ABSTRACT

Most systems currently used to manage roads are limited to the maintenance of paved roadways. Application of these systems to unpaved roads is often viewed as unwarranted due to the low levels of traffic normally associated with this type of road. However, unpaved roads in many developed and developing countries represent the larger portion of mileage in the network. Even at the low cost of maintenance per mile of unpaved roads, the total cost resulting from multiplying this value by the overall road mileage corresponds to a tremendous investment. In the United States, for instance, approximately fifty percent of the total road miles are unpaved in 1978, and the maintenance expenditures for 1984 amounted to $4.7B. The savings that can be derived by using a cost-effective maintenance system are clearly overlooked.

Some agencies who felt the need to optimize the blading and regravelling frequencies of their unpaved roads developed methodologies for determining the maintenance strategies that yield minimum overall cost. These procedures vary from road classification-based maintenance to economic analyses of cost components. However, assumptions made on the performance of the road and the effects of maintenance on this performance have been very restrictive. Recent studies of unpaved roads in developing countries focused on the development of more realistic models to predict the performance of unpaved roads and eventually resulted in management systems which seemed useful albeit some limitations and deficiencies when compared with existing pavement management systems. Moreover, closed form solutions exist only for the systems where the most restrictive assumptions are held.

This thesis presents a dynamic optimization approach to determining the optimum blading frequency for an unpaved road using the principles of optimal control. The study is structured into five parts. It begins with a discussion of the
properties of unpaved roads and how they differ from paved roads. The problem of maintaining unpaved roads and objectives of the study are likewise defined. The second part is devoted to functions and relationships describing the performance of unpaved roads. A literature review of existing approaches to unpaved roads' maintenance is the main focus of the third chapter with emphasis on system capabilities and limitations. The most important part of the research lies in the development of an optimization model using dynamic control theories to solve for the optimum maintenance frequency, with the final analysis dealing exclusively on this problem. In the same chapter, the model is tested for hypothetical problems and a sensitivity analysis is performed to evaluate the parameters in the model. From the results of the study, conclusions and possible research directions are described.

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Biographical Note

Roemer M. Alfelor was born in Manila, Philippines. He obtained his Bachelor of Science degree in Civil Engineering with honors from the University of the Philippines; Manila, Philippines in April 1985.

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While doing graduate studies, Roemer worked as a teaching assistant to Prof. Sue McNeil in the graduate course Transportation Infrastructure Systems. He also did independent research on the area of transportation infrastructure, specifically highways. He is currently doing research with Dr. Kenneth Maser on automated sensing of pavement conditions.
Acknowledgment

This thesis is dedicated to Prof. Sue McNell for her unwavering support on my research interests. I thank her for encouraging me to do research on unpaved roads and for constantly guiding my work even after she left MIT. Her concern and selflessness helped make my graduate studies at MIT a lot more pleasant than I anticipated it to be.

I sincerely thank my officemates, the people at Room 5-008, for sharing with me the joys and hardships of graduate education. I appreciate their readiness to lend a hand when I needed it. Many thanks also go to the members of the Filipino Students Association at MIT and my good friends at the Transportation Systems Division.

I would also like to extend my gratitude to Prof. Moshe Ben-Akiva for his concern and advice regarding my academic and research interests, to Prof. Koji Tsunokawa for helping me understand his work and to Dr. Kenneth Maser for allowing me to finish this thesis while working on his project.

Finally, I would like to thank my family; my brothers, sisters and relatives in the Philippines and abroad, and my mother especially, for their love, guidance and care.
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Chapter 1 - INTRODUCTION

1.1 Characteristics of Unpaved Roads

Roads are classified according to the materials used to construct them. This is generally dictated by the design standards set for various combinations of traffic volume, vehicle type, road use and environmental parameters. For high traffic volume roads, different layers of materials are placed on the surface of the soil. These include bituminous mixtures (asphalt) over a layer of stabilized base course or a slab of concrete on top of compacted subgrade. The choice is dependent on the type of traffic and the availability of surfacing materials. Either way the road is said to be paved. For low traffic volume roads, loose gravel or plain earth is the primary riding surface. An unpaved road can exist with in-situ materials on top such as dirt, sand or weathered rock or can have man-applied surfacing materials in the form of sand, sand-gravel or rocks (with or without a binder). These roads are not economically worth paving in that the number of vehicles expected to use them is very low.

Apart from being associated with low traffic volumes, the average speed of vehicles that travel over unpaved roads is generally low. The quality of the surface of the road prevents the drivers from operating at speeds found on paved highways, since the vibration of the vehicles increases with speed and in many cases, dust reduces the visibility on the road.

1.2 Functions of Unpaved Roads

Unpaved roads are designed to serve economic functions by providing low-cost highways for accommodating low traffic volumes. They are found in many agricultural areas where access to rural areas is needed for the transport of farm
products. Unpaved forest roads are provided where logging operations abound. Stones and minerals excavated from quarries and mining fields are carried over long distances on either earth or dirt roads. Moreover, parks and recreational areas require access roads to serve public interests. In many developing countries where agriculture is the major industry, unpaved roads constitute a significant portion of the total road mileage and thus play a major link in the overall economy. Some of the routes connecting major cities in these countries are unpaved, owing to the inadequate budgets required to transform these roads into hard-top, all-weather surfaces. While most third-world economies rely on farm-based industries, several developed countries also have substantially extensive farming activities, giving rise to a large number of earth and gravel roads. It is interesting to note that in 1978, the proportions of unpaved roads in representative developed countries ranged between 5 and 63%, while for developing countries they are as high as 70 to 97% of the network. Statistics on lengths and percentages of paved and unpaved roads in various countries is given on Table 1. The top part of the table shows the lower ratios of unpaved to total roads common in developed countries. The lower part of the table shows representative developing countries. The purpose of these roads varies with the density of development and age of the country. In Finland, the length of public road network is about 75,000 kilometers, of which 50% are gravel roads, 30% have oil-gravel surfacing and 20% asphalt concrete surfacing [2]. In Ontario, Canada where 75,000 kilometers of unpaved roads exist, it is predicted that these roads will continue to form an integral part of the total road networks for the foreseeable future [3]. It is important to note that most of the unpaved roads in the United States are managed by the Forest Service Agency, indicating the major functions these roads play in the economy.

Apart from economic functions, some unpaved roads are constructed and maintained for military interests and operations in remote areas.
Table 1 - Statistics on lengths of paved and unpaved roads in various countries (1978)
Source: [1]

<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>LENGTH OF PAVED ROADS (kms)</th>
<th>LENGTH OF UNPAVED ROADS (kms)</th>
<th>TOTAL LENGTH OF ROADS (kms)</th>
<th>RATIO UNPAVED/TOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>902,849</td>
<td>531,458</td>
<td>1,434,307</td>
<td>0.37</td>
</tr>
<tr>
<td>W. Germany</td>
<td>412,700</td>
<td>412,700</td>
<td>475,000</td>
<td>0.13</td>
</tr>
<tr>
<td>Ireland</td>
<td>87,442</td>
<td>4,872</td>
<td>92,294</td>
<td>0.05</td>
</tr>
<tr>
<td>Japan</td>
<td>405,353</td>
<td>682,901</td>
<td>1,088,254</td>
<td>0.83</td>
</tr>
<tr>
<td>Netherlands</td>
<td>86,300</td>
<td>18,130</td>
<td>104,430</td>
<td>0.17</td>
</tr>
<tr>
<td>U. S.</td>
<td>3,153,032</td>
<td>3,069,190</td>
<td>6,222,222</td>
<td>0.49</td>
</tr>
<tr>
<td>Spain</td>
<td>153,178</td>
<td>166,850</td>
<td>320,028</td>
<td>0.52</td>
</tr>
<tr>
<td>Turkey</td>
<td>28,954</td>
<td>30,453</td>
<td>59,407</td>
<td>0.51</td>
</tr>
<tr>
<td>Mexico</td>
<td>62,956</td>
<td>144,239</td>
<td>207,195</td>
<td>0.70</td>
</tr>
<tr>
<td>S. Africa</td>
<td>52,505</td>
<td>143,394</td>
<td>195,899</td>
<td>0.73</td>
</tr>
<tr>
<td>Brazil</td>
<td>80,202</td>
<td>1,464,482</td>
<td>1,544,684</td>
<td>0.95</td>
</tr>
<tr>
<td>Argentina</td>
<td>47,550</td>
<td>160,537</td>
<td>208,087</td>
<td>0.77</td>
</tr>
<tr>
<td>Peru</td>
<td>5,850</td>
<td>49,650</td>
<td>55,507</td>
<td>0.89</td>
</tr>
<tr>
<td>Bolivia</td>
<td>1,163</td>
<td>36,155</td>
<td>37,318</td>
<td>0.97</td>
</tr>
<tr>
<td>Paraguay</td>
<td>1,324</td>
<td>24,127</td>
<td>25,451</td>
<td>0.95</td>
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<tr>
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<td>4,476</td>
<td>46,034</td>
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<td>0.91</td>
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<tr>
<td>Lesotho</td>
<td>218</td>
<td>3,698</td>
<td>3,916</td>
<td>0.94</td>
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<tr>
<td>Liberia</td>
<td>603</td>
<td>7,349</td>
<td>7,952</td>
<td>0.92</td>
</tr>
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</table>
1.3 Design and Maintenance Standards

Most of the design elements in paved roads apply to unpaved roads. In practice, though, the design of an unpaved road is generally done in a non-technical manner. As Cashatt [4] describes it: 'The decisions are more likely to be based upon what works best, rather than a value in a “blue-book”.'

The major components included in design are:

1. surface and sub-surface materials
2. right-of-way width and cross-slope (crown)
3. proper side ditch and cross road drainage
4. horizontal and vertical alignment

Both the purpose and level of traffic of an unpaved road vary. Therefore, standards are specified to meet specific cases.

The usual specifications for materials contain statements to control the grading of the material, the plasticity of its fines, and the strength of the aggregate particles. Table 2 shows the gradings recommended by the American Association of State Highway Officials (AASHO) [5]. For very low-volume roads, the surface can remain the natural in-place material or foreign material may be used [4]. For an untreated surface material to form a crust, it must have a binder of controlled quality. The AASHO specifications call for a maximum liquid limit of 35, a plasticity index range from 4 to 9, and a minimum of 8% passing the No. 200 sieve if the surface course is to be maintained for several years without bituminous-surface treatment or other impervious surfacing. Crushed limestone is sometimes used because it grinds up under traffic until it has enough fines to bind itself. River sand which is essentially clean of binder can be placed on top of a road or incorporated into the roadbed, using the in-place roadbed as the binder.
According to Cashatt, the cross slope or crown is the most important design element of an unpaved road. The handbook "A Policy on Urban Highways and Arterial Streets" [7] states that gravel and turf roads require greater slopes than paved roads for satisfactory drainage, with 0.04-0.06 ft/ft. recommended for gravel and 0.08 ft/ft. for turf. Cashatt explained that once a surface gets wet enough to rut up with traffic, not only is it enough to drive on but maintenance is required to restore the cross-section. The same handbook, however, does not indicate any specific right-of-way width for highways other than freeways. Cashatt

Table 2 - Grading requirements for soil-aggregate materials (AASHO designation M147-65)

<table>
<thead>
<tr>
<th>Sieve Designation</th>
<th>Percentage by weight passing square mesh sieves</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Grading A</td>
</tr>
<tr>
<td>2 in. (50 mm)</td>
<td>100</td>
</tr>
<tr>
<td>1 in. (25 mm)</td>
<td>-</td>
</tr>
<tr>
<td>3/8 in. (9.5 mm)</td>
<td>30-65</td>
</tr>
<tr>
<td>No. 4 (4.75 mm)</td>
<td>25-55</td>
</tr>
<tr>
<td>No. 10 (2 mm)</td>
<td>15-40</td>
</tr>
<tr>
<td>No. 40 (0.425 mm)</td>
<td>8-20</td>
</tr>
<tr>
<td>No. 200 (0.075 mm)</td>
<td>2-8</td>
</tr>
</tbody>
</table>

Suitable for surfacing courses

Suitable for bases and subbases

that width is normally based upon maintenance capabilities, specifically the moldboard width of the motor grader. Sometimes the right of way width is determined by the size of the machineries used in the farm.
In any highway design the cross road drainage is a very critical design element. However, for an unpaved road, ditches are seldom sized by any formula and generally are limited in size by the right of way. A vee ditch is the most common [4]. The storm recurrence period of 2, 5 or 10 years is used when run-off calculations are used to size the ditches. Shoulder slopes vary between 2:1 and 4:1.

Sight distances are not critical for unpaved roads as they are in paved ones, owing to the very low vehicular speeds associated with the former. In fact no design speeds exist for unpaved roads. In general, sight distances frequently match those of the natural terrain, sags are filled no more than enough to cover drainage structures, and crests are cut no more than enough to provide fill for the sags [4].

Maintenance of unpaved roads involves blading using a motor grader or a modern tow-type blade to restore the original designed values, specifically that of the crown. Regravelling is also performed for gravel-surfaced roads when the gravel thickness falls below a minimum value. The New Corps of Engineers model [8] specifies an equation for the minimum allowable thickness of gravel. The effectiveness of blading is determined by the correct moisture content at which the road is bladed. The proper moisture content varies with the type of materials, e.g. a crushed stone surface should be bladed when very wet to aid in cementing the surface with the fines while a sand-surfaced road can be bladed in the rain, though this is not essential. Routine maintenance is likewise necessary to check the drainage and vegetation growth on the road. Weeds and grasses sometimes creep into the road especially during the late summer and fall. This can be a major problem when blading is done. A major difference between maintenance of paved and unpaved roads is that the former, once built, will exist for a number of years with minimal or no maintenance while unpaved roads deteriorate faster and
require frequent maintenance.

1.4 Identification of the Problem

The properties and standards applicable to the class of unpaved roads were described in the previous section. This section defines the problem to be addressed. Various approaches to analyzing the economic viability of construction and maintenance strategies exist for paved roads, with very little concern given to the lower class of roads. Pavement management systems have been used for many years, with improvements and better capabilities constantly surfacing with time. A number of management systems have been developed for unpaved roads, but many of them are theoretically and technically unappealing. Some agencies adopt very simple systems to have a "reasonable" set of rules from which management decisions can be made, without regards to the true economic consequences of such decisions. The more sophisticated systems operate on functional relationships among road condition, maintenance and vehicle operating costs and evaluate maintenance alternatives on the basis of least total costs. Optimal frequencies and timing of blading are the decision variables analyzed in such cases. These systems provide more rational strategies than the rule-based management approaches, because they are able to quantify the trade-offs associated with maintenance decisions. However, careful investigation of the assumptions and analytical procedures adopted by such systems reveal notable drawbacks.

This study investigates: (1) the accomplishment in the area of unpaved roads' maintenance and whether the present management systems provide the users, specifically the road agencies, cost-effective strategies and if they do, are the decisions optimal, (2) the requirements for a sound management system when applied in the context of unpaved roads, and (3) if it is possible to formulate a model that performs better than any of the existing systems when evaluated in terms of these requirements. In the course of this work we will be dealing with
1.5 Objectives of the Study

The purpose of this work is to develop an analytical method to determine optimal blading frequencies for unpaved roads. The dynamic optimization technique optimal control is used. This procedure was applied by Tsunokawa [9] in solving for the optimum frequency and intensity of overlay in the rehabilitation of highway pavements. Tsunokawa's formulation, however, assumed that the performance of a pavement can be expressed in terms of a single measure which is roughness, and that there is only one maintenance activity to correct for roughness at various levels. In reality, roughness is not a desirable measure of pavement condition because it does not say anything about the specific type of distress present on the surface. Deterioration of the pavement comes in various types and is corrected or remedied by maintenance activities specific to one or more types. Thus, it is not practical to use Tsunokawa's formulation for paved roads since it makes no sense to apply an overlay to a moderately rough road caused by cracks of medium severity when other alternatives, e.g. crack filling, can be performed. For an unpaved road, roughness is the primary indicator of condition and routine maintenance is limited to blading. Using performance functions derived from previous studies a dynamic optimization model is formulated using roughness as the state variable and blading frequency as the decision or control variable. The analytical procedures follow those of Tsunokawa's with some modifications. The computational requirements are reduced because there is only one control variable as opposed to two.
Chapter 2 - Performance Relationships for Unpaved Roads

This chapter focuses on the functions and measures used to describe the condition and behavior of unpaved roads as they are affected by traffic and environmental variables. The relationships defined here allow the understanding of the modes of deterioration found on unpaved roads and the underlying causes of this deterioration. The studies that have been made on performance of unpaved roads generated results which can be used to analyze the effects of different maintenance strategies on the road's performance. By comparing the economic implications of various strategies, it is possible to determine the most economical policy to be adopted.

2.1 Measures of Condition for Unpaved Roads

Deterioration of unpaved roads is manifested and quantified in terms of the following measures of condition [10]:

a. surface roughness
b. gravel loss of the wearing surface
c. rut depth
d. depth of loose surface materials

Note that these represent measures of surface condition only. By determining the extent and severity of these condition measures the condition of the road and its performance over time and under traffic is inferred. The following sections describe in detail how these surface condition data are measured in the field.
2.1.1 - Surface Roughness

Roughness is one manifestation of surface deterioration which is experienced by the driver and passengers of the vehicle travelling over the roadway. It is related to the profile of the road surface and parameters relating to the vehicle including tires, suspensions, seats etc. as well as the sensitivity of the passengers to vehicle motion (Fig. 1).

![Image of a truck experiencing roughness](image)

Figure 1 - The Roughness Phenomenon

The primary component of serviceability of the road is roughness [11], and the way it is perceived by the road user is very important. In terms of profile, roughness can be defined as the summation of variations in the surface profile. Typically, a road has a complex profile with different amplitudes and wavelengths. This profile varies with the surface type. For example, surfaces that are paved with asphalt exhibit corrugations with short wavelengths. On the contrary, unpaved roads that have surface treated, gravel or earth surfaces have mixtures of short, medium and long waves. In particular, earth surfaces have high concentrations of short wavelengths and large amplitudes.
The measures of roughness fall into three categories [12]:

a. A **profile numeric** defined directly by mathematical functions from the absolute profile of road surface elevations in one or two wheelpaths.

b. **Summary numerics** measured through response-type systems known as RTTRMS (Response-Type Road Roughness Measuring Systems) calibrated to a profile or other numeric by correlation, usually the cumulative axle-body relative displacement averaged over a given distance and expressed as a slope.

c. **Subjective ratings** of riding quality or pavement serviceability usually made by a panel of raters over a scale defined by subjective descriptions.

The differences among roughness measures are due to the way the instruments respond to the road profile and to the methods by which the data is processed. For profile numerics, the numeric represents either the measure of displacement amplitude relative to a moving average amplitude or the response of a standard vehicle through a mathematical model of the way a vehicle responds to roughness. In applying the profilometric methods, the longitudinal elevation profile of the road is measured and then analyzed to obtain one or more roughness indices. High speed and manual profilometers are in use, with the former being more popular in developed countries and the manual methods being a practical alternative in developing countries.

