Characteristics of Potential Vorticity Mixing by
Breaking Rossby Waves in the Vicinity of a Jet

by

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Abstract

Characteristics of potential vorticity (PV) mixing by breaking Rossby waves have been investigated, using simple models based on method of contour dynamics/contour surgery and analyzing upper-tropospheric data obtained from a high-resolution general circulation model (GCM) and National Meteorological Center (NMC). Particular emphasis is placed upon influence of planetary-scale flow structure on evolution of synoptic-scale waves and how the latter may feed back on the former when there is a diffuence in the planetary-scale flow.

Experiments with the simple models show many interesting results, some of the most important of which are described in the following. First of all, synoptic-scale waves with sufficiently large amplitudes in circular basic flows, which somewhat resemble the zonal-mean flow structure of the upper troposphere or lower stratosphere, break only outward, i.e., cyclonic air breaks into the low-PV region outside the vortex. Inward breaking (low-PV air breaking into the vortex) occurs only when the basic flow is contrived in an unrealistic fashion so that the magnitude of rate of strain (deformation strength) just inside the vortex is greater than that outside. In such flows, the location of the critical line (or critical points, in some cases) inside the vortex is closer to the vortex edge than that of outside. The cross-jet asymmetry in the magnitude of rate of strain and that in the location of critical lines are closely related and cannot be treated separately. The relationship between the cross-jet asymmetry in wave breaking and that in the magnitude of rate of strain or location of critical lines is confirmed by experiments with straight-jet basic flows; waves on the jet tend to break more readily toward the side of closer critical line, which is also the side of greater magnitude of rate of strain. When the basic flow has zonal asymmetry, characterized by a diffusence and a confluence, waves tend to amplify in the region of diffusence, due to the reduced local zonal velocity and stretching by the basic meridional flow associated with the diffusence. Finally, asymmetry in wave breaking can be induced by imposing stationary or steadily-propagating waves on the basic flow. The induced asymmetry is, qualitatively speaking, quite simple: the anticyclonic phase of "synoptic-scale" waves breaks more readily where there is anticyclonic curvature in "planetary-scale" flow, and vice versa. The asymmetry can also be explained by cross-jet asymmetry in the magnitude of rate of strain due to the imposed waves in the basic flow.

Output of a GCM run has been analyzed using Ertel's potential vorticity, $\eta$, on isentropic surfaces in the upper troposphere. The analyses show that the breaking asymmetry induced by planetary-scale curvature in the basic flow found in experiments with the simple models is also present in much more complex flows, although not as clear-cut as in simple flows. Synoptic-scale eddies exhibit particularly complex behavior when there is strong diffusence in the low-frequency flow, which is considered as the background flow. Such strongly diffuent low-frequency flow structure is defined as "blocking" in this study and is examined for its relationship with high-frequency synoptic-scale eddies during its life cycle. Five episodes of blocking
are examined, using $q$ forcing analyses and contour advection with surgery (CAS). The results demonstrate that the primary direct forcing of the blocking signatures is low-frequency flow advection, manifesting itself as a quasi-stationary Rossby wave amplification, breaking, and dissipation, throughout its life cycle. This life cycle is observed in time series of low-frequency $q$ as well. Synoptic-scale eddies are found to be breaking in such a way that they tend to reinforce the diffuence in the low-frequency flow, when there is a low-frequency diffuence. This pattern of high-frequency forcing is in agreement with results reported by other investigators. These results are confirmed by analyses of NMC data during two observed blocking events with one minor exception; high-frequency forcing is found to have an important direct contribution in forcing the low-frequency diffuence during the first 6 or 7 days of one episode. This exception suggests that there may be more than one mechanism to initiate a blocking. The time scale associated with quasi-stationary wave amplification, breaking, and dissipation, during these 7 events is found to be 10 to 15 days. Two events that have longer blocking duration than the rest are simply composed of more than one of this quasi-stationary wave cycle. Also, CAS was applied to 9-day data of a 15-event composite of European blocking. The results show the domination by low-frequency flow advection throughout the life cycle of the composite blocking. It also shows amplification, breaking, and dissipation of a quasi-stationary wave to be the dynamical feature of the phenomenon. These results suggest, at least for many of the observed blocking events, that blocking is a manifestation of quasi-stationary wave amplification, breaking, and dissipation, with approximately the same time scale.

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Chapter 1

Introduction

Waves supported by gradients in potential vorticity, Rossby waves, are some of the most ubiquitous motions observed in the middle-latitude atmosphere. They have typical length scales of 1000 km to 10000 km. Rossby waves are also clearly observed in meandering of oceanic fronts, such as the Gulf Stream and Kuroshio Current, with typical length scales of hundreds of kilometers. When these waves grow in amplitude to the extent such that the supporting potential vorticity gradients can no longer sustain the wave motion, fluid parcels that are initially moving in an oscillatory fashion within the waves may be advected out from the waves, resulting in irreversible transports of potential vorticity and the material contained in the fluid parcels. McIntyre and Palmer (1984) called this irreversible deformation of material surfaces due to unsustainably large amplitudes of Rossby waves, “Rossby wave breaking”. Because of the large scales of these waves, the mixing associated with Rossby wave breaking may be of significant importance in large-scale transport of potential vorticity and various materials in the atmosphere and some parts of oceans.

One of the most distinctive and significant characteristics of the breaking of planetary-scale waves on the edge of the stratospheric ploar vortex (McIntyre and Palmer,
is its asymmetry. The waves appear to break predominantly outward, thus ejecting air from the vortex into the midlatitude “surf zone” where it is mixed with ambient air (McIntyre and Palmer, 1984; Juckes and McIntyre, 1987). Corresponding intrusion of midlatitude air into the vortex seems to be less common, despite examples of this occurring during northern winter 1991/92 (Plumb et al., 1994). The polar vortex has thus been categorized as a “containment vessel” (Hartmann et al., 1989; McIntyre, 1990) in which the now-well-known chemical processes inside the polar vortices can occur without major interference from mixing with exterior air of different composition.

There appears to be a similar, if less clear-cut, asymmetry evident in the breaking of synoptic-scale waves in the upper troposphere (Hoskins et al., 1985 and references therein). The most usual fate of these disturbances is for high potential vorticity (PV) to be cut off equatorward of the jet, which is analogous to the stratospheric behavior. (An example of an equatorward “potential vorticity streamer” associated with a tropopause fold is described by Appenzeller and Davies [1992] and this kind of behavior is one of the two paradigms discussed by Thornicroft et al. [1993].) Occasionally, however, the opposite may occur, with low potential vorticity being cut off poleward, a phenomenon that is often associated with the creation of a “blocking” anticyclone (e.g., Hoskins et al., 1985). A similarly rich, but less asymmetric, spectrum of behavior is observed on the Gulf Stream, including the production of warm and cold eddies through detachment of meanders and of warm outbreaks detaching to the south (e.g., Pratt et al., [1991] and references therein).

One-sided wave breaking has been reported in several modeling studies. In results from a spherical barotropic contour dynamics model, Dritschel (1988a) noted that
thin filaments were shed in both directions from a single vorticity discontinuity in case where the discontinuity was close to the equator, but uniquely equatorward otherwise. He attributed this difference to the shear field of the basic flow around the vortex. On the basis of high-resolution model studies of the stratospheric polar vortex, Juckes and McIntyre (1987) attributed the one-sidedness of Rossby wave breaking to a corresponding asymmetry on the rate of strain in the ambient flow, a conclusion also reached by Polvani and Plumb (1992) on the basis of contour dynamics experiments. The curvature of the jet at the vortex edge produces a bias in the stain rate distribution with, under normal conditions, greater values immediately equatorward of the jet. A Rossby wave may be more likely to break into the region of larger strain. However, it is not clear that this is the critical measure of the asymmetry. Small-amplitude waves will only break near a "critical line", i.e., the nonlinear critical layer (Haynes, 1989 and references therein), within which the breaking occurs, collapses to the critical line in the limit of of small wave amplitude. In the barotropic experiments mentioned above, a critical line exists in the undisturbed state outside the vortex edge but not inside. In fact, Polvani and Plumb (1992) found, in barotropic cases, that the buckling of material contours in the presence of Rossby waves is confined exclusively to the exterior of the vortex edge, unless the breaking is vigorous enough to disrupt the vortex. In non-barotropic (shallow-water) cases, when there is also a critical line in the interior, weak buckling occurs there also but is separate from that outside. Asymmetry in the location of critical lines is related to the asymmetry of the distribution of velocity, rather than strain. However, these are closely related and difficult to separate even in these idealized flows.

In the real atmosphere, where the dynamically significant basic flow at any instance is rather ambiguous, it seems to be extremely difficult, if not impossible, to determine
locations of critical lines for given disturbances and relate them to rate of strain of the basic flow. However, there appears to be a simple relationship between asymmetry of synoptic-scale wave breaking and planetary-scale flow curvature, which is a manifestation of amplitudes of planetary-scale waves, as suggested by frequent occurrence of poleward intrusion of anticyclones during blocking. Blocking is characterized by a meridionally-split low-frequency jet, flowing around a low-frequency flow dipole (an anticyclonic cell co-existing with a cyclonic cell on its equator side) or flowing around an Ω-like low-frequency anticyclone. Synoptic-scale eddies propagating into this region of low-frequency diffluence are stretched meridionally and compressed zonally. Then their anticyclonic phases break poleward at the low-frequency ridge. The poleward intrusion of anticyclones in the region of planetary-scale anticyclonic curvature is an important part of the eddy straining mechanism of blocking suggested by Shutts (1983) and have been found in diagnostic and model studies of blocking (e.g., Nakamura and Wallace, 1990; Blackmon et al, 1986). Such a relationship between the poleward breaking of anticyclones and planetary-scale anticyclonic curvature of the flow is not restricted to the troposphere. Plumb et al. (1994) found large-scale intrusions of anticyclonic air into the winter Arctic vortex in the stratosphere when there was planetary-scale anticyclonic curvature associated with strong tropospheric blocking. One may speculate, therefore, that asymmetric breaking of synoptic-scale waves, predominantly equatorward except when there is planetary-scale anticyclonic curvature in the flow, may be controlled by curvature of the planetary-scale or low-frequency flow. On the other hand, the curvature associated with planetary-scale waves may receive reinforcement from or be forced partially by synoptic-scale eddies, as suggested by Shutts (1983).

In this study, the mechanism of blocking has been investigated in terms of potential
vorticity mixing by breaking Rossby waves. First, controlling factors of asymmetric synoptic-scale wave breaking were studied using simple models based on the method of contour dynamics/contour surgery (CD/CS). Chapter 2 describes results of experiments performed with CD/CS in a circular geometry. A circular vortex on an $f$-plane was perturbed by an oscillatory forcing of various amplitudes to study behavior of eddies in various basic flows. Chapter 3 describes results of experiments using a $2\pi$-periodic CD/CS. A basic state characterized by a straight zonal jet with controlled meridional asymmetry was perturbed by initialized waves of various amplitudes to study the effects of meridional asymmetry in the basic flow on evolution of waves. Effects of zonal asymmetry in the basic flow, associated with a diffluence and a confluence, on wave evolution were also examined. Chapter 4 describes experiments with the models used in Chapters 2 and 3 to study the effects of basic flow curvature, associated with a stationary or steadily-propagating "planetary-scale" wave, on behavior of "synoptic-scale" waves.

Second, wave breaking asymmetry was examined in more complex flows, output of a general circulation model (GCM) and the real atmospheric flows, with a focus on roles of wave breaking in the mechanism of blocking. Chapter 5 describes analyses of GCM output. Ertel's potential vorticity on isentropic surfaces was used to diagnose Rossby wave breaking and its relationships with the blocking flow in the upper troposphere during five episodes of blocking. Contour advection with surgery (CAS) was also used to confirm the results of the analyses. Chapter 6 describes CAS application to National Meteorological Center (NMC) data during two blocking events and a 15-event composite of European blocking. Finally, Chapter 7 concisely summarizes the findings of this study.
Chapter 2

Rossby wave breaking in idealized circular basic flows

Abstract

Asymmetric characteristics of Rossby wave breaking in a circular geometry are discussed in this chapter. Experiments using a contour dynamics/contour surgery code show that Rossby waves on the edge of a circular vortex break either only into the vortex or only toward the surrounding low-PV region, depending on the basic flow profile. Such asymmetry is not influenced by the way in which Rossby waves are introduced to the flow; both resonantly excited waves and waves prescribed by an initial condition exhibit the same asymmetry. This asymmetry is evidently regulated by cross-jet asymmetry in the basic flow. Major factors that control the direction of Rossby wave breaking appear to be the location of critical lines and the magnitude of rate of strain in the basic flow.
2.1 Breaking of resonantly-excited Rossby waves on an otherwise-circular vortex

2.1.1 Brief description of contour dynamics/contour surgery method

A non-periodic contour dynamics/contour surgery (CD/CS) code was used for the experiments described throughout this chapter. The CD model was originally developed by Zabusky et al. (1979) for a barotropic flow with inviscid, incompressible, and adiabatic assumptions on an f-plane. For such a flow, potential vorticity (PV), $Q$, and the streamfunction, $\psi$, are related to each other by

$$\nabla^2\psi + f = Q,$$  \hspace{1cm} \text{(2.1)}

where

$$u = -\frac{\partial \psi}{\partial y},$$ \hspace{1cm} \text{(2.2)}

and

$$v = \frac{\partial \psi}{\partial x}.$$ \hspace{1cm} \text{(2.3)}

Furthermore, potential vorticity is conserved following a fluid parcel, i.e.,

$$\frac{dQ}{dt} = 0.$$ \hspace{1cm} \text{(2.4)}

The relationship is slightly different for a quasi-geostrophic equivalent-barotropic flow with the same assumptions and is given by

$$\nabla^2\psi - \gamma^2\psi + f = Q,$$ \hspace{1cm} \text{(2.5)}

where $\gamma$ is the inverse of the Rossby deformation radius, $R_d$, which is defined by

$$R_d \equiv \frac{\sqrt{gD}}{f} \equiv \frac{1}{\gamma},$$ \hspace{1cm} \text{(2.6)}
where \( g \) is the gravitational acceleration constant and \( D \) is the mean depth of the fluid.

Suppose that the flow field has a piece-wise constant \( Q \) distribution, say \( Q_n(n = 1, 2, ..., N) \) in regions \( \mathbb{R}_n \) bounded by curves \( C_n \) with finite vorticity jumps, \( \Delta Q_n \). Then the velocity due to the \( Q \) field at any time at a point \( x \) is given by

\[
V(x) = -\sum_{n=1}^{N} \Delta Q_n \int_{C_n} G(|x - x_i|)dx_i, \tag{2.7}
\]

where

\[
G(|x - x_i|) = \frac{1}{2\pi} \log(|x - x_i|) \tag{2.8}
\]

for barotropic flows and

\[
G(|x - x_i|) = -\frac{1}{2\pi} K_0(\gamma|x - x_i|) \tag{2.9}
\]

for equivalent-barotropic flows (Dritschel, 1989). Here, \( x_i \) is the \( i \)th point on the \( n \)th contour, \( C_n \), and \( K_0 \) is the zeroth order modified Bessel function of the second kind. Note that Equation 2.9 becomes Equation 2.8 in the limit of \( \gamma = 0 \). The contours, \( C_n \), are approximated by finite number of points that are advected by the so-obtained velocity field, yielding a new distribution of \( Q \). These steps are repeated to perform time integration of desired lengths. As the flow evolves, points are inserted to maintain the desired resolution. In order to maintain reasonable computational efficiency, contour surgery (CS) developed by Dritschel (see Dritschel [1988b] for details), is employed in the model. It eliminates features that are smaller than a certain prescribed scale and features with curvature sharper than a certain prescribed value. Thus, the system is not strictly conservative; there is a loss of PV and material when CS operates. However, such loss is very small compared to what remains and
should not affect the results of experiments in any significant way. A brief summary of studies done in the past, using the method, is found in Pullin (1992).

A last note on CD/CS is in order here. Waugh (1993a) and LeGras and Dritschel (1993) compared forced and unforced flows simulated by CD/CS models and high-resolution pseudo-spectral models. They showed that agreement between the two different methods is remarkable when a sufficient number of PV jump contours are used in CD/CS models to match the undisturbed flow field in the pseudo-spectral model reasonably well. Thus, the results described in the following are not unique to CD/CS models and should be valid in a general sense, at least qualitatively.

2.1.2 Forced-wave experiments: outward breaking cases

Behavior of forced waves on a circular PV jump contour was studied, following the procedure taken by Polvani and Plumb (1992). A circular vortex of radius \( R \), centered on \( r = \sqrt{x^2 + y^2} = 0 \), with one PV jump contour, was used as the basic state (and also as the initial conditions) for the experiments described here. The values of PV inside and outside the vortex are denoted by \( Q_i \) and \( Q_o \), respectively. Waves are forced conservatively by an oscillatory topography \( h(x, y, t) \). When a weakly-varying topography \( h \) is present in a flow of the mean depth \( D \), PV is given by

\[
\nabla^2 \psi - \gamma^2 \psi + \frac{fh}{D} + f = Q. \tag{2.10}
\]

Since the equation is linear, it can be decomposed into two parts:

\[
\nabla^2 \psi_o - \gamma^2 \psi_o = Q - f \tag{2.11}
\]

and

\[
\nabla^2 \psi_f - \gamma^2 \psi_f = -\frac{fh}{D}, \tag{2.12}
\]
where \( \psi_v \) and \( \psi_f \) are streamfunctions associated with the vortex and forcing, respectively. Instead of specifying \( h \), \( \psi_f \) was prescribed by

\[
\psi_f = \psi_0 \sin\left(\frac{2\pi t}{\tau}\right) \exp\left[-\frac{(x - x_0)^2}{2\delta_x^2} - \frac{(y - y_0)^2}{2\delta_y^2}\right],
\]

(2.13)

where \( \psi_0 \) is an amplitude factor, \( \tau \) the oscillation period, and \( (x_0, y_0) \) and \( (\delta_x, \delta_y) \) are factors that characterize, respectively, the location and length scale of the topography in the coordinate directions. The total velocity, \( \mathbf{V} = \mathbf{V}_v + \mathbf{V}_f \), is calculated from \( Q(x, y, t) \) and \( \psi_f(x, y, t) \) at every time step.

In order to resonantly excite a Rossby wave of a desired wavenumber, Equation 2.4 was solved for the linear dispersion relationship for a small-amplitude wave at \( r = R \). With the following two matching conditions at \( r = R \),

\[
\psi_f(R^-) = \psi_f(R^+)
\]

(2.14)

and

\[
\int_{R-\epsilon}^{R+\epsilon} Q dr = \eta \Delta Q,
\]

(2.15)

where \( \eta \) is a small displacement of the contour from its undisturbed position, one obtains

\[
\sigma = s[v_0 - \frac{\Delta Q}{\gamma} \frac{I_0(\gamma)K_0(\gamma)}{I_0(\gamma)K_{-1}(\gamma) + I_{-1}(\gamma)K_0(\gamma)}].
\]

(2.16)

Here, \( \sigma \) is the oscillation frequency, \( s \) the wavenumber around the circle, \( v_0 \) the azimuthal velocity at \( r = R \), and \( I_n \) and \( K_n \) are the order \( n \) modified Bessel functions of the first and the second kind, respectively. Values of \( \tau \) for desired wavenumber are
given by this relationship, for specified values of $Q_i$, $Q_o$, and $\gamma$.

The experiments were constructed as follows. The radius of the undisturbed vortex was chosen to be $R = 5000$ km. The Rossby deformation radius, $R_d$, was set at 1000 km. These values were chosen to crudely represent upper-tropospheric dynamics in the middle latitudes. In the standard case, the relative vorticity outside the vortex, $r > R$, was set to zero ($Q_o = f$) to ensure decay of the jet at large $r$. The jump in PV across the contour was chosen so as to give a jet maximum of $30 \text{ms}^{-1}$. This required $Q_i := 1.4192f$ and thus $\Delta Q = 0.4192f$. The profiles of azimuthal velocity $v$ and the magnitude of rate of strain $S = |r^2 \frac{d}{dr}(\frac{v}{r})|$ for the corresponding basic flow are shown in Figure 2.1. Also shown (the straight line in the upper figure) is the line $V = \omega r$, where $\omega$ is the angular velocity of wavenumber 6 given by Equation 2.16. The critical lines, where $v = V$, are located at $r \simeq 0.68R$ and $r \simeq 1.17R$. (Note that the slope of $V$, i.e. $\omega$, depends on wavenumber and is greater for larger wavenumber.)

