ANALYSIS OF THE CAUSES OF INVENTORY AND PRODUCTION OSCILLATIONS IN A RETAIL-WHOLESALE-FACTORY DISTRIBUTION SYSTEM

by

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Submitted in Partial Fulfillment of the Requirements for the Degree of Bachelor of Science at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY June 1957

Signature of Author

Certified by: 

Accepted by: Chairman, Departmental Committee on Undergraduate Students
20 May 1957

Professor Leicester F. Hamilton
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Sir:

In accordance with the requirements of the degree of Economics, Politics, and Engineering, I herewith submit a thesis entitled "Analysis of the Causes of Inventory and Production Oscillations in a Retail-Wholesale-Factory Distribution System."

Respectfully,
ABSTRACT

ANALYSIS OF THE CAUSES OF INVENTORY AND PRODUCTION
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DISTRIBUTION SYSTEM

by

Malcolm Murray Jones

Submitted to the Department of Economics, Politics, and
Engineering on 20 May 1957 in partial fulfillment of the
requirements for the degree of Bachelor of Science.

There has been a lack of attention focused on the overall
stability of inventory and production distribution systems.
Motivated by this, a dynamic, time-sequence model of a retail-
wholesale-factory distribution system was constructed. This
model contained a variety of time lags, and had certain
inventory policies and ordering policies inherent in its
structure.

Twenty-two variations of this basic model were constructed
which contained changes in the time lags, changes in the
ordering policies, changes in the inventory policies, changes
in the production lead time, establishment of a direct report-
ing line between the retailer and factory, and elimination of
the retail sector.

It was found that changes in the time lags, changes in
the ordering policies, and the establishment of a direct
reporting system all improved the response of the system. To
the contrary, changes in the inventory policies and production
lead time either increased the oscillations or made negligible
improvements on the system's response. The eliminations of
the retail sector vastly improved the system response.

Thesis Supervisor: Jay W. Forrester
Title: Professor of the School of Industrial
Management
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I. INTRODUCTION

A. Historical Background of Inventory Management

The subjects of inventory control and production control may sound like dull and unimportant fields of research. However, it has been repeatedly demonstrated that businessmen need to know more about these matters.

In the early 1900's large inventories were a sign of wealth and businessmen used to boast proudly of their extensive stocks. However, companies holding large inventories in the early 1920's suffered substantial losses when the prices dropped sharply. Since that time businessmen have striven to maintain as small inventories as possible and still satisfy the demand for goods. They have paid very little attention, however, to the possibility of exercising any kind of scientific control of inventories. Only today when executives are beginning to scrutinize carefully all aspects of a business and to apply scientific techniques to some of the age old problems of management, have the subjects of inventory and production control received due consideration.

B. Why Maintain Inventories?

It is well known that for most businesses producing and marketing consumer goods, inventories are necessary if customers' goods are to be quickly filled. Only in the case of special orders will the consumer be willing to wait a
considerable period to time for delivery. In the majority of
instances, if the seller doesn't have what the customer
wants, he can go elsewhere and have his order filled.
Competition forces businesses to maintain inventories. The
problem facing the businessman is to decide how large an
inventory to maintain. Not only must he be able to provide
for the daily sales, but he must be able to supply extra
goods in case of fluctuations in sales. If the inventory is
large enough, the seller won't be faced with the problem of
running out of goods and temporarily or permanently losing
sales and good will to his competitors. On the other hand,
maintaining a large inventory is very costly, and often
unwise. The seller must pay for warehouse space, handling
costs, insurance, etc., besides running the risk that his
product might be superseded by an improved model, go out of
style or, in the case of a non-durable good, spoil.

C. Factors Affecting Inventories

In the simplest case, where the producer of the good
sells directly to the consumer, inventories are also directly
affected by the company's production. These joint effects
of sales and production can be expressed in this simple
equation:

\[ I_t = I_{t-1} + P_t - S_t \]

where \( P_t \) represents the amount of goods produced by the
factory and \( S_t \) the amount of goods sold to the consumer during
the period \( t \). \( I_t \) is the inventory level at the end of period
t, and \( I_{t-1} \) is the inventory level at the end of the preceding period, or at the beginning of period \( t \).

Having established this relation between inventory, sales, and production, it is logical to ask, what are the factors that affect sales and production. It is extremely difficult to discover all the factors that affect sales and next to impossible to accurately predict exactly how they affect sales. This is because the actions and motivations of the consumer who buys the goods are very difficult to determine and describe. However, some generalizations can be made. There are certain natural seasonal fluctuations in sales of goods, such as the increased demand for bathing suits in the summer and overcoats in the winter. In addition, there is a surge of buying in a wide variety of goods near Christmas. Advertising has an effect, but it is difficult to quantitatively describe it. And in the case of most common, non-luxury consumer goods, an increase in the price of the goods will decrease the quantity sold, while a decrease in the price will increase the quantity sold.

These factors help to explain the major changes and shifts of demand, but they do not account for the weekly or monthly fluctuations that can cause havoc with inventories. It is well nigh impossible to relate these fluctuations to specific causes. It is even more difficult to attempt to forecast these fluctuations. Realizing this, the researchers
have gone to statistics for help. Applying probability theory to future sales fluctuations, it is possible to make decisions that, depending on the degree of confidence desired, will be wrong only 33%, 5%, or 1% of the time. These decisions will determine the desired inventory level, and provide the basis for calculating what it costs, in terms of inventory needed, to assure consumer satisfaction for each confidence level. Applying this technique the Bell Telephone Company was able to construct the following table for one of the many items it carries in its inventory:

<table>
<thead>
<tr>
<th>Average number of years each item will go out of stock</th>
<th>Inventory Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$76,000</td>
</tr>
<tr>
<td>2</td>
<td>100,000</td>
</tr>
<tr>
<td>5</td>
<td>134,000</td>
</tr>
<tr>
<td>10</td>
<td>167,000</td>
</tr>
<tr>
<td>&quot;Never&quot;</td>
<td>276,000</td>
</tr>
</tbody>
</table>

The factors that affect production are fortunately more easily discernable. In part they consist of the plant capacity, the labor force, raw materials, and the transportation and distribution system. The problems involved in scheduling these factors so that production can continue unimpeded is a sizeable task. It becomes even more difficult when changes, particularly increases in production rates are required. However, it is possible to directly manipulate these factors and thus control production whereas direct

manipulation of the factors which influence sales is impossible.

