Aggregate Production Planning for Engineer-to-Order Products

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ABSTRACT

The contract manufacturing industry is growing and shifting from standard products to highly customized engineer-to-order (ETO) products. Different from standard products, ETO orders have more production process uncertainties because their design specifications and production process can be changed after the orders have been accepted. Such uncertainties increase production costs, the risk of late delivery and associated penalties, exposing the contract manufacturers to profitability decrease. Since every ETO production is unique, companies cannot rely solely on historical data to ensure accurate planning. The goal of the project is to solve the aggregate production planning problem of ETO orders. Usually, uncertainty is mitigated by keeping inventories. Such safety stock, however, does not work for customized products as they are usually a one-time purchase and therefore, cannot be kept on a regular basis. For customized production, buffer time and capacity can be used against process uncertainty. We formulate an Aggregate Production Planning (APP) model as deterministic, multi-product, multi-stage, and multi-period linear programming (LP). It minimizes the total production cost by balancing the in-house production, inventory holding, outsourcing, overtime hours cost, and backlogged orders penalties. Cost drivers for total production cost are analyzed for multiple scenarios with different production times. We then calculated the buffer capacity and performed constraint sensitivity analysis using the shadow price method. Based on the data analysis, we make recommendations for the sponsor company for planning horizon that we model: add an employee for one production stage and remove an employee from another, use 7% buffer capacity for the base plan to minimize the total production cost for the set of all possible scenarios, use a combination of hiring, overtime hours, and outsourcing. These recommendations lead to 12.32% cost reduction compared to the cost if the company does not use an aggregate planning model and the recommendations. Moreover, we formulate an aggregate production planning approach for the sponsor company to use in the future to ensure an optimal plan with the minimum total production cost.

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1. INTRODUCTION

1.1 Introduction to the Project

ZY Machining and Distribution Ltd., a global contract manufacturer providing customized manufacturing solutions in the heavy machinery industry, is sponsoring our project.

Since 2010 the contract manufacturing industry has experienced the following trends[1]:

1. The contract manufacturing market is highly competitive and tends to be a buyer market because the service of a contract manufacturer can be easily substituted in the global market.
2. The buyers reduce their operation costs by segmenting the components needed for their production. They are producing critical parts with high demands in-house while outsourcing non-critical parts with low demands.
3. The buyers are also inclined to use contract manufacturers' facilities and expertise to test, validate, and improve their new products.

Consequently, the sponsor company is dealing with increasingly various engineer-to-order products with low volume but a high level of customization. Figure 1 shows the differences between these ETO (customized) orders to traditional standard orders regarding order volume and variety.

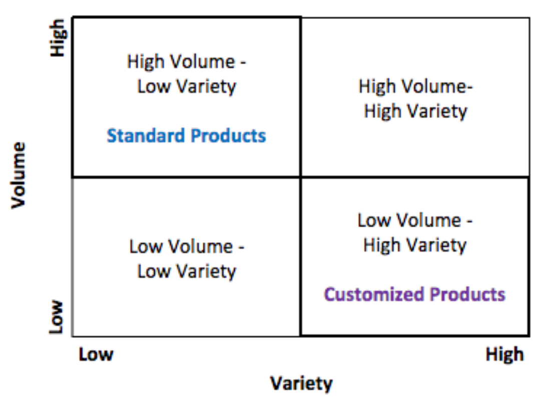

Figure 1. Product Segmentation Quadrants of Standard Products and Customized Products by Volume and Variety

[1] Interview with the operations manager of ZY Machining & Manufacturing.

The company produces both standard and customized products. About three-quarters of their orders were ETO in 2018 as shown in Figure 2. The company expects that the share of ETO products will continue to grow.

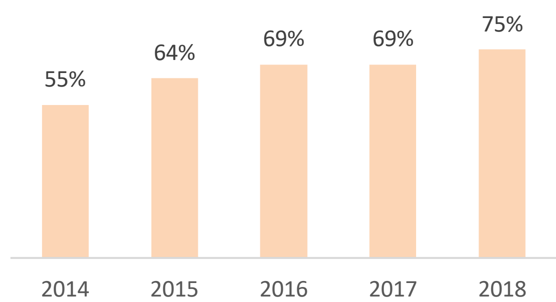

Figure 2. Share of ETO Orders of the Sponsor Company

1.2 Problem Statement

The company's biggest challenge is that the operational costs are increasing dramatically due to last-minute, unexpected production disruptions that were not planned for initially. Such disruptions are more frequent in producing customized orders because producing them involves testing and sampling to validate the production process. In many instances, the company finds problems or inconsistencies during manufacturing. In other cases, the customer updates the drawings after the production has already started. Changes to the specifications lead to uncertain delays in the production and subsequent problems in machines utilization and delivery dates. The uncertainties of the ETO production include:

- Customers request to change specifications;
- Customers request to expedite the order;
- Manufacturers proposal to improve the drawings;
- Repair due to the nature of the prototyping process.

In the sponsor company's experience, the higher the level of customization, the more likely these uncertainties will happen. As a result, the cost of producing ETO orders has been increasing over the years.

In 2018, the sponsor company reported increased revenue by 20% while profit margin decreased significantly as shown in Figure 3.

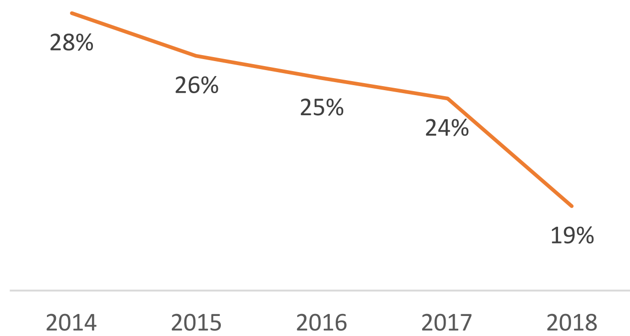

Figure 3. Profit Margin for ETO Product of the Sponsor Company

To have a better understanding of their problem, we interviewed the sponsor company's Sales and Manufacturing departments and developed a detailed process map as shown in Figure 4. The production process has five stages:

I. Stage 1: Raw Materials Purchase;

II. Stage 2: Pre-welding;

III. Stage 3: Welding;

IV. Stage 4: Machining;

V. Stage 5: Post-Machining.

We found that the aggregate planning and cost estimation processes are isolated and neither process considers potential production disruptions. As a result, sticking to the production schedule is very difficult and often impossible. The deviations from actual production to their corresponding production plans increased operational costs such as late shipment penalties cost, and last-minute repair.

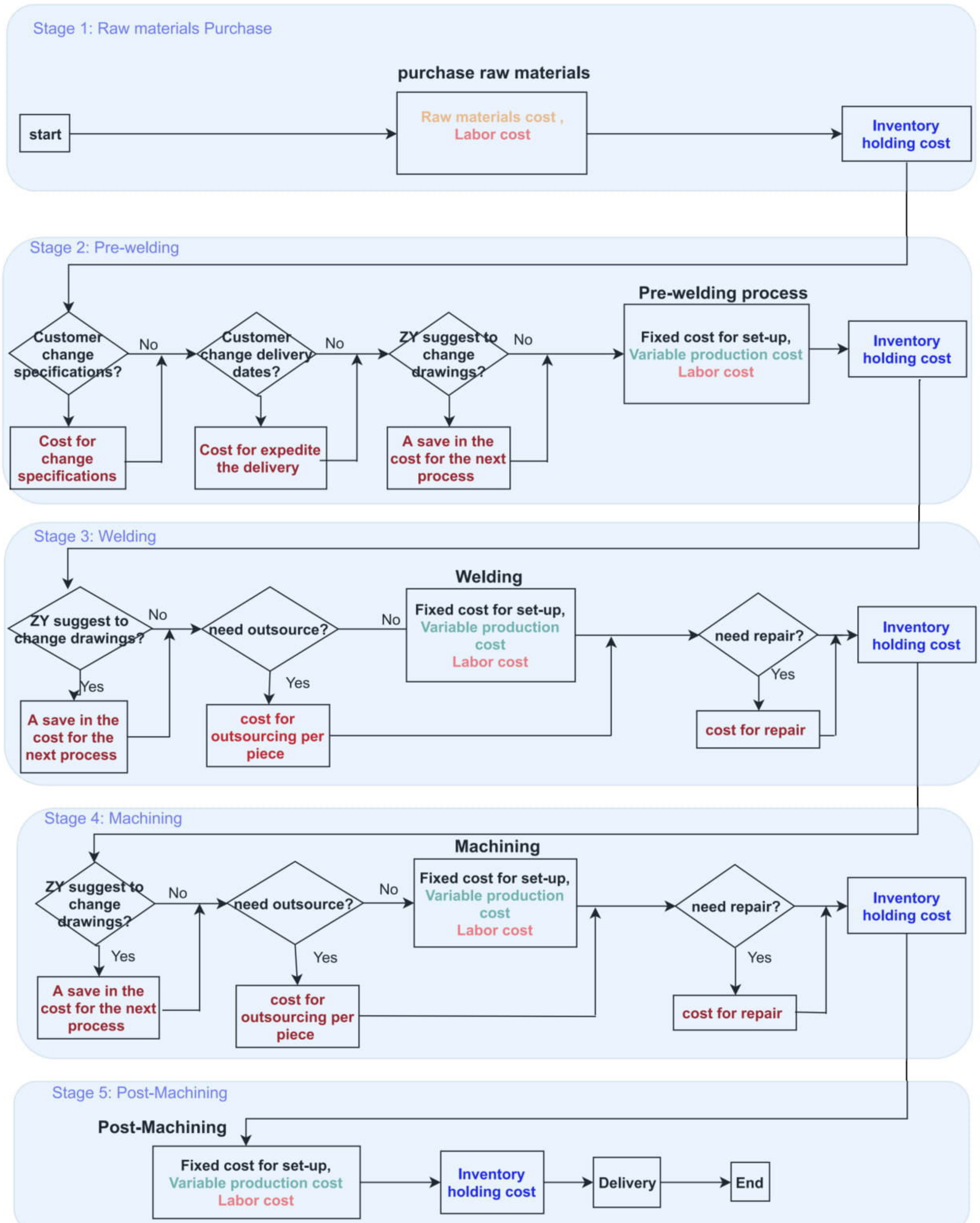

Figure 4. Detailed Process Map of ETO Products Manufacturing

Currently, the company's strategy is to increase the share of ETO products in its portfolio because its core competence is helping customers to solve their engineering and manufacturing problems. Finding the root cause for the increased production cost and reducing the total production cost by improving their production processes is crucial.

We solve the sponsor company's problem for aggregate production planning for multiple products with multi-stage production. Our project first develops a deterministic linear programming model to optimize the sponsor company's ETO production on an aggregate level. This model can give a solution with the minimum total production cost. We run the model with different production scenarios to bring recommendations for dealing with potential production disruptions. Sensitivity analysis is also conducted for insights into constrained manufacturing resources.

This project will benefit contract manufacturers by providing an aggregate production planning solution for cost minimization and dealing with possible production disruptions of the ETO orders. We believe that its benefit will go beyond the contract manufacturing industry to other sectors where multi-product customized orders are frequent in their productions.

Figure 5 shows a high-level outline of the project stages.

