The Influence of Martensitic Transformation on the Formability of 304L Stainless Steel Sheet

by

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Abstract

Sheet metal forming is a process used to manufacture many commercial and industrial products. Much research has gone into the development of materials which have excellent sheet formability in conjunction with high strength. One such material, TRansformation Induced Plasticity (TRIP) steel, utilizes the effects of the strain-induced martensitic transformation to produce both high strength and high ductility. By increasing the hardening rate of the TRIP steel through the evolution of martensite during forming, stability can be achieved during the necking process, increasing the critical strain to failure. Theory and experiment both indicate that a stress state with a higher degree of triaxiality will increase the volume fraction of martensite evolved per unit equivalent strain. The purpose of this work is to examine the influence of the martensitic transformation on the formability of 304L Stainless Steel sheet for uniaxial tension and plane strain tension, two deformation modes common in sheet metal forming operations. By comparing the hardening rates, martensite evolution, and critical strain to failure in uniaxial tension and plane strain tension, the stress state dependence of the evolution of martensite can be addressed. Through mechanical testing of the material at different temperatures and stress states, the material stress-strain behavior was obtained. The Stringfellow et al. [65] constitutive model for three-dimensional plastic deformation of transforming materials together with nonlinear finite element analysis was used to optimize the geometry of the plane-strain state specimens. Vibrating Sample Magnetometry was used to measure the percent volume fraction of martensite in the samples. It was found that a lower volume fraction of martensite evolved per unit equivalent strain in plane strain tension than in uniaxial tension, despite the higher value of triaxiality in plane strain tension. This result lead to a lower rate of hardening in the plane strain tension case, and little deviation from the expected forming limit behavior. This result can be attributed to strain state dependent microstructural development, in which fewer shear band intersections are created in plane strain tension than in uniaxial tension due to geometric constraints.

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Chapter 1

Introduction

Many manufacturing processes involve sheet metal forming operations; from the hood of an automobile to an aluminum soda can, sheet forming is used in the production of many consumer and industrial products. In order to minimize the weight or cost of the formed sheet part, the original sheet thickness can be reduced. However, a stronger material would then be required for the part to have its original structural load carrying capability. Unfortunately, for most sheet metals, there is usually an inverse relationship between the strength of a metal versus its ability to maintain stable plastic flow [50]. Mechanisms that increase the strength of a material, such as precipitated hard particles, usually do so by interfering with slip mechanisms in the parent phase. This decreases the amount of strain to failure, causing a decrease in toughness and increasing the chance for fracture around the hardened particles.

In order to avoid localization of plastic flow during forming, the critical strain to failure must be known for the material in question. The Forming Limit Diagram (FLD), developed by Keeler and Goodwin [21] in the late 1960's, showed that the critical strain to failure is a function of strain state. In general, it was found that the FLD of a material shows the critical strain to failure to be lowest in a state of plane strain, where $\epsilon_1 > 0$, $\epsilon_2 = 0$. The FLD has been a valuable tool in analyzing the trade off in material properties between the strength versus the critical strain to failure in a complicated forming process. However, it is generally desirable for a structural material to be both strong and ductile, and a material which exhibits both
of these qualities would have many practical applications.

One way to achieve this combination of properties is with a high-strength, high strain-hardening material. Considère suggested that in a simple, uniaxial tension test, the loss of stability during tensile plastic loading occurs when the point of maximum tensile load is reached [5]. At maximum load, \( dF = d(\sigma A) = 0 \), so \( \sigma A + A d\sigma = 0 \), or \( d\sigma /(-dA/A) = \sigma \). If elastic and plastic volume changes are neglected, then \( A \cdot l \) is constant, and \( l dA + A dl = 0 \), or \( -dA/A = dl/l = de \). If we define the strain hardening rate of the material, \( h \), as the slope of the stress-strain curve; i.e., \( h = d\sigma/de \), one can show that the minimum strain hardening rate necessary to maintain stability during plastic flow is given by \( h = \sigma \). If the strain hardening rate, \( h \), becomes lower than the flow stress, \( \sigma \), instability and localization give rise to a neck which results in failure of the specimen. (Figure 1-1). For most materials, \( h \) is found to decrease with increasing plastic strain. However, for certain materials the strain hardening rate, \( h \), can evolve such that it first decreases with strain and leads to a neck, but then proceeds to increase with further straining and climbs above \( \sigma \) (Figure 1-2), stabilizing the plastic flow in the neck; a second instability occurs in the necked region at a larger strain, causing final failure. Typically, the amount of plastic deformation for which \( h \geq \sigma \) will be greater than that of a material with a high yield strength and a low strain hardening rate. Locally, the advantage of a material with a high strain hardening rate is that, when it begins to neck due to an imperfection in the sheet, the material in the neck will become locally harder than the surrounding material. If the strain hardening rate is greater than the flow stress, the strain will be distributed, and the neck will propagate away from the localization instead of causing further local reduction in thickness and failure.

The extremely high strain hardening rate during plastic deformation pictured in Figure 1-2 was caused by the martensitic transformation in a metastable austenitic steel. Under the proper thermodynamic driving forces, the austenitic crystal structure of the steel undergoes a transformation to a much stronger martensitic phase. This martensitic transformation has been regularly used to harden tools through quenching and tempering for approximately 3000 years [36], but it was not until the
mid 1950's that the dependence of martensitic transformations on applied stresses were studied [1]. In the late 1960's, as the FLD was emerging in the press shop, Zackay and his colleagues [71] documented the uncommon increase in ductility in high strength austenitic stainless steels due to martensitic transformations. Figure 1-3 compares data for these “TRIP” steels in comparison with other high-strength sheet steels, demonstrating the favorable combination of both high yield strength and high elongation to failure. The acronym “TRIP”, formed from the letters of the words “TRansformation Induced Plasticity”, was given to this class of steels, describing the ability of the martensitic transformation to stabilize plastic flow and increase ductility.

Recently, the stress-state dependence of the martensite transformation in steels has been studied [70] [60] in order to understand high fracture toughness values reported for metastable alloys [34]. Comparing the martensite evolution during deformation in different stress-states, it was found that more martensite formed in tests with a higher “triaxiality”, where triaxiality is a measure of the size of the hydrostatic component of stress relative to its deviatoric component. This interest led to the formulation of a constitutive model by Stringfellow, et al., [63] for strain-induced martensitic transformation which incorporated temperature, stress-state, and strain-rate effects on the phenomenon. The model was based on the work of Olson and Cohen [47], in which the kinetics of the transformation process is assumed to be driven by nucleation of martensite embryos at slip plane intersections. The work of Stringfellow not only examined the transformation phenomenon in the stress fields of a crack tip, but also in the increasing triaxial stresses in the growing neck of a uniaxial tension specimen. The model was able to show the dramatic increase in ductility due to the increase in strain hardening rate from the martensitic transformation in the necked region.

However, all of the data with which the above model was correlated was either uniaxial tension or compression. In order to apply the Stringfellow model to sheet forming applications, it must correctly predict the response of a transforming steel in plane-strain and biaxial tension as well. The predictions offered by the Stringfellow
model suggested that, due to a higher value of triaxiality in plane strain tension than uniaxial tension, a higher hardening rate due to the presumably increased martensite transformation would occur during plane strain tension. This result would lead to an increase in the strain to failure in the plane strain tension case, significantly affecting the FLD for a transforming material. Experimental data for sheet-stretch tests in a variety of strain states [27] were available, but these tests did not provide stress-strain data needed to determine hardening rates in their analyses. A study comparing torsion and compression of 304L stainless steel was recently conducted by Miller [40], but the initial analysis of this work pointed to less martensite being formed in torsion than in compression, even though torsion has a higher value of triaxiality than compression. Miller tried to account for this discrepancy by examining the differences in microstructure between the torsion and compression samples. Miller argued that, since there is a profusion of planes of maximum shear stress in compression as compared to torsion, a constraint of slip is present in the plane-strain state of torsion. This constraint leads to lower values of strain hardening in non-transforming materials, and less martensite evolution in transforming materials as well due to the reduced numbers of slip band intersections. However, Miller's martensite measurements were incomplete, and no analysis of how the crystallographic texture may have been affected by the constrained slip condition was conducted.

The work of this thesis focuses on the effect of the state of plane strain tension versus that of uniaxial tension on the martensitic transformation due to plastic strain and its corresponding influence on the hardening rate of 304L stainless steel. By using a plane strain tension sheet specimen, we hope to determine whether the evolution of martensite is a stronger function of the triaxiality, or of the active slip systems available for nucleation of martensitic embryos. In this way, we can discover whether the state of strain, as well as the state of stress, must be accounted for in a constitutive model for martensite evolution.

First, some background is given on forming limit theory, martensite transformation kinetics, and polycrystalline texture evolution, with a motivational look at recent work on triple-phase steels.
Next, the constitutive model developed by Stringfellow et al.[63] for the isothermal, large strain deformation of multiphase materials is reviewed. This section will also discuss the self-consistent method of strain apportionment and the inclusion of the influence of triaxiality on the behavior of metastable materials undergoing transformation.

The next chapter describes the experimental methods used in this Thesis, including uniaxial and plane strain tension testing, Vibrating Sample Magnetometry to determine the evolution of volume fraction of martensite with strain, and X-Ray Diffraction to examine crystallographic differences generated by differing strain-state paths.

Next, the plane strain tension test is analyzed using Finite Element analysis. The predicted strain distribution across the specimen is correlated with the experimental distributions obtained, and the specimen stress-strain curve versus the true plane strain stress-strain responses are compared.

Finally, the Stringfellow model is used in Finite Element Modeling simulations of the experimental methods, and the results are correlated with the experimental data.

It is hoped that, with a more complete understanding of the influences of stress and strain state on the martensitic transformation, future sheet alloys can be designed to take advantage of its beneficial mechanical effects.
Figure 1-1: Flow stress, $\sigma$, and strain hardening rate, $h$, for a power-law hardening material [63].
Figure 1-2: Flow stress, $\sigma$, and strain hardening rate, $h$, for a typically transforming material [63].
Figure 1-3: The ranges of the 0.2% yield strength and the total elongation at failure (from room temperature tensile tests) that are characteristic of various classes of high-strength steels. The shaded areas represent scatter of data [71].
Chapter 2

Background

2.1 Forming Limits

Forming limits of sheet metal for in-plane strains are critical criteria in the sheet forming industry. The practical identification of these limiting strains was first developed by S. Keeler and G. Goodwin [21] in the form of a Forming Limit Diagram (FLD). The FLD is an \((\epsilon_1, \epsilon_2)\)-locus of failure strains for a given sheet material subjected to in-plane strains \(\epsilon_1 > 0\), and \(-a < \epsilon_2/\epsilon_1 < 1\), where \(-a\) is a strain ratio such that buckling does not occur. Usually, the testing is performed by proportional straining; \(\epsilon_2/\epsilon_1 = \dot{\epsilon}_2/\dot{\epsilon}_1 = \text{constant}\). Figure 2-1 depicts the first full FLD as a combination of Keeler and Goodwin data. As is generally found, this diagram shows a minimum critical strain under a state of plane strain \((\epsilon_2 = 0)\), with the critical strain increasing in both the biaxial extension and extension/contraction quadrants.

Because developing a full experimental FLD is non-trivial, much effort has been devoted to the prediction of these diagrams from basic constitutive equations for plastic flow. As the forming limit is reached, homogeneous plastic strain localizes into bands of extreme deformation, resulting in failure. Therefore, most models are based on the amount of homogeneity and stability in the straining material. Hill's criterion for localized necking along a direction of zero-extension is \(\epsilon_1^* + \epsilon_2^* = n\), where \(n\) is the power law hardening exponent \((\bar{\sigma} = K(\bar{\varepsilon})^n)\) [28]. A direction of zero-extension does not exist in the biaxial extension quadrant for materials with a smooth
yield surface if one enforces normality flow; therefore, this model cannot predict the complete FLD. However, if the notion of a yield surface vertex is considered [61], [31], this analysis can be extended to the biaxial extension region, predicting a behavior similar to experimental FLDs.

In order to account for the influence of nonhomogeneity of geometry and material behavior on the FLD, Marciniak and Kuczynski (MK) [37] introduced a linear material imperfection in the plane of the sheet. The strain ratio in this region of imperfection progresses from its initial value to zero during localization and failure. The rate of this transition is dictated by the local curvature of the yield surface at the current stress point; therefore the predictive ability of this model is also dependent on the correct assumption of a particular yield surface and hardening theory. Other modifications to this theory include accounting for evolution of internal porosity, strain-rate sensitivity, and kinematic hardening [56].

Using the MK analysis and a modified yield surface, an FLD can be predicted which is qualitatively correct for most materials, showing the lowest critical strain at plane strain. However, studies by Azrin and Backofen [4] (in tension-tension only) show that while some materials such as aluminum, aluminum-killed steel, and 430 (ferritic) stainless steel do follow the trend of a decrease in limit strain as $e_2/e_1$ goes to zero, materials such as brass are unaffected by strain ratio, and 301 Stainless Steel actually showed an increase in limit strain as strain ratio came close to plane strain (Figure 2-2). Reasons suggested for these deviations from typical FLD are differences in stacking fault energies [43] and influences of crystallographic texture on yield surfaces [32]. In high stacking fault energy materials, dislocations have more freedom to cross-slip, resulting in less work hardening, and lower ductility. As stacking fault energy decreases, partial dislocations are more widely separated, making cross-slip more difficult. As a result, more strain hardening will develop, allowing for stabilization of any localization, and generally increasing the total ductility [43]. Although this is also true for the stainless steel, a possibly more dominant phenomenon – transformation to martensite – is occurring in this composition of stainless, which may also help to explain its atypical behavior.
An FLD for 304 SS, a composition of stainless with slightly less Cr and Ni than 301 SS, (Figure 2-3) from Hecker [25], displays a combination of the two behaviors discussed above. A distinct maximum in the limit strain occurs in the biaxial-stretching portion. It was proposed by Hecker et al. [27] that the reason for the downturn in the limit strain after the maximum is that more martensite is formed in biaxial tension per unit principal surface strain than in uniaxial tension. The initial rapid transformation in the biaxial state cannot be sustained to large strains, and premature local plastic instability results. In the case of the 301 SS in the study by Azrin and Backofen [4], this austenite composition is even more unstable, causing transformation to occur even earlier during the strain history. Therefore, the constant downturn throughout the biaxial-stretching portion of the 301 SS FLD could be understood as the local maximum from the 304 SS FLD shifting to the left and downward, until it coincides with the plane-strain limiting strain. In order to understand better how the martensite affects formability, the next section will discuss in detail the martensitic transformation and the importance of the strain-induced regime.

2.2 Martensitic Transformation Evolution

Martensitic transformations are defined as any distortive, diffusionless, structural change in a crystalline material [12]. These types of transformations can be found in a variety of materials, and are responsible for the unique behavior of "shape memory alloys" [51]. In iron-based alloys, the specific phase transformation is that from a metastable γ-austenite with an fcc structure, to a α'-martensite, a much harder phase with a bcc, bct or hcp structure [33]. At high temperatures, the γ-phase is in stable equilibrium. As temperature decreases, the γ-phase becomes metastable and can transform to α'-martensite if additional energy in the form of mechanical work is used to overcome the free energy barrier. At very low temperatures, the transformation is spontaneous due to favorable thermodynamic driving forces.

Figure 2-4 shows a schematic representation of the conditions for transformation. Below a martensite start temperature, $M_s$, the transformation is spontaneous, form-
ing wide, zig-zagging plates indicative of an autocatalytic nucleation process [57]. Just above the $M_s$ temperature, martensite will nucleate at high energy sites in the parent austenite if the applied stresses provide the additional driving force required for transformation. The martensite formed in this manner is said to be “stress-assisted”, and has the same nucleation and growth process that results in the spontaneous formation of normal martensite during the cooling of unstressed and unstrained austenite [39]. As the temperature increases, the amount of stress required to assist transformation increases linearly until it reaches the yield stress [8]. The temperature at which the yield stress provides the needed driving force for transformation is the $M_s^\sigma$ temperature. Above $M_s^\sigma$, the stress required for transformation exceeds the yield stress, causing plastic deformation through the motion of dislocations, or crystallographic slip. This deformation creates new nucleation sites in the parent austenite [45]. These embryos for nucleation take the form of shear band intersections such as stacking faults, twins, and ε-martensite [35], [41]. The martensite formed at these new sites is referred to as “strain-induced” [39]. This strain-induced transformation can occur between $M_s^\sigma$ and $M_d$, where $M_d$ is the temperature at which the failure strain of the material is not high enough to induce any significant transformation.

Although it is of the same crystal structure as stress-assisted martensite, the strain-induced martensite is more finely distributed throughout the parent austenite, and takes the form of thin laths instead of plates [45]. Below the $M_s^\sigma$ temperature, the martensite which transforms is very coarse, resulting in a brittle product. Above $M_s^\sigma$, where transformation is dominated by strain-induced evolution, the product of the transformation yields tough, high strength materials [71] with remarkable increases in tensile elongation [9]. It is this strain-induced martensitic transformation which can affect the FLD of a metastable material, as mentioned in Chapter 1.

“TRIP” steels utilize the strain-induced transformation regime to stabilize plastic flow and increase ductility. Steels in this class are low-carbon metastable austenitic stainless steels, with a high chromium and nickel content [71]. Studies by Angel [1] had shown previously that the stability of these steels during deformation was dependent on temperature and composition. As discussed in the previous section, as the
temperature decreases from the \( M_d \) temperature, the material becomes more unstable due to the decreasing size of the free energy barrier to transformation. Also, Angel found that an increase in the C, N, Ni, Cr, Si, Mn, and Mo contents each tend to stabilize the austenite by decreasing the \( M_d \) temperature. What was not investigated by Angel was the influence of stress-state on the stability of the steels.

