The Investment and Financing Decisions of Liquidity Constrained Firms

by

David Bradley Gross

A.B. Economics, Stanford University, 1991

Submitted to the Department of Economics in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy at the Massachusetts Institute of Technology

June 1995

© 1995 David B. Gross. All rights reserved.

The author hereby grants to MIT permission to reproduce and to distribute publicly paper and electronic copies of this thesis document in whole or in part.

Signature of Author.....................................................................................................................

Department of Economics
May 24, 1995

Certified by.................................................................................................................................

Ricardo J. Caballero, Professor of Economics
Department of Economics
Thesis Advisor

Certified by.................................................................................................................................

Jeremy Stein, J.C. Penney Professor of Management
Sloan School of Management
Thesis Advisor

Accepted by...............................................................................................................................

Richard S. Eckaus, Ford International Professor of Economics
Department of Economics
Chairman, Departmental Committee on Graduate Studies

ARCHIVES
MASSACHUSETTS INSTITUTE
OF TECHNOLOGY

JUN 12 1995
The Investment and Financing Decisions of Liquidity Constrained Firms

by

David B. Gross

Submitted to the Department of Economics on May 24, 1995
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

Abstract

This thesis develops and tests the predictions of a structural model of the investment and financing decisions of firms that find it costly to raise external finance. Chapter I presents the theoretical model in which optimal policy functions are derived for investment, retained earnings, borrowing, and dividend payments over time, as a function of the amount of financial resources which are held inside the firm. These policy functions demonstrate how the firm should dynamically manage the flow of funds entering and leaving the company to pursue both its current investment objectives as well as to minimize the possibility of being financially constrained or going bankrupt in the future. Firms hold buffer stocks of liquid assets which generate low current returns but enable the firm to invest in the future without accessing external capital markets. All companies will act as if they are constrained at certain endogenously determined points of time, depending upon the amount of financial resources held inside the firm. This chapter provides an alternative model to the neoclassical model which can be empirically estimated and the effects of financial constraints quantitatively interpreted.

In Chapter II, the optimal policy functions are estimated nonparametrically on a panel of firms without imposing any of the functional forms generated by the theoretical model. The highly nonlinear shapes of the estimated policy functions closely match the predictions of the theory. When firms have significant internal financial resources and are unconstrained, they invest until the capital stock equals its desired level. When firms are somewhat constrained, a large fraction of each extra dollar of internal financial resources is invested. When firms are highly constrained, they borrow to prevent their capital stocks from falling further. At this point the firm's capital stock has fallen to approximately 50% of its desired level. Direct estimation of the borrowing policy function confirms that firms do most of their borrowing when internal financial resources are low. Firms hold significant buffer stocks of cash when they are unconstrained, only distributing a small fraction of cash windfalls to shareholders. Estimation results which allow for adjustment costs to capital, foreseeable future investment opportunities, and long term debt do not eliminate the robust relationship between financial resources and firm decisions.
Chapter III explores the relationship between the dynamic and cross sectional dimensions of financial constraints. Unlike the existing literature, Chapter III argues that different cash flow coefficients for different samples of firms are not necessarily due to differences in investment behavior. Instead, differences in the in sample distribution of cash flow combined with nonlinear policy functions can generate the observed regression results. Since small firms have a more volatile distribution of financial resources and the derivative of the policy function for capital is convex, small firms will have a higher average sensitivity to cash flow. This result is true even though small firms' investment behavior is statistically equivalent to the behavior of large firms, conditional on the level of financial resources.

Chapters IV and V investigate the effects of microeconomic financial constraints on aggregate activity. Chapter IV considers an economy composed of a large number of financially constrained firms of the type studied in Chapter I. Financial constraints which generate nonlinear policy functions at the micro level provide a business cycle propagation mechanism for real shocks. Aggregate shocks shift the distribution of financial resources which charges the relative number of constrained versus unconstrained firms. As more firms become unconstrained aggregate demand increases, further shifting the distribution of financial resources thereby generating persistence in the business cycle. If firms have nonlinear policy functions then the level of aggregate investment or output will be a function of the distribution of financial resources within the economy. Chapter V estimates the microeconomic policy functions of individual firms using only aggregate sectoral data and data on the distribution of financial resources across firms. Time series changes in the distribution of financial resources have significant explanatory power for aggregate economic activity, and more stringently, in a way which is consistent with the microeconomic decisions of individual firms. The estimated policy functions using aggregate data closely match the theoretical predictions from Chapters I and IV and the results using microeconomic data from Chapter II.

Thesis Advisor: Ricardo J. Caballero, Professor of Economics
Thesis Advisor: Jeremy Stein, J.C. Penney Professor of Management
Acknowledgments

I owe a debt of gratitude to the many people who helped me during my years at MIT. First I would like to thank all of the professors who trained me to become an economist. Olivier Blanchard has been a personal role model for how to be a good macroeconomist. His research and discussions always seem to reach that elusive balance between technical rigor and real world relevance. Richard Eckaus provided me with a wonderful opportunity to work at the Central Bank of Portugal for a summer. He taught me how difficult and important it is to apply economic theory to actual policy problems. I would like to thank Daron Acemoglu for reading and commenting on my thesis. His insight and friendship will always be appreciated. I would especially like to thank Jeremy Stein for advising me on my thesis. He constantly reminded me of the differences in perspective between corporate finance and macroeconomics. His suggestions greatly improved the quality of my work.

I would like to thank Ricardo Caballero most of all among my teachers for his assistance at every stage of my thesis. He was always generous with his time, encouraged me when I was frustrated, and pushed me to do the best work that I could. On many occasions he uncovered the central feature of a problem and steered me in the right direction. He has profoundly affected every aspect of my work.

I would also like to thank all of my classmates at MIT. I learned more from our numerous discussions, study groups, and marathon work sessions than I did in the class room. I also made some great friends. There are too many people to individually mention but I want to particularly acknowledge Nick Souleles and Neal Rappaport. I will always remember your friendship and help during these years.

Among my Boston friends away from MIT, I want to especially thank my roommates over the past four years. Bruce, Adam, Jon, and Josh were always there for me, no matter what I needed. You actually helped make graduate school fun.

Above all, I would like to thank my family. You always were able to say just what I needed to hear, or to listen when I needed to talk. You encouraged me in my work without adding anymore pressure. Your support has been so strong and constant, that I can't imagine how I could have gotten here without you.

Thank you all.
Contents

Abstract ........................................................................................................................................... 2

Acknowledgments .............................................................................................................................. 4

I. Dynamic Investment and Financing Decisions
   of Endogenously Constrained Firms ............................................................................................. 6

II. Estimation of Microeconomic Policy Functions ........................................................................ 43

III. Cross Sectional Differences in Investment Behavior ............................................................... 79

IV. Aggregate Investment in a Financially Constrained Economy ............................................... 88

V. Empirical Aggregation .................................................................................................................... 106

References ..................................................................................................................................... 122
Chapter I. Dynamic Investment and Financing Decisions of
Endogenously Constrained Firms

I. Introduction

This chapter develops a structural model of the investment and financing decisions of firms that find it costly to raise external finance. When it is expensive or impossible for companies to borrow money, they will dynamically manage their investment, retained earnings, and dividend policies to optimally balance potential future borrowing costs with the cost of reducing current dividends. By keeping a quantity of liquid buffer stock assets, the firm can reduce the probability that it will be constrained in the future and need to raise external finance. In addition, by changing the mix between risky capital and safe cash, the firm can alter the distribution of its future financial resources and in particular, change the probability of bankruptcy. Sometimes companies will act in a risk-averse manner to reduce the discrete cost of bankruptcy, while at other times they will act in a risk-loving manner to exploit the convexity that limited liability induces.

The model tracks the flow of funds entering and leaving the firm, deriving optimal policy functions for the firm's capital stock, cash stock, new borrowing, and dividend payments as a function of the current financial resources held inside the firm. In Chapter II, these policy functions are estimated nonparametrically on a panel of companies, confirming many of the model's predictions. By providing an alternative model for the standard neoclassical model which can be empirically estimated, the effects of financial constraints can be quantitatively interpreted. Endogenous financial and investment policies generate strong predictions about the dynamic behavior of optimizing firms.
There is an extensive literature on the problem of investment in the presence of financial constraints. The theoretical literature has focused on the static investment and financing decisions facing firms and has carefully modeled the form of the financial contract given a well-defined information structure.\(^1\) These papers endogenously derive the excess costs which firms must pay to raise external capital. However, these models have typically ignored the rich dynamic strategies which businesses can follow to avoid raising external finance.\(^2\) In addition, they are frequently too stylized to implement empirically.

The empirical literature has taken the form of the financial contract as exogenous, and looked for evidence that financial variables affect investment. One strand has used firm characteristics to split companies into constrained and unconstrained groups. Linear regressions then show that cash flow is a more important predictor of investment for constrained firms than for unconstrained firms.\(^3\) Another set of papers has used Euler equation methods to test whether businesses that are predicted to be constrained violate the first-order condition for optimal investment.\(^4\) They find that constrained firms do violate the first-order condition, and that cash flow or debt capacity helps to explain the extent of the violation.

While the Euler equation papers are able to reject the neoclassical model, they do not propose or test a fully specified alternative model of firm behavior in the presence of financial constraints. Chapters I and II attempt to propose and test such a model in order to provide a structural, quantitative interpretation of how financial constraints affect firm

---

\(^1\) See for example Meyers and Majluf (1984), Leland and Pyle (1977), Gale and Hellwig (1985), and Townsend (1979).

\(^2\) Two exceptions are Lucas and McDonald (1992) and Gertler (1992). Lucas and McDonald study how changes in the financial position of banks affect loan and reserve decisions in a dynamic asymmetric information context. Gertler derives the optimal financial contract when borrowers and lenders can contract for multiple periods.

\(^3\) See for example Fazzari, Hubbard, and Petersen (1988) and Hoshi, Kashyap, and Scharfstein (1991).

behavior. This should help to interpret the economic significance of the strong
evidence of financial constraints already in the literature.

This paper starts with the observation that even the largest corporations that are
the least likely to be financially constrained, tend to invest out of retained earnings rather
than using public debt or equity markets. The natural explanation is that there is some
friction that leads all firms to prefer private, internal capital for most investment projects.
Most of the existing literature has focused on the cross-sectional dimension of financial
constraints; specifically that certain firms have more or less access to external capital
markets. Both the theoretical and empirical sections of this thesis investigate the time
series dimension of financial constraints. Companies will dynamically manage their
portfolios of capital and liquid assets to protect themselves against bankruptcy risk while
at the same time insuring that they have sufficient resources to undertake profitable
investments. Rather than exogenously assuming that some firms are constrained, while
others face perfect capital markets, Chapter I argues that all firms will act as if they are
constrained at certain points of time, depending upon the amount of financial resources
held inside the firm. Even when a company is not constrained because it has liquid assets,
the possibility of being constrained or needing to raise costly external finance in the
future will change current behavior.

5 Bond and Meghir (1994) also propose a structural, dynamic model of firm decisions in the presence of
financial constraints. In their model, the differential tax treatment of dividends, capital gains, and new share
issues creates a wedge between internal and external sources of funds. Firms will be in different regimes
depending upon how constrained they are. Using Euler Equation methods, they test their model and find
mixed results.
6 Milne and Robertson (1994) construct a model in continuous time which generates some results which are
similar to the theory section of this paper. In their model, firms hold liquid assets rather than paying out all
cash flow as dividends, and behave in a risk averse manner, in order to avoid bankruptcy.
7 See Chapter III for a discussion of the relationship between the cross section and time series dimensions of
financial constraints.
8 Greenwald and Stiglitz (1993) also present a model in which the possibility of bankruptcy causes the firm
to behave in a risk averse manner.
Starting with a model which is similar to the frameworks used in the Euler equation and buffer stock consumption literatures, numerical methods are employed to solve for the firm's optimal policy functions. Full information (for the stylized world of the model) is derived about how companies should invest, borrow, pay dividends, and keep liquid stocks of cash in different states of nature. The policy functions illustrate how endogenous financial policies dynamically alter real firm decisions. In Chapter II, these optimal policy functions are statistically compared to the behavior of real firms to assess the economic significance of financial constraints.

The remainder of this Chapter presents and solves the theoretical model of an individual firm. In Section II, the baseline theoretical model is described and optimal policy functions are solved for numerically. Comparative statics results and simulations are also presented. Section III extends the theoretical model to allow for borrowing, dividend smoothing, foreseeable future investment opportunities, and adjustment costs to capital. All of these extensions are explored empirically in Chapter II. Section IV concludes.

---

9 See Deaton (1991) and Carroll (1992) for examples of the buffer stock consumption literature.
II. Theory

A. Baseline Model

Consider a firm which maximizes the expected net present value of future dividends by choosing investment and dividends. For the sake of exposition, I will initially describe a baseline model with the following simplifications.

1. The firm is completely liquidity constrained so that it can no longer borrow. It inherits a stock of debt $B$ and pays sufficient interest so that the principal remains constant.
2. There are no adjustment costs from changing the capital stock.
3. Future productivity shocks are independent and identically distributed.
4. The firm has no incentive to smooth dividends.

In Section III, each of these assumptions will be relaxed. The firm’s constrained optimization problem for the baseline case can be stated as,

$$\max_{I_t, div_t} E_t \sum_{s=1}^{\infty} \left( \frac{1}{1 + \theta} \right)^{s-t} div_s$$

subject to

$$K_{t+1} = (1 - \delta) K_t + I_t$$
$$M_{t+1} = (1 + r) M_t + \pi(z_t, K_t) - pI_t - iB - div_t$$
$$M_{t+1} \geq 0,$$
$$div_t \geq 0,$$
$$K_{t+1} \geq 0.$$  

Notation is for the most part standard. $Div_t$ equals the dividend payment of the firm at the end of period $t$, $I_t$ is end of period investment, $K_t$ equals the undepreciated capital stock at the beginning of the period, $M_t$ equals the cash stock at the beginning of the period before
interest has been earned and \( B \) equals the constant stock of debt. Profits, \( \pi \), are a function of a stochastic productivity or demand shock \( z_t \), and a decreasing returns to scale function of the capital stock. In all numerical calculations the parameterization \( \pi = z_t K_t^\alpha \), \( 0 < \alpha < 1 \), is used with the distribution of \( z_t \) assumed to be i.i.d. normal. Note that this implies that profits can be negative during some periods. Chapter II will provide evidence that profits are indeed negative a large fraction of periods for the firms in the sample. \( r \) is the interest rate on safe cash, \( \theta \) is the discount rate or the required return for equity, \( i \) is the interest rate on debt, \( p \) is the price of capital, and \( \delta \) is the depreciation rate.

The first two constraints in equation (2) are accounting identities governing the accumulation of the capital and cash stocks. The evolution of the capital stock is standard, with exponential depreciation and no adjustment costs other than the fact that capital must be installed before the stochastic shock \( z_t \) is observed.\(^{10}\) Cash balances, \( M_t \), equal the principal plus interest on current liquid assets, plus current profits, minus investment, interest on debt, and dividends. The timing of the firm's information and decisions is summarized below.

\[
\begin{array}{c|c|c|c|}
\text{inherits} & \text{produces, observes } z_t, & \text{chooses } I_t, \\
K_t, M_t & \text{generates } \pi, (1-\delta)K_t, (1+r)M_t & M_{t+1}, div_t \\
t & & t+1
\end{array}
\]

The substantive assumptions relate to the structure of the financial market. Specifically, it is assumed that the discount rate \( \theta \), is greater than \( r \), the rate of return on cash.\(^{11,12}\) The high discount rate can be thought of as imposed on managers by

\(^{10}\) Note that this implies that there is no rental market for capital, since the firm will have a positive probability of default.

\(^{11}\) Later it will be shown that the interest rate on debt \( i \), is also greater than \( r \), reflecting the possibility of default. However, since the firm cannot borrow in the current discussion, this assumption is not necessary.
shareholders for agency reasons. If shareholders are concerned about managers wasting
resources, they will force managers to pay out cash as dividends even though it will be
shown that cash inside the firm will be worth more than cash outside the firm.\textsuperscript{13,14}
Alternatively, the high discount rate can be thought of as reflecting a risk premium over
the safe rate of return on cash.

In order for a solution to exist and the no borrowing constraint to be meaningful,
we must impose the additional constraints that cash balances and dividends are positive.
If cash balances were allowed to be negative, the firm could effectively borrow as much
as it wished at the safe interest rate. Since there is a positive probability of bankruptcy,
this could not be an equilibrium. If dividends were allowed to be negative, the firm could
effectively issue as much equity as it wished and the model would be equivalent to the
neoclassical investment problem with $\theta$ as the relevant interest rate. There is a large body
of evidence that firms are reluctant to issue equity, perhaps because it is viewed as signal
that the firm is overvalued.\textsuperscript{15} In practice, equity finance accounts for less than 5\% of total
new external finance.\textsuperscript{16} Hence, by assuming that dividends cannot be negative, we are
assuming that the firm cannot issue equity. The assumptions of nonnegative cash balances
and dividends combined with the assumption of no borrowing imply that the only sources
of finance for investment are cash balances and current profits.

\textsuperscript{12} The high discount rate in this model is analogous to the high discount rate in the buffer stock
consumption literature, which makes consumers impatient and stops them from accumulating arbitrarily
large amounts of wealth.
\textsuperscript{13} See Easterbrook (1984) and Jensen (1986) for a more complete discussion.
\textsuperscript{14} Note that $\theta > r$ is not sufficient by itself to make cash inside the firm worth more than cash outside the
firm. Without some restrictions on the flow of funds into the firm, cash balances would be set equal to zero,
dividend policy would be irrelevant, and the Modigliani - Miller theorem would still apply. Combined with
restrictions on the flow of funds into the firm however, $\theta > r$ implies that the firm will pay dividends rather
than accumulating cash indefinitely.
\textsuperscript{15} See for example Asquith and Mullins (1986).
\textsuperscript{16} See for example Friedman (1982), Srin Vasâpan (1986), and Whited (1992).
B. Dynamic Programming

The model can be restated and solved using dynamic programming. Letting \( V \) be the value function, and substituting out dividends and investment, the firm's problem can be expressed as

\[
V(K_t, M_t; B) = \max_{K_{t+1}, M_{t+1}} \pi(z_t, K_t) - pK_{t+1} + p(1 - \delta)K_t + (1 + r)M_t - M_{t+1} - \frac{1}{1 + \theta} E_t V(K_{t+1}, M_{t+1}; B)
\]

subject to

\[
\begin{align*}
M_{t+1} & \geq 0, \\
K_{t+1} & \geq 0, \\
M_{t+1} + pK_{t+1} & \leq \pi(z_t, K_t) + p(1 - \delta)K_t + (1 + r)M_t - iB.
\end{align*}
\]

The value of the firm today equals the flow proceeds from current decisions plus the expected discounted value of the firm tomorrow. This stationary problem with no debt decision or adjustment costs to capital can be reduced to a single state variable in the financial resources available to the firm at the end of period \( t \). Define the current financial resources, \( x_t \), as

\[
x_t = \pi(z_t, K_t) + p(1 - \delta)K_t + (1 + r)M_t - iB.
\]

Then the firm must decide how to allocate \( x_t \) among next period's capital stock and cash balances, with the residual being paid out as dividends. In a single state variable the functional equation can be written as

\[
V(x_t) = \max_{K_{t+1}, M_{t+1}} x_t - pK_{t+1} - M_{t+1} + \frac{1}{1 + \theta} E_t V(x_{t+1})
\]
subject to

\[ M_{t+1} \geq 0, \]
\[ K_{t+1} \geq 0, \]
\[ M_{t+1} + pK_{t+1} \leq x_t. \] (7)

In addition we impose the bankruptcy condition that if current financial resources ever fall to zero, the firm defaults and shuts down forever. This implies that

\[ \text{if } x_t \leq 0, \text{ then } \forall \tau \geq t, \quad V(x_{\tau}) = 0, \]
\[ K_{t+1} = 0, \]
\[ M_{t+1} = 0, \]
\[ x_{t+1} = 0. \] (8)

Note that since the value function cannot fall below zero, even when \( x_t \) is negative, the firm only faces limited liability from its production decisions. This will have important implications for firm decisions.

