ECONOMIC MODELS

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PROFESSOR L. F. HAMILTON
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DEAR PROFESSOR HAMILTON:

IN ACCORDANCE WITH THE REQUIREMENTS FOR GRADUATION, I HEREWITH SUBMIT A THESIS ENTITLED, "ECONOMIC MODELS". IT HAS BEEN MADE POSSIBLE BY THE ENCOURAGEMENT AND SUGGESTIONS OF PROFESSOR JAY FORRESTER WHO SERVED AS THE CHAIRMAN OF THE THESIS COMMITTEE, AND WHO HAS MADE MANY CONSTRUCTIVE CRITICISMS OF BOTH THE PLAN OF INVESTIGATION AND THE PRESENTATION OF THE RESULTS. A SPECIAL DEBT IS OWED TO ED HOLLAND, DICK BENNETT, AND MANY OTHERS IN THE SCHOOL OF INDUSTRIAL MANAGEMENT, AND THE DEPARTMENT OF ECONOMICS. THANKS IS GIVEN ALSO TO PROFESSOR ALEXANDER OF THE SCHOOL OF INDUSTRIAL MANAGEMENT WHO SERVED ON THE THESIS COMMITTEE.

SINCERELY YOURS,

JOHN T. PETTIT, SLOAN FELLOW
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"AND TRUTH IS KNOWLEDGE OF THINGS
AS THEY ARE, AND AS THEY WERE, AND
AS THEY ARE TO COME; -----"

D + C 93:24.

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INTRODUCTION

THERE ARE TWO BASIC APPROACHES TO ECONOMIC MODELING; NAMELY, THE HISTORICAL APPROACH AND THE THEORETICAL APPROACH.

THE APPROACH OF THE THEORISTS IS TO FIRST, PROPOSE A SET OF ASSUMPTIONS, AND SECOND, TO EXAMINE THE CONSEQUENCES OF THE GIVEN SET OF ASSUMPTIONS. THE THEORIST WOULD THEN REGARD HIS THEORY AS BEING VALID FOR ALL TIME. SINCE THE THEORY IS VALID FOR THE GIVEN SET OF ASSUMPTIONS, ASSUMING AN OTHERWISE WELL CONSTRUCTED THEORY, THEN THE THEORY WOULD BE VALID FOR ANY AGE IN WHICH THE ASSUMPTIONS WERE VALID. THE THEORIST WOULD NORMALLY START WITH ASSUMPTIONS LEADING TO AN EXTREMELY SIMPLE SYSTEM. AFTER EXHAUSTIVE STUDY OF THE SIMPLER MODEL, THE ASSUMPTIONS WOULD BE RELIEVED ONE AT A TIME IN AN ENDEAVOR TO ARRIVE AT AN UNDERSTANDING OF A MORE COMPLEX SYSTEM.

ONE OF THE MAIN DRAWBACKS TO THE THEORETICAL APPROACH HAS BEEN THE DIFFICULTY IN EXAMINING EVEN THE SIMPLEST OF ASSUMPTIONS, DUE TO THE STATE OF THE DEVELOPMENT OF THE ANALYTICAL TOOLS BEING USED. TODAY, HOWEVER, MANY POWERFUL TOOLS EXIST WHICH HAVE NOT AS YET BEEN EXHAUSTIVELY APPLIED BY THE ECONOMIC THEORIST.

THE APPROACH TAKEN BY THE HISTORIAN IN ECONOMIC MODELING HAS BEEN SO GENERAL AND BROAD, AND SO PERSONALLY BIASED THAT IT HAS FAILED GENERALLY TO FURNISH THE MODERN THEORIST WITH MUCH IN THE WAY OF COMPLETE AND OBJECTIVE FACTS ABOUT THE ECONOMICS OF THE PAST. AS FAR AS THE THEORIES BASED UPON GENERALIZATIONS OF HISTORY, THEY ARE BETTER KNOWN FOR THEIR FAILURES THAN FOR THEIR SUCCESSES IN EXTRAPOLATION.

THE HISTORIAN HAS TRADITIONALLY PROJECTED THE WORKING ASSUMPTIONS FROM HIS ERA TO THE FUTURE, WHEN IN REALITY THE BASIC

ASSUMPTIONS OF THE ECONOMY HAVE CHANGED. THE FAILURES TO FORESEE THE CHANGES IN ASSUMPTIONS, AND IN HAVING THE ANALYTICAL MECHANISMS TO EVEN ROUGHLY EXAMINE THEIR CONSEQUENCES appear to have ~~HAS~~ LEFT ECONOMIC THEORY GENERALLY AT A STANDSTILL FOR A GREAT MANY YEARS. SOME EXAMPLES OF THEORIES THAT WITHERED ON THE VINE FOR LACK OF THE ANALYTICAL TOOLS TO EXTEND THEM AND THE MECHANISMS FOR CHANGING THE BASIC ASSUMPTIONS ARE: THE ANTIQUATED MARGINAL UTILITY AND MATHEMATICAL THEORIES OF JEVONS, MENGER, AND WALRAS; THE MERCHANTILIST WRITERS OF THE SIXTEENTH AND SEVENTEENTH CENTURIES, INCLUDING MUN, CHILD, LAW, ETC.; THE CLASSICAL SCHOOL AND THEIR LAISSEZ-FAIRE ECONOMICS, INCLUDING ADAM SMITH, MILL , SAY, BASTIAT, SENIOR, RICARDO, AND MANY OTHERS; ETC.

A STUDY MADE IN 1939, BY A GROUP OF ECONOMISTS AT OXFORD, Ref.- Zweig, "Economic Ideals", pg 2. SHOWED THAT THE ENTREPRENEUR CONTROLS HIS PRICING ACCORDING TO HIS CONCEPT OF A JUST PRICE AND A JUST BARGAIN BASED ON HIS TOTAL COSTS, AND NOT ON A CRITERION OF PROFIT MAXIMIZATION AS REASONED BY THE MARGINAL UTILITY SCHOOL OF THEORISTS. THE CANONIST SCHOOL OF THE MIDDLE AGES CHOSE THE ASSUMPTION THAT MEN WORK TO MAINTAIN THEIR TRADITIONAL STANDARD OF LIVING, BUT NOT TO ACCUMULATE INDEFINITE RICHES.

THE THEORIES AND MODELS OF ECONOMIES ARE BASED ON NUMEROUS AND SUNDRY ASSUMPTIONS. THESE ASSUMPTIONS CHANGED IN THEIR BASIC NATURE WITH TIME. INSOFAR AS HISTORY PROVIDES AN ECONOMY WHERE A GIVEN SET OF ASSUMPTIONS COINCIDE WITH THOSE FOR A GIVEN THEORY, AN EMPIRIC TEST OF THE THEORY IS PROVIDED. EACH AND EVERY ECONOMY THAT HAS EVER EXISTED COULD SERVE AS THE EMPIRIC TEST OF A THEORY BASED UPON THE SAME ASSUMPTIONS THE ECONOMY OPERATED UNDER PROVIDED DATA HAD BEEN ADEQUATELY RECORDED. THEORIES CAN ONLY BE RELIABLY CONSTRUCTED WHERE A SUFFICIENTLY HIGH DEGREE OF INFORMATION IS AVAILABLE ABOUT THE UNDERLYING ASSUMPTIONS OF THE ECONOMY, AND THEN SUFFICIENT

STATISTICAL DATA IS AVAILABLE TO SHOW THE CONSEQUENCES OF THE PARTICULAR ASSUMPTIONS.

UNFORTUNATELY, THE HISTORIANS DID NOT RECORD THESE ECONOMIES IN SUFFICIENT DETAIL TO PROVIDE THE DATA NECESSARY TO CONSTRUCT RELIABLE THEORIES UNTIL IN VERY RECENT YEARS, THUS, ANY MODELS CONSTRUCTED TODAY OF HISTORICAL ECONOMIES HAVE NO QUANTITATIVE TESTS AVAILABLE AS TO THEIR ACCURACY.

THE LITERATURE OF ECONOMICS IS FULL OF STUDIES REPRESENTING AN EXTREMELY LARGE NUMBER OF COMBINATIONS AND PERMUTATIONS OF ASSUMPTIONS, EACH INDIVIDUAL CONCENTRATING ON WHAT TO HIM ARE THE MOST INTERESTING ASSUMPTIONS, WITH LITTLE HEADWAY BEING GAINED IN PROVIDING A METHOD FOR UNDERSTANDING OR TESTING THE CONSEQUENCES OF THESE VARIOUS ASSUMPTIONS WITHIN ANY TYPE OF A UNIVERSAL FRAMEWORK OF PROCEDURE OR APPROACH. IN EVERY GENERATION THERE IS A GREAT DEAL OF AGREEMENT AMONG THE ECONOMISTS OF THE DAY AS TO WHICH ASSUMPTIONS ARE MOST IMPORTANT AND INTERESTING. FROM THESE ASSUMPTIONS, VERBAL GENERALIZATIONS ARE MADE, AND IT IS THESE GENERALIZATIONS WHICH APPEAR TO BE INTERPRETED IN LATER DAYS BY HISTORIANS AND THEORISTS ALIKE AS THE LIVING IDEAS OF THAT DAY. WHEN THESE GENERALIZATIONS AND IDEAS ARE CONSTRUED AND INTERPRETED IN TERMS OF THE LIFE AND EXPERIENCE OF THE INTERPRETER, A CONCEPT IS SET FORTH ABOUT THE ECONOMY BEING CONSIDERED. THE PRESERVATION OF THE SALIENT FEATURES OF THESE HISTORICAL ECONOMIES, THEIR BASIC PREMISES, THE ASSUMPTIONS AND CONDITIONS ACCEPTED BY THE PEOPLES LIVING IN THESE ECONOMIES THEN REQUIRES AN EXTREMELY HIGH DEGREE OF SOPHISTICATION IN HISTORY TO DRAW ANY VALUE FROM THE HISTORICAL APPROACH TO MODELING, EXCEPT VERY GENERAL WORD DESCRIPTIONS. THESE MAY OR MAY NOT HAVE ANY VALUE AT ALL.

THE WRITERS OF THE DAY, IN ANY GIVEN ERA, NOT ONLY PRESENT THE FACTS, BUT COLOR THEM, AND IN SO DOING, SHAPE THE PATTERNS OF

SUBSEQUENT ECONOMIC THOUGHT. THEY ARE USUALLY THE ONES WHO NOT ONLY WROTE, BUT WHO HELPED SHAPE THE REALITY AND THE THOUGHTS OF THEIR DAY. SUCH WAS THE ROLE OF WRITERS LIKE ADAM SMITH, RICARDO, JOHN STUART MILL, NASSAU SENIOR, KARL MARX, SCHMOLLER, KOSCHER, WAGNER, BRETANO, KNIES, BUCHER, MAX WEBER, JEVONS, MENGER, WALRAS, WIESER, BÖHM-BAWERK, RICHARD CANTILLON, MATHUS, QUESNAY, MIRABEAU, TURGOT, GOSSEN, RODBERTUS, EDGEWORTH, MARSHALL, SPENCER, THE CONTEMPORARY JOHN MAYNARD KEYNES, AND MANY OTHERS TOO NUMEROUS TO MENTION. THE BIAS, AND COLORING OF OBSERVATION FROM ST. THOMAS AQUINAS TO SAINT-SIMON IN HIS PREDICTIONS OF THE ASCENDANCY OF SCIENCE AND TECHNOLOGY WERE MOTIVATED BY OTHER THAN THE OBJECTIVE RECORDING OF THE ECONOMIC FACTS OF THEIR DAY. THUS, OBJECTIVE FACTS ARE EXTREMELY DIFFICULT TO OBTAIN FROM THE HISTORICAL APPROACH.

IT WAS ORIGINALLY INTENDED THAT THIS STUDY WOULD DEVELOP ALONG HISTORICAL LINES, BUT AS THOUGHTS WERE GATHERED TOGETHER, THE LACK OF A REFERENTIAL FRAMEWORK, AND OF A PROCEDURE FOR READILY EXAMINING THE CONSEQUENCES OF VARIOUS COMBINATIONS OF ASSUMPTIONS BECAME MORE AND MORE APPARENT, AS DID THE ABSENCE OF RELIABLE DATA TO COMPARE THE THEORY AGAINST. SUCH A FRAMEWORK APPEARED PLAUSIBLE BY MAKING USE OF THE KNOWLEDGE OF ELECTRONIC COMPUTER PROGRAMMING, DECISION MAKING TECHNIQUES, CONTROL SYSTEM TECHNIQUES SUCH AS USED IN SERVO SYSTEMS INVOLVING FEEDBACK, AND FLOW DIAGRAM TECHNIQUES SUCH AS USED IN ELECTRONICS, HYDRODYNAMICS, AND COMPUTER PROGRAMMING.

WHILE IT IS RECOGNIZED THAT THE MODELS DEVELOPED IN THIS STUDY ARE EXTREMELY CRUDE AND PRIMITIVE, IT IS HOPED THAT SOME OF THE TECHNIQUES AND PROCEDURES SET FORTH MIGHT FIND INTEREST AND APPLICATION BY WORKERS IN THE FUTURE IN THE BUILDING OF MODELS FLEXIBLE ENOUGH TO BE READILY TESTED UNDER CHANGING ASSUMPTIONS, SOPHISTICATED ENOUGH TO GIVE INSIGHT INTO THE

INHERENT DYNAMIC INSTABILITIES CAUSED BY PHASE LAGS AND FEEDBACK IN THE SYSTEM, CLOSE ENOUGH TO REALITY TO BE OF PRACTICAL VALUE IN POLICYMAKING, AND COMPREHENSIVE ENOUGH TO PERMIT ACCURATE PREDICTION.

THE IDEA OF CLOSED LOOP FLOWS IN ECONOMIES, AND USED IN THE FLOW DIAGRAMS IS AN OLD CONCEPT, DATING BACK TO THE EARLIEST DAYS OF ECONOMIC THOUGHT. IT WAS GIVEN PROMINENCE IN THE PHYSIOCRATIC DOCTRINE OF THE CIRCULATION OF NET PRODUCT AS DESCRIBED IN "TABLEAU ECONOMIQUE", AND WAS HAILED BY THE PHYSIOCRATS AS ONE OF THE THREE GREATEST DISCOVERIES OF THEIR AGE. THE OTHER TWO WERE CONSIDERED AS THOSE OF MONEY AND PRINTING. THE CONCEPT OF CIRCULAR FLOWS WAS ELUCIDATED BY DUPONT DE NEMOURS (1771) IN HIS DISCUSSION OF "NOTHING STANDS ALONE AND EVERYTHING HOLDS TOGETHER. FROM WEALTH SPRING CULTURE, CULTURE INCREASES WEALTH, THIS GROWTH OF WEALTH INCREASES POPULATION. THIS INCREASE OF POPULATION KEEPS UP THE VALUE OF WEALTH ITSELF." MARX DESCRIBED MANY TYPES OF CLOSED FLOW CIRCUITS IN HIS FORMULAE, SUCH AS, "COMMODITY, GOES TO MONEY, GOES TO COMMODITY", "LABOR POWER, GOES TO INCOME, INCOME GOES BY WAY OF CONSUMPTION INTO LABOR POWER", ETC. SISMONDI GAVE HIS WORD DESCRIPTIONS OF CLOSED FLOW CIRCUITS IN TERMS OF THE NATIONAL INCOME FROM ONE YEAR DETERMINING THE NATIONAL EXPENDITURE FOR THE NEXT YEAR. THE GREAT EMPHASIS ON CLOSED CIRCULAR FLOWS IS PRESENT IN THE WORKS OF THE CURRENT ECONOMIC INVESTIGATORS TO A LARGE EXTENT, SUCH AS KEYNES, SAMUELSON, HICKS, HARROD, METZLER, TUSTIN, PHILLIPS, KALECKI, TINBERGEN, RAGNAR, FRISCH, LUNDBERG, BÖHI, OPPELT, GEYER, FÖHL, TISCHNER, HENN, FORSTNER, LESEMANN, TO MENTION BUT A FEW.

MUCH HELP HAS BEEN RECEIVED BY THE SUGGESTIONS AND IDEAS CONTRIBUTED BY JAY FORRESTER, ED HOLLAND, BERNARD MULLER-THYME, AND FROM THE LECTURES OF CHARLES KINDLEBERGER, ROBERT BISHOP, FRANCIS BATOR, AND OTHERS OF THE M.I.T. SCHOOLS OF INDUSTRIAL

MANAGEMENT AND department of ECONOMICS. FOR THIS GUIDANCE AND INSTRUCTION I AM EXTREMELY GRATEFUL. GRATEFUL ACKNOWLEDGEMENT IS ALSO GIVEN TO RICHARD BENNETT AND ED ROBERTS FOR THE USE OF THEIR S.I.M. ROUTINE FOR THE I.B.M. 704 COMPUTER, AND THEIR ASSISTANCE IN CODING MODEL 3A. COMPUTATIONS WERE CARRIED OUT AT THE M.I.T. COMPUTATION CENTER ON THE I.B.M. 704 COMPUTER.

THIS PAPER PRESENTS THE RESULT OF A STUDY DIRECTED TOWARD THE FINDING OF AN APPROACH TO THE PROCEDURAL AND THEORETICAL STUDY OF ECONOMICS, UNDERTAKEN AS THE THESIS PART OF THE SLOAN FELLOWSHIP PROGRAM IN INDUSTRIAL MANAGEMENT AT M.I.T., 1958.

JOHN T. PETTIT - SLOAN FELLOW

CHAPTER I

MODELING DEFICIENCIES

PART I - A DISCUSSION OF SOME OF THE DEFICIENCIES IN THE EXISTING NATIONAL INCOME MODELS, AND MODELING TECHNIQUES.

SOME OF THE MORE GLARING DEFICIENCIES NOTED IN THE MODELING OF NATIONAL INCOME ECONOMIES WILL BE LISTED HERE, WITH LITTLE DISCUSSION, SINCE THE PRIMARY PURPOSE OF THIS PAPER IS NOT THAT OF JUDGING PAST PERFORMANCE, BUT RATHER TO PROPOSE A DIFFERENT APPROACH. AS A RESULT, NO EFFORT WILL BE MADE TO CONSTRUCT AN EXHAUSTIVE LIST OF DEFICIENCIES, BUT RATHER TO POINT OUT WHAT APPEARS TO THE WRITER TO BE SOME OF THE GLARING DEFICIENCIES. A NEW METHOD OF MODEL BUILDING WILL THEN BE PROPOSED, AND APPLIED TO SOME SIMPLE CASES.

A - CLOSED LOOPS

THERE APPEARS TO BE WIDESPREAD FAILURE IN THE CONSIDERATION OF THE FLOWS OF TANGIBLES, AND OF MONEY, IN CONSIDERING WHERE THE GOODS, SERVICES, OR MONEY CAME FROM, AS WELL AS WHERE THEY GO, IN SUCH A MANNER THAT THERE IS COMPLETE ACCOUNTABILITY. FOR EXAMPLE, ASSUME ONLY ONE FACTOR OF PRODUCTION OF A GOOD E AND LET THAT ONE FACTOR BE LABOR. LABOR IS CONVERTED TO GOODS E, WHICH ARE IN TURN CONSUMED BY THE HOUSEHOLD, WHICH FURNISHES THE LABOR. IN THE MONEY SECTOR, THE MONEY RECEIVED BY THE HOUSEHOLD FOR LABOR EITHER GOES TO BUSINESS AS PAYMENT FOR GOODS, OR IS SAVED. THAT WHICH IS SPENT ON CONSUMPTION IS EITHER PUT IN THE BANK, OR PAID OUT FOR LABOR TO THE HOUSEHOLD, ETC., EVERY DOLLAR HAVING A SOURCE AND A DISPOSITION. THIS CAN BE EXPRESSED AS A PRINCIPLE CALLED "THE PRINCIPLE OF THE

ACCOUNTABILITY OF TANGIBLES", AND CAN BE EXPRESSED AS "ANY TANGIBLE FLOWING IN A SYSTEM MUST HAVE AN ORIGIN AND DESTINATION". THIS MAY APPEAR TO BE A TRITE PRINCIPLE BUT THERE HAVE BEEN MANY MODELS PROPOSED WHICH VIOLATE THIS PRINCIPLE (SEE HARROD, HICKS, BODENHORN, METZLER, ETC.), SOME OF WHICH WILL BE DISCUSSED MORE FULLY LATER ON.

B - INFORMATION - DECISION NETWORK

MOST OF THE PROPOSED MODELS IN THE LITERATURE FAIL TO CONNECT THE MONEY SECTOR WITH THE PRODUCTIVE SECTORS. THE CONNECTION BETWEEN THESE SECTORS IS INFORMATION AND DECISIONS, AND THE RATES OF FLOW OF ALL TANGIBLES ARE CONTROLLED BY DECISIONS. SOME OF THESE DECISIONS ARE INDEPENDENT OF OTHER DECISIONS, WHILE MOST OF THEM ARE DIRECT CONSEQUENCES OF OTHER DECISIONS, THE LINKAGE BEING THE FLOW OF INFORMATION. THESE FLOWS OF INFORMATION, HOWEVER, APPEAR TO HAVE RECEIVED RELATIVELY LITTLE TREATMENT COMPARED TO THEIR IMPORTANCE. THIS IS ALSO TRUE OF THE DECISION MAKING PROCESS.

C - FLOWS VS. MAGNITUDES

IN ECONOMIC DISCUSSIONS, THE MAGNITUDE OF A VARIABLE IS SPOKEN OF MORE OFTEN THAN NOT INSTEAD OF THE RATE OF FLOW. THIS HAS LED TO GROSS ERRORS IN THE RESULTING DISCUSSIONS SINCE THE DIFFERENCE BETWEEN TWO EXTREMELY LARGE FUNCTIONS IS SUBJECT TO EXTREMELY LARGE ERRORS FOR SMALL ERRORS IN THE ORIGINAL FUNCTIONS. THIS DIFFICULTY CAN BE AVOIDED IN PART BY THE USE OF FLOW DIAGRAMS TO REPRESENT THE SYSTEM, SINCE THE CONSISTENCY OF FLOWS IS EASIER TO HANDLE, AS ARE THE UNITS, AND ELIMINATES THE INTRODUCTION OF LARGE ERRORS DUE TO THIS DIFFERENCING OF TWO LARGE QUANTITIES.

D - DECIDED VS. ACTUAL FLOWS (INTENDED, UNINTENDED, VS. ACTUAL).

DECISIONS DETERMINE THE FLOW OF TANGIBLES IN ANY ECONOMIC SYSTEM. MANY TIMES, A RATE OF FLOW IS DECIDED UPON, BUT THAT DECISION DOES NOT IMMEDIATELY BRING FORTH THE DESIRED RESULT, SO THE DECISION IS CHANGED, AND TOO MUCH FLOW OCCURS. ONCE AGAIN THE DECISION IS CHANGED, AND THERE OFTEN OCCURS A HUNTING, OR AN INSTABILITY IN THE SYSTEM CONSEQUENTLY. THIS TYPE OF A MECHANISM HAS RECEIVED RELATIVELY LITTLE ATTENTION. THIS POINT WILL BE DEALT WITH AT MUCH GREATER LENGTH, HOWEVER, LATER ON IN THIS PAPER.

E - TIME DELAYS

WHEN SPEAKING OF A DYNAMIC ECONOMIC MODEL, THE FLUCTUATIONS IN MANY CASES ARE CAUSED BY THE TIME DELAYS INHERENT IN THE SYSTEM, SUCH AS, THE TIME DELAY BETWEEN PLANTING AND HARVESTING A CROP, OR DECIDING TO PRODUCE AND ACTUALLY PRODUCING. THESE DELAYS ARE UNDOUBTEDLY OF MAJOR IMPORTANCE IN DETERMINING THE CHARACTERISTIC RESPONSES OF ANY AND ALL TYPES OF DISTURBANCES TO THE SYSTEM, YET LITTLE IF ANY ATTENTION HAS BEEN GIVEN TO THESE TIME DELAYS OR THEIR EFFECTS ON THE REST OF THE SYSTEM.

F - IMPERFECT FLOW OF INFORMATION

THE FLOW OF ACTUAL INFORMATION IN A REAL ECONOMY IS FAR FROM A SMOOTH FLOW. BALANCE SHEETS ARE FIGURED MONTHLY, OR EVEN YEARLY, SALES FIGURES ARE REPORTED WEEKLY, MONTHLY, OR EVEN YEARLY, THE CENSUS IS TAKEN EVEN LESS OFTEN, YET THIS REPORTING OF FINANCIAL AND STATISTICAL DATA INFLUENCES THE MAKING OF DECISIONS. THIS MAY BE ONE OF THE CHARACTERISTICS OF OUR MODERN DAY ECONOMY, AND ECONOMIES IN THE PAST MAY HAVE HAD A

MORE NEARLY CONTINUOUS FLOW OF THE INFORMATION ON WHICH THE INDIVIDUAL BASED HIS DECISIONS. NEVERTHELESS, IN ANY CONSIDERATION OF MODELS TO BE APPLIED TO PRESENT DAY ECONOMIES, THE IMPERFECT FLOWS OF INFORMATION MUST BE TAKEN INTO ACCOUNT.

G - TOO MANY VARIABLES

MANY WRITERS ASSUME THAT THE VARIABLES IN A MODEL ARE "FAR TOO NUMEROUS TO MENTION OR CONSIDER". WHILE THERE ARE UNDOUBTEDLY MANY VARIABLES AFFECTING A GIVEN ECONOMIC DECISION, IT IS DOUBTED WHETHER IN THE FINAL ANALYSIS THESE VARIABLES ARE INDEPENDENT, OR WHETHER THE DECISION IS NOT DETERMINED BY ONE OF THE VARIABLES, AND THE EFFECTS OF ALL THE OTHERS ARE OF A MUCH LESSER ORDER OF MAGNITUDE.

H - BASIC ASSUMPTIONS

A RELATIVELY SMALL AMOUNT OF CONSIDERATION IS GIVEN TO THE BASIC ASSUMPTIONS ON WHICH THE MODEL IS CONSTRUCTED, AND TO THE VALIDITY AND PURPOSE OF THESE ASSUMPTIONS. THE MAJOR EFFORT RATHER APPEARS TO BE GIVEN TO THE RESULTS DERIVED FROM THE ASSUMPTIONS. THESE DISCUSSIONS OF RESULTS ARE NO BETTER THAN THE VALIDITY OF THE ASSUMPTIONS.

MANY MODELS REFLECT NOTHING MORE THAN THE ASSUMPTIONS SET FORTH. (EXAMPLE: DOMAR'S, METZLER'S, HARROD'S, MODELS, ETC.) THE MODEL ITSELF, CONTAINS NO MECHANISM IN AND OF ITSELF, AND THE SOLUTION OF THE EQUATIONS GIVE NO INFORMATION NOT IMPLICITLY ASSUMED.

I - EMPIRICAL CONSEQUENCES

MUCH CONCERN IS GIVEN TO THE EMPIRICAL CONSEQUENCES RATHER THAN THE SOUNDNESS OF THE MODELING PROCEDURE, THE SOUNDNESS

OF THE PREMISES, OR THE SOUNDNESS OF THE ANALYTICAL TOOLS BEING APPLIED.

THE ECONOMIC MODEL BUILDERS APPEAR TO BE SO EAGER TO APPLY THE EMPIRIC TEST THAT THEY FAIL TO GIVE THE NECESSARY AND SUFFICIENT CONSIDERATION TO THE FUNDAMENTAL THEORY BEFORE APPLYING IT TO THE NUMERICAL DATA. IN OTHER CASES, (EXAMPLE: TINBERGEN MODEL) THE MODEL BUILDER IS ATTEMPTING TO APPLY NUMERICAL DATA BEFORE HE HAS A THEORY. THIS RESULTS IN A GREAT DEAL OF EFFORT BEING SPENT ON STATISTICAL EXERCISES OF DUBIOUS VALUE. THE RESULTS, AT BEST, GIVE AN EMPIRICAL FORMULA TO FIT THE PAST DATA.

J - GENERAL OVER-SIMPLIFICATION

A SYSTEM SHOULD BE BROKEN DOWN INTO EASILY MANAGED COMPONENTS, BUT IN ENDEAVORING TO FIT THESE COMPONENTS TOGETHER INTO A COMPLEX, OVER-SIMPLIFICATION CAN ONLY REMOVE THE BASIC VALUES TO BE DERIVED FROM FORMING THE COMPLEX.

WHEN THE SOLUTION CAN BE PRESENTED IN CLOSED FORM, SOME GREAT DEAL OF SATISFACTION IS EXPRESSED. THUS, A GREAT DEAL OF EMPHASIS IS PUT INTO SIMPLIFICATION OF THE MODELS TO THE POINT WHERE THE MODEL CAN BE EXPRESSED IN TERMS OF ONE OR TWO SIMPLE EQUATIONS, SOLVABLE IN CLOSED FORM, WITHOUT REGARD TO THE CONSIDERATION THAT TO DO SO, IN MOST CASES, THE MODEL BUILDER HAS TO ASSUME THE HEART OF THE PROBLEM OUT OF THE MODEL.

HOW FAR COULD THE ELECTRONICS ENGINEERS HAVE DEVELOPED THEIR MODELS, IF THIS APPROACH HAD BEEN TAKEN. THEY WOULD HAVE BEEN LIMITED FROM THE OUTSET TO

1) NO CIRCUIT DIAGRAMS,
2) NO COMPLEX SYSTEMS,
3) ONLY SOLUTIONS EXPRESSABLE IN CLOSED FORM,
4) NO SYSTEMS INVOLVING MORE THAN TWO OR THREE SIMPLE DIFFERENTIAL EQUATIONS.

K - Maximizing Variables

A GREAT DEAL OF EFFORT HAS BEEN GIVEN TO THE MAXIMIZING OF VARIABLES SUCH AS OUTPUT, PROFITS, ETC., WHEN TO DO SO, ASSUMES AND ANTICIPATES AN ALREADY FULL GROWN AND SOPHISTICATED THEORY, BEFORE THIS THEORY HAS BEEN EVEN REMOTELY DEVELOPED OR EXAMINED IN ITS BEGINNING EMBRIONIC STATE. THE PROCESS OF MAXIMIZING VARIABLES IN A MODEL IS USEFUL AND EXTREMELY VALUABLE ONCE THE MODEL IS DEVELOPED TO THE POINT WHERE THE TRUE NATURE OF THE VARIABLES IS APPARENT.

L - Mathematical Techniques

THE FIELD OF ECONOMICS APPEARS TO HAVE MORE THAN ITS FAIR SHARE OF CONTRIBUTORS* WHO DEMONSTRATE INADEQUATE, WRONGFULLY APPLIED, INACCURATE, AND INAPPLICABLE MATHEMATICAL TECHNIQUES AND TOOLS, AND WHO ARE OBVIOUSLY ILL EQUIPPED AND LACKING IN THE SOPHISTICATION NECESSARY TO HANDLE THE TOOLS WITH WHICH THEY ARE PLAYING. MANY TIMES, THEOREMS AND CONCEPTS ARE LIFTED FROM CLASSICAL AND QUANTUM MECHANICS, AND ASSUMED TO APPLY IN ECONOMICS TO GIVE APPARENT PRESTIGE TO AN OTHERWISE TRITE CONSIDERATION. WHILE THESE DUBIOUS TECHNIQUES WILL WORK AMONG COLLEAGUES WHO ARE UNSCHOOLED IN THE USE OF THE MATHEMATICAL TOOLS, THEY CAN ILL AFFORD THE LIGHT OF DAY. SOME OF THE WAYS THE ECONOMISTS CAN RID THEMSELVES OF THESE MALPRACTITIONERS IS TO TIGHTEN UP THE STANDARDS ESTABLISHED IN THEIR PROFESSIONAL LITERATURE, IN THEIR TEACHING PRACTICES AND REQUIREMENTS, AND IN THEIR EXPECTATIONS FROM THEIR COLLEAGUES, AND THE LEVEL OF MATHEMATICAL AND SYSTEMS TRAINING SHOULD BE RAISED AMONG THE STUDENTS IN THE FIELD.

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*FOOTNOTE: ONE OF THE MOST STRIKING EXAMPLES OF WRONGFULLY APPLIED AND INAPPLICABLE MATHEMATICAL TECHNIQUES HAS BEEN IN THE STUDY OF BUSINESS CYCLES, WHERE MANY SUPPORTERS CAN STILL BE FOUND FOR JEVONS AND MOORES (SUNSPOT - WEATHER

CROP THEORIES, THE ASTRONOMICAL THEORIES WHERE CORRELATION TECHNIQUES ARE APPLIED TO BUSINESS CYCLE DATA TO SHOW THAT THE PLANETARY MOVEMENTS EFFECT BUSINESS CYCLES DIRECTLY, ETC. MANY ECONOMISTS HAVE SPENT LIFETIMES MANIPULATING STATISTICS TO PROVE UNFOUNDED CORRELATIONS.

ONLY IN THE COMPLETE ABSENCE OF A THEORY OF CAUSE AND EFFECT CAN THESE CORRELATIONS HAVE ANY SUBSTANTIAL EFFECT. SAMUELSON, IN HIS "INTRODUCTORY ANALYSIS OF ECONOMICS", MENTIONS ONE INGENEOUS WRITER WHO DROPPED THE SUNSPOT THEORY "BECAUSE THEY DID NOT GIVE AN EIGHT-YEAR CYCLE, AND SET FORTH THE BELIEF THAT WHENEVER THE PLANET VENUS CAME BETWEEN THE SUN AND THE EARTH, THIS CAUSED MAGNETIC ABSORPTIONS AND AN EIGHT-YEAR BUSINESS CYCLE." SAMUELSON MAKES AN INTERESTING OBSERVATION IN COMMENTING ON THIS EXAMPLE WHEN HE STATES, "UNFORTUNATELY, THE FIELD OF ECONOMICS HAS NOT THE CLASSIC SIMPLICITY OF PHYSICS OR MATHEMATICS. IN THOSE FIELDS A CRANK WHO THINKS HE CAN SQUARE THE CIRCLE OR BUILD A PERPETUAL-MOTION MACHINE CAN BE SHOWN UP, IF NOT TO HIS OWN SATISFACTION, AT LEAST TO THAT OF EVERY COMPETENT OBSERVER. IN ECONOMICS, IT IS NOT QUITE SO EASY TO DEMONSTRATE THAT SUNSPOT THEORIES OF THE CYCLE ARE ALL MOONSHINE, ESPECIALLY IF THEIR PROPONENTS ARE WILLING TO SPEND A LIFETIME MANIPULATING STATISTICS UNTIL THEY PRODUCE AGREEMENT."

THE ECONOMISTS WILL BE FACED WITH THIS DILEMMA AS LONG AS A VACUUM EXISTS WITH RESPECT TO A FUNDAMENTAL CAUSE AND EFFECT THEORY OF THE PHENOMENON BEING OBSERVED.

ONE OF THE PRINCIPLE DANGERS IN THE BUILDING OF ECONOMIC MODELS IS THE TENDENCY OF MOST RESEARCHERS WITH A PHYSICAL SCIENCE, OR ENGINEERING BACKGROUND, TO START WITH A TYPE OF MODEL ALREADY FAMILIAR TO THEM. THEY THEN TRY TO FORCE THE ECONOMIC MODEL TO CONFORM TO THE EQUATIONS OF THE PHYSICAL MODEL. IT IS POSSIBLE THAT PHYSICAL ANALOGIES ARE SIMILAR. HOWEVER, THE PHYSICAL SYSTEM SHOULD BE CAREFULLY TAILORED TO THE ECONOMIC MODEL RATHER THAN VICE VERSA.

M - SUMMARY

THERE APPEARS TO HAVE BEEN LITTLE EFFORT GIVEN TO THE BASIC NEED FOR A THEORY, BASED UPON THE INTERACTION OF INFORMATION AND DECISIONS, AND THE EFFECTS THEY HAVE ON THE FLOWS OF THE

TANGIBLE FACTORS AND PRODUCTS OF PRODUCTION. AS WELL AS THE MONEY FLOWS WHICH ARE, IN MANY CASES, ONLY THE EVIDENCE OF WHAT HAS HAPPENED AFTER IT HAS HAPPENED.

THERE ALSO APPEARS TO BE LITTLE EFFORT GIVEN TO THE STUDY AND SYNTHESIS OF ECONOMIC SYSTEMS BY TECHNIQUES SIMILAR TO THOSE USED IN THE FLOW DIAGRAMS OF ELECTRONICS, HYDRODYNAMICS, AND OTHER COMPLEX SYSTEMS ANALYSES INVOLVING FLOWS OF A MUCH LESSER ORDER OF MAGNITUDE OF COMPLEXITY THAN THOSE IN ECONOMICS. MUCH COULD BE CONTRIBUTED HERE BY THE EXPERTS IN INFORMATION FLOW, AND THE TECHNIQUES ALREADY DEVELOPED IN THE COMPUTER FIELD. A FEW OF THE BETTER EXISTING MODELS WILL BE BRIEFLY DISCUSSED IN THE FOLLOWING SECTIONS.

PART 2 - DISCUSSION OF SOME DYNAMIC GROWTH MODELS.

SECTION I - HARROD'S GROWTH MODEL[1]

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[1] HARROD, R. F., ECONOMIC ESSAYS (NEW YORK, 1953), PG. 254.

A - BASIC ASSUMPTIONS

HARROD'S GROWTH MODEL IS BASED ON THREE PRINCIPAL ASSUMPTIONS:

1) SAVINGS ARE DETERMINED BY INCOME,
2) INVESTMENT IS DETERMINED BY THE CHANGE IN INCOME, AND
3) SAVINGS EQUALS INVESTMENT.

THESE ASSUMPTIONS GIVE THE FOLLOWING EQUATIONS FOR THE SYSTEM:

1) $S(T) = \alpha_1 Y(T)$
2) $I(T) = \alpha_2 [Y(T) - Y(T-1)]$
3) $S(T) = I(T)$, WHERE

S = SAVINGS, Y = INCOME, I = INVESTMENT, α_1 & α_2 ARE CONSTANTS. THESE THREE EQUATIONS CAN BE SOLVED FOR Y(T), S(T), AND I(T). THE FLOW DIAGRAM FOR THIS MODEL IS GIVEN IN FIGURE 2.

B - DISCUSSION

MODELS OF THE TYPE DEVELOPED BY HARROD FAIL TO INCLUDE ANY REAL LINKAGE BETWEEN THE ACTUAL PRODUCTIVE PROCESSES, THE FACTORS OF PRODUCTION, AND THE INCOME OF THE HOUSEHOLDS. THE ENTIRE CONCERN IS WITH THE MONEY SECTOR EXCLUSIVELY. EVEN IN ITS CONSIDERATION OF THE FLOWS OF MONEY, THE MODEL IS INCOMPLETELY DETERMINED IN THAT,

1) NO CONSIDERATION IS GIVEN OF THE MONEY FLOW FOR CONSUMPTION,
2) THE SAVINGS, INVESTMENT, AND INCOME SPOKEN OF AT TIME T ARE NOT RELATED TO THE RATES OF FLOW.

