EFFECT OF SUPERPOSED THROUGHFLOWS ON MOTION
INDUCED BY ENCLOSED ROTATING DISKS

by

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ABSTRACT

Effect of Superposed Throughflows on Motion
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Submitted to the Department of Civil Engineering on May 18, 1962 in partial fulfillment of the requirements for the degree of Master of Science.

This thesis is a study of the flow field induced by an enclosed rotating disk with superposed throughflow. The experimental program consisted of both a qualitative investigation using smoke injection for flow visualization and quantitative measurement of the pressure and velocity distribution at various combinations of throughflow rate and disk speed. Measurements of the fluid drag on the disk were also made.

The experimental program was carried out using smooth disks of 18-1/8 inch diameter which rotated in a housing that had provision for variable axial spacing. Air was used as the test fluid. Use was made of a blower to provide variable rates of throughflow around both sides of the disk. Primary emphasis was placed on turbulent flow during the quantitative phase of the test program.

Pressure and velocity measurements indicate that the effect of throughflow is of primary importance. A reduction in the tangential component of velocity and the radial pressure gradient with increasing rates of throughflow is observed and correlated with a theoretical analysis. An increase in the resisting torque is also detected when throughflow is superposed. The data appear to indicate that this increase in torque may be correlated with a so-called throughflow number.

Observation of the transition to turbulence indicates that this phenomenon is primarily a function of the local Reynolds number, \((\omega r^2)/\nu\), with a secondary effect by the axial spacing.

Of primary concern is the observation of periodic fluctuations under certain conditions. Correlation of the measured fluctuation frequency was obtained with the angular speed of the disk.
ACKNOWLEDGEMENTS

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A special word of thanks must go to Mr. William D. Ernst, Research Assistant at the Hydrodynamics Laboratory, for his sizable contribution to this investigation. The design and construction of the electronic circuitry for velocity measurements was carried out by him. He also was of great assistance in the collection of experimental data.

Mr. Wallace Fleming, of the machine shop staff, is also acknowledged for his helpful advice and expert work during the construction phase of the test program.
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LIST OF NOTATIONS

A = a constant.
a = disk radius, ft.
c = disk radial tip clearance, ft.
c_{1,2,...} = numerical constants.
c_m = torque coefficient, defined by \( M = \frac{1}{2} \rho \omega^2 a^5 \).
g = acceleration due to gravity, ft/sec^2.
K_o = \( \beta/\omega \), ratio of angular velocities.
K = U/\omega r, ratio of angular velocities with throughflow.
M = frictional torque, ft-lb.
n = integer exponent.
p = pressure intensity, lb/ft^2.
P = total pressure intensity, lb/ft^2.
Q = volumetric rate of flow, ft^3/sec.
r = radius, ft.

\( r \) = subscript "r" referring to radial component.
R = Reynolds number, as defined; if not defined, \( R = \) Disk Reynolds number \( = \frac{\omega a^2}{v} \).
s = axial clearance between disk and endwall, ft.

\( s \) = subscript "s" referring to stationary endwall.
\( t \) = subscript "t" referring to tangential direction.
u = absolute tangential velocity component.
U = absolute tangential velocity component in core.
v = absolute radial component, ft/sec.

v_o = reference velocity near the rotating disk, ft/sec.
v_o* = reference velocity near the stationary endwall, ft/sec.
w = axial component of velocity, ft/sec.
LIST OF NOTATIONS (continued)

$W =$ total velocity in the core relative to the disk surface.

$W' =$ total velocity in the core, unless otherwise defined,

$$W' = (u^2 + v^2)^{1/2}.$$  

$x =$ numerical constant.

$y =$ distance normal to stationary endwall, ft.

$Y' =$ factor in turbulent boundary layer thickness.

$z =$ distance normal to rotating disk, ft.

$\alpha(\alpha) =$ angle of local velocity, degrees.

$\beta(\beta) =$ angular velocity, radians/sec.

$\Delta(\Delta) =$ difference between quantities.

$\delta(\delta) =$ disk boundary layer thickness, ft.

$\delta'(\delta') =$ numerical factor in laminar boundary layer thickness.

$\chi(\chi) =$ non-dimensional distance.

$\eta(\eta) =$ dimensionless distance from solid boundary.

$\phi(\phi) =$ stationary endwall boundary layer thickness, ft.

$\theta =$ angular displacement about disk centerline, radians.

$\mu(\mu) =$ dynamic viscosity of fluid $\frac{lb\text{-sec}}{ft^2}$.

$\nu(\nu) =$ kinematic viscosity of fluid $ft^2/\text{sec}$.

$\pi(\pi) =$ 3.14...

$\rho(\rho) =$ mass density of fluid, slugs/ft$^3$.

$\tau(\tau) =$ shearing stress, lb/ft$^2$.

$\tau_0 =$ boundary shearing stress, lb/ft$^2$.

$\tau_r =$ radial component of shearing stress, lb/ft$^2$.

$\tau_t =$ tangential component of shearing stress, lb/ft$^2$.

$\omega =$ angular velocity of disk, radians/sec.
Fig. 1 Definition Sketch
I. INTRODUCTION

A. Description of Disk Problem

The study of flow about an enclosed rotating disk is of importance from both a practical and theoretical viewpoint. Of practical interest is the desire to accurately predict the power loss associated with the rotating components of turbomachinery. An example of such components that are analogous to a rotating disk is the shrouding of a pump impeller or turbine runner.

Considerable interest is being directed to the problem of leakage through the seals of a hydraulic turbine. Certain portions of these seals are also analogous to an enclosed rotating disk. Because of the difference in head across the runner of the turbine, a flow through the seals results. The effect on the power loss associated with the net flow through the seals is not clearly understood. A better understanding of this problem is of importance, since the overall efficiency of a turbomachine is dependent on the power loss in seals and shrouding.

The fluid mechanics of the flow about a rotating disk is part of the more general problem of three-dimensional boundary layer flow. When a disk is rotated in a fluid there is a thin zone close to the disk in which, due to the centrifugal forces, there is a pumping action causing fluid to move radially outward. The combination of a radial flow and the induced rotating flow produces a skewed boundary layer. By skewed it is meant that the total velocity vector does not lie in a single plane. Such a boundary layer on the disk has the following form:
Note that the plane in which the total velocity vector lies is warped. When the disk is enclosed in a housing there is a net flow radially inward along the stationary endwalls of the housing. This results in a similar skewed boundary layer on the stationary surfaces. The fluid which is pumped radially outward receives a net increase in angular moment whereas the fluid which returns along the endwalls receives a net loss in radial momentum. The end result is an equal and opposite torque on the disk and housing respectively. When there is a net flow through the system the boundary layers are modified and become more skewed. The flow circulation just mentioned may or may not exist.

Skewed boundary layers exist on many rotating and non-rotating surfaces other than the simple case mentioned above. Examples are; flow over a delta type aircraft wing, the stationary and rotating blades of gas compressors and turbines, flow in ducts where there is a change in flow direction, etc. In fact, because of curvilinear motion, skewed boundary layers may be expected wherever centrifugal forces are generated.

Therefore, it may be summarized that the purpose of this study is twofold:

1. to obtain information useful in the design of turbomachinery,
2. to study a simple form of the complex three-dimensional boundary layer.

B. **Scope of Investigation**

This thesis presents a study of an enclosed rotating disk with superposed throughflow using air as a test medium.

As indicated in the literature survey in the next section, there have been a number of previous investigations of flow about both free and enclosed rotating disks. However, a thorough investigation of the effects of throughflow on the rotating flow field induced by the rotating disk has not been published. The investigation, of which this thesis is a part, has such a thorough study as its objective. The results reported here provide a general picture of the important changes in the flow pattern which are induced by the superposed throughflow.

The experimental set-up used in this study is a modification of that used by Daily and Nece (4). It consists of a smooth disk which rotates in a cylindrical housing. The housing is closed at each end by circular cover plates. Axial spacing between the disk and the cover plates may be varied. Provision is made for admitting fluid into the clearance spaces between the disk and endwalls and for collecting the fluid through an annular slot at the outer periphery of the disk.

The experimental program was carried out in two steps:

1. a qualitative survey using flow visualization techniques,

2. a comprehensive program of velocity, pressure, and torque measurements.

For the first phase of the study a transparent section was used at the front of the housing. Smoke injection was the medium for flow visualization. Observation of the flow field was carried out over a
range of disk speed such that both laminar and turbulent flow were observed. The rate of flow through the system was varied in discrete steps from zero to about 60 cfm. Visual observation of the flow pattern in general, the effect of throughflow on transition to turbulence, and the measurement of periodic fluctuations were made during the initial phase of the test program.

Both the qualitative and quantitative phases of the experimental program were confined to using air as the test fluid. Pressure measurements were made to determine the effect of throughflow on the radial pressure gradient and to check the symmetry of the flow field. This was accomplished using 6 wall taps, 2 at each of three radii. The taps at each radius were spaced 90° apart. Measurements of pressure distribution were confined to turbulent flow. The data obtained are plotted in non-dimensional form for four spacings and for several combinations of disk speed and rate of throughflow.

The variation of both tangential and radial components of velocity across the gap between the stationary wall and the rotating disk was determined at three different radii for two disk speeds, four rates of throughflow, and two different spacings. Selection of the axial spacings used was made such that both close clearance flow and the case where separate boundary layers exist on the rotating disk and the endwall were included. Data are presented in non-dimensional form. The resulting data are compared to present theory, with appropriate modifications.

Some torque data were obtained at four different spacings. A correlation of the measured torque is obtained with the rate of throughflow.

Observation of the transition to turbulence, made during the
qualitative phase of this study, is correlated with the local Reynolds number. Measurement of the frequency of periodic fluctuations, which were also observed during the initial phase of this study, was accomplished using a smoke tracer technique. A linear correlation of the measured frequency with the disk speed was obtained.
II. SURVEY OF THE LITERATURE

A. Theoretical Development of Flow about a Rotating Disk

The specific problem of laminar flow about a rotating disk in an infinite fluid medium is one of the few exact solutions of the Navier Stokes equations. Von Karman (9) carried out this solution using the normalized variables; \( F(\zeta) = \frac{W}{\omega R} \), \( G(\zeta) = \frac{V}{\omega R} \), \( H(\zeta) = \frac{W}{\sqrt{\nu \omega}} \), and \( P(\zeta) = \frac{p(z)}{\rho \sqrt{\nu \omega}} \). \( \zeta \) is the non-dimensional distance, \( z \frac{\omega}{\sqrt{\nu}} \). Substitution into the Navier Stokes equations, which are simplified because of axial symmetry, results in a system of four simultaneous ordinary differential equations. The appropriate boundary conditions obtained from the no-slip condition at the disk are:

\[
\begin{align*}
\zeta &= 0, F = 0, G = 1, H = 0, P = 0 \\
\zeta &= \infty, F = 0, G = 0
\end{align*}
\]

Von Karman obtained an approximate solution of these equations. It was shown by him that the variation of tangential and radial components of velocity is significant only in a thin layer of fluid close to the disk. Therefore, the rotating disk problem may be classed as a boundary layer flow problem. The boundary layer thickness, \( \delta \), was computed by Von Karman to be:

\[
\delta = 2.58 \sqrt{\frac{\nu}{\omega}}.
\]

Neglecting edge effects this may be expressed for a finite disk as:

\[
\delta = 2.58 \frac{a}{R^{1/2}} \quad \text{[1]}
\]

where \( a = \) disk radius and \( R = \) Reynolds number = \( \frac{\omega a^2}{\nu} \). Also neglecting
edge effects, a torque coefficient defined as \( c_m = \frac{M}{\frac{1}{2} \rho \omega^2 a^5} \), \( M \) being the torque on both sides of the disk, was computed by Von Karman to be:

\[
c_m = \frac{3.68}{R^{\frac{1}{2}}}.
\]

In the same paper, Von Karman (9) also obtained a solution for the drag and boundary layer thickness on a rotating disk in an infinite medium when the flow is turbulent. He used the integrated momentum equations for the radial and tangential directions. Radial pressure gradient was assumed to be zero. A \( 1/7 \) power law was assumed for the tangential component of velocity and a modified \( 1/7 \) power law was used for the radial component. Substituting the assumed velocity profiles and using the Blasius expression for shear stress, two simultaneous equations in terms of a fictitious velocity, \( c_o \), and boundary layer thickness, \( \delta \), result. The equations are satisfied by the substitutions:

\[
c_o = c_1 \omega
\]

\[
\delta = c_2 \frac{r}{\sqrt{\nu}}
\]

The solution is then:

\[
\delta = \frac{0.526 r}{(\frac{c_o}{\sqrt{\nu}})^{\frac{1}{7}}}
\]

\[
c_m = 0.146 R^{\frac{1}{2}}
\]

The constant 0.526 was actually reported by Von Karman as 0.462 and later shown to be a misprint.
Cochran (1) later obtained a more precise solution of the laminar flow about a disk using numerical integration. The value of the moment coefficient obtained by him is:

\[ c_m = 3.87 R^{-1/2} \]  \[ 7 \]

Goldstein (6) was also concerned with turbulent flow about a rotating disk in an infinite medium. He followed a method similar to Von Karman's, but used a logarithmic expression for the velocity profiles. The numerical constants were adjusted to fit the experimental data available. An expression for the torque coefficient which is valid for higher disk speeds is obtained as:

\[ \frac{1}{\sqrt{c_m}} = 1.97 \log_{10}(R \sqrt{c_m}) + 0.03 \]  \[ 8 \]

Lately Srinivasas (17) has corrected the Goldstein expression by using constants in the logarithmic velocity expression that are evaluated close to the wall. A slightly different expression results:

\[ \frac{1}{\sqrt{c_m}} = 2.495 \log_{10}\left(\frac{a}{k_s} c_m\right) + 1.365 \]  \[ 10 \]

A solution for the flow about an enclosed disk was accomplished by Schultz-Grunow (13). He first obtained a correction to the laminar close clearance case, where a linear velocity distribution is assumed between the rotating disk and the stationary endwall. This results in an expression of the form:

\[ c_m = \frac{2\pi}{\frac{s}{a} R} + 2 R \left(\frac{a}{s}\right)^3 \left[ 0.01461 + \frac{s^2}{a} 0.1256 \right] \]  \[ 11 \]
The first term in this expression is obtained neglecting radial velocities. The second term is a correction accounting for the existence of radial and axial components of velocity. A solution for the case of separate boundary layers on the rotating disk and enclosing walls was also obtained by him using an appropriate momentum analysis. Assumed velocity profiles were used for both the laminar and turbulent cases considered. It was also postulated that the fluid outside the boundary layers rotates as a solid body at a fraction, \( K \), of the angular velocity of the disk. The results are:

a) for the Laminar Case

\[
\delta = \frac{2.17a}{R^{1/2}} = \text{constant} \tag{13}
\]

\[
K = \frac{\delta}{\omega} = 0.538 \tag{14}
\]

b) for the Turbulent Case

\[
c_m = 0.0622 R^{-1/5} \tag{15}
\]

\[
\delta = \frac{1025 r}{(\frac{\omega r^2}{\gamma})^{1/5}} \tag{16}
\]

\[
K = \frac{\delta}{\omega} = 0.512 \tag{17}
\]

Soo (16) carried out a solution for laminar flow over an enclosed rotating disk for the case when the clearance is small. In a fashion similar to Schultz-Grunow he arrived at:
\[ c_m = \frac{2\pi}{(\frac{2}{a})R} \]  \[ [18] \]

A correction to equation [18] in terms of a Reynolds number, \( \frac{\omega s^2}{v} \), is also computed. It was postulated by him that for low rates of through-flow the radial flow circulation pattern will not be affected. A theoretical solution for laminar flow with small amounts of through-flow indicates that the location of maximum velocity is shifted from the centerline of the gap between the disk and stationary endwall toward the disk surface for increasing Reynolds number, \( \frac{\omega s^2}{v} \). A solution for turbulent flow, when the clearance, \( s \), is small, is given as:

\[ c_m = 0.0622 \left( \frac{2}{a} \right)^{-1/4} R^{-1/4} \]  \[ [19] \]

Daily and Nece (4, 5) were the first to postulate the existence of four regimes of flow for an enclosed rotating disk. They are:

1) Regime I - Laminar flow in close clearance with negligible radial and axial components of velocity. The velocity distribution is linear from \( \omega r \) at the disk to zero at the endwall. The resulting expression for torque coefficient is:

\[ c_m = \frac{2\pi}{s^2} \]  \[ [18] \]

as has been obtained by Schultz-Grunow and Soo.

