AN INVESTIGATION OF LOW
AUDIO-FREQUENCY PARAMETRIC AMPLIFICATION

by

EDWARD ALFRED PATRICK

S. B., Massachusetts Institute of Technology
(1960)

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

January, 1962

Signature of Author

Department of Electrical Engineering, January 15, 1962

Certified by...

Thesis Supervisor

Accepted by...

Chairman, Department Committee on Graduate Students
Dedication

To Patricey, Maureen, and little Eddie.
Acknowledgement

The author wishes to express his gratitude to Professor Robert P. Rafuse for his assistance and supervision during this thesis work. Expressions of gratitude are also due to Dr. Paul R. Johannessen and Alan Helgesson, both of Sylvania's Applied Research Laboratory, for their interest in this work, and to Louise Juliano for being the typist.

Finally, my deepest appreciation is expressed to Patricey.
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Submitted to the Department of Electrical Engineering on January 15, 1962 in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

Considerable interest has been given to the application of the semiconductor capacitor diode for use in single-sideband upconverter, low noise parametric amplifiers. The single-sideband nature of these amplifiers causes them to be unpractical for amplifying low-audio frequencies. It is shown in this thesis that successful low-audio frequency amplification results when a double-sideband upconverter is used. The theoretical bandwidth extends down to dc and can be made to include the entire audio spectrum; furthermore, the theoretical excess noise figure is of the order of $10^{-4}$. This small excess noise figure results since $1/F$ noise from a back-biased diode is negligible; such a back-biased diode (or diodes) is the heart of the device. An analysis is performed to determine impedances, gains, bandwidth, and noise performance. Details are given for designing optimum, low audio frequency double-sideband upconverter amplifiers. Finally, two symmetric circuits are discussed (one is a two diode symmetric circuit and the other is a four diode symmetric circuit) and experimental results are compared with theory for each circuit.

Thesis Supervisor: Robert P. Rafuse
Title: Assistant Professor of Electrical Engineering
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LIST OF SYMBOLS

$\Delta F$ - bandwidth of pump tank

$\omega_b$ - signal port, break frequency

$\omega_{s,n}$ - signal frequency at which the noise figure rises to infinity.

$Q_s$ - complex amplitude of signal frequency charge

$Q_p$ - complex amplitude of pump frequency charge

$Q_i$ - complex amplitude of lower-sideband frequency charge

$Q_u$ - complex amplitude of upper sideband frequency charge

$\omega_s$ - signal frequency

$\omega_p$ - pump frequency

$\omega_i$ - lower sideband frequency

$\omega_u$ - upper sideband frequency

$E_s$ - complex amplitude of voltage across diode at $\omega_s$

$E_u$ - complex amplitude of voltage across diode at $\omega_u$

$E_i$ - complex amplitude of voltage across diode at $\omega_i$

$E_0$ - complex amplitude of source voltage

$Z_0$ - external impedance constraint on diode at $\omega_s$

$Z_u$ - external impedance constraint on diode at $\omega_u$

$Z_i$ - external impedance constraint on diode at $\omega_i$
$R_o$ - real part of $Z_o$ - the source resistance

$R_L$ - real part of $Z_u$ and $Z_i$ when $Z_u = Z_i$

$R_s$ - series resistance of diode

$Z_{\omega_s}$ - incremental input impedance at $\omega_s$

$Z_{\omega_u}$ - incremental output impedance at $\omega_u$

$Z_{\omega_i}$ - incremental output impedance at $\omega_i$

$Z_{d,\omega_s} = Z_{\omega_s} - R_s$

$Z_{d,\omega_u} = Z_{\omega_u} - R_s$

$Z_{d,\omega_i} = Z_{\omega_i} - R_s$
CHAPTER I: INTRODUCTION

1.1 Discussion of the problem:

This thesis work was undertaken to determine if parametric amplification can be accomplished "successfully" at low audio frequencies. Of special interest is the possibility of d.c. parametric amplification, since a back-biased diode should be relatively free of 1/F noise. Such 1/F noise has a power density spectrum proportional to the square of the average current. In a back-biased silicon diode this current can be very small, in the order of $10^{-10}$ to $10^{-12}$ amperes.

Initially, this author experimented with the usual single-sideband upconverter as an amplifier for low audio frequencies. It was found that the single-sideband upconverter could not be used for amplifying d.c. and was, in general, not practical for amplifying audio frequencies. The difficulty arises because the signal frequency is too low, yielding generated sidebands "too close" to the pump frequency for practical filtering. This is best demonstrated in Fig. 1.1, where the pump frequency, $\omega_p = 1$ mc and the signal frequency, $\omega_s = 1$ kc. The sidebands are located at 1000 kc $\pm 1$ kc. Therefore, a sideband tank with a $Q > 1000$ is needed to filter out a sideband current. Even if such a $Q$ is obtained, there is still a noise problem, since not all of the pump and other sideband currents can be filtered out. These unwanted currents can greatly deteriorate the signal-to-noise ratio, especially for small signals.
Figure 1.1 Demonstrates that the sidebands are "close" to the pump when $\omega_s$ is a "Low" Frequency.

Note that the higher $\omega_p$ becomes, the larger is the $Q$ required to filter off a sideband for a given signal frequency bandwidth. Since power gain is proportional to the ratio of sideband frequency to signal frequency, it is desirable to make $\omega_p$ as large as possible. But since the required $\omega_p$ increases as $\omega_p$ increases, we run into practical problems which are not easily resolved.

The author experimented with quartz crystal filters as sideband filtering elements. Even though 80 db of rejection can be obtained with a crystal filter, it is not sufficient to eliminate the flow of pump current (since the pump current is much larger than either sideband current); this pump current deteriorates the signal to noise ratio. Furthermore, the signal bandwidth is reduced to a few cycles when a single-crystal filter is used.
It is not to be implied that a single-sideband upconverter cannot be made to work when the signal frequency is at low audio frequencies. It is stated as an experimental result that this author was not successful in building a low audio frequency, single-sideband upconverter which had a workable combination of gain, noise figure, and bandwidth. One further practical limitation of the single-sideband upconverter for amplifying low audio frequencies is that it requires a tuned circuit at signal frequency. A lowloss, wide-bandwidth tuned circuit at low audio frequencies is not easy to achieve in practice.

In summary, then, any attempt to amplify low audio frequencies with the usual single-sideband upconverter is not practical for the following reasons:

1. the sidebands are too close to \( \omega_p \) for good filtering.
2. a high quality filter is not easily obtained at very low audio frequencies.

A solution to both of these problems is found in the double-sideband upconverter. Once we derive the appropriate diode equation of motion in this chapter, we will proceed in Chapter II with an analysis of the double-sideband upconverter.

1.2 The diode model for low frequencies:

The work of Rafuse and Penfield has used a theoretical diode model consisting of a resistance \( R_s \), in series with a voltage (or charge)
variable depletion-layer capacitance. There is substantial experimental justification for the use of this model in the usual parametric devices where the large shunt resistance of the diode is negligible. At low audio frequencies, however, the magnitude of the impedance of the depletion layer capacitance is "large"; in fact, it becomes infinite at zero frequency. For this reason, the shunt resistance should be included in the low frequency diode model.* Such a model with the shunt resistance ($r_p$) included is shown in Fig. 1.2. In this model $C(v)$ is the depletion layer capacitance

![Diagram](image)

**Figure 1.2:** Low Frequency diode model for parametric calculations.

One approach used by Rafuse and Penfield is to look at the equations of motion for the depletion layer capacitance while temporarily considering the resistor $R_s$ as part of the external circuitry. The same approach is used in this thesis with the addition that $r_p$ is also temporarily considered as part of the external circuitry.

For the sake of continuity, the equations of motion for the depletion layer capacitance will be developed here, although they do appear in the references. One of these equations of motion, which is the result of

*We will see in Chapt. II that this shunt resistance can be neglected when the signal circuit is untuned.
assuming charge-constraint on the depletion layer, reduces to a simple form for the abrupt junction. In section 2.3 we will develop this appropriate equation of motion and then use it for subsequent analyses.

The depletion-layer capacitance of a back-biased diode can be calculated by differentiating the voltage dependent depletion-layer charge on either side of the depletion layer. This depletion-layer capacitance is thus an incremental relationship,

\[ C(v) = \frac{dq}{dv} = -\frac{d}{dv} [eNl] \]  \hspace{1cm} (1.1)

where,

- \( e \) - electronic charge
- \( N \) - appropriate depletion layer charge density
- \( l \) - appropriate depletion length, \( K, (\Phi + v)^{-1} \)
- \( \Phi \) - contact potential
- \( v \) - external negative voltage applied across diode

The constants \( e, N \) and \( l \) will subsequently be replaced by one constant.

Using the above expression for \( l \), Eq. 1.1 becomes

\[ C(v) = K_2(\Phi + v)^{-1} = K_2(\Phi + v)^{-1} = \frac{dq}{dv} \]  \hspace{1cm} (1.2)

Since we will be working with a series model it is more convenient to invert Eq. 1.2 to get an elastance

\[ S(v) = \frac{dv}{dq} = \frac{1}{K_2} (\Phi + v)^{-1} \]  \hspace{1cm} (1.3)
The constant $\gamma$ is set by the doping distribution as:

\[ \gamma = \frac{1}{2} \] for an abrupt junction

\[ \gamma = \frac{1}{3} \] for a graded junction

Eq. 1.2 and Eq. 1.3 are plotted in Fig. 1.3.

---

**Figure 1.3:** Voltage variable depletion layer capacitance and elastance as functions of voltage.

Henceforth, in anticipation of charge constraints on the diode, we will work with the depletion-layer elastance rather than the depletion-layer capacitance. We can express $K_2$ in terms of the maximum elastance when...
\( v = V_B \), the nominal elastance when \( v = v_0 \), or the elastance when \( v = 0 \).

These definitions as equations are:

\[
S_{\text{max}} = \frac{1}{K_2} (\theta + V_B) \triangleq S(V_B) \quad (1.4)
\]

\[
S_0 = \frac{1}{K_2} (\theta + V_0) \triangleq S(V_0) \quad (1.5)
\]

\[
S_d = \frac{1}{K_2} (\theta + 0) \triangleq S(0) \quad (1.6)
\]

Using Eq's. 1.4, 1.5 and 1.6 successively in Eq. 1.3, we get:

\[
S(v) = S_{\text{max}} \left[ \frac{\theta + v}{\theta + V_B} \right] \quad (1.7)
\]

\[
S(v) = S_0 \left[ \frac{\theta + v}{\theta + V_0} \right] \quad (1.8)
\]

\[
S(v) = S_d \left[ \frac{\theta + v}{\theta} \right] \quad (1.9)
\]

Any one of the three equations above is appropriate, if the constraints on the diode are voltages. Since, as mentioned before, we anticipate charge constraints, an elastance as a function of charge is required.
1.3 Equations of motion with charge constraints:

To obtain an \( S(q) \) relationship, we first integrate one of the \( S(v) \) equations, and then differentiate the resulting \( v = v(q) \) relationship with respect to \( q \). We pick Eq. 1.7 for development. Integrating Eq. 1.7 we get:

\[
(\mathcal{Q} + V_B) \int_0^\gamma (\mathcal{Q} + v)^{-\gamma} \ dv = S_{\text{max}} \int_{q_0}^q dq
\]

(1.10)

where \( q_0 \) is the charge when \( v = 0 \); it is the bracketed expression in Eq. 1.1:

\[
q_0 = \frac{K_0}{\mathcal{Q}} (\mathcal{Q})^\alpha = \frac{\mathcal{Q}}{S_d (1 - \gamma)}
\]

(1.11)

Performing the integration of Eq. 1.10,

\[
(\mathcal{Q} + V_B)^\gamma \left[ \frac{(\mathcal{Q} + v)}{(-\gamma + 1)} \right]^v = q S_{\text{max}}
\]

(1.12)

Rearranging Eq. 1.12 and then differentiating,

\[
\frac{dv}{dq} = S_{\text{max}} \left[ \frac{S_{\text{max}} (1 - \gamma)}{\mathcal{Q} + V_B} \right]^{1-\gamma} \frac{1-\gamma}{1-\gamma} (q + q_0)
\]

(1.13)

Note that the total charge on the diode at \( V_B \) is

\[
Q_B + q_D = \frac{V_B + \mathcal{Q}}{S_{\text{max}} (1 - \gamma)}
\]

(1.14)

Thus,
\[ S(q) = \frac{dv}{dq} = S_{\text{max}} \left[ \frac{q + q_0}{q_B + q} \right]^{\frac{\gamma}{1-\gamma}}, \quad \gamma \neq 1 \quad (1.15) \]

If we had started with Eq. 1.8 instead of Eq. 1.7, we would have for the elastance,

\[ S(q) = \frac{dv}{dq} = S_0 \left[ \frac{q + q_0}{q_B + q_0} \right]^{\frac{\gamma}{1-\gamma}} \quad \gamma \neq 1 \quad (1.16) \]

where \( q_0 \) is the total charge at \( v_0 \), the average diode bias voltage

\[ q_0 = \frac{\mathcal{Q} + v_0}{S_0(1-\gamma)} \quad (1.17) \]

### 1.4 Closure:

The final equation of motion for the diode, with \( R_s \) and \( r_p \) assumed temporarily to be part of the external circuitry, is an elastance which is a function of charge. For the abrupt junction, where \( \gamma = 1/2 \), we see that Eq. 1.15 and Eq. 1.16 are in simple forms. To a small degree, Eq. 1.16 is the simplest:

\[ S(q) = \frac{dv}{dq} = S_0 + \frac{S_0}{q_0} q \quad (1.18) \]

It should be pointed out that \( S_0 \) is not the reciprocal of the so-called "nominal capacitance" specified by the varactor diode manufacture at some external negative voltage. This would be a "ball-park" approximation which is only about 35% correct. Rashe shows in Reference No. 2 that,

\[ \frac{V_0 + \mathcal{Q}}{V_B + \mathcal{Q}} = m_0^2 + 2 m_1 \quad (1.19) \]
where,

\[ m_0 = \frac{S_0}{S_{\text{max}}} \quad , \quad m_1 = \left| \frac{S_1}{S_{\text{max}}} \right| \]

are Fourier coefficients.

In Chapt. II of this thesis we will find that \( S_1 = \frac{S_0 I_p}{q_0 J_{\omega_p}} \) in terms of the notation of Eq. 1.18.

