Computational assessment of turbine rim seal system parametric variation on hot gas ingestion and flow pattern

by

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B.S., Boston University (2012)

Submitted to the Department of Aeronautics and Astronautics in partial fulfillment of the requirements for the degree of Master of Science in Aeronautics and Astronautics at the

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Abstract

A design of experiments (DOE) is carried out to assess the turbine rim cavity system parametric variation on hot gas ingestion, flow pattern, and turbine stage efficiency. The parameters focused on are purge to main mass flow rate ratio, axial gap to rim radius ratio, radial gap to rim radius ratio, normalized axial position of the blade leading edge, and internal purge cavity radius ratios. The results were used to formulate a non-dimensional sealing parameter, $\Psi$, that has a threshold value of $\Psi = 2.3 \cdot 10^{15}$, beyond which there is only a marginal variation in ingestion penetration depth. This non-dimensional sealing parameter is given as a function of rim seal geometry, purge mass flow rate ratio, Rotational Reynolds number, purge flow Reynolds number, rim seal Reynolds number, and Rossby number. The non-dimensional sealing parameter reflects the physical effects associated with rim seal geometry, flow characteristics, and operating parameters. The computed flow field demonstrates the dominant role of vortical structures in the rim cavity flow on effective flow area distribution and hence the ingestion penetration depth. Quantitative attributes of the vortex, such as non-dimensional circulation, maximum vorticity, height to width ratio, and normalized vortex center position, scale with the non-dimensional sealing parameter. As a result, the vortex attributes scale with ingestion penetration depth. The implication is that the sealing parameter potentially provides a guideline for selecting rim seal configurations and operating space to yield marginal levels of hot gas ingestion. The variation in turbine stage efficiency is approximately linear with purge mass flow rate ratio, where a decrease of 0.7% in efficiency is observed for every 1% increase in purge mass flow rate ratio. This result is in accord with published results to-date.

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# Contents

## 1 Introduction

1.1 Background and Motivation .................................................. 25
1.2 Literature review .............................................................. 30
1.3 Technical Objectives ............................................................ 34
1.4 Contributions and Findings .................................................... 35
1.5 Organization ......................................................................... 36

## 2 Technical Approach

2.1 Geometry Characterization ..................................................... 37
2.2 Steady RANS CFD Model Setup ............................................. 38
2.3 CFD Spatial Convergence - Grid Study .................................. 40
2.4 CFD Temporal Convergence .................................................. 45
2.5 Ingestion Calculation - Seal Effectiveness .............................. 45
     2.5.1 Penetration Depth ...................................................... 48
2.6 Effective Flow Area ................................................................ 50
2.7 Stage Efficiency Calculation ................................................ 54
2.8 Design of Experiments (DOE) Setup ...................................... 55
     2.8.1 Baseline Geometry Definition and Characterization .......... 56
     2.8.2 Initial Latin Hypercube DOE - Design Space Exploration ... 57
     2.8.3 Sub-DOE 1: Isolating Key Geometric Variables ............... 58
     2.8.4 Sub-DOE 2: Varying purge mass flow rate ratio with Fixed Geometry .......................................................... 58
2.9 Summary .............................................................................. 60
3 Results

3.1 Initial DOE Post-Processing ........................................ 61
3.2 Sub-DOE 1 Results: Isolating Key Geometric Variables .......... 63
  3.2.1 Impact of Rim Seal Geometry and Purge Flow on Ingestion
         Penetration Depth ........................................... 65
3.3 Non-dimensional Sealing Parameter .............................. 69
3.4 Effects of Rim Seal Geometry and Purge Flow on Ingestion .... 74
  3.4.1 Ingestion versus Non-Dimensional Sealing Parameter ..... 74
  3.4.2 Impact of Purge Flow on Purge Cavity Flow Structures ... 78
  3.4.3 Impact of Upper Trench Vortex Circulation on Ingestion .. 92
  3.4.4 Upper Trench Circulation and Purge Cavity Effective Flow Area 94
  3.4.5 Role of Rim Seal Geometry on Viscous Loss in the Purge Cavity 102
  3.4.6 Summary of Rim Seal Geometry and Purge Flow on Ingestion 106
3.5 Changes in Turbine Stage Efficiency Accompanying Purge Flow Introduction ........................................... 108
  3.5.1 Impact of shear layer mixing at interface between trench exit
         and main gas path on stage efficiency ...................... 111
3.6 Summary .............................................................. 112

4 Conclusions .............................................................. 115

4.1 Summary ............................................................... 115
4.2 Key Findings .......................................................... 116
4.3 Future Work ........................................................... 117

A Additional Parametric Assessments ................................ 119

A.1 Impact of Rotational Reynolds Number on Ingestion ........... 119
A.2 Preliminary Assessments on Impact of Unsteadiness on Flow Features and Ingestion .......................................... 120
List of Figures

1-1 Simple Brayton Cycle T-s diagram. The T-s diagram shows that to improve thermal efficiency, the compressor pressure ratio and/or the turbine inlet temperature can be increased. 27

1-2 Secondary flow circuit and nomenclature for wheelspace between turbine disk and stator[4]. 28

2-1 Wheelspace geometrical configuration [4]. 38

2-2 Regions of the rim wheelspace cavity referred to as the upper trench region, purge cavity, purge circuit, and the inner wheelspace cavity. 39

2-3 Simplified sketch of the CFD air solid domain with the inlets, outlets, and stage one blade highlighted. 41

2-4 Zoomed in view of the purge cavity domain to highlight the grid. 43

2-5 Plot of blade mid-span blade pressure loading and circumferential pressure variation in the upper trench. 44

2-6 View of the monitor points in the upper trench. 45

2-7 Plot of the CO$_2$ mass fraction versus timestep at a monitor point in the upper trench. The plot demonstrates the temporal convergence of the mass fraction at this location after approximately 1000 timesteps. 46

2-8 Design Point 1 Temporal Convergence. 46

2-9 Design Point 42 Temporal Convergence. 47

2-10 Design Point 125 Temporal Convergence. 47

2-11 Cut planes in the purge cavity used to calculate seal effectiveness. 49
2-12 Representative plot of local seal effectiveness versus cut plane in the purge circuit. Cut plane 1 is the stage one blade hub and cut plane 30 is in the inner wheelspace cavity.

2-13 Streamwise coordinate (s-coordinate) in the purge circuit used to measure ingestion penetration depth. The total depth of the purge circuit, $D_{total}$, is also shown.

2-14 Diagram to demonstrate the idea of using radial velocities at the angeling cut plane to aid in calculating the effective flow area.

2-15 Diagram showing the combination of radial and axial velocity profiles to determine the effective flow area. Table 2.4 identifies the dominant velocity profile used to determine the effective flow area for each region.

2-16 Diagram showing the full cut planes in the purge cavity, the isoclip planes clipped by setting axial/radial velocity and vorticity threshold values, and a visualization of the streamlines for a case where $C_c = 0.8%$.

2-17 Selected geometric input variables for analytical DOE.

3-1 Variable Influence Profiler for Ingestion Penetration Depth. The results show that purge mass flow rate ratio, radial gap ratio, axial gap ratio, and radius ratio at the leading edge hub of the stage one blade account for 94% of the variation in ingestion penetration depth.

3-2 Variable Influence Profiler for Stage Efficiency. The VIP shows the statistical significance of the input variables analyzed in the DOE. The results show that purge mass flow rate ratio accounts for 99% of the variation in turbine stage efficiency.

3-3 Non-dimensional ingestion penetration depth versus axial gap ratio for varying purge mass flow rate ratio. The plot shows the dependence of ingestion penetration depth as a function of both axial gap ratio and purge mass flow rate ratio.
3-4 Non-dimensional ingestion penetration depth versus radial gap ratio for varying purge mass flow rate ratio. The plot shows the dependence of ingestion penetration depth as a function of both purge mass flow rate ratio and radial gap ratio.

3-5 Non-dimensional ingestion penetration depth versus $G_{r3}$ for varying purge mass flow rate ratio. The plot shows the dependence of ingestion penetration depth as a function of both purge mass flow rate ratio and $G_{r3}$.

3-6 Non-dimensional ingestion penetration depth versus presently defined non-dimensional sealing parameter, $\Phi = \frac{U}{\Omega D}$. The trend of increasing rotational Reynolds number is not captured in $\Phi$.

3-7 Least square fit of ingestion penetration depth versus non-dimensional sealing parameter residuals. For values of $\Phi > 0.06$, the ingestion penetration depth variation is within $+/−5\%$. For parameters $\Phi < 0.6$, the curve fit can be off by as much at $+/−16\%$.

3-8 Purge cavity areas used in formulating the non-dimensional sealing parameter.

3-9 Non-dimensional ingestion penetration depth versus non-dimensional sealing parameter, $\Psi$. The plot shows that the non-dimensional ingestion penetration depth, $D_{pen}$, has a dominant functional dependence on the non-dimensional sealing parameter, $\Psi$.

3-10 Least square fit of ingestion penetration depth versus non-dimensional sealing parameter residuals. For values of $\Psi > 2.3 \cdot 10^{15}$, the ingestion penetration depth variation is within $+/−5\%$. For parameters $\Psi < 2.3 \cdot 10^{15}$, the curve fit can be off by as much at $+/−15\%$. 

11
3-11 Projection of the streamlines in the r-z plane is shown in the LHS figure. The RHS figure shows the r-θ view of the velocity vectors for the streamlines in the upper trench of the purge cavity. The LHS plot shows the toroidal vortex structure in the upper trench in the r-z plane. The RHS plot shows that the vector arrows of the streamlines in the upper trench are almost entirely in the circumferential direction.

3-12 Surface streamlines demonstrating the areas of the purge cavity with no flow separation for a 0.015 purge mass flow rate ratio case.

3-13 Effective flow area at the location of the upper trench vortex for purge mass flow rate ratios of 0.015 and 0.008 (C_e = 1.5% and C_e = 0.8%). This plot shows the width of the upper trench vortex changing size as a function of purge mass flow rate ratio. As a result, the effective flow area distribution of the purge flow is impacted.

3-14 Streamlines in the purge cavity for varying purge mass flow rate ratios (0.005-0.015). This plot demonstrates the vortices that form and change in width and height in the upper trench, near the stage one blade angelwing, and above and below the angelwing. The values of the purge mass flow rate ratio in C_e are shown.

3-15 Non-dimensional ingestion penetration depth versus non-dimensional sealing parameter with streamline plots of the upper trench vortex. This plot is used as an aid to identify flow structure patterns in the three regions identified from the ingestion penetration depth versus non-dimensional sealing parameter plot.

3-16 Progression of the upper trench vortex streamlines for the identified regions from the ingestion penetration depth versus non-dimensional sealing parameter curve. Non-dimensional sealing parameter increases from left to right and demonstrates the grouping of upper vortex sizes for the three ingestion penetration depth curve regions.
3-17 Diagram showing the angelwing vortex characterization. The diagram on the left indicates the angel wing vortex that is assessed. The diagram on the right highlights the vortex center radius, height, and width. .......................... 87

3-18 Plots of angelwing vortex center radius, width, height/width ratio, and maximum vorticity. Vortex center radius versus purge mass flow rate ratio shows how the radius increases as the purge mass flow rate ratio increases. Conversely, the width, aspect ratio, and maximum vorticity of the angelwing vortex decrease as the purge mass flow rate ratio increases. .......................... 88

3-19 Non-dimensional ingestion penetration depth versus non-dimensional sealing parameter for the fixed geometry and varying purge mass flow rate ratio DOE. The plot shows that for a fixed geometry with varying purge mass flow rate ratio, a trend similar to that from Figure 3-9 is observed. .......................... 89

3-20 Diagram highlighting the location of the upper trench vortex. ....... 91

3-21 Non-dimensional circulation versus non-dimensional sealing parameter. For $\Psi < 2.3 \cdot 10^{15}$, the circulation is greater than 30 and the ingestion penetration depth is greater than 5%. For $\Psi > 2.3 \cdot 10^{15}$, the circulation remains below 30, and correspondingly, the ingestion penetration depth near these values is $<5\%$. .......................... 93

3-22 Non-dimensional circulation and ingestion penetration depth versus non-dimensional sealing parameter. The plot shows that $\Gamma_{ND}$ follows a similar trend as that seen for ingestion penetration depth. ....... 93
3-23 Non-dimensional ingestion penetration depth versus non-dimensional sealing parameter. Data points are scaled by non-dimensional upper vortex circulation. Large data points indicate high $\Gamma_{ND}$. There is a clear grouping of data points in this plot for non-dimensional sealing parameters $\Psi > 2.3 \cdot 10^{15}$ where $\Gamma_{ND} < 30$ and the ingestion penetration depth is $< 5\%$. There is also a clear grouping of data points with $\Psi = 0.5 - 2 \cdot 10^{15}$ where $\Gamma_{ND} > 30$ and the ingestion penetration depth is $> 5\%$.

3-24 Cut planes used to evaluate the effective flow area distribution near the upper trench vortex.

3-25 Effective flow area versus non-dimensional circulation at $r/b = 1.011$, $r/b = 1.007$, and $r/b = 1.004$ for the fixed geometry and varying purge mass rate ratio DOE. The effective flow area distribution at the three radial cut planes near the upper trench vortex trend similarly.

3-26 Effective flow area at the upper trench versus non-dimensional sealing parameter. Each data point is scaled by the value of $\Gamma_{ND}$ of the upper trench vortex. Large data point size corresponds to larger $\Gamma_{ND}$. For $\Gamma_{ND} > 30$, the effective flow area of the trench is reduced to less than 70\% of the total geometrical area. When the effective flow area is $> 70\%$ of the total geometrical area, $\Gamma_{ND} < 30$, and corresponds to regions where the ingestion penetration depth is $< 5\%$.

3-27 Effective flow area versus non-dimensional circulation of the upper trench vortex for the fixed rim seal geometry and varying purge mass flow rate ratio DOE. Data points are scaled by ingestion penetration depth. Data points larger in size indicate increased ingestion penetration depth. For effective flow areas $> 60\%$ of the total area at $r/b = 1.004$, $\Gamma_{ND}$ remains between 20 – 30 and the ingestion penetration depth is $< 5\%$. For effective flow areas between 20 – 60\% of the total area at $r/b = 1.004$, $\Gamma_{ND}$ increases from 30 – 80 and the ingestion penetration depth increases.
Streamlines for cases with non-dimensional sealing parameters $\Psi = 1.96 \cdot 10^{15}, 2.2 \cdot 10^{15}$, and $2.71 \cdot 10^{15}$. Ingestion penetration depth, effective flow area at the upper trench vortex, and non-dimensional circulation of the upper trench vortex are shown. Figure highlights how the extent of flow separation along the stage one blade angelwing increases as $\Psi$ decreases.

Total pressure coefficient ($C_{pt}$), $\Gamma_{ND}$, and effective flow area at $r/b = 1.004$ versus non-dimensional sealing parameter ($\Psi$) for the fixed geometry and varying purge mass flow rate ratio DOE. The plots demonstrate that the total pressure coefficient switches from a negative to positive value at $\Psi = 1.5 \cdot 10^{15}$, indicating the hot gas from the main flowpath is entering the upper trench. The ingested hot gas results in a higher total pressure at this location.

Purge cavity normalized entropy generation rate per unit volume ($\left( \frac{T_{\text{in}}S_{\text{visc}}}{\rho V^2} \right)$) for a purge mass flow rate ratio of $C_e = 1.2\%$. The highest rate of entropy generation is observed near the stage one blade angelwing and the upper trench area where the two most prominent vortical structures in the purge cavity have been identified.

Purge cavity viscous loss, $T\Delta s/\Delta h_t$, for all DOE points versus non-dimensional sealing parameter. Large data point size corresponds to increased ingestion penetration depth. The viscous loss versus non-dimensional sealing parameter shows a similar trend to the ingestion penetration depth versus non-dimensional sealing parameter curve. Similarly, the highest viscous loss is observed for non-dimensional sealing parameters between $\Psi = 0.5-2.3 \cdot 10^{15}$, which is where the ingestion penetration depth and upper trench vortex non-dimensional circulation is highest.
3-32 Change in stage efficiency versus purge mass flow rate ratio. Change in stage efficiency is with respect to the baseline case defined in Section 2.8.1. The results show that there is a 0.7% decrease in stage efficiency for every 1% increase in purge mass flow rate ratio.

3-33 Curve fit residuals for the least square linear fit of change in stage efficiency versus purge mass flow rate ratio. The results show that the fit captures the variance in stage efficiency to within $+0.12/-0.08\%$.

3-34 Non-dimensional ingestion penetration depth and change in stage efficiency versus non-dimensional sealing parameter. A negative $\Delta e$ corresponds to a decrease in stage efficiency. Maximizing stage efficiency corresponds to low non-dimensional sealing parameters where the ingestion penetration depth is greatest.

3-35 Change in loss generated from mixing out of the purge-main flow shear layer versus purge mass flow rate ratio. Change in loss generated from mixing out of the purge-main flow shear layer is with respect to the baseline case defined in Section 2.8.1. The results show that the trench shear layer loss accounts for half of the total change in stage efficiency.

A-1 Non-dimensional ingestion penetration depth versus non-dimensional sealing parameter. The plot highlights changes in rotational Reynolds number. Changes in $Re_\phi$ and its impact on ingestion penetration depth is captured with the non-dimensional sealing parameter, $\Psi$.

A-2 Non-dimensional ingestion penetration depth versus non-dimensional sealing parameter highlighting steady and unsteady case 93. The plot shows the ingestion penetration depth is reduced by 5% when accounting for unsteadiness of the trench shear layer and the data point still follows the ingestion penetration depth curve.
A-3 Difference in streamlines pattern from time-averaged unsteady computed flow and from steady run. The streamlines plots are used to show that the upper trench vortex structure has changed between the steady and unsteady simulation. The size of the vortex is qualitatively different; the cross-sectional area in the r-z plane encompassing the upper trench vortex is 17% smaller for the unsteady solution.
List of Tables

2.1 Mesh size inputs for ANSYS Workbench Mesher normalized by the depth of the upper trench \((D_T)\). ..................................................... 42
2.2 Node and element counts for the \((3)\) grids. ................................. 42
2.3 Comparison of aerothermal parameters, stage efficiency, and ingestion penetration depth for the grid study. Stage pressure ratio, temperature ratio, and stage \(\Delta h_t\) are normalized by their respective value for the baseline grid. Change in stage efficiency is \(\Delta \varepsilon\). Change in ingestion penetration depth is non-dimensionalized by the total depth of the wheelspace cavity as discussed in Section 2.5.1. ................................. 44
2.4 Dominant velocity profile used to determine the positive flow area for each region defined from Figure 2-15. ................................................. 53
2.5 Baseline values for the axial and radial gap normalized to the annulus inner radius \(r_{inner}\). ................................................................. 56
2.6 Variation of input variables with respect to the baseline geometry. ... 58
2.7 Sub-DOE input parameter values for varying purge mass flow rate ratios isolating a critical geometric parameter. ............................. 59
2.8 Sub-DOE Input parameter values for baseline geometry with varying purge mass flow rate ratio. ....................................................... 59
3.1 Input rim seal variables and assessed outputs for the DOE. ............... 62
3.2 Statistical significance of the assessed rim seal variables on ingestion penetration depth. ................................................................. 63
A.1 Rotational Reynolds number for assessed cases. ............................. 120
Nomenclature

Greek

\( \alpha \) Ratio of purge mass flow rate injected at the FOS inlet and the swirl component of the purge mass flow

\( \epsilon \) Stage efficiency

\( \Gamma \) Circulation

\( \gamma \) Ratio of specific heat capacities

\( \mu \) Dynamic viscosity

\( \Omega \) Angular velocity of rotating disk

\( \omega \) Vorticity

\( \Phi \) Traditional non-dimensional sealing parameter

\( \Psi \) Proposed non-dimensional sealing parameter

\( \rho \) Density

\( \theta \) Circumferential coordinate

\( \varepsilon \) Sealing effectiveness

Subscripts & Superscripts

\( a \) Mainstream flow path/gas path
c  Purge flow air
s  Static
T  Trench cavity
t  Stagnation
wa  Work-averaged

**Symbols**

$m$  Mass flow rate

$A_{x,\text{effective}}$  Geometrical effective flow area

$A_{x,\text{ratio}}$  Effective flow area ratio

$A_{x,\text{total}}$  Total area of a cut plane in the purge circuit

$b$  Radial distance from the seal to the engine centerline

$c$  Tracer gas concentration

$C_c$  Non-dimensional purge mass flow rate ratio ($\frac{m_x}{m_a} \cdot 100\%$)

$c_p$  Specific heat capacity at constant pressure

$C_w$  Non-dimensional flow rate ($\frac{m}{\mu_b}$)

$C_{pt}$  Total pressure coefficient

$CCW$  Counterclockwise vortex.