2) Regime II - Separate laminar boundary layers on the rotating disk and stationary endwalls. A secondary radial outflow exists close to the disk and a radial inflow exists close to the stationary endwall. The fluid in the space between the boundary layers is assumed to rotate as a solid body at some fraction of the disk speed. The theoretical analysis made by Daily and Nece indicated:
\[ c_m = \frac{c_1}{\frac{R}{2}} \]  \[ \delta = \delta' \frac{a}{\frac{R}{2}} \]  \[ 0.0420 \left( \frac{s}{a} \right)^{\frac{-\frac{3}{4}}{4}} \]  \[ \frac{\gamma^2}{(\mu^2)^{\frac{1}{5}}} \]

The values of the constants \( c_1 \) and \( \delta' \) are functions of the spacing relative to the disk radius, \( s/a \), and are tabulated in reference (5).

3) Regime III - Turbulent flow in close clearance. Radial outflow is considered negligible. This corresponds to the turbulent flow solution as given by Soo. Nece and Daily found the theoretical solution to be:

\[ \frac{\gamma^2}{(\mu^2)^{\frac{1}{5}}} \]

However, Soo's equation, [19], is found to more closely fit the experimental data.

4) Regime IV - Separate turbulent boundary layers on the rotating disk and stationary endwall. Radial outflow exists close to the disk and radial inflow close to the stationary endwall. Solution for the torque coefficient and the boundary layer thickness is of the form:

\[ c_m = \frac{c_1}{\frac{R}{5}} \]  \[ \delta = \frac{\gamma^2}{(\mu^2)^{\frac{1}{5}}} \]  \[ \left( \frac{\mu^2}{v} \right)^{\frac{1}{5}} \]

The constants \( c_1 \) and \( \gamma \) are functions of the spacing, \( s/a \), and are tabulated in reference (4).

Correlation of the above theory was obtained by Daily and Nece.
through a comprehensive program of torque measurements.

B. Survey of Experimental Studies

There are abundant references to experimental work dealing with flow about a rotating disk. Many experimental studies were made in an attempt to obtain empirical relationships to express the power loss associated with a rotating disk in a fluid. Such references will not be considered in this survey.

Of interest to this study is the experimental work which has been done to define the overall problem at hand.

Picha and Eckert (12) performed an experimental study of the air flow between two coaxial disks. It was reasoned that if the flow in the core, i.e. the flow outside the boundary layers on the disk, was known, then the integrated momentum equations could be solved for boundary layer thickness and shear stress. The study was carried out with the aid of smoke injection as a flow visualization tool and with velocity measurements. Both shrouded and unshrouded disks were used in the experimental program. The disks were also rotated with arbitrary velocities.

It was found that, when the disks were rotated with equal and opposite speeds, the core was stationary. Rotating the disks in the same direction results in a very stable core at the disk centerline. A study of the flow between two unshrouded disks rotating at the same angular speed indicated that the core consisted of a forced vortex at the inner radii and a free vortex at the outer radii. Other results indicated that the angular speed of the core relative to the angular speed of the disk is relatively constant over a wide range of Reynolds
number. Large random velocities were also observed in the core when the disks were unshrouded.

Further studies were made on the same configuration, as used by Picha and Eckert, by Walsh and Hartnett (21). Velocity profiles in the boundary layer flow on the disks were obtained for the tangential and radial directions. Experimental correlation with the theory of Cochran (1) was obtained for laminar flow on both a single disk and opposed disks. Measurements in the turbulent boundary layers indicated the boundary layer thickness to be 50% thinner than predicted by Von Karman for the free disk case. A 1/7 power law was found to give a satisfactory fit to measurements of the tangential component in a turbulent boundary layer on the rotating disk.

Theodorsen and Regier (20) have made some experiments on a 24-inch diameter disk. Transition to turbulence was deduced from a plot of the local drag coefficient, which in turn is related to the torque coefficient, as a function of the Reynolds number. A sudden increase in the drag coefficient, indicating transition, is noticed at a Reynolds number, $\frac{\omega a^2}{\nu}$, of about 310,000. This value was confirmed with hot-wire measurements. A study was made of the effect of roughness on transition. Measurements indicated a maximum reduction of the transition Reynolds number to 220,000. An early onset of turbulence was best achieved by a high pressure air jet directed at the center of the disk. It is interesting to note that a Reynolds number, using the theoretical laminar boundary layer thickness, $\delta$, as a length parameter, will have the value $\frac{\omega a \delta}{\nu} = 1.4h_0$ at transition. This is close to the value for pipe flow.

Further interest in the problem of instability of a laminar
boundary layer on a rotating disk was generated by Gregory, Stuary, and Walker (7). Their interest in flow about a rotating disk was aroused because a very simple model of the three-dimensional boundary layer exists on the rotating disk. The China Clay method was used by them to visualize the transition to turbulence in the flow close to the disk surface. A system of vortices, having equiangular spirals for their axes, was observed. This observation was correlated with a theoretical analysis of the nature of instability in the flow. A value of the transition Reynolds number of \(2.8 \times 10^5\) was obtained.

Smith (15) also investigated the nature of instability on a smooth steel disk rotating in air. He obtained a value of the Reynolds number at transition of \(3.1 \times 10^5\). This compares favorably with the values of \(2.8 \times 10^5\) and \(3.1 \times 10^5\) obtained by Gregory, Stuart, and Walker (7) and Theordorsen and Regier (20) respectively.

Maroti, Deak, and Kreith (10) have done some experiments using a plastic disk rotating between two stationary disks. Smoke injection and cotton tufts were used to visualize the flow. When the diameter of the stationary disks was greater than the diameter of the rotating disk a periodically fluctuating flow field was observed. A linear correlation of the fluctuation frequency with the disk speed was obtained. Axial spacing was observed to effect the variation of frequency with the disk speed.

C. Other Related Work

A literature survey of this type would not be complete without consideration of the work being done on the three-dimensional boundary layer in general.
Taylor (19) presents a general outline of the structure of a three-dimensional boundary layer. Of interest is the concept that three-dimensional boundary layers consist of two portions; an outer, collateral portion and an inner quasi-collateral portion. The quasi-collateral portion may be made collateral by rotating the axis of reference. Reference to experimental work by Johnston (8) and Senoo (14) illustrates an interesting facet of the structure of a three-dimensional boundary layer. If the cross flow is plotted as a function of the main flow a triangular shape plot occurs. The data follow a line with a positive slope close to the wall and a line with negative slope out to the edge of the boundary layer. Such a plot has the form:

\[ \frac{v}{U} \]

\[ \frac{u}{U} \]

A more elaborate survey of the various solutions of skewed boundary layer flow is given by Cooke and Hall (3). In this paper the full boundary layer equations in orthogonal curvilinear coordinates are described, along with the corresponding momentum integral equations. Several examples of solution of these equations for laminar flow are given. A description of the analogy to the axially symmetric boundary layers is also explained. Such an analogy exists when the cross flow velocities are small compared to the main flow velocities. The inertia
terms for the cross flow become small compared to the other terms in the equation for the main flow direction. This reduces the momentum equation in the main flow direction to that of the axially symmetric boundary layer.

Of particular interest is the description of the empirical techniques required for turbulent flow. Basically these methods boil down to making appropriate assumptions for shear stress, shape factor, and velocity profiles. Substitution of these expressions into the momentum integral equations will lead to a solution for boundary layer growth, from which the numerical value of shear stress can be determined.

A more thorough analysis of the calculation of a skewed turbulent boundary layer is given by Cooke (2). He explains in more detail the techniques as set forth in the paper by Cooke and Hall.

In a recent paper, Olsson (11) performed a successful calculation of the boundary layer on a stationary blade of an axial flow compressor. In this particular case the cross flow was small compared to the main flow and the momentum integral equations are greatly simplified.

The literature indicates that a great deal of work is required on the skewed boundary layer before the reliance on data from two-dimensional boundary layer flow may be done away with.
III. THEORETICAL CONSIDERATIONS

A. General Method of Analysis

1. Introduction

Of primary concern in the study of flow about an enclosed rotating disk is the determination of the torque felt by the disk through fluid friction. A simple analysis is appropriate to show what variables are important.

Consider a unit element on the disk:

![Diagram of a unit element on a disk]

The torque $\Sigma M$ applied on the disk is:

$$\Sigma M = \tau_{ot} r^2 \, dr \, d\theta$$

The torque applied over both sides of the disk is:

$$2 \Sigma M = 2 \int_0^a \int_0^{2\pi} \tau_{ot} r^2 \, dr \, d\theta$$

$$\tau_{ot} = \frac{c_f}{2} \rho \, U^2 \quad U \sim \omega r$$

$$\therefore \tau_{ot} = \frac{c_f'}{2} \rho \, \omega^2 \, r^2$$
\[ 2M = 2 \int_{0}^{a} \int_{0}^{2\pi} \frac{c_\theta}{2} \rho \omega^2 r^4 \, dr \, d\theta = 2\pi c_\theta \rho \omega^2 a^5 \]

or

\[ 2M = \frac{c_m}{2} \rho \omega^2 a^5 \quad [25] \]

Dimensional analysis shows that,

\[ c_m = F(R, s/a, \frac{Q}{\omega a^3}) \]

To illustrate the computation required for \( c_m \), consider the following model:

1. Separate turbulent boundary layers on the disk and stationary housing surfaces.

2. A core between the boundary layers with radial outflow of velocity \( V \) and a tangential velocity component \( U \). Both components are assumed to be constant across the width of the core.

Following Von Karman (Ref. 9) and Daily and Nece (Ref. 5), a momentum analysis may be made:

From conservation of momentum the following may be written:
\[ \rho r \left[ \int_0^\delta \frac{u^2}{r} \, dz \right] \, dr - \tau_{or} \, r \, dr - \frac{\partial \vartheta}{\partial r} \, r \, \delta \, dr = \]

\[ \frac{\partial}{\partial r} \left[ \rho \int_0^\delta v^2 r \, dz \right] \, dr - \rho V \frac{\partial}{\partial r} \left[ \int_0^\delta v r \, dz \right] \, dr \]  \[ [26] \]

The first term on the left is the centrifugal force on the element. The second and third terms are the opposing shear force and pressure force respectively. The two terms on the right represent the net momentum flux through the element.

A second condition is obtained by considering the rate of change of angular momentum through the system; stating \( M = \Delta \rho Q \, V \, r \):

\[ \tau_{ot} \, r^2 \, dr = \frac{\partial}{\partial r} \left[ \rho \int_0^\delta u r \, v^2 \, dz \right] \, dr - \rho V r \frac{\partial}{\partial r} \left[ \int_0^\delta v r \, dz \right] \, dr \]

\[ [27] \]

Similarly, the equations for the stationary wall are:

\[ \rho r \left[ \int_0^\delta \frac{u^2}{r} \, dz \right] \, dr - \tau_{or} \, r \, dr - \frac{\partial \vartheta}{\partial r} \, r \, \vartheta \, dr \]

\[ = \frac{\partial}{\partial r} \left[ \rho \int_0^\delta v^2 r \, dz \right] \, dr - \rho V \frac{\partial}{\partial r} \left[ \int_0^\delta v r \, dz \right] \, dr \]

\[ [28] \]

\[ \tau_{ot} \, r^2 \, dr = \frac{\partial}{\partial r} \left[ \rho \int_0^\delta u r \, v^2 \, dz \right] \, dr - \rho V r \frac{\partial}{\partial r} \left[ \int_0^\delta v r \, dz \right] \, dr \]

\[ [29] \]
Note that the direction of the shear stress and the radial velocity are assumed in the same direction as on the rotating disk.

Daily and Wece (4) carried out the computation of these equations in the following steps:

1. Assume solid body rotation in core with no radial outflow, i.e. \( V = 0 \); also the terms involving \( \rho UR \frac{\partial}{\partial r} [ \int_0^\delta v r ds] dr \) are assumed equal to zero. The pressure gradient may be written as \( \frac{\partial P}{\partial r} = \rho K_o^2 \omega^2 r^2 \), \( K_o \) being a constant.

2. Assume that the tangential velocity on the stationary wall and the tangential velocity defect on the rotating disk follow the \( 1/7 \) power law. Also use a modified \( 1/7 \) power law for the radial velocity. The following forms of the velocity profile were then used:

On the rotating disk,

\[ u = \omega r [1 - \zeta^{\frac{1}{7}} + K_o \zeta^{\frac{1}{7}}] \tag{30} \]

\[ v = v_o (\zeta)^{\frac{1}{7}} (1 - \zeta)^{\frac{1}{7}} \tag{31} \]

\[ \zeta = \frac{z}{\delta} \quad v_o = \zeta \omega r \], a fictitious quantity which makes \( v = f(\omega r) \) as would be expected.

On the stationary wall,

\[ u = K_o \omega r (\zeta)^{\frac{1}{7}} \tag{32} \]

\[ v = v_o^* (\zeta)^{\frac{1}{7}} (1 - \zeta)^{\frac{1}{7}} \tag{33} \]

\[ \zeta = \frac{r}{\delta} \quad \text{and} \quad v_o^* = A \omega r \], a fictitious quantity.
3. Assume that the shear stress at the solid boundaries of the system follow the expression for pipe flow:

\[ \tau_o = 0.0225 \rho W \left( \frac{\gamma}{6} \right)^{3/4} \]  \[34\]

where

\[ W = (v_o^2 + (1-K)^2 \omega^2 r^2)^{1/2} \]  \[35\]

and

\[ W' = (v_o^2 + K^2 \omega^2 r^2)^{1/2} \]  \[36\]
on the stationary disk.

Then \( \tau_{or} = \frac{v_o}{W} \tau_o \) and \( \tau_{ot} = \frac{(1-K)\omega r}{W} \tau_o \) on the rotating disk. On the stationary end wall \( \tau_{or}' = \frac{v_o}{W} \tau_o' \) and \( \tau_{ot}' = \frac{K\omega r}{W} \tau_o' \).

4. Evaluate equations [26] through [29] using the assumed expressions for velocity, shear stress, and pressure gradient. Note that expressions for boundary layer thickness of the form \( \delta = \gamma r^{3/2} \) and \( \phi = B r^{3/2} \) will satisfy the equations. It is interesting to note at this point that the boundary growth on a free disk in turbulent flow is also of the form \( \delta = c r^{3/2} \). Note that the radial pressure gradient on a free disk is assumed zero.

