3PL Inventory (M bottles)	1.41	1.41	1.41	1.41	1.41	9.45

Table 5: Sensitivity impact on setups and inventory

However, even at four times the original setups costs, the model still does not produce at max capacity and the decrease in setups is only 6%. Figure 12 shows the change in production per period as setup and holding costs vary[18]. The variation in setups is shown in Figure 13.
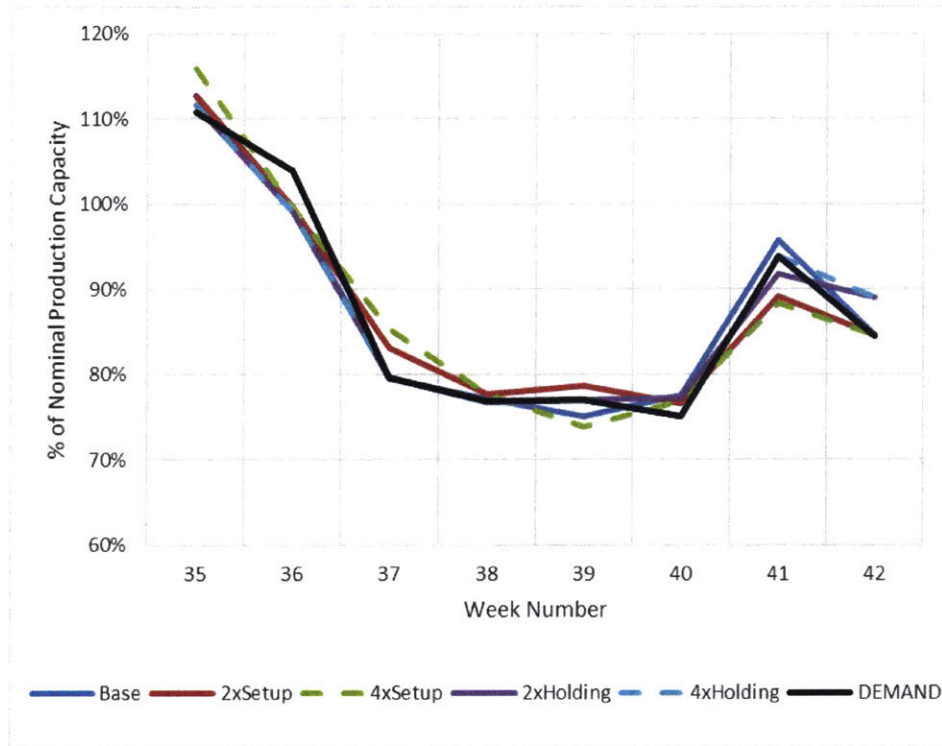

Figure 12: Sensitivity Scenarios Production Comparisons

[18] As previously noted, the initial period has a capacity in excess of nominal to match actual production data.
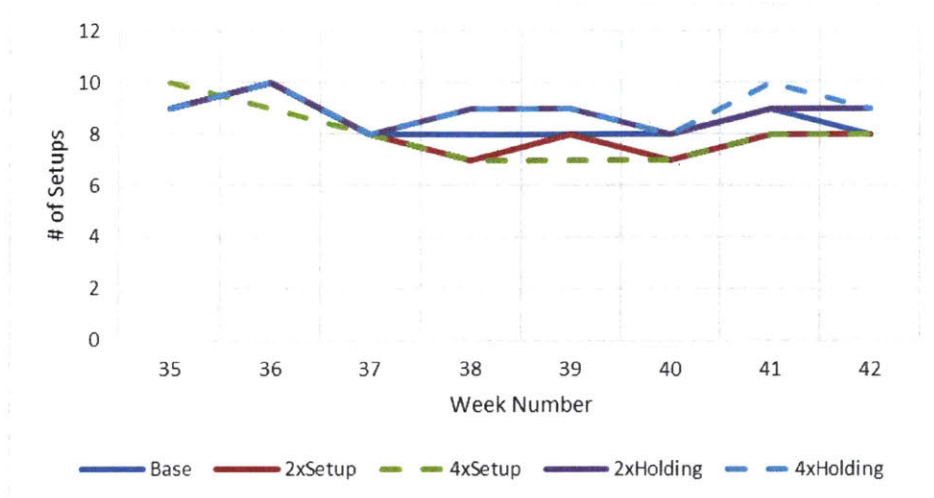
Figure 13: Sensitivity Scenarios Comparison of number of setups per period

