ESSAYS

IN THE THEORY OF

INTERNATIONAL CAPITAL MOVEMENTS

by

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ABSTRACT

The thesis consists of five major essays relating to the theory of capital movements.

The first essay considers the Transfer Problem, with and without impediments; it further generalizes the analysis of recent authors on the subject, presenting the results both geometrically and algebraically; it separates income and substitution effects, describes the type of market implicit in the "Leontief Effect," and shows the qualifications which must be made to the criteria developed by previous authors in the cases of variable production. The section on transport costs analyzes, possibly for the first time, the geometric determination of trading equilibrium, distinguishes between the f.o.b. and c.i.f. terms of trade, and uses these results to derive criteria for changes in the terms of trade after transfer when transport costs exist. The section on tariffs presents an alternative to Professor Samuelson's geometric method of showing the direction of bias given to the terms of trade by tariffs.

The second essay shows the relationship between the terms of trade, the exchange rate, and the balance of trade as alternative equilibrating variables; it derives criteria for the effects on the terms of trade or the balance of trade, of various commercial policies, including tariffs and exchange rate adjustment.

The third part is an essay on the optimum tariff; it applies the Stackelberg analysis to the theory of tariff retaliation and shows various possible means by which tariffs can be optimally reduced; a new technique of analysis is introduced in the relationship between the
three types of diagrams -- the trading quadrant, the Stackelberg tariff-subsidy chart and the world utility transformation schedule.

The fourth part shows the relationship between commodity and factor movements, enunciates the proposition that any impediment to trade stimulates a factor movement, and any impediment to a factor movement stimulates trade; a tariff argument is developed.

Finally, the fifth essay considers the effects of capital transfers on the terms of trade; impediments to both commodity and factor movements are considered.

The study can also be considered an exercise in, and a contribution to, technique in international trade theory. Throughout, a consistent single method is employed to derive results, geometrically and algebraically; and the technique will, perhaps, help to simplify and unify methods usually employed in international trade theory.

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* TABLE OF CONTENTS *

I. THE TRANSFER PROBLEM
   A. No Impediments
      1. Fixed Production
         a. Stable Market
         b. Unstable Market
      2. Variable Production
         a. Increasing Costs
         b. Constant Costs
         c. Decreasing Costs
   B. Impediments
      1. Transport Costs
         a. Geometric Representation
         b. The Terms of Trade
         c. The Transfer Problem
      2. Tariffs

II. TARIFFS, THE TERMS OF TRADE AND THE BALANCE OF PAYMENTS MECHANISM
    A. Tariffs and the Terms of Trade
    B. Tariffs and the Balance of Trade
    C. Export Taxes and the Balance of Trade
    D. Tariffs in Both Countries and the Balance of Trade
    E. Devaluation and the Balance of Trade

III. OPTIMAL TARIFF REDUCTIONS
IV. COMMODITY MOVEMENTS AND FACTOR MOBILITY

V. CAPITAL MOVEMENTS

A. No Impediments
   1. Loan
   2. Gift or Tribute

B. Capital Transfers
   1. Impediments to Capital Movements
   2. Impediments to Commodity Movements
   3. Fixed Coefficients
PREFACE

This study began as an attempt to present a theory of capital movements within the context of modern developments in the theory of international trade, and to determine the factors affecting changes in the terms of trade after a capital movement had taken, or is taking, place. The initial idea was to distinguish between "income transfers" and "capital transfers:" income transfers were intended to refer to the analysis associated with the Transfer Problem, the problem of effecting a flow of income from one country to another by means of changes in the balance of trade; capital transfers were to involve shifting a stock of productive resources from one country to another, a change in the international division of factor endowments brought about by, say, a capital levy or confiscation of capital resources by a foreign country.

It was intended to separate these types of capital movements for analytic purposes, only to co-ordinate them again in a more general theory by relaxing some of the assumptions which permitted a separation which is, in the real world, at best an indistinct, and at worst a specious, distinction. It was hoped to show how capital transfers can be effected by income transfers through the rate of saving, as alluded to in the last part of this work.

This is, then, what this study should have been; it has turned out that the last step in the analysis has not yet been accomplished.
It became more apparent during the writing that the last step was a work in itself; that the present analysis has prepared at least a considerable portion of the groundwork for that analysis, but that the fundamental alteration of the assumptions of the static model used here into the dynamic assumptions necessary for this further analysis would detract from the cohesiveness of the present study.

It is with the foregoing in mind that the five essays included here should be considered. Since the limitations of these essays are only too apparent, I shall attempt to outline some of the contributions it is hoped have been made.

The first essay considers the Transfer Problem, with and without impediments; it further generalizes the analysis of recent authors on the subject, presenting the results both geometrically and algebraically; it separates income and substitution effects, describes the type of market implicit in the "Leontief Effect," and shows the qualifications which must be made to the criteria developed by previous authors in the cases of variable production. The section on transport costs analyzes, possibly for the first time, the geometric determination of trading equilibrium, distinguishes between the f.o.b. and c.i.f. terms of trade, and uses these results to derive criteria for changes in the terms of trade after transfer when transport costs exist. The section on tariffs presents an alternative to Professor Samuelson's geometric method of showing the direction of bias given to the terms of trade by tariffs.
The second essay shows the relationship between the terms of trade, the exchange rate, and the balance of trade as alternative equilibrating variables; it derives criteria for the effects on the terms of trade or the balance of trade, of various commercial policies, including tariffs and exchange rate adjustment.

The third part is an essay on the optimum tariff; it applies the Stackelberg analysis to the theory of tariff retaliation and shows various possible means by which tariffs can be optimally reduced; a new technique of analysis is introduced in the relationship between the three types of diagrams -- the trading quadrant, the Stackelberg tariff-subsidy chart and the world utility transformation schedule.

The fourth part shows the relationship between commodity and factor movements, enunciates the proposition that any impediment to trade stimulates a factor movement, and any impediment to a factor movement stimulates trade; a tariff argument is developed.

Finally, the fifth essay considers the effects of capital transfers on the terms of trade; impediments to both commodity and factor movements are considered.

The study can also be considered an exercise in, and a contribution to, technique in international trade theory. Throughout, a consistent single method is employed to derive results, geometrically and algebraically; and the technique will, perhaps, help to simplify and unify methods usually employed in international trade theory.
The above claims are suggested to compensate, in part, for some obvious limitations. First, the analysis is entirely static, and a theory of capital movements must be dynamic; second, the essay considers only a two-country, two-factor, two-commodity model; third, the part on capital transfers, while considering impediments, assumes linear, homogeneous production functions, identical in both countries.

The author would be more reluctant to concede, however, the possible criticism that the analysis is classical, not Keynesian. The present assumptions were chosen on two grounds: first, Keynesian analysis can be more easily developed from a classical analysis than vice versa; second, the employment conditions in the world today suggest price rather than employment adjustments.

My debts are numerous: in writing this work I had the counsel and guidance of Professors Harry G. Johnson, Charles F. Kindleberger, James E. Meade, and Dr. S. A. Ozga; the product would have been much inferior without their help. Mr. Johnson very substantially contributed to the geometric representation of the transfer problem with impediments; Mr. Ozga very patiently checked a large part of my work in an earlier draft, both for accuracy of analysis and for expositional felicity; from Professor Meade I learned several lessons -- the chapters on Transport Costs and Commodity Movements and Factor Mobility owe much to him; Mr. Kindleberger's help was of a general nature during the year I spent at M.I.T., in guiding my reading, and of a specific nature in his comments and criticisms of the thesis in the summer of 1956 when
most of the writing took place. The author, of course, accepts the embarrassment of any mistakes.

I should like to express gratitude as well for general stimulation in Economics to Professors Paul A. Samuelson, Robert L. Bishop, Robert Solow, Lionel C. Robbins, W. W. Rostow, D. F. Gordon, P. W. Cartwright and J. A. Crumb, the last of whom introduced me to Economics at the University of British Columbia.

Finally, I wish to thank Miss Beatrice Rogers for help that was much more than the typing, and Miss Terry Jaeger for inserting my equations in the final manuscript.
I. THE TRANSFER PROBLEM
A. No Impediments

This section will present an exposition of various criteria used to determine changes in the terms of trade following a transfer of income from one country to another when trade impediments are absent. It will consider first the direction of change in the terms of trade, the problem to which most writers on the transfer problem have directed their attention; the limitations and necessary qualifications to this criterion will be pointed out. Later we shall derive, geometrically and algebraically, the criterion determining the degree to which the terms of trade change; and it will be demonstrated that this general criterion is not subject to the exceptions which exist in the use of the other criterion, the latter being a special case.

Throughout, our analysis deals with the transfer problem under classical assumptions -- all multiplier effects are thus ruled out. There are two countries, A and B, exporting X and Y, respectively; full employment is assumed throughout, so a decrement in expenditure in one country is compensated by an increment in expenditure in the other country and the difference between domestic expenditure and national income (in the free trade case) is the balance of trade. The first section deals with transfer when production is fixed; later sections will consider variable production under conditions of increasing, constant and decreasing costs.
Fixed Production

First we shall discuss the type of geometric analysis now used in the literature on the transfer problem.\(^1\) Consider first Figure I, the familiar box diagram apparatus, enclosing a system of community indifference curves for each country. World production is fixed: A initially possesses OL of X and ON of Y, and B initially possesses OM of Y and OK of X. With these initial endowments it is profitable for both countries to trade, and trading equilibrium is at P, determined by the intersection of A's and B's offer curves, \(O_a\) and \(O_b\) respectively. Because there are no impediments to trade, the equilibrium point P must lie on the contract curve KK, the locus of points where A's indifference curves are tangent to B's indifference curves. The terms of trade are given by the line OP.

Now assume that B must make a transfer to A equal to OT in terms of X. The point T represents the new endowment position of A and B; a new set of offer curves originating at T will determine the new trading equilibrium and any change in the terms of trade. Any change in the terms of trade will depend on the changes in demand resulting from the


increase in A's domestic expenditure and the decrease in B's domestic expenditure, each of which must be equal to OT, the transfer made from B to A.

The simplest way to develop geometrical criteria is to consider the effects of these expenditure changes at constant prices. If at constant prices transfer results in an excess demand for either commodity, the price of that commodity must rise; and an excess demand for one commodity implies an excess supply of the other commodity, so a criterion in terms of the demand for one commodity can easily be translated into a criterion in terms of the demand for the other commodity. The terms of trade can remain unchanged only when the transfer results in zero excess demand at constant prices.

B's terms of trade will improve or worsen, depending on whether there is an increase or a decrease in the demand for B's export good. Now the increase in demand for B's good following a transfer of one unit of income from B to A is simply A's marginal propensity to import; and the decrease in demand for B's good is B's marginal propensity to consume. But B's marginal propensity to consume is equal to unity minus B's marginal propensity to import because of the identity of income and expenditure; so the criterion in terms of the demand for B's good can be translated into a criterion involving only the marginal propensities to import. The criterion which results is that B's terms of trade will improve or worsen, depending on whether the sum of the marginal propensities to import is greater or less than unity.
Turn again to Figure I. At constant prices A moves up her expenditure line, $E_aE_a$, to $U$ where an A-indifference curve is tangent to the new income line; and B moves back along its expenditure line, $E_bE_b$, to $V$ where a B-indifference curve is tangent to the new income line. At $U$ and $V$ it is clear that an excess demand for B's good exists, equal to the vertical distance between $U$ and $V$; and that an excess supply of A's good exists equal to the horizontal distance between $V$ and $U$. The terms of trade of B therefore must improve in the situation drawn in Figure I, where $E_aE_a$ is steeper than $E_bE_b$.

Imagine now that A's expenditure line becomes flatter and B's expenditure line becomes steeper, pivoting around $P$. It can be seen that there will always be an excess demand for B's good as long as $E_aE_a$ is above $E_bE_b$. But as they move closer together to the northwest of $P$, this excess demand becomes smaller until they coincide, when the excess demand is eliminated. But they can only coincide on KK, since only on KK can the indifference curves of both countries have the same slope. The terms of trade remain unchanged then only when the expenditure lines coincide with the contract curve, which must be the case whenever the expenditure lines have the same slope. As $E_aE_a$ becomes flatter than $E_bE_b$, an excess demand for A's good and an excess supply of B's good results.

B's terms of trade will improve or worsen then, depending on whether the slope of A's expenditure line is (arithmetically) larger or smaller than the slope of B's expenditure line. But the slope of
A's expenditure line is the ratio of A's marginal propensity to import to her marginal propensity to consume; and the slope of B's expenditure line is the ratio of B's marginal propensity to consume to her marginal propensity to import. Letting \( \Pi \) and \( c \) denote marginal propensities to import and consume and subscripts denote countries, then the terms of trade of B, the transferring country, will improve or worsen, depending on whether

\[
\frac{\Pi_a}{C_a} > \frac{C_b}{\Pi_b}
\]

i.e., depending on whether the product of the marginal propensities to import is greater or less than the product of the marginal propensities to consume. The higher are import propensities, the more likely are the terms of trade to turn in favor of the transferring country, against the orthodox direction.

Figure II shows geometrically that this criterion is the same as the criterion in terms of the sum of the marginal propensities to import. The critical value of the criterion (when the terms of trade do not change) is:

\[
\frac{\Pi_a}{C_a} - \frac{C_b}{\Pi_b} = 0
\]

which is equal to

\[
\frac{R^P}{S_P} - \frac{Q^P}{S_P} = \frac{R^P}{S_P} = 1 - \frac{S_Q}{S_P} = \frac{R^P}{S_P} + \frac{S_Q}{S_P} - 1
\]
But \( \frac{\Delta P}{\Delta P} = \Pi_a \) and \( \frac{\Delta Q}{\Delta P} = \Pi_b \), so the criterion becomes whether
\[
\Pi_a + \Pi_b - 1 > 0.
\]

This is a criterion very commonly used, and there are many variants of it.\(^1\) It is, however, not general even under conditions of fixed production, and it provides no clue as to whether the terms of trade change must be large or insignificant in order to restore equilibrium. We shall discuss first the qualification which must be made to the criterion for even predicting correctly the direction of change of the terms of trade, and then proceed to derive a criterion which is general.

It was implicitly assumed in Figures I and II that the market was in stable equilibrium; in the stable case an excess demand for one commodity is eliminated by a rise in its price, since an increase in price shifts world expenditure away from that commodity. In the unstable case, however, an increase in the price of a commodity shifts expenditure onto that commodity, so an increase in price aggravates, rather than palliates, the disequilibrium. For equilibrium to be regained in the unstable case, an excess demand for a commodity can be eliminated only by a fall in its price, since only a fall in its price will shift world expenditure away from it. There are difficulties arising from this explanation when a dynamic analysis is introduced, but a discussion of these problems would be out of place here.\(^2\)

---

\(^1\)See, for example, Samuelson, Part I, p. 286, f. 1, or Johnson, Economics, loc. cit., p. 114.

\(^2\)In the unstable case an increase in the price of a commodity will shift expenditure onto that commodity and, depending on the dynamic process assumed, may or may not lead to a further disequilibrating increase in price.
We shall now examine the case of transfer in such an unstable market as that described above. Figure III illustrates this case. Equilibrium is initially at $P$, an unstable position, flanked by two positions of stable equilibrium, $R$ and $Q$. A transfer of $OT$ of $X$ is made from $B$ to $A$; at constant prices demands in $A$ and $B$ move along the expenditure lines to points $V$ and $U$. Since $E_aE_b$ is steeper than $E_aE_a$ in the figure, the sum of the marginal propensities to import is less than unity, creating at constant prices an excess demand for $A$'s good. Because the sum of the price elasticities is less than unity, the excess demand for $A$'s good can be relieved only by a fall in its price relative to the price of $B$'s good. Instead of the terms of trade moving against the paying country as dictated by the slopes of the expenditure lines, they move in the opposite direction.

Now from further examination of Figure III we can show that the terms of trade of the paying country not only improve but must improve so much that, as a result of transfer, $B$'s real income increases and $A$'s real income decreases! The positive income effect from the improvement in $B$'s terms of trade is more than sufficient to offset the negative income effect of the transfer itself. We can demonstrate this by proving that the new equilibrium position $P'$ must lie on the contract curve to the southeast of $P$ — i.e., we shall show that the slopes of the $A$ and $B$ indifference curves at $P'$ on the contract curve extend through $T$ on the $X$ axis.
The transfer is OT which creates an excess demand for A's good, causing the relative price of A's good to fall. The relative price line pivots around T until it is tangent to the indifference curves of both countries at P'. Now consider the price line from T through P; since the tangent to the indifference curves at P extends through the origin O, a line from T to P must be tangent to A and B indifference curves, away from the contract curve; and, since the indifference curves must be convex, a B indifference curve must be tangent to TP above, and an A indifference curve tangent below, the contract curve. Therefore at the price ratio TP, an excess demand for A's good still exists and the price of A's good must continue to fall -- i.e., equilibrium must be on the contract curve to the southeast of P. Q.E.D.

