AN INVESTIGATION OF PRICE PROPAGATION IN
AN INDUSTRIAL PRODUCTION-DISTRIBUTION SYSTEM

by

Gary Lee Benton

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Signature of Author

Department of Electrical Engineering

Certified by

Thesis Supervisor

Accepted by

Chairman, Departmental Committee on
Graduate Students
AN INVESTIGATION OF PRICE PROPAGATION IN
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Gary Lee Benton

Submitted to the Department of Electrical Engineering
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requirements for the degree of Master of Science.

ABSTRACT

The existing theory on industrial price behavior is essentially
confined to an equilibrium type of analysis. Unfortunately, this theory
provides very little insight into the dynamic behavior of prices that is
observed in the real world. In spite of this apparent lack of theory on
dynamic price behavior, there are many analytical economists who are
attempting to derive equilibrium price information from the dynamic,
statistical data that have been collected from industry. A typical example
of this effort is the economists' attempt to derive static supply and
demand schedules from the monthly data on existing price levels and
purchasing rates.

Up to the present there has been little attempt to prove that the
static and dynamic behavior of prices are really related, due, principally,
to the lack of proper analytical tools for such an investigation. With
the recent development of large digital computers, however, a powerful
tool for investigating the dynamic behavior of industrial and economic
systems has become available to researchers.

The field of Industrial Dynamics, under the supervision of Professor
Jay W. Forrester, has laid the necessary foundation for the study of
industrial and economic systems through the use of simulation techniques
on a digital computer. By utilizing these techniques, the author has
attempted to investigate the relationship between dynamic and static price
behavior in a production-distribution system. An attempt has been made
to uncover some of the basic factors which influence this relationship;
and, equally as important, the author has attempted to create an awareness
of the powerful analytical tools which are now available, with the hope
of motivating further research into this problem.

In Chapters II and III of this thesis, a pricing mechanism has been
incorporated into a five-stage production-distribution model which includes
a consumer, retailer, distributor, manufacturer, and a raw material sup-
plier sector. A static or equilibrium demand schedule is formulated for
the consumer sector as well as an equilibrium supply schedule for the raw
material supplier. Through a series of simulation runs, a "feeling" is
acquired for the dynamic behavior of the resulting system as well as for
the basic factors which tend to influence this behavior. Following this is an initial investigation into the relationship between dynamic and static price behavior, with an attempt to pin down the factors which influence both the quantity and quality of this relationship.

In Chapter IV the five-stage model is expanded to include a more realistic consumer sector. By formulating such a sector, the consumer's decision to purchase finished goods is no longer based simply on a theoretical, static demand curve, but rather on the actual needs of the consumer and on his potential ability to make these necessary purchases. A further investigation into the problem of relating static and demand price information follows, with an attempt to utilize the results of Chapter III in explaining the results obtained with the expanded model.

A statement of the author's conclusions is formulated in Chapter V accompanied by suggested methods for further research into the problem of dynamic price behavior.

Thesis Supervisor: Jay W. Forrester
Title: Professor of Industrial Management
August 24, 1959

Professor Philip Franklin
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Professor Franklin:

In accordance with the requirements for graduation, I herewith submit a thesis entitled "An Investigation of Price Propagation in an Industrial Production-Distribution System."

I should like to thank Professor Forrester for suggesting the topic of this thesis and for engaging in many helpful discussions.

Sincerely yours,

Gary L. Benton
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CHAPTER I

INTRODUCTION

Statement of Problem

The theory of supply and demand has been well developed by theoretical economists over the past seventy years beginning with Alfred Marshall, who was one of the first to set down this concept in orderly and concise form (ref. 1). Almost all basic economic textbooks that are accepted and used by educators today begin by introducing the reader to the concept of supply and demand. This concept is relatively simple to understand, and most important of all, it is compatible with the intuitive feeling that most laymen have about the relationship between price and quantity. Given the demand schedule (or curve) for a given product or service and the corresponding supply schedule, one can see quite readily that only certain combinations (usually only one) of price and quantity satisfy both the buyer and the seller. Assuming no change in the demand or supply schedule, it is apparent that price and quantity will eventually settle down to one of these equilibrium conditions. If the demand curve should shift as a result of a change in the buyers' income (or some other factor affecting demand), a new equilibrium price and quantity will be determined, and again the price and quantity will tend towards this equilibrium condition. The textbooks are often careful to point out that the short-run demand schedule may be considerably different from the long-run demand schedule, and therefore one must be careful when he considers the dynamic behavior of prices.
To say that one must be careful in studying dynamic price behavior is putting it very mildly indeed. The truth of the matter is that very little theory exists on how price and quantity approach an equilibrium situation, or for that matter, whether an equilibrium situation is ever reached in the real world. One sees that the essential shortcoming of classical supply and demand theory is its "static" viewpoint of price behavior. The existing theory can only predict the conditions of static equilibrium and this, unfortunately, provides very little insight into the dynamic behavior of prices.

This shortcoming is brought to the foreground when one attempts to incorporate price information into a dynamic model such as that discussed in Jay Forrester's Harvard Business Review article, "Industrial Dynamics--A Major Breakthrough for Decision Makers" (ref. 2). By using a digital computer for simulation studies, Professor Forrester has developed an entirely new field of industrial research, that is, one in which the dynamic behavior of industrial and economic systems may be simulated and analyzed with any degree of accuracy desired and with a considerable saving of time over all previous methods. Although there has been notable progress in Industrial Dynamics in analyzing the flows of information, materials, orders, manpower, cash, advertising efforts, and even research and development decisions, the area of price behavior has been left virtually untouched. The author feels that this is due to the lack of understanding about the factors which influence the dynamic behavior of prices, as reflected by the small quantity of theory that exists in
economic literature on this subject. That the amount of literature is small can be verified by quoting some typical treatments of this subject by leading texts on basic economics.

In his text, "Price Theory," J. S. Bain refers to the problem of dynamic price behavior as follows: (ref. 3)

A further characteristic of conventional economic theory, which may be noted briefly, concerns its treatment of time. The phenomena with which economics treats are aspects of human behavior as it evolves and changes through time. Explanation of a whole process of development of behavior through time by means of assumptions and deductive logic is of course very difficult, particularly so far as the determinants of behavior may progressively change through time. Much of economic theory as we know it today has been designed to simplify the explanatory task by inquiring mainly into what performance would be with some given and fixed set of determinants, after some sort of stable balance or regular pattern of behavior had been reached in such a given situation. It does not investigate very much the implications of changing determinants, or the reasons for their change, nor does it emphasize the process through which a stable balance would be reached with given determinants. The "end result" of the response of behavior to given determinants is what is underlined. This sort of theory is often referred to as "equilibrium" theory—referring to the final equilibrium result of given determinants. Having this characteristic, it, of course, is not satisfactory for explaining certain processes of change. But in suggesting where we are tending to arrive in a given set of circumstances, it emphasizes perhaps the most important single sort of information we desire to have. Economic theory treating with the character of behavior processes through time is as yet relative undeveloped, and for the moment we have to lean heavily on the aid the "equilibrium" variety of theory can give us. Since we are dealing here with the basic elements of theory, we will confine our attention largely in this work to theory of the equilibrium variety.

We see that Mr. Bain has an acute awareness of the fact that dynamic price theory is relatively undeveloped.
R. H. Leftwich, in his text *The Price System and Resource Allocation* (ref. 4), dismisses the problem of dynamic economic behavior by stating that

Price theory usually assumes a stable economy—one free of major fluctuations up or down—and reasonably full employment of resources. We shall use these assumptions throughout—not because fluctuations and unemployment are unimportant, but because the structure of price theory can be established in a more unequivocal and a simpler way when these two assumptions are made.

In spite of the apparent lack of theory on dynamic price behavior, there are a great number of analytical economists today who attempt to correlate the dynamic and the static (or equilibrium) behavior of prices. That is, by observing the data that have been recorded in industry over the past one hundred years, they are attempting to derive some pertinent information on the static or equilibrium relationship that exists between prices and purchasing rates. In essence, they are trying to derive information about the supply and demand curves (as they are defined in most economic texts) from the dynamic, statistical data that has been generated by the real world. These economists would probably be the first to admit to the possibility that an equilibrium condition is never reached in the real world, but yet they operate on the premise that some relationship must exist between the dynamic behavior of price and quantity and the theoretical equilibrium supply and demand curves.

Are these economists attempting to do something that is futile? If not, then under what conditions will their investigation be fruitful? What are the methods they should employ in their investigation such that
they maximize the amount of useful information derived in a minimum amount of time and effort? These are probably the questions that every analytical economist asks himself when he attempts such an investigation, but up to the present, he has found very few valid answers existing in the classical theory of economic behavior. This is not surprising, for the tools which would provide him with a means of answering his questions have only been developed in the last decade. The tools, in this instance, are comprised of a digital computer accompanied by a proven method of research into the dynamic behavior of economic and industrial systems. With the availability of an IBM 704 digital computer and the method of simulation analysis developed by Professor Forrester in the field of Industrial Dynamics, the economist can now investigate this relationship between static and dynamic price behavior with a maximum degree of accuracy and a minimum expenditure of time and effort.

The purpose of this thesis, then, is to point the direction in which such an investigation might proceed, for such an investigation may take several years of intense, unified research to complete. It is the hope of the author, however, that some of the questions asked by the economists about the relationship between dynamic and static price behavior will be answered by the research in this thesis. If this thesis accomplishes nothing more than creating an awareness of the powerful tool of simulation analysis made available by the field of Industrial Dynamics, it will have made an important contribution to the field of economic research by providing the impetus for further investigation.
Method of Investigation

The effort to establish a relationship (or the lack thereof) between dynamic price behavior and existing equilibrium price theory will proceed along two methods of approach. The first will be an effort to construct a model of a production-distribution system which includes a pricing mechanism based on an "equilibrium" supply curve of raw materials at one end of the system and an equilibrium demand curve for finished units at the other. A simplified flow diagram of this system is shown in Fig. 1.

IRC—Income Rate of Consumers ($/week)

RRR—Requisitions Received at Retail (units/week)

PGR—Price of Goods at Retail ($/unit)

SSR—Shipments Sent by Retail (units/week)
RMOS--Raw Material Orders received by Supplier (tons/week)

PMS--Price of raw Materials at Supplier ($/ton)

RMSS--Raw Material Shipped by Supplier (tons/week)

The basic production-distribution model used between the consumer and supplier sectors will be that formulated by J. W. Forrester in Memo D-34 (ref. 5) with a factory inventory of raw materials added. It is suggested that the reader refer to this memo and Memo D-46 (ref. 8) for the basic philosophy employed in creating such a model.

The only external input to the system will be the income rate of consumers, IRC, which will be assumed to influence the demand curve for finished goods in the consumer sector. The specific assumptions made about each part of the system are set forth as the equations for the model are formulated in Chapters II and III.

Having formulated such a model, we can set the system in "motion" by changing the input quantity, IRC. Under such assumptions, this will shift the customer demand curve to some new position, thereby creating a new equilibrium price and quantity condition towards which the system will tend. The resulting simulation will provide us with data about the price levels and the quantities bought and sold at each time the computer prints out information. This data can then be plotted on a graph similar to that in Fig. 2, and we can examine the relationship between the assumed static (or equilibrium) supply and demand curves and the resulting time-varying values of price and quantity generated by the system.
Our goal in these investigations will be to determine how much information about the assumed demand and supply curves will be carried by the time-varying quantities and prices that are generated by the model. It is possible that there may be some factors within the model itself which, when changed, produce a noticeable change in the amount of "static" information carried by the dynamic data. If such factors can be identified both qualitatively and quantitatively, we may be able to determine some of the conditions under which a transformation from dynamic to static information is valid. An example of such a factor is the average delay in consumers responding to a change in price. This delay certainly exists in the real world and can be easily incorporated into our model by assuming that the actual purchasing rate of the consumer lags the purchasing rate.
determined by the demand curve by some average delay. If this delay is changed, it is fairly obvious that the generated pattern of price and quantity will also change, although the amount and the type of change is not obvious from a knowledge of the delay alone. If we can reach some conclusions about the effects of this delay, we may be able to define the degree to which it affects the amount of static information carried by the dynamic data. If this delay were found to have a marked effect on the static information conveyed to us, we would then want to investigate more completely the actual factors which determine the consumers' response delay. This brings us to the second method of approach in this investigation.

As a result of having performed the simulation outlined above, the author is aware that a more complete model of the consumer sector will be needed. Such a sector would fulfill two requirements. First, it would take into account some of the actual factors which determine the consumers' delay in responding to price changes and secondly, it would allow us to discard the simple demand curve previously assumed, since the consumers' decision to purchase would no longer depend simply on a static curve, but rather upon his need for finished goods and his ability to purchase them.

In Chapter IV of this thesis, then, a more complete consumer sector is created, and the assumptions about it are stated as the equations are formulated. The static (or equilibrium) relationship between price and purchasing rate will be determined both by an equilibrium analysis and by a series of simulation runs. Once the equilibrium demand curve of the consumer sector is determined, we will replace the simple demand curve
that was assumed previously with our more complete consumer sector. The total system will again be set into "motion" and the generated price and quantity information will be examined as in Chapter III by investigating the type and amount of static information which is conveyed and by attempting to identify the factors which influence it.

In the conclusion of this thesis (Chapter V), an attempt will be made to summarize those factors which tend to influence the "static" information that is conveyed by a plot of dynamic values; conclusions about the validity of attempting to relate dynamic to static information will be discussed; and suggestions will be offered for directions of further investigation into this problem.
CHAPTER II
EXPANSION OF BASIC MODEL

The basic model, as formulated by Professor Forrester in Memo D-34 (ref. 5), will first be expanded to include an inventory of raw materials at the factory sector and a supplier sector which produces the raw materials in the form desired by the factory. The equations for the basic model appear in Appendix I.

Factory Inventory of Raw Materials

The flow diagram in Fig. 3 indicates the factors which will be incorporated in the raw material inventory at the factory. To simplify the dimensions of variables in our equation, we will assume that the number of tons of raw materials used in producing one finished unit at the factory is constant. If we were building a model of the refrigerator industry, for example, we might assume that the same number of tons of sheet steel were used in each refrigerator produced. This would probably be a very reasonable assumption.

Under this assumption of a constant number of tons of raw materials per finished unit, we could conveniently refer to a quantity of raw materials in terms of the number of finished units that could be produced from it. Thus we will refer to quantities of raw material in our model in terms of the units they will produce. The number of tons actually contained in any particular level of raw materials could be easily determined by multiplying that level by the constant number of tons per finished unit.
FACTORY RAW MATERIAL INVENTORY SECTOR
FACTORY MANUFACTURING SECTOR

Figure 3
There will always be some level of materials in transit to the factory from the raw material supplier. This level will be increased by raw materials shipped by the supplier and decreased by raw materials received by the factory.

\[ MTF.K = MTF.J + (DT)(RMSS.JK - RMRF.JK) \]  \hspace{1cm} \text{Eq. 1L}

MTF--raw Materials in Transit to the Factory (units)
RMSS--Raw Materials Shipped by Supplier (units/week)
RMRF--Raw Material Received at Factory (units/week)
DT--computational time interval specified by the simulator (weeks)

The letter L following the equation number refers to the type of variable that is defined by the equation. L stands for level, R for rate, A for auxiliary, and F for special function. A complete discussion of the distinction implied by these variable types is included in Appendix II.

The rate of receiving raw materials at the factory will be assumed to lag the rate of shipping raw materials by the supplier by some average delay, DRM. A third-order delay equation will be assumed to apply (see Appendix III).

\[ RMRF.KL = FDELAY3 (MTF.K, DRM) \]  \hspace{1cm} \text{Eq. 2R}

RMRF--Raw Material Received by the Factory (units/week)
MTF--raw Materials in Transit to the Factory (units)
DRM--average Delay in Raw Materials being transported to the factory (weeks)
The level of raw material inventory at the factory will be increased by materials received and decreased by materials used in production at the factory.

\[ \text{RAF.K} = \text{RAF.J} + (\text{DT})(\text{RMRF.JK} - \text{RMUF.JK}) \]  
Eq. 3L

\text{RAF} -- \text{Raw materials inventory Actually at the Factory (units)}

\text{RMRF} -- \text{Raw Materials Received by the Factory (units/week)}

\text{RMUF} -- \text{Raw Materials Used at the Factory (units/week)}

The rate of using raw materials, \text{RMUF}, will of course be equal to the rate of manufacturing finished units, \text{MOF}.

\[ \text{RMUF.KL} = \text{MOF.JK} \]  
Eq. 4R

\text{RMUF} -- \text{Raw Materials Used at the Factory (units/week)}

\text{MOF} -- \text{Manufacturing Orders at the Factory (units/week)}

The decision to order new raw materials will not be based on the instantaneous rate of raw material usage at the factory but rather on the average or "smoothed" rate of material usage.

\[ \text{RMSF.K} = \text{RMSF.J} + (\text{DT})(1/\text{DRUF})(\text{RMUF.JK} - \text{RMSF.J}) \]  
Eq. 5L

\text{RMSF} -- \text{Raw Material usage rate "Smoothed" at the Factory (units/week)}

\text{DRUF} -- \text{Delay in smoothing Raw Materials Used at the Factory, the smoothing time constant (weeks)}

\text{RMUF} -- \text{Raw Materials Used at the Factory (units/week)}

The desired level of raw materials inventory at the factory will be based on the smoothed rate of raw material usage, \text{RMSF}. That is, the
factory might say that if RMSF is constant at 1000 units/week, it would like to have four weeks of inventory on hand at any time. This would mean 4 weeks times 1000 units/week or 4000 units of raw materials desired in inventory.

\[ RDF.K = (ARF)(RMSF.K) \]  

Eq. 6A

RDF--Raw material inventory Desired at the Factory (units)

ARF--constant denoting the number of weeks of "smoothed" Raw material usage rate desired in Factory inventory (weeks)

RMSF--Raw Material usage rate "Smoothed" at the Factory (units/week)

When a decision to purchase raw materials is made by the factory, a number of orders will be sent into the order processing sector of our model. This processing sector of our model includes such factors as the clerical delay in sorting and typing orders and the delay in mailing orders from the factory to the supplier. The level of orders in process, then, will be increased by the decision to purchase materials at the factory and decreased by the orders received by the supplier.

\[ OPRS.K = OPRS.J + (DT)(RMDF.JK - RMOS.JK) \]  

Eq. 7L

OPRS--Orders in the Process of being prepared for and mailed to the Raw material Supplier (units)

RMDF--Raw Material Decision to purchase at the Factory (units/week)

RMOS--Raw Material Orders received by the Supplier (units/week)
The rate of orders being received by the supplier will be assumed to lag the rate of purchase orders being created at the factory by some average delay, DCMF. A third-order delay equation will be assumed to apply.

\[ \text{RMOS}_{KL} = F\text{DELAY}_3 (\text{OPRS}_K, \text{DCMF}) \]  \hspace{1cm} \text{Eq. 8R}

- **RMOS**—Raw Material Orders received by Supplier (units/week)
- **OPRS**—Orders in the process of being prepared for and mailed to the Raw material Supplier (units)
- **DCMF**—Average Delay in receiving orders for raw materials resulting from the Clerical and Mailing process at the Factory (weeks)

At any time, the factory will be aware of the level of orders and raw materials in the "pipeline." This pipeline constitutes orders for raw materials that have been initiated by a purchasing decision at the factory, RMDF, but not yet filled by raw material orders received by the factory inventory, RMRF. This pipeline is made up of orders in process at the factory, unfilled orders at the Supplier, and materials in transit to the factory.

\[ \text{RLAF}_K = \text{OPRS}_K + \text{UOS}_K + \text{MTF}_K \]  \hspace{1cm} \text{Eq. 9A}

- **RLAF**—Raw material orders in the pipeline (Actual) at the Factory (units)
- **OPRS**—Orders in the Process of being prepared for and mailed to the Raw material Supplier (units)
- **UOS**—Unfilled Orders at the Suppliers (units)
- **MTF**—Raw Materials in Transit to the Factory (units)
The desired or necessary level of orders in the pipeline will be based on the "smoothed" rate of raw material usage at the factory, RMSF, and the total average delay encountered in the pipeline.

\[ RLDF.K = (RMSF.K)(DCMF + DFS.K + DRM) \]  

**RLDF**--level of Raw materials in the pipeline Desired (or necessary) at the Factory (units)

**RMSF**--Raw Material usage rate "Smoothed" at the Factory (units/week)

**DCMF**--average Delay in receiving orders for raw materials resulting from the Clerical and Mailing process at the Factory (weeks)

**DFS**--average Delay (variable) in Filling orders at the Supplier (weeks)

**DRM**--average Delay in Raw Materials being transported to the factory (weeks)

The decision to purchase raw materials at the factory will be governed by the following equation.

\[ RMDF.KL = RMUF.JK + (1/DRIF)(RDF.K - RAF.K + RLDF.K - RLAF.K + UOF.K - UNF.K) \]  

**RMDF**--Raw Material purchasing Decision at the Factory (units/week)

**DRIF**--average Delay in adjusting the Raw Material Inventory at the Factory (weeks)

**RDF**--Raw material inventory Desired at the Factory (units)

**RAF**--Raw material inventory Actually at the Factory (units)

**RLDF**--level of Raw materials in the pipeline Desired at the Factory (units)

**RMUF**--Raw Materials Used in production at the Factory (units/week)
RLAF--level of Raw materials in the pipeline (Actual) at the Factory (units)

UOF--Unfilled Orders for finished units at the Factory (units)

UNF--level of Unfilled Orders for finished units Normally at the Factory (units)

This equation states that the factory will purchase raw materials in order to replace raw materials that are being used, RMUF, and to make up for some fraction, 1/DRIF, of the difference between the desired level and the actual level of materials in its inventory and pipeline. The unfilled orders term, UOF, is included to cover the situation in which the material supplier has run out of materials completely. If such were the case, consumers would eventually stop purchasing, due to an infinite delay in filling orders at retail. There would then be some level of unfilled orders at retail, UOR, equal to the unfilled orders at the distributor, UOD, and also equal to those at the factory, UOF. Correspondingly, there would be an equivalent level of unfilled orders at the supplier, UOS, which would constitute the total level of orders in the pipeline, RLAF, at the factory. Thus in equation 11R we see that in the extreme situation just mentioned, the rate of raw material usage, RMUF, would fall to zero, as would the actual level of materials in the factory inventory, RAF. The desired level of materials in inventory and pipeline would also fall to zero since the smoothed rate of material usage, RMSF, would fall to zero with the actual usage rate, RMOF (Equations 6A and 10A). Thus the only non-zero term would be the actual level of material orders in the pipeline, RLAF. If the unfilled orders term, UOF, were not included,
the rate of material purchasing, RMDF, would become negative and begin to cancel the unfilled orders at the supplier. This could conceivably happen in the real world, but we wish to make the assumption that once an order has been placed, the sector which placed it will wait indefinitely for the order to be filled. Thus under the extreme situation of the Supplier running out of materials, we see that the unfilled order term, UOF, will just equal the actual pipeline term, RLAF, and thereby allow the material purchasing rate, RMDF, to be zero. The last term, UNF, is included so that under normal steady-state conditions the actual level of raw materials in inventory, RAF, will equal that desired, RDF. If UNF were not included, RDF and RAF would differ by an amount equal to UOF in steady-state, and this is not what we mean when we use the term, "normal steady-state conditions." Under equilibrium conditions, then, we see that the raw material purchasing rate, RMDF, will just equal the rate at which materials are being used, RMUF, and the level of materials in inventory and pipeline will just equal that desired.

