Cellular Power Control in a Fading Environment

by

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Abstract

We develop power control algorithms that maintain reliable communication links for mobile users in cellular radio systems. The algorithms are designed to achieve good performance in the presence of both cochannel interference and signal fading. We develop a narrowband fading model that is derived from a standard characterization of fading multipath channels. The development and analysis of power control algorithms for various cellular network scenarios are based on this fading model. We study simple scenarios, such as the single cell network and the two cell network, in order to examine the effects of both intracell and intercell interference. An extension to a general multiple cell network is presented.

Power control algorithms for the single cell network are based on minimum mean squared error (MMSE) estimation. The performance of the estimation algorithms depends on the fading rate. In contrast, the multiple cell network requires the use of an iterative power control algorithm in order to alleviate complexity. The performance of the iterative algorithm depends on the rate of change of the channel, the rate of convergence of the algorithm, and the amplitude range over which the channel varies.

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Chapter 1

Introduction

Power control has increasingly become an important issue in cellular radio communication systems. In order to accommodate as many users as possible, cellular systems are designed so that a number of users share the same frequency spectrum. As a result, users cause interference to other users. The objective of power control is to minimize this cochannel interference. This is achieved by continually adjusting the transmit power of each user to the lowest level that still permits reliable communication. Therefore, maximizing system capacity, measured in terms of the number of users a system can support, depends critically on efficient power control.

The most common approach to power control is to define the quality of the transmission links by the carrier-to-interference ratio (CIR). Reliable communication is possible only when the CIR of a particular user is above some specified threshold. Consequently, the goal of power control is to find the minimum transmit power of a particular user such that its CIR constraint is satisfied. This is exactly the focus of several works found in the literature. The difference between these works is the method by which mobile users are assigned to base stations. Nettleton and Alavi [2] and Zander [6] consider the case where each user is assigned to the closest base station. Hanly [1] considers the case where each user is assigned to multiple base stations to achieve diversity gains. All of these approaches basically reduce the problem to an eigenvalue problem. Yates [5] combines these different base station assignment schemes and fits them into a general framework. Another assignment scheme that is included in this framework is minimum power assignment, where each user is assigned to the base station at which its CIR is maximized.
The motivation for this research is the fact that past analyses on power control neglect the fading aspects of cellular radio channels, and focus only on interference caused by other users. It is well known that cellular radio channels are also subject to time-varying multiple paths, which account for signal fading. Standard power control algorithms considered in past research may actually be detrimental when fading is present. In this work, we develop power control algorithms that achieve good performance in a fading environment.

The organization of the thesis is as follows. In Chapter 2, we provide an overview of cellular communication systems and a characterization of cellular radio channels. In addition, we derive a suitable fading model to be used for the development of power control algorithms and we explain power control implementation issues. As stated before, the objective of the power control algorithms we consider is to regulate each user's transmit power to the minimum power level such that each user achieves a minimum common carrier-to-interference ratio (CIR). The next two chapters will be concerned with developing PCAs for various cellular network scenarios. The flow will be to start with a simple scenario and gradually add complexity. Insights gained from the simpler scenario will be beneficial in handling the more complex scenario. In Chapter 3, we focus on intracell interference by considering a single cell network. In Chapter 4, we introduce intercell interference by considering a two cell network. Key results obtained from the analysis of the single cell network can be extended to the two cell network. However, additional issues are introduced in the analysis of the two cell network. In Appendix A, we present an extension of the results obtained from the analysis of the two cell network to a general multiple cell network.

Throughout the PCA development, we only consider the base station assignment scheme in which each user is assigned to the closest base station. When analyzing the performance of the PCA in the presence of fading, the power control update rate will stay fixed while the fading rate varies. In addition, the focus will be exclusively on the uplink direction, transmission from the mobile user to the base station.
Chapter 2

Background

The purpose of the first two sections of this chapter is to give an introduction to some fundamental issues involved in cellular communication networks. Standard network configurations are explained in Section 2.1. The characterization of cellular radio channels is described in Section 2.2. The purpose of the last two sections is to provide the necessary mechanism for developing and analyzing the performance of power control algorithms in a fading environment. A suitable fading model is derived in Section 2.3. The implementation issues associated with the power control algorithms we develop are explained in Section 2.4.

2.1 Cellular Network Configurations

2.1.1 Network Structure

The standard approach to providing communication services to mobile users located within a certain bounded geographical area is to divide the area into a number of cells. The hexagonal cell pattern shown in Figure 2-1 is the most common approximation to the division of the area. It is typical to think of this cluster of seven cells as being replicated several times and overlaying the entire area. Within each cell is a centrally located base station, which is the interface between a mobile user and another mobile user or a user connected to the Public Switched Telephone Network (PSTN). The base stations are all connected to a central Mobile Switching Telephone Office (MSTO), which in turn is connected to the PSTN. Each base station is responsible for providing communication links for all the users located within its cell boundary.
2.1.2 Multiple Access Techniques

The channel from the mobile user to the base station, which is referred to as the uplink or reverse link, is a multiple access channel since there are several transmitters communicating with a single receiver. In contrast, the channel from the base station to the mobile user, which is referred to as the downlink or forward link, is a broadcast channel since there is one transmitter communicating with several receivers. Here we will focus on the uplink and describe standard multiple access techniques for providing communication links to the mobile users. The communication resource for a cellular network is a particular band of the frequency spectrum. We will denote by $W$ the width of this allocated spectrum. Each multiple access technique differs in how this allocated spectrum is used.

The traditional multiple access technique is to isolate users by providing each with disjoint communication links. This is the approach used by Frequency Division Multiple Access (FDMA) and Time Division Multiple Access (TDMA) systems. For FDMA systems, the spectrum $W$ is divided into $N$ disjoint frequency slots $W_1, W_2, \ldots, W_N$ of width $W/N$. A user is allowed to transmit only within its assigned frequency slot $W_n$, $1 \leq n \leq N$. Similarly, for TDMA systems, a periodic time frame of length $T$ is divided into $N$ disjoint time slots $T_1, T_2, \ldots, T_N$ of width $T/N$. A user is allowed to transmit only within its assigned time slot $T_n$, $1 \leq n \leq N$, using the whole spectrum. A graphical explanation of FDMA and TDMA is shown in Figure 2-2 and Figure 2-3, respectively.

Another multiple access technique is to allow users to share the whole communication resource. This is the approach used by Code Division Multiple Access (CDMA) systems. Users are allowed to transmit using the whole spectrum at all times. The bandwidth of each user’s information signal is spread by multiplying the signal by a high bandwidth
pseudonoise sequence, which is referred to as a signature code and is unique to each user. In order to decode a particular user, the received signal is multiplied by that user's signature code. The user's signal will be despread while the signals from the other users will not be despread and will only contribute to the noise.

Cellular networks which employ CDMA will be referred to as spread networks since the transmission bandwidth is much greater than the bandwidth of the information signal. In other words, the transmission bandwidth is much greater than the data rate of the user. Cellular networks which employ FDMA or TDMA will be referred to as unspread networks. The criterion we are using to classify the different multiple access techniques is the number of degrees of freedom of the transmitted signal. The number of degrees of freedom of a signal is defined as the number of samples required to completely specify it. The number of degrees of freedom of a signal frequency-limited to \( W \) Hertz and time-limited to \( T \) seconds is \( 2WT \). By comparison, CDMA signals have \( 2WT \) degrees of freedom, whereas FDMA and TDMA signals have \( 2WT/N \) degrees of freedom.

### 2.1.3 Interference

Limiting interference among users is an extremely important consideration in designing cellular networks. For cellular unspread systems, the base station assigns disjoint communication links to each user. If adjacent cells are allowed to use the same communication
Figure 2-3: TDMA System with $N = 5$

links, a severe amount of interference may result. This could lead to unreliable communication links with a high error rate. The most common method to combat this is to configure the system so that no adjacent cells use the same communication links. However, cells that are not adjacent to each other may still use the same communication links. A cellular FDMA system configured in this way is shown in Figure 2-4. It shows two clusters of cells, where each cluster uses the entire allocated spectrum $W$. The allocated spectrum is divided into seven disjoint frequency bands $W^1, W^2, \ldots, W^7$, and each cell within a cluster is assigned one of these frequency bands. Within each cell, the frequency band $W'$ is further subdivided into disjoint frequency slots to be assigned to users located in that cell. A cellular TDMA system works in a similar fashion. This technique is generally referred to as reuse. The basic idea is that if users using the same link are separated far enough apart, the attenuation of their signals due to free space power loss will result in negligible interference.

For cellular spread systems, the base station assigns the entire spectrum to each user. There is no need to divide the allocated spectrum $W$. This type of system is shown in Figure 2-5. We see that interference originates both from within the cell and outside the cell. Instead of the statistics of a few users, the statistics of a large number of users are important, and the “law of large numbers” holds. Therefore, the total amount of interference
experienced by any user is just the average of all the users’ received power multiplied by
the number of users. Acceptable signal quality is maintained as long as the ratio of the
received power to the interference power is above some threshold. In contrast, for cellular
unspread systems, the “law of small numbers” holds. As a result, situations of appreciable
interference have a much more damaging effect on the signal quality [4].

We see that interference among users occurs in both unspread and spread systems. In
either case, power control is essential in further limiting this cochannel interference in order
to achieve more reliable communication.

2.2 Cellular Radio Channels

In a typical cellular environment, the transmitted signal from a mobile user may be re-
ceived at the base station via multiple paths. This is due to reflections caused by buildings,
cars, etc. located in the vicinity of the mobile user and the base station. Associated with
each path is a corresponding strength factor and propagation delay. Both of these pa-
rameters are time-varying since the surrounding environment of a particular mobile user is
constantly changing. Constructive and destructive interference of the multiple paths causes
the received signal amplitude to vary. In other words, multiple paths lead to signal fading.
Thus, channels that are typically encountered in a cellular network are described as fading
multipath channels.
2.2.1 Characterization of Fading Multipath Channels

We present a basic overview of the important parameters that characterize fading multipath channels. A more in-depth explanation can be found in [3]. The *multipath spread* is the difference in propagation delay between the shortest and longest path. It ranges from 100 nanoseconds inside buildings to 30 microseconds in rural areas. The *coherence bandwidth* is the reciprocal of the multipath spread. It has the same order of magnitude as the minimum frequency separation between two sinusoids that are affected independently by the channel. From the figures above, the coherence bandwidth ranges from a minimum of around 30 kHz outdoors to a maximum of 10 MHz indoors. Time variations of the channel result in a spectral broadening of the transmitted signal. The *Doppler spread* is the amount of spectral broadening. It is on the order of 100 Hz. The *coherence time* is the reciprocal of the Doppler spread. A slowly changing channel has a large coherence time or, equivalently, a small Doppler spread.

