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Mid-frequency Acoustic Scattering from Finite Internally-loaded Cylindrical Shells Near Axial Incidence

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B.S., Webb Institute of Naval Architecture (1986)

Submitted to the Department of Ocean Engineering in partial fulfillment of the requirements for the degree of

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Abstract

In this thesis, I interpret monostatic and bistatic acoustic scattering measurements for finite cylindrical shells in water to determine the influence of internal structures and the endcap. I study the interaction of a transient acoustic pulse with the endcap and investigate elastic wave scatter at both a slope discontinuity and at a ring of mass plus stiffness. The shell configurations include an empty shell, a shell with four internal ring stiffeners, a duplicate stiffened shell with sprung internals, and an internally-loaded shell with an external constrained layer damping treatment. The scattering measurements were conducted using transient acoustic pulses in the mid-frequency range \((2 < ka < 11)\) where acoustic wavelengths are similar to the cylindrical shell radius \(a\), and \(k\) is the acoustic wavenumber in water.

The scatter is discussed in terms of axisymmetric compressional and flexural waves, which are dominant near axial incidence. Incidence to 25° relative to the shell axis is considered, where 25° delineates a transitional region of incidence beyond which forced wave scatter becomes important. Near axial incidence, the initial part of the backscatter is caused by radiation from the endcap of compressional waves whose energy decays rapidly with time. The onset of the flexural response is indicated by a significant reduction in the decay rates, where the transition time equals the flexural wave group delay to the first discontinuity beyond the insonified endcap.

The frequencies considered encompass a transitional range in terms of the mechanism for acoustic coupling at the endcap. At higher frequencies \((ka > 9)\), the interaction is a local to the spherical section of the endcap and compressional waves are excited via trace-matching. At lower frequencies, the endcap moves as a piston and excites both compressional and flexural waves. The rings primarily reflect the incident structural waves; however, strong coupling between neighboring rings creates pass bands at certain combinations of frequency and frame spacing. The sprung internals only weakly influence the scatter because they are essentially decoupled from the shell motions except at frequencies near the ring resonances.

Thesis Supervisor: Professor Ira Dyer
Title: Weber-Shaughness Professor of Ocean Engineering
To Karen Jean

you have always believed.
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Chapter 1

Introduction

In structural acoustics, a question that is not yet fully understood is what does and what does not affect mid-frequency acoustic scatter. The mid-frequency regime is comprised of acoustic wavelengths which are similar in scale to characteristic structural dimensions of the scatterer, such as the shell diameter or spacing between structural discontinuities. As a means of identifying the important mechanisms affecting the scatter, a series of experimental scattering measurements were conducted. The use of experiments is motivated by the difficulty of applying standard numerical and analytical methods in this frequency regime to non-canonical structures, where the structures of interest are finite thin cylindrical shells with internal structure.

In this thesis I investigate the scatter of an acoustic plane wave, incident upon a shell submerged in water. The direction of plane wave incidence is near axial or near bow incidence. In this thesis, near axial incidence is defined as $\phi_i \leq 25^\circ$ where $\phi_i$ is the angle of incidence relative to the axis of the cylindrical shell. The excitation bandwidth extends approximately from 1/2 to 3 times the shell ring frequency, $f_r$, where

$$f_r = c_p/2\pi a,$$  \hspace{1cm} (1.1)

$c_p$ is the shell compressional wave speed, and $a$ is the cylinder radius. The bandwidth corresponds to a normalized frequency range, $2 < ka < 12$, where $k$ is the wavenumber.
in water. I interpret scatter from four different shell models to distinguish the effects of various internal complexities. The four shell models are an empty shell, a ring stiffened shell with unequal ring spacing, a ring stiffened shell with the same unequal ring spacing and resiliently mounted internals, and the latter shell with an external constrained layer damping treatment over the exterior cylindrical surface.

There is a long history of interest in the radiation and scattering response of ocean going vehicles like surface ships and submarines. For the case of submarines, this interest stems from strategic requirements of both stealth operation and capability of detection and classification. Active sonar provides a means of identification and near axial incidence represents a probable incidence regime, particularly during a tracking or pursuit scenario. With respect to monostatic target strength, \( \phi_i < 25^\circ \) is an incidence region characterized by large backscatter. The shell models used in the experiments are not intended to represent scale submarines, rather they are intended to elucidate fundamental scattering phenomena which affect the total scattering response. For example, internal structural loading can strongly modify dispersion properties of the structural waves. In practice these internals provide significant loading; they take the form of framing, internal machinery or piping, and may have vastly different attachment conditions – frames may be directly welded to the shell, while machinery and decks may be resiliently attached to other internal structure. For the models the internal mass is as great as three times the shell mass and includes both directly connected stiffeners and sprung wave-bearing rods. Structural coupling allows energy transfer between the internal structures, the shell and the acoustic medium.

Because this thesis draws upon experimental data, the shells considered are necessarily finite in length. This is particularly important near bow incidence where the shell is initially struck by the incident sound wave at the endcap, which is the source of both the geometric scatter and the subsequent elastic wave radiation. In this incidence regime the important structural waves are axisymmetric compressional and flexural waves. These waves scatter at discontinuities in the shell geometry on the endcaps as well as at discontinuities caused by the attachment of the internal structure to the
shell. The discontinuities cause wave conversion, providing both radiation and energy storage mechanisms.

In the time domain, the elastic response has a very long extent compared with the initial pulse. The initial scatter from the four models is the same, but subsequent scatter contains information that identifies structural differences between the four shells. This thesis shows how the scatter manifests these differences.

1.1 Previous Work

In the acoustics literature, the scatter from shells in water has been studied extensively. Both canonical and non-canonical geometries have been considered, with spheres and infinite cylinders most frequently studied. The bulk of shell research has focused on the dynamics of and scatter from empty shells. Naturally these canonical geometries lend themselves most readily to analytical analyses. The motivation for looking deeply at the behavior of these rather simple geometries is the tractability of the problem, and the relative ease of decomposing the scatter on a phenomenological basis for interpretation. The fundamental understanding gained by examination of these structures becomes the foundation for understanding more complicated structures.

The scattering function for both the empty spherical shell and the infinite cylindrical shell can be determined exactly using modal series [1]. To extract physical insight, alternate asymptotic forms of the exact formulation have been introduced for both cylinders [2][3] and spheres [4][5]. For example, the Sommerfeld-Watson transform is a means of transforming the exact series to an integral form, where the singularities of the integrand correspond to the various scattering phenomena. In this formulation, the total scatter can be analyzed as the summation of these contributions. These methodologies have been applied extensively to study spherical scatter [4] [6] and cylindrical shell scatter [7], illustrating the importance of the membrane waves in the mid-frequency range at beam incidence for the cylindrical shell, and all incidence angles for the spherical shell.
At beam incidence, where the endcap contributions are less important, the infinite cylindrical shell provides a good approximation of scatter from a finite shell, particularly for long cylinders [8]. This is not the case at oblique incidence because the infinite cylinder model does not properly represent either the geometrically backscattered contribution or wave reflections at the endcaps. The infinite cylindrical shell formulation is essential, however, for achieving an understanding of mechanisms for helical wave excitation, propagation and radiation within the coincidence angular region [9][2][10][11], where coincidence describes an incidence region in which trace matched excitation of elastic waves is possible.

Near bow incidence ($\phi_i = 0^\circ$), where the endcap response becomes primarily important, it is necessary to consider finite targets, which generally have shapes that are mathematically non-separable. In lieu of exact analyses, numerical \( T \)-matrix methods have been used to calculate the $\phi_i = 0^\circ$ scatter from solid cylinders with hemispherical endcaps [12][13], from spheroids [14][15][16], and from spheroidal shells [17]. Additionally, $\phi_i = 0^\circ$ experiments have been conducted with finite cylinders [18] and cylindrical shells [19][20].

The usual method of interpreting finite target scatter for $\phi_i = 0^\circ$ is resonance analysis, which correlates the structure of the resonance scattering function, both in the frequency domain (form function) and the bistatic angle domain, to wave processes on the shell. The resonance scatter function is equivalent to what I refer to as the elastic response - essentially the scattered pressure with the geometrically scattered contribution (background) subtracted. The resonances correspond to wave closure around the large meridional circumference. Resonance scattering theory (RST) was introduced by Flax [21] as a formalism to identify and classify mechanisms causing the resonance phenomena. For separable geometries (spheres), the location of resonances can be extracted by finding the poles of each term in the modal series. In the case of finite cylinders, experimental techniques have been developed to isolate the resonance response from the geometrically scattered response [22].

With the resonance analysis of the previous references, however, no consideration
is paid to wave conversion on the scatterer. In other words the only energy loss mechanism is acoustic radiation. The geometries considered by these authors present only a weak surface inhomogeneity (discontinuity in curvature for a cylinder with hemispherical endcaps), or no inhomogeneity, to the scattering process. Therefore, the only important length scale is the large meridional circumference, which is a path along the length of the cylindrical shell and around the endcaps. Maze et al. [18] note, however, that scatter at the discontinuities in curvature may complicate the resonance structure of a finite cylinder with hemispherical endcaps.

Recently several authors have considered the influence of internal attachments on the scatter from cylindrical shells [23][24][25][26][27]. These studies, in most cases, consider internal loads within an *infinite* circularly cylindrical shell. These scattering models simplify to 2D for the case of beam incidence and longitudinal structure with constant cross-section. 2D problems have been studied where the infinite cylinder contains an internal plate [23][24][25][26], sprung masses [27] and directly mounted masses and stringers [28]. The conclusions are that the primary influence of the internal structures is to provide wave conversion mechanisms for the shell waves at the attachment points. For example, conversion between flexural waves and membrane waves provides a mechanism for flexural waves to contribute to the scatter. It is also shown that dissipation within the internal structures does not substantially modify the scatter [27]. The analysis of scatter from cylinders with transverse attached structure [29] shows that the backscatter is comprised of forced wave scatter at the bulkhead plus radiation from the reflected and transmitted elastic shell waves.

### 1.2 MIT Structural Acoustics Program

The MIT Structural Acoustics Program is funded by the Office of Naval Research for basic research in scattering from shells submerged in water. The goals of the program are broad and are summarized by the opening statement in this thesis, namely what affects the acoustic scatter at mid-frequencies? As part of the program, a series of
monostatic and bistatic scattering tests were proposed for target shells of various levels of internal complexity. An experimental approach was adopted because scattering measurements could be conducted with physical models built to a desired level of complexity. The data from these scattering measurements could not be generated by other methods for the frequencies of interest given these model complexities. The experiments were conducted by the Physical Acoustics branch at the Naval Research Laboratory in the Building 71 Acoustic Facility.

Previous Studies with the Scattering Measurements.

Several theses that interpret portions of the scattering measurements have been produced by the MIT structural acoustics program.

The doctoral thesis of Corrado [9] interprets the monostatic and bistatic scatter from three different shell models and focuses on the physics of the scatter in the beam incidence and helical wave regimes, $60 \leq \phi_i \leq 90^\circ$. His work shows the primary importance of shear wave excitation and radiation in these incidence regions. His work also shows that the ring stiffeners smear the patterns observed for the empty shell backscatter and reduce the measured decay rates. Although I will be considering a different range of incidence angle ($0 \leq \phi_i \leq 25^\circ$), I take advantage of many of the ideas within his thesis, particularly the use of the time domain for extracting physical insight. The data collection procedures and the test facility specifications are described in detail within the appendices of his thesis.

The doctoral thesis of Bondaryk [30] has drawn extensively upon beamforming techniques using the synthetic array created by the bistatic data. He uses bow incidence, bistatic scattering measurements as a reference for the development of a wave model of the axial distribution of pressure on the shell surface. Beamforming analyses in conjunction with the wave model provide insight into the properties of the elastic waves, including estimates of propagation speeds, attenuation, and reflection coefficients.

Maximum Likelihood beamforming techniques were used by Mackovjak [31] to
characterize wave propagation on the shells late in time. Sewon Park [32] studied the effects of internal loading on observed decay rates at beam incidence.

1.2.1 Model Configurations

A series of three cylindrical shells that form the basis of the scattering analysis were designed based upon guidelines provided by Dyer [33]; a fourth model was designed by Klausbruckner [34]. The shells represent a progression of complexity and are intended to systematically identify important scattering features associated with the different aspects of the shell configurations. The simplest shell is empty. The second shell is identical to the empty shell except that it contains four equally spaced rings. The third shell is identical to the second except that it additionally contains resiliently mounted, wave bearing internals. These will be referred to respectively as the empty, ringed and internalled shells. The fourth shell was designed for the purpose of damping helical shear waves, shown to be important by Corrado [9] among others. The method of damping is a constrained layer treatment and was applied to the internalled shell. This shell will be referred to as the sandwich shell. The shell parameters are summarized in Table 1.1.

The shells were all constructed by the Naval Research Laboratory in Washington, D.C. The empty shell is shown in Fig. 1-1. Its dimensions are representative of each of the four shells. The shell material is nickel and was selected for its durability and its corrosion resistance. From Table 1.1, where the shell material properties are listed, it is seen that nickel has properties similar to steel. The shell is thin, 21 mils (0.05cm) everywhere, and has an approximate thickness to cylinder radius ratio of 1%. The shell has a high aspect ratio, 8:1. Although least in complexity, one feature makes this shell far from canonical. The endcaps represent the junction of three geometries – cylindrical, conical and spherical – and the junctions introduce 26° slope discontinuities.

The ringed shell contains four equally spaced axisymmetric ring stiffeners, as shown in Fig. 1-3. The unequal spacing removes effects associated with precise pe-
<table>
<thead>
<tr>
<th>Shell Parameters</th>
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<tbody>
<tr>
<td><strong>General Properties</strong></td>
</tr>
<tr>
<td>Overall shell length, $L$</td>
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<tr>
<td>Cylinder length, $L_{cyl}$</td>
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<tr>
<td>Cylinder radius, $a$</td>
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<td>Shell thickness, $h$</td>
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<td>Thickness to radius, $h/a$</td>
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<td><strong>Shell Properties</strong></td>
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<tr>
<td>Shell material</td>
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<td>Transverse shear wave speed, $c_s$</td>
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<tr>
<td>Young's modulus, $E_s$</td>
</tr>
<tr>
<td>Density, $\rho_s$</td>
</tr>
<tr>
<td>Poisson's ratio, $\nu$</td>
</tr>
<tr>
<td><strong>Ring Properties</strong></td>
</tr>
<tr>
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<tr>
<td>Mass ratio (rings:shell)</td>
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<td>Inner diameter</td>
</tr>
<tr>
<td>Outer diameter</td>
</tr>
<tr>
<td>Width</td>
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<tr>
<td><strong>Sprung System Properties</strong></td>
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<tr>
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<tr>
<td>Density, $\rho_r$</td>
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<tr>
<td>Loss Factor, $\eta_r$</td>
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<tr>
<td>Wave bearing rods</td>
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<td>Delrin rod compressional wave speed</td>
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<td>Mass Ratio (sprung system+rings:shell)</td>
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<td>Symmetry</td>
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<tr>
<td><strong>Constrained Viscoelastic Layer Properties</strong></td>
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<tr>
<td>Density, $\rho_v$</td>
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<tr>
<td>Loss Factor, $\eta_v$</td>
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</tbody>
</table>

Table 1.1: Shell parameters
Figure 1-1: External shell dimensions. Dimensions in cm.

Riodicity. The bow incidence direction with respect to the ring spacing is shown in the same figure. The rings are massive – their total mass is equal to the mass of the shell. The rings are also nickel, Ni-200. The joining condition for the rings is that the width of the connection is the same as the shell thickness. This was accomplished by welding the two structures using laser welding techniques familiar to NRL.

The internalled shell contains the same four rings, but each ring supports additional resiliently mounted structures. These sprung internals break the axisymmetry present in the first two shells, providing instead quadrant symmetry as shown in Fig. 1-4. This additional structure has the form of wave-bearing rods, which extend from the first to the last ring. The rods are delrin interspersed with stainless steel masses and provide new regions for elastic wave propagation and elastic energy storage. The speed of compressional waves on the delrin (1625 m/s) is slow compared with membrane wave speeds on the cylindrical shell (5270 m/s for compressional waves). The lowest axial resonance frequency of the internal system is approximately one-tenth of the shell ring frequency, Eq. 1.1. The mass of the rods plus the resilient material (but not including the rings) is equal to twice the empty shell mass.

A fourth shell was also tested; it is actually the internalled shell with a constrained-layer damping treatment applied to the outer surface of the cylindrical section. I will refer to this shell as the sandwich shell. The treatment was applied to damp helical shear waves. An illustration of the constrained layer damping treatment is shown in Fig. 1-5. The external shell layer is 21 mils (.05cm) thick nickel to match the thickness of the internalled shell. The thickness of the viscoelastic layer 0.1cm is twice that of the shell. Details of the design goals are summarized in an internal report [34].
Figure 1-2: Endcap dimensions. Dimensions in cm.

Figure 1-3: Location of the internal rings. Dimensions in cm. The bow direction is defined as incidence from the left.

Figure 1-4: Configuration of the internalled shell. This is a representative section, for example the bay between the 2\textsuperscript{nd} and 3\textsuperscript{rd} rings. The delrin rods extend to the adjacent rings.
Figure 1-5: Configuration for constrained layer treatment. The added layer was faired to the endcaps with epoxy.

The relative masses of the three shells are 1 (Empty): 2 (Ringed): 4 (Internalled): 5 (Sandwich).

1.2.2 Experiment

Monostatic and bistatic scattering measurements were conducted at the Building 71 Acoustic Facility at the Naval Research Laboratory located in Washington, D.C. The scattering measurements were made over a frequency range of $10 - 47kH_2$, corresponding to a non-dimensional frequency range of $2.3 < ka < 11.0$, where $k$ is the acoustic wavenumber, and $a$ is the circularly cylindrical shell radius. The shells were excited with transient acoustic pulses generated by a $3m$ long 84-element line array source. The experimental geometry is shown in Fig. 1-6. The bistatic scattered pressure was measured at a $2m$ distance from the shell center, typically at $1^\circ$ increments for $360^\circ$. The incidence angle, $\phi_i$, is measured with respect to the shell axial, thus bow incidence is equivalent to $\phi_i = 0^\circ$. The observation angle (or radiation angle), $\phi_s$, is also measured with respect to the shell axis, thus $\phi_s = \phi_i$ corresponds to the monostatic receiver. For $\phi_i = 0^\circ$, the forward scatter is $\phi_s = 180^\circ$. The pressure reported at each receiver position represents an average of 100 realizations of the total field, $p_t$. The incident pressure, $p_i$, is obtained with the target removed, also with 100 realizations. To get the scattered pressure, the incident field is coherently subtracted from the total field, $p_s = p_t - p_i$. Statistics of the subtraction process have been reviewed [9], and show the noise floor is generally a function of the coherent subtraction process rather than ambient or electrical noise.
Figure 1-6: Schematic of the bistatic scattering measurement. Scattering measurements were performed by the Naval Research Laboratory Physical Acoustics Branch in the Building 71 tank facility.

A summary of the monostatic and bistatic tests completed at the NRL facility is given in Table 1.2.

1.3 Approach

Experimental methods bridge the frequency gap between numerical and asymptotic methods, especially for complex structures. Numerical methods are limited to lower frequencies due to computational burdens of modeling short wavelength processes such as flexural wave propagation. Additionally, modelers find it difficult to account for certain structural elements such as viscoelastic treatments, whose properties are not well known. With a physical model, these characteristics are automatically included as part of the experimental process. Asymptotic methods of analysis, like statistical energy analysis (SEA) and ray tracing, are based upon assumptions which become invalid at lower frequencies. SEA requires the structural system to have high
Overview of MIT/NRL Test Program

<table>
<thead>
<tr>
<th>Aspect Angles</th>
<th>Empty (3600)</th>
<th>Ringed (3700)</th>
<th>Internalled (4000)</th>
<th>Sandwich (5100)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 to 360 by 1</td>
<td>0 to 360 by 1</td>
<td>0 to 360 by 1 $^2$</td>
<td>0 to 360 by 1</td>
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<tr>
<td><strong>Observation Angles</strong></td>
<td><strong>Bistatic Tests</strong></td>
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<tr>
<td>0</td>
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<td>0 to 360 by 1/2</td>
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<td>5</td>
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<tr>
<td>90</td>
<td></td>
<td></td>
<td>0 to 360 by 1</td>
<td></td>
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</tbody>
</table>

1. Angles measured with respect to shell axis
2. Internalled Shell monostatic data measured at 3 roll angles (0, 22.5, 45)

Table 1.2: Summary of monostatic and bistatic scattering measurements completed at the Naval Research Laboratory Building 71 tank facility.
modal overlap, which diminishes with reduced frequency. Ray tracing methods are traditionally applied to understand localized interactions on simple structures in frequency ranges where structural wavelengths are small compared with typical radii of curvature and typical length scales. The frequencies studied in this thesis, however, comprise acoustic wavelengths which are comparable to these dimensions. The structures studied in this thesis also represent significant complexity, both in terms of shell geometry and internal loading. For this reason, monostatic and bistatic scattering measurements provide the necessary basis for extraction of the essential physics of the scattering process.

The scattering experiments produced a vast body of data. Relevant to this thesis are the bistatic measurements taken at incidence angles within 25° of bow incidence, and the monostatic measurements also investigated by Corrado [9]. The bow incidence bistatic measurements have also been investigated by Bondaryk [30] and Mackovjak [31].

With a minimal level of data processing such as filtering and deconvolution, significant insight can be drawn from presentation of the data in combined time, angle, and frequency domains. Patterns emerge that characterize the fundamental scattering events. Further processing uncovers more information; for example, beamforming recasts the measured scatter as a distribution of sources along the axial length of the shell. It is also necessary to introduce scattering models to understand what is suggested by the data. As an example, a traveling wave on the shell is characterized by its measured group speed and phase speed and will be identified by comparison with the appropriate shell dispersion equations. By matching specific aspects of the observed scatter with scattering models, a picture of the complete process can be pieced together. These comparisons test our understanding of the various scattering phenomena, so models which fail to predict measurements may be discarded or perhaps improved to incorporate other important phenomenon.

Traditionally the acoustics community has presented scattering results in the frequency domain in terms of resonances, target strength, or the form function. While
there are advantages to interpretation in the frequency domain, particularly for narrow-band or time harmonic excitation, there are also advantages with the time domain. I have found the time domain to be the more intuitive domain of analysis for transient scatter, where the spatial length of the acoustic pulse is small compared with the overall length of the shell. Time of arrival of the scattered pressure can identify propagation paths of elastic waves and concurrently from what position on a shell they originated. The disadvantage of the time domain for transient analysis is that interpretations must be made in a broad band sense. Naturally, there is a duality between the time and frequency domain, via the Fourier transform, and we take advantage of both. Recent papers [35] have demonstrated techniques for simultaneous time-frequency analysis of acoustic scatter, which have proven useful for looking at dispersion and other strongly frequency-dependent processes. While I have used one of these techniques (the Wigner transform), I have also found it useful to filter the data into smaller frequency bandwidths, typically equal to one-third of the experimental bandwidth, to demonstrate the frequency dependence.

1.4 Overview of Thesis

In Chapter 2, I explore patterns in the monostatic and bistatic time domain data which have relatively simple interpretation and provide useful insight into the scattering process for the four shells. The monostatic data are presented in the time domain as contours of normalized pressure as a function of incidence angle. The monostatic field can be divided into incidence regimes classified by their dominant physical scattering mechanisms. Roughly, these divide the monostatic field into four parts which include beam incidence, a helical wave regime near beam incidence, an endcap coupling regime at or near bow incidence, and a discontinuity scatter regime between bow incidence and the helical wave regime. The helical wave regime was discussed by Corrado [9]. Corrado also considered beam incidence as did Park [32]. The discontinuity scatter regime is characterized by reduced scatter levels that are dominated by the geometrically scattered return and scatter of the forced wave at impedance disconti-
nuities. The at or near bow incidence regime is characterized by endcap scattering that my thesis shows to be the predominant mechanism for transduction. Group delays near bow incidence are independent of incidence angle for $\phi_i < 25^\circ$, indicating that the shells are unable to differentiate between plane wave excitation precisely at or near axial incidence.

I next consider the bistatic data for near bow incidence. These data are presented in the time domain as contours of the scattered pressure as a function of observation angle. Based on the observed patterns, it is clear that the endcaps are the primary source of radiation, and the geometrically scattered contribution is the dominant bistatic feature. Compressional waves on the cylindrical shell are the secondary bistatic radiation source. The initial part of the scatter is the same for each of the three shells, indicating that the first returns are not affected by the internal loading. The spatial distribution of the scatter is modified by the internals for all observation angles. In particular, the backscatter is enhanced by the internal loading, although the bistatic scatter from the ringed and internalled shells are essentially the same.

In Chapter 3, I investigate the initial interaction of the incident sound wave with the endcap. At higher frequencies ($ka > 8$), the geometric scatter is well modeled as that from an elastic spherical shell that has the same dimensions and properties as the spherical section of the endcap (spherical cap). Compressional waves are excited via trace-matching on the spherical cap. A transition to a different interaction mechanism occurs where the initial interaction of the incident sound wave with endcap is no longer local to the spherical cap. The localization is quantified in terms of a finite glint with respect to the surface of the spherical cap, where the glint size is defined as the Fresnel width of the incident plane wave with respect to the spherical cap. At lower frequencies, the size of the glint exceeds that of the spherical cap and the backscatter requires a different model. I find that a flat disk model (circular with diameter equal to the transverse diameter of the spherical cap) predicts the backscatter and coupling at lower frequencies ($ka < 8$). The reflection coefficient is calculated for a circular disk which has motions impeded by a conical shell. The motion of the circular disk excites
both compressional and flexural waves at the slope discontinuity, which connects the spherical cap to the conical shell.

In Chapter 4, I analyze wave conversion mechanisms on the shell. The wave conversion is caused by the slope and curvature endcap discontinuities and the shell loading by the internal structures. The analysis is restricted to the axisymmetric shell waves, where several approximations are made to estimate the scatter at the slope and curvature discontinuities. First, the slope discontinuity is treated as more important than the change in curvature, because the slope change is a discontinuity in the first spatial derivative, while the curvature change is a discontinuity in the second spatial derivative. Second, the junction is approximated as two semi-infinite plates connected at a slope discontinuity in order to simplify the analysis. The analysis gives estimates of the scattering coefficients for an elastic wave normally incident to the slope discontinuity and shows there is significant coupling between compressional and flexural waves at the junction.

I also study the scattering process at an eccentric ring of mass plus stiffness attached to a fluid loaded infinite cylindrical shell, which is a direct extension of the work of Corrado [9]. The infinite cylinder model is more tractable than a finite model and isolates the scattering effects at the ring. Axisymmetric flexural and compressional waves are prescribed to be incident upon the ring. The method makes use of asymptotic integration techniques to isolate specific wavenumber contributions and estimates reflection, transmission and conversion coefficients, in addition to radiation at the ring. The stiffness terms coupled with the ring mass create two ring resonance frequencies located near the ring frequency, which dramatically affect the ring dynamics and create frequency bands of high transmission and conversion. It is shown that a single ring is not sufficient to fully model the interaction, rather it is necessary to consider the coupling of subsequent rings, in which pass and stop bands are created based on the ring spacing.

In Chapter 5, I investigate the axial distribution of pressure on the surface of the four shells as a function of time and frequency, using beamforming techniques on the
bistatic data. I employ a conventional beamformer using spatially windowed sections of the synthetic bistatic array centered in the beam radiation direction with respect to the shell axis [30]. The coupling analysis of Chapter 3 and the scattering analyses of Chapter 4 are used to explain many features found in the axial distributions. There is sufficient time resolution in the incident pulse to discriminate among different propagating wave types. Subsonic wave propagation cannot be identified because the waves do not radiate to the bistatic array. The empty shell analysis reveals that two compressional waves are launched from the endcap. The first wave is excited by the incident acoustic pulse, while the second compressional wave is launched following a delay which corresponds to the group delay of a flexural wave traveling on a conical shell. The slope discontinuity is responsible for the conversion of the flexural wave to a compressional wave. The reflection coefficients at the slope discontinuity are consistent with calculations made in Chap. 4. Much later in time, energy which had been stored in flexural waves on the cylindrical shell is converted to radiating compressional waves.

Wave propagation is strongly impeded by the internal structures. The observed axial distribution of the scatter shows that the rings contain wave energy within bays in some frequency bands, and allows transmission in others. For $ka < 5$, the ring resonances provide for strong transmission near ring frequency. Also the ring spacing provides a second mechanism for energy transfer between subsequent bays. The energy distributions of the ringed and internalled shells are the same for $ka > 5$, indicating that the resiliently mounted structures are poorly coupled to the shell dynamics. This is due to the large impedance mismatch between the shell and the ring that prevents a significant transfer of energy to the internals. To a lesser degree, the poor coupling is due to the isolation between the interior dynamical system and the rings through the viscoelastic mounts.

Near $\phi_i = 0^\circ$ the empty shell can only weakly differentiate the arrival direction of the incident acoustic wave. As incidence angle increases, coupling of the incident sound wave to elastic waves at the far endcap becomes more significant relative to
$\phi_i = 0^\circ$. The internally loaded shells are more sensitive to incidence angle because the forced wave couples more strongly to shell waves at the ring stiffeners as the incidence angle increases.

In Chapter 6, I show that of all the radiation sources identified in Chap. 5, only radiation from the insonified endcap contributes to the backscatter for $\phi_i = 0^\circ$. Other radiation sources identified in the beamforming analysis, such as radiation at the rings, are only important at larger observation angles. The scatter of the forced wave from the rings becomes a competing source of backscatter for incidence angles greater than $\phi_i \simeq 25^\circ$. I estimate the coupling coefficient that relates the amplitude of the elastic waves excited by the incident sound wave to the pressure of the incident sound wave. The coupling coefficient may also be interpreted as a radiation coefficient for the endcaps.

I compare the decay rates in the backscatter to attenuation estimates for compressional and flexural wave processes and find that at early times the scatter is driven by compressional wave processes, while at later times the scatter is driven by flexural wave processes. The transition time for onset of radiation from processes driven by flexural waves is approximately the time for a flexural wave at mid-band to propagate to the far endcap for the empty shell, and to the first ring for the internally-loaded shells.

Finally in Chapter 7, I summarize the contributions of this thesis.

The Appendices contain the analytical developments related to the axisymmetric wave dynamics used throughout this thesis. Appendix A presents the dynamic equations for the three shell geometries which are used to derive the fluid loaded dispersion relations. Appendix B reviews a methodology [9] for calculating the scattering coefficients for an elastic ring connected to an infinite cylindrical shell excited by an axisymmetric compressional or flexural wave.
Chapter 2

Overview of Scatter Data

2.1 Overview

In this chapter, I interpret patterns in the monostatic and bistatic data to illustrate a few prominent scatter mechanisms that are best introduced by their radiation patterns. I begin by reviewing the monostatic scatter, which has been discussed extensively by Corrado [9]. The monostatic data provide an overall picture of the character of the scatter, the relative importance of different aspect regions, and the effect of internal loading in each region. These data can be used not only to emphasize differences between the shell models but also to identify structures which have no effects. I consider the monostatic scatter for a quadrant of incidence angles, $0 \leq \phi_i \leq 90^\circ$. Overall this quadrant can be divided into four distinct regions, each characterized by a different dominant mechanism. The four regions can be generally classified as follows.

- **Beam Incidence**: This region extends from $90 \leq \phi_i \leq 85^\circ$ and is dominated by a large geometric return followed by rapidly decaying coincident compressional wave radiation.

- **Helical Wave Radiation**: This region extends from $\phi_i = 85^\circ$ to the shear wave cutoff near $\phi_i = 60^\circ$ for an empty cylindrical shell and near $\phi_i = 40^\circ$ for the
internally loaded cylindrical shells. This region is dominated by coincident shear wave radiation from waves reflected at the far endcap and at internal discontinuities. The coincident compressional wave radiation is less important because the compressional waves lose most of their energy via radiation at the far endcap. The helical membrane waves are well coupled to the acoustic field via trace-matching conditions along the cylindrical and conical shell.

- \textit{Discontinuity Scatter}: This region extending from the shear wave cutoff to $\phi_i \approx 25^\circ$ is characterized by relatively low level scatter. The dominant returns in this region are the geometric return and scatter of the forced elastic wave from surface discontinuities and internal loads directly connected to the shell. Elastic shell waves are only weakly excited at structural discontinuities relative to those excited by trace-matching in the helical region.

- \textit{Bow Incidence}: This region extending from $0 \leq \phi_i < 25^\circ$ is dominated by radiation from axisymmetric elastic waves, which predominantly interact with the acoustic field via transduction at the endcaps.

Two of these regions, \textit{beam incidence} and \textit{helical wave}, were the subject of prior theses [9][32]. The region of scatter which is the subject of this thesis is \textit{bow incidence}. The bow incidence region is significant for its surprisingly large angular width and its relatively large scatter magnitude. Bistatic measurements conducted for $\phi_i \leq 25^\circ$ are interpreted to gain a fuller understanding of the evolution of the scattering process. The measurements are used to demonstrate the significance of the coupling process between the endcaps and the acoustic field and strongly suggest, for the empty shell, axisymmetric compressional wave propagation between the endcaps. This propagation path is notably impaired in the other shells by the internal structures. I have chosen to look at $\phi_i = 25^\circ$ because the interpretation of the monostatic data suggests that this is a transitional angular region where axisymmetric coupling is becoming less important, while scatter from discontinuities is becoming more important. Even at $\phi_i = 25^\circ$, however, the bistatic data reveal axisymmetric compressional waves to
be well excited, which indicates that in some sense the shells cannot distinguish the direction of arrival for a plane wave near bow incidence.

2.2 Measurement Variability and the First Response

Although the quality of the data is excellent, data variability must be considered. Thus, before getting involved with the data directly, it is necessary to establish some basis for comparison by estimating the variability in the measurements. Variability can be associated with measurement noise, differences in calibration, or differences between the models.

The noise in these experiments is exceptionally low and has been shown to be primarily due to the clutter subtraction process [9]. The resulting noise floor for these measurements, following a deconvolution process, is in the vicinity of $-85$ to $-90\ dB\ re\ 2m$ for Gaussian bandlimited time domain signals. The peak scatter following a deconvolution process has a magnitude generally above $-60\ dB\ re\ 2m$ or in excess of $25\ dB\ SNR$. Therefore the variability due to noise can only be stated in fractions of a $dB$.

More important differences may arise due to calibration errors in the scattering measurements or differences between the target shells caused by the construction process. The shells were built by NRL using advanced techniques such as laser welding to reduce the construction variability, however, there are some small differences between the shells. Measured differences have been documented in terms of out of round and flat spots [36].

As a means of assessing the variability of the data, I choose to compare the initial part of the scatter from each of the shells at bow incidence, $\phi_i = 0^\circ$. I will argue that given the parameters of the measurement, this initial response should be identical for each shell at bow incidence. Based upon this argument, I conclude that there is no obvious variability due to the model construction, but there appears to be a $1\ dB$ and $3\ dB$ increase in two of the monostatic measurements relative to remaining
<table>
<thead>
<tr>
<th>Shell Configuration $ka_s (ka)$</th>
<th>Monostatic Data</th>
<th>Bistatic Data $\phi_i = 0^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty</td>
<td>$&lt; 1 , dB$</td>
<td>$&lt; 1 , dB$</td>
</tr>
<tr>
<td>Ringed</td>
<td>$&lt; 1 , dB$</td>
<td>$&lt; 1 , dB$</td>
</tr>
<tr>
<td>Internalled</td>
<td>$1(+) , dB$</td>
<td>$&lt; 1 , dB$</td>
</tr>
<tr>
<td>Sandwich</td>
<td>$3 , dB$</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2.1: Likely calibration error in the monostatic and bistatic measurements. The error is based on comparing magnitude of the peak monostatic geometric scatter for $\phi_i = 0^\circ$.

two monostatic measurements and three bow incidence bistatic measurements. The differences between the remaining measurements are less than $1 \, dB$. The results are summarized in Table 2.1. An explanation of how the calibration errors were determined is given in Sec. 2.2.3.

### 2.2.1 Post Processing the Measured Timeseries

To show how the calibration errors are determined, it is necessary to discuss first how the experimental data are processed.

In order to compress the excitation pulse for better time resolution, a deconvolution process was applied to the measured scatter to estimate the system transfer function, which is given by

$$H(\omega, \phi_i, \phi_s) = \frac{P_s(\omega, \phi_i, \phi_s)}{P_i(\omega)}, \quad (2.1)$$

where $P_s(\omega)$ and $P_i(\omega)$ are the Fourier transforms of $p_s(t)$ and $p_i(t)$. The transfer function is then Gaussian filtered to shape the pulse and reduce out-of-band noise;

$$H_g(\omega) = H(\omega)G(\omega). \quad (2.2)$$

The Gaussian filters are defined by their $6 \, dB$ down bandwidth. The time domain representation of the deconvolved filtered scatter, $h_g(t)$, is computed by using the inverse Fourier transform. The processed timeseries of the empty shell $\phi_i = 0^\circ$ backscatter
Figure 2-1: Backscatter timeseries for empty shell at $\phi_i = 0^\circ$ with corresponding envelope for $2.75 < ka < 10.0$ and $r = 2m$ from target center. The amplitude is $p_s/p_i$.

is shown in Fig. 2-1 with its corresponding envelope where the Gaussian filter has a bandwidth $2.75 < ka < 10.0$. The envelope is equal to twice the real part of the demodulated timeseries. To interpret the full dynamic range of the scatter, it is useful to use a log-compressed signal envelope representation of the processed timeseries as shown in Fig 2-2. This makes it possible to interpret lower energy portions of the time signal, and to determine decay rates. Since the data are always processed in this manner, it is convenient to let

$$p_s(t) = h_g(t) \ .$$

(2.3)

To be consistent, the incident pressure must be given by

$$P_i(\omega) = G(\omega) \ ,$$

(2.4)

where $G(\omega)$ is the Gaussian filter applied to the timeseries of interest.
Figure 2-2: Envelope of backscatter timeseries for empty shell at $\phi_i = 0^\circ$, for $2.75 < ka < 10.0$ and $r = 2m$ from target center.