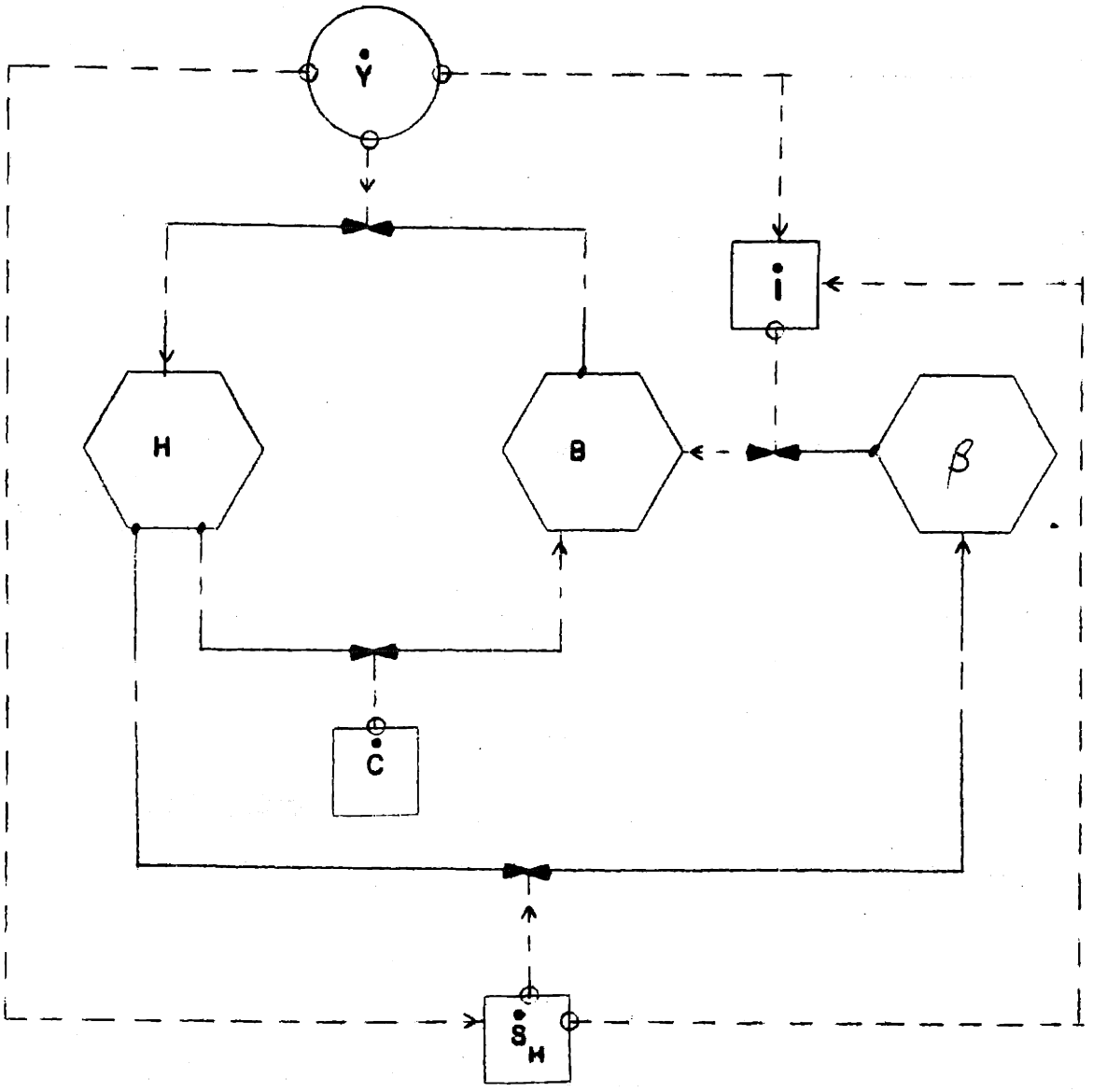
IT IS SUSPECTED THAT WHAT HARROD WAS REALLY REFERRING TO IS

IS THE RATES OF FLOW OF THESE MONEYS IN THE MONEY SECTOR, AND NOT THE TOTAL SAVINGS, INVESTMENT, OR INCOME AT T.

THE OBVIOUS DEFICIENCIES OF THIS MODEL WOULD TEND TO NEGATE ANY FURTHER ANALYSIS, UNLESS THE MODEL ITSELF WERE FIRST COMPLETED, AND THE ACTUAL LINKAGES BY FLOW OF INFORMATION, BETWEEN THE PRODUCTIVE PROCESSES, AND THE FLOWS OF THE OTHER TNAGIBLES INVOLVED IN THE SYSTEM, ARE SPECIFIED. THESE SPECIFICATIONS CAN BE MADE ON THE SAME LEVEL OF SIMPLICITY AS IS DONE IN THE PRESENT ASSUMPTIONS, BUT THE ASSUMPTIONS MADE IN THE OTHER SECTORS DETERMINE THE VALUES OF THE FLOWS IN THE MONEY SECTOR.

IN THIS MODEL, HARROD IS BASICALLY MAKING STILL ANOTHER UNVOICED ASSUMPTION. THAT IS, THAT THE INCOME OF THE HOUSEHOLD IS DETERMINED BY SAVINGS AND INVESTMENT. INCOME IS DETERMINED BY THE PRODUCTIVE PROCESS, AND ONCE INCOME IS DETERMINED, THE LEVEL OF SAVINGS AND HENCE INVESTMENT ARE DETERMINED. IT IS TRUE THAT IN SOME CASES INCOME IS MADE POSSIBLE BY THE PAST INVESTMENT, BUT ONLY THROUGH THE PRODUCTIVE PROCESS. IN OTHER WORDS, WHICH COMES FIRST, THE HORSE OR THE CART. THE ASSUMPTIONS OF CAUSE AND EFFECT WOULD SEEM MORE JUSTLY APPLIED IN THAT INSTANCE, IN DETERMINING THE MANNER IN WHICH THE PRODUCTIVE DECISIONS ARE MADE, RATHER THAN BY TRYING TO INTUITIVELY FORECAST WHAT THE ULTIMATE EFFECT WILL BE ON MONEY FLOWS WITHOUT THE BENEFIT OF EXPLICITLY INCLUDING THE PRODUCTIVE PROCESSES. THE CONCLUSION THEREFORE WITH RESPECT TO HARROD'S MODEL IS THAT IT DOES NOT IN REALITY GIVE ANY INFORMATION WITH RESPECT TO INCOME, BUT MERELY ASSUMES SOME RELATIONSHIP BETWEEN SAVING, INVESTMENT, AND INCOME, ONCE INCOME HAS BEEN DETERMINED.

IT IS INTERESTING TO NOTE THAT HICKS, IN HIS REVIEW OF HARROD'S BOOK CONTINUES TO COMPOUND AND CLOUD THE ERRORS MADE BY HARROD



HARROD'S MODEL

FIGURE 2

BY SUGGESTING MORE COMPLICATED RELATIONS[2] BETWEEN C & Y, AND I & Y, AND SUGGESTS SPECIFICALLY ADDING MORE PHASE LAGS.[3]

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[2]HICKS, J. R., "MR. HARROD'S DYNAMIC THEORY," ECONOMICS, MAY 1949, XVI, PGS. 106-121.

[3]HICKS, J. R., "A CONTRIBUTION TO THE THEORY OF TRADE CYCLES," (OXFORD, 1950), PGS. 109-110.

THIS MIGHT MAKE AN INTERESTING EXERCISE, BUT THE DEPARTURE IS SO FAR FROM PHYSICAL EXPERIENCE THAT IT WOULD BE for the writer HARD TO JUSTIFY ON THE BASIS OF INTUITION OR EXPERIENCE.

SECTION 2 - METZLER'S MODEL[1]

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[1]METZLER, L. A., "THE NATURE AND STABILITY OF INVENTORY CYCLES," REVIEW OF ECONOMIC STATISTICS, AUGUST 1941, XXIII, PGS. 113-129.

A - BASIC ASSUMPTIONS

THE MODEL CONSIDERED BY METZLER CONTAINED THE FOLLOWING BASIC ASSUMPTIONS:

1) SAVINGS ARE DETERMINED BY INCOME,
2) INVESTMENT EQUALS SAVINGS,
3) THE INCOME OF THE HOUSEHOLD EQUALS THEIR OUTPUT ON CONSUMPTION,
4) THE OUTPUT OF THE HOUSEHOLD ON CONSUMPTION IS THE SUM OF INTENDED INVESTMENT PLUS THE GOODS PRODUCED FOR CONSUMPTION,
5) INTENDED INVESTMENT, AND THE PRODUCTION OF CONSUMER'S GOODS BOTH DEPEND UPON THE LEVEL OF INCOME PRODUCERS EXPECT DURING THE CURRENT PERIOD.

ACCORDING TO METZLER, THESE ASSUMPTIONS LEAD TO THE FOLLOWING EQUATIONS:

1) $S(T) = E_1 Y(T)$.

2) $S(T) = I(T)$.

3) $Y(T) = I_P(T) + C_P(T)$.

4) $C_P(T) = (1-E_1)\ Y_E(T)$.

5) $I_P(T) = E_2[Y_E(T) - Y_E(T-1)] - (1-\alpha_1)$ $[Y_E(T) - Y(T)]$.

WHERE S = SAVINGS, Y = INCOME, C = CONSUMPTION, AND I = INVESTMENT, E_1, E_2, AND α_1 ARE CONSTANTS. TO THIS SET OF FIVE EQUATIONS, METZLER ADDS A SO-CALLED EXPECTATION FUNCTION WHICH, "TELLS HOW PRODUCERS DETERMINE THE LEVEL OF EXPECTED CURRENT INCOME". HE THEN EXPRESSES THIS EXPECTATION FUNCTION AS,

6) $Y_E(T) = E\ [Y(T-1),\ Y(T-2),\ - - - -\ Y(T-N)]$.

WHERE E IS CONSTANT. THE SUBSCRIPT E IS USED TO EXPRESS EXPECTED VALUES IN THE CASE OF INCOME AND THE SUBSCRIPT P EXPRESSED EXPECTED VALUES FOR CONSUMPTION OF INVESTMENT.

B - <u>DISCUSSION</u>

THE MODEL AS GIVEN BY METZLER IS QUITE DEVOID OF MEANING. IN THE FIRST PLACE, ALL THROUGH HIS DISCUSSION HE MIXES HIS UNITS. HE SPEAKS OF ADDING INTENDED INVESTMENT TO GOODS PRODUCED FOR CONSUMPTION, WHEN IT APPEARS AS THOUGH WHAT HE MEANS IS, THE RATE OF MONEY BEING SPENT FOR CONSUMPTION. HIS TENDENCY IS TO SPEAK ALTERNATIVELY ABOUT THE FLOW OF TANGIBLE GOODS AND SERVICES, AND THE MONEY FLOWS, BUT NOWHERE ARE THE TWO SEPARATED.

WHEN IT COMES TO WRITING THE EQUATIONS, HE WRITES THEM

APPARENTLY ENTIRELY IN TERMS OF THE MONEY FLOWS. AT THE END, ALMOST AS AN AFTERTHOUGHT, AND CERTAINLY BECAUSE HE NEEDS ONE MORE EQUATION TO HAVE AS MANY EQUATIONS AS UNKNOWN, HE THROWS IN A SO-CALLED EXPECTATION FUNCTION.

ASSUMING METZLER'S EQUATIONS, THE FLOW DIAGRAM WOULD BE IMPOSSIBLE TO CONSTRUCT WITHOUT CLARIFICATION OF UNITS. HERE, AS IN THE HERROD MODEL, THERE APPEARS TO BE NO DETERMINATION OF INCOME, BUT ONLY DETERMINATION BY ASSUMPTION OF THE OTHER VARIABLES ONCE INCOME IS KNOWN. THIS, OF COURSE, ASSUMES CONSISTENCY OF UNITS IN THE MODEL.

SECTION 3 - SAMUELSON-HICKS MODEL[1]

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[1] SAMUELSON, P. A., "INTERACTIONS BETWEEN THE MULTIPLIER ANALYSIS AND THE PRINCIPLE OF ACCELERATION," REVIEW OF ECONOMIC STATISTICS, 1939, 21, PGS. 75-81.

SAMUELSON, P. A., "A SYNTHESIS OF THE PRINCIPLE OF ACCELERATION AND THE MULTIPLIER," JOURNAL OF POLITICAL ECONOMY, 1939, 47, PGS. 786-97.

HICKS, J. R., "A CONTRIBUTION TO THE THEORY OF THE TRADE CYCLE," OXFORD, 1950, CHAPTERS IV, V, AND VI.

A - BASIC ASSUMPTIONS

THE MODEL CURRENTLY REFERRED TO AS THE SAMUELSON-HICKS MODEL WAS ORIGINALLY A FORMULATION OF THE SO-CALLED MULTIPLIER-ACCELERATOR INTER-ACTION IN DISCRETE PERIODS GIVEN BY SAMUELSON IN 1939, AND DISCUSSED WITH SIMPLIFYING ASSUMPTIONS BY HICKS IN 1950. ESSENTIALLY THE SAME MODEL WAS DEVELOPED IN CONTINUOUS FORM BY OTHER WRITERS.

THE ASSUMPTIONS USED ARE AS FOLLOWS:

1) CONSUMPTION $\dot{C}$ AND INDUCED INVESTMENT $\dot{\mathcal{I}}$ ARE FUNCTIONS OF INCOME $\dot{Y}$,
2) THE RELATIONS BETWEEN $\dot{C}$ AND $\dot{Y}$, AND $\dot{\mathcal{I}}$ AND $\dot{Y}$ INVOLVE DISTRIBUTED TIME LAGS.

THE MODEL SAYS NOTHING ABOUT SAVINGS.

THE FLOW DIAGRAM OF THIS MODEL IS GIVEN IN FIGURES 3 AND 4. THE EQUATIONS FOR $\dot{C}$ AND $\dot{I}$ ARE GIVEN AS

$$1)\quad \dot{C}_T = \gamma_0 + c_1 \dot{Y}_{T-1} + c_2 \dot{Y}_{T-2} - - - -$$

$$2)\quad \dot{\mathcal{I}}_T = V_1(\dot{Y}_{T-1} - \dot{Y}_{T-2}) + V_2(\dot{Y}_{T-2} - \dot{Y}_{T-3}) - - - -$$

WHERE $\Sigma c_i = c$, AND $\Sigma V_i = V$ AND V_i & c_i ARE CONSTANTS.

ALSO,

$$3)\quad \dot{Y}_T = \dot{C}_T + \dot{I}_T = \dot{C}_T + \dot{\mathcal{I}}_T + \dot{A}_T,$$ WHERE $\dot{A}_T$ IS AUTONOMOUS INVESTMENT. INVESTMENT $\dot{I}_T = \dot{\mathcal{I}}_T + \dot{A}_T$.

B - <u>DISCUSSION</u>

THERE ARE TWO WAYS TO LOOK AT THIS MODEL. THE FIRST IS ALONG THE LINES OF THE DISCUSSIONS OF HARROD'S AND METZLER'S MODELS. THIS GOES AS FOLLOWS. THERE appears to EXIST NO REAL MECHANISM IN THE MODEL ITSELF FOR DETERMINING $\dot{Y}$. WHAT THE RELATIONSHIPS GIVEN APPEAR TO SAY IS THAT, GIVEN $\dot{Y}_T$, $\dot{Y}_{T-1}$, $\dot{Y}_{T-2}$, - - - - $\dot{Y}_{T-N}$, THEN BY ASSUMPTION, THE VALUES OF $\dot{C}$ AND $\dot{I}$ CAN BE CALCULATED PROVIDED OF COURSE THAT YOU ASSUME THE VALUES OF THE CONSTANTS c_i AND V_i.

THE FORMULATIONS AND DISCUSSIONS GIVEN BY SAMUELSON AND HICKS DO NOT, OF COURSE, TAKE THIS POINT OF VIEW. THEY SPEAK OF CONSUMPTION PLANS, AND INVESTMENT PLANS BEING REALIZED. BY THIS IT IS INFERRED THAT THERE IS A DECISION MADE BY A HOUSEHOLD TO CONSUME. THEY CONSUME GOODS AND SERVICES, NOT MONEY. THE GOODS AND SERVICES CONSUMED WERE PURCHASED AT A PRICE WITH

DOLLARS. THE TOTAL AMOUNT OF THESE DOLLARS GOING FOR CONSUMPTION THEN REPRESENT WHAT WAS CALLED CONSUMPTION. THE DECISION TO CONSUME DEPENDS ON INFORMATION ABOUT INCOME, AND DOES NOT appear to DETERMINE INCOME DIRECTLY. SINCE THE MODEL CONTAINS NO INFORMATION ABOUT THE DETERMINATION OF INCOME, THE DISCUSSION PERTAINING TO IT WILL BE GIVEN LATER. THE INVESTMENT PLANS BEING REALIZED INFERS A DECISION TO INVEST, AND THAT THIS DECISION IS BASED ON INFORMATION ABOUT INCOME. THIS PART OF THE MODEL THEN APPEARS REASONABLE, AS IT IN NO WAY INFERS THAT INCOME IS DETERMINED BY THE INVESTMENT IN THE CURRENT PERIOD OF TIME.

THE DECISIONS TO CONSUME AND TO INVEST WOULD AFFECT THE FLOWS OF MONEY FROM THE HOUSEHOLD TO THE BUSINESS AND FROM THE BANK TO THE BUSINESS, BUT UNTIL THE BUSINESS DETERMINES TO PRODUCE MORE GOODS AND SERVICES, THE INCOME TO THE HOUSEHOLD WILL NOT BE EVEN A REAL EXPECTATION ON THE PART OF THE BUSINESS.

ONE OF THE IMPORTANT FACTORS TO BE POINTED OUT IN THIS MODEL IS THE DIFFERENCE THAT MUST BE MADE BETWEEN THE PROCESS OF CONSUMING GOODS, THE PROCESS OF CREATING GOODS, AND THE MONEY FLOWS WHICH OCCUR SUBSEQUENT TO THESE PROCESSES. IT IS OBVIOUS THAT YOU CANNOT ADD GOODS AND DOLLARS, YET IN A GENERIC SENSE THIS IS WHAT IS BEING DISCUSSED. A FLOW DIAGRAM IS PRESENTED IN FIGURE 3, IN CONTRAST TO THE ONE IN FIGURE 4. FIGURE 4 IS THE FLOW DIAGRAM OF THE VERBAL DESCRIPTIONS GIVEN BY SAMUELSON AND HICKS OF THEIR MODEL, WHILE FIGURE 3 IS THE FLOW DIAGRAM OF THE EQUATIONS PRESENTED FOR THEIR MODEL.

THE DISCUSSIONS AND CRITICISMS HERE ARE NOT SO MUCH CONCERNED WITH THE FORM OF THE FUNCTIONAL RELATIONSHIPS AS WITH THE FUNDAMENTAL METHOD OF MODELING AND ITS APPLICABILITY TO REAL ECONOMIC SITUATIONS. THUS, DISCUSSIONS OF THE MODELS OF KAHN WHERE HE ESSENTIALLY SETS OF EQUATION 2) SUCH THAT

$\gamma_i = 0, i \geq 2$. HICKS ELEMENTARY CASE WHERE HE PUTS $\gamma_i = 0$, $i \geq 2$, PHILLIPS, WHERE γ_i IS A GEOMETRICALLY DECREASING PROGRESSION, GOODWIN'S NON LINEARITY, KALECKI'S FORCED TIME DELAYS, AND DAMPING FACTORS, ETC. WILL ADD LITTLE AT THIS POINT, SINCE THEY ARE SPECIAL CASES OF THE SAMUELSON-HICKS MODEL. IF INVESTMENT AND CONSUMPTION BEING FUNCTIONS OF $\dot{Y}$ IS GRANTED, THIS STILL DOES NOT DETERMINE $\dot{Y}$. $\dot{Y}$ IS DETERMINED AS A RESULT OF THE DECISION, AND THE ENSUING EFFECT ON THE ACTUAL FLOWS OF TANGIBLES MADE IN THE PRODUCTIVE PROCESS. ONCE $\dot{Y}$ HAS BEEN DETERMINED IT IS THEN POSSIBLE TO CALCULATE ITS EFFECT ON $\dot{C}$ AND $\dot{I}$, UNDER THE CONDITIONS AND ASSUMPTIONS GIVEN IN THE MODEL BEING CONSIDERED.

THE MODELS OF HARROD, DOMAR, SAMUELSON, HICKS, KALECKI, GOODWIN, PHILLIPS, METZLER, SODENHORN, AND OTHERS OF THIS TYPE, HAVE THE SAME FUNDAMENTAL SHORTCOMING, THEY DO NOT DETERMINE $\dot{Y}$, BUT ONLY ALLOW THE VARIABLES CONSIDERED TO BE DETERMINED ONCE $\dot{Y}$ IS KNOWN. ONE APPARENT REASON FOR THE FAILURE TO BRING THIS OBJECTION INTO FOCUS LONG BEFORE THIS HAS BEEN THE ABSENCE OF A TECHNIQUE OR DEVICE FOR SHOWING THE FLOWS OF FACTORS OF PRODUCTION, GOODS, AND SERVICES, AND THE INFORMATION FLOWS AND DECISION NETWORKS THAT LINK THESE TO THE MONEY FLOWS.

THERE IS STILL A SECOND MANNER OF LOOKING AT THE SAMUELSON-HICKS MODEL WHICH WOULD APPLY EQUALLY WELL TO ALL OF THE OTHER MODELS MENTIONED. THAT IS TO ASSUME THAT BY THE PROCESS OF SETTING UP EQUATIONS FOR ALL OF THE VARIABLES IN THE TANGIBLE FLOW SECTIONS, AND BY ASSUMING VARIOUS CRITERIA FOR DECISIONS IN THAT SECTOR, AND THEN ELIMINATING THESE VARIABLES BETWEEN THE VARIOUS EQUATIONS THAT THE RESULT WOULD BE THE GENERAL TYPES OF EQUATIONS ARRIVED AT UNDER THE ASSUMPTIONS OF THE MODEL. IN OTHER WORDS, THIS WOULD IN ESSENCE BE ASSUMING THE FORM OF THE RESULTING THREE FUNCTIONS $\dot{C}$, $\dot{I}$, AND $\dot{A}$, WITH

SAMUELSON-HICKS MODEL - ACCORDING TO THE EQUATIONS.

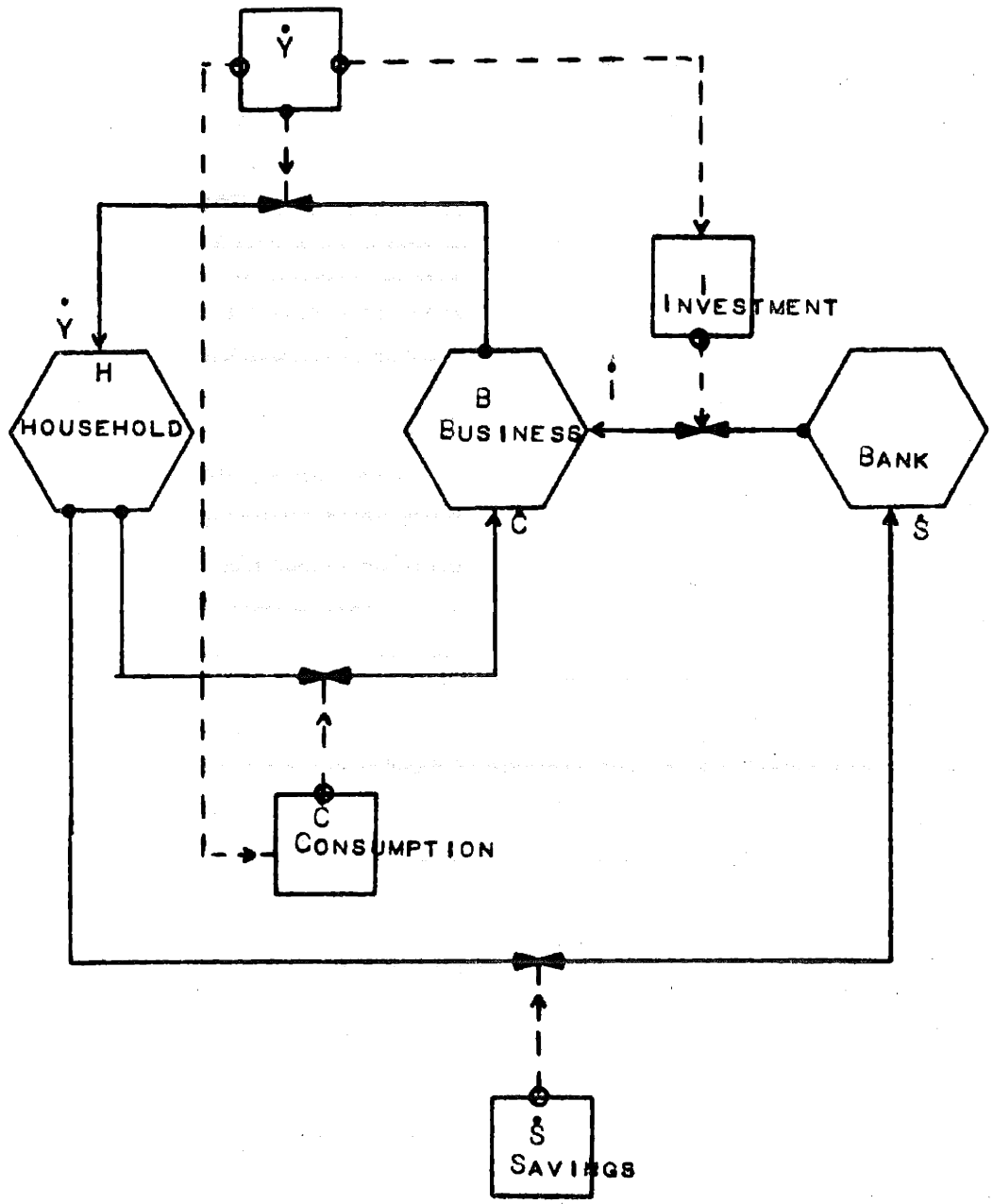
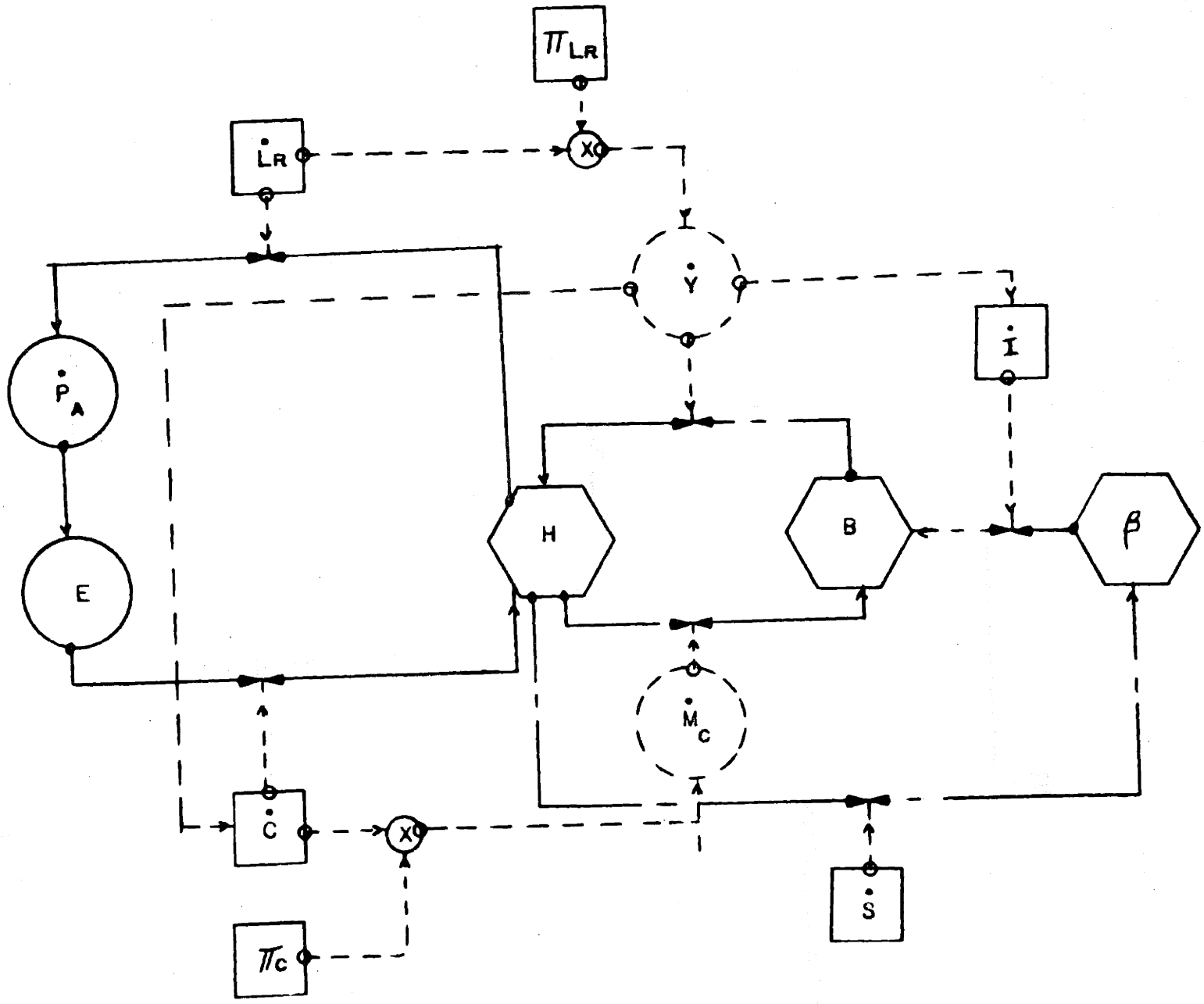


FIGURE 3.



SAMUELSON-HICKS MODEL - ACCORDING TO THEIR DISCUSSION.

FIGURE 4.

UNKNOWN COEFFICIENTS c_{ij} AND V_i . WERE THIS TO BE TRUE, THE CONSTANTS c_i AND V_i COULD BE EXPRESSED IN TERMS OF THE PARAMETERS OF THE OTHER SECTORS. IN A SENSE, IF THIS APPROACH WAS ASSUMED TO BE THE MANNER IN WHICH THESE MODELS WERE MEANT, THEN THE MAIN PROBLEM WOULD STILL LIE AHEAD IN THE PHYSICAL INTERPRETATION OF THE c_i AND V_i AND IN THE DEVELOPMENT OF A THEORY IN THE PRODUCTION SECTORS WHICH WOULD EXPRESS THEM EXPLICITLY IN TERMS OF THE BASIC PARAMETERS OF THE PRODUCTION SECTORS.

SECTION 4 - A GENERAL DISCUSSION OF DEFICIENCIES IN THE ABOVE MODELS.

A - CAUSE AND EFFECT

IN THE MODELS OF HARROD, HICKS, METZLER, ETC. DISCUSSED EARLIER, THE RATE OF MONEY FLOW $\dot{Y}$ (T) WAS NEVER ACTUALLY DETERMINED BY THEIR MODELS. THE FLOWS IN THE MONEY SECTOR COME ONLY AFTER THE FACT. THESE FLOWS OF MONEY ARE STRICTLY INDICATIVE OF ACTION THAT HAS ALREADY TAKEN PLACE. THE HEART OF THE PROBLEM, HOWEVER, IS TO BE FOUND IN THE INTERPLAY BETWEEN INFORMATION AND DECISIONS, SINCE INFORMATION CONTROLS THE RATES OF FLOW IN THE PRODUCTIVE SECTORS, AS WELL AS IN THE MONEY SECTORS, AND THE ACTUAL FLOWS OF THE FACTORS OF PRODUCTION INTO THE PRODUCTIVE PROCESS, AND THE ACTUAL FLOWS OF GOODS AND SERVICES INTO THE HANDS OF THE CONSUMER DETERMINE THE FLOWS OF MONEY. AS A CONSEQUENCE, ANY STUDY NOT INVOLVING THE CAUSES, OR ANY MODEL NOT GIVING PRIMARY IMPORTANCE TO THE CAUSES, CAN HAVE LITTLE MORE THAN ÆSTHETIC VALUE.

THE ASSUMPTIONS GOING INTO ANY MODEL CAN BE MADE SIMPLE. SIMPLIFIED ASSUMPTIONS AND APPROXIMATIONS ARE ALL RIGHT IN THE

CASE WHERE YOU HAVE AN ENTIRE SYSTEM, AND IT CAN BE ASCERTAINED THAT THE SYSTEM IS A MEANINGFUL WHOLE. COMPLEX SYSTEMS ARE MOST OFTEN BROKEN DOWN INTO SMALL SECTORS THAT CAN BE ANALYZED SOMEWHAT INDEPENDENTLY OF THE SYSTEM FROM WHICH THEY ARE TAKEN. WHEN THIS IS DONE, IT IS NECESSARY TO BE SURE THE SEGMENTS SEPARATED OUT ARE INDEPENDENT OF THE SYSTEM AS A WHOLE, AND WHAT INPUTS AND OUTPUTS ARE REASONABLE. WHEN A COMPONENT, TO BE ANALYZED, IS DEPENDENT ON OTHER COMPONENTS, THE OTHER COMPONENTS CANNOT BE IGNORED. FOR EXAMPLE, THE DECISION OF NATURE AS TO HOW MUCH HARVEST WILL BE REAPED FOR A GIVEN PRODUCTIVE EFFORT CAN BE EXAMINED INDEPENDENT OF MONEY FLOWS, BUT TO STUDY THE PRODUCTIVE FLOW WITHOUT A HARVEST WOULD BE DEVOID OF MEANING.

IN THE PRODUCTION SECTOR OF AN ECONOMIC MODEL, SIMPLIFYING ASSUMPTIONS CAN BE MADE, AS WELL AS APPROXIMATIONS, BUT TO LEAVE OUT, OR IGNORE FOR EXAMPLE, THE HARVEST, RENDERS THE MODEL USELESS AND MEANINGLESS. THUS, IN THE SYNTHESIS OF ANY SYSTEM, IT MUST BE INITIALLY ASCERTAINED JUST EXACTLY WHAT CONSTITUTES THE MEANINGFUL ENTITY, AND THE MEANINGFUL WHOLE. THEN, AND ONLY THEN, CAN MEANINGFUL SIMPLIFYING ASSUMPTIONS AND APPROXIMATIONS BE MADE.

THE CLASSICAL FORMULATION IN ECONOMIC THEORY IS BASICALLY AN ANALOG APPROACH. FOR A GROSS LOOK AT A SYSTEM THIS MIGHT SUFFICE IF THE INVESTIGATOR IS SURE OF WHAT HE IS REALLY INTERESTED IN. THEORY IN THIS AREA CAN ONLY BE TESTED BY THE EMPIRICAL TEST WHICH DEPENDS UPON QUESTIONABLE MEASURING RODS.

AN ALTERNATIVE APPROACH COULD BE THAT OF ANALYZING EACH TRANSACTION OF THE INDIVIDUAL, THE FIRM, THE GOVERNMENT, ETC., AND BY STATISTICALLY SUMMING OVER ALL INDIVIDUALS, FIRMS, GOVERNMENTS, ETC. TO ARRIVE AT THE COMPOSITE RELATIONSHIPS PREVIOUSLY DERIVED BY THE ANALOG APPROACH. IN THIS SAME

MANNER, TESTS FOR VALIDITY CAN ALSO BE FORMULATED BASED ON MORE FUNDAMENTAL, AND IN MANY CASES, LESS TRICKY INTUITION. RELATIONSHIPS ESTABLISHED SHOULD CORRESPOND TO THOSE DERIVED BY THE ANALOG METHOD. HOWEVER, WHERE THIS IS NOT THE CASE, THE GREATER DETAIL SHOULD SHED GREATER LIGHT ON THE AREAS OF DIVERGENCE.

B - SYNTHESIS OF COMPLEX SYSTEMS

AS IN ALL THEORIZING, THERE IS A TENDENCY TO WANT TO BEGIN WITH THE FINAL RESULT, AND TO FIND A SIMPLE, ALL INCLUSIVE, MAGIC FORMULAE TO GIVE THIS RESULT. TRUE, THERE ARE CASES WHERE THIS HAS BEEN DONE, BUT THESE ARE THE EXCEPTION RATHER THAN THE RULE. TO DATE, NO SIMPLE SUBSTITUTE OR FORMULAE HAS BEEN FOUND TO EXPLAIN THE BEHAVIOR OF A COMPLEX WEAPON SYSTEM, OR A COMPLEX ELECTRONIC GADGET, LET ALONE A RADIO OR TELEVISION SET. IT IS STILL NECESSARY TO BREAK SYSTEMS UP INTO SMALLER COMPONENTS WHOSE BEHAVIOR CAN BE UNDERSTOOD. ACTUALLY, IN THE CASE OF ELECTRONICS, SIMPLE BUILDING BLOCKS WERE FORMED, AND THEN COMPLEX SYSTEMS SYNTHESIZED FROM THESE. COULD ONE IMAGINE THE CONSTRUCTION OR ANALYSIS OF AN ELECTRONIC SYSTEM WITHOUT THE USE OF A CIRCUIT DIAGRAM? YET ALL THROUGH ECONOMICS, THE USE OF FLOW DIAGRAMS APPEAR TO BE ASSIDUOUSLY AVOIDED.

IN THE ECONOMIC MODELING TO-DATE, THE USUAL STARTING POINT IS NOT THE INDIVIDUAL COMPONENT, BUT THE ENTIRE COMPLEX SYSTEM. THIS APPROACH IS ALL RIGHT BY WAY OF OBSERVING EFFECT, BUT TO GET AT THE CAUSE IT IS NECESSARY TO BEGIN BY DIVIDING THE SYSTEM INTO ITS COMPONENTS, AND THEN SYNTHESIZING SEGMENTS ONE BY ONE UNTIL A MEANINGFUL WHOLE IS CONSTRUCTED. THIS COULD VERY EASILY INVOLVE HUNDREDS OF VARIABLES AND DO SO EVEN MORE SIMPLY THAN EXISTING MODELS HANDLE THREE OR FOUR VARIABLES. THE ASSUMPTIONS WOULD THEN BE MUCH CLOSER TO REALITY, AND BE MORE INTUITIVELY SATISFYING.

C - THE EMPIRIC APPROACH

FOR ANY THEORY THERE NEEDS TO BE CHECK POINTS ALONG THE WAY, WHERE MEASUREMENTS OF ACTUAL BEHAVIOR UNDER THE ASSUMED CONDITIONS OF THE MODEL ARE MADE. THIS POINT FOR ECONOMIC THEORY AS A WHOLE STILL SEEMS A LONG DISTANCE AWAY, AS OTHER FUNDAMENTAL TECHNIQUES OF SYSTEM SYNTHESIS HAVE NOT YET BEEN WORKED OUT. YET THERE HAS BEEN AND STILL IS A GREAT DEAL OF PIECE-MEAL WORK BEING DONE ALONG THE LINES OF STATISTICAL WORK WITHOUT A FUNDAMENTAL THEORY FOR ANALYSIS. AT BEST, THESE EFFORTS WILL LEAD TO EMPIRICAL FORMULAE TO APPROXIMATE THE DATA CURRENTLY AVAILABLE. HOW DOES THE ANALYST KNOW, AT THIS POINT OF DEVELOPMENT, WHAT DATA IS PERTINENT, OR HAS HE JUST ASSUMED THAT ALL NUMERICAL DATA IS PERTINENT?

ONCE A SYSTEM IS SYNTHESIZED, HOWEVER, AN EMPIRIC TEST SHOULD BE SOUGHT OUT WHICH WILL PERMIT THE OBSERVATION AND CORRECTION OF THE THEORY AND ITS BASIC ASSUMPTIONS. IT APPEARS THAT ECONOMISTS TO DATE HAVE CONCENTRATED MORE ON NUMERICAL CURVE-FITTING THAN ON BASIC THEORIZING. CONSIDER, AS JUST ONE EXAMPLE, THAT MODEL OF TINBERGEN. IN ESSENCE, HIS MODEL AMOUNTS TO A REGRESSION ANALYSIS WHERE EACH VARIABLE CONSIDERED IS ASSUMED TO BE A FUNCTION OF ALL OTHER VARIABLES IN THE SYSTEM. THE RESULTANT PARAMETERS MAY OR MAY NOT BE PERTINENT, SINCE ONE CAN NEVER BE SURE WHETHER ALL OF THE IMPORTANT VARIABLES WERE INCLUDED IN THE FIRST PLACE, OR JUST WHAT THE NATURE OF THE FUNCTIONAL RELATIONSHIPS OF ONE VARIABLE TO ANOTHER ARE, OR WOULD BE UNDER DIFFERING ASSUMPTIONS. IN OTHER WORDS, THERE IS NO CAUSE AND EFFECTS MECHANISM PRESENT.

ONCE A SUITABLE THEORY HAS BEEN FORMULATED, THE ROLE OF THE EMPIRIC TEST CAN NOT BE OVER-EMPHASIZED. ANY THEORY MUST HAVE ITS CHECK POINTS IN REALITY, AND THEREFORE CONTROLLED OBSERVATION IN TEST SITUATIONS SHOULD THEN BE CONSIDERED.

D - Complexity of the Economic System

The complexity of the existing economics systems present in the world today appears at first glance to far exceed the complexity of the most sophisticated electronic, or engineering systems. Even going back in history to more primitive cultures, the systems were extremely complex. Therefore, it is suggested that in the building up of a model, the techniques already developed by the engineers and physicists for simulation and analysis on the high speed computers, be used.

