Human Physical Interaction with a Circular Constraint

by

James Russell Hermus


Submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of

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Abstract

Despite large feedback delays, and many degrees of freedom, humans are incredibly dexterous and excel at physical interaction with complex objects. In this work we developed an upper limb crank turning experiment to study the human controller used to manage physical interaction. Subjects turned a crank with and without visual feedback, in two directions (clockwise and counterclockwise), and in three speed conditions (slow (0.075 rev/s), medium (0.5 rev/s), and fast (2 rev/s)).

We made several predictions about the dependent measures including: mean speed, standard deviation of speed, coefficient of variation of speed, mean normal force, and standard deviation of normal force. We hypothesized that subjects should perform the best at slow speeds where the effect of feedback delays, inertial dynamics, and muscle noise decrease. Notably, subjects became more variable at slow speeds, and exerted significant nonzero normal force in the slow condition. At slow speeds, increased speed variability and compressive normal forces cannot be explained by biomechanics – suggesting they result from neural control.

Next, the zero-force trajectory was computed. The zero-force trajectory allows for the peripheral biomechanics to be ‘subtracted’ to ‘reveal’ the underlying neural commands, expressed in terms of motion. We detected a coincidence of curvature and velocity extrema in the zero-force trajectory. Furthermore, this observation was robust to changes in impedance parameters. This finding is exciting. Even though the hand was confined to a circular path, when the peripheral biomechanics were subtracted, the same velocity curvature relationship seen in unconstrained movements was revealed. Lastly, the increased variability at slow speeds was present in the zero-force trajectory. This indicates that the increased variability at slow speeds is a result of neural control, not biomechanics; this finding is consistent with previous research in unconstrained motion.

Thesis Supervisor: Neville Hogan
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Chapter 1

Introduction

Human neural pathways and muscles are orders of magnitude slower than computer processors and robotic actuators, yet humans outperform robots in tasks requiring agile or dexterous movements [Hogan, 2017, Kandel et al., 2013]. Neural action potentials propagate at speeds which rarely exceed 100 m/s, orders of magnitude slower than electronic communication. Not only is the neural transmission slow, human muscles contract at frequencies less than 10 Hz, while their robotic counterparts easily achieve movements well beyond 100 Hz. In the face of these delays, the speed, agility, and precision observed in everyday activities such as soccer – maintaining a stable posture while rapidly manipulating a ball and avoiding opposing players – would seem impossible. This is the paradox of human performance.

Delays aside, humans excel at complex manipulation tasks such as tool use. Current industrial robotic systems quickly and repeatedly perform tasks that require minimal contact – picking, placing, spot welding, and painting. However, without a precise model of the system to be manipulated, current robotic approaches struggle to manage physical interaction tasks. Managing simple curved constraints is a good example. Activities such as turning a steering wheel, or opening a door, are aspects of everyday life which humans perform effortlessly, while robotic systems often struggle. In the Defense Advanced Research Project Agency (DARPA) robotics challenge [Banerjee et al., 2015, Knoedler et al., 2015], even the best humanoid robots in the world were unable to consistently accomplish the task goal of: turning a steering
wheel, pressing a gas pedal, or opening a door. Despite tremendous feedback delays and minimal knowledge of the systems they interact with, humans excel at tasks requiring agile and dexterous movements. Studying control strategies that underlie this paradox of human performance may enable advances in robotic control and human rehabilitation.

1.1 Unconstrained Motion

To learn fundamental concepts about human motor control, simple but sufficient models have been proposed. These models have then been iteratively refined as new observations have been made. When studying the human circuitry and controller, it is rarely possible or ethical to ‘open the box’ – enabling direct measurement of signals from neurons. In addition, models of human biomechanics are incomplete. Currently, it is not feasible to measure or compute exact quantities such as muscle force, inertial properties, muscle moment arms, and neural activation signals. Thus, researchers rely on the outputs of the motor system which can be observed or estimated, in order to make inferences about the processes that generate them. However, modeling these parameters is notoriously difficult. Human muscles are highly nonlinear [Zahalak, 1990] and the neural control system is complex and largely uncomprehended [Kandel et al., 2013]. Researchers search for ways to ‘peel back’ biomechanics to ‘reveal’ information about the underlying controller. One way this can be done is by looking for outputs of the motor system which are invariant to the activities or action [Hogan, 2017]. In the last 50 years, several key ‘invariants’ have been observed in the human motor system; these ‘invariants’ have established a basis for our knowledge of human motor control.

1.1.1 In what coordinate system do humans plan movement?

The human body has hundreds of degrees of freedom. If we focus on the reaching movements of the hand, the upper limb has 9 degrees of freedom. However, only 6 degrees of freedom are required to specify the position of the hand – thus, the system
is redundant. Yet, humans seem to solve this problem effortlessly. To gain insight into how humans solve the redundancy problem, researchers have investigated movement trajectory formation.

The human limb is comprised of rotational joints actuated by muscles that contract to produce joint torque. To move the hand, one could plan movements in endpoint, joint, or muscle length coordinates. Several studies have been cleverly designed to tease out clues to understand the coordinate frame humans use to plan movements.

In a study by Morasso [Morasso, 1981] subjects’ hands were confined to a plane. Subjects’ shoulders and elbows were fixed – only rotation about the elbow and shoulder was allowed. Subjects were instructed to perform point to point reaching movements. The hand endpoint trajectory, and the joint angular trajectories were computed. The joint trajectories changed markedly for different movements. The hand endpoint paths were relatively straight, with smooth bell-shaped velocity profiles. The observation of simple and consistent straight paths in hand endpoint space indicated that subjects formulated motor commands in terms of hand coordinates.

In a study by Lackner and Dizio [Lackner and Dizio, 1994] subjects were instructed to perform forward reaching to a goal. During the task the subjects could not see their hand. When unperturbed, subjects made straight reaching movements. However, when the entire room was rotated, producing Coriolis forces on the limb, their hand paths were distorted. After multiple repetitions the subjects again began making straight hand paths. This indicates that humans plan movements in endpoint space.

In another study, subjects grasped a manipulandum which induced a curl field at the hand [Shadmehr and Mussa-Ivaldi, 1994]. When the curl field was turned off, subjects made straight reaching movements. However, when the curl field was turned on, their hand paths were distorted. With practice, despite the presence of the curl field, subjects began to produce straight movements in Cartesian space. These straight movements in the presence of a force field are referred to as motor adaptation. This supports the idea that humans plan movements in endpoint coordinates. Interestingly, when the curl field was removed, subjects produced the mirror image of the effect observed when the curl field was first experienced – before motor adap-
tation had occurred. Notably, the motor adaptation was observed in other sections of the work space that had not been explored.\footnote{The effects of motor adaptation were not generalized to all regions of the work space.} This observation suggests that motor adaptation may result from a learned internal model of the limb dynamics, not simply a relationship between visited states and experienced forces.

In a fourth study, subjects received visual feedback about their hand position. The subjects’ hands were occluded, and visual feedback of their hand position was provided on a digital display. In certain trials, a nonlinear transformation was applied to the visual feedback, such that when the subjects made straight lines with their hands, curved lines were observed in the visually perceived space – the digital display. With practice, while the nonlinear transformation was applied, subjects made curved trajectories in real hand space in order to produce straight trajectories in visually perceived space – the digital display [Flanagan and Rao, 1995]. This demonstrated that subjects do not only plan movements in endpoint coordinates, they plan movements in visually perceived endpoint coordinates.

These unconstrained motion studies of multi-joint reaching clearly document the importance of hand kinematics. When force fields or visual distortions are applied, humans spontaneously adjust muscle force to restore straight hand paths in visually perceived space.

1.1.2 Minimum-Jerk

In point to point unconstrained motions, approximately straight-line hand paths with smooth bell-shaped velocity profiles are observed. A model that accurately explains the qualitative and quantitative features of human reaching movements was proposed by Hogan [Hogan, 1982, Hogan, 1984b] and later confirmed experimentally [Nelson, 1983, Flash and Hogan, 1985]. When performing unconstrained movements, the hand trajectory was described extremely well by minimizing the squared jerk, the derivative of the acceleration (plots of example movements can be seen in Figure 1-1). To calculate the mean squared jerk trajectory for a planar 2D movement, the cost function, Cost, must be minimized over a time interval from 0 to the final time, \( t_f \).
This can be written as,

\[
Cost = \frac{1}{2} \int_0^{t_f} \left( \left( \frac{d^3 x}{dt^3} \right) + \left( \frac{d^3 y}{dt^3} \right) \right) dt 
\]

(1.1)

where \(x(t)\) and \(y(t)\) are a function of time \(t\) and Cartesian coordinates in hand space. This observation was robust to movement amplitude, duration, initial/final position translation, and rotation of the line joining the two points. This model required that the movements were relaxed and well within the limits of the human motor system. Consistent with the previous observation that humans plan movements in hand coordinates, this model only predicted the correct hand trajectory when the minimum jerk cost function was formulated in endpoint coordinates, not in joint coordinates. This model is competent to explain the observations, but does not imply that the brain contains the machinery to perform optimization, nor does it imply the brain uses optimization to plan movement trajectories.

![Figure 1-1](image)

Figure 1-1: The position (a), velocity (b), and acceleration (c) profiles for a 6-inch minimum-jerk motion of 700 ms duration between two equilibrium positions. Horizontal axes represent time in seconds; vertical axes represent (a) degrees, (b) degrees per second, and (c) degrees per second\(^2\). Figure from Hogan [Hogan, 1984b].

### 1.1.3 Velocity Curvature Relationship

In a study of unconstrained two joint upper limb reaching Abend et al. [Abend et al., 1982] reported that human hand trajectories exhibit a coincidence between curvature peaks and velocity valleys in drawing movements – velocity decreases when the curvature increases as seen in Figure 1-2.
Figure 1-2: The subject was instructed to move using a curved path to reach the target. (a) The hand path is illustrated by plotting the hand location every 10 ms. Arrows indicate direction of movement along the trajectory. (b) The hand speed and curvature profile are plotted. The curvature profile is the shallower of the two. Arrowheads indicate points of local curvature maxima along the trajectory; these curvature maxima are also denoted by vertical lines over the curvature profiles. Figure from Abend et al. [Abend et al., 1982].

When studied in explicitly elliptical trajectories, the detected velocity-curvature relationship was described by a two-thirds power law [Lacquaniti et al., 1993], such that the covariation between instantaneous angular velocity, $A(t)$ and the curvature, $C(t)$ was expressed as,

$$A(t) = KC(t)^{2/3} \tag{1.2}$$

where K is a constant. The law can also be written in terms of a one-third power law relating the instantaneous tangential velocity, $V(t)$ and the radius of curvature, $R(t)=1/C(t)$, expressed as

$$V(t) = KR(t)^{1/3} \tag{1.3}$$

The power law seemed to hold for drawing ellipses and several other geometric figures (such as a lemniscate); however, the reason for this relationship was not clear. Viviani and Terzuolo [Viviani and Terzuolo, 1982] suggested the relationship to be a result of central computational constraints, which occur during the translation
of movement trajectories into the appropriate motor parameters. However, others wondered if this relationship could simply be a result of biomechanics. It intuitively makes sense that the limb slows down, to some extent, when accelerating around a corner because the arm is an inertial body actuated by muscles – finite force actuators with limited stiffness.

Two cleverly designed studies strongly support that the power law is not a result of biomechanics. Massey et al. [Massey et al., 1992] performed an experiment where subjects grasped a 3D isometric handle and exerted forces continuously to draw circles, ellipses, and lemniscates with and without visual feedback. Interestingly, the two-thirds power law was observed even when the hand did not move, indicating that inertial biomechanics is not the cause of the curvature velocity relationship.

Research has shown movement planning requires the cooperative interaction of large neuronal populations [Georgopoulos et al., 1986]. Schwartz [Schwartz, 1994] used a population vector method to transform neuronal activity to the spatial domain. This information was then used to visualize motor cortical representation of the hand trajectories made by monkeys as they drew spirals. In this cortical representation, a power law relating speed and curvature was observed. The findings of Massey et al. and Schwartz provide strong evidence that the power law relating velocity and curvature results from neural commands – not biomechanics.

Later research indicated that minimum jerk trajectory formation predicts the two-thirds power law relationship [Viviani and Flash, 1995]. Moreover, a study by Huh et al. [Huh and Sejnowski, 2015] minimized jerk during trajectory formation when a larger variety of paths were prescribed. They predicted, and then subsequently observed, different power laws, or alternatively a combination of power laws, depending on the prescribed hand path (see Figure 1-3). This curvature/velocity relationship appears to be a limitation of the human control system, and is hypothesized to be imposed by the controller used by humans to plan movements.
1.1.4 Movement Intermittency

Movement intermittency, the inability to move smoothly and continuously, has been observed in slow human movements [Hogan et al., 1999]. In addition, studies with stroke patients during rehabilitation have documented ‘fragmented’ movements composed of highly stereotyped sub-movements [Krebs, 1997]. This is vastly different from normal human movements, which are usually quite smooth and follow a minimum jerk trajectory [Flash and Hogan, 1985]. These findings are not new. For more than 100 years researchers have documented movement intermittency during cyclical movements [Crossman and Goodeve, 1983, Woodworth, 1899], eye saccades [Collewijn et al., 1988], and slow movements [Vallbo and Wessberg, 1993]. At slow speeds, these observations are unexpected. As speed decreases, all effects which make
motion control difficult – feedback delays, inertial dynamics, and muscle noise – also decline. Therefore, this variability cannot result from the human biomechanics. Thus, this unexpected observation appears to be a result of the neurologic controller.

Doeringer and Hogan [Doeringer and Hogan, 1998] designed a single joint experiment in order to determine where the source of this intermittency is introduced into the system. They saw three possible explanations. First, the intermittency was in the feedback pathway (Figure 1-4a). Second, the intermittency could not be bypassed and was part of the forward pathway (Figure 1-4b). Lastly, the intermittency was in the forward pathway but could be bypassed (Figure 1-4c). They occluded the hand in order to test if the intermittency resulted from the visual feedback pathway. They also provided explicit visual feedback about hand velocity on a display. They then asked the question, could subjects eliminate intermittency if they were made explicitly aware of their errors via visual feedback. They found that in single joint movements subjects were unable to eliminate the intermittency, and that the intermittency did not result from the presence of visual feedback, thus indicating that the intermittency was not in the visual feedback pathway, and could not be bypassed, as represented by Figure 1-4b.