5. Using the computed value of boundary layer thickness evaluate the shear stress and the torque on the rotating disk and stationary wall in terms of \( K_o \) and \( R \). Assume that the boundary layer thickness on the cylindrical portion of the casing is equal to that at the end of the rotating disk and evaluate the shear stress and torque in terms of \( K_o \), \( R \), and the spacing, s/a.

6. Note that the torque on the rotating disk is equal to the
total torque on the housing, i.e. the torque on the cylindrical portion plus the stationary end walls. Plot the variation of torque on the rotating disk as a function of $K_o$, holding $R$ constant. On the same diagram plot the torque on the housing for various values of $s/a$ as a function of $K_o$. The intersection of the two curves is the solution of $K_o$ for any particular value of $s/a$. The solution for $c_m$ may then be given in terms of $s/a$ and $R$. It has the form $c_m = \frac{\text{constant}}{R^{1/2}}$, where the constant is a function of $s/a$. The following table was computed by Daily and Wece:

<table>
<thead>
<tr>
<th>$s/a$</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.073</td>
</tr>
<tr>
<td>0.10</td>
<td>0.078</td>
</tr>
<tr>
<td>0.15</td>
<td>0.083</td>
</tr>
<tr>
<td>0.20</td>
<td>0.087</td>
</tr>
<tr>
<td>0.25</td>
<td>0.090</td>
</tr>
<tr>
<td>0.30</td>
<td>0.093</td>
</tr>
</tbody>
</table>

If the previous analysis is considered with the additional variable, throughflow, superimposed it can be seen that it will not be applicable to the present problem without modification. Note in equation [28] that the direction of the radial velocity component in the boundary layer and the corresponding wall shear are unknowns since it is still possible to have a flow circulation similar to that for zero throughflow or to have the radial flow in the outward direction throughout the axial spacing, as is the case for high rates of throughflow. The core is now composed of a flow field having both radial and tangential components,
V and U respectively. Note that U no longer has the form, $K_0 \omega r$, where $K_0$ is a constant. Note also that the radial component of flow is now a function of both the rate of throughflow and the centrifugal effects. Inspection of equations [26],[27],[28], and [29] will show that they are no longer satisfied by a simple expression of boundary layer thickness of the form $\delta = Y r^{3/5}$. The following sections are intended to shed some light on the structure of the flow field so that the previously proposed model, described by equations [26], [27], [28], and [29] may be solved for particular situations. The various components required in the model will be analyzed separately.

B. Torque Variation with Throughflow

A simple radial momentum analysis can be made to determine the order of magnitude of the increase of torque with throughflow over the corresponding value without throughflow.

Consider that the throughflow enters the housing without any tangential whirl and leaves with some tangential whirl, $K_0 \omega r$, at the disk periphery, $r = a$. Using the expression:

$$M = \Delta \rho Q V r$$

we may state that,

$$M \text{ increase} = \rho K Q \omega a^2 - 0$$

the torque without throughflow is

$$M = \frac{c_m}{2} \rho \omega^2 a^5$$
The percent increase in torque is,

$$\% \text{ Increase} = \frac{M_{\text{inc}}}{M} \times 100 = \frac{\rho K Q \omega a^2}{c_m} \frac{c_m}{\frac{R}{2} \rho \omega^2 a^5}$$

$$\% \text{ Increase} = \frac{2K}{c_m} \frac{Q}{\omega a^3}$$  \[39\]

This expression may be evaluated using $K = K_0$, the value for zero throughput, and $c_m$ from the previously obtained torque data for zero throughput.

Using the experimentally derived Daily-Nece expression for $c_m$ in turbulent flow:

$$c_m = \frac{0.102[s/a]}{R^{1/5}}$$  \[40\]

we obtain,

$$\% \text{ Increase} = \frac{K}{0.051} \frac{R}{\frac{1}{5}} \frac{\frac{1}{10}}{\frac{Q}{s/a} \omega a^3}$$  \[41\]

Example:

When $s/a = .2207$, $K_0 = 0.4$

$$\% \text{ Increase} = 9.13 \frac{R}{\frac{1}{5}} \frac{Q}{\omega a^3} \times 100$$

C. Prediction of Pressure Distribution

For the case of zero throughput the agreement of observed pressure distribution with the theoretical distribution for solid body core rotation is excellent [Ref. (4)]. The theoretical expression is
derived by neglecting any gradients in pressure or shear in the $z$ direction.

\[ p + \frac{\partial p}{\partial r} dr \]

\[ [p - (p + \frac{\partial p}{\partial r} dr)]r dz = -\rho r dr \ dz \frac{u^2}{r} = -\rho r \frac{K^2 \omega^2 r^2}{r} dr \ dz \]

and

\[ \frac{\partial p}{\partial r} = \rho K^2 \omega^2 r \]  

\[ [4.2] \]

With $K = K_0$ a constant for solid body rotation we may integrate from $r = a$ to $r$ to obtain the following:

\[ p_r - p_a = \rho \frac{K^2 \omega^2}{2} [r^2 - a^2] \]  

\[ [4.2a] \]

If we define the pressure at $r = a$ equal to zero, then the pressure at $r = 0$ is,

\[ p_0 = -\rho \frac{K^2 \omega^2}{2} a^2 \]  

\[ [4.3] \]

Using $p_0$ as a reference pressure the following is obtained:

\[ \frac{p_r}{p_0} = 1 - \left( \frac{r}{a} \right)^2 \]  

\[ [4.4] \]
Note that by this equation $p_r/p_0$ plots as a straight line versus $[1 - (r/a)^2]$.

The parameter $[1 - (r/a)^2]$ is useful to use when comparing experimental pressure data with throughflow. Departures of measured $p_r/p_0$ from a straight line will indicate effects of throughflow on the internal fluid circulation and rotation.

D. Equations for Flow in Core

1. For low rates of throughflow, a useful expression for $U$ may be written such that $U = K \omega r$ where $K = f(r)$, if the following assumptions are made:

   a. Fluid enters without any whirl.

   b. A uniform velocity distribution exists throughout the core, i.e. there is no variation in velocity over the width of the core.

   c. The Blasius expression for shear stress holds.

   d. The total velocity in the core is proportional to $\omega r$. That is, we assume $W = [(1-K)^2(\omega r)^2 + \nu^2]^{1/2} \approx \omega r - U$.

   e. Boundary layer thickness, $\delta(r)$, is unchanged with throughflow.

   f. The percentage increase in wall shear with throughflow is the same over the entire disk.

Now using starred quantities to denote those with throughflow, we may write:

$$\tau_t = 0.0225 \rho \frac{W}{\delta} \frac{\nu}{\delta}$$  \[45\]

$$\delta = c_1 r^n$$

$$\tau_t = \left[ \frac{\nu}{c_1} \right] .0225 \rho \frac{W}{r}$$  \[46\]
\[ W = (1 - K) \omega r \quad [47] \]

Let
\[
\left[ \frac{\tau^*}{\tau} \right]_a \sim \left[ \frac{\tau^*}{\tau} \right]_a \approx \frac{\rho Q K_0 \omega a^2 + \frac{c_m}{2} \rho \omega^2 a^5}{\frac{c_m}{2} \rho \omega^2 a^5} \quad [48]
\]

Holding \( R \) constant:
\[
\left[ \frac{\tau^*}{\tau} \right]_a \approx \frac{\rho Q K_0 \omega a^2}{\frac{c_m}{2} \rho \omega^2 a^5} + 1 \approx c_2 \frac{Q}{\omega r^3} + 1 \quad [49]
\]

State,
\[
\left[ \frac{\tau^*}{\tau} \right]_r \approx c_2 \frac{Q}{\omega r^3} + 1 \quad [50]
\]

Using [46] and [47] in [50], we obtain:
\[
\frac{[1 - K]}{[1 - K_0]} \frac{\gamma_4}{\gamma_4} \approx c_2 \frac{Q}{\omega r^3} + 1 \quad [51]
\]

or
\[
\frac{[1 - K]}{1 - K_0} \frac{\gamma_4}{\gamma_4} \approx c_2 \frac{Q}{\omega r^3} \quad [52]
\]

With a solution for \( K \) we may calculate \( U = K \omega r \).

2. Approximate computation of \( U \) for high rates of throughflow -
   a. Let \( \delta' = c_1 r^m \).
   b. Assume \( W = [(1 - K)^2 \omega^2 r^2 + V^2]^{1/2} \approx V \quad [53] \)
For the rotating disk:

\[ \tau_t = 0.0225 \rho W \left( \frac{y}{h^*} \right) \frac{y}{h^*} \cos \alpha \]  

[54]

where

\[ \cos \alpha = \frac{(1 - K)\omega r}{W} \]  

[55]

For a rough approximation, let \( W \approx c_2' \frac{Q}{r^2} \), where \( c_2' \) is a function of \( s \), the axial spacing. This is only good for relatively large values of \( r \).

\[ \tau_t' \approx \rho [c_2' \frac{Q}{r^2}] \left( \frac{y}{h^*} \right) \frac{y}{h^*} \frac{(1 - K)\omega r^3}{c_2' Q} \]  

[56]

\[ \tau_t' \approx \rho [c_2' \frac{Q}{r^2}] \left( \frac{y}{h^*} \right) \frac{y}{h^*} (1 - K)\omega r \]  

[57]

The corresponding shear stress without throughflow may be stated as:

\[ \tau_t = 0.0225 \rho \left( \frac{y}{h^*} \right) \left( 1 - K_0 \right) \frac{y}{h^*} (\omega r) \frac{y}{h^*} \]  

[58]

The ratio of the shear stresses is:

\[ \frac{\tau_t'}{\tau_t} \approx \frac{\frac{c_2' \frac{Q}{r^2}}{0.0225 (\frac{y}{h^*}) (1 - K_0) (\omega r) \frac{y}{h^*}}}{\frac{c_2' Q}{0.0225 (\frac{y}{h^*}) (1 - K_0) (\omega r) \frac{y}{h^*}}} \]  

[59]

Let

\[ \frac{\tau_t'}{\tau_t} \approx \frac{\frac{c_m}{2} \rho \omega^2 a^5}{\frac{c_m}{2} \rho \omega^2 a^5} \]  

[60]
Note that we are still assuming that the fluid leaves with the same whirl that it has at the disk periphery with zero throughflow.

Let

\[ \rho Q K_0 \omega a^2 \gg \frac{c_m}{2} \rho \omega^2 a^5 \quad \text{and} \quad \delta \approx \delta' \]

Then,

\[ \frac{\tau_t'}{\tau_t} = \frac{\rho Q K_0 \omega a^2}{c_2' \frac{Q}{\omega a^3}} = \frac{c_3'}{\frac{Q}{\omega a^3}} = \frac{c_4'}{\omega a^3} \quad [60a] \]

Substituting \( \delta = \delta' \) into equation [59] and assuming the same variation in shear stress at any radius the following is obtained:

\[ \frac{\tau_t'}{\tau_t} = c_3' \frac{1 - K}{\frac{Q}{\omega a^3}} \frac{\frac{Q}{\omega a^3}}{c_2' \frac{Q}{\omega a^3}} = c_2' \frac{Q}{\omega a^3} \quad [61] \]

Rearranging terms the following is obtained:

\[ \frac{1 - K}{\frac{Q}{\omega a^3}} = c_4' \frac{\frac{Q}{\omega a^3}}{c_2' \frac{Q}{\omega a^3}} \]

Equation [62] is only a rough indicator of what might be expected and is only valid over a limited range of \( Q \) and \( r \).

As before with a solution for \( K \) a solution for \( U = K \omega r \) may be obtained.
IV. EXPERIMENTAL EQUIPMENT AND INSTRUMENTATION

A. General Description of Apparatus

As already noted, the test facility used is a modification of the general set-up used by Daily and Nece, as described in references 4 and 5. The facility consists of a cylindrical housing enclosed by cover plates at the front and rear. An 18-1/8 inch diameter disk, which rotates inside the housing, is driven through a 2-inch shaft by a 5/7-1/2 hp DC motor. Instrumentation for measurement of velocity, pressure, temperature, and torque is provided. Figures 1, 2, and 3 illustrate the facility in its present modified form.

For the current studies provision was made to superpose a radial throughflow in the space between the rotating disk and the endwalls. Air was used as the test fluid. Visual studies were permitted by making the outboard endwall of transparent 1-inch thick lucite. Direct visual or photographic observations were made possible by using kerosene smoke as a tracer, which could be injected at discrete points through the transparent cover plate.

The modifications for throughflow superposition include:

1) For both sides of the disk, provisions for admitting fluid through an annulus surrounding the 2-inch drive shaft and for withdrawing fluid through an annular slot at the outer radius of the enclosing chamber.

2) A blower which delivers air through two separately metered lines to the two sides of the disk.

These modifications are described fully in the following sections.
Fig. 1 View of Test Set-up

Fig. 2 Sketch of Test Set-up
Fig. 3 Detail of Test Chamber

Fig. 4 Detail Drawing of Shaft Disk Assembly
Fig. 5 View of Rear Cover Plate

Fig. 6 View of Spacer Ring
B. Details of Housing

The external housing consists of the machined cast iron casing securely bolted to the base frame of the test stand. The actual test chamber is formed by a rear circular cover plate, a cylindrical spacer sleeve of "T" section, and a front circular cover plate. The casing is assembled in the following order: (Reference should be made to Fig. 3.)

1) Segmental spacers which bear in a slot in the housing.
2) The rear cover plate bears on the segmental spacers.
3) Spacer sleeve bears on the rear cover plate.
4) The front cover plate then bears on the sleeve spacer. The whole assembly is held together by cap screws which bolt the front cover plate to the front of the cast iron housing. Tightening of the cap screws compresses the whole assembly into a tight fit. Sealing is accomplished by rubber "O" rings located in grooves in the front and rear cover plates.

A view of one of the spacer sleeves is given in Fig. 6. It consists of a cylindrical top portion on which the sealing is accomplished, a web, and a bottom cylindrical portion whose width was 1/4-inch less than that of the top portion. This provided an annular space of 1/8-inch between the bottom of the sleeve and the front and rear cover plates through which throughflow is collected in the grooves formed by the top and bottom portion of the spacer ring and the web. The web of the spacer rings is slotted at 8 equally spaced points. At these points the throughflow passes into holes drilled and tapped for standard 1/2-inch pipe around the outside periphery of the spacer ring. These holes match with slots in the cast iron housing and the throughflow is exhausted to the atmosphere.
A view of the rear cover plate is shown in Fig. 5. It is machined from a bronze casting and is 1/2-inch thick. Stiffening is accomplished with two 60° segments, 1-inch thick, and by a spiral casing which is attached to the cover plate. A 2-7/8 inch hole is bored through the cover plate and casing to provide for the 2-inch shaft. This leaves a 7/16-inch annular space for throughflow to enter. Flow enters in a tangential direction into the casing and spirals over the drive shaft into the test chamber through the annular space provided. Sealing of the shaft is provided at the rear of the spiral casing with a standard spring loaded seal.

The front cover plate is constructed of 1-inch thick lucite and is stiffened by 4 ribs which attach to a 5-inch diameter hub at the center. The front cover plate bolts to the housing at the ends of the 4 ribs. The cover plate and hub are bored out to 2-7/8 inches at the center. Throughflow enters at the front of the hub. A 2-inch dummy shaft, described later, extends into the hub so that the throughflow also passes through a 7/16-inch annular space in the front cover plate. The lucite surfaces were carefully polished with jewelers rouge to facilitate photography. After this, polishing with wax also insured that small scratches would not show up in the photographs.

