Toward a Noise-Free Interferometric Fiber Optic Gyroscope

by

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Submitted to the Department of Electrical Engineering and Computer Science in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Computer Science and Engineering at the

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Abstract

There is no fundamental principle of quantum physics that states that a sensor output must contain noise. This Thesis describes steps taken toward the realization of a noise-free interferometric fiber optic gyroscope (IFOG). The discussion is structured around five categories of noise typically found in IFOGs. The five categories of noise discussed are those arising from vacuum fluctuations (quantum noise), thermally induced acoustic modes, nonlinearity, reflections, and light-source intensity fluctuations. The primary steps taken in this Thesis toward the goal of eliminating the noise in all these categories, including the quantum noise, are the development and experimental demonstration of a high-repetition rate short-pulse fiber laser, the asynchronous soliton mode-locked (ASM) laser; the testing of a self-phase-stabilized quadrature fiber-optic squeezer; and the development and testing of a new IFOG design that can effectively employ quadrature-squeezed light, the orthogonal polarization fiber optic gyroscope (OPFOG). The ASM laser is a harmonically mode-locked fiber laser that produces picosecond solitons at a repetition rate of 1GHz with no need for modulator drive frequency stabilization. The self-phase-stabilized squeezer is a design for a fiber-optic squeezer that inherently maintains a stable phase between the squeezed vacuum and local oscillator. The OPFOG is a stable IFOG design that requires no phase bias in the fiber ring and is insensitive to light-source intensity noise. Also, other applications were found for some of the technology developed for this Thesis. These applications include the proposal for the use of asynchronous modulation for noise suppression in soliton communication systems; the development and experimental demonstration of additive-pulse limiting, a way to stabilize pulse energies in harmonically mode-locked lasers and to reduce the intensity noise in an optical beam: the development and experimental demonstration of a new type of optical pulse storage ring; and the proposal for a pulse-excited IFOG.

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Chapter 1

Introduction

1.1 Gyroscopes

Putting a telescope in space is a difficult and risky venture, and the initial problems with the Hubble space telescope are an example. Recently, there is a serious proposal for hanging a telescope off of a very high, tethered balloon (see Fig. 1-1). This telescope could achieve close to the performance of a telescope in orbit, such as the Hubble, for about 1/10 the cost. Plus, if there is a problem, the telescope can be hoisted down, fixed, and hoisted back up.

The best angular resolution a telescope looking into outer space can obtain is approximately given by the diffraction limit, \( \lambda/d \), where \( \lambda \) is the optical wavelength and \( d \) is the telescope aperture diameter. Thus for a 1-m telescope in the visible the best angular resolution is about 500nrad.

However, a telescope, especially one on a balloon, will definitely move around in angle much more than 500nrad due to winds, mechanical vibrations, thermal changes, etc. To obtain a stable image in the presence of these fluctuations, such telescopes often need to find a few bright stars near the place where they are looking in outer space. They can lock onto the stars and with the use of servos can obtain a stable image. The Hubble finds three stars and uses them to stabilize the image. However, very often there are not three bright stars close enough to the object one wishes to look at. This is one place where a gyroscope is needed.

A gyroscope is a sensor that measures rotation with respect to its locally inertial frame. There is no such thing as a universal inertial frame, but according to the theory of general relativity, a local inertial frame can always be found.[1] The local inertial frame is the free-falling frame, the frame in which no acceleration forces are experienced. For example, a person standing on the earth is not in an inertial frame while a person in orbit around the earth is. Luckily, the far-away stars are barely moving with respect to a local inertial frame near earth. Thus if a telescope is controlled such that a gyroscope reads that it has no angular movement, it is also very nearly stationary with respect to the stars.

Gyroscopes are used in two main roles. One is in inertial navigation. In this role there are enough gyroscopes to measure all three dimensions of rotation (note that
Figure 1-1: Concept of a balloon-borne telescope for viewing the cosmos.
one two-degree-of-freedom mechanical gyroscope can measure two rotation dimensions. The gyroscopes keep track of how much rotation the mounting platform has undergone. When combined with accelerometers, the system, provided it knows its starting position and velocity, can keep track of where the platform is going. Note that this role of inertial navigation is becoming less necessary when used on earth. This is because the Global Positioning System (GPS), which uses satellites to tell an object the object's location, can substitute for inertial navigation in many cases. The other role for gyroscopes is platform stabilization, as described above for the case of image stabilization for a telescope. In this role the gyroscopes, when attached to a platform, can sense movement of the platform and, via servos, can command the platform to be still.

There are several types of gyroscopes. The first gyroscopes were spinning wheels, and most practical systems still use these types of gyroscopes. These mechanical gyroscopes still provide the best sensitivity of any type of gyroscope (both long-term and short-term performance). The mechanical gyroscopes have some weak points, though. The main point is the fact that they have moving parts. The moving parts can cause the gyroscopes to have a limited lifetime. The Hubble telescope was outfitted with mechanical gyroscopes, which it uses for steering the telescope from sector to sector, and already a few of them have died. Another point is that mechanical gyroscopes have a spin-up time, are often sensitive to linear acceleration, and have a small dynamic range.

Another successful type of gyroscope is the ring laser gyroscope (RLG). An RLG is a ring laser lasing simultaneously in both directions. Rotation with respect to its local inertial frame causes the cavity lengths for the two counter-propagating beams to be different (this is called the Sagnac effect[2]) resulting in different lasing frequencies for the counter-propagating beams. The difference frequency is measured and is proportional to the rotation rate. RLGs are used in military and commercial aircraft for inertial navigation. RLGs have the main weak point that they are expensive. Also, most RLGs have to be dithered mechanically to break out of the lock-in region. This dithering can disturb the platform the RLGs are on. Like most optical gyroscopes, the RLG is insensitive to linear acceleration and has a large dynamic range.

Probably the next most successful gyroscope is the interferometric fiber optic gyroscope (IFOG). Like the RLG, the IFOG utilizes the Sagnac effect to measure rotation. The currently most successful IFOGs use the design called the minimum configuration (Fig. 1-2).[3, 4] We refer to this design as the conventional IFOG. Light enters the system from a light source, part of which is lost through a coupler. The light is then split by a 50/50 beam splitter to travel in opposite directions around the fiber ring and recombines at the beam splitter. The fiber is coiled with many turns which magnifies the Sagnac effect. The returning light is measured by a detector. If the fiber coil rotates, the Sagnac effect results in a nonreciprocal phase shift between the two lights (one light has a different phase than the other upon their return to the beam splitter). When a phase modulator is placed asymmetrically in the ring and is run at a frequency close to half the inverse of the round-trip time of the light, and the detector output is measured via lock-in detection, the Sagnac phase can be sensitively measured. Environmental changes to the fiber are experienced by both
Beams and are canceled out.

The main advantages of the IFOG are that it can be made relatively cheaply and, like the RLG, is insensitive to linear acceleration and has a large dynamic range. The main weak point is that its current performance is about an order of magnitude behind the RLG.

In this Thesis we show that the conventional IFOG powered by an incoherent light source (the usual light source for IFOGs) has too much noise to easily meet the image stabilization requirements for a large telescope (about 1-m aperture or larger). We propose a design for a new IFOG that can, in principle, be noise-free. We now describe what we mean by noise.

1.2 Noise

We define noise as the unpredictable signals in a sensor output that cannot be distinguished from the signal of the desired measurement. There is no fundamental principle of quantum physics that states that a sensor output must contain noise. If only one quantum observable is being measured, then that observable can be measured with infinite accuracy.

In any realistic system, noise is present. This thesis discusses five main categories of noise encountered in IFOGs and ways that the noise can be diminished, if not eliminated. These categories are noise caused by vacuum fluctuations, thermally excited
acoustic modes, nonlinearity, reflections, and light-source intensity fluctuations.

1.3 Steps

The main product of this Thesis are steps taken toward the goal of a noise-free IFOG. The most fundamental noise is quantum noise, and most of the steps taken were toward reducing this noise. The main steps taken are the testing of a self-phase stabilized quadrature squeezer for stable quantum-noise reduction, the development of a quiet laser that outputs short pulses at a high repetition rate for performing the squeezing, and the development of a new design for an IFOG that can readily take advantage of quadrature-squeezed light to reduce its noise.

1.4 Thesis roadmap

We now describe the arrangement of this thesis. Chapter 2 discusses the noise arising from vacuum fluctuations. To reduce this noise optical squeezing is required, and the need for the self-phase-stabilized squeezer and optical pulses are discussed. Chapter 3 discusses the noise from thermally excited acoustic modes in the fiber and shows the requirement for the pulses to be at a high repetition rate. Chapter 3 then presents fiber laser designs for achieving the high-pulse-repetition rate with pulses suitable for squeezing, including asynchronous soliton mode locking. Chapter 4 discusses the noise arising from nonlinearity and the use of chromatic dispersion to reduce the noise. Chapter 5 discusses the noise from reflections in the gyroscope and the use of bandwidth and incoherence to reduce the noise. Chapter 6 discusses the noise arising from light-source intensity fluctuations and shows the need for a new IFOG, the orthogonal polarization fiber optic gyroscope. Chapter 7 ties the ideas of Chapters 2-6 together and presents a design for a proposed nearly noise-free IFOG. Finally, some of the technologies discovered while working on this project can be used in other applications, and these uses are discussed in Chapter 8.
Chapter 2

Noise from vacuum fluctuations

The first noise we discuss is the one that is the most difficult to eliminate: noise from vacuum fluctuations. Vacuum fluctuations add Poissonian statistics (shot noise) to a system. The way to reduce noise from vacuum fluctuations is optical squeezing.[5]

2.1 Quantization of the field

Before discussing the vacuum fluctuations, we briefly review the quantization of the electromagnetic field.

The orthogonal mode solutions to the wave equation for a wave propagating in the $z$-direction are

$$\phi_k = \sum_k (A_k \exp(-jk_z z + j\omega t) + \text{c.c.}) \quad (2.1)$$

in which $\phi$ is a scalar function representing a polarized light wave and $A$ represents the scalar complex amplitude of the light wave.

According to quantum mechanics observables are represented by operators. Thus we change $\phi$ to an Hermitian operator $\hat{\phi}$. We work in the Heisenberg picture, in which operators, instead of states, evolve with time.

As in the classical case, we write $\hat{\phi}$ in the basis of orthogonal mode solutions. Thus

$$\hat{\phi}_k = \sum_k (\hat{a}_k \exp(-jk_z z + j\omega t) + \text{h.c.}) \quad (2.2)$$

The operators $\hat{a}$ and $\hat{a}^\dagger$ do not commute

$$[\hat{a}_l, \hat{a}^\dagger_m] = \delta_{lm} \quad (2.3)$$

We see that $\hat{a}$ and $\hat{a}^\dagger$ are annihilation and creation operators, respectively. Thus we see that the average photon number in a mode $k$ is

$$\bar{n}_{\alpha_k} = <\hat{a}^\dagger_k \hat{a}_k> \quad (2.4)$$
and the fluctuations in photon number in a mode $k$ are

$$\sigma_{n_{ak}}^2 = \langle \hat{a}_k^\dagger \hat{a}_k \hat{a}_k^\dagger \hat{a}_k \rangle - \langle \hat{a}_k^\dagger \hat{a}_k \rangle^2$$  \hspace{1cm} (2.5)

These results can be compared to the classical case. The average power in a mode $k$ is $A_k^2 A_k$, and the fluctuations in the field in a mode $k$ are zero. From this, one can see that quantum mechanics predicts a noise not predicted by classical mechanics.

### 2.2 Optical loss

Consider a light source that outputs light with no photon number noise. In other words, if we measure the light with a perfect detector, we measure no noise. Now suppose there is some loss.

Loss can always be represented by a beam splitter as shown in Fig. 2-1. If $\hat{a}$ represents the quantized input at one frequency as defined in Eq. (2.2) to the system that is undergoing the loss then the output is represented by[6]

$$\hat{c} = p \hat{a} + q \hat{b}$$  \hspace{1cm} (2.6)

in which $p$ and $q$ are chosen to be real. $p^2$ is the power transmissivity of the system, and $\hat{b}$ represents the vacuum fluctuations that necessarily enter the system because of the loss. $\langle \hat{b} \rangle = 0$.

$p$ and $q$ obey

$$p^2 + q^2 = 1$$  \hspace{1cm} (2.7)

$\hat{a}$, $\hat{b}$, and $\hat{c}$ are annihilation operators for their respective modes. The average photon number in $\hat{c}$ is given by

$$\overline{n}_c = \langle \hat{c}^\dagger \hat{c} \rangle$$  \hspace{1cm} (2.8)

which is equal to

$$\overline{n}_c = p^2 \overline{n}_a$$  \hspace{1cm} (2.9)
The fluctuations in \( n_c \) are given by

\[
\sigma_{n_c}^2 = \langle \hat{c}^\dagger \hat{c} \hat{c}^\dagger \hat{c} \rangle - \langle \hat{c}^\dagger \hat{c} \rangle^2
\]

(2.10)

which is equal to

\[
\sigma_{n_c}^2 = p^2[p^2 \sigma_{n_a}^2 + (1 - p)^2 \bar{n}_a]
\]

(2.11)

From this, one can see that as the transmissivity \( p^2 \) goes to zero, \( \sigma_{n_c}^2 \) approaches \( \bar{n}_c \), which is Poissonian statistics (shot noise). Even if \( n_a \) originally contained no noise (i.e., \( \sigma_{n_a}^2 = 0 \)), the output contains noise.

### 2.3 Optical gain

Optical gain also adds noise via vacuum fluctuations to a sensor. Some sensors include optical gain, such as Er-doped fiber, after the sensor in an effort to boost the signal. This section shows that the addition of gain after the sensor only degrades the signal-to-noise-ratio (unless the gain is needed to boost the signal above the electrical noise of the detection system).

Gain is described by\([5, 6]\)

\[
\hat{c} = p\hat{a} + q\hat{b}^\dagger
\]

(2.12)

where \( \hat{a} \) represents the input light, \( \hat{c} \) represents the amplified light (amplified by a factor \( p^2 \)), and \( \hat{b} \) represents vacuum (\( \hat{b} \) is necessary for the conservation of commutator brackets\([7, 5]\)). \( p \) and \( q \) are chosen to be real. This relation can be derived from the construction of an amplifier by two squeezers in the two arms of an interferometer. Note that this equation is different in a resonator. In a resonator, the output \( \hat{c} \) travels around the resonator (if it is a ring resonator) and becomes the new \( \hat{a} \). In the end, \( \hat{c} \) can exist only at frequencies \( k \) near the resonant frequencies of the resonator.

\( p \) and \( q \) satisfy

\[
p^2 - q^2 = 1
\]

(2.13)

\[
p > 1
\]

(2.14)

The average photon number of \( \hat{c} \) is

\[
\bar{n}_c = p^2 \bar{n}_a + p^2 - 1
\]

(2.15)

and thus \( p^2 \) is the power gain of the system. One sees that an additional \( p^2 - 1 \) photons per mode \( k \) are added. This is the spontaneous emission. One finds that the fluctuations in \( n_c \) are given by

\[
\sigma_{n_c}^2 = p^4 \sigma_{n_a}^2 + p^2(p^2 - 1)(1 + \bar{n}_a)
\]

(2.16)

As one can see, the fluctuations of \( \hat{c} \) are even higher than Poissonian. This is a necessary consequence that the amplification of a signal must decrease its signal-to-noise ratio. Like the case for squeezing, this is a matter of course since noise is defined by that in the sensor output which is indistinguishable from the desired signal.
If the initial light enters with Poissonian statistics \( \langle \sigma_n^2 \rangle = \bar{n}_a \) and \( \bar{n}_a \gg 1 \) then

\[
\sigma_{n_c}^2 \approx \bar{n}_c (2p^2 - 1)
\]  

(2.17)

Thus the output, if the input has only shot noise, has noise approximately \( 2p^2 - 1 \) times its shot-noise level.

We just showed how the entering of vacuum fluctuations via loss or gain into a system adds noise. Next we show another way that vacuum fluctuations can enter a system.

### 2.4 Balanced detection

All IFOGs use some form of balanced detection. Balanced detection consists of either two detectors or one detector that has its input switched between two signals at a certain frequency. The conventional IFOG employs the latter. The sensor output is the difference current between the two detectors or the difference current between the switched signals. The powers on the detectors or switched detector are matched when no signal is present. In this way, intensity fluctuations of the light source (if the sensor is linear) at frequencies lower than the switching rate are common to the detectors and are subsequently canceled.[8] A measurement interferometer using two-detector balanced detection is shown in Fig. 2-2.

An optical sensor necessarily has two inputs. One is for the optical field and the other is empty and hence vacuum fluctuations enter there. The effect of the vacuum fluctuations on the system noise is different here than for the case of loss since in loss some light is thrown away while here all the light is kept.

We now calculate the noise resulting from the system of Fig. 2-2. The sensor in Fig. 2-2 is an interferometric sensor that measures the phase difference \( \phi \) between the two arms of the first interferometer. This sensor could be, for example, an IFOG. If the inputs to the sensor are \( \hat{a} \) and \( \hat{b} \) as labeled, one can easily show that the output difference current is

\[
\Delta \hat{i} = e \left\{ (\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b}) \sin(\phi) + (\hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a})[\sin^2(\phi/2) - \cos^2(\phi/2)] \right\}
\]

(2.18)

where \( e \) is the charge of an electron. Note that (2.18) assumes detectors with perfect quantum efficiency. If \( \hat{a} \) represents the input light and \( \hat{b} \) is vacuum, one sees that \( \Delta \hat{i} = e\bar{n}_a \sin(\phi) \) and thus measures the angle \( \phi \).

To calculate the noise of \( \Delta \hat{i} \) we use the frequency domain. The power spectral density of \( \Delta \hat{i}(t) \) is given by[9]

\[
|\Delta \hat{i}(\omega)|^2 = \frac{1}{2\pi} \int dt \left[ \frac{1}{2\pi} \int dt \Delta \hat{i}(t) \Delta \hat{i}^\dagger(t) \right] \exp(-j\omega t)
\]

(2.19)

where

\[
\Delta \hat{i}(\omega) \equiv \frac{1}{2\pi} \int dt \Delta \hat{i}(t) \exp(-j\omega t)
\]

(2.20)
Figure 2-2: A measurement interferometer.
When $\phi = 0$, for low frequencies ($\omega = 0$), and if $\hat{b}$ is vacuum one finds

$$|\Delta \bar{\mathbf{i}}(\omega)|^2 = \frac{e^2}{2\pi \hbar \alpha} = \frac{e}{2\pi \hbar} \bar{\mathbf{i}}$$

(2.21)

where $\bar{\mathbf{i}}$ is the average of the sum of the currents in both detectors or just the current if using only one detector. This is Poissonian statistics (shot noise). Thus one finds that shot noise in the output current is obtained regardless of the intensity characteristics of the light source. This should be qualified with the fact that if the detector is a switched detector rather than two detectors, light-source intensity noise at and above the switching frequency plays a role.

To reduce this noise one cannot leave the vacuum port empty. Instead, one must inject "squeezed vacuum" into the empty port, a topic of the next section.

### 2.5 Optical squeezing

The characteristic of a squeezed state is that its quantum uncertainty in one observable is lower than that of a coherent state. This characteristic can permit certain detections to have noise below the standard quantum limit. There are two main types of squeezing that we discuss. These types are photon number squeezing and quadrature squeezing.

#### 2.5.1 Photon-number squeezing

Photon-number squeezed light has reduced uncertainty in its photon number. This type of squeezed light is measured by a single detector into which the entire light beam is entering. Perfectly photon-number squeezed light has all of its photons evenly spaced in arrival time such that the detector receives a steady stream of photons. If a single detector measures a photon stream in a coherent state (the state exiting a system with large loss) the result is shot noise. However, if a detector measures a photon stream that is photon-number squeezed, the noise is below the shot-noise level.

Photon number squeezing has been achieved in a constant-current-driven semiconductor laser[10] and has been proposed to be achieved via an imbalanced nonlinear interferometer[11, 12].

The conjugate observable to photon-number squeezed light is the photon phase. A phase-space diagram of photon-number squeezed light is shown in Fig. 2-3. The uncertainty is shown by the shaded area. As one can see, the amplitude is known with much certainty while the phase is known with much uncertainty.

#### 2.5.2 Quadrature squeezing

Quadrature squeezing is the most common squeezing. The squeezed observable is one of the quadrature components. Essentially, quadrature squeezing creates two beams of light from a single beam that are quantum correlated. When two detectors measure
two beams that originated from one beam split by a 50/50 beam splitter and the detector signals are subtracted, the result is shot noise. When two detectors detect two quantum-correlated beams created by a quadrature squeezer and the detector signals are subtracted, the noise is below the shot-noise level.

Since Slusher et al.[13] achieved squeezing in a sodium beam in 1985, many groups have achieved optical quadrature squeezing through various methods, such as optical parametric oscillation and amplification[14, 15, 16] and four-wave mixing.[17] Both Shelby et al.[18] and Levenson et al.[19] have accomplished cw squeezing by the use of four-wave mixing in optical fibers.

Quadrature squeezing with a balanced nonlinear interferometer[9] is useful for sub-shot-noise interferometric measurements because it is broadband and, in principle, has no net power loss. By a nonlinear interferometer we imply that the interferometer has Kerr nonlinear media in each arm such that each arm experiences intensity-dependent phase-shifting. Because of the broadband squeezing of the scheme, pulses can be used for squeezing. The main advantage of using pulses instead of cw is that a larger nonlinear phase shift is obtained for a given average power. Pulsed squeezing experiments in fiber have been reported by Bergman and Haus[20], Rosenbluh and Shelby[21], and Doerr et al.[22]

A phase-space diagram of quadrature squeezed light is shown in Fig. 2-4. Quadrature squeezed light has no meaning unless it is compared with another light beam. One can see that quadrature squeezing can be viewed either as squeezed vacuum
Figure 2-4: Phase-space diagram of a quadrature squeezed state shown before and after a 50/50 beam splitter.

Figure 2-5 shows a quadrature squeezer connected to an interferometric sensor that measures the angle $\phi$. The phase-space diagrams indicate how the quantum uncertainty is reduced in the measurement. $\kappa$ represents the nonlinear medium, $\Psi$ represents the phase adjustment of the squeezed vacuum relative to the LO for the best noise reduction, and $\pi/2$ is the phase shift of the LO with respect to the signal from the measurement interferometer to achieve maximum sensitivity in the output.

2.5.3 Quadrature squeezing apparatus

As mentioned above, quadrature squeezing is most conveniently performed in a nonlinear interferometer. We choose fiber for the nonlinear medium in the squeezer since fiber has very low loss. This low loss is necessary since loss adds Poissonian noise to a system.

A fiber ring reflector squeezer is usually implemented using a 50/50 beam splitter or coupler. The apparatus is shown in Fig. 2-6. The drawback to this design is that the LO and squeezed vacuum must travel different paths to get to the detector. Thus some sort of electronic feedback circuit must be employed to achieve stable noise reduction.[23]

There is another scheme, proposed by Shirasaki[24], in which the LO and squeezed vacuum travel the same path using orthogonal polarizations (see Fig. 2-7). The light
Figure 2-5: Diagram of a squeezer followed by a sensor.
enters the scheme through a special polarization beam splitter (SPBS). A SPBS is a PBS that, in addition to reflecting 100 percent of the $s$-polarization, also reflects some portion of the $p$-polarization. The $p$-polarized light exiting the SPBS then is rotated 45° by the half-wave plate and is split into two orthogonal polarizations by the PBS. Each polarization enters the same axis of the polarization-maintaining (PM) fiber in the ring such that both beams recombine and pass through the half-wave plate again. Before entering the ring, the $s$-polarization is empty (contains ordinary vacuum). On return from the ring, the $s$-polarization contains squeezed vacuum. All of the $s$-polarized light is then reflected towards the detectors along with some of the $p$-polarized light. Some waveplates then adjust the relative phase between the $p$-polarized (LO) and $s$-polarized (squeezed vacuum) lights before they reach the detector PBS and are detected with homodyne detection. As one can see, the relative phase between the LO and squeezed vacuum is self-stabilized in this design. This self-stabilization is necessary in order to use squeezing in a high-sensitivity sensor such as an IFOG.