According to both theory and experiment [49], there is a shear and a dilatational component to the transformation strain. Therefore, a state of hydrostatic tension should provide a driving force for transformation, reducing the stability and increasing \( M_d \). Hecker et al. conducted an extensive study in the early 1980's to experimentally determine the effects of strain state on transformation in a 304 stainless steel [27], implementing the experimental methods of determining FLDs to achieve the various strain paths. Results from that study reported a slight increase in martensite transformation versus equivalent plastic strain for biaxial tension tests over uniaxial tension tests (Figure 2-5). At the same time, compression testing of a metastable phosphocarbide-strengthened austenitic steel yielded significantly lower martensite transformation versus equivalent plastic strain than uniaxial tension [70] (Figure 2-6). From these findings, a constitutive model was developed by Stringfellow et al. [63] which incorporated the influence of the triaxiality of the stress state with the Olson-Cohen martensite evolution kinetics. In the Stringfellow model, the triaxiality of the stress state would cause an increase in the volume fraction of martensite transforming during deformation, i.e., that more martensite would transform in a biaxial tensile stress state than in a uniaxial tensile stress state, and more in a uniaxial state than in compression. Torsion (simple shear) would produce a state of plane strain, and a triaxiality lying between those of uniaxial tension and compression.

However, actual torsion experiments of Powell et al. [52] produced a martensite evolution curve similar to compression tests. When plots of the equivalent stress versus equivalent strain are examined for the different strain-state tests [40], [26] (Figures 2-7, 2-8), the plane-strain torsion test falls below the uniaxial tension test, indicating that the hardening rate in plane-strain deformation is lower than in simple tension. This result demonstrates a deviation from isotropic hardening laws used to
describe plastic behavior. The phenomenon of "plane-strain softening" at large strains has been found to occur for other fcc materials such as brass and aluminum [68] [69] [19]. The underlying reason for this decrease in strain hardening may be affecting the martensite evolution.

Recent developments in polycrystalline large strain plasticity models along with experimental evidence suggest that strain hardening characteristics can be influenced by the evolution of crystallographic texture in fcc metals undergoing large strain deformation, and that the texture evolution is dependent on strain state [3], [10]. In the next section, some background on how strain state can influence the crystallographic texture is given.

2.3 Crystallographic Texture and Strain State

In a polycrystalline material, individual grains with varying crystallographic orientation are separated by grain boundaries. The extent to which the individual grains share a common orientation or set of orientations is described as crystallographic "texture". Metals at large strains will develop a preferred orientation or texture which is dependent on the mode of deformation [20]. Due to the crystallographic nature of slip, one expects that, at large deformations, differences in hardening may arise from differing deformation modes, since the mean inclination of the active slip planes and directions will vary differently with different states of strain. Much work has been devoted to the prediction of this geometric effect, going back to the work of Taylor [66] in 1938. A brief review can be found in Asaro and Needleman [3].

For a randomly oriented fcc polycrystal one can relate the macroscopic tensile flow stress, $\sigma$, to the critically resolved shear stress for slip, $\tau_c$, and the macroscopic tensile equivalent plastic strain, $\epsilon$, to the accumulated shear strain on all activated slip systems, $\gamma_c$, through the average Taylor factor, $\tilde{M}$, as $\sigma = \tilde{M}\tau_c$ and $d\epsilon = \sum d\gamma_c/\tilde{M}$. The Taylor factor was shown to vary with deformation mode by Bishop and Hill [7]. They found that for tension $\tilde{M}_t$ is 3.06. In the case of torsion, $\tau = \tau_c \cdot \tilde{M}_t$ and $d\gamma = d\gamma_c/\tilde{M}_t$, where $\tilde{M}_t$ was found to be 1.65. Expressing these equations as effective
stress and strain (equivalent to tension) gives

$$\sigma^{eff} = \bar{M}_t \cdot \tau_c = (\bar{M}_t/\bar{M}_r) \cdot \tau$$ (2.1)

$$d\varepsilon^{eff} = d\gamma_c/\bar{M}_t = (\bar{M}_r/\bar{M}_t) \cdot d\gamma.$$ (2.2)

For the Bishop and Hill values of $\bar{M}_t$ and $\bar{M}_r$, these relations become $\sigma^{eff} = 1.85\tau$ and $d\varepsilon^{eff} = d\gamma/1.85$, compared with the von Mises relations $\sigma^{vm} = \sqrt{3}\tau$ and $d\varepsilon^{vm} = d\gamma/\sqrt{3}$. This predicts an effective stress approximately 7% higher than that of von Mises. As strains increase, the Taylor factor increases in tension, but decreases in shear, predicting an effective stress 28% higher than that of von Mises at a true strain of 2.0 [20]. Work on shear band formation in fcc metals by Dillamore et al. [16] can explain this discrepancy as a "geometrical softening" due to textures in which smaller amounts of cumulative shears on the grain slip systems are required to achieve a given strain. Since $\bar{M}_t$ changes with strain, this idea provides an important link between deformation-induced anisotropic material behavior caused by texture and observed strain-path dependence of metal strain hardening. As the Olson-Cohen model for martensite evolution is tied to the amount of slip band intersections, a reduced amount of slip for a given strain would also correspond to a decrease in the amount of martensite evolved.

## 2.4 Recent Work

### 2.4.1 Triple-Phase Steels

Dual-phase steels, produced by intercritical annealing, consist of a ferrite parent matrix with interspersed martensite for strengthening [48]. These steels are used extensively in the automotive industry, but further increases in strength and ductility are needed to extend their application. Alloying with Si and isothermal holding at bainite transformation temperatures has been found to cause any intercritically annealed austenite to become retained in a bainite form [38] [6], creating a "triple-phase" steel.
The retained austenite is expected to transform to martensite during forming of the sheet, causing a related increase in strain hardening rate which would defer necking instabilities. However, these expectations are hampered by the variability of the stability of the austenite formed in the bainite structure due to alloying, processing, and temperature of deformation [59].

Recently, at the Colorado School of Mines, an effort has been made to characterize the influence of isothermal holding on bainite formation and mechanical properties of dual-phase steels. Their studies show that increased amounts of retained austenite in the microstructure do indeed enhance ductility due to transformation of the austenite to martensite during plastic strain [54] [55]. However, all of the above testing was in uniaxial tension. The work of this thesis should indicate that, since stress state and strain state can affect the evolution of martensite, the actual ductility of intercritically annealed, isothermally held dual-phase steels during forming operations could be different from that predicted solely by uniaxial tension tests.
Figure 2-1: The first published, full Forming Limit Diagram (FLD) showing the combined data of Goodwin and Keeler [21].
Figure 2-2: Experimental FLD of a 301 stainless steel showing decreasing major strain, $\varepsilon_1$, with increasing strain ratio, $\varepsilon_2/\varepsilon_1$ [4].
Figure 2-3: FLD for 304 stainless steel, showing downturn in limiting strain in the biaxial tension region [25].
Figure 2-4: Schematic representation of the interrelationships between stress assisted (below $M_s^o$) and strain-induced (above $M_s^o$) martensitic transformation [46].
Figure 2-5: Comparison of actual volume fraction of martensite formed at room temperature during uniaxial and balanced biaxial tension [27].
Figure 2-6: Comparison of model predictions of Stringfellow [63] for volume fraction martensite, $f$, vs equivalent plastic shear strain, $\gamma^p$ with experimental data from Young for a metastable austenitic steel [70].
Figure 2-7: Uniaxial Compression and Hollow-Bar Torsion of 304L stainless steel at room temperature [40].
Figure 2-8: Equivalent stress versus equivalent strain for compression and torsion experiments conducted on OFHC copper [26].
Chapter 3

Constitutive Modeling

3.1 Introduction

In order to investigate the ability of current theory to predict the influence of temperature, stress-state, and strain-state on the behavior of metastable, austenitic steels, the implementation of a constitutive model developed by Stringfellow, et. al. [65] was used in a finite element program suitable for analysis of boundary value problems. This model is based on the Olson-Cohen [47] model for strain-induced martensitic transformation kinetics, in which the stress-state sensitivity of the transformation process has been incorporated. In the Stringfellow model, the volume fraction of martensite is a functional of the temperature, plastic strain, and the local stress state histories.

To determine the stress-strain properties of the evolving two-phase composite, the Stringfellow model uses a "self-consistent method" to partition the strains in the two phases. This is an improvement from the model developed by Narutani, Olson and Cohen [42], which assumed that the strains in the two phases are equal using a Voigt method. The Narutani model required a correction accounting for the transformation strain indirectly, and was prone to developing numerical difficulties when used in modeling of boundary value problems [65].

An isotropic, viscoplastic, self-consistent model is used to determine the plastic strain rate in each phase as a function of the total inelastic strain rate and volume
fraction of each phase. In the section below, the mathematical formulation of the constitutive model and its implementation into ABAQUS via the UMAT subroutine will be developed. Further details of the constitutive development can be found elsewhere [65, 64].

3.2 Preliminaries

Before discussing the constitutive model, some notation conventions are introduced.

3.2.1 Notation

The tensor notation of Gurtin [23] is adopted for describing the three-dimensional constitutive relationships of this model. Scalars are written in italic type (e.g., \(a, \sigma\)). Vectors are written in boldface lowercase Roman (e.g., \(e, t\)) and second order tensors are written in boldface uppercase Roman and boldface lowercase Greek (e.g. \(T, \sigma\)). Fourth order tensors are written as capitalized calligraphic letters (e.g., \(I, \mathcal{L}\)). Given an orthonormal vector basis, \(e_i\), first, second, and fourth order tensors can be defined as follows:

\[
a = a_i e_i
\]  

(3.1)

\[
A = A_{ij} e_i \otimes e_j,
\]

(3.2)

where repeated indices indicate summation, and

\[
\mathcal{C} = C_{ijkl} e_i \otimes e_j \otimes e_k \otimes e_l,
\]

(3.3)

where the tensor product \(a \otimes b\) of two vectors \(a\) and \(b\) is a second order tensor, defined such that for any vector \(v\),

\[
(a \otimes b)v = (b \cdot v)a,
\]

(3.4)

where the dot product of vectors \(b\) and \(v\), \(b \cdot v\), is defined to equal \(b_i v_i\). Second order tensors are linear maps which assign vectors to vectors. Similarly, fourth order tensors are linear maps which assign second order tensors to second order tensors,
etc. The following notation is used to denote this mapping:

\[ u = A[v], \]  
\[ B = C[A]. \]  

(3.5)  
(3.6)

Some standard operators are defined as follows: The inner product \( A \cdot B \) of two second order tensors \( A \) and \( B \) is a scalar, and can be written in component form as follows:

\[ A \cdot B = \sum_{i,j} A_{ij} B_{ij}. \]  

(3.7)

The tensor product \( A \otimes B \) of two second order tensors \( A \) and \( B \) is a fourth order tensor which is defined such that for any second order tensor \( V \),

\[ (A \otimes B)V = (B \cdot V)A. \]  

(3.8)

In component form,

\[ (A \otimes B)_{ijkl} = A_{ij} B_{kl}. \]  

(3.9)

Two standard identities can now be defined. The second order identity, \( I \), is defined such that

\[ I = \delta_{ij} e_i \otimes e_j, \]  

(3.10)

where \( \delta_{ij} \) is the Kronecker delta. The fourth order identity, \( \mathcal{I} \), is defined such that

\[ \mathcal{I} = \frac{1}{2} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) e_i \otimes e_j \otimes e_k \otimes e_l. \]  

(3.11)

Identity tensors have the property that they map vectors or tensors into themselves:

\[ I[u] = u \]  

(3.12)

\[ \mathcal{I}[A] = A \]  

(3.13)

for all \( u \) and all \( A \).
Finally, three common tensor operations used in this thesis are as follows: The deviatoric part of $A$ is the linear operator that assigns to each tensor $A$ a tensor $A'$ and satisfies
\[
A' = A - \frac{1}{3} \text{tr}A, \tag{3.14}
\]
where $\text{tr}A$ is the trace of $A$, the linear operator that assigns to each tensor $A$ a scalar $\text{tr}A$ and satisfies
\[
\text{tr}A = \sum_i S_{ii}. \tag{3.15}
\]
The magnitude of $A$ assigns to each tensor $A$ a scalar $\|A\|$ such that
\[
\|A\| = \sqrt{A^T \cdot A}. \tag{3.16}
\]

### 3.2.2 Kinematics

Using the above notation, definitions of some standard kinematical quantities are given below which are used in the derivation of the three dimensional constitutive model for a material during strain-induced martensitic transformation. Let $x$ be the position of a particle in the current configuration at time $t$; then:

\[
v(x, t) \quad \text{spatial description of velocity},
\]
\[
L(x, t) = \frac{\partial}{\partial x} v(x, t) \quad \text{velocity gradient},
\]
\[
D = \frac{1}{2}(L + L^T) \quad \text{stretching},
\]
\[
W = \frac{1}{2}(L - L^T) \quad \text{spin},
\]
\[
\sigma(x, t) \quad \text{Cauchy (true) stress},
\]
\[
\rho_0(x) \quad \text{Mass density in reference undeformed configuration},
\]
\[
\rho(x, t) \quad \text{Mass density in current configuration},
\]
\[
T(x, t) = \frac{\rho}{\rho_0} \sigma \quad \text{Kirchhoff stress},
\]
\[
\dot{A}(x, t) = \frac{\partial}{\partial t} A(x, t) \quad \text{material time derivative of } A,
\]

The following definitions are also used frequently throughout the remainder of the
text. The Mises equivalent tensile stress, $\bar{\sigma}$, is defined as follows:

$$\bar{\sigma} = \sqrt{\frac{3}{2} \mathbf{T}' \cdot \mathbf{T}'}$$  \hspace{1cm} (3.17)

where $\mathbf{T}'$ is the deviatoric stress tensor. The hydrostatic stress, or negative pressure, is defined to be

$$p = -\frac{1}{3} \text{tr}\mathbf{T}.$$  \hspace{1cm} (3.18)

The equivalent plastic strain, $\bar{\varepsilon}$, is given by

$$\bar{\varepsilon} = \int \frac{2}{3} \dot{\varepsilon}' \cdot \dot{\varepsilon}' dt,$$  \hspace{1cm} (3.19)

where $\dot{\varepsilon}'$ is the deviatoric plastic strain rate tensor.

### 3.3 Evolution Kinetics

The evolution of martensite with plastic strain follows a sigmoidal pattern, as shown in Figure 3-1. This pattern has been observed repeatedly in experiments with metastable steels [1], [42], [27]. The characteristic "S"-shape of the true stress-strain curve, shown in Figure 3-2, is due to the hardening of the steel from the evolving martensite phase.

As the temperature decreases towards $M_s$, the upturn in flow strength becomes more dramatic, reflecting the higher rate of martensite evolution at lower temperatures.

A one-dimensional model for the formation of martensite in the strain induced regime was developed by Olson and Cohen [47] in order to capture the above phenomenon of martensite transformation. Their model is based on the assumption that strain-induced nucleation occurs predominantly at shear band intersections. Stringfellow et al. [65] incorporated pressure sensitivity into the model through a triaxiality term. The incorporation of this pressure sensitivity term required that the Olson-Cohen evolution laws be reformulated into an incremental form due to the dependence of the martensite evolution on a stress-state history. The following is a description of the evolution equations, which depend on plastic strain, temperature, and stress-
The rate of increase in the volume fraction of martensite, \( \dot{f} \), is proportional to the rate of increase in the number of martensite embryos per unit austenite volume, \( \dot{N}_m \):

\[
\dot{f} = (1 - f) \bar{v}_m \dot{N}_m, \tag{3.20}
\]

where \( \bar{v}_m \) is the average volume per martensitic unit, which is assumed to be constant. The factor \( 1 - f \) represents the decreasing volume fraction of austenite available for transformation.

The martensite embryos are formed from only those shear band intersections which will act as a nucleation site. The number of operational shear band intersections per unit volume, \( N_I \), is given as the number of shear-band intersections per unit volume, \( N_I \), multiplied by the probability, \( P \), that a shear-band intersection will act as a nucleation site. Thus, \( \dot{N}_m \) is given as

\[
\dot{N}_m = P \dot{N}_I + N_I \dot{P} H(\dot{P}), \tag{3.21}
\]

where \( H(\dot{P}) \) is the Heaviside step function, reflecting that the transformation is irreversible. \( N_I \) can be defined as

\[
N_I = \frac{f_I}{\bar{v}_I} \tag{3.22}
\]

where \( f_I \) is the volume fraction of shear-band intersections, and \( \bar{v}_I \) is the average volume for a shear-band intersection. The parameter \( f_I \) is assumed to be related to the volume fraction of austenite occupied by shear-bands, \( f^{ab} \), through a power-law expression of the form

\[
f_I = C (f^{ab})^{r_I}, \tag{3.23}
\]

where \( C \) is a geometric constant, and the exponent \( r_I = 2 \) models a random orientation of shear-bands. Olson and Cohen pointed out that these shear bands tend to be initially parallel, and thus the likelihood of intersections increases, so that the exponent, \( r_I \), tends to be higher than two (typically 4 to 5).
Consistent with observations for the case when shear bands are mainly hcp ε-martensite [47], the volume fraction of shear-bands, $f^{sb}$, evolves with plastic shear strain in the austenite, $\gamma_a$, according to the following relationship:

$$f^{sb} = (1 - f^{sb})\alpha \dot{\gamma}_a.$$  \hfill (3.24)

Or, upon integrating,

$$f^{sb} = 1 - e^{-\alpha \gamma_a},$$  \hfill (3.25)

where $\alpha = \dot{\alpha}(T)$ is a strain-independent constant which represents the rate of shear band formation, $df^{sb}/d\gamma_a$, at low strains. Since lower levels of stacking-fault energy promote shear-band (planar slip) deformation [43], $\alpha$ should increase with decreasing stacking fault energy. The dependence of stacking fault energy with temperature causes $\alpha$ to be temperature-dependent.

