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THE INTERACTION OF TEXTILE STRUCTURES

AND HIGH SPEED TEXTILE PROCESSES

by

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B.S., Massachusetts Institute of Technology (1948)

M.S., Massachusetts Institute of Technology (1955)

SUBMITTED IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF DOCTOR OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June, 1959

Signature of Author..............................

Department of Mechanical Engineering, February 24, 1959

Certified by..............................

Thesis Supervisor

Accepted by..............................

Chairman, Departmental Committee on Graduate Students
ABSTRACT

The Interaction of Textile Structures and High Speed Textile Processes

by Henry Merriam Morgan

Submitted to the Department of Mechanical Engineering on
March 4, 1959, in partial fulfillment of the
requirements for the degree of
Doctor of Science

A study has been made of the interaction which
takes place in certain high speed textile processes at points of
machine-fiber contact. Generalized treatments are given of the
frictional generation of force in fibers and the resultant visco-
elastic behavior. The specific mechanical actions of weaving,
cold-drawing and sewing have been analyzed qualitatively and quan-
titatively. The effects of these actions on textile structures is
discussed in terms of the frictional and viscoelastic behavior.
Instrumentation has been developed for in-process measurements of
the mechanical and thermal effects which occur during cold-drawing.

Thesis Supervisor: Stanley Backer

Title: Associate Professor of Mechanical Engineering.
ACKNOWLEDGMENTS

No one can complete a doctoral program, and particularly a thesis, without the help and inspiration of many people. My debt of gratitude begins with Dr. Walter J. Hamburger and the other members of the staff at Fabric Research Laboratories, Inc. for their early and continued stimulation. Prof. Edward R. Schwarz, who has started so many others in the field of textile research, provided much needed encouragement and overall guidance. Prof. Ernest Rabinowicz has been most helpful in keeping the thesis on the track. I particularly acknowledge the help given by Prof. Stanley Backer who suggested the general nature of the research and without whose aid and confidence there would be no thesis. Above all I acknowledge the patience and digital dexterity of my wife, Gwen G. Morgan.
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I. INTRODUCTION

Proper design of new processing equipment and efficient use of new fibers on existing machines requires that we understand the dynamic interaction between textile materials and textile processes. This thesis on the Interaction of Textile Structures and High Speed Textile Processes was undertaken to develop means of achieving this understanding.

During processing, textile fibers are subjected to varying applied stresses and strains, caused by machine - material interactions. For example, the rubbing of yarn over a guide produces stresses at the point of the frictional interaction. The development of a whirling balloon\(^1\) during yarn spinning is a result of many dynamic interactions including aerodynamic drag. In another case, abnormal strains may occur if a high speed flow of fiber or yarn is suddenly halted because of material faults or machine failure.

A recent survey conducted in the Textile Division\(^2\) has pointed up the importance and probable magnitude of transient strains produced by rather minor malfunctions during processing, such as the presence of a slub or temporary interruption of material flow. These transient strains increase in magnitude as the process velocity increases. However, their role can be completely assessed only when more is known about
the normal or steady state operating conditions upon which they are superimposed. The normal operation of three processes: weaving, cold-drawing, and high speed sewing, are studied in this thesis.

The concept of machine - material interaction which is developed represents a departure from the traditional studies of machine actions. Previously, the studies of actions were concerned strictly with the kinematics and dynamics of the machine itself. Cam motions, rotary motions, reciprocating actions, feed and take-up mechanisms have all been studied and analyzed by mechanical engineers for several generations. These studies usually neglect, and rightfully so, the presence or absence of the textile material which is being processed. The sheer mass of past and present textile machines is so out of balance with the few grams of fiber which are being operated upon that there is seldom any significant reaction by the fiber on the machine. For this reason, this study deals exclusively with what happens to the textile material as a result of the interaction. It is anticipated that future machinery, designed for operation at higher and higher speeds, will be less massive; and a time will come when the action of the material on the machine will be more important. Of course, the action of material on machine becomes important as the mass of the structure is increased. As examples, consider the massive drum of a carding engine whose rotational inertia is so great that it is
oblivious to the presence of fiber even when loaded, and then consider a woven fabric which must be held to dimension on a tenter frame. In the latter case, a significant reaction is induced in the machine, but the process is no longer one which can be called high speed. In general as the textile structure becomes more organized, the mass of material being processed increases. The mass flow remains reasonably constant and hence the absolute flow velocity decreases. In addition, as the structure becomes larger it can exert a greater reaction force on the machine.

Schwarz has introduced the use of symbols to portray the multiplicity of actions which occur in all textile processes. These action symbols are confined to actions which directly involve the fiber yarn or fabric undergoing processing. By the use of action symbols, processes, operations and sub-processes are reduced to their essentials. The clarity and simplicity of the "action" approach as exemplified by symbols constitutes the greatest aid for the textile machinery designer to reappraise the evolutionary machine monsters and replace them with revolutionary machine midgets.

If one were to consider the effect of these basic machine actions on the fiber, it could be seen that the mechanical actions are all variations on the theme of force transmission
to the fiber, - struck by, striking against, gripping, restraining, pulling. These particular mechanical actions produce the interactions which we be considered in the following pages. The interactions produce surface and body forces which are a consequence of machine velocity, fiber inertia, contact areas, surface roughness, bulk properties, etc. Some of these factors are fiber properties and some are machine and process properties. The combination of all factors must be considered in the machine - material interaction. In order to define quantitatively and analyze the effects of these interactions, we must be able to delineate both the fiber and machine behavior at the moment of the interaction.

Chapters II and III discuss two important phases of the broad study of fiber properties. Chapter II deals with the subject of friction which is the means of the development of normal longitudinal forces by lateral and shear stresses at the point of action. Frictional or surface forces are again the consequence of both machine and fiber properties. The treatment of Chapter II includes the effects of normal and longitudinal stresses, contact areas, lubrication and sliding velocity. The surface forces can be considered to be local forces confined to the contact points. In addition, one must consider the bulk forces which exist some distance from the local action point. The well-known principle of St. Venant
allows one to neglect the particulars of the contact area and
to treat the bulk forces away from the action point as an in-
dependent problem.

The discussion of Chapter III contains a general
consideration of viscoelastic behavior with an original analy-
sis of stress-relaxation effects when fibers are subjected to
constant strain rates. The mechanical properties of fibers
must be related to the times of the measurements of these prop-
erties. A general discussion of fiber properties must include
a general description of the time-dependent variations of prop-
erties. The properties which correspond with the time scale
of the process action are those which are involved in the machine-
fiber interaction. An attempt has been made in Chapter III to
generalize the variation in properties with time for a number
of fibers. In addition, some consideration has been given to
the cause of the viscoelastic nature of organic fibers.

A quantitative analysis of an interaction can
not be made from a knowledge of fiber properties alone. The
machine actions must be defined both qualitatively and quanti-
tatively. The velocities and other details of striking, pull-
ing, sliding, etc. must be known at each action point of a
process. Ideally one must go back to the symbolic flow diagram
and add quantitative values to the action symbols. In Chapter
V, the author has studied quantitatively one process with very
simple machine actions, which combine with fiber properties to
give a complicated and sensitive process. This process is the
cold drawing of undrawn nylon multifilament yarn. A simple
machine process was used to illustrate the principle of quan-
titative machine - fiber interaction. In Chapters IV and VI,
the actions of more complicated processes are discussed with-
out quantitative measurements. The study of these processes
present certain difficult problems of measurement and instru-
mentation, but the interaction analysis can be built upon the
foundations of the action symbolism and the fiber property
analyses of Chapters II and III.

References
1. Grishin, P.F., "Balloon Control," Special Publication of
   TMM (Research) Limited, 1956 (pages 1 - 74).

2. Morgan, H.M. "Transient Stresses in Textile Structures and

3. Schwarz, E.R., "Basic Actions as an Approach to Textile
II. FRICTION

When considering the interaction of textile yarns with the moving parts of textile machinery, it is apparent that the physical contacts produce so-called "frictional forces." The forces which are set up in a yarn and are transmitted along its free length are originally induced by the machine-fiber-contact. The treatment of forces in yarns and fibers must be separated into two areas. The first is the immediate region of local contact, and the second is the region far removed from the contact. The first problem is that of friction at an interface. This part of the interaction problem will be treated in this chapter. The second part, which deals more generally with the viscoelastic properties of textile materials, will be considered in a later chapter.

A very extensive review of the literature on this problem of the interaction of fibers with other fibers as well as with foreign hard surfaces was compiled by Finch as part of his Sc.D. thesis (1950) entitled, "Inter Fibre Stress and Its Transmission". This work reviewed the pertinent published work up to that time, 1950, in the field of metallic and non-metallic friction. It would be as much a mistake to attempt to repeat so complete a survey as it would be to ignore it. Instead, this chapter on friction
will attempt to start where Finch left off and to review the more recent work on the theories of friction as well as to develop a modification of the adhesion theory.

Most of the important recent contributions to frictional theory have come out of the Research Laboratory of the Physics and Chemistry of Surfaces, Department of Physical Chemistry, Cambridge, England. This work has been led by Bowden and Tabor. Their book on the "Friction and Lubrication of Solids" stands out as a milestone in this subject. In this book, the adhesion theory of friction is reviewed. It can be simply stated that the friction of materials may be due to the adhesion at contact points "and represents the force necessary to shear these small junctions." Bowden considers as a rough approximation that the frictional force may be $F = \mu sA$, where $A$ is the contact area and $s$ the shear strength of the junctions. Since the coefficient of friction is defined as the ratio of the frictional force to the normal force, i.e., $\mu = \frac{F}{W}$, and since the normal force $W$ and friction force $F$ are usually proportional to the contact area, it is often considered that $\mu$ is a constant. Unfortunately this is not the case with plastics and fibers; in fact, the relationship of normal force, contact area, and coefficient of friction has been the subject of several researches in the past years.
Shearing at the interface often occurs within the boundaries of one or both of the materials, resulting in the transfer of fragments. The notable exception is Teflon, or polytetrafluoroethylene, where the unusually low values of coefficient of friction are accompanied by slippage at the interface.

Pascoe and Tabor\textsuperscript{13} in discussing friction in polymers and in particular fibers, attempt to correlate the coefficient of friction with an estimated area of contact $A$. They make the following assumptions:

1. "The friction arises from the shearing of junctions formed at the region of real contact..."

2. "The area of real contact is the same as $A$ (estimated area)."

3. "The shear strength of the junctions is a constant, $s$, for any one material and is of the same order of magnitude as the shear strength of the polymer..."

They then write the basic equation:

$$\mu = s \frac{A}{W}$$

and substitute experimental relations for $A$ as a function of $W$ and fiber diameter, which leads to the relationship for nylon
\[ \mu = c W^{-0.26} D^{0.52} \]

King and Tabor\(^5\) in discussing the friction of plastics previously attempted a simpler derivation, in which they assumed \( \mu \) to be equal to \( s/p \). Again \( s \) is the shear strength of the material, and \( p \) is the yield pressure. They studied four materials at different temperatures. In general, \( \mu \) and \( s/p \) varied in the same manner, but the agreement with experimental data was not good. \( \mu \) was higher than \( s/p \) for polythene, Kel-F and Perspex, but lower for Teflon (P.T.F.E.).

Empirically, it has been found that \( \mu \) varies with the normal load \( W \) in the following manner:

\[ \mu = a + \frac{b}{W^\alpha} \]

where \( \mu \) is of the order of \( .5 \) to \( 1 \). By fitting data to the above equation with various values of \( \alpha \), \( a \), and \( b \), Gralen found the following values of the sum of squares of deviations.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( a )</th>
<th>( b )</th>
<th>Sum of Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>.27</td>
<td>.60</td>
<td>0.0031</td>
</tr>
<tr>
<td>0.7</td>
<td>.20</td>
<td>.60</td>
<td>0.0024</td>
</tr>
<tr>
<td>0.5</td>
<td>.10</td>
<td>.65</td>
<td>0.0030</td>
</tr>
</tbody>
</table>
The best fit is said to be for $\alpha = 0.7$, but it is not a clear cut choice by any means. Similar variations of $\mu$ with $W$ have been reported for many types of materials including glass and metals.

One of the most informative papers in this field has been that by Chapman and Menter$^2$. They have observed the attrition of the surface of a nylon monofilament subjected to one rub by a platinum slider at varying normal loads. The resulting wear strongly supports the adhesion theories of wear, as material has been sheared off within the boundary and away from the surface of contact. (Photographs showing these results are shown in Figures 2.1 - 2.4, and detailed discussion of the results is given later.)

In all of the papers dealing with the yield pressure and shear strength of materials, it is considered that flow will take place when the tangential or frictional stress equals the shear strength of the sample. However, consideration of the combined stresses, i.e. compression and shear, by Mohr's Circle shows that a shear stress greater than the tangential stress occurs on a different plane.

McFarlane and Tabor$^{14}$ have applied the Von Mises theory of plasticity for two dimensions:

$$p^2 + 3s^2 = r^2$$
Figures 2.1 to 2.4 reproduced from Chapman and Menter.

"The effect of frictional wear on a 50\textmu diameter monofilament of drawn nylon. A flat platinum slider has been rubbed once along the fibre at various loads."\textsuperscript{2}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.1.png}
\caption{Undamaged fibre}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.2.png}
\caption{Damage at a load of 40 g.}
\end{figure}
FIGURE 2.3
"Damage at a load of 160 g."

FIGURE 2.4
"Damage at a load of 200 g."

where \( p \) and \( s \) are the normal and shear stresses and \( Y \) is the elastic limit or yield stress of the material. They discuss the case in which \( p = Y \) and hence no tangential shear stress is required to produce flow, since the plastic flow stress has already been reached. For the three dimensional case, they have used:

\[
p^2 + \alpha s^2 = k^2
\]

and found that their experiments with steel on indium give a value of \( \alpha = 3.3 \). This approach ultimately leads to an equation relating the coefficient of friction and an adhesion coefficient, \( D \),

\[
\mu^2 = 0.3D^2 - 0.3
\]

They point out that the same relationship will hold if a Tresca-Mohr or maximum shear stress criterion is used instead of the Von Mises criterion.

One of the possible errors in McFarlane and Tabor's analysis is their assumption that the normal and shear stresses are uniform over the entire area of contact. It can be shown that this assumption is valid only when true plastic flow has occurred over the entire area.
In the case of contact between elastic cylindrical bodies, Poritsky has calculated the variation of the normal stress. This variation is also described in the better known Hertzian approach.

In the approach that follows, a combined two dimensional stress theory is developed, using a maximum shear stress criterion. Let us take an element on the surface of a fiber in contact with a hard plate. There is a normal force, \( W = \int p dA \), and a frictional force, \( F = \int f dA \), acting over the entire surface of contact. In addition, there can be a tension force, \( T = \int \sigma dA^* \) acting over the cross section of the fiber. The quantities \( p \), \( f \), and \( \sigma \) are the normal stress, shear stress, and tensile stress acting on our element or unit cube. \( A = \) the contact area, and \( A^* = \) fiber cross sectional area. By the adhesion theory of friction, plus the observations of Chapman and Menter, let us assume that deformation flow will take place when some limiting value of shear stress is developed. It is possible that adhesion may be so low that slippage takes place at the interface between the fiber and hard surface. This slippage is not covered by the adhesion theory. In this discussion, it is assumed that the combined stresses set up by compression, tension, and shear produce a shear stress in some plane which exceeds the shear flow stress, \( S_0 \), of the material.
By use of Mohr's circle in two dimensions, it can be shown that the combined stresses produce a maximum shear stress given by the following relationship:

\[
S^* = \sigma^2 + \left(\frac{p + \sigma}{2}\right)^2
\]  
(2.1)

Flow will take place when \( S^* = S_o \). Solving for the friction stress, we get

\[
\sigma = \left[S_o^* - \left(\frac{p + \sigma}{2}\right)^2\right]^{1/2}
\]  
(2.2)

The equation for coefficient of friction now becomes

\[
\mu = \frac{\int \sigma f \, dA}{\int \sigma p \, dA} = \frac{\int \left[S_o^* - \frac{1}{4}(p + \sigma)^2\right]^{1/2} \, dA}{\int \sigma p \, dA}
\]  
(2.3)

For many cases, it is quite difficult to evaluate this equation except by qualitative means. One evaluation can be made in the case of an ideal plastic material. Here, \( \sigma = \sigma_o \); \( p = p_o \); and \( S = S_o \); all constants. This resolves to \( \mu = F/W \) or

\[
\mu = \frac{\left[S_o^* - \frac{1}{4}(p_o + \sigma_o)^2\right]^{1/2}}{p_o} = \left[\frac{S_o^*}{p_o} - \frac{1}{4}(1 + \frac{\sigma_o}{p_o})^2\right]^{1/2}
\]  
(2.3a)

It can be seen that if \( \left(\frac{\sigma_o}{p_o}\right)^2 \approx 3D^2, \sigma_o = 0 \)

(where \( D = "\text{adhesion coefficient}" \)
Equation 2.3a becomes

\[ \mu^* = 0.3D^3 - 0.25 \]

which is very similar to the equation developed by McFarlane and Tabor.

Another case which can be treated is that of an elastic cylinder (fiber) in contact with a hard plate or surface. This case has been treated by Hertz (see Finch) and Poritsky. The normal stress, \( p \), is given as a function of the position on the interface of contact. The width of the strip of contact is \( 2r \), and the distance from the center line is \( x \). The relationship is

\[ p(x) = \frac{2P}{\pi r^2} \sqrt{r^2 - x^2} \]  
(2.4)

where \( P \) = normal force per unit length. At the center \( p = p_{\text{max}} = \frac{2P}{\pi r^2} \). The mean pressure is given by \( p = \frac{P}{2r} \). Now using Equation 2.3 and substituting in Equation 2.4 we get:

\[ \int_{-r}^{+r} \left[ S^* - \frac{1}{4} \left( 6 + \frac{2 \beta}{\pi r^2} \sqrt{r^2 - x^2} \right) \right] \frac{1}{2} d\alpha \]

\[ \mu = \int_{-r}^{+r} \frac{\frac{2P}{\pi r^2} \sqrt{r^2 - x^2}}{\int_{-r}^{+r} \frac{\frac{2P}{\pi r^2} \sqrt{r^2 - x^2}}{d\alpha}} \]  
(2.5)

If we take a qualitative look at Equation 2.5, we can get an idea of how the coefficient of friction can vary with different factors.
(a) $S_0$ is the shear strength and, for fibers, is dependent upon the rate of deformation. In general, $s$ increases with speed; hence $\mu$ can be expected to increase with speed.

(b) $\tau$ is the tensile stress applied to the yarn or fiber. Increases in $\tau$ will result in a decrease in $\mu$.

(c) $p$ is the normal stress. An increase in $p$ will cause a decrease in $\mu$.

All of these observations are consistent with experimental observations$^{11,12,16,17,18}$.

Equation 2.5 can be integrated for the elastic case, if we let $\tau = 0$. The denominator integrates directly to the normal load, $W = F$. The numerator becomes:

$$\int_{-r}^{+r} \left[ S_0 - \frac{1}{4} \cdot \frac{p}{n \cdot r^4} (r^2 - x^2) \right] \frac{1}{2} d\lambda$$

or, in dimensionless form,

$$\frac{p \cdot l}{n} \int_{-1}^{+1} \left[ \left( \frac{n \cdot S_0 \cdot r^4}{p^2} \right) - 1 \right] + \left( \frac{x}{r} \right)^2 \frac{1}{2} d\lambda$$

which integrates to:
\[
\frac{W}{\pi} \left[ \frac{s_0 \pi r}{P} + \frac{1}{2} \left( \frac{s_0 \pi r^2}{P^2} - 1 \right) \ln \left( \frac{\frac{s_0 \pi r^2}{P^2} + 1}{\frac{s_0 \pi r^2}{P^2} - 1} \right) \right]
\]  
(2.7)

From Poritsky\textsuperscript{15} we can substitute

\[ P_{\text{max}} = \frac{2P}{\pi r} \]

into Equation 2.7 and rewrite Equation 2.5

\[
\begin{aligned}
\mu &= \frac{1}{\pi r} \left\{ \frac{2s_0}{P_{\text{max}}} + \frac{1}{2} \left[ \left( \frac{s_0}{P_{\text{max}}} \right)^2 - 1 \right] \ln \left( \frac{\frac{4s_0}{P_{\text{max}}} + 1}{\frac{4s_0}{P_{\text{max}}} - 1} \right) \right\}
\end{aligned}
\]  
(2.8)

The significance of the quantity \( \frac{2s_0}{P_{\text{max}}} \) can be seen from a Mohr circle diagram. Under the conditions of a compressive stress only, plastic flow will occur when

\[
\frac{2s_0}{P_{\text{max}}} = 1
\]  
(2.9)

For values of \( P_{\text{max}} \) less than \( 2s_0 \), the stresses will be elastic; and for the plastic case \( P_{\text{max}} \) can not exceed \( 2s_0 \). Equation 2.8 gives an expression for the coefficient of friction in terms of a quantity which represents a dimensionless yield pressure ratio.
Table 2.1 lists values of the coefficient of friction calculated from Equation 2.8.