Traveling waves of number 6 were generated on this basic state through the topographic forcing given by Equation 2.13, centered at $(x_0, y_0) = (-R, 0)$ with decay length scale $(\delta_x, \delta_y) = (0.3R, 0.2R)$. The amplitude factor was varied to obtain variety of wave evolution. Wave evolution characteristics observed in these experiments can be broadly categorized into two, non-breaking and breaking. In experiments using piece-wise homogeneous initial PV distributions, Rossby wave breaking was identified by irreversible transport of material with one value of PV into a region characterized by another value of PV. This criterion was also used in experiments with $2\pi$-periodic CD/CS described in Chapters 3 and 4. When the forcing amplitude is smaller than a certain critical value, the forced waves simply propagate around the edge of the vortex (Figure 2.2). Beyond the critical value, the waves grow downstream of the forcing, distort into a thick streamer that is tilted “SW-NE” by the ambient shear, and then
Figure 2.1: Basic flow profiles of (a) the azimuthal velocity, \( v \), (thick curve) and \( V = \omega r \), where \( \omega \) is the angular velocity of wavenumber 6 given by Equation 2.15 (thin straight line) and (b) the magnitude of rate of strain, \( S \), plotted as a function of \( \frac{r}{R} \). Values of PV inside and outside the vortex are 1.4192\( f \) and 1.0\( f \), respectively. Plotted velocities and rate of strain are non-dimensionalized by \( \frac{\nu}{fR} \) and \( \frac{S}{f} \), respectively.
form a cut-off outside of the jet, ejecting vortex material into the "surf zone" (Figure 2.3). This "irreversible large-scale deformation of the material contour" (McIntyre and Palmer, 1985), Rossby wave breaking, was asymmetric for all values of the amplitude factors used; waves broke only outward (vortex material actively breaking into the surrounding low-PV region). When the amplitude was very large and flow became highly turbulent, low-PV material was occasionally entrained into the vortex as a by-product of re-merger with the main vortex of eddies shed earlier by breaking. These characteristics are in agreement with results reported by Polvani and Plumb (1992). (What Polvani and Plumb referred to as "microbreaking" never occurred in these experiments due to, presumably, lack of extremely high resolution required for producing it and relatively short durations of integration.) The same wave evolution characteristics were observed for all the wavenumbers, from 4 to 10, with the same basic state. Also, when the basic state was changed to $Q_i = 1.419 f$ and $Q_o = 0.9998 f$, with $u$, $V$, and $S = |r \frac{d}{dr}(u_r)|$ as shown in Figure 2.4, wave evolution characteristics for various amplitudes were the same as those mentioned above.

The conclusion reached by Juckes and McIntyre (1987), Dritschel (1988a), and Polvani and Plumb (1992) that the one-sidedness of wave breaking is due to asymmetry in shear or rate of strain of the ambient flow seems to hold in these experiments. However, there is also cross-jet asymmetry in the distance of the critical lines from the PV jump contour. As shown in Figures 2.1 and 2.4, the critical line outside the vortex is considerably closer to the contour than that of inside. It has been demonstrated that small-amplitude waves break only near a critical line; the nonlinear critical layer (Haynes, 1989 and references therein), within which finite-amplitude wave breaking occurs, collapses to the critical line in the limit of small wave amplitude. This is clearly seen in Figure 2.5, which shows evolution of small-amplitude waves on a dynamic (non-zero PV jump) contour at $r = R$ and 2 passive (zero PV jump) contours initially
Figure 2.2: Evolution of a No.6 Rossby wave generated by the oscillatory forcing in the basic state shown in Figure 2.1. The undisturbed vortex is circular and described by a single PV contour at $r = R$. The forcing is centered on $(x, y) = (-R, 0)$. The amplitude factor is subcritical and does not induce breaking. Reading across and down, the first frame is at $t = 0$, with successive frames at intervals of 12.5 days.
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Figure 2.3: Same as Figure 2.2, except that the forcing amplitude is larger and supercritical, resulting in vigorous breaking downstream of the oscillatory forcing.
Figure 2.4: Same as Figure 2.1, except that the values of PV inside and outside the vortex are $1.419f$ and $0.9998f$, respectively.
Figure 2.5: The kinematics around the vortex edge for the same configuration as shown in Figures 2.2 and 2.3, but with much weaker forcing. The contours are initially located at $r = 0.7R, 1.0R, 1.2R$, the inner and outer ones being passive (zero PV jump). The passive contours are initially located near the critical lines shown in Figure 2.1. Time interval between two successive frames is 5 days.
located at $r = 0.7R$ and $r = 1.2R$, very close to the critical lines shown in Figure 2.1. (However, this is not classified as Rossby wave breaking according to the criterion and is meant to demonstrate material mixing at the critical lines.) The conditions are identical to those of Figures 2.2 and 2.3, except that the forcing is weaker. Both contours buckle, the outer contour first and more vigorously, presumably reflecting its closer proximity to the dynamically active contour.

2.1.3 Forced-wave experiments: inward breaking cases

To test the dependence of breaking characteristics on the flow asymmetry, the basic flow was contrived to possess strain and velocity profiles very different from those shown in Figures 2.1 and 2.4, while keeping the value of $\Delta Q$ the same as before. This was done by adding a constant value of PV, $Q_a$, to the entire field.

When waves of sufficiently large amplitude are generated in these modified basic states, inward breaking indeed occurs for $Q_a$ larger than about $0.003f$. Below this value, large-amplitude waves still break only outward, even when there is no critical line outside the vortex. Figure 2.6 shows profiles of $v$, $V$, and $S$ of a basic state with $Q_i = 1.4202f$ and $Q_o = 1.001f$ (that is $Q_a = 0.001f$). Notice that there is no critical line outside the vortex for wavenumber 6, and hence no breaking of small amplitude waves outside the vortex (Figure 2.7). However, with finite disturbances on this flow, critical points (what Polvani and Plumb (1992) refer to as stagnation points) are likely to develop outside the vortex, where $v$ and $V$ are very close to each other, as well as inside. In such a case, a typical distance between critical points and the contour is estimated to be roughly the same for inside and outside the vortex. However, the magnitude of rate of strain is still considerably larger just outside the contour than inside. Evolution of large-amplitude waves excited on this basic state is shown in Figure 2.8. Clearly, breaking is outward.
Figure 2.6: As in Figure 2.1, but for $Q_1 = 1.4202f$ and $Q_o = 1.001f$, i.e., $Q_a = 0.001f$. There is a critical line inside the vortex, at around $r = 0.75R$, but not outside, $(r > R)$. However, $v$ and $V$ are very close to each other at around $r = 1.25R$. 

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Figure 2.7: The kinematics around the vortex edge with a propagating small-amplitude wave of No. 6. The otherwise-circular basic state flow profile is shown in Figure 2.6. The contours are initially located at $r = 0.75R, 1.0R, 1.25R$, the inner and outer ones being passive (zero PV jump). The passive contours are initially located near the critical lines shown in Figure 2.6. Time interval between two frames is 8 days.
Figure 2.8: As in Figure 2.7, but with stronger forcing and without passive contours. Evidently, the perturbation is large enough to create a critical layer outside the vortex, leading to breaking waves there. Time interval between two frames is 5 days.
When \( Q_a = 0.00314f \), the basic state \( S \) is almost symmetric at \( r = R \), \( S(R^+) \) being slightly greater than \( S(R^-) \) (Figure 2.9). (The difference in rate of strain across the contour, \( S(R^+) - S(R^-) \), changes sign at \( Q_a \approx 0.0035f \).) Also, there is no critical line outside the vortex and \( u \) and \( V \) are far apart for \( r > R \). When waves of sufficiently large amplitude were generated in this basic flow, they did break inward (Figure 2.10). Small-amplitude waves on this basic flow, as in the case shown in Figure 2.7, break only near the critical line, which exists only inside the vortex here (Figure 2.11).

### 2.2 Discussion

From these examples, it is difficult to conclude which of locations of critical lines and strain asymmetry is more important than the other in determining the direction of breaking. For example, breaking is uniquely outward in the case shown in Figure 2.8 despite the absence of a critical line outside the vortex, although critical layer is likely to develop when the waves are of finite-amplitude. In the case shown in Figure 2.10, \( S(R^+) \) is still slightly greater than \( S(R^-) \), while waves break only inward. However, at small but finite distance away from the contour, say, at \( r = R \pm 0.1R \), reverse is true. Moreover, the distribution of strain and the location of critical lines in the undisturbed flow are intimately related since, if

\[
s = r \frac{d}{dr} \left( \frac{u}{r} \right),
\]

then

\[
v(r) = v(r_0) + r \int_{r_0}^{r} \frac{1}{r'} s(r') dr'
\]

and so critical lines are found where

\[
r = \frac{1}{\omega} [v(r_0) + r \int_{r_0}^{r} \frac{1}{r'} s(r') dr'].
\]
Figure 2.9: As in Figure 2.6, but for \( Q_i = 1.42234f \) and \( Q_o = 1.00314f \), i.e., \( Q_a = 0.00314f \).
Figure 2.10: Evolution of No. 6 wave generated by the oscillatory forcing in the basic state shown in Figure 2.9. The amplitude is supercritical and leads to inward breaking of Rossby waves. Time interval between frames is 3.0 days.
Figure 2.11: The kinematics around the vortex edge with a propagating small-amplitude wave of No. 6. The otherwise-circular basic state flow profile is shown in Figure 2.9. The contours are initially located at $r = 0.6R, 1.0R, 1.2R$, and $1.4R$, of which all but that at $r = 1.0R$ are passive. Time interval between frames is 4.0 days.
For this reason, it is very difficult to distinguish, if it is possible at all, between strain asymmetry and velocity asymmetry in categorizing the nature of the breaking in these experiments. This is also true in linear geometry discussed in Chapter 3.

The breaking characteristics described above remain valid when the forcing is given in a different fashion. Figures 2.12 and 2.13 show evolution of initially-prescribed waves, given by

\[ r = R(1 + A\cos\theta), \]

(2.20)

with \( Q_i \) and \( Q_o \) identical to those shown in Figures 2.3 and 2.10, respectively, and \( A = 0.3 \). As in time-dependent oscillatory forcing cases, waves break only outward and inward, respectively. When \( A \) is smaller than critical values, the waves do not break, as expected from results of transiently-forced cases and a linear stability theory (Lamb, 1945). Also, experiments with multiple PV jump contours show that when the jet is slightly smoothed over a finite but narrow layer, the behavior of the waves is qualitatively very similar to that in single PV jump contour cases (Figures 2.14 and 2.15). However, there are differences in details between single and multiple PV jump contour cases. The multiple contour cases exhibit more complex behavior than single contour ones, which shall remind us of limitation of applicability of the results from the single contour experiments to the real geophysical flows, such as subtropical jet in the atmosphere. Wave breaking on a broad jet may be far more complex than those cases considered here. If the PV gradient is continuous and there are critical zones on either side of the jet maximum, some breaking is almost certain to occur in both directions though with perhaps different intensities. In fact, Thornroft et al. (1993) show one example of a baroclinic wave life cycle where cessation of breaking and formation of a reflecting layer leads to very different behavior.
Figure 2.12: Evolution of No. 6 wave given in the initial condition by Equation 2.19. The values of PV inside and outside the vortex are the same as those in Figure 2.3. Time interval between frames is 2.4 days.
Figure 2.13: As in Figure 2.12, but with the values of PV inside and outside the vortex are the same as those in Figure 2.10. Time interval between frames is 1.6 days.
Figure 2.14: Evolution of large-amplitude wave on a narrow jet represented by 5 contours with equal PV jump, each one being a fifth of $0.4192f$. The innermost and outermost values of PV are the same as those in Figure 2.3. Time interval between frames is 1.5 days.
Figure 2.15: As in Figure 2.14, but with the innermost and outermost PV values being the same as those in Figure 2.10. Time interval between frames is 1.5 days.
Chapter 3

Rossby wave breaking in idealized $2\pi$-periodic basic flows

Abstract

This chapter describes results of experiments, using a $2\pi$-periodic CD/CS model in a linear geometry. A basic state with a straight jet at a fixed location, say $y = 0$, has symmetry about $y = 0$ in flow and strain profiles. Experiments with controlled meridional asymmetry added to the basic flow show that the cross-jet asymmetry in the magnitude of strain (or shear) and location of critical lines influences wave evolution characteristics considerably. It is shown that waves tend to break more readily in regions of greater shear and toward the closer critical line. The effect of basic flow asymmetry is particularly strong when wave amplitudes are very close to the breaking threshold value. When the amplitude of a wave is supercritical (with respect to breaking threshold in a symmetric basic flow), the larger the wave amplitude, the smaller the effect of the basic flow asymmetry, and vice versa. On the other hand, the effect of basic flow asymmetry is manifested more clearly for larger amplitude waves, when the amplitudes are subcritical. In addition to the meridional asymmetry, zonal asymmetry in the basic flow, characterized by a diffluence and a confluence, is examined for its influence on wave evolution. This type of asymmetry introduces zonal asymmetry into evolution of propagating waves. Amplification of propagating waves due to locally-reduced basic zonal flow and meridional flow component is shown to cause localized breaking in the diffuent region.
3.1 Experiments with a symmetric basic flow

For the experiments to be described in this chapter, a $2\pi$-periodic CD/CS was used throughout. The basic state consists of a straight potential vorticity (PV) discontinuity at $y = 0$. Unless a modification is applied to the basic flow, it is symmetric with respect to $y = 0$. However, the system is still essentially the same as that in a circular geometry briefed in Chapter 2. One may envisage that the $2\pi$-periodic version represents a very short portion of a circular contour of a vortex with very small deformation radius (relative to the radius of the vortex) so that the curvature and effects of far-away point vortices may be neglected.

The procedure taken here is to prescribe a sinusoidal wave of a form,

$$\eta(x, t = 0) = A \cos \left( \frac{2\pi x}{L} \right), \quad (3.1)$$

on the PV contour at $y = 0$. All the results described here are with $L = \pi$, i.e., wavenumber is 2. Oscillatory forcing similar to that used in Chapter 2 did not work well because of some unresolved technical difficulty. This difference in the type of forcing should not matter as far as the purpose of these experiments is concerned. Indeed, it was shown in Chapter 2 that transiently forced waves and initially prescribed waves break in the same direction.

Below a certain critical non-dimensional amplitude, $A_c/L$, which depends on the value of deformation radius, waves did not break and simply propagated on the contour (Figure 3.1, for example). As $A$ was increased to near-critical value, an “oscillation” in the shape of the contour was observed for all values of deformation radius tested here. For values of non-dimensionalized wavelength, $L/R_d$, less than about 1.2, near-critical-amplitude waves typically showed a repeated change in its shape between a
sinusoidal curve and sawtooth-like shapes (Figure 3.2). For greater values of $L/R_d$, the oscillation was between a sinusoidal curve and $\Omega$-like shapes (Figure 3.3). These shapes, sawtooth-like and $\Omega$-like, are similar to steadily-propagating states found by S. Meacham (personal communication) for large and small values of deformation radius, respectively. Although the exact mechanism of this oscillation is not clear, it appears that the flow, which is not in the steadily-propagating state initially, approaches the steadily-propagating state as the wave propagates and overshoots, then again approaches it and overshoots, regaining the original shape.

As observed in experiments with a circular vortices, waves break when the initial amplitude, which serves as the forcing here, exceeds a certain threshold value. The threshold value depends on the deformation radius and exhibits an intriguing transition at around $L/R_d = 1.2$. Figure 3.4 summarizes most of the results of experiments with the symmetric basic flow. It categorizes wave evolution into 5 types, A, B, C, D, and E, and shows which one of the five types an experiment falls in, as a function of non-dimensionalized amplitude, $A/L$, and wavelength, $L/R_d$. Type A represents non-breaking waves that remain almost sinusoidal, including the sawtooth-like shape, such as those shown in Figures 3.1 and 3.2. Type B is given for filamentary breaking, such as that shown in Figure 3.5. Type C is assigned to non-breaking characterized by development of the $\Omega$-like shape shown in Figure 3.3. Type D represents breaking of another kind shown in Figure 3.6. Finally, Type E is assigned to yet another kind of breaking shown in Figure 3.7. The solid line drawn, thus, separates the breaking and no-breaking domains, indicating approximate values of the critical amplitude as a function of $L/R_d$. Note that the flow symmetry about $y = 0$ results in cross-jet symmetry in wave breaking, which is most likely to be impossible in the circular geometry cases discussed in Chapter 2.
Figure 3.1: Evolution of a barotropic wave with $A/L = 0.159$, which is considerably smaller than the critical amplitude for breaking. This type of evolution is categorized as Type A. The units for time, $T$, in this and following figures is $f_0^{-1}$. $\lambda$ is a parameter that indicates the degree of cross-jet basic flow shear asymmetry and is zero for basic flows with cross-jet symmetry.
Figure 3.2: Evolution of a barotropic wave with $A/L = 0.255$, which is just below the critical amplitude for breaking. The shape of the wave "oscillates" between a sinusoidal curve and sawtooth-like shape. This type of evolution is categorized as Type A.
Figure 3.3: Evolution of a wave with $A/L = 0.828$ and $L/R_d = 1.225$. The shape of the wave "oscillates" between a sinusoidal curve and $\Omega$-shape. This type of evolution is categorized as Type C.
There are a few features in Figure 3.4 that are worth some attention. The most noticeable feature is the discontinuous increase in the critical amplitude for breaking, accompanied by a change in the type of wave evolution at around $L/R_d = 1.2$. When $L/R_d$ is less than or equal to about 1.1, waves either propagate in a manner characterized by Type A or generate filamentary breaking. The value of the critical amplitude increases as $L/R_d$ increases. This trend reverses at around $L/R_d = 1.2$. The critical amplitude becomes smaller and then levels off as $L/R_d$ increases. This feature suggests that there may be two totally different mechanisms of wave breaking for any value of $L/R_d$, but only the dominant one determines the critical amplitude in these experiments. There is indeed a change in the breaking characteristics at around $L/R_d = 1.2$. For $L/R_d$ less than about 1.2, Type B breaking occurs as the amplitude is incresed. For $L/R_d$ greater than about 1.2, it is Type D which occurs first. Type B breaking appears to be caused by deformation of the material contour by the large-scale flow shear around critical points. Type D, on the other hand, appears to be caused by a completely different mechanism, perhaps by shear-induced instability of approaching two contours (Figure 3.6). Treating Type E as yet another different kind of breaking may seem somewhat subjective. It is obviously different from Type B; the material contours in Types B and E are sheared in opposite directions, suggesting different types of advection mechanism operating in the two. Distinction between Type D and Type E may be made by noting that Type E occurs about twice faster than Type D for the same value of $L/R_d$. Also, Type E breaking appears to be dominated by wavelength-scale advection, while Type D seems to depend on smaller-scale flow associated with shear-induced instability. However, the separate classification of Types D and E must be interpreted to be tentative.
Figure 3.4: A summary of wave evolution characteristics of experiments with the $2\pi$-periodic CD/CS, plotted as a function of $A/L$ and $L/R_d$. The solid curves separate breaking and no-breaking domains. See text for details.
Figure 3.5: Evolution of a barotropic wave with $A/L = 0.318$. The wave breaks by shedding filaments. This type of evolution is categorized as Type B.
$A/L = 0.859 \quad \lambda = 0.00$

Figure 3.6: Evolution of a wave with $A/L = 0.859$ and $L/R_d = 1.225$. This type of evolution is categorized as Type D.
Figure 3.7: Evolution of a wave with $A/L = 1.114$ and $L/R_d = 1.257$. This type of evolution is categorized as Type E.
3.2 Experiments with controlled asymmetry in the basic flow

3.2.1 Meridional asymmetry

In order to investigate effects of basic flow asymmetry on wave evolution characteristics, the basic flow with a straight jet was modified such that the PV jump at the contour is the same, while the flow field has asymmetry about $y = 0$. This was done by adding constant value of PV everywhere. In practice, this requires addition of zonal velocity given by

$$U_* = \lambda \frac{\Delta Q}{2} y$$

for barotropic flows and

$$U_* = \lambda \frac{\Delta Q}{2} \sinh(\gamma y)$$

for quasi-geostrophic equivalent-barotropic flows. The parameter $\lambda$ is a constant that controls the strength of the additional flow. The additional flow introduces additional shear, since

$$\frac{dU_*}{dy} = \lambda \frac{\Delta Q}{2}$$

for barotropic flows and

$$\frac{dU_*}{dy} = \lambda \frac{\Delta Q}{2} \cosh(\gamma y)$$

for equivalent-barotropic flows. Since the magnitude of shear at $y = 0^\pm$ in symmetric basic flow is

$$\left| \frac{dU}{dy} \right| = \left| \frac{\Delta Q}{2} \right|$$
for both barotropic and equivalent-barotropic flows, $U$, is that flow which adds a fraction, $\lambda$, of the otherwise-symmetric basic state shear to one side of the contour and subtracts from the other. For example, with $\lambda = 0.5$, the modified basic flow shear at $y = 0^-$ and $y = 0^+$ are 50% greater and smaller, respectively, than that in symmetric flow.