The main purpose of these last remarks is to indicate that there is a consistent relationship between Eq. 1.18 and the charge-constraint equations of motion used in the references. For instance, according to Rafuse, a maximum value of \( m_1 = 1/4 \) results for maximum sinusoidal pumping with \( \gamma = 1/2 \). Also, if \( S_0 \) has been picked as an elastance near the center of the elastance vs. voltage curve, we have that \( S_{\text{max}} = 2 S_0 \).

Combining these last two remarks gives,

\[ \frac{S_1}{S_0} = \frac{1}{2} \quad (\text{for maximum pumping and } \gamma = 1/2) \quad (1.19) \]

We will use Eq. 1.18 in Chapt. II as the equation of motion representing the diode in performing a theoretical analysis of the double-sideband upconverter. It is not to be implied here that Eq. 1.18 is necessarily the "best" form of the charge constraint equation of motion.

The author's choice of the charge-constraint equation of motion in terms of \( S_0 \) is partly due to the previous statement that it is the simplest and partly due to his historical acquisition of that equation.
CHAPTER: II
THEORETICAL ANALYSIS OF
THE DOUBLE-SIDEBAND UP_CONVERTER

2.1 Introduction:

At the 1960 International Solid State Circuits Conference in Philadelphia, a double-side band upconverter for amplifying low frequencies was presented. The circuit used by the author of that paper was a two-diode symmetric circuit. The two-diode symmetric circuit was experimented with at the beginning of this thesis. The results of the experiments with the two-diode symmetric circuit indicate that gain is flat down to dc, and the noise Figure can be low; however, there were left unanswered questions concerning general optimization procedure, impedance levels, bandwidth, etc. In this chapter we will derive equations which should serve to answer many of these questions.

2.2 Equations of motion

We closed Chapter I with Eq. 1.18 chosen as the appropriate equation of motion for the diode; this choice was made in anticipation of charge-constraints on the diode (or diodes) in the double-sideband upconverter. With further anticipation, we assume that:

1) the external circuitry is such as to give charge constraints at \( \omega_p, \omega_s \), and sideband frequencies.

2) the pump tank is a rectangular filter with bandwidth \( \Delta F \).

3) the signal circuit is not necessarily tuned.
These assumptions are partially represented by the circuit of Figure 2.1 below.

![Circuit Diagram]

Figure 2.1: This circuit demonstrates current constraints (and thus charge constraints) at $\omega_p$, $\omega_u$, and $\omega_4$ because of the tuned pump tank, and at $\omega_s$ because of the low-pass filter of $R_o$ in series with the back-biased diode.

Note from Fig. 2.1 that the pump-sidebands branch will load the signal branch as $\omega_s$ increases from zero. This loading prevents the maximum theoretical bandwidth, based on the previous three assumptions, from being realized. Signal-branch loading also occurs in the two-diode, symmetric circuit; for this reason we will find in Chapter V that the experimental bandwidth for the two-diode symmetric circuit is less than the theoretical bandwidth. During the course of this thesis work, it was found that the four-diode symmetric circuit can be made to realize the theoretical bandwidth. This four-diode symmetric circuit and the two-diode symmetric circuit can both be analyzed with the theory to be developed in this chapter.
Assuming charge constraints, we use Eq. 1.1 to obtain the voltage across the diode,
\[ v(q) = \int S(q) \, dq = S_o q + \frac{S_o}{q_o^2} q^2 \]  \hspace{1cm} (2.1)

The assumed charge constraint is,
\[ q(t) = \begin{bmatrix} Q_e & Q_p \end{bmatrix} \begin{bmatrix} j\omega t & j\omega t \end{bmatrix} \] + complex conjugates
\hspace{1cm} (2.2)

Using Eq. 2.2 in Eq. 2.1 we have the voltage across the diode at signal frequency,
\[ E_s = S_o Q_s + \left[ \frac{S_o}{2q_o} 2Q_p Q_s^* + 2Q_u Q_p^* \right] \]

or in terms of currents,
\[ E_s = \frac{S_o I_s}{j\omega} + \frac{S_o}{q_o} \left[ \frac{I_p I_p^*}{\omega} + \frac{I_u I_u^*}{\omega} \right] \] \hspace{1cm} (2.3)

at the upper-sideband frequency,
\[ E_u = S_o Q_u + \frac{S_o}{2q_o} \left[ 2Q_p Q_s \right] \]

or, in terms of currents,
\[ E_u = \frac{S_o I_u}{j\omega} - \frac{S_o}{q_o} \frac{I_p I_s}{\omega} \] \hspace{1cm} (2.4)

and at the lower-sideband frequency,
\[ E_l = S_o Q_1 + \frac{S_o}{2q_o} 2Q_p Q_s^* \]

or, in terms of currents,
and at the lower-sideband frequency,

\[ E_1 = S_0 Q_1 + \frac{S_o}{2q_o} 2q_p Q_s^* \]

or, in terms of currents,

\[ E_1 = \frac{S_o I_1}{j\omega_1} + \frac{S_o I_p I_s^*}{q_0 j\omega_p} \]

(2.5)

If we define,

\[ S_I = \frac{S_o I_p}{q_0 j\omega_p} \]

(2.6)

Eq's. 2.3, 2.4, and 2.5 become

\[ E_s = \frac{S_o I_s}{j\omega_s} - \frac{S_I I_1}{j\omega_1} + \frac{S_I I_u}{j\omega_u} \]

(2.7)

\[ E_u = \frac{S_o I_u}{j\omega_u} + \frac{S_I I_s}{j\omega_s} \]

(2.8)

\[ E_i = \frac{S_o I_1}{j\omega_1} - \frac{S_I I_s^*}{j\omega_s} \]

(2.9)

Eq's. 2.7, 2.8, 2.9 can be put into the following general matrix for the double-sideband upconverter:

\[
\begin{bmatrix}
E_s \\
E_u \\
E_i
\end{bmatrix} = \begin{bmatrix}
S_0 \\
S_I \\
S_1
\end{bmatrix} \begin{bmatrix}
\frac{1}{j\omega_u} \\
\frac{1}{j\omega_u} \\
\frac{1}{j\omega_i}
\end{bmatrix} \begin{bmatrix}
S_I \\
S_o \\
S_1
\end{bmatrix} x \begin{bmatrix}
I_s \\
I_u \\
I_i^*
\end{bmatrix}
\]

(2.10)

The circuitry external to the diode is assumed to impose the following general constraint matrix on the diode:
\[
\begin{bmatrix}
E_s \\
E_u \\
E_i
\end{bmatrix} = \begin{bmatrix}
(Z_0 + R_s) & 0 & 0 \\
0 & (Z_u + R_s) & 0 \\
0 & 0 & (Z_1 + R_s)
\end{bmatrix} \times \begin{bmatrix}
I_s \\
I_u \\
I_i
\end{bmatrix}
\] (2.11)

Matrix 2.10 represents the equations of motion for the diode with charge constraints. Matrix 2.11 represents the actual external constraints on the diode. All constraints used in the previous equations and in the two matrices are defined in the List of Symbols.

2.3 Current gains and input Impedance

We use the constraints on \(I_u\) and \(I_i\) from the general constraint matrix 2.11 to obtain the matrix

\[
\begin{bmatrix}
E_o \\
0 \\
0
\end{bmatrix} = \begin{bmatrix}
\frac{S_0}{J\omega_s} & \frac{S_1}{J\omega_u} & -\frac{S_1}{J\omega_i} \\
\frac{S_1}{J\omega_s} & \frac{S_0}{J\omega_u} + Z_u + R_s & 0 \\
\frac{S_1^*}{J\omega_s} & 0 & (-\frac{S_0}{J\omega_i} + Z_1^* + R_s)
\end{bmatrix} \times \begin{bmatrix}
I_s \\
I_u \\
I_i
\end{bmatrix}
\] (2.12)

the equations represented by this matrix are the current gains and the input impedance; the equation for upper-sideband current gain is

\[
\frac{I_u}{I_i} = -\frac{S_0}{J\omega_s} \frac{1}{\frac{S_0}{J\omega_u} - Z_u + R_s} \tag{2.13}
\]
and, for the lower sideband,

\[
\frac{I_1^*}{I_s} = -\frac{S_1^*}{j\omega_s} \frac{1}{\left(-\frac{S_0}{j\omega_1} + Z_1^* + R_s\right)} \tag{2.14}
\]

The equation, resulting directly from matrix 2.10, for the input impedance is

\[
\frac{E_s}{I_s} = \frac{S_0}{j\omega_s} + \frac{S_1^*}{j\omega_1} \frac{I_u}{I_s} - \frac{S_1}{j\omega_1} \frac{I_1^*}{I_s} \tag{2.15}
\]

Substituting Eq's. 2.13 and 2.14 into Eq. 2.15

\[
Z_{d\omega_s} = \frac{E_s}{I_s} = \frac{S_0}{j\omega_s} + \frac{|S_1|^2}{\omega_u \omega_s} \frac{1}{\left(-\frac{S_0}{j\omega_1} + Z_1^* + R_s\right)} - \frac{|S_1|^2}{\omega_s \omega_1 \left(-\frac{S_0}{j\omega_1} + Z_1^* + R_s\right)} \tag{2.16}
\]

The general input impedance is given by Eq. 2.16, where the sidebands are not necessarily tuned.

2.4 Output impedances

If the two sideband currents are filtered from the diode through two separate tuned circuits, there is no question as to what is meant by an incremental output impedance for a particular sideband. When, however, the same tuned circuit (in this thesis the pump tank) is used to filter off both sidebands, there is some question as to what is meant by a particular sideband output impedance. We will resolve this problem with a qualitative argument.
First we consider the three-port case, where it is assumed that each sideband is separately filtered out. To determine the sideband output impedances in this case, we constrain the diode at signal frequency by the external signal frequency circuitry, with the signal source voltage equal to zero. The resulting equation of motion is

\[
\begin{bmatrix}
0 \\
E_u \\
E_1^\ast
\end{bmatrix} = \begin{bmatrix}
\frac{S_o}{j\omega_s} + Z_o + R_s & S_1^\ast j\omega_u & -\frac{S_1}{j\omega_1} \\
\frac{S_1}{j\omega_s} & S_o j\omega_u & 0 \\
\frac{S_1^\ast}{j\omega_s} & 0 & -\frac{S_o}{j\omega_1}
\end{bmatrix} \begin{bmatrix}
I_s \\
I_u \\
I_1^\ast
\end{bmatrix}
\] (2.17)

The equations represented by this matrix are,

\[
Z_{d,u} = \frac{E_u}{I_u} = \frac{S_o}{j\omega_u} + \frac{S_1}{j\omega_s} \frac{I_s}{I_u}
\] (2.18)

\[
Z_{d,1} = \frac{E_1^\ast}{I_1} = \frac{S_o}{j\omega_1} - \frac{S_1}{j\omega_s} \frac{I_s^\ast}{I_1}
\] (2.19)

\[
\left(\frac{S_o}{j\omega_s} + Z_o + R_s\right) I_s + \frac{S_1^\ast}{j\omega_u} I_u - \frac{S_1}{j\omega_1} I_1^\ast = 0
\] (2.20)

where \(I_s\) and \(I_s^\ast\) are reverse current gains.**

\[
\frac{I_s}{I_u} \quad \frac{I_s^\ast}{I_1}
\]

Equations for the reverse current gains are easily obtained under the assumption of separate filters for each sideband. These equations result from Eq. 2.20 by successively setting \(I_u = 0\) and then \(I_1 = 0\).

**These reverse current gains are not the reciprocal of the forward current gains.**
However, in order to solve Eq. 2.20 for the double-sideband case, when both sideband currents flow in the same filter, it at first looks as if another equation is required. A way to solve this apparent problem is to note that the matrix 2.17 is a linear matrix; therefore superposition holds. That is, the current $I_s$ is the sum of the currents caused by $I_u$ and $I_i$ each flowing separately. This means that Eq. 2.20 is really two equations:

$$\frac{I_u}{I_s} = -\frac{\frac{S_o}{jw} + Z_o + R_s}{S_1} \frac{jw}{S_1}$$  \hspace{1cm} (2.21)$$

$$\frac{I_i}{I_s} = \frac{\frac{S_o}{jw} + Z_o + R_s}{S_1} \frac{jw}{S_1}$$  \hspace{1cm} (2.22)$$

Using these last two equations in Eq's. 2.18 and 2.19 gives

$$Z_{dw_u} = \frac{S_o}{jw} + |S_1|^2 \frac{1}{\omega_s \omega_u} \cdot \frac{1}{\frac{S_o}{jw} + Z_o + R_s}$$  \hspace{1cm} (2.23)$$

$$Z_{dw_i} = \frac{S_o}{jw} - |S_1|^2 \frac{1}{\omega_s \omega_i} \cdot \frac{1}{\frac{-S_o}{jw} + Z_o^* + R_s}$$  \hspace{1cm} (2.24)$$

These last two equations are the output impedances at the respective sidebands. They apply to the two-port case as well as the three-port. The reason why they apply to the two port, where both sidebands are filtered out by the same filter, is that $Z_{dw_u}$ and $Z_{dw_i}$ are two separate equations in terms of the frequency of separation from $\omega_p$. $Z_{dw_u}$ applies
only for upper-sideband frequencies and $Z_{d\omega_1}$ for lower. Since an output impedance is always measured at a specific frequency or as a function of frequency, these two equations together give the output impedance as a function of frequency.