**TABLE 2.1**

Calculated Values of Friction Coefficient

<table>
<thead>
<tr>
<th>$\frac{2S_o}{P_{max}}$</th>
<th>$\frac{P_{max}}{2S_o}$</th>
<th>$\mu \pi$</th>
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<td>1</td>
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<td>3.00</td>
<td>.33</td>
<td>3.89</td>
<td>1.238</td>
</tr>
<tr>
<td>4.00</td>
<td>.25</td>
<td>4.92</td>
<td>1.566</td>
</tr>
<tr>
<td>5.00</td>
<td>.20</td>
<td>5.92</td>
<td>1.884</td>
</tr>
<tr>
<td>10.00</td>
<td>.10</td>
<td>10.98</td>
<td>3.495</td>
</tr>
</tbody>
</table>

These data are plotted in Figure 2.5.
FIGURE 2.5. Calculated values of friction coefficient as a function of $\frac{P_{\text{max}}}{2S_0}$. 
Gralen, Olafson, and Lindberg\textsuperscript{19} have treated the problem of friction in a somewhat similar manner, starting with the Hertz equations which are similar to those of Poritsky. They mention two additional equations which are useful. The first gives the width of the area of contact between two cylinders:

\[
W = \frac{4W_R \times 0.91}{\pi l}
\]

(2.10)

"where \(R\) is the radius of each of the cylinders (if equal) and \(E\) is the modulus of elasticity of the material.\textsuperscript{19}\]

The second is

\[
P_{max} = \frac{WE}{\pi lR \times 0.91}
\]

(2.11)

"If the radii of the cylinders are different, \(R_1\) and \(R_2\), \(R\) in the above formulas is substituted by the harmonic means of the radii:\textsuperscript{19}\]

\[
\frac{2}{R} = \frac{1}{R_1} + \frac{1}{R_2}
\]

(2.12)

They go on to develop a theory of friction based on the simple adhesion theory in combination with the Hertz equations for the area of contact. Their final equation for the coefficient
of friction splits into one for the elastic region of strain,

$$\mu = \alpha + \frac{\beta LR}{W}$$  \hspace{1cm} (2.13)

and one for plastic strain,

$$\mu = \gamma \sqrt{\frac{LR}{W}}$$  \hspace{1cm} (2.14)

In these equations,

$$\alpha = \frac{S}{P_{\text{max}}} \quad , \quad \beta = \frac{aS_{\text{max}}}{E} \quad , \quad a = 0.78$$

$$\gamma = \frac{2.15 \times S}{VE}$$

$S$ is the force per unit area which is necessary to shear junctions and corresponds to the term $F$ as used earlier rather than the shear strength of the material $S_o$. Their data fits their equations when $\alpha = 0.257, \beta = 3.67 \times 10^2$ for nylon fibers (3 den, diameter $\approx 19 \mu$).

For Terylene fibers, the constants were $\alpha = .266$ and $\beta = 12.7 \times 10^2$.

They also have calculated values of $S$ and $P_m$ for the nylon fibers, $0.06 \times 10^4 \text{ gm/mm}^2$ and $0.20 \times 10^4 \text{ gm/mm}^2$ respectively. They give these values as the measures of the strength of the weld (s) and the yield pressure ($P_m$).
Comparing their data with the values of yield pressure given previously, \(0.8 \times 10^4 \text{ gms/mm}^2\) and \(0.3 \times 10^4 \text{ gms/mm}^2\), it can be seen that their test conditions are within the elastic region. Their values of \(S\) and \(P_m\) do not in fact correspond to the measures of strength and yield pressure, but are the frictional stresses and normal pressures of their experiment.

In a further note, Olafsson\(^{20}\) points out:

"...Later work by Howell, et al.\(^{10}\) and Lincoln\(^{21}\) has shown that the frictional forces for textile materials may be explained by considering the deformations to be of a wholly elastic character. For large values of \(W\)
\[
\left[ \frac{2S_o}{P_{\text{max}}} = 1 \right]
\]
(a plastic deformation may occur (experiments are not decisive), but for small values of \(W\) the deformation should be elastic as the theory in (19) also indicates.

".....But all experiments justify the opinion that the mechanism of fiber friction is a shearing at the contact surfaces, the frictional force being proportional to the surface area, and this area being determined by the viscoelastic properties of the fibers."

While it is difficult to integrate Equation 2.5 when \(\varphi\) is not equal to zero, it is possible to show qualitatively that a non-zero value will tend to reduce the
coefficient of friction. If we square the quantity \((\sigma + \rho)\) which appears in Equation 2.3, we get \(\sigma^2 + 2\sigma \rho + \rho^2\).

The difficulty in integrating Equation 2.5 comes from the cross product, \(2\sigma \rho\). As a first approximation, let us ignore the difficulty, and set:

\[
\sigma^2 + 2\sigma \rho = c^2 \sigma^2
\]  

(2.15)

where

\[
c^2 = 1 + \frac{2 \rho}{\sigma}
\]

If we let \(C\) be an adjustable constant, the tensile stresses will combine with the term \(S_0^2\) to form a new factor, \(S_0^2 - \frac{c^2}{4} \sigma^2\). This can now be introduced into all the subsequent equations replacing \(\frac{4S_0^2}{P_{\text{max}}}\) by \(\frac{4S_0^2 - C^2 \sigma^2}{P_{\text{max}}}\). This might then be considered to reduce the coefficient of friction as the tension is increased, as described above and shown in Figure 2.5a.

Another interesting comparison can be made using Equation 2.8 if we apply the series expansion for the logarithmic term and let \(\frac{2S_0}{P_{\text{max}}} = Y\)

\[
\ln \left(\frac{v_e}{v_{e-1}}\right) = 2 \left[ \frac{1}{Y^2} + \frac{1}{3} \left(\frac{1}{Y^2}\right)^2 + \frac{1}{5} \left(\frac{1}{Y^2}\right)^3 + \frac{1}{7} \left(\frac{1}{Y^2}\right)^4 + \cdots \right]
\]

It can be shown that for values of \(\mu\) greater than .6 or
FIGURE 2.5a. Coefficient of friction as a function of tension.
\( Y^2 > 2 \), Equation 2.8 simplifies to

\[
\mu = \frac{1}{\pi} \left[ Y + 1 - \frac{1}{Y^2} \right] = \frac{1}{\pi} + \frac{1}{\pi} (Y - \frac{1}{Y^2})
\]  

(2.16)

We can now introduce the Hertzian relationship of Equation 2.11, as related to the problem of Gralen, et al., which gives

\[
Y^2 = \left( \frac{2S_0}{P_{c0}} \right)^2 = \left( \frac{\pi R \times 0.9}{WE} \right) (2S_0)^2 = \frac{B^2}{W}
\]  

(2.17)

or

\[
Y = \frac{B}{W^{1/2}}
\]

Substitution of Equation 2.17 in Equation 2.16 gives

\[
\mu = \frac{1}{\pi} + \frac{B}{\pi W^{1/2}} - \frac{W}{\pi B^2} = a' + \frac{b'}{W^{1/2}} - c' W
\]  

(2.18)

For the case of a cylinder in contact with a hard plane, the above equation would be

\[
\mu = a' + \frac{b'}{W^{1/3}} - c' W^{4/3}
\]  

(2.19)
The coincidence of these equations with those derived by Graelen \(^{18}\) is striking.

\[
\mu = a + \frac{b}{W^n} \quad \text{(Graelen)}
\]

\[
\mu = \alpha' + \frac{b'}{W^n} - c'W^n \quad \text{(Morgan)}
\]

The constants which Graelen reported were \(a = .20; \ \alpha = .7;\) and \(b = .60;\) or \(a = .27; \ b = .60;\) and \(\alpha = 1.\) The constants derived here would be

\[
\begin{align*}
\alpha' &= .318 \\
b' &= \frac{250}{\pi} \sqrt{\frac{\pi LRx0.91}{E}} \\
m &= .50 \ \text{and} \ 0.67
\end{align*}
\]

depending on test conditions. \(b^1\) and \(c^1\) involve "sticky" dimensions and have not been evaluated. One of the advantages of the formulation developed in this paper is the fact that all the relationships are non-dimensional when they are kept in their simplest form.

Graelen, Olafsson and Lindberg have evaluated their Equation 13 for a range of values of \(l, \ R, \) and \(W.\) Their range corresponds to values of \(Y\) between about 1.01 and 1.5.
It can be seen that two definite cases exist. In one, the coefficient of friction is substantially constant and is not a function of the normal force. In the second, the coefficient is not constant and is very definitely a function of the normal force. The first case can be thought of as occurring when plastic flow occurs as a result of the normal pressure's exceeding a given value. It will be shown that the work of Chapman and Menter\textsuperscript{2} involved these conditions. Hence the constancy of $\mu$ which they observed is not surprising.

The second case is the one in which normal pressures are less than the yield value. Under these conditions, elastic deformations occur. In any given experiment, when the expected behavior of the coefficient of friction is sought, it becomes necessary to know which of the two cases is involved. It will be shown that the choice depends upon the normal pressures encountered. An estimate has been made of the normal loads involved in the experiments of Chapman and Menter\textsuperscript{2} and Pascoë and Tabor\textsuperscript{13}. These normal pressures must be evaluated in terms of the yield pressures of the materials. Yield pressures have been calculated for a number of monofilaments from the data of Coffin\textsuperscript{8}. In addition, Coffin's experimental technique has been used to obtain the same information on undrawn nylon 66 fibers which were used in the cold drawing experiments to be described later.
### TABLE 2.2

Lateral Deformation of Nylon - Partially Oriented

<table>
<thead>
<tr>
<th>Area (in)</th>
<th>Force (gms)</th>
<th>Force/Area (in x 10^-4)</th>
<th>Dynes/cm² (in x 10^7)</th>
<th>Gpd</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 x 10^-4</td>
<td>10</td>
<td>.23 x 10^4</td>
<td>22.5 x 10^7</td>
<td>.22</td>
</tr>
<tr>
<td>40</td>
<td>21</td>
<td>.53 &quot;</td>
<td>51.9 &quot;</td>
<td>.52</td>
</tr>
<tr>
<td>54</td>
<td>31.5</td>
<td>.58</td>
<td>56.8</td>
<td>.57</td>
</tr>
<tr>
<td>64</td>
<td>41.5</td>
<td>.65</td>
<td>63.7</td>
<td>.63</td>
</tr>
<tr>
<td>68</td>
<td>52</td>
<td>.76</td>
<td>74.5</td>
<td>.74</td>
</tr>
<tr>
<td>82</td>
<td>62.5</td>
<td>.76</td>
<td>74.5</td>
<td>.74</td>
</tr>
<tr>
<td>94</td>
<td>73</td>
<td>.78</td>
<td>76.4</td>
<td>.76</td>
</tr>
<tr>
<td>110</td>
<td>87</td>
<td>.79</td>
<td>77.4</td>
<td>.77</td>
</tr>
</tbody>
</table>

Diameter 9 mils Loading Rate 1.61 gms/min.

<table>
<thead>
<tr>
<th>Area (in)</th>
<th>Force (gms)</th>
<th>Force/Area (in x 10^-4)</th>
<th>Dynes/cm² (in x 10^7)</th>
<th>Gpd</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 x 10^-4</td>
<td>48</td>
<td>.80 x 10^4</td>
<td>.975 x 10^-4</td>
<td>.78</td>
</tr>
<tr>
<td>120</td>
<td>95</td>
<td>.79</td>
<td></td>
<td>.77</td>
</tr>
<tr>
<td>170</td>
<td>140</td>
<td>.82</td>
<td></td>
<td>.80</td>
</tr>
<tr>
<td>220</td>
<td>188</td>
<td>.85</td>
<td></td>
<td>.83</td>
</tr>
<tr>
<td>270</td>
<td>235</td>
<td>.85</td>
<td></td>
<td>.83</td>
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<tr>
<td>340</td>
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<td>.82</td>
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<td>.80</td>
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<tr>
<td>420</td>
<td>323</td>
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<td>.75</td>
</tr>
<tr>
<td>460</td>
<td>375</td>
<td>.82</td>
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<td>.80</td>
</tr>
<tr>
<td>560</td>
<td>422</td>
<td>.75</td>
<td></td>
<td>.73</td>
</tr>
<tr>
<td>620</td>
<td>470</td>
<td>.76</td>
<td></td>
<td>.74</td>
</tr>
</tbody>
</table>
TABLE 2.3
Lateral Deformation of Polyethylene

<table>
<thead>
<tr>
<th>Area (sq in)</th>
<th>Force (lbs)</th>
<th>F/A</th>
<th>Stress (gpd)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$110 \times 10^{-4}$</td>
<td>10.5</td>
<td>$0.95 \times 10^4 \times 1.23 \times 10^{-4}$</td>
<td>0.11</td>
</tr>
<tr>
<td>165</td>
<td>21.0</td>
<td>0.13</td>
<td>0.16</td>
</tr>
<tr>
<td>260</td>
<td>35</td>
<td>0.13</td>
<td>0.16</td>
</tr>
<tr>
<td>350</td>
<td>45</td>
<td>0.13</td>
<td>0.16</td>
</tr>
<tr>
<td>400</td>
<td>55.5</td>
<td>0.14</td>
<td>0.17</td>
</tr>
<tr>
<td>475</td>
<td>66.0</td>
<td>0.14</td>
<td>0.17</td>
</tr>
<tr>
<td>510</td>
<td>76.5</td>
<td>0.15</td>
<td>0.18</td>
</tr>
<tr>
<td>560</td>
<td>86.5</td>
<td>0.15</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Diameter 12 mils
Loading Rate - 7.24 gms/min

<table>
<thead>
<tr>
<th>Force (lbs)</th>
<th>F/A</th>
<th>Stress (gpd)</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>48</td>
<td>$0.19 \times 10^4$</td>
</tr>
<tr>
<td>440</td>
<td>95</td>
<td>0.22</td>
</tr>
<tr>
<td>720</td>
<td>140</td>
<td>0.19</td>
</tr>
<tr>
<td>930</td>
<td>188</td>
<td>0.20</td>
</tr>
<tr>
<td>1080</td>
<td>235</td>
<td>0.22</td>
</tr>
<tr>
<td>1450</td>
<td>280</td>
<td>0.19</td>
</tr>
<tr>
<td>2020</td>
<td>323</td>
<td>0.16</td>
</tr>
<tr>
<td>2600</td>
<td>375</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Diameter 12 mils
Loading Rate 20.2 gms/min

<table>
<thead>
<tr>
<th>Force (lbs)</th>
<th>Stress (gpd)</th>
</tr>
</thead>
<tbody>
<tr>
<td>480</td>
<td>0.18</td>
</tr>
<tr>
<td>940</td>
<td>0.19</td>
</tr>
<tr>
<td>2000</td>
<td>0.18</td>
</tr>
</tbody>
</table>


### TABLE 2.4

Lateral Deformation of Dacron

Diameter 9 mils  Loading Rate 1.61 gms/min.

<table>
<thead>
<tr>
<th>Area</th>
<th>Force</th>
<th>F/A</th>
<th>gpd</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 x 10^{-4}</td>
<td>10.5 gms</td>
<td>.58 x 10^{-4} (x .825 x 10^{-4})</td>
<td>.48</td>
</tr>
<tr>
<td>31</td>
<td>21</td>
<td>.68</td>
<td>.56</td>
</tr>
<tr>
<td>45</td>
<td>31.5</td>
<td>.70</td>
<td>.58</td>
</tr>
<tr>
<td>59</td>
<td>41.5</td>
<td>.70</td>
<td>.58</td>
</tr>
<tr>
<td>70</td>
<td>52</td>
<td>.74</td>
<td>.61</td>
</tr>
<tr>
<td>79</td>
<td>62.5</td>
<td>.79</td>
<td>.65</td>
</tr>
<tr>
<td>92</td>
<td>73</td>
<td>.79</td>
<td>.65</td>
</tr>
<tr>
<td>111</td>
<td>87</td>
<td>.78</td>
<td>.65</td>
</tr>
</tbody>
</table>

Diameter 9 mils  Loading Rate 7.24 gms/min.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>48</td>
<td>.60</td>
<td>.50</td>
</tr>
<tr>
<td>140</td>
<td>95</td>
<td>.68</td>
<td>.56</td>
</tr>
<tr>
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<td>.82</td>
<td>.68</td>
</tr>
<tr>
<td>240</td>
<td>188</td>
<td>.78</td>
<td>.65</td>
</tr>
<tr>
<td>260</td>
<td>235</td>
<td>.90</td>
<td>.75</td>
</tr>
<tr>
<td>350</td>
<td>280</td>
<td>.80</td>
<td>.66</td>
</tr>
<tr>
<td>380</td>
<td>323</td>
<td>.85</td>
<td>.70</td>
</tr>
<tr>
<td>450</td>
<td>375</td>
<td>.83</td>
<td>.69</td>
</tr>
<tr>
<td>540</td>
<td>423</td>
<td>.78</td>
<td>.65</td>
</tr>
<tr>
<td>580</td>
<td>470</td>
<td>.81</td>
<td>.67</td>
</tr>
<tr>
<td>670</td>
<td>513</td>
<td>.77</td>
<td>.64</td>
</tr>
</tbody>
</table>

Ave. 180 Ave. .66

38
Coffin\textsuperscript{8} built and described an experimental apparatus similar to that of Howell and Mazur\textsuperscript{10}. He measured the relationship between force and contact area for different monofilaments with diameters of 9-12 mils. He also made a preliminary investigation of the effect of rate of loading on the force-contact area relationship. Some of his data is tabulated in Tables 2.2 - 2.4. The data contained in these Tables has been calculated in a slightly different way from that of Coffin, in order to make an estimate of the yield pressure in terms of gms/mm\textsuperscript{2} and the better known textile terms of grams per denier.

The lateral yield pressures for three of the fibers studied by Coffin are listed in Table 2.5.

\textbf{TABLE 2.5}

\textbf{Average Lateral Yield Pressure}

\begin{tabular}{|l|l|l|}
\hline
\textbf{Fiber and Conditions} & \textbf{Yield Pressure} \\
\hline
Dacron (1.61 gms/min) & .79 \times 10^4 \text{ gms/mm}^2 & .65 \text{ gpd} \\
Dacron (7.24 gms/min) & .80 \times 10^4 \text{ gms/mm}^2 & .66 \\
Nylon (1.61 gms/min) & .79 \times 10^4 \text{ gms/mm}^2 & .77 \\
Nylon (7.24 gms/min) & .80 \times 10^4 \text{ gms/mm}^2 & .78 \\
Polyethylene (1.61 gms/min) & .14 \times 10^4 \text{ gms/mm}^2 & .17 \\
Polyethylene (7.24 gms/min) & .20 \times 10^4 \text{ gms/mm}^2 & .25 \\
Polyethylene (20.2 gms/min) & .18 \times 10^4 \text{ gms/mm}^2 & .22 \\
\hline
\end{tabular}
Pascoe and Tabor\textsuperscript{13} have given values for the yield pressure of nylon and polyethylene blocks, \(0.73 \times 10^4\) gms/mm\(^2\) and \(0.13 \times 10^4\) gms/mm\(^2\) respectively. These values are in very close agreement with those calculated from Coffin's data. However, their value for Terylene, \(1.6 \times 10^4\) gms/mm\(^2\), is about twice that of the Dacron fiber listed above. These two are presumably the same chemically.

In spite of the one discrepancy, it is assumed that the yield pressures for these three materials has been established. Thus, any time the normal pressure is in excess of these values, it will be expected that plastic yielding will have occurred and that the coefficient of friction will be nearly constant. When normal pressures are lower than these values, \(\mu\) will vary with the normal pressure.

The range of normal pressures which have been covered by Pascoe and Tabor\textsuperscript{13} has been estimated to be \(0.4\) to \(10^3\) gms/mm\(^2\), a figure well below the plastic yield pressure. Consequently, it is not surprising to find the large variations of \(\mu\) which they report.

The other set of conditions has been met in Chapman and Menter's\textsuperscript{2} data. While they do not give their conditions in terms of contact pressures and areas, it is possible to estimate them. Their loading conditions are given in terms of the normal load. In addition, since the length of the platinum slider is given, the force per unit
length of fiber can be calculated. It is estimated that the contact width is in the range of $\frac{1}{4}$ to $\frac{1}{2}$ the fiber diameter at light loads and is equal to the fiber diameter at the heavier loads. Thus for a fiber with a diameter of 25 the loading conditions are estimated in Table 2.6.