Substituting for $f^{sb}$ in (3.23) and using (3.22),

$$N_I = \frac{C}{\bar{\theta}_I} (1 - e^{-\alpha \gamma_a})^{\tau_I},$$  \hfill (3.26)

or, differentiating,

$$\dot{N}_I = \left[\frac{\tau_I C \alpha}{\bar{\theta}_I} e^{-\alpha \gamma_a} \left(1 - e^{-\alpha \gamma_a}\right)^{\tau_I - 1}\right] \dot{\gamma}_a.$$  \hfill (3.27)

The probability parameter, $P$, is determined assuming that there exists a Gaussian distribution of shear band intersection potencies (where “potency” is defined to the the minimum thermodynamic driving force at which a given nucleation site can be activated). Thus, $P$ is given as a cumulative probability distribution function,

$$P = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{g} \exp \left[ -\frac{1}{2} \left( \frac{g' - \bar{g}}{s_g} \right)^2 \right] dg',$$  \hfill (3.28)

where $\bar{g}$ is the dimensionless mean of a given probability distribution function and $s_g$ is its standard deviation (both of which are fit to experimental data). $P$ is taken to be a function of temperature and stress state through the argument of the distribution function, $g$. The parameter $g$ is a normalized new thermodynamic driving force,
defined as:

\[ g = g_0 - g_1 \Theta + g_2 \Sigma, \]  

(3.29)

where \( g_0, g_1, \) and \( g_2 \) are dimensionless constants. A normalized temperature, \( \Theta \), is related to the absolute temperature, \( T \), according to

\[ \Theta = \frac{T - M_{s,ut}^\sigma}{M_{d,ut}^\sigma - M_{s,ut}^\sigma}, \]  

(3.30)

where \( M_{s,ut}^\sigma \) and \( M_{d,ut} \) are the absolute \( M_s^\sigma \) and \( M_d \) temperatures of uniaxial tension. The parameter \( \Sigma \) represents a ratio of the volumetric and deviatoric stress invariants:

\[ \Sigma \equiv \frac{-p}{\bar{\sigma}} \equiv \frac{-p}{\sqrt{3\bar{\tau}}} \]  

(3.31)

Here, \( p \) is pressure and \( \bar{\tau} \) is the equivalent shear stress, as defined previously. The variable \( \Sigma \) is the "triaxiality" of the stress state. The rate of change of the probability function, \( \dot{P} \), under isothermal conditions (\( \dot{\Theta} = 0 \)), is thus given by

\[ \dot{P} = \frac{g_2}{\sqrt{2\pi s_g}} \exp \left[ -\frac{1}{2} \left( g' - \bar{g} \right)^2 \right] \Sigma, \]  

(3.32)

where it can be easily shown that

\[ \dot{\Sigma} = \Sigma \left( \frac{\dot{p}}{p} - \frac{\dot{\tau}}{\bar{\tau}} \right). \]  

(3.33)

Substituting for \( \dot{N}_m \) in (3.20) using (3.21), (3.27), and (3.28), we obtain the following expression for \( \dot{f} \):

\[ \dot{f} = (1 - f) \left( A_f \dot{\gamma}_a + B_f \dot{\Sigma} \right), \]  

(3.34)

where

\[ A_f = \alpha \beta_0 r_1 (1 - f^s) (f^a)^{r_i-1} P; \]  

(3.35)

\[ B_f = \frac{g_2}{\sqrt{2\pi s_g}} \beta_0 (f^{a})^{r_i} \exp \left[ -\frac{1}{2} \left( \frac{g' - \bar{g}}{s_g} \right)^2 \right] H(\dot{\Sigma}), \]  

(3.36)
where $\beta_0 = C\bar{\nu}_m/\bar{\nu}_I$.

### 3.3.1 Discussion

In the model described above for the evolution of strain-induced martensite in a metastable material, the volume fraction of martensite is a function of temperature, plastic strain, and stress state. The plastic strain affects the number of shear band intersections, and the stress state affects the probability that these intersections will become martensite embryos. Temperature affects both the shear band production, through $\alpha(T)$, and the probability distribution.

The assumption that the driving force of transformation is affected by triaxiality is derived from the dilatation in the crystallographic structure during the transformation from austenite to martensite,

$$\Delta V = \frac{(V_m - V_a)}{V_a},$$  \hspace{1cm} (3.37)

where $V_a$ and $V_m$ are the unstressed relative volumes occupied by the austenite and martensite phases, respectively. Although Leal [34] experimentally found that $\Delta V \approx 0.02$ to 0.05 in austenitic steels, no experimental work has been conducted to correlate this parameter with $g_2$, the sensitivity of the probability function on the triaxiality.

Of special consideration in this work are the geometric constants, $C$ and $r_I$. These parameters determine the number of shear band intersections given a number of shear bands. The parameter $C$ was originally meant to describe the dependence of the number of shear band intersections on austenite grain size, shear band morphology, and initial shear band orientation [47]. The exponent, $r_I$, reflects the reality that shear bands will tend to be initially parallel until secondary shear systems begin to operate. If the strain state produced during deformation affects the number of active slip systems in the material, then either $C$ or $r_I$ or both should be affected.

This model for the evolution of martensite was coupled with a set of constitutive equations to describe the stress-strain behavior of the composite austenite-martensite matrix. The next section will describe this formulation and conclude with the two
scalar equations which define the evolution of stress with strain for a transforming material.

3.4 Large Strain Deformation

An isotropic hypoelastic formulation is used to define the evolution of stress state as a function of the rate kinematics. Since large strains are of concern, the model is frame-indifferent. The evolution equation for the average stress in the austenite-martensite composite, $\bar{T}$, is given by

$$\bar{\mathbf{T}} = \mathcal{L}^e [\mathbf{D} - \mathbf{D}^p],$$  \hspace{1cm} (3.38)

where the stretching tensor, $\mathbf{D} = \mathbf{D}^e + \mathbf{D}^p$, which is the symmetric part of the velocity gradient tensor, $\mathbf{L}$, has been decomposed into its elastic and plastic parts. The Jaumann derivative of stress, $\bar{\mathbf{T}}$, is defined as

$$\bar{\mathbf{T}} = \mathbf{T} - \mathbf{W} \mathbf{T} + \mathbf{T} \mathbf{W}$$  \hspace{1cm} (3.39)

where $\mathbf{W} = \frac{1}{2}(\mathbf{L} - \mathbf{L}^T)$ is the spin tensor. The 4th order isotropic elasticity tensor, $\mathcal{L}^e$, is defined as

$$\mathcal{L}^e = 2G\mathbf{I} + \left(K - \frac{2}{3}G\right)\mathbf{1} \otimes \mathbf{1},$$  \hspace{1cm} (3.40)

where $G$ is the shear modulus, $K$ is the bulk modulus, and $\mathbf{I}$ and $\mathbf{I}$ are the second and fourth order identity tensors, respectively. It is assumed that the elastic properties of the austenite ($\mathcal{L}^e_a$) and martensite ($\mathcal{L}^e_m$) are equal, so that $\mathcal{L}^e = \mathcal{L}^e_a = \mathcal{L}^e_m$.

The stretching tensor can be decomposed into deviatoric and hydrostatic parts

$$\mathbf{D} = \mathbf{D}' + \frac{1}{3}(\text{tr} \mathbf{D}) \mathbf{1}$$  \hspace{1cm} (3.41)

or

$$\mathbf{D} = \frac{1}{\sqrt{2}} \dot{\gamma}^T \mathbf{M} + \dot{\epsilon}_v^T \mathbf{1}$$  \hspace{1cm} (3.42)

where $\dot{\gamma}^T = \sqrt{2} \parallel \mathbf{D}' \parallel$ is the total equivalent shear strain rate, $\mathbf{M} = \mathbf{D}' / \parallel \mathbf{D}' \parallel$ is the
unit tensor coaxial with $\mathbf{D}'$, and $\dot{\epsilon}_V^p = \frac{1}{3} \text{tr} \, \mathbf{D}$ is the total dilatation rate. The plastic stretching tensor can similarly be decomposed into deviatoric and hydrostatic parts

$$\mathbf{D}^p = \mathbf{D}^p' + \frac{1}{3} (\text{tr} \, \mathbf{D}^p) \mathbf{1}$$

(3.43)

or

$$\mathbf{D}^p = \frac{1}{\sqrt{2}} \dot{\gamma}^p \mathbf{N} + \dot{\epsilon}_V^p \mathbf{1}$$

(3.44)

where $\dot{\gamma}^p = \sqrt{2} \parallel \mathbf{D}^p' \parallel$ is the equivalent plastic shear strain rate, averaged over the two phases, $\dot{\epsilon}_V^p$ is the rate of plastic dilatation, and $\mathbf{N}$ is the unit deviatoric tensor coaxial with $\mathbf{D}^p'$. In this isotropic model, $\mathbf{D}^p'$ is chosen to be coaxial with $\mathbf{S}$, so that $\mathbf{N}$ is given by

$$\mathbf{N} = \frac{\mathbf{S}}{\sqrt{2} \dot{\tau}}.$$  

(3.45)

Using Equations 3.38- 3.45, the time rates of the pressure, $p$, and equivalent shear stress, $\dot{\tau}$, can be expressed as

$$\dot{p} = -\frac{1}{3} \mathbf{T} \cdot \mathbf{1} = -K(\dot{\epsilon}_V^T - \dot{\epsilon}_V^p)$$

(3.46)

$$\dot{\tau} = \frac{1}{\sqrt{2}} \mathbf{T} \cdot \mathbf{N} = G(\beta \dot{\gamma}^T - \dot{\gamma}^p)$$

(3.47)

where $\beta = \mathbf{M} \cdot \mathbf{N}$. The above equations can be completed by expressing $\dot{\epsilon}_V^p$ and $\dot{\gamma}^p$ – and thus $\mathbf{D}^p$ – in terms of $\dot{\tau}$, $\dot{\tau}$, the volume fraction of martensite, $f$, and the hardninesses of the two phases, $s_a$ and $s_m$. In order to obtain these relationships, a further decomposition of the plastic stretching is suggested

$$\mathbf{D}^p = \mathbf{D}^{\text{slip}} + \mathbf{D}^{\text{nucl}}$$

(3.48)

where the deviatoric tensor $\mathbf{D}^{\text{slip}}$ is the part of the plastic stretching tensor which is due to slip in the austenite and martensite phases and $\mathbf{D}^{\text{nucl}}$ is an additional inelastic strain rate resulting from the transformation process.

This "nucleation" strain is defined similarly to the effects of softening from void
nucleation, where a strain rate term is included that is proportional to the rate of increase of void volume fraction. Here, the plastic softening due to martensitic nucleation is

$$D^{\text{nuc}} = f \left\{ \frac{1}{\sqrt{2}} AN + \frac{1}{3} \Delta V 1 \right\}. \quad (3.49)$$

Since we assume that $D^{\text{slip}}$ is deviatoric, then $\varepsilon'_V = \Delta V f$, accounting for the positive transformation volume change. The deviatoric term models the transformation shape change, where $N$ is in the direction of the applied stress deviator, from Equation 3.45. The coefficient $A$ reflects the combined effect of the shape strains over an isotropic distribution of nucleation sites. Those shape strains not aligned with $N$ tend to cancel, leaving a net shape strain much less than the total shape strain for all transforming sites. The driving force for transformation is higher for sites that can transform in the direction of the applied stress; these sites are therefore transformed early in the deformation process, and $A$ is higher in these early states and decreases as the population of favorably oriented sites is depleted. In a refinement of the Stringfellow model [58] a simple trilinear dependency of the factor $A$ on the martensite volume fraction is assumed, approximating a cumulative probability distribution function with a dimensionless mean, $\bar{f}$, and a standard deviation, $s_f$, such that the value of $A$ would range from an initial value, $A_0$, to a final value, $0.1A_0$. Accounting for the deviatoric shape change due to transformation allows the effect of transformation softening to be accurately predicted by the model.

### 3.5 Self-Consistent Method of estimating composite stress-strain behavior

Since the hardness of the martensite is much higher than that of the austenite phase, using a Voigt model assuming that the strain levels in the martensite and austenite are equal to the average macroscopic strain would lead to an overestimate of the composite stress. Therefore, a self-consistent method [29] is used in the Stringfellow model. In this model, each phase is represented by a single ellipsoidal inclusion embedded in an
infinitely extending equivalent matrix which has properties representing the average of the phases. For a uniform stress or strain rate at infinity, the conditions of the inclusion phase can be determined.

If we define the shear strain rate due to slip, \( \dot{\gamma}^{\text{slip}} = \sqrt{2D} / \| D^{\text{slip}} \| \) and separate it according to its individual phases,

\[
\dot{\gamma}^{\text{slip}} = f\dot{\gamma}^m + (1 - f)\dot{\gamma}^a,
\]

where \( \dot{\gamma}^a \) and \( \dot{\gamma}^m \) represent average equivalent shear strain rates due to slip in the austenite and martensite phases, respectively. Using the relations developed for the nucleation strain, we find that the equivalent plastic shear strain rate can be written as

\[
\dot{\gamma}^p = f\dot{\gamma}^m + (1 - f)\dot{\gamma}^a + A\dot{\gamma}.
\]

By apportioning the strain with an isotropic, viscoplastic self-consistent model, a more realistic composite stress can be obtained. Assuming both phases are isotropic, viscoplastic, and incompressible, we have

\[
\tau_i = \mu_i\dot{\gamma}_i
\]

where \( \tau_i = \sqrt{\frac{1}{2}S_i \cdot S_i} \) is the equivalent shear stress, \( \mu_i \) is an "effective viscosity", and \( \dot{\gamma}_i = \sqrt{2D_i \cdot D_i} \) is the equivalent plastic shear strain rate of the \( i \)th phase. The relationship for a composite consisting of and isotropic distribution of each phase in the form of spherical inclusions embedded in a homogenized equivalent medium becomes

\[
\tau = \bar{\mu}\dot{\gamma}^p
\]

where barred quantities are the response of the composite material.

From Eshelby inclusion theory [17] for an isotropic, incompressible, spherical inclusion of shearing modulus \( \mu_i \), embedded in an infinitely extended incompressible
isotropic matrix of shearing modulus $\bar{\mu}$, the following equations apply:

$$\dot{\gamma}_i = \frac{5\bar{\mu}}{3\bar{\mu} + 2\mu_i} \dot{\gamma}_p$$  \hspace{1cm} (3.54)$$

$$\tau_i = \frac{5\mu_i}{3\bar{\mu} + 2\mu_i} \bar{\tau}.$$  \hspace{1cm} (3.55)$$

Although this solution is precise only for a linear viscous matrix, Stringfellow et al. use this model with non-linear viscous materials, interpreting $\tau_i$ and $\dot{\gamma}_i$ as volume averages within the inclusion.

The "self-consistency" of the strain rates in the two phases is enforced by Equation 3.50. Since self-consistency of the stresses is required, a similar correlation is imposed to determine the macroscopic equivalent shear stress, $\bar{\tau}$:

$$\bar{\tau} = f\tau_m + (1 - f)\tau_a,$$  \hspace{1cm} (3.56)$$

where $\tau_a$ is the stress level in the austenite, and $\tau_m$ is the stress level in the martensite.

Finally, to close the equations, the rate dependence of the individual phases can be expressed in a power law viscoplastic constitutive equation,

$$\frac{\dot{\gamma}_i}{\dot{\gamma}_0} = \left( \frac{\tau_i}{\tau_0} \right)^M,$$  \hspace{1cm} (3.57)$$

where $M$ is taken to be the same for both phases. If the composite material behaves as

$$\frac{\dot{\gamma}_p}{\dot{\gamma}_0} = \left( \frac{\bar{\tau}}{\bar{\tau}_0} \right)^M,$$  \hspace{1cm} (3.58)$$

then $\dot{\gamma}_a$ and $\dot{\gamma}_m$ can be expressed in terms of $f$, $s_a$, $s_m$, $\bar{\tau}$, $M$, and $\dot{\gamma}_0$.

The hardening behavior of the two phases must be known to express $s_a$ and $s_m$ in terms of $\gamma_a$ and $\gamma_m$. Therefore we use the relation

$$\left( \frac{s_i}{s_i^*} \right)^{n_i} = \gamma_i + \gamma_i^*,$$  \hspace{1cm} (3.59)$$

to describe the hardening behavior in the phases, where $\gamma_i^*$ is a reference strain and
$s^r_i$ is a reference shear strength.