2.2.2 Target Strength in the Time Domain

Target strength is a quantity defined in the frequency domain and relates monostatic scattered intensity to incident intensity at some reference distance, typically $1m$ or $1yd$. This is directly related to the transfer function discussed above and given by

$$T(\omega, \phi_i) = 20 \log (|P_{s}(\omega, \phi_i, r)|/|P_{i}(\omega)|) + \Delta, \ dB \ re \ 1m \ . \ (2.5)$$

Here $\Delta$ is a factor to correct for spreading losses due to the location of the receiver at $r = 2m$. The reference distance is generally reported with respect to the center of the target.

Integrated Target Strength

I am often interested in calculating the total scattered energy in an arbitrary radiation direction. The energy is given by the integral of the squared pressure in time, suitably scaled. In this thesis exact calibration of the energy units is ignored since it is always normalized. For example, the integrated result can be normalized to provide a value which represents integrated value of target strength. In other words this is the target
strength averaged over the experimental frequency band. The normalized integration represents the ratio of energy in the scattered and incident signals and is given by

\[ T(\phi_s) = 10 \log \left( \frac{\int P_s^2(\omega, \phi_s, r) d\omega}{\int P_i^2(\omega) d\omega} \right) + \Delta dB \text{ re } 1m \ . \quad (2.6) \]

by Parseval’s Theorem, this can equivalently be stated in the time domain

\[ T(\phi_s) = 10 \log \left( \frac{\int p_s^2(t, \phi_s, r) dt}{\int p_i^2(t) dt} \right) + \Delta dB \text{ re } 1m \ . \quad (2.7) \]

where \( p_s(t) \) and \( p_i(t) \) are now defined by Eq. 2.3 and the inverse Fourier transform of Eq. 2.4

**Event Target Strength**

Finally, I find it useful to consider the energy in a single event, for example the geometric return. One means for determining the energy in a single event is to integrate the pressure over the duration of that event. This is difficult in the measurements, because events are not sufficiently separated. The ratio of the peak magnitudes of the event and the incident pulse give a useful approximation of event energy. Specifically,

\[ T(t_o, \phi_s) = 20 \log (|p_s(t_o, \phi_s, r = 2m)|/|p_i(t = 0)|) + \Delta, dB \text{ re } 1m \ . \quad (2.8) \]

Here \( t_o \) is the arrival time of the event peak. The weakness of this technique is that it assumes the scattered pulse retains the shape of the incidence pulse. This is approximately true for the geometric scatter and for the compressional wave energy that radiates without first being converted to flexural waves. Several different Gaussian filters are used throughout this thesis so the peak values for the incident pressure are tabulated in Table 2.2
<table>
<thead>
<tr>
<th>Gaussian Bandwidth</th>
<th>Peak Magnitude of Incident Sound Pulse</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.75 &lt; ka &lt; 10$</td>
<td>-18 dB</td>
</tr>
<tr>
<td>$4 &lt; ka &lt; 9$</td>
<td>-21 dB</td>
</tr>
<tr>
<td>$2 &lt; ka &lt; 5$</td>
<td>-25 dB</td>
</tr>
<tr>
<td>$5 &lt; ka &lt; 8$</td>
<td>-25 dB</td>
</tr>
<tr>
<td>$8 &lt; ka &lt; 11$</td>
<td>-25 dB</td>
</tr>
</tbody>
</table>

Table 2.2: Peak amplitude of the incident pressure in the time domain for different Gaussian bandwidths

2.2.3 The Geometric Scatter as an Error Gauge

I refer to the portion of the backscatter which first arrives within time equal to the incident pulse duration as the *initial response*. The initial response includes the geometric scatter but may also include elastic contributions due to scatter within the first *interrogation length* of the pulse. I will describe the interrogation length in more detail in Sec. 3.2. Let it suffice here to state that at bow incidence the interrogation length is short enough that the internal loading *cannot* influence the initial response at bow incidence. This has an important implication. The initial response from each of the target shells must be the same, barring differences due to construction or experimental error. Thus, the initial response provides a useful metric for determining this error. I use the event target strength of the first peak arrival as the metric for comparison, where the first peak is the peak geometric scatter. The target strength of the peak monostatic geometric scatter has been discussed [9] and compared to the geometrical acoustics predictions for the target strength of the relevant geometric scattering surfaces (either the cylindrical shell, conical shell, or spherical shell).

**Monostatic Peak Geometric Scatter**

To appreciate how the initial response, in particular the geometric contribution to the initial response, manifests itself in the monostatic and bistatic data, I have contoured the envelopes of the empty shell backscatter $p_{s}(t, \phi_i)$ in Fig. 2-3 as a function of
time vs. incidence angle, $0 \leq \phi_i \leq 90^\circ$. The contours are plotted with a dynamic range greater than 30 $dB$, at 3 $dB$ intervals. 1200 $\mu$s is approximately the duration of uncontaminated monostatic scatter data.

The acoustic propagation delay of the backscatter varies with $\phi_i$ due to the geometry of the experiment. This can be deduced from Fig. 1-6 where the line source is closer to the endcap at bow incidence ($\phi_i = 0^\circ$) than to any part of the shell at beam incidence ($\phi_i = 90^\circ$), so the geometric scatter will have a shorter propagation delay at $\phi_i = 0^\circ$ than at $\phi_i = 90^\circ$. It is convenient to remove this acoustic propagation delay, setting the time origin as the arrival time of the peak geometric scatter for all $\phi_i$. The target strength of the geometric scatter (initial target strength) can be taken directly from Fig. 2-3 by accounting for the Gaussian filter ($2.75 < ka < 10$) used to shape the impulse as given by Table 2.2 in Eq. 2.8. The spreading corrections $\Delta$ total 6.5 $dB$.

I have plotted the initial target strength for all 4 shells in Fig. 2-4. The upper figure represents the monostatic initial target strengths calculated from the processed data, but does not show an equal response at $\phi_i = 0^\circ$. The differences are explained as measurement calibration error, and therefore I collapse the curves to get the lower figure by subtracting a 3 $dB$ offset from the sandwich shell initial target strength and a 1 $dB$ offset from the internalled shell target initial strength. These are the calibration errors reported in Table 2.1.

In collapsed form, the empty, ringed, and sandwich shells exhibit almost identical peak geometric scatter amplitudes for all $\phi_i$. The internalled shell exhibits a 3 $dB$ increase in the initial target strength with respect to the other shells for $40 < \phi_i < 90^\circ$. A possible cause is the increased mass and stiffness offered by the internal structures. However, it is not evident why the ringed shell is not similarly affected. The sandwich shell is also not affected. One possible explanation is that the internal structure of the sandwich shell floats within the outer constraining layer, decoupled from incident acoustic field. Since the thickness of the outer shell layer is equal to the thickness of the empty shell, the acoustic plane wave cannot distinguish between the empty shell
Figure 2-3: Contours of stacked monostatic Gaussian bandlimited impulse response for empty shell measured at \( r = 2m \), \( 2.75 < ka < 10.0 \). Arrivals are stacked so \( t = 0 \) corresponds to the arrival of the peak geometric scatter. The incidence angle range, \( 0 \leq \phi_i \leq 90^\circ \), is bow incidence through beam incidence.
Figure 2-4: Integrated target strength of the peak geometric scatter for all four shells over a frequency band $2.75 < ka < 10.0$. Upper figure represents the values extracted directly from the data. Lower figure has been collapsed so the target strengths for each model are equal near bow incidence. The curves represent the empty (solid), ringed (dashed), internalled (dash-dot) and sandwich (dots) shells.
and the sandwich shell, thus the geometric scatter is the same.

Bistatic Geometric Scatter at Bow Incidence

As I did for the empty shell monostatic data, I have contoured the envelope of the bow incidence bistatic scattered pressure as a function of time and observation angle, \( \phi_s \), where observation angle extends \( 0 \leq \phi_s \leq 180^\circ \). This result is shown in Fig. 2-5. The contours represent 5 \( dB \) intervals in pressure level providing a dynamic range of more than 50 \( dB \). The noise floor is -90 \( dB \) re 2\( m \). The peak level is -32 \( dB \) re 2\( m \), located in the forward direction and is caused by the shadow forming wave. \( t = 0 \) corresponds to the arrival of the peak geometric scatter at the monostatic receiver. Another feature is a tail extending from the shadow wave to \( \phi_s = 140^\circ \) and \( t = 250\mu s \) which is a remnant of the incident acoustic pulse \( p_i(t) \). In contrast with monostatic presentations, the timeseries at other observation angles are not shifted to account for propagation delays. It is not as convenient to do this for the bistatic data, because the forward scatter includes arrivals which precede the geometric scatter.

The peak geometric scatter is at \( t = 0 \) in backscatter and \( t = 580\mu s \) in forward scatter. The difference in arrival time is due to the differences in acoustic path length from the insonified endcap. The difference in path length for \( \phi_s = 180^\circ \) is equal to the overall shell length, hence the additional delay is given by \( t = L/c_o \). I have plotted the bistatic initial target strength for bow incidence in Fig. 2-6 as a function of observation angle for the empty, ringed and internalled shells to show they are identical at all observation angles. This supports the conclusion of Sec. 2.3.1 that the geometric scatter is necessarily the same for each shell at \( \phi_i = 0^\circ \). This also demonstrates that calibration errors are less than 1 \( dB \) for the bistatic bow incidence data as reported in Table 2.1.
Figure 2-5: Contours of $\phi_t = 0^\circ$ bistatic Gaussian bandlimited impulse response for empty shell measured at $r = 2m$, $4 < ka < 9$. $t = 0$ corresponds to the arrival of the peak geometric scatter. The observation angle range, $(0 \leq \phi_s \leq 180^\circ)$, is backscatter through forward scatter.
Figure 2-6: Integrated target strength of the peak geometric scatter over $4 < ka < 9$ for $\phi_i = 0^\circ$ bistatic scatter. Curves are drawn for the empty shell (solid), ringed shell (dashed) and internalled shell (dash-dot)

### 2.2.4 Subtraction of the Geometric Response

The scattered pressure, $p_s$, can be considered the sum of a geometric component and an elastic component

$$p_s(t, \phi_i) = p_g(t, \phi_i) + p_e(t, \phi_i) ,$$

where $p_g(t, \phi_i)$ is the geometric return and $p_e(t, \phi_i)$ is the elastic response. Several authors have considered methods for estimating the geometric contribution to the scatter [37][38] for spherical and cylindrical shells. They show that the geometric backscatter from thin shells at low frequencies approaches a value close to that from a pressure release surface. With increasing shell thickness and increasing frequency the geometric backscatter approaches a value similar to that from a rigid boundary. These authors present expressions appropriate for the transitional mid-frequency band, one of which [37] I reproduce in Sec. 3.3. That expression, Eq. 3.5, gives the geometric contribution from an elastic spherical shell. I can gain an estimate of the monostatic geometric return from each of the target shells by setting the dimensions and material properties of the spherical shell to those of the spherical sector of the endcap, which I discuss in Sec. 3.3. In Fig. 2-7, I compare an incident Gaussian pulse to the estimated
Figure 2-7: Comparison of incident Gaussian pulse and estimate of the geometric backscatter from a spherical shell \((2.75 < ka < 10)\). The estimate is governed by Eq. 3.5. The amplitudes are normalized by the incident pressure.

geometric backscatter from a spherical shell. The geometric response is approximately of opposite sense to the incident pulse.

In many of the subsequent presentations of data, the geometric scatter has been estimated and subtracted from the scattered pressure. The previous analysis has shown that the geometric return contains no information about the internal structures. It is assumed that the geometric return has the opposite sense of the incident pulse, where the geometrically scattered wave amplitudes, \(P_g(\phi_i)\), and propagation delays, \(t_g(\phi_i)\) are estimated using matched filtering. This approach is taken from Bondaryk [30], where the matched filtering is performed prior to deconvolution to take advantage of the pulse signature, and the resulting timeseries is given by

\[
p_e(t, \phi_i) = p_s(t, \phi_i) - (P_g(\phi_i)/P_i)p_i(t - t_g(\phi_i)) . \quad (2.10)
\]

I have contoured the resulting monostatic elastic response for the empty shell in Fig. 2-8 and the resulting \(\phi_i = 0^\circ\) bistatic elastic response for the empty shell in Fig. 2-16. While the subtraction process is not perfect, 10 dB reductions in the geometric response are typical. The impact can be dramatic, with some features better revealed in time. Most importantly, subsequent comparisons of the elastic response for the target shells will be more representative of the effects of various internal structural configurations.
2.3 Monostatic Data

2.3.1 Empty Shell

The four distinct scattering regions described in the introduction of this chapter are readily distinguished in Fig. 2-8. Near beam incidence, the scatter is comprised of strong rapidly decaying compressional wave radiation, as can be determined by considering the periodicity of the peaks. The region extending from $\phi_i = 85$ to $\phi_i = 60^\circ$ is the helical wave regime, where the incident acoustic wave coincidently excites shear and compressional membrane waves along the length of the cylindrical shell [9]. These waves propagate to the far end of the shell and reflect in the backscatter direction. The large well defined periodic features are created by radiation of the reflected waves. The delay prior to the onset of the period structure is given by the time for the forward propagating helical waves to reach the far endcap and reflect. The region from $\phi_i = 60$ to $\phi_i = 25^\circ$ is substantially less energetic because neither shear, compressional nor flexural waves can be coincidently excited. The features which are present in this region generally appear to be extensions of more important features in the bow incidence region. For $\phi_i < 25^\circ$, these features increase monotonically in magnitude to a maximum at $\phi_i = 0^\circ$. In this region, the time delays of these features are independent of $\phi_i$. As I will show in section 2.4.1, the bow incidence region is characterized by strong coupling of the acoustic field to axisymmetric compressional and flexural waves on the endcap but not on the cylindrical shell.

2.3.2 Internally Loaded Shells

I contour the monostatic impulse response of the ringed and internally loaded shells in Figs. 2-9 and 2-10 to show how the internals have affected the elastic response. The envelopes of the impulse responses are displayed as functions of incidence angle vs. time. The scatter is again stacked so $t = 0$ represents the arrival of the geometric contribution which has been estimated and subtracted. While many of the features
Figure 2-8: Contours of stacked monostatic Gaussian bandlimited elastic response for empty shell measured at $r = 2m$, $2.75 < ka < 10.0$. The geometric scatter has been subtracted. Arrivals are stacked so $t = 0$ corresponds to the arrival of the peak geometric scatter. The incidence angle range, $0 \leq \phi_i \leq 90^\circ$, is bow incidence through beam incidence.
observed in Fig. 2-8 remain, it is apparent that the internal structures have a dramatic effect on the elastic scattering response.

The internal structure has disturbed much of the regularity of the helical wave backscatter. The sharp transition in incidence angle from the helical wave incidence region to the diffraction regime has disappeared. The incidence regime extending from $\phi_i = 60^\circ$ to $\phi_i = 25^\circ$ is substantially more energetic with new patterns emerging, which have been shown [9] to be caused by direct scatter of the forced wave from each of the rings as shown schematically in Fig. 2-11. The expected arrival times of these scattering bursts are drawn as a function of incidence angle in Fig. 2-12 and match the arrival structure observed in Figs. 2-9 and 2-10. I have also plotted in Fig. 2-12 the anticipated arrival structure for scattering bursts off the bow and stern endcap slope discontinuities. In all 4 monostatic presentations, a return from the cone/cylinder junction is discernible immediately following the geometric scatter. The magnitude of this contribution is the same for the empty and ringed shells but is 3 $dB$ greater for the internalled shell. I regard this discrepancy in the same light as the difference in the first part of the scatter from the four shells. That is, there should be no difference in this contribution for the four shells because it is not affected by the internal structure. This increase is 2 $dB$ greater than the increases found in the geometric scatter.

The observed angle dependence of the forced wave scatter magnitude may be equal to the radiation pattern for either a dipole or an axial quadrupole. The radiation from a ring is calculated in Sec. 4.2.2, where the model predicts a dipole radiation pattern for excitation by a flexural wave. In the next section, the angle dependence of the radiation patterns observed in the monostatic data related to the force wave scatter are shown to be similar to that of a dipole.

The features in the bow incidence region $(0 < \phi_i < 25^\circ)$ have an arrival structure independent of $\phi_i$. In contrast with the empty shell, there are more features with greater magnitude for longer time. For the empty shell, these features can be discerned even for $\phi_i > 25^\circ$; however, with the internally loaded shells, the levels of the direct ring scatter have equal or greater importance. The onset of important forced
Figure 2-9: Contours of stacked monostatic Gaussian bandlimited elastic response for ringed shell measured at $r = 2m$, $2.75 < ka < 10.0$. The geometric scatter has been subtracted. Arrivals are stacked so $t = 0$ corresponds to the arrival of the peak geometric scatter. The incidence range, $0 \leq \phi_i \leq 90^o$, is bow incidence through beam incidence.
Figure 2-10: Contours of stacked monostatic Gaussian bandlimited elastic response for internalled shell measured at $r = 2m$, $2.75 < ka < 10.0$. The geometric scatter has been subtracted. Arrivals are stacked so $t = 0$ corresponds to the arrival of the peak geometric scatter. The incidence range, $0 \leq \phi_i \leq 90^\circ$, is b/cw incidence through beam incidence.
Figure 2-11: Direct scatter from ring stiffeners. This figure illustrates the bistatic scatter from ring IV.

Figure 2-12: Arrival times at \( r = 2 \) m for direct scatter of the forced wave. Arrivals are shifted to correlate with the monostatic contours; \( t = 0 \) is the time of the peak geometric scatter. Rings (solid), cone/cylinder discontinuity (dashed), cone/sphere discontinuity (dotted)
wave backscatter marks the incidence transition region where the backscattered elastic response cannot be considered exclusively comprised of radiated contributions from the insonified endcap.

The integrated elastic target strengths for the shells have been plotted in Fig. 2-13, where the sandwich shell result has been included for later discussion. The four curves correspond to the empty shell (solid), ringed shell (dashed), internalled shell (dash-dot) and sandwich shell (dotted). This is a notation which will be maintained throughout the thesis when the shell responses are being compared directly. The empty shell curve most dramatically demonstrates the four scattering zones. The target strengths for the internally loaded shells are consistently greater than those for the empty shell. The target strength for the internalled shell is consistently 2-3 dB greater than for the ringed shell, but possibly this difference is experimental error. The most prominent difference between these curves is the noticeable lack of a sharp transition between the helical wave region and the diffraction region for the internally loaded shells.

The elastic target strength decreases monotonically from $\phi_i = 0^\circ$ within the bow incidence region with the 6 dB down point located near 20°. In the case of the empty shell, the target strength continues to decrease to about $\phi_i = 40^\circ$ because there are no backscatter sources additional to the endcap radiation. For the internally-loaded shells where the additional backscatter sources are directly from the rings, the transition shifts to $\phi_i = 30^\circ$.

2.3.3 Internally Loaded Shell with External Sandwich Shell

A fourth shell configuration was also tested with the intention of examining methods of achieving reductions in the helical wave scatter. To create the sandwich shell, the internalled shell was modified externally along the length of its cylindrical section with a constrained layer damping treatment. The endcaps were left untreated. The intent of the treatment was to reduce the backscatter levels in the helical wave regime by damping shear waves. Another goal was to reduce the backscatter levels in the inci-
Figure 2-13: Integrated target strength of the monostatic elastic response for all shells $3 < ka < 10$. Curves represent empty shell(solid), ringed shell(dashed), internalled shell(dash-dot) and sandwich shell(dotted).

dence region neighboring the helical wave regime possibly by decoupling the internal structure from wave propagation on the external shell.

The effectiveness of the treatment can be surmised from Fig. 2-13 by comparing the sandwich shell target strength with the internalled shell target strength; in particular, reductions in the helical wave region are a topic for a separate research [39]. Overall, relative to the internalled shell, there are reductions in the backscatter at incidence angles on the order of $3 \, dB$. The quality of the reductions can be better appreciated by observing the time evolution of the monostatic response of the sandwich shell, which is contoured in Fig. 2-14. The same processing has been applied as in the previous monostatic representations. Some dramatic changes have occurred in comparison with the internalled shell scatter. The reduction in the helical wave regime qualitatively can be seen both in the reduction in the density of bright colored features early in time ($t < 400\mu s$) and increased attenuation later in time ($t > 600\mu s$).

In the diffraction region ($25 < \phi_i < 60^\circ$), the most dramatic features are the well defined contributions due to the direct acoustic scatter from each ring and from the slope discontinuities which show up distinctively across the time/incidence plane.
Figure 2.14: Contours of stacked monostatic Gaussian bandlimited elastic response for internalled shell with constrained layer treatment measured at $r = 2m$, $2.75 < ka < 10.0$. The geometric scatter has been subtracted. Arrivals are stacked so $t = 0$ corresponds to the arrival of the peak geometric scatter. The incidence range, $0 \leq \phi_i \leq 90^\circ$, is bow incidence through beam incidence.
The incidence angle dependence of these contributions is now easily distinguished and the levels are the nearly the same as those of the ringed and internalled shells. The magnitude of the direct ring backscatter decreases with decreasing incidence angle consistent with a dipole or quadrupole representation of the process.

The arrival times for forced wave scatter at each ring shown in Fig. 2-12 match with the peak features in the monostatic displays. I can therefore plot the magnitude of the ring scatter as a function of incidence angle for each of the four rings and for each shell. The magnitude of the ring scatter for the ringed and internalled shells is not a smooth function of angle and it is difficult to assess the angle dependence. I suspect the rapid variations are caused by interference with elastic waves which travel within the shell bays and radiate. However the direct ring scatter from the sandwich shell is a relatively smooth function with angle and lends itself more readily to analysis. I have plotted the integrated target strength of the direct scatter from each of the four rings of the sandwich shell in Fig 2-15. The levels are plotted over an angular range of \( 20 \leq \phi_i \leq 70^\circ \) in which the patterns are discernible to the eye. Two additional curves are compared with the data. These are least squares estimates which fit a \( \sin^2 \phi_i \) and a \( \sin^4 \phi_i \) angle dependence to the data. The distribution in angle for dipole radiation is \( p \sim \sin \phi_s \) where \( \phi_s \) is the radiation angle [40]. The radiation, however arises from the interaction force at the ring caused by the incident sound wave. This force must also have an angle dependence which I assume is also given by \( F \sim \sin \phi_i \). The angle dependence of the monostatic ring backscatter will have a dependence given by the product \( pF \sim \sin^2 \phi \) or as plotted in Fig. 2-15 \( \sim 20 \log \sin^2 \phi \). A similar argument leads to a \( \sin^4 \phi \) angle dependence for an axial quadrupole. Given these considerations, the monostatic data correlate better with a dipole representation of the scatter. This is consistent which the model introduced later in Chap. 4.

I return to the monostatic representation of the sandwich shell and observe that the first 200\( \mu s \) remain essentially unaffected by the treatment. In the bow incidence region there are significant reductions in later-in-time scatter levels. Subsequent features present for the ringed and internalled shells are almost entirely eliminated. These two
Figure 2-15: Radiation pattern of the scatter of the incident sound wave from the rings. The bold curves correspond to least squares estimates of the measured angle dependence. The $\sin^2 \phi_i$ curve corresponds to a dipole-like angle dependence, while the $\sin^4 \phi_i$ corresponds to an axial quadrupole-like angle dependence.

Observations provide some insight into the scattering process near bow incidence. Consider the first $200\mu s$ of backscatter for all four shells. The three shells with internal structure exhibit the same three large magnitude features which are approximately periodically spaced, while the empty shell exhibits only the first feature. Comparing the shape of the first peak, which all four shells share, with Fig 2-12 shows the contribution is bounded temporally by the arrival structure for scatter from the cone/cylinder slope discontinuity. In other words, it is caused by an elastic response within the endcap and hence must be the same for all four shells. I can not explain the second feature in the empty shell response except to note that it too must be caused by an elastic response within the endcap due to its early arrival. Finally, the additional arrivals for the internally loaded shells must be related to the internal structure. The timing of these arrivals approximately matches the time for a compressional wave to travel to the first ring, reflect and travel back to the endcap. Note that a constrained layer damping treatment is particularly effective for attenuating flexural waves and much less effective for damping compressional waves. Given that compressional waves drive this response, the constrained layer would not have substantial effect on the initial
attenuation rates. On the other hand, the treatment should have a substantial effect on the attenuation of flexural waves. Given the reductions later in time, it is reasonable to conclude the later time backscatter from the internally loaded shells involve flexural wave processes in a significant way.

2.4 Bistatic Data

2.4.1 Empty Shell at Axial Incidence

I now examine the structure of the $\phi_i = 0^\circ$ bistatic scatter for the empty shell to investigate how the elastic waves contribute to the scatter. The resulting field without the geometric contribution for the empty shell is contoured in Fig. 2-16 in which I have reduced the dynamic range to $30 \, dB$ by reducing the peak contour level and shrinking the contour increment to $3 \, dB$.

Due to imperfect subtraction, a significant residual of the shadow forming wave remains. Another feature, which is carried over from Fig. 2-5 is a tail extending from the shadow wave to $\phi_s = 140^\circ$ and $t = 250\mu s$ which is a remnant of the incident acoustic pulse $p_i(t)$. At smaller observation angles, $p_i$ is removed quite well by gating because its time of arrival precedes that of $p_s$.

Patterns in the scatter contain information about scattering centers, beampatterns and mechanisms of energy transport. Characteristic $S$ and partial $S$ patterns dominate the structure, with each pattern corresponding to localized scatter. The patterns are a consequence of the experimental geometry. Fig. 2-17 (a) and (b), demonstrate their source, identifying backward $S$ structures as caused by radiation from the near endcap and forward $S$ structures as caused by radiation from the far endcap. The partial $S$ structures are due to the directivity pattern of each scattering event such that energy outside the main lobe is below the noise floor.

In the forward scatter, some contributions arrive before the shadow forming wave. The earliest forward arrival has its radiation locus at the far endcap which is evident
Figure 2-16: Contours of $\phi_i = 0^\circ$ bistatic Gaussian bandlimited elastic response for empty shell measured at $r = 2m$, $4 < ka < 9$. The geometric scatter has been subtracted. $t = 0$ corresponds to the arrival of the peak geometric backscatter. The observation angle range, $0 \leq \phi_s \leq 180^\circ$, is backscatter through forward scatter.
Figure 2-17: Patterns in the bistatic scatter. The upper figure shows the $S$-shaped pattern that is caused by radiation from the near endcap (and geometric scatter at the near endcap shown in the illustration). The middle figure shows the reverse $S$-shaped pattern caused by radiation from the far endcap. The lower figure shows the highly directive radiation from compressional waves.
from its spatial pattern. The delay of 175 $\mu$s from $t = 0$ equals the one-way time for an axisymmetric compressional wave to travel the shell length. The interpretation is simple - the acoustic wave couples to a compressional wave at the near endcap, then the compressional wave propagates to the far endcap and radiates. In the process, the compressional wave travels along each of the three shell geometries and past four slope discontinuities. The compressional wave is supersonic and radiates as it propagates along the cylindrical shell. As Fig. 2-17 (c) suggests, radiation from this forward propagating compressional wave should be observed centered at an angle $\phi_s = 90^\circ + \theta_l$, where $\theta_l$ is defined by the trace matching condition

$$\theta_l = \sin^{-1} \frac{c_o}{c_p} = 16^\circ .$$  \hspace{1cm} (2.11)

The radiation from the initial compressional wave is a yellow feature located at $t = 350\mu s$ and $\phi_s = 106^\circ$ in Fig. 2-16. Backward propagating compressional waves can also be observed weakly at $\phi_s = 74^\circ$. There is evidence of compressional waves traveling in both directions although the levels are low compared with other features near the shell axis.

Bistatic scattering contributions in Fig. 2-16 appear to be caused by one of two processes, radiation from an endcap and radiation from a leaky (radiating) compressional wave propagating along the cylindrical shell. The endcap radiation is axially directed with significant angular beamwidth. In general, the axially directed scatter levels are significantly greater than the compressional wave radiation levels. To quantify this, I calculate the integrated target strength of the elastic scatter as a function of observation angle (Eq. 2.7).

The integrated target strength of the elastic response averaged for $4 < ka < 9$ is plotted in Fig. 2-18. The target strength is strongest on the shell axis in the forward and backward directions. From the bistatic contours, these forward/backward levels were attributed to radiation from the endcaps. The forward scatter is larger, although this is partially skewed by residual contributions from the imperfectly removed geometric contribution to the scatter. The local peak centered at $\phi_s = 106^\circ$ is caused
by radiation from the forward propagating compressional waves. However there is no visible indication at $\phi_i = 74^\circ$ of radiation from backward propagating waves. This is consistent with estimates for for the compressional wave reflection coefficient at the endcap, which will be derived in Chap. 4. Overall, the target strength in the middle third of the observation range is 10 $dB$ below the backscatter, showing that the compressional wave radiation as observed at $\phi_s = 90 \pm \theta_i$ is a weak process relative to endcap radiation, which dominates the forward and back directions.

In summary, the endcaps are the dominant means of acoustic scatter. The cylindrical shell is not an important transducer for acoustic coupling, rather the cylindrical shell primarily provides a path for energy transfer between the endcaps.

2.4.2 Internally Loaded Shells at Axial Incidence

The time evolution of the $\phi_s = 0^\circ$ bistatic scatter for both the ringed and internalled shells are contoured in Figs. 2-19 and 2-20. As in Fig. 2-16, the geometric scatter has been subtracted in each case using the matched filter method already described. The bistatic observation angles range from backscatter ($\phi_s = 0^\circ$) to forward scatter ($\phi_s =$
180°) and the envelopes correspond to the Gaussian impulse response bandlimited to 
4 < ka < 9. The time origin is set to the arrival time of the peak geometric scatter.

The scattering patterns of the two internally loaded shells are very similar. In fact, 
there are no strong differences and the similarity raises the question, why don't the 
internal wave-bearing structures leave a significant fingerprint at φt = 0°. I'll return 
to this question in Chapter 5 after spending a portion of Chapter 4 considering the 
dynamics of the ring stiffeners. At this point it is sufficient to comment that the large 
impedance of the ring stiffeners allows only weak energy transmission to the modes 
of the internal structure particularly away from resonance frequencies of the ring. 
For the remainder of the discussion in this section, the comments which refer to the 
internalled shell response apply equally to the ringed shell.

Many of the features described for the empty shell bistatic field are also apparent in 
scatter from the internally-loaded shells. These features include the shadow forming 
wave and the 'S'-shaped patterns which have a peak magnitude at φs = 0°. The 
bistatic field is dominated by axially directed radiation with neglect of the residual 
of the shadow forming wave. In contrast with the empty shell, these contributions 
are almost exclusively from the insonified endcap and directed to backscatter. Much 
less of the bistatic field can be attributed to radiation from the far endcap. In fact 
the strong forward scattered first arrival caused by the initial compressional wave 
on the empty shell is almost negligible. Another strong feature in the empty shell 
response associated with the initial compressional wave, centered at φs = 104° and 
t = 350μs, is also absent. Still, forward and backward compressional wave radiation 
can still be detected via diffuse energy around the observation directions, φs = 90 ± θt, 
later in time. In combination, these observations lend themselves to the following 
interpretation. The compressional wave that freely propagates the length of the empty 
shell scatters at the rings of the internalled shell so that little energy is transmitted, 
but most energy is reflected back towards the insonified endcap, leading to increased 
levels in backscatter. Some energy also scatters directly to the acoustic field.

As a quantitative means of comparing the total scattering strengths of the elastic
Figure 2-19: Contours of $\phi_i = 0^\circ$ bistatic Gaussian bandlimited elastic response for ringed shell measured at $r = 2m$, $4 < ka < 9$. The geometric scatter has been subtracted. $t = 0$ corresponds to the arrival of the peak geometric scatter. The observation angle range, $0 \leq \phi_o \leq 180^\circ$, is backscatter through forward scatter.
Figure 2-20: Contours of $\phi_i = 0^\circ$ bistatic Gaussian bandlimited elastic response for internalled shell measured at $r = 2m$, $4 < ka < 9$. The geometric scatter has been subtracted. $t = 0$ corresponds to the arrival of the peak geometric scatter. The observation angle range, $0 \leq \phi_o \leq 180^\circ$, is backscatter through forward scatter.
Figure 2-21: Comparison of $\phi_i = 0^\circ$ integrated target strength for the 3 shells, $4 < ka < 9$. The geometrically scattered return has been subtracted.

responses of the three shells, the integrated bistatic target strengths are plotted in Fig. 2-21 as a function of observation angle. For comparison, the empty shell result is overlaid as a solid line. The effect of the ring stiffener is enhancement in the backscattered direction of approximately 2 $dB$, with little or no enhancement of the beam directed scatter. For all three shells, $40 < \phi_s < 140^\circ$, is region of relatively weak bistatic scatter, while the axially-directed scatter is always spatially dominant. From these observations, I conclude that the two processes which contribute to the beam-directed scatter, leaky wave radiation and radiation at the rings, are both weakly coupled to the acoustic field relative to radiation from the endcaps.

2.4.3 Bistatic Results away from Axial Incidence

Small Deviations from Axial Incidence

The bistatic data discussed to this point has been exclusively measurements for $\phi_i = 0^\circ$. Due to the axisymmetry of this configuration, in theory it is possible completely neglect non-symmetric elastic shell modes and their influence. However, even a slight perturbation of the axisymmetry, either due to misalignment or non-symmetries in
the structure is a potential source for coupling to higher order vibration modes. As an example, it is difficult experimentally to excite an axisymmetric bar in a purely symmetric manner \[41\][42]. Inevitably the non-symmetric flexural wave response is also excited. With appreciation of this type of sensitivity, bistatic measurements were conducted at \( \phi_i = 5^\circ \). Mackovjak \[31\] used array processing to study the late arrival structure for the empty shell both at \( \phi_i = 0^\circ \) and \( \phi_i = 5^\circ \), and found increased energy levels later in time for the \( \phi_i = 5^\circ \) relative to \( \phi_i = 0^\circ \), which he attributed to flexural response in the \( n = 1 \) mode.

Decomposition of the incident plane acoustic wave into sum of integer order Bessel functions \[43\] illustrates the angle dependence of the higher order terms;

\[
p_i(z, r, \phi_i, \theta) = P_i e^{ikz} [J_0(k_r r) + 2 \sum_{n=0}^{\infty} i^n \cos(n \theta) J_n(k_r r)] .
\]  

(2.12)

Here \( k_r = k \sin \phi_i \), and \( k_s = k \cos \phi_i \), \( \theta \) is the circumferential dependence, and time dependence has been suppressed. The \( J_0 \) term is axisymmetric and maximum for \( \phi_i = 0^\circ \), where all other terms vanish, thus for a perfectly constructed axisymmetric target at \( \phi_i = 0^\circ \) only the \( n = 0 \) modes can be excited. The higher order modes must be coupled to higher order terms in Eq. 2.12, where \( \phi_i \)-dependence of the these terms provides a clue to how the shell will respond to oblique incidence. The \( n = 1 \) response of the cylindrical shell is proportional to the second term in the expansion, \( J_1(k_r a) \). Yet, even if the \( J_1 \) term becomes large, its importance must be weighed relative to the shells modal impedance. In the case of the bar referenced above, the impedance of the \( n = 1 \) (flexural) mode is low compared with the \( n = 0 \) (compressional) mode so small variations from purely symmetric excitation cause an important flexural response.

Contour plots of bistatic scatter for \( \phi_i = 0^\circ \) and \( \phi_i = 5^\circ \) incidence were found to be nearly indistinguishable so the \( \phi_i = 5^\circ \) contour is not reproduced here. To demonstrate, the backscatter envelopes of the Gaussian impulse responses (\( 4 < ka < 9 \)) are compared in Fig. 2-22, where the \( \phi_i = 0^\circ, \phi_s = 0^\circ \) and the \( \phi_i = 5^\circ, \phi_s = 0^\circ \) timeseries exhibit only small differences out to \( 10^3 \mu s \). From Eq. 2.12, the \( n = 0 \) (axisymmetric term) is slowly varying near \( \phi_i = 0 \), compared with the higher order
Figure 2-22: Comparison of the empty shell backscatter envelopes at $\phi_i = 0^\circ$ and $\phi_i = 5^\circ$ incidence for $4 < ka < 9$ and distance $r = 2m$ from target center. The $\phi_i = 5^\circ$ envelope is bold terms. Since the backscatter is insensitive to changes in $\phi_i$, the axisymmetric response must dominate the scatter, while contributions from higher order response modes must be unimportant.

Larger Deviations from Axial Incidence

For $\phi_i = 25^\circ$, the bistatic scattered pressure reveals some new mechanisms, but generally demonstrates that many of the same mechanisms remain unchanged. The bistatic scatter at $\phi_i = 25^\circ$ for the empty shell is contoured in Fig. 2-23. As in previous figures, the geometric scatter has been estimated and removed to reveal the elastic response more clearly. The alignment of the time axis is such that the time origin corresponds to the first arrival at $\phi_s = 0^\circ$ (the monostatic receiver is located at $\phi_s = 25^\circ$). The contour levels are the same as for Fig. 2-16 ($\phi_i = 0^\circ$) so the magnitude of the response can be compared directly.