$CW$  Clockwise vortex.

$D_T$  Depth of trench

$D_{\text{pen}}$  Non-dimensional ingestion penetration depth normalized by the total depth of the purge circuit, $D_{\text{total}}$
$D_{pen}$  Non-dimensional ingestion penetration depth

$D_{total}$  Total depth of the purge circuit

$D_{x,\epsilon=95\%}$  Depth into the purge circuit where the seal effectiveness is 95%

$G$  Gap ratio ($\frac{W_f}{r_{inner}}$)

$G_c$  Seal-clearance ratio ($\frac{S_c}{r_{inner}}$)

$G_r$  Radius ratio ($\frac{r}{r_{inner}}$)

$G_{zLE}$  Axial location of S1B LE relative to trench ratio ($\frac{z_{LE}}{r_{inner}}$)

$h$  Enthalpy

$h_{vo}$  Height of the angelwing vortex

$L_{overlap}$  Blade angelwing axial overlap

$P$  Pressure

$r$  Radial coordinate.

$r_1 - r_4$  Internal purge flow radii

$r_{inner}$  Inner radius of annulus

$r_{vo}$  Center radius of the angelwing vortex

$Re_\phi$  Rotational Reynolds number ($\frac{\rho U_b r^2}{\mu}$)

$Re_{c,b}$  Rim seal Reynolds number ($\frac{\rho U_b}{\mu}$)

$Re_c$  Reynolds number of cavity purge flow ($\frac{\rho U_b r}{\mu}$)

$Ro$  Rossby number ($\frac{U}{\Omega r}$)

$s$  Entropy

$S_{visc}^{m}$  Viscous entropy generation rate per unit volume
$s_c$ Seal clearance gap height

$T$ Temperature

$U$ Bulk mean velocity of the purge flow

$V$ Velocity in the stationary frame

$W_4$ Mass flow rate at station 4 entering the turbine from the combustor.

$W_T$ Width of trench

$w_{\text{expand}}$ Turbine work.

$w_{\text{vo}}$ Width of the angelwing vortex

$x$ Streamwise position in purge cavity.

$z$ Axial coordinate.

$z_{\text{LE}}$ Axial location of S1B LE relative to trench
Chapter 1

Introduction

1.1 Background and Motivation

High thermal efficiency, which directly impacts SFC, is a key requirement for customers of high performance aviation and power generation gas turbines. Modern gas turbines operate following the Brayton cycle. Gas turbines have five main components: compressor, combustor, high pressure turbine, low pressure turbine, and exhaust. The engine cycle is as follows: air enters the compressor and passes through the high-pressure compressor (HPC), whereby the total pressure and total temperature increase across the component. The combustor further increases the total temperature of the gas at constant pressure. Air exits the combustor and is expanded through the turbine, where work is extracted from the hot gas to drive the compressor. For an aircraft gas turbine, any remaining energy in the fluid after going through the turbine is expelled as high velocity air through an exhaust duct. For power generation gas turbines, the remaining energy is expanded through a low pressure turbine that drives a generator. A simplified T-s diagram of the Brayton cycle is shown in Figure 1-1. The ideal thermal efficiency of the Brayton cycle is given by Equation 1.1 [12].

\[ \eta_c = 1 - \frac{T_{t,a}}{T_{t,max}} \quad (1.1) \]

The T-s diagram in Figure 1-1 demonstrates how a higher \( T_{t,max} \) can be achieved
by increasing the compression ratio to operate on a higher constant pressure line across the combustor. A higher $T_{t,\text{max}}$ can also be achieved by increasing the allowable turbine inlet temperature. Operating at a higher $T_{t,\text{max}}$ improves the thermal efficiency as the ratio of $\frac{T_{t,a}}{T_{t,\text{max}}}$ approaches 0 for increasing $T_{t,\text{max}}$ and a set inlet temperature $T_{t,a}$. Today’s gas turbine engines set records for the highest compression ratios. Although the increased HPC pressure ratio leads to better cycle efficiencies, it also increases turbine inlet temperatures, ultimately increasing the stress in the materials used in the hot sections of the turbine. To combat this temperature-derived material stress, engine designers must incorporate cooling mechanisms, such as cooling air flow. However, cooling air flow comes at a significant cost. Air that is extracted from the compressor for cooling does not contribute to work extracted by the turbine, since it bypasses the combustor. In fact, turbine efficiency decreases as much as 0.8% for every percent of purge flow [20].

The purge circuit, located between the stage one nozzle guide vane and the stage one blade, is a critical subsystem for which there is a lack of quantitative enablers for sizing the purge flow rate with minimal acceptable hot gas ingestion. High pressure air from the compressor is needed to eject flow though cooling holes, slots, and cavities. The only air that has a high enough pressure to accomplish this at the first turbine stage is compressor discharge air. Unfortunately, using compressor discharge air for cooling is extremely costly since there is a large amount of work put into it by the compressor, it bypasses the combustor, and it is unavailable to expand through the turbine. Therefore, limiting the amount of compressor discharge air used for purge flow at the stage one purge cavity would have a substantial impact on improving the performance of the turbine component.

A typical geometry of a turbine stage is a stage 1 nozzle guide vane (NGV), which is stationary, and a stage 1 turbine blade, which is attached to a disk that rotates. An axial gap is required between the stationary component and the rotating component. This clearance path between the rotor and stator makes up the forward wheelspace cavity and is an important feature of the secondary flowpath of the engine. A typical wheelspace cavity is illustrated in Figure 1-2.
Figure 1-1: Simple Brayton Cycle T-s diagram. The T-s diagram shows that to improve thermal efficiency, the compressor pressure ratio and/or the turbine inlet temperature can be increased.

The hot gas from the combustor enters the first NGV and can reach a temperature of $2500^\circ F$ [12] with an overall pressure ratio of 50 with respect to the fan inlet [1]. It is critical that air at this temperature does not penetrate into the wheelspace cavity. However, Owen et. al. [19] have shown that the pressure distribution in the main gas flowpath and the secondary flow path can result in a negative pressure that causes hot gas in the main flowpath to be ingested into the wheelspace cavity. To mitigate ingestion, compressor discharge air is purged into the gas path between the rotor and stator, through the rim seal passage. Typical values of purge flow air are around 3% of the gaspath mass flow rate [23]. One of the critical functions of the purge air is to manage the temperatures of the turbine hardware. Since turbine materials are typically nickel-based alloys [25] with thermal limits below $2000^\circ F$ [28], hot gas that
is ingested into the purge cavity can have a negative impact on the life of the turbine hardware. The turbine hardware material properties experience a debit when exposed to the high gas path temperatures. It is demonstrated that a Nickel-Chrome alloy with temperature capability up to nearly 1000°C, sees a 50% reduction in fatigue capability for an increase of 200°C [24]. To protect turbine hardware and to ensure the life requirement of the turbine is met, rim seals play a crucial role in the thermal management of the cavity. Sealing effectiveness quantifies the performance of a rim seal by indicating how well the purged air reduces ingestion into the wheelspace cavity. To improve the sealing effectiveness of the purge flow between the rotor and stator, turbines employ rim seals of varying geometries that provide a smaller clearance gap or a more elaborate path that the purge air must travel through. While purge air is crucial for controlling ingestion levels, it has been shown to have a negative impact on the performance of the gas turbine. Therefore, one goal in secondary flow design is to minimize the amount of purge air extracted from the compressor while also reducing the amount of wheelspace ingestion.

The technological importance of managing hot gas ingestion in high performance modern turbine rim cavities has led to an extensive amount of research. The re-
search aims to establish the physical factors that drive hot gas ingestion through a combination of experiments, computations, and modeling. The ultimate goal in rim seal design is to establish a framework that enables effective management of hot gas ingestion and achieve an optimal engine efficiency with acceptable turbine durability. The configuration of this system is also subject to constraints that include assembly, manufacturing, and operational clearances. There are empirical relationships and rules for determining sealing effectiveness of rim seals, but many of these have been formulated using data or assumptions that can introduce a level of uncertainty, especially for new design spaces. For example, the model put forth by Owen et. al. [17], is based on inviscid flow equations and a discharge coefficient is used to account for viscous losses. The flow through purge cavities is inherently viscous, thus these models do not reflect the flow through the purge cavity. The empirical models can also mask the physics that may be driving ingestion. Other threads of research have shown the importance of non-geometrical rim seal features on hot gas ingestion. Past work at MIT by Berg [4] and Catalfamo [6] on the GE Power Hot Gas Ingestion Rig (HGIR), has shown the importance of excitations at frequencies corresponding to the natural modes of the wheelspace system. The frequencies can lead to large responses in pressure and seal flow rate, which is shown to impact ingestion. The research presented in this thesis constitutes a new thread in formulating a framework for establishing the rim seal geometric and flow drivers for hot gas ingestion. The objective of the framework is to enable an effective management of hot gas ingestion to achieve optimal engine efficiency with acceptable turbine durability. Specifically, this work consists of first implementing a set of Reynolds-Averaged Navier-Stokes (RANS) Computational Fluid Dynamics (CFD) simulations of flow in a rim-cavity wheel space configuration. The simulations vary the key input geometries and operating parameters. Second, this thesis determined the link of the hot gas ingestion manifestation to both the attributes of the resulting flow pattern and the input parameters, as well as the operating metrics of the system. Together, this work could lead to the determination of a rim seal configuration that minimizes hot gas ingestion. The Technical Approach section provides details on this new approach for managing
hot gas ingestion in high performance turbines.

The next section reviews the research literature that is publicly available.

1.2 Literature review

Wheelspace flow physics have been the subject of substantial research for over 40 years. In 1970, Bayley et. al. [3] published a paper that explored the measurements of pressure distribution, frictional moment, and cooling air flow necessary to prevent the ingress of hot gases over the turbine disk for a range of rotational speeds, mass flow rates, and different geometries. There have also been a number of experimental test rigs commissioned and studied to assess the sealing effectiveness of turbine rim seals. The following section reviews relevant existing literature on ingestion and turbine rim seals.

Owen et. al. [18] formulated and developed a 1D steady, incompressible, and inviscid orifice model that leverages empirical constants to account for viscous effects that are derived from experimental data. Owen et. al. break ingestion into several ingestion modes; rotationally induced (RI), externally induced (EI), and combined ingress. RI ingress occurs as the mainstream flow past the stationary nozzle guide vanes and rotating turbine blades creates an unsteady non-axisymmetric variation in pressure in the annulus. The variation in pressures causes ingress and egress to occur through the parts of the seal clearance where the external pressure is higher and lower, respectively, than that in the wheel-space [17]. EI ingress is caused by the circumferential distribution of pressure created by the blades and vanes in the turbine annulus. EI ingress occurs in regions where the external pressure is higher than that in the wheel-space, and egress occurs where it is lower [19]. The ingestion modes are discussed extensively in a series of several papers [17][18][19]. The main flow model used to describe the ingestion modes is an orifice model, which is derived from incompressible flow equations. The orifice model is found to provide good correlation to computed values of seal effectiveness. However, the orifice model requires empirical constants to match experimental data, and the orifice model does not identify the flow
mechanisms that cause ingestion. The orifice model lumps the effect of these flow features into empirical constants, which are determined experimentally. This type of model is useful for determining the minimal amount of purge air required to prevent ingestion. However, experimental data to calibrate the model is not typically available during the conceptual or preliminary design stage of a new turbine rim seal. Therefore, the empirical model has little impact on influencing how a rim seal is designed in early stages. Thus, for a turbine designer, a method for determining how changes in a rim seal design, by changing parameters that are in their control (geometry, purge flow, etc.), impacts hot gas path ingestion would be useful for influencing the rim seal design at early stages of turbine design.

Catalfamo [6] interrogated a computational model of the GE Power hot gas ingestion rig (HGIR) wheelspace to assess the response of the wheelspace to external stimuli in the turbine main flow path. Catalfamo found that a circumferential variation in the external flow path pressure field can lead to ingestion with the ratio of disturbance wavelength to the trench depth emerging as a key parameter. Catalfamo also showed that excitation at frequencies corresponding to the natural modes of the wheelspace system can lead to large responses in pressure and seal flow rate, which ultimately impacts ingestion. Berg [4] interrogated high response pressure data from the HGIR that shows the existence of cavity modes in the rim-seal-wheelspace cavity, including shallow cavity modes and Helmholtz resonance. The sensitivity of the cavity modes to cavity aspect ratio, purge flow ratio, and flow angle was assessed. To further probe the experimental data, Berg performed computational simulations to compare with. These simulations suggest that increasing purge flow ratio mitigates shallow cavity modal response and increasing primary flow angle reduces the shallow cavity modal response. Subsequent computational parametric assessments suggest that increasing purge flow and primary flow angle could provide a stabilizing effect on the response, but found that further experimental tests were needed to quantify the effects of cavity modes on hot gas ingestion. The work completed by Berg was useful in characterizing the geometric and aerothermal parameters of a rim seal, and the computational simulation approach for a parametric assessment of critical input
parameters aided in guiding the research presented in this thesis. The work completed by Catalfamo [6] and Berg [4] on the HGIR inspired additional research questions that provided a basis for the research presented in this thesis.

Zhang et al. [27] performed a similar assessment as proposed in this thesis by investigating the impact of purge mass flow rate ratio, rotor speed, and annulus pressure ratio. Zhang defines annulus pressure ratio as the ratio of stagnation pressure between the inlet of the first stage nozzle guide vane and the exit of the second stage blade. Zhang et al. found that the sealing effectiveness improves with increasing purge mass flow rate ratio and increasing rotor speed, through Rotational Reynolds number. Zhang et al. found that sealing effectiveness was insensitive to the annulus pressure ratio. Both Zhang et al. and the work done in this thesis use a similar method for calculating the seal effectiveness; the method calculates an area-average of the tracer gas concentration in the purge cavity at different cut planes throughout the purge cavity. Zhang et al. highlighted the importance of the vortical flow structures in the purge cavities, but do not have a quantitative way of showing the impact of these structures on ingestion.

Sangan et al. [22] investigated the performance of single and double seals by measuring the CO₂ gas concentration in the rim-seal region of the University of Bath rig to calculate the variation of concentration effectiveness with sealing flow rate. They demonstrated that there is a benefit of using a double seal because it keeps the gas confined in the outer wheelspace radially outward of the inner seal; in the inner wheelspace (radially inward of the inner seal) the effectiveness is shown to be higher than that of a single seal. Sangan et al. also concluded that the sealing effectiveness for the rotationally induced ingestion case was significantly greater than that for the externally induced case.

Daniels et al. [9] performed an experimental investigation on a United Technologies Research Center rig to determine the sealing effectiveness of four rim seal models by varying two key flow conditions: radial gap and the axial overlap of the seal. The main results of the investigation indicated that decreasing the radial gap of the seal produces an improvement in seal effectiveness. They also concluded that the tracer
gas technique employed to determine seal effectiveness is an accurate alternative to pressure measurements or flow visualization techniques. This work was instrumental in demonstrating the validity of using the $CO_2$ tracer gas as a means of quantifying ingestion, and it suggests the importance of key seal characteristics, such as radial gap and axial overlap. Although Daniels et. al. were instrumental in demonstrating the importance of these key seal characteristics, they do not describe how the modulation of these characteristics impacts the resulting geometry and physical flow features.

Zhou et. al [29] determined the impact of three cavity rim configurations, and found that the rim cavity aspect ratio is an important geometric parameter. It was determined that larger depth to width aspect ratios contribute more to ingestion. Zhou also scrutinized the instantaneous fluid velocity at low purge flows using particle image velocimetry (PIV) to identify the regions of ingestion and egress around the circumference of the rim seal region. Three-dimensional, time-dependent numerical simulation of the rim seal cavity showed qualitatively similar characteristics as the PIV maps, demonstrating the utility of numerical simulation velocity contours for identifying regions of ingestion.

Bunker et. al [5] utilized a wheelspace sector cascade rig to gather data on flow and thermal effects in a non-rotating environment. The experimental rig is used as a tool to investigate both the basic geometry and flow effects that work together to maintain the bulk of the correct flow physics in the absence of rotation. The rig is also used as validation data for the unsteady CFD modeling efforts described in part two of the paper. The experimental results show that the peak-to-peak circumferential pressure distribution variations immediately aft of the vane are as much as 18%, which is shown to be key in the resulting forcing of hot gas inboard of the rim seal. The blade leading edge bow wave is also found to have an equal or even greater influence in generating this peak-to-peak variation than the vane trailing edge wake. In part two of the paper, Laskowski et. al [14] used CFD simulations to study the turbine wheelspace cooling interactions with the vane/nozzle trailing edge wake, trench shear layer, and blade/bucket leading edge bow wave. A computational model was developed to study the mechanisms responsible for hot gas ingestion into the wheel-space cavity of a high
pressure turbine. It was determined that steady-state CFD simulations could not capture the unstable shear layer over the rim seal axial gap (trench) that causes hot gases to be ingested. It is thus inferred that an unsteady analysis is required. This work shows the importance of unsteady effects when modeling ingestion with CFD.

Zlatinov [30] found that increasing purge flow results in an increase in stage loss. This rigorous assessment of stage loss was completed for a number of purge flow design parameters, such as purge mass flow rate ratio, swirl ratio, and purge flow injection angle. This thesis builds upon Zlatinov's work by investigating the rim seal of a representative gas turbine engine and assessing the sensitivity of stage efficiency as a result of changing rim seal geometric and flow parameters.

The review of existing literature indicates there has been a substantial effort to understand and characterize rim seals to mitigate hot gas ingestion. Efforts have shown that models can be developed to accurately account for ingestion when experimental data is available; the importance of certain rim seal geometry parameters (aspect ratio, axial overlap, radial gap, etc.) on ingestion, and the importance of unsteadiness. There is however, a lack in existing knowledge of traceability in the variation in rim seal geometries and purge flow to ingestion and to the flow structures in the purge cavity.