By substituting for $\dot{\gamma}^P$ from (3.51), $\dot{f}$ from (3.34) into (3.46, 3.47), and simplifying, we obtain

$$\dot{\tau} = G'[\beta \dot{\gamma}^T - f \dot{\gamma}_m + (1 - f)\dot{\gamma}_a - A(1 - f)A'_f \dot{\gamma}_a + K'C_f \dot{e}_T]$$  \hspace{1cm} (3.60)

$$\dot{\rho} = -K'[\dot{e}^T - \Delta \nu(1 - f)(A_f \dot{\gamma}_a + D_f \dot{\tau})]$$  \hspace{1cm} (3.61)

where

$$C_f = B_f / \sqrt{3\tau}$$
$$D_f = B_f p / \sqrt{3\tau^2}$$
$$G' = G/[1 + G(1 - f)AD'_f]$$
$$K' = K/[1 + K(1 - f)\Delta \nu C_f]$$
$$A'_f = A_f[1 - K'(1 - f)\Delta \nu C_f]$$
$$D'_f = D_f[1 - K'(1 - f)\Delta \nu C_f].$$

A solution of these two equations coupled with the evolution equations for $f$, $s_a$, and $s_m$ provides a complete description of the constitutive model for a transforming material.

For most metallic fcc materials, the yield strength does not depend greatly on temperature [13]. However, in the case of stainless steels, the large fraction of alloying elements would result in solid-solution strengthening, a mechanism which interferes with dislocation motion and is temperature dependent through the activation energy needed to overcome the obstacle. Since the Stringfellow model does not account for changing yield strength with temperature, the predicted flow stress per unit strain must be shifted if such a dependence occurs.

### 3.6 Summary

When implemented in the finite element program ABAQUS, the constitutive model described above was written as a user-material subroutine, or UMAT. Twenty-three
material parameters are required to fully describe a material for this formulation. In
general, they can be broken up into three categories.

Material properties for austenite and martensite phases

The parameters $K$, $G$, $n_a$, $s^*_a$, $\gamma^*_a$, $n_m$, $s^*_m$, $\gamma^*_m$, $M$, and $\dot{\gamma}_0$ are all determined from
material testing. $K$ and $G$ are the bulk and shear modulus, respectively. The values
of $n_a$, $s^*_a$, and $\gamma^*_a$ describe the hardening behavior of the austenite phase, and $n_m$, $s^*_m$, and $\gamma^*_m$ describe the hardening behavior of the martensite phase. The parameters
$M$ and $\dot{\gamma}_0$ describe the viscoplastic behavior of the composite material for different
strain rates.

Properties for martensite evolution

The parameters $r_I$, $\alpha$, $\beta_0$, $\bar{g}$, $s_g$, $g_0$, $g_1$, and $g_2$ all control the evolution of the volume
fraction of martensite, $f$, and are determined from the experimental curves generated
from magnetometry, x-ray diffractometry, or microscopy. The formation of shear
bands and their intersections is controlled by $r_I$, $\alpha$, and $\beta_0$. The probability that
these shear band intersections will transform to martensite is controlled by $\bar{g}$, $s_g$
$g_0$, $g_1$, and $g_2$. The triaxiality, $\Sigma$, also affects the martensite evolution, but is not a
constant parameter in this model since $\Sigma = -p/\bar{\sigma}$.

Transformation strain parameters

Finally, the parameters $\Delta_V$, $A_0$, $\bar{f}$, and $s_f$ describe the effect of the shear and dilational components of the transformation strain on the composite stress-strain behavior. The dilational parameter, $\Delta_V$, should be determined experimentally from the
changes in crystallographic geometry from the parent austenite phase to the martensite phase. The parameters controlling the shear component of the transformation
strain, $A_0$, $\bar{f}$, and $s_f$, could be found through a careful microscopy study of the orienta-
tions of martensite laths as the volume fraction of martensite in the matrix increases.
In the following chapter, an experimental study is conducted to investigate further the effect of triaxiality on the transformation process, specifically in the case of forming processes, where uniaxial tension and plane strain tension are common deformation modes. First, the parameters discussed above are fit to uniaxial tension data collected at various temperatures. Next, the Stringfellow model is used to predict the behavior in plane strain tension, and these predictions are compared with experimental plane strain tension data.
Figure 3-1: Volume fraction of martensite versus plastic strain in uniaxial tension for 304 stainless steel [1]. Note the sigmoidal pattern of the evolution curves, and increasing volume fraction with decreasing temperature.
Figure 3-2: True stress versus true strain in uniaxial tension for 304 stainless steel [27]. Note the characteristic "S"-shape of the curve due to strain-induced martensitic transformation.
Chapter 4

Experimental Methods

4.1 Introduction

In order to quantify the effects of temperature and stress state on the mechanical behavior and transformation kinetics of a metastable, transforming steel, an experimental program was conducted. This program consisted of isothermal, mechanical testing in uniaxial tension and plane strain tension at several different temperatures. The volume fraction of martensite in the tested samples was determined by Vibrating Sample Magnetometry (VSM). Finally, an X-Ray Diffraction study was conducted to determine the differences in texture evolution between uniaxial tension and plane strain tension at large strains.

4.2 Material

The material chosen for the present experimental study was 304L stainless steel (SS304L) sheet, obtained from Avesta Sheffield Inc. This material is widely used in industry, primarily for its corrosion resistance. The "L" designation refers to the decreased carbon content used in welding applications to reduce the amount of carbide precipitates in the weld zone. As mentioned in Chapter 2, carbon is a strong austenite stabilizer. Therefore, this low-carbon alloy was also chosen with the desire that it be more unstable than standard 304, in order to produce significant amounts
<table>
<thead>
<tr>
<th>Element</th>
<th>C</th>
<th>Cr</th>
<th>Ni</th>
<th>Si</th>
<th>S</th>
<th>Mo</th>
<th>Cu</th>
<th>Mn</th>
<th>P</th>
<th>N</th>
</tr>
</thead>
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<td>0.022</td>
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<td>9.31</td>
<td>0.42</td>
<td>0.002</td>
<td>0.35</td>
<td>0.39</td>
<td>1.39</td>
<td>0.021</td>
<td>0.042</td>
</tr>
</tbody>
</table>

Table 4.1: Composition of 304L Stainless Steel

of martensite during straining at room temperature.

The material was cold-rolled, softened, descaled, and pinched-passed to a final thickness of 0.047 inches (1.2 mm). A micrograph showing grain structure through the thickness of the sheet is shown in Figure 4-1. The etchant used was 15 mL HCl (hydrochloric acid), 30mL HNO₃ (nitric acid), 45 mL CH₃COOH (acetic acid), and 21 drops of glycerine. Some bands of martensite (dark) have formed in the austenite matrix due to prior cold work. The average grain size was determined to be 10 μm. The chemical composition of the SS304L used in this study is presented in Table 4.1, as provided by Avesta Sheffield.

Due to the high amounts of alloying elements such as Mo and Mn, this alloy is slightly more stable than standard 304, despite its low carbon content. However, as will be shown, the martensite evolution curves for the room temperature tests were well defined, even though the total martensite volume fraction which evolved was low.

4.3 Mechanical Tests

4.3.1 Uniaxial Tension Specimen Design

The uniaxial tension specimen used in the present study is shown in Figure 4-2, consistent with ASTM A370, showing a gauge length of 3 in. (76 mm), gauge width of 0.5 in. (12.7 mm), with a radius of 0.6 in. (15.2 mm). This specimen design was used for room temperature and higher temperature testing, as well as determining the R-Ratio.

The uniaxial tension specimen used for cold temperature testing at -40°C is shown in Figure 4-3, showing a gauge length of 0.625 in. (15.9 mm), a gauge width of 0.125 in. (3.2 mm), with a radius of 0.187 in. (4.8 mm), as described in the
ASTM standard E646. This subsize specimen with "pin-ends" was used in order to accommodate the test apparatus for immersion in cooling fluids.

For all temperatures tested, specimens were machined with the axial direction at 0, 45 and 90 degrees from the rolling direction to investigate anisotropy in the stress-strain behavior due to the rolling process.

4.3.2 Plane Strain Tension Specimen Design

The plane strain specimen used in the present study is shown in Figure 4-4. This design was adapted from that of Appleby, et al.[2]. A summary of the other designs studied is given in Appendix A. The plane strain tension specimen was designed to create a state of strain such that the strain is zero in one of the three principal directions, i.e., $\epsilon_{22} = 0$. If incompressibility holds, the strains are equal and opposite in the other two principal directions, i.e., $\epsilon_{11} = \epsilon$, $\epsilon_{33} = -\epsilon$. As noted in Chapter 2, this state of strain is critical for localization and strain to failure of most sheet metals. The plane strain strain-state was achieved experimentally in this study through added constraint in the width direction via welded support “tabs” and high width to gauge length and thickness ratios. However, even with this constraint, the width of the specimen was not entirely in plane strain. The absence of stress along the edges of the specimen caused the material close to the edges to behave as if in uniaxial tension. Considerable analysis is required to determine the actual plane strain stress-strain behavior from the strain distributions across the specimen width. Therefore, the optimal geometry for this specimen was determined through Finite Element Modeling (FEM) analyses. A discussion of this analysis is given in Chapter 5.

The middle sheet of the specimen was electro-chemically gridded with 0.0500 in. (0.0127 mm) circles on 0.0625 in. (0.0159 mm) centers in order to measure the principal and transverse strains across the width of the specimen. Two parallel welds were made in a "sandwich" assembly of sheets with a gauge section of 0.236 in. (6.0 mm), and then the specimens were machined. Electron-Beam (EB) welding was used, as the heat-affected zone was smaller and the location and straightness of the weld superior to electrical-resistance welding. The EB weld was approximately 0.02 inches
(0.5 mm) wide. The specimens were machined to a length-to-width ratio of 1:8 for
20°C tests (Figure 4-4) and 1:6 for −40°C tests (Figure 4-5. The specimens tested at
20°C were spot-welded in the tab region so that the EB weld would not be required
to transfer all of the load to the gauge section when the specimen was tested in
the hydraulic wedge grips of the testing machine. The specimens tested at −40°C
used "pin-ends" in order to accommodate the test apparatus for immersion in cooling
fluids.

4.3.3 Mechanical Testing

Uniaxial and plane strain tension testing at 20 degrees Celsius was conducted on
an Instron 8501 hydraulic test machine in displacement control, located in the MIT
Mechanics of Materials Laboratory. The crosshead velocity was constant, and pro-
duced an initial approximate equivalent strain rate of $1 \times 10^{-4}$ sec$^{-1}$. Strain in the
gauge section was monitored using an Instron 2620 – 824 extensometer. Strain in
the plane strain tension specimen was additionally measured from photographs of the
circular-gridded gauge region during the test and directly from the specimen at the
end of a test using a measuring microscope. It was estimated that the diameters of
the grid circles could be read to ±0.002 in. (0.005 mm) so the probable errors in
the strains are ±0.04 (±4 percent). Strain and load data were collected from the
extensometer and load cell using DOS-PC running Notebook software. Higher tem-
perature uniaxial tension testing at 100°C and 150°C were conducted on an Instron
1125 screw-driven test machine in displacement control with an Instron Model 3111
oven fixture, also located in the Mechanics of Materials Laboratory. Speed of the
crosshead for these tests was also constant, producing an initial approximate strain
rate of $1 \times 10^{-4}$ sec$^{-1}$. A type K thermocouple was tack-welded just outside the gauge
section of each specimen to verify the correct test temperature. Strain and load data
for the high temperature testing was collected in the same manner as at the room
temperature testing.

Uniaxial and plane strain tension testing at −40°C was conducted on a Materials
Testing Systems (MTS) servo-hydraulic test machine in displacement control, located
at the MIT Plasma Fusion Center (NW22). As in the previous testing, the speed of the crosshead was constant, producing an initial approximate equivalent strain rate of $1 \times 10^{-4}$ sec$^{-1}$. The grip structure of the test machine was modified such that the loading fixture could be submerged in various cooling fluids. The temperature of the specimen was varied by using dry ice with an alcohol bath, using a type K thermocouple tack-welded to the specimen to verify the correct test temperature. A Sheppic type SE-0.5 cryogenic extensometer was used to monitor the strain in the gauge section. Strain in the plane strain tension specimen was additionally measured from the circular-gridded gauge region directly at the end of a test. Strain and load data for the low temperature testing was collected on a Macintosh PC using MTS software.

**R-Ratio Testing**

The $R$-Ratio of a sheet material is a measure of the anisotropy of its plastic deformation behavior. Differences in the behavior of the sheet material can arise when it is strained in different orientations relative to a rolling direction. This anisotropy can be caused by preferential elongation of grains, a strong crystallographic texture, or a well-defined dislocation substructure. The $R$-Ratio is defined as the ratio of the width strain, $\varepsilon_{22}$, to the thickness strain, $\varepsilon_{33}$, during a uniaxial tension test in the 1-direction. In this study, the width strains were measured during the test with a micrometer, while axial strain measurements from an extensometer were converted to the corresponding thickness strain, assuming incompressibility, ie. $\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = 0 \rightarrow \varepsilon_{33} = - (\varepsilon_{11} + \varepsilon_{22})$.

**4.3.4 Data Analysis**

The axial engineering strain, $e_{11}$, was obtained from the extensometer output or photographed grids, and the axial engineering stress, $S_{11}$, was determined from the
load cell output, using the definitions

\[ e_{11} = \frac{\Delta u}{u_0} \text{ and } S_{11} = \frac{P}{A_0}, \]  

(4.1)

where \( u_0 \) is the initial gauge length, \( \Delta u \) is the change in gauge length, \( P \) is the load, and \( A_0 \) is the initial cross-sectional area of the specimen in the reduced section. The true axial strain, \( \varepsilon_{11} \), and the true axial stress, \( \sigma_{11} \), are then defined by

\[ \varepsilon_{11} = \ln(1 + e_{11}) \text{ and } \sigma_{11} = S_{11} \cdot (1 + e_{11}) \]  

(4.2)

respectively.

**Plane-Strain Stress-Strain Formulation**

As mentioned previously, the width constraint for the plane strain specimen is not complete near the edge of the specimen. Figure 4-6 shows the strain distributions across the half-width of a plane strain tension specimen at 20°C. The axial strains are quite uniform, but the transverse strains across the width do not remain at zero strain. Near the edge of the specimen, the absolute value of the ratio of the transverse strains to the axial strains (\(|\varepsilon_{22}/\varepsilon_{11}|\)) is close to 0.5, a strain state that develops under uniaxial tension for an isotropic material.

Using this observation, a computer program in FORTRAN was developed to determine approximately the stress and strain of the plane-strain region in the center section of the specimen by removing the effect of the uniaxial tension region. Other plane strain specimen designs suffer from this “edge effect”, requiring similar reduction of the experimental data [67], [22]. The method used here is based on that of Wagoner [67]. The source code for the procedure outlined below appears in Appendix B.

Before the program was implemented, certain observations were made. First, the width of the specimen along the transverse centerline was divided into three regions—the center section, which has a strain state close to plane strain, and two edge sections
with strain states near uniaxial tension. The boundary of the plane strain section was set where \( |\varepsilon_{22}/\varepsilon_{11}| = 0.2 \), where \( \varepsilon_{11} \) and \( \varepsilon_{22} \) are the true component strains measured from the circular grid in the axial and transverse directions, respectively. Next, a percentage of the gauge section was defined in which the state of strain is approximately uniaxial tension. It was found experimentally that this percentage grew approximately linearly from close to zero at the beginning of a test to a final value at failure, the final value being dependent on the gauge length-to-width aspect ratio of the specimen. Figure 4-7 plots \( |\varepsilon_{22}/\varepsilon_{11}| \) across the half-width of a plane strain tension specimen with an aspect ratio of 1:8, tested at 20°C, showing how the region of plane strain decreases with increasing axial strain. Figure 4-8 plots \( |\varepsilon_{22}/\varepsilon_{11}| \) versus a normalized distance from the edge of the specimen for specimens with aspect ratios of 1:8 and 1:6, tested to an average axial strain of 0.30 at 20°C. The specimen with the larger aspect ratio (1:6) has a smaller region of plane strain. Also, the average axial strain in the uniaxial region was found to be approximately 80% of the average axial strain in the central plane strain region (Figure 4-6). Finally, the axial strain measurements for small strains was calculated from either the cross-head displacement adjusted for the compliance of the test structure or an extensometer attached to the specimen above and below the gauge section, in the tab region. For large strains (greater than 0.10), the axial strain measurements were taken from photographs of the specimen, or the specimen itself, by averaging the axial strains in the plane strain region. The axial strain in the plane strain region and the total load output from the load cell were the two inputs to the computer program developed.

The general steps the program performs are as follows:

1) For a given plane strain axial strain, a percentage of the gauge section is defined in which the state of strain is approximately uniaxial tension.

2) The average axial component of strain in the uniaxial tension region of the specimen in calculated as 80% of the plane strain axial strain.

3) The average cross-sectional area of the deformed specimen in the uniaxial tension region is calculated assuming isotropy and incompressibility.

4) The load supported by the uniaxial tension region is then calculated. The
average true stress in the uniaxial tension region is determined using the average axial component of strain in the uniaxial tension region and the stress-strain behavior obtained from the uniaxial tension tests. Multiplying this stress by the calculated cross-sectional area in uniaxial tension results in the load in the uniaxial tension region.

5) The average true axial stress in the center (plane strain) section is calculated after subtracting the estimated load supported by the edge (uniaxial) sections from the total recorded load. The remaining load is divided by the central region's deformed cross-sectional area to obtain its average axial stress, $\sigma_{11}$.

By comparing the uncorrected axial stress versus axial strain curve for the entire specimen response and the corrected axial stress versus axial strain curve for the plane strain region of the specimen, we can observe how the corrected curve falls close to the theoretical plane strain tension behavior assuming isotropy and isotropic hardening of a von Mises yield surface. Figure 4-9 plots both the uncorrected and corrected stress-strain curves against the experimental uniaxial tension curve and the theoretical plane strain tension curve. We see that the correction is critical in order to find the plane strain stress-strain behavior at high strains.