Experiments were performed in the fashion described in the previous section, with a few different values in $L/R_d$. Qualitative results, as far as the effect of the added shear on wave breaking is concerned, are the same for all the values of $L/R_d$ tested in the experiments. Figure 3.8 shows evolution of a barotropic wave with $A/L = 0.318$ and $\lambda = 0.05$. As shown in Figure 3.5, the wave breaks in a symmetric fashion when $\lambda = 0.0$. In this flow, shear in the basic flow is greater for $y < 0$ than $y > 0$ due to the added background flow. However, the magnitude of the introduced asymmetry is not sufficient to suppress wave breaking in $y > 0$. When $0.12 \leq \lambda$ for the same wave, the introduced asymmetry in the basic flow shear suppresses wave breaking in $y > 0$, resulting in one-sided breaking (Figure 3.9). The same effect is observed with considerably smaller additional shear in experiments with smaller deformation radii. For example, with $L/R_d = 1.037$ and $A/L = 0.637$, the transition from two-sided breaking to one-sided breaking occurs between $\lambda = 0.02$ and $\lambda = 0.05$ (Figures 3.10 and 3.11). For $L/R_d < 1.3$, the effect of the additional shear on wave evolution becomes even more dramatic. For instance, at $L/R_d = 3.142$, a subcritical (in symmetric basic flow) wave with $A/L = 0.700$ breaks with additional shear of $\lambda = 0.005$, producing large secondary vortices (Figure 3.12).

Breaking of subcritical-amplitude waves occurs with additional shear in all the values of $L/R_d$ investigated. In barotropic cases, a subcritical wave with $A/L = 0.255$,
very close to the breaking threshold, begins to break with additional shear of only $\lambda = 0.005$ (Figure 3.13). Even when the amplitude is reduced to $A/L = 0.095$, the wave begins to break one-sidedly with additional shear of $\lambda = 0.40$ (Figure 3.14).

Many experiments were run for the barotropic case to obtain a picture of the effect of the additional shear on wave evolution. Figure 3.15 summarizes the results of the experiments with barotropic flows. It shows that there are three wave evolution regimes for $0 < A/L < 1$ and $0 < \lambda < 0.5$. Wave evolution is categorized into three types: “T” for two-sided breaking, “O” for one-sided breaking, and “N” for no-breaking. The solid curves are drawn at the approximate boundaries between no-breaking and one-sided breaking and between one-sided breaking and two-sided breaking. The dashed line shows the breaking threshold in symmetric flows. Above this threshold amplitude, therefore, the waves break either two-sidedly or one-sidedly. Whether a wave of a certain amplitude breaks one-sidedly or two-sidedly depends upon the degree of asymmetry in the basic flow shear, i.e., the value of $\lambda$. The required shear asymmetry is greater for waves with larger amplitudes.

On the other hand, below the breaking threshold amplitude for symmetric basic flows, waves either break one-sidedly or do not break at all, depending on the value of $\lambda$. In this range of wave amplitude, the required asymmetry in the basic flow shear for wave breaking is greater for smaller amplitudes. It is also noteworthy that even an extremely small asymmetry in the basic flow shear results in one-sided breaking of a wave with an amplitude just below the symmetric flow threshold.

From a different point of view, Figure 3.15 shows that there are two threshold amplitudes for a given non-zero value of $\lambda$. Below the first threshold, waves do not break.
$A/L=0.318 \quad \lambda =0.05$

Figure 3.8: Evolution of a barotropic wave with $A/L = 0.318$ and $\lambda = 0.05$. 
Figure 3.9: Evolution of a barotropic wave with $A/L = 0.318$ and $\lambda = 0.12$. 
Figure 3.10: Evolution of a wave with \( A/L = 0.637 \), \( L/R_d = 1.037 \), and \( \lambda = 0.02 \).
Figure 3.11: Evolution of a wave with $A/L = 0.637$, $L/R_d = 1.037$, and $\lambda = 0.05$. 

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Figure 3.12: Evolution of a wave with $A/L = 0.700$, $L/R_d = 3.142$, and $\lambda = 0.005$. 
$A/L=0.255 \quad \lambda = .005$

Figure 3.13: Evolution of a barotropic wave with $A/L = 0.255$ and $\lambda = 0.005$. 
$A/L = 0.095 \quad \lambda = 0.40$

Figure 3.14: Evolution of a barotropic wave with $A/L = 0.095$ and $\lambda = 0.400$. 
Between the first and the second thresholds, waves break only one-sidedly, toward the side of greater basic flow shear. Beyond the second threshold, waves break into both sides of the jet, overcoming the asymmetry in the basic flow shear.

3.2.2 Zonal asymmetry

The experiments in the previous section illustrated effects of cross-jet asymmetry in the basic flow shear and locations of critical lines on wave evolution. Another potentially important asymmetry in the basic flow is that in the east-west direction. In particular, effects of diffluent and confluent jet, as observed in jet entrance and exit regions in the Northern Hemisphere, on propagating eddies are likely to be important in evolution of eddies in these regions. Stretching and deformation effects of such basic flows, as demonstrated by Shutts (1983) and Haines and Marshall (1987) for example, may have important consequences in Rossby wave breaking and thus vorticity mixing in the regions.

To study effects of such zonal asymmetry, a non-divergent and irrotational deformation flow was added to the barotropic basic flow. Since the added flow is irrotational and non-divergent, it neither changes the prescribed PV distribution nor violates the mass conservation in barotropic flows. This was done by defining a deformation streamfunction,

$$\psi_d = -\Psi_d \sin(kx) \sinh(ky), \quad (3.7)$$

where $\Psi_d$ is a constant. From this definition, the deformation flow is obtained;

$$U_d = A_d \sin(kx) \cosh(ky) \quad (3.8)$$

and
Figure 3.15: Plot of wave evolution characteristics for barotropic flows as a function of $A/L$ and $\lambda$. “T”, “O”, and “N” stand for two-sided, one-sided, and no-breaking, respectively.
\[ V_d = -A_d \cos(kx) \sinh(ky), \quad (3.9) \]

where the amplitude factor \( A_d \) is \( \Psi_d \). The deformation streamfunction with \( \Psi_d = 1 \) and \( k = 1 \) is shown in Figure 3.16. When this flow is added to the basic flow, jet is enhanced in \( 0 < x < \pi \) and weakened in \( \pi < x < 2\pi \). The new basic flow with this deformation flow has confluence and diffuence with enhanced and reduced zonal flow, respectively, as well as finite meridional velocity. In running experiments with the additional deformation flow, the model was slightly modified. When unmodified, the jet at the PV jump contour is zero in barotropic flows in this model. To make visualization of effects of the diffluent jet easier, a constant zonal velocity, \( U_0 \), was added, so that the jet at the contour is eastward and waves propagate eastward. The choice of \( U_0 \) is made such that it is on the order of the phase speed of a linear wave of number 1 to ensure eastward propagation of shorter waves. The total streamfunction with this modification and the deformation flow with \( A_d = 0.3 \) is shown in Figure 3.17.

Series of experiments were performed, varying the value of \( A_d \) and using two different forms of disturbances on the PV jump contour: sinusoidal waves of number 6 and localized wave-like disturbances. When \( A_d = 1 \), \( U_d \) locally cancels \( U_0 \) at the peak of the diffluent region.

Figure 3.18 shows evolution of No. 6 wave with \( A/L = 0.191 \) and \( A_d = 0.2 \). The same wave simply propagates without any noticeable change in its shape when \( A_d = 0 \). It breaks weakly when \( A_d = 0.1 \), but does not break when \( A_d = 0.05 \), although the shape is slightly deformed by the additional flow. The localized two-sided breaking shown in Figure 3.18 occurs, because of the additional deformation flow. When \( A_d \)
Figure 3.16: Contours of the deformation streamfunction, $\psi_d$, with $\Psi_d = 1$. 

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Figure 3.17: Contours of the total streamfunction of a basic flow modified with the deformation flow, $A_d = 0.3$, and a constant eastward flow, $U_0 = 1$. 

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is increased, the wave breaks more vigorously in the diffluent region (Figure 3.19), trapping nearly all the eddy enstrophy there. The amplification of the propagating wave in the diffluent region is due to two effects associated with the deformation flow. One is the reduced zonal flow in the diffluence. Held (personal communication) has shown that the amplitude of a propagating wave, whose wavelength is small relative to the zonal scale of the background flow variation, on a single PV jump contour is inversely proportional to $\sqrt{U(y = 0)}$, i.e., the smaller the zonal flow, the larger the amplitude, and that $U(y = 0)A^2$ is conserved. Although his result is, strictly speaking, valid for small-amplitude waves, it unambiguously states the sense of the effect of basic zonal flow variation on the amplitude of propagating waves. The second effect is meridional stretching of troughs and ridges by the deformation flow.

The stretching effect by the meridional velocity in the diffluent region is evident when $A_d$ has a moderate value, as shown in Figure 3.19, while it is too small to be apparent in Figure 3.18. When the wave amplitude is subcritical, but very close to the breaking threshold, wave breaking is induced by small values of $A_d$. A marginally-subcritical-amplitude wave with $A/L = 0.239$, which propagates in a manner shown in Figure 3.2, breaks two-sidedly with only $A_d = 0.05$. On the other hand, a wave of smaller amplitude requires greater $A_d$ for breaking. For example, the smallest value of $A_d$ required to induce breaking of a wave of $A/L = 0.143$ is somewhere between 0.1 and 0.2. One should note that waves with the same value of $A/L$, but different wavelength, do not behave in the same way, because the deformation flow strength varies with $x$ and $y$. For instance, a No. 2 wave with $A/L = 0.143$ has a dimensional amplitude 3 times as large as that of a No.6 wave with the same $A/L$. Its troughs and ridges, therefore, extend meridionally farther away than a No. 6 wave, being exposed to stronger deformation.
The aforementioned effects of the additional deformation flow are symmetric with respect to $y = 0$ and, thus, do not cause one-sided breaking. When sufficiently strong background shear, used in the previous section, is introduced, it can overcome the effects of the deformation flow, resulting in one-sided breaking (Figure 3.20). The minimum background shear asymmetry required for one-sided breaking increases with the strength of the deformation flow. This may be explained easily by the breaking characteristics described in the previous section, noting that larger $A_d$, and thus stronger meridional stretching and reduced zonal flow in the diffluent region, is tantamount to an effective increase of the wave amplitude.

Similar experiments were performed with a localized disturbance of various amplitudes. Effects of the deformation flow and shear asymmetry on such disturbances are qualitatively the same as those described above for No. 6 waves. One difference between these two types of propagating perturbation observed in these experiments is the dependence of evolution of localized disturbances on their initial location. The dependence observed is that a disturbance tends to break more readily when its initial location is upstream of the diffuence, and vice versa. Figures 3.21 and 3.22 illustrate this dependence. The only difference in the initial conditions shown in these figures is the location of the disturbance; it is located upstream of the diffuence in Figure 3.21, while it is in the middle of the diffuence in Figure 3.22. The disturbance in Figure 3.21 becomes zonally-compressed and meridionally-stretched as it enters the diffluent region and breaks. That in Figure 3.22, on the other hand, is advected out of the diffluent region and becomes zonally-stretched and meridionally-compressed as it passes through the confluent region, where the jet is enhanced by the deformation flow. As it re-enters and passes through the diffluent region, it merely, more or less, regains its original amplitude and does not break. This behavior can be readily explained by the Held's theoretical prediction of the relationship between the amplitude
Figure 3.18: Evolution of No. 6 wave with $A/L = 0.191$ and $A_d = 0.2$. The diffuence in the region $\pi < x < 2\pi$ is weaker than that shown in Figure 3.17.
Figure 3.19: Evolution of No. 6 wave with $A/L = 0.191$ and $A_d = 0.5$. The diffuence in the region $\pi < x < 2\pi$ is stronger than that shown in Figure 3.17. Note strong meridional stretching in the diffuient region, which is not noticeable in Figure 3.18.
Figure 3.20: Evolution of No. 6 wave with $A/L = 0.191$. The basic flow has deformation flow with $A_d = 0.2$ and background shear asymmetry with $\lambda = 0.2$. The shear asymmetry overcomes the effective increase of the wave amplitude due to meridional stretching in the diffluent region and suppresses breaking on one side.
Figure 3.21: Evolution of a localized wave-like disturbance in a basic flow with deformation flow, $A_d = 0.7$. The initial location of the center of the disturbance is $x = 1$. 

$A/L = 0.082$ \hspace{1cm} $A_d = 0.70$
$a/L = 0.082, \quad Ad = 0.70$

Figure 3.22: Same as Figure 3.21, except that the initial location of the center of the disturbance is $x = 5$.  

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of a propagating wave and the basic zonal flow.

3.3 Discussion

As expected from the absence of cross-jet asymmetry in the basic flow structure imposed by the geometry, wave evolution on the PV front, including breaking, is symmetric when there is no additional background flow. When the initial wave amplitude, which serves as the forcing in these experiments, exceeds the threshold value, the wave breaks symmetrically with respect to the basic state jet, as shown in Figure 3.5. Lindzen and Schoeberl (1982) theoretically predicted the upper limit of Rossby wave amplitude. The system used in these experiments is based on idealized PV distribution with infinite gradient at a single PV jump contour and is not capable for confirming their prediction.

For the range of values of $L/R_d$ examined here, there are five distinguishable types of wave evolution in the absence of meridional or zonal asymmetry in the basic flow, as described in Section 3.1. A very intriguing feature is observed at $L/R_d = 1.225$ (Figure 3.4). At this value, which lies in the middle of the region of abrupt changes, wave evolution characteristics possess double-criticality. As the initial amplitude is increased, waves begin to break in filament, as in Type B. This first criticality exists at around $A/L = 0.52$. As the initial amplitude is increased further, at around $A/L = 0.82$, waves stop breaking altogether and exhibit behavior Type C. With a further increase in amplitude, at around $A/L = 0.85$, waves again begin to break, this time however, in a different fashion, Type D. This observation indicates that there are totally different wave breaking mechanisms, dominant one of which determines the critical amplitude in the experimental setting used here. The presence of this double-criticality is supported by results of stability calculations also. Meacham
has calculated the growth rate of a small perturbation on steadily-propagating states of various values of amplitude and deformation radius and found that there is a "window" of near-zero growth rate over a certain range of amplitude when deformation radius and wavelength are comparable (personal communication). The growth rate increases as the amplitude of the steadily-propagating state is increased. With further increase in amplitude, the growth rate begins to decrease, finally reaching almost zero. As the amplitude is increased further, the growth rate becomes finite again. The "window" of no-breaking, sandwiched between the two different breaking regimes, may be a manifestation of these stable large-amplitude steadily-propagating states.

The introduced asymmetry in the basic flow induces asymmetry in Rossby wave breaking. In cases of cross-jet or meridional asymmetry in the basic flow, the sense of this asymmetry in wave breaking is in agreement with the results observed in experiments with a circular vortex in Chapter 2. Waves tend to break more readily to the side of greater basic flow shear or the magnitude of rate of strain. The side of greater basic flow shear is also the side on which the closer critical line is located, since the flow speed drops more rapidly there. Thus, the inseparable relationship between asymmetry in the basic flow shear and location of critical lines, as discussed in Section 2.2, still exists in the $2\pi$-periodic linear geometry. When this basic flow asymmetry is made sufficiently large, waves break one-sidedly, in agreement with the results shown in Chapter 2. To put it another way, asymmetry in the basic flow shear may be overcome by large wave amplitudes, leading to two-sided breaking.

These breaking characteristics may be readily understood in terms of the relationship between the onset of breaking and the occurrence of critical or stagnation points in
the flow (Pullin et al., 1989; Polvani et al., 1989). If the basic flow is symmetric, critical points exist on both sides of a distorted contour and move toward the contour together (though at different phases of the wave). When wave amplitude is increased sufficiently, these points move across the contour and induce two-sided breaking. (Flierl found that propagation speed of waves in this system is influenced noticeably by wave amplitudes in such a way that larger amplitudes push away locations of critical points from the jet (personal communication). This appears to be the reason why the threshold amplitude is greater than the distance of critical lines for linear waves.) In an asymmetric basic flow, however, at least if the wave amplitude is the same on either side of the contour, the critical point on the side with the nearer undisturbed critical line is closer and will be reached by the contour at lower amplitude. Thus, waves break to the side of the greater basic flow shear, when the amplitude-shear asymmetry relationship falls in the range of “O” shown in Figure 3.15. When the amplitude is large, i.e., the amplitude-shear asymmetry relationship falls in the range of “T” in Figure 3.15, the critical points on both sides of the distorted contour move across the contour, although at different phase of the wave. This results in wave breaking toward both sides of the contour.

The effect of the additional background shear on wave evolution can be dramatic, depending on the strength of the added shear and wave amplitude. When the amplitude is near the breaking threshold but subcritical, wave evolution is very sensitive to a slight additional shear in the basic flow. This is clearly illustrated in Figure 3.15. A barotropic wave with \( A/L = 0.255 \), which is just below the breaking threshold, breaks with only \( \lambda = 0.005 \), introduction of 1% difference in the cross-jet shear in the basic flow. This sensitivity may also be readily understood in terms of the relationship between the onset of breaking and the occurrence of critical or stagnation points in the flow. For weak wave amplitude, these points are distant from the PV contour and
barotropic wave with $A/L = 0.255$, which is just below the breaking threshold, breaks with only $\lambda = 0.005$, introduction of 1% difference in the cross-jet shear in the basic flow. This sensitivity may also be readily understood in terms of the relationship between the onset of breaking and the occurrence of critical or stagnation points in the flow. For weak wave amplitude, these points are distant from the PV contour and no breaking occurs. As wave amplitude is increased, these points and the distorted contour approach each other and are presumed to be very close when the amplitude is close to the breaking threshold. Thus, even a slight additional uniform shear in the basic flow moves the critical points inside the distorted contour on the side of increased basic flow shear and away from the contour on the side of decreased basic flow shear, resulting in one-sided breaking.

The results presented here suggest that the cross-jet asymmetry in Rossby wave breaking on a sharp jet characterized by a single discontinuity in PV is caused by asymmetry in the basic flow structure, particularly that in locations of critical lines and rate of strain or shear. Pratt et al. (1991) studied breaking of varicose and mixed sinuous-varicose disturbances on a multi-contour jet and found that breaking occurred toward the weaker PV gradient. Their interpretation is actually the same as that described above in essence, since, at least in the cases they considered, the nearer critical line is found on the same side of the jet as the weaker PV gradient.

When zonal asymmetry is introduced into the basic flow by adding a deformation flow to it, it naturally induces zonal asymmetry in wave breaking characteristics. Waves tend to amplify in the region of diffuence due to locally reduced zonal flow and meridional stretching by the meridional component of the deformation flow. This leads to breaking of subcritical-amplitude (in basic flow without meridional or zonal
asymmetry) waves in the diffluent region, when the diffuence is sufficiently strong. In terms of locations of stagnation points, they become closer to the contour in the region of diffuence, because of the reduction in the zonal flow. Thus, the breaking of subcritical-amplitude waves due to the addition of deformation flow may be understood in terms of the change in the location of critical points in the flow as well. When both the zonal and meridional asymmetries are introduced into the basic flow, their net effect on wave evolution is determined simply by the competition of the aforementioned effects, i.e., the stronger one overwhelms the other.

