PROTON REACTION CROSS SECTIONS

by

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Doctor of Philosophy

Abstract

The angular distribution of elastically-scattered protons from vanadium and copper, the \([(p, p') + (p, \alpha)]\) cross-sections for vanadium and copper, and the \((p, n)\) cross-sections for \(\text{V}^{51}\) and \(\text{Cu}^{65}\) have been measured at 7.5 Mev. The ratio of elastic scattering to Rutherford scattering was measured using a comparison technique, and is given from 130° to 150° for vanadium and from 50° to 143° for copper. The \([(p, p') + (p, \alpha)]\) cross-sections were determined by observation of scattered particles at several angles. They are:

\[
\begin{align*}
\text{vanadium} & \quad [(p, p') + (p, \alpha)] : 134 \text{ mb} \\
\text{copper} & \quad [(p, p') + (p, \alpha)] : 266 \text{ mb}
\end{align*}
\]

The \((p, n)\) cross-sections for \(\text{V}^{51}\) and \(\text{Cu}^{65}\) were determined by measuring the induced \(\text{Cr}^{51}\) and \(\text{Zn}^{65}\) activities respectively. They are:

\[
\begin{align*}
\text{V}^{51} & \quad (p, n) : 555 \text{ mb} \\
\text{Cu}^{65} & \quad (p, n) : 537 \text{ mb}
\end{align*}
\]

These results are shown to be in qualitative agreement with optical-model predictions.

The preparation and standardization of sources of \(\text{Cr}^{51}\), \(\text{Mn}^{54}\), and \(\text{Zn}^{65}\) by x-gamma coincidence methods is described.

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INTRODUCTION

The last few years have seen the evolution of an increasingly detailed picture of the atomic nucleus, as continually refined techniques have provided data of a precision and diversity not previously possible. From this picture has emerged a unifying theory, capable of correlating the structure of nuclei, the interactions with projectile particles, and the basic nucleon-nucleon forces: the optical model. (For a brief resume of the historical background, see Appendix I). The name derives from the analogy to the approximate treatment of the propagation of light in a medium, where the effect of the medium is represented by an index of refraction (Fes-58). In essence, the optical model describes the interaction of a nucleon with a nucleus, composed of \( A \) nucleons, in terms of a single-particle potential, the optical potential \( V \). This potential replaces the exact description of the interaction process which requires the consideration of interactions between the specified nucleon and each of the other \( A \) nucleons as well as the interactions of the \( A \) nucleons with themselves. Such a description is valid outside the nucleus even if the projectile "coalesces with the nucleus as soon as direct contact is made" (Boh-36), corresponding to a totally "black" nucleus. For many reactions, as will be noted shortly, the nucleus actually corresponds more nearly to a "cloudy crystal ball." Inside the nucleus, this simplification of a single-particle potential will be valid only to the extent that a nucleon can be considered moving undisturbed in the nucleus, independent of the motion of the other \( A \) nucleons. That is, the mean-free-path for collision must be much larger than the dimensions of the nucleus.
Each nucleon then moves in an average potential created by the remaining A nucleons. The validity of this description has been most clearly demonstrated in the striking success of various bound-state aspects of the optical model referred to as shell models, independent-particle models, etc. While the primary interest in the present work is in the optical potential seen by projectiles in scattering and reaction experiments, the proper context of the optical model requires consideration of the potential for bound-state nucleons as well.

The Bound State

In correlating the properties of stable nuclei, it was early found (Els-34) (May-49) that nuclei containing certain numbers of \( N \) neutrons and \( Z \) protons could be singled out as showing marked distinction from the systematically varying trends of neighboring nuclides. These discontinuities in the regularity of properties occurred for values of \( N \) and/or \( Z \) which became known as "magic" numbers: 2, 8, 20, 50, 82, 126. Evidence for the importance of these numbers in the Aufbau of nuclei has multiplied since the presentations of Mayer and Jensen (May-55) and Feenberg (Fee-55), but originated in four types of observations: Nuclei with magic-number, \( N \) (or \( Z \)), have more isotopes (or isotones) than neighboring nuclides. They show a greater abundance in nature than neighbors. The last particle in the magic configuration is tightly held, as seen from separation energies. The addition of one particle to a magic configuration is not favored, as exemplified by the observed low probability for capturing a neutron into a neutron-magic configuration and the low binding energy of particles added to such configurations.
These correlations, along with numerous other more recent observations (Cor-55) (Ell-55) (Law-60), indicate that a magic number of neutrons or protons represents a configuration analogous to a closed electron shell in the case of an atom which gives rise to the chemical inertness of the rare gases. In each case, an energy gap between two successive states produces the observed shell effects.

While the systematic variations of nuclear properties exhibit clearly shell-structure characteristics, the justification of such behavior has been more obvious in the case of atomic electrons than for nucleons. The observation of shell-structure in a system implies that the constituents move independently of one another in a force field. In the case of the atomic electrons, the primary source of the force is the charge of the nucleus, which, to a good approximation, can be considered a fixed point-charge, unaffected by the motion of the electrons. The interactions of the electrons of a multi-electron atom with themselves can, to first approximation, be neglected; each electron moves in the force field of a point charge shielded by the average field of the other electrons.

In the case of nuclei, it is not obvious how the two-body forces of free nucleons produce an average potential. It has not always been clear that the nucleons could move in the small region of the nucleus without violent collisions with one another. Whereas the coulomb force between electrons acts at all distances and varies smoothly with interelectron distance, the nucleon-nucleon force acts only over a short range, and within that distance is much stronger than the coulomb force. Further, unlike the atomic case, the size of nuclei is very nearly proportional to the number of nucleons in it. (The radius is proportional to the cube root of A (Sta-58)).
In view of these latter considerations, it had been anticipated that any individual nucleon inside a nucleus would undergo frequent collisions, requiring a chaotic description of the motion of the nucleons. That this is evidently not the case has required a re-evaluation of the motion expected of nucleons in a potential well. The fact that nucleons are in some sense packed "closely together," i.e., the internucleon distance is short with respect to the force or matter distribution ("radius") of the free nucleon, is not the important criterion in determining the probability of internucleon interactions. Rather it is that the mean-free-path for nucleon-nucleon scattering be small compared with the radius of the nucleus. A scattering event means that two nucleons have changed their momentum states. Since it is expected that in the ground state of the nucleus all the single-particle states with energy less than some set value, the Fermi energy, will be filled, it has been suggested (Wei-51) that the Pauli principle will inhibit collisions: states into which the particles might scatter are already occupied. This invocation of the Pauli principle, that two identical fermions cannot be in the same quantum state, provides the basis for our present understanding of the success of the independent-particle model of the nucleus; each nucleon moves independently in an average potential created by the other nucleons.

Using a spherical potential it has been possible to correlate and predict (Law-60) (Ell-55) an increasing variety of nuclear properties: the spin and parity of ground state and many excited states of both stable and unstable nuclides; the magnetic moments of ground states; the probabilities for transition by gamma emission between two states of a given nucleus; the
probability for transition by beta decay between two adjacent isobars; and the occurrence of closed shells at magic numbers.

At the same time, from detailed investigations of these properties have come inferences regarding this potential; it must have a "rounded edge," a shape intermediate between a square well and an harmonic oscillator potential; and some spin-orbit force is necessary (Hax-50)(May-49). A satisfactory form has been (Ross-56):

\[ V(r) = V_0 f(r) + \left( \frac{k}{2Mc} \right)^2 \lambda V_0 \frac{1}{r} \frac{df}{dr} \sigma \cdot \tilde{l} \]  
\( f(r) = \left[ 1 + \exp \left( \frac{r - R_0}{a} \right) \right]^{-1} \quad R_0 = r_0 \frac{A}{3} \]

where \( f(r) \) is a form factor, \( \frac{k}{2Mc} \) is Planck's constant divided by 2\( \pi \), \( c \) is the velocity of light in vacuo, \( M \) is the mass of the nucleon, \( \sigma \) and \( \tilde{l} \) are the spin and orbital angular momenta respectively of the particle, and \( V_0, \lambda, a, \) and \( r_0 \) are parameters discussed more fully in the next two sections.

The notion of the independent motion of particles in the nucleus will not be completely valid because of the presence of a finite amount of two-body interactions or collisions in the nucleus. The close approach of nucleons will lead to fluctuations from the average potential seen by a nucleon. These effects manifest themselves in the need to consider configuration-interaction or mixing of single-particle states in explaining some data (Ell-55). The success to date of the model is based on the observations that such effects are quite small, and need by considered only as greater
precision is demanded in fitting theory to observation. The effect of collisions can be incorporated into the model by making the potential complex: the imaginary part then produces the effect of configuration-interaction by removing a particle from a particular single-particle state. A complex-well depth of less than 0.5 Mev (El1-55) compared with a real-well depth, \( V_o \), of 43 Mev (Ross-56a) is sufficient to explain these bound-state effects.

Nuclear Size

Further support for the potential-well model of the nucleus comes from studies of nuclear size (Hof-56) (For-54) (Sta-58) (Sco-56) (Hil-57). Three sizes may be defined, corresponding to the distribution of charge (protons), the distribution of matter (nucleons), and the distribution of force (the potential). Although a precise determination of the radial dependence of any of these distributions appears unlikely (Hil-54) (Coo-55), it has been possible to infer values for root-mean-square (rms) radii, for the half-height radius where the distribution drops to half its value at the origin, and for the surface thickness, or distance over which the distribution drops from 0.9\( D \) to 0.1\( D \), where \( D \) is the value at the origin. At present it appears that all three distributions are very similar (Wil-55) (Dre-55), with the potential distribution extending a few fermis (1f = 10^{-13} \text{ cm}) beyond the charge and matter distribution (Ber-56). The data can be correlated well using the form factor of Eqn. (2), although other form factors are equally satisfactory (Sco-56) (Gla-58). The rms charge-distribution radius, determined from electron scattering (Hof-58) and mu-meson x-ray spectra (For-54) is about 1.2 f. Nuclear force radii (see Table I) are also about 1.2 f.
Using the potential well of Eqn. (1), Ross, Mark, and Lawson (Ross-56) (Ross-56a) showed that the radial dependence of nucleon density paralleled the radial dependence of the potential and reproduced the observed matter density. Self-consistent field calculations have been successfully made using this same density distribution (Tal-56) (Gre-56).

**Nuclear Reactions**

The application of the principles of the independent-particle model which has given rise to the successes in correlating the properties of nuclear structure outlined above to the description of nuclear reactions has followed a route indicated in Appendix I. The underlying concept, presented first by Weisskopf (Wei-56), is that the reaction takes place in three stages. Initially we have a target nucleus (A nucleons) and a projectile moving in the potential field of the target nucleus. This entrance channel we call the independent-particle (IP) stage. As the projectile approaches the nuclear surface or boundary of the nuclear potential it will be acted upon by the nuclear force. The particle may be reflected from the potential boundary; it may enter the region of the potential and be scattered, or it may be internally reflected from the boundary before escaping. All this is designated shape-elastic (SE) scattering. Once inside the potential boundary, it may be removed from this entrance channel by transferring part of its kinetic energy to internal energy of the target. This may occur, for example, through activation of collective modes of nuclear excitation (rotations and vibrations of the nucleus as a whole) through interactions with the nuclear surface, and through direct interactions with one or more of the A constituent nucleons. This removal of the
projectile from the entrance channel, by any means, gives the compound-system (CS) stage. The disappearance of a particle from the entrance channel is a mathematical consequence of the imaginary term in the potential. A specific case of the CS results in those situations where the projectile, by means of multiple collisions inside the nucleus, completely loses its identity, and we then have the compound nucleus (CN). The formation of the CN occurs only after the projectile has disseminated completely its incident kinetic energy among all the A target nucleons. The system is then in an excited state, the properties of which depend upon the energy, momentum, angular momentum, and parity, but not upon the mode of formation.

The final stage of the reaction consists of the break-up of the CS: the projectile (or some other particle or group of particles ejected by the CS interaction) in an exit channel and the target nucleus in an excited state. In the case of CN formation, the final stage consists of the products formed by break-up of the CN. In some instances, the incident particle may emerge from the CN into the entrance channel, giving rise to compound-elastic (CE) scattering.

This picture contains within its framework all the possible projectile-nucleus interactions. The optical potential \textit{per se} enters the picture only in the IP stage, where it describes the effect of the nucleus upon the projectile.

The observation that internucleon forces outweigh the coulomb force for small distances does not imply that the internucleon force will necessitate multiple violent collisions, prerequisite to CN formation, when a projectile encounters the nuclear boundary. To the contrary, the nucleon-
nucleon force may be considered to be a weak one: the simplest two-nucleon configuration, the deuteron, is not held together with sufficient force to withstand the centrifugal action of a rotational excitation.

Basis for the Potential

The increasing success of the various aspects of the optical model or independent particle model has provided impetus to attempts to derive the potential from fundamental considerations. For the bound state, the relationship between nuclear matter, i.e., nucleons in an infinitely extended nucleus, and the two-nucleon force has been examined in detail by Brueckner and his associates (c.f. Bru-58). While the application to finite nuclei, i.e., the inclusion of a nuclear surface, has not yet been accomplished, results thus far have been considered quite promising (Pei-59). For scattering and reactions, the existence of an optical potential, explicitly energy dependent and non-local, has been shown by Feshbach (Fes-58), and formalisms have been developed utilizing the potential (Fra-53) (Fes-60). In the limit of sufficiently low or sufficiently high energies this potential can be tied in with other descriptions and is directly related to the phenomenological potential.

At low, but unbound, excitation energies of the target nucleus well-defined quasi-stable states exist, distinctly separated in energy. These are seen, for example, in the case of neutron projectiles as sharp resonance peaks in the observed absorption of neutrons by matter as a function of bombarding energy. These resonances signify the formation of a long-lived quantum state of all \( A + 1 \) particles (the CN state). As the
excitation energy is increased these levels are found more closely spaced; for sufficient energy they merge together and become indistinguishable. The optical model at low energies averages over these resonances; it predicts the effect of the nucleus averaged over a sufficient energy interval to include many levels. The resonances are fluctuations away from the average, and are not directly predictable from the model. This averaging process (Fri-55) leads from the Breit-Wigner (Bre-36) and Kapur-Peierls (Kap-38) formalisms of resonances to the optical model (Fes-54)(Fes-58).

At high excitation energies, the mean-free-path for collision becomes large and the wavelength of the projectile becomes short with respect to the nuclear radius. The interaction then may be considered as taking place between the projectile and a nucleon imbedded in the nucleus (Gol-48). In this limit the nucleus becomes nearly transparent (Ser-47), and the optical potential can be related to the two-nucleon force directly (Rie-56) (Wat-53) (Tak-55).

Various estimates for the value of the imaginary term have been made using a variety of assumptions (Lan-55) (Fra-53), and reasons have been advanced for expecting the imaginary potential to be concentrated near the surface of the nucleus at low bombarding energies (Gom-59)(Ver-58)(Lem-59)(Har-59)(Kik-59).

The Phenomenological Potential

For purposes of obtaining a description of scattering and reaction phenomena, the optical model may be considered as a phenomenological potential, whose parameters are to be adjusted for optimum agreement with
experiments. In discussing the application of this model it is convenient
to define a cross section, \( \sigma(u) \), for an event, \( u \), as (\( \text{\textsc{m}-55, p. 317} \)):

\[
\sigma(u) = \frac{\text{the number of events, } u, \text{ per unit time per target nucleus}}{\text{the number of incident particles per unit time per unit area}}
\]  
(3)

Description of the interaction of an incident particle with a target
nucleus in the formulation of the optical model necessarily precludes the
prediction of many-particle effects which produce, for example, the strong
resonance absorption of particles at sharp bombarding energies. Neither
can the model predict the course of the reaction following the absorption
of a particle. The predictions of the model are:

a) The cross section for elastic scattering from the potential--
shape-elastic (SE) scattering--as a function of solid angle \( d\Omega : d\sigma_{SE}/d\Omega \).

b) The angular dependence of the polarization of projectiles elasti-
cally scattered from the potential, \( P_{SE}(\Theta) \).

c) The cross section for formation of the compound system, \( \sigma_{cs} \).

The following quantities can be observed experimentally:

a) The cross section for elastic scattering as a function of solid
angle \( d\sigma_{E}/d\Omega \).

b) The angular dependence of the polarization projectiles elasti-
cally scattered, \( P_{O}(\Theta) \).

c) The cross section for reactions, \( \sigma_{R} \).
These two sets of quantities differ because of the possibility that the projectile, after forming a compound nucleus, may emerge into the entrance channel, giving rise to compound elastic (CE) scattering. At sufficiently low energies, where individual resonances are resolved, the cross section for this process, $\sigma_{\text{CE}}$, can be directly evaluated. At sufficiently high energies, when many modes of decay for the CN are possible, emission into the entrance channel becomes less important and can be neglected. The relationship between the observable cross sections and the predicted cross sections is:

$$\sigma_E = \sigma_{SE} + \sigma_{CE}$$  \hspace{1cm} (4)$$

$$\sigma_R = \sigma_{CS} - \sigma_{CE}$$  \hspace{1cm} (5)$$

The relationship between the measured and predicted polarization is not so simple. However, since scattering from the CN will be unpolarized to the extent that a statistically random distribution of phases occurs (many levels occur in the energy increment observed), one can write:

$$P_\circ(\theta) = P_{SE}(\theta)$$  \hspace{1cm} (6)$$

The general form of the optical potential most commonly used is (Eck-30)(Bet-35)(Woo-54)(Ste-55):

$$V(r) = V_c(r) - V_o f(r) - i W_o f'(r) + \left( \frac{\hbar^2}{2m} \right) \frac{1}{r} \frac{df}{dr} \hat{\sigma} \cdot \hat{l}$$  \hspace{1cm} (7)$$

where $f(r)$ and $f'(r)$ are form factors giving the spherically symmetric
dependence of the well on the radial coordinate, \( r \); \( V_c \) is the coulomb potential (\( V_c = 0 \) for neutrons), usually taken as the potential of a uniformly charged sphere; \( V_o, W_o, V_s, \) and \( W_s \) are well-depth parameters; \( \mu \) is the pi-meson rest mass; and \( \vec{S} \) and \( \vec{L} \) are the spin and orbital angular momenta respectively of the projectile. The most common form for \( f(r) \) is given in Eqn. (2). \( f'(r) \) is usually set equal to \( f(r) \) or to \( g(r) \), where:

\[
g(r) = \exp \left[ -\left( r - R_0 \right)^2 b^{-2} \right].
\]  

(8)

The parameter \( a \) in \( f(r) \) determines the diffuseness of the potential well, the well-surface thickness. The parameter \( b \) in \( g(r) \) determines the width of a gaussian-shaped distribution centered at the radius, \( R_0 \). Typical values for the parameters used in calculations at energies comparable to the present work are presented in Table I.

