TWO DIMENSIONAL FILTERING TO REDUCE THE
EFFECT OF QUANTIZING NOISE IN TELEVISION

by

DONALD NORMAN GRAHAM

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
January, 1962

Signature of Author

Department of Electrical Engineering, January 22, 1962

Certified by

Thesis Supervisor

Accepted by

Chairman, Departmental Committee on Graduate Students
TWO DIMENSIONAL FILTERING TO REDUCE THE EFFECT OF QUANTIZING NOISE IN TELEVISION

by

DONALD NORMAN GRAHAM

Submitted to the Department of Electrical Engineering on January 22, 1962 in partial fulfillment of the requirements for the degree of Master of Science

ABSTRACT

The treatment of an image transmission system as one operating on a two dimensional signal is investigated in an attempt to reduce the visual effect of the quantizing process in pulse code modulation transmission of television. Using the spatial frequency response of the visual system as a weighting function on the effect of quantizing noise, optimum two dimensional predistortion and restoration filters are designed.

These lead to a calculated improvement equivalent to doubling the number of quantizing levels for a picture of a close up of a face with less improvement for more detailed scenes. A computer simulation of the system was felt unwarranted in view of the extensive computation time and the relatively small predicted gain. The tools of analysis however might prove useful in investigating other image transmission systems.

Thesis Supervisor: William F. Schreiber
Title: Associate Professor of Electrical Engineering
ACKNOWLEDGEMENTS

The author wishes to thank the Bell Telephone Laboratories for allowing him to carry out this work under the M.I.T. Co-operative Program. Thanks in particular go to M. Karnaugh for suggesting the problem and aiding in its development, Professor W. F. Schreiber for his helpful suggestions, guidance, and sympathy, Messrs. E. G. Kimme and F. F. Kuo for help with the analysis of the problem, O. J. Tretiak for help with the two dimensional Fourier transform, and Messrs. H. S. McDonald, P. P. Giordano and L. B. Jackson for their aid in converting picture material to and from the IBM 7090 magnetic tape format.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>TITLE</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>LIST OF TABLES AND FIGURES</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>SECTIONS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>Statement of the Problem</td>
<td>6</td>
</tr>
<tr>
<td>II</td>
<td>History</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Reduction of Bandwidth</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Statistical Coding</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Non-Statistical Coding</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Two-Dimensional Signal Analysis</td>
<td>13</td>
</tr>
<tr>
<td>III</td>
<td>Analysis of the Problem</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>Two Filter System</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Wiener Hopf Filter</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>Improvement</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>Calculation of Spectra</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>Autocorrelation</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>Two Dimensional Fourier Transform</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Weighting Function</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>Picture Format</td>
<td>23</td>
</tr>
<tr>
<td>IV</td>
<td>Observations</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>Signal-to-Noise Ratio</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Autocorrelation and Spectral Power Density</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Optimum Filters</td>
<td>27</td>
</tr>
<tr>
<td>V</td>
<td>Conclusions</td>
<td>28</td>
</tr>
<tr>
<td>VI</td>
<td>Suggestions for Future Work</td>
<td>30</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td></td>
<td></td>
</tr>
<tr>
<td>APPENDIX I</td>
<td>Two-Filter Optimization</td>
<td>34</td>
</tr>
<tr>
<td>APPENDIX II</td>
<td>Wiener Hopf Filter Optimization</td>
<td>38</td>
</tr>
</tbody>
</table>
LIST OF TABLES AND FIGURES

TABLE I  Weighted Error Powers  27

FIGURES

1  Two-Filter Block Diagram  41
2  Quantizer Represented as a Noise Source  42
3  Wiener Hopf Filter  43
4  Library Girl  44
   a) Original  44
   b) Quantized  44
   c) Quantizing Noise
5  a) Crowd  45
   b) Close Up  45
6  Diagrammatic Representation of the Mach Phenomenon  46
7  Spatial Frequency Response of the Visual System  47
8  Signal Autocorrelation  47
9  Signal Power Density Spectra  48
10 Noise Autocorrelation  48
11 Noise Power Density Spectra  48
12 Optimum Filters  Crowd, Noise from Detail  49
13 Optimum Filters  Library Girl  49
14 Optimum Filters  Close up, Noise from Face  50
15 Impulse Response of Optimum Filters
I STATEMENT OF THE PROBLEM

In the search for a less expensive communication channel, much work of late has been done in the field of digital transmission, specifically pulse code modulation. Digital transmission inherently implies a quantizing procedure and hence a finite number of levels at the output. If this technique is applied to television and too few quantizing levels are used, an annoying "stair step" effect occurs in a slowly varying field as the output jumps from one level to the next.\(^1\) It was toward the reduction of the visual effect of this quantizing noise that this work was directed. If successful, a picture of acceptable quality could be transmitted with fewer levels, hence at a lower bit rate.

If a picture is considered as a two dimensional signal, it is possible to extend the present signal processing theories to two dimensions and thus attack the problem by conventional methods. If the quantizer is thought to be a source of two dimensional "noise" whose spectrum can be measured, the system shown in Figure 2 presents a method for best filtering the noise from the output. This represents an extension of Costas' work on a time varying signal to two dimensions.\(^2\)
The only valid means of evaluating image quality in a commercial system is by subjective testing, and because of this, some subjective criterion should be used in optimizing the system rather than the minimum mean square error. In developing a model that is still mathematically tractable, one method would be the use of a two dimensional, spatial frequency noise sensitivity function determined by extensive subjective testing. As this has not as yet been measured, the best substitute seems to be the subjective spatial frequency response of the visual system, Figure 7, measured by two persons at the Kodak Research Laboratories. To obtain the subjectively "best" picture, the noise should be at spatial frequencies where it is least easily seen. Using this information, the input predistortion and output restoration filters can be designed to minimize the visual effect of the quantizing noise while least affecting the transmitted signal.