Several new features can be noted in the bistatic scatter relative to bow incidence scatter. The specular peak (this is a residual of the geometric subtraction) has shifted to $\phi_s = 155^\circ$ which is the supplementary angle to $\phi_i = 25^\circ$. There is also a more
Figure 2-23: Contours of bistatic Gaussian bandlimited elastic response for empty shell at 25° incidence measured at $r = 2m$, $4 < ka < 9$. The geometric scatter has been subtracted. $t = 0$ corresponds to the arrival of the peak geometric scatter. The observation angle range, $0 \leq \phi_s \leq 180^\circ$, is relative to the shell axis, where 25° is the monostatic observation angle.
significant contribution due to direct scatter from the far endcap (it is colocated with the specular peak and has the characteristic $S$ shape of a far endcap scattering process), although it does not contribute to the backscatter. In comparison with $\phi_i = 0^\circ$, there is an overall reduction in the scatter levels; however, many of the features remain. One feature that remains is the forward radiation of the initial axisymmetric compressional wave from the far endcap. Its peak levels are less than 3 $dB$ below those for $\phi_i = 0^\circ$. The radiation beampatterns are again maximum on the shell axis (at $\phi_s = 0$ and at $\phi_s = 180^\circ$), indicating that axisymmetric processes are still predominant. Diffuse patterns caused by compressional wave radiation in both the forward and backward directions can also still be identified.

Almost all of the above comments also apply to the $\phi_i = 25^\circ$ internalled shell bistatic elastic scatter, which is contoured in Fig. 2-24 as compared with the $\phi_i = 0$ bistatic scatter. One difference particular to the internally-loaded shells is contributions from direct scatter of the forced wave at the rings. The process is the same one described for the monostatic scatter and illustrated by Fig. 2-11. Beads of color trace a line of arrivals extending from the specular peak down to $\phi_s = 0^\circ$. Those most clearly visible correspond to scatter from the $2^{nd}$, $3^{rd}$ and $4^{th}$ rings. These three curved arrival structures coincide with $\phi_s = 0^\circ$ at $t = 450$, $t = 700$, and $t = 850 \mu s$.

In summary, most of the bistatic features observed at $\phi_i = 0^\circ$ remain unchanged at $\phi_i = 25^\circ$ except for a reduction in overall magnitude which is less than 3 $dB$.

## 2.5 Summary

The data sets are shown to be very reliable with respect to variability between measurements and between models. Differences of less than 1 $dB$ are usual. The only data sets which show greater variability are the monostatic sets for the internalled shell and sandwich shell which exhibit spurious levels of approximately 1 $dB$ and 3 $dB$. No corrections are made in subsequent analyses; however, the data should be considered the same where 1 $dB$ differences are observed.
Figure 2-24: Contours of bistatic Gaussian bandlimited elastic response for internalled shell at 25° incidence measured at \( r = 2m \), \( 4 < ka < 9 \). The geometric scatter has been subtracted. \( t = 0 \) corresponds to the arrival of the peak geometric scatter. The observation angle range, \( 0 \leq \phi_\circ \leq 180^\circ \), is relative to the shell axis, where 25° is the backscatter angle.
The monostatic data reveal that the bow incidence region is significant in terms of the scattered pressure levels. A unique characteristic of the bow incidence region is that the arrival structure in the time domain is predominantly independent of incidence angle. This suggests that the scattering response cannot distinguish the arrival direction of an incident sound wave within this region, $0 \leq \phi_i < 25^\circ$. This was demonstrated for small angular deviations by comparison of the $\phi_i = 0^\circ$ and $\phi_i = 5^\circ$ backscatter. Beyond $\phi_i = 25^\circ$, direct scatter of the forced elastic wave begins to contribute to the backscatter.

The calibration of variability was based upon a comparison of the initial part of the scatter at $\phi_i = 0^\circ$ which is identical for the four shells. This shows that the internal structures do not influence the initial part of the scatter. Therefore it is convenient to subtract this part of the scatter to isolate the *elastic response*, which depends strongly on the internal structures. I estimate and subtract the geometric contribution to the initial part of the scatter. This subtraction is especially useful for comparing the bistatic energy distributions where the geometric contribution to the scatter is dominant.

The bistatic data show that the primary radiation patterns are directly on axis of the shells. This shows that the dominant radiation mechanism is transduction via the endcaps. In particular the backscatter is comprised almost exclusively of radiation from the insonified endcap. Radiation levels in the coincident direction for compressional wave radiation and in the beam direction are $10 \, dB$ below the on-axis levels. The arrival structure in the bistatic data show that a compressional wave is excited by the incident sound pulse at the insonified endcap which radiates from the far endcap in the empty shell but never reaches the far endcap for internally-loaded shells.
Chapter 3

Interaction of Incident Sound Wave with the Endcap

3.1 Overview

In this chapter, I discuss the initial interaction of the incident sound wave with the endcap. At the endcap, the incident sound wave scatters to acoustic waves and to elastic shell waves. In the analysis, I will show how the geometry of the endcap affects these processes.

I begin the chapter by considering the energy which is geometrically scattered to acoustic waves. and show that the geometric scatter, for $ka_s > 7$ ($ka > 8$), is well modeled by the scatter from a spherical shell with the same properties as the spherical sector of the endcap ($a_s, h, c_p, \rho_s, \nu$). For lower frequencies, where the wavelength in water is larger than the diameter of the spherical sector of the endcap, the geometric scatter is no longer local to this sector but also involves the conical shell. At these frequencies, I find the geometric scatter is better modeled by the scatter from a circular disk equal to the projected cross section of the spherical section of the endcap.

These two models are used to describe the mechanisms for the excitation of elastic waves. Based upon the spherical shell model, trace matching of the incident sound
field to compressional waves on the spherical shell is the predominant elastic wave excitation mechanism for $ka_s > 7$. Trace matching, however, loses its relevance at low frequencies because the necessary region for coupling exceeds the dimensions of the spherical section of the endcap. For lower frequencies, a piston-like motion of the spherical sector of the endcap (modeled as a circular disk) pumps energy into both flexural and compressional waves at the cone/cylinder discontinuity.

The description of the coupling process also applies to the radiation of elastic waves from the endcap. By reciprocity, these two must be equivalent. Therefore, the angular radiation patterns from the endcap can be stated explicitly.

3.2 Initial Insonification Regions

In Chapter 2, I stated that the initial part of the scatter from each of the shells must be identical for $\phi_i = 0^\circ$. I now discuss the interrogation length of the incident sound wave relative to the distance from the point of initial contact to the first ring. The region of the shell that contributes to the initial part of the scatter is related to this interrogation length. For the analysis within this chapter, however, I am more concerned with describing the geometric component of the initial part of the scatter. The insonification region that contributes to the geometric component is better described in terms of a finite-sized glint, which is a measure of the extent of the coherently excited portion of the endcap.

Interrogation Length

The interrogation length of the incident acoustic pulse qualitatively describes the extent of the target that may contribute to the first part of the backscattered response within time equal to the pulse duration. For example, for an acoustic pulse used to interrogate a volume of distributed scatterers, the interrogation length is given by $c_o \tau/2$ [44], where $c_o$ is the speed of sound in water and $\tau$ is a measure of the temporal extent of the incident pulse.
The incident pulse used to excite the shell is broadband and of short duration. Gaussian filters are used in the post processing to shape the pulse and reduce the pulse duration. For various bandwidths used in the subsequent analyses, the temporal pulse lengths are $\tau = 21\mu s$ for $2.75 < ka < 10.0$, $\tau = 29\mu s$ for $4 < ka < 9$ and $\tau = 49\mu s$ for $5 < ka < 8$. I define the temporal pulse length as the time separation between 3 $dB$ down points of the pulse. For the $2.75 < ka < 10$ Gaussian pulse, the interrogation length equals 1.6$cm$, meaning that scatter of the incident sound wave from any portion of the shell within 1.6$cm$ of the initial point of contact can contribute to the initial part of the scatter $t < \tau$. This interrogation length includes the entire spherical sector the endcap and includes the sphere/cone discontinuity.

In the case of an elastic scatterer, one may consider an alternative interrogation length to be given by $c_g\tau/2$, where $c_g$ is the group speed of the elastic wave of interest. If I consider compressional waves having a group speed $c_p = 5270m/s$, the alternate interrogation length increases to 5.5$cm$. With this description, the $t < \tau$ response for $2.75 < ka < 10$ includes the contributions from any portion of the shell within 5.5$cm$ of the initial point of contact. Because the compressional wave path must conform to the geometry of the endcap, the axial extent of the interrogation length is somewhat less than 5.5$cm$ as shown in Fig. 3-1. Pulses with less bandwidth have proportionally longer interrogation lengths. In all cases, however, the interrogation length is a distance smaller than the distance from the point of contact to the first ring stiffener. Therefore the scatter within $t < \tau$ does not include energy which has scattered from the internals. This same argument holds for the entire bow incidence.
Figure 3-2: Events that contribute to the initial response. 1. geometric scatter from the spherical sector of the endcap, 2. radiation of elastic waves reflected from the (a) sphere/cone and the (b) cone/cylinder junctions, and 3. acoustic scatter of the forced elastic wave from the sphere/cone junction.

region, $\phi_i < 25^\circ$. Events that contribute within $t < \tau$ are illustrated in Fig. 3-2. For the remainder of this chapter, I primarily discuss the geometric contribution.

Finite Glint

A different insonification region that proves useful as a descriptive tool with respect to the geometric return may be described as a finite glint. At high frequencies ($ka \gg 1$) the glint describes a region of a scatterer that lies normal to an incident sound wave, such that the incident sound wave is specularly reflected in the direction of observed scatter. For a spherical scatterer, in the limit of vanishingly small wavelengths, the glint is a dimensionless point on the spherical surface. For finite $ka$, the glint has dimensions that are wavelength dependent, corresponding to the Fresnel width of the incident sound wave, and describes a coherently excited region of the surface. This region is that which I refer to as a finite glint and that which causes the geometric scatter.

For the case of monostatic scatter, the angular extent of a finite glint on a spherical surface, from its center to the glint edge, can be estimated from the Fresnel width of an incident ray as illustrated in Fig. 3-3 with respect to the endcap. In the illustration, the glint has been defined as the region of the spherical sector of the endcap from
Figure 3-3: Geometry which defines the extent of a glint on a spherical surface

the point of initial contact to the circle at which the phase difference of the incident
plane wave is less than \(\pi/2\), a criterion I justify in Sec. 3.4. The angular sector of the
spherical part of the endcap included in this width can be derived directly from the
geometry of Fig. 3-3;

\[
\cos \theta_g = 1 - \frac{\pi}{4ka_s}.
\]  

(3.1)

I find it helpful to divide the measured frequency range into three equal bandwidth
regions, henceforth referred to as lower-\(ka\) (2 < \(ka\) < 5), mid-\(ka\) (5 < \(ka\) < 8) and
higher-\(ka\) (8 < \(ka\) < 11) to appreciate the frequency dependence of the scatter. The
radius of the spherical sector of the endcap, \(a_s\), is different from the radius of the
spherical shell, \(a\); therefore, these same frequency bands are equivalently lower-\(ka\)
(1.7 < \(ka_s\) < 4.2), mid-\(ka\) (4.2 < \(ka_s\) < 6.7) and higher-\(ka\) (6.7 < \(ka_s\) < 9.2).

The angular width described by Eq. 3.1 at the center frequency of each of these
bands is given in Table 3.1. The angular half-width of the spherical sector of the
endcap, \(\theta_s \approx 40^\circ\), so the assumption that the geometric component of the backscattered
signal is local to the spherical sector of the endcap is valid in the higher-\(ka\) band for
this endcap geometry. In contrast, the size of the finite glint in the lower-\(ka\) band
greatly exceeds the dimensions of the spherical sector of the endcap and I do not
expect the geometric component to be well modeled by the scatter from a spherical
shell.
### Table 3.1: Angular dimension of a finite glint on a spherical shell

<table>
<thead>
<tr>
<th>Center Frequency $ka_s$ ($ka$)</th>
<th>Extent of Glint $\theta_g$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.9 (3.5)</td>
<td>61</td>
</tr>
<tr>
<td>5.5 (6.5)</td>
<td>44</td>
</tr>
<tr>
<td>8.0 (9.5)</td>
<td>36</td>
</tr>
</tbody>
</table>

#### 3.3 Models for the Initial Scatter

Based on Table 3.1 I expect that the spherical section of the endcap has the primary role in determining the geometric component of the scatter at higher-$ka$, where the size of the finite glint is less than the size of the spherical section of the endcap. To show this, I compare the initial backscatter from the empty shell with the initial backscatter from a spherical shell, which has the same properties as the spherical section of the endcap. At mid- and lower-$ka$, the initial interaction is no longer local to the spherical sector of the endcap. Therefore, I suggest that the geometric component is better modeled by the scatter from a flat circular disk of radius $a_c = 0.03 m$ (the radius at the sphere/cone junction).

#### 3.3.1 Spherical Shell Model for the Initial Scatter

The exact solution for the pressure scattered backward from an elastic spherical shell can be stated using a normal mode series [1] as

$$p_{sc}(ka_s, \tau) = P_i \sum_{n=0}^{\infty} \frac{(2n + 1)(-i)^n h_n(ka)}{h_n'(ka_s)} \left[ j_n'(ka_s) \frac{\rho_o c_o}{(ka)^2 (Z_n + z_n) h_n'(ka_s)} \right],$$

(3.2)

where $Z_n$ and $z_n$ are the modal mechanical and modal radiation impedance respectively, and $h_n$ is a spherical Hankel function of the first kind and order $n$ and $j_n$ is a spherical Bessel function of order $n$. The primes denote differentiation with respect to argument. The backscatter from a rigid sphere is given simply by setting $Z_n = \infty$, 81
while the backscatter from a pressure release sphere is given by setting $Z_n = 0$.

The modal mechanical impedance for an elastic shell is given by [1]

$$Z_n = -\frac{i\rho_s c_p h}{\Omega a} \frac{[\Omega^2 - (\Omega_n^{(1)})^2][\Omega^2 - (\Omega_n^{(2)})^2]}{[\Omega^2 - (1 + \beta^2)\gamma_n]} ,$$

(3.3)

where $\Omega_n^{(1)}$ and $\Omega_n^{(2)}$ are the non-dimensional modal resonance frequencies of the spherical shell and are the roots to the homogeneous spherical shell dispersion equation (shown in Appendix A, Eq. A.16).

The modal radiation impedance is given by [1] as

$$z_n = i\rho_s c_o \frac{h_n^{(1)}(ka)}{h_n^{(1)}(ka)} .$$

(3.4)

To show the effects of elasticity, the monostatic bandlimited impulse response from an elastic sphere is compared with that from a pressure release and rigid sphere in Fig. 3-4 at a distance $r = 1.62$ (this is the measurement distance from the origin of the target spherical cap to the monostatic receiver) and for a frequency band $2.3 < ka_s < 8.4$. The elastic spherical shell is given the same material properties as those of the endcap ($a_s = .0465m$, $h = .00053m$, $\rho_s = 8800kg/m^3$, $c_p = 5270m/s$, $\nu = .31$). The impulse responses are each synthesized from their respective form functions (Eq. 3.2) using standard Fourier techniques with Gaussian filters applied to shape the pulse. The amplitudes shown are $p_s/p_i$.

The backscatter from the rigid and pressure release spheres are essentially the same except with opposite sense. The delayed response associated with the rigid sphere can be attributed to a creeping wave contribution [45]. The first part of the elastic shell backscatter is more like that of the pressure release case, although the magnitude of the initial peak is reduced relative to the pressure release backscatter. Part of the incident acoustic wave energy scatters directly to the acoustic field and part couples to the elastic shell response which reduces the geometric response. For the rigid and pressure release spheres, where the elastic component is zero, the backscattered energy is fully contained within the geometric response (with the exception of the creeping
wave contribution).

**Estimate of the Geometric Contribution**

The exact solution for the backscatter from a spherical shell of necessity includes both the geometric contribution and subsequent additional elastic contributions. The additional contributions are not relevant to the study of the target shells, so it is useful to consider an approximate representation of the geometric contribution. The geometric contribution from an elastic body is a function of the local impedance [38] or entrained mass [37] within the region of initial contact. The approximate modal expression for this geometric contribution has been derived by Werby [37] as

\[
p_g(k_a, r, \phi_s) = P_i \sum_{n=0} (2n + 1)(i)^n b_n P_n(\cos \phi_s) h_n(kr) .
\]  

(3.5)

Here \( b_n \) is the modal coefficient that accounts for the entrained acoustic mass/unit area \( m_n \) and the shell mass \( M_s \). Specifically,

\[
b_n = \frac{4\pi k a_s^3 [m_n/M_s] j_n(k a_s) - j'_n(k a_s) }{4\pi k a_s^3 [m_n/M_s] h_n(k a_s) - h'_n(k a_s) } ,
\]  

(3.6)

\[
m_n = -\rho c/\omega \ \text{Im}\{i h_n(k a)/h'_n(k a)\} ,
\]  

(3.7)

\[
M_s = \frac{4}{3} \pi \rho_s a_s^3 [1 - (1 - h/a)^3] .
\]  

(3.8)

The estimated geometric contribution is compared with the exact backscatter in Fig. 3-5 and confirms that the geometric return indeed has opposite sense compared with the incident pulse. Its amplitude is slightly less than that of the pressure release response due to the elasticity of the shell. The comparison also shows that the geometric contribution for the exact backscatter is well resolved relative to later arrivals and it is appropriate to compare the initial part of the spherical shell scatter with the initial part of the measured \( \phi_i = 0^\circ \) backscatter.
Figure 3-4: Backscattered timeseries from 3 spherical scatterers (a) rigid, (b) elastic shell, and (c) pressure release, Gaussian bandlimited to $2.3 < ka_s < 8.4$. The radius of each sphere is $a = a_s$. $r = 1.62m$ is equal to the distance from the origin of the spherical sector of the endcap to the monostatic receiver, $\phi_i = \phi_s = 0^\circ$. The properties of the spherical shell match those of the empty shell. The amplitude is $p_s/p_i$. 
Figure 3-5: Comparison of exact backscatter from spherical shell and an approximation of the geometric contribution (2.3 < ka < 8.4). The geometric contribution has the form of a pressure release response with modified amplitude. The differences seen here are largely due to inaccuracy of the geometric representation for ka < 4

Comparison of Data with Spherical Shell Backscatter

Here I compare the measured empty shell backscatter with the equivalent spherical shell backscatter to help understand the degree to which the initial interaction is a local process with respect to the empty shell spherical sector of the endcap. The degree of localization is related to the size of the finite glint already discussed.

For comparison with the empty shell backscatter, the spherical shell modal summation is computed at a distance $r = 1.6m$ (equal to the distance from the the origin of the spherical sector of the endcap to the monostatic receiver). The two timeseries are compared in Fig 3-6, where the monostatic returns are initially the same. There are small differences which I expect because the spherical shell model should only prove useful at higher frequencies, so it is more instructive to compare the backscatter in different frequency bands. Reduced bandwidth envelopes of the initial $\phi_i = 0^\circ$ backscatter are compared in Fig. 3-7 to determine the frequency dependence of the backscatter.

In the higher-ka region, the magnitude and shape of the initial scatter from the
Figure 3-6: $\phi_i = 0^\circ$ backscatter for empty shell and equivalent spherical shell for a Gaussian bandlimited incident pulse, $2.75 < ka < 10$ ($2.3 < ka_s < 8.4$). The amplitude, $p_s/p_i$, is measured at $r = 2m$ from the target center.

Figure 3-7: Comparison of envelopes of empty shell and spherical shell initial scatter for Gaussian bandlimited backscatter (a) $2 < ka < 5$ ($1.7 < ka_s < 4.2$), (b) $5 < ka < 8$ ($4.2 < ka_s < 6.7$) and (c) $8 < ka < 11$ ($6.7 < ka_s < 9.2$). The amplitudes are $p_i/p_s$ and are measured at $r = 2m$ from target center.
empty shell and sphere are initially identical, but diverge rapidly for \( t > 0 \). I suspect the difference is caused by an interfering subsequent scattering event from within the first interrogation length of the empty shell. In the mid-\( ka \) band, the shape of the initial scatter again diverges for \( t > 0 \). In the lower-\( ka \) band, the initial scatter never matches that from that from a spherical shell since the levels differ by as much as 8 dB (or equivalently the time is in error by about 20 \( \mu s \)). To first order, however, one might argue that the spherical shell model works reasonably well for the entire frequency range. It is necessary to consider the angle dependence to make a more substantive conclusion about the correct model for the geometric response. But first I introduce a different model that accounts for the conical shell dynamics.

### 3.3.2 Circular Disk Model for the Initial Scatter

At lower frequencies the acoustic wavelengths are large compared with both the diameter and axial extent of the endcap; therefore it is possible to neglect its curvature. The incident pressure integrated over the surface of the cap produces a net axial velocity in the shell axis direction that must drive the spherical sector of the endcap like a piston. This motion would be significantly different from that of the spherical shell because the former also involves the conical shell which provides an impedance to this motion. I therefore consider the endcap to behave like a circular disk at lower frequencies and neglect the curvature of the spherical sector of the endcap. The general expression for target strength for a perfectly reflecting circular disk is given by [46][43]

\[
T(\phi_i, \phi_s) = 10 \log \frac{S^2}{\lambda^2 r_{ref}^2} + 20 \log \left| \frac{2J_1(ka \gamma) \cos \phi_s}{ka \gamma} \right| .
\]  

(3.9)

Here \( \gamma = \sin \phi_i + \sin \phi_s \), where the angles of interest are shown in Fig. 3-8. Also \( S \) is the disk area, \( \lambda \) is the wavelength in water, and \( r_{ref} \) is the reference distance.

A perfectly reflecting disk yields a geometric response that is much higher than that found in the measured data. Also a perfectly reflecting disk does not account for the motion of the endcap due to the incident sound wave which transmits energy to
Figure 3-8: Geometry for the reflection of an incident wave by a circular disk

\[
\phi_i + \phi_r = e^{i k z} + r e^{i k z}
\]

Figure 3-9: Interaction model for a sound wave incident upon a circular disk of mass and generalized impedance

the shell. It is therefore necessary to modify Eq. 3.9 with a reflection coefficient, \( r_d \). A model for the estimation of \( r_d \) is shown in Fig. 3-9 as a 1D system. A circular disk of radius \( a_c \) is located at \( z = 0 \) and is given mass equal to the mass of the spherical end-cap, plus its radiation mass. Stiffness and damping parameters are related to the impedance of the conical shell against which the disk must push. The pressure field, suppressing the time dependence is given by

\[
p = p_i + p_r , \tag{3.10}
\]

where

\[
p_i = P_i e^{i k z} , \quad p_r = r_d P_i e^{-i k z} , \tag{3.11}
\]
and the disk moves with displacement

$$u = U_o ,$$

(3.12)

and the unknown parameters are $r_d$ and $U_o$. There is a dynamic and a kinematic boundary condition at $z = 0$ which provides two equations. The sound pressure acts on a cross section $\pi a_c^2$, thus, the dynamic equation is

$$\pi a_c^2 p \bigg|_{z=0} = m \ddot{u} + C \dot{u} + K u + Z_a \dot{u} ,$$

(3.13)

where $C$ and $K$ are respectively frequency dependent damping and stiffness parameters associated with the conical shell. $Z_a$ is the radiation impedance for a circular piston which is given by [1]

$$Z_a = R_a - i \omega M_a ,$$

(3.14)

and $R_a$ and $M_a$ are respectively the acoustic radiation resistance and entrained fluid mass, which can be approximated for $ka > 1$ as

$$R_a = \rho c \pi a_c^2 , \quad M_a = 2 \rho a_c^3 / (ka_c)^2 .$$

(3.15)

The kinematic equation is

$$v \big|_{z=0} = v_i + v_r \big|_{z=0} = \dot{u} ,$$

(3.16)

where $v$ is the particle velocity of the sound field which is a sum of the incident $v_i$ and reflected $v_r$ velocity fields. The velocity is expressed in terms of the sound pressure by the momentum equation

$$\rho \dot{v} = \frac{\partial p}{\partial z} ,$$

(3.17)

which yields

$$v_i = p_i / \rho c_o , \quad v_r = -p_r / \rho c_o .$$

(3.18)

Eqs. 3.16 and 3.18 are combined to give an expression for the disk displacement in
terms of the pressure at $z = 0$,

$$u = -\frac{P_i}{\rho c_o} (1 - r_d) . \tag{3.19}$$

This expression combines with Eq. 3.10 and 3.13 to give an expression for the reflection coefficient that simplifies after routine algebra to

$$r_d = \frac{(\varepsilon_R - 1) + i\varepsilon_I}{(\varepsilon_R + 1) + i\varepsilon_I} , \tag{3.20}$$

where

$$\varepsilon_R = \frac{C + R_a}{\rho c_o \pi a_c^2} , \quad \varepsilon_I = \frac{-\omega(m + M_a) + K/\omega}{\rho c_o \pi a_c^2} . \tag{3.21}$$

To demonstrate that Eq. 3.20 has the correct behavior consider that for large frequency or large endcap mass, $r_d \simeq 1$ which is the rigid or perfectly reflecting asymptote. Also, at resonance of the endcap $\varepsilon_I = 0$, the reflection is pressure release $r_d = -1$ for no radiation resistance $\varepsilon_R = 0$.

It is still necessary to relate $C$ and $K$ to the conical shell impedance. Locally the conical shell behaves in a manner similar to a cylindrical shell of the same diameter and thickness [47], so I approximate the conical shell reaction with that of a cylindrical shell of the same radius and thickness. The disk provides an axial velocity on the target axis and because the conical shell has a vertex angle (equal to the slope discontinuity, $\varphi = 26^\circ$), the displacements of the conical shell in both the axial and radial directions have components along the target axis

$$u = u_{cone} \cos \varphi + w_{cone} \sin \varphi . \tag{3.22}$$

Based on the above expressions, the impedance seen by the circular disk is approximately related to two conical shell impedance terms by

$$Z_d = \frac{1}{\cos^2 \varphi/Z_{comp} + \sin^2 \varphi/Z_{flex}} , \tag{3.23}$$
Figure 3-10: Reflection coefficient for a plane sound wave incident upon a circular disk whose motion is impeded by the axial impedance of an elastic conical shell.

where $Z_{comp}$ and $Z_{flex}$ can be gained from the expressions for the cylindrical shell compliance $s$ that are derived in Appendix B, where $s = i/\omega Z$. For this calculation $Z_{comp} = i/\omega s_{11}$ and $Z_{flex} = i/\omega s_{22}$. The impedance seen by the circular disk due to the conical shell is equivalent to terms in Eq. 3.13 expressed by

$$Z_d = C + iK/\omega \quad (3.24)$$

The magnitude of $r_d$ is shown in Fig. 3-10, where it is clear that reflection coefficient is much less than one, indicating that part of the incident sound wave energy couples to elastic wave energy on the shell.

I can combine $r_d$ with the target strength expression (Eq. 3.9) as an estimate of the geometric response at lower frequencies. In Fig. 3-11, I compare the target strengths for a spherical shell both perfectly reflecting and elastic (Eq. 3.5) and for a circular disk (Eq. 3.9) modified by $r_d$. The perfectly reflecting sphere target strength is

$$T = 20 \log a/2r, dB \quad (3.25)$$

For comparison, I have plotted the frequency averaged target strength of the geometric
Figure 3-11: Comparison of the target strength of the measured peak geometric return with the target strength of representative models. The symbols correspond to the peak values found in each measurement band (Fig. 3-7) plotted at the center frequency of each band.

return from the empty shell in each of three frequency bands (Fig 3-7). It is clear from this figure that the geometric response is well modeled by an elastic sphere in the upper frequency bands, while the circular disk is 2 dB off in the lower frequency band where presumably it should work best.

3.4 Angle Dependence of the Initial Scatter

I consider the same two models with some modifications in order to describe the angle and frequency dependence of the geometric return in different frequency ranges. At higher frequencies, the scatter is proportional to the overlap between the glint and the spherical sector of the endcap. At lower frequencies, the disk model shows that the angle dependence of the scatter is proportional to the projected radius of the spherical part of the endcap.

One consideration which must be noted in the comparison of radiation patterns is that the measured data angles are always reported with respect to target center,
Figure 3-12: Radiation angles used to plot data and geometric beampatterns

whereas the radiation models are derived with respect model center. This difference leads to discrepancies between the apparent radiations angle observed in the data and those predicted by the models as seen in Fig. 3-12. To account for this, the radiation angles for the models, $\phi_o$ can be corrected to equivalent angles relative to target center.

Empty Shell Angle Dependence

The narrow-band integrated target strength of the geometric return is defined in Eq. 2.8 where both $p_i$ and $p_s$ are bandpass filtered with narrowband Gaussian filters. I have plotted the monostatic target strength of the geometric return in Fig. 3-13. The 3 $dB$ down point is located at $11^\circ$ for the lower-$ka$ band, at $12^\circ$ for mid-$ka$ and at $21^\circ$ for higher-$ka$.

I have plotted the bistatic target strength of the geometric return in Fig. 3-14. The bistatic angle dependence of the geometric return does not vary rapidly with $\phi_s$. The 3 $dB$ down point is located at $23^\circ$ for the lower-$ka$ band, at $18^\circ$ for mid-$ka$ and at $21^\circ$ for higher-$ka$.

Monostatic Angle Dependence of Spherical Shell Model

Fig. 3-13 shows that the monostatic geometric return diminishes with increasing incidence angle, yet the backscatter from a spherical shell is independent of incidence angle, so I must modify the model to account for the geometry of the endcap.
Figure 3-13: Integrated target strength of the empty shell monostatic peak initial return for $\phi_i = 0^\circ$, averaged over three equal bandwidths. The frequency ranges are $2 < ka < 5$ (solid), $5 < ka < 8$ (dashed) and $8 < ka < 11$ (dot-dash).

Figure 3-14: Integrated target strength of the empty shell monostatic peak geometric scatter for $\phi_i = 0^\circ$, averaged over three equal bandwidths. The frequency ranges are $2 < ka < 5$ (solid), $5 < ka < 8$ (dashed) and $8 < ka < 11$ (dot-dash).
Overlap of Finite Glint with Endcap

With respect to the size of the finite glint, the difference between the three bands is that the glint is smaller than the spherical sector of the endcap only at higher-\(ka\) band. In the case of a spherical target, as the glint rotates around the sphere due to a change in the incidence angle, it always lays upon an identical surface - so there is no angle dependence. The same finite glint laid upon the spherical sector of the endcap can only rotate for a limited angular range before the surface changes dramatically as shown in Fig. 3-15. With no change in the surface, the geometric return is unaffected; therefore, for a glint which is smaller than the spherical sector of the endcap (higher-\(ka\)), there is a small incidence range for which the initial response does not diminish. This angular range of incidence would approximately equal the angular difference between the extent of the glint and the extent of the spherical sector of the endcap, \(\theta_s - \theta_g\). For the higher-\(ka\) band, this difference is approximately 4\(^\circ\). I expect the beampattern of the geometric backscatter to be proportional to the overlap of the glint and spherical sector of the endcap, where the overlap is represented by the cross hatched region in Fig. 3-15.

Because the \(\pi/2\) phase criterion for the angular extent of the glint can be thought by some to be arbitrary, I calculate the overlap of the glint with the spherical sector of the endcap as a function of incidence angle, and for a phase criteria from \(\pi/6\) to \(\pi/2\), at \(ka_s = 8.0\), the center frequency in the higher-\(ka\) band. To determine the overlap, I
first define the glint and spherical sector of the endcap dimensions as shown in Fig. 3-3, where \( \theta_s \) is defined by Eq. 3.1. I then uniformly discretize the surface defined by the finite glint and perform a rotation, \( \phi_i \), of the surface. This is easily accomplished by mapping the discrete surface to a Cartesian representation,

\[
x = \{x, y, z\} ,
\]

shown in Fig. 3-15. The rotation \( x \rightarrow x' \) is simply stated as

\[
x' = \{x \cos \phi_i, y \sin \phi_i, z\} .
\]

The overlap is described by those discrete surface points for which

\[
x \cos \phi_i > a_s (1 - \cos \theta_s) ,
\]

where the right side of the inequality represents the edge of the spherical sector of the endcap.

The parametric result is plotted in Fig. 3-16, from which it is evident that the \( \pi/2 \) criterion indeed best matches the data (Fig. 3-13).

The overlap normalized by the glint area for \( \pi/2 \) has been calculated at \( ka_s = 2.9, 5.5, \) and 8.0 and plotted in Fig 3-17. The \( \phi_i \)-dependence of geometric scatter is reasonably approximated for \( ka_s = 8.0 \) (\( ka = 9.5 \)). The 3 \( dB \) down point occurs at \( \phi_i = 22^\circ \), which matches the \( \phi_i = 21^\circ \) value from the measurement (Fig. 3-13). This is the region where I would expect this model to work best due to the size of the glint. The comparison is poor at lower frequencies.

**Bistatic Angle Dependence of Spherical Shell Model**

For a perfectly reflecting sphere with \( ka_s \gg 1 \), the bistatic target strength is given by [43]

\[
T(\phi_s) = \frac{a_s^2}{4r^2} [1 + \cot^2 \frac{\pi - \phi_s}{2} J_1^2(ka \sin \pi - \phi_s)] .
\]

\[ (3.29) \]
Figure 3-16: Parametric study of the effect of phase criterion on the finite glint characteristics. Relative overlap of the glint and the spherical sector of the endcap as a function of incidence angle.

Figure 3-17: Relative overlap of finite glint and the spherical sector of the endcap. Curves at mid-band in each frequency range: $ka_s = 2.9$ (solid), $ka_s = 4.2$ (dashed), $ka_s = 8.0$ (dash-dot)
In the vicinity of backscatter (\(\phi_s = 0\)), the second term is small and the target strength is approximately independent of \(\phi_s\). The bistatic target strength of the geometric response shown in Fig. 3-14 is seen to diminish with increasing \(\phi_s\). One means for explaining the angle dependence is to once again use the idea of overlap of a finite glint with the spherical sector of the endcap. The rotation angle of the glint equals the angle which bisects \(\phi_i\) and \(\phi_s\), which for bow incidence is simply \(\phi_s/2\). Fig 3-17 can be used determine the predicted bistatic angle dependence by replacing \(\phi_i\) on the horizontal axis with \(\phi_s/2\). The 3 dB down point for the model is at \(\phi_s/2 = 22^\circ\) and the data show the 3 dB down point as \(\phi_s = 21^\circ\).

**Monostatic Angle Dependence of Flat Circular Disk Model**

The angle dependence of scatter from a circular disk is stated explicitly in Eq. 3.9. I calculate the target strength, frequency averaged over the same three bands and normalized to 0 dB at \(\phi_i = 0^\circ\) (normal incidence to the disk) for comparison to the measured data. The resulting frequency averaged radiation patterns for a circular disk of radius \(a_c\) are shown in Fig. 3-18. The result is perhaps somewhat surprising at first glance, which is that the model best matches the mid-\(ka\) band (The 3 dB down point of the measured data is 12° whereas the model predicts 10°) rather than the lower-\(ka\) band. Apparently the curvature can be neglected for \(ka_s < 7\). This indicates that the geometric response at lower frequencies involves contributions from the conical shell. To check this, the same frequency averaged radiation patterns for a disk of radius \(a\) are plotted in Fig. 3-19. The lower-\(ka\) curve correlates well with the measured data (The 3 dB down point of the measured data is 11° whereas the model predicts 10°). This result indicates that an additional criteria for determining transition frequencies may be to compare the size of the incident sound wave to the diameter of the feature of interest. The acoustic wavelength equals the diameter of the sphere/cone junction at \(ka \approx 6\) which is in the mid-\(ka\) band, and equals the diameter of the circularly cylindrical shell at \(ka \approx 3\) which is in the lower-\(ka\) band.
Figure 3-18: Monostatic beampattern for a perfectly reflecting circular disk of radius $\alpha_c$ at oblique incidence. The curves represent the angular dependence averaged over three frequency bands and can be compared directly with Fig. 3-13. The frequency ranges are $2 < ka < 5$ (solid), $5 < ka < 8$ (dashed) and $8 < ka < 11$ (dot-dash).

Figure 3-19: Monostatic beampattern for a perfectly reflecting circular disk of radius $a$ at oblique incidence. The curves represent the angular dependence averaged over three frequency bands and can be compared directly with Fig. 3-13. The frequency ranges are $2 < ka < 5$ (solid), $5 < ka < 8$ (dashed) and $8 < ka < 11$ (dot-dash).
Bistatic Angle Dependence for Backscatter from a Circular Disk

Figure 3-20: Bistatic angle dependence of scatter from a perfectly reflecting circular disk of radius $a_c$ at oblique incidence. The curves represent the angular dependence averaged over three frequency bands. The curves have been normalized to 0 dB on-axis target strength and can be compared directly with Fig. 3-13. The frequency ranges are $2 < ka < 5$ (solid), $5 < ka < 8$ (dashed) and $8 < ka < 11$ (dot-dash).

Bistatic Angle Dependence of Flat Circular Disk Model

The conclusions reached in the previous section should be consistent with bistatic radiation patterns. To show this I compare the explicit bistatic angle dependence for the geometric scatter from a circular disk (Eq. 3.9) with measured bistatic patterns previously shown in Fig. 3-14. The frequency averaged radiation patterns for a circular disk of radius $a_c$ are plotted in Fig. 3-20. The model compares well in the mid-$ka$ band (the 3 dB down points for the model and data are 16 and 18º respectively) which is consistent with the monostatic result. The same model predicts a radiation pattern which also matches well with the lower-$ka$ data. Needless to say, the radiation pattern for a disk of radius $a$ is does not match the data in the lower-$ka$ band (the 3 dB down points for the model and data are 17 and 23º respectively).