1.3 Technical Objectives

The overall goal of this thesis is to quantitatively establish the traceability of changes in hot gas ingestion and turbine stage efficiency to purge cavity geometry and the resulting flow pattern. With this understanding, a purge flow system can be achieved that limits the amount of purge flow necessary to safely operate the turbine and improves turbine performance. To achieve this goal, the following specific objectives are pursued.

1. Identify a scaling parameter relating purge mass flow rate ratio and key rim seal geometric variables that have a statistical impact on ingestion.
2. Identify and quantify the physical flow feature(s) in the purge trench cavity that drive ingestion.

3. Identify the impact of purge flow and key rim seal geometrical parameters on turbine stage efficiency and understand the physical driver(s) for the loss.

4. Formulate a simplistic preliminary platform to demonstrate the potential impact of pattern recognition on turbine aerothermal system technology.

1.4 Contributions and Findings

The key findings consist of:

1. The derivation of a non-dimensional sealing parameter, \( \Psi \), which incorporates the rim seal geometry, purge mass flow rate ratio, Rotational Reynolds number, purge flow Reynolds number, rim seal Reynolds number, and Rossby number. The non-dimensional ingestion penetration depth is shown to have a strong dependence on the non-dimensional sealing parameter. The results also indicate a threshold value at \( \Psi = 2.3 \cdot 10^{15} \), beyond which there is only a marginal variation in ingestion penetration depth.

\[
\Psi = \frac{m_{c,\text{total}}}{\rho b \Omega (A_1 + A_2)} \frac{Re_c Re_{\phi} Re_{c,b}}{Ro} \tag{1.2}
\]

2. The non-dimensional circulation of the upper trench vortex is shown to correlate with the ingestion penetration depth and the non-dimensional sealing parameter derived from the geometric and flow variables. Non-dimensional circulation values \( > 30 \) result in increased ingestion penetration depth, and non-dimensional circulation values \( < 30 \) result in decreased ingestion penetration depth.

3. The presence of vortical structures in the purge cavity reduces the effective flow area distribution in the purge cavity. This is demonstrated at the upper trench vortex. The effective flow area at a radius ratio of \( r/b = 1.004 \) near the upper
trench vortex is shown to correlate with ingestion penetration depth and the non-dimensional sealing parameter.

4. The stage efficiency of the turbine has a dominant dependence on the purge mass flow rate ratio with weak dependence on changes in rim seal geometry. For every 1\% increase in purge mass flow rate ratio, the efficiency decreases by 0.7\%. The penalty can mostly be traced to the shear layer mixing loss associated with the introduction of the purge flow into the main flow path.

5. A preliminary platform for incorporating the use of pattern recognition to identify flow patterns in the purge trench cavity has been established.

1.5 Organization

This thesis is organized as follows: First, the research framework and methods used are presented in Chapter 2. Chapter 3 evaluates the data obtained through the steady RANS CFD. This evaluation provides a framework for using pattern recognition as a way of studying the purge cavity flow features, investigates the impact of vortices on ingestion, and synthesizes the findings from the computational results. Chapter 4 describes the key findings of this thesis and provides recommendations for future work.
Chapter 2

Technical Approach

To establish causality between the geometric configuration of the rim seal cavity to both purge flow and hot gas path ingestion, an analytical design of experiments (DOE) was used and a systematic post-processing approach was developed. The key aspects of this approach include a method to obtain spatially and temporally converged CFD solutions for many runs, a systematic variation of key input parameters in a DOE, and a robust method of post-processing the results. The subsequent sections will describe the process in detail.

2.1 Geometry Characterization

Figure 2-1 shows the key geometrical features and designations that are used in the analysis of the rim seal. The width of the trench ($W_T$) and the depth of the trench ($D_T$) can be used to geometrically define the upper trench region. The ratio of these parameters, called the upper trench aspect ratio, $D_T/W_T$, are important considerations for characterizing a rim seal geometry, as concluded by Zhou et. al [29]. The overlap of the stage 1 blade angelwing ($L_{overlap}$), the radius of the seal/angelwing from the engine centerline ($b$), and the seal clearance gap height ($s_c$) are three additional parameters that are used to describe the rim seal configuration. $W_T$ and $s_c$ will be referenced as the axial gap and radial gap, respectively, going forward. Two other important rim seal geometry ratios considered are the axial and radial gap ratio. Both
the axial and radial gap ratios are normalized by the annulus inner radius. Figure 2-2 shows the nomenclature used for the various regions of the rim wheelspace cavity.

![Figure 2-1: Wheelspace geometrical configuration][4].

### 2.2 Steady RANS CFD Model Setup

The computational tool used for this investigation was CFX version 17.0. Steady RANS simulations of the DOE were executed using CFX. A three-dimensional air solid model representative of the turbine stage consists of a blade only domain with both the upstream and downstream rotor cavities included. The air solid was modeled with an unstructured grid consisting of triangular prism inflation layers and a tetrahedron global mesh. The grid was generated using ANSYS Workbench 16.1. A grid study was performed and will be described in the following section. The blade only domain is modeled as a sector, so periodic boundary conditions in the \( \theta \)-direction (circumferential-direction) are employed.

The computational model has two inlets: the main gas path inlet, and the inlet to the forward outer seal (FOS). Figure 2-3 highlights the air solid domain with the inlets, outlets, and stage one blade. The FOS inlet is the inlet for the purge mass flow. The model also has two outlets: the main gas path outlet and the outlet through the aft seal in the downstream rotor cavity. Representative radial velocity profiles and radial stagnation pressure distributions at the exit of the stage one nozzle.

[4]: "Figure 2-1: Wheelspace geometrical configuration"
Figure 2-2: Regions of the rim wheelspace cavity referred to as the upper trench region, purge cavity, purge circuit, and the inner wheelspace cavity.
guide vane were provided from a CFD simulation of the coupled nozzle and blade domain. These boundary conditions are imposed at the main gas path inlet, and a mass flow condition is applied to the FOS inlet. Purge mass flow at the FOS inlet is assigned as a percentage of the flow going through the main gas path. Station 4 in the engine refers to the location between the exit of the combustor and inlet to the turbine [12]. As such, the gas path mass flow rate will be presented as \( W_4 \) and purge mass flow as a ratio of the gas path in %. The main gas path outlet exits to an ambient static pressure by applying the pressure condition at the outlet with radial equilibrium. The radial equilibrium constrains the averaged static pressure within radial circumferential bands. The band-averaged pressure satisfies radial equilibrium between the radial pressure gradient and the centrifugal force calculated using the band-averaged density and circumferential velocity.

For all computations, the \( k - \omega \) Shear Stress Transport (SST \( k - \omega \)) turbulence model was used with wall integration. As a result, the grid was iterated on until \( y^+ \) values of less than 1 were achieved throughout the domain. The turbulent boundary conditions consist of turbulent kinetic energy and eddy frequency. These are applied to the inlet of the stage one blade as radial profiles. As a result, this modeling does not account for any wake deficits from the nozzle guide vane. The SST \( k - \omega \) model is chosen for this particular application since it was designed to give accurate predictions of the onset and the amount of flow separation under adverse pressure gradients by the inclusion of transport effects into the formulation of the eddy-viscosity. The adequacy of this model has been demonstrated in a large number of studies [2].

### 2.3 CFD Spatial Convergence - Grid Study

To ensure spatial convergence, a grid study was performed on one of the selected DOE geometries. To delineate between the different grids studied, the following nomenclature is selected for this study: baseline grid, a finer grid that has a 20% decrease in element sizing parameters with respect to baseline, and a coarser grid that has a 20% increase in element sizing parameters with respect to baseline. The
Figure 2-3: Simplified sketch of the CFD air solid domain with the inlets, outlets, and stage one blade highlighted.
The baseline grid was derived by iterating on sizing parameters until a grid was achieved that had \( y^+ \) values less than one; aspect ratios at the wall between 500-1000, which is suitable for RANS; and a background mesh spacing that is approximately five trailing edge diameters. The values that are changed for the coarser and finer grids are the element sizing parameters. These sizing parameters consist of max face size, max size, and face sizing. These parameters control the maximum size that the size function will return to the mesher for surfaces/faces and overall element size. Table 2.1 shows the inputs used for the element sizing parameters for the baseline, finer, and coarser grid. Table 2.2 shows the node and element counts. The inflation layer sizing parameters were set based on the initial baseline grid to ensure that a \( y^+ \) value of \(<1\) is achieved throughout the domain. A sample of the baseline grid used in this study is shown in Figure 2-4.

Table 2.1: Mesh size inputs for ANSYS Workbench Mesher normalized by the depth of the upper trench \( (D_T) \).

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>20% Finer</th>
<th>20% Coarser</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Face Size</td>
<td>0.68</td>
<td>0.54</td>
<td>0.82</td>
</tr>
<tr>
<td>Max Size</td>
<td>0.68</td>
<td>0.54</td>
<td>0.82</td>
</tr>
<tr>
<td>Face Sizing</td>
<td>0.068</td>
<td>0.054</td>
<td>0.082</td>
</tr>
<tr>
<td>Aft Seal Outlet Face Sizing</td>
<td>0.034</td>
<td>0.027</td>
<td>0.041</td>
</tr>
<tr>
<td>Inflation First Layer Height</td>
<td>1.36E-04</td>
<td>1.36E-04</td>
<td>1.36E-04</td>
</tr>
<tr>
<td>Inflation Max Layers</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Growth Rate</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Table 2.2: Node and element counts for the (3) grids.

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Fine</th>
<th>Coarse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Nodes</td>
<td>3,412,571</td>
<td>5,096,290</td>
<td>2,471,738</td>
</tr>
<tr>
<td>Number Elements</td>
<td>8,676,980</td>
<td>13,037,428</td>
<td>6,254,181</td>
</tr>
<tr>
<td>% Change in Node from Baseline</td>
<td>0</td>
<td>49</td>
<td>-28</td>
</tr>
<tr>
<td>% Change in Elements from Baseline</td>
<td>0</td>
<td>50</td>
<td>-28</td>
</tr>
</tbody>
</table>

The grid was studied for a purge mass flow rate ratio of 0.02 with the three different grids. Another comparison was done for the baseline and fine grids with a lower purge mass flow rate ratio of 0.015. The objective of these studies was to determine the
grid's influence on the aerothermal parameters of the turbine stage. The results of the study are presented in Table 2.3. The change in stage efficiency, for example, is <0.03% between the finer and baseline grid, indicating good resolution for calculating stage efficiencies. An accuracy of approximately 0.1% is considered acceptable for this study. Stage pressure and temperature ratios show <0.06% variation between meshes. Stage $\Delta h_t$, which is a measure of the turbine work output, demonstrates <0.3% variation between grids. The normalized ingestion penetration depth\(^1\), which is a key parameter for comparing different geometries, varies by 0.75% between the baseline and coarse grid. Ingestion penetration depth does not vary between the baseline and fine grid. The results show that the baseline mesh is sufficient for achieving grid convergence for the computed flow field.

In addition to key aerothermal parameters of the turbine, the blade aerodynamic loading and the circumferential pressure variation in the upper trench region of the rim seal were compared for the different grids. The plots in Figure 2-5, show that the blade aerodynamic loading is not impacted by the three grid resolutions and the

\(^1\)Ingestion penetration depth is defined and characterized in Section 2.5.1
Table 2.3: Comparison of aerothermal parameters, stage efficiency, and ingestion penetration depth for the grid study. Stage pressure ratio, temperature ratio, and stage $\Delta h_t$ are normalized by their respective value for the baseline grid. Change in stage efficiency is $\Delta \epsilon$. Change in ingestion penetration depth is non-dimensionalized by the total depth of the wheelspace cavity as discussed in Section 2.5.1.

<table>
<thead>
<tr>
<th>% Change Compared to Baseline Grid</th>
<th>$C_e = 2.0%$</th>
<th>$C_e = 1.5%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage Pressure Ratio, (%)</td>
<td>-0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>Stage Temperature Ratio, (%)</td>
<td>-0.014</td>
<td>0.017</td>
</tr>
<tr>
<td>Stage $\Delta h_t$, (%)</td>
<td>-0.14</td>
<td>0.30</td>
</tr>
<tr>
<td>Change in Stage Efficiency, $\Delta \epsilon$, (%)</td>
<td>-0.032</td>
<td>0.020</td>
</tr>
<tr>
<td>Ingestion Penetration Depth, $D_{pen}$, (%)</td>
<td>0.000</td>
<td>0.754</td>
</tr>
</tbody>
</table>

The results of the grid study show that the baseline meshing parameters are adequate with acceptable variation in aerothermal parameters, blade loading, and ingestion penetration depth. The baseline meshing parameters are therefore used for each design point in the DOE.

Figure 2-5: Plot of blade mid-span blade pressure loading and circumferential pressure variation in the upper trench.
2.4 CFD Temporal Convergence

A sample of DOE cases were post-processed to ensure that the CFD solutions reached temporal convergence. The root mean square residuals of the momentum, mass, heat transfer, and turbulence equations were kept below $8 \cdot 10^{-5}$. In addition, monitor points, which are shown in Figure 2-6, were added throughout the purge cavity. The monitor points were used to inspect the variation of static pressure and CO$_2$ tracer gas concentration as a function of iteration number to ensure the solution in the purge cavity has achieved convergence. Figure 2-7 shows the variation in CO$_2$ mass fraction at one of the monitor points in the trench as a function of timestep. The solutions typically converged after 3000 iterations, however, all cases were run for a minimum of 4000 iterations.

![Figure 2-6: View of the monitor points in the upper trench.](image)

Figures 2-8 and 2-10 show that the CO$_2$ concentrations and static pressure measurements at the monitor points from Figure 2-6 all approach 0% change with respect to the previous timestep. Thus, both the CO$_2$ concentrations and the cavity pressures have temporally converged.

2.5 Ingestion Calculation - Seal Effectiveness

The introduction of this thesis stresses the importance of the secondary air system in preventing the ingress of hot mainstream gas into the wheelspace cavity between the
Figure 2-7: Plot of the \( CO_2 \) mass fraction versus timestep at a monitor point in the upper trench. The plot demonstrates the temporal convergence of the mass fraction at this location after approximately 1000 timesteps.

(a) Change in \( CO_2 \) concentration as a function of iteration number. Plot shows the temporal convergence of \( CO_2 \) concentration in the trench.

(b) Change in normalized static pressure, \( P_s \), as a function of iteration number. Plot shows the temporal convergence of \( P_s \) in the trench.

Figure 2-8: Design Point 1 Temporal Convergence.
(a) Change in \( \text{CO}_2 \) concentration as a function of iteration number. Plot shows the temporal convergence of \( \text{CO}_2 \) concentration in the trench.

Figure 2-9: Design Point 42 Temporal Convergence.

(b) Change in normalized static pressure, \( P_s \), as a function of iteration number. Plot shows the temporal convergence of \( P_s \) in the trench.

Figure 2-10: Design Point 125 Temporal Convergence.
stationary and rotating disks. Any hot mainstream gas that enters this cavity exposes turbine hardware to higher temperatures that they are not designed for. Ingestion is defined as the amount of hot air from the mainstream flowpath entering the inner wheelspace [19].

To quantify ingestion, Owen et. al [19] defined and use the effectiveness given as a ratio of the concentration of hot mainstream gas at a given location to the concentration of cooling sealant gas. Typically, the concentrations are determined by seeding the mainstream gas and/or sealant gas with a tracer gas, like $CO_2$. This ratio is defined by the expression in Equation 2.1. Here, $c_s$ is the concentration of the tracer at a given location in the purge cavity, $c_a$ is the concentration in the hot mainstream gas, and $c_c$ is the concentration in the sealant gas. For the purposes of this work, the CFD model seeds the FOS inlet with a known concentration of $CO_2$ gas, while the mainstream gas consists entirely of air. With the concentration of $CO_2$ known for the FOS inlet and the mainstream inlet, Equation 2.1 can be used to calculate the seal effectiveness. The purge cavity is sectioned in the CFD solution and the seal effectiveness is calculated at each cut plane. Figure 2-11 shows an example of the cut planes for a given design point. Section 2.5.1 goes into detail of how the effectiveness values at each cut plane are used to quantify ingestion for each design point analyzed.

$$\varepsilon = \frac{c_x - c_a}{c_c - c_a}$$

(2.1)

### 2.5.1 Penetration Depth

The penetration depth is used to compare the ingestion levels of the different seal geometries and purge mass flow rate ratios. For the purposes of this study, a sealing effectiveness of 95% is used as the cutoff value to define how far into the cavity the hot gas has penetrated. 95% is used based on conclusions by Owen et. al.[18] where they state that determining the flow rate that minimizes ingestion should be based on a near-sealed condition. Owen et. al. suggest a seal effectiveness of 95% rather than 100%, since it is difficult to determine the exact flow rate where ingress either
Figure 2-11: Cut planes in the purge cavity used to calculate seal effectiveness.

starts or ceases to exist. A sample test case is used to plot local seal effectiveness versus cut plane in the cavity, and is shown in Figure 2-12. Each point represents the seal effectiveness at a given cut plane, and the red square shows where the seal is considered to be purged with a seal effectiveness greater than 95%. The red square represents a location in the purge cavity where the hot gas from the mainstream has mixed sufficiently with the purge flow to reach a seal effectiveness of 95%. As such, the seal is no longer considered purged at this area. At this location, there could be a debit to the life of the turbine hardware if this position corresponds with an undesirable location. For example, if the penetration depth is inside the trench (cut plane 1-7), this level of ingestion may be acceptable depending on the requirements. However, if the penetration depth is inside the inner wheelspace cavity (cut plane >30), this could result in a turbine life debit and may be unacceptable. The ingestion penetration depth, a single value for each design point, is used as a quantitative measure of ingestion for determining the effectiveness of the different rim seal geometries.

When referring to ingestion penetration depth in later sections, the cut plane number is not used to identify the distance of ingestion. A streamline coordinate distance will be used to measure the penetration depth of ingestion with respect to
Figure 2-12: Representative plot of local seal effectiveness versus cut plane in the purge circuit. Cut plane 1 is the stage one blade hub and cut plane 30 is in the inner wheelspace cavity.

the exit of the purge cavity trench. The streamline path and the total depth of the purge circuit along the streamline coordinate, $D_{total}$, is shown in Figure 2-13. The ingestion penetration depth will be reported as $D_{pen}$ and is non-dimensionalized by the total depth of the purge circuit, $D_{total}$. Equation 2.2 expresses the non-dimensional ingestion penetration depth that is reported for each DOE case. Here, $D_{x,\varepsilon=95\%}$ is the depth into the purge circuit where the seal effectiveness is 95%.

$$D_{pen} = \frac{D_{x,\varepsilon=95\%}}{D_{total}} \cdot 100$$  \hspace{1cm} (2.2)

2.6 Effective Flow Area

The concept of effective flow area is used to quantify the resultant purge flow that travels to the mainstream gas path. Radial velocity, axial velocity, and vorticity in the purge cavity are used to determine the effective flow area at a given location in the purge cavity. Figure 2-14 demonstrates the idea of using radial velocity at the
Figure 2-13: Streamwise coordinate (s-coordinate) in the purge circuit used to measure ingestion penetration depth. The total depth of the purge circuit, $D_{total}$, is also shown.
angelwing cut plane to determine the effective flow area at this location. The gray contours in the purge cavity represent areas where the radial velocity is positive. At the angelwing cut plane, radial velocity vector arrows are added to signify where the change from positive to negative radial velocity occurs. Section-AA is then used to show in a circumferential cut plane how this idea can be used to determine an effective flow area by calculating the area of the region where the radial velocity is positive.