The most popular measures of roughness using statistics from profile are the **Root-Mean Square Deviation (RMSD)** and the **Quarter Car Index (QI)**. The former was developed by the British Transport and Road Research Laboratory (TRRL) using an instrument for statically measuring profile called the TRRL beam [13]. The beam measures elevation profile and is processed in discrete sections equal in length to a standard baseline. A regression line is then estimated for the profile length yielding an equation of the form: \( y = mx + b \); where \( x \) is the longitudinal distance, \( y \) is the estimate of the profile elevation at position \( x \), and a
and \( b \) are determined by a least squares fit. At each position, there will be deviation between the measured elevation value and the estimate from the regression line. The RMSD is used as a roughness index. The Quarter Car Index, on the other hand, was developed by Brazilian researchers as a means for using profiles with rod and level to calibrate RTRMS (Figure 2 shows the RTRMS model which yields the quarter car index). Rod and level surveys are run on calibration sections to see if and how their profiles are changing [14].

For the response type systems, a vehicle is instrumented with a road meter which produces a roughness reading as a result of the vehicle motions that occur while traversing the road. RTRMS provide means to acquire roughness data using relatively low-cost equipment. The main disadvantage of these systems is that the roughness measure is intimately tied to the vehicle response, which varies among vehicles and likewise with time, vehicle condition and weather. Therefore the RTRMS measures are less accurate in general, and require fairly accurate calibration to convert the measures to a standard scale.

![Figure 2 - The Quarter Car Model](image-url)
The earliest RTTRM device was developed by the Bureau of Public Roads (now FWHA) and was called BPR Roughometer [15]. This simple trailer is towed by a vehicle at relatively high speeds (20 to 30 mph) over a pavement (Fig. 3). A seismic mass is attached to a single wheel trailer and the total vertical movement of the wheel running on the pavement relative to the seismic mass is accumulated through a mechanical integrator. The resultant movement in inches/mile of pavement is referred to as pavement roughness. This device is very simple to operate. However, it is highly susceptible to changes in temperature and the conditions of its bearings and mechanical components. Also, it has a resonant frequency which, if excited by a large component of a corresponding wavelength of the pavement, produces erroneous results. A similar instrument (with single

Figure 3 - The Bureau of Public Roads Roughometer

wheel trailer) was standardized by TRRL. It is called the Bump Integrator Trailer [16] which measures the total vertical movement of a wheel relative to its mounting frame as the wheel travels at a uniform speed along the road. This was used in vehicle operating cost studies in Kenya, Caribbean and India and in several developing countries. Usual application is vehicle-mounted sensor calibrated to
one of several trailer units. The scale ranges from a low positive value upward to about 16,000 (for poor unpaved roads). Finally, there are a number of road meters used to measure roughness. The most popular is the Mays road meter [17] because it is simple and cheap. It is capable of producing an acceptable measure of roughness while traversing a road section at a normal vehicle operating speed. This instrument is calibrated using a Surface Dynamics Profilometer which is a sophisticated but expensive device that accurately measures the profile of the road over which it passes.

To provide a common quantitative basis with which to reference the different measures of roughness for both instrument calibration and for comparison of results, the World Bank initiated the International Road Roughness Experiment (IRRE) in Brazil in 1982. The experiment resulted in the establishment of the International Roughness Index (IRI), an independent profile-related index appropriate as a reference scale for all profilometric and response systems [18]. It mathematically summarizes the longitudinal surface profile of the road in a wheel track, representing the vibrations induced in a typical passenger car by road roughness.

2.1.2 - Gravel Loss

For a gravel-surfaced road, traffic and environment act together to cause reduction in gravel thickness. This change in gravel height measured over a period of time is called gravel loss. Measurement procedures for both tangent and curve sections were based on techniques developed in the Kenya Study [16]. For tangent sections, at least three benchmarks are established on the extremes of a 50-meter road section called an interval which is located immediately before and adjacent to the transition section. Measurement grids are located by referencing these benchmarks. A benchmark consists of a 13 mm. diameter and 50 cm. long steel bar which was hammered into the subgrade, leveled with the top of the subgrade (approximately 15 cms. below surfacing level) and concreted into the surface. This
ensured that only gravel loss would be monitored since the benchmark moves with the pavement structure as the road settles. The area's width is defined by the gravel surface between the side drains, but not including the drains. The rate of gravel loss was computed from the difference in the average elevation of all grid points over time.

For curve sections, the grid encompassed the entire subsection for the 80 m. long curve sections (Fig. 4). This procedure was adopted because of the difficulty in establishing identical grids in each measurement cycle since the influence of roadway grade or superelevation of slight variations in grid placement could cause large discrepancies. The larger number of observations therefore permitted a better estimate of gravel loss.

2.1.3 - Rut Depth

Ruts decrease the serviceability of the road because they cause displacements of the vehicle in all directions. The operating speed of these vehicles is substantially reduced because the vibration increases with speed. From the user's point of view ruts are a problem. The same is true when viewed from the agency's perspective. Ruts act as drainage channels and when it rains prevents the water from running off the roadway (Fig. 5), which causes some drainage problems and which could deteriorate the road more quickly. The trapped water carries with it some of the materials on the surface, thereby making the ruts deeper by scouring the surface of the channel. At times the road becomes totally impassable. The instrument used to measure rut depth is called a rut depth gauge (Fig. 6). The bottom cross bar was fixed (only used for unpaved roads) to eliminate the influence of localized depressions on the measurement of rut depth.

Data were collected in Brazil using this instrument. In this study [19], ruts were defined as those positions on the road on which majority of the vehicles travel.
Figure 4 - Grid Layout for Measuring Gravel Loss on Curved Sections

Source: (25)
These areas were usually demarcated by the absence of loose material. The maximum rut depth at each measurement point was measured by moving the gauge transversely across the road within the wheel track until a maximum reading was obtained.

![Diagram of Ponding Water](image)

**Figure 5 - Water Ponding in Ruts**

2.1.4 Depth of Loose Surface Materials

Loose materials on the road will lead to loss of traction and was found to increase fuel consumption for a wide spectrum of vehicles in the Kenya study [10]. Therefore the amount of loose materials on the surface must be controlled. The procedure for measuring this is by laying a 1 X 0.25 m. frame on the road and all loose materials are swept into a calibrated cylinder and recorded as the average depth of loose materials.

2.1.5 Condition versus Performance

While measures of condition describe the state in which the road is, it does not contain any information about the performance of the road. The interaction of traffic with the highway and the effects of this interaction on road users costs,
Figure 6 - Rut Depth Gauge
agency costs and deterioration are the most common measures of performance. An unpaved road may be very rough with potholes everywhere along its length, but its performance cannot be judged unless we know how much traffic uses it, how important the traffic is, and how much it deteriorates with the traffic. Performance is gauged not only at the interaction of said parameters at a fixed point in time but extends to significant time periods. The following sections describe the studies made on the performance of unpaved roads.

2.2 Studies on Unpaved Roads Performance

When the need for effective maintenance strategies for managing unpaved roads was realized, a number of studies took place to analyze their performance. Most of these studies were conducted in developing countries where high percentages of unpaved roads are found and thus are most likely to benefit from an effective maintenance system. These studies led to the development of equations predicting various modes of surface deterioration as functions of parameters that are found to influence them. In the same manner, vehicle operating cost studies were made resulting in the estimation of functions relating the different components of vehicle operating cost to road and environmental conditions.

The majority of research on unpaved roads was carried out as part of a larger study that included paved roads. These were all done in developing countries with the objective of developing a rational quantitative basis for decision-making in those countries. The World Bank realized that situations in developing countries, e.g. economic, labor and technology, are different from those of developed countries where pavement management systems exist and thus concluded that these systems are not appropriate for use in entirely different environments. For this reason the World Bank initiated a collaborative research with institutions in several countries, and took a share in funding such research. The original research was conducted in Kenya and included both paved and unpaved roads. In the following sections, results of the studies on unpaved roads
2.2.1 The Kenya Study

The British Transport and Road Research Laboratory (TRRL) in collaboration with the Kenya Ministry of Works and the World Bank conducted a pioneering study and began the development of basic measurement methodologies thereby establishing simple statistical relationships. In its research on road deterioration for unpaved roads [10], the four modes of deterioration discussed earlier were measured and quantified. The results of these studies are reported in this section.

2.2.1.1 Roughness

Roughness was measured using a 5th wheel bump integrator supplemented by a vehicle-mounted integrator unit. In analyzing surface roughness, climate and other factors were not included in this study. Also, no correlations were found between roughness and particle size or roughness and vertical/horizontal curvature. The final equations that were statistically determined for roughness have only cumulative traffic as the parameter, with the equations depending on the type of surfacing material:

\[
R = 3250 + 84*T - 1.82*T^2 + 0.016*T^2
\]  \hspace{1cm} (1)

for lateritic gravel roads

\[
R = 3250 + 785*T
\]  \hspace{1cm} (2)

for sand-clay earth roads

\[
R = 3250 + 1255*T
\]  \hspace{1cm} (3)

for tracks or non-engineered roads

where \( R \) = roughness in mm/1km
T = cumulative traffic volume in both directions since last grading (thousands of vehicles)

2.2.1.2 - Gravel Loss

Gravel loss was recorded as vertical loss in mm. of materials from the road surface. The measurement technique was described in section 2.1.2. The parametric prediction of gravel loss led to the following equation for lateritic gravel roads:

\[ GL_a = 0.94 \times Ta^2 \times (4.2 + 0.092 \times Ta + 3.5 \times R_1^2 + 1.88 \times VC) \]
\[ Ta^2 + 50 \]  
(4)

where
- \( GL_a \) = annual gravel loss (mm.)
- \( Ta \) = annual traffic volume in both directions (thousands of vehicles)
- \( R_1 \) = annual rainfall measured in meters
- \( VC \) = rise and fall, vertical curvature (%)

Climate was found to be insignificant because data taken at approximately 3 monthly intervals showed no significant differences during wet and dry seasons.

It must be noted that actual loss of gravel measured in study is greater than the predicted annual gravel loss by 37.5%.

2.2.1.3 - Rut Depth

Rut depth measurements were made for each track in mm. using a 2 m. straightedge and a wedge calibrated in mm. Also, rut depths were measured from the profiles of the road surface which were plotted as part of the gravel loss
assessment. Analysis of the data led to prediction equations which contain only traffic as parameter:

\[
RD = 11 + 0.23\ast T - 0.0037\ast T^2 + 0.000073\ast T^3 \tag{5}
\]

for lateritic gravel roads

\[
RD = 14 + 1.2\ast T \tag{6}
\]

for sand-clay earth roads and tracks

where \( RD \) = rut depth (mm.)

\( T \) = as defined in previous section

2.2.1.4 - Loose Surface Materials

The method of measuring surface roughness was described in Section 2.1.4. It was theorized that loose surface materials results from the attrition of the road surface by the action of traffic and rainfall. However, the final equation was only a function of traffic:

\[
LD = 1.5 + 14\ast e^{-0.23\ast T} \tag{7}
\]

for lateritic gravel roads

\[
LD = 1.5 + 14\ast e^{-0.23\ast T} \tag{8}
\]

under grading with \( LD \geq 10.0 \) mm.

\[ = 1.0; \text{under wet grading} \]

for sand-clay earth roads

\[
LD = 1.5 + 14\ast e^{-0.23\ast T} \tag{9}
\]

for tracks or non-engineered roads

where \( LD \) = depth of loose materials in mm.
2.2.1.5 - Vehicle Operating Cost Relationships

In the Kenya Study, road surface deterioration parameters (roughness, rut depth, gravel loss) and vehicle operating costs can be reduced to a common denominator -- cumulative traffic volume, on the condition that road geometric and environmental parameters remain the same. These special relationships were used to develop unified relationships that allow direct computations of the vehicle operating costs (VOC) as a function of cumulative traffic volume, eliminating all intermediate steps that require estimation of vehicle speeds and component costs as functions of surface condition parameters. The unified VOC expressions were obtained by regressing composite VOC with cumulative traffic for three road surface types (see Table 3) [19].

2.2.2 The Caribbean Study

Due to the limited range of vertical and horizontal geometry present in the essentially rolling terrain in Kenya, the relationships for estimating vehicle performance were limited to maximum gradients of 8% and maximum horizontal curvatures of 250° per kilometer, and it was not possible to isolate the effects of road geometry on any operating cost component other than fuel. The smaller-scale Caribbean study was designed and undertaken by TRRL as a complementary effort to further study the effects of geometry and to extend the range of the relationships for hilly and mountainous terrain [20]. As such, it was intended to extend the vehicle operating cost functions derived in the Kenya Study to include the effects of road geometry and poor bituminous surfaces. The results of the study are described in the following subsections with variables as follows:

\[ Y = \text{vehicle speed (km/hr)} \]
\[ FC = \text{fuel consumption (ml/km)} \]
\[ PC = \text{parts cost per kilometer} \]
\[ VP = \text{cost of an equivalent new vehicle} \]
# Table 3. Vehicle Operating Cost Equations from Kenya Study (US$/1000 kms.)

**Source:** (10)

<table>
<thead>
<tr>
<th>Road Surface and Vehicle Type</th>
<th>VOC Estimation Equation</th>
<th>Maximum Ta</th>
<th>No. of Observations</th>
<th>r²</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Laterite</strong>&lt;sup&gt;b&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Light Goods Vehicle</td>
<td>213.32 + 0.0094T&lt;sup&gt;2&lt;/sup&gt;</td>
<td>100</td>
<td>19</td>
<td>0.985</td>
<td>1.85</td>
</tr>
<tr>
<td>Single Unit Truck</td>
<td>392.44 + 0.0216T&lt;sup&gt;2&lt;/sup&gt;</td>
<td>100</td>
<td>19</td>
<td>0.991</td>
<td>3.27</td>
</tr>
<tr>
<td>Medium Truck Trailer</td>
<td>692.98 + 0.0422T&lt;sup&gt;2&lt;/sup&gt;</td>
<td>80</td>
<td>19</td>
<td>0.984</td>
<td>8.53</td>
</tr>
<tr>
<td>Heavy Truck Trailer</td>
<td>861.93 + 0.0557T&lt;sup&gt;2&lt;/sup&gt;</td>
<td>80</td>
<td>19</td>
<td>0.984</td>
<td>10.95</td>
</tr>
<tr>
<td><strong>Sand-Clay</strong>&lt;sup&gt;c&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Light Goods Vehicle</td>
<td>207.02 + 12.87T - 0.25T&lt;sup&gt;2&lt;/sup&gt;</td>
<td>30</td>
<td>17</td>
<td>0.987</td>
<td>6.72</td>
</tr>
<tr>
<td>Single Unit Truck</td>
<td>385.96 + 18.56T - 0.27T&lt;sup&gt;2&lt;/sup&gt;</td>
<td>30</td>
<td>17</td>
<td>0.994</td>
<td>8.39</td>
</tr>
<tr>
<td>Medium Truck-Trailer</td>
<td>676.29 + 34.73T - 0.44T&lt;sup&gt;2&lt;/sup&gt;</td>
<td>30</td>
<td>17</td>
<td>0.994</td>
<td>16.96</td>
</tr>
<tr>
<td>Heavy Truck-Trailer</td>
<td>833.83 + 51.40T - 0.79T&lt;sup&gt;2&lt;/sup&gt;</td>
<td>30</td>
<td>17</td>
<td>0.994</td>
<td>22.49</td>
</tr>
<tr>
<td><strong>Track</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Light Goods Vehicle</td>
<td>206.86 + 21.51T - 0.8T&lt;sup&gt;2&lt;/sup&gt; + 0.003T&lt;sup&gt;4&lt;/sup&gt;</td>
<td>30</td>
<td>17</td>
<td>0.986</td>
<td>7.42</td>
</tr>
<tr>
<td>Single Unit Truck</td>
<td>384.54 + 31.04T - 1.0T&lt;sup&gt;2&lt;/sup&gt; + 0.004T&lt;sup&gt;4&lt;/sup&gt;</td>
<td>30</td>
<td>17</td>
<td>0.991</td>
<td>10.32</td>
</tr>
<tr>
<td>Medium Truck-Trailer</td>
<td>675.07 + 58.37T - 1.9T&lt;sup&gt;2&lt;/sup&gt; + 0.008T&lt;sup&gt;4&lt;/sup&gt;</td>
<td>30</td>
<td>17</td>
<td>0.992</td>
<td>19.23</td>
</tr>
<tr>
<td>Heavy Truck-Trailer</td>
<td>831.15 + 86.49T - 3.0T&lt;sup&gt;2&lt;/sup&gt; + 0.012T&lt;sup&gt;4&lt;/sup&gt;</td>
<td>30</td>
<td>17</td>
<td>0.990</td>
<td>28.99</td>
</tr>
</tbody>
</table>

<sup>a</sup> Cumulative Traffic Volume between Gradings in both directions ( '000 vehicles)

<sup>b</sup> Applies to gravel roads with at least 2 cm of laterite surface

<sup>c</sup> Applies to earth and gravel roads with less than 2 cm of laterite surface
2.2.2.1 Vehicle Speed and Fuel Consumption

For cars:
\[ Y = 67.6 - 0.078*RS - 0.067*F - 0.024*C - 0.00087*R \]  
for roads < 5.0 m wide, \( \Delta Y = -8.1 (5.0 - W) \) and \( r^2 = 0.90 \)
\[ FL = (24 + (969/V) + 0.0078*V^2 + 1.33*RS - 0.63*F + 0.0029*F^2)*1.08; \ r^2 = 0.95 \]  
(10)

For light vehicles:
\[ Y = 62.6 - 0.085*RS - 0.087*F - 0.022*C - 0.00066*R \]  
for roads < 5.0 m wide, \( \Delta Y = -7.0 (5.0 - W) \) and \( r^2 = 0.91 \)
\[ FL = (72 + (949/V) + 0.0048*V^2 + 1.118*(GVW*RS) - 1.18*F + 0.0057*F^2)*1.08; \ r^2 = 0.96 \]  
(11)

For trucks:
\[ Y = 51.9 - 0.222*RS - 0.122*F - 0.017*C - 0.00106*R + 0.559*PW \]  
for roads < 5.0 m wide, \( \Delta Y = -6.2 (5.0 - W) \) and \( r^2 = 0.49 \)  
(14)
\[
FL = (29 + (22119/V) + 0.0203*V^2 + 0.848*(G/Y/W*R*S) - 2.6*F \\
+ 0.0132*F^2) * 1.13 ; r^2 = 0.96
\]

2.2.2.2 - Oil Consumption

Cars ------------------ 1.2 liters/1000 kms.
Light Vehicles ------- 1.8 liters/1000 kms.
Trucks and Buses --- 4.0 liters/1000 kms.