D. A New Approach to Inventory Management

The factors influencing production and sales listed above are what might be called microscopic factors, i.e., small individual factors which comprise the whole. The approach used in this study is to analyze the inventory and production control problem from a macroscopic standpoint. That is, analyze the system as a whole, rather than the individual elements such as sales, advertising, production rates, etc. A great deal of work has been done investigating the microscopic factors of customer order sizes, inventory levels, reorder points, production rates, sales forecasts, minimizing stock room costs, etc. However, in the attention to detail the total has been neglected. It's like not "seeing the forest for the trees." In particular this study is concerned with the stability of the entire distribution system, (i.e., its ability to damp out oscillations that may be introduced) and the effect of the time lags, ordering policies, and inventory policies inherent in the system upon its stability. This is a fresh approach, and as of the writing of this thesis, the author knows of only one unpublished work on this topic. This a paper released November 15, 1956, by the General Electric Company (See Bibliography.) However, under the direction of Professor
Jay W. Forrester of the School of Industrial Management, there has been considerable research work done in this area throughout the last year and it is possible that some manuscripts may be shortly forthcoming.

The General Electric paper assumed only a single-loop system. That is, the producer of the good sells it directly to the consumer. In order to be more realistic, a multi-loop system such as the retail-wholesale-factory distribution system commonly used by numerous American industries was analyzed in this study.
II. CONSTRUCTION OF A DYNAMIC MODEL

This analysis of such a multi-loop system is aided by the use of a dynamic model which describes the flow of information and goods around the closed loops and the coupling of one loop to another. Page 8 shows a schematic representation of the basic model. It is a simple retail-wholesale-factory distribution system. Dotted lines represent the flow of information and solid lines the flow of good between the specified points. The fact that there is only one retail box and one wholesale box does not mean that this model represents just one retail store and one wholesale store. For the sake of simplicity, all retailers and wholesalers are each lumped together in one box which represents the aggregate of all the retailers and wholesalers. Likewise this model is not restricted to just one factory. The factory box represents the aggregate of all the factories being considered.

A. Retail Loop

At the bottom of the page are the consumers who buy items from the local retailers. Each retailer is able to give immediate delivery to the consumer because of his inventory of goods. The policy governing this inventory is that it shall be equal to the algebraic sum of the last eight sales. When this sale is transacted, it is recorded both by the sales department and the inventory control department of the
Model of a Retail-Wholesale-Factory Distribution System
retail store. They send reports to the retail purchasing and accounting department. There is a two week time lag between the original consumer sale and the date the retail purchasing department sends an order to its wholesaler for goods. This two week lag consists of the reporting time from sales and inventory to accounting, the time to prepare the actual figures for purchasing, and the time that the purchasing department takes to make a decision to order.

There is then a one week delay between the time the retail purchasing department issues the order and the time it is received by the wholesale sales department. It then takes a week for the wholesale sales department to notify its inventory department to ship the goods back to the retail stores. There is a one week lag between the time the wholesaler ships and the retailers receive the goods. Thus the overall time lag between a sale to the consumer and the replenishment of the retailer's inventory for that sale is five weeks. This does not mean that shipments arrive only every five weeks. They come each week but are based on sales figures five weeks old.

B. Wholesale Loop

The time lags in the wholesale loop are the same as in the retail. It takes accounting two weeks to digest the information from the wholesale sales and inventory departments and to issue an order to the factory warehouse. There are the corresponding one week lags between issue of the order by
the wholesaler and its receipt at factory, between wholesale sales and inventory departments, and between the shipment of goods from the warehouse inventory and their receipt by wholesale distributors. Thus there is a five week lag between the time the wholesaler receives the retailer's orders for goods and when he receives his supply of goods from the factory warehouse. However, since there is a week's lag between the receipt of the retailers' order and the consequent shipments of the ordered goods from the wholesaler's inventory, the replacement of goods from the factory warehouse comes only four weeks after the goods have been shipped by the wholesaler. There is no relation between this time lag and the policy of the wholesaler to keep his inventory equal to the algebraic sum of the last four weeks orders. It is purely coincidental, since all the numbers were arbitrarily picked. Likewise it is coincidental that the retailers base their inventory on twice as many weeks sales as the wholesalers. None of the numbers in this system were picked because of their absolute meaning or algebraic relation to each other. The purpose of this study is to note how changes in these constants affect the response of the system, not to study the constants themselves.

C. Factory Loop

A look at the factory warehouse loop will complete the description of the model. There is a two week lag between
the time the warehouse receives an order from the wholesaler and when the warehouse sends an order to the production department of the factory. This order is based on inventory data that is one week old. Once the factory receives an order at its production department it takes twelve weeks or three months until the warehouse receives the goods in its inventory. This twelve week lag reflects the time it takes to schedule the production, order the raw materials, secure the labor, etc. It also includes material's handling time, that is the time it takes to get the goods from production line to warehouse. Like the wholesalers, the warehouse's policy is to keep inventory equal to the algebraic sum of the last four weeks of sales. Note that inventory receives goods each week that are based on sales fourteen weeks ago.

D. Ordering Criteria

All that is lacking to make this model complete is a description of the criteria used by the retailers, wholesalers, and the warehouses to determine what they will order from the next higher level. It is assumed that they each use the same criteria. This criteria is as follows: Orders to the next higher level equal the sum of 1) orders received from lower level, 2) actual change of inventory from the previous week's level (i.e., difference in shipments and orders placed), 3) the desired change in level of inventory based on policy of keeping inventory equal to the sum of the last n weeks sales, 4) the desired change in the quantity of
goods on order to next higher level (i.e., the goods in the pipeline between ordering point and receiving point).

E. **Summary**

The description of this basic model can be summarized as follows:

A. Desired inventory levels:
   1. Retail - sum of last eight weeks sales
   2. Wholesale - sum of last four weeks sales
   3. Warehouse - sum of last four weeks orders

B. Accounting lags (i.e., time between a sale and the subsequent ordering action) - two weeks at all levels.

C. Delivery lags (time between the ordering and receipt of goods):
   1. Retail - three weeks
   2. Wholesale - three weeks
   3. Warehouse - twelve weeks

D. Ordering criteria at all levels based on:
   2. Actual change in inventory level due to difference between orders placed to next level and goods shipped to lower level.
   3. Desired change in level of inventory and goods on order.