An important implication of the demonstrated effects of the diffluent basic flow on evolution of propagating waves must be noted here. Diffluent structures in planetary-scale flow tends to develop in certain parts of the atmosphere, particularly over the western North Atlantic and the western North Pacific. Although the strength of the southern branches of the jet in the diffluent regions tends to be weaker than that in the northern branch, the aforementioned effects of the diffuence on propagating waves should be present in these regions. The upstream ends of these diffluent regions are associated with active baroclinic eddy generation or amplification, because of the large land-sea temperature contrast (during winter) and large baroclinicity maintained by oceanic currents. The barotropic amplification effects may be dwarfed by these baroclinic effects and, hence, may be undetectable in reality. However, it seems possible that small-amplitude waves entering a pre-existing planetary-scale diffuence amplify, resulting in stronger cyclonic circulation in the troughs, whose synoptic-scale upward motion may trigger large-scale release of latent heat, which tends to help further growth of the waves. In this sense, the barotropic amplification effects of planetary-scale diffuence on propagating synoptic-scale waves may not be negligible.
Chapter 4

Rossby wave breaking under influence of stationary or steadily-propagating waves in the basic flow

Abstract

This chapter describes asymmetry of Rossby wave breaking induced by curvature in the basic flow. The experiments are done with the $2\pi$-periodic and non-periodic CD/CS codes described in Chapters 2 and 3. In the $2\pi$-periodic cases, the basic flows consist of a stationary wave of number 1, which is made stationary by adding a suitable background velocity. Disturbances of wavenumber 5 with various amplitudes are superimposed on the basic flow in the initial condition. In the non-periodic cases, the basic states consist of a steadily-rotating vortex with wavenumber 3. As in the circular vortex cases described in Chapter 2, an oscillating mountain is used to excite eddies on the vortex. Experiments in the two configurations show that the direction of Rossby wave breaking is influenced by curvature, which is accompanied by cross-jet and zonal asymmetries in the magnitude of rate of strain, of the basic flow. The observed asymmetry in wave breaking is such that cyclonic/anticyclonic eddies break at cyclonic/anticyclonic phase of the basic flow.
4.1 $2\pi$-periodic cases

In Chapters 2 and 3, wave breaking was studied in basic flows characterized by a zonally-oriented PV jump contour. In the real atmosphere and oceans, however, the zonal PV gradient is not necessarily zero. Especially, the upper troposphere in the Northern Hemisphere is characterized by the presence of long waves which is associated with finite curvature in the time mean flow and non-zero zonal PV gradient. The curvature introduces cross-jet and zonal asymmetries in the magnitude of rate of strain and locations of stagnation points, thus potentially affecting evolution of synoptic-scale eddies. In order to study the effects of curvature in the basic flow on Rossby wave breaking, the $2\pi$-periodic CD/CS used in Chapter 3 was initialized with sinusoidal waves of number 5, superimposed on a stationary wave of number 1, which characterized the basic flow in the experiments. A constant background velocity was added to cancel propagation of wavenumber 1. To do this, Equation 2.4 was solved for a linear dispersion relationship with two matching conditions at $y = 0$, similar to Equations 2.14 and 2.15. Then one obtains

$$\sigma = kU_{(y=0)} - \frac{k\Delta Q}{2\sqrt{k^2 + \gamma^2}}, \quad (4.1)$$

where $k$ is the wavenumber in the $x$-direction here.

The background velocity, $V_0$, was determined from the linear dispersion relationship and is given by

$$V_0 = \frac{\Delta Q}{2\sqrt{1 + \gamma^2}} - U_{(y=0)}. \quad (4.2)$$

For the sake of clarity, the amplitudes of the stationary wave of number 1 and disturbance wave of number 5 are denoted by $A$ and $a$, respectively, in this chapter. $L$ is used for the wavelength of the disturbance wave. The results shown here are from experiments with $\gamma = 1$ (so that the scale of a disturbance is on the order of the
deformation radius), but experiments with other values of $\gamma$ showed qualitatively the same results.

As observed in cases with purely zonal background flow mentioned in Chapter 3, waves with amplitudes, $\alpha$, that are smaller than a certain critical value do not break and propagate along the wavy basic flow (Figure 4.1). When $\alpha$ is greater than the first critical value, the waves break in an asymmetric fashion; cyclonic/anticyclonic disturbances break along the trough/ridge of the wavy basic flow (Figure 4.2). This first critical value of $\alpha$ is, when it is non-dimensionalized by wavelength, somewhere between 0.056 and 0.080, with $A = 0.5$. When $\alpha$ is greater than the second critical level, the waves break without bias in the location (Figure 4.3). The non-dimensionalized value of the second critical amplitude is between 0.159 and 0.199 with $A = 0.5$. (Note that this second critical amplitude is determined rather subjectively by judging from the locations where the disturbances develop large curvature and become irreversible.) Since the magnitude of rate of strain in this basic flow is larger on the cyclonic/anticyclonic side of the PV jump contour than the anticyclonic/cyclonic side at the ridge/trough (Figure 4.4), wave breaking characteristics mentioned above agree with the argument presented in Chapters 2 and 3. With this finite-amplitude stationary wave as the basic flow, it is difficult to talk about the influence of critical lines. However, at the trough of this basic flow the critical point on the anticyclonic side of the jet is closer to the jet than is the critical point on the cyclonic side of the jet, and vice versa. Thus, critical points for smaller-scale waves in this basic flow are also likely to create the described asymmetry in wave breaking.

When $A$ is increased to 1.0, the asymmetry in wave breaking is enhanced. The first critical level is not affected noticeably, while the second critical level is increased, thus widening the range of asymmetric breaking. With $A = 1.0$, twice as large as
$a/L = 0.056$

Figure 4.1: Evolution of wave No. 5 with $a/L = 0.056$ in a basic flow consisting of a stationary wave No.1 with $A = 0.5$. Rossby deformation radius is 1. During this experiment, the disturbance never developed sharp curvature or irreversible features.
Figure 4.2: As in Figure 4.1, except that $a/L = 0.159$. The disturbance developed sharp curvature and broke off. The sharpening of disturbance troughs/ridges occurred while they were traveling through the trough/ridge of the stationary wave.
Figure 4.3: As in Figure 4.1, except that $a/L = 0.477$. The disturbance developed sharp curvature without any noticeable bias in the location and broke off.
Figure 4.4: Contours of the magnitude of rate of strain in the basic flow used in Figures 4.1, 4.2, and 4.3.
Figure 4.5: Evolution of a Gaussian-shaped disturbance on a large-amplitude steadily-propagating state calculated by Meacham. Here, $L$ is an approximate width of the disturbance at its bottom. $a$ is the height of the disturbance at its center, which is located at $x = \pi$. Rossby deformation radius is 1. A constant flow field was added to make the steadily-propagating $\Omega$-shaped wave stationary.
those shown in the figures, the first and the second critical values of \( a \) are between 0.056 and 0.080 and between 0.199 and 0.239, respectively. In an extreme case of large-amplitude stationary wave, such as that shown in Figure 4.5, this asymmetry in breaking becomes hard to overcome by increasing the amplitude of disturbances, presumably because of the enhanced curvature, which is accompanied by a greater asymmetry in the magnitude of rate of strain, in the basic flow.

### 4.2 Non-periodic cases

As discussed in Chapter 2, a circular basic flow geometry has complex effects on wave breaking characteristics, making evaluation of the effects of the cross-jet asymmetry of the basic flow on wave breaking difficult. However, since the real atmospheric disturbances of interest, synoptic-scale eddies, are sufficiently large to feel the effects of the earth’s curvature, it seems important to check whether the associated curvature in the basic flow significantly influences wave breaking characteristics in a circular geometry. The main effect of the earth’s sphericity on wave evolution in middle latitudes comes from the curvature in the west-east direction, when the \( \beta \)-effect has been absorbed into the dynamics by piece-wise uniform PV fields, separated by finite jumps. This has been demonstrated by Saravanan (1994), who reproduced overall features of the two paradigms presented by Thornicroft et al. (1993) by three-dimensional CD/CS on an \( f \)-plane. (However, if the \( \beta \)-effect were present, the basic flow with the same PV jumps would be different, since the \( \beta \)-effect must be absorbed by the shear in the basic flow.)

The non-periodic CD/CS mentioned in Chapter 2 was initialized with a steadily-rotating vortex of wavenumber 3 and forced by an oscillating mountain similar to that described in Chapter 2. The steadily-rotating solutions were calculated by D. Waugh, using a method based on a numerical procedure developed by Dritschel (1994). The
basic flow profile of a steadily-rotating state with a finite amplitude wave is illustrated in Figures 4.6, 4.7, and 4.8, in which contours of flow speed, flow vectors, and contours of magnitude of rate of strain, respectively, are plotted. The values of PV inside and outside the vortex are 2f and f, respectively. Rossby deformation radius is 0.2R, same as that used in the experiments discussed in Chapter 2. In the limit of zero-amplitude, i.e., circular vortex, the basic flow is similar to that shown in Figure 2.1. When a circular vortex of this PV distribution is forced by a wave maker, waves break only outward as described in Chapter 2. However, when the basic state has sufficient anti-cyclonic curvature and is forced by a sufficiently large-amplitude wave maker, waves do break inward (Figure 4.9). In this particular case shown in Figure 4.9, the oscillatory topographic forcing was located at slightly downstream of the peak of a trough and was rotated at the same rate as the angular speed of the basic state, so that the forcing remains more or less (the forced disturbances did modify the rotation rate to some extent) at the same phase of the rotating wave of number 3. Some experiments were done with the oscillatory forcing fixed or rotating in the direction opposite to that of the basic state. These experiments showed that inward breaking still occurs.

Figures 4.6 through 4.8 suggest that this result, inward breaking along the ridge and outward breaking along the trough, is consistent with the results presented in the preceding chapters. Observing the tightness of contours at the peaks of ridges and troughs in Figure 4.6, one can see that the shear of the azimuthal velocity is slightly larger inside/outside the vortex at ridges/troughs. This is a reflection of slightly sharper drop in the azimuthal velocity on the side of greater shear (Figure 4.6). Thus, perturbation flow of the same magnitude tends to create stagnation points more readily on the side of greater shear, causing preferential occurrence of breaking there. In addition to this shear asymmetry at the ridges and troughs, there is some
contribution to asymmetry in the magnitude of rate of strain from relatively large curvature of the flow, as illustrated by the velocity vectors shown in Figure 4.7. The magnitude of rate of strain shows clear asymmetry at the ridges and troughs (Figure 4.8). At the peak of the ridge, the magnitude of rate of strain just inside the vortex is roughly twice as large as that outside in this particular case. The asymmetry is much more conspicuous at the trough, with the sign reversed. This difference in the degree of asymmetry is probably responsible for relatively weak inward breaking along the ridges compared to vigorous outward breaking at the troughs, observed in all the experiments performed with a steadily-rotating wave of number 3 (e.g., Figure 4.9).

It turned out that the smaller the anticyclonic curvature of the basic flow, the larger the forcing must be to induce inward breaking. This trend can be also explained by the results described in Chapters 2 and 3. Figures 4.10 through 4.13 show wind vectors and the magnitude of rate of strain of two steadily-rotating states, one with greater curvature associated with the wave of number 3 than the other. Clearly, the magnitude of rate of strain is considerably greater just inside ridges than outside and vice versa, when the steadily-propagating wave amplitude and, thus, the basic flow curvature are greater. When the amplitude of the steadily-propagating wave is small, the asymmetry in the magnitude of rate of strain across the PV jump contour is small, particularly at ridges (Figure 4.13). When the amplitude is very small, such as the case shown in Figures 4.12 and 4.13, forced waves do not break inward at all unless the forcing is so strong that the entire vortex is disrupted, as in cases of circular basic states. It is difficult to discuss relevance of location of critical lines in these basic flows with finite-amplitude waves, since propagation speeds of small-scale disturbances are not uniform on the edge of the vortex. Although qualitative, sense and magnitude of the basic flow curvature seem to be good predictors for the direction of wave breaking in these idealized geometry; when there is large curvature in the basic flow, waves
Figure 4.6: Contour plot of flow speed for a steadily-rotating vortex with wave number 3.
Figure 4.7: Flow vectors for the steadily-rotating vortex shown in Figure 4.6.
Figure 4.8: Contour plot of the magnitude of rate of strain for the steadily-rotating vortex shown in Figure 4.6.
Figure 4.9: Evolution of forced disturbances on the steadily-rotating vortex shown in Figure 4.6. In this particular experiment, the oscillatory topographic forcing was initially located at the upper-right trough and was rotated at the angular speed of the rotation of the vortex.
break outward/inward where the curvature is cyclonic/anticyclonic.

4.3 Discussion

The results presented in this chapter are no surprise, given the findings shown in Chapters 2 and 3. They are less clear-cut than those in the preceding chapters, due to the way the flow asymmetry is introduced in the experiments here, imposing finite-amplitude stationary or steadily-propagating waves on the basic state PV front. In particular, the relationship between the direction of wave breaking and critical lines presented in Chapters 2 and 3 is no longer clear here. However, the relationship between the breaking direction and asymmetry in the magnitude of rate of strain, which was shown to be inseparable from asymmetry in the locations of critical lines in the preceding chapters, still holds. It implies that the relationship between the direction of breaking and asymmetry in locations of critical points is likely to be the same as that discussed in Chapters 2 and 3. When the steadily-propagating or stationary waves have sufficiently large amplitude so that the magnitude of rate of strain inside the vortex at ridges is greater than that outside, forced or initialized disturbances break into the vortex at the ridges, and vice versa. The results may be summarized succinctly by noting that anticyclonic curvature in the basic flow tends to create a favorable condition for the anticyclonic phase of a wave to grow and break and vice versa. With a sufficiently strong anticyclonic curvature in the basic state, even with a PV distribution inside and outside the vortex being identical to that produced only outward breaking in a circular case, low-PV material breaks into the vortex at ridges of the steadily-rotating wave of No. 3 (Figure 4.9). It suggests that poleward breaking of synoptic-scale anticyclones may occur more readily at ridges of planetary-scale flow in the real atmosphere.
Figure 4.10: Same as Figure 4.7 except for the larger amplitude of the steadily-rotating wave number 3.
Figure 4.11: Contour plot of the magnitude of rate of strain for the steadily-rotating vortex shown in Figure 4.10.
Figure 4.12: Same as Figure 4.7 except for the smaller amplitude of the steadily-rotating wave number 3.
Figure 4.13: Contour plot of the magnitude of rate of strain for the steadily-rotating vortex shown in Figure 4.12.
Chapter 5

Rossby wave breaking in a GCM and its role in "atmospheric blocking"

Abstract

Output of a GCM run is analyzed for Rossby wave breaking characteristics in a complex system, in which diabatic effects, baroclinic flow, etc are present. Particular emphasis is placed on the role of wave breaking in a phenomenon often referred to as "atmospheric blocking" or "persistent anomaly". Five episodes of blocking are analyzed, using isentropic Ertel's potential vorticity and the method of contour advection with surgery (CAS). The analyses show that the five episodes simulated by the GCM are dominated by advective processes of low-frequency flow, manifested by quasi-stationary wave amplification and breaking. Breaking of synoptic-scale eddies in the blocking region is also observed to be reinforcing the diffluent low-frequency flow, as shown in diagnostic and modeling studies. However, the direct contribution of high-frequency flow is somewhat smaller than that of low-frequency flow throughout life cycles of the five events. These results demonstrate that "persistence" of this phenomenon arises from simply the time scale of the dominant process, quasi-stationary wave amplification, breaking, and dissipation.
5.1 Rossby wave breaking generated by a GCM

The results shown in the preceding chapters are illustrative of the effects of the basic flow on wave evolution in a relatively clear-cut manner, because of the simplicity of the model and experiment configurations. However, as suggested by model studies of relatively more complex flows (e.g., Nakamura, 1993; Whitaker and Snyder, 1993), the real atmospheric and oceanic flows are capable for creating flows that may not be studied by simple models such as those used in the preceding chapters. To study characteristics of wave breaking and their role in a phenomenon often referred to as "atmospheric blocking" or "persistent anomaly", output of a GCM, Geophysical Fluid Dynamics Laboratory (GFDL) R30-realistic, was analyzed. Diagnoses are based on Ertel's potential vorticity on isentropic surfaces in the upper troposphere. Because of its clear relationship with the flow field and its conservative nature in adiabatic-inviscid flows, isentropic Ertel's potential vorticity is a powerful tool in studying the large-scale dynamics of the atmosphere and oceans (Hoskins et al, 1985). Especially in studying Rossby wave breaking, as illustrated in preceding chapters, these characteristics of Ertel's potential vorticity are very useful. Thus, the diagnoses described in this chapter are based almost entirely on isentropic Ertel's potential vorticity.

5.1.1 Data

The data consist of the output of an experiment with a GCM at GFDL. The model is a new version of their spectral R30 (rhomboidal truncation at the 30th harmonics) with a realistic topography. The horizontal resolution is approximately 3.75 degrees by 2.25 degrees in longitude and latitude, respectively. There are 14 \( \sigma \) levels in the vertical: 0.015, 0.05035, 0.1009, 0.17065, 0.2569, 0.3549, 0.460, 0.5682, 0.6755, 0.77695, 0.86605, 0.9353, 0.97865, and 0.99665. The topography is an improved version, which is almost free of the "Gibbs phenomenon" (artificial undulation of the surface due to spectral representation of the topography) in the east-west direction.
(Navarra et al., 1994). Sea surface temperature (SST) is linearly interpolated in time between two monthly-mean SST fields. In addition to the gravity wave drag, there is a weak diffusion in the top layer to reduce reflection of long waves at the upper boundary. Surface flux parameterization is based on bulk formulae in a neutral planetary boundary layer. The solar forcing has seasonal variation but no diurnal variation.

This model was used for a 30-year integration at GFDL, storing its output once a day from the 16th year of the integration. Analyses of the daily data, using potential vorticity as the primary diagnostic tool, produced extremely noisy results, which were likely to be meaningless, because of the inadequate sampling frequency. The model was, therefore, re-run for the 16th year, during which there are 5 episodes of blocking, generating a set of 5-times-per-day data of the surface pressure and temperature and wind at all 14 $\sigma$ levels, from January 1 through December 31.

Using the aforementioned data, wind and Ertel's potential vorticity, $q$, on isentropic surfaces were calculated. Here, $q$ is defined by

$$ q = -g \frac{\partial \theta}{\partial p} (\zeta + f), \quad (5.1) $$

where $g$ is the gravitational acceleration, $\theta$ is potential temperature, $p$ is pressure, and $\zeta$ and $f$ are relative and planetary vorticities, respectively. This is a form with the hydrostatic approximation. It was assumed that potential temperature and wind vary linearly with $\ln p$ in calculating $\frac{\partial \theta}{\partial p}$ and $\zeta$ on the chosen isentropes. $\frac{\partial \theta}{\partial p}$ was calculated by $\ln p$-weighted interpolation, using two values calculated from three $\sigma$ levels closest to the isentrope. (Cubic spline interpolation would have been more desirable, but did not work very well with these data, perhaps due to the relatively small number of data point in the vertical.) Wind at two $\sigma$ levels closest to the isentrope was used.
to interpolate the value onto the isentrope. Then, its curl was taken to calculate $\zeta$. Here, $g$ was set at $9.81\text{ms}^{-2}$ and $f$ was calculated at each Gaussian latitude used in the model.

Ertel’s potential vorticity, $q$, was calculated on several isentropes in the upper troposphere in both the Northern and Southern Hemispheres. Perhaps due to lack of vertical resolution and perfect vertical mixing within each layer of the model, isentropes that lie mostly above the tropopause were characterized by locally-up-gradient mass-weighted time- and zonal-mean $q$ flux. The surfaces of $320^\circ K$ and $310^\circ K$ were chosen for the Northern and Southern Hemispheres, respectively, for the full diagnoses, since these surfaces consistently show down-gradient mass-weighted time- and zonal-mean $q$ flux and never intersect the lower boundary of the model. In terms of pressure, these surfaces lie somewhere between 200mb and 600mb. Thus-obtained $q$ data set is by no means a substitute for the real atmospheric data. However, it serves as a high-quality data set through which one can extract information on complex fluid dynamical processes, because the original data are known with high-accuracy at every grid point and are completely internally-consistent.

5.1.2 Examples of outward and inward breaking

The one-year time series of $q$, calculated in the manner described in the previous section, were examined for Rossby wave breaking, manifested by irreversible deformation of $q$ contours. Since the waves studied here are supported by continuous $q$ gradient, which is spread over large distances, it is not as straightforward as it is in CD/CS flows to define Rossby wave breaking. In the current and the following chapters, Rossby wave breaking is identified by formation of cut-off cyclones or anticyclones on scales of hundreds of kilometers or greater, including finite-thickness filaments that are mixed into regions characterized by different values of $q$. This is a qualitative, however, clear-cut and convenient way to identify breaking events in the complex
flows generated by the GCM and the real atmosphere. Figure 5.1 shows contours of raw $q$ (thin contours) superimposed on lowpass-filtered $q$ (thick contours), which will be denoted by $\tilde{q}$, from Feb 1 through Feb 8 in the Northern Hemisphere. The lowpass filter is a 151-point filter (i.e., uses 30.2 days of data to calculate the filtered quantity at one time frame) that retains low-frequency flow with periods of 10 days or longer, including the time mean. It is essentially the same as that used by Lau and Lau (1984). The contour interval is 1 PVU.