We conclude this discussion of squeezing with a few notes.

1. Squeezing is destroyed by loss, which can be understood from the loss analysis above. It is because loss adds Poissonian noise to a system.

2. Placing nonlinearity in or after the sensor, unlike placing it before, can never help to increase the signal-to-noise ratio. This is a consequence of the fact that noise is defined as that which is indistinguishable from the desired signal. In fact, nonlinearity must be avoided as much as possible in a sensor, otherwise noise in the
output in increased.[25, 26]

3. A concern of squeezing with pulses is the chromatic dispersion in the fiber. Ref. [27] shows via simulation that squeezing is not necessarily reduced by large temporal and spectral distortions of the pulse caused by dispersion and can even be enhanced in certain cases.

2.6 Summary

Most sensors, such as an IFOG, contain Poissonian noise (shot noise) from the input of vacuum fluctuations. One can reduce this noise via optical squeezing. If measuring photon number, one uses photon-number squeezing. If measuring phase, as in an IFOG, one uses quadrature squeezing. It is best to use pulses for the squeezing because they provide higher nonlinearity for a given average power and because they avoid stimulated Brillouin scattering. However, the need for pulses to eliminate the quantum noise demands attention to another noise, which is discussed in the next chapter.
Chapter 3

Noise from thermally excited acoustic modes

In this chapter we discuss the noise created by thermally excited acoustic modes in the material through which the light is propagating. This noise significantly limits what types of sources can be used in IFOGs.

The results calculated here should allow a researcher to quickly obtain an order-of-magnitude estimate for the acoustic noise in a given experiment. The acoustic noise is analyzed in more detail in Refs.[28, 29].

3.1 Acoustic modes

The atoms in a material vibrate when at a temperature greater than 0K. In a finite-size material, these vibrations exist as acoustic modes defined by the material boundaries.[30] As realized by Mandel’shtam, these acoustic modes are the same things as Einstein’s “formal waves” of density and Debye’s thermal elastic waves.[30] These acoustic modes create fluctuations in the dielectric tensor through the photoelastic effect, causing a passing light beam to undergo what is termed Brillouin scattering. When the acoustic wavelength is much smaller than the optical beam width the light is scattered in all directions, as described by the Einstein formula. When the acoustic wavelength is larger than the optical beam width the light is phase modulated and depolarized. When the material cross-section size approaches that of the beam size the individual long-wavelength acoustic mode frequencies can be distinguished in the phase-modulated and depolarized lights. This quantization of the acoustic mode frequencies has been termed “guided acoustic-wave Brillouin scattering” (GAWBS) by Shelby, Levenson, and Bayer.[28]

The phase modulating and depolarizing fluctuations result in timing, phase, and intensity noise in lasers and sensors.[31] The lasers and sensors most susceptible to acoustic noise are those in which the light propagates mostly through a material, such as in optical fiber lasers and optical fiber sensors.
3.2 Calculation of index of refraction fluctuations due to the acoustic modes

First we calculate the index fluctuations created by the acoustic modes in a material. Suppose the material is isotropic, elastic, and has a rectangular cross section of dimensions $a$ and $b$, with an optical beam of diameter $d$ passing through the center (see Fig. 3-1). The length $L_{\text{piece}}$ of the section of material is much greater than the cross-section dimensions.

As an approximation, considering only the longitudinal vibrations, the displacement $u$ of the particles in the medium obeys the relation\[32\]

$$\nabla^2 u = \frac{\rho}{E} \frac{d^2 u}{dt^2}$$ (3.1)

where $\rho$ is the density and $E$ is Young’s modulus of the material. The spatial derivative of $u$ must be zero normal to the surface boundaries. Thus the acoustic mode solutions to (3.1) are

$$u_x = A_{xlm} \cos\left(\frac{l\pi}{a} x\right) \cos\left(\frac{m\pi}{b} y\right) \sin(\omega_{lm} t)$$ (3.2)

$$u_y = A_{ypq} \cos\left(\frac{p\pi}{a} x\right) \cos\left(\frac{q\pi}{b} y\right) \sin(\omega_{pq} t)$$ (3.3)

in which $l$, $m$, $p$, and $q$ are non-negative integers, and $\omega$ is the mode frequency obeying the relation

$$\omega_{lm}^2 = \frac{E}{\rho} \left[ \left(\frac{l\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 \right]$$ (3.4)
According to the equipartition hypothesis each mode has an average energy $k_BT$, where $k_B$ is Boltzmann's constant and $T$ is the absolute temperature.\cite{28, 29} Thus

$$A_{xlm} = \sqrt{\frac{8k_BT}{L_{piece}\omega_{lm}^2ab}} \quad (3.5)$$

The index of refraction changes caused by the acoustic modes are given by\cite{33}

$$\Delta n_x = -\frac{n^3}{2} (p_{11} \frac{du_x}{dx} + p_{12} \frac{du_y}{dy}) \quad (3.6)$$

$$\Delta n_y = -\frac{n^3}{2} (p_{12} \frac{du_x}{dx} + p_{11} \frac{du_y}{dy}) \quad (3.7)$$

$$\Delta n_{yx} = -\frac{n^3}{2} (p_{44} \frac{du_z}{dy}) \quad (3.8)$$

$$\Delta n_{xy} = -\frac{n^3}{2} (p_{44} \frac{du_y}{dx}) \quad (3.9)$$

in which $\Delta n_x$ and $\Delta n_y$ are the index changes along the x- and y-axes, respectively, and $\Delta n_{yx}$ and $\Delta n_{xy}$ are the index changes along the axes 45° to the x- and y-axes. The $p$ are the photoelastic constants for the material.

Now we compute the mean square fluctuations in index that the optical beam experiences. In computing the mean square phase fluctuations we consider each acoustic mode as independent. In the actual case, the $u_x$ and $u_y$ modes are coupled because of a term that we are neglecting in (3.1) and material geometry (for example, the circular cross-section of an optical fiber). However, it can be easily shown that the final total mean square fluctuations calculated for coupled modes is the same as that reached for the uncoupled modes. Also, the total number of modes must be the same for both the coupled and uncoupled cases.

Hence the mean square fluctuations in index that the optical beam experiences are (adding both $\cos(\omega t)$ and $\sin(\omega t)$ modes)

$$\overline{(\Delta n_x)^2} = \frac{2n^6k_BT}{L_{piece}Eab} \left\{ \frac{p_{11}^2 G_{lm}}{p_{12}^2 H_{pq}} \right\} \quad (3.10)$$

$$\overline{(\Delta n_y)^2} = \frac{2n^6k_BT}{L_{piece}Eab} \left\{ \frac{p_{12}^2 G_{lm}}{p_{11}^2 H_{pq}} \right\} \quad (3.11)$$

$$\overline{(\Delta n_x - \Delta n_y)^2} = \frac{2n^6k_BT}{L_{piece}Eab} (p_{11} - p_{12})^2 \left\{ \frac{G_{lm}}{H_{pq}} \right\} \quad (3.12)$$

where

$$G_{lm} = \frac{\left(\frac{l}{a}\right)^2}{\left(\frac{l}{a}\right)^2 + \left(\frac{m}{b}\right)^2} \quad (3.13)$$
\[ H_{pq} = \frac{\left( \frac{\varepsilon}{\tilde{\varepsilon}} \right)^2}{\left( \frac{\varepsilon}{\tilde{\varepsilon}} \right)^2 + \left( \frac{\tilde{\varepsilon}}{\varepsilon} \right)^2} \]  

(3.14)

for \( l, p \leq 2a/d \) and \( m, q \leq 2b/d \) and \( l, q \) odd and \( m, p \) even, otherwise the fluctuation is zero. The bounds on \( l, m, p, \) and \( q \) arise from the fact that when the acoustic wavelength is smaller than the beam width, the index change across the beam approximately averages to zero. The corresponding acoustic modes do not contribute to phase fluctuations of the beam. Instead, these high frequency acoustic modes diffract the light and cause scattering in all directions. This loss is extremely small and is neglected here.

Note that the approximation was made that the index change is constant across the beam until the cutoff condition. Also note that the 45° contributions \( \Delta n_{xy} \) and \( \Delta n_{yz} \) are neglected. This is allowable if we restrain our axes of observation to the \( x- \) and \( y- \) axes.

There are approximately \( ab/(2d^2) \) acoustic modes contributing to phase fluctuations of the beam. This number is simply obtained from mode-counting. Note that in an experiment, it will most likely appear that there are fewer modes than this. This is because although the fluctuations of the uncoupled modes are roughly the same magnitude for each mode, in the case where modes are coupled together because of material geometry and characteristics, some modes may cause significantly more phase fluctuations than others. Thus in a measurement a few modes may stand out while the others are buried in some other noise. However, as mentioned earlier, the total mean square fluctuations are the same whether the modes are coupled or not.

The total mean square fluctuations are the sums of the mean square fluctuations from the modes, so

\[ \sum (\Delta n)^2 = \sum (\Delta n_x)^2 \approx \sum (\Delta n_y)^2 \approx \frac{n^6 k_B T (p_{11}^2 + p_{12}^2)}{2L_{\text{piece}} Ed^2} \]  

(3.15)

\[ \sum (\Delta n_x - \Delta n_y)^2 \approx \frac{n^6 k_B T (p_{11} - p_{12})^2}{L_{\text{piece}} Ed^2} \]  

(3.16)

For example, for fused silica \( (n=1.45, p_{11}=0.121, p_{12}=0.270, E = 7.25 \times 10^{10} \text{N/m}^2) \) at 300K,[34, 28, 32]

\[ \sum (\Delta n)^2 \approx \frac{2.32 \times 10^{-32} m^3}{L_{\text{piece}} d^2} \]  

(3.17)

\[ \sum (\Delta n_x - \Delta n_y)^2 \approx \frac{1.18 \times 10^{-32} m^3}{L_{\text{piece}} d^2} \]  

(3.18)

It is a satisfying result that the total mean square index fluctuations are approximately independent of the material cross-section size. This means that a light beam experiences approximately the same total phase noise whether passing through a thick block or a thin fiber of the same material and length, as long as the beam size is the same in both cases.
The lowest contributing acoustic mode frequency is

$$\omega_{\text{lowest}} \approx \sqrt{\frac{E}{\rho \alpha}} \quad (3.19)$$

(or with $a$ replaced by $b$ if $b$ is larger) and the highest contributing acoustic mode frequency is

$$\omega_{\text{highest}} \approx \sqrt{\frac{8E}{\rho d}} \quad (3.20)$$

Note that although the above analysis was done for a rectangular cross-section, the result should be approximately the same for a circular cross-section, such as an optical fiber.

For a fused silica fiber ($\rho = 2.20 \times 10^3 \text{ kg/m}^3$) of 125$\mu$m outer diameter and a 9$\mu$m mode-field diameter, $\omega_{\text{lowest}}/(2\pi) \approx 23 \text{ MHz}$ and $\omega_{\text{highest}}/(2\pi) \approx 900 \text{ MHz}$. A typical measured $\omega_{\text{lowest}}/(2\pi)$ for such a fiber is 20MHz, in good agreement. It is difficult to say exactly what the measured $\omega_{\text{highest}}/(2\pi)$ is, but 900MHz is reasonable.

### 3.3 Calculation of phase fluctuations in optical fiber

We now wish to calculate the phase noise a beam experiences traveling through an optical fiber. Consider a small piece of fiber of length $L_{\text{piece}}$ short enough such that the acoustic mode is coherent throughout the piece and that the acoustic mode period is much longer than the light travel time through the piece. The phase noise experienced by the light traveling through the piece is given by

$$\sum (\Delta\phi)^2_{\text{piece}} = \left( \frac{2\pi L_{\text{piece}}}{\lambda_0} \right)^2 \sum (\Delta n)^2$$

$$\sum (\Delta\phi_x - \Delta\phi_y)^2_{\text{piece}} = \left( \frac{2\pi L_{\text{piece}}}{\lambda_0} \right)^2 \sum (\Delta n_x - \Delta n_y)^2 \quad (3.22)$$

$\lambda_0$ is the free-space wavelength of the light. Assuming that all the pieces of the fiber are incoherent (such that $L_{\text{piece}}$ is the coherence length of the acoustic modes) then a fiber of total length $L$ has the total phase noise

$$\sum (\Delta\phi)^2 = L_{\text{piece}}L \left( \frac{2\pi}{\lambda_0} \right)^2 \sum (\Delta n)^2$$

$$\sum (\Delta\phi_x - \Delta\phi_y)^2 = L_{\text{piece}}L \left( \frac{2\pi}{\lambda_0} \right)^2 \sum (\Delta n_x - \Delta n_y)^2 \quad (3.24)$$

The above calculation was for cw light. For light consisting of short pulses with a spectral bandwidth greater than the highest contributing acoustic mode, one can
easily show that
\[ \frac{(\Delta \phi)^2}{B} \approx L_{\text{piece}} L \left( \frac{2\pi}{\lambda_0} \right)^2 \frac{\sum (\Delta n)^2}{\omega_p/(2\pi)} \]  \hspace{1cm} (3.25)

\[ \frac{(\Delta \phi_x - \Delta \phi_y)^2}{B} \approx L_{\text{piece}} L \left( \frac{2\pi}{\lambda_0} \right)^2 \frac{\sum (\Delta n_x - \Delta n_y)^2}{\omega_p/(2\pi)} \]  \hspace{1cm} (3.26)

in which \( B \) is the measurement bandwidth (in Hz) and \( \omega_p \) is the pulse repetition frequency. Note that (3.25) and (3.26) hold only for the case of pulses with repetition rates lower than the highest contributing acoustic frequency. For pulse repetition rates higher than this or for cw light, the acoustic noise is not convolved down to frequencies lower than the lowest contributing acoustic frequency. If most optical sensors normally look only at frequencies less than the lowest contributing acoustic frequency, we assume that the acoustic noise is not a problem when high pulse-repetition rates or cw is used. This is why so much effort described in Chapter 2 was made towards obtaining pulse repetition rates a GHz or greater.

3.4 Calculation of noise in an interferometer

The acoustic modes do not cause significant noise unless there is an interferometer, as in a gyroscope or in additive-pulse mode locking (APM, discussed later). There are basically two types of interferometers: the two arms consist of spatially separated paths (as in a ring reflector) or the two arms consist of orthogonal polarizations (as in polarization APM). One can show that the intensity fluctuations \( \Delta P \) in the interferometer output power \( P_{\text{out}} \) from one of the output ports are given by

\[ \Delta P \approx \frac{\sin(\phi_b)}{1 + \cos(\phi_b)} \frac{\sqrt{2\Delta \phi}}{P_{\text{out}}} \]  \hspace{1cm} (3.27)

for the ring reflector case (\( \Delta \phi \) is a nonreciprocal phase shift resulting from the fast fluctuations, and (3.27) does not hold for a very short ring), and

\[ \Delta P \approx \frac{\sin(\phi_b)}{1 + \cos(\phi_b)} (\Delta \phi_x - \Delta \phi_y) P_{\text{out}} \]  \hspace{1cm} (3.28)

for elliptical polarization followed by a polarizer. \( \phi_b \) is the interferometer phase bias.

The total noise measured by a detector is the shot noise plus the acoustic noise, provided that the statistics of the light are Poissonian before entering the interferometer and the interferometer is linear. The power spectrum level seen by a detector measuring the power from one port of the interferometer is then

\[ \frac{(\Delta i)^2}{B} = 2e\bar{\nu} + \left( \frac{\eta e}{h\nu} \right)^2 \left( \frac{\sin(\phi_b)}{1 + \cos(\phi_b)} \right)^2 \left\{ \frac{2\Delta \phi}{B} \left( \frac{\Delta \phi_x - \Delta \phi_y}{B} \right)^2 \right\} P_{\text{detect}}^{-2} \]  \hspace{1cm} (3.29)

where the upper equation is for a ring reflector and the lower is for polarization interference. For a balanced detector looking at the output of the interferometer with
power transmissions $T_{signal}$ and $T_{LO}$ for the signal and LO respectively,

$$\frac{(\Delta i)^2}{B} = 2e\bar{i} + \left(\frac{\eta}{hv}\right)^2 \frac{T_{signal}}{T_{LO}} \left\{ \frac{2(\Delta \phi)^2}{(\Delta \phi_x - \Delta \phi_y)^2} \right\} \overline{P_{detect}}^2$$  \quad (3.30)

where $\bar{i}$ and $\overline{P_{detect}}$ are the sum of the currents and powers on both detectors respectively (or just one detector if using switched balanced detection). $\eta$ is the detector quantum efficiency, $h$ is Planck's constant, and $\nu$ is the frequency of the light (in Hz).

One can write the above equations as

$$\frac{(\Delta i)^2}{B} = 2e\bar{i}(1 + X)$$  \quad (3.31)

where $X$ represents the "excess noise", which in this case is the acoustic noise. Hence,

$$X_{single} = \left(\frac{\sin(\phi_b)}{1 + \cos(\phi_b)}\right)^2 \frac{(2\pi)^2 n^6 k_B T}{2 E d^2 \lambda_0^2} \left\{ \frac{p_{11}^2 + p_{12}^2}{(p_{11} - p_{12})^2} \right\} \frac{L}{[\omega_p/(2\pi)]} \frac{\eta \lambda_0}{\hbar c_0} \overline{P_{detect}}$$  \quad (3.32)

and

$$X_{balanced} = \frac{T_{signal}}{T_{LO}} \left(\frac{(2\pi)^2 n^6 k_B T}{2 E d^2 \lambda_0^2} \left\{ \frac{p_{11}^2 + p_{12}^2}{(p_{11} - p_{12})^2} \right\} \frac{L}{[\omega_p/(2\pi)]} \frac{\eta \lambda_0}{\hbar c_0} \overline{P_{detect}} \right)$$  \quad (3.33)

Fortunately, $L_{piec}$ has canceled out. The upper equation is for a ring reflector and the lower is for polarization interference.

For fused silica at 300K with $\lambda_0=1.55\mu m$,

$$X_{single} = \left\{ \begin{array}{l} 2.98 m/(W \sec) \\ 0.755 m/(W \sec) \end{array} \right\} \left( \frac{\sin(\phi_b)}{1 + \cos(\phi_b)} \right)^2 \frac{\eta \overline{P_{detect}} L}{[\omega_p/(2\pi)]d^2}$$  \quad (3.34)

$$X_{balanced} = \left\{ \begin{array}{l} 2.98 m/(W \sec) \\ 0.755 m/(W \sec) \end{array} \right\} \frac{T_{signal}}{T_{LO}} \frac{\eta \overline{P_{detect}} L}{[\omega_p/(2\pi)]d^2}$$  \quad (3.35)

### 3.5 Examples

We now consider some examples of the acoustic noise to get a feel on how big this noise is. In all the examples, $T=300K$.

#### 3.5.1 Polarization noise due to the acoustic modes in an isotropic fiber

The experiment consisted of launching elliptically polarized light into a 25m piece of SMF-28 fiber (standard telecommunications fiber). The light was 450fs pulses at a repetition rate of 41MHz from an APM Er-doped fiber laser ($\lambda_0 = 1.55\mu m$). The light exiting the fiber passed through a half-wave plate and a PBS, which split the light to
two detectors. The half-wave plate was used to adjust the intensities on each detector so that they were equal. The measured \(X\) of the noise of the difference current is 0.9. The calculated \(X\) for this case \((d = 10 \mu m, P_{\text{detect}} = 0.52 \text{mW}, \eta = 0.85, T_{\text{signal}} = T_{\text{LO}})\) is 2. This is in reasonable agreement considering all the approximations in the theory.

### 3.5.2 Intensity noise due to the acoustic modes in a polarization-APM fiber laser

Consider the fully self-starting polarization-APM laser of Fig. 3-2, which produces 450fs pulses at a repetition rate of 41MHz. The APM was biased such that the intensity noise in the laser was diminished by additive-pulse limiting (APL, discussed later) in the output except for shot noise, the acoustic noise, and some gain noise. Using the system parameters \(\lambda_0 = 1.55 \mu m, d = 8.2 \mu m, L = 4.7 m, P_{\text{detect}} = 0.9 mW, \phi_b \approx 114^\circ\) (30 percent output coupling), \(T = 300 K\), and \(\eta = 0.85\), one finds that the predicted \(X = 2.3\) due to the acoustic noise. It is estimated that the total average power loss in one cavity round trip is about 6dB. Thus the gain noise necessary to overcome this loss results in an \(X\) of about 6 (see Eq. (2.17)). This gives a total \(X\) of 8.3.

The measured \(X = 9.4\) which is in reasonable agreement.
3.5.3 Noise due to the acoustic modes in a ring-reflectorsqueezer

A self-stabilized ring-reflectorsqueezer [24] was constructed with \( L = 50 \text{m} \) of fiber with \( d = 5.7 \mu\text{m} \). The light source was the polarization-APM laser described in the previous section. \( T_{\text{signal}}/T_{\text{LO}} = 5 \) and \( P_{\text{detect}} = 106 \mu\text{W} \). This results in a calculated \( X \) of 50, implying that the acoustic noise is 17\,dB above shot noise. The measured acoustic noise was 12\,dB above shot noise. However, no spatial filter fiber was employed (see Chapter 7), and thus the interference of the two beams returning from the ring was imperfect. This could account for the measured acoustic noise being lower than the calculated value. Needless to say, with this much acoustic noise no quantum noise reduction could be measured.

3.5.4 Intensity noise due to the acoustic modes in a figure-eight APM fiber laser

Consider a figure-eight APM fused-silica Er-doped fiber laser [36] made with PM fiber with \( \phi_b = 66^\circ \), ring reflector length \( L = 10\text{m} \), \( \omega_p/(2\pi) = 13\text{MHz} \), \( d = 9\mu\text{m} \), \( P_{\text{detect}} = 1\text{mW} \), and \( \eta = 0.85 \). One finds that \( X = 10 \) due to the acoustic modes. This result and that for the polarization-APM laser show that fiber lasers with low pulse repetition rates contain a significant amount of excess intensity noise in their output.

3.5.5 Noise due to the acoustic modes in a fiber gyroscope

Consider a conventional IFOG that is using short pulses at repetition rate \( \omega_p \) as its source. For a coil length of \( L = 1\text{km} \), \( \lambda_0 = 1.55\mu\text{m} \), \( d = 6\mu\text{m} \), \( \phi_b = \pi/2 \), \( T_{\text{signal}}/T_{\text{LO}} = 1 \) (always true for a conventional IFOG), \( P = 50\mu\text{W} \), \( \eta = 0.85 \), and \( T = 300\text{K} \), one finds that \( X = 3.5 \times 10^9 \text{sec}^{-1}/(2\pi)/\omega_p \). For a pulse repetition rate of 40\,MHz (typical for a fiber laser with one pulse in the cavity), \( X = 88 \). As one can see, if one wishes to use pulses in a high-sensitivity fiber optic gyroscope, one must use a repetition rate greater than the highest contributing acoustic frequency.

3.6 Mode-locked lasers

In Chapter 2 it was explained that it is best to use pulsed light for reducing the quantum noise, and the above discussion points out that these pulses must be at a high repetition rate to avoid the aliasing of the acoustic noise. This section discusses lasers that can produce the desired optical pulses.

A laser that outputs light consisting of coherent spectral modes is generally referred to as a "mode-locked" laser [37]. The spectrum consists of many resonator modes of the laser cavity with their relative phases locked together. In the time domain the light appears as a periodic signal, in the form of pulses. There are essentially two categories for achieving mode locking: active and passive mode locking.
3.6.1 Active mode locking

Active mode locking can be described in two different, but equivalent, ways. The first way is to look at it in the time-domain.

In an active mode-locked laser (for example, see Fig. 3-3) there is a modulator, either amplitude or phase, that is modulated at the same frequency as the round-trip frequency of the light in the cavity (harmonic mode locking, running the modulator at a multiple of the round-trip frequency, is considered later). When the modulator is turned on, the optical field in the cavity is modulated. Every time the light passes through the modulator it becomes a shorter and shorter pulse. If the modulator is an amplitude modulator, the pulse wings directly experience higher loss than the peak. If the modulator is a phase modulator, the pulse wings are frequency-shifted and experience higher loss because of filtering. The pulse stops getting shorter and reaches a steady-state when its lengthening due to filtering (because the spectrum has gotten larger) balances out the pulse-shortening by the modulator.