The amount of inventory held in the plant increased from an average of 5% to 14% of plant capacity when setup cost was increased by a multiple of four. Inventory in Table 4 refers to the total bottles held over all time periods in the time horizon[19]. Figure 14 illustrates the change in plant inventory across the scenarios.
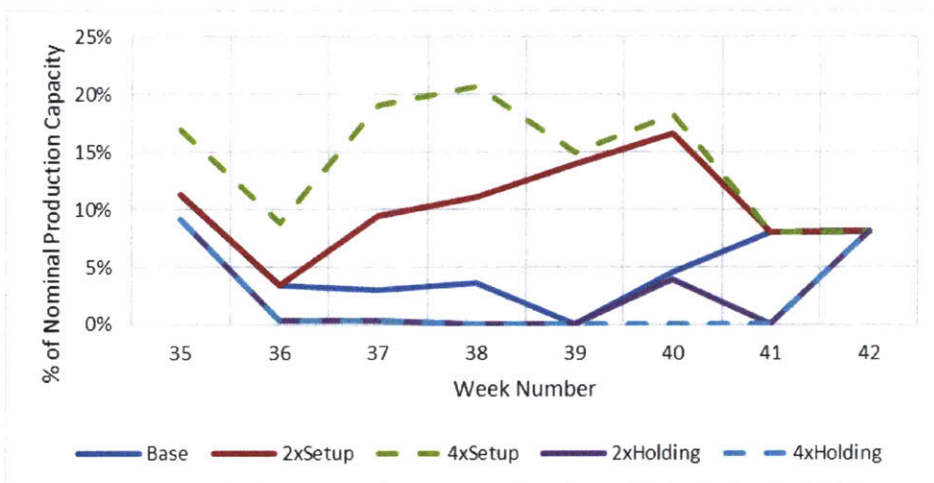

Figure 14: Sensitivity Scenarios Inventory Comparisons per period

In addition to affecting the number of setups in a period, the change in relative supply chain cost changes the average production lot size. This can be seen when looking at the increase in

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[19]It does not refer to the cumulative amount of inventory held at any particular time.

production for the four times setup cost scenario in week 37, but no corresponding change in setups is seen in Figure 13. This means that each setup produced a larger batch size which led to the increase in inventory noted in Figure 14. In fact, the average lot size as a percentage of capacity increased from 91% in the base case to 97% in the four times setup cost scenario. These results indicate the model is relatively insensitive to the exact relationship between holding cost and setup costs for Niagara. The 6% increase in lot size for a 400% variation in the relationship can still unlock cost savings for Niagara as it outperforms the actual 2015 production plan that was benchmarked.

The sensitivity analysis results show that the cost savings are insensitive to changes in setup and holding costs. Even if setup or holding cost inputs are misrepresented by a factor of four, the model results are an improvement with respect to actual production data for the representative region.

5 Conclusion

5.1 Summary of Findings

Supply chain cost reduction has a large impact on gross margin for CPG companies that typically have low margins (Investopedia, 2015). A typical CPG company with 5% gross margin can expect to double its profits with only a 5% reduction in supply chain costs (OByrne, 2016). Thus this thesis presents a new model that can be used to improve current planning processes of a CPG company to lower supply chain costs and drive higher profits. The new model addresses unique features, not currently captured in the literature, of a CPG company that leases warehouse space from a third party. The model was developed specifically to meet the needs of Niagara Bottling, but the features addressed are common to many CPG companies.

