Low-Frequency, Low-Amplitude MEMS Vibration Energy Harvesting
by
Ruize Xu
Submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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Abstract

Vibration energy harvesters work effectively only when the operating conditions match with the available vibration source. Typical resonating MEMS structures cannot be used with low-frequency, low-amplitude and unpredictable nature of ambient vibrations. Bi-stable nonlinear oscillator based energy harvesters are developed for lowering the operating frequency while widening the bandwidth, and are realized at MEMS scale for the first time. This design concept does not rely on the resonance of the MEMS structure but operates with the large snapping motion of the beam at very low frequencies when proper conditions are provided to overcome the energy barrier between the two energy wells of the structure. A fully functional piezoelectric MEMS energy harvester is designed, monolithically fabricated and tested. An electromechanical lumped parameter model is developed to analyze the nonlinear dynamics and to guide the design of the multi-layer buckled beam structure. Residual stress induced buckling is achieved through the progressive control of the deposition along the fabrication steps. Static surface profile of the released device shows bi-stable buckling of 200μm which matches very well with the design. Dynamic testing demonstrates the energy harvester operates with 35% bandwidth under 70Hz at 0.5g, operating conditions that have not been met before by MEMS vibration energy harvesters.

Thesis Supervisor: Sang-Gook Kim
Title: Professor of Mechanical Engineering
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Chapter 1

Introduction

1.1 Motivation

Humans gain strength by harnessing power from nature. From ancient watermills and windmills, to modern solar park, wind farm, energy in various forms has been sought and converted to aid human work and empower human race. Energy has been the driving force of the human civilization: Industrial Revolution as an eminent example of the explosive power of the vast employment of fossil fuel transformed the way goods are made. A different demand on energy has been rising in our increasingly digitized world besides the vigorous energy demand increase.

Vastly different objects are gaining ability to "sense" the world, such as temperature, air pollution, vibration, light intensity, chemical gradient, human vital signs, thanks to the fast decreasing cost of the electronic devices for sensing, computation, data storage and transmission, driven by well-developed micro-fabrication. The sensing ability combined with processing of large amount of data and learning of the data will provide humans unthinkable new ability to control the environment and enhance human life. According to Cisco’s prediction, more than 26 billion devices will be connected in a big network [21], the so-called Internet of Things (IoT). Massive number of sensors are to be embedded/distributed to gain data. Among a variety of challenges, a reliable energy source with minimal maintenance will become a bottleneck to deploy the application. The current solution, battery and wired power, requires
either periodic replacement or costly long wires, limiting the feasibility and cost of the application of large number of wireless sensors, distributed in a large terrain or embedded in inaccessible sites.

Meanwhile, the power consumption of the microelectronic devices such as sensors and microprocessors have been decreasing. Although most Central Processing Units (CPUs) consume power in Watts to tens of Watts, CPUs designed for low-power devices consume much less power, such as Intel’s XScale, under 1W. State-of-the-art sensors dissipated power in microWatts. Led by Defense Advanced Research Projects Agency (DARPA), the Near Zero Power RF and Sensor Operations (N-ZERO) is aiming to transform the efficiency of sensors, and achieve less than 10 nW of power consumption. The advancement of sensing and computation capabilities will help make the Internet of Things more efficient and effective if and only if we can harvest nW to microW power from the environmental sources. Energy harvesting from the ambient environment, such as solar, thermal, chemical, wind and flow, human motions, vibrations is an attractive candidate of the power source for the growing number of connected things. If the power could be generated on site, the IoT devices could have an almost perpetual power.

Vibration as a ubiquitous power source can be found in civil structures, machines, human motions etc. It can provide energy while other forms of energy are not available such as there is no sunlight in embedded system. The kinetic energy in vibrations could be converted into electricity, through electromagnetic, electrostatic or piezoelectric effects. Even though there have been tremendous advances in vibrational energy harvesting, challenges still remain with the existing technologies, which is evident with the fact the there are no commercially applicable and successful MEMS energy harvesting devices.

The power source of the massive number of sensors should be manufactured with high volume and low cost to match with the low cost of the system. The size of the energy harvester needs to allow it to fit into the small sensing unit. With the cost and size compatible to the current sensors and processors, the practical effectiveness and requirements of the energy harvesters should then be considered as follows. The
generated power need to cover the power dissipation of the loads, and tens to hundreds of microwatts power output is desirable. The energy harvesters also need to be operated in ambient environment, which brings the requirement of low-frequency, low-amplitude, wide-bandwidth operating conditions.

1.2 Thesis Objective and Contribution

This work aims to achieve MEMS-scale energy harvesting from wide bandwidth (>20%), low-frequency (<100Hz) and low-amplitude (<0.5g) input vibrations, which has been considered as the performance specifications of MEMS energy harvesters for real-world application.

As the ambient vibrations are low-frequency and low-amplitude in a wide bandwidth, an effective energy harvester should operate under these conditions. The match of operating conditions to the ambient vibrations brings up challenges to the current energy harvesting techniques. The linear resonators absorb energy from ambient vibrations effectively at their resonant frequencies, but their gain-bandwidth trade off limits their application in wide-band ambient vibrations. Moreover, the small size of the MEMS devices increases the resonant frequency significantly compared to macro-scale devices. The mono-stable nonlinear oscillators widen the bandwidth significantly, while the operating frequency range is increased by the stiffening effect.

In order to tackle the above challenges that the conventional resonance based harvesters cannot achieve, a novel concept of energy harvesting based on a bi-stable nonlinear oscillator is developed. Bi-stable oscillators have been reported at macro-scale to widen the bandwidth. However, the large-amplitude oscillations at very low frequencies of the bi-stable oscillators are independent from the device size, which makes it favorable for low-frequency vibration energy harvesting at MEMS scale. In inducing the bi-stability at MEMS scale, the buckled beam based approach is advantageous. The simple mechanical structure does not need magnets, which are typically required for macro-scale devices to induce buckling, and is compatible to general MEMS fabrication process as well as the PZT sol-gel deposition. Residual
stress, a pseudo material property of micro-fabricated thin films, is troublesome and typically avoided in MEMS devices, is employed to induce buckling in a clamped-clamped multi-layer beam. With extensive characterization and monitoring during fabrication, the desired bi-stable buckling is achieved.

A meso-scale buckled beam with PZT patches is firstly built and tested. The comparison between the buckled and unbuckled configurations demonstrates the enhancement in power generation at lower frequencies of the buckled beam based device.

To realize potential large-volume, low-cost manufacturing of this technology, the energy harvester is built with MEMS monolithic fabrication process. Residual stress in micro-fabricated thin films, typically detrimental to MEMS structures and is canceled out in MEMS devices, is intentionally introduced in MEMS multi-layer beam structure to induce desired buckling. Feedback control of the stress is implemented after each deposition to minimize the deviation of the fabricated structure's buckling to the target. A micro buckled beam oscillator with four layers is fabricated to verify the feasibility of the stress control scheme. Extensive characterization of the micro-fabricated thin films, including the deposition rate and residual stress are done during the fabrication. The fabricated devices show buckling in both transverse and longitudinal directions due to the bi-axial stress. Low-frequency (<100Hz) and low-amplitude (<0.5g) oscillations are observed.

An electromechanical lumped model with closed-form lumped parameters is built to analyze the nonlinear dynamics and to guide the design. The model is solved analytically using harmonic balance method, so the frequency response of both inter-well and intra-well oscillations are obtained. Combined with Melnikov’s theory, which estimates the input amplitude threshold of the bi-stable system, the analytical modeling is able to predict the frequency response and amplitude requirement for any buckled doubly clamped multi-layer beam design at each frequency. Parametric sweep of the design parameters such as the thickness of each layer can be done with the model to optimize the design and match the target frequency and amplitude. The piezoelectric coupling enables the model to estimate the generated electrical power. An equivalent circuit model is also constructed with both linear and nonlinear turns ratio, for future
A PZT embedded fully functional MEMS energy harvester is fabricated. The buckled beam array design eliminates the corrugations in transverse direction of the beam while preserves the buckling in longitudinal direction. The central proof mass couples the beam array and constrains the rotational mode of the suspended structure during oscillations. The surface profile scan demonstrates successfully implemented bi-stable buckling with designed amount. Power measurements demonstrate wide bandwidth operation, with low-frequency, low-amplitude vibrations, and verify the design concept. The design, modeling and fabrication and the state-of-the-art operating condition, all contribute to realizing a MEMS-scale real-environment applicable energy harvester and potential commercialization.

1.3 Thesis Organization

This thesis is organized to present the design, modeling, fabrication and testing of three generations of prototypes from meso-scale to MEMS scale. Chapter 2 reviewed previous works on vibration energy harvesting, with emphasis on the new trends such as the bi-stable systems based design and efforts in lowering the operating frequency. Chapter 3 presents the design concept of bi-stable buckled beam based energy harvester and residual stress induced buckling. A meso-scale energy harvester prototype is reported to verify some theoretical predictions and the enhancement of the new design. Lumped parameter model of the nonlinear bi-stable oscillator based energy harvester with multi-layer and residual stress embedded and an equivalent circuit model are described in Chapter 4. To further verify the design targets including the operating conditions and feasibility of stress control in MEMS fabrication, the second-generation prototype, a micro buckled beam with four layers of different materials is built and presented in Chapter 5. Leveraging on the lumped parameter model and the testing results of the micro buckled beam oscillator prototype, Chapter 6 presents the optimization on the design parameters and the design of the PZT embedded full energy harvester. Detailed fabrication and testing of the third-generation prototype
- PZT integrated energy harvester, are reported in Chapter 7 and the matching between the design goals and the test results are demonstrated. Chapter 8 summarizes the work and suggests the future work.
Chapter 2

Background

2.1 Vibration Energy Harvesting

A vibration energy harvester converts kinetic energy in vibrations to usable electric energy. The vibrations come from the ambient environment, and depending on the applications, the vibration source could be civil structure, machinery, human body and anything that vibrates and can provide energy to the parasitic energy harvesters. The energy harvesters are typically mounted on the vibration source, so the motion of the frame/package of the energy harvester is coupled to the motion of the vibration source, then the mechanical oscillator inside the harvester device moves relatively to the frame/package. The relative mechanical movement is transduced into electrical signals through electromagnetic, piezoelectric or electrostatic effects. The generated electrical signal could then be stored or consumed by the electrical load.

A basic configuration of the vibration energy harvester could be a cantilever beam with embedded piezoelectric material and electrodes, or a moving magnets and surrounding metallic coils. However, real environment applicable energy harvesters have more complications such as the effectiveness of absorbing the energy from the ambient environment, the efficiency of converting the mechanical energy into electrical energy, and the effectiveness of exploiting the harvested energy by the electrical circuit. The total efficiency of the energy harvesters depends on all three stages. The efficiency of the energy transduction largely depends on the quality of the piezoelec-
tric material for the piezoelectric energy harvesters, or the magnets and coils in the electromagnetic energy harvesters. New materials with better coupling efficiency are being developed [22–24]. Advanced electrical circuits for exploiting the power generation capability and manage power management for power storage and release are investigated [25–27]. How to effectively absorb energy from the ambient vibrations lies in the mechanical oscillator design, and is the scope of this work.

2.1.1 Working Principle

Mechanical Resonator Based Energy Harvesters

The most fundamental and popular strategy to absorb kinetic energy from the host is to utilize the mechanical resonance. Assume the host is an infinite energy source, which is a typical case since the energy harvesters are often much smaller and lighter than the host, resonance amplifies the energy transferred to the parasite from the host. A general linear resonator could be represented as a mass-spring-damper system as shown in Figure 2-1. Assume the inertial frame is excited by an external sinusoidal vibration of the form \( z(t) = z_0 \sin(\omega t) \). The mass and spring then transfer the vibrations from the frame to the mechanical oscillator inside the frame. Different transduction mechanisms convert the mechanical energy in the mechanical oscillator
through the damping. The dynamic equation of the system is,

\[ m\ddot{x}(t) + b\dot{x}(t) + kx(t) = -m\ddot{z}(t) \]  \hspace{1cm} (2.1)

where \( m \) is the proof mass, \( b \) is the damping coefficient, \( k \) is the spring stiffness and \( t \) is the time. The damping includes both the mechanical loss and the electrical load. The steady state solution is,

\[ x(t) = \frac{\omega^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + \left(\frac{b}{m}\right)^2}} z_0 \sin(\omega t - \phi) \]  \hspace{1cm} (2.2)

where \( \phi \) is the phase angle,

\[ \phi = \tan^{-1}\left(\frac{b\omega}{k - \omega^2 m}\right) \]  \hspace{1cm} (2.3)

At resonance, maximum extractable electrical power is achieved,

\[ P = \frac{\zeta_e}{4\omega_n (\zeta_e + \zeta_m)^2} mA^2 \]  \hspace{1cm} (2.4)

where \( \zeta_e \) and \( \zeta_m \) are the damping ratio due to electrical and mechanical damping respectively, and \( A \) is the input acceleration amplitude. The extractable power is inversely proportional to the resonant frequency at a fixed acceleration amplitude \( A \). The extractable power is also proportional to \( A^2 \), which limits the energy available for conversion with weak vibrations whatever the specific design is chosen. The output power is proportional to the proof mass, so it is obvious a large proof mass is desirable for energy harvesting. Finally, the term composed of the mechanical and electrical damping ratio implies that the maximum power is achieved when the electrical damping matches the mechanical damping. When the electrical damping is equal to mechanical damping (\( \zeta_e = \zeta_m \)), the maximum electrical power that can be generated is,

\[ P_{e,\text{max}}(\omega_n) = \frac{mA^2}{16\omega_n \zeta_m} \]  \hspace{1cm} (2.5)
Equation 2.5 represents the theoretical maximum of extractable power which can be dissipated in the electrical load.

A simple and widely used linear resonator based energy harvester design has a cantilever beam structure. Beeby et al. [2] developed cantilever based electromagnetic energy harvester that resonates at 52Hz. It is still the state of the art in terms of energy density and normalized energy density (energy density normalize by input vibration’s amplitude squared). At 60mg input vibration level, the harvester generates 46μW power, which gives high power density of 300μW/cm³. However, based on a cantilever beam, the linear resonance limits the harvester’s bandwidth to ~1%. The magnets and coils in this device complicate the fabrication. Strictly speaking, like many other reported small scale prototypes, this prototype is not a MEMS device: only the cantilever beam is micro-fabricated, while other parts including the base, tungsten mass, magnets and coils need to be assembled.

At MEMS scale, cantilever beams are the simplest structure and have been made [11]. A different shape of cantilever could be designed to reduce the stiffness and the resonant frequency, such as a S-shaped cantilever [28]. However, these designs all suffer from very narrow bandwidth, which is inevitable for linear resonance based design. Modifications could be made on cantilevers to widen the bandwidth such as an array of cantilever beams that resonate at different frequencies [29] so that device’s overall bandwidth is widened; or cantilevers are attached to other structures such
as buckled beams [30, 31] to widen the bandwidth using the buckled structure and transfer the energy to the cantilever beam. These designs suffer from large size, from the redundant beams and larger auxiliary structure, and may suffer from significant inefficiency.

Transduction Mechanisms

The absorbed kinetic energy needs to be converted to electricity to power electronic devices. Various mechanisms are used for the transduction. The widely used mechanisms include electrostatic [27, 32–35], electromagnetic [2, 36–41] and piezoelectric [1, 9, 11, 12, 42–44]. Electrostatic generators have the basic structure as a capacitor. With an external voltage source, charge accumulates on the capacitor. When the overlap area of the two plates or the distance between the two plates changes due to applied vibration, the work done against the electrostatic force is converted into electric current. It is easy to integrate electrostatic harvesters into microsystems, but input charge is needed for operation, and its low current (due to output impedance and voltage are high) makes it not suitable as power sources [45]. Electromagnetic generators generate electricity from the relative motion of a magnet to a coil based on Faraday’s law. They do not require a voltage source to operate, but the output voltage is relatively low, and drops as the size of the generator decreases [1, 46]. It is found that below 0.5cm$^3$, the piezoelectric mechanism has an advantage over the electromagnetic mechanism [47]. Moreover, it is still a challenge to implement the assembly of magnets in MEMS scale [2]. Piezoelectric materials convert mechanical strain energy to electrical energy through piezoelectric effect. The piezoelectric generators require no voltage source to operate and produce high voltage output. Piezoelectric materials possess high efficiencies from 40% of bulk PZT [20] to > 80% of PMN-PT [23, 48], which are favorable for energy harvesting purpose. Detailed comparison on the three transduction mechanisms could be found in [12, 20, 45] and will not be repeated here. Two relatively new transduction mechanisms are discussed in the following.

Electret based energy harvesting has been developed in recent years. Electret is
Deposition | Spin coating | Spin coating |
<table>
<thead>
<tr>
<th></th>
<th></th>
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<tbody>
<tr>
<td>Functionalization</td>
<td>Charge implantation at elevated temperature</td>
<td>Poling</td>
</tr>
<tr>
<td>Stability</td>
<td>Thermal</td>
<td>Mechanical</td>
</tr>
<tr>
<td>Advantage</td>
<td>Decoupled from the mechanical movement</td>
<td>No assembly needed</td>
</tr>
</tbody>
</table>

Figure 2-4: Electret versus piezoelectricity based energy harvesting.

the electrostatic analog of magnets (a dielectric material that has a quasi-permanent electric charge or dipole polarization). The motivation of using electret is to overcome the drawback of requiring external voltage source that the conventional electrostatic approach has. Fluorinated polymer materials or SiO$_2$ based materials such as waxes, polymers, resins, Teflon etc could be used as the electret. Corona charging, thermal poling, contact charging, and electron-beam irradiation are used for poling electrets to inject initial charge. The charge on electret is supposed to be preserved during the lifetime of the device, so that no external voltage source is needed. An example of successfully fabricated electret based devices is shown in Figure 2-3. Electret based approach still have some challenges such as it needs to increase the surface charge density, long term stability issue, deterioration at elevated temperature and difficulty in MEMS scale assembly. This approach is comparable to piezoelectricity based
approach, and they are compared in the Figure 2-4. Some other works on electret can be found in [3, 49-51].

Electrification effect has been used for energy conversion in triboelectric nanogenerator (TENG). TENGs also target at the weakness of the conventional electrostatic based energy harvesting. Instead of adding an external voltage source, TENGs generate charge with contact electrification and then use the electrostatic induction to convert the energy. Electrification is a phenomena that a material becomes electrically charged after it gets into contact with a different material [52]. The charged electrodes can then be mechanically driven periodically to induce electrical potential and generate electricity as other electrostatic energy harvesters. The output power is not reported in [52-54], but TENGs typically have a flexible substrate such as polydimethylsiloxane (PDMS) film, they have been developed as shoe sole insert energy harvester [53] and human body implantable energy harvester [54].

2.1.2 Piezoelectricity

Piezoelectricity was discovered by the brothers Pierre and Jacques Curie in 1880. The direct piezoelectric effect, which is when a material is subjected to a mechanical deformation, electrical surface charges are generated, was firstly found. The inverse effect, which is mechanical deformation is generated by applying electrical potential was later found by Lippmann. Piezoelectricity is the outcome of the lack of center inversion symmetry in atomic structure [55]. Crystals that possess piezoelectricity could be found in nature, such as quartz, tourmaline and sodium potassium tartrate.

Polycrystalline materials could also exhibit piezoelectricity, such as lead zirconate titanate (PZT), which has perovskite crystal structure (Figure 2-5). Above a temperature known as the Curie point, the crystallites exhibit cubic symmetry so the positive and negative charge coincide and there are no dipoles. However, below the Curie point, the tetragonal symmetry shows in the crystallites and the positive and negative charges separate. The dipoles align in a specific direction in a region called Weiss domain. Within the domain, the dipoles form a net dipole moment, or polarization (dipole moment per volume). In a material, the polarization in Weiss domains
are randomly oriented and hence there is no piezoelectric effect exhibited. Therefore, a process called poling is treated to the material to align the dipoles in different domains. A strong electric field is applied to the material at an elevated temperature (below the Curie point) during poling. The dipoles in different domains tend to align with the applied electric field. After removing the electric field, there are still many dipoles remain that arrangement (remnant polarization), and hence the material obtains the piezoelectric effect. The experimental details on poling process are covered in section 7.3.2.

Piezoelectric effect couples the mechanical and electrical domains. Mathematically, it connects mechanical stress, which is a second-order tensor, to electrical polarization, which is a first-order tensor, via a third-order tensor. With symmetry, this tensor can be reduced. The mechanical and electrical coupling of linear piezoelectricity can be described by the piezoelectric constitutive equations [56]:

\[ S_{ij} = s_{ijkl}T_{kl} = d_{kij}E_k \]  

\[ D_i = d_{ikl}T_{kl} + \varepsilon_{ik}^T E_k \]  

where \( S_{ij} \) is the strain component, \( T_{kl} \) is the stress component, \( E_k \) is the electric field component, \( D_i \) is the electric displacement component, \( s_{ijkl}^E \), \( d_{kij} \) and \( \varepsilon_{ik}^T \) are the elastic, piezoelectric, and dielectric constants respectively. The subscripts \( i, j, k, l \) run over integers 1, 2, 3. The superscripts \( E \) and \( T \) denote at constant electric field and at constant stress respectively. Different piezoelectric materials used in
Table 2.1: Properties of common piezoelectric materials. [20]

<table>
<thead>
<tr>
<th>Property</th>
<th>PZT-5H (10⁻¹² C/N)</th>
<th>PZT-5A (10⁻¹² C/N)</th>
<th>BaTiO₃ (10⁻¹² C/N)</th>
<th>PVDF (10⁻¹² C/N)</th>
<th>AlN (10⁻¹² C/N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{33}$</td>
<td>593</td>
<td>374</td>
<td>149</td>
<td>33</td>
<td>5.5</td>
</tr>
<tr>
<td>$d_{31}$</td>
<td>274</td>
<td>171</td>
<td>78</td>
<td>23</td>
<td>3</td>
</tr>
<tr>
<td>$k_{33}$</td>
<td>0.75</td>
<td>0.71</td>
<td>0.48</td>
<td>0.15</td>
<td>0.33</td>
</tr>
<tr>
<td>$k_{31}$</td>
<td>0.39</td>
<td>0.31</td>
<td>0.21</td>
<td>0.07</td>
<td>0.18</td>
</tr>
<tr>
<td>Relative Permittivity $\varepsilon_r$ (10⁻¹² C/N)</td>
<td>3400</td>
<td>1700</td>
<td>1700</td>
<td>12</td>
<td>9</td>
</tr>
</tbody>
</table>

micro-fabrication include quartz, zinc oxide, aluminum nitride, lead zirconate-titanate (PZT), Pb(Mg₁/₃Nb₂/₃)O₃-PbTiO₃ (PMN-PT) and polymer piezoelectrics such as poly-vinylidene fluoride (PVDF). PZT is employed for our energy harvesters in that it has high piezoelectric coefficients and thus will have high output voltage and power. Table 2.1 shows the properties of some common piezoelectric materials.

2.2 Energy Harvesting From Wide-Bandwidth Vibrations

Even though linear resonators greatly enhance the effectiveness of energy absorption, they suffer from drawback of very limited bandwidth compared to the rather wide bandwidth of the ambient vibrations: little mismatch between the resonant frequency of the energy harvester and the frequency of the source vibrations dramatically decreases the generated power. The challenge renders the conventional energy harvester designs ineffective in real environment.

In pursuing wide-bandwidth vibration energy harvesting, various approaches have been sought, such as employing multiple resonators [29,57], frequency tuning [58–62], parametric resonance [6,63], nonlinear resonance [5,7,8,16,64–68]. These mechanisms have benefits on some aspects but at the same time faces challenges and limitations.

The natural frequency of a beam based resonator can be tuned by changing the axial tension of a beam through manipulating magnets [58,59]. Beam dimensions [60], proof mass [61] have also been tuned mechanically to widen the bandwidth. However, frequency tuning inevitably consumes power, the tuning efficiency is low and the tuning range is limited [62]. Another design to widen the bandwidth is the device
Magnetic force has been introduced to provide nonlinear restoring force. Mann and Sims [5] proposed an electromagnetic energy harvester that uses the magnetic levitation to achieve wide bandwidth. The design has a center magnet, and two magnets on two sides with the poles oriented to repel the center magnet, the suspending center magnet thus have a nonlinear restoring magnetic force (Figure 2-6). Another design is to fix a magnet at the tip of a cantilever and place a fixed iron stator near the cantilever tip to achieve Duffing mode resonance [64]. The magnetic forces between the magnets and the iron stator create a nonlinear spring, whose nonlinearity is determined by the strength of the magnets and the size of the air gap between the magnets and the iron. The energy harvesters in [5, 64] are electromagnetic but in macro scale. [65] demonstrated an electrostatic energy harvester utilizing a mechanical softening spring to increase the bandwidth. The nonlinearity is achieved by using angled suspension beams. When the spring deflects in one direction, a compressive axial force first builds up, beyond a certain displacement, the axial force changes to
tension; in the other direction, axial force is always tensile. The asymmetric force-displacement relationship in the suspension therefore has a nonlinear spring softening behavior.