**TABLE 2.6**

Lateral Deformation of Nylon

(From Chapman and Menter$^2$)

<table>
<thead>
<tr>
<th>Force</th>
<th>Force/length</th>
<th>Force/area (est.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 gms</td>
<td>50 gms/mm</td>
<td>$2.4 \times 10^4$ gms/mm$^2$</td>
</tr>
<tr>
<td>160</td>
<td>200</td>
<td>$.4 \times 10^4$</td>
</tr>
<tr>
<td>200</td>
<td>250</td>
<td>$.5 \times 10^4$</td>
</tr>
<tr>
<td>320</td>
<td>400</td>
<td>$.8 \times 10^4$</td>
</tr>
</tbody>
</table>

In the last three cases, the plastic yield pressure is approached and equalled. It is therefore not surprising that a large amount of permanent deformation has resulted$^2$. (See Figures 2.1 - 2.4)

One additional set of experiments has been made using single filaments of undrawn nylon 66, the material used in the studies of cold drawing. The filaments were approximately 55 $\mu$ in diameter. They were tested on Coffin's apparatus to observe the lateral yield pressure.
The results of the tests are given in Table 2.6. For this sample of nylon, the yield load appears to be somewhat less than that given above, or in the range of \( 3 \times 10^4 \) gms/mm², which is about 3 gms/den. It is interesting to compare this yield stress with that measured in tension for the same sample. (See Chapter V.) Comparing the stresses at comparable times, i.e. 1 minute, we find for the tensile yield stress a value of 29 gpd. This is a good indication that the undrawn material is reasonably isotropic. It also suggests that the same variation of yield stress with time might be expected in the lateral direction as was found in the longitudinal direction.

One area remains for discussion and that is the order of magnitude of normal pressure which might be expected in the normal running of a yarn over a guide or other such contact point. It can be shown that the normal force per unit length of contact which occurs under such conditions is equal to the running tension divided by the radius of curvature of the bend. However, this does not yet tell us the normal pressure, since the contact width is still unknown.

In the experiment of Table 2.6, it was found that the contact width remained relatively constant, while the contact length increased with the applied force above the yield pressure. On the other hand, if the contact length is fixed, as in the case of Chapman and Menter's experiment,
the contact width must grow with increasing applied force once the yield pressure is reached. In the case of Table 2.6, the contact width remained at about 1/3 the fiber diameter. As an estimate then, let us consider that the contact width will be about 1/3 the fiber diameter at and near the yield stress.

As for the running tensions normally encountered in operation, it is safe to assume that they will be less than the breaking tension of the yarn. In fact, a safer estimate is about .5 to 1 gram per denier. This corresponds to about $.5 \times 10^4$ to $1 \times 10^4$ gms/mm$^2$ of fiber cross section. (If we consider a $25 \mu$ nylon fiber, this force is about 5 grams.) If the contact width is taken as $1/3$ d, this force corresponds to $6.65 \times 10^2$ gms/mm. In order that a normal pressure equal to the yield pressure ($.3 \times 10^4$ gms/mm$^2$) be reached, the $25 \mu$ fiber supporting 1 gm/den must be bent over a radius of curvature given by

$$\rho = \frac{.65 \times 10^2 \text{ gms/mm}}{.3 \times 10^4 \text{ gms/mm}^2} = 2.2 \times 10^{-2} \text{ mm}.$$ 

or over a radius of curvature which is less than the radius of the fiber. If higher forces are being used, the critical radius of curvature will be larger. However, this brief estimate suggests that, under normal conditions, as a fiber or yarn is being passed over a curved guide of some sort,

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it is not likely that the lateral yield pressure will be exceeded. This means that the normal operating conditions for yarns and fibers is in the region of more elastic lateral deformations and less constancy of the coefficient of friction.

The above analysis applied in the case of dry friction, where there is no lubrication. Actually there is usually a certain amount of lubrication present, either intentionally or unintentionally. When lubrication is present, slippage will generally take place in the lubrication layer. The coefficient of friction will then represent the properties of the lubricant. Since most lubricants are fluids, their behavior is that of a normal fluid\(^6,7\). In general, the coefficient of friction increases with speed.

Examples of the effects of lubricants of various sorts have been given by Roder, and are included in Figures 2.6 and 2.7. Another example of the variation of the coefficient of friction with velocity comes from the study for cold drawing to be described later. In this case, undrawn nylon of the type described above was wrapped around a snubbing pin and stretched. During this process, measurements were made of the yarn tension entering the snubbing pin and the yarn tension leaving the pin. Since the pin was one inch in diameter, and since the maximum tension was in the range of .6 to .7 gpd, the normal pressure was well below the yield pressure. The frictional behavior would
FIGURE 2.6. Variation of coefficient of friction with speed.

FIGURE 2.7. Variation of coefficient of friction with speed.

then be expected to be related functionally to the tensions and speeds involved.

Table 2.8 and Figure 2.8 illustrate the results of these experiments. It is difficult to analyze these data for the effect of the tension on the coefficient. The simple belt friction formula was used to calculate $\mu$ and this formula encompasses the entire gamut of tensions. The values of $\mu$ obtained very strongly suggest that the lubricant is playing the dominant role in this case. Pascoe and Tabor have shown that the values of $\mu$ to be expected from unlubricated nylon in the range of low normal pressures is of the order of magnitude of .5 to 1.5. Chapman and Menter have shown that $\mu$ is .3 in the high pressure range. The values of $\mu = .18$ to .22 must be the result of good lubrication from the spin finish.

A sample of undrawn Terylene without any finish of any sort was run through the drawing machine with disastrous results. The lack of finish increased the coefficient of friction to an unmanageable value and allowed excessive static to develop. The role of the anti-static lubricant was dramatically illustrated.

In summary, this short chapter was designed to introduce another concept of friction based on a maximum shear stress theory. While it was integrated for the plastic case only, it was developed for the elastic case as well.

47
<table>
<thead>
<tr>
<th>Feed roll vel.</th>
<th>Draw roll vel.</th>
<th>Coeff. of friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>.14 ft/sec.</td>
<td>.7 ft/sec.</td>
<td>.142</td>
</tr>
<tr>
<td>.18</td>
<td>.9</td>
<td>.150</td>
</tr>
<tr>
<td>.28</td>
<td>1.4</td>
<td>.158</td>
</tr>
<tr>
<td>.38</td>
<td>1.9</td>
<td>.170</td>
</tr>
<tr>
<td>.48</td>
<td>2.4</td>
<td>.172</td>
</tr>
<tr>
<td>.60</td>
<td>3.0</td>
<td>.192</td>
</tr>
<tr>
<td>.78</td>
<td>3.9</td>
<td>.200</td>
</tr>
<tr>
<td>.98</td>
<td>4.9</td>
<td>.212</td>
</tr>
<tr>
<td>1.28</td>
<td>6.4</td>
<td>.226</td>
</tr>
<tr>
<td>1.60</td>
<td>8.0</td>
<td>.246</td>
</tr>
</tbody>
</table>
FIGURE 2.8. Coefficient of friction variation during cold-drawing.
Consideration was given to the lateral yield stress of a few representative fibers and the role this stress has in friction. It was further shown that plastic deformation, and the resulting relative constancy of the coefficient of friction, represent a small minority of the cases of sliding friction normally encountered in textile processing. Brief mention was made of the importance and the role of lubricants. It is safe to say that all commercial applications involving textile processing are controlled by the lubricant used on the textile yarn. Any further details of the role of friction in processing thus depends upon a detailed knowledge of the nature of the lubricant. The role of lubricants is particularly striking in the sewing operation, in which many high velocity contacts are made and broken with each cycle.
References for Chapter II


III. VISCOELASTIC BEHAVIOR

Introduction

It is well known that the mechanical properties of textile fibers and yarns are dependent upon the times involved in determining the properties. The term viscoelasticity has been used to cover time-dependent mechanical behavior.

A viscoelastic system is one which exhibits behavior characteristics of both elastic and viscous components. If such a system is strained to a constant length, the supported stress in the system will decay or "relax". Such a system also exhibits properties which depend upon the rate of testing.

The simplest models for such behavior have been the Maxwell and Voigt models, which are merely series and parallel combinations, respectively, of an elastic spring and a viscous dashpot. A characteristic of such combinations is the phenomenon of relaxation. Relaxation occurs when the time scale of the experimental test is of the order of magnitude of the ratio of the viscosity coefficient of the viscous component to the elastic constant of the spring. This ratio is called the relaxation time in the case of a series or Maxwell model and a retardation time in the case of a parallel
or Voigt model. A simple system has a single value of relaxation or retardation time. Rather than carry both terms in the following discussion, the single term relaxation time will be assumed to stand for this ratio, even though we may be talking about a Voigt model. Two models are used because the differential equations for a Maxwell model are best applied when the strain or strain rate is the independent variable, and the equations for the Voigt model are most convenient when the stress or strain rate is controlled.

It has been observed that very few real materials actually behave as either of these simple systems. One exception, perhaps, is silicone putty, which does show a fairly discrete relaxation time. Real materials, however, are best described by a large combination of simple models. The description is then referred to as a distribution of relaxation times.¹²⁶⁷ Briefly, however, if a material exhibits relaxation or rate dependency over a wide range of testing times, it must be considered to have a wide distribution of relaxation times. A material such as silicone putty has its rate dependency, and hence its relaxation, confined to a narrow time scale.

A true relaxation test is normally performed by elongating a test specimen to a given length and observing the behavior of the supported load as a function of time. While this is not a very valuable technique for the present
purpose, i.e., study of the interaction, it does furnish a
cue to the general time-dependent behavior of the materials
in which we are interested. (For a good summary, see Meredith's
Mechanical Properties of Textile Fibers, \textsuperscript{2} Chapter III.)

The clue needed is the fact that the stress re-
relaxation takes place over many orders of magnitude of time.
The time axis must be plotted as the logarithm in order to
observe the entire behavior. Thus, while a simple model will
relax over two cycles of log time, actual materials continue to
relax over as many as 10 to 12 cycles of log time. It is this
spreading out of stress in the function of relaxation times which
makes it necessary to introduce the concept of a distribution.

As long as a test time is within this distri-
bution or spectrum, rate dependency and relaxation occur. If
the test is made at times faster than the fastest relaxation
time in the material, the material will behave purely elasti-
cally. If the test is made at slower times than the slowest
relaxation time, the material will appear to be viscous. While
tests can be made on silicone putty at rates faster than,
slower than, and with the relaxation spectrum, textile fibers
are always used at times within the spectrum. Even though very
wide ranges in test times are employed, it can be shown that,
at room temperatures, relaxation effects always occur.

A mathematical expression for the above statements
can be given by

\[ \sigma = \sigma(\varepsilon, t) \]
\[ \varepsilon = \text{strain} \]
\[ t = \text{time} \quad (3.1) \]

It is the usual custom to determine the change in stress supported by a tensile specimen by varying the strain, time, or both in a prescribed fashion. Changes in stress are given by:

\[ \frac{d\sigma}{dt} = \frac{\partial \sigma}{\partial \varepsilon} \frac{d\varepsilon}{dt} + \frac{\partial \sigma}{\partial t} \frac{dt}{\partial t} \quad (3.2) \]

A particularly convenient variation which is often used in testing is to vary the strain at a constant rate, i.e. \( \frac{d\varepsilon}{dt} = R \). Then

\[ \frac{d\sigma}{d\varepsilon} = \left( \frac{\partial \sigma}{\partial \varepsilon} \right)_{dt=0} + \left( \frac{\partial \sigma}{\partial \varepsilon} \right)_{\varepsilon=0} \frac{dt}{d\varepsilon} \quad (3.3) \]

\[ = \left( \frac{\partial \sigma}{\partial \varepsilon} \right) + \frac{1}{R} \left( \frac{\partial \sigma}{\partial \varepsilon} \right) \]

The stress-strain-time relationship can be defined as a three-dimensional surface. The complete definition has been treated in numerous places. The development

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which follows has the primary aim of utility in the specific problem of the interaction of fibers and processes.

**Stress Relaxation at Constant Strain Rate**

While a great deal has been written about distribution of relaxation times and the mechanical behavior of polymers, most of this published work has dealt with data obtained from either stress-relaxation at constant strain or from dynamic vibratory tests. Smith\(^{12}\) has written one of the few papers in which the test technique involved the treatment of data obtained at a constant strain rate. Since constant strain rates are so widely used in materials testing, this approach deserves wider attention.

The mathematical development is derived from the familiar equation for the behavior of a distribution of Maxwell units:

\[
\frac{d\mathcal{E}}{dt} = \frac{1}{E(\tau)} \frac{d\tau'(\tau)}{dt} + \frac{\tau'(\tau)}{\tau E(\tau)} \tag{3.4}
\]

where \(\mathcal{E}\) is the strain, \(\tau'\) the partial stress, \(E(\tau)\) the partial modulus and \(\tau\) the relaxation time. (For a short exposition of this treatment see Physical Chemistry of High Polymeric Systems by H. Mark and A.J. Tobolsky, p. 334, Interscience, N.Y., 1950.) This equation is set equal to zero for stress relaxation at constant strain. For the case
of constant strain rate, it is equal to a value, $R$.

In the general form, Equation 3.4 gives as the stress-strain relationship:

$$
\sigma = R \int_{0}^{\infty} E(\tau) \left[1 - e^{-t/\tau}\right] d\tau
$$

(3.5)

A very common type of distribution function which has been useful in the description of the behavior of fibrous polymers is the so-called box distribution:

$$
E(\tau) = \begin{cases} 
\frac{E_0}{\tau}, & (\tau_2 < \tau < \tau_m) \\
0, & (\tau < \tau_2, \tau > \tau_m)
\end{cases}
$$

(3.6)

When this distribution is applied to stress-relaxation at constant strain, $E_0$, the stress-time relation has been shown to be:

$$
\sigma = E_0 F_0 \left[ E_i(-t/\tau_2) - E_i(-t/\tau_m) \right]
$$

(3.7)

where $E_i(-t/\tau_2)$ and $E_i(-t/\tau_m)$ are exponential integral functions. Application of the box distribution to Equation 3.5 gives:

$$
\sigma = R E_0 \left\{ \tau_m (1 - e^{-t/\tau_m}) - \tau_2 (1 - e^{-t/\tau_2}) \\
+ t \left[ E_i(-t/\tau_2) - E_i(-t/\tau_m) \right] \right\}
$$

(3.8)
In the particular case of a very broad distribution of relaxation times and a test time somewhere well within the limits, $\tau_2 \ll t \ll \tau_m$, Equation 3.8 becomes:

$$\Sigma = R T E_0 \{1 + E_i \left( -\frac{t}{\tau_2} \right) - E_i \left( -\frac{t}{\tau_m} \right) \}$$

(3.9)

With the exception of the value of unity, the right hand side of Equation 3.9 is equivalent to that of Equation 3.7. It has been shown that when $\tau_m \gg \tau_2$, the relaxation function involving the exponential integral functions reduces to:

$$\Sigma = E_o E_0 \left[ -0.577 - \ln t + \ln \tau_m \right]$$

(3.10)

Thus, Equation 3.9 reduces to:

$$\Sigma = R T E_0 \left[ 1 - 0.577 - \ln t + \ln \tau_m \right]$$

(3.11)

The factor of 1 corresponds to a shift along the time scale of 1 when stress, $\sigma$, is plotted versus $\ln t$, and $1/2.303$ when stress is plotted versus $\log_{10} t$. It should be pointed out, also, that the time, $t$, is to be interpreted differently in the two equations. The $t$ in Equations 3.7 and 3.10 refers to the time after the constant strain is reached. The initial
time of straining the sample is neglected. In Equations 3.9 and 3.11, \( t \) is the elapsed time of the constant strain rate test, i.e., the time required to reach a given strain, \( \varepsilon_0 \).

Stress relaxation curves at constant strain rate can be constructed from stress-strain curves performed at different rates by connecting points of equal strain on a stress-log time plot. Figure 3.1 pictures stress-relaxation data obtained on undrawn nylon 66 by both constant strain and constant strain-rate techniques. The two time scales used have been shifted by the factor 1/2.303. Further examples are shown in Figures 3.2, 3.3 and 3.4. The data used in these figures are taken from published curves of Smith, McCrackin and Schieffer \(^{16}\). The fact that the relaxation lines are reasonably straight justifies the use of a box distribution of relaxation times.

The disadvantage of relaxation tests at constant strain is the fact that the short time data is only as good as the speed at which the test strain is reached. In order to achieve short loading times, it is necessary to start straining quickly and stop straining quickly. Tests of this sort often produce annoying vibrations. With the technique described above, the reliability of the data depends only on the quick straining. Thus, half of the experimental difficulties are
FIGURE 3.1. Stress relaxation of undrawn nylon at 10% extension.

--- at constant strain rate
--- at constant strain
FIGURE 3.2. Stress relaxation at constant strain rate for Fortisan. Replotted from data of Smith, McCrackin and Schiefer.\textsuperscript{3}
FIGURE 3.3. Stress relaxation at constant strain rate for fiberglass. Replotted from data of Smith, McCrackin and Schiefer.  

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FIGURE 3.4. Stress relaxation at constant strain rate for drawn nylon. Replotted from data of Smith, McCrackin and Schiefer.

eliminated. The acquisition of reliable data at shorter times than have been reported previously is possible by the study of stress relaxation at constant strain rate.

It will be noticed that the slopes of lines of different elongations may differ, i.e., higher elongations have higher slopes, indicating a variation in the behavior of different loads, but all of the lines are essentially straight. Thus, interpolations within this time range can safely be made on these graphs, and the load elongation behavior determined at intermediate strain rates.

In addition, while it has not been previously stated, the value of the negative slope of the stress relaxation versus time curve is directly proportional to the height of the distribution function. For example, if there is no relaxation occurring, the slope of the stress relaxation function will be zero over a part of the time scale. Hence, there are no relaxation times which occur in this portion of the scale. On the other hand, a steep slope is indicative of a great deal of relaxation; and hence a large percentage of the relaxation times are included in the corresponding section of the time scale. A slow linear decrease in stress is best depicted by the "box distribution" function, in which the height of the box is constant over a very broad time range. As has been mentioned, three parameters determine
the characteristics of a box distribution. These are $E_0$, $\tau_\varepsilon$ and $\tau_m$. An example of their applicability is shown by the following equation, relating them to what Andrews and Tobolsky\(^8\) call the tensile viscosity:

$$\eta_t = \int_0^\infty \tau E(\tau) d\tau$$

For the box distribution, this equation simplifies to:

$$\eta_t = E_0(\tau_m - \tau_\varepsilon)$$

If, as in all the cases under discussion here, $\tau_m$ is very much larger than $\tau_\varepsilon$, then the latter can be neglected. It is now possible to evaluate textile materials in terms of the tensile viscosity and their time-dependent behavior.

Large values of $E_0$ which result from high negative slopes of the relaxation curves are thus associated with a high tensile viscosity. This implies, by the very nature of the term, large variations in behavior with varying strain rate. On the other hand, small values of slope produce smaller values of tensile viscosity and are the result of smaller variations of properties with time.

The physical significance of $\tau_m$ is that it represents the longest value of the variable relaxation time.
At the other end of the relaxation spectrum, the high speed behavior of materials is controlled by the value of $\tau_e$, which is the shortest relaxation time. Despite the considerable experimental work\textsuperscript{6,7} which has attempted to measure this value, it has as yet not been determined for such materials as nylon, Dacron, or Orlon. Dynamic tensile tests completed in microseconds\textsuperscript{6} and oscillatory experiments conducted at frequencies in the megacycle range\textsuperscript{7} have failed to determine $\tau_e$. These materials continue to be viscoelastic down to very short loading times, and the phenomenon of purely elastic response has not been observed.

It might be pointed out that the value of $E_o$ has been found to be associated with the sharpness of molecular weight fractionation. A sharp MW fraction would yield a high value of $E_o$, and a very broad fraction would yield a low value.

A discussion by H. Pelzer\textsuperscript{9} in terms of electric circuit analogues, treats the observed constancy of loss per cycle in terms of a mechanical model with a constant value of tensile viscosity and variable values of spring constants or modulii. The constancy of loss per cycle is merely a consequence or different way of expressing a box distribution of relaxation times. It is interesting that dynamic measurements, impact tests at varying strain rates, slow speed tests
and stress relaxation tests all complement one another to furnish a uniform general and consistent picture of the time-dependent behavior of textile fibers. This picture is one best described by a very broad "box distribution".

Calculations of $E_o$ and $\tau_m$ have been made from the stress relaxation curves of Figures 3.2, 3.3 and 3.4. These are given in Tables 3.1, 3.2 and 3.3 along with quantities which will be referred to later.

