Multiple Scattering of Microwaves from Soil Surface and Vegetation Canopies

by

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ABSTRACT

A multiple scattering radiative transfer model is developed for use in global retrieval algorithms of microwave geophysical parameters. This attempts to both improve soil moisture and vegetation attenuation and scattering parameter retrievals under moderate to dense canopy covers and capture the full microwave vegetation signature. The model is developed using a ray tracing method of first-order interactions with canopy constituents larger than 21 cm (e.g. woody biomass). It introduces only one new variable, $\omega_1$ representing scattering from these constituents, making it almost as computationally efficient to implement as the commonly used tau-omega (zeroth-order) model. With concerns for vegetation cover changes with climate change, there is motivation to monitor vegetation cover properties and respective available surface water availability. The retrieval of microwave vegetation attenuation and scattering parameters provides a unique opportunity to monitor vegetation parameters in addition to commonly used optical remote sensing of vegetation techniques. The model is applied within the recently developed and hereby enhanced Multi-Temporal Dual Channel Algorithm (MT-DCA) framework. The algorithm is implemented over Africa using SMAP 36 km brightness temperature radiometric measurements using both zeroth and first-order radiative transfer models. The first-order radiative transfer model is determined to be more sensitive to surface emission resulting in an improved surface emission signature from retrievals. The retrieved $\omega_1$ are also greatest in forests noting the presence of woody biomass and resulting in first-order emission contributing to 5% of the total emission. Consequently, changes from zeroth to first-order retrievals occur primarily in vegetated regions where $\omega_1$ is non-zero. Non-zero $\omega_1$ additionally results in improved fit of parameters to SMAP measurements. Without a comprehensive forest in-situ measurement campaign, it is inconclusive whether the introduced first-order radiative transfer model improves retrievals over SMAP and SMOS baseline retrievals. Additionally, further work in developing global retrieval algorithms aimed at retrieving both surface and vegetation microwave parameters amongst moderate to dense vegetation is encouraged.

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1. Introduction

1.1 Background

Vegetation activity and underlying surface soil moisture (SM) are key boundary conditions for estimating the carbon, energy, and water fluxes between the land surface and atmosphere. They directly influence surface albedo, evapotranspiration fluxes, and carbon sequestration at the surface recently bringing much attention to their role on atmospheric carbon dioxide concentrations and climate change. With concerns for vegetation cover changes due to direct anthropogenic intervention and climate change, ecology communities are interested in monitoring measures of vegetation health (e.g. greenness, wet and dry biomass) and respective surface water availability (Saatchi et al. 2011). There is thus motivation to measure the geophysical properties of vegetation covered biomes. However, in-situ measurement networks are sparse and larger scale networks are impractical with current ground measurement methods. Therefore, satellite remote sensing observations have become the most practical method to globally monitor geophysical properties of vegetation covered surfaces despite low temporal sampling frequency and low resolution. They make use of radiative transfer (RT) theory which attempts to model electromagnetic radiation propagation including absorption, scattering, and emission through media at specific spectra to ultimately deduce physical properties of the media. The media in this case are Earth’s surface and overlying vegetation cover. A brief summary of SM monitoring, vegetation parameter monitoring, and studies which address retrieving geophysical parameters under vegetation canopies follows.

ESA launched the Soil Moisture and Ocean Salinity (SMOS) satellite in 2009 and NASA launched the Soil Moisture Active Passive (SMAP) satellite in 2015 to provide global measurements of SM using brightness temperature ($T_b$) measurements of the Earth’s surface at L-Band frequency (1.4 GHz) (Entekhabi, Njoku, et al. 2010; Kerr et al. 2010). This takes advantage of the sharp contrast in dielectric constants of dry and wet surfaces to obtain accurate SM measurements at relatively low resolution (approximately 40 km). The choice of measuring L-Band microwave emissions is primarily due to their longer wavelength. Their emission is from greater surface depths (approximately 5 cm at L-Band) and are almost unaffected by light canopy and cloud cover since their wavelength is larger than these media. However, increasing the measurement wavelength for deeper measurement depth is subject to large uncertainties due to significant Faraday rotation in the ionosphere.

In the case of vegetation monitoring, NASA’s Moderate Resolution Imaging Spectroradiometer (MODIS) satellite was launched in 1999 providing vegetation parameter measurements such as normalized difference vegetation index (NDVI), a measure of vegetation greenness, and leaf area index (LAI), a measure of vegetation crown structure. Additionally, NASA launched the Orbiting Carbon Observatory-2 (OCO-2) in 2014 to obtain global measurements of solar induced fluorescence (SIF). Note that these are only two of a suite of many vegetation observing satellites. These satellites measure at an optical frequency (~100 THz) which provides information about the vegetation crown, but measurements are subject to atmospheric contamination due to the short wavelength. The SMOS mission and recently developed algorithm for the SMAP mission, on the other hand, implement a zeroth-order RT model (discussed in detail in sections 1.2 to 1.4) to obtain both vegetation optical depth ($\tau$) in
addition to SM. At this frequency, \( r \) provides a within-canopy vegetation health parameter. From here on, the study focuses on microwave measurements due to the opportunity to characterize SM and vegetation properties from satellite-measured L-Band surface emissivity.

At microwave frequencies, vegetation canopies significantly attenuate, scatter, and emit leading to lower sensitivity of \( T_B \) measurements to SM. This is exacerbated when canopy constituents larger than the measurement frequency (21 cm for L-Band) are present which causes significant emission scattering and further reduction in SM sensitivity. Both the SMOS and SMAP missions require SM retrievals at 0.04 cm\(^3\)/cm\(^3\) (1-sigma) accuracy at under 5 kg/m\(^2\) vegetation water content (VWC) (Entekhabi, Njoku, et al. 2010; Kerr et al. 2010). This VWC limit is set due to the low level of confidence in SM under forested conditions, or approximately 30% of Earth’s land surface.

Many studies evaluated the effects of vegetation cover on passive microwave emissions with much of the pre-2004 work summarized in Pampaloni (2004). Shutko (1982) measured emissivities from aircraft of wide and grain leaf crops as well as bush and wood covered surfaces finding greater canopy attenuation with increasing frequency. Ulaby, Razani, and Dobson (1983) found a reduction in SM sensitivity with increased canopy using aircraft \( T_B \) measurements over bare, wheat-stubble, and corn covered fields. Guglielmetti et al. (2008) draw the same conclusion using ground measurements over a forest canopy. Specifically, Guglielmetti et al. (2007) find that tree branches are the largest contributors to surface signal reduction using an upward looking radiometer at L-Band (1.4 GHz) and RT model developed in Mätzler (1994). This same conclusion was previously drawn using aircraft radar measurements (N. S. Chauhan, Lang, and Ranson 1991). Additionally, forest floors have significant litter layers which Schwank et al. (2008) find significantly attenuate surface emission especially in wetter conditions. Using aircraft measurements, Macelloni et al. (2001) find emissivity is sensitive to woody biomass as opposed to LAI at L-Band. Grant et al. (2007) noted the difficulty in solely determining the effects of SM on emissivity for a forest covered surface. However, Lang, Utku, and Matheais (2001), conclude that it is possible to measure SM in forests finding 10 K variations in aircraft \( T_B \) measurements under a dense forest canopy as long as accurate \( T_B \) and physical temperature measurements are available. N. Chauhan (1999) and Della Vecchia et al. (2010) also come to this conclusion.

In order to determine SM under canopy cover, a simple zeroth-order RT model, known in the passive microwave remote sensing community as the tau-omega model, was developed which accounts for surface and vegetation emissions Mo et al. (1982). This requires information about the attenuation and scattering properties of the vegetation medium. These parameters include \( \tau \), which represents canopy extinction, and \( \omega \), or single scattering albedo, which represents canopy extinction due to scattering. Though it performs well for bare and lightly vegetated surfaces, it is not reliable for parameter retrievals in forests. This is because the tau-omega model neglects canopy scattering which is only a valid assumption when the canopy constituents are smaller than the emission wavelength (21 cm).

Despite limitations of the tau-omega model when used for densely canopy covered surfaces, some have determined that vegetation canopy parameters can be calibrated to match
measured emission from canopy covered surfaces (Saleh et al. 2003; Santi et al. 2009). Ferrazzoli, Guerriero, and Wigneron (2002) and Kurum (2013) explain that these vegetation parameters become effective parameters and can account for higher order scattering within the canopy. Kurum (2013) quantified this using an effective omega to mitigate an emission darkening effect resulting from unrealistically high $\omega$. He found that lower effective albedos would correct for this and match forest emissions.

Noting the limitations of the zeroth-order RT model, numerous studies developed models for surfaces with varying $SM$ and temperature profiles (Njoku and Kong 1977), grass covered surfaces (Saatchi, Le Vine, and Lang 1994), leaf litter covered surfaces (Schwank et al. 2008), snow covered surfaces (Proksch et al. 2015; Wiesmann and Mätzler 1999), and forest covered surfaces (Ferrazzoli and Guerriero 1996; Karam 1997; Kurum et al. 2011; Macelloni et al. 2001). The discrete numerical models developed for forested surfaces are summarized here.

Karam (1997) developed a physical second-order RT model which discretizes the canopy into two vegetation layers, one with branches, leaves, and stems, and another with trunks. It requires inputs of branch size distribution and represents coniferous and deciduous canopy types differently (Karam et al. 1992). Karam et al. (1992) finds that only the two canopy scattering terms have significant magnitudes amongst the second order scattering terms and that the simulated model results compare well with measurements for a frequencies between 1 GHz and 10 GHz.

Ferrazzoli and Guerriero (1996) created the Tor Vergata model, a discrete numerical model, based on RT theory using a matrix doubling algorithm to compute emissivity. The model characterizes the canopy as a single layer partitioning into crown (further discretized into leaves, needles, twigs, and branches), trunk, and soil components. It requires in-situ measurements of canopy geometry and allometric relations to determine other physical quantities of the canopy. They find satisfactory agreement with deciduous and coniferous forest emissivity measurements and ultimately conclude that though the zeroth-order RT equations are not sufficient to account for multiple scattering, effective vegetation parameters can be used to correct this (Ferrazzoli, Guerriero, and Wigneron 2002; Paloscia et al. 2008).

Kurum et al. (2011) developed a first-order RT model which uses the zeroth-order $T_B$ as an exciting source to determine the first-order $T_B$. The resulting equation is a first-order tau-omega model which is modified to include the addition of zeroth-order terms and first-order scattering terms. The first-order scattering terms represent emission pathways which scatter once from canonical structures in the canopy, such as branches and trunks. The first-order scattering term is significant when trees are present in the canopy and the addition of this term results in a much closer correspondence between measured and modelled emissivities for forest plots (Kurum et al. 2011; Kurum 2013).

Macelloni et al. (2001) utilized a discrete element first-order radiative transfer model developed by Tsang et al. (1985) which also compartmentalized the canopy into a crown, trunk, and surface layer. Emissivity was computed from bistatic scattering coefficients using the conservation of energy assumption. They used four different types of branch distributions to
characterize the crown layer and used allometric equations for tree geometry inputs into the model. They find that their model fits the experimental data well over tree plots in Tuscany, Italy and emissivity contribution is mostly due to primary and secondary branches.

Though these models provide much closer agreement between basin-scale measured and modelled $T_B$, they require complex parameterization and extensive measurements of the vegetation canopy. Thus, they become impractical and computationally expensive for use as a global retrieval algorithm for SMOS and SMAP measurements. Ferrazzoli, Guerriero, and Wigneron (2002) used the developed Tor-Vergata model to calibrate the tau-omega model using effective vegetation parameters, but this requires forest measurements such as branch biomass to infer the parameters. Truong-Loi, Saatchi, and Jaruwatanadilok (2015) created a retrieval algorithm for radar measurements and compared successfully with aircraft measurements though global retrieval has not been attempted. This provides motivation for a passive microwave retrieval algorithm that accounts for higher-order soil-canopy interactions, while remaining as simple to invert as the tau-omega model without using ancillary information. Here, an algorithm is developed which retrieves $SM$, vegetation attenuation, and vegetation scattering parameters from $T_B$ measurements using an introduced first-order RT model. These microwave vegetation properties represent time variant and invariant properties of the canopy allowing a means to monitor global vegetation cover in addition to commonly used satellite-based optical vegetation measurements. First, a full derivation of the tau-omega model and discussion of commonly used retrieval algorithms follows.

1.2 Zeroth-order RT (tau-omega) Model

The zeroth-order RT model neglects first-order scattering effects of the canopy and is used in global scale $SM$ retrievals in SMOS and SMAP missions due to its ease of implementation. This derivation begins with the scalar RT equations for upward and downward propagating intensities (Ulaby and Long 2014) given by:

$$\frac{d}{dz} I^+(\mu_s, \phi_s, z) = -\frac{\kappa_e}{\mu_s} I^+(\mu_s, \phi_s, z) + \frac{\kappa_a}{\mu_s} J_a(\mu_s, \phi_s, z) + \mathcal{F}^+(\mu_s, \phi_s, z)$$

$$-\frac{d}{dz} I^-(-\mu_s, \phi_s, z) = -\frac{\kappa_e}{\mu_s} I^-(-\mu_s, \phi_s, z) + \frac{\kappa_a}{\mu_s} J_a(-\mu_s, \phi_s, z) + \mathcal{F}^-(\mu_s, \phi_s, z)$$

where $I^+(\mu_s, \phi_s, z)$ is upward and $I^-(-\mu_s, \phi_s, z)$ is downward propagating intensity, $\mu_s$ is the cosine of the incidence angle (or $\theta$), $\kappa_e$ is the extinction coefficient (Np/m), $\kappa_a$ is the absorption coefficient (Np/m), $\phi_s$ is the azimuth angle, $z$ is the height above the canopy layer bottom (m), $J_a$ is the absorption source term accounting for self-emission, $\mathcal{F}^+$ and $\mathcal{F}^-$ are the source functions for upward and downward propagation, respectively. In this derivation, the bottom of the vegetation layer is $z = -d$ and top is $z = 0$. On the right hand side of (1) and (2), the first term represents extinction in the layer, the second represents layer self-thermal emission, and the final term represents scattering contribution. Using Rayleigh-Jeans approximation to Planck’s Law, the intensity is related to $T_B$ and absorption source term related to canopy physical temperature such that (1) and (2) can be written as follows (Ulaby and Long 2014):

$$\frac{d}{dz} T_B^+(\mu_s, \phi_s, z) = -\frac{\kappa_e}{\mu_s} T_B^+(\mu_s, \phi_s, z) + \frac{\kappa_a}{\mu_s} T_c + \mathcal{F}^+(\mu_s, \phi_s, z)$$

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\[- \frac{d}{dz} T_B^- (-\mu_s, \phi_s, z) = - \frac{\kappa_e}{\mu_s} T_B^- (-\mu_s, \phi_s, z) + \frac{\kappa_a}{\mu_s} T_c + F^- (-\mu_s, \phi_s, z) \tag{4}\]

where $T_B^+$ is the upward and $T_B^-$ is the downward propagating $T_B$. Additionally, $T_c$ is the physical temperature of the layer. Multiplying both sides of (3) by $\exp(\kappa_z/\mu_s)$ and integrating in terms of $z'$ between $z$ and $-d$ as well as multiplying both sides of (4) by $\exp(-\kappa_z/\mu_s)$ and integrating in terms of $z'$ between $0$ and $z$ results in, respectively:

\[ T_B^+ (\mu_s, \phi_s, z) = e^{-\kappa_e (z+d)} T_B^+ (\mu_s, \phi_s, -d) + \int_{-d}^{z} e^{-\kappa_e (z'-z')} \left[ \frac{\kappa_a}{\mu_s} T_c + F^+ (\mu_s, \phi_s, z') \right] dz' \tag{5}\]

\[ T_B^- (-\mu_s, \phi_s, z) = e^{\kappa_e z} T_B^- (-\mu_s, \phi_s, 0) + \int_{0}^{z} e^{\kappa_e (z'-z')} \left[ \frac{\kappa_a}{\mu_s} T_c + F^- (-\mu_s, \phi_s, z') \right] dz' \tag{6}\]