The history of the efforts going into the understanding of the economic complex shows that the efforts have been extremely meager, usually of secondary interest to the person doing the research,[1] and enshrouded with political intrigue

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[1] Most of the great advances in economics have been made by persons considering economics as a sideline. Examples, (to mention but a few) of the principal occupations of some of the greatest contributors to economic theory are given in the following table:

Name	Major Occupation
Thomas Aquinas	Priest
Aristotle	Natural Philospher
Bentano	Historian & Sociologist
Bentham	Man of Affairs
Bodin	Man of Affairs
Bücher	Historian & Sociologist
Nicholas Copernicus	Astronomer
Hume	Man of Affairs
John Maynard Keynes	Business man and mathematician
Knies	Historian & Sociologist
Frederic List	Journalist & Politician
Locke	Man of Affairs
Malthus	Moral Philosopher & Historian
Alfred Marshall	Mathematician, Ethics & Philosophy
Karl Marx	Politician
John Stuart Mill	Logician & Philosopher
Nicole Oresme	Bishop of Lisieux
Pareto	Engineer

NAME	MAJOR OCCUPATION
LEPLAY	SOCIAL REFORMER
FRANCOIS QUESNAY	PHYSICIAN
DAVID RICARDO	WEALTHY STOCKBROKER
RODBERTUS	LANDLORD AND FARMER
ROSCHER	HISTORIAN & SOCIOLOGIST
SCHMOLLER	HISTORIAN & SOCIOLOGIST
SISMONDE	HISTORIAN
ADAM SMITH	HISTORIAN & MORAL PHILOSOPHER
THORNTON	BUSINESS MAN AND BANK DIRECTOR
TOOKE	BUSINESS MAN AND BANK DIRECTOR
WAGNER	HISTORIAN & SOCIOLOGIST
MAX WEBER	HISTORIAN & SOCIOLOGIST
ETC.	

AND IMPLICATIONS. ECONOMIC THEORY HAS HAD AN EXTREMELY HIGH EMOTIONAL CONTENT TRADITIONALLY BECAUSE OF ITS POLITICAL IMPLICATIONS. THUS, THE THEORIES HAVE NOT BEEN NECESSARILY AS OBJECTIVELY APPROACHED AS HAVE BEEN THE THEORIES IN THE PHYSICAL SCIENCES, YET OUR MENTAL, PHYSICAL, AND SPIRITUAL LIVES AND ACTIONS ARE INSEPARABLY CONNECTED WITH OUR ECONOMIC POLICIES. RESEARCH IN THE PHYSICAL SCIENCES IS A SEARCH FOR TRUTH, AND THE SEARCH FOR TRUTH HAS BEEN FOLLOWED IN THE PATHS OF NATURE. ONE CANNOT PUT NATURE IN A WHEELBARROW AND PUSH IT AROUND. IT MUST BE SOUGHT OUT IN ITS OWN PATHS. THE QUESTION ARISES, HOWEVER, AS TO WHETHER OR NOT THE PURPOSE OF ECONOMIC RESEARCH IS TRULY AN OBJECTIVE ONE OF SEEKING OUT PURE KNOWLEDGE, AND THEN APPLYING IT, OR WHETHER IT IS TO MAINTAIN THE STATUS QUO, OR TO SERVE THE INTERESTS OF SPECIAL INTEREST GROUPS. IF THE OBJECTIVE OF RESEARCH IN ECONOMICS IS THE PURSUIT OF PURE KNOWLEDGE, THEN MUCH MORE EFFORT AND EMPHASIS SHOULD BE PUT INTO RESEARCH ON AS BASIC A BASIS AS IS POSSIBLE, AND APPLY TO ECONOMIC RESEARCH THE SAME RULES THAT APPLY TO THE PHYSICAL SCIENCES. THIS WOULD IMPLY SUPPORTING THIS RESEARCH WITH LARGE AMOUNTS OF GOVERNMENT AND PRIVATE FUNDS IN ORDER TO TAKE SOME OF THE GUESSWORK OUT OF THE OPERATION OF OUR NATIONAL AND BUSINESS ECONOMIES.

CHAPTER II

A METHOD OF NOTATION AND REPRESENTION FOR ECONOMIC MODELS.

PART I - NOTATION

A - FLOW DIAGRAMS SYMBOLISM

THE SYSTEM OF NOTATION PRESENTED HERE REPRESENTS THE COMBINATION AND REPRESENTATION DIAGRAMATICALLY OF THE FLOWS AND INTERACTIONS BETWEEN,

1) TANGIBLE FACTORS OF PRODUCTION,
2) TANGIBLE GOODS,
3) TANGIBLE VARIABLES SUCH AS WEATHER,
4) MONEY,
5) INFORMATION AND DECISION NETWORK.

THE NOTATION ALLOWS FOR THE REVISING PERIODICALLY OF A DECISION WHEN THE ACTUAL RESULT OF THE DECISION DIFFERS FROM THE INTENDED RESULT.

A SUMMARY OF THE SUMBOLISM USED IN THE FLOW DIAGRAM IS GIVEN IN FIGURE 5.

B - SUBSCRIPTS

THE SUBSCRIPT E IS USED TO DESIGNATE THE PRODUCTION SECTOR OF THE ECONOMY PRODUCING E, G FOR THE PRODUCTION SECTOR PRODUCING G, AND K FOR THE PRODUCTION SECTOR PRODUCING K, IN THE CASE WHERE THREE SECTORS ARE ASSUMED.

THE SUBSCRIPTS L_R AND L REFER TO LABOR AND LAND RESPECTIVELY.

FLOW DIAGRAM NOTATION

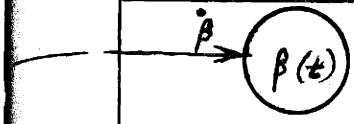
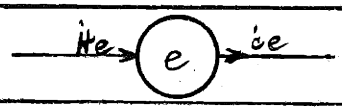
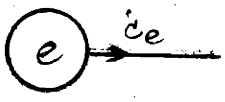
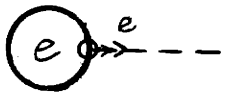
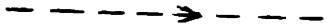
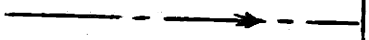
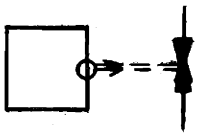
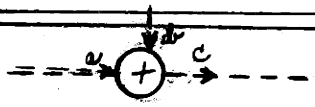
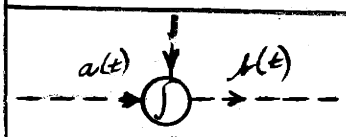
SYMBOL	REPRESENTS	PROPERTIES
	AN INSTITUTION	Σ TANGIBLE INPUTS = Σ TANGIBLE OUTPUTS
	AN ACCUMULATION OF MONEY	$\beta(t) = \beta(0) + \int_0^t \dot{\beta}(t)\,dt$.
	AN ACCUMULATION OF A TANGIBLE VARIABLE	$e(t) = e(0) + \int_0^t [\dot{H}_e - \dot{c}_e]\,dt$.
	TANGIBLE OUTPUT	THE OUTFLOW DIMINISHES THE ACCUMULATION OF
	INFORMATION OUTPUT	INFORMATION CAN BE TAKEN FROM ANY TYPE OF FLOW, VARIABLE, OR ACCUMULATOR.
	INFORMATION VARIABLE	THIS SYMBOL IS USED TO REPRESENT ANY VARIABLE IN THE INFORMATION SECTOR. INPUTS AND OUTPUTS MUST BE INFORMATION
	FLOW OF TANGIBLES	ALL TANGIBLE FLOWS ARE RATED. (QUANTITY PER UNIT TIME.)
	FLOW OF INFORMATION	THE FLOW OF INFORMATION IS A DIRECT TRANSFER AND IS NOT NECESSARILY A RATE OF FLOW.
	FLOW OF MONEY	ALL MONEY FLOWS ARE RATES. (DOLLARS PER UNIT TIME.)
	DECISION	A DECISION IS BASED ON THE INPUT OF INFORMATION.
	VALVE	TANGIBLE FLOWS ARE METERED BY A DECISION. THE RATE OF FLOW OF A TANGIBLE IS ALWAYS METERED BY A DECISION.
	ADDER	A + B = C
	MATHEMATICAL OPERATION	B (T) = ∫ADT. THE OPERATION SPECIFIED COULD BE −, X, ÷, OR ANY OTHER MATHEMATICAL OPERATION.

FIGURE 5.

C - VARIABLES

THE NOTATIONS AND DEFINITIONS OF TERMS USED FOR VARIABLES THROUGHOUT THE MODELING ARE SUMMARIZED IN "D" OF THIS SECTION.

D - DEFINITIONS OF TERMS

$\bar{A}_K(T)$ = THE AVERAGE AGE OF THE CAPITAL EQUIPMENT.

$\dot{B}_B(T)$ = THE NET RATE OF BORROWINGS BY THE BUSINESS.

$\dot{B}_H(T)$ = THE RATE OF BORROWING BY THE HOUSEHOLD.

$B_H(T)$ = BORROWINGS HELD BY THE HOUSEHOLD.

B = BUSINESS.

β = BANK.

(T) = THE AMOUNT OF MONEY HELD BY THE BANK.

$\dot{C}(T)$ = RATE OF CONSUMPTION.

$\ddot{C}(T)$ = THE RATE OF CHANGE OF THE RATE OF CONSUMPTION.

$\bar{\dot{C}}_E(T)$ = THE HABITUAL RATE OF CONSUMPTION OF E.

$\bar{\dot{C}}_G(T)$ = THE HABITUAL RATE OF CONSUMPTION OF G.

$\dot{C}_E(T)$ = THE RATE OF CONSUMPTION OF E.

$\dot{C}_G(T)$ = THE RATE OF CONSUMPTION OF G.

$\dot{d}(T)$ = THE RATE OF DEPOSITS OF CASH BY THE BUSINESS, IN THE BANK.

$\dot{D}(T)$ = THE RATE OF RETIREMENT OF CAPITAL EQUIPMENT K.

$\dot{\delta}(T)$ = THE RATE OF DIVIDENDS BEING PAID.

E = 1ST CONSUMER GOOD.

G = 2ND CONSUMER GOOD.

$\dot{G}(T)$ = THE RATE OF CHANGE OF INVENTORY OF G.

$\gamma_E(T)$ = THE PRESENT MARKUP FOR PROFIT IN PRICING E.

$\gamma_G(T)$ = THE PRESENT MARKUP FOR PROFIT IN PRICING G.

$\gamma_K(T)$ = THE PRESENT MARKUP FOR PROFIT IN PRICING K.

H = HOUSEHOLD.

$\dot{H}_E(T)$ = THE RATE OF HARVEST OF E.

$\dot{H}_G(T)$ = THE RATE OF HARVEST OF G.

$\dot{H}_K(T)$ = THE RATE OF HARVEST OF K.

$\dot{i}(T)$ = THE INTEREST RATE CHARGED ON LOANS.

$\bar{i}(T)$ = THE HISTORICAL RATE OF INTEREST.

$\dot{i}_H(T)$ = THE RATE OF INTEREST BEING PAID BY THE HOUSEHOLD.

$\dot{I}(T)$ = THE RATE OF INVESTMENT OF MONEY IN THE BUSINESS.

K = A CAPITAL GOOD.

$\dot{K}_A(T)$ = THE ACTUAL RATE OF USAGE OF K IN ALL SECTORS.

$\dot{K}_D(T)$ = THE TOTAL RATE OF USAGE OF K DECIDED ON IN ALL SECTORS.

$\dot{K}(T)$ = THE TOTAL AVAILABLE USAGE OF CAPITAL EQUIPMENT.

$\dot{K}_W(T)$ = THE AMOUNT OF WASTED USE OF K. (IDLE TIME)

$\dot{K}_{EA}(T)$ = THE ACTUAL RATE OF USE OF $\dot{K}_E$ IN $\dot{P}_E$.

$\dot{K}_{ED}(T)$ = THE RATE OF USE OF $\dot{K}_E$ IN $\dot{P}_E$ DECIDED ON.

$\dot{K}_{GA}(T)$ = THE ACTUAL RATE OF USE OF $\dot{K}_G$ IN $\dot{P}_G$.

$\dot{K}_{GD}(T)$ = THE RATE OF USE OF $\dot{K}_G$ IN $\dot{P}_G$ DECIDED ON.

$\dot{K}_{KA}(T)$ = THE ACTUAL RATE OF USE OF $\dot{K}_K$ IN $\dot{P}_K$.

$\dot{K}_{KD}(T)$ = THE RATE OF USE OF $\dot{K}_K$ IN $\dot{P}_K$ DECIDED ON.

$\dot{L}R_E(T)$ = THE RATE OF USE OF LABOR IN $\dot{P}_E$.

$\dot{L}R_G(T)$ = THE RATE OF USE OF LABOR IN $\dot{P}_G$.

$\dot{L}R_K(T)$ = THE RATE OF USE OF LABOR IN $\dot{P}_K$.

$\dot{L}_E(T)$ = THE RATE OF USAGE OF LAND IN $\dot{P}_E$.

$\dot{L}_G(T)$ = THE RATE OF USAGE OF LAND IN $\dot{P}_G$.

$\dot{L}_K(T)$ = THE RATE OF USAGE OF LAND IN $\dot{P}_K$.

$\dot{M}_C(T)$ = THE RATE OF MONEY BEING PAID FOR CONSUMPTION.

$\dot{M}_{LR}(T)$= THE RATE OF MONEY BEING PAID FOR LABOR.

$\dot{M}_L(T)$ = THE RATE OF MONEY BEING PAID FOR LAND.

$\dot{M}_K(T)$ = THE RATE OF MONEY BEING PAID FOR CAPITAL.

$\dot{M}_E(T)$ = THE RATE OF MONEY BEING PAID FOR $\dot{P}_E$.

$\dot{M}_G(T)$ = THE RATE OF MONEY BEING PAID FOR $\dot{P}_G$.

$\dot{M}_K(T)$ = THE RATE OF MONEY BEING PAID FOR $\dot{P}_K$.

$\dot{M}_{LRE}(T)$= THE RATE OF MONEY BEING PAID FOR LABOR IN $\dot{P}_E$.

$\dot{M}_{LE}(T)$= THE RATE OF MONEY BEING PAID FOR LAND IN $\dot{P}_E$.

$\dot{M}_{KE}(T)$= THE RATE OF MONEY BEING PAID FOR EQUIPMENT IN $\dot{P}_E$.

$\dot{M}_{LRG}(T)$= THE RATE OF MONEY BEING PAID FOR LABOR IN $\dot{P}_G$.

$\dot{M}_{LG}(T)$= THE RATE OF MONEY BEING PAID FOR LAND IN $\dot{P}_G$.

$\dot{M}_{KG}(T)$= THE RATE OF MONEY BEING PAID FOR EQUIPMENT IN $\dot{P}_G$.

$\dot{M}_{LRK}(T)$= THE RATE OF MONEY BEING PAID FOR LABOR IN $\dot{P}_K$.

$\dot{M}_{LK}(T)$= THE RATE OF MONEY BEING PAID FOR LAND IN $\dot{P}_K$.

$\dot{M}_{KK}(T)$= THE RATE OF MONEY BEING PAID FOR EQUIPMENT IN $\dot{P}_K$.

$\dot{P}_{EA}(T)$= THE ACTUAL RATE OF PRODUCTIVE EFFORT GOING TO E.

$\dot{P}_{GA}(T)$= THE ACTUAL RATE OF PRODUCTIVE EFFORT GOING TO G.

$\dot{P}_{KA}(T)$= THE ACTUAL RATE OF PRODUCTIVE EFFORT GOING TO K.

$\dot{P}_{ED}(T)$= THE RATE OF PRODUCTIVE EFFORT DECIDED ON IN E, PRODUCING, E.

$\dot{P}_{GD}(T)$= THE RATE OF PRODUCTIVE EFFORT DECIDED ON IN G PRODUCING, G.

$\dot{P}_{KD}(T)$= THE RATE OF PRODUCTIVE EFFORT DECIDED ON IN K PRODUCING, K.

$\dot{P}_H(T)$ = THE RATE OF PAYBACK OF LOANS BY THE HOUSEHOLD.

$\pi_E(T)$ = THE PRICE OF E TO THE CONSUMER.

$\pi_G(T)$ = THE PRICE OF G TO THE CONSUMER.

$\pi_K(T)$ = THE PRICE OF K TO THE USER.

$\pi_{LRE}(T)$ = THE PRICE OF LABOR IN PRODUCTION OF E.

$\pi_{EG}(T)$ = THE PRICE OF LABOR IN PRODUCTION OF G.

$\pi_{LRK}(T)$ = THE PRICE OF LABOR IN PRODUCTION OF K.

$\pi_{LE}(T)$ = THE PRICE OF LAND IN PRODUCTION OF E.

$\pi_{LG}(T)$ = THE PRICE OF LAND IN PRODUCTION OF G.

$\pi_{LK}(T)$ = THE PRICE OF LAND IN PRODUCTION OF K.

$\dot{R}(T)$ = THE RATE OF MONEY BEING SET ASIDE AS A RESERVE BY THE BANK.

$\rho(T)$ = THE % RESERVE HELD BY THE BANK ON ALL TIME DEPOSITS.

$\dot{S}_H(T)$ = THE RATE OF SAVINGS BY THE HOUSEHOLD.

$W(T)$ = THE WEATHER.

$\dot{Y}(T)$ = THE GROSS INCOME OF THE HOUSEHOLD.

$\dot{Y}_D(T)$ = THE DISPOSABLE INCOME OF THE HOUSEHOLD AT TIME T.

E - USEFUL FUNCTIONS

THE FLOW DIAGRAMS ARE USED TO INDICATE THE FLOWS OF TANGIBLES, INFORMATIONS, DECISIONS, MONEY, ETC. THE EXACT RELATIONSHIP BETWEEN VARIABLES ARE OFTEN ASSUMED. IN MANY CASES INVOLVING A DECISION, FOR EXAMPLE, THE EFFECTS OF TRYING DIFFERENT POLICY ASSUMPTIONS WOULD BE DESIRABLE. TO FACILITATE THE CHOICE OF FUNCTIONS IN SUCH CASES, A LIST OF FUNCTIONS HAVING USEFUL SHAPES ARE SUMMARIZED IN FIGURE 6.

SOME USEFUL FUNCTIONS

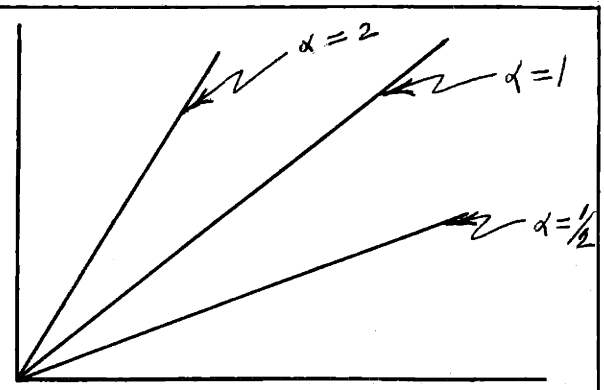
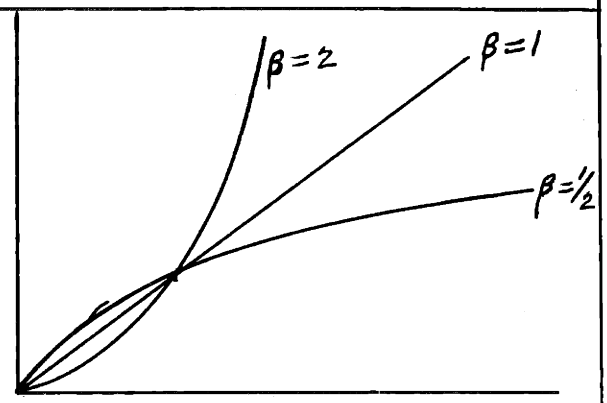
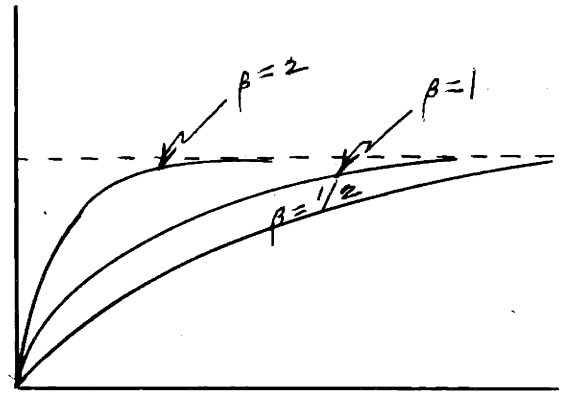
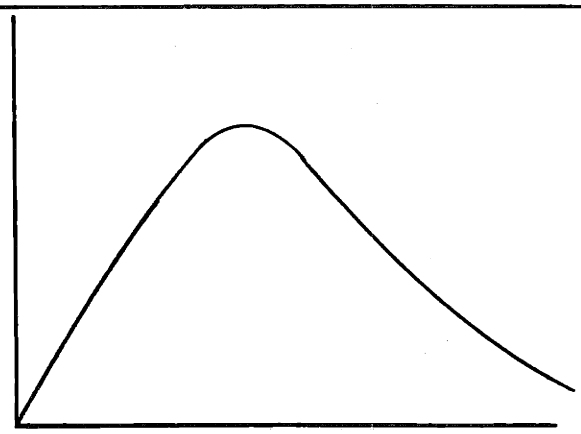
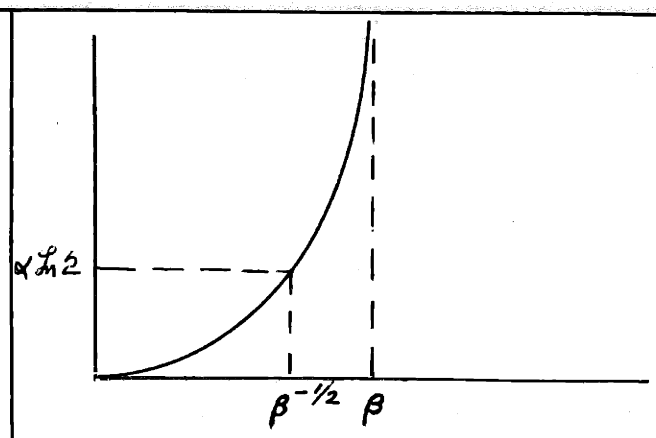
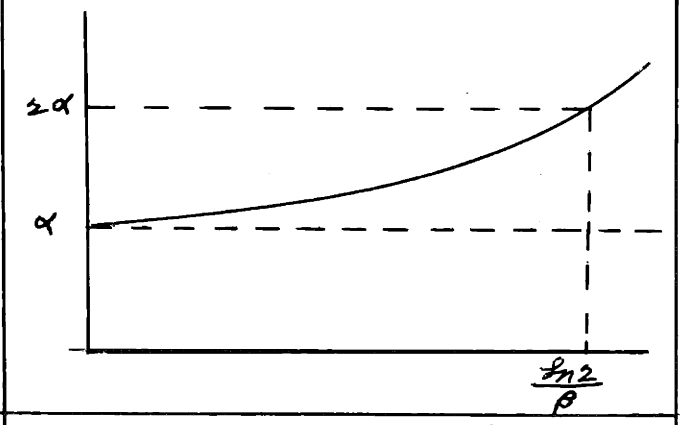
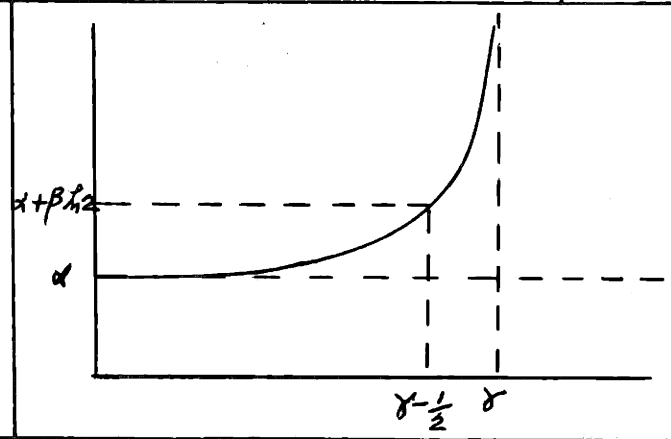
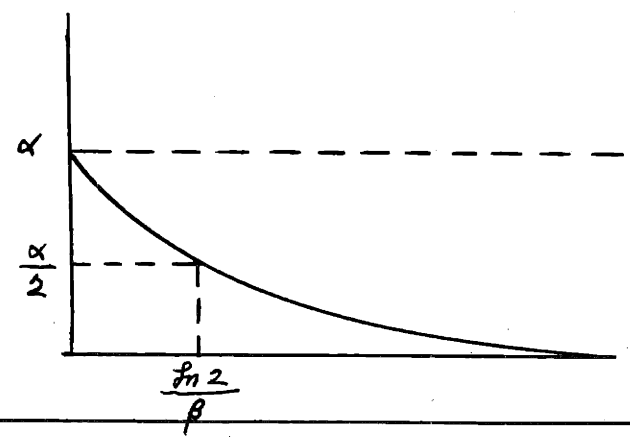
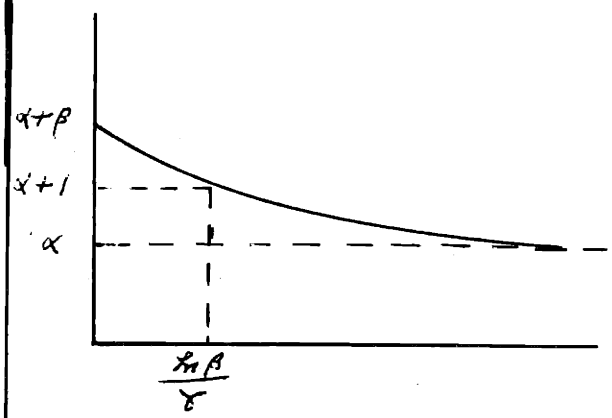
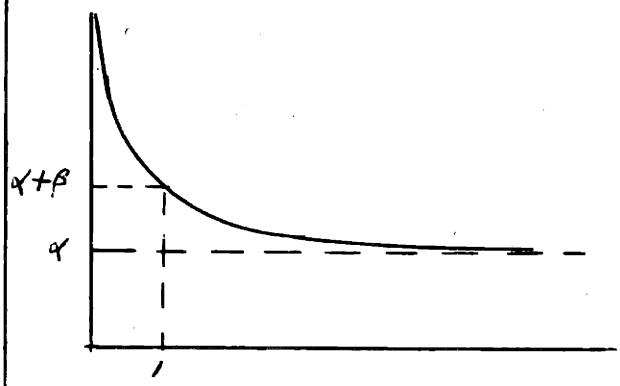
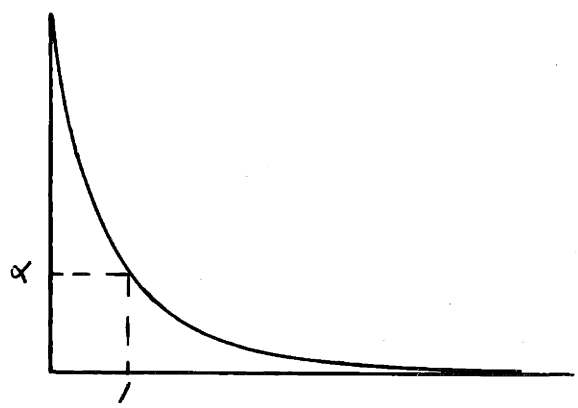
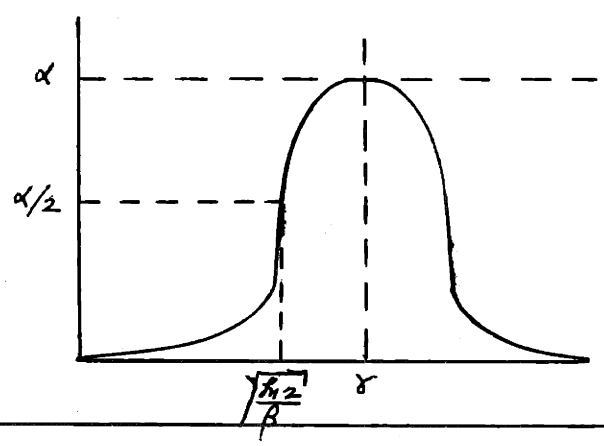
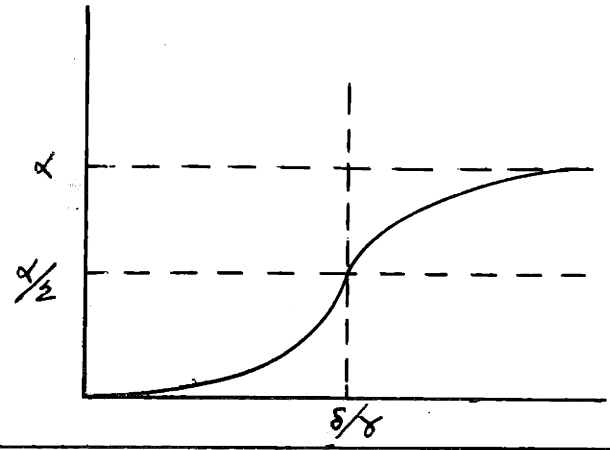
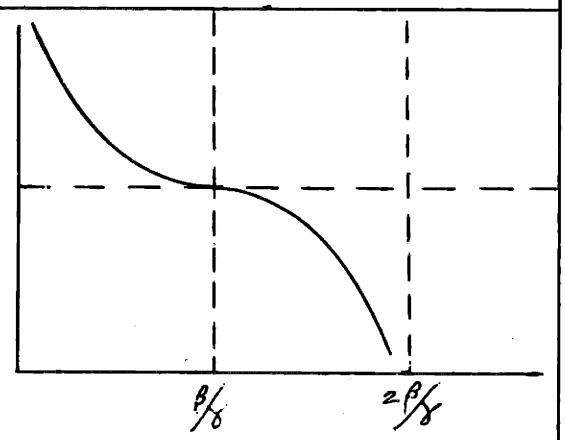
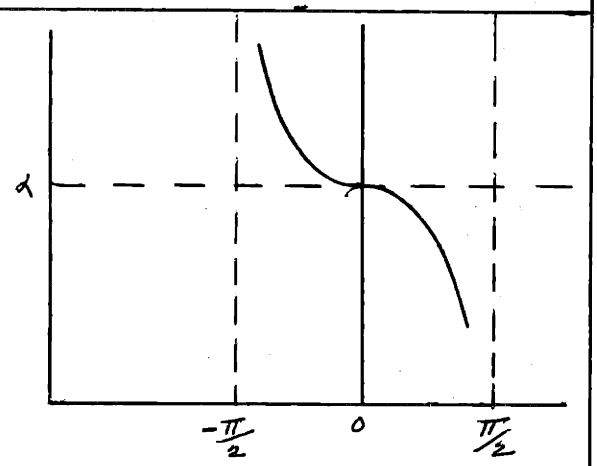
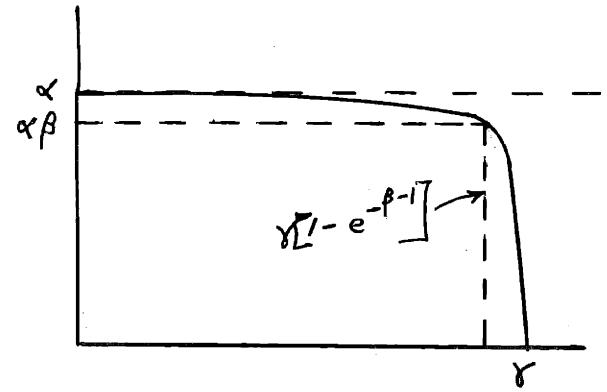
$y = \alpha x$	
$y = \alpha x^{\beta}$	
$y = \alpha(1 - e^{-\beta x})$	
$y = \alpha\left[\frac{1}{1+\beta x} - e^{-\gamma x}\right]$	

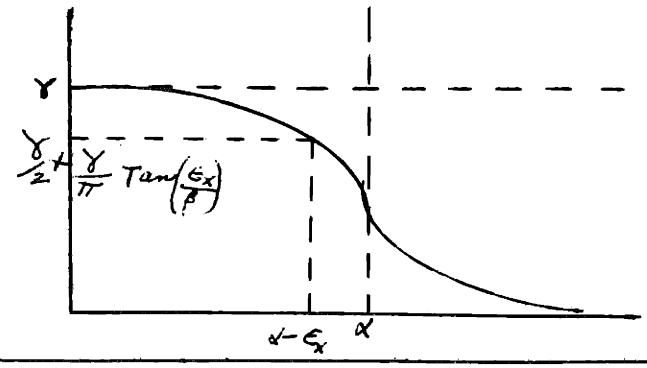
FIGURE 6.

$y = \alpha \ln\left(\frac{1}{\beta - x}\right)$	
$y = \alpha e^{\beta x}$	
$y = \alpha + \beta \ln\left(\frac{1}{\gamma - x}\right)$	
$y = \alpha e^{-\beta x}$	

$y = \alpha + \beta e^{-\gamma x}$	
$y = \alpha + \frac{\beta}{x}$	
$y = \frac{\alpha}{x}$	
$y = \alpha e^{-\beta(x-\gamma)^2}$	

$y = \frac{\alpha}{\pi}\left[\tan^{-1}(\gamma t - \delta) + \frac{\pi}{2}\right]$	
$y = \alpha + \tan\left[\frac{\beta - \gamma x}{\beta} \cdot \frac{\pi}{2}\right]$	
$y = \alpha + \beta \tan(-x)$	
$y = \left[\frac{\alpha}{1 + \frac{1}{\beta}\ln\left(\frac{1}{1 - \frac{x}{\gamma}}\right)}\right]$	

$$y = \frac{\gamma}{\pi}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{x-\alpha}{\beta}\right)\right]$$



PART 2 - METHOD OF MODELING

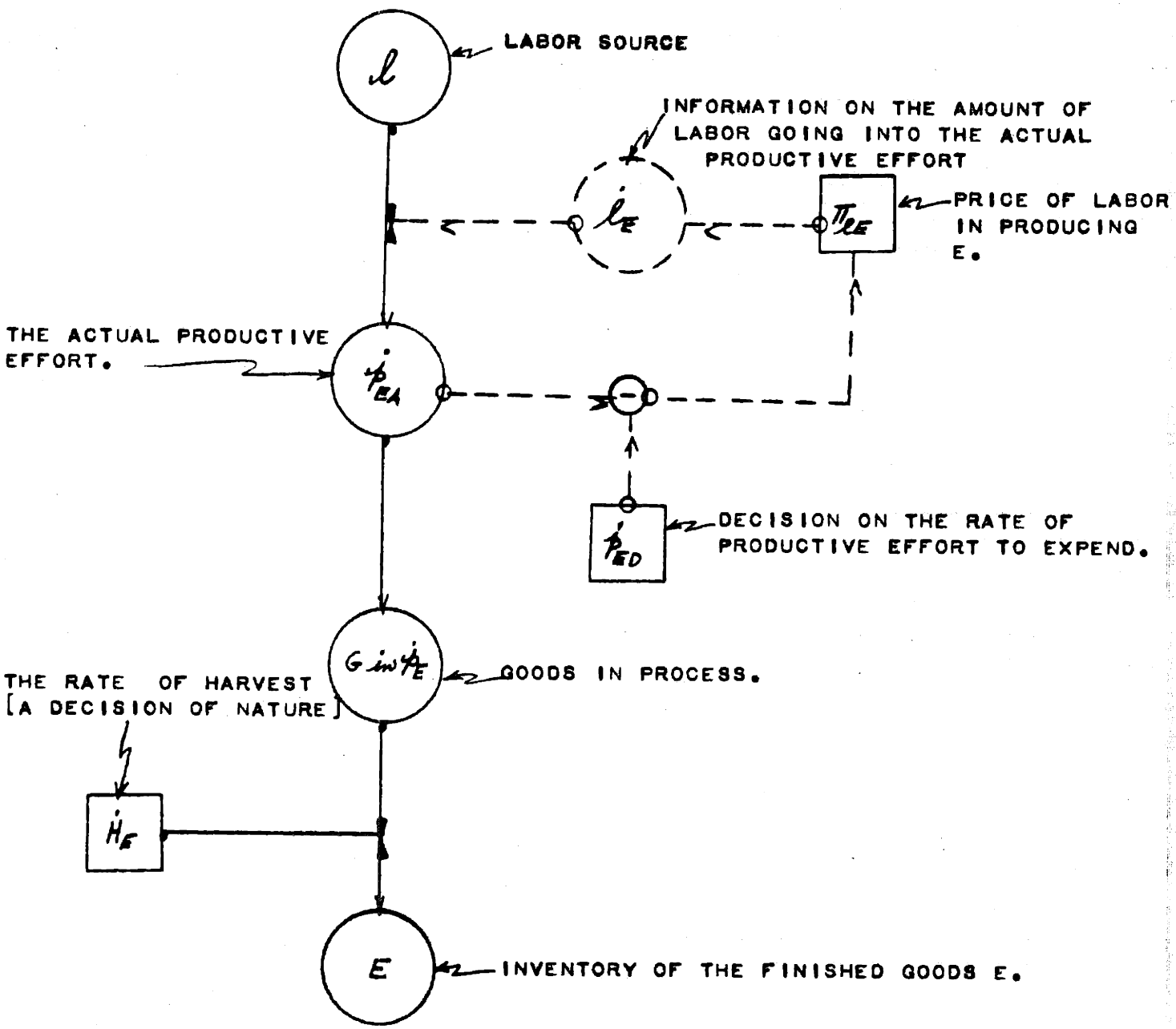
A - EXAMPLE

AN EXAMPLE OF THE NOTATION AND GENERAL MODELING APPROACH CAN BE SEEN IN THE CASE OF A DECISION TO EXPEND A CERTAIN AMOUNT OF PRODUCTIVE EFFORT AT A GIVEN TIME T. LET THE DECISION TO EXPEND PRODUCTIVE EFFORT IN PRODUCING A GOOD BE DESIGNATED BY $\dot{P}_{ED}(T)$. INFORMATION OF THIS DECISION IS USED TO DETERMINE THE PRICE OFFERED FOR LAND AND LABOR. THIS IS DONE AS FOLLOWS: IF THE ACTUAL RATE OF PRODUCTIVE EFFORT AT TIME T, $\dot{P}_{EA}(T)$ IS EQUAL TO DECIDED RATE $\dot{P}_{ED}(T)$ THEN NO CHANGE IS MADE IN THE PRICE OFFERED TO THE FACTORS OF PRODUCTION. IT IS ASSUMED THAT THE RATE AT WHICH THE FACTORS ARE AVAILABLE AT TIME T IS DETERMINED BY THE PRICE WILLING TO BE PAID FOR THESE FACTORS AT A TIME $(T-\phi)$. THE REASON FOR THE PHASE LAG BEING THAT PEOPLE DO NOT CHANGE THEIR LIVES OR HABITS, OR MAKE SHIFTS IN THEM IMMEDIATELY. THEY MUST BECOME CONDITIONED TO ANY CHANGE.

IF THE ACTUAL RATE OF PRODUCTIVE EFFORT $\dot{P}_{EA}(T)$ IS LESS THAN $\dot{P}_{ED}(T)$, THEN THE PRICE OFFERED TO THE FACTOR WILL BE INCREASED BY AN AMOUNT PROPORTIONAL TO THE DIFFERENCE BETWEEN $\dot{P}_{ED}(T)$ AND $\dot{P}_{EA}(T)$. THE CHANGE IN PRICE MAY BRING FORTH TOO MUCH ACTUAL PRODUCTION AT TIME $(T + 1)$, AND SO ANOTHER ADJUSTMENT IS MADE.