We now wish to place a constraint on our model which exists in the real world, namely, that the factory cannot remove more materials from its inventory than it actually has on hand. This is accomplished by calculating the negative inventory rate, NIRM.

\[
NIRM_K = \frac{RAF_K}{DT}
\]

Eq. 12A

NIRM--Negative Inventory Rate of using raw Materials at the factory (units/week)

RAF--Raw materials Actually in inventory at the Factory (units)
DT—computational time period specified by the simulator (weeks)

NIRM will be the maximum rate at which raw materials can be removed from inventory over the next computational time interval, DT.

Since the material usage rate, RMUF, is equal to the manufacturing rate, MOF, and this rate is, in turn, determined by the decision to manufacture at the factory, MDF, we wish to recognize two factors which might limit the values that MDF could assume. In essence, we wish to recognize the capacity limitations of the factory due to 1) a limited level of capital equipment and factory workers and 2) a limited level of raw material inventory from which to produce finished goods. The first of these two constraints has already been incorporated by Professor Forrester in the form of equation 74N (Appendix I). This equation defines the quantity, ALF, which is the constant specifying the manufacturing capacity limit at the factory. We now wish to interpret ALF as being the capacity limit due to machines and labor, since the capacity limit due to available raw materials has already been represented by NIRM, the negative inventory rate at the factory. With this interpretation, then, the following quantity is defined:

\[ \text{AMLF}_K = \text{NIRM}_K, \text{ if } \text{NIRM}_K < \text{ALF} \]
\[ = \text{ALF}, \text{ if } \text{NIRM}_K \geq \text{ALF} \]

\textbf{AMLF}—quantity denoting \textit{Manufacturing capacity and material Limiting rate at the Factory (units/week)}

\textbf{NIRM}—Negative Inventory Rate for raw Materials at factory--limiting rate due to available materials (units/week)
ALF--constant specifying manufacturing capacity Limit at Factory due to available machines and labor (units/week)

We will now change Professor Forrester's equation 46R to the following:

\[ MDF.KL = \begin{cases} MWF.K & \text{if } MWF.K < AMLF.K \\ AMLF.K & \text{if } MWF.K \geq AMLF.K \end{cases} \quad \text{Eq. 14R} \]

- **MDF**--Manufacturing Decision at the Factory (units/week)
- **MWF**--Manufacturing rate Wanted at the Factory (units/week)
- **AMLF**--quantity denoting Manufacturing capacity and material Limiting rate at Factory (units/week)

This completes the raw material inventory sector at the factory except for the specification of initial conditions and necessary parameters.

**Initial Conditions**

Initial conditions are required by DYNAMO (ref. 6) for all level, L, equations, and for all rates that appear on the right-hand side of auxiliary, A, or rate equations, R. It is desirable to start the system out in equilibrium, for if this were not done, it would be very difficult to separate the effect of a change in the inputs to the system from the transient effects caused by the system being out of equilibrium initially. The initial conditions, then, will be based on equilibrium rates and levels.

The level of materials in transit to the factory, under equilibrium conditions, will be given by the product of the average transportation
delay, DRM, and the rate of raw material usage at the factory, RMUF.

\[ MTF = (DRM)(RMUF) \]  
\text{Eq. 15N}

MTF--raw Materials in Transit to the Factory (units)
DRM--average Delay in Raw Materials being transported to the factory (weeks)
RMUF--Raw Materials Used at the Factory (units/week)

The level of raw material inventory, RAF, will just equal that desired.

\[ RAF = (ARF)(RMUF) \]  
\text{Eq. 16N}

RAF--Raw material inventory Actually at the Factory (units)
ARF--constant denoting the number of weeks of "smoothed" Raw material usage rate desired at the Factory (weeks)
RMUF--Raw Materials Used at the Factory (units/week)

The level of orders in process will be that necessary in view of the average delay in orders being prepared for and mailed to the supplier.

\[ OPRS = (DCMF)(RMUF) \]  
\text{Eq. 17N}

OPRS--Orders in the Process of being prepared for and mailed to the Raw material Supplier (units)
DCMF--average Delay in receiving raw materials resulting from the Clerical and Mailing process at the Factory (weeks)
RMUF--Raw Materials Used at the Factory (units/week)

The initial manufacturing rate, MOF, required in equation 4R will be equal to the equilibrium rate of requisitions received by the factory for finished units, RRF.
MOF = RRF

MOF--Manufacturing Orders at the Factory (units/week)
RRF--Requisitions Received at the Factory (units/week)

The raw material usage rate, RMUF, needed in the initial condition equations 15N, 16N, and 17N will be equal to the equilibrium rate of manufacturing finished units, MOF.

RMUF = MOF

RMUF--Raw Materials Used at the Factory (units/week)
MOF--Manufacturing Orders at the Factory (units/week)

The smoothed rate of raw material usage, RMSF, needed in equations 6A and 10A will be equal to the equilibrium rate of material usage, RMUF.

RMSF = RMUF

RMSF--Raw Material usage rate "Smoothed" at the Factory (units/week)
RMUF--Raw Materials Used at the Factory (units/week)

Parameters (Constants)

Reasonable values for the parameters of the material inventory sector will be assumed, with the understanding that should the system show an unusual sensitivity to any one of these constants, a closer examination could be made of its value in the real world.

The average delay in materials being shipped to the factory is assumed to be three weeks.
DRM = 3 weeks

The number of weeks of "smoothed" raw material usage desired in inventory is assumed to be four.

ARF = 4 weeks

A smoothing constant of eight weeks is assumed for the rate of material usage.

DRUF = 8 weeks

The average delay in adjusting the material inventory at the factory is assumed to be four weeks.

DRIF = 4 weeks

Finally, the average delay in processing raw material orders is estimated at two weeks.

DCMF = 2 weeks

**Raw Material Supplier Sector**

The supplier sector to be incorporated in our model is to represent the aggregation of suppliers that furnish raw materials to the aggregation of manufacturers. This sector will be characterized by a manufacturing division, an inventory of raw materials, a backlog of unfilled orders, and several other factors that are similar to the factory sector of the basic production-distribution model. The manufacturing process at
the supplier might be considered as either the process of preparing basic materials into semi-finished articles or as the actual accumulation and transportation of the basic materials themselves. An example of the former might be the production of semi-finished steel for refrigerators, whereas the latter might be the mining and transportation of copper or coal or some other basic material essential to the factory's manufacturing process.

System Equations

Referring to the flow diagram in Fig. 4, the level of unfilled orders for raw materials is increased by incoming orders and decreased by raw materials shipped by the supplier.

\[
\text{UOS}.K = \text{UOS}.J + (\text{DT})(\text{RMOS}.JK - \text{RMSS}.JK) \quad \text{Eq. 21L}
\]

UOS -- Unfilled Orders at Supplier (units)
RMOS -- Raw Material Orders received by Supplier (units/week)
RMSS -- Raw Materials Shipped by Supplier (units/week)

The inventory level of raw materials is increased by materials received from the production (or mining) sector and decreased by materials shipped by the supplier.

\[
\text{IAS}.K = \text{IAS}.J + (\text{DT})(\text{RMRS}.JK - \text{RMSS}.JK) \quad \text{Eq. 22L}
\]

IAS -- Inventory of raw materials Actually at the Supplier (units)
RMRS -- Raw Materials Received by inventory at the Supplier (units/week)
RMSS -- Raw Materials Shipped by the Supplier (units/week)
The "smoothed" rate of receiving raw material orders at the supplier is defined by the following equation.

\[
\text{ROSS.K} = \text{ROSS.J} + (\text{DT})(1/\text{DROS})(\text{RMOS.JK} - \text{ROSS.J}) \quad \text{Eq. 23L}
\]

ROSS--Raw Material Order rate "Smoothed" at Supplier (units/week)

DROS--constant Determining smoothing rate of Raw material Orders at Supplier, the smoothing time constant (weeks)

RMOS--Raw Material Orders received at Supplier (units/week)

The inventory desired at the factory will be based on the "smoothed" rate of raw material orders received at the supplier.

\[
\text{IDS.K} = (\text{AIS})(\text{ROSS.K}) \quad \text{Eq. 24A}
\]

IDS--Inventory Desired at Supplier (units)

AIS--constant denoting the number of weeks of "smoothed" material orders desired in Inventory at Supplier (weeks)

ROSS--Raw material Order rate "Smoothed" at Supplier (units/week)

The decision to produce (or mine) raw materials at the supplier will be assumed to insert production orders into the sector labeled OPS. This level of orders in production will be decreased by raw materials received into inventory at the supplier.

\[
\text{OPS.K} = \text{OPS.J} + (\text{DT})(\text{PDS.JK} - \text{RMRS.JK}) \quad \text{Eq. 25L}
\]

OPS--Orders in Production at Supplier (units)
PDS--Production Decision at Supplier (units/week)

RMRS--Raw Materials Received into inventory at Supplier (units/week)

Thus the pipeline of materials that have been ordered into production but not received into inventory is made up solely of orders in production, OPS.

\[ \text{LAS}.K = \text{OPS}.K \quad \text{Eq. 26A} \]

LAS--pipeline of materials (Actual) in the production process at Supplier (units)

OPS--orders in Production at Supplier (units)

The desired or necessary level of materials in the pipeline, LAS, will be determined by the average delay in producing materials and the "smoothed rate" of receiving material orders at the supplier.

\[ \text{LDS}.K = (\text{DPS})(\text{ROSS}.K) \quad \text{Eq. 27A} \]

LDS--pipeline Desired at Supplier (units)

DPS--average Delay in Producing materials at the Supplier (weeks)

ROSS--Raw material Order rate "Smoothed" at Supplier (units/week)

The rate of receiving materials into inventory at the supplier will be assumed to lag the decision to produce by the average production delay, DPS. A third-order delay equation will be assumed to apply.

\[ \text{RMRS}.KL = F\text{DELAY}\text{3} (\text{OPS}.K, \text{DPS}) \quad \text{Eq. 28R} \]

RMRS--Raw Materials Received at Supplier (units/week)
OPS--Orders in Production at Supplier (units)

DPS--average Delay in Producing raw materials at Supplier (weeks)

The decision to produce materials will depend upon the rate of receiving material requisitions and the difference between the desired and actual level of materials in inventory and in the pipeline at the supplier.

\[ PWS.K = RMOS.JK + \frac{1}{DRPS}(IDS.K - IAS.K + LDS.K - LAS.K + UOS.K - UNS.K) \]

Eq. 29A

PWS--Production Wanted at Supplier (units/week)

RMOS--Raw Material Orders received by Supplier (units/week)

DRPS--average Delay in adjusting Raw material inventory and Pipeline at Supplier (weeks)

IDS--Inventory Desired at Supplier (units)

IAS--Inventory (Actual) at Supplier (units)

LDS--pipeline Desired at Supplier (units)

LAS--pipeline (Actual) at Supplier (units)

UOS--Unfilled Orders at Supplier (units)

UNS--Unfilled orders Normally at Supplier (units)

The quantities UOS and UNS are included for the same reasons which motivated the inclusion of UOF and UNF in equation II.R.

The actual production decision will be based upon the supplier's ability to produce, i.e., his limiting production capacity.

\[ PDS.KL = PWS.K, \text{ if } PWS.K < ALS \]

\[ = ALS, \text{ if } PWS.K \geq ALS \]

Eq. 30R
PDS--Production Decision at Supplier (units/week)
PWS--Production rate Wanted at Supplier (units/week)
ALS--constant denoting production capacity Limit at Supplier (units/week)

The shipments sent from inventory at the supplier will depend upon the level of unfilled orders and the average delay in filling orders.

\[ STS.K = \frac{UOS.K}{DFS.K} \]  
\text{Eq. 31A}

STS--Shipment rate Tried at Supplier (units/week)
UOS--Unfilled Orders at Supplier (units)
DFS--average Delay (variable) in Filling orders at Supplier (weeks)

The variable delay in filling orders will be assumed to be made up of a minimum delay in handling orders and a variable delay due to "out-of-stock" materials. This delay due to an "out-of-stock" condition will depend upon the ratio of desired to actual inventory.

\[ DFS.K = DHS + (DUS)(IDS.K/IAS.K) \]  
\text{Eq. 32A}

DFS--average Delay (variable) in Filling orders at Supplier (weeks)
DHS--minimum Delay in Handling orders at Supplier (weeks)
DUS--normal average Delay in filling Unfilled orders due to "out-of-stock" items at Supplier (weeks)
IDS--Inventory Desired at Supplier (units)
IAS--Inventory (Actual) at Supplier (units)

The reader is referred to Forrester's Memo D-34 (ref. 5), pages 18 to 22, for a thorough discussion of this average delay equation.
II-21

The actual shipping rate at the supplier will be limited by the total level of inventory it has on hand. We thus keep track of the negative inventory rate at the supplier.

\[
\text{NIS}.K = \frac{\text{IAS}.K}{\text{DT}} \quad \text{Eq. 33A}
\]

\begin{align*}
\text{NIS} & \quad \text{Negative Inventory rate at Supplier (units/week)} \\
\text{IAS} & \quad \text{Inventory (Actual) at Supplier (units)} \\
\text{DT} & \quad \text{computational time interval chosen by the simulator (weeks)}
\end{align*}

The shipping rate at the supplier will thus be determined by the following equation.

\[
\text{RMSS}.\text{KL} = \begin{cases} 
\text{STS}.K, & \text{if } \text{STS}.K < \text{NIS}.K \\
\text{NIS}.K, & \text{if } \text{STS}.K \geq \text{NIS}.K
\end{cases} \quad \text{Eq. 34R}
\]

\begin{align*}
\text{RMSS} & \quad \text{Raw Materials Shipped by Supplier (units/week)} \\
\text{STS} & \quad \text{Shipment rate Tried at Supplier (units/week)} \\
\text{NIS} & \quad \text{Negative Inventory rate at Supplier (units/week)}
\end{align*}

The only quantity which lacks an equation is \text{UNS}, the normal or necessary level of unfilled orders at the supplier.

\[
\text{UNS}.K = (\text{ROSS}.K)(\text{DHS} + \text{DUS}) \quad \text{Eq. 35A}
\]

\begin{align*}
\text{UNS} & \quad \text{Unfilled orders Normally at Supplier (units)} \\
\text{ROSS} & \quad \text{Raw material Order rate "Smoothed" at Supplier (units/week)} \\
\text{DHS} & \quad \text{minimum order-filling Delay due to handling at Supplier} \\
\text{DUS} & \quad \text{normal average Delay in filling Unfilled orders due to "out-of-stock" items at Supplier (weeks)}
\end{align*}
Initial Conditions

The initial conditions will again be calculated with the desire to have our system in equilibrium at the start of a simulation.

The level of unfilled orders will be determined by the normal order-filling delay and the "smoothed" rate of receiving requisitions at the supplier.

\[
UOS = (ROSS)(DHS + DUS) \quad \text{Eq. 36N}
\]

\begin{align*}
UOS & \text{--Unfilled Orders at Supplier (units)} \\
ROSS & \text{--Raw material order rate "Smoothed" at Supplier (units/week)} \\
DHS & \text{--minimum Delay in filling orders due to Handling at Supplier (weeks)} \\
DUS & \text{--normal average Delay in filling Unfilled orders due to "out-of-stock" items at Supplier (weeks)}
\end{align*}

The initial level of orders in production will be determined by the product of the equilibrium rate of receiving requisitions by the supplier and the average delay in production.

\[
OPS = (DPS)(RMOS) \quad \text{Eq. 37N}
\]

\begin{align*}
OPS & \text{--Orders in Production at Supplier (units)} \\
DPS & \text{--average Delay in Production at Supplier (weeks)} \\
RMOS & \text{--Raw Material Orders received by Supplier (units/week)}
\end{align*}

The level of inventory will be determined by the number of weeks of "smoothed" order-receiving rate desired in IAS.

\[
IAS = (AIS)(ROSS) \quad \text{Eq. 38N}
\]
IAS—Inventory Actually at Supplier (units)
AIS—constant determining desired Inventory at Supplier (weeks)
ROSS—Raw material Order rate "Smoothed" at Supplier (units/week)

The rate of receiving raw material requisitions will be defined initially as follows:

\[ RMOS = RMSI \quad \text{Eq. 39N} \]

RMOS—Raw Material Orders received at Supplier (units/week)
RMSI—Raw Material orders received by Supplier Initially (units/week)

The "smoothed" rate of receiving orders will just equal the actual rate initially.

\[ ROSS = RMOS \quad \text{Eq. 40N} \]

ROSS—Raw material Order rate "Smoothed" at Supplier (units/week)
RMOS—Raw Material Orders received by Supplier (units/week)

The initial rate of receiving material orders will be set equal to the initial rate of receiving requisitions at retail.

\[ RMSI = RRR \quad \text{Eq. 41N} \]

RMSI—Raw Material orders received by Supplier Initially (units/week)
RRR—Requisitions received at Retail (units/week)

We must also define the maximum rate of production due to capacity limitations at the supplier. It will be set abnormally high during our
initial investigations on price behavior.

\[ ALS = (1000)(RMOS) \quad \text{Eq. 42N} \]

ALS\--constant denoting the production capacity Limit at Supplier (units/week)

RMOS\--Raw Material Orders received at Supplier (units/week)

**Parameters (Constants)**

We will again attempt to pick reasonable values for our system parameters. The "smoothing time" constant at the supplier will be chosen as eight weeks.

\[ \text{DROS} = 8 \text{ weeks} \]

The average delay in adjusting the raw material inventory at the supplier will be picked as four weeks.

\[ \text{DRPS} = 4 \text{ weeks} \]

The average delay in production at the supplier will be chosen as six weeks.

\[ \text{DPS} = 6 \text{ weeks} \]

The minimum handling delay and the average delay due to "out-of-stock" items will be assumed to be one week each.

\[ \text{DHS} = 1 \]

\[ \text{DUS} = 1 \]

Finally, the number of weeks of "smoothed" raw material orders desired in inventory at the supplier will be chosen as four.

\[ \text{AIS} = 4 \text{ weeks} \]
CHAPTER III
PRICING MECHANISM

Supply Schedule

Having expanded the basic model to include an inventory of raw materials at the factory and a raw material supplier sector, we can now proceed to the formulation of a pricing mechanism. At the start of our investigation, we will accept and employ the equilibrium price theory which exists presently in formulating the supply and demand schedules for the supplier and consumer sectors of our model. Later, in Chapter IV, we will take a deeper look into these concepts by discarding the theoretical demand curve at the consumer end of the model and replacing it with a more realistic consumer sector whose purchasing rate is determined by the consumer's physical need for products and his financial ability to purchase them. In essence, we will first employ a hypothetical demand curve, whereas later we shall investigate some of the physical factors which determine the shape and position of such a curve.

Our first consideration, then, will be the formulation of hypothetical supply and demand curves. It will be assumed that there exists at the supplier some supply relationship which determines the price per ton (or per unit) of raw materials corresponding to any rate at which the supplier receives requisitions from the factory. Rather than assuming that the supplier's price depends directly upon the rate of requisitions received, RMOS, we will assume that his instantaneous price is determined by the average rate of receiving orders, or in our model, the "smoothed" rate,
ROSS. This would appear, intuitively, to be a more reasonable assumption, since the actual rate, RMOS, would vary considerably from day to day, whereas the "smoothed" rate, ROSS, would be a better indication of the general "level" of raw material sales at any time. The supply schedule, then, will be similar to that pictured in Fig. 5.

**FIGURE 5**  
**TYPICAL SUPPLY SCHEDULE**

PRM--Price of Raw Materials at supplier ($/unit)

ROSS--Raw material Order rate "Smoothed" at Supplier (units/week)

Economists have the feeling that such a supply curve generally slopes upward to the right as shown in Fig. 5 due to the principle of "diminishing returns." That is, over a period of time the efficiency of the supplier in producing a ton of raw materials will tend to decrease as
the rate of production increases. Likewise, an increase in the production cost per ton is assumed to result from this decrease in efficiency of production. Assuming, then, that in most cases the price charged by the supplier reflects his production costs, it follows that the curve relating his average "level" of sales activity to the price he charges for materials would tend to slope upward to the right.

The author, at this point, wishes to make a distinction between "long-run" and "short-run" supply (and demand) curves. We will not, incidentally, use the same definitions employed by most economists in distinguishing between short- and long-run curves, due to some of the new problems which arise from employing these curves in a dynamic model. The phrase, "long-run supply curve" will be synonymous with the phrase "equilibrium supply curve," whereas a "short-run supply curve" will refer to the curve that is traced out by the instantaneous values of price and quantity generated by our model. In other words, the long-run supply (or demand) schedule represents the values of price and quantity which can be sustained in equilibrium, whereas the short-run schedule is an indication of the dynamic, instantaneous values which price and quantity will assume at any time. The former represents the "static" price information which is incorporated in the model, while the latter is the dynamic price information which is generated through a simulation run.

The purpose of this thesis, as stated in Chapter I, is to investigate the relationship which exists between these two types of price information. The definitions presented above should not be confused with the definitions that are usually presented by economic texts (see ref. 9). In accord with
our own definitions, then, we will proceed with the formulation of the long-run or equilibrium supply curve to be incorporated in our model.

We will presently accept the economists' feelings about the long-run supply curve sloping upwards to the right and inquire about the exact shape of the curve in various situations. The exact shape will probably differ from time to time and from product to product depending upon the economic and industrial environment which exists. We might, of course, try different shapes, as time permits, to gain a feeling for how the shape influences the dynamic behavior of the system. At the start, however, we will employ a linear or straight line curve to represent the supply schedule. This is probably not a bad assumption if we only "operate" over a small range of the curve, for any type of curve appears to be nearly linear when one looks at a very small portion of it.

Our initial supply curve, then, will be that shown in Fig. 6.
The following equation applies:

$$PRM_K = ARZ(1 + ROSS JK/ART)$$

Eq. 43A

PRM--Price of Raw Materials at the supplier ($/unit)

ROSS--Raw material Order rate "Smoothed" at Supplier (units/week)

ARZ--constant denoting price of Raw materials when the "smoothed" rate of orders is Zero.

ART--constant denoting "smoothed" Rate of material orders at which the price is Two times ARZ (units/week)

Note that the constants ARZ and ART refer to extreme conditions under which we assumed we would not operate. These constants are quite permissible, however, for they simplify the formulation of a linear supply curve, in spite of our assumption that the price will never be driven to the level of ARZ. It should be mentioned here that the use of this linear supply curve is in conflict with the philosophy of having plausible extreme conditions as set forth in Memo D-46 (ref. 8). We will attempt to correct this situation later, however, after we gain a feeling for the dynamic response of our system.

**Concept of Price Flexibility**

It should be apparent from observation that the slope or the "steepness" of the supply curve will have a great influence on the dynamic behavior of our system. If the curve were very "steep," this would imply that a small change in the "smoothed" rate of material orders, ROSS, would cause a large change in the price, PRM. In other words, the "steepness" of the supply curve is a measure of the sensitivity of price to a change
in the "smoothed" rate of material orders, ROSS. Economists are wary of using "steepness" as the only measure of price sensitivity, however, for the following reason. Figures 7A and 7B represent exactly the same supply curve, the only difference between the curves being the scaling used along the horizontal axis.

