THE STABILITY AND TRANSIENT RESPONSE OF
INDUSTRIAL ORGANIZATIONS

by

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ABSTRACT

Many firms and industries exhibit large fluctuations in sales, inventories, labor force and production output. These are caused by the information feedback nature of their operations. This paper is a presentation of the analysis of one such industry using the tools of servomechanisms theory and the techniques of time simulation. A systematic approach to the improvement of the system's behavior is also included.

The analysis indicates that the industry's present methods of making decisions for ordering, hiring, production scheduling and factory priority have created an oscillatory though stable system which is overly sensitive to disturbances with periods of from one to two years. In addition, inventories are being used to amplify variations in order rate, rather than to smooth them. Thus, production output varies more widely than sales.

It is found that the behavior can be improved through the untangling of conflicting decisions and the development of procedures which reduce sensitivity and use inventories more advantageously. At the same time, the improved decisions recognize the potential problem of product obsolescence and, therefore, provide sufficient protection against a sudden decline in product usage.

The limitations of the use of a linearized model for the frequency analysis are discussed along with the problems of parameter estimation, model acceptability, decision aggregation, implementation of the results in a specific situation and generalizations of the results to other environments. It is suggested that this is just a first step toward improvement and that many extensions are possible.

Thesis Supervisor: Jay W. Forrester
Title: Professor of Industrial Management
May 21, 1961

Professor Philip Franklin
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Professor Franklin:

In accordance with the requirements for graduation, I herewith submit a thesis entitled "The Stability and Transient Response of Industrial Organizations".

I should like to take this opportunity to thank Professor Jay W. Forrester for his patience and constructive criticism during the preparation of this document. I am also indebted to the Sprague Electric Company, which sponsored the research project upon which the thesis is based and cooperated to the fullest extent with our studies of their operations.

The model simulations and frequency analyses were made possible through the use of the IBM 709 digital computer at the Massachusetts Institute of Technology Computation Center.

Sincerely yours,

Willard Russell Fey
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CHAPTER I
INTRODUCTION

Many firms and industries experience large fluctuations in sales, inventories, profits, production, labor force, and order backlogs. These oscillations may exhibit growing or declining amplitudes, and they may be superimposed on rising or falling trends. Such are the familiar characteristics of servomechanism (information feedback) type systems of which industrial organizations are but one class. Recognition of the feedback nature of these systems leads to a different kind of analysis and different solution conclusions than are usually found in economic theory. The following is a servomechanism-oriented analysis of an electronic component industry which shows this type of fluctuating behavior. Since the industry is neither linear nor mathematically simple, the formal tools of feedback theory are not powerful enough to solve the problems in a satisfactory way. Therefore simulation techniques must be used in conjunction with the formal theory to produce acceptable, but nonoptimum, improvements.

The objectives, then, of this study are these:

1. to show the servomechanism type nature of economic systems,

2. to analyze and modify a given economic situation using both time simulation techniques and feedback control theory to improve substantially the system's behavior,
3. to show the close relationship between the two analysis procedures as well as the complementary nature of the results each provides,
4. to extend and formalize some of the existing analysis and synthesis techniques to handle more complex, nonlinear situations, and
5. to point out weak points and problems in the theory as applied to complex economic systems and in the analysis of this particular industry.

To reach these goals, the presentation is organized into five chapters which deal with a description of the component industry (II); a derivation of a mathematical model of the industry's structure (III); an analysis of the model using simulation techniques and feedback control theory (IV); the synthesis of a better model (V); and conclusions about a better decision structure for the industry, the interpretation and useful extent of the analysis techniques, and unresolved problems (VI).

The study indicates that a different set of decision policies, if employed by the manufacturing firms in the industry, could greatly improve the stability and profitability of their operations. With regard to the analysis techniques, there is evidence that the combined use of time and frequency studies is more powerful than either alone and that a number of principles are useful in the creation of better systems. A survey of the problem areas indicates that there is ample room for more extensive study.
CHAPTER II
DESCRIPTION OF AN ECONOMIC SYSTEM

The economic situation to be described is an industry that manufactures and uses one type of high quality electronic component. The information upon which the following description and model were based was obtained from the Sprague Electric Company, North Adams, Massachusetts, which sponsored the research project.

This chapter describes the structure of the industry and the dynamic behavior that it has experienced to serve as a background for the derivation of the model in the following chapter.

II.1 Structure of an Electronic Component Industry

An understanding of this system requires a knowledge of the characteristics and uses of the product and the parts of the system which influence its response. These are discussed in the two following sections.

II.1.1 Product Characteristics

This product line of high quality electronic components has been active for over ten years. It is in the mature phase of its life cycle. That is the time after research, development, and intensive market expansion when the industry's operating structure has become relatively fixed and the extent of the market has been clearly defined. Sales volume is still rising, but at a less rapid rate. The profitability, although still substantial, has begun to decline. New competitors are only slowly entering the market, and competition centers on speed of delivery, rather than price.
The units are used in large military and industrial systems in which reliability and accuracy are required. Since these systems are large and varied, the line must be broad. It includes some five thousand different active catalogue items where catalogue item differences may refer to tolerance, voltage rating, mounting style, electrical value, or temperature rating. Furthermore, in most cases the component cost is a small fraction of the total cost of each system, so price is not a critical problem for the customers.

There are many customers, whose total weekly order volume fluctuates widely. The individual order size varies from 1 to 20,000 units. The requested delivery time may vary from two weeks ago to a year in the future, although about half of the orders request delivery as soon as possible. In addition to a large week-to-week variation, the sales rate seems to exhibit a long term cycle with a period of two years, even though there is little, if any, such variability in the rate at which the units are used in the systems.

There are many competitors in the industry, with none controlling more than one quarter of the total business. The factors in the competition are quality, speed and reliability of delivery, and price in that order of importance. It is easiest and least expensive to compete actively on the basis of delivery time, so that is the means usually chosen. Most of the competing manufacturers have inventories of finished units which usually stock less than 10 percent of the total active catalogue items. The larger company inventories can supply more than 50 percent of units ordered, but most firms have small inventories and the business is primarily made-to-order. Due
to the high quality requirements and the breadth of the product line, almost all of the manufacturing work is done by hand, although a few machines are used. This means that production output is directly related to the size and efficiency of the work force.

II.1.2 Sectors of the System

It is necessary to study the economic environment of the firms as well as their internal functions. Thus consideration is given to the following industry sectors:

a. component users (customers)
b. component manufacturers
c. raw material suppliers
d. capital equipment manufacturers
e. the labor market
f. capital (money) suppliers

These sectors are connected through the flows of orders, materials, information, men, money, and capital goods as shown in Figure 1. It is the interaction of these flows, within and between these sectors, that produces the actions of the industry.

The following flows (see Figure 2 for the important ones) produce the industry behavior. The process starts when a company or the military orders a group of high quality electronic systems from a system manufacturer. This manufacturer, who does not have an inventory of finished systems, must design the system (if it has not been ordered before), make a parts list, check this against parts inventories, schedule production, manufacture the system, and ship the product to the buyer. The manufacturer must make sure that
Figure 1. Major sectors of the component industry.
Figure 2. Detailed diagram of the principal parts of the system.
employers, parts, and capital equipment come together at the proper time, so that time is not wasted in waiting. If employees, capital equipment, or parts are needed, they must be ordered. If money is needed to buy equipment or expand inventories of materials, it must be obtained. Seeing whether the proper parts are on hand and maintaining a buffer inventory for quick access on rush orders leads to the placing of orders for electronic components with one of the component manufacturers. The selection of the component supplier is influenced by quality, speed and reliability of delivery, price, habit (convenience), and desire to have more than one source, as well as less rational forces. The timing of an order depends on the expected delivery time and the time at which the customer will need the units.

These are one customer's operations with respect to one order. In fact, there are many customers, each continually receiving orders. Each operates differently in terms of quantities and timing, although each must perform all of the functions described in roughly the same sequence. If they did not, the necessary systems could not be produced.

The orders from the customers are handled in the following manner by the components manufacturers. The sales department receives the order, records it, checks customer's credit, and sends it to the engineering department. There, units are designed or specifications are verified and manufacturing orders are made. The orders proceed to manufacturing, where an inventory search is performed, if the order is for an inventory item. Orders that can be filled from
inventory are sent through a short final process and packaged. Then, they are taken to shipping and sent to the customer.

Orders that cannot be filled from inventory, because either they are not stock items or the inventory is too low in that item, go to production to be made-to-order. The order is scheduled and then proceeds through the manufacturing process and is shipped to the customer.

Several actions must be taken to support this manufacturing process. A decision must be made to order units to replenish inventory. These inventory orders go through the same manufacturing process that made-to-order customer orders use.

A decision must be made and appropriate action taken to hire or lay-off production workers. A decision to hire leads to an increase in the productive work force only after delays in obtaining the people and training them. A decision to reduce workers leads to a decrease in work force only after the required notice delay.

There must also be decisions to reorder raw materials and to increase or replace capital equipment. These lead to the acquisition of the required items after appropriate delays. In raw material ordering, these delays are short.

II.2 Dynamic Behavior

This system shows several types of dynamic behavior. These are oscillations at several characteristic frequencies which are superimposed on an essentially constant average sales level. The low frequency variation has a period of from one to two years and is
industry wide; the middle frequency oscillations, which are more irregular, have a period of three to five months and are internal to the manufacturers. There are also high frequency random fluctuations which range in period from two days to two weeks that are dominant in the sales data of the component manufacturers.

II.2.1 Low Frequency Oscillations

The low frequency component of the behavior is produced by the following sequence of events.

Suppose that system orders to the component customers (extreme left of Figure 2) begin to increase. The customers will soon begin to increase their orders for components. As the component suppliers' sales rise, their unfilled order backlog in manufacturing increases because the labor force that produces the output has not yet been changed. In this period, input orders exceed output. As unfilled backlog grows, the factory lead time increases, as does total service time. If the customers must order ahead on the basis of the service time they expect to encounter in order to keep their production lines in operation, then even if the input system orders were normal, the customers would continue to increase their component orders because the service delay is increasing. They continue to order at a rate higher than normal until their information indicates that the delay has declined. This information may indicate a change as long as ten weeks after the variation actually occurred.

The rising orders and backlog induce the component companies to hire production workers. However, there are delays between the increase in sales and the desire to hire and between the desire to hire
and the acquisition of fully productive employees. As a result, a sizable backlog may have accumulated before enough workers are hired and trained to make the production rate equal the order rate and thereby stabilize backlog. Further hiring then raises production above orders, so that backlog can be reduced toward an acceptable size. As the backlog decreases, the service delay drops and the customers reduce their orders. The orders continue to fall until the companies lay-off enough workers to stabilize the backlog. This usually occurs at a backlog and employment level which are below normal. With the below normal work force, production cannot keep up with the customers' basic needs and soon backlog begins to rise. Thus the cycle starts again. Due to the delays in hiring, sending orders, getting information about delay and manufacturing, this cycle takes from one to two years to repeat itself. This cycle is most clearly seen in data for one manufacturer's inventory shown in Figure 3 as a percent deviation from normal for three years, 1958-1960. The more erratic and higher frequency oscillation is also present in this data.

II.2.2 Middle Frequency Oscillations

Disturbances in the number of orders received by the manufacturers cause the companies' internal variables to oscillate in the middle frequency range (three to five months). An increase in manufacturer sales causes a decline in inventory. The reduction in inventory is corrected quickly by sending inventory orders into the factory. The increased backlog induces hiring to reduce it. If the increase in sales does not persist, the inventory correction will
Figure 3. Inventory variation for a component manufacturer as a percentage of the average.
have over-adjusted the labor force and the factory backlog will be reduced below normal before a compensating layoff is ordered. This oscillation has a period of three to five months, which is about the delay in having an inventory imbalance corrected through the inventory order-increased backlog-hiring-increased production loop.

An additional problem arises if the desired inventory used in the inventory ordering decision is based on a short-term average of sales. In this case, inventory orders are generated to raise the inventory level, so more orders are written than are necessary just to correct the declining inventory. Additional amplification is thereby introduced by the short-term variation in desired inventory. Therefore the shorter term fluctuation is produced within the manufacturer's operations by his short-term adjustment to changes in sales.

II.2.3 High Frequency Random Disturbances

When a plot of weekly sales data for one component manufacturer is first viewed, the dominant characteristic is the large-week-to-week variation which effectively hides the other modes of behavior. The analysis and model do not determine the cause of this noise, but its existence is recognized because it is the factor which excites the middle frequency modes of behavior of the manufacturing sector. Therefore, although the noise is not generated internally by the model, appropriate action must be taken to minimize its effect.
CHAPTER III

A MATHEMATICAL MODEL OF THE INDUSTRY

A mathematical representation of the relationships discussed in Chapter II is described here. Only the factors which significantly influence behavior are included. Therefore the relationships which were selected and the reasons behind these selections are taken up first. A brief discussion of the mathematical form precedes the derivation of the complete equation set. The procedure used to determine the parameter values and the method of ascertaining the acceptability of the model for the purposes of the study occupy the final two sections of this chapter.

III.1 Extent of the Model

In a model it is necessary to include a minimum amount—that is, enough to represent the major forces, but not enough to obscure the important lessons with secondary details. The system parts used here are the customers, the manufacturers, and the labor market. The representation of these sectors is based on the flows of orders, materials, information, and employees. This is shown in Figure 2. Notice that military and industrial system users, raw materials, capital equipment, money suppliers, and money flows have been omitted. Although this is an industry model (i.e., individual manufacturers are not considered separately), certain of the conclusions can be extended to be applicable to individual firms.

The military and civilian users of large-scale systems were not included in the study because they did not seem to be affected by
the operations of the industry. Thus their orders could be treated
as an independent input.

Money flows (costs, prices, etc.) are also excluded. The in-
vestigation concerns itself with the relationships that produce the
dynamic behavior of this line. Costs and prices did not appear to
be major influences in the dynamic decision making. This is not to
say that the industry is not interested in profits. It is to say
that in the day-to-day decisions that guide the system's over-all
behavior (production scheduling, hiring of manufacturing personnel,
ordering for inventory, and which orders to fill from inventory),
cost or profit considerations are not conspicuous. Unquestionably,
they will be consciously considered under unusual circumstances.
Capital equipment decisions and other major policy decisions are
influenced by costs even under normal conditions. But the majority
of the operating decisions in this line are made on the basis of
need (we are out of terminals, so order some more), not on the basis
of economic selection.

The omission of prices is less easily rationalized. To say
that prices have no influence on the firm's sales volume would be
untrue. However, there are two reasons for not including prices.
First, the principal influence of prices is to shift the customers'
demand from one manufacturer to another (the customers' total demand
is very inelastic). If individual companies are not considered, such
a shift of orders on the basis of price differentials cannot be con-
sidered. Secondly, price is the least important of the competitive
factors. The customers sometimes make systems for the government
on a cost plus fixed-fee basis. If any of the customers' costs can be passed on to the government, component prices lose some of their importance. In addition, the cost of these components is usually a very small portion of the systems' total cost. Thus incremental changes in component prices are not worth much of the customers' time to worry about. They are more interested in getting the units quickly, so they do not have to shut down production lines.

The company's raw materials are neglected because they do not significantly limit decision making. Materials are neither bulky nor expensive, so sizable inventories can be kept. In addition, it takes only a short time to receive materials after ordering.

Finally, capital equipment is omitted because the bulk of the production process is performed by hand. Only one operation is done by machine, and this is not a limiting factor. The existing machine capacity is so far above normal operations that it would become a factor only under extremely unusual conditions.

The study is thus focused on the customers, the manufacturers, and the labor market on the basis of two principles:

1. Omit all unnecessary detail.

2. Include only those functions which operate within the influence of the important information feedback loops.

III.2 Equation Characteristics

An attempt has been made to quantitatively represent the relationships in this system with difference equations for simulation and equivalent differential equations for frequency analysis. The
equations relate the aggregate flows of orders, materials, men, and information.

III.2.1 Equation Form

The difference equations used in the simulation are no higher than first-order and must be solved sequentially (not simultaneously in matrix form). Each variable is represented by an equation, and a value for each variable is found at each solution time. These solution times are separated by the computational time interval, DT (Delta Time). The simulation solution is calculated and plotted by the DYNAMO compiler program for the IBM 709 Digital Computer. The equations are nonlinear when necessary to represent the actual operation.

Two principal types of variables, "levels" and "rates", are used. A third equation type, "auxiliaries", is used for convenience. Levels are variables that measure quantities which would be countable if the system were brought to rest. Inventories, number of employees, and unfilled order backlogs are common examples. A single time subscript indicates that the level variable takes on its value at a specified instant of time (K is taken as the present instant; J is the instant preceding K with a time DT separating J and K), as shown in Figure 4.

Rates are representations of the flows of orders, materials, men, or information. They are in terms of the amount of the flow per unit of time. Thus orders per week, men hired per week, and units depleted from inventory per week are examples of rates. If time could be made to stand still, all rates would be zero. The
Figure 4. Levels and rates as computed for the model.

double letter time subscript for rates indicates that the variable takes on and holds its value during the time interval specified (JK or KL). The levels at the present time, K, are calculated using previous rates and levels. Then the rates for the KL interval are calculated on the basis of the K level values and previous rate values. After each solution time the J, K, and L labels are shifted ahead one DT, so that K always refers to the present.

In order to facilitate the visualization of combinations of variables that have some physical significance in their consolidated form, "auxiliary" variables are used. Desired inventory, which is sometimes represented as a number of weeks times average sales, is an auxiliary. Auxiliaries, like levels, have a single letter time subscript and their values apply only at the calculation instant.

The format for variable notation and time subscripts is the following. Five or fewer letters preceding a period represent the mnemonic variable code, and the one or two letters following the period make up the time subscript. Thus, IFAC.K means Inventory,
Finished, Actual at Customer calculated at the present (K) instant of time. Variables with no time notation are constants or parameters that do not change with time.

Since these difference equations are to be simulated, initial values must be provided as a starting point for the variables represented by first-order (level) equations. These values are determined by assuming that no changes were occurring in any of the variables before the beginning of the simulation and solving for the values of the variables which satisfy that condition. In this way all of the variation during the simulation can be attributed to changes in the input function. In the equation derivation an initial value is specified for each variable, even though the simulation does not require initial conditions for algebraic relations. This is done for the reader's convenience.

Part of the analysis is based on linear servomechanisms theory. Therefore each difference equation must be represented in linear, continuous form. The Laplace transform of the linear, continuous counterpart of each difference equation is given in the equation derivation. This is called the transformed equation. The transformed equation is identified with a T after the equation number. Level, rate, and auxiliary equations are indicated by an L, R, and A, respectively, after the equation number. Initial condition equations are represented by an N.

A model cannot explicitly account for every individual decision and transaction taking place in a system of this size. The aggregate effect of similar activities must be represented. Thus the model
does not distinguish between orders for different catalogue items or between units produced at the two plants. This aggregation is a difficult problem, but it was done on an intuitive basis after a study of the individual actions.

The equations were not derived with optimization techniques in mind because optimization is not a part of this analysis. Thus great flexibility is allowed in representing the relationship.

III.2.2 Nonlinearities

A model, system, or set of equations is said to be linear if all the equations that relate its variables are linear. An equation that relates \( y(t) \), a dependent variable, and its first \( n \) derivatives, \( y^{(n)} = \frac{d^n}{dt^n} y(t) \), to an explicit time function \( x(t) \), the "driving" function, must be of the following form to be linear.

\[
c_n(t)y^{(n)} + c_{n-1}(t)y^{(n-1)} + \ldots + c_1(t)y^{(1)} + c_0(t)y = x(t)
\]  

Here \( t \) is time, the independent variable, and the \( c_i(t) \)'s are the coefficients of \( y(t) \) and its derivatives. These coefficients can only be constants or explicit functions of time if linearity is to be maintained. If the right-hand side of Equation 1 is a function of \( m \) implicit time variables \( x_1, x_2, \ldots, x_m \) (other system variables that are not represented as explicit time functions) and their first \( n \) derivatives, it must be of the following form to be linear.
\[ x(t) = a_{00}(t) + a_{10}(t)x_1(t) + a_{11}(t)x_1(t) + \ldots + a_{ln}(t)x_l(t) + \ldots + a_{20}(t)x_2(t) + a_{21}(t)x_2(t) + \ldots + a_{2n}(t)x_2(t) + \ldots + a_{m0}(t)x_m(t) + a_{m1}(t)x_m(t) + \ldots + a_{mn}(t)x_m(t) \]

Here the \( a_{ij}(t) \) coefficients of the \( x \)'s and their derivatives can only be constants or explicit functions of time, and \( x^{(n)}(t) = \frac{d^n}{dt^n} x(t) \). Therefore, products, ratios, or powers other than the first of the \( x \)'s or their derivatives are not linear forms.

The most important property of linear equations or systems is that of superposition. This principle states that if an independent input \( i_1(t) \) causes the system to produce an output \( o_1(t) \) and if the input \( i_2(t) \) produces an output \( o_2(t) \), the introduction of the input \( K_1 i_1(t) + K_2 i_2(t) \), where \( K_1 \) and \( K_2 \) are constants, produces an output of the form \( K_1 o_1(t) + K_2 o_2(t) \). This characteristic makes it possible to analyze the system behavior for any one input independently of all other inputs and for any scale range. An extension of this principle implies that the output of a linear system cannot include frequency components which were not present in the input signal. This principle and others have led to the development of many formal analysis and synthesis procedures which make it possible to learn quickly a great deal about the behavior and potential improvements of linear systems.

Nonlinear system analysis has not produced any general analysis procedures, even though a few particular cases have been solved.
formally. This is because of the great difficulty in dealing with systems in which changes in the scale of the input can cause major changes in system behavior, frequency components which are not present in the input can be generated in the system, and the response to one input component may be directly related to the response to all other parts of the input.