The results of this thesis not only established that the proposed formulation (P) is suitable for Niagara's business structure, but also provided compelling evidence of why the model should be implemented. The literature review and methodology section established that the formulation captures the following key supply chain cost features of Niagara's operations:

- Changing demand due to seasonality.

- Multiple non-identical machines that can produce multiple items.

- Productive capacity constraints that account for lost production during machine setups.

- Complex setup cost structure due to double-echelon setup hierarchy.

- Double-echelon inventory warehouse cost structure for a company owned warehouse and a 3PL warehouse.

- Inventory build targets for plant and 3PL warehouses

- Transfer cost for moving items into the 3PL warehouse.

The results section provided evidence that the formulation is capable of finding near optimal solutions for typical problem sizes required for Niagara's operations. In addition, it showed that when benchmarked against the existing manual planning process, the model solutions had a 23%

reduction in the number of setups, an 11% reduction in inventory, and a 73% reduction in the number of transfers between the plant and 3PL warehouses. When using the cost parameters discussed in the methodology section, this resulted in a 41% reduction in total inventory holding and setup costs over the planning period.

These results are interesting in that they suggest improvements can be made in every area by using the model for planning. The manual planning benchmark showed that the model is able to better leverage flexibility to meet demand fluctuations to lower total number of setups. The reduction in inventory is remarkable considering the larger lots sizes and fewer setups and underscores the potential of using the model for planning. The ability of the model to reduce transfer events so dramatically is not surprising since this cost is not directly captured in the existing manual planning process.

The sensitivity analysis demonstrated the robustness of the model to changes in relative costs. The analysis showed that a 300% increase in holding costs relative to setup costs only yielded a 6% increase in setups. On the other hand, a 300% increase in setup costs only increased the average plant inventory level from 5% to 14% of capacity. When compared to the actual production data for the representative region, the model shows improved performance even if relative costs were misrepresented by a factor of four in either direction. This provides strong evidence that the model can reduce costs even if cost structure is not known with great accuracy.

In addition to lowering costs using assets as-is, a company can also use the model to the test cost effectiveness of changes to assets. This can be achieved by conducting further sensitivity analysis on costs if the company invested in improvements to machine efficiencies, holding costs, plant storage capacities, and setup times.

5.2 Further Research Opportunities

The models presented in this thesis provide valuable insight for Niagara's supply chain. However, further research in several areas can extend the utility of the model.

This thesis model assumes deterministic demand based on production forecasts. However, forecasts are inherently uncertain, thus one area of further research is incorporating stochasticity

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such that the model accounts for uncertainty.

Improvements to the implementation of the transfer cost can improve usability of the model. Accurate transfer costs requires iterative runs of the model to approximately match the assumed transfer size for the cost input with that of the actual solution transfer size. An implementation of transfer cost that uses the actual number of bottles in each event to derive the cost would simplify this process and improve accuracy.

Other model extensions to consider are inclusion of backorders and safety stock levels. Backorders allow for periods in which there is not enough capacity to meet demand and strategic inventory build was not sufficient. This allows the model to solve and identify the most cost effective items to place on backorder. Safety stocks are held to meet demand uncertainty. Adding this feature improves the ability of the model to capture a common manufacturing feature more accurately.