The other way to view active mode locking is in the frequency domain.[37] Assume that right when the modulator is turned on the laser is lasing at a single frequency. The modulation creates sidebands around the single frequency that fall into the next resonator mode. These then lase and create additional sidebands. The expansion of the spectrum is finally halted by filtering in the laser. All the spectral modes are locked together in phase because the modes are constantly injecting sidebands into adjacent modes.

3.6.2 Resonator modes

The mode locking brings up an important point. Why does the laser lase only in its "resonator modes", the frequencies that equal $c_0/(2nL)$ for a linear cavity or $c_0/(nL)$ for a ring cavity, where $c_0$ is the speed of light in vacuum, $n$ is the index of refraction inside the cavity, and $L$ is the cavity length? Even if the laser is below threshold, the emitted spontaneous emission is localized around the resonator modes. How do the atoms in the gain medium "know" what sort of cavity they are in?

One way to answer these questions is provided by quantum mechanics. When a
laser is first turned on it must start up from spontaneous emission. Even when it has been running for a long time, because of finite loss in the cavity, whatever was originally lasing eventually dies out as it is slowly replaced by spontaneous emission. This is why, after turning off the modulator in an actively mode-locked laser, the laser soon after stops pulsing and returns to its production of incoherent lasing modes.

The spontaneous emission is actually stimulated emission of vacuum fluctuations in the ring.[38] Vacuum fluctuations can successfully exist in the ring only if they fit with an integer number of wavelengths (i.e., they obey the resonator condition), otherwise they interfere with themselves destructively.

Another way to understand the mode structure of the spontaneous emission is from Eq. (2.12). In a resonator, the input vacuum \( \hat{b} \) is a time-delayed and phase-shifted version of \( \hat{c} \), and thus \( \hat{c} \) exists only at certain frequencies set by the resonator.

This is why a laser can only sustain lasing at its resonator modes.

### 3.6.3 Passive mode locking

Passive mode locking is similar to active mode locking except that the pulses amplitude and phase modulate themselves. The self-amplitude modulation is provided by saturable absorption, which is an intensity-dependent intracavity transmission with less loss for higher intensities. In passive mode locking, then, fluctuations of the field envelope of higher intensity experience less loss and thus grow. The envelope eventually becomes a short pulse with the saturable absorption pulse shortening balanced by filtering. Figure 3-4 shows a passively mode-locked laser.

The pulses produced by passive mode locking are generally significantly shorter than those produced by active mode locking, since in active mode locking the pulse shortening is very weak once the pulse becomes much shorter than the modulation window. On the other hand, in passive mode locking, the pulse shortening gets stronger as the pulse gets shorter because of the larger peak intensity of the shorter pulse. However, when the pulse width approaches the response-time of the absorber, as in the case of a slow saturable absorber, the pulse shortening becomes weak.

One way to achieve saturable absorption is through an imbalanced nonlinear interferometer. By a nonlinear interferometer we mean an interferometer with nonlinear
media in each arm. The nonlinear media cause the electromagnetic field to undergo a phase shift that changes with intensity. This is because the index of refraction of the medium \( n \) changes with intensity \( I \) \( (n = n_0 + n_2I) \). An example nonlinear medium is glass, such as in optical fibers. This type of nonlinear medium is called a Kerr medium, the same medium used for squeezing in optical fibers. The power exiting a port of an imbalanced nonlinear interferometer changes with the input power. The interferometer can be phase-biased so that higher intensity increases the transmission through the interferometer. When this interferometer is placed in a laser cavity passive mode locking occurs. Passive mode locking via an imbalanced nonlinear interferometer is termed additive-pulse mode locking (APM).[39, 40] A thorough discussion of APM in fiber lasers is given in Ref. [41].

### 3.7 Harmonically mode-locked lasers

A very convenient and potentially quiet light source is the pulsed fiber laser. In this section we focus on the Er-doped fiber laser. Er-doped fiber lasers, which lase in the wavelength region 1.5 to 1.6\( \mu \)m (a good wavelength for squeezing and sensors since the fiber loss is at a minimum at these wavelengths), can produce pulses via active or passive mode locking. 114q Because of the limited allowed doping of the gain fiber, these lasers generally have cavity lengths on the order of 5m. They thus have pulse repetition rates on the order of 40MHz when there is only one pulse in the cavity. However, one requires a pulse repetition rate of 1GHz or greater to avoid the aliasing of the acoustic noise in an IFOG. To achieve such a high pulse repetition rate in a fiber laser, harmonic mode locking must be used, in which there are several pulses in the cavity at once.

Harmonic mode locking is usually achieved via an active mode locker.[42, 43, 44, 45, 46] This is because the pulses in a purely passively mode-locked laser are usually independent of one another. One exception is the “self-ordering” effect[47], in which multiple pulses in a passively mode-locked laser influence one another through the shedding of continuum, causing a purely passive type of harmonic mode locking. However, self-ordering cannot be relied upon to generate stable pulse-streams. Self-ordering is discussed briefly in the last section of this chapter.

### 3.7.1 Additive-pulse limiting

**Theory**

Unless certain measures are taken with harmonic mode locking of a fiber laser, significant pulse-to-pulse energy fluctuations result because of the slow relaxation time of the Er gain. Er gain has a relaxation time (upper state lifetime; Er-doped fiber gain is a three-level system when pumped with 980nm light) on the order of 10ms.[44] Pulses with temporal spacings less than the relaxation time see the same gain (but, of course, do not necessarily receive the same energy) as they pass through the Er-doped fiber, provided that the electron-level populations in the Er-doped fiber have reached steady-state. When the Er-doped fiber is not in steady-state, the pulses may not
Figure 3-5: Pulses of unequal energies passing through an Er-doped fiber amplifier.

all receive the same gain. An example of pulses passing through an Er-doped fiber amplifier is shown in Fig. 3-5.

The microwave spectrum of the current of a fast detector measuring the pulse train from a harmonically active mode-locked Er-doped fiber laser with no special measures taken often appears as shown in Fig. 3-6. In this case, the modulator is driven at 1GHz. One can see the 1-GHz mode and its harmonics, but one can also see many closely spaced spaced modes corresponding to the resonator modes. The presence of these modes in the microwave spectrum implies that the pulse energies are fluctuating from pulse-to-pulse. There is essentially no stabilizing force against these fluctuations occurring naturally in the laser.

The energy fluctuations have been stabilized by employment of a Fabry-Perot interferometer with spacing equal to half the pulse spacing[43] (see Fig. 3-7). However, this scheme requires feedback stabilization to interferometrically maintain the correct mirror spacing.

The pulse energy stabilization can also be accomplished via an intensity-dependent loss in the cavity such that higher intensity light experiences higher loss (a reverse saturable absorber). This is the focus of this section.

As mentioned earlier, one way to achieve intensity-dependent transmission is to use the interference of two pulses combined with nonlinearity. When the transmission increases with increasing intensity, the action is APM and is used to produce short pulses in fiber lasers.[36, 48, 49, 50] When the transmission decreases with increasing intensity, the action is called additive-pulse limiting (APL).[51] and is used here to stabilize pulses. Note that similar limiting techniques have been applied to lasers for suppression of unwanted laser power fluctuations.[52, 53, 54, 55]

An example of an implementation of APM or APL is nonlinear polarization rotation.[48] Fig. 3-8 illustrates the principle. An elliptical polarization (which is the sum of two orthogonal polarization modes) rotates in an isotropic Kerr medium,
Figure 3-6: Microwave spectrum of the current of a fast detector measuring the output from a harmonically mode-locked Er-doped fiber laser at 1GHz with no pulse energy stabilization (vertical scale is 10dB/division).

Figure 3-7: Pulse energy equalization using a Fabry-Perot interferometer.
such as an isotropic optical fiber, by an amount proportional to its intensity. With appropriate adjustment of the birefringent elements and a polarizer, we can bias the system for increased transmission through the polarizer with increasing intensity (APM) or for decreased transmission with increasing intensity (APL).

When harmonic active mode locking is applied to a laser with an intensity-dependent loss, there are two distinct regimes of operation: APM and APL. Because the Er gain is slow, all the pulses in the cavity draw from the same pool of energy. APM and APL define how the energy is doled out. When the laser is biased in the APM regime, higher-intensity pulses experience less loss, so the intracavity energy is concentrated into only a few pulses in the cavity rather than filling all the windows of transmission created by the modulator. However, when the laser is biased in the APL regime, higher-intensity pulses experience greater loss, so the laser minimizes its pulse-intensity fluctuations. As a result, the laser oscillates with pulses of equal energy in every modulator transmission window. The pulse-energy stabilizing effect of APL is illustrated in Fig. 3-9.
Figure 3-10: Experimental setup of additive-pulse limiting laser.

Experiment

A harmonically active mode-locked fiber ring laser with APL was constructed and is shown in Fig. 3-10. The 6.6m fiber ring is mode locked by a Ti:LiNbO₃ Mach-Zehnder waveguide modulator. The fiber leading into and out of the modulator is polarization-maintaining (PM) fiber. There is an isolator/polarizer in the ring to ensure unidirectional operation. The output of the isolator/polarizer is aligned to a PM birefringence axis so that the light passes through the modulator with the correct polarization. The rest of the laser consists of non-PM fiber and contains a 10 percent output coupler, a wavelength-division multiplexer (WDM) coupler to couple in the 980nm wavelength pump, Er-doped fiber, and polarization controllers (PCs) to adjust the birefringence in the cavity. The non-PM section has nonlinear polarization rotation and is followed by a polarizer in the polarizer/isolator. Thus the laser is capable of APM or APL, adjusted via the PCs. The net chromatic dispersion of this laser is calculated to be close to 0 at 1550nm wavelength.

The modulator was driven by a radio-frequency (RF) sinusoidal source. When no RF power was applied, with appropriate adjustment of the PCs the laser output power could be nearly extinguished; and the laser could be made to self-Q switch. These behaviors indicate that the laser is capable of polarization APM or APL. However, the laser could not at first be made to passively mode lock with APM. This is probably due to enhancement of the spontaneous emission noise caused by the high loss in the modulator and the isolator (together they inflict approximately 4dB of loss) and
filtering caused by the birefringence of the LiNbO$_3$. The LiNbO$_3$ acts as a filter because the LiNbO$_3$ crystal axes are not perfectly aligned to the PM fiber axes, and since there is a polarizer in the laser, the LiNbO$_3$ acts as a birefringent filter (a filter with a periodic structure). The $\Delta n$ of LiNbO$_3$ is 0.079. The length of the waveguide is about 10cm. Thus, the LiNbO$_3$ acts as a sinusoidal filter with a period of 40GHz. It requires a strong pulse-shortening force for the pulse bandwidth to break past the bandwidth of the weak filtering (a pulse with a spectrum wider than about 20GHz in this case experiences higher loss than cw due to the filtering, and thus to achieve pulses with spectra larger than 20GHz the modulator must force cw to have an even higher loss than the pulse). However, after rebuilding the laser with the inclusion of a birefringent filter with a large bandwidth, self-starting passive mode locking was obtained (described later).

When an RF signal is applied to the modulator (one side of the Mach-Zehnder was driven) at a frequency equal to an integer multiple of the laser round-trip frequency, the laser produces a pulse train. With the polarization controllers adjusted for APM, the laser tends to oscillate with fewer pulses than dictated by the modulator. These pulses are not short, as one would normally expect with APM, again because of filtering from the modulator birefringence. Figure 3-11(a) shows an example of an APM pulse train obtained with the modulator running at 510MHz (17 times slots in the ring). Figures 3-11(b)-(d) show the same APM pulse train as the pump power is successively decreased. As one can see, the intracavity energy has gone into only a few pulses in the cavity, the number of pulses determined by the APM bias point and the pump power.

Figure 3-12 shows the same case as in Fig. 3-11 but with the polarization controllers adjusted for APL. Although the waveform resolution is limited by the bandwidth of the oscilloscope, the presence of one pulse per modulation period is apparent. Figure 3-13 shows an APL pulse train viewed with much better resolution on a sampling scope with the modulator running at 1.05GHz (35 pulses in the ring). The microwave spectrum measured from the current of a fast detector of this pulse train is shown in Fig. 3-14. As one can see, the resonator modes between the harmonic 1-GHz modes are greatly suppressed, implying in this case that the pulse-to-pulse intensity noise is less than 0.05 percent. When the pump power was reduced below the point at which APL is effective (a launched pump power of 220mW with a fiber laser output of 180$\mu$W), fluctuations increased. We achieved long-term stability of APL by placing the laser in a thermally insulated box.

In the APL case at 1.05GHz, the pulses typically had an optical spectrum full-width-at-half-maximum (FWHM) of approximately 12GHz (see Fig. 3-15), although it varied from approximately 10 to 14GHz depending on the laser operating point. Assuming Gaussian, transform-limited pulses, the 12-GHz spectral width corresponds to pulses with an intensity FWHM of 37ps. The peak intensities of the APL pulses were too low for effective pulse autocorrelation, but direct time-resolved measurement showed the pulses to be shorter than our detector apparatus response time of 80ps. The APM pulses had spectra roughly twice as broad as those for the APL pulses. Note that these spectral widths are likely determined by the birefringent filtering of the modulator which, as mentioned earlier, has a calculated f/see spectra! range of
Figure 3-11: APM pulse trains as the pump power was successively decreased from (a) to (d). The modulator frequency was 510MHz (17 times the round-trip frequency). The horizontal scale is 5ns/division.
Figure 3-12: APL pulse train for the same situation as in previous figure.

Figure 3-13: 1.05-GHz APL pulse train shown on a sampling scope. The horizontal scale is 200ps/division.
Figure 3-14: Microwave spectrum of the 1.05-GHz APL pulse train measured from the current of a fast detector. The vertical scale is 10dB/division, and the resolution bandwidth is 300kHz.
Figure 3-15: 1.05-GHz APL pulse train optical spectrum measured with a scanning Fabry-Perot interferometer. The horizontal scale is 4.5GHz/division.

approximately 40GHz resulting in a FWHM of approximately 20GHz.

Computer simulation

A computer simulation of a harmonically active mode-locked laser with APM and/or APL was run. The program kept track of the complex amplitude of the light in the cavity. Upon each pass through the cavity the light underwent the following:

1. Fiber propagation including nonlinearity and second-order chromatic dispersion
2. Intensity-dependent loss (the APM or APL)
3. Active modulation (multiplication by a sinusoid)
4. Gain with saturation
5. Spectral filtering

The simulation confirmed the APL property of pulse-energy fluctuation suppression. Using the same parameters as the laser used in the experiment with the modulator running at 1GHz, it was found that the number of round-trips necessary for APL to stabilize an initially uneven pulse train ranges from about 20 to 200 round trips, depending strongly on the APL bias point. The APL effect is strongest when the transmission is near the minimum (not where the intensity-dependent transmission slope has the largest negative value). This is because the differential transmission is the largest there. An example from the computer simulation is shown in Fig. 3-16. As one can see, four pulses of initially unequal energies ended up with equal energies.

Other methods to achieve reverse-saturable absorption

Another way besides APL to achieve an intracavity transmission that decreases with increasing pulse energy is to use solitons combined with a filter. Higher energy solitons
Figure 3-16: Computer simulation of a harmonically mode-locked laser with APL. Four pulses initially of unequal energies end up with equal energies because of APL.

have a larger bandwidth and thus see higher loss after passing through the filter. A harmonically mode-locked laser that appears to use this principle was constructed by Takara /em et al. No nonlinear polarization rotation can occur in this laser because all the fiber in the laser is PM.[56]

One could also achieve APL using an appropriately phase-biased nonlinear ring reflector[36] or the same effect as APL by using a second-harmonic generating crystal.

Spectral characteristics of harmonically active mode-locked lasers

Consider a harmonically mode-locked laser mode locked at the $n$-th harmonic of the fundamental round-trip frequency of the laser resonator. Harmonic mode locking implies that each resonator mode is coupled to the resonator modes that are an integer multiple of $n$ resonator modes away. Thus a harmonically mode-locked laser contains $n$ independent sets of coupled resonator modes, called “supermodes”. [57]

Consider a harmonically active mode-locked laser with $n=2$, i.e., with two pulses in the cavity. The spectrum and corresponding temporal picture for the two different possible supermodes are shown in Figs. 3-17 and 3-18. The figures are drawn using the complex plane. Since these two supermodes are independent, the actual condition in the laser is a linear superposition of these two supermodes. There are several important things to note about these laser pulse solutions.

1. For $n = 2$, in one supermode the pulses are all in phase, and in the other the
Figure 3-17: The spectral (left) and temporal (right) pictures of one supermode in a laser harmonically mode locked at $n=2$.

Figure 3-18: The spectral (left) and temporal (right) pictures of the other supermode in a laser harmonically mode locked at $n=2$. 
pulses are all in anti-phase. For the general case of \( n > 2 \) and if only one supermode is excited all the pulses have the same phase difference between them.

2. For \( n = 2 \), if both supermodes are excited the phase differences between pulses are unequal. If the two supermodes do not have exactly a \( \pi/2 \) phase difference between them, the two pulses do not have the same energy. For the general case of \( n > 2 \), if more than one supermode is lasing the pulses have different energies unless only certain sets of supermodes with certain phase relations are lasing. Thus, in an APL laser, which suppresses pulse-to-pulse energy fluctuations, only certain sets of supermodes can lase simultaneously, and the supermodes must have a certain phase relation between them.

3. If one looks at the pulse train with a detector (looking at the pulses directly or at the microwave spectrum), one cannot tell what the pulse phases are. Only by an interferometric interaction between pulses, such as in a Fabry-Perot interferometer, can one observe the relative pulse phases.

With this knowledge and by looking at Fig 3-15 (the optical spectrum of an APL laser measured with a scanning Fabry-Perot interferometer) one sees that only one supermode is lasing. This implies that all the pulses have an equal phase difference between them. This is remarkable because, in the time domain, one imagines all the pulses in the cavity to be essentially independent. Their timing is fixed by the modulator, but their phases ought to be random. Why does not more than one supermode lase (provided that the combination keeps all the pulse energies equal)?

This effect can be understood by reviewing work done on frequency-modulated (FM) lasers.\([58, 59]\) An FM laser contains a phase modulator that is run slightly asynchronously from the resonator round-trip frequency. Most lasers without tight filtering can lase in two or more single resonator modes at once. This is permitted by slight inhomogeneities in the gain (not all of the atoms providing the gain are coupled together),\([58]\) which can occur in Er-doped fiber. In an FM laser, for a resonator mode to lase several other frequencies must also lase. This spreading encourages more competition for the gain, and weak filtering prefers one frequency spread over the other. Basically, there is too much competition in an FM laser for two independent modes to lase simultaneously (ensuring a quiet laser).

The same holds true for a harmonically mode-locked laser (or at least for a harmonically mode-locked laser containing a filter). There is too much competition for the gain for two supermodes to lase simultaneously. Weak filtering will give preference to one supermode over the others. This is why only one supermode lases and why all the pulses must have an equal phase difference between them.

One would then expect to be able to control which supermode is lasing by changing the filtering in the laser. This was accomplished experimentally. In Fig. 3-19 is shown spectra taken from a harmonically active mode-locked fiber ring laser with APL. First, the spectrum shown in Fig. 3-19(a) was taken. Then the fiber in the laser was moved. The laser mode-hopped to a new supermode, as can be seen from Fig. 3-19(b). Note that changing the waveplates in the laser (the laser was built using bulk components), the filter, nor the modulator frequency would cause the laser to mode hop. Only touching the fiber would cause a mode-hop. This implies that the weak filtering which preferentially prefers one supermode over the other is most
likely provided by tiny reflections in the fiber due to fiber splices and/or Rayleigh scattering. This also explains the time-domain picture of why the pulses all have an equal phase difference between them. It is because light is injected from one pulse to other by these reflections, causing all the pulses to acquire the same phase relative to one another.

Because the selection of the lasing supermode appears to depend on tiny effects in the fiber, small environmental changes cause supermode hopping. With strong APL, the pulse intensities should not be affected by the mode-hopping. Without the presence of APL, a supermode hop can cause large pulse intensity fluctuations. Shan and Spirit have avoided the supermode-hopping by dithering the cavity length at a kHz rate with a piezoelectric crystal.[44] It is not well understood, but their explanation is that the dithering prevents spatial hole burning in the Er-doped fiber, keeping the gain more homogeneous spatially and thus encouraging stronger competition for the gain (the lasing supermode does not experience increased loss from spatial hole burning). They did not intentionally employ APL, but they must have at least had the absence of APM.

3.7.2 Harmonic mode locking with semiconductor gain

We have just explored the aspects of harmonic mode locking with a gain medium with a slow response time (Er-doped fiber gain). We now consider a gain medium with a fast response time (semiconductor gain).

Fast-gain media

An example of a fast-gain medium is a semiconductor amplifier (SA). A pump-probe trace of a bulk InGaAs 1.55μm semiconductor gain, measured by Hall, is shown in Fig. 3-20.[60] There is a very fast gain recovery on the order of picoseconds and a fast gain recovery on the order of 500ps. For high repetition rate pulse trains, this gain can recover between pulses and thus provide an energy stabilizing force to the pulse energies (Fig. 3-21).

However, if one builds a laser with only fast gain, the pulses are usually long and distorted. This is because fast gain is essentially an anti-mode locking force for pulses longer than the gain response time (higher light intensities receive less gain for pulses longer than the gain response time). The fast gain also injects a significant amount of noise into the pulses because the gain is unevenly distributed to the pulse (the leading edge of the pulse usually acquires the most gain).

Experiment with fast gain only

Figure 3-22 shows a laser with fast-gain as the only gain medium.[61] The gain is an InGaAs semiconductor anti-reflection-coated on both facets and coupled via fiber lenses and is driven by an offset oscillating current at 1GHz. Figures 3-23 and 3-24 show the pulse train and spectra respectively from this laser. The pulses are quite long (approximately 150ps) and the spectrum is highly chirped (the periodic structure
Figure 3-19: Optical spectrum of the pulses from a 1-GHz APL laser measured with a scanning Fabry-Perot interferometer showing the lasing of different supermodes.
Figure 3-20: Pump probe trace taken by Hall of InGaAs semiconductor laser diode gain measured at 1.5μm wavelength with 100fs pulses from a color-center laser.

Figure 3-21: Pulse energy equalization using a semiconductor amplifier.
Figure 3-22: Experimental setup of the fiber ring laser employing only semiconductor gain.

in the spectrum on the left in Fig. 3-24 is due to imperfect anti-reflection coatings on the semiconductor facets). The spectrum on the right in Fig. 3-24 is a narrower view of the optical spectrum (the Fabry-Perot mirrors were moved further apart). Because one cannot see any structure with a 1-GHz periodicity in the right-hand spectrum of Fig. 3-24, there is no evidence of the lasing of a single supermode, indicating that the phase difference is not equal from pulse-to-pulse. This lack of phase coherence is most likely due to the noise injected by the fast gain. The energy is very stable from pulse-to-pulse.

This laser source may be useful with some sensors since it can provide a large optical bandwidth with relatively high power. The incoherence between pulses is beneficial to most sensors, also, as is discussed in Chapter 5.

Experiment with fast and slow gain

To obtain shorter pulses with pulse-to-pulse energy stabilization, one can combine fast and slow gain.[62] Figure 3-25 shows such a laser. Again the semiconductor is driven by an offset oscillating current at 1GHz, and the semiconductor serves as the active mode locker.[63, 64] Figures 3-26 and 3-27 show the pulse train and spectra respectively from this laser. The pulses are much shorter (approximately 15ps) than with the laser with pure fast gain, and the spectrum is much less chirped (note that about a 20m piece of $\beta''=-20\text{ps}^2/\text{km}$ fiber was added to the cavity in order to obtain these unchirped pulses since the dispersion from the semiconductor is positive). There is also phase coherence between pulses. Pulse-to-pulse energy fluctuations were small, but not as small as with APL or pure fast gain. The fluctuations are most likely caused by noise from the fast gain.