Equivalent Measures of Strain and Stress

For an isotropic and isotropically hardening material, the data for equivalent stress versus equivalent strain under different states of stress should coincide. In this study, however, the kinetics of the martensite evolution depend upon the stress state, and the volume fraction of martensite in the specimen affects the hardening rate, as seen in Chapter 3. Therefore, in order to examine this effect on the stress states of uniaxial tension and plane strain tension, all stress-strain data was converted to a von Mises equivalent measure.

The von Mises equivalent plastic strain measure is defined as

$$\bar{\varepsilon} = \int \sqrt{\frac{2}{3} \dot{\varepsilon}' \cdot \dot{\varepsilon}'} dt,$$  \hspace{1cm} (4.3)
where $\dot{\varepsilon}$ is the deviatoric plastic strain rate tensor, and $dt$ is the time increment.

In a state of uniaxial tension, the width and thickness strains, $\varepsilon_{22}$ and $\varepsilon_{33}$, are related to the axial strain, $\varepsilon_{11}$, by the equations

$$
\varepsilon_{22} = -\frac{1}{2} \varepsilon_{11} \quad \text{and} \quad \varepsilon_{33} = -\frac{1}{2} \varepsilon_{11},
$$

(4.4)

assuming incompressibility and isotropy. If these assumptions hold, the von Mises equivalent plastic strain measure reduces to the principal axial component of strain, $\bar{\varepsilon} = \varepsilon_{11}$.

In a state of plane strain tension, the width and thickness strains, $\varepsilon_{22}$ and $\varepsilon_{33}$, are related to the axial strain, $\varepsilon_{11}$, by the equations

$$
\varepsilon_{22} = 0 \quad \text{and} \quad \varepsilon_{33} = -\varepsilon_{11},
$$

(4.5)

assuming incompressibility. If these assumptions hold, the von Mises equivalent plastic strain measure reduces to $\bar{\varepsilon} = (2/\sqrt{3})\varepsilon_{11}$. While incompressibility is a good assumption for most metals undergoing plastic deformation, the assumption of isotropy for sheet must be verified by conducting an R-Ratio test.

The von Mises equivalent stress measure is defined as

$$
\bar{\sigma} = \sqrt{\frac{3}{2} T' \cdot T'},
$$

(4.6)

where $T'$ is the deviatoric stress tensor.

For the deformation state of uniaxial tension, where $\sigma_{11} = \sigma$, and all other components of the stress tensor are zero, the von Mises equivalent stress measure reduces to the principal axial component of the stress, $\bar{\sigma} = \sigma_{11}$.

In plane strain tension, where

$$
\sigma_{22} = \frac{1}{2} \sigma_{11} \quad \text{and} \quad \sigma_{33} = 0,
$$

(4.7)

the von Mises equivalent stress measure reduces to $(\sqrt{3}/2)\sigma_{11}$. 

65
During necking, however, increasing triaxiality in the necked region occurs. This phenomenon is usually corrected by using the Bridgman (1945) correction factor, to achieve an average axial stress across the minimum section. However, since, in this study, the stress state was desired to remain constant throughout the test, data was considered only until the onset of necking.

**Hardening Rate**

As mentioned in Chapter 1, for the case of uniaxial tension, if we define a hardening rate, \( h \), as \( h = d\sigma_{11}/d\varepsilon_{11} \), then we can show that instability in the form of necking occurs when

\[
h = \sigma_{11}. \tag{4.8}
\]

In equivalent stress and strain for the uniaxial tension case, this relation becomes

\[
\bar{h} = \bar{\sigma}, \tag{4.9}
\]

where \( \bar{h} = d\bar{\sigma}/d\bar{\varepsilon} \). However, in plane strain tension, as shown in the previous section, \( \bar{\sigma} = (\sqrt{3}/2)\sigma_{11} \), and \( \bar{\varepsilon} = (2/\sqrt{3})\varepsilon_{11} \). Therefore, if the stability criterion is to be expressed in terms of equivalent stress and strain, then \( \bar{h} = (3/4)h \), and the criterion for the plane strain tension case becomes

\[
\frac{2}{\sqrt{3}}\bar{h} = \bar{\sigma}. \tag{4.10}
\]

By plotting \( \bar{h} \) versus equivalent strain for uniaxial tension, \( \frac{2}{\sqrt{3}}\bar{h} \) versus equivalent strain for plane strain tension, and equivalent stress versus equivalent strain for both cases, we can compare the equivalent strain at which the stability criterion is satisfied for both cases.

**4.3.5 Mechanical Test Results**

The above mechanical testing was performed according to a test matrix shown in Table 4.2.
<table>
<thead>
<tr>
<th>Test Type</th>
<th>Orientation</th>
<th>Temperature (°C)</th>
<th>Equivalent Strain Level</th>
</tr>
</thead>
<tbody>
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<td>Uniaxial Tension</td>
<td>0°, 45°, 90°</td>
<td>−40°, 20°</td>
<td>10%, 20%, 30%, 40%</td>
</tr>
<tr>
<td>Uniaxial Tension</td>
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<td>100°</td>
<td>10%, 20%, 30%, 40%</td>
</tr>
<tr>
<td>Plane Strain Tension</td>
<td>0°, 45°, 90°</td>
<td>20°</td>
<td>10%, 20%, 40%</td>
</tr>
<tr>
<td>Plane Strain Tension</td>
<td>45°</td>
<td>−40°</td>
<td>10%, 20%, 40%</td>
</tr>
</tbody>
</table>

Table 4.2: Mechanical Test Matrix for 304L Stainless Steel

**R-Ratio**

R-Ratio measurements for the 0°, 45°, and 90° orientations to the rolling direction were found to be 1.06, 0.94, and 1.26, respectively. These R-Ratios imply that the material is relatively isotropic in the plane of the sheet and through the thickness. Therefore, earlier assumptions about isotropy when calculating equivalent stress and strain measures should suffice for this material.

**Uniaxial Tension Tests**

Multiple tests were conducted to various strain levels to assure consistency in the data as well as to provide material for the VSM and X-ray diffraction studies. Figure 4-10 shows the equivalent stress-strain results for the uniaxial tension tests at −40°, and 20° Celsius, for 0°, 45°, and 90° orientations to the rolling direction of the sheet. The 20° results for different orientations were very similar to others for 304L SS shown in Figure 4-11. In Figure 4-12, the equivalent stress-strain results for the uniaxial tension tests at −40°, 20°, and 100° Celsius for the 90° orientation are plotted. The hardening rates were high for this material, even at higher temperatures where little strain-induced martensite was forming. When fit to a power-law hardening rule, the hardening exponent at 100°C was \( n = 0.28 \), and at 20°C, \( n = 0.36 \). The data at −40°C deviated from a simple power-law hardening rule due to a sudden increase in volume fraction of martensite. The result of the pronounced strain-induced martensite evolution was the typical sudden increase in hardening around \( \varepsilon = 0.17 \).

By plotting the equivalent hardening rate, \( \tilde{h} \) of the equivalent stress-strain curves at the three temperatures against the equivalent stress-strain behavior in Figure 4-
13, the stability of the deformation process can be determined. Since the hardening rate for 304L was high even at 20°C, the criterion for instability in uniaxial tension, \( \bar{h} = \bar{\sigma} \), was not met until strain levels of over 0.40. The characteristic "transforming" hardening behavior at \(-40^\circ C\) did show an initial softening and sudden increase in hardening, as shown previously for a TRIP steel in Figure 1-2. However, \( \bar{h} > \bar{\sigma} \) during the softening effect, so that there is no sequence of necking instability, reestablishment of stability, growth of a neck, and final instability at failure as in the TRIP steel. However, if the yield strength of the 304L were increased by 1000 MPa to the level of a typical high strength steel, as pictured in Figure 4-14, then the favorable effect of the hardening rate of the elongation to necking is apparent. At 100°C, necking would occur at an equivalent strain of 0.11. At 20°C, necking would occur at an equivalent strain of 0.17. At \(-40^\circ C\), a neck would begin around an equivalent strain of 0.05, propagate throughout the specimen, and then neck again to failure at an equivalent strain near 0.33, almost twice the level of strain to necking found at 20°C.

**Plane Strain Tension Tests**

Figure 4-15 shows the equivalent stress-strain results for the plane strain tension tests at 20° and \(-40^\circ C\) Celsius, for 0° and 45° orientations, respectively, superimposed on the uniaxial tension test data for those temperatures and orientations. At 20°, the plane strain tension equivalent stress-strain response fell slightly below that of the uniaxial tension equivalent stress-strain curve, especially for lower strains. At \(-40^\circ C\), the equivalent stress-strain curve for plane strain tension was definitely lower than that of uniaxial tension. This response does not agree with the prediction of the Stringfellow model, which would suggest that, due to a higher triaxiality in a state of plane strain tension, a larger volume fraction of martensite per unit equivalent strain would form, which would increase the hardening rate and result in a higher equivalent stress per unit equivalent strain.

Figure 4-16 compares the equivalent hardening rate, \( \bar{h} \) for uniaxial tension and \( 2/\sqrt{3}\bar{h} \) for plane strain tension, with the equivalent stress-strain behavior. As mentioned previously, when \( \bar{h} = \bar{\sigma} \) in the uniaxial tension case, or \( 2/\sqrt{3}\bar{h} = \bar{\sigma} \) in the
plane strain tension case, instability in the form of a neck will ensue. At 20°C, not much difference in the equivalent hardening rate was found. Both the uniaxial tension and plane strain tension underwent stable deformation to an equivalent strain of 0.40 before maximum load was reached and necking began. At −40°C, Figure 4-17 shows a significant difference in hardening behavior. In plane strain tension, the initial softening due to rapid martensite transformation around an equivalent strain of 0.10 was less dramatic, as was the hardening at an equivalent strain of 0.25. Again, due to the low flow strength of 304L, both the uniaxial tension and plane strain tension specimens deformed without necking to a strain of 0.45. If the yield strength of the stress-strain response of the 304L is again increased by 1000 MPa to the level of a typical high strength steel, as in Figure 4-18, the effect of stress state on critical strain to necking can be addressed more clearly. Here, both the uniaxial and plane strain tension case would go through the process of necking instability, reestablishment of stability and growth of the neck, and finally, a second instability point, leading to failure. However, the plane strain tension case would neck to failure at an equivalent strain of 0.32, slightly before the uniaxial tension case at 0.33. Again, according to the predictions made by Stringfellow [63], the plane strain tension case should have a higher hardening rate and a higher critical strain to necking than the uniaxial tension case. This discrepancy will be discussed in Chapter 7.

Forming Limit Diagram

A Forming Limit Diagram (FLD) was created from the mechanical tests performed in uniaxial tension and plane strain tension at 20°C and −40°C, shown in Figure 4-19. The criterion used for failure was the major axis strain at maximum load. Normally, the criterion is evaluated by sight, stopping the test at the moment when a localized neck forms in the sheet. However, since the −40°C tests were performed in a cooling bath, visual verification of the necking process was not possible. Therefore, the more conservative criterion of critical strain at maximum load, when diffuse necking begins, was used to create the FLD.

The FLD still has the traditional shape of Figure 2-1, with the lower critical
strain to failure in plane strain, where the minor strain equals zero. This result is expected after the discussion above concerning the stability of the deformation processes for the two stress states and temperatures. Since the plane strain tension case did not have a significant increase in its hardening rate, but instead actually produced a softer response, the critical strain in plane strain was expected to remain lower than the uniaxial tension case. Also, since the yield strength of the 304L was low, instability occurred at high strains for both uniaxial tension and plane strain tension at both 20°C and -40°C. This resulted in the two FLD's for the two temperatures to practically coincide, although the -40°C diagram falls slightly above the 20°C diagram. However, more data should be collected before this difference can be said to be significant.

4.4 Vibrating Sample Magnetometry

Due to the difference in magnetic properties of the austenite and martensite phases, vibrating sample magnetometry could be used to determine the volume fraction of martensite evolved in the specimen. A review of the theory and experimental methods of determining magnetic properties of materials can be found in Hoselitz [30]. A vibrating sample magnetometer (VSM) consists of a coil system in which a voltage, V, is induced proportional to the magnetization, J, of an oscillating, magnetized sample. As the specimen is magnetized to saturation, that is, as all of the magnetization vectors of the individual grains in a specimen are rotated to a position determined by an outside magnetic field, H, the VSM produces a magnetization curve for the material (Figure 4.6), plotting magnetization, J, in Tesla (T) or Gauss (G), versus field strength, H, in Oersted (Oe). If a material is ferromagnetic, as is martensite, the magnetic portion of the sample saturates as the outside magnetic field strength is increased, and the magnetization curve becomes linear. By extrapolating the linear portion back to zero field strength, the saturation magnetization, J_s, can be found at the intersection of this linear portion with the magnetization axis. A purely austenitic material is not ferromagnetic and does not exhibit a saturation value; therefore, we
can determine the volume fraction of martensite in a VSM sample by the saturation magnetization.

The specific saturation magnetization per unit mass, $j_s$, in emu/gram, (or $10^2 \mu_B$/atom, where $\mu_B$ is the Bohr magneton), can be determined from the equation

$$j_s = \frac{J_s}{4\pi \rho_m}$$  \hspace{1cm} (4.11)

where $\rho_m$ is the mass density of the martensite phase,

$$\frac{1}{\rho_m} = \frac{1}{\rho_{Fe}} N_{Fe} + \frac{1}{\rho_{Cr}} N_{Cr} + \frac{1}{\rho_{Ni}} N_{Ni} + \ldots$$  \hspace{1cm} (4.12)

where $N_i$ is the weight fraction of the $i_{th}$ component of the composition of the alloy.

The volume fraction of martensite in the sample is the ratio of $j_s$ for the sample determined experimentally, and the calculated $j_s$ of the martensite phase. The value of $j_s$ for the martensite phase was determined from the given composition of 304L used and an experimentally derived equation given by Steinhaus, Kussmann and Schoen [30] to be approximately 173.1 emu/gram at room temperature. This value was within the range of saturation values given in the literature (175 - 154 emu/g) for 304 SS [27].

4.4.1 Specimen Preparation

The specimens were taken from the gauge section of the uniaxial and plane strain tension specimens at various amounts of strain. Specimens were taken across the transverse axis of the plane strain specimen to examine martensite distribution. The specimens were sectioned into approximately 1mm $\times$ 1mm $\times$ 3mm shards using a Buehler diamond saw at low cutting speeds to ensure minimal deformation due to machining. One specimen was tested from the unstrained material to record pre-straining martensite levels in the sheet.
4.4.2 VSM Testing

The specimens for this study were analyzed using a VSM in the Francis Bitter National Magnet Laboratory with a 2 inch magnet bore Bitter Magnet and a 200 kOe field capacity at room temperature. The conversion from volts to emu for this VSM was 2.041 emu/Volt. By dividing emu for the sample by the mass in grams, the $j_s$ was determined for each sample. The $j_s$ was divided by 173.1 emu/gram to obtain the volume fraction of martensite present in the sample.

4.4.3 VSM Results

Figure 4-21 shows the volume fraction of martensite evolution versus equivalent strain curves for uniaxial and plane strain tension tests at 20° and −40° Celsius. All curves were fit to the sigmoidal pattern predicted by the Olson-Cohen model for martensite evolution with strain. The unstrained sheet material had a baseline volume fraction of martensite of 0.9% due to the cold-rolling process. The uniaxial tension test at 100°C produced a negligible volume fraction of martensite above the baseline value, even during necking of the sample. For both uniaxial and plane strain tension, the volume fraction of martensite evolved at 20°C above the baseline value is negligible until a strain level of approximately 0.40. At that strain, the volume fraction was around 3%. The equivalent stress-strain behavior in Figure 4-15 reflected this finding by the similarity in hardening rate for the two stress-states at 20°C. At an equivalent strain level before localized necking, around 0.60, the volume fraction of martensite in uniaxial tension reached only 11%. However, in plane strain tension, at an equivalent strain of 0.60, the volume fraction of martensite was slightly less, at approximately 8%. It is quite clear for the −40°C curves that the plane strain tension tests produced less martensite per increment of equivalent strain than the uniaxial tension tests, although both undergo rapid increases in martensite evolution between equivalent strains of 0.10 to 0.30. At an equivalent strain of 0.60, the central plane strain region of the plane strain tension specimen had a volume fraction of martensite of 56%, while the uniaxial tension specimen had a volume fraction of martensite of 96%. These results
are consistent with the equivalent stress-strain behavior in Figure 4-15, which showed a dramatic increase in hardening between 0.10 to 0.30 equivalent strain, and higher equivalent stress per unit equivalent strain in uniaxial tension than in plane strain tension.

4.5 X-ray Diffraction Study

It was suspected in this study that the development of preferred orientation of crystallographic planes, or crystallographic texture, would occur due to the large amount of strain before failure. As discussed earlier, work by Miller [40] on large strain plasticity with 304L implicated the softening effects of texture evolution in explaining differences in the stress-strain behavior of compression and torsion, although most of his study was directed at deformation-induced substructures. Crystallographic textures are often represented by pole figures. A thorough description of the theory and experimental details behind texture representation from pole figures can be found in Cullity [14]. A pole figure is a projection, in two dimensions, showing the distribution of directions normal to the particular crystallographic plane over a set of grains in a material. The distribution is obtained by reflecting an X-ray beam off the surface of a sample at various angles, and measuring the intensity of the reflected beam. If the material is sufficiently textured, peaks of intensity are found at angles that describe a given crystallographic plane, satisfying the Bragg Law, given as

\[ n\lambda = 2d\sin \theta, \]  

(4.13)

where \( \theta \) is angle made between the incident beam and the specimen surface, \( n \) is an integer, \( \lambda \) is the radiation wavelength (1.542 Å for copper \( K_\alpha \)) and \( d \) is the distance between crystallographic planes. Once the \( \theta \) angles have been found for the desired crystallographic planes, the specimen is tilted through an angle \( \alpha \) and rotated through an angle \( \beta \) for each plane as shown in Figure 4.6. The resulting pole figure shows the variation in intensity through the angle \( \alpha \) (from the center to the edge of the figure)
and the angle $\beta$ (circumferentially around the figure).