Since the vegetation canopy-air boundary is diffuse with a very low reflectivity, the downward propagating $T_B$ at the top of the canopy is zero:

\[ T_B^- (-\mu_s, 0) = 0 \tag{7} \]

Also, due to ground emission and reflection of downward propagating $T_B$, the upwelling $T_B$ at the bottom of the vegetation canopy is expressed as:

\[ T_B^+ (\mu_s, -d) = T_B^- (-\mu_s, -d) \Gamma_{coh}^p (\mu_s) + \left( 1 - \Gamma_{tot}^p (\mu_s) \right) T_s \tag{8} \]

where $\Gamma_{coh}$ is the coherent reflectivity, $\Gamma_{tot}$ is the addition of coherent and incoherent reflectivities, and $p$ is an index for either horizontal (H-polarized) or vertical polarization (V-polarized). Additionally, $T_s$ is the physical temperature of the surface. The zeroth-order solution to the RT equations is obtained by assuming that the medium is weakly scattering with single scattering albedos of less than 0.2. This sets the source functions equal to zero. Solving (6) at $z = -d$ using this assumption and the upper layer boundary condition (7) results in:

\[ T_B^- (-\mu_s, \phi_s, -d) = (1 - \gamma)(1 - \omega) T_c \tag{9} \]

or the downward propagating $T_B$ at the bottom of the canopy where $\omega$ is the single scattering albedo and $\gamma$ is transmissivity known as Beer’s law defined respectively as:

\[ \omega = \frac{\kappa_s}{\kappa_e} \tag{10} \]

\[ \gamma = e^{-(\kappa_e d/\mu_s)} \tag{11} \]

where $\kappa_s$ is the scattering coefficient (Np/m). Note that $\kappa_e$ equals the addition of $\kappa_s$ and $\kappa_a$.

Inserting (8) into (5) and solving at $z = 0$ results in the upwelling $T_B$ at the top of the canopy, or the tau-omega model:

\[ T_{BP}^{0th} = T_B^+ (\mu_s, \phi_s, 0) = \gamma_p (1 - r_p) T_s + (1 - \gamma_p) (1 - \omega_p) T_c + \gamma_p r_p (1 - \gamma_p) (1 - \omega_p) T_c \tag{12} \]

where $r_p$ is called the rough surface reflectivity which is commonly modelled as:
\[
\Gamma_p = \Gamma_{coh}^p = \Gamma_{tot}^p = \Gamma^p e^{-h \cos N \theta} \tag{13}
\]

where \(\Gamma^p\) is the p-polarized Fresnel reflectivity, or the reflectivity of a specular surface, and \(h\) is the surface roughness parameter. At lower frequencies, it is assumed that only the coherent reflectivity contributes to the total reflectivity. \(N\) is set to zero in this study consistent with the SMAP baseline algorithm discussed in 1.3.

1.3 Retrieval Algorithms

Using the tau-omega model, \(SM\) can be determined from \(T_B\) measurements noting that \(SM\) is obtained via \(\Gamma^p\) expressed for H-Pol and V-Pol respectively, as:

\[
\Gamma_h = \left| \frac{\cos \theta - \sqrt{\varepsilon - \sin^2 \theta}}{\cos \theta + \sqrt{\varepsilon - \sin^2 \theta}} \right|^2 \tag{14}
\]

\[
\Gamma_v = \left| \frac{\varepsilon \cos \theta - \sqrt{\varepsilon - \sin^2 \theta}}{\varepsilon \cos \theta + \sqrt{\varepsilon - \sin^2 \theta}} \right|^2 \tag{15}
\]

where \(\varepsilon\) is the soil dielectric constant. Three models have been developed that relate \(SM\) and \(\varepsilon\): the Dobson (Dobson et al. 1985), Wang and Schmugge (Wang and Schmugge 1980), and Mironov soil dielectric models (Mironov, Kosolapova, and Fomin 2009). The SMOS mission uses land cover classification to decide between using the Dobson and Mironov models (Kerr et al. 2010). The SMAP mission currently uses the Mironov model which only requires ancillary information about the clay fraction of the soil (Entekhabi, Njoku, et al. 2010).

Vegetation microwave optical depth (\(\tau\), in units of Np) is defined as:

\[
\tau_p = \kappa_e d \tag{16}
\]

Additionally, Jackson and Schmugge (1991) find a linear relationship between \(\tau\) and \(VWC\):

\[
\tau_p = bVWC \tag{17}
\]

where \(b\) is a static parameter related to the vegetation type and frequency. Radiometer antennas measure both H-polarized and V-polarized \(T_B\) at a known frequency and incidence angle. This provides two measurements and unknowns of \(\varepsilon, \tau, \omega, T_S, T_C, \) soil texture (e.g. clay fraction, sand fraction, etc.) and \(h\). Both the SMOS and SMAP satellites measure the surface at 6 am local time when the physical temperature of the canopy and soil are in approximate thermodynamic equilibrium (Entekhabi, Njoku, et al. 2010; Kerr et al. 2010). This sets \(T_S\) and \(T_C\) equal to one another, eliminating one unknown. Additionally, both missions use ancillary soil texture and surface roughness information leaving only \(\varepsilon, \tau, \omega, \) and \(T_S\) as unknowns.

Henceforth, in the SMAP project suite of \(SM\) products, either the single channel algorithm (SCA), dual channel algorithm (DCA), or the land parameter retrieval model (LPRM) (Meesters, De Jeu, and Owe 2005) is used to invert the tau-omega model to retrieve \(SM\) (and sometimes \(\tau\) and \(\omega\)).

The SCA uses only either the H-polarized or V-polarized \(T_B\) to determine \(\varepsilon\), and hence \(SM\), provided ancillary information about \(\tau, \omega, \) and \(T_S\) are available. The SMAP mission
currently applies the SCA inputting available $VWC$ and $b$ parameter information into (17) for a temporally and spatially varying $r$ based on Normalized Difference Vegetation Index (NDVI) information. $\omega$ is determined from MODIS International Geosphere Biosphere Programme (IGBP) classifications and is assumed to be time-invariant which is supported by field studies (Wigneron et al. 2004). Finally, the physical temperature is obtained from a numerical weather prediction model. This results in a fully determined problem obtaining a $SM$ value from only either H-polarized or V-polarized information.

The DCA uses both H-polarized and V-polarized $TB$ measurements to obtain two unknowns in the tau-omega model: commonly $\epsilon$ and $\tau$, assuming $\tau$ is polarization independent. However, H-polarized and V-polarized $TB$ measurements are correlated especially if taken at the same incidence angle and will provide less than two degrees of information (DOI) only allowing robust retrieval of one parameter (Konings et al. 2015). The SMOS mission increases the DOI by measuring H-polarized and V-polarized $TB$ at multiple incidence angles allowing robust retrieval of $\epsilon$ and $\tau$ for each dual-polarized satellite observation. A new SMOS retrieval algorithm, SMOS-INRA-CESBIO (SMOS-IC), calibrated modelled $SM$ with in-situ $SM$ from various sites with different IGBP classifications to retrieve $\omega$ (Roberto Fernandez-Moran et al. 2017). They find higher $\omega$ than originally used in SMOS and SMAP baseline algorithms (R. Fernandez-Moran et al. 2016). The SMAP mission only measures at a single incidence angle making the DCA inversion of the tau-omega model underdetermined. This results in noisy retrievals of soil dielectric constant and $\tau$ (Konings et al. 2015; Konings et al. 2016).

Like the DCA, the LPRM retrieves both $\epsilon$ and $\tau$, assuming $\tau$ is polarization independent. A manipulation of the tau-omega model is used to iteratively solve for $\tau$ and $\epsilon$ simultaneously employing a spatially and temporally static, known $\omega$ and microwave polarization difference index (MPDI), a relationship between H-polarized and V-polarized $TB$ (Meesters, De Jeu, and Owe 2005). Neither the SMOS nor SMAP mission use this as their baseline algorithm.

Recently, a multi-temporal dual channel algorithm (MT-DCA) was developed to retrieve $\epsilon$ and $\tau$ simultaneously, circumventing the DCA’s lack of DOI for the SMAP mission due to correlated $TB$ measurements at a single incidence angle (Konings et al. 2016). It does so by using $TB$ measurements from temporally adjacent passes, increasing the DOI upper limit to 3.72 and allowing robust retrieval of three parameters between the two passes. Note that the DOI is not sufficient if H-polarized and V-polarized $TB$ measurements are equal. Since temporal dynamics of $VWC$ are much slower than $SM$ changes over the three day SMAP revisits, the MT-DCA makes the assumption of constant $\tau$ between adjacent passes. Thus, it retrieves $\epsilon$ for each pass with a single constant $\tau$ between adjacent passes. This separates the time scales of $\epsilon$ and $\tau$ noting that $SM$ rapidly changes on daily timescales while $\tau$ changes on the order of a few days (due to slower internal $VWC$ processes within the trunks, branches, and leaves).

The MT-DCA separately retrieves a temporally constant $\omega$ in which retrieved values of $\omega$ are effective values rather than theoretical single scattering albedo values (Kurum 2013; Konings et al. 2017; Wigneron et al. 2017). While $\epsilon$ and $\tau$ (largely functions of moisture) change on the order of days, it is assumed that $\omega$ changes minimally on monthly to yearly time scales. This is because $\omega$ is primarily a function of the dry, static canopy structure. Wigneron et al. (2004) has
previously shown that $\omega$ can be assumed to be approximately constant even over croplands where vegetation changes rapidly. It assumes polarization independent $\tau$ and $\omega$. Konings et al. (2017) states that this assumption is acceptable at coarse spatial resolutions where vegetation anisotropy is reduced and emission does not have a detectable, preferential orientation. $\tau$ and $\omega$ also become depolarized in vegetated regions due to random orientation of trees largely in alignment with the depolarization of $T_B$ as shown in Fig. 1. However, this depolarization also reduces DOI which may invalidate the MT-DCA approach to retrieve two $SM$ values and a constant $\tau$ from adjacent over passes in (18). Also, the polarization independence assumption may be invalid in the presence of organized rows of vegetation (e.g. agricultural fields, managed forest). Nonetheless, retrievals are attempted with no masks applied due to these limitations.

Konings et al. (2016) retrieved $SM$, $\tau$, $\omega$ using three years of Aquarius satellite $T_B$ observations and showed that MT-DCA retrievals were less noisy compared to LPRM retrievals. Konings et al. (2017) employed the MT-DCA to retrieve $SM$, $\tau$, and $\omega$ from the first year of SMAP microwave radiometric observations finding similarities with SMOS DCA $SM$ and $\tau$ retrievals. The MT-DCA is selected for this analysis primarily for its ability to retrieve vegetation parameters ($\tau$, $\omega$) in addition to $SM$ without ancillary information or measurements with multiple incidence angles.

1.4 Multi-Temporal Dual Channel Algorithm Optimization

The MT-DCA contains two simultaneous “inner” and “outer” loop optimizations. The inner loop optimization is the DCA optimization completed over each pair of adjacent observation passes (approximately three days apart for SMAP) where $\epsilon$ from the first pass ($\epsilon_1$), $\epsilon$ from the second pass ($\epsilon_2$), and a constant $\tau$ between the passes are simultaneously varied until simulated $T_B$ using (12) are closest to the H-polarized and V-polarized $T_B$ measurements from adjacent passes. This retrieves optimal values of $\epsilon_1$, $\epsilon_2$, and $\tau$. For a year of SMAP measurements, this optimization is repeated over each of the approximately 120 pairs of adjacent passes over a

Fig. 1. Depolarization with Vegetation Height. $T_B$ and LIDAR data are from SMAP (Chan et al. 2016) and GLAS (Simard et al. 2011) instruments, respectively.
year. This minimization of difference between measured and forward modeled $T_B$ is accomplished using sum of squared differences between two passes as shown in the cost function:

$$\min_{X = \epsilon_1, \epsilon_2, \tau} J(X) = \sum_{t=1}^{2} \sum_{p=h,v} (T_{Bp}^{obs} - T_{Bp}^{model}(X))^2$$

(18)

where $J$ is the sum of squared differences in observed and measured $T_B$ and is the cost function to be minimized as a function of $\epsilon_1, \epsilon_2,$ and $\tau$. $T^{obs}$ is the observed $T_B$ and $T^{model}$ is the modeled $T_B$ by evaluating (12) using different values of $\epsilon_1, \epsilon_2,$ and $\tau$. (18) will be referred to as the DCA cost function henceforth. Except for the first and last overpass in the series, each overpass will have two retrieved values of $\epsilon$, from (18) applied to each overpass with the overpass before and after. These two values of $\epsilon$ for each overpass are averaged noting that these values were found to change minimally between overpasses. This optimization is nested within the outer loop optimization. The outer loop minimization determines the constant value of $\omega$ to use for each inner loop minimization of all pairs of adjacent passes. The cost function for this optimization is:

$$\min_{\omega} J_0 = \sum_{n=1}^{T} \left[ \min_{X = \epsilon_1, \epsilon_2, \tau} J(X) = \sum_{t=1}^{2} \sum_{p=h,v} (T_{Bp}^{obs} - T_{Bp}^{model}(X))^2 \right]$$

(19)

where $J_0$ is the sum of $J$ over $T$ observations (approximately 120 for one year of SMAP observations). (19) will be referred to as the $\omega$ cost function henceforth. $T$ can be set to a lower value than the number of observations to retrieve a temporally dynamic omega over the time span, $T$. However, in this study and in Konings et al. (2017), $T$ is set to the total number of observations meaning $\omega$ is effectively constant. In this study, possible values of $\omega$ are limited to increments of 0.01. This discretization increases the computational efficiency of the optimization at the expense of less precise values of $\omega$. $\omega$ is retrieved in a model selection technique in which the $\omega$ with the lowest $J_0$ from (19) is optimal. Since it was observed that $J_0$ becomes very large (poor fits of modeled $T_B$ to measured $T_B$) with multiple local minima generally at $\omega$ values greater than 0.15, $\omega$ is increased from 0 in increments of 0.01 until the $\omega$ cost function no longer continues to decrease. This improves the computational efficiency of the algorithm by reducing the number of times (19) is evaluated. The retrieved $\omega$ is 0.01 less than the $\omega$ value where the cost function begins to increase.

In this study, a first-order RT model is developed to account for first-order scattering within the vegetation canopy in an attempt to both improve SM and $\tau$ retrievals in forests and retrieve time variant and invariant microwave vegetation properties as another means for vegetation monitoring. This equation only introduces an additional time-invariant variable to the tau-omega model in (12) allowing its use in forward models for the algorithms in section 1.3. In addition, the MT-DCA is enhanced to retrieve a more robust constant $\omega$. Finally, the enhanced MT-DCA is implemented using SMAP microwave radiometric observations at 36 km postings over Africa using the tau-omega model and the first-order RT model developed here. Section 2 poses the motivating questions. Section 3 displays the methods for both the first-order scattering model and enhanced MT-DCA. Section 4 shows the behavior of the first-order RT model, results of the MT-DCA implementations at 36 km for Africa, and SM and $\tau$ validation results. The discussion and conclusions follow in Sections 5 and 6, respectively.
2. Research Questions

1) How can first-order scattering terms be included in a retrieval algorithm?
2) Is the first-order RT model more sensitive to the surface?
3) How does the enhanced MT-DCA change $SM$, $\tau$, and $\omega$ retrievals?
4) In which biomes does first-order scattering occur and how significant is it?
5) Does the first-order algorithm provide any meaningful information about vegetation properties?