This intermittency is believed to result from the composition of a motion plan made up of discrete submovements [Rohrer and Hogan, 2003]. In a study of point to point reaching movements by Park et al. [Park et al., 2017], they demonstrated that it is hard for humans to execute slow smooth rhythmic movements, instead they ‘default’ to a different approach, composing motion using overlapping submovements.

In summary, studies of unconstrained motions at preferred speeds have found that humans move in generally straight-line paths, planned in visually perceived hand coordinates. Humans move with bell shaped velocity profiles which are well described by minimizing the jerk of the hand. As predicted by minimum jerk, a relationship between velocity and curvature is observed in the hand trajectory. Curiously, human movements unexpectedly become jerky at slow speeds.
Figure 1-4: Three possible locations for an ‘intermittency generator’. In the top diagram, (a) the generator is in the feedback path, (b) the generator is in the forward path, (c) the generator is in the forward path but can be bypassed. Figure from Doeringer and Hogan [Doeringer and Hogan, 1998].

1.2 Constrained Motion studies

There are few studies that have investigated tasks involving curved constraints. Russell and Hogan [Russell and Hogan, 1989] performed studies on vertical crank turning, and documented non-zero radial forces exerted by subjects. With the wrist joint immobilized and shoulder constrained, the task of turning a crank has one degree of freedom. Yet the human arm has more than six muscles which may be used to execute the crank turning task. This results in muscle redundancy. Several works have investigated optimality criteria as a solution to muscle redundancy. Svinin et al. [Svinin et al., 2001] developed a dynamic model of the crank rotation task. They wanted to know if humans modulate the rotational stiffness of the crank and arm system to introduce an additional constraint. Ohta et al. [Ohta et al., 2004] used the same model to investigate optimality criteria, and to explain how humans deal
with muscle redundancy. Two cost functions explained observed velocity, force, and muscle activation during crank turning. One cost function combined hand contact force and joint torque with a weighting coefficient. The other cost function combined hand contact force and muscle force with a weighting coefficient.

A concave hemispherical surface constraint implemented using a joystick has been investigated as well; this work suggests subjects use a feedforward controller [McIntyre et al., 1995], prefer to move along the geodesic path [Berman et al., 2014], and that limb impedance is changed between unconstrained and constrained motion [Damm and McIntyre, 2008].

In a study by Koeppen et al. [Koeppen et al., 2017], subjects interacted with a circular constraint imposed by a robotic manipulandum. They predicted that normal forces would be observed if subjects minimized muscular effort to perform the task – consistent with previous observations. However, at certain crank positions muscles transition from lengthening to shortening or vice versa – a joint reversal (see Figure 1-5). When a joint reversal occurs, the muscle cannot contribute to the task. Therefore, Koeppen et al. predicted that the muscle should be inactive at the reversal points. As subjects turned at an extremely slow speed, 13.3 seconds per revolution, the system was considered quasi-static. Thus, they predicted that there should not be a difference between clock-wise (CW) and counter clock-wise (CCW) turning directions. Contrary to these two predictions, they observed that muscles were not inactivated at reversal points, and that there was a significant effect of turning direction. Thus, they concluded that humans do not minimize muscular effort to perform the crank turning task.
Figure 1-5: Colored radial lines depict reversal positions for the 3 muscle types. Dark and light shading denotes two distinct reversals for each muscle group. At these positions along the circular constraint, the respective muscle cannot generate tangential force. If subjects are minimizing muscle effort to perform this task, Koeppen et al. predicted that the respective muscles would be deactivated at these positions along the constraint. Figure from Koeppen [Koeppen et al., 2017].

1.3 Zero-Force Trajectory

Physical interaction is inherently bidirectional. Forces exerted on the constraint are equal and opposite the forces exerted on the hand. One way to describe the dynamics of interaction uses a mechanical impedance operator $Z\{\cdot\}$ [Hogan, 1985a, Hogan, 1985b]. The force $F(t)$ time function can be computed from the displacement time function $\Delta x(t)$, $F(t) = Z\{\Delta x(t)\}$. The displacement is defined as $\Delta x(t) = x_0(t) - x(t)$ where $x(t)$ is the actual hand position and $x_0(t)$ is a zero-force trajectory (Figure 1-6). The zero-force trajectory is the path the hand would take if zero force impeded motion, i.e. the constraint was removed. If a model for the interaction dynamics is assumed, then it can be used to ‘subtract off’ or ‘peel back’ peripheral biomechanics and reveal underlying neural influence, expressed as motion – the zero-force trajectory.
1.4 Overview of Thesis

This introductory chapter has presented the motivation for studying constrained motion and reviewed the relevant background – the research that comprises our knowledge of human upper limb motor control. Despite large feedback delays, and many degrees of freedom, humans are incredibly dexterous and excel at physical interaction with complex objects. Humans plan movements in visually perceived hand coordinates. When moving within the limits of the controller, human movement profiles follow a minimum jerk trajectory, resulting in a coincidence of curvature and velocity extrema. At slow speeds human movements become intermittent or jerky. Previous constrained motion research has been performed on crank turning. The major findings were that subjects exerted non-zero normal forces, and did not minimize muscular effort to perform the task. Physical interaction is inherently bidirectional. One way to understand this interaction is by defining an impedance operator and computing a zero-force trajectory. Thus, using the zero-force trajectory we may be able to ‘remove’ the peripheral biomechanics, and ‘reveal’ the underlying neural control influence.

After discussing the motivation and background, we will move on to discuss the experimental design. Chapter 2 explains the motivation for the crank turning experiment. It goes on to describe the experimental protocol; subjects turned a crank with and without visual feedback at three speed conditions: very slow (0.075 rev/s), close
to the preferred speed (0.5 rev/s), and about as fast as subjects could go (2 rev/s). They also turned in both the clockwise and counterclockwise directions. We made several predictions about ‘conventional’ dependent measures: mean speed, standard deviation of speed, coefficient of variation of speed, mean normal force, and standard deviation of normal force. We hypothesized that subjects should perform the best at slow speeds where the effect of feedback delays, inertial effects, and muscle noise decrease. Therefore, we expected subjects’ coefficient of variation to decrease as speed decreased. We expected normal force to approach zero at slow speeds. Notably, subjects became more variable as their speed decreased, and exerted significant nonzero normal force even in the slow condition. At slow speeds, increased speed variability and compressive normal forces cannot be explained by biomechanics – suggesting they result from neural control.

Chapter 3 re-motivates the computation of the zero-force trajectory. This approach allows for the peripheral biomechanics to be ‘subtracted’ and to ‘reveal’ the underlying neural commands, expressed in terms of motion. Next, Chapter 3 discusses the model of peripheral biomechanics that we used to compute the zero-force trajectory, and its justification. We detected the presence of peak coincidence between curvature and velocity in the zero-force trajectory. Furthermore, this observation was robust to changes in impedance parameters. This observation is exciting; although the hand was confined to a circular path, when the peripheral biomechanics were subtracted, they revealed the same velocity curvature relationship seen in unconstrained movements. This relationship is believed to be a result of neural control. Lastly, the increased variability at slow speeds was present in the zero-force trajectory, even after the peripheral biomechanics were subtracted. This indicates that the increased variability at slow speeds in the zero-force trajectory is a result of neural control, not biomechanics. This finding is consistent with previous research in unconstrained motion. Chapter 3 explains the computation of the zero-force trajectory, quantifies the velocity curvature relationship in the zero-force trajectory, and reports the increased speed variability at slow speeds in the zero-force trajectory. These observations are believed to be a result of neural control.
Chapter 4 presents the conclusions from this work, then it goes on to discuss future research. First, we hope to implement an approach to measure upper limb mechanical impedance during constrained motion. Secondly, we want to use these methods to study constrained motion tasks which require significant work production.
Chapter 2

The Crank Turning Experiment

2.1 Why a crank?

A crank-turning experiment was developed to investigate human control of constrained motion. The study examined the effect of speed and turning direction on arm movements while interacting with a crank — a horizontal planar circular constraint. The simple crank is advantageous as its motion is opposed by constant inertia and negligible friction. Consequently, at constant speed the task requires no mechanical work. In addition, with a constant curvature, velocity fluctuations cannot result from the two-thirds power law relationship between path curvature and velocity. Nevertheless, this task presents coordinative challenges. Unconstrained reaching requires subjects to compensate for multi-joint interaction torques due to ‘open-chain’ inertial dynamics, which have been reported to be represented in a frame of reference external to the body [Casadio et al., 2015, Criscimagna-Hemminger et al., 2003, Harris and Wolpert, 1998, Kluzik et al., 2008]. The kinematic constraint in the crank-turning task requires subjects to manage multi-joint coordination of ‘closed-chain’ inertial dynamics, which are substantially different from ‘open-chain’ inertial dynamics.

Unconstrained motion studies of multi-joint reaching have clearly documented the importance of hand kinematics. When force fields or visual distortions are applied, humans spontaneously adjust muscle force to restore straight hand paths, at least approximately [Flanagan and Rao, 1995, Lackner and Dizio, 1994, Shadmehr and
Mussa-Ivaldi, 1994, Wolpert et al., 1995]. Based on this evidence, a reasonable strategy to control circularly-constrained motion is to try to move in the path defined by the constraint, at constant speed, with zero normal force. Imperfections are expected because of effects such as ‘closed-chain’ inertial dynamics, limited neuromuscular response time, and activation-dependent motor noise – all of these effects decline as speed decreases. Moreover, at slower speeds, more time is available to execute feedback corrections, indicating that deviations from ideal performance due to limited response time should vanish completely as speed goes to zero. **Hypothesis:** We predict ‘imperfection’ in velocity (i.e. variability of speed), and the magnitude of the normal force should both decline as speed decreases.

Purely from a control perspective, we expect performance to improve as subjects slow down. However, previous observations indicated that this may not actually be the case. We knew that subjects may struggle to move slowly, and that nonzero normal force may be observed. These findings motivated the choice to vary both speed, direction, and visual feedback in this experiment. In many unconstrained reaching studies that reported movement intermittency, the researchers only noted the intermittency in hand trajectories after the fact. They did not specifically design experiments to understand potential causes of the variability. Thus, similar to the single joint study of Doeringer and Hogan [Doeringer and Hogan, 1998], in this work we tested whether intermittency could be eliminated when subjects were allowed complete awareness of the variability via visual feedback.

To establish a baseline, subjects were first asked to turn a crank at their preferred ‘comfortable’ speed. Then, subjects were asked to turn the crank at different constant speeds, while visual velocity feedback was provided. In this chapter, we are interested in observing the ‘conventional’ dependent measures: mean speed, standard deviation of speed, coefficient of variation of speed, mean normal force, and standard deviation of normal force. Thus, even though we do not have a hypothesis to explicitly motivate the investigation of velocity and force patterns with respect to position, we generally report these observations. Lastly, we reported the co-activation between biceps and triceps to look for evidence of a change in impedance between conditions. We found
that position dependent patterns in velocity and force were observed across subjects. We also observed co-activation regardless of turning speed or direction.

2.2 Methods

These experimental methods were developed and implemented by Doeringer [Doeringer, 1999]. Ten healthy right-handed male subjects were recruited for the study. Participants were informed about the experimental procedure and signed an informed consent form. All procedures were approved by MIT’s Institutional Review Board.

2.2.1 Apparatus

The crank used in this experiment is shown in Figure 2-1a. The crank arm was mounted on a high precision incremental optical encoder/interpolator set (Gurley Precision Instruments encoder #8335-11250-CBQA, interpolator #HR2-80QA-BRD) with a resolution of 0.0004 degrees per count. A six-axis force transducer (ATI Model 15/50) was attached to the end of the crank, with a handle mounted on it. A spool managed the force transducer cable.

![Figure 2-1](image)

Figure 2-1: (a) Crank hardware used in data collection. (b) Subject position where $e$ denotes the tangential direction, $n$ the normal direction and $\theta$ the crank position.

During the experiment, the subject’s arm was occluded from view by a wooden
structure, which did not limit the range of motion. The upper arm was suspended in
the plane of the crank, by a canvas sling connected to the ceiling using a steel cable.
The subject sat in an aluminum backed chair, while the shoulder was constrained by
a harness attached to the back of the chair. The subject was positioned such that the
crank, with radius 10.29 cm, was well within the work space of the arm. The planar
configuration and the sign conventions are depicted in Figure 2-1b.

Data acquisition was controlled by a computer running the QNX real-time oper-
ating system on an Intel Pentium 100 processor. The encoder, sampled at 200 Hz,
was connected to a set of counters and connected to the computer via digital I/O.
The ATI force transducer’s signal, sampled at 100 Hz, was processed by its embedded
controller and input to the computer through the digital I/O. Surface electromyog-
raphy (EMG) electrodes (Delsys, Boston MA) were used to record biceps and triceps
muscle activation. The signals were sampled at 200 Hz, amplified, and then input to
the computer. The visual display, also generated by the computer, was output to a
17-inch monitor (311 x 238 mm, resolution 1280 x 1024, 76 Hz) which was mounted
approximately 75 cm from subject’s eyes.

2.2.2 Protocol

Preferred ‘Comfortable’ Speed Trials

At the start of the experiment, subjects performed 20 preferred ‘comfortable’ speed
trials, 10 trials in the clockwise (CW) direction and 10 in the counterclockwise (CCW)
direction. In these trials, subjects were instructed to turn the crank at a ‘comfort-
able’ speed. Subjects were not provided any visual feedback during the preferred
‘comfortable’ trials, and each trial lasted 8 seconds.

