Backroom space allocation in retail stores
by
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Abstract
Space is one of the most scarce, expensive, and difficult to manage resources in urban retail establishments. A typical retail space broadly consists of two areas, the customer facing frontroom area and the backroom area, which is used for inventory storage and other support activities. While frontrooms have received considerable amount of attention from both academics and practitioners, backrooms are an often neglected area of retail space management and design.

However, the allocation of space to the backroom and its management impact multiple operational aspects of retail establishments. These include in-store labor utilization, delivery schedules, product packaging, and inventory management. Therefore, the backroom area directly affects the performance of the store because it impacts stock-outs, customer service levels, and labor productivity. Moreover, extant literature suggests that backroom related operations contribute to a large fraction of the total retail supply chain costs. Thus, optimizing the management of backroom spaces is an important lever for store performance improvement.

We address the gap in the extant literature related to space management of retail backrooms by investigating the following three questions: First, what is the effect of pack size on inventory levels and space needs in the backroom? Second, how can a given backroom space be efficiently utilized through optimal inventory control? Third, what is the optimal amount of space that should be allocated to the backroom in a given retail establishment?

To address the first question, we evaluate the effect of two discrete pack sizes, order pack size (OPS) and storable pack size (SPS), on inventory levels and storage space requirements in the backrooms. While SPS drives the space needs for a given inventory level, OPS drives the amount of excess inventory and therefore, the space needs. Using inventory theory and probability theory, we quantify the amount of excess inventory and the expected stock-out probability for a given OPS in the case of a normally distributed demand.

To address the second question, we discuss an inventory-theoretic approach to efficiently manage a given backroom space within a limited service restaurant. Specifically, we formulate a mathematical optimization model using mixed-integer linear programing with the objective of maximizing store profit. Applying this optimization model to real store data in collaboration with a major US retailer reveals cost implications related to constrained backroom space and the sensitivity of backroom space requirements to changes in OPS and SPS. The proposed model can serve as a decision support tool for various real-world use cases. For instance, the tool can help the retailers to identify (i) items whose contribution to the store profit does not justify their space needs in the backroom, and (ii) stores that are constrained in their profitability growth by backroom space limitations.
To address the third question, we introduce the notion of interdependency between the frontroom and the backroom of a retail establishment. Such interdependencies yield non-trivial trade-offs inherent to the optimal retail space allocation. Demand can be lost due to unavailability of inventory (or inventory stock-out), which is a result of scarce amount of backroom space, or due to unavailability of sufficient frontroom space (or space stock-out). Furthermore, constrained backroom spaces increase in-store labor cost and the ordering costs incurred per unit of revenue generated in a retail establishment. The strategic decision model formulated in this chapter accounts for revenue, inventory cost, labor cost and ordering cost to determine the optimal amount of backroom space that should be allocated within a retail establishment. Sensitivity analyses with respect to the change in input parameters is used to connect the backroom space allocation and its impact on store profit to the different supply chain levers that can be managed by the retailers.

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Chapter 1

Introduction

1.1 Research motivation

Space is one of the most difficult resources to manage in urban retail establishments due to heavily constrained space availability, high real estate cost, and strong competition by other commercial and private uses. Moreover, emerging retail trends such as omni-channel business models along with increasing competition in the industry requires even greater emphasis on operational efficiency (Hübner et al., 2013). Operational efficiency pertains to activities that improve store performance, such as reduced costs or higher customer service levels (Lee et al., 1997). Therefore, increasing operational efficiency in a store entails an optimized use of limited resources, such as retail space (Pires et al., 2017).

A typical retail space consists of two primary areas, the frontroom and the backroom. The frontroom is the customer facing area, where activities related to customer-staff interaction occur. The backroom is the non-customer-facing part of a retail establishment, used for receiving goods, storing inventory, staging and preparing products along with the staff working area and storage of maintenance equipment such as cleaning supplies or fixtures.

In this thesis, we will focus on the backroom area, which uniquely connects the upstream and the downstream functions of the retail supply chain. Items that are delivered to a store from distribution centers (DCs) or suppliers are stored in the backroom before they reach the customers by transitioning through the frontroom. In the case of grocery retail establishments, backrooms are used for storage of any inventory that cannot be accommodated in the limited frontroom shelf space. This is typically referred to as “overflow inventory” (Eroglu et al., 2013). In restaurant retail establishments, the backroom functions as the sole initial
storage area for inventory that is delivered to the store. In either case, however, growing product assortment and services along with promotional strategies have increasingly made backroom storage space necessary for sustained retail growth.

The backroom space is linked to operations at different echelons of the retail supply chain. First, at the store level, backroom space affects inventory planning (e.g., optimal end-item assortment and optimal inventory levels are affected by backroom capacity), labor productivity and allocation (e.g., smaller backrooms are more likely to be congested areas, reducing labor productivity) and operations planning decisions (e.g., time allocated for reviewing current inventory levels and placing orders is affected by extent to which backrooms are well-managed). Second, at the level of local distribution network that replenishes the store, backroom space allocation and management is connected to capacity planning (i.e., defining the quantity and positioning of the inventory) at the DCs, delivery frequency (i.e., smaller backroom spaces implies more frequent replenishments), minimum order quantities and delivery packaging or order pack size of the items (e.g., large pack sizes correspond to excess inventory, which has larger space implications in the backroom). Third, at the national network level, strategic decisions related to the network configuration, supplier selection and strategies for new market opportunities (e.g., omni-channel distribution) are tied to the backroom space in a store.

Because the backroom is a transition point between the upstream supply chain and downstream customers, it is a crucial determinant of store performance. For example, the out-of-stock levels observed at a store have been found to be a function of backroom size as well as the total store size (Milicevic and Grubor, 2015). And because a large proportion of retail supply chain costs are caused by in-store operations, which primarily occur in the backroom (Hübner et al., 2013; Sternbeck and Kuhn, 2014), this space and its management directly affect store profit.

Despite its potential financial impact and its importance as a critical link in the retail supply network, backroom space allocation and management is an often neglected area in both academic research and industrial practice (Das and Caplice, 2017; Pires et al., 2017). This motivates our interest in research about the allocation and efficient use of backroom space in a retail establishment for improved store profitability.
1.2 Research questions

The overarching research question of this thesis is: What is the optimal backroom space allocation within a retail establishment? This is depicted in Figure 1-1. We focus on Limited Service Restaurants (LSRs). LSRs, or quick service restaurants, are characterized by a relatively limited menu, reduced and fast service, pre-payment by customers, and higher customer turns than a full-service restaurant (Ottenbacher and Harrington, 2009; Robson, 2013). The main peculiarity of LSRs, as compared to grocery stores, is that many of the Stock Keeping Units (SKUs) that go into the backroom have to be assembled into end-items that are sold to the customer. This attribute complicates both cost accounting and inventory management. Moreover, labor in a LSR is typically shared between the backroom and the frontroom areas, as employees constantly interact with the consumers and prepare their orders.

![Figure 1-1: The functional areas in a retail establishment of a LSR](image)

In order to answer the overarching research question it is important to consider the possible interdependencies between the frontroom and the backroom. We note that in our research, the area in LSRs that is used for preparation of the end-items is considered to be a part of the frontroom. Frontroom and backroom spaces are closely linked to each other but contribute towards different aspects of store performance. While the demand generated is correlated with the size of the frontroom, the backroom carries the inventory required to support the fulfillment of that demand. Additionally, store support operations in the backroom also affect demand fulfillment. For example, labor time required for backroom space related activities, such as searching, picking and preparing the item, impacts fulfillment of customer demand in LSRs. Thus, while the frontroom and the backroom both affect revenue, the latter has additional implications on the store cost. These include the labor cost, fixed ordering cost, and inventory cost.
Backroom space allocation is a strategic decision. It is helpful to model stores on an aggregate space level i.e. frontroom and backroom, instead of on a more detailed SKU level. We expect the labor cost incurred for a certain amount of revenue to increase with smaller backroom space, as it is more likely to be congested and mismanaged. Ordering cost is also expected to increase with smaller backroom spaces because of higher replenishment frequencies required to generate a certain amount of revenue. On the other hand, inventory costs are related to the SKUs used by the store and the demand observed. Therefore, it is important to investigate the inventory costs related to the SKUs that flow through a given backroom space. After a strategic allocation of space to the backroom, understanding the costs related to the inventory in it also enables the retailer to optimally utilize the given backroom space.

To this end, we start by examining a simpler system in which we externalize the effect of the frontroom and concentrate on the backroom space alone. We assume a stationary demand distribution that is not affected by the frontroom space and investigate the efficient utilization of a given backroom space. This can be achieved by determining optimal inventory levels for the SKUs that are stored in the backroom space. Since retail establishments function in a multiple SKU and a constrained space environment, there are four challenges associated with determining optimal inventory levels for a given amount of backroom space.

First, these SKUs could have different inventory replenishment cycles or review periods. Second, they could be perishable items, which are combined with other SKUs in the store to prepare the end-item and are therefore subject to waste. Since the quantity wasted has a cost as well as a space implication, they should be accounted for in the backroom space utilization problem. Third, in a LSR there is a distinction between an end-item and the SKU. This requires us to translate end-item properties like the demand and the service level to equivalent measures on the SKU level. Fourth, these SKUs are delivered in different pack sizes that affect backroom space requirements, inventory levels, and costs. Additionally, there are two different pack size types for the backroom space utilization problem: packaging in which the SKUs are ordered (order pack size) and the intra pack size in which they are stored (storable pack size). For example, croissants come in order pack size of 36 units with 4 intra packs of 9 units each.

While traditional inventory theory can be used to address most of the above-mentioned complications, it does not account for multiple types of pack sizes. Therefore, it is imper-
ative to explore the effect of the two levels of pack size on backroom space requirements. We determine the pack size effect by focusing on a single SKU level. The results of this investigation are then extended towards a multiple-SKU setting for the backroom inventory control model.

In summary, this thesis addresses and investigates three related research questions:

1. What is the effect of pack size on inventory levels and space needs in the backroom?
2. How can a given backroom space be efficiently utilized through optimal inventory control?
3. What is the optimal amount of space that should be allocated to the backroom in a given retail establishment?

1.3 Research gap and contributions

1.3.1 Research gap

This research is motivated by the desire of the retail industry to optimize space and to leverage the current space for new business models. However, existing literature suggests that backroom space allocation and management have traditionally been ignored in supply chain planning and design decisions (Das and Caplice, 2017; Eroglu et al., 2013; Pires et al., 2017). This gives rise to the primary research gap that is addressed by this thesis.

Most of the extant literature on retail space management is concentrated on frontroom space (Corstjens and Doyle, 1981; Urban, 1998) and the associated research is done in the context of grocery retail stores. Since there are peculiarities of LSRs, which are also related to backroom space allocation and management, the existing literature on grocery retail stores is not directly transferable. For instance, existing literature focuses on determining optimal allocation of frontroom display space to items in a grocery retail store. However, the “displayed units” concept is not applicable to most of the items sold in a restaurant.

Additionally, to the best of our knowledge, the extant literature related to improving restaurant performance is focused on maximizing revenue by manipulating price or meal duration. This field is referred to as Restaurant Revenue Management or RRM (Kimes et al., 1999). RRM, however, does not consider the effect of the backroom space and its related costs, or its interdependency with the frontroom space. Finally, traditional inventory theory
does not consider the effect of pack sizes that drive inventory costs and space requirements in the backroom, which is also a research gap that we address.

1.3.2 Research contributions

The work presented in this thesis contributes to three different areas of literature: inventory management, retail space management, and restaurant performance management, in the following ways.

First, it contributes to inventory management literature by (i) incorporating the effect of pack sizes, (ii) distinguishing inventory levels that affect cost and space, (iii) accounting for inventory related key distinctions associated with a backroom of a LSR, and (iv) extending existing closed-form expressions for inventory policy related quantities such as on-hand inventory and order quantity to be applicable to a space-constrained LSR environment.

As mentioned earlier, traditional inventory theory models do not account for the effect of pack sizes. The failure to take into account this effect renders the recommendations derived from traditional methods inaccurate in real-world scenarios. There are two levels of pack size which are pertinent to our research: order pack size and storable pack size. To capture their effect on backroom space design and management, an approximation factor to adjust the inventory levels as a function of these two pack sizes is proposed. Approximations of the pack size effect are then used for tactical decisions on backroom space utilization.

Our research differentiates two inventory levels for the purpose of modeling cost and space requirements: average inventory, which drives holding cost, and beginning inventory, which drives space requirements. These two levels of inventory are then incorporated in an optimization model to maximize backroom space utilization.

The work presented in this thesis advances extant research in inventory management by addressing multiple non-trivial trade-offs related to backroom space and inventory management within a LSR in a unified optimization model framework for profit maximization. The related factors include (i) distinction between end-items and SKUs, (ii) SKU waste, and (iii) the above-mentioned effect of two levels of pack size. The distinction between end-items, which are sold to the customer, and the SKUs, which are stored in the backroom, entails translating end-item characteristics, like service level or demand, to equivalent level of the SKUs. We investigate the trade-offs in the optimization model while maximizing for profit instead of minimizing for cost, since backroom space decisions have a direct effect on revenue.
and are closely linked to the demand generating frontroom space. Moreover, in-store labor and ordering costs are driven by backroom space, as well as the revenue target for the store.

The optimization model proposed for backroom space utilization determines the optimal inventory levels for the SKUs. Since LSR backrooms function in a multiple-SKU setting and in a space-constrained environment, our model allows for inventory levels below the average demand, (i.e., target service level of less than 50%). To this end, we extend mathematical formulations for inventory policy related quantities (Silver et al., 2016), such as on-hand inventory and order quantity, to also make them applicable to our generalized setting.

Second, our research contributes to the retail space management literature by (i) focusing on backroom space, (ii) accounting for backroom and frontroom interdependencies for space allocation in a retail establishment, and (iii) researching in the context of LSR retail establishments.

As mentioned earlier, backroom space has rarely been considered in the literature, which includes retail space management research. Further, our conversations with a major US LSR retailer reveal that frontrooms in a retail store continuously receive more attention in terms of improving operations and allocating space than backrooms. One of the possible reasons for the focus on the frontrooms is the conception that this part of the retail space creates most of the value to the retailers since it generates demand (Pires et al., 2017). However, our research suggests some evidence that this notion alone is not enough to maximize store performance. Rather, simultaneously accounting for backroom area in retail space allocation and inventory planning decisions may help retailers improve their store profit.

Moreover, because most of the extant literature related to inventory and supply chain planning in the retail industry is focused on grocery retail, this research contributes to the retail management literature by modeling for LSR retail establishments.

Third, our research contributes to restaurant performance management literature by (i) extending the scope of space consideration in restaurants to include backroom space, and (ii) accounting for costs and therefore profit yield from the restaurant space as compared to considering only revenue.

Our research contributes to the restaurant performance management literature by accounting for the effect of total store size and backroom space. This adds to the research on space management that is predominantly focused on determining the seating capacity (optimum number and type of tables/seats) of the frontroom area in restaurant retail es-
While the restaurant performance literature, to the best of our knowledge, focuses on revenue maximization, this research includes costs and incorporates their interdependency with the split between frontroom and backroom in these establishments.

1.4 Thesis overview

The remainder of this thesis is organized as follows.

In Chapter 2, we investigate the effect of pack sizes on the inventory levels of SKUs that are stored in the backroom. Both order and storable pack sizes are integer multiples of the sellable pack size, which is assumed to be 1 unit for our analysis. The order pack size of a SKU affects the amount of inventory that is carried in the store and therefore the space requirement. We analytically derive a formulation for the excess inventory that is carried due to order pack sizes greater than 1, for a periodic review, order-up-to level policy and deterministic demand. We also derive an approximation for the excess inventory that is carried for the case of uncertain demand. While excess inventory reduces the stock-out probability when demand is uncertain, it also requires a larger space for storage in the backroom, which needs to be traded off when making a decision on the ordering amounts in a constrained space environment of a retail store. Storable pack size, on the other hand, affects the amount of space required for a given inventory level, which is also formulated for a single SKU in this chapter.

In Chapter 3, we account for the fact that the performance of a retail establishment is often constrained by its backroom space. We formulate a mixed-integer linear programming model to maximize a retail establishment’s profit through optimal inventory control for a given backroom space. Since the analysis is conducted for a LSR, it is necessary to develop models that translate the end-item properties to that of the SKUs, which are then integrated into the optimization model. Applying the optimization model to real LSR data reveals important managerial implications related to the sensitivity of the backroom space requirements due to changes in order pack size and storable pack size. Additionally, we quantify the cost of having a constrained backroom space.

In Chapter 4, we design a strategic decision model for the optimal allocation of space to the backroom in a retail establishment. This model specifically accounts for the interde-
dependencies between the backroom and the frontroom areas. The profit function is extended beyond the focus on product profit, which is employed in the third chapter, to also include the labor cost and the ordering cost. We also conduct additional analysis to investigate the sensitivity of the backroom space to the different parameters of the profit function.

In Chapter 5, we conclude with a summary of the research discussions from the previous chapters and present potential future directions of the research.
Chapter 2

Analyzing the effect of pack size on inventory and space needs in a retail store backroom

2.1 Introduction

Retail managers are often constrained by pack sizes when planning for inventory that goes into the backrooms of the stores (Das and Caplice, 2017). These constraints manifest themselves in the quantity that managers are forced to order in (order pack size, OPS) and the way that the items have to be stored in the backroom (storable pack size, SPS). The OPS and the SPS are integer multiples of the sellable or consumable pack size (CPS) of a SKU. CPS is the level at which a SKU is consumed. Typically, the relation between the three is: \( \text{OPS} \geq \text{SPS} \geq \text{CPS} \). For example, croissants in a store come in OPS of 36 eaches (sellable units) with 4 storable packs of 9 eaches in a pack, where the CPS is an each. The different pack size types have a variety of impacts on the retail store backrooms. These pack size constraints individually as well as collectively can have a significant impact on the allocation and management of retail store space. Certain research studies have identified specific effects of order pack sizes on the retail supply chain, including (a) the frequency of store replenishment, (b) the magnitude of “overflow inventory” which does not fit into the retail shelf space and hence has to be stored in the backrooms, (c) stacking efficiency in retail stores that primarily drives the operational cost in the store and, (d) the retail market
share (Eroglu et al., 2013; Van Zelst et al., 2009; Waller et al., 2008).

However, the effect of pack size has rarely been considered in academic literature that is concerned with inventory optimization in a retail supply chain (Waller et al., 2008; Wen et al., 2012). The failure to take into account the effect of pack size renders the recommendations derived from traditional methods inaccurate in real world scenarios. For instance, the suggested order quantities with and without an OPS and SPS constraint consideration may differ (Wagner, 2002). Furthermore, inventory management systems in retail stores often do not account for pack sizes when suggesting orders (Interviews during store visits, 2015). Therefore, store managers tend to make “systematic corrections” to the order size that is suggested by these automated systems (van Donselaar et al., 2006). The lack of OPS consideration could lead to inefficient ordering decisions at the store level, which affects both the inventory and the space requirements in the store. Similarly, not considering SPS in ordering decisions can lead to a mismatch between backroom capacity and the space required in it leading to inefficient utilization of the storage space.

We present some relevant empirical evidence related to OPS and SPS using data from our industry partner, which is a major US retailer and referred to as Deltaco in the thesis. A single store backroom has multiple SKUs, which have different order and storable pack sizes. For example, in Figure 2-1, we use the data available to us for 126 Deltaco stores to show the distribution of OPS for approximately 1,000 different SKUs that are ordered and carried in the stores. Figure 2-2 shows the corresponding SPS to OPS ratio for the SKUs.

In the first part of the study, we will focus on the effect of OPS alone and assume SPS and CPS to be equal to 1. In particular, we explore and quantify the specific effect of the OPS on the inventory level and resultant space requirements in the store backroom and the service level achieved. Later, we will discuss the effect of SPS on the backroom space requirements for a given amount of inventory level.

Intuitively, one would expect that when a store manager is constrained to order in integer multiples of OPS, an OPS of greater than 1 unit results in inflated inventory levels as compared to being allowed to order in single units or eaches. Therefore, traditional inventory policies are no longer accurate. To illustrate this, we examine a periodic review and order-up-to level replenishment policy to analyze the effect of pack size. This commonly used inventory policy (Silver et al., 2016) is also used by Deltaco. When OPS is greater than 1 and the store manager plans for inventory (referred to as planned inventory) without
accounting for the OPS effect (or in other words with the assumption that OPS=1) the store could end up carrying excess inventory (referred to as actual inventory).

Figure 2-1: Histogram for order pack size at Deltaco

Distribution of observed SPS to OPS ratio of the SKUs

Figure 2-2: Histogram for ratio of storable pack size to order pack size at Deltaco

This inflated inventory level can have a variety of effects on store operations. On one hand, when the demand is stochastic, the excess amount of inventory can help hedge against uncertainty and yield a lower probability of stock out. However, this benefit comes at a cost of excess storage space requirements in the store along with a larger holding cost due to
excess inventory on hand.

In summary, when the OPS of a SKU is greater than one, there is a trade-off between improved service level and higher space requirements, increased holding costs and increased waste in case of perishable items, with increasing OPS values in a constrained store space environment. For the sake of simplicity, in this chapter, we focus on exploring the improved service level and the higher space requirements. So, we first quantify the excess amount of inventory and the consequent effect on storage space needs for a single SKU, which can then be used to model for optimal order-up-to levels in a retail store facing storage space constraints while dealing with multiple SKUs, as shown in chapter 3.

The remainder of the chapter is organized as follows. In section 2.2, we quantify the effect of OPS on inventory levels for deterministic demand case. Section 2.3 presents an analytical derivation of the effect of OPS when demand is uncertain as well as discusses an approximation of this effect that provides us with a closed-form expression to determine inventory levels under an OPS constraint. Section 2.4 quantifies and discusses the effect of OPS on the probability of stock-out and the expected number of stock-out units. A part of this section also describes the implied adjustment to the planned safety stock level as a result of the OPS constraint. Section 2.5 discusses the trade-off between the excess space requirement as compared to the reduction in probability of stock-out under an OPS constraint. This is followed by discussion of the effect of SPS on the space requirements in section 2.6. Finally, the chapter presents conclusions to wrap up the discussions along with potential future work in section 2.7.

2.2 Quantifying the effect of OPS on inventory

In this section, we evaluate the effect of OPS on the inventory carried in the store when the store manager is constrained to order in multiples of the OPS instead of eaches. As noted earlier, the replenishment policy used for the analysis is a periodic review and order-up-to level or (R,S) policy. This means that for each SKU, the store manager checks the inventory position and replenishes the inventory at the store to at least an ‘S’ level every review period R. The degree of inflation of the inventory and the required adjustment to the expected ‘S’ level depends on whether the product demand is deterministic or stochastic. Additionally, the setting in which the analysis is conducted is characterized by a constant
consumption rate of the SKU, lost sales scenario and an infinite time horizon. The stores are open during daytime business hours and the orders are placed at the closing of the business, which are then delivered overnight. Deliveries, therefore, have a 0 lead-time, i.e. they are instantaneous.

We examine the effect of OPS on the ending inventory in case of deterministic demand and stochastic demand. Ending inventory is defined as the amount that is available on hand at the end of a review period and before an order is placed for the next review period. The observed effect of OPS on the ending inventory is then used to analytically derive the average inventory on hand and the beginning inventory during the review period.

Beginning inventory is the inventory level that is observed right after receiving delivery of the order. A limited service restaurant retailer like Deltaco sometimes processes and combines the SKUs that are ordered before they are sold. All the SKUs that are delivered to a Deltaco restaurant have to be stored in the backroom. Hence, beginning inventory level is used to determine the space requirement in the backroom of the retail store.

The beginning inventory when ordered in eaches (or when OPS=1) is expected to be equal to S. However, they take different values over multiple time periods when the stores are constrained to order in integer multiples of OPS while maintaining at least an inventory level of S at the beginning of every review period, due to the excess ending inventory.

2.2.1 Notation list

The following list defines the notations used for the derivation of the effect of OPS on beginning inventory.

- $S$: order-up-to level
- $X_n$: beginning inventory for review period n. This is the inventory level before the store opens on day n and also includes the inventory that is delivered on day n
- $Y_n$: ending inventory for review period n. This is the inventory level after the store is closed for business on day n
- $O_n$: number of order packs ordered and received at beginning of review period n
- $O_{en}$: equivalent number of sellable units ordered and received at the beginning of review period n
- $\eta_o$: order pack size or OPS
- $Z$: random variable for stationary demand distribution.
• \( \mu \): mean of the distribution of daily demand

• \( R \): review period for a SKU

• \( m \): Greatest common divisor (GCD) of mean demand over review period and the order pack size, i.e. GCD of \((\mu \times R, \eta_0)\). For example, GCD(110,20) = 10.

• \([\cdot]\): The ceiling function of a positive number, which is also defined as the smallest integer greater than or equal to the number

2.2.2 Dynamics of the process of ordering and the resultant change in inventory levels

The process dynamics for ordering and the change in inventory levels under an OPS constraint are as follows:

\[
O_{cn} = \begin{cases} 
\left\lceil \frac{S - Y_{n-1}}{\eta_0} \right\rceil & \text{if } Y_{n-1} < S \\
0 & \text{otherwise}
\end{cases} \quad (2.1)
\]

\[
O_{cn} = O_{cn} \times \eta_0 \quad (2.2)
\]

\[
X_n = Y_{n-1} + O_{cn} \quad (2.3)
\]

\[
Y_n = \max(X_n - Z, 0) \quad (2.4)
\]

Equation 2.1 states that orders can only be placed in integer multiples of OPS with the decision to have inventory at least up to the planned order-up-to level on the shelf. Equation 2.2 shows the corresponding number of sellable units given a certain amount that is ordered. Equations 2.3 and 2.4 show the relation between the beginning inventory, ending inventory and the demand observed.

2.2.3 Effect of OPS on beginning inventory in the case of deterministic demand

In the deterministic demand case, the beginning inventory when \( OPS = 1 \) is equal to the demand over the review period, denoted by \( \mu \times R \). Figure 2-3 shows the change in the inventory levels for a fixed review period when OPS is greater than 1. We note that with a change in the review period, the plot is simply scaled to the given review period and the observed behavior of the change in inventory level over time remains the same.

Specifically, Figure 2-3 illustrates the effect of OPS on the amount of cycle stock and the beginning inventory. The cycle stock and beginning inventory can be derived by observing the patterns in the ending inventory. The average ending inventory is a function of the demand over the review period and the order pack size of the SKU. The average cycle stock
Figure 2-3: An example to illustrate the effect of order pack size on amount of inventory carried. $n_0=100$ units/case, $\mu=80$ units/week and $R = 1$ week. In this case $m = 20$.

and the average beginning inventory are given by equations 2.5 and 2.6, respectively. A few observations about the ending inventory and an analytical derivation for equations 2.5 and 2.6 are provided below.

\[
\text{Average cycle stock} = \frac{\mu \times R}{2} + \frac{n_0 - m}{2} \tag{2.5}
\]

\[
\text{Average beginning inventory} = \mu \times R + \frac{n_0 - m}{2} \tag{2.6}
\]

where $m$ is the GCD($\mu \times R, n_0$)

- **Property 1. Ending inventory value:** Ending inventory in any time period is an integer multiple of $m$.

- **Property 2. Ending inventory range:** Ending inventory takes all values from a fixed set that contains integer multiples $(x)$ of $m$ where $x \in \{0, 1, 2, \ldots, \frac{n_0}{m} - 1\}$

- **Property 3. Ending inventory cycle:** The number of review periods over which the ending inventory takes these fixed values is given by $\frac{n_0}{m}$ (equal to 5 review periods in the example shown in Figure 2-3). In the last review period of this cycle, the beginning inventory is equal to demand over review period ($X_5$ in the example shown) and hence
the amount ordered is 0. This also implies that the amount ordered repeats itself over this fixed number of review period cycles. However, the sequence of the values of ending inventory (and also the amount ordered) is not the same over this cycle.

For deterministic demand, when $\text{OPS} > 1$, the average beginning inventory in a review period is the sum of demand over review period and the average extra inventory carried, as derived below. To derive equation 2.6, we use the relation between beginning and ending inventory along with the properties of ending inventory observed from simulation to derive the average amount of beginning inventory in case of deterministic demand:

$$X_n = Y_{n-1} + O_{en} = Y_{n-1} + \eta_o \times \left[ \frac{S - Y_{n-1}}{\eta_o} \right]$$

The equation states that the beginning inventory in a period $n$ is the sum of the ending inventory from the previous time period and the order quantity in $n$.