C. Numerical Solution

Currently, there is no known analytic solution for this problem, or others like it. However, the optimal policy functions \( K_{t+1}(x_t) \) and \( M_{t+1}(x_t) \) can be solved for numerically. Since \( \theta > 0 \) and the constraint set (7) is compact and convex, equation (6) defines a contraction mapping for the function \( V \). Given the concavity assumption for the profit function \( \pi \) and the assumptions about the stochastic shock \( z \), a solution to (6) – (8) exists and is unique.\(^{17}\) Standard numerical dynamic programming techniques are used to solve for the value function and the optimal policy functions.\(^{18}\) The state space is discretized into a grid of points. Starting with an initial guess for the value function \( V_0 \), the right hand side

\(^{18}\) See Judž (1991) and Bertsekas (1988) for references to numerical dynamic programming.
of equation (6) is maximized with \( V_0 \) in the place of \( V \), subject to the constraints and the bankruptcy condition. The expectation is computed numerically by determining the transition probability for each future state, given the current state and policy decisions. The solution of this maximization problem provides policy functions, \( K_0(x) \) and \( M_0(x) \) which correspond to the initial guess \( V_0 \). Holding \( K_0 \) and \( M_0 \) fixed, a new value function \( V_1 \) can be obtained by iterating equation (9).

\[
V^{t+1}(x) = x - pK_0(x) - M_0(x) + \frac{1}{1+\theta}E_xV^t(x_{t+1})
\]  

(9)

Since (9) defines a contraction mapping, this procedure will converge to the value function \( V_1 \) which corresponds to the value of following policies \( K_0 \) and \( M_0 \). This "policy function iteration" procedure is repeated until \( K(x) \), \( M(x) \), and \( V(x) \) converge to the true optimal policy functions and the value function. For a given discretization, this procedure is guaranteed to converge in a finite number of steps, and for a fine enough grid, this will arbitrarily approximate the continuous functions.

Depending on the values of the parameters, the policy functions can have very different shapes. Figure 1 shows the optimal policy functions \( K_{t+1}(x_t) \) and \( M_{t+1}(x_t) \) for one intuitive case. The solution can be characterized by three regimes base 1 on how constrained the firm is. In regime 1 the firm is unconstrained; in regime 2 the positive dividend constraint binds; while in regime 3 both the positive dividend and the positive cash balances constraints bind.

1. \( x_t > x^{**} \): \( K_{t+1} = K^* \), \( M_{t+1} = M^* \), \( \text{div}_t = x_t - pK_{t+1} - M_{t+1} \)
2. \( x^{**} > x_t > x^* \): \( K_{t+1} = K(x) \), \( M_{t+1} = M(x) \), \( \text{div}_t = 0 \)
3. \( x_t < x^* \): \( K_{t+1} = x_t/p \), \( M_{t+1} = 0 \), \( \text{div}_t = 0 \).
When \( x \) is large, the firm is unconstrained so it sets its capital stock and its liquid assets equal to target levels and pays the residual out as dividends.\(^{20}\) When \( x \) falls in regime 2, the firm pays no dividends and allocates some of its financial resources to the capital stock, while holding some in reserve for next period as a buffer, in case it receives a bad shock and is more constrained. If \( x \) is sufficiently small, the expected marginal product of capital is so large that the firm puts all of its financial resources in the capital stock.

Several conclusions can be drawn from this simple case. First, it is incorrect to think of firms as either being constrained or unconstrained. A company's status will change over time depending upon the realization of past stochastic shocks. While it is

\(^{19}\) Parameters values: \( \bar{z} = 1, \text{SD}(z) = 1.5, B = 0, r = 0.02, \theta = 0.12, \alpha = 0.35, \delta = 0.1. \)

\(^{20}\) Milne and Robertson (1994) generate the continuous time analog of this result. In their model, dividends follow a "bang - bang" policy in which dividends are zero until financial resources reach a cutoff level. At that level the rate of dividend payment becomes arbitrarily large to hold internal financial resources constant.
true that some firms may be more or less likely to be constrained depending upon particular parameter values, all companies' behavior will change because of the possibility of being constrained in the future. If there is no chance for a particular firm to ever be constrained, then that firm would have no incentive to hold any liquid assets which are not needed for day to day operations. Section III will show that this result remains true even in the more realistic case when firms can borrow, but at a higher interest rate than the safe rate. This suggests an empirical strategy which recognizes that firms' status will change over time.21

Another way to express the fact that the degree of financial constraints for a firm changes over time can be seen by inspection of Figure 1. The investment behavior of a firm depends very much on which regime it is in. That is, the relationship between cash flow and investment is highly nonlinear.22 In regime 3, when the firm has a low value of \( x_t \), each extra dollar of financial resources goes into next period's capital stock. In regime 1 however, when financial resources are significant, the firm has target values for the capital stock and cash balances. Hence, additional internal financial resources have no effect on investment. Previous empirical analyses which used linear regressions on different samples of firms are estimating complicated averages of underlying structural parameters. Not only will the estimated slope depend on the shape of the policy function for each sample of firms, but it will also depend on the in sample distribution of cash flow for each sample. It is difficult to interpret quantitative parameter estimates from these reduced form studies.

---

21 See also Bond and Meghir (1994) for an application to investment and Hajiivassiliou and Ioannides (1991) for an application to consumption.
Figure 2. Value Function

Figure 2 shows the value function corresponding to the policy functions of Figure 1. Several facts can be noted from the figure. For most values of \( x_t \), the value function is concave. This is because of the concavity of the profit function and the presence of financial constraints.\(^{23}\) This concavity in the value function causes the firm to behave in a risk-averse manner by holding buffer stocks of cash. When \( x_t \) is small, \( V'(x_t) \gg 1 \). An extra dollar of financial resources inside the firm is worth much more than a dollar outside the firm. Once \( x_t > x^{**} \) so that the capital and cash stocks are at their desired levels, none of the financial constraints bind. The envelope condition then implies that \( V'(x_t) = 1 \). That is, once financial resources are large enough so that the firm pays dividends, cash inside the firm is worth the same as cash outside the firm on the margin. For all excess financial resources, the company behaves in a risk-neutral manner, paying out the residual as dividends.

\(^{23}\) Grossman and Vila (1992) have a model in which borrowing constraints are sufficient to generate risk averse behavior from risk neutral agents through a similar mechanism.
The presence of bankruptcy alters the shape of the value function in two important ways. First, limited liability puts a floor on how far the value of the firm can fall. No matter how bad a negative shock the firm receives, it always has the option to default and pay zero.\(^{24}\) The presence of limited liability induces a convexity in the value function for small values of \(x\). Thus it is possible for the firm to choose to behave in a risk-loving manner to exploit this convexity.

The second effect of bankruptcy is to cause the value function to be discontinuous at zero. The irreversibility of bankruptcy induces an option-like incumbency premium for active firms. When \(x_t\) is positive, no matter how small, there is a chance that the firm will not go bankrupt and lose all future profits.\(^{25,26}\) This discontinuity can cause companies to behave in an extremely risk-averse manner, in order to avoid the fixed cost of bankruptcy.\(^{27}\) For other parameter values however, the discontinuity can cause companies to behave in a risk-loving manner. Firms which must make large interest payments may take on extra risk to avoid the otherwise near certain proposition of bankruptcy. While the possibility of bankruptcy complicates the analysis of financial constraints, it cannot be ignored. If companies never default, financial constraints would not exist and firms could borrow at the risk-free rate. Financial constraints and bankruptcy must be studied simultaneously in order to understand the investment and financing decisions firms make to avoid being constrained.

\(^{24}\) Note that it is not important for either the floor of the value function or the bankruptcy point to be located at zero. What is important is the existence of some fixed numbers for these parameters.

\(^{25}\) The discontinuity can also be viewed as the discrete financial cost of declaring bankruptcy and reorganizing. See Asquith, Gertner, and Scharfstein (1992) for an empirical estimate of the magnitude of these costs.

\(^{26}\) Note that while experiencing bankruptcy causes the firm to lose its discrete incumbency premium, the effects of bankruptcy are not limited to this fixed cost. The possibility of bankruptcy changes firm behavior for all values of financial resources, particularly as \(x_t\) gets small.

\(^{27}\) Greenwald and Stiglitz (1993) similarly argue that the possibility of bankruptcy will cause firms to behave in a risk averse manner.
Several general principles help to explain the internal investment and financing strategies of companies. The firm will choose how to allocate its internal financial resources between capital and cash in order to optimally balance the risk and return inherent in its portfolio. Investing in capital is equivalent to investing in a risky asset whose marginal return decreases with greater investment; investing in cash is equivalent to holding the risk-free asset. Hence, the firm's decision between capital and cash appears to be a classical portfolio allocation problem with a decreasing returns to scale risky asset. The presence of bankruptcy complicates this basic interpretation, however. Since the value function is not continuous at zero, marginal analysis is inappropriate for solving the firm's portfolio allocation problem. Even though the expected marginal product of capital is infinite as financial resources approach zero, the firm faces the possibility of losing its discrete incumbency premium if it goes bankrupt. The relevant comparison is not

28 Parameters values: $\bar{\xi} = 1$, SD($\xi$) = 1.5, $B = 0$, $r = 0.02$, $\theta = 0.04$, $\alpha = 0.35$, $\delta = 0.1$. 
between the expected marginal returns of capital and cash, but between their total returns. In some situations, like in Figure 3, the firm will chose to sacrifice the high marginal return on capital for a greater probability of avoiding bankruptcy. Hence in Figure 3, the firm holds a large proportion of its financial resources as cash when \( x_t \) is small. In other cases, like in Figure 1, the firm will act in a risk-loving manner, initially investing all of its financial resources in capital.

As the previous discussion illustrates, different parameter values will generate very different optimal policy functions. Firm behavior can be classified into three categories based on the shape of the optimal policy function for cash. Figure 4(a-c) show the three types of behavior in order of increasing risk-aversion. The most risk-loving behavior a firm can follow is by holding no cash as in Figure 4(a). In this situation, the firm would like to take a short position in liquid assets but the positive cash balances constraint binds over the entire range of internal financial resources. In Figure 4(b), the firm holds all of its financial resources as capital when \( x_t \) is small while in Figure 4(c) the positive cash balances constraint never binds.

For all three types of firm behavior, the dividend decision can be separated from the internal portfolio allocation problem. This can be seen by considering \( pK_{t+1} + M_{t+1} \), the amount of financial resources that the firm chooses to hold internally for next period and not pay out as dividends. In all three cases, \( pK_{t+1} + M_{t+1} = x_t \) until \( x_t \) reaches some critical value, \( x^{**} \), when all residual financial resources go to dividends. The firm only pays dividends once its capital stock and liquid assets have reached their desired levels, \( K^* \) and \( M^* \).
Figure 4(a). Optimal Policy Functions for Capital and Cash Balances. *Risk Loving Case.*

Figure 4(b). Intermediate Risk Case.

Figure 4(c). Risk Averse Case.

---

29 Parameters values: $\bar{x} = 1$, $SD(x) = 1.5$, $B = 0$, $r = 0.02$, $\theta = 0.16$, $\alpha = 0.35$, $\delta = 0.1$.

30 Parameters values: $\bar{x} = 1$, $SD(x) = 1.5$, $B = 0$, $r = 0.02$, $\theta = 0.12$, $\alpha = 0.35$, $\delta = 0.1$. 

22
D. Comparative Statics

Table 1 gives comparative statics for the optimal policy functions as $\theta$, the discount rate for equity, is changed. The second column of the table refers to the categories in Figure 4.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>Graph Category</th>
<th>Incumbency Premium</th>
<th>$K^*$</th>
<th>$M^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>(c)</td>
<td>27.5</td>
<td>4.39</td>
<td>4.25</td>
</tr>
<tr>
<td>0.06</td>
<td>(b)</td>
<td>11.7</td>
<td>3.99</td>
<td>2.61</td>
</tr>
<tr>
<td>0.12</td>
<td>(b)</td>
<td>5.11</td>
<td>3.24</td>
<td>0.80</td>
</tr>
<tr>
<td>0.16</td>
<td>(b)</td>
<td>3.78</td>
<td>2.92</td>
<td>0.00</td>
</tr>
<tr>
<td>0.25</td>
<td>(a)</td>
<td>2.42</td>
<td>1.60</td>
<td>0.00</td>
</tr>
</tbody>
</table>

As $\theta$ decreases, the incumbency premium increases. This is because a low discount rate makes the firm more forward looking, increasing the present value of future profits. Hence the firm is more concerned about bankruptcy when $\theta$ is low and less concerned with current dividends, so it behaves in a more risk-averse manner and holds more total resources inside the firm. Hence both $K^*$ and $M^*$ rise and the optimal policy functions proceed from type (a) to type (c). In the limit, as $\theta \rightarrow r$, $M^* \rightarrow \infty$, since the firm no longer has any incentive to pay dividends. Table 2 gives similar comparative statics varying the stock of debt, B.

---

31 Parameters values: $\bar{z} = 1$, $SD(z) = 1.5$, $B = 0$, $r = 0.02$, $\theta = 0.04$, $\alpha = 0.35$, $\delta = 0.1$.

32 Other parameters values: $\bar{z} = 1$, $SD(z) = 1.5$, $B = 0$, $r = 0.02$, $\alpha = 0.35$, $\delta = 0.1$. 

---
Table 2. Comparative Statics For Debt.\textsuperscript{33}

<table>
<thead>
<tr>
<th>( B )</th>
<th>Graph Category</th>
<th>Incumbency Premium</th>
<th>( K^* )</th>
<th>( M^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>(c)</td>
<td>27.5</td>
<td>4.39</td>
<td>4.25</td>
</tr>
<tr>
<td>1.00</td>
<td>(b)</td>
<td>22.4</td>
<td>4.42</td>
<td>4.69</td>
</tr>
<tr>
<td>2.00</td>
<td>(b)</td>
<td>17.9</td>
<td>4.45</td>
<td>5.04</td>
</tr>
<tr>
<td>5.00</td>
<td>(b)</td>
<td>7.43</td>
<td>4.56</td>
<td>6.18</td>
</tr>
<tr>
<td>10.0</td>
<td>(b)</td>
<td>0.91</td>
<td>4.58</td>
<td>5.32</td>
</tr>
<tr>
<td>12.0</td>
<td>(a)</td>
<td>0.35</td>
<td>4.98</td>
<td>0.00</td>
</tr>
<tr>
<td>15.0</td>
<td>(a)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The comparative statics of debt are more complicated than for the discount rate. Increasing the stock of debt raises the interest payments that must be paid each period, making bankruptcy more likely. Hence, the incumbency premium falls and the firm becomes more risk-loving for low \( x_t \) as debt increases. However, \( K^* \) and particularly \( M^* \) do not change monotonically with debt. At first, for low levels of debt, increasing debt increases \( M^* \). This is because when financial resources are large enough that the firm is unlikely to go bankrupt, the higher the interest payments, the greater the level of internal financial resources required to protect the firm against being constrained. However, once debt becomes extremely large, the option value of operating approaches zero. This causes \( M^* \) to fall since the firm is less concerned about bankruptcy. Eventually, debt is so large that the firm pays everything out as a dividend \( (K = M = 0) \) and declares bankruptcy.\textsuperscript{34}

Other comparative static exercises can be studied for the standard deviation of the stochastic shock, the risk-free rate, the concavity of the profit function, or any other parameter of interest.

\textsuperscript{33} Other parameters values: \( \tilde{z} = 1 \), \( SD(z) = 1.5 \), \( r = 0.02 \), \( \theta = 0.03 \), \( \alpha = 0.35 \), \( \delta = 0.1 \), \( i = 0.10 \).

\textsuperscript{34} A similar mechanism will determine an endogenous debt limit when the firm can borrow. This is discussed in Section III.
Simulations help to illustrate several robust predictions which emerge from all optimal policies. Figures 5 and 6 show particular sample paths over time of realized values of the capital stock, cash balances, dividends, cash flow, and investment for a firm which did not go bankrupt following optimal policies. Even with a complete prohibition on borrowing, it is apparent from Figure 5 that the firm is able to substantially smooth its capital stock through retained earnings. Of course the smoothing is asymmetric, given that the firm can always pay out a dividend to reduce its capital stock and cash balances.\footnote{This is similar to the buffer stock consumption case, when an individual can always reduce his wealth by consuming.} Capital and retained earnings are correlated, with both achieving their maxima at the same time, but dividends are counterfactually very volatile in the baseline model. In the next section, a modification of the model will be suggested to smooth dividends over time.

Looking at Figure 6, the relationship between cash flow and investment is highly nonlinear. When cash flow is negative, investment is also negative and closely tracks cash flow. However, when cash flow is large and positive, investment does not track cash flow. Regression results on Monte Carlo data also illustrate the nonlinearity. A simple linear regression of investment on cash flow generates a statistically significant coefficient of approximately 0.3. However, regressing investment on cash flow interacted with dummies for whether cash flow was small or large generated significant coefficients of 0.9 when cash flow was small, but only 0.1 when it was large. Instead, following Fazzari, Hubbard, and Petersen (1988), the sample can be split into groups of firms with high and low dividend payments over a period of time. The high dividend group has a cash flow coefficient of only 0.07 while the low dividend group has a coefficient of 0.7.
Figure 5. Simulation of Capital, Cash, and Dividends

![Graph showing simulation of capital, cash, and dividends over time]

Figure 6. Simulation of Investment and Cash Flow

![Graph showing simulation of investment and cash flow over time]
Note that these results were obtained with *a priori* identical firms whose only difference is their history of stochastic shocks. Similar simulation results can be obtained with firms with different structural microeconomic parameters. Hence, the model is able to reproduce the qualitative features of several existing empirical studies. However, these linear regressions obscure many of the underlying relationships within the model. In particular, the cash flow coefficients are not directly related to the slope of the optimal policy functions. In Chapter II, the model will be tested directly by estimating the optimal policy functions nonparametrically.