Visual Feedback Trials

Thereafter, the experiment comprised six blocks of 30 trials each. Subjects were
instructed to turn the crank at a constant speed. A trial was defined as an event
in which a subject turned the crank for a given time interval, at a specified target
speed, and in a specified direction. A block was defined as 30 trials with the same
direction and target speed. Subjects were instructed to turn the crank at a constant
speed. In each trial, subjects attempted to turn the crank at one of three constant
target speeds (slow: 0.075, medium: 0.5, and fast: 2.0 rev/s), and in either CW or
CCW directions. The order of the speed and direction blocks was randomized across
subjects. The three speeds were selected to cover a significant range: 0.075 rev/s
was extremely slow (required over 13 seconds per revolution), 0.5 rev/s was close to
subjects’ preferred speed, and 2.0 rev/s was close to the fastest subjects could turn the
crank. Visual feedback was provided to the subject by a computer monitor displaying
the target tangential speed, as well as their real-time hand tangential speed. Seven
trials in each block were ‘blind’ catch trials, in which visual feedback of actual hand
speed was removed, though the display of target speed was retained.

When visual feedback was provided to the user on a display, the horizontal axis
was time, and the vertical axis was crank speed. Speed, \( v \), was estimated using a
backward finite difference algorithm,

\[
v = \frac{x_i - x_{i-1}}{\Delta t} = (x_i - x_{i-1})f
\]

(2.1)

where \( x \) denotes angular position, \( \Delta t \) the time between successive samples, and \( f \)
the sampling frequency. The relationship between crank motion and the screen display
was re-scaled for every block; the desired speed was always displayed as a continuous
horizontal line in the middle of the screen. The width of the screen corresponded
to the time of the trial, which was a function of the desired crank speed. In the
slow speed conditions, each trial lasted 45 seconds; in the medium speed conditions,
each trial lasted 16 seconds; in the fast speed conditions, each trial lasted 4 seconds.
This yielded 8 turns of the crank for the fast and medium conditions, but only about
3.4 turns of the crank for the slow condition. The duration of the slow speed trials
was chosen as a compromise between acquiring adequate data and avoiding subject
fatigue.
2.2.3 Muscle Co-activation

EMG measurements from biceps and triceps were used to compute a muscle co-activation ratio. Modulation of muscle co-activation during a task correlates with changes of limb impedance. The co-activation ratio was computed in several steps. First, the mean of each EMG signal was subtracted and the absolute value of the result was computed. The resulting signal was filtered using a centered seven-sample moving average filter. Finally, the signal was normalized by its maximum value. These normalized signals were used to compute a co-activation ratio,

$$\text{muscle co-activation} = \min \left( \frac{\text{activation}_{\text{triceps}}}{\text{activation}_{\text{biceps}}}, \frac{\text{activation}_{\text{biceps}}}{\text{activation}_{\text{triceps}}} \right)$$

The co-activation ratio is 1 when both muscles are equally active. When one muscle is activated and the other is not active, the co-activation ratio is zero. This co-activation measure was computed at each time point, then averaged over time to yield a mean co-activation for each trial.

2.2.4 Dependence on Crank Angle

Crank speed and normal force were computed with respect to crank angle by ‘binning’ the data. As the different target speeds were sampled at the same rate, the data were interpolated to the largest number of samples. In this case, the number of samples at the slowest target speed, 0.075 rev/s, was 9001 samples. The interpolation was performed using a piece-wise cubic Hermite interpolating polynomial [Fritsch and Carlson, 1980]. After interpolation, the speed or force profile was binned into 360 sections corresponding to crank angle $0^\circ$ to $360^\circ$. Then the mean of each bin was taken, resulting in a speed, force, or muscle activation profile with respect to crank position.
2.2.5 Statistical Analysis

Dependent measures submitted to statistical analysis were: mean speed, standard deviation of speed, coefficient of variation of speed, mean normal force, standard deviation of normal force, and mean muscle co-activation. The coefficient of variation of normal force was not computed, as the mean normal force frequently passed through zero. In all trials, the first 1.5 seconds were discarded to eliminate transients.

Significance values of post-hoc sample $t$-tests and paired-sample $t$-tests were adjusted using the Sidák-Bonferroni procedure, where the original significance level was defined as $\alpha$, the number of $t$-tests was $m$, and the corrected Sidák-Bonferroni significance values were $\alpha_{SID} = 1 - (1 - \alpha)^{1/m}$. All statistical analyses were performed using the SPSS statistical software package (SPSS Inc., Chicago IL); the significance level was set to 5%.

Trials at Preferred ‘Comfortable’ Speed

To characterize subjects’ preferred behavior in the crank turning task, we tested whether turning direction influenced the dependent measures, and whether the mean normal force was significantly different from zero. The speed mean, standard deviation, and coefficient of variation were computed for the initial 10 trials performed at the preferred ‘comfortable’ speed. Then these quantities were averaged to produce a mean speed, standard deviation, and coefficient of variation for each subject. Mean force, standard deviation of force, and mean co-activation ratio were similarly averaged for each subject. Paired-sample $t$-tests were carried out to test for significant effects of turning direction on mean speed, normal force, and mean co-activation. Student’s $t$-tests were also used to detect whether the mean normal force was significantly different from zero.

Trials with Visual Feedback

For the trials performed with visual feedback, 30 trials were collected for each combination of direction and speed. In the first of these trials, subjects often did not achieve
the task goal; they frequently turned in the wrong direction, did not complete a full

cycle, or turned at the wrong speed. The first trial in each block was discarded. The

seven ‘catch’ trials, without visual feedback, were omitted from initial statistical anal-

ysis. The speed mean, standard deviation, and coefficient of variation were computed

each of the remaining trials. To quantify the influence of speed and direction, a

linear mixed model was employed (see Equation 2.3). The model was tested using

analysis of variance (ANOVA).

The linear model which represents the observed dependent measure \( Y_{i,j,k} \) was

expressed,

\[
Y_{i,j,k,l} = \mu_T + \alpha_j + \beta_k + \gamma_l + (\alpha\beta)_{j,k} + (\alpha\gamma)_{j,l} + (\beta\gamma)_{k,l} + (\alpha\beta\gamma)_{j,k,l} + E_{i,(j,k,l)} \tag{2.3}
\]

where the grand mean is \( \mu_T \), the fixed effect of speed is \( \alpha_j \), where \( j \) is an index

from 1 to 3, the fixed effect of direction is \( \beta_k \), were \( k \) is an index from 1 to 2, the

random effect of subject is \( \gamma_l \), where \( l \) is an index from 1 to 10, and the stochastic

sampling effect \( E_{i,j,k,l} \), where \( i \) is an index from 1 to 22 (representing the multiple

trials).

**Trials with No Visual Feedback**

To test the effect of visual feedback, an additional factor for visual feedback, \( \delta_m \), was

included in the model. This factor had two levels: visual feedback (the trial before

the catch trial) and no visual feedback (the catch trial), indexed by \( m \).

\[
Y_{i,j,k,l,m} = \mu_T + \alpha_j + \beta_k + \gamma_l + \delta_m + (\text{all interaction terms}) + E_{i,(j,k,l,m)} \tag{2.4}
\]
2.3 Experimental Results

2.3.1 Trials at Preferred ‘Comfortable’ Speed

The preferred ‘comfortable’ speed trials were conducted to establish a ‘baseline’, and quantify human performance without visual feedback. A representative subject’s turning speed, normal force, and co-activation are plotted with respect to crank position in Figure 2-2. While speed fluctuates about 0.5 rev/sec, systematic variations with angular position were evident. Further, it was noted (Figure 2-2b) that the subjects’ force alternated between tension (positive) and compression (negative) depending on the crank position.

In these trials, one subject turned the crank much faster than the others. His data were excluded from subsequent analysis of the preferred ‘comfortable’ speed trials. For the remaining nine subjects there was no significant difference between mean speed in the CW (0.37 ± 0.11 rev/s), and CCW (0.40 ± 0.10 rev/s) conditions (p = 0.539). Additionally, no significant difference was detected between mean normal force in the CW (-0.58 ± 1.65 N), and CCW (-0.12 ± 1.43 N) conditions (p = 0.462). Furthermore, the mean normal force was not significantly different from zero in the CW condition (p = 0.325, \( \alpha_{SID} = 0.0253 \)), or CCW condition (p = 0.811, \( \alpha_{SID} = 0.0253 \)). Finally, subjects co-activated the biceps and triceps during the task of turning the crank in both the CW (0.46 ± 0.04), and CCW (0.49 ± 0.10) conditions. A significant effect of turning direction on mean co-activation was not detected (p = 0.249).
Figure 2-2: Representative data from one subject (a) tangential speed, (b) normal force, and (c) muscle activation with respect to crank position. Red lines indicate CW trials and blue lines indicate CCW trials. Each line represents binned speed or force values for one of the ten trials. The shading indicates one standard deviation from the mean across trials.

These dependent measures are plotted against mean speed in Figure 2-3. There is no obvious effect of direction in any of these measures. Figure 2-3a suggests that speed standard deviation declines with decreasing speed, consistent with the predic-
tion that deviations from the desired velocity decrease as speed declines. As speed decreases, unmodeled inertial effects become less significant, and more time is available to implement feedback correction. Figure 2-3b suggests that speed coefficient of variation may vary with mean speed. These results are further investigated in the trials with visual feedback.
Figure 2-3: Dependent measures with respect to mean speed: (a) standard deviation of speed, (b) coefficient of variation of speed, (c) mean normal force, (d) standard deviation of normal force, and (e) mean muscle co-activation.
2.3.2 Trials with Visual Feedback

In the trials where visual feedback was provided to the subject, they were instructed to turn at one of three speeds – slow, medium, and fast. They also turned in both clockwise (CW) and counterclockwise (CCW) directions. In each of these conditions, there were qualitatively evident patterns in the velocity, normal force, and co-activation with respect to crank position. Plots of crank speed and normal force with respect to position can be seen in Figures 2-4, and 2-5. To investigate the influence of target speed, a linear mixed model was developed and tested using ANOVA. Figure 2-6 displays the dependent measures with respect to target speed and direction. A summary of the statistical analysis is presented in Table 2.1.

Mean Speed

A main effect of target speed on all dependent measures was observed. It was the only significant factor which affected mean speed, indicating that as expected, subjects could successfully perform this task.

Standard Deviation of Speed

A significant interaction between speed and direction was detected for the standard deviation of speed. To investigate the main effect of speed the CW and CCW data were combined. In order to perform this analysis we averaged over trial. Two paired-sample t-tests were conducted to compare the standard deviation of speed at different instructed speeds. There was a significant difference in the scores for the slow and medium conditions ($p < 0.001, \alpha_{SID} = 0.0253$), and the medium and fast conditions ($p < 0.001, \alpha_{SID} = 0.0253$). This supports the clear observation in Figure 2-6, that target speed was the predominant factor influencing the standard deviation of speed.

Coefficient of Variation of Speed

For the speed coefficient of variation, a significant interaction was detected between the speed and direction conditions. To understand the true effect of direction and
Figure 2-4: Mean velocity plots binned by position. The different colored lines represent the different subjects.
Figure 2-5: Mean normal force plots binned by position. The different colored lines represent the different subjects.
speed, paired-sample t-tests compared the speed coefficient of variation between the slow and medium target speed condition, and between the medium and fast target speed conditions; this was done for both the CW and CCW conditions. The difference between the slow and medium speed was statistically significant for both CW and CCW directions \((p < 0.001, \alpha_{SID} = 0.0127)\), while the difference between the medium and fast conditions for CW \((p = 0.168, \alpha_{SID} = 0.0127)\), and CCW \((p = 0.001, \alpha_{SID} = 0.0127)\) directions was only statistically significant in the CCW case. The lack of a significant effect between the medium CW and the fast CW conditions accounts for the disordinal interaction detected by the ANOVA. This observation is clearly visible in Figure 2-6, which illustrates that the effect of speed is far more prominent than the effect of direction.

However, contrary to our hypothesis, the prominent effect of speed on the coefficient of variation detected by the ANOVA, clearly shows an increased coefficient of variation at slow speeds. This effect cannot be explained by biomechanics. Factors that the human controller must compensate for such as inertial effects, limited neuromuscular response time, and activation-dependent motor noise all decline as speed decreases.

**Mean Normal Force**

A significant main effect of target speed, and an interaction between speed and direction was observed for the mean normal force (Figure 2-6d). There was no significant effect of direction. To interpret the main effect of speed on the mean normal force, the CW and CCW data were combined for post-hoc analysis. Two paired-sample t-tests were conducted to compare the mean normal forces. There was a significant difference in the scores for the slow and medium conditions \((p < 0.001, \alpha_{SID} = 0.0253)\), and the medium and fast conditions \((p < 0.001, \alpha_{SID} = 0.0253)\). The large difference between the medium and fast condition was clear (See Figure 2-6). However, the post hoc tests showed that the significant effect of speed detected by the ANOVA was not only a result of the large change between the medium and fast speed, but interestingly, the much smaller change between the slow and the medium speed was significant as well.
Consistent with our hypothesis, the post hoc test confirmed the main effect of speed detected by the ANOVA. Specifically, as speed increased a statistically significant increase in mean force was observed. However, the observed magnitude of the mean normal force was not anticipated, and required further investigation using one-sample \( t \)-tests.

One-sample \( t \)-tests were performed to determine whether the mean normal force was statistically different from zero. We found that the mean normal force was significantly less than zero in the slow condition \((p = 0.001, \alpha_{SID} = 0.0170)\), was not significantly different from zero in the medium condition \((p = 0.241, \alpha_{SID} = 0.0170)\), and was significantly greater than zero in the fast condition \((p < 0.001, \alpha_{SID} = 0.0170)\). A positive increase in normal force at fast target speeds was expected. When turning close to the upper limits of human performance, 2 rev/sec, subjects were not expected to be able to compensate for the high frequency, large inertial forces. When the task was well within the limits of the human controller, at the slow and medium speeds, we predicted a normal force close to zero. The medium and fast cases were consistent with these predictions, but the negative (compressive) normal force observed at the slowest speed was not anticipated. At the slowest speed, all effects which might make normal force deviate from a desired value decline precipitously. Negative (compressive) normal force tends to de-stabilize the position of the handle. This motivated a simulation of the system passive dynamics to determine if mean negative normal forces were a result of dynamics or a result of neural control (see Appendix A).