If the system of interest is nonlinear, two approaches to an analysis may be possible. The first is to represent all the nonlinearities as they are and simulate the system's behavior under many different input conditions in an attempt to gain an understanding of how the system operates. This is the experimental approach. Based on the understanding thus gained, changes can be made in the system. Simulations of the new systems using the same inputs can be compared, and the best system can then be chosen on the basis of the comparison. This method can always be used.

The second method is to linearize the system and proceed with the usual linear analysis. This is done by taking Taylor series expansions of the equations about the operating values of the variables and neglecting the higher than first-order powers of the variables. When this is done, the variables used are "incremental" variables rather than total variables. Thus the variable represents the deviation from the average (or steady-state) value rather than the total absolute value which includes the average value. An incremental variable satisfies the equation,

\[ \text{Total value} = \text{average or steady-state value} + \text{incremental value} \]

This method is usable as long as the excursions of the variables
remain in the relatively linear region near the average value. As the deviation becomes greater, the assumption of linearity is less accurate and the modes of behavior caused by the nonlinear relations are not represented in the system's linear counterpart. Thus for small signals the approximation may be good, but larger inputs are not handled properly by the linearized model. Thus linearized incremental analysis is sometimes called small signal analysis.

In the equation derivation that follows, both the total nonlinear equation and the incremental linear equation are presented so that analysis methods can be used. A later section indicates the accuracy of the linearized model in this case.

III.3 Model Equations

The equations for the three sectors—customers, manufacturers, and labor market—are derived in this section. Each equation has four different forms which are present together in this order.

1. The nonlinear total difference equation.

2. The linear, incremental difference equation that corresponds to 1 (included only if it is different from 1)

3. The Laplace transform of the differential form that corresponds to 2.

4. The value of the variable in its steady-state condition.

The simulations are obtained from the 1, 2, and 4 forms. The transfer functions and frequency analysis use form 3.
III.3.1 Customer Equations

Seven equations are needed to describe the customers' behavior. These represent the independent system orders, the inventory of components, the components on order, the component ordering policy, and three concepts (desired component inventory, desired components on order, and average system order level) used in the ordering decision. One other quantity used in this decision (expected delivery delay) is generated in the manufacturer sector. Figure 5 is a flow diagram of the customer equations. This diagram includes all of the customer relations, and the text equation numbers correspond to the numbers on the diagram. A diagram of the flows between model sectors is shown in Figure 6.

III.3.1.1 Independent Input

The independent input is the rate, RREC, at which the military and industry order electronic systems. The equation permits the use of a sinusoidal variation, with amplitude ASWC and period PSWC, a step change (of height ASAC) or a ramp increase or decrease with a slope of SRAC as a test input. The steady-state value of the input is STST.

\[
\begin{align*}
RREC_{KL} &= (1)(STST) + (ASWC)\sin((2\pi) \\
&\quad \quad \quad \quad \quad (TIME.K)/PSWC) \times \text{STEP}(ASAC, TSAC) \\
&\quad \quad \quad \quad \quad + \text{RAMP}(SRAC, TRAC) \\
RREC_{KL} &= (ASWC)\sin((2\pi)(TIME.K)/PSWC) \times \text{STEP}(ASAC, TSAC) \\
&\quad \quad \quad \quad \quad + \text{RAMP}(SRAC, TRAC) \\
RREC &= ((ASWC)(2\pi)/PSWC)/(s^2 + (2\pi)^2/(PSWC)^2) \\
&\quad \quad \quad \quad \quad + (ASAC/s) + (SRAC/s^2)
\end{align*}
\]
Figure 5. Flow diagram for the customer sector equations.
Figure 6. Material, order, information and labor flows between model sectors.
RREC—Requisitions Received in Estimating and scheduling department at Customer (units/week)
STST—Steady State input to the customer
   = 100,000 (units/week)
ASWC—Amplitude of the input Sine Wave at the Customer
   = 10,000 (units/week)
SIN—DYNAMO Notation for the SINE wave function
PI—π, a mathematical constant = 3.1416...
TIME—TIME (weeks)
PSWC—Period of the Sine Wave at the input to the Customer
   = 52 (weeks)
STEP—DYNAMO Notation for the STEP function
ASAC—Amplitude of input Step A at the Customer
   = 20,000 (units/week)
TSAC—Time that Step A starts, at the Customer
   = 5 (weeks)
RAMP—DYNAMO Notation for the RAMP function
SRAC—Slope of input RAMP at Customer
   = 200 (units/week)
TRAC—Time that RAMP starts at Customer
   = 5 (weeks)
s—Laplace frequency variable (1/weeks)

III.3.1.2 Inventory

The customers' inventory, IFAC, is augmented by the units, UAIC, that have been received by the customers. Depletion of inventory is due to withdrawals, RREC, for production. The present inventory level, IFAC.K, equals the inventory last period, IFAC.J, plus additions, UAIC, minus withdrawals, URMIC, since last period. Additions and withdrawals are rates (units per time period), and the computational interval, DT, is expressed in terms of the same time period. Multiplying UAIC minus RREC by DT gives the number of units by which the inventory changed during the computational interval. It is assumed that the DT is short enough so that all rates can be assumed steady during the computing interval.
Since in steady state the inventory cannot be changing, the additions to inventory, UAIC, and the withdrawals, RREC, must equal the steady-state input, STST. For this to be possible, the actual inventory must equal the desired inventory in the ordering equation.

\[ IFAC.K = IFAC.J + (DT)(UAIC.JK - RREC.JK) \]

\[ IFAC = (UAIC - RREC)/s \]

\[ IFAC = (AIDC)(STST) \]

**IFAC**—Inventory Finished Actual at Customer (units)

**DT**—Delta Time, the computation interval = 0.2 (week)

**UAIC**—Units Arriving at Inventory at Customer (units/week)

**RREC**—Requisitions Received in Estimating and scheduling department at Customer (units/week)

s—Laplace frequency variable (1/weeks)

**IFDC**—Inventory Finished Desired at Customer (units)

**AIDC**—constant defining Inventory Desired at Customer = 10 (weeks)

**STST**—Steady State input to the customer = 100,000 (units/week)

### III.3.1.3 Actual Unfilled Orders

A variable considered in determining the number of units to order is the number of open orders, RUSC. It is equal to the difference between units ordered, RSPC, and units received in inventory, UAIC, summed over time.

\[ RUSC.K = RUSC.J + (DT)(RSPC.JK - UAIC.JK) \]

\[ RUSC = (RSPC - UAIC)/s \]

\[ RUSC = (DSOC)(STST) \]

**RUSC**—Requisitions Unfilled by Supplier for Customer (units)

**DT**—Delta Time, computation interval = 0.2 (week)

**RSPC**—Requisitions Sent to Purchasing department at Customer (units/week)
UAIC—Units Arriving at Inventory at Customer (units/week)
s—Laplace frequency variable (1/weeks)
DSOC—Delay in Supplier delivery in steady state at Customer (weeks)
STST—Steady State input to the customer = 100,000 (units/week)

III.3.1.4 Reorder Decision

The customers' ordering decision is their most important characteristic. There are three considerations in making this decision. The first is that the number of components required by the systems that have been ordered must be obtained. The second is to adjust the inventory to its desired level. The third is to keep enough orders in the supply pipeline so the continuity of the production process can be maintained. The ordering equation handles these objectives by setting the ordering rate equal to the incoming system orders plus a fraction, 1/DCOC, times the difference between desired, IFDC, and actual, IFAC, inventory plus the same fraction times the difference between desired, RUDC, and actual, RUSC, pipeline orders. An additional term, NRDC, is included to provide a random disturbance.

A negative order rate implies cancelled orders. Since the real system handles cancellations differently than it handles new orders, the model is not accurate in this respect. However, the cancellation rate is negligibly small.

\[ RSPC_{KL} = RREC_{JK} \times \left( \frac{1}{DCOC} \right) (IFDC_{K} - IFAC_{K} + RUDC_{K} - RUSC_{K}) + NRDC_{K} \]

\[ RSPC = RREC + \left( \frac{1}{DCOC} \right) (IFDC - IFAC + RUDC - RUSC) \]

RSPC—Requisitions Sent to Purchasing department at Customer (units/week)
RREC—Requisitions Received in Estimating and scheduling department at Customer (units/week)
DOOC—Delay in Changing the Order rate at Customer = 3 (weeks)
IFDC—Inventory Finished Desired at Customer (units)
IFAC—Inventory Finished Actual at Customer (units)
RUDC—Requisitions Unfilled Desired by Customers (units)
RUSC—Requisitions Unfilled at Supplier from Customer (units)
STST—Steady State input to the customer
       = 100,000 (units/week)
NRDC—Noise in Reorder Decision at Customer (units/week)

Note: NRDC is determined by taking a random sample from the rectangular distribution shown in Figure 7 and holding it for one week.

![Probability of NRDC](image)

Figure 7. Probability distribution for noise in customers' ordering decision

WNRC—amplitude of Weekly Noise in Reorder decision at Customer
       = 80,000 (units/week)

III.3.1.5 Desired Inventory

The purpose of the customers' component inventory is to provide a buffer between incoming orders and production. It follows that the customer would not keep an inventory if he could rely on instantaneous delivery. Therefore the desired inventory depends on the
average service time. Desired inventory is also a function of the anticipated production rate. Obviously, it would not make any difference how long it took to get a unit, if it were never needed. Therefore we have assumed that desired inventory is proportional to the product of exponentially smoothed sales, RRSC, and a constant number of weeks, AIDC. This constant is related to the average service time and the customer's idea of a safety factor, based on the variability of incoming orders.

\[
\begin{align*}
IFDC.K &= (AIDC)(RRSC.K) \\
IFDC &= (AIDC)(RRSC) \\
IFDC &= (AIDC)(STST)
\end{align*}
\]

- IfDC--Inventory Finished Desired at Customer (units)
- AIDC--constant defining Inventory Desired at Customer = 10 (weeks)
- RRSC--Requisitions Received and Smoothed at Customer (units/week)
- STST--Steady State input to the customer = 100,000 (units/week)

III.3.1.6 Desired Units on Order

The ideal pipeline quantity, RU DC, is assumed to be the amount necessary to sustain production if average orders, RRSC, and the average time, DSIC, required for the company to fill customers' orders continue at their current levels.

\[
\begin{align*}
RU DC.K &= (RRSC.K)(DSIC.K) \\
RU DC.K &= (STST)(DSIC.K) + (DSOC)(RRSC.K) \\
RU DC &= (STST)(DSIC) + (DSOC)(RRSC) \\
RU DC &= (STST)(DSOC)
\end{align*}
\]

- RU DC--Requisitions Unfilled Desired by Customer (units)
- RRSC--Requisitions Received and Smoothed at Customer (units/week)
DSIC—Delay in Supplier delivery used in Ideal pipeline at Customer (weeks)
DSOC—Delay in Supplier delivery in steady state at Customer (weeks)
STST—Steady State input to the customer
   = 100,000 (units/week)

III.3.1.7 Average Input Orders

The desired inventory, IFDC, is computed in the model by taking an exponentially weighted average, RRSC, of past orders. This average is also used in calculating the number of units that the customer feels it is desirable to have outside of his inventory in the process of being filled.

\[ RRSC.K = RRSC.J \cdot \left( \frac{DT}{DSRC} \right) (RREC.JK - RRSC.J) \]

\[ RRSC = RREC/(1+(DSRC)(s)) \]

\[ RRSC = STST \]

RRSC—Requisitions Received and Smoothed at Customer (units/week)
DT—Delta Time, computation interval
   = 0.2 (week)
DSRC—Delay in Smoothing Requisitions at Customer
   = 10 (weeks)
RREC—Requisitions Received as the Exogenous input at Customer (units/week)
s—Laplace frequency variable (1/weeks)
STST—Steady State input to the customer
   = 100,000 (units/week)

III.3.2 Manufacturer Equations

The component manufacturers receive component orders from the customers, and in return send finished units and service delay information to the customers. The manufacturers also hire and lay-off employees to control their production output. The internal mechanism which makes this possible is shown in Figure 8. The equations which represent the system describe the following functions.
Figure 8. Flow diagram of the manufacturer sector equations.
1. Units filled from inventory
2. Units made-to-order
3. Total shipments
4. Finished inventory
5. Inventory reorder
6. Desired inventory and pipeline orders
7. Average sales used in desired inventory
8. Backlog of inventory orders in-process
9. Production of inventory units
10. Backlog of customer orders in-process
11. Production of customer units
12. Actual service delay
13. Service delay used by customer

III.3.2.1 Units Filled From Inventory

Incoming orders are filled from inventory, if possible, or, if units are not available in inventory, they are made to order. The proportion of the orders that can be filled from inventory is related to the mix of catalogue items in inventory relative to the mix in the incoming order flow, and it depends on the total size of the inventory. The experience of the test manufacturer seems to indicate that within normal inventory swings the filling proportion remains roughly constant. Therefore a constant fraction of the incoming order rate is taken as the rate, RCIS, at which orders are filled from inventory.

\[
\text{RCIS} = (\text{AISS}) \times (\text{RSPC.JK})
\]

\[
\text{RCIS} = (\text{AISS}) \times (\text{RSPC})
\]

\[
\text{RCIS} = (\text{AISS}) \times (\text{STST})
\]
RCIS--Requisitions from Customer to be filled from Inventory at Supplier (units/week)  
AISS--constant fraction of orders Initially filled from Stock at Supplier  
   = 0.7 (dimensionless)  
RSPC--Requisitions Sent to Purchasing department at Customer (units/week)  
STST--Steady State input to the customers  
   = 100,000 (units/week)

III.3.2.2 Units Made-to-Order

Orders that cannot be filled from inventory are sent to the factory to be made to order. This make-to-order rate, RCMS, is equal to the total order rate, RSPC, minus the rate at which orders are filled from inventory, RCIS.

RCMS.KL=(1-AISS)(RSPC.JK)  
RCMS=(1-AISS)(RSPC)  
RCMS=(1-AISS)(STST)

RCMS--Requisitions from Customer to be Made to order at Supplier (units/week)  
AISS--constant fraction of orders Initially filled from Stock at Supplier  
   = 0.7 (dimensionless)  
RSPC--Requisitions Sent to Purchasing department at Customer (units/week)  
STST--Steady State input to the customers  
   = 100,000 (units/week)

III.3.2.3 Total Shipments

Customer orders may be filled from inventory or made to order. The sum of the two output flows represents the total shipments to the customer. These shipments are delayed in the mails and in the shipping department. This delay is represented by a backlog, BCSS, of units in transit, an average delay, DCSS, and the output of the delay, UAIC, which is also the input to the customers' inventory.
It is also convenient to include the following delays at this point so they will be together.

1. Sales department delay
2. Delay in mailing orders to the manufacturers
3. Delay in customers' purchasing and receiving departments
4. Delay in searching inventory for the orders

The two equations that make up this delay are for the backlog, BCSS, and the output, UAIC.

\[
\begin{align*}
BCSS & = BCSS_{J} + (DT)(RCIS_{JK} + USMS_{JK} - UAIC_{JK}) \\
BCSS & = (RCIS_{+USMS_{-UAIS}}) / s \\
BCSS & = (DCSS)(STST) \\
UAIC_{KL} & = BCSS_{K} / DCSS \\
UAIC & = (RCIS_{+USMS}) / (1 + (DCSS)(s)) \\
UAIC & = STST
\end{align*}
\]

BCSS—Backlog of Customer orders in Shipping and processing operations at Supplier (units)
DT—Delta Time, computation interval = 0.2 (week)
RCIS—Requisitions from Customer to be filled from Inventory at Supplier (units/week)
USMS—Units Sent to shipping from Manufacturing at Supplier (units/week)
UAIC—Units Arriving at Inventory at Customer (units/week)
s—Laplace frequency variable (1/weeks)
DCSS—Delay in Customer order processing and Shipping at Supplier = 8 (weeks)
STST—Steady State input to the customers = 100,000 (units/week)

III.3.2.4 Finished Inventory

The inventory equation is a simple accounting relation. The inventory at the present time equals the inventory at the beginning
of the last period plus what was received from manufacturing, URIS, during the period minus what was taken from inventory, RCIS.

\[ IFAS.K = IFAS.J + (DT)(URIS.JK - RCIS.JK) \]
\[ IFAS = (URIS - RCIS)/s \]
\[ IFAS = (ADIS)(STST) \]

IFAS—Inventory, Finished, Actual at Supplier (units)
DT—Delta Time, computation interval = 0.2 (week)
URIS—Units Received at Inventory at Supplier (units/week)
RCIS—Requisitions from Customer to be filled from Inventory at Supplier (units/week)
ADIS—constAnt for Desired Inventory at Supplier = \( \frac{R}{W} \) (weeks)
STST—Steady State input to the customer = 100,000 (units/week)

III.3.2.5 Inventory Reorder Decision

The inventory ordering rate, UMSS, has two parts. The first, the inventory correction part, is a fraction, \( \frac{1}{ASRS} \), of the difference between desired inventory and pipeline, IFDS, and actual inventory and pipeline, IFAS+UOIS. The second, the inventory replacement part, is the amount that was shipped from inventory, RCIS, during the previous period.

A negative value for the ordering rate means that orders are cancelled. It is realistic to represent the cancellations of inventory orders in this way, to a limited extent.

\[ UMSS.KL = RCIS.JK + (1/ASRS)(IFDS.K - IFAS.K - UOIS.K) \]
\[ UMSS = RCIS + (1/ASRS)(IFDS - IFAS - UOIS) \]
\[ UMSS = (AISS)(STST) \]

UMSS—Units to be Made for Stock at Supplier (units/week)
RCIS—Requisitions from Customer to be filled from Inventory at Supplier (units/week)
ASRS—constant for Speed of inventory Reorder at Supplier
   = 1 (week)
IFDS—Inventory, Finished, Desired (pipeline included) at Supplier (units)
IFAS—Inventory, Finished, Actual at Supplier (units)
UOIS—Units on Order for Inventory at Supplier (units)
AISS—constant fraction of orders Initially filled from Stock at Supplier
   = 0.7 (dimensionless)
STST—Steady State input to the customers
   = 100,000 (units/week)

III.3.2.6 Desired Inventory and Pipeline Orders

The inventory that the manufacturers wish to carry is proportional to the average sales level, RESS. The proportionality constant is the turn-over period or the number of weeks of sales desired, ADIS.

The desired pipeline is of the same form as desired inventory. The constant in this case equals the normal production time, DCMS.

IFDS is the total desired in inventory and pipeline.

\[
\text{IFDS} = (\text{ADIS} + (\text{AISS})(\text{DCMS}))(\text{RESS}) \tag{45A}
\]

\[
\text{IFDS} = (\text{ADIS} + (\text{AISS})(\text{DCMS}))(\text{STST}) \tag{46T}
\]

\[
\text{IFDS} = (\text{ADIS} + (\text{AISS})(\text{DCMS}))(\text{STST}) \tag{47N}
\]

IFDS—Inventory, Finished, Desired (pipeline included) at Supplier (units)
ADIS—constant for Desired Inventory at Supplier
   = 8 (weeks)
AISS—constant fraction of orders Initially filled from Stock at Supplier
   = 0.7 (dimensionless)
DCMS—Delay desired in Customer Manufacturing at Supplier
   = 4 (weeks)
RESS—Requisitions from sales and Engineering Smoothed for inventory decision at Supplier (units/week)
STST—Steady State input to the customers
   = 100,000 (units/week)
III.3.2.7 **Average Sales for Desired Inventory**

Manufacturers' sales are exponentially averaged with a time constant of DESS weeks for the desired inventory.

\[
\text{RESS} \cdot K = \text{RESS} \cdot J + (\text{DT})/(1/\text{DESS})(\text{RSPC} \cdot JK - \text{RESS} \cdot J)
\]

\[
\text{RESS} = \text{RSPC}/(1+(\text{DESS})(s))
\]

\[
\text{RSL} = \text{STST}
\]

RESS---Requisitions from sales and Engineering Smoothed for inventory decision at Supplier (units/week)
DT---Delta Time, computation interval
= 0.2 (weeks)
DESS---Delay in information from Engineering, Smoothed, at Supplier
= 10 (weeks)
RSPC---Requisitions Sent to Purchasing department at Customer (units/week)
s---Laplace frequency variable (1/weeks)
STST---Steady State input to the customers
= 100,000 (units/week)

III.3.2.8 **Backlog of Inventory Orders in-Process**

The inventory order backlog in the factory, UOIS, is a simple level with an output of production of inventory units, URIS, and an input of units ordered for inventory, UMSS.

\[
\text{UOIS} \cdot K = \text{UOIS} \cdot J + (\text{DT})(\text{UMSS} \cdot JK - \text{URIS} \cdot JK)
\]

\[
\text{UOIS} = (\text{UMSS} - \text{URIS})/s
\]

\[
\text{UOIS} = (\text{AISS})(\text{DCMS})(\text{STST})
\]

UOIS---Units on Order for Inventory at Supplier (units)
DT---Delta Time, computation interval
= 0.2 (week)
UMSS---Units to be Made for Stock at Supplier (units/week)
URIS---Units Received at Inventory at Supplier (units/week)
s---Laplace frequency variable (1/weeks)
AISS---constant fraction of orders Initially filled from Stock at Supplier
= 0.7 (dimensionless)
DCMS--Delay desired in Customer Manufacturing at Supplier
= 4 (weeks)

STST--Steady State input to the customers
= 100,000 (units/week)

III.3.2.9 Production for Inventory

The production output of inventory units is determined by defining the factory priority system and relating this to the proportion of total output which was made for inventory. In the subject company, customer units and inventory units receive equal priority. Therefore, on the average, the output would be expected to split in proportion to the relative backlogs of inventory and customer orders in process.