**TABLE 3.1**

Parameters of Box Distribution - Nylon^16

<table>
<thead>
<tr>
<th>% Elongation</th>
<th>$E_o$ (gpd)</th>
<th>$\log \tau_m$</th>
<th>$E_2$ (gpd)</th>
<th>$\tan \delta$ at 1 min.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.44</td>
<td>4.25</td>
<td>3.82</td>
<td>.17</td>
</tr>
<tr>
<td>4</td>
<td>2.03</td>
<td>4.79</td>
<td>3.18</td>
<td>.15</td>
</tr>
<tr>
<td>6</td>
<td>1.35</td>
<td>7.45</td>
<td>2.12</td>
<td>.094</td>
</tr>
<tr>
<td>8</td>
<td>.95</td>
<td>11.65</td>
<td>1.50</td>
<td>.059</td>
</tr>
<tr>
<td>10</td>
<td>.98</td>
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<td>12</td>
<td>1.10</td>
<td>13.25</td>
<td>1.73</td>
<td>.049</td>
</tr>
<tr>
<td>14</td>
<td>.86</td>
<td>15.55</td>
<td>1.36</td>
<td>.039</td>
</tr>
</tbody>
</table>
### TABLE 3.2

Parameters of Box Distribution - Fortisan

<table>
<thead>
<tr>
<th>% Elongation</th>
<th>$E_0$ (gpd)</th>
<th>$\log \gamma_m$</th>
<th>$E_2$ (gpd)</th>
<th>$\tan \delta$ at 1 min.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.51</td>
<td>11.55</td>
<td>10.2</td>
<td>0.059</td>
</tr>
<tr>
<td>2</td>
<td>4.34</td>
<td>13.75</td>
<td>6.80</td>
<td>0.049</td>
</tr>
<tr>
<td>3</td>
<td>4.70</td>
<td>11.75</td>
<td>7.37</td>
<td>0.059</td>
</tr>
<tr>
<td>4</td>
<td>3.80</td>
<td>14.25</td>
<td>5.97</td>
<td>0.059</td>
</tr>
<tr>
<td>5</td>
<td>4.55</td>
<td>12.25</td>
<td>7.15</td>
<td>0.057</td>
</tr>
</tbody>
</table>

### TABLE 3.3

Parameters of Box Distribution - Fiber Glass

<table>
<thead>
<tr>
<th>% Elongation</th>
<th>$E_0$ (gpd)</th>
<th>$\log \gamma_m$</th>
<th>$E_2$ (gpd)</th>
<th>$\tan \delta$ at 1 min.</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5</td>
<td>5.6</td>
<td>18.65</td>
<td>8.9</td>
<td>0.037</td>
</tr>
<tr>
<td>1.0</td>
<td>7.5</td>
<td>13.85</td>
<td>11.8</td>
<td>0.049</td>
</tr>
<tr>
<td>1.5</td>
<td>15.9</td>
<td>6.25</td>
<td>24.9</td>
<td>0.113</td>
</tr>
</tbody>
</table>
Correlation with Other Dynamic Tests

Equation 3.11 can be modified slightly, to make it compatible with the terminology of Equations 3.1 - 3.3, i.e.:

\[ \bar{\sigma} = Rte_0 \left[ \ln \left( \frac{1.53 \tau_m}{t} \right) \right] \]  

(3.12)

or:

\[ \bar{\sigma} = E_0 \left[ \ln (1.53 \tau_m) - \ln t \right] \]  

(3.13)

Therefore, assuming \( E_0 \neq f(\varepsilon) \),

\[ \left( \frac{\partial \bar{\sigma}}{\partial \varepsilon} \right)_{t} = E_0 \ln \left( \frac{1.53 \tau_m}{t} \right) \]  

(3.14)

\[ \left( \frac{\partial \bar{\sigma}}{\partial t} \right)_{\varepsilon} = - \frac{E_0}{t} = -R \overline{E_0} \]

Substitution of Equations 3.14 in Equation 3.3 gives:

\[ \frac{d\bar{\sigma}}{d\varepsilon} = E_0 \ln \left( \frac{1.53 \tau_m}{t} \right) - E_0 \]  

(3.15)

The quantity \( \left( \frac{\partial \bar{\sigma}}{\partial \varepsilon} \right)_{t} \) has sometimes been called the instantaneous modulus of elasticity, since it requires that the slope of the stress-strain curves be measured at
constant time, i.e. instantaneously. The quantity, $E_0$, we have defined as the height of the distribution function. It has also been shown to be related to the imaginary part of the complex modulus. For example, if

$$E^* = E_1 + iE_2$$ (3.16)

it can be shown that the negative slope of the relaxation line, $E_0$, is given by

$$E_0 = \frac{2}{\pi} E_2$$ (3.16a)

Thus, the observed slope of the stress-strain curve for a test at constant strain rate is

$$\frac{d\sigma}{d\epsilon} = E_{\text{inst.}} - \frac{2}{\pi} E_2$$ (3.17)

In terms of the complex modulus defined in Equation 3.16, $E_1$ represents that part of the modulus where the stress and strain are in phase. This will be given by $\frac{d\sigma}{d\epsilon}$. Thus, from Equation 3.15:

$$E_1 = E_0 \ln \left( \frac{56 \tau_m}{t} \right)$$ (3.18)
In the discussion of viscoelastic behavior in terms of a complex modulus, it is useful to write the relationships in terms of a phase angle, $\delta$, where

$$\tan \delta = \frac{E_2}{E_1}$$  \hspace{1cm} (3.19)

Substitution of Equations 3.16a and 3.18 into Equation 3.19 yields

$$\tan \delta = \frac{\sqrt{\frac{\tau}{2}}}{\ln(\frac{56\tau_{TM}}{t})}$$  \hspace{1cm} (3.20)

Thus, the tangent of the phase angle, or the tangent of the loss angle, is given in terms of one material constant, $\tau_{TM}$, and the time at which the tangent is measured.

Various values of $\tan \delta$ are given in Table 3.4 for an assortment of fiber types. The method and conditions of the measurement are listed together with the reference.

It is interesting that the many different methods of measuring $\tan \delta$ produce results which are in such good agreement in their orders of magnitude. The differences which do occur are primarily the result of differences in the test technique as to time, static strain and dynamic strain. The similarities substantiate the claim made earlier that tests run under widely varying conditions "complement
one another to furnish a uniform, general and consistent picture of the time-dependent behavior of textile fibers." It was also claimed that this consistent picture is best described by a very broad "box distribution of relaxation times."

Let us look at some of the consequences of a "box distribution". It was defined earlier by Equation 3.6. The definition requires a constant value of $E_0$ and consequently implies a constant value of $E_2$. In addition, the fact that $E_0$ and $E_2$ have finite values makes it essential that $E_1$ increase with decreasing time, Equation 3.18. Thus, $\tan \delta$, which is equal to the ratio of $E_2$ to $E_1$, must decrease with decreasing time. This fact is also apparent from Equation 3.20.

On the other hand, it has also been stated that the viscoelastic behavior of high polymers can be represented by a constant damping ($\tan \delta$) factor. The constancy of $E_2$ and $\tan \delta$ are incompatible since $E_1$ must vary with time. However, if $\tan \delta$ is small, $.01 - .03$, it is difficult to observe a difference in $\tan \delta$ constant and a "box distribution" over 5 cycles of time. (See Figure 3.5) However, when $\tan \delta$ is $.05$ or greater, the $E$ versus $\log t$ plot is no longer linear. The "box distribution" must then be replaced by a so-called "wedge distribution" which has a larger
FIGURE 3.5. Variation of normalized modulus with log time for different values of tan $\delta$. Lines are straight, points are calculated.
height at the short end of the time scale and levels off at
the long end. Examples of this distribution are shown in
the papers by Fujimo, et al. 7

Examination of the data of Table 3.4 reveals
that for the majority of test conditions "box distributions"
are adequate representations of the relaxation time spectra,
since in most instances tan \( \delta \) is less than .05.

Influence of Fiber Morphology on Viscoelastic Behavior

It has been shown that the stress-strain re-
lationship depends primarily upon the value of \( E_1 \). Since
the loss angle tangent, tan \( \delta \), is equal to \( E_2/E_1 \) and
is in the neighborhood of \( \frac{1}{10} \) to \( \frac{1}{5} \), \( E_2 \) has a secondary
contribution to the value of the complex modulus. It has
been further shown that the slope of the stress relaxation
curve can be related to this value, \( E_2 \).

Lemiszka 16 has shown that a wide variety of
fibers differing greatly in chemical type have essentially
the same value of the relaxation slope. An interesting ques-
tion is raised: why is the viscoelastic behavior as repre-
sented by the imaginary part of the modulus so similar for
such a wide variety of materials? As a clue to this similarity,
it is suggested that we look at fiber morphology and the similarities which exist in the fibrillar, or fine structure of fibers.

George\textsuperscript{19} has suggested that "many of the macro-mechanical properties of filamentous solids can be related in a remarkably simple way to the geometric and dynamic properties of microdomains existing within the filaments. These microdomains are characterized by elements of length which are large compared with the interatomic distances within the solid, but probably are quite small compared with the cross-sectional dimensions of the filament."

He goes on to say, "Regardless of elementary microscopic structural class, there seems to be an abundance of data of a variety of types pointing to the existence of one or more types of intermediate structures which can exist in a solid state. Examples of such intermediate structures are grains in metals, crystallites and micelles in natural fibers, internal stress or strain gradients, free surfaces in grain boundaries.

"The term 'domain' is used herein to refer to an intermediate structural configuration which
is characterized by dimensions intermediate between the macroscopic and microscopic dimensions of the solid. One goal of studies of these domains is the determination of the 'spectrum' of domain sizes which can exist and the properties of the solid which are related to the elements of the 'spectrum' singly and in combinations.

However, George does not specifically define the dimensions of his "domain".

Yumoto, in discussing the cold drawing of polymers, refers also to a similar type of behavior.

"The author believes that the deformation of polymer can be regarded as that of the complex structure made of strong and weak networks. The former, which is tentatively called the primary network, is constituted from bundles of chain molecules and contains the arranged regions (micelles) together with their boundaries. The latter, which is tentatively called the secondary network, is constituted from the single chain molecules which are entangled with each other and fill the space among the primary networks. On deforming such a complex network, the micelle behaves like a rigid
body and the flow in the primary network must occur only at the boundaries of the micelles. The measured yield stress is the sum of individual resistances caused against flow in the primary and the secondary networks and the stress caused in the primary network should be expected to hold the leading proportion. The behavior of the secondary network, however, has an important function in determining the deforming features of the entire complex network."

The dependence of the viscoelastic behavior of polymers on some form of lateral order has been pointed out by Yoshitomi, Nagamatsu, and Kosiyama\textsuperscript{15}:

"It is well known, however, that the position of the longer end of this box distribution depends on the molecular weight of the specimen. It is nearly proportional to the 3.4\textsuperscript{th} power of the molecular weight, and it disappears when the molecular weight of a specimen is small. For example, this box distribution disappears in the case of polyisobutylene when its molecular weight is less than 1,700,000, while the molecular weight of nylon 6 used in this experiment is in the order of several thousands. The existence of this modified box distribution in nylon 6, therefore, suggests that the large clusters
effective in the mechanical behavior are existent in the specimen, and the apparent molecular weights of these clusters are much greater than the real molecular weight of nylon 6. The formation of these clusters may be attributed to the joining of short chain molecules in the crystalline region.

Ribi\textsuperscript{21} has written about the similarity between fibrils found in plant and bacterial cellulose and fibrils found in viscose and polyamide fibers:

"Also in fibers of polyamides a system of submicroscopic fibrils was found to form a structural unit. As is known, such a fiber is formed when the fibrous molecules are crystallized from a melt with artificial orientation. Even in polyethylene, plastic bundles of fibrils were observed. The fibrils of these two synthetic products are morphologically identical to those of cellulose and chitin. All samples investigated were dispersed by means of ultrasonic waves before electron microscopy.

"Hence the submicroscopic fibril is not specific for cellulose, but is obviously a quite general morphological element which may be formed when linear macro-molecules order themselves in a lattice. Indeed, it might be expected that molecules which
are chemically different but of similar structure, and therefore crystallizing in the same type of lattice, would form colloidal aggregates of the same shape and size.

"From the investigations on viscose rayons and polyamide fibers, it may be concluded that there is already a pervading system of the fine fibrils before the orientation is high enough to give an ideal fiber diagram. An orientation to a higher degree may then be due to the elementary fibrils' orienting themselves parallel to one another. In the case of the viscose rayon, the crystallinity does not increase thereby."

Keller, in one of his excellent discussions of the growth of spherulites, has said:

"It is obvious that the existence of such complex structural units must be taken into account whenever the physical properties measured on the bulk sample are related to the molecular structure, which is particularly important for technical applications."

Hsiao and Sauer\textsuperscript{23} discuss the rod-like substructure of polymers:

"Most rods appear to be of the order of 1 micron
in length and perhaps one-tenth of a micron in thickness....For polystyrene, the extended length of the polymer chain is of the order of $1.6 \times 10^3 \text{Å}$ or roughly 1/6 of a micron. Thus, the length of the rodlets is approximately four to six times the length of the polymer chains. The width of the domain is, on the other hand, some 100 times larger than the approximate distance across adjacent chains (10 Å) and hence if the domain is interpreted as a larger conglomeration of more or less parallel chains, it should contain several thousand or more, depending on how closely they are packed.

"....The natural configuration of polystyrene molecules, at least under certain environments, is one of more or less oriented, intertwined cylindrical domains."

(N.B. These domains are probably amorphous since this is not isotactic polystyrene.)

All of these ideas of a substructure which is fibrillar in nature suggest a new approach to the viscoelasticity of fibers. This approach is that the fibrils, domains, or micelles, as they may be alternatively called according to the likes and dislikes of the particular author, could
account for the deformable viscoelastic properties of these polymers. This domain may be a single crystallite, or it may be a multiple collection of crystalline and amorphous regions such as exist in a spherulite. It may be a cortical cell as is found in keratin fibers. The important concept is that this substructure exists as coherent units which react more or less elastically in a viscous surrounding.

Such a theory of viscoelasticity could be built up about a model of elastic rods of varying size and shape, variously oriented in a viscous medium. The relaxation times are the times required for revolving, moving, or extending these rods in the viscous surrounding. The deformability of the domains, spherulites, or micelles, will play an important role, since these domains, it is conjectured, will be associated with the values of $E_1$. However, the values of $E_2$ are thought to be associated with the viscous medium which surrounds the elastic rod. The explanation of the similarity of values of $E_2$, which have been found for a wide number and wide variation in fiber types, can be associated with the similarity in this interdomain region.

Let us turn to other fields where extensive work on the effects of domains and their boundaries have been considered. The introduction of coordinated aggregates recalls the domain theories of ferro-magnetism, which
further recall the hysteresis so well known in studies of ferro-magnets. The domain and spherulite regions in polymers are bound tightly together with definite boundaries. Thus, the similarity between polymer domains and a magnetic domain becomes obvious.

An additional field in which much work has been concerned with relaxation effects\textsuperscript{25} is the field of ultrasonic measurements in liquids and bulk materials. Here one usually talks about the absorption of ultrasonic energy in the medium under study. Fry\textsuperscript{26} has said:

"The term 'relaxation' refers to processes in which the time intervals required for energy exchange between various degrees of freedom of a system are not negligible compared with a period of the acoustic measurements.

"Relaxation effects, for materials with single relaxation frequencies, yield a quadratic dependence at low frequencies, a constant value at high frequencies, and no intermediate band which corresponds to experimental data on tissue. Calculations based on the action of the viscous forces to convert acoustic energy into heat show that in a homogeneous medium the absorption coefficient is proportional to the square of the frequency. It has also been shown that the viscous forces acting upon a fluid media
in a suspension of uniform particles in the medium, yields a relaxation type of frequency dependence of the absorption coefficient. It will be shown that the viscous forces acting between a suitably chosen distribution of suspended particles or structure elements and a suspending liquid can account for a linear relation between the acoustic absorption coefficient and the frequency over a wide frequency range."

Fry goes on to show that with a suitable distribution of particle sizes in a medium of a constant viscosity one can achieve results which are consistent with the type of box distribution function which has been described earlier. It is interesting to point out that the present author has previously disagreed with this general approach. However, in the light of more recent evidence, he has reversed his opinion.

In a discussion of the viscoelastic behavior of colloidal suspensions, Kruyt says,

"The size of the particles does not have a very typical behavior on plastic behavior and gel formation. Shape on the contrary is a very important factor. The more elongated the particles, the more typical their plastic properties. It will be evident that, in order to form a coherent network, a much smaller amount of material is needed with rod-shaped
or plated-shaped particles, than with round or cubic ones. Moreover, if long-range attractive forces were to play a part in the network formation, they also would be favored by an elongated shape of the particles."

(N.B. In Kruyt's terminology "plasticity is characterized by a viscosity which decreases with increasing rate of shear and usually by the presence of an actual yield stress." i.e. 
\[ \eta \omega = \dot{\gamma} \] 

Mooney\textsuperscript{29} has taken a giant step forward in the understanding of the rheology of raw rubber by adapting Eyring's theory of viscosity to much larger flow units. The Eyring theory is essentially an atomic theory with displacements and flow units of atomic dimensions. Mooney comes to the conclusion that in the flow of raw rubbers the flow units are of the order of 2 to 30 microns in diameter. These he calls "supermolecular rheological units". His treatment is quantitative in contrast to the purely qualitative discussion being given here. Mooney's analysis could well be extended to the rheology of fibers where the rheological unit is a fibril of the order of a micron in thickness and many microns in length.

The similarity between Mooney's theory and that presented here can be seen in the following quotation:\textsuperscript{29}
"Consequently it appears that, so far as rubbers are concerned, a viscosity theory assuming homogeneity of the materials is unrealistic...

"In the revised theory the macroscopic viscosity is assumed to result from the friction of rheological units as they slide over each other. The frictional force is treated as a consequence of temporary molecular attachments across the boundaries of the units. From the moment of attachment until release by thermal activation, the stress on the attachment build up at a rate which is proportional to the relative sliding velocity of the two rheological units and to the dynamic elastic modulus of the units and is inversely proportional to the unit diameter and to the number of cross attachments per unit area."

The rheological unit is considered to be completely elastic, with all frictional (viscous) effects occurring at the boundaries. "Stress relaxation within the rheological units can presumably be ignored." This is basically the same consideration which Pelzer has arrived at phenomenologically.

Pelzer has described the constancy of the loss angle tangent in terms of electrical network theory. He has proposed three types of networks which would give a suitable type of loss behavior. He has gone on to suggest the three
corresponding mechanical models for his three electrical models. One of these corresponds to elastic discs of varying sizes suspended in a viscous material with a constant viscosity coefficient. He shows that this type of model can give the type of behavior which we have come to associate with viscoelastic fibers.

The viscous component, or imaginary part of the modulus, can be associated with the reaction which takes place between rheological units, while the real part of the complex modulus can be related to the chemical structure which constitutes these units. The similarity in the fibrillar structure which exists among the many types of fibers is consistent with the similarity in the viscoelastic behavior of these materials. The wide distribution in relaxation times which we have come to describe almost universally with a logarithmic box function can then be attributed to the variation both in the size of the domains as well as the variation in size along the length of the domain, i.e. their shape. This is particularly true in the case of oriented fibrils, in which we thing of elongated spindle-shaped domains which start out from a point and increase through the width of the spindle cell, or domain, and narrow down to a point again. In terms of the modulus per unit cross-sectional area, the spindle cell, or spindle-shaped domain, will have a constant modulus. However, if the relaxation depends upon the spring constant of this elastic material in a viscous medium, it is
seen that the spring constant is a function of the cross-sectional area whereas the modulus was not. Thus variations in the spring constant exist along the length of a spindle-shaped domain.

This shape effect may account for the variation in relaxation times which Lemiszka has observed between oriented and non-oriented materials. In undrawn materials the domain will be more nearly spherical, and there will be a lesser variation in the spring constant across adjacent sections of the rheological unit. This spherical shape will result in a narrower distribution of relaxation times which is consistent with the results of relaxation experiments previously described both by the author and by Lemiszka. It can then be expected that the breadth of a box distribution of relaxation times will increase on drawing.

The breadth of box distribution function can be inferred either from the value of tan $\delta$ or by the value of $T_m$. Table 3.5 includes the results of stress relaxation at constant strain experiments of Lemiszka as well as original results from stress-relaxation tests at constant strain rate. The undrawn material of Figures 3.5a, 3.5b, and 3.5c becomes drawn during the straining. The value of tan $\delta$ decreases sharply and the value of $T_m$ increases sharply as the material becomes oriented. At the present time it is not known whether the size of these units
Fig. 7. The change of relaxation spectrum of quenched and undrawn polycaproamide with drawing and heat treatment and that of highly drawn one with boiling in water and heat treatment.

Fig. 14. Upper part: the change of relaxation spectrum of undrawn polyvinyl alcohol with drawing and that of highly drawn one with heat treatment. Middle part: the change of relaxation spectrum of undrawn polyvinyl alcohol with chemical treatment, using boric acid, and with drying.

FIGURE 3.6. Relaxation spectra.

can be controlled to the extent that the shape factor is controlled by drawing.