Parametric resonance based system can be outlined as: an oscillatory system begins to oscillate with its free frequency \( f \) when one of the system parameters vary at frequency \( 2f \). Examples that can be found in life include pendulum with a variable length, and a swing, which consists in a periodic raising and lowering center of gravity of the body of the person on the swing. The motivation of the parametric energy harvesters is to maximize the power output by mechanically amplifying displacement amplitude, and to broaden the operational frequency bandwidth with the nonlinear characteristics of its resonant peak. The recent works indicate that the parametric resonance could be used to lower the working frequency. To achieve parametric resonance, the excitation frequency \( \omega \) needs be approximately \( 2\omega_0/n \), where \( \omega_0 \) is the natural frequency of the resonator and \( n \) is an integer defining the order of parametric resonance \([63]\). As can be seen, when the number \( n \) increases, the excitation frequency that makes high order resonance will decrease \((1/n)\), which lowers the frequency. Some challenges and limitations of parametric resonance include: a prerequisite of a non-zero initial displacement is needed to "push" the system out of its stable equilibrium \([63]\); the excitation amplitude needs to overcome an initiation

Figure 2-7: Power spectrum of a parametric resonance based energy harvester at acceleration of 5.1 m/s\(^2\) \([6]\).
threshold prior to accessing the parametric resonant region [63]; the accessibility of the higher orders is increasingly difficult with higher damping [6]; the frequency bandwidths are significantly narrower in the higher order resonance (lower frequencies) [6], can be seen in Figure 2-7; the third and higher orders need longer transient build-up time (> 1 minute). In sum, this method needs to overcome the challenges to lower the input threshold and to decrease damping to realize the higher order (lower frequency) modes. In implementation, the researchers have tried to decrease the damping and the initiation threshold, by using vacuum packaging and adding more springs to amplify the displacement. Yet it still needs to face its fundamental limitation of the high order resonance. This approach may be only practical for very specific and non-time variant excitation frequencies.

Bi-stable nonlinear oscillator based energy harvesters have been investigated for widening the bandwidth [7, 8, 16, 66-68]. Bi-stable magnetoelastic structure was first investigated by Moon and Holmes [16]. The device consists of a ferromagnetic cantilevered beam with two magnets located symmetrically near the free end. When subjected to harmonic base excitation, the ferromagnetic beam is attracted by both magnets and could have bi-stable oscillations. Erturk et al. [7, 66] used this device, attach two piezoceramic layers to the root of the cantilever and obtain a bimorph generator (Figure 2-8a). An order of magnitude larger power output over a wide frequency range is observed with this device. Bi-stable nonlinear oscillator may also be implemented with specially designed mechanical structures. Bi-stable nonlinear
oscillator based energy harvesters in [8, 69] were realized by exerting an axial compression and forming a buckled configuration. A piezoelectric beam is clamped on both ends on a base that is excited vertically. One of the two clamps is free to move to compress and buckle the clamped beam as shown in Figure 2-8b.

2.3 MEMS-Scale Energy Harvesting

Energy harvesters with a small footprint may target different applications from macro-scale energy harvesters, such as the harvesters need to be embedded, where the space for the power source is limited; or to power a large number of sensor nodes, so a comparably small size is desirable. The macro-scale devices reported in literature can easily achieve milli-Watts to Watts power, and the frequency can be much lower than smaller devices. The low operating frequency could be due to larger structures such as beams and heavier proof mass are used; and the high power may be attributed to larger magnets or piezoelectric patches. These options are not available to MEMS-scale devices. Therefore, comparison between MEMS based energy harvesters and macro-scale devices are not meaningful. Small-scale energy harvesters reported in the literature are quite often not completely based on MEMS, assembly is typically involved, so these devices are referred as MEMS-scale only.

Najafi et al. [9] developed a cantilever based vibration energy harvester (Figure 2-9a). The device utilizes thinned bulk-PZT (PZT-5A, 20μm) to bond on SOI wafer. Due to the cantilever based design with a relatively large beam length (7mm), and
attached heavy tungsten proof mass (0.3g), the device can resonate $155\text{Hz} \sim 165\text{Hz}$, the bandwidth is 4% with 0.1g input and 8% with 1.5g input. Cantilever based MEMS-scale devices can also be found in [2,11,70,71]. New designs at MEMS scale are targeting to widen the bandwidth, lower the frequency or harvester energy from multiple directions: by using parylene as the spring material, a MEMS generator based on electret can resonate below 50Hz (linear resonance) [72]; disk-shaped mass suspended by concentric circular springs (Figure 2-9b) can resonate in two dimensions in the device plane, with resonance frequency around 370Hz; a hybrid MEMS energy harvester has both piezoelectric and electromagnetic transduction (linear resonance, $< 2\%$ bandwidth) [71]; [12,73] both are in MEMS scale and utilize the stretching in beams to obtain nonlinear stiffening response to widen the bandwidth.

A PZT based MEMS power-generating device was developed in our group [11]. The basic structure was a cantilever with a proof mass at the end (Figure 2-10a). The top electrode was patterned into an interdigitated shape to employ the 33 mode of the piezoelectric transducer. This new configuration made the device generate 20 times higher voltage than that of the 31 mode design. At the resonant frequency of 13.9 kHz (Figure 2-10b), the harvester generated $1\mu\text{W}$ of continuous electrical power at 2.4Vdc. The corresponding energy density is $0.74m\text{Wh/cm}^2$, which is still the
Figure 2-11: Wide bandwidth nonlinear oscillator based energy harvester developed by Hajati [12, 13]. (a) Image of the device on a US quarter coin. (b) Mechanical response of the system as a function of excitation frequency. Curve 1 shows the theoretically extractable power of the nonlinear Duffing resonator based on the mechanical analysis, assuming ideal electric loading. Curve 2 depicts the generated power based on the measured open-circuit voltage.

Hajati [12, 13] developed a novel nonlinear resonance based MEMS energy harvester, which achieved an ultra-wide bandwidth of > 20%, a maximum peak-to-peak voltage of 1.5V, a maximum extractable power of 45µW (Figure 2-11b), and high power density of 2W/cm³. More than one order of magnitude improvement in comparison to the devices previously reported in both bandwidth and power density convinces us that this is a promising way to harvest minute energy and to be employed for real-world applications. The harvester has four of doubly clamped beam resonators arranged perpendicular to each other. With each beam’s size of 6mm × 6mm × 5.5µm(L × W × H), the device’s size is about a US quarter coin (Figure 2-11a). At large deflection, a net stretching besides bending is activated, resulting in non-linear resonance, which changes the dynamic response and results in a tilted peak that possesses a wider bandwidth (Figure 2-11b). Although the new design manifests superiority, the jump-down frequency is about 1.3kHz and the excitation amplitude is 4 ~ 5g. The discrepancy from operating conditions and the real ambient state of the art, and compares to the values of lithium ion batteries. However, the bandwidth of the device is very limited as can be seen in Figure 2-10b. The input frequency and amplitude are also too high (13.9kHz and 10g) to be used for typical applications.
environment (<100Hz and < 0.5g) still exists.

2.4 Toward Low-Frequency Operation

Low-frequency operation (<100Hz) of vibration energy harvesters has attracted much attention in the past few years. The main reason is that the ambient vibrations typically have low frequency spectrum while small-scale energy harvesters tend to have much higher resonance frequency due to size effect. Strategies to lower the operation frequency include designing new geometries [10, 14, 28, 74-79], using soft materials [72,78,80], and up-conversion mechanisms [15,30,31,81-83]. The geometry design and soft material approaches are straightforward for lowering the resonance frequency: New geometries such as zigzag beam [79] or S-shaped beam [14] (Figure 2-12a) lower the stiffness of the structure.

The so called up-conversion mechanisms is widely used and will be discussed here. As an example, piezoelectric material’s power density is proportional to the frequency of the alternating strain, and is critically limited at low frequencies such as <50Hz. The up-conversion mechanism can increase the piezoelectric element’s vibrating frequency while absorbing the vibration’s energy at lower frequencies. To implement this, an up-conversion based energy harvesters typically have two sets of mechanical resonators: one has low resonance frequency and is used to absorb energy from ambient vibrations by matching the resonance frequency to the ambient vibration’s
frequency; while the other one has the transducer attached to convert the vibrational energy into electricity. The energy must be transferred from the low-frequency resonator to the high-frequency resonator to complete the energy absorption-conversion cycle. Various coupling methods between the two resonators have been developed: impact [15] can be induced to transfer the energy (Figure 2-12b); buckling has been employed as an up-conversion mechanism in [31]; a rotary mass with magnetic catch-and-release has been designed [81]; and multi-mode buckled beam has been implemented [30].

The up-conversion method inherits some shortcomings from its design. The two sets of resonators introduced complexity in the device fabrication and assembly is typically required. One of the resonator needs to resonate at low frequency increases the size of that resonator and hence the whole device’s size. Moreover, the impact or magnetic force based coupling between the two resonators is prone to suffer from significant energy loss and leads to device’s low efficiency. Due to these limitations, there is no up-conversion based MEMS scale energy harvester available yet.

2.5 Discussion and Motivation

This chapter reviews previous works on vibration energy harvesting. More specifically, a trend from increase the generated power (linear resonance, different transduction mechanisms etc.), to widen the bandwidth (frequency tuning, multiple resonators, nonlinear oscillation etc.), to lower the operating frequency (geometry, soft material, up-conversion) has been depicted. This trend is not random, but results from the evolution of the vibration energy harvesting to closer to real-environment applications.

Furthermore, our goal has been set, to develop low-cost, high-power energy harvesters that operates in a wide bandwidth at low frequencies and amplitude. To achieve the goal, this work leverages on low-cost, high-volume monolithic MEMS fabrication, high-efficiency and micro-fabrication compatible thin film PZT. New design based on residual stress induced buckled beam oscillator are then proposed in the next chapter.
Chapter 3

Design Concept, Meso-Scale
Prototype and Preliminary Testing

3.1 Design Concept

3.1.1 Bi-Stable Nonlinear Oscillator Based Design

Linear resonators, have been widely adopted in energy harvester designs due to the amplification of the small vibration from the environment. However, the usefulness of a linear resonator is highly dependent on matching the resonant frequency of the energy harvester to the frequency of the input vibrations. The frequency response decreases significantly when the input vibration's frequency is off from the resonant frequency. This gain loss becomes more serious when the system's damping is low; meanwhile, it is desirable that energy harvesters minimize the energy loss and have a high gain. This brings up the so called gain-bandwidth dilemma: the energy harvester suffers from either a small bandwidth for a high gain, or a low gain for a wide bandwidth. This trade off implies the linear resonator based energy harvester may be suited only for limited applications where the input vibrations are narrow-band.

As reviewed in Chapter 2, nonlinear oscillator based approach widens the bandwidth of energy harvesters without sacrificing the low energy loss (damping) requirement. Mono-stable Duffing oscillator exhibits a wide bandwidth resulting from its
nonlinear stiffness, which is a function of the oscillator's deflection. The equivalent stiffness varies with the deflection so that the resonant frequency "tracks" the input vibration's frequency with this negative feedback loop [12]. This approach has shown more than an order of magnitude improvement in bandwidth and successful implementation at MEMS scale. However, the demonstrated mono-stable oscillator based MEMS device operates with inputs of high frequency and large amplitude. The device presented in [13] was excited at 4g's and reached peak performance at around 1400Hz, while the typical ambient vibrations are <100Hz and <1g. With low-frequency or low-amplitude inputs, the nonlinearity of the harvester could not be activated and hence its improvement is compromised.

To gain insight on the high operational frequencies of the previous nonlinear oscillator based MEMS energy harvester, it is worth estimating the resonant frequency of a simplified mechanical nonlinear oscillator. The clamped-clamped beam based mono-stable nonlinear oscillator has an equivalent stiffness and resonant frequency [12],

\[ f = \frac{1}{2\pi} \sqrt{\frac{k_L + 3k_N w^2}{m}} \]  

(3.1)

where \( k_L \) is the linear stiffness and \( k_N \) is the nonlinear stiffness, \( m \) is the central proof mass, \( w \) is the deflection amplitude of the beam mid-point. The linear and nonlinear stiffnesses of a clamped-clamped one-layer beam with rectangular cross section are:

\[ k_L = \frac{\pi^4}{6} \left[ \frac{EWH^3}{L^3} \right] \]  

(3.2)

\[ k_N = \frac{\pi^4}{8} \left[ \frac{EWH}{L^3} \right] \]  

(3.3)

The nonlinear oscillator is designed to be operated at large amplitude to fully utilize the nonlinearity to widen the bandwidth. At large deformation, the stretching strain in the clamped-clamped beam dominates the total strain [12], and it could be calculated as,
\[
S_s = \frac{1}{L} \int_{-L/2}^{L/2} \frac{1}{2} \left( \frac{dw}{dx} \right)^2 dx = \frac{\pi^2}{4L^2} w^2
\]  

(3.4)

Assume the maximum strain is limited by the yield strain of the material (to minimize fatigue), then the maximum deflection amplitude is bounded:

\[
w^2 < \frac{4\varepsilon_y L^2}{\pi^2}
\]

(3.5)

Maximizing the proof mass in a typical design, we assume the proof mass is cubic with the width as large as the beam’s width \( W \), the length \( \alpha L \) a proportion \( \alpha \) of the beam length, and the thickness \( \beta H \) is \( \beta \) times larger than the thickness of the beam. Then the proof mass is simply,

\[
m = \rho WLH\alpha\beta
\]

(3.6)

where \( \rho \) is the mass density of the proof mass. Neglecting the linear stiffness and inserting the maximum deflection amplitude squared in 3.5 and proof mass in 3.6 into 3.1, the equivalent resonant frequency becomes,

\[
f = \frac{1}{4L} \sqrt{\frac{3}{2}} \sqrt{\frac{E\varepsilon_y}{\rho\alpha\beta}}
\]

(3.7)

\( \varepsilon_y \) is typically in the order of \( 10^{-3} \), and \( \rho \) and \( E \) (typical materials used in MEMS device (silicon, silicon dioxide, silicon nitride etc.) are typically \( 10^3 (kg/m^3) \) and \( 10^{10} (Pa) \); to maximize the proof mass, assume \( \alpha \sim 10^{-1} \) and \( \beta \sim 10^2 \), the length of the MEMS beam \( L \) at most could be in the order of \( 10^{-3} \sim 10^{-2} \text{m} \), then the varying resonant frequency peaks at as high as \( 10^3 \sim 10^4 \text{Hz} \). It should be noted that the estimation omits the linear stiffness and the piezoelectric coupling which increases the effective stiffness and hence the resonant frequency, so the estimation is the lower bound of the working frequency of clamped-clamped beam based mono-stable oscillator. The high operating frequency of the clamped-clamped beam at MEMS scale from the estimation renders the parameter optimization approach ineffective in achieving low-frequency operation such as \( < 100 \text{Hz} \) or even \( < 10 \text{Hz} \).
Bi-stable nonlinear oscillation has been chosen to tackle the low-frequency operation challenge. As reviewed in Chapter 2, previous works have used it for widening the bandwidth. Compared to linear resonators, bi-stable nonlinear oscillators can have much wider bandwidth. But in contrast to mono-stable nonlinear oscillators, I have found that bi-stable oscillators could have much lower operational frequency, which will be analyzed in more detail below. Moreover, the proposed implementation of the bi-stable system, residual stress induced buckling, is compatible with micro-fabrication of energy harvesters.

The first bi-stable oscillator is investigated by Moon [16], and the prototype is illustrated by Figure 3-1. A steel beam is clamped on one end in a rigid frame. Two magnets are mounted near the other end of the beam on the base of the frame. The magnetic force is strong enough to pull the beam to bend toward either of the magnets, so the beam can stay near one of the magnets stably. But the straight down position is unstable since any little perturbation will result the beam to bend to the side due to the difference of the magnetic force from the two magnets. When the frame is driven by an oscillatory force, depending on the amplitude of the force, the
beam jiggles near one magnet or oscillates back and forth between the two magnets.

Bi-stable nonlinear oscillators have two stable equilibria in contrast to mono-stable oscillators’ one stable equilibrium. By looking at the potential energy of the system as shown in Figure 3-2, we can see a bi-stable system has characteristic double-well shaped energy potential with a hump in between. The hump is an energy barrier, corresponding to the unstable equilibrium separates the two stable equilibria of the system, which have lower potential energy. Statically, the bi-stable system stably stays in either the lowest energy states (bi-stable) or in the unstable state (on the hump).

Dynamically, with an input oscillatory force, the system could oscillate at small amplitude around one of the stable equilibria within one well, or cross the hump and oscillate between the two wells back and forth at large amplitude. The first mode is the so called intra-well oscillation. The second is more complicated since the system could cross the hump some of the time and have chaotic oscillations, or it could oscillate periodically and have the so-called inter-well oscillations. Different modes of oscillation are affected by the input excitation as well as initial conditions. When
the input energy is high enough, the system could overcome the energy barrier and have inter-well oscillations; when the input energy is low, at steady state, the system cannot cross the energy barrier and is trapped in one well. To determine whether the input energy could lead to inter-well oscillation needs nonlinear dynamics theories and will be discussed in section 6.1.2. The three modes of vibrations are depicted by the time domain responses (Figure 3-3), which were obtained by numerically integrate the nondimensional governing equation,

\[ \ddot{w} + 2\zeta \dot{w} - w + \delta w^3 = -\dot{x} \]  

(3.8)

where \( w \) and \( x \) are the displacement response and input respectively, \( \zeta \) is the damping constant defined as \( \zeta = b/(2m\omega) \) (\( b \) is the damping coefficient and \( \omega \) is the linear resonant frequency \( \omega = \sqrt{(k_L/m)} \)), and \( \delta \) is the ratio of the nonlinear and linear stiffness: \( k_N/k_L \), and the time differentiation is with respect to the nondimensional time \( \tau = \omega t \).

The nonlinear differential equation 3.8 is solved analytically using harmonic balance method, so that the frequency responses are obtained. By considering the two oscillation modes: small-amplitude oscillations around one of equilibria (intra-well), and large-amplitude snap between two stable equilibria (inter-well), a varying bias is introduced as an auxiliary parameter. Two branches of the solutions corresponding to the two modes of oscillations could thus be derived. As shown in Figure 3-4, the intra-well and inter-well oscillations have the softening and stiffening curves which correspond to low-energy input and high energy input respectively. The stiffening response is similar to the frequency response of the mono-stable nonlinear oscillator. The main difference of the bi-stable and mono-stable stiffening response is that the even at low frequencies, the bi-stable system’s stiffening response still has large amplitude. Sweeping the frequency up, both mono-stable and bi-stable system have jump down, beyond which, the response is weak. Before jump down, both systems have wide bandwidth. The softening response has the peak tilting to the left (low frequency end). At the first look, the toward lower frequency tilt of the softening
Figure 3-3: Time domain responses and phase portraits of three modes of oscillations of a bi-stable system. The force $F$ determines the input energy level, and is varied in the series plots to demonstrate how the input energy affect the system's state. (a) (b) Intra-well oscillations. (c) (d) Chaotic vibrations (e) (f) Inter-well oscillations.
Figure 3-4: Two modes of oscillations are simulated from the nondimensional governing equation. Blue and red lines are the softening response and stiffening response respectively.

response may be a good solution for low-frequency energy harvesting, however, the amplitude of response is smaller than the stiffening response, which leads to lower strain energy and hence lower power output. Therefore, the stiffening response at very low frequencies is the target operational mode. As will show in the following sections, a meso-scale prototype at very low frequency has been fabricated and tested to verify the predictions qualitatively. The algebraic solution of the nonlinear governing equations are presented in Chapter 4.

The enhancement of the bi-stable oscillator on the low-frequency operation is illustrated in Figure 3-5, which shows the nondimensional frequency responses of the same bi-stable system with varying linear stiffness and a fixed nonlinear stiffness (this is the case of the proposed buckling induced bi-stable system presented in the following sections). The frequency response shifts to lower frequency with higher amplitude when the linear stiffness switching from positive to negative. With the negative stiffness’s amplitude becomes larger, the response is in ultra-low frequency range with significantly larger deflection amplitude, which characterizes the large-
amplitude snap through of bi-stable nonlinear oscillators. The shift of the frequency response to desirable direction (lower frequency and larger amplitude) by tuning the stiffnesses provides the critical design knob for the new generation energy harvester.

3.1.2 Buckling Induced Bi-Stability

Bi-stable oscillator could be constructed with buckled beams [31], magnets and magnetoelastic structure [66] or pre-shaped structures [84]. Magnets based bi-stable oscillators are typically in meso-scale and built with assembly of different mechanical parts. To maintain a low cost of the manufacturing of the energy harvesters, we would like to build the system with monolithic fabrication process and hence rule out the magnets based approach. Pre-shaped MEMS beam structure utilizes DRIE to define curved beam structure and achieved bi-stability [84]. However, the in-plane structure is not compatible with the PZT deposition process, which needs to cover the top surface of the sample but not the sidewall, and to maximize the coated area. Other
means to achieve bi-stability could be post-fabrication process such as bending the frame where the beams anchored to induce buckling. This approach needs auxiliary devices and precision control after the fabrication.

A clamped-clamped multi-layer buckled beam is chosen as the bi-stable oscillator design. The buckling from mechanical force in the beam structure eliminates the use of magnets and its associated assembly. The out-of-plane motion of the beam make the coating of piezoelectric material on top surface possible. Furthermore, the proposed residual stress induced buckling (presented in the next section) is compatible with the low-cost monolithic micro-fabrication process and does not introduce more expensive post-fabrication buckling mechanisms such as special packaging or precision control.

For a beam with an axial load, the static equilibrium equation is,

\[
EI \frac{d^4w}{dx^4} + P \frac{d^2w}{dx^2} = 0
\]  

(3.9)

with the characteristic equation,

\[
\lambda^4 + \frac{P}{EI} \lambda^2 = 0
\]

(3.10)

Solving for \(\lambda\), we obtain the general solution,

\[
w(x) = c_1 \sin(\omega x) + c_2 \cos(\omega x) + c_3 x + c_4
\]

(3.11)

where \(c_1\) to \(c_4\) are constants, and \(\omega = \sqrt{\frac{P}{EI}}\) where \(E\) and \(I\) are the Young’s modulus and moment of inertia of the beam respectively, \(x\) and \(w\) are the longitudinal and vertical coordinate respectively, and \(P\) is the axial load. By applying the boundary conditions of the clamped clamped beam: \(w(0) = 0, w(L) = 0, w'(0) = 0, w'(L) = 0\), we obtain the constants:

\[c_2 = -c_4\]

(3.12)
\[ c_3 = -c_1 \omega \] \hspace{1cm} (3.13)

\[
\begin{bmatrix}
\sin(\omega L) - \omega L & \cos(\omega L) - 1 \\
\omega \cos(\omega L) - \omega & -\omega \sin(\omega L)
\end{bmatrix}
\begin{bmatrix}
c_1 \\
c_2
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \hspace{1cm} (3.14)

The non-trivial solution of the equations require the determinant of the matrix to be zero, and the solution is,

\[ \omega = \frac{2n\pi}{L} \] \hspace{1cm} (3.15)

Plugging in \( \omega = \sqrt{\frac{P}{EI}} \), and \( n = 1 \) (the first buckling mode), the critical buckling load is,

\[ P_c = \frac{4\pi^2 EI}{L^2} \] \hspace{1cm} (3.16)

where the equivalent flexural rigidity \( EI \) of a multi-layer beam with \( i \)-th layer has Young’s modulus \( E_i \) and distances from upper and lower surface to the neutral axis \( H_{U,i} \) and \( H_{L,i} \),

\[ EI = \sum_{i=1}^{n} \frac{1}{3} W E_i (H_{U,i}^3 - H_{L,i}^3) \] \hspace{1cm} (3.17)

Each layer of the clamped-clamped beam is a different material with some degree of residual stress, tensile or compressive. If the total compression in some layers is larger than the total tension introduced by other layers, the whole beam would be in compression. If the total compression is larger than the critical buckling load (3.16), buckling will happen. For buckled beams with a rectangular cross section, if the width is much larger than the thickness (a typical case in MEMS-scale beams), the buckling will first happen in the direction with less flexural rigidity (equivalently, with higher critical buckling load), namely the direction perpendicular to the beam’s top surface. When the stress distribution is symmetric, the buckling could happen in two opposite directions and the buckled beam becomes a bi-stable system. Design
parameters including the thickness and residual stress of the multi-layer beam need to be guided and optimized with the analytical model which will be covered in Chapter 6.

### 3.1.3 Residual Stress Induced Buckling

Residual stress plays an important role in MEMS structures. Tensile stress increases the stiffness of a doubly clamped beam, and hence increases the resonant frequency of the beam resonator. In contrast, high compressive residual stress causes buckling, which is undesirable if it is not designed. Therefore, careful residual stress balancing is typically needed to avoid excessive stress and unexpected outcome. In this project, residual stress is sought to play a positive role of inducing buckling in micro-structures.

The origins of residual stress in micro-fabricated thin films can be divided into two categories: extrinsic and intrinsic [85]. A number of micro-fabrication processes are carried out at high temperatures. The mismatch in thermal expansion of the deposited film and the substrate results in a thermal stress. It can be calculated as [86],

$$
\sigma_{th} = \frac{E_{\text{film}}}{1 - \nu_{\text{film}}} \int_{T_0}^{T_{\text{dep}}} (\alpha_{\text{film}} - \alpha_{\text{sub}}) dT
$$

where $\sigma_{th}$ is the residual stress caused by thermal effect, $E_{\text{film}}$ is the Young’s modulus of the film, $\nu_{\text{film}}$ is the Poisson’s ratio of the film, $\alpha$ is the thermal expansion coefficient, $T$ is the temperature. And it is assumed that $E$ and $\nu$ are independent from temperature.

It has been found that during the deposition of thin films, stresses are developed not due to extrinsic causes, and they are called intrinsic stresses. The non-equilibrium conditions of the deposition give rise to these stresses [17], and they are related to the deposition conditions such as the substrate temperature, deposition rate, film thickness, background chamber ambient [86], the film and substrate materials, the deposition technique [18].