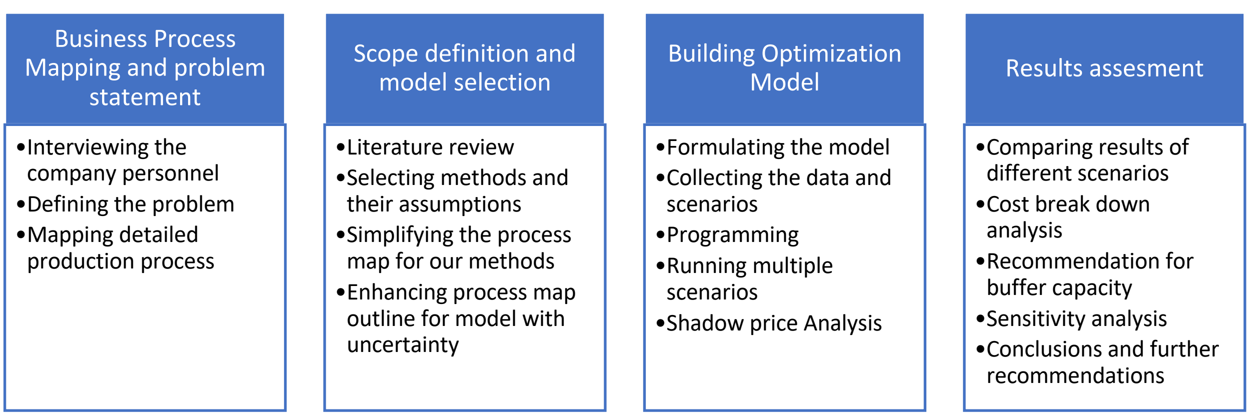

Figure 5. High Level Project Plan

2. LITERATURE REVIEW

We identify potential methods of production planning optimization for engineer-to-order products by conducting literature review.

Aggregate production planning models (APP) are widely used for mid-term optimization (Gupta and Maranas, 2003). APP models can be classified into two groups: deterministic and stochastic models (Gupta and Maranas, 2003). Stochastic models can use scenario-based or distribution-based methods (shown in Figure 6).

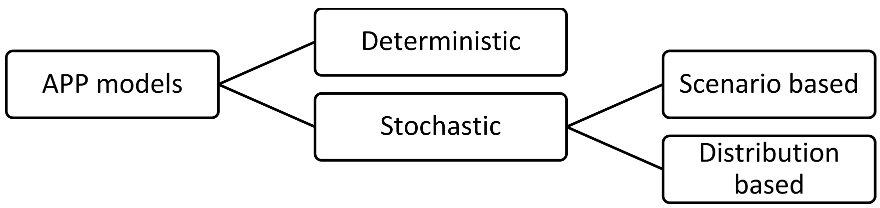

Figure 6. High-Level Overview of APP Model Types

2.1 Aggregate Production Planning Challenges for Engineered-to-Order Products

Key differences between standard and customized products (Lander and Liker 2007) are shown below in Table 1.

Characteristics	**Standard products**	**Customized products**	**Challenge**
Demand Variability	Low	High	Demand varies by projects and
Cycle time	Fixed	Varies by product	Cycle time unique for each project
Product variety	Low	High	Hard to use safety stock for raw materials and finished goods
Production runs	Long with stable batch size	Short with variable batch size	Hard to plan
Job of operators	Repetitive	Different for each product	Hard to standardize
Work knowledge	Structured (routine procedures	Tacit (unstructured undocumented	Hard to standardize
Specialization of people and equipment	High	Low	Equipment is multipurpose
Quality standards	Metric driven	Metric and aesthetic	Need customized quality procedure
Feedback loops in production	Short	Unpredictable	Hard to learn from past actions
Predominant paradigm	Steady and efficient production	Customization of few products	Low process discipline, no standard methods

Table 1. Difference Between Standard and Customized Production

To account for the uncertainties and variabilities of producing standard products, companies use a combination of inventory or time buffers (Hopp and Spearman, 2000). Use of finished goods inventories is not feasible for ETO production because in most cases, the same ETO order will not be repeated in the near future. Raw materials and finished goods stock, therefore, cannot be used to prepare for uncertainties.

2.2 Deterministic Multi-Stage Aggregate Production Planning Models

The linear programming (LP) approach was first proposed by L. Kantorovich to solve the basic mathematical problem in 1939 (Kantorovich 1960). For deterministic production planning optimization, LP approach is widely adopted (M.R. Feylizadeh and M. Bagherpour, 2011). Models have been increasingly complex over time by adding different assumptions and constraints (Yves Pochet, 2000).

We use Gabbay's (1979) formulation for constraining the inventory and demand in multi-stage production as a basis of our multi-stage and multi-product model. Khakdaman et al. (2013) optimized a company' manufacturing process whose structures similar to our sponsor company. Khakdaman et al. (2013) published another paper proposing a deterministic multi-objective model using robust optimization to incorporate supplier's processes and customer's uncertainty. These models helped us to enhance our initial formulation to account for all required constraints and costs.

2.3 Stochastic LP Models

Customization has various types of uncertainties, such as quality uncertainty and supply lead-time uncertainties (Mula et al., 2006). Uskonen and Tenhiala (2011) argued that deterministic models were not sufficient for customized product APP optimization because customization adds new risks.

Kira et al. (1997), therefore, proposed a distribution-based approach to account for the demand uncertainty by including over- and under-productions and their corresponding penalties to the objective function. They also analyzed the impact of different types of demand distributions on the costs. This research inspires us to use buffer capacities for over- or under-estimation of the production times. Biazzi

(2018) proposed a model to account for demand uncertainty by adding a safety stock to constraints. This research provides insight for using scenario playing, and in the future, it can be used when the sponsor company collects enough data to generate a more normalized pattern for their production uncertainties. For our capstone project, we assume uniform distribution for the possible disruptions because the sponsor company has not collected enough data.

For the scenario-based approach, Thompson and Davis (1990) propose a combination of linear programming and simulation in an integrated model. We will also use scenario playing method in our analysis.

2.4 Sensitivity Analysis with Shadow Prices

A shadow price analysis of LP model constraints is a technique to support important managerial decisions. A shadow price shows how much the objective function value will change if we add one unit to a constraint resource. In business, this sensitivity is more important than the actual optimal cost (Alkubaisi, 2017) because it provides insights about the most efficient investment to improve the company's capacity. For cost optimization, shadow price analysis shows how the objective function will change when one of the model constraints changes by a unit (Benidris et al., 2011).

Based on the review we identify that aggregate production planning models (APP) can be used to solve our problem, and the further directions for considering production disruptions will be scenario playing. Sensitivity analysis will also be used for recommendations for the sponsor company's decision to invest in enhancing its manufacturing capacities.

3. METHODOLOGY

In this chapter, we develop a model to identify the optimal aggregate production plan for ETO production without possible process disruptions. We then analyzed the influences of setting internal buffers for production uncertainties regarding total cost and cost breakdown to bring recommendations for the sponsor company to deal with possible production disruptions.

3.1 Aggregate Production Planning LP Model

An LP model is developed as a fundamental framework for the aggregate production planning process of the sponsor company's integrating internal production, inventory, and outsourcing. We simplified the company's detailed process map (showed in Section 1.2 in Figure 4) to the schematic representation of the processes to be modeled and optimized. The simplified map that shows the essential processes involved in the production planning process as a basis to formulate the LP model by removing tactic production schedule details (shown below in Figure 7). This model assumes deterministic production time to build a basic framework for the aggregate production planning of the engineer-to-order products.

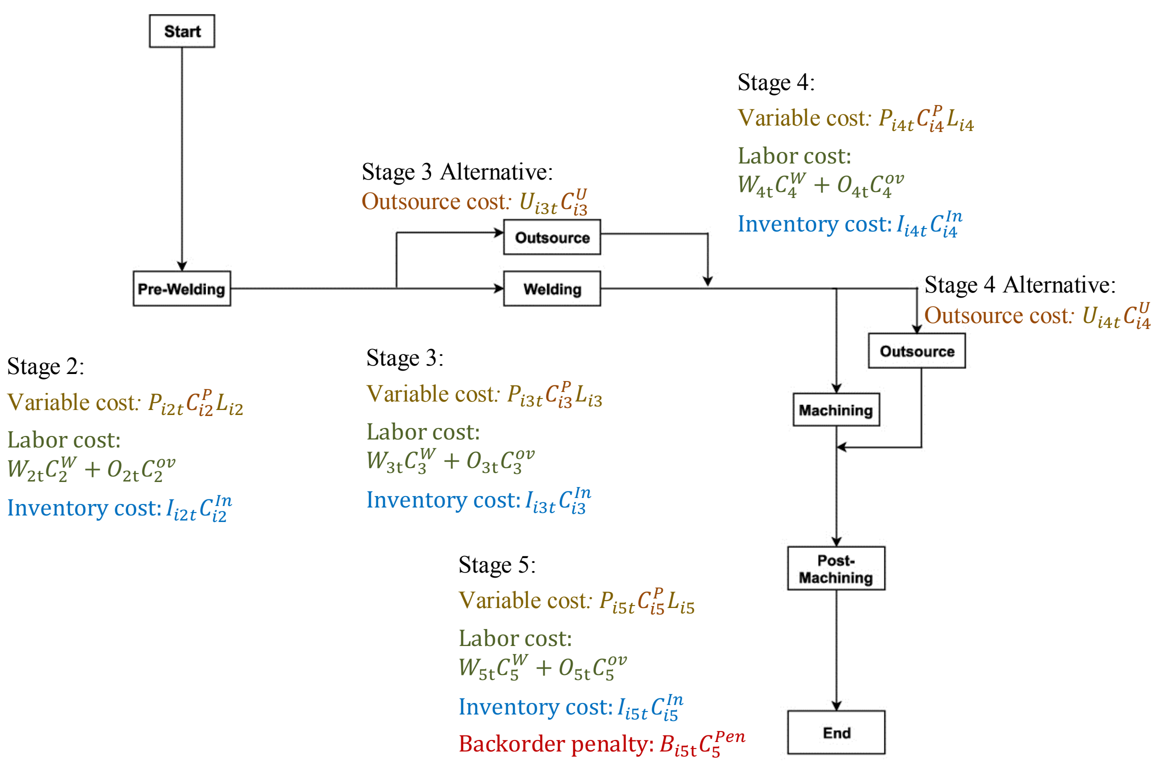

Figure 7. Simplified Process Map for the Basic Model

We use the following cost incurred in the production process:

1. Variable in-house production cost
2. Labor cost, including weekly paid salary and overtime labor cost paid hourly
3. Inventory holding cost for work-in-progress (WIP)

4. Outsourcing production cost

5. Backlogged order cost for failing in delivering on time

Note that buying new equipment is not included in the model as it is not a production planning decision.