**III. Theoretical Extensions**

In this section of the paper, the baseline model is extended to explore the robustness of the predictions of the model. Part A examines whether the possibility of borrowing eliminates the sensitivity of firm decisions to internal financial resources. Part B considers a specification in which the firm has an incentive to smooth dividend payments over time. In Part C the stochastic process for productivity shocks is generalized to allow for serial correlation. This is potentially important because if productivity is serially correlated, then internal financial resources could be proxying for future investment opportunities rather than representing the current financial position of the firm. Finally, in Part D the model is extended to allow for adjustment costs in the stock of capital. In Chapter II, each of the extensions to the model will be investigated empirically.
A. Borrowing Decision

The baseline theoretical model can be extended to allow for borrowing. The principal question which will be addressed is whether the possibility of borrowing eliminates the sensitivity of firm decisions to internal financial resources. In particular, if it were possible for the firm to borrow as much as it wanted without any friction, then investment would only be a function of the future prospects of the firm, and would be independent of current financial variables. It will be shown that this is not the case, and that the optimal policy function for capital and cash retain their basic shapes. In addition the optimal policy function for borrowing will be derived and shown to be a decreasing function of internal financial resources.

Borrowing can be added to the model in two different ways. The simpler case, which will be presented first, is when the firm is restricted to one period debt.\(^{36}\) In this case the model can still be stated in a single state variable. The functional equation for the value of the firm can be written as

\[
V(x'_i) = \max_{K_{t+1}, M_{t+1}, B_{t+1}} x'_i - pK_{t+1} - M_{t+1} + B_{t+1} + \frac{1}{1+\theta} E_t V(x'_{i+1})
\]  

(10)

subject to

\[
egin{align*}
M_{t+1} & \geq 0, \\
K_{t+1} & \geq 0, \\
\overline{B} & \geq B_{t+1} \geq 0, \\
M_{t+1} + pK_{t+1} & \leq x'_i + B_{t+1},
\end{align*}
\]

(11)

where

\[
x'_i = \pi(z_i, K_i) + p(1-\delta)K_i + (1+r)M_i - \left(1+i(x'_{i-1})\right)B_i.
\]

(12)

\(^{36}\) One period debt is equivalent to callable long term debt with a variable interest rate.
Now the firm has the option of borrowing an amount $B_{t+1}$ each period at the possibly state contingent interest rate $i(x'_t)$. If the internal financial resources of the firm, $x'_t$, are not observable to outside lenders, then $i(x'_t)$ will equal a constant $i$ which will be greater than $r$ because of the possibility of default. If $x'_t$ can be observed, then lenders will charge an interest rate based on the expected probability of bankruptcy given $x'_t$. If lenders are risk neutral, competitive, and have an opportunity cost of funds equal to the risk free rate, then a zero profit condition implies that $i(x'_t)$ will solve

$$1 + r = (1 + i(x'_t)) \text{Prob}(x'_{t+1} \geq 0 \mid x'_t).$$

Clearly, $i(x'_t)$ is decreasing in $x'_t$. Market power, risk aversion, or financial constraints among banks will cause them to charge a higher rate.

Equation (11) shows that there is an additional constraint that the firm cannot borrow more than $\overline{B}$. This maximum debt limit can be derived endogenously based on a moral hazard argument. Specifically, suppose the firm could borrow as much as it wanted. Then if the firm borrowed too much, it could not credibly commit to repaying the debt. Instead, it would be optimal for the firm to liquidate all of its assets and pay a dividend to shareholders equal to $B_{t+1} + x'_t$. It can be shown that the maximum amount of debt which the firm can credibly commit to repay for all values of $x'_t$ is equal to $V(0^+)$, the size of the discontinuity in the value function at zero, or equivalently the incumbency premium for active firms. Intuitively, the incumbency premium, which represents the option value of continuing to operate, serves as collateral to prevent the firm from declaring bankruptcy. Banks will only lend if the value of continuing to operate is greater than the amount borrowed.
We are now in a position to summarize why the firm is sensitive to internal financial resources. First, the assumption of no equity issue limits the flow of capital into the firm. Second, the fact that equity markets require the discount rate for dividends to be greater than safe rate implies that the firm will not accumulate cash indefinitely. Finally, the possibility of bankruptcy, combined with an endogenous debt capacity, limits the amount that the firm can borrow. Elimination of any of these conditions would provide a source of funds on the margin which would remove the sensitivity of investment to financial variables. In particular, if there was no bankruptcy or limited liability, then the firm could always borrow as much as it wanted at the safe rate. Hence, the firm would set its capital stock equal to the neoclassical optimum, its dividends equal to the constant value of expected profits, the value function would be linear, and \( i = \theta = r \).

Figure 7 presents the optimal policy functions for capital, and borrowing as a function of the internal financial resources of the firm, with \( i(x') \) set according to equation (13). Several things can be noted from the figure. First, the general shape of the policy functions for capital (and cash) remain the same. Both are still related positively to internal financial resources when the firm is somewhat constrained, and level off at target values when financial resources are large. However, when financial resources become small enough, the firm is willing to pay the excess cost of borrowing to prevent its capital stock from falling further. Hence, the optimal policy function for capital levels off when financial resources are small, and borrowing increases. Figure 8 shows the optimal policy functions for capital for three different constant values for the borrowing rate, \( i(x') \). The closer the borrowing rate, \( i \), is to the safe rate, \( r \), the more willing the firm is to borrow, and the flatter the optimal policy function for capital.
Figure 7. Optimal Policy Function For Capital with Borrowing Single Period Debt, Endogenous Interest Rate.\textsuperscript{37}

Figure 8. Optimal Policy Function For Capital with Borrowing Single Period Debt, Exogenous Interest Rates.\textsuperscript{38}

\textsuperscript{37} Parameters values: $\bar{z} = 1$, $SD(z) = 1.5$, $r = 0.04$, $\theta = 0.20$, $\alpha = 0.25$, $\delta = 0.1$

\textsuperscript{38} Parameters values: $\bar{z} = 1$, $SD(z) = 1.5$, $r = 0.02$, $\theta = 0.12$, $\alpha = 0.35$, $\delta = 0.1$
In the above specification, there are no adjustment costs to changing the stock of debt. All debt is single period, with the option of being rolled over. As a result, it is never optimal for the firm to hold positive stock of debt and cash at the same time. A more realistic specification would recognize that most debt is longer term, and cannot be costlessly changed. To model this, suppose that the firm can only borrow by issuing infinite maturity perpetuities.\(^{39}\) In this case, the current stock of debt becomes an additional state variable. The functional equation for the value of the firm can be written as

\[
V(x_t, B_t) = \max_{K_{t+1}, M_{t+1}, \Delta B_{t+1}} \quad x_t - pK_{t+1} - M_{t+1} + \Delta B_{t+1} + \frac{1}{1+\theta} E_t V(x_{t+1}, B_{t+1})
\]

subject to

\[
\begin{align*}
M_{t+1} &\geq 0, \\
K_{t+1} &\geq 0, \\
B_{t+1} &\leq B = V(0^+, 0), \\
\Delta B_{t+1} &\geq 0, \\
M_{t+1} + pK_{t+1} &\leq x_t + \Delta B_{t+1}.
\end{align*}
\]

Now the firm chooses the amount of new borrowing each period equal to \(\Delta B_{t+1}\).

Figure 9(a) shows the optimal policy function for capital, which is now a surface because there are two state variables. Figure 9(b) shows horizontal cross sections of the surface, with each line representing a different stock of debt. Once again the distinctive “flat-increasing-flat” shape of the optimal policy function for capital remains along the financial resources dimension. As the current stock of debt rises, the firm approaches its borrowing capacity, so its becomes less willing to borrow and less able to smooth its capital stock. In addition, a higher current stock of debt reduces the incumbency

\(^{39}\)No new conceptual issues arise with long term debt of fixed, finite maturity. However, solving the model becomes more complicated because the entire distribution of debt maturities must be tracked at all times.

32
premium, making the firm more risk-loving and thereby increasing the desired capital stock.

B. Dividend Smoothing

Figure 5 shows the time path for dividends using simulated data from the baseline model. It is clear from the figure that the simulated dividends are very volatile. This is because in the baseline model, the firm views dividends as the residual use for excess financial resources. Empirically, this is not an accurate description of firm behavior. In a classic article, Lintner (1956) gave evidence that firms try to smooth dividends over time, only changing them in extraordinary circumstances.

\[ z = 1, \quad SD(z) = 1.5, \quad r = 0.02, \quad \theta = 0.12, \quad i = 0.15, \quad \alpha = 0.35, \quad \delta = 0.1. \]

In the baseline model dividends are distributed as a truncated normal, with \( \text{div}_t = \max (x_t - pK^* - M^*, 0) \).
A theoretical explanation for dividend smoothing can be based on a simple signaling model of dividends.\textsuperscript{42} Suppose that managers have more information about the future prospects of the firm (\textit{i.e.} the mean of \( z \)) than shareholders. Outside investors will be unable to determine whether an increase in dividends represents a temporary cash windfall or a permanent increase in firm profitability. Hence, temporary fluctuations in dividends will cause the share price to fluctuate. This may be undesirable both for the firm and for shareholders, given the underlying premise of this paper that there are frictions in financial markets.

While formally modeling dividend signaling is beyond the scope of this paper, a desire to smooth dividends can be easily incorporated into the model's specification. Suppose that instead of maximizing the expected present discounted value of dividends, managers maximize the expected present discounted value of some increasing but concave function of dividends.

\[
\max_\theta \mathbb{E}_t \sum_{s=t}^{\infty} \left( \frac{1}{1+\theta} \right)^{s-t} U(\text{div}_s) \\
\text{with} \\
U' > 0 \\
U'' < 0.
\]

All of the numerical techniques in Section II can be easily duplicated for this specification. Using an isoelastic smoothing function for \( U \), Figure 10 shows the optimal policy functions for cash and dividends for the baseline model and the dividend smoothing model.\textsuperscript{43} Since \( U \) is concave, the firm will try to smooth its dividend payouts in Figure 10(b) by holding cash windfalls inside the firm and only gradually paying them

\textsuperscript{42} For a formal model, see Miller and Rock (1985), Bhattacharya (1979).

\textsuperscript{43} The optimal policy function for capital is similar to the baseline case.
out to shareholders. As the concavity of $U$ increases, the dividend policy function becomes flatter while a larger fraction of excess financial resources are held as cash. In Section VI of the paper, evidence will be presented that this provides a better description of actual firm behavior.

C. Serially Correlated Productivity Shocks and Future Investment Opportunities

The principal goal of the empirical literature on the relationship between investment and financial variables has been to determine whether financial variables affect investment because of access to capital. An alternative explanation for the positive correlation between investment and cash flow is that current cash flow is proxying for future investment opportunities. Cash flow and investment may be high at the same time, because when times are good for the firm, they are likely to continue to be good, so the firm wishes to expand.

---

44 Parameters values: $\bar{z} = 1$, $SD(z) = 1.5$, $B = 0$, $r = 0.02$, $\theta = 0.12$, $\alpha = 0.35$, $\delta = 0.1$. Let $\gamma$ be the coefficient of relative risk aversion for $U$. $\gamma = 0$ in Figure 10(a) and $\gamma = 0.6$ in Figure 10(b).
Two approaches have been taken to try and rule out this explanation. The first is to include Tobin's average $q$ in regressions of investment on cash flow to control for future investment opportunities. A second approach has been to look for natural experiments or other situations where it is possible to control for biases induced by the future profitability effect. For example, Hoshi, Kashyap, and Scharfstein (1991) compare the behavior of two similar samples of Japanese firms whose only difference is whether they belong to a *keiretsu*, or large industrial group. The firms who belong to a *keiretsu* should have better access to external capital and therefore a lower cash flow coefficient for financial reasons, while having the same bias from future profitability reasons. Similarly, Lamont (1993) uses cash flow shocks from firm subsidiaries in unrelated industries to isolate the financial component of cash flow shocks from the future profitability component. While the natural experiment approach provides very clean tests to separate these two explanations, it is not possible to use this approach in this paper because of the difficulty in finding such exogenous shocks except in special circumstances.

The approach taken in this paper is to use the structural features of the model to investigate exactly how a knowledge of future profitability will change firm behavior. Later, in the empirical section, it will be possible to determine whether future profitability can explain the observed relationship between internal financial resources and firm decisions. It is easy to modify the model to capture the future profitability effect. Suppose that the stochastic shock, $z_t$, is serially correlated. Then if the firm receives a positive shock today, both $x_t$ and the expected value of $z_{t+1}$ will be high. Since the firm expects to be profitable next period, it will invest so $K_{t+1}$ will also be high. Hence, there will be a positive association between $x_t$ and $K_{t+1}$ which must be separated from the effects of financial constraints.
Solution of the model with serially correlated productivity shocks requires an additional state variable, the current value of the stochastic shock $z_t$. The functional equation for the value of the firm can be written as

$$V(x_t, z_t) = \max_{x_t, M_{t+1}} x_t - pK_{t+1} - M_{t+1} + \frac{1}{1+\theta} E_t V(x_{t+1}, z_{t+1})$$ (17)

subject to

$$M_{t+1} \geq 0,$$
$$K_{t+1} \geq 0,$$
$$M_{t+1} + pK_{t+1} \leq x_t,$$
$$z_{t+1} = \rho z_t + (1 - \rho)(1 + \epsilon_{t+1}),$$
$$-1 < \rho < 1, \quad \epsilon_t \sim \text{i.i.d. } N(0, \sigma^2).$$ (18)

The stochastic shock is parameterized as an AR(1) with unconditional mean 1, variance $\sigma^2_t$ and correlation coefficient $\rho$.

Figure 11 shows the optimal policy function for capital with $\rho = 0.15$ along with horizontal and vertical cross sections. Looking in the dimension of financial resources, the distinctive concave shapes remains, with the capital stock set to a target level when $x_t$ is sufficiently large. In the dimension of the stochastic shock, $K_{t+1}$ is increasing in $z_t$ because of the future profitability effect. When $z_t$ is high, $z_{t+1}$ is expected to be high, so if the firm is not constrained, it sets $K_{t+1}$ high. Note also that there is no concavity in the dimension of the stochastic shock. This will be important in the empirical section when we try to separate the effects of financial constraints from the effects of knowledge of future investment opportunities.
Figure 11. Optimal Policy Function For Capital. Serial Correlation Case.\textsuperscript{45}

\textbf{(a) Policy Surface}

\textbf{(b) Horizontal Cross Sections}

\textbf{(c) Vertical Cross Sections}

\textsuperscript{45} Parameters values: \( \bar{z} = 1, \) SD(\( z \)) = 1.5, \( B = 0, \) r = 0.02, \( \theta = 0.12, \) \( \alpha = 0.35, \) \( \delta = 0.1, \) \( \rho = 0.15. \)
D. Adjustment Costs

There is an extensive literature on the role of adjustment costs in explaining investment dynamics.\textsuperscript{46} In the context of this model, costs of adjusting the capital stock will make $K_t$ an additional state variable. This will help to separate the roles of the liquid and nonliquid components of the firm's internal financial resources. The functional equation for the value of the firm with adjustment costs can be written as

$$V(x_t, K_t) = \max_{k_{t+1}, m_{t+1}} x_t - pK_{t+1} - M_{t+1} - C(K_t, K_{t+1}) + \frac{1}{1+\theta} E_t V(x_{t+1}, K_{t+1})$$ (19)

subject to

$$M_{t+1} \geq 0,$$

$$K_{t+1} \geq 0,$$

$$M_{t+1} + pK_{t+1} + C(K_t, K_{t+1}) \leq x_t.$$ (20)

The adjustment cost function $C(K_t, K_{t+1})$ can have a number of different specifications. For example,

$$C(K_t, K_{t+1}) = \begin{cases} 0 & I_t \geq 0 \\ \infty & I_t < 0 \end{cases}$$ Irreversible Investment

$$= aK_t \quad I_t > 0$$

$$= aK_t \quad I_t = 0$$ Fixed Cost of Adjustment

$$= aK_t \quad I_t < 0$$

\textsuperscript{46} See for example Jorgenson (1963) and Bertola and Caballero (1990).
Figure 12 shows the optimal policy function for capital with adjustment costs in the quadratic costs case. In the financial resources dimension, the policy function has the traditional shape, while in the current capital stock dimension, the policy function is increasing. This is because the firm now wishes to only change its capital stock slowly. To try and keep the total resources inside the firm relatively constant, the firm compensates for the adjustment costs to capital by holding more cash when its capital stock is low. This can be seen in Figure 13 which shows the optimal policy function for cash balances which is decreasing in the current capital stock.\textsuperscript{47}

\textsuperscript{47} Policy functions for the irreversible investment and fixed cost cases are available from the author. Since these nonconvex adjustment costs are most important at the plant level rather than the firm level, standard convex adjustment costs provide a better fit to the firm level data in this paper.
Figure 12. Optimal Policy Function For Capital Quadratic Adjustment Costs.\textsuperscript{48}

Figure 13. Optimal Policy Function For Cash Balances Quadratic Adjustment Costs

\textsuperscript{48} Parameters for Figures 12 and 13: $\bar{z} = 1$, $SD(z) = 1.5$, $B = 0$, $r = 0.02$, $\theta = 0.12$, $\alpha = 0.35$, $\delta = 0.1$, $a = 1$. 
IV. Conclusion

This chapter proposed a structural model of the investment and financing decisions of financially constrained firms. Rather than assuming that some firms are constrained, while others face perfect capital markets, all companies act as if they are constrained at certain endogenously determined points of time, depending upon the amount of internal financial resources. Firms dynamically manage the flow of funds entering and leaving the company to pursue both current investment objectives as well as to minimize the possibility of being constrained or going bankrupt in the future.