2.5 Equivalent two-port model

We have calculated $Z_{d\omega_1}$, $Z_{d\omega_u}$, $Z_{d\omega_s}$, and the two current gains from the signal port to each sideband port. Within this last statement is the direct implication of a three-port model; however, in practice the two-diode symmetric circuit and the three-diode symmetric circuit are both physical two-port devices. The appropriate two-port theoretical model would be a slight modification of the three-port model; we simply combine the sideband ports into one port, indicating that the gain and output impedances change from one sideband frequency to the other. But this is nothing new since each sideband gain and output impedance is already a function of frequency. The equivalent three-port which results directly from the theory of this chapter is shown in Fig. 2.2. It is worthwhile to point out that $R_{\omega_u}$ and $R_{\omega_1}$ have nothing to do with current gain; they are the real parts of the incremental output impedances. The current gains are given directly by Eq's. 2.13 and 2.14. In a sense, we may think of the output at each sideband as a current source which depends on $I_s$ and the appropriate load, among other things. For this reason $I_u$ and $I_1$ are represented as current sources in Fig. 2.2. As for the signal port - the case indicated is that in which the reactance $jx_o$ could be used to
tune the signal port. It turns out, however, that we do not want to tune the signal port when amplifying low frequencies. So, for low frequency amplification, we set $jx_0 = 0$. One further point, note that $R_{\omega_s}$, $R_{\omega_u}$, and $R_{\omega_1}$, as defined, include the resistance of the diode, $R_s$.

$$I_u = -\frac{S_1}{j\omega_s} \frac{1}{S \left( \frac{S}{j\omega_u} + Z_u + R_s \right)} I_s$$

$$I^*_1 = -\frac{S^*}{j\omega_s} \frac{1}{-S \left( \frac{S}{j\omega_1} + Z^*_1 + R_s \right)} I_s$$

$R_{\omega_s} =$ real part of input impedance

$R_{\omega_1} =$ real part of output impedance at $\omega_1$

$R_{\omega_u} =$ real part of output impedance at $\omega_u$

Figure 2.2: Equivalent three port for the double-sideband upconverter; an equivalent two port results when we use the same filter for both sidebands, such that $R_u = R_1 = R_L$
2.6 The untuned signal port-tuned sideband ports.

Reactance-tuned conditions will now be assumed for the two sidebands, but not for the signal circuit. There is no difficulty in tuning the sidebands, in practice at a frequency of several kc to several hundred mc. Since, however, the signal frequency is to be low audio including dc, it is impossible to assume in general that the signal circuit is tuned.

The tuned sideband assumption is a good assumption in the sense that, if \( \omega_s \) is small, then \( \omega_u \) and \( \omega_i \) will not be far away from the center frequency (\( \omega_p \)) of the pump tank. The only restriction found in making the assumption of tuned sidebands is that a very important feature about noise performance is not observed if this assumption is made. This point will be brought up in Chapter III when we consider the noise performance of the double-sideband upconverter.

If, then, we assume tuned sidebands, but an untuned signal port, Eq. 2.16 for the input impedance reduces to

\[
Z_{d\omega_s} = R_{d\omega_s} = \left( \frac{S_1}{R_S + R_L} \right) \omega_s \left[ \frac{1}{\omega_u} - \frac{1}{\omega_i} \right] = \left( \frac{S_1}{R_S + R_L} \right) \omega_s \left[ \frac{\omega_s - \omega_u}{\omega_u \omega_i} \right]
\]

(sideband ports tuned, signal port untuned)

Also if \( \omega_s \ll \omega_p \), then \( \omega_u \approx \omega_i \approx \omega_p \) and Eq. 2.25 reduces to

\[
Z_{d\omega_s} \approx -\frac{2S_1}{(R_S + R_L)\omega_p^2}
\]

(sideband ports tuned, signal port untuned and \( \omega_s \ll \omega_p \))

* We are assuming that the pump tank filter is coincidental with the sideband filters.
2.7 The signal-port break frequency - $\omega_b$

Using the low-frequency diode model of Fig. 1.2, the signal port of the double-sideband upconverter can be represented as in Fig. 2.3.

![Diagram of signal-port model](image)

**Figure 2.3** Representation of Signal port for calculating the signal port break frequency, $\omega_b$.

From Fig. 2.3, the ratio of $I_s$ to $E_o$ is seen to be

$$\frac{I_s}{I_s} = \frac{r_p j\omega_s}{j\omega_s (R_o R_p + R_o R + R_p R_s) + S_o (R_o + r_p)}$$  \hspace{1cm} (2.27)

For almost any diode (but especially for a good varactor diode) $r_p \approx 10^{12}$ ohms when "back-biased". It will turn out later, when we optimize for high power gain and good noise performance, that $R_o \approx 10^5$ ohms.

Indeed, $r_p \gg R_o$.

Therefore,

* The equivalent two port is implied in Fig. 2.3.
\[ \frac{I_s}{E_o} \approx \frac{j\omega_s}{(R_o + R_s)(j\omega_s + \frac{S_o}{R_o + R_s})} \quad \text{if} \quad r_p \gg R_o \quad (2.28) \]

The equations for the magnitude of a transconductance from a signal voltage \( E_o \) to a sideband current are obtained from Eq's. 2.13, 2.14 and 2.28:

\[ \left| \frac{I_u}{E_0} \right| = \left| \frac{I_1}{E_0} \right| = \frac{S_1}{(R_o + R_s) \left[ \omega_s^2 + \left( \frac{S_o}{R_o + R_s} \right)^2 \right]^{1/2} (R_L + R_s)} \quad (2.29) \]

where we have assumed,

1. sidebands tuned
2. signal port untuned
3. \( \omega_s \ll \omega_p \)
4. \( r_p \gg R_o \)

Equation 2.29 is the fundamental gain equation for the double-sideband upconverter. With \( R_L \), the load, we can use Eq. 2.29 to compute voltage gain and an appropriately defined power gain. These calculations, including noise performance calculations, are left for Chapter IV.

We will conclude this chapter by noting that Eq. 2.29 predicts constant gain at frequencies, including d.c., below a break frequency defined as

\[ \omega_b \triangleq \frac{S_o}{R_o + R_s} \quad (2.30) \]
CHAPTER III
POWER GAIN AND NOISE
PERFORMANCE OF THE DOUBLE-
SIDEBAND UPCONVERTER

3.1 Analysis of the synchronous detector:

The current gain equations (Eq's. 2.29) give the currents flowing at
each sideband, and these sidebands contain the signal information. It
was shown qualitatively in Chapt. II that single-sideband filtering will
not work effectively when \( \omega_s \) is at low audio frequencies. It is therefore
necessary to use a form of detection other than single-sideband filtering.

Detection of signal information from the two sidebands can be
accomplished by envelope detection or synchronous detection. For envelope
detection, we first produce amplitude modulation (in practice we don't
produce it, since some pump current is always present in the output) by
permitting pump current to flow with the sideband currents to the load.
It was found experimentally, during this thesis work, that a good noise
figure results for the low frequency, double-sideband upconverter when an
envelope detector is used; but this noise performance was not optimum in
this author's opinion (\( F = 1.6 \)). An investigation of the forms of
detection shows that a synchronous detector should degrade the noise per-
formance less than an envelope detector. Furthermore, the theoretical
assumption of a lossless, noiseless synchronous detector makes it possible
to define analytically a power gain for the double-sideband upconverter.
For these reasons then, a synchronous detector is assumed to be the first stage following the double-sideband upconverter. In actuality, the detector is an integral part of double-sideband upconversion, not really a "following stage." We therefore call the combined double-sideband up-converter with the detector, the double-sideband upconverter amplifier.

In order to analyze the synchronous detector, we must carefully note that there are both correlated and uncorrelated current components in the sideband currents. The various components of signal power and noise power, which enter the detector, are defined below.

\[
\begin{align*}
W_{s,1} & \quad \text{lower sideband signal power} \quad \text{correlated} \\
& \quad \text{(originates in signal port)} \\
W_{s,u} & \quad \text{upper sideband signal power} \quad \text{correlated} \\
& \quad \text{(originates in signal port)} \\
W_{n,ii} & \quad \text{lower sideband noise power} \quad \text{uncorrelated} \\
& \quad \text{(originates in lower sideband port)} \\
W_{n,uu} & \quad \text{upper sideband noise power} \quad \text{uncorrelated} \\
& \quad \text{(originates in upper sideband port)} \\
W_{n,is} & \quad \text{lower sideband noise power} \quad \text{correlated} \\
& \quad \text{(originates in signal port)} \\
W_{n,us} & \quad \text{upper sideband noise power} \quad \text{correlated} \\
& \quad \text{(originates in signal port)} \\
W_{n,is} & \quad \text{upper sideband signal-source noise-correlated} \\
W_{n,us} & \quad \text{lower sideband signal-source noise-correlated}
\end{align*}
\]

It is necessary to make the above distinction between correlated and uncorrelated inputs to the synchronous detector because of the cross-product terms which are in the equation for detected power output.
An assumed ideal synchronous detector is shown in Fig. 3.1.

\[ e_2 = C \cos \left( \frac{\omega_p t}{2} \right) \]

**Figure 3.1** Equivalent representation for a synchronous detector in terms of a noiseless multiplier.

If \( e_1 \) is sideband information of the form:

\[ e_1 = A \cos (\omega_p + \omega_s)t + B \cos (\omega_p - \omega_s)t \]  \hspace{1cm} (3.1)

then

\[ e_{out} = C \left[ \frac{A}{2} \cos(\omega_s t) + \frac{B}{2} \cos(\omega_s t) \right] + C \left[ \frac{A}{2} \cos(2\omega_p + \omega_s)t + \frac{B}{2} \cos(2\omega_p - \omega_s)t \right] \]  \hspace{1cm} (3.2)

This "high" frequency information is filtered out.

After filtering,

\[ e_{out} = C \left[ \frac{A}{2} \cos \omega_s t + \frac{B}{2} \cos \omega_s t \right] \]  \hspace{1cm} (3.3)

From Eq. 3.1, the input power to the synchronous detector can be defined as
\[ W_1 = \frac{e_1}{R_L} = \frac{A^2}{2R_L} + \frac{B^2}{2R_L} \quad (3.4) \]

The detector power output can be computed from Eq. 3.3 as

\[ W_{\text{out}} = \frac{e_{\text{out}}}{R_F} = \frac{C^2}{R_F} \left[ \frac{A^2}{2^2} + \frac{B^2}{2^2} + \frac{\overline{AB}}{4} \right] \quad (3.5) \]

Note that \( \overline{AB} = \overline{A} \overline{B} \) is sidebands are correlated

\( \overline{AB} = 0 \) if sidebands are uncorrelated

From Eq. 2.29, \( A^2 = B^2 \) for the double-sideband upconverter; therefore,

\[ W_{\text{out}} = \frac{C^2}{R_F} \left[ \frac{A^2}{2^2} + \frac{B^2}{2^2} + \frac{\overline{AB}}{4} \right] \quad \text{sidebands correlated} \quad (3.6) \]

\[ = \frac{C^2}{R_F} \left[ \frac{A^2}{4} \right] = \frac{C^2}{R_F} \left[ \frac{B^2}{4} \right] \quad \text{sidebands uncorrelated} \]

The following assumptions are made concerning the synchronous detector:

(1) it has a "correlated Power Gain" of 1 such that \( \frac{C^2}{R_F} = 2/R_L \)

(2) the synchronous detector introduces negligible noise. This means

\[ \text{Noise Figure of synchronous detector} \ll 1 \]

Preceeding stage power gain.

If assumptions (1) and (2) above hold, we can consistently define power gain and noise figure for the double-sideband upconverter.
3.2 Definition of "correlated power gain" for correlated signals:

We define the correlated Power gain, $G_c$, as the ratio of sideband power (with signal information) going into the synchronous detector to the exchangeable signal power from the source. Furthermore, we define the correlated power gain of the synchronous detector as being unity, so that $G_c$ is then the correlated power gain for the overall system.

To calculate $G_c$ we note that the exchangeable power from the source is

$$\text{Exchangeable source power} = \frac{R_o^2}{4R_o} \quad (3.7)$$

Using Eq's. 2.29, 3.4 and 3.7 we have:

$$G_c = 4R_o \left| S_1 \right|^2 \left( \frac{R_L}{(R_L + R_s)^2} \right) \left( \frac{1}{(R_o + R_w)^2} \right) \left[ \omega_s^2 + \left( \frac{2}{R_o + R_w} \right)^2 \right] \quad (3.8)$$

From Eq. (3.5) the uncorrelated power gain, $G_u$, is seen to be

$$G_u = \frac{1}{2} G_c \quad (3.9)$$

3.3 Definition of an "excess noise figure" as a figure of merit for the double-sideband upconverter - synchronous detector combination.

We will start with a signal-to-noise definition of noise figure, and end-up with noise figure in terms of the various power inputs to the synchronous detector. Then we will use the results of Chap. I to compute an excess noise figure.

The signal and noise powers at the input and output terminals are,
\[ S_1 \text{ - signal power at input terminals} \]
\[ N_1 \text{ - noise power at input terminals} \]
\[ S_2 \text{ - signal power at output terminals} \]
\[ N_2 \text{ - noise power at output terminals} \]

A basic definition for noise figure is
\[
F = \frac{S_1}{N_1} = \frac{S_1}{S_2} \cdot \frac{N_1}{N_2} \tag{3.10}
\]

It follows from the definition of correlated power gain that \( S_2 = G_c S_1 \).

Therefore,
\[
F = \frac{N_2}{G_c N_1} \tag{3.11}
\]

Note that \( N_1 \) produces correlated sideband noise. This means that \( G_c N_1 \) is the "ideal" noise at the output of the synchronous detector if the system were noiseless. We can then write,
\[
F = \frac{\text{(Ideal noise out)}}{\text{(Ideal noise out)}} + \frac{\text{(Excess noise out)}}{\text{(Ideal noise out)}} \tag{3.12}
\]

The second term of Eq. 3.12 is the excess noise figure. Using the second term of Eq. 3.12 for the excess noise figure, and the definitions of input power to the synchronous detector, we get
\[
(F-1) = \frac{\frac{1}{4} (W_{ii} + W_{uu}) + \frac{1}{2} (W_{is} + W_{us})}{\frac{1}{2} (W_{is} + W_{us})} \tag{3.13}
\]
\[ = \frac{(W_{ii} + W_{uu}) + 2(W_{is} + W_{us})}{2(W_{is} + W_{us})} \]
3.4 Calculation of excess noise figure for tuned sidebands-untuned signal

For the moment we will assume that both sidebands are tuned; we will see in Sec. 3.5 that the tuned assumption hides the fact that oscillations can occur, due to the negative output impedance at the lower sideband frequency.