During this period, there are several occurrences of wave breaking, both outward and inward. Over the eastern North Pacific, 120°W - 150°W, there are two events of outward breaking, shedding high-$q$ air toward the equator in a manner that resembles the breaking shown in Figure 2.3. Although there is some indication of weak inward breaking around 160°W between Feb 4 and Feb 5, vigorous intrusion of low-$q$ air is not seen there during this period. In contrast, there are both outward and inward breakings over the North Atlantic. The first event is occurring on Feb 1 over the western North Atlantic between 30°W and 60°W, showing a sharp ridge and a trough moving poleward and equatorward, respectively. The ridge and trough are then successively cut off by Feb 3, transporting low-$q$ air poleward and high-$q$ air equatorward. On Feb 3, another event of inward breaking is beginning to occur at around 60°W, following the formation of cut-off cyclone downstream. This breaking event also produces a cut-off anticyclone seen from Feb 4 till the end.
Figure 5.1: Contour plot of Ertel potential vorticity, $q$, on a 320°K surface in the Northern Hemisphere from Feb 1 through Feb 8. Lowpass-filtered field (thick contours) is superimposed on unfiltered field (thin contours). The contour interval is 1 PVU. The lowest latitude shown is 18°N. Outward and inward breakings are observed through contours of the unfiltered $q$. See text for more details.
Figure 5.1 continued.
Figure 5.2: Contour plot of $q$ on a 310°K surface in the Southern Hemisphere from Jul 16 through Jul 23. Lowpass-filtered field (thick contours) is superimposed on unfiltered field (thin dashed contours). The contour interval is 1 PVU. The highest latitude shown is 18°S. Only outward breaking is observed. See text for more details.
Figure 5.2 continued.
Figure 5.2 shows contours of $q$ (thin dashed contours) superimposed on $\tilde{q}$ (thick solid contours), from Jul 16 through Jul 23 in the Southern Hemisphere. (In the Southern Hemisphere, the smaller the value of $q$, i.e., large negative values, the more cyclonic is its motion.) There are several wave breaking events occurring during this period. For example, on Jul 16 at around 60°W, a deep trough is about to move away from the vortex. It is elongated and forms a thick filament by Jul 17. It has broken away and been mixed into the “surf zone”, by Jul 18. There is another major outward breaking, occurring between 150°W and 150°E from Jul 20 through Jul 22. Associated with this outward breaking event, there appears to be some anticyclonic air moving into the vortex. However, it never penetrate deep into the vortex and, thus, shall not be regarded as inward breaking. Despite the vigorous eddy activity during this period, manifested by large-amplitude undulation of $q$ contours, there is no inward breaking occurring.

This absence of inward breaking in Figure 5.2 may be attributed to the lack of anticyclonic curvature in the low-frequency flow, which may be regarded as a “background flow” or temporal “basic flow”. The low-frequency flow, which is nearly parallel to the $\tilde{q}$ contour, does not show any significant large-scale anticyclonic curvature in Figure 5.2. In contrast, the $\tilde{q}$ contours shown in Figure 5.1 suggest strong anticyclonic curvature in the lowpass flow over the North Atlantic, 0° to 60°W. It is indeed this region where inward breaking is occurring, as mentioned earlier. This relationship between the curvature of low-frequency flow and breaking direction is observed in all the other periods examined (not shown). Thus, the breaking characteristics observed in experiments with a single potential vorticity jump contour, described in Chapters 2 and 4, are still true in a broad sense; no breaking into the vortex unless there is a significant anti-cyclonic curvature in the background flow. Of course, the flow indicated in Figure 5.1 is much more complex than that in Figure 4.7. The main
difference between these two flows is that one in Figure 5.1 has two branches of jet associated with the diffuence, while that in Figure 4.7 has only one jet. The southern branch of the jet in the diffuential region of Figure 5.1 is characterized by a large cyclonic curvature, creating a mirror image of the northern branch. This feature is likely to make evolution of propagating eddies quite different from that in a basic flow with a single jet branch such as that in Figure 4.7.

5.2 “Blocking” episodes produced by the GFDL GCM

An atmospheric phenomenon referred to as “blocking” or “persistent anomaly”, characterized by a strongly diffuential pattern associated with a dipole or an $\Omega$-shape in low-frequency flow, has eluded many attempts by researchers to pinpoint its mechanism. The large meridional component of the low-frequency flow, which is “persistent” with respect to the time scale of synoptic-scale eddies, tends to advect traveling weather systems along the northern and southern flanks of the diffuence, therefore, “blocking” propagation of these systems into the region of the low-frequency anticyclone. Its “persistence” (with respect to time scales associated with typical synoptic-scale systems) tends to create anomalous (with respect to the climatology) weather patterns in the affected regions. The first account of this phenomenon was given by Rex (1950a, 1950b). Since then, there have been many attempts to systematically document characteristics of this phenomenon to learn its disposition and to, perhaps, infer its mechanism. Some of the most exhaustive studies of this kind are given by Dole and Gordon (1983) and Dole (1986). Lindzen (1986) argued that there is nothing “persistent” or “anomalous” about this phenomenon, because the natural time scale to which this phenomenon should be measured against is that of quasi-stationary planetary waves, whose moderate changes in amplitude can account for the amplitude of most “persistent anomalies”. His argument will be indeed validated in the
current and the following chapters, at least for those events examined here. For convenience, "blocking" will be used, in the current and the following chapters, to refer to low-frequency flow patterns with a large meridional component associated with a dipole or an $\Omega$-shape in $\tilde{q}$.

From the perspective of synoptic-scale wave breaking, blocking is a rather spectacular display of breaking activity, manifested by deep troughs and ridges being elongated meridionally and subsequently breaking in the region of low-frequency diffuence (Figure 5.1). As described in the previous section, poleward breaking of anticyclonic air may occur more readily during such an event, because of the considerable large-scale anticyclonic curvature of the low-frequency flow. Blocking events in the GCM output were examined for the roles of such vigorous wave breaking in formation, maintenance, and dissipation of the phenomenon. Five episodes of blocking were identified subjectively by visual inspection of the $\tilde{q}$ time series. The criterion used to identify a blocking event is that the low-frequency jet is split and flows around either an anticyclone-cyclone dipole, such as that shown in Figure 5.1, or an $\Omega$-like structure with an anticyclone, for 10 days or longer without interruption. Since contours of constant $\tilde{q}$ may be considered as quasi-streamlines of the low-frequency flow, contour plots of $\tilde{q}$ were checked visually to identify such flow structures. Since there is no universally-accepted definition of blocking or general recognition of blocking as a special phenomenon, this rather arbitrary criterion serves well the purpose of this study. These five cases were analyzed by $q$ forcing diagnoses and Contour Advection with Surgery (CAS). The analyses demonstrate that the five episodes are dominated by low-frequency flow advection from the onset till decay. All the five episodes manifest themselves as quasi-stationary planetary wave amplification, breaking, and dissipation, which may be used as a general criterion for defining blocking.
5.2.1 Identified blocking episodes

Figures 5.3 through 5.7 show time series of $\bar{q}$ during the five blocking episodes. The first three are in the Northern Hemisphere and the other two in the Southern Hemisphere. The orientation of the maps in Figures 5.3 through 5.5 is the same as that in Figure 5.1. In Figures 5.6 and 5.7, it is the same as that in Figure 5.2. All the episodes are characterized by a diffluent structure, which is quasi-stationary or slowly-propagating during these periods. These five episodes will be referred to as E1, E2, E3, E4, and E5.

During all of these five episodes, there are events of inward and outward breakings in the diffluent region, following large-amplitude undulation of $q$ contours. The pattern of breaking is such that zonally-compressed and meridionally-stretched troughs and ridges break equatorward and poleward, respectively. Examples are shown in Figure 5.1 and described earlier. Mahlman (1979) and Blackmon et al. (1986) found similar breaking patterns associated with "explosive" cyclogenesis during the onset stage of blocking events simulated by GCMs. Also, Nakamura and Wallace (1990, 1993) found synoptic-scale eddy behavior that implies this breaking pattern in their composite analyses.

This pattern of wave breaking is an essential part of a theory of the mechanism of blocking that has been examined by models and diagnoses of observational data. The theory attributes blocking formation and maintenance to forcing by transient high-frequency eddies. Shutts (1983) demonstrated that transient eddies propagating into a diffluent basic flow are zonally compressed and meridionally stretched as they enter the diffluent region and subsequently split into two by the diffluent jet. In this process of wave breaking in the diffluent region, cascading eddies deposit vorticity in such a way that it tends to maintain the diffluent basic flow. In fact, the picture of this eddy
stretching and splitting mechanism illustrated by Shutts (1983) is observed in the data used here as well (Figure 5.8), when the $q$ field is decomposed into high-frequency (oscillation period between 2 and 6 days) and low-frequency (oscillation period longer than 10 days) parts. He further demonstrated that transient eddy forcing can, by itself, generate the diffluent mean flow in a nonlinear model. Haines and Marshall (1987) suggested a stationary dipole vortex structure embedded in a diffluent flow, "modon", as a prototype of blocking and demonstrated that eddy forcing generated upstream of the dipole tends to reinforce the existing diffluent structure. Analyzing observational data during strong blocking events, Illari and Marshall (1983), Illari (1984), and Shutts (1986) found eddy forcing pattern predicted from the results of these model studies, supporting the eddy straining mechanism of blocking.

However, the results described in Chapters 2 through 4 suggest that it may be the diffluent basic flow which creates such an eddy forcing pattern, by controlling evolution characteristics of synoptic-scale waves, although generation of such a basic flow may well require initial or continuous eddy forcing in reality. In fact, this possibility is not denied by the results of these model and diagnostic studies, as Shutts (1986) alluded to it.
Figure 5.3: Contour plot of $\bar{q}$ on a 320$^\circ$K surface in the Northern Hemisphere from Jan 16 through Feb 11. The contour interval is 1 PVU. The orientation and area of the maps are the same as those in Figure 5.1. A diffluent structure develops and remains in the right-lower quadrant of the maps. The diffuence weakens on Jan 28, but re-develops within a few days. The anti-cyclonic curvature remains fairly strong throughout this period. This episode will be called E1.
Figure 5.3 continued.
Figure 5.4: Same as Figure 5.3, but for a period from Mar 10 through Mar 27. A diffluent structure develops in the right-lower quadrant of the maps, then slowly propagates eastward. This episode will be called E2.
Figure 5.4 continued.
Figure 5.5: Same as Figure 5.3, but for a period from Dec 1 through Dec 16. A diffluent structure develops in the right-lower quadrant of the maps, then slowly propagates eastward. This episode will be called E3.
Figure 5.5 continued.
Figure 5.6: Contour plot of lowpass-Ertel potential vorticity on a 310°K surface in the Southern Hemisphere from May 5 through May 18. The contour interval is 1 PVU. The orientation and area of the maps are the same as those in Figure 5.2. A diffluent structure develops in the left-upper quadrant of the maps, then slowly propagates eastward. This episode will be called E4.
Figure 5.6 continued.
Figure 5.7: Same as Figure 5.6, but for a period from Jun 1 through Jun 13. A diffusive structure develops in the left-upper quadrant of the maps, then slowly propagates eastward. This episode will be called E5.
Figure 5.8: Contour plot of lowpass (thick contours) and bandpass (thin contours) Ertel potential vorticity on a $320^\circ$K surface in the Northern Hemisphere from Feb 1 through Feb 8. The contour intervals for the lowpass and bandpass fields are 1 PVU and 0.5 PVU, respectively. Stretching and splitting of synoptic-scale eddies in the low-frequency diffuent region are clearly seen. They are associated with breaking of synoptic-scale waves shown in Figure 5.1.
Figure 5.8 continued.
5.2.2 Potential vorticity forcing during the blocking events

In order to estimate contributions from high- and low-frequency eddies and the time mean flow to the forcing of blocking, the time-mean tendency of \( q \) associated with these flows were calculated. Figures 5.9 through 5.13 show the time-mean tendency of \( q \) due to eddies, \(-\bar{V} \cdot \nabla \bar{q}^*\), the time mean flow, \(-\bar{V} \cdot \nabla \bar{q}\), high-frequency eddies, \(-\bar{V}' \cdot \nabla \bar{q}'\), and low-frequency eddies, \(-\bar{V} \cdot \nabla \bar{q} + \bar{V} \cdot \nabla \bar{q}^*\), for all five cases. Here, \( V \) is the vector wind, the overbar and the asterisk denote the time mean and deviation from the mean, respectively. Tilde and prime indicate lowpass and bandpass, respectively. The high-frequency cut-off for lowpass is 10 days, as before, and cut-offs for bandpass are 2 and 6 days. The rather wide gap between the lowpass cut-off and the low-frequency cut-off of bandpass is meant for ensuring complete separation of low- and high-frequency components. Direct comparison of the actual \( \frac{\partial q}{\partial t} \), calculated by finite differencing in time, with \(-V \cdot \nabla q\) and \(-\nabla \cdot Vq\) shows that the former gives noticeably better approximation to \( \frac{\partial q}{\partial t} \). Therefore, the tendency calculation presented in this chapter is given by these forms, rather than flux convergence. It will be demonstrated in Section 5.2.3 that neglecting \(-\bar{q} \nabla \cdot \bar{V}\) most likely does not affect the tendency calculation presented in this section. Contours of \( \bar{q} \) are also shown by thick solid lines. The contour interval for \( \bar{q} \) is 1 PVU. Tendencies due to the total, high-frequency, and low-frequency eddies have been smoothed by taking 9-point average. The contour interval for \(-\bar{V} \cdot \nabla \bar{q}\), \(-\bar{V} \cdot \nabla q^*\), \(-\bar{V}' \cdot \nabla q'\), and \(-\bar{V} \cdot \nabla \bar{q} + \bar{V} \cdot \nabla \bar{q}^*\) is 0.2 PVU/day. Dashed lines are for negative (anticyclonic forcing in the Northern Hemisphere and cyclonic forcing in the Southern Hemisphere) values. Periods over which these quantities were calculated are not necessarily the same as those shown in Figures 5.3 through 5.7, in order to retain the characteristic diffusiveness in the mean flow and to evaluate the phase relationships between the forcings and the diffusiveness.

In the five events, there are some signs of anticyclonic eddy forcing in the blocking
Figure 5.9: Contour plot of the time mean $q$ forcing by the total eddies, the time mean flow, high-frequency eddies, and low-frequency eddies, superimposed on the time mean $q$, for E1. The quantities were calculated over a period from Jan 16 through Feb 13. The contour interval for the forcings (thin contours) is 0.2 PVU/day. Contours of $\bar{q}$ (thick contours) are also shown with an interval of 1 PVU. Negative values are dashed.
Figure 5.10: Same as Figure 5.9, but for E2. The quantities were calculated over a period from Mar 16 through Mar 28.
Figure 5.11: Same as Figure 5.9, but for E3. The quantities were calculated over a period from Dec 1 through Dec 16.
Figure 5.12: Same as Figure 5.9, but for E4. The quantities were calculated over a period from May 6 through May 15.
Figure 5.13: Same as Figure 5.9, but for E5. The quantities were calculated over a period from Jun 4 through Jun 13.
ridges, although there are signs of cyclonic eddy forcing as well. In E1, there is a fairly large area of anticyclonic forcing by the eddies slightly upstream of the peak of the blocking ridge. In E2, there is only a very small area of anticyclonic eddy forcing near the peak of the ridge near 75°N. In E3, it appears to be cyclonic eddy forcing, which occupies most of the ridge, but there is some anticyclonic eddy forcing upstream of the peak of the ridge. In E4, there is an elongated area of weak anticyclonic forcing upstream of the ridge, while there is a large area of cyclonic eddy forcing near the peak of the ridge. There is also a region of weak cyclonic forcing in the northern half of the diffuence. In E5, anticyclonic eddy forcing occupies a large area from the upstream to the downstream end of the blocking structure. The presence of the anticyclonic $\bar{q}$ forcing due to the total eddies in the vicinity of the time-mean blocking ridge in E1, E4, and E5 is in agreement with diagnostic studies of observational data and GCM output (Illari and Marshall, 1983; Illari, 1984; Mullen, 1986), although there are some differences among these studies in the phase relationship between the eddy forcing and the blocking ridge. The eddy forcing pattern observed in three of the five cases, thus, has some tendency to reinforce the mean flow.

When only high-frequency eddy contribution is kept by bandpass-filtering the data, the magnitude of $\bar{q}$ forcing decreases considerably in all of the Northern Hemispheric cases, E1, E2, and E3. It implies that a major portion of the net eddy forcing during these periods in the Northern Hemisphere comes from low-frequency eddies (low-frequency flow minus the time-mean flow). The reduction is not as dramatic in the Southern Hemispheric cases as is in the Northern Hemispheric ones, perhaps partly due to the shortness of the averaging periods, 10 days. High-frequency forcing is still anticyclonic in the vicinity of the time-mean blocking ridge in E1, E4, and E5, as found by Shutts (1986), Mullen (1987), and Holopainen and Fortelius (1987), but is not necessarily more organized with respect to the blocking signature than the total
eddy forcing, perhaps except for that in E4. High-frequency forcing in E4 shows a clear dipole structure in the diffluent region of the block, anticyclonic and cyclonic forcing along the poleward and equatorward branches, respectively, of the jet. This pattern remarkably resembles those shown in idealized model studies, such as those by Shutts (1983) and Haines and Marshall (1987).

The time-mean $q$ forcing due to low-frequency eddies is somewhat greater in magnitude than that due to high-frequency eddies in the Northern Hemispheric cases. In the Southern Hemispheric cases, this is not true. In E1, most of the anticyclonic forcing near the peak of the blocking ridge comes from low-frequency eddies. In E2, despite the absence of the total eddy anticyclonic forcing, the low-frequency eddy forcing shows areas of anticyclonic forcing at the diffuence and near the peak of the ridge. Also in E3, the low-frequency eddy forcing shows small areas of weak anticyclonic forcing at the diffuence and inside the ridge, where the total eddy forcing is cyclonic. Thus, it appears to be low-frequency eddies which seem to be more in favor of forcing the blocking ridge than high-frequency eddies in the Northern Hemispheric cases. In the Southern Hemispheric cases, the opposite seems to hold. Low-frequency eddies show strong cyclonic forcing in the vicinity of the block in E4. They appear almost as strong as are high-frequency eddies in forcing a part of the blocking ridge in E5, but are confined to relatively small area of the ridge.

The time-mean tendency due to the time-mean flow advection is of comparable magnitude as that due to the total eddy. They also, in general, tend to cancel each other locally, reducing the total forcing of $\bar{q}$. In terms of its relationship with blocking ridges, the forcing due to the time-mean flow is found to be contributing positively as much as or even more than the total eddies in E1, E4, and E5. In these cases, the time-mean flow advection shows strong anticyclonic forcing in large areas in the vicin-
ity of the blocking ridges. When the $\bar{q}$ tendencies due to eddies and the mean flow in E2 and E3 are compared, it is the time-mean flow forcing which shows somewhat more favorable pattern with respect to the blocking ridges. Although not conclusive, these plots suggest that it may be the time mean flow or low-frequency flow which is relatively more important in forcing of blocking. In fact, Nakamura (1990) found that low-frequency advection of vorticity, in general, dominates evolution of composite blockings in the Northern Hemisphere. The sense of $\bar{q}$ forcing by the mean flow is different from that found by Mullen (1987), who studied absolute vorticity budget on a 300mb surface in an NCAR GCM. He found cyclonic forcing by the mean flow in a large area just west of the blocking ridge line and anticyclonic forcing farther west. The cause of this difference is not clear, but is presumably attributed to the difference between isentropic $q$ and isobaric absolute vorticity. The coarseness of the horizontal (7.5 degrees by 4.5 degrees in longitude and latitude, respectively) and vertical (9 $\sigma$ levels) resolutions of the GCM used in his study may also be responsible for the difference.

To examine further the phase relationships between the blocking signature in $\bar{q}$ and its forcings, $\frac{\partial \bar{q}}{\partial t}$ due to high-frequency flow, $-V' \cdot \nabla q'$, and low-frequency flow, $-\bar{V} \cdot \nabla \bar{q}$, were calculated at each time frame during a part of E1, E2, E4, and E5. Since calculation of these quantities requires filtering of filtered quantities, the first half of E1 and the entire period of E3 were excluded due to lack of data. Figures 5.14 through 5.21 show $-V' \cdot \nabla q'$ and $-\bar{V} \cdot \nabla \bar{q}$, superimposed on $\bar{q}$, during the latter half of E1, E2, E4, and E5, every other day. The contour interval for $-V' \cdot \nabla q'$ and $-\bar{V} \cdot \nabla \bar{q}$ is 0.2 PVU/day. Since the quantities at a particular time, rather than time averages, are shown, the phase relationships between the forcing and $\bar{q}$ are relatively clearer.
than those in Figures 5.9 through 5.13.