![Graphs showing price vs. quantity](image)

We see that the curve in Figure 7A appears to be considerably "steeper" than that in Figure 7B, in spite of the fact that both curves represent the same supply schedule.

Economists have overcome this difficulty by defining price sensitivity in a slightly different way. They define a quantity known as price "flexibility" which is a better measure of the relative sensitivity of price to changes in the quantity demanded. The "flexibility" of price is defined
as the per cent change in price due to a one per cent change in quantity at any particular point on the supply curve. The "flexibility" of supply will always be a positive number, since the supply curve slopes upward to the right (i.e., an increase in quantity demanded will cause an increase in price). The following equations define the "flexibility" of supply explicitly.

\[
\text{Average Flexibility} = \frac{\% \text{ change in price}}{\% \text{ change in quantity}}
\]

or \[
F = \frac{\text{change in price/price}}{\text{change in quantity/quantity}}
\]

\[
F = \frac{\Delta P}{P} \frac{\Delta Q}{Q}
\]

\[
F = \frac{Q}{P} \frac{\Delta P}{\Delta Q}
\]

The actual "flexibility" at a given point in the supply curve is the limit of the average "flexibility" as \(\Delta Q\) approaches zero.

\[
F = \text{Limit} \frac{Q}{P} \frac{\Delta P}{\Delta Q} \quad \text{as} \quad \Delta Q \to 0
\]

In accord with elementary calculus, the following equation is derived.

\[
F = \frac{Q}{P} \frac{dP}{dQ}
\]

The quantity \(dP/dQ\) is just the slope of the supply curve at the point \((P, Q)\) - Figure 8.

Under the foregoing definition, then, a "flexibility" of two at some point on the supply curve would imply that a one per cent increase in quantity would cause a two per cent increase in price.
We can now derive the "flexibility" of price at any point on our linear supply curve.

\[ \text{PRM}. K = \text{ARZ}[1 + \text{ROSS}. JK/\text{ART}] \]  
\[ \text{Eq. 43A} \]

or

\[ P = (\text{ARZ}) + \left(\frac{\text{ARZ}}{\text{ART}}\right) Q \]

\[ \frac{dP}{dQ} = \frac{\text{ARZ}}{\text{ART}} \]
\[ F = \frac{Q}{P} \left( \frac{ARZ}{ART} \right) \]

\[ F = \frac{Q}{ARZ \left[ 1 + \frac{Q}{ART} \right]} \left( \frac{ARZ}{ART} \right) \]

\[ F = \frac{Q}{ART + Q} \quad \text{or} \quad F = \frac{ROSS.JK}{ROSS.JK + ART} \quad \text{Eq. 44} \]

We see immediately that the price "flexibility" is not constant over the whole supply curve, but rather it increases as we move "up" the curve. The slope or "steepness" of the curve is constant, but not the "flexibility" as we have defined it. The reason for this should become clear to the reader after a few moments of contemplation.

Later we may wish to expand our supply curve equation to take account of some of the physical factors which might influence the long-run supply schedule. Such a factor might be the state of the raw material inventory at the supplier, since the supplier might cut prices if he is heavily over-stocked or raise them if his inventory becomes abnormally depleted. His availability of labor or machines might also influence the supplier's price, although these factors would probably be reflected in the level of his inventory sooner or later.

This discussion points up the need for further investigation into the actual factors which determined the long-run supply curve, such as inventory conditions, labor, capital equipment, and cash availability, as well as other industrial and economic factors. An expansion of our supplier sector would probably be necessary to accomplish this in an adequate fashion. For the present, however, we will accept equation 43A as
representing the long-range supply schedule. We can now proceed to the formulation of the mechanism by which the price of raw materials is "propagated" through the system to become reflected in the price of finished goods at retail.

**Price Propagation**

The basic assumptions behind our price "propagating" mechanism will be the following:

1) That the price of finished units at the factory, distributor, and retailer is formulated on a "cost-plus" basis.

2) That the difference between price and cost at the factory, distributor, and retailer reflects the operating or overhead costs and the desired levels of profit at each sector.

3) That the "overhead and profit margin" at each sector is constant with respect to time, although this margin may vary from sector to sector.

4) That therefore we can, for the sake of convenience, refer to the price of raw materials at the supplier sector in terms of the price that will be charged at retail for the finished goods produced from these raw materials.

In essence, then, we are assuming that the price of raw materials is actually "propagated" through the factories, distributors, and retailers and is altered only by the "profit plus overhead" margins which are assumed constant at each of these sectors.

In addition, we are reducing the number of equations needed in our model by taking all of these constant "profit plus overhead" margins into account in the price of raw materials at the supplier. The price of raw materials at the supplier, PRM, thus represents the price that will be charged at retail for the finished goods produced from these materials.
That the price, PRM, is "passed down" through the system with the actual flow of finished goods is an implicit assumption in our model. This might be quite accurate if the price of a finished refrigerator were stamped on it at the factory. The delays in "propagating" price information through the system would thus be exactly the same as those which applied to the flow of finished units.

In reality, price information is probably "propagated" through the system in much less time than is required for a finished unit to travel from the factory to the retailer. For the sake of generality, then, we will follow separately the flow of price information and the flow of raw materials and finished units, although the two may be interrelated.

There will be two distinct types of information delays recognized in our system; first, the delay incurred in passing price information from one sector to another, and second, the delay incurred in placing this received price information into effect at any one sector. We will distinguish between the price information sent from one sector and that received at the following sector by referring to the former as the price of units and the latter as the cost of units at the corresponding sectors. The first set of equations should clarify these concepts.

Information on the cost of raw materials to the factory will be assumed to be delayed by some average time, DSF. A third-order delay function will be assumed to apply here. (Appendix III)

\[ CRM.K = \text{DELAY}_3 (AUX1.K, \text{DISF}) \] \hspace{1cm} \text{Eq. 45A}

\[ AUX1.K = AUX1.J + (DT)(PRM.J - CRM.J) \] \hspace{1cm} \text{Eq. 46L}

Fig. 9 represents this information flow diagram.
CRM -- Cost of Raw Materials to the factory ($/unit)

PRM -- Price of Raw Materials at the supplier ($/unit)

AUX1.K -- Auxiliary quantity (l) necessary in formulating a third-order delay function ($ weeks/unit)

DISF -- Average Delay in passing price Information from Supplier to Factory (weeks)

The author apologizes for the use of an auxiliary quantity with the bizarre dimensions of dollar weeks per unit, but it is necessitated at the present due to the lack of a thorough understanding of the process of delaying information. A point of confusion arises when one realizes that there are two factors which must be accounted for when information is delayed; namely, the amount of information which has been sent out but not received and the frequency at which this information is transmitted. One sees, then, that in the case of prices, it is not only
necessary to keep track of the amount of price change that is contained in an information delay, but also the number of messages that have been sent out on price information. This is obviously a problem which lies in the realm of Information Theory and has yet to be resolved completely by the Industrial Dynamics research group. Until such time, however, we are forced to use some sort of artifice in representing this information delay. The particular artifice which the author has chosen requires the use of auxiliary quantities like AUX1 which have rather bizarre dimensions.

Having thus defined the relationship between the cost of materials at the factory and the price of materials at the supplier, we must next formulate the mechanism by which the cost of materials becomes reflected in the price of finished goods at the factory. We shall recognize three more delays encountered by price information at the factory: 1) the delay in applying the cost of materials received at the factory to those materials used in production, i.e., the delay associated with "passing" prices through the factory's raw material inventory, 2) the delay in applying the cost of materials used in production to the finished units shipped to the factory inventory sector, IAF, and 3) the delay associated with "passing" price information through the factory inventory. The information which emerges at this point in the system is, of course, the price of goods at the factory. See Fig. 10.

The author has chosen to represent the mechanism by which price information is passed through inventories by an averaging process; i.e., the price of materials at the factory is determined by the average cost of the materials in the factory inventory. This does not appear, at first
CRM—Cost of Raw Materials to the factory ($/unit)

CMPF—Cost of raw Materials used in Production at the Factory ($/unit)

CGF—Cost of Goods received at Factory inventory ($/unit)

PGF—Price of Goods shipped from the Factory ($/unit)

At a glance, to be a particularly reasonable assumption. If we assume, however, that price information is passed through inventory with the actual physical units, it might be quite accurate to assume that the average price of all goods being sold from all factory inventories would be approximately equal to the average cost of the total goods in inventory. If price information does not flow through inventory with the actual physical units, this type of mechanism would, of course, be invalid. For the present, however, we shall accept the validity of the following equations:
RIV.K = RIVF.J + (DT)(VIRF.JK - VORF.JK) \hspace{1cm} Eq. 47L

VIRF.KL = (RMRF.JK)(CRM.K) \hspace{1cm} Eq. 48R

VORF.KL = (RMUF.JK)(CMPF.K) \hspace{1cm} Eq. 49R

CMPF.K = RIVF.K/RAF.K \hspace{1cm} Eq. 50A

RIVF--Raw material Inventory Value at Factory ($)

VIRF--Value flowing Into Raw material inventory at Factory ($/week)

VORF--Value flowing Out of Raw material inventory at Factory ($/week)

CRM--Cost of Raw Materials to the factory ($/unit)

CMPF--Cost of Materials used in Production at Factory ($/unit)

RAF--Raw Material inventory (Actual) at Factory (units)

RMRF--Raw Materials Received by Factory (units/week)

RMUF--Raw Materials Used in production at the Factory (units/week)

The flow diagram appears in Fig. 11.
We shall continue by assuming a third-order delay relationship between the cost of materials used in production and the cost of finished goods received at the factory.

\[ CFG.K = FDELAY3(AUX2.K, DIM) \quad \text{Eq. 51A} \]
\[ AUX2.K = AUX2.J + (DT)(CMPF.J - CFG.J) \quad \text{Eq. 52L} \]

**CFG**--Cost of Goods to Factory ($/unit)

**CMPF**--Cost of Materials used in Production at Factory ($/unit)

**AUX2**--Auxiliary variable (2) necessary in defining a third-order delay function ($/weeks/unit)

**DIM**--average Delay in transmitting price Information through the Manufacturing sector (weeks)

The price of goods at the factory will again be determined by the average value of goods in inventory.

\[ IVF.K = IVF.J + (DT)(VIF.VK - VOF.VK) \quad \text{Eq. 53L} \]
\[ VIF.KL = (SRF.JK)(CFG.K) \quad \text{Eq. 54R} \]
\[ VOF.KL = (SSF.JK)(PGF.K) \quad \text{Eq. 55R} \]
\[ PGF.K = IVF.K/IAF.K \quad \text{Eq. 56A} \]

**IVF**--Inventory Value at Factory ($)

**VIF**--Value flowing Into inventory at Factory ($/week)

**VOF**--Value flowing Out of inventory at Factory ($/week)

**SRF**--Shipments Received by Factory (units/week)

**SSF**--Shipments Sent by Factory (units/week)

**CFG**--Cost of Goods to Factory ($/unit)
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PGF--Price of Goods at Factory ($/unit)

IAF--Inventory (Actual) at Factory (units)

The total flow diagram for the supplier and factory pricing mechanism appears in Fig. 12.

The pricing mechanism for the distributor and retailer sectors follows from an extension of the previous concepts.

\[ \text{CGD} \cdot K = \text{FDELAY} \cdot 3 \cdot (\text{AUX} \cdot 3 \cdot K, \text{DIFD}) \]  
\[ \text{AUX} \cdot 3 \cdot K = \text{AUX} \cdot 3 \cdot J + (\text{DT})(\text{PGF} \cdot J - \text{CGD} \cdot J) \]

CGD--Cost of Goods to Distributor ($/unit)
PGF--Price of Goods at Factory ($/unit)

AUX3--AUXiliary variable (3) necessary to define a third-order delay function ($ weeks/unit)

DIFD--average Delay in passing price Information from Factory to Distributor (weeks)

\[ \text{IVD} \cdot K = \text{IVD} \cdot J + (\text{DT})(\text{VID} \cdot J K - \text{VOD} \cdot J K) \]  
\[ \text{VID} \cdot K L = (\text{SRD} \cdot J K)(\text{CGD} \cdot K) \]  
\[ \text{VOD} \cdot K L = (\text{SSD} \cdot J K)(\text{PGD} \cdot K) \]  
\[ \text{PGD} \cdot K = \text{IVD} \cdot K / \text{IAD} \cdot K \]

IVD--Inventory Value at Distributor ($)

VID--Value flowing Into inventory at Distributor ($/week)

VOD--Value flowing Out of inventory at Distributor ($/week)

SRD--Shipments Received by Distributor (units/week)

SSD--Shipments Sent by Distributor (units/week)

CGD--Cost of Goods to Distributor ($/unit)
PGD--Price of Goods at Distributor ($/unit)

IAD--Inventory (Actual) at Distributor (units)

The following equations apply to the retail sector:

\[ \text{CGR}.K = \text{FDELAY}_3 (\text{AUX}.K, \text{DIDR}) \]  
Eq. 63A

\[ \text{AUX}.K = \text{AUX}.J + (\text{DT})(\text{PGD}.J - \text{CGR}.J) \]  
Eq. 63L

\[ \text{CGR} \text{-- Cost of Goods to Retail ($/unit)} \]

\[ \text{PGD} \text{-- Price of Goods at Distributor ($/unit)} \]

\[ \text{AUX} \text{-- Auxiliary variable (4) necessary to define a third-order delay function ($ weeks/unit)} \]

\[ \text{DIDR} \text{-- average Delay in transmitting price Information from Distributor to Retail (weeks)} \]

\[ \text{IVR}.K = \text{IVR}.J + (\text{DT})(\text{VIR}.JK - \text{VOR}.JK) \]  
Eq. 65L

\[ \text{VIR}.KL = (\text{SRR}.JK)(\text{CGR}.K) \]  
Eq. 66R

\[ \text{VOR}.KL = (\text{SSR}.JK)(\text{PGR}.K) \]  
Eq. 67R

\[ \text{PGR}.K = \text{IVR}.K/\text{IAR}.K \]  
Eq. 68A

\[ \text{IVR} \text{-- Inventory Value at Retail ($)} \]

\[ \text{VIR} \text{-- Value flowing Into inventory at Retail ($/week)} \]

\[ \text{VOR} \text{-- Value flowing Out of inventory at Retail ($/week)} \]

\[ \text{SRR} \text{-- Shipments Received by Retail (units/week)} \]

\[ \text{SSR} \text{-- Shipments Sent by Retail (units/week)} \]

\[ \text{CGR} \text{-- Cost of Goods to Retail ($/unit)} \]

\[ \text{PGR} \text{-- Price of Goods at Retail ($/unit)} \]

\[ \text{IAR} \text{-- Inventory (Actual) at Retail (units)} \]

The flow diagram for the distributor and retail pricing mechanism appears in Fig. 13.
**Consumer Demand**

Once again we shall assume some long-run demand schedule for the consumer sector which determines the equilibrium or long-run purchase rate which will be sustained at any given price at retail, PGR.

Economists have the feeling that the long-run demand curve slopes downward to the right as pictured in Fig. 14.

\[\text{RER} \quad \text{(Units/Week)}\]

\[\text{PGR ($/Unit)}\]

**FIGURE 14**

RER--Requisitions (long-run or Equilibrium) received at Retail (units/week)

PGR--Price of Goods at Retail ($/unit)

The downward slope of the demand curve results from what economists call "diminishing utility"; that is, as goods become more expensive, the utility of the goods to the consumer decreases. A specific example of this "diminishing utility" principle might be the present high sales rate
of Fords and Ramblers versus the relatively lower sales rate of Rolls Royce automobiles. The exact shape of the demand curve would, of course, vary from product to product and would tend to change over time due to changes in the needs of the consumer and his ability to purchase goods.

It is again important to formulate a measure for the sensitivity of consumers' purchasing rate, RER, to a change in the price of goods at retail, PGR.

Elasticity of Demand

A concept similar to that of "flexibility" of supply has been formulated for demand schedules. Economists have defined the "elasticity" of demand as follows:

\[
\text{Average Elasticity} = - \frac{\% \text{ change in quantity}}{\% \text{ change in price}}
\]

or \[
\overline{E} = - \frac{\text{change in quantity/quantity}}{\text{change in price/price}}
\]

\[
\overline{E} = - \frac{\Delta Q/Q}{\Delta P/P}
\]

\[
\overline{E} = - \frac{P}{Q} \frac{\Delta Q}{\Delta P}
\]

Thus \[
E = \text{Limit} - \frac{P}{Q} \frac{\Delta Q}{\Delta P} \quad \Delta P \to 0
\]

\[
E = - \frac{P}{Q} \frac{dQ}{dP}
\]

Eq. 69

Once again, the quantity \(dQ/dP\) is just the slope of the demand curve at the point \((Q, P)\). See Fig. 15.

The minus sign has been included in the definition of elasticity in order to make \(E\) a positive number (Eq. 69). This is necessary, since the
slope of the demand curve, \( \frac{dQ}{dP} \), will always be a negative number under the assumption that the curve slopes downward to the right. An elasticity of demand equal to two, then, implies that a one percent increase in price will cause a two percent decrease in the quantity demanded.

**Consumer Response Delay**

In our initial investigation of dynamic price behavior, we will attempt to incorporate a factor in our model which determines how price and quantity will move from one equilibrium condition to another. In the consumer sector, this factor will be realized by assuming some average delay in the consumer responding to a shift in the long-run demand curve. Specifically, this will be accomplished by assuming that the consumers' demand curve
determines the long-run or equilibrium purchase rate, RER, which will be
sustained at any given price, POR. The actual purchase rate, RRR, will
be assumed to "approach" this equilibrium rate in the manner described
by a first-order delay function. (Appendix IV.)

\[
RRR_{KL} = RRR_{JK} + (DT)(1/DERR)(RER_{JK} - RRR_{JK}) \quad \text{Eq. 70R}
\]

RRR—Requisitions Received by Retail (units/week)

RER—Requisition rate (Equilibrium) at Retail determined
by long-run demand curve (units/week)

DERR—average Delay in consumers adjusting to the
Equilibrium Rate of Requisitions (weeks)

This equation, in essence, states that there will be some average
delay, DERR, in the consumer responding to a change in price, PGR, or to
a shift in his long-run demand schedule, since either factor will affect
the long-run equilibrium rate, RER. In the real world this delay might
be caused by several factors, such as the time required for consumers to
become aware of a price change or - in the case of a shifting demand
curve - the time required for a consumer to make a purchasing decision
and put it into effect by submitting an actual requisition at retail.
The effect of including the delay, DERR, on the behavior of the system
will be investigated in the latter part of this chapter. We must next
formulate the demand schedule to be used in the system, with the under-
standing, of course, that this demand curve represents the relationship
between the equilibrium requisition rate, RER, and the price of retail
goods, PGR.
Low-Run Demand Curve

The first set of simulations to be run will employ a linear demand curve as shown in Fig. 16.

\[ \text{RER} = \text{AD1} \left[ \text{IRC} \cdot \text{JK} - (\text{AD2}) \cdot \text{PGR} \cdot \text{K} \right] \quad \text{Eq. 7LR} \]

RER---Requisitions received (Equilibrium) at Retail (units/week)
IRC---average Income Rate of Consumers ($/week)
PGR---Price of Goods at Retail ($/unit)
AD1, AD2---constants determining linear Demand curve, 1 and 2, (units/$) and (units/week) respectively
We have made the position of the demand curve a function of the income rate of consumers under the assumption that an increase in income will shift the curve upwards and to the right (Fig. 17). Such a shift implies that the new quantity, $Q_1$, demanded at the price $P_0$ will be greater than the quantity, $Q_0$, demanded before the increase in income occurred. The income rate, IRC, will be the input variable to the system and will be used to "excite" it initially.

Note that we are assuming that the slope or "steepness" of the curve is not changed by an increase in consumer income. This assumption is motivated by our intuitive feeling about the dynamic behavior of our model. That is, we are quite sure that the slope or "steepness" of the supply and demand curves has a marked effect upon the dynamic response of our model; in fact, we can see from inspection that the stability or instability
of our "information-feedback" system depends upon the "steepness" of both
the supply and demand curves. In exciting the system into "motion," then,
we wish to keep the relative stability of our system constant for any
given simulation run; otherwise we would be changing too many factors in
our system at once and would unduly complicate our task of analysis.

For the linear demand schedule formulated in equation 71R, we can
now calculate the "elasticity" of demand at any point on the curve.
Assuming that consumer income, IRC, remains constant after its initial
change (which causes the demand curve to shift), we can compute the
"elasticity" of demand as follows:

\[ E = - \frac{P}{Q} \frac{dQ}{dP} \quad \text{Eq. 69} \]\n
or

\[ E = - \frac{PGR}{RER} \cdot \frac{d(RER)}{d(PGR)} \]

and since

\[ RER = AD1[IRC - (AD2)PGR] \]

thus

\[ \frac{d(RER)}{d(PGR)} = -(AD1)(AD2) \]

\[ E = - \frac{(PGR)}{(RER)} \cdot \frac{d(RER)}{d(PGR)} \]

\[ = \frac{PGR}{RER} (AD1)(AD2) \]

\[ = \frac{(PGR)(AD1)(AD2)}{AD1[IRC - (AD2)PGR]} \]
or

$$E = \frac{(PGR)(AD2)}{IRC - (PGR)(AD2)}$$  \hspace{1cm} \text{Eq. 72}

Thus for any constant value of consumer income, IRC, we can compute the "elasticity" of demand at any price, PGR. As in the case of a linear supply curve, the "elasticity" of demand as expressed in Eq. 72 is not constant but tends to decrease as we move "down" the demand curve. Note that once again we will only operate over a small portion of this linear demand curve, thus eliminating the possibility of negative prices or negative purchasing rates.