The deposited film consists of small crystallites at the beginning, and when these
crystallites coalesce, a residual stress will be generated. Figure 3-6 shows this process: when the crystallites grow, the gap between them decreases, and cohesion between them increases. This cohesion continues increase to close the gap and develop the tensile stress [87].

Grain boundary reduction results in densification of the deposited film (Figure 3-7), which also gives rise to tensile stresses. The analysis on the nature of this stress formation mechanism is given by [17]. The bi-axial stress is,

$$\sigma_{xx} = \sigma_{yy} = \frac{E}{1 - \nu} \Delta v \left( \frac{1}{L_0} - \frac{1}{L} \right)$$

(3.19)

where spherical grains with initial grain size $L_0$ increase to $L$, the excess volume per unit of grain boundary is $\Delta v$. The films deposited by sputtering usually have compressive residuals stress [18]. These stresses are caused by "shot peening". Sputtering gas pressure, ratio of molecular masses of target and sputtering ions affect the developed residual stress. There are many other causes of residual stresses, such as at low temperature, vacancy annihilation changes the volume and results in in-plane stresses; impurity like oxygen tends to reduce the tensile stress in metal films; phase transformation leads to volume change and results in residual stress; trapped gas can gives rise to compressive stresses.

Residual stress have various effects on MEMS structures. The asymmetric stress distribution generates bending moment, which curls a cantilever up or down. Figure 3-8 shows an example [11]: the initially fabricated cantilever 3-8a has a bottom layer with compressive stress and top layer with tensile stress and curled drastically. After
switching to another deposition method so that the bottom layer has a much smaller compressive stress, the cantilever curled much less (Figure 3-8b). By adding SiN_x, which has a high elastic modulus (313GPa), a near flat cantilever beam (Figure 3-8c) was obtained. This painful process illustrates the challenges in stress control of MEMS fabrication. Figure 3-9 gives another example more relevant to the proposed structure of this project: a fixed-fixed beam structure broke after release due to the asymmetric residual stress. The curled up device shows the bending moment resulted from the asymmetric residual stress.

Large tensile stress could drastically increase the resonant frequency of a clamped-clamped beam [12]: The designed resonant frequency of the doubly clamped beam resonator was 700 Hz, while the experimental results showed resonance at 2.7 kHz. From nano-indentation, the stiffness of the beam was found to be 5300N/m, and the residual stress in the beam was 100 MPa.

When a large compressive residual stress is developed in a fixed-fixed beam, the equilibrium position is out of plane (buckling). [55] analyzed the compressive stress in a doubly clamped beam. The physical origin of buckling is the vanishing of the lateral
stiffness at a critical level of stress, the Euler buckling limit (more detailed analysis of the multi-layer clamped-clamped beam is given in 5.3.1). Figure 3-10 shows a buckled beam with high positive curvature [18]. That is due to the high compressive stress at the surface of the polysilicon beam.

Residual stress could be measured by various means, including specially fabricated structures, XRD and wafer bow measurement. During micro-fabrication, a quick characterization of the wafer bow could be implemented before and after the film deposition to calculate the stress of the deposited film. The principle is simple: A layer of thin film with residual stress deposited on a substrate causes the substrate to curve. The correlation between the deposit thickness and the resulting curvature can be used to estimate the residual stress. In practice, laser reflection system is used to measure the bow of a wafer with displacement resolution down to 1nm [18]. If the substrate is much thicker than the film, the stress in the film could be calculated using the Stoney equation [88],

$$\sigma_f = \frac{E_s H_s^2}{6 R H_f (1 - \nu_s)}$$

(3.20)

where $\sigma_f$ is the residual stress in the film, $E_s$ and $\nu_s$ are the Young’s modulus and the Poisson’s ratio of the substrate respectively, $H_s$ and $H_f$ are the thicknesses of the
substrate and the film respectively, $R$ is the radius of curvature.

Consider a clamped-clamped beam, the linear stiffness is (derivation in section 4.1),

$$k_L = \left[ \frac{2\pi^4W}{3L^3} \sum_{i=1}^{n} c_{33,i}^E (H_{i}^3 - H_{L,i}^3) \right] + \left[ \frac{\pi^2W}{2L} \sum_{i=1}^{n} T_{0,i}H_{i} \right]$$  \hspace{1cm} (3.21)

where the first part is the bending stiffness and the second part is the stiffness due to residual stress. If we make a quick comparison of the two terms by considering a beam of single layer, the ratio of the two stiffnesses (bending stiffness/residual stress stiffness) is,

$$\frac{k_b}{k_F} \propto \frac{E}{T_0} \left( \frac{H}{L} \right)^2$$  \hspace{1cm} (3.22)

The thickness of MEMS structures is typically in microns ($10^{-6}$m), if the beam’s length is in millimeters ($10^{-3}$m) and the material’s elastic modulus is in the order of $10^2$ GPa, then when the residual stress gets to $10^{-1}$ MPa, the stiffness due to residual stress will be comparable to the bending stiffness. This explains the unexpected high resonant frequency of the resonator presented above, but more importantly, this indicates compressive stress in micro-fabricated thin films which is typically $10^1 \sim 10^2$
MPa is sufficient to induce buckling.

3.2 Meso-Scale Prototype

The design concept of the new generation vibrational energy harvester can be summarized as follows: a buckled clamped-clamped beam based bi-stable oscillator enhance the power generation at lower frequency compared to the same sized mono-stable oscillator based energy harvester with similar wide bandwidth; In addition, the initial buckling could be induced by residual stress in a controlled way in thin films at MEMS scale. To validate the design concept, a meso-scale experimental setup is built first. The mono-stable nonlinear oscillator and bi-stable oscillator based energy harvesters are compared to confirm our conceptual design. The characteristics of the bi-stable oscillator based energy harvesters that the theoretical analysis predicts, including the two modes of oscillations are also verified. The implementation of the residual stress induced MEMS buckled structure is validated by a MEMS buckled-beam oscillator prototype presented in Chapter 5.

3.2.1 Design and Fabrication

The design of the meso-scale prototype is similar to the MEMS device design, which is based on a clamped-clamped beam structure. The prototype consisted of a U-shaped base, a strip of metal as the beam, two fasteners on the base to clamp the beam, a piece of metal as a proof mass and two PZT patches. The dimensions of the base are much larger than the thickness of the clamped beam (> 10^2), so that we can treat the base as a solid block and ignore its deformation during vibrations. The beam is a thin strip of metal in rectangular shape. Two pieces of metal clamps are screw fastened on the top ends of the base to clamp the metal beam. A piece of metal cylinder is clamped or glued in the middle of the beam as the proof mass. Two PZT patches are glued on the top surface of the clamped beam on both sides of the proof mass to transduct the strain energy in the beam into electricity so the effectiveness of the structure as an energy harvester could be tested.
The same prototype could be made to be a mono-stable or a bi-stable oscillator: the assembly of the clamped-clamped beam, proof mass and the PZT patches is made first. The initially flat beam could then be placed on the base with the proof mass at the center and both ends of the beam clamped. The flat clamped-clamped beam is a mono-stable oscillator when subjected to vibrations, with the single equilibrium state of the flat position. When the external vibration is strong enough so that the central proof mass exerts a large force on the beam, the beam deforms with large displacement and stretches longitudinally in that the two ends are clamped. The beam oscillator would show nonlinear frequency response due to the stiffening effect of the stretching, as the MEMS device in [12] did. To modify the device to a bi-stable oscillator, the flat beam assembly was firstly clamped on one end, an force perpendicular to the top surface of the beam could be exerted on the central proof mass manually so that the beam deflect out of plane, then the other end was clamped without removing the force deflecting the beam. After both ends of the beam being securely clamped, the beam still keeps the initial deformation. The pre-shaped beam could buckle up or down, which is confirmed by applying a force on the beam so it could deflect to both directions and stay in the deflected states (the beam is large enough so that the buckling could be observed by eyes). The two buckled positions are statically stable so that the device would act as a bi-stable oscillator when subjected to dynamic load.

Two materials of the metal beams are tested: a stainless steel sheet (57.6mm × 20.2mm × 0.2mm), and an aluminum sheet (57.6mm × 20.2mm × 0.076mm). Cylindrical tungsten proof mass of 21.9g (diameter: 19mm, thickness: 4mm) and 16.5g (diameter: 19mm, thickness: 3mm) are attached to the steel beam and aluminum beam respectively. To transduct the mechanical vibration into electrical power, two PZT fiber-based energy harvesters (Smart Materials Corp., material properties in [17]) composed of PZT fibers and epoxy are glued by epoxy (3M DP460) on the beams on both sides of the proof mass symmetrically. The energy harvesters are wired so that they could be directly connected to load resistors for power measurement. The two setups are shown in Figure 3-11.
Figure 3-11: Photos of the meso-scale prototypes mounted on the electromagnetic shaker. (a) A steel beam of 57.6mm × 20.2mm × 0.2mm, two PZT fiber-based energy harvester patches and the center tungsten proof mass of 21.9g. (b) An aluminum beam of 57.6mm × 20.2mm × 0.076mm, two PZT fiber-based energy harvester patches and the center tungsten proof mass of 16.5g.

3.3 Testing and Discussion of Results

The device is tested dynamically to obtain the frequency response. Both mono-stable and bi-stable configurations are compared to verify the power enhancement at low frequencies of the bi-stable design. A laser vibrometer is also employed to extract the mode shape of the deformation for the lumped model simplification. Varying input vibration’s frequency at fixed amplitude and varying input vibration’s amplitude at fixed frequency are carried out to characterize the bi-stable system.

To conduct the dynamic testing, the device is firstly mounted on an electromagnetic shaker (Labworks ET-126) by screwing the base to the top plate of the shaker. The input vibration is monitored by an accelerometer (Analog Devices ADXL335) mounted on the base of the device. The signal from the accelerometer is fed into a computer using a data acquisition board (National Instruments USB-6210 DAQ). Since the acceleration amplitude of the vibration varies with the frequency, an Labview (v11.01) program is developed to tune the amplitude of the vibration to the set value automatically by employing a feedback loop so that fixed amplitude could be achieved at varying frequencies. After the input vibration’s frequency and amplitude

65
as the input parameters of the Labview program are set, sweeping the excitation frequency at fixed amplitude or sweeping the excitation amplitude at fixed frequencies could be done. A single-point laser vibrometer (Polytech CLV) is used to measure the deflection of the buckled beam’s vibration. The vibrometer is set to shoot the laser perpendicular to the top surface of the beam. By measuring the displacement on different spots along the beam, we could construct the mode shape of the deformation (Figure 3-12), which is very close to our sinusoidal trail function used in lumped model presented in Chapter 4.

By using the same device, the bi-stable configuration and the mono-stable configuration of the beams are tested at high input acceleration amplitude of 3g to compare the stiffening responses. The input vibration’s amplitude is chosen by first conducting a series of testing at different amplitudes and correlate the amplitude that enables large-amplitude vibrations. The low amplitude testing (1g) presented below is found not able to activate the large-amplitude vibrations in the same way. The input vibration’s amplitude of 3g is much higher than our design goal (0.5g), which is due to the
large thickness of the metal sheet and associated linear stiffness compared to MEMS device even the proof mass is made of high-density tungsten. But for design concept verification, the amplitudes of the input vibrations are appropriate. Load resistors are connected to the PZT patches (1MΩ on the steel beam device, 2MΩ on the aluminum beam device) so the voltage across the resistors are measured to calculate the power generated. Figure 3-13 and Figure 3-14 show the comparison between the mono-stable and bi-stable configurations for the steel beam device and aluminum beam device respectively. It can be identified quickly that the power gradually increases with the sweeping frequency for all the high-g testing. The frequency responses manifest the wide bandwidth of the nonlinear oscillations compared to linear resonator’s sharp peak (single resonant frequency). For the steel beam device, the bi-stable configuration’s power jumps up from 60Hz to 70Hz while the mono-stable configuration does not. The jump indicated the bi-stable configuration is triggered to high-energy inter-well oscillation from the low-energy intra-well oscillations, so the larger strain in the beam generated more power. The aluminum beam device’s bi-stable configuration showed enhancement in power generation compared to its mono-stable configuration in a wide frequency range (from 10 to 90Hz). The jump down of the devices are not reached due to the excessive input power needed to the shaker might damage the shaker. Bi-stable configurations of both devices showed higher power output with the same input and same material, solely due to the initial buckling. This is analyzed in section 3.1.1, the bi-stable system’s frequency response is shifted to the lower frequency form the mono-stable system’s response.

The bi-stable and mono-stable configurations are also tested at low input acceleration amplitude of 1g. The input acceleration amplitude is set well below the critical amplitude so that inter-well oscillation can not be triggered. Figure 3-15 compares the bi-stable response and the mono-stable response of the steel based device. The softening response of the bi-stable configuration is comparable to the mono-stable response but with wider bandwidth and higher amplitude. Figure 3-16 shows the softening response of the bi-stable system at low-g input. As predicted by the analytical model, the bi-stable configuration devices have softening response at low g’s
Figure 3-13: Average power responses of mono-stable and bi-stable configurations of the steel beam device. The input is 3g for the same beam in two configurations. It is clear that the bi-stable system outputs higher power from 70Hz. The jump-down has not been reached due to the limit of the shaker.

Figure 3-14: Average power responses of mono-stable and bi-stable configurations of the aluminum beam device. The input is 3g for the same beam in two configurations. It is clear that the bi-stable system outputs higher power at lower frequencies (10~90Hz). The jump-down has not been reached due to the limit of the shaker.
Figure 3-15: Comparison of the frequency response of the bi-stable configuration and the mono-stable configuration of the steel beam device. The testing is done by sweeping the frequency up and down slowly at 1g. Compared to the mono-stable configuration, the softening response of the bi-stable configuration even produces more power with wider bandwidth.

(the input energy is not enough to overcome the energy barrier). The jump up during the forward frequency sweep and jump down during the backward frequency sweep clearly show the hysteresis.

The inter-well mode of the bi-stable energy harvester is a preferred design target since it generates much higher power than the intra-well mode. The intra-well softening response could be employed at low excitation amplitude for energy harvesting. To exploit the higher hysteresis branch showed in sweeping frequency testing in real vibrations, controlled actuation may be needed. When designing bi-stable energy harvesters, the targeting frequency range should match the frequency range from jump-up to jump-down frequencies, for either inter-well mode or intra-well mode operation.

Except for the hysteresis in frequency sweep, bi-stable system's hysteresis when sweeping input excitation amplitude lowers the g-requirement for high power output of the nonlinear systems. By fixing the input vibration frequency, the acceleration amplitude has been swept from low to high (forward sweep) and high to low (backward sweep) for both mono-stable and bi-stable configurations. Figure 3-17 shows
Figure 3-16: Frequency response of the aluminum beam device. The testing is done by sweeping the frequency up and down slowly at 1g. Jump up and jump down clearly show the hysteresis and verify the theoretical analysis of intra-well oscillation.

the comparison at 60Hz. The results show that the average power of the mono-stable configuration increases steadily in the whole process without jump or hysteresis, while the bi-stable configuration jumps at high g during forward sweep and then maintains the high output power in a wide g-interval during backward sweep until jump down at low g. The steady high power during the backward sweep is one order of magnitude higher than the mono-stable system’s power at the same frequency and same amplitude, which could help energy harvesting at low g. Due to the double-well potential with an energy barrier, a bi-stable system oscillates intra-well with small amplitude until the input acceleration is high enough to overcome the energy barrier to excite the large-amplitude inter-well oscillation, which results in the jump up. When the system is already oscillating inter-well, the system keeps the large-amplitude inter-well oscillation and maintains high power output when the input acceleration amplitude is reducing. This verifies the sensitivity to initial conditions of a bi-stable nonlinear oscillator.

More tests on the input acceleration amplitude sweep have been done at different frequencies. As Figure 3-18 shows, the bi-stable system has been tested by holding the vibration frequency at fixed frequencies from 30Hz to 100Hz every 10Hz with
Figure 3-17: Comparison of mono-stable and bi-stable configurations when varying the input vibration amplitude at 60Hz. The hysteresis of the bi-stable system helps to maintain the high power output at reduced input acceleration amplitudes.

swept input vibration amplitude from 0.5g to 4.5g and back. Hysteresis exists at all the tested frequencies except for 30Hz since the input acceleration amplitude is not high enough to excite the system to inter-well oscillation. During backward sweep, the system’s high power generation is maintained in acceleration amplitude intervals. These intervals are better presented by Figure 3-19, in which the lower point of each bar denotes the jump-down amplitude and the upper point denotes the jump-up amplitude. The high power g-intervals are different at each frequency, and get quite wide around 60Hz and 70Hz. In general, the high-g requirement (3 ~ 4g) of this bi-stable system is lowered to ~ 1g. This is due to the responses are dependent to initial conditions of the system. To utilize the property of low-g operation in real ambient vibrations, an actuation and control unit may be necessary. If future research shows the generated power compensates the power consumption of the system, this property of a bi-stable nonlinear oscillator provides a potential opportunity for low-g vibration energy harvesting.
Figure 3-18: Hysteresis of bi-stable nonlinear oscillations when sweeping the input acceleration amplitude at various fixed frequency. (a) The input acceleration amplitude is swept forward (blue line) and backward (red line) at fixed frequencies from 30Hz to 100Hz for the same bi-stable system. The high power output is maintained until jump down during backward sweep.

Figure 3-19: The intervals between jump up and jump down at each frequency. The lower point of each bar is the jump-down acceleration amplitude and the upper point is the jump-up acceleration amplitude.
3.3.1 Summary

Meso-scale buckled beam oscillator based energy harvesters are built and tested. The prototype is similar to the MEMS scale design, which is a clamped clamped beam with central proof mass and PZT patches on both sides of the mass. The same prototype is made to have buckled (bi-stable) and unbuckled (mono-stable) configurations. Dynamic testing with high $g$ (3g) and low $g$ (1g) are done on both configurations for comparison. At low $g$, the buckled device (bi-stable) shows softening response when sweeping the input vibration’s frequency forward and backward, which agrees with the prediction of the analytical model. At high $g$, the buckled configuration shows higher power output at lower frequencies than the unbuckled configuration of the same device, which proves the shift of the frequency response (stiffening) predicted by the analytical model. The shift of the frequency response is designed to be used in MEMS scale device for low-frequency energy harvesting.
Chapter 4

Lumped Parameter Model of Buckled Beam Based Energy Harvester

4.1 One Degree-of-Freedom Electro-Mechanical Lumped Parameter Model

Analytical model provides important intuition and guideline in design and optimization. This chapter is devoted to the modeling of the buckled beam bi-stable oscillator based piezoelectric energy harvester. Some previous theoretical works on the modeling of the bi-stable oscillators or energy harvesters include: [89] investigated a bi-stable Duffing oscillator with electromechanical coupling, but the simulations are only in the time domain; [68] has formulated a PZT patched cantilever beam harvester with magnetic force induced bi-stability; [8] modeled a buckled beam bi-stable energy harvester, while the interest is put on the stochastic excitation. Composites based bi-stable plates were modeled in [90,91]. Even though our design is based on bi-stable oscillator, there are some features of the specific design that need to be modeled: the device is targeting to work at low frequencies with continuous harmonic vibrations; it will be implemented by MEMS fabrication with multi-layer structure; the buckling is induced by the residual stress of the micro-fabricated thin films; the piezoelectric element converts the mechanical energy into electricity. To capture these facets of
the design, we develop the theoretical framework with an electromechanically coupled lumped model, which incorporates the multi-layer structure and residual stress. The model is solved analytically by harmonic balance to obtain the frequency response of the energy harvesters, which is of our primary interest.

The energy harvester we model has a clamped-clamped beam structure of a stack of thin films including structural layer, seed layer, piezoelectric layer and passivation layer (Figure 4-1). A heavy proof mass is concentrated at the middle of the beam to capture the external vibration and excites the whole beam to oscillate out-of-plane. Piezoelectric elements work in 33 mode with top interdigitated electrodes, coupling the electrical response with mechanical deformation. The multi-layer beam is designed to buckle by incorporating compressive residual stress in the micro-fabricated thin films. Statically, the beam either buckles up or down (two equilibria), and the dynamics becomes complex when the system is continuously excited in post-buckling regime. To simplify the analysis of the complex system, the beam’s vibration mode has been assumed and a one degree-of-freedom model can be constructed. The non-homogeneous cross section beam structure has been taken into account by considering the different thickness and material properties of each layer. Furthermore, residual
stress of each layer is built in as part of the stiffness of the beam and induces buckling. The electrical and mechanical domains are coupled with linear and nonlinear coupling, so that the generated electrical signal can be obtained.

The lumped parameter model is formulated by Lagrange’s method. In classical mechanics, the Lagrangian is defined as,

\[ \mathcal{L} = KE - PE \]  \hspace{1cm} (4.1)

where \( KE \) is the kinetic energy of the system, \( PE \) is the potential of the system. In this energy harvester as in many other vibration energy harvesters, the proof mass is much heavier than the beam’s distributed mass, so that the kinetic energy of the system is simplified as that of the center-concentrated proof mass,

\[ KE = \frac{1}{2} mw^2 \]  \hspace{1cm} (4.2)

where \( m \) is the proof mass, \( w \) is the time derivative of the beam center displacement, i.e. the velocity of the proof mass. To find out the thermodynamic potential of the system including the piezoelectric material, we start by considering the electrical enthalpy volume density:

\[ \hat{H}_e = \frac{1}{2} T_3 S_3 - \frac{1}{2} E_3 D_3 \]  \hspace{1cm} (4.3)

and the piezoelectric constitutive equations in 33 mode [56],

\[ T_3 = e_{33}^E S_3 - E_3 e_{33} \]  \hspace{1cm} (4.4)

\[ D_3 = e_{33} S_3 + \varepsilon_{33}^S E_3 \]  \hspace{1cm} (4.5)

where \( T_3, S_3, D_3, E_3 \) are the stress, strain, electric displacement and electric field in 3-direction respectively; \( c_{33}, e_{33} \) and \( \varepsilon_{33}^S \) are the elastic modulus, piezoelectric constant, and permittivity of the piezoelectric material; the superscripts \( E \) and \( S \) denote the parameters are at constant electric field and strain respectively. Substitute \( T_3 \) and
Figure 4-2: The beam has \( n \) layers of thin films in different material with various thicknesses.

\[
D_3 \text{ into equation 4.3, and add the strain energy contributed by the residual stress } T_0, \\
\int_0^{S_3} T_0 ds = T_0 S_3: \\
\tilde{H}_e = \frac{1}{2} e_{33}^E S_3^2 - e_{33} E_3 S_3 - \frac{1}{2} \varepsilon_{33}^S E_3^2 + T_0 S_3 \\
(4.6)
\]

The Lagrangian of the system can now be evaluated by integrating the enthalpy density layer by layer over the beam’s volume,

\[
L = \frac{1}{2} m \dot{w}^2 - \sum_{i=1}^{n} \iiint_{v_i} \tilde{H}_{e,i} dv \\
(4.7)
\]

where \( v_i \) is the volume of \( i \)-th layer and \( n \) is the total number of layers.

The strains developed in the beam need to be evaluated before carrying out the integrations in equation 4.8. The total strain \( S_T \) developed in the beam has two components: bending strain, which changes linearly across the beam thickness, and axial strain due to large deflection,

\[
S_T = -z \frac{d^2 \dot{w}}{dx^2} + \frac{1}{L} \int_{-L/2}^{L/2} \frac{1}{2} \left( \frac{d \dot{w}}{dx} \right)^2 dx \\
(4.8)
\]

where \( L \) is the beam length. The strain is calculated from the neutral axis of the beam (Figure 4-2). It should be stressed that the beam composes multilayers with
various elastic properties, the neutral axis therefore does not coincide with the mid-plane. The formula for calculating the position of the neutral axis (distance from the bottom surface) of a general $n$-layer beam is,

$$h_{neutral} = \frac{\sum_{i=1}^{n} E_i H_i \sum_{j=1}^{i} H_j - \frac{1}{2} \sum_{i=1}^{n} E_i h_i^2}{\sum_{i=1}^{n} E_i h_i}$$

(4.9)

The beam vibrates up and down in $z$-axis, and by assuming that it vibrates predominantly in one mode, simplification can be made when evaluating the lumped parameters. The first buckling mode of the beam is adopted, which satisfies the boundary conditions of clamped-clamped beam and has been verified as the vibration mode shape at the largest deflection in experiment (Figure 3-12).

$$w(t) = \frac{1}{2} \left( 1 + \cos \frac{2\pi x}{L} \right)$$

(4.10)

where $w(t)$ is the deflection of the beam center varying with time.