2.2.2.3 - Parts Consumption/Maintenance Labor/Tire Consumption

\[
PC = (-5.501 + 0.00262*R) * K * V * P * 10^{-11} ; r^2 = 0.91
\]
Labor Costs = Parts Cost * 0.45

Tire Consumption:
For cars and light vehicles:
\[
\text{Tires/km} = (-0.0601 + 0.0000764*R) * 10^{-3} ; r^2 = 0.81
\]
For trucks:
\[
\text{Tires/km} = (0.0706 + 0.0000135*R) * G * 10^{-4} ; r^2 = 0.9
\]

2.2.3 - The Brazil Study

This research took place between 1975 and 1984 and is probably the largest study in scope because it took advantage of the results from the Kenya study, used more resources than the other, employed more sophisticated theoretical and statistical methodologies to cover broader range of road conditions and traffic characteristics [21]. Likewise, better and more sophisticated measuring instruments were used. As opposed to the simple linear correlation forms employed in the prediction of vehicle speeds and fuel consumption in both the Kenyan and the Caribbean studies, the new models developed in the Brazil
research are based on probabilistic concepts of limiting physical constraints and desired speeds.

The road deterioration and maintenance relationships developed to predict roughness and loss of gravel were based on material properties, geometry, rainfall and traffic rather than the type of material as in Kenya.

2.2.3.1 - Roughness

The instruments used to measure roughness in the Brazil study were the Maysmeter and the G.M. Profilometer. The functions of these instruments were described in section 2.1.1. From roughness data on 30 unpaved sections, two regression equations were analyzed: one to predict roughness as a function of time within a blading period given the roughness after blading and the other predicts roughness immediately after blading. These equations are:

$$Q_l = F^* \exp(D(0.00461 + 0.00477T + 0.00094G + 5.2*10^{-6}ADT$$

$$+ 0.9832/R - 0.005777S - 5.5*10^{-6}T*ADT + 0.003792T*S$$

$$- 4.24*10^{-5}T*R - 0.1871*G/R - 5.35*10^{-6}F - 0.0081*F/R))$$

(20)

where $Q_l$ = roughness at time $D$ (in counts/km)
$D$ = number of days since last blading
$F$ = roughness after blading
$T$ = type of wearing course:
0 for laterite
1 for quartzite
$G$ = absolute value of grade, in percent
$ADT$ = average daily traffic in both directions
$R$ = radius of the curve, in meters
$S$ = season dummy variable
0 for dry season

38
1 for wet season

\[ F = 31.0 + 18.7T - 1.84G + 0.0392 \times ADT + 14.3S + 554.7GIR \\
+ 2330.6S/R + 0.2726L \]  \hspace{1cm} (21)

where \( L \) = roughness before blading

Fig. 7 shows the roughness data points and predicted roughness over time using the equations above for a study section in Brazil.

2.2.3.2 - Rut Depth

In measuring rut depth, the AASHO type gage as described in 2.1.3 was used. The apparatus is graduated to read rut depth with an accuracy of 1 mm. Rut depth

![Graph showing roughness data points and predicted roughness over time for a study section in Brazil](image-url)
was measured at two to three week intervals in case of minimal maintenance programs and every two or three days on high frequency maintenance sections. The models developed to predict rut depth used the same 30 road sections with laterite or quartzite gravel course as follows:

\[
\Delta RD = \exp(D(0.00481 + 1 \times 10^{-5} \times ADT - 0.66631R - 0.002496S - 1 \times 10^{-5} \times ADT \times T + 0.002749 \times T \times L \\
+ 0.01289 \times S \times T - 4.9024 \times S/R + 0.004371 \times S \times G)) \quad (22)
\]

where \( \Delta RD \) = mm. change in rut depth

\( L \) = lane dummy variable

\( 0 \) for downhill lane

\( 1 \) for uphill lane

other variables are as defined in Sect. 2.2.3.1

The above equation was used to calculate the rut depth at time \( D=0 \) (immediately after blading). Another equation was estimated that predicts the mean rut depth immediately after blading. These two equations were used in generating rut depths at any time after blading.

2.2.3.3 - Gravel Loss

The procedure adopted for measuring gravel loss was similar to the one used in Kenya. A grid of points 1 m. apart across the road's width and 5 meters apart along the length of the road was levelled at three monthly intervals relative to fixed benchmarks. Then the change in height of the grid points relative to the benchmarks was calculated over time which when averaged represents a change in the level of the road surface, which corresponds to the thickness of gravel lost.

In estimating the models to predict gravel loss, the same 30 sections were
used, resulting in the following:

$$\Delta GH = B \times 0.0046 \times ADT - 213.81 \times R - 0.467 \times 81.6 \times T +$$

$$213.81 \times R - 0.0043 \times ADT \times R^2 - 0.0082 \times ADT \times R^3) + D \times (0.058 - 0.04161 \times R^2 - 0.1322 \times R^3)$$ (23)

where \( \Delta GH \) = change in gravel thickness (mm)

\( B \) = number of bladings

\( R_2 \) = transverse location variable

\( \begin{align*}
1 & \text{ if location is 2 m from the road edge} \\
0 & \text{ otherwise}
\end{align*} \)

\( R_3 \) = transverse location variable

\( \begin{align*}
1 & \text{ if location is 3 m from the road edge} \\
0 & \text{ otherwise}
\end{align*} \)

Table 4 shows the gravel loss values generated using the equation.

<table>
<thead>
<tr>
<th></th>
<th>0 Blading</th>
<th>2 Bladings</th>
<th>6 Bladings</th>
<th>12 Bladings</th>
<th>0 Blading</th>
<th>2 Bladings</th>
<th>6 Bladings</th>
<th>12 Bladings</th>
</tr>
</thead>
<tbody>
<tr>
<td>250 days</td>
<td>-0.4</td>
<td>-4.0</td>
<td>-17.9</td>
<td>-35.7</td>
<td>-0.4</td>
<td>-18.1</td>
<td>-48.4</td>
<td>-94.8</td>
</tr>
<tr>
<td>250 m Curve</td>
<td>-0.4</td>
<td>-7.7</td>
<td>-23.0</td>
<td>-46.0</td>
<td>-0.4</td>
<td>-17.8</td>
<td>-53.5</td>
<td>-107.9</td>
</tr>
<tr>
<td>250 m Curve</td>
<td>-0.4</td>
<td>-1.3</td>
<td>-3.9</td>
<td>-7.7</td>
<td>-0.4</td>
<td>-11.5</td>
<td>-14.4</td>
<td>-26.7</td>
</tr>
<tr>
<td>250 m Curve</td>
<td>-0.4</td>
<td>-3.0</td>
<td>-9.0</td>
<td>-18.0</td>
<td>-0.4</td>
<td>-13.2</td>
<td>-19.5</td>
<td>-34.0</td>
</tr>
<tr>
<td>250 m Curve</td>
<td>-0.4</td>
<td>-5.0</td>
<td>-17.9</td>
<td>-35.7</td>
<td>-0.4</td>
<td>-16.1</td>
<td>-48.4</td>
<td>-94.8</td>
</tr>
<tr>
<td>250 m Curve</td>
<td>-0.4</td>
<td>-14.2</td>
<td>-42.6</td>
<td>-83.2</td>
<td>-0.4</td>
<td>-24.4</td>
<td>-73.1</td>
<td>-126.1</td>
</tr>
<tr>
<td>250 m Curve</td>
<td>-0.4</td>
<td>-11.3</td>
<td>-19.9</td>
<td>-37.7</td>
<td>-0.4</td>
<td>-17.5</td>
<td>-24.4</td>
<td>-49.4</td>
</tr>
<tr>
<td>250 m Curve</td>
<td>-0.4</td>
<td>-9.5</td>
<td>-28.6</td>
<td>-57.2</td>
<td>-0.4</td>
<td>-19.7</td>
<td>-34.1</td>
<td>-61.8</td>
</tr>
</tbody>
</table>

Table 4 - Generated Values of Gravel Loss from the Brazil Study
2.2.3.4 - Loose Materials

Again, the instrument used in the Kenya study was used to collect data for depth of loose materials. It was found out that the time effect on the thickness of loose materials was strong, but no prediction equations were estimated.

2.2.3.5 - Vehicle Operating Costs

The vehicle operating costs relationships developed in Brazil differ from the ones estimated in Kenya and the Caribbean in that the formulations are based on generally-accepted principles of vehicle mechanics and driver behavior [22]. Likewise, these relationships used the most explanatory variables. Other studies relied more on statistical correlation of associated variables through multiple linear regression. The general expressions for vehicle operating costs are explained in the following sections.

2.2.3.5.1 - Fuel Consumption

\[ FL = 500 \times a \times [(UFC_u/Y_u) + (UFC_d/Y_d)] \]  \hspace{1cm} (24)

where

- \( FL \) = average round trip fuel consumption in liters/1000 vehicle-kilometers
- \( UFC_u \) = predicted unit fuel consumption for uphill segment in ml/sec.
- \( UFC_d \) = predicted unit fuel consumption for the downhill segment in ml/sec.
- \( a \) = multiplicative factor, obtained from calibrating the mechanistic fuel prediction models to the road user cost survey data in Brazil
- \( Y_u \) = average uphill speed
- \( Y_d \) = average downhill speed
The unit fuel consumption costs for uphill and downhill segments are functions of vehicle powers on the uphill and downhill road segments, the calibrated engine speed and the vehicle type. Average speeds are controlled by the vertical gradient, engine power, road curvature, roughness and skid resistance. Another parameter, the desired speed, represents the operating speed of the vehicle in the absence of constraints based on the vertical grade, curvature and ride severity and which also affects the average speeds. It was found out that for unpaved roads fuel consumption depends heavily on roughness whereas gradient exerts more influence in the case of paved roads.

2.2.3.5.2 - Others

Expressions for tire consumption, maintenance parts consumption, maintenance labor, lubricants, crew requirements, vehicle depreciation, overhead, passenger delays and cargo handling costs were also formulated. The formulations and estimation results are documented in the Vehicle Operating Costs Manual of the HDM [22].

2.2.4 - The India Study

Because of the significant difference between road and traffic conditions in India and those of other countries, models estimated elsewhere were found inapplicable. To address this issue, the Central Road Research Institute (CRRI-New Delhi, India) was commissioned in 1976 by the World Bank and the government of India to undertake a Road User Cost Study [23]. Apart from the three basic studies on vehicle speeds, controlled experiments on fuel-speed relations and a comprehensive user cost study, pilot studies on simulation modelling of congested traffic flows, road accident costs and the value of time savings were included. A rich data base was collected but it was not possible to fully exploit this data base due to limitations in resources and time. The coefficients for given factors
across alternative model forms had considerable instabilities which caused ambiguities and contradictions. It was then realized that further data collection is required.

2.2.5 - The South Africa Study

The performance and deterioration of unpaved roads were evaluated in South Africa. The approach was based on the Brazil study, with the same set of independent variables used to predict deterioration and effects of maintenance. The relationships formulated in the Brazil study were calibrated to South African conditions [24, 25]. Aside from the equations predicting the performance of unpaved roads, the concept of road passability was also investigated. A summary of the data used in the study is given in Table 5. The equations developed from the study are summarized in the following subsections.

<table>
<thead>
<tr>
<th>Variable*</th>
<th>Mean</th>
<th>SD</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade (%)</td>
<td>3.8</td>
<td>2.6</td>
<td>0</td>
<td>8.2</td>
</tr>
<tr>
<td>Curvature (1/rad) on curved sections</td>
<td>0.0039</td>
<td>0.0009</td>
<td>0.0025</td>
<td>0.0055</td>
</tr>
<tr>
<td>Road width (m)</td>
<td>9.8</td>
<td>1.09</td>
<td>7.0</td>
<td>12.0</td>
</tr>
<tr>
<td>Material properties</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent passing the 0.42-mm sieve</td>
<td>53</td>
<td>22</td>
<td>24</td>
<td>98</td>
</tr>
<tr>
<td>Percent passing the 0.074-mm sieve</td>
<td>36</td>
<td>24</td>
<td>10</td>
<td>97</td>
</tr>
<tr>
<td>PI (%)</td>
<td>11</td>
<td>6</td>
<td>0</td>
<td>33</td>
</tr>
<tr>
<td>Liquid limit (%)</td>
<td>32</td>
<td>9</td>
<td>20</td>
<td>62</td>
</tr>
<tr>
<td>ADT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Passenger cars</td>
<td>88</td>
<td>64</td>
<td>11</td>
<td>288</td>
</tr>
<tr>
<td>Buses</td>
<td>7</td>
<td>7</td>
<td>0</td>
<td>29</td>
</tr>
<tr>
<td>Pickups</td>
<td>37</td>
<td>29</td>
<td>4</td>
<td>115</td>
</tr>
<tr>
<td>Two-axle trucks</td>
<td>56</td>
<td>93</td>
<td>1</td>
<td>435</td>
</tr>
<tr>
<td>Trucks and trailer combinations with more than two axles</td>
<td>15</td>
<td>18</td>
<td>0</td>
<td>66</td>
</tr>
<tr>
<td>Gravel loss</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of observations</td>
<td>604</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time of observation relative to start of observation or regaveling (days)</td>
<td>238</td>
<td>211</td>
<td>0</td>
<td>1099</td>
</tr>
<tr>
<td>No. of bldings relative to start of observation or regaveling</td>
<td>2.3</td>
<td>3.3</td>
<td>0</td>
<td>23</td>
</tr>
<tr>
<td>Roughness measurements</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roughness (Q11 counts/km)</td>
<td>117</td>
<td>61</td>
<td>15</td>
<td>445</td>
</tr>
<tr>
<td>No. of days since blading for last observation in each blading period</td>
<td>75</td>
<td>70</td>
<td>1</td>
<td>661</td>
</tr>
<tr>
<td>No. of vehicle passes since blading for last observation in each blading period</td>
<td>16,080</td>
<td>17,880</td>
<td>63</td>
<td>136,460</td>
</tr>
<tr>
<td>Rut-depth measurements</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rut depth (mm)</td>
<td>11.1</td>
<td>8.6</td>
<td>0</td>
<td>75</td>
</tr>
<tr>
<td>No. of days since blading for last observation in each blading period</td>
<td>61</td>
<td>66</td>
<td>1</td>
<td>661</td>
</tr>
<tr>
<td>No. of vehicle passes since blading for last observation in each blading period</td>
<td>12,490</td>
<td>14,030</td>
<td>21</td>
<td>86,700</td>
</tr>
</tbody>
</table>

Table 5 - Unpaved Roads Data Summary for the South African Study

44
2.2.5.1 - Roughness

\[
LDQ = D \left[ 0.4314 - 0.1705T_2 + 0.001159 \times NC + 0.000895 \times NT - 0.000227 \times NT + S(-0.1442 - 0.0198 \times G + 0.00621 \times SV - 0.0142 \times PI - 0.000617 \times NC) \right] \quad (25)
\]

where

- \( LDQ \) = change in the natural logarithmic value of roughness
- \( D \) = number of days in hundreds since last blading or since regravelling for gravel loss;
- \( T_2 \) = surfacing type dummy variable
  \[ \begin{align*}
  1 & \quad \text{if surfacing is clay} \\
  0 & \quad \text{if laterite or quartzite}
  \end{align*} \]
- \( NC \) = average daily car and pick-up traffic in both directions
- \( NT \) = average daily bus and truck traffic in both directions
- \( G \) = absolute value of grade in percent
- \( S \) = season dummy variable
  \[ \begin{align*}
  0 & \quad \text{if dry} \\
  1 & \quad \text{if wet}
  \end{align*} \]
- \( SY \) = percentage of surfacing material passing the 0.074 mm. sieve
- \( PI \) = plasticity index of surfacing material (percent)

This model has an \( r^2 \) value of 0.26 with a sample size consisting of 8276 observations. It was argued that the low correlation coefficient maybe accepted because of the large observation set but this is not necessarily so.

The expression predicting roughness after blading is:

\[
LRA = 1.4035 - 0.0239 \times W - 0.0048 \times SY + 0.01694 \times PI + 0.6307 \times LRB + 0.1499 \times T_1 + 0.3096 \times T_2 + 0.0002 \times NT + 0.2056 \times ABS
\]
\[ 0.01183^*P1\times BS \]  

where

\( LRA \) = natural logarithm of roughness (QI or counts/km) after blading

\( W \) = road width (meters)

\( LRB \) = natural logarithm of roughness (QI) before blading

\( T_1 \) = surfacing type dummy variable

\[ = \begin{cases} 1 & \text{if surfacing is quartzite} \\ 0 & \text{if laterite or clay} \end{cases} \]

\( BS \) = dummy variable for season during which blading occurred

\[ = \begin{cases} 0 & \text{in dry season} \\ 1 & \text{in wet season} \end{cases} \]

The sample set consisted of 1308 observations and the model has an \( r^2 \) value of 0.61.

2.2.5.2 - Rut Depth

An inspection of the data showed that rut depth after blading is not equal to zero, which agreed with the Kenya study. Three seasonal variables were included in the model because the rate of change in rut depth is different for the period of transition from dry to wet season. The equations are as follows:

For Dry Season: \( (S_1 = 0, S_2 = 0) \)

\[ \text{DRD} = D (9.78 - 1.033^*P1 - 0.192^*P1 - 3.63^*T_2 + 0.0302^*NC + 0.0198^*NT - 3.27^*RO - 3.04^*NTIP + 0.4603^*RO - 3.25^*LIR + 0.0364^*L\times SV) \]  

(27)
For the Transition Period: \( S_1 = 1, S_2 = 0 \)

\[
DRD = D \left( -83.76 + 3.658A - 0.192PI - 3.62T_2 - 0.1147NC \\
+ 0.1249NT - 3.27RO - 3.04NTIR + 0.46\times RO \\
- 325L/R + 0.0364L*SV + 6.874W \right) \tag{28}
\]

For Wet Season \( S_1 = 1, S_2 = 1 \):

\[
DRD = D \left( 9.78 - 1.033A - 0.192PI - 3.63T_2 - 0.0109NC \\
+ 0.0198NT - 3.27RO - 3.04NTIR + 0.46\times RO \\
- 325L/R + 0.0364L*SV \right) \tag{29}
\]

where

\( DRD \) = change in rut depth with time (mm)

\( S_1 \) = transition from dry to wet season dummy variable: first two months of wet season or until first blading in this period

\( ( S_1 = 1, \text{otherwise } S_1 = 0 ) \)

\( S_2 \) = wet season dummy variable: after first blading in first two months of wet season or after two months in wet season

\( ( S_2 = 1, \text{otherwise } S_2 = 0 ) \)

\( RO \) = wheelpath dummy variable (\( RO = 0 \) for external wheelpath, \( RO = 1 \) for internal wheelpath)

\( L \) = lane dummy variable

\( = 0 \) for uphill lane , \( 1 \) for downhill lane

\( R \) = radius of horizontal curvature (m)

For the rut depth immediately after blading, the following model was estimated:
FRD = 5.46 + 3.80*T₁ + 4.74*T₂ - 0.0158*NT + S₂(0.178*G - 587.4*R
- 0.118*PI + 3*T₁ + 0.0226*NC - 0.0134*NT)  (30)

where FRD is the rut depth after blading (mm) and the other variables are as defined earlier.