This completes the description of the basic model of a retail-wholesale-factory distribution system. This model
was constructed by Professor Jay W. Forrester of the School of Industrial Management and is not based on any particular industry but reflects a general picture of what might actually exist. As stated previously, there is no need for this model to be based on actual circumstance, just as long as it doesn't violate reality, for we are not interested in the model per se, but rather the effect that changes in the constants of the model will have on its response when disturbances are introduced into the system.
III. ANALYSIS OF THE SYSTEM

With the basic model clearly delineated, a number of questions immediately arise. First, how stable is the system, or what oscillatory characteristics does it have? Secondly, what affect do the various constants in the system have on the response of the system? Specifically, what is the effect of time lags on the stability of the system? What effects do the inventory policies of the retailer, wholesaler, and factory have on the system? What effects do their ordering policies have on the stability of the system? If these questions are answered, it should then be possible to answer the basic question underlying this whole investigation -- What are the causes of inventory and production oscillations in a retail-wholesale-factory distribution system?

A. The Basic Model

To answer the first question, the response of the basic model, hereinafter called model A, was determined. These results, plotted in graphs IA and IB, clearly indicate the oscillatory characteristics of the inventory and production cycles. The curves shown in those graphs will be used throughout this report to provide a basis of comparison for changes in the basic model.

* * *

The second question is more complex, and to provide an
answer it is necessary to individually consider the effects of the time lags, inventory policies, and ordering criteria on the stability of the system.

B. Changes in Time Lags

Starting with the effect of time lags on the system response, a thorough, systematic study was conducted. Seven different models were tested which were identical to model A except for individual changes in various time lags. The changes were made as follows:

1. The two one week lags representing the mailing of orders from the retailer to wholesaler and wholesaler to factory were eliminated. (Model B)

2. The one week reporting time lags from the sales department to inventory department on the wholesale and factory level were eliminated. (Model C)

3. The one week delivery time lags between the factory warehouse inventory and wholesale inventory, and between wholesale inventory and retail inventory departments were eliminated. (Model D)

4. The three types of lags described in 1, 2, and 3 were all eliminated simultaneously. (Model E)

5. The two week processing and accounting time lags found in the retail, wholesale and factory levels were cut to one week's duration. (Model F)

6. The two week processing and accounting time lags were eliminated completely. (Model G)
7. All time lags -- mailing, reporting, delivery, processing and accounting -- mentioned in 1, 2, 3, 5, and 6 above were simultaneously eliminated.

(Model E)

The results of this study are illustrated in three graphs. Graph II is a comparative plot of the response of models B, C, and D. It is appropriate to compare these three models since they each eliminated a one week lag of a different sort. Graph III is a comparative plot of models A, F, and G. This shows relative to model A, the effect of cutting down and finally eliminating the processing time lags. Graph IV is a comparative plot of models A, E, and H, and illustrates the considerable improvement in response of the system when several time lags are simultaneously eliminated.

C. Changes in Inventory Policies

The next area investigated was the effect of changing the inventory levels of the retail, wholesale, factory inventory departments. The following individual changes were made, everything else remaining the same as in model A:

1. Reduce the retail inventory level to the sum of four week's sales. (Model I)

2. Reduce the retail inventory level to the sum of three weeks, leave the wholesale at four weeks, and increase the factory inventory level to the sum of five weeks past sales. (Model J)
3. Increase the factory warehouse inventory level to the sum of six week's sales. (Model K)

4. Increase the factory warehouse inventory level to the sum of eight week's sales. (Model L)

Graphs V and VI show the results of these changes.

Graph V is a plot of models I and J illustrating the effect of a change at the retail level on the entire system's response.

Graph VI is a plot of the factory output curves for models A, K, and L. Note that for models K and L the retail and wholesale response will be unaffected by the change in the factory loop and hence the response for K and L will be identical to model A in the retail and wholesale loops.

D. Changes in Ordering Policies

The third area investigated concerned the effect of changing the ordering criteria in the system and noting how it affects the system's response. Instead of adding 1) orders received from lower level, 2) actual change in inventory from the previous weeks level, 3) desired change in inventory based on policy of keeping inventory equal to the sum of last n weeks sales, and 4) the desired change in the quantity of goods on order to next higher level, directly together as was done previously, two models were constructed in which all the criteria were the same as model A except for the specific changes noted below:
1. On the retail level, items 3 and 4 above, the desired change of inventory and goods on order, were lagged two additional weeks before being added together to give the desired order. On the wholesale and factory level these items were lagged an additional four weeks before being added together in purchasing and accounting department to give final order. (Model M)

2. Item 4 the desired change in quantity of goods on order was lagged five weeks on the retail level, and four weeks on the wholesale level before being added together with the other items. On the factory level, item 3, the desired change in inventory was lagged eight weeks. (Model N)

The determination of the number of weeks to lag the above items was based on experience with former models, and was believed to be the combination producing the minimum amplitude of oscillation in inventories and factory output, other things being held constant.

The results of these changes are shown in graph VII which plots the response of model M and N relative to model A.

*     *     *

This completes the investigation of the major areas singled out in the second question asked at the beginning of this section. However, there are several additional ways in which the basic system could be modified to lessen the ampli-
tude of inventory and production oscillations.

E. Direct Reporting to Factory

It is possible that the retailer might report consumer sales directly to the factory and thus allow the factory to pre-schedule any change in production rate. To do this model A was modified so that there was a direct information flow channel, having a one week time lag, from the retail sales department to the factory production and accounting department. When the production and accounting department received this information, they went thru the regular steps of calculating the increased order to be issued, based on depletion of existing inventory and desired increase in inventory to higher level corresponding to new sales rates. From these calculations it was apparent that this increased order would be negligible compared to the large order due to arrive from the wholesaler in a few weeks. From previous calculations it was determined that the time lags in the system amplified about nine times any original disturbance induced at the retail level. Hence, it was decided that the direct reporting information could not directly be scheduled, but that the amplitude of increased orders should be measured and then multiplied by a suitable constant. Four models, 0, P, Q, and R were tested in which this constant multiplication factor was 2, 4, 6, and 8, respectively.

There still remains to be described the manner in which this advance information was to be scheduled. The factory
output and the warehouse inventory are the items to be stabilized. But how is this to be done? It was decided to use a very simple rule. The factory production line would immediately begin to increase its output on the basis of the increase in retail orders multiplied by the suitable constant. Once the orders reached a maximum, it would hold output at this maximum until the orders from the wholesaler began to arrive through the regular channel. The wholesale order and the factory output level would then be compared and if the factory output level were higher, orders remained at that level. However, if the wholesale orders exceeded the factory output level, a new rule would be used.