The various incremental impedances were derived in Chap. II. Here, we will denote with primes, these incremental impedances with the nominal reactive terms, \( \frac{S_o}{j\omega_s} \), \( \frac{S_o}{j\omega_u} \), and \( \frac{S_o}{j\omega_1} \) subtracted off:

\[
Z_s = R_s - \frac{2 |S_1|^2}{(R_s + R_L)\omega_p^2}
\]

\[
Z_{s1} = R_s - \frac{|S_1|^2}{\omega_s j (\frac{S_o}{j\omega_s} + Z_o^* + R_s)} = R_s - \frac{|S_1|^2}{\omega_s j(R_o + R_s)}
\]

If sig. port is tuned.

\[
Z_{su} = R_s + \frac{|S_1|^2}{\omega_s j (\frac{S_o}{j\omega_u} + Z_o + R_s)} = R_s + \frac{|S_1|^2}{\omega_s j(R_o + R_s)}
\]

The squared-magnitude sideband current gains are (if we assume that the sidebands are tuned)

\[
\left| \frac{I_u}{I_s} \right|^2 = \frac{|S_1|^2}{\omega_s^2 (R_s + R_L)^2}
\]

\[
\left| \frac{I_1}{I_s} \right|^2 = \frac{|S_1|^2}{\omega_s^2 (R_s + R_L)}
\]
Thermal noise sources in series with $R_o$, $R_s$, and $R_L$ are:

\[
e_0^2 = 4kT_oR_o\Delta F
\]

\[
e_i^2 = e_u^2 = 4kT_oR_o\Delta F
\]

\[
e_d^2 = 4kT_dR_s\Delta F
\]

and

\[V_{FS} = \text{Noise of the } 1/F \text{ type in the signal port.}\]

where $T_o$ - source temperature in $^0K$

$T_L$ - load temperature in $^0K$

$T_d$ - diode (series $R$) temperature in $^0K$

$\Delta F$ - the appropriate incremental positive frequency bandwidth in c.p.s.

Noise currents which originate in the sideband ports come from $R_L$ and $R_s$; we ignore the one caused by $R_L$ since noise from $R_L$ is not held against the double-sideband upconverter. In reality, however, $R_L$ delivers some noise to the detector and should be included.

The noise currents originating in the sideband ports are, therefore,

\[
e_{ii}^2 = \frac{T_dR_s}{|R_L + Z_{d,1}|^2} \cdot 4k\Delta F_{ii} \tag{3.14}
\]

\[
e_{uu}^2 = \frac{T_dR_s}{|R_L + Z_{d,1}|} \cdot 4k\Delta F_{uu} \tag{3.15}
\]
Since the same tank is used for both sidebands, \( \Delta F_{uu} = \Delta F_{ii} \); since this tank is the pump tank, we have \( \Delta F_{uu} = \Delta F_{ii} = \Delta F \).

Another amplifier thermal noise current originates in the signal port and is due to \( R_s \) at \( \omega_s \); we also assume a noise of the \( \frac{1}{F} \) type \( V_{FS}^2 \).

Therefore,

\[
\overline{I_{ss}^2} = \frac{TdR_s \cdot \frac{4k\Delta F_{ss}}{s} + V_{FS}^2}{(R_o + R_{\omega_s})^2 + \left(\frac{S_o}{\omega_s}\right)^2}
\]

(3.16)

where \( \Delta F_{ss} \) is the actual signal bandwidth of the upconverter. It turns out when we optimize that \( \Delta F_{ss} \) should be equal to \( \Delta F \) so that the pump tank determines the amplifier bandwidth. This is required in order that the signal bandwidth not be less than the noise bandwidth. Therefore,

\[
\Delta F_{ss} = \Delta F_{uu} = 2\Delta F_{ii} = \Delta F
\]

(3.17)

Using the equations for current gain, the portion of sideband noise current due to signal port noise current is

\[
\overline{I_{is}^2} = \overline{I_{ss}^2} \left| \frac{I_i}{I_s} \right|^2 = \frac{|S_1|^2}{\omega_s^2(R_s + R_L)^2} \left[ \frac{TdR_s \cdot \frac{4k\Delta F}{s} + V_{FS}^2}{(R_o + R_{\omega_s})^2 + \left(\frac{S_o}{\omega_s}\right)^2} \right]
\]

(3.18)

\[
\overline{I_{us}^2} = \overline{I_{ss}^2} \left| \frac{I_u}{I_s} \right|^2 = \frac{|S_1|^2}{\omega_s^2(R_s + R_L)^2} \left[ \frac{TdR_s \cdot \frac{4k\Delta F}{s} + V_{FS}^2}{(R_o + R_{\omega_s})^2 + \left(\frac{S_o}{\omega_s}\right)^2} \right]
\]

(3.19)
Finally, there is a signal-source noise current flowing in the signal port:

\[
\frac{I_{s_0}^2}{I_{is_0}^2} = \frac{T_o R_o \cdot 4k\Delta F}{(R_o + R_{w_s})^2 + \left(\frac{S_{o}}{\omega_s}\right)^2}
\]

This signal-source noise produces a noise current in the sideband port given by

\[
\frac{I_{is_0}^2 + I_{us_0}^2}{2} = \frac{2|S_1|^2}{\omega_s^2(R_s + R_L)^2} \cdot \frac{T_o R_o \cdot 4k\Delta F}{(R_o + R_{w_s})^2 + \left(\frac{S_{o}}{\omega_s}\right)^2}
\]

Since all of the sideband noise currents flow in the same load, \(R_L\), Eq. 3.13 for (F-1) reduces to

\[
(F-1) = \frac{\left[\frac{I_{i1}^2}{I_{i1}^2} + \frac{I_{u1}^2}{I_{u1}^2}\right] + 2\left[\frac{I_{i1}^2}{I_{i1}^2} + \frac{I_{u1}^2}{I_{u1}^2}\right]}{2\left[\frac{I_{i1}^2}{I_{i1}^2} + \frac{I_{u1}^2}{I_{u1}^2}\right]}
\]

Using Eq.'s. 3.14, 3.15, 3.18, 3.19, 3.21, and 3.22, we get

\[
(F-1) = \frac{4kT_d\Delta F}{\omega_s^2(R_s + R_L)^2}
\]

\[
\left[\frac{R_s}{R_1 + Z_{w1}}^2 + \frac{R_s}{R_1 + Z_{w1}}^2 + 2\cdot2\frac{|S_1|^2}{R_s} \cdot \frac{2\cdot2\frac{|S_1|^2}{R_s}}{\omega_s^2(R_s + R_L)^2} \cdot \left[(R_o + R_{w_s})^2 + \frac{S_{o}}{\omega_s} - \lambda_{m_o}Z_o\right]^2\right]
\]

\[
\frac{2\cdot2\frac{|S_1|^2}{V_Fs}}{(R_L + R_s)^2 \omega_s^2 \left[(R_o + R_{w_s})^2 + \frac{S_{o}}{\omega_s} - \lambda_{m_o}Z_o\right]^2}
\]

divided by the expression on the following page:
\[ 4kT_o \Delta F \left[ \frac{2|S_1|^2 R_0}{\omega_s^2 (R_s + R_L)^2 \left[ (R_o + R_s \omega_s)^2 + \frac{S_o}{\omega_s^2} - \frac{Z_o}{\omega_s} \right]^2} \right] \]

simplifying,

\[ (F-1) = \]

\[ \frac{T_d}{T_o} \frac{R_s}{R_o} + \frac{T_d}{T_o} \frac{R_s}{R_o} \left[ \frac{\frac{1}{(R_s + Z_{\omega_1})^2} + \frac{1}{(R_s + Z_{\omega_1})^2}}{2 |S_1|^2} \frac{\frac{1}{(R_s + Z_{\omega_1})^2} + \frac{1}{(R_s + Z_{\omega_1})^2}}{2 |S_1|^2} \right] + \frac{V_{F_s}^2}{4kT_o \Delta F R_o} \]

This reduces to,

\[ (F-1) = \frac{T_d}{T_o} \frac{R_s}{R_o} + \frac{V_{F_s}^2}{4kT_o \Delta F R_o} + \frac{T_d}{T_o} \frac{R_s}{R_o} \left[ \frac{\omega_s^2 (R_s + R_L)^2 \left( (R_s + R_s \omega_s)^2 + \frac{S_o}{\omega_s^2} - \frac{Z_o}{\omega_s} \right)^2}{4 |S_1|^2} \right] \]

\[ \left[ \frac{1}{(R_s + Z_{\omega_1})^2} + \frac{1}{(R_s + Z_{\omega_1})^2} \right] \]

The several assumptions made in deriving Eq. 3.35 are:

1. untuned signal port
2. tuned sidebands
3. we assumed a $1/F$ noise source - $V_{F_s}$. 
Note also that (F-1) is a function of \( \omega_s \), except at low frequencies where
\[
\frac{S_o}{\omega_s} \gg (R_o + R_s).
\]
This frequency dependence means that (F-1) is more correctly a spot noise figure. Recall the definition of \( \omega_b \); when \( \omega_s \) is well below \( \omega_b \), (F-1) is independent of \( \omega_s \). Since the pump tank bandwidth is to determine the upconverter's bandwidth, we must have \( \omega_b \) well above \( \Delta F \). A fourth assumption, then, is

(4) \( \omega_b \) much larger than \( \Delta F \) which means \( \omega_s \) will be well below \( \omega_b \).

With assumption (4) the excess noise figure reduces to:

\[
(F-1) = \frac{T_d R_s}{T_o R_o} + \frac{V_{fs}^2}{4 k T_o \Delta F R_o} + \frac{T_d R_s}{T_o R_o} \left[ \frac{S_1}{2} \left( \frac{R_s + R_L}{4} \right) \right] \left[ \frac{1}{(R_L + Z_{\omega_1})^2} + \frac{1}{(R_L + Z_{\omega_u})^2} \right]
\]

(3.26)

It appears at first as if (F-1) is made small by making \( R_o \gg R_s \); this is indeed the case as long as the two bracketed expressions in Eq. 3.26 remain small. We will find in Chap. IV that these bracketed expressions increase in magnitude when \( R_o \) increases beyond a particular value. The equations for gain and (F-1) will be plotted in Chap. IV.

3.5 Deterioration of noise performance due to lower-sideband oscillations.

Note that the last bracket of Eq. 3.26 would become infinite if \( Z_{\omega_1} = -R_L \). Since \( R_{\omega_1} \) is, in general, negative, it is worthwhile investigating this possibility.
Of course \( Z_{\omega_1} \neq -R_L \) since \( Z_{\omega_1} \) has an imaginary part and \( R_L \) is pure real. Remember that in writing Eq. 3.26 it was assumed that the sidebands were tuned. This is an incorrect assumption, in the sense that a tuned sideband condition is not realized over the entire bandwidth, \( \Delta F \). More correctly, we write the first term in the last bracket of Eq. 3.26 as 
\[
\frac{1}{|R_L + Z_{\omega_1} + j\omega L|}
\]
where \( L \) is the external inductance used to tune the pump and sidebands. Inserting the relation for \( Z_{\omega_1} \), this last expression is:

\[
\frac{1}{R_L + R_s + \frac{S_0}{j\omega_1} + j\omega L - \left( \frac{|S_1|^2}{\omega_1 \omega_s} \cdot \frac{1}{\omega_s} - \frac{S_0}{j\omega_s} + \frac{R_o+R_s}{S_0} \right)}
\]

(3.27)

The bracketed expression on the right of the denominator of Eq. 3.27 can be solved for real and imaginary parts:

Real Part = \[
\frac{|S_1|^2}{(R_o+R_s)} \left[ \frac{(R_o+R_s)^2 + (\frac{S_0}{\omega_s})^2}{\omega_s} \right] = \frac{|S_1|^2 \omega_s}{(R_o+R_s) \omega_s \left[ \omega_s^2 + \left( \frac{S_0}{R_o+R_s} \right)^2 \right]}
\]

Im. Part = \[
-\frac{|S_1|^2}{\omega_s^2 \omega_1} \frac{S_0}{(R_o+R_s)^2 + (\frac{S_0}{\omega_s})^2} = -\frac{|S_1|^2 S_0}{(R_o+R_s)^2 \omega_s \left[ \omega_s^2 + \left( \frac{S_0}{R_o+R_s} \right)^2 \right]}
\]

So, \((F-1)^{1/2}\) has a term which is proportional to:
\[ \frac{1}{R_L + R_s} \left[ \frac{|S_1|^2 \omega_s}{(R_0 + R_s) \omega_1 \left[ \omega_s^2 + \omega_b^2 \right]} \right] + \left[ \frac{-S_o}{j \omega_1} + \omega_1 L + \frac{|S_1|^2 S_o}{(R_0 + R_s)^2 \omega_1 (\omega_s^2 + \omega_b^2)} \right] \]

\[ = \left[ \frac{1}{(R_L + R_s) \omega_1 \left[ \omega_s^2 + \omega_b^2 \right]} \right]^{2/3} \left[ -\frac{|S_1|^2 \omega_s}{(R_0 + R_s)} \right]^{2/3} + \left( \frac{-S_o}{\omega_1} + \omega_1 L + \frac{|S_1|^2 S_o}{(R_0 + R_s)^2 \omega_1 (\omega_s^2 + \omega_b^2)} \right)^2 \]

indicates magnitude.

The question is - will the denominator of the above expression have a zero for some \( \omega_s \)? Start out by assuming that the real part goes to zero for some \( \omega_s \), and then see if the imaginary part will consistently go to zero. We will do this for the case where \( \omega_s \) is well below \( \omega_b \).

The real part (left hand expression in the above denominator) is zero when

\[ \omega_1 = \frac{(R_0 + R_s)}{(R_L + R_s)} \frac{1}{\omega_s} |S_1|^2 \]

and the imaginary part, for this value of \( \omega_1 \), is

\[ \text{Im. part} = \frac{S_o}{\omega_s} \frac{(R_L + R_s)}{(R_0 + R_s)} \left[ \frac{|S_1|^2}{S_o^2} - \frac{|S_o|^2}{S_o^2} \right] + \frac{L (R_0 + R_s)}{(R_L + R_s)} \frac{|S_1|^2}{S_o^2} \omega_s \]

This imaginary part is zero when

\[ \omega_s^2 = \left[ \frac{|S_o|^2}{S_1^2} - \frac{|S_1|^2}{S_o^2} \right] \left( \frac{S_o}{L} \left| S_1 \right|^2 \right) \frac{\left( R_L + R_s \right)^2}{\left( R_0 + R_s \right)^2} \]
Since $\omega_p$ is the center frequency of the pump tank, we have $\omega_p = \sqrt{S_0/L}$.