Another laser was constructed that used a LiNbO$_3$ modulator for the modulation
Figure 3-23: Oscilloscope trace of the pulses from the pure fast-gain laser. The SA was modulated at 1GHz.

Figure 3-24: Optical spectra of the pulses from the pure fast-gain laser measured with a scanning Fabry-Perot interferometer. The spectrum on the right was taken with the Fabry-Perot mirrors further apart than for the spectrum on the left.
Figure 3-25: Experimental setup of the fiber ring laser employing both fast gain and slow gain.

Figure 3-26: Oscilloscope trace of the pulses from the hybrid-gain laser. The SA was modulated at 1GHz.
Figure 3-27: Optical spectra of the pulses from the hybrid-gain laser measured with a scanning Fabry-Perot interferometer.

and pumped the fast gain with cw current (see Fig. 3-28).[62] Figures 3-29 and 3-30 show the pulse train and spectra respectively from this laser. The spectrum, as with the APL laser, was limited by the birefringent filtering of the modulator. These pulses are about 60ps and were very stable in energy pulse-to-pulse. This system was also very stable against modulator mistiming.

3.7.3 Additive-pulse mode locking/limiting

The previously described lasers result in pulsewidths on the order of 10ps and higher. However, for optical squeezing in fibers, we would like pulse widths 1ps or shorter in order to squeeze in a reasonable length of fiber. One possibility is to combine APM with active mode locking to obtain short pulses at a high repetition rate.

As was shown in the APL section, sometimes when a harmonically active modelocked laser is in the APM regime there is more than one pulse in the cavity. This case arises from additive-pulse mode-locking/limiting (APM/L). In APM/L the intracavity transmission increases with increasing light intensity and then begins to level off (see Fig. 3-31).[65] In its realization via nonlinear polarization rotation, this characteristic is achieved by adjusting the birefringences in the laser such that higher intensities cause the polarization to rotate into the transmission axis of the polarizer and then past it with further intensity increase.

The existence of several pulses in the cavity in the presence of APM/L can be understood from a computer simulation of harmonic active mode locking with APM/L shown in Fig. 3-32. With the modulator on and the additive-pulse effect set as
Figure 3-28: Experimental setup of the fiber ring laser employing both fast and slow gain and containing a LiNbO$_3$ amplitude modulator.

Figure 3-29: Oscilloscope trace of the pulses from the hybrid-gain laser with the LiNbO$_3$ amplitude modulator. The LiNbO$_3$ modulator was run at 1GHz.
Figure 3-30: Optical spectra of the pulses from the hybrid-gain laser with the LiNbO$_3$ amplitude modulator measured with a scanning Fabry-Perot interferometer.

Figure 3-31: Intracavity transmission versus intracavity intensity with APM/L.
described above, pulses that happen to have higher energy experience less loss and hence grow in energy. Because of gain saturation and the long gain-relaxation time of Er-doped fiber, the total energy available to the pulses is roughly a constant. Hence the large pulses rob energy from the smaller pulses. The large pulses stop growing as they approach the peak of the intensity-dependent transmission curve. The laser ends up with a fixed number of pulses, i.e. “1”s, that stay in their time slots.

Because the pulses stay in their time slots, a harmonically mode-locked laser with APM/L can be used as an optical memory.[66] This operating region of the laser is not so useful for optical sensors, though, and the optical memory is further discussed in Chapter 8.

APM/L should theoretically be able to provide many short pulses in the laser cavity. To make the pulses evenly spaced (so as to make a 1GHz or higher pulse train) one can use the active modulator. However, obtaining pulses with durations much shorter than that provided by active mode locking alone is difficult while active mode locking is present. The difficulty arises from

1. Weak birefringent filtering in the modulator. A strong pulse shortening force is required for the spectral width of the pulse to exceed the FWHM of the filtering.

2. Spontaneous emission noise. The loss in the laser from the modulator adds spontaneous emission noise, making it more difficult to obtain short pulses.

3. Imperfect timing of the amplitude modulation. With amplitude modulation, the timing must be increasingly synchronous to the desired harmonic of the laser resonator as the pulses become shorter in duration. Essentially, the mistiming of the modulator from the pulse round-trip time can be no more than a pulse width, otherwise the pulse is destroyed.[67]

Despite these difficulties, the laser of Fig. 3-33 is able to obtain short pulses using APM/L with active mode locking present. This laser has an open-air section in which are waveplates for polarization control (they replace the fiber PCs), a quartz birefringent plate of thickness 12T (T≈0.5mm) serving as a birefringent filter of FWHM 21nm, and a polarization beam splitter (PBS) serving as the polarizer for the APM or APM/L action. The rejected light from the APM exits the laser via the PBS and is used as the output. Using this rejected power as the output allows for a higher output power from the laser with minimum disturbance to the laser.[68] Note that the pulses emitted from this port are not significantly different from the pulses circulating in the cavity since in mode locking the pulse changes per pass are usually small.[40]

The important difference between this laser and the previously-described APL laser is the 21nm FWHM filter. This filter eliminates much spontaneous emission noise and smooths out the gain. With the filtering and with no RF drive to the modulator, this laser can generate self-starting passively mode-locked short pulses.

This laser can also generate short pulses in the presence of amplitude modulation. However, the modulator frequency must be tuned very precisely to a harmonic of the cavity round-trip frequency. Because of environmental changes which lead to ring-length changes, the pulses would last only a few minutes before the amplitude modulation frequency slipped too far from synchronism.

The modulator timed the short pulses. This can be seen from Figs. 3-34 and 3-35, which compare the pulse timing characteristics without and with the amplitude
Figure 3-32: Computer simulation of laser with slow gain and APM/L.
Figure 3-33: Experimental setup of APM laser with synchronous amplitude modulation.

modulation on. The timing of the pulses by the amplitude modulation reduced the timing jitter of the pulses. The optical spectra of these pulses are shown in Fig. 3-36.

Usually when timing with the amplitude modulator only one pulse could exist in the cavity. Occasionally, though, up to three pulses could exist in the cavity at once (see Fig. 3-37). These are, of course, timed by the modulator. To use this as a source for a sensor, one would like to have enough pulses to fill all the time slots. Carruthers et al. have achieved this at 500MHz with a complicated laser.[69]

In other efforts to obtain short pulses at a high repetition rate, some researchers have abandoned modulation altogether, such as using a sub ring[70] (obtained short pulses at 125MHz) or external feedback in a figure-eight laser[71] (obtained short pulses at 143MHz), but these lasers have not been reported to have produced high pulse-repetition rates stably. Another method is the self-ordering effect[47] (obtained short pulses at 914MHz), but this effect is not reliable.

We have found a relatively simple and stable way to obtain short pulses at a high repetition rate. The method uses modulation but abandons synchronous modulation and instead uses, surprisingly, asynchronous modulation.

3.7.4 Asynchronous soliton mode locking

As explained earlier, to get a high pulse-repetition rate in a fiber laser, one needs to have many pulses in the cavity at once. As shown above, it is not easy to control many pulses in the cavity via amplitude modulation when the pulses are short in duration. The following describes a laser that can easily control many short pulses in the cavity via asynchronous phase modulation.
Figure 3-34: Long-term microwave spectrum with peak-hold of the current from a fast detector measuring the output of the APM laser with no external amplitude modulation. Resolution bandwidth is 100Hz, vertical scale is 10dB/division.
Figure 3-35: Long-term microwave spectrum with peak-hold of the current from a fast detector measuring the output of the APM laser with external amplitude modulation. The timing jitter is reduced over that of the previous figure.

Figure 3-36: Optical spectrum of the timed APM pulses.
Figure 3-37: Three passively mode-locked pulses timed by a synchronous amplitude modulator (horizontal scale is 1ns/division).

Experiment

The system of Fig. 3-38 is a fiber ring laser with polarization transformers (the wave-plates), an isolator/polarizer, an Er-doped fiber amplifier, and a 12T thick quartz plate serving as a birefringent filter with a FWHM of 21nm (as in the previous section). In addition, the laser contains a bulk LiNbO\textsubscript{3} phase modulator resonant at 1GHz giving a maximum peak phase shift of approximately 0.5rad (this was measured by sending cw light through the phase modulator from an Er-doped fiber laser and observing the created sidebands with a Fabry-Perot interferometer). A Polarcor polarizer is at the input of the phase modulator and is aligned to a crystal axis. Without the polarizer, weak birefringent filtering effects from the modulator crystal prevent the mode of operation described in this section. All the fibers in the ring have negative dispersion, with an average $\beta'' = -16ps^2/km$. The laser is pumped with approximately 210mW of 980nm from a Ti:Sapphire laser. The APM rejection port is used as the output, as in the laser of the previous section, in order to achieve an output power as high as possible with minimum disturbance to the cavity.[68]

With the phase modulator undriven and appropriate adjustment of the polarization transformers, passively mode-locked subpicosecond pulses can be produced via polarization APM. With the phase modulator driven at the 28th harmonic of the laser round-trip frequency (the frequency closest to the resonant frequency of the phase modulator) and the polarization transformers set for APL, 40ps pulses at the repetition rate of the modulator are obtained. When the laser is set for APM and is driven as above, either the laser acts as an APM/L storage ring with 40ps pulses, or a few subpicosecond pulses develop that move independently of the phase modulator timing, behavior similar to that reported by Davey et al.[72]

The fact that the short pulses can move independently of the phase modulator
Figure 3-38: Asynchronous soliton mode locking laser used in the experiment.

timing points out a significant difference between amplitude modulation and phase modulation when applied to short pulses. In active mode-locking theory, the difference between amplitude and phase modulation is simply the pulse width and the amount of chirp on the pulse.[37, 73] However, when passive mode locking is present, the short pulses generally cannot move through an amplitude modulation window but generally can move through a phase modulation window. When a short pulse is asynchronous with amplitude modulation, it must experience a high loss as it passes through the minimum of the modulation and is generally subsequently destroyed. However, with asynchronous phase modulation, the pulse can survive the point of highest loss, the point during which the pulse is frequency-shifted the most, since the pulse can fight against the frequency shift by using its nonlinearity (the pulse can move itself in frequency via four-wave mixing) and thus not experience such a high loss.

To summarize, when the modulator is running within about ±200Hz of a harmonic of the ring round-trip frequency (the active mode locking regime), one cannot achieve a stable sequence of short pulses, except for the self-ordering effect. This breakdown of control is due to the fact that a modulator’s ability to control pulse timing is rapidly degraded for pulses shorter than those produced by active mode locking alone, as discussed in the previous section.

However, with a detuning of the phase modulator drive frequency by ±5-20kHz from the 28th harmonic, a pulse stream of short pulses develops with 28 regular intervals between pulses, despite the asynchronicity of the modulator. When the drive to the modulator is turned off, the pulses disintegrate. This form of mode locking we call “asynchronous soliton mode locking” (ASM).[74]

Figure 3-39 shows the scope trace of the regularly spaced pulses. There is not always one pulse in every “time slot”, and examples are shown in Figs. 3-40 and 3-41. In Fig. 3-40, the pump power was lowered so as to make one pulse disappear. In Fig. 3-41 the pump power was increased so there was an extra pulse (the large pulse is
two pulses close together. The case of exactly one pulse in every time slot is usually obtained by adjusting the polarization transformers or by increasing the pump power until all the time slots are filled and then decreasing it, causing the pulses that have a time slot to die first. A filled pulse pattern is maintained for time periods on the order of an hour. After that period of time, sometimes a pulse disappears or an extra pulse is added. Because the modulator is already asynchronous, no modulator frequency stabilization is necessary. To demonstrate the asynchronicity of the modulator drive, the modulator drive signal was mixed with the signal from a fast detector measuring the pulse train, and the result was a clean 15kHz sawtooth (see Fig. 3-12).

Figure 3-13 shows an autocorrelation trace (pulsed width 1.6ps assuming sech) and the optical spectrum of the pulses (FWHM 2.0mm). Note that, most likely because of the filter and the relatively long soliton period of the solitons, the spectrum contains no 'Kelly' sidebands. Figure 3-14 shows a microwave power spectrum of the current from a fast detector measuring the pulse train. As one can see, nearly all of the energy is in the 1-GHz mode and its harmonics, with the peaks of the other resonator modes well below (scale 10dB/div). This spectrum proves that there are almost no pulse-to-pulse energy fluctuations, and the pulse spacing is very even, despite pump power noise consisting of approximately 5 percent intensity fluctuations. Figure 3-15 shows expanded views of the 1-GHz and 10-GHz modes in the microwave spectrum. The sidebands around the modes are in units of approximately 11kHz away from the amount of detuning present when these spectra were taken. Note that the heights of the sidebands increase going from the 1-GHz to the 10-GHz mode. This implies that the modulation creating the sidebands is phase modulation rather than amplitude modulation, which is what is expected.
Figure 3-40: Oscilloscope trace of the 1-GHz ASM pulse train with one pulse missing (10ns/division).

Figure 3-41: Oscilloscope trace of the 1-GHz ASM pulse train with an extra pulse (10ns/division).
Figure 3-40: Oscilloscope trace of the 1-GHz ASM pulse train with one pulse missing (10ns/division).

Figure 3-41: Oscilloscope trace of the 1-GHz ASM pulse train with an extra pulse (10ns/division).
Figure 3-42: Output from a mixer that has as inputs the 1-GHz modulation drive and the current from a fast detector measuring the pulse train.

Figure 3-43: Autocorrelation trace of the pulses from the all-negative dispersion ASM laser with the corresponding optical spectrum in the upper right-hand corner.
Figure 3-44: Microwave power spectrum of the signal from a fast detector measuring the ASM pulse train (10dB/division, center frequency 3.6GHz, span 3.8GHz, resolution bandwidth 3MHz.

Figure 3-45: Expanded views of the microwave power spectrum from a fast detector measuring the ASM pulse train. (a) is centered at 1GHz and (b) is centered at 10GHz.
Figure 3-46: Autocorrelation trace of the pulses from the diode-pumped ASM laser with the corresponding optical spectrum in the upper right-hand corner.

**Diode-pumped version**

For optical sensors, it is important that the light source be quiet. Diode pumping the laser with a single-stripe diode generally makes for a much quieter laser than pumping with a large-scale high-power Ti:Sapphire laser (which is essentially because a large scale laser is more sensitive to the environment and also because the Ti:Sapphire is itself pumped by a large-scale Argon ion laser). A typical single-stripe diode operating at 980nm produces a maximum power of approximately 90mW. To operate the ASM laser successfully at this pump power, the net cavity dispersion was decreased so as to lower the necessary soliton energy. This was accomplished by replacing the 3m of $\beta'' = -13ps^2/km$ Er-doped fiber by 1m of $\beta'' = +75ps^2/km$ Er-doped fiber\[77\] followed by 2.1m of $\beta'' = -22ps^2/km$ fiber. This brought the average dispersion down to $\beta'' = -3.3ps^2/km$. This laser produced 1.0ps pulses at 1GHz with an average output power of 0.9mW with only about 40mW of 980nm pump. The autocorrelation and spectrum of the pulses are shown in Fig. 3-46.

**Theory**

First we want to calculate what happens to a pulse as it undergoes asynchronous phase modulation. Most of the soliton perturbation theory part of this section was worked out by Haus.[78] The equation for a pulse with complex envelope $a(t,T)$, where $t$ is a short-term time variable (on the scale of the pulse) and $T$ is a long-term time variable (on the scale of several cavity round-trips), in a mode-locked laser is
given by \cite{31}

\[
T_R \frac{\partial}{\partial T} a = \left[ -l + g \left( 1 - \frac{1}{\Omega_g} \frac{\partial}{\partial t} + \frac{1}{\Omega_g^2} \frac{\partial^2}{\partial t^2} \right) + jD \frac{\partial^2}{\partial t^2} + (\gamma - j\delta)|a|^2 \right] a + T_R S(t, T)
\]

(3.36)

\(l\) is the amplitude loss per pass in the laser, \(g\) is the amplitude gain per pass, \(\Omega_g\) is the filter bandwidth, \(D\) is the dispersion parameter (equal to \((1/2)\beta''l_f\) where \(l_f\) is the fiber length), \(\gamma\) is the self-amplitude parameter (APM action), \(\delta\) is the self-phase modulation parameter, \cite{40} \(T_R\) is the cavity round-trip time, and \(S\) is an externally injected disturbance. In this case, the external disturbance is the phase modulation

\[
S(t, T) = jM \sin \left[ \Phi(t, T) \right] a(t)
\]

(3.37)

\(M\) is the peak phase modulation depth. We must assume that the modulation slips out of synchronism, hence \(\Phi\), the phase of the modulation in a frame moving with the circulating energy in the cavity, appears in the modulation function. \(\Phi\) is given by

\[
\Phi(t, T) = \omega_M t + (\omega_M - \omega_P) T - \omega_M \Delta t(T)
\]

(3.38)

in which \(\omega_M\) is the modulation frequency, \(\omega_P\) is the pulse repetition frequency, and \(\Delta t\) represents a possible timing shift experienced by the pulse envelope \(a\).

The pulses in the ASM laser are primarily passively mode-locked and are maintained by the combined actions of chromatic dispersion, Kerr nonlinearity, and APM action. They are assumed to be soliton-like, and thus their behavior can be described by soliton perturbation theory, in which solitons are described in terms of four parameters: energy \(w\), phase \(\theta\), timing \(t\), and frequency \(p\). \cite{31} We use this perturbation theory to find the perturbations on the soliton: energy change \(\Delta w\), phase change \(\Delta \theta\), timing change \(\Delta t\), and frequency change \(\Delta p\).

It can be shown from (3.36) that the timing and frequency changes obey the following coupled differential equations

\[
T_R \frac{d}{dT} \Delta p = -\frac{4}{3} \frac{g}{\Omega_g^2 \tau^2} \Delta p + T_R S_p(T)
\]

(3.39)

\[
T_R \frac{d}{dT} \Delta t = -2|D| \Delta p + T_R S_t(T)
\]

(3.40)

The driving source \(S_t\) is zero for phase modulation and \(S_p\) is found from

\[
T_R S_p(T) = \frac{1}{2} \int_{\text{over pulse}} dt \left[ f_p(t) S(t, T) + \text{c.c.} \right]
\]

(3.41)

in which \(f_p\) is the adjoint soliton perturbation theory function for frequency. Eq. (3.41) becomes

\[
T_R S_p(T) = \frac{1}{2} \int_{\text{over pulse}} dt \left\{ \left[ -j \frac{2}{\omega_0 \tau} \tanh \left( \frac{t}{\tau} \right) \right] jM \sin \Phi(t, T) \right\} A_0^2 \text{sech}^2 \left( \frac{t}{\tau} \right)
\]

(3.42)
\( w_0 \) is the pulse energy, and \( \tau \) is defined by

\[
a_s(t) = A_0 \text{sech}\left( \frac{t}{\tau} \right)
\]

(3.43)

where \( a_s \) is the unperturbed pulse envelope. (3.42) reduces to

\[
T_R S_p(T) \approx M \omega_M \cos(\Delta \omega T)
\]

(3.44)

where

\[
\Delta \omega = |\omega_M - \omega_P|
\]

(3.45)

The equation for \( \Delta p \) thus becomes

\[
\frac{d}{dT} \Delta p = -\frac{4}{3} \frac{g}{\Omega_g^2 \tau^2 T_R} \Delta p + \frac{M \omega_M}{T_R} \cos(\Delta \omega T)
\]

(3.46)

As one can see, (3.46) predicts a frequency excursion of the pulse. This frequency excursion must be sinusoidal at the frequency \( \Delta \omega \). Thus, letting

\[
\Delta p(T) = |\Delta p| \sin(\Delta \omega T + \phi)
\]

(3.47)

one finds

\[
|\Delta p| = \frac{M \omega_M}{\sqrt{(T_R \Delta \omega)^2 + \left( \frac{4}{3} \frac{g}{\Omega_g^2 \tau^2} \right)^2}}
\]

(3.48)

This gives the frequency excursion of the envelope of the soliton. It is important to note that we are looking at a time-scale on the order of \( T \). On this time scale the soliton envelope appears to undergo a frequency excursion while the resonator modes making up the soliton do not. If we looked on an even longer time scale we would of course no longer see the spectral excursion of the soliton envelope and instead see sidebands around the resonator modes from the spectral excursion (as see in the microwave spectrum of Fig. 3-45).

The spectral excursion can be measured with a Fabry-Perot interferometer. Figure 3-47(a) shows the pulse spectrum as seen on a scanning Fabry-Perot interferometer, and Fig. 3-47(b) shows the Fabry-Perot interferometer output when the Fabry-Perot is stationary and positioned with a transmission peak on the slope of the spectrum. The spectrum is found to oscillate at \( \Delta \omega \) with a peak-to-peak deviation in this case of about 4GHz, although peak-to-peak deviations as high as 25GHz have been seen. Using Eq. (3.48) for \( M = 0.25 \) and the other parameters as estimated from the laser, the calculated peak-to-peak frequency excursion of the pulse is 30GHz, which is in somewhat good agreement.

One can imagine a frequency response plot of \( \Delta p \) versus \( \Delta \omega \). There is a corner frequency at \( \Delta \omega = 4g/(3\Omega_g^2 \tau^2 T_R) \) beyond which the pulse resists the frequency excursion (via its nonlinearity).

The fact that the spectrum of the pulses oscillates at \( \Delta \omega \) implies that the pulse energy must oscillate slightly at \( 2\Delta \omega \) because of filtering. This is shown in the low-
Figure 3-47: Optical spectrum of the ASM laser (all-negative dispersion system) as measured with a Fabry-Perot interferometer. (a) the Fabry-Perot is scanning showing the complete spectrum. About 125GHz/division. (b) the Fabry-Perot is stationary and biased on the slope of the spectrum, and the voltage scale has been decreased.

The frequency power spectrum of the current of detector measuring the pulse train as shown in Fig. 3-48. One also sees energy oscillation at $\Delta \omega$, which is most likely due to asymmetry in the filtering and/or residual amplitude modulation in the phase modulator (which could be caused by an imperfectly aligned polarizer at the input to the modulator). The higher-frequency broad spectral lines are most likely caused by the Er-doped fiber gain dynamics.

Low-intensity (cw) light also undergoes a frequency excursion on the time scale $T$ because of the phase modulation (on a very long time scale the asynchronous modulation spreads out the cw energy to sidebands). This modulation is also sinusoidal at frequency $\Delta \omega$ and has a peak $M \omega_M/(T \Delta \omega)$. As one can see, this is the long-pulse limit of the soliton frequency excursion. The cw frequency excursion is greater than the soliton excursion because the soliton, via its nonlinearity, resists the frequency push. Figure 3-49 illustrates the movements of the pulse and cw spectra.

The intracavity transmission of cw light over one cycle of $\Delta \omega$ is approximately

$$T_{cw}^N = \prod_{m=1}^{N} F \left[ \frac{M \omega_M}{T \Delta \omega} \cos \left(2\pi \frac{m}{N}\right) \right]$$

(3.49)
Figure 3-48: Low-frequency power spectrum of the current from a slow detector measuring the output of the ASM laser (all-negative dispersion system).