**Scope of Experiments**

In the present work we are concerned with the optical model as derived from the low energy approximation. Consequently, only studies for bombarding energies less than 100 Mev will be considered. The model has been quite successful in interpreting scattering and reaction experiments (see Appendix I), but it has frequently been noted (Gla-57) (Hin-57) (Gla-57a) (Mel-57) that the experimental data can be fitted equally well using a variety of radial form factors \( f(r) \) and \( g(r) \) (Eqn. (7)), and a range of values for the optical model parameters. For example, the angular distribution of elastically scattered protons shows a pronounced diffraction pattern (see Fig. 1). The angular position of the maxima and minima in intensity determine the parameter combination \( \psi_{r_0}^{n} \) (Eqn. (7)) where \( n \) varies from 2 at low energies to 3 at high energies (Nod-60).
\[ V(r) = V_c(r) - V_o f(r) - i W_s f'(r) + \left[ \frac{\hbar}{mc} \right]^2 (V_s + i W_s) \frac{df}{dr} \]  

\[ f(r) = \left[ 1 + \exp \left( \frac{r-R_o}{\delta} \right) \right]^{-1} \]

\[ g(r) = \exp \left[ - \left( \frac{r-R_o}{\delta} \right)^2 \right] \]

\[ R_o = R_o A^{1/3} \]

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Elements</th>
<th>( f' )</th>
<th>( E )</th>
<th>( V_o )</th>
<th>( W_o )</th>
<th>( V_s )</th>
<th>( W_s )</th>
<th>( \xi_0 )</th>
<th>( a )</th>
<th>( b )</th>
<th>Ref.</th>
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<tbody>
<tr>
<td>( (p,n) )</td>
<td>A 50 120</td>
<td>g</td>
<td>5.5</td>
<td>44</td>
<td>8.7</td>
<td>35</td>
<td>0</td>
<td>1.25</td>
<td>.65</td>
<td>.98</td>
<td>(Alb-59)</td>
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<td>18 elements</td>
<td></td>
<td></td>
<td>4.0</td>
<td>47</td>
<td>7.0</td>
<td>35</td>
<td>0</td>
<td>1.25</td>
<td>.65</td>
<td>.98</td>
<td></td>
</tr>
<tr>
<td>Elastic protons</td>
<td>V, Fe, Co</td>
<td>f</td>
<td>3.9-6.4</td>
<td>63</td>
<td>5.0</td>
<td>0</td>
<td>0</td>
<td>1.20</td>
<td>.40</td>
<td></td>
<td>(Pre-59)</td>
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<tr>
<td></td>
<td>Cr</td>
<td></td>
<td>3.9-6.4</td>
<td>55</td>
<td>5.0</td>
<td>0</td>
<td>0</td>
<td>1.33</td>
<td>.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic protons and</td>
<td>Cu(^{(a)})</td>
<td>f</td>
<td>10</td>
<td>61.5</td>
<td>8.5</td>
<td>3.5</td>
<td>1.0</td>
<td>1.20</td>
<td>.526</td>
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<td>(Nod-60)</td>
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<td>polarization</td>
<td></td>
<td></td>
<td>to 50.9</td>
<td>7.7</td>
<td>4.0</td>
<td>0</td>
<td></td>
<td>1.30</td>
<td>.513</td>
<td></td>
<td>(Hin-57)</td>
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<tr>
<td>Elastic protons</td>
<td>Cu(^{(b)})</td>
<td>f</td>
<td>10</td>
<td>62</td>
<td>8.6</td>
<td>0</td>
<td>0</td>
<td>1.20</td>
<td>.52</td>
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<td>to 48</td>
<td>7.9</td>
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<td>.50</td>
<td></td>
<td>(Hin-57)</td>
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<td>g</td>
<td>7.0</td>
<td>45.5</td>
<td>9.5</td>
<td>9.5</td>
<td>0</td>
<td>1.25</td>
<td>.65</td>
<td>.98</td>
<td>(Bjo-58)</td>
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<tr>
<td>Non-elastic neutrons</td>
<td>Fe, Al, Pb</td>
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<td></td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>Elastic neutrons</td>
<td>Fe(^{(c)})</td>
<td>f</td>
<td>2.5-7.0</td>
<td>39</td>
<td>8.0</td>
<td>0</td>
<td>0</td>
<td>1.45</td>
<td>.35</td>
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</tbody>
</table>

\( a \) Also analyzed: Ar.
\( b \) Also analyzed: Al, Ar, Ni, Sn.
\( c \) Also analyzed: Sn, Bi, Zr.
Table I shows the range of parameters which provided equally satisfactory fits to data on the elastic scattering and polarization of 10-Mev protons on copper and argon.

It has not been possible to distinguish whether the imaginary part of the potential should be distributed throughout the nucleus or whether it should be concentrated on the surface, although there are indications of a surface absorption from analyses of inelastically-scattered proton angular-correlations (Gri-59)(McC-59) and neutron-capture resonances (Kha-59). To look at the optical model in detail clearly requires utilizing the maximum amount of information obtainable from the model.

The analysis of neutron scattering and reaction cross-sections up to 30 Mev has provided the most complete basis to date for the determination of the optical potential (Bjo-58). With neutrons it has been possible to obtain the elastic scattering cross-section as well as the total cross section. The latter comes directly from attenuation measurements.

The application of the optical model to charged particle experiments has been primarily confined to elastic scattering observations, due in part to the simplicity of the experiments and in part to the large amount of information contained in an angular distribution. Protons are the most useful charged particles for probing the real and the imaginary parts of the potential, since composite particles (deuterons, alpha particles, etc.) are easily broken up in contact with the nucleus: they are affected primarily by the imaginary part of the potential (Che-57).

Recently several workers have supplemented the wealth of elastic scattering data with measurements of total cross sections: At 61 Mev
the attenuation of a proton beam by a thin carbon target was measured (Mey-60); also, attenuation at 34 Mev for carbon, aluminum, iron, tin, and lead has been measured (Goo-59). Observations of the pulse spectra due to protons attenuated in a scintillation detector were used to obtain the cross section on carbon from 10-to 68-Mev (Bur-59). At low energies the energy loss of charged particles is primarily an atomic effect, and attenuation measurements cannot be used. It then becomes necessary to measure the separate reactions which occur. At 23 Mev the total reaction cross-section on uranium has been obtained from measurements of the separate contributing reactions (Coh-59). Qualitative agreement with the optical model parameters listed in Table I was reported on each of these.

The region of 5-to 10-Mev proton energy is of particular interest in medium weight nuclei, since at this energy the proton is reaching the top of the coulomb barrier, and the interaction should therefore be more sensitive to the form of the nuclear potential. Elastic scattering experiments on medium weight nuclei have been performed at 7.5 Mev (Wal-57), at 9.4 Mev (Gre-57), at 9.8 Mev (Hin-57), at 5.7 Mev (Tak-57), and 6.4 Mev (Pre-59). Extensive studies of (p,n) reaction cross-sections have been reported for medium weight nuclei at 6.8 Mev (Bla-51)(Bla-51a) and at 5.5 Mev (Alb-59). Recent work has been done on five isotopes from 4-to 6.5-Mev (Tak-60), and inelastic scattering has been reported at these energies (Sew-59).

It is important that the various observations mentioned above be made at one energy on one nucleus in view of the possible dependence of the optical potential on target spin (Fes-58) and the known dependence on
target A and the projectile energy (Fes-58). It was in this framework that the present studies of copper and vanadium were undertaken, providing the elastic scattering and total reaction cross-sections for 7.55-Mev protons.

The Present Experiments

Table II shows proton reactions which are energetically possible on V$^{51}$ and Cu$^{65}$ at this energy, with the energy, Q, released in the reaction. The emission of deuterons or multiple particles is not energetically possible. The (p, γ) reaction is always small compared with the (p,n) reaction above the threshold for neutron emission (Eva-55, p. 463) (Bla-53, p. 490), and it has been neglected in the present calculations. The (p,γ) reactions reported prior to 1953 (Hol-53) provide no reliable estimates of the cross section for medium weight nuclei (Del-39) (Liv-41) (Hib-45) (Lei-47) (Dub-53).
The few quantitative activation measurements indicate a (p,γ) cross-section of the order of 1 mb at 7.55 Mev for both Cu$^{65}$ and V$^{51}$: Cohen (Coh-55) found (p,γ) cross-sections of 0.3-to 1.3-mb for Fe$^{54}$, Ni$^{60}$, and Zn$^{65}$, independent of energy from 8-to 16-Mev. Carver (Car-59), bombarding Zn$^{64}$ with 3.5-to 4.5-Mev deuterons, found the sum of the (d,n) and (d,p) cross-sections to be larger than the (d,γ) cross-section by $2 \times 10^3$.

The total reaction cross-section was obtained from separate measurements of the $[(p,p') + (p,\alpha)]$ cross-section and the (p,n) cross-section. The (p,n) reaction cross-section was measured by observing the reaction products formed, while the $[(p,p') + (p,\alpha)]$ cross-section was determined by comparing the inelastically scattered particle flux with the
TABLE II

NUCLEAR REACTION Q VALUES, MEV

<table>
<thead>
<tr>
<th>Target</th>
<th>Process</th>
<th>( (p, \gamma) )</th>
<th>( (p, n) )</th>
<th>( (p, d) )</th>
<th>( (p, \alpha) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma^{51} )</td>
<td>( \gamma^{51} )</td>
<td>+10.51</td>
<td>-1.53</td>
<td>-8.81</td>
<td>+1.17</td>
</tr>
<tr>
<td>( \text{Cu}^{65} )</td>
<td>( \text{Cu}^{65} )</td>
<td>+8.90</td>
<td>-2.13</td>
<td>-7.68</td>
<td>+4.34</td>
</tr>
</tbody>
</table>

Reference: (Ash-59).
elastically scattered proton flux at several angles. A comparison technique was used to determine the ratio of the differential cross section for elastic scattering to the differential cross section for Rutherford scattering. Previous work provided data on polarization of protons from copper (Dar-59).
EXPERIMENTAL PROCEDURES AND RESULTS

The present work was conducted using the Markle cyclotron and emergent beam apparatus, described in Appendix II, as a source of a focussed beam of accelerated particles. The work is most naturally divided, for descriptive purposes, into the separate experiments which observed elastic scattering, inelastic scattering, and (p,n) reactions.

Elastic Scattering

The elastic scattering experiments followed the method developed by Waldorf (Wal-56) (see Appendix IV). The method consists of comparing the flux of protons scattered elastically at some angle \( \Theta \) from a copper or vanadium target to the flux of protons scattered elastically from a gold target at the same angle. The protons were observed in a movable scintillation detector (see Appendix II) denoted the p-s counter. A measure of the flux of incident particles came from the number of protons scattered elastically at a fixed angle \( 25^\circ \) into a monitor detector (see Appendix II). The spectrum of pulses from the p-s counter was obtained using a twenty-channel pulse-height analyzer, and the number of elastically scattered protons was obtained by summing the counts in the appropriate channels. The pulses from the monitor counter went to an integral discriminator and scaling circuit set to accept only pulses corresponding to elastically scattered protons. The twenty-channel analyzer and the monitor scaler were turned on simultaneously and automatically turned off when \( 2^n \) (where \( n \) was a preset integer from 15 to 19) counts had been recorded by the
monitor scaler. Thus, the data which were obtained represent the flux \( \phi(\Theta) \) of particles scattered elastically into the p-s counter at angle \( \Theta \) per \( 2^n \) particles scattered elastically into the monitor counter. A comparison of the flux from a target of element, \( X \), \( \phi_x(\Theta) \), to the flux from a gold target, \( \phi_{Au}(\Theta) \), gives directly the ratio of the actual differential cross section for elastic scattering to the Rutherford cross-section \( f_x(\Theta) \) if the scattering follows the Rutherford formula at the monitoring angle (Wal-56). Then:

\[
f_x(\Theta) = \frac{\phi_x(\Theta)}{\phi_{Au}(\Theta)}
\]  

(9)

Should the scattering at the monitor angle, 25°, not follow the Rutherford formula, the experimental ratios must all be divided by the factor \( f_x(25^0) \) to give the correct value for \( f_x(\Theta) \). In the present work, the value for \( f_x(90^0) \) was determined using an independent method (see Appendix VII), and the curve of \( f_x(\Theta) \) versus was normalized to this point.

Scattering from 0.25 mil copper was observed over a range of center-of-mass (C. M.) angles of 50° to 143°. Scattering from 0.05 mil copper was observed from 31° to 143°. The values measured for \( f(\Theta) \) are tabulated in Table VII, Appendix IV and shown in Fig. 1. The value interpolated for \( f(90^0) \) for the 0.25 mil target agreed, within experimental error, with the value determined independently during bombardments in the Faraday cup (see Appendix VII) of \( f(90^0) = 0.69 \), so the scattering at the monitor angle was assumed to be Rutherford scattering. These two sets of targets give two distinct angular distribution curves, each differing from the curve of Waldorf (Wal-56) (taken with 0.05 mil copper). The maximum difference
occurs between $80^\circ$ and $110^\circ$, when the 0.05 mil curve is lower by about 10%, the 0.25 mil curve lower by about 20%, than the Waldorf curve. The reason for the observed differences is not completely understood. There may be some consistent experimental error, or the scattering may be quite sensitive to energy. The average projectile-energy is slightly less in the thicker target by about 0.07 Mev. In addition, the spread of energies of particles available for reactions is greater by about 0.13 Mev in the thicker target. The 0.25 mil target results are used for comparison with the optical model predictions, since it was this thickness of target that was used for inelastic scattering and \((p,n)\) reaction studies.

The scattering from 0.5 mil vanadium was observed from $13^\circ$ to $150^\circ$. The observed counting-rate ratios did not approach unity at small angles, indicating that the scattering from vanadium at the monitor angle of $25^\circ$ did not follow the Rutherford scattering formula. Further, ratios for angles near $90^\circ$ were not, within experimental error, consistent with the value, \(f(90^\circ) = 0.43\), determined independently from bombardments in the Faraday cup. For these reasons, the curve was normalized to fit this value of \(f(90^\circ)\), and all ratios were multiplied by a normalizing factor. From inspection of the unnormalized curve near $90^\circ$, this factor was estimated to be $0.77 \pm 0.03$. The resulting values of \(f(\Theta)\), (the observed counting ratios multiplied by 0.77) are tabulated in Table VII, Appendix IV and shown in Fig. 2.

From the repeatability of results, the precision of individual points is about 3% for vanadium and about 4% for copper (see Appendix V). A possible systematic error estimated as 1% is present in the selection of
elastic scattering in the recorded spectrum. The angles are in error by less than 1°, neglecting any mechanical backlash in the counter arm.

Inelastic Scattering

The observed scattering is actually the sum of \((p,p')\) and \((p,\alpha)\) processes, since the detector did not distinguish alpha particles from protons. For convenience, we shall refer to both types of particles as "inelastic" particles.

The scattered particles were observed in the p-s counter, and the output pulses from the detector were analyzed and displayed using a 256-channel analyzer. The number of inelastic particles, \(C_i\), and the number of elastic protons, \(C_e\), were computed from the pulse-height distribution-spectra after making corrections for detector, gamma ray, and electron background. The particles detected in the p-s counter included particles which had been scattered by the aluminum housing and the lead aperture of the detector. These particles were observed in the pulse-height distribution-spectra as a uniform distribution in all the inelastic-particle channels. The effect was most pronounced at small angles where relatively few particles are inelastically scattered from the targets. From the data at small angles, aperture scattering was determined to contribute 10% of the inelastic counts, and the final cross sections were corrected for this. Spectra were obtained at several detector angles, and for each angle the quantity:

\[
k = \frac{C_i}{C_e} f(\Theta) \sin^{-4}(\Theta/2)
\]

was determined, where \(f(\Theta)\) is the ratio-to-Rutherford factor determined
from the elastic scattering experiments. \( k \) is proportional to the differential cross section for inelastic scattering. It was assumed that the differential cross section is isotropic (Sw-59), and the total cross section, \( \sigma \), was obtained by multiplying the average \( k \) by 4\( \pi \) times the Rutherford cross-section at 180\( ^{0} \) times the correction, \( c \), for aperture scattering:

\[
\text{copper: } \sigma = 240(k)_{av} c \text{ mb } \tag{11}
\]
\[
\text{vanadium: } \sigma = 151(k)_{av} c \text{ mb } \tag{12}
\]

in units of millibarns (1mb = \( 10^{-27} \text{ cm}^2 \)).