II  HISTORY

The concept of treating a picture or an optical process as a two dimensional analog of an electrical signal or system is not new, nor is the idea of reducing the necessary bandwidth for picture transmission. The latter field, however, seems to have been more thoroughly explored.
Reduction of Bandwidth

In an attempt to transmit television signals by pulse code modulation, it was soon found that 64 or more levels of quantized brightness were necessary to produce an output picture which was subjectively as good as the original.\(^1,4\) This represents six to seven binary bits of information for each sample point. For commercial quality 525 line television with a 4 mc bandwidth, this corresponds to 48 to 56 million bits per second.

If a television signal is analyzed and its conditional entropy evaluated, this will be bound on the information content of the signal. The second order conditional entropy was measured by several individuals who found it to be somewhat less than two bits per sample.\(^5,6\) The conditional entropy was naturally higher for detailed pictures, lower for predominately flat areas. This figure indicating a possible reduction in bandwidth, led to the application of much time and effort toward implementing these predicted savings.

An obvious step would be to reduce the number of quantizing levels but the "stair step" effect caused by quantizing noise when too few levels are used originally led people to search in other directions to achieve a bit rate reduction. Many tried to make some use of the statistics of the television signal.
Statistical Coding

An investigation of the first order probability distribution showed that, on the average, each level is equally likely; for predominately dark pictures occur as often as those which are predominately bright. Finding that the answer did not lie there, some thought was given to the second order distribution.

One method lies in what was called linear prediction in which the transmitted signal is the difference between the actual signal and that predicted from some combination of previous sample values. The original is restored by an integration procedure at the receiver. If the previous sample is chosen as the predicted value of the new sample, the resulting system is commonly known as a differential quantizer. Other more general possibilities are "slope" prediction in which the new value is predicted to lie on the same slope of brightness as the last two samples, "previous line" prediction, and "planar" prediction.

The signal resulting from these operations still occupies the same bandwidth as the original but now has a probability distribution peaked around zero. Thus through efficient coding the average number of bits per sample can be reduced. A disadvantage of this method lies in the fact any error incurred in transmission will affect more than one sample at the output due to the necessary integration.
Graham proposed an "alternate mode differential quantizer" in which the predictor would use "previous line" or "previous sample" prediction depending on which produced a smaller error signal when coding the previous sample. With this system, good picture quality was obtained with three bits of information per sample.

Another example of statistical coding is "run length coding" in which the transmitted signal represents a sample value and the number of successive samples at the value. In addition to the problems of storage involved in long runs, an error in transmission can have a disasterous effect on the final picture.

Many of the above ideas have only been simulated on a computer because of the difficulty in instrumenting a system for each new idea. Because of this, little attention seems to have been paid to the engineering aspects of the problem or to the characteristics of the eye which is, after all, the ultimate receiver and judge of the merits of a particular idea.

One of the eye's characteristics which is particularly well suited to television is that at an edge, it is tolerant toward errors in the magnitude of the step and in fact prefers a bit of "overpeaking". This is undoubtedly due to the fact that the eye does the same thing in viewing an edge in normal vision. A representation of this phenomenon is shown in Figure 6. This subject enters the field of "non-statistical" coding.
Non-statistical Coding

The first attempt to make some use of what is known about the eye and the subjective properties of a television picture seems to have been the concept of using a tapered quantizer for differential quantizing which gives fine steps for small prediction errors and coarse steps for large error signals. At an edge, the error signal will be large but the eye cannot judge the exact magnitude of the jump, hence coarse quantizing is permissible.

The separation of the low and high frequency components of the video signal for different treatment is the first step in one method of coding described by Kretzmer. Advantage was taken of the fact that the eye is tolerant of errors in the magnitude of large changes, or high frequency, by using only coarse quantization of the high frequency component.

The signal is passed through a low pass filter to provide the "lows" signal. This is then subtracted from the delayed original signal to provide the "highs" signal. Because the eye is less sensitive to error for large changes in brightness, a tapered quantizer is used for the "highs". The "lows" can be transmitted through a regular narrow band channel or quantized into a six or seven bit code as discussed by Goodall and Carbery. This operation results in good picture quality with a reduction of about 1/2 in channel capacity requirements.
The high frequency noise causes spots that twinkle in detail, but the rest of the picture is free from snow due to the narrow bandwidth.

Schreiber and Knapp make use of this idea as well as the statistical nature of the output of a differential quantizer. This signal is particularly suited to run length coding because of the long runs at zero in sections of low detail. In run length coding, the length of each run is transmitted. Here, the coordinate and value of each brightness change are transmitted. Actually, this is run end position coding.

For the most significant digit, the average run length is about 40 picture elements. A nine digit code would be required to specify any of the 500 points along one line. Since this would be inefficient for short run lengths, the system adopted used a five digit code which allowed runs up to 32 elements. If the runs between 1 and 32 are not equally likely, this will not be the most efficient code from an information theory standpoint, but it is much easier to instrument.

For the complete gray scale, their estimate was 60 runs per line or 945,000 runs per second. As Kretzmer found, three digits suffice to encode the brightness of each edge while five digits are necessary to specify the position.
These figures, for a low frequency cutoff at 350kc., correspond to a total bit rate of 
\((4.9 + 7.55) \times 10^6\) bits per second. This gives a saving of greater than four to one over the bit rate required for ordinary pulse code modulation.

As a supplement to this system, a "synthetic highs" generator was developed to create the high frequency component from the decoded pulse amplitude modulated edge pulses at the receiver.\(^1^0\) A tapped delay line was used to create a high frequency edge signal that was symmetric in that it rose before, dropped sharply at the edge, then returned to zero. This was then added to the "lows" to give the original signal.

This method gave the best picture quality by far of any of the systems discussed. In pictures of much detail and low contrast, the picture was actually improved in sharpness due to the "overpeaking" of the "highs".