Properties 1, 2 and 3 of ending inventory imply that:

$$Y_{n-1} \in m \times \{0, 1, 2, \ldots, \left(\frac{\eta_o}{m} - 1\right)\}.$$  

The ending inventory takes all the values in the set. Since this pattern is repeated every $\frac{\eta_o}{m}$ review periods, the average beginning inventory during this time period is also the long run average of the beginning inventory in an infinite horizon setting. The average of beginning inventory is hence given by:

$$(\frac{1}{m}) \times \left[(0 + \eta_o \times \left[\frac{\mu \times R}{\eta_o}\right]) + (m + \eta_o \times \left[\frac{\mu \times R - m}{\eta_o}\right]) + (2m + \eta_o \times \left[\frac{\mu \times R - 2m}{\eta_o}\right]) + \cdots + ((\eta_o - m) + \eta_o \times \left[\frac{\mu \times R - (\eta_o - m)}{\eta_o}\right])\right]$$

$$= \frac{m \times (0 + 1 + 2 + \cdots + (\frac{\eta_o}{m} - 1))}{(\frac{\eta_o}{m})} + \left(\frac{\eta_o}{m}\right) \times \left[(\frac{\mu \times R}{\eta_o}) + (\frac{\mu \times R - m}{\eta_o}) + (\frac{\mu \times R - 2m}{\eta_o}) + \cdots + \left(\frac{\mu \times R - (\eta_o - m)}{\eta_o}\right)\right]$$

$$= \left[m \times \left(\frac{\eta_o - m}{2}\right) + \left(\frac{\eta_o}{m}\right) \times \left[(\frac{\mu \times R}{\eta_o}) + (\frac{\mu \times R - m}{\eta_o}) + (\frac{\mu \times R - 2m}{\eta_o}) + \cdots + \left(\frac{\mu \times R - (\eta_o - m)}{\eta_o}\right)\right]\right]$$

We use the following relation from (Graham et al., 1994) to obtain the value for the second part of the equation: For any positive value $y$ and integer values of $x$,

$$\left[\frac{x}{y}\right] + \left[\frac{x-1}{y}\right] + \left[\frac{x-2}{y}\right] + \cdots + \left[\frac{x-y+1}{y}\right] = x$$

Therefore, the average beginning inventory becomes:
Alternatively, the average inventory or cycle stock can also be derived by using the ending inventory properties that are observed. The average ending inventory as shown above is as follows:

\[
\frac{m\cdot(0+1+2+\ldots+\frac{n_0-1}{m})}{(\frac{n_0}{m})}
\]

Additionally, since every review period has a constant and deterministic demand rate, the average cycle stock is given by

\[
\frac{\mu\cdot R}{2}
\]

Therefore, cumulative average cycle stock is given by

\[
\frac{\mu\cdot R}{2} + \frac{m\cdot(0+1+2+\ldots+\frac{n_0-1}{m})}{(\frac{n_0}{m})} + \frac{m\cdot(0+1+2+\ldots+\frac{n_0-1}{m})}{(\frac{n_0}{m})}\]

We can also use the ending inventory properties to derive the maximum beginning inventory, which is defined as the largest quantity that beginning inventory attains during any review period.

\[
Y_n = X_n - \mu \cdot R
\]
\[
X_n = Y_n + \mu \cdot R
\]
\[
X_{n,max} = Y_{n,max} + \mu \cdot R
\]
\[
X_{n,max} = (n_0 - m) + \mu \cdot R
\]

2.3 Effect of OPS on beginning inventory for stochastic demand: the case of normally distributed demand

We now move toward analyzing the excess inventory that is held due to an OPS greater than 1 and when demand is stochastic. In preparation for quantifying the excess inventory we first derive two properties about the beginning and ending inventory. These properties are then
used to derive the excess amount of inventory as a result of OPS on an illustrative example of stochastic demand, that is normally distributed. This is followed by a discussion of a proposed approximation of the average excess inventory that is mathematically convenient. Summarily, the following points are discussed in this section:

- for a given SKU, the beginning inventory level varies between planned order up-to level $S$ and a level of $S+\eta_0-1$.

- The beginning inventory can be approximated by a uniform distribution when analyzed in an infinite horizon setting. Therefore, for a given SKU, the excess inventory carried on average due to an OPS greater than one can be approximated to a function of the OPS alone. The excess amount of inventory (beyond the inventory level that is carried when $\text{OPS} = 1$) when $\text{OPS}$ is greater than 1 is approximated by $\frac{\eta_0 - 1}{2}$, where $\eta_0$ is the OPS of the SKU.

### 2.3.1 Additional notation list

Besides, the notation introduced in section 2.2.1, the following list defines additional notation used for the derivation of the effect of OPS on beginning inventory for the case of normally distributed demand

- $k$: planned service level without considering the OPS effect (or equivalently considering $\eta_0 = 1$)

- $k'$: service level actually achieved due to the OPS being greater than 1 (the order-up-to level is the same irrespective of order pack size consideration)

- $Z$: random variable for stationary demand distribution.

- $f_U(u)$: probability density function (PDF) of random variable $U$

- $F_U(u)$: cumulative distribution function (CDF) of random variable $U$

- $\text{erf}(.)$: Gauss error function

- $\mu$: mean of the demand distribution

- $\sigma$: standard deviation of the demand distribution
2.3.2 Properties of beginning and ending inventory

- **Property 1.** Maximum and minimum beginning inventory: In a \((R,S)\) replenishment policy inventory system, the beginning inventory ranges from \(S\), the planned order-up-to level, to \(S + \eta_o - 1\). Proof is provided below.

**Claim 1:** The minimum value of beginning inventory, \(X_n\), is \(S\).

**Proof.** We have to prove that \(X_n \geq S\). We prove this by contradiction. That is, let us suppose that \(X_n < S\). We will also use the relation between beginning and ending inventory as shown in equations 1, 2 and 3.

If \(X_n < S \implies Y_{n-1} + O_{en} < S \implies \left(Y_{n-1} + \eta_o * \begin{cases} \frac{S-Y_{n-1}}{\eta_o} & \text{if } Y_{n-1} < S \\ 0 & \text{otherwise} \end{cases} \right) < S\)

For the case when \(Y_{n-1} < S\)

\[ Y_{n-1} + \eta_o * \frac{S - Y_{n-1}}{\eta_o} < S \]

\[ \frac{S - Y_{n-1}}{\eta_o} < \frac{S - Y_{n-1}}{\eta_o} \]

Since the above statement is not true for all values of \(Y_{n-1} \geq 0\), by contradiction we have proven that \(X_n \geq S\) for the case that \(Y_{n-1} < S\)

The case that \(Y_{n-1} > S\), implies that \(O_{en} = 0\),

\[ Y_{n-1} + 0 < S \]
\[ Y_{n-1} < S \]

Therefore, by contradiction, we have proven that \(X_n \geq S\), which can be seen as the lower bound of the distributions in Figure 2-14. \(\square\)

**Claim 2:** The maximum value of beginning inventory is \(S + \eta_o - 1\).

**Proof.** We have to prove that \(X_n \leq S + \eta_o - 1\). We will prove this by contradiction again. That is, let \(X_n > S + \eta_o - 1 \implies X_n \geq S + \eta_o\)

\[ X_n \geq S + \eta_o \implies Y_{n-1} + O_{en} \geq S + \eta_o \implies Y_{n-1} + \eta_o * \begin{cases} \frac{S - Y_{n-1}}{\eta_o} & \text{if } Y_{n-1} < S \\ 0 & \text{otherwise} \end{cases} \geq S + \eta_o \]

For the case that \(Y_{n-1} < S\):

\[ Y_{n-1} + \eta_o * \frac{S - Y_{n-1}}{\eta_o} \geq S + \eta_o \]

\[ \frac{S - Y_{n-1}}{\eta_o} \geq \frac{S - Y_{n-1}}{\eta_o} + 1 \text{ which is not true} \]

37
Since the above statement is not true for \( Y_{n-1} \geq 0 \), by contradiction we have proven that \( X_n \leq S + \eta_o - 1 \) for the case that \( Y_{n-1} < S \).

Now, for the case that \( Y_{n-1} > S \):

\[
\begin{align*}
Y_{n-1} + 0 & \geq S + \eta_o \\
X_{n-1} - Z & \geq S + \eta_o \\
Z & \leq X_{n-1} - (S + \eta_o) \text{ which is not possible}
\end{align*}
\]

The last statement is not possible because demand is greater than or equal to 0. Therefore, by contradiction, we have proven that \( X_n \leq S + \eta_o - 1 \). In other words, the maximum level of beginning inventory during any review period \( n \) is less than or equal to \( S + \eta_o - 1 \).

Alternatively, it can also be proven by noticing that if the claim of \( Y_{n-1} + O_{cn} \geq S + \eta_o \) is true, then \( Y_{n-1} \geq S + (1 - O_{cn}) \cdot \eta_o \) where \( O_{cn} \) is the number of order packs. Since \( O_{cn} \) is greater than or equal to 0, it should be true for \( O_{cn} = 1 \) which is not the case. It is because a \( O_{cn} = 1 \) implies that the ending inventory from the previous period is greater than or equal to \( S \) and therefore \( O_{cn} \) should be equal to 0.

**Property 2.** Distribution of ending inventory: For normally distributed demand, the distribution of the difference between beginning inventory and demand (denoted as \( X-Z \)) in a \((R,S)\) system with an \( OPS \) constraint is approximately normal. This is shown in section 2.8.3 of the appendix. The ending inventory, however, is \( \text{max}(0, X-Z) \) since its distribution is truncated at 0.

### 2.3.3 Analytical derivation of the distribution of beginning inventory with normally distributed demand

In this part of the chapter, we will demonstrate the process of deriving the distribution of beginning inventory when \( OPS \) is greater than 1 and for an illustrative example of a normally distributed demand. The two properties from section 2.3.2 are used here in the following way: property 1 is used to determine all possible values that the beginning inventory can take while property 2 along with equation 2.1 is used to evaluate the probability of each possible value of beginning inventory.
The histogram in Figure 2-14 shows the actual distribution of beginning inventory obtained from simulation. In this section, we illustrate the derivation of the actual distribution of beginning inventory by using an instance of normally distributed demand. We will use properties 1 and 2 to arrive at the probability distribution function for the beginning inventory. The simulation discretizes the value of demand by using only integer values. Therefore, we use a continuity correction factor (Devore, 2011) when determining the probability distribution of ending and beginning inventory. Assuming normally distributed demand, if the ending inventory was not truncated, then property 2 can be used to conclude that it is approximately normally distributed as well. Given a beginning inventory value, we know that un-truncated ending inventory is given by: $Y_{n\text{-untruncated}} = X_n - Z$. Therefore, $Y_{n\text{-untruncated, min}} = X_{n, \text{min}} - Z_{\text{max}}$. Conversely, $Y_{n\text{-untruncated, max}} = X_{n, \text{max}} - Z_{\text{min}}$.

Since $Z$ (demand) is normally distributed, and knowing that more than 99% of the demand values lie within 3 standard deviations of the mean, we approximate $z_{\text{max}} = \mu + 3\sigma$ and $z_{\text{min}} = \mu - 3\sigma$. We now use property 1 to note that $X_{n, \text{min}} = S$ and $X_{n, \text{max}} = S + \eta_0 - 1$. Therefore, $Y_{n\text{-untruncated, min}} \approx S - (\mu + 3\sigma)$ and $Y_{n\text{-untruncated, max}} \approx (S + \eta_0 - 1) - (\mu - 3\sigma)$. Since demand is normally distributed in this case, ending inventory is also normally distributed with the minimum and maximum values as shown above. Let us denote $\mu_{\text{Endinv\text{-untruncated}}}$ and $\sigma_{\text{Endinv\text{-untruncated}}}$ as the mean and the standard deviation of the untruncated ending inventory distribution. Therefore, again using the normal distribution property nearly all values lie within 3 standard deviations of the mean, we obtain the following that:

\[
\mu_{\text{Endinv\text{-untruncated}}} \approx \frac{Y_{n\text{-untruncated, min}} + Y_{n\text{-untruncated, max}}}{2} \quad \text{and} \\
\sigma_{\text{Endinv\text{-untruncated}}} \approx \frac{Y_{n\text{-untruncated, max}} - \mu_{\text{Endinv\text{-untruncated}}}}{3}.
\]

This helps us calculate the probability distribution of the "observed" ending inventory and the truncated ending inventory distribution. The truncated ending inventory ranges from 0 to $Y_{n\text{-untruncated, max}}$. The probability that the truncated ending inventory takes a value of 0 is given by the sum of probabilities of all values of untruncated ending inventory of less than or equal to 0. Therefore, \(\text{Prob}(Y_n = 0) = \text{Prob}(Y_{n\text{-untruncated}} \leq 0) = \text{CDF}(Y_{n\text{-untruncated}} = 0)\). Since we already know the distribution of the untruncated ending inventory, the CDF of the distribution at 0 is also known. It is also easy to check that \(\text{Prob}(Y_n = y) = \text{Prob}(Y_{n\text{-untruncated}} = y)\) where \(y\) ranges from 1 to $Y_{n\text{-untruncated, max}}$. Since we know the truncated ending inventory distribution, the distribution of beginning
inventory can be derived by using equations 1 and 2. Since $X_n$ is bounded by the minimum and maximum values as stated in property 1, and the distribution and the possible values of the ending inventory are known, the beginning inventory distribution is also known. We illustrate the derivation of the beginning inventory distribution by using the parameters as shown in table 2.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Derived from</th>
<th>Value in this example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\mu, \sigma)$</td>
<td>both given</td>
<td>(70, 5)</td>
</tr>
<tr>
<td>$\eta_0$</td>
<td>given</td>
<td>10</td>
</tr>
<tr>
<td>$S$</td>
<td>calculated</td>
<td>72</td>
</tr>
<tr>
<td>$(X_{n,min}, X_{n,max})$</td>
<td>both calculated</td>
<td>(72, 81)</td>
</tr>
<tr>
<td>$(D_{min, estimated}, D_{max, estimated})$</td>
<td>both calculated</td>
<td>(55, 85)</td>
</tr>
<tr>
<td>$(Y_{n-untruncated,min}, Y_{n-untruncated,max})$</td>
<td>both calculated</td>
<td>(-13, 26)</td>
</tr>
<tr>
<td>$(\mu_{Endinv-untruncated}, \sigma_{Endinv-untruncated})$</td>
<td>both calculated</td>
<td>(6.5, 6.5)</td>
</tr>
<tr>
<td>$Y_{n,min}$</td>
<td>known</td>
<td>0</td>
</tr>
<tr>
<td>$Y_{n,max}$</td>
<td>$Y_{n-untruncated,max}$</td>
<td>26</td>
</tr>
</tbody>
</table>

Table 2.1: Parameter values for derivation of beginning inventory distribution

This implies that $\text{Prob}(Y_n = 0) = \text{Prob}(Y_{n-untruncated} \leq 0)$ and

$\text{Prob}(Y_n = y(= 1, 2 \cdots Y_{n,max})) = \text{Prob}(Y_{n-untruncated} = y)$ where $Y_{n-untruncated}$ is normally distributed with a continuity correction factor due demand discretization as mentioned above. Figure 2-4 shows the actual probability values of ending inventory from simulation and from the analytical method described here.

![Figure 2-4: Probability distribution for ending inventory of analytical method and from simulation.](image)

For the beginning inventory, the calculation can be done for all discrete values of be-
ginning inventory between the derived minimum and maximum values. For illustration, we show the probability of observing the most frequently occurring beginning inventory value in this example, which is 80 units.

\[
P(X_n = 80) = P(Y_{n-1} + \eta_n \times \left[ \frac{S - Y_{n-1}}{\eta_n} \right] = 80)
= P(Y_{n-1} + 10 \times \left[ \frac{72 - Y_{n-1}}{10} \right] = 80)
= P(Y_{n-1} + 7.2 - \frac{Y_{n-1}}{10} = 8)
\]

Since \(Y_{n-1} \in [0, 26] \Rightarrow \frac{Y_{n-1}}{10} = Y'_{n-1} \in [0, 2.6]\). Therefore, the possible values of \(Y'_{n-1} = 0, 1, 2\). This implies that:

\[
P(X_n = 80) = P(Y'_{n-1} = 0) + P(Y'_{n-1} = 1) + P(Y'_{n-1} = 2)
= P(Y_{n-1} = 0) + P(Y_{n-1} = 10) + P(Y_{n-1} = 20)
= 0.178 + 0.053 + 0.007
= 0.238
\]

All other \(P(X_n)\) values are calculated the same way. The probability values are normalized to make the sum of probabilities over the set of possible values of beginning inventory equal to 1. The probability distributions of beginning inventory from simulation and from the analytical method are plotted in Figure 2-5.

![Figure 2-5: Probability distribution of analytical method and from simulation for beginning inventory.](image-url)
2.3.4 Approximating the beginning inventory distribution as a uniform distribution

There are many cases in which knowing the beginning inventory as a function of OPS and order-up-to level can be useful in the field of retail operations management. One such instances is the determination of the optimal amount of storage space that should be allocated to a SKU within a backroom of a retail store that deals with multiple SKUs. This is also known as the backroom utilization problem.

When modeling for backroom space utilization in an infinite horizon, average beginning inventory is an appropriate level for calculating space requirements in the store because there is a low likelihood of the inventory level of all the SKUs being at the maximum level at the same time, given the uncertainty in demand associated with them. Therefore, it is helpful to approximate the distribution of beginning inventory instead of employing the analytical method described in section 2.3.3 to estimate the average amount of beginning inventory for the sake of mathematical convenience as long as the departure from the exact amount is not large.

Based on a combination of observations from simulation and calculations from the analytical method, we propose to approximate beginning inventory using a uniform distribution. The inventory levels that we are interested in can be approximated to:

(a) average beginning inventory during a review period, given by:

\[ \mu R + \text{safety stock} + \frac{\eta_0 - 1}{2} \]  

and (b) average inventory on hand during a review period, given by:

\[ \frac{\mu R}{2} + \text{safety stock} + \frac{\eta_0 - 1}{2} \]  

Average beginning inventory shown in equation 2.7 can be used to determine the space needs for a SKU and the average inventory on hand shown in equation 2.8 drives the holding cost associated with carrying the SKU in the store backroom.

In the case study with normally distributed demand presented in section 2.3.3, the average beginning inventory from the simulation is 76.8, the results of the analytical method and uniform distribution approximation are 76.9 and 76.5 respectively. In the following section, we discuss the accuracy of the uniform distribution approximation. This is followed
by a discussion of how the average probability of stock-out over a review period with an OPS constraint can be derived by assuming that the beginning inventory is uniformly distributed.

2.3.5 Discussion of the extent of accuracy of the uniform distribution approximation for beginning inventory

In this section, we discuss the accuracy of the uniform approximation by comparing the resulting values of beginning and average inventory on hand as well as the expected probability of stock-out from the formula and the observed values. Several simulations were run, with different values of OPS and demand. The results suggest that we can achieve a good accuracy of the uniform distribution approximation for both the average inventory levels and the probability of stock out derived as a result of it (by using equation 2.10 which is derived later in section 2.4).

The results shown in this section pertain to a specific pack size, OPS=24 units, which is also the most commonly occurring OPS for SKUs in our case-study dataset (see Figure 2-1). We calculate the average values for inventory on-hand, beginning inventory and the probability of stock-out from a Monte-Carlo simulation for multiple combinations of mean and a standard deviation of demand. The mean of demand ranges between 10 to 150, increasing in steps of a single unit. The coefficient of variation (CV), that also implies the standard deviation, ranges between 0.1 and 0.4, increasing in steps of 0.1. The CV values are capped at 0.4 in order to decrease the likelihood of negative values of demand that are generated by using a mean and a standard deviation in the simulation. The planned order-up-to level for each combination of mean and standard deviation of demand is calculated for cycle service levels ranging between 0.6 to 0.9, increasing in steps of 0.1.

For the purpose of illustration, Table 2.2 compares the average of the different parameters that are obtained from the simulation to the calculated values from the formula for five different values of mean demand:

Figure 2-6 compares the simulation results with the values from the formula derived with a uniform distribution approximation for average beginning inventory and average inventory as shown in equations 2.7 and 2.8.

Each point represents the simulation and analytical result for a different mean demand. The low root mean square error (RMSE) and mean absolute percentage error (MAPE) values reported in table 2.3 confirm that the formula is a good approximation of the simulation
Table 2.2: The effect of OPS on inventory and stock out probability for normally distributed demand results. The error estimates in the table are obtained by comparing the formula and the simulation results across all possible combinations of mean, standard deviation, and cycle service level in the ranges that are mentioned earlier.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Root Mean Squared Error (RMSE)</th>
<th>Mean Absolute Percentage Error (MAPE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beginning inventory average</td>
<td>0.97</td>
<td>0.6%</td>
</tr>
<tr>
<td>Average inventory</td>
<td>2.72</td>
<td>2.1%</td>
</tr>
<tr>
<td>Probability of stockout</td>
<td>0.0002</td>
<td>6.7%</td>
</tr>
</tbody>
</table>

Table 2.3: Error estimate of the different parameters when comparing simulation and formula results.

Figure 2-6: Comparing the results obtained from the formula and simulation-Normal.
2.4 The effect of OPS on expected probability of stock-out

A stock-out (SO) is defined as an event when the demand is greater than the beginning inventory in a given period. In this thesis, we use the cycle service level metric, which is defined as the fraction of replenishment cycles when a stock-out does not occur (Silver et al., 2016). We will investigate the effect of OPS on stock-out in two different ways: (a) change in probability of stock-out, which is discussed in section 2.4.1 and (b) change in the number of units of stock-out, which is shown in section 2.4.2. As a reminder, in this chapter the random variables for demand and beginning inventory distributions are denoted as Z and X respectively.

When order pack size is equal to one, we know that the probability of SO is simply equal to (1-cycle service level). In this section, we explore the effect of having an OPS>1 on the probability of stock out denoted as Pr(SO). Mathematically, Pr(SO) for any order pack size is given by:

\[ Pr(SO) = Pr(Z > X) \] (2.9)

When the OPS is equal to 1, the beginning inventory is a constant value and equal to the order-up-to level in a (R,S) replenishment policy. However, for OPS greater than 1, the beginning inventory (X) is a random variable. We illustrate the application of equation 2.9 by using the example of normally distributed demand. By approximating the beginning inventory to being uniformly distributed between \( S \) and \( S + \eta_o - 1 \), i.e. \( X \sim U[S, S + \eta_o - 1] \), we can show that:

\[
Pr(SO)_{normal} = \begin{cases} 
1 - \frac{1}{2\sigma(\eta_o-1)} \int_{S}^{S+\eta_o-1} \left( 1 + erf \left( \frac{x-\mu}{\sigma \sqrt{2}} \right) \right) dx & \eta_o > 1 \\
1 - F_N \left( \frac{S-\mu}{\sigma} \right) & \eta_o = 1
\end{cases}
\] (2.10)

where \( F_N \) is the cumulative distribution function of a standard normal variable with a mean and standard deviation of 0 and 1 respectively. The derivation of equation 2.10 is shown in the appendix in section 2.8.1.
Comparing the expected probability of stock-out when OPS is greater than 1 to OPS equal to 1

Again, under an (R,S) inventory replenishment policy, the probability of stock-out when OPS is equal to 1 and with uncertain demand is given by (1-cycle service level). However, an OPS greater than 1 results in carrying excess inventory as compared to the calculated inventory level when OPS = 1. So, the effective probability of stock-out is lower than the intended probability of stock-out. In order to illustrate this, we define the following two cases:

- Case 1: The Pr(SO) is derived from the planned service level when the order pack size constraint is ignored. This is equivalent to assuming that OPS is equal to 1.
- Case 2: The Pr(SO) that is achieved when the store still makes the ordering decisions based on the assumptions of case 1, but is constrained to order in integer multiples of OPS.

Figure 2-7 illustrates the relation between Pr(SO) in cases 1 and 2 for different coefficients of variation of a normally distributed demand. The Pr(SO) in case 1 is simply (1-cycle service level), which is shown on the X-axis. The Pr(SO) in case 2, which is shown on the y-axis, is calculated by using equation 2.10. It might be helpful to note that the results for Pr(SO) from simulation yield similar values as equation 2.10. The mean for demand over review period and the OPS in this illustration are set to 20 and 12 units respectively.

Figure 2-7 illustrates a typical relation between the Pr(SO) with and without the OPS constraint. For a given mean demand and OPS, the slope of the lines becomes steeper with increasing variability. Mathematically, it means that for a given OPS and a mean of demand, the following relation holds for Pr(SO)case-2:

\[
\sigma_a \leq \sigma_b \implies Pr(SO)_{case-2,\sigma_a} \leq Pr(SO)_{case-2,\sigma_b}
\] (2.11)
Figure 2-7: Probability of stock-out over different scenarios of variability. The figure shows results for normally distributed demand with a mean of 20 units over a review period. The OPS is set at 12 units.

Conversely, the effect of changing order pack sizes while keeping the variability of demand constant is illustrated in Figure 2-8. For a given mean and standard deviation of demand, the slope of the lines becomes steeper with decreasing OPS. Mathematically, it means that for a given value of \( Pr(SO)_{\text{Case-1}} \):

\[
\eta_{o,a} \geq \eta_{o,b} \implies Pr(SO)_{\text{Case-2,}\eta_{o,a}} \leq Pr(SO)_{\text{Case-2,}\eta_{o,b}} \tag{2.12}
\]

In other words, a larger OPS results in excess inventory that can be used to hedge against demand uncertainty for the same order-up-to level. On the basis of these two results, one might infer that the store manager should try to achieve a balance between carrying enough inventory to achieve a desired service level target and not carrying too much because of the pack size effect that leads to undesirable waste and space needs.
Figure 2-8: Probability of stock out over different scenarios of order pack size (OPS). The figure corresponds to results run for normally distributed demand with a mean of 20 units and a standard deviation of 6.

We are able to reproduce these results with empirical demand data. The demand distribution for a very popular product sold by Deltaco is shown in Figure 2-9. The effect of a change in Pr(SO) with a change in OPS for this product is shown in Figure 2-10.

Figure 2-9: Empirical demand distribution of a SKU in a retail store of Deltaco
2.4.2 Expected number of stock-out in units

The expected number of units of stock-out can be also be derived by using the uniform distribution approximation of the beginning inventory.

Mathematically, \( E[SO|\eta_0 > 1] = E[max(Z - X, 0)] \), where \( Z \) is a normally distributed random variable for demand with a mean \( \mu \) and a standard deviation of \( \sigma \). \( X \) is the random variable for beginning inventory that is uniformly distributed between \([S, S+\eta_0] \). If we denote \( E[SO]_{\text{normal}} \) as the expected units of stock-out for a normally distributed demand, then:

\[
E[SO]_{\text{normal}} = \begin{cases} 
\frac{1}{2 \eta_0 - 1} \int_{0}^{\infty} w \left[ \text{erf} \left( \frac{S+\eta_0 - 1 - \mu}{\sqrt{2\sigma}} \right) - \text{erf} \left( \frac{S+\mu - \eta_0}{\sqrt{2\sigma}} \right) \right] dw & \eta_0 > 1 \\
\sigma \cdot G_N \left( \frac{S - \mu}{\sigma} \right) & \eta_0 = 1 
\end{cases}
\]

(2.13)

where \( G_N(.) \) is the unit normal loss function (Silver et al., 2016). The detailed derivation of the expected number of stock-out units when OPS >1 is shown in the appendix in section 2.8.2.
2.4.3 Implied adjustment to the planning safety stock level due to OPS constraint

An alternative way of accounting for the effect of excess inventory due to an OPS > 1 is to adjust the safety stock level. If $SS_{planned, average}$ is the long run average service level target for a SKU in the store and $SS_{effective}$ is the adjusted safety stock level ordered, we can write the following relationship:

$$\mu \times R + SS_{planned, average} = \mu \times R + SS_{effective} + \frac{b_0 - 1}{2}$$

Therefore,

$$SS_{effective} = SS_{planned, average} - \frac{b_0 - 1}{2}.$$
required. If the excess amount of inventory carried is more than the planned safety stock level, then the store manager could decide to carry a negative safety stock amount. On the other hand, if the excess amount falls short of the amount of safety stock that is desired the additional safety stock is simply the difference between the two amounts.

2.5 Trade-off between the excess space requirement and the reduction in expected probability of stock-out due to OPS effect

As discussed earlier, when a store manager decides on the amount of space to allocate to a specific SKU, she has to consider the trade-off between the benefit of excess inventory to reduce the stock-out costs and the corresponding increased space requirements. The change in the expected probability of stock-out and the space requirement corresponding to the expected beginning inventory level is shown below. As a reminder, the estimates for the planned values correspond to the case when the order-up-to level is determined by ignoring the OPS constraint. The observed values result when the store still makes the ordering decisions based on the order-up-to level in the planned case but is constrained to order in integer multiples of OPS.

The percentage difference between the expected stock-out probability in the two cases is defined as:

\[
\left( \frac{Pr_SO_{planned} - Pr_SO_{observed}}{Pr_SO_{planned}} \right) \times 100
\]

where \(Pr_SO\) is calculated from equation 2.10.

On the other hand, the excess amount of inventory that is a result of the OPS, which is denoted as \(n_o\), constraint also implies a larger storage space need in the retail store backroom. As mentioned earlier, for a limited service restaurant like Deltaco, all the SKUs that are delivered to its stores have to be stored in the backroom before they are processed and sold. Hence, the beginning inventory level is used to determine the space requirement for a SKU in the retail store backroom.