These results present a somewhat ambiguous picture of the geometric scatter in the lower-$ka$ band. The monostatic angle dependence is not well modeled by the local interaction of the sound wave with the spherical section of the endcap, and in some
manner involves the conical section of the endcap. The bistatic angle dependence for lower-$ka$ correlates well with the radiation pattern of a circular disk of radius $a$. It is simply not clear how much of the conical section is involved. It is probably safe to state that below some frequency, the geometric return involves the entire cross section of the cylindrical shell. An educated guess is that this frequency region falls below $ka = 3$, where the acoustic wavelengths are larger that the shell diameter.

3.5 Acoustic coupling to Elastic Waves

The endcap is the predominant site of acoustic-elastic coupling for two reasons. First, it is the only region of the shell with geometry satisfying trace matching conditions. Second, the surface inhomogeneities provide a mechanism for the forced elastic wave to scatter to free elastic shell waves. In the case of the empty shell, nowhere along the circularly cylindrical portion do either of these conditions exist, although the ringed and internalled cylindrical shells have discontinuities at the ring attachments.

For the higher frequency model, the spherical shell, the mechanism for elastic wave generation is trace-matching. In the following section, I describe the conditions for trace-matching, and show they are satisfied for $ka > 7$. For the lower frequency model, the excitation of shell waves is via the scatter at the slope discontinuity connecting the spherical shell and the conical shell. As the incident sound field drives the spherical section of the endcap like at piston, energy is converted to free elastic waves at the end of the conical shell.

3.5.1 Spherical Shell Model: Trace-Matched Excitation

Trace velocity matching at the fluid-solid interface provides a powerful coupling mechanism to membrane waves, and can occur at surface locations where the trace acoustic velocity, $c_o/\sin \theta_l$, matches the shell wave phase speed,

$$\sin \theta_l = c_o/c_p \ .$$ (3.30)
Figure 3-21: Schematic of the coupling region for trace matched membrane wave excitation.

This can also be restated as the location where the tangential acoustic wavenumber, \(k_\theta = k \sin \phi_i\), shown in Fig. 3-21, equals the spherical shell compressional wavenumber.

The phase speed of compressional waves on a spherical shell can be found analytically at the discrete modal frequencies of a sphere using the fluid loaded thin spherical shell dispersion relation (Appendix A, Eqn. A.17). The modal phase speeds are given by

\[
c_p n = \frac{c_p \Omega_n}{n + 1/2},
\]

which are plotted in Fig. 3-22, where \(\Omega_n\) is the dimensionless natural frequency of the mode \(n\). For \(ka < 7\), the compressional waves become highly dispersive in which the phase speed increases and the group speed decreases with decreasing frequency. The first four modal frequencies and modal phase speeds for the for the fluid loaded sphere are given in Table 3.2

The trace matching angle, \(\theta_i\), describes a circle inscribed on the spherical shell. Asymptotically, this matching condition is a curve of zero thickness; however, for finite acoustic wavelengths, the coupling line has a wavelength and curvature dependent width which defines a patch as illustrated in Fig. 3-21. A coupling half-width, \(\theta_p\), has been described by Marston [48] in small angle approximation for high frequencies as

\[
\theta_p \approx (2\pi/ka \cos \theta_i)^{1/2}.
\]
Figure 3.22: Phase speeds on the spherical shell. Solid line is with external fluid loading, dashed line is in-vacuo.

<table>
<thead>
<tr>
<th>Mode Order m</th>
<th>Natural Frequency $\Omega_m = \omega a_s/c_p$ (ka)</th>
<th>Modal Phase Speed $c_{p_n}$, m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.26 (5.3)</td>
<td>13303</td>
</tr>
<tr>
<td>1</td>
<td>1.79 (7.3)</td>
<td>6306</td>
</tr>
<tr>
<td>2</td>
<td>2.65 (10.8)</td>
<td>5597</td>
</tr>
<tr>
<td>3</td>
<td>3.60 (14.6)</td>
<td>5460</td>
</tr>
</tbody>
</table>

Table 3.2: Normal modes for a fluid loaded spherical shell. Parameters are $h/a_s = .011, \nu = .3, \rho_s/\rho_o = 8.8, c_p/c_o = 3.54$. The normal modes are those due to compressional waves.
For clarity, the traced matched coupling region is sketched in Fig. 3-21 as a heavy curve. Marston uses a phase criterion of \( \pi \) between the ray that arrives at the trace-velocity matching line and a ray that arrives at the edge of the coupling patch, to define the coupling half-width.

The patch size, \( \theta_p \), is a measure of the localization of the coupling region. The small angle approximation in Eq. 3.32 is not uniformly valid in the frequency range considered here, so this equation must be more fully derived. In addition, I modify the phase criterion to be consistent with the definition of a glint, reducing the phase difference over the half-width of the patch to \( \pi/4 \). Over the full patch, this corresponds to \( \pi/2 \), consistent with the finite glint. To define the patch illustrated in Fig. 3-21, the axial distance from the phase match point \( \theta_l \) to the edge of the coupling patch \( \theta_l + \theta_p \) is denoted as \( \lambda/8 \), where \( \lambda \) is the sound wavelength in water. Therefore, from geometrical considerations, we have

\[
a_s[\cos \theta_l - \cos(\theta_l + \theta_p)] = \pi/4k \ . \tag{3.33}
\]

This can be expanded using \( \cos(A + B) = \cos A \cos B - \sin A \sin B \) to give

\[
\cos \theta_l - \cos \theta_l \cos \theta_p + \sin \theta_l \sin \theta_p = \pi/4ka_s \ . \tag{3.34}
\]

Finally, I divide through by \( \cos \theta_l \) to get the patch size

\[
\cos \theta_p - \tan \theta_l \sin \theta_p = 1 - \frac{\pi}{4ka_s \cos \theta_l} \ . \tag{3.35}
\]

In the previous section, the data showed that the initial interaction of the incident plane wave with the endcap was local only in the higher-\( ka \) band. I use the same criterion to determine whether or not the trace matched coupling can also be considered local. The half-width of the patch defining the transition from a local response will be the angular width from the trace-matching angle, \( \theta_l \), to the conical junction, \( \theta_s \). From Fig. 3-22, the compressional phase speed at mid-band is 6300 \( m/s \) which gives \( \theta_l = 14^\circ \) using Eq. 3.30. With use of \( \theta_p = 40 - \theta_l = 26^\circ \) in Eqn. 3.35, this gives \( ka = 7 \) as the
normalized transition frequency. For \( ka < 7 \), the patch extends beyond the surface of the spherical cap which must hinder the coupling efficiency of the trace-matching mechanism. For \( ka > 7 \) the patch completely resides within the spherical sector of the endcap.

At this point it is possible only to conclude that trace-matching is definitely a viable mechanism for the excitation of compressional waves only in the higher-\( ka \) band. This statement does not preclude trace-matched compressional wave excitation at lower frequencies but suggests that the coupling efficiency is reduced. It also does not preclude the existence other wave generating mechanisms such as the \textit{pump} mechanism discussed below at higher-\( ka \).

### 3.5.2 Circular Disk Model: Pump Mechanism

The model for the pump-like excitation of the endcap in terms was developed in Sec. 3.3. In the development, I derived an expression for the displacement of the circular disk, \( u \). I also showed the displacement of the disk is proportional to the combination of the axial displacement of the conical shell \( u_c \) and the radial displacement of the conical shell \( w_c \) at the point of attachment

\[
u = u_{cone} \cos \varphi + w_{cone} \sin \varphi .
\]

The cone displacements can be expressed explicitly by recognizing that

\[
\frac{u_{cone}Z_{comp}}{\cos^2 \varphi} = \frac{w_{cone}Z_{flex}}{\sin^2 \varphi},
\]

which gives

\[
u_{cone} = u/ \cos \varphi(1 + [Z_{comp}\sin^2 \varphi]/[Z_{flex}\sin^2 \varphi]) ,
\]

\[
w_{cone} = u/ \sin \varphi(1 + [Z_{flex}\cos^2 \varphi]/[Z_{comp}\cos^2 \varphi]) .
\]
Power injection into an elastic structure is given by the expression,

\[ \Pi = \text{Re}\{pu^*S\} = \text{Im}\{pu^*\omega S\} \quad (3.39) \]

where \( S \) is the cross-sectional area of the conical shell, the \(*\) denotes complex conjugate and power is injected to propagating elastic waves. Thus only the part of the conical shell velocity which is in-phase with the acoustic pressure contributes to the elastic waves which propagate from the junction. The in-phase displacements are plotted in Fig. 3-23. These displacements are used in Chap 4, where I estimate the radiation from a ring based upon a prescribed incident displacement field. In Chap 6, I compare these predicted magnitudes for initial compressional waves with estimates based upon the data (see Fig. 6-6), and find it predicts the magnitudes well for \( ka < 9 \). For \( ka > 9 \), the disk model predicts a compressional wave magnitude which is low by 4 \( dB \) indicating that trace-matching accounts for more than half of the coupling at the higher frequencies.

The analysis shows that the incident sound wave couples to both compressional and flexural waves due to the presence of the slope discontinuity. In contrast, the trace-matching discussed in the previous section excites only compressional waves. The compressional waves, however, will scatter to flexural waves at the same slope discontinuities as the analysis of Chap. 4 will reveal.

### 3.6 Radiation from Endcap

Some analytical machinery has been developed to describe the initial endcap scattering and coupling process. The same machinery can be used to describe the radiation from the endcap. Since the elastic wave propagation and scattering has not yet been addressed, it is not yet appropriate to examine radiation patterns in the data. The data are plotted later in Sec. 6.4. However, this is a good place to discuss the expected radiation patterns based on the prior discussion. At higher frequencies, where trace matching to the compressional waves is the likely coupling mechanism, the primary
Figure 3-23: Magnitude of displacements for the circular disk which are in-phase with the acoustic pressure, and therefore propagate away from the cone/sphere junction in the form of free elastic waves. The amplitudes are in picometers ($10^{-12} m$).
radiation mechanism is trace-matched compressional wave radiation. At lower frequencies, where trace-match is not important, a piston motion of the endcap will be the primary radiation mechanism.

Phase Matched Radiation

Williams and Marston [6] to define the radiation pattern for compressional waves on a spherical elastic body by describing a process which they label axial focusing. The beampattern is equal to that of a ring source whose radius is given by the locus of trace matching (From Fig. 3-21, this locus \( a_s \sin \theta_i \).) The frequency range in which this process is discussed is \( ka > 30 \), or in other words a range in which the coupling patch is small compared with the radius of the sphere. In the frequency range of interest here, \( ka \simeq 10 \), the patch is nearly the size of the spherical sector of the endcap. Therefore, it is more appropriate to compare the radiation pattern with that of a circular disk which has a radius equal to \( a_c \). The angle dependence for radiation from a disk of radius \( a_c \) was plotted in Fig 3-20.

Lower Frequency Radiation

At lower frequencies the coupling mechanism has been described by the displacement of circular disk. This is the same model suggested above. The bistatic radiation pattern is embedded in the expression for target strength Eq. 3.9 and follows directly from setting \( \phi_i = 0 \) and considering the \( \phi_s \)-dependence. This is exactly the manner in which the initial bistatic scatter was discussed, so that the radiation pattern is given by

\[
p(\phi_s) \sim \cos \phi_s J_1(ka_o\phi_s)/(ka_o\phi_s) ,
\]

where \( a_o \) is a generic radius which depends on the frequency. At higher frequencies \( a_o = a_c \), but the analysis of the initial scatter data suggests that at lower frequencies \( a_o \sim a \) where \( a \) is the radius of the circularly cylindrical shell.
3.7 Summary

The initial scatter is simply that portion of the scatter which arrives within $t < \tau$, where $\tau$ is equal to the duration of the transient excitation. At or near bow incidence, the initial scatter is not influenced by the internal structures which are located further than the interrogation length away from the point of initial contact.

The geometric contribution is an important component of the initial scatter. At bow incidence, the geometric contribution is primarily due the interaction of the incident sound wave with the spherical section of the endcap. At higher-$ka$ ($ka > 6.7$) the initial interaction is local to spherical section of the endcap as defined by Fresnel region of the incident sound wave, so the resulting geometric return is well described by the backscatter from a spherical elastic shell. At lower frequencies this is not the case since the spatial extent of the initial interaction is not local to the spherical cap. To properly model the interaction, the influence of the conical shell must be accommodated in the analysis.

I conclude that the transition frequencies are those at which the acoustic wavelengths are the same as the dimensions of the geometric feature of interest. The incident sound wavelength is equal to the diameter of the spherical sector of the endcap at $ka \simeq 6$ and equal to the diameter of the cylindrical shell at $ka \simeq 3$. For $ka > 6$ the geometric response is local to the spherical sector of the endcap, and for $ka < 3$ the geometric response involves the entire endcap.

The endcap is the primary location for acoustic coupling to elastic waves near bow incidence. Phase matched excitation of compressional waves is a powerful mechanism for coupling between the incident sound wave and free elastic shell waves; however, the description of the coupling region becomes obscured at lower frequencies, since the size of coupling region is proportional to wavelength of the incidence sound wave. An alternative means for exciting compressional waves is demonstrated by the displacement of the spherical section of the endcap in a piston motion which excites both compressional and flexural waves at the sphere/cone junction. At this point in the
thesis, it is not possible to conclude which mechanism is more important in regard to the excitation of elastic waves. However, in Chap. 6, after developing the necessary scatter coefficients, I will show that for $ka < 9$, the trace matching is less important and the compressional wave amplitudes are predicted by the disk model, while for $ka > 9$ the trace-matching is more important.
Chapter 4

Wave Conversion and Scatter

4.1 Overview

One of the primary goals of this thesis is to evaluate the effect of inhomogeneities and internal structures on the acoustic backscatter. While much can be extracted directly from the data, it is useful at this point to step away from the data and consider the type of scattering processes which could occur. Chapter 5 applies these processes to the measured scattering to show that the slope discontinuities are important as a mechanism for wave conversion between flexural and compressional waves, and the rings (particularly the ring closest to the insonified endcap) are important as a mechanism to reflect forward propagating waves in the backward direction.

I present two interaction models which are intended to represent relevant scattering phenomena within the data. The two models represent the scatter at a slope discontinuity, and the scatter at a ring stiffener within a cylindrical shell. The latter is analyzed in both the frequency and time domains.
4.2 Scatter at a Slope Discontinuity

4.2.1 Plate Model

In this section I analyze the scatter at a slope discontinuity connecting two semi-infinite flat plates as shown in Fig 4-1. The two-plate discontinuity is a simplification of that which is present at the junction of both the circularly cylindrical shell and the cone, and the cone and the spherical shell. I consider both compressional and flexural waves normally incident to the junction and calculate the reflection, transmission and conversion coefficients. This problem is also simplified substantially by neglecting acoustic coupling and by including the effects of fluid loading only approximately.

Slope discontinuities occur at two locations for each endcap. Both discontinuities are approximately 26°. The junction of the cone and the spherical shell has a combined discontinuity in slope and curvature, although curvature is a weaker discontinuity because of its second derivative compared to the first derivative. The connection of two semi-infinite flat plates is only an approximation of the physical reality, however this model should provide reasonable estimates of the scatter coefficients especially above the shell ring frequency, where the shell behaves locally as a flat plate. The flat plate equations also neglect the curvature-induced coupling between the flexural and compressional waves.

I incorporate the fluid loading in an approximate way late in the analysis, but neglect radiation coupling to the acoustic field. Thus acoustic losses at the junction are taken as zero. I am encouraged to use this limit by the analysis of a semi-infinite plate with fluid loading [49], that demonstrates that for a flexural wave incident upon a free plate edge, acoustic scatter is small and to leading order, the magnitude of the reflection is unity. Similarly, an analysis of a fluid loaded infinite plate with an attached transverse plate shows the radiation to the acoustic field by a flexural wave at the transverse plate field is at least an order of magnitude less powerful than scatter to elastic plate waves [50].

Because the shell scattering process is axisymmetric, the waves are modeled nor-
mally incident to the plate junction, effectively reducing the system to 1-D. The solution is straightforward, in which I solve a system of equations constrained by the boundary conditions at the plate junction: continuity of forces and moment, and displacements and rotation [51]. The interaction configuration is given in Fig. 4-2, and the connection condition between the two plates is clamped. A clamped connection transmits in-plane and normal forces and moments. The incident wave on plate 1 arrives at the discontinuity with unit amplitude. The angular opening between the two plates is $2\alpha_j = 154^\circ$.

In-Vacuo Coefficients

I begin with the in-vacuo thin plate equations to characterize the in-plane $u$ and out-of-plane $w$ displacement fields. One deficit implicit in using thin shell equations is that they overstate the flexural stiffness of the plate due to neglect of transverse shear. These effects become more important as the flexural wavelength decreases leading to differences between thin plate theory and thick plate theory [1] or the full 3D elastic equations [52]. This does not cause difficulties for the frequencies considered in this model; however, this is a potential source of error in terms of applied moments. A moment applied to a plate as a pair of forces separated by a distance $\delta$ gives rise to large transverse shear for $\delta$ becoming vanishingly small. This difficulty is not addressed in this analysis except to point out here that the clamped condition assumed
in this calculation may need to be relaxed to properly account for the interaction moments. For this model it is assumed the transmitted moment is applied over a distance of several shell thicknesses. The in-vacuo plate expressions can be written as a pair of decoupled equations;

\[
\frac{E_s}{1 - \nu^2} \frac{\partial^2 u}{\partial x^2} + \rho_s \omega^2 u = 0, \\
D_s \frac{\partial^4 w}{\partial x^4} - \rho_s h \omega^2 w = 0, 
\]

(4.1)

where \( D_s = E_s h^3 / 12(1 - \nu^2) \), \( E_s \) is the Young's modulus of the plate, \( h \) is the plate thickness and \( \nu \) is the plate Poisson's ratio. The dispersion relations for the compressional and flexural waves respectively are given by

\[
k_p^2 = \frac{(1 - \nu^2) \rho \omega^2}{E} = \frac{\omega^2}{c_p^2}, \\
k_f^4 = \frac{\rho h \omega^2}{D_s} = \frac{12 \omega^2}{h^2 c_p^2}. 
\]

(4.2)

For a compressional wave arriving at the junction, the in-plane and out-of-plane displacements on the two plates are given by,

\[
u_1(x) = e^{ik_p x} + R_{pp} e^{-ik_p x}, \\
w_1(x) = R_{pf} e^{-ik_f x} + R_{pe} e^{ik_f x}, \\
u_2(x) = T_{pp} e^{ik_p x}, \\
w_2(x) = T_{pf} e^{ik_f x} + T_{pe} e^{-ik_f x}, 
\]

(4.3)

where the time dependence \( e^{-i\omega t} \) is understood. There are six unknown reflection (\( R \)) and transmission (\( T \)) coefficients with subscripts \( p, f \) and \( e \) corresponding to the propagating compressional and flexural waves and evanescent flexural nearfield, where the two subscripts indicate the incident and scattered wave types respectively. It is necessary to retain terms for the evanescent field to properly match boundary conditions, however no power is transmitted to the evanescent field.
Figure 4-2: Forces at the junction of two plate system

The transmitted forces and moments shown in Fig. 4-2 are directly related to spatial derivatives of the displacement field at the junction and are given by

\[
T = Eh \frac{\partial u}{\partial x}, \\
V = D_s \frac{\partial^3 w}{\partial x^3}, \\
M = D_s \frac{\partial^2 w}{\partial x^2}.
\]  

(4.4)

Continuity of force and moment, and displacement and rotation at the interface yields a 6x6 matrix system of equations which reduce after some algebra to the reflection and transmission coefficients for the propagating waves. Specifically,

\[
R_{pp} = \frac{1}{2} \left[ \frac{N_1}{D_1} + \frac{N_2}{D_2} \right], \\
T_{pp} = \frac{1}{2} \left[ -\frac{N_1}{D_1} + \frac{N_2}{D_2} \right], \\
R_{pf} = \frac{N_3}{D_1} - \frac{N_3}{D_2}, \\
T_{pf} = \frac{N_3}{D_1} + \frac{N_3}{D_2},
\]

(4.5)

and

\[
N_1 = (1 + i) \cos^2 \alpha_j - 2 \mu \sin^2 \alpha_j, \\
N_2 = 2 \sin^2 \alpha_j - (1 - i) \mu \cos^2 \alpha_j, \\
N_3 = \sin \alpha_j \cos \alpha_j,
\]
\[ D_1 = (1 + i) \cos^2 \alpha_j + 2\mu \sin^2 \alpha_j , \]
\[ D_2 = 2 \sin^2 \alpha_j + (1 - i)\mu \cos^2 \alpha_j . \]

where \( \mu = k_f^3 h^2 / 12k_p \), and the \( D_i \) are not to be confused with the flexural rigidity \( D_3 \). These coefficients represent the ratio of the transmitted or reflected wave amplitude to the incident wave amplitude, which in this case is a compressional wave.

Similar closed form equations can be derived for flexural wave incidence by first modifying the displacement fields in plate 1 given in Eq. 4.3 to account for the incident flexural wave as

\[ u_1(x) = R_p e^{-ik_px} , \]
\[ w_1(x) = e^{ik_f x} + R_f e^{-ik_f x} + R_e e^{ik_f x} . \]  
(4.6)

The forms of the transmission fields in plate 2 are unchanged and are equal to the relationships given in Eq. 4.3. After more algebra, the scattering coefficients for an incident flexural wave are given by

\[ R_{fp} = 2\mu \left[ \frac{N_3}{D_1} + \frac{N_3}{D_2} \right] = 2\mu T_{pf} , \]
\[ T_{fp} = 2\mu \left[ \frac{N_3}{D_1} - \frac{N_3}{D_2} \right] = 2\mu R_{pf} , \]
\[ R_{ff} = 2 \left[ \frac{N_4}{D_1} + \frac{N_5}{D_2} \right] , \]
\[ T_{ff} = 2 \left[ \frac{N_4}{D_1} - \frac{N_5}{D_2} \right] , \]  
(4.7)

and

\[ N_4 = 2 \sin^2 \alpha_j - (1 - i)\mu \cos^2 \alpha_j , \]
\[ N_5 = (1 - i)\cos^2 \alpha_j - 2\mu \sin^2 \alpha_j . \]
Approximation to Fluid Loading

The target shells are heavily fluid loaded on one side, so it is necessary to consider the fluid loading effect in at least some approximate manner. This has been done exactly [53]; however, I assume that only the flexural waves are coupled to the fluid, thereby neglecting the Poisson coupling of the compressional wave, thus radiation damping is taken as zero. The effect of the fluid loading is to modify the flexural wavenumber dispersion relation, which for frequencies below coincidence, is given by [1] as

$$k_{f \text{fluid}} = k_{f \text{in vacuo}} \left[ 1 + \frac{\epsilon}{\Omega^{1/2} (1 - \Omega)^{1/2}} \right]^{1/4} \quad , \quad (4.8)$$

where $\epsilon = \rho_0 c_0 / \omega_c \rho_s h$, $\Omega = \omega / \omega_c$ and $\omega_c$ is the coincidence frequency, defined as $\omega_c = c_0^2 (\rho_s h / D_s)^{1/2}$.

Conversion to Power Coefficients

For physical interpretation, I find it useful to convert the coefficients to a ratio of scattered to incident power. In this way, it is possible to gain better understanding of the energy partitioning within the system. In the case of a flexural wave, the expression for power is given by

$$P_f = \frac{1}{2} \text{Re}\{M(-i\omega \beta^*)\} + \frac{1}{2} \text{Re}\{V(-i\omega w^*)\} \quad , \quad (4.9)$$

where $\beta$ is the plate rotation $\partial w / \partial x$ and $M$ and $V$ are from Eqs. 4.4. With use of a displacement field of the form $w(x) = Ae^{ikfx}$, one gets the power for a flexural wave of amplitude $A$ as

$$P_f = |A|^2 D w k_f^3 \quad . \quad (4.10)$$

In the case of a compressional wave, the expression for power is given by

$$P_p = \frac{1}{2} \text{Re}\{T(-i\omega u^*)\} \quad , \quad (4.11)$$
where $T$ is given by Eq. 4.4. With use of a displacement field of the form $u(x) = Ae^{ik_p x}$, one gets the power for a compressional wave of amplitude $A$ as

$$P_p = |A|^2 \frac{6}{k^2} Dk_p \omega . \quad (4.12)$$

From Eqns. 4.10 and 4.12, I define the conversion between a coefficient representing an amplitude ratio ($A$) to a coefficient representing a power ratio ($P$) as

$$P_{fp} = 2\mu |A_{fp}|^2 ,$$
$$P_{pf} = (2\mu)^{-1} |A_{pf}|^2 ,$$
$$P_{pp} = |A_{pp}|^2 ,$$
$$P_{ff} = |A_{ff}|^2 , \quad (4.13)$$

where $\mu$ has been previously defined as

$$\mu = \frac{k_f^3 k^2}{12k_p} .$$

4.2.2 Results

The junction scatter coefficients, without and with (approximate) fluid loading, for an incident compressional wave, Eq. 4.5, are plotted as a function of frequency in Fig. 4-3. The coefficients for fluid loading were calculated by applying the modified flexural wavenumber, Eq. 4.8 in these equations. Incorporation of the modified wavenumber has simply shifted the coefficients down in frequency. This is consistent with a slower flexural wave speed due to the inertial effects of the fluid loading. The downward shift in normalized frequency is approximately a difference of $ka = 3$.

For physical interpretation, I have found it easier to consider the scattering coefficients in terms of scattered power. The coefficients using Eq. 4.5 and 4.7 are converted to power using Eq. 4.13. These are plotted for an incident compressional wave in Fig. 4-4. The upper curves represent the reflection and transmission of the
Figure 4-3: Scatter amplitude coefficients at angled plate junction (a) in vacuo and (b) with approximate fluid loading for an incident compressional wave at normal incidence angle. The coefficients represent the ratio of the scattered wave amplitude to the incident wave amplitude. The slope discontinuity is 26° for nickel plates with $h = .5mm$. Although the plates are flat, the frequency normalization pretends the shell of interest is that which is in this thesis.
Figure 4-4: Conversion power coefficients at angled plate junction with fluid loading on one side for an incident compressional wave (a) reflection/transmission (b) conversion. The solid line is reflection and the dashed line is transmission. The coefficients represent the ratio of the scattered wave power to the incident wave power.

incident wave, while the lower curves represent the conversion to reflected and transmitted flexural waves. The principal scattering process is transmission of the incident compressional wave, with small reflection of the compressional wave (10 dB down at mid-band). What these figures also show is that significant energy conversion takes place at a slope discontinuity.

Scatter power results for an incident flexural wave are plotted in Fig. 4-5. The flexural wave response is approximately the same as the compressional wave response. The transmission coefficient is the same but the reflection coefficient is 4 dB lower. The conversion coefficients are the same except the flexural waves convert more power to the reflected compressional wave than the transmitted compressional wave.

For both incident compressional and flexural waves, nearly half (40%) of the incident wave power is converted to the other wavetype. This has a major effect on
Figure 4-5: Conversion power coefficients at angled plate junction with fluid loading on one side for an incident flexural wave (a) reflection/transmission (b) conversion. The solid line is reflection and the dashed line is transmission. The coefficients represent the ratio of the scattered wave power to the incident wave power.
the scatter as shown in Chap. 5. In effect, the junction acts as a damping mechanism for a single wave type, stripping almost half its energy and converting it to the other wavetype.

In summary, the reaction at the slope discontinuity is primarily transmission of the incident plate wave. However, the junction provides a mechanism which strongly couples the flexural and compressional waves. The reflection process is relatively weak but in Chap. 5 I will show it is not negligible.

4.3 Scatter at an Eccentric Ring: Frequency Analysis

4.3.1 Ring Interaction Model - Single Ring

In this section I analyze the elastic wave scatter from a ring eccentrically attached to an infinite cylindrical shell, as shown in Fig. 4-6. This configuration is used to quantify the scattering phenomena that occur at the junction of the shell and ring, and to distinguish the ring interaction from other phenomena such as finite end scatter, multiple interactions and bay resonances. The ring-shell junction is excited by an incident axisymmetric compressional or flexural wave which represents, for example, the arrival of shell waves generated at the endcap by the incident acoustic wave.

The rings have a dramatic influence upon the distribution of elastic wave energy on the target shells as was seen indirectly in Chap. 2 and will be seen more clearly in Chap. 5. I choose to model the interaction of an elastic wave with one of the ring stiffeners because this interaction has an important effect on the energy distribution near axial incidence.

I do not consider the interaction of the incident sound wave with the ring. As seen in Chap. 2 and as I will further demonstrate in Chap. 5, this interaction does not contribute significantly to the backscatter in the bow incidence region. This interaction does, however, become increasingly important for \( \phi_i > 25^\circ \).
I calculate the reflection, transmission and conversion coefficients for an axisymmetric elastic shell wave incident upon the ring. I also estimate the contribution to the acoustic field by radiation at the ring.

Several investigators have studied the dynamics of transverse stiffeners attached to a cylindrical shell. Harari [54] and Hodges et al [55] studied periodic ribs attached to a cylinder in-vacuo. Harari's formulation can be applied as well for periodic stiffeners spaced over a finite attachment region. Guo [29] has developed a formalism for determining the scatter from a transverse bulkhead attached to a fluid-loaded cylindrical shell. Following the work of Guo, Corrado [9] formulated the problem of scatter from a ring of mass, without stiffness, attached to a fluid loaded cylindrical shell.

Over the measurement frequency band, there are ring resonances that should be considered. Harari, Hodges et al, and Guo each included the dynamics of the internal structure in some fashion; however, I choose to extend the work of Corrado, which corresponds to the ring/shell system of interest, because the ring elastic properties can be readily included in his formulation.

The ring interaction model is shown in Fig. 4-6. A single ring is located at $z = 0$. The ring is attached to the shell along a circle with a clamped joining condition, so the junction can transmit forces and moments. The matching conditions at the shell/ring interface are continuity of displacement and rotation, and continuity of force and moment. The shell dynamics are represented by the Donnell thin shell
theory [1] simplified to include only the axisymmetric terms. The cylinder is infinite, but otherwise has the same dimensions and material properties as the shell targets. The incident waves are axisymmetric shell waves traveling in the positive z direction. Fluid loading has been accommodated in the shell equations. A similar difficulty arises with the use of Donnell theory, which I discussed earlier with respect to thin plate theory. That is, thin shell theory overstates the transverse stiffness of the shell. In this model, this could cause errors with respect to the moment coupling between the shell and the ring. For the analysis in this thesis, I assume that the moment is applied across an axial distance large enough that the clamped condition is appropriate. I have chosen this condition because the exterior of the ring has a flat surface several thicknesses wide which fits snugly against the inner surface of the cylindrical shell. Given a small attachment width, an alternate and equally simple joining condition could be pinned connection, in which no moment is transmitted through the junction. This would avoid the difficulty of overstating the transverse stiffness, but would also neglect any coupling due to the interaction moment.

Model for the Ring

The ring has inherent mass and stiffness which make the ring a resonant structure. The resonance frequencies of the lowest order motions of the ring lie within the measurement frequency band and I account for those stiffness terms which give rise to these resonances. The two axisymmetric resonances which fall within the measurement frequency band are the hoop resonance and the rolling resonance. The motions associated with these resonances are shown in Fig. 4-7. To include these resonances in the ring model, two stiffness parameters are incorporated into the ring model: the hoop stiffness and the rolling stiffness. The hoop stiffness causes a resonance at $ka = 3.2$ that is close to the shell ring frequency ($ka = 3.5$), and the rolling stiffness provides a rolling resonance somewhat below the ring frequency, at $ka = 2.7$.

In the measurement frequency range, the bulk compressional and shear wavelengths are large compared with the dimensions of the ring cross section. Therefore
Figure 4-7: Resonance motions of the ring within the measurement band

the higher order thickness resonances of the ring may be neglected. The bending resonances found in the work of Harari and Hodges et al are not included because the ring is very thick (almost square in cross-section). For both the ring and shell, two translations and one rotation are considered: translation in the axial and radial directions and rotation about the circumferential axis.

As I have noted, the ring is assumed to be clamped to the shell along a circle as shown in Fig. 4-6. Therefore the shell can transmit forces and moments to the ring at the attachment. I have chosen a clamped joining condition because the real shells are constructed by welding the rings directly to the shell. The hoop motion of the ring is strongly coupled to the radial shell motions, while the rolling motion is well coupled to the rotational shell motions. Both are motions predominantly associated with flexural wave propagation. Due to the eccentricity of the attachment, the rotation of the ring is also coupled to axial motion of the shell at the ring outer surface. Therefore the rolling motion of the ring will also be well coupled compressional wave motions.

The model for the ring is shown in the form of a free body diagram in Fig. 4-8. The resulting forces which are transmitted from the shell to the ring are applied at the attachment point which is the center of the ring outer surface. The forces are applied directly to the ring because the joining condition is clamped. The stiffness parameters of the ring are represented by equivalent linear and torsional springs with spring constants, $k_r$ and $k_\theta$ respectively.
Figure 4-8: Free body diagram showing the applied forces and motions of the ring. The rolling and hoop stiffness have been represented by equivalent springs.

4.3.2 Method of Solution

The method of solution is to first separately determine compliance matrices for the shell and the ring at the attachment, where the compliance matrix is a transfer function between displacements and forces. The compliance matrix $S$ for the shell is readily obtained in the wavenumber domain [9], where the relationship is

$$\hat{u}_s = [S]f$$ \hspace{1cm} (4.14)

Here $\hat{u}_s$ is the shell displacement vector in the wavenumber domain, and $f$ is the interaction force vector to be derived in Sec. 4.3.3. Eq. 4.14 can be evaluated at the ring attachment location by applying the inverse wavenumber transform (Eq. B.8). The resulting form for the shell compliance equations is

$$u_s = [s]f + u_{inc}$$ \hspace{1cm} (4.15)

where $u_s$ is shell displacement vector, $s$ is shell compliance matrix derived in Sec. B.1 and $u_{inc}$ is a vector that prescribes the displacement field of the incident elastic waves. All terms are evaluated at the attachment, $z = 0$.

The compliance matrix for the ring follows directly from its free-body diagram,
Fig. 4-8, and the resulting form of the ring compliance equations is

\[ u_r = [r]f \tag{4.16} \]

where \( u_r \) is the ring displacement vector at the outer surface, \( r \) is the ring compliance matrix derived in Sec. 4.3.3 and \( f \) is the interaction force vector. The two systems are joined by applying the condition that the displacements are equal at the attachment point \( z = 0 \). Specifically,

\[ u_{inc} = [r - s]f \tag{4.17} \]

Inversion of the combined compliance matrix yields a frequency dependent set of equations for interaction forces as a function of the prescribed incident displacement field. The resulting expression is

\[ f = [r - s]^{-1}u_{inc} \tag{4.18} \]

The excitation has the form of a free propagating compressional or flexural wave. In general the prescribed incident wave has three components

\[ u_{inc} = \{u_{inc}, w_{inc}, w'_{inc}\}^T \tag{4.19} \]

For a given free elastic wave, the displacement components are directly related, therefore it is only necessary to prescribe one component of the incident wave. The axial, radial and rotational components of the axisymmetric displacement field are related by

\[ u_{inc} = i\xi/\nu\alpha w_{inc} \],

\[ w'_{inc} = i\alpha w_{inc}/a \tag{4.20} \]

where

\[ \xi = 1 + \beta^2\alpha^4 - \Omega^2 + F_u E_f k^2 a^2 \tag{4.21} \]

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These expressions follow directly from Eq. A.7, where \( \alpha = k_z a \) and \( k_z \) is the axial wavenumber of the incident elastic wave.

By replacing the ring with the resulting interaction forces, the problem now becomes one of determining the response of an infinite homogeneous cylindrical shell subject to axisymmetric ring forces and moments. There are two methods which could be followed at this stage in order to get to the scatter.

One approach is to incorporate the resulting forces within the Donnell equations and to solve for the shell displacement fields in the wavenumber domain, where the scattered pressure is related to the radial displacement field via the momentum equation. A second approach is to use asymptotic methods to analyze the total scatter in terms of individual contributions such as single wavetypes. This is the approach used here and reviewed in Appendix B. More detailed discussions of these methods are given by Felsen [45], Corrado [9] and Guo [29].