**Two Dimensional Signal Analysis**

Mentz and Gray\(^1^3\) applied a two dimensional Fourier transform to determine the spatial frequency response of an arbitrarily shaped scanning aperture, the Fraunhofer diffraction pattern of an opening being its two dimensional Fourier transform. Elias and others noted that an impulse response of a filter in the space domain need not be restricted to be zero over any region because the entire picture is present simultaneously.\(^1^4\) This implies that any filter is realizable
without delay, that is a shift in the space domain. Others also have made use of this tool in an analysis of television scanning and optical systems; however, there seem to have been few if any concrete applications of this concept in the field of image transmission as signal processing.\textsuperscript{15,16,17,18,19,20}

This work treats an image as a two dimensional signal to which some "noise" is added by the quantizing process. The problem lies in determining how this signal should be two dimensionally filtered to give the subjectively "best" picture at the output.

III ANALYSIS OF THE PROBLEM

The optimum filters of the system shown in Figure 1 can be calculated if the signal power spectrum, $\Phi_{SS}$, the noise power spectrum, $\Phi_{NN}$, and the weighting function, $W$, are known. To enable analysis it is necessary to assume that the noise-signal crosscorrelation is a negligible, second order effect, or that the quantizing noise is not a function of the input signal. Comparing Figures 4(a) and (c) it can be seen that the noise in the face has a much lower frequency component than in the detailed background so that this assumption is not strictly true. As the number of quantizing levels increased, this becomes more valid. Measuring small sections
of the picture will permit an evaluation of this assumption and a means to circumvent it. Figure 2 shows a model of the system incorporating this assumption which will be used for the analysis.

In pulse code modulation quantizing with a restricted number of levels it is necessary to spread the levels to accommodate the incoming signal. The noise peak power then goes up as the signal peak power at the input to the quantizer increases. The assumption that this also applies to the average powers $P_{s1}$ and $P_n$, is inherent in holding the signal to noise ratio, $R$, at the quantizer independent of the system optimization.

**Two Filter System**

From Appendix 1, the weighted error power, $e_w$, is given by Equation A1.2.

$$e_w = \int \int W (|H_1 H_2 - 1| + |H_2| + |H_2|) d\omega_x d\omega_y$$

The optimum $H_1$ and $H_2$, that is those which give a minimum weighted error power, can be determined by the calculus of variations. From Equations A1.15 and A1.16, the optimum filters are:
\[ |H_1|^2 = 0, \quad \frac{P_{sl}}{P_s} \left[ \frac{(1+1/R) \sqrt{W \Phi'_s \Phi'_{nn}}} {\Phi'_s \int \int \sqrt{W \Phi'_s \Phi_{nn}}} - \frac{\Phi'_{nn}}{R \Phi'_s} \right] \]

\[ H_2 = \frac{H_1}{\frac{P_s \Phi'_{nn}}{P_{sl} R \Phi'_s} + |H_1|^2} \]

where \( \int \int \Phi'_s = \int \int \Phi'_{nn} = 1 \), the primes denoting normalized power spectra. The weighted error power with optimum filtering is given by Equation A1.17.

\[ e_{wo} = \frac{P_s \int \int \sqrt{W \Phi'_s \Phi_{nn}}}{1+R} \]^2

Wiener Hopf Filtering

Another possible filtering system is shown in Figure 3. The analysis of this system for time varying signals leads to the Wiener Hopf equation for the optimum filter \( H \). In the two-dimensional case, the spectrum factorization is not necessary because there is no restriction on the impulse response of the filter. In fact the result is equivalent to that for infinite delay in the one-dimensional case.

From the analysis in Appendix 2, the optimum filter is given by Equation A2.9.
\[ H = \frac{\phi_{ss}}{\phi_{nn} + \phi_{ss}} = \frac{1}{1 + \frac{\phi'}{\phi_{nn} R \phi_{ss}}}, \]

Comparison with Equation A1.16 shows that this filter is equivalent to \( H_2 \) when \( H_1 \equiv 1 \). This comes about because the variations of \( H_1 \) and \( H_2 \) were assumed to be independent thus yielding the optimum \( H_2 \) for any \( H_1 \). Since the two filter problem yields optimum values, the weighted error power for Wiener filtering, \( e_{ww} \), will always be greater than that for the two filter method, \( e_{wo} \). However, there could arise a case where the input filter would give only a marginal gain.

The weighted error power for the Wiener filter is, from Equation A2.11:

\[ e_{ww} = \frac{P_s}{R} \int \int W \phi_{nn}' \frac{\phi_{nn}}{1 + \frac{\phi'}{\phi_{nn} R \phi_{ss}}}, \]

**Improvement**

To assure that a minimum point was reached in the analysis of Appendix 1, we must compare the weighted error powers to find the improvement over no filtering, \( H_1, H_2 \equiv 1 \). The weighted error power with no filtering is:

\[ e_{wn} = \frac{P_s}{R} \int \int \phi_{nn}' W \]
The ratio of error powers gives the improvement realized by optimum filtering:

\[
\frac{e_{wn}}{e_{wo}} = \frac{(1+1/R) \int \int \Phi_{nn}' W}{\left[ \int \int \sqrt{W \Phi_{nn}' \Phi_{ss}'} \right]^2}
\]

By the Schwartz inequality,

\[
\geq \frac{(1+1/R) \int \int \Phi_{nn}' W}{(\int \int \Phi_{nn}' W)(\int \int \Phi_{ss}')} = 1 + \frac{1}{R}
\]

This improvement ratio will become larger as the weighted noise spectrum becomes more orthogonal to the signal spectrum.

