Design, Optimization, and Performance of an Adaptable Aircraft Manufacturing Architecture

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Abstract

The cost and time required to develop aircraft have grown strongly over time, to the point where aircraft have become prohibitively expensive and are outpaced by ever-evolving mission needs. To address this problem, this thesis presents and explores an aircraft platform architecture called “Adaptable Aircraft Manufacturing” or “AAM”, which features common tooling geometry that enables the creation of any composite aircraft (within a reachable subspace) on demand. To prove the feasibility of this architecture, a family of aircraft was constructed using a single set of AAM tooling. This thesis also optimizes the AAM geometry and quantifies the inefficiencies incurred by its adoption. This family optimization problem is both logically and computationally complex since the constraints AAM places between the variants cannot be prescribed by the designer, but arise as a result of the optimization gradients that exist between variants. A sequential-process framework which clarifies the relations and points of adjustability available in aircraft manufacturing is presented. A signomial-programming (SP) aircraft optimization code that is capable of simulating the inefficiencies generated by the AAM geometry was developed. The SP mathematical structure and the GPkit codebase were selected due to their compatibility with the constraint-heavy geometric rules that describe AAM and for the rapid speed of computation, which is necessary due to the scale of the optimization problem. To quantify the performance of AAM, a series of explorations are conducted whereby the performance of an AAM-family of aircraft is compared against a fleet of Individually-Optimal (IO) aircraft. These explorations are conducted along the axes of payload size, cruise speed, mission scope, and market bias to gain an understanding of how (and by how much) the AAM constraints affect both the performance and the geometry of the aircraft it produces. The results show that, for total-mission-cost-minimizing fleets of three designs each, the AAM fleet is between 10 and 20% more costly, but only require between 30% and 80% the tooling as an IO fleet.

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For Percy and Orion.

Anything worth doing is worth doing badly.

- GK Chesterson

All progress depends on the unreasonable man.

- George Bernard Shaw

If it doesn't get done in a hurry, it doesn't get done at all.

- myself
# Table of Contents

1  Introduction .................................................................................................................. 13

1.1  Motivation .................................................................................................................. 13

1.2  Tooling and Platforms................................................................................................ 14

1.2.1  Tooling as a driver of cost and schedule ............................................................... 14

1.2.2  Product platforms .................................................................................................. 16

1.3  Prior Work in Family Optimization Computation .................................................... 18

1.4  Adaptable Aircraft Manufacturing: Goals and Thesis Layout ................................ 19

1.4.1  Thesis structure ...................................................................................................... 19

2  Adaptable Aircraft Manufacturing (AAM) .................................................................. 20

2.1  Composite Aircraft Manufacturing .......................................................................... 20

2.2  Geometry of Adaptable Aircraft Manufacturing ..................................................... 23

2.2.1  Common molds and adjustable-layup ................................................................. 23

2.2.2  Adjustable-layup applied to wings and tails ......................................................... 24

2.2.3  Adjustable-layup applied to fuselages ................................................................. 27

2.2.4  Integration challenges in AAM structures ............................................................. 28

2.3  AAM Proof of Concept: “FAST” Project ................................................................ 28

2.3.1  Lessons from FAST and the problem of designing AAM families ....................... 30

2.4  Way Forward .............................................................................................................. 31

3  Robustness and Adaptability Framework ................................................................. 32

3.1  Classical Optimization Formulation ......................................................................... 32

3.1.1  Example of classical optimization: basic airfoil design problem ....................... 33

3.2  Uncertainty, Robust Designs, and Robustness ......................................................... 34

3.2.1  Robust designs and robustness ............................................................................. 34

3.2.2  Robustness in the airfoil design example ............................................................. 34

3.2.3  Asymmetry in robustness ...................................................................................... 36

3.3  Passive and Active Robustness and Adaptability ..................................................... 36

3.3.1  Adaptable robustness in the airfoil design example ............................................. 36

3.3.2  Costs of adaptability ............................................................................................. 38

3.3.3  Rate of uncertainty and adaptability .................................................................... 38

3.4  Framework to Describe Adaptability/Process Sequences ...................................... 39

3.4.1  “Process Sequence” framework ........................................................................... 39

3.4.2  Asset space mapping abstraction .......................................................................... 40

3.4.3  Process sequence map abstraction ........................................................................ 40
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4.4</td>
<td>Framework applied to the airfoil example</td>
<td>41</td>
</tr>
<tr>
<td>3.4.5</td>
<td>Uncertainty, robustness, and adaptability within framework</td>
<td>41</td>
</tr>
<tr>
<td>3.5</td>
<td>Framework Applied to Aircraft Manufacturing Chains</td>
<td>42</td>
</tr>
<tr>
<td>3.6</td>
<td>Representing Other Designs</td>
<td>43</td>
</tr>
<tr>
<td>4</td>
<td>AAM Performance Quantification</td>
<td>45</td>
</tr>
<tr>
<td>4.1</td>
<td>Baseline Mission Overview</td>
<td>45</td>
</tr>
<tr>
<td>4.1.1</td>
<td>Baseline driving requirements</td>
<td>46</td>
</tr>
<tr>
<td>4.2</td>
<td>Total System Cost Metric</td>
<td>47</td>
</tr>
<tr>
<td>5</td>
<td>Geometric and Signomial Programming</td>
<td>48</td>
</tr>
<tr>
<td>5.1</td>
<td>Computational Challenges in Designing Aircraft Families</td>
<td>48</td>
</tr>
<tr>
<td>5.2</td>
<td>Geometric Programming</td>
<td>48</td>
</tr>
<tr>
<td>5.3</td>
<td>Signomial Programming</td>
<td>49</td>
</tr>
<tr>
<td>5.4</td>
<td>Coding Setup for Optimization Problem</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>Models</td>
<td>51</td>
</tr>
<tr>
<td>6.1</td>
<td>Fitting Procedure for Complex Data</td>
<td>51</td>
</tr>
<tr>
<td>6.2</td>
<td>Atmospheric Models</td>
<td>52</td>
</tr>
<tr>
<td>6.3</td>
<td>Aircraft configuration</td>
<td>52</td>
</tr>
<tr>
<td>6.4</td>
<td>Airframe Material Assumptions</td>
<td>53</td>
</tr>
<tr>
<td>6.5</td>
<td>Common Molding Geometry Mathematics</td>
<td>54</td>
</tr>
<tr>
<td>6.5.1</td>
<td>Wings and tails</td>
<td>54</td>
</tr>
<tr>
<td>6.5.2</td>
<td>Fuselages</td>
<td>56</td>
</tr>
<tr>
<td>6.6</td>
<td>Wing Models</td>
<td>57</td>
</tr>
<tr>
<td>6.6.1</td>
<td>Wing aero: lift model</td>
<td>58</td>
</tr>
<tr>
<td>6.6.2</td>
<td>Wing aero: induced drag model</td>
<td>58</td>
</tr>
<tr>
<td>6.7</td>
<td>Wing Discretization for Load and Response Structures</td>
<td>59</td>
</tr>
<tr>
<td>6.7.1</td>
<td>Wing aero: airfoil performance model</td>
<td>60</td>
</tr>
<tr>
<td>6.7.2</td>
<td>Wing bending loads model</td>
<td>61</td>
</tr>
<tr>
<td>6.7.3</td>
<td>Wing bending structure: spar</td>
<td>63</td>
</tr>
<tr>
<td>6.7.4</td>
<td>Wing torsion loading cases</td>
<td>64</td>
</tr>
<tr>
<td>6.7.5</td>
<td>Wing structure: torsion box</td>
<td>66</td>
</tr>
<tr>
<td>6.7.6</td>
<td>Torsion stiffness constraint</td>
<td>66</td>
</tr>
<tr>
<td>6.7.7</td>
<td>Wing root joiner mass model</td>
<td>67</td>
</tr>
<tr>
<td>6.8</td>
<td>Tail Models</td>
<td>68</td>
</tr>
<tr>
<td>6.8.1</td>
<td>Tail sizing</td>
<td>68</td>
</tr>
</tbody>
</table>
List of Figures

Figure 1.1: US combat aircraft unit price over time [1] with inflation overlay .................................................. 13
Figure 1.2: Years from program start to initial operational capability over time [2] ........................................... 13
Figure 1.3: Max, median, and mean age of designs in Air Force service [3] .......................................................... 14
Figure 1.4: Boeing data for cost over time for the development of a large commercial jet [4] ................................. 15
Figure 1.5: Trends in composite material use in aircraft [1] .................................................................................. 16
Figure 1.6: Boeing 737NG Family .................................................................................................................... 17
Figure 1.7: Antares 20E sailplane [5] ................................................................................................................ 17
Figure 1.8: Extended wing molding strategy for the Antares 1S/T and 20E sailplanes [6] ................................. 18
Figure 1.9: BWB structural breakdown [7] ........................................................................................................ 18
Figure 2.1: Bagged layup schematic ................................................................................................................... 20
Figure 2.2: Wing skin molds for the Boeing 777X [9] .......................................................................................... 21
Figure 2.3: Cirrus SR22 fuselage molding [10] ..................................................................................................... 21
Figure 2.4: Boeing 787 nose section production [11] ........................................................................................... 22
Figure 2.5: Adjustable-layup strategy .................................................................................................................. 23
Figure 2.6: Cylindrical abstraction of primary aircraft structural geometries ........................................................ 23
Figure 2.7: Layout of conventional wing construction .......................................................................................... 24
Figure 2.8: Sailplane wing internal component integration .................................................................................. 24
Figure 2.9: Detailed wing structure of Airbus ALCAS wing box [12] ..................................................................... 25
Figure 2.10: Common wing tooling concept ....................................................................................................... 25
Figure 2.11: Spar mold shift to compensate for shell thickness ............................................................................. 26
Figure 2.12: Wing planform comparison between three airliners ......................................................................... 26
Figure 2.13: Wing planform comparison between three large UAVs ................................................................. 27
Figure 2.14: Applying adjustable-extent to fuselages allows the production of a variety of lengths .................. 27
Figure 2.15: Three aircraft constructed from common tooling (left to right: A,B,C) ........................................... 28
Figure 2.16: Wing commonality between FAST A and B .................................................................................... 29
Figure 2.17: Molds for FAST project (tail: left, wing: right) ............................................................................. 29
Figure 2.18: FAST A (spine in white box) .......................................................................................................... 30
Figure 3.1: Airfoil example: baseline performance ............................................................................................... 33
Figure 3.2: Airfoil example: small uncertain Cl scope ......................................................................................... 35
Figure 3.3: Airfoil example: large uncertain Cl scope ........................................................................................... 35
Figure 3.4: Airfoil example: large uncertain Cl range causing infeasibility ...................................................... 36
Figure 3.5: Airfoil example: adaptable-robustness of a2 .................................................................................... 37
Figure 3.6: Airfoil example: adaptable-robustness of a1 .................................................................................... 38
Figure 3.7: Variable-space mapping abstraction .................................................................................................. 40
Figure 3.8: Unit "block" of PSM .......................................................................................................................... 41
Figure 3.9: PSM for simple, non-adaptable airfoil in lift-drag optimization problem ........................................... 41
Figure 3.10: PSM for flapped airfoil in lift-drag optimization problem ................................................................. 41
Figure 3.11: PSM for baseline manufacturing-through-operation chain .............................................................. 42
Figure 3.12: PSM for Adaptable Aircraft Manufacturing ....................................................................................... 42
Figure 3.13: PSM for AAM with aircraft reconfigurability ...................................................................................... 43
Figure 3.14: PSM for an example building-block aircraft platform ....................................................................... 44
Figure 4.1: AAM performance quantification procedure .................................................................................... 45
Figure 4.2: Baseline mission flight profile .......................................................................................................... 46
Figure 7.18: Result of cruise-speed-varying mission set with $\delta speed = 5$......................................................... 95
Figure 7.19: Taper angle comparison between AAM and IO fleets......................................................... 97
1 INTRODUCTION

This chapter introduces the problem of cost and development duration in aircraft design and manufacturing, background on tooling and platforms, and the goals and layout of this thesis.

1.1 Motivation

The cost of aircraft and the time required to develop them have grown strongly over time, as shown in Figure 1.1. The unit cost of aircraft has risen approximately 9% yearly since 1975, exceeding all other measures of inflation [1].

![Figure 1.1: US combat aircraft unit price over time [1] with inflation overlay](image)

Whereas other industries such as automotive engineering have seen an increase in development speed, aircraft development (both military and commercial) have become increasingly slow since the post-WWII period, as shown in Figure 1.2 from DARPA [2].

![Figure 1.2: Years from program start to initial operational capability over time [2]](image)
The combination of high cost and slow development has significant implications for satisfying new missions. First, many aircraft development projects are abandoned due to lack of funding. Second, even if an aircraft project survives to completion, the development can take so long that the mission context will have changed by the time the aircraft is ready for service. For example, the request-for-proposals for the F-22 Raptor was issued in 1986, and the aircraft entered service in 2005: 19 years later [2]. Over that period, the nature of aerial warfare had changed from the air supremacy of the Cold War to asymmetric and electronic warfare in the Middle East, and many of the F-22’s features remain unused.

These effects may also be quantified at the fleet level. Figure 1.3 from the RAND Corporation shows the trends for design-age of US Air Force vehicles over time [3].

![Figure 1.3: Max., median, and mean age of designs in Air Force service][3]

As Figure 1.3 shows, the design-age of aircraft in the US Air Force is rising steadily, and aircraft require constant retrofitting and modernization programs to remain current. Retrofitting existing aircraft for new missions results in suboptimal matching between designs and missions since the base airframes carry all the original tradeoffs into their new roles.

In summary, aircraft development is often prohibitively expensive and is outpaced by ever-changing mission needs. Decreasing aircraft development cost and increasing development speed would provide benefits - for both individual projects and the fleet as a whole. This thesis explores one novel approach to achieving this goal.

### 1.2 Tooling and Platforms

While much of the cost and schedule for new aircraft is the result of increased use of software and electronics, hardware manufacturing still represents a large investment for new aircraft and is an area of opportunity due to its potential for commonality.

#### 1.2.1 Tooling as a driver of cost and schedule

"Tooling" is a generic term for the hardware necessary to convert raw materials into a final product, and is one of the main drivers of aircraft development cost and duration. Depending on the material of the part being constructed, tooling may be comprised of molds, dies, jigs, fixtures, or other devices. In this
usage, "tooling" is distinct from "machines" such as mills and lathes which, without alteration, can produce a multitude of components.

Tooling is a necessary precursor to producing airframe components. For the vast majority of vehicles, the design and production of tooling is orders of magnitude more expensive and time-consuming than producing a single unit. For example, analysis of Boeing data by Markish in Figure 1.4 shows the normalized labor hours vs. the normalized development time for one large commercial jet program [4].

The study shows that tooling design (yellow) and fabrication (teal) accounts for roughly 45% of development cost and dominates labor-hours for 50% of the schedule [4]. During a typical design-to-manufacture schedule for an airliner, tooling design and fabrication would take approximately 4 years. In contrast, the production of a single unit of a finalized design only takes a few weeks.

Most tooling is specific to a single aircraft design's geometry, so even subtle changes in an aircraft's external lines (such as airfoil tweaks) requires the creation of new tools. As a result, design changes late during development usually involve substantial growths in cost and schedule.

The prevalence of tooling-driven project cost and schedule is likely to grow. According to a study by the RAND Corporation, much of the price increase in today's aircraft stems from the desire for higher performance, which is partly achieved through the increased use of advanced materials such as composites (as shown in Figure 1.6) [1].
Composite materials, unlike metals, cannot be machined from billet or pressed from sheet stock. Instead, they must be molded. Manufacturing composite airframes concentrates development cost and duration into molds, fiber-placement machines, and autoclaves.

### 1.2.2 Product platforms

A product “platform” strategy is one in which some common technology or components are shared between “variants” within a product “family”. Each variant serves a different market or mission need, but since they share a subset of their requirements, these common requirements may be fulfilled with common components.

Designing for commonality between variants introduces constraints and reduces the available design space. Therefore, commonality almost invariably reduces the performance of each variant (in comparison to what may be achieved individually). In return, product platforming provides several benefits:

Firstly, commonality can reduce development cost and duration for the family as a whole. Design and tooling costs for common components can be amortized over a large number of units, and economies of scale and the learning curve effect reduce the costs associated with common components. The time spent developing common components does not have to be repeated for additional variants.

Secondly, once the platform’s manufacturing architecture has been constructed, new variants may be developed more rapidly and at lower cost than starting from scratch.
1.2.2.1 Platform architectures
For the purposes of this work, the “architecture” of a platform is defined as the manner in which the variants share commonality. “Architecture” is akin to the “configuration” of the platform, and does not necessarily specify any of the dimensions or mechanical details of the variants.

Most hardware platforms use a “building-block” architecture in which some components are interchangeable between variants in the family. One example of such an architecture would be an automaker’s range of cars that share the same engine and transmission. Another example is a set of electric motors, batteries, and control boards that are common in a line of power tools.

1.2.2.2 Platform architectures in aeronautical engineering
Most modern airliners (such as the Boeing 737NG family shown in Figure 1.6) are designed using a building-block architecture in which wings, tails, and sections of the fuselage are common between variants while segments called “plugs” are added to the fuselage in front and behind the wing to increase passenger capacity. This strategy is often called fuselage “stretching”.