Therefore,

$$\omega_{\text{noise}}^2 = \omega_p^2 \left[ \left| \frac{S_0}{S_1} \right|^2 - \left| \frac{S_1}{S_0} \right|^2 \right] \frac{S_0^2}{S_1^2} \frac{(R_L + R_s)^2}{(R_0 + R_s)^2}$$

(3.28)

This frequency, (abrev.) $\omega_{s,n}$, is the signal frequency at which thermal noise current from the lower sideband port will be amplified to the output load, $R_L$, with "infinite gain." At or near this frequency the noise performance deteriorates. This deterioration might show up experimentally as an oscillation at or near $\omega_{s,n}$. 
CHAPTER IV
DESIGN TECHNIQUES

4.1 Summary of necessary relations:

The final equations, derived in Chap. III, for impedances, gains, excess noise figure, etc. are reproduced below for convenience:

\[ S_0 \text{ - average elastance at } v = v_0. \]

\[ S_1 = \frac{S_0}{q_0} \frac{I_p}{j\omega_p} \]  \hspace{2cm} (2.6)

\[ Z_{d',\omega_s} = R_{d',\omega_s} \approx -\frac{2 |S_1|^2}{(R_s+R_L)\omega_p^2}, \text{ if } \omega_u \approx \omega_1 = \omega_p \]  \hspace{2cm} (2.26)

\[ Z_{d',\omega_u} = \frac{S_0}{j\omega_u} + j|S_1|^2 \frac{1}{\omega_s \omega_u} \cdot \frac{1}{\left(\frac{R_s}{j\omega_s} + Z_0 + R_s\right)} \]  \hspace{2cm} (2.23)

\[ Z_{d',\omega_1} = \frac{S_0}{j\omega_1} - |S_1|^2 \frac{1}{\omega_s \omega_u} \cdot \frac{1}{\left(\frac{R_s}{j\omega_s} + Z_0^* + R_s\right)} \]  \hspace{2cm} (2.24)

\[ \left| \frac{I_u}{E_o} \right| = \left| \frac{I_1}{E_o} \right| = \frac{S_1}{(R_s+R_w_s) \left[ \frac{\omega_s^2 + \left(\frac{S_0}{(R_s+R_w_s)^2}\right)^2}{(R_L+R_s)^2} \right]^{1/2}}, \text{ sidebands tuned} \]  \hspace{2cm} (2.29)

\[ G_c = \frac{4R_o |S_1|^2 R_L}{(R_L+R_s)^2(R_o+R_w_s)^2 \left[ \omega_s^2 + \left(\frac{S_0}{(R_o+R_w_s)^2}\right)^2 \right]} \]  \hspace{2cm} (3.8)
\[ G_u = \frac{1}{2} G_c \]  

(3.9)

\[ \omega_b = \frac{S_o}{R_o + R_w} = \text{signal frequency breakpoint} \]  

(2.30)

\[ (F-1) = \frac{T_d R_s}{T_o R_o} + \frac{V_{fs}^2}{4kT_o R_o\Delta F} + \frac{T_d R_s}{T_o R_o} \left[ \frac{\omega_s^2(R_s + R_L)^2}{4} \left\{ \frac{(R_o + R_w)^2 + \left(\frac{S_o}{\omega_s}\right)^2}{S_1} \right\} \right] \left[ \frac{1}{(R_L + Z_{w_1} + j\omega_l)} + \frac{1}{(R_L + Z_{w_u} + j\omega_l)} \right] \]

sidebands assumed untuned

\[ (F-1) = \frac{T_d R_s}{T_o R_o} + \frac{V_{fs}^2}{4kT_o \Delta FR_o} + \frac{T_d R_s}{T_o R_o} \left[ \frac{S_1^2 (R_s + R_L)^2}{S_o} \right] \left[ \frac{1}{(R_L + Z_{w_1} + j\omega_l)} + \frac{1}{(R_L + Z_{w_u} + j\omega_l)} \right] \]

If \( \omega_s \) is much smaller than \( \omega_b \).

\[ \omega_{s'n}^2 = \omega_p \left[ \frac{|S_o|^2}{S_1} - \frac{|S_1|^2}{S_o} \right] \frac{S_o^2}{S_1^2} \frac{(R_L + R_s)^2}{(R_o + R_s)^2} \]  

(3.23)

\[ \frac{S_1}{S_o} = \frac{1}{2} \]  

(1.19)
4.2 Theoretical performance:

In this section we will be concerned with the theoretical behavior of $R_s$, $G_c$, voltage gain, $\omega_b$, $(F-1)$, and $\omega_sN$. In general, we want to make $G_c$, $\omega_b$, and $\omega_sN$ as large as possible while we keep $R_s$ and $(F-1)$ as small as possible. By plotting curves for these parameters, we can view how they simultaneously depend on the other circuit parameters. In Sec. 4.3, these theoretical curves will be used to design low audio frequency, double-sideband upconverters which are optimized according to the requirements for good low frequency amplification. Then, in Sec. 4.4, we will select from the amplifiers of Sec. 4.3 those amplifiers which can be built in practice.

Before plotting curves for the above parameters, we will decide on practical values for $R_s$ and $S_1$, to be used in these equations. Most of the experiments to be reported in this thesis involved P.S.I. V-47 diodes* at $\omega_p = 1$ mc.; fairly accurate values to use for $S_1$ and $R_s$ for such a diode when $\omega_p = 1$ mc. are:

$$S_1 \approx \frac{1}{2} \times 10^{10} \frac{1}{\mu \cdot \mu \cdot F}$$

$$R_s \approx 4 \text{ v.}$$

These values for $S_1$ and $R_s$, along with $\frac{S_1}{S_0} = \frac{1}{2}$, will be assumed to hold throughout the rest of the thesis. The reader can note the effect of varying these parameters on the curves we plot; in this sense, there is no loss of generality by specifying $S_1$, $R_s$, and $\frac{S_1}{S_0}$ as constants.

* These diodes are made by Pacific Semiconductor Incorporated and have a specified capacitance of $47 \mu \cdot \mu \cdot F$ at $v_o = -4 \text{ v.}$
A curve for \( R_d \omega_s \) is shown in Fig. 4.1 (C.F. Table 4.1), where \( R_L \) and \( f_p = \frac{\omega_p}{2\pi} \) are parameters; note that \( R_d \omega_s \) can be made "small" by:

a) increasing \( \omega_p \)

b) increasing \( R_L \)

c) decreasing \( \frac{|S_1/S_0|}{1} \)

A "small" \( R_d \omega_s \) is desired since the signal frequency breakpoint \( \omega_b \) increases as \( R_d \omega_s \) decreases. When \( R_d \omega_s \ll R_0 \), \( \omega_b \approx \frac{S_0}{R_c} \); this curve for \( \omega_b \) is plotted in Fig. 4.2 with the assumption that \( R_d \omega_s \ll R_0 \). One justification for making this assumption is that the equations for \( G_c \), (F-1), and \( \omega_b \) are greatly simplified; another justification is that there does not appear to be any loss of "amplifier goodness" by making the assumption .*

Note that although increasing \( R_L \) decreases \( R_d \omega_s \), it also decreases \( G_c \), which is undesirable. This leads us logically to a further inspection of \( G_c \) to see how it depends on the other circuit parameters.

In Fig. 4.3, Eq. 3.8 for \( G_c \) is plotted assuming that \( \omega_s \) is much less than \( \omega_b \). Note that this curve has a maximum, with respect to \( R_L \) as the variable, when \( R_L = R_s \). We therefore conclude that maximum exchangeable power gain is obtained when \( R_L = R_s \). At this point it is desirable to define a voltage gain in terms of the peak amplitude of the envelope at the input to the synchronous detector. The reason for this is that one way we can experimentally check the upconverter's operation is by observing the

* C.F. Tables 4.3, 4.4, and 4.5
\begin{array}{|c|c|c|c|c|}
\hline
R_L & f_p = 10^5 & f_p = 10^6 & f_p = 10^7 & f_p = 10^8 \\
\hline
10 & - 9 \times 10^6 & - 9 \times 10^4 & - 9 \times 10^2 & - 9 \\
100 & - 1.21 \times 10^6 & - 1.21 \times 10^4 & - 1.21 \times 10^2 & - 1.21 \\
1000 & - 1.26 \times 10^5 & - 1.26 \times 10^3 & - 1.26 \times 10 & - 0.126 \\
10,000 & - 1.26 \times 10^4 & - 1.26 \times 10^2 & - 1.26 & - 0.0126 \\
100,000 & - 1.26 \times 10^3 & - 1.26 \times 10 & - 0.126 & - 0.00126 \\
\hline
\end{array}

Table 4.1: \( R_d, \omega_s \) with \( \omega_p \) and \( R_L \) as parameters; \( R_S = 4 \Omega, S_1 = \frac{1}{2} \times 10^{10} \)

"envelope voltage gain". So, we define voltage gain for the double-sideband upconverter,

\[
\text{V.G.} = \frac{\text{Peak amplitude of envelope of voltage across } R_L}{E_0} \\
\Delta \frac{E_o}{E_o} = \left| \frac{\frac{R_L}{R_L + R_S}}{S_1} \right| \text{ if } \omega_s \text{ is much less than } \omega_b \quad (4.1)
\]

A normalized plot of Eq. 4.1 v.s. \( R_L \) is shown in Fig. 4.4.

Finally, we consider those equations indicating how the noise performance depends on the circuit parameters - Eq. 3.26 for (F-1) and Eq. 3.28 for \( \omega_{s,n} \). We will assume, as usual, that \( \omega_s \) is much less than \( \omega_b \). Note that the first two terms of (F-1) are negligible if \( R_o \gg R_s \); however as \( R_o \) increases without limit, the third term "blows-up." On the other hand, as \( R_o \) decreases below \( R_s \)(F-1) increases and "blows up" when \( R_o = 0 \).
Figure 4.1: Theoretical variation of $R_d, \omega_s$ vs. $R_L$ for $R_s = 4 \, \Omega$, $S = \frac{1}{2} 10^{1/\mu_{uf}}$
Figure 4.2: Theoretical Plot of $S_0$ against $e^{R_0}$ when $R_0 = R_c$.

\[ S_0 \approx \frac{1}{R_0} \]

\[ f_b = \frac{\omega}{2\pi} \]
Figure 4.3: Theoretical Normalized Power Gain vs. $R_L$

$(\omega_s \text{ well below } \omega_c)$
Figure 4.4: Theoretical
Normalized Power Gain vs. $R_L$
($\omega_s$ well below $\omega_0$)
The excess noise-figure curve is therefore of the form shown in Fig. 4.5.

(F-1)

(F-1) $\rightarrow \infty$

$R_o \gg R_s$

$\omega_s$ much less than $\omega_b$

Operate here for good noise performance

$R_s$

$R_o \rightarrow \infty$

Figure 4.5: Excess noise figure vs. $R_L$ as predicted by Eq. 3.26

In order to assure that (F-1) is at the minimum indicated in Fig. 4.5, we need require only that:

a) $R_o \gg R_s$ - causes first two terms of (F-1) to be negligible.

b) $\omega_s$ much less than $\omega_b$ - causes third term to be negligible if (a) and (c) are satisfied.

c) $\omega_s$ much less than $\omega_{s,n}$.

Requirement (c) is a result of Eq. 3.26; this requirement must always hold in order to have good noise performance.

The only theoretical curve left to plot is that for $\omega_{s,n}$. Tables for $f_{s,n} = \frac{\omega_{s,n}}{2\pi}$ are shown below, where $\omega_p$, $R_L$, and $R_o$ are parameters.
The entries in the tables are for \( f_{s,n} \) as given by Eq. 3.28.

TABLE 4.2a

<table>
<thead>
<tr>
<th>( f_{s,n} )</th>
<th>( f_p = \frac{\omega_p}{2\pi} )</th>
<th>( R_L )</th>
<th>( R_O )</th>
</tr>
</thead>
<tbody>
<tr>
<td>32 cps</td>
<td>1 Mc</td>
<td>4Ω</td>
<td>10^6</td>
</tr>
<tr>
<td>56</td>
<td>1 Mc</td>
<td>10Ω</td>
<td>10^6</td>
</tr>
<tr>
<td>400</td>
<td>1 Mc</td>
<td>100Ω</td>
<td>10^6</td>
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<tr>
<td>4000</td>
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<td>1000Ω</td>
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</tr>
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</table>

TABLE 4.2b

<table>
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<tr>
<th>( f_{s,n} )</th>
<th>( f_p = \frac{\omega_p}{2\pi} )</th>
<th>( R_L )</th>
<th>( R_O )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3200 cps</td>
<td>100Mc</td>
<td>4Ω</td>
<td>10^6</td>
</tr>
<tr>
<td>5,600</td>
<td>100Mc</td>
<td>10Ω</td>
<td>10^6</td>
</tr>
<tr>
<td>40,000</td>
<td>100Mc</td>
<td>100Ω</td>
<td>10^6</td>
</tr>
<tr>
<td>400,000</td>
<td>100Mc</td>
<td>1000Ω</td>
<td>10^6</td>
</tr>
</tbody>
</table>

TABLE 4.2c

<table>
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<tr>
<th>( f_{s,n} )</th>
<th>( f_p = \frac{\omega_p}{2\pi} )</th>
<th>( R_L )</th>
<th>( R_O )</th>
</tr>
</thead>
<tbody>
<tr>
<td>32,000</td>
<td>10Mc</td>
<td>4Ω</td>
<td>10^5</td>
</tr>
<tr>
<td>56,000</td>
<td>10Mc</td>
<td>10Ω</td>
<td>10^5</td>
</tr>
<tr>
<td>40,000</td>
<td>10Mc</td>
<td>100Ω</td>
<td>10^5</td>
</tr>
<tr>
<td>400,000</td>
<td>10Mc</td>
<td>1000Ω</td>
<td>10^5</td>
</tr>
</tbody>
</table>

TABLE 4.2d

<table>
<thead>
<tr>
<th>( f_{s,n} )</th>
<th>( f_p = \frac{\omega_p}{2\pi} )</th>
<th>( R_L )</th>
<th>( R_O )</th>
</tr>
</thead>
<tbody>
<tr>
<td>320,000</td>
<td>100Mc</td>
<td>4Ω</td>
<td>10^5</td>
</tr>
<tr>
<td>56,000</td>
<td>100Mc</td>
<td>10Ω</td>
<td>10^5</td>
</tr>
<tr>
<td>40,000</td>
<td>100Mc</td>
<td>100Ω</td>
<td>10^5</td>
</tr>
<tr>
<td>400,000</td>
<td>100Mc</td>
<td>1000Ω</td>
<td>10^5</td>
</tr>
</tbody>
</table>

TABLE 4.2g

<table>
<thead>
<tr>
<th>( f_{s,n} )</th>
<th>( f_p = \frac{\omega_p}{2\pi} )</th>
<th>( R_L )</th>
<th>( R_O )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,200</td>
<td>1 Mc</td>
<td>4Ω</td>
<td>10^4</td>
</tr>
<tr>
<td>5,600</td>
<td>1 Mc</td>
<td>10Ω</td>
<td>10^4</td>
</tr>
<tr>
<td>40,000</td>
<td>1 Mc</td>
<td>100Ω</td>
<td>10^4</td>
</tr>
<tr>
<td>400,000</td>
<td>1 Mc</td>
<td>1000Ω</td>
<td>10^4</td>
</tr>
</tbody>
</table>

TABLE 4.2h

<table>
<thead>
<tr>
<th>( f_{s,n} )</th>
<th>( f_p = \frac{\omega_p}{2\pi} )</th>
<th>( R_L )</th>
<th>( R_O )</th>
</tr>
</thead>
<tbody>
<tr>
<td>32,000</td>
<td>10 Mc</td>
<td>4Ω</td>
<td>10^4</td>
</tr>
<tr>
<td>56,000</td>
<td>10 Mc</td>
<td>10Ω</td>
<td>10^4</td>
</tr>
<tr>
<td>40,000</td>
<td>10 Mc</td>
<td>100Ω</td>
<td>10^4</td>
</tr>
<tr>
<td>400,000</td>
<td>10 Mc</td>
<td>1000Ω</td>
<td>10^4</td>
</tr>
</tbody>
</table>

Tables 4.2a, 4.2b, and 4.2c are shown plotted in Figure 4.6,
Tables 4.2d, 4.2e, and 4.2f are shown plotted in Figure 4.7.
Tables 4.2g, 4.2h, and 4.2i are shown plotted in Figure 4.8.
Figure 4.6: Theoretical plot of $f_{\text{noise}}$, frequency at which $f_e \to \infty$. (with $R_L$ and $f_p$ as parameters).