3. Methodology

3.1 First-order RT Model

Instead of assuming the source functions are negligible in (5) and (6) resulting in the zeroth-order solution to the RT equations, Kurum et al. (2011) solves the RT equations analytically to the first-order resulting in the tau-omega model plus a first-order scattering term. The first-order scattering term is comprised of four terms which represent emission pathways that include scattering once from woody biomass (branches, trunks, etc.). Hornbuckle and Rowlandson (2008) find the same first-order emission pathways in their formulation. These four terms represent upward ground emission, upward vegetation emission, downward vegetation emission, and downward vegetation emission with ground reflection. Each term includes two corresponding emission pathways: one which scatters upwards and another which scatters downward and is reflected upward from the surface. These first-order terms introduce many more variables to the RT model that are a function of canopy dimensions among other variables. This requires extensive parameterization of canopy properties and in-situ measurements of the canopy. While this analytical model is insightful and effective for basin-scale canopy studies, it is difficult to implement globally as a forward model compared to the tau-omega model. Since the tau-omega model is an insufficient model in densely vegetated canopies, a simple forward model which includes first-order scattering vegetation effects without extensive parameterization of the canopy with many variables in addition to $r$, $\tau$, and $\omega$ is developed here.

Instead of analytically solving the RT equations in (5) and (6), approximations to the eight scattering terms can be developed using a ray tracing method of Kurum et al. (2011) determined pathways which considers interpretation of the tau-omega model terms, referred to as zeroth-order emission terms henceforth. Fig. 2 displays the tau-omega model terms from (12) in terms of emissivity, which is defined as $T_B/T_S$. The physical representation of each term is discussed here.
Fig. 2. Zeroth-order RT model emission pathways. Arrows represent emission pathways which “collect” reflection and attenuation terms forming the full term in terms of emissivity at end of pathway above the diffuse air-canopy layer. Note that each term is polarization dependent.

The $\gamma(1-r)$ term represents surface emission which is attenuated by the vegetation layer. Since $r$ represents reflection from the surface, $(1-r)$ is emission which is absorbed by the rough surface. Assuming the surface is a blackbody, $(1-r)$ is also the surface emission commonly referred to as soil emissivity. The surface emission signal is attenuated by the vegetation canopy by a factor of $\gamma$.

$(1-\omega)(1-\gamma)$ represents vegetation canopy emission. Mathematically, $(1-\omega)$ is equal to the fraction of extinction due to absorption in (10). Again, using the blackbody assumption, the absorption is equal to the emission. Physically, $\omega$ represents scattering of emission from particles within the collective vegetation layer in all directions. Evaluated in the limit as extinction goes to infinity ($\gamma$ approaches zero), $\omega$ becomes the reflectivity of the vegetation cloud. $(1-\omega)$ is thus the emission absorbed by the particles within the vegetation medium and is hence emitted due to the blackbody assumption. Since $\gamma$ is the fraction of emission transmitted through the canopy, $(1-\gamma)$ represents the fraction of emission lost within (and hence emitted by) the canopy. The multiplication of $(1-\omega)$ and $(1-\gamma)$ is the aggregated emissivity by the canopy in all directions. In this case, it represents upward vegetation emission which is unattenuated as it likely does not traverse the full height of the canopy layer.

Finally, $r\gamma(1-\omega)(1-\gamma)$ is the downward portion of $(1-\omega)(1-\gamma)$ emission which is reflected from the ground obtaining a multiplicative factor, $r$. It is then attenuated by the canopy layer by a factor of $\gamma$ as the pathway traverses the entire height of the layer.

As noted, this model only considers weak scattering from non-woody biomass within the canopy. The first-order emission pathways with one bounce from the scattering medium, defined as canopy constituents with dimensions larger than 21 cm (for L-Band), can be characterized in a simple model using these zeroth-order emission terms and a few key assumptions. 1) All first-order scattering takes place at the same height in the canopy where the center of mass of the
scattering medium is concentrated. 2) Upward (downward) vegetation emission that is scattered from the woody biomass at the canopy center is emitted from the bottom (top) of the canopy. 3) The scattering medium is symmetrical, reciprocal, and homogenous and scattering from this medium is defined by the first-order scattering coefficient, \( \omega_1 \). These assumptions are in addition to zeroth-order RT model assumptions used in Section 1.2, namely that the air-canopy boundary is diffuse, atmospheric emission entering the top of canopy is negligible, and canopy and soil temperature profiles are uniform.

The first and second assumption inherently result in emission pathways that only traverse a fraction of the layer and will consequently be only partially attenuated. Since the height of the canopy is denoted by \( d \), the location of the scattering is denoted by \( f'd \), where \( f \) is the fractional height of scattering medium above the ground surface. Thus, an emission pathway originating at the bottom of the canopy and interacting with the scattering medium is denoted as a variation from (11):

\[
yf = e^{-\left(\kappa f'd/\mu_s\right)}
\]

Additionally, the second assumption is a simplification that upward (downward) vegetation emission that interacts with the scattering medium must originate from below (above) the scattering medium. The final assumption states that \( \omega_1 \) is angle independent and thus upward and downward scattering from the scattering medium are equal. Additionally, \( \omega_1 \) is distinguished from the single scattering albedo definition in that \( \omega_1 \) represents scattering in all directions from woody biomass and \( \omega \) represents scattering in all directions from particles in the homogenous vegetation medium (e.g. non-woody biomass).

These assumptions and emission terms can be visualized schematically as in Fig. 3 where zeroth-order emission does not interact with the scattering medium while first-order emission does. The location of scattering medium interaction is at \( f'd \) above the surface. Note that the medium here is typically vegetation, but the same physics discussed here can apply to a soil medium with intrusions that cause volume scattering.
Fig. 3. Canopy Schematic of Zeroth and First-Order Emission Pathways. Note that this visual does not include all first-order emission pathways.

Using the terms in Fig. 2. and aforementioned assumptions, the eight combinations of first-order emission pathways are characterized into the simplified and approximate first-order terms in Fig. 4. As an example, consider the two ground emission pathways in Fig. 4A. Ground emission occurs with \((1-r)\) at the black dot, is attenuated by \(\gamma_f\) for traveling \(fd\) through the medium, and is scattered both upwards and downwards from the scattering medium resulting in collection of \(\omega_1\). In the downward scattering case, it is again attenuated by \(\gamma_f\), reflected from the surface gaining a factor of \(r\) and is fully attenuated by the length of the layer obtaining a factor \(\gamma\). This creates one first-order scattering term: \(\omega_1\gamma_f^2(1-r)\). In the upward scattering case, the emission is attenuated by \(\gamma_f^2\) creating another first-order scattering term: \(\omega_1\gamma_f^2(1-r)\).

Fig. 4. First-order scattering pathways. A: Ground emission and upward vegetation emission scattering pathways. B: Downward vegetation emission scattering pathways with and without surface reflection. Arrows represent emission pathways which “collect” reflection, attenuation,
and scattering terms forming the full term in terms of emissivity at end of pathway above canopy. Note that each term can be polarization dependent.

Like the first-order analytical solution to the RT equations in (Kurum et al. 2011), these first-order scattering terms are added to zeroth-order emission terms in (12):

\[ T_{B}^{1st} = T_{B}^{0th} + \omega_1 \left[ T_{s} \left[ \gamma (2f+1)(1-r) + \gamma (1-r) \right] + T_{c} \left[ \gamma (2f+1)r(1-\gamma)(1-\omega) + \gamma (1-\gamma)(1-\omega) + 2\gamma^2 r(1-\gamma)(1-\omega) + \gamma^2(1-f)(1-\gamma)(1-\omega) + \gamma^2 r^2 (1-\gamma)(1-\omega) \right] \right] \]

(21)

and is referred to here as the first-order RT model where \( T_{B}^{0th} \) is obtained from (12). All terms are polarization dependent. Note that (21) becomes (12) when \( \omega_1 \) is zero where \( \omega_1 \) thus serves as a factor which controls the magnitude of first-order scattering terms.

In some cases, the woody biomass distribution with height is known as is the case from localized ground or remote sensing studies (Krofcheck et al. 2016). This distribution can be considered as the probability density function (PDF) of \( f \) and the first assumption of concentrated scattering medium can be relaxed. (21) can thus be represented using the expected value rule accordingly:

\[ E[T_{B}^{1st}] = \int_{0}^{1} g(f) T_{B}^{1st} df \]

(22)

where \( g(f) \) is the PDF of \( f \) where \( f \) is bounded between zero and one. Note that this integration assumes that \( r, \gamma, \omega, \) and \( \omega_1 \) are independent of \( f \). This assumption should be sufficient as these parameters are shown to be only weakly dependent on \( f \) in 4.4. The beta distribution is recommended for use as it is bounded between 0 and 1 and distribution skewness can be varied using beta distribution parameters. In this case, (22) can be solved numerically. Some canopies can additionally be modeled with a uniform distribution where (22) takes the analytical form:

\[ E[T_{B}^{1st}] = T_{B}^{0th} + \omega_1 \left[ T_{s} \left[ \frac{\gamma r (1-r)(y^2-1)}{2\ln(\gamma)} + \gamma (1-r) \right] + T_{c} \left[ \gamma (1-\gamma)(1-\omega) + 2\gamma^2 r(1-\gamma)(1-\omega) + \left( \frac{(1-\gamma)(1-\omega)(y^2-1)}{2\ln(\gamma)} \right) (y^2 r^2 + y r + 1) \right] \right] \]

(23)

where \( g(f) = 1 \). However, in this study, the first assumption is applied in which the PDF of \( f \) is a Dirac delta function where \( g(f) = \delta(f-f_0) \). This states that the scattering medium is entirely concentrated at \( f_0 \).

3.2 MT-DCA Enhancement

In order to minimize the DCA cost function in (18), the MT-DCA uses an unconstrained gradient search function (called "fminsearch" in MATLAB) using eight spatially distributed initial conditions across the three dimensional search region. The MT-DCA selects the lowest \( J \) in the DCA cost function in (18) for each time step of the eight initial conditions. Summing each of these minimum \( J \) values for each value of \( \omega \) in each 0.01 increment in (19) results in the \( J_0 \) cost function as shown Fig. 5 (red line). Note that SMAP L1C enhanced 9 km data at Equal-Area
Scalable Earth-2 (EASE-2) grid pixel (1040, 2130) is used to produce this figure and is a representative example. As shown here from the unconstrained search, the optimal \( \omega \) is 0.09. However, with increasing \( \omega \), emissivity decreases, meaning \( \varepsilon_1 \) and \( \varepsilon_2 \) need to decrease to match the measured \( T_B \). This hinders the ability of the unconstrained gradient search to converge to physically realistic values of \( \varepsilon_1, \varepsilon_2, \) and \( \tau \) and resulted in the MT-DCA producing large \( J_0 \) values as shown in Fig. 5 (red line) for \( \omega \) greater than 0.10. Two enhancements were made as follows:

A) A constrained gradient search function (called “fmincon” in MATLAB) replaced the unconstrained gradient search function (“fminsearch”). Due to its more robust gradient search, the number of multi-start initial conditions was reduced to four.

B) The dielectric constants were constrained with a lower bound when \( SM \) is zero generated through the Mironov soil dielectric model. The Mironov model computes a soil dielectric constant from the measurement frequency, soil clay fraction (cf), and \( SM \). With \( SM \) set to zero and measurement frequency constant (1.4 GHz), the lower bound of soil dielectric constant is ultimately a function of clay fraction.

These revisions result in a smooth, convex cost function as shown in Fig. 5 (black line) with a single global minimum. Ultimately, this results in lower values of \( J \) in (19) meaning an improved fit of modeled \( T_B \) to measured \( T_B \). Optimal \( \omega \) generally increases as a consequence. Increasing \( \omega \) reduces \( \varepsilon_1 \) and \( \varepsilon_2 \). Testing this procedure in multiple regions across the globe revealed occurrences of low dielectric constants that produced unphysically low \( SM \) values (less than zero). Thus, Step B was added to properly constrain \( SM \) to always be greater than zero. This did not change the configuration of the enhanced cost function in Fig. 5.

![Fig. 5. MT-DCA \( \omega \) cost function comparison](image)

### 3.3 MT-DCA Optimization of \( \omega \) and \( \omega_l \)

With the addition of \( \omega_l \) in (21), an adjustment to the MT-DCA optimization framework discussed in 1.4 and 3.2 is required to now retrieve both \( \omega \) and \( \omega_l \). Like \( \omega \), \( \omega_l \) is assumed to be constant over a year as it is also a function of the dry, static canopy structure. Therefore, the algorithm is modified to retrieve temporally constant \( \omega \) and \( \omega_l \) simultaneously within a two
dimensional search space (instead of one dimensional as in Fig. 5) by updating (19) with (24) when the first-order RT model is used:

\[
\min_{\omega, \omega_1} J_1 = \sum_{n=1}^{\tau} \left[ X = \xi_1, \xi_2, \tau \right] J(X) = \sum_{t=1}^{\tau} \sum_{p=h,v} (T_{\text{obs}}^{bp} - T_{\text{model}}^{bp}(X))^2 \]  

(24)

where \( J_1 \) is the first-order cost function. While possible values of \( \omega \) and \( \omega_1 \) are discretized into 0.01 increments from 0 to 0.6 within the algorithm, it is computationally expensive to evaluate the function in (24) at all combinations of \((\omega_1, \omega)\) to determine the combination with the minimum \( J \) summation value. Thus, a search method is outlined here to determine the optimal \((\omega_1, \omega)\) combination by evaluating the behavior of individual, example cost functions. A simulated cost function is shown in Fig. 6 where artificially true values of H-polarized and V-polarized \( T_B \) were inserted as \( T_{\text{obs}}^{bp} \) and nominal values of \( T_S, h, SM, \theta, \) and \( \tau \) determined \( T_{\text{model}}^{bp} \) for all pairs of \((\omega_1, \omega)\). A smooth, thin valley appears within the cost function containing the optimal \((\omega_1, \omega)\) pair. A sensitivity analysis showed that observed changes of \( T_B \) on the order of 1.5 K were required to change \( \omega_1 \) by 0.01 and on the order of 2.5 K to change \( \omega \) by 0.01 showing a limited sensitivity of these parameters to radiometer noise (~1.1 K).

Actual cost functions were computed for 40 different pixels for all combinations of \( \omega \) and \( \omega_1 \) between 0 and 0.7 using (24) in the MT-DCA. An example is shown Fig. 7 which has approximately the same \( \tau \) as Fig. 6. By assessing the shape of the cost functions as a function of \( \tau \), a search procedure was developed as follows:

A) The MT-DCA zeroth-order implementation using (12) is used to obtain the optimal zeroth-order retrievals of \( SM, \tau, \) and \( \omega \). Retrieved \( \tau \) is averaged over the time series.