The process of adjustment in this unstable market now becomes clearer. Because the sum of the marginal propensities to import is less than unity, the transfer creates an excess demand for the good of the receiving country; because the sum of the price elasticities of demand for imports is also less than unity, the market can be cleared only by a fall in the price of that good. But if the price of A's good falls, A's income decreases and B's income increases. The income effects of the price change work in the opposite direction to those created directly by the transfer; the improvement in B's terms of trade shifts demand away from A's good, reducing its excess demand. At P, the original equilibrium, direct income effects have been cancelled out (the terms of trade change up to P in increasing B's income has
just cancelled out the direct income effect from the transfer), but there is an income and substitution effect resulting from the price change. At $P$, the price of $A$'s good after the transfer has fallen in relation to the price of $A$'s good before the transfer; there is still an excess demand for $A$'s good at $P$ which necessitates a further price change in the same direction. At $P'$ the income effects in reducing the excess demand for $A$'s good have caught up with the substitution effects in increasing the demand for $A$'s good.\footnote{1}

The above analysis illustrates that the degree to which the terms of trade change depends on substitution as well as income effects, so it is clear that the general criterion must involve price elasticities as well as import propensities. In the derivation of this more general criterion we shall make considerable use of "expenditure-compensated elasticities," which differ from Marshallian elasticities in that they adjust domestic expenditure to compensate for income effects. This "expenditure-compensated elasticity," $E$, is smaller than the Marshallian elasticity, $E'$, by the size of the marginal propensity to spend on imports.\footnote{2}


If $x = x(D, p)$ is the demand for $X$ as a function of domestic expenditure and the price of $X$, then
\[ -\frac{p}{x} \frac{dx}{dp} = \frac{p}{x} \frac{dx}{dx} \frac{dD}{dp} - \frac{p}{x} \frac{dx}{dp} \]
But $\varepsilon' (\equiv \frac{p}{x} \frac{dx}{dp})$ is the partial elasticity of demand and $p \frac{dx}{dD} (\equiv \pi)$ is the marginal propensity to spend on imports, so
\[ -\frac{p}{x} \frac{dx}{dp} = -\pi \cdot \frac{dD}{x dp} + \varepsilon' \]
Now if $D$ is adjusted to compensate for the price change, then
\[ \frac{dD}{x dp} = 1 \]
We shall now derive the criterion showing the degree of change in the terms of trade. In Figure IV equilibrium is initially at P. A transfer of OB is made from B to A and the new equilibrium moves to the point R; the terms of trade of B worsen as a result of transfer because the sum of the marginal propensities to import is less than unity and the market is stable.

Now draw through R a line parallel to OF and extend this line to C on the X-axis. A and B indifference curves are tangent to this line at F and G, and these points indicate the changes in expenditure in A and B at constant prices. Then the marginal propensity to import in A is:

$$\Pi_a = \frac{SV}{RU}$$

and the marginal propensity to import in B is:

$$\Pi_b = \frac{VU - TR}{RU}$$

A's expenditure-compensated elasticity of demand for imports is:

$$\epsilon_a = \frac{RS}{RM} / \frac{CM}{CM} = \frac{RS}{RM} \cdot \frac{CM}{CB}$$

(Footnote 2 continued)

in which case 

$$-\frac{p}{x} \cdot \frac{dx}{dp} (\equiv \epsilon)$$

is the expenditure-compensated elasticity of demand. Then 

$$\epsilon + \Pi = \epsilon'$$
and B's expenditure-compensated elasticity of demand for imports is:

\[
\varepsilon_b = \frac{T_G}{Bm} \left[ \frac{CM}{RM} \right] = \frac{TR}{RM} \cdot \frac{CM}{CB}
\]

Define the transfer as a proportion of the new level of imports,

\[
\ell = \frac{OB}{MB}
\]

and the change in the terms of trade as

\[
\nu = \frac{BC}{MB}
\]

For brevity, let the sum of the marginal propensities to import be represented as \( \Pi_{ar} \) and the sum of the expenditure-compensated elasticities of demand as \( \varepsilon_{ar} \).

Then

\[
1 - \Pi_{ar} = \frac{RU - SU - \nu U + TR}{RU} = \frac{ST}{RU}
\]

and

\[
\varepsilon_{ar} = \frac{CM}{CB} \left( \frac{RS + TR}{RM} \right) = \frac{CM}{CB} \cdot \frac{ST}{RM}
\]

so

\[
\varepsilon_{ar} + \Pi_{ar} - 1 = \frac{CM}{CB} \cdot \frac{ST}{RM} - \frac{ST}{RU}
\]

\[
\nu \left( \varepsilon_{ar} + \Pi_{ar} - 1 \right) = \frac{BC}{MC} \cdot \frac{MC}{MB} \left[ \frac{CM}{CB} \cdot \frac{ST}{RM} - \frac{ST}{RU} \right].
\]
\[ V(\varepsilon_{ar} + \Pi_{ar} - 1) = \frac{ST}{RU} - \frac{ST}{RU} \cdot \frac{BC}{MC} \cdot \frac{MC}{MB} \]

\[ = \frac{ST}{RU} \left( \frac{RU}{RM} - \frac{BC}{MC} \right) \frac{MC}{MB} \]

\[ = \frac{ST}{RU} \left( \frac{OC}{MC} - \frac{BC}{MC} \right) \frac{MC}{MB} \]

\[ = \frac{ST}{RU} \cdot \frac{OB}{MC} \cdot \frac{MC}{MB} = \frac{ST}{RU} \cdot \frac{OB}{MB} \]

\[ = (1 - \Pi_{ar}) \beta \]

\[ \therefore \frac{V}{\beta} = \frac{1 - \Pi_{ar}}{\varepsilon_{ar} + \Pi_{ar} - 1} \]

But \( \varepsilon_{ar} + \Pi_{ar} \) is the sum of the elasticities of demand for imports, so the criterion can be written

\[ \frac{V}{\beta} = \frac{1 - \Pi_{ar}}{\varepsilon_a' + \varepsilon_b' - 1} \]

\(^1\text{This is the formula used by Meade in the Balance of Payments Mathematical Supplement, loc. cit., p. 153.}\)
The terms of trade of the receiving country, A, will improve, remain the same or deteriorate as the expression on the right is positive, zero or negative. The familiar criterion, the numerator, is valid only when the denominator, the sum of the elasticities reduced by unity is positive. But even when it is positive, the price effects in the denominator must be taken into consideration in determining the exact change in the terms of trade.

Whenever the sum of the marginal propensities to import is greater than unity, the sum of the price elasticities must also be greater than unity. The term \(-\Pi_{av}\) is the correct criterion then whenever \(\Pi_{av}\) is greater than unity. But when \(\Pi_{av} < 1\), the sum of the elasticities can be either greater or less than unity, depending on the size of the substitution effects.

Whenever the sum of the elasticities is less than unity, we know that the sum of the marginal propensities to import is less than unity. In the unstable case then the terms of trade of the receiving country must deteriorate.

We can show these relationships more explicitly by deriving another criterion from the one previously presented, as is done in the following way:

\[
\frac{\sqrt{\frac{1 - \Pi_{av}}{\varepsilon_{av} + \Pi_{av} - 1}}}{\frac{1}{\varepsilon_{av} + 1 - \Pi_{av}}} = \frac{1}{\varepsilon_{av}} \frac{1}{1 - \Pi_{av}} - 1
\]
The criterion is then whether the denominator is positive or negative — i.e., whether
\[ \frac{\varepsilon_{av}}{1-\Pi_{av}} \geq 1 \]
From this criterion it is clear that if \( \Pi_{av} > 1 \), B's terms of trade must improve, since \( \varepsilon_{av} \) is always positive. If \( \Pi_{av} = 1 \), the denominator is infinite, so no change in the terms of trade takes place; and if \( \Pi_{av} < 1 \), then, clearly, B's terms of trade will improve or worsen, depending on whether substitution effects, \( \varepsilon_{av} \), are less than or greater than income effects \( 1-\Pi_{av} \). This can, of course, be derived directly from Figure IV. The criterion is whether \( \varepsilon_{av} \geq 1-\Pi_{av} \)

i.e., whether
\[ \frac{ST}{RM} \cdot \frac{CM}{BC} \geq \frac{ST}{RU} \]
\[ \frac{ST}{RU} \cdot \frac{OC}{CM} \cdot \frac{CM}{BC} \geq \frac{ST}{RU} \cdot \frac{OC}{BC} \geq 1 \]
The critical value, where \( \varepsilon_{av} = 1-\Pi_{av} \), is the familiar case where the sum of the price elasticities is equal to unity, in which case no change in the terms of trade will eliminate the excess demand for A's good; an "infinite" change in the terms of trade is required. The transfer at constant prices results in an excess demand for A's good, the price of A's good rises but expenditure cannot be shifted away from A's good, so no change in relative prices will restore equilibrium (within the relevant range).
More generally it can be said that the larger are the price elasticities the less the terms of trade will have to change in order to restore equilibrium, since a small price change will involve a large elimination of excess demand. Also the more the sum of the marginal propensities to import differs from unity, the larger the excess demand that must be eliminated by the price change. But income effects work in two ways: first, directly through the changes in expenditure induced by the transfer; second, indirectly through the income effect implicit within the price change. If \( \Pi_1 < 1 \), the larger is \( \Pi_1 \), the smaller is the excess demand to be eliminated, and the larger is the income effect from the price change — so up to the critical value of unity, the larger is \( \Pi_1 \), the smaller is the change in the terms of trade required; both income effects operate in the same direction here. If \( \Pi_1 > 1 \), however, the income effects operate in opposite directions. The larger is \( \Pi_1 \) the greater is the excess demand that must be eliminated, necessitating a larger price change; but, on the other hand, the larger is \( \Pi_1 \) the larger is the income effect associated with the price change so the greater is the sum of "total" elasticities, and the more effect a given price change has. It is in the separation of these effects that the last criterion derived becomes most useful.

In this section we have used one price ratio as the terms of trade without any explicit recognition of the rate of exchange between currencies.
This assumption produces the same result as if currency units were chosen so as to make the rate of exchange initially unity and kept fixed, while factor-cost prices are allowed to vary. It could instead be assumed that factor-cost prices were kept constant by monetary policy and that the relative price change takes place through variations in the exchange rate, the latter being the terms of trade if commodity units are chosen so as to make the price of commodities equal to unity.

Because of the simplifications of the model used here, it makes no difference through which price change disequilibrium is eliminated. If both the exchange rate and prices were allowed to vary, an indeterminacy would result as to which ratio had changed but the underlying real equilibrium would be the same, no matter which price varied. The terms of trade is the product of the ratio of export prices and the exchange rate; it is changed in the terms of trade which are significant, not changes in the exchange rate or relative prices. In the rest of our analysis we shall sometimes assume constant factor prices and sometimes a constant exchange rate.

Mathematical Analysis

It will be convenient in this and other sections to derive various criteria both geometrically and algebraically, to check the results and to explore new simplifications in technique. Here we shall assume that monetary authorities adjust the quantity of money so that the factor-cost price of exports is constant in each country, and that quantity units
are chosen so that prices are equal to unity. Also we assume that currency units are chosen so that the exchange rate is initially unity, but that the exchange rate is allowed to fluctuate freely. Finally, we assume that the balance of trade is initially zero.

Use the following notation:  
- $b$ = the balance of trade 
- $e$ = the exchange rate 
- $I_a$ = A's imports 
- $I_b$ = B's imports 
- $\pi_a$ = A's marginal propensity to import 
- $\pi_b$ = B's marginal propensity to import 
- $D_a$ = Domestic expenditure in A 
- $D_b$ = Domestic expenditure in B 
- $\xi_a'$ = "total" elasticity of demand for Imports in A 
- $\xi_b'$ = "total" elasticity of demand for Imports in B

Differentiate the balance of trade equation

$$ B = I_b - I_a \cdot e \quad \text{with respect to } B $$

to get

$$ 1 = \frac{dI_b}{dB} - I_a \cdot \frac{de}{dB} - \frac{dI_a}{dB} $$

Imports are a function of domestic expenditure and the exchange rate, i.e.,

$$ I_a = I_a(D_a, e) $$

$$ I_b = I_b(D_b, e) $$
so \[ \frac{dI_a}{dB} = \Pi_a \cdot \frac{dD_a}{dB} + \frac{2I_a}{2e} \cdot \frac{de}{dB} \]

and \[ \frac{dI_b}{dB} = \Pi_b \cdot \frac{dD_b}{dB} + \frac{2I_b}{2e} \cdot \frac{de}{dB} \]

Because income equals expenditure,
\[ \frac{dD_a}{dB} = -\frac{dP_a}{dB} = 1 \]

so \[ \frac{dI_a}{dB} = -\Pi_a + \frac{2I_a}{2e} \cdot \frac{e}{I_a} \cdot I_a \cdot \frac{de}{dB} \]

and \[ \frac{dI_b}{dB} = \Pi_b + \frac{2I_b}{2e} \cdot \frac{e}{I_b} \cdot I_b \cdot \frac{de}{dB} \]

where \[ \frac{2I_a}{2e} \cdot \frac{e}{I_a} = -\xi_a' \]

and \[ \frac{2I_b}{2e} \cdot \frac{e}{I_b} = \xi_b' \]

Then substituting,
\[ 1 - \Pi_a - \Pi_b = \xi_b' \cdot I_b \cdot \frac{de}{dB} + \xi_a' \cdot I_a \cdot \frac{de}{dB} - I_a \cdot \frac{de}{dB} \]

Since the balance of trade is initially zero and \( e = 1 \), then \((I_b = I_a) = I \)

\[ \frac{I \cdot de}{dB} = \frac{1 - \Pi a}{\xi_a' + \xi_b' - 1} = \frac{1 - \Pi a}{\xi a + \Pi a - 1} \]

which is the same as our geometric criterion. It will be remembered that our geometric criterion assumes \( B \) as a proportion of the new level of imports.
Variable Production

Up until now we have assumed that production was fixed, an assumption equivalent to that of rectangular production blocks. In this section we shall investigate the effects of altering this assumption, and allowing production to vary under conditions of increasing (opportunity) costs, constant costs and decreasing costs. It will be demonstrated that the criterion developed in the last section is valid in all these cases when the elasticities have been re-defined to include production changes.

In their analyses of the transfer problem, Professors Samuelson and Johnson\(^1\) state that the criterion for the direction of change in the terms of trade is independent of production conditions, since production will move in the direction dictated by changes in demand. This is, of course, indisputable; but it is not correct to say, in the general case, that the changes in prices will move in the same direction with fixed and with variable production unless a further assumption is made about the nature of cost conditions. Consequently, their criteria are criteria for changes in demand only and will coincide with the criteria for changes in the terms of trade only when production is fixed or opportunity costs are increasing. The latter assumption may be sufficiently general to justify their argument, but it should have been made more explicit.

\(^1\)Samuelson notes the case of Ricardian constant costs and incomplete specialization. See Part I, p. 301, f. 1 suggested by Professor Viner. Johnson writes: "... and since changes in production will depend on changes in the terms of trade, it does not matter whether we assume production in each country is fixed or variable." *Economica*, loc. cit., p. 113.
First, the distinction must be made between cases of complete and cases of incomplete specialization. No matter what assumption is made concerning cost conditions, if both countries are completely specialized before and after the transfer, the criterion must be the same as for fixed production, since production does not in fact vary. For this reason we shall not consider the cases of increasing, decreasing and constant costs when complete specialization exists before and after the transfer.

It is possible that one or both countries could be completely specialized before the transfer but not after the transfer, or they could be completely specialized after but not before the transfer. This case becomes more likely the greater the change in the terms of trade required to eliminate any excess demand for a commodity caused by the transfer. But in this analysis we are concerned primarily with criteria for changes in the terms of trade when the transfer is small; for this reason we shall not consider these perhaps less usual cases. The analysis does not yield sufficiently interesting results to make the venture worth while, although it does not present any particular difficult problems. The analysis will, then, be confined to those cases where at least one country is incompletely specialized.

When production is variable, it is no longer possible to use an enclosed box diagram. Instead, we shall find it convenient to use reciprocal demand-and-supply curves when these curves are derived from Meade-type trade-indifference curves\(^1\) rather than consumption-indifference curves.

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\(^1\)For a more detailed exposition of the derivation of trade indifference curves, see Meade, J. E., Geometry, \textit{loc. cit.}, Chapter II.
such as those used in the preceding diagrams. Trade indifference curves are derived from consumption indifference curves, and the transformation curve, by sliding the production block along successive consumption indifference curves, the origin of the production block tracing out the appropriate trade indifference curve corresponding to any consumption indifference curve. Thus, a trade indifference curve adjusts any consumption indifference curve for production changes; a given movement along any trade indifference curve corresponding to a given price change will then trace out the production as well as the consumption changes caused by that price change. The offer curve derived from these trade indifference curves is thus capable of describing the supply of exports and the demand for imports when production is variable.

**Increasing Costs**

We shall consider first the case of incomplete specialization under increasing costs. It can be shown that the criterion for changes in the terms of trade is exactly the same as before, when our terms are re-defined to allow for production changes. In Figure V equilibrium is initially at P and the terms of trade are given by the line OP; consumption in A is at $C_a$ in the northwest quadrant, and consumption in B is at $C_b$ in the southeast quadrant. National income in A is OA, of which OD is spent on consumption of X and DA is spent on consumption of Y; national income in B is OB in terms of Y, of which OE is spent on Y and EB is spent on X.

Suppose B makes a transfer to A equal to OT of X. At constant prices A moves up along its expenditure line $E_a^E_a$ and B moves back
along its expenditure line $E_aE_b$. Because $E_aE_a$ is steeper than $E_bE_b$, the sum of the marginal propensities to import is greater than unity, creating, at constant prices, an excess demand for B's good. Now, at constant prices, production in A and B does not change. But because of the excess demand for B's good, the price of $Y$ tends to rise; and when the price of $Y$ rises, production equilibria in A and B move along the transformation curves to produce more $Y$ and less $X$. In this stable case, then, production moves in the same direction as prices — hence, the criterion for the direction of change in the terms of trade is the same as that for fixed production.