One problem in the formulation of this equation is that if the backlogs are low, the output may be greater than the backlogs and thereby produce negative backlogs. This can happen because the total output is determined by work force alone with an assumption being that order backlogs will always be large enough.

\[
URIS=\frac{PCMS \cdot K}{(UOIS \cdot K)(BVMS \cdot K+UOIS \cdot K)}
\]

\[
URIS=\frac{(AISS)(PCMS)+(1-AISS)(UOIS \cdot K)-(AISS)(BVMS \cdot K)}{DCMS}
\]

\[
URIS=((AISS)(PCMS)+(1-AISS)(UOIS \cdot K))/DCMS \frac{(BVMS)}{DCMS}
\]

\[
URIS=\frac{(AISS)(STST)}{}\]

URIS--Units Received at Inventory at Supplier (units/week)
PCMS--Production Capability in Manufacturing at Supplier (units/week)
BVMS--Backlog in Variable Manufacturing delay for customer units at Supplier (units)
UOIS--Units on Order for Inventory at Supplier (units)
AISS--Constant fraction of orders Initially filled from Stock at Supplier
= 0.7 (dimensionless)
DCMS -- Delay desired in Customer Manufacturing at Supplier
e = 4 (weeks)
STST -- Steady State input to the customers
= 100,000 (units/week)

III.3.2.10 Backlog of Customer Orders in Process

The customer backlog in the factory, BVMS, is a level with an input of customer orders not fillable from inventory, RCMS, and an output of production of customer units, USMS.

\[ BVMS \cdot K = BVMS \cdot J + (DT) (RCMS \cdot JK - USMS \cdot JK) \]
\[ BVMS = \frac{RCMS - USMS}{s} \]
\[ BVMS = (1 - AISS) (DCMS) (STST) \]

BVMS -- Backlog in Variable Manufacturing delay for customer units at Supplier (units)
DT -- Delta Time, computation interval
= 0.2 (weeks)
RCMS -- Requisitions from Customer to be Made to order at Supplier (units/week)
USMS -- Units Sent to shipping from Manufacturing at Supplier (units/week)
s -- Laplace frequency variable (1/weeks)
AISS -- constant fraction of orders Initially filled from Stock at Supplier
= 0.7 (dimensionless)
DCMS -- Delay desired in Customer Manufacturing at Supplier
= 4 (weeks)
STST -- Steady State input to the customers
= 100,000 (units/week)

III.3.2.11 Production for Customers

The total production output, PCMS, is determined by the work force and the constant productivity. The part of this output which is for customers, USMS, is the difference between total output and output for inventory, URIS.
USMS.KL=PCMS.K-URIS.JK
USMS=PCMS-URIS
USMS=(1-AISS)(STST)

USMS--Units Sent to shipping from Manufacturing at Supplier
(units/week)
PCMS--Production Capability in Manufacturing at Supplier
(units/week)
URIS--Units Received at Inventory at Supplier (units/week)
AISS--constant fraction of orders Initially filled from
Stock at Supplier
= 0.7 (dimensionless)
STST--Steady State input to the customers
= 100,000 (units/week)

III.3.2.12 Actual Service Delay

In addition to the units that flow to the customer from the manu-
facturers, there is a flow of information about the average lead time
or service delay that is being provided. This delay is the average
difference between the time of the placement of orders by the cus-
tomers and the time that the finished units arrive at his inventory.
It is calculated in two parts. The first is the value of the total
delay in all the processing departments, DCSS. This is assumed to
be a constant.

The second part is variable based on the delays experienced by
units shipped from inventory and those made to order. A unit shipped
from inventory has a zero delay associated with it. A unit that was
made to order, as a first approximation, has experienced a delay equal
to the customer in-process backlog, BVMS, divided by the production
rate for customer orders, USMS. The inventory filling delay and the
delay in production, when weighed by the respective proportions of
the total output which came through each channel, equal the average
variable delay, DMCS.
DMCS.K = $\frac{USMS.JK}{USMS.JK + RCIS.JK} \frac{EVMS.K}{USMS.JK} (64A)$

DMCS.K = $\frac{EVMS.K}{STST} - \frac{(1-AISS)(DCMS)(USMS+RCIS)}{STST} (65A)$

DMCS = $\frac{EVMS}{STST} - \frac{(1-AISS)(DCMS)(USMS+RCIS)}{STST} (66T)$

DMCS = DCSS + (1-AISS)(DCMS) (67N)

DSOC = DCSS + (1-AISS)(DCMS) (68N)

**DMCS—Delay in Manufacturing Customer units at Supplier (weeks)**

**DCSS—Delay in Customer order processing and Shipping at Supplier**

= 8 (weeks)

**USMS—Units Sent to shipping from Manufacturing at Supplier (units/week)**

**RCIS—Requisitions from Customers to be filled from Inventory at Supplier (units/week)**

**BVMS—Backlog in Variable Manufacturing delay for customer units at Supplier (units)**

**STST—Steady STate input to the customers**

= 100,000 (units/week)

**AISS—constant fraction of orders Initially filled from Stock at Supplier**

= 0.7 (dimensionless)

**DCMS—Delay desired in Customer Manufacturing at Supplier**

= 4 (weeks)

**DSOC—Delay in Supplier delivery in steady state at Customer (weeks)**

**III.3.2.13 Service Delay Used by Customer**

The actual service delay, DMCS, is the average delay which units flowing into the shipping delay have actually experienced. The customers do not know this information at this time. However, at a later time when they receive the orders, the customers do learn what this value was. To approximate this information delay, a first-order delay is used. The time constant in this delay, DDIC, is equal to the delay in the processing department, DCSS, plus a time necessary for the customer to become aware of the value and use it.
DSIC.K = DSIC.J + \left(\frac{\text{DT}}{\text{DDIC}}\right)(\text{DMCS}.J - \text{DSIC}.J)  

\[ 69L \]

\[ \text{DSIC} = \frac{\text{DMCS}}{1 + (\text{DDIC})(s)} \]  

\[ 70T \]

\[ \text{DSIC} = \text{DCSS} + (1 - \text{AISS})(\text{DCMS}) \]  

\[ 71N \]

\[ \text{DSIC} = \text{Delay in Supplier delivery used in Inventory reorder at Customer (weeks)} \]

\[ \text{DT} = \text{Delta Time, computation interval} = 0.2 \text{ (weeks)} \]

\[ \text{DDIC} = \text{Delay in Delay Information at Customer} = 10 \text{ (weeks)} \]

\[ \text{DMCS} = \text{Delay in Manufacturing Customer units at Supplier (weeks)} \]

\[ \text{s} = \text{Laplace frequency variable (1/weeks)} \]

\[ \text{DCSS} = \text{Delay in Customer order processing and Shipping at Supplier} = 8 \text{ (weeks)} \]

\[ \text{AISS} = \text{constant fraction of orders Initially filled from Stock at Supplier} = 0.7 \text{ (dimensionless)} \]

\[ \text{DCMS} = \text{Delay desired in Customer Manufacturing at Supplier} = 4 \text{ (weeks)} \]

III.3.3 Labor Market Equations

The labor market is described by eight equations. Each equation represents one of these variables.

1. Production Capability
2. Labor Decision
3. Desired Factory Employment
4. Actual Factory Employment
5. Desired Base Work Force
6. Average Sales
7. Employees Needed to Correct Factory Backlog
8. Desired Production Backlog

The labor decision is the principal part of this sector, and it is of critical importance to the entire model because the work force
that it controls directly influences production output. A flow dia-
gram for the equations of this sector is shown in Figure 9.

III.3.3.1 Production Capability

The production rate, PCMS, is equal to the number of manufac-
turing employees, MEMS, times their average productivity, AMUS.
Productivity is assumed to be constant and independent of the number
of workers, MEMS. It is assumed that an adequate supply of capital
equipment of homogenous quality is always available, that raw mate-
rials are always available, and that there is no overtime work. Thus
employment is the only quantity that influences output.

PCMS\cdot K=(AMUS)\cdot (MEMS\cdot K) \hspace{1cm} \text{(72A)}
PCMS=(AMUS)\cdot (MEMS) \hspace{1cm} \text{(73T)}
PCMS=STST \hspace{1cm} \text{(74N)}

PCMS—Production Capability in Manufacturing at
Supplier (units/week)
AMUS—constant for conversion of Man-weeks to Units
at Supplier
\hspace{1cm} = 400 \text{ (units/man-week)}
MEMS—Men Employed in Manufacturing at Supplier (men)
STST—Steady State input to the customers
\hspace{1cm} = 100,000 \text{ (units/week)}

III.3.3.2 Labor Change Decision

The desired rate of change in the labor force, MLHS, has been
represented as a fraction, 1/DLSS, times the difference between the
desired number of employees, MDMS, in manufacturing and the actual
number, MEMS. The constant, DLSS, is analogous to the time constant
of a first-order delay or an exponentially weighted moving average
and is called an adjustment time constant.
Figure 9. Flow diagram of the labor market equations.
MLHS.KL = (1/DLSS)(MDMS.K - MEMS.K) 75R
MLHS = (1/DLSS)(MDMS - MEMS) 76T
MLHS = 0 77N

MLHS -- Men to be Laid-off or Hired at Supplier (men/weeks)
DLSS -- Delay in the change of Labor force Size at Supplier = 10 (weeks)
MDMS -- Men Desired in Manufacturing at Supplier (men)
MEMS -- Men Employed in Manufacturing at Supplier (men)

III.3.3.3 Desired Factory Employment

The total number of employees desired in manufacturing, MDMS, is the sum of two factors:

1. the number of men, MDSS, required to sustain production at the average sales rate, RSLS, and
2. the number of men, MDBS, needed to adjust the backlog to its desired level.

MDMS.K = MDSS.K + MDBS.K 78A
MDMS = MDSS + MDBS 79T
MDMS = STST/AMUS 80N

MDMS -- Men, total Desired in Manufacturing at Supplier (men)
MDSS -- Men Desired to Sustain production at average sales rate at Supplier (men)
MDBS -- Men Desired to adjust Backlog at Supplier (men)
STST -- Steady State input to the customers = 100,000 (units/week)
AMUS -- constant for conversion of Man-weeks to Units at Supplier = 400 (units/man-week)

III.3.3.4 Actual Factory Employment

The number of employees working in the factory at any time is the number that worked there last period plus those hired during the
period minus those laid off during the period. The natural attrition rate is assumed to be zero, so all changes in employment are attributed to the decision. For simplicity, the delays in hiring and training new people and in giving notice to those being laid off are not represented explicitly. However, these delays are included by increasing the adjustment time in the hiring decision.

\[
\text{MEMS}.K = \text{MEMS}.J + (\text{DT})(\text{MLHS}.JK)
\]
\[
\text{MEMS} = \text{MLHS}/s
\]
\[
\text{MEMS} = (\text{STST})/\text{AMUS}
\]

**MEMS**—Men Employed in Manufacturing at Supplier (men)

**DT**—Delta Time, computation interval

= 0.2 (weeks)

**MLHS**—Men to be Laid-off or Hired at Supplier (men)

**s**—Laplace frequency variable (1/weeks)

**STST**—Steady State input to the customers

= 100,000 (units/week)

**AMUS**—constant for conversion of Man-weeks to Units at Supplier

= 400 (units/man-week)

**III.3.3.5 Desired Base Work Force**

The number of men, **MDSS**, needed to sustain production at the average sales level, **RSLS**, is that level divided by average productivity, **AMUS**. Productivity is presumed to be a constant.

\[
\text{MDSS}.K = \text{RSLS}.K/\text{AMUS}
\]
\[
\text{MDSS} = \text{RSLS}/\text{AMUS}
\]
\[
\text{MDSS} = \text{STST}/\text{AMUS}
\]

**MDSS**—Men Desired to Sustain average production at Supplier (men)

**RSLS**—Requisitions Smoothed for Labor decision at Supplier (units/week)

**AMUS**—constant for conversion of Man-weeks to Units at Supplier

= 400 (units/man-week)

**STST**—Steady State input to the customers

= 100,000 (units/week)
III.3.3.6 Average Sales

The average sales, RSLS, used in the desired base work force term is obtained by taking an exponential average of sales, RSPC, with a smoothing time of DSLS.

\[
\text{RSLS}.K = (\text{RSLS}.J + (\text{DT})(1/\text{DSLS})(\text{RSPC} \cdot \text{JK} - \text{RSLS}.J)) \\
\text{RSLS} = \frac{\text{RSPC}}{1 + (\text{DSLS})(s)} \\
\text{RSLS} = \text{STST}
\]

RSLS--Requisitions Smoothed for Labor decision at Supplier (units/week)
DT--Delta Time, computation interval = 0.2 (weeks)
RSPC--Requisitions Sent to Purchasing department at Customer (units/week)
DSLS--Delay in Smoothing sales for Labor decision at Supplier = 10 (weeks)
s--Laplace frequency variable (1/weeks)
STST--Steady State input to the customers = 100,000 (units/week)

III.3.3.7 Employees Needed to Correct Factory Backlog

The difference between the actual factory backlog, BVMS+UOIS, and the desired backlog, BNMS, represents an excess or deficit of units in the factory. If there are too many orders for the available employees, more men must be hired to reduce the excess. If there are too few orders, employees are partially idle and the excess people should be reduced. The factor, DCBS, determines how fast any excess is to be worked off. The productivity, AMUS, determines how many employees, MDBS, are necessary to correct the backlog imbalance within the adjustment time, DCBS.

\[
\text{MDBS}.K = \frac{(\text{BVMS} \cdot K + \text{UOIS} \cdot K - \text{BNMS} \cdot K)}{(\text{AMUS})(\text{DCBS})} \\
\text{MDBS} = \frac{(\text{BVMS} + \text{UOIS} - \text{BNMS})}{(\text{AMUS})(\text{DCBS})}
\]
MDBS—Men Desired to adjust the customer Backlog at Supplier (men)
BVMS—Backlog in the Variable Manufacturing delay for customer units at Supplier (units)
UOIS—Units on Order for Inventory at Supplier (units)
BNMS—Backlog, Normal, for Men at Supplier (units)
AMUS—constant for conversion of Man-weeks to Units at Supplier
    = 400 (units/man-week)
DCBS—Delay in labor Change due to Backlog at Supplier
    = 7 (weeks)

III.3.3.8 Desired Production Backlog

The desired backlog, BNMS, of units in-process for customers and inventory is that backlog which can be manufactured in a time equal to DCMS weeks, given the current production rate, PCMS.

BNMS.K=(DCMS)(PCMS.K)
BNMS=(DCMS)(PCMS)
BNMS=(DCMS)(STST)

BNMS—Backlog, Normal, of Men at Supplier (men)
DCMS—Delay in information about Customer units in Manufacturing at Supplier
    = 4 (weeks)
PCMS—Production Capability in Manufacturing at Supplier (units/week)
STST—Steady State input to the customers
    = 100,000 (units/week)

III.4 Parameter Estimation

The equations include many parameters that must be given numerical values before a simulation can be made. These parameters represent the average delay times in all of the flow channels and operating departments, the average sales rate of the company, the number of weeks used in the calculation of moving averages, the average production lead times, the number of weeks of inventory normally carried,
the average proportion of incoming orders for inventory items, the average fraction of the orders for inventory items that are filled from inventory, the time required to order the discrepancy between desired and actual inventories, employees, and backlogs, and the average productivity in manufacturing. The methods used to obtain these parameters have varied considerably depending on the type of parameter and the sector involved. Most of the parameters pertaining to the manufacturer and labor sectors are based on data from one of the manufacturers. The constants of the customer sector are based on estimates by personnel of the manufacturing company. After determining the parameters, they were scaled to produce a steady-state ordering rate of 100,000 units per week. The pattern of behavior was not disturbed by this adjustment.

The company parameters, where possible, are based on samples of orders that have passed through the functions, as opposed to regression type methods that deal with the aggregate variables. For example, to find the average delay experienced by orders in the sales department, a sample was selected of these orders, which are date-stamped on receipt in sales and stamped on receipt in the following department. The distribution of the sample delays was found and also the average value. This average became the time constant in the delay. This procedure was followed for several time periods to test time variability.

Sometimes it was necessary to estimate parameters based on the testimony of the decision maker and/or other people associated with the situation. An example of this is the time constant determining
how fast a difference between desired and actual employees is corrected. There is no existing data on either a micro or aggregate level. The value was obtained from discussions with everyone related to the situation. A reasonable range around this value was determined by these people and the researchers. These values were then tried to see if the model was sensitive to the parameter. In this case, differences were significant, so the most probable value was used. But important simulations were rerun with other values.

In some cases aggregate variables must be used to determine the parameter. Productivity is such a "constant". The company has standard times for each of the production operations; hence a standard time should be indicated for a sequence of these operations. As with many time and cost standards, however, the accuracy of such standards is not good. An easier alternate method also of questionable accuracy with additional dynamic problems is to use a sample of production output rate divided by employees at each data interval (weekly output and weekly man-hours) for a year or more. In this case, changing the value has very little influence on the model's performance, so extensive study was not necessary.

The customer parameters are based on estimates by company personnel familiar with the customers' ordering patterns and on studies of similar functions in the company. Reasonable ranges were then chosen and used to test the model's sensitivity. Only a few of the customer parameters turned out to be sensitive and thereby deserving of further study. The most important of these is the time the customer needs to become aware of and use the information about service delay.
The last parameter is the computational interval (DT). This constant must be chosen small enough so that the assumption that rates are constant during the solution interval does not introduce large errors. In the simulations, DT was set at one fifth of a week.

### III.5 Model Acceptability

This model was derived with the objective of representing the causal mechanisms which produce the two principal oscillatory modes of behavior which the system exhibits. This is to be done without distorting the model's response to high-frequency random disturbances which the model does not produce itself. The judgment of whether the model meets the objectives is made on the basis of two types of verification. These are affirmative answers to the following two questions:

1. Does the model structure fit the real situation?
2. Does the model exhibit the desired modes of behavior and for the proper reasons?

The model's structure—that is, the variables represented, the available information sources, the delays, the decision functions, and the production function—does represent the real operation of this line. People who have worked in the line for many years accept the structure as realistic. In addition, there are no apparent logical inconsistencies.

The behavior of the model is very similar to that of the system. Both long and short-term fluctuations with approximately the right amplitudes result in the expected variables. In addition, the phase relations between the model variables are like those in the actual system.
Therefore, relative to the objectives of this study, the model is representative of the real system's operation.
CHAPTER IV
MODEL ANALYSES

An analysis of the foregoing industry model is undertaken to develop an understanding of the kind of behavior that the system produces and its cause. This is important to establish the information feedback nature of this economic system and to serve as a basis for improving the operations of the industry.

The industry exhibits two different modes of behavior. The first is a long-term cycle produced by the industry's sensitivity to long period disturbances. To understand this characteristic, an analysis of the entire feedback system is required. The other is an erratic higher frequency cycle which is produced by the manufacturer sector's sensitivity to shorter period cycles. The excitation for this behavior is high frequency random fluctuations which occur in the customers' ordering and procurement process. This characteristic can only be studied by isolating the manufacturer sector and analyzing it separately. Therefore, two analyses, one for the complete system and one for the manufacturing sector, are presented.

IV.1 Analysis Procedure

The model is a large set of nonlinear differential equations to which there are no closed form solutions. There are two ways to analyze such a system. The first is to approximate the differential equations with difference equations and simulate the equations' response on a digital computer. In this way the nonlinearities can be preserved and the accuracy of the approximation can be made as high as necessary by shortening the solution interval and using
advanced numerical analysis techniques. The model can then be the subject of experiments to deduce its behavior, but no analytical methods or solutions are possible. The second procedure involves a formal servomechanism theory analysis of the linear incremental counterpart of the original nonlinear equation set.

The simulation studies described in this chapter include the response to transient inputs, parameter sensitivity studies, and sinusoidal steady-state characteristics. The transient response is determined for various transient input functions (step, noise, ramp, sinusoid). These point out the natural frequency of oscillation, the approximate damping, the phase relationships between variables, the relative percentage variations of the variables, and the size of the ultimate state error to a ramp input. The parameter sensitivity study is performed by varying the constants in the model from one simulation to the next to see if the behavior pattern is critically dependent on any of them. A step input is usually used because of the large amount of information contained in the step response. The sinusoidal steady-state analysis requires that sinusoids of different frequencies be used as inputs to the model. Then characteristics of the behavior (e.g., the gain and phase between variables) can be plotted versus frequency. These also indicate how many cycles are required before the sinusoidal steady-state condition is reached.

The frequency analysis is performed mathematically by taking the Laplace transforms of the linearized incremental equations which approximate the original nonlinear set and forming the system transfer function. Such a function relates the behavior of one variable to the behavior of any other. Usually the transfer relation is taken
from the input to the principal variable of interest. The mathematical expression will include only system parameters and the Laplace frequency variable, \( s \), in the form of a constant times a polynomial in \( s \) divided by a polynomial in \( s \).

\[
\text{Transfer function } \frac{E(s)}{I(s)} = \frac{s^n + a_{n-1}s^{n-1} + \cdots + a_1s + a_0}{s^m + b_{m-1}s^{m-1} + \cdots + b_1s + b_0}
\]

\( E(s) \)--Laplace transform of the important variable
\( I(s) \)--Laplace transform of the input time function
\( K \)--Real constant related to the system's parameters
\( a_i \)--Real constants related to the system's parameters
\( b_i \)--Real constants related to the system's parameters
\( m \)--Integer greater than \( n \) for realizable systems
\( n \)--Integer
\( s \)--Laplace frequency variable

An alternative form of the transfer function is

\[
\text{Transfer function } = (K) \frac{(s+z_1)(s+z_2) \cdots (s+z_n)}{(s+p_1)(s+p_2) \cdots (s+p_m)}
\]

\( K \)--Real constant related to the system's parameters
\( s \)--Laplace frequency variable
\( z_i \)--Constants (real or complex) related to the system's parameters
\( p_j \)--Constants (real or complex) related to the system's parameters

The \( p \)'s are called the poles of the system, the \( z \)'s are the zeros.