This endogeneity of financial policy blurs the traditional definition of a financially constrained firm as a firm which is unable to access external capital markets. Even firms which do not need external funds for current investment may act in a constrained manner today, because it is more important for them to build a buffer against future negative shocks. Conversely, firms may be reluctant to take on debt because it will increase the probability of bankruptcy in the future. In order to quantitatively interpret the effects of financial constraints, it is essential to recognize both the forward looking nature of endogenous financial decisions and the changing status of firms over time.
Chapter II. Estimation of Microeconomic Policy Functions

I. Introduction

This chapter attempts to estimate microeconomic policy functions for the investment and financing decisions of individual firms. Policy functions for next period's capital stock, cash stock, borrowing, and dividends are estimated as a function of the total financial resources of the firm. These policy functions are then compared to the predictions of the theoretical models of financial constrained firms presented in Chapter I. The estimation results can be interpreted in two ways. First, as a reduced form data description of how firms manage their sources and uses funds. The second, preferred interpretation of the results is as an estimation of the optimal policy functions from the structural model presented in Chapter I. The estimated policy functions provide a test of the theoretical model, as well as allow a structural, quantitative interpretation of the effects of financial constraints.

A disadvantage of estimating policy functions over first order conditions, like in the Euler Equation literature, is that there is a larger potential for misspecification. The structural model generates highly nonlinear policy functions which may or may not accurately describe the way firms behave. If the model does not fit the data, then trying to estimate underlying microeconomic parameters is futile. The approach taken in this chapter is to estimate the model nonparametrically, allowing the data to choose appropriate shapes for the policy functions. While the model provides a structural interpretation of the policy functions, a particular specification is not imposed on the data.
If the estimated policy functions resemble the theory's predictions, this will be taken as confirmation of the model.¹

Kernel regression estimates of the policy functions are found to be extremely close to the predictions of the theory. The results are robust to changes in sample and data definitions, and are statistically different from the way an unconstrained firm would behave. Extending the specification to allow for adjustment costs, dividend smoothing, and serial correlation in productivity does not eliminate the sensitivity of firm decisions to internal financial resources.

The firm's capital stock for next period is estimated nonparametrically as a function of the firm's current internal financial resources.² When financial resources are large, the firm is unconstrained, and invests until its capital stock equals the desired level. Hence, in this region the estimated policy function for capital is flat. When the firm is somewhat constrained, a large fraction of each extra dollar of internal financial resources is invested, resulting in a steeply increasing policy function. When firms are highly constrained, they are willing to access external capital markets to prevent their capital stocks from falling further. At this point, the firm's capital stock has fallen to approximately 50% of its desired level.

Direct estimation of the policy function for the borrowing decision confirms these results. The firm tends to borrow only when it is constrained and internal financial resources are low. Dividend payments and cash balances are both increasing in the internal financial resources of the firm with the great majority of cash windfalls kept inside the firm rather than distributed to shareholders. This is consistent with an agency

---

¹ Other papers have found evidence of nonlinearities in response to macroeconomic shocks. Firms seem to contract more during recessions then they expand during booms. See for example, Gertler and Hubbard (1988), Kashyap, Lamont, and Stein (1994), and Gertler and Gilchrist (1994).

² The firm's internal financial resources equal profits plus last period's depreciated capital stock, plus cash balances, minus interest on debt. It plays the same role as cash flow in many existing studies.
view of the world in which managers wish to run large firms, as well as with a model in which managers wish to smooth dividends over time. Estimation results which allow for adjustment costs to capital, foreseeable future investment opportunities, or long term debt do not eliminate the robust relationship between financial resources and firm decisions.

Section II discusses data definitions and characteristics of the sample of firms used in the panel. Section III presents the empirical specification, Section IV displays the estimation results, and Section V concludes. A Data Appendix supplies complete variable definitions.

II. Data

A. Definitions and Sources

Most of the data are taken from the Compustat database, which includes quarterly and annual data on a large set of accounting variables. There are several conceptual issues that arise in trying to construct the accounting counterparts of the economic variables in the model. In cases when there is no preferred specification, alternative definitions were explored to investigate the robustness of the procedure. The data appendix will provide precise definitions and procedures.

The capital stock was measured in two different ways. Most economic studies have constructed the capital stock from investment data using the perpetual inventory method. The advantage of this approach is that the constructed capital stock is not affected by accounting rules for depreciation which are not always related to true economic depreciation. A disadvantage of the perpetual inventory method is that the rate of depreciation must be constant, or at least exogenous to the conditions of the firm.
Unusual circumstances which cause the firm to completely scrap a machine for example, will be captured in the accounting data, but not in the constructed data. Since these unusual circumstances are precisely the times when the firm is most likely to be financially constrained, the preferred specification will use the book value of capital.

The liquid assets variable $M_t$ can be measured in two ways depending on how the model is interpreted. Focusing on the buffer stock aspects of the model, $M_t$ can be defined in a restrictive sense as only the most liquid assets, cash and equivalents. It is these liquid assets which can be most easily converted to other uses when the firm needs a source of funds. Instead, focusing on the financing aspects of the model, $M_t$ can be measured as any asset that is more liquid than capital. If the point of the model is to completely track all the resources that flow into and out of the firm, then all firm assets should either be classified as capital or cash in the stylized world of the model. Following this interpretation, variables like inventories and accounts receivable would be counted as part of $M_t$. The preferred specification will use the cash and equivalents definition because the broader definition will vary widely depending upon the accounting practices of the firm.

Profits are measured as net income after adding back depreciation and interest payments. Other studies have used before tax income and excluded parts of income such as extraordinary items. While some of these components of income may reflect accounting rules which are not related to economic profits, many will have a direct impact on the financial status of the firm. For example, a fire in a plant, which is classified as an extraordinary item, will have a direct impact on the amount of total financial resources available to the firm. Since this study is about financial constraints, the broadest definition of profits, net income plus depreciation and interest payments, was used.
Finally, it is not clear what frequency of data sampling should be used. Some firm decisions take place at the annual level, while others take place every quarter. Section IV will show the results of the estimation procedure using different definitions and frequencies for several of the variables. The model is sufficiently robust that alternate definitions and time periods do not change any of the conclusions.

B. Sample Selection and Summary Statistics

The estimation procedure uses 44 quarters of data from 1983:I through 1993:IV and 20 years of annual data from 1974 to 1993. Firms were selected if their primary SIC code was between 2000 and 5999, eliminating firms in the government, financial, agricultural, mining, and services sectors. Many firms do not report complete data for all of the variables which are used in the estimation. In order to obtain the largest possible sample of firms, companies were only eliminated if they have missing values in more than half of the observations. Using an unbalanced panel greatly increases the available data, resulting in a more representative sample with a larger number of small and medium sized firms. Finally, a small number of firms were eliminated if they reported a significant merger or acquisition during the sample, or if they had large outliers in any of the variables. The remaining sample consists of 1684 firms in the quarterly sample and 3094 firms in the annual sample.

---

3 A smaller sample of manufacturing firms with SIC codes between 2000 and 3999 was examined, with similar results.
4 An earlier version of the paper used a balanced panel with a smaller sample of firms and achieved similar results.
5 Firms were eliminated if they reported a capital stock of zero, or if their reported capital or cash stocks were more than five times the fitted value described in the next section.
Table 1. Summary Statistics For Quarterly Data: 1983I - 1993IV
Millions of 1987 Dollars

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Stock - Book Value: $K_t$</td>
<td>918.5</td>
<td>35.3</td>
<td>3533.</td>
</tr>
<tr>
<td>Capital Stock - Constructed: $K_t$</td>
<td>1167.</td>
<td>44.3</td>
<td>5091.</td>
</tr>
<tr>
<td>Cash Stock: $M_t$</td>
<td>85.79</td>
<td>3.96</td>
<td>456.8</td>
</tr>
<tr>
<td>Financial Resources: $x_t$</td>
<td>1027.</td>
<td>45.4</td>
<td>3957.</td>
</tr>
<tr>
<td>Profits $\pi_t$</td>
<td>58.81</td>
<td>3.11</td>
<td>256.1</td>
</tr>
<tr>
<td>Dividends $div_t$</td>
<td>26.19</td>
<td>0.00</td>
<td>132.3</td>
</tr>
<tr>
<td>Debt: $B_t$</td>
<td>484.6</td>
<td>23.6</td>
<td>2643.</td>
</tr>
<tr>
<td>Interest Payments $iB_t$</td>
<td>13.25</td>
<td>0.73</td>
<td>68.07</td>
</tr>
</tbody>
</table>

Table 2. Summary Statistics For Annual Data: 1974 - 1993
Millions of 1987 Dollars

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Stock - Book Value: $K_t$</td>
<td>715.2</td>
<td>36.5</td>
<td>3048.</td>
</tr>
<tr>
<td>Capital Stock - Constructed: $K_t$</td>
<td>742.1</td>
<td>40.3</td>
<td>3472.</td>
</tr>
<tr>
<td>Cash Stock: $M_t$</td>
<td>71.78</td>
<td>4.37</td>
<td>395.0</td>
</tr>
<tr>
<td>Financial Resources: $x_t$</td>
<td>841.9</td>
<td>50.9</td>
<td>3463.</td>
</tr>
<tr>
<td>Profits $\pi_t$</td>
<td>156.7</td>
<td>11.1</td>
<td>672.8</td>
</tr>
<tr>
<td>Dividends $div_t$</td>
<td>31.41</td>
<td>0.70</td>
<td>144.1</td>
</tr>
<tr>
<td>Debt: $B_t$</td>
<td>394.2</td>
<td>23.9</td>
<td>2314.</td>
</tr>
<tr>
<td>Interest Payments $iB_t$</td>
<td>38.21</td>
<td>2.50</td>
<td>196.4</td>
</tr>
</tbody>
</table>

Tables 1 and 2 provide summary statistics for some variables of interest. All variables in the table and in the estimation are deflated with the GDP deflator. As the tables indicate, the average company is very large with a capital stock of between 700 million and 1.2 billion dollars, depending upon the measure of the capital stock. However, the median firm is much smaller, with a capital stock of only about 40 million dollars, or more than 20 times as small.\textsuperscript{6} Hence, there is substantial heterogeneity within

\textsuperscript{6} Using a balanced panel more than doubles the sizes of both the average and median firms.
the sample. Since most of the variables are always positive, the large standard deviations indicates that the size distribution of firms is highly skewed to the right, with a few very large firms. Cash represents about 9% of internal financial resources, capital represents about 81%, while profits net of interest represent about 10% of internal financial resources. Firms hold debt equal to about half of their financial resources. The average firm in the quarterly sample is larger than in the annual sample because larger firms are more likely to file quarterly reports.

One variable which is not restricted to be positive is profits. In the theory section, the multiplicative stochastic shock was parameterized as a normal random variable to allow for this possibility. By allowing the stochastic shock to be negative, the firm has a greater incentive to hold liquid assets because there is a higher probability that it will be constrained or go bankrupt. The data confirm the fact that even for the firms within the sample, quarterly profits are often negative. For the annual sample, profits net of interest payments are negative 10% of the time, while for the quarterly sample they are negative 12% of the time. It is during these episodes of negative profits that companies are most likely to be financially constrained.
III. Empirical Specification

A. Normalization

Until this point, firms have been treated as identical, except for the history of their stochastic shocks. Casual observation indicates that there are important differences across firms which persist over time. The most obvious example of this is differences in firm size. In order to normalize for scale as well as growth rate differences, assume that the stochastic shock, \( z \), is composed of a normally distributed, i.i.d. random variable \( \varepsilon \) multiplied by a firm specific scale parameter \( c \) and a firm specific growth parameter \( \mu \).

\[
z_{i,t} = c_i \mu_i \varepsilon_{i,t}, \quad c_i > 0, \quad \mu_i > 0, \quad \varepsilon_{i,t} \sim \mathcal{N}(1, \sigma^2).
\] (1)

A large value for \( c_i \) implies that the expected marginal product of capital for firm \( i \) is high. Hence the firm will choose to hold more resources inside the firm, and will be observed to be large. Similarly, if \( \mu_i > 1 \), the firm will expect to be more profitable in the future, making dividends today less attractive. Note also that since \( \varepsilon \) is distributed normally, the stochastic shock as well as profits will take on negative values during some periods.

In order to eliminate the effects of firm size on the estimation procedure, measured variables must be normalized in some way which removes the effects of \( c_i \) and \( \mu_i \). This normalization variable must be both theoretically valid and empirically implementable. It will be shown that the firm's desired capital stock, \( K_{i,t}^* \), will serve this purpose. First, define the frictionless capital stock, \( K_{i,t}^f \), to be the desired level of capital
in a neoclassical model with $\theta$ as the interest rate. This is the capital stock that the firm would choose in the current model, as uncertainty goes to zero.

$$\sigma^2 \downarrow 0 \Rightarrow K_{i,t} \to K'_{i,t} = \left( \frac{\alpha c_i \mu_i}{\alpha c_i \mu_i} \right)^{\frac{1}{\theta}}$$

(2)

Now, normalize all variables in the theoretical model by this frictionless value. That is, dividing both sides of equation (5) from Chapter I by the frictionless capital stock, shows that all independent references to $c$ and $\mu$ disappear.

$$\frac{x_i}{K'_{i,t}} = \frac{c_i \mu_i}{\alpha c_i \mu_i} \left( \frac{K_{i,t}}{K'_{i,t}} \right)^{\theta} + p(1-\delta) \frac{K_{i,t}}{K'_{i,t}} + (1+r) \frac{M_{i,t}}{K'_{i,t}} - i \frac{B}{K'_{i,t}}.$$  

(3)

Letting tildes denote normalized values, equation (3) can be rewritten in exactly the same form as equation (5) from Chapter I.

$$\bar{x}_i = \bar{\pi}(e_{i,t}, \bar{K}_i) + p(1-\delta) \bar{K}_i + (1+r) \bar{M}_i - i \bar{B}.$$  

(4)

Similarly, equation (6) from Chapter I can be rewritten as

$$\bar{V}(\bar{x}_i) = \max_{\bar{K}_{i,t}, \bar{M}_{i,t}} \bar{x}_i - p \mu_i \bar{K}_{i+1} - \mu_i \bar{M}_{i+1} + \frac{1}{1+\theta} \mu_i \bar{E}, \bar{V}(\bar{x}_{i+1})$$

(5)

---

7 Note that in equation (5) there remain some references to $\mu_i$ because variables dated $t$ and $t+1$ both appear. Hence, if firms have different growth rates, their normalized optimal policy functions should be different. However, this is no different than the situation for any of the other underlying microeconomic parameters. In practice, restricting the growth rate to be the same across firms, or even eliminating it from the specification (i.e. $\mu = 1$) did not change any of the conclusions from the estimation procedure.
Hence, the frictionless capital stock provides a theoretically valid normalization variable.

In order to obtain an estimate of both the frictionless and desired capital stocks, regress the observed capital stock on the frictionless capital stock in logs for each firm.

\[
\log K_{i,t} = \log K_{i,t}^f + \eta_{i,t} \tag{6}
\]

or

\[
\log K_{i,t} = \frac{1}{1-\alpha} \log \frac{\alpha c_i}{p(\theta + \delta)} \left( \frac{1}{1-\alpha} \log \mu_i \right) t + \eta_{i,t} \tag{7}
\]

This fitted value from a regression of the log of the capital stock on a constant and a trend, \( \bar{K}_{i,t} \), provides an estimate of both the frictionless and desired capital stocks to first order. This can be seen by examining a first order Taylor approximation in logs of the desired and fitted capital stocks around the frictionless capital stock.

\[
K_{i,t}^* = K_{i,t}^*(\sigma^2, \cdot), \quad \bar{K}_{i,t} = \bar{K}_{i,t}(\sigma^2, \cdot)
\]

\[
K_{i,t}^* (0, \cdot) = \bar{K}_{i,t} (0, \cdot) = K_{i,t}^f
\]

\[
K_{i,t}^* = K_{i,t}^f \exp \left( \frac{\sigma^2}{K_{i,t}^*} \frac{\partial K^*}{\partial \sigma^2} \right), \quad \bar{K}_{i,t} = \bar{K}_{i,t}^f \exp \left( \frac{\sigma^2}{\bar{K}_{i,t}^*} \frac{\partial \bar{K}}{\partial \sigma^2} \right)
\]

\[
K_{i,t}^* = c_1 \bar{K}_{i,t} = c_2 K_{i,t}^f
\]

Hence, an estimate of the desired capital stock, \( \hat{K}_{i,t}^* \), can be obtained by using the fitted value from a regression in logs for each firm of the capital stock on a constant and trend, normalized by a constant chosen so that \( K^* = 1 \).

Now, normalize all variables in the model by this estimate of the desired capital stock. Letting tildes denote normalized values, the normalized values for capital and cash for example equal

52
\[ \tilde{K}_{i,t} = \frac{K_{i,t}}{\bar{K}_{i,t}}, \quad \tilde{M}_{i,t} = \frac{M_{i,t}}{\bar{K}_{i,t}}. \]

These normalized variables are then used in the estimation procedure. To summarize, all observed data variables are normalized by dividing by a constant times the fitted value for the frictionless capital stock. This "detrending" procedure is theoretically justified given the identification assumption for the stochastic shock.

One consequence of the normalization procedure is that the empirical results isolate the dynamic effects of financial constraints. Since all variables are measured in percent deviations from the firm specific desired capital stock, all variation in \( \tilde{K} \) comes from temporary deviations within the firm. This is the analog of looking at the within dimension of a fixed effects panel regression. Hence, any evidence of financial constraints is not because of systematic cross sectional differences between firms, but because of changing status within a firm.

\[ \tilde{K}_{i,t+1} = f(\tilde{K}_{i,t}) + \xi_{i,t}. \quad (8) \]

The procedure provides an estimate for the entire function \( f \). Similar regressions are estimated for the other optimal policy functions. All regressions use the Epanechnikov
(quadratic) kernel. In simple terms, kernel regression smoothes the data by averaging points which are close together. The kernel is a symmetric weighting function which gives more weight to nearby points. The bandwidth controls the degree of smoothing by regulating the range of points that are averaged together. A larger bandwidth results in greater smoothing. Since kernel regression is just a formal technique for averaging nearby points, it imposes almost no structure on the estimated shape of the optimal policy functions. If the estimated optimal policy functions resemble the theory's predictions, this will be taken as strong confirmation of the model.

Uniform, asymptotic confidence intervals are placed around all estimated functions at the 5% level of significance. Formally,

$$\text{Prob}\left(f(\tilde{x}) \leq f(\bar{x}) \leq \bar{f}(\bar{x}), \ \forall \bar{x} \in X\right) = 0.95$$

The uniformity condition implies that 95% of the time, the entire true function will lie within the bands, for the domain pictured. Pointwise confidence intervals can also be constructed and will have a smaller width. Note that the pretest procedure of normalizing the variables by the fitted value for the desired capital stock is not taken into account in constructing the standard errors. Since these values are estimates, allowing for this uncertainty should increase the width of the confidence intervals.

---

8 The Epanechnikov Kernel with bandwidth h and distance u is $$\frac{3}{4} h(1-u^2) I(|u| \leq |h|)$$ where I is the indicator function.

9 Bandwidths were chosen informally to eliminate high frequency variation from the graphs rather than with a deterministic statistical procedure like cross-validation. In practice, a value of 0.3 was used for most estimations. Since there are no formal hypothesis tests in the paper, choosing the bandwidth visually is not a problem.