If \(v_s\) denotes the space occupied by a unit SKU and \(S\) is the planned order-up-to level, the amount of space required when \(OPS=1\) is equal to \(S \times v_s\). On the other hand, when \(OPS > 1\), the amount of space occupied on an average is \((S + \frac{n_o - 1}{2}) \times v_s\). Therefore, the
percentage difference in the space required when OPs \( > 1 \) as compared to when OPs \( = 1 \) for a given order-up-to level is defined as:

\[
\text{Space added} = \left( \frac{\text{Avg. BI}_{\text{OPs} > 1} - \text{BI}_{\text{OPs} = 1}}{\text{BI}_{\text{OPs} = 1}} \right) \times 100
\]

\[
= \left( \frac{\eta_o - 1}{2S} \right) \times 100
\]

where \( \eta_o \) is the order pack size, \( S \) is the order-up-to level and BI denotes the beginning inventory.

Figure 2-12 illustrates this trade-off for a popular item sold in a Deltaco restaurant, with a mean demand over review period of 70 units and a standard deviation of 15 units, for specific values of intended or planned cycle service level. The X-axis shows the percentage difference between the expected stock-out probability and the Y-axis shows the added space requirement due to the excess inventory that is carried as a result of the order pack size being greater than 1, as a percentage of the space requirement for a planned beginning inventory level.

Each point in Figure 2-12 corresponds to a value of the percentage change in expected \( \text{Pr}(S\text{O}) \) and the percentage change in space requirement for a given OPs and a planned cycle service level. The plot suggests that for the same reduction in stock-out probability, the space requirement change for a planned cycle service level \( A \) is higher than that of planned cycle service level \( B \), if \( A < B \).

For a given order-up-to level, an increase in the OPs increases the percentage change in space requirement linearly while it decreases the expected probability of stock-out non-linearly. Therefore, if the penalty cost for stocking out is large enough to compensate for the increased space requirement, then a large OPs can be beneficial for the store. Such an analysis can be used by the store manager to understand the implications of a given OPs constraint on two dimensions: level of service and the space requirement. Moreover, at the corporate level such a trade-off chart can be referred to when evaluating the impact of the change of OPs for a given SKU: for instance, for a planned service level of 60%, a change of OPs from 30 to 40 units implies a space requirement increase of 6% while improving the probability of stock-out by 9%. On the other hand, the same magnitude of change of OPs but from 90 to 100 units has a much smaller improvement in probability of stock-out of 1% as compared to the increase in the space requirement of 7%.
Additional illustrations of the trade-off between the increased space requirements and the decrease in stock-out probability with changing OPS, for the case of two mean and standard deviation values of normally distributed demand, is shown in section 2.8.5 of the appendix.

2.6 The effect of SPS on space

As a reminder, storable pack (or also called inner/intra pack) size is the number of sellable units that fit into a single inner pack. The storable pack size directly affects the amount of space used. The space occupied is given by number of equivalent storable packs correspond-
ing to the average beginning inventory. It is the ceiling function of the average beginning inventory divided by the storable pack size. Therefore, for a given inventory level \( I \), the number of storable packs is given by \( \lceil \frac{I}{\eta} \rceil \). The space occupied as a result is given by:

\[
\lceil \frac{I}{\eta} \rceil \times \text{space occupied by a single storable pack}
\]

The storable pack size does not have an effect on the amount of inventory carried in the backroom (if there is enough space to accommodate all the required inventory across all the SKUs) and hence the expected profit obtained does not change with a change in the storable pack size if there is a single SKU. However, the space occupied in the backroom could increase with increase in the storable pack size for the SKUs in Deltaco. In that case, the value of the space, if defined as the profit obtained per unit space of the backroom, decreases as storable pack size increases.

Chapter 3 uses the discussion presented in this chapter to formulate the beginning inventory level for a SKU as a function of its OPS and the resultant space requirement as a function of its SPS.

2.7 Conclusion

In this chapter, we investigate pack size, which is an important driver of retail store backroom inventory and space needs. There are two out of three pack sizes of a SKU that are relevant to space management in stores: order and storable pack sizes, which are integer multiples of the third called the sellable pack size of a SKU. This chapter provides a discussion on the effects of order pack size on the inventory and that of the order and storable pack size on the space needs in a store backroom. We derive and quantify the effect of order pack size on the excess inventory carried in the store and the expected probability of stock-out as well as the expected units stock-out for a single SKU. We also discuss the trade-off that the store manager has to potentially make when faced with order pack size constraints.

The discussion from this chapter can inform decisions on two levels: (i) on the store level and, (ii) on the logistics system level that includes the central distribution center and the stores that are replenished by it.

On the store level, the knowledge about the amount of excess inventory can be used to identify the “worst offenders” of backroom space. These are the SKUs that require large storage space due to excess inventory because of \( \text{OPS} > 1 \) while simultaneously yielding a low
profit for the space occupied. The total space requirement for a SKU, as shown in section 2.5, is a function of not just the beginning inventory level but also the space occupied by a unit of the SKU. The space requirements could increase with SPS when it is greater than 1, depending on how the product of the equivalent number of storable packs for a given inventory level and the space of each storable pack changes. Alternatively, the analysis can be used to determine optimal order-up-to level for a store that deals with multiple SKUs which are competing for storage space in a constrained environment. The optimal order-up-to level is different from the level suggested by traditional inventory policies that ignores OPS constraint because of the excess inventory that is a result of the constraint. Therefore, analyses and insights discussed here are building blocks for an optimization model for backroom space capacity allocation within a retail store in a constrained space and a multiple SKU environment. This will be presented in Chapter 3.

At the system level, the discussion from this chapter can be used to determine optimal OPS that minimizes the system level cost, which in turn includes the costs associated with multiple retail stores and the central distribution center serving them. The benefit of an OPS>1 is observed at the distribution center and can be quantified as savings in handling costs component, which is driven by the process of breaking supplier pack size to the store OPS level (Wen et al., 2012). On the other hand, an OPS>1 also implies larger storage space requirements and hence potentially larger backroom space in the retail stores along with increased inventory holding costs with increasing OPS. One can estimate the cost of having a larger backroom space within a retail store as the opportunity cost which results from cutting into the revenue generating space of the store or the frontroom. In other words, it is the cost of having too much space allocated to the backroom and not having enough frontroom space in the store to support the revenue that could be potentially generated from the inventory carried in the backroom space. Moreover, for very small retail store sizes, an OPS>1 can contribute to increased in-store logistics costs in the store, primarily resulting from congested backroom spaces. Therefore, the optimal OPS for the system can be determined by trading off savings observed in the central distribution center and the space related costs at the store level. Similarly, there are different system level implications for SPS. A large SPS corresponds to a lower packaging cost and also reduce the probability of breakage of SKUs during transit as compared to a SPS of one. On the other hand, larger SPS frequently have larger space implications at the store for a given inventory level. These
factors can be traded off to determine the optimal SPS at the system level.

Finally, there are multiple extensions to the work presented in this chapter. One possible extension is to focus on extending the analysis to quantify the inflated inventory amount when items are perishable, hence accounting for waste as a function of the shelf life and the OPS of a SKU. This work can also be leveraged to quantify the effect on inventory level due to OPS constraints for other types of commonly observed stochastic demand distributions and inventory replenishment policies adopted by the stores.

2.8 Appendix

2.8.1 Derivation of expected probability of stock out

In this part of the appendix, we derive the probability of stock out when OPS is greater than 1 and demand is normally distributed by using probability theory (Bertsekas and Tsitsiklis, 2002). As discussed in the chapter, we use a uniform distribution approximation for the beginning inventory to derive \( Pr(SO) \).

\[
Pr(SO)_{\text{normal}} = Pr(Z > X) \\
= 1 - Pr(X >= Z) \\
= 1 - \int \int_{x \geq z} f_X(x) f_Z(z) dz dx \\
= 1 - \int_{-\infty}^{\infty} \int_{-\infty}^{x} f_X(x) f_Z(z) dz dx \\
= 1 - \int_{-\infty}^{\infty} f_X(x) F_Z(x) dx \\
= 1 - \int_{s}^{s+\eta_0-1} \frac{1}{\eta_0 - 1} F_Z(x) dx \\
= 1 - \int_{s}^{s+\eta_0-1} \frac{1}{\eta_0 - 1} \frac{1}{2} \left( 1 + erf \left( \frac{x - \mu}{\sigma \sqrt{2}} \right) \right) dx \\
= 1 - \frac{1}{2 \times (\eta_0 - 1)} \int_{s}^{s+\eta_0-1} \left( 1 + erf \left( \frac{x - \mu}{\sigma \sqrt{2}} \right) \right) dx
\]

We use the following relation for the CDF of the normal distribution in the above derivation (Weisstein, 2002):
\[ F_Z(z) = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{z - \mu}{\sigma \sqrt{2}} \right) \right) \]

Therefore,

\[
Pr(SO)_{\text{normal}} = \begin{cases} 
1 - \frac{1}{2^{\eta_0-1}} \int_S^{S+\eta_0-1} \left( 1 + \text{erf} \left( \frac{x - \mu}{\sigma \sqrt{2}} \right) \right) dx & \eta_0 > 1 \\
1 - F_N\left( \frac{S-\mu}{\sigma} \right) & \eta_0 = 1 
\end{cases}
\]

### 2.8.2 Expected number of stock outs

We will figure out the expected number of stock outs when demand is normally distributed, waste is not considered. \( E[SO|OPS = OPS] = E[\max(Z - X, 0)] \) where \( Z \) is normally distributed random variable for demand with a mean \( \mu \) and a standard deviation of \( \sigma \). \( X \) is the random variable for beginning inventory that is between \([S,S+OPS-1]\).

We also use convolution theory from probability theory which states that if \( W = A+B \), where \( A \) and \( B \) are two continuous random variables:

\[ f_W(w) = \int_{-\infty}^{\infty} f_A(a) f_B(w-a) \, da. \]

This implies that if \( W = Z+(-X) \), we can use the convolution theory to write the probability density of \( W \) as follows:

\[ f_W(w) = \int_{-\infty}^{\infty} f_Z(z) f_{-X}(w-z) \, dz = \int_{-\infty}^{\infty} f_Z(z) f_X(z-w) \, dz. \]

Since \( X \) is only defined between \( S \) and \( S+OPS-1 \), we figure out the bounds within which \( x-w \) is defined and that is simply \( S \leq z-w \leq S + OPS - 1 \). Therefore, the second probability density is defined between \( S + w \leq z \leq S + w + (OPS - 1) \) and we can rewrite the convolution formula as:

\[ f_W(w) = \int_{-\infty}^{\infty} f_Z(z) f_X(z-w) \, dz = \int_{S+w}^{S+w+(OPS-1)} f_Z(z) \frac{1}{OPS-1} \, dz. \]
\[ f_W(w) = \int_{S+w}^{S+w+(\text{OPS}-1)} f_z(z) \frac{1}{\text{OPS}-1} \, dz \]
\[ = \frac{1}{\text{OPS}-1} \int_{S+w}^{S+w+(\text{OPS}-1)} f_z(z) \, dz \]
\[ = \frac{1}{\text{OPS}-1} \left[ F_Z(S + w + (\text{OPS} - 1)) - F_Z(S) \right] \]
\[ = \frac{1}{\text{OPS}-1} \left[ \frac{1}{2} \left( 1 + \text{erf}\left( \frac{S + w + (\text{OPS} - 1) - \mu}{\sqrt{2\sigma}} \right) \right) - \frac{1}{2} \left( 1 + \text{erf}\left( \frac{S + w - \mu}{\sqrt{2\sigma}} \right) \right) \right] \]
\[ = \frac{1}{2(\text{OPS}-1)} \left[ (\text{erf}\left( \frac{S + w + (\text{OPS} - 1) - \mu}{\sqrt{2\sigma}} \right) - \text{erf}\left( \frac{S + w - \mu}{\sqrt{2\sigma}} \right)) \right] \]

(2.15)

\[ E[X\mid \text{OPS = OPS}] = E[\max(Z - X, 0)] \]
\[ = E[\max(W, 0)] \]
\[ = \int_0^\infty w \ast f_W(w) \, dw \]
\[ = \int_0^\infty \frac{1}{2(\text{OPS}-1)} \left[ (\text{erf}\left( \frac{S + w + (\text{OPS} - 1) - \mu}{\sqrt{2\sigma}} \right) - \text{erf}\left( \frac{S + w - \mu}{\sqrt{2\sigma}} \right)) \right] \, dw \]
\[ = \frac{1}{2(\text{OPS}-1)} \left[ \int_0^\infty w \left[ \text{erf}\left( \frac{S + w + (\text{OPS} - 1) - \mu}{\sqrt{2\sigma}} \right) - \text{erf}\left( \frac{S + w - \mu}{\sqrt{2\sigma}} \right) \right] \, dw \right] \]

(2.16)

### 2.8.3 Proof of the distribution of un-truncated ending inventory distribution

**Proof.** We need to find the PDF of ending inventory before it is truncated at 0. We use the continuity approximation factor which implies that \( \left[ \frac{S - Y_{n-1}}{\eta_0} \right] \approx \frac{S - Y_{n-1}}{\eta_0} + \frac{1}{2} \) to derive the distribution.

\[ f(Y_n) = f(X_n - Z) \]
\[ = f(Y_{n-1} + \eta_0 \ast \left[ \frac{S - Y_{n-1}}{\eta_0} \right] - Z) \]
\[ \approx f(Y_{n-1} + \eta_0 \ast (\frac{S - Y_{n-1}}{\eta_0} + \frac{1}{2}) - Z) \]
\[ \approx f((S + \frac{\eta_0}{2}) - Z) \]

\( \square \)
The un-truncated ending inventory, which is the difference between the beginning inventory and normally distributed demand, is also normally distributed because the resultant distribution is a shifted normal distribution.

2.8.4 Beginning inventory distribution from simulation

Figure 2-13 shows histograms for beginning inventory, with deterministic demand, from a simulation of 1,000 review periods. Figure 2-14, shows histograms for the beginning inventory from a simulation of 1,000 review periods with normally distributed demand. The four histograms correspond to arbitrarily selected mean values of demand and a standard deviation of three units for a normal distribution. The OPS and the service level for the uncertain demand is arbitrarily set to 15 units and 60% respectively, for all scenarios. While Figure 2-14 illustrates outcome for specific parameter values, similar behavior can be observed for many different types of demand distribution and service levels in the simulation.

Figure 2-13: Histogram for beginning inventory with deterministic demand
Figure 2-14: Histogram for beginning inventory with normally distributed demand
2.8.5 Trade-off between the space requirement and expected probability of stock-out due to OPS effect: additional examples

Figure 2-15: Illustrations for trade-off for space requirement and stock-out probability
Chapter 3

Improving the profit of a retail establishment through optimal backroom inventory control

3.1 Introduction

A large proportion of supply chain costs in the retail industry are accounted for by in-store operations (Broekmeulen et al., 2004; Hübner et al., 2013). This proportion could be as high as 40%-50% of the operational costs incurred in the retail supply chain (Van Zelst et al., 2009; Reiner et al., 2013; Kuhn and Sternbeck, 2013). Backroom related operations contribute towards a major portion of them (Sternbeck and Kuhn, 2014; Sternbeck, 2015). These operations include receipt of orders, put away of these orders, and making trips into and out of the backroom to retrieve items. The complexity of backroom operations arises because they are closely linked with store planning (frequency of ordering, packaging size etc.) and upstream supply chain (network, sourcing strategies etc.).

Backroom space and its management has been linked to the performance of the retail store and the supply chain associated with it. One of the aspects discussed in literature is related to retail stock-outs due to misplacement and unavailability of SKUs inside the store. Effective management of backroom spaces and related planning & operations have been cited as one of the important factors contributing to this phenomenon (DeHoratius and Ton, 2008; Ehrenthal and Stölzle, 2013; Gruen and Corsten, 2002; McKinnon et al., 2007).
The additional handling process of unpacking to storable units in the backroom (which is also observed in the stores we visited for our research) adds to the ineffectiveness as well as the cost of the in-store logistics operations. The inefficiencies that arise due to moving stock between the retail shelf in the frontroom and the backroom also affects the cost and performance of a retail store (Ehrenthal and Stölzle, 2013). Backroom spaces in a retail store also influence the labor allocation and hence the costs associated with it (Berman and Larson, 2004; Interviews during store visits, 2015).

Despite the potential financial and store performance impact that backroom space and its management has, existing literature suggests that they have typically been ignored in supply chain planning and design decisions (Eroglu et al., 2013). Further, our conversations with a major US retailer (Interviews during store visits, 2015) reveal that in retail industry, frontrooms in a retail store continuously get much more attention in terms of improving operations and allocating space as compared to space management in the backroom. As far as we are aware, academic research related to retail store space also primarily focuses on frontroom or area visible to the customer.

The aim of our research is to address this gap by improving backroom space management within a constrained retail establishment through an inventory theoretic approach. We formulate an optimization model with the objective of improving the retail establishment’s profit through optimal backroom inventory control. The optimal backroom space capacity management and allocation in a retail space can be accomplished through different levels of efficient planning decisions. The levels, namely strategic, tactical and operational, pertain to the planning decisions based in supply chain management literature that are distinguished based on the time horizons and type of decision made (Bender et al., 2002; Vidal and Goetschalckx, 1997).

In this chapter, we present a tactical decision model, which is meant to be run over mid-term time periods (e.g. every quarter depending upon seasonality or trends in the demand data) as compared to long-term periods for strategic and short-term periods for operational decisions. The mixed-integer linear program presented in this chapter is an infinite time horizon model for a retail establishment that deals with multiple end-items/SKUs and is constrained by inventory storage space in the backroom as well as other factors like the SKU order/storable pack sizes and the temperature zones in which the SKUs have to be stored. The replenishment policy is a periodic review and an order-up-to level policy. The
model therefore tries to integrate the effect of different aspects of a retail backroom, such as size and functional capability requirements like temperature controlled storage along with supply chain and store attributes to enable a better informed planning and design of backroom space and logistics. Existing stores with an allocated backroom space can use this model to improve store profit under the above-discussed constraints.

It is also important to note that the work presented in this chapter does not account for the effect of frontroom. More specifically, the revenue function in this chapter is driven by demand that is considered to be a stationary distribution and not dependent on the frontroom space. The effect of the frontroom space on demand is included in Chapter 4 for the strategic backroom space allocation decision.

Furthermore, as mentioned earlier, in this research we specifically focus on building a model for limited service restaurant (LSR) retail establishments. LSRs are interesting case studies for several reasons. LSRs, or quick service restaurants, are characterized by a relatively limited menu, limited and fast service, pre-payment by customers and higher customer turns than a full service restaurant (Ottenbacher and Harrington, 2009; Robson, 2013). The main peculiarity of LSR (as compared to grocery stores) is that many of the SKUs that go into the backrooms have to be assembled into end-items before they are sold to the customer. This characteristic results in a need to translate the decisions that are made on the end-item (items that are sold to consumers) level to the individual component SKUs stored in the backrooms. Moreover, the labor in the retail establishment of a LSR is typically shared between the backroom and the customer facing frontroom space. In other words, the employees that constantly interact with the consumer are also involved in backroom related operations such as replenishing the frontroom with inventory or unpacking and put-away of the SKUs that are delivered from the distribution center in the backroom inventory storage space. Since employee and customer interaction plays a key role in demand generation and fulfillment in a LSR, efficient backroom space management is important to avoid issues such as congested spaces that could result in increased employee time spent in the backroom instead of focusing on the customer.

The rest of the chapter is organized as follows. After a literature review on the importance of a retail establishment’s backroom space management and planning in section 3.2, the chapter introduces the case study and the related backroom space terminology in section 3.3. This is followed by a discussion of the specific research question and the analysis.
framework along with the model flow line that lays out the plan and building blocks for the optimization model in section 3.4. In section 3.5, the chapter discusses the four sub-models, two of which are regarding the translation of end-item demand and service level to that of the ordered SKUs that are stored in the backroom. In the other two models, we introduce the process of accounting for waste and recap of the pack size effect discussed in Chapter 2. We then move on to introduce the formulation of the optimization model in section 3.6. Later, in section 3.7, we provide a discussion on the results obtained by using the optimization model and the data available with us from our industry partner. Finally, in section 3.8, we conclude with a summary of the discussions and the potential extensions for future research.

### 3.2 Literature review on the effect of retail backroom space on store performance

Backrooms have been discussed in the context of multiple aspects related to a retail store. These aspects include store inventory planning decisions (Eroglu et al., 2013; Ronen and Goodhart, 2008), labor productivity, labor allocation and as a source of major handling cost (Berman and Larson, 2004; Interviews during store visits, 2015; Sternbeck and Kuhn, 2014), the effect on upstream retail supply chain (Sternbeck and Kuhn, 2014) and new retail opportunities that strongly link backrooms and store performance (Tompkins International, 2014). The extant literature that have considered backrooms primarily focus on grocery or supermarket retail stores. Hence backroom spaces are only considered as "extra storage" space for spillover inventory that does not fit into the frontroom space (Eroglu et al., 2013). Therefore the "backroom effect", which is driven by this excess inventory, has been modeled as a function of order pack size, shelf space in the front room and reorder point in the inventory cost model (Eroglu et al., 2013). Further, the process of storing this excess inventory in the backroom is seen as a non-value adding process (Sternbeck and Kuhn, 2014).

Additionally, as far as we are aware, most of the studies focus on minimizing cost rather than maximizing profit. However, optimizing for profit is important when modeling for space requirements in the backroom space because of the direct links with the service level for the demand and hence the store performance. As an illustration, we used store level data from our industry partner, to reveal the relationship between total space, revenue and volume of
SKUs stored in the backroom. Figure 3-1 shows that the volume of inventory of the annual ordered quantity for the revenue generating SKUs (not the miscellaneous SKUs held in the backroom for servicing and maintaining the store) that moves through the backroom of any given store is highly correlated with the annual revenue of the store. This is an expected result. However, it is an interesting contrast to Figure 3-2, which is explained as follows.

![Figure 3-1: Backroom cubic volume flow through a store is positively correlated with the revenue generated to a considerable extent.](image1)

![Figure 3-2: Small retail space (thus typically a small backroom) does not mean smaller SKU volume flow through the store/backroom](image2)

Backroom space requirement is proportional to the volume of inventory of the annual ordered quantity, given that a SKU has the same review period across all stores in our dataset. Thus, in case of the stores of our industry partner and many other retailers operating in an urban environment, the size of a store is often not correlated with the total inventory throughput as seen in Figure 3-2. As a consequence, especially the small, high-volume stores can become severely space constrained, with store managers struggling to find sufficient backroom storage space to accommodate demand. This typical discrepancy between the space requirement and space availability motivates our interest in the backroom space utilization problem. It is against this backdrop that we propose an optimization model that captures trade-offs that need to be made between different products in constrained storage spaces in order to maximize profit.

Finally, our work also differs from the existing literature by focusing on LSR retail
establishments, which have not been researched in regards to backroom space management. This results in accounting for backrooms as not just the “extra storage” area but rather the primary storage area. By optimizing for profit, we also try to understand how the properties of the end-items, specifically the demand and the service level, are translated to that of the SKUs that are stored in the backroom space. Another important aspect of this research is that we specifically account for three levels of pack sizes (the level at which SKUs are ordered, stored, and sold to the customer) in our model.

3.3 Case study for the research

The research presented here is inspired by the space management problems faced by our industry partner. As mentioned in Chapter 2, it is a major LSR retailer and is referred to as Deltaco in this thesis. Deltaco has two types of retail establishments: company owned and licensed. However, we only deal with company owned establishments for our analysis because space design, ordering decisions and store operations is not under the retailer’s control for the licensed establishments.

Deltaco has a combination of two types of SKUs that can be ordered by the store manager. The first type of SKUs are processed and combined with other SKUs before they are sold to the customer (also known as the ingredient SKUs). The second type of SKUs are sold in the form that they are ordered (or the non-ingredient SKUs). On the other hand, the end-items, which are sold to the customer, can either be non-ingredient SKUs (or one-to-one end-items) or some combination of ingredient SKUs (processed end-items). Figure 3-3 shows examples of different types of SKUs and end-items that can be found in a store of Deltaco. Furthermore, each end-item falls into one of eight end-item categories.
The backroom space of a Deltaco store is used for storing inventory and also has allocated areas for supporting miscellaneous retail space activities like the workspace station for employees and a space for cleaning supplies. While we explicitly model for the inventory storage area of the backroom, the rest of the space is out of scope in this study. However, we dedicate a constant proportion of the available backroom space to the miscellaneous activities and only optimize for the rest of the space for inventory storage purposes.

There are three types of equipment used in these backrooms for inventory storage corresponding to different temperature zones: (frozen, refrigerated and ambient). These are referred to as zones in the chapter. The three types of equipment have different storage capacities and differ in the amount of space occupied in the backroom. The SKUs stored in a zone may have different delivery patterns, distribution center source, and delivery frequencies. Specifically for Deltaco, the non-ingredient SKUs are found across all the temperature zones. However, the ingredient SKUs are only found in the ambient and the refrigerated zones of the backroom.

3.4 Description of the specific research problem

In this chapter, we design an optimal inventory control policy for a given retail establishment, with a given backroom space and with the objective of maximizing a store’s retail space profit. The decision variable of the optimization model is the optimal service level for different end-items. This translates to the service level requirements of the SKUs that flow...
through the retail backroom and directly affect the utilization of space.

In this chapter, the service level of an end-item ($\gamma$) is defined as the multiplier of its mean demand that gives the equivalent amount of SKU inventory that should be available in the backroom. In other words, if the mean of end-item demand during a time period is $\mu$ then the amount of inventory that is available for sale at the store is equivalent to $\gamma \times \mu$.

The space allocated to different zones in the backroom is proportional to the space occupied by the inventory of the different SKUs that are held in it. There are two levels of space constraints relevant to the model: the total backroom space and the space available within a standard equipment for each temperature zone.

### 3.4.1 Problem setting

Deltaco’s retail establishments use a periodic review with an order up-to level or (R,S) policy for inventory replenishment. This means that for each SKU, the store manager checks the inventory position every review period ‘R’ and replenishes the inventory at the store up-to at least ‘S’ units. The manager is constrained to order in multiples of order pack size. When the order arrives at the store, the SKUs are stored in the storable packs in the backroom. An order pack typically consists of multiple storage packs.

We model the backroom space allocation within a single store that handles multiple SKUs competing for space in a constrained environment. Different SKUs can have different replenishment cycles or review periods. Deltaco has three types of replenishment cycles: one day, one week or two weeks. The orders are received by the store before it opens for business on a given day. Additionally, the orders placed during the day are replenished the following business day. Any unsatisfied demand is considered as lost sales, a scenario which is usually observed in LSRs. Moreover, we assume a stationary normal distribution of the end-item demand during a review period.

The revenue obtained at a Deltaco store is limited by the demand and the amount of inventory available for consumption. The revenue of a processed end-item is limited by the inventory of its ingredient SKUs. On the other hand, the revenue for the frozen end-items, which are one-to-one, is a function of the amount of its inventory that is available in the backroom.

The space occupied in the backroom is obtained in two steps. The amount of space allocated to a SKU is calculated as the volume of space occupied by its average beginning
inventory within each equipment in the backroom. The total volume of space occupied across all SKUs within an equipment is then translated to the number of racks, freezers or refrigerators required. This number is then translated to the area allocated to the inventory storage space of the backroom. Stacking process and stacking orientation of the SKUs in the backrooms is not considered in the model.

3.4.2 Analysis framework

In order to determine the optimal number of different equipment types in the zones of a given backroom space within a LSR retail establishment, we formulate a mixed-integer linear program. Figure 3-4 illustrates the framework of the analysis, which includes the inputs and sub-models that are integrated with the optimization model.

The sub-models include the pack size effect model and the waste model. Additionally, there are two other other models that are used to translate end-item property to SKU level property. These include the service level translation model and the demand model that simply translates the end-item demand to SKU level demand using the bill of materials. The arrows in the model flow diagram indicate the direction of information flow and the different colors refer to the decision variable, the inputs, the sub-models and the profit function components.

In the following sections, we describe the pack size, waste, demand translation and the service level translation sub-models. The pack size effect sub-model uses the derivation of the inventory levels as a function of the order pack size, order up-to level 'S', and the effect of the storable pack size to derive the amount of space occupied by a SKU in the backroom, as described in Chapter 2. This is followed by the formulation of the optimization model using the sub-models and the space constraints.
3.5 Discussion of the different sub-models of the optimization model

In this section we will discuss the four key sub-models that are incorporated into the optimization model. These sub-models include the (1) pack-size effect model, (2) service level translation (3) demand translation and (4) waste model.