During the time that the factory was operating at a level determined by retail orders, the factory was creating a surplus of goods, since the warehouse orders had not yet increased to the level of production. This surplus is used to decrease the amplitude of the orders demanded by the wholesaler a few weeks later. That is, when the wholesale order exceeds the factory output level, the factory output remains the same and the deficit between output scheduled and orders received is subtracted from the surplus output previously scheduled. This practice continues until the surplus of previously scheduled factory output is eliminated. Then the system reverts back to the basic rules followed in model A of scheduling factory output on the basis of whole-
sale orders. Graph VIII shows the change in response of the factory output for models 0, P, Q, and R.

F. Changing the Production Lead Time

No mention was previously made about the possibility of changing the production lead time. Model A contained a twelve week lead time. This was modified in the following ways:

1. Reduce the lead time to six weeks. (Model S)
2. Reduce the lead time to nine weeks. (Model T)
3. Increase the lead time to fifteen weeks. (Model U)

The results of this investigation are shown in Graph IX which compares the factory output curves for models S, T, and U, relative to model A.

G. Elimination of the Retailer

The final area investigation consisted of constructing one model in which the retail sector was eliminated. This change, model V meant that the wholesaler sold directly to the consumer who got immediate delivery of his goods, thus eliminating the one week lag between the wholesale sales and inventory departments. The only other change from the system embodied in model A is that the warehouse inventory was increased to the algebraic sum of six week's sales rather than four week's sales. Graph X shows the response of the wholesale and factory loops for model V vs. model A.

H. Obtaining a Solution

This completes the description of all the various
modifications made in the basic model. All that remains is to describe how the models are analyzed and how the results are obtained.

It is assumed that the system is operating with consumer orders at a steady state level of 1000 units per week. This means that retail, wholesale, and factory orders are also 1000 units per week, and all inventories are at the desired levels. A disturbance is introduced at the retail level. For four consecutive weeks sales increase \(2\frac{1}{2}\%\) or 25 units per week and then level off at 1100 units per week. This represents only a 10\% increase in four week's time.

In accordance with the ordering criteria explained previously, the retailer will immediately increase his orders. This increase in consumer sales will be propagated up thru the wholesale and factory loops and, because of the structure of the models, the amplitude of the oscillation of orders will increase with each loop. Eventually the factory production will start to increase and the depleted warehouse inventory will be replenished. Several months after the initial disturbance is introduced the system will reach equilibrium at the new level of 1100 units per week.

For a more detailed explanation of the actual calculations and method used to obtain the response of each model see the Appendix.
IV. DISCUSSION OF RESULTS

A. Model A

Graphs IA and IB presented on the next two pages give a complete description of the response of the basic system called model A. Note first the curves of physical inventory. The retailer experiences a dip in inventory due to the immediate depletion of his inventory caused by the increase in sales and the delay in replenishing the inventory. The inventory reaches its new steady-state level by week sixteen, eleven weeks after the rise in consumer sales has leveled off.

The wholesale inventory is depleted but then becomes over-stocked and finally reaches equilibrium by week twenty-three, eighteen weeks after the consumer sales have stopped increasing. The reason that the inventory level exceeds its equilibrium level is because the ordering policies call for more goods than necessary.

The warehouse inventory suffers the worst fluctuations of all. Because of the increased wholesale orders, its stock is almost depleted. But then, as the factory begins to ship goods, the inventory builds up rapidly to far above the steady-state level. It then falls almost as rapidly to slightly below the steady-state level, and finally levels off at week forty-eight. This is forty-three weeks after the end of the initial consumer disturbance.
GRAPH IA

Oscillations of Retail and Wholesale Inventory and Warehouse Inventory for Model A
GRAPH IB
Oscillations of Orders at Wholesale, Orders at Factory & Factory Output for Model A
A look at Graph IB will help to explain some of the oscillations in the inventories. Consumer sales are plotted showing the linear increase of twenty-five units a week from week one to week five. As a result of this, the retailer issues an increased order to the wholesaler. Note how this builds up and then gradually falls off to the steady-state. This is what replenishes the retail inventory.

On the basis of these orders the wholesaler calculates his orders and sends these to the factory warehouse. Note the rapid rise of these orders which go well above the steady-state level. This is what causes the wholesale inventory to overshoot its equilibrium level. These orders then fall below the equilibrium level, which causes the wholesale inventory to gradually fall back to equilibrium, and then return to the equilibrium level.

The actual orders to the factory production line are not plotted because they are identical to the factory output curve except that they are shifted twelve weeks backward towards the origin. The rapid increase in these orders and factory output are the severest of all. They reach a peak almost four times the steady-state level. This is what causes the warehouse inventory to go shooting up. The factory orders and output fall almost as rapidly to zero and stay at zero for three weeks. This causes the warehouse inventory to fall. The factory output then gradually rises following a very non-linear path until it exceeds the steady-state level, and then finally settles down to the steady-state level at week
forty-nine. This allows the warehouse to come back up to its steady-state level.

Note that the retail inventory curve has one major inflection point, the wholesale curve two, and the warehouse curve three. The same is true in regard to orders. The orders issued by the retailer (order at wholesale) has one major inflection point, the orders issued by the wholesaler (order at warehouse) two, and order at factory (factory output curve shifted back) has three. This pattern is followed throughout all the various model changes and is inherent in the general system's response of a three loop model.

These curves vividly illustrate why there is an interest in damping the oscillatory response of this system. Of particular practical importance are the fluctuations of the retail and wholesale inventory and the factory output. The large fluctuation in inventory means that at one moment the store-keeper's shelves are practically bare and the next moment he has so many goods he doesn't know where to put them. Thus, the store-keeper doesn't know how much space to save for inventory. Furthermore, the costs of rapidly shifting quantities of goods as the inventories fluctuate are considerable and will cause profits to dip.

The severe oscillations of factory output are probably the most serious. Practically speaking, the factory capacity will place an upper limit on production and thus chop off the
top of the factory output curve. This would not only reduce the positive amplitude of oscillation, but would also mean that the negative swing would not be as large either. There would still be rapid changes in factory output and this would cause sizeable scheduling problems of labor, raw materials, production rates, etc. In fact, these scheduling difficulties might mean that practically such a rapid rate of increase and/or decrease of factory output is impossible.

However, rather than construct a model with these limitations inherent the general form was retained because, as stated previously, this study is principally interested in analyzing the effect that changes in the system have on the response of the system, not the response of the system per se.