![Figure 1.6: Boeing 737NG Family](image)

This “stretching” architecture is used by both the Boeing 737 family and Airbus A320 family, but these two families are incompatible due to differences in the specific dimensions of their system interfaces.

A less-common, adjustable-tooling-based platform architecture is used in the Antares 18S/T and 20E sailplanes (Figure 1.7). Instead of installing additional sections during aircraft assembly, to manufacture the longer-wingspan version, a 1-meter, constant-chord mold section is placed between the outboard wing and the root (Figure 1.8). The resulting wings are structurally identical outboard of the red segment.

![Figure 1.7: Antares 20E sailplane](image)
The Boeing and Antares examples in the section above have different architectures: whereas the Boeing 737 variants share the same wings and tails, the Antares sailplane variants share the same fuselages. In both, however, the architectures are varied in primarily one dimension – fuselage length in the case of the Boeing architecture and wingspan in the Antares architecture.

1.3 Prior Work in Family Optimization Computation

The idea of sharing airframe components between variants has been explored by many researchers in the past; two of these efforts are highlighted below.

In 2003, Willcox and Wakayama published their study on the simultaneous optimization of two blended-wing-body (BWB) aircraft that carry different numbers of passengers. In their design, the inner wing, outer wing, and winglets (as shown in Figure 1.9) are common between the variants while the centerbodies are permitted to be different. This research highlighted the scaling and numerical conditioning issues that occur when multiple vehicles are optimized simultaneously. [7]

In 2016, Jansen and Perez published a study investigating the optimization of a family of aircraft for a range of markets, characterized by both passenger count and flight range. Discrete and binary decision variables are used to control aircraft design. To deal with the mixed-continuous/discrete nature of this optimization problem, the researchers used a Particle-Swarm-Optimization (PSO). [8]
In these previous works, the architecture is designed such that a component of the aircraft (a wing, tail, fuselage segment, etc.) is either common or unique. In contrast, the work in this thesis focuses on an architecture that permits continuous-variability in the geometry of aircraft components.

1.4 Adaptable Aircraft Manufacturing: Goals and Thesis Layout

This thesis explores a platform architecture called “Adaptable Aircraft Manufacturing” (or “AAM” for short). AAM is designed to reduce aircraft manufacturing cost and duration for composite aircraft while providing designers a large degree of freedom in important aerodynamic and structural variables so that a large range of missions may be accomplished without retooling.

1.4.1 Thesis structure

First, Chapter 2 discusses the physical design of AAM, its compatibility with composite structures, the degrees of freedom each variant can have, and the constraints it places between variants. This chapter also describes the FAST project, which acts as a physical proof-of-concept to demonstrate the feasibility of the architecture. Chapter 2 concludes by discussing the challenges of designing an AAM family and sets the scope for the remainder of this work, which is to quantitatively explore the performance tradeoffs generated by AAM.

To evaluate the architecture’s performance, it is necessary to optimize the families enabled by the architecture. To organize this type of adaptable-robust optimization problem, Chapter 3 presents a framework which defines and clarifies the connections and dependencies inherent in the design of the full lifecycles of aircraft platforms - from tooling, production, vehicle, through operation to satisfy a variety of missions.

Chapter 4 presents the AAM performance quantification procedure.

Like many adaptable-robust optimization problems, the design of AAM families requires the optimization of a complex web of relations consisting of thousands of variables. Chapter 5 discusses the choice of using Geometric and Signomial Programming.

Since Signomial Programming is a relatively new system for the optimization of aircraft, many new models and relations were developed to capture the tradeoffs inherent in AAM. Chapter 6 presents these models.

Chapter 7 presents a series of “Explorations” into the performance of AAM as it responds to mission scenarios.

Chapter 8 presents a discussion of the results, recommendations about how AAM might be useful to aircraft development companies, and recommends the direction of future development.
2 Adaptable Aircraft Manufacturing (AAM)

"Adaptable Aircraft Manufacturing" (AAM) is a platform architecture\(^1\) for composite aircraft in which a single set of molds is used to manufacture the primary structures of multiple variants. The degrees of freedom within the architecture enables designers to adjust the aircraft’s geometry during manufacturing, which allows a variety of missions to be accomplished without retooling.

To understand how this architecture functions, it is first necessary to understand conventional composite manufacturing. This chapter first describes conventional composite manufacturing processes. From this baseline description, the strategies used in AAM, its degrees of freedom, and its constraints are described.

For the purposes of this work, conventional tube-and-wing designs shall be considered, though the concepts presented herein are scalable to other vehicle configurations as well, such as flying wings or blended-wing-bodies.

2.1 Composite Aircraft Manufacturing

Advanced composites (fiberglass, carbon fiber, aramid, or other fibers bonded within an epoxy or other polymer matrix) are used in aircraft for their high strength- and stiffness-to-weight ratios, ability to provide tailored orthotropic material properties, compatibility with complex three-dimensional shapes, and other properties. Unlike sheet metals (which can be bent and shaped with jigs and ribs, machined, or forged into shape), constructing composite structures requires tooling investment in the form of molds. The molds are necessary to support and shape the plies during the layup process and constitute the largest per-design tooling investments for the production of composite airframes.

The most common method for molding components is a "bagged layup". This technique is ubiquitous in both small and large aircraft due to its compatibility with complex part geometries, tolerance to imperfect ply placement, and uneven ply thicknesses. In addition, a minimum of one mold may be used to produce a single part. Figure 2.1 below shows a schematic and layer order for a typical bagged layup.

\[\text{Figure 2.1: Bagged layup schematic}\]

\(^1\) "architecture" in this thesis is defined as the configuration in which the variants share commonality, not the specific measurements of the designs
As shown in Figure 2.1, the interface between the mold and the plies forms the “tooled surface”, which is precisely controlled during mold manufacturing. The external surfaces of wings, fuselages, and tails are almost invariably formed by this tooled surface while the “untooled surface” is typically exposed to the inside of the structure where surface finish is inconsequential. A mold-release (commonly a spray-on coating or wax) is applied to enable the removal of the cured layup from the mold. For example, Figure 2.2 below shows the wing skin mold for the Boeing 777X [9].

Figure 2.2: Wing skin molds for the Boeing 777X [9]

Soft, uncured plies of composite material are then laid on top of the molds either by hand or by layup machines. The layout and order of layup components are often referred-to collectively as the “layup schedule”. The layup frequently includes multiple types of fiber materials, cores to form sandwich panels, hard points, and other components as exemplified in Figure 2.3. The green-tinted sections are fiberglass laminate while the orange-tinted sections reveal the cores that stiffen the shell.

Figure 2.3: Cirrus SR22 fuselage molding [10]
Peel ply and breather fabric are then layered on top of the layup. The peel ply enables the separation of the layup plies from the breather. The interface between the peel ply and the layup forms the "untooled surface" which is significantly less precise in dimension than the tooled surface due to variations in ply or core thicknesses. The mechanical compliance of the peel ply, breather, and bag ensures compression during the curing process despite these variations.

In preparation for the curing process, air is removed from the bag using a vacuum pump. Figure 2.4 below shows the peel ply, breather, and bag layers being applied to the nose section of a Boeing 787.

![Figure 2.4: Boeing 787 nose section production](image)

For a composite layup to achieve its maximum strength, pressure and heat must be applied (using an autoclave) to compress the plies together and cure the epoxy. The combination of internal vacuum and external pressure minimizes the presence of voids in the layup, squeezes out excess epoxy, and minimizes the interlaminar distance. The layup assembly is heated and cooled following a temperature profile designed to flow and then solidify the epoxy and lock the fibers together. The interaction of fibers and hardened matrix generates the part's strength, stiffness, and other properties.

After removing the parts from the molds, post-processing steps such as cutting, fastening, and bonding may be used to finalize and join parts together to create aircraft components. Jigs can be used to align components for assembly, or parts may be self-aligning due to their geometry.
2.2 Geometry of Adaptable Aircraft Manufacturing

2.2.1 Common molds and adjustable-layup

Unlike conventional platform architectures that reuse whole components (such as the wing or tails) between variant vehicles, AAM is based on an "adjustable-layup" strategy that may be applied during the production of every component. The adjustable-layup concept is illustrated in Figure 2.5.

![Adjustable-layup strategy](image)

As shown in the figure, "adjustable-layup" is composed of two separate degrees of freedom: "adjustable-extent" and "adjustable-schedule". These changes in the layup procedure may be achieved either manually by technicians or by reprogramming the machines in automated fiber-placement systems.

"Adjustable-extent" refers to the degree of freedom that a layup may terminate before the end of the mold in the spanwise axis (for wings and tails) or lengthwise axis (for fuselages). This degree of freedom leverages the fact that standard wing, tail, and fuselage structures can be described as cylindrical shells of arbitrary cross-section with axial stiffeners, as shown in Figure 2.6. Terminating and capping structures at stations along the extruded axis produces components compatible with the loads on these structures. Instead of having discrete breakpoints, the extent of the layup may be controlled as continuous variables.

![Cylindrical abstraction of primary aircraft structural geometries](image)

"Adjustable-schedule" refers to the degree of freedom in the position, composition, and number of plies that may be placed during the layup process. This degree of freedom leverages the fact that the peel ply,
breather, and bag layers are compatible with any number and composition of plies a designer chooses during manufacturing in a bagged layup.

The sections below describe AAM's compatibility with typical aircraft structures and the degrees of freedom and constraints it generates at both the tooling and vehicle production stages.

2.2.2 Adjustable-layup applied to wings and tails
Discussions in this section regarding "wings" also apply to tails. For the purposes of this work, trapezoidal planform geometries are considered. While other planforms are possible, a trapezoidal planform is reasonable for most aircraft and is illustrative of the concept.

AAM's adjustable-layup strategy may be applied to the primary load-carrying structures (skins, spars, and shear webs) of conventional hollow wing designs since these components are composed of slender, tapered elements that run spanwise from the wing root to the wing tip (as shown in Figure 2.7).

![Figure 2.7: Layout of conventional wing construction](image)

Examples of this construction technique may be seen in Figure 2.8 (a sailplane wing prior to final closure) and in Figure 2.9 (the ALCAS wing design from Airbus [12]).

![Figure 2.8: Sailplane wing internal component integration](image)
Because wings are usually tapered, the adjustable-extent strategy enables a single set of molds to generate wings of different planform geometries. For example, Figure 2.10 below shows two simple wings being constructed from a single mold. By varying the wing-tip to wing-root extent of the layups, a single mold can be used to produce wings with a range of wingspan, aspect ratios, and taper ratios.

In addition to changeability in planform, applying the adjustable-schedule concept enables designers to change the number and composition of plies on both wing skins and spars. Changing ply count also changes the thickness of the shells. To maintain the correct local airfoil thickness, the spar layup may be shifted, as illustrated in Figure 2.11.
Figure 2.11: Spar mold shift to compensate for shell thickness

Therefore, AAM provides the designer the freedom to choose shell and spar thickness distributions, wing aspect ratio, wing area, and taper ratio (within a design space that is a function of the mold geometry).

Because the external mold lines are set at the time of mold manufacture, the wing planform taper angle, airfoil distribution, and twist distributions become common constraints between all variants. The mathematics describing the mold and wing geometry spaces are discussed in detail in Section 6.5.

By surveying existing aircraft, we can observe that constraining the taper angle between variant wings is reasonable for aircraft of similar speed and maneuverability requirements that scale in payload. Two such examples are shown in Figure 2.12 and Figure 2.13.

Figure 2.12: Wing planform comparison between three airliners

Figure 2.12 shows three airliners that cover a nearly doubling of maximum take-off weight (MTOW) but have nearly identical taper angles and all use transonic airfoils. Figure 2.13 shows the same type of overlay for three endurance-focused military UAVs that span over an order of magnitude in weight and a tripling of speed (though staying firmly within the subsonic regime).
For conventional tube-and-wing aircraft, aerodynamic forces integrate from the wingtips inwards to the root, and the taper angle mediates this trade between the wing’s aerodynamic loading and the wing’s structural properties. The observation that independently-designed vehicles converge to the same taper angle suggests that locking taper angle between variants that operate in similar flight regimes should generate a low penalty for the performance of the family.

While the adjustable-layup concept may be used on spanwise wing components, other components such as ribs and hard-points still require their own tooling, though these components and their tooling may be reused between variants if they are present at shared spanwise stations.

2.2.3 Adjustable-layup applied to fuselages
Fuselages in conventional tube-and-wing aircraft can be broken down into three primary components: nosecone, center tube, and tail cone. Applying adjustable-extent to the center tube enables AAM to achieve the same fuselage-stretching flexibility that is commonly practiced in the design of airliners, as illustrated in Figure 2.14 where the center tube (blue) is varied in length.

The diameter of the fuselage, external shape of the nose cone, and external shape of the tail cones are set during tooling manufacturing and are common between all variants. By altering the layup schedule of the nosecone, center tube, and tail cone, the fuselage strength and stiffness may be altered depending on the
mission's needs. For the purposes of this work, it is assumed that the center tube mold is created monolithically using a tool that is at least as long as the longest tube of the family.

### 2.2.4 Integration challenges in AAM structures

Since wing, fuselage, and tail geometries are adjustable in an AAM molding system, the interface geometry between these components incur efficiency penalties compared to aircraft manufactured using conventional tooling. First, a single-piece carry-through spar design is not possible due to the adjustability of the wing root location on the mold, necessitating the manufacturing of root joiners. Second, interface geometries between the wing, tail, and fuselage must be manufactured per-variant, which due to their variable nature, is expected to be more difficult and labor-intensive. Third, the wing- and tail-to-fuselage juncture geometry may generate more interference drag compared to conventional designs (assuming no additional tooling is used to create fairings for each variant). Fourth, there are components that would still require per-variant tooling to be generated, such as ribs to terminate the wing and tail tips and landing gear support structures.

For the purposes of this study, a 10% interference drag increment is assumed for AAM aircraft, and the mass required to join wings is estimated based on root spar properties.

### 2.3 AAM Proof of Concept: “FAST” Project

To demonstrate the feasibility of AAM in a physical implementation, the “Flexible Aircraft System Testbed” (“FAST”) project was conducted. In this project, three electrically-powered aircraft were manufactured from common tooling to fulfill three different missions, and flight testing was conducted to determine their performance.

Figure 2.15 shows the three completed aircraft, and Figure 2.16 highlights the wing mold commonality used on the A and B aircraft.

![Figure 2.15: Three aircraft constructed from common tooling (left to right: A,B,C)](image)
Aircraft A was designed for a conventional runway takeoff and landing, and was designed for long-endurance loiter. Aircraft B was designed for tool-less assembly to mimic the requirements of small, backpack-deployed military UAS. Aircraft C was designed to carry two spanwise-separated small payloads to fulfill a stereoscopic imaging mission.

One set of molds was used to manufacture the wings, and a second set of molds was used to manufacture both the horizontal and vertical tails (as shown in Figure 2.17). In addition to manufacturing different wing and tail planforms, the layup schedules were altered between aircraft to demonstrate the adjustable-schedule concept, as visible in the differently-colored composite wing construction shown in Figure 2.15.

In addition to the common wing and tail molding, a spine structure, highlighted Figure 2.18, was used. The fuselage, wing, and landing gear attach to the spine with friction-mounted collars to enable the rapid
mounting and adjustment of aircraft components for testing. While the spine system is feasible for small aircraft, it is not a scalable solution for larger aircraft due to the cube-square law and is therefore not used in the computation investigation performed within this thesis.

![FAST A (spine in white box)](image)

Table 2.1 below shows the high-level geometric properties of these aircraft, showing a 4.5-fold difference in flight weight and a 3.5-fold difference in wing area between the smallest (B) and largest aircraft (C).

<table>
<thead>
<tr>
<th></th>
<th>FAST A</th>
<th>FAST B</th>
<th>FAST C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wingspan</td>
<td>130 in</td>
<td>72 in</td>
<td>170 in</td>
</tr>
<tr>
<td>Length</td>
<td>67 in</td>
<td>44 in</td>
<td>93 in</td>
</tr>
<tr>
<td>Wing area</td>
<td>7.7 ft²</td>
<td>3.3 ft²</td>
<td>11.67 ft²</td>
</tr>
<tr>
<td>Wing aspect ratio</td>
<td>15.2</td>
<td>10.9</td>
<td>17.2</td>
</tr>
<tr>
<td>Flight weight</td>
<td>16.1 lb</td>
<td>5.5 lb</td>
<td>25 lb</td>
</tr>
</tbody>
</table>

Table 2.1: FAST aircraft geometries

Through flight-testing, the following performance estimates are obtained. While these aircraft are not in any sense optimal, their performance is nonetheless reasonable for small electric vehicles of this class.