It can be seen from an examination of the trading quadrant in Figure V that a criterion for changes in the terms of trade can be established in the same way as that which was established for fixed production in Figure IV. There will, however, be a difference: the denominator, the sum of the price elasticities reduced by unity, will have a different value, since the elasticities now include changes in production as well as consumption. It is simple to demonstrate that in the case of increasing costs the substitution effects will be larger. An argument of the following nature will suffice: suppose that transfer causes at constant prices an excess demand for $Y$. The price of $Y$ will rise and consumers will substitute $X$ for $Y$, while producers will produce more $Y$ and less $X$. An increase in the price of $Y$ will induce B to offer more $Y$ to A for two reasons instead of one (B's consumers consume less and her producers produce more). The expenditure-compensated elasticity
of a trade indifference curve must be greater than the corresponding expenditure-compensated elasticity of the consumption indifference curve from which the trade indifference curve is derived.

The sum of the marginal propensities to import, as shown on consumption indifference curves, must be the same as that shown on trade indifference curves at any given price ratio, since production will be the same in both cases -- in Figure V the slope of $C_aU'$ must be the same as the slope of $E_aE'_a$, and $V'C_b$ must have the same slope as $E_bE'_b$.

The criterion for changes in the terms of trade is unchanged, then, with variable production, when expenditure-compensated elasticities are re-defined to include changes in production: it is

$$\frac{dv}{d\alpha} = \frac{1 - \Pi_{ab}}{\varepsilon_{uv} + \Pi_{uv} - 1}.$$  

The effect of introducing variable production is to lessen the degree to which the terms of trade must change to eliminate any excess demand. It almost goes without saying that variable production contributes to stability.

**Constant Costs**

It is perhaps desirable to justify the possible assumption of constant opportunity costs, although the late Frank Graham\(^1\) considered

this the rule rather than the exception. There are three commonly considered constant cost cases which can be analytically justified. These are:

(a) A single factor of production

(b) Two factors of production; but, where at the points on the production function where marginal products are equalized, both commodities are produced with the same factor proportions (assuming linear homogeneous production functions)

(c) Two factors of production where increasing opportunity costs due to changing factor proportions are just offset by economies of scale.

In the absence of any economies to scale, constant costs is the limiting case of increasing opportunity costs. An analysis in terms of constant costs could also be justified as an approximation to very high supply elasticities, in which case an analysis of constant costs acquires more significance than the above "curiosities" seem to suggest.

There is a further possibility which will not be closely examined at present, to which a brief reference is justified. If fixed coefficients (rectangular isoquants) were considered realistic, the resulting offer curve would be a series of straight lines of infinite elasticity separated by limbo ratios; if the transfer does not shift consumption from one of those straight line areas, the resulting solution is the same as that under constant costs.

We shall assume now that both A and B produce S and Y under conditions of constant opportunity cost, and that A is large relative to B.
In Figure VI equilibrium is initially at $P$, and the terms of trade are OP. A's consumption equilibrium is at $C_a$ in the northwest quadrant, and B's consumption equilibrium is at $C_b$ in the southeast quadrant. B makes a transfer to A equal to OT in terms of $X$; B's expenditure shifts along its expenditure line $E_bE_b$ to $V$ -- in the consumption quadrant from $C_b$ to $V'$. A's trade indifference curves, however, have a straight-line segment, so it is impossible to tell where A's expenditure line is from the trade quadrant; but in the northwest (A's consumption) quadrant we can tell which way A's consumption moves at constant prices in relation to B's expenditure path. Clearly, $C_{a}U'$ is steeper than $E_bE_b$, indicating an excess demand for B's good.

But instead of this causing an improvement in B's terms of trade, the latter do not change. The excess demand for B's good is eliminated by a movement along A's production block to an equilibrium where A produces more $Y$ and less $X$. Since this movement is effected at constant opportunity cost, no change in the terms of trade can take place. The demand criterion is in this case inadequate for predicting the direction of change in the terms of trade. The general criterion

$$\frac{dV}{dB} = \frac{1 - \Pi_{ab}}{\xi_{ab} + \Pi_{ab} - 1}$$

is correct in this as in all other cases. The expenditure-compensated elasticity of A's demand for imports is infinity, since it is measured along a trade indifference curve with a straight line segment. Since $e_a$ is infinite, the criterion is zero; no change in the terms of trade
should take place according to the criterion, and no change actually does take place.

**Decreasing Costs**

In a later chapter we shall discuss the reasons underlying the case of decreasing costs. For the moment we shall assume that they exist and analyze the case of transfer under decreasing costs. Again, the case of complete specialization in both countries is the same as that of fixed production. We shall assume incomplete specialization in country A.

When X and Y are produced under conditions of decreasing costs, the trade indifference curves derived by sliding the decreasing cost production block along successive indifference curves are no longer convex in a single direction. Over the section appropriate to incomplete specialization they curve in the opposite direction, and result in the normal convexity only under conditions appropriate to complete specialization. The offer curves derived from these trade indifference curves have an unorthodox shape, intersecting at least three times, producing one point of stable equilibrium flanked by two points of stable equilibria.

Figure VII shows the two offer curves, \( O_a \) and \( O_b \), intersecting at points \( W \) in the southwest quadrant, and \( Q \) and \( P \) in the northeast quadrant. \( Q \) is unstable and \( W \) and \( P \) are stable.\(^1\) At \( W \), the case of

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"perverse" specialization, country A specializes in the production of Y and country B specializes in the production of X; at Q both A and B are incompletely specialized, representing a trade pessimum rather than a trade optimum. At F, the point of equilibrium which we shall consider, country B is completely specialized in the production of Y and A is incompletely specialized, producing both X and Y.

There are three contract curves, KK, K'K', and K''K'', passing through points P, Q and W. At W the trade indifference curves of A and B are tangent with their normal convexity; at Q both A and B curves have "perverse" convexity and at P, because B is completely specialized and A is incompletely specialized, B's trade indifference curve has normal convexity but A's has perverse convexity, both being concave downward.

Now let us consider the point P. We know that both A and B curves must be convex upward and that at P the curves are tangent. But it is necessary to determine whether B's trade indifference curve lies "inside" or "outside" of A's trade indifference curve. This can be determined in the following way: There are only three contract curves, none above KK, so I_a and I_b cannot be tangent above P. Now I_b is convex downward. If I_b were outside of I_a, then I_a and I_b must be tangent somewhere to the northeast of P. Therefore I_b must lie inside of I_a. It is easily seen from Figure VII that with I_b lying inside of I_a, I_a and I_b are not tangent above P. Q.E.D.
Suppose that B makes a transfer to A equal to OT of X. At constant prices B moves back along its expenditure line $E_b F_b$, and A moves up along its expenditure line $E_a F_a$ to $U$ and $V$, respectively. Since both $I_a$ and $I_b$ are convex upward, expenditure lines must lie on the same side of the contract curve. In Figure VII we have drawn them below the contract curve, with $E_b F_b$ steeper than $E_a F_a$, indicating at constant prices an excess demand for A's good. It is clear, however, from an examination of the indifference curves at R that the price of A's good will fall rather than rise, indicating another exception to the use of strictly demand criteria. We shall find that when the general criterion is used, however, it correctly predicts that B's terms of trade must improve.

In Figure VIII a transfer of OB is made from B to A and equilibrium moves from P to R in the trading quadrant. The criterion

$$\frac{dV}{dB} = \frac{1 - \Pi_{ab}}{E_{ab} + \Pi_{ab} - 1}$$

is equal to

$$\frac{RU - RT - SU}{RU} = \frac{\frac{ST}{RM} \cdot \frac{MC}{BC}}{\frac{ST}{RU}} = \frac{\frac{ST}{RM} \cdot \frac{MC}{BC} - \frac{ST}{RU}}{\frac{ST}{RM} \cdot \frac{MC}{BC} - 1} = \frac{1}{\frac{OC \cdot MC}{MC \cdot BC} - 1}$$

$$= \frac{1}{\frac{OC}{BC} - 1} = -\frac{BC}{OC + BC} = -\frac{BC}{OB}.$$
It is to be noted that the difference between this result and that of Figure IV is that the sum of the expenditure-compensated elasticities is now negative:

$$\xi_b = \frac{RS}{RM} \left/ \frac{BC}{CM} \right. \quad \text{and} \quad \xi_a = -\frac{RT}{RM} \left/ \frac{BC}{CM} \right.$$  

($\xi_a$ is negative because A imports less $Y$ as the price falls)

The sum of the expenditure-compensated elasticities is therefore

$$\frac{CM}{BC} \left( \frac{RS}{RM} - \frac{RT}{RM} \right) = -\frac{ST}{RM} \cdot \frac{CM}{BC}$$

$$\frac{RT}{RM} > \frac{RS}{RM} \quad (\text{always because } \Gamma_b \text{ lies inside } \Gamma_a).$$

The criterion then indicates the correct change in the terms of trade.

Up to this point we have considered the transfer problem in all its real, simplified aspects. We have derived a criterion for changes in the terms of trade which applies to all cases and which shows not only the direction of change but also the quantitative intensity of that change. The special cases of instability, constant and decreasing costs do not necessitate any alteration in the criterion.

We shall now proceed to consider transfer and the criteria for changes in the terms of trade when impediments are present.
B. Impediments

Transport Costs

Transport costs have been somewhat neglected in international trade theory, although their importance in determining the pattern of world trade is well understood. As no systematic geometric representation of transport costs has been presented in the literature, we must digress for a moment from the transfer problem to show how transport costs can be illustrated in familiar international trade models. First, we shall show how offer curves must be modified in order to account for transport costs; second, we shall discuss new meanings that must be attached to different terms of trade concepts, arising from our assumptions regarding transport costs; and finally, we shall apply this method to the transfer problem.

To avoid the introduction of a transport industry, we shall assume that transport costs are met by a proportion of one or both of the goods traded being used up en route. If, for example, we assumed that transport costs absorb the good of the exporting country, then only a fraction of the good exported by one country would be received as imports by the other. If transport costs involved the good of the importing country, then some of the importing country's good would be used up for every unit of the good exported by the other country. In the general case, some of both goods would be used up as costs of transport.

1This assumption has been used by Professors Samuelson and Johnson in their analyses of the transfer problem. See Samuelson, Part II, and Johnson, *Economics*, loc. cit.; *idem, J.P.E.*, loc. cit.
This assumption is a drastic, but extremely useful, simplification. If we thought in terms of a transport industry, the assumption that the good of the exporting country is used up as transport costs would imply that some of the exporting country's resources, instead of being applied to the production of the export good, were used instead in the transport industry. The commodities X and Y "wasted" as transport costs could be considered the returns paid to the transport industry; but this interpretation implies that these returns do not affect the market equilibrium, since the consumption of factors in the transport industry is not included in the community indifference map. This particular problem could be avoided if it were assumed that some outside country handled the transport services, using, however, the resources of one or both of the trading countries; these resources expressed in terms of commodities would be the quantities used up as costs of transport — i.e., the third country would trade transport services for the quantity of X and Y used up as transport costs. It would, however, have to be assumed that the third country did no further trading with A and B.

Geometric Representation

For the moment assume that there are no costs of transporting Y, but that there are costs of transporting X. Suppose that the costs of transporting X involve a proportion of X (T_X), used up en route to B; K_X is the proportion which reaches B so that K_X plus T_X equal 1. In
Figure I, \( O_a \) and \( O_b \) are the offer curves of A and B, respectively.

Draw the transport-modified or net offer curve \( O_a' \) in such a way that its \( X \)-value is always a constant proportion \( (K_x) \) of the \( X \)-value of \( O_a \) for any given quantity of \( Y \). For any value of \( Y \) the \( X \)-value of \( O_a \) represents the quantity exported by A and the corresponding \( X \)-value of \( O_a' \) represents the quantity received by B -- the difference is used up as transport costs.

Equilibrium is reached where A's transport-modified offer curve, \( O_a' \), intersects B's offer curve at \( Q_b \). A exports \( LQ_a \) of \( X \) in return for which she receives \( HQ_b \) of \( Y \), so \( a \) is the price ratio in A. B exports \( HQ_b \) of \( Y \) in return for which she receives \( LQ_b \) of \( X \), so \( b \) is the price ratio in B. \( Q_bQ_a \) of \( X \), the difference between A's exports and B's imports, is used up as transport costs.

In economic terms this means that for \( HQ_b \) exports of \( Y \), B will receive \( LQ_a \) of \( X \) minus freight charges of \( LQ_b \) of \( X \) after freight charges have been deducted. This is the same as saying that the c.i.f. price of OH imports of \( X \) is \( HQ_b \) while the f.o.b.\(^1 \) price is \( HJ \), since \( JQ_b \) is the \( Y \)-value (at A's price ratio) of \( Q_aQ_b \) of \( X \) used as transport costs.

We now consider the case where the good of the importing country is the transport good; some \( Y \) is used up for every unit of \( X \) that moves from A to B. In Figure II the slope of \( OK \) measures the amount of \( Y \) needed to transport one unit of \( X \) from A to B. To find the equilibrium

\(^1\)C.i.f. prices are intended here to include, f.o.b. prices to exclude, transport costs.
point, draw $O_a'$ by adding the $Y$-value of $OK$ to the $Y$-value of A's offer curve for any given value of $X$ — in other words, the line $OK$ is added vertically to $O_a$.

$O_a'$ intersects $Q_b$ at $Q_b$, so this is B's trading point. At $Q_b$ B's consumers pay $HQ_b$ of $Y$, c.i.f., or $HQ_a$ of $Y$, f.o.b., for $OH$ of $X$.

B's exports, $HQ_b$, are larger than A's imports, $HQ_a$, by the amount of $Y$ used up in transporting $OH$ of $X$.

Figure III shows the equilibrium when some of both $X$ and $Y$ is required to transport $X$. $O_a'$ is A's offer curve net of transport costs incurred in $X$, and $O_a''$ is A's offer curve net of all transport costs. $O_a'''$ intersects B's offer curve at $Q_b$, so this is B's trading equilibrium.\(^1\) At $Q_b$, B exports $HQ_b$ of $Y$ and imports $OH$ of $X$. But $PQ_b$ of $Y$

\(^1\)It becomes important now to distinguish very carefully whether the transport cost is used up at the point of departure or on arrival of the exported good. Just as in a tariff theory a redistribution of tariff proceeds to consumers must take into account tariffs paid on tariffs, so the assumption in transport cost theory that part of the export goods is used up after it reaches its destination involves the problem of transport costs required to ship the transport costs! If $T_x$ is the ratio of transport costs used up and $T$ the total costs paid when the total costs of shipping $T_x$ etc., are considered, then

$$T = T_x + T_x^2 + T_x^3 + \cdots = \frac{T_x}{1 - T_x} = \frac{T_x}{K_x}.$$  

The whole problem can be sidestepped if the size of the proportions is adjusted to accord with the actual assumption made. This is assumed done in all examples here so that no transport costs are required to ship transport costs. In Fig. III, for example, it is assumed that the $X$ transport costs are used up when $X$ leaves A and that the transport costs incurred in $Y$ are paid only on the amount of $X$ that reaches B.
and \( PQ_a \) of \( X \) are used up as transport costs, so \( A \)'s trading equilibrium is at \( Q_a \), where she exports \( LQ_a \) of \( X \) in return for \( HP \) of \( Y \). The price ratio in \( A \) is \( a \); the price ratio in \( B \) is \( b \). \( B \) must pay \( JQ_b \) in terms of \( Y \) as costs of transport, \( PQ_b \) of \( Y \) used directly, and \( PJ \) of \( Y \) to pay for \( PQ_a \) of \( X \). Thus the c.i.f. price of \( OH \) of \( X \) in \( B \) is \( HQ_b \), and the f.o.b. price is \( HJ \).

The meaning of the new offer curve \( O_a''' \) in relation to \( O_a \) can now be made clearer. \( O_a''' \) describes offers of \( X \) delivered in \( B \) at different price ratios and thus can be called the "c.i.f." offer curve. \( O_a \) describes offers of \( X \) before transport costs have been deducted and can be called the "f.o.b." offer curve.

Up until now we have considered only the costs of transporting \( X \). If there are costs of transporting \( Y \) as well, both offer curves must be modified in the same way as described above, depending on which goods are used up as real costs of transport. To keep the geometry simple, we shall return to the original assumption that only the good of the exporting country is the transport good.

\( O_a' \) and \( O_b' \) in Figure IV are the offer curves of \( A \) and \( B \) modified to allow for transport costs incurred in the good of the exporting country -- that is, some of \( X \) is used up in shipping \( X \) and some of \( Y \) is used up in shipping \( Y \). The curves intersect at \( P \). \( A \)'s trading equilibrium is at \( Q_a \), where she exports \( LQ_a \) of \( X \) in return for \( OL \) of \( Y \); her domestic price ratio is therefore \( a \). \( B \)'s trading equilibrium is at \( Q_b \), where she exports \( HQ_b \) of \( Y \) in return for \( OH \) of \( X \); \( B \)'s domestic price ratio is therefore \( b \).
A exports $LQ_a$ of $X$ but $B$ receives only $LP$, the difference, $PQ_a$, being used up as costs of transporting $X$. $B$ exports $HQ_b$ but only $HP$ reaches $A$, $PQ_b$ being used up as costs of transporting $Y$. $Q_a$ and $Q_b$ must be points of equilibrium, since they lie on the offer curves of $A$ and $B$, respectively, and the difference between the exports and imports of each country is just used up as transport costs.