3.5.1 Accounting for the effect of pack sizes in the analysis

As discussed in Chapter 2, there are two levels of pack size: order packs (OPS) and storable packs (SPS) that affect retail storage space and operations planning and are integer multiples of sellable packs (CPS). CPS is defined as (a) the pack size level in which a SKU is sold to the customer for non-ingredient SKUs (e.g. eaches for a muffin) or (b) the lowest manufacturing pack size unit for an ingredient SKU (e.g. gallon for milk). For the purposes of this study,
CPS is set to one. Moreover, SKUs are stored in the backroom in storable packs till the last sellable unit within it is used. Therefore, the SKUs are using the space equivalent of a SPS or integer multiple of it until the last sellable unit. So, while OPS affects the inventory levels, the SPS affects the corresponding space requirements. Therefore, the OPS and SPS of a SKU are pertinent to the backroom space capacity planning model.

We use the approximation factor for the amount of excess inventory under an (R,S) replenishment policy as derived in Chapter 2 in order to model the average inventory and beginning inventory levels of a SKU. The space occupied by a SKU is given by the number of equivalent storable packs corresponding to its beginning inventory level. For a given beginning inventory level 'I' and storable pack size \( \eta_s \), the number of storable packs is given by \( \lceil \frac{I}{\eta_s} \rceil \), where \( \lceil x \rceil \) denotes the ceiling function of x. The space occupied as a result is given by \( \left( \lceil \frac{I}{\eta_s} \rceil \right) \times v \), where v is the space occupied by a single storable pack. For a SKU that has a mean daily demand of \( \mu \), a review period of R and an OPS of \( \eta_0 \), the relevant inventory levels that we are concerned with in our model, if \( S \geq \mu \) are as follows:

1. Average inventory level during a review period cycle (\( \bar{I} \)): This is the average inventory held during a review period and is affected by the OPS. This inventory level is used to compute the holding cost during the review period when demand is uncertain.

\[
\bar{I} = \frac{\mu \ast R}{2} + \text{SS} + \frac{\eta_0 - 1}{2} \text{ excess due to OPS}
\]

2. Inventory affecting the storage space- Average beginning inventory: There are two ways of obtaining the storage space or the number of storable packs that are carried during a review period. One could plan for space usage in terms of the average beginning inventory during a review period or the absolute maximum of the beginning inventory observed during a review period, as defined below. When modeling for backroom space utilization in an infinite horizon, average beginning inventory is an appropriate level for space calculation requirements in a store since there is a low likelihood of the inventory level of all the SKUs being at the maximum level at the same time when demand is uncertain. Therefore, we define the average beginning inventory as follows:

Average beginning inventory (\( \bar{I}_{\text{beg}} \)): This is the average of the peak inventory after replenishment, which happens before the store opens for business, over the review
period cycle

\[ \overline{I_{beg}} = \mu \ast R + SS + \frac{\eta_{i}-1}{2} \]

The equivalent number of storable packs is given by:

\[ \left[ \frac{\overline{I_{beg}}}{\eta_{i}} \right] = \left[ \frac{\mu \ast R + SS + \frac{\eta_{i}-1}{2}}{\eta_{i}} \right] \]

### 3.5.2 Sub-model for demand translation process

Demand is observed at the level of end-items. This demand needs to be translated to the SKU level for the implementation in the optimization model. This is achieved by using the bill of materials available from Deltaco. Thus, for a SKU i the demand is given by:

\[ D_{i} = \sum_{j=1}^{u_{i}} w_{i,j} \ast D_{j} \]

- \( D_{j} \): random variable for demand distribution for end-item j
- \( w_{i,j} \): quantity of SKU i that is used in end-item j as obtained from the bill of materials
- \( u_{i} \): set of end-items that contain ingredient SKU i

### 3.5.3 Sub-model for service level translation

The service level of the end-items is the decision variable of the optimization model. The service level of the ordered SKUs, which translates to its inventory level and therefore the costs and the space requirements, can be obtained as a function of the service level of all the end-items that use the particular SKU.

In this section, we establish the relationship between service level of the end-item and that of the SKUs, which is also another sub-model for the optimization model. In order to so, we introduce the following notation. Let us consider a SKU i that is used in \( u_{i} \) end-items. Additionally, let:-

- \( k_{i} \): service level factor for the ordered SKU i. The product of service level factor and the standard deviation of the demand is equal to the safety stock amount carried (Silver et al., 1998)
- \( \sigma_{i} \): standard deviation of demand of the ordered SKU i
\[ \mu_i: \text{mean demand of ordered SKU } i \]

\[ k_j: \text{service level factor of the end-item } j \text{ with } j \in u_i \]

\[ \sigma_j: \text{standard deviation of demand of end-item } j \text{ with } j \in u_i \]

\[ \mu_j: \text{mean of demand of the end-item } j \text{ with } j \in u_i \]

\[ w_{ij}: \text{quantity of ordered SKU } i \text{ in end-item } j \]

\[ D_j: \text{random variable for demand for end-item } j \]

\[ D_i: \text{random variable for demand for ordered SKU } i \]

As shown in the previous section, the demand for the ordered SKU \( i \) is given by \( D_i = \sum_{j=1}^{u_i} w_{ij} * D_j \). Since demand for the end-items is normally distributed, the demand for a SKU is also normally distributed with the mean and the standard deviation of the resulting distribution as \( D_i \sim N(\sum_{j=1}^{u_i} w_{ij} * \mu_j, \sqrt{\sum_{j=1}^{u_i} (w_{ij} * \sigma_j)^2}) \).

Thus, the safety stock level for each end item \( j \) is given by \( \mu_j * \sigma_j \). Therefore the safety stock level for SKU \( i \), that is an ingredient for end-item \( j \), is given by \( w_{ij} * \mu_j * \sigma_j \). This implies that the total safety stock level for SKU \( i \), given all the end-items \( j \) that use it is given by \( \sum_{j=1}^{u_i} w_{ij} * \mu_j * \sigma_j \). Therefore,

\[
k_i * \sigma_i = \sum_{j=1}^{u_i} w_{ij} * k_j * \sigma_j
\]

\[
k_i * \sqrt{\sum_{j=1}^{u_i} (w_{ij} * \sigma_j)^2} = \sum_{j=1}^{u_i} w_{ij} * k_j * \sigma_j
\]

\[
k_i = \frac{\sum_{j=1}^{u_i} w_{ij} * k_j * \sigma_j}{\sqrt{\sum_{j=1}^{u_i} (w_{ij} * \sigma_j)^2}}
\]

### 3.5.4 Sub-model for waste

Waste is defined as any outflow of the SKU that does not generate revenue. The quantity wasted occupies space in the backroom and also adds to inventory related costs, namely the purchase and the holding costs. There could be multiple sources of waste in a LSR, including spillage, exceeding the shelf life of the SKU, food donations or even offering end-items for free during promotions.

We estimate the average waste percentage of different SKU categories from our data, which is then attributed to all the SKUs in the category for the purposes of our analysis.
average waste percentage ($\beta$) for a SKU is defined as the amount wasted as a percentage of average quantity ordered over a review period.

For a given SKU let,

- $\beta$: average waste percentage
- $Q_w$: average quantity wasted
- $Q$: average ordered quantity
- $E[US]$: expected units sold during a review period

Since only a portion of SKU quantity ordered generates revenue and the rest is lost to waste:

$$Q * (1 - \beta) = E[US]$$

But, by definition the average waste percentage is given by:

$$\beta = \frac{Q_w}{Q}$$

Using the above equations, the average quantity wasted is given by:

$$Q_w = \beta * Q$$

$$= \left(\frac{\beta}{1 - \beta}\right) * E[US]$$

3.6 Optimization model

3.6.1 Overview of the optimization model

As discussed earlier, the constrained optimization model presented in this chapter maximizes expected profit for a single LSR over all the SKUs that move through its backroom and all the end-items that are offered. It is a continuous time infinite horizon model. SKUs are stored in different temperature zones and can have different review periods.

The multiple review periods aspect is handled by maximizing the expected profit over a time period $T$, where $T$ is the lowest common multiplier of all the possible review periods across all SKUs. For instance, in case of Deltaco, since the review period for the SKUs can be one day or one week (7 days) or two weeks (14 days), $T$ is set to two weeks (14 days).

Other structural details of the optimization model are as follows:
Unit of analysis: single retail establishment of the LSR, with a given total retail space and backroom space

Objective: maximize expected profit obtained in a given time period T

Decision variables: multiplier of the mean demand of end-item j that indicates the service level of the end-item. This variable is denoted as $\gamma_j$ and is always non-negative. This is then used to translate it to service level of each SKU i that is stored in the backroom in each temperature zone. The translation process between service level of the end-items and the SKUs used is described in section 3.5.3.

Moreover, the relationship between the decision variable and the service level factor $k_j$ is given by: $\gamma_j \mu_j = \mu_j + k_j \sigma_j$, where $\mu_j$ and $\sigma_j$ are the mean and the standard deviation of end-item demand respectively.

Objective function components:

1. Revenue
2. Lost sales cost
3. Direct SKU purchase cost
4. Cost of waste
5. Inventory holding cost

Assumptions

1. Infinite horizon model
2. Periodic review and order up-to level policy
3. Stationary demand distribution
4. Independent demand distributions for individual end-items within an end-item category
5. Instantaneous replenishment
6. Demand over review period is assumed to be normally distributed
7. No seasonality of demand
8. No restrictions on how SKUs are packed in the backroom
9. Constant rate of consumption during a review period
3.6.2 Additional notations

For a LSR retail establishment:

- $X$: total retail space (sq.ft.)
- $\tau$: proportion of total retail space that is available for backroom inventory storage
- $T$: integer time period which is lowest common multiplier of review periods across all SKUs (days)
- $z$: zones in the store (frozen (f), refrigerated (r), ambient (a))
- $\delta^z_e$: number of equipment used for the specific zone $z$ in the store
- $z_{cap}$: the capacity of each piece of equipment in zone $z$ (boxes (f), shelves (r) and cubic feet (a))
- $z_{a,e}$: the area occupied by an equipment in zone $z$ (sq.ft.)
- $n$: number of SKUs

For a SKU $i$:

- $\gamma_i$: service level of SKU
- $\mu_i$: mean daily demand (CPS/day)
- $\sigma_i$: standard deviation of daily demand (CPS/day)
- $R_i$: review period for SKU (days)
- $k_i$: service level factor for SKU
- $h_i$: annual holding cost rate
- $\eta_{o,i}$: OPS of a SKU (e.g. CPS/case)
- $\eta_{s,i}$: SPS of a SKU (e.g. CPS/intra-pack (IP))
- $v_{i_z}$: space equivalent of SKU in zone $z$ (cu.ft./IP or shelves/IP or boxes/IP)
- $c_{i}$: cost per unit for SKU ($$/SKU)
• \( \delta_{i,SP} \): number of storable packs equivalent to the inventory carried for SKU in zone \( z \)

• \( L(k_i, Z) \): standard normal loss function value for service level factor \( k_i \)

• \( \beta_i \): SKU waste as a percentage of quantity ordered

• \( u_i \): number of end-items that use SKU \( i \)

• \( w_{i,j} \): quantity of SKU \( i \) in end-item \( j \)

For an end-item \( j \):

• \( \gamma_j \): service level of end-item

• \( \mu_j \): mean daily demand (CPS/day)

• \( \sigma_j \): standard deviation of daily demand (CPS/day)

• \( k_j \): service level factor for end-item

• \( p_j \): selling price per sellable unit of end-item (\$/CPS)

• \( L(k_j, Z) \): standard normal loss function value for service level factor \( k_j \)

• \( \text{Rev}_j \): expected daily revenue for end-item \( j \) (\$/day)

For end-item category:

• \( \alpha_{y-y'} \): coefficient indicating the relation between sales of end-item category \( y \) and end-item category \( y' \)

• \( m_{\text{cat}=y} \): number of end-items in category \( y \)

• cat: end-item category

### 3.6.3 Optimization model

Equations 3.1 through 3.11 depict the optimization model for this chapter.
The objective function as shown in equation 3.1 and the constraints that are shown between equations 3.2 to 3.11 are described in sections 3.6.4 and 3.6.5.
3.6.4 Derivation of the objective function

For this section we define the order-up-to levels and the corresponding service level factors for the end-item and the SKU as follows.

The order-up-to level for an end-item \( j \) is given by \( \gamma_j \ast \mu_j \). This implies that the service level factor of an end-item \( j \) can be written as \( k_j = \frac{(\gamma_j-1)\ast\mu_j}{\sigma_j} \). Similarly, order-up-to level for SKU \( i \) when \( \eta_{0,i}=1 \), is equal to \( \gamma_i \ast \mu_i \ast R_i \). However, when \( \eta_{0,i}>1 \), order-up-to level for SKU \( i \) is given by \( \gamma_i \ast \mu_i \ast R_i + \frac{\eta_{0,i}-1}{2} \). The effective service level factor for SKU \( i \) is defined as \( k_i = \frac{(\gamma_i-1)\mu_i R_i + \frac{\eta_{0,i}-1}{2}}{\sigma_i \sqrt{(R_i)}} \). The effective service level factor for the SKU, therefore, adjusts for the excess inventory that is carried because of the OPS effect.

Hence, the components of the objective function become:

- **Expected revenue** is the sum of the revenue across the unique end-item categories during the time period \( T \). The revenue that is obtained from a specific category, \( 'y' \), is derived by calculating the minimum of two quantities. The first quantity is the cumulative revenue of all the end-items that belong to category \( y \). The second quantity is the sum of the revenue obtained from the remaining categories adjusted by a correlation factor, \( '\alpha_{y,y}' \), which is explained below.

*Correlated end-item category revenue:* The revenue obtained from different end-item categories are correlated with each other. Without the consideration of these correlations, the model could potentially suggest to carry highly skewed inventory levels for SKUs pertaining to a specific end-item category. For instance, with the retailer data available to us, the model, without the consideration of the correlations suggests to carry only ambient and refrigerated SKUs and no frozen SKUs. It might be helpful to note that the frozen SKUs are typically less profitable on an average and are also one-to-one end items. Therefore the specific end-item category that corresponds to frozen SKUs is eliminated from the optimal solution when backroom space is constrained and the end-item category correlations are not considered.

Therefore, in order to account for these correlations, we use a regression model to explain the expected revenue for an end-item category as a function of the revenue from other end-item categories that are offered in the store. Therefore, the optimization model constraints the total revenue obtained from an end-item category by the *minimum* of the revenue that can be obtained from the inventory carried for the SKUs.
in the specific category and the revenue that can be obtained from the inventory carried for the SKUs belonging to the other categories that are correlated with it and are offered in the LSR.

For the purposes of the analysis shown in this chapter, a couple of assumptions are made for estimating the relations between the revenue obtained from different end-item categories.

- Consideration of correlations between end-item categories instead of correlations between individual end-items: while individual end-items within a category are probably substitutable, we consider complementarity between different end-item categories. Modeling for relations between individual end-items is left for future research.

- We fit zero intercept regression models to explain the sales of an end-item category as a function of the sales of the other end-item categories. This implies that we assume a perfect complementarity between the end-item categories if there is indeed a significant relationship that exists between them.

Therefore, by assuming the two points discussed above and using our data to model for the relations between the revenue from the point of sales information of the end-item categories, we are preserving the current proportion of sales across these categories offered in a LSR.

There are eight end-item categories in the LSRs of Deltaco, denoted as Cat-A, Cat-B and so on up to Cat-H. These categories are based on the company’s definition of end-item types. Moreover, categories A, B, C and D are all beverages and processed end-items while categories E, F, G and H are all one-to-one end-items where E is a beverage end-item and F,G and H are food end-items. Multiple linear regression was used to explain revenue of a specific category as a function of sales of other end-item categories, the results of which are shown below:

\[
\text{Revenue}_{\text{Cat-A}} = 4.03 \times \text{Revenue}_{\text{Cat-E}}. \quad (R \text{ squared}= 81\%)
\]

\[
\text{Revenue}_{\text{Cat-B}} = 7.68 \times \text{Revenue}_{\text{Cat-F}} + 0.95 \times \text{Revenue}_{\text{Cat-G}}. \quad (R \text{ squared}= 83\%)
\]

\[
\text{Revenue}_{\text{Cat-C}} = 0.83 \times \text{Revenue}_{\text{Cat-F}} + 2.05 \times \text{Revenue}_{\text{Cat-G}}. \quad (R \text{ squared}= 71\%)
\]

\[
\text{Revenue}_{\text{Cat-D}} = 3.88 \times \text{Revenue}_{\text{Cat-F}} + 0.62 \times \text{Revenue}_{\text{Cat-E}}. \quad (R \text{ squared}= 71\%)
\]
In this chapter, we choose to implement the regression models whose R-squared values were found to be greater than 70%. Category H was not found to be a significant variable in explaining the relationship between the processed and the one-to-one end-item categories. This category also had the lowest sales contribution amongst all the categories during the time period that is considered for our analysis.

- **Purchase cost** during a review period for SKU i is given by:

  \[ c_i \cdot E[\text{units sold for SKU } i] = c_i \cdot (\mu_i \cdot R_i - \sigma_i \cdot \sqrt{R_i} \cdot L(k_i, Z)) \]

  Purchase cost during time period T is therefore:

  \[ \frac{T}{R_i} \cdot c_i \cdot (\mu_i \cdot R_i - \sigma_i \cdot \sqrt{R_i} \cdot L(k_i, Z)) \]

- **Cost of waste:** given that waste is defined as a percentage of average quantity ordered over review period, the cost of waste is, as shown in section 3.5.4, given by:

  \[ c_i \cdot \frac{\beta_i}{1-\beta_i} \cdot (\mu_i R_i - \sigma_i \sqrt{R_i} \cdot L(k_i, Z)) \]

- **Inventory holding cost** during a single review period is driven by two types of inventory on hand, inventory that is available for sales, which is derived in appendix 3.9.1, and inventory that is wasted, which is derived in section 3.5.4 of the chapter:

  \[ c_i \cdot \text{holding rate} \cdot (E[\text{inventory on hand}]_{\text{sales}} + E[\text{inventory on hand}]_{\text{waste}}) \]

  \[ = c_i \cdot (h_i \cdot \frac{R_i}{365}) \cdot (\mu_i R_i + \frac{\sigma_i \cdot R_i}{2} - 0.5 \cdot (\mu_i R_i - \sigma_i \sqrt{R_i} \cdot L(k_i, Z)) + 0.5 \cdot \frac{\beta_i}{1-\beta_i} \cdot (\mu_i R_i - \sigma_i \sqrt{R_i} \cdot L(k_i, Z))) \]

  Inventory holding cost over time period T is therefore equal to:

  \[ \frac{T}{R_i} \cdot c_i \cdot (h_i \cdot \frac{R_i}{365}) \cdot (\mu_i R_i + \frac{\sigma_i \cdot R_i}{2} - 0.5 \cdot (\mu_i R_i - \sigma_i \sqrt{R_i} \cdot L(k_i, Z)) + 0.5 \cdot \frac{\beta_i}{1-\beta_i} \cdot (\mu_i R_i - \sigma_i \sqrt{R_i} \cdot L(k_i, Z))) \]
3.6.5 Constraints of the optimization model

The constraints of the optimization model as indicated by equations 3.2 through 3.11 are described below:

- **Equation 3.2**: Revenue of an end-item $j$ is driven by the expected units sold that accounts for lost sales under an uncertain demand scenario. The daily expected revenue is as follows:

  $$ p_j \cdot E[\text{units sold for end-item } j] = p_j \cdot (\mu_j - \sigma_j \cdot L(k_j, Z)) $$

  The derivation of the expected units sold is shown in section 3.9.2 of the appendix.

  Therefore, the revenue over the $T$ time period is given by:

  $$ T \cdot p_j \cdot (\mu_j - \sigma_j \cdot L(k_j, Z)) $$

- **Equation 3.3**: Service level factor for SKU $i$ can be expressed as the function of the service level factor of the end-item $j$ by using the translation model described in section 3.5.3.

- **Equation 3.4**: Service level factor for end-item $j$ can be expressed as the function of the decision variable or service level of end-item $j$, which is denoted by $\gamma_j$.

- **Equation 3.5**: Similarly, the service level factor for SKU $i$ can be expressed as the function of the service level of SKU $i$, which is denoted by $\gamma_i$. This equation accounts for the excess inventory that is carried due to an OPS $> 1$.

- **Equation 3.6**: Binary variable that indicates if an excess amount of inventory, which is a function of OPS, is carried. This constraint ascertains that an excess amount due to OPS is only carried if there is a non-zero amount of SKU $i$ inventory or in other words when $\gamma_i > 0$

- **Equation 3.7**: This equation gives the inventory of a SKU in storable packs.

- **Equation 3.8**: This equation gives the amount of equipment for each temperature zone. It is designed to consider the limited capacity of the individual equipment in the different temperature zones.

- **Equation 3.9**: This constraint makes sure that the total space occupied by all the equipment in the LSR does not exceed the capacity of the backroom space that can be used for inventory storage.
• Equation 3.10: This constraint is added to make sure that the number of storable packs and equipment are non-negative and integers.

• Equation 3.11: This constraint is added to make sure that the service level of the SKUs and the end-items are non-negative. In other words, this constraint ascertains that the SKU inventory carried cannot be negative.

The optimization model presented above can be written as a mixed-integer linear program. We use piecewise linearization (Bertsimas and Tsitsiklis, 1997) along with Special Ordered Set (SOS) constraint (Beale and Tomlin, 1970; Guide, 2012) to linearize the loss function in the expected units sold component. We also implement optimal partitioning for the standard normal loss function as proposed by Rossi et al. (Rossi et al., 2014). Details of the linearization of the loss function is provided in section 3.9.3 of the appendix. The binary constraint that indicates whether any excess amount is carried for a SKU, is linearized by implementing big-M conditions.

3.7 Results

The optimization model presented in this chapter can help retailers plan for their inventory in the store backroom and hence efficiently utilize the available backroom space, given pack size and space constraints. Data for Deltaco stores is used for the purpose of illustrating the impact on the annual expected profit, when given a total retail space and the demand of end-items in the establishment. The annual expected profit is obtained by scaling the profit obtained over the ‘T’ time period over the number of days that the LSR is open for business in a fiscal year.

We use information for 30 LSRs of Deltaco in the Massachusetts region to evaluate the optimal equipment requirement for a given backroom space and demand information related to a retail establishment. Point of sales data from the months of March to May are used to estimate the demand for the different end-items offered in each of the 30 LSRs. The time period of March to May was chosen in order to get a representative set of end-items that are regularly sold in the establishment, rather than being offered only during a promotional period. The number of end-items currently served across these stores vary between 207 to 381, which need around 190 SKUs in the backroom. Moreover, we also use a constant
proportion of 70% of the available backroom space that can be used for inventory storage. This proportion amount is an assumption and is expected to be validated in the future.

End-item demand is an input to the model. However, the data that is available to us only provides information for the quantity that is sold for each end-item, which can be obtained from the point-of-sales (POS) table. A number of factors including the ones that are backroom related can affect the lost demand. One such factors is the space available in the backroom for inventory storage. Therefore, it is important to account for demand in the model for a backroom capacity allocation problem.

Different sophisticated statistical techniques can be used to infer demand from the POS data. However, as a starting point, demand for an end-item is considered to be a constant proportion of the sales observed at the store, more specifically 1.15 times of the end-item sales. The 30 LSRs differ in the end-items that are offered, their demand distribution parameters and the total retail space and hence the backroom space available. The end-items offered in the Deltaco LSRs are categorized into eight different categories, denoted as Category A-H. Categories E,F and G correspond to food items while the rest are beverages that are either processed or one-to-one end-items. k-means clustering was used to segment the 30 LSRs into three types depending on the mean of the daily demand across the end-item categories, the result of which is shown in Figure 3-5. In the following sections, we discuss some results related to the effect of space and the demand on the annual expected profit, the cost of having a constrained backroom space and finally the effect on the store performance with change in the OPS and SPS of the SKUs that are stored in the LSR’s backroom.
3.7.1 Annual expected profit obtained with a given backroom space

Table 3.1 shows the optimal expected annual profit that can be obtained in a specific LSR with a given total retail space, 20 percent of which is allocated to the backroom. The corresponding number of equipment required in different temperature zones is shown next to it. The LSRs in the table are arranged in increasing order of the total retail space.

The amount of profit that can be obtained in the LSR is either limited by the demand and/or the backroom space available to support fulfillment of the demand. For instance, LSRs SID_5 and SID_6 have the same total retail space of 1,000 sq.ft. and hence also the allocated amount of backroom space available for inventory storage, but have different demand levels as indicated by the clusters that they belong to. More specifically, SID_5 belongs to cluster 3 while SID_6 belongs to cluster 1. Therefore, the expected annual profit that can be generated by SID_5 is more than that of SID_6 when both of the establishments have 200 sq.ft. of backroom space available. On the other hand, LSRs SID_1 and SID_22 belong to the same cluster, 2, but have very different expected annual profit outcomes because SID_1 is severely limited by space as compared to SID_22 in order to be able to support the same level of demand.
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<th>LSR cluster</th>
<th>Total retail space (in sq.ft.)</th>
<th>No. of ambient equipment</th>
<th>No. of frozen equipment</th>
<th>No. of refrigerated equipment</th>
<th>Expected annual profit (in mm $)</th>
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Table 3.1: Optimization model results for the expected annual profit calculated across all the 30 LSRs. Backroom space is 20% of the total retail space.
3.7.2 The cost of a constrained backroom space in a LSR

In this section, we use the information for a single LSR (SID_19) in order to illustrate the effect of constrained backroom spaces on the expected annual profit and the corresponding equipment requirement across the different temperature zones. SID_19 was chosen because it is representative of the total retail space and the demand levels for a LSR in the Massachusetts region.

The optimization model was run for incremental scenarios of backroom space for SID_19, which has a total retail space of 1,585 sq.ft. The backroom space was varied between 5% (79 sq.ft.) to 30% (476 sq.ft.) of the total retail space in steps of 2.5% of the space or around 40 sq.ft.

Initially, for a very constrained backroom space, the LSR is unable to support the demand observed. As more space is allocated to the backroom, the expected annual profit increases till it reaches a point after which any increase in the amount of backroom space available is not useful because of the level of external demand observed. For SID_19, increasing the backroom space from 119 sq.ft to 159 sq.ft for the a given demand level, yields a profit improvement of nearly 117%, which could also be interpreted as the opportunity cost of designing a LSR with a constrained backroom space.

This improvement, however, is expected to be affected by other costs that are not included in our optimization model. Specifically, increasing the amount of backroom space implies reducing the frontroom space, which generates demand and therefore could potentially reduce the extent of improvement observed. On the other hand, other costs like in-store logistics cost that is driven by congested backroom spaces could result in a larger improvement of profit when backroom space is increased from 119 sq.ft to 159 sq.ft.
Figure 3-6: Illustration of the effect of constrained backroom space in a LSR: Expected profit change in SID_19

Table 3.2: Illustration of the effect of constrained backroom space in a LSR: Equipment requirement across different temperature zones in SID_19

The increase in the annual expected profit when the allocated backroom space increases
is because the LSR is able to offer additional end-items since it now has the storage space to carry the inventory required. This is illustrated in Figure 3-7, which compares the change in service level ($\gamma_j$) for different end-items when the backroom space available increases from 119 sq.ft to 159 sq.ft. The change in the service level is indicated by the change in the $\gamma$ value or the level above the mean demand for a specific end-item.

By increasing the backroom space from 7.5% to 10% the end-items behave differently. Group-1 (G1) end-items which were not not able to be offered are now available in the assortment. Group-2 end-items (G2) can now be offered at a higher service level because allocating the extra space to carry more SKU inventory corresponding to this group enhances the expected profit for LSR. Group-3 (G3) end-items, on the other hand, have no change in their service level. Finally, a small number of end-items that fall into Group-4 (G4) are now available at a decreased service level to optimally use the increased backroom space.

![Figure 3-7: The change in the service level of the end-items ($\gamma_j$) with increase in the backroom space available](image)

G2: Store can increase the service level for end-items in group 2.

G1: Store can now offer end-items in group 1.

G4: Store chooses to provide a lower service level for the end-items in group 4.

G3: Store continues to provide the same service level for the end-items in group 3 (that fall on the 45 degree line).
3.7.3 The effect of pack sizes on the performance of a LSR with a constrained backroom space

As discussed earlier, there are two levels of pack size, namely the OPS and the SPS, that are relevant to the utilization and management of the backroom space within a LSR. The SPS has a direct effect on the space requirement in the backroom for a given amount of SKU inventory carried. On the other hand, while space requirements could potentially also change with changes in OPS, the reason for this effect is different than for SPS. Adjustments to the OPS change the amount of inventory carried in the backroom and, therefore, the space required for storage as well as profits. Thus by extension, both the pack size levels (SPS and OPS) affect the amount of equipment required in the backroom as well as the annual expected profit of a LSR.

Figure 3-8 illustrates the effect on the expected annual profit with the change in pack sizes. Specifically, the plot in the figure shows the change of profit over three scenarios when compared to the profit obtained under current SPS and OPS levels for the SKUs, which is also known as the base case scenario. The three other scenarios are defined in the following way. Scenario-1 is the case when the SPS for only SKUs from the paper category is set to one and OPS is at the current level. Scenario-2 is the case when the SPS of all SKUs is set to one and OPS is at the current level. Finally, scenario-3 is when the OPS and the SPS of all SKUs is set to one.