**Reflection and Transmission Coefficients**

The reflected and transmitted shell waves correspond to the poles in the integrand of the wavenumber transform of radial displacement (Eq. 4.14). Each pole contribution corresponds to a single wavenumber, so it is possible to isolate and estimate reflection and transmission coefficients of individual elastic waves (see Sec. B.4.) The transmitted wave field may be expressed as

\[
w_T = \frac{a^2}{c_p^2 \rho_s} \left\{ \frac{(-F_z S_{21} + iF_r S_{22} + iM_n S_{23}/a)}{\partial D/\partial k_z} \right\} + \delta(k_z a - k_{zinc} a) \bigg|_{k_z a = \alpha_r},
\]

(4.22)

where \( \alpha_r \) is the normalized wavenumber root of the shell axisymmetric dispersion equation which corresponds to the transmitted wave, and \( k_{zinc} \) is the axial wavenumber of the prescribed incident elastic wave. The radial displacement of a reflected wave at the ring is

\[
w_R = -\frac{a^2}{c_p^2 \rho_s} \frac{(F_z S_{21} - iF_r S_{22} - iM_n S_{23}/a)}{\partial D/\partial k_z} \bigg|_{k_z a = \alpha_r}.
\]

(4.23)
Radiation at the Ring

The method of stationary phase is used to get an expression for the contribution due to radiation from the rings (see Sec. B.5). The resulting expression is

\[
p_s(R, \phi_s) = \frac{kaE_f e^{ikR}}{\pi R} \frac{\hat{w}^* k_{zz}}{\sin \phi_s H_0'(ka \sin \phi_s) D(k_{zz})},
\]  

(4.24)

where \( \phi_s = 90^\circ \) is the beam direction. Here \( \hat{w}^* \) is related to the wavenumber description of the displacement at the ring/shell attachment derived in Appendix B

\[
\hat{w}^* = iF_z S_{21} + F_r S_{22} + M S_{23}/a.
\]  

(4.25)

4.3.3 Compliance Matrices

Ring Compliance Matrix

The ring compliance matrix relates the interaction forces at the attachment to the motion of the ring. The interaction forces and sign conventions are given by the free body diagram, Fig. 4-8. From the free body diagram, it is straightforward to write the equations of motion for the ring in matrix form as

\[
\begin{bmatrix}
  u \\
  w \\
  \Omega_r
\end{bmatrix} =
\begin{bmatrix}
  r_{11} & 0 & r_{13} \\
  0 & r_{22} & 0 \\
  r_{31} & 0 & r_{33}
\end{bmatrix}
\begin{bmatrix}
  F_z \\
  F_r \\
  M_\theta
\end{bmatrix},
\]  

(4.26)

where

\[
\begin{align*}
  r_{11} &= \frac{1}{m_r \omega^2} - \frac{h^2}{4(I_\omega \omega^2 - k_\theta)}, \\
  r_{13} = r_{31} &= \frac{h}{2(I_\omega \omega^2 - k_\theta)}, \\
  r_{22} &= \frac{1}{m_r \omega^2 - k_r}, \\
  r_{33} &= \frac{1}{I_\omega \omega^2 - k_\theta}.
\end{align*}
\]  

(4.27)
Here $r_{ij}$ are compliance terms which represent the displacement of the ring in the direction $i$ due to a force in the direction $j$. $I_o$ is the rotational inertia and $m_r$ is the mass of the ring per unit circumferential length.

Shell Compliance Matrix

The shell compliance matrix has been previously derived [9], and I review the derivation in Appendix B. The resulting form of the shell compliance matrix is

$$\begin{bmatrix}
  u \\
  -iw \\
  -iw'a
\end{bmatrix} =
\begin{bmatrix}
  s_{11} & 0 & s_{13} \\
  0 & s_{22} & 0 \\
  s_{31} & 0 & s_{33}
\end{bmatrix}
\begin{bmatrix}
  F_z \\
  -iF_r \\
  -iM/a
\end{bmatrix} +
\begin{bmatrix}
  u_{inc} \\
  -iw_{inc} \\
  -iw'_{inc}a
\end{bmatrix},
\tag{4.28}
$$

where $u_{inc}$, $w_{inc}$ and $w'_{inc}$ describe the displacement field of the prescribed incident axisymmetric shell wave and the compliance terms are defined by Eq. B.4.

Interaction Forces

The influence of the ring is represented by the three ring forces/moments applied to the shell middle surface. These interaction forces are found by equating the displacement components of the shell, Eq. 4.28 and the ring, Eq. 4.26 to satisfy the kinematic boundary condition. Combination of Eq. B.11 and Eq. 4.26 gives expressions for the interaction forces as

$$F_z = \frac{1}{dt} \left\{ (-s_{33} + a^2 r_{33}) u_{inc} - (is_{13} + ar_{13}) w'_{inc} a \right\}, \tag{4.29}$$

$$M/a = \frac{1}{dt} \left\{ (is_{13} - ar_{13}) u_{inc} + (-s_{11} + r_{11}) w'_{inc} a \right\}, \tag{4.30}$$

$$F_r = \frac{-1}{s_{22} - r_{22}} w_{inc}, \tag{4.31}$$
where
\[
d_t = (s_{11} - r_{11}) (s_{33} - a^2 r_{33}) - s_{13}^2 - a^2 r_{13}^2.
\] (4.32)

This effectively replaces both the ring and the incident shell waves with three ring forces and moments applied to the shell at the ring attachment location.

4.3.4 Ring of Mass without Stiffness vs. Ring of Mass with Stiffness

In Fig. 4-9, I compare the reflection coefficients for the ring of mass used by Corrado [9] and the coefficients for the ring of mass plus stiffness derived in this thesis. These show differences which can be attributed to the elastic properties of the ring. In the upper figure, the reflection coefficients for both ring types for an incident compressional wave approach 0 \(dB\) above \(ka = 4\), independent of the elastic properties. Below \(ka = 4\) the differences are significant as the ring of mass without stiffness continues to behave as a highly reflecting body, while the ring mass plus stiffness becomes poorly reflecting, and non-reflecting at \(ka = 2.7\), the location of the rolling resonance which is well coupled to the axial motions of the compressional wave.

In the lower figure, the reflection coefficients for an incident flexural wave have the same general properties; differences exist below \(ka = 4\) and diminish at higher frequencies. It is evident that the dynamics of the ring of mass plus stiffness near the two resonance frequencies are very different from the ring of mass only, while above the resonance region, the two systems behave almost identically. This is characteristic of the behavior of a resonant system whose resonance frequency is \(f = f_0\), that is, for \(f \gg f_0\) the system is mass-controlled and the stiffness is not important.

Without damping, the impedance of a spring-mass system goes to zero at resonance or equivalently the interaction force goes to zero. The result is that the ring is invisible to the incoming wave and there is no reflection. This explains the dips found at resonances in the ring of mass plus stiffness curves: at resonance the incoming shell waves propagate past the ring unimpeded. The compressional waves are well coupled
Figure 4-9: Comparison of reflection coefficients for a ring of mass and a ring of elasticity: (a) incident compressional and (b) incident flexural wave. The solid lines are the ring coefficients for mass plus stiffness and the dashed lines are ring coefficients for mass only. The coefficients represent the ratio of the scattered wave amplitude to the incident wave amplitude.

to the rolling resonance, $ka = 2.7$ and the flexural waves are well coupled to both the rolling and hoop resonance $ka = 3.5$. Above these resonances, the ring behaves as a large mass impedance, thus the two sets of curves converge. At higher frequencies outside this frequency band, where the higher order modes of the ring can be excited, the curves should diverge again at the higher order ring thickness resonances neglected in this analysis.

4.3.5 Results

Reflection and Transmission of Shell Waves

The reflection (solid line) and transmission (dashed line) coefficients for a compressional wave incident upon a ring are shown in Fig. 4-10. The coefficients represent
Figure 4-10: Reflection and transmission coefficients at ring for an incident compressional wave. The coefficients represent the ratio of the scattered wave amplitude to the incident wave amplitude. The solid line is reflection and the dashed line is transmission.

the ratio of radial displacements of the reflected or transmitted waves to the incident wave. Above $ka = 4$, the ring becomes an almost rigid boundary reflecting most of the incident energy in the form of compressional waves. This is due to the large impedance discontinuity controlled by the mass of the ring. Around the ring rolling resonance, $ka = 2.7$, is a region of high transmission. The ring accepts no power at resonance, simply following the shell displacement so that the wave propagates past the ring essentially unimpeded. At resonance, the interaction force reduces to zero. This is a steady state interpretation which neglects transient effects associated with a pulse waveform; however I'll consider the effect of transients in the Sec. 4.4.

The implication for axial incidence is that above $ka = 4$, the ring will act as a reflector of compressional waves, essentially trapping the compressional wave energy between the endcap and the first ring. As I showed in the previous chapters, the endcaps provide the dominant loci for acoustic coupling, particularly compressional waves. Since the incident sound wave couples to the shell primarily at the insonified endcap, elastic waves can only reach the far end of the shell via propagation down the shell axis. By containing most of the energy in the first bay between the endcap and the first ring the shell has been effectively shortened.

The reflection (solid line) and transmission (dashed line) coefficients for an incident
Figure 4-11: Reflection and transmission coefficients at ring for an incident flexural wave. The coefficients represent the ratio of the scattered wave amplitude to the incident wave amplitude. The solid line is reflection and the dashed line is transmission.

flexural wave are shown in Fig. 4-11. The coefficients represent the ratio of the radial displacements for the scattered and incident waves. The flexural reflection coefficient is not as large as the compressional reflection coefficient. This may be explained by the presence of the fluid which partially short circuits the ring and allows some of the fluid-borne flexural energy to bypass the junction [49]. This is consistent with higher transmission coefficients for the flexural wave than for the compressional wave. The influence of both ring resonances are apparent at lower-\(ka\). The flexural wave has a predominantly radial displacement field and therefore is coupled to the hoop resonance. Additionally, flexural wave propagation includes a rotational field which couples to the rolling resonance. In contrast with the compressional wave coefficients, the regions of high transmission have very narrow bandwidths. The bandwidths are proportional to the radiation resistance of the ring/shell junction with respect to the incident and scattered waves.

Conversion of Shell Waves

The rings also provide a mechanism for wave conversion. Some of the incident compressional wave energy will be converted to flexural wave energy, some will be directly converted to acoustic waves. The same is true for an incident flexural wave. These results are shown in Fig. 4-12, showing the reflected (solid) and transmitted (dashed)
Figure 4-12: Conversion coefficients at ring for (a) incident axisymmetric compressional and (b) incident flexural waves. The coefficients represent the ratio of the scattered wave amplitude to the incident wave amplitude. The solid line is reflection and the dashed line is transmission.

conversion coefficients for both wavetypes. The conversion coefficients represent the ratio of the displacement of the converted wave normalized by the displacement of the incident wave, where flexural waves are described by their radial component and compressional waves are described by their axial component. It is somewhat difficult to interpret these figures due to vastly different wave impedances. However, by comparison of the compressional wave conversion coefficients with those for scatter at a slope discontinuity, Fig. 4-3, the conversion ratios are generally 20 $dB$ less at the ring. The ring conversion ratios are highest near the ring rolling resonance because the rolling motion of the ring is coupled to both shell waves, via rotation for the flexural wave and axial displacement for the compressional wave.
Effect of Sprung Internal Structures

The analysis does not include consideration of any additional modifications to the shell dynamics caused by other internal structures. Guo [25] reports that the imprint of the internals is only visible for light loading. For moderate to heavy loading, the only effect of the internals is the loading at the attachment point, which approaches the behavior of a rigid boundary. The internals are only weakly excited by the shell vibrations due to large impedance differences between the shell and rings. This may be the condition which explains why there are only small differences in the scattering from the internalled shell and ringed shell. Since there can only be weak excitation of the rings, the rings cannot transfer significant energy to the sprung internals.

One potential influence of the sprung internals is to modify the ring dynamics near resonance, either by shifting the location of the resonance in frequency or by damping the ring response. Further analysis in Sec. 5.3.3 indicates that the primary influence of the sprung internals is to damp the ring resonance.

Radiation to the Acoustic Field

I first look at the frequency dependence of the ring radiation in the beam direction. I choose to look in the beam direction because the dipole-like radiation pattern observed in the measured data indicates this to be the peak radiation direction. The beam directed scatter is given by setting $\phi_s = 90^\circ$ in Eq. 4.24, where $\phi_s$ is the radiation direction relative to the shell axis. The radiation coefficients for incident compressional and flexural waves are shown in the beam direction over the experimental frequency range in Fig. 4-13. The radiated pressure is normalized by the incident sound wave pressure.

The expression for the radiated pressure was derived in terms of the displacement amplitudes of an incident free elastic wave. It was necessary, therefore, to use displacement amplitudes properly related to the incident sound wave pressure. In Chap. 3, I estimated the initial response on the endcap due to the arrival of a sound wave. I plot-
ted the displacement of the spherical sector of the endcap as a function of frequency in Fig. 3-23. I use this displacement to be the initial axial displacement amplitude, $u$ of the compressional wave. This amplitude is modified by propagation from the endcap to the ring and for which three factors can be accounted: radiation, scatter at the slope discontinuities, and spreading loss. I neglect the radiation reduction as negligible (This is discussed further in Sec. 5.2). The reduction in amplitude due to scatter can be estimated from the scatter coefficients given by Eq. 4.5. The compressional wave suffers a $5\,dB$ reduction in amplitude due to scatter at the slope discontinuities. I assume the flexural wave is created by conversion of the initial compressional wave at one of the two slope discontinuities (the conversion coefficient is also given by Eq. 4.5). The third factor is spreading loss which accounts for an increase in the shell diameter from the narrow end of the conical shell to the cylindrical shell. The spreading factor is simply $(a_c/a)^{1/2}$.

In Fig. 4-13, the total scattered pressure due to the incident flexural wave is more than $20\,dB$ greater than for incident compressional wave. This may be understood by considering how each of the interaction forces contributes to the radiated pressure. I've decomposed $\hat{w}(k_z)$ into individually forced components based on Eq. 4.25. Substituting individual terms for $\hat{w}(k_z)$ into Eq. 4.24 yields the lower three curves in Fig. 4-13 which show the field in the beam direction is essentially due to the radial interaction force. The contributions to the pressure due to both the axial interaction force and moment are negligible. This is an expected result given the radiation properties of compact multipole sources. For example, the moment may be considered as a pair of normal ring forces applied to the shell in opposite direction (a tesseral quadrupole). Along the line $\phi_s = 90^\circ$, the contributions from this force pair cancel.

The differences in the contribution from the flexural and compressional wave may now be understood by the fact that the displacement field of a flexural wave is predominantly radial while the displacement field of a compressional wave is predominantly axial. From Eq. 4.31, the radial interaction force is coupled only to the radial displacement of the incident elastic wave.
There are two nulls in the pressure field. There is a narrow null at the hoop resonance \((ka = 3.2)\) regardless of wavetype, because the radial interaction force goes to zero at resonance. There is also a broader null at \(ka = 2.7\). The null, however is not related to the ring rolling resonance which occurs at this frequency, rather it is related to a null in the shell compliance function \(s_{22}\).

Since the beam directed scatter may not be entirely representative of the ring radiation characteristics, I’ve plotted the total radiated far field at mid-band \((ka = 7)\) over half of the radiation space \((0 < \phi_s < 90^\circ)\) in Fig 4-14 to explore the angle dependence. The total field is shown in the upper figure and has been decomposed into individual contributions in the lower three figures. The amplitudes are known to be inaccurate when the stationary phase point is near a pole in the dispersion relation [56]. This occurs when the stationary phase point \((k_{zs} = k \cos \phi_s)\) is very close the coincident direction \((\phi_s \sim 74^\circ\) for axisymmetric compressional waves). The inaccuracy causes a narrow spike in the pressure distribution that appears in each of the plots.

Once again the radiation due to incident flexural waves is dominated by contributions from the radial interaction force; however, this in not true for the radiation due to incident compressional waves. The contributions from the ring moment and axial ring force become more significant for \(\phi_s < 90^\circ\) and reach maximum values near \(\phi_s = 45^\circ\) and \(\phi_s = 74^\circ\) respectively. For comparison, the peak value of the contribution from the radial force is at \(\phi_s = 90^\circ\). These maxima can be explained readily. As I stated previously, the ring moment can be represented as a force pair for which a tesseral quadrupole radiation pattern could be expected. Similarly the radial force should radiate either as either a dipole or a lateral quadrupole. I will explore these models in the next paragraph. The axial force yields a radiation peak in the coincident direction or the direction of phase-matched radiation for a compressional wave. In other words the incident elastic wave scatters to a compressional wave via the axial interaction force, and the scattered compressional wave subsequently radiates. Since the incident compressional wave has a primary displacement component
Figure 4-13: Decomposition of far field pressure in the beam direction caused by radiation from a ring for an incident compressional wave (solid curves) and an incident flexural wave (dashed curves). (a) Total field (b) Due to the resulting radial ring force, (c) due to the resulting axial ring force, (d) Due to the resulting ring moment. The ring forces are axisymmetric. The radiated field is normalized by the incident pressure. The amplitudes of the incident flexural and compressional waves are based upon the coupling coefficients estimated in Chap. 3, corrected for transmission losses from the endcap to the first ring.
<table>
<thead>
<tr>
<th>Multipole</th>
<th>Angular Dependence</th>
</tr>
</thead>
<tbody>
<tr>
<td>dipole</td>
<td>$\sin \phi_s$</td>
</tr>
<tr>
<td>quadrupole (axial)</td>
<td>$\sin^2 \phi_s$</td>
</tr>
<tr>
<td>quadrupole (tesseral)</td>
<td>$\sin 2\phi_s$</td>
</tr>
</tbody>
</table>

Table 4.1: Radiation patterns for compact multipole sources

In the axial direction, the axial force dominates the interaction. The result is that the ring radiation from a incident flexural wave exceeds that of a compressional wave at all radiation angles except within 5° of the coincidence direction. To get a sense of the importance of these radiation levels relative to the measured data, consider that a pressure level in the frequency domain of -62 dB re 2m corresponds to a peak level of -80 dB re 2m in the time domain for a Gaussian pulse, $2.75 < ka < 10$. In other words these contributions are observable in the measured scatter but weak compared the coincident radiation and particularly compared with radiation from the endcaps. To appreciate this, consider the bistatic time domain contours in Fig. 2-19 or 2-20. Along the coincident observation angles ($\phi_s = 74, 106^\circ$) the peak levels are near -72 dB re 2m, whereas the peak levels at 90° are closer to -81 dB re 2m. These contributions both weak relative to the peak levels on the shell axis $\phi_s = 0^\circ$, where peak levels approach -60 dB re 2m.

The radiation patterns associated with the radial ring force and moment appear to be related to compact multipole radiation patterns. This is relatively simple to check. The angle dependence for three compact multipole sources is shown in Table 4.1 [40]. In Fig. 4-15, I compare these angle dependence terms with the radiation patterns normalized to a peak value of one (0 dB). In the upper figure, the dipole pattern is more representative of the radiation from a normal ring force than an axial quadrupole. In the lower figure, the tesseral quadrupole is representative of the radiation pattern from a ring moment.
Figure 4-14: Decomposition of far field pressure at $ka = 7$ caused by radiation from a ring for an incident compressional wave (solid curves) and an incident flexural wave (dashed curves). (a) Total field (b) Due to the resulting radial ring force, (c) due to the resulting axial ring force, (d) Due to the resulting ring moment. The ring forces are axisymmetric. The radiated field is normalized by the incident pressure. The amplitudes of the incident flexural and compressional waves are based upon the coupling coefficients estimated in Chap. 3, corrected for transmission losses from the endcap to the first ring.
Figure 4-15: Ring radiation patterns due to an applied axisymmetric radial ring force and ring moment. The radial ring force radiation pattern (solid) is compared with a dipole pattern $\sin \phi_s$ (dashed) and a lateral quadrupole pattern $\sin^2 \phi_s$ (dash-dot). The ring moment radiation pattern (solid) is compared with a tesseral quadrupole radiation pattern $\sin 2\phi_s$ (dashed).
4.3.6 Ring Interaction Model - Multiple Rings

The effects on wave propagation of spatially periodic structures attached to a shell or plate is well documented. Where the structural wavelengths are the same or small compared with the spacing of the periodic structures, alternating pass and stop frequency bands are formed. Free elastic wave propagation occurs in the passbands, while only evanescent waves exist in the stop bands. This phenomenon has been observed for 1D systems [57], for plates [58], and for cylindrical shells in vacuo [55][54] and with fluid loading [59].

An illustrative 1D example is a beam with attached masses supporting compressional wave propagation [57]. Consider a beam with thickness equal the shell thickness and with periodically-spaced attached masses having the same cross-section as the rings but symmetrically rather than eccentrically attached to the beam. The spacing of the masses for this example is equal to the spacing between the first and second shell rings \((l = 23.5cm)\). This is a standard problem [57] for which the inverse of the transmission coefficient is given by

\[
\frac{1}{|t|} = \cos k_p l - \frac{m}{2m'} k_p l \sin k_p l ,
\]

(4.33)

where \(m\) is the mass of each periodic mass, \(m'\) is the mass per unit length of the beam, and \(k_p\) is the compressional wavenumber. Wherever this expression yields \(|t| \geq 1\), it corresponds to a passband of the periodic system. Insertion of the shell parameters in Eq. 4.33 yields the transmission curve shown in Fig. 4-16. The transmission through a single mass is compared and shows that the effect of the periodically spaced rings is to create passbands even though the transmission through a single ring is small. These passbands are located in wavenumber by the relationship

\[
k_p l = n\pi .
\]

(4.34)

Thus, the passbands occur where an integer number of compressional half-wavelengths equals the ring spacing. Between the passbands exist stopbands in which the local
Figure 4-16: Transmission coefficient for compressional waves on a bar with spatially periodic masses (solid line). The spacing is 23.5\textit{cm} and the relative mass of the bar and the periodic masses are equal to those of the shell and the rings. The frequency scale assumes normalization by the radius of the cylindrical shell. The transmission through a single mass (dashed line) is compared.

The minimum in the transmission coefficient is 6\textit{dB} less than that for a single ring.

The rib spacing for the internally loaded shells is clearly not spatially periodic as seen in Fig. 1-3. The coupling between adjacent rings continues to remain important, but the irregularity modifies the stop and passband behavior of a periodic system. The irregularity causes localization of the structural wave energy where the strength of the localization depends on the width of the passbands and the frequency shifts associated with the irregularity [60][61][59]. The phenomenon of energy localization is referred to as \textit{Anderson} localization and is a coherent phenomenon that gives rise to exponential spatial energy decay even in the passbands. Strong localization occurs where the ratio $T/R$ is low, where $T$ and $R$ are the transmission and reflection coefficients for a single ring.

To understand the coherent interaction of multiple rings and the effects of localization due to irregularity, I formulate the problem in the following manner. I first derive the expression for the coupling of two rings separated by the length of the first shell bay, 23.5\textit{cm}. I then consider the additional influence of an third ring separated from the first two by the length of the second shell bay, 19.0\textit{cm}.

For the problem of two rings, I consider the amplitude just beyond the first ring as the sum of the first compressional wave transmission plus multiple reflections between
the first and second ring. This can be stated as

\begin{align*}
v_t &= v_i T_{rpp} + v_i R_{rpp}^2 T_{rpp} e^{2ikpl} + v_i R_{rpp}^4 T_{rpp} e^{4ikpl} + \ldots, \\
v_t &= v_i T_{rpp} \sum_{n=0} \frac{R_{rpp}^{2n} e^{2nkpl}}{1 - (R_{rpp} e^{ikpl})^2},
\end{align*}

(4.35)

where \(v_t\) is the transmitted wave amplitude, \(v_i\) is the incident wave amplitude and \(e^{2nkpl}\) accounts for the phase accumulation for a compressional wave round trip within the first bay. \(k_p\) is the compressional wavenumber which is complex to account for radiation damping and \(l\) is the distance between rings. Eq. 4.35 can be written as

\[
\frac{v_t}{v_i} = T_{rpp} \left\{ \frac{1}{1 - (R_{rpp} e^{ikpl})^2} \right\}.
\]

(4.36)

In Eq. 4.36 it is seen that the new transmission coefficient \(v_t/v_i\) is given by the single ring coefficient modified by a term controlled by the spacing of the rings and the reflection coefficient. Insertion of the single ring reflection and transmission coefficients into Eq. 4.36 gives the coefficients in shown Fig. 4-17. The flexural wave transmission is found in an identical manner using the flexural wavenumber and reflection coefficients. In this case, the flexural wavenumber is modified to include structural damping of \(\eta = .001\). In comparison with the 1D model of a periodic system, the passbands for the two ring system lie in nearly the same frequency ranges. The minimum transmission coefficients which lie between the passbands are again 6 dB below the single ring coefficients. The passbands are more closely spaced in the case of flexural waves because the wavelengths are substantially shorter.

The reflection from a pair of rings is given by a similar formulation. The coefficient is equal to the sum of the first reflection plus the reflection from the second ring plus multiple reflections from within the first bay. The final expression is given by

\[
\frac{v_r}{v_i} = R_{rpp} \left[ 1 + T_{rpp} \left\{ \frac{1}{1 - (R_{rpp} e^{ikpl})^2} \right\} \right].
\]

(4.37)

Once again the reflection coefficient is the single ring coefficient modified by a function which depends on the distance between the rings. The modified reflection coefficients
Figure 4-17: Transmission coefficients for compressional and flexural waves on a cylindrical shell with two rings (solid line). The two rings are spaced 23.5 cm apart. The transmission through a single ring (dashed line) is compared.
Figure 4-18: Reflection coefficients for compressional and flexural waves on cylindrical shell with two rings (solid line). The two rings are spaced 23.5 cm apart. The reflection from a single ring (dashed line) is compared.

are plotted in Fig. 4-18.

Next, a third ring is included in the analysis to assess its influence. An expression for the reflection and transmission coefficients at each of the rings can be derived in a similar manner. To simplify the expressions, higher order terms in $T$ are discarded. The expression for transmission through the first ring that results from the same type of derivation as Eq. 4.36 is

$$\frac{v_t}{v_i} = T_{rpp} \left[ \frac{1}{1 - (R_{rpp} e^{ikp l_1})^2} \right] \left\{ 1 + T_{rpp}^2 \left[ \frac{(R_{rpp} e^{ikp l_2})^2}{1 - (R_{rpp} e^{ikp l_2})^2} \right] \right\}. \quad (4.38)$$

Here $l_1$ is the length of the first bay and $l_2$ is the length of the second bay. This expression is simply that given by Eq. 4.36 plus higher order terms in $T_{rpp}^2$, where only additional terms of order $T_{rpp}^2$ are included. The transmission coefficient through the first ring based on the coupled three ring model is shown in the upper curve of

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Figure 4-19: Transmission coefficients for compressional waves on cylindrical shell with three rings (solid line). The upper figure is the transmission through the first ring and the lower figure is transmission through the first and second ring. The two rings are spaced $l_1 = 23.5 \text{cm}$ and $l_2 = 19.0 \text{cm}$. The coefficients for incoherent transmission through one and two rings (dashed line) are compared.

Fig. 4-19. This curve is essentially identical to that shown is Fig. 4-17 and shows the third ring is only weakly coupled to the energy in the first bay.

A similar expression can be derived for transmission through the second ring. The previous analysis shows that is sufficient to retain only those terms of the same order in $T_{pp}$. The resulting expression for transmission through the first two rings is thus

$$\frac{v_l}{v_i} = T_{pp}^2 \left\{ \frac{1}{1 - (R_{pp}e^{ikl_2})^2} \right\} \left\{ \frac{e^{ikl_1}}{1 - (R_{pp}e^{ikl_1})^2} \right\}. \quad (4.39)$$

The curve of transmission through the second ring shows that the irregularity of the ring spacing has strongly diminished the compressional wave transmission. The high transmission bands of the first ring due to the bay resonance are essentially removed.
The \( ka = 5 \) transmission band remains but with a strongly reduced peak transmission coefficient. The additional peak at \( ka = 6.4 \) reflects a resonance in the second bay. Note that the high transmission near the ring resonance frequencies is not affected by the irregular ring spacing.

**Summary of Ring Scattering Results**

Overall the ring analysis shows that the most prominent characteristic of the rings is reflection of incident compressional waves which then radiate to the far field primarily in the coincident radiation direction. In terms of the finite shells studied here, these compressional waves also propagate to the endcaps where they radiate even more efficiently as seen in the Chap. 2 bistatic data. It is also important, however to consider the effects of multiple rings which are strongly coupled and create *passbands* not identified by the single ring analysis. These passbands permit energy to reach the subsequent rings. The irregularity of the ring spacing combined with the narrow passband bandwidths cause strong localization of these passbands within the first pair of rings. In Chap. 5, beamforming analysis reveals that both the passbands and the localization can be observed in the measured data.

The ring radiation due to an incident flexural wave is a relatively weak process compared with the coincident radiation, although the radiation lobe is much broader in angle. The ring radiated pressure levels due to an incident flexural wave are most important in the beam direction and follow a dipole-like radiation pattern. The peak radiation from the interaction moment is 20 \( dB \) down from the peak radiation due to the radial interaction force, and the angular distribution of pressure follows a lateral quadrupole-like radiation pattern. No study was made of the radiation from the rings due to coherent interaction. It is known that the coherent interaction of multiple rings can cause strong Bloch wave radiation [62]. An investigation of the monostatic frequency domain data for the ringed and internalled shells (see Corrado [9] pages 252 and 253) does not reveal strong Bloch wave radiation features. Two factors may explain this. First the number of rings is small; therefore, the enhancement of the
radiation due to multiple scatterers may be small. Second the irregular spacing of the rings has been seen to cause the rings to couple in pairs causing localization of energy within single bays; therefore, there may not be coherent radiation from all of the rings.

4.4 Scatter at an Eccentric Ring: Temporal Analysis for Multiple Rings

The ring interaction analysis of Sec. 4.3.5 is most easily interpreted in the frequency domain, where it is apparent that regions of high transmission exist near resonant modes of the attached structure. What is of interest here is interpreting these transmission and reflection coefficients in the time domain. This is a rather simple exercise, which involves multiplying the scatter coefficients by some filter and transforming the product to the time domain.

I generated several waveforms using Fourier synthesis to study the transient response. These include the three narrowband pulses \((2 < ka < 5), (5 < ka < 8),\) and \((8 < ka < 11),\) and a broad band pulse \((2.75 < ka < 10).\) The lowest frequency band \((2 < ka < 5)\) includes the resonance frequencies in the ring model as does the broad band signal. I illustrate the scatter of a transient wave by plotting the time histories of the incident elastic wave field and each of the scattered wave fields. An axisymmetric elastic wave scatters to four distinct elastic waves at the ring: (1) a reflected compressional wave (2) a transmitted compressional wave (3) a reflected flexural wave and (4) a transmitted flexural wave.

The scatter of a transient incident compressional wave for the lower frequency band is plotted in Fig. 4-20. This is the frequency range which has the highest transmission coefficients, \(T_{rpp}\). The magnitude of the reflected and transmitted compressional waves, shown in the middle two figures, are approximately equal, although their shape is different. The higher rate of zero crossings near \(t = 0\) in the reflected wave indicates that this wave is comprised of higher frequency energy than the transmitted wave, as we should expect given the frequency of highest transmission, \(ka = 2.6,\)
is well below the center frequency of this pulse, $ka = 3.5$. The ring has effectively partitioned the energy on a frequency selective basis and as a result, spreading of the pulse has taken place. The spreading of the pulse can be described by the combination of energy temporarily stored in the motion of the ring which subsequently couples to the shell waves and energy which reflects back and forth within the subsequent bay. The former is effectively the ring-up time or equivalently the ring-down time for a 1-D resonant system. This time is inversely proportional to the $Q$-factor of the resonance and proportional to its bandwidth. The ring down time of the resonance can also be interpreted as radiation loss of energy in the ring motion coupling to propagating elastic waves in the shells. Thus, the loss factor is given by

$$\eta_r \approx \Delta f_{3dB}/f_o , \quad (4.40)$$

where $\Delta f_{3dB}$ is the bandwidth of the resonance, and $f_o$ is the resonance frequency. Thus the envelope of ring-down can be described by the expression:

$$e(t) \sim \exp\{\eta_r\omega t\} . \quad (4.41)$$

I can make similar comments regarding the scatter of a incident transient flexural wave for the lower frequency band, which is plotted in Fig. 4-21. Here again the energy is partitioned such that the lower frequency energy in the incident pulse is more efficiently transmitted than the higher frequency energy. The flexural wave is less efficiently transmitted than the compressional wave.

I have plotted the amplitudes of the scattered transient compressional waves in the next higher frequency band in Fig. 4-22. These results are different from those in the lower frequency band. First, the amplitude of the transmitted compressional wave is significantly less than the amplitude of the reflected compressional wave. In fact, most of the scattered energy is contained in the reflected compressional wave which is not distorted with respect to the incident pulse. This indicates that the ring behaves approximately as a rigid boundary with respect to reflection of the incident wave. The
Figure 4-20: Timeseries of the elastic waves scattered by a ring of mass plus stiffness for an incident transient compressional wave ($2 < ka < 5$). Amplitudes are normalized by the peak magnitude of the incident wave.
Figure 4-21: Timeseries of the elastic waves scattered by a ring of mass plus stiffness for an incident transient flexural wave ($2 < k\lambda < 5$). Amplitudes are normalized by the peak magnitude of the incident wave.
Figure 4-22: Timeseries of the elastic waves scattered by a ring of mass plus stiffness for an incident transient compressional wave ($5 < ka < 8$). Amplitudes are normalized by the peak magnitude of the incident wave.

energy which is transmitted is continually reflected back and forth in the second bay, where the observed period ($t \sim 90 \mu \text{s}$) is the time for a compressional wave to make one round trip within the second bay. This frequency band is far enough above the ring rolling resonance that the interaction is mass controlled and no energy can be stored in the ring.

The same results are seen for flexural wave incidence and also at the higher frequency band. Rather then reproducing those figures here, I have a summary of each of the scattered wave energies relative to the incident wave energy listed in Table 4.2 and Table 4.3. These coefficients will prove useful in Chap. 5, where I analyze the measured data.
Figure 4-23: Timeseries of the elastic waves scattered by a ring of mass plus stiffness for a broadband incident transient compressional wave \((2.75 < ka < 10)\). Amplitudes are normalized by the peak magnitude of the incident wave.

Finally, I have plotted the amplitudes of the scattered broadband transient compressional waves in Fig. 4-23. The incident wave is primarily reflected. There is a small amount of distortion which is associated with the features observed in the lower frequency band. The periodic reflections in the subsequent bay are again easily observed.

In summary, the ring of mass plus stiffness primarily reflects the incident transient wave. In the lower frequency band, there can be significant time spreading due to energy being stored in the motion of the ring. The time spreading is proportional to the bandwidth of the resonance. As expected, the incident elastic waves are most efficiently transmitted in the lower frequency band which includes the ring resonances.
<table>
<thead>
<tr>
<th>Frequency Band</th>
<th>Reflected Compress. $dB$</th>
<th>Transmitted Compress. $dB$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.75 &lt; ka &lt; 10$</td>
<td>-1</td>
<td>-8</td>
</tr>
<tr>
<td>$2 &lt; ka &lt; 5$</td>
<td>-3</td>
<td>-7</td>
</tr>
<tr>
<td>$5 &lt; ka &lt; 8$</td>
<td>-0</td>
<td>-11</td>
</tr>
<tr>
<td>$8 &lt; ka &lt; 11$</td>
<td>-0</td>
<td>-18</td>
</tr>
</tbody>
</table>

Table 4.2: Relative energy of the elastic compressional waves scattered by multiple rings of mass and stiffness due to an incident compressional wave. The levels represent the ratio the scattered wave energy to the incident compressional wave energy.

<table>
<thead>
<tr>
<th>Frequency Band</th>
<th>Reflected Flexural $dB$</th>
<th>Transmitted Flexural $dB$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.75 &lt; ka &lt; 10$</td>
<td>-1</td>
<td>-10</td>
</tr>
<tr>
<td>$2 &lt; ka &lt; 5$</td>
<td>-2</td>
<td>-6</td>
</tr>
<tr>
<td>$5 &lt; ka &lt; 8$</td>
<td>-1</td>
<td>-12</td>
</tr>
<tr>
<td>$8 &lt; ka &lt; 11$</td>
<td>-2</td>
<td>-13</td>
</tr>
</tbody>
</table>

Table 4.3: Relative energy of the elastic compressional waves scattered by multiple rings of mass and stiffness due to an incident compressional wave. The levels represent the ratio the scattered wave energy to the incident flexural wave energy.
The effect of the subsequent rings is to trap the transmitted waves so that a resonance builds up between the first and second ring. The resonances correspond to the peaks seen in the transmission curves for a multiple ring system.

4.5 Summary

In this chapter two wave conversion mechanisms were analyzed. These included scatter of a normally incident plate wave at a slope discontinuity and scatter of an axisymmetric shell wave at an elastic ring attached to a cylindrical shell. Compressional and flexural waves were the only excitation mechanisms considered. The predominant reaction at the slope discontinuity is transmission of the incident wave while the predominant reaction at the ring is reflection of the incident wave.

The approximate partitioning of energy at a slope discontinuity is 55% transmitted (-3 dB), 10% reflected (-10 dB) and 35% (-5 dB) converted to the other wavetype. At the ring stiffener generally more than 80% (-1 dB) of the wave energy is reflected. This analysis indicates that the slope discontinuity is a far more efficient mechanism for wave conversion.

Elastic properties were included in the ring dynamics and it was found that these only modified the scattering coefficients near resonance frequencies of the ring. There are two ring resonance frequencies in the measurement band and both are near the cylinder ring frequency. Near resonance, the ring is weakly coupled to the shell motions and the structural wave are efficiently transmitted. Transient analysis shows that these resonances must be charged up, where the charge up time is proportional to the coupling between the ring and the shell. The time duration of excitation pulses studied are generally much shorter than these characteristic times. Therefore, there must be a reaction at the ring due to the arrival of a shell wave. The reaction distorts the pulse shape by transferring energy to the ring motion which subsequently radiates back into the shell. The pulse is further distorted by coupling between successive rings where the coupling creates frequency bands of high transmission or pass bands. The pass bands

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are located in frequency where an integer number of structural half-wavelengths equal the ring spacing. The irregular spacing of the rings causes strong localization of the structural waves except near the ring resonance frequencies.

Acoustic radiation was neglected in the slope interaction model, but was considered for the ring interaction. Compressional waves contribute most effectively via radiation in the coincident direction. The acoustic radiation from the ring is weak compared with coincident radiation. In the case of flexural waves, the acoustic radiation from the ring is a stronger mechanism for coupling to the acoustic field than flexural wave scatter to radiating compressional waves. The analyses neglected the effect of coherent radiation from multiple rings which has been shown to cause Bloch wave radiation [62]; however, Bloch wave radiation is not considered to be important for these shells.

4.5.1 Comparison with Transmission Line Estimates

The same scatter coefficients were estimated by Bondaryk [30] using a transmission line model of the axisymmetric wave propagation. The coefficients were derived to match the beamformer output for each shell. Scatter coefficients were estimated at a ring stiffener and at a slope discontinuity. The assumptions of the transmission line analysis are

- The scatter coefficients are independent of frequency

- The scattered waves have either the same sense or opposite sense with respect to the incident wave, i.e. no phase information is retained.