The relationship between the price ratios $a$ and $b$ is a constant, as shown in the following way. If $K_x$ is the proportion of $X$, and $K_y$ is the proportion of $Y$ used up as transport costs, then

$$K_x K_y = \frac{HP}{HQ_b} \frac{LP}{LQ_a} = \frac{OL}{LQ_a} \left( \frac{HQ_b}{OH} \right) = \frac{P_x^A}{P_y^A} \left( \frac{P_x^A}{P_y^A} \right),$$

where $P_x^A$, $P_y^A$, $P_x^B$, $P_y^B$ are the prices of $X$ and $Y$ in $A$ and $B$, respectively. The difference in the slopes is always a constant, $K_x K_y$.

This suggests an alternative way of finding the equilibrium, a method which is exactly analogous to that which Lerner made famous in his analysis of tariff equilibrium. Given the offer curves $Q_a$ and $Q_b$, a pencil $aOB$ can be rotated around $O$ until two points are found on $Q_a$ and $Q_b$ between which transport costs will just consume the difference between exports and imports of each country. This equilibrium could only be reached at $Q_a$ and $Q_b$ in Figure IV.

The equilibrium in Figure I is the same as that for a tariff imposed by $A$, where $A$'s government spends all the proceeds of the

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tariff on the home good. The equilibrium in Figure II is the same as that where A's government spends all the proceeds of the tariff on imports, while Figure III shows the equilibrium if the government spends some of the proceeds to buy $PQ_a$ of $X$ and the remainder to buy $PQ_b$ of $Y$. In Figure IV the tariff analogy would be that where both A and B imposed tariffs and each spent the proceeds on their home good.

Up to this point we have considered various assumptions as to how the real costs of transport are paid and the way this affects the equilibrium trade position. Further combinations can be made, but they become considerably complicated without yielding very interesting results; they will not be further pursued. Instead, we shall consider the different terms of trade concepts arising from our new analysis.

The Terms of Trade

Clearly there is no longer a unique meaning to the commodity or net barter terms of trade. Should they be considered one of the domestic price ratios or an intermediate ratio?

We shall distinguish four measures of the terms of trade arising from the previous analysis. First, there are the two domestic ratios

$$\frac{P_x^A}{P_y^A} \text{ in } A$$

and

$$\frac{P_x^B}{P_y^B} \text{ in } B.$$ 

The ratio $\frac{P_x^A}{P_y^A}$ is the ratio of the home price of $X$ to the c.i.f. price of $Y$, or A's domestic price ratio; $\frac{P_x^B}{P_y^B}$ is the ratio of the c.i.f. price of $X$ to the home price of $Y$, or B's domestic price ratio. In Figure IV these domestic price ratios are $a$ and $b$. 
There are two intermediate ratios. In Figure IV, OF' measures the terms of trade when prices are calculated f.o.b. This can be deduced in the following way. For HQ of X, country B will receive OH of X after transport costs have been deducted. Since transport costs are PQa of X (which equals QbP') the f.o.b. price ratio must be OF'. Or for LQa exports of X, country A receives HP of Y c.i.f. or HQb of Y f.o.b. Thus the exchange ratio at f.o.b. prices, which I shall call the f.o.b. terms of trade, is OF'.

OF represents the c.i.f. terms of trade. B would receive LQa of X f.o.b. in exchange for HP of Y c.i.f. in A. But after PQa of X is deducted for transport costs, B would receive only LP. Similarly, for OH of X c.i.f. in B, A would receive HQb of Y f.o.b. or HP of Y c.i.f. Thus OF must be the terms of trade valued at c.i.f. prices.

If TTa, TTb, TTfob, and TTcif represent these four different terms of trade measures, the relationship among them is

\[
\frac{1}{K_x} \cdot T \cdot T_a = \frac{K_y}{K_x} \cdot T \cdot T_{fob} = T \cdot T_{cif} = K_y \cdot T \cdot T_b.
\]

The c.i.f. terms of trade will equal the f.o.b. terms of trade only when the proportions Kx and Ky are equal. The c.i.f. and f.o.b. terms of trade always lie in between domestic price ratios, consequently it

1Algebraically, the f.o.b. terms of trade, \( \frac{P_x^A}{P_y^B} \), equals

\[
\frac{P_x^A}{P_y^B} = \frac{P_x^A}{P_y^B} \cdot \frac{H_P}{L_Q} = \frac{H_P}{K_y} \cdot \frac{L_Q}{Q_a}.
\]

The c.i.f. terms of trade are

\[
\frac{P_x^A}{P_y^B} = \frac{H_P}{K_y} \cdot \frac{L_Q}{Q_a} = \frac{H_P}{L_P} \cdot \frac{L_Q}{Q_a}.
\]
is clear that the terms of trade calculated at f.o.b. or c.i.f. prices will always be more favorable than when calculated at domestic price ratios.

Empirical studies generally use import prices c.i.f. and export prices f.o.b. As Professor Haberler points out, this method makes it possible for the terms of trade of both countries to improve simultaneously following a lowering of freight costs. In Figure IV this means that the domestic price ratios are squeezed more closely together as $K_x$ or $K_y$ becomes larger. Further, the c.i.f. terms of trade need not move in the same direction as the f.o.b. terms of trade following a change in costs of transport. In empirical studies, then, it becomes very important to make explicit which terms of trade have been computed and the degree to which changes in costs of transport have influenced the value of the measure used.}

---


2However, I am unable to understand why Professor Haberler says that the "correct procedure" in empirical studies is to include the "price of transport services in the computation of the terms of trade" (ibid., f. 7). Since the c.i.f. terms of trade have a different meaning than any other measure of the terms of trade, they answer a different, but not necessarily a more relevant, question. If, for example, the effect of changes in the terms of trade on A's national income were in question and transport services were provided by B, the relevant terms of trade should use c.i.f. import prices and f.o.b. export prices, i.e., the price ratio $\text{loco A's port of export}$. It would not matter whether the change in the terms of trade were caused by a reduction in transport costs or an improvement in productivity in B. But these considerations depart from the assumptions of a two country--two commodity world.
It is now possible to turn to the more interesting application of this analysis, to the Transfer Problem.

The Transfer Problem

When trade involves transport costs, there are two important effects that must be considered and their direction of influence on the terms of trade following transfer ascertained. If it is assumed that the export good is used as the transport good, the increased demand in A for B's good will involve an additional increase in demand for B's good used as transport costs. This is a factor tending to make the terms of trade turn in favor of the paying country. When some of both goods are used up as transport costs, nothing can be said about the direction of any bias given to the terms of trade without further empirical knowledge about the proportions in which they are combined.

There is a second factor which must be considered. Transport costs incurred in either good raise the c.i.f. price of imports and hence reduce the volume of imports, and also the average physical propensity to import. However, it cannot be said that the higher price of imports reduces the marginal propensity to import unless an additional assumption about consumers' behavior is introduced (if the marginal propensity were assumed to equal the average propensity this would suffice), although some writers have by "symmetrical ignorance" lent support to that argument.¹

¹See the interesting discussion of this point in Samuelson, Part I, pp. 293-298.
As in the no-impediment case, it is simplest to develop geometric criteria by considering the effects of transfer at constant prices.\textsuperscript{1} If at constant prices demand conditions are satisfied after the appropriate changes in resources used as transport costs are considered, then no change in the terms of trade will be necessary; this, then, provides the critical value of the criteria. It will be possible to show that the introduction of transport costs does not necessitate any modification in the fundamental methods used in developing geometric criteria.

Figure V illustrates the transfer problem when transport costs are incurred in the good of the exporting country. \( Q_a \) and \( Q_b \) are the initial equilibrium trading positions of A and B on their respective offer curves. The pre-transfer price ratios in A and B are \( a \) and \( b \), and \( e \) represents the f.o.b. terms of trade.

Now suppose that B makes a transfer to A equal in value to \( ON \) of B's good c.i.f. The costs of delivering \( ON \) of \( Y \) to A are \( NM \), so B must export \( OM \). After the transfer is made, the new offer curves of A and B start at their new endowment positions, \( N \) and \( M \), respectively.

\textsuperscript{1}Professor Samuelson writes: "Our graphical analysis . . . fails to handle the real transport case, because then the final consumption points for the two countries do not coincide, instead differing by a vector representing the amount of goods actually used up in transport" (Part II, p. 282). This section shows that the problem is nevertheless amenable to purely graphical analysis; the methods employed here were suggested by Professor Johnson.
To find the direction in which the terms of trade change, consider the change in demand at constant prices. Draw a' from A's new endowment position, N, parallel to a; and draw b' from B's new endowment position, M, parallel to b. Consumers in A, at constant prices after transfer, would move up a' to the point where a' is tangent to an A indifference curve; this point is Q_a'. Consumers in B, at constant prices after transfer, would move up b' to the point where b' is tangent to a B indifference curve; this is Q_b'. Draw Q_a'Q_a' and extend it to the X-axis at E_a; and draw Q_b'Q_b' and extend it to the Y-axis at E_b. These are the expenditure lines of A and B at constant domestic prices and show the changes in demand for X and Y, as a result of transfer. But Q_a'Q_a' and Q_b'Q_b' do not include the changes in quantities of X and Y required for transport costs. To show the total changes in quantities demanded, we must modify the expenditure lines to allow for transport costs.

Now at E_a, where no Y is imported, the transport costs of Y imports are zero; at Q_a transport costs are P_Q_a; and at Q_a' transport costs are Q_a'P_a'. This new line, P_P_a', is the transport-modified expenditure line and shows the total effect of the change in A's demand for X and Y at constant prices. Similarly, P_P_b' is B's transport-modified expenditure line at constant prices. Now draw e' parallel to e from M; clearly the points P_b' and P_a' must lie along this line since the horizontal distance between b' and e' measures the costs of transporting X and the vertical distance between a' and e' measures the costs of transporting Y.
If the pre-transfer price ratios are to remain unchanged, the horizontal distance between Q_a' and Q_b' must be exactly equal to the amount of X used up as transport costs, and the vertical distance must exactly equal the Y used up as transport costs -- otherwise Q_a' and Q_b' are not equilibrium positions and the terms of trade would have to change. For the terms of trade to remain unchanged, then, P_a' must coincide with P_b'; only then will Q_b' and Q_a' differ by the amount of transport costs incurred. Clearly, P_a' can only coincide with P_b' when PP_a' and PP_b' are the same line.

The geometric criterion is thus established. If PP_a' is flatter than PP_b', the terms of trade of B, the paying country, will deteriorate; if PP_a' coincides with PP_b', the terms of trade will not change; and if PP_a' is steeper than PP_b', B's terms of trade will improve.¹ Figure V

¹This is the geometric form of Professor Samuelson's criterion, as can be shown as follows. Our criterion is whether the slope of

\[
PP_a' \geq \text{slope of } PP_b'.
\]

The slope of PP_b' is (in Fig. V)

\[
\frac{E_bF}{FP} = \frac{E_bF}{FQ_b} \cdot K_x.
\]

But \( \frac{E_bF}{FQ_b} \) is the slope of \( Q_bQ_b' \), which is the ratio of B's marginal propensity to consume to her marginal propensity to import, \( \frac{c_b}{m_b} \), so the slope of

PP_b' is \( K_x \cdot \frac{c_b}{m_b} \).

Similarly the slope of PP_a' is \( \frac{1}{K_y} \cdot \frac{m_a}{c_a} \). Therefore the terms of trade of B (the paying country) will improve or worsen as

\[
\frac{1}{K_y} \cdot \frac{m_a}{c_a} \geq K_x \cdot \frac{c_b}{m_b}, \quad \text{i.e., as } \quad m_a m_b \geq K_x K_y c_a c_b.
\]

It is easy to see that, since \( K_x \) and \( K_y \) are less than unity, the effect of transport costs in X, the good of the exporting country, is to lower the right hand argument, increasing the probability that the terms of trade of B, the paying country, will improve. On the other hand, the higher price of imports will have changed marginal import propensities, so no definite conclusions are possible.
illustrates the case where B's terms of trade will improve. When PP_a' is steeper than PP_b', it means that the ratio of A's increase in demand for B's good to A's increase in demand for her own good is greater than the ratio of B's decrease in demand for her own good to her decrease in demand for A's good, after transport costs have been allowed for. Because P_a' is north of P_b', there is an excess demand for Y, and because Q_a' is east of Q_b', there is an excess supply of X. Therefore the price of Y must rise and the price of X must fall -- i.e., B's terms of trade must improve in order to restore equilibrium.

PP_a' is steeper than Q_a'Q_a' and PP_b' is flatter than Q_b'Q_b', so transport costs incurred in the exporting good unambiguously tend to make PP_a' steeper than PP_b', i.e., to increase the probability that the terms of trade move in favor of the paying country, the direction contrary to the orthodox presumption. This is the direct influence of transport costs when the exporting country's good is used. Against this must be set the indirect effect; transport costs raise the price of imports and so alter the marginal propensity to import, although the direction of change cannot be determined a priori.

When transport costs are incurred in both goods, the likelihood of a meaningful "presumption" is further reduced. In Figure VI we assume that some of both X and Y is used up in transporting X; but to keep the geometry simple we assume that costs of transporting Y are zero. The initial equilibrium is the same as that of Figure III. In Figure VI, A and B are initially in equilibrium at Q_a and Q_b; a and b
are the domestic price ratios but \( a \) is also the f.o.b. terms of trade
and \( b \) is also the c.i.f. terms of trade, since there are no costs of
transporting \( Y \).

\( B \) makes a transfer of OM of \( Y \), all of which reaches \( A \), since
there are no costs of transporting \( Y \). At unchanged prices \( A \) moves
up its price line \( a' \) to \( Q_a' \), where it is tangent to an \( A \) indifference
curve; \( B \) moves up its price line \( b' \) to \( Q_b' \). Draw \( Q_aQ_a' \) and \( Q_bQ_b' \) and
extend them to the \( S \) and \( Y \) axes.

But now when we draw the transport-modified expenditure lines
we find that as \( A \) imports more \( Y \), no transport costs must be paid;
but that as \( B \) imports less \( X \), there is a decrease in demand for both
\( X \) and \( Y \) used as transport costs. In Figure VI the transport-modified
expenditure line of \( A \) is the same as the normal expenditure line,
\( Q_aQ_a' \); but the transport-modified expenditure line of \( B \) is \( Q_bP_b' \),
which may be steeper or flatter than \( Q_bQ_b' \), depending on the propor-
tions in which \( X \) and \( Y \) are used in transporting \( X \). The geometric
criterion\(^1\) is the same as before (whether \( Q_aQ_a' \) is steeper or less

\(^1\) This can be converted into an algebraic criterion in the following
way. Our general geometric criterion is whether the slope of \( PP_a' \) is
the slope of \( PP_b' \). With zero costs of transporting \( Y \), this is equi-
valent to whether the slope of \( QQ_a' \left( \frac{M_a}{C_a} \right) \) is the slope of \( QQ_b' \).

Now the slope of \( Q_aP_b' = \frac{\varepsilon_k G}{\varepsilon_k Q_a} = \frac{\varepsilon_k G}{\varepsilon_k R} \cdot \varepsilon_k = \kappa_x \left( \frac{\varepsilon_k F}{\varepsilon_k R} + \frac{\varepsilon_k G}{\varepsilon_k R} \right) \).

But \( \frac{\varepsilon_k F}{\varepsilon_k R} \) is the slope of \( Q_bQ_b' \) or \( \frac{C_b}{m_b} \), and \( \frac{\varepsilon_k G}{\varepsilon_k R} \)\(( = \frac{k_H}{OH}) \) is the amount
of \( Y \) required to transport a unit of \( X \), \( a_x \). Then the slope of
\( PP_b' = \kappa_x \left( \frac{C_b}{m_b} + a_x \right) \) so our criterion becomes whether
\( \frac{M_a}{C_a} \kappa_x \left( \frac{C_b}{m_b} + a_x \right) \).
steep than $Q_aP_b$) but now nothing at all can be said about the direction of bias to the criterion unless more specific assumptions are made about the proportions in which $X$ and $Y$ are combined as transport costs.

There is also doubt about the direction in which the higher price of imports influences marginal propensities to import, although if what Professor Samuelson calls the "Basic Convention" is subscribed to -- that is, if it is assumed that the marginal propensity to import is more likely to equal the average propensity to import than any other value -- then higher import prices must reduce marginal import propensities.

In summary, nothing a priori can be said about the effect of transport costs on criteria for changes in the terms of trade. Even

(Footnote 1 continued)

The proportion $K_x$ reduces, and the proportion $a_x$ raises, the right hand argument, so these effects make it impossible to determine the bias given to the criterion.

When transport costs of both $X$ and $Y$ are involved, the terms of trade of the paying country, $B$, will improve, stay the same or deteriorate as

$$\frac{1}{k_xk_y} \geq \left( \frac{c_b}{n_b} + a_x \right) \left( \frac{c_a}{n_a} + a_y \right)$$

which includes Professor Samuelson's criterion as a special case when $a_x$ and $a_y$ are zero.
if one granted the "symmetrical ignorance" argument (the present writer does not), and could thus establish a "presumption" that higher import prices reduce marginal propensities to import, favoring the orthodox result, empirical work would be inescapable in deciding which country's resources are used up as costs of transport.