B) Assuming \( \tau \) from (12) will change marginally when the first-order RT model in (21) replaces the zeroth-order RT model in (12) (which is found to be the case in 4.3), if \( \tau \) is greater than 0.2 then continue to Step C. If it is less than 0.2, then continue to Step E.

C) \( \omega_1 \) is held constant at each value of 0, 0.06, 0.13, 0.2, 0.35, and 0.5 while \( \omega \) is varied to find the optimal combination of \( \omega \) and \( \omega_1 \) at the respective constant \( \omega_1 \). \( \omega_1 \) constant at 0 is effectively the zeroth-order implementation and pulls this information from Step A. This assumes that the \( \omega \) cost function is smooth and convex at these constant \( \omega_1 \) as in Fig. 5. This is generally true for these 40 pixels and is shown in Fig. 8 for the same example pixel as in Fig. 7. Nonetheless, a gradient search for the optimal \( \omega \) using (24) with constant \( \omega_1 \) is completed at each value of \( \omega_1 \). The gray vertical lines in Fig. 7 correspond to this search. This results in an optimal pair of \( \omega \) and \( \omega_1 \) at each constant value of \( \omega_1 \).

D) The \((\omega_1, \omega)\) pair with the lowest \( J_1 \) computed in (24) determined in Step C is then used as the starting point for a gradient search. Due to the known shape of the 40 evaluated cost functions, the search continues by increasing \( \omega_1 \) holding \( \omega \) relatively constant in one case and decreasing \( \omega_1 \) holding \( \omega \) relatively constant in another case. The horizontal gray “boxes” in Fig. 7 depict this search. The lowest \( J_1 \) is selected as the optimal \((\omega_1, \omega)\) pair. If \( J_1 \) is not less than \( J_0 \), then the zeroth-order optimal pair of \((0, \omega)\) is selected.

E) For low \( \tau \) (e.g. bare soil regions), the resulting cost function contains a valley, oriented primarily along the \( \omega \) axis with low values of \( \omega_1 \). In this case, Steps C and D are repeated.
with \( \omega \) held constant in Step C and a gradient search increasing and decreasing \( \omega \) holding \( \omega_i \) relatively constant in Step D.

F) Any pixel that retrieves \( \omega \) and/or \( \omega_i \) greater than 0.6 are flagged noting that the MT-DCA was unable to obtain optimal retrievals. In these cases, the cost function "valley" continues into unphysically high \( \omega \) and/or \( \omega_i \) without finding an optimal pair.

![Simulated Log Cost Function](image)

Fig. 6. Simulated log cost function using nominal inputs

![Log Cost Function 36 km (EASE2 = 196, 525; Lat/Lon = 2.12, 15.87)](image)

Fig. 7. MT-DCA Example Pixel Cost Function
Fig. 8. Intermediate $\omega$ search with $\omega_i$ held constant (Step C). $\omega_i = 0.13$ is optimal in this case.

When the optimization selects a non-zero $\omega_i$, this can physically be interpreted as an indicator of upwelling emission scattered from woody biomass. Non-zero $\omega_i$ also mathematically means a reduction in the summation of $J$ from (19) to (24), or, in other words, a better fit between modeled and measured $T_B$. This is because the zeroth-order implementation finds the prior best fit to the data for each pixel ($J_0$). Thus, in order to optimally select a non-zero value of $\omega_i$, $J_i$ in (24) must be lower than $J_0$ in (19). Thus, $\omega_i$ serves as a mathematical adjustment parameter which can selectively remove the effects of first-order scattering with $\omega_i$ equal to zero, or indicate woody biomass scattering effects (or at least a better mathematical fit to observations) with a non-zero $\omega_i$.

3.4 MT-DCA Africa Retrieval, Retrieval In-situ Comparison, and Ancillary Datasets

The MT-DCA was implemented using zeroth-order (routine discussed in 1.4) and first-order (routine discussed in 3.4 and MATLAB script available in Appendix) RT models over Africa from $38^\circ$N to $35^\circ$S and $18^\circ$W to $55^\circ$E using the first year of SMAP data from April 1st, 2015 to March 31st, 2016 at 36 km EASE-2 grid postings. These implementations will be referred to henceforth as zeroth-order and first-order implementations, respectively. Africa was chosen due to its diversity of non-frozen biomes across the continent. Here, SMAP L1C 36 km passive $T_B$ and L3 physical temperature products, derived from Numerical Weather Prediction model (O’Neill 2012), are the primary inputs (Chan et al. 2016). Clay fraction at 36 km EASE-2 postings are also input to convert $e$ to $SM$. Pixels with greater than 5% water fraction are masked via water fraction ancillary data. $N$ in (13) is set to 0. Finally a constant surface roughness parameter ($h$) equal to 0.13 is used to avoid imposing an ancillary spatial pattern on retrievals. $f$ is set to 0.5 assuming that the scattering medium is entirely concentrated at half of the canopy height which should be sufficient for application over a wide range of biomes in Africa. Section
4.4 shows that retrievals are weakly dependent on \( f \). Additionally, resulting retrievals are compared to IGBP land cover classification and GLAS LIDAR (Simard et al. 2011) vegetation height gridded to 36 km EASE-2 grid.

The sensitivity of the mean retrieved parameters to \( f \) in (21) is evaluated by implementing the first-order algorithm over a region within the Congo Basin of 1.7°S to 5.1°S and 25.7°E to 28.8°E which included 96 EASE-2 grid pixels and a gradient from north to south of tropical evergreen forest to woody savanna. A Dirac delta function PDF of \( f \) equal to 0.2 to 0.8 in 0.1 increments was input into the MT-DCA first-order implementation.

In order to compare the enhanced zeroth-order and first-order \( SM \) retrievals with ground measurements, the first-order and zeroth-order MT-DCA routines were implemented over pixels with SMAP core calibration/validation in-situ sites at 9 km resolution (Colliander 2017). This is to compare with the publicly available 9 km in-situ measurements spanning from April 1st, 2015 to October 31st, 2016. These calibration/validation sites include calibrated sensor measurement networks which can be upscaled to the scale of the SMAP radiometer footprint (Colliander et al. 2017). The 36 km inputs in the zeroth and first-order implementations of this study were replaced with SMAP L1C enhanced 9 km \( T_B \) observations using Backus Gilbert optimal interpolation (Chaubell et al. 2016) and SMAP L3 9 km physical temperature observations. Additionally, the SMAP SCA baseline 9 km parameters were obtained for cross comparison. In-situ measurements with quality flags were removed.

There is no known method to directly validate \( \tau \), \( \omega \), and \( \omega_1 \). However, zeroth-order and first-order temporal mean \( \tau \) retrievals are compared to an above ground biomass (AGB) in-situ dataset obtained from point measurements acquired from 2000 and after (Avitabile et al. 2016). Additionally, they will be compared to Avitabile et al. (2016) \( AGB \) estimated map which combines the in-situ measurements with Saatchi et al. (2011) and Baccini et al. (2012) \( AGB \) estimations. While sub-seasonal changes in \( \tau \) are likely due to changes in \( VWC \), annual mean \( \tau \) should be proportional to \( AGB \) which is slowly changing on annual time scales especially within Africa’s forested biomes. \( \omega \) and \( \omega_1 \) are even more challenging to validate and will only be compared to values used in SMAP and SMOS baseline algorithms (Wigneron et al. 2017).

4. Results

4.1 First-Order RT Equation Behavior

The effects of simulated first-order scattering on emissivity are first evaluated with a comparison between simulated outputs from zeroth and first-order RT models. Four different scenarios of high and low \( r_p \) and \( \gamma \) are evaluated in Fig. 9. In all cases, since the first-order scattering term is multiplied by a factor of \( \omega_1 \), the emissivity linearly increases with \( \omega_1 \) while the zeroth-order emissivity remains unchanged. The increased emissivity with \( \omega_1 \) counteracts the darkening effect of zeroth-order emissivity typically caused by high \( \omega \) in more vegetated regions (Kurum 2013). Higher \( \gamma \) leads to greater deviations in zeroth and first-order tau-omega model emissivities with changing \( \omega_1 \). Generally, the highest emissivities result from the lowest values of \( r_p \) and \( \gamma \). When \( r_p \) and \( \gamma \) are reduced, a greater amount of emission is attenuated (and absorbed) within the canopy and absorbed by the surface during surface reflection, respectively. However, more emission is thus emitted from the canopy and surface. These competing effects show that
low $r_p$ and $\gamma$ can create unphysical emissivities greater than 1 with increased values of $\omega_1$. This generally occurs when $\omega_1$ becomes unphysically much larger than $\omega$. This did not occur in any MT-DCA retrievals using (21) primarily due to the algorithm’s objective function of matching observed $T_B$ which are always lower than $T_S$.

Fig. 9. Simulated effects of first-order scattering on total emissivity.

Next, the scaling of first-order scattering terms in (21) in comparison to zeroth-order terms is evaluated in Fig. 10 with respect to incidence angle using nominal inputs values for all other parameters. For both H and V polarizations, the zeroth-order vegetation emissivity contributes most significantly to total emissivity with a significant contribution from the total emissivity of first-order scattering terms. Fig. 11 displays individual first-order scattering term emissivities as a function of incidence angle. Generally, the first-order scattering term is dominated by first-order ground scattering contribution followed by upward and downward scattered vegetation emission. In comparison to Kurum et al. (2011) analytical first-order scattering emissivity results, introduced first-order scattering terms are generally half the magnitude of analytically computed emissivities and tend to decrease instead of increase with incidence angle.
Fig. 10. Contribution of aggregated zeroth (vegetation and surface) and first-order terms to emissivity with respect to incidence angle.

Fig. 11. Contribution of individual first-order scattering terms to emissivity with respect to incidence angle. Upward and downward scattered vegetation emission have the same magnitude.

The analysis in Figs. 10 and 11 is repeated as a function of SM and is shown in Figs. 12 and 13. For medium to high SM values, the zeroth-order vegetation emissivity is dominant and tends to increase with SM while the zeroth-order ground emission term tends to decrease with increased SM. The first-order scattering term is not sensitive to SM overall. However, individual first-order scattering terms are sensitive to SM as shown in Fig. 13 at an incidence angle of 40°. As in Fig. 11, the ground emission term is typically the dominant first-order scattering term. Finally, the total magnitude of the first-order scattering term is less than half the magnitude of and is less sensitive to SM than Kurum et al. (2011) analytically derived first-order scattering terms.
Fig. 12. Contribution of aggregated zeroth and first-order emission terms to emissivity with respect to $SM$.

Fig. 13. Contribution of individual first-order scattering terms to emissivity with respect to $SM$.
Upward and downward scattered vegetation emission have the same magnitude.

Since both SMAP and SMOS missions flag retrieved parameters in regions with $VWC$ greater than 5 kg/m$^2$, the vegetation and ground emission terms of both zeroth-order and first-order RT models are evaluated in Fig. 14 to determine if the first-order RT model can retrieve $SM$ from surfaces under higher $VWC$. At the nominal values listed in Fig. 14, the first-order vegetation emission surpasses the first-order ground emission at approximately the same or lower $VWC$ than the intersection of zeroth-order terms. While decreasing $SM$ and increasing $\omega$ marginally increase the $VWC$ of this intersection, the first-order RT model does not necessarily strengthen confidence in parameter retrievals in regions with greater than $VWC$ of 5 kg/m$^2$. 
In order to comparatively evaluate the sensitivity of the zeroth and first-order RT models to SM, the SM sensitivity is computed for each model at different values of SM through:

$$SM \text{ Sensitivity Difference} = \left| \frac{\partial T_B}{\partial SM} \right|_1 - \left| \frac{\partial T_B}{\partial SM} \right|_0$$

(25)

where the subscript refers to the RT model order (e.g. subscript 1 uses (21) and 0 uses (12)). Simply, a more “sensitive” model is one that responds with a larger change in $T_B$ with the same unit change of SM. The absolute value removes the effect of the opposite sign of changing SM and $T_B$. The magnitude of change in sensitivity between first-order and zeroth-order RT models is ultimately of concern. Thus, positive values of (25) can be interpreted as an enhanced sensitivity to SM with (21) over (12). Fig. 15 plots the SM sensitivity difference with respect to SM at different nominal values of $\tau$. Similarly, Fig. 16 shows SM sensitivity difference with respect to $\tau$ at nominal values of SM. To summarize these results, only at high values of $\tau$ are there enhancements to SM sensitivity for all values of SM. At lower values of $\tau$, only medium to high values of SM show SM sensitivity enhancements. Fig. 16 additionally shows that at $\tau<0.05$ and $\tau>0.75$, any value of SM will still result in an improvement to SM sensitivity. Consider the fact that SM can increase on the order of 0.05 m$^3$/m$^3$ after a rain event. Thus, the first-order RT model in (21) will decrease by 0.1 K more than the zeroth-order model (12). This sensitivity improvement is marginal and less than the typical ~1 K satellite-based radiometer measurement error. However, it is promising that larger sensitivity improvements are made at higher values of $\tau$ or in moderate to highly vegetated regions.
Algorithm Sensitivity to Soil Moisture

![Graph showing algorithm sensitivity to soil moisture](image)

Fig. 15. First and zeroth-order SM sensitivity comparison with respect to SM. Solid and dashed lines are for SM sensitivity enhancements at $\tau = 0.25$ Np and $\tau = 0.75$ Np, respectively. The dotted line is a reference for when the RT models have the same SM sensitivity.

![Graph showing SM sensitivity difference](image)

Fig. 16. First and zeroth-order SM sensitivity comparison with respect to $\tau$. Solid and dashed lines are for SM sensitivity enhancements at $SM = 0.1$ m$^3$/m$^3$ and $SM = 0.3$ m$^3$/m$^3$, respectively. The dotted line is a reference for when the RT models have the same SM sensitivity.

4.2 MT-DCA Africa Zeroth-Order Implementation

Fig. 17 shows the results of the zeroth-order implementation where the expectation operator in this context is the temporally averaged variable over each pixel time series. Additionally, “New” refers to the enhanced MT-DCA procedure discussed in 3.2 while “Old” is the original MT-DCA procedure as discussed in 1.4 and (Konings et al. 2016). The spatial patterns in Figs. 17A and 17C are as expected where the surface is wetter and more vegetated at the equator with these conditions showing a decreasing gradient away from the equator. This largely reflects the regional climate. Fig. 17E shows high $\omega$ everywhere, but the bare surfaces
(e.g. Sahara and Kalahari Deserts) with no significant correlation with vegetation density (not shown). SMAP L1C $T_B$ is not water body corrected where pixels near water bodies (e.g. coastal regions) may have $T_B$ measurements that are biased low (due to low water $T_B$). This can result in a positive bias in $\omega$ and $SM$. This can explain artificially wet surfaces and high $\omega$ in coastal regions as shown in Figs. 17A and 17E, respectively. This is especially the case for the coastal deserts in Somalia.