3.2 Model Assumptions

In the LP model formulation, we use the following assumptions:

- General assumptions:
  - Multi-stage production;
  - Stages have a strict sequence for every product;
  - Multiple products;
  - Multi-period APP model is executed with weekly time buckets;
- Demand:
  - Demand is deterministic, known for each product, for the entire period of order lead time;
  - We assume that demand is the same for products on every production stage and it is the same product entering the 1st stage and coming out of the last stage;
- Outsourcing:
  - Outsourcing capacity is constrained by product, production stage, and week;
- Production time:
  - Estimated production time is different by product by stage;
  - During one week a product can go through all stages both internally and with outsourcing;
- Backorders:
  - Backorders are allowed;
  - The penalty is assigned for late delivery by product, calculated on week and item basis;
- Labor:
  - Hiring people is very expensive but possible; we do not include the fixed one-time cost of hiring into the total cost calculation as it is not a regular cost and should be considered as an investment;
  - Labor is assigned strictly to every stage and cannot be moved between stages;
  - Cost of labor is different by production stage;
- Capacity constraints:
  - Capacity is limited by units of equipment by stage;

The following items are <u>not included</u> in the model:

- Product transportation times to and from outsourcing production facilities;
- Production set-up time and cost;
- Fixed production costs;
- Raw materials cost and purchasing cost;
- Raw materials inventory holding costs or arrival lead time;
- Constraints for inventory holding space;
- Transportation cost.

3.3 Model Formulation

In this section, we formulate the linear program. Table 2 shows indices notations for the products, production stages, and time periods. Decision variables are shown in Figure 3, and notations of all the input data from the sponsor company are shown in Table 4.

Index	Description	Maximum Value	Value Range
I	Product	*N*	$1 \le i \le N$
S	Production stage	*S*	$1 \le s \le S$
T	Time period	*T*	$0 \le t \le T$

Table 2. Indices for LP Model

Decision Variable	Description	Indices Range
A_{st}	number of employees to hire at start of period t in stage s	$\forall t$ $\forall s$
W_{st}	number of employees in the end of period t in stage s	$\forall t$ $\forall s$
O_{st}	number of overtime hours to work in period t in stage s	$\forall t$ $\forall s$
I_{ist}	number of units of inventory of product i in the end of period t in stage s	$\forall i$ $\forall t$ $\forall s$
P_{ist}	number of units to manufacture internally in period t in stage s of product i	$\forall i$ $\forall t$ $\forall s$
U_{ist}	number of units to outsource to contract manufacturer in period t in stage s of product i	$\forall i$ $\forall t$ $\forall s$
B_{it}	backlog by product by week	$\forall i$ $\forall t$

Table 3. Decision Variables for LP Model

Parameter	Description	Measurement
D_{it}	demand for product i with shipment date in period t	Units
L_{is}	estimated production time for stage s for product i (hours/unit)	Hours/unit
$W_{s\,0}$	size of workforce at start of planning period for stage s (week 0)	Number of employees
H	number of hours each employee can work per week	Hours/person/week
O^{max}	maximum hours of overtime allowed per employee per week	Hours/person/week
U_{is}^{Max}	maximum weekly available outsourcing production capacity for product i for stage s	Units/product/week
P_s^{Max}	available production equipment by stage	Units of equipment
I_{is0}	amount of Inventory at start of planning period for product i for stage s (week 0)	Units
C_{is}^{P}	variable production cost for product i at stage s (utility and equipment depreciation cost)	￥/hour
C_s^{Ov}	cost of an overtime hour for stage s	￥/hour
C_s^{W}	cost of employee for stage s	￥/person/week
C_{is}^{In}	inventory holding cost per stage s per product i	￥/unit/week
C_{is}^{U}	cost of outsourcing product i in a stage s	￥/unit
C_i^{Pen}	penalty per product i	￥/unit/week
z	coefficient of internal capacity used	Dimensionless

Table 4. Input Data for LP Model

The objective is to find the aggregate production plan with minimal cost over the planning horizon. The objective function is subject to several constraints explained after the model formulation. Therefore, the objective function (1) is the following:

$$Min\ z = \sum_i \sum_s \sum_t \left(P_{ist} L_{is} C_{is}^{P} I_{ist} + I_{ist} C_{is}^{In} + U_{ist} C_{is}^{U}\right) + \sum_s \sum_t (W_{st} C_s^{W} + O_{st} C_s^{Ov}) + \sum_i \sum_t (B_{it} C_i^{Pen}) \quad \forall s,\ \forall t \in \{1 \dots T\} \ (1)$$

s.t.

$$\sum_i (P_{ist} L_{is} - (HW_{st} + O_{st}) * z \leq 0 \quad \forall s,\ \forall t \in \{1 \dots T\} \ (2)$$

$$\sum_i (P_{ist} L_{is} - (P_s^{Max} * 24 * 6) * z \leq 0 \quad \forall s,\ \forall t \in \{1 \dots T\} \ (3)$$

$$W_{st} - W_{s\,t-1} - A_{st} = 0 \quad \forall s,\ \forall t \in \{1 \dots T\} \ (4)$$

$$O_{st} - O^{max} W_{st} * z \leq 0 \quad \forall s,\ \forall t \in \{1 \dots T\} \ (5)$$

$$U_{ist} - U_{is}^{Max} \leq 0 \quad \forall i,\ \forall s,\ \forall t \ (6)$$

$$I_{is\,t-1} - I_{ist} + P_{ist} + U_{ist} - (P_{is+1\,t} + U_{is+1\,t}) = 0 \quad \forall i,\ \forall s \in \{1 \dots S-1\},\ \forall t \in \{1 \dots T\} \ (7)$$

$$I_{is\,t-1} - I_{ist} + P_{ist} + U_{ist} - D_{it} - B_{it-1} + B_{it} = 0 \quad \forall i,\ s=S,\ \forall t \in \{1 \dots T\} \ (8)$$

$$A_{st}\ W_{st}\ I_{ist}\ P_{ist}\ U_{ist} O_{st}\ B_{it} \geq 0 \quad \forall s,\ \forall t \ (9)$$

$$0 < z < 1 \quad (10)$$

Constraint (2) constraints that the number of works needed for producing the products internally must be lower than the total available work hours including both regular hours and overtime. Constraint (3) ensures that the maximum running time for each equipment is 24 hours a day and 6 days a week. Constraint (4) is a conservation flow of the number of employees at every stage for period (t-1) and period t. Constraint (5) defines the maximum possible overtime hours per week. Constraint (6) is an outsourcing capacity constraint indicating the capacity limits of sub-suppliers. Constraints (7) and (8) are inventory conservation flow and demand constraints. Constraint (7) is an inventory conservation flow ensuring product movement from stage to stage and allows to calculate inventories of semi-finished products at every stage and between the stages. Constraint (8) is an inventory conservation flow and a demand constraint for products from the last production stage (the only stage that can satisfy customer demand). This constraint also includes backlog. Constraints (9) sets all decision variables non-negative. Constraint (10) sets range for the coefficient z which is used to determine the share of available total internal capacity (labor and equipment for planning purposes which can be any between 0% and 100%). Constraint (10) is used only when we are calculating the high-level optimum buffer capacity in Section 4.6.

3.4 Production Scenarios Analysis

We set up a base scenario as the benchmark to assess the influences of production disruptions according to the company's estimations. 15 alternative scenarios of production time are created by the company based on previous experience. Table 5 shows the scenarios we use in our analysis.

Scenario #	Changes from the Base Scenario	Description	Products Affected	Probability
0 (base)	N/A	Baseline		6.25%
1	Stage 2 reduced 20%	Found a way to use welding equipment more efficiently	All	6.25%
2	Stage 3 reduced 20%	After production start it turned out that less time required	All	6.25%
3	Stage 4 reduced 20%	After production start it turned out that less time required	All	6.25%
4	Stage 5 reduced 20%	After production start it turned out that less time required	All	6.25%
5	Stages 2, 3, and 4 and 5 increased 30%	Underestimated production time	All	6.25%
6	Stages 2, 3, and 4 and 5 increased 50%	Underestimated production time	All	6.25%
7	Stage 4 increased by 100%	Problems with equipment and underestimated production time	All	6.25%
8	Stage 2 increased by 70%	Re-work required	4 and 5	6.25%
9	Stage 3 increased by 80%	Re-work required	17 and 20	6.25%
10	Stage 4 increased by 70%	Re-work required	7 and 8	6.25%
11	Stage 5 increased by 70%	Re-work required	10 and 11	6.25%
12	Stage 3 increased by 50%	Specification change	1 3 7 and 20	6.25%
13	Stage 4 increased by 20%	Specification change	1 2 17	6.25%
14	Stage 3 increased by 20% and Stage 4 increased by 30%	Specification change	1 2 3 4 5 6 and 7	6.25%
15	Stage 2 increased by 10% and Stage 5 increased by 30%	Specification change	10 11 12 13 14 and 15	6.25%

Table 5. Alternative Production Scenarios for Expected Cost Calculation

We run both the base scenario and the 15 disruption scenarios to calculate the total expected cost for all the scenarios. Since the sponsor company does not have systematic record of how frequent these scenarios are happening, we assume a uniform distribution for all 16 scenarios when calculating the expected total production cost for all the scenarios.

3.5 Model Efficiency Assessment

To assess the benefit of using our proposed APP model for company's aggregate production planning process, we run all 16 scenarios in three different manufacturing options:

Option 1: Hiring, working overtime, and outsourcing is allowed. This option represents the company's current setting (using overtime and outsourcing for production disruptions).

Option 2: Outsourcing and overtime are not allowed, but hiring is allowed. This option could be used for the regular internal manufacturing planning because overtime and outsourcing in general are more expensive.

Option 3: No hiring allowed, but overtime hours and outsourcing is allowed. This option represents the urgent reaction under production disruptions: all deviations from the initial plan can be covered only by overtime hours and outsourcing because hiring is impossible with such short notice.

Option 3 represents the extreme situation when the company react on urgent situations by using last-minute production plan changes. Both Option 1 and 2 can be used as a planning solution depending on which one has a lower total cost. Difference between total expected cost for option 3 and minimum total expected cost between Option 1 and 2 is the cost benefit of proposed aggregate production planning model use.

3.6 Buffer Capacity

Within the optimal planning option (which can be either Option 1 or 2), we solve the model for 16 scenarios with different costs and their corresponding production plans. The total expected production cost is calculated based on scenarios probabilities of the scenarios happening while the sponsor company could have only one production plan to address the possible disruptions. A solution summarizing all the scenarios, therefore, is critical in this project.

Setting buffer capacities for the base scenario is a potential method to deal with the production disruptions. Buffer capacity provides more flexibility because the production plan can be reorganized faster using the internal buffer capacities upon production disruptions, resulting in a lower cost. In this report, we suggest a high-level solution using the total expected cost of all the scenarios.

To estimate the size of buffer capacity, we use the total expected cost as the target cost for the base scenario, and calculate the buffer size with which the base scenario costs the company the same amount of money as the total expected cost for all the possible scenarios.

Introducing buffer capacity to the base scenario will cost the company additional costs compared to the base scenario without buffer capacity, because the company is paying for the idle capacities that are waiting for disruptions. Therefore, by targeting the total expected cost of all the scenarios to be matched, we ensure that the company is not setting aside too much buffers, staying within the total expected cost.

3.7 Shadow Price Analysis

The company has capacity constraints both on workforce and equipment. To collect insights for improving the company's capacity, we conduct a shadow price analysis on each decision variable. The analysis shows what cost reduction the company can receive by adding one unit of constraint resource. Moreover, the same analysis can show excess capacities which can be removed or reallocated. This helps the company make decisions on what to invest to decrease their production cost in the most efficient way.

To estimate impact of constraint change, we run the model before and after suggested changes and compare cost differences.

4. DATA ANALYSIS AND RESULTS

In the previous chapter, we formulated a deterministic APP LP model, created production disruption scenarios, and then conducted a shadow price analysis. In this chapter, the results are discussed.