**Demand Curve with Constant Elasticity**

The second set of simulation runs to be made will employ a different type of demand curve; namely, one which has the same "elasticity" at all points on the curve. Such a curve is known to analytical economists as a hyperbolic demand function. A hyperbolic demand curve with a constant elasticity equal to one is shown in Fig. 18.
The following equation defines this demand schedule.

\[(\text{PGR.K})(\text{RER.KL}) = (P_0)(Q_0)\]

PGR--Price of Goods at Retail ($/unit)

RER--Requisitions received (Equilibrium) at Retail
(units/week)

\[(P_0, Q_0)--\text{desired values of Price and Quantity through which the demand curve will pass--specified by the simulator. ($/unit) and (units/week), respectively.}\]

A general demand function with any constant elasticity, \(E\), which passes through any desired point, \((P_0, Q_0)\), is represented by Eq. 73.

\[(\text{PGR.K})^E(\text{RER.KL}) = (P_0)^E(Q_0)\]  \hspace{1cm} \text{Eq. 73}

\(E\)--constant Elasticity of demand specified by the simulator (dimensionless)

It is an easy task to show that this demand schedule has a constant elasticity equal to \(E\):

\[E_d = - \frac{P \ dQ}{Q \ dP}\]  \hspace{1cm} \text{Eq. 69}

or

\[E_d = - \frac{(\text{PGR})}{(\text{RER})} \cdot \frac{d(\text{RER})}{d(\text{PGR})}\]

now

\[\text{RER} = (P_0)^E(Q_0)(\text{PGR})^{-E}\]  \hspace{1cm} \text{Eq. 73}

thus

\[
\frac{d(\text{RER})}{d(\text{PGR})} = - E(P_0)^E(Q_0)(\text{PGR})^{-(E+1)}
\]
\[ E_d = - \frac{(PGR)}{(RER)} \cdot \frac{d(RER)}{d(PGR)} = \frac{(PGR)}{(RER)} \cdot E(p_0)^E(q_0)(pgr)^{-(E+1)} \]

\[ E_d = \frac{E(p_0)^E(q_0)(pgr)^{-E}}{(p_0)^E(q_0)(pgr)^{-E}} \]

\[ E_d = E \quad \text{Q.E.D.} \]

We again wish to make the position of the demand curve a function of the consumer income rate, IRC. This is accomplished readily as follows.

\[ \text{RER.KL} = \frac{(p_0)^E(q_0)}{(pgr.k)^E} \left[ 1 + \frac{\text{IRC.K - IRI}}{\text{AIRD}} \right] \]

or

\[ \text{RER.KL} = \frac{(aec)}{(pgr.k)^E} \left[ 1 + \frac{\text{IRC.K - IRI}}{\text{AIRD}} \right] \quad \text{Eq. 74R} \]

RER--Requisitions received (Equilibrium) at Retail (units/week)

PGR--Price of Goods at Retail ($/unit)

E--constant Elasticity specified by the simulator (dimensionless)

IRC--Income Rate of Consumer ($/week)

IRI--Income Rate of consumer Initially ($/week)

AEC--constant which determines a demand curve with constant elasticity, E, for the Consumer ($)^E$/week

AIRD--constant which specifies the increase in consumer Income Rate that will Double the initial quantity demanded ($/week)

The constant AEC is, of course, determined by the following equation:
AEC = (P₀)Ε(Q₀)

The meaning of the constant AIRD is clarified by Fig. 19.

The shift in the demand curve resulting from a consumer income increase equal to AIRD $/week will cause the quantity which is demanded at the price P₀ to be just twice the quantity, Q₀, demanded before the shift in demand occurred. This can be shown to be a linear relationship; that is, if consumer income increases by AIRD/2 $/week, the new quantity demanded at the price P₀ will be (1.5)(Q₀) units/week; likewise, an increase of 2(AIRD) will cause the quantity demanded at P₀ to be 3(Q₀). See Fig. 20.

We have thus derived a very flexible demand curve; that is, we can pass a curve of any constant elasticity, Ε, through any point, (P₀, Q₀), and specify, precisely, the amount by which the curve will shift as a
result of any specified increase in consumer income, IRC. We have also formulated a demand curve which avoids negative prices or purchasing rates (as might occur with a linear demand curve). This curve is considerably more "realistic" than the linear curve too, since it assumes that there will always be a few purchases when the price is very high and that the purchasing rate will tend to increase indefinitely as the price of goods approaches zero. In other words, the behavior of our system in the extreme regions of the demand curve will be more in accord with our intuitive feeling about actual consumer behavior and will also be compatible with the philosophy of plausible extreme conditions set forth in Memo D-46 (ref. 8).

In spite of the very flexible demand curves, both linear and hyperbolic, that we have formulated, there is still a great need for the construction of a consumer sector which accounts for the actual physical
factors which determine the consumers' long-run demand schedule. Such a consumer sector will be constructed in Chapter IV, but first we shall take advantage of the flexible demand curves formulated in equations 71R and 74R to gain a "feeling" for the dynamic behavior of our system.

We must first complete our pricing mechanism by stating initial conditions and values for the system parameters.

**Initial Conditions**

All variables defined by level equations require initial conditions. This includes the auxiliary variables in equations 46L, 52L, 58L, and 64L. In accord with the usual third-order delay functions, the initial level in the delay equals the input variable times the average delay:

\[
\begin{align*}
AUX1 & = (PRM)(DISF) \\
AUX2 & = (CMPF)(DIM) \\
AUX3 & = (PGF)(DIFD) \\
AUX4 & = (PGD)(DIDR)
\end{align*}
\]

\text{Aux1, 2, 3, 4—Auxiliary variables (1, 2, 3, 4) necessary to define third-order delay functions (\$ \cdot \text{weeks/unit})}

\text{PRM—Price of Raw Materials at supplier (\$/unit)}

\text{CMPF—Cost of raw Materials used in Production at Factory (\$/unit)}

\text{PGF—Price of Goods at Factory (\$/unit)}

\text{PGD—Price of Goods at Distributor (\$/unit)}

\text{DISF—average Delay in passing price Information from Supplier to Factory (weeks)}
DIM--average Delay in transmitting price Information through the factory Manufacturing sector (weeks)

DIFD--average Delay in passing price Information from Factory to Distributor (weeks)

DIDR--average Delay in transmitting price Information from Distributor to Retail (weeks)

The initial value of raw materials and finished goods in the factory, distributor, and retail inventories will be determined by the equilibrium price of raw materials and the initial level of goods and materials in these inventories.

\[
\begin{align*}
RIVF &= (RAF)(PRMI) & \text{Eq. 80N} \\
IVF &= (IAF)(PRMI) & \text{Eq. 81N} \\
IVD &= (IAD)(PRMI) & \text{Eq. 82N} \\
IVR &= (IAR)(PRMI) & \text{Eq. 83N}
\end{align*}
\]

RIVF--Raw material Inventory Value at Factory ($)

IVR--Inventory Value of finished goods at Factory ($)

IVD--Inventory Value at Distributor ($)

IVR--Inventory Value at Retail ($)

PRMI--Price of Raw Materials Initially ($/unit)

The initial rate of materials and goods shipped and received at each sector will just be equal to the initial value of requisitions received at retail, RRR.

\[
\begin{align*}
RMRF &= RRR & \text{Eq. 84N} \\
RMUF &= RRR & \text{Eq. 85N} \\
SRF &= RRR & \text{Eq. 86N}
\end{align*}
\]
SSR = RRR  
SRD = RRR  
SSD = RRR  
SRR = RRR  
SSR = RRR  

RMRF--Raw Materials Received by Factory (units/week)
RMUF--Raw Materials Used at Factory (units/week)
SRF--Shipments Received by Factory (units/week)
SSF--Shipments Sent by Factory (units/week)
SRD--Shipments Received by Distributor (units/week)
SSD--Shipments Sent by Distributor (units/week)
SRR--Shipments Received by Retail (units/week)
SSR--Shipments Sent by Retail (units/week)

The initial requisition rate, RER, determined by the demand curve will just be equal to the actual rate, RRR, since the system is assumed to start in equilibrium.

RER = RRR  

RER--Requisitions received (Equilibrium) at Retail (units/week)
RRR--Requisitions Received at Retail (units/week)

The initial value of RRR must be specified.

RRR = RRI  

RRR--Requisitions Received at Retail (units/week)
RRI--Requisitions Received at retail Initially (units/week)
The equation for the supply curve requires an initial value for the "smoothed" rate of receiving raw material orders at the supplier, ROSS.

\[ \text{ROSS} = \text{RRR} \quad \text{Eq. 94N} \]

ROSS—Raw material Orders "Smoothed" at Supplier (units/week)
RRR—Requisitions Received at Retail (units/week)

The demand curve equations require an initial value for consumer income rate, IRC.

\[ \text{IRC} = \text{IRI} \quad \text{Eq. 95N} \]

IRC—Income Rate of Consumer ($/week)
IRI—consumer Income Rate Initially ($/week)

**System Input**

The input variable, IRC, which will "excite" the system, is determined by the following equation.

\[ \text{IRC.KL} = \text{IRI} + \text{IRCC.K} \quad \text{Eq. 96R} \]

IRC—Income Rate of Consumers ($/week)
IRI—Income Rate of consumers Initially ($/week)
IRCC—Income Rate of Consumers Change that "excites" the system ($/week)

The equation for IRCC will depend upon the type of "excitation" desired for the system. IRCC might be a step, a ramp, or perhaps a "noisy" or random function. The exact type of input function used in each simulation run will be specified as we attempt to analyze our results.
Parameters

The values of ARZ and ART (Eq. 43A) for the supply curve will depend upon the "flexibility" of supply desired and the initial price and quantity through which the curve is passed. Rather than specifying here the values that will be used, we shall state in the analysis of results the initial "flexibility" and initial price and quantity used in each case. This will also be the policy followed in determining AD1 and AD2 (Eq. 71R) or E, AEC, and AIRC (Eq. 74R) for the demand curve.

The average delays in the price "propagating" mechanism will be assumed to be approximately equal to the corresponding delays that apply to the flow of raw materials or goods in each sector. In other words, we will assume for the present that price information does flow with the actual raw materials or goods in the system.

\[
\begin{align*}
\text{DISF} & = 3 \text{ weeks} \\
\text{DIM} & = 6 \text{ weeks} \\
\text{DIFD} & = 2 \text{ weeks} \\
\text{DIDR} & = 1 \text{ week}
\end{align*}
\]

DISF—average Delay in passing price Information from Supplier to Factory (weeks)

DIM—average Delay in passing price Information through the Manufacturing sector of the factory (weeks)

DIFD—average Delay in passing price Information from Factory to Distributor (weeks)

DIDR—average Delay in passing price Information from Distributor to Retail (weeks)

The average delay, DERR, in consumers responding to a change in price
or to a change in their long-run demand schedule will be varied as follows:

\[ \text{DERR} = 0, 10, 20, \text{or } 40 \text{ weeks} \]

This completes the formulation of the pricing mechanism in our system.

**Simulation Procedure**

By a series of simulation runs, we will now see how the "flexibility" of supply and the "elasticity" of demand affects the stability of our pricing-production-distribution model. In the process, we will also examine the quality and quantity of "static" information which is conveyed by the dynamically generated data and investigate the factors which influence this information, such as the average delay in consumer response, as well as the "flexibility" of supply and the "elasticity" of demand.

**Results of Simulation Runs**

The first set of simulation runs was made in order to gain a "feeling" for how the "flexibility" of supply and the "elasticity" of demand influence the stability of our model. We should clearly understand what is meant by a "stable system," however, before proceeding any further.

A leading text on feedback control theory defines a stable system as follows (ref. 7): "A stable system is one whose output is bounded when the inputs to the system are bounded." In laymen's terms this means simply that a stable system is one which does not "blow up" unless the inputs to the system "blow up." This definition is quite different from the layman's understanding of a stable system, for he would probably say that any system which oscillates is an unstable system. This is untrue,
according to our definition, unless the oscillations become larger and larger in amplitude. If the oscillations tend to die out as time progresses, then the system is definitely a stable one.

We can clarify this concept of stability by an example. If the inputs to several systems are characterized by a step function as shown in Fig. 21A, a typical response of a stable system is shown in Fig. 21B, that of an unstable system is shown in Fig. 21C, and the response of a system which lies on the borderline between being stable and unstable is shown in Fig. 21D.

The input to our system in the first set of runs will be characterized by a step function as shown in Fig. 21A, since the income rate of consumers will jump from an initial value to some higher value, thus causing the long-run demand curve to shift instantaneously also. We will consider our system to be stable if the consumers' purchasing rate, RRR (which is viewed as the output of our system), tends to settle down or "die out" to some new equilibrium value. If RRR tends to increase indefinitely in amplitude, the system will be known to be unstable. The other possibility, of course, is that RRR may tend to oscillate indefinitely with constant amplitude. This is just the borderline situation described by Fig. 21D.

Linear demand and supply curves were used exclusively in the first set of runs, and the final equilibrium values of price and quantity (PGR and RRR) were the same in each simulation. The only quantities which were varied from run to run were the "flexibility" of supply, the "elasticity" of demand, and the initial value of consumer income, IRI. The initial value of consumer income was varied so that the increase in the equilibrium
value of consumer purchases, RRR, resulting from a shift in the demand curve, was the same in every run. This helps to simplify our task of analysis, for otherwise the increase in the equilibrium value of RRR (as determined by the intersection of the supply and demand curves) would be confusingly different in every run. This should become clearer to the reader as we attempt to plot the dynamic response of our system for various values of "flexibility" and "elasticity."

In all of the first runs made, the consumers' average delay in responding to changing prices or to a changing demand schedule, DERR, is assumed
to be zero. The effect of increasing this delay will be investigated later, after we gain a "feeling" for the part that supply "flexibility" and demand "elasticity" play in determining the stability of our system.

In Fig. 22 the dynamic variation of consumer purchases, RRR, is plotted for various values of supply "flexibility" and a constant value of demand "elasticity." The quantity RRR is plotted in terms of its percentage variation around its initial value, RRI, since the qualitative nature of the consumers' purchasing rate is more important in this initial investigation than the absolute values which RRR assumes during the simulations.

It is apparent, from Fig. 22, that as the "flexibility" of supply increases from zero to one, the dynamic response of RRR to a step input of consumer income, IRC, becomes less and less damped in nature. If the "flexibility" is zero, this implies that the price of raw materials, PRM, is constant regardless of the quantity of raw materials ordered per week. Accordingly, in our system, when the consumer demand schedule shifts, the price of goods at retail, PGR, would continue to remain constant, thereby causing the consumers' purchasing rate, RRR, to remain constant also after its initial shift. As the "elasticity" of demand increases toward one, we note that the consumers' purchasing rate becomes more and more sensitive to price changes. It is not surprising, therefore, that the system becomes more and more oscillatory as the "demand" elasticity increases.

When the "elasticity" of demand and "flexibility" of supply are both unity, we see that consumer purchases tend to oscillate indefinitely, neither growing nor dying out with time. This result is not unexpected
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either, since we know from our definitions that a one percent change in raw material orders will cause a one percent change in the price of raw materials at the supplier, while a one percent change in prices at retail will cause a one percent change in the consumer purchasing rate, RRR. These two effects appear to just "balance" one another, such that the consumer purchasing rate, RRR, will tend to oscillate indefinitely.

When the supply "flexibility" is greater than one (with demand "elasticity" equal to unity), we see that the supplier's sensitivity to a change in the "smoothed" material order rate is so great that the system becomes unstable, i.e., RRR is seen to oscillate with increasing amplitude as time progresses. One might say that the supplier's pricing policy "over-compensates" for the changes in the raw material order rate which he sees, thereby causing the consumer's purchasing rate to fluctuate with even greater change in amplitude - eventually resulting in the system blowing up (unless, of course, there exists some physical constraints on the system, such as the impossibility of RRR going negative). Before drawing definite conclusions about the effect of varying supply "flexibility" on the stability of our system, we should examine the effect of changing the demand "elasticity."

Fig. 23 demonstrates the effect of holding supply "flexibility" constant at unity and varying the demand "elasticity." We note the same type of relationship existing between "elasticity" and the stability of the system as existed in the case of varying "flexibility." It is becoming clearer, now, that the stability of the system is determined by both the "elasticity" and the "flexibility" incorporated in the system. The exact relation between the two which determines the stability of the system
FIGURE 23

STEP RESPONSE
Flexibility = 1
Elasticity = 0.5, 1, 2

$\Delta x = 100 \times (\% \text{ of initial value})$

$G = 2$
$G = 1$
$G = 0.5$
$G = 0.2$
$G = 0.1$

1 year 2 years 3 years
became apparent to the author after a careful examination of Figures 22 and 23 and after a closer look at the interdependence of demand and supply as formulated in our model.

When either "elasticity" or "flexibility" have the value zero, the response of RRR to a step change in IRC is an immediate jump to its new equilibrium value, with no oscillations present whatsoever. With this knowledge and the information conveyed by Figures 22 and 23, it becomes apparent that the stability of our system (as judged by the output variable, RRR) depends upon the product of the supply "flexibility" and the demand "elasticity." This relationship is defined more completely by the following:

If \((F)(E) < 1\), RRR will have a stable response to a step input of IRC.

If \((F)(E) > 1\), RRR will have an unstable response to a step input of IRC.

If \((F)(E) = 1\), RRR will respond to a step input of IRC with sustained oscillations.

Where \(F\) = "flexibility" of supply, \(E\) = "elasticity" of demand.

This relationship might be pictured graphically as shown in Fig. 24.
It is interesting to note that it is actually the slope or "steepness" of the demand and supply curves which determines the stability of the system. This can be shown to be true as follows:

By definition  \[ F_s = \frac{Q_s}{P_s} \left(\frac{dP}{dQ}\right)_s = \text{Flexibility of Supply} \]

\[ E_d = -\frac{P_d}{Q_d} \left(\frac{dQ}{dP}\right)_d = \text{Elasticity of Demand} \]

(The subscripts s and d refer to the supply and demand curves respectively)

\[ (F_s)(E_d) = -\frac{Q_s}{P_s} \frac{P_d}{Q_d} \left(\frac{dP}{dQ}\right)_s \left(\frac{dQ}{dP}\right)_d \]

Since in equilibrium, the price and quantity are determined by the intersection of the supply and demand curves, then \[ P_s = P_d \]

and \[ Q_s = Q_d \]

Thus \[ (F_s)(E_d) = -\left(\frac{dP}{dQ}\right)_s \left(\frac{dQ}{dP}\right)_d \]

or

\[ (F_s)(E_d) = -(\text{slope of the supply curve}) \times (\text{slope of the demand curve}) \]

Q.E.D.

Thus we see that the stability of the system is actually determined by the product of the slopes of the demand and supplies curves at their point of intersection.
Viewing Price Stability

We might now consider the price of goods at retail, PGR, as the output of the system (rather than RRR) and check to see whether the stability of its response to a step input of consumer income, IRC, is also determined by the relationship shown in Fig. 24.

The time response of PGR (retail price) is plotted in Figures 25 and 26. The "elasticity" and "flexibility" are varied in the same manner used to investigate the consumers' purchasing response, RRR. It should be apparent to the reader that the relative stability of the price response is also determined by the product of "flexibility" and "elasticity" in the same manner described by Fig. 24. This is not a surprising result to the person familiar with feedback-control theory, for it is often true of a feedback system that if one of its outputs has a stable response, all of its outputs have stable responses. This is why we can usually speak of a system being stable rather than having to explain the degree of stability of each output variable in the system.

The qualitative relationship defined by Fig. 24, then applies to our whole system and each of its output variables (under the assumption that there is no delay in consumers responding to a change in price or to a change in their demand schedule).

Consumer Response Delay

We wish also to obtain a "feeling" for the dynamic behavior which results from increasing the consumer response delay, DERR. Intuitively, we might guess that a longer delay in the consumers' response would tend to "dampen" the response of the system to a step input of income, IRC.
STEP RESPONSE
Flexibility = 1
Elasticity = .1, .2, .5, 1, 2
DEPR = 0

FIGURE 74

X = 100 \times (\% of initial value)

Weeks

1 year 2 years 3 years
The results observed in Fig. 27 prove that our intuition is correct. As DERR increases, the system response does appear to become more "damped" in nature, but the resulting oscillatory or transient response seems to require a much longer time to die out than previously. In essence, we are changing the "natural frequencies" of our whole system. We would no longer expect our model to generate purchasing cycles of approximately a year (as occurred in Figures 25 and 26 when DERR = 0), but rather, we would expect to find cycles with a considerably longer period of oscillation, depending, of course, upon the magnitude of DERR.

Unfortunately we cannot draw any general conclusions about the effect of DERR on the stability of the system. The only thing we can say from examination of Fig. 27 is that the natural frequencies of the system are markedly affected by a change in the delay DERR.

Static Demand Information

Having acquired a feeling for the effect which "elasticity," "flexibility," and the consumer reaction delay, DERR, have on the dynamic behavior of our system, we can proceed to the problem of deriving static or equilibrium information from the dynamically generated data. In the next set of runs, we have attempted to show how these factors influence the quality and the quantity of static information contained in the dynamic response of our system. In each of these runs the demand curve has again been shifted by introducing a step increase in consumer income, IRC. As a result of this shift, the price and quantity at the retail end of our system tend to move from one equilibrium condition to another (the equilibrium positions being determined by the intersection of the supply and
STEP RESPONSES

Flexibility = 1
Elasticity = 4

$H(x) = \frac{x}{100}$ (% of initial value)

---

FIGURE 27
demand curves). In each case we have plotted the dynamic response of price and quantity (PGR and RRR) in an attempt to show how this dynamic information is related to the static demand and supply curves which exist in the system.

In Figures 28, 29, and 30, the consumer response delay is assumed to be zero, and the "flexibility" of supply is varied while holding the elasticity of demand constant at unity. We see that the quality of static information conveyed in the dynamic response is very good, for the price and quantity tend to oscillate along the demand curve exclusively. The quantity of static information conveyed, however, appears to be a function of the "elasticity" of demand and the "flexibility" of supply, for as the "flexibility" is increased from .1 to unity, the dynamic data traces out more and more of the static demand curve.

This is an expected result in terms of our "feeling" for how "elasticity" and "flexibility" affect the stability of our system, since we know that greater and greater oscillations will occur as the product of "elasticity" and "flexibility" is increased from zero to infinity.

The fact that price and quantity tend to oscillate along the demand curve will next be shown to result from our assumption that DERR is zero. In Figures 31, 32, and 33 we have chosen the "flexibility" and "elasticity" such that the system is known to be stable; that is, the price and quantity will definitely settle down to some new equilibrium condition. However, the consumer response delay, DERR, has been increased to 10, 20, and 40 weeks respectively.

We see now that the quality of static information conveyed is markedly
Elasticity = .25
Flexibility = .5
DERR = 10 Weeks

DERR = 20 Weeks

DERR = 40 Weeks
changed, for as DERR increases from zero to infinity, the amount of information conveyed about the demand curve tends to decrease, while more and more information is provided about the supply curve. It should be apparent that if DERR were an extremely long delay, the price and quantity would tend to "creep" slowly up the supply curve until a new equilibrium condition is reached. We see, then, that the delay DERR can cause the quality of static information conveyed to go from one extreme to the other. When DERR is zero, we receive excellent information about the demand curve, and in the other extreme of DERR being infinite, we receive excellent information about the supply curve. Somewhere in between, as shown in Fig. 32, the quality and quantity of static information conveyed appears to be extremely poor. Unfortunately this is probably the range of values which DERR has in the real world, which would tend to make us a bit dubious about the validity of deriving pertinent static information from statistical data. We would not jump to any rash conclusions at this point, however, for we have only examined a very simple case in which the supply and demand curves are assumed to be linear and time invariant; and more important, we have only examined the situation of moving from one equilibrium condition to another. It is very doubtful whether any of these simple situations ever exist in the real world (as mentioned in the introduction), and as a result we are made even more aware of the need for replacing our simple demand curve with a more realistic consumer sector.

It might be valid, however, to reason that if the quantity and quality of static information is often very poor under extremely simple conditions,
we would hardly expect an improvement to occur when the system is made as complex as the real world. The proof of this reasoning, of course, is "in the pudding," and we will refrain from reaching any general conclusions until a more realistic situation has actually been examined.

**Hyperbolic Demand Curve - Nonequilibrium Situation**

Before leaving this investigation of a simple demand and supply curve situation, we will attempt to incorporate the constant "elasticity" demand curve described by Equation 74R. We will, at the same time, attempt to inject one further note of realism by setting up a situation in which no equilibrium condition exists. This will be accomplished without a confusing degree of complexity by assuming that the "elasticity" of demand remains constant but the position of the demand curve tends to fluctuate with time. This might be analogous to a situation in which there is a seasonal demand for a product. An example might be ice cream, water skis, fuel oil, or some other product whose sales rate tends to follow a seasonal pattern.