The MT-DCA enhancements discussed in 3.2 ultimately caused generally higher $\omega$ especially outside of the Congo Basin and reduced $\omega$ in the Kalahari Desert and minor portions of the Sahara Desert. Increased $\omega$ creates a darkening effect (reduction) on modeled $T_B$. In order to match measured $T_B$, $SM$ and $\tau$ are decreased to increase the modeled $T_B$ explaining the decrease in $SM$ and $\tau$ in regions with increased $\omega$. The opposite sequence occurs in regions with decreased $\omega$. On average, the MT-DCA enhancements tend to reduce $SM$ and $\tau$ and increase $\omega$ across Africa.
Fig. 17. A: Mean SM. B: Mean SM change. C: Mean \( \tau \). D: Mean \( \tau \) change. E: Mean \( \omega \). F: Mean \( \omega \) change. “New” refers to the enhanced MT-DCA procedure discussed in 3.2 while “Old” is the original MT-DCA procedure as discussed in 1.4 and (Konings et al. 2016).
4.3 MT-DCA Africa First-Order Implementation

In order to place the following results in context, it is worth first discussing the resulting constant $\omega_1$ value (Fig. 18). This factor controls the magnitude of the first order scattering term and, thus, where it is zero, the same retrievals as the zeroth-order implementation are obtained. Initial inspection shows that forested regions of Africa have a higher, non-zero $\omega_1$. This is confirmed by Table 1 where the median $\omega_1$ in the tropical evergreen forests is 0.05 and in Fig. 19 where taller vegetation typically has a higher $\omega_1$. Further, $\omega_1$ has a median of zero in grasslands, savannas, and shrublands where woody vegetation is sparse or nonexistent. Surprisingly, $\omega_1$ is generally non-zero in bare areas and is addressed further in Section 5.5.

Ultimately, non-zero $\omega_1$ was selected for a given pixel only when $J_1$ from (24) was less than $J_0$ from (19). In other words, non-zero $\omega_1$ is selected only when first-order scattering terms improve the fit between measured and modeled $T_B$. Fig. 20 shows the $J_1$ and $J_0$ difference which is normalized into units of K improvement per observation. While this improvement is generally marginal (~0.02K), it is significant in bare regions (up to -1.2 K) as shown in Table 2.

![Map of Africa showing the distribution of $\omega_1$ values](image)

Fig. 18. $\omega_1$. As noted in 3.3, pixels that could not determine optimal values of $\omega$ and/or $\omega_1$ were masked.

34
Table 1. $\omega_I$ distribution binned based on each IGBP land cover classification. Land cover classes with a regional presence less than 1% were removed due to low sample size.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Evergreen Broadleaf Forest</th>
<th>Open Shrublands</th>
<th>Woody Savannas</th>
<th>Savannas</th>
<th>Grasslands</th>
<th>Croplands</th>
<th>Mosaic</th>
<th>Bare</th>
</tr>
</thead>
<tbody>
<tr>
<td>10th</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>0.05</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
</tr>
<tr>
<td>90th</td>
<td>0.266</td>
<td>0.17</td>
<td>0.275</td>
<td>0.11</td>
<td>0.12</td>
<td>0.17</td>
<td>0.13</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Fig. 19. Joint density of $\omega_I$ and LIDAR vegetation height.
Fig. 20. Change in residual $J$ from zeroth to first-order. $J_I$ from (24) and $J_0$ from (19) were subtracted, divided by the number of observations, divided by two (to average between H and V polarization), and square rooted to convert to units of K improvement per observation.

Note that the color axis was adjusted in an attempt to show the relative scale of change in residual values between the Congo Basin and Sahara Desert. However, dark red values in the Sahara and Kalahari Deserts are as low as -1 K per observation.

Table 2. Normalized residual difference between first and zeroth-order implementations binned based on each IGBP land cover classification. Land cover classes with a regional presence less than 1% were removed due to low sample size.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Evergreen Broadleaf Forest</th>
<th>Open Shrublands</th>
<th>Woody Savannas</th>
<th>Savannas</th>
<th>Grasslands</th>
<th>Croplands</th>
<th>Mosaic</th>
<th>Bare</th>
</tr>
</thead>
<tbody>
<tr>
<td>10th</td>
<td>-0.014</td>
<td>-0.027</td>
<td>-0.003</td>
<td>-0.005</td>
<td>-0.280</td>
<td>-0.016</td>
<td>-0.020</td>
<td>-1.257</td>
</tr>
<tr>
<td>50th</td>
<td>-0.003</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>90th</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Fig. 21 displays the annual mean first-order implementation results for $SM$, $\tau$, and $\omega$ and their respective changes from zeroth-order implementation results. Again, only pixels with non-zero $\omega_I$ resulted in changes in $SM$, $\tau$, and $\omega$. $SM$ and $\tau$ primarily increased in areas with taller vegetation while $\omega$ generally increased in bare areas as supported by Fig. 22. The relatively larger percent changes in $\omega$ in the bare regions are likely the cause of the significantly improved fits to the measured $T_B$ (Fig. 20). In fact, $\omega$ is now largest in the bare areas as shown in Fig. 22 and Table 3 whereas it was near zero in the zeroth-order implementation (Fig. 17E). The overall
effect of the first-order scattering terms is to increase $SM$, $\tau$, and $\omega$ with the spatial distribution of changes largely a function of vegetation structure.

Additionally, the change in standard deviation between time series of first and zeroth-order implementations for $SM$ and $\tau$ is displayed in Figs. 23A and 23B, respectively. There appears to be an increase in $SM$ temporal variability in forested areas. On the other hand, $\tau$ temporal variability appears to be reduced in both bare and forested regions. This is supported by the boxplots in Fig. 24 where the temporal variation is binned as a function of vegetation height. Thus, this can be interpreted as more sensitivity to the surface and less sensitivity to the overlying vegetation with the first-order RT model.
Fig. 21. A: Annual mean SM. B: Annual mean SM change. C: Annual mean τ. D: Annual mean τ change. E: ω. F: ω change. The subscript “1” refers to the first-order implementation using (21) and subscript “0” refers to the zeroth-order implementation using (12). “1rst” and “0th” are used
for effective scattering albedo subscripts in place of “1” and “0” to eliminate confusion with the first-order scattering coefficient, $\omega_1$.

![Box plots](image)

Fig. 22. First and zeroth-order difference binned as a function of LIDAR vegetation height. A: $SM$ difference. B: $\tau$ difference. C: $\omega$ difference.

Table 3. $\omega$ (from first-order implementation) distribution binned based on each IGBP land cover classification. Land cover classes with a regional presence less than 1% were removed due to low sample size.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Evergreen Broadleaf Forest</th>
<th>Open Shrublands</th>
<th>Woody Savannas</th>
<th>Savannas</th>
<th>Grasslands</th>
<th>Croplands</th>
<th>Mosaic</th>
<th>Bare</th>
</tr>
</thead>
<tbody>
<tr>
<td>10th</td>
<td>0.08</td>
<td>0.08</td>
<td>0.09</td>
<td>0.09</td>
<td>0.08</td>
<td>0.10</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>50th</td>
<td>0.11</td>
<td>0.16</td>
<td>0.12</td>
<td>0.13</td>
<td>0.21</td>
<td>0.25</td>
<td>0.17</td>
<td>0.58</td>
</tr>
<tr>
<td>90th</td>
<td>0.24</td>
<td>0.63</td>
<td>0.57</td>
<td>0.60</td>
<td>0.61</td>
<td>0.63</td>
<td>0.59</td>
<td>0.67</td>
</tr>
</tbody>
</table>
To evaluate the ratio between $\omega_1$ and $\omega$, a quantity is introduced here as $\psi (\psi = \omega_1/\omega)$ and spatial distribution is shown in Fig. 25. With $\omega_1$ representing scattering from woody vegetation
and κ representing scattering from non-woody vegetation cloud, ψ captures the ratio between these two types of scattering as a static vegetation structure parameter. The spatial distribution closely resembles the distribution of annual mean τ (Figs. 17C and 21C). Therefore, the highest ψ are primarily in forests and lowest values in regions with sparse vegetation cover as shown in Figs. 26 and Table 4. Fig. 27 shows that ψ and annual mean τ are weakly related with a Pearson correlation coefficient of 0.51 (p<0.001).

![Image](image_url)

**Fig. 25.** ψ spatial distribution

![Image](image_url)

**Fig. 26.** Joint density of ψ and LIDAR Vegetation Height
Table 4. \( \psi \) distribution binned based on each IGBP land cover classification. Land cover classes with a regional presence less than 1% were removed due to low sample size.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Evergreen Broadleaf Forest</th>
<th>Open Shrublands</th>
<th>Woody Savannas</th>
<th>Savannas</th>
<th>Grasslands</th>
<th>Croplands</th>
<th>Mosaic</th>
<th>Bare</th>
</tr>
</thead>
<tbody>
<tr>
<td>10th</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
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</tr>
<tr>
<td>50th</td>
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<td>0.00</td>
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<td>0.00</td>
</tr>
<tr>
<td>90th</td>
<td>1.11</td>
<td>0.32</td>
<td>0.67</td>
<td>0.26</td>
<td>0.27</td>
<td>0.37</td>
<td>0.38</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Fig. 27. \( \psi \) and temporal mean \( \tau \) relationship. Blue points are all woody vegetation IGBP classes while yellow points are non-woody vegetation IGBP classes.

Finally, the relative magnitude of zeroth and first order emission terms are shown here. The ratio of first-order scattering \( T_B \) (only \( T_B \) of first-order terms in (21)) to total \( T_B \) (all terms in (21)) is displayed in Figs. 28 and 29 for H and V polarizations, respectively. While Fig. 18 indicates the presence of first-order scattering, these figures quantify the relative contribution of first-order scattering to the SMAP radiometer’s \( T_B \) signal. The first-order scattering terms range from 0% to 40% of the \( T_B \) signal with most pixels between 5% and 10% in woody biomass regions (not shown). This translates to a median of 11 K for first-order scattering terms as shown in Figs. 28B and 29B which is significant.

In 4.1, the first-order RT model in (21) is shown to be more sensitive to the surface. This is because the first-order scattering terms add more interactions with the surface (e.g. increased presence of surface reflectivity) and it adds two surface emission pathways as shown in Fig. 4A. In Fig. 30A, the ratio of surface emission first-order scattering terms (e.g terms with \((1-\gamma)\)) to total first-order scattering emission is displayed at H-polarization. Conversely, Fig. 30B displays the ratio of vegetation emission first-order scattering terms (e.g terms with \((I-\gamma)/(I-\omega)\)) to total first-order scattering emission. Figs. 30C and 30D display the respective V-polarization equivalent. Due to the large amount of signal extinction in forests, the first-order surface emission percentage is \(\sim 25\%\). However, the \( T_B \) signal is still partitioned to additional surface emission terms within the first-order algorithm achieving more sensitivity to the surface.
Building on this further, a metric which determines extra sensitivity to the surface between zeroth and first-order RT models is defined as surface contribution difference:

\[
\text{Surface Contribution Difference} = \frac{T_s y(1-r) + \omega_1 T_s [y^2(1-r) + y(1-r)]}{(21)} - \frac{T_s y(1-r)}{(12)}
\] (26)

The first term is all surface emission terms in (21) divided by the total first-order emission from (21). The second term is the zeroth-order equivalent of this with the single surface emission term divided by the total zeroth-order emission. Thus, the fact that two additional first-order scattering terms from surface emission have non-zero emissivities means that the first-order optimization routine now includes a greater contribution from surface terms using the first-order model in (21) than the zeroth-order model in (12). This is shown in Fig. 31 where positive values of surface contribution difference, though marginal, can be interpreted as increases in surface sensitivity. As expected from the results in 4.1, this occurs in wetter regions with more vegetation (e.g. Congo Basin) where the surface signal increases by 2%.

Fig. 28. A: Ratio of first-order scattering \( T_{BH} \) to total \( T_{BH} \). B: First-order scattering \( T_{BH} \) distribution only in regions with woody biomass. Note that 26% percent of woody biomass pixels do not have any first-order scattering.
Fig. 29. A: Ratio of first-order scattering $T_{BV}$ to total $T_{BV}$. B: First-order scattering $T_{BV}$ distribution only in regions with woody biomass. Note that 26% percent of woody biomass pixels do not have any first-order scattering.

Fig. 31. Surface contribution difference from (26). A: H-polarization. B: V-Polarization

4.4 MT-DCA Retrieval Sensitivity to $f$

Each parameter is averaged over the entire region for comparison between different values of $f$ as shown in Fig. 32. Generally, pixels with zero $\omega_f$ at $f = 0.5$ remain with $\omega_f$ equal to zero at all other values of $f$. Consequently, the other retrieved parameters do not change. Pixels with nonzero $\omega_f$ at $f = 0.5$ change slightly at other values of $f$. Thus, only pixels that exhibit first-order scattering show a weak sensitivity to $f$ showing some finer scale sensitivity of retrievals to $f$. 

45
Fig. 32. Mean parameter retrieval with varied concentration of woody biomass. Parameter for each row is denoted on y-axis to left. Mean parameters of all pixels in focus region are in the left column. Pixels with zero \( \omega_1 \) originally at \( f = 0.5 \) (39% of region) from the first-order implementation in 4.3 are compared in the center column while pixels with nonzero \( \omega_1 \) originally at \( f = 0.5 \) (61% of region) are compared in the right column.

4.5 In-situ Soil Moisture Comparison

In order to compare the SMAP baseline (SCA), zeroth-order implementation (MT-DCA\(_0\)), and first-order implementation (MT-DCA\(_1\)) retrieved SM with the in-situ measured SM, statistics including bias, root mean square error (RMSE), unbiased RMSE, and Pearson’s correlation coefficient are used here for intercomparison (Entekhabi, Reichle, et al. 2010). The bias is the temporally averaged algorithm-derived SM product minus the temporally averaged in-situ SM. Correlation is Pearson’s correlation coefficient between the algorithm SM product and in-situ SM measurements. The root mean square error (RMSE) and unbiased RMSE (ubRMSE) are shown, respectively, as:

\[
RMSE = \sqrt{E \left[ (SM_{Algorithm} - SM_{In-situ})^2 \right]} 
\]  
\[
ubRMSE = \sqrt{RMSE^2 - Bias^2} 
\]

Table 5 displays the resulting comparison between each algorithm-derived SM and in-situ SM measurements at each calibration/validation site that made their data publically available. Note that the calibration/validation sites are primarily grasslands, croplands, natural mosaic, or a
combination of these. Thus, they are largely sites with less woody vegetation and possibly a large polarization difference where the removal of \( o \) and \( r \) polarization assumption in the MT-DCA may be insufficient. Additionally, care must be taken to interpret the in-situ observations as only another point of comparison and not "truth" as there is measurement error, error in capturing the large scale spatial heterogeneity in the pixel, and error in upscaling point measurements to a 9 km scale.

All three algorithms compare similarly to the in-situ measurements at each site. Thus, if the SCA algorithm has a low level of comparison with in-situ measurements, then the MT-DCA implementations compare similarly as well. This is case because all three algorithms are derivations of the same SMAP L1C \( T_B \) measurements. Additionally, \( SM \) from the first-order implementation differs from the zeroth-order implementation at only 4 of the 13 sites. This is expected since the sites contain less woody vegetation and would be expected to have low or zero \( o_i \). Therefore, it is difficult to determine if \( SM \) from the first-order implementation compares more closely to in-situ measurements than the SMAP baseline algorithm or zeroth-order MT-DCA implementation. The sites with poor comparisons such as Fort Cobb, South Fork, and Kenaston have non-zero \( o_i \) in the first-order algorithm possibly meaning that the effects of vegetation may be creating a difference between algorithms and in-situ measurements. A comprehensive forest in-situ measurement campaign would need to be conducted to make further assertions. Nonetheless, example time series with auxiliary information are shown for REMEDHUS and Fort Cobb sites as shown in Figs. 33 and 34.