The three important things that derive from this analysis are the values of the poles as literal functions of system parameters, the magnitude of the transfer function as a function of \( \omega \), the radian frequency, and the ultimate state responses to step and ramp inputs as literal functions of the system's parameters. Once these are known, it is easy to find the stability of the system relative to the system's parameters, the range of frequencies to which the system is particularly sensitive (bandwidth), the ability of the system to reject high
frequency random fluctuations (high frequency cut-off point), the
amplitude of response to different frequency inputs (amplification),
and the ultimate state errors for ramp and step inputs as functions
of system parameters. This, coupled with the simulation results,
gives a complete picture of the system's nature.

IV.2 Industry analysis

The complete industry system is analyzed using simulation to
determine transient response, parameter sensitivity and experimental
frequency response. Feedback theory will provide the pole-zero
configuration, the gain function and the ultimate state error to a
ramp.

IV.2.1 Transient Response

The transient response of the complete system is found by
simulating its behavior for several transient inputs. These basic
inputs are the step, the ramp, the sine wave and a random disturbance
(noise). The most common transient input, the impulse, is difficult
to use and interpret in a discrete, nonlinear system and therefore
is not used.

IV.2.1.1 Step Response

The system's response to a step input of height 20 percent
of the steady state order rate applied at the customers' input, HASC,
is shown in Figure 10.

The most important feature of the response is its oscillatory
nature. This oscillation is damped (declining) and has a period of
58 weeks. The damping is enough to allow a second peak in production
which is 9 percent of the first. The oscillation resulting from a
Fig. 10. Present Industry's Step Response.
Orders

Service Delay

Inventory Orders

Time (Weeks)

Step Response.
step input indicates that the industry system is sensitive to input variations with periods between 50 and 70 weeks. That the oscillation decays to its steady state condition indicates that the system is stable and that it will not continue to exhibit fluctuating behavior unless disturbances continue to impinge on the system. This does not necessarily mean that a seasonal cycle must be present in the input. A disturbance can be applied in the form of random noise variations in decisions or by periodic operations within the firm or industry which have not been modeled (e.g., vacations for all employees within a short period each summer or physical inventory counts which once a year cause an inventory adjustment due to discrepancies between actual inventory and the inventory records).

The input step height at the customers' input was twenty percent of the steady state input rate. However, the customers' orders to the manufacturers reach a rate 33 percent above the steady state. Thus, the customers' ordering overshoots the input change. In addition the manufacturers' employment and production capability overshoot customers' ordering rate and reach a peak rate 52 percent above the steady state. This occurs thirteen weeks after the peak in manufacturers' sales. Notice also that, at first, manufacturers' inventory declines until the production rate equals the sales rate. As production rises above sales, the inventory begins to build up and continues to rise until production falls enough to again equal sales. The timing and magnitude of the relationships between sales, production (or work force) and inventory and inventory ordering determines whether the production peak will be higher or lower than
the sales peak. If it is higher, amplification is said to have taken place. If the production peak is lower than peak sales, attenuation is present. In this case the system amplifies variations in the input.

One other interesting fact is that ordering for inventory, \textit{ULSS}, reaches a peak \textit{before} sales reaches its peak. This in-phase characteristic of inventory ordering is the basic reason for the production overshoot. It means that at the same time that the work force is being increased to handle the additional sales, many inventory orders are also entering the factory. This added pressure requires that more people be hired to replace and increase inventory.

The delivery delay, \textit{DLC3}, varies in response to changes in factory backlog, \textit{EWLS}. Its peak value is 8 percent of the average delay. Since its value is used in the customers' ordering decision, its variation causes an increase in the peak of the ordering rate.

Therefore, the step response indicates the following.

1. The system is oscillatory with a 50 to 70 week period.
2. It is stable and so must be continually excited.
3. Once started, fluctuations disappear after the second cycle.
4. Variations at the input to the manufacturers are amplified by that sector because of the rapid adjustment of inventory and the delayed reaction of the hiring decision which leads to overadjustment of employment.
5. Amplification of variations in the customers' input occurs because customers use information about the service delay in their ordering decision and because they also try to correct inventory in the same way the manufacturer does.

IV.2.1.2 Ramp Response

A ramp input tests the system's ability to continuously adjust to a unidirectional signal. The response to a ramp input which declines at a rate of ten percent per year behaves as follows. After an initial period during which inventory rises and sales fall, all variables except delay decline at a steady rate which is equal to the downward slope of the ramp. The important characteristics of this response are the length of time after the ramp begins that inventory begins to fall and the amount that inventory must remain above desired inventory to make the work force decline at the same percentage rate as that of the ramp. Here, it takes 21 weeks before inventory begins to decline. The difference between actual and desired inventory must be 37,500 units to produce the necessary rate of decline in the work force.

In an industry where obsolescence is a potential problem, it is necessary to have the inventory adjust fairly rapidly to a continuous sales downtrend so that unsalable inventories do not accumulate. Therefore, these two characteristics are measures of the protection which is built into the system.

IV. 2.1.3 Response to a Sinusoid

Any input to an industrial system can be represented as the sum of many sine waves of different frequencies and amplitudes. This
is true of a step, a seasonal variation or random noise. Since the system will probably be more sensitive to sine waves whose frequencies fall within a specific range, the response will tend to follow these and reject others. In the presence of such selectivity it is important to know how the system would respond to a sine function whose frequency lies within the sensitive range. Figure 11 shows a simulation of the model using as its input a sinusoid with a ten percent amplitude and a period of one year.

Two characteristics of the sine response are of particular importance. These are the transient behavior before the cycle begins to repeat itself and the sinusoid steady state performance after repetition has started. Notice first that all of the variables oscillate in an approximately sinusoidal manner with periods equal to the period of the input sine wave. However, the amplitude of the variation increases from the first cycle to the second. For example, the first peak in production is 22 percent, while the second is 39 percent. The third peak is about equal to the second, so the system is in sinusoidal steady state beginning with the second cycle.

The amplitudes of the variables after the second cycle indicate the system's amplification for inputs whose period is one year. Employment has an amplitude of forty percent or four times the input amplitude. Inventory varies by 25 percent and the delay has a nine percent swing. The manufacturers' sales have a magnitude of 22 percent.

The phase relationships are also important. It is particularly interesting that the phase between and the amplitudes of the manufacturers' sales and production produce the inventory variation. The minimum inventory occurs when the production rate curve crosses
Fig. 11. Present Industry's Sine Response
the sales rate from below at 65 weeks in Figure 11. During the period when production is higher than sales (weeks 65 to 93), inventory increases by an amount almost equal to the area between the production and sales curves. Starting at week 93 and ending at 119 the circumstances are reversed. Sales is higher than production, causing inventory to fall. Therefore, the phase of inventory relative to sales is determined by the times at which sales and production intersect. The amplitude of the inventory variation is related to the shapes of the sales and production curves (area between) between the crossovers. This means that both the amplitude and phase of inventory will be changed either by a change in the amplitude of production or sales or by a change in the phase between these two.

In Figure 11 it can be seen that production lags sales by ten weeks and inventory lags sales by 26 weeks. The inventory ordering leads sales by two weeks and helps to cause a substantial variation in production.

The conclusions to be drawn from the sine response are these.

1. Two cycles are required before sinusoidal steady state is reached.

2. Both production and inventory fluctuate substantially more than sales. This indicates that inventory is acting as an amplifier instead of as a buffer between sales and production.

* The inventory change actually is equal to the area between the shipment rate and production. However, the lag between sale and shipment is small in this case so the discussion is based on the more familiar sales variable.
3. Sales fluctuate more widely than the customers' input. This means that the service delay information is also causing amplification.

IV.2.1.4 Response to a Random Input

In many systems it is impossible to excite all modes of behavior from any one input mode. This is true of this system. It would not be a problem if, in the real system, the only outside disturbances arose at this point. However, noise is present in every decision function. Thus, the system can exhibit adverse characteristics which cannot be deduced from the preceding study. Therefore, it is necessary to introduce noise into the system at places that will excite the previously undiscovered sensitive areas. The most logical place to insert noise is in the customers' ordering decision.

A simulation of 10 years of behavior with a steady input and noise in the customers' ordering decision is shown in Figure 12. The output is somewhat erratic. However, there is evidence of both the 60 week total system cycle and a higher frequency oscillatory component with a period of from 16 to 30 weeks. The latter is attributable to the oscillatory nature of the manufacturing sector. In both cases the phase relationships between the variables are as described in previous sections. The rapid damping which was particularly evident in the step response is again present.

IV.2.2 Parameter Sensitivity

The model contains several constants whose values may influence the behavior. The primary sensitive parameters are the labor adjustment time, DLSS, the backlog adjustment time, DCBS, the inventory
Production

Inventory

Fig. 12. Present Industry's Response to Noise in Order
adjustment time, ASRS, and the time, DDIC, that it takes for the customer to become aware of and use service delay information.

The labor adjustment time, DLSS, is the most critical parameter in the system. This is reasonable because it is a major part of the decision which controls work force and, thereby, production output. Since the way in which production output adjusts to the sales rate determines the dynamic behavior of the service delay and inventory, the hiring decision along with the customers' ordering decision is the determinant of the system's behavior. The way in which DLSS influences the system is that short values produce a stable system with small, rapidly-damped fluctuations and large values encourage large, slowly declining variations. Figure 13 shows the step response of Figure 10 with the single change that DLSS is equal to 25 weeks instead of ten. Notice the longer-period, less-damped, larger-amplitude fluctuations. On the other hand, for DLSS shorter than ten weeks the oscillations become smaller in size, more heavily damped and higher in frequency.

The backlog adjustment time, DCBS, also influences behavior. At first glance it might be expected that reductions in DCBS would also stabilize the system. However, the opposite is true here. Long values of DCBS produce lower amplitude and lower frequency responses. Figure 14 shows the step response with DCBS equal to forty weeks instead of six. In this simulation the stabilizing effect of long values of DCBS becomes evident. However, the generalization does not hold for very low values of DCBS. In fact, once the value of DCBS becomes less than that of the normal delay in the factory backlog, DCBS, further reductions in DCBS stabilize the response. At the
Fig. 14. Present Industry's Step Response with Long Backlog Adjustment Time.
upper end of the scale, once the backlog adjustment becomes slower than the total system's response time, an unstabilizing effect sets in. This happens because the backlog is not adjusted as fast as the customers adjust their ordering to the change in backlog. Thus a backlog build up due to customers ordering ahead can outrun the backlog term in the hiring decision which is attempting to reduce the backlog. Therefore, at some high value of DCBS, the natural frequency of the hiring feedback (which is normally higher than that of the complete system) can approach the natural frequency of the complete loop. When this happens the two loop responses will reinforce each other and tend to produce instability.

Due to the amplification inherent in the inventory ordering decision, the inventory adjustment time, ASRS, also affects the behavior. This parameter tends to encourage stabilization as it is lengthened. However, this is done at the expense of inventory stability and can only be used within limits. This is true because as ASRS is increased, the inventory fluctuation increases. As inventory varies, the proportion of units that can be filled from inventory will change (in the real system), although the model does not recognize this effect. Since the service delay and therefore customer ordering are dependent on the proportion of units filled from inventory, large fluctuations can be introduced into manufacturers' sales if the inventory becomes very low. Thus, at large values of ASRS, control over inventory can be lost and cause instability.

Figure 15 shows the system's step response when ASRS is forty weeks. The response is oscillatory, but lower peak values of all variables (including inventory) are obtained. It should be remembered
Fig. 15. Present Industry's Step Response with Long Inventory Adjustment Time.
that the reverse effect described above cannot occur in this model because of the constancy of the split fraction. However, longer values of \( \Delta SR \) than forty weeks are needed to cause this type of trouble.

The customers' awareness delay, DDIC, influences the amplification of the system with respect to disturbances with periods in the sixty to seventy week range. As this delay is increased, the system becomes more stable and the response frequency decreases. Short values of DDIC imply that the customers quickly adjust their orders in the pipeline and can thereby build up larger backlogs.

IV.2.3 Sinusoidal Steady State

The system has disturbances of many frequencies imposed upon it. To each of these the system responds differently with respect to the amplitudes of and phases between the variables. Since some frequencies may be more prevalent than others in the inputs and because the system may be more sensitive to frequencies with certain ranges, it is important to know how the system reacts to these different frequencies. In a previous section the meaning of the amplitude and phase relationships was discussed. In this section a presentation of the experimental data for different frequencies is presented.

The critical system variables are manufacturer production, sales, inventory and service delay. The amplitudes of these and the phase angles between sales and inventory and between sales and production are shown in Figures 16, 17 and 18. These were taken from simulations like the one shown in Figure 11, but with the indicated input frequencies. The data for these figures is shown in Appendix A.4.1.
Figure 16. Percentage amplitude of production and sales versus input period.

Figure 17. Percentage amplitude of inventory and service delay versus input period.
Figure 18. Phase between production and inventory and sales versus input period.
It is clear that within the range of input periods studied (26 weeks to 156 weeks) amplification is in evidence. However, it is largest at a period of between one and two years. On a percentage basis inventory varies less than production.

The phases between the variables also vary with frequency. The lag of production behind sales diminishes rapidly as the period increases. For a three year period input, production and sales are exactly in phase. The inventory lag after sales also tends to decrease. This lag of 249 degrees for a 26 week period drops to 69 degrees for a three year period.

These graphs show the system's sensitivity to inputs in the range from one to two year periods. They also indicate that at lower frequencies the inventory (which would lag sales by 270 degrees if production were constant) is not performing its function of reducing production variations.

IV.2.4 Frequency Analysis

There are three concepts in servomechanism theory which are helpful and practical in this analysis. These are the system pole-zero configuration, the gain function and the ultimate state response to a ramp input. The first two of these will be developed for this system. The ramp response will be considered in the design of a better system.

IV. 2.4.1 Poles and Zeros of the System's Transfer Function

The transfer function which describes the response of the manufacturers' production to an impulse input to the customer sector is derived in Appendix A.2.2. It is of the form
\[ T(s) = \frac{\text{third order polynomial} \cdot ((\text{DDIC})(s)+1)((\text{C4})(s^2)+\text{C3}+1)}{((\text{DSRC})(s)+1)(\text{seventh order polynomial})} \]

**Transfer function**
- DDIC—Delay in Delay Information at Customer 10 (weeks)
- C4—Coefficient in second order zero
- C3—Coefficient in second order zero
- DSRC—Delay in Smoothing Requisitions at Customer 10 (weeks)

The "poles" of this function are those values of \( s \) which make the denominator equal to zero. The form of these poles determines the type of system response. Negative real poles correspond to declining exponential responses to impulse inputs. Positive real poles imply rising (unstable) exponential behavior. Complex conjugate pole pairs give oscillatory behavior with a declining amplitude when the real part is negative and an expanding amplitude when the real part is positive.

For this function there are eight poles. Six are negative real and so imply stable, non-oscillatory responses to any of the transient input types (step, impulse and ramp). The last two poles form a complex conjugate pair with a negative real part. Therefore, the damped oscillatory nature of the system derives from this pole pair. The nature of the \( s \)-plane pole diagram is shown in Figure 19.
The six negative real poles act to smooth the time response. Each corresponds to a first order delay whose time constant equals the reciprocal of the value of the pole (after neglecting the minus sign). For example, the first real pole is at \( s = -0.1 \) radius/week. The time constant for the delay that it represents is \( 1/0.1 \) or ten weeks. All of the other simple poles represent time constants that are shorter than ten weeks.

The seventh order denominator polynomial cannot be factored. If it could, the damping constant and frequencies could be expressed as literal functions of the system's parameters.

IV. 2.4.2 Gain Characteristics

Amplification is the characteristic of systems which causes the response to a sine wave input to have a larger amplitude than the input. In general, the amount of amplification will be different for different input frequencies. The plot of the ratio of output amplitude divided by input amplitude versus frequency is called the gain curve. It is of interest to know what the size of the response will be at different frequencies because any arbitrary input is a summation of different frequency sinusoids. Therefore, if it is known that the input is predominantly made up of certain frequencies and the system exhibits high gain for those frequencies, the expected responses will be very large. Since the shape of the gain curve can be changed by altering system parameter values, it may be possible to reduce its sensitivity to the prevalent inputs.

In Figure 20 are shown four gain curves. The upper one is the gain of the present industry model from the customer input to manufacturers' production. The other three describe the present
Fig. 20. Gain for Present Industry and Improved Industry and Manufact
Industry and Manufacturer and for
and Manufacturer.
manufacturer sector, the proposed industry model and the proposed
manufacturer sector. The present industry gain curve is high with a
peak of 3.36 when the input has a period of 83.7 weeks. Therefore,
for a sine with about a one and one half year period, the manufacturers'
production will vary almost three and one half times as widely as the
input. This is not a desirable condition.

A second important characteristic is the sharpness of the
peak. The higher and sharper is the gain curve the more acutely is the
system tuned to a small band of frequencies. The higher is this
sensitivity, the more predictable will be the system's behavior and the
greater will be the fluctuations. The usual measure of sensitivity is

Q. Q is defined as the frequency of the gain peak divided by the band-
width. The bandwidth is the width of the frequency range for which
the gain curve is higher than .707 times the peak gain. The bandwidth
for this system is .078 radians per week and includes input periods
from 52 weeks to 150 weeks. The Q for this system is then .85. This
is a low value by engineering standards where the operating frequencies
are much higher. However, it indicates a fairly high degree of sen-
sitivity for an economic system.

There is a great deal of high frequency noise (periods of 1
day to 6 months) generated within this system. Therefore the ability
of the system to reject such variations is of particular importance.
A measure of this sensitivity is the cut off frequency. That is the
frequency at which the gain becomes less than a specified value. Usually
in engineering work, the value of gain is taken to be .707. At this
point only half of the input power is reflected at the output.
Another method is to ask what is the gain at the lowest noise frequency that interests the analyst. If input periods up to 26 weeks are considered noise, the gain at this point would be of interest. The gain of this system at that period is 0.83.

From the system's gain function, it would be expected that the system would select from the input any power at frequencies that fall within the 50 to 150 week range, and amplify them substantially. It would also be expected that this response would persist with some tenacity. The high frequency rejection is not good either, so the system is likely to follow random disturbances too closely.

IV.3 Manufacturer Sector Analysis

In economic systems, large random disturbances are present in most decisions and operations. It is likely then that all modes of behavior of the system will eventually be excited. Since all of the characteristics are not evident from an examination of the system from the one exogenous input point, it is necessary to study other parts of the system, particularly the internal information feedback loops. The manufacturer sector contains such an internal loop and, therefore, it must be studied by itself.

IV.3.1 Transient Response

Like the complete system, the manufacturing sector is oscillatory in nature. The period, fifty weeks, is shorter than that of the overall system. The fluctuation is also damped. The second production peak in the step response is less than one percent of the first peak. Overshoot is also present, but to a lesser extent.

The important characteristic of this subsystem is that it has a tendency to oscillate at a higher frequency than the complete system
and with a smaller amplitude.

IV.3.2 Frequency Analysis

The transfer function for the manufacturer sector (as derived in Appendix A.1.1 is much simpler than that of the industry. There are three negative real poles and one second order complex pole pair. The analytical expressions for the damping, oscillation frequency and tendency to oscillate are easily found.

The tendency to oscillate is determined by the value of the function

$\frac{(DCMS + DCBS)^2}{(4)(DLSS)(DCBS)}$

DCMS—Delay desired in Customer Manufacturing at Supplier = 4 (weeks)
DCBS—Delay in Changing backlog at Supplier = 6 (weeks)
DLSS—Delay in the change of Labor force Size at Supplier = 10 (weeks)

As this function increases, the stability does also. For values less than one, the system will oscillate. It is clear from this function that the system becomes more stable as DLSS decreases and DCMS increases. Figure 21 shows the shape of this function versus DCBS. A minimum occurs at DCBS = DCMS with a value of DCMS/DLSS. Therefore, a) oscillation is possible if DCMS is less than DLSS and b) an increase in DCBS tends to stabilize the response unless DCBS is less than DCMS, in which case the reverse is true.
Figure 21. Tendency for the present manufacturing sector to oscillate versus backlog adjustment time.

The damping time constant for the oscillation is $TD$.

$$TD = \frac{2(DLSS)(DCBS)}{DCMS + DCBS}$$

$TD$—Time to Damp oscillations (weeks)
$DLSS$—Delay in the change of Labor force Size at Supplier
   = 10 (weeks)
$DCBS$—Delay in Changing Backlog at Supplier
   = 6 (weeks)
$DCMS$—Delay desired in Customer Manufacturing at Supplier
   = 4 (weeks)

This expression indicates that the cycles die out more quickly if DCMS is large, DLSS is short and DCBS is short. For the standard system parameters TD is 12 weeks.

Oscillation occurs with a period of TN.
\[
TN = (2\pi) \sqrt{\frac{(4)(DLSS)^2(DCBS)^2}{(4)(DLSS)(DCBS) - (DCMS + DCBS)^2}}
\]

TN—Time for Natural oscillation (weeks)

DLSS—Delay in the change of Labor force Size at Supplier  
\[= 10 \text{ (weeks)}\]

DCBS—Delay in Changing Backlog at Supplier  
\[= 6 \text{ (weeks)}\]

DCMS—Delay desired in Customer Manufacturing at Supplier  
\[= 4 \text{ (weeks)}\]

Thus the period increases as DLSS, DCMS, and DCBS increase. This holds, however, only if the denominator of the argument of the square root is positive. A negative value means that the poles are no longer complex and no oscillation will take place. The standard parameter values give a period of 52 weeks.