B - FLOW DIAGRAMS

DIAGRAMATICALLY, THIS MUCH OF THE MODEL CAN BE SHOWN IN FIGURE 7, ASSUMING ONLY ONE FACTOR OF PRODUCTION, LABOR = $LR_E(T)$, AND THE PRICE FOR LABOR $=\pi_{LRE}(T)$ AS



TANGIBLE FLOWS DIAGRAM - EXAMPLE

FIGURE 7

THE DIAGRAM IS NOT USED TO EXPRESS THE EXACT RELATIONSHIPS BETWEEN THE VARIABLES, OR BETWEEN THE DECISIONS. IN THIS DIAGRAM THERE ARE THREE DECISIONS SHOWN,

1) RATE OF PRODUCTIVE EFFORT,
2) THE PRICE OFFERED FOR LABOR, AND
3) THE RATE OF HARVEST. (THIS IS ASSUMED TO BE A DECISION OF NATURE IN THIS EXAMPLE)

C - DETERMINING RELATIONSHIPS BETWEEN VARIABLES

TO DEMONSTRATE THE METHOD TO BE USED IN DETERMINING THE RELATIONSHIPS BETWEEN THE VARIABLES IN THE FLOW DIAGRAM, THE EXAMPLE WILL BE CONTINUED.

THE PRICE $\pi_{LRE}(T)$ IS ASSUMED TO BE DIRECTLY PROPORTIONAL TO $[\dot{P}_{ED}(T) - \dot{P}_{EA}(T)]$. THUS, $\pi_{LRE}(T) = \alpha_1[\dot{P}_{ED}(T) - \dot{P}_{EA}(T)]$, WHERE α_1 IS A CONSTANT.

THE AMOUNT OF LABOR CALLED FORTH BY A GIVEN INCREASE IN PRICE DECREASES AS THE $\dot{LR}_E$ INCREASES. SO,

$d\dot{LR}_E = \alpha_2 [\alpha_3 - \dot{LR}_E] \; d\pi_{LRE}(T - \phi_1)$, WHERE ϕ_1 IS A CONSTANT PHASE LAG, AND α_2 AND α_3 ARE CONSTANTS.

THIS GIVES, $\dot{LR}_E(T) = \alpha_3 - \alpha_4 E^{-\alpha_4 \pi_{LRE}(T - \phi_1)}$.

BUT WHEN $\pi_{LRE} = 0$, $\dot{LR}_E = 0$, SO $\alpha_3 = \alpha_4$. FINALLY THEN,

$$\dot{LR}_E(T) = \alpha_3 [1 - E^{-\alpha_2 \pi_{LRE}(T - \phi_1)}]$$

THE PRODUCTIVE EFFORT IS ASSUMED TO BE DIRECTLY PROPORTIONAL TO THE RATE OF LABOR $\dot{LR}_E(T)$.

D - SUMMARY OF EQUATIONS

$$1)\quad \pi_{LRE}(T) = \alpha_1 \left[\dot{P}_{ED}(T) - \dot{P}_{EA}(T)\right]$$

$$2)\quad \dot{LR}_E(T) = \alpha_3 \left[1 - E^{-\alpha_2 \pi_{LRE}(T-\phi_1)}\right]$$

$$3)\quad \dot{P}_{EA}(T) = \alpha_5 \dot{LR}_E(T)$$

E - SOLVING THE EQUATIONS

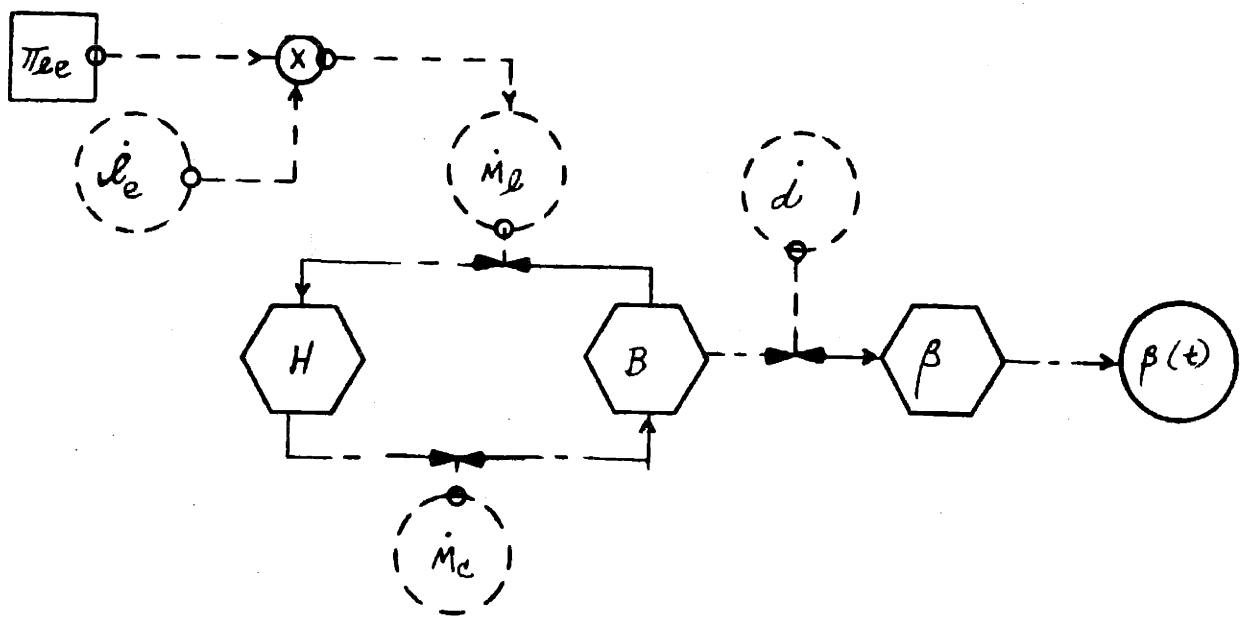
THE METHOD OF DETERMINING THE $\dot{P}_{ED}(T)$ WILL BE DISCUSSED IN THE CONSIDERATION OF SPECIFIC MODELS. AT THIS POINT IF $\dot{P}_{ED}(T)$ IS GIVEN, THEN $\pi_{LRE}(T)$ CAN BE FOUND FROM

$$\pi_{LRE}(T) = \alpha_1 \left\{ \dot{P}_{ED}(T) - \alpha_5 \cdot \alpha_3 \left[1 - E^{-\alpha_2 \pi_{LRE}(T-\phi_1)}\right] \right\}$$

ONCE $\pi_{LRE}(T)$ IS DETERMINED, $\dot{LR}_E(T)$ CAN BE DETERMINED FROM EQUATION 2), AND THEN $\dot{P}_{EA}(T)$ CAN BE DETERMINED FROM EQUATION 3). THE DETERMINATION OF $\pi_{LRE}(T)$ AND $\dot{LR}_E$ NOW MAKE IT POSSIBLE TO DETERMINE THE RATE OF MONEY FLOWING TO THE PERSON DOING THE LABOR.

THE MONEY SECTOR CAN NOW BE PARTIALLY DIAGRAMED FOR PURPOSES OF ILLUSTRATION IN FIGURE 8.

IF THE ORDER OF SOLVING THE EQUATIONS AT A GIVEN TIME IS TAKEN TO CORRESPOND TO THE NATURAL FLOWS OF TANGIBLES IN THE SYSTEM, THE SOLUTION OF ANY VARIABLE, OR ANY DECISION FUNCTION SHOULD DEPEND ON ALREADY DETERMINED VALUES OF OTHER VARIABLES ONLY. THEREFORE, NO SIMULTANEOUS SOLUTIONS SHOULD BE REQUIRED. THIS IS AN EXTREMELY IMPORTANT POINT SINCE DECISIONS IN ACTUAL ECONOMIES DO NOT PRESUPPOSE SIMULTANEOUS SOLUTIONS IN ORDER TO ARRIVE AT A RESULT. THEY MAY BE MADE AT RANDOM, OR IN AN IN-ATELY PERVERSE MANNER, BUT MORE LIKELY THAN NOT, THERE IS A PRIMARY CRITERIA OR TWO THAT DETERMINE THE DECISION ON AN AVERAGE.



MONEY FLOWS DIAGRAM - EXAMPLE

FIGURE 8.

CHAPTER III

PRESENTATION OF FOUR DYNAMIC MODELS

INTRODUCTION: GENERAL TYPES OF MODELS CONSIDERED IN THIS CHAPTER.

CHAPTER III WILL CONSIST OF THE APPLICATION OF THE METHOD OF ANALYSIS DESCRIBED IN CHAPTER I, PART 4, TO SPECIFIC DYNAMIC ECONOMIES WHERE GOODS AND SERVICES ARE PRODUCED AND CONSUMED.

THE METHOD OF APPROACH PRESENTED HERE IS APPLICABLE TO ANY ECONOMIC ENTITY, WHETHER IT BE A BUSINESS FIRM, A FAMILY, OR AN ECONOMY AS A WHOLE.

TO ARRIVE AT A MODEL OF A REAL ECONOMY WITH ALL OF ITS DETAIL WOULD INVOLVE MORE TIME THAN IS AVAILABLE FOR THE PRESENT STUDY. HOWEVER, TO BUILD UP TO A REAL ECONOMIC SITUATION, IT IS NECESSARY TO FIRST CONSIDER THE PRODUCTION OF ONE GOOD, BY ONE FAMILY, USING A SINGLE FACTOR OF PRODUCTION. THE NEXT CONSIDERATION WOULD BE ONE GOOD, ONE FAMILY, AND TWO PRODUCTIVE FACTORS., ETC. UNTIL THE MODEL CONTAINS N GOODS, M FAMILIES, AND P PRODUCTIVE FACTORS. THE SAME PROCEDURE COULD THEN BE FOLLOWED INCLUDING MONEY, THEN BUSINESS CREDIT, THEN CONSUMER CREDIT, THEN DISTRIBUTION OF INCOME EFFECTS BROUGHT ABOUT BY THE GOVERNMENT, THEN INNOVATION, ETC. AFTER THE BUILD-UP TO THREE GOODS, THREE PRODUCTIVE FACTORS, AND THREE FAMILIES, THERE ARE CERTAIN REGULARITIES THAT APPEAR WHICH MAKE EXTENSION TO THE CASE OF N, M, AND P RELATIVELY SIMPLE. THE CASE WHERE N, M, AND P ARE EQUAL TO TWO STILL CONTAINS THE ELEMENTS OF BEING A SPECIAL CASE, BUT N, M, AND P EQUAL TO THREE WILL BEGIN TO HAVE PROPERTIES WHICH CAN BE GENERALIZED. NO ATTEMPT TO GENERALIZE WILL BE MADE IN THIS PARTICULAR PAPER.

A PRELIMINARY LIST OF THE BUILD-UP OF MODELS SUGGESTED IS GIVEN IN FIGURE 9. FOR THE PURPOSES OF THIS PAPER, ONLY THE FOLLOWING MODELS ARE DISCUSSED FURTHER:

1) MODEL 1, [1 GOOD, 1 FAMILY, 1 PRODUCTIVE FACTOR].
2) MODEL 2, [1 GOOD, 1 FAMILY, 2 PRODUCTIVE FACTORS].
3) MODEL 2A, [1 GOOD, 1 FAMILY, 2 PRODUCTIVE FACTORS, MONEY].
4) MODEL 12C, [2 GOODS, 1 FAMILY, 3 PRODUCTIVE FACTORS, MONEY, BUSINESS CREDIT, AND CONSUMER CREDIT.

IN READING THESE PRESENTATIONS IT SHOULD BE KEPT IN MIND THAT THE METHODS OF MAKING DECISIONS, AND THE FACTORS INVOLVED IN THE DECISIONS ARE THIS WRITER'S OPINION OF HOW THESE DECISIONS ARE MADE. SINCE IT IS INTENDED THAT THESE FORMULATIONS BE TESTED NUMERICALLY ON THE ELECTRONIC COMPUTER, DEFINITE RELATIONSHIPS ARE ESTABLISHED AND RANGES DETERMINED FOR ALL PARAMETERS. HOWEVER, IN THE FUTURE, MANY VARIATIONS OF POLICY IN THE MAKING OF DECISIONS IN THE MODELS COULD BE EXAMINED VERY RAPIDLY. ALSO, THE EFFECTS OF VARYING THE VARIOUS PARAMETERS CAN BE STUDIED FAIRLY READILY.

THE MODELS DISCUSSED HERE ARE DYNAMIC MODELS, IN THAT THEY ARE DESIGNED TO DESCRIBE EVENTS PROGRESSING IN TIME. THEY CONTAIN REGENERATIVE LOOPS, AND PHASE LAGS SIMILAR TO THOSE EXISTING IN ACTUAL ECONOMIC SYSTEMS. IT IS THEREFORE EXPECTED THAT THE SOLUTIONS OF THESE PROBLEMS WILL CONTAIN CERTAIN INSTABILITIES, AND CYCLES SIMILAR TO BUSINESS CYCLES, THE EXACT NATURE OF THE CYCLES BEING DETERMINED BY THE ASSUMED VALUES OF THE PARAMETERS AND THE ASSUMED DECISION CRITERIA. BUT THESE QUESTIONS AND MANY OTHERS WILL HAVE TO BE INVESTIGATED IN LATER STUDIES.

Model 3	Goods	People	Productive Factors	A Money	B Business Credit	C Consumer Credit	D Distribution of Income	* A	B	C	D
1	1	1	1					✓			
2	1	1	2					✓	✓		
3	1	1	3								
4	1	2	1								
5	1	2	2								
6	1	2	3								
7	1	3	1								
8	1	3	2								
9	1	3	3								
10	2	1	1								
11	2	1	2								
12	2	1	3							✓	
13	2	2	1								
14	2	2	2								
15	2	2	3								
16	2	3	1								
17	2	3	2								
18	2	3	3								
19	3	1	1								
20	3	1	2								
21	3	1	3								
22	3	2	1								
23	3	2	2								
24	3	2	3								
25	3	3	1								
26	3	3	2								
27	3	3	3								

* A CHECK MEANS THIS MODEL IS INCLUDED IN THIS STUDY.

FIGURE 9.

IT IS THE PURPOSE OF THIS PAPER TO POINT OUT A TECHNIQUE FOR DYNAMIC SYNTHESIS OF A SYSTEM, AND TO APPLY THIS TECHNIQUE TO SOME RELATIVELY SIMPLE SYSTEMS WHICH COULD FORM THE BASIS FOR THE FUTURE BUILDING OF MORE COMPLEX MODELS.

THE MODELS DISCUSSED WILL BE CONSTRUCTED WITH THE ULTIMATE OBJECTIVE OF OBTAINING TRIAL SOLUTIONS FOR THE VARIOUS EQUATIONS ON HIGH SPEED ELECTRONIC COMPUTERS, FOR DIFFERENT VALUES OF THE PARAMETERS.

PART I - MODEL I

1 GOOD

1 FAMILY

1 PRODUCTIVE FACTOR

A - GENERAL DISCUSSION AND FLOW CHART

THE BASIC MODEL CONSISTS OF THE FOLLOWING ASSUMPTIONS:

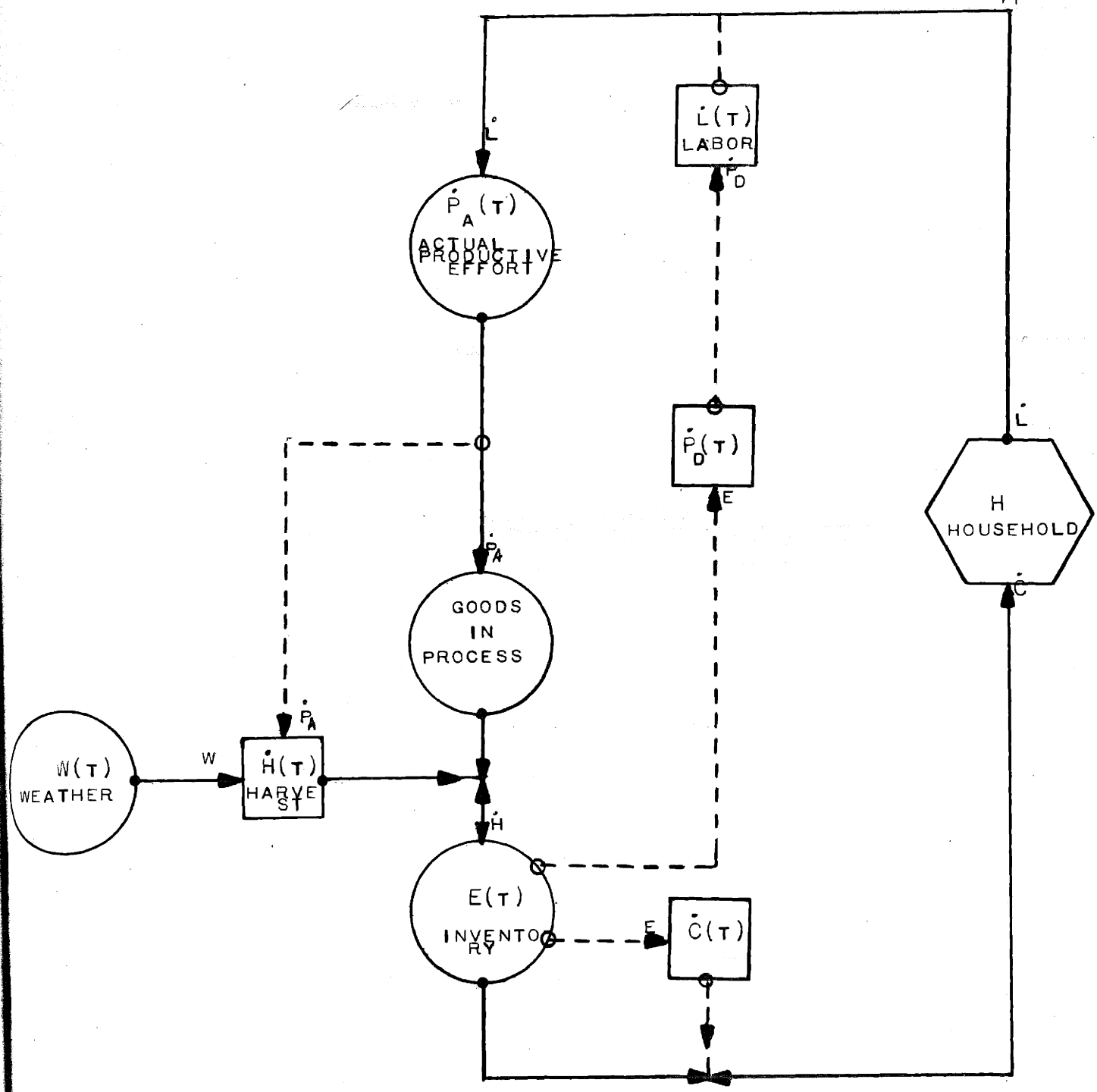
1) 1 GOOD
2) 1 FAMILY
3) 1 PRODUCTIVE FACTOR.

THE FLOW CHART SHOWS THE HOUSEHOLD H AS THE CONSUMER OF THE GOOD PRODUCED E, AND SOURCE OF LABOR $\dot{L}R$. THE LABOR GOES INTO THE ACTUAL RATE OF PRODUCTIVE EFFORT $\dot{P}_A$, WHICH IN TURN CONTRIBUTES TO THE GOODS IN PROCESS. THE GOODS IN PROCESS ARE METERED INTO INVENTORY BY THE DECISION OF NATURE WHICH CONTROLS THE HARVEST. THE GOODS ARE THEN METERED OUT OF INVENTORY INTO THE HOUSEHOLD FOR CONSUMPTION BY THE DECISION TO CONSUME. THE DECISION TO EXPEND PRODUCTIVE EFFORT BRINGS ABOUT THE DECISION TO APPLY LABOR, WHICH GOES INTO THE ACTUAL RATE OF PRODUCTIVE EFFORT, UND SO WIEDER.

THE FLOW CHART IS GIVEN IN FIGURE 10.

B - DERIVATION AND DISCUSSION OF VARIABLES AND DECISIONS

THE DECISION TO PUT IN PRODUCTIVE EFFORT IS MADE AT TIME T ON THE BASIS OF INFORMATION ABOUT THE LEVEL OF INVENTORY AT $(T - \phi_C)$, AND THE PRODUCTIVE EFFORT PUT IN AT TIME $(T - \phi_C)$. THE PRODUCTIVE EFFORT DECIDED ON AT TIME T, $\dot{P}_D(T)$, IS ASSUMED TO BE DIRECTLY PROPORTIONAL TO $\dot{P}_A(T - \phi_P)$, AND TO DECREASE EXPONENTIALLY FROM A NORMAL LEVEL OF INVENTORY AS $E(T)$ DECREASES.



FLOW CHART - MODEL I

| GOOD
| FAMILY
| PRODUCTIVE FACTOR

FIGURE 10.

THESE ASSUMPTIONS RESULT IN THE RELATIONSHIP

$$\dot{P}_D(T) = \alpha_1 \dot{P}_A(T - \phi_P) E^{-\alpha_2 E (T - \phi_C)},$$

WHERE $\alpha_1 = \dot{P}_{MAX}/\dot{P}_N$, $\alpha_2 = \frac{1}{EN} \ln (\dot{P}_{MAX}/\dot{P}_N)$, $\dot{P}_N$ IS A NORMAL RATE OF PRODUCTION, AND $\dot{P}_{MAX}$ IS THE MAXIMUM POSSIBLE RATE OF PRODUCTIVE EFFORT.

THE DECISION TO PUT IN LABOR RESULTS FROM $\dot{P}_D$. THE HIGHER THE $\dot{P}_D$ THE GREATER THE $\dot{LR}$, UNTIL THE MAXIMUM $\dot{LR}_{MAX}$ IS BEING EXPENDED. THUS,

$$\dot{LR}(T) = \alpha_3 [1 - E^{-\alpha_4 \dot{P}_D(T)}], \text{ WHERE}$$

$\alpha_3 = \dot{LR}_{MAX}$, $\alpha_4 = \frac{\ln 2}{\dot{P}_{D1/2}}$, AND $\dot{P}_{D\ 1/2}$ IS THE VALUE OF $\dot{P}_D(T)$ AT WHICH $\dot{LR} = \frac{\dot{LR}_{MAX}}{2}$.

THE ACTUAL PRODUCTIVE EFFORT $\dot{P}_A(T)$ IS DIRECTLY PROPORTIONAL TO $\dot{LR}(T)$, GIVING $\dot{P}_A(T) = \alpha_5 \dot{LR}(T)$, WHERE

$$\alpha_5 = \frac{\dot{P}_{AO}}{\dot{LR}_O},$$ $\dot{P}_{AO}$ AND $\dot{LR}_O$ ARE THE VALUES OF $\dot{P}_A(T)$ AND $\dot{LR}(T)$ AT $T = 0$.

THE RATE OF HARVEST IS A FUNCTION OF THE WEATHER AND $\dot{P}_A(T)$. THE HIGHER THE $W(T)$ THE LESS THE ATTENUATING EFFECT OF WEATHER ON THE HARVEST. THUS,

$$H(T) = P_A(T - \phi_E) \quad [1 - E^{-\alpha_6 W(t)}], \text{ WHERE}$$

$\alpha_6 = \frac{\ln 2}{W\ 1/2}$, AND $W\ 1/2$ IS THE VALUE OF WEATHER AT WHICH $[1 - E^{-\alpha_6 W(T)}] = 1/2$.

THE CHANGE IN INVENTORY $\dot{E}(T)$ IS THE RATE OF HARVEST $\dot{H}(T)$ MINUS THE RATE OF CONSUMPTION $\dot{C}(T)$.

$$\dot{E}(T) = \dot{H}(T) - \dot{C}(T), \text{ OR}$$

$$E(T) = E_0 + \int_0^T [\dot{H}(T) - C(T)]\, dT$$

THE RATE OF CONSUMPTION IS ASSUMED TO BE PROPORTIONAL TO THE LEVEL OF INVENTORY. THUS,

$\dot{C}(T) = \alpha_7 E(T - \phi_C)$, WHERE $\alpha_7 = \dfrac{\dot{C}_0}{E_0}$, AND $\dot{C}_0$ AND E_0 ARE THE VALUES OF $\dot{C}(T)$ AND $\dot{E}(T)$ AT $T = 0$.

C - <u>THE SUMMARY OF EQUATIONS</u>

1) $P_D(T) = \alpha_1 P_A (T - \phi_P)\, E^{-\alpha_2 E (T - \phi_C)}$

2) $\dot{L}_R(T) = \alpha_3 [1 - E^{-\alpha_4 \dot{P}_D(T)}]$

3) $\dot{P}_A(T) = \alpha_5 \dot{L}_R(T)$

4) $\dot{H}(T) = \dot{P}_A(T - \phi_E)\, [1 - E^{-\alpha_6 W(T)}]$

5) $\dot{C}(T) = \alpha_7\, E(T - \phi_C)$

6) $E(T) = E_0 + \int_0^T [\dot{H}(T) - \dot{C}(T)]\, dT.$

D - <u>SUMMARY OF CONSTANTS</u>

FROM FUNCTION	CONSTANT
$\dot{P}_D$	$\alpha_1 = \dot{P}_{MAX}/\dot{P}_N$
	$\alpha_2 = \dfrac{1}{E_N} \ln \left(\dfrac{P_{MAX}}{P_N}\right)$
$\dot{L}_R$	$\alpha_3 = \dot{L}_{R_{MAX}}$
	$\alpha_4 = \mathcal{L}_N{}^2/P_D{}^{1/2}$

D - SUMMARY OF CONSTANTS (CONTINUED)

FROM FUNCTION	CONSTANT
$\dot{P}_A$	$\alpha_5 = \dot{P}_{AO}/\dot{L}R_O$
H	$\alpha_6 = LN2/W\ 1/2$
$\dot{C}$	$\alpha_7 = \dot{C}_O/E_O$

E - SOLVING THE EQUATIONS

THE SOLUTION OF THE EQUATIONS 1) THRU 6) DEPENDS ON THE VALUES OF THE VARIABLES AT T = O. THE FUNCTIONS CONTAINING PHASE LAGS WHERE VALUES ARE NEEDED FURTHER BACK THAN T = O, CAN BE ASSUMED TO HAVE NORMAL VALUES, FOR ALL ARGUMENTS LESS THAN ZERO. ONCE THE PARAMETERS ARE DECIDED ON, THE CONSTANTS CAN BE CALCULATED. THEN,

$$\left.\begin{array}{l} \dot{P}_D(o) \\ \dot{L}R(o) \\ \dot{P}_A(o) \\ \dot{H}(o) \\ \dot{C}(o) \\ E(o) \end{array}\right\} F(o)$$

THEN, F(1), F(2), ----------F(N).

THE ORDER OF COMPUTATION IS IMPORTANT, AS EACH CALCULATION DEPENDS ON THE VALUES CALCULATED BEFORE. THE RESULT FOR ANY GIVEN VARIABLE IS A TIME SEQUENCE OF VALUES. THE CLOSER THE INTERVALS OF TIME THE MORE NEARLY CONTINUOUS THE FUNCTIONS WILL BE WITH TIME.

PART 2 - MODEL 2

1 GOOD

1 FAMILY

2 PRODUCTIVE FACTORS

A - GENERAL DISCUSSION

IN THE MODEL HERE DESCRIBED THERE IS ONE GOOD, ONE FAMILY, TWO PRODUCTIVE FACTORS, AND THERE IS NO MONEY. THERE CAN BE TRADE OF GOODS, BUT THIS CYCLE IS NOT DISCUSSED OR SEPARATED OUT. THERE ARE FIVE DECISIONS WHICH DETERMINE THE DYNAMIC CHARACTERISTICS OF THE SYSTEM,

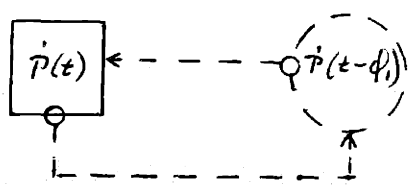
1) THE RATE OF LABOR TO BE USED, (MAN HOURS/WEEK/PERSON),
2) THE RATE OF LAND TO BE USED, (ACRES/PERSON),
3) THE RATE OF PRODUCTIVE EFFORT TO BE EXPENDED, (UNITS OF GOODS/WEEK/PERSON),
4) THE RATE OF HARVEST PER SEASON, (UNITS OF GOODS/WEEK/PERSON),
5) THE RATE OF CONSUMPTION, (UNITS OF GOODS/WEEK/PERSON).

DECISIONS ARE BASED ON INFORMATION, BUT THE RESULT OF THE DECISION IS A METERING OUT OF A FLOW OF TANGIBLE EFFORT OR PRODUCT. IN THE PRESENT MODEL, THE PRINCIPAL DECISION IS MADE OF HOW MUCH PRODUCTIVE EFFORT IS TO BE USED. THE DECISIONS ON RATES OF LAND AND LABOR ARE THEN DEPENDENT ON THIS DECISION. IT IS ASSUMED THAT THE PRODUCTIVE EFFORT DECIDED ON EQUALS THE ACTUAL EFFORT. THUS, THE DECISIONS OF HOW MUCH LAND, AND HOW MUCH LABOR ARE TO BE USED ARE DEPENDENT DECISIONS. THE DECISION OF HARVEST RATE IS ALSO A DEPENDENT DECISION, WHILE RATE OF CONSUMPTION IS AN INDEPENDENT DECISION SINCE IT IS NOT DIRECTLY DEPENDENT ON ANY OF THE OTHER DECISIONS.

B - DIAGRAM

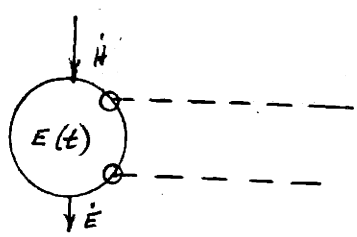
THE DIAGRAM OF THE MODEL 2, GIVEN IN FIGURE 11, IS DESIGNED TO SHOW THE FLOW CHARACTERISTICS OF THE MODEL. THE TANGIBLE FLOWS ARE CONTINUOUS BLACK LINES WHILE INFORMATION FLOWS ARE DASHED BLACK LINES. THE DECISIONS ARE IN SQUARE BOXES, AND THE CIRCULAR BOXES CONTAIN THE VARIABLES OF THE SYSTEM. THE RELATIONSHIPS BETWEEN INPUTS, OUTPUTS, AND THE VARIABLE ITSELF ARE FUNCTIONAL RELATIONSHIPS DEPENDENT ON THE NATURE AND PROPERTIES OF THE VARIABLES. IN SOME CASES THESE RELATIONSHIPS ARE KNOWN, WHILE IN OTHER CASES ASSUMPTIONS WILL HAVE TO BE MADE ABOUT THE SYSTEM, IN ORDER TO DETERMINE THE NATURE OF THE FUNCTIONAL RELATIONSHIP.

AN EXAMPLE OF ONE TYPE OF VARIABLE IS $\dot{P}(T-\phi_1)$

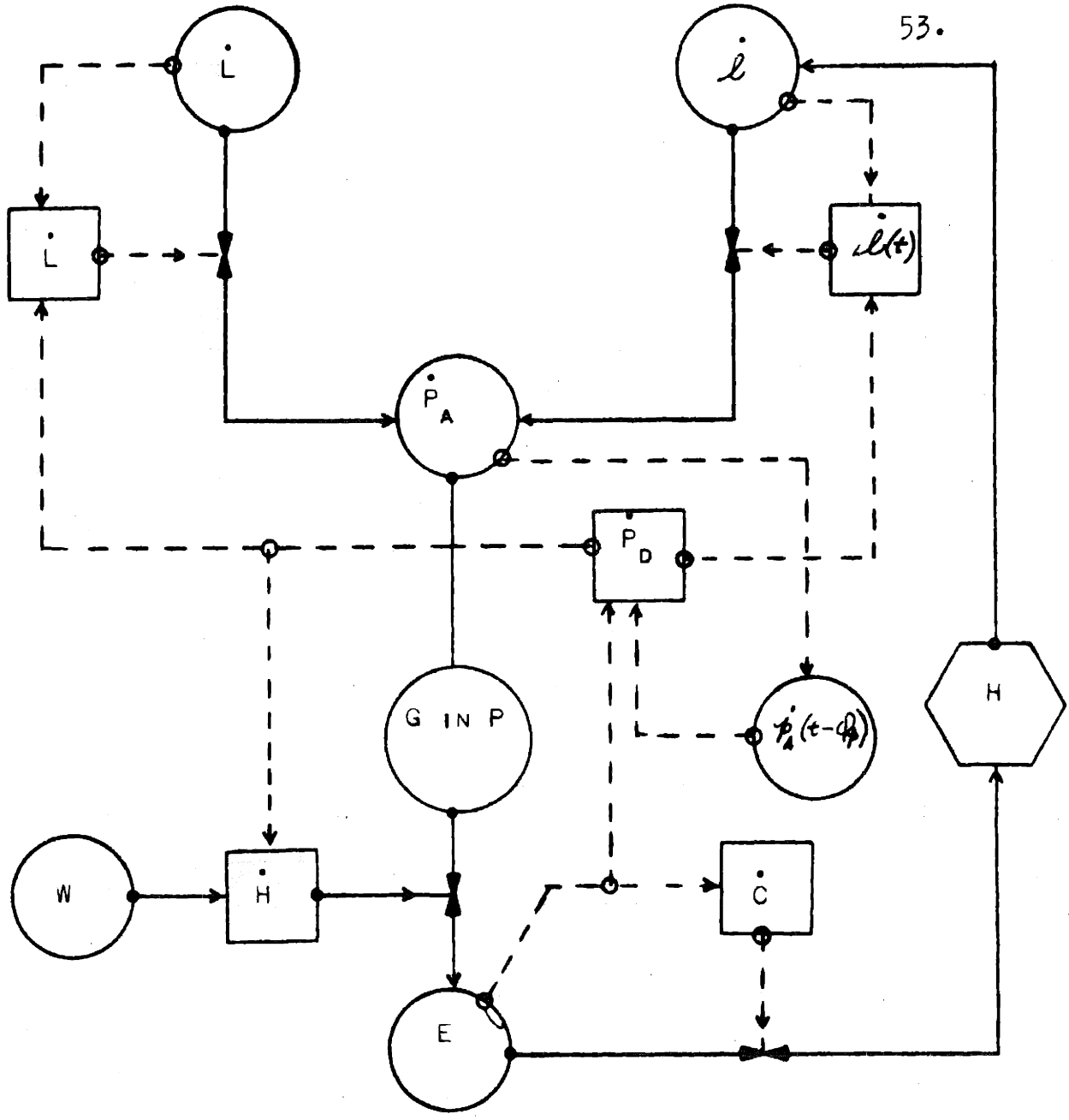


THIS MEANS THAT $\dot{P}(T-\phi_1)$ IS DEPENDENT ON THE FLOW OF INFORMATION FROM $\dot{P}(T)$, WHILE THE DECISION OF $\dot{P}(T)$ IS DEPENDENT ON THE INFORMATION $\dot{P}(T-\phi_1)$.

AN EXAMPLE OF ANOTHER TYPE OF RELATIONSHIP IS



THIS MEANS THAT THE INVENTORY $E(T)$ IS DEPENDENT ON THE RATE OF HARVEST $\dot{H}(T)$, AND THE RATE OF CONSUMPTION. THE DASHED LINES INDICATE THAT INFORMATION ON $E(T)$ HAS AN INFLUENCE ELSEWHERE IN THE SYSTEM.



FLOW DIAGRAM - MODEL 2

1 GOOD

1 FAMILY

2 PRODUCTIVE FACTORS

[VARIABLES ARE ALL AVERAGE PER PERSON].

FIGURE 11.

THE VALUES OF ALL FLOWS IN THE SYSTEM CAN BE DETERMINED AT ANY TIME T, AS WELL AS THE VALUES OF THE VARIABLES IF THE MANNER OF MAKING THE DECISIONS IS KNOWN. THUS, THE APPROACH TO BE TAKEN IN EXAMINING THE NATURE OF THE SYSTEM IS

1) TO SPECIFY THE MANNER IN WHICH DECISIONS ARE MADE, AND THEN
2) TO DETERMINE THE RESULTANT EFFECTS ON THE VARIABLES OF THE SYSTEM.

C - PRODUCTIVE EFFORT

THE FIRST IN THE SEQUENCE OF DECISIONS MADE BY AN INDIVIDUAL IN THIS ECONOMY IS, "HOW MUCH PRODUCTIVE EFFORT SHOULD I PUT IN?" THE DECISION IS BASED ON INFORMATION REGARDING THE LEVEL OF HIS INVENTORY, AND THE AMOUNT OF PRODUCTIVE EFFORT HE PUT IN LAST YEAR AT THIS SAME TIME. A UNIT OF PRODUCTIVE EFFORT IS DEFINED BY THE COMBINATION OF LAND AND LABOR REQUIRED TO PRODUCE A UNIT OF GOODS FOR INVENTORY. THUS,

$$1)\quad \dot{P}_D(T) = \dot{P}_D[E\,(T-\phi_C),\ \dot{P}_A(T-\phi_P)] \text{ WHERE}$$

$\dot{P}_D(T)$ IS THE PRODUCTIVE EFFORT DECIDED ON AT TIME T, $E(T)$ IS THE INVENTORY, $\dot{P}_A(T)$ IS THE ACTUAL PRODUCTIVE EFFORT AT TIME T, AND ϕ_P AND ϕ_C ARE CONSTANTS.

THE ASSUMPTION IS MADE THAT ALL LABOR BEING USED IN THIS MODEL GOES INTO PRODUCTIVE EFFORT, AND THAT NONE GOES FOR CAPITAL FORMATION SUCH AS, THE CLEARING OF NEW LAND, ETC.

ASSUME* THAT IT IS POSSIBLE TO EXPRESS THE FUNCTIONAL

---

*FOOTNOTE: NO EFFORT WILL BE MADE AT THIS POINT TO FORTIFY THIS ASSUMPTION.

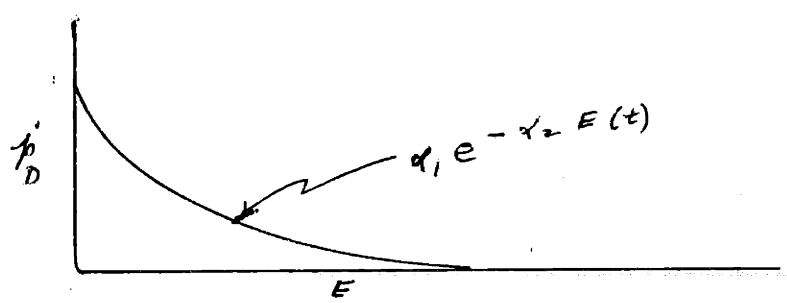
RELATIONSHIP GIVEN IN EQUATION 1) AS THE PRODUCT OF TWO FUNCTIONS, ONE OF WHICH IS A FUNCTION OF $E(T)$ ALONE, AND

ONE OF WHICH IS A FUNCTION OF $\dot{P}_A(T-\phi_P)$ ALONE. THEN

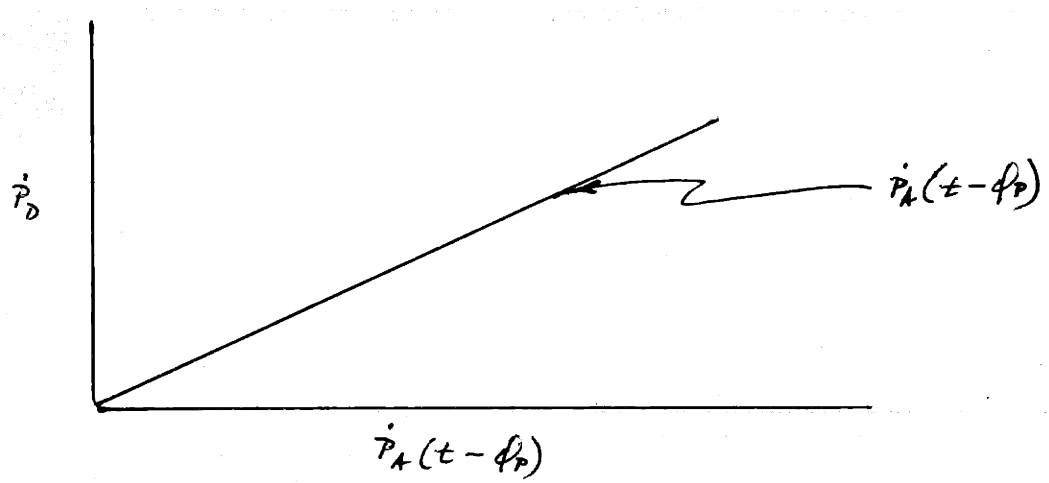
$$\dot{P}_D(T) = f_1\ [E(T-\phi_C)],\ f_2\ [\dot{P}_A(T-\phi_P)],$$

NONE OF THE INDEPENDENT VARIABLES, SAY $\dot{P}_A(T-\phi_P)$ WERE HELD CONSTANT, A CHANGE IN THE OTHER $E(T)$, WOULD GIVE A CORRESPONDING CHANGE IN THE DEPENDENT VARIABLE $\dot{P}_D(T)$. BY SO DOING, AN EFFORT WILL BE MADE TO CONSTRUCT PLAUSIBLE FUNCTIONAL RELATIONSHIPS FOR f_1 AND f_2 OF EQUATION 2). THE FUNCTIONS WILL BE CHOSEN TO FIT THE EXTREME VALUES OF THE VARIABLES, AND ARBITRARY CONSTANTS WILL BE LEFT WHICH WILL LATER BE INTERPRETED IN TERMS OF THE ESSENTIAL PARAMETERS OF THE MODEL.