**Calculation of Spectra**

For ease of calculation and computation, it was chosen to arrive at the signal and noise spectra from the two dimensional Fourier transform of their autocorrelation functions. From the standpoint of symmetry, it seems reasonable to assume that these functions are circularly symmetric, that is there is no preferred direction of correlation in the average picture. This has been validated experimentally by measuring the autocorrelation optically. The term "average" however raises an argument as to what is the average picture. The
picture of the girl in the library was chosen because of its detailed background and close up of a face which shows the bothersome effect of quantizing quite readily.

**Autocorrelation**

To estimate the autocorrelation function, a section to the picture was shifted with respect to the entire picture keeping the aperture area constant. This is in fact a measure of the crosscorrelation between a section of the picture and its neighboring area. The dc level is not of interest and hence the mean value of the samples in the area must be subtracted.

\[
\phi_{11} = \frac{1}{2T} \frac{1}{2L} \int_{-L}^{L} \int_{-T}^{T} f(x,y)f(x+T_x,y+T_y) \, dx \, dy
\]

Subtracting the mean value \( v_1 \), we obtain

\[
s_1(x,y) = f(x,y) - v_1 \quad s_2(x+T_x,y+T_y) = f(x+T_x,y+T_y) - v_2
\]

\[
\phi'_{11} = \phi_{11} - v_1 v_2
\]

\[
= \frac{1}{2L} \frac{1}{2T} \int_{-L}^{L} \int_{-T}^{T} s_1(x,y)f(x+T_x,y+T_y)
\]

In terms of the sample values, \( f_{1,j} \):
\[ \varphi_{k,m} = \frac{1}{N_{1}N_{2}} \sum_{i=1}^{N_{1}} \sum_{j=1}^{N_{2}} (f_{1}, j - v_{1})f_{1+k, j+m} \]

A computer program was written to perform this two-dimensional autocorrelation on an area representing \( \left( \frac{1}{9} \times \frac{1}{9} \right) \) of the picture. While this area was adequate to get a large enough sample of the random process to estimate the autocorrelation function of the quantizing noise, such was not the case for the original signal. In fact, the calculations gave values of the estimated autocorrelation function which were greater for some shifts than at the origin and hence were meaningless. To surmount this difficulty two alternate paths were chosen. The autocorrelation functions obtained optically by Kretzmer for the two pictures shown in Figure 5 were used.22 The second method used to obtain the autocorrelation function of the signal was to compute the correlation in the x direction only of the entire "Library Girl" picture. This was a large enough section to obtain a valid estimate of the autocorrelation function in a reasonable computation time.

**Two Dimensional Transform**

The two-dimensional Fourier transform of a circularly symmetric function, known as the Hankel transform, is easily derived.23
\[
F(\omega_x, \omega_y) = \left( \frac{1}{2\pi} \right)^2 \iint f(x,y) e^{-j(x\omega_x+y\omega_y)} dx dy
\]

\[
x = r \cos \theta \quad y = r \sin \theta
\]

\[
\omega_x = \omega_r \cos \varphi \quad \omega_y = \omega_r \sin \varphi
\]

\[
dx dy = r \, dr \, d\theta
\]

\[
x\omega_x + y\omega_y = \omega_r r \cos(\theta-\varphi)
\]

\[
F(\omega_r, \varphi) = \left( \frac{1}{2\pi} \right)^2 \int_0^\infty r f(r) dr \int_0^{2\pi} e^{-j\omega_r r \cos(\theta-\varphi)} d\theta
\]

Holding \(\varphi\) constant and integrating over an entire period,

\[
\int_0^{2\pi} e^{-j\omega_r r \cos(\theta-\varphi)} d\theta = \int_0^{2\pi} e^{j\omega_r r \cos \theta} d\theta
\]

\[
= 2\pi J_0(\omega_r r)
\]

where \(J_0\) is the zeroth order Bessel function.

\[
F(\omega_r) = \frac{1}{2\pi} \int_0^\infty r f(r) J_0(\omega_r r) dr
\]

The inverse transform is shown to be

\[
f(r) = 2\pi \int_0^\infty \omega_r F(\omega_r) J_0(\omega_r r) d\omega_r
\]

A computer program was written for the IBM 7090 to perform this integration numerically using the trapezoidal rule. The transform of the autocorrelation function yields
the power density spectrum. The inverse transform is used to calculate the impulse response of the optimum filters.

The region near dc in the spectrum is very sensitive to truncation of the autocorrelation function and hence may be in error. In fact, truncation of the function at \( r = a \), that is multiplication by a "window", corresponds to smoothing in the frequency domain with the function

\[
S(\omega_r) = \int_0^a \omega_r J_0(\omega_r r) dr = \frac{a}{\omega_r} J_1(a\omega_r)
\]

**Weighting Function**

The measurement of the spatial frequency response of the visual system is based on the Mach phenomenon as discussed by Lowery and DePalma and shown in Figure 6.\(^3\) This shows that the subjective response to an edge tends to accentuate that edge. The subject was asked to match the brightness of a small window to that of the edge presented just below it. The subjective curve was determined by moving the window to a new position and having the subject again adjust the brightness of the small test section. The response curve is then the ratio of the transform of the object curve to that of the subjective curve. This led to a similar response curve for several types of edges.
The assumption that this response curve and hence the impulse response, are circularly symmetric is open to some doubt because the two eyes are in a horizontal plane. A modification to account for variations in the visual system could be made in further work as more data becomes available.

**Picture Format**

A slow flying spot scanner and photomultiplier were used to provide a picture signal. The signal was sampled, quantized to 1024 levels and placed on a tape suitable for processing on the IBM 7090 computer. A 225 line format was chosen so that the line structure would not be bothersome enough to interfere with an evaluation of quantizing noise and yet have few enough samples to enable computer operations on the picture in a reasonable time. In correspondance with television practice, 152 samples per line were chosen to give equal horizontal and vertical resolution in the output picture for a 1 to 1 aspect ratio, a Kell factor of 0.7 and equal percentage horizontal and vertical blanking intervals. The Kell factor, open to wide variations, results from the fact that if the scanning is treated as a vertical sampling process, no low pass filter smooths the output in the vertical direction. Thus to achieve equal resolution, more samples are needed in the vertical direction. Twelve of the 152 samples were taken by the blanking interval giving,
by the sampling theorem, a maximum frequency of 70 cycles per picture height in the output. The signal was band limited to this value and then double sampled to enable a more accurate measurement of the autocorrelation function.