Observations were made from about 20\( ^{0} \) to 140\( ^{0} \). The values for \( k \) are shown in Table III. The values for angles smaller than 60\( ^{0} \) are omitted, since the scattering at these angles was used to estimate the aperture-scattering factor. These smaller angles are also more sensitive to errors in measuring \( \Theta \). The average values for \( k \) are:

\[
\text{copper: } (k)_{av} = 1.23 \pm 0.09 \tag{13}
\]
\[
\text{vanadium: } (k)_{av} = 0.99 \pm 0.09 \tag{14}
\]

where the errors are the computed standard errors. The cross sections are therefore (using \( c = 0.90 \)):

\[
\text{copper: } [(p, p') + (p, \alpha)] : 266 \pm 27 \text{ mb } \tag{15}
\]
\[
\text{vanadium: } [(p, p') + (p, \alpha)] : 134 \pm 13 \text{ mb } \tag{16}
\]
<table>
<thead>
<tr>
<th>C.M. Angle</th>
<th>Inelastic Counts: $C_i \times 10^4$</th>
<th>Elastic Counts: $C_o \times 10^4$</th>
<th>$C_i/C_o$</th>
<th>$f_x(\Theta)$</th>
<th>$\sin^4(\Theta/2)$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>34.9°</td>
<td>2.53</td>
<td>25.46</td>
<td>0.100</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>56.7°</td>
<td>3.44</td>
<td>15.32</td>
<td>0.224</td>
<td>0.77</td>
<td></td>
<td></td>
</tr>
<tr>
<td>71.2°</td>
<td>6.95</td>
<td>18.91</td>
<td>0.368</td>
<td>0.48</td>
<td>0.115</td>
<td>1.54</td>
</tr>
<tr>
<td>(a) 92.9°</td>
<td>4.53</td>
<td>9.95</td>
<td>0.455</td>
<td>0.70</td>
<td>0.276</td>
<td>1.15</td>
</tr>
<tr>
<td>(b) 92.9°</td>
<td>9.02</td>
<td>21.36</td>
<td>0.422</td>
<td>0.70</td>
<td>0.276</td>
<td>1.07</td>
</tr>
<tr>
<td>107.2°</td>
<td>4.71</td>
<td>10.77</td>
<td>0.438</td>
<td>1.10</td>
<td>0.420</td>
<td>1.15</td>
</tr>
<tr>
<td>143.0°</td>
<td>1.87</td>
<td>12.27</td>
<td>1.522</td>
<td>0.66</td>
<td>0.809</td>
<td>1.24</td>
</tr>
</tbody>
</table>

**COPPER**

**VANADIUM**

<table>
<thead>
<tr>
<th>C.M. Angle</th>
<th>Inelastic Counts: $C_i \times 10^4$</th>
<th>Elastic Counts: $C_o \times 10^4$</th>
<th>$C_i/C_o$</th>
<th>$f_x(\Theta)$</th>
<th>$\sin^4(\Theta/2)$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.0°</td>
<td>2.73</td>
<td>25.92</td>
<td>0.105</td>
<td>0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>56.9°</td>
<td>2.64</td>
<td>19.10</td>
<td>0.138</td>
<td>0.96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>71.4°</td>
<td>4.68</td>
<td>16.32</td>
<td>0.287</td>
<td>0.52</td>
<td>0.116</td>
<td>1.29</td>
</tr>
<tr>
<td>93.1°</td>
<td>4.93</td>
<td>9.96</td>
<td>0.495</td>
<td>0.52</td>
<td>0.278</td>
<td>0.93</td>
</tr>
<tr>
<td>107.5°</td>
<td>1.61</td>
<td>4.83</td>
<td>0.333</td>
<td>1.08</td>
<td>0.423</td>
<td>0.85</td>
</tr>
<tr>
<td>128.9°</td>
<td>5.55</td>
<td>7.56</td>
<td>0.737</td>
<td>1.29</td>
<td>0.928</td>
<td>1.02</td>
</tr>
<tr>
<td>143.1°</td>
<td>1.67</td>
<td>2.68</td>
<td>0.622</td>
<td>1.15</td>
<td>0.810</td>
<td>0.85</td>
</tr>
</tbody>
</table>

(a) Observed the transmitted beam.
(b) Observed the reflected beam.
The error of 10% is the root-mean-square error calculated from the following error estimates:

a) The uncertainty in the gamma-radiation background subtracted from the inelastic scattering is 2%.

b) The uncertainty in the dividing point between elastic and inelastic scattering pulses recorded in the analyzer is 2%.

c) The uncertainty in the elastic-scattering cross sections is 5%.

d) The uncertainty in $\sin^4(\Theta/2)$ is 2%, due to errors in the zero-angle and errors in relative angle measurements.

e) The correction for aperture scattering is good to 3%.

f) The standard error in $(k)_{av}$ is 8%.

For comparison with optical model predictions, it is necessary to have the total reaction or non-elastic cross section. In the case of vanadium, cross section (16) is added directly to the $(p,n)$ reaction cross-section to obtain the total non-elastic cross section for comparison with the optical model. In the case of copper, however, the observed cross section is $\bar{\sigma}$, the average cross section for the two isotopes Cu$^{65}$ and Cu$^{63}$, whereas we are interested only in Cu$^{65}$. To evaluate the contribution of Cu$^{65}$ to the cross section, two assumptions might be made, lacking data at 7.5 Mev on the relative scattering. Most simply, we might assume that the cross sections are the same for both isotopes. Then the Cu$^{65}$ cross-section would be:

\[ \text{Estimate } \#1: \quad \text{Cu}^{65} \left[ (p,p') + (p,\alpha) \right] : 266 \pm 27 \text{ mb} \quad (17) \]
Another estimate can be made from data (Maz-57) on the inelastic scattering of protons from the separate isotopes at 6.51 Mev. (No data at nearer energies are available.) From the published data, the ratio, \( C_i / C_e \) was calculated for each isotope. The Cu\(^{65}\) data were given for 50\(^\circ\) and the Cu\(^{63}\) data for 90\(^\circ\). Assuming \( f_{65}(50^\circ) = f_{63}(90^\circ) \), the ratio of the Cu\(^{65}\) to the Cu\(^{63}\) cross-section for \((p,p')\) from this data is 0.23 \( \pm \) 0.10. The error is the estimate of the error introduced by assuming \( f_{65}(90^\circ) = f_{65}(50^\circ) \).

In terms of the average cross section \( \bar{\sigma} \) for the two isotopes, \( \bar{\sigma}(Cu^{65}) = 0.37\bar{\sigma} \).

To apply this to the present experiment, the \((p,\alpha)\) cross-section must first be subtracted from the experimental cross section to obtain the average \((p,p')\) cross-section. The data of Fulmer (Ful-60) on the \((p,\alpha)\) cross-section from 8.5-to 23-Mev on natural copper was extrapolated to 7.5 Mev, giving a value of 25 mb for the \((p,\alpha)\) cross-section. This was subtracted from the experimental value of 266 mb given above, and the resulting cross section for \((p,p')\) was multiplied by 0.37, giving a \((p,p')\) cross-section for Cu\(^{65}\) of:

\[
\text{Estimate #2:} \quad \text{Cu}^{65}(p,p') \quad 89 \pm 44 \text{ mb} \quad (18)
\]

where the error is the uncertainty in the \( f(\Theta) \) correction which is the limiting error. The \((p,\alpha)\) cross-section for Cu\(^{65}\) was taken to be the average cross section for the two isotopes, taken from Fulmer:

\[
\text{Estimate #2:} \quad \text{Cu}^{65} (p,\alpha) \quad 25 \pm 10 \text{ mb} \quad (19)
\]

The error is an estimate of the validity of assuming the \((p,\alpha)\) cross-sections are the same for Cu\(^{65}\) and Cu\(^{63}\). This gives:
Estimate #2:  \[ \text{Cu}^{65} \left( p, p' \right) + \left( p, \alpha \right) \] 114 \pm 45 \text{ mb} \tag{20} 

where the error is the rms error. The true cross section might be expected to lie between the two estimates above from the following argument. The Q value (see Table II) for the \( p, n \) reaction inhibits neutron emission more for \( \text{Cu}^{63} \) than for \( \text{Cu}^{65} \). As the bombarding energy is increased, this becomes less important, and the \( p, n \) cross-sections become closer numerically. Assuming equal amounts of CS formation in each isotope requires that proton emission be more important for \( \text{Cu}^{63} \) than \( \text{Cu}^{65} \), as observed at 6.5 MeV. As the bombarding energy is increased, neutron emission is less inhibited by the Q value, and will increase at the expense of proton emission. Thus, the \( \text{Cu}^{65} \) and \( \text{Cu}^{63} \) \( (p, p') \) cross-sections should approach each other numerically. The average of the two estimates above has been taken as the best estimate for \( \text{Cu}^{65} \), with the error taken as half the difference between the two estimates. This gives:

**Observed:**  \( \text{V}^{51} \left( p, p' \right) + \left( p, \alpha \right) \] 134 \pm 13 \text{ mb} \tag{21}

**Estimate #3:**  \[ \text{Cu}^{65} \left( p, p' \right) + \left( p, \alpha \right) \] 192 \pm 75 \text{ mb} \tag{22}

The copper error value is half the difference between Estimate #1 and Estimate #2.

Previously reported studies of inelastic proton scattering have been primarily confined to scattering from single resonance levels, either to determine the energies of the levels (Bue-60) or the angular distribution of particles from particular levels (Sew-59). Since levels are more widely separated in the lighter elements (Bur-57)(Kin-57), most experimental work has been done with them.
Resolved energy levels have been observed for $^{51}$V (Maz-58) and $^{63}$Cu and $^{65}$Cu (Maz-57) for excitation energies up to 4 Mev. Some observations have been made of the energy spectra of protons emitted from bombardments at energies exceeding 20 Mev (Kin-57).

The $^{51}$V (p,p') cross-section from 4.5-to 6.5-Mev has recently been estimated (Tak-60) from the work of Seward et al. (Sew-59), and this estimate extrapolates to a value of 150-to 200-mb at 7.55 Mev.

\[(p,n) \text{ Reaction}\]

When Eqn. (3), which defines the (p,n) reaction cross-section, is integrated over the area of the target and the time duration of the bombardment, we obtain:

$$\sigma(p,n) = \frac{N}{QS} \quad (23)$$

where $Q$ is the number of bombarding protons, $N$ is the number of (p,n) reactions occurring during bombardment, and $S$ is the number of atoms per cm\(^2\) in the target. Since the half-life of each species studied is long compared to the duration of bombardment (245-d Zn\(^{65}\) and 27.75-d Cr\(^{51}\) compared with 1-hour bombardment time), $N$ may be set equal to the activity at the end of bombardment divided by the decay constant.

Table IV shows the various products which can be formed by the energetically possible reactions induced by 7.55-Mev protons on the constituents of natural vanadium and natural copper. In the case of vanadium, the low relative abundance of $^{50}$V, coupled with the preponderance
TABLE IV  
PROTON-INDUCED REACTIONS

<table>
<thead>
<tr>
<th>Target</th>
<th>$^{50}$</th>
<th>$^{51}$</th>
<th>Cu$^{63}$</th>
<th>Cu$^{65}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abundance</td>
<td>0.25%</td>
<td>99.75%</td>
<td>60.1%</td>
<td>30.9%</td>
</tr>
<tr>
<td>Reaction Product</td>
<td>(p,n)</td>
<td>stable</td>
<td>27.75 d Cr$^{51}$</td>
<td>40m Zn$^{63}$</td>
</tr>
<tr>
<td></td>
<td>(p,$\alpha$)</td>
<td>stable</td>
<td>stable</td>
<td>stable</td>
</tr>
<tr>
<td></td>
<td>(p,t)</td>
<td>27.75 d Cr$^{51}$</td>
<td>stable</td>
<td>stable</td>
</tr>
</tbody>
</table>

Reference (Ste-56)
of \((p,n)\) over competing reactions at this energy, means that the target activity will be, to a satisfactory approximation, only \(^{51}\text{Cr}\) from the \(^{51}\text{V} (p,n) ^{51}\text{Cr}\) reaction. In the case of copper, once the \(^{63}\text{Zn}\) activity from \(^{63}\text{Cu} (p,n) ^{63}\text{Zn}\) has been allowed to decay, the activity will be exclusively \(^{65}\text{Zn}\) from the \(^{65}\text{Cu} (p,n) ^{65}\text{Zn}\) reaction.

Prior to all bombardments for reaction cross-section determinations, the cyclotron was used exclusively for protons or alpha particles for several days. This was to eliminate the possibility of deuteron contamination in the proton beam, since deuterons are not always completely separated from protons by operating conditions alone. Using the elastic scattering into the monitor as a record, at no time did possible contamination reach as much as 0.5%.

The targets were rectangular metal foils held in aluminum target frames at an angle of 45° to the beam and contained in a Faraday cup (see Appendix VIII). These targets were weighed and measured, and the actual thickness was corrected for the target angle to obtain the effective target thickness, \(\rho\), in \(\text{mg/cm}^2\).

The beam of hydrogen molecule-ions emerging from the cyclotron was broken up into protons by an aluminum foil over the front slit in the exit tube. (See Appendix II) This slit, together with a collimator of lead inserted into the beam a few centimeters before the Faraday cup, ensured that the entire beam entering the cup would pass through the center of the target foil.

The current reaching the Faraday cup was integrated with an electronic integrating-circuit (see Appendix IX) to give the total charge, \(q\), in
microcoulombs which passed through the target. At the same time, particles scattered into an opening in the side of the cup at a mean angle of $90^\circ$ were observed with a monitor detector (see Appendix II). By counting the elastically scattered particles, an additional check on the particle accumulation was possible.

Following bombardment, the target foils were removed from the Faraday cup, placed in glass vials, dissolved in 2.0 ml of a suitable acid (see Appendix VI), and the activity was compared with the activity of calibrated $\text{Zn}^{65}$ and $\text{Cr}^{51}$ sources under identical conditions. The standard sources were prepared during the course of this work and were calibrated using coincidence-counting techniques (see Appendix XI). The activity of each target was corrected for decay back to the end of bombardment.

The results of five bombardments on copper and four bombardments on vanadium (see Appendix VII) are presented in Table V. The precision of the results is seen to be 1.3% for the copper and 4.0% for the vanadium. An rms error is determined from the following:

a) The standard error in comparing the targets to the standards is 1% (counting statistics and imprecision in the preparation of identical standards).

b) The standard error of the coincidence-counting is 3% (counting statistics, background corrections, imprecision in the preparation of identical standards, comparison with the N. B. S. (Appendix XI)

c) Neglecting electron contamination of the cyclotron beam, deuterons in the incident beam, and errors in the calibration of the current integrator gives an estimated error of 1%.
<table>
<thead>
<tr>
<th>Bombardment Time</th>
<th>Collected Charge (Microcoulombs)</th>
<th>Monitor Detector Counts $\times 10^4$</th>
<th>Target Thickness (mg/cm$^2$)</th>
<th>Activity at end of Bombardment (dis/sec)</th>
<th>Cross Section (millibarns)</th>
</tr>
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<tbody>
<tr>
<td>2240 May 27, 1960</td>
<td>(124)$^a$</td>
<td>3.26</td>
<td>7.43</td>
<td>$1.49 \times 10^4$</td>
<td>533</td>
</tr>
<tr>
<td>1740 June 7</td>
<td>196</td>
<td>5.24</td>
<td>7.42</td>
<td>$2.57 \times 10^4$</td>
<td>582</td>
</tr>
<tr>
<td>1203 June 8</td>
<td>196</td>
<td>5.12</td>
<td>7.54</td>
<td>$2.43 \times 10^4$</td>
<td>543</td>
</tr>
<tr>
<td>1830 June 11</td>
<td>196</td>
<td>5.16</td>
<td>7.45</td>
<td>$2.48 \times 10^4$</td>
<td>560</td>
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**VANADIUM**

**COPPER**

<table>
<thead>
<tr>
<th>1240 May 26, 1960</th>
<th>196</th>
<th>8.07</th>
<th>5.61</th>
<th>$5.09 \times 10^2$</th>
<th>543</th>
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<tr>
<td>1505 May 27</td>
<td>294</td>
<td>12.3</td>
<td>5.66</td>
<td>$7.49 \times 10^2$</td>
<td>527</td>
</tr>
<tr>
<td>1715 May 27</td>
<td>589</td>
<td>10.8</td>
<td>2.54</td>
<td>$6.89 \times 10^2$</td>
<td>539</td>
</tr>
<tr>
<td>2020 May 27</td>
<td>294</td>
<td>11.9</td>
<td>5.65</td>
<td>$7.56 \times 10^2$</td>
<td>534</td>
</tr>
<tr>
<td>2035 May 11</td>
<td>294</td>
<td>12.1</td>
<td>5.75</td>
<td>$7.82 \times 10^2$</td>
<td>543</td>
</tr>
</tbody>
</table>

$^a$ Obtained from monitor counts.
d) The uncertainty in isotopic abundance is estimated as 1% (Str-58).
e) The uncertainty (Str-58)(Way-55)(Dze-58)(Gle-59) in half-lives is estimated as 0.5%.
f) The cos 45° correction for the target angle introduces a maximum error of 1%.