It might be thought that the crystallite could be a rheological unit. However, Fujino, Kawai, Horino, and Miyamoto\textsuperscript{30} have shown that increasing the crystallinity of polyethylene terephthalate fibers has little effect on its distribution of relaxation times. On the other hand, the relaxation spectrum is increased disproportionately in the time range of $10^{-3}$ to $10^0$ seconds. This same effect is shown of polycapramide and polyvinyl alcohol. (See Figure 3.6) The experimental data of Fujimo et al.\textsuperscript{30} seem to support the qualitative theory which has been presented here, although these authors suggest a slightly different interpretation.

While no direct experimental evidence has been introduced to support this concept of viscoelasticity, it is suggested that no theory of viscoelasticity can be valid which considers only the molecular structure and the fiber structure with consideration of the intermediate fibrillar or domain structure. Any theory of viscoelasticity must account for the one characteristic fact which has been found to exist for all fibers, the very broad distribution of relaxation times.
References for Chapter III


11. Alfrey, T. "Mechanical Behavior of High Polymers" N.Y. Inter-science (1948)


20. Yumoto, H. "Studies on the Cold-Drawing of High Polymers: The Quasi-Static Drawing of Polycopramide"
   Part II Ibid. p. 141-147 (1956)
   Part III Ibid. p. 353-360 (1956)


IV. WEAVING

One of the few cases in which the concepts of viscoelasticity have been applied to a textile process is the work of Greenwood and Cowhig\(^1\) in their study of weaving. These workers have related the stress relaxation of warp and woven fabric tensions to the movement of the cloth fell during a loom stoppage. In addition, Greenwood and Vaughn\(^2,3\) have made a study of the beat-up force as a function of several loom variables, e.g. loom speed and shed timing. They measured the force-time relationship for the beat-up cycle. These are examples of investigations into machine-fiber interactions. The following discussion will attempt to analyze their results in terms of the viscoelastic concepts developed in Chapter III.

First let us consider Greenwood and Vaughn's study of the beat-up force. They applied strain guages to the back of the reed in order to obtain measured values of beat-up force. This ingenious use of strain guages does not introduce any new variable and allows actual in-process measurements to be made. The force-time measurements allow us to analyze the amplitude of the forcing function as well as its frequency.

The action of the beat-up in relation to the overall warp-tension cycle can be seen in Figure 4.1, taken from Greenwood and Cowhig. At a loom speed of 190 picks per minute the frequency of the complete tension cycle is 3.1 cycles per second. However, the time
FIGURE 4.1 - "Typical Warp Tension Cycle"

duration of the beat-up force pulse is much shorter than the complete tension cycle, and has been found to be approximately 30 milliseconds.\textsuperscript{3} Thus, the beat-up cycle can be thought of as having a frequency of approximately 33 cycles per second, while the overall warp tension cycle is approximately ten times slower than the beat-up cycle.

For the fabric which Greenwood and Vaughn\textsuperscript{3} studied, it was found that the beat-up force had a peak of about 15 to 20 grams per 140 denier warp end. Thus the beat-up cycle can be considered to have a force amplitude of approximately .15 grams per denier at an intermittent frequency of 33 cycles per second. This beat-up cycle is superimposed upon the entire warp-tension cycle produced by the shedding. The measured values of the warp-tension for the shed cycle were between 20 - 27 grams per 140 denier end. It can be seen from Figure 4.1 that the tension in the unwoven warp and in the woven fabric are identical with the exception of the short time during which the beat-up force is active. During this time the warp tension is increased to a value greater than the fabric tension. The total beat-up force is, as indicated, equal to the difference between the fabric tension and the warp tension. While the warp tension varied between 20 - 27 grams, as indicated, the corresponding fabric tension on the other side of the beat-up reed varied from 5 to 25 grams, approximately. Thus the tension variations on the fabric side are more severe than on the warp side of the fell. This difference is particularly severe when bumping occurs.
Since Greenwood and Vaughn's measurements are so complete, the stress cycles of the warp fabric tensions are well defined. Thus in order to apply the data which can be obtained in the manner of Chapter III to estimate such things as yarn growth, or hysteresis energy stored per cycle, simple experimental data can be made on yarns in which the weaving stress cycles are reproduced. In other words, the action of the machine has been defined quite explicitly and this machine action can be applied to hypothetical or real warp and fabric materials in order to reproduce the machine fiber interaction. The stress cycle frequency of interest is in the range of 3 to 33 cycles per second. This range has been quite adequately covered experimentally by Tipton. Thus a great deal of fiber data is currently available in the literature and can be directly applied to this problem.

Another interesting application of the principles of viscoelastic behavior has also been made by Greenwood and Cowhig. In this case, they have applied the principle of stress relaxation to the warp and fabric in a loom when the loom is temporarily shut down for one reason or another. If the stop takes place with a closed shed, the warp tension and the fabric tension are equal and both are equal to a tension value considerably above the minimum fabric tension which occurs during beat-up. In the stopped condition, the tension in both fabric and warp will relax to some value less than that which occurred when the stoppage was made. The exact nature of the stress
relaxation and/or creep depends on the nature of the stoppage as well as the type of tensioning device used on the loom. If constant tension is maintained during a stoppage, then creep will occur, and the position of the fell is apt to move. Fell position can move either away from or toward the weaver. This motion of the fell will cause a setting on streak when weaving is resumed.

Let us consider a hypothetical case of loom stoppage, where a constant tension is maintained. It is assumed that the stoppage occurred with a closed shed in such a way that the fabric tension equaled the warp tension. It is important to mention at this time that equal fabric and warp tensions do not necessarily mean the same yarn tension in both warp and fabric, since the presence of any crimp in the fabric will in fact produce a higher axial tension in the yarn as it lies in the fabric than in the same yarn as it lies straight in the warp. Thus, the tension and hence strain condition of the yarns in the woven fabric are apt to be higher than the tension and/or strain in the warp. The motion of the fell, or weave point, will now depend upon whether the fabric grows at the expense of the warp or vice versa. If the fell position moves away from the weaver, the fabric has grown at the expense of the warp. This is in fact the usual case in practice.

If, on the other hand, the tensioning device maintains a constant length during stoppage and the stress is allowed
to relax, a different type of action can take place. The overall process must follow a line of constant total length. The fell will move only if the fabric or warp grows at the expense of a contraction in the other member. As indicated previously, the yarn tensions in the fabric are apt to be higher than the yarn tensions in the warp. The work of Chapter III has shown that in the low strain region, which is the region which is important during weaving, the higher the tension the faster the relaxation. Thus, it might be expected that greater relaxation would occur in the fabric than in the warp. The fabric will then extend as the warp contracts, and the fell will again move away from the weaver. Wasterberg and Nordhammer have questioned the analysis made by Greenwood and Vaughan, and they suggest that the tension might actually increase during a stoppage because of so-called stress restoration effects. They have analyzed warp tension cycles produced on an Instron tensile tester, and have shown that stress restoration can in fact occur if a stoppage is made at a low force point in the force-time cycle. While this is an interesting and worth while observation, it does not occur in practice according to Greenwood and Vaughan. The reason given is that a stoppage usually occurs at higher than minimum values of tension. An interesting observation by these authors is the fact that the position of the cloth fell might be maintained by leaving the beat-up
reed against the fabric in such a way that the tendency of the fabric to relax more than the warp will be overcome. In other words, the beat-up can be used to produce an unbalance in the fabric and warp tensions. Under proper conditions, no growth or motion of the fell will then occur.

In all of these analyses, if a complete understanding of the behavior of the fabric and the warp is to be achieved, it is important to know the stress-strain-time surface of the warp after it has gone through the multiple load cycling which occurs in practice. Also, we must know the same facts for the yarns in the fabric as they exist in the loom state; that is, as the fabric is held in width with a minimum fabric crimp. In this case the modulus of the fabric may not be as different from the modulus of the warp as has been indicated by Waesterberg and Nordhammer. In spite of this reservation, which is in the nature of a refinement, the application of the viscoelastic phenomenon of stress relaxation and/or stress restoration is a good example of machine fiber interaction. Thus we see that two types of loading cycles can be important in weaving: the one involving the dynamic loading which we commonly think of when considering weaving, and the other the stress relaxation type of behavior occurring when the loom is shut down.
References for Chapter IV

   "Part II: Disturbed Weaving Conditions" J.T.I. 47 p. T255 (1956)


V. COLD DRAWING

Introduction

The process of cold drawing fibers around a snubbing pin was selected as a process in which the machine actions are simple and easily described. These actions consist of two rolls, each moving at a constant velocity but one moving more slowly than the other, with a velocity ratio which is called the draw ratio. The yarn is few on the slower moving roll, called the feed roll, then wrapped around a snubbing pin from which it is carried to the draw roll. The only moving parts on which the yarn and the machine interact are these two rolls and the pin. It is assumed that in practice there is no relative motion between the yarn and either the feed or draw roll, the only relative motion occurring between the yarn and the snubbing pin. While the machine action is simple, it will be shown that the interaction which is produced can be extremely complex in undrawn materials which are subjected to this process. The properties of the materials almost completely control the conditions of draw and hence the process. However, there are enough machine variables to make this an extremely valuable example of the principle of machine-fiber interaction.
Apparatus and Experimental Procedures

The test apparatus used is based on that described in the basic patent by Babcock,\textsuperscript{1} and later used with modifications by Marshall and Thompson\textsuperscript{2}. In some respects it is similar to the commercial equipment such as that made by Whitin used for the drawing of certain meltpun fibers. The equipment (See Figure 5.1) consists of two rollers of different diameters which can be driven at adjustable angular velocities. In this equipment, the feed roll had a diameter of 1 inch and the draw roll had a diameter of 4 inches. If both rollers had been driven at the same angular velocity, there would have been a draw ratio of 4 to 1. For added control on the draw ratio, sprocket gears were used in the drive system so that additional draw ratios could be achieved by varying the number of teeth in the gears driving each roll. The combination of the diameter difference and the number of teeth in the gear permitted a range of peripheral speeds. The ratio of the faster surface speed, draw roll velocity, to the slower surface speed, feed roll velocity, is called the machine draw ratio. This nomenclature is used to distinguish it from the natural draw ratio of the fiber to be described later.

By gearing the two rollers together, as described, with a chain and sprocket gears, a positive control of the draw ratio was achieved. In addition, the absolute
values of the two rolls were controlled by a variable speed
motor driving the chain. Thus, once the draw ratio was set
by the appropriate choice of gears, the process velocity
could be varied over the range covered by the variable speed
motor. Two idling rollers were canted to the feed and draw
rolls as described by Babcock. These were driven by the
multiple wraps of yarn as seen in Figure 5.1. Canting of
the idler roll controlled the motion of the yarn across the
feed and draw rolls and prevented any snagging or breakage
due to the overlapping of successive sections of yarn.

Between the two rolls, the yarn was wrapped
around a snubbing pin as shown. This pin controlled the
tension build-up to that tension at which drawing took place,
and served to control the point of draw. In this particular
assembly, the snubbing pin was in fact a torsion gauge which
measured the difference between the feed tension \( T_1 \) and the
draw tension \( T_2 \). In order to get a torque which was easily
measurable, the diameter of the snubbing pin was made much
larger than that usual in commercial practice. According
to Babcock, the pin should be less than \( 3/8 \) inches in dia-
meter in order to control the point of draw effectively.
All of the data reported in this thesis was obtained on a
pin with a diameter of approximately 1 inch. The details
of the snubbing pin torsion gauge are given in the Appendix.
FIGURE 5.1. General view of cold-drawing machine showing draw and feed rolls, torsion gauge, draw pin and traveling microscope with thermocouple.
Since the torsion pin records only the difference in tension $T_2 - T_1$, it was necessary to make an independent measurement of either $T_2$ or $T_1$ in order to define both of them. It was thought that the drawing process would be least affected by making an independent measurement of $T_1$. This measurement was made, using a tensiometer described and designed by Victory\textsuperscript{3}. The signals from the torsion pin and the Victory tensiometer were calibrated and recorded on a dual channel Sanborn strain gauge unit. The variations in the two tensions could thus be studied as functions of the process variables.

At the outset of the study, the major points that were to be investigated were the time-dependent increase of tension around the pin and the frictional characteristics of the yarn-pin-velocity combination. Three samples of undrawn yarn were acquired. A sample of nylon 66 was furnished by the Chemstrand Corporation of Decatur, Alabama. A sample of Caprolan, or nylon 6, was supplied by the Allied Chemical Corporation, and a sample of Terylene (polyethylene terephthalate) was furnished by Canadian Industries, Ltd. Unfortunately the Terylene was requested without spin finish, since it was thought that such a material would be of more interest in the frictional studies. As it turned out,
the lack of finish made the material so wild that it was completely unmanageable. The drawing studies were therefore conducted on the twonylons only.

To repeat, the original purpose was to investigate the effect of draw ratio, draw velocity, pin diameter and pin surface. However, the effect of draw ratio and velocity turned out to be the major sources of interest. As in most such investigations, new avenues of research proved enticing, and a study not originally planned was made of the behavior of the draw point in terms of its position, movement, and the temperature which was developed. In this case, the neck, or draw point, was controlled at a position off the snubbing pin, in the free space between the pin and the draw roll. At the start of the investigation, it was assumed that the draw point, or neck, would be confined to a position on the pin. Thus, it was very surprising that the first trial runs showed the neck to be stable at a point in this free space between the draw roll and the snubbing pin. Later experiments showed, in fact, that the neck could be moved away from or toward the pin. By suitable adjustments of the draw ratio and draw velocity, this result was obtained. The neck could also be held on the pin. Such effects have been described for polyethylene terephthalate by Marshall and Thompson. Babcock has described the commercial significance of motion of the draw point:
"It has been discovered that variations in the physical characteristics of cold drawn linear polyamide filaments are caused by cyclical movements or periodic shiftings of the draw point....It has been found by experience that if the draw point is maintained within a zone of not in excess of ½ inch along the length of the thread, substantial elimination of variations in physical characteristics of the filaments in yarn will be effected....The snubbing pin has a diameter up to ⅝ inch and preferably has a diameter of from 1/8 to ½ inch....By use of the drawing apparatus it is found that the draw point is located on the surface of the snubbing pin and that comparatively little shifting of the draw point occurs....Any shifting of the draw point is maintained within an area of ½ inch of the surface of the snubbing pin. Although it is preferred that the draw point be localized within a fixed zone, it has been found that sustained or comparatively long movements of this zone can be tolerated,.....if the speed of movement of the zone does not exceed 2 percent of the linear velocity of the undrawn yarn."

Marshall and Thompson² have also described the motion of the neck in their experiments using a flat
plate heater instead of a snubbing pin. They have shown that
the neck can be made to advance, retreat, or stay fixed,
depending upon the draw ratio and draw velocity. The
results of their experiments are reproduced in Figure 5.2.
It must be remembered that their process differed from
that described here. They used a heater plate instead of
a snubbing pin to control the drawing process. It should
also be pointed out that they worked with polyethelene ter-
phthalate which has a higher second order transition tempera-
ture than the polyamide fibers studied here. In their case,
it was necessary to use a hot plate in order to achieve
proper drawing conditions.

In order to understand the cold drawing pro-
cess, let us look at a section of undrawn yarn which is
pulled off the cake and follow it as it goes through the
apparatus. The undrawn material is pulled off the package
at a very light tension. It is then fed onto the feed roll,
across the idler roll, back to the feed roll several times,
all at fairly light tension. The tension is only sufficient
to secure positive drive; no slippage between the yarn and
the roller is allowed. In commercial practice, as well as
in the practice described here, the yarn is next wound around
the snubbing pin two or three times, then it goes on through
free space and eventually onto the draw roll where positive
**Figure 4.** Draw-roll and draw-point velocities on a two-roller drawing machine at 20° C. Amorphous polyethylene terephthalate; initial birefringence $3.1 \times 10^{-3}$ and filament cross-section $6.7 \times 10^{-4} \text{cm}^2$. $R = (a) 1.55$, $(b) 2.00$, $(c) 3.11$, $(d) 4.00$, $(e) 4.65$, $(f) 5.14$, $(g) 6.00$, $(h) 8.00$.

**Figure 5.2.** Taken from Marshall and Thompson.

action is again secured so that no slippage occurs between the yarn and the draw roll. If we look at a section of yarn from the tensional point of view, we see that the tension $T_1$ between the feed roll and the snubbing pin is small. The tension builds up in the section of yarn as it is dragged around the snubbing pin. The tension increases exponentially with the angle of wrap, according to the well known belt friction formula. Around the pin, or at least around the early part of the pin, the material is in the so-called Hookean portion of a stress-strain curve. If we know the tension build-up, we can assume a value of modulus and calculate the strain, or extension, as a function of the angle of wrap. The material increases in tension until it either leaves the snubbing pin before necking, or necks on the snubbing pin and leaves the snubbing pin after necking.

Let us first look at the case in which necking occurred in the free space between the snubbing pin and the draw roll. In this case, the tension increased exponentially as we have described, in such a way that it reached a maximum value in the yarn just as it left the snubbing pin. The tension then remained constant in the free length of yarn between the snubbing pin and the draw roll. While the material may not have reached the yield extension as it left the snubbing pin, the effect of the extremely short time, constant force, creep condition produced a yield extension,
and hence necking, at some point in the free space between the pin and the draw roll.

In most of the experiments which are to be described, the necking occurred within an inch of the snubbing pin. The material was brought up to just below the yield load and extension on the pin. The yield extension was then reached a short time after leaving the snubbing pin. In this way, the neck position was maintained in the free space between the snubbing pin and the draw roll.

The value of the yield load and the position of the neck can be shown to be dependent upon the diameter of the pin and the draw and feed velocities. This is due to the yield load, which is known to be a function of the rate at which the load build-up occurs, and the machine variables mentioned which control the loading time.

Now let us go back to the section of the material under observation. It left the pin at a tension which was determined by the total angular wrap and the coefficient of friction between the yarn and the snubbing pin. The drawing force was established by the properties of the material and the loading time history. Once the drawing force was established and the drawing took place in the free section of the yarn, the value of $T_2$ in our belt friction formula was fixed. The input tension, or the tension of $T_1$
was determined by the coefficient of friction and the drawing
tension. In this experiment, $T_1$ adjusted itself to the process
and was not controlled. As a matter of fact, neither $T_1$
not $T_2$ were controlled by any frictional or tensional devices
other than the snubbing pin. The values of $T_2$ and $T_1$ were
thus completely established by (a) the yield load under the
particular loading time conditions, and (b) the value of the
coefficient of friction between the yarn and the pin. It
must be pointed out that the coefficient of friction is a
function of the velocity of the process. Thus, we have a
good practical example of the interaction of the material
and process.

As pointed out by Marshall and Thompson\(^2\),
the neck is stable, when it is moving neither away from nor
toward the snubbing pin, but remaining fixed in space.
Under these conditions, the machine draw ratio, $V_2/V_1$ is
equal to what is known as the natural draw ratio of the
material. The natural draw ratio of the material will be
shown to be a function of the conditions of the process;
that is, it is a function of the amount of work put into
the material and the temperature which the neck reaches.
However, both of these process conditions depend on material
properties. The temperature will be shown to be a function
of, among other things, the moisture content or the humidity of the sample. An attempt will be made to apply the first law of thermodynamics to the small section of yarn which is undergoing necking at a given time. We will set up an isolated thermodynamic system which contains the neck and the short section of yarn immediately preceding and immediately following the neck.

In order to study the first law behavior, or in order to set up an energy balance, we must know some of the following factors: the drawing force; the drawing extension, which will give us the work input; and the heat capacity and temperature of the system, which will tell us the heat which is developed. In addition, we should have some idea of the internal energy change of the system. As a point of fact, we know some, but not all of these factors, so only an approximate energy balance can be made.

Experimental Techniques and Observations

The first experiments were made to determine the machine draw ratios and draw speeds which would yield a stable neck position; and to establish the range of available process conditions in which we could match the natural draw ratio of these materials to the machine draw ratio.
It turned out that for nylon 66, machine draw ratios in the vicinity of 5 were required to match the natural draw ratio. On the other hand, with the Caprolan it was necessary to reduce the machine draw ratio to approximately 4 in order to achieve stable drawing conditions. Once the stable range for each fiber was discovered, qualitative experiments were run at, below, and above, these draw ratios for the entire range of speeds available. The same behavior as described by Marshall and Thompson\textsuperscript{2} was observed. In particular, it was found with nylon 66 that at draw ratios less than 4.67 no stable drawing occurred. In addition, at draw ratios above 5.67 no stable drawing conditions were observed. The operating range was thus kept between 5.67 and 4.67 for the velocities which were available. For nylon 6, as mentioned, the draw ratio was kept at 4.

a. Observation of the Neck

Since the position of the neck is a crucial point in the process, its continual observation was necessary. This observation was aided by the use of two plates of plane polarizing film. The polarizing film was placed in such a way that the polarizing direction was at 45 degrees relative to the fiber axis. The analyzer was placed on the other
side of the fiber, again with the plane of polarization at 45 degrees to the yarn or fiber axis, but crossed with respect to the first polarizing plate. The light source shining through the sample and through the two plates made the position of the neck quite clear. (See Figure 5.3) The neck showed up as a very sharp point when viewed in this manner. In addition, a travelling microscope was placed above the analyzer plate, (See Figure 5.4) and the approximate retardation of the sample could be observed while the process was going on. The travelling microscope gave a magnification of about 30 times the initial yarn size. Thus the neck position could be monitored either by looking through the travelling microscope or by just viewing through the polarizing plates.