<table>
<thead>
<tr>
<th></th>
<th>FAST A</th>
<th>FAST B</th>
<th>FAST C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max endurance</td>
<td>2.6 hours</td>
<td>1.5 hours</td>
<td>1.9 hours</td>
</tr>
<tr>
<td>Glide ratio</td>
<td>12.4</td>
<td>11.8</td>
<td>12.2</td>
</tr>
<tr>
<td>Climb rate</td>
<td>760 ft/min</td>
<td>700 ft/min</td>
<td>570 ft/min</td>
</tr>
<tr>
<td>Stall speed</td>
<td>27 mph</td>
<td>22 mph</td>
<td>28 mph</td>
</tr>
<tr>
<td>Dash speed</td>
<td>60 mph</td>
<td>70 mph</td>
<td>50 mph</td>
</tr>
</tbody>
</table>

Table 2.2: FAST aircraft performance

2.3.1 Lessons from FAST and the problem of designing AAM families

While the FAST aircraft proved the viability of creating multiple aircraft from the same set of molds, the experience also highlighted the challenges in performing trade-offs between multiple aircraft:

First, while AAM reduces tooling cost of primary structures, it increases the parts-count and labor in the creation of small, mechanically non-obvious components such as wing joiners, control surface attachment hardware, engine mounts, and mechanisms. For instance, on the FAST project, since the servo locations for each aircraft is different, it was not possible to include the servo pocket into the wing mold, and post-process installations were necessary, increasing both manufacturing time and installed-mass relating to these components.
Second, it becomes extremely difficult to understand what is optimal for the family when variants emphasize different metrics of performance.

Suppose a family were to be designed that includes both an endurance-focused aircraft and a speed-focused aircraft. It is not clear what the optimum tooling, aircraft, nor what the performance penalty would be. In the design of a single aircraft, the optimal design occurs at the resolution of constraints and the pressure gradients of different variables in a design. In the design of an optimal family, the gradients of each variable defining each variant interact with the gradients of the others of the family.

In effect, the question that needs to be answered is, "how expensive are the constraints that AAM places upon these aircraft?" The only way to evaluate such a proposition would be to optimize AAM for the specifics of that particular mission set.

2.4 Way Forward
The remainder of this thesis attempts to address AAM family design by laying out a logical and mathematical structure capable of generating, for any set of mission requirements, the optimal family design. The models must be of sufficient fidelity to capture the inefficiencies generated by the constraints of the architecture. This optimal-family may then be compared against a set of individually-optimal aircraft to determine the performance losses incurred by AAM constraints.
3 ROBUSTNESS AND ADAPTABILITY FRAMEWORK

As discussed in Chapter 1, creating the tooling and production architecture to build an aircraft is much more expensive and time-consuming than creating a single unit. Chapter 2 discussed both the physical implementation of AAM and motivated the necessity of optimizing an AAM family for a given mission set.

The AAM family-design problem may be considered to be an adaptable-robust optimization problem due to the presence of uncertainty (in future missions), a component which is designed and constructed only once (the manufacturing architecture), and an availability of adaptability (in the degrees of freedom within AAM).

The optimization of adaptable-robust systems is complex, and the mathematical organization of this class of problems is an active area of research and debate [13]. Most existing studies in robust optimization use a set of guiding principles and then assemble an ad-hoc structure to solve a particular problem. This chapter presents a framework designed to clarify and organize the interactions in this class of problems, firstly to enable the solution of the AAM performance problem, and secondarily in hopes that such a framework might benefit future robust optimization investigations.

First, to provide a common point of discussion, a classical optimization formulation is presented. Then, uncertainty, robustness, and adaptability are defined qualitatively. These concepts can be difficult to visualize, so an example in the form of an airfoil optimization problem is interleaved with the discussion to illustrate these concepts.

A framework is then presented which captures the dependency and time-rate features inherent in adaptable-robust problems. This framework shall be used to facilitate the optimization and evaluation of the AAM concept.

3.1 Classical Optimization Formulation

The classical optimization formulation involves the following sets and functions [14]:

\[ x \text{ the set of "decision variables" (values to be determined)} \]
\[ p \text{ the set of "parameters" (values that are imposed a-priori)} \]
\[ g(\ ) \text{ inequality constraint functions} \]
\[ h(\ ) \text{ equality constraint functions} \]
\[ U(\ ) \text{ objective function} \]

The optimization problem may be phrased as:

Find \( x \) such that:

\[ \min U(x, p) \]
\[ g(x, p) \leq 0 \]
\[ h(x, p) = 0 \]

This formulation is powerful because it can capture a wide range of problems and provides a starting point from which the design problem and solution space may be explored. In the context of aircraft design, aircraft performance requirements are captured within the parameter set \( p \) since they are imposed, and are thus not controllable by the designer.
3.1.1 Example of classical optimization: basic airfoil design problem

Suppose the goal of a designer is to design an airfoil for a wing. The designer is given the lift coefficient at which the airfoil must operate, and the goal is to minimize the drag coefficient.

In the context of wing design, the lift coefficient links the weight of the aircraft, the wing area, the air density, and the flight speed. Suppose the thickness of the airfoil is held constant for all designs, and the flow conditions (Reynolds, Mach, etc.) are identical for all operations. Airfoils change their lift coefficients by changing angle of attack, which is the only degree of freedom available at this stage. First, consider a situation in which flaps are not used.

Nomenclature:
\( \alpha \) angle of attack
\( C_l \) lift coefficient
\( C_{l_1} \) required operating lift coefficient
\( C_d \) drag coefficient
\( s \) airfoil shape

The optimization problem is phrased as:

\[
\text{Find } s, \alpha \text{ such that: } \\
\min_{s, \alpha} C_d(s, \alpha, C_l = C_{l_1})
\]

Suppose two designs \((s_1, \alpha_1)\) and \((s_2, \alpha_2)\) are being considered; their performance curves are shown in Figure 3.1. For readers unfamiliar with airfoil performance plotting conventions, the vertical axis \((C_l)\) may be considered the independent variable and the horizontal axis \((C_d)\) is the performance metric of interest. In Figure 3.1, the \(\times\) symbols denote the upper and lower stall limits of each airfoil, so the airfoil may not operate outside of the \(\times\) range.

![Figure 3.1: Airfoil example: baseline performance](image)

As shown in the figure, the more "point-design" airfoil \(s_2\) performs better (lower \(C_d\)) at the required \(C_l\) and would therefore be the preferred design. Also notable is the fact that \(s_2\)'s performance deteriorates faster than \(s_1\) as the \(C_l\) is changed. This type of local-vs-broad performance tradeoff is typical of many aerodynamic systems.
3.2 Uncertainty, Robust Designs, and Robustness

The term "uncertainty" within this framework refers to a difference between a value used during design and its real value during operation. Uncertainty may arise due to lack of knowledge of operating conditions, naturally-chaotic and unsteady processes, inaccuracy in predictive models, or other sources [15]. In contrast, "deterministic" problems are defined as those with no uncertainty.

For long-duration projects such as aircraft manufacturing, mission requirements and environment are also uncertain over time – for example, the price of fuel may change due to market forces, the payload mass may change due to the need to install new sensors, or takeoff lengths may change as operators attempt to access new airfields. Uncertainty in mission requirements may be captured by uncertainty distributions in the p-set. This type of uncertainty may be classified as "parametric" (following the classifications defined by [16]).

3.2.1 Robust designs and robustness

A design that is optimal under deterministic assumptions may perform poorly or become infeasible when parameters deviate from their deterministic values. "Robust" designs retain their performance, remain feasible, or a combination of both - despite these variations.

The quantification of robustness is complex and is dependent on the context in which it is explored. Robustness of a design is dependent on the uncertain parameter (in its identity, scope, and distribution) and the metric of performance that the designer selects.

There are two facets of robustness to consider: "sensitivity-robustness" and "feasibility-robustness" which shall be explored in the following example.

3.2.2 Robustness in the airfoil design example

The lift coefficient links lift, air density, speed, and wing area. Some or all of these parameters may vary due to weather, payloads, maneuvers, or other factors, which makes the operating lift coefficient uncertain. To capture the possible range of \( C_l \), new parameters are introduced to the optimization problem:

- \( C_{l^-} \) minimum operating lift coefficient
- \( C_{l^+} \) maximum operating lift coefficient

The range of the uncertainty will be called "scope". The optimization problem must now change to reflect the new range of values over which \( C_l \) may extend, as shown below:

\[
\text{Find } s, \alpha \text{ such that: } \min \int_{C_{l^-}}^{C_{l^+}} C_d(s, \alpha, C_l) \, dC_l
\]

For now, it is assumed that the probability distribution between \( C_{l^-} \) and \( C_{l^+} \) is uniform, so a direct integration may be used; otherwise, a probability density function would be used to weigh the drag-performance of the airfoil as a function of \( C_l \).

First, it is useful to recognize that the scope of the uncertainty affects the choice of the best solution. Figure 3.2 shows a small range of uncertainty, bracketed in orange. Over a small range of \( C_{l^-} \) to \( C_{l^+} \), \( s_1 \) still outperforms \( s_2 \), and remains the superior design.
However, if the uncertainty scope in $C_i$ is large enough, $s_1$ becomes the preferred design, as $s_1$ performs better than $s_2$ over a large range, as shown in Figure 3.3.

This behavior of $s_1$ exemplifies higher “sensitivity-robustness” than $s_2$ for this uncertain scope of $C_i$ since its performance does not degrade as quickly as $C_i$ varies away from $C_{i1}$. By comparison, $s_2$ has lower sensitivity-robustness as its performance degrades faster as $C_i$ is changed.

“Feasibility-robustness” addresses the fact that some designs may become infeasible (failing to meet all constraints) given a large-enough perturbation in a given parameter. This category also encapsulates designs in which some minimum amount of performance is required. In this example, if the scope of $C_i$ is
expanded further, $s_2$ becomes infeasible for some $C_l$ within $[C_l^-, C_l^+]$. Figure 3.4 shows this infeasible range where the scope exceeds bounds of the feasible performance space (in red).

![Figure 3.4: Airfoil example: large uncertain $C_l$ range causing infeasibility](image)

In this case, $s_1$ may be considered to have greater feasibility-robustness, as measured by its continued feasibility over a greater scope of $C_l$.

While in this airfoil example, $s_1$ has greater sensitivity-robustness and feasibility-robustness, it is important to note that one does not in general imply the other.

### 3.2.3 Asymmetry in robustness

It is important to note that both sensitivity-robustness and feasibility-robustness of a design may be asymmetric or even discontinuous. Wing structures, for instance, are typically designed up to some "design load" which is separated by a margin to its "ultimate load" beyond which point it shall break. However, the load may be decreased to 0 without failure.

### 3.3 Passive and Active Robustness and Adaptability

There are multiple ways in which robustness may be achieved. Designs may incorporate "adaptable" features in which one or multiple degrees of freedom exist which allow a designer or user to alter the system in response to variations in an uncertain parameter. This definition is in contrast to "passive" features, which achieve robustness without being adjusted. Complex systems often utilize a combination of both adaptable and passive features.

Under this definition, examples of "fully-passive" systems include most architectural installations such as bridges. Bridges are engineered with sufficient margin above their expected loads, which makes them robust to some amount of overload. However, the bridge is not adjusted in response to the loading scenario.

### 3.3.1 Adaptable robustness in the airfoil design example

In the above discussion, the airfoil shapes are passively providing robustness since their shapes do not change in response to the change in $C_l$. Suppose now that the designer considers the usage of flaps on
A deflected flap alters the flow around the airfoil and moves the low-drag "bucket" in the $C_l$ axis, and constitute an "adaptable" feature with respect to the airfoil geometry. New variables are now added to the problem:

\[ \delta_f \quad \text{flap deflection} \]
\[ s^\delta \quad \text{deflected airfoil shape, a function of } s \text{ and } \delta_f \]

With this adaptability feature, the non-deflected airfoil shape $s$ (which governs the manufacturing) now has a degree of freedom which separates it from $s^\delta$, which is being presented to the flow at any given operating point. The optimization problem is re-phrased as:

\[
\text{Find } s, \alpha, \delta_f \text{ such that:} \\
\min \int_{C_l^-}^{C_l^+} C_d(s^\delta, \alpha, C_l) \, dC_l \\
\text{subject to: } s^\delta = s(\delta_f)
\]

Figure 3.5 below shows the performance of the two designs given the ability to actuate flaps, with the dotted blue line showing the union of the minimum-$C_d$ points for all $\delta_f$ on airfoil $s_2$.

![Figure 3.5: Airfoil example: adaptable-robustness of a_2](image)

By adjusting $\delta_f$, $s_2$ becomes higher-performing than $s_1$, even over a large range of uncertainty, as shown by the dotted blue line being lower-$C_d$ than the red performance curve of $s_1$. Therefore, $s_2$ becomes more-robust to this large variation in $C_l$ by incorporating flaps, an adaptability feature.

Adding flaps to $s_1$, however, does not achieve the same effect in generating performance. Figure 3.6 shows the performance curves of $s_1$ when the same type and range of flaps are used.
The dotted line in red reveals the union of all minimum-$C_d$ points of $s_1$ with flap deflections. In this case, the addition of flaps does not significantly improve the performance of $s_1$ over the range of $C_l$-uncertainty.

This example demonstrates that adaptability features must be designed in conjunction with the base designs and that adding adaptability features does not always provide an increase in performance.

### 3.3.2 Costs of adaptability

Adaptability features usually incur system costs. In this example, the addition of a flap incurs penalties in the form of weight, pitching-moment, manufacturing cost, maintenance costs, etc. Depending on the system in question, the bookkeeping of these penalties may occur in other parts of the system or at different phases of design and manufacturing, making adaptability features a complex method of achieving robustness. Only when the improvements in performance outweigh the penalties does it make sense to invest in adaptable features. In the context of AAM, these adaptability costs manifest in the aircraft geometry constraints placed between variants and structures such as joiners, as mentioned in Chapter 2.

### 3.3.3 Rate of uncertainty and adaptability

The design of adaptable features must also consider the rate at which uncertainties vary. To illustrate this idea, consider the following two scenarios.

In the first scenario, the airfoil is used in the wing of a racing sailplane. The uncertainty in $C_l$ arises from the desire to fly at different speeds based on how thermal lift varies during a single race, so the pilot must be free to control the flap deflections in real-time, which results in the need for linkages and control levers.

In the second scenario, the airfoil is used for a series of wings on a loiter-focused UAV. The uncertainty in $C_l$ arises from differences in payload. In this application, it might be best to omit the control linkages in favor of a series of mounting holes, which reduce the costs associated with this degree of adaptability.

These two scenarios demonstrate that uncertainties and adaptability features have a rate characteristic. The costs for an adaptable feature will vary depending on the rate of response required. Systems that must adapt to faster variations are frequently more taxed by their need for more rapidly-changing
geometries, mechanisms, software, or dynamics. In addition, only adaptability mechanisms that are at least as fast as the uncertainty may provide adaptable robustness - whereas variables that are modified at a rate slower than the uncertainty may only react passively.

3.4 Framework to Describe Adaptability/Process Sequences

In the airfoil example above, the goal of the designer is to optimize the airfoil shape \( (s) \), angle of attack \( (\alpha) \), and flap deflection \( (\delta_f) \) to minimize drag for some range of lift coefficients \( (C_l) \). With the airfoil shape defined, the designer may then manufacture the tooling necessary to build the wing. During flight, the pilot may change \( \alpha \) and \( \delta_f \) to alter the behavior of the wing for various flight conditions.

In the classical optimization formulation, \( s, \alpha, \) and \( \delta_f \) constitute design variables within the set \( x \) (i.e., these are values that the designer has control over), and they are treated the same in the optimization problem. However, these variables behave very differently when manufacturing, operation, and the rate of uncertainty are considered since they have different changeability rates and costs of change.

To discuss and optimize adaptable systems in response to uncertain parameters, it is useful to capture the dependencies of the manufacturing-through-operation chain, the response rates of adaptability, and the rate of the variations to which they must respond. The following framework provides a method to organize these relationships in an intuitive manner that permits them to be investigated with clarity.

3.4.1 “Process Sequence” framework

The goal of the following framework is to map the adaptable-robust optimization problem (while capturing its features) onto the classical optimization formulation so that it may be solved in an available solver. The framework is based on four element classifications: “assets”, “processes”, “adjustments”, and “environments” which have the following variables and roles.

- **A** Asset: “Assets” are a set of values that are static unless operated-upon by some process.

- **P( )** Process function: “Processes” are functions that map one asset to another and may accept an “Adjustment”.

- **J** Adjustments: “Adjustments” act on degrees of freedom in each process to alter the mapping of one asset to another.

- **E** Environment: Processes occur within “environments” which interfaces the process with information from the world that is external to the system being designed.

\( \text{( )}^n \) Element at stage \( n \) of the chain, addressed from \( n = 1 \ldots N \).

This framework captures the chronology of manufacturing-through-operation as a chain of function mappings. Each of the elements \( (A, P( ), J, E) \) are located using a chain address \( n \). Starting from chain location 0, the downstream asset \( A^1 \) would be generated with the function:

\[
A^1 = P^0(A^0, E^0, J^0)
\]

Further downstream assets \( (A^2, A^3, \text{etc.}) \) follow suit, and chaining together these process functions enables any number of asset/process steps.
\[ A^2 = P^1(A^1, E^1, J^1) = P^1(P^0(A^0, E^0, J^0), E^1, J^1) \]

This framework is meant to organize design elements and interactions so that the dependencies and chronologies may be followed, and does not necessitate any specific optimization methodology. Conventional methods may still be used to investigate design problems, and constraints and objective functions may still be defined from any of the variables in the problem.

This framework is compatible with the classical optimization structure as discussed in section 3.1. \( A^n \) and \( J \) contain the decision variables are concatenated as \( x \). Parameters \( (p) \) that the designer does not have control over typically reside in \( E \). The chain of \( P^n(P^{n-1}(...)) \) processes becomes the \( U(\ ) \) function which must be minimized or maximized.