Figure 3-49: Illustration of sinusoidal excursions of pulse and cw spectra in an ASM laser.
and for the soliton is approximately

\[ T_{\text{soliton}}^N = \prod_{m=1}^{N} \frac{\int_{-\infty}^{\infty} F \left[ \omega - \frac{M\omega_M}{\sqrt{(T_R\Delta\omega)^2 + \left( \frac{\pi}{2} \tau \omega \right)^2}} \right] \cos(2\pi \frac{\omega_M}{\omega}) \sech^2 \left( \frac{\pi}{2} \tau \omega \right) d\omega}{\int_{-\infty}^{\infty} \sech^2 \left( \frac{\pi}{2} \tau \omega \right) d\omega} \]

(3.50)

\( F(\omega) \) is the intensity transmission function of the filter centered at the carrier frequency, and \( N \) is the number of round trips it takes to complete one cycle at \( \Delta\omega \) (\( N = 2\pi/(\Delta\omega T_R) \)). For the birefringent filter used in the laser when set for full modulation (i.e., the birefringent axes are 45° to the polarizers)

\[ F(\omega) = \cos^2 \left( \frac{\pi \omega}{2\Omega_g} \right) \]

(3.51)

From (3.49) and (3.50) we show that with the right modulation, the solitons have a higher average intracavity transmission than cw light. The resultant pulse stabilization against low-intensity background is not unlike that provided by sliding-frequency guiding filters proposed for long-distance communications[79, 80] and the continuously frequency shifted laser for soliton generation.[81]

Fig. 3-50 shows \( T_{\text{soliton}} \) (the average transmission of the pulse through the filter in the cavity per round trip) versus the pulse width (1.76\( \tau \)) using Eq. (3.50). An infinitely long pulse width corresponds to the cw case. The solid lines in plots (a)-(c) are plotted using the estimated parameters for the ASM laser in the experiment. These estimated parameters are \( \gamma=2 \), \( \Omega_g = 2\pi \times 2.6 \text{THz} \) (the birefringent filter bandwidth), \( T_R=25\text{ns} \), \( \Delta\omega = 2\pi \times 10\text{kHz} \), \( \omega_M = 2\pi \times 1\text{GHz} \), and \( M=0.5\text{rad} \). As one can see, for the solid lines, the transmission peaks at a pulse width of about 2ps. The steady-state pulse width in the laser is probably at this transmission peak. This 2ps predicted pulse width is close to the measured pulse width in the laser (1.6ps).

The dashed lines of Fig. 3-50 (a) show the change in the transmission curve for both increasing and decreasing the asynchronicity (\( T_R\Delta\omega \)) by a factor of 4. As one can see, increasing the asynchronicity weakens the discrimination of the pulse against cw, while decreasing the asynchronicity increases the discrimination. One cannot decrease the asynchronicity below a certain limit, however, or else cw can build up during the peak of the frequency excursion of the pulse and take over the pulse.

Figure 3-50 (b) shows the cases of either increasing the filter bandwidth (\( \Omega_g \)) by a factor of 2 or increasing the modulation strength parameter (\( M\omega_M \)) by a factor of 4. Increasing the filter bandwidth moves the transmission peak to a shorter pulsewidth but also weakens the discrimination against cw. Increasing the modulation strength both moves the transmission peak to a shorter pulsewidth and strengthens the discrimination against cw.

From this information, the best way to shorten the pulses and increase the discrimination against cw is to decrease the asynchronicity, increase the modulation strength parameter, and increase the filter bandwidth. This case is shown by the dotted line
Figure 3-50: Theoretical plots of average intracavity transmission of the pulse through the filter in the cavity versus pulse width (intensity FWHM).
in Fig. 3-50 (c). In a real system, the increase in the modulation strength parameter could be accomplished by using a waveguide phase modulator rather than a bulk phase modulator as used in the experiment described here. Note that the above analysis assumes the absence of APM. APM can help to produce shorter pulses with ASM.

**APM/L characteristic of ASM**

Figure 3-50 shows that there is a built-in limiting process in ASM, and that ASM is similar to APM/L. In other words, the average intracavity transmission increases for increasing pulse intensity, levels off, and then decreases with further intensity increase. This provides for stable pulses and allows for an optical memory to be built using ASM (see Chapter 8).

**Suitability of ASM for producing pulses on its own**

We have shown that ASM can cause pulses to have a lower loss than cw, thus providing a stabilizing mechanism for pulses. Because of this characteristic ASM may be a passive mode-locking mechanism. It cannot be confirmed that ASM was entirely responsible for the pulses in the above-described experiment since nonlinear polarization rotation was present and thus APM may have been present. An ideal test of whether ASM can produce short pulses is to build an ASM laser out of PM fiber so that nonlinear polarization rotation cannot occur. Such a PM fiber laser would be environmentally stable and simple and thus very desirable for use with a high-sensitivity sensor such as an IFOG.

One more criterion an ASM laser must meet in order to produce pulses on its own, however, is that $\Delta \omega$ must be large enough so that spontaneous emission cannot build up enough cw to rob a significant amount of energy from the pulse during the times when the pulse spectrum is at an extremum of its oscillation cycle.

Since the transmission fluctuation of the pulse is small, we let the pulse transmission be a constant equal to $T_{\text{soliton}}$. We assume that the gain relaxation time is much slower than $\Delta \omega$ and is thus a constant. In the steady-state the following must hold:

$$T_{\text{soliton}} T_{\text{rest}} = 1$$  \hspace{1cm} (3.52)

in which $T_{\text{rest}}$ is the transmission of the cavity per round trip not including the filtering, and $g$ is the overall power gain per round trip (different from the $g$ of the previous section). The number of photons in a pulse is given by

$$\text{number of photons per pulse} = \frac{u_0}{h \nu}$$ \hspace{1cm} (3.53)

where $\nu$ is the frequency of the light. Assuming that the noise starts from one photon, the criterion for the prevention of excess noise buildup is approximately

$$(T_{\text{rest}} g)^{N/8}(1 \text{ photon}) \ll \text{number of photons per pulse}$$ \hspace{1cm} (3.54)
where, as before, \( N \) is the number of round trips during a \( \delta \omega \) cycle \((N = 2\pi/(\Delta \omega T_R)) \). The 1/8 in \( N/8 \) has been chosen arbitrarily but with the idea that \( T_{cw} \) is above \( T_{soliton} \) for about an eighth of a cycle. Rewriting,

\[
T_R \Delta \omega \gg \frac{2\pi \log(T_{soliton})}{8 \log(\text{photons per pulse})}
\]

for about an eighth of a cycle. Rewriting,

\[
T_R \Delta \omega \gg \frac{2\pi \log(T_{soliton})}{8 \log(\text{photons per pulse})}
\]

(3.55)

For the all-dispersion ASM laser described above, photons per pulse \( \approx 4 \times 10^7 \), \( T_R = 25 \text{ns} \), \( T_{soliton} \approx 0.9951 \). Thus, for that laser, \( \Delta \omega \gg 9 \times 10^3 \text{rad/s} \approx 1.4 \text{kHz}(2\pi) \). The actual \( \Delta \omega \approx 10 \text{kHz}(2\pi) \) and so meets this criterion.

An ASM laser using all-PM fiber and the bulk phase modulator was constructed and tested. Short pulses were not obtained, but this is most likely because of weak birefringent filtering from the PM fiber, which could not be eliminated. Because of the weak modulation depth of the bulk phase modulator, the calculated increased loss for \( cw \) over a short pulse can easily be less than the loss the pulse experiences from the birefringent filtering by the PM fiber, causing the laser to favor long pulses.

We have shown that ASM provides higher transmission for pulses than \( cw \) and thus suppresses noise. ASM also performs another useful function: harmonic mode locking.

**Harmonic mode locking by ASM**

In a laser cavity, the light is confined to the modes of the resonator. On the time scale of \( T \), the asynchronous phase modulator, running near \( n \) times the laser round-trip frequency, creates sidebands around each resonator mode that fall close to resonator modes \( n \) resonator modes away. If the system were linear, because of the magnitude of the asynchronousity in ASM, the sidebands would fall too far from the resonator modes to directly couple, as in conventional active mode locking, and thus would end up destroying any pulses in the cavity. However, the soliton, because of its nonlinearity, couples the resonator modes and sidebands via four-wave mixing. This coupling to the sidebands is a direct consequence of the soliton's resistance to the frequency push. Thus every \( n \)th harmonic is locked together, just as in linear harmonic mode locking. As in the case of APL, the spectrum is divided into supermodes and only one supermode lases. In this way asynchronous modulation can harmonically mode lock a pulse train.

If the above is true, then the spectrum as measured with a Fabry-Perot interferometer should consist of a single supermode. Figure 3-51 shows the result of such a measurement. Because of the large width of the spectrum, the light was first prefiltered through an additional Fabry-Perot interferometer. As one can see, there is structure in the spectrum with a 1-GHz periodicity, most likely implying that only one supermode is lasing. The apparent fluctuations in the spectrum shows that frequency supermode-hopping may be occurring. This is not surprising considering the oscillatory nature of the pulse spectrum. If in an APL laser a supermode is really chosen because of tiny reflections in the fiber, then in an ASM laser the movement of the pulse frequency may average out the effect of reflections, resulting in more
Figure 3-51: Optical spectrum of the ASM laser measured with a scanning Fabry-Perot interferometer. The spectrum was prefiltered by an additional Fabry-Perot interferometer. Horizontal scale is about 1.6GHz/division.

supermode hopping.

Note that the pulse timing by the asynchronous modulator only holds true for light in a resonator. Asynchronous modulation cannot be used for the timing of pulses in a transmission line. In a transmission line, there is no preference for lasing in certain spectral modes as there is in a resonator. After long distance propagation through a transmission line with asynchronous modulation, the spectrum will be completely spread out, indicating that the asynchronous modulation does not help the pulse timing. In a resonator, however, asynchronously generated spectral modes die out, and the pulses, because of the coupling from the nonlinearity, have no choice but to be timed relative to one another.

3.7.5 Self-ordering of pulses in passively mode-locked lasers

In some purely passively mode-locked lasers, multiple pulses in the cavity arrange themselves so as to be evenly spaced.[47] It is proposed that this process is caused by wakefield soliton interaction.[82] These pulses are most likely incoherent with respect to each other.

A drawback to self-ordering is that repetitions on the order of a GHz or higher are difficult to achieve. Also, the self-ordering regime is often difficult to find, and one laser may exhibit self-ordering while another laser built with the same parameters may not. Also, it was found experimentally that much higher pump powers are required to obtain the same number of pulses in a pure APM self-ordered laser than an ASM laser. One interesting solution is to use ASM for achieving self-ordering. It was found that with weak ASM, self-ordering was easily obtained. This may be because asynchronous phase modulation helps to clean up low-level background radiation, allowing for the
formation of many pulses and tends to disfavor the clumping together of pulses. Of course, when the phase modulation amplitude was increased, the pulses moved to spacings as dictated by the phase modulation, as in the ASM operation described earlier.

An example of a self-ordered pulse train from the all-negative dispersion ASM laser and the corresponding microwave spectrum are shown in Figs. 3-52 and 3-53. As one can see from the microwave spectrum, the self-ordered pulses are not as stable as those achieved with APL or ASM.

3.8 Summary

Thermally excited acoustic modes add noise to an interferometric sensor. When using optical pulses, the acoustic noise can be avoided at low frequencies by using a high pulse repetition rate. One also desires short pulses for squeezing purposes. Thus this section presented asynchronous soliton mode locking (ASM). ASM allows one to have a quiet, simple, stable laser with short pulses at a high repetition rate.

Please note that concerning the above-described ASM theory, the approximation of noise as a soliton of infinite pulse width may not be valid, and further investigation needs to be accomplished.
Figure 3-53: Microwave power spectrum of the current of a fast detector measuring the self-ordered pulse train.
Chapter 4

Noise from nonlinearity in the sensor

Nonlinearity in the sensor causes both classical and quantum noise. Classical noise is discussed first.

4.1 Classical noise from nonlinearity

Because of the optical Kerr effect, the effect responsible for APM, APL, and solitons, false signals can occur in sensors.[83, 84, 85, 86] As before, we focus the discussion on IFOGs, which are especially susceptible to this type of noise because of their long fiber lengths.

From nonlinear optics, one can show that for two lightwaves propagating in opposite directions in a fiber, the change in the propagation constants $\beta$ of the beams due to the nonlinearity are[83]

$$\Delta \beta_\pm = \kappa(P_\pm + 2P_\mp) \tag{4.1}$$

in which $+$ represents travel in one direction and $-$ represents travel in the other direction. $\kappa$ is the nonlinear coefficient of the light in the fiber.

As shown by Bergh, Lefevre, and Shaw if the detection bandwidth is much smaller than the frequency of the source fluctuations, the false phase shift given by the nonlinearity is[83]

$$\Delta \phi = \kappa L (1 - 2K) \left[ \frac{\overline{P^2} - 2\overline{P}^2}{\overline{P}} \right] \tag{4.2}$$

where $P$ represents the total power in the ring, and the overline implies time average. $K$ is the power-splitting ratio of the ring coupler. If $K = 0.5$ then there is no error. However, the reason the nonlinearity is a problem is that $K$ will vary slightly with environmental changes. Also, it is difficult to make $K = 0.5$ exactly.

However, one sees another way to reduce the noise. If one makes the statistics of the light such that $(\Delta P)^2 = \overline{P}^2$ the noise is also canceled.[84, 85, 86] Thermal light (light with Bose-Einstein statistics rather than Poissonian statistics) has exactly
these statistics (see the Appendix for a discussion of incoherent light sources). It has also been shown that an amplified spontaneous emission source with a finite bandwidth has very close to these statistics after passing through a chromatically dispersive medium.[86]

If we still wish to use a pulsed source, as is needed for optical squeezing, there is still a solution to this nonlinearity problem. A coherent source can be modulated before entering the ring to have the desired intensity statistics. If the input is single-frequency cw, the simplest way is to modulate the intensity with a square wave of 50 percent duty cycle.[83] Note that the square-wave modulation frequency must be well above the detection bandwidth (above the phase modulation frequency in a conventional IFOG), but the frequency of the highest harmonic of the modulation must be below the lowest contributing acoustic frequency (to prevent aliasing of the acoustic noise).

If one is using pulses, the intensity is already modulated. One simply needs to adjust the pulse duty cycle so as to achieve the desired intensity statistics. The pulse duty cycle can be adjusted via chromatic dispersion. If the pulses have a Gaussian shape with pulse-intensity width parameter $\tau$ and spacing $T_P$, i.e.,

$$P(t) = \sum_n P_0 \exp \left\{ - \left[ \frac{(t - nT_P)}{\tau} \right]^2 \right\} \tag{4.3}$$

then the following must hold in order to cancel the intensity fluctuations

$$\frac{1}{T_P} \int_{-T_P/2}^{T_P/2} dt \exp[-2(t/\tau)^2] = 2 \left\{ \frac{1}{T_P} \int_{-T_P/2}^{T_P/2} dt \exp[-(t/\tau)^2] \right\}^2 \tag{4.4}$$

Numerically this reduces to

$$\tau \approx 0.20T_P \tag{4.5}$$

or

pulse intensity FWHM = $0.20\sqrt{4\ln 2}T_P \tag{4.6}$

Thus for a train of Gaussian pulses at 1GHz, the pulse intensity FWHM must be 330ps.

However, to reduce the backscatterer noise, one requires a large bandwidth. If the source puts out transform-limited pulses the pulses must be dispersed before they enter the coil. For a transform-limited Gaussian pulse with $\tau$ as its pulse-intensity width parameter, its new pulse-intensity width parameter $\tilde{\tau}$ after passing through a chromatically dispersive medium is

$$\tilde{\tau} = \sqrt{\tau^2 + \frac{(4\ln 2\beta''L_{\text{medium}})^2}{\tau^2}} \tag{4.7}$$

where $\beta''$ is the second-order chromatic dispersion in the medium and $L_{\text{medium}}$ is the length of the dispersive medium.

For example, if one starts with a 1GHz train of unchirped Gaussian pulses with
intensity FWHM of 500fs and a fiber with $\beta''=+100\text{ps}^2/\text{km}$, one needs a fiber length of 220m before the sensor to achieve the proper intensity statistics to eliminate the nonlinear noise. Note that in general negative-dispersion fiber cannot be used for the pulse-broadening because of soliton effects.

The above analysis was assuming that there is no dispersion in the sensor. In the case of dispersion in the sensor a more complicated analysis is necessary, but it should be possible to choose the system parameters such that the classical intensity noise is still canceled to first order.

### 4.2 Enhanced quantum noise from nonlinearity

Kerr nonlinearity causes a coupling of amplitude and phase fluctuations. Amplitude fluctuations (including amplitude fluctuations due to the input of vacuum fluctuations) in a sensor with nonlinearity cause phase fluctuations, increasing the noise in the sensor output. If squeezed vacuum is used in the sensor, the nonlinearity in the sensor destroys some of the quantum noise reduction.[25]

For a non-pulsed source, the enhanced noise is very small because of the low intensity. However, with pulsed sources, the high peak-intensities can create significant noise. As described in the previous section, the pulses can be dispersed in time in order to reduce the peak intensity.

In optical fiber squeezing it is best to employ short pulses. This section explains how dispersion can be used also in this case to reduce the enhancement of the quantum noise in the sensor while still allowing a squeezer with short pulses to reduce the quantum noise. We show that a dispersive element inserted between the squeezer and the sensor reduces the sensor nonlinearity, leaving the squeezing unaffected.[26]

#### 4.2.1 Optical squeezing

Optical squeezing, as proposed by Shirasaki and Haus, can be performed with a nonlinear interferometer.[9] The nonlinear fiber interferometer is conveniently replaced by a fiber ring reflector when pulses with a small duty cycle are used. It is in this configuration that squeezing has been demonstrated experimentally, with Gaussian pulses[20] and with solitons.[21]

For convenience, in the discussion to follow, we represent the squeezer by a nonlinear Mach-Zehnder interferometer. The two outputs of the interferometer are squeezed vacuum and the pump. These can be sent into an interferometric sensor, such as an interferometric fiber gyroscope, permitting measurement below the shot-noise level. The squeezer uses the intensity-dependent phase shift of the optical fiber to achieve its squeezing. Pulses with high peak intensity are used in the squeezer to produce the desired amount of nonlinear phase shift. The high-intensity pulses also cause nonlinearity in the sensor. This nonlinearity destroys some of the quantum noise reduction and can even raise the quantum noise above the shot noise level if the sensor nonlinearity is large. This destruction of noise suppression is especially severe for nonrectangular pulses, such as Gaussian pulses and solitons.[25] The sensor could
be made relatively less nonlinear by such methods as the use of a larger mode-field diameter in the sensor. However such adjustments are limited in scope.

4.2.2 Dispersion of pulsed squeezing for reduction of sensor nonlinearity

To increase the range of adjustment, we propose the placement of a lossless linear element with positive chromatic dispersion between the squeezer and the sensor. This dispersion lengthens the pulses in time, reducing their peak intensity in the sensor. We show that the dispersion does not affect the squeezing nor any linear characteristics of the sensor, and we calculate the noise output for Gaussian pulses and solitons. This idea of pulse stretching to avoid nonlinearities and saturation is similar to that used in radar and high-power amplifiers.

Suppose that identical linear operations with impulse response $h(t)$ operate on the inputs to the balanced detectors of a homodyne detection apparatus (Fig. 4-1). The detector outputs are subtracted and then integrated over time, giving the signal $\hat{s}$.

The operation of $h(t)$ with $\hat{a}(t)$ gives the convolution of $h(t)$ with $\hat{a}(t)$, and likewise on $\hat{b}(t)$. Using the scattering matrix of the beam splitter,[87] one obtains the input to the two detectors $h(t)\otimes\hat{a}(t)$ and $h(t)\otimes\hat{b}(t)$, respectively. Thus

$$\hat{s} = \int_{-\infty}^{\infty} dt [|h(t)\otimes\hat{a}(t)|^2 - |h(t)\otimes\hat{b}(t)|^2]$$

(4.8)

When we use Parseval's theorem,

$$\int_{-\infty}^{\infty} dt |x(t)|^2 = \int_{-\infty}^{\infty} df |X(f)|^2,$$

(4.9)
\begin{equation}
\hat{s} = \int_{-\infty}^{\infty} df |H(f)|^2 [\vert \hat{A}(f) \vert^2 - \vert \hat{B}(f) \vert^2]
\end{equation}

(4.10)

It is apparent from Eqs. (4.8)-(4.10) that \( \hat{s} \) is unchanged by the insertion of \( h(t) \) if \( |H(f)|^2 = 1 \). This condition is satisfied when the power (photon number) is unaffected by \( h(t) \) (i.e., \( h \) is lossless). Thus any linear operation that conserves photon number and that is performed on both inputs of a homodyne detector does not change the homodyne output. Note that this applies only if the integration is done over all time, but, since pulses are used, an integration over one pulse width is sufficient.

Now consider the interferometers, which have only linear operations except in the squeezer, shown in Fig. 4-2. The dispersive elements \( h \) are inserted in two ways: in Case I they immediately precede the detectors; in Case II they immediately follow the squeezer. Since the operations on the optical beams between the beam splitter and the two detector inputs are linear; since linear operations, which includes \( h \), are described in the frequency domain by multiplication of the transfer functions that are \( \mathcal{C} \)-number scalar functions of frequency; and since the operation of \( h(t) \) can be represented by \( H(f) \) times the identity matrix, the order of the operations can be changed without a change of input-output relation. Hence the operations of Cases I and II in Fig. 4-2 are equivalent. In both cases, the noise output remains unaffected by the operation \( h(t) \).

Case II is the introduction to the scheme proposed here for reducing the effect of the nonlinearity of the sensor (which was considered linear thus far) by reducing the peak intensity of the pulses in the sensor. Note that, if the sensor is a fiber ring, the dispersion does not affect the total noise due to backscattering. The amount of noise that is due to linear backscattering is determined by the pulse power spectrum, and this is unchanged by the dispersion.

Now we calculate the output of the system with dispersion for the case of pulses with a Gaussian envelope. The work done by Haus et al. on an interferometric fiber gyroscope connected to a squeezer is used here.[25]

Suppose that the pump is an unchirped Gaussian pulse with intensity pulse width parameter \( \tau_0 \), i.e.,

\begin{equation}
\Phi(t) = \kappa |a(t)|^2
\end{equation}

whether \( |a(t)|^2 \) is the light intensity in the fiber. In this case,

\begin{equation}
|a(t)|^2 = \frac{1}{2} |f_L(t)|^2
\end{equation}

(4.15)

where \( \Phi_s(t) \) is the nonlinear phase shift in the squeezer and \( \Phi_g \) is the nonlinear phase shift.
Figure 4-2: Two equivalent arrangements of squeezer, sensor, detectors, and dispersive elements.
shift in the sensor.

Next we consider the dispersion between the squeezer and the sensor. A Gaussian pulse propagating through a medium of length $L$ with simple group-velocity dispersion changes its width from $\tau_0$ to $\tau$. The peak nonlinear phase shift is proportional to the peak intensity and thus changes according to the relation

$$\Phi_g^L(0) = \Phi_g(0)(\tau_0/\tau)$$

(4.16)

where $\Phi_g^L$ is the nonlinear phase shift in the sensor.

One can define

$$\Phi_g^L(0) = r\Phi_s(0)$$

(4.17)

where $r$ is a nonlinear reduction factor given by

$$r = (\tau_0\kappa_2/\tau\kappa_1)$$

(4.18)

where $\tau_0$ is the length of the original pulse, $\tau$ is the length of the pulse after dispersion, $\kappa_1$ is the nonlinearity coefficient in the squeezer, and $\kappa_2$ is the nonlinearity coefficient in the sensor.

Figure 4-3 shows plots of the noise-reduction factor $R$ versus the nonlinearity-reduction factor $r$ for various values of $\kappa_1$. $R$ is defined as the ratio of the noise in the detector, minimized by adjustment of the phase $\psi$, and shot noise. As one can see, the plots level out as $r$ gets small, and the value that they approach is the same as if the sensor were perfectly linear. Also, one can see the saturation of the squeezing near -7dB that is due to the nonuniform phase of the Gaussian pulse.

If the squeezer and the sensor have the same nonlinearity, the noise is actually in-
creased for large $\kappa$. As Haus et al. have shown, the sensor nonlinearity can essentially be neglected if $4\Phi_s(0)^2 \ll 1$ and cw light is used. But if Gaussian pulses are used, the effect of the nonlinearity is more severe. For a saturation squeezing value of -7dB to be achieved, $r$ must be not more than $1/200$. For example, if the squeezer and the sensor are made of the same material and have the same length, the dispersion must increase the pulse width 200 times for -7dB of squeezing to be achieved. For 5dB of squeezing, $r$ is only $1/20$. For example, assuming that the squeezer and the sensor are of the same length, 1.8m of $\beta''=+20\text{ps}^2/\text{km}$ fiber could provide $r=1/10$ if the squeezed pulses were 100fs wide. Another factor of 2 could be achieved by using a larger mode-field diameter in the sensor.