Python 3.7.2 Gurobi Optimizer version 8.1.0 within Spyder 3.3.1 environment is used for results calculation. Pseudo code for the program is provided in Appendix A.

4.1 Data Input

The sponsor company provided data for analysis and optimization: all data used for multiple scenarios has the same information except production time. For all scenarios, we set the following parameters according to the operation of the sponsor company:

1. 40 hours a regular week per employee;
2. 20 hours per week maximum allowed overtime time per employee;
3. equipment can run 6 days a week and 24 hours a day;
4. outsourcing option is feasible for only Stages 3 and Stage 4.

The total number of products is 20 and the planning horizon is 20 weeks. 5 of them have already been produced partially at the beginning of the planning period, and all of them require to be delivered at or before Week 16. Weeks from 17 to 20 are analyzed to account for the possible backlog costs.

4.2 Preliminary Analysis of the Sponsor Company's Data

We conduct a preliminary analysis of the data. The statistics are shown in Table 6. Cost differences between in-house production and outsourcing are shown in Table 7. Most parts are cheaper to produce in-house, while for some stages, where the supplier has engineering or equipment advantage, the supplier (outsourced production) can make it at a lower cost.

	Demand (unit)	**Backorder Penalty (¥/(unit*week)**
Average	41.4	1000
Standard Deviation	20.1	0

(a)

	Stage 2		Stage 3		Stage 4		Stage 5	
	Average	Standard Deviation	Average	Standard Deviation	Average	Standard Deviation	Average	Standard Deviation
Outsourcing cost (¥)	N/A	N/A	386.8	143.2	482.8	464.2	N/A	N/A
Outsourcing limit (unit/week)	N/A	N/A	5	0	3	0	N/A	N/A
Salary (¥ /week)	800	0	1500	0	11400	0	800	0
Overtime (¥ /(hour*person))	30.6	0	49.2	0	53.55	0	33.15	0
Inventory Holding Cost (¥ /(week*person))	8	0	10	1.7	15.5	2.6	13.5	0
Beginning inventory (unit)	3.9	8.4	3.3	10	0	0	0	0

(b)

Table 6. Statistics of Input Data (a) by Products and (b) by Production Stages

Product / Stage	Stage 3	Stage 4
Product 1	72.7	865.8
Product 2	273.9	-1197.1
Product 3	-278.6	251.2
Product 4	471.2	604.9
Product 5	-450	0
Product 6	-87.6	136.85
Product 7	-310.7	-147.5
Product 8	-198.5	-68
Product 9	-7.2	158.3
Product 10	-341.4	-439.75
Product 11	-328.55	-48.8
Product 12	-298.6	-98.1
Product 13	-198.5	-99.5
Product 14	-184.35	-167.3
Product 15	-285.8	-284
Product 16	-64.3	-154.5
Product 17	535.9	-275.1
Product 18	911.2	-591.7
Product 19	2121.8	456
Product 20	514.1	-112.5

Table 7. Comparison of the Cost Differences between In-House Production and Outsource Cost. Numbers Marked Read Shows the Stage that is Cheaper to Produce by Outsourcing.

Table 8 shows the existing workforce and equipment constraints for the company at the beginning of the planning period. Detailed input data, costs, and demands are shown in Appendices B to G.

Stage	Available Employees	Units of Equipment
Stage 2	5	4
Stage 3	3	4
Stage 4	3	4
Stage 5	3	4

Table 8. Available Employees and Units of Equipment by Stage

4.3 Aggregate Production Plan for the Base Scenario

One important result of the planning tool is a detailed production plan for each product. Our model's result shows the aggregate production plan of each product by week and by stage that yields the minimum total cost. The detail plan includes volume of units produced in-house and outsourced, inventories kept, employees hired, overtime hours and backlogs. The total cost of the base scenario with Option 1 (company's current manufacturing option with hiring, overtime hours and outsourcing) is ￥684,833.

Taking a closer look at one product's results, the in-house production of Product 8 starts at Week 9 and finishes at Week 14 as shown in Table 9. Inventory levels for the work-in-progress are shown in Table 10. The base scenario's optimal APP schedule does not have any outsourcing and backlogs.

For labor, the optimum solution suggests we hire employees in Week 1 (shown in Table 11). Table 12 shows overtime hours by stage by week between Week 9 and Week 14 when Product 8 is being produced.

Product-Stage	Week 9	Week 10	Week 11	Week 12	Week 13	Week 14
Product 8, Stage 2	0	0	13.1	4.5	0	Complete
Product 8, Stage 3	1.1	18.1	27.2	16.1	17.5	Complete
Product 8, Stage 4	0	0	46.4	0	33.6	Complete
Product 8, Stage 5	0	0	46.4	0	33.6	Complete

Table 9. In-House Production Plan Example for the Base Scenario (Extract)

Product-Stage	Week 9	Week 10	Week 11	Week 12	Week 13	Week 14
Product 8, Stage 2	61.4	43.2	29.1	17.5	0	0
Product 8, Stage 3	1.1	19.2	0	16.1	0	0
Product 8, Stage 4	0	0	0	0	0	0
Product 8, Stage 5	0	0	46.4	46.4	8	0

Table 10. Inventories Level for Work-in-Progress Materials for the Base Scenario (Extract)

Stage	Week 1
Stage 1	2.03
Stage 2	0
Stage 3	3.35
Stage 4	1.31

Table 11. People Hiring Plan for the Base Scenario. Employees Should be Hired at Week 1 According to the Model

Stage	Week 9	Week 10	Week 11	Week 12	Week 13	Week 14
Stage 1	0	0	0	0	0	0
Stage 2	0	0	0	0	0	0
Stage 3	0	0	0	0	0	0
Stage 4	0	0	45.6	86.2	43.9	0

Table 12. Overtime Hours Planned for the Base scenario from Week 9 to Week 14 (Extract)

A detailed production plan for the Base Scenario is provided in Appendices H to K.

4.4 Results for Multiple Scenarios

We run the scenarios under the three different options explained in Section 3.5. The results are shown in Table 13. Note that cost change is equal to the difference between the cost of planning ahead using the APP Model (the less of the total cost of Option 1 and 2) and the Cost of Option 3 (reaction on changes without preliminary planning).

The results are different from our assumptions that in-house production with hiring (Option 2) is more cost efficient than using hiring, outsourcing and overtime (Option 1). In the next section, we investigate the cost drivers to understand the underlying reasons for this result.

The results indicate that the sponsor company can use either Option 1 or 2 in their regular planning activities depending on which one is cheaper. For the data set we have, Option 1 is always better, because hiring new employees in the beginning of the planning period is expensive and employees cannot be fired even if demand is low and production capacity is not fully utilized. For the extreme scenarios, the demand cannot be met due to available equipment capacity, resulting in backlogged orders and penalties. However, with other inputs it is possible that Option 2 can have lower cost. Option 3 represents the extreme case when the sponsor company urgently react on disruptions. We then compare the costs of the lower of Option 1 and 2 to that of Option 3. For any of the scenarios, Option 3 is more expensive. Range of cost reductions varies from 2.91% to 44.37% depending on the scenario, and the total expected cost for all the scenarios is 11.86% more expensive. Recall that Option 3 represents the case where the firm uses only flexible resources (overtime and outsourcing) to react to each scenario whereas lower of Option 1 and Option2 also uses less flexible and less expensive resources by allowing the hiring of regular workers. The results of our analysis show that aggregate planning with the proposed model used in advance is beneficial for the company because it minimizes the total production cost by planning less flexible and less expensive resources ahead, such as regular workforce. Cost of change (￥96,116) represents the cost of not preparing to the disruptions in advance, or can be seen as a benefit for the company if APP model is used.

Scenario #	Option 1: Overtime, Outsource, and Hiring Allowed (￥)	Option 2: Only Hiring Allowed (￥)	Option 3: Only Overtime and Outsource Allowed (￥)	Cost Change as a Result of Using the APP model (￥)	Cost Change (%)
0 (Base)	684,833	1,007,545	714,766	-29,933	-4.19%
1	661,411	984,114	685,862	-24,451	-3.56%
2	669,827	922,753	699,948	-30,121	-4.30%
3	640,489	939,652	673,258	-32,769	-4.87%
4	669,214	991,923	689,250	-20,036	-2.91%
5	816,386	1,294,663	1,211,194	-394,808	-32.60%
6	895,161	1,494,430	1,609,120	-713,959	-44.37%
7	822,813	1,530,375	856,347	-33,533	-3.92%
8	690,641	1,012,941	722,106	-31,465	-4.36%
9	691,535	1,149,169	720,922	-29,387	-4.08%
10	694,714	1,023,772	722,764	-28,050	-3.88%
11	685,491	1,008,174	717,555	-32,065	-4.47%
12	691,101	1,037,686	721,172	-30,071	-4.17%
13	730,326	1,105,609	772,906	-42,580	-5.51%
14	695,750	1,049,438	726,381	-30,631	-4.22%
15	688,264	1,011,175	722,267	-34,003	-4.71%
Expected cost	**714,247**	**1,097,714**	**810,364**	**-96,116**	**-11.86%**

Table 13. Initial Scenarios Run Results in Three Different Model Options

The total expected cost for Option 1 (￥714,247), which is the most efficient for the company, will be relevant later in Section 4.6 for buffer capacity estimation towards final optimal production plan calculation.

4.5 Detail Cost Analysis on Scenarios

We take the Base Scenario and two extreme scenarios for analysis: Scenario 6 and Scenario 7 to analyze the top cost drivers for the total production costs.

Cost components breakdown for the Base Scenario is shown in Figure 8. Option 1 has the optimum cost, which indicates that for the base scenario combination of hiring, overtime and outsource for some products and stages is cheaper than hiring new people and using only in-house production. The top cost drivers are salary and outsourcing.

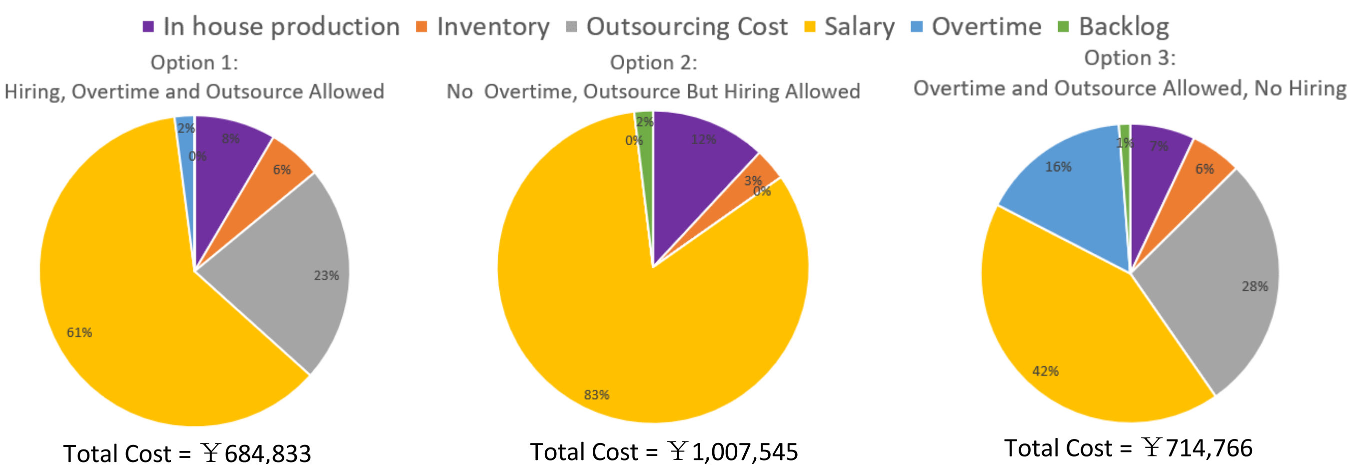

Figure 8. Cost Component Breakdown of the Base Scenario under the Three Simulation Options

A further analysis of the cost show that regular employee working hours total weekly payments for Options 1 to 3 are ￥20,873/week, ￥40,200/week, and ￥15,100/week, respectively. Option 2's labor cost for regular hours are almost as twice of that of Option1. This shows that restricting overtime and outsourcing (Option 2) can lead to large amount of labor costs. Option 3 only limits the amount of hiring, and its costs are only 4.3% higher than that of Option 1, which shows that outsourcing and overtime are not the top drivers for the base scenario.