With this in mind, it is appropriate to discuss the results of changes in the models presented in graphs II-X.

B. The Effect of Changing Time Lags

Graph II, comparing models B, C, and D, shows that the elimination of the one week reporting lag between the sales and inventory departments on the wholesale and warehouse level actually increased the oscillatory characteristics of the factory output. This may appear illogical but upon examination of the equations (9) and (15) of the system, (See Appendix III, discussion of model C and also Appendix II) it is seen that eliminating the one week time lag increases the oscillations of the system. What happens is that
GRAPH II
Oscillations of Retail and Wholesale Inventory, and Factory Output

Retail Inventory
Wholesale Inventory
Factory Output

Model B
Model C
Model D
by speeding up delivery of the goods, the change in actual inventory is made more severe.

The differences in wholesale inventory response for the three models are very slight. Model B has the least amplitude of oscillation and reaches its steady-state level one week sooner than the others. However, all three models have a better response than model A.

Comparing the factory output curves again, models B and D are identical except for a two week shift. The maximum amplitude of these curves are approximately five hundred units below that of model A. On the other hand, the amplitude of model C is fifty units above model A.

The response of the retail inventory is identical for all three models. This is because all the changes made in the system are on the wholesale and warehouse level. However, the response is better than model A because one of the three week lags in information flow between the time the retailer orders the goods and the time he receives them has been eliminated.

A word should be said about the practicality of effecting the proposed changes. The elimination of the mail lag (model B) is possible by using an improved communication system such as teletype or telephone rather than the U.S. Mail. However, this would involve a considerable expense and such an outlay would have to be weighed against the saving of costs effected
by decreasing the oscillations. The elimination of the reporting lag between the sales department and the inventory department calls for better organization and an improved communication system. This change might be harder to make and based on its affect in response does not seem advisable. Elimination of the delivery lags (model C) would call for better and faster transportation facilities or shorter distances between the wholesaler and the distributor. This does not seem at all feasible because once warehouses are built, they cannot easily be moved and the cost of using air freight instead of trucks or trains to speed up delivery would no doubt many times outway the saving effected by reduced oscillation of inventory and factory output.

Graph III compares models A, F, and G, and shows that cutting in half and finally eliminating the accounting and processing time delays decidedly improves the response of this system. The non-linearities of this system are demonstrated by the fact that the elimination of the first week's delays (model F) effect a greater absolute and percentage improvement in the system's response (compare models A and B) than the elimination of the second and final week delays (compare models F and G).

The actual elimination of both week's delays is probably very hard to accomplish in practice, especially at the factory level, where the size of the operation is much larger than
Graph III
Oscillations of Retail and Wholesale Inventory, and Factory Output

Model A
Model F
Model G
for any individual wholesale or retail establishment. However IBM's RAMAC boasts being able to do just such a thing. Its cost of rental is considerable, though, and should be weighed against any inventory and production saving cost. The elimination of one of the two week delays is probably quite possible on both the retail and wholesale level simply by enlarging the accounting department and buying a few more adding machines. Even this change, though, costs money. To effect such a change on the factory level would involve considerable expansion of labor and equipment.

Graph IV compares two rather ideal systems to model A. It is definitely impractical to think of ever setting up such a system as is represented by model E. It does, however, show the considerable improvement in response obtained by eliminating time lags and provides a goal to shoot for. In addition, it shows the oscillations that are caused entirely by the ordering policies and inventory rules of the system, thus proving that time lags are not the sole cause of instability of inventories and production cycles.

Comparing the plots of model H with model G, graph III, one can see that simultaneously eliminating the three one week lags is only a slight improvement over eliminating the two week processing and accounting lags. This seems to indicate that the lags in the accounting department have more influence on the amplitude of oscillations than information flow or
delivery lags.

C. The Effect of Change in Inventory Policies

Graph V indicates that reducing the retail inventory level does not improve the stability of the system. To the contrary, it creates quite severe secondary oscillations in the factory output that are much worse than those found in model A. The amplitude of the wholesale inventory oscillations are also larger, although they reach the steady-state level sooner. Increasing the warehouse inventory level while further decreasing the retail inventory (model J), provides only a very slight improvement in the amplitude of oscillations, the one change counterbalancing the other. There are no problems in implementing such a change; in fact, reducing the inventory level is a space and cost saving measure. However, it does not seem like an advisable move to make because of the effect on factory output.

Graph VI isolates the effect of changing just the warehouse inventory and keeping everything else constant. Increasing the inventory rule to the sum of six weeks provides some improvement in the secondary oscillations of the factory output. Increasing the inventory rule still further to the sum of the past eight weeks sales actually increases the secondary oscillations. This demonstrates that the smoothing effect of summing n weeks sales does not necessarily always improve the response of the system. There are so many other
GRAPH V
Oscillations of Retail and Wholesale Inventory, and Factory Output

Model I
Model J

Physical Inventory (in thousands)

Factory Output

Wholesale Inv.

Retail Inv.
factors involved that may negate the affect of increasing the number of weeks included in the summation.

D. The Effect of Changing Ordering Policies

Changing the ordering policies exerts quite a noticeable effect on the system's response, as is shown on Graph VII. The effect on the retail inventory is slight and the oscillations of the wholesale inventory are not reduced noticeably either. In fact, the wholesale inventory takes several additional weeks to return to equilibrium.

The factory output is considerably affected. The maximum oscillation is cut almost in half and the factory output never reaches zero as is the case with model A. In return for these benefits, it takes the new system four to ten weeks longer to return to equilibrium. This means that in the case of model M, it takes the factory over a year to completely readjust to the new output level. It is interesting to note that model N not only reaches its equilibrium level sooner, but also fluctuates to a lower level on the first upswing. On this basis it seems to respond better than model M.

There should be no appreciable cost involved in adopting either of these two policies. All it means is a change in the method of calculations. This change is not very complicated either and should not involve the hiring of any personnel or buying or new equipment.
GRAPH VII

Oscillations of Retail and Wholesale Inventory, and Factory Output

Model A
Model M
Model N

Retail Inv.
Factory Output
Wholesale Inv.
E. The Effect of Establishing a Direct Reporting System

The four models plotted in graph VIII indicate the successive reduction of the amplitude of the oscillation in the factory output effected by increasing the multiplication constant employed in the system. The improvement applies only to the first major fluctuation in factory output, and does not affect the secondary oscillations. This is due to the ordering policies inherent in the system.