Note that dispersion can also be used in the case of soliton squeezing. The dispersed squeezed solitons travel as regular pulses in the sensor. To make a rough approximation of this case, one can assume that the pulse has a Gaussian envelope and that the squeezing in the squeezer is uniform across the pulse. The pulse experiences reduced nonlinearity in the sensor. So we set

$$\Phi_s(t) = \frac{1}{2} \Phi_s(0)$$

(4.19)

leave the rest of the equations unchanged and recompute. Figure 4-4 shows the results. Now one can achieve -7dB of squeezing with $r \approx 1/20$ and -10dB of squeezing, which is the limit set by the quantum efficiency of present optical detectors, with $r \approx 1/100$.

Thus, nonlinearity destroys reduced quantum noise and can even increase it. If unsqueezed light enters the sensor, the nonlinearity enhances the quantum noise. Chromatic dispersion between the sensor and the squeezer can reduce this quantum noise increase without harming the squeezed light.
Note that accomplishing the necessary chromatic dispersion may be difficult since the relative phase between the LO and the squeezed vacuum must be maintained. One solution is to accomplish the dispersion inside the sensor, with a small sacrifice in noise because of the initial nonlinearity. The dispersion could be tailored so that the classical noise from nonlinearity is also suppressed (i.e., the pulses are over-dispersed so that, on average, the light has the appropriate intensity statistics for the noise cancellation).

4.3 Summary

Nonlinearity in the sensor adds both classical and quantum noise. If the light source is pulsed, which is necessary for the reduction of noise from vacuum fluctuations (squeezing), one can use chromatic dispersion between the squeezer and the sensor (or in the sensor) to significantly reduce the noise from the nonlinearity.
Chapter 5

Noise from reflections in the sensor

In this chapter we consider noise arising from reflections in the sensor.

5.1 Backscattering in IFOGs

An interferometric sensor is shown in Fig. 5-1 that measures the difference in path length (phase $\phi$) between the two arms of the first interferometer. The light at various points in the sensor are shown as phasor diagrams, in which the amplitude of the light is drawn in the complex plane. This could be an IFOG, for example, with the first Mach-Zehnder interferometer replaced by a ring reflector. Shown on the figure is the rotation signal, which as you can see is rotated by $\pi/2$ with respect to the LO (the light exiting the other port) by the second interferometer. The output current is proportional to the signal’s projection onto the LO. Thus the output current is proportional to $\sin(\phi)$.

In the case of the first interferometer being implemented as a ring reflector, some light is backscattered from one direction of the propagating light to the other by reflections in the fiber caused by slight inhomogeneities in the fiber. This type of scattering is termed Rayleigh scattering. As shown in Fig. 5-1 this causes some light to exit the same port as the rotation signal. This light generally has a random phase with respect to the LO. This light creates a signal in the balanced-detector difference current that is indistinguishable from the rotation signal. If the magnitude and phase of these backscattering signals did not change with time, the result would be a bias error which could be simply subtracted out. However, with environmental changes, these backscattering signals change unpredictably, thus leading to noise.[88, 89]

Rayleigh scattering or other reflections are essentially the sole contributors to backscattering noise. Scattering such as Brillouin or Raman do not contribute since the scattered light is at a different frequency. However, with a very large bandwidth source, these types of scattering could pose a problem. Also, in sensors that measure frequency, such as resonant fiber gyroscopes,[90] these types of scattering can be significant noise sources.
Figure 5-1: Measurement interferometer. The figure includes backscattered light, which would occur if the interferometer were implemented as a ring reflector.
As shown by Cutler et al. and Bohm et al.,[88, 89] the total backscattered power that propagates backward in a fiber of length \( L \) from a forward propagating beam of power \( P_\text{forward} \) is

\[
P_2 \approx \frac{1}{4} G \beta^2 \alpha_s L P_\text{forward}
\]  

(5.1)

in which \( G \) is the directivity gain of the scattering, \( \beta \) is the acceptance angle of the fiber core, and \( \alpha_s \) is the attenuation constant due to Rayleigh scattering (a fiber of length \( L \) has a transmission of \( \exp(-\alpha_s L) \) due to Rayleigh scattering). Eq. (5.1) assumes that \( \alpha_s L \ll 1 \).

In a bad case, the phase of the scattered power is \( \pi/2 \) with respect to the LO upon exiting the fiber ring. An even worse case is power backscattered from both directions with a \( \pi/2 \) phase. The corresponding balanced detector current in this worst case is

\[
\Delta i = 2 \frac{\eta e}{h \nu} \sqrt{P_\text{detect}} \sqrt{2 \frac{P_s}{L_p} L_c T_{\text{signal}}}
\]  

(5.2)

which can be rewritten as

\[
\Delta i = \frac{\eta e}{h \nu} P_\text{detect} \sqrt{T_{\text{signal}} T_{\text{LO}}} G \beta^2 \alpha_s L \frac{L_c}{L_p}
\]  

(5.3)

where \( P_\text{detect} \) is the total power reaching the detectors, \( L \) is the length of the ring, and \( T_{\text{signal}} \) and \( T_{\text{LO}} \) are the power transmissivities of the signal (in this case the backscattered power) and LO respectively after leaving the ring reflector. For a conventional IFOG \( T_{\text{signal}} = T_{\text{LO}} \).

If the light source has a finite coherence length, not all of the backscattered light is at the same frequency as the LO. In balanced detection, a signal that is at a different frequency than the LO is averaged out. This is the reason for the \( L_c/L_p \) term. \( L_c \) is to be taken as the light coherence length or \( L \), whichever is smaller, and \( L_p \) is the pulse spacing distance or \( L \), whichever is smaller. For a non-pulsed source \( L_p = L \). For a pulsed source with a finite coherence length between pulses of \( L_{pc} \), (5.3) should contain the additional term \( \sqrt{L_{pc}}/L \). The smallest \( L_{pc} \) can be is \( L_p \).

The balanced detector current for a rotation signal is

\[
\Delta i = \frac{\eta e}{h \nu} P_\text{detect} \sqrt{T_{\text{signal}} T_{\text{LO}}} \sin(2\phi_s)
\]  

(5.4)

where \( \phi_s \) is the Sagnac phase shift. Note that from convention \( \phi_s = \phi/2 \), where \( \phi \) is as in Fig. 5-1. For small \( \phi_s \), \( \sin(2\phi_s) \approx 2\phi_s \). Comparing (5.3) and (5.4) one sees that the minimum detectable \( \phi_s \) in the presence of backscatter in this worst case is

\[
\phi_{\text{min}} = \frac{1}{2} \sqrt{G \beta^2 \alpha_s L \frac{L_c}{L_p}}
\]  

(5.5)
For an IFOG,
\[ \Omega = \frac{\lambda_0 c_0}{2\pi LD} \phi_s \]  \hspace{1cm} (5.6)
where \( \Omega \) is the gyroscope rotation rate. The typical parameters for single-mode fiber are \( G=1.0, \ \beta=0.1 \text{rad} \)[88] and \( \alpha_s = 2 \times 10^{-4} \text{m}^{-1} (850 \text{nm}/\lambda_0)^4 \) (Rayleigh scattering has a \( \lambda_0^{-4} \) dependence)[89, 91] (the loss due to Rayleigh scattering is approximately 1dB/km at 840nm wavelength and 0.1dB/km at 1550nm wavelength).[92] For the parameters \( \lambda_0=1.55 \mu\text{m}, \ L=500 \text{m}, \ D=8 \text{cm} \) and excitation with an SLD with \( L_c=30 \mu\text{m}. \) one finds \( \Omega_{\text{min}}=0.2 \text{deg/hr}. \) Operation with a longer coherence source results in even worse operation.

This is the worst-case noise due to backscatter. The actual measured drift due to backscatter has turned out to be significantly lower than this value (as much as two orders of magnitude).[92] There are several reasons for this. One, as shown by Takada,[93] is that if one does a careful analysis of the backscattered light phases, one finds that the actual noise is multiplied by a factor \( \sqrt{2 |1 - 2K|} \), where \( K \) is the power splitting ratio of the ring beam splitter. As \( K \) goes to 0.5 the noise due to backscatter goes to zero. Other, more subtle, reasons, such as phase modulating with a “proper frequency”, have also been found as to why the measured backscatter noise is so low.[92]

5.2 Comparison to shot noise

Noise due to Rayleigh backscattering is not generally white and instead exists mostly at low frequencies. However, as an approximation, it can be considered white over a small region, allowing us to compare it to shot noise. We find

\[ X = \frac{\eta \lambda_0}{h c_0} \frac{P_{\text{detect}}}{T_{\text{signal}}} G \beta^2 \alpha_s \frac{L_c}{2B} L \frac{L_c}{L_P} \]  \hspace{1cm} (5.7)

It is estimated that \( B \approx 2.5 \text{Hz} \) for backscatter noise.[89] For a conventional interferometric fiber gyroscope with the same parameters as above one finds \( X = 8.5 \times 10^6 \text{W}^{-1} P_{\text{detect}} \). For a typical \( P_{\text{detect}} = 50 \mu\text{W}, \ X=423 \). Note that this \( X \) is for only low frequencies \( (f < 2.5 \text{Hz}) \).

5.3 Pulses

If pulses are used in a fiber sensor, one must use a pulse repetition rate higher than the highest contributing acoustic mode frequency in order to avoid the aliasing of acoustic noise in the fiber to low frequencies. However, this implies that if phase-coherent pulses are used, \( L_P \) is small. \( L_P \) must be on the order of 0.2m to avoid the aliasing of the acoustic noise. For backscattering noise in a ring of length \( L=1 \text{km}, \) for example, then \( X \) is 5000 times higher for such phase-coherent pulses than an incoherent source with the same coherence length.

It is desirable to reduce the backscatter noise for high-repetition rate pulses. One
possible solution is to use phase-incoherent pulses. As shown in Chapter 3, a laser employing only fast gain generates incoherent pulses with broad spectra. One can also vary the frequency of a phase-coherent pulse train before entering the sensor or squeezer or inside the sensor or squeezer[92, 89], broadening the spectral modes that the pulse spectrum consists of. This frequency varying can be accomplished with a phase modulator run asynchronously with respect to the pulse train or with a random signal. The frequency varying reduces $L_{Pe}$ and thus can significantly help to reduce noise due to scattering. Note that an ASM laser already has frequency changes from pulse-to-pulse, which is another advantage of using an ASM laser as a light source.

Another way to reduce backscatter noise when using pulses is to use the capability of pulses to generate larger bandwidths via self-phase modulation. In a fiber with carefully designed dispersion, a pulse can increase its bandwidth dramatically. In an experiment done by Takara et al., the 0.9nm bandwidth of 3ps pulses was expanded to greater than 200nm using self-phase modulation in a 3km fiber.[94] These large bandwidths would reduce the backscatter noise in optical sensors.

5.4 Summary

Reflections in the IFOG fiber ring cause noise. This noise can be minimized with the use of broad-band light sources. The noise is increased for high pulse-repetition rates, and thus the coherence between pulses may have to be reduced before entering the sensor.
Chapter 6

Noise from intensity fluctuations of the light source

This chapter considers the intensity noise in a sensor output that originated in the light source. Nearly all light sources, unless they are feedback stabilized, have noise that is frequency-dependent, falling with frequency at low frequencies, and leveling off into white noise. The noise may also contain some peaks due to relaxation oscillations, mechanical resonances, etc. First we discuss noise in different types of sources and then discuss ways to eliminate these effects. We show that true balanced detection is required for complete insensitivity to light-source intensity fluctuations, which leads to the design of the orthogonal polarization fiber optic gyroscope.

6.1 Intensity noise in the output of light sources

6.1.1 Single-frequency lasers

Since a single frequency laser lases in only one mode, there is no mode-beating noise in the output. There is, of course, $1/f^2$ noise due to gain noise and some excess white noise due to gain noise.

6.1.2 Mode-locked lasers

A mode-locked laser, like a single-frequency laser, contains no mode-beating noise since the lasing modes are locked together. Like a single-frequency laser, the output light also contains gain noise. In addition, if the laser is a fiber laser and uses APM, it has intensity noise due to the acoustic modes in the fiber (Chapter 3).

6.1.3 Incoherent sources

A broad-band incoherent source can have a significant amount of white intensity noise above shot noise. This is because of beating between the incoherent spectral modes.[95, 96, 97]
Table 6.1: Calculated spectral widths and output powers of incoherent sources at 1550nm wavelength.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\Delta \nu$</th>
<th>Average power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Er-doped fiber</td>
<td>2.6THz</td>
<td>1mW</td>
</tr>
<tr>
<td>SLD at 40mA</td>
<td>20THz</td>
<td>1$\mu$W</td>
</tr>
<tr>
<td>SLD at 140mA</td>
<td>4.3THz</td>
<td>10$\mu$W</td>
</tr>
</tbody>
</table>

It can be shown that an incoherent source with an infinite bandwidth (a thermal source) has the intensity statistics $(\Delta P)^2 = \overline{P}^2$. Thus an incoherent source of a finite bandwidth has the noise, as measured by a single detector, of[98]

$$\frac{\langle \Delta i \rangle^2}{B} = 2e \tilde{i} + \frac{\tilde{i}^2}{\Delta \nu}$$

(6.1)

Thus

$$X = \frac{\tilde{i}}{2e \Delta \nu} = \frac{\eta \lambda_0 P_{detect}}{2hc_0 \Delta \nu}$$

(6.2)

where $X$ is the excess noise as defined in Eq. (3.31). $\Delta \nu$ is defined as

$$\Delta \nu = \frac{(\int P(\nu)d\nu)^2}{\int P^2(\nu)d\nu}$$

(6.3)

in which $P(\nu)$ is the power spectral density of the optical field.

The calculated $\Delta \nu$ for the three incoherent source spectra in the Appendix (Figs. A-1 to A-3) are given in Table 6.1.

For example, for the Er-doped fiber fluorescence spectrum one finds that $X = 1.3 \times 10^6 W^{-1} P_{detect}$. Thus the noise is twice the shot noise level with $P_{detect} = 0.8 \mu$W. This white noise is obviously a problem for sensors that are susceptible to light-source intensity noise, such as conventional IFOGs.

### 6.2 Balanced detection

Balanced detection allows for the elimination of light-source intensity noise, both classically and quantum mechanically.

#### 6.2.1 Reduction/elimination of classical light-source intensity noise

A good way to eliminate sensitivity of the sensor to light-source excess intensity noise is balanced detection. There are two types of balanced detection. In the first type, which is true balanced detection, there are two detectors. In this type, the light-source
intensity noise (assuming the sensor is linear, i.e., has no intensity-dependent signal) is completely canceled.[8] In the second type there is only one detector, the input signal is modulated, and the output signal is measured via lock-in detection. This type of detection is used, for example, in conventional phase-modulated IFOGs.[99] In this type, light-source noise at the modulation frequency (typically around 100kHz) appears in the sensor output. This is why conventional IFOGs are sensitive to high-frequency light-source noise.

One of the criteria to achieve good excess intensity-noise suppression is that the travel times from the source to the two detectors must be equal. This makes balanced detection for a sensor such as an IFOG difficult since it must use a long delay in order to match the arrival time of light returning from the gyroscope.[100]

All balanced detection is essentially homodyne detection since the light from the source must be split to send to both detectors at some point in the sensor. A homodyne detector is shown in Fig. 6-1. If the inputs to the homodyne detection are $\hat{s}$ and $\hat{p}$ as shown, then the difference current, assuming perfect quantum-efficiency detectors, is $\Delta \hat{i} = \hat{s}^* \hat{p} + \hat{p}^* \hat{s}$. This is a projection of $\hat{p}$ onto $\hat{s}$.

### 6.2.2 Reduction/elimination of quantum mechanical light-source intensity noise

Balanced detection eliminates the classical light-source intensity noise. To eliminate the quantum-mechanical noise quadrature squeezing must be employed, in which squeezed vacuum is injected into the empty port of the measurement interferometer. This is true only for true balanced detection. Switched balanced detection is still sensitive to the quantum-mechanical noise of the light at and above the switching frequency. Since the conventional IFOG uses switched balanced detection, it cannot have significantly reduced quantum noise.

With the conventional IFOG, one has two main choices. One can use photon-number squeezed light. If the photon-number squeezing is perfect, the output noise
of the conventional IFOG can be at best 3dB below the shot-noise limit. One can instead use quadrature squeezed light. If the quadrature squeezed light is perfect, the output noise of the conventional IFOG can again be at best 3dB below the shot-noise limit. Although it has not been rigorously proven, the conventional IFOG is doomed to have at best a noise of about 3dB below the shot-noise limit.

Thus a stable, sensitive IFOG that uses true balanced detection needs to be developed. This is accomplished in the next section.

6.3 Orthogonal polarization fiber optic gyroscope

As mentioned in the introduction, there is no fundamental reason why a sensor output must contain noise. This chapter describes an optical sensor that is not so unrealistic and that has the theoretical potential to be noiseless. The orthogonal polarization fiber optic gyroscope (OPFOG) is an IFOG design that has no intrinsic loss and thus can employ squeezed light.[101] The OPFOG also requires no phase bias in the fiber ring and is insensitive to light-source intensity noise. We believe that the OPFOG is the first interferometric fiber optic gyroscope capable of stable, high-sensitivity measurement that contains only reciprocal optical elements.

6.3.1 Introduction

IFOGs measure rotation in inertial space and can be used for many purposes, such as inertial navigation and platform stabilization.[2] The generic configuration of the most successful interferometric fiber gyroscopes is shown in Fig. 6-2.[102, 99, 103, 104] This scheme is referred to as the conventional IFOG. Great strides have been made in the past decade improving the performance of these gyroscopes so as to bring their quality close to that required for inertial navigation.

However, despite their excellent performance, these gyroscopes have some weak points. One point is that they require a nonreciprocal (i.e., the scattering matrix for the optical modes under consideration is not symmetric[105, 106]) phase bias in the fiber ring in order to operate at maximum sensitivity. Most conventional IFOGs use asymmetrically-placed phase modulation in the ring to achieve the phase bias. However, this phase modulation can result in gyroscope drift, extra loss, high cost, high electrical power consumption, and complicated, high-frequency electronics. Another point is the appearance of the light-source intensity noise in the gyroscope output because of the switched-type of balanced detection that the conventional IFOG employs. As gyroscope light-source power is increased, the gyroscope resolution is limited by this noise.[95, 97] Finally, conventional IFOGs have significant loss for the rotation signal, limiting the gyroscope resolution as well. Because of this inherent loss, the conventional IFOG cannot take advantage of quadrature squeezed radiation to significantly increase its resolution.[25]

In 1984, Kajioka reported a different fiber gyroscope design, which we refer to as the orthogonal polarization fiber optic gyroscope (OPFOG).[107, 108] This scheme requires no phase bias inside the ring and is insensitive to light-source intensity noise.
Figure 6-2: Basic scheme of the conventional IFOG. The small circles on the polarization-maintaining (PM) fiber ends represent the stress rods of panda fiber. The fiber coupling lens are omitted for clarity. BS = beam splitter.
However, it exhibits false rotation signals to first-order from component and source-wavelength fluctuations. This version of the OPFOG is referred to as the original OPFOG.

This chapter describes a modification to the original OPFOG that eliminates its first-order instability and increases the resolution as well.

### 6.3.2 Original orthogonal polarization fiber optic gyroscope

Our concept of the original OPFOG is shown in Fig. 6-3.[107, 108] It operates via two orthogonal polarization modes. The lettered polarization diagram insets display polarization states as the light propagates through the system. The solid polarization states correspond to no ring rotation (i.e., no Sagnac effect), and the dashed polarization states correspond to ring rotation in a particular direction.

45°-polarized light impinges on a PBS (point a in Fig. 6-3). The light is separated into two modes by the PBS, and the modes propagate in opposite direction around a fiber ring and recombine at the PBS (point b). The fiber is PM, and its birefringence axes are adjusted such that both the clockwise and counter-clockwise beams travel along the same birefringence axis. If there is some ring rotation, the polarization becomes elliptical (dashed figure in b). A portion of the returning light is reflected by a beam splitter to a balanced detector (point c). Because the two counter-propagating modes can be accessed outside the ring, the phase bias necessary to achieve maximum sensitivity to the Sagnac effect can be obtained with a quarter-wave plate in front of the detector. The quarter-wave plate is adjusted with its axes aligned to the two counter-propagating modes, so that a relative phase shift between the modes of π/2 is achieved (point d). The light then enters a PBS oriented at 45° and goes into two balanced detectors. The difference current between the detectors gives the Sagnac phase directly, at its most sensitive operating point.

Besides not needing an explicit phase bias in the ring, the other main advantage of the original OPFOG is its immunity to light-source intensity noise. Because both polarization modes travel exactly the same path, but time-reversed, both “ports” of the gyroscope may be used. The conventional scheme does not share this advantage, since the unused ring port is environmentally unstable.[99, 109] The viability of both ports endows the original OPFOG with less loss and permits the use of homodyne detection. With balanced homodyne detection, the intensity noise is automatically canceled at all frequencies.[8]

Conventional IFOGs with phase modulation also cancel intensity noise, but only at frequencies lower than the modulation frequency. Conventional gyroscopes pick up intensity noise in the frequency bands near the modulation frequency and its harmonics. Broad-band incoherent sources, such as SLDs, have a white intensity noise that is significant compared to the shot noise at detector currents above approximately 10μA (depending on Δν (see Chapter 5)).[95] To solve the light-source intensity noise problem in conventional gyroscopes, Yurek et al.[95] have proposed and Moeller et al.[100] have implemented noise subtraction schemes. These schemes consist of balanced detection using the gyroscope signal and some light split off from the input path. These schemes have several drawbacks, though, such as noise is added by
Figure 6-3: Our concept of the original OPFOG. PBS = polarization beam splitter. The insets display the polarization states at the corresponding lettered locations. The solid polarization states represent the case of zero ring rotation, and the dashed represent ring rotation in one direction.
the shot noise from the additional detector, a long delay line must be used in order to match the arrival times at the detectors, and the accompanying electronics are complicated.

Although the original OPFOG requires no phase bias in the ring and cancels intensity noise, it has a disadvantage that outweighs these advantages: the system is not environmentally stable. If the quarter-wave plate birefringence or source wavelength change (i.e., the relative phase deviates from \( \pi/2 \)) because of environmental changes, the polarization states at point \( d \) in Fig. 6-3 with no ring rotation deviate from circular, resulting in first-order fluctuations in the rotation signal. These fluctuations prevent high sensitivity rotation measurement, such as is required by inertial navigation.

### 6.3.3 Modified orthogonal polarization fiber optic gyroscope

The modified OPFOG that overcomes this instability is shown in Fig. 6-4. Again, the insets display the polarization states at various points in the scheme; the dashed states correspond to the case of ring rotation. The scheme is partly based on the self-phase stabilized fiber squeezing scheme proposed by Shirasaki.[24]

The light, p-polarized, enters the apparatus through a special PBS (SPBS) (point \( a \) in Fig. 6-4). The SPBS reflects 100 percent of the s-polarization (electric field polarized perpendicularly to the table) and some of the p-polarization (a normal PBS reflects 0 percent of the p-polarization). The light then enters a PM fiber that serves as a spatial filter. The input birefringence axes of the spatial-filter fiber are at 45° to the SPBS axes, separating the p-polarized input light into two orthogonally polarized modes.

The output birefringence axes of the spatial-filter fiber are aligned with a PBS, which sends each mode in opposite directions around a fiber ring. The polarization of each mode is rotated 90° upon reflection from the ring, and the light re-enters the spatial-filter fiber.

The returning light exiting the spatial-filter fiber is of the same polarization as the input light, p-polarized (see b of Fig. 6-4). If there is any rotation there will be some light in the s-polarization, and the net polarization will be elliptical, as shown by the dashed figure at b in Fig. 6-4. In effect, the s-polarization represents the rotation signal, and the p-polarization represents its LO. 100 percent of the s-polarization and some of the p-polarization is reflected by the SPBS.