As indicated previously, the position of the neck could be made to advance, remain fixed, or retreat onto the pin. Under the conditions in which the neck was relatively fixed, minor fluctuations were observed. These fluctuations, which have been described by Babcock\(^1\), are produced by variations in the draw ratio at the neck. However, it is thought that these variations might be smoothed out in the length of yarn which is between the neck and the draw roll. A slight amount of continued draw takes place beyond the neck. In this region any minor fluctuations in draw could be evened out.
FIGURE 5.3. Neck in free space as observed between crossed polarizing plates.
FIGURE 5.4. Close-up of thermocouple tube and torsion guage.
b. Measurement of Temperature at the Neck

Since a complete understanding of the process requires a calculation of the energy balance between the change in internal energy, the work input, and the heat produced, it was necessary to measure the temperature developed at the neck. In order to do this a thermocouple was constructed using copper and constantan wires. An accurate measurement of the temperature requires that the heat capacity of the thermocouple be small in relation to the amount of heat developed; therefore, very small thermocouple wires were used. For this purpose, 3 mil wires of both copper and constantan were crossed in an X frame. The intersection of the two wires served as the hot junction point. The wires were held on a phenolic tube which fitted over the objective of the travelling microscope. The hot junction was then brought into focus in the travelling microscope, Figure 5.4, and placed against the moving yarn. In this way, an observation could be made of the contact between the yarn being stretched and the hot junction of the thermocouple. The cold junction of the thermocouple was kept at room temperature. The output from the thermocouple was fed into a recorder.

While it was an advantage to use very small wire, it was found that these wires broke after a short time of contact with the moving yarn. Attempts to use 10 mil
wire produced a thermocouple system which was strong enough to withstand the frictional rubbing on the yarn; however, it was believed that an appreciable portion of heat developed at the neck was being conducted away in the larger wires. This fact was evidenced in the temperature observed with the 10 mil thermocouple system, which was somewhat lower than that measured with a 3 mil system. It was therefore thought preferable to continue to use 3 mil wire, replacing the wires as often as necessary.

With the hot junction directly in focus to the travelling microscope, it was possible to insure direct contact of this junction with the yarn system. When operating with conditions which gave a stable neck it was possible to traverse the length of the sample and obtain a temperature distribution along the neck both after, at, and before the neck. It was found that the temperature was relatively constant from the neck to about \( \frac{1}{2} \) inch beyond the neck toward the draw roll. After this the temperature fell off gradually toward the draw roll. Behind the neck the temperature was essentially room temperature which indicated that the frictional drag of the yarn around the snubbing pin contributed little, if any, temperature rise. Thus the undrawn material being fed to the neck was considered to be at room temperature. The fact that the yield stresses measured were consistent with the values which might be pre-
dicted from the viscoelastic theory, neglecting temperature rise, suggested that the heat conductivity described by Marshall and Thompson\textsuperscript{2} played little part in initiating necking. That is to say, that the heat conductivity of the material was so low that very little effect was achieved from the heat developed in the neck flowing into the undrawn material to lower the yield load. The experimental yield or drawing load was not lowered as predicted by Marshall and Thompson.\textsuperscript{3} It was in fact greater than the slow speed room temperature drawing load. (This point will be discussed further in a following section.) Thus, it is believed that the undrawn material starts to draw at room temperature, the value of the yield load being strictly determined by the velocity considerations which have been described. A schematic diagram of the temperature measuring system is given in Figure 5.5.

c. The Effect of the Number of Wraps Around the Snubbing Pin

Initially, two wraps were taken around the snubbing pin, giving a total belt friction angle of 4 radians. This condition produced a ratio $T_2/T_1$ of approximately 10 to 1 on the basis of a coefficient of friction
FIGURE 5.5. Schematic diagram of thermocouple used in measuring temperature rise during cold-drawing.
of about .2. Later, exploratory studies were made on the effect of the number of wraps. Conditions with one wrap and three wraps were also looked at. In these two cases the drawing force remained about the same as with the two wraps; however, the feed tension, \( T_1 \), varied as predicted by the belt friction formula. In some cases it was easier to obtain a stable neck with only one wrap of \( 2\pi \) radians around the snubbing pin. In other cases, when it was desirable to take only one tension measurement, that is, to eliminate a separate reading of \( T_1 \), it was found that by using 3 wraps around the snubbing pin, \( T_1 \) was of the order of 1 or 2 percent of \( T_2 \). Thus only small errors would be introduced by neglecting \( T_1 \) or by adding in a calculated value based on the coefficient of friction and the number of wraps.

d. Frictional Behavior Around the Snubbing Pin

It is interesting to observe the variation of the coefficient of friction developed around the snubbing pin with variations in experimental conditions. Table 5.1 summarizes some of the results of the nylon 66 tests.

There is a remarkable reproducibility in the values of \( \mu \) at equivalent speeds in spite of differences in draw ratios. The variation of speed, plus the absolute
value, are apparently the result of the spin finish. This behavior is not representative of the dry friction already discussed.

As mentioned previously, the sample of Terylene was difficult to work with since it had no finish. However, one test was made at a draw ratio of 5.67. The neck promptly retreated to the pin and continued around it until the yarn broke. Table 5.2 lists the values of load and coefficient of friction. In spite of large changes in the drawing load and in spite of the position of the neck, the coefficient of friction remained reasonably constant. Since this sample was tested after a series of nylon yarns, without cleaning the pin, it is likely that some of the nylon spin finish remained to influence the results.

When the neck was on the pin, as in the last three items of Table 5.2, part of the yarn was moving at the velocity of the feed roll and part at the velocity of the draw roll. Thus, the slight increase in \( \mu \) from .194 to .210 could be the result of more and more of the yarn travelling at the higher speed.

However, two main points have been brought out. One is that the coefficient of friction depends directly upon the yarn speed. The second is that, while the coefficient of friction varies, the yarn tension changes occur
### TABLE 5.1

Average Values of Coefficient of Friction - Nylon 66

<table>
<thead>
<tr>
<th>Draw Speed</th>
<th>Neck Position or D.R.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>120</td>
</tr>
<tr>
<td>1</td>
<td>131</td>
</tr>
<tr>
<td>2</td>
<td>140</td>
</tr>
<tr>
<td>3</td>
<td>151</td>
</tr>
<tr>
<td>4</td>
<td>163</td>
</tr>
<tr>
<td>5</td>
<td>181</td>
</tr>
<tr>
<td>6</td>
<td>200</td>
</tr>
<tr>
<td>7</td>
<td>223</td>
</tr>
<tr>
<td>8</td>
<td>247</td>
</tr>
<tr>
<td>9</td>
<td>247 5.33</td>
</tr>
<tr>
<td>10</td>
<td>258</td>
</tr>
<tr>
<td>11</td>
<td>213</td>
</tr>
<tr>
<td>12</td>
<td>213 5.00</td>
</tr>
</tbody>
</table>

### TABLE 5.2

Coefficient of Friction

Terylene (D.R. = 5.67)

<table>
<thead>
<tr>
<th>Speed</th>
<th>( T_2 - T_1 )</th>
<th>( T_1 )</th>
<th>( T_2 )</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} )</td>
<td>512</td>
<td>15</td>
<td>527</td>
<td>.194</td>
</tr>
<tr>
<td></td>
<td>306</td>
<td>7.5</td>
<td>313.5</td>
<td>.204</td>
</tr>
<tr>
<td></td>
<td>358</td>
<td>10.5</td>
<td>368.5</td>
<td>.194</td>
</tr>
<tr>
<td></td>
<td>512</td>
<td>15</td>
<td>527</td>
<td>.194</td>
</tr>
<tr>
<td></td>
<td>768</td>
<td>20</td>
<td>788</td>
<td>.200</td>
</tr>
<tr>
<td></td>
<td>1025</td>
<td>25</td>
<td>1050</td>
<td>.204</td>
</tr>
<tr>
<td></td>
<td>1230</td>
<td>27.5</td>
<td>1257.5</td>
<td>.210</td>
</tr>
</tbody>
</table>
primarily on the feed roll side of the pin. The properties of the fiber and overall speed determine the drawing force, which remains reasonably constant as long as the neck is off the pin. The feed tension, $T_1$, adjusts itself to be compatible with $\mu$ and the draw tension, $T_2$.

e. Momentum Considerations

It was first considered that the force was constant through the free length of the sample between the snubbing pin and the draw roll. In other words, beginning with the unstretched portion, through the neck region, and on to the stretched portion, it was felt that the force was a constant. In order to verify this assumption, a calculation was made of the inertial force developed from the acceleration of undrawn material to drawn material. In order to do this, the material was considered as a flowing medium, and fluid flow momentum equations were applied by isolating a boundary before and after the neck. Equations for this consideration follow.

The mass flow is constant; i.e. the mass passing a plane in the undrawn section per unit time is equal to the mass passing a plane in the drawn section per unit time.

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$
If we isolate a system which encloses the neck, we have

\[ \rho_1 A_1 x_1 v_1 - \rho_2 A_2 x_2 v_2 = \Delta m v \]

or:

\[ \rho_1 A_1 x_1 - \rho_2 A_2 x_2 = m \]

\[ \Delta m v = \rho_1 A_1 x_1 (v_1 - v_2) \]

The unbalanced force will be

\[ F = \frac{\Delta m v}{\Delta t} = \frac{\rho_1 A_1 x_1 (v_1 - v_2)}{\Delta t} = \rho_1 A_1 v_1 (v_1 - v_2) \]

For \( F \) in terms of grams per denier,

\[ F_1^* = \frac{F}{\rho_1 A_1} = \frac{F}{\rho_1 A_1} (v_1 - v_2) \]

or

\[ F_2^* = \frac{F}{\rho_2 A_2} = \frac{F}{\rho_2 A_2} (v_1 - v_2) \]
Since \( v_2 \) is greater than \( v_1 \), the system acquires momentum and the unbalanced force is acting in the direction of flow. Substitution of \( D.R. = \frac{v_2}{v_1} \) into Equation 5.4 gives:

\[
\begin{align*}
|F_x^*| &= k v_1^2 (D.R. - 1) \\
|F_z^*| &= k v_z^2 \left( 1 - \frac{1}{D.R.} \right)
\end{align*}
\]

(5.5)

If \( v \) is reported in cm/sec and we want \( F^* \) in gms/den, \( k \) is \( 1.136 \times 10^{-9} \). Therefore, for the most extreme case studied, \( D.R. = 5.67 \) and \( v_2 = 250 \) cm/sec,

\[
|F_z^*| = 1.136 \times 10^{-9} \times 6.25 \times 10^4 \left( 1 - \frac{1}{5.67} \right)
\]

(5.6)

\[
= 0.59 \times 10^{-4} \text{ gms.} \frac{\text{denier}}{}
\]

The drawing force under these conditions was in the range of 4 grams per denier. Thus, the inertial force is extremely small, and the assumption of constant force through the neck is justified. In retrospect, it is easy to see that the force, due to the change in inertia of the mass of the material, should be negligible, since the inertia of the fiber itself becomes important only when we have wave propagation effects. We might consider that we have a strain wave front at the neck propagating through the material when the
material has a propagation velocity equal to the motion of the neck through the material. However, such a material would have an extremely low modulus. Hence the force developed by this large strain wave is really quite small, and is in fact negligible compared to the actual drawing force.

f. Types of Necks Formed

In actual drawing at a neck, the transition from drawn to undrawn material is very sharp. Figure 5.6 shows some of the neck formations which have been observed. Figure 5.6a is a photograph of a sample which was obtained by stopping the drawing machine slowly, allowing the sample to cool under tension and observing it under a polarizing microscope. Figure 5.6b is a sample which was drawn in such a manner that the draw point was on the pin. In this case the draw zone is no longer sharp, but drawing has taken place almost without a true neck.

Figures 5.6c and 5.6d are samples which show an interesting side effect. They were obtained by cutting the yarn at the drawn side of the free neck, while the drawing process was going on. The sample was under an appreciable tension at the time of cutting. There is a very pronounced
FIGURE 5.6. Nylon fibers with various necks.

a) Sample stopped slowly.

d) and c) Sample cut while drawing - notice internal longitudinal cracks.

b) Sample drawn with neck on pin.
set of longitudinal cracks running along the length of the
drawn section, but stopping at the neck. These cracks
developed only when samples were cut in the manner described.

A possible explanation of the cracks is that they represent a failure of the drawn fiber along the boundaries of the fundamental, rheological units or fibrils. These sections of fiber are hot, retaining a good bit of the temperature developed at the neck. The boundaries of rheological units, being more amorphous than other parts of the fiber, will be particularly sensitive to heat.

However, the fact that these flaws opened up in this manner depended upon a particular aspect of the sample preparation. When a tensile specimen was released in the manner indicated, an unloading stress wave propagated toward the fixed end. At the fixed boundary, which in this case is the partially fixed boundary at the neck, the unloading tensile wave was reflected as a compressive wave. Thus, the unsupported fiber could be under a sizable compressive stress until buckling takes place. This phenomenon is known as dynamic buckling.

The conclusion reached is that the presence of the cracks is indicative of potential weaknesses at the boundaries of the rheological units. The weaknesses become a source of failure under the applications of heat and longitudinal compression.
g. Slow Speed Force-Elongation Data

Samples of the two nylons were tested on the Instron to observe their force-deformation relationships at varying extension rates. The data for the two materials are shown in Figures 5.7 and 5.8. Figure 5.7 has the results of four extension rates applied to nylon 66. The most interesting aspect of this set of data is the great dependence of the yield load on the extension rate and the relatively minor effect of rate at the high elongations.

It will be remembered from the discussion of viscoelastic behavior that a large rate effect is associated with a large value of $R_2$, and hence a large energy loss. It is interesting that the large energy loss is associated with the yield phenomenon. Of course, this association is not unexpected, since yielding and energy loss are both associated with plastic work.

The data of Figure 5.7 has been replotted in Figure 5.9 to show the relaxation behavior which has been discussed in Chapter III. This figure shows the large effect of the loading time (i.e. extension rate) on the yield load.

In addition, Figures 5.7 and 5.8 show the consequences of the constancy of the drawing force at the neck. If the draw force is assumed to be equal to the initial yield force for a given set of conditions, it can be
FIGURE 5.7. Force-extension behavior of undrawn nylon 6-6

at four strain rates. (10, 1, .1, 01 min.⁻¹)
FIGURE 5.8. Force-extension behavior of undrawn nylon 6 at three different strain rates (10, 1, .1 min.$^{-1}$)
FIGURE 5.9. Replot of data of Fig. 5.7 for undrawn nylon showing relaxation behavior.
seen that all forces less than the yield force can not be realized in continuous drawing. Thus, an unstable region exists between the yield point and that higher extension which corresponds to an equivalent force. The natural draw ratio which exists under a given set of conditions is defined by the two points of equal force, one being the yield point. In the two figures, the dotted lines define both a sort of natural draw ratio and the constant drawing force. It can be seen that faster loading times produce higher yield loads and consequently higher natural draw ratios. Reference to Figure 5.2 shows that for stable drawing higher draw ratios require higher drawing speeds as indicated.

Experimental Results

A. Calculation of Heat Developed at the Neck

With the crossed wire thermocouple, we can measure the temperature at the neck. However, in order to use this measurement in the energy balance equation, we must know what it means in terms of the heat developed. Thus, we must know the specific heat capacity of the material in order to take the integral product of the specific heat over the temperature rise. Marx, Smith, Worthington, and Dole⁹ have published data on the specific heat of undrawn and drawn
nylon 6 as well as the same variations of nylon 66 over the temperature range of -20 to 180°C. It is difficult to say at what point we are dealing with undrawn nylon and at what point drawn nylon when we talk about the temperature rise at the neck. It is perhaps best to look at the specific heat of both these variations and make a judicious compromise between the values of the two. The variation in specific heat of these materials is shown in Figures 5.10, 5.11, and 5.12 taken from the paper of Marx et al. The heat developed over a certain temperature interval will be the area under the $C_p - T$ curve. We are starting with a room temperature of 70°F. or 21°C. and rising to 100°C. in the case of the nylon 6 and somewhat above that in the case of the nylon 66. If we integrate Marx et al.'s curves over this range we achieve our values of $\Delta Q$.

It must be pointed out that the data of Marx were determined on samples which were dried by evacuation before being placed in the calorimeter. The samples with which we are dealing start out with a normal regain for these materials at standard conditions of atmosphere and relative humidity. We must thus superimpose the effects of the added moisture on the data for the polymers themselves. It is safe to assume that the specific heat of water does not vary over the temperature range which we are studying.
Fig. 1.—Comparison of the specific heat of undrawn 6 Nylon (solid line) with that of undrawn 6-6 Nylon (dotted line) over the temperature range -20 to 80°.

FIGURE 5.10. Specific heat of undrawn nylon 6 and nylon 6-6.

Fig. 4.—Comparison of the specific heat of undrawn 6-Nylon (solid line) with that of 6-6 Nylon (dotted line) over the temperature range 70 to 170°.

FIGURE 5.11. Specific heat from Marx, Smith, Worthington and Dole.9
Fig. 2.—Comparison of the specific heat of drawn 6 Nylon (solid line) with that of drawn 6–6 Nylon, over the temperature range -20 to 80°.

Fig. 5.—Comparison of the specific heat of drawn 6 Nylon (solid line) with that of 6–6 Nylon (dotted line) over the temperature range 80 to 180°.

FIGURE 5.12. Specific heat from Marx, Smith, Worthington and Dole⁹.
When we approach 100°C. and above, in addition to adding heat to raise the temperature of the nylon and water, we must add heat to evaporate some of the moisture. The peaks and dips in specific heat curves of Marx et al. are essentially equilibrium values, whereas cold drawing is far from an equilibrium process. It might be assumed that neither the peaks nor the valleys will occur in the cold drawing process, since they represent crystallization which requires time to develop. The best approximation will then be a smooth curve through the data. In fact, the value of heat capacity increases almost linearly in the range of 20°C. to 100°C. If linearity is assumed, integration is not necessary, and an average value can be used over each interval. Thus, the following values have been assumed:

<table>
<thead>
<tr>
<th>Nylon 66</th>
<th>( C_p ) (nylon)</th>
<th>( C_p ) (nylon and water)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21°C. to 107.5°C.</td>
<td>.43 cal/gm°C.</td>
<td>.45 cal/gm°C.</td>
</tr>
<tr>
<td>21°C. to 134°C.</td>
<td>.45</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nylon 6</th>
<th>( C_p )</th>
<th>( C_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>21°C. to 81.5°C.</td>
<td>.43</td>
<td>.45</td>
</tr>
<tr>
<td>21°C. to 88°C.</td>
<td>.44</td>
<td>.46</td>
</tr>
<tr>
<td>21°C. to 100°C.</td>
<td>.44</td>
<td>.46</td>
</tr>
</tbody>
</table>

The combined value of nylon and water is based on an assumed value of .96 gms. nylon and .04 gms. water.

Calculated values of \( \Delta Q \) based on measured...
temperature changes and averaged values of $C_p$ represented only the heat quantity consumed in a temperature rise.

Other sources and sinks for heat are:

1) Evaporation of water - This requires 500 cal/gm of water evaporated or 20 cal/gm of nylon, assuming 4% moisture.

2) Phase changes in the fiber - Any increase in crystallization which may occur will create heat at the expense of a lowered energy level. This heat has been found to be small, approximately 1 to 2 cal/gms with some polymers $^6$. With nylon, little crystallization can be expected at temperatures up to $100^0$ C.

3) Release of heat of wetting - When fibers absorb water, heat is given off as the water becomes bound to the fiber. Consequently, heat is required to remove the bound water before it can be boiled off. The heat of wetting in this temperature and humidity range can be considered to be 5 cal/gm of fiber.

In the energy balance being considered here, all of these extra effects will be lumped into the term reserved for changes in internal energy. The heat term, $\Delta Q$, includes only the direct products of temperature rise.
b. Work Input

Application of the first law of thermodynamics requires the determination of the work input. The work must be defined in some way consistent with the choice of the system. In this work, the work term has been arbitrarily defined as the draw force times the extension. The reason for this choice is that the major part of the extension takes place at a constant force through the neck. It is true, however, that a small amount of extension, 5 to 10 %, takes place before the neck with a force which is not constant, but is nearly a linear function of the extension. The calculated values of \( \Delta W_{\text{arc}} \) are thus high by a few percent. Since no measurement was made of the elastic extension, it seemed better to make the over-estimate of work and collect the excess in the energy term. In fact, any elastic strain energy retained in the stretched sample should be expected not to result in heat, but to be retained as an increase in internal energy.