2.2.5.3 - Gravel Loss

The gravel thickness loss in millimeters was estimated as:

GL = D (1.71 + 0.382*G + 0.078*SV - 0.197*PI + 0.0104*N + 404.961*R)  (31)

where

GL = gravel loss
N = average daily traffic in both directions

2.2.5.4 - Wet Season Passability

It was observed from the 48 sections studied in South Africa that roads became impassable after extended periods of light rainfall. One function of the surfacing material is to provide all-weather riding surface, therefore it must be strong enough to carry traffic even when it gets wet. It was shown that the soaked laboratory California Bearing Ratio (CBR) of the surfacing material discriminates best between passability and impassability in the wet season (Fig. 8). The criteria of surfacing material properties for ensuring that a road remains passable during a wet season (assuming there is no flooding) have been developed by Visser [25] as follows:

\[ SFCBR \geq 8.25 + 3.75\times \log_{10} ADT \]  (32)

and for surfacing stability in terms of ravelling or looseness as follows:
where \( SFCBR \) = the (minimum) soaked CBR at standard Proctor laboratory compaction for ensuring passability

\( PO75 \) = the amount of material passing the 0.075 mm. sieve

\( ADT \) = average daily traffic in both directions (vehicles/day)

**Figure 8 - Criteria for Wet Season Passability in South Africa**

2.2.5.5 - Vehicle Operating Costs

No vehicle operating cost relationships were developed for South Africa.

The relationships developed in South Africa were used to develop a maintenance and design model (MDS) for unpaved roads whose main function is to evaluate alternative regravelling and blading strategies. This system will be discussed in detail in the next chapter.
2.2.6 - The Bolivia Studies

From 1981 through 1983, the Bolivian National Highway Department, Servicio Nacional de Caminos ( SNC ), conducted studies to improve their highway maintenance practices [26]. During that time the most costly maintenance activity performed by SNC was aggregate surface maintenance. For this reason objective criteria were developed to set maintenance levels for the particular group of roads. Only one measure of road condition was analyzed in the study -- roughness. Using a Maysmeter calibrated by rod and level profile system, roughness measurements were made. When these measures were compared with predictions based on the equations derived from Kenya, Brazil and South Africa studies, it was observed that Bolivian roads became rougher more quickly than predicted using the existing equations. This prompted the SNC to collect road performance information for roads with different traffic levels in Bolivia. The information collected was used to estimate performance functions which are described in the next sections.

2.2.6.1 - Roughness

Two performance equations predicting roughness following grading or regravelling were estimated which are:

\[
Q_{1} = 7.922 + 177 \times \left\{ \left( e^{(A*B*C-0.78)/1.04} \right)^{3} + e^{(A*B*C-0.78)/1.04} \right\} (34)
\]

where

\[
A = 5.8
\]

\[
B = D \times (0.0059 + 1.1e^{-5*AADT})
\]
\[ C = (575\text{AADT})^{0.4175} \]

\[ D = \text{number of days since grading} \]

\[ \text{AADT} = \text{average annual daily traffic} \]

**Roughness of the Road Following Regravelling:**

\[ QI = 29.189 + 185 \left( e^{(A \cdot B \cdot C - 5.47)/2.135} + (5 + 3(A \cdot B \cdot C - 5.47)/2.135) \right) \]

(35)

where

\[ A = 5.2 \]

\[ B = D \left( 0.00608 + 0.00001138 \cdot \text{AADT} \right) \]

\[ C = (575\text{AADT})^{0.2682} \]

\[ D = \text{number of days since regraveling} \]

\[ \text{AADT} = \text{annual average daily traffic} \]

The performance curves for unpaved roads following grading are shown on Fig. 9 for test sections in Bolivia.

![Figure 9 - Performance Curves in Bolivia](image)
2.2.6.2 - Vehicle Operating Costs

Literature studies were made in Bolivia to establish user equations that relate roadway characteristics with vehicle operating costs. The information collected was used to verify existing relationships. Most of the expressions arrived at were derived from the Brazil study with adjustments made where necessary by calibration using Bolivian data to reflect local conditions. The equations for different components of user costs are given below.

2.2.6.2.1 - Fuel Consumption

\[
\text{In } FC = 5.078 + 0.00141*QI \text{ for light vehicles} \quad (36)
\]
\[
= 5.675 + 0.00061*QI \text{ for bus} \quad (37)
\]
\[
= 5.887 + 0.00108*QI \text{ for trucks} \quad (38)
\]

where

\( FC \) = fuel consumption in liters/100 kms.
\( QI \) = roughness in counts/km

2.2.6.2.2 - Parts Consumption:

\[
\text{PC} = k0.302*\exp(5.497 + 0.00426*QI)*0.5 \text{ for light vehicles} \quad (39)
\]
\[
K0.485*\exp(5.703 + 0.00323*QI)*0.5 \text{ for bus} \quad (40)
\]
\[
(305 + 105.7*QI)*0.5 \text{ for trucks} \quad (41)
\]

where \( PC \) = parts cost per 1000 kms. in Bolivian pesos
\( K = \) vehicle age in 1000 km. units
2.2.6.2.3 - Labor Costs (in Bolivian pesos)

\[ LC = (\exp (3.33 + 0.548 \ln PC + 0.00403 Ql)) \times 0.5 \]
for light vehicles \hspace{1cm} (42)

\[ = (\exp (3.231 + 0.516 \ln PC + 0.00514 Ql)) \times 0.5 \]
for bus \hspace{1cm} (43)

\[ = (\exp (3.396 + 0.519 \ln PC)) \times 0.5 \]
for trucks \hspace{1cm} (44)

2.2.6.2.4 - Tire Consumption

\[ TL = e^{(14.6488 - 0.9432 \ln Ql)/10,000} \]
for light vehicles \hspace{1cm} (45)

\[ = 4.181 - 0.00951 Ql \]
for bus \hspace{1cm} (46)

\[ = 3.933 - 0.00951 Ql \]
for trucks \hspace{1cm} (47)

where TL = tire life in 10,000 kilometers

These expressions were incorporated in a small computer program called RSML which allows the unit costs of highway maintenance and vehicle operating cost consumables to be entered at the beginning of the program run. This program will be explained in the next chapter.

2.2.7 - Summary

In this chapter, some of the major road studies conducted on unpaved roads are reviewed. The extent and nature of the studies simply show that concerns for unpaved roads have been very high in many places in the world, specifically the developing countries. In the next chapter, how the management systems for these roads work in different places will be described. Most of the results of the studies discussed in Chapter 2 were used in the existing maintenance systems. This shall be the main focus of Chapter 3.
Chapter 3 - Review of Existing Approaches to Unpaved Roads' Maintenance

In this chapter, systems employed to set maintenance frequencies for unpaved roads are reviewed. A literature review on this area indicates that several agencies and researchers have developed specific methods for determining the most cost-effective maintenance strategy that applies to unpaved roads. The level of sophistication and computational complexity used in the analysis vary among the systems. In general, the more rigorous and accurate solution methodologies are employed where there is a great mileage of unpaved roads. This is true for most developing countries where almost all major studies on unpaved roads' performance described in the previous chapter took place. However, it is also true that developed countries would benefit from an accurate management system. A naive system either for an undeveloped country or a small county is not justified just because the total length of roads is small. An accurate management system refers to one that economically optimizes maintenance activities under the most realistic set of assumptions. Thus, any system that provides the user optimal decisions with the least restrictive assumptions satisfies our criterion of being a good management system. Other requirements are special capabilities of the system such as resource allocation and the ease of use. Two types of management systems are currently used for unpaved roads as far as this research has investigated; those that are based on road classification and those that adopt a mathematical optimization technique to arrive at optimal decision variables.

3.1 - "Road Classification" - Based Maintenance

This is a simple way to assign maintenance to a road. The procedure is to divide the roads into different classes based on characteristics such as traffic volume. For each class, a level of maintenance is defined. Some examples are given in the following subsections.
The Ontario Road Classification System [3]

The total road network of Ontario includes 75,000 kilometers of unpaved roads. The costs for maintaining the system have escalated dramatically since the mid-70's and, in recent years, municipalities have not been able to increase revenues from the taxpayers. Preliminary studies in 1980 indicate that up to 75% of the road budget of Ontario is dedicated to unpaved portion of the road network. No maintenance system was present then. It was realized that it was timely to introduce one, and a classification system based on service criteria was adopted.

In the classification system, roads are divided into 3 classes based on four quality-of-service characteristics:

a. average daily traffic (ADT)

b. visibility

c. ease of passage

d. all-season travel

The main criterion used is the average daily traffic through which the classification was made in Table 6.

<table>
<thead>
<tr>
<th>Class</th>
<th>Average Daily Traffic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250-400+</td>
</tr>
<tr>
<td>2</td>
<td>100-300</td>
</tr>
<tr>
<td>3</td>
<td>0-150</td>
</tr>
</tbody>
</table>

Table 6 - Ontario Classification System
The overlaps in the classification are taken cared by the other characteristics. For visibility, two elements are considered - passing sight distance and stopping sight distance. It is believed that the encroachment of roadside shrubs and trees and the presence of dust or lack of it, as well as changes in the geography of the road will impact on the passing and/or stopping sight distance. Ease of passage is related to rutting, corrugations and potholes which are known to affect rideability, vehicular maintenance costs, tire wear and rider comfort. The last characteristic, all-season travel, is typical of most roads that have very low ADT. It is suggested that many types of roads, i.e. recreational unpaved roads that are open only during the summer season, be allocated the smallest amount of maintenance dollars.

The purpose of the classification is to establish a basis for distributing maintenance funds. The Transportation Research Board Report on low-volume roads [27] concludes that "loose top maintenance costs increase with the road width and ADT". Because these parameters are factors in the Ontario classification system, a formula was developed that relates maintenance costs to each class in a linear manner. The total amount of maintenance costs for unpaved roads in a system is expressed as:

\[ Y = A + C_1l_1 + C_2l_2 + C_3l_3 \]  

(48)

where \( Y \) = total annual maintenance costs
\( A \) = proportion of fixed costs for maintaining unpaved roads
\( C_1, C_2, C_3 \) = annual maintenance costs of class 1, 2 or 3 roads per kilometer, respectively
\( l_1, l_2, l_3 \) = distance (kms) of class 1, 2 or 3 roads, respectively
\( L = l_1 + l_2 + l_3 \)

Given the principle of proportionality of maintenance costs by road class, i.e.
C_1 = 3C_3 = 2C_2, then C_3 can be calculated by substituting these relationships into equation (48) and solving for C_3:

\[
\frac{Y_1L}{L} = (A_1L) + C_3[(3A_1L) + (2A_2L) + (13L)]
\]  

(49)

While the approach used in this system is so simplistic that it can be easily understood by non-technical road managers and can be implemented with few available data, it is apparent that there is no attempt to come up with optimal maintenance strategies. It was not stated how the other level of service characteristics, i.e. visibility and ease of passage, can be quantified and considered in classification. The assumption on linearity of cost with average daily traffic and road width is very unrealistic. The province of Ontario may benefit from improvements to the maintenance system considering the great mileage of roads involved.

3.1.2 - Setting Maintenance Levels for Forest Service Roads

The United States Forest Service operates one of the largest low-volume road networks under the jurisdiction of a single agency in the world [28]. Approximately 260,000 miles constitute the system and the annual expenditure for construction, reconstruction and maintenance exceeds $0.5 billion. The forest roads are predominantly low-standard earth and gravel roads whose functions/purposes include:

a. timber
b. recreation
c. fire control
d. watershed protection
e. wildlife
The procedure draws on knowledge of the land, its uses and the roads that serve it, to establish an optimum maintenance level (objective level). The objective is to reduce maintenance levels while minimizing the harmful effects of these reductions. A form included a table such as that shown in Table 7 and is completed for each road in the following steps:

1. advanced information
2. appraisal
3. budget review

<table>
<thead>
<tr>
<th>Function</th>
<th>Maintenance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Σ</td>
</tr>
</tbody>
</table>

Table 7 - Appraisal Form used by the U. S. Forest Service in Evaluating Unpaved Roads

The first step involves deciding which road segments should be included in the analysis form. Roads which are candidates for capital improvements in the future are excluded. In the appraisal stage, each person individually or the appraisal group as a whole subjectively weighs the relative impact of various levels of road maintenance on various functions by assigning negative impact ratings. The scale ranges from 0 to 6 where 0 means no negative impact and 6 corresponds to intolerable road condition without the given maintenance. For
each maintenance level, the average impact rating is computed. The budget review then provides a mechanism for determining the feasibility of reducing the maintenance level, and by how many levels, to meet budget restrictions. This is done by noting on the form the lowest feasible maintenance level and the priority for level reductions.

The approach suffers from the same level of subjectivity as the assignment of Present Serviceability Index (PSI) to pavements. Maintenance levels are not objectively defined and so nor are the effects of reduced maintenance.

3.1.3 - Maintenance and Condition Rating of Gravel Roads in Finland

In Finland, the Roads and Waterways Administration (TVH) is responsible for the condition of public roads. Maintenance of unpaved roads is divided into winter and summer maintenance. The former includes removal of snow and de-icing (sanding and salting) while the latter includes dust binding, gravelling, grading and dragging. Although trips on unpaved roads account for only about 12% of the total vehicle kilometers, summer maintenance and structural improvements of these roads require more than 25% of the annual allocations reserved for the maintenance and strengthening of public roads in Finland [2].

Gravel roads are divided into five classes on the basis of average daily traffic (ADT) shown on Table 8.

The amounts of calcium chloride (CaCl₂) used annually for dust binding on gravel roads and the average amounts of crushed gravel used annually for regravelling during summer maintenance are tabulated by maintenance class. Frequencies of grading and dragging on gravel roads are also given for each class based on previous experience. The condition of the road is evaluated by
using the evaluation scale in Table 9.

<table>
<thead>
<tr>
<th>Maintenance Class</th>
<th>ADT</th>
<th>% of Total Gravel Roads</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1501-5000</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>501-1500</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>201-500</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>101-200</td>
<td>37</td>
</tr>
<tr>
<td>7</td>
<td>≤ 100</td>
<td>45</td>
</tr>
</tbody>
</table>

Table 8 - Finland Classification System

The result is a figure from 0-5 that describes road condition. If the figure given by maintenance supervisor is less than the minimum given in Table 10, the necessary maintenance work should be started to raise the condition of the road above the lower limit.

This evaluation system is similar to the Present Serviceability Index (PSI) which has been applied for paved roads. Another aggregate measure called the Unsurfaced Road Condition Index (URCI) was developed by the U.S. Army Corps of Engineers [30] which basically resembles the Pavement Condition Index (PCI) applied to pavements in terms of methods for condition rating. The URCI has not been used so far for unpaved road maintenance. Aggregate measures of road condition do not objectively represent the extent and nature of maintenance needs. They are not recommended for determining maintenance strategies for both paved and unpaved roads.
Table 9 - Rating Scale for Evaluation of Condition of Wearing Course on Gravel Roads [29]

<table>
<thead>
<tr>
<th>Rating</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1-5.0</td>
<td>Road surface has maintained its shape and is very even and firm; possible unevenness of surface does not affect driving comfort</td>
</tr>
<tr>
<td>3.1-4.0</td>
<td>Road surface has generally maintained its shape and is even and firm; some single holes here and there; no dust; running speed can be maintained in spite of unevenness</td>
</tr>
<tr>
<td>2.1-3.0</td>
<td>Road surface has generally maintained its shape and is mostly even and firm; local small holes and unevenness; some dust; holes and uneven spots can be avoided, or they are such that the running speed can be maintained; in giving way to overtaking or oncoming vehicles a lower running speed should be used</td>
</tr>
<tr>
<td>1.1-2.0</td>
<td>Shape of road cross-section may have changed somewhat; some &quot;washboard waves&quot; on surface; local settlements or humps marked with traffic signs; moderate dust; lower running speed sometimes needed and uneven spots must be avoided</td>
</tr>
<tr>
<td>0.1-1.0</td>
<td>Shape of road cross-section has changed in several spots; surface is uneven due to holes, &quot;washboard waves&quot;, and ravellings; settlements and humps on roads that cannot be avoided; plenty of dust; road surface must constantly be watched and running speed changed often</td>
</tr>
</tbody>
</table>
Table 10 - Lower Limit of Objective Condition

<table>
<thead>
<tr>
<th>Maintenance Class</th>
<th>Lower Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.3</td>
</tr>
<tr>
<td>4</td>
<td>2.8</td>
</tr>
<tr>
<td>5</td>
<td>2.4</td>
</tr>
<tr>
<td>6</td>
<td>2.0</td>
</tr>
<tr>
<td>7</td>
<td>1.5</td>
</tr>
</tbody>
</table>

3.2 - Mathematical Optimization Techniques

The more elaborate procedures applied in setting maintenance frequencies for unpaved roads consist of analysis of quantitative relationships established to describe a road’s performance and determination of the most economic strategy using a specified criterion as the objective. Total cost of vehicle operation and road maintenance is the objective function usually minimized. These procedures offer concrete and rational basis for decision-making, as opposed to the classification-based maintenance systems described in section 3.1 which are not founded on any quantification of costs. The main disadvantage of adopting a numerical solution technique is the extent of computation necessary and the requirement that performance functions be available for use in the economic evaluation. When no such set of relationships exists as is the case in Ontario, Finland and the United States, it might be reasonable to adopt the simple classification systems. However, as we have seen in the previous chapter, many studies have already taken place which dealt with deterioration of unpaved roads and its relationships with road user costs and maintenance expenditures. These functional relationships are utilized in the economic analyses that existing systems perform. The following sections describe the systems that have been developed so far. For each system considered, we shall briefly discuss the
assumptions used, define the working models utilized in the analysis and the performance criteria being optimized, identify the solution methodology employed and evaluate the overall effectiveness of the system for use in actual practice.

3.2.1 - Simplified Graphical Method

From the results of the Kenya Study, the TRRL came up with an economic model called Road Transport Investment Model (RTIM) whose function is to evaluate alternative investment strategies for road construction and maintenance in developing countries [31]. Both paved and unpaved roads are included in the model. However, after a road is constructed, only the costs of maintenance and road use need be evaluated. Kerali [32] proposed a simple analytical model that aims at minimizing the total costs involved after a road is constructed. The performance relationships derived from the Kenya Study predict distresses on unpaved roads as functions of cumulative traffic and surfacing materials only (Sec. 2.2.1). Moreover, the study determined that roughness was the surface condition measure by which road user costs can be predicted. Assuming that the geometric and traffic characteristics of the road remain fixed, Kerali concluded that the vehicle operating cost incurred on a given road per vehicle-kilometer will change only if the road roughness changes. Given the set of polynomial equations relating vehicle operating cost with cumulative traffic (Table 3), the problem of determining the most cost-effective strategy can be solved by simple first-order differentiation. The relationship between unit vehicle operating cost and cumulative number of vehicles is shown in Fig. 10a for a particular type of vehicle and surfacing material and a maintenance activity assumed to be repeated several times after a constant traffic interval $T_1$. The graph assumes zero traffic growth and that the effect of maintenance activity is to reduce roughness to the level immediately after the construction, totally independent of how rough the road is at any point in time. The shaded area represents the cumulative increase in vehicle operating cost due to increase in roughness. This area is equal to:
Fig. 10b shows the cumulative increase in post-construction cost (VOC + Maintenance Cost) for the traffic interval \( T_1 \). Cost of maintenance is a fixed vertical line every time maintenance is performed. A Total Cost Line (TCL) is drawn which connects the total cost coordinates after each maintenance cycle. This line passes through the origin. The slope of the TCL will depend on the shape of the excess VOC curve and on the unit cost and interval of maintenance activity. The optimum maintenance interval will therefore be given by the total cost line with the least gradient.