$R_s = 4 \, \Omega, \quad R_o = 10^6$
Figure 4.7: Theoretical plot of $f_s$ noise with $R_L$ and $f_p$ as parameters.

$R_S = 4 \Omega$, $R_O = 10^5 \Omega$

$F_p = 100 \text{ Mc}$

$F_p = 10 \text{ Mc}$

$F_p = 1 \text{ Mc}$
Figure 4.8: Theoretical plot of $f_{\text{noise}}$ - frequency at which $f_e \to \infty$ with $R_L$ and $R_P$ as parameters.
$R_S = 4 \, \Omega$, $R_O = 10^4$

- $f_P = 10 \, \text{Mc}$
- $f_P = 1 \, \text{Mc}$
- $f_P = 100 \, \text{Kc}$
4.3 Design techniques:

The approach adapted in order to design optimized double-sideband upconverter amplifiers at low audio frequencies is the following:

\[ \frac{S_1}{S_0} = \text{constant } \left( \frac{1}{2} \right) \text{ and } R_s = \text{constant } (4\Omega). \]

Then compute values for \( f_s, n, G_c, R_d, \omega_s, \) and \( \omega_b \) for different values of \( R_o, R_L, \) and \( f_p. \) There will then result required values for \( S_o, \Delta F, L, \) and pump tank \( Q \) - all of which may or may not be practical. This procedure is followed in the preparation of Tables 4.3, 4.4, and 4.5. By using tables similar to these, the designer can obtain an optimized amplifier in accordance with his required specifications.

4.4 Practical problems

The major practical problem is that the best all around double-sideband upconverter results for values of \( S_o, L, \) and pump tank \( Q \) which are either inconsistent or impractical; nevertheless, "good" amplifiers can be designed. One major problem is that general theoretical optimization requires a "large" \( \omega_p; \) this is seen from Tables 4.3, 4.4, and 4.5, where \( \omega_b, G_c, \) and \( f_{sn} \) all have their values for the largest \( \omega_p \) considered. But the largest \( \omega_p \) in these tables, \( \omega_p = 100 \text{ mc}, \) results in impractical values for tank \( Q. \) We are confronted here with the fact that optimum theoretical performance cannot be realized because of inability to obtain the high \( Q's \) required in practice.
### TABLES 4.3

Values which result when the parameters at the left are assumed:

<table>
<thead>
<tr>
<th>$R_L$</th>
<th>$f_p$</th>
<th>$f_s$, Mc</th>
<th>$G_c$</th>
<th>$R_d$, $\omega_b$</th>
<th>$S_o$</th>
<th>$L$</th>
<th>$\Delta F$</th>
<th>$Q$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>1</td>
<td>8000</td>
<td>500</td>
<td>-800</td>
<td>6000</td>
<td>160</td>
<td></td>
<td></td>
<td>Good</td>
</tr>
<tr>
<td>200</td>
<td>1</td>
<td>700</td>
<td>5000</td>
<td>-8K</td>
<td>500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>400</td>
<td>10,000</td>
<td>-10K</td>
<td>200</td>
<td></td>
<td></td>
<td></td>
<td>NO !</td>
</tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>$R_s$</td>
<td>1</td>
<td>32</td>
<td>62,000</td>
<td>$\approx$ 1M</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>10</td>
<td>7000</td>
<td>5000</td>
<td>-75</td>
<td>5000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>10</td>
<td>4000</td>
<td>10,000</td>
<td>-100</td>
<td>2000</td>
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<td></td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>560</td>
<td>50,000</td>
<td>-1K</td>
<td>300</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_s$</td>
<td>10</td>
<td>320</td>
<td>62,500</td>
<td>$\approx$ 10K</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>100</td>
<td>70,000</td>
<td>5000</td>
<td></td>
<td>20Kc</td>
<td>5000</td>
<td></td>
<td></td>
<td>Perhaps a quartz crystal can be used for the tank.</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>40,000</td>
<td>10,000</td>
<td></td>
<td>20Kc</td>
<td>5000</td>
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<tr>
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<td>100</td>
<td>5,600</td>
<td>50,000</td>
<td></td>
<td>2 Kc</td>
<td>50,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_s$</td>
<td>100</td>
<td>3,200</td>
<td>62,000</td>
<td></td>
<td>1 Kc</td>
<td>100,000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 4.3

<table>
<thead>
<tr>
<th>( s )</th>
<th>( R_L )</th>
<th>( f_p )</th>
<th>( f_{s,n} )</th>
<th>( G_c )</th>
<th>( R_d, \omega_s )</th>
<th>( \omega_b )</th>
<th>( S_o )</th>
<th>( L )</th>
<th>( \Delta F )</th>
<th>( Q )</th>
<th>( \Omega )</th>
<th>( \text{cps} \times 10^{10} \times 1/\mu \text{H} )</th>
<th>( \mu \text{h. c.p.s.} )</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{6} )</td>
<td>( 2000 )</td>
<td>( 1 )</td>
<td>( 8000 )</td>
<td>( 500 )</td>
<td>( -800 )</td>
<td>( 6000 )</td>
<td>( 160 )</td>
<td>Good</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 10^{6} )</td>
<td>( 200 )</td>
<td>( 1 )</td>
<td>( 700 )</td>
<td>( 5000 )</td>
<td>( -8K )</td>
<td>( 500 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
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<td>( 1 )</td>
<td>( 400 )</td>
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<td>( -10K )</td>
<td>( 200 )</td>
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<td>( 1 )</td>
<td>( 56 )</td>
<td>( 50,000 )</td>
<td>( -100K )</td>
<td>( 25 )</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 10^{6} )</td>
<td>( R_{S} )</td>
<td>( 1 )</td>
<td>( 32 )</td>
<td>( 62,000 )</td>
<td>( \approx 1M )</td>
<td>( 10 )</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>( 200 )</td>
<td>( 10 )</td>
<td>( 7000 )</td>
<td>( 5000 )</td>
<td>( -75 )</td>
<td>( 5000 )</td>
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<td>( 4000 )</td>
<td>( 10,000 )</td>
<td>( -100 )</td>
<td>( 2000 )</td>
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<td>( -1K )</td>
<td>( 300 )</td>
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</tr>
<tr>
<td>( 10^{6} )</td>
<td>( R_{S} )</td>
<td>( 10 )</td>
<td>( 320 )</td>
<td>( 62,500 )</td>
<td>( \approx 10K )</td>
<td>( 100 )</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( 10^{6} )</td>
<td>( 200 )</td>
<td>( 100 )</td>
<td>( 70,000 )</td>
<td>( 5000 )</td>
<td>( 20Kc )</td>
<td>( 5000 )</td>
<td>Perhaps a quartz crystal can be used for the tank.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>( 10^{6} )</td>
<td>( 100 )</td>
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<td>( 40,000 )</td>
<td>( 10,000 )</td>
<td>( 2Kc )</td>
<td>( 50,000 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>( 10^{6} )</td>
<td>( 10 )</td>
<td>( 100 )</td>
<td>( 5,600 )</td>
<td>( 50,000 )</td>
<td>( 1Kc )</td>
<td>( 100,000 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 10^{6} )</td>
<td>( R_{S} )</td>
<td>( 100 )</td>
<td>( 3,200 )</td>
<td>( 62,000 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4.4

Values which result when the parameters at the left are assumed:

<table>
<thead>
<tr>
<th>$R_L$</th>
<th>$f_p$</th>
<th>$f_s,n$</th>
<th>$G_c$</th>
<th>$R_d,\omega_s$</th>
<th>$\Omega$</th>
<th>$S_0$</th>
<th>$L$</th>
<th>$\Delta F$</th>
<th>$Q$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>1</td>
<td>7000</td>
<td>500</td>
<td>-8K</td>
<td>Toroid</td>
<td>5000</td>
<td>200</td>
<td>OK</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>4000</td>
<td>1000</td>
<td>-10K</td>
<td></td>
<td>2000</td>
<td>500</td>
<td>NO!</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>560</td>
<td>5000</td>
<td>-100K</td>
<td></td>
<td>300</td>
<td>3300</td>
<td>NO!</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_s$</td>
<td>1</td>
<td>320</td>
<td>6250</td>
<td>$\approx -1M$</td>
<td></td>
<td>100</td>
<td>10,000</td>
<td>NO!</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>10</td>
<td>70,000</td>
<td>500</td>
<td>-75</td>
<td>50,000</td>
<td>2000</td>
<td>5000</td>
<td>NO!</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>10</td>
<td>40,000</td>
<td>1000</td>
<td>-100</td>
<td>20,000</td>
<td>5000</td>
<td></td>
<td>NO!</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>5600</td>
<td>5000</td>
<td>-1K</td>
<td>3,000</td>
<td>33,000</td>
<td></td>
<td>Quartz crystal for tank</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_s$</td>
<td>10</td>
<td>3200</td>
<td>6250</td>
<td>$\approx -10K$</td>
<td>1000</td>
<td>100,000</td>
<td></td>
<td>All Q's high -perhaps use quartz crystal for filter.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>100</td>
<td>700,000</td>
<td>500</td>
<td></td>
<td></td>
<td>4000</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>400,000</td>
<td>1000</td>
<td></td>
<td></td>
<td>56,000</td>
<td>5000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>100</td>
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<td>32,000</td>
<td>6250</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
TABLE 4.5

Values which result when the parameters at the left are assumed:

<table>
<thead>
<tr>
<th>$R_L$</th>
<th>$f_p$</th>
<th>$f_{s,n}$</th>
<th>$G_c$</th>
<th>$R_d, \omega_s$</th>
<th>$\omega_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega$</td>
<td>Mc.</td>
<td>c.p.s.</td>
<td>$\Omega$</td>
<td>c.p.s.</td>
<td>$10^{10}$</td>
</tr>
<tr>
<td>$\times 10^4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>0.1</td>
<td>7000</td>
<td>50</td>
<td>-800K</td>
<td>4000</td>
</tr>
<tr>
<td>100</td>
<td>0.1</td>
<td>4000</td>
<td>100</td>
<td>-1M</td>
<td>2000</td>
</tr>
<tr>
<td>10</td>
<td>0.1</td>
<td>560</td>
<td>500</td>
<td>$\approx$ -10M</td>
<td>200</td>
</tr>
<tr>
<td>$R_s$</td>
<td>0.1</td>
<td>320</td>
<td>625</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\times 10^4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>1</td>
<td>70,000</td>
<td>50</td>
<td>-8K</td>
<td>40Kc</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>40,000</td>
<td>100</td>
<td>-10K</td>
<td>20Kc</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>5,600</td>
<td>500</td>
<td>-100K</td>
<td>2Kc</td>
</tr>
<tr>
<td>$R_s$</td>
<td>1</td>
<td>3,200</td>
<td>625</td>
<td>$\approx$ -1M</td>
<td></td>
</tr>
<tr>
<td>$\times 10^4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>10</td>
<td>700,000</td>
<td>50</td>
<td>-75$\Omega$</td>
<td>Toroid 40Kc</td>
</tr>
<tr>
<td>100</td>
<td>10</td>
<td>400,000</td>
<td>100</td>
<td>-100$\Omega$</td>
<td>&quot; 40Kc</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>8,000</td>
<td>500</td>
<td>-1K</td>
<td>&quot; 40Kc</td>
</tr>
<tr>
<td>$R_s$</td>
<td>10</td>
<td>32,000</td>
<td>625</td>
<td>$\approx$ -100K</td>
<td>&quot; 20Kc</td>
</tr>
<tr>
<td>$\times 10^4$</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>100</td>
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<td>50</td>
<td></td>
<td></td>
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<tr>
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<td>100</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td></td>
<td>500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_s$</td>
<td>100</td>
<td></td>
<td>625</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Amplifier's which can be built are indicated by an "OK" or a "good" in the tables. Note that it may be possible to use a quartz crystal in some cases where a high Q is required. Without the use of quartz crystals, many of the amplifiers listed in Table 4.5 can be built. From this table we conclude that a low frequency, double-sideband upconverter could have the following parameters:

(1) \( R_o = 10^4 \ \Omega \)

(2) \( R_L = 100 \ \Omega \) when \( R_s = 4 \ \Omega \)

(3) \( f_p = 100 \text{ kc} \) for \( \Delta F = 2 \text{ kc} \)

- \( 1 \text{ mc} \) for \( \Delta F = 20 \text{ kc} \)
- \( 10 \text{ mc} \) for \( \Delta F = 40 \text{ kc} \).