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7 Appendix: Python Code

```python
from gurobipy import *
import os
import numbers
import pandas as pd
import numpy as np
os.chdir('C:\ ')
filename = 'Input.xlsx'
Demand=pd.read_excel(filename, sheetname='Demand')
Items = Demand.Level1
Cat = Demand.Level2.unique()
NumCat = len(Cat)
Category = pd.pivot_table(Demand, values='Level1', index=None, columns=['Level2'], aggfunc =
np.count_nonzero)
Level = [[0 for i in range(2)] for j in range(NumCat)]
for k in range(NumCat):
    if k==0:
        Level[k][0] = 0
        Level[k][1] = Category[k]
    else:
        Level[k][0] = Level[k-1][1]
        Level[k][1] = Level[k-1][1]+Category[k]
del Demand['Level1']
del Demand['Level2']
D = Demand.values.tolist()
Capacity=pd.read_excel(filename, sheetname='Capacity')
Machines=list(Capacity.index.values)
R1 = Capacity.values.tolist()
SetupCost1=pd.read_excel(filename, sheetname='Setup Cost S1')
S1 = SetupCost1.values.tolist()
SetupCost2=pd.read_excel(filename, sheetname='Setup Cost S2')
S2 = SetupCost2.values.tolist()
SetupTime1=pd.read_excel(filename, sheetname='Setup Time K1')
K1 = SetupTime1.values.tolist()
SetupTime2=pd.read_excel(filename, sheetname='Setup Time K2')
K2 = SetupTime2.values.tolist()
HoldingCost1=pd.read_excel(filename, sheetname='h1')
h1 = HoldingCost1['h1'].values.tolist()
HoldingCost2=pd.read_excel(filename, sheetname='h2')
h2 = HoldingCost2['h2'].values.tolist()
Capability=pd.read_excel(filename, sheetname='Capability')
c = Capability.values.tolist()
StartingI=pd.read_excel(filename, sheetname='Starting I')
Is = StartingI['Plant'].values.tolist()
StartingW=pd.read_excel(filename, sheetname='Starting W')
Ws = StartingW['3PL'].values.tolist()
EndingI=pd.read_excel(filename, sheetname='Ending I')
Ie = EndingI['Plant'].values.tolist()
EndingW=pd.read_excel(filename, sheetname='Ending W')
```

```python
We = EndingW['3PL'].values.tolist()
PlantCost=pd.read_excel(filename,sheetname = 'Plant Storage and Transfer')
PC = PlantCost.values.tolist()
R2 = PC[0][0]
h3 = PC[1][0]


T = len(D[0])
n = len(D)
J = len(R1)
M = sum(R1[0][:])

#Quality Check Inputs
error=0
#Check consistent number of Level 1 items across variables
if (n != len(h1)):
    error=1
    print 'Error: "Demand" and "h1" have different number of level 1 items'
if (n != len(h2)):
    error=1
    print 'Error: "Demand" and "h2" have different number of level 1 items'
if (n != len(Is)):
    error=1
    print 'Error: "Demand" and "Starting I" have different number of level 1 items'
if (n != len(Ws)):
    error=1
    print 'Error: "Demand" and "Starting W" have different number of level 1 items'
if (n != len(Ie)):
    error=1
    print 'Error: "Demand" and "Ending I" have different number of level 1 items'
if (n != len(We)):
    error=1
    print 'Error: "Demand" and "Ending W" have different number of level 1 items'
if (n != len(K1)):
    error=1
    print 'Error: "Demand" and "Setup Time K1" have different number of level 1 items'
if (n != len(S1)):
    error=1
    print 'Error: "Demand" and "Setup Cost S1" have different number of level 1 items'
if (n != len(c)):
    error=1
    print 'Error: "Demand" and "Capability" have different number of level 1 items'

#Check consistent number of Level 2 items across variables
if (NumCat != len(K2)):
    error=1
    print 'Error: "Demand" and "K2" have different number of level 2 items'
if (NumCat != len(S2)):
```

```
        error=1
        print 'Error: "Demand" and "S2" have different number of level 2 items'

#Check consistent number of machines across variables
if (J != len(K1[0])):
    error=1
    print 'Error: "Capacity" and "K1" have different number of machines'
if (J != len(K2[0])):
    error=1
    print 'Error: "Capacity" and "K2" have different number of machines'
if (J != len(S1[0])):
    error=1
    print 'Error: "Capacity" and "S1" have different number of machines'
if (J != len(S2[0])):
    error=1
    print 'Error: "Capacity" and "S2" have different number of machines'
if (J != len(c[0])):
    error=1
    print 'Error: "Capacity" and "Capability" have different number of machines'

#Check consistent number of time periods across variables
if (len(D[0]) != len(R1[0])):
    error=1
    print 'Error: Demand and Capacity have different number of time periods'

#Check for missing data entries
for i in range(n):
    if h1[i] != h1[i] or h2[i] != h2[i] or Is[i] != Is[i] or Ws[i] != Ws[i] or Ie[i] != Ie[i] or We[i] != We[i]:
        error=1
        print 'Error: Data entry missing in h1, h2, Is, Ie, or We for item', Items[i]
    if Is[i] != round(Is[i],0) or Ws[i] != round(Ws[i],0) or Ie[i] != round(Ie[i],0) or We[i] != round(We[i],0):
        error=1
        print 'Error: Is, Ie, Ws, or We data entry is not an integer for item ', Items[i]
if R2 != R2 or h3 !=h3:
    error=1
    print 'Error: Data entry missing in "Plant Storage and Transfer"'
for i in range(n):
    for j in range(J):
        if K1[i][j] != K1[i][j] or S1[i][j] != S1[i][j] or c[i][j] != c[i][j]:
            error=1
            print 'Error: Data entry missing in K1, S1, or "Capability" for item', Items[i], ' on machine',
Machines[j]
for l in range(NumCat):
    for j in range(J):
        if K2[l][j] != K2[l][j] or S2[l][j] != S2[l][j]:
            error=1
            print 'Error: Data entry missing in K2 or S2 for category', Cat[l], 'on machine', Machines[j]
for t in range(T):
```