The averages of the results in Table V are:

\[ ^{65}_{\text{Cu}} \, (p,n) \, ^{65}_{\text{Zn}} : \, 537 \pm 21 \text{ mb} \]  
(24)

\[ ^{51}_{\text{V}} \, (p,n) \, ^{51}_{\text{Cr}} : \, 555 \pm 30 \text{ mb} \]  
(25)

Few precise measurements of (p,n) cross-sections in the region of 5-to 12-Mev bombarding energy have been reported. The excitation functions for 60 nuclei determined at Zürich (Bla-51) (Bla-51a) give cross sections for proton energies up to 6.8 Mev, with a stated accuracy of 10-20%. At 6.8 Mev the cross sections for elements of atomic number between 28 and 32 are all within 40% of 4 \times 10^{2} \text{ mb}. The only reported experiments at 7.5 Mev (How-58) gave a value of about 6 \times 10^{2} \text{ mb} for the Cu^{65}_{\text{Cu}} \, (p,n) \, Zn^{65}_{\text{Zn}} cross-section, based on the Zürich value at 6.5 Mev. This work also studied the cross sections for Cu^{63}_{\text{Cu}}, \, Zn^{68}_{\text{Zn}}, \text{ and Zn}^{66}_{\text{Zn}}. A study of 18 medium weight nuclei (Alb-60) at energies of 3.5-to 5.5-Mev showed cross sections of about 3 \times 10^{2} \text{ mb} at at 5.5 \text{ Mev} for elements with atomic weights from 45 to 70. An extrapolation of this data to 7.5 Mev gives an estimated cross section of about 5 \times 10^{2} \text{ mb} for vanadium and copper. The recent work at Rochester (Tak-60) on V^{51}_{\text{V}} from 4.5-to 6.5-Mev extrapolates to a value of about 450 \text{ mb} at 7.5 Mev. The present value of 555 \text{ mb} is not outside the experimental error of the extrapolation and the Rochester error.
COMPARISON OF RESULTS WITH OPTICAL MODEL PREDICTIONS

A program for the IBM-704 computer was obtained* from the Lawrence Radiation Laboratory of the University of California at Livermore, and computations were made using the potential:

\[ V(r) = V_c(r) - V_o f(r) - iW f'(r) + \left( \frac{K}{2Mc} \right) \lambda V_o \frac{1}{r} \frac{df}{dr} \delta \hat{L} \]  

(26)

with \( \lambda \) the mass of the proton and:

\[ f(r) = \left[ 1 + \exp\left(\frac{r-R_0}{a}\right) \right]^{-1} \]  

(27)

\[ f'(r) = \left[ 1 + \exp\left(\frac{r-R_0}{b}\right) \right]^{-1} \]  

(28)

The remaining terms are described in connection with Eqns. (7) and (10).

This potential differs from the potential used by recent workers elsewhere, Eqn. (7), in two respects: the spin interaction term is that used by Ross, Mark, and Lawson (Ross-56), Eqn. (1), and the form factor for the imaginary potential, \( f'(r) \), differs from the form factor for the real potential, \( f(r) \), only in the diffuseness parameter, \( b \). Previous workers have either used a surface absorption, \( g(r) \), Eqn. (8), or set \( f'(r) = f(r) \). **

A series of computations were made to fit the data of Waldorf (Wal-56) on elastic scattering of 7.5-Mev protons from copper and C. E. Darden (Dar-59) on the polarization of 7.5-Mev protons from copper. The intermediate

* The author is indebted to Dr. F. E. Bjorkland for this program.

** The author is indebted to Mrs. S. Yang-Hu for these computations.
angle elastic-scattering data was fit equally well with several combinations of parameters, although below 40°, the predictions were in uniform disagreement. Computations made using a spin-interaction parameter, \( \lambda \), of 25 to 55 were in qualitative disagreement with the polarization data, predicting too much polarization. A computation using \( \lambda = 10 \) provided a reasonable polarization. Parameters for the two computations used for comparison with the present experiments were selected from these preliminary results. They are presented in Table VI.

The energy of the protons bombarding the target has been taken as 7.55 Mev (see Appendix III) and the energies shown in the table are the calculated energies of such protons midway through the targets (Ste-59). Note that in the present determination of the (p,n) reaction cross-section a 1.88 mg/cm² aluminum foil, inserted to break up the accelerated hydrogen molecule-ions (see Appendix VIII) degraded the energy of the incident protons by 0.78 Mev (Ste-59). The (p,n) reaction cross-sections vary only slowly with energy at 7.5 Mev, as evidenced by bombardments using various thickness targets (see Appendix IV), so that the observed (p,n) cross-sections can be directly compared with the predictions of these calculations with negligible error.

The elastic scattering predictions have been drawn with the experimental data in Figs. 1 and 2. The non-elastic cross-section predictions are given along with the present experimental results in Table VII. The polarization prediction and experimental data are shown in Fig. 3.

All the predictions are in qualitative agreement with the data, although there is quantitative disagreement of as much as 20%. The effect
<table>
<thead>
<tr>
<th></th>
<th>$E$</th>
<th>$V$</th>
<th>$W$</th>
<th>$r$</th>
<th>$a$</th>
<th>$b$</th>
<th>$\lambda$</th>
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<tbody>
<tr>
<td>copper</td>
<td>29</td>
<td>7.46</td>
<td>55</td>
<td>4</td>
<td>1.25</td>
<td>0.55</td>
<td>1.2</td>
</tr>
<tr>
<td>vanadium</td>
<td>23</td>
<td>7.47</td>
<td>55</td>
<td>4</td>
<td>1.25</td>
<td>0.55</td>
<td>1.2</td>
</tr>
</tbody>
</table>

\(^{(a)}\) Terms defined in eqn. (26).
<table>
<thead>
<tr>
<th></th>
<th>Cu$^{65}$</th>
<th>$\nu^{51}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(p,n)$:</td>
<td>537±21</td>
<td>555±30</td>
</tr>
<tr>
<td>$[(p,p')+(p,\alpha)]$:</td>
<td>192±75</td>
<td>134±13</td>
</tr>
<tr>
<td>total:</td>
<td>729±78</td>
<td>689±33</td>
</tr>
<tr>
<td>predicted:</td>
<td>587</td>
<td>797</td>
</tr>
</tbody>
</table>
of changes in the optical model parameters has been discussed in detail in the papers by Glassgold et al. (Gla-57) and Melkanoff et al. (Mel-57) and these provide the basis for the present discussion of possible changes in the parameters.

The prediction for elastic scattering from copper shows an increasingly poor fit to the data with larger angles, suggesting that either $V_o$ or $r_o$ be decreased. The predicted maxima and minima are uniformly exaggerated, suggesting an increase in $W_o$ while $\alpha$ is nearly optimum. The predicted scattering from vanadium shows good agreement with the observed positions of the maxima and minima, indicating the combination $V_o r_o^2$ is optimum. The prediction fits down to the smallest angles observed. The predicted maxima is too small, while the predicted minima is too large. This could be improved by an increase in $\alpha$ or $W_o$.

The polarization predictions are nearly insensitive to all parameters except $\lambda$. The agreement with the experimental data indicates that the present value for $\lambda$, about one-fourth the value used in the bound state, (Ross-56) is about optimum.

The predicted non-elastic cross section for copper is about 20% low, while the prediction for vanadium is about 16% high. Since no attempt has been made to optimize the parameters of the equation, this agreement is considered quite good. A more detailed study of optical model predictions might show a dependence of the parameters upon $\lambda$. The effect of compound elastic scattering should be taken into consideration for a more detailed analysis (Bey-56) (Emm-56) (Wal-56) (Fes-58). A detailed analysis should also consider the predictions of a surface-absorption potential. In the present computations the imaginary-potential surface
is more diffuse than the real-potential surface, and hence absorption occurs at slightly larger distances than potential scattering.

No consideration has been given in the present work to Cu\textsuperscript{63}. The total non-elastic cross section may be estimated from the \((p,n)\) cross-section values previously published and the present inelastic scattering data. Howe (How-59) gives the \((p,n)\) cross-section at 7.5 Mev as about 300 mb, while the data from Rochester (Tak-60) from 4.0 to 6.5-Mev may be extrapolated to give about 250 mb. Adding 340 mb for the \(\left[ (p,p') + (p,\alpha) \right]\) cross-section, determined from the data presented in the present work for Cu\textsuperscript{65}, gives a total cross section of 590 to 640 mb, similar to the Cu\textsuperscript{65} value of 587 in the present work. Recent work shows that the primary difference of the elastic scattering from different isotopes of an element is a slight shift of the maxima and minima of the \(f_x(\Theta)\) versus \(\Theta\) curve (Beu-59)(Boo-59)(Bru-60). This effect is neglected in the present fitting of experimental data.

From an experimental viewpoint, several experiments are called for by the present work. The large-angle elastic scattering provides an especially sensitive test for optical-model parameters, and it is desirable to carry observations to as large angles as possible. The energy dependence of both theory and experiment should be determined. The present observations of a difference in the elastic scattering from targets of different thickness may be due to such an effect. As mentioned earlier, both the mean energy of the projectiles and the range of energy available for reaction differs in the two targets. Experiments are desirable to separate these two effects. The change of mean energy can be done by inserting an
absorber into the beam. Machine computations using the optical model show a change in the scattering with incident energy, but the predicted change is not so large as the present observations would require. A change in the energy spread available might lead to an averaging over more levels of the target nucleus.

Further work is called for on polarization, since to date no work has been reported for vanadium.
APPENDIX I

THE OPTICAL MODEL

Historical Introduction

Aspects of the present optical model are traceable back to the attempts by Bethe (Bet-35) (Bet-40), Fermi (Fer-35), and others (Fri-55) to represent the interaction of neutrons incident upon nuclei in terms of a simple single-particle potential well. The potential was made complex in order to produce reactions (Ost-36) (Bet-40). The observation of sharp resonances in the neutron cross-sections (Ama-35) which were in qualitative disagreement with the predictions of this model, both as to the sharpness of the resonances and closeness of the energy spacing, led to the suggestion of the formation of the compound nucleus (Boh-36) (Wig-36) (Boh-37). This theory was subsequently cast into mathematical form and elaborated by Bethe (Bet-37), Weisskopf (Wei-37), and others (Wei-40), and came to fruition in the Breit-Wigner formalism for resonance reactions (Wig-47). The foundations of this formalism have been subject to rigorous scrutiny in subsequent work (Kap-38) (Tho-55) (Lan-58). The results of the formalism, as expressed in the usual Breit-Wigner isolated resonance formula (Eva-55, p. 447) factors the probability for the (a,b) reaction into the product of the probability for the formation of the CN by particle a times the probability that the CN, once formed, will decay by emitting particle b. (The term particle in this context can refer to a photon, a nucleon, or a composite nuclear particle.) The most notable success of this formalism has come in the description of low energy neutron-reactions.
Along with the development of the CN picture (Sha-55), application of single-particle theories of interactions continued. A complex well was used in analyzing 18.6 Mev elastic proton scattering (Lev-52) (Gug-52). The analogy between the projectile-nucleus system and a light-wave -- glass-ball model was presented by Bethe, who connected the index of refraction, including an imaginary part to account for attenuation, in the light-wave model with an absorption coefficient in the case of the nuclear model. The first successful application of this optical analogy (Ser-47) (Fer-49) was to the early high energy experiments ($\geq 100$ Mev) where the nucleus could be considered as nearly transparent to the incident particle (Pas-50) (Byf-52) (Gat-52) (Tay-53) (Hec-54). Shortly thereafter, with the availability of data on the scattering of slow neutrons as a function of mass number $A$, showing variations now called giant resonances, came attempts to apply various single particle models (For-50) (Law-56). The result of averaging over resonances was considered (Fes-49). With the experiments by Barschall (Bar-52) on low energy neutron absorption came the indication, as shown by Feshbach, Porter, and Weisskopf (Fes-53) (Fes-54) (Ada-54), that the nucleus was partially transparent at these energies, and an optical model was applicable. This resulted in a critical evaluation of the range of validity of the CN picture (Fri-55).

The ensuing period has seen the theory of the optical model extended in many directions by numerous workers: the theoretical basis for the model has been examined (Fes-58); computational methods have been developed and refined (Gla-57); the formalism has been expanded (Fes-58a); the experimental applications have been extended to the interactions of
deuterons (Eng-60), alpha particles (Che-57) Igo-60), protons, neutrons (Moh-56) (Cro-59), mesons (Rai-59), anti-protons (Noz-57), and complex nuclei (Por-58). Calculations have been extended to non-spherical potentials (Mar-59) (Cha-58). The review of Feshbach (Fes-58) presents a complete account of the status of the optical model as of January, 1958, with a detailed discussion of the history and results of the model. More recent work has been reviewed by Glassgold (Gla-59).

The Form of the Optical Potential

It was early found that a simple square-well representation of the phenomenological potential introduced for computational simplification, while reproducing some qualitative effects (Dar-55) (Ell-56), was not quantitatively satisfactory (Cha-54) (Gat-52), and did not predict sufficiently large reaction cross-sections (Wal-55) (Emm-55). This was attributed to the reflection of the projectile at the sharp discontinuity in the potential at the nuclear boundary which prevented the particle from penetrating the boundary sufficiently (Bet-40) (Wal-55). The model was improved by the use of various forms for the well containing a sloping or diffuse edge (Woo-54) (Law-55). A further refinement to the basic central potential was introduced (Hec-54) (Mal-54) (Sno-54) (Fer-55) (Ste-55a) to explain the observed polarization of scattered particles by means of a spin-orbit interaction potential (Fer-54). This is usually taken to be a Thomas-type potential (Tho-26) (Ste-55). It has further been found that the dependence of charged-particle scattering upon the form assumed for the distribution of charge in the nucleus is negligible (Gla-55). For convenience in calculations the coulomb-force field is usually taken as that of a sphere of uniform charge density, radius R.
The recent availability of programs for machine computation of the numerical predictions of the model has allowed the use of more realistic forms for the potential well than was earlier possible (Bjo-58).

Calculations

To describe exactly the system consisting of an incident particle, \( p \), and \( A \) nucleons of a target nucleus would require one to write the solution to the many-body Schrödinger equation:

\[
E \Psi = (H_A + H_p + \sum_i V_i) \Psi
\]  \hspace{1cm} (I-1)

where \( E \) is the energy of the system, \( H_A \) is the Hamiltonian operator for the \( A \) nucleons alone, \( H_p \) is the Hamiltonian operator for \( p \) alone, and \( V_i \) is the interaction potential between \( p \) and the \( i \)th nucleon. The solution \( \Psi \) is then a function of the generalized coordinates (position, momentum, spin, angular momentum) of the \( A \) nucleons and of \( p \). The optical model replaces this many-body problem with an equivalent single-particle problem: the total effect of the \( A \) nucleons upon \( p \) is described by the optical potential \( V \), fixed in the center of mass coordinate system, and the solution \( \psi \) is a function of the generalized coordinates of \( p \) only. The appropriate equation is now:

\[
E_p \psi = (T_p + V) \psi
\]  \hspace{1cm} (I-2)

where \( E_p \) is the energy of \( p \), and \( T_p \) the kinetic energy operator for \( p \). In order to produce the effect of particle absorption (particle non-conservation)
with Eqn. (I-2), a complex potential is used. The imaginary (non-Hermitian) part of the potential causes the particle density to decrease with time (Bet-40).

Using the optical model to calculate these cross sections for protons, the mathematical problem is the following (Ray-26) (Bla-55): At very large distances from the nucleus we desire a solution to Eqn. (I-2) which behaves as a plane wave moving toward the nucleus plus a scattered wave moving radially outward from the nucleus. For large distances the Schrödinger equation contains only a coulomb potential, since the nuclear force acts only in a volume of the order of the size of the nucleus. The solutions may be written in terms of an incoming wave of angular momentum, \( L \), plus a parameter \( \eta_L \) times an outgoing wave of angular momentum \( l \). We express the effect of the nucleus upon the incident plane wave in terms of the parameters \( \eta_L \). The cross sections for scattering or reaction are expressed in terms of the \( \eta_L \) as (c.f. Fes-54 or Bla-55):

\[
\sigma_e = \sum_g \pi \lambda^2 (2\ell+1) \left| 1 - \eta_L \right|^2 \tag{I-3}
\]

\[
\sigma_r = \sum_l \pi \lambda^2 (2\ell+1) \left( 1 - \left| \eta_L \right|^2 \right) \tag{I-4}
\]

where \( \lambda \) is the DeBroglie wave length of the incident proton, \( \sigma_r \) is the reaction cross-section, and \( \sigma_e \) the elastic scattering cross section.

Within the nucleus itself Eqn. (I-2) contains, in addition to the coulomb potential \( V_c (r) \), the nuclear potential \( V_n (r) \). The solutions to the equation in this region may in principle be written directly. In
practice, a computer is needed for computations for all but the simplest forms for \( V_n(r) \).

In order for the wave function \( \psi \) of the incident proton to be physically realizable, there can be no discontinuities of either \( \psi \) or its derivative with respect to the radial coordinate \( r \), \( \psi' \). (Such a discontinuity would correspond to an infinite energy.) Denote the solution to the Schrödinger equation far outside the nucleus by \( \psi_\circ \), (coulomb wave-functions), and the solution to the equation inside the nucleus by \( \psi_i \). Then at some distance, \( p \), from the center of the nucleus, \( V_n \) will be less than some arbitrary small value \( \varepsilon \), and we can consider the potential to be only a coulomb potential.