2.2.2 Selection of Channel Model

The selection of a channel model largely depends on the characteristics of the signal being transmitted. In particular, the choice of a channel model depends on the relation between the bandwidth of the transmitted signal and the coherence bandwidth. In one case, we have narrowband signaling, where the transmission bandwidth is less than the coherence
bandwidth. In this case, the channel is modeled as a direct gain. The received signal is approximated by the transmitted signal multiplied by a complex-valued Gaussian random process, which represents the time-varying nature of the channel. Inherent in the Gaussian approximation is the assumption that a large number of paths exist, and therefore central limit theorem arguments can be applied. In another case, we have wideband signaling, where the transmission bandwidth is greater than the coherence bandwidth. In this case, the channel is modeled as a tapped delay line filter with complex-valued time-varying tap coefficients. An explanation of the development of these channel models can be found in [3].

2.3 Fading Model

2.3.1 Gauss-Markov Characterization

In developing a suitable fading model to be used for later analysis, we will focus on the narrowband signaling model. As mentioned before, in this model, the received signal is approximated by the transmitted signal multiplied by a channel gain. The channel gain is considered to be a complex-valued Gaussian random process. Since the power control algorithms we will develop update user transmit powers at discrete times, we will further consider the channel gain to be a discrete random process. As a result, we can express the channel gain $G(n)$ in the form

$$G(n) = G_R(n) + iG_I(n), \quad (2.1)$$

where $G_R(n)$ and $G_I(n)$ are the quadrature components of the channel gain. The components are independent, identically distributed real-valued Gaussian random processes whose mean and variance at time $n$ are 0 and $\sigma_g^2$, respectively. Zero mean implies that there are no fixed signal reflectors in the medium.

In order to introduce a parameter that allows us to control the fading rate, we will model each of the quadrature components as a Gauss-Markov process. The quadrature components will then have the following form:

$$G_R(n) = \alpha G_R(n-1) + Z_R(n), \quad (2.2)$$

$$G_I(n) = \alpha G_I(n-1) + Z_I(n), \quad (2.3)$$
where $|\alpha| < 1$ and $Z_R(n)$ and $Z_I(n)$ are independent white Gaussian noise processes each with mean 0 and variance $\sigma_z^2$. Defining the autocorrelation as $R_X(m) = E[X(n)X(n-m)]$, the autocorrelation of the quadrature components is

$$R_{G_R}(m) = R_{G_I}(m) = \frac{\alpha^m \sigma_z^2}{1 - \alpha^2}.$$  \hspace{1cm} (2.4)

We now designate $\alpha$ as our fading parameter. If $\alpha$ is close to one, there is high correlation of the quadrature components at successive time intervals. This corresponds to slow fading. If $\alpha$ is close to zero, there is low correlation of the quadrature components at successive time intervals. This corresponds to fast fading.

The variance $\sigma_g^2$ of $G_R(n)$ and $G_I(n)$ can be found by setting $m$ to 0 in (2.4), obtaining

$$\sigma_g^2 = R_{G_R}(0) = R_{G_I}(0) = \frac{\sigma_z^2}{1 - \alpha^2}.$$ \hspace{1cm} (2.5)

We will assume that $\sigma_g^2$ remains fixed. Therefore, when varying the fading parameter $\alpha$, $\sigma_z^2$ must be adjusted to keep $\sigma_g^2$ constant.

Since we will be dealing only with power, we will need to determine the power gain of the channel. The power gain $H(n)$ can be expressed as

$$H(n) = |G(n)|^2 = G_R^2(n) + G_I^2(n).$$ \hspace{1cm} (2.6)

The power gain of each user in the network will behave according to this model. We will assume that $H(n)$ is a stationary process. We will further assume that the power gains of each user are independent.

### 2.3.2 Power Gain Statistical Profile

Now we will derive the probability density function of $H(n)$ and its statistical characteristics, which will be useful for later analysis. As mentioned before, at a particular update time $n$, $G_R(n)$ and $G_I(n)$ are i.i.d. Gaussian random variables with mean 0 and variance $\sigma_g^2 = \sigma_z^2/(1 - \alpha^2)$. From (2.6), we can derive the cumulative distribution function of $H(n)$ as follows:

$$F_{H(n)}(h) = \int\int_{x^2 + y^2 \leq h} f_{G_R(n)G_I(n)}(x,y) \, dx \, dy.$$ \hspace{1cm} (2.7)
\[ F_{H(n)}(h) = \frac{1}{2\pi \sigma_g^2} \int_0^{\sqrt{h}} \int_0^{2\pi} e^{-(1/2\sigma_g^2)(r^2 + y^2)} r \, dr \, d\theta \]

Using polar coordinates to evaluate (2.8), we have

\[ F_{H(n)}(h) = \frac{1}{2\pi \sigma_g^2} \int_0^{\sqrt{h}} \int_0^{2\pi} e^{-(1/2\sigma_g^2)r^2} r \, dr \, d\theta \]

\[ = \left(1 - e^{-h/2\sigma_g^2}\right) u(h), \]  

where \( u(h) \) is the unit step function. The density of \( H(n) \) is then

\[ f_{H(n)}(h) = \frac{dF_{H(n)}(h)}{dh} = \frac{1}{2\sigma_g^2} e^{-h/2\sigma_g^2} u(h). \]

Thus, \( H(n) \) is exponentially distributed at a particular update time \( n \). Letting \( \lambda = 1/2\sigma_g^2 \), we have the following expressions for the density, mean, and variance of \( H(n) \):

\[ \text{Density} : \quad f_{H(n)}(h) = \lambda e^{-\lambda h}, \quad h > 0, \]

\[ \text{Mean} : \quad E[H(n)] = \frac{1}{\lambda} = 2\sigma_g^2 = \frac{2\sigma_g^2}{1 - \alpha^2}, \]

\[ \text{Variance} : \quad \text{VAR}[H(n)] = \frac{1}{\lambda^2} = 4\sigma_g^4 = \frac{4\sigma_g^4}{(1 - \alpha^2)^2}. \]

We note that since the variance \( \sigma_g^2 \) of the quadrature components \( G_R(n) \) and \( G_I(n) \) is assumed to remain constant, the mean and variance of the power gain also remain constant.

As an aside, the density of the magnitude of the channel gain \( |G(n)| \) follows immediately from the relation \( |G(n)| = \sqrt{H(n)} \),

\[ f_{|G(n)|}(g) = \frac{g}{\sigma^2} e^{-g^2/2\sigma_g^2} u(g). \]

This is a Rayleigh density and hence narrowband fading channels are usually referred to as Rayleigh fading channels.

We will find that it is useful to have expressions for the mean and second moment of
$H(n)$ conditional on $H(n-1)$. The strategy that we will take is to first condition on both $G_R(n-1)$ and $G_I(n-1)$. We will find that this will lead us to conditioning on $H(n-1)$. We will denote by $G(n)$ the random vector whose components are $G_R(n)$ and $G_I(n)$,

$$G(n) = \begin{bmatrix} G_R(n) \\ G_I(n) \end{bmatrix}. \tag{2.16}$$

Therefore, conditioning on $G(n-1)$ is equivalent to conditioning on $G_R(n-1)$ and $G_I(n-1)$.

Conditional on $G(n-1)$, the expected value of $H(n)$ is

$$E[H(n)|G(n-1)] = E[G_R^2(n) + G_I^2(n)|G(n-1)] \tag{2.17}$$

$$= E[[\alpha G_R(n-1) + Z_R(n)]^2 + [\alpha G_I(n-1) + Z_I(n)]^2|G(n-1)] \tag{2.18}$$

$$= \alpha^2 G_R(n-1) + \alpha^2 G_I(n-1) + 2\alpha G_R(n-1)E[Z_R(n)|G(n-1)] \tag{2.19}$$

$$+ 2\alpha G_I(n-1)E[Z_I(n)|G(n-1)] + E[Z_R^2(n) + Z_I^2(n)|G(n-1)].$$

We will now show that $Z_R(k)$ and $Z_I(k)$ are independent of $G(n)$ for $k > n$. This will allow us to greatly simplify (2.19). We start by combining (2.2) and (2.3) into a single equation,

$$G(n) = \alpha G(n-1) + Z(n), \tag{2.20}$$

where

$$Z(n) = \begin{bmatrix} Z_R(n) \\ Z_I(n) \end{bmatrix}. \tag{2.21}$$

Iterating (2.20), we obtain

$$G(n) = \alpha[\alpha G(n-2) + Z(n-1)] + Z(n) \tag{2.22}$$

$$= \alpha^2 G(n-2) + \alpha Z(n-1) + Z(n) \tag{2.23}$$

$$\rightarrow \sum_{m=0}^{\infty} \alpha^m Z(n - m). \tag{2.24}$$

Since the components of $Z(n)$ are white noise processes, $Z(n)$ is independent at each sample time $n$. Hence from (2.24), we see that $G(n)$ is independent of $Z(k)$, and thus of $Z_R(k)$ and
\( Z_I(k) \) for \( k > n \). Using this fact, (2.19) can be simplified to obtain

\[
E[H(n)|G(n-1)] = \alpha^2[G_R^2(n-1) + G_I^2(n-1)] + 2\alpha G_R(n-1)E[Z_R(n)] + 2\alpha G_I(n-1)E[Z_I(n)] + E[Z_R^2(n)] + E[Z_I^2(n)].
\]  
(2.25)

Noting that the white noise processes have mean 0 and variance \( \sigma_z^2 \), this becomes

\[
E[H(n)|G(n-1)] = \alpha^2[G_R^2(n-1) + G_I^2(n-1)] + 2\sigma_z^2
\]  
(2.26)

\[
= \alpha^2 H(n-1) + 2\sigma_z^2.
\]  
(2.27)

To understand what this says, let us condition on the polar coordinates of \( G(n-1) \) instead of conditioning on the rectangular coordinates \( G(n-1) \). These two conditions are equivalent. Therefore, conditioning on the magnitude \( |G(n-1)| \), or equivalently the magnitude squared \( |G(n-1)|^2 = H(n-1) \), and the phase \( \phi_G(n-1) = \arctan[ G_I(n-1)/G_R(n-1) ] \), (2.27) becomes

\[
E[H(n)|H(n-1), \phi_G(n-1)] = \alpha^2 H(n-1) + 2\sigma_z^2.
\]  
(2.28)

In other words, the power gain at time \( n \) is independent of the phase of the channel gain at time \( n-1 \), and we have

\[
E[H(n)|H(n-1)] = \alpha^2 H(n-1) + 2\sigma_z^2.
\]  
(2.29)

Following the same procedure for the conditional second moment of \( H(n) \), we have

\[
E[H^2(n)|G(n-1)]
\]  
(2.30)

\[
= E[[G_R^2(n) + G_I^2(n)]^2|G(n-1)]
\]  
(2.31)

\[
= E[[\alpha G_R(n-1) + Z_R(n)]^2 + [\alpha G_I(n-1) + Z_I(n)]^2]|G(n-1)]
\]  
(2.32)

\[
= E[\alpha^2[G_R^2(n-1) + G_I^2(n-1)] + 2\alpha[G_R(n-1)Z_R(n) + G_I(n-1)Z_I(n)] + [Z_R^2(n) + Z_I^2(n)]^2|G(n-1)]
\]  
(2.33)