Table 5. Soil Moisture In-Situ Comparison

<table>
<thead>
<tr>
<th>Site Name</th>
<th>RMSE SCA</th>
<th>MTDCAl</th>
<th>MTDACl</th>
<th>Bias SCA</th>
<th>MTDCAl</th>
<th>MTDACl</th>
<th>ubRMSE SCA</th>
<th>MTDCAl</th>
<th>MTDACl</th>
<th>Correlation SCA</th>
<th>MTDCAl</th>
<th>MTDACl</th>
</tr>
</thead>
<tbody>
<tr>
<td>REMEDHUS</td>
<td>0.042</td>
<td>0.056</td>
<td>0.053</td>
<td>-0.007</td>
<td>0.000</td>
<td>0.000</td>
<td>0.042</td>
<td>0.056</td>
<td>0.053</td>
<td>0.843</td>
<td>0.811</td>
<td>0.830</td>
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<tr>
<td>Reynolds Creek</td>
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<td>0.064</td>
<td>-0.022</td>
<td>-0.003</td>
<td>-0.003</td>
<td>0.044</td>
<td>0.064</td>
<td>0.064</td>
<td>0.541</td>
<td>0.502</td>
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</tr>
<tr>
<td>Yanco</td>
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<td>0.073</td>
<td>0.040</td>
<td>0.047</td>
<td>0.047</td>
<td>0.056</td>
<td>0.065</td>
<td>0.065</td>
<td>0.872</td>
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<td>0.061</td>
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<td>Walnut Gulch</td>
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<td>0.038</td>
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<td>-0.003</td>
<td>-0.003</td>
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<td>0.039</td>
<td>0.039</td>
<td>0.771</td>
<td>0.784</td>
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</tr>
<tr>
<td>Little Washita</td>
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<td>0.046</td>
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<td>0.013</td>
<td>0.013</td>
<td>0.023</td>
<td>0.046</td>
<td>0.046</td>
<td>0.915</td>
<td>0.863</td>
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</tr>
<tr>
<td>Fort Cobb</td>
<td>0.062</td>
<td>0.066</td>
<td>0.066</td>
<td>-0.055</td>
<td>-0.045</td>
<td>-0.030</td>
<td>0.028</td>
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<td>0.057</td>
<td>0.904</td>
<td>0.885</td>
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<tr>
<td>Little River</td>
<td>0.093</td>
<td>0.040</td>
<td>0.040</td>
<td>0.085</td>
<td>0.006</td>
<td>0.006</td>
<td>0.028</td>
<td>0.039</td>
<td>0.039</td>
<td>0.856</td>
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</tr>
<tr>
<td>South Fork</td>
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<td>-0.060</td>
<td>-0.121</td>
<td>-0.125</td>
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<tr>
<td>Monte Buey</td>
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<td>0.161</td>
<td>0.160</td>
<td>-0.015</td>
<td>-0.156</td>
<td>-0.156</td>
<td>0.057</td>
<td>0.040</td>
<td>0.041</td>
<td>0.914</td>
<td>0.784</td>
<td>0.784</td>
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<td>0.140</td>
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<td>-0.032</td>
<td>-0.113</td>
<td>-0.107</td>
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<td>0.043</td>
<td>0.043</td>
<td>0.727</td>
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</tr>
<tr>
<td>TxSON</td>
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<td>0.062</td>
<td>0.062</td>
<td>-0.028</td>
<td>0.028</td>
<td>0.028</td>
<td>0.036</td>
<td>0.043</td>
<td>0.043</td>
<td>0.838</td>
<td>0.798</td>
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</tr>
</tbody>
</table>
Fig. 33. REMEDHUS Core Cal/Val In-situ, SCA, MT-DCA Zeroth-Order, and MT-DCA First-Order Parameter Comparison.

Fig. 34. Fort Cobb Core Cal/Val In-situ, SCA, MT-DCA Zeroth-Order, and MT-DCA First-Order Parameter Comparison.
4.6 In-situ \( \tau \) Comparison

Annual mean \( \tau \) retrievals from the zeroth-order and first-order MT-DCA implementations are compared to \( AGB \) in-situ measurements and estimations in Fig. 35 (Avitabile et al. 2016). Note that only 0.6\% of the 36 km EASE-2 pixels contained in-situ AGB measurements as shown with fewer data points for comparison in Fig. 35A as opposed to Fig. 35B. Nonetheless, in both cases, both orders of \( \tau \) retrievals are significantly correlated (\( p<0.01 \)) with \( AGB \) with the first-order retrieved \( \tau \) marginally less correlated. Finally, there appears to be a superlinear relationship between \( \tau \) and \( AGB \) (apparent in Fig. 35B) where \( \tau \) begins to saturate for \( AGB \) above 350 Mg/ha. This therefore suggests that there is an upper bound of \( \tau \) that can be detected at L-Band which makes sense in the context that retrievals become unreliable under dense canopies.

![AGB In-Situ Comparison](image1)

Fig. 35. Comparison between both zeroth-order (black) and first-order (magenta) annual mean \( \tau \) and in-situ measured \( AGB \) (A) and estimated \( AGB \) (B). Pearson correlation coefficients are displayed in the legend next to the respective order of \( \tau \) retrieval.

5. Discussion and Conclusions

5.1 Algorithm Implementation and Behavior

This section addresses the first three research questions. First-order scattering is included within the retrieval algorithm using ray tracing of the first-order wave interactions with the scattering medium. The advantage is that this only produced one new variable, \( \omega_1 \), in addition to zero-order RT model parameters. Therefore, within the MT-DCA framework, the method does not require ancillary information about the vegetation canopy (e.g. allometric relationships for canopy geometry) allowing it to be used in a global retrieval algorithm. Also, additional emission terms contributing to the zeroth-order RT model do not imply higher emissivities. This only changes the relative partitioning of the measured \( T_B \) into additional emission terms since the objective function of the MT-DCA optimization routine is to match the measured \( T_B \) with forward modeled \( T_B \).

This introduced first-order RT model was determined to be more sensitive to the surface especially for high \( SM \) and \( \tau \), but only marginally with no improvement of the upper bound of
VWC under which reliable retrievals can be made. In another perspective, the greater sensitivity to the surface is described by the first-order scattering terms creating additional interactions with the surface. This additional surface emission signal is stronger further away from regions of woody biomass mostly due to additional vegetation emission terms as well (Fig. 30). Ultimately, the first-order surface emission contribution is greater than the zeroth-order contribution only in the vegetated regions of Africa quantifying the surface sensitivity improvement (Fig. 31). This results in a greater standard deviation of SM in these vegetated regions (Fig. 24).

The MT-DCA enhancement (primarily of constraining the DCA search) resulted in generally higher values of \( \omega \) and simultaneously lower mean \( SM \) and \( \tau \). This included a lower value of \( J_0 \), or an improved fit between measured and modeled \( T_B \). The constrained minimization ultimately found more robust minima at higher values of \( \omega \) leading the algorithm to generally select higher \( \omega \) within each pixel. This effect was most prevalent in The Sahel.

5.2 First-Order Algorithm Performance

While the ultimate goal within quantification of passive microwave emission multiple scattering is to determine \( SM \) under forested canopies (Kurum et al. 2011; Lang, Utku, and Matthaeis 2001), this was not an explicit goal of this study due to validation limitations, especially in Africa. However, the relative performance of zeroth-order and first-order implementations is discussed here using both physical in-situ and mathematical metrics for assessment. Physically, \( SM \) retrievals were compared to \( SM \) in-situ measurements at 13 SMAP core calibration/validation sites across the globe and \( \tau \) retrievals were compared to \( AGB \) measurements and estimations across Africa. The zeroth and first-order retrieved \( SM \) compared closely with in-situ measurements and SMAP baseline algorithm. However, their relative performance was inconclusive mainly because the in-situ sites were cropland and grassland which are not expected to exhibit first-order scattering. Additionally, high positive correlations between \( AGB \) and annual mean \( \tau \) shows that at least annual mean retrieved \( \tau \) relate to the ground measurements. However, first-order \( \tau \) does not appear to improve correlations with the \( AGB \).

Perhaps of more concern are \( \tau \) temporal dynamics which are primarily a function of \( VWC \) within primary canopy branches (Guglielmetti et al. 2007; Jackson and Schmugge 1991). However, no in-situ validation methods of this exist. This is no surprise as satellite radiometer footprint scale biomass \( VWC \) measurements are destructive and costly. Nonetheless, comprehensive measurement campaigns are required to properly validate any \( SM \) or \( \tau \) retrievals in moderate to densely vegetated surfaces.

No known methods are available to validate \( \omega \) and \( \omega_1 \) retrievals. However, it is of value to compare zeroth and first-order median \( \omega \) retrievals to \( \omega \) used in SMAP baseline, SMOS baseline, and SMOS-IC algorithm (Roberto Fernandez-Moran et al. 2017) with respect to IGBP land cover classifications as shown in Table 6. While first-order retrieved \( \omega \) are larger than zeroth-order retrieved \( \omega \), both MT-DCA retrieval algorithms result in higher \( \omega \) than what is currently used in SMAP and SMOS baseline algorithms. The MT-DCA\(_0\) and SMOS-IC algorithm derived \( \omega \) are closely related. Note that median MT-DCA retrievals shown here are only from Africa and may change if this first-order RT model and algorithm is implemented globally.
Table 6. \( \omega \) Algorithm Comparison. MT-DCA retrievals were only performed over Africa where not all land cover classifications are present (denoted by -). MT-DCA \( \omega \) are medians of the distribution of retrieved \( \omega \) within each land cover classification.

<table>
<thead>
<tr>
<th>Land Cover Classification</th>
<th>SMAP Baseline</th>
<th>SMOS Baseline</th>
<th>SMOS-IC</th>
<th>MT-DCA(_0)</th>
<th>MT-DCA(_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>'Evergreen Needleleaf Forest'</td>
<td>0.05</td>
<td>0.06 or 0.08</td>
<td>0.06</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>'Evergreen Broadleaf Forest'</td>
<td>0.05</td>
<td>0.06 or 0.08</td>
<td>0.06</td>
<td>0.08</td>
<td>0.11</td>
</tr>
<tr>
<td>'Deciduous Needleleaf Forest'</td>
<td>0.05</td>
<td>0.06 or 0.08</td>
<td>0.06</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>'Deciduous Broadleaf Forest'</td>
<td>0.05</td>
<td>0.06 or 0.08</td>
<td>0.06</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>'Mixed Forests'</td>
<td>0.05</td>
<td>0.06 or 0.08</td>
<td>0.06</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>'Closed Shrublands'</td>
<td>0.05</td>
<td>0.00</td>
<td>0.10</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>'Open Shrublands'</td>
<td>0.05</td>
<td>0.00</td>
<td>0.08</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td>'Woody Savannas'</td>
<td>0.05</td>
<td>0.00</td>
<td>0.06</td>
<td>0.11</td>
<td>0.12</td>
</tr>
<tr>
<td>'Savannas'</td>
<td>0.08</td>
<td>0.00</td>
<td>0.10</td>
<td>0.11</td>
<td>0.13</td>
</tr>
<tr>
<td>'Grasslands'</td>
<td>0.05</td>
<td>0.00</td>
<td>0.10</td>
<td>0.11</td>
<td>0.18</td>
</tr>
<tr>
<td>'Wetlands'</td>
<td>-</td>
<td>0.00</td>
<td>0.10</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>'Croplands'</td>
<td>0.05</td>
<td>0.00</td>
<td>0.12</td>
<td>0.15</td>
<td>0.18</td>
</tr>
<tr>
<td>'Urban'</td>
<td>0.03</td>
<td>0.00</td>
<td>0.10</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>'Mosaic'</td>
<td>0.07</td>
<td>0.00</td>
<td>0.12</td>
<td>0.12</td>
<td>0.16</td>
</tr>
<tr>
<td>'Frozen'</td>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>'Bare'</td>
<td>0.00</td>
<td>0.00</td>
<td>0.12</td>
<td>0.02</td>
<td>0.27</td>
</tr>
</tbody>
</table>

The retrieved parameters from the first-order algorithm fit the SMAP measured \( T_B \) better than the zeroth-order algorithm wherever \( \omega_1 \) is non-zero. Wherever \( \omega_1 \) is zero, the first-order algorithm retrieves the same \( SM, \tau \), and \( \omega \) values as the zeroth-order algorithm. Therefore, \( \omega_1 \) behaves as an on/off switch for the first-order scattering terms as can be deduced from (21). However, higher \( \omega_1 \) does not result in a monotonically better fit to SMAP measured \( T_B \). Vegetated biomes with high \( \omega_1 \) only have a marginally improved fit to measured \( T_B \) on the order of 0.05K/observation with the first-order algorithm. Curiously, bare surfaced areas with low \( \omega_1 \), on the other hand, have significant improvements in fit on the order of 0.7K/observation (Fig. 20 and Table 2). Also, Fig. 30 shows that the first-order implementation marginally increased the surface signal in Africa’s forested regions. This is supported by an increase in temporal variability of \( SM \) in forests in Figs. 23A and 24A.

Based on the assessment of these physical and mathematical metrics, it appears that the first-order algorithm mathematically performs better than the zeroth-order algorithm, but it is inconclusive whether this results in more reliable \( SM \) and vegetation retrievals. It is important to note here that the value is not placed in this algorithm producing a \( SM \) product in addition to the publicly SMAP L3 36 km. Rather, this provides a unique opportunity to extract additional geophysical information within the \( T_B \) measurements about the surface.

5.3 Vegetation Information

This section addresses the final two research question to determine if first-order scattering can extract additional information from \( T_B \) measurements about vegetation cover and biome dependence. The SMAP SCA algorithm, as mentioned in 1.3, solely retrieves \( SM \) using
ancillary inputs of $\tau$ and $\omega$ from the measured $T_{BV}$. Thus, all information within measured $T_{BV}$ is placed within the retrieved $SM$ and $T_{BH}$ information is neglected. Alternatively, the first-order implementation utilizes available measured information within $T_{BH}$ and $T_{BV}$ to maximally partition the measured signal into time varying $SM$ and $\tau$ and constant $\omega$ and $\omega_I$. Lessons learned specifically from the retrieved vegetation parameters are noted here.

1. $\omega_I$ is largest in forests (Fig. 18). This is expected because a greater portion of the forest media contain objects larger than the microwave wavelength (21 cm). Thus, $\omega_I$ can be defined as an effective parameter that represents scattering from woody biomass and can be an indicator of woody biomass with dimensions larger than 21 cm.

2. Consequently from the previous point, the emission from first-order scattering, or emission solely due to wave interactions with woody biomass within the canopy, is largest in forests and is significant. Only 26% of Africa’s biomes with woody vegetation do not have significant first-order scattering (Figs. 28 and 29). The remainder of the region has a median of 11 K of first-order emission or about 5% of the signal. This is a similar magnitude of the signal that Kurum et al. (2011) determines from first-order analytic RT solution. From this, it is clear that the SMAP radiometer definitively detects first-order emission from a 36 km footprint. Also, in not considering this emission within utilization of the tau-omega model can result in retrieval errors in these regions.

3. In relating $\omega$ and $\omega_I$ within the metric $\psi$, $\psi$ tends to be higher in vegetated regions as shown with its relation to vegetation height and land cover classification in Figs. 25 and 26 and Table 4. It is moderately correlated with $\tau$ as shown in Fig. 27. A physical interpretation of this is that $\psi$ represents the ratio of woody biomass scattering to non-woody biomass scattering. As one would expect, as there is a greater amount of AGB through $\tau$, scattering is expected to primarily come from woody biomass (in which the vegetation contains a higher fraction of woody biomass compared to its total biomass). However, it is possible that there is some mathematical compensation between $\tau$ and $\psi$ within the optimization that may arise from the separation of time scales where $\psi$ is constant over the year and $\tau$ changes approximately weekly.