For convenience, we define a logarithmic derivative (Bla-55):

\[
f(\psi) = p \left( \frac{\psi'}{\psi} \right)_{r=p}
\]

(I-5)

The continuity condition can then be written:

\[
f(\psi_\circ) = f(\psi_i) \quad \text{at} \quad r = p
\]

(I-6)

This equation connects the wave function outside the nucleus, which is written in terms of coulomb wave-functions and the \( \eta_\varepsilon \), to the wave function inside the nucleus, which is the solution to Eqn. (I-2) with \( V = V_c + V_n \). This is solved to find values for \( \psi \) and \( \psi' \) at \( r = p \), from which the \( \eta_\varepsilon \) are determined, using Eqn. (I-6).
APPENDIX II

THE CYCLOTRON

The Markle Cyclotron at M. I. T. is a fixed frequency machine designed to accelerate hydrogen molecule-ions to an energy of about 15 Mev (alternatively deuterons to 15 Mev or alpha particles to 30 Mev) in the 15-kilogauss field between two 42-inch diameter electro-magnet pole faces. The accelerated particles are deflected from their spiral orbits by a 40-kilovolt potential-difference between two parallel curved plates. The deflected particles enter a target box on the perimeter of the vacuum tank which encloses the dees, deflecting plates, and ion source. On those occasions when the cyclotron beam is used for radioisotope production, the targets are mounted on a water-cooled target-holder and inserted into this target box to intercept the deflected beam of particles. For scattering experiments, and for the experiments of this work, a slotted target-holder is inserted into the target box allowing the beam to emerge from the box into an exit tube. This tube passes through two quadrupole electro-magnets which provide controlled focussing of the emerging beam. The exit tube next passes through the 4-foot thick concrete wall of the vault which encloses the cyclotron to provide radiation shielding for personnel. The exit tube at this point is inside the scattering chamber vault.

The exit tube, after interruptions with two adjustable slits, opens into a 64 cm-diameter cylindrical scattering chamber. The perimeter of this
chamber incorporates Plexiglass windows, allowing visual observations of conditions inside the chamber. Centered in this chamber is a target-ladder holding four 2.8 cm-square target frames one above the other. An hydraulic system permits the ladder to be moved up or down from a remote panel in order to change targets rapidly. Each target is automatically centered vertically in the beam with this device. Two scintillation detectors, each with an associated multiplier-phototube, are secured onto arms pivoted about the center of the chamber. These too may be moved from the remote-control panel. The angular position of each may be accurately determined by means of a potentiometer whose center-tap position is caused to vary by rotation of the arm. A potentiometer at the remote-control panel forms the other half of a bridge circuit, and allows one, by a null adjustment, to determine the setting of the corresponding potentiometer attached to the detector arm. This setting then indicates the angular position of the detector. All observations of elastic and inelastic scattering were made in this chamber.

Diametrically across the scattering chamber from the beam entrance port is an exit port. The chamber has been aligned to ensure that the beam, when properly focussed, enters the scattering chamber through the center of the entrance port, passes through the center of a target in the target-ladder, and egresses through the center of the exit port. An additional port, the monitoring port, views the target at an angle of 25° above the exit port. A Mylar window over the monitoring port seals the scattering chamber vacuum-system from the atmosphere, and a scintillation detector (referred to as the monitor detector) can be affixed to the outside
of this port to observe the flux of particles scattered at this angle.

For the (p,n)-reaction bombardments of the present work a 33 cm-long pipe terminating in a small (22 cm-diameter) scattering chamber was attached to the exit port. For some studies a lead aperture was inserted into this pipe at the opening to the small chamber. The top of this small brass cylindrical chamber is a removable Plexiglass disc which seals the chamber from the atmosphere. A brass rod passes vertically through the center of this insulating lid and supports a Faraday cup in the center of the chamber. The construction of the support rod and the position of the hole in the chamber lid fixed the cup in the center of the beam. Opening into the side of the small scattering chamber at an angle of 90° with the beam direction was an 0.9m brass tube, closed at the remote end with an aluminum vacuum-seal. Affixed to the outside of this seal was a scintillation detector, the monitor detector, which observed particles scattered at a mean angle of 90° from a target in the center of the small chamber. As with the detector affixed to the monitoring port of the large chamber, this arrangement served to monitor the current passing through a target.

The Faraday cup used in the present work was constructed by closing one end of a 10 cm-long square brass tube, 2.54 by 2.54 cm in cross sectional area, with a plate of lead backed by brass. When the cup was positioned in the small scattering chamber, the cup axis being perpendicular to the support rod, the beam entered the open end and was stopped

* The author is indebted to Mr. Frank Fay of the cyclotron staff for the construction of this cup and the associated apparatus.
in the lead end-plate. Individual targets used for (p,n)-reaction measurements were held in the center of the Faraday cup at an angle of 45° to the incident beam and cup axis. The cup completely surrounded the target, with the exception of a 2.54 cm-diameter hole cut in the side of the cup. This opening allowed protons scattered by the target through a mean angle of 90° to escape from the cup and to be observed by the monitor detector. Because of the thinness of the foils, only a negligible fraction (less than 0.1%) of the beam was scattered out of the opening; the remainder was stopped in the walls of the Faraday cup. The Faraday cup, through the support rod, was connected by means of a coaxial cable to a current integrator (see Appendix IX) outside the scattering chamber vault.

The vacuum box of the cyclotron is evacuated by a diffusion pump located in a pit below the cyclotron, and the large scattering chamber is similarly evacuated by a diffusion pump connected directly into it. These pumps are kept in constant operation barring repairs of some contiguous portion of the cyclotron apparatus. In performing the experiments described in the present paper, valves in the cyclotron target-box, the cyclotron exit-tube, and adjacent to the entrance and exit ports of the large scattering chamber were opened, leaving the entire system open to essentially the same vacuum and allowing the particles to pass unimpeded from ion source to Faraday cup.
APPENDIX III

CYCLOTRON ENERGY DETERMINATION

A method of measuring the energy of the incident proton beam was developed as part of the present work, based on the comparison of the size of pulses produced in a scintillation detector by protons elastically scattered from carbon and protons inelastically scattered from the first excited state of C\textsuperscript{12} (Wal-53). The former lose only the energy of nuclear recoil, whereas the inelastically scattered protons give an additional 4.43 Mev to the nuclei. From classical kinematics, the energies of these two groups of particles as a function of scattering angle $\vartheta$ and incident beam energy $E_1$ are:

**Elastic:**  
\[ E = 0.851 \frac{E_i}{1 + 7 \times 10^4 (\cos^2 \vartheta - \sin^2 \vartheta) + 0.168 \cos \vartheta \sqrt{1 - 7 \times 10^4 \sin^2 \vartheta}} \]  
(III-1)

**Inelastic:**  
\[ E^* = 0.851 \frac{E_i}{1 + 7 \times 10^4 (\cos^2 \vartheta - \sin^2 \vartheta) + 0.168 \cos \vartheta \sqrt{1 - 7 \times 10^4 \sin^2 \vartheta - \frac{4.80}{E_i}}} \]  
(III-2)

By measuring the ratio of these scattered proton energies, $E/E^*$, one can determine $E_1$. The ratio is quite sensitive to $E_1$, while only slightly affected by $\vartheta$. Measuring this ratio rather than measurements of $E$ and $E^*$ separately eliminates calibration of the detector response. Since protons of different energies lose energy in proportionally different amounts, it was necessary to correct these two groups to account for the finite thickness of the target. The complicated nature of these equations makes it impossible to obtain a simple expression for $E_1$ in terms of the observed ratio.
$E/E^*$, a fortiori when corrections are made for target thickness, so a
series of graphs was plotted giving $E_1$ as a function of $(E/E^*)$ for vari-
os values of $\vartheta$ and target thickness.

In calculating the graph, the following procedure was used: an
integral value of $E_1$ was chosen. The change in energy suffered in passing
through half the thickness, $t$, of the foil was calculated and subtracted
from $E_1$. This new energy, $E_1'$, was used to calculate the energies of
elastic and inelastic scattering at $\vartheta = 30^\circ$ and $45^\circ$. Then the energy
loss suffered by each group of particles in passing through a target of
thickness $\frac{t}{2 \cos \vartheta}$ was subtracted from the respective energies. Finally,
the ratio, $R$, of these two corrected energies was found, and this was
plotted against $E_1$. Curves for various conditions are shown in Figs. 4
and 5. Values for energy degradation were obtained from Sternheimer
(Ste-60).

A variation of 0.5% in the determined value of $E_1$ will be caused by
any of the following:

a) a 10% variation in target thickness
b) a 1% variation in the observed pulse ratio $E/E^*$
c) a 10% variation in angle
d) 0.2 mg/cm$^2$ of absorber in front of the detector

A target of 1.25 mg/cm$^2$ polyethylene was used in the present work,
and the scattered protons were observed with a NaI(Tl) scintillation
detector. The output pulses from the detector were amplified and observed
with a twenty-channel pulse-height analyzer. The spectra on the analyzer
showed three peaks: two groups of protons from carbon, and protons
Figure 4

PROTON SCATTERING FROM POLYETHYLENE

Detector at 45° Various thicknesses

125 mg/cm² with 188 mg/cm² Al
absorber over detector
150 mg/cm²
100 mg/cm²
zero mg/cm²

Ratio of Pulse Heights: Elastic to Inelastic Protons from C12
Figure 5

Proton scattering from polyethylene

Detector at 30°
1.21 mg/cm² polyethylene

1.88 mg/cm² Al absorber over detector
No absorber

Ratio of pulse heights: elastic to inelastic protons from C12

Incident Proton Energy (MeV)

3.20 3.40 3.60 3.80 4.00 4.20 4.40 4.60 4.80 5.00 5.20 5.40 5.60 5.80 6.00 6.20 6.40 6.60 6.80 7.00 7.20 7.40 7.60 7.80 8.00
elastically scattered by hydrogen. Resolution, at best, was estimated to be 7% on the most energetic peak. The twenty-channel analyzer was used to determine the height of the peaks. Following the run, a pulser was used to calibrate the twenty-channel analyzer. Essentially then, two readings of the pulser dial were the observed data, and the ratio of these was used to determine $E_i$ from the suitable graph.

The following results were obtained:

October 31 : 7.57 Mev
November 14 : 7.53 Mev
            : 7.50 Mev
            : 7.46 Mev
December 29 : 7.59 Mev
            : 7.77 Mev

(Erroneous results on December 24, 1959; May 27; June 7, 12, 13, 1960)

The value selected from these was 7.55 Mev. The energy of the beam may not remain constant, since from time to time changes are made in the cyclotron itself, in addition to unavoidable expansion and contraction of the cyclotron components during the course of operation. There are several obvious sources of error in the above results, however, which may prevent the data from reflecting true energy fluctuation.

First, it is important that the pulse ratio be accurately measured. There is an uncertainty of 0.5% in calibrating the twenty-channel analyzer. The channels are of unequal width, varying at times by as much as 50%, making it difficult to determine precisely the position of the center of the peak. The maximum error this could make is about 6% in the pulse
ratio. Care must be taken that the pulser is properly linear. On two
dates giving "erroneous results" the pulser was malfunctioning.

Secondly, air or vapor in the evacuated scattering chamber will
cause decomposition of the detector crystal. The crystal was not covered,
and air was let into the scattering chamber inadvertently on several
occasions. As a result, a layer of absorber of undeterminable thickness
formed on the face of the crystal. On the last dates, the erroneous
results are attributed to the presence of about 1 mg/cm² absorber over
the crystal.

Errors in measuring the detector angle are negligible. The target
showed a tendency to wrinkle with continued use.

A more satisfactory method of measuring beam energy had previously
been developed (Wal-53) using the same principle, but with deuterons
instead of protons as incident particles. In this method the protons
from the (d,p) reaction are observed by placing a deuteron absorber in
front of the crystal. The comparison of pulse heights of protons from
the ground state and the 4.43 Mev level of C⁰ are compared with suitable
curves*, and the deuteron energy deduced. The energy of protons in the
cyclotron is just half the deuteron energy. A determination in June, 1960
using this method gave a proton energy of 7.55 Mev.

The primary advantage of the measurement using deuterons instead of
protons is the greatly decreased sensitivity of the results to the condition
of the detector. The use of an absorber directly in front of the detector
to stop the deuterons makes any film on the crystal negligible by comparison.

* The author wishes to thank Mr. Lawrence Bowen for the use of his
computations and assistance with the measurement.
APPENDIX IV

BOMBARDMENTS: ANGULAR DISTRIBUTIONS

The procedure used in obtaining angular distributions was in essence the method used previously by Waldorf (Wal-56), with one obvious difference. Whereas Waldorf observed scattering from targets which were about 2.6 cm square, this work used vertical strips, approximately 0.26 cm wide by 2.6 cm high. The narrower targets were used to improve resolution and angular definition.

The Apparatus

Two detectors were used for this work, one fixed in angle at 25° above the beam direction, designated the monitor detector, and one movable detector, designated the p-s detector. The monitor detector was affixed to the monitoring port of the 64-cm scattering chamber about 34 cm from the target. A Mylar window on the port in front of the detector sealed the chamber from the atmosphere while allowing particles to emerge into the detector. Lead apertures of various sizes were used over the face of the detector to define the solid angle subtended with respect to the target. The monitor detector was a CsI(Tl) scintillation crystal optically coupled to a multiplier phototube affixed to a rotatable arm inside the scattering chamber, about 18 cm from the target. It subtended a solid angle of about 9.4 x 10⁻⁵ steradians.

The output pulses from the detectors were amplified by preamplifiers inside the scattering chamber vault, and were further amplified and shaped by two Franklin amplifiers outside the vault in the working area. These
amplifiers produce a delay-line double-differentiated shaped-output pulse, proportional in height to the input pulse-height for normal, i.e., sufficiently small, input pulses. They also incorporate an optional discriminator circuit which provides a standard output pulse whenever the input pulse satisfies pre-selected height requirements. It can act alternatively as an integral or differential discriminator.

The linearly amplified pulses from the p-s counter were fed into a twenty-channel pulse-height analyzer, which displayed the pulse-height spectrum using mechanical registers. The monitor-counter amplifier was used as an integral discriminator, selecting pulses corresponding to elastically scattered protons. The discriminator output went to a monitor scaler, operating in conjunction with the twenty-channel analyzer, which gated the analyzer off when a pre-selected number of pulses had been recorded. Since the number of protons elastically scattered into the fixed monitor counter is directly proportional to the flux of particles which pass through the target, the analyzer records the spectrum of particles at some angle $\Theta$ for a preset amount of beam.

Determination of Zero Angle

The angle of the movable p-s detector is measured using a potentiometer arrangement (see Appendix II) which allows relative angles to be read with a precision of about $0.2^\circ$. The $360^\circ$ of rotation is measured in terms of 1000 Helipot units, as the potentiometer dial-units are designated. To determine the correct dial reading corresponding to $0^\circ$, use was made of the fact that the scattering of protons is symmetrical about
$0^\circ$ (the beam direction). A gold target was used, and the analyzer was gated off after $2^{18}$ counts were recorded by the monitor scaler. The spectrum of counts in the analyzer registers was recorded; the channels corresponding to the elastic scattering were determined by inspection; and the counts in these channels were summed to obtain the number of elastically-scattered protons. A series of these observations was made, varying the angle of the detector in increments of two Helipot units ($0.7^\circ$) from 370 to 380 units. Then an absorber was rotated over the face of the detector (using a control on the remote panel) and the counter was swung around through the beam, to the opposite side of the scattering chamber. Here observations were again made in increments of two units, from 628 to 638 Helipot units. No adjustment of the cyclotron nor of the electronics was made during this time. By plotting the elastic counts of the analyzer as a function of Helipot-dial reading, the counting rate was found to be symmetrical about a dial reading of 504.4 ± 1.0 Helipot units, which was then taken as zero angle. The entire procedure was accomplished within 20 minutes.

**Bombardment Procedure**

The studies of elastic scattering on copper were made in May, 1960, with some additional work in July, 1960. Two sets of targets were used for the earlier work: a pair consisting of 0.25 mil copper and 0.125 mil gold, and a pair consisting of 0.05 mil copper and 0.02 mil gold.

The studies on vanadium were conducted in July, 1960, using the 0.5 mil stock used for the inelastic scattering and the $(p,n)$ reaction cross-section. The 0.125 mil gold strip was used with this.
The estimates of uncertainty in the present work were made in the following way. At each angle, the analyzer spectrum was obtained from first the gold target, then the copper (or vanadium) target, and then a repeat of the gold target. The difference in the two gold bombardment counts, expressed in per cent, gave an estimate of the precision of the determination. The rms value of this error for all angles was about 3% for vanadium and about 4% for copper.

Several experimental difficulties occurred during this work. The resolution of the elastic peak became so poor for the 0.05 mil copper target at large angles that it was not possible to go beyond 140°. It was found that the counting rates showed considerable variation with time, presumably because of changing conditions of the cyclotron which cannot be controlled. On occasions, a succession of counts differed by more than 10%. This could not be attributed to counting statistics. These counts represent the number of elastic protons detected in the p-s counter for a predetermined number of similar protons detected in the monitor counter. This count could change if there were a significant change in beam energy (5% change to give a 10% count change) in the position of the beam spot and hence beam-target-detector angle, or if there were a change in the characteristics of the monitor detector. A change in the characteristics of the p-s counter would be compensated for when evaluating the counts. From observing the changes which occurred in the p-s counter gain, it may not be unreasonable to assume that changes similarly affected the monitor set-up.
The effect of changing the target angle, keeping the detector fixed, was found to be negligible over considerable range of angles.

It was observed that when the counters and target remained fixed in one place for some time, that the null-point on the angle-measuring bridge might drift by one Helipot unit. The effect of backlash in the gears is unknown, but an attempt was made to approach a setting of the detector angle from one direction every time. Often the detector overshot the desired angle, requiring some jockeying of the detector. The inaccuracy of this is small, since repeatability of points is in most cases excellent.