The Lagrange equations are,

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\xi}_i} \right) - \frac{\partial L}{\partial \xi_i} = Q_i^{Force} + Q_i^{Dissipation}$$

(4.11)

where $\xi_i$ is the $i$-th independent generalized coordinate, $Q_i^{Force}$ and $Q_i^{Dissipation}$ are the generalized external force and the generalized dissipative force. We choose the deflection of the mid-point of the beam $w$ and the output voltage $V$ as the generalized coordinates. The Lagrange equation with respect to the first coordinate $w$ is then,

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{w}} \right) - \frac{\partial L}{\partial w} = F - b\dot{w}$$

(4.12)

Evaluating the integrations in equation 4.7 and substituting into equation 4.12, the governing equation of the mechanical domain can be obtained and written in a compact form,

$$m\ddot{w} + k_L w + k_N w^3 + b\dot{w} + c_N w V_N + c_L V_L = F$$

(4.13)
where \( k_L, k_N, b, c_L, c_N \) and \( F \) are the linear stiffness and nonlinear stiffness of the beam, the mechanical damping coefficient, the linear and nonlinear electromechanical coupling, and the external excitation force respectively. These lumped parameters are functions of the device dimensions and material properties, which are useful for device design,

\[
k_L = \left[ \frac{2\pi^4 W}{3L^3} \sum_{i=1}^{n} c_{33,i}^E (H_{U,ii}^3 - H_{L,ii}^3) \right] + \left[ \frac{\pi^2 W}{2L} \sum_{i=1}^{n} T_{0,i} H_i \right] \tag{4.14}
\]

\[
k_N = \frac{\pi^4 W}{8L^3} \sum_{i=1}^{n} c_{33,i}^E H_i \tag{4.15}
\]

\[
c_L = \frac{\pi e_{33} W (H_{U,P}^2 - H_{L,P}^2) (\sin(2\pi b) - \sin(2\pi a))}{Lg} \tag{4.16}
\]

\[
c_N = \frac{\pi^2 e_{33} W H_p (b - a)}{Lg} \tag{4.17}
\]

where \( W, H \) are the width and thickness, \( a \) and \( b \) denotes the span of the electrodes on the beam, since they do not cover the whole beam (Fig.4-1), and \( g \) is the gap between two electrode fingers; the subscript \( p \) denotes the variable is associated to the piezoelectric layer. The linear stiffness has two parts: the first part is from bending of the beam and the second comes from the residual stress. More particularly, when the residual stress is negative (compressive) and large enough, the linear stiffness \( k_L \) will be negative, so that equation 4.13 becomes a characteristic bi-stable Duffing equation.

The second Lagrange equation with respect to the coordinate \( V \) is,

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{V}} \right) - \frac{\partial L}{\partial V} = \frac{\int V dt}{R} \tag{4.18}
\]

Taking time derivative of the equation gives the governing equation for the electrical domain,

\[
C_0 \left( \dot{V}_L + \dot{V}_N \right) + \frac{V_L + V_N}{R} = I_L + I_N \tag{4.19}
\]

where \( I_L = c_L \dot{\omega} \) and \( I_N = c_N \omega \dot{\omega} \) are two parts of the electrical current generated by piezoelectric element through coupling, and \( L \) and \( N \) denote linear and nonlinear
Table 4.1: Assumed Parameters Expressions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Trial Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input Force</strong></td>
<td>( F = F_0 \sin(\omega t + \phi) = m \cdot a \sin(\omega t + \phi) )</td>
</tr>
<tr>
<td><strong>Beam Mid-point Displacement</strong></td>
<td>( w = \delta + w_0 \sin(\omega t) )</td>
</tr>
<tr>
<td><strong>Current (Inter-well)</strong></td>
<td>( I = I_L + I_N = I_{0,L} \cos(\omega t) + I_{0,N} \sin(2\omega t) )</td>
</tr>
<tr>
<td><strong>Current (Intra-well)</strong></td>
<td>( I = I_L + I_N = I_{0,L} \cos(\omega t) + I_{0,N1} \sin(2\omega t) + )</td>
</tr>
<tr>
<td></td>
<td>( I_{0,N2} \cos(\omega t) )</td>
</tr>
<tr>
<td><strong>Voltage (Inter-well)</strong></td>
<td>( V = V_L + V_N = V_{0,L} \cos(\omega t - \alpha) + )</td>
</tr>
<tr>
<td></td>
<td>( V_{0,N} \sin(2\omega t - \beta) )</td>
</tr>
<tr>
<td><strong>Voltage (Intra-well)</strong></td>
<td>( V = V_L + V_N = V_{0,L} \cos(\omega t - \alpha) + )</td>
</tr>
<tr>
<td></td>
<td>( V_{0,N1} \sin(2\omega t - \beta) + V_{0,N2} \cos(\omega t - \alpha) )</td>
</tr>
</tbody>
</table>

*\( m \) and \( a \) are the proof mass and excitation amplitude (acceleration amplitude of vibrations). The subscript 0 denotes amplitude.

coupling respectively; the induced voltages on the electrical port are written in separate parts \( V_L \) and \( V_N \), due to the fact that they come from two parts of the current respectively and have different frequencies due to different coupling, and this differentiation makes the assumptions on their function simple (next section). \( C_0 \) is the internal capacitance of the piezoelectric element and is calculated as

\[
C_0 = \frac{W_{\text{eff}}L_{\text{eff}}H_p\varepsilon_{33}^S}{2g^2}
\]

where \( W_{\text{eff}} \) and \( L_{\text{eff}} \) are the effective width and length of the PZT element (area covered by the electrodes), and the number 2 in the denominator is due to the width of one finger electrode and the gap between electrodes are the same in the designed MEMS device (4\( \mu m \)).

### 4.2 Analytical Solution

The nonlinear governing equations 4.13 and 4.19 can be numerically integrated to obtain the solution in time domain, but analytical solutions provide more insights on the dynamic behavior; more importantly, the explicit relations between system parameters and the performance are significant for device design. Therefore, the heuristic harmonic balance method is adopted to approximate the frequency response
Bi-stable oscillator has characteristic double-well potential. Depending on if the oscillator has enough energy to overcome the energy barrier, it crosses the well and has the so called inter-well oscillation, or stays in one well and oscillates intra-well. To differentiate the two modes of oscillations, we assumed that when the beam oscillates around the buckled configuration (intra-well), the beam is oscillating around a non-zero bias position $\delta$ from the flat position; when the bias $\delta$ is zero, the beam oscillates around the flat position and the oscillation becomes symmetric on both sides of the flat position (inter-well). Consider the intra-well case first: plug in the assumed functions of $w$ to 4.13, and balance the terms with $\delta$ but without sinusoidal functions, we find $\delta$, which is a function of the deflection amplitude,

$$\delta = \sqrt{-\frac{k_L}{k_N} - \frac{3}{2}w_0^2}$$  \hspace{1cm} (4.21)

By assuming the external force is sinusoidal and continuous, from trigonometric relations, it is easily found that the frequency of the induced electrical current from nonlinear coupling doubles that of the current from linear coupling. Physically, it is due to the developed stretching strain has half the cycle of the bending strain. The induced electrical current and voltage are then written in different parts with different frequencies shown in Table 4.1. Unknown phase differences have been added to the sinusoidal functions and will be solved. Writing equation 4.19 into two separate equations, we can solve for $V_L$ and $V_N$ separately:

$$C_0\dot{V}_L + \frac{V_L}{R} = c_L\dot{w}$$ \hspace{1cm} (4.22)

$$C_0\dot{V}_N + \frac{V_N}{R} = c_N\dot{w}$$ \hspace{1cm} (4.23)

Multiply 4.22 and 4.23 by $\sin(\omega t)$ and $\cos(\omega t)$, integrate from 0 to $2\pi$ and set both to zeros, we obtain six equations; and multiply 4.23 by $\sin(2\omega t)$ and $\cos(2\omega t)$, integrate from 0 to $\pi$ and set both to zeros, we obtain another four equations. Solving the ten
equations, the amplitudes of voltages and the phase constants can be found,

\[ V_{0,n} = \frac{c_n R \omega}{\sqrt{1 + C_0^2 R^2 \omega^2}} w_0 \]

(4.24)

\[ V_{0,N}/V_{0,n} = \frac{c_n R \omega}{2 \sqrt{1 + 4C_0^2 R^2 \omega^2}} w_0^2 \]

(4.25)

\[ V_{0,n2} = \frac{c_n R \omega w_0}{\sqrt{2 + 2C_0^2 R^2 \omega^2}} \cdot \sqrt{\frac{2k_L}{k_N}} - \frac{3w_0^2}{k_N} \]

(4.26)

\[ \sin (\alpha) = \frac{C_0 R \omega}{\sqrt{1 + C_0^2 R^2 \omega^2}}, \cos (\alpha) = \frac{1}{\sqrt{1 + C_0^2 R^2 \omega^2}} \]

(4.27)

\[ \sin (\beta) = \frac{2C_0 R \omega}{\sqrt{1 + 4C_0^2 R^2 \omega^2}}, \cos (\beta) = \frac{1}{\sqrt{1 + 4C_0^2 R^2 \omega^2}} \]

(4.28)

It is interesting to note that the voltage due to nonlinear coupling is a function of the deflection amplitude squared, while the voltage due to linear coupling is proportional to the deflection amplitude. This indicates that when the deflection is beyond some point, the nonlinear response will dominate the total response. The deflection amplitude is not known yet and will be solved from the mechanical domain equation 4.13.

Substitute assumed functions listed in table 4.1 to equation 4.13, multiply by \( \sin(\omega t) \), integrate from 0 to \( 2\pi \), set to zero, and multiply \( \cos(\omega t) \), integrate from 0 to \( 2\pi \), set to zero, we obtain two equations:

\[ f_1 w_0^3 + f_2 w_0 = F_0 \cos (\phi) \]

(4.29)

\[ f_3 w_0^3 + f_4 w_0 = F_0 \sin (\phi) \]

(4.30)

Take the square of both sides of 4.29 and 4.30 and add them together, one equation with one unknown \( w_0 \) is formed,

\[ (f_1^2 + f_3^2) w_0^6 + 2 (f_1 \cdot f_2 + f_3 \cdot f_4) w_0^4 + (f_2^2 + f_4^2) w_0^2 - f_5 = 0 \]

(4.31)

where \( f_1, f_2, f_3, f_4, f_5 \) are functions of the known system’s parameters:
\[ f_1 = \frac{\left(-\frac{15}{4}k_N - c_N^2c_0R^2\omega^2 - \frac{75}{4}C_0^2k_NR^2\omega^2 - \frac{11}{2}C_0^2k_NR^4\omega^4 - 15C_0^4k_NR^4\omega^4\right)}{(1 + C_0^2R^2\omega^2)(1 + 4C_0^2R^2\omega^2)} \]  

\[ f_2 = \left[ \frac{-m\omega^2 - 2k_L + c_L^2C_0R^2\omega^2 - 10C_0^2k_LR^2\omega^2 - C_0c_N^2R^4\omega^4/k_N - 5C_0^2mR^2\omega^4}{(1 + C_0^2R^2\omega^2)(1 + 4C_0^2R^2\omega^2)} \right] \]

\[ f_3 = -\frac{3c_N^2R\omega}{2(1 + C_0^2R^2\omega^2)} + \frac{c_N^2R\omega}{4(1 + 4C_0^2R^2\omega^2)} \]  

\[ f_4 = b\omega + \frac{c_L^2R\omega}{1 + C_0^2R^2\omega^2} - \frac{c_N^2k_LR\omega}{k_N(1 + C_0^2R^2\omega^2)} \]  

\[ f_5 = (m \cdot a)^2 \]

The inter-well oscillation is symmetric with respect to the flat unbuckled position, and hence there is no bias in \( w(t) \). Solving 4.13, 4.22 and 4.23 in the same way as solving for the intra-well case, but with \( \delta = 0 \), we obtain the same equation as 4.31, but with different coefficients:

\[ f_1 = \frac{3}{4}k_N + \frac{C_0c_N^2R^2\omega^2}{2(1 + 4C_0^2R^2\omega^2)} \]  

\[ f_2 = -m\omega^2 + k_L + \frac{C_0c_N^2R^2\omega^2}{1 + C_0^2R^2\omega^2} \]

\[ f_3 = \frac{c_N^2R\omega}{4(1 + 4C_0^2R^2\omega^2)} \]  

\[ f_4 = b\omega + \frac{c_L^2R\omega}{1 + C_0^2R^2\omega^2} \]

\[ V_{0,L} = \frac{c_LR\omega}{\sqrt{1 + C_0^2R^2\omega^2}}w_0 \]  

\[ V_{0,N} = \frac{c_NR\omega}{2\sqrt{1 + 4C_0^2R^2\omega^2}}w_0^2 \]

Equation 4.31 is an algebraic equation and is straightforward to solve. Since the voltages are functions of \( w_0 \), the generated power can be calculated by assuming the
harvester is connected to a resistor:

\[ P = \frac{V^2}{R} \]  \hspace{1cm} (4.43)

The \( w_0 \) in 4.31 are in even order and hence the third order equation gives three solutions. By solving the equation for intra-well and inter-well cases, we will obtain two sets of solutions, which correspond to two modes of oscillations (Figure 3-4). The jumps and tilt of the two curves show clearly the two modes of oscillations have characteristic softening and stiffening frequency responses. The softening response tilts to lower frequencies, which can be used for low-frequency energy harvesting at low g’s. When the input excitation is strong enough, the beam can snap between two buckled positions and the inter-well oscillation is triggered. The stiffening response has shifted up low-frequency response, which has large amplitude and increases the power output at low frequencies. The analytical solutions have been compared to the numerical integration results at 0.5g and 3g (Figure 4-3). The governing equations have been integrated numerically from 0 to 1s with initial conditions of the system having the lowest energy (buckled position, zero velocity). The numerical amplitude has been extracted from the maximum and minimum after half a second (steady solution). The two modes of oscillations and power enhancement have also been verified by experiment.

In developing the model of the buckled beam bi-stable oscillator based energy harvester, a balance between capturing the nonlinear dynamics and the simplicity of computation is made. The primary mode of buckling is chosen for constructing the one DOF lumped model, combined with electromechanical coupling, to estimate the electrical power generation. The harmonic trial function is verified as the maximum deflection mode by the laser vibrometer’s measurement of the macro-scale prototype (Figure 3-12). The lumped model also captures the two modes of oscillations of the bi-stable system. However, the intermediate modes of the snapping could be the superposition of the higher modes of the buckling. The theoretical analysis is reported by Qu [84]. More accurate modeling with the inclusion of multiple modes could be
Figure 4-3: Comparison between analytical solutions and numerical integration solutions. (a) and (b) compares the deflection and power in frequency domain at low excitation amplitude of 0.5g so that the harvester oscillates intra-well. (c) and (d) compares the deflection and power in frequency domain at 3g so that the harvester oscillates inter-well.
developed in the future.

4.3 Lumped Circuit Model

This section presents an electromechanically coupled circuit representation of the nonlinear energy harvester. The lumped parameter model derived in the previous sections and the circuit model are essentially identical. The circuit model is convenient for future electrical circuit design for the energy harvester system to fully exploit the potential of the nonlinear system. Various aspects of the nonlinear system are discussed in this section. The main differences between the linear and nonlinear systems have been compared: the role of the nonlinearity in widening power bandwidth of nonlinear resonance based energy harvesters became evident. The scheme of varying electrical damping at each excitation frequency to optimally employ nonlinear resonance and maximize the power generation has been analyzed with the lumped model. The optimal electrical damping for maximizing power has also been discussed.

The modeled doubly clamped beam with a load at the center is the same as the model Figure 4-1 shows. At large deflection, the beam is stretched and has the nonlinear force-deflection relationship,

\[ F = k_L w + k_N w^3 \]  \hspace{1cm} (4.44)

To demonstrate the circuit model, we first construct the simplest case, which is connecting the harvester to a load resistor. The lumped circuit model is then presented in Figure 4-4. The load resistor could be replaced by more advanced circuits in the future, while the lumped parameters derived in the following steps will still be able to be used. In the mechanical domain, \( F \) is the excitation force on the oscillator, \( m \) is the concentrated mass at the beam center, \( b \) is the mechanical damping coefficient, \( k_L \) and \( k_N \) are the linear and nonlinear spring stiffness's respectively. In the electrical domain, \( C_0 \) is the internal capacitance of the piezoelectric element, \( R_0 \) is the internal resistance of piezoelectric element, \( R_L \) is the resistance of the load resistor. In the
equivalent circuit, the mechanical domain and the electrical domain are coupled by a transformer [92], and the turns ratio as a scale factor relates the force in mechanical domain to the voltage in electrical domain. It characterizes the coupling between two energy domains but remains unknown for now.

By applying Kirchhoff's voltage law (KVL) in the mechanical domain and Kirchhoff's current law (KCL) in the electrical domain, we obtain two governing equations for the two domains,

\[
F = m\ddot{w} + b\dot{w} + k_L w + k_N w^3 + nV \quad (4.45)
\]

\[
I = C_0 \dot{V} + \frac{V}{R_0} + \frac{V}{R_L} \quad (4.46)
\]

The two equations describe the two domains separately but are coupled by the turns ratio. The unknown variables \(w\), \(V\) and \(I\) can be solved when knowing the turns ratio \(n\). By comparing with the governing equations we have already derived (4.13), the turns ratio is,

\[
n = \frac{\pi e_{33} W (H_{U,P}^2 - H_{L,P}^2) (\sin (2\pi b) - \sin (2\pi a))}{L_g} + \frac{\pi^2 e_{33} W H_P (b - a) w}{L_g} \quad (4.47)
\]

It should be noted that in order to model the nonlinear oscillation based energy harvester, the lumped circuit model has been extended from the typical linear circuit: the turns ratio is not a constant as in linear resonance systems, but a function of the
deflection amplitude, similar to the nonlinear stiffness. After knowing the turns ratio, all the lumped parameters in the circuit model Figure 4-4 shows are known, and we are able to analyze the dynamics of the system or design the circuit interface to extract the energy efficiently. The analytical solution of the circuit could also be derived by using the Harmonic Balance Method as illustrated in section 4.2 and will not be repeated here. Some implications of the model will be discussed.

Compared to a linear system, the nonlinear system’s coupling between the two domains is not a constant but a function of the deflection, which results from the nonlinear stiffness in the mechanical domain. Due to the nonlinear turns ratio, the electrical damping is not linearly proportional to the resistance but depends on the deflection amplitude as well. The nonlinear relationship between the load resistance and the electrical damping is displayed by the concave curve in Figure 4-5. Another important feature of the nonlinear system is that the deflection is insensitive to the change of damping. Due to the feedback effect of the the nonlinear stiffness [93], the increased electrical damping decreases the deflection only a little. Based on this feature, varying electrical damping scheme was proposed [12]: At a certain frequency, the output power can be boosted up by increasing the electrical damping until the system is close to jump down. The direct outcome of this scheme is that the power spectrum has a much wider bandwidth. By imposing the system an optimal electrical damping at each frequency, an envelope of the maximum power has been obtained (Figure 4-6), which shows a wide bandwidth.

For linear resonator based energy harvesters, the electrical damping must match the mechanical damping to extract the maximum power from the system. However, it was believed that the nonlinear oscillator based energy harvester could inject more electrical damping and extract even more power compared to the matched impedance case [12]. This claim is analyzed as following. Figure 4-6 shows the optimal electrical damping at each frequency to push the power to the maximum at that frequency. It can be seen that the electrical damping should be much higher than the mechanical damping to maximize the power generation before getting to the global maximum power (around 130Hz). After that point, the optimal electrical damping becomes
lower. The condition for the global maximum power from the figure seems to be the equality of electrical damping and mechanical damping, and this is the case for linear systems [8]. For a nonlinear system, this could also be proved. To simplify the analysis, we can add an inductor in parallel with the capacitor and resistors in the electrical domain, the total impedance of the electrical domain would be,

$$\frac{1}{Z_{\text{total}}} = \sqrt{\frac{1}{R_0^2} + \frac{1}{R_L^2} + \left(\omega C_0 - \frac{1}{\omega L}\right)^2}$$  \hspace{1cm} (4.48)

By choosing the inductance to be \( L = 1/(C_0\omega^2) \), the internal capacitance can be canceled out. The internal resistance is usually much larger than the load resistance [94], so we can assume the internal resistor port is open. After eliminating the internal capacitance and internal resistance, the total impedance of the system becomes,

$$\frac{1}{Z_{\text{total}}} = \sqrt{Z_{be}^2 + Z_{bm}^2 + (Z_m - Z_{kL} - Z_{kN})^2}$$  \hspace{1cm} (4.49)

where the subscripts denote the source of the impedance. The power on the electrical damping is,
Figure 4-6: The maximum power envelope and the normalized optimal electrical damping at each frequency.

\[
P = \left( \frac{Z_{be}}{Z_{totalV}} \right)^2 = \left( mA \right)^2 + b_e^2 + b_m^2 + \frac{b_e}{\omega^2 \left( \omega^2 - \omega_0^2 \right)}
\]

(4.50)

where \( \omega \) is the excitation frequency and \( \omega_0 = \sqrt{k_L + \frac{3}{4} k_N w^2} \). It can be easily proved from equation 4.50 that if \( \omega = \omega_0 \), the electrical damping should match mechanical damping to maximize the power. However, if the mechanical damping in the system is too large, the maximum electrical damping might not be able to match the mechanical damping, and the optimal electrical damping for the global maximum power is the maximum that it can reach (Figure 4-7).
Figure 4-7: The normalized electrical damping at the global power maximum with increasing mechanical damping.
Chapter 5

Buckled Micro Beam Oscillator
Without PZT

5.1 Design

A buckled micro beam oscillator is designed and fabricated to validate the design of low-frequency, low-amplitude energy harvesting, as well as to verify the feasibility of the target operating frequency (<100Hz) and amplitude (<0.5g) of the energy harvester MEMS structure. The fabrication of a full energy harvester device requires relatively long micro-fabrication process with ten photolithography steps (five masks) and multiple sol-gel coating steps for ZrO$_2$, PT and PZT layer formation. Many unknowns in the fabrication, including the effects of the residual stress on the structure, the quality of the PZT thin film would complicate the implementation and troubleshooting of the full device development. Moreover, whether the target operating frequency and amplitude range could be met independently from the PZT coating.

Therefore, simplified structure is developed firstly to reduce the fabrication complexity, and to decouple the challenge to integrate PZT layer, verify the design concept before implementing the full energy harvester device. The merits of the simplified prototype is evident. The first MEMS prototype without the piezoelectric layer, its associated diffusion barrier layer and electrode layer has been developed before the full energy harvester. This prototype is to validate the balancing of residual stresses,
both tensile and compressive in each layer, resulting in a total compression that induces desirable buckling. The materials used in this first prototype could also provide data for the full device fabrication.

The structure of the device is similar to the full device’s design, which is a rectangular suspended beam with two opposite edges bonded on the silicon frame and two other edges free (Figure 5-1). Since the silicon frame was etched out of the silicon wafer, the thickness (wafer-thick: \( \sim 525 \mu m \)) is much larger than the thickness of the beam (< 3\( \mu m \)). We could use the frame as the base for mounting the device and ignore any deformation of it. The beam itself is composed of four layers: thermal dioxide, PECVD \( \text{Si}_3\text{N}_4 \) in tensile stress, PECVD \( \text{Si}_3\text{N}_4 \) in compressive stress and PECVD \( \text{SiO}_2 \) in compressive stress. Thermal oxide layer is the etch stop of the DRIE and a source of compression with little variation in stress between wafers. The tensile PECVD \( \text{Si}_3\text{N}_4 \) will not be included in the full energy harvester device but we intentionally add this layer with the tensile stress to imitate the tensile sol-gel coated thin films such as the PZT, \( \text{ZrO}_2 \) and PT in the full device to test the stress balancing strategy. The compressive PECVD \( \text{Si}_3\text{N}_4 \) and PECVD \( \text{SiO}_2 \) provide the compression in the energy harvester design. The PECVD thin films are the main control knobs due to the flexibility to tune the stress during deposition and their wide range of the available stress. These two layers will also be the passivation and structural layers in the energy harvester full device and hence the stress and growth rate are characterized extensively to assure reliable and predictable deposition in full device fabrication. Two beam layouts are designed for comparison: 15\text{mm} \times 12\text{mm} and 15\text{mm} \times 3\text{mm} (Figure 5-2). The overall size of the two devices are smaller than a quarter coin (18\text{mm} \times 18\text{mm}). A silicon proof mass on the backside of the beam (3\text{mm} \times 12\text{mm} \times 530\mu m). An optional tungsten proof mass could be attached on the silicon proof mass as an external mass.
Figure 5-1: 3D schematic of the micro buckled-beam oscillator.

Figure 5-2

Figure 5-3: Top view of the two beam layout designs.
Table 5.1: Residual Stress of Micro-Fabricated Thin Films

<table>
<thead>
<tr>
<th>Material</th>
<th>Residual Stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal Oxide (300 ~ 1000nm)</td>
<td>-300MPa ± 4%</td>
</tr>
<tr>
<td>LPCVD Nitride (200nm)</td>
<td>100MPa</td>
</tr>
<tr>
<td>LPCVD Nitride (700nm)</td>
<td>160MPa</td>
</tr>
<tr>
<td>LPCVD Nitride (1200nm)</td>
<td>250MPa</td>
</tr>
<tr>
<td>PECVD Oxide (Low Frequency)</td>
<td>-260MPa ± 40%</td>
</tr>
<tr>
<td>PECVD Nitride (75% High Frequency)</td>
<td>-240MPa ± 40%</td>
</tr>
<tr>
<td>ZrO₂ (90nm)</td>
<td>370MPa ± 15%</td>
</tr>
<tr>
<td>PT (10nm)</td>
<td>400MPa ± 15%</td>
</tr>
<tr>
<td>PZT (140 ~ 220nm)</td>
<td>650MPa ± 15%</td>
</tr>
</tbody>
</table>

5.2 Stress Control of Thin Films

5.2.1 Induce Buckling with Residual Stress

Residual stresses in micro-fabricated thin films could be either tensile or compressive, and the cause has been reviewed in Chapter 3. Table 5.1 lists the residual stresses of the thin films fabricated at MTL. In this section, we would like to discuss how to achieve buckling by controlling the residual stress and the thickness of each layer. From structural mechanics, a beam buckles if the axial load is greater than the beam’s Euler critical load, which depends on the dimensions of the beam, the material property, the initial defect, as well as the boundary condition. The beam buckling has been analyzed in 3.1.2. To adapt to the multi-layer structure with residual stress, we consider the load from the stress and the initial buckling shape.