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```
    for i in range(n):
        if D[i][t] != D[i][t]:
            error=1
            print 'Error: Data entry missing in "Demand" for item ', Items[i], ' in time period ', list(Demand)[t]
        if round(D[i][t],0) != D[i][t]:
            error=1
            print 'Error: "Demand" data entry is not an integer for item ', Items[i], ' in time period ',
list(Demand)[t]
for t in range(T):
    for j in range(J):
        if R1[j][t] != R1[j][t]:
            error=1
            print 'Error: Data entry missing in "Capacity"'

#Check Inventory and Capacity constraints are satisfied
if sum(Is) > R2 or sum(Ie) > R2:
    error=1
    print 'Error: Is or Ie exceeds plant inventory capacity'
for i in range(n):
    if (Is[i]+Ws[i]-sum(D[i])) > Ie[i]+We[i]:
        error=1
        print 'Error: Starting Inventory minus Demand exceeds Ending Inventory for item', Items[i]
Inv=sum(Is)+sum(Ws)
for t in range(T):
    Cap=0
    Dem=0
    x=0
    for j in range (J):
        Cap += R1[j][t]
    for i in range(n):
        Dem += D[i][t]
    Inv += Cap - Dem
    if t == T-1:
        Inv = Inv - (sum(Ie)+sum(We))
    if Inv < 0:
        x = t+1
        error=1
        break
if x > 0: print 'Error: Inventory and Capacity are not sufficient to meet demand starting in time period', x

#Indicate Successful Check
if error==0:
    print 'No errors found'
```

```
#Create Model
model = Model('CLSP-multimachine-MultiLevel')

#create variables
Z,Y,X,I,W,H = {},{},{},{},{},{}
for t in range(T):
    for k in range(NumCat):
        for j in range(J):
            Z[k,j,t] = model.addVar(vtype=GRB.BINARY,name="Z[%s, %s, %s]"%(k,j,t))
for t in range(T):
    for i in range(n):
        I[i,t] = model.addVar(vtype=GRB.INTEGER,name="I[%s, %s]"%(i,t))
        W[i,t] = model.addVar(vtype=GRB.INTEGER,name="W[%s, %s]"%(i,t))
        H[i,t] = model.addVar(vtype=GRB.BINARY,name="H[%s, %s]"%(i,t))
        for j in range(J):
            Y[i,j,t] = model.addVar(vtype=GRB.BINARY,name="Y[%s, %s, %s]"%(i,j,t))
            X[i,j,t] = model.addVar(vtype=GRB.INTEGER,name="X[%s, %s, %s]"%(i,j,t))