10 To ensure that the estimation procedure is consistent, a Monte Carlo study was performed using simulated data from the model, filtered by the normalization procedure. The estimated policy functions converge to the model's policy functions.

IV. Empirical Results

A. Capital Stock

Figure 1(a-d) shows the kernel regression estimates of the policy function for next period's capital stock. The different panels show the results for quarterly or annual data, and for the constructed or book value definitions of capital. Figure 2 shows the normalized optimal policy function predicted by the model with borrowing for the sake of comparison. The variables on each axis are normalized by the fitted value for the desired capital stock. The vertical axis of Figure 1 can be read in percentage terms as the amount that next period's capital stock exceeds or falls below the desired value. A value of 1 indicates no deviation. Similarly, the horizontal axis can be read as the amount of total financial resources held inside the firm, at the end of the period, relative to the desired capital stock. The dotted lines are the uniform nonparametric confidence intervals for the estimated function. They get wider near the boundaries because the data is more concentrated in the center.

A comparison of the four panels of Figure 1 illustrates the robustness of the results. The quarterly and annual splits include coverage of different sets of firms, at different frequencies, over different periods of time. (1974-1993 for the annual data, 1983-1993 for the quarterly data.) The split based on the definition of capital uses completely different data variables from different accounting statements. In all cases the shape of the policy functions remain essentially identical, with slightly different slopes, and minimums ranging from 50% - 60% of the firm's desired stock of capital. Since the results are so similar across samples, all remaining results will be presented using quarterly data and the book value of capital.
Figure 1(a). Estimated Policy Function for Capital Book Value of Capital, Quarterly.

The vertical axis is next period's capital stock, $K_{t+1}$, normalized by the desired value of capital for each firm. The horizontal axis is current internal financial resources, $x_t$, also normalized by each firm's desired level of capital. A value of 1 on the vertical axis means that capital equals its desired level. A value of 0.5 means capital equals half of the firm's desired level.

Bandwidth = 0.3. 1983I - 1993IV.
**Figure 1(b).** Estimated Policy Function for Capital Constructed Capital Stock. Quarterly.

**Figure 1(c).** Estimated Policy Function for Capital Book Value of Capital, Annual.

Bandwidth = 0.3. 1983I - 1993IV.

Bandwidth = 0.3. 1974 - 1993.
**Figure 1(d).** Estimated Policy Function for Capital Constructed Capital Stock, Annual

Bandwidth = 0.3. 1973 - 1993.

**Figure 2.** Optimal Policy Function for Capital with Borrowing

Parameters values: $\bar{z} = 1$, $SD(z) = 1.5$, $r = 0.02$, $\theta = 0.10$, $i = 0.12$, $\alpha = 0.35$, $\delta = 0.1$
A comparison of Figures 1 and 2 illustrates how similar the estimated policy functions are to the theory. The data clearly reject the null hypothesis suggested by the neoclassical theory that the optimal policy function be invariant to the level of internal financial resources. Even given the number of large firms within the sample, investment is sensitive to the relative amount of internal financial resources. Starting at the right side of the figure, the strong nonlinearity in the capital stock is present and precisely estimated. When financial resources are large, the normalized capital stock reaches its desired level of 1. This invariance of the capital stock to large amounts of financial resources remains, even when the graphs are extended to include much larger values for financial resources. As financial resources fall, next period's capital stock also falls, reaching a minimum at about half of the desired capital stock. The slope of the optimal policy function demonstrates the nonlinearity. Near the minimum of the function, the slope reaches a maximum of 0.6. In this region, about 60% of each extra dollar of financial resources is invested in capital. As financial resources increase, the slope decreases, eventually reaching zero. Hence, the effect of financial resources on investment depends very much on the level of financial resources within the firm.

At the left boundary, when the firm is most constrained, the estimated policy function levels off. This is precisely what is predicted by the extended model which includes borrowing. Once financial resources become sufficiently small, firms are willing to bear the excess cost of accessing external debt markets. This extra source of funds enables companies to maintain their capital stocks even when financial resources are very low.\textsuperscript{12}

\textsuperscript{12} Nonparametric boundary effects are another possible explanation for the shape of the optimal policy function in this region which must be ruled out. In some cases, kernel estimation at less than one bandwidth's distance from the boundary can cause bias. The reason for this is that a lack of data on the other side of the boundary causes the effective kernel to be asymmetric. If the first derivative of the function to be estimated is non zero, this asymmetry will cause bias. Since the shape of the optimal policy function in Figure 14 changes at a distance from the boundary approximately equal to the bandwidth, this appears to be
B. Cash Balances, Dividends, Borrowing, and Financial Resources

Figure 3 shows the estimated policy function for cash balances. The corresponding picture for the broader definition of liquid assets is very similar. The first thing to note is that firms hold a minimum of between 8% and 20% of their fitted capital stocks as cash at all levels of internal financial resources. This level remains roughly constant until financial resources are sufficient to provide for next period's capital and cash stocks. The cash floor may reflect period to period liquidity requirements for payroll or other short term liabilities as well as standard investment requirements to cover depreciation. Once internal financial resources become sufficiently large, cash balances increase sharply, with a slope of almost one. Furthermore, there is no indication that the policy function levels off, even for values of \( x_t \) equal to four or five times the fitted value for the capital stock.

The shape of the estimated policy function for large levels of internal financial resources is completely different from the predictions of the baseline theory. The baseline theory predicts that once the firm has sufficient resources for its investment and liquidity needs, it should pay out the residual profits to shareholders as dividends. Instead, when managers receive large cash windfalls, even of several times the size of the desired capital stock, they keep the excess financial resources as cash for future projects rather than returning it to the shareholders.\(^{13}\)

\(^{13}\) Blanchard, López-de-Silanes, and Schleifer (1993) look at a sample of firms that received large cash windfalls from lawsuits, which did not change their investment opportunity sets. They also find that firms rarely pay out the windfalls to shareholders.
Figure 3. Estimated Policy Function for Cash Balances

Bandwidth = 0.4.

Figure 4. Estimated Policy Function for Dividends

Bandwidth = 0.6.
To confirm the low rate of dividend payout, consider the optimal policy function for dividends. The baseline theory predicts that the optimal policy function should be zero until some critical \(x^*\) when it should increase linearly with a slope of one. Figure 4 shows the estimated policy function for dividends. While the graph is generally increasing, an inspection of the vertical axis demonstrates that firms only pay a tiny fraction of their resources to shareholders in any given quarter. Rather than increasing at a slope of almost one, like the estimated policy function for cash, the estimated policy function for dividends has a slope of about 0.01.

While the shapes of the estimated policy functions for both cash balances and dividends are not consistent with the baseline model, they are precisely what is predicted by the dividend smoothing model described in Chapter I, Section III(b). If managers try to smooth dividends over time, the optimal policy function for dividends will flatten, with excess financial resources stored in cash.\(^{14}\) By increasing the concavity of the smoothing function, the slope of the dividend function generated by the model will match the estimated function. Since the general shape of the optimal policy function for capital is not affected by dividend smoothing, the baseline model augmented to include dividend smoothing closely matches the estimated policy functions for capital, cash, and dividends.\(^{15}\)

---

\(^{14}\) In Chapter I, compare Figure 11(b), which shows the optimal policy functions for cash and dividends for the dividend smoothing case, with Figure 11(a), which shows the baseline model.

\(^{15}\) Another possible explanation for the shapes of the estimated policy functions for cash balances and dividends can be based on an agency argument. There is a large corporate finance literature which suggests that managers do not want to maximize firm profits, but rather try to maximize firm size or managerial perquisites. If this agency view of the world is correct, then managers would try to increase their sphere of influence by keeping as many resources in the firm as possible. See for example Jensen and Meckling (1976) and Jensen (1986).
Figure 5. Kernel Density Estimate of Financial Resources

(a) Density Estimate

(b) Distribution Function

Bandwidth = 0.3

Figure 5 shows the kernel density estimate of financial resources and the corresponding distribution function. Inspection of the figure illustrates two important points. First, there is a significant probability of being constrained within the data. Almost 6% of the observations have normalized financial resources less than 0.5. From Figure 1(a), this implies that 6% of the time, the observed capital stock is less than about 60% of its desired value. After this point the distribution function increases sharply. By the time the level of normalized financial resources reaches 1.0, 59% of the observations have been covered. This implies that for 59% of the sample, the observed capital stock is less than 80% of its desired value. Hence, for a large fraction of observations, the capital stock is significantly below its desired level. Even for the publicly traded firms within the sample, the level of internal financial resources has an economically important effect on firm decisions. The second fact which can be noted from Figure 5(a) is that the distribution of financial resources has a fat right tail. A significant number of firms hold very large levels of financial resources. This is just a restatement of the observation from Figure 3 that companies keep cash windfalls inside the firm.
Figures 6 and 7 investigate the borrowing decision more closely. It was shown earlier that the estimated policy function for capital flattens out when financial resources are small, just as the model with costly borrowing predicts. In order to directly test whether this is due to borrowing, Figures 6 and 7 present two different specifications of the firm's borrowing policy function. Figure 6 shows the estimated policy function for next period's stock of debt as a function of the firm's internal financial resources. Figure 7 shows the estimated policy function for new borrowing, or the change in the stock of debt.\textsuperscript{16} The results from both specifications are consistent with the predictions of the theory. When financial resources are low, the firm is constrained, so it chooses to hold a larger stock of debt next period, and it borrows more. This helps to confirm that borrowing is the reason that the estimated policy function for capital levels out when financial resources are low. In Figure 7, once financial resources become sufficiently large, the firm actually buys back some of its debt, resulting in negative new borrowing.

\textit{C. Multiple State Variable Results — Debt}

Kernel regression naturally generalizes to the case of multiple regressors. Rather than smoothing over intervals like in the single regressor case, the procedure smoothes the dependent variable over regions defined by the independent variables. Of course, in order to be able to display the results in three dimensions, there cannot be more than two independent variables.\textsuperscript{17} In the next few parts of the paper, the current stock of debt, the

\textsuperscript{16} If there are no adjustment costs to changing the stock of debt then Figure 6 is the correct specification. If there are adjustment costs to calling long term debt, then Figure 7 is more appropriate. In the next subsection of the paper, the estimation results for the model with the current stock of debt as a state variable will be presented. See Chapter I, Section III(a) for a complete description of the theory with borrowing.

\textsuperscript{17} In addition, as the number of independent variables increases, the rate of convergence of the estimated function declines. This is called the "curse of dimensionality." See Härdle pg. 91-95, 257-258.
Figure 6. Estimated Policy Function for Debt

Figure 7. Estimated Policy Function for Borrowing

Bandwidth = 0.3.

Bandwidth = 0.5.
current value of the stochastic shock, and the current capital stock will be included with financial resources as regressors. These regressors correspond to the additional state variables of the model from Chapter I, Section III. Two basic questions will be addressed. First, does the addition of an extra explanatory variable remove the robust features of the relationship between financial resources and firm decisions. Second, are the results of the new estimations consistent with the theoretical extensions proposed earlier.

Figure 8 shows the estimated policy function for capital including the current stock of debt as a state variable. This corresponds to the case when debt is long term and costly to adjust. Figure 9 from Chapter I shows the corresponding theoretical prediction. In the financial resources dimension in Figure 8(b), $K_{t+1}$ remains positively associated with $x_t$, with the traditional concave shape. The capital stock is also positively associated with debt, as the theory predicts. However, this association with debt may be for different reasons than in the theoretical model. In the theoretical model, as the level of debt approached the firm's debt capacity, the firm held more capital when it was unconstrained and ceased borrowing when it was constrained. Inspection of the horizontal cross section in Figure 8(b) confirms only half of this prediction. As the level of debt increases (the upper lines), the policy function tends to flatten out when financial resources are small. That is, the firm seems to borrow more when it is constrained and has a high current level of debt, rather than when it is constrained and has a low level of debt.
Figure 3. Estimated Policy Function For Capital. Long Term Debt Case

(a) Policy Surface

Bandwidth = 0.6, 1.0.

(b) Horizontal Cross Sections

(c) Vertical Cross Sections
Figure 9, which shows the estimated policy function for borrowing, confirms this hypothesis. While firms tend to borrow when they are constrained as the theory predicts, the higher their level of current debt, the more they borrow. The most likely explanation for this is some unobserved heterogeneity among firms which is correlated with their stock of debt. Firms with higher stocks of debt may have better future prospects, or cheaper access to capital markets. The later explanation is particularly convincing given that in Figure 8(a), when debt and financial resources are low, there is no tendency for the firm's policy function for capital to flatten, suggesting that borrowing is prohibitively
expensive or impossible. While these results provide more evidence for the widely observed cross-sectional heterogeneity in firm's access to capital markets, they do not eliminate the strong relationship between internal financial resources and firm decisions.

D. Future Profitability and Serial Correlation

The most serious objection to the empirical financial constraints literature is the possibility that the correlation between investment and financial variables is due to future investment opportunities rather than limited access to capital. As discussed in Chapter I, Section III(c), if the stochastic shock $z_t$ is serially correlated, then when times are good for the firm, they are likely to continue to be good, so the firm wishes to expand. Hence, $x_t$ and $K_{t+1}$ will be correlated for reasons other than financial constraints.

The existing literature has provided very strong evidence that the observed correlation between financial variables and firm decisions is due to financial constraints. By investigating case studies and natural experiments, these papers are able to find exogenous changes in financial variables which could not be related to the future profitability effect. Looking at the differential response of firms which are predicted to be constrained or unconstrained, it is possible to isolate part of the effect of financial constraints on investment. While this is an excellent strategy for rejecting the null hypothesis of perfect capital markets, it is difficult to use this methodology to determine the quantitative importance of financial constraints.

This paper takes a different approach. Until this point, the future profitability explanation was simply ignored given the existing evidence in the literature that the effect is small. In this section, three arguments will be given why the future profitability effect is unlikely to account for the observed relationship between financial resources and the
capital stock. First, the future profitability effect would not generate the observed concave shape between \( x_t \) and \( K_{t+1} \). Second, the measured rate of serial correlation is very small. Finally, using the value of \( z_t \) as an additional state variable adds little explanatory power. The combined effect of these arguments and the existing empirical literature is to indicate that the future profitability effect is probably very small.

Suppose that the future profitability effect were the only reason for the positive association between \( x_t \) and \( K_{t+1} \). Since there are no financial constraints, the capital stock would be set equal to the neoclassical optimum, conditional on the current value of the stochastic shock. That is,

\[
K_{t+1} = \left( \frac{\alpha \left[ \rho z_t + 1 - \rho \right]}{p(\theta + \delta)} \right)^{\frac{1}{1-\alpha}},
\]

and

\[
\frac{dK_{t+1}}{dx_t} = \frac{\partial K_{t+1}}{\partial z_t} \frac{\partial z_t}{\partial x_t} = \frac{\alpha \rho}{(1-\alpha) p(\theta + \delta)} \frac{K_{t+1}^{1-\alpha}}{K_t^\alpha} > 0 \quad \forall x_t
\]

Hence, if the neoclassical model was the correct specification and the previous regressions were estimated, while there would be a positive relationship between \( K_{t+1} \) and \( x_t \), there would be no tendency for the estimated policy function for capital to flatten when \( x_t \) was large. Similarly, if both effects operate like in the model of Chapter I, then

\[
\frac{dK_{t+1}(x_t, z_t)}{dx_t} \bigg|_{x_t, z_t} = \frac{\partial K_{t+1}}{\partial x_t} + \frac{\partial K_{t+1}}{\partial z_t} \frac{1}{K_t^\alpha} = 0 + \frac{\partial K_{t+1}}{\partial z_t} \frac{1}{K_t^\alpha} > 0 \quad \forall x_t.
\]
Hence, if both effects operate but the future profitability effect is ignored by the econometrician by not including $z_t$ as a state variable, the optimal policy function for capital would not flatten out for large levels of financial resources as it does in the data.\textsuperscript{18}

Suppose instead that the stochastic shock is observable to the econometrician. It is then possible to obtain more direct information about the future profitability effect. Since profits and the capital stock are both observed, then given a value of $\alpha$ and the assumed parameterization for the profit function, the current value of the stochastic shock can be explicitly computed as $z_t = \frac{\pi_t}{K_t^\alpha}$. There is no simple method to obtain an estimated for $\alpha$ since both $\pi$ and $K$ are endogenous. However, all results are insensitive to the value of $\alpha$ chosen.

Table 3 shows the estimated serial correlation of the stochastic shock for different values of $\alpha$, using quarterly data.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\rho(z_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.16</td>
</tr>
<tr>
<td>0.2</td>
<td>0.10</td>
</tr>
<tr>
<td>0.5</td>
<td>0.04</td>
</tr>
<tr>
<td>0.6</td>
<td>0.03</td>
</tr>
<tr>
<td>0.7</td>
<td>0.02</td>
</tr>
<tr>
<td>0.8</td>
<td>0.02</td>
</tr>
<tr>
<td>0.9</td>
<td>0.04</td>
</tr>
<tr>
<td>1.0</td>
<td>0.14</td>
</tr>
</tbody>
</table>

\textsuperscript{18} It might be possible to generate an unusual stochastic process for $z_t$ which could account for the observed nonlinearity between capital and financial resources. In particular if $z_t$ was an AR(1) for low and medium values of $z$, but i.i.d. with a high conditional mean for high values of $z$, then the policy function for capital would have the observed shape. However, the policy functions for all the other firm decisions would not correspond to the estimation results.
As the table illustrates, the estimated serial correlation is very low, no matter what value of $\alpha$ is chosen.\textsuperscript{19} While these estimates may be biased towards zero because of measurement error, it is difficult to imagine that this is the cause of the relationship between financial resources and the capital stock.

To further investigate the future profitability effect, the stochastic shock can be included as an additional state variable as suggested in Section III(c) of Chapter I. Figure 10 shows the estimated policy function along with horizontal and vertical cross sections, with $\alpha$ set equal to 0.7. Changing $\alpha$ does not change the results. As the figures illustrate, the addition of $z_t$ as a state variable provides little additional explanatory power. In the financial resources dimension, in Figure 10(b), the traditional shape emerges. In the dimension of the stochastic shock however, $K_{t+1}$ is actually slightly negatively related to $z_t$. The addition of the stochastic shock does not alter the shapes of any of the other estimated policy functions. It is clear that the future profitability effect is not able to account for the robust relationship between financial resources and firm decisions.

\textsuperscript{19}To ensure that the low serial correlation was not due to seasonal effects, the same calculation was performed using annual data. The serial correlation becomes even smaller and for some values of $\alpha$ is even negative.
Figure 10. Estimated Policy Function For Capital. Serial Correlation Case

(a) Policy Surface

Bandwidth = 0.4, 0.6.