### 3.4.2 Asset space mapping abstraction

The chained-process view presented above may be abstracted to a series of asset-space-mappings in which the assets are represented as points within their respective spaces, and adjustments control how processes map these assets from one space to the next. Figure 3.7 shows, conceptually, this tooling-to-performance mapping for both a single-mission aircraft and an AAM family.

![Figure 3.7: Variable-space mapping abstraction](image)

In conventional aircraft manufacturing, the tooling only creates one aircraft design, so the mapping is one-to-one between the tooling to the aircraft space. AAM, by contrast, allows the mapping from tooling space to vehicle space to be varied during manufacturing. Thus, a single point in tooling space is able to produce a large reachable area in performance space.

### 3.4.3 Process sequence map abstraction

These elements may be used to build a Process Sequence Map, or "PSM" for short. The PSM is a visual representation of the chaining of processes, and is more intuitive to understand. Figure 3.8 shows how the four elements arrange into a block. The elements interface with others both up- and downstream.
3.4.4 Framework applied to the airfoil example

The following PSM captures the airfoil-and-flap optimization problem. First, in the non-adaptable robust case, an airfoil (asset) undergoes the process (flow) under some flow environment at a user-adjustable angle of attack to achieve both lift and drag.

In the adaptable case, the geometry of the airfoil is modified by the flap deflection, so it generates a different, deflected airfoil asset before being adjusted in angle of attack, as shown in Figure 3.10.

3.4.5 Uncertainty, robustness, and adaptability within framework

Uncertainty in parameters may be captured via changes to $E$ at some stage $E^n$, and only adjustments that are faster than the perturbation in $E$ may be changed to react to the uncertainty. For many
situations, the rate of change of assets and adjustments grows monotonically faster downstream. In these situations, \( i^j \) such that \( i > n \) may be changed in response to uncertainty.

### 3.5 Framework Applied to Aircraft Manufacturing Chains

Conventional vehicle-focused design methodology generates bespoke tooling for each vehicle design. This method shall be considered the "baseline" for this discussion. The PSM for this baseline manufacturing-through-operation chain is shown in Figure 3.11.

![Figure 3.11: PSM for baseline manufacturing-through-operation chain](image)

In this baseline example, there are no degrees of freedom during the aircraft manufacturing process, so there is a one-to-one relationship between the tooling and the aircraft. Adjustability in this system is concentrated in the "operation" block that controls how the flight shall be flown. Within this block are the fuel and payload loadouts, the flight altitude and airspeed, and control inputs to the aircraft.

An interesting observation may be made by laying out the manufacturing-through-operation chain in this manner: that the changeability rate of assets monotonically falls from the initial tooling to flight performance phases. Changes that occur earlier in the chain affect all downstream processes (and therefore have larger influence), but are slower to enact – while changes downstream in the chain can react more quickly to uncertainty.

The Adaptable Aircraft Manufacturing (AAM) strategy introduces degrees of freedom in the aircraft manufacturing process, which accepts a series of "Production adjustments", as shown in Figure 3.12.

![Figure 3.12: PSM for Adaptable Aircraft Manufacturing](image)
These degrees of freedom at the manufacturing stage breaks the one-to-one relationship between tooling and aircraft. The result is that designers receive access to two sets of adaptability: one during manufacturing (which changes the aircraft) and another during operation (changing how the aircraft is flown). Controlling the aircraft manufacturing step makes it possible to re-optimize the aircraft (within the subspace accessible from existing tooling) in the timespan of months rather than years. The concept is that this increased response speed shall be a sufficient improvement to justify the performance penalty imposed by the commonality architecture.

Aircraft produced by AAM are not necessarily more robust to variations in flight-mission requirements than a conventional aircraft, but the manufacturing hardware (where the vast majority of the cost and development time is concentrated) is robust to these variations, and the adjustments enabled by AAM are sufficiently fast to respond to changes in mission requirement.

### 3.6 Representing Other Designs

The PSM abstraction may be used to represent other system-level designs as well. For instance, many racing sailplanes have wingtip extensions which may be installed before the aircraft are airborne, which would constitute a "reconfiguration" of the aircraft asset prior to the flight mission.

If a reconfigurability step is used on an AAM vehicle, a "reconfiguration" process block is added between the "aircraft" and the flight mission. Figure 3.13 shows the new elements in blue.

![Figure 3.13: PSM for AAM with aircraft reconfigurability](image)

Conventional building-block platforms may also be represented with the following map. In this case, the "tooling" asset is split between the various components, and the vehicle integration process is adjusted by "component selection" which may be of mixed-value types: for example, containing binary values to determine whether to include a plug that extends the fuselage, as is done in the Boeing 737NG platform.
Figure 3.14: PSM for an example building-block aircraft platform
4 AAM PERFORMANCE QUANTIFICATION

The primary objective of this thesis is to quantify the performance penalties resulting from the use of common tooling. To accomplish this, the methodology shown in Figure 4.1 is used.

First, a flight mission is specified, and then a mission-set is built upon this baseline mission by perturbing some of its driving requirements. Then, two rounds of fleet-optimization are conducted: first (in green), an AAM family is optimized. Second, a set of individually-optimal ("IO") aircraft (one designed per mission in the mission set, without commonality) is optimized. Both the AAM and the IO fleet are optimized for the same metric as discussed in section 4.2.

4.1 Baseline Mission Overview

The baseline mission is comprised of a simple delivery flight mission with the following profile as illustrated in Figure 4.2.
This mission profile is fairly generic and reflects typical mission-driving requirements for many aircraft in the industry. Depending on the specified numbers regarding payload size and mass, flight range, and speed, this profile is applicable to surveillance, payload delivery, or passenger delivery missions.

A single-engine, unswept, composite tube-and-wing aircraft, as shown in Figure 4.3 is assumed. The airframe assumptions and models are discussed in depth in Chapter 6.

4.1.1 Baseline driving requirements
The baseline mission approximately corresponds to a medium-scale UAS class vehicle. The primary driving requirements are shown in Table 4.1.

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payload mass</td>
<td>1000 kg</td>
</tr>
<tr>
<td>Payload volume</td>
<td>$4 \ m^3$</td>
</tr>
<tr>
<td>Takeoff length (fully loaded)</td>
<td>400 m</td>
</tr>
<tr>
<td>Flight altitude</td>
<td>5000 m</td>
</tr>
<tr>
<td>Flight range</td>
<td>2000 km</td>
</tr>
<tr>
<td>Cruise speed</td>
<td>$60 \ m/s$</td>
</tr>
<tr>
<td>Ultimate load factor</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 4.1: Baseline mission specifications
4.2 Total System Cost Metric

Nomenclature:

- $\xi_a$: specific cost of aircraft ($/kg$)
- $\xi_f$: specific cost of fuel ($/kg$)
- $C_i$: cost of unit $i$
- $C_F$: total cost of a family of vehicles
- $i$: index of variant number
- $m_a$: empty mass of aircraft
- $m_f$: mass of fuel to accomplish mission
- $n_m$: number of missions flown
- $n_v$: number of variant designs in family
- $N_i$: number of units of variant $i$ produced

To evaluate the performance of a fleet, it is necessary to optimize it for some metric. The metric of performance for each variant is the total aircraft cost over $n_m$ missions, which is calculated using equation (4.1).

$$C_i = \xi_a m_a + n_m \xi_f m_f$$

(4.1)

This metric enables the optimizer to trade off airframe mass and fuel mass in a realistic manner, avoiding both an aircraft that is small and inefficient and one that is fuel-efficient but massive and expensive to construct. By surveying the market, the following values are used for the coefficients: $\xi_f = $2.50/gallon, $\xi_a = $100/kg, and it is assumed that each aircraft has a mission lifetime of 1000 flights.

To optimize a IO fleet or AAM family, the total cost as shown in equation (4.2) is minimized. In this equation, $N_i$ reflects the number of units of variant $i$ produced.

$$C_F = \sum_{i=1}^{n_v} N_i \left( \xi_a m_{a_i} + n_m \xi_f m_{f_i} \right)$$

(4.2)
5 GEOMETRIC AND SIGNOMIAL PROGRAMMING

The optimizations are solved with a signomial program that is procedurally generated following the process-sequence framework described in Chapter 3. This chapter explains geometric programming and signomial programming in brief, and describes the benefits these computational methods have for robust-adaptable optimization. For more information regarding GP, see [17] [18] [19].

5.1 Computational Challenges in Designing Aircraft Families

There are several problems that arise when designing a family of aircraft (as opposed to a single aircraft). First, the number of design variables quickly grows as additional variants are added to the family. Exploring such a rapidly-expanding decision variable space using conventional optimizers is extremely slow.

Second, the design of AAM requires the use of a substantial number of inequalities and cross-variant constraints. For instance, the fuselage tube shared between multiple variants may be driven by the payload volume of one variant, but its tailcone length may be driven by the tail sizing of another. Depending on the algorithm and codes that are used for design, the treatment of these inequalities may generate substantial computational and manual reformulation overhead.

5.2 Geometric Programming

Nomenclature:

\[ a \quad \text{exponent} \]
\[ c \quad \text{coefficient} \]
\[ f(x) \quad \text{monomial} \]
\[ g(x) \quad \text{posynomial} \]
\[ h(x) \quad \text{signomial} \]
\[ x \quad \text{variable} \]

Geometric Programming (GP) is a type of convex optimization problem that has the following formulation:

Variables and coefficients are strictly positive. Variables may be either free, or may be set to equal some value.

\[ x, c > 0 \quad (5.1) \]

A “monomial” is defined as the product of any number of variables, each of which is raised to some exponent. The exponents are not strictly positive – they only need to be real.

\[ f(x) = c x_1^{a_1} x_2^{a_2} \ldots x_n^{a_n} \quad (5.2) \]

A “posynomial” is the summation of any number of monomials. A monomial is also a posynomial with only one term.
\[ g(x) = \sum_{k=1}^{K} f(x)_k = \sum_{k=1}^{K} c_k x_1^{a_{1k}} x_2^{a_{2k}} \ldots x_n^{a_{nk}} \quad (5.3) \]

A GP in standard form is written as:

\[
\begin{align*}
\text{minimize} & \quad g_0(x) \\
\text{subject to} & \quad f_i(x) = 1, \quad i = 1, \ldots, m \\
& \quad g_i(x) \leq 1, \quad i = 1, \ldots, n
\end{align*}
\]

Both the set of monomial and posynomial inequalities are called “constraints”. It is critical to note that only monomials may have equality constraints; posynomial constraints must be less-or-equal-to 1.

A GP problem is solved by transforming the constraints into a logarithmic space, which converts the nonlinear problem to a linear, convex optimization problem, which provides the following benefits:

1. GPs of large variable counts may be solved extremely quickly.
2. Results are guaranteed to be globally optimal.
3. No initial guesses are needed for the solution to be found.
4. Parameter sensitivities are produced as part of the solution process for free.

These qualities, in addition to GP’s natural handling of constraints as a series of inequalities, make GP an interesting method to solve adaptable-robust optimization problems.

### 5.3 Signomial Programming

As models increase in fidelity and complexity, there are many components that cannot be modeled using a strict GP. In short, due to the strict less-or-equal-to relation in \( g(x) \leq 1 \), GP is compatible with posynomials that generate costs. However, there are many relations regarding structures and aerodynamics that require the ability to track “beneficial” summed values such as the sum of distances between components.

In these cases, a more relaxed variant, Signomial Programming (SP) may be used. Whereas a “posynomial” is any summation of monomials with strictly positive coefficients, a “signomial” \( (h_i(x)) \) is permitted to use negative \( c \) (though \( x \) are still strictly positive). With the inclusion of at least one signomial, the GP is then considered Signomial Program (SP). In standard form, an SP may be written as:

\[
\begin{align*}
\text{minimize} & \quad g_0(x) \\
\text{subject to} & \quad f_i(x) = 1, \quad i = 1, \ldots, m \\
& \quad g_i(x) + h_i(x) \leq 1, \quad i = 1, \ldots, n
\end{align*}
\]

While SP do not solve as quickly as a GP and do not solve with a global-optimum guarantee, the solve times for SP are still faster than conventional gradient-based optimization solvers.
Where possible, the models are formulated as a strict-GP; SP-formulations are used where necessary. It is hoped that these models shall contribute to the growing library of design models for aircraft and enables efforts to conduct adaptable-robust optimizations for large families of aircraft.

5.4 Coding Setup for Optimization Problem
The computation code for the AAM evaluation problem is written in Python with extensive use of the GPkit and GPfit packages developed at MIT [17], which provides a symbolic method to link procedurally-generated constraints to existing GP and SP solvers. For all optimizations performed in this work, the commercial solver MOSEK was used [20].
6 MODELS

This section describes the key GP- and SP-compatible aircraft models. The models developed during this thesis effort are first-principles based with the goal of being sufficiently high fidelity to capture AAM-generated inefficiencies while staying GP- and SP-compatible.

6.1 Fitting Procedure for Complex Data

Where possible, first-principles models are used. However, there are many relations that are the result of more complex processes than cannot be reasonably replicated in a GP- or SP-compatible manner, such as drag. In these cases, GP-compatible "fits" must be generated. These fits are, in effect, GP-compatible surrogate models which relate any number of inputs (x) to a single output (y). Many modules used in this work use a mix of first-principles and fitted data.

To generate fits, the procedure shown in Figure 6.1 is used.

![Figure 6.1: GP-compatible fitting procedure](image)

To generate a fitted relation, the procedures and code developed by Hoburg [17] are used with a custom software wrapper that automates the reporting and equation generation process. As part of this report, "fit plots" are generated. In these plots, the GP-compatible fit (vertical axis) is plotted against the high-fidelity input (horizontal axis). Therefore, good fits are characterized by the tight coalescing of data points onto the 1:1 line, which denotes a perfect prediction between the input and output values.

For the remainder of this chapter, to denote an empirical fit, the notation $\equiv F$ is used. For example,

$$y \equiv F(x_1, x_2, x_3) \quad (6.1)$$

Means that a GP-compatible fit was generated to produce the relation between $y, x_1, x_2, x_3$. Select fitment plots are shown for discussion and the full fit equations are found in the Appendix.
6.2 Atmospheric Models

Nomenclature:

- $h$: altitude
- $\theta$: atmospheric property ratio
- $\xi$: altitude ratio

A new atmospheric model was developed for the troposphere to enable the prediction of atmospheric properties at altitude. The model is valid between 1m MSL to 10km MSL. This altitude range covers the majority of air travel.

Via the fitting procedure described in Section 6.1, ambient temperature, pressure, density, dynamic viscosity, and kinematic viscosity are related to their sea-level values via a nondimensional altitude ($\xi$). The result is a series of $\theta$ fits that relate the at-altitude properties with their values at sea level.

$$\xi = \frac{h}{10,000 \text{ m}}$$

$$\theta_{T,P,...} = \frac{T,P,...}{(T,P,...)_{\text{ground}}} \cong \mathcal{F}(\xi)$$

Figure 6.2 below shows these scaling values plotted against the nondimensional altitude.

![Atmospheric property ratios vs $\xi$](image)

6.3 Aircraft configuration

A low-sweep, tractor, tube-and-wing aircraft configuration is assumed with conventional spar and longeron structural elements. Figure 6.3 below provides an overview graphic of the assumed configuration.
This configuration is defined by the following assumptions:

- The center tube of the fuselage is centered above the aerodynamic center of the wing.
- The payload resides wholly within the center tube of the fuselage.
- The fuel tanks reside wholly within the wing.
- The aircraft is powered by a piston engine residing in the nose.

### 6.4 Airframe Material Assumptions

There are two types of material assumed for the airframe: unidirectional and quasi-isotropic carbon fiber reinforced plastic (CFRP). For these materials, the following properties in Table 6.1 are assumed. These values are based on testing conducted by MIT [21] and also publically available materials specifications sheets from industry [22].

<table>
<thead>
<tr>
<th>Name</th>
<th>Common symbol</th>
<th>Value (Unidirectional)</th>
<th>Value (Isotropic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus</td>
<td>$E$</td>
<td>130 GPa</td>
<td>50 GPa</td>
</tr>
<tr>
<td>Young’s Modulus (used)</td>
<td>$E$</td>
<td>65 GPa</td>
<td>25 GPa</td>
</tr>
<tr>
<td>Ultimate stress</td>
<td>$\sigma_{\text{ultimate}}$</td>
<td>1500 MPa</td>
<td>300 MPa</td>
</tr>
<tr>
<td>Stress allowable</td>
<td>$\sigma_{\text{allowable}}$</td>
<td>375 MPa</td>
<td>75 MPa</td>
</tr>
<tr>
<td>Shear stiffness</td>
<td>$G$</td>
<td>5 GPa</td>
<td>50 GPa</td>
</tr>
<tr>
<td>Shear stiffness (used)</td>
<td>$G$</td>
<td>2.5 GPa</td>
<td>25 GPa</td>
</tr>
<tr>
<td>Ultimate shear</td>
<td>$\tau_{\text{ultimate}}$</td>
<td>50 MPa</td>
<td>150 MPa</td>
</tr>
<tr>
<td>Shear allowable</td>
<td>$\tau_{\text{allowable}}$</td>
<td>12.5 MPa</td>
<td>37.5 MPa</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>1600 kg/m$^3$</td>
<td>1600 kg/m$^3$</td>
</tr>
<tr>
<td>Minimum gauge</td>
<td>$t_{\text{min}}$</td>
<td>0.5 mm</td>
<td>0.5 mm</td>
</tr>
</tbody>
</table>

*Table 6.1: Materials performance assumptions*

The unidirectional material properties are used for spar and longeron calculations, while the isotropic properties are assumed for all other structures. A safety factor of 4 is used on the ultimate stresses to
generate the “allowable” stresses to account (roughly) for fatigue margin, and a safety factor of 2 is used for stiffness.