A computer program was written to "strip the sync" leaving only video information, quantize the output to seven levels, and produce output tapes of the quantized signal and the noise or the difference between the quantized and the original signals. These three pictures are shown in Figure 4a, b, and c. A seven level linear quantizer was chosen to leave one level of a three bit system for synchronizing pulses.

IV OBSERVATIONS

Signal to Noise Ratio

Because of the seven level quantizer, the ratio of the peak signal power to the peak noise power is $7^2 = 49$. For the section of the "Library Girl" in the detailed background, the signal power was $E_{ps}^2/3.4$ while the noise power was $E_{pn}^2/3.3$ giving a signal to noise ratio of 48. $E_{ps}$ and $E_{pn}$ are the peak signal and noise voltages respectively. Although one cannot infer the probability distribution of a signal from its average power, an example of a signal whose average power is equal to $1/3$ the peak power is a triangular wave in which all values are equally likely. The noise from the face of the "Library Girl", had an average power equal to $E_{pn}^2/3.4$. When the entire picture was measured, the average noise was $E_{pn}^2/3.1$ while the signal power was $E_{ps}^2/7.6$ giving a signal to noise
ratio of 20. These figures suggest that the noise signal is uniformly distributed over its possible values while the signal for this particular picture has a distribution which is peaked around the average value or gray. The average noise power seems to remain about the same for both high and low detail pictures. The dilemma as to what to select as the signal to noise ratio for the quantizer was fortunately resolved because both 20 and 49 yielded the same optimum filters and improvement ratios within 5%. A value of 20 was chosen for the calculations of the optimum filters.

Autocorrelation and Spectral Power Density

The signal autocorrelation functions for three pictures are shown in Figure 8. The curves for "Crowd" and "Close Up" are based on an average of the x and y correlations of these two pictures obtained with an optical autocorrelator. That for the "Library Girl" is from the x autocorrelation only. As should be expected, the function for "Crowd" drops off much more sharply than the other two indicating less correlation for the same amount of shift.

The power density spectra for these three pictures are shown in Figure 9. Notice that the curve for "Crowd" is flatter and has more high frequency components than the other two. As should be expected, the spectrum for the "Library Girl" lies mostly between the other two.
Again, it should be pointed out that the curves are in serious doubt near dc because of incomplete knowledge of the autocorrelation function for large shifts. Because this curve represents the radial component of a circularly symmetric power density function, the power in a small band $\omega_r d\omega_r$ is $\omega_r d\omega_r$. The curves are only plotted to $2\pi x 70$ radians, the maximum frequency that can be present with the given sampling rate.

Figure 10 shows the autocorrelation function of three sections of quantizing noise from the "Library Girl". One of the samples was a section from the detail background to represent high frequency quantizing noise, a second section was chosen from the cheek of the "Library Girl" to give some representative low frequency noise. The third curve was calculated from the entire noise picture of the "Library Girl" in the same fashion as the third signal curve. Again it was found that on the average, the two dimensional autocorrelation is not a function of the direction of shift. Curve 1 is the autocorrelation in the x direction, curve 2 in the y direction.

A comparison of the power density spectra in Figure 11 shows that the assumption that the quantizing noise is a function of the quantizer alone is not strictly true. A signal with a large high frequency content gives rise to a noise spectrum with more power at high frequencies.
Because of this invalidation of the assumption that the quantizer adds only a fixed noise, three sets of filters were calculated, each one the optimum for a given input signal.

Optimum Filters

Three sets of filters were calculated by a computer program from the equations in the appendices. The frequency response of the filters are shown in Figures 12, 13, and 14. The three cases are "Crowd" signal, "Detail" noise; "Close Up" signal, "Face" noise; "Library Girl" entire picture signal and noise. Table 1 gives the weighted error power with no filtering, optimum $H_1$ and $H_2$, and optimum Wiener Hopf filtering. Because only one set of filters can be installed in a system, the weighted error power when each signal is passed through the other optimum filter is also given. The figures for a signal passing through its own set of filters are different due to computational errors.

<table>
<thead>
<tr>
<th>Weighted Error Power</th>
<th>Crowd</th>
<th>Close Up</th>
<th>Library Girl</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Filtering</td>
<td>.0123</td>
<td>.0223</td>
<td>.01515</td>
</tr>
<tr>
<td>Optimum $H_1$, $H_2$</td>
<td>.00515</td>
<td>.00368</td>
<td>.00417</td>
</tr>
<tr>
<td>Improvement</td>
<td>2.39</td>
<td>6.03</td>
<td>3.78</td>
</tr>
<tr>
<td></td>
<td>3.78 dB</td>
<td>7.8 dB</td>
<td>5.78 dB</td>
</tr>
<tr>
<td>Wiener Hopf Filtering</td>
<td>.0114</td>
<td>.0141</td>
<td>.0125</td>
</tr>
<tr>
<td>&quot;Library Girl&quot; Filters</td>
<td>.0066</td>
<td>.0163</td>
<td>.00394</td>
</tr>
<tr>
<td>&quot;Close Up&quot; Filters</td>
<td>.0074</td>
<td>.00374</td>
<td>.007</td>
</tr>
</tbody>
</table>
Although the average noise powers were almost equal in all three cases, the weighted error power was much higher for the "Close Up". This can be attributed to the shape of the noise power spectra in Figure 11 which shows the curve for "noise from face" peaked near the frequency of maximum visual sensitivity in Figure 7. This increased noise power also gives some validity to the choice of a noise weighting function in that it agrees with the subjective evaluation of quantizing noise, finding it much more objectionable in flat broad areas than in detail. The "Library Girl" again lies between the other two in error power. Note that with optimum two filter processing, the weighted error power for "Close Up" can be reduced below the weighted error power for the "Crowd" although it was greater before filtering.