The total axial load in the multi-layer beam is contributed by stress from all the layers,

\[ P = \sum_{i=1}^{n} T_{0,i} H_i \]  

(5.1)

Therefore, knowing the residual stress of each layer, the thickness could be designed so that the load \( P \) surpasses the critical buckling load \( P_c \) to induce buckling. The requirement for designing the buckling beam is to make the longitudinal compression diminishes the bending stiffness at the critical point, so that the linear stiffness
becomes zero. Since we have already derived the stiffness in the lumped model in Chapter 4, the critical condition would be,

$$k_L = \left[ \frac{2\pi^4 W}{3L^3} \sum_{i=1}^{n} c_{33,i}^E (H_{U,i}^3 - H_{L,i}^3) \right] + \left[ \frac{\pi^2 W}{2L} \sum_{i=1}^{n} T_{0,i} H_i \right] = 0 \quad (5.2)$$

In the post-buckling regime, the beam buckles in its first mode due to higher modes are unstable, with an amplitude $w_0$ unknown,

$$w(x) = \frac{w_0}{2} \left( 1 + \cos \frac{2\pi x}{L} \right) \quad (5.3)$$

By dropping the time derivatives and the dynamic input in the governing equation of the lumped model, we obtain a static equilibrium equation,

$$k_L w_0 + k_N w_0^3 = 0 \quad (5.4)$$

The initial buckling amplitude thus is,

$$w_0 = \sqrt{\frac{k_L}{k_N}} \quad (5.5)$$

### 5.2.2 Balance of Residual Stress

When the stress distribution of the thin films in the multi-layer stack are not be symmetric, a moment resulting from the stress distribution is induced,

$$M_r = \frac{W}{2} \sum_{i=1}^{n} T_{0,i} (H_{U,i}^2 - H_{L,i}^2) \quad (5.6)$$

The residual moment bends beam to one direction and could deteriorate the bistability. Consider the global equilibrium of a beam with a residual moment, shown in Figure 5-4,

The beam is subjected to a bending moment of the axial load $Pw(x)$, residual moment $M_r$, moment at the support $M_s$, and internal moment $M(x)$, hence the equilibrium equation is,
Figure 5-4: Beam equilibrium.

\[ M(x) + M_R - Pw(x) - M_S = 0 \]  

(5.7)

Substitute \( M(x) = -EIw'' \) and rearrange terms,

\[ w(x)'' + \frac{P}{EI}w(x) = \frac{M_R - M_S}{EI} \]  

(5.8)

The solution \( w(x) = w_h(x) + w_p(x) \) has the homogeneous and particular parts,

\[ w_h(x) = c_1 \sin \left( \sqrt{\frac{P}{EI}}x \right) + c_2 \cos \left( \sqrt{\frac{P}{EI}}x \right) \]  

(5.9)

\[ w_p(x) = \frac{M_R - M_S}{P} \]  

(5.10)

Applying the boundary conditions \( w(0) = 0, w(L) = 0, w'(0) = 0, w'(L) = 0 \):

\[ c_1 = 0 \]  

(5.11)

\[ c_2 = \frac{M_S - M_R}{P} \]  

(5.12)

\[ \sqrt{\frac{P}{EI}} = \frac{2n \pi}{L} \]  

(5.13)

Therefore, the initial buckling is,
Figure 5-5: Measured thickness (a) and residual stress (b) of the thermal oxide.

\[ w(x) = \frac{M_R - M_S}{P} \left( 1 - \cos \left( \sqrt{\frac{P}{EI}} x \right) \right) \]  
\[ (5.14) \]

Then consider when \( M_R = 0 \) (no residual moment),

\[ M_S = -\frac{Pw_0}{2} \]  
\[ (5.15) \]

To preserve the bi-stability, \( w \left( \frac{L}{2} \right) \) should be larger than zero,

\[ M_R > -\frac{Pw_0}{2} \]  
\[ (5.16) \]

In parametric sweep optimization, the residual moment is minimized.

5.3 Fabrication

The fabrication process starts with 4" <100> silicon wafers. After RCA clean, \( \sim 300nm \) thermal dioxide is grown with wet oxidation. The thermal dioxide is the bottom layer of the multi-layer beam and serves as the etch stop of the final DRIE release. The thickness and stress across 25 wafers are characterized, and the variation is within 10% and 7% respectively (Figure 5-5).

Two PECVD silicon nitride layers with compressive and tensile stress are then deposited. The dual-frequency plasma deposition of the ST Systems CVD enables
flexible stress control in a wide range from tensile to compressive. The tuning parameter is the ratio of the duration of the applied high (13.56MHz) and low (380kHz) frequency plasma. The growth rate and residual stress of the PECVD thin films have been characterized by measuring the thickness, and the wafer bow before and after the deposition. The data guides the control of the injected compression in the structure and the balance of the stress in the stack. The growth rate and stress are plotted with respect to the high frequency percentage (Figure 5-6). The monotony of the curves shows the feasibility to tune the thickness and stress to match the design targets by setting the frequency ratio.

PECVD silicon dioxide as the last layer is deposited on top as a passivation and to balance the stress in the stack. The stress of the PECVD silicon dioxide can also be controlled by tuning the plasma frequency ratio, however, all are compressive. The high-frequency recipe is chosen then since it gives higher compressive stress, of about −260MPa. The first photolithography step is to pattern the beams from the top, and thick photoresist (10μm) is coated as an etching mask to endure the long dry etching step. Reactive ion etching (RIE) is used to etch through the whole silicon dioxide and nitride stack. The photolithography (with thick resist) and RIE are repeated on the backside of the wafers to pattern the frame and proof mass, leaving opening for the deep reactive ion etching (DRIE). Finally, etching through the whole wafer thickness
from the back (DRIE) releases the beam and separates the dies, and acetone bath follows to remove the resist (Figure 5-7). The released beam structures should only have silicon dioxide and nitride but as Figure 5-7 shows, there is still residual silicon on the beam due to the nonuniform etching of the DRIE. An extra step of XeF$_2$ etching is used to gently remove the residual silicon. The fabrication process flow is shown in Figure 5-8. The details on the recipes and machine tools are similar to those for the full energy harvester fabrication and are covered in Chapter 7, so will not be repeated here.

The micro-fabricated beam structures have four layers in different residual stresses (Figure 5-9). The total thickness is $\sim 2.7\mu m$, and the aspect ratio (length/thickness) is larger than 4000:1. As Figure 5-10 shows, the transparent film is the stack of silicon dioxide and nitride, and the proof mass and frame are wafer-thick silicon covered by the thin films. To dynamically test the devices, the device should be mounted on the vibration source and the proof mass needs to have space for out-of-plane movement. A 3D printed fixture is therefore made to facilitate the mounting. The fixture is a squared frame with recessed square on top surface (Figure 5-11a), so the micro device can be inserted on top, leaving space underneath for the proof mass to move freely. The assembly (Figure 5-11b) can then be attached to the shaker for dynamic testing.
Figure 5-8: Fabrication process flow of the micro buckled beam.

<table>
<thead>
<tr>
<th>Residual Stress (MPa)</th>
<th>Thickness (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PECVD SiO₂</td>
<td>-230 640</td>
</tr>
<tr>
<td>Tensile PECVD Si₃N₄</td>
<td>80 740</td>
</tr>
<tr>
<td>Compressive PECVD Si₃N₄</td>
<td>-80 1000</td>
</tr>
<tr>
<td>Thermal SiO₂</td>
<td>-300 320</td>
</tr>
</tbody>
</table>

Figure 5-9: The cross sectional view of the multi-layer beam with measured thickness and residual stress.

Figure 5-10: Top view of the two fabricated MEMS prototypes with different widths. The aspect ratio of the wide beam is 4444:1. Both devices have backside wafer-thick silicon mass.
5.3.1 Compression Control of the Multi-Layer Structure

The total compression in the multi-layer beam structure is controlled by designing the thickness of each layer after knowing the residual stress of each material and the growth rate of the deposition. The thickness and residual stress are therefore measured for each layer. The calculation of the compression and the balancing of stress is covered in section 5.3.1. The stress control in fabrication is presented in this section.

Residual Stress Measurement

The residual stress of thin films can be measured by measuring the changes in the radius of curvature of the substrate that the thin film is deposited on. The stressed thin film causes the substrate to bow, and the stress in the thin film can be estimated by the Stoney equation 3.20. Tencor FLX-2320 at TRL scans the reflected surface of the sample with laser before and after each deposition, and calculates the residual stress of the thin films. Nanospec and Filmetrics at TRL are used for measuring the thickness of the dielectric thin films. The patterned ZrO₂, PT and PZT layers’ thicknesses are measured using surface profilometer since either Nanospec or Filmetrics has the material properties of these materials.
Feedback Control of the Stress Injection

Even though the deposition rate and residual stress of each material have been characterized extensively, there is variation in the deposited thin films in different batches. This may be due to the deposition tool is shared with all the users and the various materials deposited using the same tool changes the chamber environment significantly. A thorough etching step is added to the PECVD deposition step with a short conditioning deposition (running the deposition of the same material without loading the sample). Variation is further reduced by a feedback control scheme (as shown in Figure 5-12). There are three PECVD based control layers: the silicon oxide layer
underneath the ZrO₂, and the two passivation layers. After each control layer de-
position, the measured thickness and residual stress are fed in the thickness design
program, and the rest control layers’ thickness will be recalculated to adapt to the
change in previous control layers. In this way, the deviation of the total compression,
and linear stiffness from the designed value could be minimized with the available
facilities.

5.4 Testing

The micro-fabricated buckled beam device is to validate the design concepts that,
the residual stress in thin films could be controlled and balanced to effectively induce
buckling in micro-fabricated beam structure, and the buckled beam device could have
large-amplitude oscillations with low-frequency and low-amplitude inputs (<100Hz
and 0.5g). Even though the piezoelectric layer is omitted in this prototype, the fre-
quency response can still be obtained by measuring the deformation of the beam with
input vibrations. This testing was done with the setup illustrated by the schematic
in Figure 5-13. The device is firstly mounted on the electromagnetic shaker. The
vibration of the shaker is monitored by an accelerometer (Analog Device ADXL335),
and the input harmonic signal’s frequency and amplitude are controlled by the signal
generator (Prema ARB1000) and the power amplifier (Labworks PA138). A laser
vibrometer (Polytec Scanning Vibrometer PSV300) was employed to measure the ve-
locity of different spots on the device to calculate the relative movement of the beam
to the frame.

The wide-beam device is firstly tested with only silicon mass by sweeping the
frequency forward and backward (Figure 5-14). The input’s amplitude is close to
but does not exceed 0.2g in the sweeping frequency range. The central proof mass
and the silicon frame of the devices are scanned by the laser vibrometer during the
sweeping so the maximum deflection of the center deflection at various frequencies to
generate the frequency spectra. During the forward frequency sweep, the deflection
of the wide beam center increases gradually and jumps down at 140Hz. The system
jumps up at 100Hz during backward sweep. The spring stiffening response has a wide bandwidth below 140Hz. An external tungsten proof mass of 0.24g is then attached to the backside silicon mass of the device to go through the same frequency sweep testing (Figure 5-15). It can be observed that the frequency response shifts to much lower frequencies due to the heavier mass, and the large-amplitude region is even below 10Hz. The deflection amplitude is also significantly increased compared to the same device without the external proof mass. It can be inferred that the device with piezoelectric material will have a boosted power output due to the increased mass.

The narrow beam device is tested without attaching the external tungsten mass by sweeping the frequency (Figure 5-16). Large-amplitude responses at low frequencies were obtained, which correspond to the analytical prediction of the bi-stable device’s ultra-low frequency response due to the snap through. (The vibrometer was off during the testing, so the presented deflection amplitudes are estimated by comparing with data measured by the vibrometer and the accelerometer and scaling. The precision is subjected to the accelerometer’s precision, but the trends of the frequency response is preserved.)

To further prove the role of the compressive stress induced buckling in shifting the working frequency range of the compressed clamped-clamped beams, we also fabricate a MEMS device with the same geometry but with slightly different beam composition.
The device has a tensile LPCVD silicon nitride as the structural layer instead of the compressive PECVD silicon nitride with an additional LTO layer (this composition is similar to the previous device our group developed [12]). The thicknesses and residual stresses of other layers are comparable to the buckled plate devices but the overall stress in the structure is tensile, hence the structure is mono-stable nonlinear. The beam is flat without buckling as can be seen in Figure 5-17, and the composition is also illustrated. The same sweeping frequency test has been done using the same setup. The spring stiffening curves showed clearly the characteristics of the mono-stable nonlinear resonator (Figure 5-18). The jump-up frequencies at 0.1g and 0.2g are close to 320Hz and the jump-down frequencies are 390Hz and 430Hz. Compared to the compressed plate devices, this device has working frequency range about three time higher, while the deflection amplitude is much lower.

The size of the MEMS prototypes is large enough that the beams’ motion is visible to human eyes. High-speed camera is thus employed to capture the dynamics of the
nonlinear oscillators and provide direct visual information. The video is recorded at 1200 frames per second (FPS) and played at 30 FPS, so 40 times slower motion of the devices’ oscillations could be observed (screenshots of the videos are shown in Figure 5-19). The tests for video shooting are done at various input frequencies (10~100Hz) and amplitudes (0.05~0.5g). Important observations could be summarized: The wide-beam device’s oscillations are large and increase gradually with the excitation amplitude, while the narrow-beam devices show only little oscillations with low amplitude excitation and much stronger oscillations with higher excitation amplitude. The narrow-beam device’s oscillation from weak to strong when subjected to increasing excitation is believed to be the transition from intra-well to inter-well oscillation of a bi-stable system, since the initial buckling and the sudden snap through are easily seen from the videos. The transition is not seen with the wide-beam device, which may be due to the corrugations in the plate in transverse direction diminishes the
longitudinal buckling. More analysis on this is carried out in section 6.1.1. Another observation is the narrow-beam device shows significant rotation about the plate's longitudinal axis during large-amplitude oscillation, while the wide-beam device has no obvious rotations during oscillation.

### 5.5 Discussion of Results

The micro buckled beam oscillator design significantly reduces the fabrication cycle of the prototype device, and provides useful feedback on the device design. The dynamic tests of the wide- and narrow-beam oscillators show the operational frequencies and the amplitude match the design targets ($<100$Hz and $<0.5$g): the wide-beam devices have stiffening frequency response below 140Hz, with -3dB bandwidth of $\sim 35\%$ at 0.2g; the narrow-plate device have large oscillations below 50Hz at 0.4g. Devices using the same geometry design but with the tensile LPCVD silicon nitride as the
Figure 5-17: MEMS clamped-clamped beam structure without buckling. The multi-layer structure and the associated parameters are shown in the schematic.

Structural material are also fabricated for comparison. The no compression built-in devices have about an order of magnitude lower deflection oscillating $300 \sim 430 \text{Hz}$ with the same input excitation amplitude. The higher amplitude responses at lower frequency of the compressed beam devices demonstrate the compression enhances the clamped-clamped beam structure at MEMS scale with low-frequency input vibrations.

The fabrication of the micro buckled beam oscillator also tests the proposed residual stress induced buckling approach. The fabricated devices, even though left out the piezoelectric layer, still keeps multi-layer structure of various materials with residual tension and compression. The manipulation of the thickness and stress of the control layers succeeds in inducing buckling in beam structure at MEMS scale. The materials that will be used in full energy harvester device have been extensively characterized to provide the reference for the future fabrication.

Important lessons are learned from the development of the micro buckled beam oscillator prototype. The wide and narrow beam devices with different widths also have proportional center proof mass, and are supposed to have the same or similar responses, however, the distinction between the frequency responses of the two devices is obvious. The wide-beam oscillator shows stiffening response around 100Hz while the narrow-beam oscillator shows much lower frequency response. The distinction is mainly due to the wide beams’ residual stress in transverse direction forms corrugations and diminishes the buckling in longitudinal direction, while the narrow beams’ transverse corrugations are only near the edge and the whole beam still preserves the
Figure 5-18: The frequency response of the LPCVD nitride based device with silicon proof mass.

longitudinal buckling. This is verified by the high speed videos, which show transitions from small amplitude oscillations to large amplitude oscillations with increasing input excitation amplitude for the narrow beams and no obvious transitions for the wide beams. The high speed videos also reveals the significant rotational mode of the narrow beams' oscillations. These undesirable modes are analyzed (section 6.1) and eliminated in the full energy harvesters.
Figure 5-19: Screenshots of the high speed videos of the wide beam device (a) and the narrow beam device (b).
Chapter 6

Parameter Design and Optimization

6.1 Design Analysis

6.1.1 Modified Design

The micro-fabricated buckled beam oscillators presented in the previous chapter have demonstrated large-amplitude response with low-frequency and low-amplitude inputs. However, two phenomena deviated from the designed operational mode. First, the wide-plate oscillator shows higher operational frequencies than the narrow-plate device. The high speed videos of the wide-plate oscillator also do not show the two modes of oscillations, inter-well and intra-well, at high and low amplitude inputs respectively, while the narrow-plate oscillator did. Second, the narrow-plate oscillators show significant rotations about the longitudinal axis during the oscillation. Analysis on the phenomena and a proposed new design are presented in this section.

Corrugations in the Transverse Direction

The discrepancy of the wide- and narrow-plate designs’ oscillation modes lies in the different buckling modes of the two devices. From the photo in Figure 6-1, the wide plate has corrugations in the transverse direction from the boundary near the frame all the way to the boundary near the center proof mass, while the narrow plates only had buckling in the transverse direction near the edge of the frame and the mass with
no transverse corrugations in the middle region of the plates. The buckling in both longitudinal and transverse directions is due to the bi-axial compression in the plate structures, namely the plates' built-in residual stresses were in both longitudinal and transverse directions. The transverse corrugations of the wide plate diminished the longitudinal buckling and hence the bi-stability. The narrow plates, on the other hand, had transverse corrugations mainly near the plate edges, so it still maintained significant longitudinal buckling, and hence showed bi-stable oscillations with low-frequency, high-amplitude snap through.

The corrugations in the transverse direction need to be eliminated while the longitudinal buckling needs to be maintained, to enable bi-stable oscillation. Therefore, the new design was improved toward this goal. The strategy was to increase the critical buckling load in the transverse direction. Even though the compressive stress in longitudinal and transverse directions are close to isotropic, if it is lower than the critical load in the transverse direction but higher than the critical load in the longitudinal direction, the buckling only happens in the longitudinal direction.
Figure 6-2: Effect of boundary conditions on the buckling coefficient of rectangular plates subjected to in-plane boundary conditions. a and b are the length and width of the plate, c and ss denote clamped and simply-supported respectively [19].

It could be quickly identified that the width and the length are the critical design parameters for decoupling the buckling in transverse and longitudinal directions, since the stack’s compositions are isotropic across the plates. Analytically, the bi-axial buckling could be modeled by the double sinusoidal functions as in the examples in [19], depending on the boundary conditions. The critical loads of a rectangular plate are related to the boundary conditions as well as the width and length (as shown in Figure 6-2): reducing the dimension along the transverse direction increases the critical buckling load in that direction. The less corrugation in the narrow plate than in the wide plate also manifest this point. Therefore, the width of the wide-plate oscillator should be decreased if keeping the same length. The boundary condition combination of the designed structure was more complex than the classical examples in the references and hence the parameter design of the width was aided by finite
FEA: similar composition and stress of the fabricated device

1mm wide 0.5mm wide 0.3mm wide 0.2mm wide

Critical load: 0.9 Critical load: 3.7 Critical load: 10.3 Critical load: 22.7

Figure 6-3: Comsol model of the plate with designed composition and various width. The critical buckling load increases as the plate width decreases.

element analysis with Comsol (Figure 6-3). Various plate widths of a plate with designed composition are simulated to find the critical buckling load. As can be seen, with deceasing width, the critical buckling load increases (more difficult to buckle). The plate width of 0.4mm is finally chosen to have the critical buckling load at least 5 times higher than the compressive load to be applied.

Rotational Mode of Narrow Plate

Unlike the wide-plate oscillator, the narrow-plate oscillator showed intra-well and inter-well oscillations observed from the high speed videos at small and large amplitude inputs respectively. Nevertheless, the narrow plate also displayed rotations about the plate’s longitudinal axis during the oscillation. The rotation of the plate could induce nonuniform strain across the plate which is not effective for extracting power using the parallel interdigitated electrodes array which are perpendicular to the plate’s longitudinal axis. Therefore, the rotational mode needs to be minimized in the new design.

The rotation of the narrow-plate oscillator about the plate’s longitudinal axis is due to the asymmetric distribution of the mass about the plate’s longitudinal axis. The relatively large proof mass bonds to the bottom surface of the plate and the center of mass is about half the thickness of the mass away from the plate’s longitudinal axis. The vibration of the narrow plate is strictly along the input vibration’s direction.
without rotation only when the center of mass and the plate’s longitudinal axis perfectly aligned along the input vibration’s direction. If there is a small misalignment, which could be due to the manufacturing or perturbation of the input vibration, the inertial force on the proof mass would produce a torque about the plate’s longitudinal axis and trigger the rotation.

With the same input torque \( T \), the rotation \( \theta \) is inversely proportional to the torsional constant of the plate \( (J_T) \) and proportional to the plate’s length:

\[
\theta = \frac{TL}{GJ_T}
\]

where \( G \) is the shear modulus of the material, and \( L \) is the length of the plate. For a high aspect ratio of the narrow plate (width/thickness), the torsional constant is, \( J_T = \frac{1}{3}WH^3 \). Since the stack’s thickness is determined by the functionality and fabrication compatibility (presented in the next section), we could decrease the length of the plate or increase the width of the plate to increase the torsional constant and minimize the rotation. Nevertheless, the length of the plate is also related to the critical buckling load in the longitudinal direction and hence the frequency response, we chose to increase the width.

**New Design**

Summarizing the modifications demanded from the previous analysis, the plate requires small width to increase the critical buckling load in the transverse direction and minimize the buckling in that direction, while the oscillator as a whole should have larger width to resist the rotation. The two seemingly contradictory requirements could be decoupled by coupling multiple single beams to a parallel beam array, so the width of the beam and the width of the beam array are decoupled. The schematic of the new design is illustrated in Figure 6-4. The narrow beam could minimize the buckling in transverse direction locally, while the much larger width of the beam array including the gap between the single beams increased the torsional constant significantly to restrain the rotations.
6.1.2 Threshold of Input Vibration Amplitude

Energy Barrier

A bistable system's potential energy with respect to the deflection could be depicted by a double-well shaped curve. The two wells correspond to the stable equilibria of the system, and the local maximum between the two wells correspond to an unstable equilibrium. If we consider a buckled beam based bi-stable system with a linear and a nonlinear stiffness as in Chapter 4, its potential is,

\[ U(x)_{\text{Bi-stable}} = \frac{1}{2}k_L x^2 + \frac{1}{4}k_N x^4 \]  

(6.2)

It should be noted that the potential energy of the system is determined solely by the stiffness's, independently from the dynamic state (could be in static state too) or damping. The crest in between the two wells is the so called "energy barrier", which is a function of the linear and nonlinear stiffness:

\[ E_{\text{Barrier}} = \frac{1}{4} \frac{k_L^2}{k_N} \]  

(6.3)
As explained in Chapter 3, the bistable oscillator could oscillate within one potential well, or between the two wells. Since the inter-well oscillation gives large amplitude deflection and hence higher power, it is the ideal operational mode for energy harvesting. Determination on whether the system could overcome the energy barrier and to follow the input vibration to have the dynamic snapping becomes critical in the energy harvester design process. A quick conclusion that the input energy should be higher than the energy barrier will not serve as a sufficient design criteria to determine the mode of oscillation since the system’s state depends on its initial state as well as the damping after injecting the energy. When the system oscillates within one well already, whether the system overcomes the barrier relies on whether the input vibration provides the extra energy. The problem is complicated by the involvement of damping, which continuously consumes energy during the oscillation. This not straightforward dynamic problem has been analyzed by Melnikov theory.