Here I present a comparison of the coefficients extracted from the data by this process and the coefficients derived in this chapter. The main conclusions of this comparison are that each approach shows that the rings primarily reflect the elastic waves while the slope discontinuities primarily transmit the elastic waves.
Scatter at a slope discontinuity

The results for the scatter at a discontinuity are presented by Bondaryk using the format

\[
R = \begin{bmatrix}
R_{pp} & R_{pf} \\
R_{fp} & R_{ff}
\end{bmatrix}, \quad T = \begin{bmatrix}
T_{pp} & T_{pf} \\
T_{fp} & T_{ff}
\end{bmatrix}.
\] (4.42)

Here the coefficients are related to the ratio of the scattered and incident wave power, such that

\[
R_{pp}^2 + R_{pf}^2 + T_{pp}^2 + T_{pf}^2 = 1,
\]
\[
R_{ff}^2 + R_{fp}^2 + T_{ff}^2 + T_{fp}^2 = 1.
\] (4.43)

I use the decibel values of these coefficients for the comparison. The calculated values for the coefficients derived in Chap. 4 are averaged over the measurement frequency band \(2 < ka < 11\). The calculated coefficients are

\[
R \simeq \begin{bmatrix}
-10 & -9 \\
-7 & -13
\end{bmatrix} \text{dB}, \quad T \simeq \begin{bmatrix}
-2 & -7 \\
-9 & -2
\end{bmatrix} \text{dB}.
\] (4.44)

The transmission line results are

\[
R_{TL} \simeq \begin{bmatrix}
-9 & -8 \\
-14 & -14
\end{bmatrix} \text{dB}, \quad T_{TL} \simeq \begin{bmatrix}
-3 & -8 \\
-8 & -1
\end{bmatrix} \text{dB}.
\] (4.45)

From the comparison, I conclude that the scattering model is strongly consistent with the transmission line results. That is, the elastic wave energy is predominantly transmitted through the discontinuity. In general the estimates differ by only 1 dB. The only major discrepancy is in the conversion coefficient \(R_{fp}\), for which the transmission line model estimates a much lower value -14 dB than the scattering model -7 dB. A weakness of the experiment is that flexural waves cannot be observered directly and must be inferred by scatter to radiating waves. Therefore errors in the flexural wave estimates based on the data are most likely. I conclude that the comparison
represents very good agreement between two strongly different approaches for estimating the scatter and provides confidence that these coefficients properly represent the slope discontinuity scatter. Further comparisons of the reflection and transmission coefficients at the slope discontinuities between model and data are made in Chaps. 5 and 6 that also show good agreement.

Scatter at a ring of mass and stiffness

The same format is used to present the scatter at a ring stiffener. Here I present the coupled ring scatter coefficients averaged over the frequency band $2 < ka < 11$. I did not investigate the effect of multiple ring coupling on conversion between wavetypes, thus these coefficients are not provided. The calculated scatter coefficients averaged over the measured frequency band are

$$ R \simeq \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix} dB, \quad T \simeq \begin{bmatrix} -8 & 0 \\ -8 & 0 \end{bmatrix} dB. \quad (4.46) $$

The results from the transmission line model for the data are

$$ R_{TL} \simeq \begin{bmatrix} -3 & -10 \\ -10 & -3 \end{bmatrix} dB, \quad T_{TL} \simeq \begin{bmatrix} -6 & -10 \\ -10 & -6 \end{bmatrix} dB. \quad (4.47) $$

The transmission line results show that the data concur with the ring/shell interaction model in that the rings primarily reflect the incident elastic waves. The differences are 2 dB. The transmission line results suggest better transmission through the ring than that calculated by the model. This may indicate a weakness in the assumptions of the ring/shell interaction model. An assumption of the model is that the ring and shell are clamped, which requires that the ring displacement equals the shell displacement at the point of attachment. It is possible that locally the attachment condition between the ring and shell is more flexible than allowed by the model. A more flexible connection would permit more efficient transmission.
Chapter 5

Wave Propagation on the Shells

5.1 Overview

In this chapter, I examine how the surface discontinuities and internal structures affect the distribution elastic wave energy on the shells. I use beamforming to project the bistatic scatter to the shell surface and reveal the time evolution of the flow of energy along the length of each shell.

Before interpreting the scatter data, I perform a frequency-wavenumber analysis of elastic wave propagation for each of the three shell geometries that comprise the target shells. From the analysis, I obtain the phase speed, group speed and attenuation properties of axisymmetric compressional and flexural waves on cylindrical, conical and spherical shells.

I explore how the wave processes evolve over time with the bistatic beamforming. The resulting distributions illustrate the excitation and scattering mechanisms described in Chap. 3 and Chap. 4, for example the forward propagation of the initially excited compressional wave and its subsequent reflection at the first ring. The distributions reveal that axisymmetric compressional waves dominate the early shell response while axisymmetric flexural waves dominate the later shell response. The transition time is seen to be related to the typical distance between structural discon-
tinuities and the average group speed of the flexural waves.

For the internally-loaded shells, the rings substantially modify the axial distribution of pressure on the shell surface by inhibiting the transmission of elastic waves to the far endcap. As a result, the elastic waves are predominantly contained within the first two shell bays that extend from the insonified endcap to the second ring. Better transmission occurs in the lower frequency band, which contains the ring resonance frequencies.

To understand the propagation mechanisms near bow incidence, I examine the empty shell and internalled shell bistatic scatter at $\phi_i = 25^\circ$. I find that the same axisymmetric wave processes continue to be important. In addition there is evidence that the forced elastic wave which travels along the shell at the trace axial speed of the incident sound wave, scatters to free elastic waves and radiates at each ring.

## 5.2 Frequency Wavenumber Analysis

Before returning to the measured data, I consider the frequency and wavenumber properties of axisymmetric shell waves. The wavenumber roots of the fluid loaded dispersion relations are necessary to study the propagation and attenuation properties of these elastic shell waves. The fluid loaded axisymmetric dispersion relations can be found in Appendix A for a cylindrical shell (Eq. A.9), a conical shell and a spherical shell (Eq. A.17).

The dispersion equations are given in the form $D(\omega, \nu) = 0$, where $\nu$ is complex wavenumber which can be determined as a function of real frequency $\omega$. The complex wavenumber roots have the form $\nu = \nu_r + i\nu_i$, where the real part corresponds to elastic wave propagation and the imaginary part corresponds to attenuation. The phase speed is given by

$$C_p = \omega / \nu_r .$$  \hspace{1cm} (5.1)
The group speed is given by

\[ C_g = \frac{d\omega}{d\nu_r} . \]  \hspace{1cm} (5.2)

The spatial attenuation is given in decibels by

\[ \Delta_l = 8.69\nu_i \frac{dB}{m} . \]  \hspace{1cm} (5.3)

To get to the temporal decay rate, the phase speed (Eq. 5.1) must be factored into Eqn. 5.3 to give

\[ \Delta_t = 8.69\omega\nu_i/\nu_r \frac{dB}{s} . \]  \hspace{1cm} (5.4)

Cylindrical Shell

The axisymmetric dispersion equation for a fluid loaded thin cylindrical shell has the form \( D(\omega, k_z) = 0 \) where \( k_z \) is complex axial wavenumber, \( k_{zr} + ik_{zt} \). The dispersion relation was derived in Appendix A and is given by

\[ (1 + \beta^2\alpha^4 - \Omega^2 + F_u E_f k^2 a^2)(-\alpha^2 + \Omega^2) + \nu^2\alpha^2 = 0 . \]  \hspace{1cm} (5.5)

Here \( \alpha = k_z a, \Omega = \omega a/c_p, \beta^2 = h^2/12a^2 \) and

\[ F_u = \frac{H_0(k_r a)}{k_r a H_0'(k_r a)} , \quad E_f = \frac{c_p^2 \rho_o a}{c_p^2 \rho_s h} , \quad k_r = \left( k_o^2 - k_z^2 \right)^{1/2} . \]  \hspace{1cm} (5.6)

The phase and group speeds for axisymmetric compressional waves are found by replacing \( \nu_r \) in Eqs. 5.1 and 5.2 with the real part of the axial compressional wavenumber \( k_z \). The derivatives were estimated numerically by employing a differencing scheme. The axisymmetric compressional waves are essentially non-dispersive over the frequency band, \( 2 < ka < 12 \) as shown in the top two plots of Fig. 5-1. The temporal attenuation on the cylindrical shell is given by Eq. 5.4 as a function of frequency. The result is shown in the bottom plot of Fig. 5-1.

The same approach is used to find the phase and group speeds of the flexural waves
Figure 5-1: Frequency wavenumber properties of axisymmetric compressional waves on a cylindrical shell (a) phase speed (b) group speed and (c) attenuation
Figure 5-2: Frequency wavenumber properties of axisymmetric flexural waves on a cylindrical shell (a) phase speed (b) group speed.

on the cylindrical shell, where now I solve for the propagating flexural wavenumber roots, which are real because the flexural waves are subsonic. The flexural wave phase and group speeds are plotted in Fig. 5-2. There is necessarily some damping from mechanical losses, however no material damping was included in the equations of motions.

Conical Shell

The equations of motions for a thin conical shell with small vertex angle have been developed by Guo [47][63] to investigate membrane wave propagation. The fluid loaded conical dispersion relation is given as

\[ [1 + \beta^2 \alpha(s)^4 - \Omega^2 + F_u E_f k^2 a(s)^2 \{ -\alpha(s)^2 + \Omega^2 \} + \nu^2 \alpha(s)^2 = 0 \]  

(5.7)
where $s$ is a small parameter related to the axial position along the conical shell, $s(z) = z \tan \alpha_c$, where $z$ is axial position and $\alpha_c$ is the vertex angle of the conical shell (see Fig. 5-3). Guo's results are valid for $\alpha_c$ small. Also $a(s) = a_o + s$ is the local radius, where $a_o$ is the reference radius; here I use $a_o$ equal to the cylinder radius, $a$. Because the dispersion relation depends on axial position, it must be solved locally as function of $s$.

Guo [63] shows the axisymmetric compressional waves on the conical shell are essentially non-dispersive, with phase and group speeds approximately equal to those of the cylindrical shell. He also shows the attenuation is small over the length of the conical endcap (less than 1 $dB$ for a round trip on the conical shell.) For the purpose of further discussion, I assume that the compressional wave speeds and attenuation rates are the same as those for the cylindrical shell.

I have extended the conical shell analysis to study axisymmetric flexural wave propagation. The dispersion curves for flexural waves at five radii $a(s)$ equally spaced along the target conical shell are plotted in Fig. 5-4. The curves show the local dispersion properties at the cone/cylinder junction, the cone/sphere junction and three locations between. I have determined the group speeds by replacing $k_{zr}(s)$ for $\nu_r$ in Eq. 5.2 and applying a differing scheme. The group speed curves are plotted at the same uniformly spaced axial locations in Fig. 5-5. Once again the flexural roots are real and subsonic.
Figure 5-4: Dispersion curves for axisymmetric flexural waves at uniformly spaced locations along a conical shell. The upper curve represents the dispersion properties at the cone/cylinder junction and the lower curves are shown for decreasing local radii. The lowest curve is for the cone/sphere junction. The intermediate curves are spaced uniformly in $a(z)$.

Figure 5-5: Group speed vs. normalized frequency for flexural waves on a conical shell. The curves represent the group speed at the same axial locations plotted in Fig. 5-4.
Figure 5-6: Monostatic bandlimited spherical shell impulse response for three frequency bands: $2 < ka < 5$ (solid), $5 < ka < 8$ (dashed), $8 < ka < 11$ (dash-dot). The sphere has parameters which are identical to those of the spherical sector of the endcap.

Decay Rates for Compressional Waves on the Spherical Shell

To find the decay rates for the compressional waves which propagate on the spherical section of the endcap, I perform an analysis in the time domain by calculating the scattered pressure for a spherical shell and obtaining the decay rates in different frequency bands. This provides an average decay rate over each band.

The modal series representation of the scattered pressure was given by Eq. 3.2, from which I previously calculated the backscatter from a spherical shell in 3 frequency bands: lower-$ka$ ($2 < ka < 5$), mid-$ka$ ($5 < ka < 8$), and higher-$ka$ ($8 < ka < 11$). These figures are reproduced over an increased dynamic range and temporal scale in Fig. 5-6, to show the decay rates. Based on Fig. 5-6, it is not possible to discuss the decay rate for the lower-$ka$ because the process is dominated by flexural wave radiation for $t > 100\mu s$. This is not surprising because spherical shell phase speed curves (Fig. 3-22) show that the compressional waves are cut off for $ka < 5$.

For the mid- and higher-$ka$ bands, the decay rates are evident. A series of decaying peaks indicate repeated circumnavigations of the compressional wave. The spacing of
these peaks, $t = 62 \mu s$, corresponds to an average group speed of $4700 \text{m/s}$, which is slightly below the group speed for compressional waves on the cylindrical shell ($5270 \text{m/s}$). The decay rates for the mid- and higher-$ka$ bands are $175 \text{ dB/ms}$ and $150 \text{ dB/ms}$ respectively. Transforming these rates to radiation loss for propagation across the span of the spherical sector of the endcap gives $\Delta = 2.4 \text{ dB}$ for mid-$ka$ and $\Delta = 2.0 \text{ dB}$ for higher-$ka$.

**Overview of Wave Propagation Properties**

I have reviewed the dispersion properties of the axisymmetric compressional and flexural waves on the various shell geometries. A few general conclusions can be drawn from these results.

The reduction in radius associated with the conical shell does not have an important effect on compressional wave propagation, hence the compressional wave dispersion properties on the cylindrical and conical shell are essentially the same.

Relative to the cylindrical shell, the conical shell does have an important effect on the flexural wave propagation, which is strongly dependent on both frequency and local radius. The reduction in radius causes a shift in the frequency dependent minimum group speed, which is located slightly below the local ring frequency, given by $f_r(s) = c_p/2\pi a(s)$.

The compressional waves on the spherical shell are highly dispersive in this frequency range as was shown in Chap. 3, in contrast with propagation on the conical and cylindrical shells. But at higher frequencies ($ka > 10$) these waves approach the same propagation speeds found on the conical and cylindrical shells. At lower frequencies the compressional wave appears to approach a cutoff although this phenomenon is not studied here.

A comparison of the attenuation rates for each of the three geometries indicates that radiation from the spherical shell accounts for more than half of the total shell radiation. This can made clear by the considering the reduction in compressional
wave amplitude due to radiation for one circumnavigation of the empty shell. Compressional waves have similar propagation characteristics on both the conical shell and cylindrical shells, so that these surfaces can be considered as the same; thus, one circumnavigation of the empty shell involves 170cm of equivalent cylindrical shell and 13cm of spherical shell. At mid-band, the compressional wave decay rate on the cylindrical shell is 9 dB/ms and the group speed is approximately 5300m/s; thus the reduction in one circumnavigation due to radiation from the cylinder is approximately 3 dB. In comparison the radiation reduction for compressional waves on the two spherical caps is approximately 4-5 dB.

5.3 Beamforming Analysis at $\phi_i = 0^\circ$

5.3.1 Beamformer

The bistatic scatter has delay information which can be exploited using conventional beamformer processing techniques to determine the equivalent source distribution on the shell surface. I use a method previously used on these data [30][9][31] that is simply described as delay and sum focusing. A spatial sector of the bistatic scatter is windowed in space and back-propagated to the shell surface using the receivers as a synthetic array

$$p_{\text{surf}}(t) = \sum_{n=1}^{N} G_n r_{mn} p_{en}(t - r_{mn}/c_o),$$

(5.8)

where $r_{mn}$ is the path length from shell location $m$ to receiver $n$, $p_{en}$ is the time series of the scattered pressure at receiver $n$ that corresponds to the elastic response, $G_n$ is the array weighting function and $N$ is the number of receivers used. The beamformer output is a time history of the spatial distribution of pressure on the shell surface. The pressure distribution corresponds to radiating wave processes including compressional wave propagation and localized radiation, for example from a ring. Non-radiating flexural wave propagation cannot be seen in the output, but can be inferred by its scatter to compressional waves and radiation at shell discontinuities. Only the energy
which is directed in the aperture of the receiving array can be represented by the processing.

For most presentations, I use a 60° aperture, $60^\circ \leq \phi_s \leq 120^\circ$, to include contributions from leaky wave radiation in both the forward and backward directions with the resulting emphasis on beam-directed scatter. The sensors are uniformly weighted over the central 30° and Hanning tapered at the ends to control sidelobes. In Chap. 2, I showed that scatter outside this region can generally be attributed to endcap radiation, which has its main lobe at the shell axis, $\phi_s = 0^\circ$. Thus the endcap radiation does not contribute to the beamformer response. So, while the beamformer provides a means to visualize the flow of energy between the endcaps, it exploits the least energetic region of the scatter (see Fig. 2-21). Therefore peak features in the beamformer output may not correlate with important features in the backscatter.

In all the beamforming results presented in this chapter, geometric contribution has been estimated and subtracted using the process described in Chap. 2.

### 5.3.2 Empty Shell

I begin with the beamformer output of the $\phi_i = 0^\circ$ empty shell bistatic data, using an aperture $60 \leq \phi_s \leq 120^\circ$. The output is contoured in Fig. 5-7 and represents envelopes of the time signal plotted as a function of position along the shell length. The vertical axis corresponds to normalized distance along the shell, where the insonified endcap is at $z/L = 0$, and the far endcap is at $z/L = 1$. The slope discontinuities connecting the conical shells and the circularly cylindrical shell are located at $z/L = 0.07$ and $z/L = 0.93$. $t = 0$ corresponds to the initial interaction of the incident sound wave with the spherical section of the insonified endcap.

The bright spot in the bottom left hand corner corresponds to a remnant of the geometric contribution which remains because the subtraction process is approximate. The model for the initial interaction presented in Chap. 3 predicts that this initial interaction excites both axisymmetric compressional and flexural waves. There are
Figure 5-7: Contours of axial pressure distribution on empty shell surface for $\phi_i = 0^\circ$. The distribution represents the beamformer output for an aperture $60 < \phi_s < 120^\circ$ (perpendicular sector analysis). The data are Gaussian bandlimited to $2.75 < ka < 10.0$ and the geometric contribution has been subtracted. The axial location $z/L = 0$ corresponds to the insonified endcap. $t = 0$ is the arrival of the incident acoustic pulse at the insonified endcap.
two prominent features in the beamformer output which extend over the full shell length. The first originates at the insonified endcap at \( t = 0 \), while a second duplicate feature originates at the insonified endcap at \( t = 200 \mu s \). These correspond to the two initially excited axisymmetric waves.

**Initial Compressional Wave**

A feature with constant positive slope (linearly increasing axial distance with increasing time) emerges from the interaction and extends \( 0 < z/L < 1 \). The positive slope indicates the feature is a forward propagating wave (waves which travel from the insonified endcap to the far endcap); a negative slope indicates backward propagation. The one-way travel time along the cylindrical shell is \( t \approx 145 \mu s \), which is the group delay for an axisymmetric compressional wave traveling at \( c_p \approx 5100 m/s \). The compressional wave scatters at the cone/cylinder discontinuity (\( z/L = 0.93 \)) upon reaching the far endcap. This is evidenced by the feature which coincides with the arrival of the compressional wave. The spatial attenuation of this wave is approximately \( 10 \, dB \) versus the \( 1.5 \, dB \) predicted. This is so because the aperture effects of the radiation have not been properly accounted for in this beam-directed analysis. That is, as the compressional wave propagates to the far end of the shell, its radiation is directed to an increasingly deemphasized section of the bistatic array (see Fig. 2-17). Thus, the array taper, while reducing sidelobes, has also reduced the influence of contributions near array edges. Below I will show that the actual attenuation is the same as predicted.

A reflected compressional wave of reduced amplitude returns to the near endcap, while transmitted waves propagate forward to the spherical section of the far endcap. The reflection coefficient of \( 0 \, dB \) is not consistent with the predicted \(-10 \, dB\) from Chap. 4. This is explained in the same manner as the observed high spatial attenuation; below I will show that the reflection coefficient is the same as predicted. The main lobe of radiation from the spherical section of the endcap is not captured as it is confined to \( \phi_s \gg 120^\circ \) in the forward direction. Although not clear from Fig. 5-7, the
roundtrip time for the reflected wave to return to the insonified endcap is 310µs. I will discuss the energy transmitted to the far endcap after first taking a clearer look at the propagation characteristics of the initial compressional wave.

The added attenuation can be removed at the expense of increased sidelobes by modifying the array size and weighting. To do this, the same beamforming analysis was recomputed with an aperture centered about the radiation direction of forward propagating compressional waves, $86 \leq \phi_s \leq 126^\circ$ to isolate the forward propagating waves and to shift the region of artificial attenuation away from the radiation lobe. The result of shifting the array is that only forward wave propagation is captured. The new output is contoured in Fig. 5-8, where the measured attenuation is approximately 2 dB over the shell length and consistent with the attenuation computed analytically for an infinite cylinder (Fig. 5-1.)

Similar beamforming was conducted to isolate and better represent the backward propagating waves by centering the aperture in the backward radiation direction, $54 \leq \phi_s \leq 94^\circ$. Contours of this result are shown in Fig. 5-9 and can be used to estimate the compressional wave reflection coefficient at the cylinder/cone junction of the far endcap. The initial backward propagating compressional wave leaves the junction coincident with the arrival time of the initial forward propagating compressional wave ($t \approx 150\mu s$). Its amplitude corresponds to approximately a 10 dB reduction compared with forward propagating wave amplitude at arrival. This -10 dB reflection coefficient is consistent with the mid-band scattering coefficient analytically estimated for flat plates shown in Fig. 4-4, (reproduced in Table 5.1). Despite the low reflection coefficient, Fig. 5-9 shows that this reflected compressional wave is important compared with all subsequent backward propagating compressional waves.

**Transmitted Energy at the Far Endcap**

According to Table 5.1, the transmitted compressional wave power is -2.5 dB relative to the incident compressional wave. As just shown by the data, the reflected compressional wave is -10 dB, while the remainder is scattered to flexural waves (radiation
Figure 5-8: Contours of axial pressure distribution of forward propagation on empty shell surface for $\phi_i = 0^\circ$. The distribution represents the beamformer output for an aperture $86 < \phi_s < 126^\circ$. The data are Gaussian bandlimited to $2.75 < ka < 10$ and the geometric contribution has been subtracted. The axial location $z/L = 0$ corresponds to the insonified endcap. $t = 0$ is the arrival of the incident acoustic pulse at the insonified endcap.
Figure 5-9: Contours of axial pressure distribution of backward propagation on empty shell surface for $\phi_i = 0^\circ$. The distribution represents the beamformer output for an aperture $54 < \phi_s < 94^\circ$. The data are Gaussian bandlimited to $2.75 < ka < 10$ and the geometric contribution has been subtracted. The axial location $z/L = 0$ corresponds to the insonified endcap. $t = 0$ is the arrival of the incident acoustic pulse at the insonified endcap.
<table>
<thead>
<tr>
<th>Scattering Process Incident Compressional Wave</th>
<th>Coefficient $dB$ (POWER)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitted Compressional</td>
<td>-2.5</td>
</tr>
<tr>
<td>Transmitted Flexural</td>
<td>-7</td>
</tr>
<tr>
<td>Reflected Compressional</td>
<td>-10</td>
</tr>
<tr>
<td>Reflected Flexural</td>
<td>-9</td>
</tr>
</tbody>
</table>

Table 5.1: Mid-band scattering estimates for a compressional wave incident upon a slope discontinuity

necessarily occurs but was not included in the model). The conversion of compressional waves to flexural waves is a mechanism that provides energy transfer which, through flexural wave propagation in the structure, is a storage mechanism. It also introduces large group delays due to the low group speed of the flexural wave compared with the compressional wave. (At mid-band, $ka = 6.5$, the time for a flexural wave to propagate the length of the circular cylindrical section is $10^3 \mu s$.)

Much of the transmitted compressional wave power reaches the spherical section of the endcap, where I calculated that the compressional wave loses approximately 2 $dB$ to radiation. By the time the transmitted compressional wave circumnavigates the entire endcap, which includes the conical shell and the spherical shell, it has scattered at 4 slope discontinuities. With use of Table 5.1, the cumulative effect including radiation loss is 12 $dB$ attenuation of the initial compressional wave. The delay for this process is $t \approx 35 \mu s$, thus the second backward propagating wave that is a consequence of this circumnavigation is 2 $dB$ weaker than the initially reflected wave. This wave is clearly seen in Fig 5-9 and has a magnitude which is consistent with 12 $dB$ of attenuation.

**Duplicate Compressional Wave**

I return to Fig. 5-7 to examine the duplicate compressional wave emerging from the near endcap at $t = 200 \mu s$. The time delay is substantially shorter than the roundtrip time of the initial compressional pulse ($310 \mu s$) and substantially longer than the time
Figure 5-10: Group delay vs. normalized frequency for flexural waves on a conical shell section.

for the incident sound wave to travel the axial length of the endcap (40μs.) The long delay to the onset of the second compressional wave indicates the participation of flexural waves.

In Chap. 3, I proposed that both flexural and compressional waves are excited at the sphere/cone junction by the incident sound wave. The initially excited flexural wave travels along the conical shell from the sphere/cone junction to the cone/cylinder junction where it scatters. To confirm this hypothesis, it is necessary to demonstrate a 200μs group delay is consistent one-way propagation of an axisymmetric flexural wave on the conical shell. The group speed curves for a representative conical shell were plotted in Fig. 5-4 and can be collapsed to show group delay as a function of frequency by integrating the inverse of local group speed along the cone length,

\[ t_g(\omega) = \int_{cone} \frac{1}{c_g(\omega)} \, dl \quad (5.9) \]

The resulting group delay is shown in Fig.5-10, where delays \( t \sim 200\mu s \) correspond to wave propagation in the lower \( ka \) band. The process is highly dispersive over the full band and at the higher frequencies; delays are predicted to be as short as 50μs. Thus, the potential source of the duplicate compressional wave is identified. It is still necessary, however, to show that this wave is dominated by low frequency energy. I do this using time-frequency analysis.
Time-Frequency Analysis

I now return to Fig 5-7 show the effects of the highly dispersive propagation. Because the flexural modes are non-radiating, I cannot investigate the flexural wave dispersion directly with use of the measured data. The second compressional wave must retain a frequency dependent group delay associated with flexural wave propagation on the conical shell, preserved because the compressional wave is essentially non-dispersive. I can illustrate the delay with time-frequency analysis.

The beamformer output at any location on the shell surface gives arrival structure of energy traveling along the shell length, so I perform a Wigner [35] time-frequency analysis of a single timeseries from the beamformer output. The Wigner distribution is a high resolution nonlinear transform that can introduce many non-physical features to the output; however, I use the Wigner distribution because of its high resolution and because the expected patterns are already established. This is important because the strong artifacts associated with the Wigner transform may be dismissed if the observables are understood a priori. Reduction of the artifacts can be accomplished using low pass filtering, but this smoothing is done at the expense of reduced resolution. The Wigner distribution as applied to the shell surface pressure is given by

\[ W(t, \omega) = \int_t^\tau p_{\text{surf}}(t + \frac{\tau}{2})p_{\text{surf}}(t - \frac{\tau}{2})e^{-j\omega \tau}d\tau . \] (5.10)

A smoothed Wigner distribution is given by

\[ W_s(t, \omega) = \int_t^\tau \int_{\omega'} g(t - t', \omega - \omega')W(t', \omega')dt'd\omega' , \] (5.11)

where Gaussian smoothing is applied in the two orthogonal analysis directions

\[ g(t, \omega) = e^{-\frac{1}{2}t^2/\sigma_t^2}e^{-\frac{1}{2}\omega^2/\sigma_\omega^2} . \] (5.12)

A time history at \( z/L = 0.4 \) on the shell surface was selected for analysis, and the resulting distribution is shown in Fig. 5-11. The smoothing applied for this case

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Figure 5-11: Smoothed Wigner Distribution of shell surface pressure at $z/L = 0.41$ for empty shell at $0^\circ$ incidence, and $2.75 < ka < 10$. The smoothing is Gaussian with $\sigma_t = 6\mu s$ and $\sigma_{ka} = 0.2$. An explanation of the features is shown in Fig. 5-12.

has standard deviations, $\sigma_t = 6\mu s$ and $\sigma_\omega = .2ka$. The results are qualitative in amplitude and have been normalized to a peak value of 1.0. They should not be strictly interpreted to give information regarding the energy content of the observed features, but rather as a means to understand arrival times. I show the predicted arrival times in Figure 5-12 for both the initial and duplicate compressional waves. In Fig. 5-11, at $t = 75\mu s$, the broad band non-dispersive arrival corresponds to the initially excited compressional wave. The $75\mu s$ delay is the time required for a compressional wave to travel from the spherical section of the endcap to $z/L = 0.4$. Approximately $200\mu s$ after the first arrival there is a second arrival, which is dominated by low frequency components. The emphasis on the contributions in the lower $ka$ band may explain why the $200\mu s$ delay is such a prominent feature even though the time spread should include delays ranging from 50 to $200\mu s$. The delay pattern which is characteristic of the flexural wave dispersive effects can be discerned in the arrival structure.

In Fig. 5-11 there are several artifacts that are cross terms located between the first and second arrivals in the lower-$ka$ region which should be neglected. Additional
Figure 5-12: Expected arrival times of the initial and duplicate compressional waves at the axial location, $z/L = 0.41$. The first arrival at $t = 70\mu s$ is the initial compressional wave, while the second arrival is the duplicate compressional wave which scatters from a flexural wave on the conical shell.

...smoothing removes these cross terms but also masks the dispersion.

This analysis demonstrates that flexural waves are excited by the initial interaction of the incident sound wave with the endcap. The propagation delay for a flexural wave traveling in the conical shell near the ring frequency is large (greater than the time for a compressional wave to travel the length of the shell). Because the flexural waves are subsonic in this frequency band, they are not subject to radiation damping, and therefore provide an energy storage mechanism that can radiate at later times.

**Summary of empty shell results**

The observations of this section show that both compressional and flexural waves are important. Both waves are excited by the incident sound wave and are coupled via discontinuities in the shell. The initial compressional wave is the dominant wave process for early times ($t < 400\mu s$) and decays rapidly due to radiation and scattering losses at the endcap. Some of the scattering losses are due to conversion to flexural
waves which propagate on the shell for much longer times because there are no losses to radiation, and because scattering losses occur less frequently and at later times due to low flexural wave group speeds.

5.3.3 Internally Loaded Shells

To study the surface pressure distributions on the two internally-loaded shells, I apply the same beamforming parameters to the bistatic data applied to the empty shell bistatic data. I use the same aperture of weighted receivers $60^\circ \leq \phi_s \leq 120^\circ$ to capture the leaky wave radiation. The data are bandpass Gaussian filtered to $2.75 < ka < 10$.

The results are contoured in Fig. 5-13 and 5-14, where the axial distributions of the two shells are strikingly different from those of the empty shell but nearly identical to each other. The similarity of the two results indicates that the sprung internals are poorly coupled to the shell motions. Two explanations may account for this poor coupling. First the lowest order modes of the internal structures are located an order of magnitude lower in frequency than the lowest frequency in the excitation band. Second, the large impedance mismatch between the shell and ring only allows for weak coupling into the internals, such that the ring is effectively a restraint with respect to the motion of the shell. Due to the similarity between these results, discussions regarding the internalled shell distribution also pertain to the ringed shell.

In Figs. 5-13 and 5-14, the vertical axis corresponds to normalized axial position $z/L$. The four rings are located at $z/L = .18, .45, .67,$ and $.82$ respectively. Many of the phenomena discussed in Section 5.3.2 can be identified in the internally-loaded shell responses, including the incompletely subtracted geometric contribution, the initially excited compressional wave and the duplicate compressional wave. The initial and duplicate compressional waves can be identified colocated with scattering bursts at the cone/cylinder junction ($z/L = 0.07$) at $t = 0$ and $t = 200\mu s$ respectively. These processes are all directly related to the initial acoustic interaction with the endcap, and are necessarily the same because the endcaps are identical.
Axial Field Distribution for Ringed Shell
0 deg Incidence -- ka=2.75-10 -- aperture=60-120 deg

Figure 5-13: Contours of axial pressure distribution on ringed shell surface for $\phi_i = 0^\circ$. The distribution represents the beamformer output for an aperture $60 < \phi_i < 120^\circ$ (perpendicular sector analysis). The data are Gaussian bandlimited to $2.75 < ka < 10$ and the geometric contribution has been subtracted. The axial location $z/L = 0$ corresponds to the insonified endcap. $t = 0$ is the arrival of the incident acoustic pulse at the insonified endcap.
Figure 5-14: Contours of axial pressure distribution on internalled shell surface for $\phi_i = 0^\circ$. The distribution represents the beamformer output for an aperture $60 < \phi_i < 120^\circ$ (perpendicular sector analysis). The data are Gaussian bandlimited to $2.75 < ka < 10$ and the geometric contribution has been subtracted. The axial location $z/L = 0$ corresponds to the insonified endcap. $t = 0$ is the arrival of the incident acoustic pulse at the insonified endcap.
The distributions change dramatically compared with the empty shell distribution for increasing time and increasing distance from the endcap. When the initial compressional wave encounters the first ring, part of the energy is reflected in the backward direction toward the insonified endcap, part is transmitted to the next bay and part radiates to the acoustic field. The magnitude of the transmitted compressional wave beyond the first ring is significantly reduced as compared with the incident wave. A feature colocated with the arrival of the compressional wave at the first ring is the contribution due to ring radiation. As the transmitted compressional wave arrives at each subsequent ring it is strongly reflected and weakly transmitted, except in the pass bands (see Sec. 4.3.6), which are evident only in the frequency domain. Within each subsequent bay, the magnitude of the compressional wave is greatly reduced.

In Chap. 4, the analysis of the shell/ring interaction for compressional wave excitation addressed each of the above phenomena and predicted high ring reflection coefficients for $ka > 4$, in addition to low transmission and low level radiation. The analysis for multiple rings predicted frequency bands of relatively high transmission, passbands, at the first ring in addition to strong energy localization confined within the first two rings. This presence of passbands is not evident in the time domain and is discussed further with respect to a frequency domain representation of the radiation distribution.

The result of high reflection at the rings, as seen in Fig. 5-14, is compartmentalization of the shell energy. Energy is partitioned from bay to bay in such a way that each subsequent bay is less energetic than its neighbor; thus the rings partially decouple neighboring bays and reduce the shell length as seen by the elastic waves. The most energetic bay is the first, which extends from the insonified endcap to the first ring. The implication of this redistribution on the backscatter is discussed in Chap. 6 where the effect is shown to be enhanced backscatter and modified decay rates.
Frequency Domain Analysis

In Chap. 4, an analysis of the coherent interaction between two and also three rings showed that passbands can arise due to coupling between the rings. The analysis further revealed that the irregularity of the ring spacing causes strong spatial localization of structural wave energy. The analysis predicted passbands for the compressional waves at \( ka \sim 2.6, 5.1, 7.7 \) and 10.3. To show these, I transform the beamformer output to the frequency domain and plot the spatial distribution of pressure as a function of frequency. The result for the ringed shell is shown in Fig. 5-15 and the result for the internalled shell is shown in Fig. 5-16.

The most prominent feature in these figures is that the transmission is greatest near the ring resonance frequencies, \( ka \sim 3 \). In particular the transmission extends the full shell length and is not diminishes by the irregularity of the ring spacing. With respect to this phenomenon, a distinction between the ringed and internalled shell is that the spatial decay of radiation source strength for the internalled shell is greater in this frequency band. The increased decay is evidence that the sprung internal structures are coupled to the shell motions in this frequency band. This is discussed further in the following section on narrow band analysis.

In addition, the compressional wave pass bands associated with the coupling between the first and second ring are located at the predicted frequencies for both shells. It is also notable that the expected localization phenomenon can be observed in these figures for frequencies well above the ring resonances. That is, the irregularity of the ring spacing causes a strong spatial decay of the radiation levels away from the insonified endcap [61]. The analysis of multiple ring interaction in Chap. 4 shows that the strongest coupling is between adjacent rings for this geometry and that additional neighboring rings do not have an important effect on this coupling.

One last consideration is the aperture effect of the beamformer with respect to resolution in the distributions. The spatial resolution can be estimated from classical optics. The angular resolution for a uniform line array is approximately \( \beta \simeq \lambda/L \), where \( L \) is the array length and \( \lambda \) is the acoustic wavelength. Therefore the axial spatial
Figure 5-15: Distribution of radiation on ringed shell for $\phi_i = 0^\circ$ as a function of frequency. The distribution represents the beamformer output for an aperture $60 < \phi_i < 120^\circ$ (perpendicular sector analysis). The geometrically scattered return has been subtracted. The axial location $z/L = 0$ corresponds to the insonified endcap.
Figure 5-16: Distribution of radiation on internalled shell for $\phi_i = 0^\circ$ as a function of frequency. The distribution represents the beamformer output for an aperture $60 < \phi_i < 120^\circ$ (perpendicular sector analysis). The geometrically scattered return has been subtracted. The axial location $z/L = 0$ corresponds to the insonified endcap.
resolution is approximately given by $\Delta_x \simeq r(\lambda/L)$, where $r$ is the distance from the array to the shell. This expression gives $\Delta_x/L = 0.08$ at $ka = 5$ and $\Delta_x/L = 0.05$ at $ka = 8$; these values are consistent with Fig. 5-15 and Fig. 5-16. A second issue is the ability of the array to fully resolve the radiation process from a single bay resonance. The array aperture is $60^\circ$ or $30^\circ$ on either side of the beam direction. I consider the beamwidth and radiation directions for the single bay resonance. For compressional waves traveling in a single bay, the primary radiation lobes are steered in the coincident radiation directions, which are $\pm 16^\circ$ off the beam axis. The angular width of the radiation is approximately given by the previous expression $\beta \simeq \lambda/l$ where $l$ is the bay length. At $ka = 3$, $\beta \simeq 28^\circ$, thus the half-angle width of the radiation lobe added to the coincident radiation angle is encompassed within the aperture of the array. Array taper, however, diminishes contributions near the array edges so that the coherent wave processes may not be fully resolved in Fig. 5-15 and Fig. 5-16.