Using the same independence arguments as before and noting that \( E[Z_R^2(n)] = E[Z_I^2(n)] = \)
$3\sigma_z^4$ and $E[Z_R^3(n)] = E[Z_f^3(n)] = 0$, this becomes

$$E[H^2(n)|G(n-1)]$$

$$= \alpha^4[G_R^2(n-1) + G_f^2(n-1)]^2 + 4\alpha^2[G_R^2(n-1)\sigma_z^2 + G_f^2(n-1)\sigma_z^2]$$

$$+ [3\sigma_z^4 + 2\sigma_z^2\sigma_z^2 + 3\sigma_z^4] + 2\alpha^2[G_R^2(n-1) + G_f^2(n-1)][\sigma_z^2 + \sigma_z^2]$$

$$= \alpha^4H^2(n-1) + 8\alpha^2\sigma_z^2H(n-1) + 8\sigma_z^4. \hspace{1cm} (2.34)$$

Again, we see that this is independent of the phase $\phi_G(n-1)$, and we have

$$E[H^2(n)|H(n-1)] = \alpha^4H^2(n-1) + 8\alpha^2\sigma_z^2H(n-1) + 8\sigma_z^4. \hspace{1cm} (2.37)$$

The following is a summary of the conditional mean and conditional second moment of $H(n)$:

**Conditional Mean:**

$$E[H(n)|H(n-1)]$$

$$= \alpha^2H(n-1) + 2\sigma_z^2 \hspace{1cm} (2.38)$$

$$= \alpha^2H(n-1) + (1 - \alpha^2)E[H(n)], \hspace{1cm} (2.39)$$

**Conditional Second Moment:**

$$E[H^2(n)|H(n-1)]$$

$$= \alpha^4H^2(n-1) + 8\alpha^2\sigma_z^2H(n-1) + 8\sigma_z^4$$

$$= \alpha^4H^2(n-1) + 4\alpha^2(1 - \alpha^2)H(n-1)E[H(n)] + 2(1 - \alpha^2)^2\{E[H(n)]\}^2, \hspace{1cm} (2.40)$$

where (2.13) was used to make the dependence on $E[H(n-1)]$ more explicit.

### 2.4 Implementation of Power Control Algorithm (PCA)

The implementation of the power control algorithms we develop is as follows. At each update time $n$, the base station will determine the appropriate transmit power for each user in its cell based on the CIR constraints. This calculation can only make use of the base station’s channel measurements at the previous update times $n-1$, $n-2$, and so on. The channel measurements will include the power gains for each user in its cell and the
total received power level. The base station can measure the power gains of its own users since these users can be separated in the decoding process. We will assume that the base makes perfect channel measurements. We will also assume that the control information is passed from the base to the mobile user instantaneously so that each user transmits at the appropriate power level at exactly the same update time. The base will then make new channel measurements at the present update time for use at the next update time.
Chapter 3

PCA Development: Single Cell Network

Although the existence of just a single cell is highly unrealistic in cellular networks, we begin our PCA development with this scenario since it is enough to provide us with valuable insight into designing PCAs that perform well when fading is present. This will allow us to tackle the more realistic and complex scenario of multiple cells in a more efficient manner.

We will use the narrowband signaling model for the channel fading when developing PCAs and analyzing their performance. Since we are considering the single cell scenario, we must look at spread systems, where each user is assigned the whole spectrum allocated to a cell, i.e. transmitting CDMA-type signals, so that cochannel interference will result. Otherwise, there will be no need for power control. The narrowband signaling model is suitable for indoor spread systems since, in this case, the signal bandwidth is usually less than the coherence bandwidth. However, outdoor spread systems require the use of the wideband signaling model since, in this case, the signal bandwidth is usually greater than the coherence bandwidth.

In order to simplify the development of PCAs, only two users will be considered first. Sections 3.1 and 3.2 present the derivation and performance analysis of PCAs, respectively. Section 3.3 presents an extension of the derivation and performance analysis to an arbitrary number of users.
3.1 PCA Derivation

3.1.1 Joint Solution

Consider a single cell with two users, user 1 and user 2, as shown in Figure 3-1. The carrier-to-interference ratio (CIR) is the received power divided by the sum of the interference power and the noise power. Thus, the CIR constraints for each user are

\[
\text{CIR}_1 = \frac{H_1(n)p_1(n)}{H_2(n)p_2(n) + \sigma^2} \geq \gamma, \tag{3.1}
\]

\[
\text{CIR}_2 = \frac{H_2(n)p_2(n)}{H_1(n)p_1(n) + \sigma^2} \geq \gamma, \tag{3.2}
\]

where \(p_i(n)\) and \(H_i(n)\) are the transmit power and power gain of user \(i\), respectively, \(\sigma^2\) is the external noise power at the base, and \(\gamma\) is the minimum common CIR that is required for reliable communication. All of these quantities are positive and the power gains \(H_1(n)\) and \(H_2(n)\) are assumed to be independent.

At this stage, our goal is to determine each user's desired transmit power \(d_i(n)\), which will be the minimum power solution to the CIR constraints. The minimum power solution is obtained when (3.1) and (3.2) are satisfied with equality,

\[
\text{CIR}_1 = \frac{H_1(n)p_1(n)}{H_2(n)p_2(n) + \sigma^2} = \gamma, \tag{3.3}
\]

\[
\text{CIR}_2 = \frac{H_2(n)p_2(n)}{H_1(n)p_1(n) + \sigma^2} = \gamma. \tag{3.4}
\]
To see this, assume that both users are transmitting at some power level such that their CIR is above the threshold $\gamma$. From (3.1), we see that user 1 can still meet the CIR constraint by decreasing its power until its CIR meets the threshold with equality. Now the denominator of (3.2) decreases, and user 2 can also decrease its power until its CIR meets the threshold with equality. This cycle will continue until, in the limit, both $\text{CIR}_1$ and $\text{CIR}_2$ meet the threshold with equality.

In order to achieve the CIR constraints with equality, each user must adjust its power so that both users experience the same amount of interference. This can only happen if both users are received at the same power. Denoting the received power of user $i$ by $q_i(n) = H_i(n)p_i(n)$, we must have

\[ q_1(n) = q_2(n) = q^*(n), \]  

where $q^*(n)$ is the desired received power. Now the system of equations shown in (3.3) and (3.4) becomes a one-dimensional problem,

\[ \frac{q^*(n)}{q^*(n) + \sigma^2} = \gamma, \]  

whose solution is

\[ q^*(n) = \frac{\gamma \sigma^2}{1 - \gamma}. \]  

Hence, the desired transmit power of user $i$ is

\[ d_i(n) = \frac{q^*}{H_i(n)}, \quad i = 1, 2, \]  

where we dropped the dependence on time of the desired received power. Later we will see that this is true only in the single cell scenario and is not true in the multiple cell scenario.

We see that achieving a minimum common CIR with the minimum power level is equivalent to adjusting the power of each user such that its received power at the base station is equal to the desired received power. We will say that the solution is feasible if the desired transmit power of both user 1 and user 2 is positive and finite. Therefore, a feasible solution exists if and only if $0 < \gamma < 1$. Furthermore, note that if the external noise power were zero, the desired transmit power for each user is zero. As a result, it is important that the external noise power is positive in order to obtain sensible desired transmit powers.
Notice that the desired transmit power for user $i$ at update time $n$ depends on its power gain at the same time. However, at time $n$, the base will only have measurements of the power gains of each user at time $n-1$, $n-2$, and so on. Therefore, when determining the appropriate transmit power for each user, the base must make good estimates of the power gains based on past measurements. From estimation theory, the conditional mean minimizes the mean squared error between the estimate of the power gain and the actual power gain. In other words, the minimum mean squared error (MMSE) estimate is the conditional mean. The MMSE estimate of the power gain of user $i$, $\hat{H}_i(n)$, is

$$
\hat{H}_i(n) = \mathbb{E}[H_i(n)|H_i(n-1), H_i(n-2), \ldots] 
= \mathbb{E}[H_i(n)|H_i(n-1)] 
= \alpha_i^2 H_i(n-1) + (1 - \alpha_i^2)\mathbb{E}[H_i(n)],
$$

(3.9) (3.10) (3.11)

where the second equality is due to the Gauss-Markov fading model and the third equality is obtained from (2.39) in Section 2.3. Hence, a PCA customized to the fading condition experienced by each user is

$$
p_i(n) = \frac{q^*}{\hat{H}_i(n)} 
= \frac{q^*}{\alpha_i^2 H_i(n-1) + (1 - \alpha_i^2)\mathbb{E}[H_i(n)]}, \quad i = 1, 2.
$$

(3.12) (3.13)

It is important to note that this is not the MMSE estimate of the desired transmit power $d_i(n)$. This will be explained in more detail in the next section.

We will now develop an approximate PCA in which the fading condition of each user is categorized into two types: slow fading and fast fading. The approximate PCA is basically the PCA shown in (3.13) approximated for both slow and fast fading. The approximations will be done by looking at what happens in the limit. In the limit of very slow fading ($\alpha_i \to 1$), (3.13) becomes

$$
p_i(n) = \frac{q^*}{H_i(n-1)},
$$

(3.14)

which agrees with the fact that the power gain measurement at the previous update time provides all the information about the power gain at the present update time. In the limit
of very fast fading ($\alpha_i \to 0$), (3.13) becomes

$$p_i(n) = \frac{q^*}{E[H_i(n)]}, \quad (3.15)$$

which agrees with the fact that the power gain measurement at the previous update time provides no information about the power gain at the present update time. Therefore, the approximate PCA is

$$p_i(n) = \frac{q^*}{H_i(n)}, \quad i = 1, 2, \quad (3.16)$$

where

$$H_i(n) = \begin{cases} H_i(n-1), & \text{slow fading,} \\ E[H_i(n)], & \text{fast fading.} \end{cases} \quad (3.17)$$

The original PCA shown in (3.13) requires the base to determine the actual fading parameter for each user, whereas the approximate PCA requires the base to determine the type of fading experienced by each user. The fading parameter can be estimated from the Doppler spread, which determines how fast the channel is varying. Therefore, it is more practical to use the original PCA if the base can make accurate measurements of the Doppler spread of each user's transmitted signal. On the other hand, if the base cannot make accurate measurements of the Doppler spread, it is more practical to use the approximate PCA since the base can determine the type of fading experienced by each user by taking correlations of the past measurements of the power gains of each user. High correlation corresponds to slow fading and low correlation corresponds to fast fading. Notice that in either case, the base needs to derive the mean of the power gain of each user. One method is to just take the running average of all the past power gain measurements of each user.

From now on we will focus on the original PCA shown in (3.13). A slight modification needs to be introduced. We must introduce maximum power constraints for each user. This is important for two reasons. The first reason is that, in practice, users cannot transmit past some maximum power $p_{max}$ due to battery constraints. The second reason is the peculiar behavior of the fading model. Notice that the transmit power of user $i$ is inversely proportional to the measurement of its power gain for the slow fading case. In the discussion of the fading model in Section 2.3, the power gains were said to behave according to an exponential distribution. Therefore, arbitrarily low power gains may result. Consequently, the PCA allows for arbitrarily high transmit powers under slow fading. Hence, we need to
impose maximum power constraints for each user,

\[ p_i(n) = \min \left\{ \frac{q^*}{\hat{H}_i(n)}, p_{\text{max}, i} \right\}, \quad i = 1, 2, \quad (3.18) \]

where \( p_{\text{max}, i} \) is the maximum power constraint for user \( i \).