(b) Horizontal Cross Sections

(c) Vertical Cross Sections
E. Adjustment Costs

Figure 11 shows the estimated policy function for next period's capital stock as a function of current financial resources and the current capital stock. As discussed in Chapter I, Section III(d), this is the appropriate specification if there are costs to adjusting the capital stock. Figure 12 of Chapter I provides the corresponding theoretical prediction for the quadratic costs case. In the financial resources dimension, the traditional shape remains. This is particularly relevant for the current specification, because holding the current capital stock fixed and moving along the dimension of financial resources isolates the effects of the liquid parts of financial resources. That is, for a given value of $K_t$, each line in Figure 11(b) shows the effects of the liquid part of $x_t$ on $K_{t+1}$. Hence the relationship between financial resources and the capital stock is not only due to slow adjustment of capital. Looking in the capital stock dimension, it is clear that there is a strong positive relationship between $K_t$ and $K_{t+1}$. This is exactly what is predicted by the theoretical model with convex adjustment costs.\(^{20}\) Hence, as a large literature suggests, adjustment costs appear to be an important factor in explaining investment dynamics. However, their existence does not eliminate the fundamental relationship between internal financial resources and firm decisions.

\(^{20}\) Note that this is also consistent with irreversible investment, but not consistent with fixed costs of adjustment at the firm level.
Figure 11. Estimated Policy Function For Capital. Adjustment Cost Case

(a) Policy Surface

Bandwidth = 0.6, 0.8.

(b) Horizontal Cross Sections

(c) Vertical Cross Sections
V. Conclusion

This chapter estimated the microeconomic policy functions of firm investment and financing decisions and compared them to the predictions of the structural model of financially constrained firms presented in Chapter I. Nonparametric estimation of the policy functions closely matched the nonlinearities predicted by the model. More significantly, the effects of financial constraints are quantitatively important.

When firms are unconstrained, they invest until their capital stock equals its desired level. When firms are somewhat constrained, the capital stock is highly sensitive to the level of internal financial resources. If internal financial resources fall even further, companies borrow to prevent the capital stock from dropping to less than 50 or 60% of the desired level. Direct estimation of the borrowing policy function confirms this conclusion. Cash windfalls are held inside the firm as liquid assets rather than being distributed to shareholders as dividends. Extensions to the model to allow for long term debt, dividend smoothing, knowledge of future investment opportunities, and adjustment costs to capital do not overturn the underlying relationships between internal financial resources and firm decisions.
Data Appendix

This appendix provides Compustat definitions for all of the quarterly variables used in the paper. The corresponding codes for the annual variables are exactly the same without the trailing Q. All variables are deflated with the GDP deflator.

Capital Stock –

**Book Value.**  \( K_t = \text{Property, Plant, and Equipment, Net.} \)
\( = \text{PPENTQ + DPQ,} \text{ Beginning of period capital stock,} \)
\( \text{before depreciation.} \)

**Constructed.**  \( I_t = \text{Capital Expenditures.} \)
\( = \text{CAPXQ.} \)
\( K_t = (1-\delta)K_{t-1} + I_{t-1}, \text{ Initial values are set equal to book value.} \)
\( \delta = 0.025, \text{ quarterly} \)
\( \delta = 0.100, \text{ annually.} \)

Cash Stock –  \( M_t = \text{Cash and Equivalents.} \)
\( = \text{CHEQ}/(1+r), \text{ Beginning of period cash stock.} \)
\( r = 3 \text{ month T-Bill rate.} \)

**Profits –**  \( \pi_t = \text{Net Income plus Depreciation plus Interest.} \)
\( = \text{NIQ + DPQ + XINTQ.} \)

**Financial Resources –**  \( x_t = \pi_t + (1-\delta)K_t + (1+r)M_t - iB. \)
\( = \text{NIQ + DPQ + PPENTQ + CHEQ - DVPQ,} \)
\( \text{End of period financial resources, including preferred} \)
\( \text{dividends, DVPQ, as part of interest payments.} \)

**Dividends –**  \( \text{Div}_t = \text{Cash Dividends for common stock.} \)
\( = \text{DVQ.} \)

**Debt –**  \( B_t = \text{Total Debt.} \)
\( = \text{DTQ.} \)
Mergers – Firms were eliminated if Compustat reported a major merger or acquisition in the data comparability code.

CPRQ = "AB".

Firms were also eliminated if they reported a capital stock of zero, or if their reported capital or cash stocks were five times greater than the fitted value for capital in any quarter.

\[ \hat{K}_{i,t} = 0, \quad \hat{K}_{i,t} > 5, \quad \hat{M}_{i,t} \Rightarrow 5. \]
Chapter III. Cross Sectional Differences in Investment Behavior

Chapters I and II both focused on the dynamic dimension of financial constraints. Much of the empirical literature has focused on cross sectional issues. This chapter will explore the relation between the two. In particular, I will show that different cash flow coefficients of different samples of firms are not necessarily due to differences in investment behavior. Instead, differences in the in sample distribution of cash flow combined with nonlinear policy functions can generate the observed regression results. Empirical estimation of policy functions and the distribution of financial resources for firms in different size classes confirms this hypothesis.

The theoretical model in Chapter I investigated the optimal behavior of a single firm over time, as a function of its financial position. The current level of liquidity is the state variable which determines whether or not a firm acts in a constrained or an unconstrained manner. Hence, in Chapter I firm behavior changes over time. The only necessary difference between firms that are currently constrained and firms that are unconstrained is the current level of liquidity. Cross sectional differences between firms do matter in the model presented in Chapter I, however. If firms have different underlying structural parameters, they will have different policy functions and will therefore respond differently to stochastic shocks. However, all firms will choose optimal policies so as to spend some time in both the constrained and unconstrained regions. Hence, both dynamic and cross sectional effects potentially matter for investment behavior.

In Chapter II, the dynamic dimension of financial constraints was explored empirically, by making the strong identifying assumption that firms are identical up to fixed firm size and growth rate effects. The empirical specification allowed firms to have different average, or desired capital stocks, and different growth rates. However, firm
policy functions measured in percent deviations from their desired capital stocks were constrained to be the same. Hence, Chapter II focused on the within or dynamic dimension of the data, after eliminating the average between or cross sectional effect.

Much of the empirical literature has focused on the cross sectional dimension of financial constraints. Many papers have shown significant differences between groups of firms that have been split based on some a priori measure of access to external capital markets. These papers have argued that small firms, for example, behave differently because they have less access to external capital and are more dependent on internal cash flow to finance their investment. In the context of the framework presented here, these papers are arguing that because small firms have different underlying micro parameters, they should have different policy functions. If that were true, then the policy functions estimated in Chapter II would be the average policy function within the sample, obscuring cross sectional differences.

In order to investigate whether or not there are important cross sectional differences in firm's investment policy functions, I can apply a similar sample splitting strategy. Firms can be split into groups that are more or less likely to be constrained, and then different policy functions can be estimated for each group. More generally, I can run bivariate kernel regressions, using a cross sectional measure like size as one of the independent variables.

Figure 1 shows the estimated policy function for capital as a function of internal financial resources, $x_{i,t}$, and a measure of firm size, $c_{i}$, the log of the average capital stock. Estimation was performed using the methodology of Chapter II. Both the vertical axis, $K_{i,t+1}$, and $x_{i,t}$ are in percent deviations from the firm's desired capital stock. In the $x$

---

1 See for example Fazzari, Hubbard, and Petersen (1988) and Hoshi, Kashyap, and Scharfstein (1991).
Figure 1. Estimated Policy Function For Capital, Controlling For Firm Size

Figure 2. Estimated Policy Function For Capital

Each line represents a different size cohort.
dimension the standard concave shape emerges. Thus dynamic changes in firm liquidity change investment behavior, even after controlling for differences across firm classes. In the cross section dimension however, there is no significant effect. This can be seen more clearly in Figure 2, which shows horizontal sections of Figure 1. Each curve represents the policy function for a different size class of firms. There are no significant changes in the shapes of the policy functions for different size classes.

These results seem to contradict the results of the empirical cross literature. If large firms are less sensitive to their financial positions, then their policy functions should be closer to a horizontal line at one. How can this apparent contradiction be reconciled?

Consider a linear regression of the form typically run in the cross section literature.

\[
\frac{I_{it}}{K_{it}} = \alpha \frac{Cash_{it}}{K_{it}} + \epsilon_{it+1}
\]  

(1)

Substituting for investment, defining \( Cash_{it} = \pi_{it} + (1 + r) M_{it} - iB_{it} \), and normalizing by \( K_{it}^* \) instead of \( K_{it} \) yields,

\[
\frac{K_{it+1}}{K_{it}^*} = \alpha \left( \frac{\pi_{it} + (1 + r) M_{it} - iB_{it}}{K_{it}^*} \right) + (1 - \delta) \frac{K_{it}}{K_{it}^*} + \tilde{\epsilon}_{it+1}
\]

(2)

or

\[
\tilde{K}_{it+1} = \alpha \tilde{\epsilon}_{it} + (1 - \delta)(1 - \alpha) \tilde{K}_{it} + \tilde{\epsilon}_{it+1}
\]

(3)

Tables 1 and 2 show regression estimates of equation (3) for small and large firms, after removing individual firm fixed effects.
All coefficients are positive and highly significant. The implied estimates for δ are of the correct order of magnitude. The regressions also replicate the standard result from the sample splitting literature. There is a statistically significant difference in α between the two samples, with small firms having a larger coefficient.

Given the previous estimation of firm policy functions, it is possible to explicitly compute α for different size classes of firms. This will determine the source of the difference in cash coefficients between small and large firms. Substituting the policy function for \( \tilde{K}_{it+1} \) into equation (3) implies that

\[
\alpha = \mathbb{E}\left[ \frac{d\tilde{K}_{it+1}}{d\tilde{x}_{it}} \middle| \tilde{K}_{it} \right] = \begin{cases} \int K'(\tilde{x}_{it}) f_{1}(\tilde{x}_{it} | \tilde{K}_{it}) d\tilde{x}_{it} & \text{small firms} \\ \int K'(\tilde{x}_{it}) f_{1}(\tilde{x}_{it} | \tilde{K}_{it}) d\tilde{x}_{it} & \text{large firms} \end{cases}
\]

(4)

The cash coefficient is the average of firm sensitivity to liquidity at different levels of financial resources. Equation (4) illustrates that α depends on two things: firm policy functions, and the density of financial resources. Figures 1 and 2 showed that firm policy
functions do not differ across size classes. Hence, the only remaining candidate to explain differences in $\alpha$ as a function of firm size is the density of financial resources.

Figure 3 shows the density of financial resources for firms of different size classes. Inspection of the figure indicates that small firms have much more weight in the tails of the distribution than large firms. This result is intuitive. Large firms undertake many projects at the same time, so their financial resources as a percentage of desired capital are less volatile than small firms. Hence, to first order, the density of financial resources for small firms equals a mean preserving spread of the density for large firms.

Figure 4 shows a nonparametric estimate of $K'(x)$. Since $K'$ is convex over the majority of its domain, a mean preserving spread in $x$ will raise the average value of $K'(x)$. Hence, the cash coefficient will be higher for small firms, even though their investment behavior, contingent on their internal financial resources is identical to large firms. The reason is that small firms spend a greater percentage of time constrained. Time spent in the constrained region has a bigger effect on the size of the cash flow coefficient because of the nonlinearity in the policy function. Hence, the observed cross sectional differences between firms in this case arise from the time series properties of financial resources, not from any differences in investment behavior.

---

2 Both densities and policy functions were constructed conditional on $\bar{R}_L$. The figures show the results at a representative value of $\bar{R}_H$.

3 Nonparametric estimates of the derivative of a policy function can be computed by numerically differentiating the estimated policy function, or by differentiating the kernel weights before estimation. See Härdle (1990).
**Figure 3.** Density of Financial Resources by Size Cohort

The narrow densities, with mass concentrated in the center represent large firms. The wide densities with mass concentrated in the tails represent small firms.

**Figure 4.** Nonparametric Estimate of the Derivative of the Policy Function for Capital
It is possible to explicitly compute the integral in equation (4) for different size classes of firms. Unfortunately, results from this computation are very imprecise, and sensitive to small changes in bandwidth or conditioning procedure.\textsuperscript{4,5} In addition, the means of the distributions of financial resources in Figure 3 are somewhat bigger for smaller firms than for larger firms. One possible explanation is that smaller firms choose to hold larger buffer stocks of liquid assets. However, this would imply counterfactually that cash coefficients increase with firm size. Another possible explanation is that small firms are more likely to have missing or inconsistent data at low levels of financial resources.\textsuperscript{6} In order to eliminate this effect, and focus on the different levels of volatility of financial resources, all size classes can be constrained to have the same mean level of financial resources. Table 3 shows the implied values of $\alpha$ from equation (4) in order of increasing firm size class.\textsuperscript{7} Even though the policy function is constrained to be the same across size classes, the estimated cash coefficients fall with firm size.

\textsuperscript{4} Nonparametric estimates of the derivative of a function are much less precise than the estimates of the original function. The rate of convergence is an order of magnitude slower. Hence, small changes in procedure can have large effects.  

\textsuperscript{5} In practice, these changes have a big impact on the shape of the policy function at low levels of liquidity. If the policy function flattens out because of borrowing, then the derivative of the policy function near zero pictured in Figure 4 will drop to approximately zero. This will reduce the estimates of $\alpha$, and have the biggest impact on small firms.  

\textsuperscript{6} This was not a problem in Chapters I and II because estimates were constructed conditional on $x$.  

\textsuperscript{7} The values of $\alpha$ in Table 3 are all larger than the regression estimates in Table 1 and 2. However, the levels of the computed coefficients in Table 3 are very sensitive to the implementation procedure. (See footnotes 4 and 5.) What should be focused on in Table 3 is the comparison between small and large firms.
Table 3. Average Sensitivity of Investment to Cash

\[ \alpha_j = E_j \left( K'(\bar{x}_u) \mid \bar{K}_u \right) = \int K'(\bar{x}_u) \hat{f}_j(\bar{x}_u \mid \bar{K}_u) d\bar{x}_u \]

<table>
<thead>
<tr>
<th>Size Class</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smallest</td>
<td>0.129</td>
</tr>
<tr>
<td>2</td>
<td>0.127</td>
</tr>
<tr>
<td>3</td>
<td>0.118</td>
</tr>
<tr>
<td>4</td>
<td>0.102</td>
</tr>
<tr>
<td>5</td>
<td>0.094</td>
</tr>
<tr>
<td>6</td>
<td>0.090</td>
</tr>
<tr>
<td>Largest</td>
<td>0.087</td>
</tr>
</tbody>
</table>

Without controlling for the amount of time spent in different regimes, it is not possible to determine the source of the cross sectional difference in cash flow coefficients. The most common explanation has been differential access to external funds, which changes investment behavior. This explanation is not consistent with the observed significant difference in cash flow coefficients, when firms are split by size. Instead, it is due to a different distribution of financial resources across size classes. Small firms spend more time in the constrained region with less liquidity, hence they appear to be more sensitive to cash flow. However, contingent on their current liquidity, small firms investment behavior is statistically equivalent to the behavior of large firms.
Chapter IV. Aggregate Investment in a Financially Constrained Economy

I. Introduction

Chapters I-III have focused on the microeconomic problem of a single firm, or a small groups of firms. While the investment behavior of individual firms is certainly of interest, the main reasons that economists care about financial constraints is the possibility that they affect aggregate activity. This chapter will try to address this issue by considering an economy composed of a large number of financially constrained firms of the type previously studied. The principal question that will be investigated is how the economy responds to shocks. Can financial constraints of the previous type provide a propagation mechanism for real shocks to affect the economy over time? How will financial constraints affect the level of economic activity and the persistence of aggregate shocks?

There have been several other studies of aggregate financial transmission mechanisms.¹ This chapter will focus on a different channel, ignoring all other sources of dynamics. It will show that because the policy function for individual firm decisions are nonlinear, the level of aggregate activity will depend on the distribution of financial resources across firms at each point in time. The sum total of financial resources in the economy is not a sufficient statistic for aggregate investment. What matters for investment is how that liquidity is distributed across firms. Intuitively, what matters is the relative number of constrained versus unconstrained firms. Aggregate shocks will shift the distribution of financial resources, changing the level of aggregate activity. Since the distribution of financial resources takes time to adjust, this will provide a propagation mechanism for aggregate shocks, generating persistence in the business cycle.

¹For example, Bernanke and Gertler (1989) study a model in which changes in firm net worth affect the agency cost of borrowing money. Kiyotaki and Moore (1995) study a model in which changes in the price of collateral affect the ability of firms to borrow.
II. General Equilibrium

Consider an economy made up of a large number of financially constrained firms of the type considered in Chapter 1. Each firm will be considered to be an intermediate good producer, indexed by $i \in [0,1]$. Intermediate goods firms produce output $y_{it}$ which they sell to a final good producer.

A. Final Good Producer

The final good producer makes a composite good $Y_t$ by combining all of intermediate goods with a Dixit-Stiglitz production function.

$$Y_t = \left( \int_0^1 y_{it}^p \, di \right)^{\frac{1}{p}}. \quad (1)$$

This final good is sold back to consumers and firms for consumption and investment.

The final good producer is perfectly competitive, and sells the composite good at a constant price equal to one. The demand for good $i$ is the solution to the final good producer's maximization problem:

$$\max_{\{y_t\}} \quad Y_t - \int_0^1 p_i y_{it} \, di. \quad (2)$$

The first order condition for this problem gives the demand curve for good $i$.

Demand for good $i$: $p_i = \left( \frac{y_{it}}{Y_t} \right)^{p-1}. \quad (3)$
B. Intermediate Good Producer

Gross profits of an intermediate good firm equal revenues minus costs. Costs are assumed to be proportional to the value of output.

$$\pi_{it} = p_{it} y_{it} (1 - c_{it})$$
$$= y_{it}^\rho Y_t^{1-\rho} (1 - c_{it}).$$

(4)

Firm profits are therefore proportional to a geometric average of individual and aggregate output. Costs are stochastic, and have an idiosyncratic, i.i.d. component $z$, as well as an aggregate component $\omega$ which is assumed to follow a stationary, first order, Markov process.

$$(1 - c_{it}) = z_{it} + \omega_t.$$ 

Output of an intermediate good firm is a decreasing returns to scale function of the capital stock. The production function can be written as,

$$y_{it} = k_{it}^\alpha.$$ 

(5)

Substituting the production function and the stochastic shocks into the profit function implies that

$$\pi_{it} = (z_{it} + \omega_t)Y_t^{1-\rho} k^{\alpha\rho}.$$ 

(6)

Note that equation (6) has basically the same form as the profit function studied in Chapter 1. The stochastic shock now consists of an idiosyncratic and an aggregate component, and there is an additional aggregate demand effect.