**Standard Deviation of Normal Force**

Muscle noise increases with force production. Therefore, we predicted that the standard deviation of normal force would increase as speed increases. A significant main effect of target speed on the standard deviation of normal force was identified (Figure 2-6e). There was no significant effect of direction, nor any significant interaction. The CW and CCW data were combined for post hoc analysis. Two paired-sample \( t \)-tests were conducted to compare the standard deviation of normal force. There was
a significant difference in the scores for the slow and the medium speed (p < 0.001, \( \alpha_{SID} = 0.0253 \)), and the medium and the fast speed (p < 0.001, \( \alpha_{SID} = 0.0253 \)). Consistent with our hypothesis, post hoc analysis confirmed the main effect of speed detected by the ANOVA; as speed increased, a statistically significant increase in standard deviation of normal force was observed.

**Muscle Co-activation**

The ANOVA investigating muscle co-activation revealed two significant main effects: an effect of speed, and an effect of direction. It also detected a significant interaction between speed and direction (Figure 2-6f). Post hoc \( t \)-tests were performed to understand the main effects of speed and direction. In the clockwise condition, a paired \( t \)-test compared the slow to the medium condition (p = 0.179, \( \alpha_{SID} = 0.0073 \)), and the medium to the fast condition (p = 0.001, \( \alpha_{SID} = 0.0073 \)). Two additional tests investigated the CCW condition by comparing the slow to the medium condition (p = 0.918, \( \alpha_{SID} = 0.0073 \)), and the medium to the fast condition (p = 0.086, \( \alpha_{SID} = 0.0073 \)). The only significant effect was detected between the medium and the fast CW conditions.

The main effect of direction was tested by comparing the clockwise and counterclockwise trials at each level of the speed factor, slow (p = 0.102, \( \alpha_{SID} = 0.0073 \)), medium (p = 0.063, \( \alpha_{SID} = 0.0073 \)), and fast (p < 0.001, \( \alpha_{SID} = 0.0073 \)). A significant effect of direction was only detected between the fast conditions.

There was no evidence that direction affected a co-activation during the slow cases. Unexpectedly, the effect of turning direction became significant as speed increased. We found weak evidence that co-activation varied between tasks. In this work large variability between subjects was observed. Regardless of the effect of speed and direction, clear evidence of co-activation of the biceps and triceps was observed in all conditions. This co-activation produced a non-zero impedance at the hand, further motivating the investigation of the zero-force trajectory.
Figure 2-6: Dependent measures, (a) mean speed, (b) standard deviation of speed, (c) coefficient of variation of speed, (d) mean normal force, (e) standard deviation of normal force, and (f) mean co-activation ratio. Error bars indicate the standard deviation between subjects.
<table>
<thead>
<tr>
<th>Dependent Measure</th>
<th>Factors</th>
<th>$F$</th>
<th>Prob $&gt; F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean speed</td>
<td>Speed</td>
<td>$F_{2,0,18} = 3579.513$</td>
<td>$&lt; 0.001^*$</td>
</tr>
<tr>
<td></td>
<td>Direction</td>
<td>$F_{1,0,9,0} = 0.864$</td>
<td>0.377</td>
</tr>
<tr>
<td></td>
<td>Speed*Direction</td>
<td>$F_{2,0,18,0} = 0.621$</td>
<td>0.548</td>
</tr>
<tr>
<td>Standard deviation of speed</td>
<td>Speed</td>
<td>$F_{2,0,18} = 477.497$</td>
<td>$&lt; 0.001^*$</td>
</tr>
<tr>
<td></td>
<td>Direction</td>
<td>$F_{1,0,9,0} = 0.030$</td>
<td>0.867</td>
</tr>
<tr>
<td></td>
<td>Speed*Direction</td>
<td>$F_{2,0,18,0} = 5.317$</td>
<td>0.015</td>
</tr>
<tr>
<td>Coefficient of variation of speed</td>
<td>Speed</td>
<td>$F_{2,0,18} = 110.824$</td>
<td>$&lt; 0.001^*$</td>
</tr>
<tr>
<td></td>
<td>Direction</td>
<td>$F_{1,0,9,0} = 10.991$</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>Speed*Direction</td>
<td>$F_{2,0,18,0} = 5.857$</td>
<td>0.011</td>
</tr>
<tr>
<td>Mean normal force</td>
<td>Speed</td>
<td>$F_{2,0,18} = 258.878$</td>
<td>$&lt; 0.001^*$</td>
</tr>
<tr>
<td></td>
<td>Direction</td>
<td>$F_{1,0,9,0} = 3.232$</td>
<td>0.166</td>
</tr>
<tr>
<td></td>
<td>Speed*Direction</td>
<td>$F_{2,0,18,0} = 3.752$</td>
<td>0.043</td>
</tr>
<tr>
<td>Standard deviation of normal force</td>
<td>Speed</td>
<td>$F_{2,0,18} = 749.292$</td>
<td>$&lt; 0.001^*$</td>
</tr>
<tr>
<td></td>
<td>Direction</td>
<td>$F_{1,0,9,0} = 0.001$</td>
<td>0.978</td>
</tr>
<tr>
<td></td>
<td>Speed*Direction</td>
<td>$F_{2,0,18,0} = 0.463$</td>
<td>0.637</td>
</tr>
<tr>
<td>Mean muscle coactivation</td>
<td>Speed</td>
<td>$F_{2,0,18} = 18.842$</td>
<td>$&lt; 0.001^*$</td>
</tr>
<tr>
<td></td>
<td>Direction</td>
<td>$F_{1,0,9,0} = 21.094$</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>Speed*Direction</td>
<td>$F_{2,0,18,0} = 7.320$</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Table 2.1: Summary of results for the five ANOVAs. The * indicates a significant effect.

### 2.4 Potential Artifacts

Here we present findings related to muscle activation, passive simulation of the crank dynamics and the catch trials. The effect of each of these factors independently could have clouded our results.
2.4.1 Muscle Activation

By design, the forces required to move the crank and the limb at the slowest speeds were extremely small. Production of extremely small muscle forces might have required isolated motor unit activity. However, fully-developed interference patterns were observed in EMG signals for all speed conditions and directions, indicating a superposition of many motor unit activities. A representative example of the EMG signals can be seen in Figure 2-7.

<table>
<thead>
<tr>
<th>Speed</th>
<th>Biceps</th>
<th>Triceps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slow</td>
<td><img src="image" alt="Biceps EMG Normalized" /></td>
<td><img src="image" alt="Triceps EMG Normalized" /></td>
</tr>
<tr>
<td>Medium</td>
<td><img src="image" alt="Biceps EMG Normalized" /></td>
<td><img src="image" alt="Triceps EMG Normalized" /></td>
</tr>
<tr>
<td>Fast</td>
<td><img src="image" alt="Biceps EMG Normalized" /></td>
<td><img src="image" alt="Triceps EMG Normalized" /></td>
</tr>
</tbody>
</table>

Figure 2-7: Representative EMG signals normalized by their maximum value during clockwise turning. Each trail was normalized by its maximum value. Each plot displays a small 500 ms section. The max value in a trial is not observed in any of these windows; thus, a magnitude of one is not observed.
2.4.2 Simulation of the Crank

Grasping a crank with the hand introduces a closed kinematic chain, altering the relationship between muscle force and limb motion encountered during unconstrained motions. To assess whether this change might account for our observations, a simulation of a purely inertial model (no muscle forces) of the closed kinematic chain was performed. The model was identical to that used by Ohta et al. [Ohta et al., 2004]. The model is further explained in Appendix A. Figure 2-8, 2-9, and 2-10 show the result of simulating the crank velocity and normal force that would arise solely due to closed-chain dynamics. Negative (compressive) normal forces were not observed in the simulations. Moreover, aside from a scaling factor, the pattern of variation with respect to angle was identical for all speeds and directions. This indicates that uncompensated inertial dynamics cannot account for the observed negative mean normal forces. A figure overlaying the subjects’ velocity, normal force, passive velocity, and passive normal force can be seen in Figure 2-12. The dependent measures were computed for the passive system and are displayed in Figure 2-11. Compare Figure 2-6 and Figure 2-11. In the slow speed condition, there is a clear difference in the speed coefficient of variation and the mean normal force. Consistent with our predictions, the ‘closed-chain’ dynamics cannot result in an increased coefficient of variation at slow speeds.
Figure 2-8: View of the (a) speed and (b) normal force in the passive simulation. Note that the pattern is independent of direction, but the amplitude varies greatly between speed conditions.

Figure 2-9: Closed-chain inertia simulations of the crank velocity with respect to crank position. Note that the pattern of variation is independent of speed and direction.
Figure 2-10: Closed-chain inertia simulations of the crank normal force with respect to crank position. Note that the normal force is greater than zero in all conditions.

2.5 Effect of Visual Feedback

To investigate the effects of visual feedback, a feedback term was added to the mixed model and tested using ANOVA. Seven of the trials for each condition were selected to be pseudo random ‘catch trials’ in which visual feedback of actual speed was not provided. The catch trial, and the trial performed immediately before the catch trial, comprised the two levels of the feedback condition. Figure 2-13 displays the dependent measures with respect to target speed, and direction. A summary of the statistical analysis is presented in Table 2.2.

As expected from the foregoing, target speed had a significant main effect on all dependent measures. Direction had a significant main effect on speed coefficient of variation, and muscle co-activation. Feedback had a significant main effect on speed
Figure 2-11: Passive simulation dependent measures: (a) mean speed, (b) standard deviation of speed, (c) coefficient of variation of speed, (d) mean normal force, and (e) standard deviation of normal force. If only one line is observed, then the CCW line lies on top of the CW line.
Figure 2-12: Overlaid plot with the mean subject velocity (dotted blue lines), normal force (dotted red lines), the passive velocity simulations (solid blue line), and the passive normal force (solid red line). The shading indicates one standard deviation from the mean across subjects.
coefficient of variation, mean normal force, and muscle co-activation, but no significant main effect on mean speed, standard deviation of speed, nor on the standard deviation of normal force. A two-way interaction between speed and direction on mean normal force and muscle co-activation was observed. Two-way interactions between speed and feedback for speed coefficient of variation, and mean force were observed. A three-way interaction was observed for the mean normal force.

As may be seen in Figure 2-13b and 2-13c, while the presence or absence of visual feedback may have influenced speed standard deviation (and thereby speed coefficient of variation) and mean force, it had minimal influence on the principal effect of the primary independent variable, target speed (compare Figure 2-6a with Figure 2-13a). These results indicate that the visual feedback provided in this study was not the cause of the irregularities observed.
Figure 2-13: Dependent measures used for the mixed model ANOVA with the feedback term: (a) mean speed, (b) standard deviation of speed, (c) coefficient of variation of speed, (d) mean normal force, (e) standard deviation of normal force, (f) mean muscle co-activation. The blue lines indicate CW trials and red lines indicate CCW trials. The dashed lines indicate the pre-catch trials and the solid lines indicate the catch trials. The error bars indicate one standard deviation about the mean.
<table>
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<tr>
<th>Dependent Measure</th>
<th>Factor</th>
<th>F</th>
<th>Prob &gt; F</th>
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<td>Mean speed</td>
<td>Speed</td>
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<td>Speed*Direction</td>
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<td>Speed*Feedback</td>
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<td>Feedback</td>
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<td>Feedback</td>
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</tr>
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<td></td>
<td>Speed*Direction</td>
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<td>Speed*Direction</td>
<td>$F_{2,18} = 2.978$</td>
<td>0.076</td>
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<td>Speed*Feedback</td>
<td>$F_{2,0.18.0} = 0.020$</td>
<td>0.980</td>
</tr>
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<td>Direction*Feedback</td>
<td>$F_{1,0.9.0} = 0.260$</td>
<td>0.622</td>
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<td>Speed<em>Direction</em>Feedback $F_{2,0.18.0} = 0.126$</td>
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<td>Mean muscle coactivation</td>
<td>Speed</td>
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<td>&lt; 0.001*</td>
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<td>Direction</td>
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<tr>
<td></td>
<td>Feedback</td>
<td>$F_{1,0.9.0} = 5.362$</td>
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<td>Speed*Direction</td>
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<td>Direction*Feedback</td>
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<td>0.232</td>
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<tr>
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<td>Speed<em>Direction</em>Feedback $F_{2,0.18.0} = 0.009$</td>
<td>0.991</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2: Summary of results for the five ANOVAs investigating the effect of feedback. The * indicates a significant effect.
2.6 Effect of Trial

In this analysis we did not include trial as a factor in the ANOVA. This decision was made based on the observation that learning was not evident. This also supports the idea that the patterns observed during movement were intended by the subjects, and not an artifact of trying to learn to perform the task. The task of turning a crank is generally a skill that is considered to be ‘over trained’; crank turning is an activity of everyday life. The slopes over trial for the dependent measures (See Figure 2-14 and 2-15) were extremely small. In addition, the variance accounted for by the linear regression was small, with $R^2$ never greater than 0.1.
Figure 2-14: Plots of the velocity dependent measures with respect to trial. The different dots represent different subject dependent measures at a specific trial. The solid lines are the lines of best fit for each speed condition: red indicates slow, blue indicates medium, and green indicates fast.
2.7 Discussion

This study aimed to investigate the detailed patterns of motion and force human subjects exhibit when performing a simple constrained-motion task, turning a circular crank. We found that subjects had little difficulty performing this simple task. Nevertheless, several unexpected results were observed.
2.7.1 Preferred ‘Comfortable’ Speed Trials

When subjects turned the crank without feedback at their preferred pace, they adopted a roughly constant speed with some fluctuations that varied systematically with position (Figure 2-2). Subjects’ preferred ‘comfortable’ speed was close to that of the medium target speed, which was investigated during the visual feedback trials. During some of the motion, subjects exerted negative (compressive) normal forces; in other regions, subjects exerted positive (tensile) forces. When normal force is positive, the handle is in tension. In this case the crank is analogous to a pendulum, which is statically stable; forces evoked by small deviations from a nominal position act to restore that position. When the normal force is negative, the handle is in compression. In this case the crank is analogous to an inverted pendulum, which is statically unstable; forces evoked by small deviations from a nominal position act to drive the limb further from that position. In general, we might expect destabilizing forces to be avoided, but that was not observed.