We can make a slight improvement to the PCA shown in (3.18). Consider the situation when only one of the users experiences a “bad” channel and transmits at its maximum power. The other user, who is experiencing a “good” channel, may now decrease its transmit power to some level below the nominal level and still meet its CIR constraint. For example, let us assume that user 2 is directed to transmit at \( p_{\text{max}, 2} \). The minimum transmit power of user 1 that meets the CIR requirement is now \( \gamma [\hat{H}_2(n)p_{\text{max}, 2} + \sigma^2] / \hat{H}_1(n) \). This power level is below the nominal level of \( q^*/\hat{H}_1(n) \). The following PCA incorporates this situation for both users:

\[ p_1(n) = \min \left\{ \frac{q^*}{\hat{H}_1(n)}, \frac{\gamma [\hat{H}_2(n)p_{\text{max}, 2} + \sigma^2]}{\hat{H}_1(n)}, p_{\text{max}, 1} \right\}, \quad (3.19) \]

\[ p_2(n) = \min \left\{ \frac{q^*}{\hat{H}_2(n)}, \frac{\gamma [\hat{H}_1(n)p_{\text{max}, 1} + \sigma^2]}{\hat{H}_2(n)}, p_{\text{max}, 2} \right\}. \quad (3.20) \]

Note that if a user experiences a “bad” channel, there is no way the received power of that user will reach the desired received power. Therefore, the PCA can achieve the CIR constraint of a particular user only during periods when that user experiences a “good” channel.

There are two important points to be gained from studying the single cell scenario. First, the minimum power solution is obtained when the CIR constraints are met with equality. Second, if the CIR constraints are satisfied with equality, then each user must be received at the same power. These are both due to the symmetries of the CIR constraints. We will find that these results, with slight modification, will also apply to the multiple cell scenario.

### 3.1.2 Joint MMSE Solution

In this section, we will use a different approach to determine the transmit power of each user. It will be convenient to use vector notation. The quantities we are interested in are the power gain vector \( \mathbf{H}_n \), the transmit power vector \( \mathbf{p}_n \), the actual received power vector
and the desired received power vector \( \mathbf{q}_n^* \). Component \( i \) of each vector corresponds to the appropriate scalar quantity of user \( i \) at time \( n \). The new approach is to find, at a particular update time, the transmit power vector \( \hat{\mathbf{p}}_n \) which minimizes the mean squared error between the actual received power vector \( \mathbf{q}_n \) and the desired received power vector \( \mathbf{q}_n^* \) conditional on having a perfect measurement of the power gain vector at the previous step \( \mathbf{H}_{n-1} \). We will refer to \( \hat{\mathbf{p}}_n \) as the MMSE transmit power estimate.

The estimation problem is formulated as follows:

\[
\hat{\mathbf{p}}_n = \arg \min_{\mathbf{p}_n} \mathbb{E}\left[ \| \mathbf{q}_n - \mathbf{q}_n^* \|^2 | \mathbf{H}_{n-1} \right],
\]  

(3.21)

which can be rewritten as

\[
\hat{\mathbf{p}}_n = \arg \min_{\mathbf{p}_n} \mathbb{E}\left[ (\mathbf{q}_n - \mathbf{q}_n^*)^T (\mathbf{q}_n - \mathbf{q}_n^*) | \mathbf{H}_{n-1} \right]
\]  

(3.22)

\[
= \arg \min_{\mathbf{p}_n} \mathbb{E}\left[ \sum_{i=1}^{2} (q_i(n) - q_i^*(n))^2 | \mathbf{H}_{n-1} \right],
\]  

(3.23)

where we are still considering only two users. Note that this construction can easily be extended to an arbitrary number of users. Since the expected value of the sum equals the sum of the expected values, (3.23) becomes

\[
\min_{\mathbf{p}_n} \left\{ \mathbb{E}\left[ (H_1(n)p_1(n) - q_1^*(n))^2 | \mathbf{H}_{n-1} \right] + \mathbb{E}\left[ (H_2(n)p_2(n) - q_2^*(n))^2 | \mathbf{H}_{n-1} \right] \right\}.
\]  

(3.24)

Taking into account the fact that the power gains of each user are independent, (3.24) becomes

\[
\min_{p_1(n)} \mathbb{E}\left[ (H_1(n)p_1(n) - q_1^*(n))^2 | H_1(n-1) \right] + \min_{p_2(n)} \mathbb{E}\left[ (H_2(n)p_2(n) - q_2^*(n))^2 | H_2(n-1) \right].
\]  

(3.25)

Notice that the MMSE transmit power estimate of each user can be determined independently. Therefore, the MMSE transmit power estimate of user \( i \) must satisfy

\[
\hat{p}_i(n) = \arg \min_{p_i(n)} \mathbb{E}\left[ (H_i(n)p_i(n) - q_i^*(n))^2 | H_i(n-1) \right].
\]  

(3.26)

By taking the derivative with respect to \( p_i(n) \), setting the result to zero, and solving for
\[ \hat{p}_i(n) = q^* \frac{E[H_i(n) | H_i(n-1)]}{E[H_i^2(n) | H_i(n-1)]} \]  \hspace{1cm} (3.27)

where we used the fact that the desired received power for user 1 and user 2 are equal and constant,

\[ q_1^* = q_2^* = q^* = \frac{\gamma \sigma^2}{1 - \gamma}. \]  \hspace{1cm} (3.28)

Substituting (2.39) and (2.41) into (3.27), we obtain

\[ \hat{p}_i(n) = q^* \frac{\alpha^2 H_i(n-1) + (1 - \alpha^2) E[H_i(n)]}{\alpha^4 H_i^2(n-1) + 4\alpha^2 (1 - \alpha^2) H_i(n-1) E[H_i(n)] + 2(1 - \alpha^2)^2 \{ E[H_i(n)] \}^2}, \]  \hspace{1cm} (3.29)

which we will refer to as the MMSE-PCA. For the same reason as before, maximum power constraints also need to be incorporated in the MMSE-PCA.

We can evaluate \( \hat{p}_i(n) \) under slow and fast fading by setting \( \alpha_i \) to the appropriate value. Under the slow fading condition, (3.29) reduces to

\[ \hat{p}_i(n) = \frac{q^*}{H_i(n-1)}. \]  \hspace{1cm} (3.30)

and under the fast fading condition, (3.29) reduces to

\[ \hat{p}_i(n) = \frac{q^*}{2E[H_i(n)]}. \]  \hspace{1cm} (3.31)

By comparing the control equations of the MMSE-PCA and the original PCA derived in Section 3.1.1, we observe that they behave essentially the same way under slow and fast fading. For slow fading, the PCAs are exactly the same. However, for fast fading, there is a factor of two difference between the PCAs. This is due to the fact that here we are finding the MMSE estimate of the transmit power rather than the MMSE estimate of the power gain.

### 3.2 Performance Analysis

The performance of the PCA shown in (3.13) will be analyzed. We ignore maximum power constraints in order to simplify the analysis. Let the error \( e \) be the difference between the
actual received power of user $i$ and the desired received power,

$$e = q_i(n) - q^*.$$ (3.32)

Ideally, we want $E[e] = 0$ and $\text{VAR}[e] = 0$. The mean of the error for the PCA shown in (3.13) is

$$E[e] = E_{H_i(n-1)} [E[q_i(n) - q^*|H_i(n-1)]]$$ (3.33)

$$= E_{H_i(n-1)} \left[ E \left[ \frac{H_i(n)q^*}{\hat{H}_i(n)} - q^*|H_i(n-1) \right] \right]$$ (3.34)

$$= q^* E_{H_i(n-1)} \left[ E \left[ \frac{H_i(n)}{E[H_i(n)|H_i(n-1)]} - 1|H_i(n-1) \right] \right]$$ (3.35)

$$= q^* E_{H_i(n-1)} \left[ \frac{E[H_i(n)|H_i(n-1)]}{E[H_i(n)|H_i(n-1)]} - 1 \right].$$ (3.36)

$$= 0.$$ (3.37)

Using similar techniques, the variance of the error is

$$\text{VAR}[e] = E[e^2]$$ (3.38)

$$= E_{H_i(n-1)} [E[(q_i(n) - q^*)^2|H_i(n-1)]]$$ (3.39)

$$= q^2 E_{H_i(n-1)} \left[ E \left[ \left( \frac{H_i(n)}{E[H_i(n)|H_i(n-1)]} - 1 \right)^2 |H_i(n-1) \right] \right]$$ (3.40)

$$= q^2 E_{H_i(n-1)} \left[ \frac{E[H_i^2(n)|H_i(n-1)]}{(E[H_i(n)|H_i(n-1)])^2} - 1 \right].$$ (3.41)

Using (2.39) and (2.41), we can determine the behavior of the variance in the limit of slow and fast fading. For slow fading,

$$\text{VAR}[e] = 0,$$ (3.42)

and for fast fading,

$$\text{VAR}[e] = q^2.$$ (3.43)

Since $q^* = \gamma \sigma^2/(1 - \gamma)$, we see that the PCA will still perform well in fast fading only if the external noise power at the receiver is low and the CIR threshold required is low.
3.3 Extension to Arbitrary Number of Users

We now extend the results of the previous sections to the case where there are an arbitrary number of users in the cell. Assume there are $K$ users. The CIR constraint for user $i$ is

$$CIR_i = \frac{H_i(n)p_i(n)}{\sum_{j \neq i} H_j(n)p_j(n) + \sigma^2} \geq \gamma, \quad 1 \leq i \leq K. \quad (3.44)$$

By the same reasoning used before, we can see that the minimum power solution to these constraints is obtained when these constraints are satisfied with equality. In order to easily determine the desired transmit power of user $i$, we reduce the dimensionality of the problem to just one dimension by using the fact that each user must be received at the same power in order to meet the constraints with equality. Therefore, along with changing the inequality to equality, we can rewrite (3.44) in terms of the desired received power $q^*$ to obtain

$$\frac{q^*}{(K-1)q^* + \sigma^2} = \gamma, \quad (3.45)$$

whose solution is

$$q^* = \frac{\gamma \sigma^2}{1 - (K-1)\gamma}. \quad (3.46)$$

Hence, the desired transmit power of user $i$ is

$$d_i(n) = \frac{q^*}{H_i(n)}, \quad 1 \leq i \leq K. \quad (3.47)$$

Now a feasible solution exists if and only if $0 < \gamma < \frac{1}{K-1}$.

The derivation of the PCAs for the arbitrary number of users case is essentially identical to the derivation for the two users case. The only change is the expression for the desired received power, which depends on the number of users. Since the explanations remain unchanged, only a summary of the results will be presented.