Figure 2-14: Diagram to demonstrate the idea of using radial velocities at the angelwing cut plane to aid in calculating the effective flow area.

Figure 2-15 builds on this idea by showing a plot of the superposition of positive radial velocity, positive axial velocity, and negative axial velocity. Figure 2-15 can be used to determine the effective flow area for many cut planes in the purge cavity. The effective flow area distribution in the purge cavity is used as an aid to determine how the purge flow travels to the main gas path.

The cavity trench is broken into regions where the dominant velocity component, shown in Table 2.4, is used to create isoclips at a given radial or axial location. The isoclips extend the circumference of the analyzed sector. For example, in Region 2, the dominant velocity component that indicates the purge flow is traveling toward the main gas path, is a positive axial velocity. A vorticity threshold value is also added
Figure 2-15: Diagram showing the combination of radial and axial velocity profiles to determine the effective flow area. Table 2.4 identifies the dominant velocity profile used to determine the effective flow area for each region.

Table 2.4: Dominant velocity profile used to determine the positive flow area for each region defined from Figure 2-15.

<table>
<thead>
<tr>
<th>Region</th>
<th>Dominant Velocity Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$+V_r$</td>
</tr>
<tr>
<td>2</td>
<td>$+V_z$</td>
</tr>
<tr>
<td>3</td>
<td>$+V_r$</td>
</tr>
<tr>
<td>4</td>
<td>$-V_z$</td>
</tr>
</tbody>
</table>

when creating the isolcic in the CFD post-processor to prevent the purge cavity vortices from being included in the area calculation. Figure 2-16 shows a sample of the cut planes and how they are used to calculate an effective flow area based on the velocity and vorticity threshold values. Effective flow area is calculated at each cut plane using Equation 2.3. Here $A_{x,ratio}$ is the effective flow area ratio, where $x$ designates a certain cut plane in the trench, $A_{x,\text{effective}}$ is the effective flow area, and $A_{x,\text{total}}$ is the total area of the cut plane. The effective flow area can be used to measure how the purge air that is flowing through the purge circuit is impacted by the changes in rim seal geometry and purge mass flow rate ratio.

$$A_{x,ratio} = \frac{A_{x,\text{effective}}}{A_{x,\text{total}}}$$  \hspace{1cm} (2.3)
2.7 Stage Efficiency Calculation

In addition to calculating the penetration depth of ingestion at each DOE point, the stage efficiency of the turbine stage is calculated. It should be noted that the stage efficiency for this calculation consists of the blade domain only. The stage efficiency is used to quantify the impact of rim seal geometry and purge flow on turbine performance. The efficiency is calculated using Equation 2.4, leveraging the work from Zlatinov’s thesis [30]. Turbine efficiency, simply stated, is the ratio of the actual work produced after accounting for losses (Equation 2.6) and the ideal work generated from the turbine (Equation 2.5). In the following equations, station 1 refers to the inlet of the stage one blade, and station 2 is the exit of the stage one blade.

\[
\varepsilon = \frac{w_{\text{expand,actual}}}{w_{\text{expand,ideal}}}
\]  

(2.4)

\[
w_{\text{expand,ideal}} = c_p (T_{t1,\text{mix}} - T_{t2})
\]  

(2.5)

\[
w_{\text{expand,actual}} = c_p T_{t1,\text{mix}} \left( 1 - \left( \frac{P_{t2}}{P_{t1}} \right)^{\frac{\gamma - 1}{\gamma}} \right)
\]  

(2.6)
The inlet flow from the NGV and flow from the cooling circuit is expanded through the stage one blade. Therefore, a mixed temperature of the two streams is required in order to calculate the ideal and actual work. Young et al. [26] describe a method for calculating the stagnation temperature after the mixing of two streams by using the steady-flow energy equation. Equations 2.8 and 2.9 can be used to calculate the stagnation temperature after mixing. In Equation 2.8, $c_{p,cold}$ is the specific heat capacity of the purge flow, and $c_{p,hot}$ is the specific heat capacity of the main gas path flow. The next step to solve Equations 2.5 and 2.6 is to replace the non-uniform inlet flow into the stage one blade with an equivalent uniform flow that would produce the same work output if expanded through a turbine. This is accomplished by using the idea of work averaged pressure, developed by Cumpsty and Horlock [8]. Equation 2.7 shows the expression for work averaged stagnation pressure derived in [8]. With the mixed stagnation temperature of the main flow and cooling flow and the work averaged stagnation pressure, the ideal and actual turbine work can be determined from Equations 2.5 and 2.6. The ideal and actual turbine work are then substituted into Equation 2.4 to calculate the stage efficiency.

$$P_{wa}^{\text{wa}} = \left[ \frac{\int T_i \hat{m} \, d\hat{m}}{\int \left( \frac{T_i}{\bar{T}} \right)^{\gamma-1} \, d\hat{m}} \right]^{\frac{\gamma}{\gamma-1}} \tag{2.7}$$

$$T_{t1,mix} = \frac{T_{t1,\text{inlet}S1B}c_{p,hot}(1 - \phi_I) + \phi_IT_{t1,\text{inlet}FOS}c_{p,cold}}{(1 - \phi_I)c_{p,hot} + c_{p,cold}} \tag{2.8}$$

$$\phi_I = \frac{\hat{m}_{S1B}}{\hat{m}_{FOS} + \hat{m}_{S1B}} \tag{2.9}$$

### 2.8 Design of Experiments (DOE) Setup

The computational DOE is constructed to vary the geometric and fluid input parameters. The following sections describe the DOE setup.
2.8.1 Baseline Geometry Definition and Characterization

The baseline geometry of the rim seal is used as a reference for defining and characterizing changes to the DOE input variables. In this section, the baseline geometry will be characterized by non-dimensionalizing the rim seal axial gap, radial gap, axial position of the stage one blade leading edge, and the internal purge cavity radii by the annulus inner radius length scale. Owen et. al [17] define and use gap ratio and seal clearance ratio as non-dimensional parameters for the wheelspace rim seal. 

\[ G = \frac{W_T}{r_{inner}}, \] is the gap ratio, and \( G_c = \frac{s_c}{r_{inner}} \), is the seal clearance ratio. Here \( W_T \) is the width of the trench, or the axial gap, and \( s_c \) is the seal clearance, or the radial gap. Similarly, this is applied to the axial position of the stage one blade leading edge and the internal purge cavity radii. 

Purge mass flow is characterized as a ratio of the purge flow mass flow rate to the annulus mass flow rate, \( C_c \). Purge mass flow rate ratio, \( C_c \), reported as a percentage of annulus air, %, is injected through the FOS inlet. Equation 2.10 shows the expression used to report the non-dimensional purge mass flow rate ratio, \( C_c \). The characterization values for the baseline geometry are shown in Table 2.5.

\[ C_c = \frac{m_c}{m_a} \cdot 100[\%] \]  

\[ (2.10) \]

Table 2.5: Baseline values for the axial and radial gap normalized to the annulus inner radius \( r_{inner} \).

<table>
<thead>
<tr>
<th>Characterization</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gap Ratio, ( G = \frac{W_T}{r_{inner}} )</td>
<td>0.0132</td>
</tr>
<tr>
<td>Seal Clearance Ratio, ( G_c = \frac{s_c}{r_{inner}} )</td>
<td>0.0143</td>
</tr>
<tr>
<td>Axial location of S1B LE relative to trench, ( G_{zLE} = \frac{z_{LE}}{r_{inner}} )</td>
<td>0.0164</td>
</tr>
<tr>
<td>Radius 1 Ratio, ( G_{r1} = \frac{r_1}{r_{inner}} )</td>
<td>0.0066</td>
</tr>
<tr>
<td>Radius 2 Ratio, ( G_{r2} = \frac{r_2}{r_{inner}} )</td>
<td>0.0033</td>
</tr>
<tr>
<td>Radius 3 Ratio, ( G_{r3} = \frac{r_3}{r_{inner}} )</td>
<td>0.0082</td>
</tr>
<tr>
<td>Radius 4 Ratio, ( G_{r4} = \frac{r_4}{r_{inner}} )</td>
<td>0.0033</td>
</tr>
<tr>
<td>Purge mass flow rate ratio, ( C_c )</td>
<td>1.5%</td>
</tr>
</tbody>
</table>
2.8.2 Initial Latin Hypercube DOE - Design Space Exploration

The input rim seal variables for this study were selected based on typical variations in these variables, and to ensure the solution space is fully explored. Purge mass flow rate ratio, axial gap ratio, radial gap ratio, and axial position of the stage 1 blade are typical input variables for rim seals. The internal purge cavity radius ratios and the radius ratio at the stage 1 blade hub were selected as additional parameters of interest. Figure 2-17 shows a sample geometry with the selected geometric input variables highlighted.

![Selected geometric input variables for analytical DOE.](image)

Once the input variables were selected, the design space was assessed to determine the lower and upper limits of the selected parameters. Table 2.6 shows the lower and upper limits for each input variable. The values are reported as ratios of the annulus inner radius, $r_{inner}$. This table is used in conjunction with a Latin hypercube sampling technique to create the initial 81 geometries to assess with RANS CFD. Latin hypercube sampling is a statistical modeling method used to generate a random sample of parameter values from a multidimensional distribution. McKay et. al. [15] discuss this method in detail. One of the benefits of Latin hypercube, is that when the output is dominated by only a few of the input variables, this method ensures...
that each of those components is represented in a fully stratified manner, no matter which components might turn out to be important. This method does a sufficient job at fully exploring the design space, which is the main reason it was selected for the initial DOE studies.

Table 2.6: Variation of input variables with respect to the baseline geometry.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial location of S1B LE relative to trench, $G_{ZLE}$</td>
<td>0.0082</td>
<td>0.0247</td>
<td></td>
</tr>
<tr>
<td>Radial gap ratio, $G_c$</td>
<td>0.0032</td>
<td>0.0143</td>
<td></td>
</tr>
<tr>
<td>Axial gap ratio, $G$</td>
<td>0.0131</td>
<td>0.0329</td>
<td></td>
</tr>
<tr>
<td>Radius 1, $r_1$</td>
<td>0.0033</td>
<td>0.0197</td>
<td></td>
</tr>
<tr>
<td>Radius 2, $r_2$</td>
<td>0.0016</td>
<td>0.0148</td>
<td></td>
</tr>
<tr>
<td>Radius 3, $r_3$</td>
<td>0.0082</td>
<td>0.0164</td>
<td></td>
</tr>
<tr>
<td>Radius 4, $r_4$</td>
<td>0.0016</td>
<td>0.0066</td>
<td></td>
</tr>
<tr>
<td>Purge mass flow rate ratio, $C_c$</td>
<td>0.50%</td>
<td>2%</td>
<td>%</td>
</tr>
</tbody>
</table>

2.8.3 Sub-DOE 1: Isolating Key Geometric Variables

The purpose of the initial DOE is to thoroughly explore the design space. To isolate the impact of a specific geometric parameter, a more structured approach is taken. One geometric parameter is varied from an upper to lower bound, and each value of that parameter is run for three different purge flow values. The remaining geometric parameters are held constant at the baseline value. Table 2.7 shows the selected values run for each purge mass flow rate ratio.

2.8.4 Sub-DOE 2: Varying purge mass flow rate ratio with Fixed Geometry

The third and final DOE is completed with the baseline geometry and a finer resolution on the purge mass flow rate ratio. This is used to identify how the flow structures in the trench cavity change as the purge mass flow rate ratio changes incrementally. Table 2.8 shows the values of purge mass flow rate ratio used for this DOE.
Table 2.7: Sub-DOE input parameter values for varying purge mass flow rate ratios isolating a critical geometric parameter.

<table>
<thead>
<tr>
<th>Varying Parameter</th>
<th>Purge mass flow rate ratio, $C_c$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.50%</td>
</tr>
<tr>
<td>Axial Gap Ratio, $(G)$</td>
<td>0.0132</td>
</tr>
<tr>
<td></td>
<td>0.0198</td>
</tr>
<tr>
<td></td>
<td>0.0247</td>
</tr>
<tr>
<td></td>
<td>0.0330</td>
</tr>
<tr>
<td>Radial Gap Ratio $(G_c)$</td>
<td>0.0143</td>
</tr>
<tr>
<td></td>
<td>0.0099</td>
</tr>
<tr>
<td></td>
<td>0.0066</td>
</tr>
<tr>
<td></td>
<td>0.0033</td>
</tr>
<tr>
<td>Radius 3 Ratio, $(G_{r3})$</td>
<td>0.0082</td>
</tr>
<tr>
<td></td>
<td>0.0107</td>
</tr>
<tr>
<td></td>
<td>0.0132</td>
</tr>
<tr>
<td></td>
<td>0.0165</td>
</tr>
</tbody>
</table>

Table 2.8: Sub-DOE Input parameter values for baseline geometry with varying purge mass flow rate ratio.

<table>
<thead>
<tr>
<th>Purge mass flow rate ratio $(C_c, %)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
</tr>
<tr>
<td>1.4</td>
</tr>
<tr>
<td>1.3</td>
</tr>
<tr>
<td>1.2</td>
</tr>
<tr>
<td>1.1</td>
</tr>
<tr>
<td>1.0</td>
</tr>
<tr>
<td>0.9</td>
</tr>
<tr>
<td>0.85</td>
</tr>
<tr>
<td>0.80</td>
</tr>
<tr>
<td>0.75</td>
</tr>
<tr>
<td>0.7</td>
</tr>
<tr>
<td>0.675</td>
</tr>
<tr>
<td>0.65</td>
</tr>
<tr>
<td>0.625</td>
</tr>
<tr>
<td>0.60</td>
</tr>
<tr>
<td>0.575</td>
</tr>
<tr>
<td>0.55</td>
</tr>
<tr>
<td>0.525</td>
</tr>
<tr>
<td>0.50</td>
</tr>
</tbody>
</table>
2.9 Summary

Chapter 2 outlines the processes, methods, and calculations used to interrogate the CFD simulations from the three sets of DOE geometries. The DOE's are used to determine the impact of rim seal geometry and purge mass flow rate ratio on ingestion and stage efficiency. Section 2.5.1 describes the quantitative means of comparing each test case by measuring the depth into the purge cavity that the hot gas reaches before the seal effectiveness reaches 95%. Section 2.7 describes the means of quantifying the impact on turbine performance. The method of implementing the DOE's and assessing the computational results has been outlined. Chapter 3 presents the interrogation of the results to establish the scaling of ingestion penetration depth in terms of geometric characteristics, flow features, and purge mass flow rate ratio.
Chapter 3

Results

The results from the DOE and their implications are described and discussed in this chapter. The CFD simulations from the DOE were analyzed to determine the variation in ingestion penetration depth due to changes in the rim-seal cavity geometrical configuration and purge mass flow rate ratio. The analysis found that it is possible to scale the ingestion penetration depth results with a scaling parameter derived from geometrical, purge flow, and turbine operating parameters. Likewise, the corresponding changes of the flow pattern in the rim-seal cavity purge flow path is determined through changes in the attributes characterizing discrete vortical structures in the purge cavity. The effect of rotation and geometry result in a complex flow structure and flow separation. The role of the vortical structures in the purge cavity on the ingestion penetration depth is elucidated, followed by determining its functional dependence on the scaling parameter. The chapter ends with the quantification of loss generation in the turbine stage in response to the introduction of purge flow for all the configurations that have been assessed.

3.1 Initial DOE Post-Processing

Section 2.8.2 describes the key input rim seal variables that make up the DOE. Table 3.1 summarizes the input rim seal variables and the assessed outputs.

The matrix of input rim seal variables and critical outputs are assessed using a
Table 3.1: Input rim seal variables and assessed outputs for the DOE.

<table>
<thead>
<tr>
<th>Input Rim Seal Variables</th>
<th>Purge Mass Flow Rate Ratio, $C_c$</th>
<th>Axial Gap Ratio, $G$</th>
<th>Radial Gap Ratio, $G_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial Position of Stage One Blade LE Relative to Trench, $G_{2LE}$</td>
<td>Internal Purge Cavity Radius Ratios, $r_{1,2,4}$</td>
<td>Radius Ratio of the Stage One Blade Hub, $r_3$</td>
<td></td>
</tr>
<tr>
<td>Critical Outputs</td>
<td>Ingestion Penetration Depth, $D_{pen}$</td>
<td>Stage Efficiency</td>
<td></td>
</tr>
</tbody>
</table>

variable influence profiler (VIP), which is a data post-processing tool available in ModelCenter. ModelCenter is a commercially available software package with DOE post-processing tools, such as the VIP analysis. The initial 81 case DOE is interrogated with a VIP analysis to determine the statistically significant rim seal variables that impact both ingestion penetration depth and stage efficiency. The VIP uses the input data to construct an interpolating Kriging model of the results. The interpolating Kriging model is a regression analysis between the assessed input and output variables that can be used to estimate the output variables at unmeasured input variables [11]. Using the Kriging model, a functional analysis of variance (ANOVA) is performed to compute input variable importance estimates, main effects, and interaction effects [16][21]. The ANOVA analysis within the VIP is then used to compare the input and output variable statistical means to determine statistical significance.

The results from the VIP analysis for ingestion penetration depth ($D_{pen}$), which is calculated using Equation 2.2, are shown in Figure 3-1. Table 3.2 shows a summary of the statistical significance for each of the input rim seal variables assessed. The results from the steady RANS CFD simulation and the Kriging model indicate that the rim seal variables that have the largest statistical significance on ingestion are purge mass flow rate ratio ($C_c$), radial gap ratio ($G_r$), axial gap ratio ($G$), and the radius ratio at the leading edge hub of the stage one blade ($G_3$). Together, these four variables account for 94% of the variation in ingestion penetration depth. Table 3.2 also suggests that the internal purge cavity radius ratios ($G_{r1,2,4}$) have no statistical significance in impacting ingestion, at least for the variation in radius ratios that were chosen for this study.
Table 3.2: Statistical significance of the assessed rim seal variables on ingestion penetration depth.

<table>
<thead>
<tr>
<th>Rim Seal Variable</th>
<th>Statistical Significance on $D_{pen}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purge Mass Flow Rate Ratio, $C_c$</td>
<td>65%</td>
</tr>
<tr>
<td>Radial Gap Ratio, $G_r$</td>
<td>24%</td>
</tr>
<tr>
<td>$G_{r3}$</td>
<td>3%</td>
</tr>
<tr>
<td>Axial Gap Ratio, $G$</td>
<td>2%</td>
</tr>
<tr>
<td>$G_{zLE}$</td>
<td>&lt; 1%</td>
</tr>
<tr>
<td>$G_{r1}$</td>
<td>&lt; 1%</td>
</tr>
<tr>
<td>$G_{r2}$</td>
<td>&lt; 1%</td>
</tr>
<tr>
<td>$G_{r4}$</td>
<td>&lt; 1%</td>
</tr>
</tbody>
</table>

The results for stage efficiency, which is calculated using Equation 2.4, are shown in Figure 3-2. Figure 3-2 shows that purge mass flow rate ratio accounts for 99% of the variation in turbine stage efficiency under the conditions and constraints implemented on the DOE. Therefore, of the input rim seal variables assessed, purge mass flow rate ratio is the only significant input rim seal variable that impacts the variation in stage efficiency.