Up to this time we have considered the criterion for the direction of change in the terms of trade when real impediments to trade exist. Now, as in the case of zero impediments, we can consider the more formal and complete criterion.

In Figure VII we assume for simplicity that there are no costs of transporting Y and the costs of transporting X are incurred in X itself. Equilibrium before transfer is at $Q_a$ and $Q_b$ for A and B, respectively, $Q_b Q_a$ being the amount of X used up as transport costs. The f.o.b. terms of trade are the same as the domestic price ratio in A; the c.i.f. terms of trade are the same as the domestic price ratio in B.

---

1The "probability" that the marginal propensity to import equals the average propensity to import is the same as for any other value. Professor Viner's "arrow" analogy is irrelevant, since there are no forces known a priori directing the values of the marginal propensity to import as the judgment of the archer directs the destination of the arrow. See Viner, Studies in the Theory of International Trade (New York, 1937), pp. 324-325; and also Samuelson, Part I, loc. cit., pp. 296-298. But knowledge cannot derive from ignorance; and empirical data, or even its presumption, cannot arise from Kantian synthetic a priori judgments. In any specific case of transfer, empirical data could provide a better judgment of import propensities than the "Basic Convention" anyway. In the analysis of the transfer problem it is not possible to discuss a "presumption" as to the direction of changes in the terms of trade, but only to indicate the presumption as to the direction of bias given by real and artificial impediments.
Suppose that B makes a transfer to A equal to OM in terms of B's good. The new offer curves originating from M then determine the new equilibrium at G and T. The formula for the change in the terms of trade can then be derived as follows.

The marginal propensity to spend on imports in A \((\Pi_A)\) is \(\frac{DE}{GB}\) which is the same as the marginal physical propensity to import in A because there are no costs of transporting Y. But in B, \(\Pi_B = \frac{DC + JH}{GB}\) which is smaller than the marginal physical propensity to import in B, because AG(=CB) must be paid as transport costs. The marginal physical propensity to import in B is

\[
\Pi_B = \frac{DE}{GB} = \frac{DC + JH}{GB} \cdot \frac{GB}{GB} = \frac{\Pi_B}{K_x} \quad \text{i.e.,} \quad \Pi_B = \Pi_B \cdot K_x.
\]

Now the excess demand for A's good is \(1 - \Pi_A - \Pi_B\)

\[
= 1 - \frac{DE}{GB} - \frac{DC + JH}{GB} = \frac{HF}{GB}
\]

Define the following terms:

\[
\xi_A = \frac{EG}{GA} / \frac{RS}{AR}
\]

\[
\xi_B = \frac{HG}{GA} / \frac{RS}{AR}
\]

\[
\eta = \frac{OS}{SA}
\]

\[
\epsilon = \frac{RS}{SA}
\]

Then, \(\frac{\epsilon}{\eta} = \frac{RS}{OS} = \frac{1}{OR} = \frac{1}{RA} \quad \text{or} \quad \frac{RA}{RS} - 1 = \frac{GB}{GA} \cdot \frac{AR}{RS} - 1
\[
\frac{H_F}{GB} - \frac{H_F}{GA} \frac{AR}{RS} - \frac{H_F}{GB} = \frac{1 - T A - T B}{\varepsilon_{AB} + T A B - 1}
\]

\[
\frac{1 - T A - T B \cdot K_x}{\varepsilon_{AB} + T A + T B \cdot K_x - 1} = \frac{2}{B}
\]

Φ. E.D.

The extension to cases where there are costs of transporting both X and Y can be made, but the results fail to justify the added complexity of the geometry.
Tariffs and the Transfer Problem

We shall now consider the case of artificial impediments in the form of tariffs, and the direction in which tariffs tend to move the terms of trade following transfer. But, as in the section on transport costs, we shall first find it useful to elucidate somewhat more clearly the geometric determination of tariff equilibrium.

In the first place, some assumption must be made concerning the way in which tariff proceeds are spent. It could be assumed that the government spends the tariff proceeds entirely on home goods, entirely on imports, or divides its spending in some proportion between home goods and imports. This is the assumption Lerner\(^1\) made in his important article on export and import taxes; equilibrium is determined by rotating the pencil describing the difference in price ratios between the two countries until the government is just willing to consume the difference between private consumption of consumers in the tariff-imposing country and the exports of the other country. This technique has been well described in the literature and need not be further pursued here.

A more interesting assumption, perhaps, is that tariff proceeds are redistributed to consumers, and that consumers spend their increased income according to their taste patterns. By this method the community indifference map describes demand conditions without the necessity of further arbitrary assumptions as in the Lerner case. This assumption

\(^1\) Lerner, Abba P., op. cit.
has been used and described to some extent by Metzler,¹ Johnson² and Meade,³ and it is the assumption we shall make here. It should be noted, however, that the method used here in the analysis of the transfer problem can be applied also to the Lerner assumptions.

In Figure VIII, $Q_a$ and $Q_b$ are the offer curves of A and B; free trade equilibrium is at P. Suppose now that A's government wants to effect a new equilibrium point on B's offer curve at the point $P'$. In order that A's consumers be satisfied with this equilibrium, A must change the price ratio in A to equal the slope of the A-indifference curve through $P'$; this can be done by a tariff imposed on A's imports of an ad valorem amount equal to $\frac{AO}{OB}$.

A tariff of $\frac{AO}{OB}$ will improve A's terms of trade, which in this case are equal to B's domestic price ratio, from OP to OP'. It is clear that A's indifference curve through P must be lower than that through $P'$, so A's welfare has increased.

Consumption on A's offer curve is at $Q_a$; but tariff proceeds equal to AO in terms of X are redistributed to A's consumers, increasing their income line from QQ_a to AP'. With this increased income they move from $Q_a$ to $P'$, buying an additional $JQ_a$ of domestic goods and $JP'$ of imports. A's marginal propensity to consume X is therefore $\frac{JQ_a}{LQ_a}$, and her marginal propensity to import is $\frac{JL}{LQ_a}$.

With the tariff of $\frac{AO}{OB}$ imposed by A, A's tariff-modified offer curve must intersect $Q_b$ at $P'$. This tariff-modified offer curve is derived in Figure IX.


³Meade, Geometry, loc. cit., Chapter VI.
Suppose in Figure IX that the rate of tariff is \( \frac{AO}{OB} \), then a point on A's tariff-modified offer curve can be found by drawing a perpendicular to the X-axis at B and extending it upwards until the tangent to the indifference curve on that perpendicular extends to A. Such a point is P', which corresponds to P' in Figure VIII. In a similar way all the points on \( O_a' \) can be determined. Draw the perpendicular from any point B' on the X-axis in the trading quadrant and extend it upwards until it crosses the indifference curve tangent to a line from A', where the ratio \( OA'/OB' \) equals \( OA/OB \). Such a point is G. Similarly, the point F and all other points on \( O_a' \) can be determined.

Suppose now that both A and B impose tariffs, such that the tariff-modified offer curves (not drawn) intersect at P' in Figure X. The tariffs to which this equilibrium refer are determined by the tangent to the indifference curves of A and B through P'; the tangent to \( I_a \) meets the X-axis at K and the tangent to \( I_b \) meets the Y-axis at L. A's tariff is therefore \( \frac{KO}{ON} \left( \frac{OH}{HN} \right) \) and B's tariff is \( \frac{LO}{ON} \left( \frac{OJ}{JK} \right) \). The equilibrium on A's free-trade offer curve is at \( Q_a \), but this income is supplemented by the redistribution by A's government of tariff proceeds amounting to OK, so A's consumers consume at P'. Similarly, B's consumers originally consume at \( Q_b \), but their income is supplemented by tariff proceeds amounting to OL. The terms of trade are given by the line OP'.

We shall now use this method of representing tariff equilibrium to determine the criteria for changes in the terms of trade following
Figure 1
transfer when tariffs exist. In Figure XI\(^1\) equilibrium is initially at \(P\) where \(A\) has a tariff equal to \(\frac{10}{ON}\) and \(B\) has a tariff of \(\frac{8}{ON}\), the terms of trade are \(OP\). Country \(B\) now makes a transfer to country \(A\) equal to \(OT\) in terms of \(A\)'s good or \(OT'\) in terms of \(B\)'s good. To derive the criterion for changes in the terms of trade it is simplest to consider the changes in demand at constant prices. Draw a line \(TT'\) from \(T\) parallel to \(OP\). Also draw the expenditure lines of \(A\) and \(B\), \(E_aE_a\) and \(E_bE_b\), intersecting \(TT'\) extended at \(R\) and \(S\), respectively.

These expenditure lines show the changes in demand in \(A\) and \(B\) at constant prices. If at constant prices (tariff-inclusive prices), there is an excess demand for either commodity after transfer, the price of that commodity must rise relative to the price of the other commodity. But it is clear from Figure XI that there is zero excess demand for each commodity only when \(R\) coincides with \(S\): if \(R\) is above \(S\), that is, if the proportion of \(A\)'s marginal propensity to import to her marginal propensity to consume is greater than the proportion of \(B\)'s marginal propensity to consume to her marginal propensity to import, then \(E_aE_a\) is steeper than \(E_bE_b\), so at constant prices there is an excess demand for \(B\)'s good and \(B\)'s terms of trade must improve. Similarly, if \(E_aE_a\) is less steep than \(E_bE_b\), then \(B\)'s terms of trade must deteriorate. \(B\)'s terms of trade will improve, stay the same or worsen, then, depending on whether the slope of \(E_aE_a\) is greater than, equal to or less than the

\(^1\)I am indebted to Professor Johnson for this method of illustrating the effects of tariffs on the terms of trade. For an alternative geometric argument see Samuelson, Part II.
Figure XI
slope of $E_bE_b$. But the slope of $E_aE_a$ is $\frac{\Pi_a}{C_a}$ and the slope of $E_bE_b$ is $\frac{C_b}{m_b}$ so the criterion becomes whether $\Pi_a \geq \frac{C_a}{C_b}$, each term representing marginal physical propensities.

We must now show the direction of bias given to the criterion by the assumption that trade is to some extent impeded by tariffs. To do this it is most convenient to translate the marginal physical propensities to import into marginal propensities to spend on imports. It turns out that, as of any given marginal propensities to spend on imports, tariffs create a presumption that the terms of trade will change in the orthodox direction, against the transferring country.

Consider again Figure XII. The marginal physical propensities to import of A and B are, respectively, $\frac{RB}{RW}$ and $\frac{GF}{SF}$ -- i.e., $\frac{RB}{OT}$ and $\frac{GF}{OT'}$.

At constant prices, the excess demand for A's good is $1 - \frac{RB}{OT} - \frac{FG}{OT'} = \frac{RB + SG}{OT'}$.

Now A's marginal propensity to spend on imports is $\frac{RB}{RU}$ and B's marginal propensity to spend on imports is $\frac{SP}{DP}$, where the tangents to the indifference curves at R and S are parallel to LP and KF, respectively.

The excess demand for A's good can be written

$$1 - \frac{RB}{RU} \cdot \frac{RV}{RW} = \frac{SP}{DP} \cdot \frac{DP}{CP} = 1 - \frac{TT_A \cdot RU}{RD} - \frac{TT_B \cdot DP}{CP}.$$ 

But when the transfer is very small, then $\frac{RU}{RW} \rightarrow \frac{BV}{BW} = \frac{NH}{ON} = \frac{OM}{LM}$

$$\frac{OM}{LM} = \frac{1}{1 + t_A} \equiv \chi_A.$$
i.e., the proportion of the untaxed price of B's import to the tax-
inclusive price.

It will be instructive, now, to consider the full criterion for changes in the terms of trade when artificial impediments exist. In Figure XII equilibrium is initially at $P$ on $B$'s offer curve; but $A$ has a tariff equal to $\frac{OB}{OC}$. Now suppose that a transfer of $CT$ in terms of $A$'s good is made from $B$ to $A$. The new equilibrium moves to $G$; a $B$ trade indifference curve is tangent to $TG$ at $G$, and an $A$ trade indifference curve is tangent to $LG$ where $LG$ is drawn in such a way that $\frac{LT}{TA}$ is equal to the ad valorem rate of tariff.

Now draw through $G$ the lines $a$ and $b$ parallel to the original price lines in $A$ and $B$ and mark off the points at which they are tangent to their respective indifference curves. Such points are $Q_a$ and $Q_b$ which allow us to determine the elasticities as used in the previous chapters and the sum of the marginal propensities to import.

Define the following terms, with notation as in the previous chapters.

$$\ell = \frac{OT}{TA}$$
$$\ell_A = \frac{EF}{GA} / \frac{RT}{AR}$$
$$\ell_B = \frac{GH}{GA} / \frac{RT}{AR}$$

$$\Pi_A = \frac{EK}{GD}$$
$$\Pi_B = \frac{KC - HG}{GC}$$

$$\chi_A = \frac{GD}{GC}$$

Then, $\frac{\ell}{\ell} = \frac{RT}{OT} = \frac{1}{OR / RT - 1} = \frac{1}{\frac{OR}{AR} / \frac{RT}{AR} - 1} = \frac{1}{\frac{GC}{GA} \cdot \frac{AR}{RT} - 1}$
\[
\begin{align*}
\frac{HF}{GC} &= \frac{GC - FK - KC + HG}{GC} \\
\frac{HF \cdot AR}{6A \cdot RT} - \frac{HF}{GC} &= \frac{HF \cdot AR}{6A \cdot RT} - \frac{HF}{GC} \\
1 - \frac{TT_B}{\frac{6F}{6A} + \frac{6H}{6A}} &\quad \frac{AR}{RT} + \frac{TT_B}{\frac{6A}{6A}} + \frac{TT_A}{\frac{6A}{6A}} \chi_A - 1 \\
1 - \frac{TT_A}{TT_B} &\quad \frac{\chi_A}{E_{ab} + \frac{TT_A}{TT_B} - 1}
\end{align*}
\]

\[
= \frac{1 - TT_{ab}}{E_{ab} + TT_{ab} - 1}
\]
in terms of physical propensities.
II. TARIFFS, THE TERMS OF TRADE AND THE BALANCE OF PAYMENTS MECHANISMS
The last chapter completed the discussion of the transfer problem and criteria for changes in the terms of trade under several assumptions. The assumption implicit in the last chapter was that the government in country \( B \) taxes \( B \)'s consumers by the amount of the transfer and turns the proceeds of this tax over to \( A \)'s government; the latter then redistributes this additional income to \( A \)'s consumers. Because savings are not introduced, the domestic expenditure in \( B \) decreases by the amount of the transfer and domestic expenditure in \( A \) increases by the amount of the transfer; thus \( B \) has an export surplus and \( A \) has an import surplus equal to the amount of the transfer so the latter is exactly effected.

In this chapter we shall consider a different set of assumptions. Instead of assuming that the transfer process has already been completed, we shall show various alternatives to the above mechanism -- alternatives by which a given transfer can be effected. In particular, we shall demonstrate the effects of devaluation, import and export taxes, and subsidies on the balance of trade. It will first be desirable, however, to expand somewhat on the relationship between prices, the exchange rate and the balance of trade.

Prices, exchange rates and the balance of trade can be considered alternative equilibrating variables. In a stable market there is only

\(^1\)For a recent discussion of the transfer problem under both classical and Keynesian assumptions, see Johnson, H. G., "The Transfer Problem and Exchange Stability," loc. cit.
one exchange rate which will satisfy equilibrium if the balance of trade and domestic prices are given; if prices and exchange rates are given, the balance of trade must adjust in order to make that a consistent equilibrium; if the exchange rate and the balance of trade are given, domestic prices must adjust.

Similarly, if some outside variables are introduced, such as tariffs, any two of the above variables can be fixed and equilibrating adjustment must take place in the third. It is easily seen that the case of tariffs improving the terms of trade is a case where the balance of payments has been determined by some other factor and adjustment is allowed to take place through changes in domestic prices or through changes in the exchange rate. Similarly, tariffs operate on the balance of trade if exchange rates and domestic prices are fixed.

In the following analysis we assume that "internal stability" is maintained by the monetary authorities in each country. By internal stability we mean that each country's monetary authorities, by appropriate policies stabilize the factor cost prices of their export goods. This means that when relative prices (the terms of trade) are allowed to vary, it is always through variations in the exchange rate. The analysis could as easily be carried out in terms of variations in domestic prices with fixed exchange rates, but in terms of modern governmental commitments, the former policy, if not exactly realistic, seems more in the spirit of at least the desired norm of public policy. Assuming the savings-investment relationship is sufficient to ensure full
employment, perhaps the next most desired aim of government economic policy in the postwar world is to prevent either inflation or deflation. The peculiarity of our assumption is that internal stability is defined only with reference to the price of export goods.

Tariffs and the Terms of Trade

It is perhaps strange that, in all the literature on tariffs and the terms of trade, so little attention has been given to the degree to which the terms of trade change following a tariff. Before we consider the relationship between the tariffs and the balance of trade, we shall assume that the barter terms of trade are allowed to vary, while zero balance of trade is maintained. This assumption is easily justified if we assume a flexible exchange rate with domestic prices fixed by monetary policy, or a fixed exchange rate with flexible domestic prices or some adjustment in both domestic prices and the exchange rate.