Outsourcing and overtime costs of the three options under the Base Scenario from Week 1 to Week 20 is shown in Figure 9 and Figure 10, respectively. The outsourcing and overtime costs of Option 1 is higher than those of Option 3, indicating that a certain amount of outsourcing and overtime should be allowed under the current manufacturing capacity. To determine this, the company should analyze and compare the production cost for each product at its production stages.

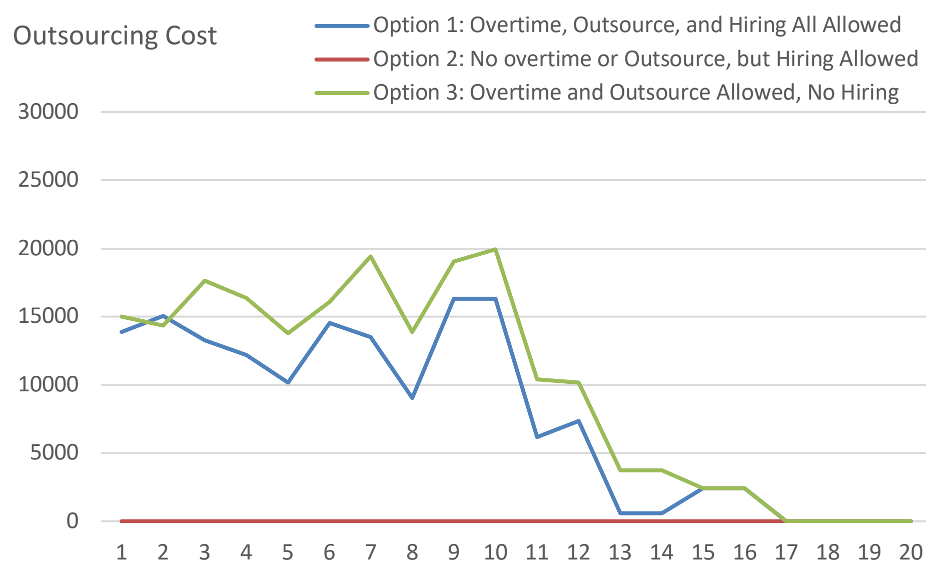

Figure 9. Comparison of the Outsourcing Costs for the Three Options for the Base Scenario from Week 1 to Week 20

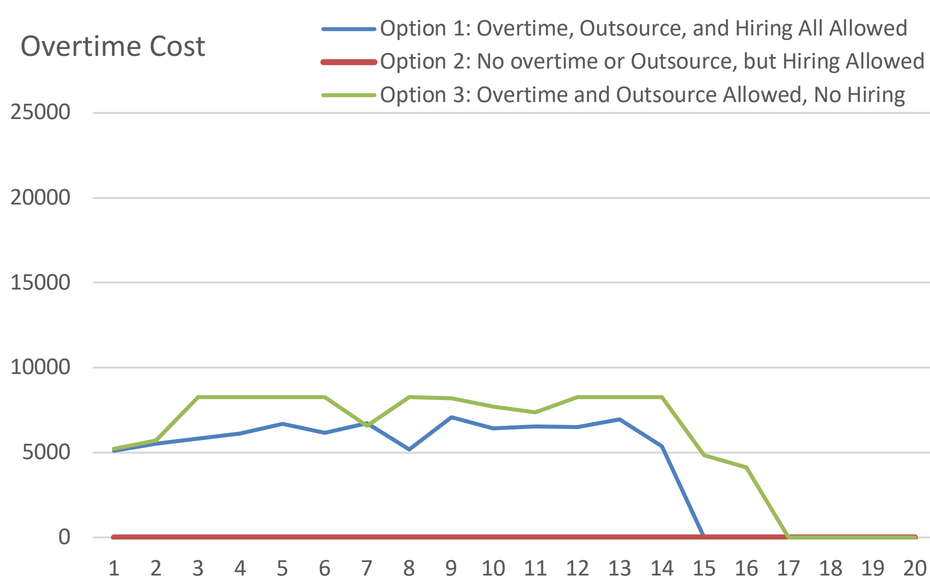

Figure 10. Comparison of the Overtime Costs for the Three Options for the Base Scenario from Week 1 to Week 20

To see the difference in costs between the base scenario and the most extreme disruption scenarios, we analyze Scenario 6 and 7.

Under Scenario 6, the production time for every stage of every product increases by 50%, which represents an extreme scenario, or there is an unexpected production shut down.

Analysis of Scenario 6 indicates that Options 2 and 3 have a lot of penalties for backlogged orders. It means that internal capacities are not enough to complete all production on time (Option 2), but only using overtime and outsourcing also is not enough (Option 3) either. Combining hiring, overtime and outsourcing (Option 1) allows to reduce volume of backlogged orders and meet almost all demand on time, still having some backlog.

Cost components breakdown for Scenario 6 is shown in Figure 11. Regular salaries under Scenario 6 for Option 1 to 3 are 26185/week, 54822/week, and 15100/week. With only in-house production allowed, salary takes 73% of the total cost for Option 2, which explains why Option 2 is significantly more expensive than Option 1.

When no hiring is allowed (Option 3), the backlog costs take 57% of the total costs, which indicates that the company's and its sub-suppliers' resources are insufficient for producing the demands under Scenario 6 without adding more employees.

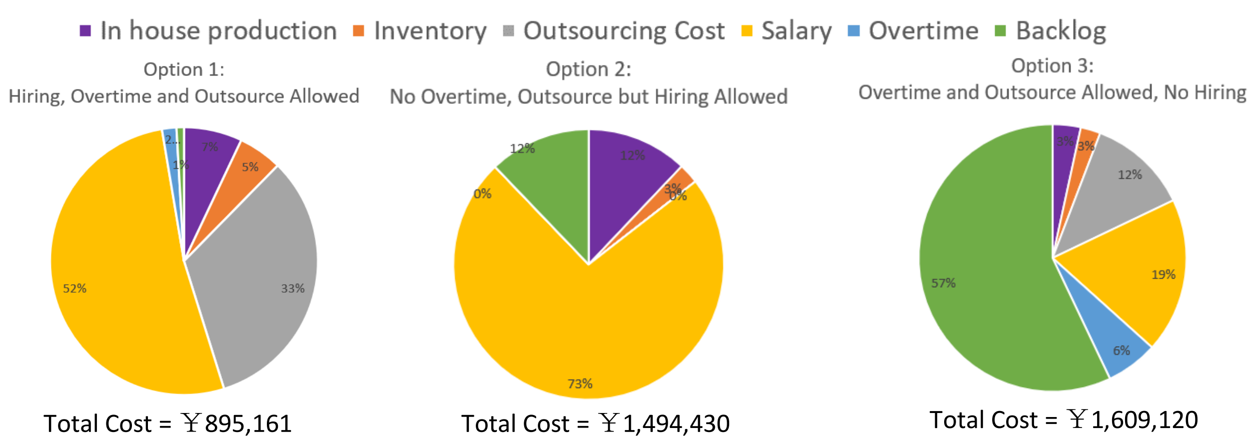

Figure 11. Cost Component Breakdown for Scenario 6 under the Three Simulation Options

Scenario 7 is the scenario when the production time of all the products at the Stage 4 increase by 100%. In this scenario, outsourcing is critical to keep costs minimum, as only additional employees hiring is not enough to ensure production due to available equipment capacity constraint.

Cost components breakdown for Scenario 7 is shown in Figure 12. Production under Option 2 has 33% of the backlog cost, which proves that the company's in-house resources for Stage 4 are insufficient to meet the production demand due to limited available equipment.

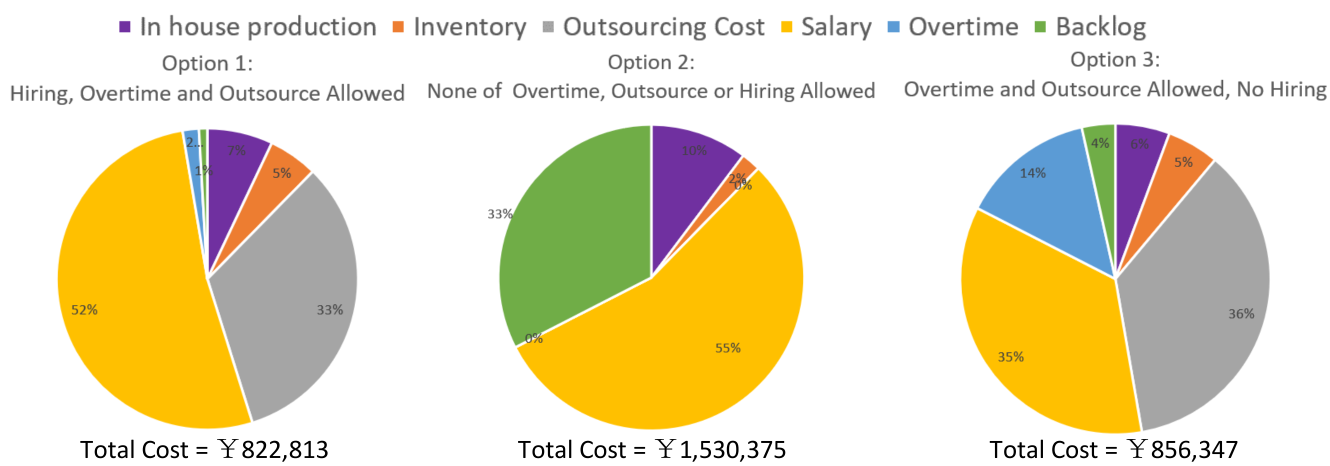

Figure 12 Cost Component Breakdown for Scenario 7 under the Three Simulation Options

Key learnings from the cost analysis:

1. Payments for the employee's weekly regular salaries take up the largest portion of the company's production cost in the optimal solutions. Penalties for backorder costs are generally low if workforce is planned in advance, but they could lead to a large portion of cost if the company does not have sufficient equipment capacity.
2. Only in-house production is not a solution for the company even if it is hiring new employees because their capacity is constrained not only by available workforce, but also by available equipment, which is more expensive and time consuming to buy and install compared to hiring new employees. New equipment installment is out of the model scope.