The results from the VIP analysis are intended to be a preliminary assessment of the results and are used to guide the focus of study.

3.2 Sub-DOE 1 Results: Isolating Key Geometric Variables

The Latin Hypercube distribution of input rim seal variables provides a quantitative measure of the statistical significance the variables have on ingestion penetration depth. The distribution also ensures the solution space is fully explored within the range of parameters defined. However, isolating and identifying the impact of each input rim seal variable on ingestion penetration depth and flow structures is technically challenging. The challenge is in identifying the rim seal variable responsible for the change when the Latin Hypercube distribution changes all rim seal variables for each case. Therefore, it is necessary to infer the dominance of a specific variable by selecting input rim seal variables with a known associated variance. The use of
Figure 3-1: Variable Influence Profiler for Ingestion Penetration Depth. The results show that purge mass flow rate ratio, radial gap ratio, axial gap ratio, and radius ratio at the leading edge hub of the stage one blade account for 94% of the variation in ingestion penetration depth.

Figure 3-2: Variable Influence Profiler for Stage Efficiency. The VIP shows the statistical significance of the input variables analyzed in the DOE. The results show that purge mass flow rate ratio accounts for 99% of the variation in turbine stage efficiency.
sub-DOE's is useful for identifying changes in flow behavior since the exact variance in input rim seal variables is known. The first sub-DOE described in Section 2.8.3 constitutes a more structured approach to determine how varying each rim seal geometry variable independently impacts ingestion penetration depth for different purge mass flow rate ratios. The second sub-DOE defined in Section 2.8.4 focuses on a fixed geometry with varying purge mass flow rate ratios. The second sub-DOE is directed at determining how changes in purge mass flow rate ratio impacts flow structures in the purge cavity.

### 3.2.1 Impact of Rim Seal Geometry and Purge Flow on Ingestion Penetration Depth

The results from the first sub-DOE are used to assess the impact of radial gap ratio ($G_r$), axial gap ratio ($G$), and purge mass flow rate ratio ($C_c$) on ingestion penetration depth as each geometrical/operating parameter is varied independently. This sub-DOE is also used to affirm the findings and parametric trends from the VIP analysis on the initial DOE. The setup of this DOE is explained in detail in Section 2.8.3. The results from this study are shown in Figures 3-3, 3-4, and 3-5.

Figure 3-3 is a plot of the ingestion penetration depth as a function of axial gap ratio for three different purge mass flow rate ratios. This plot shows the sensitivity of ingestion penetration depth to purge mass flow rate ratio ($C_c$) and axial gap ratio. At the reference axial gap ratio, which is described in Section 2.8.1, the ingestion penetration depth increases by 8% when $C_c$ is reduced by 0.8%. As the axial gap ratio is increased, the change in ingestion penetration depth increases by 14% for the same change in $C_c$. This shows the combined dependence of ingestion penetration depth on axial gap ratio and $C_c$. Another observation is the change in ingestion penetration depth as a function of axial gap ratio is more sensitive at lower values of $C_c$. For example, over the range of axial gap ratios explored for $C_c = 1.5\%$, the ingestion penetration depth only changes by 3%. However, over the same range of axial gap ratios at $C_c = 1.0\%$, the ingestion penetration depth changes by 6%. This
observation suggests that there is change in flow behavior in the purge cavity that makes the rim seal less effective. As the axial gap ratio increases, the volume of the cavity that needs to be purged increases. For the same purge mass flow rate ratio, an increase in volume will lower the velocity of the purge flow, thus reducing the effectiveness of the purge flow at purging the cavity of hot gas.

![Graph](image)

**Figure 3-3:** Non-dimensional ingestion penetration depth versus axial gap ratio for varying purge mass flow rate ratio. The plot shows the dependence of ingestion penetration depth as a function of both axial gap ratio and purge mass flow rate ratio.

Figures 3-4 and 3-5 show the ingestion penetration depth as a function of radial gap ratio and the radius ratio at the leading edge hub of the stage one blade ($G_{r_n}$). Figures 3-4 and 3-5 both show that ingestion penetration depth is dependent on purge mass flow rate ratio. Figures 3-4 and 3-5 show that ingestion penetration depth increases by 6% when lowering the purge mass flow rate ratio from $C_c = 1.0\%$ to $C_c = 0.7\%$. Figures 3-4 and 3-5 also show that a change in radial gap ratio from $G_c = 0.0033$
to $G_c = 0.0066$ impacts the ingestion penetration depth by 1%. Similarly, a change in $G_{r_{3}}$ from $G_{r_{3}} = 0.0107$ to $G_{r_{3}} = 0.0132$ impacts the ingestion penetration depth by 0.5%. The impact from these two input variables is not as pronounced as that seen for changes in axial gap ratio, for this particular geometry, but they still have a measurable impact on ingestion penetration depth.

![Graph](image)

**Figure 3-4**: Non-dimensional ingestion penetration depth versus radial gap ratio for varying purge mass flow rate ratio. The plot shows the dependence of ingestion penetration depth as a function of both purge mass flow rate ratio and radial gap ratio.

The results from the first sub-DOE suggest that radial gap ratio ($G_c$), axial gap ratio ($G$), and the radius ratio at the leading edge hub of the stage one blade ($G_{r_{3}}$) impact ingestion penetration depth. These results are in accord with the findings from the VIP analysis performed on the initial 81 case DOE. Going forward, purge mass flow rate ratio ($C_c$), axial gap ratio ($G$), and radial gap ratio ($G_c$) will be the focus of future studies. These three rim seal variables are chosen since the VIP analysis...
Figure 3-5: Non-dimensional ingestion penetration depth versus $G_{r_3}$ for varying purge mass flow rate ratio. The plot shows the dependence of ingestion penetration depth as a function of both purge mass flow rate ratio and $G_{r_3}$. 
demonstrates they account for 91% of the variation in ingestion penetration depth, and the first sub-DOE confirms these variables independently have a measurable impact on ingestion penetration depth.

### 3.3 Non-dimensional Sealing Parameter

The method for calculating ingestion penetration depth is described in Section 2.5.1. Ingestion penetration depth is calculated for all cases from the DOE's, and used to quantify ingestion for each case. A non-dimensional grouping of these parameters is formulated to determine if ingestion penetration depth is scalable by rim seal geometry, flow, and operating parameters.

Existing literature describes the use of a non-dimensional sealing parameter, $\Phi$, which is expressed in Equation 3.1 [18] [22]. Here, $C_w$ is the non-dimensional flow rate ($\frac{m}{\mu b}$), $G_c$ is the seal-clearance ratio ($\frac{c}{b}$), $Re_\phi$ is the rotational Reynolds number ($\frac{\rho \Omega b^2}{\mu}$), $U$ is the bulk mean velocity of the sealing flow at the FOS inlet, $\Omega$ is the angular velocity of the rotating disk in $\text{rad/s}$, and $b$ is the radius of the seal. $\Phi$ can be simplified to a ratio of the velocity of the purge flow and the tangential velocity component from the wheelspeed of the turbine. Figure 3-6 shows that ingestion penetration depth has a weak dependence on $\Phi$, and it also shows that $\Phi$ is unable to scale with changes in rotational Reynolds number. Figure 3-7 shows that the residuals from the curve fit of the data have variation greater than $+/- 5\%$ for over one third of the assessed range of non-dimensional sealing parameter values. The variation shows that $\Phi$ does capture the general trend, but there is scope for enhancement by incorporating additional parameters characterizing its design and operation.

$$\Phi = \frac{C_w}{2\pi G_c Re_\phi} = \frac{U}{\Omega b}$$  \hspace{1cm} (3.1)

Thus, a new non-dimensional sealing parameter, $\Psi$, is formulated in this thesis. The results from the DOE have shown that radial gap ratio, axial gap ratio, and purge mass flow rate ratio are key input rim seal variables that have a statistical impact on ingestion penetration depth. As such, these variables are important for deriving the
Figure 3-6: Non-dimensional ingestion penetration depth versus presently defined non-dimensional sealing parameter, $\Phi = \frac{U}{\Omega_b}$. The trend of increasing rotational Reynolds number is not captured in $\Phi$. 
Figure 3-7: Least square fit of ingestion penetration depth versus non-dimensional sealing parameter residuals. For values of $\Phi > 0.06$, the ingestion penetration depth variation is within $+/ - 5\%$. For parameters $\Phi < 0.6$, the curve fit can be off by as much at $+/ - 16\%$. 
sealing parameter. The derivation begins by taking a ratio of the purge mass flow rate injected at the FOS inlet and the swirl component of the purge mass flow. This ratio is represented as $\alpha$ in Equation 3.2.

$$\alpha = \frac{\dot{m}_{c, \text{total}}}{\dot{m}_{c, \text{swirl}}} \quad (3.2)$$

From Equation 3.2, $\alpha$ is multiplied in the numerator and denominator by the density at the inlet of the FOS ($\rho$), the bulk mean velocity of the sealing flow at the FOS inlet ($U$), and a characteristic length of the upper trench ($D_T$). This is shown in Equation 3.3.

$$\frac{\alpha \rho}{\rho} = \frac{\alpha \rho U}{\rho U} = \frac{\alpha \rho U D_T}{\rho U D_T} \quad (3.3)$$

Multiplying Equation 3.3 by viscosity ($\mu$) in both the numerator and denominator form the Reynolds number of the cavity purge flow. This is shown in Equation 3.4.

$$\alpha \frac{\rho U D_T}{\mu} \frac{\mu}{\rho U D_T} = \alpha \frac{\mu}{\rho U D_T} \quad (3.4)$$

To introduce the rotor wheelspeed, Equation 3.4 is multiplied by the Rotational Reynolds number squared. This is shown in Equation 3.5. The square of the Rotational Reynolds number is needed to form an additional non-dimensional group.

$$\alpha Re_c \frac{\rho \Omega b^2}{\rho U D_T} \frac{\rho \Omega b^2}{\mu} = \alpha Re_c \frac{\rho \Omega b^2}{\rho U D_T} \frac{\mu}{\rho U D_T} = \alpha Re_c \frac{\Omega b^2}{U D_T} Re_\phi \quad (3.5)$$

Equation 3.5 shows the grouping of $\Omega b/U$, which is the inverse of Rossby number. The only remaining term in Equation 3.5 is a ratio of the rim seal radius to the depth of the trench. This is shown in Equation 3.6.

$$\alpha Re_c Re_\phi \frac{\Omega b}{U D_T} = \alpha Re_c Re_\phi \frac{b}{D_T} \quad (3.6)$$

Multiplying Equation 3.6 by the coolant flow Reynolds number forms an additional Reynolds number with the rim seal radius as the length scale, $Re_{c,b}$. $Re_{c,b}$ will be referred to as the rim seal Reynolds number. This is shown in Equation 3.7.
The final non-dimensional sealing parameter consists of a ratio between \( \Theta_{\text{total}} \) to \( \Theta_{\text{swirl}} \), the purge flow Reynolds number, Rotational Reynolds number, Rossby number, and rim seal Reynolds number. Equation 3.8 shows the derived non-dimensional sealing parameter used for scaling the ingestion penetration depth.

\[
\Psi = \frac{\alpha Re_c Re_\phi Re_{c,b}}{Ro}
\]  

(3.8)

Equation 3.8 can be expanded further to incorporate the axial gap ratio and radial gap ratio. This is done by representing the swirl component of the purge flow in terms of rim seal geometry and operating parameters. Physically, the axial gap and radial gap control the overlap of the blade angelwing, the width of the trench \( W_T \), the depth of the trench \( D_T \), and the radial sealing clearance \( s_c \). Together, these rim seal geometry variables control the cross-sectional area of the rim seal. These rim seal dimensions can be used to calculate a characteristic area of the upper portion of the purge cavity. Figure 3-8 shows the areas that are calculated using Equations 3.9 and 3.10. The upper portion of the purge cavity is considered as the characteristic area since it captures the changes in the rim seal geometry as the axial gap ratio and radial gap ratio are varied. Equation 3.11 is an expression for the swirl component of the purge mass flow rate in terms of rim seal geometry, rim seal radius, rotor wheelspeed, and density at the inlet of the FOS.

\[
A_1 = L_{\text{overlap}}s_c
\]  

(3.9)

\[
A_2 = W_T D_T
\]  

(3.10)

\[
\dot{m}_{c,\text{swirl}} = \rho b \Omega (A_1 + A_2)
\]  

(3.11)

Finally, substituting Equation 3.11 into Equation 3.8 for \( \dot{m}_{c,\text{swirl}} \), yields Equation
Equation 3.12 is used as the non-dimensional sealing parameter to characterize all the rim seal configurations from the DOE's.

\[
\Psi = \frac{m_{c,\text{total}}}{\rho b \Omega (A_1 + A_2)} \frac{Re_c Re_{\phi} Re_{c,b}}{Ro} \tag{3.12}
\]

In this thesis, the DOE results demonstrate that the non-dimensional ingestion penetration depth, \(D_{\text{pen}}\), has a dominant functional dependence on this newly derived non-dimensional parameter, \(\Psi\). The dependence is discussed further in Section 3.4.1.

### 3.4 Effects of Rim Seal Geometry and Purge Flow on Ingestion

#### 3.4.1 Ingestion versus Non-Dimensional Sealing Parameter

The variation in the ingestion penetration depth in terms of the non-dimensional sealing parameter formulated in Equation 3.12 is shown in Figure 3-9 for all DOE cases. It can be inferred from Figure 3-9 that there are three distinct regions of the curve/data points. For values of the non-dimensional sealing parameter between \(0.5 - 1.5 \cdot 10^{15}\) (Region 1), the ingestion penetration depth follows a steep negative and
nearly linear slope. Between \(6 - 14 \cdot 10^{15}\) (Region 3), the penetration depth follows a shallow negative and nearly linear slope. Finally, between \(1.5 - 6 \cdot 10^{15}\) (Region 2), the curve transitions between the two bounding slopes. Of particular interest, is how the region where the non-dimensional sealing parameter value is \(2.3 - 14 \cdot 10^{15}\) sees changes in ingestion penetration depth < 5%. This suggests a threshold in ingestion penetration depth behavior at \(\Psi = 2.3 \cdot 10^{15}\). The \(\Psi = 2.3 - 14 \cdot 10^{15}\) region of the curve signifies risk adverse rim seal configurations (set by geometry and operating conditions) where ingestion is minimized. For non-dimensional sealing parameters between \(0.5 - 2.3 \cdot 10^{15}\), even the smallest change in non-dimensional sealing parameter can result in large changes in ingestion penetration depth.

![Fitted Curve](image)

Figure 3-9: Non-dimensional ingestion penetration depth versus non-dimensional sealing parameter, \(\Psi\). The plot shows that the non-dimensional ingestion penetration depth, \(D_{pen}\), has a dominant functional dependence on the non-dimensional sealing parameter, \(\Psi\).

To fit a curve to the computational data, a least square fit of a power curve is
used. Equation 3.13 shows the statistical fit equation of ingestion penetration depth versus non-dimensional sealing parameter.

\[
D_{pen} = 6.8968 \cdot 10^{25} \cdot (\Psi)^{-1.69} + 1.465 \tag{3.13}
\]

In order to demonstrate the quality of the curve fit, the \(R^2\) value and the curve fit residuals are assessed. The \(R^2\) value of the fit is 84% and indicates the amount of variation in the data that is explained by the regression analysis [7]. \(R^2\) is also known as the coefficient of determination and can be expressed as the ratio of the regression sum of squares and the total variation in \(y\), where \(y\) is the dependent variable [7]. The residuals between the curve fit values for \(D_{pen}\) and the calculated values of \(D_{pen}\) from the CFD simulations are also examined to assess the goodness of fit. Figure 3-10 shows the curve fit residuals. Figure 3-10 shows that the statistical fit captures the variance in ingestion penetration depth to \(+/- 5\%\) for non-dimensional sealing parameters \(\Psi > 2.3 \cdot 10^{15}\). For parameters \(\Psi < 2.3 \cdot 10^{15}\), the curve fit can be off by as much as \(+/- 15\%\). To account for this variation, the \(+/- 3\sigma\) fitted curves are calculated and shown in Figure 3-9. \(+/- 3\sigma\) represents a band of \(+/- 3\) standard deviations away from the mean of the data. This is selected so the bounding equations have a 99.74\% confidence that values of ingestion penetration depth will fall within the bounds for a given non-dimensional sealing parameter. Equations 3.14 and 3.15 show the equations with 99.74\% \((+/- 3\sigma)\) confidence.

\[
D_{pen,+3\sigma} = 1.8558 \cdot 10^{33} \cdot (\Psi)^{-2.143} + 0.0449 \tag{3.14}
\]

\[
D_{pen,-3\sigma} = 6.3102 \cdot 10^{19} \cdot (\Psi)^{-1.237} + 2.885 \tag{3.15}
\]

A non-dimensional sealing parameter has been developed based on rim seal geometry, purge mass flow rate ratio, Rotational Reynolds number, purge flow Reynolds number, rim seal Reynolds number, and Rossby number. This non-dimensional sealing parameter, \(\Psi\), can be used to collapse the ingestion penetration depth of over 130 CFD solutions. The next objective is to determine why the response of the curve
Figure 3-10: Least square fit of ingestion penetration depth versus non-dimensional sealing parameter residuals. For values of $\Psi > 2.3 \cdot 10^{15}$, the ingestion penetration depth variation is within $+/ - 5\%$. For parameters $\Psi < 2.3 \cdot 10^{15}$, the curve fit can be off by as much at $+/ - 15\%$. 
can be broken into the three regions discussed above. The following sections aim to answer this question.

3.4.2 Impact of Purge Flow on Purge Cavity Flow Structures

Radial gap ratio \((G_r)\), axial gap ratio \((G)\), and purge mass flow rate ratio \((C_c)\) have been identified from the DOE as the key rim input seal variables that impact the ingestion penetration depth. The next step is to determine the physical rational explaining the observations from Section 3.1 and 3.2.1, and why the identified rim seal variables have an impact on ingestion penetration depth. To begin to answer this, a multiple step process is used. The process is highlighted below:

1. Section 3.4.2.1: Qualitative investigation of the two dimensional (2D) streamlines in the purge cavity to identify flow patterns. The streamlines are obtained from the projection of streamlines on the axial-radial section. This is completed for the fixed geometry and varying purge mass flow rate ratio DOE.

2. Section 3.4.2.2: Qualitative investigation of the streamlines in the purge cavity to identify flow patterns at the three regions from the ingestion penetration depth versus non-dimensional sealing parameter curve in Figure 3-9. The assessment of streamline patterns is completed for all DOE points.

3. Section 3.4.2.3: Quantitative characterization of the stage one blade angelwing vortex. The angelwing vortex in the purge cavity is quantified to assess the hypothesis from Section 3.4.2.1 that vortices change size, strength, and position as a function of purge mass flow rate ratio.