In Figure I, F is the free-trade equilibrium, determined by the intersection of the offer curves $Q_a$ and $Q_b$. Now assume that country A imposes a tariff equal to $\frac{OB}{QM}$, moving equilibrium along B's offer curve to T, where a line ET is tangent to an A indifference curve at T. The price ratio in B which is also equal to the terms of trade is now CT.

Draw a line CT parallel to OP through T and determine the points $Q_a$ and $Q_b$, where the expenditure lines of A and B cross CT extended. Since this line represents the pre-tariff terms of trade, the propor-
tional change in the terms of trade \( (v) \) is \( \frac{OC}{TM} / \frac{CM}{TM} = \frac{OC}{CM} \).

Define the following terms:

\[
E_A = \frac{ST}{TM} \frac{BC}{CM}, \quad E_B = \frac{RT}{TM} \frac{OC}{CM}
\]

\[
t = \frac{OB}{OM}, \quad V = \frac{OC}{CM}
\]

\[
TT_A = \frac{SP}{TU}, \quad TT_B = \frac{TU - RP}{TU}
\]

so \( 1 - TT_{ab} = \frac{TU - SP - TU + RP}{TU} = \frac{RS}{TU} \)

Then, \( E_A \cdot \frac{OB}{CM} = E_A \cdot \frac{BC + OC}{CM} = \frac{ST}{TM} \cdot \frac{CM}{BC} \cdot \frac{BC}{CM} \left( \frac{BC + OC}{BC} \right) \)

\[= \frac{ST}{TM} \left( 1 + \frac{OC}{BC} \right) = \frac{RT}{TM} - \frac{RS}{TM} + \frac{OC}{TM} \cdot \frac{ST}{BC} \]

\[= \frac{OC}{CM} \left( \frac{RT}{TM} \cdot \frac{CM}{OC} \right) + \frac{OC}{CM} \left( \frac{ST}{TM} \cdot \frac{CM}{BC} - \frac{RS}{TU} \right) \]

\[= \frac{OC}{CM} \left( \frac{RT}{TM} \cdot \frac{CM}{OC} + \frac{ST}{TM} \cdot \frac{CM}{BC} - \frac{RS}{TU} \right) \]

\[= V \left( E_{ab} + TT_{ab} - 1 \right) \]
But \( \frac{\partial \log C}{\partial \log M} \rightarrow \frac{\partial \log M}{\partial \log O} \) if the tariff is smaller

i.e., it approaches \( \xi_A \cdot \xi \) so

\[
\nu (\xi_A + T\xi - 1) = \xi_A \cdot \xi
\]

\[
\frac{\nu}{\xi} = \frac{\xi_A}{\xi_A + T\xi - 1}
\]

i.e., the change in the terms of trade caused by a tariff in A is equal to A's expenditure-compensated elasticity of demand divided by the sum of the price elasticities reduced by unity.

This result is an alternative way of showing that a tariff must improve the terms of trade in the stable case, and it permits but one exception. When B's offer curve is infinitely elastic, A cannot change the terms of trade; this conclusion is implicit in the formula, since in that case \( e_b \) is infinite, so the criterion is equal to zero. Another border case is when A's offer curve is infinitely elastic; in this case both the numerator and denominator are infinite, so the terms of trade must improve by the amount of the tariff.

In a similar way it could be shown geometrically that when both A and B impose tariffs, A's terms of trade will improve or worsen, depending on whether

\[
\nu = \frac{t_a \cdot \xi_a - t_b \cdot \xi_b}{\xi_A + T\xi_A - 1}
\]

is positive or negative. They will remain unchanged only when the expression is zero -- i.e., whenever \( \frac{t_a}{t_b} = \frac{\xi_b}{\xi_a} \) This case will
be of considerable use later in our analysis of tariff reductions.

Mathematical Analysis

We shall check this result algebraically. Differentiate the

Balance of Trade equation

\[ B = I_a - I_a \cdot V \]

with respect to \( t \) to get

\[ 0 = \frac{dI_b}{dt} - I_a \cdot \frac{dv}{dt} - V \cdot \frac{dI_a}{dt} \]

Now \( I_a = I_a (D_a, v, t) \) and \( I_b = I_b (D_b, v) \)

So

\[ \frac{dI_a}{dt} = \Pi_a \cdot \frac{dD_a}{dt} + \frac{2I_a}{a(vt)} \left( 1 + \frac{dv}{dt} \right) \]

and

\[ \frac{dI_b}{dt} = \frac{2I_b}{2v} \cdot \frac{dv}{dt} \]

But since tariff proceeds are redistributed, \( \frac{dD_a}{dt} = I_a \).

Then

\[ 0 = \frac{dI_b}{dv} \cdot \frac{a}{I_b} \cdot I_b \cdot \frac{dv}{dt} - I_a \cdot \frac{dv}{dt} - \Pi_a \cdot I_a - \frac{2I_a}{a(vt)} \cdot \frac{v \cdot t}{I_a} \cdot I_a + \]

\[ - \frac{2I_a}{a(vt)} \cdot \frac{v \cdot t}{I_a} \cdot I_a \cdot \frac{dv}{dt} \]

Forming elasticities, and writing \( I_b = I_a = I \),

\[ 0 = \epsilon_b \cdot I \cdot \frac{dv}{dt} - I \cdot \frac{dv}{dt} + \epsilon_a \cdot I \cdot \frac{dv}{dt} - \Pi_a \cdot I + \epsilon_a \cdot I \]
\[ \therefore \frac{dv}{dt} (\varepsilon_a' + \varepsilon_b' - 1) = -(\varepsilon_a' - TT_a) \]

i.e.
\[ \frac{dv}{dt} = \frac{-\varepsilon_a' - TT_a}{\varepsilon_a' + \varepsilon_b' - 1} = -\frac{\varepsilon_a}{\varepsilon_a + TT_a - 1} \]

i.e.
\[ \frac{dv}{dt} = -\frac{\varepsilon_a}{\varepsilon_a + TT_a - 1} \]

which is negative because \( v \) is the reciprocal of \( A \)'s terms of trade.
Tariffs and the Balance of Trade

Our next consideration is the effect of tariffs on the balance of trade; in order that tariffs may have any effect on the balance of trade, some restriction must be made as to the degree of price flexibility, since perfect flexibility, as in the previous section, allows adjustment to take place through changes in the terms of trade rather than through changes in the balance of trade. To isolate the effects of tariffs on the balance of trade, we shall assume that export prices are fixed by monetary policy in each country and also that the exchange rate is fixed; it is immediately apparent that this is a limiting case where all adjustment takes place through the balance of trade and none through the terms of trade.

The problem can be posed in either of two ways: first, how large a tariff is required to effect a given transfer; second, how large a transfer will a given tariff effect? These problems are answered by the same criterion.

In Figure II, free-trade equilibrium is initially at $P$. We shall now determine the effect on the balance of trade of any given rate of tariff. Consider the point $S$ along A's expenditure line $E_aE_a$. Then the price ratio in $B$ that will make that an equilibrium is determined by the tangent to the $B$ indifference curve through that point. Let this price line cut the X-axis at $C$ and then draw the price ratio, $a'\!$, in $A$ the same as at $P$, $a'$ cutting the X-axis at $B$.

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1This section on tariffs and the balance of trade is based on a paper given by Dr. S. A. Ozga at the London School of Economics. See also Meade, J. E., *Balance of Payments Mathematical Supplement*, loc. cit., pp. 147-156. The argument below is an attempt to simplify, and to unify, the techniques in deriving these criteria as well as to present their economic significance.
The rate of tariff that will make S the trading point, i.e., make the new price ratio in B equal to \( b \), is then \( \frac{BH}{SM} = \frac{BC}{CM} \). Now define the following terms:

\[ \ell_B = \frac{RS}{SM} \left/ \frac{BC}{CM} \right. \], \( \Pi_B = \frac{ST}{SU} \), \( \Pi_B = \frac{SU - RT}{SU} \)

\[ \ell = \frac{OB}{BM} \]
\[ t = \frac{BC}{CM} \]

Now \( t \cdot \ell_B = \frac{RS}{RM} \)

but \( \frac{RS}{BM} = \frac{RS}{SU} \cdot \frac{SU}{SM} = \frac{SU - ST - SU + RT}{SU} \cdot \frac{OB}{BM} \)

\[ = \ell \left( 1 - \Pi_B \right) \]

\[ \therefore \ell = t \cdot \frac{\ell_A}{1 - \Pi_B} \]
\[ t = \ell \left( 1 - \Pi_B \right) / \ell_A \]

i.e., B's balance of trade will improve or worsen, depending on whether the sum of the marginal propensities to import is less than or greater than unity, following a tariff imposed by B, since \( e_B \) must be positive.

It should be clear first that the tariff affects only the prices of the tariff-imposing country, B, and the price of B's imports rises by the amount of the tariff; the terms of trade are unchanged. There is, therefore, a substitution effect in B, whose domestic price ratio has changed, but not in A, whose domestic price ratio remains unchanged.
The economic significance of the criterion can be explained in the following manner. When B imposes a tariff on A's exports, the price of X rises in B, decreasing the quantity of X imported as home goods are substituted for imports; this tends to depress the price of X in A and to increase the price of Y in B. To prevent deflation, A's monetary authorities increase domestic expenditure; and to prevent inflation, B's monetary authorities decrease domestic expenditure. Now suppose the sum of the marginal propensities to import is less than unity; then the effect of inflation in A and deflation in B is to improve B's balance of trade and to shift expenditure from B's to A's good, restoring internal stability in A and B with an export surplus for B. But if the sum of the marginal propensities to import is greater than unity, inflation in A and deflation in B shifts expenditure from A's onto B's good; the effect of the inflation of domestic expenditure in A and deflation in B is to aggravate the deflation in A and the inflation in B. Hence, internal stability in each country can be restored only by a reversal of expenditure policies in which A's monetary authorities deflate, and B's monetary authorities inflate, domestic expenditures.

Consider a special case where \( e_b \) is zero; then no change in expenditure is needed in either country, since no substitution takes place, so the change in the balance of trade is zero. This case would involve indifference curves in B that are kinked at the point of equilibrium -- the angle between the kinks being greater than the tariff.
Now consider another case where $e_b$ is positive but the sum of the marginal propensities to import is unity. This means that, when A inflates and B deflates domestic expenditure, the increase in demand for both commodities in A is exactly offset by the decrease in demand for both commodities in B, so no shift of world demand is possible by changes in domestic expenditure. A must keep inflating and B must keep deflating until the balance of trade increases an "infinite" amount.

When the marginal propensities to import in each country are zero, then, all of A's increased expenditure falls on its home good, and all of B's decreased expenditure is a reduction of spending on its home good, so the change in expenditure and hence the change in the balance of trade are exactly equal to the product of the rate of tariff and $e_b$. In this case the proportional change in the quantity of $Y$ demanded by B as a result of the tariff (the proportional change in the price of $Y$) is exactly the same as the change in the balance of trade (as a proportion of the new value of imports).

The same result holds if the sum of the marginal propensities to import is zero, but where one of the goods is inferior.

One final case is worth mentioning -- suppose \[ \frac{e_B}{1 - T_{av}} > 1 \]. This is the case where the increase in domestic expenditure in A and the decrease in domestic expenditure in B are just equal to the tariff proceeds raised in B. B's domestic expenditure, after tariff proceeds have been redistributed, is the same as it was before the tariff was imposed, increase in expenditure caused by the tariff proceeds being exactly compensated by the adverse balance of trade.
The denominator is a stability condition associated with the assumption regarding domestic expenditure and its adjustment to maintain a given level of export prices. When the sum of the marginal propensities to import is less than unity, the market under these assumptions is stable; but when it is greater than unity, the market is unstable, and a tariff worsens the balance of trade. The market is unstable at least if it is assumed that the monetary authorities in A continue to inflate and in B continue to deflate domestic expenditure to relieve an excess demand for B's good. If, however, the monetary authorities collectively guessed the correct situation, they could reverse their policies.

Mathematical Analysis

We shall derive the criterion algebraically for a tariff imposed by A's government.

Differentiate \( B = I_b - I_a \cdot e \) with respect to \( t \)

to get \( \frac{dB}{dt} = \frac{dI_b}{dt} - \frac{dI_a}{dt} \)

Now \( I_a = I_a \left( D_a, t, e \right) \) and \( I_b = I_b \left( D_b, e \right) \)

so \( \frac{dI_a}{dt} = \Pi_a \cdot \frac{dD_a}{dt} + \frac{2I_a}{2t} \) and \( \frac{dI_b}{dt} = \frac{dD_b}{dt} \cdot \Pi_b \).

Substituting, \( \frac{dB}{dt} = \Pi_b \cdot \frac{dD_b}{dt} - \Pi_a \cdot \frac{dD_a}{dt} - \Pi_a \cdot I_a + \frac{2I_a}{2t} \)

But \( \frac{dD_b}{dt} = \frac{dB}{dt} \)

\( \therefore \frac{dB}{dt} = \frac{I_a - \Pi_a}{1 - \Pi_{ab}} \) and \( \frac{dD_a}{dt} = -\frac{dB}{dt} + \frac{2D_a}{2t} = -\frac{dB}{dt} + I_a \)

\( \left[ \frac{dD_a}{dt} = I \equiv \text{tariff proceeds} \right] \)
Export Taxes and the Balance of Trade

Our next consideration will be with the effects of export taxes on the balance of trade. It cannot be assumed that export and import taxes have symmetrical effects, as in their effects on the terms of trade. With the tariff, the terms of trade cannot change under our assumptions -- the domestic price ratio rises by the full amount of the tariff. With the export tax, however, the domestic price ratio of the taxing country remains unchanged; but the price ratio in A must change. The price of B's exports must rise in A by the full amount of the export tax. The degree to which the terms of trade change, then, depends on the degree of substitution between X and Y in B, rather than A.

Consider Figure III; equilibrium is initially at P, but B imposes an export tax equal to $\frac{BC}{BM}$. What is the change in the balance of trade?

Clearly $\frac{OC}{BM}$, since the terms of trade now change by the full amount of the export tax. Define the following terms:

\[
1 - T_{ab} = -\frac{RS}{SO} \quad \epsilon_a = -\frac{RS}{SM} \left(\frac{BC}{CM}\right) \quad t_{eb} = \frac{BC}{BM} \quad \lambda = -\frac{OC}{BM}
\]

Then

\[
-\frac{\epsilon_a}{t_{eb}} = \frac{OC}{BC} = \frac{OB}{BC} + 1 = \frac{OB}{MB} \cdot \frac{MB}{MC} - 1 = \frac{SU}{SM} \cdot \frac{BM}{BC} + 1.
\]

\[
= \frac{RS}{SM} \cdot \frac{BM}{BC} + \frac{RS}{SU} = -\frac{\epsilon_a}{T_{ab}} + 1 = \frac{\epsilon_a + T_{ab} - 1}{T_{ab} - 1} = \frac{\epsilon_a}{1 - T_{ab}} - 1
\]

\[
\frac{\lambda}{Tea} = 1 - \frac{\epsilon_a}{1 - T_{ab}}
\]
i.e., an export tax imposed by $B$ will improve or worsen the balance of trade, depending on whether the expenditure-compensated elasticity of demand as a proportion of unity minus the sum of the marginal propensities to import is less than or greater than unity. The meaning of the formula can perhaps be best explained by considering a number of cases. First, if the criterion is zero, i.e., if $e_a$ is equal to unity minus the sum of the marginal propensities to import, this means that the change in spending on $Y$ in $A$ due to the higher price of $Y$ is exactly offset by the change in spending in $B$ on $Y$ due to the redistribution of the tariff proceeds of the export tax.

Suppose $1 - \Pi_a$ is negative; in this case $B$'s balance of trade must improve because the decrease in spending on $Y$ in $A$ is greater than the increase in spending on $Y$ in $B$ due to the redistributed tax proceeds ($\ell_a' > 1 - \Pi_B$). With no change in domestic expenditure, there is an excess demand for $X$ and an excess supply of $Y$. If monetary authorities in $B$ begin to inflate, and in $A$ to deflate, domestic expenditure, since $1 - \Pi_a < 0$, this would only aggravate the excess demand for $X$. Therefore, $A$ must inflate and $B$ must deflate domestic expenditure until the excess demand is eliminated, resulting in an improvement in $B$'s balance of trade.

Now consider the case where $1 - \Pi_a > 0$; then $B$'s balance of trade will improve or worsen, depending on whether, before inflation or deflation has been decided upon, the change in spending on $Y$ in $A$ ($e_a'$) is less than or greater than the change in spending on $Y$ in $B$, $1 - \Pi_B$.,
due to the tax proceeds. If $e_a'$ is greater than $1 - r_a$, then there
is an excess supply of $Y$, so to prevent deflation, $B$ inflates domestic
expenditure; and to prevent inflation in $A$, $A$'s monetary authorities
deflate domestic expenditure so $B$'s balance of trade worsens.

If, on the other hand, $1 - r_a > e_a'$, there is an excess demand
for $Y$, so $B$ must deflate and $A$ inflate to preserve the level of export
prices; in this case $B$'s balance of trade improves.

Mathematical Appendix

Assume that $A$ imposes an export tax, $t_{ea}$.