**Illustrative Example:**

Consider a gravel road with lateritic surface material and whose traffic consists of light-goods vehicles. The equation for VOC is given in Table 3 for laterites as follows:

\[
VOC = 213.32 + 0.009T^2
\]

where \( VOC \) = vehicle operating costs in $/1000 kms.
\( T \) = cumulative traffic volume (1000's)

Assuming a grading interval of \( T_1 \) vehicles, the cumulative increase in VOC is, from Equation 50, equal to:

\[
\int_{T_1}^{T_1} (\text{UnitYOC} \cdot T) \, dT = T_1 \cdot 213.32
\]

\[
= 107 \cdot T_1^2 + 0.003 \cdot T_1^3 - T_1 \cdot 213.32
\]
Figures 10a & b. Relationships between excess VOC and Maintenance Cost

Source: (32)
If we let $M =$ maintenance cost in US $/grading, the slope of the Total Cost Line (TCL) is expressed as:

$$m = \frac{107T_1^2 + 0.003T_1^3 - T_1 \times 213.32 + M}{T_1}$$

$$\frac{dm}{dT_1} = 107T_1^2 + 0.006T_1^3 - M$$

Setting $M = $ 200 and solving for $T_1$ yields $T_1 = 13.8$ or 13,800 vehicles in both directions. For an average daily traffic of 30 light goods vehicle in one direction, the optimal blading frequency is once every 230 days.

The straightforward nature of the solution presented above is attributed to the use of few explanatory variables in the prediction equations for roughness and vehicle operating costs as well as the assumption of fixed maintenance costs. The major flaw in the analysis is the assumption that roughness is brought back to the constructed value everytime maintenance takes place, no matter how high the before-maintenance roughness is. Maintenance costs should have been expressed as a function of roughness before blading (or cumulative traffic in this case) for this assumption to become acceptable. Likewise, the analysis would not have been as simple as this if costs were discounted, in which case a more exact solution would have been generated. Apart from these, the system offers a very simple mathematical approach which may be adopted by agencies who do not care a lot about precision but would not depend on rule-based type of management described in Section 3.1. However, there is a high degree of accuracy lost due to the assumptions made and these may result in suboptimal decisions. A more accurate and comprehensive analysis may be necessary.
3.2.2 - The Highway Design and Maintenance Standards Model (HDM)

The more extensive road studies in Brazil described in Section 2.2.3 resulted in a comprehensive investment model for road design, construction and management called HDM whose functions are similar to RTIM. The World Bank developed this system for use in evaluating road investment in many developing countries [33]. This model performs financial and economic analyses of user-defined alternatives for both paved and unpaved roads. An added feature is the Expenditure Budgeting Model (EBM) which finds the set of design and maintenance options that minimizes the total discounted transport costs or maximizes the net present value of the system when the user comes to consider the impact of expenditure constraints on the composition of the best possible group of alternatives. The performance functions for unpaved roads estimated in the Brazil Study are discussed in Section 2.2.3. While Kerali was able to take the vehicle operating and maintenance cost functions out of the RTIM and developed a model that gives optimum time to maintain the unpaved roads, no such attempt was made so far with the HDM relationships. In the present HDM working model, sets of design, construction and maintenance options are input as alternatives and each alternative is analyzed by calculating its life-cycle costs.

Maintenance options are entered as number of bladings per year (blading frequency). A "steady-state" roughness cycle is assumed which represents an equilibrium condition given a specific maintenance policy [34]. Fig. 11 shows the performance curves assumed by HDM for unpaved roads. Maximum and minimum values of roughness (Qlmax and Qlmin, respectively) are determined from the following equations:

$$Q_{lmax} = \text{Max} [279 - 421\times(0.5 - \text{MGDj})^2 + 0.220\times C - 9.93\times \text{RF}\times \text{MMP}; 150]$$

where \(\text{MGDj} = \) material dust gradation ratio
\(C = \) curvature
RF = rise plus fall
MMP = precipitation
j = surfacing type

\[ Q_{\text{min}} = \max \left[ 10; \min \left( 100; 4.69 \times D_{95}; (1 - 2.78 \times \text{MG}) \right) \right] \] (52)

Where:
- \( D_{95} \) = maximum particle size of material, defined as the equivalent sieve opening through which 95% of the material pass
- \( \text{MG} \) = slope of mean material gradation
  \[ = \min \left[ \text{MG}_j; 1 - \text{MG}_j; 0.36 \right] \]
- \( \text{MG}_j = (\text{MG}_{0.75} + \text{MG}_{4.25} + \text{MG}_{0.2}) / 3 \)

With a specified blading interval and known material properties, the average roughness is computed as:

\[ Q_{\text{lave}} = Q_{\text{max}} + (1 - a)(1 - b) \left[ Q_{\text{max}} - Q_{\text{min}} \right] / [(1 - a)(1 - b)] \] (53)

Where:
- \( a = 0.553 + 0.230 \times \text{MGD} \)
- \( b = \exp \left( c \times ( \text{TG2} - \text{TG1}) \right) ; \quad 0 < b < 1 \)
- \( \text{TG2} - \text{TG1} = \) interval of blading
- \( c = -0.001 (0.461 + 0.0174 \times \text{ADL} + 0.0114 \times \text{ADH} + 0.0287 \times \text{ADT} \times \text{MMP}) \)
- \( \text{ADT} = \) average daily vehicular traffic in both directions in vehicles/day
- \( \text{ADL} = \) average daily light vehicle traffic (\( \text{GYW} < 3500 \text{ kgs.} \)) in both directions (vehlday)
- \( \text{ADH} = \) average daily heavy vehicle traffic (\( \text{GYW} \geq 3500 \text{ kgs.} \))

And the steady-state roughnesses before and after blading are:
\[ Q_{\text{high}} = \frac{Q_{\text{limax}} (1-b) + Q_{\text{limin}} (1-a) b}{(1-a b)} \]  
\[ Q_{\text{low}} = \frac{Q_{\text{limin}} (1-a) + Q_{\text{limax}} a (1-b)}{(1-a b)} \]

The analysis period used in HDM is equal to one regravelling cycle at the end of which the gravel thickness falls below a pre-set minimum allowable gravel thickness. The user may specify a blading strategy as either one of the following:

(a) scheduled - fixed time interval in days between successive gradings
(b) traffic-responsive - a fixed traffic interval in number of vehicle passes between successive gradings
(c) roughness-responsive - maximum allowable roughness

In all cases the average roughness between successive gradings, \( Q_{\text{ave}} \), is computed as a function of the number of days between gradings. If only the post-construction costs were to be analyzed, an optimization model should have been formulated which would minimize the sum of user costs and maintenance costs.

The assumption of equilibrium or steady-state roughness maintained by HDM is similar to the main assumption held by the model developed in this work but the derivation is totally different. However, the approximation of the road user and maintenance costs adopted in our final model is much more accurate than the ones presented in the HDM model.

3.3.3 - Maintenance and Design System (MDS) for Unpaved Roads

A model that evaluates alternative regravelling and blading strategies for unpaved roads was developed by Visser [25] using the performance relationships estimated in the Brazil Study but calibrated for South African conditions. The
Figure 11. Prediction of Roughness Progression over time for a study section in Brazil.
criterion used in the evaluation was the total transport costs, including road maintenance and road user costs. A flowchart of this system (MDS) is shown in Fig. 12. The model is coded in Fortran and starts by reading the required input parameters as shown in Table 11. After the input data is edited, the first step is to check whether the road type would fulfill the requirements of wet-season passability for a specific road purpose. Equations 32 and 33 define the criteria used for wet-season passability. Next, regravelling strategies are evaluated for those sections with gravel surfacing. Gravel loss should not exceed the limit specified by the minimum allowable gravel thickness. Fixed increments of 25 mm. are used in evaluating alternative regravelling strategies. Computations are halted when the discounted cost for the initial gravel thickness and regravelling thickness combinations exceed 20% of the minimum strategy cost. When optimal regravelling strategy is determined, the model then generates blading alternatives which are expressed in number of bladings per year. To accommodate the growth of traffic, blading frequencies are adjusted such that the number of vehicle passes between bladings remains approximately constant during the analysis period. Annual average roughness is computed first by integration of the roughness-time
relation for every grade-curvature combination and then by obtaining the weighted average over the road link using as weights the proportion of the road

<table>
<thead>
<tr>
<th>INFLUENCE OF ADT ON TOTAL COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>BASIC DESIGN INFORMATION</td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td>LENGTH OF THE ANALYSIS PERIOD (YEARS)</td>
</tr>
<tr>
<td>LENGTH OF ROAD LINK (KM)</td>
</tr>
<tr>
<td>MINIMUM TIME BETWEEN REGRAVELLINGS (YEARS)</td>
</tr>
<tr>
<td>REGRAVELLING STRATEGY</td>
</tr>
<tr>
<td>SALVAGE VALUE OF GRAVEL SURFACING (PERCENT)</td>
</tr>
<tr>
<td>DISCOUNT RATE OR TIME VALUE OF MONEY (PERCENT)</td>
</tr>
<tr>
<td>ROAD WIDTH (M)</td>
</tr>
<tr>
<td>LENGTH OF DRY SEASON (DAYS)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TRAFFIC DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVERAGE DAILY CAR TRAFFIC AT START OF AN PERIOD</td>
</tr>
<tr>
<td>AVERAGE DAILY BUS TRAFFIC AT START OF AN PERIOD</td>
</tr>
<tr>
<td>AVERAGE DAILY MEDIUM TRUCK TRAFFIC AT START OF AN PERIOD</td>
</tr>
<tr>
<td>AVERAGE DAILY HEAVY TRUCK TRAFFIC AT START OF AN PERIOD</td>
</tr>
<tr>
<td>ANNUAL GROWTH RATE OF CAR TRAFFIC (PERCENT)</td>
</tr>
<tr>
<td>ANNUAL GROWTH RATE OF BUS AND TRUCK TRAFFIC (PERCENT)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MATERIAL INFORMATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>PERCENTAGE OF SURFACING MATERIAL PASSING 0.074 MM SIEVE</td>
</tr>
<tr>
<td>PLASTICITY INDEX OF SURFACING MATERIAL (PERCENT)</td>
</tr>
<tr>
<td>SOAKED CRQ OF GRAVEL SURFACING (PERCENT)</td>
</tr>
<tr>
<td>SOAKED CRQ OF INSITU ROADBED (PERCENT)</td>
</tr>
<tr>
<td>TYPE OF SURFACING TO BE EVALUATED</td>
</tr>
<tr>
<td>SURFACING MATERIAL TYPE</td>
</tr>
<tr>
<td>IS ROAD WITHOUT SURFACING PERMITTED TO BECOME IMPASSIBLE</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CONSTRUCTION INFORMATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVERAGE GRAVEL THICKNESS AT START OF AN PERIOD (MM)</td>
</tr>
<tr>
<td>MINIMUM ALLOWABLE THICKNESS OF ADDED GRAVEL (MM)</td>
</tr>
<tr>
<td>ROUGHNESS AT START OF ANALYSIS PERIOD (Q1*)</td>
</tr>
</tbody>
</table>

Table 11 - Input Information in Running MDS
**INFLUENCE OF ADT ON TOTAL COST**

**COST INFORMATION**

<table>
<thead>
<tr>
<th>COST INFORMATION</th>
<th><strong>Cost</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>COST OF GRAVEL SPREAD, DLR$/M³</td>
<td>12.00</td>
</tr>
<tr>
<td>COST OF MOVING REGRAVELLING EQUIPMENT TO LINK, DLRs</td>
<td>0</td>
</tr>
<tr>
<td>COST OF NEW CAR, DLRs</td>
<td>6500</td>
</tr>
<tr>
<td>COST OF NEW BUS, DLRs</td>
<td>135000</td>
</tr>
<tr>
<td>COST OF NEW MEDIUM TRUCK, DLRs</td>
<td>19000</td>
</tr>
<tr>
<td>COST OF NEW HEAVY TRUCK, DLRs</td>
<td>40000</td>
</tr>
<tr>
<td>COST OF GASOLINE, DLR$ PER LITER</td>
<td>28</td>
</tr>
<tr>
<td>COST OF DIESEL FUEL, DLR$ PER LITER</td>
<td>27</td>
</tr>
<tr>
<td>COST PER CAR TIRE, DLR$</td>
<td>50</td>
</tr>
<tr>
<td>COST PER BUS OR TRUCK TIRE, DLR$</td>
<td>250</td>
</tr>
<tr>
<td>DAILY COST TO OPERATE MOTOR-GRAVLER, DLRs</td>
<td>200</td>
</tr>
</tbody>
</table>

**ROAD GEOMETRY**

**PERCENTAGE OF LENGTH OF LINK IN EACH CATEGORY**

<table>
<thead>
<tr>
<th>CURVATURE</th>
<th>7-2</th>
<th>2-6</th>
<th>4-6</th>
<th>6-10</th>
<th>8-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>TANG AND RAD &gt; 400M</td>
<td>50</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>400M &lt; RADIUS &gt; 200M</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>200M &lt; RADIUS &gt; 100M</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>RADIUS &lt; 100M</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**PRINTOUT CONTROLS**

<table>
<thead>
<tr>
<th>PRINTOUT CONTROLS</th>
<th><strong>Required</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>IS PRINTOUT OF REGRAVELLING STRATEGIES REQUIRED</td>
<td>Y</td>
</tr>
<tr>
<td>IS PRINTOUT OF ROUGHNESS STRATEGIES REQUIRED</td>
<td>Y</td>
</tr>
</tbody>
</table>

**COMPUTED INFORMATION**

<table>
<thead>
<tr>
<th><strong>INFORMATION</strong></th>
<th><strong>Value</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>MINIMUM ALLOWABLE GRAVEL THICKNESS (MM)</td>
<td>49</td>
</tr>
</tbody>
</table>

Table 11 - Continuation

Link in each grade-curve combination. This annual average roughness is then used in the user cost computations.
Road maintenance costs were estimated using field experience and verified by discussions with the personnel associated with unpaved roads' maintenance in Texas. Road user cost elements included in the analysis are essentially those developed in the Brazil study. The total costs of maintenance and vehicle operation are then computed for each maintenance strategy. The program terminates by ordering the strategies in terms of increasing discounted total costs. Sample output from MDS run are shown on Tables 12 and 13 for both regravelling and blading strategies.

The MDS model is by far the most comprehensive model applied for management of unpaved roads. Visser made a fairly substantial contribution by developing a good model that yields cost-effective maintenance strategies. The system was also extended to the network level in which dynamic programming formulation was used to solve for the minimum total discounted transport costs over each road link group for each of the alternative maintenance strategies subject to maintenance budget constraints. The main disadvantage of using the model is that it does not give closed-form solutions. Also, the approximation of roughness by an average roughness for use in cost computation (which is performed on an annual basis and discounted annually) may not give very accurate solutions.

3.2.4 - Bolivian Maintenance System for Unpaved Roads

The Bolivian National Highway Agency (Servicio Nacional de Caminos) is using a computer program called RSML which includes the logit equations given in Section 2.2.6 (Eqns. 34 and 35) representing the performance of aggregate surfaced roads. The procedures are summarized as follows:

1. A road section is assumed to be just rehabilitated and its roughness following this activity is defined.
2. The roughness of the road for each succeeding day is predicted.
## Table 12 - Annual Data for the Different Blading Strategies using HDM

<p>| STRAT YEAR—NO. OF ANNUAL AV MAX RUT ANNUAL DIS ANNUAL DISC |
|---------------|-----------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th>NO.</th>
<th>BLADE/YR ROUGHNESS</th>
<th>DEPTH (M)</th>
<th>MAINT CST</th>
<th>USER COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>24.9</td>
<td>61.1</td>
<td>1498.6</td>
</tr>
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<td>1</td>
<td>1</td>
<td>24.9</td>
<td>55.5</td>
<td>1254.1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>24.9</td>
<td>59.4</td>
<td>1197.1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>24.9</td>
<td>46.3</td>
<td>1142.7</td>
</tr>
<tr>
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<td>5</td>
<td>24.9</td>
<td>41.1</td>
<td>1098.7</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>25.4</td>
<td>38.0</td>
<td>1041.2</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>25.4</td>
<td>34.2</td>
<td>993.8</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>25.4</td>
<td>33.2</td>
<td>948.1</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>25.4</td>
<td>29.8</td>
<td>905.5</td>
</tr>
<tr>
<td>1</td>
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<td>10</td>
<td>14.9</td>
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Table 12 - Annual Data for the Different Blading Strategies using HDM
<table>
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<tr>
<th>RANKING NO.</th>
<th>STRATEGY NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

| REGR COST   | 15703        | 15703        | 15703        | 15703        | 15703        |
| ROAD MAINT  | 694          | 599          | 515          | 789          | 423          |
| USER OPER   | 3677         | 3778         | 3696         | 3635         | 4039         |
| CLOS DELAY  | 0            | 0            | 0            | 0            | 0            |
| TOTAL COST  | 20074        | 20081        | 20114        | 20127        | 20165        |

<table>
<thead>
<tr>
<th>NUMBER OF BLADING</th>
</tr>
</thead>
<tbody>
<tr>
<td>IN 1ST YEAR   7</td>
</tr>
<tr>
<td>IN 10TH YEAR  11</td>
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<table>
<thead>
<tr>
<th>SPOT REGRAVELLING STRATEGY</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRATEGY NO. 1</td>
</tr>
</tbody>
</table>

| IN REG TH (MM)   | 75  |
| REGR TH (MM)    | 75  |
| AT TIME (YEARS) | 1,325 |
| REG TH (MM)     | 75  |
| AT TIME (YEARS) | 2,912 |
| REG TH (MM)     | 75  |
| AT TIME (YEARS) | 5,225 |
| REG TH (MM)     | 75  |
| AT TIME (YEARS) | 7,097 |
| REG TH (MM)     | 75  |
| AT TIME (YEARS) | 8,717 |

<table>
<thead>
<tr>
<th>STANDARDS OF SERVICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAX ROUGHNESS</td>
</tr>
<tr>
<td>AVG ROUGHNESS, QI</td>
</tr>
<tr>
<td>MAX RUT DEPTH (MM)</td>
</tr>
</tbody>
</table>

Table 13 - Summary of Maintenance Strategies from MDS run
3. User costs associated with traffic using the road each day, for each roughness condition, are accumulated.