Our system, then, will be set into motion from some equilibrium condition, but the demand curve will tend to fluctuate continuously over time, thereby generating sales data which may cover a large area of our supply and demand plot. As might be the situation in industry, we will assume that dynamic information is only available on a monthly basis; i.e., we will only plot the price and quantity information which exists at the end of each four-week period. From this scattering of points, then, we will attempt to derive pertinent information about the constant supply curve and the fluctuating demand curve incorporated in the model.
The results of such a run are displayed in Figures 34 and 35. In Fig. 34, the constant supply curve is plotted along with the fluctuating demanded curve (which is shown in three of its instantaneous positions). The demand curve has been assumed to fluctuate sinusoidally between the curves $D_1$ and $D_2$ with a period of oscillation of 52 weeks per cycle. The curve $D_3$ represents the "middle" position of the demand curve as it oscillates between $D_1$ and $D_2$.

The scattered points of monthly data are plotted in Fig. 35 along with a curve that appears to "fit" these points in a reasonable fashion. We see that the quality of this "fitted" curve is not too bad, for it tends to slope downwards to the right in approximately the same fashion as the "middle" demand curve $D_3$ in Fig. 34. We are somewhat surprised to see that we are gaining fairly good information about the demand curve, for the delay DERR was set equal to ten weeks in this simulation run. Our results from the previous investigation suggested that with a delay, DERR, of ten weeks, the amount of information conveyed about the demand curve would be considerably less than in the case of DERR equal to zero. This assures us, however, that we would have been foolish to draw any general conclusions from the results of our first set of simulations. It also tends to make us feel that there might be a good possibility of deriving static information in a situation where no equilibrium condition exists.

At this point, the reader, along with the author, is probably more disturbed than ever by the level of abstraction we are working on. Theoretical supply and demand curves are interesting to play with, but they fail to take adequate account of the actual physical factors which influence
Elasticity = .5
Flexibility = .5
DEAR = 10 Weeks
the buyer's purchasing decision and the seller's pricing decision.

Our motivation is thus stronger than ever to formulate a more realistic consumer sector and to make appropriate use of the existing raw material supplier sector to determine the long-run supply curve. Before doing so, however, we should review our initial investigation to see whether or not our results have been at all successful in adding to our understanding of dynamic price behavior.

Conclusions Drawn from Initial Investigation

We have definitely acquired a better "feeling" for how the "elasticity" of demand and the "flexibility" of supply influence the stability of our production-distribution system. We have been able, for our particular system, to derive an explicit, analytical relationship between the stability of the system and the "flexibility" and "elasticity" of supply and demand (Fig. 24). It should be pointed out that this relationship is only applicable to our particular system as we have formulated it. If any parameters of the system were changed, or any of the sectors were formulated differently, we should expect a different type of relationship to exist between the system's stability and the "elasticity" and "flexibility" of demand and supply. We witnessed an example of this when we changed the value of the consumer's response delay, DERR; for the stability of the system was definitely altered, although it was difficult to state the exact degree of change that occurred.

In changing the value of DERR, however, we were able to gain a "feeling" for how this delay influenced the natural frequencies of our system. The natural frequencies of a system become much more important, as any
electrical engineer knows, when we excite the system with a fluctuating input. A striking demonstration of this fact is presented by Professor Forrester in Fig. B of his *Harvard Business Review* article (ref. 2). In this example, his basic production-distribution system is excited with a random input rate of consumer purchases, RRR. In spite of this completely random input, the manufacturing rate at the factory, MOF, begins to fluctuate with very definite yearly cycles. This yearly fluctuation of production is a direct manifestation of the natural frequencies of the system and is thus determined directly by the combination of delays incorporated in the system. We will have a contact with this phenomenon when a noise component is added to the consumer sector formulated in the next chapter.

In summary, our initial investigation of dynamic price behavior disclosed some very interesting facts about the simple pricing mechanism that was formulated. We observed that the quantity of static information conveyed to us was heavily dependent upon the "flexibility" and "elasticity" of supply and demand, while the quality or nature of this information seemed to be greatly influenced by the consumer response delay, DERR. Our feeling that an equilibrium condition seldom exists in the real world motivated us next to incorporate a fluctuating demand curve in our model. The resulting dynamic information was surprisingly useful in our attempt to derive knowledge about the "static" demand curve and injected a light of hope into our investigation. We aired our growing dissatisfaction, however, in the use of such simple, abstract supply and demand curves and re-emphasized the need for formulating a more realistic consumer sector for our model.
Such a consumer sector will next be formulated in Chapter IV, and we will continue our effort to relate static and dynamic price information in the revised model.
CHAPTER IV
FORMULATION OF A CONSUMER SECTOR

Having acquired a "feeling" for the types of behavior that will occur in a model such as that discussed in Chapters II and III, we are now in a position to improve our method of investigation. That is, rather than observing and analyzing the system response that results from the incorporation of theoretical demand and supply curves, we can now attempt to recognize the actual physical factors which determine supply and demand and continue to investigate the possibility of deriving a "static" demand or supply relationship from the resulting dynamic behavior.

The models of Chapters II and III assumed simply that the purchase rate of consumers was determined by the price of goods at retail and the income rate of consumers, as expressed in equations 71R and 74R. A small note of realism was injected by assuming some average delay, DERR, in consumers reacting to a change in price level or income rate. The inadequacy of these simple assumptions in explaining the actual behavior of consumers in the real world is quite apparent; however, we admit that such an explanation of consumer behavior might be reasonable in certain very restricted cases. (The author does not claim to know of any such cases in the real world.)

We might now attempt to discard the simple static demand relationship employed in Chapter III and make some assumptions about the actual levels and rates which would affect the consumer's decisions to purchase some particular type of hard good. To aid the reader in grasping the
important concepts involved and to simplify our discussion, let us assume that we are dealing with the consumer's purchase of refrigerators (although we might apply this discussion equally as well to washing machines, dishwashers, stoves, etc.)

**Consumer Ownership Sector**

Let us postulate a consumer sector in which the total number of refrigerators in use is constant. This assumption might be rationalized by accepting the premise that old refrigerators are only discarded when they are physically replaced by new ones. If we allow for the possibility that new refrigerators might be bought for new homes, i.e., refrigerators that are not replacing old ones, we must also allow for the possibility of old refrigerators that are discarded but never replaced (because the owner died or started eating canned foods exclusively). With both possibilities existing, we could reasonably assume that one phenomenon balances the other and that the total number of units owned by the consumer sector is still constant. If we assume, in addition, that each refrigerator is owned by no more or no less than one consumer, we are assuming that the total number of consumers in the consumer sector also remains constant. (This actually assumes that consumers are lost at the same rate they are created, an assumption which unfortunately ignores the disturbing population growth these days.)

Ignoring the disparity between our model and the present birth rate then, we will proceed with the formulation of such a consumer
sector. At any time we can imagine that there will be a certain number of refrigerators (or units) in use by the consumers that are not yet "worn out." By the term "worn out," we mean that they have not reached the average age at which their performance becomes unsatisfactory to the used. When a unit is "worn out," then, we are implying that it still performs its function of cooling food, but the consumer is not satisfied with it because it fails to perform its function well enough, it does not hold enough food, it looks terrible, or perhaps its handle has fallen off and a crowbar is required to open it.

The goods in service that are not worn out, then, are increased by shipments of new units received from the retailer and are decreased by the units that become "worn out." (See flow diagram in Fig. 36)

\[ GSNW_{K} = GSNW_{J} + (DT)(SSR_{JK} - GWOC_{JK}) \]  
Eq. 97L

GSNW = Goods in Service in the consumer sector that are Not Worn out (units)

SSR = Shipments Sent by Retailer (units/week)

GWOC = Goods Worn Out in Consumer sector (units/week)

We are assuming here that the delay between shipments sent at retail and the units actually put into use by the consumer is negligible compared to the other delays in the system (probably a fourth of a week at the most).

The rate at which units "wear out" is most likely determined by the age of the units that are in service, GSNW. It is unlikely, however,
that if the average age of a unit is 200 weeks when it wears out that all of the units that are 200 weeks old today will become worn out today. More realistically, if the average lifetime of a refrigerator is 200 weeks, a few refrigerators will wear out before and a few will last longer than 200 weeks, with the majority becoming worn out somewhere close to the age of 200 weeks. This behavior is shown graphically in Figure 37. If one thousand new refrigerators exist in service at time zero, the rate at which they wear out, GWOC, will be that pictured.

If no new refrigerators are added to the initial 1000 after time zero, the number of goods in service at any time, GSNW, will appear as shown in Fig. 38.
If we now make the shipments sent by the retailer 1000 units per week, we know that after an equilibrium condition has been reached, the rate of goods wearing out, GWOC, will just equal the rate of shipments being sent by the retailer, SSR, and there will be (1000 units/week) x (200 weeks) or 200,000 units in service, GSNW, at any time. If the rate, SSR, should jump to 2,000 units per week, we know that the wearing out rate, GWOC, will approach this new input rate with an average delay of 200 weeks (See Fig. 39). The new level of goods in service, GSNW, will of course approach (2,000 units/week) x (200 weeks) or 400,000 units.

The behavior we have just described is well known to the people in Industrial Dynamics as that of a third-order delay. This is the same type of delay used in Chapter III to delay information about prices, except that here we are delaying a flow of materials, whereas previously we were delaying information about a price level.

We will thus use a third-order delay function:
GWOC\cdot KL = FDELAY3 (GSNW,K, DGW) \hspace{1cm} Eq. 98R

GWOC = Goods Worn Out at Consumer sector (units/week)
GSNW = Goods in Service that are Not Worn out (units)
DGW = average Delay in Goods Wearing out in consumer sector (weeks)

The goods that become worn out but that are not yet discarded fall into two categories—those that have not been replaced by requisitions to retail (these constitute postponed purchases) and those that have been replaced by requisitions but not yet discarded (the number of goods in this category is exactly equal to the number of unfilled orders at retail). As soon as the orders for new refrigerators are filled, the worn out refrigerators that are replaced by these units are discarded; thus the rate of discarding worn out units is exactly equal to the rate at which shipments are sent be the retailer. The following equations thus hold true:

\[ RGC\cdot K = RGC\cdot J + (DT)(GWOC\cdot JK - RRG\cdot JK) \hspace{1cm} Eq. 99L \]

RGC = Replaceable Goods in Consumer sector (units)
GWOC = Goods Worn Out at Consumer sector (units/week)
RRG = Rate of Replacing Goods in the consumer sector (units/week)

This equation says that the replaceable goods in the consumer sector, RGC, are increased by goods that wear out, GWOC, and are decreased by goods that have been replaced, RRG.

The rate of requisitions received at retail by definition is just
equal to the rate at which goods are replaced by orders:

\[ \text{RRR.KL} = \text{RRG.JK} \quad \text{Eq. 100R} \]

\( \text{RRR} = \text{Requisitions Received at Retail (units/week)} \)

\( \text{RRG} = \text{Rate of Replacing Goods at the consumer sector (units/week)} \)

The goods that have been replaced by orders but not yet discarded are increased by requisitions to retail and decreased by goods that are discarded.

\[ \text{GRNR.K} = \text{GRNR.J} + (\text{DT})(\text{RRG.JK} - \text{GDR.JK}) \quad \text{Eq. 101L} \]

\( \text{GRNR} = \text{Goods Replaced by orders but Not yet Replaced by new units (equal, by definition, to unfilled orders at retail) (units)} \)

\( \text{RRG} = \text{Rate of Replacing Goods in the consumer sector (units/week)} \)

\( \text{GDR} = \text{Goods Discarded by consumers due to Receipt of units that were ordered (units/week)} \)

The rate of goods discarded was stated to be equal to the rate at which the ordered units are received.

\[ \text{GDR.KL} = \text{SSR.JK} \quad \text{Eq. 102R} \]

We stated that the total number of units in the consumer sector was assumed to be constant, since we only discard old units when we receive new ones. Thus the total number of units owned by the consumer sector is made up of those in service but not worn out, those worn out but not replaced, and those replaced but not yet discarded.
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\[ \text{TOC}.K = \text{GSNW}.K + \text{RGC}.K + \text{GRNR}.K \quad \text{Eq. 103A} \]

TOC = Total number of units Owned by Consumers (units)
GSNW = Goods in Service that are Not Worn out (units)
RGC = Replaceable Goods at the Consumer sector (units)
GRNR = Goods Replaced by orders but Not yet Replaced by new units (units)

The quantity TOC will not be used anywhere in our system, but it will enable us to make sure that the total number of units in the consumer sector is constant.

The only rates which have not yet been defined are SSR and RRG. The former rate, SSR, will be determined in the retail sector of our model (See equation 5R in Appendix I) and the latter rate, RRG, will be determined by equations which are not yet formulated. We now proceed to the remaining part of our consumer model.

**Purchasing Decision**

The purchase rate (or replacement rate, RRG) will depend upon several factors, such as the price of units at retail, the amount of income the consumer has saved for the purchase of these units, and the length of time the consumer has postponed the replacement of a worn out unit. That is to say, his purchase rate depends upon what he will spend, what he can buy for what he will spend, and upon the length of time he has put up with a worn out unit. There may be other factors which affect these three, but the author feels that the consumer's need and his ability to buy are the most basic factors
motivating his purchases. This leaves us with a variety of ways in which
to incorporate these factors into the consumer sector, but as long as our
assumptions are reasonable, the resulting behavior of the model should
also be reasonable and refinements can be made after the general behavior
of the consumer sector is understood.

Savings Sector

Let us postulate, then, a level of income that the consumer has saved
for the purchase of a new unit. This level will be increased by some
fraction of his total income that he places into this savings account
each week and decreased by the amount that he spends on a unit each week.
It is unlikely that any given consumer places the same fraction of his
income per week into such a savings account. We notice also that if we
looked at each consumer, the amount he took out of this savings account
would be zero each week until he made a purchase, at which time he would
remove enough to cover the total price of the goods. Note that we are
making the basic assumption here that no credit exists for our consumers;
that is, all purchases must be full cash purchases, and that cash must
come from his savings held for that purpose. This assumption removes us
considerably from the world today, and we would have to expand our model
considerably to bring in this realistic factor. However, this assumption
would not be absurd if we considered an earlier period in history in which
most purchases were cash purchases and this cash came from a man's savings.
The author imagines that the days of Benjamin Franklin were not unlike
this, realizing, of course, that we would no longer speak of refrigerators
but perhaps articles of furniture or tools of a trade.

This problem which appears to exist in dealing with the level of saved income and the rate of saving and spending for an individual consumer can be simplified considerably by considering the **average** level of saved income and the **average** rates of saving and spending. We can imagine that in an aggregation of consumers, some will save a lot of money for the purchase of units because their incomes are very high and some will save very little due to small incomes. On the average, however, we can speak of those rates of saving and spending and the level of savings which are representative of most of the consumers in the consumer sector. This is the same sort of averaging process that we used in determining the average delay of refrigerators becoming worn out.

Our level and rates pertaining to income saved, then, will be an average level and average rates. The average level of income saved for the purchase of refrigerators is thus increased by the average rate of income being saved by the consumer and decreased by the average rate of savings being spent.

\[
\text{ISC}.K = \text{ISC}.J + (DT)(\text{ISRC}.JK - \text{ISGC}.JK) \quad \text{Eq. 104L}
\]

**ISC**--average Income Saved per Consumer for purchase of units ($/person)
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LSRC = \text{average Income Saving Rate per Consumer for purchase of units ($/person)}

LSGC = \text{average Income Spent on Goods per Consumer ($/week/person)}

If we postulate, now, an average income rate for the consumer sector, then the average savings rate can be considered to be a time-varying fraction of the average income rate of consumers. This fraction will probably depend upon several other factors within the consumer sector.

\[ \text{LSRC}_{KL} = (\text{FIS}_{K})(\text{IRC}_{JK}) \]  \hspace{1cm} \text{Eq. 105R}

LSRC = \text{average Income Saving Rate per Consumer for purchase of units ($/week/person)}

FIS = \text{average Fraction (variable) of Income rate Saved for purchase of units ($/week/$/week)}

IRC = \text{average Income Rate per Consumer ($/week/person)}

We now postulate that the average fraction of average income that the consumer will save will depend on his need and his ability to buy; that is, the same factors which motivate him to purchase also motivate him to make his purchases possible by saving his money. His need for units is reflected in our model by the number of purchases that have been postponed or the level or replaceable goods, RGC. There will probably be some normal level of postponed purchases or replaceable goods at which the consumer may be considered as having his needs met adequately. That is, perhaps there is normally 1/5 of the total goods owned by the consumers in the replaceable goods level, RGC. All that we are saying is that perhaps under normal circumstances most consumers hang on to their worn-out units for a certain number of
weeks - let us say 50 weeks or approximately one year. Some consumers replace them sooner, some later, but the average length of time that units stay in RGC is 50 weeks. Thus under normal conditions and under equilibrium conditions (when the rate of replacing goods is equal to the rate of goods wearing out), we would expect to find 50 weeks of the equilibrium rate of goods wearing out in RGC. To be specific, if the rate of goods wearing out were 1,000 units/week in equilibrium, we would expect to find (50 weeks) x (1,000 units/week) or 50,000 units in RGC. We would thus say that consumers consider a level of replaceable goods equal to 50,000 as being "normal."

Now before specifying the relationship between the fraction of his average income the consumer will save, ISRC, and his need for units as reflected in the level of replaceable goods, RGC, let us first inquire as to why the replaceable goods level might change. A change in RGC could be brought about by a change in either the rate of goods wearing out, GWOC, or the rate of goods being replaced by orders for new units, RRG. The rate, GWOC, of course depends upon SSR, shipments sent by retailer, which is a function of the average delay in filling orders at retail. Any change in this average delay, DFR, is brought about by an undesirable level of inventories at the retailer, a factor whose immediate cause is related to the purchasing decision made at the retailer and, of course, to the rate at which consumers decide to replace worn-out refrigerators.

We have thus followed a complete feedback loop; that is, we have shown that the level of replaceable goods, RGC, depends upon the rate
of replacing them, RRG, and the rate at which goods wear out, GWOC, but that this latter rate, GWOC, is a function of the former rate, RRG (although there is approximately a 200 week delay in GWOC responding to a change in RRG).

We have thus effectively pinned down the major cause of RGC changing over time; that is, the level of postponed orders depends almost completely on the rate at which the consumer replaces them, RRG.

Thus when the consumers' level of replaceable goods is observed to be greater or less than that considered "normal" by the consumer, we know it is a result of his having changed his replacement rate, RRG, at some previous time.

Now we can ask what the response of the consumer will be when he becomes aware of an "abnormal" level of replaceable goods. It is obvious that he will feel a greater need for new units if RGC is above its normal value and a correspondingly smaller need if RGC is below its normal value. If his need increases, we feel that he will try to remedy this situation by increasing his replacement rate, RRG (for this is just the phenomenon of "feedback" in action). However, there must have been something that limited his rate of replacement, RRG, previously, or he would not have found himself with a greater level of replaceable goods than normal (as explained in the last paragraph).

This limiting factor will be his ability to purchase new units, as reflected in his average level of income saved for this purpose. We must then ask why, if his level of income savings were normal and adequate to begin with, is it not now adequate to sustain the normal rate of replacing units, RRG? This situation could have been
brought about by a combination of several factors. First, the price of goods may have increased, thus making his normal level of savings inadequate. If the price did not change, then possibly his rate of saving income decreased, due either to a decrease in his total income rate or to a decrease in the fraction of his income he was saving.

Let us investigate each of these factors more fully. A change in the price of goods would be caused by factors that are outside of the consumer sector, except of course, a change in price might be due to a change in the consumers' requisition rate, RRR. (Here again is another feedback loop.) Be that as it may, the consumer does not have the ability to predict the change in price that will occur in the future as a result of his changing his replacement rate, RRG. All that we are saying here is that the consumer's decision to change his rate of saving income, ISRC, or to change his rate of replacing goods, RRG, will be based only on present price information and not on speculations about future prices. (This may not always be true in the real world, but it has sufficient validity to make it a reasonable assumption in our model.) We have thus established a relationship between the present price of goods and the consumers' rate of saving income (as well as his decision to replace worn out units).

A change in the average income rate of consumers is also a factor which will be assumed to be virtually unaffected by the saving or spending decisions made by the consumer. That is, the income rate may affect the consumers' saving and spending decisions but not
vice versa. This would obviously be untrue if we were formulating a model of a whole economy, for an increase in spending by consumers would eventually cause an increase in the wages of the consumers, since cash would flow in a cycle from consumers to manufacturers (or to the government) and back to the consumer again. Our model, however, assumes that the consumers of refrigerators are virtually a distinct group from the factory workers who produce the refrigerators. Stated more precisely, we are assuming the wages paid by the refrigerator industry are a negligible fraction of the average income rate of the consumers. This assumption is felt to be quite reasonable in view of the fact that refrigerators constitute a very small portion of the total goods and services sold in our economy.

We have thus established the income rate of consumers as an independent input to our system and one which we can change in our model as we study its over-all dynamic behavior.

Assuming, then, that the income rate does affect the rate of saving and spending, we must now establish what relationship exists. We have already specified the relation between income rate and saving rate as stated in equation 105R. We have not specified, however, how the fraction of income saved, FIS, is determined by other factors in the system. We made the statement, previously, that this fraction depended upon his need (as reflected in his level of replaceable goods, RGC) and his ability to buy (as reflected in the ratio of average saved income to the present price of goods). We are now in a position to state these relationships more precisely.
As the consumers' level of replaceable goods increases from zero to the total number of goods owned by the consumer sector (which is constant in our model), the fraction of his average income rate that he will save per week will probably increase. Just how much it will increase is wide-open to speculation, but we are forced to make some reasonable assumptions about this relationship if we are to retain this factor of realism in our model. After considerable thought, the author has chosen the relationship shown in Fig. 40.

![Diagram showing FIS, FIS_{max}, FIS_{norm}, RGC_{normal}, TOC, RGC (Units)]

FIS—Fraction of Income rate the consumer will Save for purchases ($/week/$/week)

RGC—Replaceable Goods in Consumer sector (units)

TOC—Total goods Owned by Consumers (unit)

The assumption made here is that the fraction saved is fairly constant if RGC is between zero and the normal level of replaceable goods.
More precisely, we are assuming that if the income rate and the price of goods is constant, the consumers will not change the fraction of income they are saving if the level of replaceable goods drops below the normal level. If RGC should drop below its normal level, this implies that consumers are not holding on to their worn out refrigerators as long as normal; or looked at another way, the total average time between purchases has become less than normal. Our assumption about the fraction of income saved thus says that consumers will not try to maintain this shorter average delay in purchases by saving at a faster rate, but will still save at a rate which will sustain the normal average delay between purchases, since this average delay is satisfactory to them (as implied by the term "normal"). Here again, this may not always be true in the real world, but it should be a reasonable assumption in most instances.

When the level of replaceable goods becomes greater than that normally accepted, however, the consumers will attempt to return conditions to their normal state by increasing their rate of savings. The exact relationship which exists might vary considerably between consumers and between various types of hard goods, but any assumption that maintains this relationship of an increasing rate of savings for an increasing level of replaceable goods should be quite reasonable.