---

2 Note that intermediate good firms will have different levels of liquidity depending upon their past shocks. Therefore they will produce different levels of output, which will be sold at different prices. So $y_{it} \neq Y_t \forall i$. 

90
C. Consumers

The representative consumer is risk neutral, with time separable utility. The consumer owns all of the firms in the intermediate good sector, so his income equals the aggregate dividend stream. The interest rate on consumer savings equals the rate of time preference. The consumer maximization problem can therefore be written as,

$$\max \sum_{t=0}^{\infty} \left( \frac{1}{1+\theta} \right) C_t$$

s.t.
$$W_t = (1+\theta)W_{t-1} - C_t + Div_t. \quad (7)$$

Given these preferences, the consumer is indifferent about the timing of his consumption. Hence the level of consumption is determined by the bank which determines the level of net consumer savings or dissaving.

$$C_t = Div_t - \Delta W_t, \quad \text{where}$$
$$\Delta W_t = W_t - (1+\theta)W_{t-1}. \quad (8)$$

Unlike consumers, firms are not indifferent about their savings decision. The bank insures that consumers take the opposite position of firm savings minus firm borrowing.\(^3\)\(^4\) Therefore,

$$\Delta W_{t+1} = -M_{t+1} + (1+r)M_t + B_{t+1} - (1+\theta)B_t$$
$$= \Delta B_{t+1} - \Delta M_{t+1} \quad (9)$$

\(^3\) Firms waste a fraction \((1-\alpha)\) of their cash balances on managerial perquisites. Hence, the net return to firm savings is \((1+r) = (1-\alpha)(1+\theta).\)

\(^4\) Banks charge intermediate good firms an interest rate so that they make zero expected profits. \((1+\theta) = \left(1+i_{lt}\right) \Pr\left[ x_{lt} \geq 0 \right].\) While \(i_{lt}\) will vary across firms and time, the average return to the bank, \(\theta,\) is fixed.
D. Policy Functions

Since consumers discount the future at a constant interest rate, and the profit function is of the form (6), the functional equation for the value of an intermediate good firm can be written in the standard form from Chapter 1, as follows.

\[ V_u = \max_{k_{it+1}, m_{it+1}, b_{it+1}} x_{it} - k_{it+1} - m_{it+1} + b_{it+1} + \frac{1}{1+\theta} E_r V_{it+1} \quad (10) \]

subject to

\[ m_{it+1} \geq 0, \]
\[ k_{it+1} \geq 0, \]
\[ m_{it+1} + k_{it+1} \leq x_{it} + b_{it+1} \quad (11) \]

where

\[ x_{it+1} = \pi_{it+1} + (1-\delta)k_{it+1} + (1+r)m_{it+1} - (1+i_{it+1})b_{it+1}. \quad (12) \]

Since the profit function now depends on the aggregate shock, \( \omega_{t+1} \), and the level of aggregate demand, \( Y_{t+1} \), the policy functions for \( k_{it+1} \), \( m_{it+1} \), \( b_{it+1} \), and \( div_{it+1} \) will also depend on the these variables and their laws of motion. But the level of aggregate output and its law of motion will depend on the entire distribution of financial resources within the economy, as will be demonstrated below. Since it is implausible to assume that an intermediate good firm is aware of this distribution, instead assume that firms only know the current value of \( Y_{t+1} \) and \( \omega_{t+1} \). Hence, policy functions for intermediate good firm decisions can be written as

\[ k_{it+1} = K(x_{it}, Y_{it+1}, \omega_{it+1}) \]
\[ m_{it+1} = M(\cdot) \]
\[ div_{it+1} = Div(\cdot) \]
\[ b_{it+1} = B(\cdot). \quad (13) \]
E. Aggregation

The level of aggregate output can be determined by aggregating the output of all of the intermediate goods producers.

\[ Y_{t+1} = \left( \int_{0}^{\infty} y_{t+1}^{p} dt \right)^{\gamma_{p}}. \]  

(14)

Substituting the production function and the policy function for capital for intermediate good firms into equation (14) implies that

\[ Y_{t+1} = \left( \int_{0}^{\infty} K(x, Y_{t+1}, \omega_{t+1})^{\alpha p} di \right)^{\gamma_{p}}. \]  

(15)

Or, in terms of the distribution of financial resources,

\[ Y_{t+1} = \left( \int_{0}^{\infty} K(x, Y_{t+1}, \omega_{t+1})^{\alpha p} f_{t}(x) dx \right)^{\gamma_{p}}. \]  

(16)

Thus, the level of aggregate output is the solution to a fixed point problem, which always admits a unique solution, because of the concavity of the production function.\(^{5}\) Hence,

\[ Y_{t+1} = \mathcal{Y}(\omega_{t+1}, f_{t}(x)). \]  

(17)

Similarly, the future distribution of financial resources can be calculated by integrating over all intermediate good firms' decisions.

\[ f_{t+1}(x) = \mathcal{X}(\omega_{t+1}, f_{t}(x)). \]  

(18)

\(^{5}\) If there were constant returns to scale in the production function (\(\alpha = 1\)), in an economy with no financial constraints, any level of aggregate output would be sustainable, if all firms believed that other firms were also going to produce at that level. In a financially constrained economy with CRS, output is restricted to be less than some maximum level of output because firms are limited in the amount they are willing to invest.
Note that because there is a positive probability of bankruptcy, there must be some birth process for new firms to enter the economy. Otherwise, the number of firms, and all aggregate activity, would shrink to zero. There are several different ways to model the birth process. For simplicity, and to avoid confounding other effects, assume that new firms enter the economy with exactly the same distribution of financial resources as currently solvent firms. This must be taken into account when solving for equation (18).\footnote{A more complete model would allow free entry of new firms, but at some cost greater than the amount of financial resources that the firm begins with.}

\section*{F. Timing}

The timing of information and decisions are illustrated below, starting at the beginning of period \( t \).

Period \( t \):

- final good sector has \( Y_{t-1} \)
- firms have \( x_{it-1} \) with cross sectional distribution \( f_{t-1}(x) \)
- \( \omega_t \) is learned
- everyone guesses \( Y_t = Y(\omega_t,f_{t-1}(x)) \)
- firms order: \( k_{it}(x_{it-1},Y_t,\omega_t) \) from final good producer
- pay out: \( div_{it}(x_{it-1},Y_t,\omega_t) \) to consumer
- save: \( m_{it}(x_{it-1},Y_t,\omega_t) \)
- borrow: \( b_{it}(x_{it-1},Y_t,\omega_t) \)
- consumer orders: \( C_t = Div_t - \tilde{\Delta}W_t \) from final good producer

Production: firms learn \( z_{it} \), and produce \( y_{it} \)

\( y_{it} \) is sold to the intermediate good sector for \( \pi_{it} \)

this generates \( x_{it},f_t(x) \), and \( Y_t \) and the process starts over.
This process result in an equilibrium in which all of the composite good is sold to either consumers or firms. This can be seen by comparing consumer and producer demand for the good with the supply of the final good producer.

\[
C_t + I_t = \left( \text{Div}_t + M_t - (1+r)M_{t-1} - B_t + (1+\theta)B_{t-1} \right) + \left( K_t - (1-\delta)K_{t-1} \right) \\
= \left( K_t + M_t + \text{Div}_t + B_t \right) - \left( (1-\delta)K_{t-1} + (1+r)M_{t-1} - (1+\theta)B_{t-1} \right) \\
= X_{t-1} - (X_{t-1} - \Pi_{t-1}) \\
= \Pi_{t-1} \\
= \int_0^1 (z_{it-1} + \omega_{t-1}) Y_{it-1}^{1-\rho} Y_{i-1} \, di \\
= Y_{t-1}(\bar{z} + \omega_{t-1}).
\]

But, \((\bar{z} + \omega_{t-1}) = 1 - \bar{c}_{t-1}\). So

\[
C_t + I_t + Cost_t = Y_{t-1}(\bar{z} + \omega_{t-1}) + Y_{t-1}\bar{c}_{t-1} \\
= Y_{t-1}.
\]

Hence aggregate demand equals aggregate supply.

\[\]

**G. Response to Shocks**

Consider the response of the economy to an aggregate shock. An increase in \(\omega_t\) has several immediate effects on expected firm profits,

\[
\pi^*_{it} = (z_{it} + \omega_i) Y_{it}^{1-\rho} k_{it}^{\text{op}}.
\]

Expected profits rise immediately for three reasons.

\(i\) Direct effect: firms are more productive, which raises expected profits.

\(ii\) Aggregate Demand effect: other firms produce more, causing \(Y_t\) to rise, which increases the demand for your good.
iii) Investment effect: $k_{it}$ increases because firms expect to earn a higher return on their capital, so they invest more.

The effects of shocks persist, because they change the distribution of financial resources within the economy, which affects aggregate demand.

$$\omega_t \uparrow \Rightarrow Y_t, K_t \uparrow \Rightarrow f_t(x) \text{ shifts to the right} \Rightarrow Y_{t+1}, K_{t+1} \uparrow \ldots$$

III. Dynamic Evolution of the Economy

In order to more fully illustrate dynamic properties of this economy, we can consider several different experiments.

A. Transition to Steady State

First suppose the economy begins with an initial distribution of financial resources that is uniformly distributed. The aggregate shock is constant for all periods, so the economy will converge to a steady state level of output, and a steady state distribution of financial resources. Figures 1 and 2 show the dynamic evolution of $Y_t$ and $f_t(x)$ corresponding to equations (17) and (18). Output converges monotonically to a higher level of output, while $f$ converges to an ergodic distribution with thinner tails than the uniform. Since $\omega_t$ is constant, all of the dynamics result from changes in the distribution of financial resources. Consider what happens as the tails of $f$ get thinner. This is like a mean
Figure 1. Aggregate Output
Transition to Steady State

Figure 2. Distribution of Financial Resources
Transition to Steady State
preserving decrease in the spread of \( x_{it} \). Since the policy function \( K(x_{it}) \) is concave, the aggregate level of capital, and output rises in the economy.\(^7\)

Intuitively, there are more firms that are very constrained (and very unconstrained) in the economy at the beginning of the experiment, when financial resources are approximately uniform. As time goes on, the number of firms with extreme levels of liquidity is reduced.\(^8\) Firms that are very unconstrained have a low marginal propensity to invest because they are near their desired capital stock. So a change in the level of liquidity of these firms has little effect on the aggregate capital stock. However, exactly the opposite is true for very constrained firms. Adding liquidity to a constrained firm, or equivalently reducing the number of constrained firms, results in a large increase in aggregate output. The combined effect is to raise the aggregate level of capital and output in the economy.

**B. Permanent Aggregate Shock**

Next consider an economy which is initially in steady state, with \( \omega_i = \bar{\omega} \). At time \( t = T \), there is an aggregate shock which increases \( \omega \) permanently to \( \omega_i = \bar{\omega} \). Figures 3 and 4 show the evolution of aggregate output and the distribution of financial resources over time. At \( t = T \), the three impact effects on expected profits raises productivity, aggregate demand, and investment, all of which cause aggregate output to increase. However, \( Y_t \) does not jump to its new steady state value because the distribution of financial resources is initially fixed. Over time, the increase in profits shifts \( f_t(x) \) to the right as firm liquidity

---

\(^7\)This is similar to the mechanism which was found to operate empirically in Chapter III, to explain cross sectional differences in cash flow regressions. In Chapter III the relevant distribution was the time series distribution of future liquidity for a given size class of firms. Here, the relevant distribution is the cross sectional distribution of current liquidity across firms.

\(^8\) Note that which firms are constrained, and which firms are unconstrained is changing over time.
Figure 3. Aggregate Output
Permanent Aggregate Shock

Figure 4. Distribution of Financial Resources
Permanent Aggregate Shock
rises. This increases the aggregate capital stock and aggregate output, and the economy converges to the new steady state.

C. Temporary Aggregate Shock

Next consider an economy which is initially in steady state, with $\omega_t = \omega$. At time $t = T$, there is an aggregate shock which temporarily increases $\omega_t$ to $\bar{\omega}$. Figures 5 and 6 show the evolution of aggregate output and the distribution of financial resources over time. The impact effect raises output and shifts the distribution of financial resources to the right. Afterwards, even though $\omega_t$ has returned to its old value, aggregate output remains higher than the steady state value. This is because the distribution of financial resources has shifted, so that there are fewer financially constrained firms.

---

9 The shift of $f_t(x)$ looks like a downward shift in figure 4 because there is more mass in the tail corresponding to large $x$, after the shock.
Figure 5. Aggregate Output
Temporary Aggregate Shock

Figure 6. Distribution of Financial Resources
Temporary Aggregate Shock
D. Markov Distribution for Aggregate Shock

Finally, suppose that $\omega_t$ follows a stationary, symmetric, two-state, first-order Markov process.

\[
\begin{align*}
P(\omega_t = \bar{\omega} | \omega_{t-1} = \bar{\omega}) &= p, \\
1 - P(\omega_t = \omega | \omega_{t-1} = \bar{\omega}) &= 1 - p \\
1 - P(\omega_t = \omega | \omega_{t-1} = \omega) &= 1 - p, \\
P(\omega_t = \omega | \omega_{t-1} = \omega) &= p.
\end{align*}
\]

Figure 7 shows the policy function for next periods stock of capital, $k_{it+1}$, conditional on $\omega_{t+1}$. The policy function illustrates the investment effect. When $\omega_{t+1}$ is in the high state, the firm's desired stock of capital is high, so it invests more if it can. Figures 8 and 9 show the dynamic evolution of $Y_t$ and $f_t(x)$ for $p = 4/5$. 
Figure 8. Aggregate Output
Markov Process for Aggregate Shock

Figure 9. Distribution of Financial Resources
Markov Process for Aggregate Shock
E. Persistence

Inspection of Figure 8 illustrates that the impact effect of an aggregate shock is an order of magnitude larger than the persistence effect. This is because all of the effects of an aggregate shock which lead to persistence also operate in the initial period. Consider the three reasons why profits rise after a positive shock: $\omega$ rises, $k_{it}$ rises, and $Y_t$ rises. Each of these effects operates in the initial period leading to an increase in output. In subsequent periods, the latter two effects continue to lead to an increase in output as firms become less constrained. However, the initial impact effect will always be much larger unless the initial distribution of financial resources is degenerate.

Actual economies have other frictions which also lead to persistence. Any friction which causes firms to delay immediately increasing their capital stocks after a positive shock will interact with financial constraints to further delay the adjustment process. One example of such a friction is adjustment costs to capital, which were discussed in Chapters I and II. This chapter ignored all other frictions and transmission mechanisms to illustrate most clearly the effects of a change in the distribution of financial resources on aggregate activity.
IV. Conclusion

This chapter has demonstrated that financially constraints which generate nonlinear policy functions will generate a business cycle transmission mechanism for real shocks. Aggregate shocks shift the distribution of financial resources which changes the relative number of constrained versus unconstrained firms. It is this distribution which determines the current level of economic activity. As more firms become unconstrained, aggregate demand increases, further increasing the level of investment. This will lead to a further shift in the distribution of financial resources, generating persistence in the business cycle.
Chapter V. Empirical Aggregation

I. Introduction

Chapter IV argued theoretically that microeconomic financial constraints can affect aggregate economic activity. The purpose of this chapter is to investigate whether firm level financial constraints of the type previously studied are empirically relevant for the macroeconomy. The central hypothesis of Chapter IV is that if firms have nonlinear policy functions then the level of aggregate investment or output will be a function of the distribution of financial resources within the economy. This chapter will attempt to estimate the microeconomic policy functions of individual firms using only aggregate sectoral data and data on the distribution of financial resources across firms. In particular, this chapter will investigate whether time series changes in the distribution of financial resources can explain aggregate economic activity, and more stringently, in a way which is consistent with the microeconomic decisions of individual firms.

The methodology which will be employed is a variant of a procedure suggested by Caballero and Engel.\footnote{See for example Caballero and Engel (1993) and Caballero, Engel, and Haltiwanger (1994).} In a model with nonconvex costs of adjusting labor, they study how the distribution of firm deviations from the desired numbers of employees affects aggregate labor demand. While all previous applications of this methodology have been to Ss type models, any nonlinear model will have similar properties. Caballero and Engel's main idea is that aggregate activity is simply the sum of individual actions. It is possible to estimate individual policy functions from data on aggregate activity by taking a discrete approximation to the convolution of individual policy functions with the distribution of the state variable. When this methodology is employed in this context,
using aggregate sectoral data and data on the distribution of financial resources across firms, the estimated policy functions for firm output are very similar to the policy functions that were estimated in Chapter II using firm level data. This provides evidence that microeconomic constraints do influence the level of aggregate activity.

II. Methodology

At the aggregate level, capital stock data is extremely smooth. It is difficult to explain the short run time series properties of such a smooth, trending series without large serial correlation problems. However, in all of the theoretical specifications in Chapters I and IV, there is a one to one correspondence between output and capital. Typically, firm output was assumed to be a concave (or constant returns to scale) function of firm capital. Hence, the policy function for firm output will inherit all of the properties of the policy function for capital. In particular, it will be flat initially, then concave, leveling off at a desired level of output.

Consider the policy function for output for an individual firm $j$. Initially, for expositional purposes, ignore differences in firm scale, and the nature of the stochastic shock. One possible specification is

$$y_{j,t+1} = z_{j,t+1}Y(x_{j,t})$$

Then aggregate output can be found by integrating over the outputs of all firms in the sector.\(^2\)

---

\(^2\) There are two principal explanations for the smoothness of capital at the aggregate level, which have been ignored. The first is measurement error, or problems with data construction. The second is adjustment costs. In order to concentrate only on financial mechanisms for aggregate activity, adjustment costs are ignored. See Chapters 1 and 2 for a discussion of adjustment costs.

\(^3\) It is also possible to use Dixit-Stiglitz aggregation as in Chapter 4. Results are qualitatively similar.
\[ Y_{t+1} = \int_0^y y_{t+1} \, dj \]
\[ = \int_0^y z_{t+1} \, Y(x) \, dx \]

Since the future value of the idiosyncratic shock is independent of the current value of financial resources, this can be rewritten as

\[ Y_{t+1} = \int_0^y \int_0^{z_{t+1}} Y(x) f_r(x) \, dx \, dj \]
\[ = \left( \int_0^{z_{t+1}} dj \right) \left( \int_0^y Y(x) f_r(x) \, dx \right) \]
\[ = \bar{z}_{t+1} \left( \int_0^y Y(x) f_r(x) \, dx \right). \]

In order to change this into a form which can be implemented empirically, discretize the policy function \( Y \) and the density of financial resources \( f \) using step functions. Figure 1 illustrates the discretization of the policy function.