A PCA which uses the MMSE estimate of the power gains is

$$p_i(n) = \frac{q^*}{H_i(n)} \quad (3.48)$$

$$= \frac{q^*}{E[H_i(n)|H_i(n-1)]} \quad (3.49)$$

$$= \frac{q^*}{\alpha_i^2 H_i(n-1) + (1 - \alpha_i^2)E[H_i(n)]}, \quad 1 \leq i \leq K. \quad (3.50)$$
If instead we minimize the mean squared error between the actual received power and the desired received power, we obtain the MMSE transmit power estimate,

\[
\hat{p}_i(n) = q^* \frac{E[H_i(n)|H_i(n-1)]}{E[H_i^2(n)|H_i(n-1)]} \\
= q^* \frac{\alpha_i^2 H_i(n-1) + \alpha_i^2 E[H_i(n)]}{\alpha_i^2 H_i^2(n-1) + 4\alpha_i^2 \alpha_i^2 H_i(n-1)E[H_i(n)] + 2(\alpha_i^2)^2 (E[H_i(n)])^2},
\]

(3.51) \hspace{1cm} (3.52)

where \(\alpha_i^2 = 1 - \alpha_i^2\). This was referred to as the MMSE-PCA. As before, maximum power constraints need to be incorporated in both PCAs.

Since the power gains are assumed to be independent, the performance analysis presented in the previous section applies here as well.
Chapter 4

PCA Development: Two Cell Network

Developing PCAs for a two cell network allows us to examine the dynamics of both intracell interference and intercell interference. Initially, we will focus on the effects of intercell interference by considering two cells with only one user per cell. Sections 4.1 and 4.2 present the derivation and performance analysis of PCAs for this case, respectively. Section 4.3 presents an extension of the derivation and performance analysis to an arbitrary number of users.

We comment that a network of just two cells is still highly unrealistic. However, it is important to obtain a deep understanding of all the issues involved in this scenario. It provides enough meat such that moving on to the multiple cell scenario, such as the common hexagonal cell pattern, is a smooth transition.

4.1 PCA Derivation

4.1.1 Joint Solution

Similar to the single cell scenario, we will first determine the desired transmit power of each user from the the CIR constraints. Figure 4-1 depicts the situation we will be studying in this section. The following notation will be used:

\[ \text{CIR}_i : \text{carrier-to-interference ratio of user in cell } i, \]
Figure 4-1: Two cells with one user per cell

\[ p_i(n) : \text{transmit power of user in cell } i, \]
\[ H_{ij}(n) : \text{power gain from user in cell } j \text{ to base } i, \]
\[ \sigma_i^2 : \text{external noise power at base } i, \]
\[ \gamma : \text{minimum common CIR for reliable communication}, \]
\[ d_i(n) : \text{desired transmit power of user in cell } i. \]

Each power gain \( H_{ij}(n) \) is assumed to be independent of each other for all \( i \) and \( j \).

The CIR constraints for each user are

\[ \text{CIR}_1 = \frac{H_{11}(n)p_1(n)}{H_{12}(n)p_2(n) + \sigma_1^2} \geq \gamma, \tag{4.1} \]
\[ \text{CIR}_2 = \frac{H_{22}(n)p_2(n)}{H_{21}(n)p_1(n) + \sigma_2^2} \geq \gamma. \tag{4.2} \]

The desired transmit power of each user will be the minimum power solution to these constraints. In determining the minimum power solution, we try to make use of the single cell results of Section 3.1.1. As in the single cell case, the minimum power solution to these constraints is obtained when these constraints are met with equality. Unfortunately, unlike the single cell case, the dimensionality of the problem cannot be reduced. The reduction in dimensionality was due to the fact that users within a cell must be received at the same power level in order to achieve equality in the CIR constraints. This result cannot be
applied to our present scenario since the users are located in different cells, which means that they do not have to be received at the same power at their respective base stations. However, this result will be helpful when we consider an arbitrary number of users located in each cell. A general rule is that, when determining the minimum power solution to the CIR constraints, the dimensionality of the problem is equal to the number of cells.

In order to determine the desired transmit power of each user, we rewrite the CIR constraints in matrix form,

\[
\begin{pmatrix}
H_{11}(n) & -\gamma H_{12}(n) \\
-\gamma H_{21}(n) & H_{22}(n)
\end{pmatrix}
\begin{pmatrix}
p_1(n) \\
p_2(n)
\end{pmatrix}
= 
\begin{pmatrix}
\gamma \sigma_1^2 \\
\gamma \sigma_2^2
\end{pmatrix},
\]

where the inequality was replaced with equality. Assuming \( G_n \) is invertible, the desired transmit power vector \( d_n \) is the transmit power vector \( p_n \) that satisfies (4.3),

\[
d_n = G_n^{-1} y \\
= \frac{\gamma}{H_{11}(n)H_{22}(n) - \gamma^2 H_{12}(n)H_{21}(n)}
\begin{pmatrix}
H_{22}(n) & \gamma H_{12}(n) \\
\gamma H_{21}(n) & H_{11}(n)
\end{pmatrix}
\begin{pmatrix}
\sigma_1^2 \\
\sigma_2^2
\end{pmatrix}.
\]

Using the same definition of feasibility mentioned in Section 3.1.1, we observe that a feasible solution exists if and only if

\[
H_{11}(n)H_{22}(n) - \gamma^2 H_{12}(n)H_{21}(n) > 0.
\]

Notice that the desired transmit power of each user is a function of the power gains of each user to its own base and the power gains of each user to the adjacent base. Using the same PCA development techniques presented for the single cell case will result in unnecessarily complex PCAs in which the base stations must communicate with each other to exchange channel measurements and power control information in order to determine the appropriate transmit power for its own user. We would like to find a PCA in which the base stations do not need to communicate with each other and only need to use their own local measurements. This will be the focus of the next section.
4.1.2 Iterative Solution

We are interested in determining a PCA in which the base stations do not need to communicate with each other. In the previous section, we derived an expression for the desired transmit powers in terms of the power gains, the external noise powers, and the minimum common CIR. We could have come up with a more direct expression for the desired transmit power from the CIR constraints shown in (4.1) and (4.2). The desired transmit power of user \( i \) is just the interference it experiences with appropriate scaling,

\[
d_1(n) = \frac{\gamma}{H_{11}(n)}[H_{12}(n)p_2(n) + \sigma_1^2] = a_1(n)p_2(n) + b_1(n),
\]

\[
d_2(n) = \frac{\gamma}{H_{22}(n)}[H_{21}(n)p_1(n) + \sigma_2^2] = a_2(n)p_1(n) + b_2(n),
\]

where

\[
a_1(n) = \frac{\gamma H_{12}(n)}{H_{11}(n)}, \quad b_1(n) = \frac{\gamma \sigma_1^2}{H_{11}(n)},
\]

\[
a_2(n) = \frac{\gamma H_{21}(n)}{H_{22}(n)}, \quad b_2(n) = \frac{\gamma \sigma_2^2}{H_{22}(n)}.
\]

This can be rewritten in matrix form as follows:

\[
\begin{bmatrix}
  d_1(n) \\
  d_2(n)
\end{bmatrix}
= \begin{bmatrix}
  0 & a_1(n) \\
  a_2(n) & 0
\end{bmatrix}
\begin{bmatrix}
  p_1(n) \\
  p_2(n)
\end{bmatrix}
+ \begin{bmatrix}
  b_1(n) \\
  b_2(n)
\end{bmatrix}.
\]

Notice that the desired transmit power of user \( i \) is expressed directly as a function of the other user’s transmit power. If each user transmits at exactly the desired power, that is \( p_n = d_n \), then from (4.10) we obtain

\[
d_n = (I - A_n)^{-1} b_n.
\]

It can be verified that this is equal to the expression for the desired transmit power shown in (4.4),

\[
d_n = (I - A_n)^{-1} b_n = G_n^{-1} y.
\]

The following iterative PCA was analyzed by Yates in [5] and was shown to converge to
the desired transmit power under certain conditions when there is no fading present:

\[
\begin{align*}
    p_0 & = p' = \text{initial transmit power vector}, \\
    p_{n+1} & = d_n = A_n p_n + b_n, \ n \geq 0.
\end{align*}
\]

This iterative PCA instructs each user to adjust its transmit power level to the desired transmit power assuming that the other user does not change its transmit power level and the power gains stay fixed. First, we will explain how this PCA converges to the desired received power when there is no fading present. Later on we will determine whether this PCA can be successfully used when fading does exist.

Before explaining the convergence, we will explain why this PCA relieves the base stations from communicating with each other. As mentioned before, each base station can measure the power gain of its own user and the total received power level. The interference seen by its user, which includes the interference from the user in the adjacent cell and the external noise power, can be determined by subtracting the received power of its user from the total received power. The base can determine the received power of its user since it can measure that user’s power gain and it knows the power level of that user from the PCA. Therefore, at each update time \( n + 1 \), the base can determine the desired transmit power of its own user at time \( n \) by using only local measurements.

**Convergence Analysis (No Fading Present)**

Assuming no fading, i.e. power gains are fixed, the iterative PCA becomes

\[
p_{n+1} = A p_n + b,
\]

since the scalars shown in (4.9) are no longer time-varying. Iterating this we obtain,

\[
p_n = A^n p_0 + \left( \sum_{k=0}^{n-1} A^k \right) b.
\]

It can be seen that this algorithm will converge to some value if and only if the magnitude of each eigenvalue of \( A \) is less than one. The eigenvalues of \( A \) are \( \lambda_1 = \sqrt{a_1 a_2} \) and \( \lambda_2 = \)
\[-\sqrt{a_1a_2}.\] Hence, this condition is

\[
a_1a_2 = \frac{\gamma^2 H_{12}H_{21}}{H_{11}H_{22}} < 1,\tag{4.16}
\]

which is equivalent to the condition of feasibility shown in (4.6). In other words, this PCA will converge as long as the condition of feasibility is satisfied. We will say that the PCA is stable in this case. Assuming this is the case, we have

\[
\lim_{n \to \infty} p_n = \left( \sum_{k=0}^{\infty} A^k \right) b
\]

\[
= (I - A)^{-1} b\tag{4.18}
\]

\[
= d. \tag{4.19}
\]

We observe that the transmit power of each user approaches the desired transmit power when fading is not present. The rate of convergence of the PCA depends upon the magnitude of the eigenvalues of $A$. A magnitude close to zero results in a very fast convergence rate, and a magnitude close to one results in a very slow convergence rate. By examining (4.16) we see that either a low required CIR or large direct power gains (from user to its own base) relative to cross power gains (from user to adjacent base) results in fast convergence.