A bronze front cover plate was used for the velocity measurements. This cover plate is essentially in the same form as it was when used by Daily and Nece (4). It is constructed of 1/2-inch thick bronze and is stiffened by two 60° segments one-inch thick.

Axial symmetry of the casing was provided by means of a dummy shaft which is shown in Figures 3 and 4. It is attached to the front of the rotating disk. Shafts of different lengths were provided for the
various spacings used. The ends of the dummy shafts are capped with a nose cone to permit a smooth transition of flow from the 2-inch delivery pipe over the shaft into the test chamber. This dummy shaft extended well into the front hub so that the same amount of whirl was imparted to the flow through the front as was imparted to the flow at the rear by the drive shaft. The drive shaft and disks are the same as reported in references 4 and 5, except that the disks are turned down to 18-1/8 inch providing a radial tip clearance of 1/16-inch between the disks and the 18-1/4 ID sleeves. In order to enable photography of the flow pattern the bronze disks were coated with a coating of glossy black paint.

Although there is more light reflection from a glossy paint than from a flat paint, the reflection is in one direction. By illuminating the flow pattern at the right angle all reflected light leaves the disk at a great enough angle so that it is out of the field of view of the camera lens.

Four sleeves and two disks were used; this gave the various s/a combinations as shown on the following table:

<table>
<thead>
<tr>
<th>Sleeve Length</th>
<th>1/2&quot; Disk</th>
<th>1/4&quot; Disk</th>
<th>1/2&quot; Disk</th>
<th>1/4&quot; Disk</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4&quot;</td>
<td>1/8&quot;</td>
<td>1/4&quot;</td>
<td>.0138</td>
<td>.0276</td>
</tr>
<tr>
<td>1-1/2&quot;</td>
<td>1/2&quot;</td>
<td>5/8&quot;</td>
<td>.0552</td>
<td>.0690</td>
</tr>
<tr>
<td>2-1/2&quot;</td>
<td>1&quot;</td>
<td>1-1/8&quot;</td>
<td>.1103</td>
<td>.1211</td>
</tr>
<tr>
<td>4-1/2&quot;</td>
<td>2&quot;</td>
<td>2-1/8&quot;</td>
<td>.2207</td>
<td>.2345</td>
</tr>
</tbody>
</table>

The use of two different disks provided additional spacings more simply than by using additional sleeves.
C. Throughflow Circuit

Throughflow is provided by a Spencer Turbine Compressor rated at 50 cfm at 20.8 inches of water. Flow from the exhaust of the turbine branches into separate 2-inch lines leading to the front and rear of the housing.

The flow is metered by separate 3/4-inch orifices in each line. Calibration of the orifice meters was done with a precisely calibrated rotometer and a large 1 on 10 inclined manometer. The calibration runs were made with the orifice meters installed in place. Readings were corrected for temperature to standard air. The flow coefficients obtained deviated less than 1% from the coefficients defined in reference 5A for this particular geometry. When in operation, the pressure difference across the orifices was read on a commercial manometer, 0 to 5 inches of water being read on a 1 on 1 inclined scale and 5 to 20 inches on a vertical scale. Pressures from 0 to 5 inches could be read to 0.01 inch, and pressures from 5 to 20 inches could be read to 0.02 inch. This corresponds to a reading error of less than 1% of the flow rate.

D. Instrumentation for Measurements

1. Velocity Measurement

   a. Description of Velocity Probe - Two different probes were used in the test program. These probes are identical to those used by Daily and Nece. A directional probe measured the orientation of the total velocity vector and a pitot-static tube determined the magnitude of the total velocity. Selection of two different probes insured the desired sensitivity to flow direction. A pitot-static tube rather than
a total head tube was used to eliminate any discrepancies between static pressure measured at the wall and that existing some distance away from the wall.

The directional probe is a two-hole cobra probe with the tubes ground off at an angle of 45° with the flow. The velocity probe is of similar dimensions having two 0.0135-inch static pressure openings, one on either side of the impact hole. The distance of 0.105 inch from the tip to the static opening is three times the 0.035-inch diameter of the individual tubes at the tip. The relative dimensions of this tube are comparable to a standard Prandtl-pitot tube. The magnitude of the effect of the stem on the velocities at the tip has been checked by Daily and Nece. This was done assuming potential flow about a circular cylinder placed perpendicular to a uniform velocity field. Results of this analysis indicate a velocity at the tip equal to 0.99 times of the free stream velocity with the probe dimensions used. Both probes were constructed as to be interchangeable in the traverse control unit. The probes are illustrated in Figure 7.

b. **Calibration of Velocity Probes** - The directional probe was previously calibrated by Daily and Nece in a 6 by 14-inch wind tunnel operated at standard conditions with a velocity of 100 feet per second. At that time, using an inclined manometer, the sensitivity of the probe was found to be 1/4 of a degree. However, using more sensitive pressure sensing equipment than previously used, the sensitivity was in some cases as high as 5 minutes of angle.

Previous calibration of the pitot-static tube in a free surface water tunnel indicated a tube coefficient very close to unity. A coefficient of unity was therefore assumed throughout this investigation.
Fig. 7 View of Velocity Probes

Fig. 8 View of Traverse Control Unit
c. Traverse Control Unit - The velocity probe traverse control unit is shown in Figure 8. (It is in essentially the same form as when used by Daily and Nece.) This unit is so constructed that the tip of either the velocity or directional probe may rotate about a point. The entire unit containing the probe rotates in any one of three machined holes in the brass front cover plate, being held against the face of the plate by clamps. The inner face of the base assembly is flush with the inside surface of the cover plate. The assembly was rotated by a worm in a bracket which is mounted on the front of the cover plate. This bracket also held a 135° segment of calibrated plate from a surveying transit. A 1-minute vernier was mounted on another bracket attached to the body of the traverse control device. Axial motion was achieved using a knurled nut and spindle device. The end of the spindle was attached to the head block of the probe. The perpendicular distance of the probe from the face of the cover plate was measured with a dial indicator attached to the end of the traverse control unit. This indicator read to 0.001 inch. Each probe was set in its own teflon bushing through which it slid. This provided both lateral support and acted as a seal. Both teflon bushings were a press fit in the control unit. Angular alignment was provided by having the head block of the probes slide in a machined slot in the traverse control mechanism.

d. Indicating Circuit - A Shaevitz diaphragm type of transducer was used as the pressure sensing device. This transducer uses a linearly variable differential transformer to detect the deflection of a bellows caused by a pressure difference.

Excitation of the primary circuit was accomplished with a Hewlett-Packard Audio Oscillator at 7.4 volts and 1000 cps. The output
signal from the secondary coils of the transformer was first amplified
10 times by a Ballantine Decade Amplifier. The amplified signal was
then rectified by a full wave rectifier constructed at the Hydrodynamics
Laboratory by Mr. W. D. Ernst. The resulting DC signal was read on a
Leeds-Northrup Galvanometer, having a 100 millimeter scale and a sensi-
tivity of 0.0028 microamperes per millimeter of deflection. By intro-
ducing different values of resistance into the galvanometer circuit
various values of pressure will cause a full scale deflection. Four
scales were used in this test program, having the ranges of pressure
(in inches of manometer fluid, specific gravity 0.812) of 0—0.0335 inch,
0—.574 inch, 0—1.730 inch, 0—10.733 inches. This corresponds to a
range of velocity in standard air of 0—10.9, 0—45.2, 0—78.4, and
0—195 feet per second. The calibration of the transducer was accom-
plished by simultaneously measuring static pressure with the transducer
and a micro-manometer. The plot of pressure vs. galvanometer reading
indicated the sensing device to be truly linear. Experimental scatter
was less than 1%. This precision indicated that some confidence may
be placed in the velocity measurements since velocity is proportional to
the square root of the pressure. For example, if the error in measuring
the pressure is ±10%, then the deviation in the velocity has the
extremes; $\sqrt{1.1} V_{act} = 1.05 V_{act}$ and $\sqrt{.90} V_{act} = .95 V_{act}$, or the
deviation in velocity is ±5%.

2. Pressure Measurement

Pressure taps 1/16 of an inch in diameter were provided at radii
of 4, 6, and 8 inches, corresponding to 0.441, 0.662, and 0.883 of the
disk radius. Several taps at each radius were provided around the cover
plate so that symmetry of the flow field could be checked. The taps were
connected by 1/4-inch tubing to an inclined manometer. Pressure could be 
read to ±0.005 inches of water. Some measurements were taken on a micro-
manometer, such that readings could be reproduced to 0.001 inches of 
water.

3. **Torque Measurement**

Semi-quantitative torque data were obtained as part of the 
present survey with air by adapting the torque measuring equipment 
previously used for measurements with liquids (Refs. 4,5). As de-
cribed in the latter references, the torque is sensed by four SR-4 strain 
gages mounted in a hollow recess in the drive shaft. This recess is 
located outboard of the nearest support bearing and beyond the sealing 
gland of the disk housing so that only torque on the rotating disk is 
measured. No correction is necessary for bearing or sealing gland 
friction. The strain gages are wired into a Wheatstone bridge circuit. 
Voltage differences produced by the circuit were transmitted from the 
rotating shaft by a slip ring arrangement and read on a Leeds-Northrup 
Galvanometer.

For adaption to air measurements where the torque magnitude is 
very low, an extra sensitive zero attenuation scale was added. This is 
shown as Scale No. 5 in Figure 10. It should be noted, however, that 
while useful indications were obtained for the survey purposes of the 
present study, the necessary corrections for strain gage misalignments, 
drift, and temperature were large compared to the measured torque 
quantities.

4. **Disk Speed and Velocity Fluctuations**

Rotational speed of the disk was measured by a new type 1531-A 
Strobotac. This instrument puts out a short duration, white light of
Fig. 9 Schematic of Smoke Generator

Fig. 10 Schematic of Torque Measuring Circuit
relatively high intensity. The Strobotac was also used to measure the frequency of predominant fluctuations. When smoke was injected into a fluctuating flow field the resulting streak line appeared to whip in and out when illuminated by a light flashing at some frequency other than that of the fluctuation. By adjusting the flashing frequency of the Strobotac, until the streak line appeared to remain stationary, the fluctuation frequency could be determined. Since this frequency of flashing could be either the true frequency of fluctuation or some multiple of it, a further check had to be made. The Strobotac is so designed that, when the scale selector dial is turned to the next range, the flashing frequency increases 6 times. After adjusting the Strobotac until the streak line appeared stationary the scale selector was switched to the next range. If 6 stationary streak lines were then observed, the original flashing frequency was then known the same as the fluctuation frequency. On the other hand, if a stationary streak line was observed and after switching to the next range only three stationary streak lines were observed, then the original flashing frequency could have been only one half that of the fluctuation frequency. For this example the streak line was being illuminated only every other cycle. By this method it was relatively easy to collect correct fluctuation data.

5. **Photographic Equipment**

A 4x5 Graphic View camera, equipped with a f6.8/90 mm Scheinder lens and Synchro-Compur P. shutter was used. Lighting was by either using the Strobotac set for single or multiple flash or by two Strobo III sealed beam electronic flash units. Films used were Kodak Royal Pan, Kodak Royal X Pan, and Polaroid 3000 film. The use of Polaroid3000 film in a special adapter for 4x5 cameras proved especially convenient, since
results could be seen 10 seconds after exposure. The Strobotac provided a 3 μ second exposure and the large electronic flash provided an exposure of \( \frac{1}{10,000} \) of a second or 100 μ seconds.

6. Smoke Generation

Smoke was generated using an apparatus developed by the MIT Gas Turbine Laboratory. A stainless steel coil, encased in insulation, is heated to a temperature of around 1100°F with a current of 30 amperes at 12 volts. Filtered kerosene drips into this coil and is mixed with air producing a dense, dry, white smoke. This piece of equipment is shown schematically in Figure 9. It was recognized that injections at 90° to the disk surface would interfere with the disk-induced flow under some circumstances. Consequently, provision was made also for directing the injected stream with a tangential component by drilling 1/32-inch ports at 30° and 45° as well as 90° to the wall of the cover plate. Ports were located at radii of 4, 6, and 8 inches.
V. EXPERIMENTAL PROCEDURE

A. General Description of the Test Program

This investigation was made using air as the test fluid exclusively.

The test program was carried out in two steps; a qualitative phase using flow visualization and an investigation of the pressure and velocity distribution in the flow field.

During the first phase of the study the casing geometry was varied in four steps. Values of the relative axial spacing, s/a, were 0.0138, 0.0276, 0.0690, and 0.2207. The clearance between the tip of the disk and the housing, c, was 1/16 of an inch. This value was not changed during the test program. The range of disk speed was zero to 2200 rpm corresponding to Reynolds numbers from zero to \( 1 \times 10^6 \). Throughflow rates used were in the range zero to 60 cfm of standard air.

Each run consisted of a particular value of disk speed, rate of throughflow, and axial spacing. Smoke was carefully injected into each of three ports. The resulting smoke trail was illuminated with the Strobotac. Observation of the flow field was made to determine whether flow was laminar or turbulent, whether a core existed and whether the flow was unsteady. Any periodic motion was measured using the technique described in the section on measurement of velocity fluctuations. The magnitude of the pressure at 6 different points was determined from readings on an inclined manometer.

Torque measurements were made using the following procedure.

Before making measurements, the electric motor was allowed to run for 1 hour at the disk speed to be used for that run. The electronic equipment was turned on and allowed to warm up during the same period. After
1 hour the motor was shut down and the torque measuring circuit was balanced. A calibration reading, which is a measure of the battery strength, was taken at this time. The motor was again turned on. After steady state was reached the fluid torque on the disk was recorded from the resulting galvanometer deflection. A detailed description of this procedure is given in Reference (4). Temperature readings were taken at the casing and the inlet piping to the orifice meters. During each run, care was taken to insure that the same rate of flow was obtained through the front and rear of the casing.

The second phase of the investigation consisted of velocity measurements. The transparent front cover plate was removed from the housing and the original brass cover plate as used by Daily and Nece was fitted to the casing. Measurements were made at three stations on the cover plate. Data were taken at two different Reynolds numbers, $2.95 \times 10^5$ and $6.9 \times 10^5$. Four rates of throughflow, 0, 6, 26, and 55 cfm were used. Velocity profiles were determined across two spacings, $s/a = 0.0138$ and $s/a = 0.0690$. The values of 0.0138 and 0.0690 are nominal values. The actual measured values were 0.0100 and 0.0665 respectively.

Following the procedure used during the qualitative investigation, the motor and electronic equipment were allowed to warm up for at least 1 hour. Oscillator output was adjusted to the required frequency and voltage. The transducer null was adjusted to as low a value as possible. This was accomplished by turning a knurled nut at the back of the instrument, moving the coils of the transformer relative to the core. At null position the transducer produced a net reading on the galvanometer for zero pressure differential. This reading was cancelled by changing the
potential on one side of the galvanometer input with an auxiliary voltage source.