4. Section 3.4.2.4: Quantitative characterization of the upper trench vortex using non-dimensional circulation for all DOE points.
3.4.2.1 Purge Flow Streamline Pattern

The 2D (radial and axial cross section) surface streamlines in the purge cavity\(^1\) provide insight into the flow structures and flow movement through the cavity. The 2D surface streamlines can be determined from the axial and radial velocity components. The circumferential velocity, or the swirl component, dominates the velocity magnitude in the purge circuit flow field. The high rotational speed of the rotor disk results in a shear force that acts to accelerate the fluid in the immediate vicinity of the rotating disk to the rotational speed of the disk. The circumferential velocity decreases away from the rotating disk to a vanishing value on the stationary stator wall. Thus, the toroidal vortices in the purge cavity have a finite circumferential velocity within the cores. Changes in the axial and radial velocities can be assessed to understand the flow structures in the purge cavity. Thus, the streamline projections are used for assessing the flow structures in the purge cavity. The toroidal vortex in the upper trench and the dominance of the circumferential velocity is shown in Figure 3-11 for a representative rim seal configuration. The streamline velocity vectors in Figure 3-11 are almost entirely in the circumferential direction, with small radial and axial velocity components at a given location in the purge cavity. The dominant circumferential velocity compared to the respective axial and radial velocity is evident even along the stator wall. Along the stator wall, the total velocity in the stationary frame is low (dark blue velocity contour) compared to the higher (light blue velocity contour) near the rotating disk. For all the streamline configuration plots, the color of the streamline is the velocity in the stationary frame normalized by the wheel speed of the rotor at the rim seal radius, \(b\).

Figure 3-14 shows the progression of 2D surface streamlines in the purge cavity as the rim seal geometry is held fixed and the purge mass flow rate ratio is varied from 0.005-0.015. For a purge mass flow rate ratio of 0.015, there is a clear counterclockwise (CCW) vortex in the upper trench region. Other than this CCW vortex, the flow separation in the purge cavity is minimal as the flow moves from the inlet of the purge cavity to the main flow path. Figure 3-12 demonstrates how there is minimal flow

\(^1\)The purge cavity region is defined in Figure 2-2 from Section 2.1.
Figure 3-11: Projection of the streamlines in the $r$-$z$ plane is shown in the LHS figure. The RHS figure shows the $r$-$\theta$ view of the velocity vectors for the streamlines in the upper trench of the purge cavity. The LHS plot shows the toroidal vortex structure in the upper trench in the $r$-$z$ plane. The RHS plot shows that the vector arrows of the streamlines in the upper trench are almost entirely in the circumferential direction.

Separation in the purge cavity for a purge mass flow rate ratio case of 0.015.

Figure 3-12: Surface streamlines demonstrating the areas of the purge cavity with no flow separation for a 0.015 purge mass flow rate ratio case.

As the purge mass flow rate ratio decreases from 0.015-0.011 ($C_c = 1.5 - 1.1\%$), a CCW vortex begins to form near the stage one blade angelwing as more of flow separates around the stage one blade angelwing and forms a re-circulation zone. As purge mass flow rate ratio reduces further, additional vortices form above and below
the angelwing as the velocity is reduced. These vortices start to obstruct the path that the purge flow travels to reach the main flow path. The vortices also act as a source of entropy generation through viscous dissipation. This idea of the vortices acting as a source of loss is explored in Section 3.4.5. Figure 3-14 also shows a progression of the vortices as the purge mass flow rate ratio varies. The size of the vortices can be quantitatively measured by the width and height of the vortex in the 2D surface streamline projections. The width and height of the vortex are determined by the dividing streamlines that bound the re-circulation zone. At purge mass flow rate ratio values between 0.005-0.0075 ($C_e = 0.5-0.75\%$), the upper trench vortex blocks nearly the entire purge cavity, and the angelwing vortex width is nearly that of the axial gap. As the purge mass flow rate ratio increases from 0.0075-0.012 ($C_e = 0.75-1.2\%$), the upper trench vortex width decreases to roughly 3/4 of the axial gap, and similarly the angelwing vortex width decreases. These observations indicate that the purge cavity vortices change size as the purge mass flow rate ratio is varied. Additionally, as the purge mass flow rate ratio increases from 0.006-0.012 ($C_e = 0.6-1.2\%$), the center of the upper trench vortex moves from the bottom of the trench to the mid-point of the trench depth. This observation highlights that the positions of the vortices in the purge cavity also change as the purge mass flow rate ratio is varied.

The observations from these streamline configurations aids in formulating the following hypotheses:

1. The trench vortices are a physical flow feature that change in size and strength as the purge mass flow rate ratio is varied. The vortices act as flow blockages that reduce the effective flow area distribution throughout the purge cavity. As a result, this impacts the ingestion penetration depth. Of particular interest is the upper trench vortex highlighted in Figure 3-13.

2. The formation of vortices in the purge cavity impacts the effective flow area distribution throughout the purge cavity. For purge mass flow rate ratios greater than 0.011 – 0.012, the formation of vortices is minimal and the effective flow area is nearly that of the cavity. For low purge mass flow rate ratios, the
Figure 3-13: Effective flow area at the location of the upper trench vortex for purge mass flow rate ratios of 0.015 and 0.008 ($C_e = 1.5\%$ and $C_e = 0.8\%$). This plot shows the width of the upper trench vortex changing size as a function of purge mass flow rate ratio. As a result, the effective flow area distribution of the purge flow is impacted.

formation of vortices acts as blockage and results in less effective flow area as more of the purge air gets entrained in the vortices. This is demonstrated in Figure 3-13, which shows how the effective flow area at the upper trench vortex for $C_e = 1.5\%$ is larger than that of the $C_e = 0.8\%$ case. The streamlines also highlight how the width of the upper trench vortex for the $C_e = 1.5\%$ case is approximately half the width of the trench. But, for the $C_e = 0.8\%$ case, the width is nearly three quarters of the trench width. The streamlines can be seen getting entrained in the vortex for the $C_e = 0.8\%$ case more than in the $C_e = 1.5\%$ case. As a result, the width and height of the upper trench vortex increase.

3.4.2.2 Parametric Trend in Streamline Pattern Variation

The assessed variation in streamline pattern as described in Section 3.4.2.1, is shown in Figure 3-14. The variation in size and position of the vortical flow structures in the
<table>
<thead>
<tr>
<th>$C_c = 1.5%$</th>
<th>$C_c = 1.4%$</th>
<th>$C_c = 1.3%$</th>
<th>$C_c = 1.2%$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
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<tr>
<td>$C_c = 1.1%$</td>
<td>$C_c = 1.0%$</td>
<td>$C_c = 0.9%$</td>
<td>$C_c = 0.85%$</td>
</tr>
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<td><img src="image5.png" alt="Image" /></td>
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<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
</tr>
<tr>
<td>$C_c = 0.8%$</td>
<td>$C_c = 0.75%$</td>
<td>$C_c = 0.7%$</td>
<td>$C_c = 0.675%$</td>
</tr>
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<td><img src="image9.png" alt="Image" /></td>
<td><img src="image10.png" alt="Image" /></td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
</tr>
<tr>
<td>$C_c = 0.65%$</td>
<td>$C_c = 0.625%$</td>
<td>$C_c = 0.6%$</td>
<td>$C_c = 0.575%$</td>
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<td><img src="image13.png" alt="Image" /></td>
<td><img src="image14.png" alt="Image" /></td>
<td><img src="image15.png" alt="Image" /></td>
<td><img src="image16.png" alt="Image" /></td>
</tr>
<tr>
<td>$C_c = 0.55%$</td>
<td>$C_c = 0.525%$</td>
<td>$C_c = 0.5%$</td>
<td></td>
</tr>
<tr>
<td><img src="image17.png" alt="Image" /></td>
<td><img src="image18.png" alt="Image" /></td>
<td><img src="image19.png" alt="Image" /></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3-14: Streamlines in the purge cavity for varying purge mass flow rate ratios (0.005-0.015). This plot demonstrates the vortices that form and change in width and height in the upper trench, near the stage one blade angelwing, and above and below the angelwing. The values of the purge mass flow rate ratio in $C_c$ are shown.
purge cavity as the purge mass flow rate ratio is changed is evident. The ingestion penetration depth versus non-dimensional sealing parameter plot shown in Figure 3-9 is used as a tool to plot the streamlines for individual cases at each of the three regions. The streamlines are plotted for all the assessed DOE cases and are used to identify flow patterns.

Figure 3-15: Non-dimensional ingestion penetration depth versus non-dimensional sealing parameter with streamline plots of the upper trench vortex. This plot is used as an aid to identify flow structure patterns in the three regions identified from the ingestion penetration depth versus non-dimensional sealing parameter plot.

Figure 3-15 shows the plots of streamlines for various test cases that are superimposed onto the ingestion penetration depth versus non-dimensional sealing parameter curve. Figure 3-15 indicates a trend for the streamlines in the upper trench region. Assessing the streamline plots for each region of the curve helps qualitatively characterize the upper trench vortex formation and progression as the non-dimensional sealing parameter increases. With the global trend identified from Figure 3-15, the streamlines from the plot are consolidated in Figure 3-16 to show the difference in the
upper trench vortex for each region. Figure 3-16 also includes the associated $\Psi$ value for each streamline plot. Starting with Region 1, the streamlines show the upper trench vortex at the base of the trench encompasses a large percentage of the trench width. The upper trench vortex center sits radially at the bottom of the trench and the width and height of the vortex are relatively large. Correspondingly, Region 1 of the curve is where the hot gas penetrates the furthest into the purge circuit\(^2\). Region 3, however, has an upper trench vortex that, while still present, has streamlines that make it along the rotor wall and to the mainstream flow path. Region 3 is also the region of the ingestion penetration depth curve where the hot gas does not protrude past the depth of the trench. Region 2 shows a progression, from $\Psi = 1.33 \cdot 10^{15}$ to $\Psi = 2.51 \cdot 10^{15}$, of the upper trench vortex diverging from the Region 1 vortex width and height and becoming more like those seen in Region 3. In Region 2, the upper trench vortex progresses from having a radial center of the vortex that is located near the bottom of the trench for $\Psi = 1.33 - 2.15 \cdot 10^{15}$ to a radial center that is located in the middle of the upper trench for $\Psi = 2.19 - 2.51 \cdot 10^{15}$.

The ideas and hypothesis formed through the inspection of the streamlines are qualitative assessments used to identify flow structure trends. However, the assessment lays the groundwork for flow structures to focus on for quantitative investigation. The quantitative assessment is continued in Section 3.4.2.4. The qualitative assessment of the streamlines in the purge cavity also highlights the potential utility of pattern recognition in solving the aerothermal system of a gas turbine.

### 3.4.2.3 Vortex Characterization

The streamline investigation shows how the upper trench vortex and angelwing vortex change as the non-dimensional sealing parameter is varied. Prior to identifying the upper trench vortex as a dominant vortex in the purge cavity, the angelwing vortex was identified to be a key flow structure. To understand if there is any merit in the qualitative observations from Section 3.4.2.2, the angelwing vortex is quantitatively characterized. The angelwing vortex is characterized by a vortex center radius, height,\(^2\)

\(^2\)The purge circuit is defined in Figure 2-2 from Section 2.1.
Figure 3-16: Progression of the upper trench vortex streamlines for the identified regions from the ingestion penetration depth versus non-dimensional sealing parameter curve. Non-dimensional sealing parameter increases from left to right and demonstrates the grouping of upper vortex sizes for the three ingestion penetration depth curve regions.
width, and maximum vorticity. The width and height of the vortex are determined by the dividing streamlines. The geometry characterization of the vortex is shown in Figure 3-17.

Figure 3-17: Diagram showing the angelwing vortex characterization. The diagram on the left indicates the angel wing vortex that is assessed. The diagram on the right highlights the vortex center radius, height, and width.

The vortex center radius, width, height/width aspect ratio, and the maximum vorticity were calculated for the 19 cases from the second sub-DOE described in Section 2.8.4. The 19 cases were run with the same geometry, and the purge mass flow rate ratio was varied from 0.005 – 0.015. Varying only $C_c$ is an effort to isolate the impact of a single rim seal variable on the flow structures in the purge cavity. The results are shown in Figure 3-18. The plots show that the size of the vortex, the vortex center radius, and the maximum vorticity change as a function of purge mass flow rate ratio. In Figure 3-18, the vortex center radius is normalized by the rim seal radius, the vortex width is normalized by the axial gap, and the vorticity is normalized by the vorticity for the case where $C_c = 0.5\%$. The plot of vortex center radius versus purge mass flow rate ratio shows how the radius increases as the purge mass flow rate ratio increases. Conversely, the width, aspect ratio, and maximum vorticity of the angelwing vortex decrease as the purge mass flow rate ratio
increases. The aberrations in the data are a result of manually post-processing the vortex characteristics. Manually determining the vortex width, height, and center radius using the dividing streamlines results in some subjectivity. However, the trend with purge mass flow rate ratio remains clear. Figure 3-19 is included to show how the ingestion penetration depth increases as the non-dimensional sealing parameter decreases for these 19 cases. The trend from the ingestion penetration depth curve for the 19 cases is similar to the observed behavior of the the width, aspect ratio, and maximum vorticity of the angelwing vortex.

![Figure 3-19: Plots of angelwing vortex center radius, width, height/width ratio, and maximum vorticity. Vortex center radius versus purge mass flow rate ratio shows how the radius increases as the purge mass flow rate ratio increases. Conversely, the width, aspect ratio, and maximum vorticity of the angelwing vortex decrease as the purge mass flow rate ratio increases.](image)

This assessment suggests there is merit to the hypothesis put forward in Section 3.4.2.1 that the size, height, width, location, and strength of the purge cavity vortices change as a function of the non-dimensional sealing parameter. The trend in ingestion penetration depth from Figure 3-19 also matches the trend observed from Figure 3-9. The similar trend in ingestion penetration depth suggests that the vortex
Figure 3-19: Non-dimensional ingestion penetration depth versus non-dimensional sealing parameter for the fixed geometry and varying purge mass flow rate ratio DOE. The plot shows that for a fixed geometry with varying purge mass flow rate ratio, a trend similar to that from Figure 3-9 is observed.
characterization results for the second sub-DOE are representative of all the DOE cases, and confirms that a similar study of the vortices should be pursued for all DOE cases. The combined effect from vorticity and size of the vortex on ingestion penetration depth has yet to be shown. However, both maximum vorticity and vortex aspect ratio monotonically decrease as a function of purge mass flow rate ratio, which is similar to the trend observed in Figure 3-19 for ingestion penetration depth as a function of non-dimensional sealing parameter. These findings aid in substantiating the hypothesis from Section 3.4.2.1 that vortices play a role in ingestion. The angel-wing vortex is shown to increase in width as purge mass flow rate ratio decreases, which will directly impact the effective flow area of the purge flow at this location. The increase in maximum vorticity as the purge mass flow rate ratio decreases is an indication that more of the purge flow is being entrained in the vortex. As more of the flow is entrained, a lesser amount of purge flow is available for purging the cavity.

The upper trench vortex can be similarly characterized as the results in Figure 3-16 would imply: note that the results shown in Figure 3-16 involve variation in axial gap ratio, radial gap ratio, internal purge circuit radius ratios, and the axial position of the stage one blade leading edge, and not just the purge mass flow rate ratio. The characterization of upper trench vortex is described in the next section.

3.4.2.4 Upper Trench Vortex Circulation

The assessment of the streamlines in the purge cavity identified the upper trench vortex as a critical purge cavity vortex. The upper trench vortex is shown in Figure 3-20. The upper trench vortex changes position, size, and strength as the purge mass flow rate ratio and rim seal geometry are varied. For the angel-wing vortex, the characterization in Section 3.4.2.3 included maximum vorticity, height to width ratio, and vortex center radius. Here, the characterization will be expanded to include circulation of the upper trench vortex to quantitatively assess the changes in strength and size, since circulation is a measure of the strength of the vortex. The circulation around a closed curve, \( C \), is defined as the line integral of velocity along a contour encompassing the vorticity [13]. This is expressed in Equation 3.16. Stokes' theorem
relates the integral over an open surface $A$ to the line integral around a surface’s bounding curve $C$. This relationship is defined in Equation 3.17 and is used in the CFD post-processor to calculate the circulation of the upper trench vortex. The circulation can be determined from Equation 3.16 or 3.17.

$$\Gamma = \oint_C u \cdot ds$$  \hspace{1cm} (3.16)

$$\Gamma = \oint_C u \cdot ds = \iint_A \omega \cdot n dA$$  \hspace{1cm} (3.17)

The circulation of the upper trench vortex is non-dimensionalized by the purge mass flow rate, the rim seal radius, and the density at the inlet of FOS as given in Equation 3.18 below.

$$\Gamma_{ND} = \frac{\Gamma}{\dot{m}_c/(\rho b)}$$  \hspace{1cm} (3.18)

The non-dimensional circulation is plotted as a function of the non-dimensional sealing parameter in Figure 3-21. The results show that $\Gamma_{ND}$ follows a similar trend
as that seen for ingestion penetration depth. In Figure 3-21, a steep negative slope characterizes non-dimensional sealing parameter values between $\Psi = 0.5 - 1.5 \cdot 10^{15}$, a transition between $\Psi = 1.5 - 4 \cdot 10^{15}$ occurs, and finally a negative shallow slope characterizes values between $\Psi = 4 - 14 \cdot 10^{15}$. For $\Psi < 2.3 \cdot 10^{15}$, the non-dimensional circulation is $> 30$, and for $\Psi > 2.3 \cdot 10^{15}$, the non-dimensional circulation asymptotes to a value of 25. Similarly, $\Psi = 2.3 \cdot 10^{15}$ is a threshold value of the non-dimensional sealing parameter, above which there is marginal change in the non-dimensional circulation of the upper trench vortex. Figure 3-22 shows non-dimensional circulation of the upper trench vortex and ingestion penetration depth as a function of non-dimensional sealing parameter on the same plot. Figure 3-22 highlights how $\Gamma_{ND}$ follows a similar trend as that seen for ingestion penetration depth. The streamlines in the purge cavity for $\Psi = 2 - 2.5 \cdot 10^{15}$ correspond with the transition from no flow separation in the purge cavity to flow separation across the stage one blade angelwing and in the bottom of the trench. The flow separation that occurs results in a re-circulation zone. The transition from no separation to re-circulation zone is first shown in Figure 3-14. The re-circulation zones result in an increase in $\Gamma_{ND}$ of the upper trench vortex as the non-dimensional sealing parameter is lowered further. An explanation for the observed trend in the variation of non-dimensional circulation with the non-dimensional sealing parameter is put forward in Section 3.4.4.