6.5 Common Molding Geometry Mathematics

Nomenclature:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR</td>
<td>aspect ratio</td>
</tr>
<tr>
<td>b</td>
<td>wingspan</td>
</tr>
<tr>
<td>c</td>
<td>chord</td>
</tr>
<tr>
<td>S</td>
<td>wing area</td>
</tr>
<tr>
<td>( y )</td>
<td>spanwise coordinate</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>full taper ratio</td>
</tr>
<tr>
<td>( \delta )</td>
<td>taper sweep</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>taper ratio (tip over root)</td>
</tr>
</tbody>
</table>

\( (\ )_m \) ( ) of mold
\( (\ )_w \) ( ) of wing
\( (\ )_r \) ( ) at root
\( (\ )_t \) ( ) at tip

6.5.1 Wings and tails

The space of available wing geometries (for a single set of molds) is described with the following figure and equations. Figure 6.4 shows the mold (orange) and a wing (blue) with control variables in the \( y \) (spanwise) axis, subscripted with either \( m \) (for mold) or \( w \) (for wing) and \( r \) and \( t \) for root and tip, respectively. \( \Delta \) and \( \delta \) represent the total taper angle and sweep angle, respectively.

\[ 0 \leq y_{mt} \leq y_{wt} \leq y_{mr} \]  

(6.4)

Figure 6.4: Trapezoidal wing geometry
\[ AR \equiv \frac{b^2}{S} \]  
(6.5)

\[ \lambda \equiv \frac{c_r}{c_r} \]  
(6.6)

\[ b = 2(y_{wr} - y_{wt}) \]  
(6.7)

\[ S = (c_r + c_t)(y_{wr} - y_{wt}) \]  
(6.8)

\[ S = \frac{y_{wr}^2 - y_{wt}^2}{2}(\tan(\delta) + \tan(\Delta - \delta)) \]  
(6.9)

\[ AR = 4 \left( \frac{y_{wr} - y_{wt}}{y_{wr} + y_{wt}} \right) \left( \frac{1}{\tan(\delta) + \tan(\Delta - \delta)} \right) \]  
(6.10)

\[ AR = 4 \left( \frac{1 - \lambda}{1 + \lambda} \right) \left( \frac{1}{\tan(\delta) + \tan(\Delta - \delta)} \right) \]  
(6.11)

Figure 6.5 below shows the available aspect ratio and wing area (nondimensionalized to \( y_{mr}^2 \)) for three taper angles and ratios.

Figure 6.5: Wing design spaces as a function of taper angle and taper ratio
The area enclosed by the envelopes denote the reachable regions for three taper ratio limits. As the figure shows, as the taper ratio limit is raised, the reachable area (in wing area and aspect ratio) shrinks.

For aircraft of moderate to high aspect ratios, a 0-length tip chord would result in unacceptable tip-stall and handling issues, so the taper ratio must also be constrained. A $\lambda \geq 0.1$ limit provides a reasonable estimate of this behavior for vehicles of high aspect ratios.

### 6.5.2 Fuselages

The fuselage consists of three sections: an elliptical nosecone, a straight center-tube, and an ogival tailcone as illustrated in Figure 6.6. The constant-cross-section is assumed to be manufactured via adjustable-extent on a single mold, which is as long as the longest variant, as shown in Figure 6.7.

*Figure 6.6: Fuselage geometry*

*Figure 6.7: Commonality scheme for fuselages*
6.6 Wing Models

Nomenclature:

$A_e$  
enclosed area

$AR$  
aspect ratio

$b$  
wingspan

$c$  
chord

$C_l$  
lift coefficient (2D)

$C_L$  
lift coefficient (3D)

$C_{ap}$  
drag coefficient (profile)

$C_D$  
drag coefficient (3D, total)

$C_{Di}$  
drag coefficient (induced)

$D$  
drag

$D_i$  
induced drag

$e$  
span efficiency

$E$  
Young's modulus

$g$  
gravitational acceleration

$G$  
material shear modulus

$h_v$  
Void height above neutral axis

$I_{yy}$  
second moment of area about the bending axis

$I$  
polar moment of inertia

$k_{Mr}$  
root bending moment ratio (vs elliptical)

$L$  
lift

$L_{max}$  
maximum lift

$m$  
mass

$M$  
bending moment

$N$  
load factor

$n$  
number of discretization segments

$q$  
dynamic pressure

$r_f$  
fuselage radius

$S$  
wing area

$\bar{\epsilon}$  
airfoil thickness ratio

$t_{cap}$  
thickness of spar cap

$T$  
torque

$T'$  
torque per unit span

$V$  
airspeed

$w$  
deflection

$y$  
spanwise coordinate

$y_{ND}$  
nondimensionalized spanwise coordinate

$\delta_F$  
flap deflection (degrees)

$\Delta$  
allowable tip deflection ratio

$\epsilon$  
strain (spanwise direction)

$\eta_{spar}$  
efficiency of spar

$\kappa$  
curvature

$\pi$  
3.14...
\( \sigma \)  Stress (spanwise direction)
\( \rho \)  air density
\( \lambda \)  taper ratio
\( \chi \)  nondimensional chord ratio
\( \phi \)  torsional angle

\( (\ )_r \)  ( ) at root
\( (\ )_t \)  ( ) at tip

6.6.1 Wing aero: lift model
The wing lift model uses the standard lift equation.

\[
L = \frac{1}{2} \rho V^2 S C_L = q S C_L \tag{6.12}
\]

For drag calculation purposes, it is assumed that the 2-D sections operate at the same lift coefficient as the wing in 3-D. This simplification is somewhat aggressive, but holds reasonably well for low-sweep, untwisted wings.

\[
C_l(y) = C_L \tag{6.13}
\]

6.6.2 Wing aero: induced drag model
To calculate the induced drag for the wing, the following definition for induced drag coefficient is used.

\[
C_{Dl} = \frac{C_L^2}{\pi e AR} \tag{6.14}
\]

Expanding out from this definition, the induced drag force is calculated as:

\[
D_l = \frac{1}{2} \rho V^2 S C_{Dl} = q^2 S^2 C_L^2 = \frac{1}{qne} \left( \frac{L}{b} \right)^2 \tag{6.15}
\]

The span efficiency factor \( e \) relates the drag of any wing or tail to the theoretical lifting-line ideal (uniform downwash, elliptical loading). Assuming simple planar, tapered wing geometries, the efficiency factor can be calculated using the aggregate planform variables as written below.

\[
e = f(AR, \lambda, \alpha_{twin}, \Lambda) \tag{6.16}
\]

Under the assumption of a free incoming stream (neglecting wash from upstream and downstream), a trapezoidal, untwisted, low sweep planform, the driving variables that determine the induced drag are solely the taper ratio and the aspect ratio of the wing or tail. Therefore, a surrogate fit is used to predict span efficiency as a function of taper ratio and aspect ratio.

\[
e \equiv \mathcal{F}(AR, \lambda) \tag{6.17}
\]
The training data used to generate the GP-compatible model is generated from the 3D-panel method implemented in the XFLR5/MIAReX code with airfoil viscosity drag contribution disabled. A population of wings was created spanning aspect ratios from 2 to 26 and taper ratios ranging from 0.01 to 1. The lift and induced drag data predicted from the model was used to create a training space that was used to fit a 9-term posynomial model. The data and fit curves are shown in Figure 6.8.

In Figure 6.8, the dots represent the span efficiency predicted for a given taper ratio and aspect ratio while the solid lines show the trends as predicted by the GP-compatible model at those same aspect and taper ratios. Maximum prediction error is 1.4% (note that the vertical axis is truncated).

These relations hold well when effects from other upstream/downstream surfaces can be neglected. For conventional configurations, downwash from the wing onto the tail is fairly uniform for designs in which the tail span is much less than the wingspan, which translates into a tail incidence offset required for pitch trim.

6.7 Wing Discretization for Load and Response Structures
Each wing (left, right) is discretized into \( n \) spanwise segments. At each station, the local Reynolds number, local drag, and loads are calculated and integrated from the tips inwards to the root to generate the loads that the structures must withstand. Figure 6.9 below shows the discretization scheme used in the simulation.
6.7.1 Wing aero: airfoil performance model

Airfoil thickness ratio is arguably the most important variable linking wing aerodynamics and structural properties. To capture this tradeoff, it is assumed that the wing uses an NACA-24xx airfoil whereby the thickness ratio of the airfoil is a free variable. The wing airfoil drag model from Hoburg and Abbeel [23] was used to predict the drag of the wing airfoil as a function of thickness, Reynolds number, and lift coefficient. Figure 6.10 below shows the polar performance for select thickness ratios.

Figure 6.10: GP-compatible airfoil model for NACA 24xx airfoils [23]
6.7.2 Wing bending loads model

For tube-and-wing aircraft, the bending moment integrates from the wingtips inwards towards the root. The result is a well-characterized, monotonically-increasing relation between spanwise location and bending moment. The model presented here nondimensionalizes the bending moment distribution in both the spanwise direction and against the root bending moment.

First, for a wing with elliptical loading, the root bending moment is approximately equal to:

\[ M_{\text{root, ellipt}} = \frac{bL_{\text{max}}}{10} \]  

(6.18)

Where the maximum wing lift is constrained by two cases: maneuvering load factor \( N \) times the weight of the aircraft and a gust load case at cruise speed, in which \( C_L = 1 \), as shown in equation (6.19).

\[ L_{\text{max}} = \max(N \cdot m \cdot g \cdot \frac{1}{2} \rho V^2 S) \]  

(6.19)

An adjustment factor may be used to relate a trapezoidal wing geometry to this elliptical baseline.

\[ M_{\text{root}} = \frac{1}{10} k_{BM} bL_{\text{max}} \]  

(6.20)

\[ k_M \approx \mathcal{F}(AR, \lambda) \]  

(6.21)

To derive this parameter, \( k_{BM} \) is fitted to bending moment data generated by a vortex-lattice code. The resulting fit (Figure 6.11) shows good agreement between the points (vortex-lattice data) and lines (GP-compatible fits).

![Bending moment ratio](image)

Figure 6.11: Bending moment ratio vortex-lattice and GP-model
The wing bending moment distribution is then nondimensionalized by span \( (y_{ND}) \) and may then be modeled as a function of the taper ratio of the wing.

\[
\frac{M(y)}{M_{root}} \equiv T(\lambda, y_{ND})
\]  

(6.22)

This effect is shown in Figure 6.12. When the taper ratio is lower (wings more triangular), the bending moment stays lower for more of the non-dimensionalized span. When the wing is more rectangular, the bending moment outboard is higher.

![Figure 6.12: Notional nondimensionalized wing bending distribution as a function of taper ratio](image)

### 6.7.2.1 Wing bending: stiffness constraint model

A spanwise-bending-curvature-limit model is used to constrain the wing in bending stiffness. This type of constraint is simple to model and ensures sufficient bending stiffness at all stations. The constraint works by setting a maximum tip deflection as a ratio of the halfspan, as shown in Figure 6.13. A \( \Delta = 10\% \) wing deflection limit is used.

![Figure 6.13: Wing spar bending stiffness limit model](image)

The curvature \( \kappa \) may then be adequately modeled as:
This relation holds extremely well over the range of typical wing deflections, as shown in Figure 6.14.

\[ \kappa_{\text{limit}} = \frac{4\Delta}{b} \]  

(6.23)

In addition to this curvature constraint, a wing-droop constraint is placed such that the wing does not hit the ground during landing. To enforce this limit, a secondary \( \Delta = 2 \frac{\tau_f}{b} \) is used.

### 6.7.3 Wing bending structure: spar

The spar properties are based on an equivalent-rectangular-spar model in which the total width of the spar, spar cap thickness, and maximum height are identical to the wing. Due to the curvature of the real spar, an efficiency factor \( \eta_{\text{spar}} \) is included so that a simple \( I_{yy} \) formulation may be used. Since the NACA-4 airfoils are simply scaled in thickness, \( \eta_{\text{spar}} \) may be estimated to be a constant. For a conservative estimate, the bending loads are assumed to be purely sustained by the spar caps.

Through FEA of thin-shelled cross-sections, an efficiency value of 0.83 is calculated for NACA-4 series airfoils, i.e., if one were to construct an airfoil-fitting spar with the same maximum height as a rectangular spar, it would have 83% of the rectangular spar's second area of inertia.

This equivalent-rectangular-spar model is governed by the following simple Euler beam bending relations, which links this spar model to the strength and stiffness constraints stated above.
The SP-compatible constraints to model wing strength are therefore:

\[
\sigma_{\text{allowable}} \geq \frac{1}{2} \frac{M \bar{r} c}{l_{yy}}
\]  

(6.27)

The stiffness requirements for the wing are formulated via the following equations:

\[
\varepsilon = \frac{\sigma}{E} = \frac{1}{2} \frac{M \bar{r} c}{l_{yy} E}
\]  

(6.28)

\[
\kappa = \frac{\varepsilon}{\bar{y}} = \frac{\varepsilon}{1/2 \bar{r} c}
\]  

(6.29)

\[
\kappa = \frac{M}{l_{yy} E} \leq \kappa_{\text{limit}} = \frac{4\Delta}{b}
\]  

(6.30)

The SP-compatible model for the stiffness constraint is therefore composed of the following equations:

\[
\frac{M}{l_{yy} E} \leq \frac{4\Delta}{b}
\]  

(6.31)

\[
l_{yy} \leq 0.27667 c \left( t_{cap}^3 + 3 t_{cap}^2 h_v + 3 t_{cap} h_v^2 \right)
\]  

(6.32)

These constraints are enforced at each span station to generate the cap thickness at each station.

6.7.4 Wing torsion loading cases

There are two primary torsional moment contributors for low-sweep wings – the nose-down pitching moment generated by the airfoil’s pitching moment coefficient, and the nose-up load generated by the center of lift being in front of the elastic center. In reality, these effects are both present and will offset each other, but it is conservative and effective to consider them separately.

The nose-down case is most severe at high speeds when \( C_t \) is nearly zero, and the torque on the wing is dominated by the airfoil pitching moment. Figure 6.16 illustrates this torsion case.
The local torque generated by the wing may be calculated as:

\[ T' = \frac{1}{2} \rho V^2 c^2 C_m \]  

(6.33)

The pitching moment \( C_m \) is modeled using NACA-4-series data and is fitted according to the fitting procedure discussed above.

\[ C_m \approx F(\tilde{t}, Re) \]  

(6.34)

The nose-up case is corresponds to the maximum-load-factor when the lift (which acts in front of the elastic center of the wing box) torques the wing in the nose-up direction. A conservative estimate uses \( C_m = 0 \), so that the aerodynamic pitching relief is neglected. This is a reasonable assumption because \( C_{\text{max}} \) far exceeds \( C_m \) for normal wing geometries.

The local torsional moment contribution in this case is calculated by the product of the local lift and the moment arm from the lift vector (assumed to be the quarter-chord) and the elastic center (which is determined by FEA to be approximately at \( x/c = 0.4 \)).

\[ T' = f(q, C_L, C_m = 0) \]  

\[ T' = \frac{1}{2} \rho V^2 c C_L (l_{EC} - l_{AC}) \approx \frac{1}{2} \rho V^2 c^2 C_L (0.4 - 0.25) \]  

(6.35)
\[ T' = 0.075 \, N \, q \, c^2 \, C_{\text{cruise}} \]  

(6.36)

In both cases, the torque load is controlled by the pitching moment due to the tail, and thus the reaction load is at the root. The torque applied at any point on the wing is therefore integrated from the tip inwards and is given by the following equation.

\[ T(y) = \int_{y_{\text{tip}}}^{y_{\text{root}}} T'dy \]  

(6.37)

To first order, the elastic response of the structure may be expected to be left/right symmetrical.

### 6.7.5 Wing structure: torsion box

As a conservative estimate, it is assumed that the spar caps do not contribute to the torsion response of the wing, and that the entire torsion box is of a constant thickness. It is assumed that the torsion box also uses the \( \chi = 0.15 \) to \( \chi = 0.65 \) segment of the wing section.

![Wing box torsion model](image)

*Figure 6.18: Wing box torsion model*

For the purposes of weight estimation for the remainder of the wing, it is assumed that the full perimeter of the airfoil uses the same skin gauge that is calculated for the torsion box.