4.5.1 X-ray Diffraction Specimen Preparation

Three specimens were prepared for X-ray diffraction. A baseline specimen with no strain, a specimen with 40% equivalent strain from a 20° C uniaxial tension test, and a specimen from the middle (plane strain region) of a specimen with 35% equivalent strain from a 20° C plane strain tension test. The specimens were cut with a Buehler diamond saw at low cutting speeds to 3 - 6 mm square. The thickness of the specimens was the thickness of the sheet, approximately 1 mm thick. The specimens were polished in the plane of the sheet, with the rolling direction oriented at an angle $\beta = 0^\circ$. All three were lightly polished with 1 $\mu$m and then 0.3 $\mu$m particle solution to remove any surface roughness that may scatter the X-ray beam.

4.5.2 X-ray Diffraction Testing

Crystallographic texture was measured by X-ray irradiation using a Rigaku RU200 diffractometer with pole figure goiniometer. Partial pole figures were generated by using the Schulz reflection method on the 111 and 220 crystallographic planes using copper $K_\alpha$ radiation.

4.5.3 Pole Figure Results

Figures 4-23, 4-25, and 4-26 show the resulting pole figures for the 304L stainless steel for the baseline, uniaxial tension, and plane strain cases. The rolling direction of the sheet is in the direction of $\beta = 0^\circ$.

The baseline pole figure (4-23) displays the common texture of an fcc sheet rolled in plane strain compression [5], with a high magnitude of reflection from the 111 planes at angles $\alpha \approx \pm 65^\circ$ and $\beta = 90^\circ, 270^\circ$. This type of texture is also seen in the plane strain compression test for copper, another fcc material, strained to a component strain $\epsilon_{33} = -0.52$ [10] shown in Figure 4-24.

The uniaxial tension pole figure (4-25) shows a texture similar to the baseline
sample. The pattern has simply become more well-defined, with stronger texture due to the higher strain. The regions of maximum intensity have elongated in the $\beta = 90^\circ, 270^\circ$ directions. Since all of the specimens, including the uniaxial tension specimen, was polished in the plane of the sheet, this pole figure cannot be compared with pole figures for most uniaxial tension tests, which are polished in the plane perpendicular to the axis of loading. This pole figure was prepared simply to compare with the baseline and plane strain tension sample to observe any major crystallographic changes in texture.

The plane strain tension specimen had the same strain state of deformation as a plane strain compression test, but a different stress state with a higher triaxiality. Here, we would expect to see an even stronger plane strain compression texture, converging to a "dumbbell like shape" as reported in Bronkhorst et al. [10]. Unfortunately, the sample size was too small (3 mm $\times$ 3 mm) to get an accurate pole figure for this case. In Figure 4-26, the pole figure shows extremely large magnitudes of reflection at four points between $\alpha = 30^\circ$ and $\alpha = 60^\circ$. This pattern suggests that only a few individual grains are reflecting all the the X-ray beam.

This preliminary X-ray diffraction study does not reveal much about the difference in texture between the uniaxial tension and plane strain tension tests. However, it does show that the crystallographic texture is affected by equivalent strains as low as 0.35, and that this sheet material still retains a typical plane strain compression texture from the rolling process.

### 4.6 Experimental Values for Model Parameters

The experimental results from the uniaxial tension testing in this section were used to assign values to the parameters in the Stringfellow constitutive model in order to see how closely the model would predict the material response in plane strain tension. These parameters are entered into the ABAQUS UMAT subroutine via a properties vector as shown in Table 4.3.

The bulk modulus and shear modulus were calculated from the standard properties
of stainless steel, with an elastic modulus, $E$, of 195 GPa, and a Poisson's ratio, $\nu$, of 0.33, where $K = E/[3(1 - 2\nu)]$, and $G = E/[2(1 + \nu)]$.

The properties of the austenite phase, $n_a$, $s^*_a$, and $\gamma^*_a$, were determined from the uniaxial tension test at 20°C since there was little martensite forming before an equivalent strain of 0.40 at that temperature. The hardening rate for the 100°C test was significantly lower than the 20°C test, even though the martensite volume fractions were extremely low in both cases. The model could not predict this difference in hardening rate, and so the 20°C test was used as the baseline. This increase in hardening rate between the 100°C and 20°C test could be due to a precipitate hardening effect from a small volume fraction of finely dispersed martensite laths. At 20°C, the martensite forming could be fine enough to interact with dislocation motion and increase hardening in proportion to the square root of the concentration. At 100°C, since little martensite formed even through necking, it is reasonable to assume that this precipitation hardening effect would not occur during elongation. This question of hardening with small volume fractions of martensite could be answered more fully with a Transmission Electron Microscope (TEM) analysis.

The properties of the martensite phase, $n_m$, $s^*_m$, and $\gamma^*_m$, could not be determined experimentally due to the extremely low $M^*_S$ temperature. Tests on 304L at temperatures as low as 4K still cannot induce transformation without plastic deformation [53]. Since it is assumed that the transformed martensite inherits the dislocation substructure from the parent austenite, the martensite formed in the strain-induced regime would already be hardened. Therefore, the martensite data was estimated from the ability of the model to fit both the 20°C and −40°C stress-strain data, given the martensite volume fraction data at each temperature. Testing at more temperatures between 20°C and −40°C would provide an even more accurate picture of this curve.

The constants $\alpha$, $\bar{g}$, $s_g$, $\beta_0$, $g_0$, and $g_1$, used to describe the martensite evolution, were determined to best fit the volume fraction of martensite evolution data for the 100°C, 20°C, and −40°C uniaxial tension tests. Since these tests were isothermal and at a constant triaxiality, the equation for evolution (3.34) is simplified. Upon
integrating, the expression for the total volume fraction of martensite becomes

$$f = 1 - \exp(-\beta_0 P (1 - \exp(-\alpha \gamma_a)^{r_I}))$$

(4.14)

where $P$ is the cumulative probability distribution function given in Equation 3.28, which is a function of temperature and triaxiality. At $-40^\circ C$, $\alpha = 4.0$; at $20^\circ C$, $\alpha = 0.6$. For both temperatures, $\beta_0 = 6.0$ and $r_I = 4.0$.

The value of $g_2$, a weighting factor for the influence of triaxiality, was taken from a fit to the tension-compression martensite data of Young [70], since an in-plane compression test of the sheet material was impractical. However, plane strain compression of the 304L would be a fairly simple test to perform, and would give a better estimate for this variable.

The value of $r_I$, an exponent which describes the evolution of the number of shear band intersections for a given number of shear bands, was set to $r_I = 4.0$, representing a scenario where slip bands are initially parallel to each other, gradually becoming more random in orientation [47].

Values for the viscoplastic response of the material, $M$ and $\dot{\gamma}_0$, were not specifically tailored to this material since all testing was conducted at a strain rate of $1 \times 10^{-4} sec^{-1}$. Therefore, the value of $M$ was set to 100 at a reference strain rate of $\dot{\gamma}_0 = 0.0001 sec^{-1}$, since the sensitivity to strain rate should be low at such a low strain rate.

The constants $A_0$, $\dot{f}$, and $s_f$, which control the softening due to transformation when the rate of the volume fraction of martensite forming, $\dot{f}$, is high, were fit to the $-40^\circ C$ equivalent stress-strain behavior.

Finally, $\Delta_V$, a measure of the dilatation from austenite to martensite, was set to zero, even though Leal [34] found experimentally that $\Delta_V \approx 0.02$ to 0.05 in austenitic steels. This was done since the Stringfellow model does not allow $\Delta_V$ to affect $g_2$, and so $\Delta_V$ has little direct effect on the stress-strain behavior. Also, plastic dilatation leads to a numerical Jacobian that is nonsymmetric, requiring more complex solver routines in ABAQUS and a severe increase in computation time. Finally, there have
<table>
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<th>Symbol</th>
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</tr>
<tr>
<td>$A_0$</td>
<td>Maximum value of softening parameter, $A$</td>
<td>0.6</td>
</tr>
<tr>
<td>$f$</td>
<td>Dimensionless mean of probability function $A(f)$</td>
<td>0.35</td>
</tr>
<tr>
<td>$s_f$</td>
<td>Standard deviation of probability function, $A(f)$</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 4.3: Contents of PROPS vector of material parameters used by ABAQUS as input to the UMAT material-law subroutine developed by Stringfellow [63] for $T = -40^\circ C$. Relevant parameters were taken from fits to the experimental data in the current thesis.

been no experimental studies to relate $\Delta_V$ and $g_2$, although theoretically we would assume that a larger $\Delta_V$ would cause an increase in the sensitivity to triaxiality.

With these model parameters, a study of the strain distributions across the plane strain specimen width was conducted to model the efficacy of the specimen for different aspect ratios, presented in Chapter 5, and the theoretical plane strain tension and uniaxial tension equivalent stress-strain responses were predicted and compared with the experimental data in Chapter 6.
Figure 4-1: Type 304L stainless steel, cold-rolled, softened, and pinched-passed. Some martensite (dark) has formed in the austenitic matrix. 1000×
Figure 4-2: Uniaxial tension sheet specimen for 20°C, all dimensions in inches. Thickness = 0.047 in. (1.2 mm).

Figure 4-3: Uniaxial tension sheet specimen for -40°C, all dimensions in inches. Thickness = 0.047 in. (1.2 mm).
Figure 4-4: Plane strain tension specimen for 20°C, all dimensions in inches. L = 0.236 in. (6.0 mm), W = 1.890 in. (48.0 mm), Thickness of sheet = 0.047 in. (1.2 mm).

Figure 4-5: Plane strain tension specimen for −40°C, all dimensions in inches. L = 0.236 in. (6.0 mm), W = 1.420 in. (36.0 mm), Thickness of sheet = 0.047 in. (1.2 mm).
Figure 4-6: Strain distributions across the half-width of a plane strain tension specimen at 20°C and an aspect ratio of 1:8 for increasing axial strain. Sheet orientation: 45° from rolling direction.
Figure 4-7: $|\epsilon_{22}/\epsilon_{11}|$ across the half-width of a plane strain tension specimen at $20^\circ C$ and an aspect ratio of 1:8 for increasing axial strain. Sheet orientation: $45^\circ$ from rolling direction.
Figure 4-8: $|\epsilon_{22}/\epsilon_{11}|$ versus the normalized distance from the edge of the specimen for specimens with aspect ratios of 1:8 and 1:6, tested to an average axial strain of 0.30 at 20°C.
Figure 4-9: Uncorrected and Corrected axial stress-strain curves from the plane strain tension specimen compared to the experimental uniaxial tension stress-strain behavior and the theoretical plane strain tension curve at 20°, assuming isotropy and isotropic hardening of a von Mises yield surface.
Figure 4-10: Equivalent stress versus equivalent strain for uniaxial tension tests conducted on 304L stainless steel sheet at 20°C and −40°C and for 0°, 45°, and 90° orientations to the rolling direction of the sheet.
Figure 4-11: Stress-strain curves for 304L rod stock and sheet material in uniaxial tension at a strain rate of $10^{-3}s^{-1}$ [62].
Figure 4-12: Equivalent stress versus equivalent strain for uniaxial tension conducted on 304L stainless steel sheet at 20°C, -40°C, and 100°C and at 90° orientation to the rolling direction of the sheet.
Figure 4-13: Equivalent stress-strain behavior in uniaxial tension for 304L plotted with the equivalent hardening rate, \( \bar{h} = d\bar{\sigma}/d\bar{\varepsilon} \). When \( \bar{h} = \bar{\sigma} \), instability occurs.
Figure 4-14: Equivalent stress-strain behavior in uniaxial tension for 304L with yield strength artificially increased by 1000 MPa to simulate the behavior of a high strength TRIP steel plotted with the equivalent hardening rate, $\tilde{h} = d\tilde{\sigma}/d\tilde{\varepsilon}$. When $\tilde{h} = \tilde{\sigma}$, instability occurs.
Figure A.15: Equivalent stress versus equivalent strain for uniaxial tension and plane strain tension conducted on 304L stainless steel sheet at 20°C, −40°C
Figure 4-16: Equivalent stress-strain behavior in uniaxial tension and plane strain tension for 304L at 20°C plotted with the equivalent hardening rate, $\dot{h} = d\sigma/d\dot{\varepsilon}$ for uniaxial tension.
Figure 4-17: Equivalent stress-strain behavior in uniaxial tension and plane strain tension for 304L at $-40^\circ C$ plotted with the equivalent hardening rate, $\dot{h} = \frac{d\dot{\varepsilon}}{d\varepsilon}$ for uniaxial tension
Figure 4-18: Equivalent stress-strain behavior in uniaxial tension and plane strain tension for 304L at $-40^\circ C$ with yield strength artificially increased by 1000 MPa to simulate the behavior of a high strength TRIP steel plotted with the equivalent hardening rate, $\tilde{h} = \frac{d\tilde{\sigma}}{d\tilde{\varepsilon}}$ for uniaxial tension.
Figure 4-19: Forming limit diagram for 304L stainless steel at 20°C and -40°C. Conservative forming limit criterion of critical strain at maximum load was used.
Figure 4-20: Magnetization curve for a typical ferromagnetic material, saturating at a saturation magnetization, $J_s$. 
Figure 4-21: Volume fraction of martensite versus equivalent strain for uniaxial and plane strain tension of 304L at 20°C and -40°C.
Figure 4-22: Schematic of X-ray diffractometer from [10].
Figure 4-23: Experimental initial crystallographic texture for cold-rolled 304L stainless steel sheet.
Figure 4-24: Experimental crystallographic texture for plane strain compression to a through-thickness true strain $\varepsilon_{33} = -0.52$. 


Figure 4-25: Experimental crystallographic texture for uniaxial tension to an equivalent true strain $\bar{\varepsilon} = 0.40$
Figure 4-26: Experimental crystallographic texture for plane strain tension to an equivalent true strain $\tilde{\epsilon} = 0.35$
Chapter 5

Simulation of Plane Strain Tension Specimen

5.1 Introduction

The plane strain tension specimen was designed in part by using finite element analysis in order to predict the strain distribution across the specimen width and the total stress-strain behavior. Two dimensional modeling in plane stress and plane strain for this type of rectangular specimen had been attempted by Appleby et al. [2] to examine overall deformation, but no attempt was made to correct the stress-strain behavior with this data. Also, since the tab region was not modeled, the width constraint boundary conditions applied at the edge of the gauge section were unrealistically rigid.

In this work, the finite element code ABAQUS was used in conjunction with the UMAT subroutine from Stringfellow [63], in order to more accurately model the geometry and loading conditions of the plane strain tension test. The modelling was then used to help determine the best geometry for obtaining plane strain tension stress versus strain data.
5.2 Shear Strength of Constraint

In order to determine the type of joining method required to attach the tabs to the middle sheet of the plane strain tension specimen, a two dimensional analysis of the shear stresses was conducted. The specimen mesh is shown in Figure 5.5. The cross-section of the specimen design was modeled using 1068 4-node two-dimensional plane strain (ABAQUS CPS4) elements. The mesh was refined toward the edge of the tab in order to accurately predict the high shear stress concentrations in this region. In order to simplify the analysis, the material parameters used were the experimental 304L stress-strain data at 20°C, with the assumption of isotropic hardening. Due to the fact that the volume fraction of martensite per unit strain was very low for the 20°C case, this assumption was acceptable. Figure 5.5 shows the shear stress distribution in the specimen at an equivalent strain of 0.50. Shear stresses over 200 MPa are predicted in the region close to the end of the tab support. Since the strongest epoxy-nylon adhesives have a maximum shear strength of 40 MPa [18], an adhesive joining method would not be able to provide the constraint needed at the tab edges. This was also experimentally found to be the case [2]. From the analysis, it was decided that a more durable joining method must be used, such as welding, since the stresses were on the order of the yield stress of the material. Therefore, an Electron Beam (EB) Weld provided the necessary constraint, having the same yield stress as the bulk material (approximately 350 MPa). Also, the tab sections were spot welded together to facilitate a more gradual transfer of shear forces to the middle sheet of the plane strain specimen.

5.3 Effect of Gauge Length to Width Aspect Ratio

In order to determine the effect of the gauge length to width aspect ratio on the strain distribution of the specimen, a three dimensional analysis of the plane strain tension specimen was conducted.
5.3.1 Practical Constraints

In order to achieve a strain state of plane strain over a large fraction of the specimen width, the aspect ratio of the gauge length to the width of the specimen should be low [11]. However, there were some practical limits to the size of the gauge section and width in our case.

First, the circle grid in the gauge section must be visible, with at least one complete row of circles visible in order to determine strain distributions across the specimen width. As mentioned previously, the diameter of these circles is 0.05 inch (1.27 mm) on 0.0625 inch (1.59 mm) centers. Also, since the EB weld is centered 0.039 inch (1 mm) from the edge of the tab, this overhang of the tab region into the gauge section cannot be allowed to obstruct the view of the middle row of circle grids. Therefore, the limiting gauge length was set at 0.234 inch (6 mm), allowing for one full row of circles to be visible in the gauge, with one-half of a row of circles on either side.