It was difficult to compare directly the 0.25 mil and the 0.05 mil copper targets, since the difference in thickness made it necessary to run the thin target a factor of 5 longer, and stability problems made this undesirable. For this reason two gold targets as well as two copper targets were used. A brief direct comparison of the two copper targets showed a definite difference in the scattering, confirming the results of the gold-copper comparison.
TABLE VIII

\( f_x(\theta) \) RATIO OF ELASTIC SCATTERING TO RUTHERFORD SCATTERING

<table>
<thead>
<tr>
<th>( \Theta )</th>
<th>Vanadium</th>
<th>0.25 mil Copper</th>
<th>0.05 mil Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.1</td>
<td>0.93</td>
<td>49.5</td>
<td>0.90</td>
</tr>
<tr>
<td>16.7</td>
<td>0.85</td>
<td>53.1</td>
<td>0.92</td>
</tr>
<tr>
<td>20.4</td>
<td>0.82</td>
<td>56.7</td>
<td>0.77</td>
</tr>
<tr>
<td>27.7</td>
<td>0.82</td>
<td>60.3</td>
<td>0.63</td>
</tr>
<tr>
<td>35.0</td>
<td>0.72</td>
<td>64.0</td>
<td>0.58</td>
</tr>
<tr>
<td>42.3</td>
<td>1.07</td>
<td>66.6</td>
<td>0.58</td>
</tr>
<tr>
<td>49.6</td>
<td>1.08</td>
<td>71.2</td>
<td>0.48</td>
</tr>
<tr>
<td>56.9</td>
<td>0.96</td>
<td>74.8</td>
<td>0.49</td>
</tr>
<tr>
<td>64.2</td>
<td>0.74</td>
<td>78.4</td>
<td>0.52</td>
</tr>
<tr>
<td>71.4</td>
<td>0.52</td>
<td>82.3</td>
<td>0.50</td>
</tr>
<tr>
<td>78.7</td>
<td>0.34</td>
<td>85.7</td>
<td>0.61</td>
</tr>
<tr>
<td>85.9</td>
<td>0.35</td>
<td>89.3</td>
<td>0.70</td>
</tr>
<tr>
<td>93.1</td>
<td>0.52</td>
<td>91.8</td>
<td>0.79</td>
</tr>
<tr>
<td>100.3</td>
<td>0.65</td>
<td>92.9</td>
<td>0.81</td>
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<td>107.5</td>
<td>1.08</td>
<td>96.5</td>
<td>0.88</td>
</tr>
<tr>
<td>114.6</td>
<td>1.43</td>
<td>100.1</td>
<td>0.99</td>
</tr>
<tr>
<td>121.8</td>
<td>1.38</td>
<td>103.6</td>
<td>1.08</td>
</tr>
<tr>
<td>128.9</td>
<td>1.29</td>
<td>107.2</td>
<td>1.10</td>
</tr>
<tr>
<td>136.0</td>
<td>1.29</td>
<td>110.8</td>
<td>1.09</td>
</tr>
<tr>
<td>143.1</td>
<td>1.15</td>
<td>114.4</td>
<td>1.16</td>
</tr>
<tr>
<td>150.7</td>
<td>0.87</td>
<td>121.5</td>
<td>1.11</td>
</tr>
</tbody>
</table>


APPENDIX V

BOMBARDMENTS: INELASTIC SCATTERING

The total cross-section, $\sigma_{n}$, for \([p,p'] + (p,c)\) processes, called "inelastic" processes here, may be written in terms of the experimentally determined quantities as:

$$\sigma_{n} = \left(\frac{Ze^{2}}{4E}\right)^{2}\int\left(\frac{C_{i}}{C_{e}}\right) f_{x}(\Theta) \sin^{-4}(\Theta/2) \, d\Omega$$

(V-1)

where the quantities in the first bracket come from the Rutherford scattering formula (c.f. Eva-55 p.643), $C_{i}$ and $C_{e}$ are the net fluxes into the solid angle $d\Omega$ of inelastically-scattered and elastically-scattered particles respectively, and $f_{x}(\Theta)$ is the non-Rutherford factor at the angle $\Theta$.

To observe the scattering, the output from the $p-s$ counter as it observed the desired target at an angle $\Theta$ was sent to a 256-channel pulse-height analyzer in the M. I. T. Nuclear Data Center. This analyzer produced a printed record of the pulse-height distribution from the detector at each angular setting. The pulses corresponding to the elastically scattered protons fell into the higher channels, allowing more than 100 channels to be used to record the spectrum of smaller pulses, corresponding to scattered protons which had undergone various inelastic interactions with the target nuclei as well as background pulses. The background pulses were due to the presence of electrons generated in the target from the break-up of the $H^{2+}$ molecule-ions and to background gamma radiation. Correction (amounting to less than 2%) was made for this background at each angle studied by inserting an aluminum absorber, about 250 mg/cm$^{2}$, between the target and the detector and recording the pulse-height spectrum under these conditions. The two spectra (inelastic scattering and background) were plotted on semi-logarithmic coordinate graph-paper. The background spectrum, when multiplied by a suitable
factor to account for the difference in duration of the two spectra observations, was found to fit the lowest-energy portion of the inelastic particle spectra within experimental error.

The counts in the channels corresponding to elastic scattering were summed to give the elastic scattering flux \( C_e \) at the angle \( \Theta \). The counts in the channels below the elastic-proton peak, down to the channels where the pulses were due essentially to pure background, were summed to give the gross inelastic flux. The background counts in these channels were summed, multiplied by the proper factor as determined graphically, and subtracted from the inelastic sum to give the net flux of inelastic particles, \( C_i \). The background for the elastic peak was observed to be negligible.

The 10% correction for detector aperture-scattering was taken from the lowest angular observation for each target. At a later date a direct observation of the incident beam was made, by lowering the beam intensity and rotating the detector directly into the incident beam. The inelastic counts under these conditions were about 6% of the elastic counts. Some data at 18° and 36° was obtained with the proton-counter, which gave an aperture scattering of about 8% for that detector.

The small angle inelastic counts are attributed to aperture scattering rather than to the presence of low-energy particle contamination of the incident beam for two reasons. The studies of the Faraday cup operation (see Appendix VIII) with different targets and a variety of operating conditions set an upper limit of about 1% on the amount of such contamination. Secondly, particles observed at small angles from copper, vanadium, and gold do not give the pulse-height distributions expected from such contamination: the Rutherford cross-section increases as the inverse square of the projectile energy, giving predominance to the lower energy particles, while the observed distribution of particles was uniform in energy within an estimated 1%. 
The results of inelastic-proton scattering measurements conducted at the University of Rochester (Sw-59) on cobalt, chromium, and titanium at energies of from 3.5 to 6.5 Mev indicate that the scattering from these elements can be considered as isotropic. With this assumption, the average value for \( k \) (Eqn. 10) over the various angles was used in determining the total cross section in order to average out random errors in angle determination and spectrum-summing techniques.

The targets used for the study of inelastic scattering were the same as those used for elastic scattering observations. The structure of the inelastic scattering, appearing as peaks in the pulse-height distribution, remained constant in position with respect to the elastic scattering peak as the observation angle was varied (except at small angles, where aperture scattering predominated) and this, along with correlation of the observed peaks with known vanadium (copper) excitation levels, (Maz-57) (Maz-58) indicated the scattering was essentially all from vanadium (copper).

The pulse-height spectra showed deuteron contaminants in the proton-beam were negligible.
APPENDIX VI

BOMBARDMENTS: (p,n) REACTIONS

Copper

The five bombardments of copper which were used for the cross-section determination were conducted in May and June of 1960. Previous series in March, 1959; April, 1959; May, 1959; July, 1959; and December, 1959 were not used due to uncertainties in the accuracy of the current-monitoring method used for these runs (see Appendix VII). An additional run in April, 1960 gave the same value when proper corrections were made.

In monitoring the beam during the copper runs, two current integrators were used (see Appendix IX). The first and last runs were made with the integrator of H. A. Enge; the other three were monitored with the El Dorado integrator. The former was calibrated during the course of this work, and observations of elastic scattering from the copper targets using each device verified that any difference was less than experimental repeatability.

The copper foils were cut in approximately 2.54 cm-square targets from stock supplied by the A. D. Mackay Company. Four were from 0.25 mil thick stock and one from 0.1 mil stock. All were weighed and measured, in order to compensate for possible non-uniformity. The four targets from the same stock showed a standard deviation of 1.5% from the average value. The purpose of bombarding the thinner foil was to observe any possible large dependence of the cross section on either beam energy or beam
energy-spread. Results of bombardments in July, 1959, using multiple
thicknesses of foil, showed that the cross section was not greatly sensi-
tive to these variables. A thorough investigation was not conducted, since
other workers have reported similar lack of sensitivity (How-58). The
lack of difference between the thick foil and the thin foil bombardments
further shows that loss of activity due to atoms recoiling out of the
target is negligible. Such an effect would be expected to decrease the
activity by less than 1% using the Bohr theory of heavy-ion stopping
(Boh-48) (Val-59).

Targets were dissolved in 2.0 ml of chromic acid and compared with
Zn$^{65}$ standard sources using a well-crystal scintillation counter. These
standards consisted of seven separate aliquots (see Appendix XI) from a
stock solution which had been calibrated by coincidence-counting (see
Appendix X). The targets and the seven standards were counted on four
occasions in July, 1959, and the counting rate from each target compared
with the average counting rate of the standard. Further comparisons with
single-standard vials and sources from the N. B. S. in May and June, 1960,
corroborated these results. The imprecision in counting (standard error)
is less than 2% on each target.

Checks were made during earlier bombardments to verify that contami-
nating activity was not present with the targets. Counting regularly over
several weeks, with further checks made after several months, failed to
show any evidence of contamination. Varying the geometry and discrim-
inator settings made no observable difference. The use of a single-
channel analyzer to scan the $\gamma$-ray pulse-height spectrum showed no
difference between target and standard activity.
The cross section was calculated using the following formula:

\[
\sigma(p,n) = \frac{R \cdot T_{1/2} \cdot A \cdot \cos 45^\circ}{Q \cdot \ln 2 \cdot N \cdot s \cdot b}
\]  
(VI-1)

where:

- \( R \) = the activity, in dis/sec, of the target at the end of bombardment.
- \( T_{1/2} \) = the half-life, in seconds, of the activity.
- \( A \) = the atomic weight, in mg/mole, of the target element.
- \( N \) = Avogadro's (Lochschmit's) number = \( 6.023 \times 10^{23} \) atoms/mole.
- \( s \) = the target thickness times density, in mg/cm\(^2\).
- \( b \) = the isotopic abundance of the desired nuclide.
- \( Q \) = the number of protons collected during the bombardment, or \( 6.281 \times 10^{12} \times q \), where \( q \) is the number of microcoulombs collected.
- \( 45^\circ \) = the angle of the target with the incident beam.

For the Cu\(^{65}\) (p,n)Zn\(^{65}\) reaction the following values were used:

- \( A = 63.5 \times 10^3 \) mg/mole
- \( b = 30.9\% \)
- \( T_{1/2} = 2.117 \times 10^7 \) sec.

which gave the following formula:

\[
\sigma(p,n) = 1.173 \times 10^3 \text{ R/sq}
\]  
(VI-2)

The results of the bombardments are tabulated in Table V.
Vanadium

Four bombardments on vanadium were made using 0.5 mil foil obtained from the A. D. Mackay Company. Target foils were cut to be approximately 2.6 cm by 1.9 cm, and were weighed and measured prior to mounting in target-holding frames. The results gave an average target thickness of 7.46 mg/cm² with a standard error of 1%.

For the three bombardments in June the Enge Integrator was used. The integrator readings for the May bombardment were erroneous, due to the neglect of the guard-ring potential, and the current for this run was calculated by comparing the scattering data recorded during this run with the scattering data recorded during the other three monitored runs. These three gave a value of $259 \times 10^3$ elastic protons per microcoulomb, with a standard error of less than 1%.

The targets were dissolved in 2.0 ml of dilute nitric acid and potassium bromate in glass vials, and the activity was compared with the activity of standard $\text{Cr}^{51}$ sources using a well-crystal scintillation counter. These sources consisted of three separate aliquots (see Appendix XI) from a stock solution which was calibrated using the coincidence-counting method (see Appendix X). The targets and three standards were counted during July, 1960, and the counting rate from each target was compared with the median counting rate of the standards.

Varying the discriminator settings and the counting geometry did not change the counting rate ratios, and this, along with observations spaced in time over one half-life, indicated that any radioactive contamination was less than 1%. The counting rates of standards and targets differed by
only about 10%, so that rate dependence of the counting was negligible.

The following values were used in determining the cross section for the V$^{51}$(p,n)Cr$^{51}$ reaction:

\[
A = 50.95 \times 10^3 \text{ mg/mole}
\]
\[
b = 99.75\%
\]
\[
T_{\frac{1}{2}} = 2.398 \times 10^6 \text{ sec.}
\]

giving the following formula:

\[
\sigma(p,n) = 33.03 \text{ R/sq}
\]

The results of the bombardments are tabulated in Table V.
APPENDIX VII

CURRENT MONITORING: SCATTERING METHOD

The determination of an absolute cross section divides into a
determination of S, Q, and N (see Equation 23). During the early
stages of this work, an attempt was made to measure the combination
(S x Q) directly by observing the flux φ of protons scattered elastically
at an angle θ from the target into a detector subtending a solid angle dΩ.
For a point target of atomic number, Z, this can be expressed, using the
Rutherford scattering formula as (c.f. Eva-55, p. 847):

\[
\phi = \left[ \left( \frac{Ze^2}{4E} \right) \sin^{-4}(\theta/2) \right] d\Omega \ S \ Q \ f(\theta)
\] (VII-1)

where e is the charge of the electron, E the energy of the incident protons,
and f(θ) is the ratio of elastic scattering to Rutherford scattering. The
factor f(θ) had been previously studied at the M. I. T. cyclotron (Wal-56)
for copper, and had been claimed to be known within 3%.

The first bombardments were conducted in the 64-cm scattering chamber
using the monitor detector, masked by a lead absorber with a 0.32-cm dia-
meter aperture to observe the scattered protons. The output from the
detector, after suitable amplification, was fed to the twenty-channel
analyzer which was set to observe the distribution of pulses corresponding
to elastically scattered protons. An additional channel of the analyzer
was used to observe all larger pulses, corresponding to deuterons or pile-
up pulses. These were usually less than 0.1% of the elastic count.
Previous work (Wal-56) indicated that copper scattering followed the Rutherford formula at this angle \( f(25^\circ) = 1 \).

A run on March 27, 1959 showed the necessity of considering the finite size of the target and of the detector aperture. Letting \( A = \) target height, \( r = \) aperture radius, \( R = \) target-detector distance, the solid angle \( d\Omega \) of Eqn. (VII-1) can be written (Day-56):

\[
d\Omega = \pi \left( \frac{r^2}{R} \right)^2 \left[ 1 + O \left( \frac{r^2}{R^2} \right) + O \left( \frac{A}{R^2} \right) \right]. \tag{VII-2}
\]

where \( O(x) \) stands for terms of the order of magnitude of \( x \). By making the distance, \( R \), sufficiently large, these corrections can be neglected.

To increase the distance, \( R \), a 92-cm brass extension tube, over 7.5 cm in diameter, was affixed to the monitoring port, and the detector was mounted at the end of this. Since the range of protons in air is about 75 cm, it was necessary to evacuate this scattering tube. This was done by sealing the end near the detector with a sheet of 1 mil brass, sealing the other end with a rubber 0-ring against the scattering chamber, and evacuating the tube with a mechanical vacuum pump. With this set-up, resolution of about 20% was observed.

The pulse spectrum observed on the twenty-channel analyzer consisted of an elastic scattering peak, confined usually to 5 or 6 channels, and a lower energy background. The spectrum was plotted on semi-logarithmic, coordinate graph paper, and the elastic peak was drawn from inspection. The counts in the channels of this peak were summed to obtain the elastic counts.
On April 8, 1959, two runs were made using this set-up and copper targets approximately 2.6 cm-square mounted in aluminum target frames. The second run was prematurely stopped when the cyclotron filament burned out, but the results obtained in the two runs differed by 50%. It was concluded that targets of smaller area should be used, both to define the scattering angle more accurately and to decrease the solid angle corrections.

On May 27 bombardments were made using horizontal copper strips 0.6 cm by 2.5 cm in the target frames. The three targets had a range of 7% in the value of the cross section determined from them, although the absolute value was too large (by 60% of the final thesis value).

Five runs on July 8 and 9 checked these results within a standard error of 9%. In addition, foils of double and triple thickness were bombarded on July 9, and gave no indication of a thickness effect.

The rather large absolute value for the cross section determined in the eight runs above prompted an investigation of the accuracy of the various constituent measurements. The monitoring quantity \((S \times Q)\) which is desired can be written, for a suitably small target size and aperture size, as:

\[
S \times Q = k \phi \frac{E^2}{R} \sin^4(\theta/2) f(\theta)^{-1} \tag{VII-3}
\]

where the constant, \(k\), contains all the numerical factors not measured directly. The fractional error, \(e\), in the measurement of factor \(x\) in
Eqn. (VII-3) contributes to the over-all fractional error, \( e \), as:

\[
e^2 = (e_\phi)^2 + (2e_\phi)^2 + (2e_r)^2 + (4\alpha e_\theta)^2 + (e_\gamma)^2
\]  

(VII-4)

On October 27, 1959 the scattering chamber was let open to the atmosphere, and measurements were made for \( \cos \Theta \) and \( R \) using a meter stick. The expected accuracy of \( \cos \Theta \) was 0.2\% and for \( R \) was 0.1\%. For \( \sin \frac{\Theta}{2} \) the accuracy was then 0.5\%. Measurements of the beam energy were made during December, 1959, and were expected to be accurate to 1\%. The aperture size was known to about one drill size, or 3\%. Assuming the energy remained within 1\% limits, the scattering was Rutherford, and the flux was accurate to 1\%, gave an over-all error in beam monitoring of about 8\%.