#### 6.7.5.1 Torsion strength constraint

The torsion box model uses the classic closed-section torsion formulation shown in the equation below.

\[ \tau = \frac{T}{2 \, t_{\text{box}} \, A_e} \leq \tau_{\text{allowable}} \]  

(6.38)

For NACA-4 series airfoils, the area enclosed by the torsion box (assumed \( \chi \) limits of 0.15 to 0.65) is 0.666 that of the entire airfoil area, which in itself is equal to 0.685 \( c^2 \), yielding the equation below:

\[ A_e = 0.666 \times 0.685 \, \bar{r} \, c^2 = 0.456 \, \bar{r} \, c^2 \]  

(6.39)

#### 6.7.6 Torsion stiffness constraint

The twist angle rate of the wing may be calculated using:
\[ \phi' = \frac{d\phi}{dy} = \frac{T}{GJ} \tag{6.40} \]

where \( J \) is the polar moment of inertia of the torsion box. To predict \( J \) in a GP-compatible manner, the inverse of its ratio (as nondimensionalized to \( c^4 \)) is fitted to FEA-generated data from SolidWorks.

\[ J = J_{\text{eff}} \, c^4 = J_{\text{eff}}^{-1} \, c^4 \tag{6.41} \]

\[ J_{\text{eff}}^{-1} = F(\bar{c}, t_{\text{box}}) \]

To constrain the wing, a maximum-allowable angular deflection (\( \phi_{\text{max}} = 1 \, \text{deg} = 0.017 \, \text{rad} \)) is used.

\[ \phi' \frac{b}{2} \leq \phi_{\text{max}} \tag{6.42} \]

### 6.7.7 Wing root joiner mass model

Due to the inability for AAM tooling to mold a single continuous spar and torsion box, a root joiner is necessary, and the weight associated with this inefficiency must be estimated. A rough estimate may be created by assuming a constant-section joiner with structure equal to that of the spar and torsion box at the root. This structure is then extruded for a length along the span as a product of the root thickness, as shown in Figure 6.19.

![Figure 6.19: Wing joiner mass estimation schematic](image)
6.8 Tail Models

The tail cross-section and spanwise loading models are identical to those developed for the wing as described in section 6.6 above, except with camber set to 0. As such, the camber-driven relationships discussed previously are neglected for the tail.

6.8.1 Tail sizing

Since tooling commonality is also considered for the tails, it is necessary to trade tail performance as a function of taper angle and taper ratio. To accomplish this, tail surface sizing is conducted with volume ratios adjusted for aerodynamic effectiveness due to aspect ratios. Without some sort of limit on the tail effectiveness, the optimizing gradient would push for extremely stubby, low-aspect-ratio tails to minimize structural mass.

The (modified) horizontal tail volume ratio equation is

\[ V_H = \frac{S_{HT}}{S_{avg}} \left( \frac{1}{2\pi} \frac{dC_L}{d\alpha_{HT}} \right) \geq 0.4 \quad (6.43) \]

\[ \frac{dC_L}{d\alpha_{HT}} = F(A_{RT}, \lambda_{HT}) \quad (6.44) \]

For cruise, it is assumed that the horizontal tail operates at a lift coefficient of 0.1. To calculate tail loads, the maximum lift coefficient of the tail is assumed:

\[ C_{l_{max_{HT}}} = 1.0 \quad (6.45) \]

The (modified) vertical tail volume ratio equation is

\[ V_V = \frac{l_{VT}}{S} \left( \frac{1}{2\pi} \frac{dC_L}{d\beta_{VT}} \right) \geq 0.04 \quad (6.46) \]

\[ \frac{dC_L}{d\beta_{VT}} = F(A_{VT}, \lambda_{VT}) \quad (6.47) \]

To calculate tail loads, the maximum lift coefficient of the tail is assumed:

\[ C_{l_{max_{VT}}} = 1.0 \quad (6.48) \]

To generate the \( \frac{dC_L}{da} \) relation, the inverse of this value was fitted to vortex-lattice generated data for planforms of a typical range of aspect and taper ratios.

\[ \frac{dC_L}{da} = \frac{dC_L^{-1}}{da} = F(A_R, \lambda) \quad (6.49) \]
6.9 Wing and Tail Commonality Constraints

The AAM-compatible wing and tail commonality constraints are enforced by setting the airfoil thicknesses of all variants to be equal:

\[ \bar{t}_1 = \bar{t}_2 = \ldots \]  \hspace{1cm} (6.50)

and by setting the taper angles between all variants to be equal:

\[ \Delta_1 = \Delta_2 = \ldots \]  \hspace{1cm} (6.51)

Thus, the wing layup thickness distribution is freely optimized by the optimizer.
6.10 Fuselage Models

Nomenclature:

- \( C_D \): 3D drag coefficient
- \( C_{D,\text{ref}} \): reference 3D drag coefficient
- \( C_{\text{f,turb}} \): flat-plate turbulent friction coefficient
- \( D \): drag
- \( E \): Young's modulus
- \( I \): second moment of area
- \( I \): polar moment of inertia
- \( l \): length
- \( \bar{l} \): length, normalized to radius
- \( M \): bending moment
- \( Re_r \): Reynolds number of radius
- \( r_{\text{fuse}} \): fuselage radius
- \( S_{\text{front}} \): frontal area
- \( T \): torque
- \( t_{\text{lon}} \): longeron thickness
- \( t_{\text{shell}} \): shell thickness
- \( \xi \): Reynolds scaling variable
- \( \Delta \): deflection limit ratio
- \( (\_)_n \): of nosecone
- \( (\_)_c \): of centertube
- \( (\_)_t \): of tailcone
- \( (\_)_p \): in pitch
- \( (\_)_y \): in yaw

The fuselage is composed of three sections (elliptical nose, straight center-tube, and ogival tail) with the dimensional definitions shown in Figure 6.20.

![Fuselage geometry variables](image)

**Figure 6.20: Fuselage geometry variables**

6.10.1 Fuselage Drag

For the fuselage, the drag is calculated with the following equation:

\[
D = \frac{1}{2} \rho V^2 S_{\text{front}} C_D
\]  

(6.52)
The drag coefficient is a function of both the shape of the fuselage and the Reynolds number of the flow. To calculate the drag coefficient, the following model is used. First, it is assumed that the fuselage operates in a Reynolds-number regime dominated by turbulent flow, and thus the Reynolds dependency may be modeled using a scaling variable $\xi$. The factor of 2 is used to roughly account for cooling drag of the engine, interference drag of the wing and tail junctures with the fuselage, and typical “real-world” mechanisms and antennas that protrude from the fuselage.

\[
C_D = 2 \xi C_{D_{\text{ref}}} \tag{6.53}
\]

\[
\xi = \frac{c_{f_{\text{turb}}}(Re_r)}{c_{f_{\text{turb}}}(Re_{r_{\text{ref}}})} \tag{6.54}
\]

\[
\xi \equiv F(Re_r/Re_{r_{\text{ref}}}) \tag{6.55}
\]

To relate $C_{D_{\text{ref}}}$ to the fuselage shape, a series of CFD calculations were performed on a unit fuselage (all geometry nondimensionalized to a radius of 1) with a known reference unit Reynolds number. A sample CFD result is shown in Figure 6.21.

![Sample fuselage CFD plot](image)

Figure 6.21: Sample fuselage CFD plot

$C_{D_{\text{ref}}}$ data was generated for the following $\bar{I}$ ranges to populate a data-space to encapsulate a reasonable breadth of fuselage geometries.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{I}_n$</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>$\bar{I}_c$</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>$\bar{I}_t$</td>
<td>1</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 6.2: Ratio bounds for fuselage CFD

Using the standard fitting procedure, a surrogate model for $C_{D_{\text{ref}}}$ was then generated, which shows reasonably good agreement between CFD and surrogate-predicted drag coefficients, as shown in Figure 6.22.

\[
C_{D_{\text{ref}}} \approx F(\bar{I}_n, \bar{I}_c, \bar{I}_t) \tag{6.56}
\]
6.10.2 Fuselage pitch and yaw bending loads
The fuselage is assumed to react to the tail loads with four longerons: two for pitch and two for yaw, clocked at 90 degrees from each other. Figure 6.23 shows the bending load schematic for a conventional aircraft in bending.

Figure 6.22: GP modeling fitment for fuselage drag (fit data vs CFD data)
First, consider a fuselage that is clamped at the wing. The tail load is transferred by the fuselage, but the load is completely reacted by the clamping force, so the fuselage section ahead of the wing is not loaded. The loading case is shown notionally in blue.

In flight, the tail load is relieved by the inertial acceleration of the airframe in response to the control load (green), while the fuselage section ahead of the wing is also loaded due to acceleration. While the maximum bending moment is lower than the bending load in the clamped case, a conservative estimate may be obtained by assuming a maximum bending moment equal to the moment generated when the wing is clamped, and estimating that this load drops off linearly to either side of the wing to 0 at the tips (red line).

This root bending moment is calculated by multiplying the force generated by the tail at a lift coefficient of 1.0 by the distance between the wing and tail's aerodynamic centers. An estimate is also made that this pull-up maneuver occurs at double the cruise speed to provide maneuvering speed margin.

\[
M_p = \left(\frac{L}{2} + l_t - 0.75 c_{HT}\right) \left(2 \rho V_{cruise}^2 s_{HT} C_L\right)
\]

(6.57)

A simple beam-bending model is used to calculate the necessary thickness in both the pitch-pair and the yaw-pair of longerons.

The strength constraint of the longerons (in pitch) is calculated using:

\[
\sigma_{allowable} \geq \sigma = \frac{M_p r_{fuse}}{I_p}
\]

(6.58)

\[
I_p \leq 0.6667 \theta_{lon} r_{fuse} \left(t_{lonp}^2 + 3 r_p t_{lonp}^2 + 3 r_p^2 t_{lonp}\right)
\]

(6.59)

\[
t_{lonp} + r_p \leq r_{fuse}
\]

(6.60)
The bending stiffness constraint of the longerons is modeled as:

\[ \kappa = \frac{M_p}{I_p E} \leq \kappa_{\text{limit}} = \frac{2 \Delta}{l_t + \frac{1}{2} l_c} \]  

(6.61)

The linear fall-off assumption for the bending moment enables a simple formulation for the mass estimation of the longerons, which is equal to half the mass that would be generated by a tube of constant cross-section.

6.10.3 Fuselage roll loads

Roll forces on an airframe are complex due to a combination of inertial relief, vehicle gyroscopic forces, and aerodynamic damping as a function of component location, but a conservative estimate may be obtained by calculating the instantaneous torque generated by an abrupt aileron input, as shown in Figure 6.24.

This model for roll torque assumes a \( C_l = 1 \) for the aileron-section of the wing, which occupies the outboard half of both the right and left wings. To provide maneuvering speed margin, a speed twice the cruise speed is considered.

The torque produced at the wing root may be calculated as:

\[ T_{\text{max}} = 2 \rho V_{\text{cruise}}^2 S_A 0.375 b C_L \]  

(6.62)

\[ S_A = 0.5 \lambda^{0.2354} S \]  

(6.63)

Similar to the bending moment model, this torque may be considered to fall off to 0 at the nose and the tail of the aircraft. In reality, the root roll moment would be lower than this value, though the local torque load may be higher near the tail, as the tail is both a large inertial and aerodynamic-damping load.
Since the cross-section of the fuselage is a circular cylinder, the strength limit of the fuselage shell may then be calculated as:

$$\tau = \frac{T}{2 t_{shell} A_e} \leq \tau_{allowable}$$  \hspace{1cm} (6.64)

$$A_e = \pi r_{fuse}^2$$  \hspace{1cm} (6.65)

Like the wing model, the deflection may be calculated as:

$$\phi' = \frac{d\phi}{dy} = \frac{T}{G J} \leq \phi'_{max}$$  \hspace{1cm} (6.66)

$$\phi' = \frac{\phi}{\frac{l_t}{2} + l_t}$$  \hspace{1cm} (6.67)

For thin-shelled cylinders, the following equation may be used to predict the polar moment of inertia.

$$J = 2 \pi r_{fuse}^3 t_{shell}$$  \hspace{1cm} (6.68)

Due to the linear fall-off assumption, the structure may also be assumed to fall off linearly, which results in the simple formulation that the mass of the torsion shell is the surface area of the fuselage multiplied by half of the maximum thickness.

**6.10.4 Fuselage commonality constraints**

To enforce AAM-commonality on fuselages, the fuselage radius \( r_{fuse} \), the nose length \( l_n \), and the tail length \( l_t \) are held constant between variants. The length of the center-tube tooling is calculated using the longest variant of a family.
6.11 Propulsion Models

Nomenclature:

\( a \) acceleration
\( D \) drag
\( h_{\text{fuel}} \) fuel heating value
\( L \) length traveled
\( m \) mass
\( P_e \) engine shaft power
\( T \) thrust
\( V \) speed
\( V_{TO} \) takeoff speed
\( \eta_p \) propeller efficiency
\( \eta_e \) engine efficiency
\( \theta_p \) air density ratio at altitude
\( \mu \) climb margin

It is assumed that the aircraft uses a propeller/reciprocating-engine propulsion system that burns fuel as a direct function of shaft power output. The propulsive model is much simpler than the airframe model so that the broad strokes of engine sizing may be integrated into the optimization.

6.11.1 Takeoff propulsion model

The takeoff model is formulated assuming a constant-acceleration takeoff in which the energetic losses to drag and friction are negligible compared to the kinetic energy of the aircraft (i.e., \( D < 1 \) during takeoff).

The engine power is calculated at the end of the acceleration roll, which provides a conservative bound on the engine power for the aircraft.

Under these assumptions, the engine power may be calculated as:

\[
P_e \eta_p = T V_{TO}
\] (6.69)

Since drag losses are assumed to be negligible from an energy standpoint, the kinematics of the takeoff roll yield the following equations:

\[
T = m_{\text{aircraft}} a
\] (6.70)

\[
P_e \eta_p = \frac{1}{2} m_{\text{aircraft}} \frac{V_{\text{takeoff}}^3}{L_{\text{takeoff}}}
\] (6.71)

\[
V_{\text{takeoff}} \geq V_{\text{stall}}
\] (6.72)

During this takeoff run, the propeller efficiency is assumed to be fairly low to take into account the low advance-ratio operating regime.

\[
\eta_{\text{takeoff}} = 0.5
\] (6.73)
6.11.2 Cruise power model

To calculate the cruise engine power, the following relations are used. It is assumed that the engine power lapse rate with altitude is the same as the air density lapse rate, as this directly relates to the amount of oxygen available. This assumption is reasonably accurate for non-turbocharged and non-supercharged engines.

\[ P_{\text{cruise}} \eta_{\text{cruise}} \theta_\rho \geq D V_{\text{cruise}} \]  
(6.74)

The cruise propeller efficiency is assumed to be 85%.

\[ \eta_{\text{cruise}} = 0.85 \]  
(6.75)

An additional parameter for climb margin is used to ensure the engine has sufficient power for typical climbing and maneuvers.

\[ P_{\text{max}} \mu \geq P_{\text{cruise}} \]  
(6.76)

\[ \mu = 0.5 \]  
(6.77)

6.11.3 Cruise range model

The cruise range follows the GP-compatible formulation ((6.78) and (6.79)) presented by Hoburg, which models the Breguet range equation [17].

\[ z = g \frac{L_{\text{range}} T}{h_{\text{fuel}} \eta_0 W} = g \frac{L_{\text{range}}}{h_{\text{fuel}} \eta_0 \left( \frac{L}{D} \right)^{-1}} \]  
(6.78)

\[ \frac{W_{\text{fuel}}}{W_{zfw}} \geq z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} \]  
(6.79)

This formulation requires the estimation of engine thermodynamic efficiency \( \eta_{\text{engine}} \), which shall be considered to be a constant 30%. The total efficiency \( \eta_0 \) of the system is the product of the engine efficiency and the propeller efficiency, as shown in (6.80).

\[ \eta_0 = \eta_{\text{prop}} \eta_{\text{engine}} \]  
(6.80)

6.11.4 Engine mass model

The engine mass is calculated using the following empirical relation by Raymer. In this equation, power is measured in Watts and the engine weight is in Newtons [24].
6.12 Aircraft Balance

While it is not yet feasible to complete a full stability and control analysis within the GPkit architecture, a simple balance equation may be used assuming the geometry shown in Figure 6.26.

![Aircraft weight balance schematic](image)

To balance this configuration (or at least to ensure the aircraft is not tail-heavy), equation (6.82) is used which forces the sum of the moments contributed from elements in front of the aerodynamic center to be greater than those from behind.

\[
W_{\text{eng}} \cdot (l_n + 0.5 l_c) + W_n \cdot \left( \frac{l_c}{2} + \frac{l_n}{2} \right) \geq W_{HT} \cdot l_{HT} + W_{VT} \cdot l_{VT} + W_t \cdot \left( \frac{l_t}{2} + \frac{l_c}{2} \right)
\]  

(6.82)
6.13 Aircraft Optimization Code Example Outputs

The design code was executed for the baseline mission (Table 6.3), generating the design shown in Figure 6.27.

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payload mass</td>
<td>1000 kg</td>
</tr>
<tr>
<td>Payload volume</td>
<td>4 m$^3$</td>
</tr>
<tr>
<td>Takeoff length (fully loaded)</td>
<td>400 m</td>
</tr>
<tr>
<td>Flight altitude</td>
<td>5000 m</td>
</tr>
<tr>
<td>Flight range</td>
<td>2000 km</td>
</tr>
<tr>
<td>Cruise speed</td>
<td>60 m/s</td>
</tr>
<tr>
<td>Ultimate load factor</td>
<td>6</td>
</tr>
</tbody>
</table>

*Table 6.3: Baseline mission*

The results produced by the optimization code seem reasonable, with an $L/D$ in the low 30’s and high aspect ratios for both the wing and horizontal tail. The payload fraction is quite high, though this is reasonable considering the carbon fiber structural assumptions, the lack of fixed mass, and the relatively short range that is specified for the baseline mission (in comparison to other large UAS of comparable size).