**Narrow Band Analysis**

Similar *narrowband* beamformer output was generated in three frequency bands for the empty shell, the ringed shell and internalled shell. In place of the individual color contours, I have shown time integrated axial distribution of pressure for each shell and for each of the three frequency bands in Fig. 5-17.

In Fig. 5-17(c), the time integrated distributions are compared for higher-$ka$ where the partitioning of wave processes from bay to bay for the two internally loaded shells is evident. There is approximately a $6 \, dB$ drop in the integrated levels between successive bays, which could be interpreted as an experimentally derived overall transmission coefficient for all wavetypes. Even though only compressional waves radiate, all wavetypes contribute to the radiation due to wave conversion and as well as radiation at the rings, which is much more efficient for the flexural waves. This transmission coefficient is more representative of predicted flexural wave coefficient $T_{rf}$ than the predicted compressional wave coefficient $T_{rpp}$, indicating that the radiation from subsequent bays primarily reflects contributions from flexural wave driven processes.
Figure 5-17: Comparison of integrated beamformer output for the three shells. (a) $2 < ka < 5$ (b) $5 < ka < 8$ (c) $8 < ka < 11$. The geometric contribution has been subtracted. The curves represent the empty shell (solid), ringed shell (dashed) and internalled shell (dash-dot). The insonified endcap is at normalized shell position $z/L = 0$. 
That the transmission is much greater than predicted by $T_{rpp}$ is evidence as well that the ring coupling provides a mechanism for more efficient energy transfer between the bays. The ring locations are easily distinguished as $3 \, dB$ increases relative to neighboring integrated levels. This enhancement is due to the radiation from the rings primarily caused by generation of a flexural wave at each ring, and its subsequent radiation, and by previously generated flexural waves interacting with the rings and also radiating. In contrast, the empty shell integrated distribution is approximately uniform along its length with increases at the endcaps indicating radiation from the slope discontinuities as well as sidelobe radiation from the spherical sector of the endcap. Also significant, is the finding that the ringed and internalled shell distribution are nearly the same except in the lower $ka$ band. This is partially consistent with my earlier qualitative observation based on Fig. 5-13 and 5-14, that the sprung internals do not significantly influence wave propagation on the shells, but now I conclude that differences do occur at lower frequencies.

A comparison of the time integrated pressure distributions is made in Fig. 5-17(a) for the lower frequency band ($2 < ka < 5$) that includes the ring resonances. As discussed in Chap. 4, this is a region of high transmission due to the ring resonances, particularly for the compressional waves and the distributions reflect this, as indicated by smaller differences between the empty, ringed and internalled shells. The bay decoupling observed for the higher frequencies is weaker. There is an approximate $3 \, dB$ drop in magnitude between successive bays which correlates well with the compressional wave transmission coefficient in this band ($T_{rpp} = -4 \, dB$). A difference of note is the increased spatial decay of the internalled shell time integrated distribution with respect to the other two shells. The increase is $5 \, dB$ over the full length of the shell. One possible explanation is damping of the ring resonance by the sprung internals since this is a frequency band in which the ring response is be largest due to resonances.
Summary of Internally Loaded Shell Results

The observations of this section suggest that the rings have dramatically modified the distribution of wave energy along the length of the shell. Primarily the rings impede the transmission of elastic wave energy to the far endcap, confining this energy within the first bay, although less so for $ka < 5$. In the pass bands provided by the ring coupling, the elastic wave energy is confined within the first two bays. This would suggest that radiation in the backward direction is enhanced because the elastic waves are redirected to the insonified endcap rather than radiating from the far endcap. (Increased backscatter is seen in the monostatic and bistatic representations of the measured data in Chap. 2).

Later in time, $t > 200\mu s$, the energy distributions are dominated by radiation from the rings, particularly for $ka > 5$. This is consistent with a flexural wave process which radiates at each ring but not within the bays. In the analysis of Sec. 6.3.2, I will show that these flexural waves are the dominant contributor to the later-in-time backscatter, where the transition is greatly reduced for internally-loaded shells ($t \approx 200\mu s$) as compared with the empty shell ($t \approx 10^3\mu s$). The transition time is related to the time for a flexural wave to travel from the insonified endcap to the subsequent discontinuity - the first ring for the internaled shells and the far endcap for the empty shell.

As an additional note, the distribution analyses identify the radiation sources which contribute to the beam-directed bistatic scatter, but these are not necessarily the important radiation sources that contribute to the monostatic scatter. For example a strong feature observed at a ring may indicate strong radiation, but as the analysis of Chap. 4 shows, the peak radiation may be observed only at $\phi_s = 90^\circ$ falling off as $\sin \phi_s$. 

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5.4 Beamforming Analysis Near Axial Incidence

The $\phi_i = 0^\circ$ beamforming analysis, centered on $\phi_s = 90^\circ$ indicates that compressional waves are the dominant wave process for early time and flexural wave processes are the dominant wave process for later times. At $\phi_i = 0^\circ$ these processes are axisymmetric, and in Chap. 3, I contend that axisymmetric coupling processes continue to be important for $\phi_i \leq \phi_o$, where $\phi_o$ corresponds to the endcap radiation beamwidth. For the endcap configuration in this thesis, the beamwidth is $\phi_o \approx 20^\circ$ but varies with frequency. In this section, I investigate the importance of axisymmetric waves relative to other wave processes near bow incidence, but slightly outside the main radiation lobe, by beamform processing the $\phi_i = 25^\circ$ bistatic data for both the empty and internalled shells.

5.4.1 Empty Shell at $\phi_i = 25^\circ$

To study the empty shell wave propagation at $\phi_i = 25^\circ$, I have applied the same beamforming parameters to the measured bistatic data as used previously. The same bistatic aperture $60 < \phi_s < 120^\circ$ is used to produce the beamformer output contoured in Fig. 5-18 and Gaussian bandlimited to $2.75 < ka < 10$.

The two compressional waves which are the dominant features in Fig. 5-7 can be discerned in Fig. 5-18. The initial compressional wave has the same group speed and modestly reduced amplitude ($< 3 \, dB$) as seen by comparing the source distribution at $\phi_i = 25^\circ$ (Fig. 5-18) with the source distribution at $\phi_i = 0^\circ$ (Fig. 5-7) The compressional wave propagates to the far endcap and scatters in a manner identical to that for $\phi_i = 0^\circ$. The duplicate compressional wave, prominent for $\phi_i = 0^\circ$ is more difficult to identify due to destructive interference. Its amplitude is approximately 6 $dB$ down compared with that for $\phi_i = 0^\circ$, but the time delay prior to its onset remains $t \simeq 200 \mu s$. The axisymmetric waves have reduced amplitudes which I predict are proportional to the angular dependence of the endcap radiation patterns.

Some differences arise for $\phi_i > 0^\circ$. The incident sound wave generates a forced
Figure 5-18: Contours of axial pressure distribution on empty shell surface for $\phi_i = 25^\circ$. The distribution represents the beamformer output for an array of observation angles located at $60 < \phi_i < 120^\circ$ (perpendicular sector analysis). The data are Gaussian bandlimited to $2.75 < ka < 10$ and the geometric scatter has been subtracted. The axial location $z/L = 0$ corresponds to the insonified endcap. $t = 0$ is the arrival of the incident acoustic pulse at the insonified endcap.
Figure 5-19: Comparison of the integrated energy distribution on the surface of the empty shell at $\phi_i = 0^\circ$ (dashed) and $\phi_i = 25^\circ$ (solid) for $3 < ka < 10$. The geometric contribution has been subtracted.

elastic wave which propagates at the trace axial wave speed given by $c_o/\cos \phi_i = 1642 m/s$ at $\phi_i = 25^\circ$. The delay for the forced wave to travel the length of the shell is $524 \mu s$. A peak feature at $t \approx 500 \mu s$ and $z/L = .93$ coincides with the arrival of the forced wave at the far endcap. A compressional wave is generated by the arrival of incident acoustic pulse at the far endcap that propagates backward toward the near endcap. This compressional wave is of relatively low magnitude as compared with the initial compressional wave excited at the insonified endcap. Its magnitude which is equal, however, to that of the backward propagating compressional wave caused by scatter of the initially excited compressional wave at the far endcap.

Two new initial features appear in the distribution that emanate from the insonified endcap and have low group speeds relative to the compressional wave. Both processes only contribute over a fraction of the shell length. I find these difficult to interpret.

The conclusion from this analysis is that the same axisymmetric waves are excited even for $\phi_i > 0^\circ$ and few new mechanisms are introduced. The one noticeable mechanism is excitation of compressional waves at the far endcap by the forced elastic wave. Overall the wave processes do not change significantly. This can be shown in an integrated sense by comparing the integrated pressure distributions at $\phi_i = 0^\circ$ and $\phi_i = 25^\circ$ shown in Fig. 5-19.
5.4.2 Internalled Shell at $\phi_i = 25^\circ$

The internalled shell axial field distribution for $\phi_i = 25^\circ$ is contoured in Fig. 5-20. I apply the same beamformer parameters as for Fig. 5-18. The comparison with the $\phi_i = 0^\circ$ axial field distribution (Fig. 5-14) reveals that while energy distribution at $\phi = 0^\circ$ is concentrated within the first bay, the energy distribution for $\phi_i = 25^\circ$ spans the length of the shell. The pressure distributions integrated over time are compared in Fig. 5-21. The primary difference is that the peak levels located at each ring stiffener are more than 5 $\text{dB}$ greater for $\phi_i = 25^\circ$, and the rings are sharply distinguished.

This is apparent as well in Fig. 5-14 where a discernible feature is the four rows of scattering bursts axially colocated with the four ring stiffeners ($z/L = .18, .45, .67, \text{ and } .82$). In Fig. 5-20, the onset time for each row of bursts coincides with the arrival of the forced elastic wave rather than the arrival of the initial compressional wave. (These arrival times correspond to the initial red peak at each ring.) The interpretation is that the forced elastic wave scatters to free elastic waves at each ring. The more important elastic waves are flexural rather than compressional as evidenced by the low levels within the bays relative to the levels at the rings. Thus the subsequent bursts from the ring are caused by flexural waves reflecting back and forth within the bays with radiation resulting from each interaction. The train of ring bursts decays slowly because the primary flexural wave loss mechanism is radiation at the rings. I have plotted the time history at the third ring ($z/L = .67$) in Fig. 5-22 to show the decay. The arrival of the forced elastic wave at the ring causes the initial peak burst at $t \approx 350 \mu s$ and marks the onset of flexural wave activity. (The earlier compressional wave arrival ($t \approx 110 \mu s$) is -15 $\text{dB}$ relative to the peak burst.) I estimate the flexural wave decay by combining the broadband flexural wave scatter coefficients from a ring found in Tables 4.2 and 4.3 with an average interaction rate. The decay rate suggested by the dashed line is 5 $\text{dB}/\text{ms}$. The broadband reflection coefficient for an incident flexural wave is $R_{eff} = -1 \text{ dB}$ and the transmission coefficient is $T_{eff} = -10 \text{ dB}$. The average decay rate can be estimated using an interaction rate based on the mid-band flexural wave group speed ($C_g = 700 \text{ m/s}$) and average bay spacing ($l = L/5$),
Figure 5-20: Contours of axial pressure distribution on internal shell surface for $\phi_i = 25^\circ$. The distribution represents the beamformer output for an aperture $60 < \phi_i < 120^\circ$ (perpendicular sector analysis). The data are Gaussian bandlimited to $2.75 < ka < 10$ and the geometric scatter has been subtracted. The axial location $z/L = 0$ corresponds to the insonified endcap. $t = 0$ is the arrival of the incident acoustic pulse at the insonified endcap.
Figure 5-21: Comparison of the integrated energy distribution on the surface of the internalled shell at $\phi_i = 0^\circ$ (dashed) and $\phi_i = 25^\circ$ (solid) at $3 < ka < 10$. The geometric contribution has been subtracted.

Figure 5-22: Time history of the equivalent source amplitude at the third ring stiffener for the internalled shell at $\phi_i = 25^\circ$ ($2.75 < ka < 10.0$)

where the loss per interaction is considered to be conversion to any non-flexural wave process (energy not accounted for by $R_{rf}$ and $T_{rf}$). The resulting expression for the temporal decay rate of the flexural wave process is

$$\Delta_t = (0 \Theta R_{rf} \Theta T_{rf}) \frac{C_g}{l}, \text{ dB/s}$$  \hspace{1cm} (5.13)

where $\Theta$ denotes logarithmic subtraction. These parameters yield a 4 dB/ms decay rate that correlates very well with the data and confirms that the wave process driving the rings is predominantly flexural wave driven.

Since the backscatter depends primarily upon the response within the first bay,
much of this flexural wave energy contributes only weakly to the backscatter. The ring radiation due to an incident flexural wave is maximum in the beam direction, with a dipole-like radiation pattern, so the contributions to $\phi_s \leq 25^\circ$ are small. These contributions become more important with increasing incidence angle $\phi_i$ and increasing observation angle $\phi_s$. Therefore, there is no conflict in accepting that although the energy distribution levels are larger for $\phi_i > 0^\circ$, the monostatic scatter is greatest at $\phi_i = 0^\circ$.

5.5 Summary

The incident sound wave excites a compressional wave at the insonified endcap which scatters to flexural waves at slope discontinuities and the ring. The compressional wave radiates efficiently at the spherical sector of the endcap and decays rapidly, while the flexural waves store the energy which is slowly converted to radiating compressional waves at the slope discontinuities and radiation at the rings. As a result, axisymmetric compressional waves are the dominant elastic wave process at early times while axisymmetric flexural waves become more important at later times. The transition time is given by the time for a flexural wave at mid-band to travel to the first shell discontinuity beyond the insonified endcap.

The transition time for the empty shell is $t \sim 10^9 \mu s$ which equals the time for a flexural wave at mid-band to travel from the insonified endcap to the far endcap. The transition time for the internally-loaded shells is $t \sim 200 \mu s$ which is the time for a flexural wave at mid-band to travel to the first ring. The presence of the rings greatly reduces the distance elastic waves travel (on average) between discontinuities. As a result, energy is converted from one wavetype to another more frequently in an internalled target.

The rings incorporated in the internally-loaded models studied are highly reflective individually over most of the frequency band; however, the coupling between the rings increases the transmission at frequencies in which distance between the rings is an
integer number of half-wavelengths. Because the rings are not periodically spaced, strong localization of the structural wave causes energy to be confined within the first two shell bays. In the lower-\(ka\) band, which includes the ring resonances, energy is transmitted more efficiently to the far endcap.

Only small differences could be observed comparing the wave propagation on the ringed and internalled shells. Above the resonance frequencies of the rings, the energy distributions are nearly identical. At lower frequencies which include the ring resonances, there is greater attenuation of the elastic wave energy as a function axial distance for the internalled shell. This indicates that the sprung internals damp the motions of the rings near resonances. Away from this resonance band the sprung internals are essentially decoupled from the shell motion.

For \(\phi_i > 0^\circ\), the scattering process evolves in the same manner. The main differences are that the elastic waves initially excited on the endcap have reduced amplitude, while the forced elastic wave scatters more efficiently to free elastic waves at the rings. These free elastic waves are primarily flexural waves which are trapped within the bays. These waves radiate at the rings, but the ring radiation pattern (dipole) is such that they may not contribute well to the backscatter. I explore this in the next chapter.
Chapter 6

Comparison of Scattering Models with the Measured Backscatter

6.1 Overview

In this chapter, I combine the observations taken from chapters 3, 4 and 5 to interpret the properties of the backscatter near bow incidence for each of the shells. The beamforming analysis of Chap. 5 illustrates the time evolution of the radiation sources and proves to be useful for identifying the elastic wave processes. However, the beamformer only identifies those radiation sources which contribute to the beam direction $60 < \phi_s < 120^\circ$. These sources do not necessarily contribute to the monostatic scatter near bow incidence. Therefore it is necessary to consider the radiation patterns for the wave processes identified, and to determine how important each one is in terms of the backscatter.

6.2 Transduction at the Endcap

The primary radiation source for the backscatter is the endcap. This was made evident by the bistatic time domain contours shown in Chap. 2. The transduction between
the acoustic medium and the endcap is discussed in detail in Chap. 3, where I note that the description of the launching process (coupling of the incident sound wave to elastic waves) is equivalent to the radiation process. There, a circular disk attached to a conical shell serves as model for the initial interaction of the incident sound wave with the endcap. One of the parameters I estimate using this interaction model is the displacement amplitude for each of the axisymmetric elastic waves excited by the interaction. The circular disk model is intended to be representative primarily at lower frequencies where trace-matching becomes unimportant and where the acoustic wavelengths are large compared with the axial extent of the spherical sector of the endcap. At this point, the bistatic data provides another means to estimate the coupling coefficient for the initially excited compressional wave. The empty shell bistatic forward scatter has a first arrival which precedes the arrival of the shadow forming wave that is caused by radiation of the initially excited compressional wave from the far endcap. The temporal separation is large enough to well resolve the target strength of this process.

Reciprocity provides a tool to estimate the coupling coefficient $C_m$ for a compressional wave excited on the endcap by the incident sound wave, where $C_m$ may be defined in the frequency domain as the ratio of the compressional wave in-plane velocity $V(\omega)$ at the point of excitation to the incident sound pressure amplitude $P_i(\omega)$. Specifically,

$$C_m = \frac{V}{P_i}, \quad (6.1)$$

where the frequency dependence is understood.

Reciprocity

The reciprocity principle for a linear elastic/acoustic system is illustrated by Fig. 6-1 and states that a source and a receiver may be interchanged without changing the ratio of the response to the source strength. For the problem at hand, the acoustic source to be the incident plane wave and the response is the compressional wave in-plane shell velocity. Interchanging the source and receiver gives the elastic source as the in-plane
Figure 6-1: Reciprocity requires that the pressure does not change when the source and receiver are interchanged.

stress associated with a propagating compressional wave and the acoustic response as the fluid particle velocity. The relationship is

$$\frac{V(x_2)}{p(x_1)} = \frac{V(x_1)}{\sigma(x_2)},$$  \hspace{1cm} (6.2)

where $V(x_2)$ is the shell in-plane velocity due to an acoustic pressure $p(x_1)$ and $V(x_1)$ is the acoustic particle velocity due to an in-plane shell stress at $x_2$. This expression of reciprocity is a point to point relationship but the interaction of interest is plane wave excitation. This can be addressed by requiring the acoustic source is in the far field of the endcap such that the excitation can be considered to be locally a plane wave, where an additional correction must be incorporated to account for spreading loss.

I modify the reciprocity relationship to estimate the coupling coefficient. Because the two endcaps on each shell are identical, the endcaps can also be interchanged. In other words, referring to Fig. 6-2, I can rewrite the reciprocity relationship as

$$\frac{V_1}{P_1} = \frac{V_2}{\sigma_2},$$  \hspace{1cm} (6.3)

where the source and receiver are not only interchanged but also placed on the opposite ends of the shell. Here $V_1$ is the shell in-plane velocity at the spherical section of the endcap due to the incident sound wave, $\sigma_2$ is the in-plane stress of a compressional wave at the far endcap, and $V_s$ is the acoustic particle velocity in the forward scatter direction, where again the frequency dependence is understood. The acoustic particle
velocity \( V_s \) is directly related to the acoustic pressure \( P_s \) via the momentum equation as given by the simple expression in the far field

\[
P_s = \rho c_0 V_s . \tag{6.4}
\]

In a similar manner, the compressional wave stress \( \sigma_2 \) is proportional to the in-plane shell velocity,

\[
\sigma_2 = \rho_s c_p V_2 . \tag{6.5}
\]

Eq. 6.3 can now be rewritten as

\[
V_1 V_2 = \frac{P_1 P_s}{\rho c_0 \rho_s c_p} , \tag{6.6}
\]

where \( P_s \) is in the far radiation field of the stern endcap.

**Spreading Correction**

Since there is no spreading loss for a plane acoustic wave, Eq. 6.6 must be modified to account for the spreading loss from the far endcap to the receiver in the forward scatter direction. The spreading correction for spherical spreading from a spherical surface of radius \( a_s \) is \( r/a_s \) where \( r \) is the distance from the sphere origin to the receiver. This changes Eq. 6.6 to

\[
V_1 V_2 = \frac{r}{a_s} \frac{P_1 P_s}{\rho c_0 \rho_s c_p} . \tag{6.7}
\]

This is valid in the frequency domain for \( kr > 1 \).
Figure 6-3: Gaussian bandlimited forward scattered impulse response of the empty shell at bow incidence ($2.75 < ka < 10.0$). The distance from the target center to the bistatic receiver is $2m$. The initial packet at $t = 200\mu s$ is the forward radiated initial compressional wave. The peak response near $t = 600\mu s$ is the shadow forming wave. The amplitude is $p_s/p_i$.

**Empty shell forward radiation**

The initially excited compressional wave radiates in the forward direction $\phi_s = 180^\circ$ and arrives at the receiver prior to the geometric shadow-forming wave due to the higher group speed of the shell compressional wave. Its contribution is easily identified in the bistatic contours for the empty shell at bow incidence Fig. 2-16). The forward radiation from the compressional wave is temporally well separated from subsequent contributions, as shown in Fig. 6-3 where the compressional wave arrival is located at $t = 200\mu s$ (the shadow wave is at $t = 580\mu s$). This contribution is windowed and extracted for analysis, and the target strength, $T_c$ of the radiated first compressional wave shown in Fig. 6-4 is given by

$$T_c(\omega) = 20\log|P_s(\omega, 100 < t < 300\mu s)|/|P_i(\omega)|, dB \text{ re } 2 m.$$  \hspace{1cm} (6.8)

where the reference is from the target center.
Figure 6-4: Target strength of the empty shell forward scattered signal at bow incidence measured over $100 < t < 300$ $\mu$s. The time duration captures the radiation of the initial compressional wave. No spreading correction is included, therefore the target strength represents the magnitude at the bistatic receiver.

Shell Transfer Function

The only unknowns in Eq. 6.7 are $V_1$ and $V_2$, which are related by a transfer function $H = V_2/V_1$ that accounts for the effects of wave propagation from location 1 to location 2 (See Fig. 6-2.) The transfer function accounts for the attenuation due to radiation and the scatter at the slope discontinuities. Expressions for losses due to radiation and discontinuity scatter are found in Sec. 5.2 and Sec. 4.2.1.

The transfer function can be expressed in terms of a transmission line as in Fig. 6-5 where the amplitudes shown represent the compressional in-plane velocities immediately beyond each slope discontinuity. From Fig. 6-5 we get

$$|H| = \left| \frac{V''}{V_1} \right| \left| \frac{V'''}{V''} \right| \left| \frac{V_2}{V'''} \right|,$$

where

$$\left| \frac{V_j}{V_{j-1}} \right| = \mu_t e^{-k_{zi}p_j},$$

and $\mu_t$ is the transmission coefficient for propagation across a slope discontinuity, $p_j$ is the path length between locations $j - 1$ and $j$, $k_{zi}$ is the imaginary part of the complex compressional wave root of either the conical or cylindrical shell dispersion relationship, and $e^{-k_{zi}p_j}$ accounts for the radiation loss (see Sec. 5.2). In Chap. 5, I
Figure 6-5: Transmission line representation of the transfer function from the insonified endcap to the far endcap for the empty shell

show the radiation losses from a conical shell or a cylindrical shell are approximately equal. Thus in logarithmic form, Eq. 6.9 becomes

$$20 \log |H| = -\Delta_t P - 0.6\Delta + n20 \log \mu_t ,$$ \hspace{1cm} (6.11)

where $\Delta_t$ is the spatial radiation attenuation (dB/m) (Eq. 5.3) for a cylindrical shell, $P = .84m$ is the total path length on the conical plus cylindrical shell, $0.6\Delta$ is the total radiation loss from 60% of the spherical shell section and $n = 4$ is the total number of slope discontinuities. The spatial attenuation, $\Delta_t$, can be replaced with the temporal attenuation $\Delta_t c_p$ (Eq. 5.4) where $\Delta_t$ is plotted in Fig. 5-1. The expression $20 \log \mu_t$ is is equivalent to the transmission coefficient across a slope discontinuity, $T_{pp}$ (Eq. 4.5).

I combine Eq. 6.7 and Eq. 6.9 to get

$$\left( \frac{V_1}{P_1} \right)^2 = \frac{P_s r}{P_i a |H|} \frac{1}{\omega} ,$$ \hspace{1cm} (6.12)

where $\omega = \rho c_o \rho_s c_p$. If we substitute Eq. 6.1, 6.8 and 6.11 into the above expression and change to logarithmic notation, we get

$$20 \log C_m = \{ T_c + \Delta_t c_p P + 0.6\Delta - nT_{pp} + 20 \log r/a - 20 \log \omega \}/2.$$ \hspace{1cm} (6.13)

Eq. 6.13 yields an experimental estimate of the coupling coefficient which is plotted (solid line) in Fig. 6-6. The highest coefficients are at the upper end of the frequency range and diminish slowly to a broad minimum $6 < ka < 9$. For $ka < 5$, the
Figure 6-6: Estimate of coupling coefficient, $C_m$. The coupling coefficient represents the ratio of the incident pressure to the in-plane velocity of the excited compressional wave. The coefficients are referenced to an incident pressure of 1$\mu$Pa. The input in-plane velocity estimated using the disk model (dashed line) (Eq. 3.19) is plotted for comparison.

Amplitude increase and fluctuates. The analysis in Chap. 3 indicates that the coupling mechanisms change between the higher and lower frequencies. Fig. 6-6 indicates that the high frequency trace-matching mechanism is a more efficient mechanism than the low frequency mechanism for exciting compressional waves, but at $ka = 2$ it climbs within 5 $dB$ of the higher frequency limit.

A coefficient based on the in-plane velocity calculated using the circular disk and conical shell model (dashed line) is compared with the experimental estimate. The two estimates give nearly the same results (< 1 $dB$ difference) for $6 < ka < 9$. Below $ka = 6$, the disk model does not predict the dips at $ka = 3.5$ and $ka = 5$ which may be caused by interference due to multiple radiation sources. The first dip near $ka = 3.5$ is located near the shell ring frequency. The second dip at $ka = 5$ is near the ring frequency of the conical shell close to the spherical section of the endcap. The ring frequencies are characterized by large radial motions damped by fluid loading and it is possible that wave scatter at the endcap discontinuities interferes with the compressional wave radiation. The disk model and the data agree within 1
\( dB \), however, for \( ka < 3 \). The disk model predicts low amplitudes by 5 \( dB \) at higher-\( ka \). The increased coefficient is interpreted to mean that trace-matching is important for \( ka > 9 \), and that the two models are equally important only in a small frequency range \( 8 < ka < 9 \).

Thus, aside from the dips, where the disk model overestimates the coupling by as much as 9 \( dB \) at \( ka = 5 \), the disk model predicts the data within 1 \( dB \) for \( ka < 9 \) where the model is expected to be valid.

### 6.3 Monostatic Radiation

The coupling coefficient provides a means to analyze the radiation quantitatively. In Chap. 3, I discussed the initial return both in terms of its on-axis magnitude and its angle-width or radiation pattern. In the same way, I separate the radiation problem into its on-axis magnitude and its beampattern. I first examine the \( \phi_i = 0^\circ \) monostatic scatter and then investigate the effects of changing the incidence angle and the radiation angle. I examine the envelopes of the \( \phi_i = 0^\circ \) monostatic timeseries for each of the shells, shown in Figs. 6-7 to 6-9. The envelopes represent the Gaussian band-limited impulse responses for each shell for a frequency range \( 2.75 < ka < 10.0 \).

For each of the timeseries, I have indicated the decay rates at both early (\( t < 200\mu s \)) for the internally-loaded shells) and later times (\( t > 200\mu s \)). In Chap. 5, I discuss the evolution of these two distinct temporal regions in terms of the elastic wave propagation. I suggest that strong attenuation of the initial backward propagating compressional waves is caused by frequent scattering at discontinuities and radiation from the endcap which leads to a rapid decay of compressional wave energy on the shells. This process both contributes to the observed scatter and energizes the flexural wave processes. In turn the flexural waves store energy in the form of non-radiating waves which are slowly converted to radiating compressional waves via interactions with various shell discontinuities. The flexural waves also radiate at the rings.
Figure 6-7: Backscatter envelope of the Gaussian bandlimited impulse response for the empty shell at $\phi_i = 0^\circ$, $(2.75 < ka < 10.0$ and $r = 2m$ from target center). The envelope is target strength in $dB$ re $2m$ from the target center.

Figure 6-8: Backscatter envelope of the Gaussian bandlimited impulse response for ringed shell at $\phi_i = 0^\circ$, $(2.75 < ka < 10.0$ and $r = 2m$ from target center). The envelope is target strength in $dB$ re $2m$ from the target center.
Figure 6-9: Backscatter envelope of the Gaussian bandlimited impulse response for sandwich shell at $\phi_i = 0^\circ$, $(2.75 < ka < 10.0$ and $r = 2m$ from target center). The envelope is target strength in $dB$ re $2m$ from the target center.

6.3.1 Comparison of Well Resolved Arrivals with Scatter Models

In the analysis to follow, I interpret the backscatter in the time domain for a few well resolved contributions. I take advantage of the time resolution which the experiment provides and the non-dispersive nature of the compressional waves. This necessarily averages the response over the frequency band for the following 3 scenarios.

- I calculate the contribution from the initial compressional wave which propagates to the far endcap of the empty shell, reflects from the slope discontinuity and radiates from the insonified endcap.

- I calculate the contribution from the initial compressional wave which reflects at the first ring of the internally-loaded shells and radiates from the insonified endcap.

- I calculate the contribution for the initial compressional wave which radiates at the first ring. I also approximately consider the contribution for the initial flexural wave which radiates from the first ring.
Beyond these identifiable contributions, the resolution necessary for identification of specific contributions is lost with the evolution of time so it is more useful to discuss the backscatter in terms of decay rates.

**Initial Compressional Wave Circumnavigation of Empty Shell**

The empty shell $\phi_i = 0^\circ$ backscatter is shown in Fig. 6-7. The early returns are compressional wave driven, while energy stored in flexural waves arrives much later in time.

I show that the peak arrival beginning at $t \simeq 310 \mu s$ is due to radiation from the initially excited compressional wave which reflects at the far endcap slope discontinuity. The most important reflection at the far end takes place at the cone/cylinder discontinuity as shown by the beamforming analysis of Chap. 5. The process can be described in parts: acoustic coupling to the insonified endcap, compressional wave propagation to the cone/cylinder discontinuity at the far endcap, reflection from the discontinuity, compressional wave propagation back to the insonified spherical cap, and radiation. The group delay for this process was discussed in Chap. 5 and given as $t \simeq 300 \mu s$. Eq. 6.13 can be modified and rearranged to give an expression for the target strength of this process. The equation relating the scatter to the incident pressure is thus

$$T_c = 40 \log C_m + 20 \log \omega - \Delta t c_p P - 0.6 \Delta + n_r R_{pp} + n_t T_{pp} - 20 \log R/a_s.$$  \hspace{1cm} (6.14)

Here $n_t$ is the number of slope discontinuities through which transmission occurs (4) and $n_r$ is the number of slope discontinuities at which the wave is reflected (1). $R_{pp}$ is the reflection coefficient for an incident compressional wave at a slope discontinuity. Also $r = 1.62 m$ from the endcap (2 m from target center) and $P = 1.6 m$. The mid-
band $ka = 6.5$ values for each of the parameters are

\[ 20 \log C_m = -135 \text{ dB re 1Pa} \quad \text{(Fig. 6-6)}, \]
\[ R_{pp} = -10 \text{ dB} \quad \text{(Fig. 4-4)}, \]
\[ T_{pp} = -2.5 \text{ dB} \quad \text{(Fig. 4-4)} \]

Insertion of the mid-band parameters into Eq. 6.14 gives $T_c = -54 \text{ dB re 2m}$ from target center. This is the frequency domain target strength which must be transformed to the time domain for comparison with Fig. 6-7. The data are Gaussian bandlimited $2.75 < ka < 10.0$; therefore, the conversion of $T_c$ to the time domain simply involves calculating the magnitude of the filter impulse response, $|g(t_o)|$ given in Table. 2.2. Then the peak magnitude for radiation of the initial compressional wave which circumnavigates the empty shell is

\[ 20 \log p_s(t) = T_c - 18 = -72 \text{ dB re 2m}. \quad (6.15) \]

This is very near to the measured target strength observed in Fig. 6-7 at $t \approx 300 \mu s$.

A second round trip of the compressional wave gives a peak level for a second return which is below the noise floor. In other words, the compressional wave contribution has decayed to a level below the noise floor within two axial circumnavigations of the shell. The signals for $300 < t < 600 \mu s$ must be caused by compressional waves which are delayed due to scatter to flexural waves which propagate only on the endcaps and subsequently scatter back to radiating compressional waves. For example the duplicate compressional wave observed in Fig. 5-7 generates a backward propagating compressional wave which arrives at $t \approx 500 \mu s$.

**Initial Compressional Wave Reflection at First Ring**

The initial part of the elastic response ($t < 200 \mu s$) for the internally loaded shells is characterized by a sequence of rapidly decaying peaks that do not occur in the empty shell response. The spacing of these peaks approximately equals the roundtrip time
for a compressional wave to run from the endcap to the first ring and return.

I investigate the peak level of the radiation from first ring scattered arrival using Eq. 6.14 replacing \( R_{pp} \) with \( R_{rpp} \). The path length for this process is \( P = .30m \). Here \( n_t = 4 \), \( n_r = 1 \), and \( R_{rpp} = -1 dB \) (Table 4.2). Otherwise, I use the same mid-band \((ka = 6.5)\) parameters as before (Eq. 6.3.1) in Eq. 6.14 to get

\[
20 \log p_s(t) = -59 dB \text{ re } 2m .
\]

This matches the \( t = 100\mu s \) peak arrival denoted in Figs. 6-8 and 6-9.

**Radiation from the First Ring**

In Chap. 4, the radiation coefficients are estimated for both flexural and compressional wave incidence. The coefficients are normalized relative to the incident sound pressure. These curves are based on the elastic wave displacements estimated from the disk and cone interaction model. At mid-band in the beam direction, the target strengths for the radiation are shown in Fig. 4-14, and where \( T_c = -65 dB \text{ re } 1m \) for the initial flexural wave and \( T_c = -90 dB \text{ re } 1m \) for the initial compressional wave. I apply cylindrical spreading and a Gaussian filter to these frequency domain target strengths to get

\[
20 \log p_s(t) = -86 dB \text{ re } 2m
\]

for the flexural wave and

\[
20 \log p_s(t) = -111 dB \text{ re } 2m
\]

for the compressional wave. By applying the dipole radiation pattern to these expressions to get the levels in the bow incidence direction, both of these contributions fall below the noise floor. In other words, although the perpendicular sector beamforming analysis suggests observable levels of radiation at the rings, this radiation does not contribute to the backscatter near axial incidence.
<table>
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<tr>
<th>Shell Configuration</th>
<th>Initial Decay Rate $dB/ms$</th>
<th>Later Decay Rate $dB/ms$</th>
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</thead>
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<td>—</td>
</tr>
<tr>
<td>Ringed</td>
<td>64</td>
<td>8</td>
</tr>
<tr>
<td>Internalled</td>
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<td>8</td>
</tr>
<tr>
<td>Sandwich</td>
<td>62</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 6.1: Backscattered decay rates for the 3 shells insonified at bow aspect

### 6.3.2 Decay Rates

Most of the contributions to the backscatter cannot be interpreted individually due to dispersion, multiple scattering and interference. It is possible, however, to explain the observed decay rates listed in Table 6.1.

#### Decay Rates of the Empty Shell

The early response has been shown to be driven by the compressional wave. One scenario with which to estimate the expected decay rate for the empty shell is to consider a compressional wave which circumnavigates the shell. The loss mechanisms with respect to the compressional wave energy are transmission at the slope discontinuities, and radiation. In one round trip on the circular cylindrical section, the compressional wave is transmitted four times, $T_{pp} = -2.5 \ dB$ per transmission, and radiates from the cylindrical plus conical shells ($\Delta c_p P = 3 \ dB$, where $P = 1.68m$), and from both spherical shell endcaps ($\Delta = 5dB$). The round trip time, $t_{RT} = 0.35ms$, yields a decay rate of 50 $dB/ms$. This is much greater than the decay rate observed in Fig. 6-7 (33 $dB/ms$). This may be partly explained by considering that compressional waves scatter to flexural waves that store energy and reradiate by scatter at subsequent discontinuities on the near endcap. As shown in Sec. 5.2, the delay associated with this process is 50 to 200$\mu s$ which falls well within the duration of the initial decay process. A more important consideration, however, is to allow that, as for rings, the scatter at multiple slope discontinuities is a coherent process. Therefore, the sequential losses at
<table>
<thead>
<tr>
<th>Discontinuity</th>
<th>Delay (microsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cone/Cylinder</td>
<td>37</td>
</tr>
<tr>
<td>Ring I</td>
<td>73</td>
</tr>
<tr>
<td>Ring II</td>
<td>162</td>
</tr>
<tr>
<td>Ring III</td>
<td>234</td>
</tr>
<tr>
<td>Ring IV</td>
<td>319</td>
</tr>
</tbody>
</table>

Table 6.2: Delays for the initially excited compression wave to travel to shell discontinuities and return to the insonified endcap. Arrival times are reduced relative to the first arrival from the insonified endcap.