It is useful to take a closer look at how the samples of the transmit power actually evolve. This will lead us to a method for improving the convergence rate, which we will describe later. Let us focus on user 1. By symmetry, the results will be similar for user 2. For $n \geq 1$, we have

\[
p_1(n + 1) = d_1(n) = a_1p_2(n) + b_1
\]

\[
= a_1d_2(n - 1) + b_1
\]

\[
= a_1[a_2p_1(n - 1) + b_2] + b_1
\]

\[
= a_1a_2p_1(n - 1) + a_1b_2 + b_1.
\]

Hence, the evolution of the transmit power for user 1 is

\[
p_1(0) = p_1'
\]

\[
p_1(1) = a_1p_2' + b_1
\]

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\[ p_1(n) = a_1 a_2 p_1(n-2) + a_1 b_2 + b_1, \quad n \geq 2, \]

where \( p'_1 \) and \( p'_2 \) are the initial transmit power of user 1 and 2, respectively. Notice that the even and odd samples of the transmit power evolve independently. Let us look at the behavior of the even samples or odd samples with a normalized time scale. Letting \( e_1(n) \) denote the even samples and \( o_1(n) \) denote the odd samples, we have

\[
\begin{align*}
  e_1(0) &= p'_1 \\
o_1(0) &= a_1 p'_2 + b_1 \\
  \begin{cases} 
    e_1(n) \\ 
    o_1(n) 
  \end{cases} &= a_1 a_2 \begin{cases} 
    e_1(n-1) \\ 
    o_1(n-1) 
  \end{cases} + a_1 b_2 + b_1, \quad n \geq 1.
\end{align*}
\]

Both \( e_1(n) \) and \( o_1(n) \) are governed by a first-order difference equation with an initial condition. The solution is

\[
\begin{align*}
  \begin{cases} 
    e_1(n) \\ 
    o_1(n) 
  \end{cases} &= \begin{cases} 
    e_1(0) \\ 
    o_1(0) 
  \end{cases} - \frac{a_1 b_2 + b_1}{1 - a_1 a_2} (a_1 a_2)^n + \frac{a_1 b_2 + b_1}{1 - a_1 a_2} \\
  &= \begin{cases} 
    e_1(0) \\ 
    o_1(0) 
  \end{cases} - d_1 (a_1 a_2)^n + d_1, \quad n \geq 0, \quad (4.21)
\end{align*}
\]

where \( d_1 \) is the desired transmit power for user 1. The relation \( d_1 = \frac{a_1 b_2 + b_1}{1 - a_1 a_2} \) can be verified from (4.11). Therefore, both the even and odd samples of \( p_1(n) \) will converge exponentially to the desired transmit power if and only if

\[ a_1 a_2 < 1, \quad (4.23) \]

which is the same condition for convergence stated before.

**Improving Convergence (No Fading Present)**

We will now show how to develop a smarter algorithm which converges in much less time. The basic idea is to divide the PCA into two stages. In the first stage, we will use the
Table 4.1: First stage of two-stage PCA

<table>
<thead>
<tr>
<th>( n )</th>
<th>( p_1(n) )</th>
<th>base 1 measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( p_1(0) )</td>
<td>( d_1(0) )</td>
</tr>
<tr>
<td>1</td>
<td>( d_1(0) )</td>
<td>( d_1(1) = (a_1a_2)p_1(0) + (a_1b_2 + b_1) )</td>
</tr>
<tr>
<td>2</td>
<td>( d_1(1) )</td>
<td>( d_1(2) = (a_1a_2)p_1(1) + (a_1b_2 + b_1) )</td>
</tr>
</tbody>
</table>

iterative PCA just discussed, where the transmit power at a particular update time is equal to the desired transmit power at the previous update time. The first stage will be carried out in only a finite number of iterations. The number of iterations depends on the time it takes the base to determine the relevant parameters of the channel such that it can instruct its user to transmit at the desired power from that point on. In the second stage, each user will be instructed to transmit at exactly the right power level. We are still assuming fixed power gains here.

The two-stage PCA is demonstrated as follows. We will again focus on user 1. Table 4.1 highlights the first stage. Starting from time 0, it shows the transmit power of user 1 and the desired transmit power measurements of base 1. Note that the desired power at a particular time is a function of the transmit power at the previous time. At time \( n = 2 \), base 1 can determine \( a_1a_2 \) and \( a_1b_2 + b_1 \) by solving the set of linear equations

\[
p_1(0)(a_1a_2) + (a_1b_2 + b_1) = d_1(1), \tag{4.24}
\]

\[
d_1(0)(a_1a_2) + (a_1b_2 + b_1) = d_1(2), \tag{4.25}
\]

where \( d_1(0) \) was substituted for \( p_1(1) \). The solution is

\[
a_1a_2 = \frac{d_1(2) - d_1(1)}{d_1(0) - p_1(0)}, \tag{4.26}
\]

\[
a_1b_2 + b_1 = \frac{d_1(2) - d_1(0)}{p_1(0)} \frac{d_1(1)}{1 - \frac{d_1(0)}{p_1(0)}}. \tag{4.27}
\]

In the second stage, base 1 will instruct its user to transmit at exactly the desired transmit power level,

\[
p_1(n) = \frac{a_1b_2 + b_1}{1 - a_1a_2} = d_1, \ n \geq 3. \tag{4.28}
\]
The power of user 2 is controlled in exactly the same manner. Notice that this algorithm converges by the fourth iteration.

If \( p_t(0) = d_t(0) \) for either user 1 or user 2 or both, then that user happened to transmit at exactly the right power level initially. The base assigned to that user can detect this and instruct its user to keep its power at that level from that point on.

### 4.2 Performance Analysis

In this section we will determine the performance of the iterative PCA introduced in the previous section. We have seen that the PCA converges to the desired transmit power when there is no fading present. Now we will examine its behavior when fading is present.

#### 4.2.1 Error Dynamics

The performance criterion we are interested in is how well the transmit power trajectory \( p_n \) tracks the desired transmit power trajectory \( d_n \). This will be measured by the error vector \( e_n \), which is defined to be the difference between the actual transmit power vector \( p_n \) and the desired transmit power vector \( d_n \) at a particular update time \( n \),

\[
e_n = p_n - d_n.
\]  \hspace{1cm} (4.29)

The error will be affected both by the time-varying channel and the PCA. Consider the trajectory of \( p_n \) as the channel changes. Let us assume the channel starts out at a particular state, in which the power gains remain fixed for some period of time. Starting from some initial transmit power, \( p_n \) will start to converge to the desired transmit power, which is specified by the power gains associated with the particular channel state. When the channel changes to a new state, \( p_n \) will now start to converge to the new desired transmit power specified by the new channel state. We observe that the error will largely be determined by the rate at which the channel changes and the rate of convergence of the PCA.

The coherence time gives us an indication of the rate at which the channel changes. We will assume that the channel remains in a particular state over a time interval of length equal to the coherence time \( N \). To be more specific, the channel is in state \( m \), \( m = 0, 1, \ldots \), over the time interval \( [mN, (m + 1)N - 1] \). We will characterize the channel by the pair \((A_n, b_n)\), where the matrix \( A_n \) and the vector \( b_n \) are determined by the power gains and
are defined in Section 4.1.2. A channel in the $m$th time interval will be characterized by the pair $(\bar{A}_m, \bar{b}_m)$, where $\bar{A}_m$ and $\bar{b}_m$ are fixed over this interval. Throughout the analysis, we will assume that a feasible desired transmit power vector exists in each interval. This translates into the magnitude of each eigenvalue of $\bar{A}_m$ being less than one, which means the PCA is always stable.

Initially, we will focus on the behavior of the error during the initial time interval $[0, N - 1]$. The channel is characterized by the pair $(\bar{A}_0, \bar{b}_0)$, which determines the desired transmit power vector for this interval,

$$\bar{d}_0 = (I - \bar{A}_0)^{-1} \bar{b}_0 = \left(\sum_{k=0}^{\infty} \bar{A}_0^k\right) \bar{b}_0. \quad (4.30)$$

The transmit power trajectory $p_n$ will behave according to the PCA law,

$$p_{n+1} = \bar{A}_0 p_n + \bar{b}_0, \quad (4.31)$$

which after iterating becomes

$$p_n = \bar{A}_0^n p_0 + \left(\sum_{k=0}^{n-1} \bar{A}_0^k\right) \bar{b}_0, \quad (4.32)$$

where $p_0$ is the initial transmit power vector. Hence, the error during the initial interval is

$$e_n = p_n - \bar{d}_0 = \bar{A}_0^n p_0 - \left(\sum_{k=0}^{\infty} \bar{A}_0^k\right) \bar{b}_0 \quad (4.33)$$

$$= \bar{A}_0^n p_0 - \bar{A}_0^n \left(\sum_{k=n}^{\infty} \bar{A}_0^k\right) \bar{b}_0 \quad (4.34)$$

$$= \bar{A}_0^n (p_0 - \bar{d}_0) \quad (4.35)$$

$$= \bar{A}_0^n e_0, \quad (4.36)$$

where $e_0$ is the initial error. As expected, the eigenvalues of $\bar{A}_0$ play a key role in the behavior of the error. Our earlier assumption of the existence of a feasible desired transmit power means that the error will always approach zero. The closer the magnitudes of the eigenvalues are to zero, the faster the error approaches zero, and the closer the transmit power trajectory is able to track the desired transmit power. A longer coherence time allows
the transmit power trajectory to track the desired transmit power as close as possible. In addition, note that if the initial transmit power happens to be close to the desired transmit power, the error will be small throughout the entire interval.

The error during subsequent intervals is determined by three factors: the coherence time of the channel, the dynamics of the previous time interval, and the amount of change in the channel between time intervals. For example, let us consider the next time interval \([N,2N – 1]\). If the coherence time is long or the convergence rate is fast in the initial interval, then the transmit power will be approximately equal to the desired transmit power for that interval. If, in addition, there is only an incremental change in the power gains between state 0 and state 1, the initial transmit power for the next interval will again be approximately equal to the desired transmit power for that interval, and the error will be small throughout the next interval. This is the ideal case. The worst case is a short coherence time, slow convergence rate, and a large change in the channel, which would lead to a large error. It can be seen that, in general, the error is affected by the interplay between the factors mentioned above.

In Section 4.1.2, we presented an improved convergence algorithm where the transmit power is “pinned” to the desired transmit power by the fourth iteration. Using this algorithm will achieve good error performance in a time-varying channel if \(N > 4\). In each time interval, the error is zero after 4 time steps, and the transmit power trajectory tracks the desired transmit power trajectory exactly from that point on. However, we note that this is special to the case of only one user per cell. This does not generalize to the case of an arbitrary number of users per cell.