When negative normal force is exerted on a crank, the simple crank becomes another example of a stick with a fixed end. To stabilize this system subjects must generate a stiffness normal to the directed force. From the work of Rancourt and Hogan [Rancourt and Hogan, 2001] this stiffness is given by,

\[ K \geq \frac{F}{R} \]  

(2.5)

where \( F \) denotes the force exerted and \( R \) denotes the length of the stick (seen in Figure 2-16). Thus, the system could become unstable if the proper hand stiffness is not generated. The compressive forces we observed were always less than 5 N. The crank radius was 0.1029 m. This results in a minimum required stiffness for stability which is less than 48.6 N/m. Based on the measurements of Mussa-Ivaldi et al. [Mussa-Ivaldi et al., 1985] the stiffness of the hand was greater than 150 N/m. Instability of this magnitude could be compensated by hand stiffness.
2.7.2 Visual Feedback Trials

If the subjects attempted to learn the path defined by the constraint, and tried to move around the crank with constant velocity, due to the complexity of ‘closed-chain’ inertial dynamics, decreased time for sensory feedback, and signal-dependent motor noise, speed and force deviations should increase with increasing speed; this was observed. A single main effect of target speed on mean speed was detected. This showed that subjects could perform this undemanding task independent of the direction. Mean speed error was close to zero. Speed standard deviation increased significantly as target speed increased. This is consistent with the challenges of faster movements, resulting in increasing deviation from ideal behavior. Turning direction did not introduce differences.
More importantly, speed and force deviations should decline to zero as the movement slows. This was the reason we included such an extremely slow speed – 0.075 rev/sec, requiring 13.33 seconds to complete each revolution. At this speed, fluctuations due to inertial mechanics, feedback delays, and signal-dependent noise decline precipitously. For example, any inertial perturbations at the slow speed of 0.075 rev/sec are over 44 times smaller than at the medium speed of 0.5 rev/sec. For this reason, the speed coefficient of variation should have declined with speed; in fact, it increased. The speed coefficient of variation increased substantially at the slowest speed; it was almost double that of the medium speed movements (29% vs. 15%) and two-and-a-half times greater than that of the fast movements (29% vs. 12%). The trend of increasing coefficient of variation with decreasing speed can be seen in Figure 2-6c.

If the increased variability at slow speeds resulted from the forces exerted by the muscles, the variabilty must be an effect of either the neural commands sent to the muscles or the noise properties of the muscles themselves. Yet, current descriptions of muscle physiology cannot explain this behavior. Muscle noise has been found to be signal-dependent [Faisal et al., 2008, Harris and Wolpert, 1998, Jones et al., 2002, Salmond et al., 2016, Schmidt et al., 1979, Slifkin and Newell, 1999, Sternad et al., 2011] and the largest muscle activation would be expected at high speeds. At slow speeds lower muscle activation may be expected. If very few motor units, or even individual motor units, are activated a noise increase would be expected. However, EMG measurements showed that even in the slow speed condition, a full interference pattern was observed—indicating the combined activity of many motor units. This indicates that the increase in speed coefficient of variation is not the result of individual motor unit recruitment.

Target speed was found to be the only factor which significantly affected normal force mean and standard deviation. We predicted subjects would ideally exert a mean normal force with an average of zero. Post-hoc paired-sample t-tests detected a significant difference between target speed conditions. As expected there was a significant increase in force mean and standard deviation from the medium to the fast
target speed. An increase of normal force with target speed might be attributed to limitations of the biological controller. The fast target speed, 2 rev/s, was close to the upper limit at which humans can perform. Normal force fluctuations may increase because compensation for the high frequency and high magnitude inertial forces exceed the response time of the neuro-mechanical periphery. This might account for the increase in force fluctuations at higher speeds (Figure 2-6e). However, this would be expected regardless of the control strategy used. Moreover, fast cyclic motions would evoke centrifugal forces. If not compensated, they might account for the substantially positive mean radial forces we observed in the fast condition (Figure 2-6d).

Most of the variation of force standard deviation with speed was due to the fast target speed. Interestingly, there was also a significant difference between the slow and medium target speed conditions. This observation is not consistent with subjects trying to move in the path defined by the constraint. Radial force standard deviation for the slow and medium target speeds, though statistically different, was essentially the same, consistent with what might be expected of an imperfect motion controller. However, mean normal force was less than zero at both speeds, and this effect reached statistical significance at the slowest target speed. Significant negative mean normal force, observed in the slow target speed condition, is especially intriguing. It cannot be reconciled with a motion controller attempting to maintain zero radial force. A deviation from zero radial force might contribute to robustness, i.e. the control strategy might be to ‘err on the side of stability’. However, that would imply a positive mean radial force, which is stabilizing. Negative mean normal force is destabilizing. Why our subjects chose this strategy is at present unclear.

If knowledge of the path defined by the constraint increases with experience, and subjects tried to move with minimal normal forces, then the mean normal forces would be expected to decrease with turning experience. However, this was not observed (see Figure 2-15). Observed normal forces may be intended by the neuromotor system; suggesting they do not represent errors which the biological controller is trying to minimize.

The passive simulations of the crank provide insight to the effects of inertial dy-
namics on the task. Most importantly the simulations demonstrate that inertial effects can not result in negative normal forces during this planar motion (see Figure 2-10). Secondly, the position dependent variations in velocity and radial force were not just a result of inertial mechanics. The effect of inertial mechanics would predict the same position dependent velocity and force pattern regardless of speed or direction. However, this was not observed. The model also quantitatively demonstrated that the slow speed condition is quasi-static. The maximum radial force in the slow passive simulations was 0.11 N, while a peak force of 5 N and 79 N was observed in the medium and fast cases, respectively. There was a tremendous decrease in the normal force during the slow condition. The normal force in the slow speed was 2.2% of that observed in the medium case and 0.01% of that observed in the fast case.

2.7.3 Effect of Visual Feedback

The visual feedback provided to the user on the computer display was not the cause of the observed speed irregularities. Target speed was the only significant factor that affected the mean speed, standard deviation of speed, and standard deviation of normal force. Multiple main effects or interactions were detected for the coefficient of variation of speed and the mean normal force. Nonetheless, in Figure 2-13c and 2-13d, it is clear that the effect of feedback does not change the observations of interest; increased coefficient of variation and negative compressive normal forces were both observed at slow speeds. Finally, co-activation was observed regardless of visual feedback. As discussed before, we reasonably expected co-activation to be minimized and to decline with speed. However, this was not observed.

2.7.4 Comparison to Previous Crank Studies

Russel and Hogan [Russell and Hogan, 1989] investigated a loaded crank with a length of 20.3 cm. Their crank was oriented vertically. They instructed subjects to move from the bottom to the top of the crank, pause briefly, and then return to the bottom. Subjects repeated this motion for 20 seconds at a comfortable speed. Consistent with
our findings, they observed significant positive and negative radial forces during the partial rotation of the crank. Interestingly, they observed differences between turning directions. However, as the crank was oriented vertically, it was unclear if these effects were a result of subjects compensating for gravity.

A study that more closely resembles our work was performed by Ohta et al. [Ohta et al., 2004]. Ohta et al.’s study used a 0.05 m radius crank (approximately half the radius used for our study), imposed constant damping 0.3 N-m/s/rad, and instructed subjects to turn for one revolution, at their preferred speed in the clockwise direction. Each subject was instructed to start at different initial crank positions. Large variations in velocity were observed because subjects completed only one revolution, starting and ending at rest. For this reason, velocity and reaction forces cannot be directly compared to our study. In both studies, subjects exerted both positive and negative normal forces over the course of a single revolution. In our trials, we observed much smaller radial forces at “preferred” speeds than reported by Ohta et al. In their study, subjects exerted normal forces which were greater than 10 N, while in our study subjects exerted forces around 4 N (in the preferred ‘comfortable’ speed condition). This difference in observed normal force might be expected to result from the difference in crank damping – our experiment had no damping and negligible friction.

In the study performed by Koeppen et al. [Koeppen et al., 2017], subjects turned a virtual crank (radius 0.1 m) at a speed of 0.075 rev/s – the same as the slow condition in our study. In our study, the normal force observed at slow speeds was consistent with their observations. Positive and negative normal forces of less than 5 N were observed. However, when the limb was in positions where a muscle could not contribute to the task, that muscle was not deactivated. This indicates that subjects did not minimize effort to perform the task. They also unexpectedly observed that subjects produced different muscle activation in different directions. Consistent with their observations, we also observed significant effects of direction and significant interactions between speed and direction, which were not anticipated. We observed a significant effect of direction for the coefficient of variation and the muscle co-
activation. Additionally, we detected an interaction between speed and direction for
the standard deviation of speed, coefficient of variation of speed, mean normal force
and muscle co-activation. At present, the reason for these interaction terms has not
been explained. A controller trying to move along the path defined by the constraint,
with zero normal force, cannot explain why at slow speeds a significant effect of
direction would be observed.

2.8 Conclusion

In this chapter, we tested several hypotheses, based on the assumption that sub-
jects tried to generate an approximately circular hand path, at constant speed, with
zero normal force. Observations in the preferred ‘comfortable’ speed condition were
consistent with this hypothesis. In the preferred ‘comfortable’ speed condition, the
mean normal force was not significantly different from zero. However, in this study
both extremely slow and extremely fast speeds were investigated beyond the medium
speed (close to the preferred ‘comfortable’ speed). At high speeds where subjects
approached the limits of the neuromuscular system, subjects were expected to strug-
gle; this was observed. In the fast condition, the standard deviation of speed, the
standard deviation of radial force, and the mean radial forces all were larger positive
values than observed in the medium speed conditions. Interestingly, at extremely
slow speeds the coefficient of variation in speed increased. This was not expected as
all effects which might cause variability decrease as speed decreases: inertial effects,
feedback delays and motor noise. Surprisingly, at this extremely slow speed subjects
exerted a statistically significant negative mean normal force. To our knowledge these
observations at slow speeds, negative normal force and increasing coefficient of vari-
atation, cannot be explained by biomechanics or muscle physiology – suggesting these
observations are a result of the control strategy used by humans.
Chapter 3

Crank Model

In the previous chapter, we qualitatively reported that position dependent error was observed in the tangential velocity and normal force. These patterns were consistent across subjects. The ‘conventional dependent measures’ were not developed to investigate patterns with respect to crank position. In fact, the cyclical correlation between adjacent positions, in combination with the repeated measures design, violates the assumptions of many standard statistical approaches. To determine if these effects were a result of poor inertial compensation, the passive dynamics of the system were simulated. Nonetheless, the simulated passive velocity and force profiles could not explain the patterns exhibited by the subjects. We wondered if these unexplained observations resulted from neural activity or biomechanics. This question motivated the computation of the zero-force trajectory.

3.0.1 An approach to Model Physical Interaction

Physical interaction with a constraint is inherently bidirectional. Forces exerted on the constraint are equal and opposite to the forces exerted on the hand. One way to describe the dynamics of interaction is by using a mechanical impedance operator $Z\{\cdot\}$ [Hogan, 1985a, Hogan, 1985b]. A general actuator model based on nonlinear equivalent networks was proposed by Hogan [Hogan, 2014]. There are two ways to model the bidirectional interaction dynamics, via the Thévenin (impedance in series)
or Norton (impedance in parallel) equivalent network (see Figure 3-1). Hogan demonstrated that although muscles are commonly perceived as controllable force generators (Thévenin), the requirement to identify forward path and interaction dynamics from external measurements favors the Norton form. Thus, the Thévenin approach should not be used. Modeling the muscle as a motion-source (Norton) equivalent network still allows for force to be specified as the output variable; however, the mechanical impedance determines the output force in response to deviations from a nominal motion. The Norton equivalent network is unambiguously identifiable. This is because the Norton source is a zero of the interaction-port operator, whereas the Thévenin source is not. Consequently, in this work we have used the Norton equivalent network approach. A broad class of systems, not just the human limb, can be modeled using the Norton form in order to gain insight about how the system is controlled.

Using the Norton approach, the force, \( F(t) \), time function can be computed from the displacement time function \( \Delta x(t) \), \( F(t) = Z\{x(t)\} \). The displacement is defined as \( \Delta x(t) = x_0(t) - x(t) \), where \( x(t) \) is the actual hand position, and \( x_0(t) \) is a zero-force trajectory. In this simple case, \( f = k(x_0 - x) \), the impedance, \( Z\{\cdot\} \), is a linear stiffness. The zero-force trajectory is the path the hand would take if zero force impeded motion, i.e. the constraint was removed. If a model for the interaction dynamics is assumed, then it can be used to ‘subtract off’ or ‘peel back’ peripheral biomechanics, and reveal underlying neural influence expressed in terms of motion – the zero-force trajectory.
3.0.2 Damper Connectivity

A model of the upper limb impedance often consists of a stiffness and a damping term. However, unlike stiffness, which is usually defined as a displacement relative to the zero-force trajectory, damping can be defined relative to either the zero-force trajectory, or to a constant velocity – usually zero. The choice of the damping term fundamentally changes the transfer function. To understand this idea, we can look at two simple linear mass spring damper systems.

![Diagram of a mass spring damper system with damping](image)

Figure 3-2: Diagram of a mass spring damper system with damping (a) relative to zero velocity, and (b) relative to the zero-force trajectory velocity.

The damping relative to zero velocity is displayed graphically in Figure 3.2a. From the equation of motion,
\[ m\ddot{x} = k(x_0 - x) + b\dot{x} \quad (3.1) \]

we can take the Laplace transform and solve for the transfer function,

\[ \frac{X(s)}{X_0(s)} = \frac{k}{ms^2 + bs + k}. \quad (3.2) \]

The damping relative to the zero-force trajectory velocity is displayed graphically in Figure 3.2b. From the equation of motion,

\[ m\ddot{x} = k(x_0 - x) + b(\dot{x}_0 - \dot{x}) \quad (3.3) \]

we can take the Laplace transform and solve for the transfer function,

\[ \frac{X(s)}{X_0(s)} = \frac{bs + k}{ms^2 + bs + k}. \quad (3.4) \]

This example shows that when the damping is defined relative to the zero-force trajectory, a zero is added to the system. McIntyre and Bizzi [McIntyre and Bizzi, 1993] found that damping relative to the zero-force trajectory enhances the control following ability of the limb during single-joint movements. This improvement is consistent with the expected effect of adding a zero to the system. Thus, it also improves reference tracking for two-joint movements. In addition, muscle spindles are attached to muscle in parallel and actively provide feedback about the change in position and velocity of the muscle. Damping relative to the zero-force trajectory is consistent with muscle spindle physiology and was used in the subsequent calculations.