3.4.3 Impact of Upper Trench Vortex Circulation on Ingestion

Figure 3-21 shows the variation in $\Gamma_{ND}$ of the upper trench vortex with non-dimensional sealing parameter. To elucidate the regions of high non-dimensional circulation correspond to greater ingestion penetration depth, the curve from Figure 3-9 is re-created and each data point is scaled in size by the non-dimensional circulation value. In other words, a plot with scaled data points is used to denote the magnitude of the upper trench vortex non-dimensional circulation: data points that are large in size correspond to high $\Gamma_{ND}$ values, and data points small in size are values with low $\Gamma_{ND}$. Figure 3-23 shows the variation in ingestion penetration depth with non-dimensional sealing parameter. It is evident that the grouping of data points with $\Psi > 2.3 \cdot 10^{15}$
Figure 3-21: Non-dimensional circulation versus non-dimensional sealing parameter. For $\Psi < 2.3 \cdot 10^{15}$, the circulation is greater than 30 and the ingestion penetration depth is greater than 5%. For $\Psi > 2.3 \cdot 10^{15}$, the circulation remains below 30, and correspondingly, the ingestion penetration depth near these values is <5%.

Figure 3-22: Non-dimensional circulation and ingestion penetration depth versus non-dimensional sealing parameter. The plot shows that $\Gamma_{ND}$ follows a similar trend as that seen for ingestion penetration depth.
have $\Gamma_{ND}$ values $< 30$, which is consistent with Figure 3-21. There is also the grouping of data points with $\Psi = 0.5 - 2.3 \cdot 10^{15}$ where $\Gamma_{ND} > 30$, which corresponds with increased ingestion penetration depth. The implication is the non-dimensional circulation of the upper trench vortex is an indicator of the ingestion penetration depth.

3.4.4 Upper Trench Circulation and Purge Cavity Effective Flow Area

To quantify why the circulation of the upper trench plays a critical role in impacting the ingestion penetration depth, the effective flow area at the upper trench vortex is assessed. The method used to calculate the effective flow area is described in Section 2.6.

3.4.4.1 Effective Flow Area Distribution near Upper Trench Vortex

The effective flow area at a position in the purge cavity is defined in Section 2.6 to be the flow area that the purge flow travels through in the direction of the mainstream gas path. To determine a representative effective area near the upper trench vortex to use for all DOE points, an effective area distribution is calculated at three different radial cut plane locations. The cut planes used for this analysis are located at $r/b = 1.011$, $r/b = 1.007$, and $r/b = 1.004$ and are shown in Figure 3-24. Here, $r$ is the radial coordinate from centerline and $b$ is the rim seal radius. Figure 3-25 is a plot of effective flow area as a function of non-dimensional circulation for the fixed geometry and varying purge mass flow rate ratio DOE. This plot demonstrates that the effective flow area distribution at the three radial cut planes near the upper trench vortex trend similarly. The effective flow area distribution is nearly identical for upper trench vortex $\Gamma_{ND}$ values between 30-80. At $\Gamma_{ND} = 30$, the effective flow area begins to increase as the non-dimensional circulation decreases. The cut plane at $r/b = 1.004$ is selected as the representative cut plane to evaluate the effective flow area for all DOE points. The assessment of the streamline configurations in Section
Figure 3-23: Non-dimensional ingestion penetration depth versus non-dimensional sealing parameter. Data points are scaled by non-dimensional upper vortex circulation. Large data points indicate high $\Gamma_{ND}$. There is a clear grouping of data points in this plot for non-dimensional sealing parameters $\Psi > 2.3 \cdot 10^{15}$ where $\Gamma_{ND} < 30$ and the ingestion penetration depth is $< 5\%$. There is also a clear grouping of data points with $\Psi = 0.5 - 2 \cdot 10^{15}$ where $\Gamma_{ND} > 30$ and the ingestion penetration depth is $> 5\%$. 
3.4.2.1 demonstrates how the upper trench vortex moves radially as the purge mass flow rate ratio is varied. Therefore, the cut plane at $r/b = 1.004$ is used to ensure the variation in upper trench vortex radial position is captured.

![Cut planes for upper trench vortex](image)

Figure 3-24: Cut planes used to evaluate the effective flow area distribution near the upper trench vortex.

### 3.4.4.2 Effective Flow Area at Upper Trench Vortex

Figure 3-26 is a plot of the effective flow area at the upper trench vortex as a function of non-dimensional sealing parameter. The data points in Figure 3-26 are scaled by $\Gamma_{ND}$ for the upper trench vortex. In other words, the size of the data point reflects the magnitude of the non-dimensional circulation of the upper trench vortex. For non-dimensional sealing parameters $\Psi < 2.3 \cdot 10^{15}$, the effective flow area is reduced to less than 70% of the total area at $r/b = 1.004$. For $\Psi < 2.3 \cdot 10^{15}$, $\Gamma_{ND} > 30$ for the upper trench vortex, and the ingestion penetration depth is $> 5\%$. When the effective flow area is $> 70\%$ of the total area at $r/b = 1.004$, $\Gamma_{ND} = 25 - 30$ for the upper trench vortex, $\Psi > 2.3 \cdot 10^{15}$, and the ingestion penetration depth is $< 5\%$. The described behavior of the effective flow area at the upper trench vortex suggests that a threshold effective area exists at $\Psi = 2.3 \cdot 10^{15}$. The threshold value at $\Psi = 2.3 \cdot 10^{15}$ for effective flow area is in accord with the threshold non-dimensional circulation of
Figure 3-25: Effective flow area versus non-dimensional circulation at $r/b = 1.011$, $r/b = 1.007$, and $r/b = 1.004$ for the fixed geometry and varying purge mass rate ratio DOE. The effective flow area distribution at the three radial cut planes near the upper trench vortex trend similarly.
the upper trench vortex and the ingestion penetration depth.

Figure 3-26: Effective flow area at the upper trench versus non-dimensional sealing parameter. Each data point is scaled by the value of $\Gamma_{ND}$ of the upper trench vortex. Large data point size corresponds to larger $\Gamma_{ND}$. For $\Gamma_{ND} > 30$, the effective flow area of the trench is reduced to less than 70% of the total geometrical area. When the effective flow area is $> 70\%$ of the total geometrical area, $\Gamma_{ND} < 30$, and corresponds to regions where the ingestion penetration depth is $< 5\%$.

One of the hypotheses put forward in Section 3.4.2.1 is that the effective flow area and circulation of the upper trench vortex are correlated. The effective flow area at the upper trench vortex as a function of $\Gamma_{ND}$ is plotted for the fixed geometry and
varying purge mass flow rate ratio DOE in Figure 3-27. Figure 3-27 is used as an aid to summarize the physical rational for this hypothesis. Here, each data point is scaled by the ingestion penetration depth. The plot indicates that the effective flow area at the upper trench vortex correlates with the non-dimensional circulation of the upper trench vortex, and this is directly linked to the ingestion penetration depth. As the effective flow area increases from 60-100% of the total area at \( r/b = 1.004 \), \( \Gamma_{ND} \) remains between 20 – 30 and the ingestion penetration depth < 5%. For effective flow areas between 20-60% of the total area at \( r/b = 1.004 \), \( \Gamma_{ND} \) increases from 30 – 80 and the ingestion penetration depth increases monotonically over this interval. The results suggest that as \( \Gamma_{ND} \) of the upper trench vortex increases from more of the purge flow getting entrained into the vortex, the effective flow area at the location of the upper trench vortex is reduced. The results from the initial DOE suggest that this transition occurs near \( \Psi = 2.3 \cdot 10^{15} \). The non-dimensional sealing parameter value of \( \Psi = 2.3 \cdot 10^{15} \) also corresponds with the observed behavior in the streamline configurations from Figure 3-14. The streamline configurations are observed to transition from no flow separation to the formation of re-circulation zones along the stage one blade angelwing. The flow separation along the stage one blade angelwing is shown in Figure 3-28. As \( \Psi \) decreases, the re-circulation zone along the angelwing increases, non-dimensional penetration depth increases, effective flow area at the upper trench vortex decreases, and the non-dimensional circulation of the upper trench vortex increases. The relative size of the re-circulation zone is assessed based on the dividing streamlines that bound the re-circulation zone. The transition from minimal to no flow separation along the stage one blade angelwing occurs between \( \Psi = 2.2 - 2.7 \cdot 10^{15} \), which suggests that the observed threshold value of \( \Psi = 2.3 \cdot 10^{15} \) is an indication of the change from no flow separation to flow separation in the purge cavity.

The investigation of ingestion penetration depth, non-dimensional circulation of the upper trench vortex, and the effective flow area at \( r/b = 1.004 \) have identified a change from no flow separation to the formation of re-circulation zones near \( \Psi = 2.3 \cdot 10^{15} \). The total pressure coefficient, \( C_{pt} \), is calculated for the fixed geometry and
Figure 3-27: Effective flow area versus non-dimensional circulation of the upper trench vortex for the fixed rim seal geometry and varying purge mass flow rate ratio DOE. Data points are scaled by ingestion penetration depth. Data points larger in size indicate increased ingestion penetration depth. For effective flow areas $> 60\%$ of the total area at $r/b = 1.004$, $\Gamma_{ND}$ remains between 20 - 30 and the ingestion penetration depth is $< 5\%$. For effective flow areas between 20 - 60\% of the total area at $r/b = 1.004$, $\Gamma_{ND}$ increases from 30 - 80 and the ingestion penetration depth increases.
\[ \Psi = 1.96 \times 10^{15} \quad \Psi = 2.2 \times 10^{15} \quad \Psi = 2.71 \times 10^{15} \]

In Figure 3-28: Streamlines for cases with non-dimensional sealing parameters \( \Psi = 1.96 \cdot 10^{15}, 2.2 \cdot 10^{15}, \) and \( 2.71 \cdot 10^{15} \). Ingestion penetration depth, effective flow area at the upper trench vortex, and non-dimensional circulation of the upper trench vortex are shown. Figure highlights how the extent of flow separation along the stage one blade angle wing increases as \( \Psi \) decreases.
varying purge mass flow rate ratio DOE to understand the role of total pressure in the purge cavity. The total pressure coefficient is calculated between the cut plane at \( r/b = 1.004 \) and the injection point of the purge flow at the FOS inlet. The expression for \( C_{pt} \) is given in Equation 3.19. The mass flow average absolute total pressure at the cut plane and the density at the inlet of the FOS is used for these calculations.

\[
C_{pt} = \frac{P_{t,r/b=1.004} - P_{t,FOS_{inlet}}}{\frac{1}{2} \rho \left( b \Omega \right)^2}
\] (3.19)

The total pressure coefficient versus non-dimensional sealing parameter is plotted in Figure 3-29. For values of \( \Psi \) between \( 2.3 - 7 \cdot 10^{15} \) (Regime 1) the behavior of the total pressure coefficient is nearly linear with constant slope. The plot shows a change in behavior of \( C_{pt} \) at \( \Psi = 2.3 \cdot 10^{15} \), where it transitions to a steeper and more negative slope (Regime 2). At \( \Psi = 1.5 \cdot 10^{15} \), \( C_{pt} \) transitions from a negative to positive value. This plot suggests that in Regime 1, the decrease in total pressure is minimal as the flow travels from the FOS inlet to the upper trench. At \( \Psi = 2.3 \cdot 10^{15} \), the total pressure begins to rise through the purge cavity. The transition from negative to positive \( C_{pt} \) at \( \Psi = 1.5 \cdot 10^{15} \) signifies the point where the driving total pressure at the FOS inlet can no longer purge the cavity up to \( r/b = 1.004 \). The driving total pressure at the FOS inlet no longer being able to purge the cavity is evident by the ingestion penetration depth versus non-dimensional sealing parameter plot in Figure 3-29. Figure 3-29 shows that at \( \Psi = 1.5 \cdot 10^{15} \), the ingestion penetration depth is 8.2%. An ingestion penetration depth of 8.2% is located at the bottom of the upper trench. The results suggest that at \( \Psi = 1.5 \cdot 10^{15} \), hot gas has penetrated into the purge cavity, which causes the total pressure at this location to increase.

### 3.4.5 Role of Rim Seal Geometry on Viscous Loss in the Purge Cavity

It has been shown that the upper trench vortex circulation and the total pressure coefficient between the plane at \( r/b = 1.004 \) and the FOS inlet have similar trends to the ingestion penetration depth. The non-dimensional circulation of the upper
Figure 3-29: Total pressure coefficient \( (C_{pt}) \), \( \Gamma_{ND} \), and effective flow area at \( r/b = 1.004 \) versus non-dimensional sealing parameter (\( \Psi \)) for the fixed geometry and varying purge mass flow rate ratio DOE. The plots demonstrate that the total pressure coefficient switches from a negative to positive value at \( \Psi = 1.5 \cdot 10^{15} \), indicating the hot gas from the main flow path is entering the upper trench. The ingested hot gas results in a higher total pressure at this location.

trench vortex impacts the effective flow area in the purge cavity at this location, thus reducing the effectiveness of the purge flow. The total pressure coefficient is also demonstrated to change from a negative to positive value. The change in sign of \( C_{pt} \) is an indicator that hot gas from the main flow path is being ingested into the upper trench. The change in \( C_{pt} \) and \( \Gamma_{ND} \) of the upper trench vortex suggests that there would be an increase in viscous dissipation in the purge cavity, which results in increased viscous loss in the purge cavity.

To quantify the increased viscous dissipation due to the vortical structures in the purge cavity, the loss accounting method of volumetric entropy generation that Zlatinov [30] used for flowpath loss accounting is leveraged. The entropy generation rate per unit volume, \( S_{gen}'' \), is introduced by Zlatinov to trace entropy generation to the flow features responsible. For the purposes of this thesis, the viscous entropy generation per unit volume, \( S_{visc}'' \), is leveraged to quantify the loss incurred from the presence of vortical structures in the purge cavity. Contour plots of \( S_{visc}'' \) are also used to confirm that the vortical structures directly impact the ingestion penetration.
Equation 3.20 [30] represents the tensor notation expansion of the equation for $S''_{\text{visc}} = \frac{1}{T} \tau_{ij} \frac{\partial V_i}{\partial x_j}$ and is used to calculate the viscous entropy generation rate per unit volume.

$$S''_{\text{visc}} = \frac{\mu_{\text{eff}}}{T} \left[ 2 \left( \left( \frac{\partial V_x}{\partial x} \right)^2 + \left( \frac{\partial V_y}{\partial y} \right)^2 + \left( \frac{\partial V_z}{\partial z} \right)^2 \right) \right] + \frac{\mu_{\text{eff}}}{T} \left[ \left( \frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x} \right)^2 + \left( \frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x} \right)^2 + \left( \frac{\partial V_y}{\partial z} + \frac{\partial V_z}{\partial y} \right)^2 \right] + \frac{\mu_{\text{eff}}}{T} \left[ -\frac{2}{3} \left( \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \right)^2 \right] \tag{3.20}$$

Figure 3-30 is a plot of normalized entropy generation rate per unit volume in the purge cavity. In Figure 3-30, $S''_{\text{visc}}$ is non-dimensionalized as $(T\tau_{ij}S''_{visc})/(\frac{\epsilon V^3}{2b})$. The plot shows the increased rate of entropy generation near the stage one blade angelwing and in the upper trench. The stage one blade angelwing and upper trench locations correspond with two of the main vortical structures identified in the purge cavity. The use of volumetric entropy generation shows the majority of the viscous loss in the purge cavity can be attributed to these vortices.

By integrating the entropy generation rate per unit volume over the volume of the purge cavity, the entropy production can be calculated using Equation 3.21 and used to quantify the viscous loss. The viscous loss in the purge cavity is then expressed as $T\Delta s/\Delta h_t$.

$$\Delta s = \frac{1}{m_c} \iint_V S''_{\text{visc}} dV \tag{3.21}$$

A plot of the the viscous loss versus non-dimensional sealing parameter is shown in Figure 3-31. In this plot, the data points are scaled by the corresponding ingestion penetration depth. The plot shows that for non-dimensional sealing parameter values of $\Psi = 0.5 - 2.3 \cdot 10^{15}$ there is increased loss in the purge cavity ($> 0.025$), and this region also corresponds with relatively high values of non-dimensional circulation of the upper trench vortex and ingestion penetration depth. For non-dimensional
Figure 3-30: Purge cavity normalized entropy generation rate per unit volume \(\left(\frac{\left[T_{t_2}^n + \frac{S_{v_{avg}}}{(\rho U^3)}}{\frac{L^3}{2\theta}}\right]}\) for a purge mass flow rate ratio of \(C_e = 1.2\%\). The highest rate of entropy generation is observed near the stage one blade angelwing and the upper trench area where the two most prominent vortical structures in the purge cavity have been identified.
sealing parameter values of $\Psi > 2.3 \cdot 10^{15}$, the normalized viscous loss is $< 0.03$ and asymptotes to 0.012.

Quantifying the entropy production and loss in the purge cavity due to viscous dissipation identifies a correlation between viscous loss in the purge cavity, non-dimensional sealing parameter, and ingestion penetration depth. Increased entropy generation from the presence of vortical structures in the purge cavity results in increased viscous loss in the purge cavity and increased penetration of the hot gas. The assessment of the viscous loss in the purge cavity is specific to the purge cavity region only, and is used as a comparison between cases to quantify the viscous loss from vortices in the purge cavity. The implication of this finding is a rim seal configuration exists that limits the amount of viscous loss in the purge cavity, and reduces the ingestion penetration depth.

3.4.6 Summary of Rim Seal Geometry and Purge Flow on Ingestion

The DOE results from the 133 CFD simulations were used to identify $C_c$, $G$, $G_e$, $Re_\phi$, $Re_c$, $Re_{c,b}$, and $Ro$ as the key input rim seal variables that set the ingestion penetration depth. The results were used to determine a single non-dimensional sealing parameter that correlates with non-dimensional ingestion penetration depth. The established non-dimensional sealing parameter is given below in Equation 3.22.

$$\Psi = \frac{m_{c,\text{total}}}{\rho \Omega (A_1 + A_2)} \frac{Re_c Re_{c,b}}{Ro}$$  \hspace{1cm} (3.22)

By analyzing the streamline configurations in the purge cavity, the upper trench vortex is identified as a flow structure that impacts the effective flow area of the purge flow at the location of the upper trench vortex. The upper trench vortex is characterized by non-dimensional circulation, which is shown to trend with the effective flow area of the purge flow at the location of the upper trench vortex. As the geometry and purge mass flow rate ratio of the rim seal change, the non-dimensional circulation of the upper trench vortex and the effective flow area at $r/b = 1.004$ is
Figure 3-31: Purge cavity viscous loss, $T\Delta s/\Delta h_t$, for all DOE points versus non-dimensional sealing parameter. Large data point size corresponds to increased ingestion penetration depth. The viscous loss versus non-dimensional sealing parameter shows a similar trend to the ingestion penetration depth versus non-dimensional sealing parameter curve. Similarly, the highest viscous loss is observed for non-dimensional sealing parameters between $\Psi = 0.5 - 2.3 \cdot 10^{15}$, which is where the ingestion penetration depth and upper trench vortex non-dimensional circulation is highest.
altered when $\Psi < 2.3 \cdot 10^{15}$. The change in upper trench vortex non-dimensional circulation is a result of the formation of re-circulation zones in the purge cavity. Both the upper trench vortex non-dimensional circulation and effective flow area are shown to correlate with ingestion penetration depth. Computed results show that the non-dimensional circulation and the non-dimensional ingestion penetration depth have a functional dependence on a single non-dimensional sealing parameter, namely the non-dimensional sealing parameter given in Equation 3.22. The resultant scaling indicates that there is a threshold value of $\Psi = 2.3 \cdot 10^{15}$, beyond which the ingestion penetration depth has a marginal dependence on the non-dimensional sealing parameter. Therefore, the scaling rule provides insight into rim seal configurations that are less prone to ingestion. Viscous entropy generation rate per unit volume is used to calculate the viscous loss in the purge cavity. The viscous entropy generation rate per unit volume shows that the vortices in the upper purge cavity region are responsible for a majority of the entropy generation in the purge cavity. The viscous loss in the purge cavity from the vortical structures is strongly dependent on the non-dimensional sealing parameter and ingestion penetration depth. As $\Gamma_{ND}$ of the upper trench vortex increases, the viscous loss in the purge cavity circuit also increases. As the non-dimensional circulation of the upper trench vortex increases, the effective flow area at the upper trench vortex decreases, which leads to an increase in ingestion penetration depth.