4. User costs are accumulated to some roughness threshold (i.e. a level of roughness that will activate a maintenance response).

5. The maintenance sequence specified was two gradings followed by rehabilitation. This makes up a complete cycle. (Fig. 13)

6. Ten cycles are simulated for each roughness threshold and the combined user and maintenance costs for that roughness threshold are determined.

7. The same analysis was performed for a range of roughness thresholds.

A sample output from RSML is shown on Table 14 for an annual average daily traffic (AADT) of 100. It can be seen that the optimal frequency is given by column which is 4.3/year or approximately two gradings and two regravellings annually.

![Figure 13 - RSML gravel road maintenance activity sequence](image)

There are many drawbacks in this system. First, it assumes that the road returns to the initial condition after regravelling or grading. The definition of the cycle as consisting of two gradings and one regravelling does not represent the...
ideal strategy of maintaining the roads. In most cases, regravelling is done after more than two bladings since gravel loss rate is quite slow. The deterioration equations used for predicting roughness do not include other parameters like climate, geometry, curvature and material properties which are important determinants of roughness. Finally, the regravelling strategy depends on the arbitrary roughness threshold level assumed, which does not make sense because gravel loss determines the need for regravelling instead of roughness.

### Rehabilitation and Grading

<table>
<thead>
<tr>
<th>MAX MAINTENANCE</th>
<th>USER COSTS</th>
<th>TOTAL COSTS</th>
<th>TOTAL ADJUSTED</th>
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<td>COSTS</td>
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### Summary

This chapter describes the types of maintenance systems and the level of analysis incorporated in them. It clearly differentiates between systems used in developed and developing countries. The more accurate mathematical models used in developing countries are either unrealistic because of the assumptions maintained in the analysis or do not give closed-form solutions. Hence, the need for an alternative analytical technique which does not have any...
of the limiting properties of the described models to solve the unpaved roads' maintenance problem is apparent.

In the next chapter, an alternative technique is proposed which uses dynamic optimization models. It will be shown how the same problem may be treated and solved in a dynamic optimization framework.
Chapter 4 - Development of a Dynamic Control Model for Optimal Blading Frequencies

4.1 Dynamic Optimization

Most of the real systems are dynamic in the sense that their behavior changes over time. Any dynamic system is characterized by its condition or state, and this condition changes with time. System performance is influenced by condition. Thus, both are functions of time. When the system does not perform as well as it should, decisions are made which may change the condition of the system in order to better its performance. Associated with such an action is a corresponding cost which the decision maker incurs. If the system is to operate for a specific length of time, a set of decisions (collectively termed 'policies') would have to be made at different times within this period. The goal of dynamic optimization is to develop a set of decisions (pertaining to time and nature of action) which optimizes the overall performance of the system over the analysis period. The measure of performance depends on the type of the system and the objectives defined by the decision-maker(s).

Several dynamic optimization techniques are currently employed depending on the type of the system under investigation and the level of accuracy desired. Dynamic programming is one approach that has been applied to discrete-time, discrete-condition problems. The discrete times are called stages and the conditions are referred to as states. At any time (stage), the system can be in any range of specified conditions (states). Transition from one state to another during the discrete time interval involves a gain or loss of a given performance parameter. The transition may be deterministic or probabilistic, which likewise depends on the nature of the system being analyzed. For the multi-stage process, dynamic programming solves for the path which the state or condition must trace in order to minimize or maximize the objective function. In the more general case of dynamic optimization problems where time is treated as continuous variable,
the process is called a "continuous multi-stage decision process". The calculus of variations furnishes the most interesting and significant example of continuous multi-stage decision process. The simplest problem that can arise is that of maximizing the integral:

\[\pi = \int F[t, x(t), x'(t)] \, dt\]  \hspace{1cm} (56)

subject to \(x(t) \geq 0\), \(x(0) = x_0\)

where

\(x(t) = \) value of state variable at time \(t\)  
\(x'(t) = \) rate of change of state variable at time \(t\)

Solution to this problem involves finding the state variable \(x^*(t)\) which maximizes the objective function \(\pi\). The extension of this problem involves constraints on the derivatives of the functions (e.g. state variable). The maximum principle for optimal control is a generalization of these calculus of variation problems and will be discussed later.

For any dynamic optimization problem, the principle of optimality is stated as [35]:

"An optimal policy has the property that whatever the initial state and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

This principle assists in the solution of the problem based on stages in both the discrete and continuous cases.

4.2 Dynamic Optimization Models for Highway Systems

For project level analysis of highway maintenance and rehabilitation, several dynamic optimization approaches have been adopted. Existing Pavement Management Systems (PMS's) proposed by highway researchers to set
maintenance strategies and policies are based on the solution of the dynamic optimization problem using one of the available techniques. The current PMS's work on the a set of agency-defined standards which are not founded on a quantified economic basis. Although this type of system is now considered impractical for pavement management, it is still widely used in unpaved roads as described in the previous chapter. For this class of roads, dynamic optimization techniques have the advantage of giving a closed-form solution to the highway maintenance problem compared with many simulation models like the Maintenance and Design System developed for South Africa (24,25) or the HDM models. Apart from this, we are able to model the interaction between maintenance and deterioration and their effects on performance more realistically in the dynamic optimization framework, thereby avoiding such restrictive assumptions as constant roughness after blading or constant maintenance cost which the mathematical models described in the previous chapter had to assume to be able to solve the problem. Dynamic optimization, therefore, is not only attractive for paved roads but would similarly be the most effective tool for managing unpaved roads.

A review of highway literature indicates extensive studies dealing with dynamic optimization of routine maintenance and rehabilitation of pavements. Probabilistic dynamic programming was used by Carnahan [36] in determining optimal maintenance decisions for a pavement system. The methodology was directed towards enhancement of PAVER (CERL) which is a pavement management system developed by Shahin and Kohn [37] whereby the state variable (condition) was treated as discrete variable (PCI ranges) and transitions from one state to another were assumed to follow a Markovian process. Baita [38, 39] formulated a dynamic control model using the principles of optimal control to compute the optimum time for rehabilitation of either flexible or concrete pavements. The objective function used in the analysis was the difference between road user utility and the sum of road user costs (vehicle operation and travel time) and overlay costs. A solution to the maximization problem with road
condition and traffic as state variables and time to rehabilitation as the control variable can be generated. The jump in the performance function resulting from application of maintenance is difficult to model in the dynamic control framework because of the discontinuity over time of the state and control variables. This difficulty constrained Balta to consider only single overlay in his formulations. In a later study, Tsunokawa [9] recognized this difficulty and proposed a practical procedure for approximating the discontinuous performance function by a continuous curve, thereby solving the multiple maintenance problem using optimal control.

Tsunokawa's work was devoted to the optimization of the frequency and thickness of overlays (rehabilitation) for highway pavements using roughness as the measure of condition. In terms of applicability, Tsunokawa's model may not find its place in actual highway practice because roughness as a measure of pavement condition does not specifically define the types of distress on the pavement. Deterioration on paved roads is manifested in different forms and severity of distress. The kind of maintenance activity that corrects a particular form of surface distress is not limited to a single technology which Tsunokawa assumed to be overlay. Realizing this, Tsunokawa proposed further research on the development of a solution procedure for problems with multiple maintenance technologies and vector descriptors of road condition and maintenance intensities, which will be of greater value for managing paved roads. However, in its current form Tsunokawa's model does not seem very attractive for pavement maintenance.

A more practical application of Tsunokawa's analytical model is in setting optimal blading frequencies for unpaved roads. For this type of road, the use of roughness as a measure of condition is justified because routine maintenance is generally limited to one activity which is blading. Moreover, blading corrects all the surface distresses that affect vehicle operation (i.e. ruts, potholes) which are all determinants of roughness. Therefore, the nature of
deterioration and maintenance of unpaved roads eliminates the need to consider a vector of condition measures and maintenance technologies that Tsunokawa had to deal with.

4.3 The Optimal Control Theory

The principles of optimal control were developed in the 50's by Pontryagin and his co-workers to generalize the solution of all calculus of variation problems. In optimal control problems, variables are divided into two classes, state variables and control variables [40]. The simplest form of the control problem is that of choosing the continuous control function $u(t)$, $t_0 \leq t \leq t_1$:

$$\max \pi = \int_{t_0}^{t_1} f(t, x(t), u(t)) \, dt$$  \hspace{1cm} (57)

subject to

$$x'(t) = g(t, x(t), u(t))$$  \hspace{1cm} (58)

$t_0, t_1, x(t_0) = x_0$ fixed; $x(t_1)$ free

The functions $f$ and $g$ should be continuously differentiable functions. The control function $u(t)$ is required to be piecewise continuous with time and affects both the performance function $\pi$ through its own value and the change in the state variable $x(t)$. Equation 58 is called the state equation. Solution to this type of optimization problem involves forming the Hamiltonian function $H$ as follows:

$$H(t, x(t), u(t), \Omega(t)) = f(t, x, u) + \Omega^\top g(t, x, u)$$  \hspace{1cm} (59)

where $\Omega(t) = \text{adjoint, auxiliary or costate variable}$
The Hamiltonian function is similar to the Lagrangian equation used in solving a non-linear programming problem, with $\Omega$ as the Lagrangian multiplier or shadow price. As such, this adjoint variable represents the marginal contribution of the change in the state variable to the performance or objective function.

To determine the optimal control variable $u^*(t)$, the derivative of the Hamiltonian function with respect to $u$ is set equal to zero, which is the optimality condition:

$$\frac{dH}{du} = f_u + \Omega^* g_u = 0 \quad (60)$$

subject to $x(t_0) = x_0$ and $\Omega(t_1) = 0$

The mathematical proofs and derivations for the optimal control solution are not included here, but the reader is referred to Kamien and Schwartz [40].

4.4 Optimal Control Model for Highway Systems

The extension of the general control problem to highway maintenance poses some problems due to the discontinuity in both the state and control variables. Fig. 14 shows the state variable (roughness) as a function of time under a maintenance strategy (overlay) which Tsunokawa used in his model. The times $t_1$, $t_2$ and $t_3$ represent the points in the life of the facility when an overlay is placed on the pavement. It can be seen that the reduction $G$ in the roughness after the first overlay is a function of both the roughness before the overlay $s(t_1^-)$ and the intensity of the overlay $w_1$. To be able to transform the problem into one which can be solved using optimal control technique, Tsunokawa approximated the sawtooth roughness curve in Fig. 14 by a
continuous average roughness curve that passes through all the midpoints of roughness $s(t)$.

Figure 14 - Sawtooth Roughness Trajectory Curve

the spikes (Fig. 15). Referring to this figure, we can see that the difference between roughnesses ($\Delta AB$) of the two points A and B on the midpoints of the spikes is equal to:

$$\Delta AB = \int_{t_1}^{t_2} R(t) \, dt - 1/2 \times [G(w_1, R(t_n^+)) + G(w_{n-1}, R(t_{n-1}^-))]$$

where

- $\dot{R}(t)$ = deterioration rate of the pavement
- $G(w_{n-1}, R(t_{n-1}^-))$ = reduction in roughness due to an overlay of $w_{n-1}$ at roughness $R(t_{n-1}^-)$
- $\tilde{G}(w_n, R(t_n^+))$ = inverse of $G$ obtained from rewriting $x = y - G(w, y)$ as $y = x + \tilde{G}(w, x)$;

equal to the reduction in roughness needed to bring roughness to condition $R(t_n^+)$
Finally, to reformulate the problem into a standard optimal control problem, the discrete overlay application time $t_n$'s and maintenance intensities $w_n$'s, where $n$ is the $n$th overlay, were considered to be generated by some underlying functions of time. The concepts of maintenance application rate $h(t)$ and maintenance intensity rate $w(t)$ were employed such that:

\[
\int_{t_{n-1}}^{t_n} h(t) \, dt = 1 \quad (62)
\]

and

\[
\int_{t_{n-1}}^{t_n} w(t) \, dt = w_n \quad (63)
\]

Using these relationships, the approximation of the average roughness curve by the roughness trend curve was derived as follows:

\[
\int h(t) \Delta AB = \int \dot{R}(t) \, dt - \int h(t) / 2 \left( G(w(t), R(t)) + G(w(t), R(t)) \right) \, dt
\]

\[
= \int [ \dot{R}(t) + h(t) K(w, R) ] \, dt \quad (65)
\]
where $K(w,R) = \text{overlay impact function}$
\[
K(w,R) = \left[ G(w,R) + G'(w,R) \right] / 2
\] (66)

For a small interval $dt$, the roughness trend curve has a slope equal to:
\[
\dot{S} = dS/dt = \ddot{R}(t) - h(t)*K(w(t), R(t))
\] (67)

This rate of change in roughness for the roughness trend curve may then be substituted into the Hamiltonian function for a general objective function of the form
\[
\begin{align*}
\min H &= C(s(t)) + h(t)^*M(w(t)) + z(t)^*[ F(S(t)) - h(t)^*K(w(t),S(t)) ] \\
\text{subject to:} & \quad h_1 \leq h \leq h_2
\end{align*}
\] (68)

where $C(s(t)) = \text{user cost function}$
$h(t) = \text{overlay application rate}$
$M(w(t)) = \text{maintenance cost function}$
$w(t) = \text{overlay intensity rate}$
$z(t) = \text{current value adjoint variable}$
$F(S(t)) - h(t)^*K(w(t),S(t)) = S = g(s,t,h)$
$h_1, h_2 = \text{minimum and maximum application rates}$

The solution to this minimization problem requires the formation of the Lagrangian $L$ with multipliers $\mu_1$ and $\mu_2$:
\[
L = C(S(t)) + h(t)^*M(w(t)) + z(t)^*[ F(S(t)) - h(t)^*K(w(t),S(t)) ] \\
+ \mu_1 ( h_1 - h ) + \mu_2 ( h - h_2 )
\] (69)
For the constrained minimum with respect to \( h \) and \( w \), the conditions are:

\[
\begin{align*}
\frac{\partial L}{\partial h} &= M(w) - z K(w, S) - \mu_1 + \mu_2 = 0 \\
\mu_1 &\geq 0, \quad \mu_1 (h_1 - h) = 0 \\
\mu_2 &\geq 0, \quad \mu_2 (h - h_2) = 0 \\
\frac{\partial L}{\partial w} &= h^2 (M_w(w) - z K_w(w, S)) = 0
\end{align*}
\]

(70) \hspace{1cm} (71) \hspace{1cm} (72) \hspace{1cm} (73)

These conditions are equivalent to:

\[
\begin{cases}
h_1 \\
h \\
h_2
\end{cases}
\begin{cases}
> \\
= \\
<
\end{cases}
\begin{cases}
Hh = M(w) - z K(w, S) = 0
\end{cases}
\]

(74) \hspace{1cm} (75) \hspace{1cm} (76)

In reality, upper and lower bounds do not exist only for application rate \( h(t) \). The overlay intensity rate \( w(t) \) is also governed by minimum and maximum values and this should have been accommodated in the model.

Equation 74 means that if the change in total cost with respect to the application rate \( h(t) \) is greater than zero, then it is best to apply overlay at the lowest frequency (\( h_1 \)). Conversely, if the marginal value of the total cost decreases with the application rate (Eq. 76), maintenance should be done as often as possible. The solutions defined by these two equations are called bang-bang controls. When the term \( Hh \) is equal to zero, \( h \) assumes values between \( h_1 \) and \( h_2 \) and is termed as a singular control solution.

Tsunokawa assumed that the solution approaches steady-state conditions, which means that the roughness \( S(t) \) and adjoint variable \( z(t) \) both attain constant values over time (\( \dot{S} = \dot{z} = 0 \)). This equilibrium assumption is called a
saddle-point and is true for infinite time horizon problems which Tsunokawa assumed in his analysis. The curves that satisfy the equilibrium condition defined above are called stable branches.

For the discounted value optimization problem, the canonical equations used by Tsunokawa are the following:

\[
\begin{align*}
\frac{dz}{dt} &= z - dH \cdot dS = (1 - F(S) + h^aK_3(w, S)) \cdot z - C_3(S) \quad (77) \\
\frac{dS}{dt} &= S - dH \cdot dz = F(S) - h^aK(w, S) \quad (78)
\end{align*}
\]

To solve for the steady-state equations, he expressed w as a function of s and z, \( w(w(S, z)) \), using the optimality condition (Eq. 73) and substituted this expression into the canonical equations to determine steady-state values of S and z. The procedures for solving steady states of bang-bang and singular control for the pavement maintenance problem are described in the following subsections.

4.4.1 Steady-States of Bang-Bang Control

Given the expression for w as a function of s and z (solving Eq. 73 for w), solve Equations 77 and 78 for S and z, using either \( h_1 \) or \( h_2 \). Whichever solution satisfies Eq. 74 (for \( h_1 \)) or Eq. 76 (for \( h_2 \)) is the steady-state solution. The corresponding value of w may be determined from the equation \( w = w^*(S, z) \).

4.4.2 Singular Control Solution

In the case of singular control, \( H_1 = 0 \). If this condition holds, then it must be true that \( dH_1/dt \) or \( H_1 = 0 \) (the converse is not true). Therefore if we differentiate Eq. 75 with respect to t, we get the following expression:
\[
d\frac{dH_h}{dt} = \dot{H}_h = M_w \dot{w} - z k - z k_w \dot{w} - z k_s \dot{S} = 0 \tag{79}
\]

Using Eq. 73 for \( h = 0 \), then \( H_h \) can be simplified to:

\[
\dot{H}_h = -z k - z k_s \dot{S} = 0 \tag{80}
\]

Substituting the canonical equations 77 and 78, we obtain the following:

\[
\dot{H}_h = (-iK + KF_g - hKg) \dot{z} + KCS - FKgz + hKgZ = 0 \tag{81}
\]

\[
= (-iK + KF_g - FKg) \dot{z} + KCg = 0 \tag{82}
\]

Note that \( \dot{z} = 0 \) and \( \dot{S} = 0 \) satisfy Eq. 80. This means that all singular control solutions are steady-states. The loci of the intersection of \( Hh=0 \) and \( \dot{H}h=0 \) determine the singular control solution. This solution must satisfy the condition that \( h_1 < h < h_2 \).

4.4.3 Boundary Control Problem

Since the initial roughness \( S(0) \) is equal to some value \( S_0 \), we have to find an initial value for the roughness trend curve \( S_0E \) which corresponds to \( S_0 \). This requires defining a set of \( S_0E \) values in the range \([S_0E_{min}, S_0E_{max}]\) and determining if unique stable branches exist for all these values. Stable branches or curves are those that converge to the steady-state solutions and are determined using phase diagrams. With unique stable branches for the defined range, the optimal trajectories (set of optimal control solution) are found. All the \( S_0E \)'s have corresponding \( S_0 \)'s in the sawtooth curve which we defined earlier. The
object is to find the initial roughness trend value $S_0E$ that corresponds to the true initial roughness.