Each run consisted of two parts; a determination of flow direction and a measurement of the total velocity. First the directional probe was placed in the traverse control mechanism. The tip of the probe was brought as close as possible to the surface of the mechanism which is flush with the wall when installed. Careful measurement with a depth micrometer was made of the distance from the outer edge of the tip to the wall. Half the thickness of the probe is subtracted from this measured value, giving the distance from the wall to the centerline of the probe tip. This distance was set on the Ames dial at the other end of the control mechanism. In this manner the actual distance from the wall to the centerline of the probe may be read directly on the Ames dial. A minimum reading of 0.020 inch was attainable. The assembled traverse control mechanism, with probe in place was placed in a machined hole in the cover plate and rotated until index marks on the cover plate and the unit were lined up. This indicated that the probe was aligned in the tangential direction. The vernier plate in the protractor was then adjusted to read zero. Tygon tubing was used to connect the pressure taps of the probe to the pressure ports of the transducer. Readings of flow direction were taken at even increments of distance from the wall. At each setting the probe was rotated until a zero pressure differential was read on the galvanometer. The resulting angle of inclination was recorded. After every two or three readings, a check of the galvanometer zero reading was made by disconnecting the pressure leads to the transducer. The electronic circuitry remained exceptionally stable throughout this investigation and no corrections for drift were necessary.
The procedure used for the total velocity probe was essentially the same as for the directional traverse. At each point the probe was oriented at the measured angle obtained with the directional probe and the stagnation pressure read on the galvanometer. Care was taken to exactly reproduce the same rate of disk speed, throughflow, and fluid temperature as was used during the directional traverse. The procedure for reduction of data is given in the following section.

B. **Data Reduction**

1. **Velocity Data**

The raw velocity data consist of the flow direction in degrees and a stagnation pressure reading in millimeters of galvanometer deflection. Since the pressure sensing system was linear, it was relatively easy to reduce the galvanometer reading to a value of total velocity. Each scale on the galvanometer could be expressed as $h = Bx$, where $h$ is pressure head in feet of air at 60°F, $x$ is the galvanometer deflection in millimeters, and $B$ is the constant obtained from calibration of the transducer. The tube coefficient, expressed as:

$$c_p = \frac{P - p}{\frac{V^2}{2}} \quad [P - p \text{ being the stagnation pressure}]$$

was assumed to be unity. The velocity at any point measured is then:

$$v = \sqrt{2gh}$$

or

$$v = \sqrt{2gB/x}$$
Therefore, the total velocity is obtained by multiplying the square root of the galvanometer reading by an appropriate constant. The value of velocity obtained was corrected for temperature since the temperature in the chamber was different from 60°F. The constant, \(\sqrt{2gB}\), is expressed in terms of feet of air at 60°F, whereas the measured pressure should be in terms of air at the measured temperature in the casing. A simple correction is obtained from the equation of state:

\[
\frac{P}{\gamma} = h_{60}^0 = (520)R \quad (R \text{ is the gas constant})
\]

\[h_t = RT = R[t + \frac{460}{520}] \]

where \(t\) is temperature °F.

\[
\frac{v_t}{v_{60}^0} = \frac{h_t}{h_{60}^0} = \frac{t + 460}{520}
\]

At temperatures of 80° to 90°, which were encountered during the test program, this correction amounts to from 2 to 3%. The total velocity was reduced to its tangential and radial components using the measured value of the skew angle, \(\alpha\). The tangential component is obtained by multiplying the total velocity by \(\cos \alpha\), and the radial component by multiplying by \(\sin \alpha\). The radial and tangential components of velocity were normalized with respect to velocity at the disk surface, \(\omega r\). The data are presented as plots of skew angle, \(\alpha\), radial velocity, \(\frac{V}{\omega r}\), and tangential velocity, \(\frac{V}{\omega \alpha r}\), versus the relative distance from the wall, \(\frac{V}{s}\).

2. **Pressure Data**

The raw pressure data were plotted as inches of manometer fluid
versus \((a^2 - r^2)\) for each value of \(R\). This results in a straight-line plot for the zero throughflow pressure distribution. The resulting smooth curves were extrapolated to \(r = 0\) and \(r = a\). The data were reduced by subtracting the value of pressure at \(r = a\). This results in shifting all the curves so that they originate from the same point, defining the pressure at the disk periphery as equal to zero. A sample extrapolation plot is shown in Figure 20. The data are made non-dimensional by dividing the abscissa \((a^2 - r^2)\) by \(a^2\), giving \([1 - \left(\frac{r}{a}\right)^2]\) and the values of pressure by the corrected value of pressure, \(p_o\), at \(r = 0\) for the parabolic pressure distribution at zero throughflow. The deviation from the pressure distribution for a forced vortex, \(\Delta p\), is also plotted versus \([1 - \left(\frac{r}{a}\right)^2]\) for each value of source flow on the same figure.

3. **Torque Data**

As previously mentioned, a zero attenuation scale was incorporated into the torque measuring circuit. Before use, the strain gage bridge was calibrated for the four less sensitive attenuation scales. The four scales, 1-4, are for resistances of 5 K, 2.5 K, 1 K, and 0.5 K, respectively. The zero attenuation scale 5 leaves only the resistance of the galvanometer in the circuit. The four lower scales were calibrated by applying a known torque and reading the resulting galvanometer deflection. The calibration curves obtained are straight lines, the torque being equal to the galvanometer reading multiplied by a scale factor which is constant for each resistance.

The scale factor for scale 5 was established by two independent procedures. First, using the manufacturers specified galvanometer resistance, the scale factor was computed. Second, using the calibration results for scales 1-4. The sum of the circuit resistance and galvanometer
resistance was plotted versus scale factor on log-log paper. The resulting straight line was extrapolated to the case of galvanometer resistance alone. The two methods gave the same result which was used to convert scale 5 data to torque values.

The scale factors from the torque strain gage calibration results depend on the strength of the 6 volt automotive battery used in the circuit. To account for changes in battery strength during a test period, checks were made each day before and after an experiment. Changes were not observed during any one test run, but gradual changes over a period of several days did occur. To check on the battery strength, the battery output was impressed across a known resistor and the current read on the galvanometer. By comparing this reading with the value obtained during the torque strain gage calibration, corrections could be made for changes by multiplying the recorded data by the ratio:

<table>
<thead>
<tr>
<th>Original Calibration Reading Before Tests</th>
<th>Calibration Reading During Tests</th>
</tr>
</thead>
</table>

This check was always performed using the scale 4 setting.

Due to a slight misalignment of the strain gages, there is a difference in reading at various positions of rotating of the shaft. During rotation of the shaft without load, a net reading results. This value must be subtracted from the readings. Correction curves were obtained as a function of rpm for all combinations of disks and dummy shafts by taking readings at various rotational speeds with the housing off. These readings were corrected for drift, battery strength, and air torque on a free disk.

On scale 5, the rotational correction was of the same order of
magnitude as the air torque. It was decided that an attempt at a precise calibration and rotational correction for scale 5 would be in vain. Therefore, rotational corrections were extrapolated from the rotation calibrations of the other four scales. This is justified, since all data presented as a percent increase with thoroughflow number are thereby cancelling out any errors in the extrapolation procedure. At each value of disk rotation a torque reading was taken for no thoroughflow and several rates of thoroughflow. The data are presented as the percent increase of torque over the corresponding value of torque without thoroughflow versus thoroughflow number, $\frac{Q}{ωa^3}$.

4. **Fluctuation Data**

As previously mentioned, periodic fluctuations were measured with the Strobotac illuminating a streakline made visible with smoke. Correlation of the fluctuation frequency was obtained with the disk speed. Data are presented as fluctuation frequency, cycles per second, versus disk rotation in revolutions per second.

5. **Transition Data**

Transition was observed at each radii by injecting smoke when the flow was laminar and gradually increasing the disk speed until the steady smoke streak was observed to break up into a randomly fluctuating streakline. The disk speed was then reduced until a steady laminar flow was again observed. The local Reynolds number $\frac{ωr^2}{v}$ was recorded at the transition point both for increasing disk speed and decreasing disk speed. Data are reported as spacing, s/a, versus local Reynolds number, $\frac{ωr^2}{v}$. 
VI. EXPERIMENTAL RESULTS AND DISCUSSION

A. Smoke Visualization Observations

The use of smoke as a flow visualization tool was found to be especially convenient in this study. Some orientation is required to clearly understand the meaning of the photographs in this thesis. First it should be emphasized that the smoke trail observed with an instantaneous photographic exposure is a streakline, i.e. an instantaneous line-up of fluid particles all of which passed the same point in the fluid space some time earlier. It is a streamline only if the motion is steady.

The several items of information attainable with this technique are:

1. Laminar versus turbulent flow.
2. Flow in the boundary layer on the rotating disk.
3. Flow in the midspace between the disk and stationary wall boundary layers which do not merge.
4. Direction of the flow velocity relative to the disk velocity.
5. Transition from laminar to turbulent flow.
6. Appearance of fluctuations identified with observed periodicitites.

Examples of several of these are illustrated as follows:

a) Laminar boundary layer flow at the disk with close spacing and merged boundary layers (Regime I). Shown by arrows in Figure 12.

b) For the same geometry and Reynolds number as Figure 12, except with a superposed throughflow, Figure 13 shows a laminar boundary layer (A) at the disk which persists with turbulent throughflow (B) midway between
Fig. 12 Laminar Flow $R = 9.9 \times 10^4$, $Q = 0$, $s/a = 0.0138$

Fig. 13 Laminar Boundary Layer on Disk (A) with turbulent throughflow in rotating core (B) $R = 9.9 \times 10^4$, $s/a = 0.0138$, $Q = 24$ cfm.
Fig. 14 Example of Periodic Fluctuations, s/a = 0.0690

Fig. 15 Example of Periodic Fluctuations, s/a = 0.0690
Fig. 16 An Example of a Representative Fluctuation at Three Stages of Time in the Cycle

Fig. 17 An Example of Steady Laminar Flow at Disk (A) with a turbulent core (B), s/a = 0.0690, \( R = 1.7 \times 10^4 \)
the disk and stationary wall.

c) Laminar flow in the rotating core that occurs in the mid-
space between disk and wall boundary layers with zero throughflow. This
is shown by the arrow in Run h3 in Figure 11.

d) Direction of core flow velocity relative to the disk velocity
with superposed throughflow is shown by Run h6 in Figure 11. Superposed
on this photograph is a set of coordinate axes at each point of smoke
injection and a line showing core flow direction.

e) Whipping streaklines in the core flow which appeared for some
combinations of Reynolds number and throughflow are illustrated in
Figures 14-16. The three arrows in Figure 16 point to the streakline at
three successive instants of time obtained by triple flash exposure.

Before introducing any quantitative results, a brief discussion
of the general flow pattern is in order. Figure 11 is a representative
display of the effects of throughflow. The ratio of spacing to disk
radius, s/a, is 0.0690 for all photographs in this display. Reading from
left to right the rate of throughflow is 0, 6, 16, 26, 36, and 55 cfm.
Looking from bottom to top the respective Reynolds number, R, are,
$1 \times 10^4$, $1.6 \times 10^5$, and $6.7 \times 10^5$. This spacing and range of Reynolds
numbers correspond to the zero-throughflow Regimes II (laminar) and IV
(turbulent), namely a rotating core of fluid between the two boundary
layers on the stationary endwall and on the rotating disk. Regime III,
merged turbulent boundary layers, does not exist at this spacing.

Consider the core flow in the midspace between the disk and casing
wall. At low R(bottom row) the flow is laminar at zero throughflow.
When a rate of throughflow of only 6 cfm is superposed the flow pattern
is drastically altered. Note that this core flow is turbulent and the
amount of skew has changed substantially at all radii. As the throughflow rate is increased to 16 cfm, a large amount of turbulent mixing of the smoke is evidenced by the rather indistinct character of the smoke trail. With increasing rates of throughflow beyond 16 cfm the scale of the turbulence and consequent mixing increases to the point that the smoke trail is no longer visible at the inner radius. Note that when the throughflow rate is a maximum \( \dot{Q} = 55 \) cfm we have practically a pure "source" flow and the effect of rotation is small. This is shown by the streak lines indicating essentially radial outflow. If the flow had both radial and significant tangential velocity components a streakline would be oriented in some type of spiral. Probably each point on such a spiral would have a total velocity vector whose angle of skew relative to the tangential direction decreased with increasing radius.

This follows since with increasing radius the radial component of the total velocity decreases while the tangential component would normally increase.

A turbulent throughflow does not mean the boundary layer flow is turbulent. This is illustrated in Figure 13, which is a photograph of two streaklines made by injecting smoke into the boundary layer on the disk and then decreasing the smoke rate in such a manner that smoke is injected into the core also. As mentioned previously, there is a laminar boundary layer persisting at the disk \( (A) \) and a turbulent throughflow \( (B) \) in the midspace between the disk and the stationary wall. Visual observation confirmed the fact that the smooth circular streaklines are very close to the disk and the streaklines indicating turbulent flows are in the core.

The effect of a given increment in throughflow is more pronounced
at lower Reynolds number (i.e. disk speed) than at higher values. For example, compare the photographs for the low throughput, \( Q = 6 \text{ cfm} \).

At the low Reynolds number, \( 1 \times 10^4 \), we already noticed a definite influence of throughput on skew at all radii. At \( R = 6.7 \times 10^5 \) the core flow and skew angle appear to be essentially the same as for zero throughput at the outer radii.

While in the case of \( Q = 16 \text{ cfm} \) the flow pattern is affected over the entire radius, the effect is more noticeably pronounced at \( R = 1 \times 10^5 \) than at \( R = 1.6 \times 10^5 \).

With increased throughput the increased turbulence and turbulent mixing of the smoke makes the streaklines less distinct. Nevertheless, the same effect of a greater influence of throughput at the lower disk speed can be discerned.

It will be noted that at any particular \( Q - R \) combination the throughput effect is most pronounced at the inner radii. Note that at \( Q = 55 \text{ cfm} \) and \( R = 1.6 \times 10^5 \) there is a large effect of the throughput at radii less than 6 inches. However, this effect is not as great as is shown in the picture immediately below where the Reynolds number is less but the throughput is the same.

The top row, where the Reynolds number is the highest, illustrates not only the throughput effect on skew and scale of turbulence, but also a fluctuation phenomenon. When there is no throughput the flow is "steady" and turbulent. A definite and abrupt change from the steady flow pattern in the core is noted when a low rate of throughput is superposed. At the inner radius a periodic fluctuation is noted. Observation with the Strobotac flashing at a frequency slightly different from the fluctuation frequency permits this phenomenon to be observed in
slow motion. The smoke trail appears to whip out and then curl back in on itself. Such an observation leads one to believe that the smoke particles are being caught up in a vortex. It appears that this vortex has a counterclockwise sense of rotation about an axis perpendicular to the disk surface and translates in a circular path past the point of smoke injection. When the source flow rate is increased to 16 cfm this periodicity extends out to the radius of the middle injection port. This whipping motion is more pronounced at the higher rate of flow and the smoke trail from the 4-inch radius port will curl out to about 6 inches and then back to the smoke injection port. Smoke injected at the middle port tends to curl in under the smoke trail from the 4-inch port. This is even more evident at $Q = 26$ cfm as seen in the fourth picture from the left in the top row. With further increase in throughflow the fluctuation tends to lose its definite periodicity and become random. Thus at large flow rates the periodic motion is not observed. When the flow rate is 55 cfm the turbulent mixing is so high that the smoke trail from the inner radius is not observable and the smoke trail at $r = 6$ inches is also faint. It is interesting to note that throughout the range of Reynolds number and rates of throughflow at which flow periodicity is observed, there is no noticeable fluctuation at the outer portion of the disk, i.e. $\frac{V}{a}$ greater than 0.66.

As a consequence of the relative importance of throughflow changing with increasing throughflow, three separate categories of conditions might be identified, namely:

a) Low throughflow rate, high disk speed. Radial velocities negligible compared to tangential velocities.

b) Both radial and tangential components of velocity of the
same order of magnitude.

c) High throughput rate, low disk speed. Tangential velocities small compared to radial velocities. It appears that a theoretical treatment of the problem might be likewise separated into three such classes.