#integrate new variables
model.update()

#set objective
obj1 = LinExpr()
obj2 = LinExpr()
obj = LinExpr()
for t in range(T):
    for i in range(n):
        coef4 = [S1[i][j] for j in range(J)]
        var4 = [Y[i,j,t] for j in range(J)]
        obj1 += h1[i]*I[i,t] + h2[i]*W[i,t] + h3*H[i,t] + LinExpr(coef4, var4)
    for l in range(NumCat):
        coef4 = [S2[l][j] for j in range(J)]
        var4 = [Z[l,j,t] for j in range(J)]
        obj2 += LinExpr(coef4, var4)
obj = obj1 + obj2
model.setObjective(obj,GRB.MINIMIZE)

#add constraints for ending inventory positions
for i in range(n):
    model.addConstr(I[i,T-1]==Ie[i], name="Inventory_End")
    model.addConstr(W[i,T-1]==We[i], name="3PLInventory_End")

#add constraint for Plant inventory capacity
for t in range(T):
    coef1 = [1 for i in range(n)]
    var1 = [I[i,t] for i in range(n)]
    model.addConstr(LinExpr(coef1,var1)<=R2,name="CapacityR2")
```

```python
#add constraints for relationship between production, inventory, and demand as well as transfer event
    for i in range(n):
        coef1 = [1 for j in range(J)]
        var1 = [X[i,j,t] for j in range(J)]
        if t > 0:
            model.addConstr(I[i,t]+W[i,t]==LinExpr(coef1,var1)-D[i][t]+I[i,t-1]+W[i,t-1],
name="Inventory[%s,%s]"%(i,t))
        else:
            model.addConstr(I[i,t]+W[i,t]==LinExpr(coef1,var1)-
D[i][t]+Is[i]+Ws[i],name="Inventory[%s,%s]"%(i,t))
#add constraint that Inventory is always positive
        model.addConstr(I[i,t]>=0,name="Plant_Inventory_Bound")
        model.addConstr(W[i,t]>=0,name="WH_Inventory_Bound")
        if t > 0:
            model.addConstr((W[i,t]-W[i,t-1]) <= M*H[i,t])
        else:
            model.addConstr((W[i,t]-Ws[i]) <= M*H[i,t])


#add constraints for Production Capacity, and Setup event for category level.
for t in range(T):
    for j in range(J):
        coef1 = [1 for k in range(n)]
        var1 = [X[k,j,t] for k in range(n)]
        coef2 = [K1[k][j] for k in range(n)]
        var2 = [Y[k,j,t] for k in range(n)]
        coef3 = [K2[l][j] for l in range(NumCat)]
        var3 = [Z[l,j,t] for l in range(NumCat)]
        model.addConstr(LinExpr(coef1,var1) + LinExpr(coef2,var2) + LinExpr(coef3,var3) <= R1[j][t], name =
"Capacity[%s,%s]"%(j,t))
        for a in range(NumCat):
            coef1 = [1 for i in range(Level[a][0],Level[a][1])]
            var1 = [Y[i,j,t] for i in range(Level[a][0],Level[a][1])]
            model.addConstr(LinExpr(coef1,var1)<=R1[j][t]*Z[a,j,t],name="Category_Event{%s,%s,%s]"%(l,j,t))
for t in range(T):
    for j in range(J):
        for i in range(n):
            model.addConstr(X[i,j,t]<=M*Y[i,j,t],name="Setup")
            model.addConstr(X[i,j,t]<=M*c[i][j],name="Capability")
            if c[i][j] == 0:
                model.addConstr(Y[i,j,t]==0)
            model.addConstr(X[i,j,t]>=0, name="Production_Bound")

model.setParam("TimeLimit", 3600.0)
model.optimize()
```