### 4.2.2 Small Perturbation Error

The purpose of this section is to determine the incremental error obtained by the iterative PCA. To do this we make two assumptions. First, we assume that each user transmits at the desired power at a particular update time \(n\). Second, we assume a small perturbation of the power gains in one time step,

\[
H_{ij}(n + 1) = H_{ij}(n) + \tilde{H}_{ij}(n), \quad \forall i,j,
\]

where \(\tilde{H}_{ij}(n)\) denotes the perturbation.
Following this setup, we have
\[ p_n = d_n, \] (4.39)
where \( d_n \) must satisfy
\[ G_n d_n = y. \] (4.40)

Now after the perturbation of the power gains, the desired transmit power at time \( n + 1 \) must satisfy,
\[ \left( G_n + \tilde{G}_n \right) \left( d_n + \tilde{d}_n \right) = y, \] (4.41)
where
\[ \tilde{G}_n = \begin{bmatrix} \tilde{H}_{11}(n) & -\gamma \tilde{H}_{12}(n) \\ -\gamma \tilde{H}_{21}(n) & \tilde{H}_{22}(n) \end{bmatrix}, \] (4.42)
and
\[ \tilde{d}_n = \begin{bmatrix} \tilde{d}_1(n) \\ \tilde{d}_2(n) \end{bmatrix}. \] (4.43)

Note that (4.41) takes into account the fact that the desired transmit power vector must also be perturbed in order to maintain equality. Since \( p_n = d_n \), the iterative PCA will instruct each user to keep its power at the same level at the next time step,
\[ p_{n+1} = d_n = p_n. \] (4.44)

The incremental error vector \( \tilde{e} \) will be defined to be the difference between the desired transmit power vector and the actual transmit power vector at time \( n + 1 \),
\[ \tilde{e} = d_{n+1} - p_{n+1} = (d_n + \tilde{d}_n) - d_n = \tilde{d}_n. \] (4.45)

From (4.41) we can solve for \( \tilde{d}_n \) as follows:
\[ (G_n + \tilde{G}_n)(p_n + \tilde{d}_n) = y \] (4.46)
\[ G_n p_n + \tilde{G}_n p_n + (G_n + \tilde{G}_n) \tilde{d}_n = y \] (4.47)
\[ \tilde{d}_n = -(G_n + \tilde{G}_n)^{-1} \tilde{G}_n p_n. \] (4.48)

We can find a good approximation for \( (G_n + \tilde{G}_n)^{-1} \) by using the assumption of small
perturbations, i.e. \( \tilde{G}_n \) \( \ll 1 \). We first rewrite \( (G_n + \tilde{G}_n)^{-1} \) as follows:

\[
(G_n + \tilde{G}_n)^{-1} = G_n^{-1}((G_n + \tilde{G}_n)G_n^{-1})^{-1} = G_n^{-1}(I + \tilde{G}_nG_n^{-1})^{-1} \tag{4.49}
\]

Now, using the assumption of small perturbations,

\[
(I + \tilde{G}_nG_n^{-1})^{-1} \approx (I - \tilde{G}_nG_n^{-1}) \tag{4.51}
\]

since

\[
(I - \tilde{G}_nG_n^{-1})(I + \tilde{G}_nG_n^{-1}) = I - \tilde{G}_nG_n^{-1}\tilde{G}_nG_n^{-1} \approx I. \tag{4.53}
\]

As a result,

\[
(G_n + \tilde{G}_n)^{-1} \approx G_n^{-1}(I - \tilde{G}_nG_n^{-1}), \tag{4.54}
\]

and the error vector can be approximated as

\[
\hat{e} \approx -G_n^{-1}(I - \tilde{G}_nG_n^{-1})\tilde{G}_n\pi_n \tag{4.55}
\]

\[
= -G_n^{-1}\tilde{G}_n\pi_n + G_n^{-1}\tilde{G}_nG_n^{-1}\tilde{G}_n\pi_n \tag{4.56}
\]

\[
\approx -G_n^{-1}\tilde{G}_n\pi_n, \tag{4.57}
\]

where the small perturbation assumption was used once again.

Now assume that \( \tilde{H}_{ij}(n) \) has mean 0 and variance \( \sigma_{ij}^2 = \rho s_{ij} \). The parameter \( \rho \) is introduced so that the behavior of the incremental error is more apparent. The mean error is then

\[
E[\hat{e}] = -G_n^{-1}E[\tilde{G}_n\pi_n] = 0. \tag{4.58}
\]

The error covariance is

\[
K_{\hat{e}} = E[\hat{e}\hat{e}^T] \tag{4.59}
\]

\[
= E[G_n^{-1}\tilde{G}_n\pi_n(G_n^{-1}\tilde{G}_n\pi_n)^T] \tag{4.60}
\]

\[
= G_n^{-1}E[\tilde{G}_n\pi_n(\tilde{G}_n\pi_n)^T](G_n^{-1})^T. \tag{4.61}
\]
Now,
\[
\tilde{G}_n p_n = \begin{bmatrix}
\tilde{H}_{11}(n)p_1(n) - \gamma \tilde{H}_{12}(n)p_2(n) \\
\tilde{H}_{22}(n)p_2(n) - \gamma \tilde{H}_{21}(n)p_1(n)
\end{bmatrix},
\] (4.62)
and
\[
E[\tilde{G}_n p_n (\tilde{G}_n p_n)^T] = \begin{bmatrix}
p_1^2(n)\sigma_1^2 + \gamma^2 p_2^2(n)\sigma_{12}^2 & 0 \\
0 & p_2^2(n)\sigma_{22}^2 + \gamma^2 p_1^2(n)\sigma_{21}^2
\end{bmatrix}
\] (4.63)
\[
= \begin{bmatrix}
\rho[p_1^2(n)s_{11} + \gamma^2 p_2^2(n)s_{12}] & 0 \\
0 & \rho[p_2^2(n)s_{22} + \gamma^2 p_1^2(n)s_{21}]
\end{bmatrix},
\] (4.64)
which is diagonal since the set of perturbations \{\tilde{H}_{ij}(n)\} are independent and zero mean. Hence,
\[
K_{\tilde{e}} = G_n^{-1} \begin{bmatrix}
\rho[p_1^2(n)s_{11} + \gamma^2 p_2^2(n)s_{12}] & 0 \\
0 & \rho[p_2^2(n)s_{22} + \gamma^2 p_1^2(n)s_{21}]
\end{bmatrix} (G_n^{-1})^T.
\] (4.65)

This result verifies two observations that are intuitive. First, as ρ approaches zero, i.e. the variance of the power gain perturbation approaches zero, the error covariance goes to zero. Second, in general, the error covariance matrix is a matrix where all entries are nonzero, which means that the errors for user 1 and user 2 are correlated. This makes sense since each user’s power solution to the CIR constraints are coupled.

### 4.2.3 Combined Error Analysis

We are now in the position to merge the error dynamics analysis and the small perturbation error analysis in order to obtain an explicit expression for the error that results after a channel state transition. We focus on the transition between state 0 and state 1, which occurs at time $N$. The transmit power at this time can be expressed as
\[
p_N = \tilde{d}_0 - e_N,
\] (4.66)
where $\tilde{d}_0$ is the desired transmit power for state 0 and
\[
e_N = \tilde{A}_0^N e_0.
\] (4.67)
Due to the state transition, we have

\[ d_N = \tilde{d}_1, \quad (4.68) \]

where \( \tilde{d}_1 \) is the desired transmit power for state 1. Assuming a small perturbation of the power gains, we have

\[ \tilde{d}_1 - \bar{d}_0 = \bar{e}_{0 \rightarrow 1}, \quad (4.69) \]

where \( \bar{e}_{0 \rightarrow 1} \) denotes the incremental error due to the state transition. This has the exact same characteristics as the incremental error described previously. Hence, the initial error during time interval 1 is

\[ e_{\text{init}1} = d_N - p_N \quad (4.70) \]

\[ = \bar{e}_{0 \rightarrow 1} + e_N. \quad (4.71) \]

The mean and covariance of this error for a given \( e_N \) is then

\[ E[e_{\text{init}1}|e_N] = e_N, \quad (4.72) \]

\[ K_{e_{\text{init}1}|e_N} = K_{\bar{e}_{0 \rightarrow 1}}, \quad (4.73) \]

where \( K_{\bar{e}_{0 \rightarrow 1}} \) has the same form as the covariance shown in (4.65). The characteristics of this error depend on the coherence time, the rate of convergence during the initial interval, and the amount of change in the power gains. We observe again that a long coherence time (\( N \) large), a fast rate of convergence during the initial interval (magnitude of eigenvalues of \( \tilde{A}_0 \) close to zero), and a small change in the power gains (\( \rho \) approaching zero) result in a small initial error during time interval 1. The same analysis can be applied to subsequent channel state transitions.

For a more comprehensive error analysis, the characteristics of the steady-state error need to be determined. However, this requires us to obtain the probability distribution of \( e_N \), which is a difficult problem.
4.3 Extension to Arbitrary Number of Users Per Cell

The results of the previous sections will now be extended to the case where there are an arbitrary number of users in each cell.

4.3.1 Joint Solution

Assume cell $i, i = 1, 2$, has $K_i$ users. Some of the notation introduced in Section 4.1.1 needs to be modified as follows:

$$\text{CIR}_i^k : \text{carrier-to-interference ratio of user } k \text{ in cell } i,$$
$$p_i^k(n) : \text{transmit power of user } k \text{ in cell } i,$$
$$H_{ij}^k(n) : \text{power gain from user } k \text{ in cell } j \text{ to base } i,$$
$$d_i^k(n) : \text{desired transmit power of user } k \text{ in cell } i.$$

The CIR constraints for a particular user $k$ in cells 1 and 2 are

$$\text{CIR}_1^k = \frac{H_{11}^k(n)p_1^k(n)}{\sum_{l \neq k} H_{11}^l(n)p_1^l(n) + \sum_{i=1}^{K_2} H_{12}^i(n)p_2^i(n) + \sigma_i^2} \geq \gamma, \ 1 \leq k \leq K_1,$$  \hspace{1cm} (4.74)

$$\text{CIR}_2^k = \frac{H_{22}^k(n)p_2^k(n)}{\sum_{l \neq k} H_{22}^l(n)p_2^l(n) + \sum_{i=1}^{K_1} H_{21}^i(n)p_1^i(n) + \sigma_2^2} \geq \gamma, \ 1 \leq k \leq K_2.$$  \hspace{1cm} (4.75)

As stated before, the desired transmit power for each user is obtained from the minimum power solution to the CIR constraints. There are two simplifications that can be made. First, the minimum power solution is obtained when the constraints are met with equality. Second, the dimensionality of the problem can be reduced to two when the constraints are met with equality. The reason is that users within a cell must be received at the same power level in order to achieve equality in the constraints. Hence, at the minimum power solution, we have,

$$H_{ii}^k(n)p_i^k(n) = q_i(n), \ 1 \leq k \leq K_i,$$  \hspace{1cm} (4.76)
where \( q_i(n) \) denotes the received power of each user in cell \( i \). Rewriting the CIR constraints in terms of the received power, we have

\[
\frac{q_1(n)}{(K_1 - 1)q_1(n) + \sum_{l=1}^{K_i} \frac{H_{1l}^k}{H_{11}^k} q_2(n) + \sigma_i^2} = \gamma,
\]

(4.77)

\[
\frac{q_2(n)}{(K_2 - 1)q_2(n) + \sum_{l=1}^{K_1} \frac{H_{2l}^k}{H_{11}^k} q_1(n) + \sigma_2^2} = \gamma,
\]

(4.78)

where the inequality was replaced with equality. In matrix form this becomes

\[
\begin{bmatrix}
1 - \gamma(K_1 - 1) & -\gamma \sum_{l=1}^{K_2} \frac{H_{1l}^k}{H_{11}^k}
\end{bmatrix}
\begin{bmatrix}
F_n
\end{bmatrix}
\begin{bmatrix}
q_1(n) \\
q_2(n)
\end{bmatrix}
= \begin{bmatrix}
\gamma \sigma_1^2 \\
\gamma \sigma_2^2
\end{bmatrix}.
\]