**Figure 1.** Discretized Policy Function for Output
Mathematically,

\[ Y(x) = \beta_l, \quad x \in \left[ x^{(l)}, x^{(l+1)} \right), \quad l \in \{0, \ldots, n-1\}. \]  \hspace{1cm} (4)

Define the empirical cross sectional density, \( \{f_{l,t}\} \) as

\[ f_{l,t} = P \left[ x \in \left[ x^{(l)}, x^{(l+1)} \right] \right]. \]  \hspace{1cm} (5)

Then,

\[ Y_{t+1} = \overline{z}_{t+1} \left( \sum_{l=0}^{n-1} \beta_l f_{l,t} \right) \]  \hspace{1cm} (6)

So the shape of the policy function for individual firms can be found by estimating the \( \beta_l \) coefficients. This can be implemented by fitting data on the microeconomic distribution of financial resources, \( \{f_{l,t}\} \), to aggregate output data. Note that the \( \beta \)'s can only be estimated up to scale because of the parameter \( \overline{z} \). In Section IV, the precise procedure for implementing this will be discussed, taking into account differences in firm size, industry effects, and the nature of the stochastic shock.

III. Data

Data for the distribution of financial resources was taken from the Compustat database using the normalization methodology discussed in Chapter II. Figure 2 plots \( f_{l,t} \) as a function of time, for a 6 point discretization, ranging from a minimum for \( \overline{x} \) of 0 to a maximum of 2.4. As the figure illustrates, there is substantial time series variation in the quantiles of the distribution.
In order to provide a sufficient number of observations for the regression analysis, sectoral data at the 2-digit SIC level were used. Separate measures of \( \{f_{l,t}\} \) are available by sector, since Compustat provides principal 4-digit SIC codes for all firms. Using the twenty manufacturing sectors with SIC codes between 20 and 39, and annual data from 1974-92 provides 380 observations.

**IV. Specification and Results**

In order to implement the methodology suggested in Section II, we must take into account firm and sector level heterogeneity. Consider the following general policy function for firm \( j \) and sector \( i \).

\[
y_{j,i,t+1} = \exp\left( A_j + \mu_i t + \omega_{i,t+1} + \eta_{i,i,t+1} + z_{j,i,t+1} \right) \nu(\bar{x}_{j,i})
\]

(7)
$A_j$ is a fixed firm size effect, $\mu_i$ is a sector specific trend, $\omega_{t+1}$ is an aggregate shock, $\eta_{i,t+1}$ is a sector shock, and $\varepsilon_{i,t+1}$ is an idiosyncratic shock. Aggregate output for sector $i$ can be found by integrating over the outputs of all firms in the sector, just like in Section II.

$$Y_{i,t+1} = \int_{j \in \text{(sect i)}} \exp \left( A_j + \mu_i t + \omega_{t+1} + \eta_{i,t+1} + z_{j,t+1} \right) Y(\tilde{x}, j) dj$$

$$= \exp \left( \mu_i t + \omega_{t+1} + \eta_{i,t+1} \right) \left( \int_{j \in \text{(sect i)}} \exp \left( A_j + z_{j,t+1} \right) dj \right) \int_0^1 Y(\tilde{x}, j) f_j(\tilde{x}) d\tilde{x}$$

$$= \exp \left( \mu_i t + \omega_{t+1} + \eta_{i,t+1} \right) \left( \sum_{l=0}^{l \leq l} \beta_l f^{i}_{l,t} \right)$$

(8)

Or in logs,

$$\ln Y_{i,t+1} = \alpha_i + \mu_i t + \omega_{t+1} + \ln \left( \sum_{l=0}^{l \leq l} \beta_l f^{i}_{l,t} \right) + \eta_{i,t+1}.$$  

(9)

There are several things to note about equation (9). The $\alpha$'s are fixed effects, the $\mu$'s are sector specific trends, the $\omega$'s are aggregate shocks, and the $\eta$'s are sector specific stochastic shocks, which are assumed to be orthogonal to all of the regressors. This equation cannot be estimated by OLS because of the nonlinearity of the $\beta$'s, but it can be estimated using maximum likelihood. Finally, the $\beta$'s can only be estimated up to a normalization, just like in Section II. This is because doubling the scale of the policy function $Y(\alpha)$ is equivalent to changing all of the firm specific scale parameters, $A_j$. Hence, all results will be shown normalized by the maximum value of $\beta$.

Three different sets of results of varying degrees of generality will be presented. All results will include sector fixed effects and will be corrected for first order serial
correlation. Specification 1 will include an economy wide trend, with no sector specific trends or fixed time effects. Specification 2 will include sector specific trends, but again no fixed time effects. Finally, Specification 3 will include fixed time effects. The specifications will be presented at differing discretizations for robustness.

**Specification 1:**

\[
\ln Y_{i,t+1} = \alpha_i + \mu t + \ln \left( \sum_{l=0}^{n-1} \beta_l f_{i,t}^l \right) + \eta_i, t+1.
\]

In order to estimate this equation, take first differences to eliminate the fixed effects. The equation which is estimated by maximum likelihood is

\[
\ln \frac{Y_{i,t+1}}{Y_{i,t}} = \mu + \ln \left( \sum_{l=0}^{n-1} \beta_l f_{i,t}^l \right) + \tilde{\eta}_{i,t+1}.
\]

(10)

Clearly, all of the \( \beta \)'s are not identified, since multiplying them by a constant does not change the right hand side of equation (10). So the maximum likelihood procedure estimates \( \frac{\beta_i}{\beta_0} \) by normalizing \( \beta_0 \) to equal 1. When the policy functions are graphed, the maximum value of \( \beta \) is used for the normalization so that the maximum of the policy function is at 1. Tables 1 and 2 show the estimation results for 5 and 10 point grids.

All of the coefficients in both specifications are highly statistically significant, and of similar magnitudes. More importantly, the shapes of the estimated policy functions closely match both the theoretical predictions from Chapter I, and the empirical estimation results using micro data in Chapter II. Figures 3 and 4 show the estimated policy functions from Specification 1.

---

4 Estimation is by maximum likelihood, allowing the error to follow an AR(1) process. The econometric procedure for allowing for serial correlation in a nonlinear model with an additive error is the same as in the linear case.
### Table 1. Specification 1
Maximum Likelihood Estimation
\( n = 5, xmax = 2. \)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 )</td>
<td>0.8652</td>
<td>0.1269</td>
<td>6.820</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>1.1007</td>
<td>0.1483</td>
<td>7.422</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>1.1769</td>
<td>0.1609</td>
<td>7.313</td>
</tr>
<tr>
<td>( \beta_4 )</td>
<td>1.0896</td>
<td>0.1640</td>
<td>6.643</td>
</tr>
<tr>
<td>( \beta_5 )</td>
<td>1.1723</td>
<td>0.1715</td>
<td>6.834</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.0163</td>
<td>0.0039</td>
<td>4.164</td>
</tr>
<tr>
<td>( \rho )</td>
<td>-0.1404</td>
<td>0.0553</td>
<td>-2.538</td>
</tr>
</tbody>
</table>

\( F(5,333) = 3.0919 \quad p \text{ value} = 0.0096 \)

Estimation in first differences, allowing for first order serial correlation. \( \beta_0 \) is normalized to equal one.

### Table 2. Specification 1.
Maximum Likelihood Estimation
\( n = 10, xmax = 2. \)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 )</td>
<td>0.5725</td>
<td>0.1799</td>
<td>3.183</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.7050</td>
<td>0.1482</td>
<td>4.756</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>0.7288</td>
<td>0.1199</td>
<td>6.078</td>
</tr>
<tr>
<td>( \beta_4 )</td>
<td>0.8387</td>
<td>0.1278</td>
<td>6.563</td>
</tr>
<tr>
<td>( \beta_5 )</td>
<td>0.9653</td>
<td>0.1446</td>
<td>6.675</td>
</tr>
<tr>
<td>( \beta_6 )</td>
<td>0.9454</td>
<td>0.1446</td>
<td>6.540</td>
</tr>
<tr>
<td>( \beta_7 )</td>
<td>1.0116</td>
<td>0.1542</td>
<td>6.562</td>
</tr>
<tr>
<td>( \beta_8 )</td>
<td>0.9791</td>
<td>0.1583</td>
<td>6.186</td>
</tr>
<tr>
<td>( \beta_9 )</td>
<td>0.9024</td>
<td>0.1773</td>
<td>5.090</td>
</tr>
<tr>
<td>( \beta_{10} )</td>
<td>0.9991</td>
<td>0.1527</td>
<td>6.545</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.0167</td>
<td>0.0039</td>
<td>4.286</td>
</tr>
<tr>
<td>( \rho )</td>
<td>-0.1386</td>
<td>0.0558</td>
<td>-2.484</td>
</tr>
</tbody>
</table>

\( F(10,328) = 3.4122 \quad p \text{ value} = 0.0003 \)

Estimation in first differences, allowing for first order serial correlation. \( \beta_0 \) is normalized to equal one.
Figure 3. Policy Function for Output
2-Digit Manufacturing Sectors
Specification 1. $n = 5$.

Figure 4. Policy Function for Output
2-Digit Manufacturing Sectors
Specification 1. $n = 10$. 
While there is some noise in the estimates, the policy functions consistently show the robust concavity, leveling off at a maximum, which was demonstrated using micro data in Chapter II. Furthermore, there appears to be a flattening or an increase in the policy function at low levels of liquidity, just like in the micro estimation. This is exactly what is predicted by the model with borrowing. These qualitative results are robust to changes in the discretization, or the maximum value of x.

The estimated value of the aggregate trend, μ, is equal to about 1.6% per year and is highly statistically significant in both discretizations. The estimated value of ρ is about -0.14 and also significant in both discretization.\(^5\) Since this serial correlation coefficient is from estimation in first differences, this is evidence of a high degree of persistence in the residual of the original model, equation (9). This suggests that other mechanisms beside financial constraints are also important for explaining aggregate dynamics.

The F statistic at the bottom of each of the tables tests whether all of the estimated coefficients are equal to β₀ which is normalized to one. This test can be thought of in two ways. From an economic point of view, testing whether all of the coefficients equal one is a test of the null hypothesis that there are no financial constraints. If the neoclassical model were true, then firm output should be independent of liquidity, so the estimated policy function should be a horizontal line at one. The test that all of the coefficients are equal to one can also be interpreted in a purely statistical fashion. Since \(\left\{f_{i,t}^i\right\}\) is a discretization of a density, \(\left\{f_{i,t}^i\right\}\) sums to one for each \(i\) and \(t\). So under the null, \(\sum_{i=0}^{\infty} \beta_i f_{i,t}^i = 1\). Hence a test that all of the β's are equal to one is equivalent to a test that the distribution of financial resources has more information than a constant. For both discretizations, we can reject the hypothesis at the 1% level.

---

\(^5\) Although statistically significant, the absolute magnitude of ρ is small. Hence, in Specification 1, ignoring the serial correlation does not have a qualitative impact on the shape of the estimated policy function.
Specification 2: \[ \ln Y_{i, t+1} = \alpha_i + \mu_i t + \ln \left( \sum_{j=0}^{q-1} \beta_i f_{t,i}^j \right) + \eta_{i, t+1}. \]

The second specification is similar to Specification 1, except now the trend is allowed to differ by sector. This is important because there was substantial heterogeneity in the growth experiences of different manufacturing sectors during this period. Some industries, for example tobacco, experienced declines. In order to estimate this equation, take second differences to eliminate both the fixed effects and the fixed trend effects.

The equation which is estimated by maximum likelihood is

\[ \Delta^2 \ln Y_{i, t+1} = \Delta^2 \ln \sum_{j=0}^{q-1} \beta_i f_{t,i}^j + \tilde{\eta}_{i, t+1}. \] (11)

A similar normalization procedure is used for the \( \beta \)'s, and first order serial correlation in \( \tilde{\eta} \) is controlled for. Tables 3 and 4 give the estimation results for the two discretizations.

The coefficients are again highly significant, and of similar orders of magnitude. Since the estimation is now in second differences, the estimated serial correlation is much higher, equal to about -0.6.\(^6\) Figures 5 and 6 plot the shapes of the estimated policy functions. Again the policy functions have the traditional concave shape, closely resembling the estimation results from Specification 1. The initial flattening for low values of \( x \) is apparent in both discretizations, and takes the form of a sharp drop in Figure 6 to a normalized value of \( \beta_1 = 0.47 \). However, this estimate is not extremely precise, with a one standard deviation increase in \( \beta_1 \) raising the normalized value to 0.64. The F test that all of the \( \beta \)'s are equal to one is again rejected at a high level of significance.

\(^6\) Since \( \rho \) is fairly large in absolute value, ignoring serial correlation changes the estimated coefficients. Once serial correlation is taken into account, the results are very close to the estimates in first differences in Specification 1.
Table 3. Specification 2.
Maximum Likelihood Estimation

\[ n = 5, \text{xmax} = 2. \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 )</td>
<td>0.9661</td>
<td>0.1439</td>
<td>6.715</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>1.2076</td>
<td>0.1677</td>
<td>7.202</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>1.3239</td>
<td>0.1853</td>
<td>7.146</td>
</tr>
<tr>
<td>( \beta_4 )</td>
<td>1.1781</td>
<td>0.1809</td>
<td>6.511</td>
</tr>
<tr>
<td>( \beta_5 )</td>
<td>1.2544</td>
<td>0.1839</td>
<td>6.713</td>
</tr>
<tr>
<td>( \rho )</td>
<td>-0.5912</td>
<td>0.0446</td>
<td>-13.25</td>
</tr>
</tbody>
</table>

\[ F(5,297) = 2.4844 \text{ p value} = 0.0317 \]

Estimation in second differences, allowing for first order serial correlation. \( \beta_0 \) is normalized to equal one.

Table 4. Specification 2.
Maximum Likelihood Estimation

\[ n = 10, \text{xmax} = 2. \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 )</td>
<td>0.4986</td>
<td>0.1711</td>
<td>2.914</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.6790</td>
<td>0.1526</td>
<td>4.448</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>0.7908</td>
<td>0.1280</td>
<td>6.179</td>
</tr>
<tr>
<td>( \beta_4 )</td>
<td>0.8791</td>
<td>0.1360</td>
<td>6.466</td>
</tr>
<tr>
<td>( \beta_5 )</td>
<td>1.0359</td>
<td>0.1548</td>
<td>6.690</td>
</tr>
<tr>
<td>( \beta_6 )</td>
<td>1.0401</td>
<td>0.1564</td>
<td>6.649</td>
</tr>
<tr>
<td>( \beta_7 )</td>
<td>1.1034</td>
<td>0.1650</td>
<td>6.689</td>
</tr>
<tr>
<td>( \beta_8 )</td>
<td>1.0323</td>
<td>0.1633</td>
<td>6.323</td>
</tr>
<tr>
<td>( \beta_9 )</td>
<td>0.9538</td>
<td>0.1830</td>
<td>5.212</td>
</tr>
<tr>
<td>( \beta_{10} )</td>
<td>1.0545</td>
<td>0.1579</td>
<td>6.678</td>
</tr>
<tr>
<td>( \rho )</td>
<td>-0.5994</td>
<td>0.0446</td>
<td>-13.45</td>
</tr>
</tbody>
</table>

\[ F(10,328) = 3.2812 \text{ p value} = 0.0005 \]

Estimation in second differences, allowing for first order serial correlation. \( \beta_0 \) is normalized to equal one.
**Figure 5.** Policy Function for Output
2-Digit Manufacturing Sectors
Specification 2. \( n = 5 \).

**Figure 6.** Policy Function for Output
2-Digit Manufacturing Sectors
Specification 1. \( n = 10 \).
Specification 3: \[ \ln Y_{t,t+1} = \alpha_i + \omega_{t+1} + \ln \left( \sum_{l=0}^{n-1} \beta_{i} f_{l,t} \right) + \eta_{t,t+1}. \]

The final specification includes fixed time effects, to allow for the possibility of aggregate shocks.\(^7\) The results are similar if sector specific trends are also included. In order to eliminate the fixed sector effects, we can first difference, just like in Specification 1. In order to eliminate the fixed time effects, difference each group from the equation for the aggregate economy. The equation to be estimated by maximum likelihood is

\[ \Delta \ln Y_{t,t+1} = \Delta \ln \sum_{l=0}^{n-1} \beta_{i} f_{l,t} - \Delta \ln \sum_{l=0}^{n-1} \beta_{i} f_{l,t} + \hat{\eta}_{t,t+1}. \]  

(12)

The estimates for this specification are somewhat volatile, so Table 5 gives the estimation results for a fairly coarse discretization, \(n = 4\). Figure 7 plots the estimated policy function.

The shape of the estimated policy function once again seems to match the previous specifications. However, the flattening of the policy function for large values of \(x\) is no longer as obvious. This is most likely due to imprecision in the estimates. In fact, the F test is no longer statistically significant. Thus we are unable to reject the hypothesis that the policy function is a horizontal line at one. Hence, while the point estimate of the policy function in Specification 3 is of the appropriate shape, restricting attention to sector deviations from the aggregate economy induces too much imprecision in the estimates to make reliable inferences.

\(^7\) See Chapter IV for a theoretical description of a dynamic transmission mechanism for aggregate shocks.
Table 5. Specification 3.
Maximum Likelihood Estimation
\( n = 4, xmax = 2. \)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 )</td>
<td>0.9366</td>
<td>0.1108</td>
<td>8.452</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.9863</td>
<td>0.1146</td>
<td>8.609</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>1.0835</td>
<td>0.1401</td>
<td>7.731</td>
</tr>
<tr>
<td>( \beta_4 )</td>
<td>1.1004</td>
<td>0.1478</td>
<td>7.446</td>
</tr>
<tr>
<td>( \rho )</td>
<td>-0.1088</td>
<td>0.0537</td>
<td>-2.025</td>
</tr>
</tbody>
</table>

\( F(4,317) = 0.7258 \) \( p \) value \( 0.5744 \)

Estimation in two first differences over time and over the aggregate, allowing for first order serial correlation. \( \beta_0 \) is normalized to equal one.

Figure 7. Policy Function for Output
2-Digit Manufacturing Sectors
Specification 3. \( n = 4. \)
V. Conclusion

This chapter has demonstrated that financial constraints at the microeconomic level are important for the macroeconomy. Using only aggregate data and data on the distribution of financial resources across firms, it is possible to back out the shape of the policy function for individual firm decisions. This policy function is strikingly similar to the policy function which was estimated in Chapter II using firm level data, with a completely different methodology. Moreover, the aggregate results are consistent with the financial transmission mechanism which was suggested in Chapter IV. While the high degree of persistence suggests the possibility of some omitted factors, the distribution of financial resources across firms explains a large part of aggregate activity. An extra dollar of liquidity has a greater effect in the hands of a currently constrained firm than an unconstrained firm. Financial constraints are therefore both theoretically and empirically important for individual firms, sectors of the economy, and the entire macroeconomy.
References