The establishment of such a system would not involve any large expense. It would simply mean that the retail sales department would send a copy of every consumer sale's slip to the factory in addition to notifying its own purchasing and accounting department. It would mean a considerably more complicated ordering system at the factory level, and this might necessitate the hiring of more employees or using more calculating machines.

F. The Effect of Changing the Factory Lead Time

The effect of varying the factory lead time from six weeks to fifteen weeks is demonstrated in Graph IX. The interpretation of the graph may be confusing as it shows that reducing the factory lead time to six weeks (model S) almost eliminates the secondary oscillations at the expense of increasing the number of weeks when the factory is completely shut down. The same results are true for a factory lead time of nine weeks (model T), while increasing the lead
GRAPH IX

Oscillations of Factory Output

- Model A
- Model S
- Model T
- Model U
time to fifteen weeks, decreases the number of weeks the factory is shut down, but increases the secondary oscillations. In no case does the change in lead time affect the first maximum oscillation of the system.

In view of the considerable capital outlay, in terms of larger plants and improved production facilities, needed to reduce the production lag, a change in production lead time does not seem at all justified with such a minor improvement in factory output response.

G. The Effect of Eliminating the Retail Sector

As would be expected Graph X shows that there is considerable improvement in factory output and inventory response with just a wholesale and factory loop. The difficulties involved in establishing enough wholesale outlets and supplying them from a few factories probably are sizeable enough to make such a system just a dream.
GRAPH X

Oscillations of Wholesale and Warehouse Inventory, and Factory Output for Model V
V. CONCLUSIONS

Although the previous results did not take into account any actual cost figures, and it is assumed that all companies operate on the philosophy of maximizing profit, it seems apparent that decreasing the oscillations of factory output and inventories should decrease costs and thus benefit a company. Operating on this basis the following conclusions seem apparent:

1. With the exception of one previously discussed, reduction of time lags found in any system will improve the response of the system. However, there are increased costs associated with such changes, and these would have to be weighed against the operating savings effected due to the improved system's response before such a change is made.

2. In only one case, model K, did changes in the inventory policies improve the response of the system. In all other instances, the oscillations, particularly of factory output increased. It therefore seems that inventories should be as large as possible on the retail level, and that changes in the warehouse inventory may or may not improve the response of the system.

3. Changes in the ordering policies of a system should be investigated, for it has been shown that they can
materially affect the amplitude of oscillation of factory output with only a slight increase in time required to reach equilibrium.

4. The establishment of a direct reporting link between the retailer and factory involves little capital expense, only a slight increase in operating costs, and in return, considerably improves the response of the factory output.

5. Changes in the production lead time which would involve major capital expenditures have questionable value in improving the response of the factory output.

6. Elimination of the retailer sector thereby forcing the wholesaler to sell directly to the consumer, vastly improves the response of the system, but raises countless other problems which rule out its probability of being introduced.

7. It is hoped that these results will provide a guide for anyone interested in improving the response of such a three loop coupled system as was investigated in this study. It has been definitely shown that the oscillations in such a system can be influenced by various changes in the system.
APPENDIX I

The calculations underlying the results presented in the previous pages are mathematically simple but time consuming. They consist of the numerical solution of a set of difference equations which describe what happens in the model. These equations readily lend themselves to an iterative solution by a digital computer, but since the author had no knowledge of programming, they were solved entirely by hand. These models were for the case where the shortest time interval was one week.

Each variation of the basic model has its own set of equations, but they are similar in form to the equations describing general model A which are presented below.

First, it is necessary to define a multitude of terms so that the equations will be meaningful.

Retail Loop:

\( O_{R_t} \) - orders placed by the retailer to the wholesaler at the end of the week \( t \).

\( S_{R_t} \) - retail sales or consumer orders for the week \( t \).

\( \Delta I^R \) - actual change in inventory due to the difference between shipments and receipts of goods.

\( \Delta I^I \) - desired change in inventory and quantity of goods on order based on the policy of keeping inventory equal to the sum of the last \( n \) weeks' sales.

\( P_{I_{R_t}} \) - physical inventory of the retailer at the end of week \( t \).

\( R_{D_t} \) - goods received from the wholesale distributor during the week \( t \).
Wholesale (Distributor) loop:

\[ \text{OD}_t \] - orders placed by the wholesaler during the week \( t \).

\[ \text{SD}_t \] - wholesale sales during the week \( t \).

\[ \Delta I_D \] - actual change in inventory due to the difference between shipments and receipts of goods.

\[ \Delta I_D' \] - desired change in inventory and quantity of goods on order based on the policy of keeping inventory equal to the sum of the last \( n \) weeks' sales.

\[ \text{PI}_{Dt} \] - physical inventory of the wholesaler at the end of week \( t \).

\[ \text{R}_{wt} \] - goods received from the factory warehouse during the week \( t \).

Wholesale loop:

\[ \text{OD}_t \] - orders placed by the warehouse to the factory during the week \( t \).

\[ \text{SD}_t \] - warehouse sales during the week \( t \).

\[ \Delta I_W \] - actual change in inventory due to the difference between shipments and receipts of goods.

\[ \Delta I_W' \] - desired change in inventory and quantity of goods on order based on the policy of keeping inventory equal to the sum of the last \( n \) weeks' sales.

\[ \text{PI}_{wt} \] - physical inventory of the warehouse at the end of the week \( t \).

\[ \text{RF}_t \] - goods received from the factory during the week \( t \).

Note that the subscript \( R \) refers to the retail loop, \( D \) to the wholesale (distributor) loop, and \( W \) to the warehouse loop, and \( F \) to the factory.

In order to indicate a time lag a constant is subtracted from the subscript \( t \). For example: \( S_{R_{t-2}} \) indicates the retail sales two weeks prior to the week \( t \) now being considered, and \( O_{R_{t-3}} \) indicates retail order placed three weeks prior to the
week $t$. The expressions containing the summation terms are not easily discernable and further explanation will be presented after listing the equations.