It is observed also that very large variations in the skew angle can occur in a very short distance. This can be seen in Figure 13 where the flow in the boundary layer at the rotating disk is essentially in the tangential direction. A short distance further out the skew angle is approximately $60^\circ$. This is much larger skew than observed for zero throughput. Furthermore this skew effect increases with throughput rate. This suggests that the methods used in the analysis of the zero throughput case (e.g. 6,9) based on two-dimensional flow type boundary layers may require modification in applying to the throughput case.

Observation of the physical size of the random turbulent fluctuations in a streakline will give some idea of the "scale" of the turbulence or the mean size of the eddies composing the turbulent flow field. It was always observed that the "scale" of the turbulence was larger with increased throughput. This is probably due to the geometry of the entrance section of the test apparatus. The annulus through which the flow enters is probably a separation zone with large eddies being formed at this point. The convection of these eddies radially outward produces a large scale turbulence. In Figure 35 some velocity data taken at $Q = 56.0 \text{ cfm}$ indicates a backflow at the inner radius close to the stationary wall. This suggests the formation of a vortex ring close to the entrance section. The influence of this vortex formation at the entrance on the observed periodic flow is a point for speculation.
B. Velocity Data

1. General Discussion

Figures 18 through 33 display the velocity distribution in the space between the disk and the stationary endwall. Reading from top to bottom on each plot the variation of skew, radial velocity, and tangential velocity are plotted versus the relative distance from the stationary endwall, y/s. The data for each particular combination of Reynolds number, rate of throughput, and spacing are given on the same plot for three different relative radii, r/a = 0.469, 0.648, and 0.828.

Before discussing the data in detail a general survey will be useful. Note that without throughput there is a net radial flow outward close to the disk and a net flow inward close to the wall. For the wider spacing a core exists with essentially zero radial outflow and a constant value of u/ωr. This agrees with the concept of solid body rotation as set forth in the theoretical treatment of the problem. Note that the radial velocity increases rapidly close to the wall. Since the probe was thick compared to the boundary layer thickness it was possible to get readings of velocity close enough to the wall to clearly define the peak of the radial velocity profile. It is important to note that this peak is very close to the wall. With an increase in throughput the boundary layer is thickened on the rotating disk as shown in Figure 29. This process continues with increasing throughput until the boundary thickness varies inversely with radius, which is displayed in Figure 31. Note that when there is a net flow through the system, a reduction of the tangential components of velocity occur in the core. This is also true for the close clearance flow. The largest reduction in tangential velocity occurs at the inner radius. It is this reduction in core
Fig. 18 Velocity Data
s/a = 0.0138
R = 2.95 x 10^5
Q = 0

Fig. 19 Velocity Data
s/a = 0.0138
R = 6.9 x 10^5
Q = 0

Fig. 20 Velocity Data
s/a = 0.0138
R = 2.95 x 10^5
Q = 6 cfm

Fig. 21 Velocity Data
s/a = 0.0138
R = 6.9 x 10^5
Q = 6 cfm
Fig. 22  Velocity Data  
$s/a = 0.0138$
$R = 2.95 \times 10^5$
$Q = 25.7$ cfm

Fig. 23  Velocity Data  
$s/a = 0.0138$
$R = 6.9 \times 10^5$
$Q = 25.7$ cfm

Fig. 24  Velocity Data  
$s/a = 0.0138$
$R = 2.95 \times 10^5$
$Q = 52.0$ cfm

Fig. 25  Velocity Data  
$s/a = 0.0138$
$R = 6.9 \times 10^5$
$Q = 52.0$ cfm
Fig. 30  Velocity Data
s/a = 0.0690
R = 2.95 x 10^5
Q = 25.7 cfm

Fig. 31  Velocity Data
s/a = 0.0690
R = 6.9 x 10^5
Q = 25.7 cfm

Fig. 32  Velocity Data
s/a = 0.0690
R = 2.95 x 10^5
Q = 52 cfm

Fig. 33  Velocity Data
s/a = 0.0690
R = 6.9 x 10^5
Q = 52 cfm
Fig. 34 Velocity Data $R = 0$

\[ s_a = 0.0138 \]
\[ Q = 51.2 \text{ cfm} \]
- $s_a = 0.469$
- $s_a = 0.648$
- $s_a = 0.828$

Fig. 35 Velocity Data $R = 0$

\[ s_a = 0.0690 \]
\[ Q = 56.0 \text{ cfm} \]
- $s_a = 0.469$
- $s_a = 0.648$
- $s_a = 0.828$
Fig. 36 Radial Velocity Profile
Laminar Flow

Fig. 37 Tangential Velocity Profile
Laminar Flow
Fig. 38 Turbulent Tangential Velocity Profiles on Disk, s/a = 0.0690

Fig. 39 Turbulent Tangential Velocity Profiles on Disk, s/a = 0.0690

Fig. 40 Core Velocity Reduction

Fig. 41 Core Velocity Reduction
Fig. 42  Core Velocity Reduction

\[
\left(\frac{1-K}{1-K_a}\right)^{\frac{1}{2}} - 1 = 10^{-1} \left( \frac{Q}{\omega r^3} \right)
\]

- $r_a = 0.648$
- $r_a = 0.828$

- $s_a = 0.0552$
- $R = 6.9 \times 10^5$
velocity with increasing throughflow that is the mechanism for the observed decrease in the radial pressure gradient. Equation [52] expresses this core reduction in the form:

\[
\frac{\frac{1}{1-K}}{\frac{1}{1-K_0}} - 1 = c_1 \frac{Q}{\omega r^3}
\]  

[52]

Examination of this equation will indicate that as Q increases K must decrease. Similarly with a decrease in r, K must decrease. This is exactly what is observed. In Figure 40 the measured values of \(\frac{\frac{1}{1-K}}{\frac{1}{1-K_0}} - 1\) are plotted versus throughflow number, \(\frac{Q}{\omega r^3}\). The data have a slope of 1 for low rates of throughflow which is exactly what [52] predicts. However, there is a difference in the measured proportionality factor from what is predicted theoretically. The important fact to be gleaned from this is that a theory based on pipe flow data for shear stress is able to predict the trend. It would then be logical to believe that the tangential velocity profile may be expressed in terms of a 1/7 power law. Figure 38 illustrates that to a rough approximation this is true. The velocity relative to the disk has been plotted dimensionlessly as a function of the dimensionless distance z/\delta from the disk. It is important to note that the velocity distribution does not follow a definite power law. However, the Reynolds number is still rather low. With increasing throughflow, and hence increasing turbulence, the data tend to more closely follow the expected 1/7 power law. At the outer edge of the boundary layer the data for all rates of throughflow do roughly follow a 1/7 power law.

2. \(s/a = 0.0138, \quad R = 2.95 \times 10^5\)

Before discussing results with the small spacing, consideration
should be given to the fact that the clearance, \( s \), is 0.091 inch and the probe thickness is 0.035 inch. This will certainly lead to errors due to local acceleration of the flow around the probe. However, the data indicate the difference between close clearance and separate boundary layer flow.

Reference should be made to Figures 18, 20, 22, and 24. The first thing that meets the eye on inspection of Figure 18 is the fact that flow is definitely of the close clearance type, i.e. the boundary layers on the rotating disk and stationary endwall are merged. No core flow exists.

Inspection of the radial profile indicates that greater radial outflow and radial inflow velocities exist at the inner radii. In other words, the radial velocity does not appear to be a unique function of \( y/\delta \), where \( \delta \) in this case is one half the clearance space, \( s/2 \). Further inspection will show that the amount of fluid circulating is a constant from the inner to the outer portion of the disk. It should be borne in mind when inspecting the graph the amount of flow out or in is expressed by the integral:

\[
Q = 2\pi \int_0^\delta r v \, dz
\]

or

\[
Q = 2\pi \omega r^2 \int_0^\delta \left[ \frac{v}{\omega r} \right] dz
\]

Q being plus in the boundary layer on the rotating disk and of the same magnitude and minus at the stationary wall.
If we assume \( v = F(z) \) to be the same except for a constant then it is readily seen that for a constant rate of flow, \( Q \), the normalized radial velocity, \( v/\omega r \), must vary inversely as the radius squared, i.e. \( v/\omega r \propto \frac{1}{r^2} \). Examination of the profiles will show this to be true. This is exactly opposite to what is found for the large spacing and is also contradictory to the present approximate theory.

The superposition of a low rate of throughflow, \( Q = 6.2 \text{ cfm} \), influences the flow field to a great extent. This is shown in Figure 20. Note that at \( r/a = 0.469 \), the radial velocity profile closely resembles that for pipe flow. At this point the radial velocities are very large in comparison to the tangential components. With increasing radius the velocities decrease as would be expected, but the peak of maximum radial velocity shifts closer to the disk with increasing radius. A reduction in the tangential component of velocity at the center of the clearance space is evident. This reduction is greatest at the inner radius. Steeper velocity gradients result on the disk and lesser velocity gradients at the stationary wall.

An increase in the throughflow rate to 25.6 cfm [Figure 22] results in a significant change in the flow. There is no radial flow inward anywhere in the chamber. The radial velocity profile at \( r/a = 0.469 \) resembles that of fully developed pipe flow. With increasing radial distance the point of maximum radial velocity shifts towards the disk surface. A further reduction in the tangential velocities outside the boundary on the rotating disk is also evident. At \( r/a = 0.469 \), \( \frac{v}{\omega r} \) is reduced to zero outside the rotating disk boundary layer. Note also that the boundary layer has thickened considerably. Further scrutiny indicates that the boundary layer thickness appears to decrease with increasing
radius. Computations of boundary layer thickness which appear in section 5 of this discussion indicate that this trend may be predicted theoretically. It should be mentioned at this point that throughout this investigation the boundary layer thickness of the radial profile always appeared to be greater than for the tangential profile. In other words when there are imperceptible gradients of velocity in the tangential direction there are perceptible gradients in the radial direction.

A further increase in throughflow rate to 52.0 cfm brings about even further change in the flow field, as shown in Figure 2h. Note that there is a still greater increase in the tangential velocity gradients on the rotating disk. However, the mechanism for the increased gradient is now a decrease in the boundary layer thickness, since the velocity outside the boundary layer is already zero. The radial velocity profiles resemble those for pipe flow throughout the clearance space between the rotating disk and the stationary endwall. Note that the peak of the profile still tends towards the rotating disk with increasing radius.

The data indicate that, as would be expected for close clearances and low disk speed, that the effect of throughflow is of paramount importance. With no throughflow a circulation exists with outward flow at the disk and inward flow at the wall. The quantity of flow circulated appears to be a constant with radius indicating negligible axial flow except close to the axis of rotation and close to the disk periphery where the flow must change direction. With throughflow superposed this circulation is disrupted. There is indication that an axial flow now exists over the whole disk since the peak radial velocities approach the disk with increasing radius.
3. \( s/a = 0.0138, \ R = 6.9 \times 10^5 \)

In conjunction with this portion of the discussion, reference should be made to Figures 19, 21, 23, and 25.

When there is no net flow through the system a close clearance type of flow exists such that the boundary layer thickness is constant at \( s/2 \). There is a similar circulation of radial flow as was observed at the lower Reynolds number. Note that again the rate of flow is the same over the disk and roughly of the same magnitude as for \( R = 2.95 \times 10^5 \). Since velocity measurements could not be made any closer to the disk, it is difficult to judge the actual magnitude of any circulation flow. One discrepancy that exists is the fact that the tangential components of velocity are not similar. It would be supposed from the theory that the profiles would be similar, having a velocity of \( 1/2\omega r \) at the center of the space between the disk and the wall. Note that there is a reduction in the centerline velocity with decreasing radius. Although all precautions were taken to entirely seal the housing in which the disk rotates when taking data for zero throughput flow, the tangential velocity profiles seem to indicate that there was a slight amount of fluid pumped through the system. If such a leakage occurred it could not be detected through examination of the radial velocity profiles.

The introduction of a low rate of throughput, 6.4 cfm, produces a reduction in the tangential components of velocity outside of the boundary layer on the rotating disk. This is shown in Figure 21. As previously mentioned equation [52] predicts this reduction of tangential velocity. It is interesting to note that the best agreement with equation [52] was obtained with this spacing, even though the theoretical expression was developed on the basis that a core flow exists. The
reduction of tangential velocity, expressed as \[ \frac{1}{\frac{1}{1-K_o} - 1} \], is plotted as a function of the throughflow number, \( \frac{Q}{\omega r^3} \), in Figure 40. Note that the data are linear with a slope of unity for throughflow numbers less than about 0.015. This is the region in which the theory would be expected to hold. The data indicates an equation of the form:

\[
\frac{1}{\frac{1}{1-K_o} - 1} = 55.0 \frac{Q}{\omega r^3}
\]

whereas the theoretical expression is:

\[
\frac{1}{\frac{1}{1-K_o} - 1} = 172.0 \frac{Q}{\omega r^3}
\]

The agreement is good except for the constant which can be adjusted by using a different value of \( K_o \) and the torque coefficient. The fact that the agreement in general is so good whereas it is not for the wider spacing, is probably due to the fact that over the range of agreement the boundary layer thickness remains fairly constant. This is one of the assumptions that was made in the derivation of equation [52].

The data obtained for \( R = 2.95 \times 10^5 \) do not seem to fit the theory. This is not to be expected because of the low Reynolds where the flow is not fully turbulent. A further fact to be gleaned from Figure 40 is that for high throughflow number, the data appear to follow a new line with a slope, 0.306, or the core reduction is proportional to \( [1 + \frac{Q}{\omega r^3}] \) raised to \([0.386]^{[\frac{1}{7}]} = 0.22 \). The theory, expressed by equation [62], predicts a dependency on \( [\frac{Q}{\omega r^3}]^{0.25} \). This is certainly not an agreement with theory but it does indicate the feasibility of joining two separate solutions for high and low throughflow rate.
Inspection of the data for higher rates of throughflow will indicate other interesting features of this problem. Note that at \( Q = 23.7 \text{ cfm} \) in Figure 23 that there is no radial inflow close to the stationary wall. The radial velocity profile is not symmetrical, with the point of maximum radial velocity occurring close to the disk. Further reduction in the tangential components of velocity is also evident as would be expected.

Further increase in the throughflow rate to 51.2 cfm produces some interesting changes in the flow field as shown in Figure 25. At \( r/a = 0.469 \) the radial velocity profile has the appearance of that observed for pipe flow. With increasing radius the profile flattens out and the point of maximum velocity shifts closer to the disk surface. This phenomenon is not a function of the geometry. Reference to Figure 34, where the velocity data obtained for zero disk speed is plotted, will show that within reason the velocity profile is symmetrical at all radii. Therefore this shifting of the radial velocity profile with increasing radius has to be associated with the influence of the rotating disk.

It is interesting to note that a 1/7 power law may be deduced for the tangential component of velocity, even though there is not sufficient velocity data in the boundary layer. This may be inferred from the fact that the shear stress on the disk is apparently predicted from the Blasius expression for shear stress, from which in turn a 1/7 power law may be deduced.

\[
4. \quad s/a = 0.0690, \quad R = 2.95 \times 10^5
\]

For this portion of the discussion reference should be made to Figures 26, 28, 30, and 32.