Thus the calculated values of \( \Delta W \) can be expected to be greater than the calculated values of \( \Delta \Phi \) for two reasons:

1) the recognized error in considering \( \Delta W = \text{Force} \times (D.R - 1) \). This error produces values which are 1 to 2 cal/gm high at most.
2) the retained elastic strain energy. The elastic strain energy has been estimated to be 1 - 2 cal/gm.

c. Change in Internal Energy

Since no direct measurement has been made of the internal energy change, $\Delta E$ will be estimated from the difference in $\Delta W$ and $\Delta Q$. Some of the types of changes which have been classed as energy changes have been alluded to above. While they represent energy changes of the system, e.g. loss of water, residual strain energy, etc., these changes are not what is normally considered as an internal energy change. The normal energy change is the change associated with the stability of the highly oriented molecular structure. The structural change must be one of energy loss by the system, and it is accomplished with a decrease in the entropy of the molecular system.

A measure of the internal energy change could be made from density, infrared and X-ray measurements. However, these determinations were not made. Jackl has estimated the energy change in cold-stretching PVC to be between 1 and 3 calories per gram. While these values can not be immediately applied to the problem at hand, the order of magnitude is certainly the same.
d. Energy Balance

An energy balance has been attempted for the results of tests at experimental conditions listed in Table 5.3.

<table>
<thead>
<tr>
<th>Fiber</th>
<th>Draw Ratio</th>
<th>Draw Speed (cm/sec)</th>
<th>Draw Force (gms)</th>
<th>Draw Max. Temp. °C</th>
<th>ΔQ (cal/gm)</th>
<th>ΔW (cal/gm)</th>
<th>ΔW-ΔQ (cal/gm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nylon 66</td>
<td>5</td>
<td>93</td>
<td>498</td>
<td>107.5</td>
<td>39</td>
<td>45.4</td>
<td>6.4</td>
</tr>
<tr>
<td>&quot;</td>
<td>5</td>
<td>93</td>
<td>509</td>
<td>107.5</td>
<td>39</td>
<td>46.4</td>
<td>7.4</td>
</tr>
<tr>
<td>&quot;</td>
<td>5.33</td>
<td>245</td>
<td>594</td>
<td>107.5</td>
<td>39</td>
<td>58.4</td>
<td>19.4</td>
</tr>
<tr>
<td>&quot;</td>
<td>5.67</td>
<td>245</td>
<td>622</td>
<td>134.2</td>
<td>53</td>
<td>66.1</td>
<td>13.1</td>
</tr>
<tr>
<td>Nylon 6</td>
<td>4.24</td>
<td>245</td>
<td>480</td>
<td>100</td>
<td>36.3</td>
<td>33.7</td>
<td>-2.6</td>
</tr>
<tr>
<td>&quot;</td>
<td>4</td>
<td>245</td>
<td>493</td>
<td>88.2</td>
<td>30.8</td>
<td>32.1</td>
<td>1.3</td>
</tr>
<tr>
<td>&quot;</td>
<td>4</td>
<td>21</td>
<td>482</td>
<td>81.5</td>
<td>27.1</td>
<td>31.5</td>
<td>4.4</td>
</tr>
</tbody>
</table>

In every case except one, ΔW-ΔQ is positive.

At the lower temperatures, i.e., 100° C. and below, the average of the three nylon 6 experiments is ΔW - ΔQ = +1.0 cal/gm. For these three cases it can be concluded that (1) nylon and water are being heated up; (2) the assorted energy changes, i.e., inverse heat of wetting (36.5 cal/gm), crystallization and internal energy change and strain energy are essentially balanced; and (3) little or no water is
being evaporated.

As the draw ratio and draw force go up, more work is supplied to the yarn and the temperature goes up. It can also be noticed that the excess of $\Delta W$ over $\Delta Q$ increases. These draw conditions produce a considerable amount of visible vapor which leaves the neck. Thus, it has been concluded that the discrepancy between work and heat is primarily the result of the evaporation of water. While it might be thought that all water would evaporate at 100$^\circ$C., bound water behaves differently. In fact, nylon 66 retains about 1% water at 130 - 140$^\circ$ C. The increase in $\Delta W - \Delta Q$ in the first three cases, which all reach the same temperature is indicative of more and more evaporation. The decrease in $\Delta W - \Delta Q$ at the highest temperature can not be accounted for. One possible explanation is that a greater decrease in true internal energy has occurred due to greater crystallization at the higher temperature; but this reasoning is purely speculative.

It is interesting to note that the highest value of $\Delta W - \Delta Q$, 19.4 cal/gm, is about equal to the heat required to evaporate 4% water, i.e., 20 cal/gm. The role of water in cold drawing has added significance over that previously ascribed to it by Yumoto$^{14}$. When temperatures reach and exceed 100$^\circ$ C., the water acts as a buffer and limits further temperature increases. Thus, it is expected that the natural draw ratios which can be achieved
under various conditions of speed, will be profoundly affected by the presence of moisture.

Table 5.3 illustrates a further interesting point. The samples of nylon 66 had to be drawn at higher draw ratios at the available speeds than the samples of nylon 6. As time passed, some of the experiments were repeated. The natural draw ratios of nylon 66 decreased and approached those of nylon 6.

As pointed out in the chapter on viscoelasticity, the yield load varies pronouncedly with the time used in applying the yield load. This variation was discussed in terms of distribution functions of relaxation times and complex dynamic modulii. In the case of undrawn yarns the rate dependence yield load changes with time. A freshly prepared undrawn nylon yarn which is amorphous has a very high value of $E_o$ and a high value of $\tan \delta$. Crystallization is believed to occur spontaneously with aging; and as a consequence, the negative slope of the relaxation lines, $E_o$, decreases.

In order to see how this aging affects the draw ratio, we must remember one basic fact of drawing at a neck. The fact is that the force is constant through the neck. The undrawn yarn reaches its yield load as has been described. The natural draw ratio is determined by the extension on the time-temperature surface that is produced.
by the same force in the drawn portion of the yarn. If the yield load is decreased, e.g. by aging, it is reasonable to expect a lower extension in the drawn portion and a lower natural draw ratio. Similarly, higher draw ratios can be achieved in a given sample by increasing the drawing speed. The decrease in the drawability can be directly related to the change in tensile properties, and it is not necessary to discuss the decrease in terms of crystallization.

Discussion of Literature

This section is a commentary upon the existing literature on the subject of cold drawing. The results of the present experiments are evaluated in terms of the current theories. Since most of the work on cold-drawing is probably being held in confidence by those fiber producers who are financially dependent upon the commercial practice, reference can be made only to the few articles in the public domain.

The most widely quoted work is that of Marshall and Thompson who have treated the process in two ways: a) by a calculated adiabatic load-extension curve which has a negative slope, and b) by "an alternative process of extension at constant tension in a shoulder, coupled with exchanges of heat along the specimen by conduction, (which) appears to be possible at a lower tension than that needed for pure adiabatic extension."
The first of these two suggested processes can not be valid since the dynamic behavior of a neck requires the force to be essentially constant from the drawn to the undrawn sections, including the neck. The second process is inconsistent with the measured tensions of this thesis. Also, the dimensions of the neck required for this process are too small to be reasonable. At slower drawing conditions, the neck can be of a reasonable size. However, the consequences of a lowered tension are a lower drawing temperature, because of heat loss, and less plastic work.

Hockway introduced the concept of hydrostatic tension at the neck and a consequent lowering of the melting point. This approach is certainly valid and justified, but he has evidently made an error in his calculation of yield load, because of a misapplication of Bridgman's equations. In addition, he has projected the lowered melting point as well beyond his calculated value. A reasonable lowering of the melting point would be 40°C. to a value for nylon 66 of 225 - 230°C. This temperature is well above the temperatures reached in a neck. The increased temperatures continue the softening action of the amorphous regions which began at the second order transition temperature (-60°C.), but cannot be thought of as contributing to the melting of any crystalline portions of the fiber.
Bridgman's analysis of the stress conditions existing at a neck "indicated that the stress system could be considered as a uniform axial tensile stress with a superimposed hydrostatic tension." The hydrostatic tension increased from zero at the surface to a maximum at the axis. As a consequence of his assumptions, the maximum shear stress was constant across the section. Shear flow could therefore start anywhere across the section, but brittle failure would initiate at the center because of the maximum tensile stress.

The high elongations which are a consequence of necking and drawing in fibers can hardly be associated with brittle failure. Thus, Bridgman's analysis for brittle failure can not be readily applied to the drawing of fibers. Parker has questioned Bridgman's analysis on the grounds that the failure of necked specimens at normal and high temperatures is really a shear failure. The material deforms and breaks as a consequence of high shear stresses and not because of high normal stresses. Parker's experimental evidence is that "the shear stress as well as the normal stress reaches a maximum value at the center of the section." The variation in the stresses is given in Figure 5.13. It can be seen that the shear stress is constant from the axis to 30% of the radius. This region will be most susceptible to shear flow and flaw formation. The development of internal fissures described by Hookway may not be due to hydrostatic tension,
Fig. 3.5 The stress distribution in the neck of a 2-in. diameter annealed mild steel tensile specimen just preceding fracture.

FIGURE 5.13. Stress distribution in the neck from Parker.

Parker, E., "Brittle Behavior of Engineering Structures".
John Wiley & Sons (1957).

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but are more likely to be the result of shear failure occurring at a line of maximum shear stress. The shear flaw is then drawn in such a way that it appears in the fully drawn section of the fiber as a tubular channel.

Hookway calculated the hydrostatic tension from Bridgman's equations. He also calculated values for the quantity F, which is the constant value of normal tension superimposed upon the hydrostatic tension. In the absence of necking, F would equal .

However, Hookway took $1/3$ of F and added it to his calculated value of hydrostatic tension to get a quantity he calls the total hydrostatic tension. The justification for doing this is questioned. From Hookway's calculated values of hydrostatic tension, a lowering of the melting point of $63^\circ C.$ was postulated. Correcting his calculations by eliminating the value of $1/3 F$, reduces the melting point lowering to only $45^\circ C.$ It is difficult to justify Hookway's statement:

"In view of the approximations made in this calculation one should not expect anything better than an order of magnitude solution for the drop in melting point due to hydrostatic tension. The change in melting point could be anywhere between 20 and $200^\circ C.$, i.e., there is a possibility of melting at the neck due to a combination of hydrostatic tension and temperature."
The suggestion of a change in melting point of 20 to 200°C. and the possibility of melting at the neck due to a combination of hydrostatic tension and temperature is not supported by the force and temperature measurements made on the continuous drawing machine. The yield stress is consistent with that expected from simple viscoelastic effects, i.e., it increases with decreasing time, instead of being consistent with temperature effects alone. This fact is not compatible with the idea of melting at the neck, since melting would be accompanied by a decrease in the drawing force. The drop in the drawing force which occurs with necking at slow speeds is completely eliminated in continuous high speed drawing. The softening due to conduction of the developed heat would lower the drawing force, which in turn would reduce the work input and hence produce less heat. The production of less heat would lower the temperature, which would then raise the yield load and drawing force, and so on until an equilibrium condition is reached.

However, the foregoing postulation of any lowering of the drawing force at all is inconsistent with the experimental results. The rate of heat flow has been calculated from the equations originally proposed by Marshall and Thompson.²

In its simplest form, the heat flow equation is:

\[ 5.5 \]
$$\frac{\Delta H}{\Delta t} = \Delta Q \rho A v = \frac{\Delta Q}{\Delta x} = C_p \Delta \theta e A v = \frac{k A}{\Delta x}$$

(5.7)

From which,

$$\Delta x = \frac{k}{C_p e v}$$

For the conditions of these experiments,

$$k = 0.5 \times 10^{-3} \text{ cal sec}^{-1} \text{ cm}^{-1} \text{ oC}^{-1}$$

$$C_p = 0.44 \text{ cal gm}^{-1} \text{ oC}^{-1}$$

$$\rho = 1.14 \text{ gm cm}^{-3}$$

$$v = 25 - 250 \text{ cm/sec}^{-1}$$

$$\Delta x = 4 \times 10^{-5} \text{ to } 4 \times 10^{-6} \text{ cm}.$$  

The neck dimension must be in the order of .4 to .04 microns, or from 40 to 400 millimicrons. A neck of this size is much smaller than what was found. It must therefore be concluded that the heat is not flowing fast enough to raise the temperature of the filaments before they yield.

Jackl's suggesting "that since high polymers do not draw at a neck above a certain temperature, termed the softening temperature, the temperature in the flow zone must be equal to or slightly greater than the softening temperature," must be disputed. The inability to draw at a
neck at an elevated temperature is a result only of the lowering of the yield point to the extent that the same load can not be supported at a higher elongation at the existing temperature and rate of extension. Certainly flow is taking place in the amorphous regions primarily, and elevated temperatures increase the amorphous mobility. The lowering of the yield load with temperature and the ultimate removal of the so-called "upper and lower yield" phenomenon are associated with the removal of a potential energy barrier which resists flow. The reason for necking is the presence of such a barrier, not the a priori presence of the softening temperature.

Muller,10,11 and his co-workers have unsuccessfully tried to measure temperatures developed at the neck on bulk samples of plastics which are extended relatively slowly. They attribute their lack of success to faulty measuring technique rather than to the absence of the high temperatures. Their attempts at measuring a heat balance by calorimetry have been more successful. They have assumed that because irreversible work has been performed on their sample, there must be a consequent production of heat in the form of a temperature rise. However, the slowness of their tests may allow the heat to dissipate at a lower temperature than that reached in an adiabatic extension. These authors' ability to measure a heat balance and their inability to measure predicted temperatures seem to support this criticism of
their work.

Newman\textsuperscript{12} has reported the results of experiments in which he used the Instron Tensile Tester to cold draw a bulk sample of amorphous polyethylene terephthalate. His "drawing" conditions are a 1.0 inch sample length and a cross-head speed of 0.1 in/min. He relates these results to dynamic shear modulus measurements made at different temperatures. He then matches the modulii obtained in these two ways, i.e. cold drawing and dynamic shear, and calculates a temperature developed at the neck. He further states:

"Since the mechanical damping \( g = \left( \frac{T_i}{C_i} \right) \), Figure 2, goes through a maximum at a glass temperature (or melt temperature), the conversion of work into heat is a maximum at this point. . . . This maximum should, therefore, represent a self balancing position to which the system strives; temperatures in excess of the maximum will result in diminished heat production and cooling."

Newman's drawing speed (0.1 in/min) is so slow that it is difficult to believe that any great temperature rise was effected. The speed is far too slow for an adiabatic extension process. In addition, his comments about the self balancing position could be true only if a temperature rise is allowed to propagate back into the undrawn section to reduce the yield load and hence reduce the work input at a given
draw ratio. While Newman's reasoning is interesting, it does not fit the experimental measurements on nylon which have been described.

Foster and Heap\textsuperscript{13} have studied the drawing of amorphous polyethylene terephthalate in heated baths of silicone oil. While this process is not cold drawing, they introduce a very interesting result, i.e., that a minimum or "characteristic stress" must be reached before the drawing will produce orientation. At varying temperatures between 90 and 120° C. the "characteristic stress" is constant at .02 grams per denier. This stress requirement is not found in the drawing of nylon, where the degree of orientation is entirely related to the strain\textsuperscript{15}. The concept of the rheological unit can be used to explain this result. In a polymer system such as amorphous polyethylene terephthalate, the intermolecular force is the Van der Waals force. This type of force is easily weakened by elevation of the temperature. Thus, the viscosity of the medium surrounding a rheological unit is softened in such a way that the unit cannot be elongated unless the "characteristic stress" is present.

The plotting of Foster and Heap's data on semilog paper (Figure 5.14) shows that their value of "characteristic stress" is just above the lower toe of an exponential stress relaxation function. The implication is that the

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FIGURE 5.14. Stress vs. time showing "characteristic stress" replotted from Foster and Heap, "High-temperature Tensometry"
Brit. J. Appl. Phys. 8 p. 400-402 (1957)
elastic portion of the system, i.e. the interior of the rheological unit, has passed the strain to the viscous portion. Orientation is possible only at higher strain rates since the slower rates produce loading times in excess of even the longest relaxation times.

In conclusion, the concept of cold drawing which has been introduced here is different from all other concepts that have been proposed. The major point of departure is the emphasis placed on the time-dependence of the yield load. The development of heat through energy conversion from plastic work has also been considered. Previously, the role of moisture has been confined to its effects on the original properties of the undrawn material. That role is also considered here, but the additional importance of water as a temperature buffer has been demonstrated for the first time. Values of work input and heat output have been calculated for a number of examples. The discrepancy between the two increases as more and more water is evaporated.

Inherent in all of the observations of cold drawing is the fundamental interaction of fiber properties and machine action. The friction coefficient which controls the tension distribution around the pin has been shown to vary with the machine action of total velocity. The ability to draw at a given draw ratio depends upon the properties of the fiber, its moisture content, and the availability of a wide
enough range of draw speeds. There is probably no other textile process in which a greater control is required of both fiber property and machine action in order to achieve the desired commercial product.
References for Chapter V


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    J. Appl. Phys. 8 p. 400-402 (1957)

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    II Ibid. p. 141-147 (1956)
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VI. SEWING

The process of high speed sewing offers another interesting example of machine fiber interaction. In this process, the sewing thread undergoes a complex repetitive loading cycle. An attempt will be made in this chapter to detail the machine actions which are involved in this important process. This will be done both statically and dynamically; that is, at very low speeds as well as at high speeds with the use of high speed motion picture equipment. While the machine actions have been defined, no force measurements have been made on the sewing thread as it is being processed. Some of the difficulties encountered in making such measurements will be outlined. The purpose of this chapter is similar to that of Chapter IV, which was to show how the principles of friction and viscoelasticity can be applied to a specific process, once machine actions have been described.

While the process of cold drawing discussed in the previous chapter is a steady tension, flow problem. The sewing action consists of severe cyclical applications and removals of force. The sewing action is anything but steady and continuous. It can be represented in either of two ways: (1) by considering the sewing thread to be under zero static stress with a loading and unloading cycles applied, or (2) by considering the thread to be under a given static stress with an oscillating stress superimposed in such a way that the stress varies between a maximum value and zero. Observation of the sewing action of high speed
machines by high speed motion pictures shows that the thread is actually slack over a large part of the cycle. Hence, it is probably better to describe the stress cycle in sewing by the first alternative given above.

The problem of sewing has been treated very infrequently in the literature. In fact, only two references have been found which deal with this problem at all. One is the Master's thesis by Schranze, who studied the actions of the various machine parts as well as described some of their functions in forming a stitch. Schranze's work contributes little that can be used in this study of machine fiber interaction. The second reference appeared as a paper presented by Hardy to the Thread Institute and hence has received only narrow circulation. His work is an attempt to actually measure sewing tensions and the cyclical forces which are developed. Unfortunately, because of the limitations of the tension measuring equipment which was used, the measurements were made at 1/10 of the top speed of a commercial industrial sewing machine. A determination of the forces developed at 500 stitches per minute does not furnish information which is directly applicable to the forces which will be developed at 5000 stitches per minute. Thus, while Hardy's data is interesting, it has little value in the analysis of tensions in high speed sewing.

Because of the complexity of the actions in a single sewing cycle, the first study of the thread motion was made by
slowly turning the machine through a cycle of operation. This
was done on a Singer Sewing Machine Company Model Number 600-1.
The motion of the needle was measured as a function of the angle
of shaft rotation. In addition, the motion of a point on the
thread was coordinated with the needle motion (Figure 6.1).
There are three lines on the figure, two representing the posi-
tion of the thread point, and one the needle position as a function
of angular rotation. The position of the lowest point of the needle
travel was arbitrarily defined as the origin. It can be shown
that the needle motion is very close to a sine curve with a fre-
quency of 83 cycles per second when sewing at 5000 stitches per
minute. The maximum needle velocity is about 25 feet per second.

Let us follow the motion of the thread point, rep-
resented by the upper curve of the two. At the start of the cycle
the thread has just been carried through the fabric by the needle.
As the needle retracts, the thread forms a loop which is hooked
by the bobbin carriage beneath the bed of the machine. The thread
is carried down by the rotation of the bobbin hook. The thread
is moving down at a velocity estimated at 100 feet per second,
while the needle is moving up with a maximum velocity of 25 feet
per second. Thus there is a differential velocity of approximately
125 feet per second between the thread and the needle during this
part of the cycle. The thread reaches its lowermost point when
it is carried around the bobbin carriage. At this point, it appears
FIGURE 6.1. Motion of needle and thread for static motion of Singer Model 600 – 1.