IT IS REASONABLE TO ASSUME THAT IF $\dot{P}_A(T-\phi_P)$ WERE HELD CONSTANT THAT AS $E(T)$ INCREASED $\dot{P}_D(T)$ WOULD DECREASE UNTIL AT SOME EXTREMELY LARGE $E(T)$, $\dot{P}_D(T)$ WOULD GO TO ZERO. THUS,



FOR A CONSTANT $E(T)$ IT IS REASONABLE TO ASSUME THAT THE RATE OF PRODUCTIVE EFFORT PUT IN THIS YEAR WILL BE PROPORTIONAL TO THE PRODUCTIVE EFFORT PUT IN AT THE SAME TIME LAST YEAR.



THEN, $\dot{P}_D(T) = \alpha_1 \dot{P}_A(T - \phi_P)\, E^{-\alpha_2 E(T - \phi_C)}$, WHERE $\alpha_1 = \dot{P}_{MAX}/\dot{P}_N$,

AND $\alpha_2 = \frac{1}{E_N} \ln\left(\frac{\dot{P}_{MAX}}{\dot{P}_N}\right)$, E_N IS THE NORMAL LEVEL OF INVENTORY, $\dot{P}_N$ IS THE NORMAL RATE OF PRODUCTIVE EFFORT, AND $\dot{P}_{MAX}$ IS THE MAXIMUM RATE OF PRODUCTIVE EFFORT POSSIBLE.

D - <u>LAND AND LABOR</u>

THE AMOUNT OF PRODUCTIVE EFFORT DECIDED UPON WILL DETERMINE THE AMOUNTS OF LAND AND LABOR TO BE USED. THE GREATER THE PRODUCTIVE EFFORT DECIDED ON, THE GREATER THE RATE OF USE OF LAND AND LABOR, EXCEPT THAT THE AMOUNTS OF LAND AND LABOR ARE LIMITED. THEREFORE,

$$\dot{LR}(T) = \alpha_3 \left[1 - E^{-\alpha_4 \dot{P}_D(T)}\right], \text{ AND}$$

$$\dot{L}(T) = \alpha_5 \left[1 - E^{-\alpha_6 \dot{P}_D(T)}\right], \text{ WHERE}$$

$\alpha_3 = \dot{LR}_{MAX}$, $\alpha_4 = \frac{\ln 2}{\dot{P}_{DLR\ 1/2}}$, $\alpha_5 = \dot{L}_{MAX}$, $\alpha_6 = \frac{\ln 2}{\dot{P}_{DLR\ 1/2}}$,

$\dot{P}_{DLR\ 1/2}$ IS THE VALUE OF $\dot{P}_D(T)$ AT WHICH $\dot{LR}(T) = \frac{\dot{LR}_{MAX}}{2}$, AND

$\dot{P}_{DL\ 1/2}$ IS THE VALUE OF $\dot{P}_D(T)$ AT WHICH $\dot{L}(T) = \frac{\dot{L}_{MAX}}{2}$.

THE ACTUAL PRODUCTIVE EFFORT IS ASSUMED TO BE DIRECTLY PROPORTIONAL TO THE RATE OF LAND AND LABOR GOING INTO THE PRODUCTIVE EFFORT. THUS,

$$\dot{P}_A = \alpha_7 \dot{LR}(T) \cdot \dot{L}(T), \text{ WHERE}$$

$\alpha_7 = \frac{\dot{P}_{AO}}{\dot{LR}_O \cdot \dot{L}_O}$, $\dot{P}_{AO}$, $\dot{LR}_O$, AND $\dot{L}_O$ ARE THE VALUES OF $\dot{P}_A(T)$, $\dot{LR}(T)$, AND $\dot{L}(T)$ AT $T = 0$.

E - RATE OF HARVEST

THE RATE OF HARVEST IS A DECISION OF NATURE, BUT BASED UPON THE PRODUCTIVE EFFORT $\dot{P}_A(T)$ PUT IN AT SOME PRIOR TIME $(T-\phi_E)$. A REASONABLE ASSUMPTION WOULD BE THAT

$$\dot{H}(T) = \dot{P}_A(T-\phi_E)\ [1-E^{-\alpha_8 W(T)}]$$

WHERE $\alpha_8 = \dfrac{\ln 2}{W\ 1/2}$, AND W 1/2 IS THE VALUE FOR WEATHER AT WHICH ONLY HALF AS MUCH HARVEST WOULD OCCUR FOR A GIVEN PRODUCTIVE EFFORT AS WOULD OCCUR FOR IDEAL WEATHER FOR PRODUCTION.

F - RATE OF CONSUMPTION

THE RATE OF CONSUMPTION AT ANY TIME T IS ASSUMED TO BE DEPENDENT ON THE RATE OF CONSUMPTION THE PERSON IS USED TO, INCREASED OR DECREASED WHEN THE LEVEL OF INVENTORY IS EXTREMELY HIGH OR EXTREMELY LOW. THE USUAL RATE OF CONSUMPTION $\dot{C}_N(T)$ IS

$$\dot{C}_N(T) = \int_1^N \frac{\dot{C}\ (T-\xi)}{N}.$$

THE CHANGE OF $\dot{C}$ WITH E CAN BE GIVEN BY

$$\dot{C} \propto [1+\mathrm{TAN}\ [\ \frac{E(T-\phi_C) - E_N}{E_N} \cdot \frac{\pi}{2}\ ]],$$ WHERE E_N IS WHAT THE INDIVIDUAL WOULD CONSIDER A NORMAL INVENTORY, AND $\dot{C} \geq \dot{C}$ MIN, WHERE $\dot{C}$ MIN IS THE MINIMUM SUBSISTANCE LEVEL, AND ϕ_C IS A CONSTANT.

THEREFORE,

$$\dot{C}(T) = [1+\mathrm{TAN}\ (\ \frac{E\ (T-\phi_C)\ -E_N}{E_N} \cdot \frac{\pi}{2}\ )]\int_1^m \frac{\dot{C}(T-\xi)\ d\xi}{N}.$$

G - CHANGE IN INVENTORY

FROM THE FLOW CHART, AN ADDITIONAL RELATIONSHIP CAN BE SEEN TO BE $\dot{E}(T) = \dot{H}(T) - \dot{C}(T)$.

H - SUMMARY OF EQUATIONS, $[\dot{P}_0, \dot{L}R, \dot{L}, \dot{P}_A, \dot{H}, \dot{C} \geq 0]$

1) $\dot{P}_D(T) = \alpha_1 \dot{P}_A(T - \phi_P)\, E^{-\alpha_2 E(T-\phi_C)}$

2) $\dot{L}R(T) = \alpha_3 [1 - E^{-\alpha_4 \dot{P}_D(T)}$

3) $\dot{L}(T) = \alpha_5 [1 - E^{-\alpha_6 \dot{P}_D(T)}$

4) $\dot{P}_A(T) = \alpha_7 \dot{L}R(T) \cdot \dot{L}(T)$

5) $\dot{H}(T) = P_A(T - \phi_E)\, [1 - E^{-\alpha_8 W(T)}$

6) $\dot{E}(T) = \dot{H}(T) - \dot{C}(T)$, $\therefore E(T) = E_0 + \int_0^T [\dot{H}(T) - \dot{C}(T)]\, dT$.

7) $\dot{C}(T) = [1 - \mathrm{TAN}\left(\frac{E(T-\phi_C) - E_N}{E_N}\right) \cdot \pi/2\,] \cdot \int_1^m \frac{\dot{C}(T-\xi)}{N}\, d\xi \qquad \dot{C} \geq \dot{C}_{MIN}$.

I - SUMMARY OF CONSTANTS

FUNCTION	CONSTANT
$\dot{P}_D$	$\alpha_1 = \dot{P}_{MAX}/\dot{P}_N$
	$\alpha_2 = \frac{1}{E_N} \ln\left(\frac{\dot{P}_{MAX}}{\dot{P}_N}\right)$
$\dot{L}R$	$\alpha_3 = \dot{L}R_{MAX}$
	$\alpha_4 = \ln 2/P_{DLR}\ 1/2$

I - SUMMARY OF CONSTANTS (CONTINUED)

FUNCTION	CONSTANT
$\dot{L}$	$\alpha_5 = \dot{L}_{MAX}$
	$\alpha_6 = \ln 2/\dot{P}_{DL}\ 1/2$
$\dot{P}_A$	$\alpha_7 = \dot{P}_{AO}/\dot{L}R_O\dot{L}_O$
$\dot{H}$	$\alpha_8 = \ln 2/W\ 1/2$
	TIME DELAYS
$\dot{P}_D$	ϕ_P
$\dot{H}$	ϕ_E
$\dot{P}_D, \dot{C}$	ϕ_C

J - METHOD OF SOLVING THE EQUATIONS

THE ADDITIONAL ASSUMPTIONS ARE MADE THAT

$\dot{P}_A\ (-\phi_P) = \dot{P}_N,\ \dot{C}(-\phi) = \dot{C}_O,\ E\ (-\phi_C) = E_N$.

ASSUMING THE VALUES OF THE PARAMETERS ARE KNOWN, THE CONSTANTS CAN BE CALCULATED. ONCE THE CONSTANTS ARE DETERMINED, AND W(T) IS GIVEN, THE FOLLOWING ORDER OF COMPUTATION IS FOLLOWED, MAKING USE OF THE NECESSARY VALUES FROM PRIOR COMPUTATIONS,

$$\left.\begin{array}{l} \dot{P}_D \ (o) \\ \dot{L}_R \ (o) \\ \dot{L} \ (o) \\ \dot{P}_A \ (o) \\ \dot{H} \ (o) \\ \dot{C} \ (o) \\ E \ (o) \end{array}\right\} = F \ (o)$$

THEN F(1), F(2) --------- F(N).

THIS IS A STRAIGHT REPETITIVE PROCESS. WHILE IT COULD BE A DIFFICULT CHORE TO OBTAIN ANY CLOSED FORM SOLUTIONS TO THESE EQUATIONS AS FUNCTIONS OF TIME, THE NUMERICAL SOLUTION ON A HIGH SPEED COMPUTER IS RATHER A SIMPLE MATTER.

PART 3 - MODEL 2A

1 GOOD

1 FAMILY

2 PRODUCTIVE FACTORS

MONEY

A - GENERAL DISCUSSION AND FLOW CHARTS

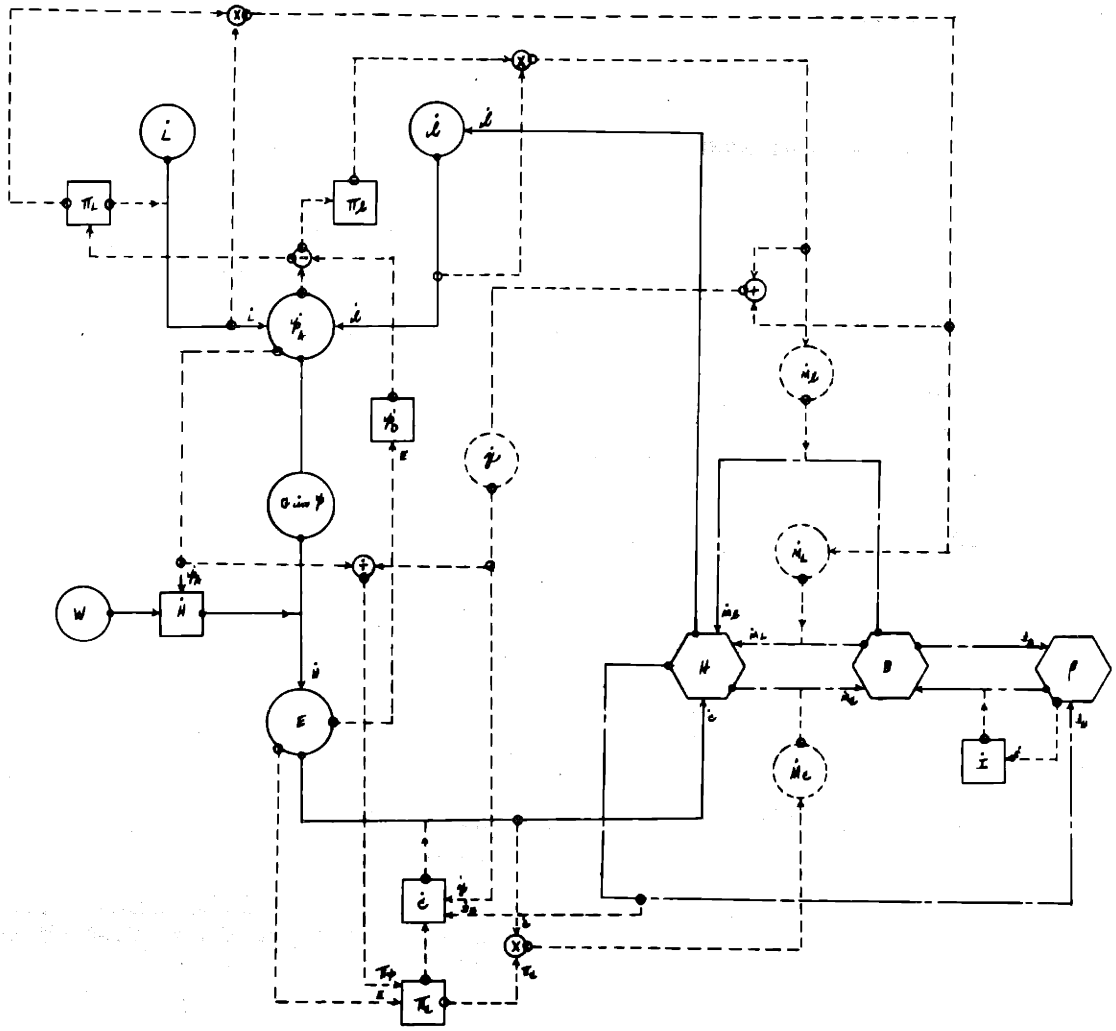
MODEL 2A IS CONSTRUCTED FOR ONE FAMILY, ONE GOOD, TWO PRODUCTIVE FACTORS, AND MONEY. THE MODEL IS IDENTICAL TO MODEL 2 WITH THE ADDITION OF MONEY. IT IS ASSUMED THAT THE HEAD OF THE FAMILY SERVES IN THE DUAL CAPACITY AS HEAD OF THE BUSINESS, AND HEAD OF THE FAMILY. THIS SEPARATION OF THE FUNCTIONS OF A HOUSEHOLD H, AND A BUSINESS B ALLOWS THE MONEY FLOWS TO BE SEEN, EVEN THOUGH THE FUNCTIONING MAY INVOLVE SIMPLY TAKING MONEY OUT OF ONE POCKET AND PUTTING IT IN ANOTHER.

THE MAIN PURPOSE OF THIS MODEL IS TO DEVELOP THE MECHANISM FOR DETERMINING PRICE BEFORE DEVELOPING A MORE COMPLEX MODEL.

THE FLOW DIAGRAM IS GIVEN IN FIGURE 12. A ZERO SUBSCRIPT ON A VARIABLE INDICATES THE VALUE TAKEN BY THE VARIABLE AT T = TO.

THERE ARE FIVE INDEPENDENT DECISIONS.

1) THE DECISIONS AS TO HOW MUCH PRODUCTIVE EFFORT SHOULD BE EXPENDED AT TIME T IS $\dot{P}_D(T)$,
2) THE RATE OF HARVEST AT TIME T, A DECISION OF NATURE, = $\dot{H}(T)$,
3) THE RATE OF CONSUMPTION = $\dot{C}(T)$,
4) THE RATE OF SAVINGS BY THE HOUSEHOLD = $\dot{S}_H(T)$,
5) THE RATE OF INVESTMENT IN THE BUSINESS = $\dot{I}(T)$.



FLOW DIAGRAM - MODEL 2A

1 GOOD

1 FAMILY

2 PRODUCTIVE FACTORS

MONEY

[VARIABLES ARE ALL AVERAGE PER PERSON]

FIGURE 12.

WHERE THE FUNCTIONAL RELATIONSHIPS BETWEEN THE VARIABLES AFFECTING ANY DECISION ARE UNKNOWN, THE FOLLOWING ASSUMPTION WILL BE MADE: FOR A DECISION DEPENDENT UPON N VARIABLES THE DECISION FUNCTION IS ASSUMED TO BE THE PRODUCTS OF N FUNCTIONS, EACH OF WHICH IS A FUNCTION OF ONLY ONE OF THE N VARIABLES. THE NATURE OF THE FUNCTIONAL RELATIONSHIPS WILL THEN BE DETERMINED BY ASSUMING ALL BUT ONE OF THE INDEPENDENT VARIABLES AS CONSTANT, AND THEN VARYING THE ONE. FUNCTIONS WILL BE CHOSEN SUCH THAT THEIR BEHAVIOR IS PROPER AT THE LIMITING VALUES, AND SUFFICIENT ARBITRARY CONSTANTS WILL BE LEFT IN THE FUNCTION TO DESCRIBE POSSIBLE INTERMEDIATE VALUES.

A LISTING OF SOME POSSIBLE FUNCTIONS ARE GIVEN IN FIGURE 6. BY THE ACTUAL SELECTION OF THE FUNCTIONS FROM THIS TABLE, THE NATURE OF THE ASSUMED LIMITING VALUES IS OBVIOUS. A FURTHER DISCUSSION WILL THEN BE REQUIRED TO DETERMINE THE VALUES OF THE CONSTANTS.

B - PRODUCTIVE EFFORT

THE FIRST DECISION FOR AN ENTREPRENEUR IN THE SEQUENCE OF DECISIONS IS, "HOW MUCH PRODUCTIVE EFFORT SHOULD BE PUT INTO BUILDING MY INVENTORY?" THIS DECISION IS BASED ON INFORMATION ABOUT THE LEVEL OF INVENTORY ONLY. BY AN ARGUMENT SIMILAR TO THAT GIVEN IN DETAIL[1] IN MODEL 2, THE RELATIONSHIP CAN BE WRITTEN AS,

$$\dot{P}_D(T) = \alpha_0 E^{-\alpha_1 E(T-\phi_C)}, \text{ WHERE } \alpha_0 = \dot{P}_{MAX},$$

$\alpha_1 = \ln 2/E\ 1/2$, AND $\dot{P}_D(T)$ IS THE RATE OF PRODUCTIVE EFFORT DECIDED UPON, $E\ 1/2$ IS THE VALUE OF E WHEN $\dot{P}_D = \frac{\dot{P}_{MAX}}{2}$.

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[1] SEE EQUATION 7), MODEL 1.

C - RATE OF HARVEST

THE HARVEST DECISION FOLLOWS AS, [2]

---

[2] SEE EQUATION 5), MODEL 2.

$\dot{H}(T) = [1 - E^{-\alpha_2 W(T)}] \cdot \dot{P}_A (T - \phi_E)$, WHERE ϕ_E IS CONSTANT, AND $\alpha_2 = \frac{\ln 2}{W_{1/2}}$.

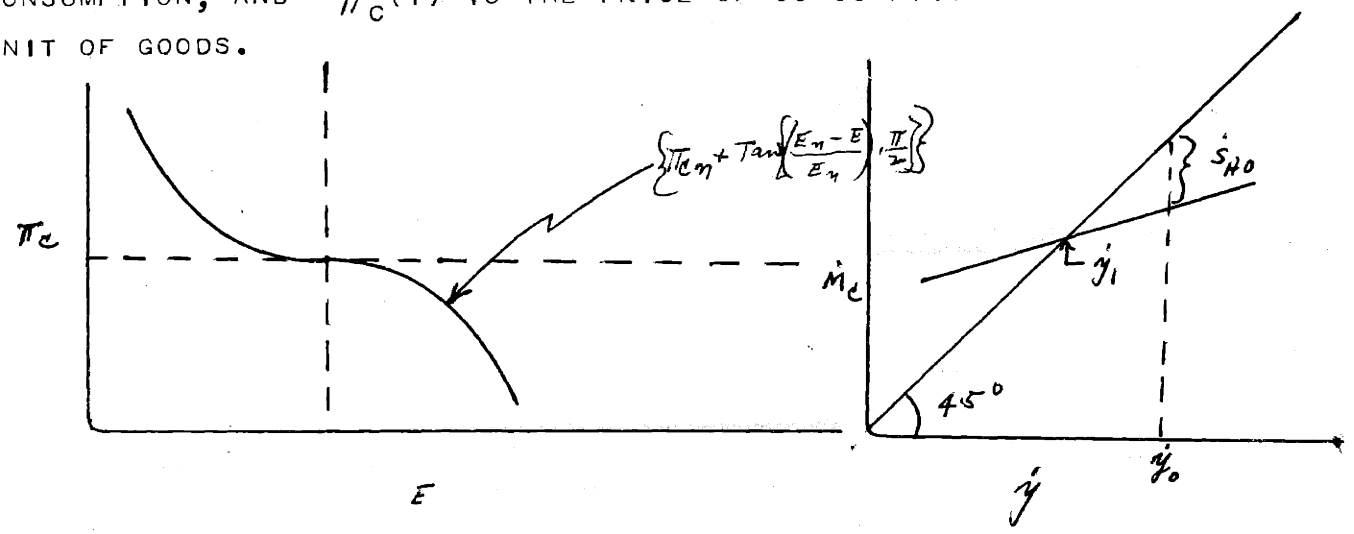
D - RATE OF CONSUMPTION

THE DECISION ON RATE OF CONSUMPTION DEPENDS ON PRICE, AND INCOME. THE LOWER THE PRICE OF CONSUMPTION THE GREATER THE RATE OF CONSUMPTION, AND VICE VERSA. THE HIGHER THE INCOME, THE HIGHER THE RATE OF CONSUMPTION. IT WOULD BE REASONABLE TO ASSUME THEN THAT

$$\dot{C}(T) = \frac{\alpha_{11} \dot{Y} + \alpha_{12}}{\pi_C}, \quad S_H > 0$$

$$\dot{C}(T) = \dot{Y}/\pi_C, \quad S_H \leq 0$$

WHERE $\dot{Y}(T) = \dot{M}_L(T) + \dot{M}_{LR}(T)$, AND WHERE $\dot{C}(T)$ IS THE RATE OF CONSUMPTION, AND $\pi_C(T)$ IS THE PRICE OF CONSUMPTION PER UNIT OF GOODS.



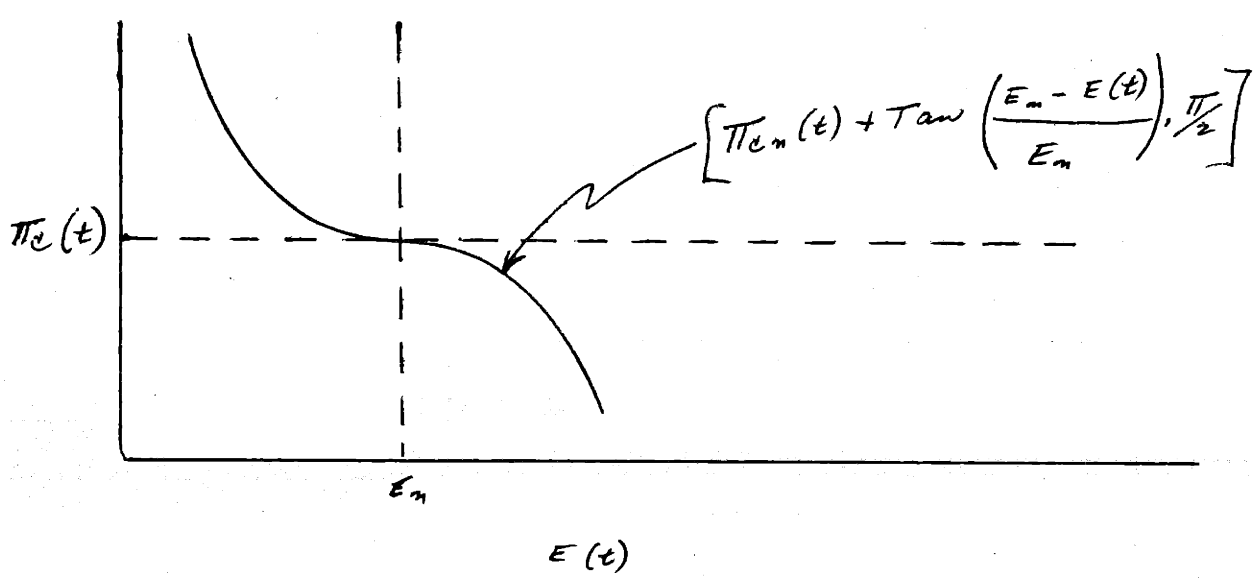
AT $\dot{Y} = \dot{Y}_1$, $\dot{M}_C = \dot{Y}_1$, AND AT $\dot{Y} = \dot{Y}_0$, $\dot{M}_C = \dot{Y}_0 - \dot{S}_{HO}$.

THIS GIVES $\alpha_{11} = 1 - \dfrac{\dot{S}_{HO}}{(\dot{Y}_0 - \dot{Y}_1)}$,

AND $\alpha_{12} = \dfrac{\dot{Y}_1 \, \dot{S}_{HO}}{(\dot{Y}_0 - \dot{Y}_1)}$

E - <u>PRICE OF CONSUMPTION</u>

THE PRICE AT WHICH AN ITEM WILL TRADE IS THAT PRICE AT WHICH ITS SUPPLY AND DEMAND ARE JUST EQUAL. SINCE ALL SUPPLY GOES INTO INVENTORY, AND ALL PURCHASES GO OUT OF INVENTORY, THE LEVEL OF AVAILABLE INVENTORY IS THE DIRECT MEASURE OF SUPPLY AND DEMAND AT THE EXISTING PRICE. IF, FROM WHAT IS CONSIDERED A NORMAL INVENTORY, THE LEVEL OF INVENTORY BEGINS TO RISE, THEN SUPPLY IS IN EXCESS OF DEMAND, AND IF THE LEVEL OF INVENTORY BEGINS TO FALL, THEN DEMAND IS IN EXCESS OF SUPPLY. A REASONABLE FUNCTION FOR PRICE VS. INVENTORY WOULD THEN BE (see figure) WHERE $\pi_C(T)$ IS THE PRICE PER UNIT OF CONSUMPTION



AT TIME T, AND $\pi_{CN}(T)$ IS THE NORMAL PRICE OF CONSUMPTION, BASED ON COST PLUS PROFIT, E_N IS THE NORMAL INVENTORY.

PRICE CHANGES TEND TO LAG CHANGES IN INVENTORY SINCE ENTREPRENEURS ARE RELUCTANT TO CHANGE PRICES IMMEDIATELY. THUS,

$$\pi_C(T) = [\pi_{CN}(T) + T_{AN}\left(\frac{E_N - E(T-\phi_C)}{E_N} \cdot /2\right)]$$

WHERE $\pi_{CN}(T) = \frac{Y(T)}{\dot{P}_A(T)}(1+\gamma)$, WHERE γ IS THE NORMAL PROFIT MARK-UP.

F - <u>LAND AND LABOR</u>

THE RELATIONSHIP BETWEEN LAND AND LABOR FOR A GIVEN PRODUCTIVE EFFORT DECIDED UPON $\dot{P}_D(T)$ WAS DEVELOPED IN EQUATION 8) OF MODEL 2. THE AMOUNT OF LAND AND LABOR AVAILABLE IS A FUNCTION OF THE PRICE OF LAND π_L AND PRICE OF LABOR π_{LR}. THE LAND AND LABOR ARE ASSUMED TO BE FURNISHED BY THE INDIVIDUAL. THE INDIVIDUAL IS CONSIDERED TO BE AN ENTREPRENEUR IN THAT HE HIRES LAND AND LABOR AT THE EXISTING PRICE. IT IS ASSUMED THAT PAYMENT IS MADE TO HIMSELF, AT THE EXISTING PRICES, FOR HIS OWN CONTRIBUTIONS TO PRODUCTION, AS WELL AS TO OTHER MEMBERS OF THE FAMILY. IT IS ALSO ASSUMED THAT THE SUPPLY OF LAND AND LABOR DEPENDS ON THE PRICES OFFERED FOR THEM AT A PRIOR PERIOD OF TIME. THE ACTUAL RATE OF PRODUCTION $\dot{P}_A(T)$ IS A FUNCTION OF LAND AND LABOR USAGE. THE ARGUMENT FOLLOWS FROM MODEL 2 AND IS GIVEN BY

$$\dot{P}_A(T) = \alpha_4 \dot{L}R(T) \cdot \dot{L}(T), \text{ WHERE}$$
$$\alpha_4 = \dot{P}_{AO}/\dot{L}R_O \cdot \dot{L}_O.$$

BUT, THE LAND AND LABOR AVAILABLE ARE DETERMINED BY THE

PRICES OFFERED. SINCE THE AMOUNTS OF LAND AND LABOR DO NOT RESPOND IMMEDIATELY TO CHANGES IN PRICES. THEREFORE,

$$\dot{L}(T) = \alpha_5[1 - E^{-\alpha_6 \pi_L (T-\phi_L)}],$$

$$\dot{L}_R(T) = \alpha_7[1 - E^{-\alpha_8 \pi_{LR}(T-\phi_{LR})}], \quad \alpha_5 = \dot{L}_{MAX},$$

$$\alpha_7 = \dot{L}_{RMAX}, \quad \alpha_6 = LN2/\pi_{L\ 1/2}, \text{ AND } \alpha_8 = \frac{LN\ 2}{\pi_{LR\ 1/2}}.$$

ϕ_L IS THE LAG IN TIME FROM THE CHANGE OF PRICE TO THE CHANGE IN RATE OF LAND CORRESPONDING TO THE CHANGE IN PRICE. SIMILARLY, ϕ_{LR} IS THE LAG FOR LABOR.

G - PRICES OF LAND AND LABOR

THE PRICE AT TIME T FOR EITHER LAND OR LABOR DEPENDS ON THE PRICE SELECTED AT (T-1), PLUS AN AMOUNT TO CORRECT FOR THE ACTUAL RATE OF PRODUCTION NOT CORRESPONDING TO THE RATE DECIDED UPON. THUS,

$$\dot{\pi}_{LR}(T) = \alpha_9[\dot{P}_D(T-1) - \dot{P}_A(T-1)], \quad \alpha_9 = \pi_{LRO}/\dot{P}_{AO},$$

$$\dot{\pi}_L(T) = \alpha_{10}[\dot{P}_D(T-1) - \dot{P}_A(T-1)], \quad \alpha_{10} = \pi_{LO}/\dot{P}_{AO}.$$

H - CHANGES IN INVENTORY

FROM THE FLOW DIAGRAM IT CAN BE SEEN THAT THE RATE OF CHANGE OF INVENTORY EQUALS THE RATE OF HARVEST, LESS THE RATE OF CONSUMPTION.

$$\dot{E}(T) = \dot{H}(T) - \dot{C}(T).$$

I - MONEY FLOWS

THE MONEY FLOWS IN THE MONEY SECTOR ARE DETERMINED BY PRICE,

AND FLOWS OF GOODS AND SERVICES. THUS,

$$\dot{M}_L(T) = \pi_L(T) \cdot \dot{L}(T).$$

$$\dot{M}_{LR}(T) = \pi_{LR}(T) \cdot \dot{L}_R(T).$$

$$\dot{M}_C(T) = \pi_C(T) \cdot \dot{C}(T).$$

WHERE $\dot{M}_L$ IS THE RATE OF MONEY FLOW FROM THE BUSINESS FOR LAND,

$\dot{M}_{LR}$ IS THE RATE OF MONEY FLOW FROM THE BUSINESS FOR LABOR,

AND $\dot{M}_C$ IS THE RATE OF MONEY FLOW TO THE BUSINESS FOR CONSUMPTION.

ASSUMING RATE OF INVESTMENT AT TIME T EQUALS SAVINGS AT TIME $(T - \phi_2)$, THEN

$$\dot{S}_H(T) = \dot{M}_L(T) + \dot{M}_{LR}(T) - \dot{M}_C(T),$$

$$\dot{S}_B(T) = \dot{M}_C(T) - \dot{M}_L(T) - \dot{M}_{LR}(T) + \dot{I}(T), \text{ AND}$$

$$\dot{I}(T) = \dot{S}_H(T - \phi_2) + \dot{S}_B(T - \phi_2).$$

WHERE $\dot{S}_H$ IS THE RATE OF SAVINGS OF THE HOUSEHOLD,

$\dot{S}_B$ IS THE RATE OF SAVINGS OF THE BUSINESSES,

$\dot{I}$ IS THE RATE OF INVESTMENT IN THE BUSINESSES,

AND ϕ_2 IS A CONSTANT.

J - <u>SUMMARY OF DEFINITIONS AND EQUATIONS</u>

1) $\beta(T)$ = AMOUNT OF MONEY IN THE BANK AT TIME T.

$$\beta(T) = \dot{S}_B(T) + \dot{S}_H(T) - \dot{I}(T).$$

2) $\dot{C}(T)$ = RATE OF CONSUMPTION AT TIME T.

$$\dot{C}(T) = \alpha_{11}\dot{Y} + \alpha_{12} \quad , \quad S_H > 0.$$

$$\dot{C}(T) = \dot{Y}/\pi_C \quad , \quad S_H \leq 0.$$

3) $\dot{H}(T)$ = RATE OF HARVEST AT TIME T.

$$\dot{H}(T) = [1 - E^{-\alpha_2 W(T)}].$$

4) $E(T)$ = INVENTORY AT TIME T.

$$\dot{E}(T) = \dot{H}(T) - \dot{C}(T).$$

5) $\dot{I}(T)$ = RATE OF INVESTMENT AT TIME T.

$$\dot{I}(T) = \beta(T - \phi_I) \quad \text{FOR } \beta(T - \phi_I) > 0$$

$$\dot{I}(T) = 0 \quad \text{FOR } \beta(T - \phi_I) \leq 0.$$

6) $\dot{L}R(T)$ = RATE OF LABOR GOING INTO PRODUCTIVE EFFORT AT TIME T.

$$\dot{L}R(T) = \alpha_7 [1 - E^{-\alpha_8 \pi_{LR}(T - \phi_{LR})}]$$

7) $\dot{L}(T)$ = RATE OF LAND GOING INTO PRODUCTIVE EFFORT AT TIME T.

$$\dot{L}(T) = \alpha_5 [1 - E^{-\alpha_6 \pi_L(T - \phi_L)}]$$

8) $\dot{M}_C(T)$ = RATE OF MONEY GOING TO THE BUSINESS FOR CONSUMPTION BY THE HOUSEHOLD AT TIME T.

$$\dot{M}_C(T) = \pi_C(T) \cdot \dot{C}(T).$$

9) $\dot{M}_{LR}(T)$ = RATE OF MONEY GOING FOR LABOR AT TIME T.

$$\dot{M}_{LR}(T) = \pi_{LR}(T) \cdot \dot{L}R(T)$$

10) $\dot{M}_L(T)$ = RATE OF MONEY GOING FOR LAND AT TIME T.

$$\dot{M}_L(T) = \pi_L(T) \cdot \dot{L}(T)$$

11) $\dot{P}_A(T)$ = THE ACTUAL RATE OF PRODUCTIVE EFFORT AT TIME T.

$$\dot{P}_A(T) = \alpha_4 \dot{L}R(T) \cdot \dot{L}(T).$$

12) $\dot{P}_D(T)$ = THE RATE OF PRODUCTIVE EFFORT DECIDED TO BE EXPENDED AT TIME T.

$$P_D(T) = \alpha_0 E^{-\alpha_1 E(T-\phi_C)}.$$

13) $\pi_C(T)$ = THE PRICE PER UNIT OF CONSUMPTION GOODS AT TIME T.

$$\pi_C(T) = \frac{\dot{Y}(T)}{\dot{P}_A(T)}(1+\gamma) + \mathrm{TAN}\left[\frac{E_N - E(T-\phi_C)}{E_N} \cdot \pi/2\right].$$

14) $\pi_{LR}(T)$ = THE PRICE FOR A UNIT OF LABOR AT TIME T.

$$\pi_{LR}(T) = \alpha_9[\dot{P}_D(T-1) - \dot{P}_A(T-1)].$$

15) $\pi_L(T)$ = THE PRICE OF A UNIT OF LAND AT TIME T.

$$\pi_L(T) = \alpha_{10}[\dot{P}_D(T-1) - \dot{P}_A(T-1)].$$

16) $\dot{S}_B(T)$ = THE RATE OF SAVINGS OF MONEY BY THE BUSINESS.

$$\dot{S}_B(T) = [\dot{M}_C(T) + \dot{I}(T)] - [\dot{M}_L(T) + \dot{M}_{LR}(T)].$$

17) $\dot{S}_H(T)$ = THE RATE OF SAVINGS OF MONEY BY THE BUSINESS.

$$\dot{S}_H(T) = \dot{Y} - \dot{M}_C.$$

18) $\dot{Y}(T)$ = RATE OF MONEY GOING FOR LAND AND LABOR AT TIME T.

$$\dot{Y}(T) = \dot{M}_L(T) + \dot{M}_{LR}(T).$$

THE ONLY OUTSIDE VARIABLE ACTING ON THIS SYSTEM IS THE WEATHER W(T). IT IS ASSUMED THAT W(T) IS GIVEN. IN ADDITION, THERE IS THE CONSTANT γ_E APPEARING IN EQUATION 13). THIS IS THE NORMAL MARKUP IN PRICE OVER COST TO ALLOW FOR A PROFIT.

THERE ARE FIVE PHASE LAGS ALLOWED FOR IN THE SYSTEM. THESE ARE DESIGNATED BY ϕ_E, ϕ_L, ϕ_{LR}, ϕ_C, AND ϕ_I. THE CONSTANTS, TO BE DETERMINED BY THE BOUNDARY CONDITIONS ARE α_1 THRU α_{11} INCLUSIVE.

IF THE CONSTANTS, α_i AND ϕ_i ARE SPECIFIED FOR A GIVEN SYSTEM, AND γ, AND W(T) ARE SPECIFIED, ALL OF THE OTHER 18 VARIABLES OF THE SYSTEM CAN BE EXPRESSED AS FUNCTIONS OF TIME, AND THIS COMPLETELY DEFINES THE ECONOMIC SYSTEM GIVEN IN THE MODEL.

K - <u>METHOD OF SOLVING THE EQUATIONS</u>

THE ADDITIONAL ASSUMPTION IS MADE THAT ALL VARIABLES HAVE NORMAL VALUES FOR T<0. ASSUMING THE VALUES OF THE PARAMETERS ARE KNOWN, AND THE CONSTANTS HAVE BEEN CALCULATED. THEN, FOR A GIVEN W(T), THE FOLLOWING ORDER OF COMPUTATION IS FOLLOWED, MAKING USE OF THE RESULTS FROM PRIOR COMPUTATIONS WHERE CALLED FOR AT T = 0:

$$\left\{ \dot{P}_D, \pi_{LR}, \pi_L, \dot{L}_R, \dot{L}, \dot{P}_A, \dot{H}, \dot{M}_{LR}, \dot{M}_L, \dot{Y}, \dot{S}_H, \pi_C, \dot{C}, E, \dot{M}_C, \dot{S}_B, \dot{I}, \beta' \right\} = F(0).$$

THEN COMPUTE,

F(1), F(2) -------- F(N). THIS GIVES THE VALUES OF EACH OF THE VARIABLES AS A FUNCTION OF TIME.