Note that when throughflow is equal to zero the profiles are
similar. The boundary layer thickness is a constant over the disk. Such a situation is typical of laminar flow. Figures 36 and 37 compare the radial and tangential velocity profiles respectively with the theoretical profiles on a free disk. Of interest is the similarity of the radial profile with the theory of a free disk. Note that the variation and magnitude are very similar. There is a rather large experimental scatter when \( z/\delta < 0.2 \). In this particular case this corresponds to a distance of around 0.02 inch from the disk. When compared with the probe thickness of 0.035 inch, the experimental scatter is not an unexpected result. The important point to observe is that the radial velocities are of the same magnitude. For the free disk the variation of the tangential component of velocity, which gives rise to the radial secondary flow, varies from 0 to \( \omega r \) through the boundary layer. The equivalent variation through the boundary layer on the enclosed rotating disk is from \( \omega r \) to \( 2\omega r \). Therefore it would seem reasonable that the secondary flow would vary in a different fashion. On Figure 37 the non-dimensional tangential velocity component relative to the rotating disk, \( [1-(\omega /\omega r)][1-K] \), is plotted as a function of the relative distance through the boundary layer, \( z/\delta \). The theory for a free disk, due to Cochran, is also plotted on the diagram. For the free disk the normalised velocity relative to the disk reduces to \( 1 - \frac{u}{\omega r} \). The agreement is only fair, for the data seem to indicate a steeper velocity gradient near the wall than for the theory. As was the case with the radial profile the experimental scatter is large close to the wall. However, with appropriate corrections to the data to account for the discrepancy that the velocity appears to fall to zero at some distance from the wall, it would seem that the free disk profile is a fair
approximation to the observed velocity distribution for the enclosed disk.

It is interesting to note at this point that the observation of laminar flow at this high a Reynolds number is not in agreement with the observations made with a smoke tracer. The smoke injection studies indicate that transition to turbulence should occur somewhere between $r/a = 0.648$ and $0.828$ where the local Reynolds number, $\frac{\omega r^2}{v}$, is $1.24 \times 10^5$ and $2.03 \times 10^5$ respectively. The flow, however, appears to be laminar over the whole disk. In Figure 59 the points of observed transition are plotted on a diagram of $a/s$ vs. the local Reynolds number. For this spacing $a/s$ is $15.037$. Following across at $1.5.037$ the intersection of the experimental curve is at $\frac{\omega r^2}{v} = 1.3 \times 10^5$ or transition should be at around $r/a = 0.648$. If the dotted curve on the same diagram is used, which is the experimental curve shifted to correspond with the observed point of transition as found by Gregory, Stuart, and Walker (7) on a free disk, a value of the local Reynolds number of $2.64 \times 10^5$ is deduced. The flow therefore should be laminar at the three points of measurement as observed. All this seems to indicate that the probe has less effect on the transition to turbulence in the boundary layer flow than does a fairly high velocity jet of smoke. In some slight fashion this correlates with the observations of Theodorsen and Regier on a free disk (20). They noticed that a jet of air had a greater effect on transition to turbulence than did the addition of roughness protrusions on the surface of the disk.

Further inspection of Figure 26 will indicate that the boundary layer thickness on the stationary wall is approximately twice that on the rotating disk. This trend is predicted by the theory. The actual value
predicted by Daily and Nece (4) is $\phi/\delta = 1.70$. This is certainly fair agreement. Another point to be made is the typical hump in the tangential velocity profiles. This hump is not predicted by the theory, but has been previously observed (4). Another fact that should be brought out at this point is that if the velocity profiles are similar and the boundary layer thickness is constant, then the radial flow in the boundary layers is increasing with increasing radial distance. Similarly the flow inward on the stationary wall is decreasing with decreasing radius. This leads to the assumption that there must be an axial flow towards the disk over the whole disk area.

As shown in Figure 28 the introduction of a low rate of through-flow, $Q = 6.2$ cfm, produces a distinct change in the flow pattern. This was also evident in the smoke studies as Run 12 in Figure 11 indicates. Note that the boundary layer thickness has remained constant over the whole disk, indicating that flow is still laminar. There is a radial flow outward in the core at $r/a = 0.469$. At $r/a = 0.648$, however, all radial flow outward is in the boundary layer on the rotating disk. The amount of radial inflow is also significantly greater with increasing radial distance. It must therefore be deduced that an axial flow toward the disk occurs at the outer radii. A typical reduction in the tangential component of velocity in the core is evident. The theory as expressed by equation [52] predicts that this should be so. However, at this low Reynolds number there is no agreement with the theoretical expressions. At this point flow is still laminar in the boundary layer and no agreement with a theory based on turbulent flow should be expected.

An increase in the rate of throughflow to 25.7 cfm produces an even greater reduction in the tangential component of velocity in the
core. This is shown in Figure 30. The greatest reduction from that at
the lower rate of flow now occurs at \( r/a = 0.648 \). A reduction in the
core rotation to close to zero at \( r/a = 0.469 \) has already occurred with
a lower rate of flow. An increase in the boundary layer thickness is
evident indicating that the flow has become turbulent. The maximum
radial velocities still occur close to the disk. A radial inflow exists
at the outer portion of the stationary wall with no inflow being observed
at \( r/a = 0.469 \). Therefore an axial flow towards the disk over the outer
portion of the disk appears to be present.

Increasing the flow rate to \( Q = 56.2 \text{ cfm} \)
produces still further change in the flow field. Backflow along the sta-
tionary wall has essentially disappeared. It is interesting to note at
this point that the observed periodic fluctuations disappear at approxi-
mately the same rate of throughflow as does the radial inflow.

The radial profile now approaches that of pipe flow. However,
the point of maximum radial velocity occurs closer to the disk with
increasing radius. The core rotation is essentially zero throughout the
chamber as the tangential velocity profile plotted in Figure 32 indicates.
The velocity profiles are characteristic of turbulent flow.

Observation of the velocity field at this spacing and Reynolds
number permit several conclusions to be made about the flow. A laminar
flow exists at Reynolds numbers as high as \( 3 \times 10^5 \). This laminar flow
can exist even though there is a turbulent flow in the core. An axial
flow in the core towards the disk can be deduced for all rates of radial
outflow. This is different from what is deduced for flow in close
clearance.
5. \( s/a = 0.0690, \ R = 6.9 \times 10^5 \)

The velocity profiles for this spacing and disk speed are plotted in Figures 27, 29, 31, and 33.

The difference in the boundary layer flow at the higher disk speed is evident. Boundary layer thickness increases with increasing radial distance, which is the expected result for turbulent flow. The boundary layer on the disk at zero throughflow is approximately 50\% thicker than predicted theoretically. This comparison is made in Table III. When the tangential velocity relative to the disk, \([1-(\frac{\omega r}{\omega})]/[1-K]\), is plotted versus the relative distance, \(z/\delta\), through the boundary layer, it becomes evident that the velocity profiles are similar at the outer radii. Such a correlation is made in Figures 38 and 39. At \(z/\delta > 0.3\) the agreement with a 1/7 power law is very good. For \(z/\delta < 0.3\) there is a deviation from the 1/7 power law. Measurements of the boundary layer on a free disk made by Welsh and Hartnett (21) follow a similar pattern. The profile observed at \(r/a = 0.469\) is not similar to that observed at the outer radii. This is to be expected since the local Reynolds number at this point is \(1.52 \times 10^5\), indicating that a zone of transition or laminar flow is present at this point.

As was indicated by the flow visualization study, the superposition of a low rate of throughflow, as shown in Figure 29, has little effect on the mean flow pattern when the disk rotation is high. A significant radial inflow is evident along the stationary wall. All radial outflow including the net flow through the disk chamber is in the boundary layer at the rotating disk. Note that a significant reduction in core rotation is already evident. It is this reduction in core rotation which is the mechanism for the decrease in the pressure gradient with increasing
throughflow. The decrease of core rotation as a function of throughflow number is plotted in Figure 41. The form of the equation is not quite the same as was observed for the close clearance case. The empirical equation for low throughflow numbers at \( r/a = 0.828 \) is of the form:

\[
\frac{1 - K}{1 - K_0} - 1 = 28\left(\frac{Q}{\omega a^3}\right)^{1.308}
\]

However, some data taken at \( r/a = 0.648 \) appear to have the form:

\[
\frac{1 - K}{1 - K_0} - 1 = 43\left(\frac{Q}{\omega a^3}\right)
\]

This is the form of the equation predicted by equation [52]. Further investigation of the reduction in core rotation was made at \( s/a = 0.0552 \). A similar result occurs. The observations indicate that at \( r/a = 0.828 \) the core velocity reduction is of the form:

\[
\frac{1 - K}{1 - K_0} - 1 = 17\left(\frac{Q}{\omega a^3}\right)^{1.270}
\]

whereas data taken at \( r/a = 0.648 \) agree with the theory and have the value:

\[
\frac{1 - K}{1 - K_0} - 1 = 40\left(\frac{Q}{\omega a^3}\right)
\]

This is an unexpected result. However, the theory is based on the assumption that the boundary layer growth remains substantially unchanged with the superposition of throughflow. This is certainly not the case as is evident by inspection of Figures 27, 29, 31, and 33. However, the empirical expressions should be of some use since they are valid over a range of
throughflow number that is of engineering importance.

Besides the reduction in core rotation, as has already been discussed, the rate of throughflow has a noticeable effect on the boundary layer growth. An interesting effect is shown in Figure 31, where $Q = 25.7 \text{ cfm}$. Note that the boundary layer thickness is decreasing with increasing radius. A boundary layer thickness as such is not evident in the radial velocity profiles. However, using the definition of boundary layer thickness as being $99\%$ of that in the core for the tangential velocity profile, a comparison of theoretical and actual boundary layer thickness on the rotating disk may be made. Using theory as developed for zero throughflow the boundary layer thickness in Regime IV has the form:

$$\delta = \frac{Y r}{[\omega r^2]^{rac{1}{5}}}$$

The constant $Y$ is a function of $K$. By substituting the observed value of $K$ into $Y = f(K)$ we may obtain values of the thickness $\delta$. Comparison is made of the theoretical and measured values in Table III.

It can be seen from this table that a rough approximation can be made of the boundary layer thickness using the theory for zero throughflow.

Another observation to be made at this spacing is the persistence of a radial inflow even when the net rate of flow through the system is high. The amount of backflow decreases with decreasing radius, again indicating an axial flow in the core towards the disk. The influence of the rotating disk is also evident in the shifting of the point of maximum velocity towards the disk with increasing radius.

It is interesting to note that at high Reynolds numbers a $1/7$ power
### TABLE III

Computed and Actual Boundary Layer Thickness

\[ R = 6.9 \times 10^5 \quad s/a = 0.0665 \]

<table>
<thead>
<tr>
<th>( q )</th>
<th>( r/a )</th>
<th>( \frac{\omega r^2}{v} )</th>
<th>( K )</th>
<th>Computed</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.469</td>
<td>1.52 \times 10^5</td>
<td>0.207</td>
<td>0.105</td>
<td>0.138</td>
</tr>
<tr>
<td>0.648</td>
<td>2.90 \times 10^5</td>
<td>0.127</td>
<td>0.203</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.828</td>
<td>4.74 \times 10^5</td>
<td>0.119</td>
<td>0.237</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.6</td>
<td>0.469</td>
<td>1.52 \times 10^5</td>
<td>0.207</td>
<td>0.162</td>
<td>0.171</td>
</tr>
<tr>
<td>0.648</td>
<td>2.90 \times 10^5</td>
<td>0.127</td>
<td>0.203</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.828</td>
<td>4.74 \times 10^5</td>
<td>0.127</td>
<td>0.203</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>0.469</td>
<td>1.52 \times 10^5</td>
<td>0.038</td>
<td>0.308</td>
<td>0.370</td>
</tr>
<tr>
<td>0.648</td>
<td>2.90 \times 10^5</td>
<td>0.189</td>
<td>0.270</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.828</td>
<td>4.74 \times 10^5</td>
<td>0.341</td>
<td>0.203</td>
<td></td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>0.469</td>
<td>1.52 \times 10^5</td>
<td>0.038</td>
<td>0.308</td>
<td>0.370</td>
</tr>
<tr>
<td>0.648</td>
<td>2.90 \times 10^5</td>
<td>0.081</td>
<td>0.303</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.828</td>
<td>4.74 \times 10^5</td>
<td>0.200</td>
<td>0.270</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

law approximately satisfies the observed velocity distribution in the tangential direction. An approximation of the observed boundary layer thickness is possible using the theory developed for separate turbulent boundary layers and zero throughflow. A prediction of the observed change in core reduction is possible using momentum principles and a shear stress expression developed for pipe flow. Such conclusions should be useful in the design of turbomachinery where turbulent flow is prevalent.

### C. Pressure Data

Figures 44 through 55 display the variation of the radial pressure distribution with increasing amounts of throughflow. The data are plotted...
Fig. 43 Sample Pressure Extrapolation

Fig. 44 Pressure Data

Fig. 45 Pressure Data

Fig. 46 Pressure Data

Fig. 47 Pressure Data

Fig. 48 Pressure Data
dimensionlessly versus the quantity \(1 - \left(\frac{r}{a}\right)^2\) in two ways. The top portion of each graph indicates the deviation from the parabolic pressure distribution which exists without throughflow, namely:

\[
\frac{P_r}{P_0} \bigg|_{Q=0} - \frac{P_r}{P_0} \bigg|_{Q}
\]  

[63]

where

\[P_o = -\frac{P}{2}(K_o \omega a)^2\]

and \(K_o = \) value for \(Q=0\).

The bottom portion of each graph is the pressure defect from the pressure at \(r=a\), namely:

\[
\frac{P_r - P_a}{P_0} \bigg|_{Q}
\]

[64]

For a parabolic pressure distribution equation [63] is equal to zero and equation [64] is equal to \([1 - \left(\frac{r}{a}\right)^2]\).

With zero throughflow the pressure varies as for a solid body, or forced vortex, type of core rotation. This agrees with the findings of Reference (4). With throughflow there is a continuous departure from this parabolic distribution giving reduced radial gradients until at the maximum throughflow rate, \(Q \approx 55 \text{ cfm}\), the pressure is essentially constant in the radial direction.

The mechanism for this trend can be shown by considering the expression for the pressure gradient. Neglecting the effect of the radial velocities the radial gradient may be written:
\[ \frac{dp}{dr}_Q = \rho \left( \frac{K_0 \omega r}{2} \right)^2 \]

where
\[ K = \frac{U}{\omega r} = F(r). \]

The radial pressure gradient for a forced vortex is given by:

\[ \frac{dp}{dr}_Q = 0 = \rho \frac{(K_0 \omega r)^2}{2} \]

where
\[ K_0 \text{ is a constant.} \]

The radial pressure gradient with throughflow may then be written as:

\[ \frac{dp}{dr}_Q = \left[ \frac{K}{K_0} \right]^2 \frac{dp}{dr}_Q = 0 \]

Reference, for example to Figure 52, will indicate a typical trend for the variation of the radial pressure gradient with throughflow. Note that for any constant value of radial distance that there is a consistent reduction in the pressure gradient with increasing throughflow. Inspection of equation [66] will show that for \( \frac{dp}{dr}_Q \) to be less than \( \frac{dp}{dr}_Q = 0 \), then \( K \) must be less than \( K_0 \). Therefore, a decrease in \( K \) with \( Q \) is noted. Further inspection will show that for constant \( Q \), \( \frac{dp}{dr}_Q \) approaches \( \frac{dp}{dr}_Q = 0 \) for increasing radius, i.e. decreasing \( [1 - (r/a)^2] \). This leads to the observation that \( K \) increases with increasing radius. This is exactly what was determined from the velocity measurements. As previously mentioned the function \( \frac{Q}{\omega r^3} \) was shown to be proportional to \( \frac{Q}{\omega r^3} \). It can be seen that an increase in \( Q \) means a decrease in \( K \) and an increase in \( r \) means an increase in \( K \). This observation is made
neglecting the effect of the radial velocities. For high disk speed the primary mechanism for the pressure distribution is the core rotation.