(Two parallel lines represent thread motion for two stitches. Single line represents needle motion.)
that a tension peak should develop. Immediately following the
tension peak, the thread slips off the bobbin carriage, but is still
engaged by the bobbin hook. An instantaneous retraction takes
place before the take-up mechanism in the upper part of the machine
begins to control the motion of the thread. During this free con-
traction the lower portion of the thread can move faster than the
upper portion and the thread will go into compression, buckle and
go slack. Because of the uncontrolled free retraction, the actual
velocity of the thread during this part of the cycle cannot be
estimated from the static method used. It can be said, however,
that the actual velocity might be higher than the 190 feet per
second figure indicated.

In the ascending part of the cycle, thread motion
is controlled by the upper, or rotary take-up mechanism above the
needle. Again the thread motion and the needle motion are out of
phase. When the thread reaches its highest point, the newly formed
stitch is pulled into the fabric. It might be expected that a
second tension peak will develop. At this point the needle is
already moving down and shortly thereafter the needle carries the
thread back into the fabric for a new stitch. It is only over
this small portion of the cycle that the needle and thread actually
move together. During the rest of the cycle, the thread is pulled
either down by the bobbin hook or up by the take-up mechanism.
In both cases, the thread motion is considerably different from
and usually opposite to the direction of the needle motion.
Throughout the sewing cycle, thread is run over many guides and machine parts. However, the most severe actions seem to take place between: (a) the thread and the needle, and (b) the thread and the fabric being sewn. No observations were made of the interaction between the bobbin hook and the thread nor of any interactions which occur beneath the bed of the machine. It must be emphasized that these interactions certainly are important and should not be neglected.

The relative velocities which occur between the thread and the needle are higher than those which occur in any other common textile process, reaching values close to 200 feet per second. Referring to the chapter on Friction, it should be remembered that very high coefficients of friction occur at high velocities. This fact is extremely important, since the forces which are developed in the sewing thread are entirely due to the friction between the thread and the machine parts. This is true both in the tensioning device, as well as in the take-up mechanism, and also at the point of contact between the thread and the bobbin hook below the bed of the machine.

Further studies of the machine action have been made by taking high speed motion pictures of a sewing cycle. In this case a Fairchild camera was focussed on the upper part of a Singer Sewing Machine Model 401. The machine was sewing at 2500 stitches per minute and the camera was taking pictures at 1750 frames per second. Thus, it was possible to divide one
revolution of the sewing machine into 42 separate photographs. These are shown in Figure 6.2. Again the origin for the motion is the point of lowest penetration of the needle. This sequence of pictures shows that the sewing thread is slack for most of the cycle and is buckled into widely distorted forms most of the time. It is possible to estimate tension points by observing when the sewing thread appears to grow taut. The thread first appears to be taut in Frame 18, which is equivalent to approximately 140 degrees revolution. Reference to Figure 6.1 (which admittedly describes the thread motion of a different machine) indicates that this might correspond to the tension peak predicted when the thread is at its lowest point after being carried around the bobbin carriage. The thread goes slack immediately thereafter, and in the next sequence of pictures appears to buckle quite violently. The next tension point appears to begin around Frame 30 and continues in the next frame. If we look at the left portion of the machine, near the tension discs, it is possible to see that the tension check spring has been deflected. The significance of this deflection will be discussed later in detail. Shortly after Frame 32, the thread is seen to go slack again, and it remains slack through the rest of the cycle until the tension peak occurring in Frame 17 reoccurs. The second tension peak, that corresponding to Frames 30 and 31, corresponds to the tension peak shown in Figure 6.1 called peak 2. It is during this tensioning that new thread is fed into the system to take the place of that which has just been put into the stitch.
FIGURE 6.2. High speed pictures of Singer Sewing Machine 401 operating at 2500 stitches per minute. - Camera operating at 1750 frames per second.
FIGURE 6.2 (cont.)

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From consideration of the thread motion and the tension peaks developed as estimated from this series of photographs, it is possible to outline some of the difficulties which might be encountered in introducing a tension probe to try to measure sewing tensions. In the first place, the tension peaks seem to develop over a very short portion of the sewing cycle. In the two instances, the tension peaks seem to exist only over two, at the most three, frames of the sequence of pictures. It will be remembered that these frames are spaced 1/1750 of a second apart, and thus the time between exposures is slightly more than 1/2 millisecond. Therefore, duration of the peaks is in the range of 1 to 1 1/2 milliseconds. Any tension recording device must have a high enough resonant frequency to measure accurately these very short peaks. The second difficulty which must be considered in measuring tensions is what will be the effect of placing the tension probe on the sewing thread. If the probe is placed between the take-up mechanism and the needle, which certainly appears to be the most desirable position, it will inhibit the free motion which is seen to occur in the normal sewing operation. It is possible that inhibition of this free motion will contribute tension levels and tension peaks which are different from those which occur when the probe is absent. Some workers have been known to place a tension measuring device back near the position of the check spring, that is, near the disc tensioning device. It can be seen in Frame 31 and those immediately thereafter, that the
tension condition back at the disc can be considerably different from the tension between the take-up wheel and the needle. The additional friction which occurs at the different points of contact can produce wide variations in tension along the length of the thread. Hardy placed a three-wheel brush tensiometer between the take-up mechanism and the needle. This tensiometer puts the sewing thread through a widely different path than it occupies in its normal position. (See Figure 6.3) His measured tensions, which are shown in Figure 6.4 could be greatly affected by the introduction of this probe. In spite of these difficulties in determining quantitatively the sewing thread tensions, let us try to piece together a qualitative picture of the sewing thread tension cycle from the static thread motion of Figure 6.1, the dynamic motions of the forty odd frames from the high speed motion camera, and the data of Figure 6.4 of Hardy's.

First, quoting Hardy: "As the take-up lever rises to retract the loop, a slight tension is applied to the thread; but immediately thereafter the loop is cast free from the hook assembly and the tension again falls to zero. After the loop has been tightened around the bobbin thread, however, the tension starts to increase again, and this causes the check spring to deflect. A slight inflection at point (B) occurs when the check spring has deflected completely.....i.e., when the thread is stretched directly from the tension discs to the adjacent guide posts..... The momentum of the spring then carries it slightly past the thread and the tension is momentarily reduced. Following this, continued
FIGURE 6.3. Position of tension probe used by Hardy.

Hardy, H.E., Jr., Paper presented at 24th annual meeting of the Thread Institute, November 7, 1957.
FIGURE 6.4. Sewing thread tension traces measured by Hardy.

Hardy, H. B., Jr., "A Research Approach to Improved Thread Performance", presented at the 24th annual meeting of the Thread Institute, November 7, 1957.
upward motion of the take-up lever continues to increase the tension in the thread (in order to set the stitch) until this tension becomes sufficient to pull new thread through the tension discs. Thus, the magnitude of the peak tension at (C) is directly determined by the pressure between the discs (and the frictional properties of the thread)." (The references in this quotation are to the points (A), (B), and (C) of Figure 6.4.)

"The take-up lever then starts to descend, allowing the thread to go slack. At the same time, the needle is entering the fabric with a new loop of thread, and the tension at (D) is caused simply by friction against the guides, as the thread is pulled below to enlarge the loop, so it can pass around the bobbin case. With this adjustment, however, the loop never becomes quite large enough to pass freely around the bobbin; and the tension at (E) in Figure 4 represents the deflection of the check-spring as the thread is 'stretched' and forced around the bobbin case. With the thread under tension in this manner, it has actually been observed with a stroboscope, at high speeds, to snag on the bobbin case....."

While Hardy has used still a third Singer machine, one which does not have a rotary take-up mechanism, we can still compare some of the actions. His actions, beginning with point (A) of Figure 6.4, begins at an angular rotation of about 160 - 170° as shown on Figure 6.1. The tension peaks included by points (B) and (C) appear to come much more quickly in Hardy's work than
evidenced either by Figure 6.1 or by the high-speed photographs, Figure 6.2. However, the difference in the origin of the two scales could be shifted accordingly. It might be best to consider that tension peaks (B) and (C) are those which correspond to tension peak 2 of Figure 6.1. The tension peak (D) of Hardy starts as the thread is being pulled down through the fabric by the lower bobbin mechanism. It is possible that development of this tension is apparent only because of the introduction of the tension probe, since a tension at this point was not observed in the high-speed motion pictures. It is therefore possible that Hardy's peak (B) is an artifact of his testing equipment. The long slack time which is seen to occur between points (D) and (E) are verified by the high-speed motion pictures. Tension peak (E) is then that which corresponds to that of Figure 6.1 occurring at about 140° rotation. Hardy mentions a second deflection of the check spring. A close look at Frames 15, 16, and 17 shows no check spring deflection when sewing on the double rotary machine at 2500 stitches per minute. It is possible that this peak develops so quickly that the tension spring does not have time to respond because of its low resonant frequency. At high sewing speeds the motion of the check spring is not a reliable indicator of thread tension. It will respond only when the tension remains for a reasonable length of time.

The combination of the high-speed motion picture study, the static study, and Hardy's data gives us an indication
of the type of stress cycle which is imposed on the thread during sewing. This cycle has at least one large peak, which occurs when the thread is pulled into the stitch. There can be other minor tension peaks which are developed depending upon the setting of the machine and the speed at which the sewing is done. No estimate has been made of the relative importance of these secondary peaks. At extremely high speeds they could reach the magnitude of the main peak although no data is available to show this. The stress cycle might be considered to be a single or multiple peak of about a millisecond’s duration, which occurs over a small part of the sewing cycle. The loading on the thread is far from being a sinusoidal cyclical motion, but instead is a pulse loading. This must be considered when we try to apply the viscoelastic data to the sewing problem. In addition, the velocities at which the sewing thread moves over guides is much faster than the velocities at which frictional data has currently been measured. In order to know what frictional forces are developed in practice, it is obvious that tests must be made in the 50 - 200 feet per second velocity range. Reference to the chapter on Friction shows that we can expect the coefficient of friction to be higher at the high velocities, but just how much higher we don’t know.

The design of a tensiometer which will measure accurately the magnitude of tensions developed at high speeds of 5000 stitches per minute is a very difficult problem. The probe must be so designed that there is a minimum of distortion of the
sewing thread. In addition, the resonant frequency characteristics should be sufficient to measure the short tension pulses which seem to occur. It is possible that a transducer of the piezoelectric type with a resonant frequency close to 10 kilocycles could perform satisfactorily. This type of gauge has a high output and might prove to be quite satisfactory in this application. The transducers which have been used to date have all been bonded, wire-resistance strain gauges attached to a beam of some sort. These gauges in the opinion of the author are inadequate to quantitatively study sewing tensions. It is recommended that a study of sewing tensions be made with a tensiometer replacing one of the guides between the needle and the take-up mechanism. A minimum amount of distortion would then occur. This tension measurement should be made in conjunction with high-speed motion pictures similar to those in Figure 6.2. It can be seen that a complete study of sewing will thus involve complex and expensive equipment. It is recommended that this type of research be conducted in industrial laboratories where more funds are available. However, industrial research is not published as quickly as academic research, and it might perhaps be better that this equipment be made available to academic institutions so that theses could be written on data obtained with suitable equipment.
References for Chapter VI


VII. GENERAL CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

At the end of a thesis, it is advisable to review in some detail the specific accomplishments which have been made, as well as to outline specific lines for future research. In a broad thesis such as this, a little progress in a number of areas replaces a large advance in a single area. Specifically, the major contributions have been made in the study of cold-drawing; however, minor original contributions have been made in the combined stress-yield theory of friction and in the discussion of the viscoelastic behavior of fibers. Generally, a new concept has been introduced for handling problems of machine-fiber interactions.

The friction theory developed in Chapter II was an analytical attempt to rationalize some of the observed deviations from Amonton's Law. Most of the recent work in fiber friction has involved the empirical fitting of mathematical formulæ to experimental data. The theory which has been proposed was developed from simple stress analysis and did not require the introduction and adjustment of non-constant "constants". In addition, a small amount of experimental data was presented on the lateral stress behavior of undrawn nylon 66.

In Chapter III, the concept of stress relaxation at constant strain rate was developed. Since many textile operations involve the interaction of fiber and machine with a constant
velocity of relative motion, data interpreted in terms of this concept is particularly applicable to processing problems.

The similarities and dissimilarities between various types of dynamic tests were also discussed in Chapter III and it was shown how the data obtained by vibratory tests could be interpreted in terms of stress relaxation tests and vice versa. The resultant generalization of the viscoelastic behavior of textile materials should help to clarify some of the confusion about the applicability of various dynamic data. It is believed that a positive and original contribution has been made toward a general understanding of viscoelastic behavior.

An exploratory, qualitative discussion of the role of the rheological unit in the determination of viscoelasticity was also given. While this concept of micro- and macroscopic rheological units was somewhat speculative, it might be a start toward the understanding of the similarities of viscoelastic behavior among widely different materials. Certainly, a great deal more must be done to establish the shapes and sizes of rheological units and to relate the time relaxation constants which are associated with the different morphological units.

Additional experimental techniques were developed which aided the cold-drawing studies. For example, the use of two crossed polarized plates made it possible to follow clearly the position of the draw point. The ability to observe the neck made it possible to control effectively the position of the neck.
The crossed wire thermocouple was used to measure the temperature developed at a free neck. Again no published data exists on the direct measurement of neck temperatures. These temperatures are usually calculated from calorimetry or otherwise inferred from indirect measurements. The measurement of the maximum temperatures is considered to be an original contribution.

The force-temperature measurements were combined to establish an energy balance at the neck. It was observed that a large discrepancy existed between the work input and the heat energy associated with the measured temperature rise. This discrepancy was attributed to the evaporation of the adsorbed water.

Only preliminary and cursory investigations were made on other processes. The detailed study of each additional process could amount to a separate thesis of itself. Of these processes, one of the more challenging in the field of interaction between fiber and machine is that of high speed sewing. The textile material in this process is subjected to more severe conditions than in practically any other operation. The combined action of high speed cyclical loading at high temperatures and stress levels produces more failures per time of operation than in any other process. Specifically, several types of measurements should be made. These are:

(1) tension measurements near the needle with a high natural frequency force gauge which contacts the thread without introducing a new variable;
(2) temperature measurements on the section of thread which passes back and forth through the needle.

Both of these measurements could be made with variations of the instruments used in the cold-drawing studies. However, the challenge that exists is finding the proper placement of the gauges to minimize their perturbing effect on the process. The tension device might be incorporated in the lower thread guide which is just above the needle. The thermocouple should be placed between this gauge and the needle eye.

The greatest difficulty in the study of sewing tensions is in making sure the many machine adjustments are properly made. Otherwise the measurements would have very little meaning. A complete study must include the effects of controlled variations in the timing and tensioning devices. These variations should be defined by someone with a great deal of practical sewing experience. In addition, careful control must be maintained of the textile variables associated with the thread, such as fiber type, thread construction, and particularly the thread finish.

The analytical and experimental details of the friction and viscoelastic work of this thesis can be applied directly to the sewing problem. They can also be applied to many other processes through the intelligent application of Schwarz's "slubrys". The process under study should be broken down into the symbolic sequence of actions with quantitative values of machine-fiber
relative motion given wherever possible. Once the action velocity is established, the proper time scale for determining the material properties is known. Most of the challenge in further work on machine-fiber interaction lies in ingenious applications of instruments for measurement. The rapid improvements in measuring techniques which have been sparked by high speed missile work should provide valuable tools for these studies. The standard instruments of today, e.g. high gain amplifier, high output piezoelectric transducers and memory oscilloscopes can be used to much greater advantage in the textile field by qualified research personnel.
APPENDIX

Design of Torsion Gauge

The torsion gauge was designed according to the suggestion of J. Katz with two beams at 90° connected to a cylinder which served as the drawing pin. (See Fig. A.1) Since a torsion on this system will produce a maximum shear stress in the middle of the larger side of each of the rectangular beams, gauges mounted at 45° should give pure tension and compression. The shear stress in each segment is given by:

\[ T_{\text{max}} = \frac{M_t}{\alpha b c^2} \]

where \( M_t \) = applied torque
\[ \alpha = 0.333 \] and is function of \( c \) and \( b \)
\( c \) = thickness of beam
\( b \) = width of beam

For the case chosen, \( b = 1" \)
\[ c = 0.020" \]

The torques to be measured are in the order of .1 - .5 in pounds on each side.

For \( M_t = 1 \text{ in. lb.} \)
\[ T_{\text{max}} = \frac{1}{0.333 \times 1 \times 3 \times 3 \times 10^{-4}} = 3.3 \times 10^2 \text{ psi} \]

For Al, \( G = 3 \times 10^6 \text{ psi} \)
\[ \gamma = \frac{T}{G} = 1.1 \times 10^{-4} \]

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FIGURE A.1

Design of Torsion Gauge - 1
Four SR4 gauges, type A-7, were mounted at 45° to the axis of the beams in such a manner that two were in torsion and two in compression. Two arms of the conventional bridge were in each beam. The Sanborn Strain Gauge Amplifier and Recorder were used in the standard manner.

Sample calibration data is given below.

**Electrical Calibration = 19 mm. at x1 attenuation**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>X 1</td>
<td>100 gms.</td>
<td>14.5 mm.</td>
<td>14.5 mm./100 gm.</td>
</tr>
<tr>
<td>X 1</td>
<td>200</td>
<td>29</td>
<td>14.5</td>
</tr>
<tr>
<td>X 1</td>
<td>300</td>
<td>44</td>
<td>14.6</td>
</tr>
<tr>
<td>X 4</td>
<td>300</td>
<td>11</td>
<td>3.7</td>
</tr>
<tr>
<td>&quot;</td>
<td>500</td>
<td>18</td>
<td>3.6</td>
</tr>
<tr>
<td>&quot;</td>
<td>800</td>
<td>29</td>
<td>3.6</td>
</tr>
<tr>
<td>&quot;</td>
<td>1000</td>
<td>36</td>
<td>3.6</td>
</tr>
<tr>
<td>&quot;</td>
<td>1200</td>
<td>43</td>
<td>3.6</td>
</tr>
</tbody>
</table>

The behavior is essentially linear over the range of operation, 200 - 800 gms.
The natural frequency of the torsion gauge was calculated as follows: (Timoshenko, "Strength of Materials" p. 289-290)

\[ \mu \frac{\partial^2 \phi}{\partial t^2} = G I_p \frac{\partial^2 \phi}{\partial x^2}, \quad \phi = \Theta l \]
\[ \Theta = \frac{M}{C} \]
\[ C = \beta b c^3 \]
\[ \beta = \frac{1}{3} \]

For a loaded shaft,

\[ \omega_m = \sqrt{\frac{I}{I_s g}} \]
\[ R = \frac{C}{l} = 18 \text{ in. lbs.} \]
\[ I = \frac{\pi r^4 p a}{2} = 26 \times 10^{-6} \text{ in lb. sec}^2 \]
\[ I_s = \text{moment of inertia of beam} \]
\[ = \frac{M (b^2 + c^2)}{12}; \quad M = \rho l b c \]
\[ = 1.25 \times 10^{-6} \text{ in lb sec}^2 \]

Since there are two beams supporting the load, the factor \( k \) for the given system will be twice that given above, or 18 in. lbs. \( \times 2 = 36 \) in. lbs.

\[ \omega_m = 115.0 \text{ sec}^{-1} \]

and

\[ f_m = 183 \text{ cps} \]

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The calculated natural frequency is well above that of the Sanborn Recorder. If higher values of \( f_n \) are required, the mass of the pin would have to be reduced, since its moment of inertia is a controlling factor. A second gauge was designed, Fig. A.2, which had a substantially reduced moment of inertia of the pin. In addition, the value of \( k \) was increased by shortening the length of the beams. However, this gauge was never used.

Reference

FIGURE A.2

Design of Torsion Gauge - 2
NOTE: FLATS MUST BE \perp WITHIN \pm 1°

SLIP FIT TO BRACKET

.06 R. (TYP)

.030 PARALLEL

POLISH

(2) .070 D. HOLES

MAT'L: HARD ALUM.
SCALE: 1:1

REQ'D: 1 PC.
BIographical outline

Name: Henry M. Morgan

Born: Honolulu, Hawaii, September 28, 1925

Married: June 26, 1946; five children

Education: S.B. in Physics, Mass. Institute of Technology 1943-1948

Professional Experience:

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c. Study of the relationship of fiber properties to
   molecular structure.
d. Instrumentation for high speed tensile testing.
e. Relation between chemical treatments and mechanical
   properties of protein fibers, including human hair.
f. Effect of drawing rate on cold drawing of nylon in-
   cluding measurement of drawing tension and temperature.
g. Interaction of textile materials and textile processes.
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Member, National Academy of Sciences, -National Research Council, Advisory Board on Quartermaster Research and Development, Committee on Fabrics for Body Armor 1952-1958
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Memberships:
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The Fiber Society, Inc.
Society of the Sigma Xi.
Society of Rheology

Publications:


Reference: American Men of Science.