The exact relationship assumed in our model is pictured in Figure 41.
FIS--Fraction of Income Saved by consumers for purchases ($/week/$/week)

RGC--Replaceable Goods in Consumer sector (units)

AFSN--constant denoting Fraction of income Saved when RGC is less or equal to its Normal level ($/week/$/week)

TOC--Total goods Owned by Consumers (units)

ANRG--constant denoting Normal level of Replaceable Goods (units)

AFS1, AFS2--constants determining relationship between
Fraction of income Saved and level of replaceable goods, 1 and 2 (units)

The two equations which specify the two straight-line segments of this curve are the following:

\[
FIS.K = FISP.K \quad \text{if} \quad RGC.K \geq ANRG \\
= AFSN \quad \text{if} \quad RGC.K < ANRG
\]

\[
TSC.KL = (PGR.K)(RRR.JK)
\]

Eq. 106A

Eq. 107R
FISP—Fraction of Income Saved when an abnormally high level of Postponed purchases or replaceable goods exists ($/week/$/week)

RGC—Replaceable Goods (or postponed purchases) at Consumers (units)

FIS—Fraction of Income Saved by Consumers at any time ($/week/$/week)

AFSl, AFSl—constants determining the relationship between the Fraction of income Saved and the level of replaceable goods (units)

AFSN—constant denoting Fraction of income Saved when RGC is less or equal to its Normal level ($/week/$/week)

ANRG—constant denoting Normal level of Replaceable Goods (units)

Note that we have not restricted the values that RGC can have to be less than TOC, the total goods owned by consumers. We know that this is unnecessary if we have formulated our model correctly, for we have assumed that there are no more or no less than TOC goods in the consumer sector at any time. We need only worry in the event that TOC.K is observed to be other than constant at all times.

The reader might now ask why we have made the fraction of income saved a function of need only (as reflected in RGC), since we have emphasized that this fraction depends upon both the consumer's need and his ability to purchase. We respond to this question by pointing out that we have not yet incorporated the consumer's ability to purchase goods, and that when we do, the level of replaceable goods, RGC, will depend inversely upon the consumers ability to purchase.

That is, if his ability to purchase increases, his level of replaceable
goods RGC will tend to decrease, since his replacement rate, RRG, will increase. If on the other hand his ability decreases, his replaceable goods level will increase due to a decreased replacement rate. Thus, in effect, since the fraction of income saved depends upon the level of replaceable goods, RGC, and RGC in turn depends upon the consumer's ability to buy, then the fraction of income saved will be a function of both the consumer's need and his ability to buy.

Having thus rationalized our assumptions about the average rate of income saved, ISRC, we now turn to the average rate of income spent, ISGC. As stated before, if we dealt with each individual consumer, his spending rate would be zero each week until he removed an amount of cash adequate for the purchase of a new unit. Since we agreed to eliminate this problem by observing the average rate of spending, we will proceed as follows.

If we assume that savings are removed at the time new units are ordered, (i.e., that since savings are committed at the time of purchase, they are considered to be "spent" at the same time), then there is no question about the following equation:

\[ TSC_{KL} = (PGR_K)(RRR_{JK}) \]  

\textbf{Eq. 107 R}

\textbf{TSC.--Total Savings Spent by Consumers ($/week)}

\textbf{PRG.--Price of Goods at Retail ($/unit)}

\textbf{RRR.--Requisitions Received at Retail (unit/week)}

This equation calculates the total cash expenditures per week made by consumers for new units. The total number of consumers at any time that are in a position to purchase new units are those that
have goods in service but not worn-out and those that have worn-out
units that have not been replaced by orders. Since we have postulated
one and only one consumer per refrigerator in the consumer sector, the
following equations hold:

\[ \text{NGNR}.K = \text{GSNW}.K + \text{RGC}.K \quad \text{Eq. 108A} \]
\[ \text{NCNR}.K = (\text{ACG})(\text{NGNR}.K) \quad \text{Eq. 109A} \]

NGNR--Number of Goods Not Replaced by orders in consumer
sector (units)

GSNW--Goods in Service but Not Worn out (units)

RGC--Replaceable Goods in Consumer sector (units)

NCNR--Number of Consumers that have Not Replaced goods
by orders (persons)

ACG--constant denoting number of Consumers per Good
(persons/unit)

We will of course set ACG equal to one in accordance with our assump-
tion, although we have the flexibility to change this constant if we
wish.

We now postulate that the average rate of income spent on units is
determined by dividing the total spending rate, TSC, by the total number
of consumers that are in a position to purchase, NCNR.

\[ \text{ISGC}.K = \frac{\text{TSC}.K}{\text{NCNR}.K} \quad \text{Eq. 110A} \]

ISGC--average Income Spent on Goods per Consumer ($/week/person)

TSC--Total savings Spent on goods by Consumers ($/week)

NCNR--Number of Consumers that have Not Replaced goods by
orders (persons)
What equation 11OR says is that when 1000 consumers spend 100 dollars each per week (a total of 100,000 dollars per week), our model assumes that if there are 250,000 consumers in a position to purchase, then the average level of savings is decreased by 100,000/250,000 or .4 dollars per week per consumer. Another way of looking at this is to realize that we are effectively "spreading-out" the amount of savings that a consumer spends for a new unit, so that instead of spending 100 dollars one week and zero dollars every week for 249 weeks, he spends an average of 100/250 or .4 dollars per week every week. This assumption may be quite accurate when dealing with an aggregation of consumers.

**Ability To Purchase**

We now come to the problem of determining the ability of a consumer to purchase a new unit and how this ability affects his purchasing decision. If we maintain our concept of averages, the ability of a consumer to purchase will depend upon the ratio of his average savings to the price of goods at retail.

\[
PPC.K = \frac{ISC.K}{PGR.K}
\]

Eq. 111A

**PPC**—Potential Purchases per Consumer (units/person)

**ISC**—average Income Saved for purchases per Consumer ($/person)

**PGR**—Price of Goods at Retail ($/unit)

Note that we are assuming that the potential consumer purchases, PPC, is calculated on the present price and not a future price. This assumption is reasonable if we use this piece of information to determine the present purchasing rate, RGC.
We must now formulate the mechanism whereby the average consumer makes a decision to purchase. Our previous assumption still holds: that is, his purchase rate will depend on his need (as reflected in RGC) and on his ability to purchase, PPC. We now make one further assumption about this decision. Not only will his purchase rate depend upon his need and his ability to purchase, but also upon his desired ability to purchase. We will define his "normal" purchase rate in terms of the average length of time he will take (or the average delay required) to replace those goods that need replacing. The average delay, ther, in replacing worn-out units will be determined by the ratio of his potential purchasing ability to his desired purchasing ability. If we allow for the possibility that the consumer's desired purchasing ability may vary with economic conditions that are external to our system, then the following equation holds:

\[ \text{DVRG}_K = \left( \frac{1}{\text{PPDC}_K} \right) (\text{DRG})(\text{PPC}_K) \]  

**Eq. 112A**

**DVRG**--average Delay (Variable) in Replacing worn-out Goods in consumer sector (weeks)

**DRG**--average Delay in Replacing worn-out Goods under satisfactory conditions, i.e., when consumers' potential purchasing ability is equal to that desired (weeks)

**PPC**--Potential Purchases per Consumer (unit/person)

**PPDC**--Potential Purchasing power Desired per Consumer (units/person)

This functional relationship is shown graphically in Fig. 42.
We see that if the consumer's desired purchasing ability should double and his potential purchasing ability should remain constant, then the average delay in replacing the worn out goods that exist in the consumer sector will also be doubled. On the other hand, should both his desired and potential purchasing ability double at the same time, this average delay will stay constant—a quite reasonable assumption in terms of our feelings about actual consumer behavior.

We are now in a position to complete our consumer sector by defining the rate of replacing worn-out goods, RRG. Having specified how the average delay in replacing goods, DVRC, is determined, the replacement rate is simply the following:
$$RRG.KL = \frac{RGC.K}{DVRG.K}$$  \text{Eq. 113R}  

RRG--Rate of Replacing worn-out Goods in consumer sector (units/week)  

RGC--Replaceable Goods in Consumer sector (units)  

DVRG--average Delay (Variable) in Replacing Goods in consumer sector (weeks)  

Equations 99L, 112A, and 113R together will be recognized by Industrial Dynamics people as constituting a first-order delay function whose average delay varies with time.  

Our consumer sector is now complete (except for initial conditions and values of constants), since the sector will provide an output rate of requisitions, RRR, if it is provided with the three required inputs, IRC, income rate of consumers; PGR, price of goods at retail; and PPDC, potential purchasing ability desired by consumer.  

**Initial Conditions**  

Initial conditions are required by DYNAMO (ref. 6) for all level equations and for any rates on the right-hand side of an auxiliary or rate equation. Since we desire to start our model in equilibrium as done in Chapter III, we will calculate the initial values of levels
and rates in our system in terms of the assumed equilibrium price, PRMI, and the equilibrium purchasing rate, RRI.

The first level encountered is the goods in service but not worn-out, GSNW. The normal steady state level of this variable is given by the product of the equilibrium shipment rate from retail and the average delay in goods wearing out.

\[ GSNW = (DGW)(SSR) \]  

Eq. 115N

GSNW—Goods in Service but Not Worn-out in consumer sector (units)

DGW—average Delay in Goods Wearing out in consumer sector (weeks)

SSR—Shipments Sent from Retail (units/week)

The initial value of replaceable goods in the consumer section will likewise be equal to the product of the normal average delay in replacing worn-out units and the equilibrium rate of shipments sent from retail.

\[ RGC = (DRG)(SSR) \]  

Eq. 116N

RGC—Replaceable Goods at Consumer sector (units)

DRG—average Delay in Replacing Goods when conditions are normal (weeks)

SSR—Shipments Sent by Retail (units/week)

The initial value of goods replaced by orders but not yet received, GRNR, will be given by the product of the equilibrium, shipping rate, SSR, and the normal average delay in filling orders at retail, DFR.
GRNR = (SSR)(DUR + DHR)  \hspace{1cm} \text{Eq. 117N}

GRNR—Goods Replaced by orders, but Not yet Received (units)

SSR—Shipments Sent by Retail (units/week)

DUR—Delay, average, in filling Unfilled orders at Retail caused by out-of-stock items when inventory is "normal" (weeks)

DHR—Delay due to minimum order Handling time required at Retail (weeks)

The initial level of average income saved per consumer will be assumed to be equal to the price of goods at retail, i.e., the initial level of consumer's desired purchasing ability will be assumed to be one unit per person.

\[ ISC = \frac{PGR}{ACG} \hspace{1cm} \text{Eq. 118N} \]

ISC—Income Saved per Consumer ($/person)

ACG—constant denoting number of Consumers per Good in the consumer sector (persons/unit)

PGR—Price of Goods at Retail ($/unit)

The initial rate of replacing goods, RRG, will just be equal to the initial rate of requisitions received at retail, as will be the initial shipment rate from retail.

\[ RRG = RRR \hspace{1cm} \text{Eq. 119N} \]

\[ SSR = RRR \hspace{1cm} \text{Eq. 120N} \]

RRG—Rate of Replacing Goods in the consumer sector (units/week)

SSR—Shipments Sent from Retail (units/week)

RRR—Requisitions Received at Retail (units/week)
The initial rate of requisitions at retail is specified as RRI.

\[ \text{RRR} = \text{RRI} \quad \text{Eq. 121N} \]

RRR--Requisitions Received at Retail (units/week)
RRI--Requisition Rate at Retail Initially (units/week)

The initial rate of total spending by consumers will be the product of the initial price of goods at retail and the initial rate of requisitions received at retail.

\[ \text{TSC} = (\text{PGR})(\text{RRR}) \quad \text{Eq. 122N} \]

TSC--Total Spending rate by Consumers ($/week)
PGR--Price of Goods at Retail ($/unit)
RRR--Requisitions Received at Retail (units/week)

Finally, the initial price of goods at retail will be specified as PRMI.

\[ \text{PGR} = \text{PRMI} \quad \text{Eq. 123N} \]

PGR--Price of Goods at Retail ($/unit)
PRMI--Price of Raw Materials Initially at supplier ($/unit)

**Parameters (constants)**

There are two average delays in the consumer sector to be picked. Let us assume that the average lifetime of a refrigerator before it wears out is 200 weeks and that consumers are willing, under normal circumstances, to hold onto worn-out refrigerators for an average of 50 weeks.
DGW = 200 weeks
DRG = 50 weeks

We have stated previously that there is one and only one consumer per refrigerator in the consumer sector.

ACG = 1 person/unit

We will assume, now, that the initial price of raw materials and the initial rate of requisitions are 100 $/unit and 1000 units/week respectively.

FRMI = 100 $/unit
RRI = 1000 units/week

There are four remaining constants which determine the relationship between the fraction of income saved and the level of replaceable goods in the consumer sector. The normal level of replaceable goods will be the same as the initial level determined by the product of DRG and SSR. This is 50 weeks times 1000 units/week or

ANRG = 50,000 units

We will next assume that the normal fraction of income saved per week is 1/250 of the average income rate of the consumer. (At a price of 100 $/unit and an income rate of 100 $/week, this fraction, 1/250, will provide enough for one unit in 250 weeks, which is the normal delay between purchases).

AFSN = .004

The constants AFS1 and AFS2 have been chosen such that if the level of replaceable goods is twice its normal size or 100,000 units the fraction of income saved will be three times that saved under normal circumstances or 3/250.
AFS1 = 6,250,000 units
AFS2 = 25,000 units

Input Variables

The input variables to our consumer sector are the income rate of consumers, IRC, and the purchasing power desired by the consumer, PPDC. In any simulation run the following equations will be used.

\[
IRC.KL = IRI + IRCC.K \quad \text{Eq. 124A}
\]

\[
PPDC.K = AIPP + CPPD.K \quad \text{Eq. 125A}
\]

IRC--Income Rate of Consumers ($/week per person)
IRI--Income Rate Initially of consumers ($/week per person)
IRCC--Income Rate Change, as a function of time, at Consumers ($/week per person)
PPDC--Potential Purchasing ability Desired per Consumer (units/person)
AIPP--constant denoting Initial value of Potential Purchasing ability desired per consumer (units/person)
CPPD--Change, as a function of time, in the Potential Purchasing ability Desired per consumer (units/person)

The constants IRI and AIPP will be specified as 100 $/week/person and 1 unit/person respectively.

IRD = 100 $/week/person
AIPP = 1 unit/person

The only other equations required are those specifying IRCC and CPPD. These might be step functions, ramp functions or perhaps sinusoidally varying functions depending upon the type of input desired.
The runs to be made using this consumer sector will either have various input functions for IRCC and CPPD, or else the parameters of the system will be changed while using the same input functions.

**Summary of Proposed Behavior of Consumer Sector**

It is essential, at this point for the reader to gain an over-all view of the behavior of the consumer sector as a whole information-feedback system. As stated by Professor Forrester, "An information-feedback system is a system of counter-balancing influences. An error in one factor is balanced within the system by self-induced changes in other factors." (ref. 5). It is essential that we go back now and see how the combination of assumptions we have made about the consumer sector produces this information-feedback phenomenon.

Referring to Fig. 36, if the system is in equilibrium initially, we have assumed that the average income saved per consumer is 100 dollars per person, the total goods in service but not worn-out is 200,000 units, the number of replaceable units is 50,000 units and, if DHR + DUR is 1.4 weeks, the number of goods replaced by orders but not discarded is 1400 units. The level of purchasing ability is initially 1 unit per person, and is equal to the desired level of purchasing ability, PPDC. The initial fraction of income saved is 1/250th; the number of consumers who are potential purchasers is GSNW + RGC or 250,000 people, and the average delay in replacing worn-out units is DRG or 50 weeks. Let us reason through the behavior of the system in response to a 10 per cent step decrease in the income rate of consumers, IRC.

The first effect that would be felt would be a decrease in the
level of income saved by consumers, since the imput rate of savings would be less than the output rate of spending. The consumer would then become aware of a reduced level of potential purchases, PPC. Under the assumptions we have made, he would thus reduce his rate of replacing worn-out units by increasing the average delay, DVRG, in making replacements. Very shortly this would result in an increased level of replaceable goods, RGC, since the input rate of goods wearing out, GWOC, would not decrease appreciably for a period of at least 200 weeks. As the level of replaceable goods, RGC, increase, then, the consumer becomes aware of an increasing need and will begin to increase the fraction of his income, FIS, that he saves for purchases.

We have thus followed a complete feedback loop, for a decrease in income level has been counter-balanced by a self-induced increase in the fraction of income saved and eventually the system will come to a new equilibrium condition (assuming the system is not inherently unstable) in which the level of savings, ISC, is just sufficient to sustain some constant purchasing rate, RRR. It is not yet obvious to the author what this new purchasing rate will be, nor what level of savings will be sustained in equilibrium, but these quantities could be calculated from the equations of the system alone or determined by an actual simulation run on the computer.

We see, immediately, that there will be some rather long transients in our consumer sector, since a change in shipment rate SSR is not felt in the rate of goods wearing out, GWOC, for an average time of
200 weeks. This tells us that the "natural" frequencies of our system contains a very low fundamental frequency on the order of 1 cycle per 200 weeks. This means simply that we may have to run our model for at least 500 weeks if we are to observe the total behavior of the system in response to a change in the input variables.

Now that the consumer sector has been formulated and a preliminary understanding of its information feedback properties has been acquired, we can proceed to the problem of generating price and quantity data by submitting simulation runs.

It is hardly obvious what relationship exists between the price of goods at retail, PGR, and the rate of requisitions received at retail, RRR. It is not obvious from just looking at the consumer sector what the equilibrium or static values of price and quantity will be, and it is even less obvious what the dynamic value of price and quantity will be under different input conditions. Our goal now is to examine both the static and dynamic values of price and quantity and investigate the possibility of deriving one from the other.

Static Demand Curve for Consumer Sector

The long-run or equilibrium demand schedule for the consumer sector can be calculated from the equations of the system alone. The simple requirements that assure equilibrium of the system are 1) that the inputs to the system are constant and 2) that the input rate equals the output rate for each "level" variable in the system. By assuming, then, that the inputs to the consumer sector are constant, we can set up "input equals output" equations and determine the equilibrium
purchasing rate, RRR, for any constant input price, PGR. To save
time and space, the author will present only the final expressions
for the equilibrium relationship between price, PGR, and quantity, RRR.

When the consumer's income rate, IRC, is constant at 100 $/week and
his desired ability to purchase, PPDC, is constant at 1 unit/person,
the following equations apply:

\[ RRR = \frac{251,400}{2.5(PGR)+1.4} \text{ when } PGR \leq 100 \text{ $/unit} \quad \text{Eq. 126} \]

\[ \frac{RRR}{251,400-1.4(RRR)} = \frac{226,400-201.4(RRR)}{62,500 (PGR)} \text{ when } PGR \geq 100\text{$/unit} \quad \text{Eq. 127} \]

The necessity of two equations results from an assumption about
the fraction of income, FIS, that the consumer will save. That is,
it can be shown that a price, PGR, of less than 100 $/unit results in
a level of replaceable goods, RGC, of less than 50,000 units. Cor-
respondingly, if PGR is greater than 100 $/unit, RGC will be greater
than 50,000 units. In the former case, we have assumed that FIS is con-
stant at .004, while in the latter, FIS is assumed to increase linearly
with an increase in RGC.

Equation 127 appears to be unusually complex, since it requires
the solution of a quadratic equation of the form \(Ax^2 + Bx + C = 0\).
This complexity, however, is an inherent property of the consumer sector
we have formulated and must therefore be accepted as such.

Equations 126 and 127 are plotted together in Fig. 43.
Demand Schedule for Consumer Sector

ING = $100 \$/Week

PFC = 1 Unit/Person

FIGURE 1.3
We now turn to the problem of checking our theoretically derived demand curve by means of several simulation runs.

**Derivation of Long-Run Demand Curve Through Simulation Techniques**

We will now take advantage of the model we've formulated and determine the equilibrium price and quantity by means of several simulation runs. The few additional equations we will need will be formulated next.

The input values of IRC, PPDC and PGR can be inserted into the model by using equations 124A and 125A and the following equations:

\[ \text{PGR}.K = \text{PGI} + \text{PGRC}.K \]  
Eq. 128A

- **PGR**—Price of Goods at Retail ($/unit)
- **PGI**—Price of Goods Initially at retail ($/unit)
- **PGRC**—Price of Goods at Retail Change as a function of time ($/unit)

We have already specified PGI as being equal to PRMI or 100 $/unit. The change in price will be a step increase or decrease.

\[ \text{PGRC}.K = \text{FSTEP} (\text{PI}, 1) \]  
Eq. 129A

- **PGRC**—Price of Goods at Retail Change ($/unit)
- **PI**—Price Increase ($/unit)

This equation says that an increase of PI $/unit will occur at the first week after the simulation begins.

We will make several runs with the following price increases:

- PI = 50, 40, 30, 20, 10, -10, -20, -30, -40, -50 $/unit

The input variables IRC and PPDC will assumed to be constant
at 100 $/week/person and 1 unit/person respectively. This is accomplished as follows:

IRCC.K = 0

CPPD.K = 0

IRCC--Income Rate Change as a function of time, at Consumers ($/week/person)

CPPD--Change, as a function of time, in the Potential Purchasing ability Desired per consumer (units/person)

We need only provide an unfilled order backlog at retail to complete this system. Unfilled orders will be increased by requisition received at retail and decreased by shipments sent by retail.

\[
UOR.K = UOR.J + (DT)(RRR.JK - SSR.JK)
\]

UOR--Unfilled Orders at Retail (units)
RRR--Requisitions Received at Retail (units/week)
SSR--Shipments Sent by Retail (units/week)

As in the basic production-distribution model, we assume that the shipment rate from retail is the following:

\[
SSR.K = \frac{UOR.K}{DFR}
\]

SSR--Shipments Sent by Retail (units/week)
UOR--Unfilled Orders at Retail (units)
DFR--Delay (average) in Filling orders at Retail (weeks)

Unlike the basic model, however, we assume that DFR is constant, since we will be interested in the equilibrium rates and levels only. Since under equilibrium conditions \( DFR = DHR + DUR \) and since DHR and DUR
are 1 week and .4 weeks respectively, we then specify DFR and 1.4 weeks:

$$\text{DFR} = 1.4 \text{ weeks}$$

The flow diagram for our equilibrium model is shown in Fig. 44.

This model, then, will permit us to determine the equilibrium values of RRR for any constant input price, PGR.

Compatibility of Analytical and Simulated Results

Having made several simulation runs with the model just formulated, the author found that the equilibrium price and quantity determined from each run fell exactly on the analytically derived curve of Fig. 43. This, of course, assures us that equations 126 and 127 have been derived correctly.
Figure 43, in itself, is a very interesting result, for we have the feeling that this curve is far less fictitious than those that were assumed in Chapter III. This is true because the consumer's decision to purchase is now based upon the actual physical factors which determine his need for finished goods and his ability to purchase them.

We are pleased to see that this demand curve slopes downward to the right, for it tends to reassure us that the economists' theory of "diminishing utility" is valid. The author would like to suggest that someone continue this type of investigation further, for it might be possible to build up a useful set of theorems about the relationship between the actual physical factors which influence the consumers' purchasing decision and the long-run demand schedule which results from incorporating these factors in a dynamic model.
Results of Incorporating Consumer Sector in Production-Distribution Model

Having completed the formulation of a more realistic consumer sector and the derivation of one of its equilibrium demand schedules, we are now in a position to observe its behavior when incorporated in the total system. The price of goods at retail, PGR, will provide one of the necessary inputs to the consumer sector, and the consumers' purchasing rate, RRR, will in turn provide the only necessary input to the retail sector. The consumers' income rate, IRC, and their desired purchasing ability, PPDC, will be held constant at 100 $/week and 1 unit/person respectively, thereby making the curve of Fig. 43 representative of their long-run demand schedule.