It is interesting to note that the characteristics of the complex poles are influenced only by the backlog adjustment time, the labor adjustment time and the normal production delay. Other parameters do not affect the fluctuating component of the response.

The gain curve for this sector as shown in Figure 20 relates manufacturer production to sales (the sector's input). The relationship between this curve and the industry curve is that the industry has a higher and sharper peak that occurs at a lower frequency. These relationships imply several things. Firstly, the broader bandwidth (smaller \( q \)) means that the response of the manufacturer is less predictable than the industry. The manufacturer reacts more strongly to a wider range of inputs and follows random fluctuations more readily. Thus, the high frequency components in the manufacturers' behavior should be more erratic than the long term cycle. This appears to be the case.

Secondly, since the industry gain is higher than the company gain,
the addition of the customer sector is important. Therefore, a study of the manufacturer alone would miss a major part of the system's dynamics. This reinforces the principle that all of the influential feedback loops must be included in the model. In addition it means that steps taken to improve one sector without a knowledge of the entire system may result in larger problems than they cure. For example, lengthening the backlog adjustment parameter, DCBS, seems to be a good idea from the sector point of view. The larger DCBS, the better. However, the industry analysis indicates that, for the whole system, lengthening DCBS beyond a certain point reduces stability.

Thirdly, the fact that the sector has a different natural frequency and broader band response indicates that the study must not over concentrate on the entire system. This might be justified if the only disturbances arose at the customer input. However, random signals throughout the system can excite this sector's natural response frequencies.

IV.4 Limits of the Linearized Model

The frequency analysis was based upon a linear set of equations derived from the original nonlinear ones. Any conclusions based on the linear equations must include a knowledge of the limits of this approximation. These limits must be based on any errors in the characteristics which are used to develop the conclusions. In this case these characteristics are the shape of the gain function and the pole positions. Since the actual poles and gain are not known and in nonlinear systems are not defined, the determination of any error is difficult.

In Figure 22 is shown a plot of the percentage difference between
the corresponding values of four important variables in the step responses of the linear and nonlinear models. These variables are sales, production, inventory and service delay. None of these differences ever exceeds three percent. However, the instant by instant time values are not the critical factors. Of more importance are comparisons of the damping factors, the oscillation frequencies and the peak values. These agree within 0.2, 1.5, and 2.8 percent respectively. Thus, the pattern of the system's behavior is only slightly disturbed even though the disturbance, a twenty percent step, is much more severe than would be anticipated in the actual system.

Changes in parameters (e.g., backlog adjustment time, labor adjustment time, inventory adjustment time and normal production delay) produce the same types of changes and of very nearly the same magnitude in frequency, damping and amplitude in the two models. Sine wave inputs to the two models produce amplitudes of variation in the principal variables which differ by less than five percent.

These results indicate that the linear model is more than adequate to serve as a basis for understanding the nature of the system's behavior and for comparison with improved versions of the system. The latter is true because the expected improvement is much greater than errors in the linear approximation.

IV.5 Summary of Results

The analysis of the model shows the following characteristics of the industry and the manufacturers.

1. The industry and the manufacturer sector are oscillatory with a period of one to two years, but stable. The fluctuation is caused by slow labor and fast inventory adjustments and by the
rapid correction of backlogs. In addition the customers' use of delay information in their ordering decision produces oscillation. Thus the psychological tendency to wait and see before laying off or hiring, to quickly adjust inventory for fear of future outages or high levels of inventory which could be hard to sell, to quickly submit to the customers' pressure to reduce the delay (backlog) and to place more orders as the service delay increases in order to safeguard against production line shut downs all contribute to the system's cyclical nature.

2. The manufacturer sector has a wider bandwidth and a higher natural frequency of oscillation than the industry. Thus, high frequency random disturbances in various decisions will produce erratic cycles with a period of twenty to forty weeks in the manufacturer's production and inventory. These are higher in frequency than the sector's peak gain because the majority of the random power is at the higher frequencies.

3. The inventory is not being used as a buffer between sales and production. Instead it is amplifying sales variations and thereby forcing production to vary more widely than sales. As the oscillation period approaches eighty weeks the inventory is furthest from its proper phase
of lagging sales by 270 degrees.

4. The linearized model is an acceptable representation of the nonlinear system relative to the gain curves and pattern of transient responses. This would be expected in a system whose basic nature is that of oscillation about a steady average value. The linear representation would not be expected to yield useful results in a system whose pattern included growth.
CHAPTER V
SYSTEML IMPROVEMENT

The analysis presented in Chapter IV has two objectives. The first is to show the feedback nature of economic systems and the applicability of the concepts of servomechanisms theory to their analysis. The second purpose is to develop an understanding of the causes of an industry's behavior to serve as a basis for the synthesis of a decision structure which will produce better, but not necessarily "the best", behavior.

The sequence of steps to be followed in finding improved decision policies are these.

1. Define the criteria upon which "improvement" is to be judged.
2. Translate these criteria into behavior characteristics or changes in characteristics.
3. Relate the changes in behavior characteristics to possible changes in system structure and changes in parameters.
4. Find the best of the potential changes by analyzing the system as it would be with each change.

Each of these steps is taken up in a section of this chapter.

V.1 Criteria for Judging Improvement

The definition of standards for judging improvement requires the isolation of the economic objectives of an economic unit which has control of at least one of the four managerial decisions (customer ordering, inventory filling at the manufacturer, inventory reorder and

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hiring). For this purpose the manufacturer sector is viewed as a single company (although this is not actually the case), and it is from the manufacturer's point of view that the improvement is undertaken and evaluated.

Since the manufacturer is taken to be a corporation, the usual corporate profit objective will be assumed to apply for this situation. This means that profits on both a long term and short term basis are important. Thus contributions to profit are improvements, and profit reductions are to be avoided. Profit rate can be defined as the rate of revenue inflow minus the rate at which expenses arise.

\[
\text{Profit rate} = \left( \frac{\text{Sales rate}}{\text{Price}} - \frac{\text{Unit cost}}{\text{Unit}} \right) - \text{Other non-unit costs}
\]

Total profit acquired within the period between time \( t_1 \) and time \( t_2 \) equals the integral of the profit rate between the two time limits.

\[
\text{Total profit} = \int_{t_1}^{t_2} (\text{Profit rate}) \, dt
\]

In the components industry model, unit price is assumed to be independent of the dynamics of the other variables and constant. Thus the variables that could affect profit are unit cost, unit sales rate, and costs that are not related to sales rate. These latter costs are not the accountants' fixed costs. They include costs that are related to changes in other variables (e.g., interest on the investments in finished, semi-finished, and raw material inventories), as well as the supposedly constant overhead and administrative expenses.

The profit objective then implies that improvement can take any of the following forms.

1. Increased sales rate (as long as the revenue
from the increased sale exceeds the unit cost

2. Decreased unit cost

3. Decreased non-unit costs

The converse of any of these is a deterioration in performance.

It must be noted that the dynamics of the change in any of these factors is of major importance. Thus a balance must be struck between short-term and long-term improvements, if a conflict arises. This usually presents itself as a sacrifice in current profit rate to achieve a higher total profit in the long run.

V.2 Behavior Characteristics that Reflect Improvement

In order to achieve the desirable results of increased sales, decreased unit costs, and decreased non-unit costs, the dynamic behavior of three variables is of primary importance. These variables are employment level, inventory of finished units, and service delay.

V.2.1 Employment Level

Employment is the key variable in the manufacturer sector because it controls production output. There are two characteristics of employment which are closely related to costs. These are the average work force needed to produce a given output and its stability.

The average work force required to produce a given output is a measure of productivity and thereby a measure of labor cost per unit, assuming constant wage rates. Therefore, if productivity could be increased, unit costs would decline.

The other aspect of employment is its tendency to fluctuate. The wider and faster are such oscillations, the greater will be the costs. This relation between cost and amplitude and frequency of oscillation occurs for several reasons. First, there are costs
associated with the process of hiring and laying off people. There are costs of training new people and of low productivity during the training period. The faster changes are needed and the greater the change, the more difficult and expensive it is to accomplish.

Secondly, since rapid changes are hard to accomplish, the use of overtime work is usually increased in the upswing part of the cycle, and the in-process backlog of work to do is insufficient to keep all the workers busy at all times on the downswing. Both overtime and insufficient work are expensive.

Thirdly, fluctuations in employment tend to make the workers insecure. This can cause demands for higher wages to compensate for possible unemployment, and it can make it difficult in the future to obtain needed men at a reasonable wage.

The desirable characteristics of the work force are then

1. to reduce average work force needed for a given output, and
2. to reduce the amplitude and increase the period of employment fluctuations.

Movements in an opposite direction with respect to any of these characteristics will be detrimental.

V.2.2 Inventory Level

An inventory of finished units represents a sizable cash investment. It also requires space to house it and people to administer its use. Therefore the reasons for maintaining it must be worthwhile. An inventory is usually held for three purposes.

1. To provide fast service for the customers

(and thereby to obtain a competitive advantage)
2. To act as a buffer between sales rate and employment so that fluctuations in employment level can be reduced and costs thereby reduced.

3. To improve productivity by (a) allowing the use of large production lots and (b) to allow the choice of a sequence for producing different types of units which is most efficient.

Within the demands of these three objectives, it is desirable to have both the average inventory (i.e., average investment) and the peak inventory as small as possible. Peak inventory is of particular importance not so much because the storage facilities must be large enough to handle the peak inventory, but because, in an electronic component industry, obsolescence is a major potential danger. It is also desirable that the peak inventory (i.e., the peak cash drain) occur when the cash position is favorable. This would prevent the necessity for either external financing or internal austerity programs.

Therefore the desirable characteristics of inventory are these.

1. Low average inventory level.
2. Low peak inventory level.
3. Inventory must achieve its three objectives.
4. Periods of rising inventory should coincide with periods of improving cash position.

V.2.3 Service Delay

The analysis in the previous chapter indicates that the varying nature of the information about service delay which is used in a positive feedback sense in the customers' ordering decision is an important cause of the system's long-term fluctuating behavior.
Since this fluctuation contributes to the size of employment and inventory fluctuations, it is desirable to reduce any variations in the delay and thereby eliminate the cycle.

Variations in service delay are produced either by variations in the relative proportion of the outgoing shipments which came from inventory or by changes in the manufacturing delay experienced by units made-to-order. Since the latter is the principal problem, it is desirable to stabilize the manufacturing delay for customer orders.

Although the analysis does not show it, it is also important to keep the average value of the delay as low as possible. The internal competitive nature of the industry is not studied here. However, there actually is a competitive advantage to having a short delay. A short delay will increase the sales volume of the manufacturer possessing it. Thus it is important not to increase the average service delay in an attempt to make it stable.

Therefore the desirable characteristics of the service delay are these.

1. Low average service delay (i.e., a low manufacturing delay for customer orders).
2. Small variations in the manufacturing delay.
3. The higher the frequency of any oscillations, the better.

V.3 Search for a Better System

There are two ways of approaching the problem of improving the system. These involve either changing the parameters in the existing system or changing the policy structure. The following sections describe why the latter method was chosen, how a better system was selected, and the mathematical form of the new policy equations.
V.3.1 Parameter Modification or Policy Revision?

The existing policies were not rationally developed in the light of system interactions to achieve clearly defined objectives. Instead, they grew up out of necessity and in most cases do not have a formal description. Thus there could be no confidence that anything near the full potential of a broad perspective approach could be realized by an exhaustive study of the existing parameters. In addition, several of the old decisions were in conflict. No amount of parameter changing could resolve this.

In a broader sense, moreover, it is important to develop an understanding of the method of changing policies, since this is a more general and powerful solution procedure. Thus both because of the nature of the problem and to learn more about the synthesis process, the policies of the manufacturer will be studied for possible modifications.

V.3.2 Decision Conflicts

Since there are an infinite number of possible policy changes, the search for the best one or even a good one will not be easy. In fact, because of the complexity and nonlinearity of the system and the system's objectives, it is doubtful that any set of policies could be proved to be optimum even if it were; it is equally doubtful that the optimum set could be derived. Thus the optimization techniques developed in linear systems theory based on minimum squared error between desired and actual output are not useful both because of the system's complexity and nonlinearity, and the multiplicity of desired output characteristics, none of which relate to minimum error. Therefore, formal synthesis techniques are not applicable, and some other approach must be developed.
The procedure that was used consists of five steps. These are

1. Define what it is that must be controlled.
2. Define the basis upon which control is desired.
3. Given the realities of the physical nature of the product, develop nonconflicting decisions to control the desired variables at the point where they can be most directly controlled.
4. Eliminate any remaining flexibility within the system by defining decisions which further the most important possible objective.
5. Adjust parameters to obtain improved behavior as viewed in the time domain and, if possible, the frequency domain.

The first two steps are self-explanatory and have been taken up in the preceding section. Thus three variables were found to be important, and the desirable nature of the average value, stability, and frequency of oscillation of each has been stated.

The third step is that of developing nonconflicting policies to effect the proper control. Nonconflicting decisions are ones that in fact control what they are supposed to control without interfering with or relying in any way on other decisions. This refers to decisions within the selected economic unit only. For example, in the original system it is clear that the hiring decision and the inventory reorder decision are in conflict. The inventory reorder decision cannot control inventory. Only the production rate, which is determined by the work force, can control inventory. Thus the inventory decision can only send orders to the factory which compete with customer orders for the
time of the available workers.

It is often possible to classify explicit control decisions according to whether they are generation or allocation decisions. This is useful because the existence of conflicts may become clear in this way. A generation decision is one which creates an order for a factor which contributes to the production of an output for the organization. An allocation decision is one which divides an existing total. Therefore, there must be just as many generation decisions as there are factors.

In the case of the given system, there is one generation decision for the customer (ordering decision) and one for the manufacturer (hiring decision). All other decisions can only allocate existing flows to different channels. Thus the inventory reorder decision cannot generate output for inventory; it can only temporarily divert output from made-to-order customer units to inventory units.

A reorganization of the decision structure can leave loose ends in the operation which must be tied with decisions which are selected on the basis of their compatibility with the system's objectives. For example, if it is conceded that an inventory reorder decision cannot control inventory, then the control must be shifted to the place that can exert the necessary control. In this case, that is the hiring decision. However, the question of setting inventory orders into the factory remains as an operational problem. Inventory orders must still be generated, but no longer on the basis of the inventory level. Thus such a loose end must be eliminated by creating a decision to write the orders on another basis.

Once the new formal rules have been selected, the numerical values
of any parameters must be found by an analysis of the new system.

V.3.3 A Better Policy Structure

A better policy structure must be based on the effective control of the three important variables—inventory, employment level, and service delay. The behavior objective of each is to reduce its average value and improve stability.

The first question is how can each variable be controlled? Employment level is the only one that can be controlled directly. The inventory and delay can only be influenced indirectly by controlling production output. Production output can only be regulated by varying the work force which produces it. Thus the real control rests solely in the hiring decision. Such a situation leads to the conclusion that the employment level decision should be based on the inventory and customer backlog which produces the service delay, as well as on a normal work force needed to sustain continuity of production at the average sales level.

The hiring decision must be reformulated to achieve these ends. The new decision equation can be written in the same form as before.

\[ MLHS_K = \frac{1}{DLCS} (MDHS.K - MEMS.K) \]  

- **MLHS** --- Men to be Laid-off or Hired at Supplier (men)
- **DLCS** --- Delay in Labor Change at Supplier  
  
- **MDHS** --- Men Desired in Manufacturing at Supplier (men)
- **MEMS** --- Men Employed in Manufacturing at Supplier (men)

However, the desired work force, MDSS, will be the sum of three terms. These are the basic work force, MDSS, the men needed to correct inventory, MIAS, and the men, MBAS, needed to adjust the backlog to obtain the desired delay.

\[ MDSS_K = MDSS.K + MIAS.K + MBAS.K \]
MDSS = MDSS + MIAS + MBAS

MDSS = STST/AMUS

MDSS—Men Desired in Manufacturing at Supplier (men)
MIAS—Men Desired to Sustain production at average sales rate at Supplier (men)
MIAS—Men for Inventory adjustment at Supplier (men)
MBAS—Men for Backlog Adjustment at Supplier (men)
STST—Steady State input to the customers
AMUS—Constant for conversion of Man-weeks to Units at Supplier

= 100,000 (units/weeks)
= 400 (units/man week)

The basic work force need not change from the average sales divided by productivity formulation. But since the averaging time constant may not be the same as in the old system, a new averaging equation is necessary.

MDSS = RLSS/K/AMUS

MDSS = RLSS/AMUS

MDSS = STST/AMUS

RLSS = RLSS.J + (DT)(1/DSSS)(PSIC.JK-RLSS.J)

RLSS = RSFC/(1 + (DSSS)(s))

RLSS = STST

MDSS—Men Desired to Sustain production at the average sales level at Supplier (men)
RLSS—Requisitions for Labor decision Smoothed at Supplier (units/week)
AMUS—Constant for conversion of Man-weeks to Units at Supplier

= 400 (units/man-week)
STST—Steady State input to the customers

= 100,000 (units/week)
DT—Delta Time, computation interval

= 0.2 (weeks)
DSSS—Delay in Sales Smoothing at Supplier value to be determined (weeks)
RSFC—Requisitions Sent to Purchasing department at Customer (units/week)
s—Laplace frequency variable (1/weeks)

The inventory correction term can be of the standard form with
the error between desired inventory and actual inventory being divided by the adjustment time, DIAS, and the productivity. The desired inventory, as in the present system, can be a constant, ADIS, times average sales, RISS.

\[
\text{MIAS.K} = \frac{(\text{ADIS})(\text{RISS.K}) - \text{IFAS.K})}{(\text{AMUS})(\text{DIAS})} \\
\text{MIAS} = \frac{(\text{ADIS})(\text{RISS}) - \text{IFAS}}{(\text{AMUS})(\text{DIAS})} \\
\text{DIAS} = 0 \\
\text{RISS.K} = \text{RISS.J} + (\text{DT})(1/\text{DISS})(\text{RSFC.JK-RISS.J}) \\
\text{RISS} = \text{RSFC}/(1 + (\text{DISS})(s)) \\
\text{RISS} = \text{STST}
\]

MIAS—Men for Inventory Adjustment at Supplier (men)  
ADIS—constant for Desired Inventory at Supplier  
\(= 8\) (weeks)  
RISS—Requisitions for Inventory term Smoothed at Supplier (units/week).  
IFAS—Inventory, Finished, Actual at Supplier (units)  
AMUS—constant for conversion of Man-weeks to Units at Supplier  
\(= 400\) (units/man-week)  
DIAS—Delay in Inventory Adjustment at Supplier value to be determined (weeks)  
DT—Delta Time, computation interval  
\(= 0.2\) (weeks)  
DISS—Delay in Inventory term Smoothing at Supplier value to be determined (weeks)  
RSFC—Requisitions Sent to Purchasing department at Customer (units/week)  
s—Laplace frequency variable (1/weeks)  
STST—Steady State input to the customers  
\(= 100,000\) (units/week)

The backlog correction term is only necessary if the delay is longer than an acceptable upper limit. The maximum acceptable backlog, BIAS, is the acceptable delay, DAUS, times the shipment rate, UAIC. The adjustment term then equals the difference between the actual backlog, RUSC, and the acceptable backlog divided by the backlog adjustment time, DBAS, and the productivity. This term will only be
used if the above difference is positive.

\[
BMAS.K = (DAUS)(UAIC.JK)
\]

\[
BMAS = (DAUS)(UAIC)
\]

\[
BMAS = (DAUS)(STST)
\]

\[
MBAS.K = \begin{cases} 
(RUSC,K-BMAS.K)/((DBAS)(AMUS)) & \text{if } RUSC > BMAS \\
0 & \text{if } RUSC \leq BMAS
\end{cases}
\]

\[
MBAS = (RUSC-BMAS)/((DBAS)(AMUS))
\]

MBAS = 0

BMAS—Backlog, Maximum, Acceptable at Supplier (units)
DAUS—Delay Acceptable as an Upper limit at Supplier
= 12 (weeks)
UAIC—Units Arriving at Inventory at Customer (units/week)
STST—Steady State input to the customers
= 100,000 (units/week)
MBAS—Men for Backlog Adjustment at Supplier (men)
RUSC—Requisitions Unfilled by Supplier for Customer (units)
DBAS—Delay in Backlog Adjustment at Supplier
value to be determined (weeks)
AMUS—constant for conversion of Man-weeks to Units at Supplier
= 400 (units/man-week)

Once the labor decision has been modified to include an inventory adjustment term, the meaning of the old inventory reorder decision disappears. Therefore, a new inventory ordering rule must be defined to compliment the labor decision. Such can occur if the inventory orders are generated at a rate which will keep the backlog of in-process orders at the most efficient level. This is the level at which every employee always has work to do, but not so much work that aisles and work areas become cluttered or that the prospect of the large load causes over-tension or depression. The inventory order generation rate should equal the rate at which inventory units leave the factory (to maintain a steady flow) plus an adjustment term to keep the backlog at the proper level. The efficient backlog, EPES, will be proportional
to the work force. The difference between the efficient backlog and the actual backlog, UOIS + BVMS, when divided by the backlog adjustment time, SDIS, (= 1 week) will determine the rate at which inventory orders will be generated to keep the backlog at its efficient level.