Threshold Estimation Using Melnikov Method

The complex dynamics of the bistable system is analyzed by Melnikov’s method. The principle of Melnikov’s method is to measure the separation between the stable and unstable manifolds in phase space. The Melnikov function is derived and a closed form input threshold for the buckled beam oscillator is obtained to aid the optimum design. [95,96] provide analysis of similar problems, and the analysis here will follow the same fashion. To simplify the analysis, a mechanical bistable system without electro-mechanical coupling is considered. The governing equation of the bistable system is,

\[ m\ddot{x} + c\dot{x} + k_L x + k_N x^3 = f(t) \]  \hspace{1cm} (6.4)

where \( m \) is the mass, \( x \) is the displacement, \( c \) is the mechanical damping coefficient, \( k_L \) and \( k_N \) are linear and nonlinear stiffness respectively, \( f(t) \) is the input force as a function of time. Rewriting equation 6.4 in a perturbed state space form,
\[
\dot{x} = f(x) + \epsilon g(x, t)
\]  \hspace{1cm} (6.5)

where \(\epsilon\) is a small factor and the vectors are defined as,

\[
x = \begin{bmatrix} x \\ y \end{bmatrix}
\]  \hspace{1cm} (6.6)

\[
f(x) = \begin{bmatrix} y/m \\ k_L x - k_N x^3 \end{bmatrix}
\]  \hspace{1cm} (6.7)

\[
\bar{g}(x, t) = \begin{bmatrix} 0 \\ -cy + f \end{bmatrix}
\]  \hspace{1cm} (6.8)

For \(\epsilon = 0\), the Hamiltonian system is,

\[
m\ddot{x} + k_L x + k_N x^3 = 0
\]  \hspace{1cm} (6.9)

By writing \(y = m\dot{x}\) and \(\dot{y} = -k_L x - k_N x^3\), and integrating the separable differential equation \(\frac{dx}{dy} = \frac{y/m}{x(-k_L - k_N x^2)}\), we obtain

\[
\frac{y^2}{m} = -k_L x^2 - \frac{1}{2} k_N x^4 + C
\]  \hspace{1cm} (6.10)

where \(C = 0\) so the homoclinic paths to reach the origin. Separate variables again,

\[
\int 1/\left[ x \sqrt{-\frac{k_L}{m} - \frac{1}{2} \frac{k_N}{m} x^2} \right] dx = \int dt
\]  \hspace{1cm} (6.11)

and the homoclinic trajectory and its time derivative in the right half portion of the phase space are,

\[
x(t) = \frac{1}{\sqrt{b}} \text{sech}(\sqrt{at})
\]  \hspace{1cm} (6.12)

\[
y(t) = \frac{\sqrt{a} \text{sech}(\sqrt{at}) \tanh(\sqrt{at})}{\sqrt{b}}
\]  \hspace{1cm} (6.13)
where \( a = \frac{k_L}{m} \) and \( b = \frac{k_N}{m} \).

The Melnikov function could be written as,

\[
M = \int_{-\infty}^{\infty} -cy^2 + f(t)y dt
\]  \hspace{1cm} (6.14)

where \( f(t) \) is an input harmonic force \( f(t) = f_0 \cos(\omega(t + t_0)) \). Insert \( y(t) \) and \( f(t) \) into the integral:

\[
M = -\frac{ac}{b} I_1 + \sqrt{\frac{a}{b}} f_0 \cos(\omega t_0) I_2 - \sqrt{\frac{a}{b}} f_0 \sin(\omega t_0) I_3
\]  \hspace{1cm} (6.15)

where \( I_1, I_2 \) and \( I_3 \) are,

\[
I_1 = \int_{-\infty}^{\infty} \text{sech}^2(\sqrt{a}t) \tanh^2(\sqrt{a}t) dt = \frac{2}{3\sqrt{a}}
\]  \hspace{1cm} (6.16)

\[
I_2 = \int_{-\infty}^{\infty} \text{sech}^2(\sqrt{a}t) \tanh^2(\sqrt{a}t) \cos(\omega t) dt = 0
\]  \hspace{1cm} (6.17)

\[
I_3 = \int_{-\infty}^{\infty} \text{sech}^2(\sqrt{a}t) \tanh^2(\sqrt{a}t) \sin(\omega t) dt = \frac{\pi \omega \text{sech}(\frac{\pi \omega}{2\sqrt{a}})}{a}
\]  \hspace{1cm} (6.18)

Substitute \( I_1 \) to \( I_3 \) to equation 6.15 and let \( M(t_0) = 0 \), and rearrange the equation, we can find the critical force \( f_0 \) to be,

\[
f_0 = \frac{\omega_0^2 c}{\sqrt{b} \, 3\pi \omega \text{sech}(\frac{\pi \omega}{2\omega_0})}
\]  \hspace{1cm} (6.19)

where \( \omega_0 = \sqrt{k_L/m} \) and \( b = k_N/m \), and hence the input acceleration’s amplitude threshold is,

\[
a_{\text{cri}} = \frac{k_{LC}}{k_{N}^{1/2} m^{3/2} 3\pi \omega \text{sech}(\frac{\pi \omega}{2\omega_0})}
\]  \hspace{1cm} (6.20)

The obtained closed-form threshold of input vibration amplitude is a function of frequency so that combined with the frequency response obtained from the lumped parameter model, the mode of oscillation could be analyzed (next section). It should
be noted that the first term of the threshold is a function of the linear and nonlinear stiffness, damping and the proof mass. These can be identified as the key design parameters for designing the low-amplitude bi-stable oscillator based energy harvester.

**Implication of the Input Threshold**

Melnikov function measures the distance between stable and unstable manifolds in the first order and is inherently conservative [95]. However, the closed-form solution provides the trend of the threshold by varying the frequency and damping. It also helps to determine the operational frequency range since the analytical solution of the lumped model solved by harmonic balance could not provide sufficient information. This is particularly important in that the operational frequency and amplitude are the main design goals. Here is an example on using the Melnikov method together with the lumped model to obtain the operational frequency range at a specific input amplitude. Figure 6-5 shows the frequency response of a bi-stable energy harvester with an initial buckling of 200µm and 0.26g proof mass. Assuming the input amplitude is 0.5g and the damping coefficient is 0.015Ns/m, the lumped model could then be solved by the harmonic balance method. The red line shows the inter-well oscillations, but before the jump down, both inter-well and intra-well responses overlap, due to the fact that when solving the nonlinear model, the harmonic balance method assumes trial functions for both modes, and solves for both solutions mathematically. But physically, the system could be in one state. To find the physical solution, we need information on whether the input is sufficient to activate the inter-well oscillation, so the Melnikov method is used to find the input amplitude threshold at each frequency (red line with the y-axis on the right). Now if we specify an input amplitude, say 1g, and draw a horizontal line, there will be an intersection of this amplitude threshold line and the Melnikov’s solution. At frequencies higher than the intersection, the input amplitude is larger than the Melnikov’s threshold, and the system will have enough energy to activate and sustain at the inter-well oscillations, while below the intersection, Melnikov’s threshold is larger than the input amplitude, so the input energy is not sufficient to sustain the inter-well oscillation and the sys-
Figure 6-5: True oscillation frequency response from lumped model solution and Melnikov method. Initial buckling of 200 $\mu m$, with tungsten mass of 0.26g.

The system will choose the intra-well oscillation. So between the intersection frequency and the jump-down frequency, the system would have large amplitude inter-well oscillation, and below the intersection frequency and higher than the jump-down frequency, the system could only have small amplitude intra-well oscillation. The operational frequency response is highlighted with the yellow line. Therefore, the design should make the large amplitude oscillation match our target operational frequency range, at the target input amplitude. This is carried out by computer simulations presented in parameter design section.

6.2 Designing Parameters for Low-Frequency, Low-Amplitude Energy Harvester

After modifying the design presented in section 6.1.1, the design process then focuses on choosing the design parameters to realize the design goals. The design parameters are mostly the dimensions of the device since the layout and material selection have been determined from the design concept and the fabrication process. With the
analytical model developed in Chapter 4, the design parameters are lumped so that their relationships to the design goals could be discovered. And a design flow was constructed to optimize the design parameters with the design constraints satisfied.

6.2.1 Design Parameters

With the modified design the design parameters are mostly the dimensions of the device, among which, some of them are predefined due to the functionality requirement such as the sequence of the stack, and the compatibility to the fabrication process such as the thickness of the spin-coated PZT thin films. The parameters to be determined include the thickness's and residual stress's of the control layers.

6.2.2 Design Goals and Constraints

Design Goals

The design goals are summarized by the inequalities:

\[ \text{Power} > 5\mu W \]
\[ \text{Voltage} > 1V \]
\[ \text{Bandwidth} > 20\% \]
\[ \text{Operational Frequency} < 100Hz \]
\[ \text{Operational Amplitude} < 0.5g \]

Material Properties

Constraints

Design constraints come from the limits of materials, such as the strain limit, the compatibility to fabrication process, such as the limitations on the film’s thickness, and the consideration on layout, such as the footprint limit. The constraint from the material is mainly the strain developed during the oscillation is smaller than the yield strain of the materials. Material properties are listed in Table 6.1 and 6.2.
Table 6.1: Mechanical Properties of the Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s Modulus (GPa)</th>
<th>Poisson’s Ratio</th>
<th>Tensile or Fracture Strength (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal Silicon Dioxide (wet)</td>
<td>70</td>
<td>0.17</td>
<td>0.11</td>
</tr>
<tr>
<td>LPCVD Silicon Nitride</td>
<td>290</td>
<td>0.27</td>
<td>5.5 +/- 0.8</td>
</tr>
<tr>
<td>PECVD SiO₂</td>
<td>85 +/- 4</td>
<td>0.25</td>
<td>9.52</td>
</tr>
<tr>
<td>PECVD SiN</td>
<td>160</td>
<td>0.25</td>
<td>2.4</td>
</tr>
<tr>
<td>ZrO₂</td>
<td>240</td>
<td>0.27</td>
<td>/</td>
</tr>
<tr>
<td>PZT</td>
<td>60</td>
<td>0.3</td>
<td>0.06-0.12</td>
</tr>
</tbody>
</table>

Table 6.2: Piezoelectric Properties of PZT

<table>
<thead>
<tr>
<th>Material</th>
<th>d_{33} (pC/N)</th>
<th>k_{33}^2</th>
<th>ε_r</th>
</tr>
</thead>
<tbody>
<tr>
<td>PZT</td>
<td>274</td>
<td>0.45</td>
<td>1200</td>
</tr>
</tbody>
</table>

Fabrication Constraints

The thermal oxide and LPCVD thin films have limitations on the thickness, since the deposition process slows down significantly with excessive time, and more importantly, the films are more probable to crack with increasing thickness. The PECVD silicon dioxide layer underneath the ZrO₂ should be at least 300nm to be effective as a diffusion barrier. The ZrO₂, PT and PZT layers are deposited by sol-gel spin coating, hence, the thickness of each layer is pre-fixed. To achieve the best quality of PZT thin films, extensive experiments have been done on trying various parameters such as the spin speed, pyrolysis temperature and time, and the number of layers of each material. It has been found that at least one layer of ZrO₂ is needed as the diffusion barrier, at least two layers of PT layers are necessary to provide abundant lead, and three to four layers of PZT could be deposited without cracking or other defects. The gold electrode layer should be at least 100nm thick to withstand the wire bond process.

\[ H_{\text{Thermal SiO}_2} < 2\mu m \]
\[ H_{\text{LPCVD SiN}} < 2\mu m \]
\[ H_{\text{PECVD SiO}_2} > 300nm \]
$80nm < H_{ZrO_2} < 250nm$  
$8nm < H_{PT} < 15nm$  
$80nm < H_{PZT} < 400nm$  
$H_{Au} > 100nm$  
$-400MPa < T_{PECVD SiN} < 300MPa$  
$-300MPa < T_{PECVD SiO_2} < 0MPa$

**Layout Considerations**

We have limited the device dimension smaller than a quarter coin size, to be easily embedded into other systems, and to be consistent with the previous generation device for comparison. The PZT wet etching may cause severe undercut that can be 3 to 10 times bigger than the etching depth. Number of contact pads are determined by the number of beams and how the beams are connected. To increase the active area and consider the mask making expense, the electrodes has been chosen to be $4\mu m$. To insure high voltage output and also to improve the poling and piezoelectric properties, gap distance between electrode fingers of $4\mu m$ is also used. Contact pads should have a minimum size for wire bonding.

*Device Length* $< 19mm$  
*Device Width* $< 19mm$  
$d_{electrode} = 4\mu m$  
$d_{gap} = 4\mu m$  
$L_{pad} > 100nm$

**6.2.3 Design Flow**

Each design parameter affects the design goals, and the complex relations are illustrated by Figure 6-6, which shows the lumped parameters connecting the design parameters and the design goals are coupled. Relying on the lumped parameter model, analytical form of key parameters such as the input amplitude threshold and the resid-
ual moment, and the design constraints, a design process flow has been constructed to systematically optimize the design and realize the design goals.

At the beginning of the design process, some design variables could be set from the functional requirements so that the number of design variables is reduced and the design process can be simplified. The sequence of the materials in the thin film stack structure is mainly determined by the functional requirements, such as the active piezoelectric layer and its associated diffusion barrier, etch stop, structural layer for supporting the proof mass etc. The material selection is based on the experience from the previous designs of the energy harvesters as well as the materials available at MTL. More details of the material selection are described in the fabrication section 7.2.

The constraints from the fabrication limitations were also considered to minimize the design variables, such as the thickness of the PZT and associated diffusion barrier and seed layer. Thicker PZT film increases the volume of the active transduction material and consequently, increases the power output but the increased thickness introduces potential cracking and other mechanical defects. The sol-gel coated $ZrO_2$ and PT layers are similar in this sense, and their thickness's were set to optimize the quality of the PZT after extensive experiments. The residual stresses of most of the materials are also relatively fixed, such as the sol-gel coated materials, thermal oxide, LPCVD silicon nitride.
With the residual stresses characterized by extensive experiments on dummy samples, the main design parameters left are the thicknesses of the thermal oxide layer, silicon nitride layer, two PECVD silicon dioxide layers and one PECVD nitride layer. Only five independent parameters were then need to be determined. Since the residual stress control of the buckling beam based design including total compression and stress balancing (analyzed in section 5.3.1) are critical to the design goals and the fabrication, the design of these parameters started firstly by checking the total compression injected by looking for linear stiffness smaller than zero, then minimized the residual moment.

The optimal parameter searching process is carried out by a Matlab program (see Appendix A) which parametrically sweeps each independent design variable and calculates the linear stiffness and the residual moment and compare to the design criteria to find the matched design combinations. Since the number of the design variables is small and the parameter values were discrete numbers in a range set by the constraints, there is not much computation burden and the time for running the program is only a couple of seconds. The design parameter space then gives the lumped parameter values such as the linear and nonlinear stiffnesses, which were fed into the lumped parameter model. The simulation results showing the frequency responses of displacement, strain, voltage and power with the input amplitude threshold obtained in section 6.1.2, to make sure the design goals are satisfied. If not, the independent design parameters of the stresses of the PECVD control layers could be altered in their large range and the design programs were run again. The design flow is summarized in Figure 6-7.

The design parameter space is typically large, and the solutions to achieve the required design specifications are not unique. Multiple design parameter solutions make the fabrication process flexible and much more robust to variations introduced by the deposition tools. For example, when each PECVD layer has been deposited, the real thickness and stress may depart from the designed values, even great effort has been spent on characterizing the depositions on dummy samples. With the simple design flow, the rest design parameter could be recalculated in seconds to give the
Design goals (power, operating frequency range, operating amplitude)

Preset the active layer and the structural layer (material, thickness's, residual stress)

Parametrically search design space of control layers’ thickness and residual stress to minimize the residual moment

Calculate the lumped parameters from the beam composition design

Check if the design’s lumped parameters matches the design goals from analytical model

Deposition (real-time stress control)

Figure 6-7: Design scheme based on analytical model and parametric search.
same design specifications with new set of the rest design parameter values. This repetitive process has been done after each deposition to tune the whole design on track, until the last deposition so that the deviation from the initial target could be minimized. This is also the basis of the feedback stress control, which is covered in section 5.3.1.
Chapter 7

MEMS Buckled Beam Based Energy Harvester

7.1 Design

The design of the MEMS energy harvester is guided by the analysis on the previous buckled beam oscillator prototype, and the parameter optimization presented in Chapter 6. An array of parallel slender beams coupled by a central proof mass has been designed to eliminate corrugations in the transverse direction on the beams and keep the longitudinal buckling; the effective width of the beam array is much larger than the width of a single beam to resist the rotational mode during oscillation. The 3D rendition of the design is illustrated in Figure 7-1.

The beam composition of the MEMS energy harvester is different from that of the buckled beam oscillator. The piezoelectric layer and its associated seed layers are added to generate electricity. The sol-gel spin coated layers have high tensile stress and the thickness of PZT is limited to less than 500nm before cracking or other defects starting to emerge. Aluminum is designed as the electrodes first since it is inert to the XeF$_2$ etching, however, the wire bonder is only compatible to gold. Ti/Au replaces the Aluminum later. Compressive PECVD SiN is designed to be the structural layer and a main source of compression. The stress in PECVD SiN inverses however, after rapid thermal annealing of PZT. Thicker LPCVD SiN and
Figure 7-1: 3D rendition of the buckled beam based energy harvester. The buckling of the beam is exaggerated.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness</th>
<th>Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>PECVD Nitride</td>
<td>800nm</td>
<td>-200MPa</td>
</tr>
<tr>
<td>Electrodes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pt</td>
<td>10nm</td>
<td>400MPa</td>
</tr>
<tr>
<td>ZrO₂</td>
<td>70nm</td>
<td>370MPa</td>
</tr>
<tr>
<td>PZT</td>
<td>240nm</td>
<td>650MPa</td>
</tr>
<tr>
<td>PECVD Oxide</td>
<td>400nm</td>
<td>-300MPa</td>
</tr>
<tr>
<td>LPCVD Nitride</td>
<td>300nm</td>
<td>-250MPa</td>
</tr>
<tr>
<td>Thermal Oxide</td>
<td>750nm</td>
<td>170MPa</td>
</tr>
<tr>
<td>Structural layer</td>
<td>1000nm</td>
<td>-300MPa</td>
</tr>
<tr>
<td>Passivation layer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proof mass</td>
<td>530μm</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7-2: Beam composition design of the MEMS energy harvester.
thermal dioxide are used instead as the structural layer and compression source. The
design of the beam composition is depicted in Figure 7-2.

7.2 Fabrication

7.2.1 Deposition of Structural Layer

Thermal Dioxide

Thermal oxide is the first layer of the beam stack structure. It is the etch stop for DRIE since its etching rate is much lower than the etching rate of silicon [97]. The compressive residual stress (~300MPa) is a main source of compression in the current design since thick layer (up to 2μm) could be achieved without much difficulty. To balance the residual stress in the stack, compressive top and bottom layers are necessary since the piezoelectric layer and its buffer layer have high tensile stress in the middle of the stack. The growth is well predicted by theory and the stress is consistent (Figure 5-5). Thermal oxide is also very stable during annealing process. Experiment with thermal oxide sample shows, after 6min rapid thermal annealing (total length of the PZT annealing), the residual stress of thermal oxide only changes less than 3%.

<table>
<thead>
<tr>
<th>Equipment</th>
<th>tube 5D-ThickOX at ICL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recipe</td>
<td>wet at 1050°C</td>
</tr>
<tr>
<td>Duration of Deposition</td>
<td>3hrs 6mins</td>
</tr>
<tr>
<td>Target Thickness</td>
<td>1000nm</td>
</tr>
</tbody>
</table>

LPCVD Silicon Nitride

The samples after thermal oxidation need to be transferred to the low pressure chemical vapor deposition (LPCVD) furnace within 4 hours to minimize the contamination. LPCVD silicon nitride has high fracture strain compared with other materials (PECVD nitride and oxide), and is suitable for structural layer to support the heavy
proof mass. The residual stress can be tuned by changing the gas ratio during deposition, however the recipe of LPCVD silicon nitride is fixed at MTL for consistent deposition purpose. The residual stress of the MTL's LPCVD silicon nitride is tensile (at 740nm, the stress is \(\sim170\text{MPa}\)), therefore, the thickness of the silicon nitride is designed so the total compression can still induce buckling and the oscillation frequency can reach the design target.

<table>
<thead>
<tr>
<th>Equipment</th>
<th>VTR at ICL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature/ Pressure</td>
<td>775°C/250 mTorr</td>
</tr>
<tr>
<td>Gas Flow</td>
<td>25 sccm (NH_3), 250 sccm (H_2SiCl_2)</td>
</tr>
<tr>
<td>(sccm = standard cubic centimeters/sec)</td>
<td></td>
</tr>
<tr>
<td>Target Thickness</td>
<td>750nm</td>
</tr>
</tbody>
</table>

**PECVD Silicon Dioxide**

An oxide layer is necessary to prevent the chemical reaction between silicon and the lead that diffuses from the active layers [11, 12]. Therefore, a thin layer of PECVD silicon oxide is deposited following the LPCVD silicon nitride. The residual stress of the silicon oxide is compressive and can be tuned, contributing compression to the beam stack.

<table>
<thead>
<tr>
<th>Equipment</th>
<th>STS-CVD at TRL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recipe</td>
<td>LFSIO</td>
</tr>
<tr>
<td>Gas Flow</td>
<td>1420 sccm (N_2O), 392 sccm (N_2), 12 sccm (SiH_4)</td>
</tr>
<tr>
<td>Power</td>
<td>60W</td>
</tr>
<tr>
<td>Duration of Deposition</td>
<td>3.5mins</td>
</tr>
<tr>
<td>Target Thickness</td>
<td>300nm</td>
</tr>
</tbody>
</table>

The residual stress of the PECVD dioxide is found to change after rapid thermal annealing of PZT. Even the annealing is four to five cycles of 1min annealing, the temperature is 700 °C. To better predict the real stress in the structure, experiments are done on characterizing the stress with the annealing time. The stress in PECVD dioxide after annealing can be found in Figure 7-3a.
Figure 7-3: Residual stress change due to rapid thermal annealing. (a) PECVD silicon oxide. (b) PECVD silicon nitride.

PECVD Silicon Nitride

PECVD silicon nitride is firstly designed to be the structural layer since its stress can be easily tuned (Figure 5-6). However, the residual stress changes drastically after rapid thermal annealing of PZT (Figure 7-3b), from compressive stress to tensile stress. The first batch devices with PECVD silicon nitride layer have not enough compression and no buckling is induced. This layer is then replaced by thicker thermal oxide and LPCVD nitride layers.

| Equipment | STS-CVD at TRL |
| Recipe    | MFSIN          |
| Gas Flow  | 40 sccm NH₃, 1960 sccm N₂, 40 sccm SiH₄ |
| Power     | 30W            |

ZrO₂

ZrO₂ is the diffusion barrier/buffer layer for PZT [11,12]. Same as the PZT thin film in this device, ZrO₂ is deposited through sol-gel spin coating. The sol-gel is ZrO₂ dissolved in organic solvent and is purchased from Mitsubishi Materials. In the fabrication process, it has been found that ZrO₂ thin film is easy to crack after pyrolysis (Figure 7-4a). The cracks, even at sub-micron level, detaries the subsequent PZT thin films and result in unusable PZT. Extensive experiments on the coating condition
Figure 7-4: Microscopic images of patterned ZrO$_2$ thin films. (a) Cracked ZrO$_2$ thin film. (b) Crack-free ZrO$_2$ thin film.

were carried out to establish a reliable recipe for depositing crack-free thin film. Parameters in the coating process including temperature and duration of the pyrolysis, spin speed and ramp speed of the spin coater, volume of each dispense, and number of layers have been tuned. The final recipe chooses the highest spin speed of the coater and one layer deposition to reach the thinnest coating (thinner film is less likely to crack), with appropriate temperature settings (see the recipe below). Coating should also be done with low humidity (preferably <45%). The recipe produced crack-free coating of ZrO$_2$ repeatedly in three years and the coatings showed no defects.

<table>
<thead>
<tr>
<th>Solution</th>
<th>9%wt ZrO$_2$ sol-gel by Mitsubishi Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin Coating</td>
<td>500rpm(10secs) - 4000rpm(30secs)</td>
</tr>
<tr>
<td>Drying/Pyrolysis</td>
<td>80 °C(60secs)/300°C(300secs)/150°C(60secs)</td>
</tr>
<tr>
<td>Thickness</td>
<td>~ 90nm(one coat)</td>
</tr>
<tr>
<td>Etch</td>
<td>7:1 BOE solution:DI water = 8:13 (10secs)</td>
</tr>
<tr>
<td>Annealing</td>
<td>700°C(60secs)</td>
</tr>
</tbody>
</table>

**PT**

PT (PbTi) as a seed layer is added between the diffusion barrier ZrO$_2$ and PZT. The lead rich seed layer provides abundant lead during the high-temperature annealing process, so the PZT’s lead loss from diffusion is compensated. The higher yield rate of healthy PZT has been observed with the added PT. The deposition process is the
same to the PZT deposition.

Solution $1\text{wt}\% \text{PbTi} 125/100 \text{ (PT)} E1$ sol-gel (Mitsubishi Materials)

Spin Coating $500\text{rpm}(5\text{secs}) - 3000\text{rpm}(30\text{secs})$

Drying/Pyrolysis $180^\circ\text{C}(60\text{secs})/390^\circ\text{C}(300\text{secs})/180^\circ\text{C}(60\text{secs})$

Thickness $10 \sim 15\text{nm}$ (two layers)

Etch DI Water:HCL:BOE(7:1)=10:3:1 (dip in the etchant and spray with DI water)

Annealing $700^\circ\text{C}(60\text{secs})$

**PZT**

PZT is the active material in the energy harvester device and converts the mechanical energy into electrical energy; the quality of the PZT thin film is critical to the performance of the device. The PZT thin film is deposited with sol-gel spin coating as well, followed with pyrolysis, the process of heating the coated sample to burn the organic solvent, wet etching, which patterns the PZT, and annealing, in which the amorphous PZT crystallize to polycrystalline perovskite phase.

The deposition parameters such as the pyrolysis and annealing temperatures affect the quality of the PZT and were extensively investigated by previous researchers [98–103], the new recipe is mainly based on the previous recipe [12]. Two steps to increase and decrease temperature are to minimize the possibility of thermal shock during pyrolysis. The etching time should be controlled well to avoid under-etching (PZT residue at unwanted spots, and hard to remove after annealing) or over-etching (delamination could happen). A simple way to implement the etching is to dip the sample into the etchant and pull out to spay wash. White residue still on the wafer is a sign of under-etching, and more dipping should be done. The dipping should stop when there is no white residue after spray rinse. The spin coating-pyrolysis should be repeated multiple times to reach a desirable thickness (each layer is 70–80 nm), however, it is found that annealing multiple coatings results in sever degradation of PZT (Figure 7-5a), and the new recipe changes the repeated cycle to coating-pyrolysis-annealing, so that each layer of PZT is pyrolyzed, patterned and annealed
Figure 7-5: Microscopic images of patterned and annealed PZT with SEM profile images. (a) Degraded PZT with hillock. (b) Good PZT coating.
before coating the next layer. The much thinner PZT of each layer becomes stable after annealing and a whole stack of PZT suffers no degradation or cracking (Figure 7-5b).