Differentiate the balance of trade equation

$$B = t_{ea} \cdot I_b - I_a \cdot e$$

with respect to $t_{ea}$.

$$\frac{dB}{dt_{ea}} = I_b + \frac{dI_b}{dt_{ea}} - \frac{dI_a}{dt_{ea}}.$$  

Now $I_b = I_b(D_b, t_{ea}, e)$, so

$$\frac{dI_b}{dt_{ea}} = \Pi_b \frac{dD_b}{dt_{ea}} + 2I_b \frac{dD_b}{d(t_{ea})} \frac{d(t_{ea})}{dt_{ea}},$$

$$\frac{dI_a}{dt_{ea}} = \Pi_a \frac{dD_a}{dt_{ea}}.$$  

But

$$\frac{dD_b}{dt_{ea}} = \frac{dB}{dt_{ea}}.$$  

and

$$\frac{dD_a}{dt_{ea}} = -\frac{dB}{2t_{ea}} + \frac{2D_a}{2t_{ea}} = -\frac{dB}{dt_{ea}} + I.$$
Substituting,

\[
\frac{dB}{dt_{ea}} = I_b + \pi_b \cdot \frac{dB}{dt_{ea}} + \frac{\partial I_b}{\partial \tau_{ea}} - \pi_a \frac{d\theta_a}{dt_{ea}} \\
= I_b + \pi_b \frac{dB}{dt_{ea}} - \varepsilon_b' \cdot I_b + \pi_a \frac{dB}{dt_{ea}} - \pi_a \cdot I_a
\]

\[
\frac{dB}{dt_{ea}} = \frac{-\varepsilon_a' - \pi_a + 1}{1 - \pi_a \cdot \varepsilon}
\]

\[
\frac{dB}{dt_{ea}} = 1 - \frac{\varepsilon_b}{1 - \pi_a \cdot \varepsilon}
\]

**Tariffs in Both A and B**

We shall now consider the case where both A and B impose tariffs -- not for pedantic completeness but because this result will prove to be of considerable importance in a later chapter. The analysis is quite simple and establishes the commonsense result that the change in the balance of trade due to tariffs in both A and B is the sum of changes due to A's tariff and B's tariff.

Figure IV illustrates this proposition. Equilibrium is initially at P under free trade with zero balance of trade. Assume that both A and B impose tariffs and that equilibrium moves to T. Draw a line from
T parallel to OP reaching the X-axis at B and then draw lines to the X-axis tangent to the A and B indifference curves at T.

Define the following terms:

\[ b = \frac{OB}{BM}, \quad t_a = \frac{BC}{BM}, \quad t_b = \frac{BD}{DM} \]

\[ \varepsilon_a = -\frac{ST}{TM} \cdot \frac{BC}{BM}, \quad \varepsilon_b = \frac{RT}{TM} \cdot \frac{BD}{DM} \]

\[ 1 - \Pi_{ab} = \frac{RS}{TW} \]

Then,

\[ \varepsilon_a \cdot t_a = \frac{ST}{TM} \quad \text{and} \quad \varepsilon_b \cdot t_b = \frac{RT}{TM} \]

\[ \therefore \varepsilon_a \cdot t_a - \varepsilon_b \cdot t_b = \frac{RS}{TM} \]

\[ B = \frac{RS}{TM} = \frac{RS}{TW} \cdot \frac{TW}{TM} = \frac{RS}{TW} \cdot \frac{OB}{BM} = \left(1 - \Pi_{ab}\right) \cdot B \]

\[ \therefore B \left(1 - \Pi_{ab}\right) = \varepsilon_a \cdot t_a - \varepsilon_b \cdot t_b \]

So

\[ B = \frac{t_a \cdot \varepsilon_a - t_b \cdot \varepsilon_b}{1 - \Pi_{ab}} \]

A proposition which will prove to be of considerable use later is that the tariff changes required to keep the balance of trade constant must meet the condition that

\[ t_a \varepsilon_A - t_b \varepsilon_B = 0 \]

i.e., that

\[ \frac{t_a}{t_b} = \frac{\varepsilon_B}{\varepsilon_A} \]
so tariffs must be reduced in proportion to each other as the reciprocal of the expenditure-compensated elasticities of demand.

Devaluation and the Balance of Trade

It will not be necessary to dwell extensively on the effects of devaluation on the balance of trade, since the result derived is the reciprocal of that derived for the transfer problem without impediments. It will be useful, however, to explain the meaning of the criterion in a different way than has been done previously.

In Figure V equilibrium is initially at P and the exchange rate is OP. Suppose that B appreciates the exchange rate by the amount $\frac{BE}{BM}$. What is the change in the balance of trade?

Define the following terms:

$$\varepsilon_a = \frac{RS}{SM} / \frac{BC}{CM}, \quad \varepsilon_b = \frac{ST}{SM} / \frac{BC}{CM}$$

$$1 - TTal = -\frac{RT}{SW}$$

$$\beta = \frac{OC}{BM}, \quad \epsilon = -\frac{BC}{BM}$$

Then,

$$\epsilon ( \varepsilon_a + \varepsilon_b ) = \frac{RS + ST}{SM} \frac{RT}{SM}$$

But

$$\frac{RT}{SM} = \frac{RT}{SW} \cdot \frac{SW}{SM} = \frac{RT}{SW} \cdot \frac{OB}{BM} = \frac{RT}{SW} \left( \frac{OC}{BM} - \frac{BC}{BM} \right) = \left( 1 - TTal \right) \left( \beta + \epsilon \right)$$

$$\epsilon \cdot \varepsilon_{al} = \left( 1 - TTal \right) \left( \beta + \epsilon \right)$$

So

$$\frac{\beta - \epsilon}{\epsilon} = \frac{\varepsilon_{al}}{1 - TTal} - 1 = \frac{\varepsilon_{al} + TTal - 1}{1 - TTal} = \frac{\varepsilon_a + \varepsilon_b'}{1 - TTal} - 1$$
The numerator is the familiar condition that the sum of the price elasticities reduced by unity must be positive for a depreciation of the exchange rate to improve the balance of trade. But it is clear from the criterion and from our diagram that an appreciation of the exchange rate improved B's balance of trade in spite of the fact that the sum of the price elasticities is greater than unity. In our criterion, which is the criterion used by Professor Meade extensively, the denominator being negative, caused the balance of trade to change adversely. What economic meaning can be given to this?

This question is answered by our assumption regarding monetary policy in A and B. It will be remembered that the assumption used is that monetary authorities stabilize export prices by inflation or deflation of domestic expenditure. Let us consider their action in the case illustrated in Figure V.

The appreciation, before monetary policy is undertaken, creates an excess demand for A's good and an excess supply of B's good. The initial tendency is therefore for monetary authorities in B to inflate and in A to deflate domestic expenditure. But this policy, instead of relieving the excess supply of B's good, aggravates it because \( 1 - \frac{1}{\tau_{AB}} \) is greater than zero. Therefore this policy must be reversed; B must deflate and A must inflate until the zero excess demand and supply are eliminated and this expenditure policy results in a worsening of A's trade balance.
Now consider the other form of the criterion, \[ \frac{d\theta}{d\epsilon} = \frac{\epsilon_{al}}{1 - \Pi_{al}} - 1. \]

This breaks up the elasticities into substitution and income effects. The substitution effects unequivocally act in one direction; they increase the demand for \( X \) in our example. The expression, \( 1 - \Pi_{al} \), illustrates the income effect; \( 1 - \Pi_{la} \) shows the increased demand for \( X \) in A and \( \Pi_{b} \) shows the increased demand for \( X \) in B. The difference between the change in demand for \( X \) in A and the change in demand for \( X \) in B describes the net income effect. When this net income effect is exactly equal to the sum of the substitution effects, a devaluation cannot change the terms of trade. This is the familiar case where the sum of the price elasticities is equal to unity.

If substitution effects are greater than income effects, then devaluation is successful; if substitution effects are less than income effects, it is unsuccessful.
III. OPTIMAL TARIFF REDUCTIONS
The following section is an essay on optimal tariff reductions; but it makes clear certain relationships between subsidies and tariffs by which the effects of transfer can be duplicated. It also presents income transfers as a preferable substitute for retaliation, under certain circumstances.

Considerable attention has been given recently to the Bickerdike proposition that tariffs improve the terms of trade, and, from an initial free trade equilibrium, increase welfare, except in the singular case where the foreign offer curve is infinitely elastic. This proposition has been revived by Kaldor and expanded by others; and formulae have been derived for the tariff which will maximize social welfare, the optimum tariff. ¹

The initial analyses were conducted under the assumption that the foreign country would not react to this optimum tariff policy


conducted by the home country. But Scitovsky¹ and later Johnson² have refined the proposition considering the effects on national welfares of retaliation. Johnson, in particular, has developed the theory as a problem in duopoly;³ the solution to the problem of retaliation he considers is arrival at a Cournot equilibrium.

In this section we shall refine still further this proposition and show the equilibrium under a variety of assumptions -- not only for its own sake but also as the prelude to the succeeding discussion of optimal tariff reductions.

Consider Figure Ia, where the trading quadrant is divided into six sections,⁴ equilibrium in any of which results from different commercial policies in A and B. The area can first be divided into two parts -- that southwest of KK, where imports are relatively more expensive than in the country of their origin, and that part northeast of KK, where exports are cheaper abroad than at home.


³Johnson, H. G., ibid. See especially footnote 5, p. 146; "suggested by comments from Mr. Solomon Adler and Dr. Robert Solow."

⁴See Meade, J. E., Geometry, loc. cit., Figure XXVI.
Next consider the offer curves $O_a$ and $O_b$. An equilibrium in (1) means that both countries have tariffs; an equilibrium in (4) means that both countries have subsidies. In (5) and (6) country A has an import tariff and B has an export subsidy; but in (5) the subsidy in B is greater than the tariff in A, so the tariff-inclusive price of A's imports is less than the domestic price; similarly, in (6) the tariff is larger than the subsidy. Similarly, (2) and (3) indicate equilibria where B has a tariff and A has a subsidy, but in (2) the tariff is larger than the subsidy, and in (3) the subsidy is larger than the tariff.

It is convenient to describe these conditions in a diagram like that of Figure Ia, where $t_a$, $t_b$, $s_a$, and $s_b$ refer to tariffs and subsidies in A and B. Draw KK to connect points where tariffs in A equal subsidies in B and where subsidies in A equal tariffs in B—the angle of this line must be forty-five degrees. Now mark off the areas one to six as shown in the figure. Each of these areas then describes the mixture of subsidies and tariffs that will correspond with the appropriate number in Figure Ia.

It is immediately clear that KK in Figure Ib corresponds to the contract curve in Figure Ia; the latter is the locus of all points where marginal rates of substitution are the same in A and B, and they can only be equal under free trade or in the case of a tariff in one country being exactly offset by a subsidy in the other. KK must, of course, pass through the origin, F, in Figure Ib; this origin corresponds to the free trade point F in Figure Ia.
Now consider Figure II. A imposes an optimum tariff, determined by the indifference curve tangent to the foreign offer curve, at $P_a$. The tariff $\frac{AO}{OB}$ (which equals the reciprocal of the elasticity of $O_b$ reduced by unity) produces the tariff-modified offer curve $O_{a'}$. If $B$ now wishes to maximize social welfare, taking $A$'s tariff as given, it must now impose a tariff determined by $P_{b'}$ where a $B$ indifference curve is tangent to $O_{a'}$. $O_{b'}$ is $B$'s tariff-modified offer curve resulting from the tariff required to make $P_{b'}$ the equilibrium. $A$ will now find it profitable to impose a higher tariff if the indifference curve tangent to $O_{b'}$ is tangent to $O_{b'}$ to the left of $P_{b'}$. Retaliation then follows this pattern if each country each time takes the tariff in the other country as given.

In Figure III we show how, under that assumption, the final equilibrium is determined. Draw the tariff-modified offer curves for $B$ corresponding to every ad valorem rate of tariff, $O_{b}$, $O_{b'}$, $O_{b''}$, $O_{b'''}$, etc. Now connect the points $P$, $P'$, $P''$, etc., where successive $A$ indifference curves are tangent to the series of tariff-modified offer curves in $B$. The locus of these points shows the reactions of $A$'s government to the different levels of tariffs in $B$; and, following Johnson, can be called $A$'s reaction curve, $R_a$.

In Figure IV we have drawn both countries' reaction curves; Cournot equilibrium is determined by their intersection at $P$, as can be shown in the following way. Draw the tariff-modified offer curves of
A and B which go through P; in Figure IV this corresponds to \( t_{2a} \) and \( t_{4b} \) for A and B, respectively. Now, since P is on both reaction curves, an A indifference curve must be tangent to \( t_{4b} \), and a B indifference curve must be tangent to \( t_{2a} \) at P. Therefore, neither country has any incentive to change its tariff; given the tariffs imposed by the other country, both countries have imposed optimum tariffs, the tariffs being in this example \( \frac{AO}{OB} \) for A and \( \frac{OJ}{JB} \) for B. The terms of trade are given by the line OP.

We now reproduce the quadrant showing the levels of tariffs and subsidies, and in this quadrant are introduced reaction curves formed in the same way as in the treatment of duopoly problems. The four quadrants are filled with indifference curves, corresponding to, but of a different shape than, those in the trade diagram. We should expect first that, as equilibrium moves south, B's welfare increases corresponding to lower rates of tariffs in A, subsidies being considered negative tariffs; and as equilibrium moves west, A's welfare increases corresponding to lower rates of tariff in B. We know also that A and B indifference curves must be tangent on the contract curve, a negatively sloping forty-five degree line through the origin.

It is not possible to determine unambiguously the direction of the reaction curves; if A's best social welfare policy lies in lowering tariffs when B raises tariffs, then A's reaction curve, indicating the locus of vertical tangents to A's indifference curves will have a negative slope -- but if a higher tariff in B makes a higher tariff in A
optimal, then it will have a positive slope. Depending on the elasticities of the different tariff-modified offer curves in the trade quadrant, either of these results may prevail.

Assume the reaction curves are drawn as in Figure V, and initially equilibrium is at point F, the free trade point. Now suppose B imposes an optimum tariff based on the assumption that A keeps her tariff at zero; clearly the highest indifference curve for B is at \( P_b \), where an indifference curve is tangent to the X-axis. This point corresponds to \( P_b \) in Figure IV. Now suppose that A retaliates to this tariff on the assumption that B keeps her tariff at \( F P_b \); A's tariff in this case would be point Q, corresponding to Q in Figure IV. If A and B now keep adjusting their tariffs to the optimum level, each time on the assumption that the foreign tariff is constant, the level of tariffs will move along the spiral, \( P_b, Q, M, N, \) etc., to the point C; and the trade equilibria in Figure IV will move in the trading quadrant in a corresponding way until the point P is reached, indicating tariff equilibrium under these assumptions.

Professor Johnson has pointed out that, with certain elasticity combinations, a country may gain from tariffs over free trade despite retaliation. In Figure V the indifference curves through C indicate that both countries have lost by the tariff war compared with the tranquility of the free trade optimum; but, as the foregoing statement suggests, this is by no means a necessary case. It is true that both countries could gain simultaneously by a movement onto the contract curve.
but the free trade position may not be included within the range of mutual gain.

This suggests immediately a condition of which much more will be made later in the analysis of optimum tariff reductions -- if a principle of mutual gain in tariff reductions is agreed upon, and it is desired at least to attain the efficiency of the contract curve, it may be necessary to consider international transfers of income to compensate for the concession of free trade made by the country which would lose by that policy alone.

First, however, we shall consider other policy assumptions than the one previously made, that each country treats the foreign tariff as a datum. If one country, let us say B, finds out from past experience the reaction curve of A, then it may treat A's reaction curve as a datum rather than A's tariff. In this case B can make the most of her welfare position by imposing that tariff determined by the point of tangency of a B indifference curve with A's reaction curve, $S_b$, in both Figures IV and V. This point must be better for B, but worse for A, than the Cournot point C, but it may or may not be better for B than the free trade point F.

Similarly, we can draw a Stackleberg point for A, on both diagrams, $S_a$. If both countries perceive the other's Cournot reaction curve and assume (erroneously) that the other reacts along it, both countries may impose the tariff corresponding to its Stackleberg point; in this case the expectations of each government have turned out wrongly, and they
may therefore revise their expectations, try some bluffing policy, try to outlast the foreign government or any one of the many fashionable solutions to duopoly problems. If they both retain their Stackleberg tariffs, they may move farther from equilibrium to D.

The above policies are possible outcomes of tariff retaliation; but there is some futility in pursuing possible cases as these when empirical examination of past tariff policies do not indicate that wars of this type have developed. Further, it is doubtful that many, if any, countries have even imposed tariffs designed to maximize social welfare; the motives of protection, employment and revenue perhaps deserve first place among the apologies for protection. Further consideration of the line of thought suggested here would mean interesting intellectual exercise but could hardly be excused other than as a pedantic, sterile creation of further empty boxes.

What has been useful in the theory of tariff retaliation, inter alia, is a descriptive way to portray various possible means of reducing tariff barriers, and the assumptions and conditions necessary to make this optimal. As we turn to a discussion of these tariff reductions, we shall not begin even from the elegant symmetry of a Cournot position, but rather assume a fact which exists in the real world -- tariffs in both countries of any given size. The following section will suggest various criteria for optimally reducing these barriers and improving national, or world, welfare.
In the discussion of optimal tariff reductions, we can consider a number of different conditions to which the size of the reductions must adhere. Free trade may make one of the two countries worse off; so an immediate condition could be that both countries must gain by the tariff reductions. This would limit the degree to which tariffs can be reduced, unless further policies involving subsidies or transfers are introduced. If these are not relied upon, world allocation cannot be efficient; equilibrium cannot then be on the contract curve.