Interestingly, the optimizer chooses a semi-spherical nose ($L_n = 1$), possibly due to the drag improvements with longer nosecone ratios not being worth the increase in additional weight and the system-wide repercussions it generates.
7 AAM EXPLORATIONS AND DISCUSSION

Nomenclature

- $\phi$: market bias factor
- $\bar{c}_a$: aircraft cost by mass ($100/\text{kg}$)
- $\bar{c}_f$: fuel cost by mass ($2.50/\text{kg}$)
- $C_F$: cost of fleet
- $n_v$: number of variant designs within fleet
- $n_m$: number of missions flown, per unit (1000)
- $N_i$: number of units of variant $i$ produced
- $m_{pay}$: payload mass
- $m_a$: aircraft empty mass
- $m_f$: fuel mass
- $M_0$: baseline mission
- $M_1$: perturbed mission 1
- $M_2$: perturbed mission 2
- $M_3$: perturbed mission 3
- $v_{pay}$: payload volume
- $\delta_0$: scope factor in ( )
- $V_{cruise}$: cruise speed
- $\Delta$: wing taper angle

( )^#: referring to mission # or aircraft that satisfies mission #

The goal of this work is to explore the penalties incurred by the adoption of the AAM architecture. To accomplish this, the procedure shown in Figure 7.1 is used.

![Figure 7.1: AAM performance evaluation procedure](image-url)

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80
By optimizing both the AAM family and the IO fleet for the same metric and comparing the performance of both sets of aircraft, the performance penalty arising from the AAM architecture is quantified.

This quantification methodology is used to interrogate the AAM family along several axes of interest. A sweep of parameters is called an “Exploration”.

7.1.1 Scope vs. performance

“Scope” is a concept referring to the breadth of the mission-set that must be satisfied by the variants of a family. It is a general rule in platform engineering that as the scope of the mission set is broadened, the family should become increasingly less efficient than a set of individually-optimal solutions. To control this concept numerically, the “scope factor” $S$ is used.

7.2 Payload Explorations

The first set of Explorations (numbered 1-4) focus on payload size. In these Explorations, both the payload mass and volume are scaled by the scope factor $S_{pay}$, and a family size ($n_v$) of 3 is considered.

The payload Explorations follow the procedure outlined below:

1. The baseline mission $M_0$ is perturbed by $S_{pay}$ to generate $M_1$, $M_2$, and $M_3$ which set the driving requirements for aircraft #1, #2, and #3, respectively.
   a. Mission $M_1$ is generated with $m_{pay1} = (m_{pay0}) \times S_{pay}^{-1}$
   b. Mission $M_1$ is generated with $v_{pay1} = (v_{pay0}) \times S_{pay}^{-1}$
   c. Mission $M_2$ is identical to $M_0$
   d. Mission $M_3$ is generated with $m_{pay3} = (m_{pay0}) \times S_{pay}$
   e. Mission $M_3$ is generated with $v_{pay3} = (v_{pay0}) \times S_{pay}$

For instance, Table 7.1 below shows a mission set that is generated by perturbing the payload mass and volume with $S_{pay} = 2$.

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Mission 1</th>
<th>Mission 2</th>
<th>Mission 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payload mass</td>
<td>500 kg</td>
<td>1000 kg</td>
<td>2000 kg</td>
</tr>
<tr>
<td>Payload volume</td>
<td>2 m$^3$</td>
<td>4 m$^3$</td>
<td>8 m$^3$</td>
</tr>
<tr>
<td>Takeoff length (fully loaded)</td>
<td>400 m</td>
<td>400 m</td>
<td>400 m</td>
</tr>
<tr>
<td>Flight altitude</td>
<td>5000 m</td>
<td>5000 m</td>
<td>5000 m</td>
</tr>
<tr>
<td>Flight range</td>
<td>2000 km</td>
<td>2000 km</td>
<td>2000 km</td>
</tr>
<tr>
<td>Cruise speed</td>
<td>60 m$/S$</td>
<td>60 m$/S$</td>
<td>60 m$/S$</td>
</tr>
<tr>
<td>Ultimate load factor</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

*Table 7.1: Perturbed mission set in payload mass*

If the scaling factor is set to 1, all missions of the set are identical and the members of the family are each identical to an individually-optimal aircraft.

2. As described in Chapter 4, the cost of the fleet is calculated and minimized as:

$$C_F = \sum_{i=1}^{n_v} N_i \left( \bar{c}_a \, m_{a_i} + n_m \, \bar{c}_f \, m_{f_i} \right)$$ (7.1)
a. The "market bias factor" $\delta$ is used in the same way as $\delta_{\text{pay}}$, but applied to the number of units of each variant that is produced (rather than the mission specifications that variant satisfies). For the purposes of this thesis, 3 variants are considered, and the market bias factor shifts system weighting to either mission 1 or mission 3.

$$N_1 = \delta^{-1}$$
$$N_2 = \delta^0 = 1$$
$$N_3 = \delta^{-1}$$

3. Two fleets (AAM) and (IO) are optimized under the models described in Chapter 6 for the fleet cost metric $C_F$.

4. The performance of these two fleets and their variants are then compared, so that the tradeoffs between the variants may be understood.

The payload explorations may be visualized in $\delta_{\text{pay}}$-$\delta$ space as shown in Figure 7.2.

![Payload Explorations map](image)

*Figure 7.2: Payload Explorations map*

The first Exploration sweeps $\delta_{\text{pay}}$ while keeping $\delta = 1$, which means that there is no skewing of weights between the different variants. Exploration 2 sweeps $\delta$ at $\delta_{\text{pay}} = 5$ to explore what happens to airframe geometry and efficiency as the market bias is shifted from one variant onto another. Exploration 3 sweeps $\delta_{\text{pay}}$ at an interesting $\delta$ point identified in Exploration 2. Exploration 4 revisits Exploration 1, but tweaks the AAM assumptions.
7.2.1 Exploration 1: Family performance vs. payload scope
In the first Exploration, the scope factor $S_{pay}$ is swept between 1 and 10 while the market bias factor $\mathcal{D}$ is fixed at 1. The family/fleet performance is summarized in Figure 7.3 below.

![Family performance ratios vs scope factor](image)

This figure shows that the performance of the AAM fleet (when measured as total cost, fuel mass, or airframe mass compared against the IO fleet) starts near 1 at $S_{pay} = 1$ and does not rise above 20%, even as the scope factor is raised to 10.

The data also reveals that the mold area ratio starts at around 33% at $S_{pay} = 1$. This is because the 3 AAM aircraft are reusing the same mold geometry whereas the IO aircraft are assumed to each have their own mold. What may not be intuitive is that the mold ratio increases as the scope is increased, meaning that the total tooling savings are projected to decrease as the breadth of missions is increased.

To understand this effect (and also to understand the tradeoffs between variants as $S_{pay}$ is increased), consider the following analysis. First, let us consider the trivial case where $S_{pay} = 1$. Because the scaling factor is 1, all three missions are identical, and we would expect that all aircraft within the same fleet are identical.

Indeed, as shown in Figure 7.4, this is the case. This figure shows two groups: AAM aircraft on the left and IO aircraft on the right, in order from #1 to #3, with both a top view (centered) and a side-view (upper) of each aircraft. This same format will be used for all subsequent aircraft geometry outputs.
As shown in Figure 7.4, the AAM #1, #2, and #3 aircraft are effectively identical, and these aircraft are nearly identical to IO #1, #2, and #3. As shown by the red lines, the AAM aircraft are marginally larger than the IO aircraft, due to the additional mass of joiners and slightly increased interference drag estimate.

At the $\delta_{pay} = 3$ case, the payload mass of aircraft #1 in each fleet is reduced to 333 kg, and the payload of aircraft #3 is increased to 3000 kg. As Figure 7.5 shows, the AAM aircraft share a common fuselage radius, and while the largest AAM vehicle is nearly the same size as the IO vehicle, AAM #1 and #2 are starting to become larger than their IO counterparts.
As $S_{pay}$ is increased to 5 (in Figure 7.6), we note that AAM #1 is becoming significantly larger than IO #1, while AAM #3 and IO #3 are still fairly close in geometry. This AAM-family solution is the optimum given the cost objective, of which AAM #3 is the largest contributor, leading to the architecture design swinging towards its optimum more-so than the other two.

![Figure 7.6: $S_{pay}$=5, $S_{pa}$=1: AAM and IO fleet comparison](image)

At $S_{pa} = 10$, this scaling factor reflects a 100-fold change in magnitude between the payload requirements of the small and large aircraft. As shown in Figure 7.7, AAM #1 is now drastically oversized, and has the same fuselage as AAM #2. AAM #3, by virtue of being the largest variant, dominates this cost-minimizing family design.
To track how the different variants evolve as mission scope is expanded, consider Figure 7.8. In this figure, the colored lines link the points, which represent the AAM aircraft (#1 to the left, #2 in the center, and #3 to the right). As the scope is increased, the mass ratio of AAM #1 increases dramatically.

Another way of visualizing this data is presented in Figure 7.9, which shows both the empty masses and fueled masses for AAM and IO #1 and #3 vs. their payloads. While IO #1’s airframe and fueled mass decrease monotonically as payload is decreased, an increasing scope factor $\delta$ causes the mass of AAM #1 to increase as $\delta_{pay} > 3$, to the point where it is about 5.5 times bigger than IO #1 at $\delta_{pay} = 10$. This growth of AAM #1’s weight is the result of the common fuselage radius and the weight and drag repercussions it generates. In contrast, AAM #3’s mass curve is very similar to that of the IO aircraft.

The decrease in tooling savings at large $\delta_{pay}$ is explained by the dominance of the large aircraft on tooling cost – there is proportionally less surface area to be saved on the smaller variants.
Empty Mass ratio

\[
\text{Empty Mass ratio} = \frac{m_{\text{Baseline}}}{m_{\text{Baseline}} + \Delta m}
\]

Figure 7.8: Empty mass ratio between AAM and IO vs scope

Figure 7.9: Aircraft mass vs payload carried (illustrations on left are to-scale)
7.2.2 Exploration 2: Family and variant Performance vs market bias

Exploration 2 focuses on the response of the variants to changes in the market bias factor $\delta$. A $\delta < 1.0$ shifts greater weight onto the performance of the smaller aircraft by increasing the value of $N_1$ and decreasing the value of $N_3$. By re-running the optimizations for a range of $\delta$, it becomes possible to see how performance is traded between variants.

A range of $\delta$ between 1.0 and 0.1 was used. $\delta_{pay}$ is held constant at 5 for this exploration. Because the IO aircraft share no constraints, $\delta$-scaling the mission count does not affect their optimum designs. Figure 7.10 below shows the geometries of the optimal AAM families at several $\delta$ values. As the market bias factor shifts the weighting towards AAM #1, we can see that AAM #3's fuselage must shrink significantly to accommodate the increased optimization weight on the performance of the #1 aircraft.

Figure 7.10: AAM optimal aircraft as $\delta$ is varied, vehicles are to-scale.

The inefficiencies generated in aircraft #2 and #3 may be tracked as the increased mass ratio (in comparison to IO #2 and #3). These ratios are presented in Figure 7.11.

Figure 7.11: Relative performance of AAM variants against IO vs $\delta$
Figure 7.11 presents both the fuel and empty-mass ratios between the AAM aircraft and their IO counterparts. This ratio effectively measures how far AAM #2 and #3 stray from their respective optimal designs; the higher the ratio, the farther the design is from the optimal IO solution.

As $\phi$ is decreased, both the structural mass and fuel mass of AAM #1 decreases. This makes sense, as the optimizer is shifting priority to AAM #1. However, what is interesting is that AAM #2 and AAM #3 do not suffer greatly due to this shift, at least until $\phi = 0.3$ or so. In fact, AAM #2 becomes closer to the optimal IO #2 design until $\phi = 0.2$. After this point, both AAM #2 and #3 diverge from their optima (with #3 suffering very aggressive inefficiency past this point, to the point where it becomes nearly 2x more massive than the IO aircraft that accomplishes the same mission).

This increase in the mass ratio of AAM #3 may be explained by revisiting Figure 7.10 in which the fuselage of AAM #3 is increasing in length while simultaneously decreasing in diameter. The fuselage structure is sized for bending and torsional strength and stiffness, and the increased fineness ratio increases both the loads and decreases the available structural cross-section that must respond to that load, thereby increasing the mass of the structure greatly.

It is also instructive to track how the mold area changes as $\phi$ is changed. Again, there is a precipitous decrease in molding efficiency at $\phi < 0.2$ due to the need for the fuselage tooling to extend in length aggressively to satisfy its volume requirements.

![Relative mold area vs $\phi$](image)

Figure 7.12: Relative mold area vs $\phi$

While it is tempting to look at the $\phi$-sweep and conclude that one should “tune” the relative weights between the variants to shift performance back towards the variants that are more heavily-taxed by inefficiencies generated by the platform architecture, it must be noted that the family-optimum only moves this way so long as the $N_i$ generated by $\phi$-scaling reflects reality. There is no sense in decreasing the performance of a larger aircraft by 50% in exchange for an increase of performance of 50% of a smaller aircraft, as that reflects a net loss for the family.
7.2.3 Exploration 3: Family performance vs scope and market bias

The third Exploration performs a sweep of $\delta_{pay}$ with $\delta = 0.2$ to see how the variants' scope-wise performance shift as a result of market bias. Figure 7.13 below presents the #1 and #3 aircraft empty mass vs. the mission payload for both the $\delta = 1$ (blue) and $\delta = 0.2$ (green) cases. The IO cases are shown in red, and do not change as a result of $\delta$.

![Figure 7.13: Aircraft empty mass vs payload carried, for $S$-sweeps and $\delta = 1$ and $0.2$](image)

As the figure shows, the use of the market bias factor shifts the performance of the AAM #1 variant closer to IO #1 and the performance of AAM #3 away from IO #3.
7.2.4 Exploration 4: Common wings and tails but unique fuselages

As identified in the graphical outputs of Explorations 1-3, it appears that the fuselage's commonality is the primary mechanism that puts the performance of AAM #1 and #3 at odds with each other. In contrast, the wing planform shape reached by the optimizer is extremely consistent over these optimizations. Ergo, a fourth Exploration was completed, varying $\delta_{\text{pay}}$ while keeping $\delta = 1$, but with a partial- AAM implementation such that only the wings and tails are common, but the fuselages are permitted to be unique to each variant.

Running the design algorithm for this partial-AAM architecture yields a family that is nearly identical in design and performance to the IO family, as shown in the center block of Figure 7.14. Due to the removal of the fuselage constraint, AAM #1 is able to be nearly the same size as the IO aircraft.

![Figure 7.14: Full and partial-AAM and IO aircraft for $\delta_{\text{pay}} = 5$, $\delta = 1$](image)
As the mass trend in Figure 7.15 shows, there is a drastic improvement in mass-efficiency for AAM #1. However, with the removal of the shared-tooling reduction in molding surface area, this partial-AAM would not achieve the same degree of tooling savings as the full-AAM implementation, as shown in Figure 7.16.

![Empty mass of #1 and #3 variants vs payload carried](image)

Figure 7.15: Empty mass of variants #1 and #3 vs payload carried for full and partial AAM
Whether the 10% cost reduction in manufacturing and service cost of these aircraft is worth the 10% increase in tooling cost is dependent on the relative expense and time-criticality of the manufacturing, which are beyond the scope of this study. However, it is important to note that these trade-offs exist, both at the level of family scope design but also in the details of how an architecture is implemented.
7.3 Cruise Speed Explorations

The first set of Explorations was conducted to investigate how AAM’s inefficiencies scale with payload size (and therefore fuselage shape and size). The Speed Exploration described in this section varies the cruise speed requirement to generate families that must span across a growing breadth of flight speeds, which forces the variants to trade and co-optimize a wing taper ratio that must compromise between low- and high-speed flight.

7.3.1 Cruise speed exploration setup

The scaling factor is applied somewhat differently in this Exploration because reducing the cruise speed by a large factor is unrealistic in real-world missions. Instead, the mission speed requirements are calculated with equations (7.2) through (7.4). Mission 1 is therefore always equal to the baseline, whereas Mission 2’s speed requirement is simply the average of the baseline and scaled speeds.

\[ V_{\text{cruise}1} = V_{\text{cruise}0} \]  
\[ V_{\text{cruise}3} = S_{\text{speed}} V_0 \]  
\[ V_{\text{cruise}2} = \frac{1}{2} (S_{\text{speed}} V_{\text{cruise}1} + V_{\text{cruise}3}) \]  

Therefore, for example, at a \( S_{\text{speed}} = 3 \), the perturbed missions are shown in Table 7.2.

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Mission 1</th>
<th>Mission 2</th>
<th>Mission 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payload mass</td>
<td>1000 kg</td>
<td>1000 kg</td>
<td>1000 kg</td>
</tr>
<tr>
<td>Payload volume</td>
<td>4 m³</td>
<td>4 m³</td>
<td>4 m³</td>
</tr>
<tr>
<td>Takeoff length (fully loaded)</td>
<td>400 m</td>
<td>400 m</td>
<td>400 m</td>
</tr>
<tr>
<td>Flight altitude</td>
<td>5000 m</td>
<td>5000 m</td>
<td>5000 m</td>
</tr>
<tr>
<td>Flight range</td>
<td>2000 km</td>
<td>2000 km</td>
<td>2000 km</td>
</tr>
<tr>
<td>Cruise speed</td>
<td>60 m/s</td>
<td>120 m/s</td>
<td>180 m/s</td>
</tr>
<tr>
<td>Ultimate load factor</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

*Table 7.2: Speed perturbation example*

It must be noted that while the structural relations hold, this analysis at high speeds becomes dubious due to transonic effects which have not been captured inside the AAM optimization code. Therefore, a maximum of \( S_{\text{speed}} = 5 \) is used.
7.3.2 Exploration 5: Cruise speed

As $S_{\text{speed}}$ is varied from 1 to 5, the performance ratios of AAM to IO fleets are shown in Figure 7.17. Both the total tooling area and performance ratios are relatively insensitive over this speed range, and the total cost is only around 10% while the tooling in the AAM family is less than half that of the IO fleet.