\[ EPBS \cdot K = (DMES)(AMUS)(MENS \cdot K) \]
\[ EPBS = (DMES)(AMUS)(MENS) \]
\[ EPBS = (DMES)(STST) \]
\[ UMSS \cdot KL = URIS \cdot JK + (1/SDIS)(EPBS \cdot K - UOIS \cdot K - BVMS \cdot K) \]
\[ UMSS = URIS + (1/SDIS)(EPBS - UOIS - BVMS) \]
\[ UMSS = (AISS)(STST) \]

EPBS—Efficient Production Backlog at Supplier (units)
DMES—Desired Manufacturing Backlog for Efficiency at Supplier
\[ = 4 \text{ (weeks)} \]
AMUS—constant for conversion of Man-weeks to Units at Supplier
\[ = 400 \text{ (units/man-week)} \]
MENS—Men Employed in Manufacturing at Supplier (men)
STST—Steady State input to the customers
\[ = 100,000 \text{ (units/week)} \]
UMSS—Units to be Made for Stock at Supplier (units/week)
URIS—Units Received at Inventory at Supplier (units/week)
SDIS—Scheduling Delay for Inventory orders at Supplier
\[ = 1 \text{ (week)} \]
UOIS—Units on Order for Inventory at Supplier (units)
BVMS—Backlog in Variable Manufacturing delay for customer units at Supplier (units)
AISS—constant fraction of orders Initially filled from Stock at Supplier
\[ = 0.7 \text{ (dimensionless)} \]

The changes in these two decisions can potentially achieve all but one of the behavior objectives. This one problem is that service delay or, more accurately, the manufacturing delay for customer orders, is not controlled. As long as the factory priority system is such that neither type of order is distinguishable, the customer orders will
sometimes wait for inventory orders to be finished. Thus the manu-
facturing delay will vary with the backlogs of the two kinds of orders.
If a priority system is instituted so that customer orders are always
made first, the dependence of the service delay on the backlog of in-
ventory orders will be eliminated. In addition, if the usual backlog
of customer orders is small compared to the work force, the delay for
manufacturing customer units should approach the actual working time
for the physical processes, DHLS. Therefore, the waiting time in
production queues which causes the variation in the service delay as
well as its large value will disappear for customer orders and thereby
achieve the last objective. The changed priority system causes
the output equations to change.

\[ USLS \cdot KL = \frac{EVLS \cdot K}{DHLS} \]
\[ USLS = \frac{RCLS}{(1 + DHLS(s))} \]
\[ USLS = (1 - AISS)(STST) \]
\[ URIS \cdot KL = ICIS \cdot K - (EVLS \cdot K/DHLS) \]
\[ URIS = RCLS - (EVLS/DHLS) \]
\[ URIS = (AISS)(STST) \]

USLS—Units Sent to shipping from Manufacturing at Supplier
(units/week)
EVLS—Backlog in Variable Manufacturing delay for
customer units at Supplier (units)
DHLS—Delay in Production, Minimum, at Supplier
\( = 2 \) (weeks)
RCLS—Requisitions from Customer to be made to order at
Supplier (units/week)
s—Laplace frequency variable (1/weeks)
AISS—Constant fraction of orders Initially filled from
Stock at Supplier
\( = 0.7 \) (dimensionless)
STST—Steady State input to the customers
\( = 100,000 \) (units/week)
Unis—Units Received at Inventory at Supplier (units/week)
PCLS—Production Capability in Manufacturing at Supplier (units/week)

Given the changes in the labor decision, the inventory ordering decision, and the factory priority system, the oscillations and average values of employment level, inventory, and delay can be reduced through the proper selection of the parameters in the new structure. The following analysis describes this selection process.

It must be made clear that these new rules may not be "the best". In fact, more sophisticated formulations for desired inventory, sales averaging, and variable adjustment times for inventory and backlog can be developed to improve the system further. This presentation is an example of the approach and as such is content to describe a few simple changes that make a substantial difference in system behavior.

V.3.4 Parameter Selection

Parameter selection is the final phase of the synthesis process. This requires the isolation of important parameters that are in fact controllable, the definition of the basis on which they should be selected and the selection itself.

V.3.4.1 Important Parameters

The parameters which it is possible to control are the following.

1. DLS—Delay in Labor Change at Supplier (weeks)
2. DIS—Delay in Inventory term Smoothing at Supplier (weeks)
3. DSS—Delay in Sales Smoothing at Supplier (weeks)
4. DIS—Delay in Inventory Adjustment at Supplier (weeks)
5. SDIS—Scheduling Delay for Inventory orders at Supplier (weeks)
6. DBAS—Delay in Backlog Adjustment at Supplier (weeks)

7. DMES—Desired Manufacturing Backlog for Efficiency at Supplier (weeks)

The parameters that are set by the nature of the operation are these.

DCSS—Delay in Customer order processing and Shipping at Supplier
   = 8 (weeks)

AMUS—constant for conversion of Man-weeks to Units at Supplier
   = 400 (units/man-week)

DPAS—Delay in Production, Minimum, at Supplier
   = 2 (weeks)

AISS—constant fraction of orders Initially filled from Stock at Supplier
   = 0.7 (dimensionless)

ADIS—constant for Desired Inventory at Supplier
   = 8 (weeks)

DAUS—Delay Acceptable as an Upper limit at Supplier
   = 12 (weeks)

All customer parameters are assumed to be unchangable. Of the seven uncertain parameters, two -- the scheduling time for inventory orders, SDIS, and the most efficient factory backlog, DMES -- must be selected on the basis of their effect on productivity rather than on system dynamics. The scheduling delay should be short to be sure the in-process backlog is always sufficient to keep all of the workers busy. The DMES must be chosen to provide the most efficient backlog. Values of one week for SDIS and four weeks for DMES are used in the analysis. The behavior is not dependent on these values.

Two other parameters can be selected without recourse to comparative simulation studies or gain curves. The adjustment time for backlog, DBAS, is not necessary because the backlog adjustment term of the labor decision is not used in this analysis. This is
because it is only activated in very unusual situations in its non-linear form. The normal dynamic behavior is insensitive to it.

The labor adjustment time, DLCS, should be made as short as possible. This is because lengthening this parameter has no advantages. A larger DLCS produces less stability, higher gain and longer adjustment to a ramp input. In this case, since it seems unreasonable to adjust employment more often than once a week, DLCS is set equal to one week.

Therefore, the inventory adjustment time DIAS, and the smoothing time constants for base work force, DSSS, and desired inventory, DISS, are the important parameters for selection.

V.3.4.2 Basis for Selection

The three important parameters must be selected in accordance with the objectives described earlier in the chapter. These are translated into the following characteristics that can be obtained from the poles of the transfer function, the gain curve, the ultimate state response to a ramp input and simulations for step and sine wave inputs.

The poles of the transfer function for the industry and the supplier indicate the tendency of the system to oscillate and the frequency of any fluctuations. This tendency is measured by the discriminant \((B^2/4\Lambda)\) of any second order poles. The discriminant and stability increase together. The frequency is determined by the magnitude of the imaginary part of the poles if they are complex. Large discriminants and low frequencies are desirable.

The gain from input to production and the simulation for a sine input indicate the amplitude of fluctuations in production and how these are related to the amplitude and phase of inventory variations.
The gain also shows the system's ability to reject high frequency noise components. In Figure 23a, sales, production and inventory are shown.

The inventory low is reached at time $t_1$ where the production rate equals the sales rate. Inventory rises during the period that production is greater than sales (or until time $t_2$). The total inventory rise, $\Delta P$, is equal to the area between the production and sales curves. In Figure 23b three possible production curves are shown with a given input sine wave. Two of the production curves differ in amplitude, but not in phase with respect to sales. The other is constant production. Both the amplitude and the phase of inventory relative to sales are different for the three output curves. Notice that the phase of inventory can be shifted ninety degrees by changing from large fluctuations to constant production. In the process, the inventory variation can decline. The constant production is a desirable extreme, but is difficult to accomplish. However, the inventory variation that would occur if production were constant is useful for comparison purposes.

The total inventory variation would be the integral of the sales sine wave from zero to its period divided by 2.

$$\text{Total Inventory Variation} = \int_{0}^{T} 2 (K) \sin\left(\frac{2\pi t}{T}\right) dt = \frac{KT}{\pi}$$

- $K$—Amplitude of sales curve (units/weeks)
- $\pi$—Mathematical constant 3.1416 (dimensionless)
- $T$—Period of sine wave (weeks)
- $t$—time (weeks)
Figure 23a. Theoretical relation between sales, production and inventory.

Figure 23b. Relation between sales and three types of production.
Since the amplitude of the fluctuation is one half the total variation, and the amplitude of the sales curve can be expressed as a percent, FR, of the steady state rate, and the average inventory is ADIS times the steady state sales rate, the percentage amplitude of inventory, PAI, can be written as

\[ PAI = \frac{(PR)(T)}{(2\pi)(ADIS)} \]

PAI—Percentage Amplitude of Inventory (dimensionless)
FR—Percentage amplitude of sales Rate (dimensionless)
T—Period of sine wave (weeks)
\(\pi\)—Mathematical constant
\(\pi = 3.1416\) (dimensionless)
ADIS—constant for Desired Inventory at Supplier
\(\equiv 8\) (weeks)

This says that, for a given percentage sales variation and a given number of weeks of inventory desired, the percentage inventory amplitude will rise in direct proportion to the period of the input, if production is held constant. Notice also in figure 23a that as production shifts to the right relative to sales the area between the curves increases and inventory will fluctuate more widely. When the new decision parameter values yield a percentage inventory variation which is close to the theoretical value over the important frequency range, it can be assumed that further improvement will be difficult to achieve.

Another important characteristic of the gain curve is its height in the high frequency noise region above \(w\) equals .24 radians per week (26 week period). The lower the curve in this region, the better.

The last important gain factor is the frequency at peak gain. The amount of input power increases as the frequency increases (due to the nature of the random variations in the system and the lack of
seasonal usage). Therefore, it is important that the peak gain occur at a low frequency so that most of the potential amplification arises at frequencies with little input power.

Thus, the object is to reduce the height of the gain curve without causing production to lag sales more than it now does. It is also important to hold the high frequency gain at low levels and to have peak gain occur at a low frequency.

Finally, the step and ramp responses give important information about the system's ability to adjust to signals that are contrary to the assumptions about the reasonably constant average level of usage. In an industry where the product could become obsolete with a resulting large loss in unsold inventories, protection must be provided for an unexpected change in the nature of the input signal. The analysis then must compromise between a system which performs well under normal conditions and one which adjusts well to potentially costly abnormal conditions. The important characteristics of the step and ramp responses are a) the time lag between the step rise (or fall) in sales and the following low (or peak) in inventory and b) the number of units by which actual inventory lags desired inventory for a ramp input with a given slope. From the point of view of insurance against a step or ramp decline, a short inventory lag behind a step change and a small ultimate state error to a ramp input are desirable. Since a compromise must be made, arbitrary limits will be set on the lag and the error. Therefore, the design will require that the ultimate state ramp error between desired and actual inventory not exceed fifty percent of the normal inventory (i.e., 400,000 units) for a ten percent ramp and the inventory shall not lag the step response by more
than half a year (26 weeks). Within these constraints, the normal performance should be made as good as possible.

V.3.4.3 Parameter Selection

The first problem is to find which parameters affect the stability of the system. The transfer function for the revised decisions as derived in Appendix A.1.4 indicates eight poles for this system. Three are first order. Three arise from a third order factor which is independent of the parameters in question. These three poles are all negative real. The last two come from the second order factor and are a function of DLCS and DIAS only. The discriminant for this pair is DIAS divided by \((4)(DLCS)\). Increases in DIAS and decreases in DLCS cause the discriminant and the stability to rise. Therefore, for stability reasons DIAS should be long. This is reasonable because one would expect the system to resist oscillation if inventory is adjusted slowly.

A second important characteristic is the peak gain. This can be plotted as a function of changes in the three parameters. This is done in Figure 24a based on simulations of the gain curve for the new decisions derived in Appendix A.2.4. The data for these plots appears in Table 2. The parameters are varied from the basic values of DIAS, DISS and DSSS of 60, 30, and 25 weeks, respectively. The diagram is arranged so that the peak gain curve for each parameter passes through the gain for the basic values. Thus changes in the parameters are comparable. Notice that DISS has little influence on the peak gain. Changes in DIAS and DSSS produce about the same change in gain. Both reduce peak gain as they are increased.
Figure 24a. Industry gain versus variations in DIAS, DSSS and DISS.

Figure 24b. Gain at 26 week period versus DIAS, DSSS and DISS.
The rejection of high frequencies is also important. The gain at a period of 26 weeks is plotted versus the three parameters in Figure 24b. The result is that this measure is insensitive to changes in DIAS and a little more sensitive to DSSS than to DSS+ per unit change.

Another important factor is the period of oscillations at the peak. This characteristic is plotted versus the three parameters in Figure 25. In this case, the period is insensitive to DIAS. Increases in DIAS and DSSS both increase the period.

For protection against a sudden decline in product usage it is necessary to have a minimum ultimate state error between desired and actual inventory for a ramp input. The relation for this error is derived in Appendix A.3.

\[
\text{ERRI}_{\text{ult}} = (\text{DIAS})(192)(\text{DSSS} + 8.4).
\]

If this is limited to 400,000 units, the values of the DIAS and DSSS must satisfy the following relation

\[
\text{DIAS} = \frac{2080}{\text{DSSS} + 8.4}
\]

On the basis of these considerations the best values of DSSS and DIAS are not specified. However, if a long value of DIAS is selected (for stability reasons and to reduce the system's sensitivity to DIAS) the necessary value of DSSS is determined. Therefore, let DIAS be sixty weeks. For this value, DSSS should be 25 weeks. Since the system is relatively insensitive to changes in DIAS when DIAS is long it can be any value between 20 and 40 weeks.

V.3.4.4 **Improved System Performance**

Using the guides of the previous section the three parameters are set at the following values.
Figure 25. Oscillation period at the peak gain versus DIAS, DSSS and DISS.
1. DIAS—Delay in Inventory Adjustment at Supplier 60 (weeks)

2. DSSS—Delay in Sales Smoothing at Supplier 25 (weeks)

3. DISS—Delay in Inventory term Smoothing at Supplier 30 (weeks)

The performance using these values can be shown by the gain curve, the step response and the response to a one year sine wave.

The gain curves for the improved industry and manufacturer sector are shown in Figure 20 with the curves for the present system. A major reduction in gain and an increase in period at the peak gain are indicated. High frequency cut-off is also much sharper.

The step response shown in Figure 26 shows no tendency to oscillate or to overshoot. Inventory falls to a lower level than in the present system and is not corrected as quickly. However, it begins to rise within the 26 week limit specified earlier.

The sine wave response in Figure 27 shows a substantial improvement over the original system. All fluctuations show large reductions including inventory. The phase of inventory relative to sales is almost 270 degrees, as it would be for constant production. The theoretical constant production variation for inventory with a sales amplitude of 15.7 percent and a period of 52 weeks is 16.3 percent. In this system inventory varies by 17.2 percent or very close to the theoretical value. In addition, the reduction in delay variation has cut the sales amplitude from 21 percent to 15.7 percent.

In the process of improving the normal behavior, the protection against obsolescence has declined. This is unavoidable with this linear set of decisions. However, it is possible to introduce nonlinear functions into the inventory adjustment delay, the smoothing
Fig. 26. Improved Industry's Step Response
Orders

Service Delay

Inventory

Hiring

Step Response

Time (Weeks)
Fig. 27. Improved Industry's Sine Response.
process for base work force and the desired inventory which will increase the system's ability to adjust to long term changes in product usage.

This loss in protection is somewhat offset by two factors which do not show up clearly in the results. These are

a) the labor productivity will increase and the use of overtime will decline through the smoother operations and more careful factory loading and

b) the service delay will decline and encourage increased sales for any firm that adopts the new procedures.

V.4 Summary of the Improvement Process

A better system from the point of view of the manufacturer is desired. Therefore, the corporate objective of high profits is translated into a desire for low average values and small fluctuations in manufacturer's employment level, inventory and service delay. To accomplish this, the hiring decision is reformulated to include inventory, unfilled order backlog and average sales level to conform to the realities of the system. These are that service delay and inventory can only be controlled by the adjustment of the production rate. Then, inventory ordering should only be done to keep the factory operating at full capacity at all times, not as an inventory control. The average service delay can be reduced by changing the priority system in production to one of customer orders first.

With the new structure determined, the final step is the selection of values for the inventory adjustment time, the sales smoothing time
constant for desired inventory which will improve the system. A balance is struck between improved normal operations and protection against unforseen changes in basic product usage. The important considerations in the selection are the peak gain, the frequency at the maximum gain, the value of gain at a period of 26 weeks, the relation between the inventory variation and its theoretical constant production amplitude, the tendency of the transfer function poles to oscillate, the ultimate state error between desired and actual inventory to a ramp input, and the time required after a sudden change in sales for the inventory to begin to recover. On the basis of these factors, the values of the three parameters came out to be longer than those usually used in industry.

The new system's transient response to a step input did not tend to oscillate and production did not rise higher than peak sales. Inventory showed nearly a 270 degree phase lag with respect to sales and the peak gain was substantially reduced. The protection against a ramp decline in usage was not as good, but nonlinear controls can be introduced into the inventory adjustment delay and the basic desired work force to improve this. Improvements that do not appear clearly in the results are increased labor productivity and reduced service delay.
CHAPTER VI
CONCLUDING REMARKS

After the analysis and theoretical improvement of any system, three questions must be answered. These are:

1. Of what practical use is the procedure and the results?
2. How generally applicable are the approach and conclusions?
3. What difficulties are still unresolved and what influence could they have on the results and their usefulness?

This chapter attempts to answer these three questions as well as pointing out areas for further investigation.

VI.1 Practical Usefulness of the Results

This study should be useful in two ways. The first of its contributions is in helping companies and industries with problems like those described to understand their performance, and in the case of the sponsoring company to improve it. The second is that, in the process of analyzing an economic situation in terms of feedback control concepts and improving behavior using frequency characteristics (e.g., gain, oscillation periods, bandwidth, etc.), it may be possible to influence others to use these tools for the solution of their economic problems. In other words, it is important to communicate the knowledge that economic systems are a type of servomechanism and to establish confidence in the ability of feedback control theory and simulation techniques to prevail over major economic problems.

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The usefulness of the results of any improvement or optimization oriented analysis in a particular situation is determined by the gain associated with solving the problems under study, the degree to which the results do in fact solve the problems, the severity of any adverse effects that are a by-product of using the results and the ability to impose the results on the system. Thus, the practicality can disappear if the problems are not worth solving or solved incorrectly, if more harm than good is done, or if the results cannot be made to work in the system. This situation appears to pass these four tests.

The objectives of this work strike at the principal causes of lost profit — erratic behavior, low productivity and uncompetitively long delivery delay. If the results can be made to operate effectively, the yearly cost savings and gains through increased sales could be expected to reach from ten to twenty percent of sales revenue. This is certainly a worthwhile gain.

The results are based on a realistic representation of the causes of the problems. This was the conscious aim of the work at every stage. The model behavior exhibits the modes of behavior present in the real system and those persons who best understand the operations accept the model as realistic. Finally, the results are reasonable and have specific characteristics which relate to each problem.

The results possess one fault which is related to the system's behavior under unexpected conditions. The adjustment to a long term change in the level of business is not as rapid as in the original system. However, this problem is not expected to arise for some time, and there are extensions of the results that would eliminate this problem while preserving the improved normal behavior. Before the
introduction of the results into an operating system, nonlinear controls should be developed to improve the flexibility in the face of long term changes in usage.

The actual modification of the real system to conform to the suggestions should not be difficult. The new system does not demand unusual information or the dislocation of employees or operating functions. It primarily requires a change in the order in which work is done and in the way in which already existing data are used in relatively simple decision functions. Thus, the analysis has attacked worthwhile problems in a reasonable manner without introducing extraneous problems which cannot be easily solved. It has done it in such a way that sensible, readily usable results have been obtained. These are the general virtues of a broad perspective approach to major problems based on the concepts of information feedback systems.

VI.2 Generality of the Approach and Conclusions

The approach to the solution of aggregate managerial problems using the concepts of feedback theory is almost universally valid because the causal mechanisms in economic systems are organized in an information feedback manner. However, a formal servomechanisms analysis is possible only if two conditions are satisfied. Firstly, the system must be simple enough so that the transfer function is obtainable without major risk of error. Secondly, the necessary linearization must not eliminate or distort any nonlinear effects which contribute to the pattern of the system's behavior in an important way. In any case, the experimental simulation techniques are limited only by the memory capacity of the large digital computers and by the analyst's ability to understand the experimental
results. With the present large size and continued expansion of computer storage, the latter becomes the important constraint.

Naturally, the conclusions are less general than the techniques. Not all firms are manufacturers dealing in a product which is in a phase of almost constant average sales with service delay the principal consideration in customer ordering. Fewer still have an inventory of any size and only one meaningful factor (not necessarily labor) which produces their output. Probably only a very few companies satisfy all of these conditions and in addition have internal decisions and parameters which are the same as those in the study. In fact, the indiscriminant use of these results in a system that has not been analyzed in a similar manner, could prove to be costly. However, it is not for the specific proposals or the parameter values that this study is useful to anyone other than the company that sponsored the project. Its importance relates to the qualitative ideas about the nature of economic systems, the factors which can produce modes of behavior which are common in industry and to the demonstration that there are techniques that can produce major improvements in very simple ways.

VI.3 Areas of Further Investigation

Additional study areas can be divided into two classes — those related to the model itself and the questions that it answers and those related to the procedure in obtaining the model or the answers. The first group includes the following.