Each discontinuity may not be simply be additive, rather something more complicated as shown for the rings.

The group speed of flexural waves on the circularly cylindrical shell at mid-band is approximately 700 m/s; therefore the one-way travel time for flexural waves on the cylindrical section (.74 m) is approximately 1050 μs. Thus the late arrival packet, \( t > 10^3 \mu s \) corresponds to initially excited flexural waves traveling to the far endcap and scattering to compressional waves (or conversely, the initial compressional wave scattering to a flexural wave at the cone/cylinder discontinuity at the far endcap.) We saw this more clearly in the beamforming analysis of Sec. 5.3.2.

**Decay Rates of the Internally Loaded Shells**

In the case of backscatter from the ringed and internalled shells, decay rates are suggested for \( t < 200 \mu s \) by dashed lines in (Figs. 6-8 and 6-9). A second decay rate is suggested for \( t > 200 \mu s \).

The initial decay is caused by backward and forward propagation of the compressional waves within the first bay. Once again the loss mechanisms with respect to the compressional wave energy are scatter at the discontinuities, and radiation. The scattering losses include scatter to flexural waves at both the slope discontinuity and the ring, and transmission through the ring. In one propagation length within the first bay, the compressional wave reflects once at the ring \((R_{rpp} = -1 dB\) per reflec-
tion), it transmits through 4 slope discontinuities \(T_{pp} = -2.5 \, dB\), radiates from the cylindrical and conical sections \(\Delta t_{cp}P = 1 \, dB\) and \(P = 0.77m\), and radiates from the spherical sector of the endcap \(\Delta = -2 \, dB\). In Chap. 4, I show that the first and second ring are strongly coupled; therefore, I take the average path to include propagation to the second ring. The round trip time, \(t_{RT} = P/c_p\) is \(t \sim 0.16ms\). A decay rate of 87 \(dB/ms\) is calculated using these parameters. As with the empty shell, this estimate is vastly different than the initial decay rates observed in Figs. 6-8 (64 \(dB/ms\)) and 6-9 (65 \(dB/ms\)) for the ringed and internalled shells respectively. Here again the reduction in the measured decay rates relative to estimates may be tied up in not properly considering the combined interaction of the slope discontinuities and the rings as a complicated coherent process. Despite the difficulty in accurately predicting the initial decay, this calculation is evidence of why the initial decay rates for the internally loaded shells are much greater than that of the empty shell. The cause is an increased frequency of wave scattering interactions at both the rings and the slope discontinuities.

Beyond the initial part of the elastic response, there is a secondary response with reduced decay rate for \(t > 200\mu s\). The transition time \(\tau\) corresponds to the one way travel delay for a flexural wave to propagate from the spherical section of the endcap to the first ring. The ring provides a mechanism which introduces stored flexural wave energy to the scatter earlier in time via reflection and wave conversion. The subsequent decay rates are greatly reduced to 8 \(dB/ms\) for both the ringed and internalled shells indicating that flexural waves play the primary role later in time. This observed decay rate is consistent with a flexural wave process. Recall that in Sec. 5.4, I estimated the decay rate for a flexural wave contained within a bay confined by two rings. Here I make the same calculation for the first bay which extends from the near endcap and the first ring. From Table. 4.3, the ring reflection coefficient is \(R_{rff} = -1 \, dB\). The remainder of the energy is transmitted to the subsequent bay, scattered to compressional waves which radiate, or scattered directly to the acoustic field. Similarly, when the flexural wave interacts with a slope discontinuity, the sum of its transmission and reflection coefficients is \(T_{ff} \oplus R_{ff} = -2 \, dB\) where \(\oplus\) is a
logarithmic additive operator. I therefore assume an average of 1.8 $dB$/interaction loss, where the lost energy is that energy which is scattered to wave fields which radiate. The flexural wave speed at mid-band is 700$m/s$ (Fig. 5-2). I take the average propagation length between discontinuities with the first bay as 10$cm$ (The distance from the cone/cylinder junction to the ring). The average group speed gives $i$ interactions/$ms$, or equivalently a decay of 10 $dB/ms$, consistent with the measurements.

**Summary of Decay Rate Analysis**

The results of the previous analysis are summarized in Table 6.1. Overall the analysis suggests that the initial decay rates for all four shells are primarily controlled by compressional wave processes. However the measured decay rates are less than those predicted by radiation and scatter models. I believe the models used to predict these decays are too simple, and that coherent interactions between each of the shell discontinuities must be considered to fully describe the backscatter. Flexural wave processes become primarily important following a delay approximately equal to the one-way propagation time for a flexural wave to propagate to the first important structural discontinuity beyond the insonified endcap at mid-band ($ka = 6.5$). The first discontinuity for the internally-loaded shells is the first ring stiffener while for the empty shell it is the far endcap. The decay rates for the ringed and internalled shell are essentially the same, indicating that the sprung internals provide neither additional damping nor additional energy storage mechanisms.

### 6.4 Radiation Patterns

I now consider the effect of increasing the incidence angle and the radiation angle. All of the analyses in the thesis show that the scatter in the bow incidence region is predominantly from endcap radiation. Although other portions of the shell also radiate (as seen clearly in Chap. 5), they do not contribute to the backscatter. Thus, the radiation pattern is the same as the endcap beampatterns already described in
Figure 6-10: Bistatic radiation patterns for the empty(solid), ringed(dashed) and internalled(dash-dot) shells for $\phi_i = 0^\circ$ ($2.75 < ka < 10$).

Sec. 3.6. The bistatic radiation patterns for $\phi_i = 0^\circ$ are shown in Fig. 6-10 and are the same for all the shells out to the 3 $dB$ down point at $\phi_s = 15^\circ$. These radiation patterns are similar to the predicted bistatic radiation patterns based on a circular disk array plotted in Fig. 3-20 where the 3 $dB$ down point is at $\phi_s = 16^\circ$. Beyond $\phi_s = 15^\circ$, the internally loaded radiation patterns diverge from the empty shell curve. A new radiation mechanism contributes for the internally-loaded shell backscatter for $\phi_s > 15^\circ$ which is shown to be forced wave scatter from the rings.

To investigate the possibility of new contributions, I have plotted the $\phi_i = 25^\circ$ monostatic timeseries for the empty shell and internalled shell in Fig. 6-11 and Fig. 6-12 respectively. The empty shell backscatter is very similar to the bow incidence backscatter. There is a substantial reduction in the first part of the response but the prominent feature remains a series of moderately decaying peaks which diminish to the noise floor for $t > 800\mu s$. The late flexural wave arrivals are now in the noise.

The internalled shell backscatter also has features very similar to those at bow incidence. The most likely mechanism to contribute to the backscatter other than radiation from the endcap is the forced wave radiation from the rings that was shown to be important in the beamforming analysis. The arrival times for forced wave
Figure 6-11: Backscatter envelope of the Gaussian bandlimited impulse response for empty shell at $\phi_i = 25^\circ$, $(2.75 < ka < 10.0$ and $r = 2m$ from target center). The envelope is target strength in $dB re 2m$ from the target center.

Figure 6-12: Backscatter envelope of the Gaussian bandlimited impulse response for internalled shell at $\phi_i = 25^\circ$, $(2.75 < ka < 10.0$ and $r = 2m$ from target center). The envelope is target strength in $dB re 2m$ from the target center. The vertical dashed lines indicate the arrival times for forced wave scatter from the rings.
<table>
<thead>
<tr>
<th>Discontinuity</th>
<th>$\phi_i = 0^\circ$ delay (microsec)</th>
<th>$\phi_i = 25^\circ$ delay (microsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cone/Cylinder</td>
<td>84</td>
<td>39</td>
</tr>
<tr>
<td>Ring I</td>
<td>210</td>
<td>152</td>
</tr>
<tr>
<td>Ring II</td>
<td>526</td>
<td>438</td>
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<td>Ring III</td>
<td>779</td>
<td>669</td>
</tr>
<tr>
<td>Ring IV</td>
<td>949</td>
<td>826</td>
</tr>
<tr>
<td>Cone/Cylinder</td>
<td>1075</td>
<td>942</td>
</tr>
</tbody>
</table>

Table 6.3: Delays for the forced elastic wave to travel to shell discontinuities and scatter to the monostatic receiver. Arrival times are reduced relative to the first arrival from the insonified endcap.

radiation are listed in Table 6.3, and in Fig. 6-12 vertical dashed lines correspond these arrival times. Clearly the arrival which coincide with the dashed lines compete in importance with the endcap radiation. For large incidence angles, these forced wave contributions will increase by $\sin^2\phi_i$ as shown in Chap. 2, whereas the radiation from the endcap falls off with the with the beampattern. $\phi_i = 25^\circ$ represents a transition where these contributions are equally important.

6.5 Summary

I have shown that the circular disk coupling model introduced in Chap. 3 properly predicts the displacement amplitudes of the initially excited compressional wave over most of the frequency band except for dips shown by the data at $ka = 3.5$ and $ka = 5$. The dips may correspond to destructive interference caused by additional radiation near the shell ring frequencies. At higher frequencies the disk model predicts a low value indicating that trace-matched compressional wave excitation becomes more important for $ka > 9$.

Based on the observations of Chap. 4 and Chap. 5, the rings inhibit the propagation of the axisymmetric waves to the far end of the shell, redirecting most of the elastic wave energy back towards the insonified endcap. The result of the ring reflections is

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increased peak levels in the backscatter early in time as compared with the empty shell. Both compressional and flexural waves contribute to the backscatter but at different times. A transition time $\tau$ equal to the time for a flexural wave at mid-band to propagate to the first ring, separates these two regions. For $t < \tau$, compressional waves contribute with large peak amplitudes but decay rapidly due to radiation attenuation and scatter to flexural waves. For $t > \tau$, flexural waves contribute via scatter to radiating compressional waves with lower peak amplitudes. The decay rates in the flexural wave response region are greatly reduced due to low group speeds which yield low interaction rates and less frequent conversion to radiation. The observed decay rates for both the ringed and internalled shells are the same, indicating that the sprung internals are not important at bow incidence over most of the measured frequency band.

Although radiation from the rings is well resolved in the perpendicular sector beamforming analysis, this radiation is not detected in the backscatter. Two causes for this are low ring radiation levels relative to endcap radiation levels and the dipole beampattern of the ring radiation which gives levels that are 7.5 dB down as compared with the beam direction at $\phi_i = 25^\circ$. There is, however, evidence of contributions from the forced wave scatter at the rings for $\phi_i = 25^\circ$ where the levels are comparable to the later in time flexural response levels.
Chapter 7

Conclusions

7.1 Summary of Results

I have shown that the backscatter from finite cylindrical shells excited by a sound wave near axial incidence is caused by three dominant sources, the geometric response, radiation from axisymmetric compressional waves and and radiation from axisymmetric flexural waves which are converted to compressional waves. The insonified endcap is the source by which each of these components contributes to the backscatter. The geometrically scattered contribution is the first arrival. This is followed by radiation from compressional wave driven processes which decay rapidly. The backscatter then transitions to radiation from flexural wave driven processes, where the transition time equals the group delay for a flexural wave at mid-band to travel to the first structural discontinuity beyond the insonified endcap. The extent of the region where the axisymmetric structural waves remain predominant extends for $\phi_i = 0^\circ$ to $\phi_i \leq 25^\circ$, where for $\phi_i > 25^\circ$, the scatter of the forced elastic waves at the ring stiffeners becomes important relative to the axisymmetric wave radiation.

The endcaps are a critical mechanism of the scattering process near axial incidence. The endcaps determine the magnitude of the geometric scatter and provide the primary location for coupling of the incidence sound wave to elastic shell waves. The frequencies
considered represent a transitional frequency range in terms of the mechanism for this coupling. At higher frequencies \((ka > 9)\), the interaction is local to the spherical sector of the endcap, where the criterion for localization is that it is equal to the Fresnel width of the incident sound pulse relative to the endcap dimensions. In this frequency range, the geometric scatter is equivalent to that from a spherical shell of the same dimensions. In addition, the coupling mechanism is the same as that for a spherical shell, that is, trace-matched excitation of compressional waves. Where the Fresnel width is large compared to the dimensions of the spherical section of the endcap, it marks a transition in the manner in which the shell responds. At lower frequencies \((ka < 8)\) the interaction involves the entire endcap. The curvature of the spherical sector of the endcap can be neglected, so the interaction is well represented as a flat circular disk coupled to a conical shell. The motion of the circular disk injects energy into both axisymmetric compressional and flexural waves at the slope discontinuity which connects the disk to the circular shell.

Peak levels of target strength are found for observation angles \(\phi_s < 15^\circ\). This is an indication of the radiation pattern for the endcap. The mechanisms for radiation are reciprocal to the coupling mechanisms just described. Thus the radiation pattern is equivalent to that of a circular plane array which has a diameter equal to the projected diameter of the spherical sector of the endcap. At mid-band, the 3 \(dB\) down radiation beamwidth is \(\phi_s = 16^\circ\).

The endcap has discontinuities in both slope and curvature. The effect of the slope discontinuity provides a significant mechanism for conversion between axisymmetric compressional and flexural waves. Thus energy in the compressional waves can be stored in flexural waves, and energy in the flexural waves can radiate via conversion to compressional waves.

I extended the work of Corrado [9] to consider effects of stiffness on the scatter from a ring stiffener. The stiffness of the ring is important near resonance frequencies of the ring (which are located near the shell ring frequency); above those the interaction is mass controlled. In the mass controlled frequency range a single ring stiffener is highly
reflective both for compressional and flexural waves inhibiting wave propagation to the far endcap. However, the coupling of subsequent rings provides a mechanism for the creation of transmission bands that are located in frequency where elastic half-wavelengths are equal to the ring spacing. The irregular spacing of the rings is found to cause localization of the structural waves near the insonified end of the shell. The rings strongly redistribute the location of radiation sources along the length of the shell and thereby enhance the backscatter. At resonance, the ring cannot apply a significant force to the shell, thus the shell and rings decouple permitting efficient transmission of the elastic waves across the attachment.

Radiation from the rings is a weak process relative to other radiation mechanisms. The radiation pattern from the rings is primarily dipole-like where the important interaction force is the radial ring force. The interaction moment provides for much weaker radiation which has a radiation pattern that is the same as a lateral quadrupole. Therefore the ring radiation can be detected in the beam direction, but this radiation gives a negligible contribution to the backscatter. The scatter of the forced wave at the rings begins to contribute significantly to the backscatter for $\phi_s > 25^\circ$.

The sprung internals only affect wave propagation near ring resonances. This is explained by two observations. The large mass loading of the ring yields a strong impedance mismatch between the shell and the rings. Thus, energy transfer to the internal structures is necessarily weak. Also, the lowest order resonance frequencies of the sprung internals are an order of magnitude below the lowest frequency in the measurement band, which further decouples the two systems.

7.2 Contributions

- The high quality of the NRL scattering data has provided a unique opportunity to make reliable comparisons of scatter models with the measured scatter. The accuracy of the measurements, which exhibit generally less than 1 $dB$ variance, requires that models also predict the measurements within 1 or 2 $dB$. As a
result quantitative deterministic models representing various components of a complex scattering process could be directly validated with the data.

- Proper modeling of the initial interaction of the incident sound wave with the endcap must account for the Fresnel width of the incident sound wave relative to the endcap dimensions. A transition occurs where the Fresnel width defined by a $\pi/2$ phase criterion, is larger than the size of the endcap. Where the Fresnel width is small compared to the size of the endcap, the interaction is a localized phenomenon relative to the geometry of the endcap. Where the Fresnel width is large, the interaction involves the dynamics of the entire endcap.

- The coupling to axisymmetric elastic waves via the endcap is predominant within 20° of axial incidence. This represents approximately one-quarter of the total angular incidence region.

- The elastic properties of the internal structures are important only at resonance, otherwise, the attachment condition is of primary importance. The stiffness of the rings give rise to a frequency band of high transmission with respect to compressional waves due to the creation of a rolling resonance at which the rings partially decouple from the shell motions.

- The presence of multiple rings must be considered with respect to the elastic wave propagation. The coupling of adjacent rings leads to transmission bands near frequencies at which an integer number of elastic half-wavelengths equals the spacing of the rings. If the rings are not strictly periodic, the irregularity causes localization [60] such that the spatial distribution of wave energy decays exponentially away from the region of localization. The localization is strong for systems like the one studied here, where the reflection coefficients for a single ring are large and the frequency shift due to irregularity is large compared with the bandwidth of the passbands. The irregularity does not modify the transmission near the ring resonances.

- Within the radiation lobe of the endcap, the only important contribution to the
the scatter is radiation from the endcap. Other radiation sources such as ring radiation are substantially weaker. For the shell configurations studied, the ring radiation levels are 30 \( dB \) below the peak endcap radiation levels. Ring radiation contributes in a more important way near the beam radiation direction due to its dipole radiation pattern. Outside the endcap radiation lobe, forced wave scatter from the impedance discontinuities becomes an important contribution to the backscatter.
Appendix A

Shell and Plate Equations

Thin shell theories are available corresponding to each of the geometries which make up the target shells. These equations are widely available in the literature and therefore are not developed here. The use is thin shell equations in this frequency range is justified because the basic assumptions necessary for their use are satisfied. The important considerations are

- The elastic wavelengths are large compared with the shell thickness ($\lambda_s > 20h$).
  In these cases the transverse shear and rotatory inertia may be neglected.

- The radii of curvature are large compared with the shell thickness.

This thesis exclusively considers axisymmetric motions of the shells, so I present the resulting forms of the equations with appropriate simplifications to account for axisymmetry. Additionally fluid loading is included as a modification to the in-vacuo equations.

A.1 Donnell thin cylindrical shell equations

For the cylindrical shell, the Donnell thin shell differential equations [1] are used. An assumed harmonic time dependence $e^{-i\omega t}$ is suppressed. As I am only interested in
axisymmetric shell motions, all derivatives with respect to circumferential angle may be neglected. Additionally, axisymmetric circumferential motions are decoupled from radial and axial motions, as well as from the fluid and may also be neglected. The two coupled equations that result from these considerations are

\[
\frac{\partial^2 u}{\partial z^2} + \frac{v}{a} \frac{\partial w}{\partial z} - \frac{\ddot{u}}{c_p^2} = 0 , \quad (A.1)
\]

\[
\frac{\nu}{a^2} \frac{\partial u}{\partial z} + \frac{w}{a^2} + \beta^2 \alpha^2 \frac{\partial^4 v}{\partial z^4} + \frac{\ddot{w}}{c_p^2} - \frac{p_s(1 - \nu^2)}{Eh} = 0 . \quad (A.2)
\]

Here \( \beta^2 = h^2/12a^2 \) and \( p_s \) accounts for external loading which acts normal to the shell surface. It is assumed that the displacements are of the form

\[
\begin{align*}
   u &= U e^{ik_z z} , \\
   w &= W e^{ik_z z} .
\end{align*}
\]

The differential equations are most readily manipulated in algebraic form by separation of variables and using the wavenumber transform in axial wavenumber, \( k_z \), which is given by

\[
\hat{f}(k_z) = \int_{-\infty}^{\infty} f(z) e^{-ik_z z} dz . \quad (A.3)
\]

The resulting linear algebraic equations are

\[
\begin{align*}
   [-\alpha^2 + \Omega^2] U + iv\alpha W &= 0 , \quad (A.4) \\
   iv\alpha U + [1 + \beta^2 \alpha^4 - \Omega^2] W &= \frac{a^2}{c_p^2 \rho_s} p_s , \quad (A.5)
\end{align*}
\]

where \( \alpha = k_z a \) and \( \Omega = \omega a/c_p \). Fluid loading can be accommodated in Eq. A.5 using the momentum equation,

\[
\frac{\partial p_s}{\partial r} = -\rho_s \frac{\partial \dot{w}}{\partial t} , \quad (A.6)
\]

so that Eq. A.5 may be be rewritten as

\[
iv\alpha U + [1 + \beta^2 \alpha^4 - \Omega^2 + F_u E_f k^2 a^2] W = 0, \quad (A.7)
\]
where
\[
F_u = \frac{H_0(k_r a)}{k_r a H'_0(k_r a)} , \quad E_f = \frac{c_p^2 \rho_o a}{c_p^2 \rho_s h} , \quad k_r = (k_o^2 - k_z^2)^{1/2} . \tag{A.8}
\]

The dispersion relation is obtained from the determinant of the displacement coefficients from Eqs. A.4 and A.7
\[
(1 + \beta^2 \alpha^4 - \Omega^2 + F_u E_f k^2 a^2)(-\alpha^2 + \Omega^2) + \nu^2 \alpha^2 = 0 . \tag{A.9}
\]

The complex axial wavenumber roots, \( k_{zr} + k_{zi} \), which satisfy the dispersion equation can be found numerically using a root finder. The resulting dispersion relations are discussed in Chapter 5.

### A.2 Conical shell equations

The equations of motion for a thin conical shell in-vacuo have been formulated assuming thin shell theory and small vertex angle, \( \alpha_c \) and using the method of multiple scales [47]. The reference coordinates for a conical shell are shown in Fig. A-1. The resulting equations are similar in form to the Donnell cylindrical shell equations. For axisymmetric modes, the leading order equations simplify to

\[
[\eta^2(s) - k_p^2 a^2(s)] U + i \nu \eta(s) W = 0 , \tag{A.10}
\]

\[
i \nu \eta(s) U + [\beta^2 \eta^4(s) + 1 - k_p^2 a^2(s)] W = 0 . \tag{A.11}
\]

Here \( a \) and \( \eta \) are normalized functions of axial length times a small parameter, \( s(z) = z \tan \alpha_c \). Thus \( a(s) = a_o + s \) is the local radius, where \( a_o \) is the reference radius; here \( a_o \) is equal to the cylinder radius. To relate terms to the Donnell equations, note that \( k_p^2 a^2(s) \) is a local equivalent to the \( \Omega^2 \) term in the cylindrical shell equations and \( \eta(s) \) represents the local solution for axial wavenumber. For the leading order solution, \( U \) corresponds to a displacement along the shell axis, and \( W \) corresponds to displacement normal to the shell axis. Fluid loading is accommodated in the manner described for
the cylindrical shell modifying Eq. A.11. By including fluid loading, Eqs. A.10 and A.11 can now be rewritten in the form of Eqns. A.4 and A.7,

\[
[a^2(s) - \Omega^2(s)] U + i\nu \alpha(s) W = 0 \,,
\]

(A.12)

\[
i\nu \alpha(s) U + [\beta^2 a^2(s) + 1 + \Omega^2(s) + F_u(s)E_f(s)k^2 a^2(s)] W = 0 \,.
\]

(A.13)

The dispersion relation takes the same form as for the cylindrical shell but as a function of axial position along the conical shell. Consequently, the same methods can be applied to determine the conical shell dispersion relations.

## A.3 Spherical shell equations

The axisymmetric differential equations of motion for a spherical shell are rather cumbersome and not printed here. Rather I present the algebraic form of the two coupled equations [1] as

\[
[V_n \cdot (\nabla^2 - (1 + \beta^2) \gamma_n) + \beta^2 \gamma_n + 1 + \nu] V_n - [\beta^2 \gamma_n + 1 + \nu] W_n = 0 \,,
\]

(A.14)

\[-\lambda_n [\beta^2 \gamma_n + 1 + \nu] V_n + [\Omega^2 - 2(1 + \nu) - \beta^2 \lambda_n \gamma_n] W_n = -\frac{a^2}{c_p^2 \rho_s \rho_s} \rho_s \,,
\]

(A.15)
where \( \tau_n = \lambda_n + \nu - 1 \), \( \lambda_n = n(n+1) \), and \( \Omega = \omega a/c_p \) is the non-dimensional frequency parameter. Also \( p_{s_n} \) is the modal contribution to the external loading in the radial direction.

The determinant of the displacement coefficients gives the dispersion relation for the spherical shell in vacuo (i.e. setting \( p_{s_n} = 0 \)). Specifically,

\[
\begin{align*}
\Omega^4 &- [1 + 3\nu + \lambda_n - \beta^2(1 - \nu - \lambda_n^2 - \nu \lambda_n)]\Omega^2 + \\
(\lambda_n - 2)(1 - \nu^2) + \beta^2[\lambda_n^3 - 4\lambda_n^2 + \lambda_n(5 - \nu^2) - 2(1 - \nu^2)] &= 0. \quad (A.16)
\end{align*}
\]

Fluid loading can be readily incorporated in the dispersion relation by relating the radial surface velocity to the fluid pressure using the momentum equation. The dispersion equation can be rewritten as

\[
(1 - \xi_n)\Omega^4 - [1 + 3\nu + \lambda_n - \beta^2(1 - \nu - \lambda_n^2 - \nu \lambda_n) + \xi_n(1 + \beta^2)\tau_n]\Omega^2 + \\
(\lambda_n - 2)(1 - \nu^2) + \beta^2[\lambda_n^3 - 4\lambda_n^2 + \lambda_n(5 - \nu^2) - 2(1 - \nu^2)] = 0, \quad (A.17)
\]

where \( \xi_n \) is the fluid loading parameter given by

\[
\xi_n = -\frac{i}{c_p \rho_s h \Omega} \frac{1}{z_n}, \quad (A.18)
\]

and \( z_n \) is the modal radiation impedance

\[
z_n = i\rho_c c_o \frac{\hat{h}_n(ka)}{\hat{h}'_n(ka)} . \quad (A.19)
\]

To simplify the solution of Eq. A.17, since \( \beta^2 << 1 \) for thin shells, terms including \( \beta^2 \) can be neglected. Also, the radiation impedance, \( z_n \) can be decomposed into its real and imaginary components as

\[
z_n = r_n - i\omega m_n, \quad (A.20)
\]

where \( r_n \) can be interpreted as the radiation resistance and \( m_n \) is the accession to
inertia due to the added mass of the fluid, so that Eq. A.17 can be rewritten

\[
(1 + \frac{m_n}{h \rho_s}) \Omega^4 + i \frac{a}{h c_p \rho_s} \frac{r_n}{\Omega^3} - [2(1 + \nu) + \Upsilon_n (1 + \frac{m_n}{h \rho_s})] \Omega^2
- i \frac{a}{h c_p \rho_s} \Upsilon_n \Omega + (\lambda_n - 2)(1 - \nu^2) = 0 .
\]  \hspace{1cm} (A.21)

The undamped natural frequencies can be found by setting \( r_n = 0 \) and solving for \( \Omega \).
Appendix B

Ring and Shell Interaction

The development of this section follows a similar development by Corrado [9]. The equations are simplified to consider only axisymmetric \((n = 0)\) excitation. Additionally, elastic terms are added to the ring equations. The elastic terms account for the stiffness of the ring and have their most important effects at the resonances of the ring.

The geometry of the ring interaction is shown in Fig. B-1 to establish a reference system. Without loss in generality, the ring is placed at \(z = 0\). The connecting condition is clamped, so all forces and moments are fully transmitted from the shell to the ring. At the point of attachment, the boundary condition is continuity of axial and radial displacement and continuity of rotation.

Figure B-1: Coordinate reference for shell/ring dynamic model

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The solution technique is to first decompose the shell-ring system into two components and establish the compliance matrices for each component – an infinite cylindrical shell and an elastic ring. These systems can then be coupled via the above boundary condition. This problem is formulated for the case of excitation by an incident elastic wave; thus for a prescribed incident wave, the resulting interaction forces and moment at the ring can be found. The interaction forces are then applied to cylindrical shell at \( z = 0 \) using the Donnell equations. Residue calculus is used to determine the reflection and transmission coefficients of the prescribed incident wave. The method of stationary phase is used to determine the radiation from the ring. These methods are discussed here briefly; more detailed discussions can be found in the work of Corrado [9] and Guo [29].

### B.1 Shell Compliance Matrix

Time harmonic excitation of the form \( e^{-i\omega t} \) is assumed and suppressed for all of the analysis. Thin shell theory [1] is used to describe the shell motions. The linear algebraic form of the Donnell equations are given by Eqs. A.4 and A.7 and are rewritten here in matrix form

\[
\begin{bmatrix}
\alpha^2 - \Omega^2 & -i\nu \alpha \\
 i\nu \alpha & \left[ 1 + \beta^2 \alpha^4 - \Omega^2 + F_u E_f k^2 a^2 \right]
\end{bmatrix}
\begin{bmatrix}
\dot{u} \\
\dot{w}
\end{bmatrix} = \frac{a^2}{c_p^2 h \rho_s}
\begin{bmatrix}
F_z \\
F_r - ikz M
\end{bmatrix}.
\] (B.1)

The symbols have previously been defined in Appendix A. Forcing terms appear on the right hand side of the fluid-loaded shell equations and account for the interaction forces between the shell and the ring. These forces and moments are also axisymmetric and are applied at \( z = 0 \). In the spatial domain, they are defined as \( F_z \delta(z) \), \( F_r \delta(z) \), and \( M \delta'(z) \).

Rearranging the equations to give displacements as a function of force defines the shell compliance matrix \( S \) in the wavenumber domain, where I use the term compliance...
to describe the ratio of displacement to force. The coupled equations are

\[
\begin{bmatrix}
\dot{u} \\
-i\dot{\omega}
\end{bmatrix} = \frac{a^2}{c_p^2 h \rho_s \bar{D}} \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} \begin{bmatrix}
F_z \\
-i(F_r - ik_z M)
\end{bmatrix},
\]  

(B.2)

where \(D\), the dispersion relation for axisymmetric compressional and flexural waves on the cylindrical shell, is the determinant of the matrix given by

\[
D = (1 + \beta^2 \alpha^4 - \Omega^2 + F_u E_f k^2 a^2)(-\alpha^2 + \Omega^2) + \nu^2 \alpha^2, \quad (B.3)
\]

and the compliance terms are given by

\[
S_{11} = 1 + \beta^2 \alpha^4 - \Omega^2 + F_u E_f k^2 a^2, \\
S_{12} = S_{21} = -\nu \alpha, \\
S_{22} = \alpha^2 - \Omega^2.
\]  

(B.4)

It is useful to separate \(F_r\) and \(M\) and include an equation describing the rotations of the shell about the \(\theta\)-direction. The rotation, \(\ddot{\omega}\), is related to the radial displacement by \(\ddot{\omega} = \partial \dot{\omega} / \partial z = ik_z \dot{w}\) or

\[
\ddot{\omega}a = i\alpha \dot{w}.
\]  

(B.5)

This yields three equations,

\[
\begin{bmatrix}
\dot{u} \\
-i\dot{\omega} \\
-i\omega^2 \alpha
\end{bmatrix} = \frac{a^2}{c_p^2 h \rho_s \bar{D}} \begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
S_{21} & S_{22} & S_{23} \\
S_{31} & S_{32} & S_{33}
\end{bmatrix} \begin{bmatrix}
F_z \\
-iF_r \\
-iM/a
\end{bmatrix}.
\]  

(B.6)

The new compliance terms follow directly from the relations above as

\[
S_{13} = S_{31} = i\alpha S_{12}; \\
S_{23} = S_{32} = i\alpha S_{22}; \\
S_{33} = -\alpha^2 S_{22}.
\]  

(B.7)
The compliance at the shell/ring junction in the spatial domain is

\[ f(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(k) e^{ikz} dk, \tag{B.8} \]

with solutions sought at \( z = 0 \). The transform variable of interest is

\[ \hat{f}(k) = \frac{a^2}{c^2_p h \rho_s} \frac{1}{D} S_{ij}(k) \tag{B.9} \]

so the transform of interest is

\[ s_{ij}(0) = \frac{1}{2\pi} \frac{a^2}{c^2_p h \rho_s} \int_{-\infty}^{\infty} \frac{S_{ij}}{D} dk_z. \tag{B.10} \]

Here, \( s_{ij} \) is the compliance at the attachment representing the ratio of the displacement in the direction \( i \) due to a force at the attachment in the direction \( j \). \( S_{12}, S_{21}, S_{23}, \) and \( S_{32} \) are odd functions in \( k_z \) and integrate to zero; therefore these terms can be neglected. As a result, the three equations governing the shell displacements at \( z = 0 \) are

\[
\begin{pmatrix}
  u \\
  -i w \\
  -i w' a
\end{pmatrix} =
\begin{bmatrix}
  s_{11} & 0 & s_{13} \\
  0 & s_{22} & 0 \\
  s_{31} & 0 & s_{33}
\end{bmatrix}
\begin{pmatrix}
  F_z \\
  -i F_r \\
  -i M/a
\end{pmatrix} +
\begin{pmatrix}
  u_{inc} \\
  -i w_{inc} \\
  -i w'_{inc} a
\end{pmatrix}. \tag{B.11}
\]

Here \( u_{inc}, w_{inc} \) and \( w'_{inc} \) describe the displacement field of the prescribed incident axisymmetric wave. The rotation is proportional to the radial displacement as shown in Eq. B.5. The axial displacement is also directly proportional to the radial displacement by

\[ u_{inc}/w_{inc} = i(1 + \beta^2 \alpha^4 - \Omega^2 + F_u E_f k^2 a^2)/(\nu \alpha). \tag{B.12} \]

This comes directly from the homogeneous cylindrical shell equation of motion in the radial direction.
B.2 Ring Compliance Matrix

The ring compliance matrix is developed in the text (see Eq. 4.26)

B.3 Interaction Forces

The interaction forces are developed in the text (see Eq. 4.31)

B.4 Reflection and Transmission Coefficients

The reflection and transmission coefficients represent the ratio of the radial displacement of the incident wave and the scattered wave. The radial displacement field at the ring is given by Eq. B.11. For a known incident wave amplitude, the expressions for the interaction forces are given by Eq. 4.31. The total displacement field is predominantly comprised of contributions from wavenumber roots of the dispersion equations. For the axisymmetric scenario, the contributing roots correspond to the compressional and flexural waves. In the wavenumber domain the displacements are given by Eq. B.6 as

\[ \hat{w}(k_z) = \frac{a^2}{c_p^2 h_{\rho_s}} \frac{1}{D} \left\{ i F_z S_{21} + F_r S_{22} + M S_{23}/a \right\} . \]  

(B.13)

The wavenumber transform, Eq. B.10, gives the radial displacement field at the ring attachment as

\[ w(0) = \frac{1}{2\pi} \frac{a^2}{c_p^2 h_{\rho_s}} \left\{ i F_z \int_{-\infty}^{\infty} \frac{S_{21}}{D} dk_z + F_r \int_{-\infty}^{\infty} \frac{S_{22}}{D} dk_z + M \int_{-\infty}^{\infty} \frac{S_{23}}{D} dk_z \right\} . \]  

(B.14)

The real part of the integrand is dominated by free wave roots of the shell dispersion relation. By closing the contour, the contributions from each of the propagating free waves are proportional to the residues of the wavenumber integral. The radial
displacement of a transmitted wave at the ring is

\[
\omega_T = \frac{a^2}{c_p^2 h \rho_s} \left\{ \frac{-F_z S_{21} + i F_r S_{22} + i M_n S_{23}/a}{\partial D/\partial k_z} \right\} + \delta(k_z a - k_{zinc} a) \bigg|_{k_z a = \alpha_r} ,
\]

(\text{B.15})

where \( \alpha_r \) is the normalized wavenumber root of the shell axisymmetric dispersion equation corresponding to the transmitted wave of interest and \( k_{zinc} \) is the axial wavenumber of the prescribed incident wave. The radial displacement of a reflected wave at the ring is

\[
\omega_R = -\frac{a^2}{c_p^2 h \rho_s} \left( \frac{F_z S_{21} - i F_r S_{22} - i M_n S_{23}/a}{\partial D/\partial k_z} \right) \bigg|_{k_z a = \alpha_r} .
\]

(\text{B.16})

\section{B.5 Direct Acoustic Scatter from Ring}

The pressure radiated from the ring can be expressed in terms of the total radial displacement field at the ring (Eqn. B.13) via the momentum equation,

\[
\hat{p}_s = \frac{\omega^2 \rho_o}{k_r} \frac{H_o(k_r R)}{H'_o(k_r a)} \hat{w} .
\]

(\text{B.17})

The inverse wavenumber transform (Eq. B.8) recasts the wavenumber form of the pressure in an integral form which can be solved at \( z = 0 \) as

\[
p_s(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{p}_s e^{ik_z z} dk_z .
\]

(\text{B.18})

After some algebra that includes replacing the Hankel function in the numerator with its large argument form, the integral expression becomes

\[
p_s(z, r) \simeq i \frac{k^2 a^2 E_f}{2\pi} e^{-i \pi/4} \int_{-\infty}^{\infty} \sqrt{\frac{2}{\pi k_r r}} \frac{e^{ik_r r + i k_z z}}{k_r a H'_o(k_r a) D} \hat{w}_* dk_z .
\]

(\text{B.19})

Note that \( \hat{w}_* \) is given by

\[
\hat{w}_* = i F_z S_{21} + F_r S_{22} + M S_{23}/a .
\]

(\text{B.20})
The method of stationary phase can be used to provide an approximate expression for the scattered pressure [9]. This requires a new coordinate system $z = R \cos \phi_s$ and $r = R \sin \phi_s$, where $\phi_s = 90^\circ$ is normal to the shell axis. The stationary phase point can be shown to be

$$k_{zs} = k \cos \phi_s .$$  \hspace{1cm} (B.21)

The resulting expression for the radiation from the ring is

$$p_s(R, \phi_s) = \frac{k a E_f e^{ikR}}{\pi} \frac{\hat{w}^*(k_{zs})}{R \sin \phi_s H'_0(ka \sin \phi_s) D(k_{zs})} .$$  \hspace{1cm} (B.22)
Bibliography