The width of the 20°C specimens could not be wider than the width of the hydraulic wedge grips, which was 2 inches (50.8 mm). At −40°C, load restrictions on the test machine required that the specimen be no wider than 1.42 inches (36 mm). With a limiting gauge length of 0.234 inch (6 mm), the lowest aspect ratios allowed are 1:8 for 20°C, and 1:6 for −40°C.

5.3.2 Finite Element Modeling

Despite the physical constraints noted above, the plane strain tension specimen was modeled with gauge length-to-width aspect ratios of 1:10, 1:8, 1:6, and 1:4 in order to examine the variability of the strain distributions and stress-strain behavior.

The specimen mesh for the 1:8 aspect ratio is shown in Figure 5.5. Symmetry about the centerline and principal axes allowed us to model only one eighth of the specimen. Symmetry conditions are applied at nodes along the back face, mid-plane, and center-plane, while the nodes along the top face of the “tab” region of the specimen are constrained to have only equal axial displacements. This mode of displacement is intended to simulate the boundary conditions encountered by the
specimen when pulled in a hydraulic gripping test apparatus. The mesh utilized 1544
8-node 3-D reduced-integration brick (ABAQUS C3D8R) elements. The reduced in-
tegration scheme prevents mesh locking, an unrealistic stiffness that arises due to
the constraint of incompressibility. The mesh was refined at the outside edge of the
specimen in order to adequately model the width strains in this region.

Material parameters, incorporated into the analyses via a “properties” vector of
the UMAT subroutine [63] are given in Table 4.3. The model material is based,
as closely as possible, on the stress-strain and martensite evolution data in the ex-
perimental section of this thesis. For this analysis, the temperature parameter was
set to 20°C. Solutions were obtained under displacement control, with a constant
normalized displacement rate, in order to obtain an initial equivalent strain rate of
1 × 10⁻⁴ sec⁻¹. All analyses were run on a Hewlett Packard Series 735 computer using
version 5.3 of the ABAQUS finite element code, together with an updated version of
the UMAT subroutine implementation, provided by Socrate [58], of the constitutive
model developed for TRIP steels.

5.3.3 Results

Strain distributions of the axial and transverse strains along the mid-plane in the
gauge section are shown in Figure 5-4 for a specimen with an gauge length-to-width
aspect ratio of 1:8 at approximate axial strain in the plane strain region of 0.10, 0.20,
0.30, and 0.40. While the axial strains are quite uniform across 90% of the specimen,
the absolute value of the transverse strains deviates from a value close to zero near
the middle of the sheet to one-half the value of the axial strains at the edge. This
state of strain at the edge is the same state that develops in uniaxial tension. The
state of strain in the middle of the specimen, where the transverse strains are close
to zero, is the state which develops in plane strain tension. By examining the ratio of
transverse to axial strains in Figure 5-5, we can determine a fraction of the specimen
which is mainly in plane strain tension, and mainly in uniaxial tension. The criterion
used to determine a plane strain tension strain state was the fraction of the specimen
in which the absolute value of the ratio of the transverse to axial strains, |ε₂/ε₁|, was

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less than or equal to 0.2. In Figure 5-5, the model predicts a plane strain tension region of 80%. This figure also shows the prediction that the fraction of the specimen in plane strain tension will remain constant as the axial strain increases.

As the aspect ratio of gauge length to width decreases, the region of plane strain becomes a larger percentage of the total width. Figure 5-6 shows $|\epsilon_2/\epsilon_1|$ versus the normalized distance from the edge of the specimen for aspect ratios of 1:4, 1:6, 1:8, and 1:10 at an approximate axial strain in the plane strain region of 0.30. The fraction of the specimen in plane strain tension according to the above criterion is 58%, 73%, 80%, and 83% respectively.

The axial stress-strain response for the four aspect ratios was also examined. The axial true strain was calculated from the displacement of a node in the gauge length in the region of plane strain, and the axial true stress was simply the reaction force due to the imposed displacement divided by the cross-sectional area in gauge section. Figure 5-7 compares the axial stress-strain response for the four aspect ratios to the uniaxial tension and plane strain tension response predicted from simple one-element models. As the aspect ratio decreases, the axial stress-strain curve moves closer to the theoretical plane strain tension curve. From this analysis, it would seem that hardly any correction to the axial stress-strain behavior need be made for a plane strain tension specimen with an aspect ratio smaller than 1:4.

5.4 Correlation with Experimental Data

Plane strain tension specimens for 20°C were constructed according to the method described in Chapter 4, with aspect ratios of 1:8 and 1:6. By measuring the displacements in the gauge section from the circle grid, the axial and transverse strains were determined. Figure 5-8 plots the experimental data from the specimen with an aspect ratio of 1:8 against the axial and transverse strains predicted by the three-dimensional model for the same displacements of the tab region. Although there is scatter in the experimental data due to the error introduced by reading the circle grids, the general shape of the strain distribution data is similar to the predicted curves. At higher
strains (0.30, 0.40), the edge effects are more pronounced in the experimental data, causing the axial strains to become less uniform across the width of the specimen and the transverse strains to be greater than zero across a larger fraction of the specimen. The increased scatter in the data at higher strains was due to increased difficulty of accurately measuring the circle grid marks.

Figure 5-9 shows $|\epsilon_2/\epsilon_1|$ versus the normalized distance from the edge of the specimen for the experimental data and model predictions for an aspect ratio of 1:8. Here we see that, unlike the model prediction, the increasing strain level causes a decrease in the fraction of the specimen width which is in plane strain. This change of constraint with strain is most likely due to the approximation of modelling the specimen tab and sheet sections as one continuous body instead of modelling the localized EB weld region and spot welded regions connecting the tab and the sheet material. The actual boundary conditions at the edges of the specimen close to the tab constraint were probably more compliant, causing the level of constraint in the specimen to be affected by strain level.

Next, we compare the experimental strain distributions for two different aspect ratios, 1:6 and 1:8, with model predictions in Figure 5-10 for an axial strain in the plane strain region of 0.30. As predicted, the experimental data for the 1:6 specimen has a higher value of $|\epsilon_2/\epsilon_1|$ than the 1:8 specimen at a given normalized distance from the edge of the specimen. However, at a strain level of 0.30, both the 1:6 and 1:8 aspect ratio specimens have a higher value of $|\epsilon_2/\epsilon_1|$ than their predicted values at a given normalized distance from the edge of the specimen. As noted before, this due to the change in constraint in the specimen for increasing strain.

Finally, we examine the experimental axial stress-strain response for aspect ratios of 1:6 and 1:8. The axial true strain was calculated from the displacements of circle grids in the gauge length in the region of plane strain, and the axial true stress was found by dividing the total load by the estimated instantaneous cross-sectional area in gauge section (or $A_0/(1+\epsilon)$, where $A_0$ is the initial cross-sectional area, and $\epsilon$ is the value of the engineering strain in the plane strain region of the specimen). Figure 5-11 compares the axial stress-strain response for the two aspect ratios to the uniaxial
tension and plane strain tension response predicted from simple one-element models. As the aspect ratio decreases from 1:6 to 1:8, the axial stress-strain curve moves closer to the theoretical plane strain tension curve. However, these “uncorrected” axial stress-strain curves, even for the 1:8 aspect ratio, fall below the predicted axial stress-strain behavior for an aspect ratio of 1:4 (Figure 5-7).

5.5 Discussion

From finite element modelling of the plane strain tension specimen, it seemed that the uncorrected axial stress-strain curves would be close enough to the theoretical plane strain tension curve to allow the use of either a 1:6 or 1:8 aspect ratio specimen. While the strain distributions were similar to those predicted by the three-dimensional model used, the axial stress-strain behavior was well below that predicted by the constitutive model. Similar “softening” has been observed in testing by Hecker [26] and Ghosh [19] on 70-30 brass, where the hardening rate in a plane strain deformation state was lower than the hardening rate in uniaxial tension or compression. This behavior was also noted in a study of 304L by Miller [40] when testing in a state of shear (hollow torsion test). The most likely explanation for this “softened” response in plane strain is a combination of evolving crystallographic texture, microstructural, and substructural features [26]. In order that the plane strain tension curves in this work be as close to the actual stress-strain response of the material in the plane strain region of the specimen, the curves were corrected by accounting for the effect the edges in a state of uniaxial tension, as described in the previous Chapter. By using a correction technique, we can rule out the possibility that the softening behavior is due lack of a plane strain constraint in the specimen.
Figure 5-1: Two dimensional finite element mesh for plane strain tension test.
Figure 5-2: Magnitude of shear stress $S_{12}$ through the thickness of the plane strain tension specimen. Note the concentration of shear stress at the end of the tab, where the constraint needed for plane strain is most critical.
Figure 5-3: Three dimensional finite element mesh for plane strain tension test, modelling one-eighth of the entire specimen.
Figure 5-4: Finite element model prediction of the distribution of axial and transverse strains across the width of a plane strain tension specimen with a gauge length-to-width aspect ratio of 1:8 for average axial strains in the plane strain region of the specimen of 0.01, 0.02, 0.03, and 0.04.
Figure 5-5: Finite element model prediction of the absolute value of the ratio of transverse strain to axial strain across the width of a plane strain tension specimen with an aspect ratio of 1:8 for average axial strains in the plane strain region of the specimen of 0.01, 0.02, 0.03, and 0.04.
Figure 5-6: Finite element model prediction of the absolute value of the ratio of transverse strain to axial strain across the width of a plane strain tension specimen for aspect ratios of 1:4, 1:6, 1:8, and 1:10 at an average axial strain in the plane strain region of 0.30.
Figure 5-7: Finite element model prediction of the axial stress-strain response of the plane strain tension specimen for aspect ratios of 1:4, 1:6, 1:8, and 1:10, compared with predicted one-element responses in uniaxial tension and plane strain tension.
Figure 5-8: Experimental data for axial and transverse strain distribution across width of a plane strain tension specimen with an aspect ratio of 1:8, compared with finite element model prediction. (Predicted level of average axial strain in plane strain region: o=0.10, +=0.20, x=0.30, and *=0.40)
Figure 5-9: Finite element model prediction of the absolute value of the ratio of transverse strain to axial strain across the width of a plane strain tension specimen with an aspect ratio of 1:8 for average axial strains in the plane strain region of the specimen of 0.01, 0.02, 0.03, and 0.04, compared with experimental data (Predicted level of average axial strain in plane strain region: o=0.10, +=0.20, x=0.30, and *=0.40).
Figure 5-10: Finite element model prediction of the absolute value of the ratio of transverse strain to axial strain across the width of a plane strain tension specimen for aspect ratios of 1:6 (x) and 1:8 (o) at an average axial strain in the plane strain region of 0.30.
Figure 5-11: Experimental axial stress-strain response of the plane strain tension specimen for aspect ratios of 1:6 and 1:8 at 20°C, compared with the predicted one-element responses in uniaxial tension and plane strain tension.
Chapter 6

Correlation of Results

6.1 Introduction

The constitutive model reviewed in Chapter 3 was correlated with the experimental data for the equivalent stress-strain behavior and martensite evolution of 304L stainless steel in uniaxial tension and plane strain tension. The Stringfellow model assumes that the evolution of the volume fraction of martensite is dependent on temperature, strain level, and strain state. From the experimental work in Chapter 4, the values of the parameters of Stringfellow model were defined for the 304L stainless steel sheet used in this study. In this chapter, the model, incorporated into an ABAQUS UMAT subroutine [58], is used to predict the plane strain stress strain behavior for 304L stainless steel at $20^\circ C$ and $-40^\circ C$.

6.2 Model versus Experimental Data

The uniaxial tension and plane strain tension equivalent stress-strain behavior was predicted by the Stringfellow constitutive model using a simple one-element model in ABAQUS. For $20^\circ C$, all of the parameters in the PROPS vector of the UMAT (Table 4.3) described in Chapter 4 remained the same except for $\alpha$ and $T$. As mentioned previously, at $T = 20^\circ C$, $\alpha = 0.6$. Since very little martensite formed below 0.40 equivalent strain for both the uniaxial tension and plane strain tension case.
(Figure 4-21), there is correspondingly little difference in the uniaxial tension and plane strain tension equivalent stress-strain curves predicted by the model, shown in Figure 6-1. The experimental data also shows little difference in hardening behavior between uniaxial tension and plane strain tension; however, the plane strain tension stress-strain curve falls below that of uniaxial tension, as discussed previously. This response could be due to a geometric “plane strain softening”, which has been observed to occur in other fcc materials in the same strain state [26], [19],[40]. At strains approaching 0.40, the decrease in the hardening rate for uniaxial tension is slightly greater than that in plane strain tension. It could be that at strains high enough to finally produce martensite embryos, the initial effect of the martensite is first one of softening through the transformation shear strain.

For $-40^\circ C$, $T = -40^\circ C$ and $\alpha = 4.0$. Also, since the yield strength increased at the lower temperature, an effect which is not captured in the Stringfellow model, the yield strength of the model predictions for uniaxial tension and plane strain tension were increased to the experimental values ($\sim 425$ MPa). Due to the greater triaxiality of the plane strain tension stress state as compared to the uniaxial tension stress state, the model predicts a slightly greater equivalent stress per unit strain in plane strain tension than in uniaxial tension, as shown in Figure 6-2. Experimentally, the plane strain tension equivalent stress-strain behavior does exhibit the same characteristic “S”-shape curve as in uniaxial tension, but the equivalent stress-strain curve falls below that of uniaxial tension. As in the $20^\circ C$ case, this disagreement between the model and experiment could be due to a strain state affect, in which a difference in the geometry of the deformation can effect the yield and hardening behavior of a material through the evolution of crystallographic texture evolution and the morphology of the material substructure. In TEM studies by Miller [40], it was observed that the geometric orientation of slip in a state of plane strain (hollow-specimen torsion) was more planar than in a uniaxial tension case, resulting in a less “blocky” substructure. Since the constitutive model is based on the assumption that martensite forms at shear band intersections, these effects could have a dramatic impact on the evolution of martensite per unit equivalent strain in the material.
Looking back at the evolution kinetics for the volume fraction of martensite proposed, two model parameters control the variability of number of shear band intersections created per increase in number of shear bands due to geometrical constraints, $r_I$ and $\beta_0$ (Equation 3.23). If these parameters were dependent on strain state, perhaps tied to some Taylor model definition, a more realistic picture of the actual evolution of martensite could be drawn. While $r_I$ controls the rate of increase in volume fraction of shear band intersections per volume fraction of shear bands, $\beta_0$ affects the average volume fraction of shear band intersections; therefore, the parameter $\beta_0$ was adjusted to affect the overall volume fraction of martensite produced per unit equivalent strain. By decreasing the parameter $\beta_0$ from 6.0 to 1.0, the martensite evolution curve for plane strain tension at $-40^\circ C$ can be predicted. Figure 6-3 shows the model predictions of the martensite evolution curves for uniaxial tension and plane strain tension, where $\beta_0 = 6.0$ and $\beta_0 = 1.0$, plotted against the experimental data. Note that the correlation between the data and model prediction for uniaxial tension is excellent when $\beta_0 = 6.0$, whereas the correlation for plane strain tension is best for $\beta_0 = 1.0$. These results support the idea that small density of shear band intersections in plane strain results in a lower volume fraction of martensite. The equivalent stress-strain behavior predicted from forcing the model to predict the correct martensite evolution is shown in Figure 6-4. Here, good agreement is found between the model prediction and the experimental stress levels per unit strain in the plane strain tension case.

6.3 Conclusions

From comparing the behavior of 304L stainless steel in uniaxial tension and plane strain tension with the predictions made by the constitutive model for TRIP steels as developed by Stringfellow [65], it was found that the experimental plane strain tension stress-strain behavior could not be predicted by the model using the model parameters for the uniaxial tension tests. The increased triaxiality in the plane strain tension case lead to a prediction of an increased amount of hardening in that stress state as compared to uniaxial tension, while the experimental results showed a softer
response. However, if the model parameters controlling the evolution of shear band intersections are modified to reflect a possible strain state effect, then the experimental martensite evolution curve in plane strain tension can be predicted. Once the correct volume fraction of martensite per unit equivalent strain was predicted by the evolution kinetics, the stress-strain constitutive relations, using the "self-consistent" method for strain apportionment between the phases, was able to predict the equivalent stress-strain behavior.
Figure 6-1: Equivalent stress-strain behavior of 304L in uniaxial tension, plane strain tension, at 20°C, comparing prediction of constitutive model from Stringfellow et al. [65] with experimental data.
Figure 6-2: Equivalent stress-strain behavior of 304L in uniaxial tension, plane strain tension, at $-40^\circ C$, comparing prediction of constitutive model from Stringfellow et al. [65] with experimental data.
Figure 6-3: Volume fraction of martensite per unit equivalent true strain in uniaxial tension at 20°C (*), -40°C (o), plane strain tension at 20°C (x), -40°C (+), plotted against prediction of constitutive model from Stringfellow et al. [65] for uniaxial tension ($\beta_0 = 6.0$) and plane strain tension ($\beta_0 = 6.0, 1.0$).
Figure 6-4: Equivalent stress-strain behavior of 304L in uniaxial tension, plane strain tension, at $-40^\circ C$, comparing experimental data with prediction of constitutive model from Stringfellow et al. [65] after adjusting model parameters to correctly predict martensite evolution in plane strain tension; $\beta_0 = 1.0$. 
Chapter 7

Conclusions

7.1 Introduction

The research presented in this thesis has examined the influence of strain-induced martensitic transformation on the behavior of 304L stainless steel sheet in uniaxial tension and plane strain tension, two modes of deformation common to sheet metal forming. According to the kinetic model for martensite evolution in the strain-induced regime developed by Stringfellow [65], the increased triaxiality in the plane strain tension stress state should lead to an increase in the volume fraction of martensite transformed per unit equivalent strain. This increased martensite volume fraction would result in a higher hardening rate in the equivalent stress-strain behavior. With a higher hardening rate, the maximum strain before a necking instability occurs would be increased in plane strain tension, and the FLD of the 304L would look “flatter” compared to the FLD of a non-transforming material, since the critical strain to failure in plane strain tension would not be quite as low.