VI.3.1 Use of Compensation Networks

The usual servomechanism theory synthesis procedure includes the introduction of compensation networks at various places to make the
results come out as desired, rather than the reorganization of the structure on physical grounds as was done here. These are usually first or second order lead or lag circuits designed to shift phases, attenuate amplitudes, or both. The physical counterpart of these networks in this system is the basic work force term in the hiring equation and desired inventory. A nonlinear compensator would be the delay in adjusting inventory, DIAS. Notice that both desired inventory and basic work force are actually simple first order lag networks which operate on the sales input. However, this need not be so. They could be of higher order, be based on differences between long and short term averages, tuned to frequencies which are dominant in the type of signal which obsolescence would be expected to be, be based on industry indices instead of sales, be nonlinear, or constant. Desired inventory could also be related to the inventory filling fraction or to inventory itself. It is reasonable to expect that additional improvements could be made by a careful study of these factors.

VI.3.2 Analysis of Other Nonlinearities

This study has neglected the system nonlinearities in inventory filling, productivity and labor market response to requests for employees. The analysis could be extended to include these as added verification that their exclusion was not critical to the results.

VI.3.3 Competition

The competitive forces between customers and between manufacturers is not analyzed in this industry model. These interactions would be useful extensions to the present work.

VI.3.4 Customers Point of View

The synthesis was performed from the manufacturers' point of view.
Alternatively, the manufacturers' structure could be taken as given and modifications could be made to the customers' structure and decisions.

VI.3.5 Labor Market's Point of View

In this study, the labor market is represented as a passive subsystem which responds as requested, wages being ignored. However, the delays in the hiring decision include contractual lay-off notice delays, and job change priority sequences as well as the physical nature of the labor market's response time. Since these are critical to system behavior, it would be of interest to study some of the potential long term ramifications of these and other characteristics of labor agreements.

VI.3.6 Different Obsolescence Paths

The system has been designed using the ultimate state ramp error as a measure of protection against obsolescence. However, there is no guarantee that obsolescence will follow a ramp path. The product could become useless in a matter of weeks. It could also follow an exponential path or show a short period of expanded sales followed by a rapid decline. The various possibilities should be explored to be certain of the safety of the new operations.

VI.3.7 Integral, Derivative and Proportional Control

The proposed hiring decision has elements of integral and proportional control inherent in it. Production is being controlled almost proportionally by the basic work force term. Integral control is applied in the inventory term where inventory is the integral of the difference between sales and production. It might be useful to use the rate of change of the difference between sales and production
as another term in the decision. Although derivative control is usually an unstable process, proper filtering might make it helpful in following adverse trends.

VI.3.8 Product Life Cycle

This production is in the mature phase of its life cycle when the average usage level is roughly constant. It might be easier and more profitable to study the factors which lead to obsolescence and watch for them or reverse them than to build protection into a structure which is basically designed to handle fluctuations about a constant average value. Such a study might also concern itself with the factors that cause rising usage.

VI.3.9 The Multi-product Firm

The present study has concerned itself with one product line and a pool of manufacturers and a pool of customers. Both customer and manufacturer firms undoubtedly handle other products. The company problems inherent in the interacting dynamics of different product lines is an important one. This model could form the basis for the study of the managerial and factor allocation problems of a multi-product firm.

VI.3.10 Money, Raw Material and Capital Equipment Flows. The model presented here does not consider money flows, raw materials, or capital equipment. They were omitted because they "did not influence the system's dynamic behavior". However, under circumstances only slightly different from the ones described, any of these could become important. In fact, the phase relationships between finished inventory, raw material inventory and cash position could have a major effect on the solvency of a firm without influ-
encring the dynamic behavior of this line in any way. Therefore, the
inclusion of these factors would be most useful.

VI.3.11 Multidimensional Production Functions

If factors, like raw materials and capital equipment, were used
in the analysis, the equation which relates production to these factors
would have to include them all. Such a multidimensional production
function is difficult to formulate. Therefore, much work is required
to reveal reasonable functions.

VI.3.12 Use of Suboptimization

The problems and model studied here relate to the major manager-
ial problems on a system wide basis. There is no mention of the
relationship between the aggregate decision functions and the operating
actions necessary to make them work. For example, the new hiring
decision states the number of men to be hired or laid off each week.
Nothing is said about who they are, what department they should work
in, or their productivity relative to the aggregate average productivity
used in the decision. Similarly, the inventory ordering decision
states the number of inventory units that should be scheduled for
production each week. It does not say which catalogue items should
make up this total, what the production lot sizes should be or in what
order the catalogue items should be made. These questions relate to
details which do not constitute the major profit gains. However, the
proper execution of these details can make a contribution. Therefore,
it would be of interest to study the means of translating the aggregate
decisions and project goals into objective functions for operations
research analyses of these micro actions.

Procedural problems deserving attention are the following.
VI.3.13 Implementation of Results

The analysis and synthesis presented here can only succeed if the results are made to work in the organization itself. This problem of implementing the results is a major one. The failure to use results in a proper manner has doomed many otherwise successful projects. A study is needed to determine the characteristics of the people, the system and the results which decide the ease and accuracy of system modification.

VI.3.14 Aggregation

Many decisions of a similar nature are being made simultaneously within different firms. These are put together into a single equation which relates total inputs to total output. This process of aggregation is difficult to perform and harder to prove correct. Therefore, much work is required to develop good aggregation techniques and to provide verification of the methods now used.

VI.3.15 Parameter Estimation

It is stated that in this system most of the parameter values are not critical to the system's behavior and those that do present problems are studied more carefully. However, the relationship between the methods of determining the values and their meaning to system behavior has not been clarified. For example, statistical regression techniques are often used to determine parameter values. These techniques use data for the variables to obtain the values of the parameters which minimize the square of the error between the actual behavior and the behavior that is "predicted" by the equation under study. The question is, in a feedback system like the one presented, can such techniques provide the actual parameter values
and, if not, how do any errors effect the dynamic behavior.

A further problem is to determine whether or not a parameter is varying in a way that affects system performance. The variation of service delay in this system does influence behavior. Yet, service delay is often thought of as a parameter. Worse still, its percentage fluctuation is less than that of the other variables and so it is harder to measure. Therefore, the problems of estimating the variation and value of a parameter should be studied in greater detail.


APPENDIX

A.1 Derivation of the Transfer Functions

The frequency analyses in Chapters IV and V are based on system transfer functions between the input and the production variable, PCMS, for the four systems. These systems are the following:

1. Manufacturer sector of the existing system
2. Total industry as now existing
3. Manufacturer sector of the proposed system
4. Total industry as proposed.

The transfer functions for these systems will be derived in that order.

A.1.1 Transfer Function for the Manufacturer Sector of the Existing System

The transfer function is found by solving the set of Laplace transformed equations developed in Chapter III for the ratio (PCMS/RSPC) where PCMS is the production capability and RSPC is the input orders to the manufacturers' sector.

The procedure to be used is to solve the UMSS equation 43T in terms of RSPC. This is inserted in the MEMS equation 82T so that MEMS can be found as a function of RSPC. Equation 73T relates PCMS to MEMS and from this the ratio PCMS/RSPC is found. The transfer function will depend only on the values of the system's parameters and the Laplace frequency variable, s.

\[ \text{UMSS} = \text{RCIS} + \frac{1}{\text{ASRS}} ((\text{L1})(\text{RESS}) - \text{IFAS} - \text{UOIS}) \]

\[ \text{L1} = \text{ADIS} + (\text{AISS})(\text{DCMS}) \]

141

142
From the equations for RESS (49T), IFAS (40T) and UOIS (52T), UMSS can be rewritten

\[
UMSS = RCIS + \left( \frac{1 - \text{ASRS}}{L_2} \right)(L_1)(RSPC)(\frac{UMSS-RCIS}{s})
\]

\[L_2 = 1 + (DESS)(s)\]

Transposing the UMSS term on the right-hand side of equation 143 and substituting \((AISS)(RSPC)\) for RCIS from equation 28T gives

\[
UMSS = \left( \frac{RSPC}{L_3} \right) \left( \frac{L_1(s)}{L_2} + (AISS)(L_3) \right)
\]

\[L_3 = 1 + (ASRS)(s)\]

The MEMS equation can now be found as a function of RSPC. The MLHS function is from equation 76T.

\[
\frac{MLHS}{s} = \left( \frac{1}{(s)(DLSS)} \right) \frac{RSLs}{AMUS} + \frac{BVMS+UOIS-(DCMS)(PCMS)}{(AMUS)(DCBS)} - MEMS
\]

From equation 73T, PCMS equals \((AMUS)(MEMS)\) and therefore PCMS can be defined directly

\[
PCMS = \left( \frac{1}{(s)(DLSS)} \right) \frac{RSPC}{L_4} + \frac{BVMS+UOIS-(DCMS)(PCMS)}{DCBS} - PCMS
\]

\[L_4 = 1 + (DSLS)(s)\]

From equations 59T and 52T, BVMS+UOIS is found to be

\[
BVMS+UOIS = (RCMS+UMSS-PCMS)/s
\]

If this and UMSS are substituted back into equation 148, and the equation 30T, RCMS=(1-AISS)(RSPC)=(ACSS)(RSPC) is also used, PCMS can be rewritten as

\[
PCMS = \frac{(DCBS)(s)(L_2)(L_3) \cdots + (s)(L_1)(L_4) + (L_2)(L_3)(L_4)}{(L_2)(L_3)(L_4)(DLSS)(DCBS)(s^2) + (DCMS+DCBS)(s) + 1}
\]

The numerator of this function can be written in the form
Numerator = \((C_3)(s^3)+ (C_2)(s^2)+(C_1)(s)+1\)

\[C_3 = (DESS)(ASRS)(DCBS+DSLS)\]

\[C_2 = (DSLS)(ASRS+DESS+L1)+(DCBS)(ASRS+DESS)+(ASRS)(DESS)\]

\[C_1 = DCBS+DSLS+DESS+ASRS+L1\]

The poles of this transfer function are the values of \(s\) which make the denominator function go to zero. These are \(-1/DESS\) from \(L2\), \(-1/ASRS\) from \(L3\), \(-1/DSLS\) from \(L4\) and \(-\frac{(DCMS+DCBS)}{2(DLSS)(DCBS)}\) from the second-order factor.

The second-order poles may be real or complex conjugates depending on the algebraic sign of the term under the square root sign.

If these two poles are complex, the system will be oscillatory, but damped. Oscillation will take place if the following inequality is satisfied by the system parameters.

\[(DCMS+DCBS)^2 < (4)(DLSS)(DCBS)\]

An easier form for the visualization of this inequality is

\[\frac{(DCMS+DCBS)^2}{(4)(DLSS)(DCBS)} < 1\]

If the poles are complex, the frequency at which the system will fluctuate will be (in radian frequency) the magnitude of the imaginary part

\[WN = \sqrt{\frac{(4)(DLSS)(DCBS)-(DCMS+DCBS)^2}{(4)(DLSS)^2(DCBS)^2}}\]

The period of the oscillations will be
\[ TN = \frac{2}{WN} \]

This will be an exponentially damped oscillation such that the time constant of the exponential that multiplies the sinusoid will be TD, where

\[ TD = \frac{(p)(DLSS)(DCBS)}{DCBS+DCMS} \]

A.1.2 Transfer Function for the Complete Present System

This function will be derived by writing the service delay, DMCS, as a function of customer orders, RSPC, and production, PCMS. Using the relation derived in the previous section for PCMS divided by RSPC, DMCS can be written as a function only of RSPC. If this expression is substituted into the equation for RSPC, the transfer between system input, RREC, and customer orders, RSPC, is found. When this is multiplied by the RSPC to PCMS function, the result is the total system function.

From Chapter III, DMCS is written

\[ DMCS = \frac{BVMS}{STST} - \frac{(1-AISS)(DCMS)(USMS+RCIS)}{STST} \]

The backlog, BVMS, the customer production rate, USMS, the inventory filling rate, RCIS, and made-order rate, RCMS, are given by

\[ BVMS = \frac{RCMS-USMS}{s} \]
\[ USMS = (ACSS)(PCMS) \]
\[ RCIS = (AISS)(RSPC) \]
\[ RCMS = (ACSS)(RSPC) \]

If the transfer function for the manufacturer sector is written in the form
\[
\frac{PCMS}{RSPC} = \frac{ZN}{PD}
\]

\[
ZN = (DCBS)(s)(L2)(L3) + (L2)(L3)(L4) + (s)(L1)(L4)
\]

\[
PD = (L2)(L3)(L4) \left( DLSS(DCBS)(s^2) + (DCMS + DCBS)(s) + 1 \right)
\]

and this is introduced into equation 66T with the previous four equations, DMCS becomes

\[
\frac{DMCS}{RSPC} = \left( \frac{(ACSS)}{(s)(STST)} \right) \left( \frac{L5}{PD} \right)
\]

\[
L5 = PD - ZN - (DCMS)(s) \left( (AISS)(DP) + (ACSS)(ZN) \right)
\]

From equation (15T), RSPC can be written in terms of RREC as

\[
RSPC = RREC + \left( \frac{1}{DCOC} \right) \left( (AIDC + DSOC)(RRSC) - IFAC + (STST)(DSIC) - RUSC \right)
\]

Since RRSC is related to RREC by a first-order delay equation and DSIC is similarly related to DMCS, equation (166) can be rewritten in terms only of the variables RSPC, RREC, and DMCS.

\[
\frac{RSPC}{RREC} = \left( \frac{L6}{L7} \right) \left( \frac{(L7)(L6) + (AIDC + DSOC)(s)}{(L6)(L8) + (STST)(s)(DMCS/RSPC)} \right)
\]

\[
L6 = 1 + (DDIC)(s)
\]

\[
L7 = 1 + (DSRC)(s)
\]

\[
L3 = 1 + (DCOC)(s)
\]

The substitution of equation 161 into this expression gives

\[
\frac{RSPC}{RREC} = \left( \frac{L6}{L7} \right) \left( \frac{(L7)(L6) + (AIDC + DSOC)(s)}{(L6)(L8)(DP) + (ACSS)(L5)} \right)
\]

If RSPC/RREC is multiplied by PCMS/RSPC, the total transfer function is obtained.

\[
\frac{PCMS}{RREC} = \left( \frac{(ZN)(L6)}{L7} \right) \left( \frac{(L7)(L8) + (AIDC + DSOC)(s)}{(L6)(L8)(DP) + (ACSS)(L5)} \right)
\]
This function has one simple pole, L7, and a seventh-order pole. Some of the roots of the seventh-order pole are obtained by simulation. The s in the function can be replaced by -TIME.K and the total value printed out at each computation interval. The values of TIME (or s) for which the function becomes zero are the pole values.

A.1.3 Transfer Function for the Manufacturer Sector of the Proposed System

The procedure is to find inventory, IFAS, and backlog, BVMS, as functions of the input, RSPC, and production, PCMS. They are then used in the hiring decision equation to obtain production relative to input.

\[ BVMS = \frac{(ACSS)(DPMS)(RSPC)}{L9} \]

173

\[ L9 = 1 + (DPMS)(s) \]

174

This is true because BVMS is the backlog in the first-order production delay for customer made-to-order units, RCMS.

The next three equations are used to obtain IFAS in terms of PCMS and RSPC.

\[ IFAS = \frac{URIS-RCIS}{s} \]

40T

\[ URIS = PCMS-USMS \]

175

\[ USMS = \frac{(ACSS)(RSPC)}{L9} \]

176

Inventory then becomes

\[ IFAS = \frac{PCMS}{s} - (RSPC)\left(\frac{ACSS}{(s)(L9)} + \frac{AISS}{s}\right) \]

177

The new hiring decision equation is
MLHS = \left( \frac{1}{DLCS} \right) AMUS RLSS + \frac{(ADIS)(RISS) - IFAS}{(AMUS)(DIAS)} - MEMS

Since PCMS equals (MLHS)(AMUS) divided by s and RLSS and RISS are average values of input sales, RSPC, the MLHS equation can be re-formulated as

PCMS = \left( \frac{1}{s(DLCS)} \right) L10 \frac{RSPC}{L10} + \frac{(ADIS)(RSPC)}{(L11)(DIAS)} - \frac{IFAS}{DIAS} - PCMS

L10 = 1 + (DSSS)(s)

L11 = 1 + (DISS)(s)

If the inventory equation is substituted into equation 179 and the terms on the right-hand side involving PCMS are transposed, PCMS can be rewritten as


There are five poles. They occur at -1/DPMS, -1/DSSS, -1/DISS, and -(1/2 DLCS) ± \sqrt{1/DLCS} \sqrt{\frac{1}{(4)(DLCS)} - \frac{1}{DIAS}}

The condition for oscillation is that inventory adjustment time, DIAS, be less than four times the labor adjustment time, DLCS. In this event, the damping time constant is (4)(DLCS) and the oscillation frequency is \sqrt{(4)(DLSS) - DIAS \over (4)(DLCS)^2(DIAS)}.

A.1.4 Transfer Function for Complete Proposed System

The procedure is to find the delay, DMCS, in terms of RSPC. This is substituted into the equation for RSPC to obtain RSPC in terms of RREC. This is then multiplied by the manufacturer sector function PCMS/RSPC to arrive at the total system function.
The equation for DMCS is

$$\text{DMCS} = \frac{\text{BVMS}}{\text{STST}} - \frac{(\text{ACSS})(\text{DPMS})(\text{USMS}+\text{RCIS})}{\text{STST}}$$

Using equation 59T for BVMS, equation 62T for USMS and equation 28T for RCIS, DMCS becomes

$$\frac{\text{DMCS}}{\text{RSPC}} = \frac{\text{ACSS}}{\text{STST}} - \frac{(\text{AISS})(\text{DPMS})^2(s)}{L9}$$

When this is introduced into equation 167 and multiplied by \(\frac{\text{PCMS}}{\text{RSPC}}\), the resulting function is

$$\frac{\text{PCMS}}{\text{RREC}} = \frac{(L12)(L6)(\text{(AIDC+DSOC})(s)+(L7)(L8))}{(L10)(L11)(L7)(L13)((\text{DLCS})(\text{DIAS})(s^2)+\text{(DIAS})(s)+1)}$$

$$L12=(\text{DIAS})(s)(L9)(L11)+(\text{ADIS})(s)(L9)(L10)+(L10)(L11) \frac{(\text{ACSS}+\text{(AISS})}{L15}$$

$$L13=(I7)(L6)(L8)+(\text{ACSS})(\text{AISS})(\text{DPMS})^2(s^2)$$

The functions L10, L11, and L7 are first-order and provide poles at -1/DSSS, -1/DISS, and -1/DSRC. The L13 is a third-order function, which includes the parameters AISS, ACSS, DPMS, DCOC, and DDIC. None of these are parameters to be selected. Therefore the cubic equation can be solved when these parameters have their standard value. However, the second term on the right-hand side of the L13 equation is very small compared to the \(s^2\) part of the first term. If this term is neglected, the third-order pole breaks into three simple poles at -1/DPMS, -1/DCOC, and -1/DDIC. It is easier to think of the three simple poles and the omitted term moves these poles only slightly, so the numerical solution of the cubic equation is not presented.

The last two poles are the same two that are analyzed in A.4.3. Only under unusual circumstances (i.e., if DIAS is less than (4)(DLCS))
could these poles produce oscillatory behavior. Thus with reasonable parameter values all eight poles are negative and real.

A.2 Gain Functions

The gain for any transfer function is equal to the product of the gains of the zeros divided by the product of the gains of the poles. The gain of a polynomial in $s$ of degree $n$ is found by letting $s=j\omega$ in the function, where $j=\sqrt{-1}$ and $\omega=$radian frequency variable, and then taking the square root of the sum of the squares of the real and imaginary parts of the resulting function. For example, let the function be

$$(A)(s^3)+(B)(s^2)+(C)(S)+1$$

Substituting $j\omega$ for $s$ gives the new function

$$-(j)(A)(\omega^3)-(B)(\omega^2)+(j)(C)(\omega)+1$$

The square of the real part is

$$\left(1-(B)(\omega^2)\right)^2$$

The square of the imaginary part is

$$\left((C)(\omega)-(A)(\omega^3)\right)^2$$

The gain is then the square root of the sum of these two squares.

$$\text{Gain} = \sqrt{\left(1-(B)(\omega^2)\right)^2 + \left((C)(\omega)-(A)(\omega^3)\right)^2}$$

For a second-order polynomial, $A$ equals zero and the gain is

$$\text{Gain (second order)} = \sqrt{\left(1-(B)(\omega^2)\right)^2 + \left((C)(\omega)\right)^2}$$
Similarly, the first-order gain is

\[
\text{Gain (first order)} = \sqrt{1 + (C)(w)^2}
\]

Since a system gain may be made up of many such pole and zero gains, the function can become complex. However, it can be simulated and plotted using the DYNAMO program as easily as a time function by letting \( w = \text{TIME}.K \) in the function. This is the procedure that was used to derive the gain functions which were used in Chapters IV and V.

A.2.1 Gain for the Present Manufacturer Sector

The transfer function for the present manufacturer sector has a third-order zero, three first-order poles, and a second-order pole. Therefore the gain equals the gain of the third-order zero divided by the product of the gains of the poles.

Using the expressions for the gain of first-, second-, and third-order poles developed in section A.2 and the forms of the poles as derived in A.1.1, the gain can be written as

\[
\text{Gain} = \frac{\sqrt{\left[1 + (C2)(w^2)\right]^2 + (C1)(w) - (C3)(w^3)^2}^2}{(L14)(L15)(L16)\sqrt{\left[1 - (DLSS)(DCBS)(w^2)\right]^2 + (DCMS + DCBS)(w)^2}}
\]

\[
L14 = \sqrt{1 + (DESS)(w)^2}
\]

\[
L15 = \sqrt{1 + (ASRS)(w)^2}
\]
A.2.2 Gain for the Present Industry Model

The present industry transfer function has a first-, a second-, and a third-order zero and a first-order and a seventh-order pole. The total gain equals the product of the zero gains divided by the product of the pole gains.