Considering that blocking is a transient phenomenon, plotting \(-V' \cdot \nabla q'\) and \(-\bar{V} \cdot \nabla \bar{q}\) with \(\bar{q}\) is probably a better way of illustrating relationships between forcing and response than plotting the time mean fields. With information drawn from the time mean statistics, such as those in Figures 5.9 through 5.13, one cannot ascertain the disposition of various forcings with respect to the blocking flow, since their effects are spread over nearly the entire life cycle of a blocking. For example, even if the high-frequency eddy forcing is important in initiation of a blocking, it will not appear significant in the time mean picture if it is unimportant throughout the rest of the blocking event. On the other hand, even if the magnitude of the high-frequency forcing is nearly constant throughout a particular blocking event and it is important in the onset stage, it is not necessarily important after the onset if it is dwarfed by other forcings. In order to examine the roles of various forcings in blocking events, all of which are unsteady, it is crucial to study the forcings and response as functions of time, as well as space.

In general, a typical magnitude of \(-\bar{V} \cdot \nabla \bar{q}\) is somewhat greater than that of \(-V' \cdot \nabla q'\) in all of the five events. There is also a tendency for the two to oppose each other. The reinforcing phase relationship between the high-frequency forcing and \(\bar{q}\) is observed in E1, E4, and E5, when \(\bar{q}\) shows strong diffuence. For example, \(-V' \cdot \nabla q'\) is negative (anticyclonic) in much of the northern half of the diffuence and near the peak of the blocking ridge on Feb 1 (Figure 5.14), which is a pattern shown in diagnostic studies (Illari, 1984; Shutts, 1986) and model studies (Shutts, 1983; Haines and Marshall, 1987) and supported the idea of eddy forcing mechanism. Similar patterns with some mixture of cyclonic forcing are seen during E4 and E5. These signatures of positive feedback of high-frequency flow are in agreement with the results of energetics studies.
(Hansen and Chen, 1982; Hansen and Sutera 1984) as well. The areas of anticyclonic forcing are located slightly upstream of the peak of the ridge, as found by Illari (1984) and Shutts (1986), as well as near the peak of the ridges in E1 and E5. This phase relationship between \( \dot{q} \) and high-frequency forcing is not observed in E2. The high-frequency forcing in E2 is weakly anticyclonic in the blocking region when the diffluence is developing (e.g., Mar 11 through Mar 15 in Figure 5.16), but is more or less cyclonic once the block reaches its peak.

Finally, low-frequency forcing is consistently anticyclonic in the blocking region in most of the periods examined. It tends to have anticyclonic and cyclonic forcings upstream and downstream of the peak of the blocking ridge, respectively. Even in E2, whose low-frequency forcing patterns in the vicinity of the block appear less anticyclonic than the others, there are visible signs of anticyclonic and cyclonic forcings upstream and downstream, respectively, of the peak of the low-frequency ridge. The low-frequency anticyclonic forcing slightly precedes \( \dot{\alpha} \) occurs almost simultaneously with growth of low-frequency ridge (e.g., May 5 through May 15 in Figure 5.19). Thus, low-frequency forcing appears to dominate evolutions of these blocking events throughout their life cycles, in agreement with Nakamura's (1990) study of composite blockings in the Northern Hemisphere.
Figure 5.14: Contour plot of $\tilde{q}$ forcing by high-frequency flow (thin contours), superimposed on $\bar{q}$ (thick contours) during the latter half of E1. The contour interval for lowpass high-frequency forcing is 0.2 PVU/day, while that for $\bar{q}$ is 1 PVU.
Figure 5.14 continued.
Figure 5.15: Contour plot of $\bar{q}$ forcing by low-frequency flow (thin contours), superimposed on $\tilde{q}$ (thick contours) during the latter half of E1. The contour interval for lowpass low-frequency forcing and $\bar{q}$ are 0.2 PVU/day and 1 PVU, respectively.
Figure 5.15 continued.
Figure 5.16: Same as Figure 5.14, but for E2.
Figure 5.17: Same as Figure 5.15, but for E2.
Figure 5.17 continued.
Figure 5.17 continued.
Figure 5.18: Same as Figure 5.14, but for E4.
Figure 5.18 continued.
Figure 5.19: Same as Figure 5.15, but for E4.
Figure 5.20: Same as Figure 5.14, but for E5.
Figure 5.21: Same as Figure 5.15, but for E5.
Figure 5.21 continued.
5.2.3 Application of contour advection with surgery to the blocking episodes

To confirm the results of \( q \) forcing analyses described above, contour advection with surgery (CAS) developed by Waugh and Plumb (1994) was applied to the five blocking episodes, using only lowpass-filtered wind. CAS advects contours of constant value of \( q \) by given wind data to predict future locations of the contours. When the contours develop features smaller than a certain prescribed scale or features with curvature greater than a certain prescribed value, CAS smoothes out the contours by removing them and re-connecting the contours in an appropriate manner. It has shown to work quite well when adiabatic-inviscid assumption reasonably holds (Waugh, 1993b; Waugh and Plumb, 1994; Waugh et al., 1994). Since the wind data are available 5 times per day in this case, CAS is presumed to work quite well so long as it is applied for sufficiently short periods such that the adiabatic-inviscid assumption is reasonable. If these blocking events are indeed dominated by low-frequency flow advection, as indicated by the aforementioned diagnoses, and if the adiabatic-inviscid assumption is appropriate in these flows, then CAS with only low-frequency portion of the wind should be able to reproduce \( \tilde{q} \) fields during the episodes.

The previously reported results of CAS simulation (Waugh, 1993b; Waugh and Plumb, 1994; Waugh et al., 1994) are all produced by stratospheric wind, which is more slowly-varying both in time and space than tropospheric wind. It is also expected to be closer to an adiabatic-inviscid flow than tropospheric flows, since it is practically isolated from the heat sources and strong turbulence in the troposphere. These factors probably contributed in good agreement between the CAS-produced and the observed \( q \) distributions in those stratospheric cases, despite the rather low temporal (twice-daily) and spatial (4° X 5° or 2° X 5° in latitude and longitude, respectively) resolutions of wind data used. The flows considered here are significantly more rapidly-
varying both in time and space and are also not as close to an adiabatic-inviscid
flow as are stratospheric ones. However, application of CAS to the aforementioned
GCM-produced data shows that it performs reasonably well in the upper troposphere,
perhaps due to the high temporal resolution of the wind data used here. An example
of CAS simulation of $q$ field using the total wind is shown in Figure 5.22. It shows the
first 9 days of a CAS run initialized by contours of $q = 2$ PVU on Jan 1. This case was
run for only the Northern Hemispere. In these runs with the total wind, 200 time
steps were taken on one day. The corresponding GCM-produced contours are shown
in Figure 5.23. Broad agreement in the contour positions between the CAS-produced
and GCM-produced is remarkable. Some small-scale features, such as a narrow ridge
over the western part of the North America, are also reproduced by CAS quite well.
In general, CAS-produced $q$ contours compare very well with the actual ones up to
10 days or so. In some cases, however, CAS-simulated $q$ contours compare quite well
with the corresponding GCM-produced $q$ contours beyond 15 days of integration.

Some of the results of CAS runs with only lowpass wind and the actual $\tilde{q}$ during
the five blocking episodes are shown in Figures 5.24 through 5.33. For easy visual
inspection of the results, only one or two contours were used in each run. At least
two runs with different contours were done for each event to confirm consistency of
the results. Since temporal variation in lowpass wind is considerably smaller than
the total wind, by definition, only 100 time steps were taken per day for these runs.
Although there are some differences in detail, the CAS-produced and actual $\tilde{q}$ contour
positions are, broadly, in good agreement with each other in all of the five episodes.
The latter half of CAS result in E1 does not resemble the actual $\tilde{q}$ field as much as does
its first half, probably because the adiabatic-inviscid assumption does not hold over
a long period. When CAS is ran from Feb 1, rather than Jan 16, the CAS-produced
contours resemble the actual ones well (not shown). These results confirm that of $\tilde{q}$

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Figure 5.22: Output of CAS with the total wind. The initial contours were taken from $\bar{q} = 2$ PVU on Jan 1. Plotting interval is 1 day, beginning on Jan 1 and ending on Jan 9. The lowest latitude shown is 18°N.
Figure 5.23: Contours of $\bar{q} = 2$ PVU from Jan 1 through Jan 9. The lowest latitude shown is 18°N.
forcing analyses of the previous section; the five blocking episodes are dominated by low-frequency flow advection throughout their life cycles.

CAS was run for each period, using the total wind instead of lowpass wind, to check how different the CAS-generated and actual $\tilde{q}$ contours may be with the total wind advection. In general, the CAS-produced $\tilde{q}$ fields with the total wind advection do not resemble the GCM-produced ones. An example from a run with the total wind during E3 is shown in Figure 5.34. Mixing of the material inside and outside the $\tilde{q} = 2$ PVU contour is considerably more vigorous than that in a run with only lowpass wind (Figure 5.28). The CAS-produced contours have many more small-scale features than the actual $\tilde{q}$ contour (Figure 5.29) and do not resemble the actual ones even on the synoptic scale. Actually, the CAS-produced contours resemble that of the unfiltered $q$ after the first several days (not shown). To summarize, CAS with the total wind advection reproduces evolution of the $q$ field quite well, but not that of the $\tilde{q}$ field. On the other hand, CAS with only the low-frequency portion of the wind reproduces evolution of the $\tilde{q}$ field quite well, but not that of the $q$ field. In other words, advection by the rotational part of the flow dominates evolution of $q$ and $\tilde{q}$ fields.
Figure 5.24: Output of CAS with only lowpass wind for El. The initial contours were taken from $\tilde{q} = 2$ and 3 PVU on Jan 16. Plotting interval is 1 day, beginning on Jan 16 and ending on Feb 11. The lowest latitude shown is 18°N.
Contour Advection with Surgery
Infile=qlp116n

Figure 5.24 continued.
Figure 5.25: Contours of $\tilde{q} = 2$ and 3 PVU from Jan 16 through Feb 11. The lowest latitude shown is 18°N.
Figure 5.26: Output of CAS with only lowpass wind for E2. The initial contours were taken from $\bar{q} = 3$ and 4 PVU on Mar 7. Plotting interval is 1 day, beginning on Mar 7 and ending on Apr 2.
Contour Advection with Surgery
Infile=qlp307

Figure 5.26 continued.
Contour Advection with Surgery
Infile=qlp307

Figure 5.26 continued.
Figure 5.27: Contours of $\bar{q} = 3$ and 4 PVU from Mar 7 through Apr 2.
Figure 5.28: Output of CAS with only lowpass wind for E3. The initial contours were taken from $\bar{q} = 2$ PVU on Dec 1. Plotting interval is 1 day, beginning on Dec 1 and ending on Dec 16.
Contour Advection with Surgery
Infile=qlp2101

Figure 5.28 continued.
Figure 5.29: Contours of $\bar{q} = 2$ PVU from Dec 1 through Dec 16.
Figure 5.29 continued.
Figure 5.30: Output of CAS with only lowpass wind for E4. The initial contours were taken from $\bar{q} = -2$ and $-3$ PVU on May 1. Plotting interval is 1 day, beginning on May 1 and ending on May 18. The highest latitude shown is $18^\circ$S.
Contour Advection with Surgery
Infile=pvlp501

Figure 5.30 continued.

173
Figure 5.31: Contours of $\bar{\eta} = -2$ and $-3$ PVU from May 1 through May 18. The highest latitude shown is $18^\circ S$. 

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Figure 5.31 continued.
Figure 5.32: Output of CAS with only lowpass wind for E5. The initial contours were taken from $\bar{q} = -2$ and $-3$ PVU on May 28. Plotting interval is 1 day, beginning on May 28 and ending on Jun 14.
Contour Advection with Surgery
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Figure 5.32 continued.
Figure 5.33: Contours of $\bar{q} = -2$ and $-3$ PVU from May 28 through Jun 14.
Figure 5.33 continued.
Figure 5.34: Output of CAS with the total wind for E3. The initial contours were taken from $\bar{q} = 2$ PVU on Dec 1. Plotting interval is 1 day, beginning on Dec 1 and ending on Dec 16.
Contour Advection with Surgery
Infile=qlp1201

Figure 5.34 continued.
Examining Figures 5.24, 5.26, 5.28, 5.30, and 5.32, one can see that low-frequency flow advection indeed transports materials in such a way that blocking signatures in $\bar{q}$ appear to form, to be maintained, and to dissipate, over a characteristic period of low-frequency flow. In most cases, "formation and maintenance" of blocking dipoles were associated with low-$q$ material being stretched poleward with some circular motion and high-$q$ being stretched toward the opposite direction, forming a "cat's eye" (e.g., from Day 4 to Day 11 of Figure 5.24), which is a characteristic feature of critical layer Rossby waves (e.g., Haynes, 1989). Even when a cat's eye does not appear in the figures shown here but only the diffuence (e.g., Day 15 through Day 19 of Figure 5.26), it does appear when more contours are used to show the flow inside the diffuence. The cat's eye formation and dissipation occur more or less at the same location, with a slight tendency to translate eastward, especially in the Southern Hemispheric cases (E4 and E5). Of course, the resemblance of these features to the cat's eye produced by a critical layer Rossby wave does not imply that the breaking waves are in their critical layers, since these long waves have very large westward phase speed that cannot be matched by even a locally-westward low-frequency flow (Lindzen et al. 1984; Lindzen, 1986). Thus, these blocking episodes may be regarded simply as a manifestation of forced quasi-stationary Rossby wave amplification, breaking, and dissipation, in which the time scale is that of the quasi-stationary wave itself.

During E1, there appears to be two episodes of quasi-stationary wave growth and breaking. The first one begins on around Day 3 and ends on around Day 17 shown in Figure 5.24. The second one begins on around Day 17 and ends on around Day 25 shown in Figure 5.24. The two consecutive cycles of quasi-stationary Rossby wave amplification, breaking, and dissipation appear as a single blocking event with longer duration than the others and with a slight weakening during its life time. Thus, the intrinsic time scale associated with this longer blocking event is the same as that in
others. This possibility was suggested by Lindzen (1986).

5.3 Discussion

The results described above may appear contradictory to those of other observational and model studies of blocking phenomenon that put a focus on eddy or high-frequency forcing, but they are not. Indeed, the eddy or high-frequency forcing have been found to have tendency, to some extent, of reinforcing the blocking flow in a fashion described by other investigators (Figures 5.9 through 5.14, 5.16, 5.18, and 5.20). However, the results presented above unambiguously identify the eddy or high-frequency forcing as primarily a passive player in blocking flow forcing, as far as its direct contribution is concerned. This does disagree with the results presented by Iliari (1984). Iliari calculated the time-mean forcing of quasi-geostrophic potential vorticity, $q_g$, due to the mean flow and eddies during a blocking event in July of 1976 over Europe, using twice-daily NMC analyses. Her results show that the $q_g$ forcings due to the mean flow and eddies have the same order of magnitude and are off-setting with each other in the blocking ridge. In the five events studied here, the two have the same order of magnitude, but the forcing due to the time-mean flows appears to be more anticyclonic than that due to eddies in the regions of blocking ridge.

This difference between Iliari's result and those presented here may be explained by a few factors. First of all, the case she examined is in the middle of a summer, while all the cases studied here occurred during a winter or relatively cold parts of spring and fall. Since the mean flow is considerably weaker during the summer than winter (so are eddies, however), particularly in the Northern Hemisphere, it is possible that the mean flow forcing of $q_g$ during July of 1976 was indeed quite different from those shown here. The second factor that may explain the difference is the different treatment of the static stability in calculation of $q$ and $q_g$; it is allowed to vary in the vertical
and horizontal in the calculation of the former, while it is a function of only pressure in the calculation of the latter. Since the static stability has rather large horizontal variations near the edge of the jet, the mean flow forcing of \( q \) may show noticeable differences from that of \( q_g \). Of course, the differences between \( q \) on isentropic surfaces and \( q_g \) on isobaric surfaces (e.g., the latter is conserved in adiabatic-inviscid flow only within the quasi-geostrophic framework and, thus, ageostrophic effects are not resolved properly) may well be the source of the differences between her results and those presented here. Relatively low sampling frequency, twice per day, of the data used by Illari might affect the accuracy of eddy forcing, but is very unlikely to change the sign of the forcing due to the time-mean flow.

In the five cases examined here, there is no evidence of high-frequency eddies being important in the onset stage or other stages of the blocking events. As far as the direct contribution to evolution of \( \bar{q} \) field is concerned, high-frequency flow is negligible in all the five cases. Evolutions of \( \bar{q} \) is dominated by low-frequency flow advection. This fact does not contradict the fact that synoptic-scale waves develop large amplitudes during the onset or mature stage of the blocks. As shown in Chapter 3, traveling small-scale waves increase their amplitudes in a region of weak background zonal flow. If there is a diffluence in the background flow, with some meridional component, the traveling waves are zonally-compressed and meridionally-elongated by the background flow, resulting in an increase in their amplitudes. (Such a background diffluence may well be forced partly by high-frequency eddies in the real atmosphere.) If there is high baroclinicity or large temperature contrast near the beginning of the diffluence, as there tends to be in the real atmosphere, eddies that enter the region of diffluence receive both the baroclinic and barotropic effects to amplify. Thus, simultaneous occurrence of developing low-frequency diffluence and “explosive” cyclogenesis may well be triggered by the former, rather than the latter generating the former. If the
background flow is purely zonal and strong, a growing baroclinic wave is most likely to be advected zonally, rather than creating a diffuence in the background flow by itself.

Nevertheless, there may be cases in which this does not apply in reality, as will be shown in Chapter 6. Nakamura (1994) pointed out that quasi-stationary Rossby wave activity flux, defined by Plumb (1986), appears to accumulate in a region of weak low-frequency zonal flow in his composite blocking analysis, resulting in amplification of quasi-stationary wave there. When the intensity of the obstruction of the zonal flow, associated with a growing baroclinic wave, is sufficiently strong to overcome the advection by the background flow, it may initiate a blocking by weakening the local background zonal flow. Since somewhat “smoothed” SST (i.e., somewhat “smoothed” baroclinicity over the North Atlantic and the North Pacific storm tracks) is prescribed in most GCM experiments, such as that examined in this chapter, extremely strong cyclogenesis may not occur in GCM runs. Also, the observed eastward translation of three of the blocking patterns discussed here, may be due to lack of horizontal resolution to effectively simulate the positive feedback of synoptic-scale eddies. Tendency of observed blocking patterns to slowly translate westward has been noted by many authors and has been attributed to the positive feedback of breaking eddies in the diffuence. This tendency is not clearly observed in the events examined here.

The present results do not support “global” theories of blocking either. Examples of global theories are those by Charney and DeVore (1979) and Tung and Lindzen (1979). The former attributes the blocking pattern to one of multiple equilibria of the atmosphere to a given set of topographic and thermal forcings, while the latter attempts to explain the blocking as a large-amplitude stationary wave, resonantly excited by topographic and land-sea differential heating. Lindzen (1986) argued that
these "global" theories depend upon planetary-scale free waves being resonantly excited by stationary forcings, which are unattainable in the real atmosphere due to either very large westward propagation speed of these waves (Lindzen et al., 1984) or practically impossible multiple reflection of these waves in the real atmosphere. Although the GFDL GCM has a lid at the top, reflection of planetary-scale free waves at the top is, at least, damped by the "sponge" in the top layer. Also, the zonal-mean jet on the chosen isentropes typically has eastward flow of 30 to 40 m/s and, hence, is not likely to permit the external mode of planetary-scale free waves to be stationary. These factors make the global theories unlikely candidates to explain the blocking events studied here.

Some theories of blocking based on time-dependent nonlinear Rossby waves (Malguzzi and Malanotte-Rizzoli, 1985; Malanotte-Rizzoli and Malguzzi, 1987) may have some relevance to the evolutions of low-frequency flow during the five events. These theories have their own problems, as discussed by the authors, yet the time-dependent evolutions of the flow do resemble those of the low-frequency flow during the five events. If one interprets the severe truncation used by Malguzzi and Malanotte-Rizzoli (1985) as an effective removal of high-frequency eddies from the total flow, the evolution of the flow, which is internally driven, may be thought of as self-advectiong low-frequency flow evolution observed in the five blocking episodes. A drawback of this analogy is the lack of continuous low-frequency forcing in the proposed model. Low-frequency flow in the real atmosphere and that in the GCM evolve with continuous forcing from topography, diabatic heating, eddy q and heat transports, and land-sea temperature contrast.

The blocking-reinforcing tendency of high-frequency eddies found in this study support the localized nonlinear forcing due to transient eddies of idealized blocking
dipoles represented by an equivalent barotropic modon (McWilliams, 1980; Haines and Marshall, 1987; Butchart et al., 1989) or a nonlinear stationary Rossby wave (Malkoff and Malanotte-Rizzoli, 1984). As originally demonstrated by Shutts (1983), transient eddies propagating into a region of strong diffuseness become meridionally-elongated and zonally-compressed. As they travel around the region of blocking dipole, the eddies are broken and deposit vorticity in the dipole in a reinforcing manner. This nonlinear relationship between high-frequency eddies and a blocking dipole is observed in the analyses presented here. These theories do not, however, explain the mechanism through which quasi-stationary waves grow, break, and dissipate, because of the very feature that makes them analytically clear. CAS analyses show that it is the low-frequency flow advection, rather than high-frequency eddy transport, which is responsible for the direct forcing of $\tilde{q}$ during these events. It implies that a comprehensive theory of blocking, if it exists at all, must account for the forcings of quasi-stationary waves and their transience, as well as the positive feedback of high-frequency eddies.