The total system response, $H_1H_2$, in the worst case drops only .8 db across the band. The Wiener Hopf optimum filter, while giving some improvement over no filtering, by no means matches the gains with two filters.

The best over-all results seem to come from choosing "Close Up" as the signal on which to design the system. The impulse responses for $H_1$ and $H_2$ are shown in Figure 15.

V. CONCLUSIONS

The improvement by optimal filtering of the "Close Up" over no filtering is a factor of 6 or 7.8 db. If the number of quantizing levels were doubled, thus giving an extra bit of information to be transmitted, this would
increase the signal to noise ratio by 4. The noise spectrum would also shift to higher frequencies thus reducing the weighted error power somewhat. To a first approximation, optimal filtering doubles the effective number of quantizing levels for the "Close Up" with less improvement for more detailed pictures.

In investigating a system which processes the one dimensional time varying video signal and using a weighting function measured for electrical noise on a television screen, an improvement of 3.2 db was noted for the "Library Girl" by members of Department 13.2 at the Bell Telephone Laboratories. While the two weighting functions are different in nature, the comparison with 5.8 db improvement by two dimensional filtering is interesting.

$H_1$ is basically a differentiating filter whose response rises with increasing frequency. This is evidenced by the negative lobe on the impulse response, Figure 15, which subtracts the values of surrounding points from those near the origin. $H_2$ then must be an integrating filter to restore the signal at the output. If the impulse response for $H_2$ were entirely positive, the filtering could be done optically without encountering the problem of negative intensity.

It would be useful to process one picture to test the validity of the mathematical analysis and the weighting function, but the marginal gain that is predicted did not seem to warrant an hour of computation time on the IBM 7090.
The analysis of image transmission as a two dimensional process should be valuable in the treatment of more complex processing schemes and could possibly lead to a system whose capabilities indicate that it would be worth trying to implement.

VI. SUGGESTIONS FOR FUTURE WORK

Two dimensional filtering, if realized, will probably be done optically because of the problems involved with digital data handling. To process the picture format used here on the 7090 computer would require 50 million multiplications and additions plus storage and indexing, running about a half an hour for each filter. It is unfeasible to filter the signal with tapped delay lines because of the lossiness and phase distortion of currently available delay lines when delays of several scan times are required.

Optical two dimensional filtering is difficult at first glance because while the impulse response of the filter may go negative, negative intensity has no meaning. This difficulty has successfully been surmounted with a photographic process by Kelly.\(^{18}\) A special "Herschel" film is used in which the silver depositing reaction can be run in one direction by blue incoming light, and reversed by red light. If the filter is made up of red and blue transparent regions corresponding to positive and negative areas of the impulse response and placed near the plane of the object lens,
each point in the object plane passes through the filter and is focused into an area on the image plane consisting of a small replica of the filter. If the film is prefogged to the halfway point, the red light will effect the negative intensity and the resulting image will be the object, two dimensionally filtered. Actually the Herschel sensitive region in Kodak Autopositive film extends from green to infrared and has a speed slower than the conventional silver depositing process so that white light will expose the film in a normal manner. Kelly used a sharp cutting yellow filter as the negative portions of the impulse response with a neutral density filter for the positive regions.

A similar filtering setup has been suggested where two television cameras comprise the image plane, one with a red filter, one with a blue. Their outputs were then subtracted to give the desired signal. With this method, difficulties would arise in registering the two images and from non-linearities in the individual cameras but these problems have been solved in the three gun color television camera and perhaps some of the existing technology in that field could be applied here. It is possible that a method to realize this subtraction process in a single camera tube with a special mosaic could be devised.

Another field in which work could be done lies in measuring and evaluating the actual two dimensional spatial frequency noise weighting function so as to enable its inclusion in further work in image transmission and processing.


APPENDIX I
TWO FILTER OPTIMIZATION

From Figure 2, the system error, the difference between the input and output, has a power density spectrum $\Phi_{ee}$.

Al.1 \[ \Phi_{ee} = \Phi_{ss} \left| H_1 H_2 - 1 \right|^2 + \Phi_{nn} \left| H_2 \right|^2 \]

\[ + \Phi_{sn} H_2 (H_1 H_2 - 1)^* + \Phi_{ns} H_2^* (H_1 H_2 - 1) \]

Here * denotes complex conjugate; $\Phi_{ss}$, signal power spectrum; and $\Phi_{nn}$, noise power spectrum.

Assuming that the signal-noise cross power spectra, $\Phi_{sn}$ and $\Phi_{ns}$ are zero, the weighted error power is obtained by integrating the weighted error power spectrum over the entire frequency plane.

Al.2 \[ \epsilon_w = \int \int W \left[ \Phi_{ss} \left| H_1 H_2 - 1 \right|^2 + \Phi_{nn} \left| H_2 \right|^2 \right] \, dx \, dy \]

This integral must be minimized subject to the constraint that the signal-to-noise ratio, $R$, at the quantizer must remain fixed.