(4) Required pump tank Q's are reasonable.

One of the best amplifiers is the first entry in Table 4.3; the specifications are:

\[
\begin{align*}
R_o & \approx 10^6 \ \Omega \\
r_p & \approx 1 \text{ mc} \\
R_L & \approx 2000 \ \Omega \\
G_c & \approx 500 \\
\Delta F & \approx 6000 \text{ cps.} \\
Q & \approx 160
\end{align*}
\]

Before the theory was completed in this thesis, an amplifier had been built which corresponded closely to the above specifications. This
Amplifier was optimized experimentally as much as possible; but $\Delta F$ could not be realized in practice. A two-diode symmetric circuit was used, and his circuit could not realize the theoretical $\Delta F$, because of a by-pass capacitor at the input. The experimental noise figure was found to be 1.6; the excess noise figure here is assumed by the author to be caused by a noise bandwidth greater than the signal bandwidth. The next step was a change from the two-diode circuit to a four-diode symmetric circuit. This latter circuit both theoretically and experimentally has a noise bandwidth which is closer to the signal bandwidth. Both of these amplifiers will be discussed in Chap. V.
CHAPTER V
TWO EXPERIMENTAL AMPLIFIERS

1 The two-diode symmetric circuit

5.11 Analysis

In Chap. I, a basic one-diode circuit was suggested for use as a low frequency, double-sideband upconverter. It was mentioned in Chap. I that, because of loading problems, this simple one-diode circuit does not result in a good upconverter. Moreover, the amplitude of pump current in \( I \), for the one-diode circuit, is much larger than either sideband current amplitude; this is so because of the small signal nature of the analysis. Here are several reasons why a relatively large pump amplitude in the output is undesirable:

1. a relatively large pump amplitude will saturate succeeding * amplifiers.*

2. the pump amplitude must eventually be filtered-out since it adds to the output noise. If this pump signal is greatly reduced before it reaches the detector, there will be less conversion noise in the detector.

3. if an envelope detector is used, and if \( E_0 \) is of the order of micro-volts, then large voltage amplification is needed in order to operate the envelope detector. But since the upconverter has a voltage gain less than one, it is likely that an additional amplifier will be required.

Note that, since we are dealing with low audio frequency, it may be desirable to amplify before detection, because then the succeeding amplifiers are high frequency amplifiers.
Therefore, we expect that, in general, the double-sideband up-converter amplifier will be of the form indicated in Fig. 5.1.

![Diagram of double-sideband up-converter amplifier]

**Fig. 5.1 General form for the double-sideband upconverter**

By using a two-diode symmetric circuit, the pump current can theoretically be balanced out at the load, \( R_L \). In practice, however, the pump current can never be completely balanced; actually this is not undesirable since we want amplitude modulation and not suppressed carrier.

There are two disadvantages in the two-diode circuit:

1. The symmetry involves subtracting two large pump currents to produce a small resultant current in \( R_L \). This resultant current depends critically on the balance, and thus on the lumped elements in each branch of the circuit. So, in practice, this resultant pump current in \( R_L \) will not have long term
stability. This means that the d.c. operating point of the detected signal will not have long term stability unless the parameters involved in the balance are stable.

(2) There is necessarily a capacitor for by-passing pump and sideband currents across the signal port of the two-diode symmetric upconverter; this prevents the maximum bandwidth, \( \omega_b \), from being realized.

A two-diode symmetric circuit where the pump drives in parallel and the signal drives in push-pull is shown in Fig. 5.2.

![Two-diode symmetric circuit diagram](image)

**Figure 5.2** Two-diode symmetric circuit; the pump drives the diodes in parallel and signal drives the diodes in push-pull.

* Good design technique will increase this stability - but this still remains a fundamental problem.
The pump current flow in this two-diode circuit is easily analyzed by considering Fig. 5.3.

Figure 5.3

For the moment, we are interested in the theoretical requirements for balancing the pump current out of the $Z_2$ branch; writing loop equations and solving for $I_1$ and $I_2$:

\[
I_1 = \frac{\begin{bmatrix} E_1 & -Z_2 \\ E_2 & (Z_1+Z_2) \end{bmatrix}}{\begin{bmatrix} Z_1+Z_2 & -Z_2 \\ -Z_2 & (Z_1+Z_2) \end{bmatrix}} = \frac{(Z_1+Z_2)E_1 + Z_2E_2}{(Z_1+Z_2)^2 - Z_2^2}
\]

\[
I_2 = \frac{\begin{bmatrix} (Z_1+Z_2) & E_2 \\ -Z_2 & E_2 \end{bmatrix}}{\begin{bmatrix} (Z_1+Z_2) & -Z_2 \\ -Z_2 & (Z_1+Z_2) \end{bmatrix}} = \frac{(Z_1+Z_2)E_2 + Z_2E_1}{(Z_1+Z_2)^2 - Z_2^2}
\]
An ideal balance is obtained \((I_1 = I_2)\) if,

1) \(E_1 = E_2\)

2) branch impedances, \(Z_1\), are identical as indicated.

then,

\[
I_1 = I_2 = \frac{E_1}{Z_1}, \quad \text{independent of } Z_2.
\]

Of special importance for the circuit of Fig. 5.2 is that the diodes are placed in opposite directions. This is necessary in order that the generated sideband currents from each diode reinforce in the load, \(R_L\).

Another form of the two-diode, symmetric circuit is shown in Fig. 5.4; in this circuit the pump drives the diodes in parallel, while the signal drives the diodes in push-pull. Note, however, that the signal source being supplied through a transformer does not allow the signal bandwidth to extend down to d.c.

![Diagram](image)

**Figure 5.4** Two-diode symmetric circuit; pump drives the diodes in parallel and signal drives the diodes in push-pull.
The circuit of Fig. 5.2 is more general than that of Fig. 5.4 since the former has bandwidth down to d.c. The following subsection deals with experimental results for the circuit of Fig. 5.2.

5.12 Experimental vs. theoretical results:

A complete low-frequency, double-sideband upconverter amplifier is shown in Fig. 5.5. The transistor amplifier shown has a voltage gain of about 4000; this is sufficient to insure envelope detection for very low inputs to the upconverter. The only experimental results which we will report for this entire system is that the noise figure is 1.6 when \( R_O = 10^6 \Omega \) and \( R_L = 2K \). It was found that these values for \( R_O \) and \( R_L \) resulted in the lowest noise figure; this is consistent with the prediction of the first entry of Table 4.3. We can not say much more about this system of Fig. 5.5 as a whole, since the theory of previous chapters assumed a synchronous detector. What we will do is to report experimental results for \( V_G \), \( G_C \), and bandwidth for the upconverter by itself (look at output terminals of \( T_2 \) with the transistor amplifier disconnected).

If the input signal amplitude is held constant, normalized voltage gain as a function of \( R_L \) can be obtained by observing the peak amplitude of the envelope at the output terminals of \( T_2 \). By placing a known variable resistance at the output of \( T_2 \), the reflected load resistance \( R_L \) is known. Equation 4.1 indicates that this resulting measurement of \( V_G \) should have its maximum value when \( R_L = \infty \) and should be reduced to one-half this maximum when \( R_L = R_s \). In practice we must correct \( R_s \) for any stray loss
T₁ and T₂: Cambion Coil Forms LS9-4T

Transistors - 2N706

batteries - mercury

Housing - all circuitry is enclosed in a µ metal box which gives about 10 skin depths shielding at 60 c.p.s.

Signal Source Antennuator - attenuates signal by 10⁻⁵ for measurement purposes.

cₑ - necessary pump by-pass capacitor.

T₁:

Pri 300T no.38
Sec 30T no.6.T

T₂:

Pri 30 turns no.38
Sec 300 turns no.38

Figure 5.5 Two-diode, symmetric circuit, double-sideband upconverter amplifier
and wiring resistance (call this wiring resistance, \( R_w \)). Also note that because of odd harmonic symmetry, the resistor \( R_L \) looks like \( 2R_L \) to either diode. This means that we can define an effective loss by \( R_s' = R_s + R_w \) and an effective load by \( R_L' = 2R_L \). The normalized V.G. will be \( 1/2 \), therefore, when

\[
R_L' = R_2
\]
or

\[
R_L = \frac{R_s + R_w}{2}, \quad \text{relationship when normalized V.G. is } 1/2. \quad (5.1)
\]

\[
R_s' = R_s + R_w, \quad R_w - \text{wiring loss.} \quad (5.2)
\]

An experimental plot of normalized V.G. vs. \( R_L \) is shown in Fig. 5.6. Note that \( R_s + R_w \leq 4 \Omega \) experimentally. This means that for \( R_s = 4 \Omega \), \( R_w = 4 \Omega \), which is reasonable. We seem to be in agreement, in this respect, between theory and experiment.

An experimental curve for normalized voltage gain vs. \( \omega_s \) is shown in Fig. 5.7. Note that the upper-frequency break-point is determined almost completely by the 100 \( \mu \)f. signal-port, by-pass capacitor. We see at once that the signal bandwidth is much smaller than the noise bandwidth.

It was at this point that we switched our interests to the four diode, double-sideband upconverter, which promises to realize the maximum theoretical signal frequency bandwidth.

\* In order to lower the noise bandwidth for the whole system of Fig. 5.5, a low pass filter was used at the output of the detector; this is certainly not a good design technique, and may account for part of the excess noise figure.
Figure 5.8: Experimental

Plot of Peak Envelope amplitude vs. $R_L$ with

\[ \omega_p = 2\pi \times 10^6 \]
\[ R_o = 10^6 \Omega \]

(input signal = constant)

Normalized Peak Envelope Amplitude

\[ \frac{R_s + R_{wiring}}{2} \leq 4 \lambda \]

$\frac{1}{2} \, 10 \, 15 \, 20 \, 25 \, 30 \, 35 \, 40 \, 45 \, 50 \, 55 \, 60 \, 65 \, 70 \rightarrow R_L \rightarrow$
Figure 5.4: Experimental Plot of Normalized voltage gain vs. $\omega_s$ with

- $R_o = 10^6 \ \Omega$
- $\omega_p = 2\pi \times 10^6$
- $c_e = 100 \ \mu F$
- $R_L \approx 2 \ \Omega$

Flat down to d.c.
5.2 The four-diode symmetric circuit

5.21 Analysis

A four-diode, low frequency double-sideband upconverter is shown in Fig. 5.8*. Because of symmetry, the analysis of this bridge circuit reduces to a single-diode analysis (i.e., if a perfect balance is obtained, one can look in at the sideband terminals or at the signal terminals and see, in effect, one diode).

Note the directions of the dots on the transformers; because of these directions, pump current is constrained to flow clockwise around the upper mesh and counterclockwise around the lower mesh. Then, if the impedances of the four branches are identical, the voltage drops across each branch are equal. This means that, at pump frequency, opposite tie points of the bridge circuit of Fig. 5.8 are at equal potentials. The result is that no pump current flows through the load, $R_L$, or through the signal branch. Unfortunately, the diodes must be placed in the circuit such that two are pumped positively at the same time two are negatively pumped (this will be explained when we consider the sideband currents). We will assume that the balance is close enough so that we can use the one-diode approximation for the circuit of Fig. 5.8. We should keep this approximation in mind when making comparisons between the single-diode theory and the experimental results for the four diode, symmetric circuit.

The relative direction of sideband current flow through a diode is determined by the phase of $S_{1}$ for that diode, and the phase of the signal

* This circuit was brought to the attention of the author by Alan Helgesson of Sylvania's Applied Research Laboratory.
Figure 5.8  Bridge, Double Sideband Upconverter
large, $Q_s$, on the diode. This is seen when we recall from Chap. I that sideband current is,

$$S_1 Q_s$$  upper sideband current

$$-S_1 Q_s^*$$  lower sideband current

Relative phases of the $S_1$'s are indicated in Fig. 5.3 by a + or - sign. These relative phases are determined by the directions of pump current through the diodes. If, as a reference, the upper left hand diode is pumped positively (use $S_1^+$ for notation), the relative pumping phases for the other diodes are as indicated. Noting then the direction of signal voltage across each diode (and thus the direction of $Q_s$), we can indicate the direction of sideband current flow in each diode. On the diagram of Fig. 5.8, directions are indicated for the upper-sideband current in each diode. Note that upper-sideband currents flow into the load branch from the two upper diodes, and these currents consistently flow from the $R_L$ branch into the two lower diodes. It should now be obvious why the transformer dots were chosen in the direction indicated.

Because of symmetry, no sideband current flows horizontally in the circuit of Fig. 5.8. Also, we note that the sideband termination, $R_L$, "sees" one diode when looking into the bridge. This, of course, assumes ideal symmetry.

A very important advantage of this four-diode symmetric circuit over the two-diode symmetric circuit is that the signal source "looks" into
the bridge and sees one diode;" and there is no by-pass capacitor
necessary as in the latter circuit. This means that the optimum signal
bandwidth, \( \omega_b \), should be achievable.

5.52 Experimental vs. theoretical results:

The experimental work to be reported here also concerns the theoretical
values listed as the first entry in Table 4.3. Other values for the lumped
elements in the circuit of Fig. 5.8 are:

\[ L_3 \] - Cambium Radio no.2060-10,
\[ 40 - 800 \text{ mh, } Q \approx 85 \text{ at } 790 \text{ kc}. \]

\[ T_3 \text{ and } T_4 \] - turns ratio 5:1, Cambion Radio core -
\[ \text{no. LS 14-6x, } 0.2 - 1.5 \text{ mc}. \]

diodes - Four, P.S.I. V-47
bias - 4 v mercury batteries in series with
\[ 20 \text{ meg. - across each diode} \]
design care - transformers must not couple the signal circuit;
i.e., the transformers should not pass near
\[ \omega_s = \Delta F. \]

Fig. 5.9 is a plot of normalized voltage-gain vs. \( \omega_g \) for \( R_L = 100 \Omega \),
and Fig. 5.10 is a plot of normalized voltage gain for \( R_L = 1000 \Omega \). Note
that the bandpass is larger for the larger \( R_L \) - indicating that \( \omega_b \) has
increased for the larger \( R_L \). This is consistent with the theory for \( \omega_b \).