7.2 Analysis of Results

Mechanical testing was performed in uniaxial tension and plane strain tension, and the volume fraction of martensite in the samples was determined through VSM. The plane strain tension specimen was designed from a non-linear finite element analysis
which correctly predicted that approximately 70-80% of the specimen width would achieve a state of plane strain. A calculated correction to the equivalent stress-strain behavior of the total specimen gauge width allowed a conversion to equivalent stress-strain behavior for the purely plane strain region of the specimen.

It was found that a smaller volume fraction of martensite per unit equivalent strain was evolved in plane strain tension than in uniaxial tension, with a corresponding decrease in the hardening rate. This result is opposite to the prediction from the Stringfellow model, suggesting that there is another phenomenon occurring which has a first-order effect on the martensite evolution kinetics. One possible explanation could be that the difference in the deformation mode, or strain state, between uniaxial tension and plane strain tension. Differences in the evolution of crystallographic texture or substructural morphologies from the different modes of slip have been show to affect hardening rates both in theory and experiment. If the phenomenon is affecting the number of shear band intersections evolving per unit strain in the material, the martensite evolution kinetics would be effected.

When model parameters were adjusted to account for a decrease in shear band intersections forming in a plane strain tension state, the model was able to capture the equivalent stress-strain behavior of the plane strain tension test. This demonstrates that the stress-strain relations in the Stringfellow constitutive model can predict the softening/hardening effects of transformation due to strain-induced martensite evolution, as long as the kinetics of the evolution process are correct.

The actual shape of the Forming Limit Diagram (FLD) for the 304L stainless steels in the second quadrant was not significantly affected by the transformation, since the critical strain to failure in plane strain was not increased. Also, the yield strength of the material was low and the hardening was high even with very little volume fraction of martensite forming, so that instability in the deformation process did not occur until after 0.40 equivalent strain, at which point the hardening rates for both the plane strain and uniaxial tension tests were approaching the same value. Therefore, the traditional shape of the FLD for negative minor strains, with the critical strain to failure being the lowest in a state of plane strain tension, was retained. The uniaxial
tension test at 100°C, near $M_d$, did show a significant decrease in the strain to failure in uniaxial tension, indicating that the strain-induced martensitic transformation can affect the average critical strain level.

However, more data points at different strain states, especially in the first quadrant of the FLD, are needed before it can be concluded that the martensitic transformation has no effect on the overall shape of the FLD. As shown in Figures 2-2 and 2-3, the room temperature FLDs for 301 and 304 stainless steel both show a deviation from the traditional FLD shape in the first quadrant of a monotonically increasing critical major strain to failure for increasing positive minor strains. Investigating this deviation in the FLD for various temperatures would produce a clearer understanding of how the martensitic transformation affects the shape of the entire FLD.

Preliminary textural evolution studies revealed the initial texture of the cold-rolled sheet and a sharpening of this texture during uniaxial tension. However, the results for the plane strain tension texture were inconclusive due to the small sample size. Future testing will hopefully produce more meaningful crystallographic textural analysis.

One question that could be asked at this point is if texture evolution might also explain the difference in martensite evolution between simple tension and simple compression, instead of the idea of triaxiality and stress state. But testing by Hecker [27] in biaxial tension, which would be the same strain state as compression but with a higher triaxiality, produced a slightly higher volume fraction of martensite than in uniaxial tension. Since the strain state is the same, the texture evolution would be identical – only the stress state is different. Therefore, the role of both stress state and strain state must be known in order to fully explain the kinetics of strain-induced martensitic transformation.
7.3 Future work

7.3.1 Kinetics

In order to more fully understand the role of stress state and strain state in the kinetics of strain-induced martensitic transformations, further experimental studies should be conducted. By performing mechanical testing on the same 304L sheet stock in plane strain compression, a comparison of strain hardening and martensite volume fraction evolution can be made with the plane strain tension test, which is in the same state of strain, but a different stress state, with a higher triaxiality. This type of test has been performed by others [10] [26] studying deformation processes, since the plane strain compression mode of deformation is nearly equivalent to the process of cold rolling.

A more complete X-ray diffraction study should be undertaken to analyze differences in texture evolution in the sheet material, for different strain states (uniaxial tension, plane strain tension, biaxial tension), and different initial rolling directions. X-ray diffraction techniques can also be used as another method of determining volume fraction of martensite in the sample.

Finally, specimen samples should be prepared for Transmission Electron Microscopy to document substructure evolution for different strain states. By documenting the actual volume fraction of shear band intersections, $f_I$, and the volume fraction of austenite occupied by shear bands, $f_{sb}$, a relation between strain state and the model parameters $\beta_0$ and $r_I$ can developed. Additionally, by measuring lattice parameters between the austenite and martensite, as in the study of Leal [34], a correlation between the change in volume associated with transformation, $\Delta V$, can be related to the weighting factor, $g_2$, which determines the sensitivity of the probability function to triaxiality. Finally, TEM can be used to investigate the interactions between mobile dislocations in the austenite and finely dispersed martensite particles to determine if a precipitation hardening effect is contributing to the hardening rate for small volume fractions of martensite.
7.3.2 Formability

In order to document the effects of stress state and strain state on the formability of TRIP steels, more data are required to complete the FLD for 304L stainless steel. The data needed can be obtained through the traditional hemispherical punch-stretch tests. An interesting test of rectangular plane strain tension test used in this thesis would be to correlate the limit strains in the center of the specimen for various aspect ratios with the punch-stretch limit strains for the left-hand side of the FLD (negative minor strains).

Ultimately, an FLD should also be constructed for a high strength TRIP steel, since, at high strengths, the instability criterion $h \leq \sigma$ is met at an earlier equivalent strain. Therefore, the phenomenon of restabilization of a necking instability could occur, drastically effecting the average critical strain level for different temperatures. A candidate for this work would be the triple-phase steels which are being developed [54] to extend the formability limits of dual-phase ferrite-martensite steels.

Using the improved kinetics model from experimental studies or 304L, a finite element modeling study could be performed to investigate the localization phenomenon for high strength TRIP steels. Both the triaxiality and the strain path would be changing during the sheet localization process. Using an MK imperfection-based approach, the FLD for a TRIP steel could be predicted by simulating various tensile modes of deformation. In this way, the transformation characteristics of a material could be tailored, though composition, temperature, stress state, and strain state, to produce a given degree of formability.
Appendix A

A.1 Plane Strain Tension Specimens

A.1.1 Introduction

The sheet-metal forming industry has developed several varieties of plane strain tension tests over the years [44]. These specimens are designed to create a plane strain tension state, where the minor, transverse, or width strain is zero. Conventional uniaxial tension testing, which produces a negative width strain, does not provide information about the response of sheet materials when in a plane strain tension state unless the yield surface and hardening behavior of the material are known.

Plane strain testing for bulk material can be achieved by machining a groove in a wide plate to create a gauge section that is constrained across the width [11]. However, for thin sheet material, machining a uniform thickness gauge section is not only difficult, but may also produce different results from the testing of a full-thickness sheet due to possible gradients in material properties through the thickness from the rolling process [15]. Therefore, variable strain-state tests were developed in which sheet of various length-to-width ratios was stretched over a hemispherical punch [24]. These tests are dependent on not only the mechanical properties of the sheet metal, but also the sheet thickness, distribution of strain in the sheet, and interactions between the punch and the specimen, such as friction and bending. Also, since these tests were developed for constructing forming limit diagrams, the stress distribution in the sheet during testing was not needed. Therefore, the stress-strain behavior of
the sheet can be non-trivial to determine due to variable lubrication and specimen geometry.

Efforts have been made to remove the effects of friction and bending and so produce a plane strain stress-strain curve in order to examine the evolution of the yield surface for sheet materials. Three specimen types are reviewed that involve the use of a very wide, short sample, as well as various other methods of introducing constraint to the gauge section to prevent width strain.

A.1.2 Wide Sample Methods

Increasing the width of the sample and decreasing the gauge length changes the strain state from one close to uniaxial tension to that of plane strain tension, in which the minor strain approaches zero. In the rectangular sheet tension test designed by Appleby, et al.[2], samples were tested with decreasing length to width ratios (1:1, 1:2, 1:4) to approach the plane strain condition. The sample is further constrained by reinforcements welded to each side of the sample on both ends, making the sample three layers thick except in the gauge section, as shown in Figure A-1. In this type of sample, in-plane strains are measured by means of grid markings in the gauge section, and through-thickness deformations can be determined by holographic interferometry.

A similar approach was taken by Wagoner [67]. His study examined the effect of edge profile and length-to-width ratio on strain state in a tensile specimen. The specimen geometry that yielded the highest center strain at failure with a large region of plane strain is shown in Figure A-2. The approximate “plane strain region” where $\epsilon_{22}/\epsilon_{11}$ is less than 0.2, comprises about 80% of the specimen width. The length-to-width ratio in this specimen is about 1:4. However, unlike the rectangular sheet tension test, which always fails in the central, plane strain region, this Wagoner specimen tends to fail at the edges of the specimen, making it unable to measure plane-strain failure strains for certain specimen geometries. Also, special grips are used to exert a high clamping force at the inner contact lines of the specimen, providing the needed constraint to improve the plane strain condition.

Finally, a third type of wide specimen test was developed by Graf and Hosford [22].
In this test, the specimens are simply wide, short, rectangular, gridded sheets that have been formed around the ends of a cylindrical grip fixture and locked into place as shown in Figure A-3. In this simple gripping structure, the only constraints creating a plane strain condition are the length-to-width ratio (1:7.6) and friction between the specimen and the cylindrical grip. This type of specimen, although simple to prepare, does not come as close to approaching plane strain in the center section as do the two specimens discussed previously.

In all three of these specimens, the actual plane-strain stress-strain material response is calculated from the experimental loads and in-plane strains measured from grid markings. None of these samples produce a specimen which is 100% plane strain due to end-effects; the edges approach a state of uniaxial tension. Therefore, a calculation must be made to subtract the load being carried from these regions in order to determine the actual plane strain stress-strain response.
Figure A-1: Rectangular sheet tension test designed by Appleby, et al. [2].
Figure A-2: Rectangular sheet tension test designed by Wagoner [67].
Figure A-3: Rectangular sheet tension test designed by Graf and Hosford [22].
Appendix B

B.1 Plane Strain Tension Program

The following two programs calculate the plane strain tension axial true stress and true strain data from the total specimen load data and the axial true strain from the circle grid in the specimen gauge. The first program is for the tests performed at 20°C, with the stress-strain behavior for uniaxial tension represented by a relation \( \sigma_{11} = K(\epsilon_{11} + \epsilon_0)^n \). The second program is for the tests performed at -40°C, with the stress-strain behavior for uniaxial tension represented by a two-part curve fit; the complicated behavior of transformation softening and hardening required this type of function to best fit the experimental curve. The output in both cases is written to a separate file called 'psoutput'.

```
program pstrncalc

integer i,t,size
double precision load(1000),strn(1000),
& fraction,n,K,e0,E,eyield,t0,nuUT,gauge,
& utstrain,utthick(1000),utload,psload,
& pstthick(1000),nuPS,fracmax,w0,
& psarea,stress(1000)
character filelo*10, filets*10

n=0.53
K=155.0e7
e0=0.05
E=190.0e9
eyield=0.0017
t0=0.0012
w0=0.048
```
nuUT=0.5
nuPS=1.0
gauge=0.006

c************************************************************************************
cEnter Data
c************************************************************************************

write(*,*)'Please enter file size(<1000):'
read(*,*)size
write(*,*)'Please enter file for load:'
read(*,`(A') filelo
open(22, file=filelo, status='OLD')
rewind(22)
do 2 i=1,size
   read(22,*), load(i)
2 continue

write(*,*)'Please enter file for corrected true strain:'
read(*,`(A') filets
open(23, file=filets, status='OLD')
rewind(23)
do 3 i=1,size
   read(23,*), strn(i)
3 continue

write(*,*)'Please enter maximum fraction of specimen'
write(*,*)'width in uniaxial tension at maximum load:'
read(*,*) fracmax

do 4 t=1,size
   write(*,*) t

fraction= (fracmax/size)*t

calc load in region of uniaxial tension
sigma=K(e+e0)^n (from 20C UT test data)

Strain in uniaxial tension region is approx 80% that in
pstrain region.

utstrain=0.80*strn(t)
uttthick(t)=t0*(1+(exp(-nuUT*utstrain)-1))
write(*,*)'uttthick', uttthick(t)

c if strain is below elastic strain, do elastic.
c if not, do plastic.
if(utstrain.lt.eyield) then
  utload= E*utstrain*utthick(t)*
    & (1+(exp(-nuUT*utstrain)-1))*fraction*(w0/2.0)
else
  utload=K*(utstrain**e0)**n*utthick(t)*fraction*(w0/2.0)
endif
write('(*1x)',utload=' utload: ',utload*2.0)

continue

psload = load(t)*1000.0 - 2.0*utload
write('(*1x)',psload=' psload: ',psload)

psthick(t)=t0*(1+(exp(-nuPS*strn(t)))-1))
psarea=psthick(t)**(w0*(1.0-fraction))
write('(*1x)',psthick='',psthick(t)

stress(t)=psload/(psarea)
write('(*1x)',psstress=' stress(t): ',stress(t)/1.0e6

continue

write to file

open(24,file= 'psoutput',status='NEW')
do 6 i=1,size
  write(24,9)strn(i),stress(i)/1.0e6
format(F10.4,F10.4)

continue

close(22)
close(23)
close(24)
stop
end

program pstrncalcCOLD

integer i,t,size
double precision load(1000),strn(1000),
& fraction,n,K,e0,E,eyield,t0,nuUT,gauge,
& utstrain,utthick(1000),utload,psload,
& psthick(1000),nuPS,fracmax,w0,
& psarea,stress(1000)
character filelo*10, filets*10

E=130e9
eyield=0.002
ccross=0.16
t0=0.0012
w0=0.0355
nuUT=0.5
nuPS=1.0
gauge=0.006

c*************************************************************************
c Enter Data
c284 225
c*************************************************************************

write(*,*)'Please enter file size(<1000):'
read(*,*)size
write(*,*)'Please enter file for load:'
read(*,'(A)') filelo
open(22,file=filelo,status='OLD')
rewind(22)
do 2 i=1,size
   read(22,*), load(i)
   continue
write(*,*)'Please enter file for corrected true strain:'
read(*,'(A)') filets
open(23,file=filets,status='OLD')
rewind(23)
do 3 i=1,size
   read(23,*), strn(i)
   continue
write(*,*)'Please enter maximum fraction of specimen'
write(*,*)'width in uniaxial tension at maximum load:'
read(*,*)fracmax

do 4 t=1,size
   write(*,*)t

*************************************************************************
c Define PSFraction Evolution
*************************************************************************

c = (fracmax/size)*t
c calc load in region of uniaxial tension
c (curve fit from -40C UT test data)
Strain in uniaxial tension region is approx 80% that in pstrn region.

\[
\text{utstrain} = 0.80 \times \text{strn}(t) \\
\text{utthick}(t) = t0^*(1 + (\exp(-\nu UT \times \text{utstrain}) - 1)) \\
\text{write(*,'utthick',utthick(t)}
\]

if strain is below elastic strain, do elastic.
if not, do plastic.

\[
\text{if(utstrain lt. eyield) then} \\
\text{utload} = E \times \text{utstrain} \times \text{utthick}(t) \times \\
(1 + (\exp(-\nu UT \times \text{utstrain}) - 1)) \times \text{fraction} \times (w0/2.0)
\]

elseif(utstrain lt. ecross) then

\[
\text{utload} = ((\text{utstrain} - 0.0017)^*1.0e18)**(1.0/6.0) + \\
(\text{utstrain} - 0.017)^*17.0)**3*1.0e6*\text{utthick}(t) \times \\
(1 + (\exp(-\nu UT \times \text{utstrain}) - 1)) \times \text{fraction} \times (w0/2.0)
\]

endif

\[
\text{write(*,'utload:',utload*2.0} \\
\text{write(*,'fraction',fraction}
\]

continue

subtract from total load

\[
\text{psload} = \text{load(t)} \times 1000 - 2.0 \times \text{utload} \\
\text{write(*,'psload:',psload}
\]

find xsec. area of pstrn region

\[
\text{psthick}(t) = t0^*(1 + (\exp(-\nu PS \times \text{strn}(t)) - 1)) \\
\text{psarea} = \text{psthick}(t) \times (w0^*(1.0 - \text{fraction})) \\
\text{write(*,'psthick',psthick(t)}
\]

Divide pstrn load by xsection area of pstrn region

\[
\text{stress(t)} = \text{psload} / (\text{psarea}) \\
\text{write(*,'psstress:',stress(t)/1e6} \\
\text{utload=0.0} \\
\]

continue
c***********************************************************
c  Write to file
c***********************************************************

open(24,file='psoutput',status='NEW')
do 6 i=1,size
   write(24,9)strn(i),stress(i)/1e6
9    format(F10.4,F10.4)
6    continue
    close(22)
    close(23)
    close(24)
    stop
    end
Bibliography