Al.3 \[ R = \frac{\Phi_{sn}}{\Phi_{nn}} = \frac{1}{\Phi_{nn}} \int \int \Phi_{ss} \left| H_1 \right|^2 \, dx \, dy \]

By the method of Lagrange multipliers, the variational problem to be solved is

\[ \epsilon_w = \int \int \left\{ W \left[ \Phi_{ss} \left| H_1 H_2 - 1 \right|^2 + \Phi_{nn} \left| H_2 \right|^2 \right] + \frac{\lambda \Phi_{ss}}{\Phi_{nn}} \left| H_1 \right|^2 \right\} \, dx \, dy \]

Allowing variations in $H_1$ and $H_2$ independently

\[ H_1 \rightarrow H_1 + 6H_1 \]

\[ H_2 \rightarrow H_2 + 6H_2 \]
Thus we find,
\[ \epsilon_w + \delta \epsilon_w = \iint \left\{ \text{W} \left[ \Phi_{ss} \left| (H_1 + \delta H_1)(H_2 + \delta H_2) - 1 \right| + \Phi_{nn} \left| H_2 + \delta H_2 \right| \right]^2 \ight. \\
\left. + \frac{\lambda \Phi_{ss}}{P_n} \left| H_1 + H_1 \right| \right\} \, dx \, dy \]

Equation Al.4 then yields
\[ \delta \epsilon_w = \iint \left\{ \text{W} \Phi_{ss} \text{Re}(H_1 | H_2 | 2^* \delta H_2^* + H_1 | \delta H_2^* - H_1^* \delta H_2^* - H_2^* \delta H_1^*) \right. \\
\left. + \text{W} \Phi_{nn} \text{Re}(H_2 | 2 \delta H_2^*) + \frac{\Phi_{ss}}{P_n} \text{Re}(H_1 | \delta H_1^*) \right\} \, dx \, dy \]

+ higher order terms

Equation Al.5 can be rewritten to collect terms involving \( \delta H_1^* \) and \( \delta H_2^* \) separately, neglecting the higher order terms.
\[ \delta \epsilon_w = 2 \text{Re} \iint \left\{ \text{W} \Phi_{ss} \left( H_1 | H_2 | 2 \delta H_2^* \right) + \frac{\lambda \Phi_{ss} H_1}{P_n} \delta H_1^* \ight. \\
\left. + \text{W} \Phi_{ss} \left( H_2 | H_1 | 2 \delta H_2^* \right) + \Phi_{nn} \delta H_2^* \delta H_2^* \right\} \, dx \, dy \]

At an extremum, \( \delta \epsilon_w \) must be equal to zero. This is true if
\[ \text{Al.6} \quad \Phi_{ss} \left( H_1 | H_2 | 2 \delta H_2^* \right) + \frac{\lambda \Phi_{ss} H_1}{P_n} = 0 \]
\[ \text{Al.7} \quad \Phi_{ss} \left( H_2 | H_1 | 2 \delta H_2^* \right) + \Phi_{nn} \delta H_2^* = 0 \]

Equations Al.6 and Al.7 can be separately rearranged to give
\[ \text{Al.8} \quad H_1 = \frac{\text{W} \delta H_2^*}{\text{W} | H_2 | 2 + \frac{\lambda}{P_n}} \]
\[ \text{Al.9} \quad H_2 = \frac{\Phi_{ss} H_1^*}{\Phi_{nn} + \Phi_{ss} | H_1 | 2} \]
Eliminating $H_2$ from Al.8 and Al.9 gives

$$
\text{Al.10} \quad H_1 \left( \frac{\lambda}{P_n} + \frac{W_{ss}^2 |H_1|^2}{(\phi_{nn} + \phi_{ss} |H_1|^2)^2} \right) = \frac{W_{ss} H_1}{\phi_{nn} + \phi_{ss} |H_1|^2}
$$

From Al.10

$$
\text{Al.11} \quad |H_1|^2 = 0, \quad \frac{1}{\phi_{ss}} \left[ \sqrt[\lambda]{\frac{P_n}{\lambda}} \sqrt{\phi_{nn} \phi_{ss} W} - \phi_{nn} \right]
$$

To evaluate the Lagrange multiplier, $\lambda$, insert Al.11 in Al.3 assuming that $|H_1|^2 \neq 0$.

$$
R = \frac{1}{P_n} \iint \left[ \sqrt[\lambda]{\frac{P_n}{\lambda}} \sqrt{\phi_{nn} \phi_{ss} W} - \phi_{nn} \right] \, d\omega_x \, d\omega_y
$$

When one term is integrated, this gives

$$
\text{Al.12} \quad R = \frac{1}{P_n} \sqrt[\lambda]{\frac{P_n}{\lambda}} \iint \sqrt{\phi_{nn} \phi_{ss} W} \, d\omega_x \, d\omega_y - 1
$$

Solving Al.12.

$$
\text{Al.13} \quad \sqrt[\lambda]{\frac{P_n}{\lambda}} = \frac{(R+1)P_n}{\iint \sqrt{\phi_{nn} \phi_{ss} W}}
$$

Thus Al.11 can be evaluated to give

$$
\text{Al.14} \quad |H_1|^2 = \frac{1}{\phi_{ss}} \left[ \frac{P_n(R+1) \sqrt{\phi_{nn} \phi_{ss} W}}{\iint \sqrt{\phi_{nn} \phi_{ss} W}} - \phi_{nn} \right]
$$

Or in terms of normalized power spectra, $\phi_{nn} = P_n \phi_{nn}'$, $\phi_{ss} = P_s \phi_{ss}'$. 
\[ |H_1|^2 = \left( \frac{P_{s1}}{P_s} \right) \frac{1}{\phi_{ss}} \left[ \frac{(1+R/R_n)\phi_{nn}\phi_{ss} W}{\phi_{nn} \phi_{ss} W} - \frac{\phi_{nn}}{R} \right] \]

Al.9 in terms of the normalized power spectra is

\[ H_2 = \frac{H_1^*}{\frac{1}{R} \phi_{nn} + |H_1|^2} \]

Equations Al.15 and Al.16 give the optimum \( H_1 \) and \( H_2 \) for all frequencies such that \( |H_1|^2 \) remains positive. If \( |H_1|^2 \) would be negative according to Al.15, \( H_1, H_2 = 0 \).