Reducing the OPS or SPS to one is more likely sub-optimal for retailers like Deltaco when the entire retail supply chain is accounted for. This is because while smaller pack sizes are preferable in the retail establishments, they could lead to large handling or labor costs at the distribution centers. However, the purpose of using the scenarios for the analysis is to illustrate as to how we can determine an upper bound for improvement in pack sizes at the retail establishments. In other words, if a retailer aims for more realistic reductions in SPS and OPS, which will be a size that is greater than one, the LSRs should be able to obtain a portion of the benefits that are quantified in this section.

The SKUs whose SPS is changed in scenario-1 are the paper items that are essential to be made available in order to make a sale of a food or a beverage end-item. The SKUs that fall into the paper category are also identified as the 'worst offenders of space' due to their relatively large OPS and SPS in Deltaco's retail establishments. Moreover, paper SKUs are
associated with all the end-item categories that are demanded and served in a LSR.

Based on an assumed consumer preference for the availability of the SKUs that fall into the paper category, they are categorized as priority and non-priority SKUs. Priority paper SKUs, like cups, are indispensable: a customer does not buy the end-item if the LSR does not carry the SKU. The non-priority paper SKUs, like napkins, could be available only a certain number of times, specifically 4 out of 5 times in this analysis, in order to make a sale of the end-item that is associated with it. This is another parametric input to the optimization model.

It was observed that for the same proportional change in the OPS/SPS across all the SKUs, the proportion of change in the space requirement is different. Additionally, for a given SKU, change in the space requirement is not proportional to the change in the OPS/SPS. It is because the change in the space requirement is driven by other factors like the inventory carried and the space occupied by a unit storable pack of a SKU. Therefore it can be helpful to identify the “worst offenders”, or the ones that occupy large amounts of space, amongst the SKUs of the backroom.

When there is enough amount of backroom space available to support the observed demand, there is smaller change in the amount of inventory carried (in terms of the sellable units) in the backroom due to the change in pack sizes. Therefore, the equipment requirement across the different temperature zones and the expected annual profit are similar across the different cases. This is illustrated in Figure 3-8, which shows the change in the expected annual profit when the amount of backroom space available is 317 sq.ft. in a 1,585 sq.ft. retail establishment. As expected, the largest improvement in profit, which is nearly an increase of 14%, happens when the OPS and the SPS is changed to 1 for all the SKUs. This again, does not account for savings due to in-store logistics costs e.g. labor/time costs associated with breaking the order packs of a SKU into its storable packs.

Such an analysis can be used by LSR managers to quantify the potential impact of change in SPS and OPS on their space requirement and the value of this space. Of course, this impact should be traded off against other changes (e.g. handling costs or network effects) in order to understand the system wide impact before making any decisions.

On the other hand, when backroom spaces are more constrained to begin with, the LSR stands to receive much higher gains in the expected annual profit when the pack sizes are reduced. The same figure (Figure 3-8) shows the change in the expected annual profit when
the LSR has only 119 sq.ft. allocated to the backroom to support the same demand levels. As compared to the case with the current SPS and OPS, the improvement in profit is nearly equal to 83%, 100% and 147% for scenarios 1, 2 and 3 respectively. Since changing the SPS of only the paper SKUs alone accounts for a large proportion of the increase in the annual expected profit, it suggests that Deltaco could choose to focus their initial attention to the paper category alone in order to reap a substantial benefit. Furthermore, for the LSR whose data is used to illustrate results in this section, the profit obtained in the third case for a smaller backroom space of 119 sq.ft. is almost similar to that obtained with a larger allocated space of 317 sq.ft. In other words, Deltaco’s LSRs could get the same improvement in their profit from reducing the pack sizes of their SKUs as compared to the improvement in profit that is made possible by increasing the total backroom space available for inventory storage.

![Graph showing changes in annual expected profit with change in OPS and SPS of a SKU](image)

**Figure 3-8:** Change in annual expected profit with change in OPS and SPS of a SKU

The change of SPS to one unit for the SKUs in the paper category results in an increase in the expected profit because this clears up space to carry other SKUs. This results in improved service levels for the corresponding end-items. Figure 3-9 illustrates the change in the $\gamma$ factor of the end-items for the case when the SPS is set to one as compared to the
current SPS parameter for the paper SKUs. It might be helpful to know that categories A, B, C and D require 2 priority and 1 non-priority paper SKU. On the other hand, categories E and F require 1 priority & 1 non-priority paper SKU and G and H require a single non-priority paper SKU.

When the SPS of the paper products are set to one, the model increases the service level factors of the end-items from categories that could not be served because of larger SPS even though they are more profitable (higher profit margin per unit on an average) than the others, as indicated by group-2 (G2) and group 1 (G1) in the figure. At the same time, the service level of the end-items in group 4 (G4) decreases under constrained backroom spaces. Furthermore, while group-1 (G1) end-items are added to the assortment, group-5 (G5) are removed from it. Group 3 (G3) end-items, on the other hand, are offered at the same service level.

![Figure 3-9: Change in the service level of the end-items (γj) when SPS is changed to one for paper SKUs only.](image)

Similarly, increasing the OPS always results in a larger amount of inventory carried for
a given SKU in the backroom and therefore the space required also increases. The opposite
is observed when OPS is decreased. On the other hand, the direction of change in profit
depends on the demand for the end-items that use the SKU. While some SKUs benefit
from carrying excess inventory because their probability of stock out improves (decreases),
others do not benefit from carrying excess inventory due to a larger order pack size. Such
an analysis can potentially help a retail manager understand the effect of changes in OPS
on two dimensions, space and profit, in order to gauge the benefits of the change.

3.7.4 Can we infer the appropriate amount of backroom space that should
be allocated from the model results?

The optimization model can be used to evaluate the change in the profits with different
allocations of the backroom space within a retail establishment. This process is demonstrated
for one of the stores in our dataset in section 3.7.2 of this chapter. The optimization model
is implemented on all the stores from our dataset that are located in Massachusetts. In
this implementation, we calculate the expected profit corresponding to backroom space
allocations ranging between 5% and 40%, in steps of 2.5%, of the total retail space. We
find that the store profit increases with increase in the amount of backroom space. After a
certain allocation point, however, there is no significant increase in the profit with increase
in the backroom space. This behavior is observed across all stores and is also illustrated in
the results shown in Figure 3-6.

We use these results to estimate the appropriate amount of backroom space that should
be allocated in a retail establishment. To that end, we deem that the amount of backroom
space that should be allocated within a retail establishment lies (approximately) at the
point after which there is no significant improvement in the store profit with increase in the
backroom space. This suggested amount of backroom space can be compared to the current
allocation in order to determine whether the stores are backroom space constrained or have
an excess amount of it.

Therefore, we also need information on the current backroom space allocations in the
stores. Since our dataset only has information for the total retail space, we visited 19 out of
the 30 stores from our dataset in Massachusetts to measure the frontroom and the backroom
spaces. The relationship between the backroom and the total retail space is then used to
estimate the amount of backroom space in the unobserved stores. Details of the observations
and the relationship between the total retail space and the backroom space are discussed in Chapter 4.

On comparing this suggested allocation, which is determined from the optimization model results, to the current allocation, we conclude that almost all the retail stores in the Massachusetts region from our dataset should reduce their currently allocated backroom space. This comparison is shown in Figure 3-10.

![Figure 3-10: Comparison of the suggested and the current backroom space allocation in the stores when accounting for only inventory related cost](image)

However, we realize that inventory related revenue and costs is only a part of the consideration for the backroom space allocation problem. There are other backroom space related costs, labor cost and ordering cost, that are not included in this model. Furthermore, the optimization model presented in this chapter does not account for the effect of frontroom space on demand and thus on the revenue, which is crucial to the backroom space allocation problem. Therefore, these considerations form the foundation of our analysis about the determination of the optimal amount of backroom space that should be allocated within a retail establishment. Chapter 4 of this thesis incorporates the labor cost, ordering cost,
inventory cost, and the frontroom space effect on the revenue to solve this backroom space allocation problem.

3.8 Conclusion and future research

Backroom spaces in a store are a relevant, yet neglected area of retail establishments. A number of factors, including increasing urbanization and competition in the industry, have contributed to the importance of tackling the problem of backroom space management. In order to address this problem, we accomplish the following tasks through the work presented in this chapter. First, we model the SKU inventory levels and space allocation to equipment types within an available backroom space in a store that deals with multiple SKUs/end-items. Second, we conduct the analysis in the context of retail establishments that are limited service restaurants (LSR). By recognizing the industry's unique characteristic that the end-items that reach a consumer could be distinct from the SKUs that are ordered and stored in the backroom space, we implement sub-models that translate end-item demand and service level to that of the SKUs in the optimization model. Third, in this work we also analyze the effects of change in the primary drivers of space requirements, namely the SPS and the OPS when demand is uncertain and multiple SKUs are competing in a constrained backroom space scenario. We quantify this effect for a sample store in our case-study dataset. Fourth, we design the model to also suggest an assortment of end-items that should be served in the LSRs, by trading off the profit contribution and the space needs of the corresponding SKUs in the light of constrained storage spaces. This is made possible by allowing the service level parameter (denoted as \( \gamma \) in this chapter) to be 0, if needed.

There are multiple extensions for the discussed optimization model. In the immediate future, the current model can be modified to account for additional in-store operational process that impacts profit and space requirements in the backroom of a LSR. For instance, in the case of Deltaco's LSRs defrosting of the frozen end-items is an essential step before they are available for sale at the LSR. The profit is affected because the revenue and the lost sales for the frozen end-items is dependent on the amount that is defrosted, rather than the inventory that is available in the freezer. Since defrosting requires an additional equipment, which has a different capacity and space usage, it also adds to the amount of total space required if frozen end-items are offered in the LSR. The details of the derivation
of the effective service level depending on the type of policy adopted for defrosting along with the resulting modification to the optimization model can be found in section 3.9.4 of the appendix.

In yet another potential future research, the model and analysis presented in this chapter can be extended to include non-stationary demand and various types of empirical demand distributions, and to consider network effects (i.e., effects across multiple stores in a distribution network) for the identification of optimal pack sizes in order to maximize profit in constrained retail space environments.

The model was formulated for our industry partner. It can be used as a decision support tool for backroom space management and to study the effect that the changes in supply chain parameters have on the retail backrooms. For instance, it can be used to identify “worst offenders” of backroom space usage while simultaneously evaluating the value of this space usage by the revenue obtained. Since sales/demand are an input, the model can also be used to understand the relationship between planned growth in store sales, pack sizes, and future space required to achieve the growth level under pack size constraints. This model can hence be used to identify opportunity within a retail store and assess the potential growth that can be obtained by identifying space offenders without investing in modifying the current backroom space allocated. Additionally, the model can also be consulted when designing new stores.

3.9 Appendix

3.9.1 Average order quantity and inventory on-hand under a (R,S) replenishment policy

This section shows the derivation for average order quantity and inventory on-hand for a (R,S) system and when order up-to level can be less than the mean of the demand. We will use the following notations in this section for a SKU:

- \( \eta_0 \): OPS
- \( \mu \): mean daily demand
- \( S \): order up-to level
• $\sigma$: standard deviation of daily demand

• $L(k, Z)$: standard normal loss function value for service level factor $k$

• $Y$: random variable for normally distributed and stationary demand. Demand is always non-negative

• $Q_n$: order quantity in review period $n$

• $E[\text{EI}]$: expected on-hand inventory just before replenishment arrives or also called the ending inventory during a review period

• $E[\text{OQ}]$: expected order quantity during a review period

• $E[\text{OH}]$: expected on-hand inventory during a review period

• $f(.)$: probability distribution function

• $F(.)$: cumulative distribution function

Let $k = \frac{S-\mu}{\sigma}$. For the case when $\eta_0 = 1$, we can write the following:

$$Q_n = \begin{cases} S & \text{if } Y_{n-1} \geq S \\ Y_{n-1} & \text{otherwise} \end{cases}$$

$$E[\text{EI}] = \int_0^S (S - y) f_Y(y) dy = (S - \mu) + \sigma * L(k, Z)$$

because $\int_0^S S * f_Y(y) dy \approx \mu - \sigma * L(k, Z) - S * (1 - F_Y(S))$ as shown in (Silver et al., 1998)

$$E[\text{OQ}] = \int_S^\infty S * f_Y(y) dy + \int_0^S y * f_Y(y) dy = \mu - \sigma * L(k, Z)$$

By using the above equations we can derive the expected inventory on hand during a review period as follows:

$$E[\text{OH}] = E[\text{EI}] + \frac{E[\text{OQ}]}{2}$$

$$= (S - \mu) + \sigma * L(k, Z) + 0.5 * (\mu - \sigma * L(k, Z))$$

$$= S - 0.5 * (\mu - \sigma * L(k, Z))$$

Now when $\eta_0 > 1$, the beginning inventory can range anywhere between $S$ and $S+\eta_0-1$ as shown in Chapter 2. Therefore, the ending inventory can be written as:
\[ E[\text{EI}] = \int_{S}^{S+\eta_0-1} \int_{0}^{u} (u - y) * f_Y(y) f_U(u) dy du \]

However, using the approximation for the average beginning inventory as \( S + \frac{\eta_0 - 1}{2} \) from Chapter 2, the average ending inventory is approximated to

\[
E[\text{EI}] \approx \int_{0}^{S+\frac{\eta_0 - 1}{2}} \left( S + \frac{\eta_0 - 1}{2} - y \right) f_Y(y) dy = \left( S + \frac{\eta_0 - 1}{2} - \mu \right) + \sigma * L\left( \frac{S + \frac{\eta_0 - 1}{2} - \mu}{\sigma}, Z \right)
\]

Using the same approximation we can derive the expression for expected inventory on hand as:

\[
E[\text{OH}] \approx S + \frac{\eta_0 - 1}{2} - 0.5 * \left( \mu - \sigma * L\left( \frac{S + \frac{\eta_0 - 1}{2} - \mu}{\sigma}, Z \right) \right)
\]

### 3.9.2 Derivation for the expected units sold

Let \( D_R \) be the distribution of demand over the review period and \( x \) be the beginning inventory after replenishment for the review period and before any demand is observed for that period.

Statement: \( E[\min(D_R, x)] = E[D_R] - E[(D_R - x)^+] \) where \( E[(D_R - x)^+] = E[\max(D_R - x, 0)] \). By definition, the second term here is the expected units lost for a given inventory level \( x \) and demand over review period \( D_R \).

Proof:

\[
E[\min(D_R, x)] = E[D_R | D_R < x] P[D_R < x] + E[x | D_R \geq x] P[D_R \geq x]
\]

\[
E[\max(D_R - x, 0)] = E[D_R - x | D_R - x \geq 0] * P[D_R - x \geq 0] + E[0 | D_R - x < 0] * P[D_R - x < 0]
\]

\[
= E[D_R - x | D_R - x \geq 0] * P[D_R - x \geq 0]
\]

\[
= E[D_R | D_R \geq x] * P[D_R \geq x] - E[x | D_R \geq x] * P[D_R \geq x]
\]

Adding both terms gives:

\[
E[\min(D_R, x)] + E[\max(D_R - x, 0)] = E[D_R | D_R < x] P[D_R < x] + E[x | D_R \geq x] P[D_R \geq x] + E[D_R | D_R \geq x] P[D_R \geq x] - E[x | D_R \geq x] P[D_R \geq x].
\]

Therefore,

\[
E[\min(D_R, x)] + E[\max(D_R - x, 0)] = E[D_R | D_R < x] * P[D_R < x] + E[D_R | D_R \geq x] * P[D_R \geq x]
\]

\[
= E[D_R]
\]

which gives, \( E[\min(D_R, x)] = E[D_R] - E[(D_R - x)^+] \)
3.9.3 Linearizing the loss function

We use piecewise linearization of the unit normal loss function in order to linearize the loss function in the expected units sold. The segments chosen were obtained from Rossi et. al (Rossi et al., 2014). The paper provides for optimal partitioning of the standard normal loss function. However, the segments provided only capture loss function corresponding to service level of approx. 98%. The number of segments is extended to capture loss function corresponding to service level upto 100%. We also allow for the model to linearize service levels of less than 50% because a negative safety stock factor level is possible in this iteration of the optimization model. The next step is to implement piecewise linearization in the model. This is done in two steps (Guide, 2012; Bertsimas and Tsitsiklis, 1997): (i) using a weighting mechanism to pick a specific piece from the segments available; (ii) to ensure that only one segment is chosen from the available segments i.e. only upto two consecutive weights for the pieces are greater than zero.

This is done by adding the following constraints to the model. Let us introduce the following notations for demonstration of the linearization:-

- \( l \) = number of breakpoints of the first order normal loss function (and therefore the number of segments/pieces is given by \( l-1 \))
- \( x_i \) = breakpoints for each segment of the approximated function
- \( f(x_i) \) = actual value of the normal loss function corresponding to the breakpoint \( x_i \)
- \( \lambda_i \) = weight associated with breakpoint \( x_i \)
- \( f'(x) \) = approximate value of the normal loss function at \( x \) derived from piecewise linearization of the normal loss function
- \( b_j \) = binary variable corresponding to each segment formed by breakpoints \( x_i \) and \( x_{i+1} \)
- \( a_j \) = continuous variable

For a value \( x \) the linearization of the loss function yields the following constraints:-

\[
\sum_{i=1}^{l} \lambda_i \cdot f(x_i) = f'(x) \\
\sum_{i=1}^{l} \lambda_i \cdot x_i = x
\]
The constraints that are used to ensure the adjacency requirements are as follows:

\[ b_j = \begin{cases} 
1 & \text{if } (\lambda_i + \lambda_{i+1}) > 0 \\
0, & \text{otherwise} 
\end{cases} \quad \forall j = 1, 2, \ldots, l - 1 \]

\[ \sum_{j=1}^{l-1} b_j = 1 \]

This is further linearized by using the continuous variables \( a_j \) and the following equations:

\[ \sum_{j=1}^{l-1} a_j = 1 \]

\[ a_j \leq b_j \quad \forall j, i \]

\[ a_j \leq (\lambda_i + \lambda_{i+1}) \quad \forall j, i \]

\[ a_j \leq (\lambda_i + \lambda_{i+1}) - (1 - b_j) \quad \forall j, i \]

The decision variables for the piecewise linearization part are \( \lambda_i, b_j \) and \( a_j \).

### 3.9.4 Accounting for the defrosting process to derive the effect on the service level for the frozen end-items

Besides depending on the demand and the inventory available in the store, the revenue for the frozen SKUs also depends on the amount of inventory that is defrosted. In this section, we will incorporate the effect of a defrosting policy on the expected units sold and discuss how to incorporate it in the optimization model. The specific defrosting policy that is used in this section is that the store employee decides to defrost up to a level 'm' or whatever inventory is available in the freezer.

**Additional notations**

- \( X_{v,n} = \) Beginning inventory level on day \( v \) during review period \( n \)
- \( W_{v,n} = \) Ending inventory level on day \( v \) during review period \( n \)
- \( m = \) maximum shelf space available to defrost the item
- \( Y = \) stationary demand distribution with a mean \( \mu \) and a standard deviation of \( \sigma \)
- \( f_{v,n} = \) amount defrosted at the store on day \( v \) during review period \( n \)
- $S_{v,n} = \text{amount sold at the store on day } v \text{ during review period } n$

- $L(h,Z) = \text{unit normal loss function at } h$

- $E[LS] = \text{Expected lost sales}$

- $E[US] = \text{Expected units sold}$

- $X_{1,n} = \text{Beginning inventory for day } 1 \text{ of review period } n$

- $a = \lceil \frac{X_{1,n}}{m} \rceil$

- $R = \text{Review period of the frozen SKU}$

**Assumptions:**

- As a starting point, the shelf life of the defrosted item is assumed to be 1 day. This implies that the employee discards any left-over defrosted items at the end of the day

- The amount that the store decides to defrost on day $k$ is given by $\text{MIN} \ (m, X_{v,n})$

- The items can be defrosted in units of 1.

- They can only be defrosted once in a single day. In the future, the model can be extended to defrosting multiple times a day with varying shelf lives.

**Calculations**

The dynamics of the inventory levels and the sales during a review period $n$ is as follows:

$X_{v,n} = \text{MAX} \ (0, X_{v-1,n} - F_{v-1,n})$

$F_{v,n} = \text{MIN} \ (X_{v,n}, m)$

$S_{v,n} = \text{MIN} \ (Y, F_{v,n})$

$W_{v,n} = \text{MAX} \ (0, X_{v,n} - Z_{v,n})$

**Case A: When } a \leq R**

- The amount defrosted and hence also the amount available for sales during the day is equal to $m$ for the first $(a-1)$ days

- The amount defrosted and available for sales the day after that is equal to $X_1, n(a-1) \ast m$
The amount defrosted and available for sales for the rest of the review period is 0.

Ending inventory in the freezer at the end of the review period is 0.

Therefore, the expected lost sales (E[LS]) is given by

\[
E[LS] = \frac{(a-1)\max(0,D-m)+1\max(0,D-(X_{1,n}-(a-1)*m))+(R-a)*E[D]}{R}
\]

which is also equal to:

\[
E[LS] = \frac{(a-1)\sigma G(v_{a-1})+\sigma G(v_1)+(R-a)\mu}{R}
\]

where \(v_{a-1} = \frac{m-\mu}{\sigma}\) and \(v_1 = \frac{X_{1,n}-(a-1)*m-\mu}{\sigma}\). And therefore, expected units sold \(E[units sold] = \mu-E[units short]\).

**Case B: When \(a > R\) The following holds true,**

- The amount defrosted and hence also the amount available for sales during each day of the review period is equal to \(m\)

- Ending inventory in the freezer at the end of the review period can be obtained as a function of \(m\). This left-over inventory adds on to the beginning inventory for the next review period \((n+1)\)

Therefore, the expected lost sales \((E[LS])\) is given by: \(\sigma * G(v_R)\) where \(v_R = \frac{m-\mu}{\sigma}\), and the expected units sold is still \(E[US] = \mu-E[LS]\)

**Required modifications for the optimization model:** We can use the above derivations to modify the expected profit obtained for frozen items and add an additional decision variable that determines the store service level for these items. Therefore, we would make ‘\(m\)’ as an additional decision variable for all the frozen items and constrain the sum of this shelf space across all the frozen items by the total number of shelves available for defrosting.

Additionally, we can also modify the average beginning inventory for every review period based on the knowledge of the ending inventory at the end of the review period that is carried over to the next review period. In summary, the additions to the optimization model are as follows:

- \(m\) is a decision variable
• \( m \) is constrained to be less than the beginning inventory

• \( m \) is greater than 0 if the item is ordered and carried in the store

Therefore, for a frozen item \( i \) and using the same notations as earlier, the expected revenue that is obtained is given by:

\[
\begin{align*}
\begin{cases}
p_j \times T \times (\mu - \frac{(a_i - 1) \sigma L(v_{a_i - 1}, Z) + \sigma L(v_1, Z) + (R_i - a_i) * \mu_i}{R_i}) & \text{if } a_i = \left\lfloor \frac{(\gamma_i * \mu_i + \frac{n_{a_i - 1}}{2})}{m} \right\rfloor \leq R_i \\
p_j \times T \times (\mu_i - \sigma_i * L(v_{R_i}, Z)) & \text{otherwise}
\end{cases}
\end{align*}
\]

where \( v_{a_i - 1} = \frac{m_i - \mu_i}{\sigma_i} \) and \( v_1 = \frac{(\gamma_i * \mu_i + \frac{n_{a_i - 1}}{2}) - (a_i - 1) \mu_i}{\sigma_i} \) and \( v_{R_i} = \frac{m_i - \mu_i}{\sigma_i} \).
Chapter 4

Strategic allocation of backroom space in a retail establishment

4.1 Introduction

Backroom space allocation is a common and a relevant problem in retail operations. In retail establishments, this problem is related to the determination of the right balance between the space customers use for shopping or the frontroom space and the amount of space needed in the storage room to manage and stage incoming orders prior to selling them or the backroom.

There are three different levels of decision that can be made related to space allocation and management of the backroom: (i) on a macro level, the decision is to determine the proportion of the retail space that should be allocated to the backroom. This is a strategic decision that is formulated to provide guidance on backroom space allocation in newly acquired retail spaces or spaces that can be remodeled; (ii) on a meso level, the decision is related to how a given backroom space can be managed within a retail establishment, which can be framed as a tactical decision model; and finally (iii) on a micro level, the decision is to determine the daily exact order amounts that would maximize the performance of the retail establishment with a given backroom space, which is an operational decision model.

In Chapter 3, we proposed an inventory theoretic model in a multiple-SKU and constrained backroom space setting for a limited service restaurant (LSR). The model proposed in this work is a tactical decision model that recommends optimal average inventory levels for the SKUs and hence the amount of equipment corresponding to different temperature
zones that maximizes the expected profit of the LSR. The profit function components include revenue, SKU purchase cost, inventory holding cost and cost of waste.

However, the work does not account for the interdependencies between the backroom and the preparation and frontroom spaces within a LSR. It is important to capture such interdependencies because the functional areas are closely linked with each other. For instance, the model from Chapter 3 can potentially suggest that increasing the backroom space will never result in a decrease in the expected profit of the LSR. The results are expected to be different in the real world because not having enough frontroom space (when backroom space is large) limits the demand that can be generated at the store, which in turn limits the revenue that can be generated. Furthermore, in the proposed model from Chapter 3, we are determining the equipment requirements from the average optimal order-up-to inventory level for different SKUs and for a given backroom space. In contrast, in this chapter we will determine the optimal backroom space allocation within a retail establishment.

By focusing on the interdependencies between the functional spaces of a retail establishment, we avoid potentially allocating too much (or too little) backroom space. We simplify our system further by addressing the frontroom and the preparation spaces as a single space, denoted as FR. The rest of the space is, hence, the backroom (BR), as shown in Figure 4-1.

![Figure 4-1: The functional areas in a retail establishment of a LSR](image)

This chapter proposes an optimization model formulation that decides on the amount of space that should be allocated to the BR within a given retail establishment’s space by capturing the trade-offs of allocating excess or dearth amount of space to it. We accomplish this by extending the profit function to not just capture revenue and the inventory holding and purchase costs but to also include other space-dependent costs namely the labor cost and the ordering cost. Labor cost and ordering cost are expected to be a function of the BR space, the total retail space and the revenue generated at the LSR. The rationale behind this hypothesis is elaborated in the later sections of the chapter. Furthermore, similar to
the analysis of the previous chapter, the revenue is given by the product of price and the minimum of the inventory and demand. While the observed demand was considered to be an extraneous factor in the previous work, in the current work demand is modeled as a function of the FR space as well as certain store and geographic characteristics. Therefore revenue is modeled as a function of the BR and FR spaces as well as certain specific store (walk-in+drive-through combination vs. walk-in alone) and geographic characteristics (population density).

In summary, the research presented in this chapter proposes a profit maximization approach to determine the optimal size of the BR for a given retail establishment. The profit function is comprised of four components: revenue, inventory purchase and holding costs, labor cost, and ordering cost. The research presented here contributes to the retail space allocation literature by simultaneously accounting for FR and BR spaces in a retail establishment of a LSR.

The rest of the chapter is organized as follows. In section 4.2, we present some extant literature related to FR space allocation. This is followed by characterization of the different functional spaces within a LSR and the specific components of the profit function that they independently drive in section 4.3. Then we provide some background about the problem and a detailed discussion on the model formulation in section 4.4. This is followed by discussion on the data used for our analysis in section 4.5. In section 4.6, we present the results of the regression analysis that is used to estimate the parameters for the different functional forms of the four components of profit. In section 4.7, we formulate the optimization model. This is followed by discussion about the numerical enumeration method that is adopted to solve for the optimal BR space and comparison of the model solution and the observed BR space and profit in section 4.8. In section 4.9, we present some sensitivity analyses to understand the impact of the different profit components parameters on the optimal BR space allocation and expected profit. Finally, we conclude and discuss some potential future research ideas in section 4.10.

We also note that in this chapter, we use the terms 'LSR' and 'store' interchangeably unless explicitly mentioned in reference to other store types e.g. grocery retail store.
4.2 Relevant frontroom space related considerations in literature

In this section, we will discuss the extant academic literature related to the FR area in the context of retail space design. One stream of work is about modeling the effect of the displayed inventory in the FR shelves on the item demand for grocery retail stores (Corstjens and Doyle, 1981; Urban, 1998). More specifically, the research states that the demand for an item initially increases with the increase in the number of units displayed in the FR shelf. However, there is a diminishing rate of increase in demand for every additional unit displayed in the FR shelf. The academic literature, to the best of our knowledge, has concentrated on grocery type retail stores where the store inventory is split between the FR shelves and the BR. The concept therefore is not directly transferable when modeling for revenue as a function of the FR space for a LSR because of the following conceptual differences.