The V.G. was 0.707 when \( R_L = 1000 \Omega \) and \( R_o = 10^6 \Omega \); this gives a
\[ G_c = 280. \] For the case of lower bandwidth when \( R_L = 100 \Omega, G_c \approx 2700. \]
The bridge output impedance was \( \approx 150 \, \Omega \) when operating under the above conditions. Although this is higher than expected by the one-diode theory, it can be explained by practical unbalance and losses in the bridge circuit. If an ideal balance were obtained and if there were no losses we would expect an output impedance of about \( R_b = 4 \, \Omega \).

The extended bandwidth capabilities of the four-diode circuit over the two diode circuit offer good possibilities. There is still the long term stability problem of dc operating point.
Envelope p-p amplitude
Input p-p amplitude
\( R_0 = 10^6 \ \Omega, \ \omega_p = 2\pi \times 10^6 \)
\( R_L = 100 \ \Omega \)

Flat down to d.c.

\( f_b, \text{ Experimental, } \sim 1700 \text{ cps} \)
Plot of Normalized Envelope p-p amplitude

Input p-p amplitude

\[ R_0 = 10^6 \, \Omega, \quad \omega_p = 2\pi \times 10^6 \, \Omega \]

\[ R_L = 1000 \, \Omega \]

Flat down to d.c.

\[ f_c, \text{ experimental, } \approx 2 \, \text{kc} \]
CHAPTER VI
CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

The experimental results of this chapter support the theoretical results of previous chapters. Of course, not all the theory could be verified experimentally, but some of it was. For example, the normalized V.G. curve of Fig. 5.6 indicates experimentally that $R_\theta \approx 4 \Omega$, according to theory. This justified the theoretical use of the diode model in which $R_\theta$ is in series with the depletion layer (Sect. 1.2). In Fig. 5.7 the two-diode symmetric circuit is shown to have gain down to dc (and remarkably constant, as well). This indicates that the results of Sect. 2.6 and Sect. 2.7 for the untuned signal port are correct. Then, from Fig. 5.9 and Fig. 5.10 we note the increased bandwidth of the four-diode circuit as compared with the two-diode circuit. This four-diode circuit allows us to check the validity of the concept of $\omega_b$ - signal frequency breakpoint. We see from Fig. 5.10 and Fig. 5.9 that the bandwidth is determined by $\omega_b$, and that $\omega_b$ increases as $R_L$ increases (as predicted by Eq. 2.30).

It is also reported that $F = 1.6$ for the double-sideband upconverter amplifier of Fig. 5.5. According to Eq. 3.38 and the design criteria of Table 4.3, the excess noise figure, in theory, should be $(F-1) \approx \frac{R_\theta}{R_0} \approx 10^{-4}$. 
Tables of the form of Tables 4.3, 4.4, and 4.5 can be used to design low frequency, double-sideband upconverters. For instance, the first entry of Table 4.3 indicates that a good amplifier can be built with \( \omega_p = 1 \text{ mc} \) and 6000 cps. of bandwidth if \( G_c \) is sacrificed for noise performance; this sacrifice of \( G_c \) results when \( R_L \) is made \( \gg R_g \) in order to increase \( \omega_{s,n} \) and thus improve the noise performance.

One of the major practical problems to be recognized when using Tables 4.3, 4.4, and 4.5 is that the pump tank bandwidth, \( \Delta F \), must determine both the signal and noise bandwidths. This means that \( \omega_b \) must be much larger than \( \Delta F \), and \( \omega_{s,n} \) much larger than \( \Delta F \). Many of the amplifiers designed in the tables have excellent theoretical performance, except that an impractically high pump tank \( Q \) is required to make \( \Delta F \) much less than \( \omega_b \) and \( \omega_{s,n} \). This is a direct consequence of a "high" pump frequency. Some of these required \( Q \)'s can be 100,000 and higher for a 20 kc signal bandwidth; this does suggest the possibility of using quartz crystals for filters.

In the absence of such high-\( Q \) pump tanks, we will have to use those amplifiers checked as "OK" or "good" in Tables 4.3, 4.4, and 4.5.

6.2 Future work:

Work will continue on the four-diode circuit in an attempt to obtain an amplifier which has a 20 kc bandwidth and good noise performance. The results of this thesis indicate that these specifications should be
obtained; although the low frequency cut-off may not be at d.c., due to the
long-term stability problem of symmetric circuits.
Appendix A: Experimental method for measuring noise figure

1. Introduction

Noise figure measurements were made for the entire system consisting of the up-converter, amplifier, and detector. The noise performance theory concluded in Chap. III assumed a noiseless detector. Recall that a value for the experimental noise figure was given in Chap. V as 1.6 for the two-diode symmetric circuit, with amplifier and envelope detector. The purpose of this appendix is to indicate how this noise figure was measured.

A distinction is made between "spot noise figure" which is the noise figured measured in a narrow band centered at a specific frequency, and the "average noise figure", which is the noise figure measured over the entire output range of a device.

The basic block diagram used in the noise figure measurements is shown below in Fig. 1.

![Block Diagram]

Figure 1
The signal generator is,

1) A flat, white noise generator if noise figure measurements over a "large" bandwidth are desired.

2) A sinusoidal wave generator if spot noise figure is to be measured.

The methods for measuring either overall noise figure or spot noise figure are similar. First we define the output noise power from the system under test as $Q$, when no external power is applied at the system input.

We define $Q_2$ as the output power when generator power is applied at the input to the system. Also, if $P_o$ is the actual thermal noise power theoretically expected from the source resistor $R_o$, we define $n$ as

$$n = \frac{\text{generator power supplied at the system input}}{P_o}$$

If we define $W_o$ as the output noise power which would result if the system were noise free, then

$$F = \frac{Q_1}{W_o}$$

Note now that $(n-1)$ is the "excess power" supplied by the generator. Therefore

$$Q_2 = FW_o + (n-1)W_o$$

From these two equations it follows that

$$F = \frac{(n-1)Q_2}{Q_2-Q_1}$$
The experimental method used in this thesis was to supply generator power until the output power was $Q_2 = 2Q_1$ (this is a usual method). It is only necessary, then, to record $n$ and calculate $F$:

$$F = (n-1), \quad \text{if } Q_2 = 2Q_1$$

(4)

2. Power measuring meter:

A Ballentine Model 310-A voltmeter was used as the power measuring device. This meter actually responds to the average magnitude of its input signal. It is calibrated in r.m.s. for sinusoidal inputs, so it can be used directly to determine the power supplied to a system from a sinusoidal generator. Furthermore, because there is a simple relationship between the average magnitude of gaussian noise and the r.m.s. of gaussian noise, this meter can be used for measuring the r.m.s. value of band-limited, gaussian noise.

The conversion factor involved is derived as follows: first, note that the relationship between the Ballentine r.m.s. meter reading and the average magnitude of a sinusoidal input is

$$\left( \frac{\text{Average Magnitude of Sinusoid}}{\text{R.M.S. Meter Reading}} \right) = 0.90$$

(5)

second, the relationship between the r.m.s. value ($\sigma_x$) of a gaussian random variable and the average magnitude of the variable, $|\bar{x}|$, is

$$|\bar{x}| = \sigma_x \sqrt{\frac{2}{\pi}}$$

(6)

Using these last two equations together we get
\[ \mu_p \Delta = \begin{bmatrix} \text{Predicted} \\ \text{Meter} \\ \text{Reading} \end{bmatrix} = 0.886 \frac{1}{\sigma_\text{x}}, \text{ for gaussian noise} \quad (7) \]

The conversion factor, given by Eq. 7, is used when \( Q_1 \) is measured, since \( Q_1 \) is assumed to be thermal noise (with a gaussian distribution).

Constraining the noise bandwidth:

It is necessary to know the system bandwidth accurately to calculate \( P_0 \). To constrain this bandwidth, a simple low-pass filter was inserted between the system being measured and the power measuring meter. One reason why such a low-pass filter was chosen is that the equivalent noise bandwidth of such a filter can be easily determined from the filter's breakpoint. Consider the low-pass filter shown below, with a breakpoint at \( \frac{1}{RC} \).

The thermal noise bandwidth is defined as

\[ B = \frac{1}{\gamma_0^2} \int_0^{\infty} |Y(\omega)|^2 d\omega \quad (8) \]
where \( Y_0 \) is the maximum transmittance as indicated on the previous page.

For the simple (one pole) low-pass filter, \( Y(\omega) = \frac{1}{RC(\omega + \omega_r)} \), \( \omega_r = \frac{1}{RC} \)

Thus,

\[
B = \omega_r^2 \int_0^\infty \frac{1}{\omega^2 + \omega_r^2} \, d\omega = \omega_r^2 \left[ \frac{1}{\omega_r} \tan^{-1} \frac{\omega}{\omega_r} \right]_0^\infty = \frac{\pi}{2} \omega_r
\]

\[= 1.57 \omega_r. \quad (1.9)\]

We now have everything necessary to measure the noise figure as long as \( \omega_r \) is kept well below the actual bandwidth of the amplifier. This restriction is necessary in order that \( 1.57 \omega_r \) be the noise bandwidth used in calculating \( P_o \).

If a sinusoidal generator is used, it is easier to work with r.m.s. voltages instead of powers. To see this, refer to the sketch below where \( e_{rms} \) is the r.m.s. noise voltage in series with \( R_o \) and \( E_{rms} \) is the r.m.s. value of the generator voltage (it is reasonable to assume in this case that the generator impedance is negligible compared with \( R_o \).
A simple manipulation of Eq. 4 gives,

\[ F - 1 = n = \frac{E^2}{e_{\text{rms}}} = \frac{E^2}{e_{\text{rms}}} \cdot \frac{1}{4kT R_0 B} \]

where \( kT \approx 10^{-21} \) at room temperature (290°C).

4. Summary:

The method used is as follows:

1) Constrain the bandwidth by inserting a filter with a known noise bandwidth. In the case of a low-frequency amplifier (which includes dc), a low-pass filter is appropriate.

2) Record the value of \( Q_1 \), noting that a correction must be made if an average magnitude reading meter is used (correction given by Eq. 5).

3) Calculate \( n \) from a knowledge of \( R_0 \), \( b \), and \( e_{\text{rms}} \) (if a sinusoidal generator is used).

4) Compute the excess noise figure from Eq. 10.

Slight error was introduced in the experimental noise figure measurement made in this thesis since the Ballentine Model 310-A has a low frequency cut-off at about 10 cps. For the noise figure measurement indicated in Chap.V, the error is negligible on the basis that \( B \geq 700 \) cps for all measurements. On the other hand, \( 1/F \)-noise has a power density spectrum which increases as the frequency decreases. Nevertheless, a noise figure of 1.6 would seem to indicate that there is little, if any, such noise arising in the double-sideband upconverter.
Appendix B: An investigation of oscillations in the signal circuit of the two-diode symmetric circuit

1. Introduction:

Because of the negative resistance in the signal port, oscillations may take place, unless special case is exercised when designing the up-converter. We will find that the necessary requirements to prevent signal port oscillations are:

a) keep \( R_o \gg Z_{\omega_s} \)

b) keep \( \left| \frac{S_o}{j\omega_s} \right| \gg \frac{|S_1|^2}{(R_s + R_L) \omega_1 \omega_4} = |Z_{\omega_s}| \)

this is true if \( \Delta F \) is much larger than \( \omega_0 \).

2. Oscillation at a natural singularity

For this analysis, consider the equivalent circuit representing the signal port shown below:
The problem here is identical to that covered previously at the lower-sideband port. Therefore, we break the equivalent circuit at the point marked "x" and determine the natural frequency at which there is a zero.

The impedance seen at "x" is:

\[ L_j\omega + \frac{S_0}{j\omega} - \frac{S_1^2}{(R_s + R_L)\omega_1} + \frac{1}{C\left[j\omega + \frac{1}{R_C}\right]} + R_s = Z_x \]

or,

\[ j\left[\omega_L - \frac{S_0}{\omega_s} - \frac{\omega_s^2 + (\frac{1}{R_0})^2}{\omega_s^2 + (\frac{1}{R_0})^2}\right] + \frac{1}{R_0C^2}\left[\omega_s^2 + (\frac{1}{R_0})^2\right] - \frac{S_1^2}{(R_s + R_L)\omega_1} \]

Clearly, the first term can equal zero only when \( \omega_s \gg \omega_p \). On the other hand, the last term can be zero only when \( \omega_s \) is within the pass band, \( \Delta F \).

Thus, there are no natural zeroes at which the input port can oscillate.

3. Relaxation oscillations in the signal port:

Relaxation oscillations could take place if a capacitor in the signal circuit "sees" a current controlled negative-resistance or if an inductor "sees" a voltage controlled negative-resistance.

There are two capacitors in the signal circuit of the two-diode, double-sideband upconverter. The two resulting circuits for investigation are shown below.

![Diagrams](a) and (b)

Note that in the above circuits, the inductor is assumed a short circuit at \( \omega_s \), the frequency of interest. This inductor referred to is the \( L_1 \) of Fig. 5.5.
For circuit - "a", the net real part of the impedance "seen" by $C_e$ is:

$$R_o \left| \frac{1}{2} R_e \left[ \frac{S_o}{j\omega_s} - \frac{|S_1|^2}{(R_L + R_L)\omega_s \omega_1} \right] \right|$$

$$= R_o \left| R_a, \text{ where } R_a = \frac{1}{2} R_e \left[ \frac{S_o}{j\omega_s} - \frac{|S_1|^2}{(R_s + R_L)\omega_s \omega_1} \right] \right|$$

There will be no oscillations if $R_o \parallel R_a$ is positive - or $R_a$ positive; clearly, condition (b) listed in Sect. 1 of this appendix must not be violated:

$$\left| \frac{S_o}{j\omega_s} \right| \gg \left| Z_{\omega_s} \right|,$$  \hspace{1cm} (1)

For circuit - "b", the real impedance seen by $S_o$ is:

$$Z_{\omega_s} + R_o \parallel \left[ Z_{\omega_s} + \frac{S_o}{j\omega_s} \right]$$

This impedance is positive if Eq. 1 above is not violated and if

$$R_o \gg \left| Z_{\omega_s} \right|$$  \hspace{1cm} (2)

Note that the requirements set forth in Eq. 1 and Eq. 2 are already fulfilled according to the design technique of Chap. IV. So, we do not expect oscillation to arise in the signal port.
BIBLIOGRAPHY