**Retail loop:**

1. $O_{Rt} = S_{Rt-2} - \triangle I_R - \Delta I'_R$

2. $S_{Rt} = \text{Consumer Sales}$

3. $\Delta I_R = (S_{Rt-2} - S_{Rt-4})$

4. $\Delta I'_R = \left[\sum_{j=0}^{2} S_{Rt-j-2} - \sum_{j=0}^{2} S_{Rt-j-3}\right] - \left[\sum_{j=0}^{2} S_{Rt-j-2} - \sum_{j=0}^{2} S_{Rt-j-3}\right]$

5. $P_{IRt} = P_{IRt-1} - R_{Dt} - S_{Rt}$

6. $R_{Dt} = O_{Rt-3}$

**Wholesale (Distributor) loop:**

7. $O_{Dt} = S_{Dt-2} - \Delta I_D - \Delta I'_D$

8. $S_{Dt} = O_{Rt-1}$

9. $\Delta I_D = (S_{Dt-3} - S_{Dt-4})$

10. $\Delta I'_D = \left[\sum_{j=0}^{3} S_{Dt-j-2} - \sum_{j=0}^{3} S_{Dt-j-3}\right] - \left[\sum_{j=0}^{3} S_{Dt-j-2} - \sum_{j=0}^{3} S_{Dt-j-3}\right]$

11. $P_{IDt} = P_{IDt-1} - R_{Wt} - S_{Dt-1}$
(12) \[ R_{W_t} = O_{D_t-3} \]

**Warehouse loop:**

(13) \[ O_{W_t} = S_{Wt-2} \neq \Delta I_W \neq \Delta I_W' \]

(14) \[ S_{Wt} = O_{D_t-1} \]

(15) \[ \Delta I_W = (S_{Wt-3} - S_{Wt-4}) \]

(16) \[ \Delta I_W' = \left[ \sum_{j=0}^{3} S_{Wt-j-2} - \sum_{j=0}^{3} S_{Wt-j-3} \right] \neq \left[ \sum_{j=0}^{3} S_{Wt-j-2} - \sum_{j=0}^{3} S_{Wt-j-3} \right] \]

(17) \[ P_{I_{Wt}} = P_{I_{Wt-1}} \neq R_{F_t} - S_{Wt-1} \]

(18) \[ R_{F_t} = O_{W_t-12} \]

Equations 4, 10, and 16, are the ones which may seem unintelligible. The first two expressions in brackets refer to the desired inventory levels, and when subtracted give the desired change in inventory. The last two quantities in brackets relate to the goods in the pipe line, and when subtracted give the desired change in the quantity of goods on order. Thus, these equations give the desired change in inventory and goods on order for the retailer, wholesaler, and factory warehouse, respectively.

Understanding these equations it is possible to grasp a meaning of the effect of time lags, ordering policies, inventory policies, production lags, etc., on the solution of the system. The equations also show the coupling
between the retail, wholesale, and warehouse loops.

Appendix II will present an actual solution of the equations for model A. Appendix III will discuss the changes required in the equations for each model change.
APPENDIX II

The following is a week by week solution of the set of difference equations listed in Appendix I. These results pertain only to model A.

Column 1 is the input to the system that starts the whole thing going, and is represented by equation (2). Columns 2, 3, and 4 represent the three quantities on the right hand side of equation (1). From the standpoint of calculations, column 2 is column 1 lagged two weeks, column 3 is column 5 lagged two weeks, and column 4 is the sum of columns 8 and 9 lagged two weeks. Column 5 represents the solution of equation (3) without the two week time lag which introduced when these results are transferred to column 3. In terms of calculations it is column 1 minus column 2. No equation was written for column 6 since it serves as a check to see if the calculations are correct. It is calculated by taking the previous week's results adding column 10 and subtracting column 1 for that week. Column 7 is the solution of equation (5) and consists of taking the previous week's results adding column 11 and subtracting column 1. Column 8 is the solution of the first two quantities of equation (4), while column 9 is the solution of the last two quantities of equation (4). In both these, the two week lag is added when the results are added together and transferred to column 4; and column 1 is the item summed up over eight and three weeks time, respectively. Column 10
is the solution of equation (1) and represents the summation of columns 2, 3, and 4. Column 11 is the solution of equation (6) or column 10 lagged three weeks.

Columns 12-23 for the wholesale loop are almost identical to columns 1-11 except for a few minor changes. Column 12 is column 10 lagged one week and is the solution of equation (8). Column 13 is column 12 lagged one week due to the reporting delay between the wholesale sales and inventory departments. Columns 14, 15, and 16 are identical in solution form to columns 2, 3, and 4, and represent the terms on the right hand side of equation (7). Column 17 is column 13 minus column 14 and is the solution of equation (9). Columns 18 and 19 are calculated in a similar manner to columns 6 and 7 except that column 13 is the item subtracted rather than column 12. Equation (11) represents column 19. Columns 20 and 21 are calculated in similar manner to columns 8 and 9 and are represented by equation (10). Column 12 is the item used in the summation terms. Column 22 is the sum of columns 14, 15, and 16, or the solution of equation (7). Column 23 is the solution of equation (12) or column 22 lagged three weeks.

The solution form of columns 24-35 are identical to columns 12-23 just discussed. Column 24 is column 22 lagged one week as stated in equation (14). Column 25 and 26 are column 24 lagged one and two weeks, respectively. Column 27
is column 29 lagged two weeks and column 28 is the sum of columns 32 and 33 lagged two weeks. Columns 26, 27, and 28 represent the three right hand quantities of equation (13). Column 29 is column 25 minus column 26 and is the solution of equation (15). Column 30 is calculated by taking the previous weeks results adding column 34 and subtracting column 25. Column 31 is calculated by taking the previous weeks results adding column 35 and subtracting column 25 as stated in equation (17). Columns 32 and 33 are the solution of the first and second parts of equation (16) without the two week lag which is introduced when those results are transferred to column 28. Column 24 is the item used in the summation item. Column 34 is the summation of columns 24, 27, and 28 or the solution of equation (13). Column 35 is column 34 lagged 12 weeks or the solution of equation (18).

The solution of these equations for model A is presented on the following page:
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<th>Desired change in inv. &amp; on order</th>
<th>Actual change of inv.</th>
<th>Total of inv. plus goods on order</th>
<th>Physical inv.</th>
<th>Change in desired inv.</th>
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<td>19</td>
<td>Physical inv.</td>
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<td>Change in desired inv.</td>
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<td>21</td>
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<td>22</td>
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<td>24</td>
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<td>25</td>
<td>Goods shipped from warehouse</td>
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<td>To replace</td>
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APPENDIX III

Altogether twenty-two different models were constructed and tested. The changes which differentiate these models from one another were discussed in the previous section entitled, "Analysis of the System." This appendix will list the actual changes required in the method of calculations for each model, beginning with model B.