L - SUMMARY OF CONSTANTS

FROM FUNCTION	CONSTANTS	FROM FUNCTION	PHASE LAGS
$\dot{P}_D$	$\alpha_0 = \dot{P}_{D\ MAX}$	$\dot{H}$	ϕ_E
$\dot{P}_D$	$\alpha_1 = LN2/E\ 1/2$	π_C, $\dot{P}_D$	ϕ_C
$\dot{H}$	$\alpha_2 = LN2/W\ 1/2$	$\dot{L}$	ϕ_L
$\dot{C}$	$\alpha_3 = \pi_{CO}/(\dot{Y}_O - \dot{S}_{HO})$	$\dot{L}_R$	ϕ_{LR}
$\dot{P}_A$	$\alpha_4 = P_O/LR_O L_O$	$\dot{i}$	ϕ_I
$\dot{L}$	$\alpha_5 = \dot{L}_{MAX}$		
$\dot{L}$	$\alpha_6 = LN2/\pi_L\ 1/2$		
$\dot{L}_R$	$\alpha_7 = \dot{L}_{R_{MAX}}$		
$\dot{L}_R$	$\alpha_8 = LN2/\pi_{LR}\ 1/2$		
π_{LR}	$\alpha_9 = \pi_{LRO}/\dot{P}_{AO}$		
π_L	$\alpha_{10} = \pi_{LO}/\dot{P}_{AO}$		
$\dot{S}_H$	$\alpha_{11} = 1 - \dfrac{\dot{S}_{HO}}{(\dot{Y}_O - \dot{Y}_1)}$		
$\dot{S}_H$	$\alpha_{12} = \dfrac{\dot{Y}_1\ \dot{S}_{HO}}{(\dot{Y}_O - \dot{Y}_1)}$		

PART 4 - MODEL 12 C

2 GOODS

1 FAMILY

3 PRODUCTIVE FACTORS

MONEY

BUSINESS CREDIT

CONSUMER CREDIT

A - GENERAL

THE FLOW DIAGRAM FOR MODEL 12 C IS GIVEN IN FIGURE 13. THE G SECTOR IS ONE CONSUMER GOOD, AND THE E SECTOR IS A SECOND CONSUMER GOOD. THE K SECTOR REPRESENTS A CAPITAL GOOD. THE MODEL INCLUDES THREE FACTORS OF PRODUCTION, LAND, LABOR, AND K.

THE DISTINGUISHING DIFFERENCE BETWEEN LAND AND LABOR AS FACTORS, COMPARED TO EQUIPMENT K, IS THAT K IS ASSUMED TO BE LIMITED IN SUPPLY, AND RATIONING IS DONE ON THE BASIS OF RELATIVE DEMAND BETWEEN SECTORS, WHILE WITH LAND AND LABOR, THE RATIONING IS DONE BY PRICE.

THE GOOD G IS CONSIDERED TO BE A MANUFACTURED GOOD, WHILE E IS CONSIDERED TO BE AN AGRICULTURAL GOOD.

B - PRODUCTION AT MINIMUM COST

THE ACTUAL RATE OF PRODUCTION OF A GOOD WILL BE ASSUMED TO BE DONE BY COMBINING LAND, LABOR AND EQUIPMENT, SUCH THAT THE PRICE PER UNIT OF PRODUCTION WILL BE A MINIMUM. THIS MEANS THAT IF $\dot{M}_P$ = COST OF PRODUCING A UNIT OF GOODS, THAT $[\frac{\dot{P}_A}{\dot{M}_P}] = MAX$, OR D $[\dot{P}_A/\dot{M}_P] = 0$. THIS IS TRUE IF

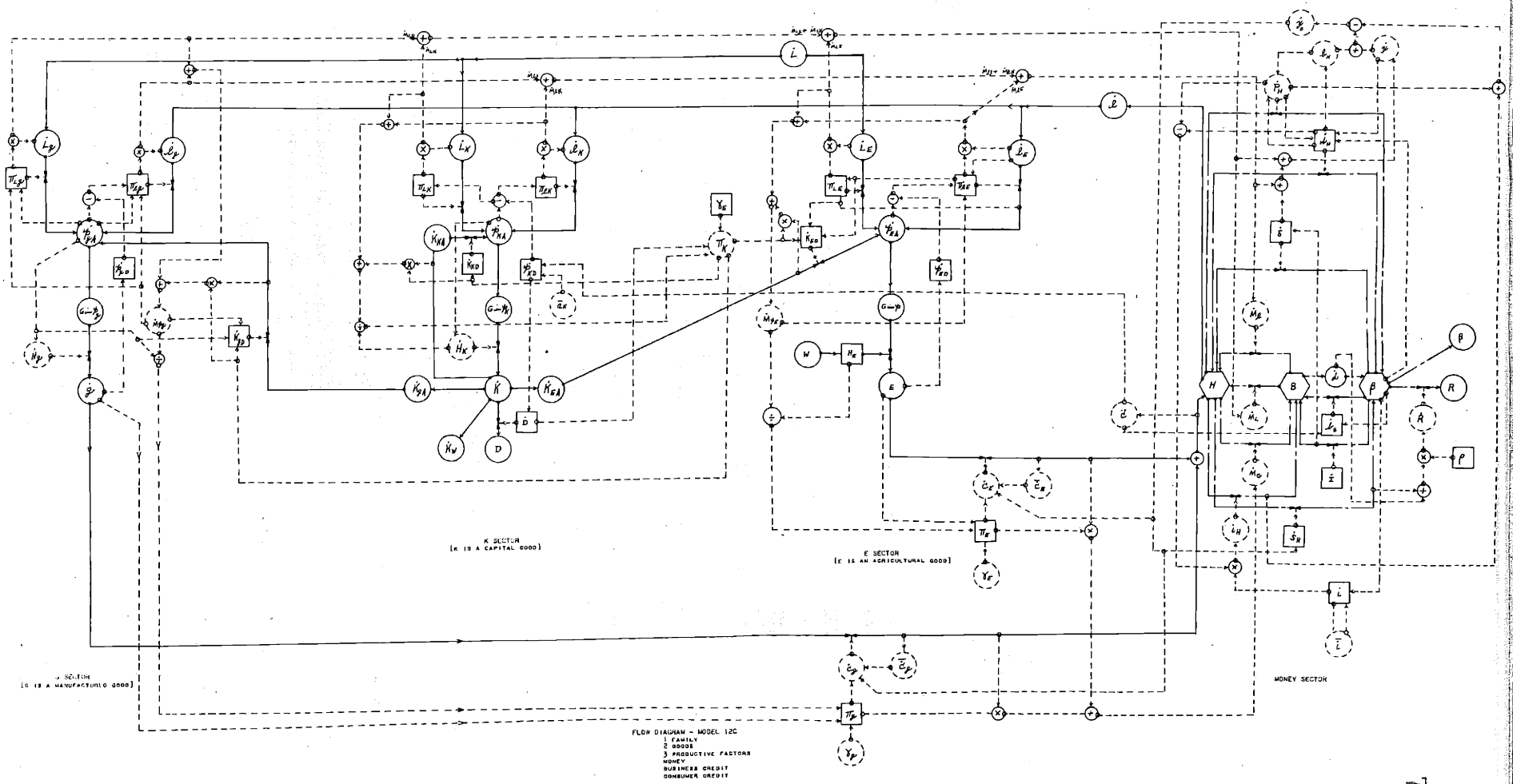

K SECTOR
[K IS A CAPITAL GOOD]
E SECTOR
[E IS AN AGRICULTURAL GOOD]
MONEY SECTOR
FLOW DIAGRAM - MODEL 12C
1 FAMILY
2 GOODS
3 PRODUCTIVE FACTORS
MONEY
BUSINESS CREDIT
CONSUMER CREDIT

FIGURE 13.

$$\frac{d\dot{P}_A}{\dot{P}_A} = \frac{d\dot{M}_P}{\dot{M}_P} \quad . \qquad \text{BUT SINCE}$$

$$d\dot{M}_P = \frac{\partial \dot{M}_P}{\partial \dot{L}} d\dot{L} + \frac{\partial \dot{M}_P}{\partial \dot{L}_R} d\dot{L}_R + \frac{\partial \dot{M}_P}{\partial \dot{K}} d\dot{K},$$

$$\text{AND } d\dot{P}_A = \frac{\partial \dot{P}_A}{\partial \dot{L}} d\dot{L} + \frac{\partial \dot{P}_A}{\partial \dot{L}_R} d\dot{L}_R + \frac{\partial \dot{P}_A}{\partial \dot{K}} d\dot{K},$$

$$\text{THEN, } \left[\frac{\partial \dot{P}_A}{\dot{P}_A \partial \dot{L}} - \frac{\partial \dot{M}_P}{\dot{M}_P \partial \dot{L}} \right] d\dot{L} + \left[\frac{\partial \dot{P}_A}{\dot{P}_A \partial \dot{L}_R} - \frac{\partial \dot{M}_P}{\dot{M}_P \partial \dot{L}_R} \right] d\dot{L}_R$$

$$+ \left[\frac{\partial \dot{P}_A}{\dot{P}_A \partial \dot{K}} - \frac{\partial \dot{M}_P}{\dot{M}_P \partial \dot{K}} \right] d\dot{K} = 0.$$

THIS IS TRUE PROVIDED THE COEFFICIENTS ARE EQUAL TO ZERO, OR

$$1) \quad \frac{\dfrac{\partial \dot{P}_A}{\partial \dot{L}}}{\dfrac{\partial \dot{M}_P}{\partial \dot{L}}} = \frac{\dfrac{\partial \dot{P}_A}{\partial \dot{L}_R}}{\dfrac{\partial \dot{M}_P}{\partial \dot{L}_R}} = \frac{\dfrac{\partial \dot{P}_A}{\partial \dot{K}}}{\dfrac{\partial \dot{M}_P}{\partial \dot{K}}} = \frac{\dot{P}_A}{\dot{M}_P}.$$

$$\text{BUT} \quad \dot{M}_P = \pi_L \cdot \dot{L} + \pi_{LR} \cdot \dot{L}_R + \pi_K \cdot \dot{K},$$

$$\text{AND} \quad \dot{P}_A = F_1(\dot{L}) \cdot F_2(\dot{L}_R) \cdot F_3(\dot{K}).$$

$$\text{THUS} \quad \frac{\partial \dot{P}_A}{\partial \dot{L}} = \frac{\dot{P}_A}{F_1(\dot{L})} \cdot \frac{\partial F_1(\dot{L})}{\partial \dot{L}}, \quad \frac{\partial \dot{P}_A}{\partial \dot{L}_R} = \frac{\dot{P}_A}{F_2(\dot{L}_R)} \cdot \frac{\partial F_2(\dot{L}_R)}{\partial \dot{L}_R},$$

$$\frac{\partial \dot{P}_A}{\partial \dot{K}} = \frac{\dot{P}_A}{F_3(\dot{K})} \cdot \frac{\partial F_3(\dot{K})}{\partial \dot{K}},$$

AND $\dfrac{\partial \dot{M}_P}{\partial \dot{L}} = \pi_L, \quad \dfrac{\partial \dot{M}_P}{\partial \dot{L}_R} = \pi_{LR}, \quad \dfrac{\partial \dot{M}_P}{\partial \dot{K}} = \pi_K.$

SUBSTITUTING IN 1) GIVES

$$2) \quad \frac{d[\ln F_1(\dot{L})]}{\pi_L \, d\dot{L}} = \frac{d[\ln F_2(\dot{L}_R)]}{\pi_{LR} \, d\dot{L}_R} = \frac{d[\ln F_3(\dot{K})]}{\pi_K \, d\dot{K}} = \frac{d[\ln \dot{M}_P]}{d\dot{M}_P}.$$

C - <u>FLOW CHART</u>

THE VARIABLES AND DECISIONS WHERE DERIVATIONS, EXPLANATIONS OR ASSUMPTIONS ARE GIVEN ARE CONTAINED IN THE FOLLOWING SECTIONS IN ALPHABETICAL ORDER. THOSE VARIABLES WHICH ARE DEFINITIONS, OR CAN BE OBTAINED DIRECTLY FROM THE FLOW CHART ARE CONTAINED IN THE SUMMARY OF EQUATIONS AT THE END.

D - <u>RATE OF BORROWING BY THE BUSINESS</u> $\dot{B}_B(T) = \dot{B}_B(\beta, \ddot{C}, \dot{i})$.

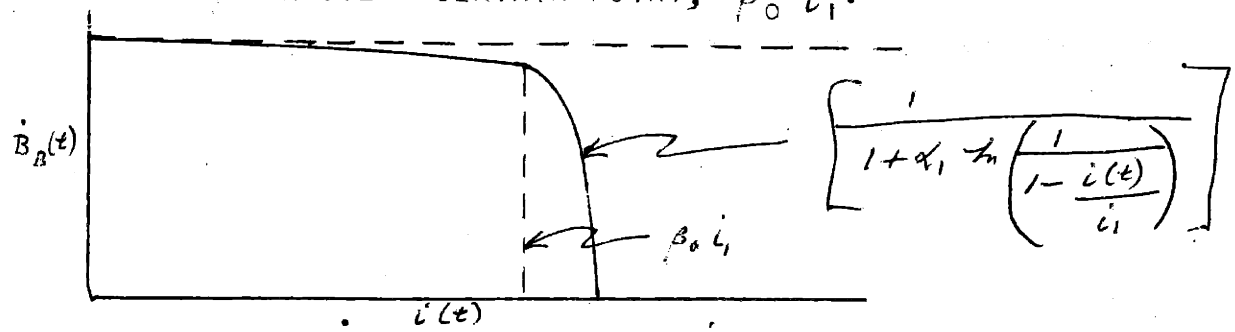
THE RATE OF BORROWINGS BY THE BUSINESS WILL BE ASSUMED TO BE DIRECTLY PROPORTIONAL TO THE AMOUNT OF MONEY HELD BY THE BANKS, PROVIDED THAT THE BANK CAN NEVER HAVE A NEGATIVE BALANCE. THE BORROWINGS ARE ALSO ASSUMED TO BE DIRECTLY PROPORTIONAL TO THE RATE OF CHANGE IN CONSUMPTION AT SOME TIME $(T - \phi_0)$ WHERE INFORMATION IS AVAILABLE.

THUS $\dot{B}_B(T) = \alpha_0 \beta(T) \ddot{C} \; (T - \phi_{B_B}), \quad \beta(T) \geq 0.$

AT TIME $T = 0$, ASSUME $\ddot{C}_0 = \ddot{C}(-\phi_{B_B})$ THEN

$\alpha_0 = \dfrac{\dot{B}_{B0}}{\beta_0 \ddot{C}_0}$, WHERE $\dot{B}_{B0}$ IS THE RATE OF BORROWINGS BY THE BUSINESS AT $T = 0$, β_0 IS THE AMOUNT OF MONEY IN THE BANK AT $T = 0$, AND $\ddot{C}_0$ IS THE RATE OF CHANGE OF THE RATE OF

CONSUMPTION AT T = 0. THE RATE OF INTEREST WILL HAVE AN EFFECT ON THE RATE OF BORROWINGS, BUT MAINLY WHEN THE INTEREST RATE GETS ABOVE A CERTAIN POINT, $\beta_0 \, i_1$.



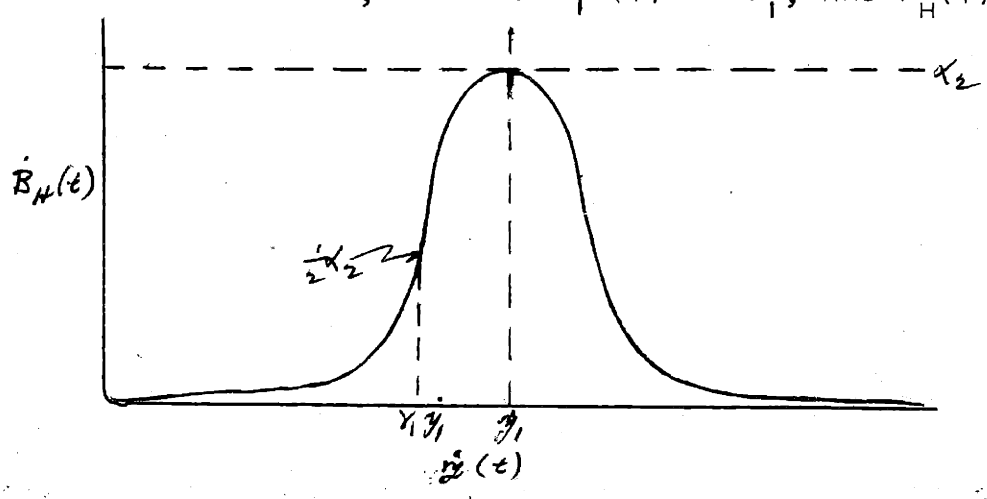
FINALLY THEN, IF $\dot{B}_B$ IS DEPENDENT ON i,

$$B_B(T) = \alpha_0 \, \beta(T) \, \ddot{C} \, (T - \phi L R_B) \left[\frac{1}{1 + \alpha_1 \, LN\left[\frac{1}{1-\left(\frac{i(T)}{i_1}\right)}\right]} \right]$$

WHERE $\alpha_1 = \left[\frac{\left(\frac{1}{\alpha_0} - 1\right)}{LN\left(\frac{1}{1-\beta_0}\right)} \right]$.

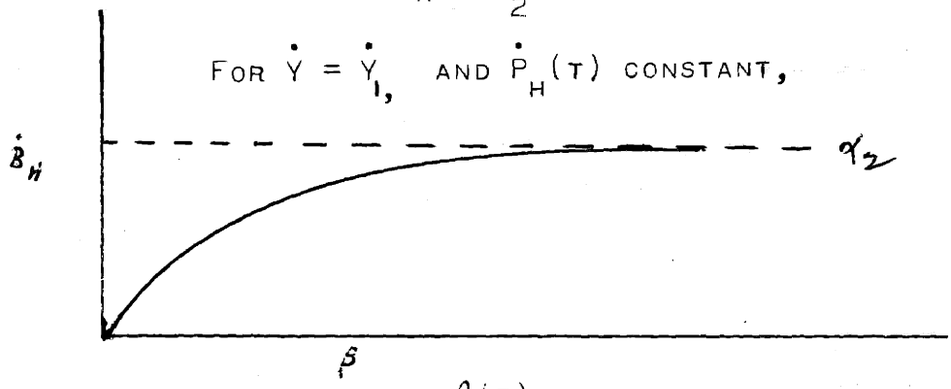
E - <u>RATE OF BORROWINGS BY THE HOUSEHOLD</u> $\dot{B}_H(T) = \dot{B}_H(\dot{Y}, \beta, \dot{P}_H)$.

IF THE MAXIMUM RATE OF BORROWINGS BY THE HOUSEHOLD α_2 WERE TO OCCUR AT AN INCOME $\dot{Y}_1$, PROVIDED THE BANK HAD THE MONEY TO LOAN, AND IF FOR $\dot{Y} > \dot{Y}_1$, THE HOUSEHOLD WOULD NOT WANT TO BORROW AS MUCH AS AT $\dot{Y} = \dot{Y}_1$, AND FOR $\dot{Y} < \dot{Y}_1$, THE ABILITY TO BORROW WOULD BE LESSENED, THEN FOR $\beta(T) >> \dot{Y}_1$, AND $\dot{P}_H(T)$ CONSTANT,



$\dot{B}_H(T) = \alpha_2 E^{-[\dot{Y}(T) - \dot{Y}_1]^2 \alpha_3}$, WHERE α_2 IS THE MAXIMUM RATE OF BORROWING BY THE HOUSEHOLD $\dot{B}_H$MAX, $\alpha_3 = \dfrac{LN2}{\dot{Y}_1^2 (\gamma_1 - 1)^2}$, $\dot{Y}_1$ IS THE VALUE OF $\dot{Y}$ AT WHICH $\dot{B}_H$ IS A MAXIMUM, AND $\gamma_1 \dot{Y}_1$ IS THE VALUE OF $\dot{Y}$ AT WHICH $\dot{B}_H = \dfrac{\alpha_2}{2}$

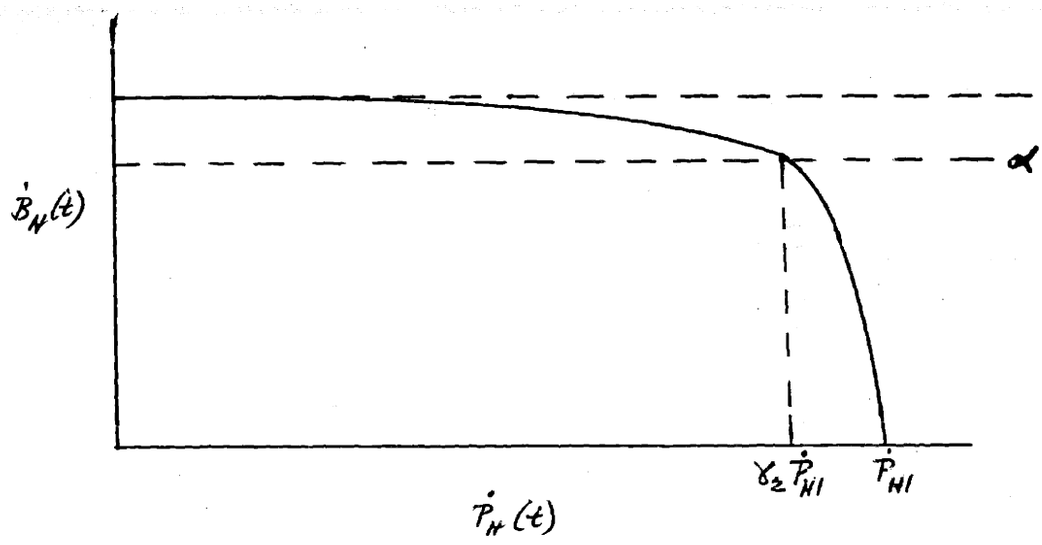
FOR $\dot{Y} = \dot{Y}_1$, AND $\dot{P}_H(T)$ CONSTANT,



$\dot{B}_H(T) = \alpha_2[1 - E^{-\alpha_4 \beta(T)}]$. THE VALUE OF α_4 CAN BE DETERMINED FROM THE CONDITION THAT $\beta(T) = 0$, $\dfrac{d\dot{B}_H(T)}{d\beta(T)} = 1$.

THIS GIVES $\alpha_4 = \dfrac{1}{\alpha_2}$.

FOR $\dot{Y} = \dot{Y}_1$, AND $\alpha_3 \beta(T) >> 1$, THE RATE OF BORROWING IS UNAFFECTED BY THE RATE OF PAYBACK UNTIL THE RATE OF PAYBACK REACHES A CRITICAL POINT $\gamma_2 \dot{P}_{H1}$, WHERE γ_2 IS CONSTANT AND $\dot{P}_{H1}$ IS THE VALUE OF $\dot{P}_H(T)$ AT WHICH $\dot{B}_H(T) = 0$. THIS GIVES



$$\dot{B}_H(T) = \alpha_2 \left[\frac{1}{1 + \alpha_5 \mathrm{LN} \dfrac{1}{1 - \dfrac{\dot{P}_H(T)}{\dot{P}_{HI}}}} \right]$$ WHERE

$$\alpha_5 = \left[\frac{\frac{1}{\alpha} - 1}{\mathrm{LN}\left(\frac{1}{1-\gamma_2}\right)} \right].$$

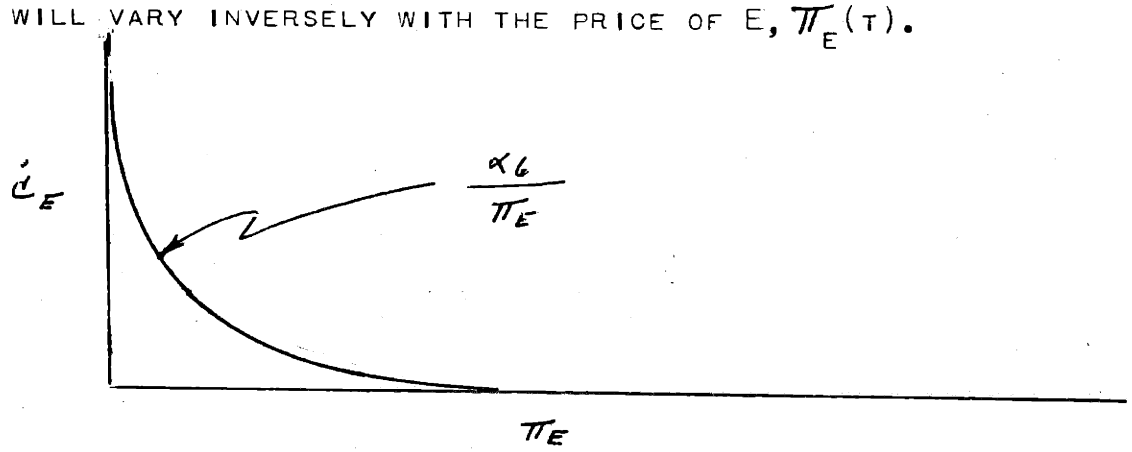
COMBINING THESE EFFECTS OF $\dot{Y}$, β AND $\dot{P}_H$ GIVES

$$\dot{B}_H(T) = \alpha_2 E^{-\alpha_3(\dot{Y}-\dot{Y}_1)^2} \cdot [1-E^{-\alpha_4 \beta(T)}] \left[\frac{1}{1+\alpha_5 \mathrm{LN} \left(\dfrac{1}{1-\dfrac{\dot{P}_H(T)}{\dot{P}_{HI}}} \right)} \right].$$

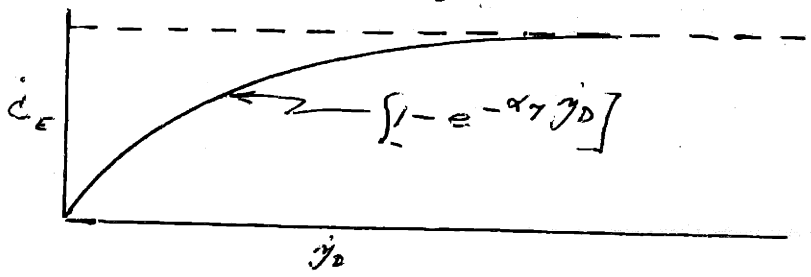
F - <u>RATE OF CONSUMPTION OF E</u>, $\dot{C}_E(T) = \dot{C}_E[\pi_E, \dot{Y}_D, \frac{1}{N}\int_0^N \dot{C}_E(T-s)\,ds]$.

THE RATE OF CONSUMPTION OF PRODUCT E DEPENDS ON THE CURRENT INCOME AVAILABLE FOR CONSUMPTION, THE PRICE OF E, AND THE HABITUAL RATE OF CONSUMPTION $\bar{C}_E(T) = \frac{1}{N}\int_1^N \dot{C}_E(T-s)\,ds$.

FOR A GIVEN AMOUNT OF CURRENT INCOME AVAILABLE FOR CONSUMPTION $\dot{Y}_D(T)$, AND GIVEN AN AMOUNT OF SAVINGS, THE RATE OF CONSUMPTION WILL VARY INVERSELY WITH THE PRICE OF E, $\pi_E(T)$.



FOR A GIVEN π_E, AND A GIVEN AMOUNT OF SAVINGS, THE RATE OF CONSUMPTION OF E INCREASES WITH AN INCREASING AMOUNT OF CURRENT INCOME. THE CHANGE IN RATE OF CONSUMPTION OF E FOR EXTREMELY LARGE VALUES OF $\dot{Y}_D$ TENDS TO GO TO ZERO.



THUS,

$$C_E(T) = \frac{\alpha_6}{\pi_E(T)} [1-E^{-\alpha_7 \dot{Y}_D (T-\phi_C)}] \cdot \int_1^N \frac{\dot{C}_E(T-\xi)\, d\xi}{N}.$$

AT T = 0, THE ASSUMPTION IS MADE THAT THE RATE OF CONSUMPTION OF E IS EQUAL TO THE HISTORICAL RATE OF CONSUMPTION, AND THE DISPOSABLE INCOME $\dot{Y}_D(0) = \dot{Y}_D(\ -\phi_C)$. THE VALUE OF $\dot{Y}_{D\ 1/2}$ IS DEFINED AS VALUE OF $\dot{Y}_D$ AT WHICH ONLY ONE HALF THE MAXIMUM RATE OF CONSUMPTION WILL TAKE PLACE.

THUS, $\alpha_7 = \dfrac{\text{LN}_2}{\dot{Y}_{D\ 1/2}}$, AND $\alpha_6 = \dfrac{\pi_{EO}}{[1-E^{-\dot{Y}_{DO}\alpha_7}]}$.

G - <u>RATE OF CONSUMPTION OF G</u>

THE ARGUMENTS USED FOR $\dot{C}_E(T)$ HOLD EQUALLY WELL FOR $\dot{C}_G(T)$, BUT WITH DIFFERENT PARAMETERS. THUS,

$$\dot{C}_G(T) = \frac{\alpha_8}{\pi_G(T)} [1-E^{-\alpha_7 \dot{Y}_D (T-\phi_C)}] \cdot \int_1^N \frac{\dot{C}_G(T-\xi)\, d\xi}{N},$$

WHERE $\alpha_8 = \dfrac{\pi_{GO}}{[1-E^{-\frac{\dot{Y}_{DO}}{\dot{Y}_{D\ 1/2}} \text{LN } 2}]}$.

H - RATE OF RETIREMENT OF CAPITAL EQUIPMENT K

$$\dot{D}(T) = \dot{D}\,(\dot{P}_{KA}, \dot{P}_{KR}, \dot{P}_{KD})$$

THE RATE OF RETIREMENT OF EQUIPMENT IS EQUAL TO THE HARVEST RATE AS A RESULT OF THE PRODUCTIVE EFFORT PUT IN ON REPLACEMENT EQUIPMENT $\dot{P}_{KR}(T)$. SINCE $\dot{H}_K(T) = \dot{P}_{KA}(T-\phi)$, THEN

$$\dot{D}\,(T) = \frac{\dot{P}_{KA}(T-\phi)\ .\ \dot{P}_{KR}(T-\phi)}{\dot{P}_{KD}(T-\phi)}.$$

I - RATE OF DIVIDENDS $\dot{\delta}\,(T) = \dot{\delta}\,(I)$

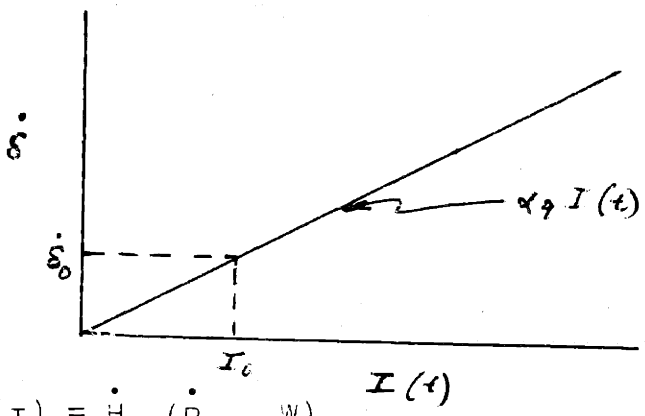
THE RATE OF DIVIDENDS IS ASSUMED TO BE PROPORTIONAL TO THE TOTAL INVESTMENT. THUS,

$$\dot{\delta}\,(T) = \alpha_9 I\,(T),$$

WHERE $\alpha_9 = \dfrac{\dot{\delta}_0}{I_0}$,

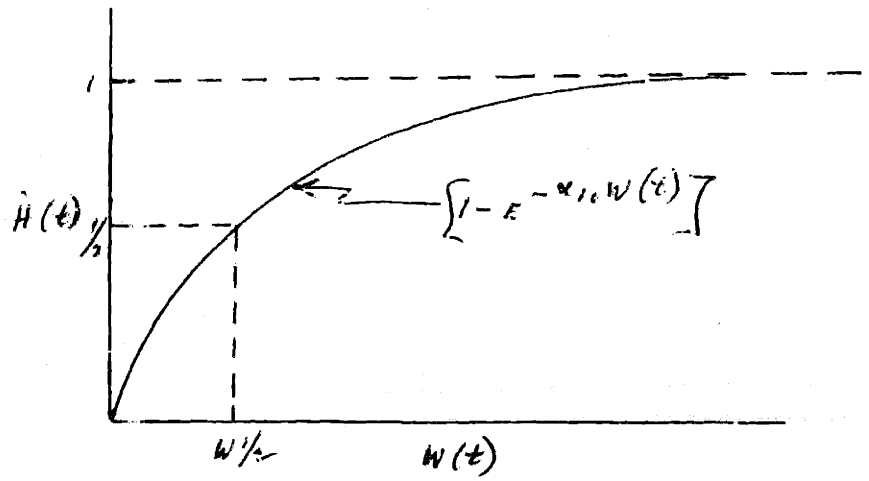
$\dot{\delta}_0 = \dot{\delta}(0)$, AND

$I_0 = I(0)$.



J - RATE OF HARVEST OF E, $\dot{H}_E(T) = \dot{H}_E\,(\dot{P}_{EA}, W)$.

THE RATE OF HARVEST OF E IS ASSUMED TO BE DIRECTLY PROPORTIONAL TO THE ACTUAL RATE OF PRODUCTIVE EFFORT AT $(T-\phi)$. SINCE E IS ASSUMED TO BE AN AGRICULTURAL GOOD, THE RATE OF HARVEST IS REALLY A DECISION OF NATURE, BUT FOR THE SAKE OF THE ARGUMENT AT THIS POINT, THE DEPENDENCE ON PRODUCTIVE EFFORT IS ASSUMED. THE WEATHER AFFECTS THE RATE OF HARVEST, IN THAT WHEN $W(T) = 0$, $\dot{H}(T) = 0$, AND $\dot{H}(T)$ INCREASES AS $W(T)$ INCREASES TO A LIMITING VALUE.



THUS,

$$\dot{H}_E(T) = [1-E^{-\alpha_{10}W(T)}]\ \dot{P}_{EA}(T-\phi_E), \text{ WHERE}$$

$$\alpha_{10} = \frac{LN2}{W\ 1/2}.$$

K - <u>RATE OF HARVEST OF G, $\dot{H}_G(T) = \dot{H}_G(\dot{P}_{GA})$</u>

THE RATE OF HARVEST OF G IS ASSUMED TO BE INDEPENDENT OF THE WEATHER, SINCE G IS ASSUMED TO BE A MANUFACTURED GOOD. $\dot{H}_G(T)$ IS ASSUMED TO BE EQUAL TO THE ACTUAL RATE OF PRODUCTIVE EFFORT AT TIME $(T-\phi_G)$.

$$\dot{H}_G(T) = \dot{P}_{GA}(T-\phi_G).$$

L - <u>RATE OF HARVEST OF K, $\dot{H}_K(T) = \dot{H}_K(\dot{P}_{KA})$</u>

THE RATE OF HARVEST OF K, $\dot{H}_K$ IS ASSUMED TO BE EQUAL TO THE ACTUAL RATE OF PRODUCTIVE EFFORT AT TIME $(T-\phi_K)$.

$$\dot{H}_K(T) = \dot{P}_{KA}\ (T-\phi_K).$$

M - <u>INTEREST RATE $\dot{\iota}(T) = \dot{\iota}(\beta, \bar{\iota})$</u>

THE INTEREST RATE IS DETERMINED BY THE AMOUNT OF MONEY IN THE BANK (T), AND THE HISTORICAL RATE OF INTEREST $\bar{\iota}(T) = \int_1^N \frac{\dot{\iota}(T-s)\,ds}{N}$. THE INTEREST RATE IS ASSUMED TO

DECREASE EXPONENTIALLY WITH β. THUS,

$$\dot{\iota}(T) = \dot{\iota}_{MAX}\, E^{-\alpha_{11}\beta(T)\int_1^N \frac{\dot{\iota}(T-\phi)\,d\phi}{N}} .$$

AT $T = 0$, ASSUME $\dot{\iota}_0 = \bar{\dot{\iota}}_0$. THEN $\alpha_{11} = [\frac{\text{LN}\,\dot{\iota}_{MAX}}{\beta_0}]$

N - <u>RATE OF INVESTMENT $\dot{I}(T) = \dot{I}(\quad \dot{S}_H)$</u>

THE RATE OF INVESTMENT IS ASSUMED TO BE DIRECTLY PROPORTIONAL TO THE RATE OF SAVING BY THE HOUSEHOLD.

$$\dot{I}(T) = \alpha_{12}\,\dot{S}(T)$$

$$\text{WHERE} \quad \alpha_{12} = \frac{\dot{I}_0}{\dot{S}_0} .$$

O - <u>USE OF CAPITAL EQUIPMENT</u>

SINCE THE ACTUAL RATE OF USE OF EQUIPMENT $\dot{K}_A(T)$ CAN NEVER EXCEED THE TOTAL EQUIPMENT AVAILABLE $\dot{K}(T)$, THE ALLOCATION BETWEEN THE VARIOUS SECTORS IS ASSUMED TO BE DONE IN SUCH A MANNER THAT THE RATIOS OF THE ACTUAL USAGE TO DESIRED USAGE IN ONE SECTOR IS EQUAL TO THIS RATIO IN ALL OTHER SECTORS.

THUS,

$$\frac{\dot{K}_{EA}(T)}{\dot{K}_{ED}(T)} = \frac{\dot{K}_{GA}(T)}{\dot{K}_{GD}(T)} = \frac{\dot{K}_{KA}(T)}{\dot{K}_{KD}(T)} = \frac{\dot{K}_A(T)}{\dot{K}_D(T)} .$$

$$\text{OR} \quad \dot{K}_{EA}(T) = \frac{\dot{K}_A(T)}{\dot{K}_D(T)} \cdot \dot{K}_{ED}(T).$$

AND SIMILARLY FOR $\dot{K}_{GA}$ AND $\dot{K}_{KA}$.

THE DESIRED RATE OF USE OF EQUIPMENT $\dot{K}_{ED}(T)$ IS DETERMINED THROUGH THE LEAST COST CRITERIA, EQUATION 1), SINCE THE $_K$ IS ASSUMED TO BE DETERMINED INDEPENDENTLY. THUS, $\dot{K}_{ED}(T)$ MUST SATISFY THE EQUATION.

$$\dot{K}_{ED}(T) = \frac{\pi_{LRE}(T)\,[1 - E^{-\alpha_{13}\dot{LR}_E(T)}]}{\alpha_{13}\,\pi_K(T)\,E^{-\alpha_{13}\dot{LR}_E(T)}}, \quad \alpha_{13} = \frac{LN_2}{\dot{LR}_{E\,1/2}},$$

SIMILARLY,

$$\dot{K}_{GD}(T) = \frac{\pi_{LRG}(T)\,[1 - E^{-\alpha_{14}\dot{LR}_G(T)}]}{\alpha_{14}\,\pi_K(T)\,E^{-\alpha_{14}\dot{LR}_G(T)}}, \quad \alpha_{14} = \frac{LN_2}{\dot{LR}_{G\,1/2}},$$

$$\dot{K}_{KD}(T) = \frac{\pi_{LRK}(T)[1 - E^{-\alpha_{15}\dot{LR}_K(T)}]}{\alpha_{15}\,\pi_K(T)\,E^{-\alpha_{15}\dot{LR}_K(T)}}, \quad \alpha_{15} = \frac{LN_2}{\dot{LR}_{K\,1/2}}.$$

THE AMOUNT OF WASTE (IDLE TIME) OF CAPITAL EQUIPMENT IS GIVEN BY $\dot{K}_W(T)$. BY DEFINITION, THE IDLE TIME IS THE TIME THE EQUIPMENT IS NOT ENGAGED IN PRODUCTIVE EFFORT. THUS,

$$\dot{K}_W(T) = \dot{K}(T) - [\dot{K}_{EA}(T) + \dot{K}_{GA}(T) + \dot{K}_{KA}(T)], \quad \dot{K}_W(T) \geq 0.$$

P - <u>RATE OF USE OF LABOR</u>

THE RATE OF USE OF LABOR IS ASSUMED TO BE DIRECTLY PROPORTIONAL TO THE PRICE OF LABOR, BUT SINCE IT TAKES TIME FOR A CHANGE OF PRICE TO BE REFLECTED IN THE AVAILABILITY OF LABOR,

$$\dot{LR}_E(T) = \dot{LR}_{MAX}[1 - E^{-\alpha_{16}\,LRE\,(T-\varphi_{LRE})}].$$

SIMILARLY,

$$\dot{LR}_G(T) = \dot{LR}_{MAX}[1 - E^{-\alpha_{17}\,LRG\,(T-\varphi_{LRG})}],$$

$$\dot{LR}_K(T) = \dot{LR}_{MAX}\,[1 - E^{-\alpha_{18}\,LRK(T-\phi_{LRK})}],$$

$$\alpha_{16} = \frac{1}{\pi_{LREO}}\,LN\left(\frac{1}{1 - \frac{\dot{LR}_{EO}}{\dot{LR}_{MAX}}}\right),$$

$$\alpha_{17} = \frac{1}{\pi_{LRGO}}\,LN\left(\frac{1}{1 - \dot{LR}_{GO}/\dot{LR}_{MAX}}\right),$$

$$\alpha_{18} = \frac{1}{\pi_{LRKO}}\,LN\left(\frac{1}{1 - \dot{LR}_{KO}/\dot{LR}_{MAX}}\right), \quad \text{AND}$$

ϕ_{LRE}, ϕ_{LRG}, AND ϕ_{LRK} ARE CONSTANTS.