1. "Displayed units" concept: shelf space management is a strategy used by grocery retail stores to improve sales and thus profits in the store (Bultez and Naert, 1988; Corstjens and Doyle, 1981; Dreze et al., 1994). The literature related to shelf space management deals with allocating display space to items in the store. However, most of the items sold in a restaurant retail establishment are not displayed. In fact, the SKUs in the BR are different from the end-items and therefore have to be combined before they can be served to the customer.

2. "Dedicated space" concept: items do not have a dedicated space. Rather, multiple end-items are associated with the space occupied by SKUs in the BR and hence indirectly with the same space in it.

3. Different discretization of the frontroom space: the discrete spaces within a LSR are different from the spaces in a grocery store. The FR space in a LSR is divided into preparation and seating, self-service and other areas related to checkout and pickup. The composition of a FR space in a grocery store is different, primarily being comprised of shelves and aisles. Moreover, grocery stores typically have more number of checkout counters as compared to LSRs.

Another stream of research that is related to our work is concerned with evaluating the performance of a restaurant and is termed as restaurant revenue management in literature.
Restaurant Revenue Management (RRM) maximizes revenue per available seat-hour by manipulating the price and meal duration (Kimes et al., 1999). More specifically, most of the work is related to deciding on the optimum number and type of tables/seats that should be available in the FR. However, the total size of a LSR and allocation of space to the individual areas within it, which affect the establishment’s performance, has not been considered.

The foundation of our work lies on the hypothesis that when comparing two LSRs with the same total retail space, the one that manages its layout optimally also attains a superior performance level. In the following section, we will discuss the effect of the two areas in a LSR, the FR and the BR, on the profit of the retail establishment.

### 4.3 Effect of the frontroom and backroom on store profit

The FR and the BR affect LSR's profit in different ways. The FR usually consists of a seating area, facilities area, billing/order receipt, pick-up area and the preparation area. FR space generates demand and therefore directly affects the revenue obtained. The preparation area within the FR directly affects the revenue obtained by supporting the fulfillment of demand that is generated by the rest of the FR.

On the other hand, the BR affects the profit of the LSR in three different ways. First, it affects the revenue because it limits inventory storage space and hence the extent of support that it can lend for fulfillment of demand generated in the FR. In other words, a single unit of demand generates revenue only when there is sufficient inventory in the BR to fulfill it.

Second, the BR space affects the ordering costs in a LSR. Ordering costs is a function of the replenishment frequency and is driven by inventory on the sales floor as well as in the amount available in the BR for a grocery retail store (Eroglu et al., 2013). Therefore, by extension, in a LSR the ordering cost is driven by the BR space, which is the storage space for most of the inventory that is received from the distribution center. If the BR space of a LSR is reduced, we would expect that it will require higher frequency of deliveries and hence incur larger ordering costs to achieve a revenue target, because of space limitations for inventory storage in the BR.

Third, it affects the labor cost in the retail space. Labor costs directly contribute towards and are a large proportion of the in-store logistics costs which in-turn could potentially
account for up to 50% of the retail supply chain costs (Kuhn and Sternbeck, 2013; Reiner et al., 2013). These in-store logistics processes contribute to a large proportion of retail supply chain costs because they are accomplished through a large share of manual activities, (a phenomenon which is also observed in the case of our partner retailer) and there are limited possibilities of using technology for these activities (Reiner et al., 2013). Therefore, in-store logistics processes could account for more than 40% of the labor hours of store employees (Reiner et al., 2013). In grocery retail stores, labor costs are synonymous with handling costs (Broekmeulen et al., 2004; Van Zelst et al., 2009) and since they contribute to large proportion of the retail supply chain costs, substantial benefit can be obtained if handling costs at the store can be reduced (Van Zelst et al., 2009).

However, most of the extant literature on retail operations are focused on inventory costs or FR shelf space allocation with explicit focus on shelf space elastic demand or planogramming decisions and with no explicit modeling of the labor cost (Cachon, 2001; Corstjens and Doyle, 1981; Dreze et al., 1994; Urban, 1998). One of the ways of reducing the retail establishment’s labor costs is to have better managed BR spaces since they significantly impact these costs.

Labor costs are driven by the specific type of operations that are conducted in a retail space. For example, in a grocery retail store the labor costs are driven by the time required for the following activities: unpacking of a case, stacking old inventory, searching for the shelf location, preparing the shelf, travel, waste disposal and stacking new inventory (Van Zelst et al., 2009). In a LSR, however, the labor costs are driven by additional activities beyond the ones that are required for inventory handling. Based on the observations that we made during visits to the retail establishments of our partner retailer, the labor activities include the following: unpacking of case, searching for the shelf location in the BR, rotating or rearranging the items on the shelves in accordance with the first-in-first-out rule, waste disposal, stacking the inventory on the BR shelves, retrieving the items from the BR intermittently and filling the temporary storage area in the preparation area of the FR, interacting with the consumers to receive their order and then prepare and deliver the order.

Thus, the labor costs in a LSR are closely linked with the BR space for various reasons. One of the reasons is that most of the labor activities related to handling and store maintenance occur in the BR space. On the other hand, the employees who are responsible for these BR related labor activities are, typically, also the ones who interact with the con-
sumers as well as prepare and deliver their orders. Therefore, we expect that the labor costs are driven by factors like congestion in the small or the mismanaged BR spaces that require longer durations to accomplish the above-mentioned labor activities and at the same time result in service delays in a LSR.

In the following three sub-sections, we present our hypotheses for the expected structure of the profit components as a function of the FR and the BR spaces along with the other parameters of a LSR based on the discussions from this section.

4.3.1 Hypothesis for revenue

Revenue is determined by the amount of demand that is converted to sales. Two types of resources affect store revenue: inventory and space. The limitation of these two resources result in two types of lost demand that we are interested in: demand lost due to unavailability of inventory (or inventory stock-out) or demand lost due to unavailability of enough FR space (or space stock-out). Therefore, revenue is a function of factors that drive demand, which is generated by the FR, and demand support through inventory carried in the BR.

In a grocery retail store, demand has been found to be a function of the amount of shelf space allocated to an item (Corstjens and Doyle, 1981; Urban, 1998). In a LSR, on the other hand, the demand is generated by the different areas (e.g. seating or facilities) of the FR.

The other determinants of demand include factors like commodity prices, geographical, demographic and other retail establishment characteristics. According to the classical demand theory, households optimize their utility function subject to a budget constraint, which is a function of the commodity prices and the total household income (Becker, 1965; Pindyck and Rubinfeld, 2005). The theory of household production as proposed by Becker (Becker, 1965) extends this classical demand theory beyond prices and income to include time constraints that could influence household purchases for consumption, which includes items like food. More specifically, Becker suggests that a household trades off between spending their limited resource, which is time, on activities that are involved in eating a meal prepared at home or choosing to outsource some aspects like preparation and cleaning or in other words purchase food away from home by visiting a restaurant (Stewart et al., 2004).

In case of our partner retailer, the commodity prices for the majority of the items sold across the US stores are the same. Therefore, we ignored the item prices when explaining the variability in demand across the different stores. However, we decided to account for the
time aspect in our analysis because it makes sense for our research and is a determinant of demand for food consumed away from home (Stewart et al., 2004). Based on the knowledge of the type of items that are sold in the LSRs of our partner retailer, we expect that the potential customers prefer visiting the stores of our partner retailer that are in close proximity to not just their place of residence but also close to their workplace or places that they visit for leisure. Therefore, we use population density around a specific radius of the retail establishment as a proxy for the potential store traffic. Moreover, we opt for the ambient population density (instead of household population density) in the region, which is defined as “a temporally averaged measure of population density that incorporates human mobility” (Sutton et al., 2003).

In addition to space and population density factors, we also include the restaurant type: walk-in, or a combination of walk-in and drive-through service LSR. For the same space, a retail establishment with walk-in and drive-through service is expected to yield a larger revenue simply because it generates and serves a larger demand that originates from two service stations. However, the improvement in revenue comes at a cost, which will be explained in later sections.

Summarily, the revenue obtained in a LSR is comprised of the following determinants.

1. Revenue driven by the location of the retail establishment. Population density is used as a proxy to account for differences in the store locations,

2. Revenue that is determined by the store layout characteristic or type (walk-in or walk-in+drive-through combination)

3. Revenue that is determined by the amount of demand that is generated by the FR of the LSR and,

4. Revenue that is limited by the inventory carried in the LSR or alternatively the BR space that is available for inventory storage.

Since 1 and 2 from the above list of determinants are input parameters for a given retail establishment, we hypothesize the change in the revenue with space allocated to the BR (and therefore an implied amount of FR space) within the store. Figure 4-2 illustrates the expected functional form of total store revenue with change in BR space in an establishment of size TR.
We hypothesize that there are two regions that lie under the curve for revenue and BR space: (i) a BR space constrained region, which limits revenue because of insufficient inventory storage space (from a → b) and, (ii) a FR space constrained region, which limits revenue because of unavailability of enough demand generating space (from b → TR). So, revenue starts increasing with increase in BR space that is available to carry inventory while having enough amount of FR space to generate demand. This region lies between a and b of BR space in Figure 4-2. Revenue decreases with relatively larger BR spaces because there is not enough FR space to generate demand. This is indicated by the region to the right of b. a sq.ft. denotes the minimum BR space required to generate a revenue and TR or the total retail space is the maximum amount of BR space that can be allocated in a retail establishment. While the BR space in Figure 4-2 is depicted as an area, we will sometimes refer to it as a percentage of the total retail store space. Thus, the ratio b/TR represents the revenue optimal BR size.

4.3.2 Hypothesis for labor cost

Restaurants are retailers that offer two customer products: time and customer experience. Therefore, managing the queue at a LSR is a “paramount issue” for the store manager (Muller, 1999). Thus, speed and the effectiveness of the service are critical for the competi-
tiveness of the LSR (Harrington et al., 2013). Waiting time for customers, which is affected by the operational processes conducted by the employees in the retail establishment, can have a significant impact on the performance of the LSRs. For instance, achieving even small improvements in the customer waiting time can significantly increase the absolute and relative market share of the retailer (Allon et al., 2011). Knutson’s study (Knutson, 1988) suggests that multiple factors, including the ones that are related to a restaurant’s employees like employee greeting, service convenience and service speed affect customer satisfaction in restaurants.

Thus in summary, time and customer experience attributes in a LSR, which affect the performance achieved by a retail establishment, are related to the labor performance and labor hours necessary to execute different activities. These labor activities include both handling inventory that includes unpacking, put-away, waste disposal etc. as well as interacting with the customers and preparing their orders. Since typically there are no dedicated employees for these tasks, it is important to effectively and efficiently allocate labor time between various activities.

Therefore, it is also important that BR spaces are well managed. If not, it could result in employees spending excessive time searching for items in the inventory storage space. This could lead to a reduction in the speed of service or employee-customer interaction time, which could eventually result in a lost sale. This problem is exacerbated during peak hours of customer demand when employees find it difficult to replenish items promptly from the inventory storage area in the BR, which are typically not organized effectively and thus also leading to misplaced SKUs (Raman et al., 2001).

Labor cost, in this research, is modeled as a function of the total store revenue, the total retail space and the BR space within it. It is hypothesized that when we consider two retail establishments with the same total retail space, the one with the smaller BR space incurs a larger labor cost to reach the same performance level. Smaller BR spaces not only incur larger handling costs but also increased opportunity costs from potential lost sales. The latter, as discussed above, is a result of employees being tied up in the small and potentially more congested BR spaces and therefore not having enough interaction with the customers to sustain their continued interest in desiring the service. Figure 4-3 illustrates the expected change in labor cost with BR space in a retail establishment.
4.3.3 Hypothesis for ordering cost

The amount of BR space allocated in a retail establishment has a significant impact on the replenishment frequency for the inventory that is stored in it. A smaller BR space needs a higher frequency of deliveries in order to fulfill a given demand level, simply because it is constrained by the storage space. Ordering cost is therefore a function of total retail space, the BR space within it and the targeted performance level. It is hypothesized that given two LSRs with the same total retail space, the one with the smaller BR space incurs a larger ordering cost to generate the same revenue. This is because it requires a higher replenishment frequency to attain the same revenue. Figure 4-4 illustrates the discussed hypothesis, which is similar to the hypothesized functional form for labor cost and BR space.
4.3.4 Hypothesis for inventory cost

Inventory cost comprises of the purchase cost and the holding cost of the SKUs that are stored in the BR and are used for generating revenue. We hypothesize that purchase cost scales with the revenue obtained. The holding cost is calculated on the value of the average inventory carried and under a periodic review and order up-to level inventory policy. The total inventory cost can thus be expressed as a function of the total revenue. Therefore, we expect the inventory cost curve as a function of the BR space to be the same as that of the revenue function, as shown in Figure 4-2, if the ratio of average inventory cost to the revenue is a constant.

4.4 Model formulation

In order to determine the optimal allocation of BR space within a given retail establishment, we will design a constrained optimization problem with the objective of maximizing expected profit. As mentioned earlier, profit is comprised of four components: (a) revenue (b) inventory cost, which includes purchase and holding cost (c) labor cost and (d) ordering cost. Therefore, profit can be expressed as a function of BR space and other relevant parameters including total retail space, population density and LSR type (walk-in (WI) or walk-in + drive-through combination (WI+DT)). A feasibility constraint is added to ascertain that the BR space allocated is positive and can take a maximum value equal to the total retail space.

We use different types of data obtained from our partner retailer and various other sources in order to derive the functional forms of the different profit components. The data obtained from our partner retailer is on the level of a single retail establishment. Regression analysis on this dataset is used to test the hypotheses presented in the previous section and to estimate the parameters for the different profit components as a function of the decision variable, the BR space.

We use dual criteria to select the regression models for the different profit components. One of them is the adjusted R-squared value that indicates the extent to which the independent predictors explain the variability in the profit components’ in the dataset. The second criteria is related to the extent to which the model can explain the observed space allocation in the stores from the dataset. Since we are designing an optimization model that
determines the optimal BR space allocation by estimating the parameters from a limited dataset of a single retailer, it is important to ascertain that the model is generalizable and accurate. We achieve this by selecting the functional forms such that the optimization model explains the BR space allocation and expected profit values that are observed in the retail establishments of our dataset. The validity of this comparison for selection of the functional forms relies on the assumption that the retail establishments of our partner retailer allocate BR space by optimizing for store profit.

The functional forms for the profit components, which will later be incorporated into the optimization model, are summarized and formalized in the discussion below.

- **Revenue** - This is the total revenue obtained when a fixed amount of FR space is allocated to a LSR with a total retail space of TR sq.ft. Therefore, the total revenue obtained is given by \( r \times FR \), where 'r' is the revenue obtained per unit FR space in the LSR. We use regression to estimate 'r' as a function of the FR and the BR spaces as well as the population density and LSR type.

- **Inventory cost** - This cost includes the inventory purchase cost and the holding cost of the items that are required to generate a certain level of revenue. The inventory cost is a function of the total store revenue and is therefore also a function of the BR space. The average inventory cost can be expressed as a function of the average profit margin of the SKUs and the annual holding cost.

- **Labor cost** - This is the cost associated with not having enough BR space to support the operations of the FR in order to generate a certain level of revenue. Labor cost is a function of the total retail space, the BR space and the revenue generated.

- **Ordering cost** - A smaller BR space also implies a larger ordering cost for a certain amount of revenue generated. We assume that the ordering cost consists of transportation, warehousing and other distribution related costs like backhauling.

Therefore, the strategic decision optimization model that maximizes expected profit, \( \pi \), in order to determine BR space (and therefore an implied amount of FR space) within a LSR of type I, population density \( \vartheta \), and with a total retail space of TR, is given by:

\[
\max \pi(FR, BR, I, \vartheta) \\
\text{s.t. } 0 \leq BR \leq TR
\]
where, \( \pi(\text{FR, BR, I, } \partial) = \text{Rev}(\text{FR, BR, I, } \partial)-\text{InvC}(\text{Rev, } \gamma)-\text{LC}(\text{Rev, BR, TR})-\text{OC}(\text{Rev, BR, TR}) \) and,

- \( \text{Rev}(\text{FR, BR, I, } \partial) = \) Revenue obtained in the LSR as a function of the FR and BR spaces along with the non-space related variables or the population density and the LSR type. The LSR type is a binary variable that indicates whether the retail establishment is a walk-in and a drive-through combination as compared to walk-in alone.

- \( \text{InvC}(\text{Rev, } \gamma) = \) Inventory cost, which is expressed as a function of the revenue obtained and the profit margin \( \gamma \).

- \( \text{LC}(\text{Rev, BR, TR}) = \) Labor cost, which is expressed as a function of the total retail space and the BR space along with the total revenue.

- \( \text{OC}(\text{Rev, BR, TR}) = \) Ordering cost, which is expressed as a function of the total retail space and the BR space along with the total revenue.

### 4.5 Data used for analysis

#### 4.5.1 Data related to space in the retail establishments

The data available with us has the information for 126 LSRs of our partner retailer which are distributed across four different cities. The total retail space for these LSRs ranges anywhere between \( \approx 300 \) sq.ft. to \( \approx 5,000 \) sq.ft. The dataset, however, is missing information on the area of the distinct spaces (FR and BR) within the retail establishment. Therefore, we visited 19 out of the 126 LSRs in Massachusetts in order to measure the area of the FR space, which is shown in Figure 4-5.

A linear regression line between the FR space and the total retail space was fitted to estimate the expected BR space in the LSR, the results of which are shown below:

\[
(\text{TR-BR}) = \frac{2 \times \text{TR}}{3} - 94.1
\]  

(4.1a)
where TR is the total retail space. This also implies that the BR space, can be expressed as:

\[
BR = \frac{TR}{3} + 94.1
\]  

(4.1b)

The relation expressed in equation 4.1a is then used across all the LSRs in the dataset to obtain the expected FR space for 126 stores by using the information for their total retail area. 122 LSRs out of the 126 are used for analysis because of the following reasons. Three out of the four LSRs that are removed were open for business in the middle of the fiscal year that is relevant to our analysis. The remaining single establishment is removed because it has a very small total retail space and was considered an outlier.

\[
FR = TR - BR
\]

\[
TR - BR = 0.67 * TR - 94.1
\]

\[
R^2 = 0.73
\]

So

\[
\begin{align*}
0 & \quad 500 & \quad 1000 & \quad 1500 & \quad 2000 & \quad 2500 \\
0 & \quad \text{Total retail space or TR (in square foot)} & \quad \text{Frontroom or FR space in square foot)}
\end{align*}
\]

Figure 4-5: Total retail space vs. frontroom space in the observed stores

The distribution of the implied ratio of FR to BR is shown in Figure 4-6. The ratio ranges between \( \approx 0.85 \) to \( \approx 1.85 \). A ratio of less than 1 implies that the BR is larger than the rest of the space in the LSR and the opposite is true when the ratio is greater than 1. The histogram in the figure plots the following relationship:

\[
\frac{FR}{BR} = \frac{\frac{2TR}{3} - 94.1}{\frac{TR}{3} + 94.1} = \frac{2 * TR - 282}{TR + 282}
\]

(4.2)
4.5.2 Other (non-space related) data for the retail establishments

The non-space related information for the retail establishments, which is used to estimate the parameters of the profit function components and is available in the dataset include (i) the salaries of employees for labor cost function, (ii) the distribution costs for ordering cost function, (iii) the cost of goods sold (COGS) for the inventory cost function, and (iv) the total revenue obtained in each retail establishment. Additionally, we also have information to indicate whether the LSR is a combination of a walk-in and a drive-through service type or a walk-in service only.

Furthermore, the population density data was collected from LandScan (LandScan Database, 2014), which gives the ambient population, averaged over 24 hours, of a region within a 1 kilometer (km)² area. More accurately, the population density obtained from LandScan is rather an average population in a cell that is approximately one square km in area. It is then averaged for all cells that lie within one km distance from the retail establishment. The distance between the retail establishment and the cells can be obtained by using the 5 digit zipcode information for the LSRs and the geocodes (latitude and longitude) of the cells.

The LandScan global population dataset was developed at Oak Ridge National Labo-
This dataset leverages multiple input variables like the roads, slopes, land cover, populated places, nighttime lights, exclusion areas, urban density factors and coastlines to provide an estimate of the population that is at potential risk during an attack or a natural disaster in a spatial resolution of a cell size of 1km^2 (Dobson et al., 2000).

At this stage, it might be helpful to make a distinction between the two types of population density information that was considered for the analysis, namely (i) the residential population density data on a zipcode level which can be obtained from US census data (Census data, 2010) and (ii) ambient population density data that is obtained from LandScan, as mentioned above. Ambient population, as compared to residential population data, incorporates human mobility and is defined as “a temporally averaged measure of population density that takes into account where people work, sleep, eat, drive, shop, etc.” (Sutton et al., 2003).

Therefore, in order to estimate the store traffic for a retailer with establishments similar to that of our partner retailer, considering ambient population density is more accurate as compared to residential population density. The latter is probably more relevant to determine potential traffic in a grocery retail store if we would assume that people decide to shop for groceries in stores that are close to their residential places. Moreover, as expected, the residential population density was found to be an insignificant variable when explaining the variability in revenue in our dataset. This was however not the case when we considered ambient population density data to explain variability in revenue obtained in the retail establishments.

In the next section, we will discuss the regression results for the functional forms of the four profit components.

4.6 Estimated functional forms of the four profit components

4.6.1 Revenue as a function of the frontroom and backroom spaces in the retail establishment

As discussed in section 4.3, we expect that when FR spaces within a retail establishment are small, the revenue obtained is limited by the demand that can be generated. Conversely, as the FR space increases (and hence the BR space decreases) the revenue is limited by the inventory available in the BR space.
Figure 4-7 depicts how revenue per unit total retail space (TR) changes with the ratio of FR to BR space in the case of 122 LSRs in our dataset. The data is normalized because of confidentiality reasons. Based on our observations from store visits and conversations with store managers, we realized that the FR space is typically larger. Therefore, we were expecting our dataset to include FR to BR ratio values that are larger than 1 (or close to 1), which is found to be true for the stores in our dataset. This is also indicated in Figure 4-7.

In order to estimate the functional form for revenue from our data, we run several models to explain revenue per unit FR in our dataset. As discussed earlier, the criteria for choosing the revenue model that is incorporated into the optimization model are (a) based on the explanatory power of the revenue regression model and (b) a functional form that explains the observed expected BR space allocation and the corresponding expected profit amount from our dataset.

Moreover, since we are aware that retail establishments for our partner retailer are subject to frequent promotions, we decided to use the monthly median values of revenue to avoid being influenced by extreme points or outliers.

Figure 4-7: Revenue per unit total retail (TR) space and the ratio of frontroom (FR) to backroom (BR) space
Revenue is found to be a non-linear function of space and the other independent variables. A multiple linear regression model on transformed functions of the dependent and the predictor variables is used to explain the variability in revenue per unit FR space across the different retail establishments. The results of the regression are shown in table 4.1.

Table 4.1: Regression results. Dependent Variable: $\log_e \left( \frac{Revenue}{FR} \right)$

| Variable          | Coefficient | Std. Error | t-value | P>|t| | 95% Confidence Interval |
|-------------------|-------------|------------|---------|------|-------------------------|
| Intercept         | 6.70        | 0.25       | 27.16   | 0.000**** | [6.21, 7.19] |
| $(\log e BR)^3$   | -0.0073     | -0.0014    | -5.04   | 0.000**** | [-0.010, -0.0044] |
| $\log e \left( \frac{FR}{BR} \right)$ | -0.691 | 0.42 | -1.65 | 0.10* | [-1.51, 0.14] |
| Population density$^{0.55}$ | 0.001 | 0.0002 | 6.53 | 0.000**** | [0.0007, 0.0013] |
| Walk-in+Drive-through vs. Walk-in only | 0.34 | 0.08 | 6.53 | 0.000**** | [0.19, 0.49] |

The coefficients of the predictor variables suggest that revenue in a retail establishment always increases with population density. Moreover, a walk-in+drive-through combination yields a larger revenue than a walk-in only store for the same total retail space and in the same location. Figure 4-8 illustrates the change in revenue as a function of the BR space in different sized retail establishments for a given amount of population density and store type, which is obtained by using the values of the parameters that are estimated from regression.
Figure 4-8: Estimated revenue function for retail establishments of different sizes and for a given population density and store type

Therefore, revenue as a function of BR space, in a LSR with total retail space of TR is given by:

$$\log_e \left( \frac{\text{Rev}}{\text{TR-BR}} \right) = \alpha_0 - \alpha_1 \left( \log_e(BR) \right)^3 - \alpha_2 \log_e \left( \frac{\text{TR-BR}}{BR} \right) + \alpha_3 \vartheta^{0.55} + \alpha_4 I$$ (4.3)

where $$\alpha_0$$ is the intercept and $$\{\alpha_1, \alpha_2, \alpha_3 \text{ and } \alpha_4\}$$ are the absolute values of the estimated coefficients of the predictor variables. Furthermore, I is the binary variable that is true when the LSR is a walk-in and drive-through combination and $$\vartheta$$ is the ambient population density. Alternatively, revenue can be expressed as a non-linear function of the predictor variables as shown in equation 4.4:

$$\text{Rev}(\text{TR, BR, I, } \vartheta) = e^{(\alpha_0 + \alpha_1 I + \alpha_3 \vartheta^{0.55})} \times (\text{TR-BR})^{1-\alpha_2} e^{\alpha_2 (\log_e(BR))^3}$$ (4.4)

On simplifying equation 4.4 further, we obtain the following expression for the revenue function:

$$\text{Rev}(\text{TR, BR, I, } \vartheta) = A(I, \vartheta) \times (\text{TR-BR})^{1-\alpha_2} e^{(\alpha_2 - \alpha_1 \log_e(BR))^3}$$ (4.5)

where $$A(I, \vartheta)$$ is a constant for a given LSR and is a function of the average ambient
population density around the zipcode that the LSR is situated in and its type depending on whether it is a walk-in+drive-through combination as compared to a walk-in only.

4.6.2 **Inventory cost as a function of space**

As discussed earlier, inventory cost is the sum of the purchase cost and the holding cost corresponding to the inventory that is used to generate a certain level of revenue in the retail establishment. In order to estimate the purchase cost, we use monthly median value of the cost of goods sold or COGS. It is assumed that COGS also includes the cost of wasted SKUs, which could be a result of spillage, product expiry or incorrect portioning of the SKUs to produce the final end-item, and that does not contribute to revenue generation.

In our dataset, COGS is found to be strongly positively correlated and linearly increasing with the revenue obtained in the retail establishment as shown in Figure 4-9 (data is normalized due to confidentiality reasons). Therefore, the purchase cost, which is a function of the profit margin and the revenue obtained, can also be expressed as a function of the total retail space and the BR space in it.

We use the relationship between purchase cost and the revenue generated in the retail establishment to derive the *average* inventory cost in the following way. We assume a periodic review inventory policy of a period ‘R’, as in the case of our partner retailer, and an annual holding rate of ‘h’, which is assumed to be equal to 25%.

\[
\text{Inventory cost} = \text{Inventory purchase cost} + \text{Inventory holding cost} \\
= \text{COGS} + \text{COGS} \times \text{Inventory holding rate} \times \frac{R}{2} \tag{4.6}
\]

Therefore, on a monthly level the inventory cost can be written as:

\[
\text{Inventory cost} = \text{COGS} + \text{COGS} \times \frac{h}{12} \times \frac{R}{2} \tag{4.7}
\]
Figure 4-9: Relationship between COGS and Revenue in our dataset

In order to express the inventory cost as a function of the revenue generated in the retail establishment and hence in terms of the BR space, we first run a 0-intercept regression model for COGS as a function of the predictor variable revenue. A 0-intercept model was fitted to account for the case that a retail establishment that does not generate a revenue, also, does not purchase inventory and hence implying that the COGS is zero. The results of the regression are shown in table 4.2. Therefore, equation 4.7 for inventory cost can be written as:

\[
\text{Inventory cost} = (\gamma_0 \times \text{Revenue}) \times \left(1 + \frac{h}{12} \times \frac{R}{2}\right)
\]

(4.8)

where \(\gamma_0\) is the coefficient for regression between COGS and Revenue.
Table 4.2: Regression results. Dependent Variable: Cost of Goods Sold

| Variable  | Coefficient | Std. Error | t-value | P>|t| | [95% Confidence Interval] |
|-----------|-------------|------------|---------|---------|--------------------------|
| Revenue   | 0.156       | 0.001      | 152     | 0.000**** | [0.153, 0.157]          |
| N         | 122         | R²         | 0.94    | Adj. R² | 0.93                     |

**** p < 0.001, *** p < 0.01, ** p < 0.05, * p ≤ 0.1

At this stage we would like to discuss about the review period, ‘R’, used in equation 4.7. In practice, LSRs like our partner retailer function in a multiple SKU and therefore in multiple review period environment. However, for a strategic allocation model where the analysis is conducted on a retail establishment level, we assume an average review period value of 1 week which is greater than or equal to the review period of majority of SKUs that are used in the LSRs of our partner retailer. Alternatively, the implied average review period can be estimated from the ordering costs and average cost per order as shown in the appendix. Because using this method adds complexity to the profit function and does not significantly impact the results, the analysis was conducted with the assumption of the above-mentioned review period of 1 week.