The polarization state after exiting the SPBS (c of Fig. 6-4) is the same as that entering it except that the p-polarization intensity has been reduced. This reduction in the LO does not affect the signal-to-noise ratio of the gyroscope, however, as is shown later. The light then passes through a quarter-wave plate whose birefringence axes are aligned with those of the SPBS. If there is no ring rotation, the polarization is unchanged. If there is ring rotation, the quarter-wave plate converts the elliptical polarization to linear (see d of Fig. 6-4), the polarization angle proportional to the Sagnac phase shift. The light then enters a PBS oriented at 45° and goes into two balanced detectors, as in the original OPFOG. The difference current between the detectors gives the Sagnac phase directly, at its most sensitive operating point.
Figure 6-4: Modified OPFOG. SPBS = special polarization beam splitter. The insets show the polarization states in the scheme.
The key difference in this modified OPFOG from the original version is the orientation of the quarter-wave plate. In the original OPFOG, the birefringence axes of the quarter-wave plate are aligned to the polarizations of the counter-propagating modes, thus providing a $\pi/2$ phase shift between the modes. Birefringence change in the quarter-wave plate or wavelength change results in first-order rotation detection errors. In the modified OPFOG, as stated above, the quarter-wave plate axes are aligned to the polarizations of the LO and the signal, providing a $\pi/2$ phase shift between them. In this case, birefringence change in the quarter-wave plate or wavelength change results only in second-order change in the scale-factor. Two other minor differences between the original and modified designs are the inclusion of the spatial-filter fiber in the modified design, and the use of the SPBS in the modified design to reduce the loss in the system.

We now further analyze the stability of the OPFOG. The key to the success of any gyroscope design is that the gyroscope must be capable of measuring the Sagnac phase shift with high sensitivity while not having environmental fluctuations cause changes that are indistinguishable from the Sagnac phase shift, to first order. We show that the modified OPFOG achieves this goal.

The stability of the OPFOG is most easily understood by the use of the Poincaré sphere (see Fig. 6-5). The light enters the gyroscope p-polarized (point 1 in Fig. 6-5). The first pass through the spatial-filter fiber rotates the polarization state on the sphere by some amount about the U-axis (the axis through the spatial-filter eigenpolarizations) (to point 2). Upon reflection from the ring, the polarization state is rotated by 180° about the Q-axis (to point 3). Any Sagnac phase shift causes rotation about the U-axis. On the Poincaré sphere we represent ring rotation by dashed, double-headed arrows. After passing through the spatial-filter fiber again, the polarization state is rotated by the same amount and in the same direction about the U-axis as when the light first passed through (to point 4), returning to its original polarization state. Reflection off the SPBS leaves the polarization state unchanged (except for a reduction in the p-polarization component). The quarter-wave plate then rotates the polarization state by 90° about the Q-axis (point 5). The balanced detector output is proportional to the difference between the angular displacements of the light polarization state from the polarization state going to each detector, shown by the circles on the U-axis in Fig. 6-5. As one can see, in the ideal case the balanced detector measures the Sagnac phase shift with no error. Of course, though, there are imperfections. We show that imperfections do not cause false rotation signals to first-order.

One imperfection is due to disturbances to the fiber (e.g., birefringence changes) in the fiber ring. However, the PBS and spatial-filter fiber cause the fiber in the ring to be accessible by only two modes, one at each fiber end. Thus, in the absence of time-variations, nonlinearities, and dc magnetic fields, i.e., when the system for these modes is reciprocal, the optical path length between these access modes is exactly the same for both propagation directions. Thus, to first order, disturbances to the fiber ring do not cause false rotation signals.

Another imperfection is due to birefringence changes between the beam splitting points of the SPBS and the ring PBS. In this section of the gyroscope, the bire-
Figure 6-5: Poincaré sphere showing polarization states in the modified OPFOG.
fringence changes can occur in three regions: in the isotropic material between the beam-splitting point of the SPBS and the spatial-filter fiber, in the highly-birefringent spatial-filter fiber, and in the isotropic material between the spatial-filter fiber and the beam-splitting point of the ring PBS. In the first and third regions, an imperfection appears as a small rotation about an arbitrary axis on the Poincaré sphere. In the second region, an imperfection appears as a small deviation of the birefringence axis of the spatial-filter fiber from the U-axis and/or a small change in the amount of rotation about this axis. Some of these birefringence changes shift point 4 on the Poincaré sphere in Fig 6-5 (the polarization state of the light returning to the SPBS). To prove that the birefringence changes do not cause false rotation signals, to first order, we must prove that point 4 always lies on the equator of the Poincaré sphere, to first order, despite the birefringence changes.

Consider the first region (between the SPBS and the spatial-filter fiber) in which there may be a small, arbitrary birefringence. The movement of the polarization state on the Poincaré sphere by the birefringence can be broken into separate rotations about the Q-, U-, and V-axes. Rotations about the Q- and U-axes correspond to linear birefringence, and rotation direction is the same for both light propagation directions. Rotation about the V-axis corresponds to either optical activity or the Faraday effect, and for each light propagation direction, rotation is the same direction for the Faraday effect and in opposite directions for optical activity. One can immediately see that any rotation about the V-axis does not cause point 4 to deviate from the equator. However, the effects of the Q- and U-axis rotations are not so obvious. Consider the following theorem:

**Theorem 1:** If the input polarization state to the second region (the spatial-filter fiber) lies near the Q-V plane on the Poincaré sphere (within an angular deviation from the Q-V plane that is much less than 1 rad), the output polarization state, after passing through the second and third regions, reflecting from the ring, and returning through the third and second regions has the opposite latitude, to first order, and lies near the Q-V plane.

If theorem 1 is true, then one can see that small rotations about the Q- and U-axes do not cause point 4 to deviate from the equator, to first order. This is because rotation about the Q-axis does not change the latitude of the polarization state to first order, and rotation about the U-axis is canceled upon the light's return through the first region. Since the birefringence is small, each rotation amount is small, and thus, to first order, the rotations can be done in any order. Therefore, if theorem 1 is true, birefringence changes in the first region do not cause false rotation signals, to first order. Next we prove theorem 1.

Consider the second region. The spatial-filter fiber causes a large rotation of the input polarization state about the U-axis. A birefringence change changes either the amount of rotation or slightly deviates the rotation axis from the U-axis.

**Theorem 2:** If the input polarization state to the third region lies near the Q-V plane on the Poincaré sphere, the output polarization state, after passing through the third region, reflecting from the ring, and returning through the third region, has the opposite latitude, to first order, and lies near the Q-V plane.

If theorem 2 is true, one can see that theorem 1 must be true. This is because
rotation of a polarization state that lies near the Q-V plane about an axis that lies near the U-axis consists of a large latitude change, which is canceled upon the light's return through the second region, and a small longitudinal change. Now we prove theorem 2.

Consider the third region, in which there may be a small, arbitrary birefringence. Reflection from the ring causes the input polarization state to rotate about the Q-axis by 180°. If the input polarization state lies near the Q-V plane, this implies that the output polarization has the opposite latitude to first order and lies near the Q-V plane. Thus, following the arguments for the first region, which are also for a small, arbitrary birefringence, theorem 2 must be true. Therefore imperfections due to birefringence changes (whether linear, elliptical, or Faraday) in the section between the beam-splitting points of the SPBS and the ring PBS do not cause first-order false rotation signals.

In the part of the OPFOG where the light travels in only one direction (from the center of the SPBS to the detectors) component drifts are more significant. Since the birefringence axes of the quarter-wave plate are aligned to the polarization of the signal and LO, the quarter-wave plate does not cause false rotation signals. Drift in the amount of birefringence of the quarter-wave plate or a change in wavelength changes the amount of rotation about the Q-axis on the Poincaré sphere, resulting in only a scale-factor change. Also, movement of the birefringence axis of the quarter-wave plate does not cause false rotation signals to first order, because of the high birefringence. However, birefringence drifts in any isotropic material between the center of the SPBS and the center of the detector PBS can result in first-order false rotation signals. This problem can be solved by insuring there is no material in between the beam-splitting points of the SPBS and the detector PBS, except for the quarter-wave plate. An OPFOG design that accomplishes this lack of material between the SPBS and the detector PBS is described in a later section.

Therefore it is theoretically possible to build an OPFOG of high sensitivity with no first-order false rotation signal.

6.3.4 Gyroscope resolution

Besides not requiring a phase bias in the ring and canceling light-source intensity noise, another advantage of the modified OPFOG is increased resolution. The modified OPFOG has no loss of the rotation signal. After the initial pass through the SPBS on the way to the ring, it has loss only in the LO, and the LO power has no effect on the signal-to-shot-noise ratio (the LO simply amplifies the signal), as long as the LO photon number is much greater than one.

We compare the resolutions of the conventional scheme, the original OPFOG, and the modified OPFOG. The comparisons ignore classical intensity noise; ignore coupling loss, fiber loss, and other such losses; assume the source light has an infinite coherence length and is polarized; assume the conventional scheme is using sinusoidal phase modulation in the ring; and assume no Faraday rotators are employed.

Let the minimum detectable Sagnac phase shift, $\phi_{\text{min}}$, be the Sagnac phase shift for a signal-to-shot-noise ratio of unity. Using the work of Moeller et al.[110] the
smallest phase shift one can detect with a conventional IFOG is

\[(\phi_{\text{min}})_{\text{conventional}} = \left[ \frac{1 + J_0(\phi_m)}{J_1(\phi_m)} \right] \left[ \frac{1}{P_{\text{detect}}} \right]^{\frac{1}{2}} \left[ \frac{\hbar v B}{4\eta} \right]^{\frac{1}{2}} \]  

(6.4)

where \( J_0 \) and \( J_1 \) are Bessel functions and \( \phi_m \) is the peak of the phase shift imparted by the oscillating phase modulation. The average optical power incident on the detector \( P_{\text{detect}} \) in terms of the total power entering the system \( P_{\text{source}} \) is

\[ P_{\text{detect}} = \left[ \frac{1 + J_0(\phi_m)}{2} \right] |r|^2 (1 - |r|^2) P_{\text{source}} \]  

(6.5)

in which \( r \) is the amplitude reflectivity of the beam splitter sending the light to the detector (see Fig. 6-2). Thus,

\[(\phi_{\text{min}})_{\text{conventional}} = \left[ \frac{\sqrt{1 + J_0(\phi_m)}}{|r|\sqrt{1 - |r|^2 J_1(\phi_m)}} \right] \left[ \frac{1}{P_{\text{source}}} \right]^{\frac{1}{2}} \left[ \frac{\hbar v B}{2\eta} \right]^{\frac{1}{2}} \]  

(6.6)

The optimum \( \phi_m \) is 2.2rad, making \( J_0(\phi_m) \approx 0.11 \) and \( J_1(\phi_m) \approx 0.56 \). The optimum \( |r|^2 = 0.5 \). Thus,

\[(\phi_{\text{min}})_{\text{conventional}} \approx 3.8 \left[ \frac{1}{P_{\text{source}}} \right]^{\frac{1}{2}} \left[ \frac{\hbar v B}{2\eta} \right]^{\frac{1}{2}} \]  

(6.7)

For the original OPFOG the difference current from the balanced detector is

\[ \Delta i = \frac{\eta e}{hv} |r|^2 (1 - |r|^2) P_{\text{source}} \sin(2\phi_s) \]  

(6.8)

where, as before, \( r \) is the amplitude reflectivity of the beam splitter sending the light to the detectors. \( \phi_s \) is the Sagnac phase shift. The total power on the detectors is

\[ P_{\text{detect}} = |r|^2 (1 - |r|^2) P_{\text{source}} \]  

(6.9)

Thus the smallest phase shift the original OPFOG can detect is

\[(\phi_{\text{min}})_{\text{original OPFOG}} = \left[ \frac{1}{|r|\sqrt{1 - |r|^2}} \right] \left[ \frac{1}{P_{\text{source}}} \right]^{\frac{1}{2}} \left[ \frac{\hbar v B}{2\eta} \right]^{\frac{1}{2}} \]  

(6.10)

The optimum \( |r|^2 = 0.5 \), and so

\[(\phi_{\text{min}})_{\text{original OPFOG}} = 2 \left[ \frac{1}{P_{\text{source}}} \right]^{\frac{1}{2}} \left[ \frac{\hbar v B}{2\eta} \right]^{\frac{1}{2}} \]  

(6.11)

For the modified scheme let \( r_p \) be the amplitude reflection of the p-polarization in the SPBS. Note that \( T_{LO}/T_{signal} = |r_p|^2 \). The difference current of the balanced...
detectors is
\[
\Delta i = \frac{\eta e}{h\nu} |r_p|(1 - |r_p|^2)P_{source}\sin(2\phi_3)
\]  
(6.12)
and the total power on the detectors is
\[
P_{\text{detect}} = |r_p|^2(1 - |r_p|^2)P_{source}
\]  
(6.13)
Thus the minimum detectable Sagnac phase for the modified OPFOG is
\[
(\phi_{\text{min}})_{\text{modified OPFOG}} = \left[ \frac{1}{\sqrt{1 - |r_p|^2}} \right] \left[ \frac{1}{P_{source}} \right]^{\frac{1}{2}} \left[ \frac{h\nu B}{2\eta} \right]^{\frac{1}{2}}
\]  
(6.14)
Note that (6.14) was made using the approximation that the LO photon number is much greater than one, and thus \( r_p \) cannot be zero. An interesting point is that \( r_p \) can be made quite small, however, resulting in low power on the detectors, preserving detector linearity, yet with a decrease in \( \phi_{\text{min}} \) (an increase in the signal-to-shot-noise-ratio).

A reasonable value for \( |r_p|^2 \) is 0.2. Using this value, one finds that the modified OPFOG requires 11 times less light-source power than the basic conventional IFOG and 3.2 times less power than the original OPFOG to achieve the same minimum detectable phase shift. Note that if the conventional scheme uses a circulator to send light to the detector, the power ratio advantage is reduced to 2.9.

### 6.3.5 Experiment

A gyroscope using the modified OPFOG scheme was constructed and tested. It was built according to Fig. 6-4, except no quarter-wave plate was used since the relative phase shift between the reflected s- and p-polarizations of the SPBS was already near \( \pi/2 \) (it was measured to be 0.38, thus a reduction of 7 percent in the scale factor was incurred).

The fiber coil consisted of approximately 80m of panda fiber and was 32.5cm in diameter. The fiber was not quadrupole wound, and so the gyroscope was highly sensitive to acoustic and thermal disturbances. The spatial-filter fiber was 1m in length and also consisted of panda fiber. The light source was an SLD with center wavelength 1.545\( \mu \)m, giving an optical power of 12\( \mu \)W after the SPBS going towards the ring. The SPBS had \( |r_p|^2 = 0.17 \) at a wavelength of 1.55\( \mu \)m. All the fiber ends were angle-polished.

The electronics consisted of a balanced detector made of two 3mm-diameter InGaAs detectors in a shunt feedback amplifier configuration[111] followed by a buffer with a gain of 10. The electronics are shown in Fig. 6-6. In this case, \( R_1 = 1.3k\Omega, R_2 = 270k\Omega, R_3 = 12k\Omega, \) and \( R_4 = 120k\Omega \). For drift measurements, the balanced detector output was connected to an external amplifier with a low-pass filter.

To calibrate the gyroscope, the ring was placed on a rotatable platform together with a Systron Donner Angular Displacement Sensor (ADS), an angular accelerometer capable of nanoradian accuracy. The scale factor of the ADS was known to be
0.50V/μrad. By dithering the platform at various frequencies and using a Hewlett-Packard signal analyzer the gyroscope scale factor was found to be 2.2V/rad/s.

To measure long-term stability, the coil assembly was placed in a large thermally-insulated box, and the rest of the optics was covered with a cardboard box. The output from the balanced detector was connected to an amplifier with a low-pass filter, which was connected to a chart recorder. Using a low-pass filter cut-off of 0.03Hz, the peak-to-peak gyroscope drift was measured to be approximately 7deg/hr over a 15-minute interval (see Fig. 6-7).

A major source of drift appeared to be air turbulence in the section between the SPBS and the detector PBS. This noise was noticeably reduced when the gyroscope was covered with the cardboard box, but was probably still significant. Birefringence changes in the glass of the SPBS and the detector PBS probably also contributed significantly to the drift.

It was verified that large drift (on the order of 50deg/hr over 2-minute intervals) resulted when wave plates were used whose birefringence axes did not line up with the rotation signal and LO polarizations, as in the original OPFOG.

6.3.6 Improved modified OPFOG

As mentioned earlier, the presence of matter between the beam-splitting points of the SPBS and detector PBS can cause false rotation signals since the matter may move
Figure 6-7: OPFOG output low-pass filtered with a cutoff of 0.03Hz with the coil at rest with respect to the earth.

with the environment. It has been shown that the quarter-wave plate, because of its orientation, does not contribute. However, small birefringence changes in the glass of the SPBS or detector PBS cause false rotation signals.

For example, consider a 0.01°C temperature change across a 1cm-thick piece of isotropic glass in the section between the beam-splitting points of the SPBS and detector PBS. This could be the glass in a cube SPBS, such as was used in the above-described OPFOG experiment. This temperature gradient in the glass could occur, for example, if part of the glass contacts metal, and the outside temperature changes. From Chapter 3

\[
\Delta n_x = -\frac{n^3}{2} p_{11} \frac{du_x}{dx}
\]  

(6.15)

The thermal expansion coefficient of fused silica (SiO₂) is 0.5×10⁻⁶/°C.[112] Thus using \(du_x/dx = 0.5\times10^{-8}\), \(p_{11} = 0.121\), and \(n = 1.45\) one finds \(\Delta n_x \approx 9\times10^{-10}\). This corresponds to an equivalent rotation rate error of

\[
\Omega = \frac{c_0 L_{\text{glass}} \Delta n}{L_{\text{coil}} D_{\text{coil}}}
\]

(6.16)

For the parameters of the above-described OPFOG experiment, \(\Omega \approx 11\text{deg/hr}\). This is close to the measured drift in the experiment.

A solution to eliminate the sensitivity to small material stresses is to replace
Figure 6-8: Design of an SPBS with no material between the light-splitting points of the SPBS and the detector PBS. $\theta_B = $ Brewster's angle.

the detector cube PBS with a birefringent wedge and replace the cube SPBS by a prism at Brewster's angle (see Fig 6-8). Immediately after the light enters the birefringent wedge, the two polarizations are separated. By using the prism and the birefringent wedge, there is no material between the beam-splitting points of the SPBS and detector PBS.

Another advantage to the prism is that, unlike a cube SPBS, the relative phase shift between the s- and p-polarizations when exiting the SPBS is exactly zero. This allows one to use a standard quarter-wave plate to achieve the desired bias. Another advantage to the prism is that it has very little wavelength sensitivity, although the direction of the output beam does change with wavelength.

### 6.3.7 Experiment

Such an improved modified OPFOG was built and is shown in Fig. 6-9. The SPBS prism is made of silicon, and its characteristics are shown in Fig. 6-10. $T_{LO}/T_{signal}$ for the silicon SPBS at $\lambda_0 = 1.55\mu m$ is 0.29. The detector PBS is a Wollaston polarizer with a 20° beam separation. The coil length is 355m with a 3in diameter. The spatial filter fiber was 1m in length. The ring PBS was a small, fiber-pigtailed bulk element. The optics for the light entering and exiting the SPBS was done with large bulk elements. The coil and the ring PBS system were on a temperature-controlled platform and covered. The rest of the optics were not temperature controlled. Walls
Figure 6-9: Experimental setup of improved modified OPFOG.
Figure 6-10: Silicon prism used in the improved modified OPFOG.

were built around the beams going into and out of the SPBS to reduce noise due to moving air.

The quarter-wave plate was a zero-order plate (two birefringent plates glued together). Its performance was far from perfect (i.e., the plates were not aligned perfectly). Another imperfection was that the silicon prism was not properly anti-reflection coated as shown in Fig. 6-10, and those two faces still had a reflection of about 26 percent in power. The reflections significantly increased the loss in the OPFOG and a strong backreflection from the SPBS was sent towards the detectors. This backreflection was minimized with slight tilting of the SPBS and blocking of the beam. Another imperfection was that the splices of the fiber-pigtailed ring PBS to the fiber coil had to be accomplished manually and thus a relatively poor extinction ratio was achieved.

Because of the higher loss in the gyroscope, the SLD source used in the previous modified OPFOG experiment could not be used here. With a properly anti-reflection-coated silicon prism the SLD could have been used. Also, one can obtain SLDs with much higher output power than the SLD used in the previous modified OPFG experiment. The SLD was tried out as the light source, but the electrical noise from the detectors was overwhelming, even with lock-in detection. Instead a 980nm diode-pumped Er-doped fiber fluorescent source was used (see Fig. A-3). The fiber was backwards pumped, to the end was spliced a fiber connector with a high reflector, and a fiber-coupled isolator (with PM fiber on its output) was used between the Er-doped fiber and the OPFOG to keep the Er-doped fiber from lasing.
The electronics are as shown in Fig. 6-6 with $R_1=1.2k\Omega$, $R_2=227k\Omega$, $R_3=12k\Omega$, and $R_4=12k\Omega$. Also, an external gain of 50 was employed.

The OPFOG was placed on a rotatable table at the Charles Stark Draper Laboratory. The results are shown in Fig. 6-11. A 3Hz low-pass filter was used in obtaining this data. Both the short-term and long-term drift is better than that of the previous modified OPFOG (remember the data from the previous modified OPFOG was taken using a 0.03Hz low-pass filter).

Figures 6-12 and 6-13 show power spectra in two different frequency ranges and scales of the gyroscope output. The flat line is the calculated shot noise level, which is determined accurately from the equation

$$\text{shot noise level [dB]} = 10 \log \left( 4e R_2 V_0 \frac{R_4}{R_3} \right)$$

(6.17)

where $V_0$ is the DC voltage level when a detector is blocked. The peaks are most likely structural resonances. As one can see, the gyroscope is shot-noise limited down to about 2Hz.

**Performance evaluation**

The advantage of the intensity noise cancellation by the OPFOG can be clearly seen. The average current in each detector for the data in Figs. 6-11 to 6-13 is $5.4\mu$A. As one can see, the OPFOG is nearly shot-noise limited down to 2Hz with this amount of detector current. Since $T_{LO}/T_{signal}$ for this OPFOG is 0.29, a conventional IFOG with the same coil length and diameter would require an average detector current of $38\mu$A to achieve the same sensitivity if the only noise was shot noise. However, the conventional IFOG is sensitive to light-source noise. For the Er-doped fiber fluorescent source used in the experiment, $\Delta
u$ is calculated to be 2.6THz (see table in Chapter 5). Thus in this case a conventional IFOG would have $X=45$ due to the light-source excess intensity noise (the noise would be 17dB above the shot-noise level). To achieve $X=1$ in this case, a conventional IFOG would require an incoherent source with a $\Delta
u$ of 120THz. This is unrealistic. Thus, the performance of the OPFOG in the experiment for frequencies 2Hz and up is significantly better than a conventional IFOG of the same dimensions employing an incoherent source. However, for very low frequencies, the current performance of the conventional IFOG is still significantly better.

To make the OPFOG valuable for most applications, the noise must reach the shot-noise level at around 0.1Hz. This should be possible with further system improvements such as better protection of the beams from moving air, a better quarter-wave plate, etc. Because of the good performance of the OPFOG at high frequencies, it may be best suited for platform stabilization applications rather than inertial navigation.
Figure 6-11: Improved modified OPFOG output. To show the calibration the OPFOG was turned one way and then the other for 1-minute time periods at a rate of 10deg/hr.
Figure 6-12: Power spectrum of the improved modified OPFOG output at rest (log scale, out to 100Hz).
Figure 6-13: Power spectrum of the improved modified OPFOG output at rest (linear scale, out to 1kHz).
6.3.8 Need for OPFOG

The following example calculation demonstrates that for applications such as platform stabilization, a typical conventional interferometric gyroscope is not good enough while an OPFOG can satisfy the requirement.

Consider a space telescope as discussed in Chapter 1. It contains several gyroscopes to stabilize itself while looking at a far-away object. Small vibrations in the structure of the satellite smear out an image unless the telescope is stabilized. As discussed before, in the visible region for a 1-m telescope aperture, the telescope can resolve 500nrad. Such a system cannot tolerate an angular jitter greater than a total of about 50nrad (1/10 of the resolution) over the frequency range 0.1 to 100Hz.