In an attempt to decrease the effect of target-size and scattering-angle uncertainty, a different monitoring set-up was tried in December, 1959. The small scattering chamber (see AppendixII) was affixed to the rear of the larger chamber in line with the beam. Around the perimeter of this cylindrical chamber are tapped exit holes, and into the hole at 90\(^\circ\) was affixed the aforementioned brass tube. The tube end near the detector was closed with an aluminum foil, and the other end was open to the cyclotron and scattering chamber vacuum-system. The targets, which were 2.6 cm-square foils as well as 1.3 cm-wide horizontal strips, were held in a ladder in the center of the chamber. The factor \( f(90^\circ) \) was taken to be 0.975 from earlier work by Waldorf (Wal-56).

The results of four bombardments on December 24 and December 29, 1959 showed a reproducibility of less than 5\% (standard error) although the absolute values for the cross section were essentially the same, and were considered somewhat large.
Following the evaluation of the December results, a current-integrator -- Faraday-cup arrangement was assembled to replace the monitoring method described above. A study of the angular distribution of protons elastically scattered from copper during May, 1960 showed that difficulties in obtaining an accurate and reliable value for the non-Rutherford factor, \( f(\theta) \), made a direct Faraday cup measurement of the current more accurate than the scattering method.

The angular distributions obtained in the present work from 0.25 mil copper differed markedly from the previously published values in the region of 90°, the desirable angle for monitoring. A 0.05 mil target appeared to give yet a third distribution, again most markedly different at 90°.

The scattering monitor was utilized in conjunction with the Faraday cup measurements of the charge, \( q \), during bombardments. The Faraday cup current reading was used in calculating the cross section, and at the same time it provided a determination of \( f(90^\circ) \) for both copper and vanadium with a precision of 2%. The following values were determined in this way, using a solid angle of \( 1.62 \times 10^{-5} \) steradians:

\[
\begin{align*}
  f(90^\circ) \text{ copper: } & \quad 0.69 \pm 0.01 \quad (\text{VII-5}) \\
  f(90^\circ) \text{ vanadium: } & \quad 0.43 \pm 0.01 \quad (\text{VII-6})
\end{align*}
\]

It is considered that the uncertainties in: (a) the angular and energy dependence of \( f(\theta) \); (b) the scattering angle, where an error in angle of 0.5% at 25° would cause an error of nearly 10% in the cross-section value; (c) determining the aperture area to better than 3%; and
(d) the beam energy, not only make it necessary to use a more direct measurement of the quantity \((S \times Q)\), but make it difficult to correct satisfactorily the data gathered on the runs described above.
APPENDIX VIII
CURRENT MONITORING: FARADAY CUP

To measure \( Q \), the number of protons reaching a target during bombardment, a Faraday cup was constructed. This was a square brass tube, 7.6 cm long, closed at one end with lead and brass, and open at the other to allow entry of particles. In the center of the tube, diagonally across it at an angle of 45°, a bracket for holding an aluminum target frame was affixed by means of machine screws. A target foil was first cemented to a frame, and then placed into the cup. A 2.54 cm-diameter hole in the side of the cup, opposite the target foil, allowed protons to be scattered at a mean angle of 90° and escape from the cup. The flux of protons escaping through the hole was calculated to be less than 0.1% of the incident flux for the thickest target used. Two identical cups were mounted vertically adjacent on the end of a brass rod which held them rigidly in the center of the small scattering chamber (see Appendix II). This rod, emerging through the plastic lid of the chamber, served as the electrical contact to the otherwise insulated cup. The dual cup arrangement allowed the set-up to be tested using a dummy gold target.

The particles scattered out of the Faraday cup at 90° were observed using the monitor detector described in Appendix VII. Measurement of the scattered flux \( \phi \) and the charge collected provided independent measurements of the beam current.

For the first measurements, the Faraday cup was connected to one terminal of a 1 microfarad polystyrene capacitor, the other terminal was grounded, and the voltage across the capacitor was read with a Kischly vacuum-tube voltmeter. For investigating various factors affecting the current to the faraday cup the twenty-channel analyzer, which observed the elastically scattered protons,
was manually gated on while the capacitor charged zero to +100 volts, then
was manually gated off and the counts were recorded. This gave the flux per
microcoulomb of charge.

The first investigation of this set-up was done on March 30 and 31, 1960.
A battery was connected to the cup to repel electrons, and the current
recorded at the cup was found to be a strong function of the potential on
the cup. A potential of at least -300 volts was needed to eliminate electron
current effects. It was also observed that identical results, within
experimental error (5 - 10%), were obtained with a different capacitor, with
a different NaI (Tl) detector, and with a CsI (Tl) detector, and with a
CsI (Tl) detector placed directly into the vacuum system. Using two apertures,
differing in cross section area by a factor of 9.5, the scattered flux was
found, within experimental error, to show the same factor. These results
showed that the counts recorded in the twenty-channel analyzer represented
the flux $\phi$. Because it was later found that several components of the
set-up were not functioning as intended on these days, quantitative conclusions
were not drawn from the data gathered. Considerable difficulty was experienced
in obtaining repeatability during these runs.

Guard electrodes at negative potentials with respect to ground around
the Faraday cup were installed to eliminate spurious electron currents either
into or out of the Faraday cup, and eliminate the need of having the cup
itself at a large negative potential. Brass rings, with 2.54 cm diameter
holes, were placed 2.54 cm from the front and side openings for this purpose.
Grid wires were at first used over the front ring, but were found, using a
scintillating screen, to cast a shadow, and so they were removed.

The proton current cannot be reliably measured by capturing the hydrogen
molecule-ion beam in the Faraday cup. Direct observation of the beam
constitution showed a partial break-up of this beam, attributed to residual gas
in the vacuum and glancing collisions with apertures. It is therefore necessary to break the molecules into constituent protons in advance of the target. The proton fraction of the charge-exchange equilibrium predominates above 0.5 Mev (Al1-58). A 1.88 mg/cm² aluminum foil was placed over the slotted target-holder in the cyclotron target-box (see Appendix II). This placement insured that the electrons produced with the break-up would be swept aside by the magnetic field of the cyclotron.

The results on April 15, with a gold target and guard rings set at a potential of -300 volts with respect to ground, gave agreement between scattering and Faraday cup within 10%. A copper target was also observed, and the results indicated f (90°) was 0.7, with 10% repeatability, that the break-up foil could be inserted either in the target box or in the exit tube to the scattering chamber with the same effect. The latter position was used for subsequent work since it gave better operating conditions, and better beam focussing. Apertures and collimators inserted either in the large scattering chamber or in front of the cup made no difference.

To eliminate the undesirable fluctuations of Faraday-cup potential associated with the use of a simple capacitor-voltmeter current monitor, the integrator described in Appendix IX was obtained in early May, 1960.

The current recorded in the Faraday cup depends on the presence or absence of electrons in the beam, both electrons which come along with the protons after the accelerated ions have been split up, and electrons which are generated whenever the beam strikes anything. It is also affected by electrons lost by the target as secondary electrons which escape from the cup. It is in general otherwise unaffected by the beam position, beam shape, or beam energy. The recorded number of elastically scattered protons depends on the beam position and shape primarily to the extent that this changes the angle of scattering. This depends on the conditions of the cyclotron fields.
The scattering is also energy dependent. It is independent of the presence or absence of electrons, and is not affected by the various electrodes surrounding the Faraday cup, so long as the beam does not strike one. Previous work showed that, with the apertures in place in the scattering chamber, the beam was well centered on the target.

Application of a negative potential to a guard ring by the scattering hole of the Faraday cup was found to decrease the positive current to the cup, as measured by the scattered counts. This effect, 10%, was attributed to secondary emission of electrons from the target, which emerge from the hole in the cup. A potential of -500 volts was sufficient to inhibit their emergence.

Two parallel plates forming a capacitor in front of the cup were used briefly to ensure that energetic electrons did not reach the cup. It was found that placing one or both of the two electrodes at ground or positive potential gave an increase in positive current into the cup, (determined by comparing the scattering and the Faraday-cup current). When the two plates were isolated from ground, and a potential of 600 volts applied across them, a similar increase in positive current into the cup was observed. The magnitude of this effect was 40%. At any particular voltage arrangement, good repeatability was achieved, even when the cyclotron variables were slightly modified. This shows that electrons out of the target were responsible for this effect, and that the application of a negative potential surrounding the Faraday cup is necessary. The difference between 300, 600, and 900 volts potential was less than the statistical variation, showing that any voltage above 500 volts is satisfactory.

With the application of -900 volts to all the electrodes, the counts recorded in the detector were: (gold target):

Average Value Experimentally: \(1.70 \times 10^3\) counts/microcoulomb \(\text{(VIII-1)}\)
The result expected in this geometry for 7.5 Mev protons was:

\[ \text{Theoretical Value: } 1.71 \times 10^3 \text{ counts/microcoulomb} \]  

The agreement was felt to be well within the experimental uncertainty, and was repeatable within about 2%. The principle source of error comes from a possible 3% non-uniformity of the gold target thickness, determined by weighing several samples.

Since the Faraday cup readings were indicative of the true proton current with this set-up, several cyclotron parameters were varied to see the effect on the counting rate of scattering. The insertion of apertures in the scattering chamber about 100 cm. in front of the Faraday cup cut the intensity by a factor of better than 20, but did not change the scattering-monitor counting, within statistical limits. Detuning the focussing magnets caused no observable change.

These results showed that the foil over the slits broke up the hydrogen molecule-ions, and the electrons were swept out of the proton beam by magnetic fields so that only pure protons arrived at the Faraday cup.
APPENDIX IX

CURRENT MONITORING: THE CURRENT INTEGRATOR

The device used to measure the charge collected in the Faraday cup was, for most of this work, a circuit designed and used by H. Enge* of the ONR-Rockefeller Generator of M. I. T. for similar measurements of their particle currents. The integration of target current with time is done by charging a 0.02 microfarad polystyrene capacitor with the target current; measuring the voltage across the capacitor gives the charge collected. In this circuit, the positive input current, by means of a positive feedback amplifier, drives the lower plate of the capacitor negative by the appropriate amount, so that the input terminal is held at nearly zero potential. When the capacitor is charged to -50 volts (one microcoulomb), a Schmitt trigger fires, operating a relay which discharges the capacitor. Simultaneously, a register records the collection of one microcoulomb. A correcting network compensates for the finite time required to discharge the capacitor, although in the present work this time was negligible.

Two adjustments can be made on the integrator: one potentiometer sets the level at which the Schmitt trigger fires, hence the number of microcoulombs per register, and another potentiometer

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*The author is indebted to Prof. Enge for the use of this integrator and a discussion of the operation of the circuit (unpublished).
sets the balance of the positive feed-back amplifier so that when there is no input current there will be no current in the output of the amplifier. When the latter is improperly set, a voltage appears on the grid of the input electrometer tube, causing output current. By placing a low or zero impedance across the input terminals, the effect of improper balance adjustment is magnified, so that adjustment can be made easily. In operation with the cyclotron, where the impedance is quite large with respect to the 0.1 megohm internal impedance of the integrator, any maladjustment becomes negligible. This was verified in operation with the cyclotron: no change in the current reading was detectable with a maximum change in balance adjustment. (For this study, the scattering from a gold target-foil was used as an independent measure of the beam current, as described in Appendix VIII.)

To calibrate the integrator, a test current was run into it and the time required to register 100 microcoulombs was determined using a stopwatch. The current was supplied by a 1.57 volt Burgess A battery in series with a 4.5 megohm wire-wound resistance and a 1.000 megohm General Radio Company decade resistance-box. This gave a current of 0.27 microamperes, comparable to the currents of 0.1 microamperes normally obtained during cyclotron bombardments. The calibrating current was accurately determined to 0.2% by measuring the voltage drop across the decade box using a Leeds and Northrup Type K potentiometer. This was essentially a bridge comparison of the decade voltage with the voltage of an Eppley Lab Inc. standard cell, using a Rubicon galvanometer as a
null indicator.

The calibration of the integrator was checked on five occasions, two of these prior to bombardments, with the cyclotron running. The results gave 0.981 microcoulombs per register, with a standard error of 0.6%.

For part of the work, an El Dorado Model C-110 current integrator was obtained for trial, from Measurements Equipment Company. The principle of this circuit is similar to the Enge design: the input voltage is held near zero, the input current charges a capacitor, and the voltage across the capacitor is read on a panel meter. When the indicator needle on this meter reaches a mechanically preset position, the capacitor is discharged and begins charging again. In operation, the device was allowed to recycle once, with 150 microcoulombs full scale deflection. Calibration of this device consisted of intercomparison with the Enge integrator and simple measuring devices. No effort was made to check the accuracy of the readings to better than 2%. Results with this integrator were identical with the other results within experimental error.

For some earlier work, a simple capacitor-voltmeter combination was used to integrate the current. Using a 1.0 microfarad polystyrene capacitor and a Kiethly vacuum-tube voltmeter, the capacitor was charged to a maximum of 100 volts during a bombardment, corresponding to 10 microcoulombs.
APPENDIX X

COINCIDENCE COUNTING

The Coincidence Method

The coincidence method (Put-50) (Mit-55) (Man-56a) utilizes the fact that, if two detectors observe the same source of activity, the counting rates, $R_i$, from each detector can be written:

$$R_i = e_i A + r_{ib}$$  \hspace{1cm} (X-1)

while the rate of coincidence counts, $R_c$, can be written:

$$R_c = e_x e_y A + r_{cb} + r_{cc}$$  \hspace{1cm} (X-2)

where $r_{ib}$ is the background counting rate, $r_{cc}$ is the rate of chance coincidences, $A$ is the rate at which the source emits detected radiation, and $e_i$ is an efficiency factor defined by Eqn (X-1). The $e_i$ describes the dependence of $R_i$ on the decay scheme, attenuation of the radiation before it reaches the detector, source-detector geometry, detector efficiency, and electronic pulse-height discrimination. For the pure electron-capture activities Cr$^{51}$ and Mn$^{54}$ (see Fig. 6), $A$ is the source disintegration rate. In the case of Zn$^{65}$, 1.7% of the disintegrations (Gle-59) are positron emissions which were not observed in the present work, and the source disintegration rate is $A/98.3$. Using Eqns. (X-1) and (X-2), $A$ can be determined from measured rates:

$$A = \frac{(R_1 - r_{ib}) (R_2 - r_{2b})}{(R_c - r_{cb} - r_{cc})}$$  \hspace{1cm} (X-3)
The electron-capture decay of either Cr\textsuperscript{51}, Mn\textsuperscript{54}, or Zn\textsuperscript{65} produces an atom in an excited state. This excitation energy may produce an x-ray. For some fixed fraction of the decays, the nucleus is left in an excited state, which may produce a gamma ray. Since every gamma ray is preceded by an electron-capture, Eqns. (X-1) and (X-2) may be applied, and the activity of the source can be determined from Eqn. (X-3) by measuring x-ray, gamma ray, and coincidence-counting rates.

The x-rays were detected in a cylindrical brass proportional counter,\textsuperscript{*} 23 cm-long by 9 cm-diameter, and housed in an electrostatic shield of steel and aluminum. A beryllium window 3 mil thick in the side of the chamber allows x-rays to enter. The counter was filled to one atmosphere with a mixture of 90\% argon - 10\% methane (Matheson Chemical Company P-10 gas) and sealed. A potential of about +3000 volts was applied to the 5-mil center wire with respect to the grounded shell.

This positive potential was supplied by a Model 15v30 high voltage supply, obtained from the Laboratory for Nuclear Science (LNS) instrument pool. The requirements for a stable voltage are quite stringent for counting of this sort, and several other types of regulated voltage supplies were tried with little success. The gas amplification depends logarithmically upon the potential applied between the center wire and shell (Cur-55), so that a change of 0.5\% in the potential with time will cause a change of nearly 12\% in the size of the output pulse from the amplifier.

The output of the counter goes through a cathode following into a Baird-Atomic Model 215 Linear Non-overloading Amplifier. The presence of background pulses more than 100 times the size of the x-ray pulses

\textsuperscript{*} The author is indebted to Professor M. Deutsch for the use of this counter and discussions regarding the proper operation of it.
made it necessary to use an amplifier with non-overloading properties. Such pulses were part of the natural background in the counter. The gain on the Baird amplifier was adjustable up to a maximum of 64, and pulses of 12 volts were obtainable corresponding to photoelectric absorption of the desired x-rays in the counting gas. The resolution was of the order of 10%, and noise due to the components of amplifier, cathode follower, and counter was of the order of 1 volt.

The gamma rays were detected in a 2 in. by 2 in. Na(Tl) scintillation crystal, placed a few centimeters from the beryllium window of the x-ray counter and extending outside the steel shield of the latter. A brass collar held the crystal in place, and was slotted to allow a sample to be inserted between the two detectors. A 3 in. by 3 in. crystal was briefly used to verify the independence of the results on the detector.

The crystal was optically coupled to a multiplier phototube with a cathode follower output, followed by a Model 501-M amplifier from the LNS instrument pool. The voltage on the multiplier phototube was supplied by a Technical Measurements Corporation Model HV4A high-voltage supply.

The output signals of the two amplifiers went to a Model 510 differential discriminator from the LNS instrument pool, modified to include two discriminators and a coincidence circuit.* The gamma-ray input went to an integral discriminator section which was set, using an oscilloscope and pulser, to select only those pulses larger than the Compton valley of the pulse-height distribution (pulses which corresponded to the complete absorption of the desired gamma rays in the crystal).

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* The author is indebted to Emil DeAgazio for the modifications of this circuit and discussions regarding the operation of the circuitry described here.
This eliminated the possibility that a gamma ray might be scattered into or out of the crystal, be counted, and, in addition, generate a pulse in the x-ray counter. The output of the x-ray amplifier went to a differential discriminator section, similarly set to respond only to pulses corresponding in height to photoelectric absorption of an x-ray in the counting gas. The two discriminators each give uniformly shaped pulses when given the proper input pulses, and these discriminator outputs went to two separate scale-of-256 binary scalers. The discriminator pulses also went to a coincidence-circuit section of the Model 510 unit which produces an output pulse whenever it receives pulses from the two channels within several microseconds of one another.