Q - <u>RATE OF USE OF LAND</u>

THE RATE OF USE OF LAND IN $\dot{P}_E(T)$ IS ASSUMED TO BE DIRECTLY PROPORTIONAL TO THE PRICE OF LAND. SINCE IT TAKES TIME FOR A CHANGE OF PRICE TO BE REFLECTED IN THE AVAILABILITY OF LAND,

$$\dot{L}_E(T) = \dot{L}_{MAX}\,[1 - E^{-\alpha_{19}\,LE(T-\phi_{LE})}].$$

SIMILARLY,

$$\dot{L}_G(T) = \dot{L}_{MAX}\,[1 - E^{-\alpha_{20}\,LG(T-\phi_{LG})}],$$

$$\dot{L}_K(T) = \dot{L}_{MAX}\,[1 - E^{-\alpha_{21}\,LK(T-\phi_{LK})}],$$

WHERE $$\alpha_{19} = \frac{1}{\pi_{LEO}}\,LN\left[\frac{1}{1 - \frac{\dot{L}_{EO}}{\dot{L}_{MAX}}}\right],$$

$$\alpha_{20} = \frac{1}{\pi_{LGO}} \; LN\left[\frac{1}{1 - \dot{L}_{GO}/\dot{L}_{MAX}}\right],$$

$$\alpha_{21} = \frac{1}{\pi_{LKO}} \; LN\left[\frac{1}{1 - \dot{L}_{KO}/\dot{L}_{MAX}}\right], \text{ AND}$$

ϕ_{LE}, ϕ_{LG}, AND ϕ_{LK} ARE CONSTANTS.

R - <u>PAYBACK OF LOANS TO THE HOUSEHOLD</u>

IT IS ASSUMED THAT ALL LOANS ARE PAID BACK IN N EQUAL INSTALLMENTS, ONE AT EACH INCREMENT OF TIME. THUS,

$$\dot{P}_H(T) = \int_0^N \frac{\dot{B}_H(T - \xi)}{N} \, d\xi$$

S - <u>ACTUAL PRODUCTIVE EFFORT</u>

THE ACTUAL RATE OF PRODUCTIVE EFFORT IN E IS $\dot{P}_{EA}(T) = \dot{P}_{EA}(\dot{L}_E, \dot{L}R_E, \dot{K}_E)$. THE ACTUAL RATE OF PRODUCTION IS ASSUMED TO INCREASE IN DIRECT PROPORTION TO EITHER LAND, LABOR, OR CAPITAL EQUIPMENT. THEREFORE,

$$\dot{P}_{EA} = \alpha_{22}, \; \dot{L}R_E(T) \cdot \dot{L}_E(T) \cdot \dot{K}_E(T)$$

SIMILARLY,

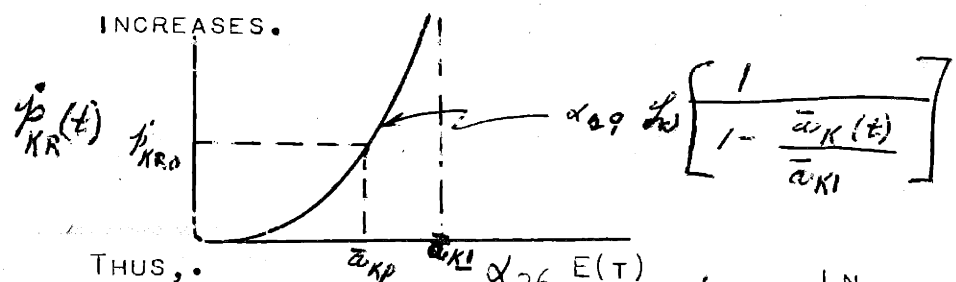
$$\dot{P}_{GA} = \alpha_{23} \; \dot{L}R_G(T) \cdot \dot{L}_G(T) \cdot \dot{K}_G(T), \text{ AND}$$

$$\dot{P}_{KA} = \alpha_{24} \; \dot{L}R_K(T) \cdot \dot{L}_K(T) \cdot \dot{K}_K(T), \text{ WHERE}$$

$$\alpha_{22} = \frac{\dot{P}_{EAO}}{\dot{L}R_{EO} \cdot \dot{L}_{EO} \cdot \dot{K}_{EO}}, \; \alpha_{23} = \frac{\dot{P}_{GAO}}{\dot{L}R_{GO} \cdot \dot{L}_{GO} \cdot \dot{K}_{GO}}, \text{ AND } \alpha_{24} = \frac{\dot{P}_{KAO}}{\dot{L}R_{KO} \cdot \dot{L}_{KO} \cdot \dot{K}_{KO}}.$$

T - PRODUCTION DECISION

THE RATE OF PRODUCTION DECIDED UPON AT TIME T IN PRODUCING E IS $\dot{P}_{ED}(T) = \dot{P}_{ED}(E)$. THE RATE OF PRODUCTIVE EFFORT $\dot{P}_{ED}(T)$ IS MAXIMUM WHEN E IS ZERO, AND DECREASES EXPONENTIALLY AS E INCREASES.



THUS, $\dot{P}_{ED}(T) = \alpha_{25} E^{\alpha_{26} E(T)}$, $\alpha_{26} = \frac{LN_2}{E\ 1/2}$, AND $\alpha_{25} = \dot{P}_{ED\ MAX}$.

SIMILARLY,

$$\dot{P}_{GD}(T) = \alpha_{27} E^{-\alpha_{28} G(T)}, \quad \alpha_{28} = \frac{LN_2}{G\ 1/2}, \quad \alpha_{27} = \dot{P}_{GD\ MAX}.$$

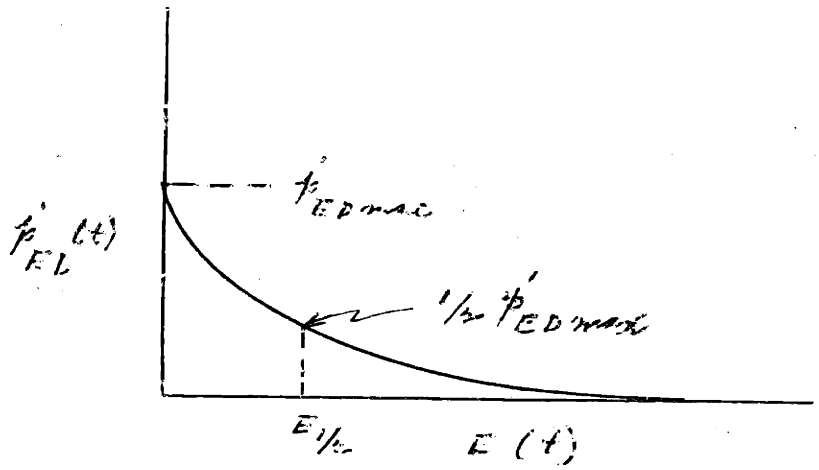
THE DECISION OF THE RATE OF PRODUCTION OF CAPITAL EQUIPMENT IS $\dot{P}_{KD}(T) = P_{KD}\ (\bar{A}_K, \dot{K}_D, \dot{K}_A, \ddot{C})$

THE DECISION $\dot{P}_{KD}(T)$ IS MADE IN TWO PARTS:

1) REPLACEMENT OF EXISTING EQUIPMENT, $\dot{P}_{KR}(T)$,
2) PRODUCTION OF NEW CAPITAL GOODS, $P_{KN}(T)$,

OR $\dot{P}_{KD}(T) = \dot{P}_{KR}(T) + \dot{P}_{KN}(T)$.

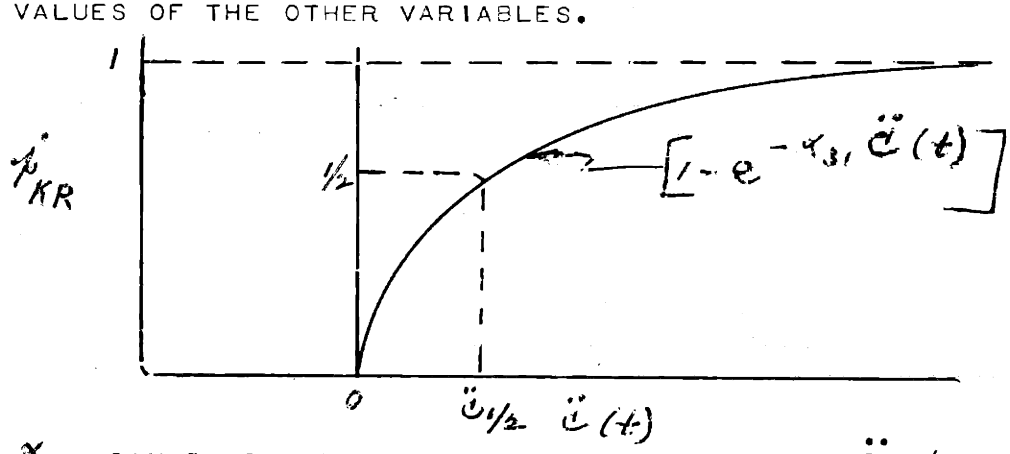
THE REPLACEMENT DECISION $\dot{P}_{KR}(T)$ IS BASED ON THE AVERAGE AGE OF THE EXISTING EQUIPMENT $\bar{A}_K(T)$, AND ON $\dot{C}$ (T) WHICH IS ESSENTIALLY THE MAJOR ELEMENT IN THE BUSINESS OUTLOOK. THE RATE OF PRODUCTION $\dot{P}_{KR} = 0$, FOR $\bar{A}_K = 0$ AND INCREASES AS $\bar{A}_K$ INCREASES TO A MAXIMUM OF $\bar{A}_{K1}$.



THE VALUE OF α_{29} CAN BE EXPRESSED IN TERMS OF THE VALUES OF THE VARIABLES AT T = 0 AS,

$$\alpha_{29} = \frac{\dot{P}_{KRO}}{LN\left[\dfrac{1}{1-(\bar{A}_{KO}/\bar{A}_{K1})}\right]}$$

THE VARIATION OF $\dot{P}_{KR}$ WITH $\ddot{C}$ IS ZERO FOR $\ddot{C} < 0$, AND INCREASES AS $\ddot{C}$ INCREASES TO A CERTAIN MAXIMUM RATE DEPENDENT ON THE VALUES OF THE OTHER VARIABLES.



α_{31} CAN BE EXPRESSED IN TERMS OF THE VALUE $\ddot{C}$ 1/2 WHERE $\ddot{P}_{KR}$ IS 1/2 THE MAXIMUM ATTAINABLE VALUE AS $\alpha_{31} = \dfrac{LN2}{\ddot{C}\ 1/2}$.

SINCE PLANS ARE NOT GENERATED IMMEDIATELY WITH CHANGES IN $\ddot{C}(T)$.

$$\dot{P}_{KR}(T) = \alpha_{29}\ LN\left[\frac{1}{1-\dfrac{\bar{A}_K(T)}{\bar{A}_{K1}}}\right] \cdot \left[1 - E^{-\alpha_{31}\ddot{C}(T-\phi_{\ddot{C}})}\right].$$

THE DESIRED RATE OF PRODUCING NEW EQUIPMENT $\dot{P}_{KN}(T)$ IS BASED ON THE $\ddot{C}(T)$, AND THE SAVINGS TO BE REALIZED BY OBTAINING THE NEW EQUIPMENT BY PRODUCERS OF E, G, AND K. SO $\dot{P}_{KN}(T) = \dot{P}_{KN}[\ddot{C}(T-\phi_{\ddot{C}}), (\dot{K}_D(T) - \dot{K}_A(T)]$. THE VARIATION OF $\dot{P}_{KN}(T)$ WITH $\ddot{C}(T)$ IS ASSUMED TO BE OF THE SAME TYPE AS THE VARIATION OF $\dot{P}_{KR}$. THE ADDITIONAL ASSUMPTION IS MADE THAT $\dot{P}_{KN}$ IS DIRECTLY PROPORTIONAL TO $[\dot{K}_D(T) - \dot{K}_A(T)]$, $\dot{P}_{KN} \geq 0$.

THUS,

$$\dot{P}_{KN}(T) = \alpha_{30}[1-E^{-\alpha_{31}\ddot{C}(T-\phi_{\ddot{C}})}]\,[\dot{K}_D(T-\phi_{PK}) - \dot{K}_A(T-\phi_{PK})].$$

WHERE $\alpha_{30} = \dfrac{\dot{P}_{KNO}}{[\dot{K}_{DO} - \dot{K}_{AO}]}$, $\dot{K}_D(T) = \dot{K}_{ED}(T) + \dot{K}_{GD}(T) + \dot{K}_{KD}(T)$, AND $\dot{K}_A(T) = \dot{K}_{EA}(T) + \dot{K}_{GA}(T) + \dot{K}_{KA}(T)$.

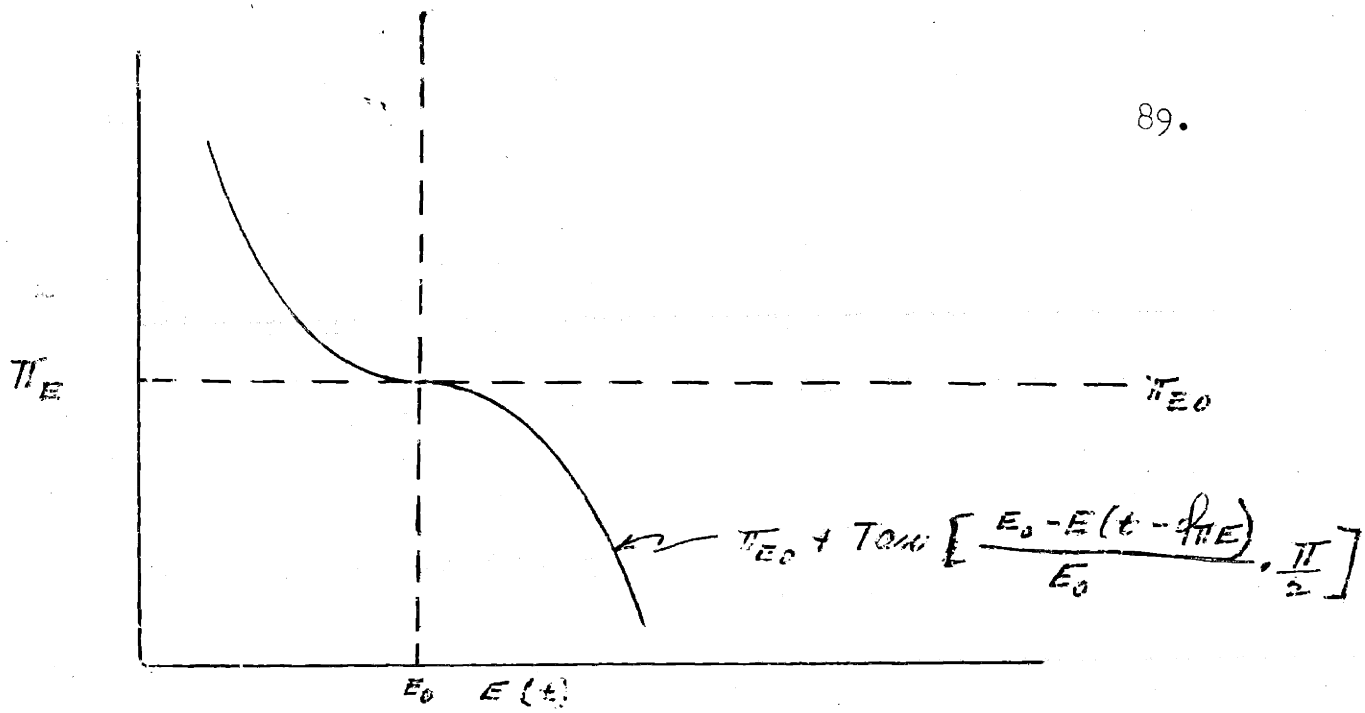
SINCE THE DECISION $\dot{P}_{KD}(T)$ IS THE SUM OF $\dot{P}_{KR}(T)$ AND $\dot{P}_{KN}(T)$,

$$P_{KD}(T) = \left\{\alpha_{29} L_N \left[\frac{1}{\left(1 - \dfrac{\bar{A}_K(T)}{\bar{A}_{KI}}\right)}\right] + \alpha_{30}[\dot{K}_D(T-\phi_{PK}) - \dot{K}_A(T-\phi_{PK})]\right\}.$$

$$[1 - E^{-\alpha_{31}\ddot{C}(T-\phi_C)}]$$

U - <u>THE PRICE OF GOODS</u>

THE PRICE OF THE PRODUCT E FOR CONSUMPTION IS $\pi_E(T) = \pi_E(\pi_{PE}, \gamma_E, E)$. FOR A NORMAL LEVEL OF INVENTORY, THE PRICE π_E IS ASSUMED TO BE DETERMINED BY THE COST OF PRODUCTION OF THE INVENTORY PLUS A NORMAL PROFIT. IF THE AVERAGE LEVEL OF INVENTORY IS HIGHER THAN NORMAL AT TIME T, THE PRICE IS LOWERED AT TIME $(T + \phi_{\pi E})$.



BUT, $\pi_{EO}(T) = \pi_{PE}(T)\ [1 + \gamma_E(T)]$, WHERE π_{PE} IS THE AVERAGE COST OF PRODUCTION OF THE EXISTING INVENTORY, AND γ_E IS THE PROFIT MARKUP, SO,

$$\pi_{PE}(T) = \int_{T1}^{T} \frac{\dot{M}_E(T)}{\dot{H}_E(T)}\, dT.$$

WHERE T_1 CAN BE DETERMINED FROM

$$\int_0^{T1} \dot{H}_E(T)\, dT = \int_0^{T} \dot{C}_E(T)\, dT.$$

THUS, $$\pi_E(T) = [1 + \gamma_E(T)] \int_{T1}^{T} \frac{\dot{M}_E(T)}{\dot{H}_E(T)}\, dT + \tan\left[\frac{E_0 - E(T - \phi_{\pi E})}{E_0} \cdot \pi/2\right],$$

WHERE E_0 IS THE NORMAL LEVEL OF INVENTORY, AND $\phi_{\pi E}$ IS A CONSTANT.

THE PRICE OF G CAN BE ARRIVED AT BY SUBSTITUTING G FOR E IN THIS FORMULAE. THIS ASSUMES FIRST IN IS THE FIRST OUT ON INVENTORY.

V - PRICE OF K. THE PRICE OF K AT TIME T IS $\pi_K(T) = \pi_K(\dot{M}_K, \gamma_K, \dot{D})$.

THE PRICE OF K AT TIME T IS EQUAL TO THE COST OF PRODUCTION PLUS A NORMAL MARKUP FOR PROFIT. WHEN EQUIPMENT IS RETIRED,

ITS COST IS TAKEN OFF OF THE COST OF THE INVENTORY. THUS,

$$\pi_K(T) = [1 + \gamma_K(T)] \int_{T3}^{T} \frac{\dot{M}_K(T)}{\dot{H}_K(T)} \, dT.$$

WHERE T_3 CAN BE DETERMINED FROM

$$\int_0^{T3} \dot{H}_K(T) \, dT = \int_0^{T} \dot{D}(T) \, dT.$$

THIS ASSUMES THAT THE FIRST EQUIPMENT PRODUCED IS THE FIRST EQUIPMENT RETIRED FROM USE. $\pi_K(T)$ IS ASSUMED TO BE THE SAME IN EACH OF THE THREE SECTORS. THIS IS AN ASSUMPTION WHICH CAN EASILY BE RELAXED FOR MORE DETAILED MODELS. FOR THIS MODEL, HOWEVER, THE $\pi_K(T)$ IS ASSUMED TO BE DETERMINED STRICTLY BY THE COST OF PRODUCTION OF K AND A NORMAL PROFIT MARKUP γ_K.

W - <u>PRICE OF LABOR. THE PRICE OF LABOR IS $\pi_{LRE}(T) = \pi_{LRE}[\dot{M}_E, \dot{P}_{EA}, (\dot{P}_{ED} - \dot{P}_{EA})]$.</u>

THE PRICE OF LABOR IS USED TO DETERMINE THE RATE AT WHICH LABOR IS USED. THE PRICE AT TIME T THEN IS EQUAL TO THE PRICE AT (T-1), PLUS A CORRECTION PROPORTIONAL TO THE DIFFERENCE BETWEEN THE RATE OF PRODUCTION DECIDED UPON AND THE ACTUAL RATE. THE PRICE AT (T-1) HOWEVER, MUST FIT THE CONDITIONS FOR MINIMUM PRODUCTION COST, EQUATION 2), SINCE IT IS ASSUMED THAT THE PROPORTIONS OF THE FACTORS OF PRODUCTION VARY ACCORDING TO THE LEAST COST CRITERIA GIVEN IN EQUATION 1).

THUS,

$$\pi_{LRE}(T) = \frac{\dot{M}_E(T-1)}{\dot{LR}_E(T-1)} + \frac{\pi_{LREO}}{\dot{P}_{EAO}} [\dot{P}_{ED}(T-1) - \dot{P}_{EA}(T-1)].$$

SIMILARLY,

$$\pi_{LRG}(T) = \frac{\dot{M}_G(T-1)}{\dot{LR}_G(T-1)} + \frac{\pi_{LRGO}}{\dot{P}_{GAO}} [\dot{P}_{GD}(T-1) - \dot{P}_{GA}(T-1)],$$

$$\pi_{LRK}(T) = \frac{\dot{M}_K(T-1)}{\dot{LR}_K(T-1)} + \frac{\pi_{LRKO}}{\dot{P}_{KAO}} [\dot{P}_{KD}(T-1) - \dot{P}_{KA}(T-1)].$$

X - <u>PRICE OF LAND</u>

THE PRICE OF LAND $\pi_{LE}(T)$ IS DETERMINED THROUGH THE LEAST COST OF PRODUCTION CRITERIA, EQUATION 1) AS,

$$\pi_{LE}(T) = \frac{\dot{LR}_E(T) \cdot \pi_{LRE}(T)}{\dot{L}_E(T)} .$$

SIMILARLY,

$$\pi_{LG}(T) = \frac{\dot{LR}_E \cdot \pi_{LRE}(T)}{\dot{L}_G(T)},$$

$$\pi_{LK}(T) = \frac{\dot{LR}_E \cdot \pi_{LRE}(T)}{\dot{L}_K(T)}.$$

Y - <u>BANK RESERVES. THE RATE OF RESERVES BEING SET ASIDE BY THE BANK IS $\dot{R}(T) = \dot{R}(\rho, \dot{S}_H, \dot{d})$.</u>

THE RESERVES HELD BY THE BANK AT ANY TIME T IS DEPENDENT ON THE % RESERVES DECIDED UPON, TIMES THE AMOUNT OF TIME DEPOSITS HELD. THUS,

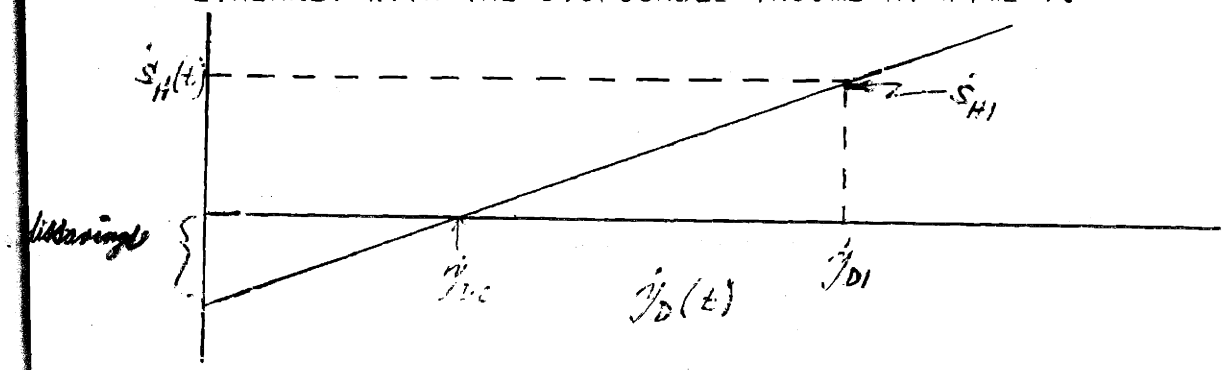
$$\dot{R}(T) = \rho(T) [\dot{S}_H(T) + \dot{d}(T)].$$

THE TOTAL RESERVES ON HAND AT TIME T ARE

$$\dot{R}(T) = \dot{R}_0 + \int_0^T \dot{R}(T) \, dT.$$

Z. - SAVINGS BY THE HOUSEHOLD

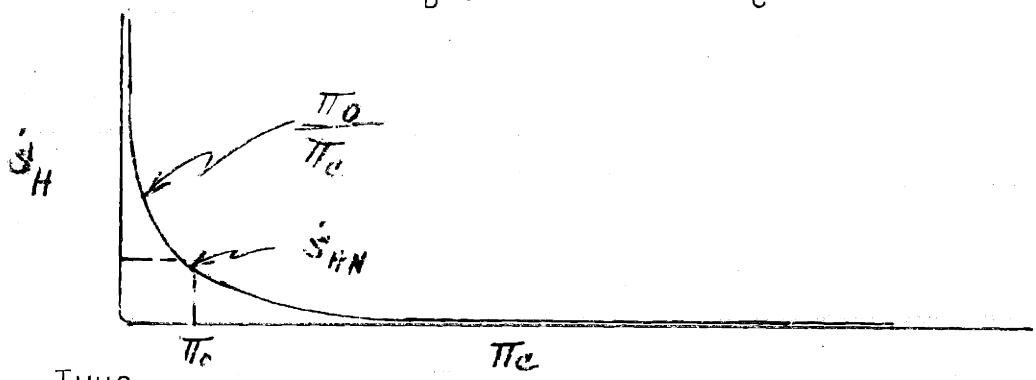
THE RATE OF SAVINGS BY THE HOUSEHOLD IS $\dot{S}_H(T) = \dot{S}_H(Y_D)$. THE RATE OF SAVINGS BY THE HOUSEHOLD IS ASSUMED TO VARY LINEARLY WITH THE DISPOSABLE INCOME AT TIME T.



THE PRICE OF CONSUMPTION $\pi_C(T)$ IS DEFINED BY π_E & π_G AS $\pi_C \cdot \dot{C} = \dot{M}_C = \pi_E \cdot \dot{C}_E + \pi_G \cdot \dot{C}_G$. THE ACTUAL CONSUMPTION IS $\dot{C} = \dot{C}_E + \dot{C}_G$. THE EXPRESSIONS YIELD THE RELATIONSHIP

$$\frac{1}{\pi_C} = \frac{1}{\pi_E} + \frac{1}{\pi_G}, \text{ OR } \pi_C = \frac{\pi_E \cdot \pi_G}{(\pi_E + \pi_G)}.$$

FOR A GIVEN DISPOSABLE INCOME $\dot{Y}_D$, THE HIGHER π_C, THE SMALLER THE SAVINGS.



THUS,

$$S_H(T) = \alpha_{32} \frac{(\dot{Y}_D(T) - \dot{Y}_{DO})}{\pi_C(T)} = \alpha_{32} \frac{(Y_D(T) - Y_{DO})(\pi_E + \pi_G)}{\pi_E(T) \cdot \pi_G(T)},$$

WHERE $\alpha_{32} = \left[\frac{\dot{S}_{HI} \cdot \pi_{CO}}{\dot{Y}_{DI} - \dot{Y}_{DO}}\right]$.

HOW ARRIVED AT

AA- SUMMARY OF DEFINITIONS AND EQUATIONS

DEFINITION

$\bar{A}_K(T)$ = AVERAGE AGE OF THE CAPITAL EQUIPMENT AT TIME T.

$$A_K(T) = \frac{\int_{T_1}^{T} (T-\xi)\, \dot{H}_K(\xi)\, d\xi}{\int_{T_1}^{T} \dot{H}_K(T)\, dT}$$, WHERE T_1 CAN BE

DETERMINED FROM $\int_0^{T_1} \dot{H}_K(T)\, dT = \int_0^{T} \dot{D}(T)\, dT$.

DERIVATION

$\dot{B}_B(T)$ = THE NET RATE OF BORROWING BY THE BUSINESS AT TIME T.

$$\dot{B}_B(T) = \alpha_0\, \rho(T) \cdot \ddot{C}(T-\phi_{BB}) \left[\frac{1}{1+\alpha_1 LN\left(\frac{1}{1-\frac{i(T)}{\dot{i}}}\right)} \right], \quad \rho(T) \geq 0 .$$

DERIVATION

$\dot{B}_H(T)$ = THE RATE OF BORROWING BY THE HOUSEHOLD AT TIME T.

$$\dot{B}_H(T) = \alpha_2\, E^{-(Y-Y_1)^2} \cdot \alpha_3 \left[1 - E^{-\alpha_4 \rho(T)}\right] \cdot \left[\frac{1}{1+\alpha_5 LN\left(\frac{1}{1-\frac{\dot{P}_H(T)}{\alpha_6}}\right)} \right]$$

DEFINITION $B_H(T)$ = BORROWING HELD BY THE HOUSEHOLD AT TIME T.

$$B_H(T) = \int_0^T \dot{B}_H(T)\, dT.$$

FLOW CHART B = BUSINESS

β = BANK

$\beta(T)$ = AMOUNT OF MONEY IN THE BANK AT TIME T.

$$\beta(T) = \underbrace{[\mathcal{D}(T) + \dot{S}_H(T) + \dot{P}_H(T)]}_{\text{INCOME}} - \underbrace{[\mathcal{I}(T) + \dot{B}_B(T) + \dot{\delta}(T) + \dot{B}_H(T) + \dot{R}(T)]}_{\text{OUTGO}}.$$

FLOW CHART $\dot{C}(T)$ = RATE OF CONSUMPTION AT TIME T.

$$\dot{C}(T) = \dot{C}_E(T) + \dot{C}_G(T).$$

DEFINITION $\ddot{C}(T)$ = RATE OF CHANGE OF RATE OF CONSUMPTION AT TIME T.

$$\ddot{C}(T) = \ddot{C}(T) - \ddot{C}(T-1).$$

DEFINITION $\bar{\dot{C}}(T)$ = THE HABITUAL RATE OF CONSUMPTION OF E AT TIME T.

$$\bar{\dot{C}}_E(T) = \int_1^N \frac{\dot{C}_G(T-\xi)\, d\xi}{N}.$$

DEFINITION $\bar{\dot{C}}(T)$ = THE HABITUAL RATE OF CONSUMPTION OF G AT TIME T.

$$\bar{\dot{C}}_G(T) = \int_1^N \frac{C_G(T - \xi)}{N} d\xi$$

DERIVATION $\dot{C}_E(T)$ = RATE OF CONSUMPTION OF E AT TIME T.

$$\dot{C}_E(T) = \frac{\alpha_6}{\pi_E(T)} [1 - E^{-\alpha_7 \dot{Y}_D(T - \phi_C)}] \int_1^N \frac{\dot{C}_E(T - \xi)}{N} d\xi.$$

DERIVATION $\dot{C}_G(T)$ = RATE OF CONSUMPTION OF G AT TIME T.

$$\dot{C}_G(T) = \frac{\alpha_8}{\pi_G(T)} [1 - E^{-\alpha_7 \dot{Y}_D(T - \phi_C)}] \int_1^N \frac{\dot{C}_G(T - \xi)}{N} d\xi.$$

FLOW CHART $\dot{\mathcal{D}}(T)$ = THE RATE OF DEPOSITS OF CASH BY THE BUSINESS TO THE BANK.

<u>INCOME</u>

$$\dot{\mathcal{D}}(T) = [\dot{M}_C(T) + \dot{\iota}_H(T) + \dot{\mathcal{I}}(T) + \dot{\mathcal{B}}_B(T)] - [\dot{M}_L(T) + \dot{M}_{LR}(T)].$$

DERIVATION $\dot{D}(T)$ = THE RATE OF RETIREMENT OF CAPITAL EQUIPMENT K, AT TIME T.

$$\dot{D}(T) = \frac{\dot{P}_{KA}(T-\phi_K) \cdot \dot{P}_{KR}(T-\phi_K)}{\dot{P}_{KD}(T-\phi_K)}.$$

DERIVATION $\dot{\delta}(T)$ = RATE OF DIVIDENDS BEING PAID AT TIME T.

$$\dot{\delta}(T) = \alpha_9 I(T).$$

DEFINITION E = 1ST CONSUMER GOOD.

$\dot{E}(T)$ = THE RATE OF CHANGE OF THE INVENTORY OF E AT TIME T.

$$\dot{E}(T) = \dot{H}_E(T) - \dot{C}_E(T).$$

FLOW CHART G = 2ND CONSUMER GOOD.

$\dot{G}(T)$ = THE RATE OF CHANGE OF THE INVENTORY OF G AT TIME T.

$$\dot{G}(T) = \dot{H}_G(T) - \dot{C}_G(T).$$

DEFINITION $\gamma_E(T)$= THE NORMAL MARKUP OVER COST FOR PROFIT IN PRICING E.

$\gamma_E(T)$= A GIVEN CONSTANT FOR THE SYSTEM.

DEFINITION $\gamma_G(T)$= THE NORMAL MARKUP, OVER COST, FOR PROFIT IN PRICING.

$\gamma_G(T)$= A GIVEN CONSTANT FOR THE SYSTEM.

DEFINITION $\gamma_K(T)$ = THE NORMAL MARKUP, OVER COST, FOR PROFIT IN PRICING K.

$\gamma_K(T)$ = A GIVEN CONSTANT FOR THE SYSTEM.

DEFINITION H = HOUSEHOLD

$\dot{H}_E(T)$ = THE RATE OF HARVEST OF E.

$$\dot{H}_E(T) = [1-E^{-\alpha_{10}W(T)}]\ \dot{P}_{EA}(T-\phi_E).$$

DERIVATION $\dot{H}_G(T)$ = THE RATE OF HARVEST OF G.

$$\dot{H}_G(T) = \dot{P}_{GA}(T-\phi_G).$$

DERIVATION $\dot{H}_K(T)$ = THE RATE OF HARVEST OF K.

$$\dot{H}_K(T) = \dot{P}_{KA}(T-\phi_K).$$

DERIVATION $\dot{\iota}(T)$ = THE INTEREST RATE CHARGED ON LOANS.

$$\dot{\iota}(T) = \dot{\iota}_{MAX}\ E^{-\alpha_{11}\beta(T)} \cdot \bar{\dot{\iota}}(T).$$

DEFINITION $\bar{\dot{\iota}}(T)$ = THE HISTORICAL RATE OF INTEREST.

$$\bar{\dot{\iota}}(T) = \int_1^N \frac{\dot{\iota}(T-s)\,ds}{N}.$$

FLOW CHART $\dot{\iota}_H(T)$ = THE RATE OF INTEREST BEING PAID BY THE HOUSEHOLD.

$$\dot{\iota}_H(T) = \dot{\iota}(T) \cdot [B_H(T-1) - P_H(T-1)].$$

DERIVATION $\dot{I}(T)$ = THE RATE OF INVESTMENT OF MONEY IN THE BUSINESS AT TIME T.

$$\dot{I}(T) = \alpha_{12} S(T).$$

DEFINITION K = A TYPE OF CAPITAL EQUIPMENT.