4.4.4 Trajectory Diagram and Optimal Decision Variables

After the range of $S_0E$'s defining unique stable branches has been defined, the trajectory diagram maybe constructed. This curve represents the values of $S$, $z$, $h$ and $w$ each time along the stable branches. The reader is referred to Tsunokawa [9] for the complete detail of constructing this curve.

With the trajectory diagram, the optimal overlay strategy in terms of real decision variables $t_0$ and $w_n$ maybe computed using Equations 62 and 63. This strategy depends on the selected value of the initial roughness $S_0E$ which defines the origin of the trajectory diagram. The time to first overlay $t_1$ is then used along with the value of the roughness trend curve at time $t_1$ ($S^m(t_1)$) to solve for the initial roughness $S_0$, which is equal to:

$$S^m(t_1) = S(t_1^--) - 1/2Q(w_1,S(t_1^--)).$$  \( 83 \)

4.4.5 Sufficient Conditions for Optimality

So far we just defined the necessary conditions for optimality. Tsunokawa proved that if the vehicle operating cost function, overlay cost function and deterioration are all convex and the effective overlay impact function $K(w,s)$ is concave, then the minimized Hamiltonian is convex, thereby solving the optimization problem.
4.5 Application of Tsunokawa's formulation to Unpaved Roads

As discussed in section 4.2, the methodology used by Tsunokawa for the pavement maintenance case finds a more suitable application in the determination of optimal maintenance frequencies for unpaved roads. The major differences in the two solutions are:

1. there is only one control variable for unpaved road which is the time to blade the road.
2. maintenance cost is a function of the roughness before blading
3. analysis is performed on a daily basis, as opposed to yearly time increment.
4. more explanatory variables are included to predict the effects of maintenance on condition deterioration and vehicle operating costs.

The road deterioration equation used in this analysis is taken from a preliminary report for the Brazil Road Studies on unpaved road deterioration and reported as follows [41]:

\[
\ln R = \ln R_0 + T (0.0070 + 1.3 \times 10^{-5} \times \text{AADT} - 0.0036 \times S - 3.5 \\
\times 10^{-5} \times R_A - 6 \times 10^{-8} \times \text{AADT} \times R_A - 0.0136 \times W \times \text{RAD})
\]

(84)

where
\[
R = \text{roughness in counts/km, using a Maysmeter}
\]
\[
R_0 = \text{roughness after blading in counts/km.}
\]
\[
T = \text{number of days since last blading}
\]
\[
\text{AADT} = \text{annual average daily traffic in both directions}
\]
\[
S = \text{season dummy variable}
\]
\[
= \begin{cases} 
0 & \text{if dry} \\
1 & \text{if wet}
\end{cases}
\]
\[
W = \text{road width in meters}
\]
\[
\text{RAD} = \text{curve radius in meters} \\
\text{RA} = \text{roughness after blading}
\]

The above equation is plotted on Figs. 16a and b for traffic volumes of 30 and 250 pcus/day respectively and with different initial roughnesses.

Assuming that the road has negligible horizontal curvature, the radius of the curve may be set at an infinitely large value. Thus, the last term of the equation may be equated to zero. The state equations given by Eq. 58 for the general optimal control problem and Eq. 67 for the case of pavement require that the rate of change of roughness with time \( S \) for the roughness trend curve be a function of the roughness at that particular time, or

\[
\dot{S} = f(S) - h(t) \cdot K(S) \tag{85}
\]

This equation assumes that the rate of change of roughness of the roughness trend curve without maintenance is equal to the rate of change of roughness of the sawtooth curve. To conveniently use the roughness progression equation defined by Eq. 84 in the state equation, its time derivative must be a function of the roughness at any time. Differentiating the said equation with respect to \( t \) yields (assuming variables are constant including the season dummy variable):

\[
\frac{dR}{dt} = R_0 \cdot \exp \left( KT \cdot K \right) \\
= R \cdot K \tag{86}
\]

where \( K = \) expression in the parenthesis

\[
= f \left( \text{fixed variables, } R, R_0 \right)
\]
Figure 16a. Roughness Progression with Time

\[ Q = 30 \text{ pcus/day} \]

Figure 16b. Roughness Progression with Time

\[ Q = 250 \text{ pcus/day} \]
This is not the functional form required for \( f(S) \) since \( R_o \) can take different values. In order to come up with the appropriate form, an equation is defined which is an approximation to the existing model. This equation is given as:

\[
R = R_o \times \exp \left( 0.0034 + 1.3e^{-5}Q \right)
\]  

(87)

The equation predicts roughness as a function of time and traffic volume. It is consistent with the general observation that the road deteriorates even without traffic. Table 15 shows the different values computed for different combinations of \( R_o, Q \) and \( T \) using the two equations. The values in the parentheses are the estimates.

Differentiating Eq. 87 with respect to \( t \) gives

\[
\frac{dR}{dt} = R_o \times \exp K_1 \times K_1
\]

\[= R \times K_1
\]

(88)

where \( K_1 \) = constant term depending on the assumed value of AADT

An equation predicting roughness after blading was estimated in the Brazil Road Studies. This equation was then calibrated to South African conditions by Visser (Eq. 26) as follows:

\[
\ln RA = 1.4035 - 0.0239W - 0.0048SV + 0.016PI
+ 0.6309RB + 0.1499T1 + 0.309T2 + 2 \times 10^{-4}Q
+ 0.2056BS - 0.011BSPI
\]

(89)

or

\[
RA = RB^{0.83}\exp K_2
\]

(90)
Table 15 - Predicted and Approximated Values of Roughness

<table>
<thead>
<tr>
<th></th>
<th>T = 50</th>
<th>T = 100</th>
<th>T = 200</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q = 30 pcu's/day</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ro = 50</td>
<td>86</td>
<td>87</td>
<td>131</td>
</tr>
<tr>
<td></td>
<td>(60)</td>
<td>(73)</td>
<td>(106)</td>
</tr>
<tr>
<td>Ro = 100</td>
<td>120</td>
<td>145</td>
<td>210</td>
</tr>
<tr>
<td></td>
<td>(120)</td>
<td>(146)</td>
<td>(213)</td>
</tr>
<tr>
<td>Ro = 150</td>
<td>165</td>
<td>180</td>
<td>218</td>
</tr>
<tr>
<td></td>
<td>(181)</td>
<td>(219)</td>
<td>(275)</td>
</tr>
<tr>
<td></td>
<td>Q = 150 pcu's/day</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ro = 50</td>
<td>70</td>
<td>98</td>
<td>192</td>
</tr>
<tr>
<td></td>
<td>(65)</td>
<td>(85)</td>
<td>(145)</td>
</tr>
<tr>
<td>Ro = 100</td>
<td>125</td>
<td>158</td>
<td>248</td>
</tr>
<tr>
<td></td>
<td>(130)</td>
<td>(170)</td>
<td>(291)</td>
</tr>
<tr>
<td>Ro = 150</td>
<td>169</td>
<td>190</td>
<td>240</td>
</tr>
<tr>
<td></td>
<td>(196)</td>
<td>(256)</td>
<td>(437)</td>
</tr>
<tr>
<td></td>
<td>Q = 500 pcu's/day</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ro = 50</td>
<td>83</td>
<td>139</td>
<td>388</td>
</tr>
<tr>
<td></td>
<td>(82)</td>
<td>(134)</td>
<td>(362)</td>
</tr>
<tr>
<td>Ro = 100</td>
<td>142</td>
<td>201</td>
<td>405</td>
</tr>
<tr>
<td></td>
<td>(164)</td>
<td>(269)</td>
<td>(724)</td>
</tr>
</tbody>
</table>
where \( RA \) = roughness in counts/km after blading
\( RB \) = roughness before blading
\( K_2 \) = constant term defined by the input variables
all other variables are as defined in Eq. 26.

If we let \( K_3 = \exp K_2 \), then the improvement in road condition or decrease in roughness \( G \) as a result of blading is just equal to:

\[
G = RB - RA = RB - K_3 \cdot RB^{0.63}
\]

(91)

The same function holds for the roughness trend curve value (\( S \)). Rewriting Eq. 89 for the roughness trend curve gives

\[
G = S - K_3 \cdot S^{0.63}
\]

(92)

Recall that the overlay impact function for the paved road is given by:

\[
K = \frac{1}{2} \cdot (G + G)
\]

and

\[
K_s = \frac{1}{2} \cdot (G_s(1-G_s) + G_s) = \frac{1}{2} \cdot (2G_s -G_s^2)(1 - G_s)
\]

(93)

(94)

Taking the derivative of Eq. 92 with respect to \( S \), we get the following expression:

\[
G_s = 1 - 0.63 \cdot K_3 \cdot S^{-0.37}
\]

(95)
Substituting this to Eq. 94 for $K_s$ and integrating with respect to $S$ results in the corresponding **blading impact function** for unpaved roads which is given by:

$$K = 0.58 \cdot S^{1.37/K_3} - 0.5 \cdot K_3 \cdot S^{0.63}$$  

(Eq. 96)

Equations 88 and 96 can now be used to define the state equation $S$ which is required for the Hamiltonian function. This state equation is written as:

$$\dot{S} = f(S) - h(t) \cdot K(S) \quad (\text{from Eq. 85})$$

$$\dot{S} = K_1 \cdot S - h(t) \cdot (0.58 \cdot S^{1.37/K_3} - 0.5 \cdot K_3 \cdot S^{0.63})$$  

(Eq. 97)

The next step is to determine the performance functions used in the analysis, which classifies the problem as either a maximization or a minimization problem. Road user utility surplus is a good measure of the consumer benefit that can be derived from providing a smooth and well-maintained roadway. This concept is consistent with economic theory of supply and demand. However, it is very difficult, if not totally impossible, to draw the demand curve for a highway based on the road users’ willingness to pay for highway service. In the first place, most highways (particularly unpaved roads) are used no matter how bad their conditions are simply because there are no alternatives available to the user, unlike in the case of the market for a certain commodity where there are many choices open to the consumer.

In solving for the optimum rehabilitation of pavements, Balla equated user utility to the largest cost that a user will tolerate and still choose the roadway. He assumed that user utility is equal to the user cost associated with 1.0 PSI and 50,000 AADT [39]. His objective function maximizes the difference between the utility and the sum of maintenance costs and vehicle operating costs. However, this is equivalent to minimizing the sum of maintenance and vehicle operating costs.
since at any time t for a given traffic volume and road condition the user utility is fixed. The lack of appropriate analytical tool to estimate the user utility derived from highway service makes it unreasonable to define an objective function similar to Bata's.

It is therefore more practical to concentrate on the road user costs and agency costs when analyzing highway services. In our objective function, we will seek to minimize the sum of maintenance costs and vehicle operating costs. The inclusion of the value of travel time in the objective function is debatable. Road condition affects the operating speed of the vehicle which in turn determines the travel time. The value that we have to attach to travel time is a function of many variables which include the purpose of the trip. There is also a time element in the value of trip time in that it differs on a case to case basis. On some occasions the trip may be very important and travel time is therefore highly valued. Since the purpose of the trip on unpaved roads ranges from mere pleasure to transport of important commodities, it is not practical to include travel time in our objective function. Comfort is even more abstract and unquantifiable a level of service variable as value of travel time. Hence, we will constrain our analysis to costs that are measurable such as maintenance and vehicle operating costs.

Based on the above analysis the objective function is formulated for the continuous optimization problem as:

$$\min \pi = \int_0^T \left[ C(S(t)) + h(t) M(S(t)) \right] e^{-it} dt$$  \hspace{1cm} (98)

subject to

$$S(0) = S_0$$

where

- $S(t)$ = roughness defined by the roughness trend curve
- $C(S(t))$ = vehicle operating cost function
- $M(S(t))$ = maintenance cost function
- $h(t) = \text{blading application rate}$
\[ i = \text{interest rate} \]

Note that the objective function is for the roughness trend curve which is used to approximate the true roughness sawtooth curve.

For simplicity in calculation, the only component included in the vehicle operating cost function is the cost of fuel consumption because it the major component of cost. The analysis can be generalized by including other cost components such as parts consumption, oil consumption or labor which are all influenced by road condition. Although the vehicle operating cost functions estimated from the Brazil study are the most accurate with respect to the number of variables used and the method of analysis employed, they are too complicated and cumbersome to deal with in the optimization solution. Thus, we make use of the regression equations developed in the Caribbean Study [20] which also include many relevant parameters and are statistically significant. The expressions for vehicle speed and fuel consumption for a passenger car are given by equations 10 and 11 in Section 2.2.2.1. These are:

for speed
\[ V = 67.6 - 0.078*RS - 0.067*F - 0.024*C - 0.00087*P \]

for fuel consumption
\[ FL = 1.08 ( 24 + 969V + 0.0076*V^2 + 1.33*RS - 0.63*F + 0.0029*F^2 ) \]

where \( V = \text{vehicle speed in kilometers/hour} \)
\( FL = \text{fuel consumption in ml/km.} \)

The value of fuel consumption is plotted against road roughness in Fig. 17 for assumed values of rise, fall and curvature.

Cost of maintenance is influenced by the productivity of the motor grader.
This productivity is measured in terms of the number of kilometer passes that a motor grader is able to blade for a given day depending on the roughness of the road to be bladed. From studies made by Visser in South Africa [25], he found that the relationship between daily productivity of a typical motor grader and the roughness before blading is represented by Fig. 18. This piecewise function, when included in our analysis is computationally difficult to deal with. We therefore estimate it using an exponential function of the form:

\[ N(RB) = 60 \times \exp(-0.009 \times RB) = 60/ \exp(0.009 \times RB) \]  

(99)

where \( N(RB) \) = number of kilometer passes /day

\( RB \) = roughness before blading in counts/km.

For a road of length \( L \) (kms.) and a daily cost of grader equal to \( CG \), the
maintenance cost is given by:

\[ M = L \times CG \times N(RB) \]
\[ = L \times CG \times \exp(0.009 \times RB) \times 60 \]  \hspace{1cm} (100)

This is the equation used in the optimization model, again substituting \( S \) to \( RB \) for the roughness trend curve.

Given the expressions for vehicle operating costs, maintenance costs and the differential equation representing the change in roughness trend curve with time, the general optimal control problem for unpaved roads is formulated using the Hamiltonian equation:

\[ H = C(S(t)) + h(t)M(S(t)) + z(t)(F(S) - h(t)K(S(t))) \]  \hspace{1cm} (101)

where \( H = \) Hamiltonian function as described in the previous chapter

\( z(t) = \) adjoint variable representing marginal valuation of
road condition
The application rate $h(t)$ assumes any value between 0 and 1. However, we cannot blade a road every day ($h=1$). Instead, we set an upper limit to the frequency with which we can do maintenance. We assume in this problem that the most frequent interval of blading that we can have is every 25 days ($h = 0.04$) or approximately once every month. When the solution requires that blading be performed more frequently than this, traffic volume is probably sufficiently high that it is more economical to pave the road to decrease the vehicle operating costs. Likewise, to avoid the computational problems that arise when very low values of $h$ are considered [9], we define a lower bound for the application rate which in this problem is set equal to 0.005 or once every 200 days. In reality a bounded control of this value is true only for very lightly trafficked roads such as the ones that are open only to users during certain periods of the year.

With these the overall objective function is:

$$
\min H = \beta(S) + h M(S) + z^\top (F(S) - h^\top K(S))
$$

subject to $h_1 \leq h \leq h_2$

which is similar to Eq. 68. Optimality conditions for this problem are:

$$
\begin{cases}
    h_1 \\
    h \\
    h_2
\end{cases}
\quad \text{if } \begin{cases}
    Hh = M(S) - z^\top K(S) > 0 \\
    Hh = M(S) - z^\top K(S) = 0 \\
    Hh = M(S) - z^\top K(S) < 0
\end{cases}
$$

In solving for the steady states of bang-bang and singular control, the value(s) of $h$ that satisfy the conditions $\dot{S} = 0$ and $\dot{z} = 0$ are fixed and optimality conditions (Eq. 103) are checked to see whether they are satisfied by the equilibrium points. The procedures are the same as those described in Sec. 4.4 for paved roads except in this problem, there is only one control variable which is
the application rate unlike in that problem where the intensity of maintenance is also treated as a control variable.

4.6 Summary

In the preceding chapter, the theory of dynamic optimization and its application to highway pavement maintenance were discussed. Emphasis was made on the optimal control technique and how it was used to solve the maintenance problem for paved roads. The application of this technique to optimum blading for unpaved roads was explored, utilizing some of the results of the studies discussed in Chapter 2. Finally, the formulation of the optimal control problem for unpaved roads was presented.

Chapter 5 takes the models developed in this chapter and applies it to hypothetical scenarios found on unpaved roads. It includes a detailed solution to the optimal control problem and a sensitivity analysis of the results.
Chapter 5 - Application of the Models

To test the models formulated in the preceding sections, the optimal control problem is solved for various combinations of traffic volume, cost of grading, fuel cost, interest rate and other relevant variables. However, for the purpose of illustrating the steps involved in the solution methodology, two case studies with different levels of traffic are first analyzed in detail. All other variables in these two scenarios are kept constant. Table 16 shows the values assumed for each of the following cases:

- **Case 1** - 15 cars, 5 trucks per day
  - or 30 passenger car units (pcu's)/day
- **Case 2** - 100 cars, 50 trucks per day
  - or 250 passenger car units/day

<table>
<thead>
<tr>
<th>Average Daily Traffic ( ADT )</th>
<th>Case 1 - 30 pcu's/day</th>
<th>Case 2 - 250 pcu's/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of Grading</td>
<td>$200/day</td>
<td></td>
</tr>
<tr>
<td>Interest Rate</td>
<td>8% per annum</td>
<td></td>
</tr>
<tr>
<td>Fuel Cost</td>
<td>28¢/liter</td>
<td></td>
</tr>
<tr>
<td>Type of Surface</td>
<td>gravel (lateritic)</td>
<td></td>
</tr>
<tr>
<td>Length of Road</td>
<td>10 kms.</td>
<td></td>
</tr>
<tr>
<td>Width of the Road</td>
<td>10 meters</td>
<td></td>
</tr>
<tr>
<td>Rise</td>
<td>5 m./km.</td>
<td></td>
</tr>
<tr>
<td>Fall</td>
<td>5°/km.</td>
<td></td>
</tr>
<tr>
<td>Curvature</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plasticity Index</td>
<td>5%</td>
<td></td>
</tr>
<tr>
<td>% Passing 0.075 mm. sieve</td>
<td>15%</td>
<td></td>
</tr>
</tbody>
</table>

Table 16 - Variables used in the Case Study
5.1 Solution of Example Applications

The analysis begins by plotting the two curves $H_h=0$ and $H_h=0$ on the $S$-$z$ plane. These curves are expressed as:

\[ H_h = M(S) - zK(S) = 0 \]  \hspace{1cm} (104)

\[ H_h = MsF - zKsF - iZ*K + Cs*K + zK*Fs = 0 \]  \hspace{1cm} (105)

where 
- $S$ = roughness in QI (counts/km)
- $z$ = adjoint variable
- $M(S)$ = maintenance cost function
- $K(S)$ = blading impact function
- $Ms = dM(S)/dS$
- $i$ = daily interest rate
- $C(S)$ = vehicle operating cost function
- $F(S)$ = given by Eq. 88
- $Fs = dF(S)/dS$

The intersection of the two curves represent the singular control solution which also obeys the steady state conditions ($\dot{S}=0$ and $\dot{z}=0$). At steady states, $S$ and $z$ both attain time invariant values corresponding to the coordinates of the intersection of the two curves.