3. The sponsor company's flexibility is limited by its hiring policy, which leads to the increased minimum cost. Once hired, employees cannot be fired according to the company's policy, and this causes abundant labor costs during the period when producing the demands requires fewer hours than that of the busiest period. This indicates that the sponsor company should consider the option to lay off employees in the future.

The combination of hiring new employees and using overtime hours and outsourcing during the aggregate production planning (Option 1 - the first column in Table 13) is the most efficient option for the sponsor company with its current constraints. In Section 4.7, we will explore the opportunities for making investments in hiring additional employees and buying more equipment as further recommendations.

4.6 Buffer Capacity Recommendation

Buffer capacity provides more flexibility because under disruptions the production plan can be reorganized faster using internal resources, resulting in a lower cost. We suggest a high-level approximation solution, explained in Section 3.6. We use total expected cost calculated based on the set of scenarios in Section 3.4. The data is shown in Table 13, Option 1 (minimum of the three options).

The total expected cost for all the scenarios is ￥ 714,247 (detailed results shown in Table 13, model Option 1). We need to generate detailed aggregate production plan with the expected total cost to ensure that the cost is minimum, but at the same time accounts for possible disruption.

To generate that plan, we use the Base Scenario (Option 1) and add a buffer capacity which will make the production cost equal to the expected cost. We run base scenario with buffer capacity from 1% to 20% with 1% increment. The total cost matches most closely the expected total cost of all the scenarios when the buffer capacity size is 7% (shown in Table 14).

Scenario	Capacity planned	Buffer capacity	Total Cost
0 (Base)	93%	7%	￥712,525

Table 14. Recommended Buffer Capacity for the Base Scenario

Having buffer capacity defined for the Base Scenario which leads to the same cost as total expected cost for all scenarios, we can generate the detailed production plan for the company using the model.

4.7 Shadow Price Analysis

The shadow price analysis is conducted to identify opportunities for the sponsor company to improve the resources allocation. In our model, we calculate the shadow prices for every scenario and also the combination of resources used (Options 1, 2, 3).

To gain insights on critical model constraints, we analyze and describe the shadow price for the Base Scenario with Option 3 (no additional employees hiring is allowed). Table 15 shows the shadow prices for the workforce constraint for the Base scenario for a 6-week horizon. The results show that the production has excessive workforce at Stage 3, while total cost could be decreased greatly if additional employees added to the Stage 5.

Stage	**W1**	**W2**	**W3**	**W4**	**W5**	**W6**
Stage 2	-5,979	-5,575	-5,151	-4,697	-4,213	-5,575
Stage 3	6,101	5,939	5,779	5,622	5,466	5,939
Stage 4	-7,693	-6,917	-6,148	-5,383	-4,623	-6,917
Stage 5	-77,172	-7,1736	-6,6234	-60,667	-55,035	-71,736

Table 15. Workforce Shadow Price Analysis Result for the Base Scenario (Option 3, no Hiring Allowed, Unit ￥)

To show the cost improvement of workforce re-allocation, we re-run the model with one less worker at Stage 3 and one more worker at Stage 5. The greatest cost benefit is seen on Option 3, when we react on disruption without ability to hire new employees. For all the scenarios, the expected total production cost reduction is 5.67% (shown in Table 16).

While the total number of employees stays the same, production cost has been decreased for all the scenarios. If the company does not use APP model in advance to plan resources, using shadow price analysis still can help substantially to improve costs by better resource allocation.

Scenario #	Total Cost with Initial Workforce Available	Total Cost after removing one employee at Stage 3 and adding one to Stage 5	Total Cost Reduction
0 (base)	714,766	701,033	-1.92%
1	685,862	673,229	-1.84%
2	699,948	682,976	-2.42%
3	673,258	659,439	-2.05%
4	689,250	688,781	-0.07%
5	1,211,194	1,001,083	-17.35%
6	1,609,120	1,274,077	-20.82%
7	856,347	840,141	-1.89%
8	722,106	709,956	-1.68%
9	720,922	706,909	-1.94%
10	722,764	709,035	-1.90%
11	717,555	701,695	-2.21%
12	721,172	708,371	-1.78%
13	772,906	756,856	-2.08%
14	726,381	711,797	-2.01%
15	722,267	705,259	-2.35%
Expected cost	**810,364**	**764,415**	**-5.67%**

Table 16. Cost Reduction after Workforce Adjustment Based on Shadow Price Analysis (Option 3: No hiring, but Overtime and Outsourcing allowed)

If we consider only in-house production with possible hiring (Option 2), then the biggest constraint is available equipment. Table 17 shows shadow price analysis for the equipment under Scenario 6, and it shows that Stage 3 is capacity constrained.

Stage	Week 5	Week 6	Week 7	Week 8	Week 9
Stage 2	0	0	0	0	0
Stage 3	-110.592	-110.662	-110.718	-110.698	-84.7406
Stage 4	0	0	0	0	0
Stage 5	0	0	0	0	0

Table 17. Equipment Shadow Price Analysis Result for the Scenario 6 (Option 2: In-House Production and Hiring)

We re-run all the scenarios after adding one unit of equipment for Stage 3, the results are shown in Table 18. The expected cost reduces by only 0.18%. This indicates that investing in new equipment at Stage 3 is not the best decision for the company if other constraints are not improved.

Scenario #	Only in-house production with regular working hours	Only in-house production with regular working hours, added one unit of equipment for Stage 3	Total Cost Reduction
Base	**1,007,545**	**1,007,545**	0%
1	984,114	984,114	0%
2	922,753	922,753	0%
3	939,652	939,652	0%
4	991,923	991,923	0%
5	1,294,663	1,290,798	-0.30%
6	1,494,430	1,476,391	-1.21%
7	1,530,375	1,530,375	0%
8	1,012,941	1,012,941	0%
9	1,149,169	1,139,962	-0.80%
10	1,023,772	1,023,772	0%
11	1,008,174	1,008,174	0%
12	1,037,686	1,037,686	0%
13	1,105,609	1,105,609	0%
14	1,049,438	1,049,438	0%
15	1,011,175	1,011,175	0%
Expected cost	**1,097,714**	**1,095,769**	**-0.18%**

Table 18. Cost Reduction after Equipment Added to Stage 3 (Option 2: Only In-House Production with Hiring)

In summary, we recommend adding one employee to Stage 5 and remove one employee from Stage 3, and no additional equipment is justified with the current demand. Also, Option 1 (use in-house production with overtime hours and outsourcing) is the most efficient one.

Table 19 shows the final costs for all scenarios and expected cost based on scenarios probability for Option 1, after workforce change recommended by shadow price analysis. Final expected total cost is 0.52% lower than that before implementing the workforce changes recommended (￥710,547), without increase in the total numbers of employees.

Scenario #	Hiring, Outsource and Overtime Allowed (￥)	Hiring, Outsource and Overtime allowed after Workforce Optimization(￥)	Total Cost Reduction (￥)
Base	**684,833**	**681,141**	-0.54%
1	661,411	657,694	-0.56%
2	669,827	662,857	-1.04%
3	640,489	636,887	-0.56%
4	669,214	669,330	2%
5	816,386	812,822	-0.44%
6	895,161	893,527	-0.18%
7	822,813	817,956	-0.59%
8	690,641	686,961	-0.53%
9	691,535	687,492	-0.58%
10	694,714	690,797	-0.56%
11	685,491	681,802	-0.54%
12	691,101	688,028	-0.44%
13	730,326	725,869	-0.61%
14	695,750	691,128	-0.66%
15	688,264	684,462	-0.55%
Expected cost	**714,247**	**710,547**	**-0.52%**

Table 19. Total Cost after Workforce Optimization (Option 1: Hiring, Overtime Hours and Outsourcing are Allowed)

To estimate the total impact of our recommendations (use of APP model with hiring, overtime hours and outsourcing, and removing one employee from Stage 3 and adding one for Stage 5) for the company, if implemented, we compare the total expected cost for Option 3 model (when the company just reacts on disruptions by using overtime hours and outsourcing, without preliminary resource planning) and Option 1 model with workforce reallocation recommended based on shadow price analysis. Cost reduction is 12.32% without any additional investments for the company (Table 20).

Cost	Condition 1: Hiring, Outsource and Overtime allowed, after workforce optimization	Condition 3: No hiring, but Overtime and Outsourcing allowed, before workforce optimization	Total Cost Reduction
Total Expected Cost	710,547	810,364	-12.32%

Table 20. Total Cost Improvement

5. DISCUSSION

In Chapter 3, we formulated the APP LP model for the company, introduced three model options and 16 scenarios, explained the methodology for data analysis, recommended the approach to buffer capacity calculation and shadow price analysis.

In Chapter 4, we analyzed model results, compared the total cost differences for all model options and scenarios, analyzed main cost drivers, calculated recommended buffer capacity and made a recommendation for workforce reallocation.

This chapter discusses our results toward a summarized recommendation for the sponsor company and future directions of the research.

5.1 Recommendation for the Sponsor Company

Based on the analysis of the provided data, our recommendations for the particular production planning dataset are:

1) Use the formulated APP LP model for cost minimization and optimal plan calculation;
2) Add 7% buffer capacity across all stages to the base scenario based on the current data set;
3) One employee should be removed from stage 3 and one employee should be added to stage 5;

These 3 actions lead to more than 12% of cost savings in the total expected production cost compared to expected cost if no actions made.

Our general recommendations for the regular practice of aggregate production planning are:

1) Use a combination of in-house production, overtime hours and outsourcing;
2) Conduct a regular cost analysis to identify the situations where outsourcing is cheaper than in-house production towards a more strategic outsourcing practice;
3) Calculate an approximate buffer capacity size based on the formulated model and multiple scenarios analysis to prepare for production time uncertainty;

4) Use total expected cost based on frequent scenarios for selling price calculation to be sure that all the costs are accounted for and the company's profitability stays within expected rates. However, extreme scenarios can be excluded for quoting accuracy. This requires the sponsor company to record the types and frequencies of production disruptions in their daily operation;
5) Use shadow price analysis when making decisions for improving the production capacity such as adding employees or equipment;
6) Consider the option of laying off employees in the future and incorporating firing costs if so.

5.2 Research limitations and Future Recommendations

Our base scenario used as a benchmark was created based on the operation manager's experience. Though the scenarios reflect some possible situations, they were created based on past experience as well. We assume a uniform distribution of each scenario because of the lack of systematic data collection. As all production is customized and no historical data is available on specific products, the quality of scenarios will depend on the manager's experience and ability to use historical data. If enough historical data is collected, products can be aggregated into groups and future scenarios can be created on a product group level instead of individual product level.

For the modeling purposes, outsourcing price is set based on a unit basis and does not depend on production time. The model can be enhanced by adding outsourcing cost calculated based on production time, as well as incorporating production uncertainties of the outsourced production.

Our method for recommended buffer capacity calculation as explained in Section 3.6, is based on an aggregate level calculation of all products and stages over a time period of 20 weeks. In the future, the expected cost calculation can be performed by every stage with scenarios probability different by stage. This will allow setting different optimal buffer capacity for different stages to account for a different level of production process time uncertainty and probabilities by stages. Another way to estimate buffer capacity and to find the optimal buffer level is to include buffer capacity in the objective function of the model.