Solution 15%wt PZT (110/52/48) E1 sol-gel (Mitsubishi Materials)
Spin Coating 500rpm(5secs) – 3000rpm(30secs)
Drying/Pyrolysis 180°C(60secs)/390°C(300secs)/180°C(60secs)
Target Thickness ~ 240nm(three layers)
Etch DI Water:HCL:BOE(7:1)=10:3:1 (dip in the etchant and spray with DI water, repeat until the PZT is fully removed)
Annealing 700°C(60sec)

Electrodes

Ti/Al (200 Å/ 1000 Å) are firstly used as the electrodes due to the aluminum is inert to the $XeF_2$ and the electrodes would not be etched during the long $XeF_2$ etching. But the wire bonder at MTL only has the gold wire and it is difficult to wire bond the
gold wire to Al contact pads after trials including elevating the chuck temperature during bonding. The electrodes are then changed to Ti/Au (200 Å/2000 Å).

Photolighography

AZ-5214 resist (2000rpm) / Exposure 1.4 secs + bake 100s at 120°C+ Flood Exposure 48secs

Metal Deposition Ti/Au(200Å/2000Å)

Equipment EbeamFP Temescal Model FC2000

Lift-Off Ultrasound cleaning in acetone bath for 10min

PECVD Silicon Nitride

The passivation layers, include PECVD silicon nitride and oxide is deposited after the electrodes deposition to protect the electrodes from dirt that could cause defects. The silicon nitride and oxide are also flexible in residual stress controlled as aforementioned, hence, they serve as stress control layers and provide compression to the beam structure. The PECVD silicon nitride has a better adhesion to the PZT and electrodes compared to PECVD dioxide, and hence is deposited first.

    Equipment      STS-CVD at TRL
    Recipe         MFSIN
    Gas Flow       40 sccm NH₃, 1960 sccm N₂, 40 sccm SiH₄
    Power          30W
    Duration of Deposition 40mins
    Target Thickness 800nm

PECVD Silicon Dioxide

XeF₂ etches silicon dioxide slower than etching the silicon nitride, and hence the silicon dioxide layer is deposited on top to provide more reliable passivation during the final release.
7.2.2 Etching of the Multi-Layer Structure

Reactive Ion Etching of Passivation and Structural Layers

The passivation and structural layers are dry etched with reactive ion etching (RIE) instead of wet etched so that the PZT will not be affected by the acid etchant. The etching rate of RIE is accurate after characterization and is suitable for time-controlled etching. Since the thicknesses of the passivation and structural layers vary from sample to sample, the etching time is calculated by first divide the thickness by the etching rate, then plus 20 ~ 30% of the calculated time (due to the non-uniformity of the PECVD coatings). The etching time is typically long (>30 min) and the chamber temperature can get high so the photoresist is burned and becomes hard to strip. Therefore, the etching time is divided into cycles of maximum 10min for each cycle, with 5min cooling down time. Thick photo resist (10um) is applied as the etching mask. An ultrasonic bath with Microstrip for > 15min is need after the etching is done to strip the photo resist.

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Plasmaquest at TRL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recipe Name</td>
<td>SiO₂ Etching, Cool-down</td>
</tr>
<tr>
<td>Gas Flow</td>
<td>O₂ 4 sccm, CF₄ 40 sccm</td>
</tr>
<tr>
<td>Power</td>
<td>400W</td>
</tr>
<tr>
<td>Etching Rate</td>
<td>2nm/s</td>
</tr>
</tbody>
</table>

Wet Etching of the Active Layer

ZrO₂ is wet etched after pyrolysis but before annealing. 1μm of SPR is sufficient as the mask for the wet etching. The SPR is removed by Microstrip in ultrasonic bath for > 8min. Rapid thermal annealing follows the removal of the photoresist.

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Acid Hood at TRL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Etchant</td>
<td>DI water: BOE (7:1) = 13:8</td>
</tr>
<tr>
<td>Duration of Etching</td>
<td>10s</td>
</tr>
</tbody>
</table>

PT and PZT are wet etched using the same etchant. The etching is fairly fast and a dip is sufficient. 1μm of SPR is used as the mask for the wet etching. The SPR
is removed by Microstrip in ultrasonic bath for > 8min. Rapid thermal annealing follows the removal of the photoresist.

**Equipment**  
Acid Hood at TRL

**Etchant**  
DI Water:HCL:BOE(7:1) = 10:3:1

**Duration of Etching**  
Dip in the solution and then rinse with DI water, repeat until no residual PT/PZT

**Backside Deep Reactive Ion Etching (DRIE)**

Wet etching is not used for etching through the silicon wafer to minimize the effect on PZT. Deep reactive ion etching is directional and can define high aspect ratio structures with straight walls. The thermal oxide layer is the etch stop of the DRIE. Since the etcher at MTL only accepts 6" wafers, the 4" wafers are mounted on 6" quartz carrier wafers first.

OLE3 is the slowest and most uniform etching recipe of the etcher and is used for the etching through process. However, the DRIE process still produces highly nonuniform etching (edge of the wafer is etched 20% more than the center). If stopping the etching when the edge is ready, the thick residual silicon in the central area is difficult to remove by slow XeF₂. While stopping when the central area if fully etched, the devices near the edge are broken by the excessive etching. The cause of the problem is found to be the plasma source is on one one side of the sample and the mounting does not transfer heat well. The wafer is rotated 90 degrees after each hour of etching so that all sides have relatively similar exposure to the ions. The mounting condition has also been extensively experimented. The channels in the photoresist between the sample and the carrier wafer that are suggested by MTL’s recipe are eliminated so that the sample wafer is in fully contact with the carrier. The risk is the trapped air can pop out so that the sample detaches from the carrier. Kapton tapes are therefore used to eliminate the risk. The mounting is shown in Figure 7-7a. The etching is much more uniform (< 5% variation on the same wafer) after taking these measures (Figure 7-7b).

The sample-carrier assembly is soaked in acetone overnight after the etching to
Figure 7-7: DRIE mounting and results. (a) Modified mounting of the sample to the carrier wafer. Fully covered photoresist with Kapton tapes are used. (b) Etching depth of DRIE on various spots on the same wafer measured after each hour of etching. Uniform etching results are obtained.

detach the sample. The released devices fall off from the wafer since a trench surrounding the device is also etch through. This separation eliminates the use of die saw, which may break the device.

<table>
<thead>
<tr>
<th>Equipment</th>
<th>STS1 at TRL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recipe Name</td>
<td>OLE3</td>
</tr>
<tr>
<td>Gas Flow</td>
<td>SF$_6$ 140 sccm, C$_4$F$_8$ 95 sccm</td>
</tr>
<tr>
<td>RF Forward Power</td>
<td>Coil 600W, Platen 140W</td>
</tr>
<tr>
<td>Camber Process Pressure</td>
<td>31 mTorr</td>
</tr>
<tr>
<td>Camber Peak-to-Peak Voltage</td>
<td>280V</td>
</tr>
<tr>
<td>Camber Bias Voltage</td>
<td>8V</td>
</tr>
<tr>
<td>Etch Rate</td>
<td>$1.6 \sim 2\mu m/min$</td>
</tr>
</tbody>
</table>
Figure 7-8: Photos of the released device. (a) Release device with annotations. (b) Released device with a US quarter coin.
Final Release Using XeF$_2$

The released devices may still have residual silicon ($< 50\mu m$) due to the non-uniform etching of the DRIE. A gentle XeF$_2$ etch without ion bombardment and excessive heat is useful for the removal of the residual silicon on the released delicate structures. Long time XeF$_2$ etch is found to etch the Au electrodes. The devices are therefore put on a carrier wafer and taped on the sides the contact pads area is not exposed to XeF$_2$.

<table>
<thead>
<tr>
<th>Equipment</th>
<th>XeF$_2$ at TRL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Etch Time</td>
<td>60s</td>
</tr>
<tr>
<td>Cycles of Etching</td>
<td>10 ~ 60 (depending on the amount of residual silicon)</td>
</tr>
<tr>
<td>Etching Rate</td>
<td>$\sim 1\mu m$/cycle</td>
</tr>
</tbody>
</table>

The release device is shown in Figure 7-8. The fabrication process is summarized in Figure 7-9.

7.2.3 Device Packaging

Initial testing of released devices including poling and P-E test could be done using probe station without device packaging. However, due to the number of contact pad (28 or 112), the space needed for the proof mass to move freely, and the required mounting onto a shaker, this kind of testing is only suitable for quick verification of the quality of the PZT but is not convenient for whole device poling and characterization, or dynamic testing. The released device is therefore packaged, wired bonded and integrated onto a testing circuit board after PZT quality screening test.

Testing Before Packaging

The released devices may suffer from low quality of PZT or defects on the inter-digitated electrodes even the beam structure is sound. To save the cost of ceramic packages and wire bonding, and effort in the packaging process, a screening process on the un-packaged devices is desirable before packaging.

The testing is measure the polarization of the PZT by applying an electric field, the so-called P-E test. A hysteresis loop (an example is shown in Figure 7-10) is
Fabrication process flow of the PZT embedded fully functional energy harvester.

Figure 7-9:
expected for good PZT with working electrodes, short circuit or open circuit due to the electrodes defects will result in irregular pattern or no signal. The testing setup consists ferroelectric tester (Radiant Technologies Inc. RT66a), high voltage interface (Radiant Technologies Inc. RT66a), voltage amplifier (Trek Model 601C) and probe station (Figure 7-11). The device rests on a flat surface and two probes contact a pair of contact pads on the devices. An hysteresis test can then be carried out with the maximum electric field of $125 \text{ kV/cm}$ to $250 \text{ kv/cm}$ in the Vision software (Radiant Technologies Inc.).

The testing setup could integrate a hotplate to heat up the device to an elevated temperature and with applied voltage, the PZT could be poled to further verify the quality. The testing parameters such as the applied voltage, poling temperature, poling duration have big impact on the quality of the PZT and are thus tested extensively. For example, too high poling voltage results in PZT breakdown (Figure 7-12) while too low voltage is not sufficient to achieve enough polarization. Since each device has as many as 52 pairs independent electrodes, a lot of testing could be done on one device. Extensive poling and P-E testing are done in this way to find the the
optimum poling and P-E testing conditions.

**Wire Bonding**

After the quality of the PZT and electrodes is verified, the device is packaged into ceramic pin grid array package (PGA12063002, Global Chip Materials, LLC). The package has 120 pins, which are more than the number of the contact pads (112 or 28), with a cavity of 17mm x 17mm. A silicon frame of 525μm thick and about the same size of the cavity is inserted in the cavity to leave space for the proof mass to move and elevate the device up for easier wire bonding. The device is then mounted on the silicon frame (Figure 7-13).

The packaged device is wired bonded to be able to extract electrical signal. Aluminum as the contact pads were used at the beginning to be inert to XeF₂ etching, but it was too difficult to bond with the gold wire. After switching to the gold contact pads, the bonding is much more reliable (Figure 7-14). Since there are more pins on the package than the contact pads on the device, only some pads on the package are active and wire bonded (Figure 7-15).
Figure 7-12: PZT breakdown when excessive electric field is applied.

Figure 7-13: Device in a ceramic pin grid array package.

Figure 7-14: Wire bonding of a packaged device.
A testing circuit board is built to easily pole and test the device. Perforated printed circuit boards (Vector Electronics 8029) have a compact size (5cm×7.5cm) are used to easily insert devices. A zip socket (Aries Electronics 169-PRS13001-12) is soldered on the circuit board instead of soldering the packaged device so that the device can be easily swapped and testing circuit board could be reused. Two 8 slide-style DIP switches (TE Connectivity ALCOSWITCH Switches 1825057-7) are positioned near the packaged device so that each unit of device (14 pairs of electrodes) could be connected or disconnected from the circuit independently. Two BNC coaxial connectors (TE Connectivity AMP Connectors 5-1814832-1) on the PCB allow the generated electrical signal be measured by low-noise shielded coaxial cables. The finished testing circuit board is shown in Figure 7-16.
Figure 7-16: Packaged device on testing circuit board. The wires connecting the device to switches and BNC connectors are soldered on the backside of the board.
7.3 Characterization and Poling of PZT

7.3.1 XRD Crystallography

X-ray crystallography (XRD) is an nondestructive technique to determine the crystal structure of materials. It is performed during the fabrication after the PZT deposition to verify the quality of the PZT. Rigaku Cr-Source RU300 Rotating Anode X-ray Generator at the Center for Materials Science and Engineering (CMSE) is used at the early stage of the fabrication process to verify the health of the PZT thin film. The tested sample has one layer of ZrO₂, two layers of PT and three layers of PZT, on top of silicon substrate. The perovskite phase PZT’s crystal peaks are superimposed on the measured data. The agreement, especially the peak at (110) orientation verifies the perovskite phase of the annealed PZT (Figure 7-17).

The XRD requires the sample fully covered with the thin film while the PZT is patterned before annealing during device fabrication, therefore the crystal structure is not checked often and the PZT might have degraded at the later stage of the fabrication process.
7.3.2 Poling

Poling is a process that rearranges the randomly oriented dipoles in piezoelectric material to form aligned polarization. A high electric field is applied at elevated temperature is the typical procedure. After packaging, the device’s all independent units are connected together and could be poled at the same time. The effectiveness of the poling affect the PZT coupling (efficiency) significantly. Therefore, extensive experiments are done to find the optimum poling conditions, including the electric field, temperature and duration. A container for enclose the packaged device during poling is self-made with a beaker insulated by fiberglass and aluminum foils, so the container can stand high temperature on hotplate and the temperature can be maintained constant in the container during poling. The packaged device with testing circuit board rest on the bottom of the beaker during poling with two high temperature resistant coaxial cables connected to the ferroelectric tester out from the beaker. A thermocouple probe for air is inserted in the beaker and monitors the air temperature during the poling. The setup is shown in Figure 7-18 and the poling condition is summarized:
Temperature  250°C (hotplate), 100°C (measured, air inside the container)
Electric Field  250 kV/cm
Duration  30min

7.3.3 P-E Testing

The P-E needs to be done after poling to verify the quality of the poled PZT. The same Radiant RT66a ferroelectric tester with high voltage interface setup is used as in the pre-packaged P-E testing. The applied alternating electric field results in the typical hysteresis loop which is depicted in Figure 7-19. The measured remnant polarization, saturation polarization, and the coercive field of the device can be estimated as $2P_r = 44 \mu C/cm^2$, $2P_s = 7.5 \mu C/cm^2$, and $2E_c = 150 kV/cm$, respectively. Both saturation and remnant polarization are much lower than the group’s previous PZT results [11, 12, 104]. This indicates the low performance of the PZT and will be discussed more in section 7.6. The parameters in the Vision software for the poling testing is summarized in table below.
7.4 Static Testing

In the static and dynamic testing sections, results of multiple devices will be shown. The devices are fabricated at different time and have different initial buckling. To avoid confusion in the later discussion, the devices are coded as device I, II, and III. These represent three batches of devices. Device I is the first batch of fabricated devices and has the PECVD SiN as the structural layer. The compressive stress in the PECVD SiN is switched to tensile during the annealing of PZT and then the whole beam loses the compression and buckling, and the compression cancels the bending stiffness so the linear stiffness is estimated to be zero. Device II is made after the troubleshooting of the device I. Thicker thermal dioxide and LPCVD SiN are used to replace the PECVD SiN as the structural layer, and hence the residual stress is well controlled and the buckling is close to the design target. Device III is a quick modification on a device from device II’s batch to inject more buckling by manually bending and compare with the device II. The main differences of these devices are listed in table 7.1.

Table 7.1: Summary of the three devices of micro-fabricated energy harvesters.

<table>
<thead>
<tr>
<th>Device Code</th>
<th>Initial Buckling</th>
<th>PZT Thickness</th>
<th>Buckling Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0</td>
<td>240nm</td>
<td>No Buckling</td>
</tr>
<tr>
<td>II</td>
<td>200μm</td>
<td>150nm</td>
<td>Residual Stress</td>
</tr>
<tr>
<td>III</td>
<td>300 ~ 400μm</td>
<td>150nm</td>
<td>Residual Stress+</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Manual Bending</td>
</tr>
</tbody>
</table>

The released devices show buckling in beams that are visible to naked eyes. To more accurately characterize the buckling and verify the bi-stability of the buckled structure, optical profiling of the surface is done. Since the frame of the device and the proof mass has the same thickness and when putting on a flat surface, the proof
mass touches the surface and restricts the beam buckling. A silicon frame is thus inserted under the device so the proof mass is suspended and the beam can buckle freely. Wyko NT9800 optical profilometer is used to scan the surface profile, and the stitching feature assembles multiple scans to cover the whole beams' surface profile.

Figure 7-20 shows the surface profile of two beams on device I. The maximum deflection of the beam (end of the beam to the central proof mass) is only $\sim 20\mu m$, which is much less than the designed $\sim 200\mu m$. The dynamic tests of this device also shows much higher operating frequency range. This is due to the lack of compression in the beam. More accurately, using the lumped stiffnesses in equation 4.14, the proof mass alone can results the same deflection, and the linear stiffness is about zero, which means the little compression cancels the bending stiffness but is not enough to induce buckling.

After replacing the PECVD SiN layer, the device II is made. The surface profile of the whole device is scanned, as shown in Figure 7-21. The 28 beams on the same device show good consistency in buckling, and the measured surface profile in Figure 7-21 matches well with the design (Figure 6-2). The device's surface is scanned first with top surface facing up, and then flipped so the beams buckle to the other direction and the bottom surface of the beams are scanned. Since the weight of the
Figure 7-21: Surface profile scan of device II. (a) Three dimensional representation of the scanned surface of the whole device. (b) Surface profile plot of four beams showing the initial deflection. The device’s profile is scanned both upside up and upside down to show the buckling in both directions.
proof mass is equivalent to 1g loading, which is higher than the threshold of the snap, in this way, we can observe the bi-stable buckling. The buckling in both directions show similar $\sim 200\mu m$ maximum deflection, which proves no significant asymmetrical stress distribution that leads to only one direction buckling. The surface profile and the large buckling in longitudinal direction also proves the modified 3rd generation design with narrow beam width effectively eliminates the transverse direction and preserves the buckling in the desired direction.

7.5 Dynamic Testing

Device I, II, III are tested dynamically with input vibrations to characterize the devices’ oscillations as well as the power generation capability. The setup is similar to the one shown in Figure 5-13. The Polytec OFV 056 vibrometer scanning head is used for this testing. The scanning head could shoot the laser at multiple points of interest on the device to generate vibration modes automatically.

Device I is tested by sweeping the input harmonic vibration’s frequency from up from 20Hz to 1000Hz, to find the operating frequency and amplitude. The input amplitude varies from 0.5g to 1g during the sweeping. The proof mass is the wafer-thick central silicon proof mass ($13.9mm \times 3mm \times 525\mu m$). Figure 7-22 shows the test results. The deflection of the device shows a peak at very low frequency $<50Hz$, which agrees with the low linear stiffness and hence low linear resonance frequency. The lower but relatively flat response between 100 $\sim$ 350Hz is due to the nonlinear resonance from the stiffening effect of the beam. The nonlinear stiffening response is more clear in the open-circuit voltage response in Figure 7-22b. Similar to the frequency response in [12], the frequency response shows gradual increased amplitude with increasing frequency on the input until jump down near 350Hz. Various resistors are connected to the harvester directly and the voltage drop on the resistor is measured to calculate the power consumed on the load (Figure 7-22c). At $2M\Omega$, the maximum power is obtained at 0.07 $\mu W$.

Device I’s test results are promising: the jump-down frequency and the operating
Figure 7-22: Dynamic testing results of device I. The input vibration’s frequency is swept from 20Hz to 1000Hz, and the amplitude varies from 0.5g to 1g. The proof mass is silicon \((13.9 \text{mm} \times 3\text{mm} \times 525\text{µm})\). (a) Deflection versus frequency. (b) Open-circuit voltage. (c) Power on various load resistors.
amplitude are about one quarter and $1/8 \sim 1/4$ of our group's previous work [12] respectively, and the bandwidth is comparably large ($\sim 20\%$ of the jump-down frequency). However, the operating frequency is higher than the design goal ($<100\mathrm{Hz}$).

No buckling induced in the device due to the loss of PECVD SiN's compression makes the device a mono-stable nonlinear oscillator.

Device II's buckling and bi-stability have been verified by the surface profile scan. Dynamic testing similar to that for device I is done. The main change in testing practice is the automatic sweep controlled by the computer is replaced by fixed frequency testing for more accurate characterization of the performance. The sweeping test has a continual signal with varying frequency, so that the initial condition at each frequency is not at the lowest energy state and affect the results. To eliminate the hysteresis effect, the device is tested with an initial static state, and the input vibration's amplitude increase to reach the target value at a constant frequency. In this way, we can make sure the device starts with lowest energy state and decide if the input energy is enough to trigger the snap, without the interference of the device's initial state.

The fixed frequency testing is done at various frequencies below $100\mathrm{Hz}$ and at different input amplitude level below $0.5g$ (Figure 7-23a). The relative displacement of the mass to the frame is calculated by subtract the displacement amplitude of the mass by the displacement amplitude of the frame. Since the vibrometer cannot scan the two spots simultaneously but only sequentially, the phase difference cannot be obtained, especially with the abrupt bi-stable snapping that may not be perfectly the same in each cycle. The estimation is an underestimation as a result, since the phase difference between the two is not always $180^\circ$. But the trend shows large amplitude snap in a wide frequency range below $100\mathrm{Hz}$.

Device II is also connected to a load resistor ($1M\Omega$) to measure the power consumed 7-23. The voltage across the resistor is measured at a sample rate of $5.2\ \text{kHz}$. The power consumed by the resistor is calculated as $P = V^2/R$. During a period of $3.2s$, the peak voltage is identified for each fixed frequency and amplitude and is used to calculate the peak power. At $0.5g$, the peak power spectrum shows $> 50\%$.
Figure 7-23: Dynamic testing results of device II. (a) Displacement of the mass relative to the frame (silicon mass). (b) The peak power of the device II (with 0.22g tungsten mass). (c) The average power of the device II (with 0.22g tungsten mass).
Figure 7-24: Frequency response of the device II with input amplitude of 0.5g. (a) Excited at 30Hz. (b) Excited at 50Hz.

of the jump-down frequency half-power bandwidth below from 30Hz to 80Hz (Figure 7-23b). The average power Figure 7-23c shows $\sim 35\%$ bandwidth of the jumped-down frequency (70Hz). The power is lower than expected due to the low coupling efficiency of the PZT and will be discussed in the next section.

The dynamic testing also verifies the rotational mode that the second generation prototype has is restrained effectively. The device is tested at constant amplitude (0.5) at 30Hz and 50Hz, as can be seen in Figure 7-24, the rotation mode double the frequency of the drive mode and has the amplitude less than 1/10th of the primary mode. The effective restriction on the rotational mode is attributed to the coupling of the beam array with proof mass (section 6.1.1).

Device III is has more initial buckling and has a higher input threshold to snap.
The device snaps through $>0.9g$ and hence is tested at $1g$. A $1M\Omega$ resistor is used to measure the power. As Figure 7-25 shows, a similar high power level is maintained in a wide bandwidth below 90Hz. The higher input amplitude leads to higher jump-down frequency than device II.

### 7.6 Discussion of Results

Among Device I, II and III, device II is the one with the designed initial buckling $\sim 200\mu m$. The static surface profile scan on the device clearly show the bi-stable buckling in two directions, and validated the residual stress induced buckling design concept. The dynamic test with input vibrations at various frequencies demonstrate the ultra-wide bandwidth of the bi-stable buckled beam based oscillator, with a half-power bandwidth of 35% of the jump-down frequency (70Hz). High energy level is maintained from 30Hz to 70Hz with input amplitude at 0.5g verifies the achievement of the designed operating condition goal ($<100Hz$, $<0.5g$). The wide-band response of a MEMS energy harvester at low frequencies ($<100Hz$) and low amplitude (0.5g) have not been achieved by previous works. The new design shows promising advance towards the real environment applicable energy harvester.
Even though Device I deviates from the original design due to the unexpected stress change in PECVD SiN, the almost zero linear stiffness (compressive stress cancels the bending stiffness) in the beam structure provides a good opportunity for understanding the enhancement of the low-frequency operation from the buckling. If we compare the frequency response of device I and II, the output energy of device I peaks above 300Hz while the similar level of energy is achieved below 70Hz in device II. The shift of the frequency response to lower frequencies is the prediction of the theoretical analysis (3.1.1), and is supported by the experiments.

The large-amplitude snapping of the bi-stable buckled beam has characteristic high velocity as well as high acceleration. The instantaneous acceleration of the snapping at each frequency are plotted in Figure 7-26. During the accelerating stage, the beam is moving from one buckled position towards the other buckled position and becomes compressed. Even though mechanical structures can sustain much higher compression than tension, the high acceleration induced fast change in strain may have effects on the fatigue life of the buckled beam and needs to be studied in the future research.