This can be written as

\[
\text{Gain} = \frac{(L17)(L18)}{(L19)} \frac{\sqrt{1+((DDIC)(w))^2}}{\sqrt{1+((DSRC)(w))^2}}
\]

\[
L17 = \sqrt{\left(1+(C2)(w^2)\right)^2 + \left((C1)(w)-(C3)(w^3)\right)^2}
\]

\[
L18 = \sqrt{\left(1-(DCOC)^2(DSRC)^2(w^2)\right)^2 + \left((DCOC+DSRC+AIDC+DSOC)(w)\right)^2}
\]

\[
L19 = \sqrt{(L20)^2 + (L21)^2}
\]

\[
L20 = 1-(CB)(w^2)+(CD)(w^4)-(CF)(w^6)
\]

\[
L21 = (CA)(w)-(CC)(w^3)+(CE)(w^5)-(CG)(w^7)
\]

\[
CA = EA + (DA)(DD) - (ACSS)(EB + (ACSS)(DCMS))
\]

\[
CB = (DDIC)(DCOC) + (EA)(DD) + (DA)(DG) - (ACSS)(DB)
\]

\[
CC = (DA)(DC) + (EA)(DG) + (DDIC)(DCOC)(DD) - (ACSS)(DE)
\]

\[
\]
CE=(DA)(DJ)+(EA)(DI)+(DDIC)(DCOC)(DC) 208
CF=(EA)(DJ)+(DDIC)(DCOC)(DI) 209
CG=(DDIC)(DCOC)(DJ) 210
EA=DDIC+DCOC-(ACSS)(AISS)(DCMS) 211
DA=1+ACSS 212
DD=DESS+ASRS+DSLS+DCMS+DCBS 213
EB=DCBS+DSLS+DESS+ASRS+ADIS+(AISS)(DCMS) 214
DG=(DLSS)(DCBS)+(DCMS+DCBS)(DESS+ASRS+DSLS)+(DF) 215
DF=(ASRS)(DESS+DSLS)+(DESS)(DSLS) 216
DB=EC+(ACSS)(DCMS)(EB) 217
EC=(DCBS)(DESS+ASRS)+(ASRS)(DESS)+(DSLS)(ASRS+DESS+L1) 218
DC=(DESS)(ASRS)(DSLS)+(DF)(DCMS+DCBS)+(DLSS)(DCBS)(DH) 219
DE=ED+(EC)(ACSS)(DCMS) 220
DH=DESS+ASRS+DSLS 221
DI=(DLSS)(DCBS)(DF)+(DCMS+DCBS)(DESS)(ASRS)(DSLS) 222
ED=(DESS)(ASRS)(DCBS+DSLS) 223
DJ=(DLSS)(DCBS)(DESS)(ASRS)(DSLS) 224

A.2.3 Gain for the New Manufacturer Sector

The new manufacturer sector transfer function has one third-order zero, three first-order poles, and one second-order pole. The gain equals the gain of the zero function divided by the product of the gains of the pole functions.

Therefore the gain expression is

\[
\text{Gain} = \frac{\sqrt{1- (K2)(w^2) + (K1)(w) - (K3)(w^3)^2}}{(L22)(L23)(L24)(L25)}
\]
K1 = DIAS + ADIS + DSSS + DISS + (AISS)(DPMS)  
K3 = (DPMS)[(DIAS)(DISS) + (ADIS)(DSSS) + (DSSS)(DISS)(AISS)]  

L22 = \sqrt{1 + (DPMS)(w)^2}  
L23 = \sqrt{1 + (DSSS)(w)^2}  
L24 = \sqrt{1 + (DISS)(w)^2}  
L25 = \sqrt{1 - (DLCS)^2(DIAS)^2(w^2) + (DIAS)(w)^2}  

A.2.4 Gain for the New Industry Model

The new industry model's transfer function has one first-order zero, one second-order zero, and one third-order zero. There are three first-order poles, one second-order pole, and one third-order pole. The product of zero gains divided by the product of the pole gains gives the total gain function.

The gain is written in the form

Gain = \frac{(L18)(L26) \sqrt{1 + (DDIC)(w)^2}}{(L27)(L23)(L24)(L25) \sqrt{1 + (DSRC)(w)^2}}  

L18 = \sqrt{(1 - (DCOC)^2(DSRC)^2(w^2)) + (DCOC + DSRC + AIDC + DSOC)(w)^2}
\[
L26 = \sqrt{\left(1 - (K2)(w^2)\right)^2 + \left((K1)(w) - (K3)(w^3)\right)^2}
\]

\[
L27 = \sqrt{\left(1 - (K5)(w^2)\right)^2 + \left((K4)(w) - (K6)(w^3)\right)^2}
\]

\[
L23 = \sqrt{1 + (DSSS)(w)}^2
\]

\[
L24 = \sqrt{1 + (DISS)(w)}^2
\]

\[
L25 = \sqrt{\left(1 - (DLCS)^2(DIAS)^2(w^2)\right)^2 + (DIAS)(w)}^2
\]

\[
K1 = DIAS + ADIS + DSSS + DISS + (AISS)(DPMS)
\]

\[
K2 = (DIAS)(DPMS + DISS) + (ADIS)(DPMS) + (DSSS)(ADIS + DISS)
\]

\[
K3 = (DPMS)\left((DIAS)(DISS) + (ADIS)(DSSS) + (DSSS)(DISS)(AISS)\right)
\]

\[
K4 = DPMS + DCOC + DDIC
\]

\[
K5 = (DDIC)(DPMS + DCOC) + (DPMS)(DCOC + (ACSS)(AISS)(DPMS))
\]

\[
K6 = (DCOC)(DDIC)(DPMS)
\]

### A.3 Derivation of the Ultimate State Ramp Error

It is important to know by how many units actual inventory will differ from desired inventory in the long run for a ramp input into the manufacturer sector. This is found by writing the error as a function of the input in the frequency domain, substituting the slope divided by $s^2$ (the transform of a ramp) for the input, and applying the final value therein.

If the difference between desired inventory and actual inventory
is noted as ERRI, the hiring decision in the proposed system can be written in terms of it

$$\text{MLHS} = \left( \frac{1}{\text{DLCS}} \right) \left( \frac{\text{RLSS}}{\text{AMUS}} + \frac{\text{ERRI}}{(\text{AMUS})(\text{DIAS})} - \text{MEMS} \right)$$

If the following equations are substituted into equation and the terms are rearranged, the result is

$$\text{MEMS} = \text{MLHS}/s$$

$$\text{PCMS} = (\text{AMUS})(\text{MEMS})$$

$$\text{RLSS} = \frac{\text{RSPC}}{\text{L10}}$$

$$\text{PCMS} = \frac{\text{ZNN}}{\text{RSPC}} = \text{PDN}$$

$$\text{ERRI} = (\text{DIAS}) \left( \frac{(\text{L10})^2 + (\text{DLCS})(s)(\text{ZNN}) - \text{PDN}}{(\text{PDN})(\text{L10})} \right) (\text{RSPC})$$

Let RSPC be a ramp input with a Laplace transform equal to the slope, SRAS, divided by $s^2$. The final value theorem states that the ultimate error, ERRI, to the input is found by multiplying the right-hand side of equation 249 by $s$ and letting $s$ go to zero. This means that, for the error to be finite, the numerator of the transfer function must have a multiplying factor of $s$. When the ZNN and PDN are expanded, the numerator is found to have a factor of $s$. The result of the application of the final value theorem is that the ultimate state error is

$$\text{ERRI}_{ult} = (\text{DIAS}) \left( \text{DLCS} + \text{ADIS} + \text{DSSS} - (\text{ACSS})(\text{DPMS}) \right) (\text{SRAS})$$

If this is written in terms of the two parameters in question, DIAS and DSSS, assuming a 10 percent ramp and the standard values of
the other parameters, the following expression results:

\[ \text{ERRI}_{\text{ult.}} = (192)(\text{DIAS})(DSSS+6.4) \]
A.4 Data for Figures 16, 17, 18, 24a, 24b, 25

The data which are used to plot the diagrams in Figures 16, 17, 18, 24a, 24b and 25 are shown in Tables 1 and 2.

A.4.1 Data for Figures 16, 17 and 18

Ten percent input sine waves of various frequencies were used as the customers' input. The resulting variations in sales, production and inventory and the phase angles between them are shown in Table 1 below.

Table 1. Amplitude and phase of sales, production, inventory and service delay versus frequency of input sine waves.

<table>
<thead>
<tr>
<th>Input Period weeks</th>
<th>Amplitude of Sales percent</th>
<th>Amplitude of Production percent</th>
<th>Amplitude of Inventory percent</th>
<th>Amplitude of Service Delay percent</th>
<th>Time between peak Sales &amp; peak Service weeks</th>
<th>Time between peak Sales &amp; peak Production weeks</th>
<th>Phase angle between Sales and Production degrees</th>
<th>Phase angle between Sales and Inventory degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>14</td>
<td>10</td>
<td>7</td>
<td>3.6</td>
<td>10</td>
<td>18</td>
<td>138</td>
<td>249</td>
</tr>
<tr>
<td>52</td>
<td>21</td>
<td>39</td>
<td>25</td>
<td>9.1</td>
<td>10</td>
<td>26</td>
<td>69</td>
<td>180</td>
</tr>
<tr>
<td>104</td>
<td>15</td>
<td>26</td>
<td>20</td>
<td>4.4</td>
<td>1</td>
<td>28</td>
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<td>20</td>
<td>16</td>
<td>2.7</td>
<td>0</td>
<td>30</td>
<td>0</td>
<td>69</td>
</tr>
</tbody>
</table>
### A.4.2 Frequency and Simulation Data

The functions plotted in Figures 23, 24 and 25 are based on data derived from gain curves for different parameter values. Table 2 below lists these results.

**Table 2. Characteristics of the proposed system's gain curve versus DIAS, DSSS and DISS.**

<table>
<thead>
<tr>
<th>DIAS weeks</th>
<th>DSSS weeks</th>
<th>DISS weeks</th>
<th>Peak Gain</th>
<th>Frequency at Peak</th>
<th>Period at Peak</th>
<th>Gain at Frequency of 0.24</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>25</td>
<td>30</td>
<td>2.031</td>
<td>0.064</td>
<td>98</td>
<td>0.973</td>
</tr>
<tr>
<td>20</td>
<td>25</td>
<td>30</td>
<td>1.890</td>
<td>0.047</td>
<td>134</td>
<td>0.614</td>
</tr>
<tr>
<td>30</td>
<td>25</td>
<td>30</td>
<td>1.775</td>
<td>0.039</td>
<td>160</td>
<td>0.488</td>
</tr>
<tr>
<td>40</td>
<td>25</td>
<td>30</td>
<td>1.685</td>
<td>0.035</td>
<td>179</td>
<td>0.425</td>
</tr>
<tr>
<td>50</td>
<td>25</td>
<td>30</td>
<td>1.613</td>
<td>0.032</td>
<td>195</td>
<td>0.387</td>
</tr>
<tr>
<td>60</td>
<td>25</td>
<td>30</td>
<td>1.556</td>
<td>0.030</td>
<td>208</td>
<td>0.364</td>
</tr>
<tr>
<td>80</td>
<td>25</td>
<td>30</td>
<td>1.471</td>
<td>0.028</td>
<td>223</td>
<td>0.330</td>
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### A.5 Notation Used in the Appendix

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>ACSS</td>
<td>Constant fraction of Customer units Specially made at Supplier = 0.3 (dimensionless)</td>
</tr>
<tr>
<td>ADIS</td>
<td>Constant for Desired Inventory at Supplier = 8 (weeks)</td>
</tr>
<tr>
<td>AIDC</td>
<td>Constant defining Inventory Desired at Customer = 10 (weeks)</td>
</tr>
<tr>
<td>AISS</td>
<td>Constant fraction of orders Initially filled from Stock at Supplier = 0.56 (dimensionless)</td>
</tr>
<tr>
<td>AMUS</td>
<td>Constant for conversion of Man-weeks to Units at Supplier = 400 (units/man-week)</td>
</tr>
<tr>
<td>ASRS</td>
<td>Constant for Speed of inventory Reorder at Supplier = 1 (week)</td>
</tr>
<tr>
<td>BCSS</td>
<td>Backlog of Customer orders in Shipping and processing operations at Supplier (units)</td>
</tr>
<tr>
<td>BNMS</td>
<td>Backlog, Normal, for Men at Supplier (units)</td>
</tr>
<tr>
<td>BVMS</td>
<td>Backlog in Variable Manufacturing delay for customer units at Supplier (units)</td>
</tr>
<tr>
<td>CA</td>
<td>Auxiliary constant for industry gain function (weeks)</td>
</tr>
<tr>
<td>CB</td>
<td>Auxiliary constant for industry gain function (weeks²)</td>
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<tr>
<td>CC</td>
<td>Auxiliary constant for industry gain function (weeks³)</td>
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<tr>
<td>CD</td>
<td>Auxiliary constant for industry gain function (weeks⁴)</td>
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<tr>
<td>CE</td>
<td>Auxiliary constant for industry gain function (weeks⁵)</td>
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<tr>
<td>CF</td>
<td>Auxiliary constant for industry gain function (weeks⁶)</td>
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<tr>
<td>CG</td>
<td>Auxiliary constant for industry gain function (weeks⁷)</td>
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<tr>
<td>C1</td>
<td>Coefficient of s in numerator of manufacturer sector transfer function (weeks)</td>
</tr>
<tr>
<td>C2</td>
<td>Coefficient of s² in numerator of manufacturer sector transfer function (weeks²)</td>
</tr>
<tr>
<td>C3</td>
<td>Coefficient of s³ in numerator of manufacturer sector transfer function (weeks³)</td>
</tr>
<tr>
<td>DA</td>
<td>Auxiliary constant for industry gain function (dimensionless)</td>
</tr>
<tr>
<td>DB</td>
<td>Auxiliary constant for industry gain function (weeks²)</td>
</tr>
<tr>
<td>DC</td>
<td>Auxiliary constant for industry gain function (weeks³)</td>
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<tr>
<td>DCBS</td>
<td>Delay in Changing Backlog at Supplier = 6 (weeks)</td>
</tr>
<tr>
<td>DCOC</td>
<td>Delay in Changing the Order rate at Customer = 3 (weeks)</td>
</tr>
<tr>
<td>DCMS</td>
<td>Delay desired in Customer Manufacturing at Supplier = 4 (weeks)</td>
</tr>
<tr>
<td>DCSS</td>
<td>Delay in Customer order processing and Shipping at Supplier = 6 (weeks)</td>
</tr>
<tr>
<td>DD</td>
<td>Auxiliary constant for industry gain function (weeks)</td>
</tr>
<tr>
<td>DDIC</td>
<td>Delay in Delay Information at Customer = 10 (weeks)</td>
</tr>
<tr>
<td>DE</td>
<td>Auxiliary constant for industry gain function (weeks³)</td>
</tr>
<tr>
<td>DESS</td>
<td>Delay in information from Engineering, Smoothed, at Supplier = 10 (weeks)</td>
</tr>
</tbody>
</table>
DF  Auxiliary constant for industry gain function (weeks^2)
DG  Auxiliary constant for industry gain function (weeks^2)
DH  Auxiliary constant for industry gain function (weeks)
DI  Auxiliary constant for industry gain function (weeks)
DIAS Delay in Inventory Adjustment at Supplier (weeks)
DISS Delay in Inventory Smoothing at Supplier (weeks)
DJ  Auxiliary constant for industry gain function (weeks^5)
DLCS Delay in Labor Change at Supplier
   = 1 (week)
DLSS Delay in the change of Labor force Size at Supplier
   = 10 (weeks)
DMCS Delay in Manufacturing Customer units at Supplier (weeks)
DSIC Delay in Supplier delivery used in Inventory reorder at
   Customer (weeks)
DPMS Delay in Production, Minimum, at Supplier
   = 2 (weeks)
DSLs Delay in Smoothing sales for Labor decision at Supplier
   = 10 (weeks)
DSOC Delay in Supplier delivery in steady state at Customer (weeks)
DSRC Delay in Smoothing Requisitions at Customer
   = 10 (weeks)
DSSS Delay in Smoothing Sales for labor decision at Supplier
   (weeks)
DT  Delta Time, computation interval
   = 0.5 (weeks)
EA  Auxiliary constant for industry gain function (weeks)
EB  Auxiliary constant for industry gain function (weeks)
EC  Auxiliary constant for industry gain function (weeks^2)
ED  Auxiliary constant for industry gain function (weeks^2)
ERRI ERRor between desired and actual Inventory (units)
ERRIult. ERRor between desired and actual Inventory, ultimate for
   ramp input (units)
IFAC Inventory Finished Actual at Customer (units)
IFAS Inventory Finished Actual at Supplier (units)
IFDS Inventory Finished Desired (pipeline included) at Supplier
   (units)
K1  Auxiliary constant for new industry gain function (weeks)
K2  Auxiliary constant for new industry gain function (weeks^2)
K3  Auxiliary constant for new industry gain function (weeks)
K4  Auxiliary constant for new industry gain function (weeks)
K5  Auxiliary constant for new industry gain function (weeks^2)
K6  Auxiliary constant for new industry gain function (weeks^2)
L1  Delay factor including ADIS and DCMS (weeks)
L2  Simple pole at -1/DMCS (dimensionless)
L3  Simple pole at -1/ASRS (dimensionless)
L4  Simple pole at -1/DSLS (dimensionless)
L5  Sixth-order numerator frequency factor in DMCS/RSPC
   (dimensionless)
L6  Simple pole at -1/DDIC (dimensionless)
L7  Simple pole at -1/DSRC (dimensionless)
L8  Simple pole at -1/DDIC (dimensionless)
L9  Simple pole at -1/DPMS (dimensionless)
L10 Simple pole at -1/DSSS
L11 Simple pole at -1/DISS
L12 Third-order numerator factor in PCMS/RREC transfer function (dimensionless)
L13 Third-order denominator factor in PCMS/RREC transfer function (dimensionless)
L14 Gain for simple pole -1/DESS (dimensionless)
L15 Gain for simple pole -1/ASRS (dimensionless)
L16 Gain for simple pole -1/DSLS (dimensionless)
L17 Gain for third-order industry zero (dimensionless)
L18 Gain for second-order industry zero (dimensionless)
L19 Gain for seventh-order industry pole (dimensionless)
L20 Real part of seventh-order s function (dimensionless)
L21 Imaginary part of seventh-order s function (dimensionless)
L22 Gain for simple pole -1/DPMS (dimensionless)
L23 Gain for simple pole -1/DSSS (dimensionless)
L24 Gain for simple pole -1/DISS (dimensionless)
L25 Gain for second-order new industry pole (dimensionless)
L26 Gain for third-order new industry zero (dimensionless)
L27 Gain for third-order new industry pole (dimensionless)
MDDBS Men Desired to adjust Backlog at Supplier (men)
MEMS Men Employed in Manufacturing at Supplier (men)
MLHS Men to be Laid-off or HIred at Supplier (men)
PCMS Production Capability in Manufacturing at Supplier (units/week)
PD Poles in Denominator of present manufacturing sector transfer function (dimensionless)
PDN Poles in Denominator of New manufacturing sector transfer function (dimensionless)

mathematical constant = 3.1416... (dimensionless)

RCIS Requisitions from Customer to be filled from Inventory at Supplier (units/week)
RCMS Requisitions from Customer to be Made to order at Supplier (units/week)
RESS Requisitions from sales and Engineering Smoothed for inventory decision at Supplier (units/week)
RISS Requisitions, for desired Inventory, Smoothed at Supplier (units/week)
RLSS Requisitions, for Labor decision, Smoothed at Supplier (units/week)
RREC Requisitions Received in Estimating and Scheduling department at Customer (units/week)
RRSC Requisitions Received and Smoothed at Customer (units/week)
RSLS Requisitions Smoothed for Labor decision at Supplier (units/week)
RSPC Requisitions Sent to Purchasing department at Customer (units/week)
RUSC Requisitions Unfilled by Supplier for Customer (units/week)

s Laplace frequency variable (1/weeks)
<table>
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<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>SRAS</td>
<td>Slope of Ramp at Supplier (units/week²)</td>
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<tr>
<td>STST</td>
<td>Steady State input to the customers = 100,000 (units/week)</td>
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<tr>
<td>TD</td>
<td>Time for Damping of oscillations (weeks)</td>
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<tr>
<td>TN</td>
<td>Time for Natural oscillation (weeks)</td>
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<tr>
<td>UAIC</td>
<td>Units Arriving at Inventory at Customer (units/week)</td>
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<tr>
<td>UMSS</td>
<td>Units to be Made for Stock at Supplier (units/week)</td>
</tr>
<tr>
<td>UOIS</td>
<td>Units on Order for Inventory at Supplier (units)</td>
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<tr>
<td>URIS</td>
<td>Units Received at Inventory at Supplier (units/week)</td>
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<tr>
<td>USMS</td>
<td>Units Sent to shipping from Manufacturing at Supplier (units/week)</td>
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<td>w</td>
<td>Radian frequency variable (radians/week)</td>
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<tr>
<td>wN</td>
<td>Radian frequency of Natural oscillations (radians/week)</td>
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<td>Zeros in Numerator of present manufacturing sector transfer function</td>
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<tr>
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<td>Zeros in Numerator of New manufacturing sector transfer function (dimensionless)</td>
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