$\dot{K}_A(T)$ = THE TOTAL ACTUAL RATE OF USAGE OF THE CAPITAL EQUIPMENT IN ALL SECTORS.

$$\dot{K}_A(T) = \dot{K}_{EA}(T) + \dot{K}_{GA}(T) + \dot{K}_{KA}(T).$$

DEFINITION $\dot{K}_D(T)$ = THE TOTAL RATE OF USAGE OF THE CAPITAL EQUIPMENT DECIDED ON IN ALL SECTORS.

$$\dot{K}_D(T) = \dot{K}_{ED}(T) + \dot{K}_{GD}(T) + \dot{K}_{KD}(T).$$

FLOW CHART $\dot{K}(T)$ = THE TOTAL AVAILABLE USAGE OF CAPITAL EQUIPMENT.

$$\dot{K}(T) = \dot{K}(T-1) + \dot{H}_K(T) - \dot{D}(T).$$

FLOW CHART $\dot{K}_W(T)$ = THE AMOUNT OF WASTE USE (IDLE TIME) OF CAPITAL EQUIPMENT.

$$\dot{K}_W(T) = \dot{K}(T) - \dot{K}_A(T), \quad \dot{K}_W(T) \geq 0.$$

DERIVATION $\dot{K}_{EA}(T)$ = THE ACTUAL RATE OF USE OF $\dot{K}_E$ IN $\dot{P}_E$.

$$\dot{K}_{EA}(T) = \frac{\dot{K}_A(T)}{\dot{K}_D(T)} \cdot \dot{K}_{ED}(T)$$

DERIVATION $\dot{K}_{ED}(T)$ = THE RATE OF USE OF $\dot{K}_E$ IN $\dot{P}_E$ DECIDED ON.

$$\dot{K}_{ED}(T) = \frac{\pi_{LRE}(T)[1-E^{-\alpha_{13}\dot{LR}_E(T)}]}{\alpha_{13}\,\pi_K(T)\,E^{-\alpha_{13}\dot{LR}_E(T)}}.$$

DERIVATION $\dot{K}_{GA}(T)$ = THE ACTUAL RATE OF USE OF $\dot{K}_G$ IN $\dot{P}_G$.

$$\dot{K}_{GA}(T) = \frac{\dot{K}_{GD}(T) \cdot \dot{K}_A(T)}{\dot{K}_D(T)}.$$

DERIVATION $\dot{K}_{GD}(T)$ = THE RATE OF USE OF $\dot{K}_G$ IN $\dot{P}_G$ DECIDED ON.

$$K_{GD}(T) = \frac{\pi_{LRG}(T)\,[1-E^{-\alpha_{14}\dot{LR}_G(T)}]}{\alpha_{14}\,\pi_K(T)E^{-\alpha_{14}\dot{LR}_G(T)}}, \qquad \alpha_{14} = \frac{LN2}{\dot{LR}_{G\,1/2}}$$

DERIVATION $\dot{K}_{KA}(T)$ = THE ACTUAL RATE OF USE OF $\dot{K}_K$ IN $\dot{P}_K$.

$$\dot{K}_{KA}(T) = \frac{\dot{K}_{KD}(T) \cdot \dot{K}_A(T)}{\dot{K}_D(T)} .$$

DERIVATION $\dot{K}_{KD}(T)$ = THE RATE OF USE OF $\dot{K}_K$ AND $\dot{P}_K$ DECIDED ON.

$$\dot{K}_{KD}(T) = \frac{\pi_{LRK}(T)\,[1-E^{-\alpha_{15}\dot{LR}_K(T)}]}{\alpha_{15}\,\pi_K(T)\,E^{-\alpha_{15}\dot{LR}_K(T)}}, \qquad \alpha_{15} = \frac{LN\ 2}{\dot{LR}_K\ 1/2}$$

DERIVATION $\dot{LR}_E(T)$ = THE RATE OF USAGE OF LABOR IN $\dot{P}_E$.

$$\dot{LR}_E(T) = \dot{LR}_{MAX}[1 - E^{-\alpha_{16}\pi_{LRE}(T-\phi_{LRE})}].$$

DERIVATION $\dot{LR}_G(T)$ = THE RATE OF USAGE OF LABOR IN $\dot{P}_G$.

$$\dot{LR}_G(T) = \dot{LR}_{MAX}[1 - E^{-\alpha_{17}\pi_{LRG}(T-\phi_{LRG})}] .$$

DERIVATION $\dot{LR}_K(T)$ = THE RATE OF USAGE OF LABOR IN $\dot{P}_K$.

$$\dot{LR}_K(T) = \dot{LR}_{MAX}[1-E^{-\alpha_{18}\pi_{LRK}(T-\phi_{LRK})}].$$

DERIVATION $\dot{L}_E(T)$ = THE RATE OF USAGE OF LAND IN $\dot{P}_E$.

$$\dot{L}_E(T) = \dot{L}_{MAX}[1-E^{-\alpha_{19}\pi_{LE}(T-\phi_{LK})}].$$

DERIVATION $\dot{L}_G(T)$ = THE RATE OF USAGE OF LAND IN $\dot{P}_G$.

$$\dot{L}_G(T) = L_{MAX}[1-E^{-\alpha_{20}\pi_{LG}(T-\phi_{LG})}].$$

DERIVATION $\dot{L}_K(T)$ = THE RATE OF USAGE OF LAND IN $\dot{P}_K$.

$$\dot{L}_K(T) = \dot{L}_{MAX}[1-E^{-\alpha_{21}\pi_{LK}(T-\phi_{LK})}].$$

FLOW CHART $\dot{M}_C(T)$ = THE RATE OF MONEY BEING PAID FOR CONSUMPTION AT TIME T.

$$\dot{M}_C(T) = \dot{M}_{CE}(T) + \dot{M}_{CG}(T).$$

FLOW CHART $\dot{M}(T)$ = THE RATE OF MONEY BEING PAID FOR LABOR AT TIME T.

$$\dot{M}_{LR}(T) = \dot{M}_{LRE}(T) + \dot{M}_{LRG}(T) + \dot{M}_{LRK}(T).$$
$$\dot{M}_{L}(T) = \dot{M}_{LE}(T) + \dot{M}_{LG}(T) + \dot{M}_{LK}(T).$$
$$\dot{M}_{K}(T) = \dot{M}_{KE}(T) + \dot{M}_{KG}(T) + \dot{M}_{KK}(T).$$

FLOW CHART $\dot{M}_{E}(T)$ = THE RATE OF MONEY GOING FOR PRODUCTION $\dot{P}_{E}(T)$.

$$\dot{M}_{E}(T) = \dot{M}_{LRE}(T) + \dot{M}_{LE}(T) + \dot{M}_{KE}(T).$$
$$\dot{M}_{G}(T) = \dot{M}_{LRG}(T) + \dot{M}_{LG}(T) + \dot{M}_{KG}(T).$$
$$\dot{M}_{K}(T) = \dot{M}_{LRK}(T) + \dot{M}_{LK}(T) + \dot{M}_{KK}(T).$$

FLOW CHART $\dot{M}_{LRE}(T)$ = THE RATE OF MONEY BEING PAID FOR LABOR IN $P_{E}(T)$.

$$\dot{M}_{LRE}(T) = \pi_{LRE}(T) \cdot \dot{LR}_{E}(T), \quad M_{LE}(T) = \pi_{LE}(T) \cdot \dot{L}_{E}(T), \quad \dot{M}_{KE}(T) = \pi_{K}(T) \cdot \dot{K}_{E}(T).$$
$$\dot{M}_{LRG}(T) = \pi_{LRG}(T) \cdot \dot{LR}_{G}(T), \quad \dot{M}_{LG}(T) = \pi_{LG}(T) \cdot \dot{L}_{G}(T), \quad \dot{M}_{KG}(T) = \pi_{K}(T) \cdot \dot{K}_{G}(T).$$
$$\dot{M}_{LRK}(T) = \pi_{LRK}(T) \cdot \dot{LR}_{K}(T), \quad \dot{M}_{LK}(T) = \pi_{LK}(T) \cdot \dot{L}_{K}(T), \quad \dot{M}_{KK}(T) = \pi_{K}(T) \cdot \dot{K}_{K}(T).$$

DERIVATION $\dot{P}_{EA}(T)$ = THE ACTUAL RATE OF PRODUCTIVE EFFORT IN PRODUCING E AT TIME T.

$$\dot{P}_{EA}(T) = \alpha'_{22} \dot{LR}_{E}(T) \, \dot{L}_{E}(T) \, \dot{K}_{E}(T).$$
$$\dot{P}_{GA}(T) = \alpha'_{23} \dot{LR}_{G}(T) \, \dot{L}_{G}(T) \, \dot{K}_{G}(T).$$
$$\dot{P}_{KA}(T) = \alpha'_{24} \dot{LR}_{K}(T) \, \dot{L}_{K}(T) \, \dot{K}_{K}(T).$$

DERIVATION $\dot{P}_{ED}(T)$ = THE RATE OF PRODUCTION DECIDED ON IN PRODUCING E AT TIME T.

$$\dot{P}_{ED}(T) = \alpha_{25} E^{-\alpha_{26} E(T)}.$$

$$\dot{P}_{GD}(T) = \alpha_{27} E^{-\alpha_{28} G(T)}.$$

DERIVATION $\dot{P}_{KD}(T)$ = THE RATE OF PRODUCTION DECIDED ON IN PRODUCING K AT TIME T.

$$\dot{P}_{KD}(T) = [\alpha_{29} \, LN\left(\frac{1}{1 - \frac{\bar{A}_K(T)}{\bar{A}_{KI}}}\right) + \alpha_{30}[\dot{K}_D(T - \phi_{PK}) - \dot{K}_A(T - \phi_{PK})]] \cdot [1 - E^{-\alpha_{31} \ddot{C}(T - \phi_C)}].$$

DEFINITION $\dot{P}_H(T)$ = THE RATE OF PAYBACK OF LOANS BY THE HOUSEHOLD.

$$\dot{P}_H(T) = \int_0^N \frac{\dot{B}_H(T - \xi)\, d\xi}{N}.$$

DERIVATION $\pi_E(T)$ = THE PRICE OF E AT TIME T.

$$\pi_E(T) = [1 + \gamma_E(T)] \int_{T1}^{T} \frac{\dot{M}_E(T)}{\dot{H}_E(T)} dT + TAN\left[\frac{E_0 - E(T - \phi_{\pi_E})}{E_0} \cdot \pi/2\right]$$

WHERE T1 CAN BE DETERMINED FROM

$$\int_0^{T1} \dot{H}_E(T)\,dt = \int_0^T \dot{C}_E(T)\,dT.$$

$\pi_G(T)$ = THE PRICE OF G AT TIME T.

$$\pi_G(T) = [1+\gamma_G(T)] \int_{T2}^{T} \frac{\dot{M}_G(T)}{\dot{H}_G(T)}\,dT + \text{TAN}\left[\frac{G_0 + G(T - \delta\pi_G)}{G_0} \cdot \pi/2\right].$$

WHERE T_2 CAN BE DETERMINED FROM

$$\int_0^{T2} \dot{H}_E(T)\,DT = \int_0^T \dot{C}_E(T)\,DT.$$

DERIVATION

$\pi_K(T)$ = THE PRICE OF K AT TIME T.

$$\pi_K(T) = [1 + \gamma_K(T)] \cdot \int_{T3}^{T} \frac{\dot{M}_K(T)}{\dot{H}_K(T)}\,dT$$

WHERE T_3 CAN BE DETERMINED FROM

$$\int_0^{T3} \dot{H}_K(T)\,dT = \int_0^T \dot{D}(T)\,dT.$$

DERIVATION $\pi_{LRE}(T)$ = THE PRICE OF LABOR IN E AT TIME T.

$$\pi_{LRE}(T) = \frac{\dot{M}_E(T-1)}{\dot{LR}_E(T-1)} + \frac{\pi_{LREO}}{\dot{P}_{EAO}} \left[\dot{P}_{ED}(T-1) - \dot{P}_{EA}(T-1) \right].$$

$$\pi_{LRG}(T) = \frac{\dot{M}_G(T-1)}{\dot{LR}_G(T-1)} + \frac{\pi_{LGO}}{\dot{P}_{GAO}} \left[\dot{P}_{GD}(T-1) - \dot{P}_{GA}(T-1) \right].$$

$$\pi_{LRK}(T) = \frac{\dot{M}_K(T-1)}{\dot{LR}_K(T-1)} + \frac{\pi_{LKO}}{\dot{P}_{KAO}} \left[\dot{P}_{KD}(T-1) - \dot{P}_{KA}(T-1) \right].$$

DERIVATION $\pi_{LE}(T)$ = THE PRICE OF LAND IN E AT TIME T.

$$\pi_{LE}(T) = \frac{\dot{LR}_E(T) \cdot \pi_{LRE}(T)}{\dot{L}_E(T)},$$

$$\pi_{LG}(T) = \frac{\dot{LR}_E(T)\ \pi_{LRE}(T)}{\dot{L}_G(T)},$$

$$\pi_{LK}(T) = \frac{\dot{LR}_E(T)\ \pi_{LRE}(T)}{\dot{L}_K(T)}.$$

DEFINITION $\dot{R}(T)$ = THE RATE OF MONEY BEING SET ASIDE AS A RESERVE BY THE BANK.

$$\dot{R}(T) = \rho(T)\,[\dot{S}_H(T) + \dot{\delta}(T)].$$

$\rho(T)$ = THE % RESERVE HELD BY THE BANK ON ALL TIME DEPOSITS.

DERIVATION $\dot{S}_H(T)$ = THE RATE OF SAVINGS BY THE HOUSEHOLD.

$$\dot{S}_H(T) = \frac{\alpha_{32}(\dot{Y}_D(T) - \dot{Y}_{DO})\,(\pi_E(T) + \pi_G(T))}{\pi_E(T) \cdot \pi_G(T)}.$$

DEFINITION $W(T)$ = THE WEATHER AS A FUNCTION OF TIME.

$W(T)$ IS ASSUMED TO BE THE ONLY EXOGENEOUS VARIABLE IN THE MODEL.

FLOW CHART $\dot{Y}(T)$ = THE INCOME OF THE HOUSEHOLD AT TIME T.

$$\dot{Y}(T) = \dot{M}_L(T) + \dot{M}_{LR}(T) + \dot{\delta}(T).$$

FLOW CHART $\dot{Y}_D(T)$ = THE DISPOSABLE INCOME OF THE HOUSEHOLD AT TIME T.

$$\dot{Y}_D(T) = \dot{Y}(T) + \dot{B}_H(T) - \dot{P}_H(T) - \dot{\iota}_H(T).$$

AB - Summary of Constants

THE CONSTANTS ARE DERIVED IN MANY CASES IN TERMS OF THE VALUES OF THE VARIABLES AT TIME $T = 0$. THIS ASSUMES, IN GENERAL, NON ZERO VALUES OF THE VARIABLES AT $T = 0$. IF THE MODEL IS TO BE TESTED, BEGINNING WITH $T = 0$, AND THE VALUES OF THE VARIABLES ARE TO BE ZERO AT $T = 0$, THEN THE CONSTANTS WILL HAVE TO BE EVALUATED AT SOME OTHER TIME SAY T_0 WHERE THE VARIABLES HAVE NON ZERO VALUES.

FROM FUNCTION	DERIVED CONSTANTS
$\dot{B}_B$	$\alpha_0 = \dfrac{\dot{B}_{BO}}{\beta_0 \dot{C}_0}$
$\dot{B}_B$	$\alpha_1 = \dfrac{\frac{1}{\alpha_0} - 1}{\left[\mathrm{LN}\,\frac{1}{1-\beta_0}\right]}$
$\dot{B}_H$	$\alpha_2 = \dot{B}_{H\ MAX}$
$\dot{B}_H$	$\alpha_3 = \dfrac{\mathrm{LN}_2}{\dot{Y}_1^{\,2}\,(\gamma_1 - 1)^2}$
$\dot{B}_H$	$\alpha_4 = \dfrac{1}{\alpha_2}$
$\dot{B}_H$	$\alpha_5 = \left[\dfrac{1/\alpha - 1}{\mathrm{LN}\,\frac{1}{1-\gamma_2}}\right]$
$\dot{C}_E$	$\alpha_6 = \left[\dfrac{\pi_{EO}}{1 - E^{-\dot{Y}_{DO}\alpha_7}}\right]$
$\dot{C}_E$, $\dot{C}_G$	$\alpha_7 = \dfrac{\mathrm{LN}_2}{\dot{Y}_{D\ 1/2}}$
$\dot{C}_G$	$\alpha_8 = \left[\dfrac{\pi_{GO}}{1 - E^{-\alpha_7 \dot{Y}_{DO}}}\right]$
$\dot{\delta}$	$\alpha_9 = \dfrac{\dot{\delta}_0}{I_0}$

FROM FUNCTION	DERIVED CONSTANTS
$\dot{H}_E$	$\alpha_{10} = \dfrac{LN_2}{W\ 1/2}$
$\dot{i}$	$\alpha_{11} = \dfrac{LN\ \dot{L}_{MAX}}{\beta_0}$
$\dot{I}$	$\alpha_{12} = \dfrac{\dot{I}_0}{\dot{S}_0}$
$\dot{K}_{ED}$	$\alpha_{13} = \dfrac{LN_2}{\dot{L}R_E\ 1/2}$
$\dot{K}_{GD}$	$\alpha_{14} = \dfrac{LN_2}{\dot{L}R_G\ 1/2}$
$\dot{K}_{KD}$	$\alpha_{15} = \dfrac{LN_2}{\dot{L}R_K\ 1/2}$
$\dot{L}R_E$	$\alpha_{16} = \dfrac{1}{\pi_{LREO}} LN \left[\dfrac{1}{1 - \dfrac{\dot{L}R_{EO}}{\dot{L}R_{MAX}}}\right]$
$\dot{L}R_G$	$\alpha_{17} = \dfrac{1}{\pi_{LRGO}} LN \left[\dfrac{1}{1 - \dfrac{\dot{L}R_{GO}}{\dot{L}R_{MAX}}}\right]$
$\dot{L}R_K$	$\alpha_{18} = \dfrac{1}{\pi_{LRKO}} LN \left[\dfrac{1}{1 - \dfrac{\dot{L}R_{KO}}{\dot{L}R_{MAX}}}\right]$

FROM FUNCTION	DERIVED CONSTANTS
$\dot{L}_E$	$\alpha_{19} = \frac{1}{\pi_{LEO}} \, LN \left[\frac{1}{1 - \frac{\dot{L}_{EO}}{\dot{L}_{MAX}}} \right]$
$\dot{L}_G$	$\alpha_{20} = \frac{1}{\pi_{LGO}} \, LN \left[\frac{1}{1 - \frac{\dot{L}_{GO}}{\dot{L}_{MAX}}} \right]$
$\dot{L}_K$	$\alpha_{21} = \frac{1}{\pi_{LKO}} \, LN \left[\frac{1}{1 - \frac{\dot{L}_{KO}}{\dot{L}_{MAX}}} \right]$
$\dot{P}_{EA}$	$\alpha_{22} = \frac{\dot{P}_{EAO}}{\dot{L}R_{EO} \, \dot{L}_{EO} \, \dot{K}_{EO}}$
$\dot{P}_{GA}$	$\alpha_{23} = \frac{\dot{P}_{GAO}}{\dot{L}R_{GO} \, \dot{L}_{GO} \, \dot{K}_{GO}}$
$\dot{P}_{KA}$	$\alpha_{24} = \frac{\dot{P}_{KAO}}{\dot{L}R_{KO} \, \dot{L}_{KO} \, \dot{K}_{KO}}$
$\dot{P}_{ED}$	$\alpha_{25} = \dot{P}_{ED\ MAX}$
	$\alpha_{26} = \frac{LN_2}{E_{1/2}}$
$\dot{P}_{GD}$	$\alpha_{27} = \dot{P}_{GD\ MAX}$
	$\alpha_{28} = \frac{LN_2}{G_{1/2}}$

FROM FUNCTION

$\dot{P}_{KD}$

DERIVED CONSTANTS

$$\alpha_{29} = \frac{\dot{P}_{KRO}}{LN\left[\dfrac{1}{1 - \dfrac{\bar{A}_{KO}}{\bar{A}_{K1}}}\right]}$$

$$\alpha_{30} = \left[\frac{\dot{P}_{KNO}}{\dot{K}_{DO} - \dot{K}_{AO}}\right]$$

$$\alpha_{31} = \frac{LN_2}{\ddot{C}_{1/2}}$$

$\dot{S}_H$

$$\alpha_{32} = \left[\frac{\dot{S}_{H1}\ \pi_{CO}}{\dot{Y}_{D1} - \dot{Y}_{DO}}\right]$$

PHASE LAGS

ϕ_{BB}
ϕ_C
$\phi_{\ddot{C}}$
ϕ_{PK}
ϕ_E
ϕ_G

ϕ_{LRE}
ϕ_{LRG}
ϕ_{LRK}
ϕ_{LE}
ϕ_{LG}
ϕ_{LK}
ϕ_E
ϕ_G
ϕ_K

AC - METHOD OF SOLVING THE EQUATIONS

THE ASSUMPTION IS MADE, AS IN THE OTHER MODELS, THAT THE VALUES OF ALL VARIABLES FOR $T<0$, ARE NORMAL VALUES. THE ORDER OF COMPUTATION AT $T = 0$ IS:

$$\{ \dot{P}_{ED}, \pi_{LRE}, \pi_{LE}, \dot{LR}_E, \dot{L}_E, \dot{M}_{LRE}, \dot{M}_{LE}, \dot{P}_{KD}, \pi_{LRK}, \pi_{LK}, \dot{LR}_K, \dot{L}_K, \dot{M}_{LRK}, \dot{M}_{LK}, \dot{P}_{GD}, \pi_{LRG}, \pi_{LG}, \dot{LR}_G, \dot{L}_G, \dot{M}_{LRG}, \dot{M}_{LG}, \dot{M}_{LR}, \dot{M}_L, \dot{Y}, \pi_K, \dot{K}_{ED}, \dot{K}_{KD}, \dot{K}_{GD}, \dot{K}_D, \dot{K}_{EA}, \dot{K}_{GA}, \dot{K}_{KA}, \dot{K}_A, \dot{K}_W, \dot{P}_{EA}, \dot{H}_E, \dot{M}_{PE}, \dot{A}_K, \dot{P}_{KA}, \dot{H}_K, \dot{K}, \dot{D}, \dot{P}_{GA}, \dot{H}_G, \dot{M}_{\varphi G}, \dot{B}_H, B_H, \dot{P}_H, \dot{\delta}, i, \bar{i}, i_H, E, G, \dot{S}_H, \pi_E, \pi_G, \dot{C}_E, \dot{C}_G, \bar{\dot{C}}_E, \bar{\dot{C}}_G, \dot{C}, \ddot{C}, \dot{M}_C, \dot{B}_B, \dot{I}, \dot{R}, R, \dot{\alpha}, \beta \} = F(0).$$

THEN COMPUTE $F(1)$, $F(2)$ ------ $F(N)$. THESE VARIABLES AS A FUNCTION OF TIME COMPLETELY DEFINE THE SYSTEM.

CHAPTER IV

APPRAISAL AND OUTLINE FOR FURTHER WORK

PART I - SOME OUTSTANDING DEFICIENCIES IN THE MODELS PRESENTED

A - DECISIONS

THESE MODELS ARE ONLY INTENDED TO BE FIRST APPROXIMATIONS. EACH OF THE DECISION MAKING PROCESSES NEEDS MORE CRITICAL EXAMINATION IN TERMS OF THE FACTORS UNDERLYING THE MANNER IN WHICH EACH DECISION IS MADE. FOR ANY REAL ECONOMIC SITUATION, THIS MAY BE A COMPROMISE, OR AN AVERAGE. FOR EXAMPLE, IN THE MODELS GIVEN, THE HARVEST DECISION IS ASSUMED TO DEPEND ON $\dot{P}_E(T - \phi_{PE})$. THIS MEANS THAT THE HARVEST WOULD BE INDEPENDENT OF THE DISTRIBUTION OF PRODUCTIVE EFFORT OVER TIME. THIS IS NOT TRUE IN ANY ACTUAL ECONOMY. HOWEVER, THE REFINEMENT IN THE MODEL IS RELATIVELY SIMPLE IN ANY PARTICULAR CASE, BUT THE INCLUSION OF ALL OF THIS TYPE OF REFINEMENT COULD HAVE OBSCURED THE PURPOSE AND INTENT OF THIS PRESENTATION, BY COMPLICATING IT UNDULY, AND OVERSHADOWING THE PROCEDURES AND TECHNIQUES.

THIS REFINEMENT CAN BE ADDED TO THE MODEL BY ASSUMING A FUNCTION $\kappa_N(T)$ WHICH REPRESENTS THE PERCENTAGE OF THE PRODUCTIVE EFFORT AT TIME T GOING INTO THE PRODUCT STARTED AT T = N. THEN,

$$\dot{H}(T) = \int_0^N \kappa_N(T - \phi) \cdot \dot{P}_E(T - \phi)\, d\phi.$$

B - INFORMATION FLOWS

INFORMATION IS ASSUMED TO FLOW AT ALL TIMES. THIS IS NOT

SO. PRESENT METHODS OF OBTAINING INFORMATION IN A REAL ECONOMY, ONLY AT BEST PROVIDES INFORMATION FOR A DEFINITE PERIOD OF TIME, AND THERE IS A LAG IN OBTAINING THIS INFORMATION. THIS EFFECT COULD BE WORKED, INTO THE MODEL AS A REFINEMENT, ONLY AFTER EXAMINING THE PRESENT MODELS FOR VARIOUS DECISION CRITERIA, AND FOR VARIOUS VALUES OF THE PARAMETERS TO UNDERSTAND THE EFFECTS OF CHANGES IN THEM. THE EFFECT OF IMPERFECT INFORMATION NEEDS TO BE BROUGHT INTO THE MODEL. THIS COULD POSSIBLY BE DONE BY EXPECTED VALUES, ANTICIPATIONS, HABITUAL PATTERNS, ETC.

C - MONEY FLOWS

THE MODELS DO NOT ALLOW FOR THE EXPANSIONS OR CONTRACTIONS OF THE AMOUNT OF MONEY IN THE SYSTEM. THIS IS A SIMPLE YET IMPORTANT ADDITION TO THE SYSTEM.

ALMOST ANY POLICY, OR CHANGE IN POLICY OF A FISCAL OR MONETARY NATURE, COULD BE ADDED EASILY TO SIMPLE MODELS OF THE TYPE GIVEN. THE VALUES OF THE PARAMETERS COULD BE FOUND IN AN ACTUAL ECONOMY BY MAKING CAREFUL CHANGES AND OF OBSERVING THE EFFECTS OF THE CHANGE AT THE MOST SENSITIVE POINT REVEALED BY THE MODEL. THIS METHOD OF MODELING COULD BE DONE FOR AN ACTUAL ECONOMY ON A TRIAL AND ERROR BASIS TO REFINE THE ASSUMPTIONS, MUCH AS IS DONE IN THE PHYSICAL SCIENCES.

D - FLOW DIAGRAMS

IT CAN BE ARGUED THAT THE FLOW DIAGRAM SHOULD CONTAIN ALL OF THE MATHEMATICAL PROCESSES INVOLVED IN THE COMBINATION OF VARIABLES. THIS WOULD UNDULY COMPLICATE THE DIAGRAM, AND MAKE ITS USE MUCH LESS UNIVERSAL. IT WOULD ADD LITTLE, SINCE THE ASSUMPTIONS CANNOT BE IMMEDIATELY APPARENT ANYWAY. THE ACTUAL RELATIONSHIPS SHOULD BE KEPT WITH THE ASSUMPTIONS FOR

ANY PARTICULAR CASE AS THAT IS THE POINT AT WHICH THEY HAVE THE MOST MEANING.

PART 2 - FURTHER WORK NEEDED

THE ONLY OUTSIDE FORCE ACTING ON THE MODELS IN THIS PAPER, AS DEFINED, IS WEATHER $W(T)$. THE SET OF DESCRIBED FUNCTIONS IN EACH CASE COMPLETELY DESCRIBES THE MODEL WHEN THEY ARE EXPRESSED AS FUNCTIONS OF TIME AND $W(T)$. THE NUMERICAL SOLUTION, AS TIME IS VARIED, CAN READILY BE OBTAINED.

THE FOLLOWING PARTICULAR STUDIES WOULD BE OF INTEREST, AND WOULD CONSTITUTE A VALUABLE EXTENSION OF THIS STUDY, WHICH WOULD HAVE BEEN INCLUDED IF TIME HAD PERMITTED:

1) OBTAIN SOLUTIONS FOR VARIOUS INITIAL VALUES OF THE PARAMETERS, AND FOR THE FOLLOWING WEATHER FUNCTIONS, $W(T)$
   - A) CONSTANT,
   - B) STEP FUNCTION,
   - C) SQUARE WAVE,
   - D) SINUSOIDAL VARIATION.
2) OBTAIN THE GROWTH IN RATE OF CONSUMPTION $\ddot{C}(T)$ FOR THE CONDITIONS GIVEN IN 1) ABOVE.
3) STUDY THE EFFECTS OF LIMITING THE AMOUNT OF LAND AVAILABLE, LABOR AVAILABLE, ETC.
4) STUDY THE EFFECTS OF CHANGING THE NORMAL VALUES OF THE PARAMETERS.
5) CHOOSE DIFFERENT DECISION CRITERIA FOR THE MODEL, AND EXAMINE VARIOUS COMBINATIONS OF THESE CRITERIA.
6) IN TERMS OF THESE MODELS AND THE ASSUMPTIONS MADE IN THEM, DISCUSS SUCH QUESTIONS AS:
   - A) IS A LITTLE BIT OF INFLATION GOOD OR BAD?
   - B) UNDER WHAT CONDITIONS CAN THE ECONOMY BUY BACK ALL IT CAN PRODUCE?
   - C) HOW IMPORTANT TO THE GROWTH $\ddot{C}(T)$ IS THE FUNCTION OF GOVERNMENT IN REDISTRIBUTING INCOME? WHAT WILL THIS DO TO INCENTIVES?
   - D) WHAT MECHANISMS COULD BE ADDED TO THE MODEL WHICH WOULD INCREASE THE STABILITY OF $\ddot{C}(T)$?
   - E) WHAT ARE THE CONSEQUENCES OF WIDELY HELD OWNERSHIP, AND CLOSELY HELD OWNERSHIP, ETC. IN THE ABSENCE OF A MECHANISM FOR THE DISTRIBUTION OF WEALTH?

F) WHAT ARE THE CONDITIONS OF MAXIMUM GROWTH? (I.E. $\dot{C}(T)$ MAXIMUM)

G) WHAT EFFECT WOULD THE ASSUMPTION OF COMPLETE AUTOMATION HAVE ON THE SYSTEM?

H) UNDER THE ASSUMPTION OF COMPLETE AUTOMATION, HOW COULD THE ECONOMY BUY BACK WHAT IT COULD PRODUCE? (SUGGESTION: THRU DIVIDENDS.)

I) WHAT ARE THE CRITERIA FOR STABILITY IN THE MODEL?

J) HOW DOES THE TIMING OF DECISIONS AND ACTIONS AFFECT THE STABILITY?

K) ETC.

ALL OF THESE QUESTIONS, AND MANY MORE CAN BE DISCUSSED, AND THE ARGUMENTS ASSISTED BY THE USE OF THE FLOW DIAGRAMS, AND THE SOLUTIONS OF THE ABOVE MODELS, AS WELL AS OF MORE SOPHISTICATED MODELS OF THIS TYPE.

THESE MODELS SHOULD BE EXTENDED TO INCLUDE N GOODS, M FAMILIES, PRODUCTIVE FACTORS, MONEY, CREDIT, DISTRIBUTION OF WEALTH, INNOVATIONS, DISTRIBUTION OF WEALTH, FOREIGN TRADE, DISTRIBUTION OF OWNERSHIP, ETC. THESE STUDIES SHOULD BE DONE ONE SECTOR AT A TIME, IN ORDER TO BUILD A SOLID FOUNDATION. SIMPLIFICATIONS COULD THEN BE MADE SUCH AS GROUPING SIMILAR TYPES OF GOODS TOGETHER, AND USING AVERAGE VALUES. VARIABLES SUCH AS CONSUMPTION COULD BECOME AVERAGE CONSUMPTION PER PERSON, AND SO FORTH.

I CANNOT HELP BUT FEEL THAT MANY OF THE OBSCURITIES WHICH APPEAR IN ECONOMICS COULD BE ELIMINATED IF SUBSTANTIAL PROGRESS WERE MADE IN THE BUILDING UP OF ECONOMIC FLOW DIAGRAMS CONTAINING CLOSED LOOPS OF TANGIBLES AND MONEY, AND INTERLATED BY DECISIONS, AND THE FLOWS OF INFORMATION. THIS APPEARS TO ME TO BE ONE OF THE MOST FRUITFUL AREAS FOR RESEARCH ON THE HORIZON TODAY SINCE ECONOMICS TOUCHES THE LIVES, HEALTH AND WELFARE OF EACH OF US IN NO SMALL WAY IN EVERYTHING WE DO.

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APPENDIX

APPENDIX A - AN EXAMPLE OF POPULATION MODEL.

CHANGES IN POPULATION ARE DEPENDENT ON THE BIRTH RATE AND THE DEATH RATE.

$$\dot{P}(T) = P_0 + \int_0^T [\dot{B}(T) - \dot{D}(T)]\, dT.$$

THE MODEL FOR POPULATION IS GIVEN IN FIGURE 14, WHERE THE VARIABLES ARE DEFINED. THIS FLOW CHART SHOWS THE TANGIBLE FLOWS THAT DETERMINE POPULATION, ALONG WITH THE DECISION NETWORK.

BIRTH RATE AND DEATH RATE ARE BOTH DECISIONS OF NATURE. THE BIRTH RATE PER PERSON WOULD TEND TO INCREASE AS THE AVERAGE RATE OF CONSUMPTION, PER PERSON INCREASED, UNTIL A CERTAIN MAXIMUM BIRTH RATE PER PERSON IS REACHED. THE BIRTH RATE IS ALSO DIRECTLY PROPORTIONAL TO THE POPULATION. THUS,

$$\dot{B}(T) = \alpha_1 P(T) \cdot [1 - E^{-\alpha_2 \dot{C}(T)}]$$

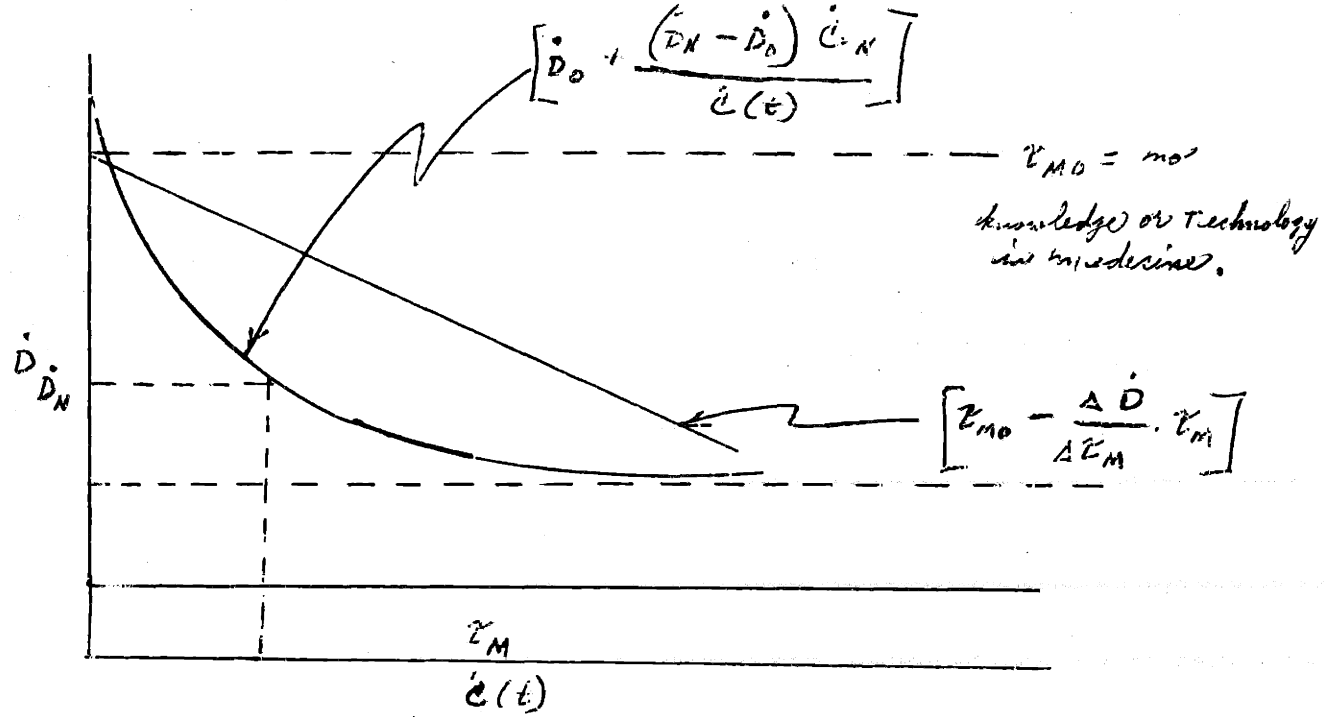
WHERE $\alpha_1 = \dfrac{\dot{B}_0}{P_0[1-E^{-\alpha_2 \dot{C}_0}]}$, $\alpha_2 = \dfrac{LN_2}{\dot{C}_{1/2}}$, $\dot{B}_0$, P_0, AND $\dot{C}_0$

ARE VALUES OF $\dot{B}(T)$, $P(T)$, AND $\dot{C}(T)$ AT $T = 0$, AND $\dot{C}_{1/2}$ IS THE VALUE OF $\dot{C}(T)$ AT WHICH

$$[1 - E^{-\alpha_2 \dot{C}(T)}] = 1/2.$$

THE DEATH RATE WOULD BE DIRECTLY PROPORTIONAL TO POPULATION, AND THE STATE OF THE MEDICAL ART. ALSO, AN INCREASE IN $\dot{C}$ WOULD BE EXPECTED TO CAUSE A DECREASE IN $\dot{D}$.

THUS,

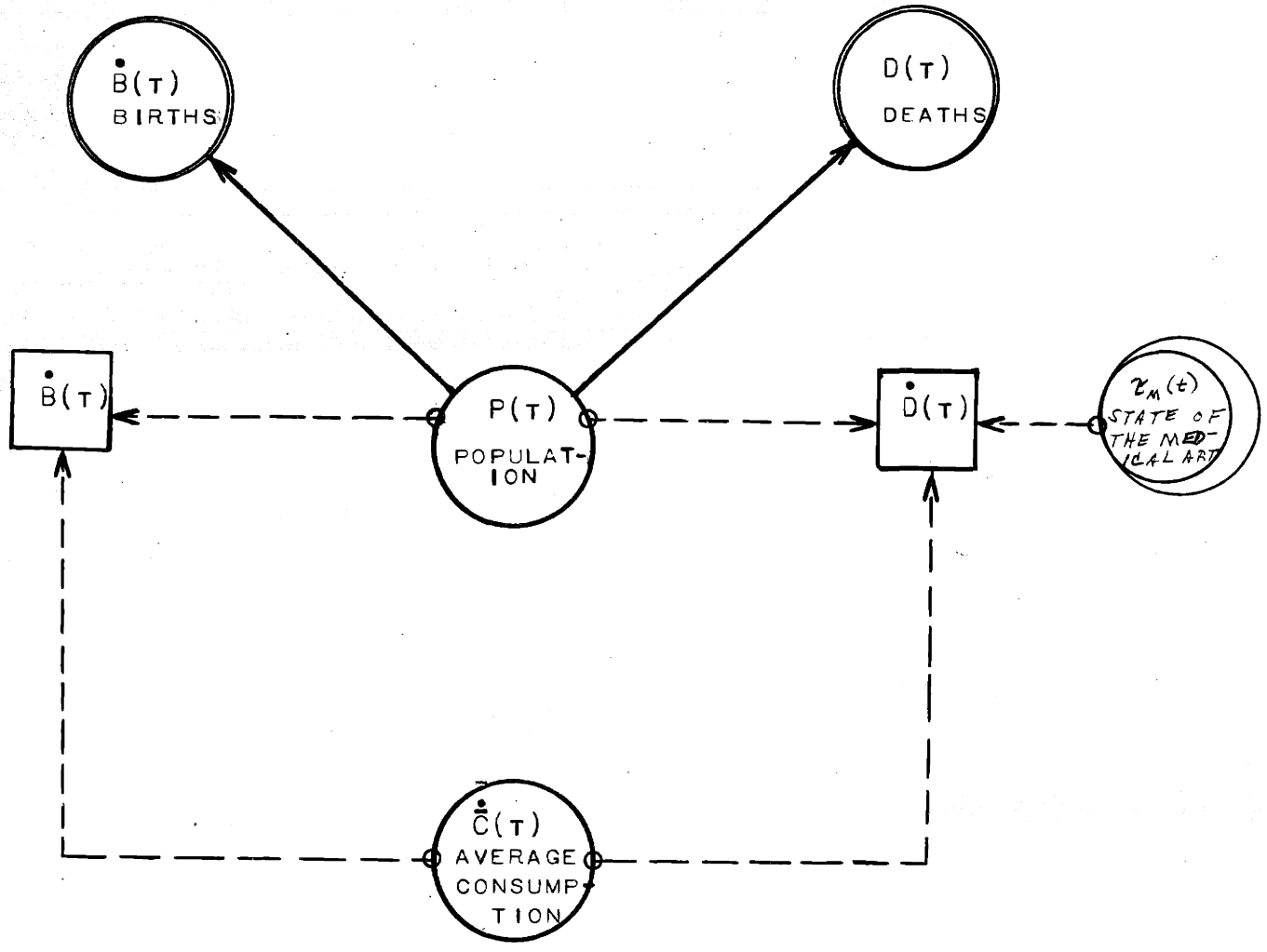


WHERE τ_{MO}, $\frac{\Delta \dot{D}}{\Delta \tau_M}$, $\dot{D}_O$, $\dot{D}_N$, AND $\dot{C}_N$ ARE THE PARAMETERS OF THE SYSTEM.. THE DEATH RATE CAN THEN BE DESCRIBED IN TERMS OF THESE THREE VARIABLES PROVIDED IT IS ASSUMED THAT NO OTHER FACTORS BUT THESE HAVE AN INFLUENCE. THUS, IF $\dot{D}_N$ = NORMAL DEATH RATE PER PERSON FOR WHAT IS CONSIDERED A NORMAL AVERAGE RATE OF CONSUMPTION PER PERSON $\dot{C}_N$, THEN

$$\dot{D}(T) = P(T).\ [\tau_{MO} - \frac{\Delta \dot{D}(T)}{\Delta \tau_M}.\tau_M].[\dot{D}_O + \frac{(\dot{D}_N - \dot{D}_O)\ C_N}{\dot{C}(T)}].$$

<u>SUMMARY OF EQUATIONS</u>:

$$1)\quad P(T) = P_O + \int_O^T [\dot{B}(T) - \dot{D}(T)]\ dT.$$



POPULATION MODEL

FIGURE 14

$$2)\quad \dot{B}(T) = \frac{\dot{B}_0}{P_0\left[1-E^{\frac{-\dot{C}_0}{\dot{C}_{1/2}}\,\mathrm{LN}\,2}\right]}\,P(T)\,.$$

$$\left[1-E^{\exp\left(\frac{-\dot{C}(T)}{\dot{C}_{1/2}}\right)\,\mathrm{LN}\,2}\right]$$

$$3)\quad \dot{D}(T) = P(T)\,.\left[\gamma_{MO} - \frac{\Delta\dot{D}(T)}{\Delta\gamma_M}\,.\,\gamma_M\right].$$

$$\left[\dot{D}_0 + \frac{(\dot{D}_N - \dot{D}_0)\;\dot{C}_N}{\dot{C}(T)}\right].$$

Discussion

The population model given here is determined primarily by the values of $\dot{C}(T)$. It was constructed to show how population models can be constructed to show the flows of information that determine the decisions of nature. The flow diagram can be used to show clearly the interdependence of variables, and to see a meaningful whole picture of a system. The rational which goes into the assumptions used in constructing a model can be more readily visualized by the use of the flow diagrams, not to mention its usefulness in instruction.

The model for population given by Malthus can be very simply expressed by a flow diagram of this type, and the differences between models can be seen more readily.