We shall retain the linear supply curve in the raw material supplier sector and choose the constants ARZ and ART in equation 43A such that the "flexibility" of supply is one half.

We will next employ the same technique used by Professor Forrester in his Harvard Business Review article (ref. 2); that is, we will inject a random noise component into the consumer's purchasing rate, RRR. This, of course, is an attempt to incorporate the same degree of uncertainty in our model that is observed in the day-by-day purchasing rates in the real world.

The following equations will provide a random input component to RRR. First, we will change equation 113R to the following:

\[ RRNA.K = RGC.K / DVRG.K \]

Eq. 134A

RRNA -- Requisitions Received at retail -- component determined by Needs of consumers' and their Ability to purchase (units/week)
RGC -- Replaceable Goods at Consumer sector (units)

DVRG -- average Delay (Variable) in Replacing worn-out Goods (weeks)

The following two equations will also be added.

\[ RRR.KL = RRNA.K + RRN.K \]  \hspace{1cm} \text{Eq. 135R}
\[ RRN.K = (ADN)NOISE \]  \hspace{1cm} \text{Eq. 136A}

RRR -- Requisitions Received at Retail (units/week)

RRNA -- Requisitions Received at retail--component determined by Needs of consumers' and their Ability to purchase (units/week)

RRN -- Requisitions Received at retail--Noise component (units/week)

No additional initial conditions are required by these three equations; however, the constant ADN must be specified. Since our system has been formulated to sustain an initial price, PQR, of 100 $/unit and an initial purchasing rate, RRR, of 1000 units/week, we might choose ADN arbitrarily as 50 units/week.

\[ ADN = 50 \text{ units/week} \]

The noise component that is being incorporated into our system will thus have the following probability density distribution. (Fig. 45)

Approximately sixty percent of this noise component will fall within plus or minus 14.5 units/week (one standard deviation). This may appear to be a rather small value for the standard deviation, but the author found it necessary to reduce the noise component, RRN, due to the large amount of amplification existing in the total system.
System Response

In order to gain a "feeling" for the response of our total system to a noisy purchasing rate, RRR, the factory manufacturing rate, MOF, is plotted with the consumer purchasing rate, RR3, in Fig. 46.

It becomes apparent to us that our total system is basically unstable; for although the noise input, RRN, is bounded, the outputs of the system, RRR and MOF, are responding with growing oscillations. Here is a perfect example of an unstable system resulting from the combination of two stable components; namely, the consumer sector and the production-distribution sector. We know from our previous investigations that both of these sectors are stable within themselves; however, the combination of the two has resulted in a system which will "blow up" in response to any input disturbance.
It is unfortunate that an unstable system has resulted, but it does not, in theory, impede our investigation of price behavior. Rather than being impeded, we have actually been challenged to explain the factors which have produced an unstable system.

In Chapter III we discovered that one of the major factors which influenced the stability of our system was the slope of both the demand and the supply curves. We were even able to derive an analytical expression for the relationship between the "flexibility" and "elasticity" and the stability of the system. We pointed out quite clearly, however, that this particular relationship would not hold if we were to reformulate our system. It is still reasonable to believe, however, that the "flexibility" and "elasticity" which exist in the new system do affect its stability, but we cannot conclude the exact relationship without making a series of simulation runs as in Chapter III. Due to a time limitation, the author was unable to investigate this instability phenomenon completely, but we should be able to indicate some of the basic factors which might have caused this condition.

Undoubtedly, the combination of delays in the total system have played a major role in causing instability to result, but their exact causal relationship is not obvious. It is very possible that the slope of the demand curve has produced instability, particularly since the slope at the starting point (PGR = 100 $/unit, RRR = 100 units/week) is discontinuous (due to our assumption about the fraction of income saved by the consumer, equation 106A). We could avoid this problem by formulating a smooth, continuous function for the fraction, FIS, although this
would probably result in a considerably more complex expression for the static demand curve. This should dramatically point out to future researchers the dangers involved in employing straight line approximations in dynamic models.

Undoubtedly, one of the more important factors which has contributed to this instability condition is the high amplification existing in our system. This amplification is very obvious in the dynamic response plotted in Fig. 46, for the manufacturing rate, MOF, has much larger peaks of oscillation than the consumers' purchasing rate, RRR. Undoubtedly, the rate of receiving raw material orders at the factory (which determines the price of raw materials) has even greater fluctuation peaks due to the amplifying factors in the factory's decision to purchase raw materials.

Without further simulation evidence, then, we can only conclude that the observed instability condition was caused by the combination of delays, amplifying factors, and functional relationships incorporated in our model.

**Price Behavior**

In spite of the unstable condition which resulted, it should still be informative to observe the price behavior generated by our model. Plotting the monthly data on prices and purchasing rates (as in Chapter III), we notice that the resultant scattering of points tends to fall along some smooth curve (Fig. 47). Although the pattern is distinct, however, the static information that is being conveyed appears to be quite erroneous in the range of prices exceeding 100 $/unit. This could be due to the instability condition which exists in the system, but the author holds the
opinion that it is the long delays in the consumer sector which are causing this erroneous information. The reasoning behind this opinion is the following. As the system gets under motion initially, the consumer's purchasing rate is determined, for the most part, by his ability to purchase, PPC, which is in turn determined by the ratio of his savings to the present price of goods at retail. As the price, PGR, starts to fluctuate (as a result of this purchasing pattern being propagated through the system), the consumer's purchasing rate, RRR, tends to follow these fluctuations with a high degree of sensitivity (since his average delay in replacing goods, DWRG, varies inversely with his purchasing ability, PPC). However, it requires a comparatively long passage of time before the consumer feels a changing degree of need and starts to do something about it by increasing the fraction of income saved, FIS. This seems to be caused by two factors: first, because we have made his fraction of income fairly insensitive to small changes in the level of replaceable goods, RGC (Fig. 41) and secondly, because of the 50-week average delay which the consumer normally takes in replacing these worn-out units.

Since the equilibrium demand curve, in the range of prices exceeding 100 $/unit, has been shown to result from our assumption about the fraction of income saved, it seems reasonable that this part of the demand curve would not be applicable to the consumer's instantaneous demand until he began to increase this fraction, FIS, and until his potential purchasing ability, PPC, began to rise with his rising level of saved income, ISG.

If this reasoning has not been too confusing to the reader, he should begin to understand why the instantaneous price and quantity, (PGR and RRR),
tend to diverge from the equilibrium demand schedule and tend to trace out, instead, a short-run demand curve having a much greater "elasticity" in response to changing prices.

To back up this reasoning, we would, of course, try increasing the sensitivity of the consumer's response to small changes in his level of replaceable goods, RGC, and perhaps cut down his average delay in replacing these goods, DVRG, and then make another set of simulation runs. A new equilibrium demand curve would result from these changes, and perhaps the instantaneous values of PQR and RRR would follow this curve more closely. In the process of this investigation it would probably be advisable to replace the function used in equation 106A by a smooth continuous curve, in an attempt to clarify the dangers which exist in using straight line approximations.

The author regrets that the limitations of time prevented a further investigation into this phase of the problem, for we are just now beginning to understand the physical factors which influence the quality and quantity of static information conveyed by dynamically generated data. It should be painfully apparent, however, that there are a multitude of factors which need to be understood much more clearly before this investigation can provide the economist with a set of useful, tangible results.
CHAPTER V

SUMMARY

Conclusions

The author is quite reluctant to draw any sweeping conclusions from the results of this initial investigation, for we have barely "scratched the surface" of the problem which confronts today's analytical economists. We have been able to demonstrate, however, that the proper tools are now available for active research into this problem, and have been able to uncover, through the use of these tools, some of the basic factors which influence the quality and quantity of equilibrium information conveyed by dynamic, statistical data.

In Chapters II and III, we found that under very simple conditions (moving from one equilibrium condition to another on hypothetical demand and supply schedules) the "elasticity" and "flexibility" of demand and supply greatly influenced the quantity of static information conveyed; while the delays in the system, particularly that of consumer's response, DERR, influenced the nature of this information. At the same time, we were able to gain a feeling for how the stability of the system was influenced by the slopes of the demand and supply curves and by the combination of delays which were incorporated in the model. By creating a situation in which no equilibrium condition ever existed, we were then surprised to find that fairly good static information was being conveyed to us by our model. This injected a light of hope into our investigation, but it re-emphasized the need for expanding our model to include some of the physical factors which influence the consumer's purchasing decision.
In Chapter IV a more complete consumer sector was formulated and one of its equilibrium demand schedules was derived analytically. When we incorporated it in the system, however, we found a condition of instability resulting and we attempted to explain this condition in terms of our previous results and in terms of the assumptions that were made in formulating this consumer sector.

In observing the dynamic price behavior in this unstable system, we found a distinct, but considerably erroneous short-run demand curve being traced out. The author ventured an opinion as to the cause of this phenomenon and indicated the type of simulation runs which would either refute or verify this opinion.

Validity of Relating Static and Dynamic Price Behavior

From the observations made in this thesis, there appears to be certain situations in which a dynamic to static transformation is both valid and fruitful. We have been unable to carry this initial investigation far enough, however, to permit us to formulate any theorems that would be immediately useful to analytical economists. It is hoped, however, that the results of this investigation will contribute to a better understanding of dynamic price behavior, and that the questions asked by today's economists will eventually be answered by further research into this problem.

Suggestions for Further Research

The author would like to suggest that further research be pursued in the direction of formulating more realistic consumer sectors. There were many factors which were necessarily omitted from our model which
probably play an important role in determining the consumer's pur-
chasing rate. Such factors as the effect of advertising, the avail-
bility of credit, the number of other hard goods which must be pur-
chased by the consumer, and the effect of style changes in a product
should all be taken into consideration when formulating such a model.

Further research into the methods by which price information is
transmitted through a system should be pursued also, for we were
forced to employ an artifice in simulating this phenomenon and as
a result, we were burdened with cumbersome variables whose dimensions
were quite meaningless.

A better understanding is also needed of the causes of instability
in a dynamic model such as ours. Although we had a vague understand-
ing of the physical factors involved, we were at a loss to state the
precise degree to which any one factor contributed to the unstable
condition of our model of Chapter IV.

These suggestions, of course, are all preliminary steps in
solving the problems which face today's analytical economists. It
should now be clearer than ever that this investigation will require
many man-hours of concentrated effort to complete. It is the author's
hope, however, that this thesis has paved the road to a better under-
standing of dynamic price behavior, and that it will, by its very
presence in contemporary economic literature, motivate researchers to
employ the powerful analytical tools now available to them.
Appendix I

The following are the equations of the basic production-distribution model as formulated by Professor J. W. Forrester in Memo D-34 (ref. 5):

Retail Sector

1L \[ UOR.K = UOR.J + DT(RRR.JK - SSR.JK) \]

2L \[ IAR.K = IAR.J + DT(SRR.JK - SSR.JK) \]

3A \[ STR.K = UOR.K/DFR.K \]

4A \[ NIR.K = IAR.K/DT \]

5R \[ SSR.KL = STR.K, \text{ if } STR.K \leq NIR.K \\
= NIR.K, \text{ if } STR.K > NIR.K \]

6A \[ DFR.K = DHR + DUR \frac{(IDR.K/IAR.K)}{} \]

7A \[ IDR.K = (AIR)(RSR.K) \]

8L \[ RSR.K = RSR.J + (DT)[1/DDR](RRR.JK - RSR.J) \]

9R \[ PDR.KL = RRR.JK + \frac{[1/DIR] [(IDR.K - IAR.K)]}{(LDR.K - LAR.K) + (UOR.K - UNR.K)} \]

10A \[ LDR.K = (RSR.K)(DCR + DMR + DFD.K + DTR) \]

11A \[ LAR.K = CPR.K + PMR.K + UOD.K + MTR.K \]

12A \[ UNR.K = (RSR.K)[DHR + DUR] \]

13L \[ CPR.K = CPR.J + (DT)(PDR.JK - PSR.JK) \]

14R \[ PSR.KL = FDELAY3 (CPR.K, DCR) \]

15L \[ PMR.K = PMR.J + (DT)(PSR.JK - RRD.JK) \]

16R \[ RRD.KL = FDELAY3 (PMR.K, DMR) \]

17L \[ MTR.K = MTR.J + (DT)(SSD.JK - SRR.JK) \]

18R \[ SRR.KL = FDELAY3 (MTR.K, DTR) \]

I-A
Distributor Sector

19L \quad \text{UOD.}K = \text{UOD.}J + (\text{DT})(\text{RRD.}JK - \text{SSD.}JK)

20L \quad \text{IAD.}K = \text{IAD.}J + (\text{DT})(\text{SRD.}JK - \text{SSD.}JK)

21A \quad \text{STD.}K = \text{UOD.}K / \text{DFD.}K

22A \quad \text{NID.}K = \text{IAD.}K / \text{DT}

23R \quad \text{SSD.}KL = \text{STD.}K, \text{if } \text{STD.}K \leq \text{NID.}K
\quad = \text{NID.}K, \text{if } \text{STD.}K > \text{NID.}K

24A \quad \text{DFD.}K = \text{DHD} + \text{DUD} \quad (\text{IDD.}K / \text{IAD.}K)

25A \quad \text{IDD.}K = (\text{AID})(\text{RSD.}K)

26L \quad \text{RSD.}K = \text{RSD.}J + (\text{DT})[1 / \text{DRD}](\text{RRD.}JK - \text{RSD.}J)

27R \quad \text{PDD.KL} = \text{RRD.}JK + (1 / \text{DRD})[(\text{IDD.}K - \text{IAD.}K) + (\text{LDD.}K - \text{LAD.}K)
\quad \quad + (\text{UOD.}K - \text{UND.}K)]

28A \quad \text{LDD.}K = (\text{RSD.}K)(\text{DCD} + \text{DMD} + \text{DFF.K} + \text{DTD})

29A \quad \text{IAD.}K = \text{CPD.K} + \text{PMD.K} + \text{UOF.K} + \text{MTD.K}

30A \quad \text{UND.}K = (\text{RSD.}K)(\text{DHD} + \text{DUD})

31L \quad \text{CPD.K} = \text{CPD.J} + (\text{DT})(\text{PDD.JK} - \text{PSD.JK})

32R \quad \text{PSD.KL} = \text{FDELAY3}(\text{CPD.K}, \text{DCD})

33L \quad \text{PMD.K} = \text{PMD.J} + (\text{DT})(\text{PSD.JK} - \text{RRF.JK})

34R \quad \text{RRF.KL} = \text{FDELAY3}(\text{PMD.K}, \text{DMD})

35L \quad \text{MTD.K} = \text{MTD.J} + (\text{DT})(\text{SSF.JK} - \text{SRD.JK})

36R \quad \text{SRD.KL} = \text{FDELAY3}(\text{MTD.K}, \text{DTD})

Factory Sector

37L \quad \text{UOF.K} = \text{UOF.J} + (\text{DT})(\text{RRF.JK} - \text{SSF.JK})

38L \quad \text{IAF.K} = \text{IAF.J} + (\text{DT})(\text{SRF.JK} - \text{SSF.JK})

39A \quad \text{STF.K} = \text{UOF.K} / \text{DFF.K}

40A \quad \text{NIF.K} = \text{IAF.K} / \text{DT}
\[ SSF.KL = STF.K, \text{ if } STF.K \quad NIF.K \]
\[ = NIF.K, \quad \text{if } STF.K \quad NIF.K \]
\[ DFF.K = DHF + DUF \ (IDF.K/IAF.K) \]
\[ IDF.K = (AIF)(RSF.K) \]
\[ RSF.K = RSF.J + (DT)[1/DRF](RRF.JK - RSF.J) \]
\[ MWF.K = RRF.JK + [1/DIF] \ [(IDF.K - IAF.K) + (LDF.K - LAF.K) \]
\[ + (UOF.K - UNF.K)] \]
\[ MDF.KL = MWF.K, \quad \text{if } MWF.K \quad ALF \]
\[ = ALF, \quad \text{if } MWF.K \quad ALF \]
\[ LDF.K = (RSF.K)(DCF + DPF) \]
\[ LAF.K = (CPF.K + OPF.K) \]
\[ UNF.K = (RSF.K)(DHF + DUF) \]
\[ CPF.K = CPF.J + (DT)(MDF.JK - MOF.JK) \]
\[ MOF.KL = FDELAY3(CPF.K, DCF) \]
\[ OPF.K = OPF.J + (DT)(MOF.JK - SRF.JK) \]
\[ SRF.KL = FDELAY3(OPF.K, DPF) \]

**Initial Conditions**

\[ RRR = RRI \]
\[ UOR = (RSR)(DHR + DUR) \]
\[ IAR = (AIR)(RSR) \]
\[ RSR = RRR \]
\[ CPR = (DCR)(RRR) \]
\[ PMR = (DMR)(RRR) \]
\[ MTR = (DTR)(RRR) \]
\[ RDD = RRR \]
\[ UOD = (RSD)(DHD + DUD) \]

I-C
63N \( IAD = (AID)(RSD) \)

64N \( RSD = RRD \)

65N \( CPD = (DCD)(RRD) \)

66N \( PMD = (DMD)(RRD) \)

67N \( MTD = (DTD)(RRD) \)

68N \( RRF = RRR \)

69N \( UOF = (RSF)(DHF + DUF) \)

70N \( IAF = (AIF)(RSF) \)

71N \( RSF = RRF \)

72N \( CPF = (DCF)(RRF) \)

73N \( OPF = (DPF)(RRF) \)

74N \( ALF = (1000)(RRI) \)

**Parameters (constants)**

- DHR = 1.0 week
- DHD = 1.0 week
- DHF = 1.0 week
- DUR = 0.4 week
- DUD = 0.6 week
- DUF = 1.0 week
- AIR = 8 weeks
- AID = 6 weeks
- AIF = 4 weeks
- DRR = 8 weeks
- DRD = 8 weeks
- DRF = 8 weeks
DIR = 4 weeks
DID = 4 weeks
DIF = 4 weeks
DCR = 3 weeks
DCD = 2 weeks
DCF = 1 week
DMR = 0.5 week
DMD = 0.5 week
DTR = 1.0 week
DTD = 2.0 weeks
DPF = 6.0 weeks
ALF = (1000)(RRI)
RRI = 1000 units/week

Definitions

The following define the variables and constants used in the equations above.

AID, proportionality constant for Inventory at Distributor, (weeks)
AIF, proportionality constant for Inventory at Factory, (weeks)
AIR, proportionality constant between Inventory and average sales at Retail, (weeks)
ALF, constant specifying manufacturing capacity Limit at Factory, (units/week)
CPD, Clerical in Process orders at Distributor, (units)
CPF, Clerical in Process manufacturing orders at Factory, (units)
CPR, Clerical in Process orders at Retail, (units)
DCD, Delay in Clerical order processing at Distributor, (weeks)
DCF, Delay in Clerical processing of manufacturing orders at Factory, (weeks)
DCR, Delay in Clerical order placing at Retail, (weeks)
DFD, Delay in Filling orders at Distributor, (weeks)
DFF, Delay (variable) in Filling orders at Factory, (weeks)
DFR, Delay in Filling orders at Retail, (weeks)
DHD, Delay due to minimum Handling time required at Distributor, (weeks)
DHF, Delay due to minimum handling time required at Factory, (weeks)
DHR, Delay due to minimum handling time required at Retail, (weeks)
DID, Delay in Inventory (and pipeline) adjustment at Distributor, (weeks)
DIF, Delay in Inventory (and pipeline) adjustment at Factory, (weeks)
DIR, Delay in Inventory (and pipeline) adjustment at Retail, (weeks)
DMD, Delay in Mail from Distributor, (weeks)
DMR, Delay in Mail from Retail to distributor, (weeks)
DPF, Delay in Production lead time at Factory, (weeks)
DRD, Delay time constant in smoothing Requisitions at Distributor, (weeks)
DRF, Delay in smoothing Requisitions at Factory, (weeks)
DRR, Delay in smoothing Requisitions at Retail, the smoothing time constant, (weeks)
DT, Delta Time, (weeks) = the time interval between solutions of the equations
DTD, Delay in Transportation of goods to Distributor, (weeks)
DTR, Delay in Transportation of goods to Retail, (weeks)
DUD, Delay, average, in Unfilled orders at Distributor caused by out-of-stock items when inventory is "normal", (weeks)
DUF, Delay, average, in Unfilled orders at Factory caused by out-of-stock items when inventory is "normal", (weeks)
DUR, average Delay in Unfilled orders at Retail caused by out-of-stock items when inventory is "normal", (weeks)
IAD, Inventory Actual at Distributor, (units)
IAF, Inventory Actual at Factory warehouse, (units)
IAR, Inventory Actual at Retail, (units)
IDL, Inventory Desired at Distributor, (units)
IDF, Inventory Desired at Factory, (units)
IDR, Inventory Desired at Retail, (units)
LAD, pipeline orders Actual in transit to Distributor, (units)
LAF, pipeline orders Actual in transit through Factory, (units)
LAR, pipeline orders Actual in transit to Retail, (units)
LDD, pipeline orders Desired (necessary) in transit to Distributor, (units)
LDF, pipeline orders Desired (necessary) in transit through Factory, (units)
LDR, pipeline orders Desired (necessary) to supply Retail, (units)
MDF, Manufacturing rate Decision at Factory, (units/week)
MOF, Manufacturing Orders into Factory, (units/week)
MTD, Material in Transit to Distributor, (units)
MTR, Material in Transit to Retail, (units)
MWF, Manufacturing rate Wanted at Factory, (units/week)
NID, Negative Inventory limit rate at Distributor, (units/week)
NIF, Negative Inventory limit rate at Factory, (units/week)
NIR, Negative Inventory limit rate at Retail, (units/week)
OPF, Orders in Production at Factory, (units)
PDD, Purchasing rate Decision at Distributor, (units/week)
PDR, Purchasing rate Decision at Retail, (units/week)
PMD, Purchase orders in Mail from Distributor, (units)
PMR, Purchase orders in Mail from Retail, (units)
PSD, Purchase orders Sent from Distributor, (units/week)
PSR, Purchase orders Sent from Retail, (units/week)
RRD, Requisitions (orders) Received at Distributor, (units/week)
RRF, Requisitions (orders) Received at Factory, (units/week)
RRI, Retail Requisitions, Initial rate, constant, (units/week)
RRR, initial value, Requisitions (orders) Received at Retail, (units/week)
RSD, Requisitions Smoothed at Distributor, (units/week)
RSF, Requisitions Smoothed at Factory, (units/week)
RSR, Requisitions Smoothed at Retail, (units/week)
SSD, Shipments Sent from Distributor, (units/week)
SSF, Shipments Sent from Factory, (units/week)
SSR, Shipments Sent from Retail, (units/week)
SRD, Shipments Received at Distributor, (units/week)
SRF, Shipments Received at Factory inventory (manufacturing output), (units/week)
SRR, Shipments Received at Retail inventory, (units/week)
STD, Shipping rate to be Tried at Distributor, (units/week)
STF, Shipping rate to be Tried at Factory, (units/week)
STR, Shipping rate to be Tried at Retail, (units/week)
UND, Unfilled orders, Normal, at Distributor, (units)
UNF, Unfilled orders, Normal, at Factory, (units)
UNR, Unfilled Normal level of orders at Retail, (units)
UCD, Unfilled Orders at Distributor, (units)
UOF, Unfilled Orders at Factory, (units)
UOR, Unfilled Orders at Retail, (measured in units of goods on order)
The following is a summary of the equations formulated in this thesis.