While this algorithm serves as a new $SM$ retrieval method in the presence of woody vegetation, this also provides a unique opportunity to monitor vegetation attributions from space via their electromagnetic properties. This includes $AGB$ and $VWC$ measured from the level of attenuation ($\tau$) and the relatively static structure of the canopy from scattering attributes ($\omega$ and $\omega_I$).

5.4 Tau-Omega Model Sufficiency

Fig. 18 and Table 1 show that first-order scattering is most significant in forests among other regions. This provides a measure of the degree to which the tau-omega model (12) is insufficient where use of (12) is sufficient when $\omega_I$ is near-zero and consequently the first-order scattering contribution to $T_B$ is low (Figs. 28 and 29). This shows multiple pixels outside of the SMAP dense vegetation mask (not shown) where the tau-omega model may be insufficient. However, depending on the application, the adjustment in retrieved parameters due to first-order scattering may not have a significant effect on results. This is especially the case when mean
changes in $SM$, for example, are on the order of $0.01 \, \text{m}^3/\text{m}^3$ away from the densely vegetated Congo Basin. Thus, (12) is generally sufficient for use outside of the Congo Basin.

5.5 Sahara Desert First-Order Scattering

The physical explanations of woody biomass presence become insufficient to explain non-zero $\omega_1 (-0.03)$ in the Sahara Desert with very significant improvements in parameter fit to measured $T_B$. From Fig. 17E, there is a clear divide in the Sahel between high $\omega$ and near-zero $\omega$. Further analysis shows that the zeroth-order implementation results in a non-smooth cost function in the Sahara desert which is mitigated by increasing $\omega_1$ as shown in Fig. 36. Since $\tau$ is near zero in this region ($\gamma$ approaches 1), $T_B$ in (12) becomes insensitive to $\omega$ which may create this nearly flat cost function. As $\omega_1$ is increased, $T_B$ in (21) may become more sensitive to $\omega$ despite $\tau$ approaching zero. This shows a limitation for optimizing $\omega$ within the zeroth-order implementation which appears to be mitigated by including first-order scattering terms. Physically, the Sahara upper soil layer may become the canopy medium and any volume scattering, or wave interactions with objects larger than the L-Band wavelength, is represented as first-order scattering and higher $\omega_1$ in the model. This is possible since the sensing depth may be greater due to a dry surface combined with an entisol soil type classification which consists of a uniform soil profile. Since the Sahara is primarily sedimentary rock (mostly sand) as shown in Fig. 37 (USGS 2017), it is possible that aggregated sedimentary rock larger than 21 cm is causing first-order scattering within the sand grain medium. While field campaign maps out the classification of sedimentary soil with the assistance of remote sensing (Ciampalini et al. 2012), the infeasibility of creating a large scale database of subsurface geology in the Sahara due to socio-political, weather, and accessibility constraints is noted (Guerrak 1988). Another likely possibility is that the MT-DCA is unable to partition between the microwave parameters constituting the surface signal in the desert and the tau-omega model is invalid. Therefore, it compensates by partitioning a portion of the signal into first-order scattering. None of these possibilities are deemed more likely than another. Additional analysis on this is beyond the scope of the study and is a case for further investigation.
Fig. 36. Sahara Desert $J_I$ Cost Function Change with $\omega$. Note that the y-axis bounds are different between the two rows.

Fig. 37. USGS Geologic Map (USGS 2017). Sedimentary, metamorphic, and igneous rock are labeled as green, purple, and red (light and dark), respectively.

6. Conclusions and Broader Impact

In this study, a first-order RT model is developed which expands upon the commonly used zeroth-order RT model (tau-omega model) within the microwave passive remote sensing community. These models partition between the surface and vegetation emission with the common goal of converting radiometer-measured $T_B$ to $SM$ globally. More recently, the MT-
DCA was developed to further partition the \( T_B \) signal between \( SM \) and microwave vegetation parameters (\( \omega \) and \( \tau \)). The MT-DCA is also enhanced here for more robust retrievals with closer retrieval fits to the measured \( T_B \). Spatial mapping of soil moisture has wide applications in improving water, carbon, and energy cycle modeling and monitoring because it is a key land surface boundary condition. Further, spatial mapping of microwave vegetation has applications in vegetation health monitoring globally especially with concerns for vegetation cover changes with climate change. However, the tau-omega model becomes insufficient for use as a retrieval algorithm in the presence of woody biomass (or any constituents larger than 21 cm) within the vegetation medium. The developed first-order RT model attempts to account for emission scattering from this woody biomass scattering medium.

While many RT equations have been developed to estimate surface emission under a dense vegetation canopy, none have been specifically developed for global implementation due to their extensive requirements of information about the canopy geometry. This first-order RT model in combination with the MT-DCA are the first attempt to retrieve passive microwave canopy and surface parameters at a continental scale while accounting for first order scattering. This first-order implementation was completed over Africa at a 36 km resolution using SMAP \( T_B \) measurements. There were no means to validate retrieved \( SM \) and vegetation parameters in forested regions where it is expected that the zeroth-order RT model produces large retrieval errors. However, it was determined that the first-order RT equation itself was more sensitive to the surface and its behavior compared relatively closely to an analytically derived first-order RT model used successfully in basin scale studies (Kurum et al. 2011). Additionally, comparison of retrievals over Africa showed that the surface signal and temporal variability of \( SM \) was marginally increased from the zeroth-order to the first-order implementation in forests.

Additionally, a new parameter representing first-order scattering from a woody biomass medium, \( \omega_f \), is introduced in the first-order RT model. This provides the opportunity to retrieve this parameter and determine where first-order scattering is significant. As expected, \( \omega_f \) is larger in forests where woody biomass is present. Consequently, relative contribution of zeroth and first-order emission showed that first-order emission is significant in forests (especially in the Congo Basin). The first-order signal is typically 5% of the total \( T_B \) signal showing that neglect of this additional emission in the zeroth-order RT model implementation could result in large retrieval errors. Finally, the two scattering parameters were related within a metric called \( \psi \) as roughly the physical representation of the ratio of woody to non-woody biomass. A relation between \( \psi \) and \( \tau \) is evident. Also, \( \psi \) is higher in forests possibly showing a means independent from \( \tau \) to characterize time-invariant forest structure from microwave scattering properties.

Additional work is required to validate retrievals especially in forested regions. This necessitates a large scale forest \( SM \) and vegetation monitoring campaign. Also, retrievals in biomes on other continents (e.g. North and South America) should be attempted for intercomparison. Additionally, a distinct non-zero first-order scattering signal is found in the Sahara with a consequently large improvement in fit between measurement and model parameters. This signal should be investigated further for its physical and/or mathematical representation. Finally, it is encouraged that first-order RT models specifically for global
geophysical parameter retrieval in the presence of medium to densely vegetated canopies are developed for use in SMAP and SMOS mission retrieval implementations. Since forests consist of 30% of the surface and play a large role in terrestrial carbon, water, and energy exchange with the atmosphere, there is motivation to obtain an accurate characterization of their temporally dynamic attributes (e.g. SM, VWC). After over 60 years of focused study, microwave remote sensing of regions with medium to dense canopies remains a challenge.

7. Appendix

7.1 MTDCA MATLAB Script

clear
close all
mfilename = 'MTDCA_SAmerica_1st_L1C_346380_250400_36km';
M_lat = [346:380];  %079:321, 434:630 Africa
M_lon = [250:400];  %150:380, 250:400 South America
Resolution = 36;
h_overwrite = 0.13; %not using ancillary data for this
MaxDay = 366; %Day Number Cutoff (e.g. 366 = March 31, 2016)
disp(['Running m-file: ',mfilename])
% Set Up Inputs
switch Resolution
    case 36
        cd('C:\Users\Entekhabi-Group\Dropbox (MIT)\SMAP Data\General Data')
        % Select Input Data: TBh, TBv, Ts, water frac, clay frac, h roughness
        TBhFileNam = 'SMAP_TB_L1C_36km_201504_201703';
        TBvFileNam = 'SMAP_TB_L1C_36km_201504_201703';
        TsFileNam = 'SMAP_Ts_L2_36km_201504_201703';
        wfFileNam = 'SMAP_L3_SM_P_T12400_36km';
        ClayFileNam = 'soil_texture_36km';
        hFileNam = 'h_roughness36km';
        % Load Region: Need to change name of variable in matfile command below
        TBhMatNam = matfile(TBhFileNam);
        TBvMatNam = matfile(TBvFileNam);
        TsMatNam = matfile(TsFileNam);
        wfMatNam = matfile(wfFileNam);
        ClayMatNam = matfile(ClayFileNam);
        hMatNam = matfile(hFileNam);
        % Note this script only uses h = 0.13. To use temporally static h, need to % add hw(il) to inputs instead of hw
        TBh = TBhMatNam.TBh_L1C_36km(M_lat,M_lon,:);
        TBv = TBvMatNam.TBv_L1C_36km(M_lat,M_lon,:);
        Ts = TsMatNam.Ts_L2_36km(M_lat,M_lon,:);
        water_f = wfMatNam.water_f(M_lat,M_lon);
        clay = ClayMatNam.clay(M_lat,M_lon);
        h = hMatNam.h(M_lat,M_lon);
        cd('C:\Users\Entekhabi-Group\Dropbox (MIT)\SMAP Data\General Data\Amazon_36km_MTDCA_0th')
        ZerothOmegaMat = matfile('MTDCA_SAmerica_0th_L1C_150380_250400_h13_36km');
        SAmericaRow = ZerothOmegaMat.FullSAmericaM_lat(:,:);
        SAmericaCol = ZerothOmegaMat.FullSAmericaM_lon(:,:);
IRow = M_lat-SAmericaRow(1)+1;
ICol = M_lon-SAmericaCol(1)+1;
omg0Init = ZerothOmegaMat.omg_SAmerica(iRow,iCol);
JCostInit = ZerothOmegaMat.omgCost_SAmerica(iRow,iCol); %SAmerica
tau0Ave = nanmean(ZerothOmegaMat.tau_SAmerica(iRow,iCol,:),3);
% load previous omega
    cd('C:\Users\Entekhabi-Group\Dropbox (MIT)\SMAP Data\General
    Data\Amazon_36km_MTDCA 1st')
    FirstOmegaMat = matfile('MTDCASAmerica_1st_L1C_150380_250400_h13_36km');
    omg1Prev = FirstOmegaMat.omg1_SAmerica(iRow,iCol);
    case 9
    % Did not use 9km inputs here
    end
%Temporal Correction
TBh = TBh(:,:,1:MaxDay);
TBv = TBv(:,:,1:MaxDay);
Ts = Ts(:,:,1:MaxDay);
    cd('C:\Users\Entekhabi-Group\Dropbox (MIT)\SMAP\9km MTDCA\MT-
    DCA\MTDCAMinCon_v2_FirstOrder')
% Frequency [GHz] % Incidence Angle [deg]
    freq = 1.4 ;
    angle = 40.0 ;
% Replace Missing Variables with NaN
    TBv(TBv<-99) = NaN;
    TBh(TBh<-99) = NaN;
    Ts ( Ts<-99) = NaN;
    water_f(water_f<-99) = NaN;
% Water and Freeze Masks (Sizes and Thresholds)
    [Nlat,Nlon,NPeriod] = size(TBh);
    water = NaN(Nlat,Nlon);
    FT = NaN(Nlat,Nlon);
    water_threshold = 0.05;
    FT_threshold = 273.15;
% Exclude Water Bodies
    water = nanmean(water_f,3);
    water(water>water_threshold) = NaN ;
    water(isnan.omg0Init)=NaN; %New mask for no omega start!
    water(isfinite.omg1Prev)=NaN; %New mask for omega' previous!
    water(isnan(tau0Ave))=NaN; %New mask for tau0!
    water(isfinite(water)) = 1 ;
% Omega Search Vector (Setting Upper Limit of 0.8)
    omegav = [0.0 : 0.01 : 0.80];
    omegav1 = [0.0 : 0.01 : 0.80];
% Create lower bound of dielectric constant vector
    kmin = real(mironov(1.4,0,clay));
% Apply Water Bodies and Freeze Masks
for i = 1 : NPeriod
    TBv(:,:,i) = TBv(:,:,i).* water;
    TBh(:,:,i) = TBh(:,:,i).* water;
    Ts(:,:,i) = Ts(:,:,i).* water;
    FT = Ts(:,:,i);
FT(FT < FT\_threshold) = NaN;
FT(isfinite(FT)) = 1;
TBv(:,:,i) = TBv(:,:,i) .* FT;
TBh(:,:,i) = TBh(:,:,i) .* FT;
Ts(:,:,i) = Ts(:,:,i) .* FT;
end

% Build Land Cells Vector (Suffix 'w')
[waterw,iwf] = sort(reshape(water,Nlat*Nlon,1));
iwf(isnan(waterw)) = [] ;
waterw(isnan(waterw)) = [] ;

Nland = length(iwf);
TBhw = NaN(Nland,NPeriod);
TBvw = NaN(Nland,NPeriod);
Tsw = NaN(Nland,NPeriod);
cfw = NaN(Nland,1);
hw = NaN(Nland,1);
kminw = NaN(Nland,1);

omgOlnitw = NaN(Nland,1);
CostInitw = NaN(Nland,1); % SAmerica

for i = 1 : NPeriod
    z = reshape( TBh(:,:,i),Nlat*Nlon,1) ;
    TBhw(:,i) = z(iwf);
    z = reshape( TBv(:,:,i),Nlat*Nlon,1) ;
    TBvw(:,i) = z(iwf);
    z = reshape( Ts(:,:,i),Nlat*Nlon,1) ;
    Tsw(:,i) = z(iwf);
end

z = reshape(clay,Nlat*Nlon,1);
cfw = z(iwf);
z = reshape(kmin,Nlat*Nlon,1);
kminw = z(iwf);
z = reshape(h,Nlat*Nlon,1);
hw = z(iwf);
hw = h_overwrite;
z = reshape(tauOAve,Nlat*Nlon,1);
tauOlnitw = z(iwf);
z = reshape(omgOlnit,Nlat*Nlon,1);
omgOlnitw = z(iwf);
z = reshape(JCostlnit,Nlat*Nlon,1); % SAmerica

CostInitw = z(iwf);
kDCA2aw = NaN(Nland,NPeriod);
kDCA2bw = NaN(Nland,NPeriod);
tauDCA2aw = NaN(Nland,NPeriod);
tauDCA2bw = NaN(Nland,NPeriod);
mvDCA2aw = NaN(Nland,NPeriod);
mvDCA2bw = NaN(Nland,NPeriod);
omgDCA2aw = NaN(Nland,NPeriod);
omgDCA2bw = NaN(Nland,NPeriod);
DCAflags = NaN(Nland,NPeriod);
omgCost = NaN(Nland,length(omegav),length(omegav1));

% Estimation by Selected Land Points
% Find Adjacent-in-Time Overpass Sequences
pass = [1:NPeriod]';
tbh = squeeze( double(TBhw(il,:))');
tbv = squeeze( double(TBvw(il,:))');
ts = squeeze( double( Tsw(il,:))');

% Passes Moving-Window Size for omega Estimation
% (Odd Integer or length(pass) for Constant)
omega_window = length(pass);