4.6.3 Estimated functional form for labor cost

Labor cost is driven by two factors, the BR space within a retail establishment and the activities that are directly or indirectly related to the space in order to support the revenue that is generated in the LSR. Therefore, we hypothesize that given two retail establishments with the same total retail space, the one with the smaller BR incurs a larger labor cost for the same revenue generated in the retail establishments. The increased labor cost stems from potentially larger congestion in the BR space for retail establishments that operate in a multiple SKU environment. This condition is worse in a restaurant, as compared to a grocery retail store, because producing an end-item depends on multiple SKU ingredients. Thus, labor cost is likely to be associated with BR space management. Moreover, smaller BRs inherently imply larger frequency of delivery and therefore the handling cost, which is a result of activities related to unpacking, packing and stacking away in the shelves, increases.

It is important to note that in this analysis we are explaining the change in labor cost when two retail establishments have the same total retail space and are generating the
same revenue but with different BR spaces. In other words, our hypothesis also suggests that when comparing the labor cost for two retail establishments with the same total retail space, it is possible that the one with a smaller BR incurs a smaller labor cost if the revenue generated is less. Thus, labor cost is measured relative to the revenue.

We conduct a regression analysis to explain labor cost per unit dollar of revenue generated in the retail space as a function of the BR space allocated within it. The results of this analysis is shown in table 4.3.

Table 4.3: Regression results. Dependent Variable: \( \frac{(\text{Labor Cost/Revenue})}{\text{TR}} \)

| Variable         | Coefficient | Std. Error | t-value | P>|t| | [95% Confidence Interval] |
|------------------|-------------|------------|---------|------|---------------------------|
| Intercept \( \times 10^3 \) | 1.4         | 0.05       | 27.7    | 0.000**** | [1.3, 1.5] |
| \( \log_e(BR) \times 10^3 \) | -0.2        | 0.008      | -24.5   | 0.000**** | [-0.21, -0.18] |

**** \( p < 0.001 \), *** \( p < 0.01 \), ** \( p < 0.05 \), * \( p \leq 0.1 \)

Figure 4-10 shows the regression fit and the dependent variable with changing BR space in the 122 retail establishments in our dataset.

Depending on the estimated parametric values for the labor cost function and when the BR space is large enough, the labor cost can be negative, which is not realistic. We prevent such a scenario by imposing a minimum (LR\(_{\text{min}}\)) on the dependent variable ratio. LR\(_{\text{min}}\) is equal to the lowest observed value in the dataset.

The functional form of the labor cost (LC) as a percentage of total revenue (Rev) with respect to the BR space in a store of size TR is shown in equation 4.9.

\[
\frac{(LC/Rev)}{TR} = \text{max}(\beta_0 - \beta_1 \times \log_e BR, LR_{\text{min}})
\]

\[\text{LC} = \text{Rev} \times \text{TR} \times \text{max}((\beta_0 - \beta_1 \times \log_e BR), LR_{\text{min}})\]

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4.6.4 Estimated functional form for ordering cost

Ordering cost is also estimated as a function of the BR space because its size can have a significant impact on the replenishment frequency. To recap, we hypothesize that as the BR reduces in a given retail space of a LSR, the ordering cost increases in order to support the generation of the same amount of revenue. We expect this to happen because in order to reach the revenue level, the LSR has to make more frequent deliveries of smaller order sizes because it is constrained by inventory storage space in the BR.

We conduct a regression analysis to explain ordering cost per unit dollar of revenue generated in the LSR in a retail establishment as a function of the BR space allocated. The results of this analysis is shown in table 4.4.
Table 4.4: Regression results. Dependent Variable: \(\frac{\text{Ordering Cost/Revenue}}{\text{TR}}\)

| Variable          | Coefficient | Std. Error | t-value | P>|t| | [95% Confidence Interval] |
|-------------------|-------------|------------|---------|------|--------------------------|
| Intercept*10³     | 0.32        | 0.01       | 24.1    | 0.000**** | [0.29, 0.35]            |
| \(\log_e(BR)\) * 10³ | -0.05       | 0.002      | -21.8   | 0.000**** | [-0.05, -0.04]           |

N 122 R² 0.80 Adj. R² 0.80

**** \(p < 0.001\), *** \(p < 0.01\), ** \(p < 0.05\), * \(p < 0.1\)

Figure 4-11 shows the regression fit and the dependent variable with changing BR space in the 122 retail establishments in our dataset.

Similar to the of labor cost function, the regression results suggest that if the BR space is large enough then the ordering cost can be negative, depending on the estimated values of the parameters. We prevent such a scenario by setting a minimum value for the dependent variable ratio to the lowest observed value for the ratio in the dataset.

Therefore, ordering cost (OC) as a percentage of total revenue (Rev) and the BR space in a store of total retail size TR is given by:

\[
\frac{(OC/Rev)}{TR} = max(\rho_0 - \rho_1 \cdot \log_e BR, OR_{min})
\]

\[
OC = Rev \cdot TR \cdot max((\rho_0 - \rho_1 \cdot \log_e BR), OR_{min})
\]
Figure 4-11: Ordering cost (OC) incurred to generate a unit of revenue (Rev) per unit total retail space (TR)

4.7 Optimization model for strategic decision of backroom space allocation in a retail establishment

The estimated functional forms for the different profit components are incorporated into the constrained optimization model that can be used to determine the BR space allocation. The model objective is to maximize the expected profit of the retail establishment. The model was formulated in section 4.4 and is also shown below:

\[
\max \pi(\text{TR}, \text{BR}, I, \partial) \\
\text{s.t. } 0 \leq \text{BR} \leq \text{TR}
\]

The estimated functional forms of the profit components are all defined only on positive values for BR space. Since the constraint takes care of the positivity of the BR space, we add a tolerance ($\epsilon > 0$) to the minimum value that the BR space can take. This ensures that
the profit function is defined for all feasible values of the BR space in a retail establishment. Therefore, the optimization model can be re-written as follows:

$$\max \pi(T_R, B_R, I, \partial)$$

s.t. $\epsilon \leq B_R \leq T_R$

We also define the following notations that will be used in this section:

- **TR**: total available store space, which is also the sum of the frontroom and backroom areas (sq.ft.)
- **BR**: decision variable for the optimization model or the backroom space (sq.ft.)
- **FR**: frontroom space (sq.ft.)
- **r**: revenue generated per unit area of the FR in the retail space ($/sq.ft.)
- **C_h**: average annual holding cost over the review period per unit $/sq.ft.$
- **\gamma_0**: ratio of COGS to revenue
- **\beta_0, \beta_1**: estimated parameters for the labor cost function ($/sq.ft.$ and $/sq.ft.$, respectively)
- **\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4**: estimated parameters related to revenue per unit FR space function or $r$ ($/sq.ft.$, $/sq.ft.$, $/sq.ft.$, \$/sq.ft. respectively)
- **\rho_0, \rho_1**: estimated parameters related to ordering cost function ($/sq.ft.$ and $/sq.ft.$, respectively)
- **I**: binary variable that indicates whether the store is walk-in and drive-through combination as compared to walk-in alone
- **\partial**: ambient population density (population per square kilometer)
- **Rev**: revenue function ($/month$)
- **LC**: labor cost function ($/month$)
- **OC**: ordering cost function ($/month$)
- **InvC**: inventory cost function ($/month$)
Expected profit as a function of the decision variable BR and other parameters is given by:
\[ \pi(\text{TR}, \text{BR}, I, \partial) = \text{Rev}(\text{TR}, \text{BR}, I, \partial) - \text{InvC}(\text{Rev}, C_h) - \text{LC}(\text{BR}, \text{TR}, \text{Rev}) - \text{OC}(\text{BR}, \text{TR}, \text{Rev}) \]

The different profit functions can be written as:

- Revenue function is derived in equation 4.5 and is given by:
  \[ A(I, \partial)(\text{TR}-\text{BR})^{1-\alpha_2}\text{BR}^{(\alpha_2-\alpha_1\times(\log \text{BR})^2)} \]

- Inventory cost function, which is derived in equation 4.8 is given by:
  \[ \gamma_0 \times \text{Rev}(\text{TR}, \text{BR}, I, \partial) * (1 + \frac{\text{h}}{12} + \frac{\text{f}}{2}) \]

- Labor cost function as derived in equation 4.9 is given by:
  \[ \text{Rev}(\text{TR}, \text{BR}, I, \partial) * \text{TR} \times \max((\beta_0 - \beta_1 \times \log \text{BR}), \text{LR}_{\text{min}}) \]

- Ordering cost function as derived in equation 4.10 is given by:
  \[ \text{Rev}(\text{TR}, \text{BR}, I, \partial) * \text{TR} \times \max((\rho_0 - \rho_1 \times \log \text{BR}), \text{OR}_{\text{min}}) \]

Therefore, the profit function becomes

\[ \pi(\text{TR}, \text{BR}, I, \partial) = [A(I, \partial)(\text{TR}-\text{BR})^{1-\alpha_2}\text{BR}^{(\alpha_2-\alpha_1\times(\log \text{BR})^2)}] \times [1 - \gamma_0(1 + C_h) - \text{TR} \times \max((\beta_0 - \beta_1 \log \text{BR}), \text{LR}_{\text{min}}) - \text{TR} \times \max((\rho_0 - \rho_1 \log \text{BR}), \text{OR}_{\text{min}})] \]

and which implies that the the constrained optimization problem is:

\[ \max [A(I, \partial)(\text{TR}-\text{BR})^{1-\alpha_2}\text{BR}^{(\alpha_2-\alpha_1\times(\log \text{BR})^2)}] \times [1 - \gamma_0(1 + C_h) - \text{TR} \times \max((\beta_0 - \beta_1 \log \text{BR}), \text{LR}_{\text{min}}) - \text{TR} \times \max((\rho_0 - \rho_1 \log \text{BR}), \text{OR}_{\text{min}})] \]

\[ s.t. \ e \leq \text{BR} \leq \text{TR} \]

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4.8 Solving for backroom space

The optimization model, which is shown in equation 4.12, is non-linear but is of a manageable size because the backroom space is bound by the total retail space. Therefore, we use an enumeration method to find the global optimum for the BR space allocation within a total retail space. We obtain a solution by increasing the BR space by a single unit (in one square foot increments) in every step of enumeration for values ranging between the minimum (ε) and the maximum possible values (TR) that it can take. In each step, we calculate the individual components of the profit function and then determine the amount of the BR space that corresponds to the maximum profit.

Figure 4-12 plots the revenue, total cost (inventory cost + labor cost + ordering cost) and the resultant profit for BR space increasing from 5 sq. ft. to 1,600 sq. ft. for a 1,600 sq. ft. retail establishment.

Table 4.5 compares the average BR space allocation and the average profit as seen in our dataset, against the results of the optimization model. The comparison shows that the optimization model can explain the observed BR space allocation and the current expected profit for an average sized retail space. This space is a walk-in only service and is situated in a location whose population density is the same as that of the average in the dataset. Additionally, the optimization model can also explain the observed average for the amount of BR space and profit across all the stores in the dataset.

Table 4.5: Comparison of the average values from the dataset and the solution from the optimization model

<table>
<thead>
<tr>
<th>Measure name</th>
<th>Average from dataset</th>
<th>Model solution (1,600 sq.ft.)</th>
<th>Average of model solution (all stores)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR space allocated (as a % of total retail space)</td>
<td>40%</td>
<td>43%</td>
<td>42%</td>
</tr>
<tr>
<td>Monthly (annualized) profit at the LSR ($)</td>
<td>$73,986 (0.89 M)</td>
<td>$74,454 (0.89 M)</td>
<td>$75,100 (0.90 M)</td>
</tr>
</tbody>
</table>
We also compare the estimates from the dataset and the solution obtained from the model for four store size ranges, the results of which are shown in table 4.6. The results in tables 4.5 and 4.6 are thus used for benchmarking the functional forms and the parameters of the different components of the profit function that is used in the optimization model.

Assuming that on an average the space in the stores from our dataset is allocated with the objective of maximizing the establishment’s profit, the results in tables 4.5 and 4.6 establish a reasonable level of model accuracy. Therefore, this model can be incorporated into a decision support tool that would enable the retailers to identify the stores which are performing poorly because of mis-allocation of BR space within them. In order to demonstrate one of the ways of using the results from the optimization model to identify the poorly performing stores, we compare the current BR space allocation in each store with the optimal BR space that is suggested from the model. We reiterate that the output from the comparison of the optimal and the currently allocated BR space in the stores from our dataset is shown for the sole purpose of demonstrating how the retailers can use the model results for strategic space allocation decision. We are limited by our current dataset
to identify the stores that have mis-allocated space within them, which can be addressed in the future by collecting additional relevant data.

Table 4.6: Comparison of the average values from the dataset and the solution from the optimization model for different store sizes

<table>
<thead>
<tr>
<th>Retail size</th>
<th>Measure name</th>
<th>Measure value from dataset</th>
<th>Measure value obtained from model solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(TR: Total retail space)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Very Small, TR:</strong></td>
<td>Average BR space allocated (as a % of total retail space)</td>
<td>47%</td>
<td>45%</td>
</tr>
<tr>
<td>&lt; 1,000 sq.ft.</td>
<td>Average monthly (annualized) profit at the LSR ($)</td>
<td>≈ 66,573</td>
<td>≈ 71,190</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.80 M)</td>
<td>(0.85 M)</td>
</tr>
<tr>
<td>Average total retail space:~ 750 sq.ft.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Small, TR:</strong></td>
<td>Average BR space allocated (as a % of total retail space)</td>
<td>41%</td>
<td>42%</td>
</tr>
<tr>
<td>≥1,000 &amp; &lt;1,500 sq.ft.</td>
<td>Average monthly (annualized) profit at the LSR ($)</td>
<td>≈ 77,684</td>
<td>≈ 74,307</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.93 M)</td>
<td>(0.89 M)</td>
</tr>
<tr>
<td>Average total retail space:~ 1,200 sq.ft.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Medium, TR:</strong></td>
<td>Average BR space allocated (as a % of total retail space)</td>
<td>39%</td>
<td>43%</td>
</tr>
<tr>
<td>≥1,500 &amp; &lt;2,000 sq.ft.</td>
<td>Average monthly (annualized) profit at the LSR ($)</td>
<td>≈ 73,357</td>
<td>≈ 75,784</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.88 M)</td>
<td>(0.91 M)</td>
</tr>
<tr>
<td>Average total retail space:~ 1,700 sq.ft.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Large, TR:</strong></td>
<td>Average BR space allocated (as a % of total retail space)</td>
<td>37%</td>
<td>37%</td>
</tr>
<tr>
<td>≥2,000 sq.ft.</td>
<td>Average monthly (annualized) profit at the LSR ($)</td>
<td>≈ 75,324</td>
<td>≈ 87,655</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.90 M)</td>
<td>(1.05 M)</td>
</tr>
<tr>
<td>Average total retail space:~ 2,400 sq.ft.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The comparison for the 122 stores in the dataset is shown in Figure 4-13. The stores whose FR space was measured are indicated separately in the plot as the “observed” stores. The stores that lie in the top half of the plot are the BR space constrained stores. The distance from the 45 degree dotted line indicates the extent to which the stores are BR space constrained or have an excess of BR space allocated within them. Therefore, most of the stores in this plot are currently BR space constrained. By changing the relative space allocation between the FR and the BR in these stores, retailers could potentially improve their profitability.
4.9 Sensitivity analysis

The proposed optimization model is used to analyze the sensitivity of the optimal BR space and profit to the change in different components of the objective function. This analysis can be used to obtain some insights related to space management in a retail establishment.

4.9.1 Sensitivity to the change in total retail space

The BR space allocation and the expected profit are evaluated for retail establishments of different sizes. These establishments are considered to be walk-in only service and situated in a location that has a population density equal to the dataset average. The results of the analysis is shown in Figure 4-14.

The total retail space is indicated in the horizontal axis of the plot and ranges between 500 sq.ft. to 5,000 sq.ft. while increasing in discrete intervals of 100 sq.ft. The vertical
axis on the left shows the optimal BR space allocation and the one on the right shows the corresponding annualized expected profit.

Figure 4-14: Optimal BR space allocation in different types of LSRs for increasing total retail space

As the total retail space increases, the BR space allocated initially increases, which indicates that a larger retail space needs a larger BR space to support the revenue that can be generated in it. After a certain point, which corresponds to a store size of 2,300 sq.ft. in the plot, increasing the total retail space does not change the recommended allocation to the BR. In other words, for relatively larger retail spaces, any excess retail space beyond a certain point is allocated to the FR. Just 10% of the stores in our dataset are greater than or equal to 2,300 sq.ft. in total retail area.

BR space that is allocated in stores larger than 2,300 sq.ft. stays constant because of the minimum that is applied to the ratio of labor cost to the total revenue per unit of total retail space, and is denoted as LRT-min in this section. Initially, the labor cost that is incurred for every dollar of revenue per unit of total retail space reduces with BR space until it reaches the amount that corresponds to LRT-min, after which the total labor cost is only
affected by revenue and the total retail space. Therefore, the total cost does not decrease at the same rate with BR space, which is illustrated in Figure 4-15.

The BR space that corresponds to LRT-min can be calculated by using the estimated regression coefficients for the labor cost function parameters shown in equation 4.9. It is equal to 898 sq.ft. for stores in our dataset. Because revenue decreases with BR space between 898 sq.ft. and total retail space, the optimal BR space for stores larger than 2,300 sq.ft. is 898 sq.ft. The optimal BR space for smaller stores is reached before 898 sq.ft. Moreover, the minimum value that is applied to the ratio of ordering cost to the total revenue per unit of total retail space is reached at a BR space of less than 898 sq.ft. However, since labor cost is larger than the ordering cost for stores in our dataset, we predominantly observe the effect of LRT-min on the resultant BR space allocation.

Figure 4-15: The change in the profit components in the different sized retail spaces

4.9.2 Sensitivity to the change in labor and ordering cost

Labor and ordering costs increase with the increase in revenue generated and decrease with the increase in the amount of BR space that is available within a retail establishment to
support a target revenue yield. In this section, we investigate the impact of the change in labor and ordering cost as a proportion of the revenue on the BR space allocation and profit.

Different scenarios are created by changing the ratio of labor cost to revenue across all stores in the dataset from its current amount and with its current BR space allocation. The current set of values correspond to the base case. Illustrations of two scenarios that are obtained by an increase and a decrease in the base case amount are shown in Figure 4-16. Figure 4-17 shows the sensitivity of the optimal BR space and the profit to the change in labor cost that is incurred per unit of revenue for different scenarios. The scenarios are indicated by the amount of change made to the base case cost. For instance, a ‘2/1’ scenario indicates that in the new scenario, a store with its current BR space allocation incurs twice the labor cost for every dollar of revenue generated in it as compared to its current amount.

As the ratio of labor cost to the revenue decreases, the optimal BR space also decreases. Expressed differently, decreasing the above ratio in a store implies that it is operating more efficiently to reach a given level of revenue and with the same amount of BR space. Therefore, the profit for the retail establishment with the optimal BR space allocation increases with decrease in the ratio of labor cost to revenue. For instance, in a 1,000 sq. ft. store, a 50% increase in the ratio, increases BR space requirement by 15% and decreases profit by 16%.
On the other hand, a 50% decrease in the ratio of labor cost to revenue, decreases BR space requirements by 25% and increases profit by 16%. The extent of impact on the BR space allocation is also dependent on the total retail space for a given amount of change in the ratio of labor cost to revenue. In a 1,500 sq.ft. store, a 50% increase (or decrease) in the ratio increases (or decreases) BR space requirement by 30% (or 23%).

Because optimal BR space allocation and the resultant profit could be significantly sensitive to the labor cost incurred as a percentage of revenue, it can be beneficial for retailers to investigate different ways of managing this cost. In this study, labor cost that is incurred to generate a revenue level is assumed to be driven by two factors: (i) execution time for SKU handling operations, (ii) and congestion in the BR. Reducing both the factors could contribute towards reduction of labor cost. Extant literature has shown that the time required for SKU handling operations in grocery retail stores can be reduced by increasing order pack size (OPS) or reducing the frequency of replenishments (Curșeu et al., 2009; Van Zelst et al., 2009). According to a study (Curșeu et al., 2009), reducing the frequency of replenishments or in other words reducing order lines, results in lower handling costs because of fixed setup times and economies of scale. However, like explained in Chapter 2, larger OPS results in excess inventory, which could potentially lead to larger amounts of waste when SKUs are perishable and therefore imply higher handling time and cost. Moreover, Curseu et.al. (2009) assumes that there is enough storage space for larger OPS, which is however not always true for stores functioning in constrained space environments, like in the case of our partner retailer. Thus, considering these trade-offs to estimate an optimal pack size can contribute towards more efficient handling operations and therefore lower labor costs. Additionally, expertise of the store employees and their working speed affects the time for SKU handling operations and hence the associated costs (Van Zelst et al., 2009). Therefore, by training employees, providing them with clear guidelines to conduct handling operations and motivating them could also decrease time and cost for handling operations (Van Zelst et al., 2009). On the other hand, BR congestion can be reduced by having more organized and clean areas. For example, changes to in-store operations like encouraging accurate labeling of the inventory storage space or maintaining clean and wider aisles could reduce congestion in the BR.
Effect of changing labor cost incurred per unit of revenue

Total retail space: 1,000 sq.ft.

Effect of changing labor cost incurred per unit of revenue

Total retail space: 1,000 sq.ft.

Similarly, ordering cost incurred to generate a certain level of revenue within a retail space also affects optimal BR space allocation and the profit obtained. Figure 4-18 illustrates this sensitivity for different scenarios of the ratio of ordering cost to revenue as compared to the base case.

The direction of the change in the optimal BR space and the expected profit is the same as that of the change in the labor cost for every unit of revenue. However, the rate of change of BR space and profit with change in the ordering cost is smaller even though both labor and ordering costs have the same functional forms. As an example, on increasing the ordering cost as a percentage of revenue for the retail stores in our dataset by twice the current amount, the BR space allocation for a 1,000 sq.ft. store increases by 12% and the profit reduces by 11%. On the other hand, reducing the ordering cost as a percentage of revenue to one-fourth of the base case, reduces BR space allocation by around 7% and improves the profit by only approximately 1%. Since the ordering cost is approximately 4 times less in the order of magnitude than the labor cost required to generate a certain level of revenue within a retail space for stores in our dataset, there is also a large difference in the sensitivity of the BR space allocation and profit to change in the two costs.
However, it can still be helpful for the retailers to investigate ways of managing ordering costs for their stores that impacts BR space allocation decisions. The ordering cost components that are related to the BR space include the frequency of replenishments and the cost for each replenishment. The frequency of replenishments for the same BR space can be reduced by using optimal inventory policies that would reduce over-ordering or waste. On the other hand, cost per replenishment can be reduced by reducing the time spent by the delivery personnel at the store. For instance, in case of our partner retailer, most of the stores receive night-time delivery or when the store employees are away. Since the SKUs in LSRs can be perishable, the delivery personnel have the additional responsibility of putting away some of the SKUs that should be stored in refrigerators or freezers. Unorganized BR spaces could increase the time required by the delivery personnel to put-away these SKUs. Moreover, sub-optimal inventory policies leads to unavailability of enough space for the perishable SKUs that results in returns, which are loaded back into the truck by the delivery personnel. This increases the time required for delivery activities and therefore could also increase the cost associated with each replenishment.

Effect of changing ordering cost incurred per unit of revenue

Figure 4-18: Sensitivity of optimal BR space and profit to change in the ratio of ordering cost to revenue
4.9.3 Sensitivity to the change in inventory cost

Inventory cost is obtained by modeling the relationship between cost of goods sold (COGS) and revenue, as shown in equation 4.8. In this section, we will evaluate the sensitivity of the BR space and profit to the change in the ratio of COGS to revenue.

Figure 4-19 shows the change in the optimal BR space allocated and profit obtained with increase in the ratio of COGS to revenue. The average profit margin, which is equal to \( \frac{\text{Revenue} - \text{COGS}}{\text{Revenue}} \), for the items that are sold in the stores, decreases with increase in the ratio of COGS to revenue. This is also depicted in the plot. Additionally, as the ratio increases, the BR space amount, after which the revenue is larger than the total costs, increases as illustrated in Figure 4-20. Therefore, the optimal BR space increases with increase in the ratio of COGS to revenue as shown in the plot of Figure 4-19.

As expected, the impact of the change in the ratio of COGS to revenue is also dependent on the total retail space. For instance, in a 1,000 sq.ft. store, the BR space allocation increases with increase in this ratio between 10\% and 90\%. However, in a 1,500 sq.ft. store the BR space allocation increases till it reaches the point after which the condition for the minimum of the ratio of labor cost to revenue per unit of total retail space is activated. While increasing the ratio of COGS to revenue continues to result in a decreased store profit irrespective of store size, BR space allocation for larger retail spaces, like a 1,500 sq.ft. store, stays unchanged after a certain point.

![Effect of changing the ratio of COGS to revenue](image)

**Figure 4-19:** Estimated change in optimal BR space with change in ratio of COGS to revenue
Since inventory cost or COGS could have a significant impact on the BR space allocation and the profit, it can be beneficial for the retailers to develop operational strategies that would enable them to manage this cost in the stores. COGS includes the materials cost or the purchase cost of the SKUs that are used by the store. As discussed in section 4.6.2, SKU usage is the sum of two quantities; quantity of the SKUs that ultimately generate revenue and quantity for the ones that are wasted. Therefore, by reducing waste, COGS could be potentially reduced to obtain the same revenue amount. Waste in a LSR could happen due to multiple reasons, two of which are spillage or product expiry. Therefore, by training employees effectively or having a clear process flow that they could follow could reduce inefficiencies in the operations related to the preparation of the end-items. Moreover, integrating the shelf-life aspect of the SKUs in determination of optimal inventory policies could reduce waste due to perishability.

![Graphs showing the change in profit components with change in the ratio of COGS to revenue](image)

Figure 4-20: Illustrating the change in the profit components with change in the ratio of COGS to revenue
4.9.4 Effect of population density and store type on space allocation and profit

Store type (walk-in only or walk-in+drive-through combination) and the ambient population density around a fixed radius of the store location are both input parameters of the optimization model. The profit function, which is derived in equation 4.11, in a store of size TR, store type I and population density \( \partial \) can be written as the product of two functions, \( \text{Rev}(I, \partial, TR, BR) \times K(TR, BR) \). \( K(TR, BR) \) is only dependent on space factors, TR and BR, and the revenue function or \( \text{Rev}(I, \partial, TR, BR) \), is dependent on both space factors as well as the population density and store type.

For a given amount of BR space in a retail establishment of size TR, the revenue and therefore the profit amount monotonically increases with increase in population density. The same is true for a store with a walk-in+drive-through combination as compared to a walk-in only service type. Therefore, the optimal BR space allocation does not change with change in store type or population density around a fixed radius for a given store size.

However, it also implies that a smaller retail establishment in a location with a higher population density can match the profitability of a larger establishment in a low population density area. Similarly, a smaller retail establishment of a walk-in+drive-through combination type can yield a profit that is comparable to a larger retail establishment with a walk-in only service type.

Figure 4-21 illustrates how a smaller LSR in a larger ambient population density area can generate a comparable or significantly larger profit than a LSR with a larger total retail space and situated in an area with a smaller population density. The high population density, in this example, is 10 times more than the low population density. For instance, the profit of a 900 sq.ft. LSR in a high population density can be similar to that of a 1,400 sq.ft. LSR in a low population density. Based on the functional form for revenue as shown in equation 4.4, increasing population density and therefore the demand also increases revenue potential. At the same time, there is an increase in the related costs to support the demand and achieve an increased revenue.
Effect of high and low ambient population density on expected profit across different sized LSRs

Figure 4-21: Estimated change in optimal BR space and profit with change in ambient population density

Similarly, Figure 4-22 illustrates two results (a) BR space allocation for either store service type is the same for a given retail space, and (b) a smaller LSR of a walk-in+drive-through combination service type can yield a comparable or larger profit than a LSR with a walk-in only service. For instance, the profit of a smaller walk-in+drive-through LSR that is only 500 sq.ft. can match that of a walk-in only type LSR of a larger retail space with a 1,700 sq.ft. area.

As discussed above, the expected profit for a walk-in and drive-through combination retail establishment is consistently larger than a walk-in only LSR type. It is because at the optimal BR space allocation, the revenue obtained in the LSR with a walk-in+drive through combination is larger than the walk-in only establishment. Moreover, even though the costs increase to support this increased revenue, this increment is constant for a given BR space and a given total retail size. It also implies that if a drive-through combination service type is added to an existing walk-in store, the BR space allocation does not need to change (assuming that the space allocation in the existing store is optimal).
The analysis presented in this section implicitly assumes that the replenishment frequency over a time period is not limited. The increased revenue, which is possible because the demand generated increases with population density or adding a drive-through service to a walk-in store, is supported by increase in inventory, labor, and ordering costs. As a result, while the profit increases, the backroom space allocation remains unchanged. However, if there is a limit on the replenishment frequency during a time period, the BR space allocation is expected to increase with increase in population density or change in service type. This is because the revenue potential is limited by the BR space inventory, which in-turn is a function of the maximum number of replenishments that can occur during a time period. This limit on the replenishment frequency can thus be considered as a new constraint in the space allocation model, which we plan to implement in the future versions.