The results presented above do not, by themselves, confirm any existing theory for blocking flows. As hinted by results reported by Illari (1984) for a summer blocking, the five cases examined here may well represent only one of several mechanisms that blocking may take. The most straightforward way to interpret the evolution of the five blocks is probably as a response of quasi-stationary waves to greater-than-average forcing of low-frequency flow. This interpretation, essentially that suggested by Lindzen (1986), easily explains the observed less frequent occurrence of blocking in the Southern Hemisphere, where large portion of the energy of the disturbances is contained in wavenumbers 1 and 2 (Tung and Lindzen, 1979), than in the Northern Hemisphere. Quasi-stationary or low-frequency flow forcing due to topography, land-sea temperature contrast, diabatic heating, and eddy $q$ and heat transports, may
tend to force particular phases of quasi-stationary waves, resulting in local amplification there. If the amplitude exceeds a certain threshold, the wave may break and dissipate, as observed in preceding chapters. As Plumb (1986) showed, both high- and low-frequency eddy activity fluxes are pointing upward from the lower to the upper troposphere in the regions of storm tracks. If these fluxes converge in the upper troposphere, as may be the case when the upper flow is weak, it may result in local amplification of the quasi-stationary wave there. The wave activity fluxes, in general, are pointing eastward in the upper troposphere between the North Pacific storm track and the North Atlantic storm track (Plumb, 1986). Thus, assuming that the GCM simulates such characteristics reasonably well, it seems possible also for the fluxes that originate in the North Pacific storm track to converge over, say, Europe, if the zonal flow there is weak, and cause local amplification of the quasi-stationary wave over Europe.

Although the direct contribution of high-frequency eddies has been shown to be negligible, their indirect contribution through low-frequency flow forcing is not likely to be small, as implied by somewhat reasonable climatological blocking simulation by climatological high-frequency eddy forcing (Metz, 1986). Effects of heat and vorticity transport in the troposphere by the high-frequency eddies on the low-frequency flow cannot be studied by examination of \( \tilde{q} \) forcing on a single isentropic surface. This is a major drawback of the approach taken in this study; one cannot identify the source of forcing at the fundamental level.
Chapter 6

Quasi-stationary Rossby wave breaking during observed “atmospheric blocking” episodes

Abstract

In order to check whether the main result of the previous chapter, domination of “blocking” mechanism by low-frequency flow advection, is also true in the real atmosphere, two episodes of blocking that have been studied by other investigators and a 15-event composite of European blocking are examined. Application of CAS to these three data sets confirms the main result of the previous chapter, except for the onset stage of one episode. CAS runs show that the direct contribution of high-frequency flow is important during the first several days of a Pacific blocking episode. These results raise a possibility that there are more than one mechanism of blocking formation. Despite this discrepancy between the observed and CAS-produced \( \tilde{q} \) field during the first several days of one episode, the results of CAS show that the strongly diffuent low-frequency flows are primarily a result of low-frequency flow advection, which manifests itself as quasi-stationary Rossby wave amplification, breaking, and dissipation.
6.1 CAS application to two observed cases of blocking

In order to check whether low-frequency flow advection dominates the evolution of low-frequency flow itself during observed events of "atmospheric blocking", CAS was applied to data produced by NMC global analyses. In order to make a comparison with results reported by others, using different diagnostic methods, two events that have been studied by Shutts (1986) and Ek and Swaters (1994) have been chosen.

6.1.1 Data

A part of NMC twice-daily global analyses, temperature and wind at 12 pressure levels in the Northern Hemisphere, were obtained from NCAR archives for two periods: Jan 1 through Mar 31 of 1983 and 1989. The data are given on regular grids with 2.5 degrees by 2.5 degrees in longitude and latitude on 12 isobaric surfaces in the vertical: 1000mb, 850mb, 700mb, 500mb, 400mb, 300mb, 250mb, 200mb, 150mb, 100mb, 70mb, and 50mb. Wind and Ertel's potential vorticity, $q$, on the 330°K surface were calculated from the data, using essentially the same procedure as that used for the GCM output in the previous chapter. The data were missing at several time frames. Linear interpolation was applied to fill in these gaps. Then, a 61-point lowpass filter (i.e., 30.5-day data are used to calculate a filtered field at one time frame) with a 10-day cut-off was applied to wind and $q$ to isolate the low-frequency components. These filtered data were used for identification of blocking pattern and CAS application. For the time mean statistics, unfiltered data were used.

6.1.2 1983 event

Shutts (1986) examined a blocking episode over Europe in February 1983 and concluded that $q$ flux due to synoptic-scale eddies is sufficiently large and has the correct
phase relationship with the blocking anticyclone to cause the observed blocking ridge. Figure 6.1 shows \( \tilde{q} \) from Feb 5 to Feb 22 of 1983, which is the period of blocking identified and examined by Shufts. There is a sign of planetary-scale diffluence over the North Atlantic on Feb 5 (right-lower quadrant of the map). The diffluence becomes stronger with time with its ridge penetrating into high latitudes. It first forms an \( \Omega \)-shaped block, then forms a dipole. By Feb 22, the strong diffluence has dissipated, leaving behind a rather chaotic and unorganized \( \tilde{q} \) distribution with a slight sign of diffluence.
Figure 6.1: Contour plot of $\bar{q}$ from Feb 5 through Feb 22 of 1983. Contour interval is 1 PVU. The orientation of the maps is the same as that in Figure 6.2.
Figure 6.1 continued.
Figure 6.2 shows $-\nabla \cdot \nabla \overline{q^*}$, $-\nabla \cdot \overline{Vq}$, and the sum of the two, superimposed on $\overline{q}$ for a period from Feb 5 to Feb 22. The flux divergence form is used in this chapter, since these data do not appear to suffer from unrealistically large divergent wind on isentropic surfaces. One must keep in mind, in interpreting the $q$ forcing statistics given here, a possibility that twice-daily data sampling frequency may be inadequate for obtaining robust results. From Figure 6.2, it is evident that the pattern Shutts found, fairly large anticyclonic forcing (minimum value being about $-2$ PVU/day) due to eddies slightly upstream of the peak of the blocking ridge over Europe, is obtained from the data used here. Since the data Shutts used were sampled 4 times a day, while those used here are given only twice a day, there are some differences between the eddy $\overline{q}$ forcing pattern obtained by him and that shown in Figure 6.2. The anticyclonic forcing due to eddies in the blocking ridge tends to reinforce the mean flow to some extent. However, $\overline{q}$ forcing due to the mean flow is considerably greater in magnitude and has more organized structures with respect to $\overline{q}$ field. In fact, the sum of the two flux convergences appears to be much the same as the flux convergence due to only the mean flow, except for some increase in the anticyclonic forcing in the middle of the blocking ridge and a reversal of forcing sign from cyclonic to anticyclonic in a very small area slightly west of the peak of the ridge. Overall, synoptic-scale $-\nabla \cdot \overline{Vq}$ field is very similar to $-\nabla \cdot \overline{Vq}$, indicating domination of $\overline{q}$ forcing by the mean or low-frequency flow, as shown in the previous chapter using GCM data.

To check whether low-frequency flow advection is the dominant direct mechanism of generating the blocking flow, CAS was run with initial $q$ contours taken from those of 2 and 3 PVU of $\overline{q}$ on Feb 1, using only lowpass wind. Contours of the actual $\overline{q}$ with values 2 and 3 PVU from Feb 1 through Feb 18 are shown in Figure 6.3. There is no strongly diffusent structure on Feb 1. On Feb 2, a tongue of contour of 2 PVU begins to move equatorward, suggesting development of some diffusence there.
Figure 6.2: Contour plot of $\bar{q}$ forcing by eddies, the time mean flow, and the sum of the two, superimposed on $\bar{q}$ over a period from Feb 5 to Feb 22 of 1983. Contour intervals for $\bar{q}$ and the forcings are 1 PVU and 0.2 PVU/day, respectively.
By Feb 5, which is Day 1 of this blocking episode according to the Shutts' choice, the contours are clearly indicating strongly diffluent flow beginning over the western North Atlantic. The ridge continues to grow, intruding as north as 70°N, until it begins to narrow and eventually forms cut-offs as it decays.

Figure 6.4 shows the result of a CAS run with only lowpass wind, beginning on Feb 1 with contours of $\tilde{q} = 2$ and 3 PVU. The plots are given at time frames corresponding to those in Figure 6.3. Although there are some differences in detail, broad agreement of $\tilde{q}$ fields in Figures 6.3 and 6.4 is very good. From the onset stage till the decay stage, CAS with low-frequency flow reproduces the overall structure of $\tilde{q}$ very well. The pattern of $q$ advection shows formation of a cat's eye, after amplification of the quasi-stationary wave. When CAS was run with the total wind, the produced contour positions do not resemble the actual ones at all. Thus, again, this blocking episode is dominated by, as far as the direct cause is concerned, low-frequency flow advection, which manifests itself as quasi-stationary Rossby wave amplification, breaking, and dissipation. Another CAS run with different $\tilde{q}$ contours shows a similar result (Figures 6.5 and 6.6).

Not only in the results of these CAS runs, but also in the actual $\tilde{q}$ time series (Figure 6.1), there are signs of quasi-stationary wave amplification, breaking, and dissipation. As the low-$\tilde{q}$ blob at the blocking ridge intrudes into the high latitudes, deep troughs associated with high-$\tilde{q}$ on both upstream- and downstream-side of the ridge become necessarily more pronounced. In this event, the trough on the downstream-side of the ridge develops anticyclonic co-rotation with the ridge as it grows and becomes cut-off from the main vortex at about the same time as the low-$\tilde{q}$ ridge forms a cut-off. After this breaking, they are broken into small-scale features, which eventually disappear in the region of diffluence as the block dissipates.
Figure 6.3: Contours of $\bar{q} = 2$ and 3 PVU from Feb 1 through Feb 18 of 1983. The orientation of the maps is the same as that in Figure 6.2.
Figure 6.4: Output of CAS with only lowpass wind, beginning on Feb 1 and ending on Feb 18. The initial contours were taken from $q = 2$ and 3 PVU on Feb 1. Plotting interval is 1 day.
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Figure 6.4 continued.

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Figure 6.5: Contours of $\bar{q} = 2$ and $4$ PVU from Feb 1 through Feb 18 of 1983. The orientation of the maps is the same as that in Figure 6.2.
Figure 6.5 continued.
Figure 6.6: Output of CAS with only lowpass wind, beginning on Feb 1 and ending on Feb 18. The initial contours were taken from $\tilde{q} = 2$ and 4 PVU on Feb 1. Plotting interval is 1 day.
Contour Advection with Surgery
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Figure 6.6 continued.
6.1.3 1989 event

Ek and Swater (1994) examined February of 1989, during which there was an episode of Pacific blocking. They identified a change in the shape of the block from an $\Omega$-like to a dipole during this episode. They suggested that the blocking flow of both shapes was nearly a free mode, citing that the net vorticity flux into a large rectangular region containing the block was nearly zero except when the block was forming or dissipating.

Figure 6.7 shows $\hat{Q}$ from Jan 30 through Mar 15 of 1989. Some sign of weak diffuence over the North Pacific is already seen on Jan 30 (lower-left quadrant of the map). The diffuence becomes readily visible in $\hat{Q}$ field by Feb 1 and forms an $\Omega$-shaped block by Feb 4. The diffuence becomes somewhat weaker in the middle part of February, but remains readily identifiable. By Feb 23, typical blocking signatures, such as an $\Omega$-shaped or dipole $\hat{Q}$ contours, are no longer seen clearly. However, there is a growing quasi-stationary wave over the eastern North Pacific (lower-left quadrant of the maps) from Feb 22 through Feb 25, eventually forming a large dipole blocking structure by Feb 27. This signature, while changing its shape, slowly moves westward, as sometimes observed in strong blocking cases. The quasi-stationary trough loses its amplitude noticeably after Feb 28, but regains its strength after Mar 4. On Mar 15, the last day for which the data were available to calculate $\hat{Q}$, there still remains a cut-off anticyclone, appearing as an $\Omega$-shaped block. Since the strong diffuence in the low-frequency flow does not disappear throughout the period from Feb 1 to Mar 15, it will be considered as a single blocking episode here.
Figure 6.7: Contour plot of $\tilde{q}$ from Jan 30 through Mar 15 of 1989. Contour interval is 1 PVU. The orientation of the maps is the same as that in Figure 6.2.
Figure 6.7 continued.
Figure 6.7 continued.
Figure 6.8 shows $-\nabla \cdot \nabla \dot{q}$, $-\nabla \cdot \dot{V} \dot{q}$, and the sum of the two, superimposed on $\dot{q}$ for a period from Feb 1 to Mar 15. As seen in the 1983 event, the total $\dot{q}$ forcing is almost entirely due to the mean flow advection, while eddy forcing tends to reinforce the blocking ridge.

Since this blocking episode is rather long and the adiabatic-inviscid assumption in CAS may not hold well over a long period, CAS was run for five separate periods within the episode, as well as for the entire period as a single run. For brevity, only results of runs with the initial contours taken from those with $\dot{q} = 2$ and 4 PVU are shown here. For comparison, contours of the observed $\dot{q} = 2$ and 4 PVU are shown from Jan 30 to Mar 15 in Figure 6.9. The first run, R1, was initialized with contours of $\dot{q} = 2$ and 4 PVU on Jan 30 and run until Mar 15. The second run, R2, was initialized by contours of $\dot{q} = 2$ and 4 PVU on Feb 8 and run until Mar 15. The third run, R3, was initialized by contours of $\dot{q} = 2$ and 4 PVU on Feb 17 and run until Mar 15. The fourth run, R4, was initialized by contours of $\dot{q} = 2$ and 4 PVU on Feb 26 and run until Mar 15. The fifth run, R5, was initialized by contours of $\dot{q} = 2$ and 4 PVU on Mar 7 and run until Mar 15. All of the five runs, R1 through R5, used lowpass wind. Some short runs were also done with the total wind.

The result of R1 is shown in Figure 6.10, from Jan 30 to Mar 15. Comparing the CAS-produced contours with those observed (Figure 6.9), one should note several points. First of all, this relatively long (compared with other events examined here) blocking event, which appears more or less continuous in the strong diffluence pattern in $\dot{q}$, in reality, consists of 3 consecutive cycles of quasi-stationary Rossby wave amplification, breaking, and dissipation. The first cycle begins on Day 1 (Jan 31) and ends around Day 12 (Feb 20), when the second cycle is beginning at nearly the same location. After small-scale chaotic features produced by the first breaking event are blown away or dissipate (by contour surgery), the second cycle reaches its maximum amplitude and
Figure 6.8: Contour plot of \( \bar{q} \) forcing by eddies, the mean flow, and the sum of the two, superimposed on \( \bar{q} \) over a period from Feb 1 to Mar 15 of 1989. Contour intervals for \( \bar{q} \) and the forcings are 1 PVU and 0.2 PVU/day, respectively.
then breaks. As its "residues" are being dissipated, the third amplification begins on Day 24 (Feb 23), again, at almost the same location. The quasi-stationary wave amplifies through Day 29 (Feb 28) or so, then begins to break, producing many small-scale chaotic features, as well as the blocking dipole observed in the actual $\tilde{q}$ contours. The third cycle finally ends with injection of a large low-$\tilde{q}$ blob into high latitude, as observed in the actual $\tilde{q}$ plot. This event, thus, resembles E1 of Chapter 5 in that more than one episode of quasi-stationary wave amplification, breaking, and dissipation cycle occur consecutively at nearly the same location, appearing as a relatively long-lived blocking. Yet, each cycle has a time scale of 10 to 15 days, the same as all other cycles found in the current study. As in the 1983 event, these cycles of amplification, breaking, and dissipation also appear in the actual $\tilde{q}$ field shown in Figure 6.7, although the details of material transport are less clear than those in CAS results.
Figure 6.9: Contours of $\bar{q} = 2$ and 4 PVU from Jan 30 through Mar 15 of 1989. The orientation of the maps is the same as that in Figure 6.2.
Figure 6.9 continued.
Figure 6.9 continued.
Figure 6.10: Contours of $\bar{q} = 2$ and 4 PVU, produced by CAS with low-frequency flow, from Jan 30 through Mar 15 of 1989. The orientation of the maps is the same as that in Figure 6.9.
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Figure 6.10 continued.
Contour Advection with Surgery
Infile=qlp13089

Figure 6.10 continued.
Contour Advection with Surgery
Infile=qlp13089

Figure 6.10 continued.
Figure 6.10 continued.
The accuracy of CAS in this run is not as good as that in runs for 1983 event. Considering the limitation of the adiabatic-inviscid assumption, rather poor performance of CAS after 10 days or so is understandable, although there is fairly good overall agreement between the CAS-produced and the actual \( \bar{q} \) contours until the end of R1. If there is a notable difference between the observed and CAS-produced \( \bar{q} \) during the first 10 or 15 days of a CAS run, however, it suggests that only the low-frequency flow advection may not be sufficient to account for evolution of \( \bar{q} \) field. Indeed, there is such a discrepancy in R1. Comparing the observed (Figure 6.9) and CAS-produced (Figure 6.10) \( \bar{q} \) contours from Jan 30 through Feb 7, one may easily note the difference in the amplitude and area of poleward-intruding quasi-stationary ridge in the two; the CAS-produced ridge is much smaller in both amplitude and area than the observed, through Feb 6. A discrepancy of this magnitude during the first 8 days of a CAS run is difficult to justify as a result of insufficient time resolution of the wind data (since only the low-frequency portion is used) and seems to suggest that low-frequency flow advection may not be the single dominant factor in \( \bar{q} \) field evolution during this period. To check this possibility, CAS was run for 9 days with the total wind, beginning with the same initial condition as in R1. The result is shown in Figure 6.11. Despite rather coarse time resolution of the wind data (the total wind at this level varies rapidly with time), CAS with the total wind produces the ridge in \( \bar{q} \) during the first 6 days of this period considerably better than does CAS with low-frequency flow. These results, thus, suggest that high-frequency eddies were also important direct forcing of \( \bar{q} \) during the onset stage of this blocking event.

The accuracy of CAS with lowpass wind in reproducing the observed \( \bar{q} \) during this blocking does show improvement when CAS is run for shorter duration. Figure 6.12 shows the first 9 days of R2, i.e., from Feb 8 to Feb 16. It does compare better with the observed (Figure 6.9) than the same period of R1 (Day 9 through Day 17 of Figure 6.10). It is quite reasonable, considering that the adiabatic-inviscid assumption in
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Figure 6.11: Contours of $\Omega = 2$ and $4$ PVU, produced by CAS with the total flow, from Jan 30 through Feb 7 of 1989. The orientation of the maps is the same as that in Figure 6.9.
Contour Advection with Surgery
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Figure 6.12: Contours of $\bar{q} = 2$ and 4 PVU, produced by CAS with low-frequency flow, during the first 9 days of R2. Day 0 and Day 9 correspond to Feb 8 and Feb 16, respectively, of 1989. The orientation of the maps is the same as that in Figure 6.9.
Figure 6.13: Contours of $\bar{q} = 2$ and 4 PVU, produced by CAS with low-frequency flow, during the first 9 days of R3. Day 0 and Day 9 correspond to Feb 17 and Feb 25, respectively, of 1989. The orientation of the maps is the same as that in Figure 6.9.
Figure 6.14: Contours of $\bar{q} = 2$ and 4 PVU, produced by CAS with low-frequency flow, during the first 9 days of R4. Day 0 and Day 9 correspond to Feb 26 and Mar 6, respectively, of 1989. The orientation of the maps is the same as that in Figure 6.9.
Figure 6.15: Contours of $\bar{q} = 2$ and 4 PVU, produced by CAS with low-frequency flow, during R5. Day 0 and Day 9 correspond to Mar 7 and Mar 15, respectively, of 1989. The orientation of the maps is the same as that in Figure 6.9.
CAS may not hold well much longer than a week or so on a $330^\circ K$ surface. This trend continues as shown in Figures 6.13 through 6.15. In these figures, only the first 9 days of each of R3, R4, and R5, respectively, are shown. The contours in these figures do compare better with those observed in the corresponding periods (Figure 6.9) than do those of R1 (Figure 6.10). However, they all show qualitatively the same evolution mechanism of the $\bar{q}$ contours as that discussed for R1, i.e., quasi-stationary Rossby wave amplification, breaking, and dissipation.