In Figure 44 the pressure distribution for the smallest spacing, s/a = 0.0138 at R = 3.3 x 10^5, is presented. The results appear to be erratic since there is not a consistent decrease in the radial pressure gradient as there is for high disk speeds at this same s/a. It would then appear that the previous argument for the reduction in radial pressure gradient does not hold. At flow rates higher than 24.2 cfm the pressure distribution tends toward what would be expected for a frictionless source flow. This indicates that the tangential components are very small and the flow is essentially radially outward. This is confirmed by the velocity measurements as shown in Figure 24 where R = 2.95 x 10^5 and Q = 52 cfm. Further emphasis to this observation is shown in Figure 47 where s/a = 0.0276 and R = 1.7 x 10^5. Apparently with the superposition of throughflow at this low disk speed the core rotation is reduced essentially to zero. The radial velocities are also small so the net result is that there is no perceptible radial pressure gradients with throughflow.

Where erratic results are obtained the measured pressure differences are small. These differences along with any error are magnified when normalized to a very small reference pressure as is the case for low disk speed. Other discrepancies can be accounted for in the fact that the atmosphere into which the throughflow was exhausted could not be considered as an infinite medium. Normal activity such as people walking about, drafts from fans, etc., produced pressure fluctuations which affected the measurements. The most consistent pressure data were obtained on Saturday and Monday mornings when the air in the laboratory was still.
D. **Torque Data**

The torque data are shown in Figure 58 as a log-log plot of percent increase over zero throughflow torque versus throughflow number, \( \frac{Q}{ma^2} \). For comparison, theoretical values predicted by equation (41) are shown for an average value of Reynolds number \( R = 4 \times 10^5 \) and the extremes of s/a (s/a = 0.0138 and 0.2207).

For any combination of s/a and R, equation (41) gives a straight line with a slope of 1:1 on log-log paper. The slope of the data in Figure 58 is also approximately 1:1, which means that the measured torque increase is proportional to \( \frac{Q}{ma^2} \) as expected. However, at any combination of \( \left[ R^2 \right]/\left[ (s/a)^{1/10} \right] \) the proportionality factor deviates from the predicted by the vertical displacement of the data from the theoretical line. The greatest deviation is noted for the data taken at s/a = 0.2207 and \( R = 4 \times 10^5 \). The apparent deviation from the theory suggests that the effective \( K_0 \) of the throughflow is not the same as the value for zero throughflow. As indicated by the velocity measurements, the basic solid body rotation of the core is modified with throughflow. Therefore some discrepancy with the theoretical expression should be expected.

There is a definite deviation from the predicted values of torque. However, the measurements were made in air. The absolute values of the torque are very small and there is room for large experimental error. More precise measurements (e.g. with liquids) will be necessary to establish more than the trend shown.

E. **Fluctuation Data**

Of the greatest significance is the observation of periodic fluctuations in the flow field of the rotating core. These fluctuations
Fig. 56 Fluctuation Data

Fig. 57 Fluctuation Data

Fig. 58 Torque Data

Fig. 59 Transition Data
are a definite, reproducible phenomenon. It is interesting to note that the maximum magnitude of the fluctuations is observed at $r/a = 0.441$, while the fluctuations observed at $r/a = 0.66$ are of a lesser magnitude. At no time during the experimental program were periodic fluctuations observed at $r/a = 0.88$. Figures 14, 15, and 16 illustrate the character of these fluctuations. As already mentioned the use of a Strobotac and smoke injection permitted a slow motion study of this phenomenon. The fluctuating streaklines observed were so definite that a stationary streakline oriented in any position within its cycle could be observed by flashing the Strobotac at the fluctuation frequency. In slow motion the smoke trail was observed to whip out and then curl in on itself. At certain rates of throughput, smoke injected at $r/a = 0.66$ was observed to curl in under the smoke injected at $r/a = 0.441$. This leads to the assumption that this phenomenon is due to a system of vortices having rotation about an axis perpendicular to the disk and translating in a circular path about the rotating shaft.

Fluctuation frequency data for two different spacings and different throughput rates are shown in Figures 56 and 57. These are plots of frequency in cycles per second versus disk speed in revolutions per second. On such a plot for any constant $Q$ the data give a straight line over a certain range of disk speed. If we assume the vortex system rotates at some constant fraction of the disk speed and consider a constant integral number of vortex cells, this straight line can be represented by:

\[ F = nK' \omega + A \]
where
\[ n = \text{an integral number of cells} \]
\[ \omega = \text{disk speed} \]
\[ K' = \text{fraction of disk speed} \]
\[ A = \text{a constant.} \]

Rearranging:
\[ F = \frac{F}{K'} = n \omega + \frac{A}{K'} \]

For \( K' \) a constant over a range of \( \omega \) values:
\[ \frac{dF}{d\omega} = \frac{1}{K'} \left( \frac{dF}{d\omega} \right) = n \]

Thus if we can evaluate \( K' \) we may obtain the number of cells \( n \). Logically \( K' \) is probably the fraction of disk speed at which the core fluid rotates. Velocity traverses across the core will give this core rotating velocity. Using the velocity data for \( s/a = 0.0690 \) and \( R = 6.9 \times 10^5 \) and equation [68] a rough idea of the number of cells may be obtained. For example, at low disk speed and \( Q = 5.4 \, \text{cfm} \), the measured slope, \( \frac{dF}{d\omega} \), is 0.537, for \( s/a = 0.0690 \). Assuming \( K' \) to be of the order of the measured \( K \) at \( r/a = 0.469 \) the value of \( n \) is deduced to be either 2 or 3 cells. A similar computation for the other lines leads to the assumption that the number of cells is probably in the range 2-4. There is some indication by this reasoning that at high throughflow rates where \( K \) is low that as many as 8 cells might exist.

In Figures 56 and 57 the several data can be approximately represented by lines having one of three slopes; 1, 2/3, 1/2. These are
in the ratio 6:4:3. For any one particular Q the data fall on two different lines of different slope, one in the low speed range and one in the higher. In Figure 56 the ratio of the slopes from low to high speed is 1:2 for $Q = 5 \text{ cfm}$ and approximately 4:3 for $Q = 15 \text{ cfm}$ and $Q = 24 \text{ cfm}$. In Figure 57 the ratio for $Q = 5 \text{ cfm}$ is 4:3.

Note also in Figure 56 for each of the two higher throughflows either of two different frequencies could be observed in certain speed ranges. Either frequency was likely to occur after shutting down the blower and starting up again. In both cases the higher frequency is 4/3 the lower.

Maroti, Deak, and Kreith (10) experimented with a partially enclosed disk. Their apparatus consisted of a disk which rotated between walls without enclosure, and therefore with free discharge, at the periphery. The stationary walls were 20% larger in diameter than the rotating disk. They observed a system of inflow and outflow regions on the disk which alternated in a peripheral direction along the disk surface. The number of inflow and outflow regions ranged from 2 to 4. In their experiments periodicity was observed with and without source flow being admitted at the axis of rotation of the disk. In the present experiments on a fully enclosed disk periodicity was only observed at relatively low amounts of throughflow (i.e. 6 to 25 cfm). It was not observed at zero throughflow or at rates of throughflow greater than 25 cfm. It should be noted that in the present tests the effect of spacing appears to have little effect on what particular flow regime is present and only a small effect on frequency. This is shown by comparing the data in Figures 56 and 57 where we observe the discontinuity in slope for any one Q is at about the same disk speed for both clearances and the
slopes themselves in the two speed ranges are close, although not identical, for the two clearances.

On Figure 57 some data of Reference 10 are plotted along with some data taken during this study. The data from Reference 10 are for \( s/a = 0.0625 \). The data from this study are for \( s/a = 0.0690 \). At first glance the data in the same speed range appear quite different from that obtained in the present tests. If a cellular flow structure is considered to be the cause of the observed velocity fluctuations in both sets of experiments, then the frequency of these fluctuations is dependent on the number of cells and the speed at which they pass the point of observation. This fluctuating phenomenon occurs in the core and is dependent on the disk speed only because the core rotates at some fraction, \( K \), of the disk speed. Therefore, in comparing different data, the independent variable that should be used is the core rotation. In Reference 10 it was reported that the core rotation for the partially enclosed disk was less than the corresponding value for a fully enclosed disk, as found in this study. Therefore, if the data are adjusted for the different values of \( K \) in the two experiments, one would expect a correlation. This can be done by expressing the slope of the data in terms of the corresponding value of \( K \). This has been done in Figure 57 using the \( K \) obtained for zero throughput flow. The MIT data have a slope of \( K/0.86 \) whereas the data from Reference 10 have a slope of \( K/0.85 \). This would seem to indicate that the same type of instability is present in both cases.

It is also interesting to note that if most of the data are extrapolated to zero in Figures 56 and 57 they would not pass through the origin, but would indicate zero frequency at some finite disk speed.
In all cases the periodic motion was observed above some finite speed at which point a definite and finite frequency was observed. This threshold speed increased as throughput was increased. A change in throughput produces a shifting of the curves but does not appear to change the slope to the extent that would be expected from the measured reduction in core rotation with increasing throughput.

From these observations it may then be concluded that in a rotating flow field a periodic type of cellular vortex motion is possible. This investigation appears to indicate that the same periodic flow field is obtained for two different geometries, i.e. the configuration used in this investigation and that used in Reference 10. It further suggests that the periodic flow observed is of a definite type which is independent of the method of introducing a disturbance which produces the instability.

F. Transition Data

The smoke tracer technique used in this study served as an indicator of transition to turbulence. As mentioned previously, both turbulent and laminar flow could be observed with this method. The local Reynolds number, $\frac{\omega r^2}{v}$, at transition was determined by first injecting smoke at a low disk speed, when the flow was laminar, and gradually increasing the disk speed until turbulent flow was observed. A second run was made by starting with turbulent flow and reducing the disk speed until laminar flow was observed. By this method a range of Reynolds number over which transition occurred could then be determined.

In Figure 59 the points at which transition was found to occur are plotted as a/s versus $\frac{\omega r^2}{v}$. The value of a/s for a free disk is zero since the spacing s is infinity. Therefore it was possible to plot on
the same diagram the value of Reynolds number at which both instability and transition was observed on a free disk (Reference 7, 15, 20). When the data obtained is plotted on this diagram they appear to form a smooth curve as shown by the solid line in Figure 59 which is asymptotic to approximately \( R = 1 \times 10^5 \) and passes through the value of instability as obtained in Reference 7. One set of observations, at \( s/a = 0.0138 \), or \( s/a = 72 \), did not indicate turbulence until a higher Reynolds number was reached than previously observed. The range of Reynolds number for this case is shown in the upper right-hand part of Figure 59. If the solid line for the previous experimental data is shifted to match the high \( R \) values the curve (as shown dotted in Figure 59) falls through the value of transition reported in Reference 7. As previously mentioned in the discussion of the velocity data, this deduction appears to be correct since laminar flow was observed in the velocity measurements at a Reynolds number higher than the transition Reynolds number obtained with the smoke studies. As deduced previously this seems to indicate that the point of transition obtained by the smoke injection technique is actually the value of instability and turbulence is started by the turbulent smoke jet.

It also should be mentioned at this point that it is observed that at Reynolds numbers where a turbulent core appears the boundary layer on the disk is still laminar. Reference to Figures 13 and 17 illustrate this phenomenon. In both cases the smooth circular streaklines are due to smoke injected into the laminar boundary layer on the rotating disk. The wavy indistinct streaklines which are oriented at some angle to the laminar flow streaklines are due to smoke which was injected into the core.

The present tests seem to indicate then a slight decrease in the
local Reynolds number at which transition occurs with decreased spacing. This effect is small and transition occurs at a value close to that for a free disk.
VII. SUMMARY OF CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

This study has presented a broad outline of the effect of a superposed source flow on the motion induced by an enclosed rotating disk. Certain features of such a flow field are of engineering significance.

A noticeable difference between flow in a close clearance and flow in a spacing which is wide enough to have separate boundary layers on the disk and enclosing walls is observed with superposed throughflow as has been reported for zero throughflow (4). In both close and wide clearance cases there is an induced circulation pattern consisting of radial outflow close to the disk and radial inflow close to the enclosing endwall.

With close clearance and zero throughflow the integral of the radially outward and radially inward circulation is the same at all radii. With wider spacing this integral increases with increasing radius. This means that an axial flow of fluid towards the disk must exist throughout most of the chamber.

A circulation of radial outflow close to the disk and radial inflow close to the endwall is observed to persist even when there are fairly high rates of throughflow.

At all rates of throughflow there is a distinct tendency for the maximum radial velocities to occur close to the rotating disk.

The superposition of a net flow radially outward in the disk chamber produces a reduction in the tangential component of velocity outside the rotating boundary layer. This results in steeper velocity
gradients at the disk surface with a resulting increase in shear stress. The observation of an increase in torque and a decrease in the radial pressure gradient with an increasing throughflow rate provides a correlation with the velocity measurements.

A simple momentum analysis using a suitable expression for shear stress at the disk surface is able to predict the reduction in core rotation for low rates of throughflow. Similarly, a simple theory for the increase in torque appears to be adequate for rough approximations.

When laminar flow was observed on the disk a rough correlation with the theoretical radial and tangential profiles on a free disk was obtained. Turbulent tangential velocity profiles were found to be similar for all rates of throughflow. A 1/7 power law approximately describes the observed velocity distribution.

It may then be concluded from the above observations that some success may be obtained in predicting surface resistance of the enclosed rotating disk using the momentum analysis as proposed in the theoretical section of this thesis.

Of particular interest is the observation of periodic fluctuations in the flow field when throughflow is superposed. This phenomenon is believed to be cellular in nature and to consist of three separate regimes of flow. This may be of engineering importance in turbomachinery since such a situation is probably a source of high energy dissipation. A periodically fluctuating flow field conceivably could produce induced vibrations in the less massive types of turbomachinery such as fans.

B. **Recommendations**

This investigation indicates that several areas of study are in
need of future work. Recommendations for future research may be itemized in the following manner:

1. **Extension of Velocity Measurements**

   It is evident from the present studies that the major effect of three-dimensionality of the boundary layer flow lies very close to the disk and housing surfaces. A probe of much smaller dimensions should be used. The new thermistor type of probe might prove very useful. Investigation should be made using water as the test fluid so that higher Reynolds numbers than were possible with air at a reasonable disk speed may be obtained. An attempt should be made to measure the magnitude of axial velocities.

   A complete program of velocity fluctuation measurements is also required to outline the nature of the observed periodic flow. Either a thermistor or hot-wire anemometer will have to be used since a rather high frequency response is a necessity for such measurements.

2. **Torque Data**

   A comprehensive program of torque measurements with liquids over a wide range of disk speed and throughput rate is in order. Emphasis should be placed on both laminar and turbulent flow and both close and wide spacings. The equipment used in this study will be adequate for such a program. Provision for such a study was made during the modifications to the test stand for this investigation.

3. **Rough Flow**

   Hydraulically rough rotating surfaces are of engineering significance. The recommendations made for velocity and torque data should also include rough disks.
4. **Superposition of Throughflow Radially Inward**

This study has been confined to a superposition of flow radially outward. The leakage through the seals of a hydraulic turbine, for example, would be analogous to a radial flow in the opposite direction to what has been studied.
REFERENCES


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