3.5 Changes in Turbine Stage Efficiency Accompanying Purge Flow Introduction

The initial DOE results infer that turbine stage efficiency has a dominant dependence on the purge mass flow rate ratio and a weak dependence on changes in rim seal geometry. The calculation for stage efficiency is described in Section 2.7 and expressed as $\epsilon = \frac{\dot{W}_{\text{expand, actual}}}{\dot{W}_{\text{expand, ideal}}}$. Changes in stage efficiency with respect to a case where the purge mass flow rate ratio is 0.015 is used to quantify the impact of purge mass flow rate
ratio on stage efficiency. The change in stage efficiency is calculated using Equation 3.23. Figure 3-32 shows the change in stage efficiency as a function of purge mass flow rate ratio is approximately linear.

\[
\Delta \epsilon = \epsilon_x - \epsilon_{1.5\%}
\]  

(3.23)

Figure 3-32: Change in stage efficiency versus purge mass flow rate ratio. Change in stage efficiency is with respect to the baseline case defined in Section 2.8.1. The results show that there is a 0.7% decrease in stage efficiency for every 1% increase in purge mass flow rate ratio.

To fit a curve to the computational data, a least square linear curve fit is used. Equation 3.24 shows the statistical fit equation for the change in stage efficiency as a function of purge mass flow rate ratio.

\[
\Delta \epsilon = -0.7 \cdot \frac{\dot{m}_c}{\dot{m}_a} \cdot 100 + 1.071
\]  

(3.24)
In order to demonstrate the quality of the curve fit, the $R^2$ value and the curve fit residuals are assessed. The $R^2$ value of the fit is 98% and indicates the amount of variation in the data that is explained by the regression analysis [7]. The curve fit residuals between the curve fit values for $\Delta \epsilon$ and the $\Delta \epsilon$ values calculated from the CFD simulations are also examined to assess the goodness of fit. Figure 3-33 is a plot of the curve fit residuals. Figure 3-33 shows that the deviation from the linear fit is $+0.12/ -0.08\%$.

Figure 3-33: Curve fit residuals for the least square linear fit of change in stage efficiency versus purge mass flow rate ratio. The results show that the fit captures the variance in stage efficiency to within $+0.12/ -0.08\%$.

Figure 3-34 shows a plot of ingestion penetration depth and change in stage efficiency as a function of non-dimensional sealing parameter. It is evident that the maximum stage efficiency occurs for low non-dimensional sealing parameters where the ingestion penetration depth is greatest. The ingestion penetration depth curve is stable over a wide range of non-dimensional sealing parameters such that between
$\Psi = 2.3 - 14 \cdot 10^{15}$, the penetration remains $< 5\%$ and does not make it past the bottom of the trench. Meanwhile, the stage efficiency varies by $0.6\%$ over the same range of non-dimensional sealing parameters, which is a significant change in turbine performance. Ultimately, the trade-off is dependent on the requirements of the turbine.

Figure 3-34: Non-dimensional ingestion penetration depth and change in stage efficiency versus non-dimensional sealing parameter. A negative $\Delta \epsilon$ corresponds to a decrease in stage efficiency. Maximizing stage efficiency corresponds to low non-dimensional sealing parameters where the ingestion penetration depth is greatest.

3.5.1 Impact of shear layer mixing at interface between trench exit and main gas path on stage efficiency

The change in stage efficiency is shown to be near-linear with purge mass flow rate ratio. To explain the near-linear relationship, the interaction of the shear layer that develops between the interface of the trench exit and main gas path is investigated. The purge mass flow that enters the main gas path acts as a jet flow entering a mainstream. The mixing of two flows results in a more pronounced shear developing separate from the shear layer that exists along a wall due to viscous forces. To show the loss mechanism mainly responsible for the change in stage efficiency is the viscous loss generated from mixing out of the purge-main flow shear layer at the trench exit,
the viscous entropy generation from the mixing of two flows is quantified. Equation 3.25, which is derived in [10], is used to calculate the viscous entropy generation. It should be noted that the loss generated from the mixing out of the purge-main flow shear layer is separate from the viscous dissipation in the purge cavity discussed in Section 3.4.5. The loss associated with the mixing out of the purge-main flow shear layer impacts the overall stage efficiency of the turbine. The loss from the viscous dissipation in the purge cavity is associated with the purge cavity region only.

\[
\Delta s_{\text{visc}} = \frac{\dot{m}_e}{\dot{m}_a} \left[ \frac{(V_{x,a} - V_{x,c})^2 + (V_{r,a} - V_{r,c})^2 + (V_{\theta,a} - V_{\theta,c})^2}{2T_a} \right] 
\]  

(3.25)

The entropy generation is non-dimensionalized as \( T\Delta s/\Delta h_t \) to show the quantitative impact of the shear layer on stage efficiency. Figure 3-35 shows the shear layer loss \( (T\Delta s/\Delta h_t) \) as a function of purge mass flow rate ratio. The results show that the viscous loss generated from mixing out of purge-main flow shear layer at the trench exit accounts for approximately half of the total change in stage efficiency as the purge mass flow rate ratio changes. This finding is consistent with Zlatinov’s thesis [30] and shows that this trend remains true for larger changes in the rim seal geometry. Another finding is that the slope of the change in stage efficiency versus purge mass flow rate ratio curve is equal to a decrease of 0.7% stage efficiency per 1% increase in purge mass flow rate ratio, which is consistent with the findings of Regina et. al [20].

3.6 Summary

The Latin hypercube DOE and results from the CFD simulations have aided in identifying the important rim seal geometrical and flow parameters that impact hot gas ingestion and turbine performance. The non-dimensional sealing parameter determined from the DOE is shown to provide a useful scaling for the ingestion penetration depth and turbine stage efficiency data. For turbine performance, purge mass flow rate ratio has the most significant impact. For every 1% increase in purge mass flow rate ratio, the turbine stage efficiency decreases by 0.7%. The results show that the viscous shear
Figure 3-35: Change in loss generated from mixing out of the purge-main flow shear layer versus purge mass flow rate ratio. Change in loss generated from mixing out of the purge-main flow shear layer is with respect to the baseline case defined in Section 2.8.1. The results show that the trench shear layer loss accounts for half of the total change in stage efficiency.
layer at the trench exit accounts for approximately half of the total change in stage efficiency. For ingestion, the results suggest that there is a non-dimensional sealing parameter with a threshold value of $\Psi = 2.3 \cdot 10^{15}$, at which point the ingestion penetration depth is only marginally impacted by further increasing the non-dimensional sealing parameter. The results show that effective flow area at $r/b = 1.004$ and non-dimensional circulation of the upper trench vortex correlate with the non-dimensional sealing parameter, and thus the ingestion penetration depth. The total pressure coefficient between the cut plane at $r/b = 1.004$ and the inlet of the purge flow at the FOS is also shown to indicate a change at $\Psi = 2.3 \cdot 10^{15}$ when the hot gas from the main flow path enters further into the upper trench and causes the total pressure at this location to increase. The results from this thesis also highlight the use of pattern recognition in conjunction with classical fluid dynamic definitions. The use of pattern recognition was used to identify the upper trench vortex as an important flow feature in the purge cavity. The use of pattern recognition helps demonstrate a platform for executing a DOE with a large number of CFD simulations to assess aerothermal system problems.
Chapter 4

Conclusions

In this chapter, a summary of this thesis is presented, the key findings are described, and suggestions are provided for future work on this topic.

4.1 Summary

The underlying objective addressed in this thesis is to identify the role of rim seal geometry and purge mass flow rate ratio on ingestion and stage efficiency. Past research at MIT by Berg [4] and Catalfamo [6] on the HGIR focused on the presence of cavity modes in the wheelspace cavity and the sensitivity of the cavity modes to rim seal parameters. It was shown that the cavity modes have an impact on ingestion. Berg and Catalfamo’s research inspired additional research questions that provided a basis for the research presented in this thesis. This thesis investigates the impact of geometric and flow rim seal parameters on ingestion and flow patterns. A design of experiments is carried out to assess the turbine rim cavity system parameter variation on hot gas ingestion, flow pattern, and turbine stage efficiency. The parameters focused on are purge mass flow rate ratio, axial gap ratio, radial gap ratio, normalized axial position of the blade leading edge, and internal purge cavity radius ratios. The work has identified that there are key rim seal variables that exist and can be altered to limit ingestion. The computed flow field also demonstrates the dominant role of vortical structures in the rim cavity flow on effective flow area distribution and hence
the ingestion penetration depth. Quantitative attributes of the vortices, such as circulation, width, height, maximum vorticity, and vortex center position, scale with a newly determined non-dimensional sealing parameter defined in Equation 4.1. As a result, the vortex attributes scale with ingestion penetration depth. The implication is that the non-dimensional sealing parameter potentially provides a guideline for selecting rim seal configurations and operating space with marginal levels of hot gas ingestion.

4.2 Key Findings

The key findings of this thesis are as follows:

1. A new non-dimensional sealing parameter was developed based on rim seal geometry, purge mass flow rate ratio, Rotational Reynolds number, purge flow Reynolds number, rim seal Reynolds number, and Rossby number. This non-dimensional sealing parameter is shown to correlate with the non-dimensional ingestion penetration depth of over 130 DOE geometries analyzed with CFD simulations. The results indicate a threshold values at $\Psi = 2.3 \cdot 10^{15}$, beyond which there is only a marginal variation in ingestion penetration depth.

$$
\Psi = \frac{\dot{m}_{c,total}}{\rho b \Omega (A_1 + A_2)} \frac{Re_c Re_0 Re_{c,b}}{Ro} \quad (4.1)
$$

2. The upper trench vortex is identified as a critical flow feature that correlates with the ingestion penetration depth and the non-dimensional sealing parameter. For $\Psi > 2.3 \cdot 10^{15}$ the non-dimensional circulation asymptotes to a value of 25, and for values of $\Psi = 0.5 - 2.3 \cdot 10^{15}$ the non-dimensional circulation increases from 30 to $> 250$.

3. The vortical structures in the purge cavity set the effective flow area distribution in the purge cavity. Specifically, the effective flow area at a radial cross section bounded by the upper trench vortex is assessed. The associated non-dimensional
circulation of the upper trench vortex and the effective flow area at $r/b = 1.004$ are shown to have a dominant functional dependence on the non-dimensional sealing parameter defined in Equation 4.1. This indirectly implies the role the upper trench vortex has on the ingestion penetration depth. The non-dimensional circulation, $\Gamma_{ND}$ of the upper trench vortex, which is a measure of the strength of the vortex, controls the effective flow area of the purge flow at the upper trench vortex. When the upper trench vortex has a $\Gamma_{ND} > 30$, the effective flow area at this location becomes constricted and prevents the purge flow from clearing the purge cavity effectively. When the upper trench vortex has a $\Gamma_{ND} < 30$, the effective flow area at the upper trench vortex increases and ingestion is limited. Similar to ingestion penetration depth, this threshold value occurs at $\Psi = 2.3 \cdot 10^{15}$.

4. Purge mass flow rate ratio ($C_c$) is the critical rim seal variable that impacts stage efficiency. The stage efficiency is found to decrease by 0.7% for every 1% increase in purge mass flow rate ratio. The penalty can mostly be traced to the shear layer mixing loss associated with the introduction of the purge flow into the main flow path. The shear layer mixing accounts for approximately half of the penalty. This finding is consistent with published results to-date.

5. A preliminary platform for incorporating the use of pattern recognition to identify flow patterns in the purge cavity is demonstrated.

4.3 Future Work

The results of this thesis have uncovered several areas of study that would provide further insight into the role of rim seal geometry, purge mass flow rate ratio, and purge cavity flow features in the wheelspace cavity on hot gas ingestion and turbine performance. Below is a summary of these threads that are of engineering importance.

1. Computational Investigation
(a) Unsteady RANS Computations: The objective of this thesis was to explore the design space of a representative gas turbine geometry and investigate how rim seal geometry and flow features in the purge cavity impact ingestion. Given the large amount of computational resources required for URANS, steady RANS was selected for the initial DOE's presented in this thesis. This thesis has proven the method and demonstrated the role of vortical structures on ingestion. Going forward, URANS calculations are necessary to assess the impact of unsteadiness on the results. This includes the unsteadiness of the shear layer over the purge trench, the interaction between the nozzle guide vane and blade, and the impact and role of turbulence at the inlet of the nozzle guide vane.

(b) Utility of the new non-dimensional sealing parameter for other turbines: This thesis has shown the ingestion penetration depth data correlates with $\Psi$ for a representative gas turbine rim seal. The non-dimensional sealing parameter accounts for rim seal geometry, turbine operating space, and purge flow characteristics. However, it is useful to understand the utility of this parameter on other gas turbine rim seal configurations. Determining the ingestion penetration depth data and the associated $\Psi$ value for other turbine rim seals is beneficial to determine if the data from other turbines agrees with the scaling shown in Figure 3-9.

(c) Artificial Intelligence (AI) and the use of neural networks: This thesis has demonstrated a simplistic preliminary platform for how AI can impact turbine aerothermal system technology. The use of streamlines plots to study patterns in the purge cavity led to the discovery of the importance of vortical structures on ingestion. Investigating how AI can be used to achieve an optimal aerothermal system and using the data to understand how and why the configuration is optimal, is a beneficial study for the advancement of aerothermal systems.
Appendix A

Additional Parametric Assessments

The assessments thus far have focused on the baseline wheelspeed, $Re_\phi = 3.315 \cdot 10^6$. It is known from Zhang et. al. [27] that wheelspeed plays a role in sealing effectiveness, since the inertial term of the Rotational Reynolds number is impacted by changes in wheelspeed. To establish the applicability of the non-dimensional sealing parameter for different rotational Reynolds number, a sub-study was performed on a case with moderate ingestion.

A second study is also performed to assess the role of unsteadiness. A complete assessment of the impact of flow unsteadiness on hot gas ingestion is outside the scope of this thesis. However, an initial unsteady calculation is performed to begin quantifying the impact of unsteadiness on hot gas ingestion, and the loss generation associated with purge flow introduction into the main flow path.

A.1 Impact of Rotational Reynolds Number on Ingestion

The non-dimensional sealing parameter derived in Section 3.3 is a function of many of the input rim seal variables that includes the wheelspeed of the rotor, $Re_\phi = 3.32 \cdot 10^6$. To assess the utility of the generated curve in Figure 3-9 for changes in wheelspeed, a representative case in which the penetration depth is larger than 20% is assessed.
at wheelspeeds of $1.5 \cdot \Omega_{ref}$, $1.1 \cdot \Omega_{ref}$, and $0.9 \cdot \Omega_{ref}$ with corresponding rotational Reynolds numbers shown in Table A.1.

Table A.1: Rotational Reynolds number for assessed cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>$Re_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline - Case 133</td>
<td>$3.32 \cdot 10^6$</td>
</tr>
<tr>
<td>$1.5 \cdot \Omega_{ref}$</td>
<td>$4.97 \cdot 10^6$</td>
</tr>
<tr>
<td>$1.1 \cdot \Omega_{ref}$</td>
<td>$3.65 \cdot 10^6$</td>
</tr>
<tr>
<td>$0.9 \cdot \Omega_{ref}$</td>
<td>$2.98 \cdot 10^6$</td>
</tr>
</tbody>
</table>

The results indicate that an increase in rotational Reynolds number decreases the ingestion penetration depth, and conversely a decrease in rotational Reynolds number results in an increase in ingestion penetration depth. Figure A-1 shows that the non-dimensional sealing parameter captures this trend. Further work is needed to ensure the applicability of the non-dimensional sealing parameter, $\Psi$, for changes in rotational Reynolds number at other cases, but the initial studies indicate good agreement with $\Psi$.

A.2 Preliminary Assessments on Impact of Unsteadiness on Flow Features and Ingestion

The results of the steady CFD simulations suggest that the non-dimensional circulation of the upper trench vortex impacts the effective flow area at the location of the upper trench vortex. The change in effective flow area at $r/b = 1.004$ and non-dimensional circulation is shown to correlate with the ingestion penetration depth. It is known from the work of Laskowski et. al. [14] that unsteady CFD is required to capture the impact of the unstable shear layer over the rim seal trench exit, and that this feature impacts ingestion. While running a DOE of unsteady computations is outside the scope of this thesis, a sample unsteady run was computed for a preliminary assessment of the role that unsteadiness plays. In the implementation of the unsteady simulation, the steady solution for case 93 was used as the initial condition for the unsteady RANS solver, and then a transient analysis was initiated for 4500
Figure A-1: Non-dimensional ingestion penetration depth versus non-dimensional sealing parameter. The plot highlights changes in rotational Reynolds number. Changes in \( Re_\theta \) and its impact on ingestion penetration depth is captured with the non-dimensional sealing parameter, \( \Psi \).

time steps.

A representative case with a moderate ingestion of > 20% is chosen for assessing the impact of flow unsteadiness. The unsteady RANS simulation was conducted with the blade only domain to capture the unsteadiness induced by the shear layer over the trench exit and the rotor pumping. The computed unsteady flow was appropriately time averaged and the ingestion penetration depth and non-dimensional circulation of the upper trench vortex were compared between the steady and unsteady solutions for case 93. The streamlines in Figure A-3 indicate a visible change in the flow structures in the trench. For case 93, flow unsteadiness leads to a 50% reduction in the upper trench vortex non-dimensional circulation and a 5% decrease in ingestion penetration depth. Figure A-2 shows a plot of ingestion penetration depth as a function of non-
dimensional sealing parameter and highlights the results from the steady and unsteady solution of case 93.

Figure A-2: Non-dimensional ingestion penetration depth versus non-dimensional sealing parameter highlighting steady and unsteady case 93. The plot shows the ingestion penetration depth is reduced by 5% when accounting for unsteadiness of the trench shear layer and the data point still follows the ingestion penetration depth curve.

The results of this initial run, show the importance of the unsteadiness induced by the shear layer over the trench exit and the rotor pumping on penetration depth. Nevertheless, the ingestion penetration depth for the initial unsteady run follows the ingestion penetration depth versus non-dimensional sealing parameter curve. Additional effort is required to fully understand the impact of unsteadiness on ingestion and the validity of the non-dimensional sealing parameter developed.
Figure A-3: Difference in streamlines pattern from time-averaged unsteady computed flow and from steady run. The streamlines plots are used to show that the upper trench vortex structure has changed between the steady and unsteady simulation. The size of the vortex is qualitatively different; the cross-sectional area in the r-z plane encompassing the upper trench vortex is 17% smaller for the unsteady solution.
Bibliography