### 6.2 Application of CAS to 15-event composite of European blocking

In order to check the generality of the results obtained from CAS runs with the GCM and observational data, CAS was applied to 9-day data of a composite of 15 European blocking episodes compiled by Nakamura (1994). Nakamura speculated, from examination of evolution of pseudo-Ertel's potential vorticity (Nakamura and Wallace, 1993) and approximate quasi-stationary wave activity flux (Plumb, 1986) of the composite data, that underlying mechanism of these blocking events may be quasi-stationary Rossby wave amplification and breaking. Here, CAS was used to verify his speculation, which agrees with the results presented in the previous and the current chapters.

#### 6.2.1 Data

The composite data were compiled from twice-daily NMC analyses by taking the 15 strongest episodes of European blocking in the period between 1965 and 1992. The composite data provide low-frequency part (fluctuation period of 8 days or longer) of geopotential height and pseudo-Ertel's potential vorticity, which will be denoted by $\bar{q}_p$, at 250 mb. The data are given on a regular grid with a resolution $2.5^\circ X 2.5^\circ$ in longitude and latitude from $20^\circ N$ to $90^\circ N$ over a period of 9 days, beginning at the onset stage. The details of compositing procedure is given by Nakamura (1994).
Two sets of approximate low-frequency wind data for 250 mb level were constructed from the geopotential height data by assuming geostrophic balance. For the first set, Coriolis parameter is a function of latitude. For the second set, Coriolis parameter is fixed at the value at 45°N. Note that \( q_p \) is not conserved on an isobaric surface (Nakamura and Wallace, 1993) and, thus, use of thus-derived wind data in CAS runs is likely to introduce larger errors than would use of \( q \) and wind on an isentropic surface. However, since \( q_p \) is conserved in a three-dimensional sense (Nakamura and Wallace, 1993) and vertical motion at 250 mb is much smaller than the horizontal motion, the errors from the use of \( q_p \) with isobaric wind in CAS may not be much more of a concern than are the errors from adiabatic-inviscid assumption in CAS.

### 6.2.2 Results

The time series of \( \tilde{q}_p \) from the composite data are shown in Figure 6.16. By Day 2 in the figure, a dipole structure is evident downstream of a diffluent region over the North Atlantic. The composite block reaches its maximum intensity on around Day 4 and begins to lose its strength on around Day 7. Figure 6.17 shows the result of a CAS run with locally-geostrophic wind, initialized by contours of \( \tilde{q}_p = 1 \) and 2 PVU. It clearly shows amplification, breaking, and dissipation of a quasi-stationary wave. As the wave breaks, it transports a large blob of low-\( \tilde{q}_p \) air poleward with anticyclonic motion, while somewhat smaller blob of high-\( \tilde{q}_p \) air is ejected equatorward with cyclonic motion. The CAS-produced \( \tilde{q}_p \) contour positions do compare well with the actual ones, shown in Figure 6.18. One potentially important discrepancy between the CAS-produced and the actual is the lack of below-1-PVU air in the CAS-produced anticyclone. The size of below-1-PVU air in the actual blocking anticyclone is not negligible. This may be due to the positive feedback of high-frequency eddies, which are not included in the wind data used to run CAS.

Figure 6.19 shows the result of another CAS run, initialized with contours of \( \tilde{q}_p = 2 \)
Figure 6.16: Contour plot of $q_e$ from Day 0 to Day 8 of a 15-event European blocking compiled by Nakamura (1994). Contour interval is 1 PVU. The orientation of the maps is the same as that in Figure 6.2.
Contour Advection with Surgery
Infile=qq.t

Figure 6.17: Output of CAS with the composite wind, beginning on Day 0 and ending on Day 8. The initial contours were taken from $q_p = 1$ and 2 PVU on Day 0.
Figure 6.18: Same as in Figure 6.16, but only contours of $\bar{q}_p = 1$ and 2 PVU.
Figure 6.19: Output of CAS with the composite wind, beginning on Day 0 and ending on Day 8. The initial contours were taken from $q_p = 2$ and 3 PVU on Day 0.
Figure 6.20: Same as in Figure 6.16, but only contours of $\bar{q}_p = 2$ and 3 PVU.
and 3 PVU. Again, the CAS-produced \( \hat{\varphi}_p \) contours compare quite well with the actual ones (Figure 6.20). With these contours, anticyclonic rotation associated with the amplification and breaking is seen, as mentioned by Nakamura (1994). This feature, again, resembles the cat's eye of a critical layer Rossby wave. When the wind derived from a constant Coriolis parameter \( f_0 = f(45^\circ N) \) is used, the CAS results still capture the observed features of synoptic scale or larger quite well (not shown). The amplification and breaking of the quasi-stationary wave are observed in the actual \( \hat{\varphi}_p \) time series as well (Figure 6.16). Thus, the result obtained from case studies using the GCM and NMC data, domination of blocking dynamics by low-frequency flow advection of quasi-stationary wave amplification, breaking, and dissipation, is supported by a data set representing 15 events of strong blocking.

6.3 Discussion

The results presented in this chapter, in general, support those of Chapter 5; evolution of blocking flows is dominated by low-frequency flow advection throughout life cycles of the blocks. One notable discrepancy was observed, however, during the onset of a Pacific block. CAS with only low-frequency flow advection did not reproduce a poleward-intruding anticyclonic air seen in the \( \hat{\varphi} \) field very well during the first 6 or 7 days of the block. On the other hand, when CAS was applied with the total wind, it reproduced the intruding ridge during this period quite well. Considering that CAS runs with the lowpass wind reproduced the actual \( \hat{\varphi} \) field reasonably well and those with the total wind did not during other blocking events, including those in the GCM data, this exceptional case may not be treated as a result of low-quality data. It is likely to suggest the importance of high-frequency flow during the onset of this episode. Thus, there is a possibility of two different mechanisms to initiate a blocking flow, suggested by the results of the current and the previous chapters; one is by an amplifying quasi-stationary wave, the other by an intense obstruction of zonal flow by "explosive" cyclogenesis. The mechanism of blocking generation demonstrated by
Shutts (1983) in a nonlinear model with initially-zonal flow may represent the latter type of blocking mechanism. Despite this possibility of an important role of high-frequency transients during the onset of the episode, low-frequency flow dominates the block evolution during the rest of the period and appears as the primary factor in the direct $\bar{q}$ forcing.

The possibility of two different initiation mechanisms raises a further question on the appropriateness of the time mean circulation diagnoses to study blocking flows. In order to identify important mechanisms at various stages of a blocking, one must study time-dependent behavior of various components of the flow. If only the time mean statistics were used in this chapter, the potential significance of high-frequency eddies in the onset of the 1989 event may have been overlooked. As discussed in Chapter 5, fundamental nature of blocking flows is their transience. All the events shown here exhibit one or more cycles of quasi-stationary wave amplification, breaking, and dissipation. The characteristics of high- and low-frequency forcings during this cycle may vary from episode to episode. In particular, those events occurring during summer may have noticeable difference from the cases examined here, as suggested by Illari's (1984) results.

As observed in E1 of Chapter 5 and the Pacific block in this chapter, the evolution of $\bar{q}$ appears as a single event of blocking with longer duration and minor interruptions, when more than one cycle of amplification, breaking, and dissipation occur consecutively over the same area. The time scale associated with this cycle in the two observed cases, five GCM-simulated cases, and a 15-event composite is roughly the same: 10 to 15 days. Such flow evolution during blocking explains why the time mean flows during blocking have been known to be unstable. The time mean flow during a blocking is, in a philosophical sense, the same as the time mean flow over a life cycle of a wave in a barotropically unstable flow. The blocking flow is never steady
or stationary. It simply evolves relatively more slowly, compared to synoptic-scale waves. Ek and Swater (1994) concluded that the 1989 event may correspond to, at its mature stage, a free-mode solution of the quasi-geostrophic equations, citing that the net vorticity flux in a box that contained the block was very small while the block was strong. From the results presented here, it is clear that the net vorticity flux in a box that contained the blocking signature should be small, because the dominant forcer, low-frequency flow, is mainly mixing the material within the area of the block and no net additional low-\( q \) air or high-\( q \) air is being brought into the block. CAS and \( \bar{q} \) time series show that this is a result of transient behavior of the blocking flow.
Chapter 7

Summary

Rossby wave breaking characteristics in flows of various profiles and complexity have been studied. Emphasis was placed on the influence of the basic or background flow structure on the evolution of transient waves and their feedback on the background flow.

First, Rossby wave evolution characteristics in idealized flows were examined, using the method of CD/CS. Various experiments were performed in two different geometries: $2\pi$-periodic and circular on an $f$-plane.

Experiments in the $2\pi$-periodic geometry were directed to illuminate effects of meridional and zonal asymmetries in the basic flow on evolution of propagating waves. In one series of experiments, sinusoidal waves of various amplitudes were prescribed in the initial conditions, in basic flows with cross-jet symmetry, varying Rossby deformation radius. As expected from the cross-jet symmetry in the basic flow, the wave evolution is symmetric with respect to the jet. These experiments show that waves break when their amplitudes are greater than certain threshold values, which depend on deformation radius and wavelength. Broadly speaking, two different types of wave breaking were observed. One type is filamentary breaking, in which the basic flow
shear seems to play a major role in deforming the material contour around the waves' critical points (e.g., Figure 3.5). This type occurs for $L/R_d$ less than about 1.2, where $L$ is wavelength and $R_d$ is deformation radius. Beyond $L/R_d = 1.2$, another or possibly two different types of breaking occur (Figures 3.6 and 3.7). They appear to be induced by material advection by the wave-induced flow itself and/or shear instability caused by approaching two contours. The threshold amplitude for wave breaking exhibits a discontinuous profile at around $L/R_d = 1.2$ (Figure 3.4), supporting the existence of at least two totally different mechanisms of Rossby wave breaking.

In another series of experiments with the $2\pi$-periodic version, effects of cross-jet (meridional) asymmetry in basic flows characterized by a single straight jet on Rossby wave evolution were studied. In these experiments, sinusoidal waves of various amplitudes were prescribed in the initial conditions, while the degree of the asymmetry in the basic flow was systematically varied from experiment to experiment. Also, a few different values of deformation radius were tested. Results of these experiments demonstrate that the cross-jet asymmetry in the basic flows induces corresponding asymmetry in wave evolution. When cross-jet meridional asymmetry in the basic flow shear is present, waves show biased tendency to amplify in the direction of greater shear. It occurs because of longer time allowed for a wave to grow due to the weaker basic zonal flow on this side. When the shear asymmetry is sufficiently large for a given wave amplitude or when the wave amplitude is sufficiently but not excessively large for a given shear asymmetry, the wave breaks only toward the side of greater shear or the magnitude of rate of strain (Figure 3.15). In other words, waves can overcome the basic flow asymmetry and break two-sidedly, if their amplitudes are sufficiently large. The side of greater shear is also the side of closer critical line in this flow configuration. Stagnation or critical points develop more readily on this side than the side of weaker shear, where critical line is farther away from the PV front.
Thus, this meridional asymmetry in the breaking characteristics may be understood in terms of the locations of critical lines, as well as in terms of asymmetry in shear or the magnitude of rate of strain of the basic flow. In fact, these two factors are intimately related and cannot be separated.

Non-divergent and irrotational deformation flows of various strengths (e.g., Figure 3.16) were added to barotropic basic flows in the $2\pi$-periodic geometry to study effects of zonal asymmetry in the basic flow on wave evolution. Addition of such deformation flows introduces a diffuence and a confluence to the basic flow (Figure 3.17) without altering the basic PV distribution or violating mass conservation. Sinusoidal waves or a localized wave packet of various amplitudes were prescribed on the straight basic PV front. The waves show tendency to gain/lose amplitude in the region of diffuence/confluence. This tendency arises from two factors: locally-reduced or locally-enhanced basic zonal flow and meridional stretching or compression by the meridional component of the basic flow. Waves that are subcritical in a symmetric basic flow break only in the diffuence when the diffuence is sufficiently strong (e.g., Figure 3.19). On the other hand, waves that are supercritical in a symmetric basic flow do not break in the confluent region when the confluence is sufficiently strong. These breaking characteristics can also be understood in terms of the locations of critical points. Critical points for a given wave are closer to the PV front in the diffuence and vice versa, because of the altered basic flow. When there are both meridional and zonal asymmetries in the basic flow, the net effect of the flow asymmetry is determined by the competition between the two asymmetries, i.e., effect of either asymmetry may prevail over the other, depending on the prescribed strength.

The effect of the cross-jet asymmetry in cases with a circular vortex, consisting of a single potential vorticity discontinuity, as the basic flow, is very similar to that
in the $2\pi$-periodic straight jet cases. Most experiments in the circular geometry were done with a transient wave maker to resonantly excite waves on the PV front, although some were done with initially prescribed waves, as in the $2\pi$-periodic cases, also. Experiments with the two different types of wave forcing show qualitatively the same results. Waves on the edge of the vortex break, when their amplitudes are supercritical, toward the side of closer critical line or greater magnitude of rate of strain (e.g., Figures 2.1 and 2.3). Although not as clear-cut as is in the case of straight jet, there is also an intimate relationship between the location of critical line and the magnitude of rate of strain in this flow configuration, which makes separation of these two effects impossible. Because cross-jet symmetry in the basic flow is impossible to attain due to the geometry, no clear-cut two-sided breaking was observed in these experiments. Unless the basic flow is contrived in an unrealistic fashion, waves break only away from the vortex.

Using these two models with a single PV front, effects of zonal and cross-jet asymmetries in the basic flow, induced by presence of a stationary or a steadily-propagating “planetary-scale” wave, on evolution of propagating “synoptic-scale” waves were studied. In the $2\pi$-periodic geometry, this was done by using a stationary wave of number 1 as the basic state and prescribing smaller waves of various amplitudes (e.g., Figure 4.2). In the circular geometry, a transient wave maker was used to excite disturbances on the basic state characterized by a steadily-propagating wave of number 3 (e.g., Figure 4.9). When the asymmetries are introduced into the basic flow in these manners, their effects on breaking of “synoptic-scale” waves become, in a qualitative sense, relatively simple. “Synoptic-scale” waves tend to break more readily when the curvature in the “planetary-scale” flow is favorable. The rule found from the experiments is that the anticyclonic phase of the “synoptic-scale” waves breaks into the vortex more readily when there is an anticyclonic curvature in the “planetary-scale” flow and vice
versa (Figures 4.2 and 4.9). This is in agreement with the results described above, since the magnitude of rate of strain inside the vortex at the "planetary-scale" ridge can be greater than that outside when the curvature is sufficiently large (Figures 4.4 and 4.11). Although not quantitative, this qualitative criterion, presence of strong planetary-scale anticyclonic curvature for occurrence of inward breaking, turned out to be reliable in analyses of the GCM and observational data, whose flow fields are much more complex than those in the CD/CS experiments.

To study Rossby wave breaking characteristics in more complex flows, 5-times-per-day output of a GFDL GCM, a new version of R30-realistic, was examined. An emphasis was placed on roles of wave breaking in dynamics of "atmospheric blocking", which is subjectively defined as a prominent diffluent structure in low-frequency flow that lasts 10 days or longer. The output was analyzed by Ertel's potential vorticity, \( q \), on isentropic surfaces in the upper troposphere. One-year time series of \( q \) show that poleward breaking of synoptic-scale waves (i.e., formation of cut-off anticyclones) occurs only when there is a considerable anticyclonic curvature in the quasi-stationary or low-frequency flow (Figures 5.1 and 5.2). The strong influence of the basic flow on wave evolution, found in CD/CS experiments, thus, appears to be applicable to much more complex flows in a broad sense. General quantification of this claim is probably very difficult if not impossible, since the key factor is the relative strength of baroclinic and barotropic forcings of synoptic-scale eddies measured against some indicator of low-frequency flow curvature. It is not quite clear how one may be able to quantify such relationships and obtain a generalized result.

Five episodes of blocking were identified in the GCM data. The relationship between the direction of synoptic-scale eddy breaking and low-frequency flow curvature holds true during the five episodes; anticyclones break into the vortex when there
is visibly large anticyclonic curvature in the low-frequency flow. Strongly-diffuent low-frequency flows, defined as blocking in this study, stretch high-frequency eddies meridionally and compress them zonally, splitting them into cut-off cyclones and anticyclones in the diffuent region (Figure 5.8). Calculation of low-frequency or the time-mean $q$ forcing by these eddies shows that they deposit potential vorticity in the blocking diffuence in a way such that the existing low-frequency diffuence is maintained (e.g., Figures 5.14 and 5.18). This observation is in agreement with patterns of the time-mean blocking flow forcing by eddies or high-frequency flow found by other investigators. It also supports results of model studies that show similar flow evolution and forcing characteristics. However, the direct forcing of the diffuent blocking flow by high-frequency flow or eddies was found to be somewhat smaller and less organized than that by low-frequency or the time-mean flow in these GCM-simulated events (for example, compare Figures 5.14 and 5.15). Thus, the role of high-frequency flow or eddies in these episodes, appears to be of secondary importance, as far as the direct contribution is concerned. The primary forcing seems to come from low-frequency flow. High-frequency flow or eddies may be important, however, through contribution to low-frequency flow forcing.

In order to further examine the result of $q$ forcing analyses, low-frequency flow being the dominant forcer of the five blocking flows, CAS was applied to the data. CAS was run with low-frequency flow and the total flow, separately, for the five events. The results confirm that low-frequency flow advection is the primary direct forcing of these events throughout their life cycles. Low-frequency $q$ distribution is reasonably well-reproduced by CAS with only low-frequency flow advection throughout the life cycles of the five episodes (e.g., Figures 5.24 and 5.25). The results of CAS runs also show strong transience, which is characterized by amplification, breaking, and dissipation of quasi-stationary waves, during the blocking events. This cycle was also
found in time series of low-frequency $q$ distribution. Each cycle has a time scale of 10 to 15 days. One episode, which lasted about one month, consisted of two cycles occurring at nearly the same location, whereas the other four episodes consisted of only one and lasted for 10 to 15 days.

To check whether the results of the GCM data analyses hold in real blocking events, two observed events, a North Atlantic blocking and a North Pacific blocking, were analyzed by isentropic $q$ diagnosis in the upper troposphere and by CAS application to the data. The original data were produced by twice-daily NMC global analyses. The time-mean $q$ forcings by eddies and the mean flow show that eddies tend to reinforce the blocking flow, but appear to be of secondary importance in the direct forcing, and that it seems to be the time-mean flow advection which dominates the dynamics (Figures 6.2 and 6.8), as in cases simulated by a GCM. Application of CAS to the data also show that low-frequency flow advection dominates the evolutions of the two blocks throughout their life cycles, in general (Figures 6.3, 6.4, 6.9, and 6.10). However, an exception was found in the North Pacific blocking; the first 6 or 7 days during the onset, it is the total flow advection, which reproduces the low-frequency $q$ distribution well, rather than the low-frequency flow advection (compare Figure 6.9 with Figures 6.10 and 6.11), implying that high-frequency eddies may be important during the onset of this episode. It suggests that there may be more than one mechanism to initiate a blocking flow in reality. As observed in the GCM-simulated cases, CAS runs and time series of low-frequency $q$ reveals the strong transience of blocking, which is characterized by amplification, breaking, and dissipation of quasi-stationary waves. The time scale of the quasi-stationary wave cycle, amplification, breaking, and dissipation, is 10 to 15 days, as is in the GCM-simulated events. The North Pacific blocking event, which lasted about 45 days, consisted of 3 cycles occurring at nearly
the same place.

Finally, CAS was applied to a 15-event composite data of European blocking. The results confirm the general results of the analyses of the five GCM-produced and two observed episodes; evolution of the composite block is dominated by low-frequency flow advection, which manifests itself as amplification, breaking, and dissipation of a quasi-stationary wave (Figures 6.19 and 6.20). Its life cycle has a time scale of 10 to 15 days, as do those observed in the other seven cases.

The results presented in this study show that transience is a fundamental characteristic of blocking. The most straightforward interpretation of the results presented here seems to be that of blocking as a manifestation of amplification, breaking, and dissipation of forced quasi-stationary waves. This interpretation readily explains why occurrence of blocking in the Northern Hemisphere is considerably more frequent than in the Southern Hemisphere, where quasi-stationary forcing of planetary-scale wave is mostly in wavenumbers 1 and 2. Although the results portray high-frequency eddies as primarily a passive player in direct forcing of low-frequency $q$ during blocking, there may be cases in which the direct contribution of eddies is important during the onset stage. It is also likely that indirect contribution of high-frequency eddies through forcing of low-frequency flow is not negligible.
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