![Figure 7.17: Fleet performance ratios between AAM and IO for speed-varying families](image)

Figure 7.17: Fleet performance ratios between AAM and IO for speed-varying families

Figure 7.18 shows the vehicle designs for both the AAM family and the IO fleet at an $S_{\text{speed}} = 5$. The taper angles of the IO aircraft vary between 0.0735 and 0.226 while all AAM aircraft share the same 0.167 taper.

![Figure 7.18: Result of cruise-speed-varying mission set with $S_{\text{speed}} = 5$](image)

Figure 7.18: Result of cruise-speed-varying mission set with $S_{\text{speed}} = 5$
It is highly instructive to take a closer look at the geometric and performance differences between AAM #1 and IO #1 for $\delta_{\text{speed}} = 5$ as shown in Table 7.3.

<table>
<thead>
<tr>
<th></th>
<th>AAM #1</th>
<th>IO #1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing taper angle</td>
<td>0.167</td>
<td>0.0735</td>
</tr>
<tr>
<td>Empty mass (kg)</td>
<td>374.1</td>
<td>360.1</td>
</tr>
<tr>
<td>Wingspan (m)</td>
<td>17.15</td>
<td>21.29</td>
</tr>
<tr>
<td>Airfoil thickness ratio</td>
<td>0.11</td>
<td>0.20</td>
</tr>
<tr>
<td>Root chord (m)</td>
<td>1.59</td>
<td>0.94</td>
</tr>
<tr>
<td>Root section thickness (m)</td>
<td>0.175</td>
<td>0.188</td>
</tr>
<tr>
<td>Cruise L/D</td>
<td>28.18</td>
<td>35.32</td>
</tr>
<tr>
<td>Fuel required for mission (kg)</td>
<td>84.03</td>
<td>65.96</td>
</tr>
</tbody>
</table>

Table 7.3: Comparison of vehicle specs for AAM #1 and IO #1 for speed-varying families of $\delta_{\text{speed}} = 5$

Despite the wing taper angle being extremely different between the two aircraft (2.27 ×), the empty mass and aerodynamic performance penalties are not enormous in this context. The AAM #1 aircraft is heavier by a factor of 1.04 and consumes more fuel by a factor of 1.27, but these are inefficiency ratios significantly lower than a 5-fold speed scope and a taper angle ratio of 2.27 might first suggest.

The underlying mechanism is that a higher taper angle improves the structural efficiency of the wing while reducing its aerodynamic efficiency due to the decrease in aspect ratio. However, the increased taper angle also permits the airfoil thickness ratio to be reduced for the same wing strength and stiffness (as shown in Table 7.3), which dampens the drag penalty from the reduced wingspan. In total, the gradient between wing taper angle and aircraft performance appears to be shallow, and it is possible to vary the taper angle without incurring large performance penalties.

By plotting the taper angles, both from the IO and AAM aircraft in Figure 7.19, we see that the AAM taper angle of the average ($M_2$) aircraft approximately aligns with the optimal taper angle. This fact, in combination with the flat performance ratio curve, supports the concept of a shallow and benign trade-off curve between wing efficiency and taper angle.
Wing taper angle vs flight speed range

Figure 7.19: Taper angle comparison between AAM and IO fleets
8 CONCLUSION AND RECOMMENDATIONS

8.1 Conclusion

The cost and time required to manufacture aircraft has grown strongly over time. To mitigate these problems, designers are increasingly investing in platform designs, which reuse system components to decrease costs and enable more rapid development times. Prior work in this field focused on commonality and uniqueness of whole airframe components (wings, fuselage segments, tails, etc.).

Adaptable Aircraft Manufacturing (AAM) is the name of an architecture for composite aircraft that leverages the natural compliance of conventional composite layups to give designers continuous-adjustability in several degrees of freedom during manufacturing. The result is the ability for a single set of tooling to generate multiple aircraft, thereby eliminating the spool-up costs and time required to respond to new missions.

A proof-of-concept project ("FAST") was undertaken in which three physical aircraft were designed, manufactured, and flight-tested. The vehicles span a 4.5-fold ratio in weight, and their performance was found to be reasonable. This process also highlighted the difficulties of quantifying the performance penalty incurred by the adoption of the architecture, which motivated a deeper, optimization-based investigation.

To evaluate the performance penalties incurred by the adoption of AAM, an optimized AAM family must be compared to a fleet of Individually-Optimal (IO) aircraft. To accomplish this optimization, a process sequence framework was defined which clarifies the points of adjustability in the manufacturing-to-mission chain. Geometric and signomial programming was selected for the solution method, and numerous GP and SP-compatible models were developed.

These efforts culminated in a series of Explorations that quantify the penalties of the AAM architecture in various scenarios of mission scope, market bias, and selection of commonality. The results from the Explorations show that the fuselage-stretching component of the architecture is very costly to the smaller aircraft in the fleet when payload is scaled, but that wing scaling is very efficient – both for payload-driven families and for speed-driven families.

In addition, the results of the optimization suggest that the sensitivity of performance to wing taper angle is shallow, which enables the tapered, adjustable-layup wing strategy to cover a wide variety of flight cases without incurring a large penalty.

8.2 AAM Usage Recommendations

The most useful application of AAM is likely the development of low-volume, time-critical aircraft such as prototypes, sensor platforms, and technology demonstrators. In these situations, the performance penalties incurred by AAM are likely a worthwhile trade for the elimination of tooling spool-up.

By contrast, in high-volume, high-mission-life, and performance-sensitive cases such as airliners, the penalties generated by AAM may not be tolerable.

8.3 Future Work Recommendations

There are numerous simplifications and omissions in the aircraft modeling effort. Increasing the fidelity of these models would enable more confident explorations of the performance of AAM in more mission cases.
Firstly, transonic effects on wing aerodynamics were neglected for this model. Transonic airfoil performance, sweep modeling, and flutter predictions would be necessary to explore these trades in the transonic regime.

Secondly, NACA 24xx (wings) and NACA 00xx (tails) airfoils were assumed, therefore only capturing the effects of thickness on drag and structural properties. A model with freedom in both camber and thickness would illuminate not only the aerodynamic vs. structural trade but also the high vs. low speed performance trade. In addition, these airfoils are assumed to be constant over the entire span, which is rarely done in today’s aircraft due to the efficiency improvements that may be achieved with airfoils that vary along the span of the wing.

Third, the fits for both wing and fuselage drag have an error distribution. It would be instructive to perform sensitivity analyses on the effect of the GP-compatible fitment error.

Fourth, the current cost model (breaking down the cost to airframe cost and fuel cost) is extremely simplistic. Greater detail in the cost model would permit better-informed trades for real-life applications.

Fifth, there are multiple components on aircraft that cannot scale between variants – such as engines, propellers, cockpits, and landing gear. It would be interesting to investigate the continuous-freedom that AAM provides the airframe along with the discrete choice freedom that characterizes these components.

Sixth, the details of propulsion performance were not explored here – trades involving propeller diameter, static thrust and cruise efficiency, maximum rotational speed, engine size, and engine efficiency were not modeled in detail. Improvements in these models would allow a more informed trade regarding the aircraft’s propulsion performance.

At a system trade-off level, additional work should be done to characterize the tooling and manufacturing process cost. In the analysis above, the tooling area is the surrogate for the tooling cost, and it is a pure output rather than part of the objective function. This was done deliberately to isolate the effects of AAM constraints on vehicle geometry and remove the uncertainties associated with estimating tool material and manufacturing costs. In real applications, however, the costs associated with these activities play a large part and should be included. With the inclusion of some estimate for tooling and production cost, it would then become possible for the model to determine the extents of scope that a family should cover, and at which point the tooling savings of a variant is overtaken by the performance penalties generated by commonality.

As these models are increased in fidelity, their breadth of applicability is necessarily narrowed. Therefore, it may be best to specialize the investigation onto real-world family problems such as a family of airliners, general-aviation aircraft, or a series of military UAS.
9 WORKS CITED


10 APPENDIX

This appendix presents the posynomial fits \((y = \mathcal{F}(x_1, x_2, \ldots))\) used in the modeling effort.

10.1 Atmospheric properties

- \(h\) altitude
- \(\theta\) atmospheric property ratio
- \(\theta_a\) sonic speed ratio
- \(\theta_p\) pressure ratio
- \(\theta_T\) temperature ratio
- \(\theta_v\) kinematic viscosity ratio
- \(\theta_d\) density ratio
- \(\xi\) altitude ratio

An altitude ratio is defined as:

\[
\xi = \frac{h}{10,000 \, \text{m}}
\]

All ratios \((\theta)\) are defined as the property (at altitude) divided by the property at ground level. For example, for density,

\[
\rho(h) = \rho_{SL} \cdot \theta_{\rho}(\xi)
\]

The following ratios are then fitted to the altitude ratio:

**Air density:**

\[
\theta_{\rho} = 0.003364784744960794 
= 0.00357028923540129 \cdot \xi^{1.114460214272049} 
+ 1.000092378763481 \cdot \xi^{1.720053062154687 \times 10^{-5}}
\]

**Kinematic viscosity:**

\[
\theta_v = 0.004902805241063856 
= 1.000091089167459 \cdot \xi^{1.667878025264792 \times 10^{-5}} 
+ 0.004244975861412132 \cdot \xi^{1.101954255212324}
\]

**Temperature:**

\[
\theta_T = 0.007055130159177537 
= 0.001734268132687297 \cdot \xi^{1.133745954218059} 
+ 1.000062780072123 \cdot \xi^{1.205086670238576 \times 10^{-5}}
\]

**Pressure:**

\[
\theta_p = 0.003178611896660296 
= 0.004155532994364745 \cdot \xi^{1.118578717325016}
\]
1.000116123678683 \cdot \xi^{2.17849375759152}

Speed of sound:

\[\theta_a = 0.01404083091281109 = 0.001725728898034604 \cdot \xi^{1.133741305114455} + 1.000062469199736 \cdot \xi^{1.199118504898769e-05}\]

10.2 Span efficiency

e_{CDI} \quad \text{span efficiency}

\lambda \quad \text{taper ratio, tip over root}

AR \quad \text{aspect ratio}

Span efficiency is calculated as a function of aspect ratio and taper ratio:

\[e_{CDI}^{1.02583551736847673} \geq 0.000347181847334128 \cdot AR^{1.986445677275891} \cdot \lambda^{-1.352261863161144} + 1.0444397383532572 \cdot AR^{-0.2201203645519972} \cdot \lambda^{0.2201203645519972} + 0.004481514943241946 \cdot AR^{2.502762266336954} \cdot \lambda^{5.061781313263068}\]

10.3 Airfoil drag coefficient

\(C_d\) \quad \text{airfoil drag coefficient}

\(C_l\) \quad \text{airfoil lift coefficient}

\(Re\) \quad \text{chord Reynolds number}

\(\bar{t}\) \quad \text{airfoil thickness ratio}

The airfoil drag model is developed by Hoburg [17]:

\[1 \geq 2.56 \cdot C_l^{5.88} \cdot Re^{-1.54} \cdot \bar{t}^{-3.32} \cdot C_d^{2.62} + 3.8E - 9 \cdot C_l^{-0.92} \cdot Re^{-1.38} \cdot \bar{t}^{6.23} \cdot C_d^{-0.57} + 0.0022 \cdot C_l^{-0.01} \cdot Re^{0.14} \cdot \bar{t}^{0.033} \cdot C_d^{-0.73} + 1.19E + 4 \cdot C_l^{0.78} \cdot Re^{-1} \cdot \bar{t}^{1.76} \cdot C_d^{-0.91} + 6.14E - 6 \cdot C_l^{5.53} \cdot Re^{-0.99} \cdot \bar{t}^{-0.52} \cdot C_d^{-5.19}\]
10.4 Wing planform root bending moment scaling factor

$k_M$ wing bending moment scaling factor

$\lambda$ taper ratio, tip over root

$AR$ aspect ratio

\[
k_M \geq 0.001212111669266 + 0.879494775656048 \times AR - 0.08219811680517 \times \lambda + 0.034680663622367 + 0.1721954085219 + 0.0118373645000936 \times AR - 0.000134475375186788 + 0.155430097289671 \times AR^{0.293978714734703} + 0.4161432548755362
\]

10.5 Wing planform distributed load factor

$M(y)$ bending moment at a station

$M_{root}$ bending moment at root

$\gamma_{ND}$ nondimensionalized spanwise coordinate (0 at tip, 1 at root)

\[
\left( \frac{M(y)}{M_{root}} \right)^{15.9401974758838} \geq
\]

\[
21.8073391359745 \times AR^{-0.518062950807365} \times \lambda + 0.2078794482594976 \times \gamma_{ND}(y) + 0.2656316472380573 \times AR^{1.68073787263664} \times \lambda + 0.947973742476176 \times \gamma_{ND}(y) + 422.3620249793271 \times AR^{-5.987905039530678} \times \lambda + 0.0932476515370553 \times \gamma_{ND}(y)
\]

10.6 Wing pitching moment coefficient

$\bar{c}$ camber ratio, = 0.02 for assumed NACA 24xx

$C_m$ airfoil pitching coefficient

$Re$ chord Reynolds number

$\bar{t}$ airfoil thickness ratio

\[
C_m^{0.1193640741696259} \geq
\]

\[
0.6666763820202701 \times \bar{c}^{0.08006736682465984} \times \bar{t} - 0.1031703746910964 \times Re^{-0.1261378845415252} + 0.5404230684497912 \times \bar{c}^{-0.03089254067108933} \times \bar{t} - 0.2580824098445388 \times Re^{-0.1094212763992742} + 0.7317004748067376 \times \bar{c}^{0.08557855172658224} \times \bar{t} - 0.9275456073762085 \times Re^{-0.1301131584364669} + 0.194614253762333 \times \bar{c}^{0.1880908730958243} \times \bar{t} - 0.05000258090660886 \times Re^{0.0542444960687792}
\]

10.7 Wing section polar moment of inertia

$J_{eff}$ polar moment of inertia normalized effectiveness

$\bar{t}$ airfoil thickness ratio

$\bar{t}_{box}$ torsional box wall thickness
$f_{\text{eff}}^{0.1489413598026966} \geq$

$0.7085536993547519 \cdot \xi^{-0.0750844170453722} \cdot t_{\text{box}}^{-0.1471318184315939}$

$+ 0.707928846840818 \cdot \xi^{-0.07707472581481598} \cdot t_{\text{box}}^{-0.1448022256406551}$

### 10.8 Lift-curve slope

$\frac{dC_l}{d\alpha}$ wing/ tail lift-curve slope

$\lambda$ taper ratio, tip over root

$AR$ aspect ratio

$$\frac{dC_l}{d\alpha} \leq$$

$0.1810662337002523 \cdot AR^{-0.01912025539878591} \cdot \lambda^{-0.003971038896335333}$

$+ 0.5090888195372596 \cdot AR^{-1.163113846477295} \cdot \lambda^{-0.04689211420147769}$

### 10.9 Fuselage drag coefficient

$C_D$ 3D drag coefficient

$C_{D\text{ref}}$ reference 3D drag coefficient

$\bar{l}$ length, normalized to radius

$Re_r$ fuselage radius Reynolds number

$Re_{\text{ref}}$ reference Reynolds number used in CFD

( ) $n$ ( ) of nosecone

( ) $c$ ( ) of centertube

( ) $t$ ( ) of tailcone

Reference drag coefficient as a function of fuselage geometry:

$$C_{D\text{ref}}^{0.9961940970551306} \geq$$

$0.002550798237812972 \cdot \bar{l}_{n}^{0.03170748105643763} \cdot \bar{l}_{c}^{0.585581954886895} \cdot \bar{l}_{t}^{0.153988593635604}$

$+ 0.002431879649743918 \cdot \bar{l}_{n}^{0.03360622160828439} \cdot \bar{l}_{c}^{0.216595202387439} \cdot \bar{l}_{t}^{0.3062428891322153}$

$+ 0.00970624668763379 \cdot \bar{l}_{n}^{0.8660726546410055} \cdot \bar{l}_{c}^{0.20991461335030463} \cdot \bar{l}_{t}^{0.156159772797256}$

$+ 0.04360464367264542 \cdot \bar{l}_{n}^{0.0546537461813091} \cdot \bar{l}_{c}^{0.2582157325901689} \cdot \bar{l}_{t}^{1.426680237054239}$

Fuselage drag scaling relations:

$$\xi = \frac{c_{f\text{turb}}(Re_r)}{c_{f\text{turb}}(Re_{\text{ref}})}$$
$$Re_{rat} = \frac{Re_r}{Re_{ref}}$$

\[ \xi^{0.05908348225025092} \geq 0.9811418822487932 \times Re_{rat}^{-0.006332827350850367} + 0.01885339101015998 \times Re_{rat}^{-0.180870990839732} \]