### 4.9.5 Comparison of sensitivity of the backroom space allocation to the change in the regression parameters

In this section, we will compare the extent of sensitivity of the optimal BR space and the resultant profit in a retail establishment to the change in the parameters of the revenue and cost functions which are estimated from regression. Given that we have a relatively small dataset to estimate these parameters, a sensitivity analysis could apprise us about the
information that should be prioritized for data collection in the future.

Three scenarios, namely the base case, low case, and high case, are considered for analysis. The currently estimated set of parametric values correspond to the base case scenario. Low and high cases correspond to a 30% decrease and increase respectively in the values of the parameters from the base case. We evaluate the effect of the change of each parameter on the BR space allocation and profit while the other parameters are set to their base case values. The different profit function components are summarized below for ease of reference to the parameters that are discussed in this section.

- **Revenue (Rev)** from equation 4.5: \(A(I, \vartheta) \ast (TR-BR)^{1-a_2}BR^{(a_2-a_1)(\log_{e}BR)^2}\), where TR is the total retail space and I, \(\vartheta\) are the LSR type and population density respectively.

- **Inventory cost** from equation 4.8: \(\gamma_0 \ast Rev \ast (1 + C_h)\), where \(C_h\) is the holding cost related constant.

- **Labor cost (LC)** from equation 4.9: \(\frac{(LC/Rev)}{TR} = \max(\beta_0 - \beta_1 \ast \log_{e}BR, LR_{\text{min}})\)

- **Ordering cost (OC)** from equation 4.10: \(\frac{(OC/Rev)}{TR} = \max(\rho_0 - \rho_1 \ast \log_{e}BR, OR_{\text{min}})\)

The two space related revenue function parameters are \(a_1\) and \(a_2\) as indicated above. Figures 4-23a and 4-23b illustrate how the revenue changes with change in \(a_1\) and \(a_2\) for a given retail space. Increasing \(a_1\) or \(a_2\) decreases the maximum revenue that can be obtained. On the other hand, increasing \(a_1\) decreases the level of BR space that maximizes revenue and increasing \(a_2\) increases the level of BR space that maximizes revenue.
Figure 4-23: Change of revenue with space dependent regression parameters

\( \beta_0 \) (and \( \rho_0 \)) is the intercept for ratio of labor cost (and ordering cost) to revenue per unit of total retail space and is therefore independent of the BR space. Increasing \( \beta_1 \) (and \( \rho_1 \)), on the other hand, implies that the labor cost (and ordering cost) per unit of revenue decreases at a higher rate with BR space.

A 1,000 sq.ft. store is used for the sensitivity analysis. Using this store size ensures that we isolate the effect of LR\(_{min}\) and OR\(_{min}\) on the base case scenario’s optimal BR space allocation. To recap, LR\(_{min}\) and OR\(_{min}\) are the minimum values imposed on the ratio of labor and ordering cost to the revenue generated per unit of total retail space. The results of the sensitivity analysis is shown in Figure 4-24.

An increase or decrease in the revenue parameter \( \alpha_2 \) and labor cost parameter \( \beta_0 \) have the largest and the second largest impact on the BR space allocation. A 30% decrease in \( \alpha_2 \) (\( \beta_0 \)) reduces BR space allocation by approximately 33% (47%) while an increase of 30%
increases the BR space allocation by approximately 54% (34%). On the other hand, the inventory cost parameter, $\gamma_0$, has the smallest impact on the BR space allocation. A 30% increase (or decrease) in $\gamma_0$ results in approximately 3% increase (or decrease) in the BR space allocation. It is also observed that while decreasing the value of labor cost parameter, $\beta_1$, by 30% results in approximately 10% increase in the BR space allocation, increasing it by the same amount decreases BR space allocation by approximately 47%. Ordering cost parameter, $\rho_1$, has a relatively smaller impact with both increase and decrease of the base case value.

Figure 4-24: Comparison of sensitivity of the optimal backroom space to space related profit function parameters that are estimated from regression
Figure 4-25: Profit corresponding to optimal BR allocation with change in space related profit function parameters that are estimated from regression

Figure 4-25 depicts the change in the expected profit that corresponds to the optimal BR space for the base case, low case, and high case scenarios. We note a peculiarity in the effect of increasing or decreasing revenue parameter $\alpha_2$, that is, the profit increases for change in either direction of the parameter. This phenomenon can be explained by recollecting the nature of impact that $\alpha_2$ has on the revenue function. As discussed earlier, increasing $\alpha_2$ increases the level of BR space at which revenue is maximized and this maximum revenue decreases. More specifically, the revenue curve becomes flatter as $\alpha_2$ increases (see Figure 4-26). However, this also means that besides reduction in revenue, there is an additional opportunity to reduce the total cost because the revenue is maximized at a much larger BR space. Therefore, depending on the extent of change in the level of the BR space that maximizes revenue, the profit could increase as $\alpha_2$ increases. The expected profit reduces with increase in $\alpha_2$ for a 5% change from its base case value. It is because the amount of reduction in the BR space that maximizes revenue is much smaller, which therefore does not result in a large enough decrease in the costs. In contrast, increasing revenue parameter $\alpha_1$ also reduces the maximum revenue possible but at the same time the BR space that maximizes revenue also reduces. Therefore, while the revenue decreases, the total cost simultaneously increases due to a smaller BR space at which revenue is maximized.
Figure 4-26: Effect of revenue regression parameter on the revenue and cost for low and high case scenarios

Comparison of Figures 4-24 and 4-25 reveals that the sensitivity of the optimal BR space and the corresponding profit for a given percentage change in the space related profit function parameters could differ significantly in magnitude. For instance, while the BR space allocation in a 1,000 sq.ft. retail establishment is most sensitive to revenue parameter $\alpha_2$, the impact on the corresponding profit with change in the parameter is one of the lowest. Rather, the same percentage change in revenue parameter $\alpha_1$ has a much larger impact on the profit. On the other hand, the impact of the labor cost parameters $\beta_0$ and $\beta_1$ are one of the largest, for both BR space allocation and profit. This implies that mis-allocating stores on the basis of incorrectly estimated revenue parameter $\alpha_2$ (and with accurate estimates of the other parameters), has smaller implications on the store performance as compared to the impact of inaccurate estimations of labor cost parameters or revenue parameter $\alpha_1$.

The comparison of the sensitivity of the BR space and the corresponding profit was done for a low amount of 5% change and also for changes ranging between 10% to 50% in steps of 10%, to the base case parameters. The overall direction of change of both the BR space allocation and the profit were the same. The parameters that the BR space and the profit were most and least sensitive to were similar to the case shown in Figure 4-24.

4.10 Conclusion and future research

Recent academic research has discussed and has started to provide some evidence regarding the importance of BR space and the related challenges in the retail supply chain. One such challenge is associated with the complex problem of space allocation and the management of the BR space in constrained urban retail environments. The complexity arises due to the
interconnectedness of the BR with the rest of the areas in the retail establishment.

In this chapter, we present a modeling framework to determine the strategic allocation of the BR space in a retail establishment for a LSR. This is accomplished by modeling the non-trivial trade-offs between having too much or too little of the BR space. More specifically, these trade-offs are modeled to provide an estimate of the change in the different profit components with unit change in the BR space allocated. These functions are then incorporated into an optimization framework that decides on the optimal BR space within a retail establishment with the objective of maximizing the expected profit.

In the course of our discussion, we have presented our hypothesis and the rationale behind what we expect for the functional forms for revenue, inventory cost, labor cost and ordering cost. We then used relevant data from our partner retailer to test these hypotheses and estimate the parameters of the different functions for the profit components. Since we are limited by the observations in the dataset, we investigated the accuracy of the model by comparing the results from it and the average values for BR space allocation and profit from our dataset. The profit function components that could explain the current allocation and performance in the stores from our data set were chosen for the optimization model.

Additional analysis provides some insights into the sensitivity of the optimal BR space to the change in the input parameters of the model, which includes, total retail space, labor cost, ordering cost, inventory cost, population density and store type. We find that the recommended BR space allocation increases with larger total store size but only up to a certain point, after which it stays unchanged. Reducing inventory cost and labor or ordering costs as a percentage of revenue in a retail establishment generally reduces the optimal BR space allocation. The impact also depends on the interactions of the different input parameters like the total retail space. Since labor, ordering and inventory costs could impact space allocation and the corresponding profit, it can be beneficial for the retailers to investigate operational strategies that they could implement in order to manage these costs. For example, pack size and ordering policies impact labor cost and ordering costs and hence are also potentially connected to the optimal BR space allocation and profit. Discussions on the other factors that affect the cost components and therefore the BR space allocated are elaborated in section 4.9 of this chapter. On the other hand, BR space allocation within a retail establishment stays unchanged with change in population density and store type. This implies that if a store, which is currently allocated with an optimal amount of BR
space, undergoes a change in its service type (i.e., walk-in to a walk-in and a drive-through combination) then space re-allocation is not required. The implication with respect to the population density is that a store with a smaller retail space (and thus a smaller BR space) located in an area of a high population density can match the profit obtained from a larger retail space (and thus a larger BR space) located in an area of a lower population density.

Perhaps the biggest limitation of our analysis is the unavailability of data related to the distinct spaces in the retail establishment. The regression models could potentially provide more accurate estimates of the parameters if we had access to information about the measurements for FR and the BR spaces for all the retail establishments in our dataset. Therefore, we also conduct analysis to investigate the sensitivity of the BR space allocation and profit to the change in the regression parameter estimates. We find that the magnitude of sensitivity of the BR space can be different from the sensitivity of the corresponding profit. Therefore, this implies that the performance implications from the mis-allocation of BR space within a retail establishment resulting from inaccurate estimates of the regression parameters can vary.

Other related data that could have added more value to the analysis is information for retail establishments with different store formats, for instance LSRs that are drive-through only. The current dataset only includes LSRs that are either walk-in only or a combination of a walk-in and a drive-through service. Acquiring additional information would enable us to attribute the specific effect on the BR space allocation to different store formats.

Further, the current optimization model can be extended to explicitly account for the preparation space (PS) in the retail establishment. This would require us to divide the currently identified FR space into FR space and PS. Accounting for this additional space would greatly enhance the applicability of the modeling framework for retail establishments in the restaurant industry. Including PS space in the model would imply that the revenue model be modified to a function of the three spaces: FR, BR and the PS. We would also expect to include an additional criteria related to the end-items that are served in the retail establishments. Specifically, we will be distinguishing between the prepared and the one-to-one end-items because the PS space directly limits the revenue generated from the sale of the prepared end-items only. Moreover, the current model can also be refined a step further to specifically account for the seating area in the FR. Academic literature related to restaurant revenue management has discussed the effect of the seating area on the demand
observed at the store. Therefore, further splitting the FR space into seating area and the rest of the area would require us to modify the revenue function.

Finally, another potential extension of this research would be to modify the optimization model for applicability to a retail establishment that caters to both online and in-store needs. This will enable us to evaluate the implications of space allocation to the distinct areas within the retail establishment in the light of omni-channel demand fulfillment strategies.

4.11 Appendix

We could also use the following process to estimate the implied average review period from the ordering costs and average cost per order by using the following steps:

\[ V(\text{delivery}) \times f(\text{delivery time period}) = OC(y) \]  

(4.13)

where \( V \) denotes the average fixed cost for every delivery, \( f \) denotes the frequency of delivery and \( OC(y) \) denotes the ordering cost as a function of BR space, which will be discussed in the next section. Therefore, equation 4.13 can be written as a function of the average review period 'R' in the following way:

\[ V \times \frac{1}{R} = OC(y) \]  

(4.14)

which also implies that:

\[ R = \frac{V}{OC(y)} \]  

(4.15)

Further, we can substitute the estimated average value of review period in equation 4.8 which implies that:

\[ \text{Inventory cost} = (\gamma_0 \times \text{Revenue}(y)) \times \left( 1 + \frac{h}{12} \times \frac{V}{2 \times OC(y)} \right) \]  

(4.16)

However, using this method adds complexity to the profit function. Moreover, the difference in the inventory cost values by setting \( R \) to be 1 week and using equation 4.16 is negligible. This is because of extremely low inventory holding cost values as compared to the COGS and therefore also the revenue. Hence we eventually decided to use one week for the review period parameter in our analysis.
Chapter 5

Conclusion

5.1 Summary of research discussions

The research presented in this thesis addresses the problem of backroom (BR) space allocation in retail establishments. This problem is of vital importance to the retail industry because of multiple factors, including large portions of supply chain costs being associated with BR space management and operations, and high real-estate cost that make retail space an expensive and a constrained resource. This thesis provides a modeling framework that can be incorporated into a decision support tool for retailers to optimally allocate space to the BR within a given retail establishment and hence improve its profitability. Specifically, it considers BR space allocation in the context of Limited Service Restaurants (LSRs). We focus on three research questions.

1. What is the effect of pack size on inventory levels and space needs in the backroom?

2. How can a given backroom space be efficiently utilized through optimal inventory control?

3. What is the optimal amount of space that should be allocated to the backroom in a given retail establishment?

The theoretical models we develop to address each of these questions are applied to a case study based on real data from a major global retailer with more than 8,000 company-owned stores across the United States alone. We have information for 126 of these stores.

In Chapter 2, we investigate the first research question, i.e., the effect of pack size
constraints on BR inventory and space requirements. The analysis is conducted on a single Stock Keeping Unit (SKU) level. Two pack sizes are considered: Order Pack Size (OPS), i.e., the size in which SKUs are ordered and Storable Pack Size (SPS), i.e., the size in which the SKUs are stored in the BR.

OPS and SPS have different implications on the BR space. SPS affects the BR space requirements for a given inventory level. For a given amount of inventory, the space occupied is greater than or equal to the product of equivalent number of storable packs and the space occupied by one storable pack. In other words, SKUs are stored in the backroom in storable packs until the last sellable unit within a storable pack is used.

OPS affects the inventory level, because stores carry excess inventory for OPS greater than one. This excess amount of inventory also increases the BR space requirements. We analyze and quantify this excess amount of inventory for a periodic review and order-up-to level inventory policy. We derive a closed-form expression for excess inventory caused by OPS effects in the simplified case of deterministic demand. Then, we propose an approximation for this excess inventory for normally distributed demand. A uniform distribution is found to be a good approximation for the beginning inventory level over a review period based on comparisons to the average estimates from Monte-Carlo simulation.

While the inventory level and the space requirements in the BR increase with increasing OPS, there is also an unintended positive side-effect of larger OPS: a reduction in stock-out probability. The excess amount of inventory helps to hedge against a larger proportion of demand uncertainty than what the planned stock-out probability would imply without considering OPS effects. We derive the change in the expected stock-out probability as a function of the planned order-up-to level, normally distributed demand parameters, and OPS. The stock-out probability is found to be non-linearly decreasing with increase in OPS.

In summary, there is a trade-off that is observed with an increase in OPS: while space requirements increase, the stock-out probability decreases. As an example, a SKU with a mean (and standard deviation) of demand over review period of 70 units (and 15 units), and for a planned cycle service level of 80%, increasing the OPS from 1 to 10 (20) increases the space requirements by 5% (11%) and decreases probability of stock-out by 28% (47%). If there were no space constraints and/or stores dealt with only a single SKU, then having a large OPS or SPS would not be a concern. Rather, larger OPS would be desirable given their potential to reduce stock-out probability (as long as the increase in the holding costs.
is not large). But in reality, stores function in a multiple-SKU and a space constrained environment. This setting is considered in our next research question.

In Chapter 3, we investigate how inventory control in a LSR affects the efficiency of space utilization of a given BR. LSRs generally face an important complication related to inventory and BR space management: the distinction between a SKU and an end-item. While the revenue is calculated on the level of an end-item that is sold to a customer, the space requirements in the BR and the inventory cost is driven by the characteristics of the SKU. However, different SKUs have different review periods, pack sizes, and purchase cost per unit. This makes the trade-offs for allocating space to the SKUs for maximum profit non-trivial. This challenge is addressed with the help of an optimization model that chooses the target service level of each end-item with the objective to maximize expected profit.

The model extends the extant academic literature by integrating four important aspects of the underlying optimization problem in a single model formulation. First, our model translates the end-item service levels to those of the SKUs. The service level of a SKU is a function of the multiple end-items that use it, the quantity in which it is used in each of the end-items, and the variability of the end-item demand. Second, we distinguish two types of inventory levels: beginning inventory and average inventory. While beginning inventory level affects space, average inventory drives the holding cost. Third, since we are dealing with a multiple-SKU setting, we account for multiple review periods by maximizing for profit over an integer time period, which is the lowest common multiplier of review periods across all SKUs. Fourth, we account for correlations between the revenues of various end-item categories to better reflect the sales patterns that are observed in reality.

The optimal end-item service levels can be translated to the service level of the SKUs, which in-turn can be used to estimate the equipment requirements in the BR by using the information on equipment capacity. The equipment requirements are then used to evaluate the BR space requirements. Thus, the optimization model provides our partner retailer with the quantity of each of the three equipment types, freezer, refrigerator and ambient, used by the stores and that maximizes the profitability of a given BR space.

There are three key take-aways from our case-study analysis. First, the recommended mix of equipment types is a function of the amount of space that is available for inventory storage, the potential end-item assortment available for sale, and the parameters of the end-item demand distribution. Second, increasing the BR space increases the profit obtained
up to a critical level, after which there is no significant improvement observed. The initial improvement happens because (a) end-items that could not be offered under a constrained BR space can be offered as the space increases, and (b) the end-items that had a lower service levels for constrained BR space can achieve higher service levels as BR space increases. Third, this analysis can also enable us to estimate the opportunity cost of having a constrained BR space. In other words, we can estimate the amount of profit that we are losing because of a small BR space.

Some of the potential real-world use cases of the optimization model for a retailer are described as follows. First, the model can be used to identify the SKUs that are the “worst space offenders” by evaluating their value contribution against their BR space usage. The value provided by the SKU could be interpreted as its revenue or profit contribution. For instance, the model can be used to estimate the improvement in profit when the SKU pack sizes, OPS and SPS, are changed. This improvement in profit could happen because of two reasons. One of the reasons is that reducing the OPS reduces the excess inventory that is carried to reach a certain level of service for the end-items, which in-turn reduces the purchase cost, holding cost and the cost of waste. The other reason being, reducing OPS or SPS potentially reduces the space requirements to attain the current service level of the end-items. Therefore, this creates an opportunity to improve profit by making additional space available to increase the service levels of the other end-items that are being offered at a lower service level or cannot be included in the end-item assortment because of constrained spaces.

Second, a retailer can also use the optimization model to analyze the interrelationships between growth targets for store sales, pack size constraints, and the resulting future space requirements to sustain this growth. Alternatively, by implementing the model on different stores with their existing BR space and the SKU characteristics like pack size, the retailer will be able to identify the stores whose growth is constrained by their BRs.

Third, the optimization model can also be used for identification of improvement opportunities from under-utilized equipment. This can enable the retailers to assess potential profit growth that can be obtained based off the current equipment available in the BR space.

Fourth, the optimization model can also be used for store delivery frequency rationalization. Recent conversation with our partner retailer suggests that there is a growing concern
about the rate at which last-mile delivery costs are increasing, which exceeds the rate of increase of sales/profit performance of the stores. Therefore, this optimization model can be used to investigate the impact on profit with change in the review period and hence the delivery frequency.

Fifth, the optimization model can also be used for SKU rationalization. In other words, it can act as a decision support tool to determine if space usage of a newly introduced SKU is justified by its superior profit contribution compared to incumbent SKUs.

In Chapter 4, we tie our previous considerations together to address the third and the over-arching research question about BR space allocation. To this end, we extend the optimization model developed in Chapter 3 to account for other BR related costs besides inventory cost, specifically labor and ordering cost, as well as the effect of frontroom (FR) space on demand and thus on the revenue. Since FR space is closely linked to the BR space in a space constrained retail environment, it is crucial to the BR space allocation problem. So, we design a strategic decision optimization model for optimal BR space allocation within a given retail establishment that maximizes its expected profit.

Using historical data from our partner retailer, we propose and estimate closed-form expressions for store revenue and cost as a function of the total retail space, BR space, ambient population density surrounding the store and store type. With respect to the impact of space allocation on store revenue, we find that there can be two regions of sub-optimal space allocation: (i) a BR constrained region, which limits revenue because of insufficient inventory storage space and, (ii) a FR constrained region, which limits revenue because of insufficient demand-generating space. Ambient population density and store type are also found to be significant explanatory variables for revenue obtained in a store. Revenue increases with increase in ambient population density because the demand potential increases. Similarly, a store with a walk-in and a drive-through combination service type has a higher revenue yield than a walk-in only store. It is because with the same amount of FR space, a store with a combination service type can potentially generate a larger demand than one with a single service type.

We compare the BR space allocation and profit for an average sized store in our dataset and the optimal values obtained from solving the model for the same store size. The results from the model solution and the dataset average are found to be approximately equal. Comparison of the dataset average for 4 subsets of stores segmented based on their sizes
i.e., less than 1,000 sq.ft, greater than or equal to 1,000 sq.ft. and less than 1,500 sq.ft., and so on are also approximately similar to the model solutions for the BR space allocation and profit. We infer that because our model is able to explain the current space allocation and profit, we establish a reasonable level of model accuracy and therefore its applicability to real-world settings.

We also analyze the sensitivity of the optimal BR space allocation and profit to changes in various input parameters. We find that increasing the total retail space increases the optimal BR area for small to moderately sized stores (less than 2,300 sq.ft.). This happens because, while larger retail spaces have an increased revenue potential, a comparable amount of BR space is also required to support this revenue generation.

Optimal BR space allocation and the resultant profit are also found to be sensitive to the labor cost and the ordering cost incurred. We analyze their effect by evaluating the proposed amount of BR space and profit for different scenarios of the ratio of labor (and ordering) cost to revenue generated. Increasing the labor or ordering cost per unit of revenue leads to more space being allocated to the BR and a lower profit. The opposite is true when the ratios are decreased. For instance, in a 1,000 sq.ft. store, a 50% increase in the ratio of labor cost to revenue increases the suggested BR space by 15% and decreases profit by 16%. On the other hand, a 50% decrease in the ratio of labor cost to revenue decreases BR space by 25% and increases profit by 16%.

As the optimal BR space allocation and profit are sensitive to labor and ordering cost, it can be helpful for the retailers to investigate strategies related to managing these costs. For example, determining optimal OPS and training employees could reduce the required time for handling operations in the store and therefore potentially reduce labor cost. Further, assuming that labor costs also increase with increased BR congestion, having more organized and cleaner BR areas can contribute towards a reduction in these cost. On the other hand, ordering cost can be managed by impacting the replenishment frequency and cost per unit of replenishment. While optimal inventory policies could reduce replenishment frequencies, arranging BRs to be “delivery-ready” for the delivery personnel so that it reduces stop times at the stores could also reduce the cost per unit of replenishment.

The optimization model from Chapter 4 can be used as a decision support for multiple use-cases for retailers. First, it can help determine the optimal BR space allocation for newly acquired retail store spaces. Second, it can be used to identify the existing stores
that require immediate attention or remodeling because their poor performance can be associated to a mis-allocation of space within them. Third, it can prescribe the right store size for a specific geography. For instance, sensitivity analysis shows that small retail stores of a specific type and in areas of higher population density can match the profit obtained from larger stores in areas of lower population density. Assuming that population density is positively correlated with retail occupancy costs, including rent, utilities, insurance etc., retailers would be inclined to choose a smaller store size in areas of higher population density. Therefore, the model can be used to determine how much smaller a store can be so that the benefit of a lower rent is not outweighed by the profit reduction incurred from reducing store size.

In conclusion, this thesis combines inventory theory, optimization, Monte-Carlo simulation, and econometric methods to tackle a problem related to the space allocation and management of a limited store space, which is a pertinent challenge in the retail industry. By focusing on the BR space, which is a neglected area of retail space optimization and design in both academia and industry, this thesis contributes towards supply chain theory and practice. By using data from a major retailer to inform or estimate model parameters when applicable, we demonstrate how the methods, findings, and the implications presented in this thesis provide relevant tools and insights for several real-world settings.

5.2 Future research

We have identified four potential directions for future research.

First, our current research accounts for the effect of two areas within a retail establishment of a LSR for optimal space allocation that maximizes the profit obtained. These two spaces are the customer facing area or the frontroom (FR) and the inventory storage area or the backroom (BR). The preparation space (PS), which also affects the revenue potential, and hence the profit of the retail establishment, by supporting the fulfillment of demand, is considered to be a part of the FR. In future, the space allocation model can be extended to split the retail establishment into the FR, PS and the BR areas. Therefore, we would account for the interdependencies between all the three functional areas, FR, BR and PS, when explaining store profit.

There is an additional consideration that stems from modeling the revenue obtained as
a function of PS, that is, the distinction between the processed (e.g., cappuccino) and one-to-one end-item (e.g., muffin) types for the space allocation problem. While the FR and the BR areas affect the revenue for the two types of end-items, the PS affects the revenue for only processed end-items. Therefore, while the revenue for one-to-one end-items should be modeled as a function of the FR and the BR, the revenue from the processed end-items should be modeled as a function of the three spaces.

Moreover, the FR can be further divided into the seating area and the rest of the FR area. Different attributes of the seating area, for instance seating capacity and the configuration or the table mix, have been shown to have an impact on the restaurant revenue yield (Kimes and Robson, 2004). Therefore, we can potentially improve the explanatory power of the revenue model by accounting for the impact of the FR seating area on the demand generated in the retail establishment of a LSR.

Furthermore, accounting for the PS and the FR seating area can enable us to modify the space allocation model to include the service type of the LSR as a decision variable instead of it being an input like in the current version of the model from Chapter 4. It can enable us to determine the right service type for a retail establishment in a specific geography that maximizes store profit. Three different service type combinations are possible for a LSR: walk-in only, drive-through only or walk-in and drive-through combination. These service types are linked differently to the functional areas of the retail establishment. For instance, while a walk-in only and walk-in and drive-through combination stores have all the three areas (FR, BR and PS), the drive-through only has the BR and PS.

Second, the results from the BR space utilization model, which is presented in Chapter 3, can be extended to model for other replenishment policies or even determine the optimal policy for constrained BR space settings. Demand estimation of the end-items should also account for the correlations between them. Currently, the correlations are modeled on the level of the end-item categories. Therefore, we only observe complementarity between the categories. By determining the correlations on the individual end-item level we can model for both substitutability and complementarity of the end-items, which could enhance the currently proposed BR space utilization model.

Third, the research could be extended to investigate combined BR space allocation and replenishment strategies for the BR that a network of retail establishments can adopt to maximize their joint performance when faced with constrained space environments. A few
examples of potential strategies is shown in Figure 5-1. In strategy A, multiple stores can share an external BR and use their entire store space for FR and PS areas. In strategy B, a store in a sub-urban area can be the hub for stores in urban areas, which are typically much more space constrained. In strategies C and D, stores could transfer items between each other or from an external party respectively, when they have inventory shortage in their constrained BR spaces.

Figure 5-1: Examples of potential replenishment strategies adopted by a network of stores with constrained BR spaces

Summarily, the considerations for determination of the optimal strategy under space constraints include decisions about (i) the physical location of the BR, which could be internal or external to the rest of the store space; (ii) the trigger for a transaction between the stores or the external party, which could be reactive (emergency) or proactive (non-emergency); and (iii) the trade-offs of having shared inventory or dedicated inventory for the stores.

As compared to the current research that considers space allocation for a given store, this extension will consider multiple stores and how their spaces can be leveraged for maximizing their joint performance. Moreover, the network level decision variables like the degree of inventory pooling between the stores or decision about the sub-urban hub for the urban
stores could affect BR space allocation. The advantage of such strategies is that the stores can potentially achieve increased service levels with lower safety stock, which also implies lower BR space requirements and therefore can be helpful in constrained space environments.

Fourth, in light of growing online retail business and the competition to deliver to customers within a short time window, it is important to understand how BRs can be leveraged for multi-/omni-channel retailing models. This includes determining BR related designing, planning and management modules to cater to multiple demand streams. In other words, this motivates the research directions that should be undertaken to re-conceptualize BR spaces beyond storage areas but rather as also being local distribution hubs with staging areas for multi-/omni-channel markets (Tompkins International, 2014). In order to accomplish this, BRs have to be equipped with additional activities and functionalities, which include, picking, packing and shipping for the online markets. This makes the BR space an even more constrained and a congested environment. Therefore, the trade-offs of having an excess or scarce amount of BR space, which are considered in the space allocation model of Chapter 4, will probably be affected. Furthermore, relevant decisions related to omni-channel business models, for example, on-the-go prioritizing of the demand stream that should be served first, emerge. These decisions need to be considered when evaluating store performance or labor allocation within it.
Bibliography


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