**Factory Inventory of Raw Materials**

1L \[ MT_F.K = MT_F.J + (DT)(RM_S.JK - RM_RF.JK) \]

2R \[ RM_RF.KL = F_DELAY3 (MT_F.K, DRM) \]

3L \[ RA_F.K = RA_F.J + (DT)(RM_RF.JK - RM_RF.JK) \]

4R \[ RM_RF.KL = MOF.JK \]

5L \[ RM_SF.K = RM_SF.J + (DT)(1/DRUF)(RM_RF.JK - RM_SF.J) \]

6A \[ RDF.K = (ARF)(RM_SF.K) \]

7L \[ OPRS.K = OPRS.J + (DT)(RM_DF.JK - RM_OS.JK) \]

8R \[ RM_OS.KL = F_DELAY3 (OPRS.K, DCMF) \]

9A \[ RL_AF.K = OPRS.K + UCS.K + MTF.K \]

10A \[ RL_DF.K = (RM_SF.K)(DCMF + DFS.K + DRM) \]


12A \[ NIRM.K = RA_F.K/DT \]

13A \[ AML_F.K = NIRM.K, \text{ if } NIRM.K < ALF = ALF, \text{ if } NIRM.K \geq ALF \]

14R \[ MDF.KL = MWF.K, \text{ if } MWF.K < AML_F.K = AML_F.K, \text{ if } MWF.K \geq AML_F.K \]

**Initial Conditions**

15N \[ MTF = (DRM)(RMUF) \]

16N \[ RAF = (ARF)(RMUF) \]

17N \[ OPRS = (DCMF)(RMUF) \]

18N \[ MOF = RRF \]
19N RMUF = MOF
20N RMSF = RMUF

Parameters (Constants)

DRM = 3 weeks
ARF = 4 weeks
DRUF = 8 weeks
DRIF = 4 weeks
DCMF = 2 weeks

Raw Material Supplier Sector

21L UOS.K = UOS.J + (DT)(RMOS.JK - RMSS.JK)
22L IAS.K = IAS.J + (DT)(RMRS.JK - RMSS.JK)
23L ROSS.K = ROSS.J + (DT)(1/DRPS)(RMOS.JK - ROSS.J)
24A IDS.K = (AIS)(ROSS.K)
25L OPS.K = OPS.J + (DT)(PDS.JK - RMRS.JK)
26A LAS.K = OPS.K
27A LDS.K = (DPS)(ROSS.K)
28R RMRS.KL = FDELAY3 (OPS.K, DPS)
29A PWS.K = RMOS.JK + (1/DRPS)(IDS.K - IAS.K + LDS.K - LAS.K + UOS.K - UNS.K)
30R PDS.KL = PWS.K, if PWS.K < ALS
            = ALS, if PWS.K ≥ ALS
31A STS.K = UOS.K/DFS.K
32A DFS.K = DHS + (DUS)(IDS.K/IAS.K)
33A NIS.K = IAS.K/DT
34R RMSS.KL = STS.K, if STS.K < NIS.K
            = NIS.K, if STS.K ≥ NIS.K
35A \( \text{UNS}.K = (\text{ROSS}.K)(\text{DHS} + \text{DUS}) \)

**Initial Conditions**

36N \( \text{UOS} = (\text{ROSS})(\text{DHS} + \text{DUS}) \)
37N \( \text{OPS} = (\text{DPS})(\text{RMOS}) \)
38N \( \text{IAS} = (\text{AIS})(\text{ROSS}) \)
39N \( \text{RMOS} = \text{RMSI} \)
40N \( \text{ROSS} = \text{RMOS} \)
41N \( \text{RMSI} = \text{RRR} \)
42N \( \text{AIS} = (1000)(\text{RMOS}) \)

**Parameters (Constants)**

\( \text{DROS} = 8 \text{ weeks} \)
\( \text{DRPS} = 4 \text{ weeks} \)
\( \text{DPS} = 6 \text{ weeks} \)
\( \text{DHS} = 1 \text{ week} \)
\( \text{DUS} = 1 \text{ week} \)
\( \text{AIS} = 4 \text{ weeks} \)

**Pricing Mechanism**

43A \( \text{PRM}.K = \text{ARZ}[1 + \text{ROSS}.JK/\text{ART}] \)
45R \( \text{CRM}.K = \text{FDELAY3} (\text{AUX1}.K, \text{DISF}) \)
46L \( \text{AUX}.K = \text{AUX1}.J + (\text{DT})(\text{PRM}.J - \text{CRM}.J) \)
47L \( \text{RIVF}.K = \text{RIVF}.J + (\text{DT})(\text{VIRF}.JK - \text{VORF}.JK) \)
48R \( \text{VIRF}.KL = (\text{RMRF}.JK)(\text{CRM}.K) \)
49R \( \text{VORF}.KL = (\text{RMUF}.JK)(\text{CMRF}.K) \)
50A \( \text{CMRF}.K = \text{RIVF}.K/\text{RAF}.K \)
CGF.K = FDELAY3 (AUX2.K, DIM)


TVF.K = IVF.J + (DT)(VIF.JK - VOF.JK)

VIF.KL = (SRF.JK)(CGF.K)

VOF.KL = (SSF.JK)(PGF.K)

PGF.K = IVF.K/IAF.K

CGD.K = FDELAY3 (AUX3.K, DIFD)


IVD.K = IVD.J + (DT)(VID.JK - VOD.JK)

VID.KL = (SRD.JK)(CGD.K)

VOD.KL = (SSD.JK)(PGD.K)

PGD.K = IVD.K/IAD.K

CGR.K = FDELAY3 (AUX4.K, DIDR)


IVR.K = IVR.J + (DT)(VIR.JK - VOR.JK)

VIR.KL = (SRR.JK)(CGR.K)

VOR.KL = (SSR.JK)(PGR.K)

PGR.K = IVR.K/IAR.K

RRR.KL = RRR.JK + (DT)(1/DERR)(RER.JK - RRR.JK)

RER.KL = AD1[IRC.JK - (AD2)PGR.K]

RER.KL = \frac{(AEC)}{(PGR.K)}E \left[1 + \frac{IRC.K - IRI}{AIRD}\right]

AEC = (P_0)^E(Q_0)

AUX1 = (PRM)(DISF)

AUX2 = (CMPF)(DIM)

AUX3 = (PGF)(DIFD)
AUX2 = (PGD)(DIDR)
RIVF = (RAF)(PRMI)
IVF = (IAF)(PRMI)
IVD = (IAD)(PRMI)
IVR = (IAR)(PRMI)
RMRF = RRR
RMUF = RRR
SRF = RRR
SSF = RRR
SRD = RRR
SSD = RRR
SRR = RRR
SSR = RRR
RER = RRR
RRR = RRI
ROSS = RRR
IRC = IRI
IRC.KL = IRI + IRCC.K

Parameters (Constants)

DISF = 3 weeks
DIM = 6 weeks
DIFD = 2 weeks
DIDR = 1 week
DERR = 0, 10, 20, or 40 weeks
Consumer Sector

97L GSNW.K = GSNW.J + (DT)(SSR.JK - GWOC.JK)
98R GWOC.KL = FDELAY3 (GSNW.K, DGW)
99L RGC.K = RGC.J + (DT)(GWOC.JK - RRG.JK)
100R RRR.KL = RRG.JK
101L GRNR.K = GRNR.J + (DT)(RRG.JK - GDR.JK)
102R GDR.KL = SSR.JK
103A TOC.K = GSNW.K + RGC.K + GRNR.K
104L ISC.K = ISC.J + (DT)(ISRC.JK - ISGC.JK)
105R ISRC.KL = (FIS.K)(IRC.JK)
106A FIS.K = FISP.K if RGC.K ≥ ANRG
            = AFS! if RGC.K < ANRG
107R TSC.KL = (PGR.K)(RRR.JK)
108A NGNR.K = GSNW.K + RGC.K
109A NCNR.K = (ACG)(NGNR.K)
110R ISGC.KL = TSC.JK/KXNR.K
111A PPC.K = ISC.K/PGR.K
112A DVRG.K = (1/PPDC.K)(DRG)(PPD.K)
113R RRG.KL = RGC.K/DVRG.K
115N GSNW = (DGW)(SSR)
116N RGC = (DRG)(SSR)
117N GRNR = (SSR)(DUR + DHR)
118N ISC = PGR/ACG
119N RRG = RRR
120N SSR = RRR
121N RRR = RRI

I-M
122N  TSC = (PGR)(RRR)
123N  PGR = PRMI
124A  IRC.KL = IRI + IRCC.K
125A  PPDC.K = AIPP + CPPD.K
128A  PGR.K = PGI + PGRC.K
129A  PGRC.K = FSTEP (PI, 1)
130A  IRCC.K = 0
131A  CPPD.K = 0
132A  UOR.K = UOR.J + (DT)(RRRJK - SSRJK)
133R  SSR.K = UOR.K/DFR
134A  RRNA.K = RGC.K/DVRG.K
135R  RRR.KL = RRNA.K + RRN.K
136A  RRN.K = (ADN) NOISE
137A  FISP.K = (1/AFS1)(RGC.K - AFS2)

Parameters (Constants)

DCW = 200 weeks
DRG = 50 weeks
ACG = 1 person/unit
PRMI = 100 $/unit
RRI = 1000 units/week
ANRG = 50,000 units
AFSN = .004
AFS1 = 6,250,000 units
AFS2 = 25,000 units
IRI = 100 $/week/person
AIPP = 1 unit/person
PI = 50, 40, 30, 20, 10, -10, -20, -30, -40, -50 $/unit
ADN = 50 units/week
DFR = 1.4 weeks

Definitions

The following define the variables and constants used in the equations above.

ACG, constant denoting number of Consumers per Good in the consumer sector (persons/unit)
ADN, constant specifying the Dispersion of the Noise (units/week)
AFS1
AFS2, constants determining relationship between Fraction of income Saved and the level of replaceable goods (units)
APSN, constant denoting Fraction of income Saved when RGC is less or equal to its Normal level ($/week/$/week)
AIS, constant determining desired Inventory at Supplier (weeks)
ALF, constant specifying manufacturing capacity Limit at Factory due to available machines and labor (units/week)
AMLF, quantity denoting Manufacturing capacity and material Limiting rate at Factory (units/week)
ANRG, constant denoting Normal level of Replaceable Goods (units)
ARF, constant denoting the number of weeks of "smoothed" Raw material usage rate desired at the Factory (units/week)
AUX1
AUX2
AUX3
AUX4, auxiliary quantities (1,2,3,4) necessary in formulating a third-order delay function ($ weeks/unit)
CGD, Cost of Goods to the Distributor ($/unit)
CGF, Cost of Goods to the Factory ($/unit)
CGR, Cost of Goods to the Retailer ($/unit)
CMPF, Cost of Materials used in Production at the Factory ($/unit)
CRM, Cost of Raw Materials to the factory ($/unit)
DCMF, average Delay in receiving orders for raw materials resulting from the Clerical and Mailing process at the Factory (weeks)
DERR, average Delay in consumers adjusting to the Equilibrium Rate of Requisitions (weeks)
DFR, average Delay (variable) in Filling orders at Retail (weeks)
DFS, average Delay (variable) in Filling orders at Supplier (weeks)
DGW, average Delay in Goods Wearing out in consumer sector (weeks)
DHS, minimum Delay in filling orders due to Handling at Supplier (weeks)
DIDR, average Delay in transmitting price Information from Distributor to Retail (weeks)
DIFD, average Delay in passing price Information from Factory to Distributor (weeks)
DIM, average Delay in transmitting price Information through the Manufacturing sector (weeks)
DISF, average Delay in passing price Information from Supplier to Factory (weeks)
DPS, average Delay in Producing raw materials at Supplier (weeks)
DRIF, average Delay in adjusting the Raw material Inventory at the Factory (weeks)
DRG, average Delay in Replacing worn-out Goods under satisfactory conditions, i.e., when consumers potential purchasing ability is equal to that desired (weeks)
DRM, average Delay in Raw Materials being transported to the factory (weeks)
DROS, constant Determining smoothing rate of Raw material Orders at Supplier, the smoothing rate constant (weeks)
DRPS, average Delay in adjusting Raw material inventory and Pipeline at Supplier (weeks)
DRUF, average Delay in smoothing Raw materials Used at the Factory, the smoothing time constant (weeks)
DUS, normal average Delay in filling Unfilled orders due to "out-of-stock" items at Supplier (weeks)
DVRG, average Delay (Variable) in Replacing worn-out Goods in consumer sector (weeks)
E, "Elasticity" of demand (dimensionless ratio)
F, "Flexibility" of supply (dimensionless ratio)
FIS, Fraction of Income Saved by consumers for purchases ($/week/$/week)
FISP, Fraction of Income Saved when an abnormally high level of Post-poned purchases or replaceable goods exists in consumer sector ($/week/$/week)
GDR, Goods Discarded by consumers due to Receipt of units that were ordered (units/week)
GRNR, Goods Replaced by orders but Not yet Replaced by new units in consumer sector (units)
GSNW, Goods in Service that are Not Worn-out (units)
GWOC, rate of Goods Wearing Out in Consumer sector (units/week)
IAS, Inventory of raw materials Actually at the Supplier (units)
IDS, Inventory Desired at Supplier (units)
IRC, average Income Rate per Consumer ($/week/person)
IRI, Income Rate Initially of consumers ($/week/person)
ISC, average Income Saved per Consumer for purchase of units ($/person)
ISGC, average Income Spent on Goods per Consumer ($/week/person)
ISRC, average Income Saving Rate per Consumer for purchase of units ($/week/person)
IVD, Inventory Value at Distributor ($)
IVF, Inventory Value at Factory ($)}
IVR, Inventory Value at Retail ($)}
LAS, pipeline of materials (Actual) in the production process at Supplier (units)
LDS, pipeline Desired at Supplier (units)
MDF, Manufacturing Decision at the Factory (units/week)
MTF, raw Materials in Transit to the Factory (units)
NCNR, Number of Consumers that have Not Replaced goods by orders (persons)
NGNR, Number of Goods Not Replaced by orders in consumer sector (units)
NIRM, Negative Inventory Rate of raw Materials at the factory (units/week)
NIS, Negative Inventory rate at Supplier (units/week)
OPRS, Orders in the Process of being prepared for and mailed to the Raw material Supplier sector (units)
OPS, Orders in Production at Supplier (units)
PDS, Production Decision at Supplier (units/week)
PGD, Price of Goods at Distributor ($/unit)
PGF, Price of Goods at Factory ($/unit)
PGR, Price of Goods at Retail ($/unit)
PRM, Price of Raw Materials at the supplier ($/unit)
PRMI, Price of Raw Materials at the supplier Initially ($/unit)
PPC, Potential Purchases per Consumer (units/person)
PPDC, Potential Purchasing power Desired per Consumer (units/person)
PWS, Production Wanted at Supplier (units/week)
RAF, Raw material inventory Actually at the Factory (units)
RDF, Raw material inventory Desired at the Factory (units)
RER, Requisitions received (Equilibrium rate) at Retail (units/week)
RGC, Replaceable Goods in Consumer sector (units)
RIVF, Raw material Inventory Value at Factory ($)
RLAF, Raw material orders in the pipeLine (Actual) at the Factory (units)
RLDF, level of Raw materials orders in the pipeLine Desired (or necessary) at the Factory (units)
RMDF, Raw Material purchasing Decision at the Factory (units/week)
RMOS, Raw Material Orders received at Supplier (units/week)
RMRF, Raw Materials received by the Factory (units/week)
RMRS, Raw Materials Received at Supplier (units/week)
RMSF, Raw Material usage rate "Smoothed" at the Factory (units/week)
RMSS, Raw Materials Shipped by Supplier (units/week)
RMUF, Raw Materials Used at the Factory (units/week)
ROSS, Raw material Order rate "Smoothed" at Supplier (units/week)
RRG, Rate of Replacing Goods at the consumer sector (units/week)
RRI, Requisitions Received at retail Initially (units/week)
RRN, Requisitions Received at retail - Noise component (units/week)
RRNA, Requisitions Received at retail - component determined by Needs of consumers and their Ability to purchase (units/week)
RRR, Requisitions Received at Retail (units/week)
STS, Shipment rate Tried at Supplier (units/week)
TOC, Total goods Owned by Consumers (units)
TSC, Total Savings Spent by Consumers ($/week)
UOS, Unfilled Orders at Supplier (units)
VID, Value flowing Into inventory at Distributor ($/week)
VIF, Value flowing Into inventory at Factory ($/week)
VIR, Value flowing Into inventory at Retail ($/week)
VIRF, Value flowing Into Raw material inventory at Factory ($/week)
VOD, Value flowing Out of inventory at Distributor ($/week)
VOF, Value flowing Out of inventory at Factory ($/week)
VOR, Value flowing Out of inventory at Retail ($/week)
VORF, Value flowing Out of Raw material inventory at Factory ($/week)
APPENDIX II

Classes of Equations

Most of the equations in this thesis which define variables or parameters are followed by an equation number and a letter which indicates the type of equation employed. There are four classes of equations which are recognized in the text of this thesis - level, L, rate, R, auxiliary, A and initial condition or constant, N. The distinction made between these four equation types is set forth in the following paragraphs. Excerpts will be drawn from Memo D-46 (ref. 8).

Level Equations

"Levels" are the varying contents of the reservoirs (stocks, inventories, balances) of the system. Levels are countable, "stationary" quantities. They would exist (like an inventory) and could be measured even though the system were brought to rest and no flows existed.

An example of a level equation is that for raw materials in transit to the factory, MTF.

\[ \text{MTF}.K = \text{MTF}.J + (\text{DT})(\text{RMSS}.JK - \text{RMRF}.JK) \quad \text{Eq. 1L} \]

MTF--Materials in Transit to the Factory (units)
RMSS--Raw Materials Shipped by Supplier (units/week)
RMRF--Raw Materials Received by Factory (units/week)

The indication "1L" on the right shows that this is the first equation in our model and that it is a level equation, "L".

II-A
Rate Equations

A rate equation determines a flow between two levels. The rate is, in turn, determined by the values of levels in the system, very often including the level from which the flow comes and the one into which it goes. The rate equations may be either of the "overt" or "implicit" decision types; there is no structural or evident difference in the equations themselves.

An example of a rate equation is that for the replacement of goods in the consumer sector formulated in this thesis.

\[ \text{RRG}.KL = \frac{\text{RGC}.K}{\text{DVRG}.K} \]  
Eq. 113R

RRG--Rate of Replacing Goods by consumers (units/week)

RGC--Replaceable Goods in Consumer sector (units)

DVRG--average Delay (Variable) in Replacing Goods in consumer sector (weeks)

Auxiliary Equations

....It is often convenient to break down a rate equation into component equations we will call "auxiliary equations." The auxiliary equation is a great help in keeping the model formulation in close correspondence to the actual system, since it can be used to separately define the many factors that enter decision-making.

An example of an auxiliary equation is that which defines the average variable delay in replacing goods in the consumer sector (in equation 113R above).

\[ \text{DVRG}.K = \frac{\text{DRG}(\text{PPC}.K)}{\text{PPDC}.K} \]  
Eq. 112A

DVRG--average Delay (Variable) in Replacing worn-out Goods in consumer sector (weeks)

DRG--average Delay in Replacing worn-out Goods under "normal" conditions (weeks)

II-B
PPC--Potential Purchases per Consumer (units/person)

PPDC--Potential Purchasing power Desired per Consumer
(units/person)

This is an auxiliary equation because the variable DVRG could be eliminated by combining equations 113R and 114A.

Initial Condition and Constant Equations

"Initial Value" equations define initial values of levels (and some rates) which must be given before the first cycle of model equation computation can begin.

We should add, also, that an "N" type equation can be used to specify values of parameters. An example of an "N" equation which is used for this purpose is that for the manufacturing capacity limit at the supplier.

\[ ALS = (1000)(RMOS) \]  \hspace{1cm} \text{Eq. 42N}

ALS--constant denoting the production capacity Limit at Supplier (units/week)

RMOS--Raw Material Orders received at Supplier (units/week)

The following equation demonstrates the use of an "N" equation to define the initial value of a variable:

\[ MOF = RRF \]  \hspace{1cm} \text{Eq. 18N}

MOF--Manufacturing Orders at the Factory (units/week)

RRF--Requisitions Received by the Factory (units/week)

There are other equation types recognized by our computer routine than just L, R, A and N; however, since they are not employed in this thesis, the reader is referred to Memo D-30 (ref. 6) for a more complete discussion.

II-C
APPENDIX III

Third-Order Exponential Delays

A third order delay function is defined by the following two DYNAMO equations (ref. 6):

\[
\text{LEV}_K = \text{LEV}_J + (\Delta T)(\text{IN}_J K - \text{OUT}_J K) \quad \text{Eq. 1 L}
\]
\[
\text{OUT}_K = \text{FDELAY3} (\text{LEV}_K, \text{DEL}) \quad \text{Eq. 2 R}
\]

LEV--Level fed by input rate IN and depleted by inflow rate OUT.

IN--INflow rate to level LEV

OUT--OUTflow rate from level LEV

DEL--average Delay in OUT following IN.

This type of delay is characterized by the impulse response pictured in Fig. 48a and the step response shown in Fig. 48b.

Third-order delays are used often in this thesis to describe the flows of materials, orders, and price information. In the latter case, the level in the delay has, as yet, no physical meaning. (See Chapter III).

III-A
The output "rate" of price information, however, does following
the input "rate" in the fashion pictured in Figures 48a and 48b. In
the case of materials or orders, however, the level in a delay
represents the materials or orders that have been sent into this de-
lay but have not yet passed through it.
APPENDIX IV

First-Order Exponential Delays

A first-order delay function is defined by the following two equations:

\[ \text{LEV}.K = \text{LEV}.J + (\text{DT})(\text{IN}.JK - \text{OUT}.JK) \]
\[ \text{OUT}.KL = \frac{\text{LEV}.K}{\text{DEL}} \]

\[ \text{LEV} \]--Level fed by input rate \( \text{IN} \) and depleted by output rate \( \text{OUT} \)

\[ \text{IN} \]--Input rate to level \( \text{LEV} \)

\[ \text{OUT} \]--Output rate to level \( \text{LEV} \)

\[ \text{DEL} \]--average DELay in OUT following IN

This type of delay has the impulse and step responses shown in Figures 49A and 49B respectively:
A first-order delay as described above is employed several times in this thesis to indicate the flows of materials and orders. There are a few equations (Chapters III and IV) in which the average delay (corresponding to DEL above) is variable rather than remaining constant. The general nature of these delays, however, is still similar to that pictured in Figures 49A and 49B except that the average delay, DEL, will tend to vary with time.
REFERENCES


5. Forrester, Jay W., Memorandum D-34: "Equations for Production-Distribution System--A Section of Industrial Dynamics Class Notes", Industrial Dynamics Group, School of Industrial Management, Massachusetts Institute of Technology, Cambridge, March 10, 1959.