### 3.0.3 Estimates of Limb Impedance

The limb impedance is required to compute the zero-force trajectory. However, to the author’s knowledge, two joint upper limb impedance has not been measured during a task requiring physical interaction with a constraint. Several studies have investigated limb impedance: in a single joint during static posture [Cannon and Zahalak, 1982, Lacquaniti et al., 1982], in two-joints during static posture [Mussa-
Ivaldi et al., 1985, Dolan et al., 1993, Osu and Gomi, 1999], in two joints during movement [Tsuji et al., 1995, Gomi et al., 1992, Bennett et al., 1992], and in two joints during physical interaction (catching a ball) [Lacquaniti et al., 1993].

**Stiffness**

Mussa-Ivaldi et al. [Mussa-Ivaldi et al., 1985] documented hand stiffness fields during two-joint arm posture. They conducted experiments where a robot manipulandum was used to apply small displacements to the hand. While the hand was held at the displaced position, the restoring force was measured before the onset of voluntary reaction. Then, they evaluated the conservative and non-conservative components of the hand’s elastic field. They determined that the hand stiffness was nearly curl free.

If a vector field has zero curl, numerically it can be represented by a matrix which is symmetric; thus, it has real orthogonal eigenvectors. This vector field can be represented graphically by an ellipsoid (see Figure 3-3), described by three parameters: the magnitude (area), shape (ratio of axes), and orientation (direction of the major axis). Interestingly, when they applied disturbances along a fixed predictable direction, subjects could increase the magnitude of the stiffness, but they were not able to change the direction or orientation of the stiffness ellipse.
Figure 3-3: Displays a simulation of constant joint stiffness. The joint stiffness has been computed from the Cartesian stiffness of the subject during hand posture in position 'reference' (Re). With all the joint stiffness terms kept constant, the model of the arm was displaced to 11 work space positions and the corresponding Cartesian stiffnesses were then computed. The five positions where the stiffness was measured were included: (D, distal; P, proximal; Ri, right; L, left). Figure from Mussa-Ivaldi et al. [Mussa-Ivaldi et al., 1985].

Damping

When investigating single joint movements, Cannon and Zahalak [Cannon and Zahalak, 1982] found that the muscle torque ranged from 3 to 30 N-m, the joint damping, $B$, was proportional to the stiffness, $K$, such that $B = \beta K$. The mean value of the proportionality constant, $\beta$, was 0.045 s for the flexors and 0.05 s for the extensors. Lacquaniti et al. [Lacquaniti et al., 1982] investigated the response of the elbow to small pseudo-random perturbations during two paradigms – a ‘do not resist’ and a ‘resist’ case. They observed $\beta$ values as small as 0.025 s in the ‘resist’ case, and as large as 0.125 s in the ‘do not resist’ case. Dolan [Dolan et al., 1993] investigated two joint limb stiffness and found that the range of the $\beta$ term was slightly larger, 0.011 s to 0.277 s.
3.0.4 Hypothesis

In unconstrained motion clear evidence of coincidence between velocity and curvature extrema has been observed. Moreover, the synchrony of velocity and curvature was even observed in isometric tasks where subjects exerted forces to manipulate a virtual object. This observation strongly indicates that this relationship is a result of neural commands. In our study, we instructed subjects to move with a constant velocity. The subjects moved around a circular constraint; thus, the hand path was required to maintain a constant curvature. If the curvature was constant, the two-thirds power law would predict a constant velocity, and subjects’ ability to move with a constant velocity should improve at slower speeds where dynamics, feedback delays, and muscle noise become insignificant. In fact, systematic variations of velocity were observed. It would be interesting if this velocity and curvature relationship arose from the zero-force trajectory, especially when the hand path was confined to move with constant curvature. **Hypothesis 1:** The zero-force trajectory will exhibit synchrony of speed and curvature extrema.

A different control input would be required at fast speeds in order to compensate for the effects of inertial dynamics. However, when the movement was well within the limits of the human controller, at medium and slow speeds, the orientation of the zero-force trajectory would be expected to be the same. **Hypothesis 2:** The orientation of the zero-force trajectory covariance ellipse will be independent of direction, when turning at slow and medium speeds.

3.1 Methods

3.1.1 Inertial Parameter Estimation

Knowledge of the subject’s inertia is required to develop a model and simulate the system. Inertia parameters were estimated based on the results of the cadaver studies of Dempster [Miller and Nelson, 1973, Plagenhoef, 1971]. The upper arm, forearm, hand, and forearm plus hand were denoted by 1, f, h, and 2 respectively. Each of the
body segments was described by the following parameters: length, $l$, mass, $m$, inertia, $I$, radius of gyration, $k_a$, and center of mass, $c$. The length $l_f$ was the distance from the elbow to the center of the fist; the length $c_h$ was the distance from the center of the wrist to the center of the hand. Link 2 was considered a combination of the forearm and hand. The hand was assumed to be a point mass at the end of the forearm.

For the upper arm,

$$m_1 = 0.028 \times \text{(Body Mass)} \quad (3.5)$$

$$c_1 = 0.436l_1 \quad (3.6)$$

$$k_1 = 0.322l_1 \quad (3.7)$$

$$I_1 = m_1k_1^2 \quad (3.8)$$

For the forearm,

$$m_f = 0.016 \times \text{(Body Mass)} \quad (3.9)$$

$$c_f = 0.436(l_f - c_h) \quad (3.10)$$

$$k_f = 0.322(l_1 - c_h) \quad (3.11)$$

For the hand,

$$m_h = 0.006 \times \text{(Body Mass)} \quad (3.12)$$

In order to combine the hand and forearm,

$$m_2 = m_f + m_h \quad (3.13)$$

$$c_2 = \frac{m_f c_f + l_f}{m_f + m_f} \quad (3.14)$$

$$l_2 = m_f k_f^2 + m_h l_f^2 \quad (3.15)$$

Crank parameters are reported in Table 3.1, and limb parameters are reported in Table 3.2.
Table 3.1: Crank inertia properties.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>CG location</td>
<td>$c_c$</td>
<td>6.48</td>
<td>cm</td>
</tr>
<tr>
<td>Inboard Mass</td>
<td>$m_c$</td>
<td>0.7005</td>
<td>kg</td>
</tr>
<tr>
<td>Inboard Inertia</td>
<td>$I_c$</td>
<td>$0.778 \times 10^{-3}$</td>
<td>kg-m$^2$</td>
</tr>
<tr>
<td>Outboard Mass</td>
<td>$m_h^+$</td>
<td>0.5063</td>
<td>kg</td>
</tr>
</tbody>
</table>

Doeringer [Doeringer, 1999] represented each of the subjects’ parameters as a vector of inertial parameters in order to estimate the mean and median vector. Dividing all vectors by their length and taking the dot products, he could determine which subject was closest to the mean and median. He found that subject number 1 was the mean and median subject based on inertial parameters. Thus, for representative simulations subject 1 will often be used.

Table 3.2: Human limb inertia properties for this experiment.

<table>
<thead>
<tr>
<th>Subject</th>
<th>$l_1$ (cm)</th>
<th>$c_1$ (cm)</th>
<th>$m_1$ (kg)</th>
<th>$I_1$ (kg-m$^2$)</th>
<th>$l_2$ (cm)</th>
<th>$c_2$ (cm)</th>
<th>$m_2$ (kg)</th>
<th>$I_2$ (kg-m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36.8</td>
<td>16.1</td>
<td>2.23</td>
<td>0.0313</td>
<td>37.5</td>
<td>23.6</td>
<td>2.25</td>
<td>0.148</td>
</tr>
<tr>
<td>2</td>
<td>36.2</td>
<td>15.8</td>
<td>2.03</td>
<td>0.0276</td>
<td>36.8</td>
<td>23.6</td>
<td>2.10</td>
<td>0.137</td>
</tr>
<tr>
<td>3</td>
<td>32.4</td>
<td>14.1</td>
<td>2.20</td>
<td>0.0239</td>
<td>33.0</td>
<td>20.7</td>
<td>2.23</td>
<td>0.114</td>
</tr>
<tr>
<td>4</td>
<td>36.2</td>
<td>15.8</td>
<td>2.61</td>
<td>0.0354</td>
<td>38.1</td>
<td>23.5</td>
<td>2.55</td>
<td>0.167</td>
</tr>
<tr>
<td>5</td>
<td>34.9</td>
<td>15.2</td>
<td>2.03</td>
<td>0.0257</td>
<td>35.6</td>
<td>22.7</td>
<td>2.10</td>
<td>0.128</td>
</tr>
<tr>
<td>6</td>
<td>29.2</td>
<td>12.7</td>
<td>2.35</td>
<td>0.0208</td>
<td>33.7</td>
<td>20.8</td>
<td>2.35</td>
<td>0.123</td>
</tr>
<tr>
<td>7</td>
<td>37.5</td>
<td>16.3</td>
<td>1.97</td>
<td>0.0287</td>
<td>37.5</td>
<td>24.0</td>
<td>2.05</td>
<td>0.140</td>
</tr>
<tr>
<td>8</td>
<td>34.3</td>
<td>15.0</td>
<td>1.97</td>
<td>0.0240</td>
<td>36.8</td>
<td>23.7</td>
<td>2.05</td>
<td>0.135</td>
</tr>
<tr>
<td>9</td>
<td>36.8</td>
<td>16.1</td>
<td>2.03</td>
<td>0.0286</td>
<td>38.7</td>
<td>24.9</td>
<td>2.10</td>
<td>0.152</td>
</tr>
<tr>
<td>10</td>
<td>34.3</td>
<td>15.0</td>
<td>2.29</td>
<td>0.0279</td>
<td>33.7</td>
<td>21.1</td>
<td>2.30</td>
<td>0.121</td>
</tr>
</tbody>
</table>
3.1.2 Computing the Zero-Force Trajectory

The arm was modeled as a two-link planar manipulator, with no gravitational effects. The shoulder joint location was modeled as a fixed point, as the thorax was assumed to be stationary. The two link manipulator dynamics coupled to the crank is more rigorously explained in Appendix B. This approach is the same as used by Ohta et al. [Ohta et al., 2004]. A diagram of the sign convention used to model the coupled crank and limb system is displayed in Figure 3-4.

![Figure 3-4: Model of crank rotation task which displays the sign convention and notation used in the computations.](image)

A model of the closed-chain system could be constructed from the sum of moments acting on the crank (Equation 3.16), the summation of moments about the shoulder (Equation 3.17),

80
\[ I\ddot{\theta} + b_c\dot{\theta} = re^T F \] (3.16)

\[ M\ddot{q} + h = \tau - J^T F, \] (3.17)

\[ \ddot{x} = J\ddot{q} + J\dot{q} = r(\ddot{\theta}e - \dot{\theta}^2n), \] (3.18)

and the kinematic relationship that equated the acceleration at the handle to the acceleration at the hand (Equation 3.18). Parameters comprising these equations include the mass matrix, \( M \), the centrifugal and Coriolis forces, \( h \), the Jacobian relating unconstrained differential arm motions to hand motions, \( J \), the radius of the crank, \( r \), the damping of the crank, \( b_c \), the inertia, \( I \), the force on the handle, \( F \), the normal unit vector, \( n \), the tangential unit vector, \( e \), and the joint torque, \( \tau \).

A relationship for \( F \) was found by combining Equations 3.16, 3.17, and 3.18.

\[ F = \left\{ JM^{-1}J^T + r^2I^{-1}ee^T \right\}^{-1}\left\{ JM^{-1}(\tau - h) + J\ddot{q} + r\dot{\theta}(\dot{\theta}n + b_cI^{-1}e) \right\} \] (3.19)

Muscle force production is a complex function of many factors, however its dominant behavior can be well described by a function of muscle length and its rate of change [Joyce et al., 1969, Rack and Westbury, 1969]. Thus, the muscles were modeled as a linear spring and viscous damping element with common motion [Hogan, 1984a]. To implement this on a two joint manipulator, the same muscle model previously used by Flash [Flash, 1987] was implemented. However, in this case we used a damping term which was defined relative to the zero-force trajectory. The single joint and multi-joint muscles are graphically displayed in Figure 3-5.
The joint torque is defined by,

\[ \tau = K(q_0 - q) + B(\dot{q}_0 - \dot{q}) \]  \hspace{1cm} (3.20)

The stiffness in units of N-m/rad is defined as

\[ K = G \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \]  \hspace{1cm} (3.21)

The viscous damping in units of N-m-s/rad is defined as
\[ B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \] (3.22)

The \( K_{11} \) and \( B_{11} \) terms are the net shoulder joint stiffness and damping, the \( K_{12}, B_{12}, K_{21}, \) and \( B_{21} \) are the two-joint parameters, and the \( K_{22} \) and \( B_{22} \) terms describe the elbow parameters. The term \( G \) is a gain term. The values for the joint stiffness and damping used were consistent with that of Flash [Flash, 1987], such that the matrix \( K \) is the joint stiffness matrix at the zero-force position,

\[ K = G \begin{bmatrix} 29.5 & 14.3 \\ 14.3 & 39.3 \end{bmatrix} \] (3.23)

and \( B \) is the joint damping matrix at the zero-force position.

\[ B = \beta K \] (3.24)

A gain, \( G \), of 1.0 was used in the slow and medium cases, and a gain of 2.0 was used in the fast case. The damping was derived from the stiffness by the multiplication of a constant factor, \( \beta \), which was 0.05 s for the slow and medium cases, and 0.1 for the fast cases. The \( \beta \) term is in units of time consistent with a first-order model of muscle impedance.

Substituting 3.20 into Equation 3.19, the equation can be manipulated to solve for \( \dot{q}_0 \)

\[ \dot{q}_0 = B^{-1}\left[MJ^{-1}\left[\left\{JM^{-1}J^T + t^2I^{-1}ee^T\right\}F - J\ddot{q} - r\dot{q}\left(\dot{\theta}(\theta + b_n) + b_nI^{-1}e\right)\right] + K(q_0 - q)\right] + \dot{q} \] (3.25)

Integrating Equation 3.25 allows for the computation of the zero-force trajectory that would result from a prescribed position, velocity, acceleration, and force.