Model B - Elimination of the mailing lag

The last two summation terms of equations (14) and (10) are reduced by one week (i.e. \( \sum_{j=0}^{t-1} \) rather than \( \sum_{j=0}^{t-2} \)). This affects directly columns 9 and 21. Also the lags expressed in equations (6) and (12) are cut to two weeks (i.e. \( t-2 \) rather than \( t-3 \)). This directly affects columns 11 and 23. The lags expressed in equations (8) and (14) were also eliminated which directly affects columns 12 and 24.

Model C - Elimination of the reporting lag between sales and inventory departments

The last two summation terms of equations (14) and (10) are reduced by one week which directly affects columns 9 and 21. Also the lags expressed in equations (6) and (12) are cut to two weeks which affects columns 11 and 23. Equations (9) and (15) are also changed so that they are identical in form to equation (3). This change directly affects columns 13, 17, 25 and 29. The last term of equations (11) and (17)
are changed to correspond with equation (5). This will directly affect columns 19 and 32.

**Model D - Elimination of the delivery lag**

As with models B and C, equations (4) and (10) are modified so that the last two summation terms are reduced by one week. This directly affects columns 9 and 21. Also the lags expressed in equations (6) and (12) are cut to two weeks which directly affects columns 11 and 23.

**Model E - Mail lags, reporting lags and delivery lags were all eliminated simultaneously**

The last two summation terms in equations (4) and (10) are completely eliminated since no pipeline lags existed. This eliminates columns 9 and 21. The lags in equations (6), (8), (12) and (14) were eliminated. Equations (9) and (15) are also changed so that they are identical in form to equation (3). These changes mean that columns 10, 11, 12 and 13; and 22, 23, 24 and 25 are identical to each other as there are no time lags separating them. Columns 17 and 29 are also affected. The last terms of equations (11) and (17) are also changed to correspond to equation (5), which directly affects columns 19 and 32.

**Model F - Accounting time cut to one week**

This change reduced the lag in the first term of equations (1), (7) and (13) to a one week lag which directly affects columns 2, 14 and 27. All time lags in equations (3), (4),
(9), (10), (15) and (16) are reduced by one week. This directly affects columns 3, 4, 16 and 28, and eliminates columns 15 and 27.

**Model G** - Eliminates the accounting lag entirely

The lag in the first term of equations (1), (7) and (13) is eliminated entirely which directly affects columns 2, 14 and 27. All time lags in equations (3), (4), (9), (10), (15) and (16) are reduced by two weeks. This directly affects columns 4, 15, 16 and 28, and eliminates column 3.

**Model H** - Eliminates all time lags

The lag in the first term of equations (1), (7) and (13) is eliminated entirely which directly affects columns 2, 14, and 27. The last two summation terms in equations (4) and (10) are completely eliminated, which eliminates columns 9 and 21. In addition, all time lags in equations (3), (9) and (15) are eliminated which eliminates columns 3, 5, 15, 17, 27 and 29. The time lags in the remaining terms of equations (4), (10) and (16) are cut two weeks which directly affects columns 4, 16, and 28. The lags in equations (6), (8), (12) and (14) are eliminated. This means that columns 10, 11, 12, 13, and 14; and 22, 23, 24, 25 and 26 are identical to each other. The last terms of equations (11) and (17) are also changed to correspond to equation (5) which directly affects columns 19 and 32.
Model I - Retail inventory cut to the sum of four weeks past sales

The only change is in equation (4) where the first two summation terms are reduced four weeks (i.e. ∑\[\sum_{j=0}^{3}\] instead of ∑\[\sum_{j=0}^{7}\]). This directly affects columns 8 and 6.

Model J - Retail inventory cut to three weeks, wholesale inventory the same and warehouse inventory increased to five weeks.

The first two summation quantities of equation (4) are reduced five weeks, which directly affects columns 8 and 6; and the first two summation quantities of equation (16) are increased one week, which directly affects columns 32 and 30.

Model K - Warehouse inventory is increased to six weeks

The only change is in equation (16) where the first two summation quantities are increased two weeks, which directly affects columns 32 and 30.

Model L - Warehouse inventory is increased to eight weeks

The only change is in equation (16) where the first two summation quantities are increased four weeks, which directly affects columns 32 and 30.

Model M - Change in ordering policy # 1

The time lags in equation (4) are all increased by two weeks, which directly affects only column 4. The time lags in equations (10) and (16) are all increased four weeks which directly affects only columns 16 and 28.
Model N - Change in ordering policy #2

The time lags in the last two terms of equations (4) and (10) are increased five weeks and four weeks, respectively. This directly affects columns 9 and 21. The time lags in the first two terms of equation (16) are increased by eight weeks which directly affects only column 32.

Models O, P, Q and R - Direct reporting with a multiplication factor of two, four, six and eight, respectively.

A new column is created at 24a. This represents the direct report of consumer sales to the factory with a one week lag. Columns 32a and 33a are also created and represent the desired increased inventory and quantity of goods on order. The calculations for these columns are identical to those for columns 32 and 33 except the consumer sales are used instead of wholesaler orders. There is no change in equation (16). The multiplication factor is applied to the sum of these two columns. These changes directly affect only column 28, and the rules outlined previously are followed.

Model S - A six week production lag

The only changes are in equation (16), where the last two summation terms are reduced six weeks, and equation (18), where the time lag is decreased six weeks. This directly affects columns 33 and 35.
Model T - A nine week production lag

The only changes are in equation (16), where the last two summation terms are reduced three weeks and equation (18) where the time lag is decreased three weeks. This directly affects columns 33 and 35.

Model U - A fifteen week production lag

The only changes are in equation (16), where the last two summation terms are increased three weeks and equation (18), where the time lags are increased three weeks. This directly affects only columns 33 and 35.

Model V - Just a wholesale-factory loop

Equations (1) - (6) are completely eliminated. This eliminates columns 1-11. Equation (8) is changed so that $S_{Dt} =$ consumer sales. Equation (9) is modified by subtracting a one week time lag from the first term on the right hand side. Equation (11) is modified by subtracting the one week time lag on the last term on the right hand side. These changes directly affect columns 13, 14, 18 and 20. Equation (16) is modified by increasing the first two summation terms two weeks, which directly affects columns 29 and 32.

*   *   *

It is important to remember that the specific changes noted above indirectly affect an entire model. Hence, one cannot modify just the columns mentioned and not any other, and hope to get the correct solution. The entire system
following the point where the change was made must be re-calculated in order to obtain the new response.
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