For the coincidence method to be valid, it is necessary that the two radiations, x-ray and gamma ray, which are simultaneously generated in the process of a decay (the finite lifetime of the excited atomic state which occurs following electron-capture is negligible) be detected within the resolving time of the apparatus. Both radiations travel to their respective detectors with the velocity of light. The gamma ray, in the detection process, produces first an excitation in the sodium-iodide crystal, which is partially transformed into light energy. The light produces electrons in the multiplier phototube which produce a current pulse. The pulse occurs within a fraction of a microsecond following the absorption of the gamma ray. On the other hand, the detection of the x-ray follows the creation of ion-electron pairs by the x-ray in an electrostatic field. The collection of the electrons at an electrode produces the current pulse. The initial velocity of these electrons will
be small for large distances from the electrode; hence, electrons produced at large distances will require more time to arrive at the electrode than electrons produced nearby. The result is that radiation attenuated in various parts of the detector will produce pulses delayed by varying amounts (up to a maximum of several microseconds under the experimental conditions used in the present work).

The initial pulses from the two detectors were shaped and lengthened in passing through the discriminator circuits. The pulses out of the gamma-ray discriminator were about three microseconds long; the pulses from the x-ray discriminator were over one microsecond long. In coincidence they produced a resolving time of 3-to 5-microseconds, which was considered long enough for all true coincidences to be observed simultaneously.

Since the differential discriminator (x-ray pulses) requires about one microsecond longer to process an input signal than the integral discriminator, a delay line was inserted into the circuit between the gamma-ray amplifier and the integral discriminator. To determine the optimum length for this delay, studies were made using various lengths of delay line varying from 0.4-to 4.0-microseconds, in increments of 0.2-to 0.4-microseconds. A value of one microsecond was selected as producing the maximum coincidence rate for a given source strength. Delay values near this appeared to produce the same coincidence rate, indicating that the selection was not critical.

The insertion of the delay line introduced some distortion in the signal from attenuation and reflections, even though an attempt was made to terminate the line with the optimum resistance. However, by placing the delay line in the circuit just preceding a discriminator, detrimental effects were eliminated.
The scalers used with this apparatus each had a discriminator. The output pulses from the Model 510 circuit corresponding to x-rays, gamma rays, and coincidences were of nearly uniform size. Occasionally there were smaller pulses from spurious sources or marginal inputs to the Model 510 circuit. Such pulses were infrequent, and the scaler discriminators were set to reject them. The coincidence scaler received the largest proportion of these non-uniform pulses, since they were caused when two background pulses occurred barely within the resolving time of the coincidence circuit. As was expected, all pulse-heights occurred with equal frequency for these pulses. The discriminators were each set, using a pulser and oscilloscope, to reject pulses below three-fourths the usual height.

Correction was made for the occurrence of two distinct events within such a short time interval (the resolving time τ) that they would be incorrectly counted as a coincidence. This resolving time depended on the settings of the various discriminators and was separately determined for each change in operating conditions. Random inputs were fed into the coincidence circuit using two independent sources. The observed rate of chance-coincidence pulses was used to correct the rate during normal counting by assuming that the rate of chance coincidences was proportional to both the x-ray and the gamma ray counting rates. The chance rates gave resolving times of 3-to 5-microseconds, in agreement with the coincidence circuit.

Although the efficiency of the counting gas in the x-ray counter for gamma rays is small, the interaction of the rays with the brass shell can
generate interfering copper and zinc x-rays. To correct for the background in the x-ray counter caused by gamma ray interactions of all types, an aluminum absorber was placed between the source and proportional detector, and the counting rate in the x-ray counter at this time gave the background. The optimum thickness of aluminum was determined by observing the x-ray rate and coincidence rate with various thicknesses of absorber. An absorber of 220 mg/cm$^2$ aluminum was used for all sources. Calculations showed that the attenuation of the gamma rays by this absorber was negligible, while the x-ray absorption was essentially complete. This correction was at most a 6% effect for Zn$^{65}$, and about 0.5% effect for Cr$^{51}$.

It was verified that the source position and detector geometry made no difference in the determined disintegration rate, after proper background corrections were made. The constancy of counting rates with time showed that the over-all set-up was capable of operating steadily over a period of several weeks.
APPENDIX XI

THE PREPARATION AND CALIBRATION OF STANDARDS

During the course of the present work, radioactive standards were prepared of Cr\(^{51}\), Mn \(^{54}\), and Zn \(^{65}\). The chemical preparation of these is described in Appendix XII. In each case the radioactivity, after being taken to dryness during the chemical processing to verify that inactive residue was satisfactorily small, was dissolved in around 1 ml of approximately 1 M HCl to provide a stock solution from which aliquots were removed for sources. From these stocks, 10-microliter aliquots were pipetted to prepare both coincidence-counting sources and sources for comparison with the cyclotron bombardment targets.

The sources for coincidence-counting were deposited in the center of a 0.4 mg/cm\(^2\) Rubber Hydrochloride film* which covered a 2.54 cm-diameter hole in a standard cardboard sample card. The micropipets were rinsed twice with 3 M HCl and the wash added in droplets surrounding the central active drop. An infra-red heat lamp was used to evaporate the solution, after which another Rubber Hydrochloride film was placed over the source. The films were heated under the heat lamp until the wrinkles inherent in them disappeared.

The sources for comparison with the targets were prepared at the same time as the coincidence sources. They were made by placing 10 microliters of stock solution, along with two micropipet rinses, into standard glass vials, and then adding 2 ml of the appropriate acid. All vials were coated inside with a Silicone compound, Beckman Dessicote 3473,

* From Originality House, 480 Avalon Avenue, Akron, Ohio.
and centrifuged before counting. Transferring one solution of Zn\textsuperscript{65} to three of the vials \textit{seriatim}, and measuring the activity in each case, indicated there was less than 0.5\% difference between the vials for gamma-ray counting purposes. Another investigation with Zn\textsuperscript{65}, done by diluting a source, indicated that a 10\% difference in solution volume made less than 0.5\% counting difference. The vial sources and coincidence sources were prepared alternately from the stock to compensate for any change in the stock solution. Although standards of various activities were prepared during the course of this work, only the sets actually used in target activity determination will be described here. They are mentioned in the order of preparation.

\begin{center}
Zn\textsuperscript{65}
\end{center}

The Zn\textsuperscript{65} standards used for the final activity determination were prepared on August 4, 1959, at which time three vials and three coincidence sources were prepared using the same 10-microliter micropipet for all sources. The coincidence sources were within 1\% of each other. The vial sources showed a range of 2\% difference, so that on August 10 and 11 additional vials were made up which bracketed the previous activity and gave a standard error in the precision of the median value of less than 0.5\%.

The coincidence source was counted from May 16 to May 20, 1960. The result, corrected for 1.5\% positron emission and for decay, was:

\begin{center}
This Lab source: \( 3.48 \times 10^3 \) dis/sec 1200 EST July 1, 1960 (XI-1)
\end{center}
The standard error in the precision of this result, including effects of backgrounds, chance coincidences, counting statistics, etc., is 1%.

For comparison purposes, two sources of Zn\textsuperscript{65} activity were obtained from the National Bureau of Standards. The first of these, obtained in December, 1959, was stated to be (Nat-59):

\textbf{N. B. S. source \#1:} \textit{485 x 10\textsuperscript{3} dis/sec/ml 0800 EST March 1, 1957 (XI-2)}

with a reported estimated accuracy of \( \pm 5\% \). From this solution two 200-microliter aliquots were removed and placed with two rinses each (6 M HCl and water) into sample vials. The micropipet was found, by weighing mercury, to be within 0.5\% of 200 microliters. Counting indicated less than 1\% difference in the activity of the two vials. To one was added 1.8 ml acid, and comparison was made with the Zn\textsuperscript{65} vials from this laboratory during December, 1959, and February, May, June, and July, 1960, giving a value of 1.17 (standard error of less than 1\%) for the ratio of this laboratory source to N. B. S. source \#1. From this ratio, the expected value of my source is:

\textbf{This Lab source, from N. B. S. \#1:} \textit{3.61 x 10\textsuperscript{3} dis/sec 1200 EST July 1, 1960 (XI-3)}

The second N. B. S. source, obtained in April, 1960, was stated (Gar-60) to contain \( 5.4214 \pm 0.0014 \) gms of solution consisting of ZnCl\textsubscript{2} in 6 M HCl (approximately 250 micrograms/ml zinc), with a total 1.114-Mev gamma-ray activity of:

\textit{194 x 10\textsuperscript{4} gamma rays/sec 1500 EST March 11, 1960 (XI-4)}
Calculating this to be a 21.2% HCl solution, with a density of 1.0905 gm/ml (ml=18) at 30°C (pipetting was done at 29.5°C, a correction of 0.02%) gives $3.90 \times 10^5$ gamma-rays/sec/ml. Assuming the decay is 48% gamma rays (Gar-60), (values for this differ by $\pm$ 6%) gives a value of $8.13 \times 10^4$ dis/sec/ml on March 11, 1960.

On June 23, 1960 the ampoule was opened and four 10-microliter aliquots were removed and pipetted with rinses into standard vials to which 2 ml of acid were then added. Several micropipets were used. The difference in the counting of the vials was less than that due to counting statistics (less than 1%). Comparing these with the sources of this laboratory gave a ratio of this laboratory source to N. B. S source #2 of 0.597, with a standard error of less than 1%. This gives:

This Lab source from N. B. E. #2: $3.43 \times 10^3$ dis/sec 1200 EST July 1, 1960 (XI-5)

The limiting uncertainty in this is anticipated to come from the uncertainty in the decay scheme. The values (XI-4) and (XI-5) are in satisfactory agreement with value (XI-1), the value used in the present work.

$\text{Mn}^{54}$

In order to provide a further check on the operation of the coincidence-counting apparatus, a Mn$^{54}$ standard was prepared for concurrent calibration by the N. B. S. On April 8, 1960, an aliquot of 800 microliters was taken from the stock solution of Mn$^{54}$, prepared as described in Appendix XII, and placed in a special glass ampoule for the N. B. S. The ampoule was filled to 3 ml, and the solution was estimated to be 1 M in HNO$_3$. At the same time, a 10-microliter aliquot coincidence-counting source was prepared.
The results of coincidence-counting this source on June 20 and 21, 1960 gave:

**This Lab source:** $3.27 \times 10^3$ dis/sec 1200 EST July 1, 1960 (XI-6)

with a standard error of about 2%.

The results of 4pi counting at the N. B. S. on the 800-microliter aliquot gave $3.08 \times 10^5$ dis/sec on April 14, which, when corrected for decay and aliquot fraction, predicts an activity of:

**This Lab source from N. B. S.:** $3.20 \times 10^3$ dis/sec 1200 EST July 1, 1960 (XI-7)

$\text{Cr}^{51}$

The $\text{Cr}^{51}$ standards used for the final activity determination were prepared on June 24, 1960. Two coincidence and three vial sources were prepared at this time, using the same technique described for Zn$^{65}$, with the micropipets used for the second N. B. S. source pipetting.

The sources were coincidence-counted on June 28, 29, and 30, 1960, giving an average value, after correcting for decay, of:

**This Lab source:** $1.20 \times 10^4$ dis/sec 1200 EST July 1, 1960 (XI-8)

with a standard error of 1% and a difference between the two sources of 1.4%.

The range of the three vial sources, attributed to preparative techniques, was 1.8%.

* The author is indebted to S. B. Garfinkel of the Radioactivity Section, National Bureau of Standards, for these results.
APPENDIX XII

RADIOCHEMICAL PREPARATIONS

Sources used for X-ray coincidence-counting should ideally be massless to avoid absorption of the X-ray in the source (Ras-56) (Put-50). For this reason, the activities used in the present work were prepared carrier-free, with a minimum amount of inactive non-volatile contaminants.
Carrier-free $\text{Cr}^{51}$ was prepared by bombarding a stack of vanadium-metal foils with 15-Mev deuterons at the Cyclotron on January 15 and March 23, 1960. The only long-lived activity produced in this way is $\text{Cr}^{51}$ by $(d,2n)$ on $\text{V}^{51}$. The $(d,p)$ reaction on $\text{V}^{51}$ produces 3.8 m $\text{V}^{52}$, the $(d,2p)$ on $\text{V}^{51}$ gives 5.8 m $\text{Ti}^{51}$, and other $\text{V}^{51}$ products are stable. Contamination from the 0.25\% abundant $\text{V}^{50}$ isotope may be neglected.

A number of attempts to remove carrier-free chromium were taken to dryness with concentrated perchloric acid, thus oxidizing the elements to $\text{V(v)}$ and $\text{Cr(vi)}$ respectively. The residue was then dissolved in a small amount of 1M nitric acid and cooled in an ice bath with diethyl ether. After about 20 minutes, a few drops of $\text{H}_2\text{O}_2$ were added and the solution was shaken. The $\text{Cr(vi)}$ partially extracted into the ether layer. The ether layer was removed and taken to dryness to give a slight white residue. The aqueous portion was then taken to dryness with perchloric acid to reoxidize the elements, and the residue was dissolved, cooled and extracted again. Repeated applications of this procedure removed most of the $\text{Cr}^{51}$ from the vanadium, giving only a slight residue, which was satisfactory for this work. The radiochemical purity of the final source was checked by observing the gamma-radiation spectrum using a 5 in. NaI(Tl) scintillation detector and shielding facilities and 256-channel pulse-height analyzer of the M.I.T. Radioactivity Center. No contamination was observable.
Carrier-free Mn$^{54}$ was prepared by bombarding a stack of vanadium metal foils with a total thickness of approximately 0.02 cm with 25 microampere hours of 30-Mev alpha particles on January 18, 1960. The only noticeable activity produced in this way is Mn$^{54}$, by (a,n) reaction on V$^{51}$. The (a,γ), (a,p), (a,pn) reactions on V$^{51}$ produce stable products, and the (a,pn) reaction on V$^{50}$, produces 140 y Mn$^{53}$, as does (a,2n) on V$^{51}$.

The target foils were dissolved in 1M HNO$_3$ on a steam bath, and iron carrier was added. The solution was made alkaline with NaOH, H$_2$O$_2$ was added, oxidizing the vanadium to V(v), and the solution was heated to coagulate the precipitate of Fe(OH)$_3$ which carried the Mn$^{54}$ as MnO$_2$ (Ha -51). The solution was then centrifuged, the precipitate dissolved in 9M HCl, and this solution was run through a column of Dowex-l anion-exchange resin. The resin held the iron while the Manganese passed directly through. The procedure of adding iron carrier, oxidizing, precipitating, centrifuging, dissolving, and passing through the iron-exchange column was repeated several times. The final effluent from the column gave a brown residue. This was taken to dryness with concentrated perchloric acid and concentrated nitric acid, leaving a slight white residue which was satisfactorily small for the present work. Observation of the gamma-ray spectrum indicated no radiochemical contamination.
For the preparation of \( \text{Zn}^{65} \), copper foils were bombarded with 7.5-Mev protons, from which the only long-lived product is \( \text{Zn}^{65} \). 38 m \( \text{Zn}^{63} \) will be formed by \((\text{p},\text{n})\) on \( \text{Cu}^{63} \), while \((\text{p},\alpha)\) gives 12.8 h \( \text{Cu}^{64} \) from \( \text{Cu}^{65} \) and 9.9 m \( \text{Cu}^{62} \) from \( \text{Cu}^{63} \).

The chemical separation of zinc from the copper target foil follows reference (Iriv-49). The foils were dissolved in 6M \( \text{H}_2\text{SO}_4 \) with the addition of \( \text{H}_2\text{O}_2 \). After heating to decompose the excess \( \text{H}_2\text{O}_2 \), \( \text{NH}_4\text{HPO}_4 \) was added. This quantitatively reduced the Cu(II) which coagulated on heating to leave the Zn(II) in solution. The solution was passed through a cation-exchange resin Dowex 50. The Zn(II) stayed on the column during an \( \text{H}_2\text{O} \) wash and was removed with 12M HCl. The eluant was diluted to 2M HCl and passed through an anion-exchange resin, Dowex 1, where the zinc was held. The activity was finally eluted with 0.1M HNO\(_3\). This eluant, upon evaporation to dryness, left only a slight residue. The activity was observed to be radiochemically pure.
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BIOGRAPHICAL NOTE

The author, a native Californian, received his early formal education in the city school-system of Bakersfield, California. His undergraduate years, from 1952 to 1956, were spent at the oldest chartered college in California, The College of the Pacific, in Stockton. At The College of the Pacific his studies were supported in part by a Crown-Zellerbach Scholarship, Mayr Foundation Scholarship, and Laboratory Assistantships in the departments of Chemistry and Physics. He was elected into the honorary scholastic society of Phi Kappa Phi and graduated with high honors, receiving a Bachelor of Science in Chemistry, June, 1956.

He arrived at the Massachusetts Institute of Technology in September, 1956. His studies at M. I. T. have been supported by a Teaching Assistantship for the first year and by a Research Assistantship for the following three years. He was elected to the honorary research society of Sigma Xi and the honorary chemical society of Phi Lambda Upsilon, and is a member of The American Physical Society. During his years at M. I. T. he served as Chairman of the Graduate House Executive Committee and as President of the M. I. T. Water Polo Club.

Since his graduation from The College of the Pacific, the author has held brief positions with the Shell Oil Company at Martinez, California; The United States Naval Radiological Defense Laboratory at San Francisco; and at Suffolk University in Boston, where he was the Instructor in Physics.