(4.79)

Assuming \( F_n \) is invertible, the desired received power vector \( q_n^* \) is the received power vector \( q_n \) that satisfies (4.79),

\[
q_n^* = F_n^{-1} \gamma
\]

(4.80)

\[
= \frac{1}{\det(F_n)} \begin{bmatrix}
1 - \gamma(K_2 - 1) & \gamma \sum_{l=1}^{K_2} \frac{H_{1l}^k}{H_{11}^k}
\end{bmatrix}
\begin{bmatrix}
\gamma \sigma_1^2 \\
\gamma \sigma_2^2
\end{bmatrix}.
\]

(4.81)

Hence, the desired transmit power of user \( k \) in cell \( i \) is

\[
d_i^k(n) = \frac{q_i^*(n)}{H_{ii}^k(n)}, \quad 1 \leq k \leq K_i.
\]

(4.82)

Now, a feasible solution exists if and only if

\[
1 - \gamma(K_1 - 1) > 0,
\]

(4.83)

\[
1 - \gamma(K_2 - 1) > 0,
\]

(4.84)

\[
\det(F_n) > 0.
\]

(4.85)

Again, we observe that the joint solution to the CIR constraints will lead to complicated PCAs since the desired transmit power of each user is a function of the power gains of each user to its own base and the power gains of each user to the adjacent base.
4.3.2 Iterative Solution

The form of the iterative solution for an arbitrary number of users is similar to the one derived in Section 4.1.2. From the CIR constraints shown in (4.74) and (4.74), we can write the desired transmit power of a particular user $k$ in cells 1 and 2 in terms of the interference that user experiences with appropriate scaling,

$$
d^k_1(n) = \frac{\gamma}{H^k_{11}(n)} \left[ \sum_{l \neq k} H^l_{11}(n) p^l_1(n) + \sum_{l=1}^{K_2} H^l_{12}(n) p^l_2(n) + \sigma^2 \right], \quad 1 \leq k \leq K_1 \tag{4.86}
$$

$$
d^k_2(n) = \frac{\gamma}{H^k_{22}(n)} \left[ \sum_{l \neq k} H^l_{22}(n) p^l_2(n) + \sum_{l=1}^{K_1} H^l_{21}(n) p^l_1(n) + \sigma^2 \right], \quad 1 \leq k \leq K_2. \tag{4.87}
$$

This can be rewritten in matrix form as follows:

$$
\begin{bmatrix}
  d_1(n) \\
  d_2(n)
\end{bmatrix} =
\begin{bmatrix}
  A_{11}(n) & A_{12}(n) \\
  A_{21}(n) & A_{22}(n)
\end{bmatrix}
\begin{bmatrix}
  p_1(n) \\
  p_2(n)
\end{bmatrix}
+
\begin{bmatrix}
  b_1(n) \\
  b_2(n)
\end{bmatrix}, \tag{4.88}
$$

where

$$
\begin{align*}
   d_i(n) &=
\begin{bmatrix}
   d^1_i(n) \\
   d^2_i(n) \\
   \vdots \\
   d^{K_i}_i(n)
\end{bmatrix},
   p_i(n) =
\begin{bmatrix}
   p^1_i(n) \\
   p^2_i(n) \\
   \vdots \\
   p^{K_i}_i(n)
\end{bmatrix},
   b_i(n) =
\begin{bmatrix}
   \gamma \sigma^2 \\
   \gamma \sigma^2 \\
   \vdots \\
   \gamma \sigma^2 \\
   \frac{H^2_{ii}(n)}{H^k_{ii}(n)} \\
   \frac{H^k_{ii}(n)}{H^k_{ii}(n)} \\
   \frac{H^{K_i}_{ii}(n)}{H^{K_i}_{ii}(n)} \\
   \frac{H^{K_i}_{ii}(n)}{H^{K_i}_{ii}(n)} \\
   0
\end{bmatrix},
\end{align*}
$$

$$
A_{ij}(n) =
\begin{cases}
   \frac{H^2_{ij}}{H^2_{ii}} & \frac{H^{K_j}}{H^k_{ii}} \\
   \frac{H^1_{ij}}{H^1_{ii}} & \frac{H^{K_j}}{H^k_{ii}} \\
   \frac{H^2_{ij}}{H^2_{ii}} & \frac{H^{K_j}}{H^k_{ii}} \\
   \vdots & \vdots \\
   \frac{H^{K_i}_{ij}}{H^{K_i}_{ii}} & \frac{H^{K_j}}{H^k_{ii}} \\
   \frac{H^{K_i}_{ij}}{H^{K_i}_{ii}} & \frac{H^{K_j}}{H^k_{ii}} \\
   \gamma & \gamma \\
   \frac{H^1_{ij}}{H^1_{ii}} & \frac{H^{K_j}}{H^k_{ii}} \\
   \frac{H^2_{ij}}{H^2_{ii}} & \frac{H^{K_j}}{H^k_{ii}} \\
   \vdots & \vdots \\
   \frac{H^{K_i}_{ij}}{H^{K_i}_{ii}} & \frac{H^{K_j}}{H^k_{ii}} \\
   \frac{H^{K_i}_{ij}}{H^{K_i}_{ii}} & \frac{H^{K_j}}{H^k_{ii}}
\end{cases}, \quad i \neq j. \tag{4.90}
$$

50
Again, the desired transmit power of user \( k \) in cell \( i \) is expressed directly as a function of all the other users' transmit power. If each user transmits at exactly the desired power, that is \( p_n = d_n \), then from (4.88) we obtain

\[
d_n = (I - A_n)^{-1} b_n. \tag{4.91}
\]

It can be verified that this gives the same expression for the desired transmit power for user \( k \) in cell \( i \) shown in (4.82).

As before, the iterative PCA is

\[
\begin{align*}
p_0 & = p' = \text{initial transmit power vector}, \quad (4.92) \\
p_{n+1} & = d_n = A_n p_n + b_n, \quad n \geq 0.
\end{align*}
\]

Assuming there is no fading, this PCA will converge to the desired transmit power as long as the magnitude of each eigenvalue of \( A_n \) is less than one. The rate of convergence will be largely determined by the dominant eigenvalue. In the presence of fading, the performance of the PCA will be affected by the coherence time of the channel, the rate of convergence during each channel state, and the amount of change between channel states.

The development and performance analysis of the iterative PCA extends in a very straightforward manner to a multiple cell network with an arbitrary number of users. Appendix A highlights this extension.
Chapter 5

Conclusion

Power control algorithms are developed for both the single cell network and the two cell network. The algorithms are designed to maintain a minimum carrier-to-interference ratio (CIR) for each user in the presence of fading. Central to the power control algorithms for the single network is minimum mean squared error (MMSE) estimation. In determining the appropriate transmit power, either the power gains are estimated or the transmit power itself is estimated based on past measurements of the power gains. The estimation procedure uses a Gauss-Markov model for the channel gain. The performance of the estimation algorithms is dependent upon the fading rate. In contrast, the power control algorithm for the two cell network uses an iterative approach since algorithms based on estimation are too complex in this case. The performance of the iterative algorithm is determined by the rate of change of the channel, the rate of convergence of the algorithm, and the amplitude range over which the channel varies. A straightforward extension of the iterative algorithm to a general multiple cell network is also presented.

The power control algorithm development presented in this work is a starting point. Some refinements to this work are in order. First, the statistical characterization of the power gains is based on the narrowband signaling model for fading multipath channels. Cellular systems that are classified as wideband signaling, such as outdoor spread systems, require a characterization of the power gains based on the wideband signaling model. For these systems, the power control algorithms need to be modified accordingly. Second, throughout the analysis, it is assumed that the base station can make perfect measurements of the power gains. Power control algorithms that take into account imperfect measurements
can be developed using a Kalman filtering approach to estimate the power gains. For this approach, a Gaussian distribution on the measurement noise is assumed.
Appendix A

Iterative PCA for Multiple Cell Network

We extend the iterative PCA introduced for the two cell network to a general multiple cell network. We assume a cellular spread system with $K$ users and $M$ cells. The following notation will be used:

\[
\begin{align*}
\text{CIR}_i & : \text{ carrier-to-interference ratio of user } i, \\
p_i(n) & : \text{ transmit power of user } i, \\
H_{ij}(n) & : \text{ power gain from user } j \text{ to base } i, \\
b_i & : \text{ assigned base of user } i, \\
d_i(n) & : \text{ desired transmit power of user } i.
\end{align*}
\]

Since a user can be located in any one of the $M$ cells, $b_i \in \{1, 2, \ldots, M\}$ for all $i$. This notation relieves us from having to identify the cell at which a user is located and prevents the resulting equations from looking unwieldy.

The CIR constraint for user $i$ is

\[
\text{CIR}_i = \frac{H_{b_i}(n)p_i(n)}{\sum_{j \neq i} H_{b_j}(n)p_j(n) + \sigma_{b_i}^2} \geq \gamma, \quad 1 \leq i \leq K.
\]  \hspace{1cm} (A.1)
From this we can write the desired transmit power of user $i$,

$$d_i(n) = \frac{\gamma}{H_{b_i}(n)} \left[ \sum_{j \neq i} H_{b_{ij}}(n)p_j(n) + \sigma^2_{b_i} \right], \quad 1 \leq i \leq K,$$

(A.2)

which can be rewritten in matrix form as follows,

$$
\begin{bmatrix}
  d_1(n) \\
  d_2(n) \\
  \vdots \\
  d_K(n)
\end{bmatrix}
= \gamma
\begin{bmatrix}
  0 & H_{b_{12}}(n) & \cdots & H_{b_{1K}}(n) \\
  H_{b_{21}}(n) & 0 & \cdots & H_{b_{2K}}(n) \\
  \vdots & \vdots & \ddots & \vdots \\
  H_{b_{K1}}(n) & H_{b_{K2}}(n) & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
  p_1(n) \\
  p_2(n) \\
  \vdots \\
  p_K(n)
\end{bmatrix}
+ \gamma
\begin{bmatrix}
  \sigma^2_{b_1} \\
  \sigma^2_{b_2} \\
  \vdots \\
  \sigma^2_{b_K}
\end{bmatrix}.
$$

(A.3)

The iterative PCA remains unchanged,

$$
\begin{align*}
p_0 &= \mathbf{p}' = \text{initial transmit power vector}, \\
p_{n+1} &= \mathbf{d}_n = A_n\mathbf{p}_n + \mathbf{b}_n, \quad n \geq 0.
\end{align*}
$$

(A.4)

As before, assuming there is no fading, this PCA will converge to the desired transmit power as long as the magnitude of each eigenvalue of $A_n$ is less than one. The rate of convergence will be largely determined by the dominant eigenvalue. In the presence of fading, the error dynamics analysis presented in Section 4.2.1 applies here as well.
Bibliography