6. CONCLUSION

This project formulated the aggregate production planning (APP) model for the sponsor company and solved it for multiple scenarios, including base and 15 alternative production scenarios; estimated the influence of setting aside internal buffer capacities; recommended changes in available workforce; and suggested improvements for the sponsor company's aggregate production planning processes.

Based on the results, we recommend the sponsor company:

1. Use a combination of in-house production, overtime hours and outsourcing for minimum production costs;
2. Implement a proposed method to calculate the buffer capacity size based on the formulated model and multiple scenarios analysis to prepare for production time uncertainty;
3. Use shadow price analysis for decisions to improve production capacities.

Based on the calculation on the provided data set, these actions 12.32% of cost savings in the total expected production cost.

We believe that the developed approach and the formulated model can be expanded as a general methodology for any industry and company involved in manufacturing of ETO products.

Future research option could be calculation buffer capacity by every production stage based on scenarios probability derived from historical data. Another direction is adding buffer capacity size to the objective function of the LP model. Finally, more constraints and inputs can be added to the model.

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APPENDICES

APPENDIX A: Pseudo Code

Part 1: ***Opt (production_data production_scenario z*** *for aggregate production function*

1. *READ* ***production_data production_scenario z***
2. *Convert the internal production capacity by the available capacity z by :*
   Maximum regular and overtime hour =
   the maximum regular and over time from ***production_data*** *x* ***z***

3. *DEFINE decision variables including*
   $$\begin{cases} \{Number\ of\ Internatal\ Production\} \\ \{Number\ of\ Outsourcing\} \\ \{Unit\ Employee\ Salary\}\{Hiring\ Cost\}\ and\ \{Unit\ Overtime\ Cost\} \\ \{Unit\ Inventory\ Cost\} \\ \{Unit\ Backorder\ Cost\} \end{cases}$$
4. *APPLY the optimization constraints including*
   $$\begin{cases} Internal\ production\ capacity\ constraint \\ Outsource\ production\ capacity\ constraint \\ Conservation\ flow\ of\ number\ of\ employees \\ Conservation\ flow\ of\ inventory \\ Employee\ working\ hours\ constriant \\ \text{All decision variables are non} - \text{negative} \end{cases}$$
5. *DEFINE objective function as the summary of the following costs:*
   $$\begin{cases} Total\ Internal\ production\ cost \\ Total\ Outsourcing\ cost \\ Total\ labor\ cost\ including\ total\ salary\ hiring\ cost\ and\ overtime\ cost \\ Total\ inventory\ cost \\ Total\ backorder\ cost \end{cases}$$
6. Optimize the model using Gurobi

Part 2*: Scenario playing model*

1. *INPUT* ***production data*** obtained from the sponsor company including:

$$\{Unit\ internal\ production\ cost\}$$
$$\{Unit\ outsourcing\ cost\}$$
$$\{Unit\ employee\ salary\}\ \{Hiring\ cost\}\ and\ \{Unit\ overtime\ cost\}$$
$$\{Unit\ inventory\ cost\}$$
$$\{Unit\ backorder\ cost\}$$
$$\{Working\ hours\ constraints\ including\ regular\ and\ overtime\ hours\}$$
$$\{Initial\ inventory\}\{initial\ number\ of\ employee\}$$
$$\{Outsoucing\ capacity\ constriant\}$$

2. *INPUT the capacity ranges* including minimum capacity C_{min} maximum capacity C_{max} and increments $C_{incr.}$

3. **while** (need APP optimization (y/n?) =y)

    INPUT ***production_scenario***

    while *($C_{min} \leq z \leq C_{max}$*

    Read z as the available capacity defined in Step 4.2.

    Run function ***Opt (production_data production_scenario z***

    $z=z+ C_{incr}$

    end

    write decision variables total cost and cost breakdown to excel

    end

4. Write the decision variables total cost total internal production cost total outsourcing cost total labor cost total overtime cost and total backorder cost.
5. End

APPENDIX B: Inventory Holding Cost by Product; Cost in Yuan per Unit/Stage/Week

Product	**Stage 2**	**Stage 3**	**Stage 4**	**Stage 5**
Product 1	8	7	20	13.5
Product 2	8	7	20	13.5
Product 3	8	7	20	13.5
Product 4	8	10	13.5	13.5
Product 5	8	10	13.5	13.5
Product 6	8	10	13.5	13.5
Product 7	8	12	16	13.5
Product 8	8	12	16	13.5
Product 9	8	10	13.5	13.5
Product 10	8	10	13.5	13.5
Product 11	8	10	13.5	13.5
Product 12	8	10	13.5	13.5
Product 13	8	10	13.5	13.5
Product 14	8	10	13.5	13.5
Product 15	8	15	20	13.5
Product 16	8	10	13.5	13.5
Product 17	8	10	13.5	13.5
Product 18	8	10	13.5	13.5
Product 19	8	10	13.5	13.5
Product 20	8	10	13.5	13.5

APPENDIX C: Initial Inventory by Product by Stage, in units

Product	Stage 2	Stage 3	Stage 4	Stage 5
Product 1	0	0	0	0
Product 2	0	0	0	0
Product 3	0	0	0	0
Product 4	0	0	0	0
Product 5	0	0	0	0
Product 6	0	0	0	0
Product 7	0	0	0	0
Product 8	0	0	0	0
Product 9	0	0	0	0
Product 10	0	0	0	0
Product 11	0	0	0	0
Product 12	0	0	0	0
Product 13	0	0	0	0
Product 14	0	0	0	0
Product 15	0	40	0	0
Product 16	0	25	0	0
Product 17	30	0	0	0
Product 18	20	0	0	0
Product 19	10	0	0	0
Product 20	18	0	0	0

APPENDIX D: Maximum Possible Outsourcing by Product, in Units per Week/Stage

Product	**Stage 2**	**Stage 3**	**Stage 4**	**Stage 5**
Product 1	0	5	3	0
Product 2	0	5	3	0
Product 3	0	5	3	0
Product 4	0	5	3	0
Product 5	0	5	3	0
Product 6	0	5	3	0
Product 7	0	5	3	0
Product 8	0	5	3	0
Product 9	0	5	3	0
Product 10	0	5	3	0
Product 11	0	5	3	0
Product 12	0	5	3	0
Product 13	0	5	3	0
Product 14	0	5	3	0
Product 15	0	5	3	0
Product 16	0	5	3	0
Product 17	0	5	3	0
Product 18	0	5	3	0
Product 19	0	5	3	0
Product 20	0	5	3	0

APPENDIX E: Cost of Outsourcing by Product, Yuan per Unit/Stage

Product	Stage 2	Stage 3	Stage 4	Stage 5
Product 1	0	750	340	0
Product 2	0	137.5	1800	0
Product 3	0	450	0	0
Product 4	0	437.5	375	0
Product 5	0	450	0	0
Product 6	0	400	200	0
Product 7	0	500	375	0
Product 8	0	350	250	0
Product 9	0	350	250	0
Product 10	0	350	450	0
Product 11	0	350	300	0
Product 12	0	350	500	0
Product 13	0	350	200	0
Product 14	0	350	200	0
Product 15	0	500	375	0
Product 16	0	150	200	0
Product 17	0	600	1500	0
Product 18	0	460	1000	0
Product 19	0	150	1000	0
Product 20	0	300	340	0

APPENDIX F: Cost of In-House Production by Product, Yuan per Unit/Hour/Stage

Product	**Stage 2**	**Stage 3**	**Stage 4**	**Stage 5**
Product 1	1.03	5.35	15.24	1.15
Product 2	1.03	5.35	15.24	1.15
Product 3	1.03	5.35	15.24	1.15
Product 4	1.03	9.83	5.83	1.15
Product 5	1.03	9.83	5.83	1.15
Product 6	1.03	9.83	5.83	1.15
Product 7	1.03	9.83	10.5	1.15
Product 8	1.03	9.83	10.5	1.15
Product 9	1.03	5.35	5.83	1.15
Product 10	1.03	5.35	5.83	1.15
Product 11	1.03	5.35	15.24	1.15
Product 12	1.03	5.35	15.24	1.15
Product 13	1.03	9.83	15.24	1.15
Product 14	1.03	9.83	5.83	1.15
Product 15	1.03	5.35	10.5	1.15
Product 16	1.03	5.35	10.5	1.15
Product 17	1.03	9.83	5.83	1.15
Product 18	1.03	5.35	5.83	1.15
Product 19	1.03	9.83	10.5	1.15
Product 20	1.03	5.35	10.5	1.15

APPENDIX G: Demand by Product, Units

Product	W 1	W 2	W 3	W 4	W 5	W 6	W 7	W 8	W 9	W 10	W 11	W 12	W 13	W 14	W 15	W 16	W 17	W 18	W 19	W 20
Product 1												16								
Product 2												20								
Product 3												50								
Product 4												20								
Product 5														40						
Product 6														20						
Product 7														20						
Product 8														80						
Product 9												30								
Product 10												50								
Product 11																50				
Product 12																60				
Product 13																80				
Product 14																25				
Product 15								60												
Product 16								50												
Product 17										60										
Product 18										40										
Product 19										20										
Product 20										36										

APPENDIX H: Aggregate Production Plan for the Base Scenario (In-House Production)

Product-Stage	W 1	W 2	W 3	W 4	W 5	W 6	W 7	W 8	W 9	W 10	W 11	W 12	W 13	W 14	W 15	W 16	W 17	W 18	W 19	W 20
Product 1 Stage 2	3.0	2.1	2.3	3.2	0.6						1.8	3.0								
Product 1 Stage 3	3.0	2.1	2.3	3.2	0.6															
Product 1 Stage 4																				
Product 1 Stage 5	0.6	1.4	1.6	1.6	3.0	3.0					1.8	3.0								
Product 2 Stage 2									5.0	5.0	5.0	5.0								
Product 2 Stage 3																				
Product 2 Stage 4											5.7	14.3								
Product 2 Stage 5											5.7	14.3								
Product 3 Stage 2	3.0	3.0	3.0	3.0	3.0	5.1	7.2		12.2	10.5										
Product 3 Stage 3	3.0	3.0	3.0	3.0	3.0	5.1	7.2		12.2	10.5										
Product 3 Stage 4												14.0								
Product 3 Stage 5	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	17.0								
Product 4 Stage 2			3.0	3.0	3.0	3.0	2.3			1.4	1.3	3.0								
Product 4 Stage 3																				
Product 4 Stage 4																				
Product 4 Stage 5			3.0	3.0	3.0	3.0	2.3			1.4	1.3	3.0								
Product 5 Stage 2														4						
Product 5 Stage 3														4						
Product 5 Stage 4														4						
Product 5 Stage 5														4						
Product 6 Stage 2							2.0		3.0	3.0	5.0	7.0								
Product 6 Stage 3							2.0		3.0	3.0	5.0	7.0								
Product 6 Stage 4																				
Product 6 Stage 5							2.0		3.0	3.0			8.9	3.1						
Product 7 Stage 2		4.5			3.2	2.3						3.0	7.0							
Product 7 Stage 3		4.5			3.2	2.3						3.0	7.0							
Product 7 Stage 4		4.5			3.2	2.3							1							
Product 7 Stage 5		4.5			3.2	2.3							1							
Product 8 Stage 2	13.1	10.9	12.5	11.8	1	4.1					13.1	4.5								
Product 8 Stage 3									1.1	18.1	27.2	16.1	17.5							
Product 8 Stage 4											46.4		33.6							
Product 8 Stage 5											46.4		33.6							
Product 9 Stage 2	3.0	3.0	3.0	3.0	3.0	3.0	6.0		6.0											
Product 9 Stage 3	3.0	3.0	3.0	3.0	3.0	3.0	6.0		6.0											
Product 9 Stage 4																				
Product 9 Stage 5	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0										
Product 10 Stage 2												5								
Product 10 Stage 3												5								
Product 10 Stage 4												5								