The power generated is lower than expected and needs to be analyzed. The low coupling of the PZT is found to be the cause. The capacitance of the device is measured with Radiant RT66a setup, to be $\sim 2.5nF$, so we can estimate the dielectric
constant of the PZT:

\[ C_0 = \varepsilon_r\varepsilon_0 A_{\text{cap}}/g \]  

(7.1)

where \( A_{\text{cap}} \) is the area of the equivalent parallel plate capacitor and \( g \) is the gap between the equivalent parallel plate capacitor. The area can be calculated as,

\[ A_{\text{cap}} = n_{\text{unit}} \cdot n_{\text{electrodes}} \cdot L_{\text{electrodes}} \cdot H_{\text{electrodes}} \]  

(7.2)

where \( n_{\text{unit}}, n_{\text{electrodes}}, L_{\text{electrodes}}, H_{\text{electrodes}} \) are the number of independent capacitor units (one unit is on half a beam with a pair of electrodes), the number of electrode fingers in one capacitor unit, the overlap length of the interdigitated electrodes, the thickness of the PZT; and the numbers are 56, 539, 0.174mm, 147nm respectively. Plug in these numbers in equation 7.1 and the relative dielectric constant is around 1500. The piezoelectric coupling coefficient could be then estimated,

\[ d_{33} = \frac{2C_0V_{OC}}{A_{\text{cap}} E_{PZT}} \]  

(7.3)

where \( d_{33} \) and \( E_{PZT} \) are the piezoelectric coefficient and Young's modulus of PZT respectively. With the measured open-circuit voltage \( V_{OC} \) of 0.12V and the estimated strain \( S \) of \( \sim 0.03\% \) from the displacement measurement, the piezoelectric constant \( d_{33} \) is about 42pC/N. The coupling factor, \( k_{33}^2 \), can be estimated,

\[ k_{33}^2 = \frac{d_{33}^2 E_{PZT}}{\varepsilon_r \varepsilon_0} \approx 0.84\% \]  

(7.4)

The coupling \( k_{33}^2 \) is defined as the ratio of the stored electrical energy to the input mechanical energy can be roughly understood as the efficiency of the PZT. The piezoelectric constant and coupling factor are much lower than the values used when designing and simulating the device \( (k_{33}^2 \approx 35\%) \). Using the estimated dielectric constant and piezoelectric constant, the lumped parameter model could simulate the system's performance, as shown in Figure 7-27.

The matching between the simulation and the test results implies that the low-
Figure 7-27: Simulated frequency response of device II using the estimated dielectric constant and piezoelectric coefficient. The input amplitude is 0.5g. The damping coefficient is chosen to match the jump-down frequency (70Hz). (a) Displacement. (b) Phase portrait. (c) Peak power.
coupling PZT is the cause of the low power output. The degradation of the PZT starts to show during the fabrication: part of the area of the wafer has defects and the coating of the PZT can only go up to two layers (147nm) without too much defects instead of designed three to four layers (260nm).

The best PZT was achieved in our group by Y. B. Jeon and the coupling factor $k_{33}^2$ reached 45%, with a thickness of 0.5$\mu$m. Assuming the high coupling could be achieved, based on the current device configuration, the performance is simulated (Figure 7-28). The comparison between the two simulation results with only the difference in piezoelectric coupling suggests the power could be improved significantly by employing better quality of PZT or other materials.

The power density of the device II is 3.59 $\mu$W/cm$^3$ (over the volume of the beams and the proof mass), the normalized power density (NPD) is the power density further normalized by the acceleration amplitude squared, a figure of merit (FOM) first used by Beeby [2] and later used in other literature [2,105,106]. Device II’s NPD is reported here: 0.15 $kgsm^{-3}$. Since various energy harvesters reported in literature are tested at different levels of vibrations, the NPD is trying to eliminate the difference in the maximum power by normalizing the power density with the acceleration amplitude squared. Even though the aim of this FOM is to reach a universal metric for comparison, it has some shortcomings: the maximum energy is proportional to the input vibration’s amplitude squared divided by the frequency; the normalization does
not fully eliminate the source vibration’s characteristics since the attainable power is proportional to mass times internal displacement range, or to volume$^{4/3}$, dividing by volume does not remove the size dependence completely and thus favors larger devices [105].
Chapter 8

Conclusions

8.1 Summary of Thesis Contributions

This thesis has enabled a new concept of vibration energy harvesting which can meet all the requirements for commercially viable energy harvesting products at MEMS scale: wide bandwidth and low-frequency operation at low-amplitude ambient vibrations. Bi-stable nonlinear oscillator is realized and demonstrated at MEMS scale for the first time. The MEMS buckled beam oscillator does not rely on the resonance of the structure but on the snapping motion with large displacement in a wide bandwidth at low frequencies. Compared to the current state-of-the-art resonant MEMS energy harvesters, this thesis shows that this energy harvester meets the wide bandwidth (>35%), low-frequency (<70Hz) and low-amplitude (<0.5g) operational requirements at the same time.

The conventional linear resonators harvest energy from ambient vibrations effectively only at their resonant frequencies, but the gain-bandwidth dilemma limits their application in widely varying ambient vibrations. The recent mono-stable nonlinear resonators have widened the bandwidth significantly, while the operating frequency range of them was also increased by the hardening effect. Bi-stable oscillators have been reported at macro-scale to resolve these issues. Their large size structure, however, could hinders the practical use of them. The MEMS buckled beam based bi-stable energy harvester is developed in this thesis for the first time, which would meet...
the requirements for energy harvesting as described above.

An electromechanical lumped parameter model of a buckled clamped-clamped multi-layer beam with piezoelectric coupling is developed to design and verify the design of MEMS buckled beam harvesters. Nonlinear oscillations in intra-well and inter-well mode are predicted from the analytical solutions of the above model using the harmonic balance method. In the design process, the linear and nonlinear stiffness's are shown to determine both the operating frequency range and oscillation amplitude, and thereby become the key design parameters. The closed-form design parameters are functions of the dimensions of the buckled beam. The dimensions of the control layers are then parametrically swept to find the frequency response that matches our designed frequency range. Melnikov theory is used to derive the threshold of the input vibration's amplitude of the bi-stable system, combined with the frequency response from the lumped model, to ensure the large-amplitude snapping at low frequencies be activated at the designed input acceleration amplitude (low-g).

Residual stress is intentionally introduced and controlled along the MEMS monolithic fabrication processes of 10 thin film layers. With extensive measurements and characterization of the deposition rate and residual stress of each thin film material in the multi-layer beam structure, thicknesses of the multiple layers of the beam are designed based on the analytical model developed (Appendix A) to incur a desirable amount of compression for buckling of about 200$\mu$m at the center of the beam. Symmetric distribution of the stress with respect to the neutral axis is also considered to ensure bi-stable buckling. The thickness and stress of each thin film deposition is monitored during the fabrication as a feedback to adjust the subsequent layer deposition, minimizing the deviation of the final fabricated device from the design. The fabricated device shows buckling matches with the designed amount within 5%.

Bi-axial residual stress induces buckling in both longitudinal and transverse directions of rectangular beams. The flat wide beam prototype showed the corrugations in the transverse direction, diminishing the effect of the longitudinal buckling. The heavy proof mass under the beam also exerts rotational mode during beam oscillation and induces non-uniform strain distribution. A new design based on slender buckled
beam array with coupled central proof mass is therefore made. The narrow beam width increases the critical buckling load in the transverse direction so that the compressive stress in that direction is smaller than the critical load, and the corrugations are eliminated. The whole beam array’s effective width is increased by the coupling of the central proof mass, and the rotational mode is effectively reduced.

Finally, a wafer-scale MEMS monolithic fabrication process was successfully completed to make and demonstrate the MEMS buckled beam oscillator for energy harvesting. Challenges in the fabrication include the thin film coating/etching/patterning of thin PZT layer over the structural membrane layer, the release of the buckled structure and the control of thermal effects on the residual stress of underlying layers. A fully functional energy harvester with PZT thin film is fabricated and tested. Static surface profile of the device demonstrates 200µm buckling on both sides, which proves the designed bi-stability of the buckled structure. Dynamic testing demonstrates the state-of-the-art operating conditions of MEMS energy harvesters of 35% bandwidth below 70Hz at 0.5g.

8.2 Future Work

The future work will focus on improving the quality of the PZT and its coupling to improve the power generation. As analyzed in Chapter 7, with piezoelectric coupling $k_{33}^2$ over 40%, > 5µW power could be expected using the current device configuration. The power density would be above 220 µW/cm³, and the normalized power density (NPD) would be $\sim 9 \text{ kgs}m^{-3}$. Other piezoelectric materials with high piezoelectric coupling such as PMN-PT may also be considered. The thickness of the PZT in this work is also smaller than designed and can be increased to further increase the power.

The power measurement is currently done with a simple resistor load. The generated power could be stored after rectifying the AC signal and connected to a capacitor or battery. The energy harvester’s power generation could be further improved by employing specially designed electrical interfaces, such as the synchronized switching harvesting on the inductor (SSHI). Integrating these circuit designs with the energy
harvester device is suggested.

In the design process of this work, the maximum strain in the multi-layer structure is designed to be smaller than the yield strain of the materials to minimize the fatigue. To deploy the energy harvesters in real-world applications, testing and validating the reliability such as the fatigue property of the beam materials are needed. The instantaneous acceleration of the structure is much larger than the input acceleration according to the testing results. Even though the beam is mainly in compression during the acceleration, the effect of the large acceleration on fatigue needs to be studied in the future. The G shock capacity of energy harvesters from impact is rarely considered or reported in the current energy harvesting research field. The energy harvester in this work operates at low input amplitude (<0.5g), while during impact the g shock could easily get to $10^2 \sim 10^3 g$). Packaging design with mechanical stops to limit the travel distance of the proof mass, or other mechanisms may be considered to increase the g-shock.
Appendix A

MATLAB Codes

A.1 Beam Composition Design

% Beam composition design of a multi-layer beam structure
close all
clear all
clc

%% Input Parameters
BeamMatAll = beam_mat; % Load materials
BeamComp_pzt = BeamMatAll('SiO2_Thermal', 'SiN', 'PECVD_SiO1', 'ZrO2', 'PT', 'PZT', 'PECVD_SiN2', 'PECVD_SiO2',:);
n_pzt = 8; % Number of layers (from bottom to top, the index goes from 1 to n)
E = BeamComp_pzt.E; % Young's modulus for each material
nu = BeamComp_pzt.nu; % Poisson's ratio
E_ = E ./ (1 - nu); % Biaxial modulus
T = BeamComp_pzt.T; % Thickness of each layer
S0 = BeamComp_pzt.T0; % Residual stress before release
W = 0.4e-3;
L = 6e-3;
%% Parametric sweep of the thicknesses
% Design space
% t1 = 300e-9:100e-9:1000e-9; % Thermal oxide
% t2 = 300e-9:100e-9:900e-9; % LPCVD nitride
% t3 = 300e-9:20e-9:400e-9; % First PECVD SiO2 layer
% t7 = 800e-9:50e-9:1500e-9; % PECVD SiN layer
% t8 = 100e-9:50e-9:1200e-9; % Second PECVD SiO2 layer

q = 1; % An index number for recording candidate combinations

%for f = 1:length(t1)
%for h = 1:length(t2)
for i = 1:length(t3)
for j = 1:length(t7)
for k = 1:length(t8)
% T(1) = t1(f);
% T(2) = t2(h);
T(3) = t3(i);
T(7) = t7(j);
T(8) = t8(k);
% Neutral axis
H = neutralaxis(T, E);
% Moment due to residual stresses
M = 0;
epsilon = sum (S0.* T) / sum (E_.* T); % Total strain of the beam due to residual stresses
S0_ = S0 - E_ * epsilon; % Residual stresses after release but before bending
for p = 1:n_pzt
M = M + S0_(p) * 1/2 * ( (H + sum(T(1:p)))^2 - (H + sum(T(1:p-1)))^2 ); % Total moment generated by residual stress
end
% Tip deflection due to the residual moment
E_I = 0;
for p = 1:n_pzt
E_I = E_I + E_(p) * 1/3 * ((H + sum(T(1:p)))^3 - (H + sum(T(1:p-1)))^3); % Equivalent E*I
end
rho = E_I / M; % Radius of curvature of the beam
theta = L / rho; % Angle subtended by the arclength L
tip_deflec = rho*(1 - cos(theta));
% Overall stress
T0 = sum(S0.*T)/sum(T);
if abs(tip_deflec) <= 10*sum(T) && T0<0 % Look for designs with small residual moment and compressive total stress
%Design_pzt(1,q) = t1(f);
%Design_pzt(1,q) = t2(h);
Design_pzt(1,q) = t3(i);
Design_pzt(2,q) = t7(j);
Design_pzt(3,q) = t8(k);
Design_M(q) = M; % Total moment generated by residual stress
Design_H(q) = H; % Neutral axis
Design_tip(q) = tip_deflec; % Tip deflection
q = q + 1;
end
end
end
end
%end
%end
% Display the parameters of optimal design in nano-meters
1e9*Design_pzt
A.2 Frequency Response from Lumped Parameter Model

Main.m
% Lumped model simulation of the bi-stable buckled beam based energy harvester

close all
clear all
clc

%% System parameters
R = 1e6; % Load resistance (Ohm)
m = 2.64e-4; % Proof mass (kg)
a = 0.5*9.8; % Excitation amplitude (m/s^2)
b = 0.013; % Damping coefficient
F0 = m*a; % Equivalent excitation force
% Frequency range
fmin = 1; %Hz
fmax = 200; %Hz
fstep = .1;
f = fmin:fstep:fmax;
flength = length(fmin:fstep:fmax);
OMEGA = 2*pi*f; % Frequency range in radian/sec

%% Load the material properties
BeamMatAll = beam_mat;
%% Beam composition
BeamComp = BeamMatAll('SiO2_Thermal', 'SiN',

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PECVD_SiO1', 'ZrO2', 'PT', 'PZT', 'PECVD_SiN2', 'PECVD_SiO2', ;

%% Calculate the lumped parameters from the beam composition
[kl, kn, C0, Cn, Cl, L, z_, H_l, H_u] = beam_cal(BeamComp);

%% Spring Stiffening Solution (Interwell)
for i = 1:length
    omega = 2*pi*f(i);
    A = 3/4*kn + 1/2*Cn^2*R^2*C0^2*omega^2/(1+4*R^2*C0^2*omega^2);
    B = -m*omega^2 + kl + Cl^2*R^2*C0/(1+R^2*C0^2*omega^2);
    C = 1/4*Cn^2*R*omega/(1+4*R^2*C0^2*omega^2);
    D = b*omega + Cl^2*R*omega/(1+R^2*C0^2*omega^2);
    E = (m*a)^2;
    W0_stiff = sqrt(roots([A^2+C^2, 2*(A*B+C*D), B^2+D^2, -E]));
    w0_stiff(:,i) = W0_stiff.*imag(W0_stiff)==0);
end

% Maximum deflection amplitudes
w0max_stiff = max(w0_stiff);

% Maximum strains
Sb_stiff = 2*pi^2/L^2*max([H_u(6), H_l(6)])*w0max_stiff; % Max bending strain
Sa_stiff = pi^2/4/L^2*w0max_stiff.*2; % Max stretching strain

% Voltage from the linear coupling
V0l_stiff = Cl*R*OMEGA.*w0max_stiff./sqrt(1 + C0^2*R^2*OMEGA.^2);
% Voltage from the nonlinear coupling
V0n_stiff = 1/2*Cn*R*OMEGA.*w0max_stiff.*2./sqrt(1 + 4*C0^2*R^2*OMEGA.^2);
% Power consumed by the load resistor
P_stiff = V0l_stiff.^2/R + V0n_stiff.^2/R;

%% Spring Softening Solution (Intrawell)
for i = 1:length

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\omega = 2\pi f(i);

A = -15/4kn + 1/2Cn^2R^2C0^2\omega^2/(1+4R^2C0^2\omega^2);

B = -m\omega^2-2*kl+Cl^2\omega^2R^2C0/(1+R^2C0^2\omega^2);

C = 1/4Cn^2R^2\omega^2/(1+4R^2C0^2\omega^2);

D = b\omega + Cl^2R^2\omega^2R^2C0/(1+R^2C0^2\omega^2);

E = (m*a)^2;

WOsoft = sqrt(roots([A^2+C^2, 2*(A*B+C*D), B^2+D^2,-E]));
w0_soft(:,i)=WOsoft.*(imag(WOsoft)==0);

end

% Maximum deflection amplitudes
w0max_soft = max(w0_soft);

% Maximum strains
Sb_soft = 2*pi^2/L^2*max([H_u(6), H_l(6)])*w0max_soft; % Max bending strain
Sa_soft = pi^2/4/L^2*w0max_soft.^2; % Max stretching strain

% Voltage from the linear coupling
V0l_soft = Cl*R*OMEGA.*w0max_soft./sqrt(1 + C0^2*R^2*OMEGA.^2);

% Voltage from the nonlinear coupling
V0n_soft = 1/2*Cn*R*OMEGA.*w0max_soft.^2./sqrt(1 + 4*C0^2*R^2*OMEGA.^2);

% Power consumed by the load resistor
P_soft = V0l_soft.^2/R + V0n_soft.^2/R;

%% Plot results

% Deflection
figure
grid minor
hold on
plot(f,w0_stiff*1e6,’r.’,’linewidth’,3);
plot(f,w0_soft*1e6,’b.’,’linewidth’,3);
xlabel(‘Excitation Frequency (Hz)’);ylabel(‘Deflection Amplitude (\mum)’);

% Power

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figure
hold on
plot(f,P_stiff*1e6,'r.', 'linewidth',3)
plot(f,P_soft*1e6,'b.', 'linewidth',3)
xlabel('Excitation Frequency (Hz)'); ylabel('Power (\muW)');

beam_cal.m
function [k_L, k_N, C0, Cn, Cl, L, z, H, H_u] = beam_cal(BeamComp)
% Calculate the lumped parameters including the linear and nonlinear
% stiffness, internal capacitance of the piezoelectric element and the
% turns ratio

%% Beam Parameters
W = 0.4e-3*28; % Width of the beam
L = 12e-3; % Length of the beam
E = BeamComp.E; % Young's modulus for each material
nu = BeamComp.nu; % Poisson's ratio
H = BeamComp.H; % Thickness of each layer
T0 = BeamComp.T0; % Residual stress
Lp_u = 0.8e-3/L; % Piezoelectric layer covers from -Lp_u*L to -Lp_u*L and Lp_u*L to Lp_u*L if the origin is at the beam midpoint Lp_u = 5.2e-3/L;
E_ = E ./ (1 - nu); % Biaxial modulus
% Piezoelectric layer properties
g = 4e-6; % Interdigitited electrodes' gap (m)
E_p = BeamComp.E('PZT'); % Elastic modulus of the piezoelectric material
H_p = BeamComp.H('PZT'); % Thickness of the piezoelectric layer
epsilon_r = 1200; % Relative permitivity
epsilon_0 = 8.85e-12; % Vacuum permitivity
d33 = 270e-12; % Piezoelectric constant
c33 = 63e9; % Elastic modulus of the piezoelectric material
\( e_{33} = d_{33} c_{33}; \) % Piezoelectric constant

\[
\begin{align*}
%% \text{Neutral Axis (from the bottom of the beam)}
\text{z}_\_ &= \text{neutralaxis}(H, E); \\

%% \text{Stiffness of the composite beam}
% \text{Calculate the upper and lower bound of each layer}
\text{for } i=1: \text{length}(H) \\
\text{H}_\text{1}(i) &= \text{z}_\_ - \text{sum}(H(1:i)); \\
\text{if } i &== 1 \\
\text{H}_\text{u}(i) &= \text{z}_\_; \\
\text{else} \\
\text{H}_\text{u}(i) &= \text{z}_\_ - \text{sum}(H(1:i-1)); \\
\text{end} \\
\text{end} \\
%
\text{Piezoelectric layer}
\text{H}_{\text{p}\_\text{u}} = \text{H}_\text{u}(6); \\
\text{H}_{\text{p}\_\text{l}} = \text{H}_\text{l}(6); \\
%
% \text{Calculate the stiffness}
\text{k}_\text{b} &= 2\pi^4 W/3/L^3 \text{sum}(E' \cdot (H_{\_\text{u}}^3 - H_{\_\text{l}}^3)); \% \text{Bending stiffness} \\
\text{k}_\text{res} &= \pi^2 W/2/L \text{sum}(T0 \cdot H); \% \text{Stiffness due to residual stress} \\
\text{k}_N &= \pi^4 W/8/L^3 \text{sum}(E \cdot H); \% \text{Nonlinear stiffness due to stretching} \\
\text{k}_L &= \text{k}_b + \text{k}_\text{res}; \% \text{Linear stiffness} \\
%
% \text{Find the internal capacitance of the piezoelectric element and the constant in the "turns ratio"}
\text{W}_\text{eff} &= 0.174e-3*28; \\
\text{L}_\text{eff} &= 4.312e-3*2; \\
% \text{Coupling coefficients}
\text{Cn} &= \pi^2 e_{33} W_{\text{eff}} H_{\text{p}} (L_{\text{p}\_\text{u}} - L_{\text{p}\_\text{l}})/L/2/g;
\end{align*}
\]
\( C_I = \pi \times e^{33} W_{\text{eff}} (H_{p_u}^2 - H_{p_l}^2) \times (\sin(2 \pi L_{p_u}) - \sin(2 \pi L_{p_l}))/Lg; \)

% Internal capacitance of the piezoelectric element

\( C_0 = W_{\text{eff}} L_{\text{eff}} H_p \epsilon_p \epsilon_0 \epsilon_{r_p}/(2g); \)

beam_mat.m

function [T_all] = beam_mat

%% Material parameters
% Young’s Modulus (Pa)
E_PZT = 63e9;
E_ZrO2 = 244e9;
E_PT = 63e9;
E_SiO2_Thermal = 70e9;
E_PECVD_SiO1 = 85e9;
E_PECVD_SiO2 = 85e9;
E_SiN = 290e9;
E_Si = 185e9;
E_PECVD_SiN1 = 160e9;
E_PECVD_SiN2 = 160e9;
E = [E_PZT; E_ZrO2; E_PT; E_SiO2_Thermal; E_PECVD_SiO1;
E_PECVD_SiO2; E_SiN; E_Si; E_PECVD_SiN1; E_PECVD_SiN2];

% Poisson’s Ratio
nu_PZT = 0.3;
nu_ZrO2 = 0.27;
nu_PT = 0.3;
nu_SiO2_Thermal = 0.17;
nu_PECVD_SiO1 = 0.25;
nu_PECVD_SiO2 = 0.25;
nu_SiN = 0.27;
nu\_Si = 0.27;
nu\_PECVD\_SiN1 = 0.253;
nu\_PECVD\_SiN2 = 0.253;
nu=[nu\_PZT; nu\_ZrO2; nu\_PT; nu\_SiO2\_Thermal; nu\_PECVD\_SiO1;
nu\_PECVD\_SiO2; nu\_SiN; nu\_Si; nu\_PECVD\_SiN1; nu\_PECVD\_SiN2];

% Thickness (m)
H\_PZT = 147e-9;
H\_ZrO2 = 131e-9;
H\_PT = 5e-9;
H\_SiO2\_Thermal = 998e-9;
H\_PECVD\_SiO1 = 303e-9;
H\_PECVD\_SiO2 = 528e-9;
H\_SiN = 741e-9;
H\_Si = 0;
H\_PECVD\_SiN1 = 800e-9;
H\_PECVD\_SiN2 = 902e-9;
H=[H\_PZT; H\_ZrO2; H\_PT; H\_SiO2\_Thermal; H\_PECVD\_SiO1;
H\_PECVD\_SiO2; H\_SiN; H\_Si;H\_PECVD\_SiN1; H\_PECVD\_SiN2];

% Residual Stress (Pa)
T0\_PZT = 650e6;
T0\_ZrO2 = 370e6;
T0\_PT = 400e6;
T0\_SiO2\_Thermal =-300e6;
T0\_PECVD\_SiO1 = -217e6;
T0\_PECVD\_SiO2 = -220e6;
T0\_SiN = 166e6;
T0\_Si = 0;
T0\_PECVD\_SiN1 = 400e6;
T0_PECVD_SiN2 = -150e6;
T0=[T0_PZT; T0_ZrO2; T0_PT; T0_SiO2_Thermal; T0_PECVD_SiO1;
T0_PECVD_SiO2; T0_SiN; T0_Si; T0_PECVD_SiN1; T0_PECVD_SiN2];

% Create materials table
MaterialNames=’PZT’;’ZrO2’;’PT’;
’SiO2_Thermal’;’PECVD_SiO1’;’PECVD_SiO2’;’SiN’;’Si’;’PECVD_SiN1’;
PECVD_SiN2’;
T_all=table(E,nu,H,T0,’RowNames’,MaterialNames);

neutralaxis.m

function [z_neutral] = neutralaxis(H, E)
    C = 0;
    for i = 1:length(H)
        C = C - E(i)*H(i)*sum(H(1:i)) + 1/2*H(i)*E(i)*H(i);
    end
    z_neutral = - C / sum(E.*H);
Appendix B

Masks

Figure B-1: Masks of the Buckled Micro Beam Oscillator Without PZT.
Figure B-2: Masks of the MEMS Buckled Beam Based Energy Harvester.
Bibliography


[36] C B Williams and R B Yates. s S oRs ACTUATORS Analysis of a micro-electric generator for microsystems dF-1 z ( O ii. 52:8–11, 1996.