The velocity and force signals were filtered with a second order Butterworth filter using a cutoff frequency of 10 Hz, except in the slow tangential force condition. The
tangential force in the slow condition was small in magnitude. At slow speeds, a large number of samples with a magnitude close to the resolution of the sensor were observed. This results in problematic step changes in the force measurements. To eliminate this artifact, the tangential force in the slow condition was filtered with a cutoff frequency of 0.5 Hz, far beyond the turning frequency of the slow task (0.075 rev/sec).

3.1.3 ‘Conventional’ Dependent Measures

Consistent with Chapter 2, we computed the speed dependent measures for the zero-force trajectory. The analysis of these parameters was considered the ‘conventional’ dependent measure analysis. These dependent measures are reported in this section for the zero-force trajectory: mean speed, standard deviation of speed, and coefficient of variation of speed.

3.1.4 Velocity and Curvature Relationship

In unconstrained motions, the relationship between curvature and velocity has been well documented in human arm trajectories. We wanted to test if this relationship was evident in the zero-force trajectory. To compute the velocity and curvature of the zero-force trajectory, the zero-force trajectory was transformed into Cartesian coordinates, $x_0$ and $y_0$. Using the methods of Dohrmann et al. [Dohrmann et al., 1988] the derivatives of the Cartesian position of the zero force trajectory were computed. Smoothing parameters of $0.03, 10^{-6}, 10^{-11}$ were used for the slow, medium, and fast trials, respectively. These derivatives were used to compute tangential speed, $V_0$, and curvature, $\kappa_0$.

\[
V_0 = \sqrt{\dot{x}_0^2 + \dot{y}_0^2} \tag{3.26}
\]

\[
\kappa = \frac{\dot{x}_0 \ddot{y}_0 - \dot{y}_0 \ddot{x}_0}{(\dot{x}_0^2 + \dot{y}_0^2)^{3/2}} \tag{3.27}
\]
Positive curvature would be consistent with a positive z rotation, according to
the right-hand rule. However, for the ease of computation the inward direction has
been defined as the positive curvature, such that the curvature has been multiplied
by minus one in the clockwise condition.

To determine if the valleys in velocity correspond to the peaks in curvature, the
following steps were taken. Local minima in tangential velocity, and local maxima in
curvature were identified. For each local minimum in tangential velocity, the nearest
local maximum in curvature was found. The signed distance between the two peaks
was normalized by the target speed, and considered to be the interpeak interval, \( \Delta \).
\( \Delta \) was computed for each subject at each condition, and the data were pooled across
trials. The 95% confidence interval for each subject in each condition was computed.

### 3.1.5 Sensitivity to Impedance Assumptions

To the knowledge of the author, limb impedance measurements during physical in-
teraction with a constraint have not been reported. Thus, in this study, the stiffness
values were based on unconstrained arm stiffness and damping measurements. Con-
sequently, the impedance parameters were varied to understand if the results were
sensitive to the impedance values used to compute the zero-force trajectory. The
impedance parameters (the gain term for the stiffness, \( G \), and the proportional damp-
ing term, \( \beta \)) were varied. When the gain and damping terms changed, the zero-force
trajectory was expected to change. However, we were interested in understanding if
the velocity curvature relationship was observed even when the zero-force trajectory
changed shape. The 95% confidence interval for \( \Delta \) was computed when the \( G \) and \( \beta \)
term were varied over a 3:1 range.

### 3.1.6 Covariance Ellipse Orientation

We wanted to understand if the zero-force trajectory consistently changed orientation
as a function of speed and direction. The zero-force trajectory covariance can be
computed,
\[
\text{cov}(x_0, y_0) = \frac{1}{N} \sum_{i=1}^{N} (x_{0,i} - \mu_{x,0})(y_{0,i} - \mu_{y,0})
\] (3.28)

where \(x_0\) and \(y_0\) are the Cartesian zero-force trajectory points, \(\mu_{x,0}\) and \(\mu_{y,0}\) are the mean Cartesian zero-force trajectory points, and \(N\) is the number of samples. This results in a matrix which is of the form,

\[
C = \begin{pmatrix}
\text{cov}(x_0, x_0) & \text{cov}(x_0, y_0) \\
\text{cov}(y_0, x_0) & \text{cov}(y_0, y_0)
\end{pmatrix}
\] (3.29)

where \(C\) is the covariance matrix. The eigenvectors of the covariance matrix were computed in order to determine the major and minor axes of the covariance ellipse. The dependent measure was the angle from the major axis. The covariance ellipse angle was always defined relative to the 3 o’clock position on the crank – consistent with the previous definition of crank angle.

As discussed in Chapter 2, the first trial for each condition was excluded. In all subsequent trials, the first 1.5 seconds were discarded to remove the transient effects induced by the initial condition specified for numerical integration. In this analysis, only complete revolutions were included.

To quantify the influence of speed and direction on the dependent measure, covariance major axis angle, a linear mixed model was employed (see Equation 3.30). The model was tested using analysis of variance (ANOVA). This was the same type of ANOVA used in Chapter 2 to test the dependent measures for the visual feedback trials.

The linear model which represents the observed dependent measure \(Y_{i,j,k}\) was expressed,

\[
Y_{i,j,k,l} = \mu_T + \alpha_j + \beta_k + \gamma_l + (\alpha\beta)_{j,k} + (\alpha\gamma)_{j,l} + (\beta\gamma)_{k,l} + (\alpha\beta\gamma)_{j,k,l} + E_{i,(j,k,l)}
\] (3.30)

where the grand mean is \(\mu_T\), the fixed effect of speed is \(\alpha_j\), where \(j\) is an index from 1 to 3, the fixed effect of direction is \(\beta_k\), were \(k\) is an index from 1 to 2, the random
effect of subject is $\gamma_l$, where $l$ is an index from 1 to 10, and the stochastic sampling effect $E_{i,j,k,l}$, where $i$ is an index from 1 to 22 (representing the multiple trials). All statistical analyses were performed using SPSS statistical software package (SPSS Inc., Chicago IL); the significance level was set to 5%.

### 3.2 Results

#### 3.2.1 ‘Conventional’ Dependent Measures

The bidirectional interaction between the forces exerted on the constraint and the force observed at the hand can be described by a mechanical impedance operator. A model of upper limb impedance was assumed, and the zero-force trajectory was computed. The effect of speed and direction observed in the hand velocity was also observed in the zero-force velocity as seen in Figure 3-7. The zero-force trajectory mean and standard deviation of speed increased with target speed. The zero-force trajectory speed coefficient of variation increased substantially at the slowest speed; it was more than double that of the medium speed movements (170% vs. 53%) and nearly five times greater than that of the fast movements (170% vs. 35%).
3.2.2 Velocity Curvature Relationship

Visually, the velocity peaks corresponded well to the curvature valleys (see Figure 3-9). In order to quantify this effect, $\Delta$ was computed. The confidence interval
plots of Figure 3-10 display the $\Delta$ parameter, which often slightly led or lagged zero. Nonetheless, all 95% confidence intervals for the $\Delta$ parameter were less than 3% of a revolution from zero. Within the precision of this computation, the curvature peaks correspond to the velocity valleys. This coincidence of extrema is consistent with observations in unconstrained movements and indicates the position dependent velocity and normal force fluctuations are a result of neural commands.
Table 3-8: A representative trial from subject 1 in each of the speed and direction conditions: zero-force trajectory (variable color line), path defined by the constraint (black dashed lines), covariance major axis direction (long red line), covariance minor axis direction (short red line).

Figure 3-8: A representative trial from subject 1 in each of the speed and direction conditions: zero-force trajectory (variable color line), path defined by the constraint (black dashed lines), covariance major axis direction (long red line), covariance minor axis direction (short red line).
Figure 3-9: Plots of a velocity and curvature for a single trial, and histograms of the $\Delta$ parameter for all trials preformed by subject 1 at each of the three speed conditions in the CW direction.
Figure 3-10: Plots of the 95% confidence interval for the $\Delta$ parameter for each subject at each condition. $\Delta$ is the error in time between peaks normalized by the target turning speed. The deviation from zero is less than 3% of a revolution in all cases.
3.2.3 Sensitivity to Impedance Assumptions

In this work, we assumed a linear time-invariant first-order model of impedance with damping proportional to the assumed stiffness. We expected that if we varied the impedance the zero-force trajectory would change. We varied the gain for the stiffness term, $G$, (slow and medium $[0.5, 1, 1.5]$, and fast $[1, 2, 3]$) and the proportionality constant $\beta$, (slow and medium $[0.025 \text{ s}, 0.05 \text{ s}, 0.75 \text{ s}]$, and fast $[0.05 \text{ s}, 0.1 \text{ s}, 0.15 \text{ s}]$) used to specify the damping. However, the peak coincidence between the curvature and velocity extrema was still observed, even when the zero-force trajectory changed shape. The 95% confidence interval for the $\Delta$ parameter was always less than 4% of a revolution from zero (see Figure 3-11 and 3-12). This indicates that the observation of peak coincidence between the curvature and velocity is not sensitive to the particular values of stiffness and damping.

3.2.4 Covariance Ellipse Orientation

The major axis angle of the covariance ellipse was computed to determine if the zero-force trajectory changed orientation with respect to speed or direction. The ANOVA plot of the mean major axis angle for each condition is plotted in Figure 3-13. A significant effect of direction was detected, while neither an effect of speed nor an interaction between speed and direction were detected. A summary of the results of the ANOVA is presented in Table 3.3. The zero-force trajectory, which is a result of neural commands significantly changes with turning direction.
Figure 3-11: Representative plots of the resulting zero force trajectory, velocity, curvature and \( \Delta \), when the stiffness gain term \( G \) is varied. These plots are from subject 1 in the slow CW condition.
<table>
<thead>
<tr>
<th>Direction</th>
<th>Slow</th>
<th>Medium</th>
<th>Fast</th>
</tr>
</thead>
<tbody>
<tr>
<td>CW</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CCW</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3-12: Plots of the 95% confidence interval for the \( \Delta \) parameter for each subject at each condition. The stiffness gain term \( G \) and the proportional damping term \( \beta \) were multiplied by a factor of 0.5, 1, or 1.5. In all cases the 95% confidence interval is far less than 4% of a revolution. The dashed lines designate the separation between subjects. Within a group of confidence intervals for a given subject, the order of the gains is listed in the legend of the CW slow plot.
Figure 3-13: Dependent measures mean angle of the major covariance ellipse. Error bars indicate the standard deviation between subjects. The only significant factor was the main effect of direction.

<table>
<thead>
<tr>
<th>Dependent Measure</th>
<th>Factors</th>
<th>$F$</th>
<th>Prob &gt; $F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean speed</td>
<td>Speed</td>
<td>$F_{2,0.18} = 0.804$</td>
<td>0.463</td>
</tr>
<tr>
<td></td>
<td>Direction</td>
<td>$F_{1,0.9} = 297.029$</td>
<td>&lt; 0.001*</td>
</tr>
<tr>
<td></td>
<td>Speed*Direction</td>
<td>$F_{2,0.18} = 2.806$</td>
<td>0.087</td>
</tr>
</tbody>
</table>

Table 3.3: Summary of results for the ANOVA. The * indicates a significant effect.

3.3 Discussion

In this chapter, we wanted to look beyond the dependent measures, speed and normal force. We computed the zero-force trajectory by using a model of the limb to ‘peel back’ peripheral biomechanics, and ‘reveal’ the consequences of underlying neural commands, expressed in terms of motion. This approach is mathematically similar to that of the ‘virtual trajectory’ in the equilibrium point hypothesis [Bizzi et al., 1982, Bizzi et al., 1984, Feldman, 1986]. However, unlike the virtual trajectory, we do not suggest that the CNS encodes this quantity. Instead, the zero-force trajectory is
a construct based on the measured force and motion, in combination with peripheral biomechanics. Computing the zero-force trajectory in the Norton form, to better understand how a system is controlled, is an approach which could be applied to a broad class of systems, not just the human limb.

3.3.1 ‘Conventional’ Dependent Measures

The dependent measures (mean, standard deviation, and coefficient of variation) which were computed for hand speed in Chapter 2, were also computed for the zero-force speed. The same trends were observed in the zero-force trajectory speed dependent measures as were observed in the hand speed. This is especially interesting as the increased coefficient of variation at slow speeds was also observed in the zero-force trajectory – yet, larger in magnitude. Computing the zero-force trajectory from the hand path and reaction forces requires inverting the system dynamics. The system dynamics acts like a low pass filter. Inverting a low pass filter increases the variability in the signal. This further supports the observation that peripheral biomechanics were not the cause of the increased coefficient of variation at slow speeds.

3.3.2 Velocity Curvature Relationship

A coincidence of velocity and curvature extrema has been observed in unconstrained reaching motions [Abend et al., 1982], in isometric tasks where subjects managed force to move a virtual point [Massey et al., 1992], and in monkey motor cortical activity [Schwartz, 1994]. This coincidence is believed to be imposed by the neurological controller. In our study, subjects were instructed to move with a constant velocity while the constraint confined their hand path to a circle. However, the hand tangential velocity was not constant. In fact, there were consistent position dependent speed fluctuations. We tested Hypothesis 1: The zero-force trajectory will exhibit coincidence of speed and curvature extrema. Strikingly, when the curvature and tangential velocity of the zero-force trajectory were computed, coincidence of extrema was observed. The representative plot of a single cycle of velocity and curvature can
be seen in Figure 3-9. The change in crank cycle between corresponding extrema was computed. The confidence interval plots of Figure 3-10 show that the $\Delta$ parameter often slightly led or lagged zero. Remarkably, all 95% confidence intervals for the $\Delta$ parameter were less than 3% of a revolution from zero. These results support hypothesis 1. This strongly indicates that the coincidence of velocity and curvature extrema is a consequence of neural commands.