**Weighted Error Power**

The weighted error power with optimum filtering can be found by inserting Al.15 and Al.16 in Al.2. This gives

\[ \epsilon_{wo} = \frac{P_s \left[ \int \int \frac{\phi_{nn} \phi_{ss} W}{\phi_{nn} \phi_{ss} W} \right]^2}{1+R} \]

The weighted error power is proportional to the input signal power due to the assumption in the model of fixed signal-to-noise ratio at the quantizer.
WEINER HOPF FILTER OPTIMIZATION

An analysis of Figure 3 in a similar manner as in Appendix I gives an error power spectrum $\Phi_{ee}$

$$\Phi_{ee} = \Phi_{ss} \left| H^{-1} \right|^2 + \Phi_{nn} \left| H \right|^2 + \Phi_{sn}(H^*-1)H$$

$$+ \Phi_{ns} H^*(H-1)$$

Here * denotes complex conjugate; $\Phi_{ss}$, signal power spectrum, $\Phi_{nn}$, noise power spectrum, and $\Phi_{ns}$, $\Phi_{sn}$, cross-power spectra. It is not necessary in this case to assume that $\Phi_{sn}$ and $\Phi_{ns}$ are zero. The weighted error power is

$$\xi_w = \iint \omega \Phi_{ee} \,dx \,dy$$

The signal-to-noise ratio constraint is $R = \frac{P_s}{P_n}$, but this does not involve H and in no way affects the system optimization. Allowing a variation in H

$$H \rightarrow H + \delta H$$

gives from A2.2

$$\xi_w + \delta \xi_w = \iint \omega \left[ \Phi_{ss} \left| H+\delta H^{-1} \right|^2 + \Phi_{nn} \left| H+\delta H \right|^2 + \Phi_{sn}(H+\delta H)(H+\delta H^{-1})^* \right. \right.$$

$$\left. + \Phi_{ns}(H+\delta H)^*(H+\delta H^{-1}) \right] \,dx \,dy$$

The variation of the function is

$$\delta \xi_w = \iint \omega \left[ (H^* \delta H + H \delta H^* - \delta H^* \delta H^*) \Phi_{ss} + \Phi_{nn} (H \delta H^* + H^* \delta H) \right.$$
For any signal, $\phi_{sn} = \phi_{ns}^*$. Making this substitution in A.24 and collecting terms yields

$$\delta e_w = \int \int \Re \left[ (H - 1) \phi_{ss} + \phi_{nn} H + \phi_{sn} H + \phi_{ns} H_0 \right] \delta H^* \, d\omega_x \, d\omega_y$$

At a stationary point $\delta e_w$ must equal zero for any $\delta H^*$, therefore

$$W \left[ (H - 1) \phi_{ss} + \phi_{nn} H + \phi_{sn} H + \phi_{ns} H_0 \right] = 0$$

For this filtering model, the weighting function becomes vacuous giving for the optimum $H$

$$H = \frac{\phi_{ss} + \phi_{ns}}{\phi_{ss} + \phi_{nn} + \phi_{nn} + \phi_{ns}}$$

Again making the assumption that the cross-power spectra are negligible gives

$$H = \frac{\phi_{ss}}{\phi_{ss} + \phi_{nn}}$$

Using the notation from Appendix 1, $\phi_{ss} = P_s \phi_{ss}'$ and $\phi_{nn} = P_n \phi_{nn}'$ yields

$$H = \frac{1}{1 + \frac{\phi_{nn}'}{\phi_{ss}'}}$$

**Weighted Error Power**

Inserting A2.8 in A2.2 gives

$$e_{ww} = \int \int \left[ \frac{\phi_{ss}}{\phi_{ss} + \phi_{nn}} - 1 \right]^2 \left[ \frac{\phi_{ss}}{\phi_{ss} + \phi_{nn}} \right]^2 \, d\omega_x \, d\omega_y$$
Collecting terms gives

\[ \varepsilon_{ww} = \iint W \left( \frac{\phi_{ss} |\phi_{nn}|^2 + |\phi_{nn}|^2 \phi_{ss}^2}{|\phi_{nn} + \phi_{ss}|^2} \right) \]

Factoring gives

\[ A2.10 \quad \varepsilon_{ww} = \iint \frac{W \phi_{ss} \phi_{nn}}{\phi_{ss} + \phi_{nn}} \]

Or in terms of the normalized functions

\[ A2.11 \quad \varepsilon_{ww} = \frac{P_s}{R} \iint \frac{W \phi'_{nn}}{1 + \frac{1}{R} \phi'_{ss}} \]
TWO FILTER BLOCK DIAGRAM

FIGURE 1

QUANTIZER REPRESENTED AS A NOISE SOURCE

FIGURE 2

WIENER–HOPF FILTER

FIGURE 3
LIBRARY GIRL

FIGURE 4
DIAGRAMMATIC REPRESENTATION OF THE MACH PHENOMENON

FIGURE 6

SINE WAVE AMPLITUDE RESPONSE OF THE VISUAL SYSTEM
AT A DISTANCE 6 TIMES THE PICTURE HEIGHT

FIGURE 7
FIGURE 8
NORMALIZED NOISE AUTOCORRELATION FUNCTION

FIGURE 9
SIGNAL SPECTRAL POWER DENSITY
FIGURE 10
NORMALIZED NOISE AUTOCORRELATION FUNCTION

FIGURE 11
NOISE SPECTRAL POWER DENSITY
Figure 12
Optimal Filters Crowd Noise From Detail

Figure 13
Optimal Filters Library Girl
OPTIMAL FILTERS CLOSE UP NOISE FROM FACE

IMPULSE RESPONSE OF CLOSE UP FILTERS CIRCULARLY SYMMETRIC ABOUT THE ORIGIN