Product 10 Stage 5												5								
Product 11 Stage 2															10.4	39.6				
Product 11 Stage 3															10.4	39.6				
Product 11 Stage 4															7.4	36.6				
Product 11 Stage 5																5				
Product 12 Stage 2													30.2	1.0	25.8	3.0				
Product 12 Stage 3													30.2	1.0	25.8	3.0				
Product 12 Stage 4													8.7	22.5	22.8					
Product 12 Stage 5																6				
Product 13 Stage 2														37.1	17.3	25.5				
Product 13 Stage 3														37.1	17.3	25.5				
Product 13 Stage 4														37.1	17.3	25.5				
Product 13 Stage 5																8				
Product 14 Stage 2															8.1	16.9				
Product 14 Stage 3															8.1	16.9				
Product 14 Stage 4																25.0				
Product 14 Stage 5																25.0				
Product 15 Stage 2							6.0	14.0												
Product 15 Stage 3							6.0	14.0												
Product 15 Stage 4	4						6.0	14.0												
Product 15 Stage 5	4						6.0	14.0												
Product 16 Stage 2								25.0												
Product 16 Stage 3								25.0												
Product 16 Stage 4								5												
Product 16 Stage 5								5												
Product 17 Stage 2					2.2	9.4	7.8	2.9	2.6	5.0										
Product 17 Stage 3	1.1	1.1	1.6	1.0	2.5	2.8														
Product 17 Stage 4	4.1	6.1	6.6	6.0	7.1	7.3	5.6	4.1	5.3	6.0										
Product 17 Stage 5	6.1	6.1	6.6	6.0	7.1	7.3	5.6	4.1	5.3	6.0										
Product 18 Stage 2							5.0	5.0	5.0	5.0										
Product 18 Stage 3																				
Product 18 Stage 4	5.0	5.0	5.0	5.0			5.0	5.0	5.0	5.0										
Product 18 Stage 5	5.0	5.0	5.0	5.0			5.0	5.0	5.0	5.0										
Product 19 Stage 2						1.1	1.8	1.1	3.0	3.0										
Product 19 Stage 3																				
Product 19 Stage 4																				
Product 19 Stage 5	3.0	3.0	3.0	1.0		1.1	1.8	1.1	3.0	3.0										
Product 20 Stage 2							3.0	5.0	5.0	5.0										
Product 20 Stage 3																				
Product 20 Stage 4			1.0	5.0	5.0	5.0	5.0	0.8	9.2	5.0										
Product 20 Stage 5			1.0	5.0	5.0	5.0	5.0	0.8	9.2	5.0										

APPENDIX I: Aggregate Production Plan for the Base Scenario (Outsourcing)

Product-Stage	W 1	W 2	W 3	W 4	W 5	W 6	W 7	W 8	W 9	W 10	W 11	W 12	W 13	W 14	W 15	W 16	W 17	W 18	W 19	W 20
Product 1 Stage 2																				
Product 1 Stage 3											1.8	3.0								
Product 1 Stage 4	0.6	1.4	1.6	1.6	3.0	3.0					1.8	3.0								
Product 1 Stage 5																				
Product 2 Stage 2																				
Product 2 Stage 3									5.0	5.0	5.0	5.0								
Product 2 Stage 4																				
Product 2 Stage 5																				
Product 3 Stage 2																				
Product 3 Stage 3																				
Product 3 Stage 4	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0								
Product 3 Stage 5																				
Product 4 Stage 2																				
Product 4 Stage 3			3.0	3.0	3.0	3.0	2.3			1.4	1.3	3.0								
Product 4 Stage 4			3.0	3.0	3.0	3.0	2.3			1.4	1.3	3.0								
Product 4 Stage 5																				
Product 5 Stage 2																				
Product 5 Stage 3																				
Product 5 Stage 4																				
Product 5 Stage 5																				
Product 6 Stage 2																				
Product 6 Stage 3																				
Product 6 Stage 4							2.0		3.0	3.0	3.0	3.0	3.0	3.0						
Product 6 Stage 5																				
Product 7 Stage 2																				
Product 7 Stage 3																				
Product 7 Stage 4																				
Product 7 Stage 5																				
Product 8 Stage 2																				
Product 8 Stage 3																				
Product 8 Stage 4																				
Product 8 Stage 5																				
Product 9 Stage 2																				
Product 9 Stage 3																				
Product 9 Stage 4	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0										
Product 9 Stage 5																				
Product 10 Stage 2																				
Product 10 Stage 3																				
Product 10 Stage 4																				

Product 10 Stage 5																				
Product 11 Stage 2																				
Product 11 Stage 3																				
Product 11 Stage 4															3.0	3.0				
Product 11 Stage 5																				
Product 12 Stage 2																				
Product 12 Stage 3																				
Product 12 Stage 4															3.0	3.0				
Product 12 Stage 5																				
Product 13 Stage 2																				
Product 13 Stage 3																				
Product 13 Stage 4																				
Product 13 Stage 5																				
Product 14 Stage 2																				
Product 14 Stage 3																				
Product 14 Stage 4																				
Product 14 Stage 5																				
Product 15 Stage 2																				
Product 15 Stage 3																				
Product 15 Stage 4																				
Product 15 Stage 5																				
Product 16 Stage 2																				
Product 16 Stage 3																				
Product 16 Stage 4																				
Product 16 Stage 5																				
Product 17 Stage 2																				
Product 17 Stage 3	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0										
Product 17 Stage 4	2.0																			
Product 17 Stage 5																				
Product 18 Stage 2																				
Product 18 Stage 3	5.0	5.0	5.0	5.0			5.0	5.0	5.0	5.0										
Product 18 Stage 4																				
Product 18 Stage 5																				
Product 19 Stage 2																				
Product 19 Stage 3	3.0	3.0	3.0	1.0		1.1	1.8	1.1	3.0	3.0										
Product 19 Stage 4	3.0	3.0	3.0	1.0		1.1	1.8	1.1	3.0	3.0										
Product 19 Stage 5																				
Product 20 Stage 2																				
Product 20 Stage 3			1.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0										
Product 20 Stage 4																				
Product 20 Stage 5																				

APPENDIX J: Aggregate Production Plan for the Base Scenario (Inventory Holding)

Product-Stage	W 1	W 2	W 3	W 4	W 5	W 6	W 7	W 8	W 9	W 10	W 11	W 12	W 13	W 14	W 15	W 16	W 17	W 18	W 19	W 20
Product 1 Stage 2																				
Product 1 Stage 3	2.4	3.1	3.8	5.4	3.0															
Product 1 Stage 4																				
Product 1 Stage 5	0.6	2.0	3.6	5.2	8.2	11.2	11.2	11.2	11.2	11.2	13.0									
Product 2 Stage 2																				
Product 2 Stage 3									5.0	1	9.3									
Product 2 Stage 4																				
Product 2 Stage 5											5.7									
Product 3 Stage 2																				
Product 3 Stage 3						2.1	6.3	3.3	12.5	2	17.0									
Product 3 Stage 4																				
Product 3 Stage 5	3.0	6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0	3	33.0									
Product 4 Stage 2																				
Product 4 Stage 3																				
Product 4 Stage 4																				
Product 4 Stage 5			3.0	6.0	9.0	12.0	14.3	14.3	14.3	15.7	17.0									
Product 5 Stage 2																				
Product 5 Stage 3																				
Product 5 Stage 4																				
Product 5 Stage 5																				
Product 6 Stage 2																				
Product 6 Stage 3											2.0	6.0	3.0							
Product 6 Stage 4											3.0	6.0	0.1							
Product 6 Stage 5							2.0	2.0	5.0	8.0	8.0	8.0	16.9							
Product 7 Stage 2																				
Product 7 Stage 3												3.0								
Product 7 Stage 4																				
Product 7 Stage 5		4.5	4.5	4.5	7.7	1	1	1	1	1	1	1	2							
Product 8 Stage 2	13.1	24.0	36.5	48.3	58.3	62.4	62.4	62.4	61.4	43.2	29.1	17.5								
Product 8 Stage 3									1.1	19.2		16.1								
Product 8 Stage 4																				
Product 8 Stage 5											46.4	46.4	8							
Product 9 Stage 2																				
Product 9 Stage 3							3.0		3.0											
Product 9 Stage 4																				
Product 9 Stage 5	3.0	6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0	3	3									
Product 10 Stage 2																				
Product 10 Stage 3																				
Product 10 Stage 4																				

Product 10 Stage 5																				
Product 11 Stage 2																				
Product 11 Stage 3																				
Product 11 Stage 4															10.4					
Product 11 Stage 5																				
Product 12 Stage 2																				
Product 12 Stage 3													21.5							
Product 12 Stage 4													8.7	31.2	57.0					
Product 12 Stage 5																				
Product 13 Stage 2																				
Product 13 Stage 3																				
Product 13 Stage 4														37.1	54.5					
Product 13 Stage 5																				
Product 14 Stage 2																				
Product 14 Stage 3															8.1					
Product 14 Stage 4																				
Product 14 Stage 5																				
Product 15 Stage 2																				
Product 15 Stage 3																				
Product 15 Stage 4																				
Product 15 Stage 5	4	4	4	4	4	4	46.0													
Product 16 Stage 2																				
Product 16 Stage 3	25.0	25.0	25.0	25.0	25.0	25.0	25.0													
Product 16 Stage 4																				
Product 16 Stage 5																				
Product 17 Stage 2	23.9	17.8	11.2	5.2		1.7	4.5	2.4												
Product 17 Stage 3					0.4	0.9	0.3	1.2	1.0											
Product 17 Stage 4																				
Product 17 Stage 5	6.1	12.2	18.8	24.8	31.9	39.1	44.7	48.8	54.0											
Product 18 Stage 2	15.0	1	5.0																	
Product 18 Stage 3																				
Product 18 Stage 4																				
Product 18 Stage 5	5.0	1	15.0	2	2	2	25.0	3	35.0											
Product 19 Stage 2	7.0	4.0	1.0																	
Product 19 Stage 3																				
Product 19 Stage 4																				
Product 19 Stage 5	3.0	6.0	9.0	1	1	11.1	12.9	14.0	17.0											
Product 20 Stage 2	18.0	18.0	17.0	12.0	7.0	2.0														
Product 20 Stage 3								4.2												
Product 20 Stage 4																				
Product 20 Stage 5			1.0	6.0	11.0	16.0	21.0	21.8	31.0											

APPENDIX K: Aggregate Production Plan for the Base Scenario (Overtime Hours)

Stage	**W 1**	**W 2**	**W 3**	**W 4**	**W 5**	**W 6**	**W 7**	**W 8**	**W 9**	**W 10**	**W 11**	**W 12**	**W 13**	**W 14**	**W 15**	**W 16**	**W 17**	**W 18**	**W 19**	**W 20**
2																				
3																44.3				
4																				
5											45.6	86.2	43.9							