% Case of Static Omega
if (omega_window == length(pass))
    % Cut Out No Overpass Days
    passo = pass;
    imiss = find(isnan(tbh(paso)))
    passo(imiss) = [];
    imiss = find(isnan(tbv(paso)))
    passo(imiss) = [];
    inomg = passo;
    % Static Omega Optimization
    % omega Estimation (Should be Identical to Block Below)
    % Starting the omega cost function at omega = 0.09 (omgStart = ko = 10)
    JCost = NaN(length(omegav),length(omegavl));
    omgStart0 = omg0Initw(il);
    ko = round(omgStart0*100+1);
    ko1 = 1;
    JCost(ko,1) = CostInitw(il);
    if tau0Initw(il)>0.2
        if tau0Initw(il)<0.8 & tau0Initw(il)>0.4
            omgStart = [0.22 0.32 0.40 0.47 0.52];
        elseif tau0Initw(il)<0.4
            omgStart = [0.25 0.37 0.5 0.6 0.6];
        else %tau>0.8
            omgStart = [0.15 0.22 0.26 0.28 0.3];
        end
        ko1v = [7 14 21 36 51];
        MinCol = NaN(size(ko1v));
        for i = 1:size(ko1v,2)
            koStart = round(omgStart(i)*100+1);
            ko = koStart;
            ko1 = ko1v(i);
            Jol = J_omega_MinCon_v2_firstOrder([1:length(paso)],tbv(paso),...
                tbh(paso),ts(paso),hw,angle,omegav(ko),omegavl(ko1),kminw(il));
            JCost(ko,ko1) = Jol;
            ko = koStart+1;
            Jor = J_omega_MinCon_v2_firstOrder([1:length(paso)],tbv(paso),...
                tbh(paso),ts(paso),hw,angle,omegav(ko),omegavl(ko1),kminw(il));
            JCost(ko,ko1)=Jor;
            if Jol>Jor
            end
        end
    end
end
while Jol > Jor
    Jol = Jor;
    ko = ko + 1;
    if ko > 68 % Stop if gets to right boundary
        i2 = 10;
        i1 = 10;
        Jol = 0;
    else
        Jor = J_omega_MinCon_v2_firstOrder([1:length(passo)], tbv(passo), ... 
                                         tbh(passo), ts(passo), hw, angle, omegav(ko), omegav1(ko1), kminw(il));
        JCost(ko, ko1) = Jor;
    end
    ko = ko - 1;
else
    ko = koStart;
end
while Jol < Jor
    Jor = Jol;
    ko = ko - 1;
    if ko < 2 % Stop if gets to left boundary
        i2 = 10;
        i1 = 10;
        Jor = 0;
    else
        Jol = J_omega_MinCon_v2_firstOrder([1:length(passo)], tbv(passo), ... 
                                         tbh(passo), ts(passo), hw, angle, omegav(ko), omegav1(ko1), kminw(il));
        JCost(ko, ko1) = Jol;
    end
    ko = ko + 1;
end
MinCol(1, i) = min(JCost(:, ko1));
end
MinCol = [JCost(round(omgStart0*100+1), 1) MinCol];

%% Search Phase 2 (tau>0.2) %%%
[ko, ko1] = find(JCost == min(JCost(:)));
k1Save = ko1;
k0Save = ko;
JCostGenSave = JCost;
JCostBest = JCost(ko, ko1);
J = 10^12;
moveRight = 4;
moveUp = 3;
moveUpOffset = 3;
% Gradient Search Right
while J > JCostBest
    i1 = 0;
    while i1 < moveRight
        i1 = i1 + 1;
        ko1 = ko1 + 1;
        if ko1 > 68 % Stop if gets to right boundary
i2 = 10;
i1 = 10;
JCostBest = 0;
else
    i2 = 0;
    while i2<moveUp
        i2 = i2+1;
        if ko>68 %stop if gets to right boundary
            i2 = 10;
            i1 = 10;
            JCostBest = 0;
        else
            if isnan(JCost(ko,kol))
                JCost(ko,kol) = J_omega_MinCon_v2_firstOrder([1:length(passo)],tbv(passo),...
                tbh(passo),ts(passo),hw,angle,omegav(ko),omegav1(ko),kminw(il));
            end
            ko = ko+1;
        end
        end
    end
%Stops horizontal search if discovers a J less than original
    if min(JCost(:,kol))<JCostBest
        i2 = 5;
        i1 = 4;
    end
    ko = ko-moveUpOffset;
end
end
J = JCostBest;
JCostBest = min(JCost(:,));
[ko kol] = find(JCost == JCostBest);
end
% Save final JCost matrix
JCostGenFinal = JCost;
if kol==1
    kol = kolSave;
end
% Gradient Search Left
JCost = JCostGenSave; % Forget about gradient search right and search left
JCostBest = JCost(ko,kol);
J = 10^12;
while J>JCostBest
    i1 = 0;
    while i1<moveRight % Moving Left in this case
        i1 = i1+1;
        kol = kol-1;
        if kol<2 % Stop if gets to left boundary
            i2 = 10;
            i1 = 10;
            JCostBest = 0;
        else
            i2 = 0;
        end
    end
while i2<moveUp
  i2 = i2+1;
  if ko<2  % Stop if gets to bottom boundary
    i2 = 10;
    i1 = 10;
    JCostBest = 0;
  else
    if isnan(JCost(ko,kol))
      JCost(ko,kol) = J_omega_MiCon_v2_firstOrder([1:length(passo)],tbv(passo),...
        tbh(passo),ts(passo),hw,angle,omegav(ko),omegav1(ko1),kminw(il));
    end
    ko = ko-1;
  end
end

% Stops horizontal search if discovers a J less than original
if min(JCost(:,ko1))<JCostBest
  i2 = 10;
  i1 = 10;
  ko = ko+moveUpOffset;
end

J = JCostBest;
JCostBest = min(JCost(:,));
[ko kol] = find(JCost == JCostBest);
end
end

else  % tau<0.2
  kov = [1 1 1 1 1 1 1 1 1 1 1];
  omgStart = 0.05;
  MinCol = NaN(size(ko));
  for i = 1:size(kov,2)
    koiStart = round(omgStart*100+1);
    ko = koiStart;
    Jol = J_omega_MiCon_v2_firstOrder([1:length(passo)],tbv(passo),...
      tbh(passo),ts(passo),hw,angle,omegav(ko),omegav1(ko1),kminw(il));
    JCost(ko,kol) = Jol;
    kol = kolStart+1;
    Jor = J_omega_MiCon_v2_firstOrder([1:length(passo)],tbv(passo),...
      tbh(passo),ts(passo),hw,angle,omegav(ko),omegav1(ko1),kminw(il));
    JCost(ko,kol) = Jor;
    if Jol<Jor
      kol = koiStart;
      while Jol<Jor  % Move Left
        kol = kol - 1;  % continue to the left of starting point
        if kol ~= 0
          Jor = Jol;
          Jol = J_omega_MiCon_v2_firstOrder([1:length(passo)],tbv(passo),...
            tbh(passo),ts(passo),hw,angle,omegav(ko),omegav1(ko1),kminw(il));
        end
      end
    end
  end
end
JCost(ko,ko1) = Jol;
else
    Jol = 1e12;
end
end
else
while Jol>Jor % Move Right
    ko1 = ko1+1;
    if ko1 <= 60
        Jol = Jor;
        Jor = J_omega_MinCon_v2_firstOrder([1:length(passo)],tbv(passo),...
            tbh(passo),ts(passo),hw,angle,omegav(ko),omegav1(ko1),kminw(il));
        JCost(ko,ko1) = Jor;
    else
        Jor = 1e12;
    end
end
end

MinCol(1,i) = min(JCost(ko,:));
end

% Search Phase 2 (tau<0.2) %%%%%%%%%%%%%%%%%%%%%
[ko ko1] = find(JCost == min(JCost(:)));
ko1Save = ko1;
koSave = ko;
JCostGenSave = JCost;
JCostBest = JCost(ko,ko1);
J = 10^12;
moveUp = 5;
% Gradient Search Up
while J>JCostBest
    i2 = 0;
    while i2<moveUp
        i2 = i2+1;
        if isnan(JCost(ko,ko1))
            JCost(ko,ko1) = J_omega_MinCon_v2_firstOrder([1:length(passo)],tbv(paso),...
                tbh(passo),ts(passo),hw,angle,omegav(ko),omegav1(ko1),kminw(il));
        end
        ko = ko+1;
        % Stops vertical search if discovers a J less than original
        if min(JCost(:,ko1))<JCostBest
            i2 = 5;
            i1 = 4;
        end
        % Force quit search if gets to upper bound
        if ko>68
            i2 = 10;
            i1 = 10;
            JCostBest = 0;
        end
    end
    J = JCostBest;
JCostBest = min(JCost(:));
[ko ko1] = find(JCost == JCostBest);
end
JCostGenFinal = JCost;
% Gradient Search down
if ko==1
ko1 = ko1Save;
ko = koSave;
% Gradient Search Left
JCost = JCostGenSave;
JCostBest = JCost(ko,kо1);
J = 10^12;
while J>JCostBest
i2 = 0;
while i2<moveUp
i2 = i2+1;
if isnan(JCost(ko,kо1))
JCost(ko,kо1) = J_omega_MinCon_v2_firstOrder([1:length(passo)],tbv(passo),...
tbh(passo),ts(passo),hw,angle,omegav(ko),omegavl(kо1),kminw(il));
end
ko = ko-1;
% Stops vertical search if discovers a J less than original
if min(JCost(:,kо1))<JCostBest
i2 = 5;
i1 = 4;
end
% Force quit search if gets to lower bound
if ko<2
i2 = 10;
i1 = 10;
JCostBest = 0;
end
end
J = JCostBest;
JCostBest = min(JCost(:));
[ko ko1] = find(JCost == JCostBest);
end
end
end
JCost = nanmean(cat(3,JCostGenFinal,JCost),3);
JCostBest = min(JCost(:));
[ko ko1] = find(JCost == JCostBest);
ogv(inomg) = ko/100-0.01;
ogv1(inomg) = ko1/100-0.01;
else % Case of Dynamic Omega
end % end of static omega look
omgCost(il,;,:) = JCost;
% Assign Optimal omega Into w Vectors
% Colon (:) Assignment and Temporary Variables Needed for parfor
omgDCA2w(il,:) = ogv;
% Last Pass to Produce Target Variable Estimates and Model TBs
[kDCA2aw(il,:), kDCA2bw(il,:), ...]
[tauDCA2aw(il,:), tauDCA2bw(il,:), ...]
tbvma, tbhma, tbvmb, tbhmb, DCAflags(il,:)] = ...

Pixel_Run_MinCon_v2_firstOrder(pass, tbv, tbh, ts, hw, angle, omgDCA2w(il,:)', omgDCA2w1(il,:)', kminw(il)) ;
% Convert Dielectric to Soil Moisture

mv = [ 0.0005 : 0.0005 : 0.60 ];
kv = real(mironov(freq, mv, cfw(il)));
mvDCA2aw(il,:) = interp1(kv, mv, kDCA2aw(il,:));
mvDCA2bw(il,:) = interp1(kv, mv, kDCA2bw(il,:));
% Command Window Display of Diagnostics

tbhm = (tbhma + tbhmb) ./ 2;
tbvm = (tbvma + tbvmb) ./ 2;
telapsed(il) = toc(tstart(il));

bias_h = nanmean(tbh - tbhm);
bias_v = nanmean(tbv - tbvm);
ubrmse_h = nanstd(tbh - tbhm);
ubrmse_v = nanstd(tbv - tbvm);
tau_ab = nanstd(tauDCA2aw(il,:) - tauDCA2bw(il,:));
k_ab = nanstd(kDCA2aw(il,:) - kDCA2bw(il,:));
end %end of il loop
display(['Total CPU Time = ', num2str(tototal, '%%7.1f'), ' [sec]'])

% Assign Estimation Land Pixels Back into Global Grid

water_f = NaN(Nlat, Nlon);
water_f(iwf) = waterw;
water_f = reshape(water_f, Nlat, Nlon);

omgDCA2 = NaN(Nlat, Nlon, NPeriod);
omg1DCA2 = NaN(Nlat, Nlon, NPeriod);

mvDCA2 = NaN(Nlat, Nlon, NPeriod, 2);
tauDCA2 = NaN(Nlat, Nlon, NPeriod, 2);
kDCA2 = NaN(Nlat, Nlon, NPeriod, 2);
TBh = NaN(Nlat, Nlon, NPeriod);
TBv = NaN(Nlat, Nlon, NPeriod);
Ts = NaN(Nlat, Nlon, NPeriod);
DCAFlag = NaN(Nlat, Nlon, NPeriod);

omgJoCost = NaN(Nlat, Nlon, length(omegav), length(omegavl));

for i = 1 : NPeriod
    x = NaN(Nlat, Nlon);
    x(iwf) = omgDCA2w(:, i);
    omgDCA2(:, :, i) = reshape(x, Nlat, Nlon);
    x = NaN(Nlat, Nlon);
    x(iwf) = omgDCA2w1(:, i);
    omg1DCA2(:, :, i) = reshape(x, Nlat, Nlon);
    x = NaN(Nlat, Nlon);
    x(iwf) = mvDCA2aw(:, i);
    mvDCA2(:, :, i, 1) = reshape(x, Nlat, Nlon);
    x = NaN(Nlat, Nlon);
    x(iwf) = mvDCA2bw(:, i);
mvDCA2(:,:,i,2) = reshape(x,Nlat,Nlon);
    x = NaN(Nlat,Nlon);
    x(iwf) = tauDCA2aw(:,:,i);
    tauDCA2(:,:,i,1) = reshape(x,Nlat,Nlon);
    x = NaN(Nlat,Nlon);
    x(iwf) = tauDCA2bw(:,:,i) ;
    tauDCA2(:,:,i,2) = reshape(x,Nlat,Nlon);
    x = NaN(Nlat,Nlon) ;
    x(iwf) = NaN(Nlat,Nlon) ;
    TBh(:,:,i) = reshape(x,Nlat,Nlon) ;
    x = NaN(Nlat,Nlon) ;
    x(iwf) = NaN(Nlat,Nlon) ;
    x(iwf) = NaN(Nlat,Nlon) ;
    TBv(:,:,i) = reshape(x,Nlat,Nlon) ;
    x = NaN(Nlat,Nlon) ;
    x(iwf) = NaN(Nlat,Nlon) ;
    x(iwf) = NaN(Nlat,Nlon) ;
    Ts(:,:,i) = reshape(x,Nlat,Nlon);
    x = NaN(Nlat,Nlon);
    x(iwf) = NaN(Nlat,Nlon);
    x(iwf) = NaN(Nlat,Nlon);
    kDCA2(:,:,i,1) = reshape(x,Nlat,Nlon);
    x = NaN(Nlat,Nlon);
    x(iwf) = NaN(Nlat,Nlon);
    x(iwf) = NaN(Nlat,Nlon);
    kDCA2(:,:,i,2) = reshape(x,Nlat,Nlon);
    x = NaN(Nlat,Nlon);
    x(iwf) = NaN(Nlat,Nlon);
    DCAFlag(:,:,i) = reshape(x,Nlat,Nlon);
end
for j = 1:length(omegavl)
for i = 1:length(omegav)
    x = NaN(Nlat,Nlon);
    x(iwf) = omgCost(:,i,j);
    omgJoCost(:,:,i,j) = reshape(x,Nlat,Nlon);
end
end
save(mfilename,'omgDCA2','omglDCA2','mvDCA2','tauDCA2','TBh','TBv','Ts','kDCA2','DCAFlag','omgJoCost','water_f','M lat','M lon','h','clay','-v7.3')
8. References


D.


USGS. 2017. General geologic map of the world. mrdatal.usgs.gov/geology. 02.05.18.