To solve for steady states of bang-bang control, the same procedures as described in Sect. 4.4.1 are followed. Each of the bounded controls ($h_1=.005$ and $h_2=.04$) is substituted into the steady state conditions to solve for the steady state values of $S$ and $z$. To determine whether the optimality conditions are met, which means that steady states of bang-bang control exist, Eq. 103 must be satisfied.
Figs. 19a and 19b show the S-z diagrams of the two curves \( Hh=0 \) and \( Hh=0 \) for cases 1 and 2 respectively. The intersection points are defined by the following singular control steady-state parameters:

**Case 1** - \( Q = 30 \ \text{pcu's/day} \)
- \( S = 150.5 \ \text{QI} \)
- \( z = 1.26 \)
- \( h = 0.0056 \) (every 180 days)

**Case 2** - \( Q = 250 \ \text{pcu's/day} \)
- \( S = 266 \)
- \( z = 1.52 \)
- \( h = 0.00737 \) (every 136 days)

Table 17 gives the values obtained for the bounded controls \( h_1 \) and \( h_2 \).

<table>
<thead>
<tr>
<th>Bounded Control</th>
<th>( S )</th>
<th>( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case 1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q = 30 \ \text{pcu's/day} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( h_1 = 0.005 )</td>
<td>175.5</td>
<td>1.70</td>
</tr>
<tr>
<td>( h_2 = 0.04 )</td>
<td>36</td>
<td>0.85</td>
</tr>
<tr>
<td><strong>Case 2</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q = 250 \ \text{pcu's/day} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( h_1 = 0.005 )</td>
<td>no steady states</td>
<td></td>
</tr>
<tr>
<td>( h_2 = 0.04 )</td>
<td>49</td>
<td>-0.74</td>
</tr>
</tbody>
</table>

Table 17 - Steady States of Bounded Controls

The bounded control solutions are plotted on Figs. 19a and 19b for the two
Figure 19a. S-z Curves for Case 1

Q = 30 pcu's/day
= 15 cars, 5 trucks
Figure 19b. S-z Curves for Case 2

\[ Q = 900 \text{ pcu's/day} = 300 \text{ cars, 200 trucks} \]
cases. Notice that these solutions do not satisfy the optimality condition for bang-bang controls for either case 1 or case 2. Thus, for both cases, the steady state solutions are singular controls.

Since the initial roughness of the road differs from the steady state initial roughness, a roughness trajectory curve need be constructed for each of the case studies (see Sect. 4.4.3). Assuming a range of roughness values (SoEmin, SoEmax) within which the initial roughness is assumed to fall, it is determined whether all values in this range actually converge to the steady state solution (called saddle point). The curves defined by such roughness values are called **stable branches**. Stable branches are actually derived by integrating the canonical equations \( S = 0 \) and \( z = 0 \) with respect to \( t \) and solving for the appropriate terms in the expressions \( S(t) \) and \( z(t) \) such that when both terms are differentiated, both \( S \) and \( z \) are time invariant. However, the expressions for \( S \) and \( z \) that we are dealing with in the unpaved roads problems do not allow such calculations to be made. Hence, the stable branches are determined by calculating \( S \) and \( z \) at different points on the \( S-z \) plane and drawing the curve(s) which converge to the steady state solutions.

The procedures defined above for determining stable branches are computationally rigorous and time consuming. Engineers and planners might find it the major drawback in the optimal control solution. However, this should not be a problem at all since the procedure of finding stable branches may actually be done numerically with the aid of the computer. Stable branches are drawn on Figs. 20a and 20b for cases 1 and 2 respectively. Note that minimum roughnesses (70 and 230) exist for the stable branches. A maximum roughness value of 300 for the roughness trend curve was arbitrarily assumed. These extreme values are the ranges to be used in the analysis.

The roughness trend curves are drawn assuming an initial roughness within the ranges defined. Such curves are shown on Figs. 21a and 21b.
Figure 20a. Stable Branches for Case 1
Figure 20b. Stable Branches for Case 2
case 1. The same curves are drawn on Figs. 22a and 22b for the second case.

To convert the roughness trend curve to the true sawtooth curve, the relationship \( \int h(t) \, dt = 1 \) is used. The discrete blading times \( (t_n)'s \) are determined from the sawtooth curve. For the first blading, the roughness trend curve passes through the midpoint of the first spike. The length of this spike is expressed as:

\[
G [ R(t_1^-) ] = R(t_1^-) \cdot R(t_1^-)^{0.63} \times K_2
\]  (106)

The initial roughness of the sawtooth curve \( (R_0) \) is then computed from the relationships:

\[
R(t_1^-) = R_0 \times \exp [ t_1 \times (0.0034 + 1.3 \times 10^{-5} \times Q) ]
\]  (107)

and

\[
Sm(t_1) = R(t_1^-) - 1/2 \times G[R(t_1^-)]
\]  (108)

where 
- \( G[R(t_1^-)] \) = reduction in roughness due to first blading
- \( R(t_1^-) \) = roughness before first blading
- \( R_0 \) = initial roughness of the sawtooth curve
- \( Sm(t_1^-) \) = roughness trend curve value at first blading
- \( K_2 \) = constant defined in Eqtn.
- \( t_1 \) = time to first blading
- \( Q \) = traffic volume (pcu's/day)

The corresponding sawtooth curves for case 1 are shown of Figs. 23a and 23b for initial roughnesses 70 and 300 QI respectively.
Figure 21. Roughness Trend Curves for Case 1
Figure 22. Roughness Trend Curves for Case 2
Figure 23a - Sawtooth Curve for Case 1 with Initial Roughness = 70 QI

Figure 23b - Sawtooth Curve for Case 1 with Initial Roughness = 300 QI
Because the optimal control solution is performed on the roughness trend curve and not the true sawtooth curve, it is necessary to compare the total costs calculated using both curves to see whether costs for the roughness trend curve reasonably approximates the actual costs. Such comparison is shown on Table 18.

<table>
<thead>
<tr>
<th>Case</th>
<th>( Q = 30 )</th>
<th>( Q = 250 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S = 70 )</td>
<td>( 21,132 ), ( 3,653 ), ( 24,785 )</td>
<td>( 240000 ), ( 12442 ), ( 252442 )</td>
</tr>
<tr>
<td>( S = 230 )</td>
<td>( 30,951 ), ( 6283 ), ( 37,234 )</td>
<td>( 270551 ), ( 16945 ), ( 287496 )</td>
</tr>
<tr>
<td>( R = 55 )</td>
<td>( a/d = 0.66 )</td>
<td>( a/d = 0.69 )</td>
</tr>
<tr>
<td>( R = 132 )</td>
<td>( b/e = 0.58 )</td>
<td>( b/e = 0.73 )</td>
</tr>
<tr>
<td>( a/f = 0.66 )</td>
<td>( c/f = 0.68 )</td>
<td>( c/f = 0.68 )</td>
</tr>
<tr>
<td>Freq.</td>
<td>every 180 days</td>
<td>every 180 days</td>
</tr>
</tbody>
</table>

Table 18 - Comparison of Total Costs Between Roughness Trend and Sawtooth Curves

From the values on the table it is apparent that the roughness trend curve consistently underestimated the true costs for the corresponding sawtooth curves. The underestimation was more pronounced for the very low level of traffic (\( Q = 30 \)).
pcu's/day) while for higher traffic the approximations were close to the true value.

In order to verify if the given solutions are the true minimum, we compare the total costs of the steady state solutions above with the costs associated with other blading frequencies. The results are shown on Table 19.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Frequency</th>
<th>Frequency</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>every 180 days</td>
<td>every 150 days</td>
<td>every 200 days</td>
</tr>
<tr>
<td></td>
<td>(optimum)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ro = 56 QI</td>
<td>37,234</td>
<td>38,963</td>
<td>43,105</td>
</tr>
<tr>
<td>Case 2</td>
<td>every 136 days</td>
<td>every 100 days</td>
<td>every 150 days</td>
</tr>
<tr>
<td></td>
<td>(optimum)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ro = 132 QI</td>
<td>287,496</td>
<td>292,426</td>
<td>298,782</td>
</tr>
</tbody>
</table>

Table 19 - Total Costs for Optimal and Non-Optimal Solutions ($)

The values shown above are not significantly different from each other. However, note that the case study is only for a 10 km. road. If the same analysis is made for the entire unpaved road network, say in Bolivia which has a total length of 36,155 kms. (Table 1), the amount that represents the difference in blading interval of 20 days (every 150 days instead of 180 days) for case 1 is equal to 3615.5($38,963-$37,234) = $6,251,200 for the entire planning horizon or $0.5 million annually using an interest rate of 8%. If traffic were to increase to 250 pcu's/day as in case 2, the amount corresponding to a difference in blading interval of 36 days (100 instead of 136) is equal to 3615.5($292,426-297,496) = $17,824,415 for the entire planning horizon or $1.42 million annually. For a developing country like Bolivia, these amounts represent a significant improvement in the overall highway economy. The impacts are emphasized by
comparing the total costs for the optimal solutions in both cases when applied to the total road network in Bolivia. These are shown on Table 20.

<table>
<thead>
<tr>
<th>Case 1a</th>
<th>a 10 km. road</th>
<th>for the whole network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 2a</td>
<td>$287,496</td>
<td>$1039 B (entire horizon)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$83.2 M (annually)</td>
</tr>
</tbody>
</table>

Table 20 - Total costs (Optimal) when either scenario apply in Bolivia

These figures give us some idea of the order of costs associated with system of unpaved roads in a developing country like Bolivia.

5.3 Sensitivity Analysis

Following the illustration of the solution to the problem, the sensitivity of the model to variations in the different parameters is explored. The objective of this exercise is to see how the solution shifts from the optimal solution of the base problems when other variables are changed and determine whether the shifts make sense intuitively based on the relationships between maintenance and vehicle operating costs. To do so, a number of hypothetical cases which differ from the base cases analyzed earlier, are defined. The steady state solutions are then calculated for these various scenarios.

Table 21 shows the results of these tests. The optimum frequency of blading is indeed a function of the different variables notably the volume of traffic.
<table>
<thead>
<tr>
<th></th>
<th>Base Case</th>
<th>100</th>
<th>100</th>
<th>100</th>
<th>100</th>
<th>100</th>
<th>100</th>
<th>100</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Cars</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of Trucks</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuel Cost ( $/liter)</td>
<td>0.22</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost of Grading ($/day)</td>
<td>200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length (km)</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% passing 0.0075 mm sieve</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plasticity Index</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest Rate (%)</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Width of Road (m)</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surfacing T1 T2</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rise (m/km)</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fall (m/km)</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blading Season</td>
<td>wet</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Curvature (deg/km)</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of Bladings/year</td>
<td>2</td>
<td>1.8</td>
<td>2.4</td>
<td>1.97</td>
<td>2</td>
<td>2.7</td>
<td>no</td>
<td>no</td>
<td>3.4</td>
</tr>
<tr>
<td>Steady State Roughness</td>
<td>150</td>
<td>175</td>
<td>124</td>
<td>158</td>
<td>152</td>
<td>266</td>
<td>sol.</td>
<td>sol.</td>
<td>162</td>
</tr>
<tr>
<td>Adjoint Variable (z)</td>
<td>1.26</td>
<td>-1.1</td>
<td>2.79</td>
<td>1.24</td>
<td>34</td>
<td>152</td>
<td></td>
<td></td>
<td>1.32</td>
</tr>
</tbody>
</table>

Table 21 - Sensitivity Analysis
and the cost of fuel. For the base case with an average daily traffic of 15 cars and 5 trucks, it can be observed that increasing the fuel cost results in less frequent optimum blading (therefore higher steady state roughness) since fuel consumption increases with speed therefore it is much better to keep the road rough to reduce the vehicle speeds. On the other hand, if we double the daily cost of grading the optimal solution is to blade more frequently so that the steady state roughness goes down. Since the maintenance cost increases with roughness before grading, it is more economical to keep the average roughness low. Interest rate is found to be insignificant in the optimal solution.

Increasing the average daily traffic to 100 cars and 50 trucks raises both the frequency of maintenance and the steady state roughness. This makes sense intuitively since doing more frequent maintenance in this case does not stop the average roughness from attaining a high value due to the traffic. When the fuel cost is increased to $1 per liter and the traffic remains the same, it is more economical to pave the road because of the high vehicle operating costs. However, decreasing the price of gasoline to $0.1 per liter shifts the optimum blading frequency to a higher value. With lower fuel prices, the vehicle speeds may be increased without considerably increasing the vehicle operating costs. The optimum solution is to blade the road more frequently so that the average roughness (and therefore the cost of maintenance) can be lowered.

Further increase in the traffic volume (300 cars and 200 trucks daily) warrants capital investment on the road by paving it.

5.4 Computer Implementation

The amount of computation involved in solving the optimal control problem requires the use of a computer. For this reason, a program was written which gives the values of the roughness (S) and the adjoint variable (z) which were plotted on the S-z graphs. The steady-state solutions (singular and bounded
controls) are also calculated by the program. In determining stable branches, however, manual computation was made although this could have been done with the computer as well. Finally, the path traced by the roughness trend curve from the initial roughness was computed using a numerical approach.

Cost computations were made with a numerical integration software called NAG.

5.5 Summary

This chapter completes the discussion on the optimal control theory and its application to unpaved roads. It was shown that the problem of selecting the optimum maintenance for unpaved roads can be solved using the models formulated. It is proposed that further analysis of the model and the solution be made.
6.1 Summary of Research

This thesis takes a large step forward in the area of maintenance management of unpaved roads by applying the concepts of dynamic optimization in setting optimal blading frequencies for unpaved roads. It recognizes the fact that unpaved roads have been given little attention in terms of research and optimization studies as compared with paved roads. Developing countries suffer the most from lack of systems to effectively maintain their unpaved roads due to the very large fraction of total road lengths in these countries which are unpaved. The problem is further magnified by the fact that in most of these countries, many primary roads on which high traffic volumes exist are unpaved, thus making the total vehicle operating costs on these roads really great. The same situation is seldom true for developed countries where unpaved roads constitute a smaller portion of the total network and these roads are almost always associated with very little traffic. For these countries, total vehicle operating costs and maintenance costs for the unpaved roads are small compared to those of paved highways. But in some countries like the United States, Japan and Finland, the fractions of unpaved roads are quite high. In Ontario, Canada, the budget for maintenance of unpaved roads has been very large. Thus, the need to effectively maintain unpaved roads is not only limited to developing countries but is a major concern in many developed countries as well.

A review of the existing systems used to manage unpaved roads indicate that the classification-based types of maintenance system are popular in developed countries. The main reason why these simple systems are adopted is the lack of performance functions with which economic analyses may be performed. Clearly this type of system is not appropriate for developing countries because of the large network of unpaved roads involved. An economic analysis is required which will
objectively define the consequences of maintenance decisions.

From the results of the road studies conducted in selected developing countries some mathematical optimization and simulation techniques have been implemented. These systems, however, either suffer from very restrictive assumptions to solve the problem ( Sect. 3.3.1 - Graphical Method ) or do not give closed-form solutions to the problem ( Sect. 3.3.3 - Maintenance and Design System ( MDS ) ). The dynamic optimization approach ties these two problems together and comes up with a closed-form solution with very few assumptions made. It has the advantage of being more realistic than the graphical analysis. Likewise, the user does not have to define maintenance strategies because the solution is defined by the problem.

The optimization technique chosen is optimal control, because the condition ( roughness ) curve is not constant between blading and this is very difficult to model in the dynamic programming framework. However, the discrete jumps in the condition function due to application of maintenance make the optimal control solution infeasible. This may be overcome by approximating the sawtooth condition curve by a continuous curve.

Such an approximation was already done by Tsunokawa when he solved for the optimal overlay frequency and intensity for paved roads. The methodology is theoretically acceptable. However, its application to paved roads is doubtful because in reality there exists several maintenance technologies and condition measures, whereas Tsunokawa limited his analysis to overlay as a maintenance activity and roughness as a condition measure. For this approach to become applicable in management of paved roads, a solution needs to be found for the case where multiple technologies and multiple conditions are considered. This is certainly an overwhelming task.

The situation is greatly simplified for unpaved roads because condition can
be appropriately defined by a single measure (roughness) and there is generally one maintenance activity involved (blading). The approximation procedure can be conveniently solved for the unpaved roads problem without fear that the model will not be used in practice.

The dynamic optimization concepts and their applications to highway maintenance are described in Chapter 4. Model formulations and the solutions to the unpaved roads problem are all discussed in the same chapter. The solution procedures are then tried for different scenarios found in reality and the results of the model runs are evaluated.

5.2 Research Recommendations

The work accomplished in this research includes a literature review on unpaved roads and the acknowledgement of the fact that an alternative solution to maintenance problems is required. The original objective of the research is the development of an alternative maintenance system software which uses dynamic optimization technique to set blading frequencies for unpaved roads. Unfortunately, the prediction equations and performance functions used in the analysis on which the optimization formulation was made are not estimated from actual data but are just approximations to the existing models. The existing models themselves have very poor explanatory power and in functional forms not suitable for the optimization formulation. Hence, a general unpaved roads management system which agencies might utilize cannot be developed at the moment. Instead, an exploratory analysis of the solution to the optimization problem was made. Further research is needed on the following areas:

1. Development of equations predicting highway deterioration, vehicle operating costs and maintenance costs which are in functional forms suitable for optimization and which are estimated from
actual data.

2. Solving the more general case of varying traffic volume which was assumed constant in the model. This will require having two state variables in the formulation, namely roughness and traffic volume.

3. Incorporating other components of vehicle operating costs such as lubrication costs, tire wear, parts consumption etc. In the analysis made, only fuel consumption was included.

4. For gravel roads, regravelling is a major maintenance activity and is equivalent to rehabilitation in paved roads. The model should be able to incorporate gravel loss such that when the thickness of the gravel falls below a certain threshold, regravelling is performed and at the same time the road is bladed even if the original model (without regravelling) does not dictate so. The procedures involve defining the discrete points in time when the road needs to be regravelled. Since gravel loss is only a function of time and other fixed variables as traffic and environment, the regravelling strategy may be included in the model by simply scheduling this activity at a point close to a blading activity.

The first requirement is the most critical in that this prevented the research from coming up with a ready-to-use maintenance system that may be adopted by highway agencies trying to optimize the blading frequency for its network of unpaved roads. The other research efforts are important in special situations when a very realistic maintenance system is needed. However, this research has demonstrated the applicability and usefulness of these techniques to the management of unpaved roads.
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