The zero-force trajectory was estimated using a model of the neuromuscular dynamics which assumed the system was time-invariant, first-order, and linear. These assumptions are demonstrably incorrect, yet nevertheless serve as a workable approximation. At the present time, to the author’s knowledge, upper limb impedance has not been measured during constrained multi-joint action. Thus, the impedance used for this model was based on static limb stiffness and damping measurements. In order to implement these assumptions, we assumed a damping relative to the reference trajectory, a curl free hand stiffness, and a damping term which was proportional to the stiffness. Given these assumptions, the regularity of the patterns that emerge is striking. Unlike Gomi and Kawato’s [Gomi and Kawato, 1996] findings, our results estimated a zero-force trajectory ‘close’ to the actual hand path, consistent with the work of Won and Hogan [Won and Hogan, 1995]. A zero-force trajectory close to the actual hand path may indicate that the limb impedance estimate was too large. In order to understand if the results were sensitive to the choice of impedance parameters, the stiffness gain, $G$, and proportional damping constant, $\beta$, were each varied over a 3:1 range. As expected, the zero-force trajectory changed shape when the impedance parameters varied. Intriguingly, even when the impedance was varied, the change of crank position $\Delta$ between corresponding velocity and curvature peaks was still less than 4% of a revolution. This supports the observation that the synchrony of velocity and curvature extrema observed in the zero-force trajectory was not an artifact of our calculations.
3.3.3 Covariance Ellipse Orientation

We predicted that at slow and medium speeds the major axis direction of the zero-force trajectory covariance ellipse should not be significantly different. However, there was a single main effect of direction on the ellipse major axis angle; this result was not expected. The reason for this observation is currently unclear. Interestingly, direction dependence was also observed by Russell and Hogan [Russell and Hogan, 1989] during vertical crank turning and by Koeppen et al. [Koeppen et al., 2017] in slow horizontal crank turning.

3.4 Conclusions

The path defined by the constraint has constant curvature; thus, systematic patterns in velocity which result from the ‘two-thirds’ power law would not be expected. Nonetheless, in Chapter 2 we observed position dependent fluctuations in hand velocity and force. In this chapter, we assumed a model of peripheral biomechanics to computationally ‘peel back’ and ‘reveal’ consequences of underlying neural commands, in the form of motion. In Chapter 2, hand trajectory speed coefficient of variation increased at slow speeds. This was unexpected. At slow speeds all effects which increase variability – feedback delays, inertial effects, and muscle noise – decrease. However, in this chapter, when the biomechanics were subtracted to compute the zero-force trajectory, the same unexpected increase in variability at slow speeds was still observed. This finding indicates that biomechanics were not the cause of this variability. In fact, the variability must result from underlying neural commands. A significant effect of direction was detected for the zero-force trajectory covariance ellipse major axis angle. The reason for this angle dependence is still unknown. Finally, the most striking result was a synchrony between velocity and curvature extrema observed in the zero-force trajectory. The finding was robust to changes in the assumed impedance, indicating that the coincidence of velocity and curvature extrema is a result of the underlying neural control.
Chapter 4

Conclusions and Future Work

4.1 Conclusions

At the start of this work we discussed how understanding fundamentals of human physical interaction could lead to development in many areas. To accomplish these goals, in this work, we set out to search for fundamental invariants in the human controller during the task of turning a crank. We successfully discovered several of these invariances.

In Chapter 2, we discovered that when humans move slowly, during constrained motion, their hand velocity becomes more variable and negative mean normal forces are exerted. We know that at slow speeds effects which make control difficult – inertial dynamics, feedback delays, and motor noise – become negligible. Thus, these observations are not consistent with the subject attempting to move around the path defined by the constraint at a constant velocity, and with zero normal force.

In Chapter 3 we assumed a model of the human biomechanics and used it to ‘subtract’ or ‘peel’ back the peripheral biomechanics and reveal the underlying neural commands, in terms of motion – the zero force trajectory. We discovered that at slow speeds the variability in the zero-force trajectory increases. Unexpectedly, a significant effect of direction on the zero-force trajectory orientation was reported. Finally, even though the hand was confined to move with a constant curvature, when the zero-force trajectory was computed we observed a coincidence of velocity and
curvature extrema. This indicates that the position dependent velocity and normal force fluctuations were the result of neural commands. Moreover, these fluctuations are consistent with the coincidence of velocity and curvature extrema reported in unconstrained motion.

4.2 Future Work

4.2.1 Measure Limb Impedance During Physical Interaction

In this work, we qualitatively observed position dependent muscle co-activation. Limb impedance varies with muscle activation; thus, the limb impedance may vary with time during execution of the task. The time varying impedance of the upper limb has not been documented during a constrained task. In our work, an instrumented crank was used as the constraint. This prevented the application of perturbations to measure the impedance of the arm during the task.

Lee and Hogan [Lee and Hogan, 2015] measured the time varying impedance of the ankle during locomotion using the Anklebot (Interactive Motion Technologies, Watertown, MA). The same approach could also be applied to the upper arm. The constraint could be simulated by an InMotion 2 (Interactive Motion Technologies, Watertown, MA) haptic robot, allowing for small stochastic force perturbations to be imposed by the handle during a movement. The interactive dynamics of the arm could be estimated by measuring the motion in response to the perturbations using the ensemble-based methods of Lee et al. [Lee and Hogan, 2015, Lee et al., 2014].

Robotic systems strong enough to simulate a mechanical constraint also induce position-dependent inertial effects. In order to move at speeds which are equivalent to the medium or fast condition, the position-dependent inertia of the robot must be eliminated. To address this challenge, we propose the use of a superimposed admittance model to compensate for the nonlinear inertia of the InMotion 2 robot, such that when the endpoint is moved the handle will act like an object of constant mass. Ideally, in the circular constraint task the inertia could be equivalent to turning
a mechanical crank – constant inertia at all positions.

In this work, we investigated the zero-force trajectory. However, when the impedance parameters were changed, the shape of the zero-force trajectory also changed. Thus, we were limited to the investigation of the curvature/velocity peak coincidence; this quantity was observed to be robust to the choice of the stiffness and damping in our model. Measurement of the limb impedance would allow for a more accurate estimate of the zero-force trajectory. It has been reported that humans modulate the limb stiffness when catching a ball [Lacquaniti et al., 1993] and when the required movement accuracy is changed [Wong et al., 2009]. We could determine if humans modulate the impedance of the arm in order to perform tasks requiring physical interaction with a constraint.

Figure 4-1: Experimental setup. The InMotion2 robot will be used to simulate the constraint. The subject will be provided with visual velocity feedback. The wrist will be braced, the elbow will be supported by a sling, and the shoulders will be strapped to the chair. Figure from Koeppen et al. [Koeppen et al., 2017].

4.2.2 Measure Impedance During Work Production

In principle, force may be produced directly (independent of motion) or indirectly (taking advantage of muscle stiffness/impedance). A paper by Chib et al. [Chib et al., 2009] reported that subjects could control force and position independently –
consistent with the idea that a ‘pure’ force control term exists. This can be expressed mathematically in Equation 4.1 and as a block diagram in Figure 4-2.

\[ f(t) = Z\{\Delta x(t)\} + f_0(t) \]  

(4.1)

The output force \( f(t) \) is the resulting force at the hand, \( f_0(t) \) is the ‘pure’ force control term, and \( Z\{\Delta x\} \) is the force resulting from the limb impedance. With this model, the only way pure force generation would be possible is if the \( Z\{\cdot\} \) term maps to zero force, or if the \( \Delta x \) term goes to zero (\( x \) is equal to \( x_0 \)).

Figure 4-2: Block diagram displaying the ‘pure’ force generator \( f_0 \) and the ‘indirect’ force production using the impedance of the limb \( Z \).

However, contrary to this observation, it has been reported that muscle impedance increases with force production [Bergmark, 1989, Cholewicki and McGill, 1995, Rack and Westbury, 1974]; thus, force control independent of motion may be difficult for humans. If ‘pure’ force generation is challenging for humans, it would imply that humans cannot perform high force, high compliance tasks. To determine if humans can produce pure force generation, damping could be added to the virtual crank while performing the crank turning task using the InMotion 2 robot to simulate the constraint. To determine if a ‘pure’ force generator exists, two correlations will be computed: the correlation between the mean magnitude of the hand reaction force vs. the mean impedance parameters, and the correlation between the hand reaction force vs. the motion of the hand. If there is no significant relationship between crank damping and impedance, it would be consistent with the idea that the
required force can increase without increasing the impedance of the limb, consistent with Equation 4.1. However, if there is a strong correlation, it would indicate that subjects cannot generate hand force without increasing the impedance of the limb, such that force generation can be modeled as $f(t) = Z\{\Delta x(t)\}$, i.e. force must be controlled indirectly.

4.2.3 Conclusions from the Future work

Physical interaction with a constraint is inherently bidirectional. Forces exerted on the constraint equal forces exerted on the hand. To understand these tasks, knowledge of the interaction dynamics is essential. Currently, time-dependent impedance estimates of the upper limb during a constrained motion task have not been documented. In the future, we hope to develop and implement an approach to measure the impedance of the upper limb during constrained motion. Then, document how the impedance of the upper limb changes as a function of speed, direction, position, and time. Next, we will investigate work production against a constraint to determine if humans can generate force independent of impedance. With quantitative measurements of human time-varying interactive dynamics, the approaches used by humans to excel at physical interaction will be better understood.
Appendix A

Modeling a Two-Link Manipulator Coupled to a Crank

The variation of motion due to closed-chain inertial dynamics was simulated. This was done by setting muscle-generated joint torques and crank damping to zero in the model defined in Chapter 3. The system was initialized at the zero degree position (Figure 3-4), with the initial angular velocities of the three target speeds: slow (0.075 rev/s), medium (0.5 rev/s), and fast (2.0 rev/s). Numerical integration was performed using MATLAB’s ode45 algorithm [Shampine and Reichelt, 1997].
Appendix B

Simulation of the Passive Dynamics

The model of the arm and crank system was constructed in the same manner as performed by Ohta et al. [Ohta et al., 2004]. Figure 3-4 displays the variables and notation used in the development of the model. The system has one degree of freedom; therefore, there is always a kinematic relationship which can be used to transform from Cartesian position, \( x = [x, y]^T \), to joint position, \( q = [q_1, q_2]^T \), and to crank position, \( \theta \), where the center of the crank is defined as \( x_c = [x_c, y_c] \).

\[
x = \begin{bmatrix} l_1 C_1 + l_2 C_{12} \\ l_1 S_1 + l_2 S_{12} \end{bmatrix} = \begin{bmatrix} r \cos \theta \\ r \sin \theta \end{bmatrix} + x_0,
\]

(B.1)

The notation \( S_1, C_1 \) denote \( \sin q_1, \cos q_1 \) and \( S_{12}, C_{12} \) denote \( \sin (q_1 + q_2), \cos (q_1 + q_2) \). The radius of the crank is \( r \), the damping of the crank is \( b_c \), and the inertia is \( I \). The upper arm denoted 1, and the forearm denoted 2 are described by length \( l_1, l_2 \), mass \( m_1, m_2 \), inertia about the z axis \( I_1, I_2 \), and center of mass distance from the joint axis \( c_1, c_2 \). The force on the handle is \( F = [F_x, F_y]^T \), with the normal unit vector, \( n \) and tangential unit vector, \( e \). The joint torque is denoted \( \tau = [\tau_1, \tau_2]^T \).

From the sum of moments acting on the crank,

\[
I \ddot{\theta} + b_c \dot{\theta} = re^TF
\]

(B.2)

summation of moments about the shoulder,
\[ M \ddot{q} + h = \tau - J^T F, \]  
(B.3)

and the kinematic relationship that equates the acceleration at the handle to the acceleration at the hand,

\[ \ddot{\theta} = J \ddot{q} + \dot{J} \dot{q} = r(\ddot{e} - \dot{e}^2 n), \]  
(B.4)

a model of the system can be constructed. Parameters comprising these equations include the mass matrix,

\[ M(q) = \begin{bmatrix} 
    m_1 c_1^2 + m_2 (l_1^2 + c_2^2 + 2l_1 c_2 C_2) + I_1 + I_2 & m_2 (c_2^2 + l_1 c_2 C_2) + I_2 \\
    m_2 (c_2^2 + l_1 c_2 C_2) + I_2 & m_2 c_2^2 + I_2 
\end{bmatrix} \]  
(B.5)

the centrifugal and Coriolis forces,

\[ h(q, \dot{q}) = \begin{bmatrix} 
    -m_2 l_1 c_2 S_2 (2 \dot{q}_1 \dot{q}_2 + \dot{q}_2^2) \\
    m_2 l_1 c_2 S_2 \dot{q}_1^2 
\end{bmatrix} \]  
(B.6)

and the Jacobian,

\[ J(q) = \begin{bmatrix} 
    -(l_1 S_1 + l_2 S_{12}) & -l_2 S_{12} \\
    l_1 C_1 + l_2 C_{12} & l_2 C_{12} 
\end{bmatrix} \]  
(B.7)

From Equations B.2, B.3, and B.4 the relationship in Equation B.8 can be shown

\[ R(\theta) \dot{\theta} + H(\theta, \dot{\theta}) = r e^T J^{-T} \tau, \]  
(B.8)

where the configuration dependent damping is

\[ R(\theta) = I + \dot{r}^2 e^T J^{-T} M J^{-1} e, \]  
(B.9)

and the configuration dependent inertia is
\[
H(\theta, \dot{\theta}) = b_c \dot{\theta} + r \mathbf{e}^T \mathbf{J}^{-1} \mathbf{T} \{ h - \mathbf{M} J^{-1} (r \dot{\theta}^2 \mathbf{n} + \mathbf{j} \mathbf{q}) \}, 
\] (B.10)

From Equations B.2, B.3, and B.4 we can also solve for \( \mathbf{F} \),

\[
\mathbf{F} = \{ \mathbf{J} \mathbf{M}^{-1} \mathbf{J}^T + r^2 I^{-1} \mathbf{e} \mathbf{e}^T \}^{-1} \{ \mathbf{J} \mathbf{M}^{-1} (\mathbf{\tau} - h) + \mathbf{J} \mathbf{q} + r \dot{\theta} (\dot{\theta} \mathbf{n} + b_c I^{-1} \mathbf{e}) \} 
\] (B.11)
Bibliography