The angular rate jitter $\Delta \Omega$ of an IFOG is

$$
\overline{(\Delta \Omega)^2} = \left( \frac{\lambda_0 c_0}{2\pi LD} \right)^2 \overline{(\Delta \phi)^2}
$$

(6.18)

where $L$ is the length of the fiber in the gyroscope, $D$ is the diameter of the fiber coil, and $\Delta \phi$ are the fluctuations of the Sagnac phase shift. The total fluctuation in angle of the gyro output from frequency $\omega_1$ to $\omega_2$ is then

$$
\overline{(\Delta \theta)^2} = \int_{\omega_1}^{\omega_2} \frac{d\omega}{\omega^2} \left( \frac{\lambda_0 c_0}{2\pi LD} \right)^2 \overline{(\Delta \phi)^2}
$$

(6.19)

If the only gyroscope noise is Poissonian noise (6.19) becomes

$$
\sqrt{(\Delta \theta)^2} = \Delta \theta_{rms} = \frac{\lambda_0 c_0}{(2\pi)^{3/2} LD} \sqrt{\frac{e}{2i \tau T_{signal}}} \left(1/\omega_1 - 1/\omega_2\right)
$$

(6.20)

Where $\bar{I}$ is the average current in the detector (the total average current if there are two detectors as in the OPFOG).

The conventional IFOG is sensitive to light-source intensity noise at the gyroscope modulation frequency. As $X$ for the light-source intensity noise (see Chapter 5) nears 1 the signal-to-noise ratio of the gyroscope no longer increases with optical power.

An IFOG in a satellite would likely employ a superluminescent light source. Some reasonable parameters for a gyroscope that might be placed in such a spacecraft are $\lambda_0=1.55\mu$m, $L=500$m, and $D=8$cm. If it uses the SLD source of Fig. A-2 ($\Delta \nu = 4.3$THz) the excess light-source noise is dominating ($X=1$) at $\bar{I}\approx1.4\mu$A. Thus the total rms angle noise from 0.1 to 100Hz is approximately 310nrad, which does not significantly improve with increased light-source power. This is too much noise. An OPFOG, on the other hand, is not limited by light-source excess noise, and so can meet the specifications.
6.4 Summary

True balanced detection is required in the sensor to completely eliminate its sensitivity to light-source intensity fluctuations. The conventional IFOG employs switched balanced detection and thus is sensitive to light-source intensity noise at its phase modulation frequency, including the light-source quantum intensity noise (photon number noise). Thus the conventional IFOG cannot be noise-free (at least not without great complication). Hence the OPFOG, which is a stable IFOG employing true balanced detection, was developed.
Chapter 7

Squeezed interferometric fiber optic gyroscope

This chapter describes a possible setup for a noise-free IFOG using the technology developed during this Thesis. The setup is shown in Fig. 7-1.

To make the squeezing realistically possible, the light source must be pulsed. Also, to avoid the aliasing of the acoustic noise, the pulse 1 repetition rate must be greater than about 1GHz. This can be accomplished by using a diode-pumped ASM laser. To increase the pulse peak power so that the squeezing may accomplished using a shorter fiber, a diode-pumped external amplifier should be used.

The pulses must be either incoherent or made incoherent with an asynchronous (to the pulse train) or randomly-driven phase modulator before entering the system in order to suppress noise due to reflections. However, the necessity of this step remains to be determined experimentally.

The light must be quadrature squeezed, which can be accomplished stably with the self-phase stabilized squeezer. The relative phase between the LO and squeezed vacuum from the squeezer must be appropriately adjusted for minimum noise via a wave plate. The LO and squeezed vacuum must then be injected into an OPFOG using a polarization-independent circulator.

The pulses must be dispersed to a duty ratio as given by Eq. (4.6) before entering the gyroscope fiber coil so as to reduce the noise due to nonlinearity in the gyroscope. Note that the nonlinearity drift problem in the squeezer is not too severe because of the phase adjustment for the squeezed vacuum. The dispersion is not trivial since both the squeezed vacuum and LO must be dispersed while maintaining their relative phase. The simplest way to do this, with some sacrifice in noise, is to place the dispersion inside the gyroscope (use positive dispersion fiber in the spatial-filter fiber and the fiber ring). If the dispersion is accomplished inside the sensor rather than before it, a birefringent plate placed before the spatial-filter fiber and with its birefringence axes aligned with those of the spatial-filter fiber can eliminate the problem of nonlinear cross-phase modulation when the pulses initially enter the spatial filter. This is because the short pulses separate in the birefringent plate before entering the spatial-filter fiber.

Finally, the light enters a balanced detection apparatus. In principle, when there
Figure 7-1: A quadrature-squeezed OPFG: a nearly noiseless fiber optic gyroscope. WP = wave plate, BP = birefringent plate, FR = Faraday rotator, \( \lambda/2 \) = half-wave plate, \( \lambda/4 \) = quarter-wave plate.
is no rotation of the gyroscope, the gyroscope output has very little noise.
Chapter 8

Other applications of technology developed in this Thesis

As often occurs in research, other applications of the technology developed during this Thesis were found. This chapter describes four such applications.

8.1 Asynchronous modulation for noise reduction in long-distance soliton transmission

The discovery of asynchronous soliton mode locking for producing short pulses in harmonically mode-locked lasers has led to a proposal for another use of asynchronous phase modulation. ASM could be used to suppress noise in soliton transmission. The noise suppression is similar to that provided by sliding-frequency guiding filters.[79] Every so often (maybe at each amplifier stage) a phase modulator could be placed in the transmission line. The phase modulators could be run asynchronously to the inverse of the travel time between modulators. The signal for the phase modulation could be supplied through a wire that runs alongside the optical fiber. The phase modulation frequency would not have to equal the bit rate and instead could be much lower. At every phase modulator would be a filter. The noise would be pushed into the wings of the filter by the asynchronous modulation while the solitons could resist the push. The advantage of this noise-suppression system over the sliding-frequency guiding system is that every station could be identical, making system repairs simpler. Note that this system is compatible with wavelength division multiplexing operation. The frequency excursion of the pulse spectrum by the asynchronous modulation would be so small as to be negligible. Note that, as explained in Chapter 2, ASM cannot help to time solitons in straight-line transmission.

8.2 Extracavity additive-pulse limiting

The discovery of additive-pulse limiting for pulse-energy stabilization in harmonically mode-locked fiber lasers leads to another use for APL. Another way to reduce light-
source intensity noise (including the shot noise) is to use limiting via self-amplitude modulation. One way to achieve self-amplitude modulation is APM and APL.

Consider the passively mode-locked laser of Fig. 3-2 using polarization APM. It uses the APM rejection port as its output. When the laser is passively mode locked, the light that stays in the laser experiences APM, while the rejected output light experiences APL. Trace (a) of Fig. 8-1 shows a typical curve of intensity that is kept inside the laser (right after the PBS) versus incident intensity on the PBS while that of trace (b) shows the corresponding curve of rejected intensity versus incident intensity. As one can see, there is a region over which intensity fluctuations inside the laser are suppressed in the output (the plateau in trace (b)).

To demonstrate the intensity-noise suppression capability, the pump (a 980nm laser diode) was weakly modulated with a 1.1kHz sinusoid while the laser was pulsing. Figure 8-2(a) shows the time domain of the laser output without special adjustment of the APM action, while Fig. 8-2(b) shows the laser output with the APM adjusted so the APL in the output is at the plateau. As one can see, the intensity fluctuation is eliminated to first order by the APL.

Acoustic noise from one round trip and some gain noise from one round trip are not eliminated by this method. Figure 8-3 shows power spectra of the laser output

Figure 8-1: Output intensity versus input intensity for the transmission (a) and rejection (b) of an APM system.
Figure 8-2: Output power of the APM laser from the APM rejection port for (a) no special adjustment and (b) adjustment for best APL in the output when the pump power is weakly modulated at 1.1kHz. Note that the DC portion is filtered out.
Figure 8-3: Power spectra of the output from the APM rejection port of the APM laser for (a) no special adjustment and (b) adjustment for best APL in the output.

without (trace (a)) and with (trace (b)) the APL noise cancelation. The solid line at the bottom of the spectrum represents the shot noise, and as one can see, there is a white-noise level above the shot noise due to acoustic and gain noises.

One can also perform APL noise reduction completely outside the cavity via a nonlinear interferometer. The experimental setup is shown in Fig. 8-4. The light passes through a special polarization beam splitter (SPBS), passes through a rotatable half-wave plate, and enters a PM fiber. At the end of the PM fiber is a quarter-wave plate and mirror, which are adjusted so the the optical fields exiting the PM fiber from each birefringence axis re-enters the PM fiber on the other birefringent axis.[22] Thus each polarization in the PM fiber has the same total path length from the entrance to the return to the entrance. Part of the light is then reflected downwards by the SPBS.

The input light consisted of approximately 450fs pulses at 41MHz. Trace (a) of Fig. 8-5 shows the power spectrum of the laser output directly, and trace (b) shows the laser output after passing through the APL device. The power on the detector was set to be the same in both cases. As one can see, the APL system effectively reduced the intensity noise. The solid line at the bottom of Fig. 8-5 is the calculated shot-noise level. The noise reduction leveled off a little above it. This level appears to be set by the acoustic noise in the fiber. If this experiment was repeated with a high-repetition rate source, the intensity noise should fall below the shot-noise level. Note that the rising of the noise in Fig. 8-5 near 100kHz is due only to detector noise.
Figure 8-4: Experimental setup for extracavity APL intensity noise reduction.

Figure 8-5: Power spectra of the current from a slow detector measuring (a) the light directly from the laser and (b) after passing through the extracavity APL system. The average power on the detector is the same in both cases.
8.3 Additive-pulse mode locking/limiting storage ring

As mentioned in Chapter 3, a harmonically mode-locked Er-doped fiber laser with APM/L maintains a pulse sequence. This feature can be used to make a pulse storage ring. A pulse storage ring is an optical device capable of memorizing a pulse sequence of "0"s and "1"s. The soliton storage ring of Nakazawa et al.[113, 114, 115] with nearly unlimited transmission distance is such a device. This section describes the proposal and experimental demonstration of a different type of pulse storage ring that uses intensity-dependent interferometric action (APM/L) to create a stronger intracavity transmission differential transmission between the "0"s and “1”s.[66]

8.3.1 Introduction

As the bit-rate of optical communications increases, the use of electronics to directly send, buffer, and receive data becomes difficult. An optical fiber pulse storage ring, an optical device that can memorize a sequence of pulses, can help accomplish these tasks. In 1993 Nakazawa et al. reported a soliton propagation experiment that maintained a soliton time-domain pattern for 180 million km.[114] In ring form, this soliton propagation system can be viewed as a pulse storage device, as can the long distance propagation systems demonstrated by Mollenauer et al.[80] and Eiselt et al.[116]

This appendix describes a pulse storage ring that operates on a different principle from that of these long distance propagation rings. The proposed ring is a unidirectional fiber ring consisting of an amplitude modulator, gain with a long relaxation time, and an additive-pulse mode locking/limiting (APM/L) action (Fig. 8-6). The amplitude modulator is used to maintain the pulse spacing, and the APM/L maintains the “0”s and “1”s.

8.3.2 Theory

In the APM/L storage ring, the APM/L is set such that the intracavity transmission increases with increasing light intensity, peaks, and then decreases. In its realization via nonlinear polarization rotation, this characteristic is achieved by adjusting the birefringences such that higher intensities cause the polarization to rotate into the transmission axis of the polarizer and then rotate past it with further intensity increase. The effect could also be produced by an appropriately phase-biased nonlinear ring reflector.

The APM/L storage ring stores pulse patterns as follows. With the modulator on and the additive-pulse effect set as described above, pulses that have higher energy see less loss and hence grow in energy. Because of gain saturation and the long gain-relaxation time, the total energy available to the pulses is roughly a constant. Hence the large pulses rob energy from the smaller pulses. The large pulses stop growing as they approach the peak of the intensity-dependent transmission curve. The laser
Figure 8-6: Basic diagram of an APM/L storage ring. PSR = pulse storage ring. PPG = pulse pattern generator.
Figure 8-7: Experimental setup. EDFA = Er-doped fiber amplifier. SA = semiconductor amplifier. PC = polarization controller. RF = radio frequency. PBS = polarization beam splitter. $\lambda/2$, $\lambda/4$ = waveplates.

ends up with a fixed number of pulses, i.e., “1”s, that stay in their time slots.

### 8.3.3 Experiment

The experimental setup (Fig. 8-7) consists of two APM/L storage rings. One acts as a pulse-pattern generator (PPG), and the other acts as a pulse storage ring (PSR). The PPG serves as a pulse-pattern source that loads the PSR. Each ring contains Er-doped fiber each pumped by 250mW of 980nm from a Ti:Sapphire laser, an isolator/polarizer to both ensure unidirectional operation and to act as a polarizer for the APM/L action, a semiconductor traveling-wave amplifier (SA; 1.55$\mu$m wavelength InGaAsP/InP, anti-reflection coated on both facets and coupled via fiber lenses, the same as described in Chapter 3) used as an amplitude modulator[63], and polarization controllers (PCs) both to control the APM/L action and to ensure that the light passes through the SA with the optimum polarization (note that the PPG uses some bulk waveplates instead of PCs). The PPG also contains a birefringent tuning plate (quartz of thickness 12T, the same one as in the ASM laser) for wavelength adjustment, and the PPG uses the APM/L rejection port as the output, allowing for high output power and a set output polarization. Both rings are of the same length, 13.4m (set equal by the variable optical delay), and most of the fiber has anomalous dispersion with $\beta'' = -22$ps$^2$/km.
Figure 8.8: Example of loading (a) oscilloscope trace of bit pattern in the PPG (started from noise). (b) oscilloscope trace of bit pattern in the PSR after being loaded from the PPG using a 90ns electrical gate. Horizontal scale is 10ns/division.

The frequency synthesizer driving the SAs was run at 1.0GHz. 66 times the round-trip trip frequency of the rings (i.e., the modulator is run synchronously). The Ti:LiNbO$_3$ Mach-Zehnder amplitude modulator acting as a gate was kept closed. The PCs of both rings were adjusted so that a random pulse pattern in each ring started from noise. The PSR was then erased by temporarily blocking the light in the open-air section of the variable optical delay in the PSR. When the PSR was set for fewer than approximately 12 “1”s, no “1”s were able to start from noise after erasure. When the variable RF delay, the input polarization, and the input power were adjusted correctly, and a 90ns electrical gate pulse was applied to the Ti:LiNbO$_3$ modulator, the pulse pattern in the PPG would load into the PSR and remain there. Figures 8-8 and 8-9 show examples of loading. The patterns lasted on the order of 15 to 30 minutes. To prove that any leakage through the LiNbO$_3$ modulator was not affecting the PSR, the pattern in the PPG could be manually blocked after loading with no effect on the PSR.

The minimum pulse energy for a “1” entering the PSR to successfully load was measured to be approximately 2.3pJ. This is much lower than the 30-60pJ pulse energy of a stored “1” in the PSR. Also, for reliable loading, the loading peak wavelength (tuned via the birefringent filter) had to be within approximately 1nm of the free-running peak wavelength of the storage ring. Furthermore, the RF delay had to be adjusted so that the loading pulses from the PPG entered the PSR modulator within about ±15 percent of a modulation cycle from the time of its peak transmission. These criteria imply that the loading of the storage ring is essentially a seeding
In other words, to load, the incoming pulses need not exactly match the parameters of a “1” in the PSR but instead need to only add enough energy to a given time slot to bring it above the lasing threshold.

### 8.3.4 Role of the semiconductor amplifiers

The SAs serve several interesting functions besides acting as amplitude modulators. During the injection current modulation cycle, the SAs go from well below transparency to a little above transparency. The “1”s pass through the SAs at the above-transparency point. Because of the fast semiconductor gain relaxation time, the SAs are most likely saturated during the amplification of the “1”s. The saturation broadens the pulses, and the steady-state pulsewidths in the rings are about 350ps (the typical pulsewidth in a similar system with only Er-doped fiber gain and a modulator is about 40ps). The long pulses are beneficial because they eliminate the need for ring-length stabilization via their ability to tolerate greater timing errors. If higher modulation speeds are used, the pulses can be proportionately shorter. It is important to note that, because of the SA gain saturation of the “1”s, the SAs are also providing some limiting action. However, we have also demonstrated that when the SA is replaced by a LiNbO₄ modulator, the ring still easily stores multiple-pulse patterns. Thus limiting by the SAs is not necessary for the storage rings to work. However, ring-length stabilization is required for long-term storage in that case.

The SAs are not in saturation when a “0” passes through, and instead they emit spontaneous emission in the “0” slots. The small pulses seen in the “0” slots in the PPG output (Figs. 8-8(a) and 8-9(a)) are amplified SA spontaneous emission.
However, the peak wavelength of the spontaneous emission (which has a FWHM of about 3nm) is about 1.5nm from the lasing wavelength. Therefore because of filtering and APM, the spontaneous emission in the "0" slots is well below the lasing threshold. In fact, the spontaneous emission from the SA at the non-lasing wavelengths takes gain from the Er-doped fiber, further suppressing lasing in the "0" slots and is one reason why the storage can be erased without "1"s spontaneously starting from noise. The ring output power was only approximately 7 percent higher when it contained "1"s than when it contained all "0"s.

### 8.3.5 Storage capability

The number of stored "1"s in the APM/L storage ring is determined by the available gain and the APM/L bias. However, there is some leeway in the number of "1"s that can be stored for a given pump power and APM/L bias. In the system described here, this was about ±2 "1"s. The maximum number of "1"s that the PSR can store stably depends on the pump power. At our maximum pump power, the PSR can store about 20 "1"s stably. When the APM/L bias is set to hold more "1"s than this, pump power fluctuations cause "1"s to appear and disappear. Strengthening the APM/L action via shorter pulses, a longer cavity, and/or replacement of the SAs (because of their limiting effects) with linear modulators could dramatically increase the leeway range and the number of storable stable "1"s.

A practical high-speed storage ring must be able to store different numbers of "1"s. Changing the cavity energy to change the number of stored "1"s is impractical, though, because of the slow gain dynamics of the Er-doped fiber. However, there are ways that the number of "1"s can be changed while keeping the average power seen by the Er-doped fiber a constant. One is to change the APM/L bias electro-optically. Another way is to inject cw light into the Er-doped fiber at a wavelength that is below threshold because of filtering. The power of this injection light can be quickly reduced as data enters the storage ring and be controlled to maintain the correct number of "1"s.

The main advantage of the APM/L storage ring over the dispersed "0" soliton storage ring of Nakazawa et al.[113] is that its differential transmission between a "1" and a "0" is generally much stronger, which increases the storage stability. For example, in a 100m long storage ring with 10ps solitons and a sinusoidal amplitude modulator running at 10GHz, a typical ratio of the intracavity transmission of a "1" to a "0" is 1.003 for the dispersed "0" soliton storage ring and is 1.1 for an APM/L storage ring. In fact, APM/L action can be added to a dispersed soliton storage ring, increasing the storage stability. Also, an APM/L storage ring, unlike the dispersed "0" soliton storage ring, does not have to contain first-order solitons. This allows, for example, the use of higher energy pulses than first-order solitons for increased storage stability.
8.3.6 Possible applications

One application for the APM/L storage ring may be a low-to-high bit-rate converter for burst transmission. To accomplish this, one uses the storage ring as a multiplexer. Figure 8-10 shows a demonstration of multiplexing (sequential loading), done manually. The storage ring provides the additional functions of standardizing multiplexed pulses in wavelength, timing, and energy. It could serve as a convenient re-sender in the case of contention at nodes in a communications system. It could also serve as a repetitive PPG producing pseudo-random ultra-high rate bit patterns for systems diagnosis.

8.3.7 Asynchronous soliton mode locking storage ring

An attractive alternative to the APM/L storage ring is to build an ASM storage ring, where the ASM replaces the APM/L. As explained in Chapter 3, ASM has a similar effect as APM/L and thus can store "1"s and "0"s. Actually, because ASM has been observed to store up to 5 solitons in one time slot, an ASM storage ring could store "5"s (or hopefully even higher) through "0"s (see Fig. 8-11). This type of storage, although not particularly useful for communications, increases the storage capacity.

Some other advantages of an ASM storage ring are
1. The ASM storage ring would require no ring-length stabilization.
2. ASM has been demonstrated to operate with low pump powers, making a diode-pumped ASM storage ring realistic.
3. The modulator does not have to be synchronized to the initial incoming data sequence since the mode locking is asynchronous.

8.4 Pulse-excited conventional interferometric fiber optic gyroscope

Even though conventional IFOGs may not be able to be noise-free, removing the classical noise from conventional IFOGs is an important task. As previously explained, the usual light-source for a conventional IFOG is a broad-band incoherent source. The incoherent source has the proper statistics to overcome noise from nonlinearity and reflections. However, the light-source intensity noise puts a limit on the ultimate signal-to-noise ratio of the conventional IFOG for a given source bandwidth.

If high-repetition rate short pulses are used instead of incoherent light, noise from nonlinearity and reflections can also be diminished or eliminated. By using a high pulse-repetition rate, the noise from the acoustic modes can be avoided. The main difference between the use of pulses and an incoherent source is that the pulses need not have classical light-source intensity noise. Eliminating the light-source classical intensity noise would allow for a shot-noise-limited conventional IFOG for high optical powers on the detector, which would be a great improvement of the existing signal-to-noise ratio of the conventional IFOG.
Figure 8-10: Demonstration of sequential loading. (a) Oscilloscope trace of bit pattern in the PPG. (b) Oscilloscope trace of bit pattern in the PSR after loading using a 90ns electrical gate. (c) Oscilloscope trace of bit pattern in the PSR after moving the RF delay by one modulation period and loading again using a 90ns electrical gate.
Figure 8-11: Oscilloscope trace of the output of an ASM laser with randomly stored "0"s through "5"s.
Appendix A

Incoherent light sources

This appendix discusses some of the characteristics of incoherent light sources, the usual light source for the conventional IFOG.

Incoherent sources emit light consisting of many incoherent modes (modes with random phases). In the time-domain, the light intensity is approximately constant with small fluctuations arising from mode-beating. The light has a finite coherence time. Coherence time is defined as follows.[117] Suppose the light can be represented by $a(t) = A(t)\exp(j\omega_0 t)$ in which $A(t)$ is a slowly-varying function representing the envelope of the light. The coherence time $\tau_c$ is defined by

$$\overline{A(t)A^*(t - \tau_c)} = \frac{1}{\exp(1)|A(t)|^2}$$

(A.1)

the overbar indicates time average. A coherence length can also be defined. It is given by $L_c = (c_0/n)\tau_c$.[91] $n$ is the index of refraction and $c_0$ is the speed of light in vacuum.

Incoherent light can be obtained from a laser lasing in many different independent modes, a thermal source (such as a blackbody), or from spontaneous emission from a gain medium. Amplified spontaneous emission from a gain medium (a laser below threshold) is the most common way to obtain incoherent broad-band light. This type of light source usually produces more optical power that can be confined to a single spatial mode than a thermal source. A typical gain medium producing amplified spontaneous emission is a semiconductor amplifier (SA) (a laser diode with antireflection coatings on both facets). This type of source is typically called a superluminescent diode (SLD). Another common gain medium for the production of incoherent broad-band light is fiber doped with an element such as erbium (Er)[118, 96], neodymium (Nd)[119, 97], or thulium (Tm). This type of source is typically called a fiber superluminescent source.

Spectra from a 1.55 $\mu$m wavelength SLD is shown in Figs. A-1 and A-2, and that from an Er-doped fiber is shown in Fig. A-3.
Figure A-1: Optical spectrum of the output of an SLD at a low current (current = 40mA).

Figure A-2: Optical spectrum of the output of an SLD at a high current (current = 140mA).
Figure A-3: Optical spectrum of the spontaneous emission from Er-doped fiber pumped with 980nm light.
Bibliography


[112] I. Meller Optics, “Catalog.”