Another possible policy decision may rest upon balance of trade considerations. If relative factor-cost prices were constant, due to a fixed exchange rate and monetary policy in fixing domestic prices, in only a special case would the balance of trade remain at zero, if that is the initial tariff equilibrium. It might be agreed, if the stabilization of domestic factor-cost prices is regarded as inviolable, that tariffs must be reduced in the context of this special case — i.e., in such a way as to leave the balance of trade unchanged.

If, on the other hand, a fluctuating exchange rate, or fluctuating factor-cost prices, ensured a zero balance of trade, then countries might insist that tariffs be reduced such that the terms of trade remained unchanged. This case, beginning with the same equilibrium, would require tariff reductions identical with those required to keep the balance of trade constant under the assumptions of the preceding paragraph; this will be demonstrated later.
It is also possible that one country, faced with the imposition of an optimum tariff by the other country, persuades the other country to relinquish her tariff in return for which she makes a transfer of an amount that will compensate for the loss sustained by the other country, of the gains from the tariff. It can be demonstrated that the transferring country can gain in this; and also that in some cases can gain more than by retaliation! It will be shown later that this transfer must be less than the transfer implicit in a subsidy equal to the optimum tariff.

Another possible assumption regarding tariff reductions is that one country may have to maintain the tariff for political reasons; but it may be amenable to those devices which would eliminate the effect of the tariff. It will be shown that a subsidy equal to the tariff can be imposed by the other country, and a transfer of income made to compensate the subsidizing country; both countries gain by this procedure.

Finally, we shall have to consider the possibility of reducing tariffs without regard to either balance of payments or terms of trade considerations; but with regard only to welfare. It may be agreed that no country should lose by a reduction in tariffs, but the division of gain must rest on international welfare considerations. Weights can be assigned to welfare increases in the two countries, and the increase in welfare can then be maximized -- in this case a limiting factor may be that neither country should lose.
Constant Terms of Trade

We shall consider first the assumption that A and B reduce tariffs in such a way that the terms of trade remain unchanged. The assumptions in this case are that export prices are fixed by monetary policy and that the exchange rate fluctuates in such a way as to maintain equilibrium in the balance of trade. It should be noted once again that the analysis is exactly the same when the exchange rate is fixed and factor cost prices are free to fluctuate with changes in demand and supply due to changes in tariffs.

It was shown in the preceding chapter that the degree to which the terms of trade change following a tariff, under the same assumptions as the above, is determined by the formula,

\[ V = \frac{t \cdot \varepsilon_a}{\varepsilon_a + \Pi_{ab} - 1} \]

and that the formula for changes in the terms of trade, when both countries impose tariffs, is

\[ V = \frac{t_a \cdot \varepsilon_a - t_b \cdot \varepsilon_b}{\varepsilon_a + \Pi_{ab} - 1} , \]

where \( t_a \) and \( t_b \) are changes in the ad valorem rates of tariff in A and B, respectively.

From the formula it is clear that changes in the terms of trade will take place as a result of tariff reductions unless the expression \( t_a \varepsilon_a - t_b \varepsilon_b \) is equal to zero -- i.e., unless tariffs are reduced in such a way that \( \frac{t_a}{t_b} = \frac{\varepsilon_b}{\varepsilon_a} \). For the terms of trade to remain unchanged, tariffs must be reduced in a ratio to each other in inverse
proportion to the ratios of the expenditure-compensated elasticities.

The criterion has been derived in the previous chapter; but it may be useful to illustrate this condition, and it is so illustrated in Figures VI and VII. In Figure VI, equilibrium is at P initially, with a tariff of \( \frac{ON}{ON} \) in A and \( \frac{OK}{KN} \) in B. A and B indifference curves are tangent to the price lines a in A and b in B; the terms of trade are OP.

Now suppose that tariffs are reduced in such a way that the terms of trade remain unchanged -- i.e., that equilibrium moves to a point such as P'. Now, since the terms of trade have not changed, the relative price change in A has been the reduction in the tariff in A, and the price change in B has been the reduction in B's tariff. Then the expenditure-compensated elasticity of demand, when the tariff reductions are small, in A is \( \frac{RP}{PM} / t_A \), and in B is \( \frac{RP}{PM} / t_B \). Then

\[ \xi_A \cdot t_a - \xi_B \cdot t_b = t_a \cdot \frac{RP}{PM} / t_a - t_b \frac{RP}{PM} / t_b = 0. \]

Now turn to Figure VII; the diagram is filled now with iso-terms of trade lines, on any one of which the terms of trade remain constant, each line being defined by the above formula. The line \( t_0 \) shows the locus of tariffs in A and B which will maintain the same terms of trade as under free trade. As the lines move northwest, A's terms of trade improve, and B's terms of trade improve as equilibrium moves onto lower lines.
Suppose equilibrium is initially at P, corresponding to P in Figure VI; $I_a$ and $I_b$ are the indifference curves of A and B through P. To get to $P'$, S reduces tariffs by $PW$ and B reduces tariffs by $PV';$ the welfare of both A and B must increase, since the volume of trade increases with no change in the terms of trade. There is opportunity for further improvement until A's tariffs have been reduced to zero, point R in Figure VII. Even at R, however, both A and B could be made better off without either country being made worse off if B further reduces her tariff and A subsidizes exports until prices in A and B are identical — i.e., equilibrium moves to the contract curve but not to the free trade point on the contract curve. The point is Q; at Q both A and B are better off than at P, but A is worse off than with free trade, and B is better off than with free trade. The terms of trade are unchanged. At Q, country B maintains a tariff of $PB$, while country A imposes a subsidy of $FC$. The point Q could, of course, be duplicated by free trade policies and a transfer from A to B large enough to move equilibrium to Q.

**Constant Balance of Trade**

Now we shall discuss the case where the terms of trade are fixed (pegged exchange rate and monetary policy to keep export prices constant), so disequilibrium must be removed by changes in domestic expenditure and the balance of trade. We can demonstrate the criterion which shows the changes in tariffs required to leave the balance of trade unchanged;
this criterion is the same as that for keeping the terms of trade constant, as would be expected -- the only difference is that in the former a disequilibrium is cleared by a price change, and in the latter by a balance of trade change.

In the previous chapter we derived a criterion for changes in the balance of trade when both A and B imposed tariffs; this was

$$\mathcal{N} = \frac{\epsilon_a t_a - \epsilon_b t_b}{1 - \Pi_{ab}}.$$ 

the numerator of which is the same as that expressing changes in the terms of trade. The criterion for tariff reductions is therefore the same as before -- i.e., \( \frac{t_a}{t_b} = \frac{\epsilon_b}{\epsilon_a} \) which can be illustrated in the same manner as that of the previous case in Figure VI.

In Figure VII, instead of iso-terms of trade lines we can substitute iso-balance of trade lines, and the tariff reductions which will keep the balance of trade constant are shown along any of these lines.

In the general case, with export as well as import taxes in A and B, the criterion becomes:

$$\mathcal{N} = \frac{\epsilon_a t_{ia} - \epsilon_b t_{ib} - \epsilon_b t_{ea} + 1 + \epsilon_a t_{eb} - 1}{1 - \Pi_{ab}}.$$ 

When the balance of trade is zero, then this becomes:

$$\frac{\epsilon_a}{\epsilon_b} = \frac{t_{ib} + t_{ea}}{t_{ia} + t_{eb}}.$$
In order to keep the balance of trade constant, export and import taxes must be reduced in the proportions indicated.

It is possible to consider some more special assumptions about the nature and extent of tariff reductions, assumptions which are not too far divorced from reality. It may be that a country must maintain a given tariff for a number of reasons -- political, social or legal; but that this country's government is willing to frustrate the content of the tariff by other means. We shall show in this part perhaps the best way in which the misallocating effect of the tariff can be avoided. More specifically, we shall show how equilibrium can in fact be made the free-trade equilibrium, while one country maintains its tariff.

Consider Figure VIII. \( O_a \) and \( O_b \) are the offer curves of A and B, intersecting at free-trade equilibrium \( F \); but country A has a tariff equal to \( \frac{O_A}{O_B} \), so equilibrium is actually at \( P \). Now suppose that A agrees to eliminate the effect of the tariff while maintaining its form. This can be done in the following way.

Draw \( F_N \), from the free-trade equilibrium to the \( X \)-axis, and mark off \( M \) on the \( X \)-axis so that \( \frac{OM}{MN} \) is equal to \( \frac{AO}{OB} \), A's original tariff; then if B puts a subsidy on her exports equal to the tariff in A, and A makes a transfer to B equal to \( OM \), then the equilibrium income in each country will be exactly the same as under free trade. That each country is in equilibrium at \( F \) can be shown as follows: \( \frac{OM}{ON} \) is the import tariff in A and the export subsidy in B, and the line \( OF \) is the price line in both A and B, since it is tangent to A and B indifference curves.
at \( F \); the net barter terms of trade are \( MF \), but what is more significant here, the gross barter terms of trade are \( OF \).

In a similar way a system of subsidies and transfers can render ineffective any tariff, and the substance, if not the form, of free trade can always be achieved. Referred to Figure VII, any point such as \( D \) can be converted into a free-trade equilibrium with, say, \( A \) maintaining its tariff, if \( B \) imposes an equivalent subsidy moving equilibrium to \( E \); and then agreeing on the transfer required to move equilibrium to \( F \), the free-trade point.

In the discussion of retaliation, an alternative to retaliation has been neglected. It will be shown that if one country, \( A \), is pursuing a social welfare policy, and this policy leads to the imposition of an optimum tariff, \( B \) may be able to do better than impose a retaliatory tariff, by bribing \( A \) to remove her tariff, with a transfer equal to or possibly greater than her gain from the optimum tariff. Such a case is shown in Figures IX and X. \( F \) is the free-trade position, but \( A \) imposes an optimum tariff, moving equilibrium to point \( P \) on \( B \)'s offer curve in Figure IX. Now the highest indifference curve \( B \) could attain through retaliation is that tangent to \( O_A' \), \( A \)'s tariff-modified offer curve, which cuts the contract curve at \( P''' \); but the \( A \) indifference curve through \( P \) goes through \( P'' \) to the southeast of \( P''' \). Therefore, if \( A \) is willing to trade a transfer for her tariff, \( B \) could exactly compensate \( A \) with \( OM \) of \( X \), the point from which equilibrium at \( P'' \) will be established; but if further incentives are needed, \( B \) could offer a larger
transfer up to the endowment position which will make P''' an equilibrium. If a larger transfer is insisted upon, B is better off to retaliate.

This is perhaps more easily seen from Figure X. A's optimum tariff moves equilibrium to P', and the indifference curve through that point crosses the contract curve at P'1; but if B retaliates, she cannot increase her income by as much as at P'1, since the indifference curve through P', the point of retaliation, crosses the contract curve at P''' to the northwest of P'1. Between P'1 and P''' there is room for bargaining. It should be noted that the room for bargaining is no longer between P4 and P'1, since from P4 to P''' country B can do better by retaliating.

From the analysis up to this point in the chapter we have shown various ways in which world efficiency could be improved, and how the effects of some tariffs could be palliated or relieved by the appropriate commercial policy or international agreement. It was suggested that free trade may be a difficult equilibrium to attain, especially if in the movement toward free trade one country's real income falls. In these cases some criterion must be established for a division of income, at least toward a division of gain of income, to select the appropriate points along the contract curve that it seems desirable to maintain.

Our analysis has discussed transfers as substitutes for the removal of tariffs. In a cynical age this seems the height of absurdity at first sight. Nevertheless, in the postwar years international income
transfers have been made and tariffs have been only moderately reduced from extreme levels; the experience of the post-war years reveals that some countries may prefer to make income transfers rather than reduce tariff barriers. "Trade, Not Aid" is perhaps an appeal against this very policy.

If trade impediments keep equilibrium off the contract curve, both countries could be made better off by a movement toward the contract curve. The latter can be considered a utility efficiency locus, representing the points where the marginal rates of substitution in each country are equal. It is clear that, as equilibrium moves along the contract curve to the northwest, B becomes worse off and A becomes better off. The criterion of national welfare is easy along this curve; world welfare is more elusive; it cannot be read from the diagram unless some more information is specified as the weighting attached to each country's welfare. If cardinal utility is assumed, a figure could be attached to each indifference curve in each country and the point on the contract curve where utility is highest becomes the "maximum maximo" — in this way changes in utility could be divided between countries. In Figure XI this would involve superimposing a series of world welfare indifference curves on the trade quadrant, and each indifference curve would have a figure associated with it as an index of its, or its actual, utility. If the point on the summit is M, this position could be obtained by A retaining a small tariff and B imposing an equivalent subsidy, so that equilibrium moves to the world optimum M — or, both
countries could remove tariffs completely and a transfer OT could be
made from B to A. In Figure XII, from the tariff diagram, if the ini-
tial point is somewhere at P and the world maximum is at M, then a sub-
sidy and a tariff could bring equilibrium to the point which maximizes
world welfare; an alternative would be to eliminate tariffs and to make
the appropriate transfer.

However, this analysis has, but need not have, rested upon assump-
tions of cardinal utility. It is unnecessary to add the utilities of in-
dividual countries; and at any rate this is an illegitimate procedure.
It is possible to formulate the solution on ordinal principles if a world
social welfare function can be determined. This has different interpre-
tations, so we must discuss briefly the meaning of this function as used
here.

Every individual, imagining himself in the role of a lesser deity,
could have a world welfare function divorced from his own utility func-
tion; this welfare function would compose his judgments on the value of
the welfares of different individuals. But, because of our assumptions,
about community-indifference curves, the world welfare function can be
simplified to evaluate national instead of individual welfares. A wel-
fare function can describe the scale of preferences based on income
distribution between the two countries.

Each individual can have a world welfare function, but we shall
assume the fiction of a world general will which pronounces judgments
on the values of different national distributions of utility. This
Figure XII
general will have a map of indifference curves, showing substitution between the welfares of A and B, with a shape similar to an individual or community indifference curve, with the proper convexity conditions, etc. In Figure XIII, A's welfare is measured on the vertical axis and B's welfare is measured on the horizontal, both on purely ordinal scales of welfare. $W_0$, $W_1$ and $W_2$ refer successively to improvements in world welfare as dictated by the world general will.

In order to find the world optimum, it is necessary to derive the utility opportunity curve, a concept used by Samuelson extensively. From the trading quadrant we know that the world utility opportunity curve will be the utility dimension of the contract curve, since an improvement for both countries can be made whenever equilibrium is off that curve. This curve is drawn in Figure XIII, convex upward, indicating diminishing marginal "utility" of income. Tangency with the highest world indifference curve is at $M$, and this represents the world optimum.

Now, on this basis, we can return to Figures XI and XII, and they can very easily be divorced from cardinal utility assumptions. To draw the indifference map in these figures under purely ordinal assumptions, it is only necessary to order the curves in accordance with the world indifference curves in Figure XIII; the results of the previous analysis then hold. It should be noted in Figure XIII that M may or may not coincide with the equilibrium resulting from free trade; it does not coincide, as drawn.
The technique of Figure XIII is expository, but it is a useful way of portraying the effects of tariff retaliation and reduction. Consider Figures XIV and XV. O' is the origin of the utility map; O'B is B's welfare without trade and O'A is A's welfare without trade. With trade the utility opportunity locus is \( U_a U_b \); this represents the utility dimension of the contract curve between \( U_a \) and \( U_b \) in XV, where the indifference curves go through the origin. Now in Figure XIV let F be the free trade point; then, if B imposes successively higher tariffs, equilibrium will move along A's offer curve. B's welfare must increase to a maximum at her optimum tariff point, and then eventually decline to the no-trade welfare position when tariffs become prohibitive. Also, as B imposes a subsidy, her welfare decreases and A's welfare increases until B's welfare is no better than without trade. The line OFK then represents the locus of equilibria along A's offer curve with different levels of tariffs and subsidies in B. Similarly, OFL represents positions along B's offer curve.

OFK and OFL then represent the effects of imposing tariffs or subsidies in A and B, respectively, when the other country has zero tariffs; in this case the only point which touches the utility efficiency locus is the free-trade point. Now a whole map of such curves can be drawn for each level of tariffs in the other country. We have drawn one such curve, OPQ, representing the welfare effects of B imposing different levels of tariffs on A's subsidy-modified offer curve, O_a' in Figure XV. This must touch the efficiency locus when the tariff
Figure XIV
imposed by B is equal to the subsidy in B, at F. Each point where this new curve crosses any other curve has a special significance related to the trading quadrant in XV. For example, at point (6) OPQ crosses OPL; point (6) can then be a point on B's offer curve (zero taxes in B and a tariff in A) or a corresponding point where the same two indifference curves that cross on B's offer curve, also cross at some other point where both A and B have subsidies. It is intuitively obvious that any point not on UU must correspond to two points in the trading quadrant, since two indifference curves that cross must cross twice. Similarly, all the points, 1, 2, 3, 4, 5, 6 in XIV have corresponding points in XV; but 1 and 2, being on the efficiency locus, have only one point, while the others have two corresponding points.

The whole area within the utility locus can be filled out with curves such as OPQ; this is done in XVI, and reaction curves are derived by marking the points where B's welfare is highest subject to A's offer curves, and where A's welfare is highest subject to B's offer curves. These reaction lines, $R_a$ and $R_b$, intersect at C, the Cournot equilibrium. If C is within the rectangle GFE, both countries lose by retaliation, but if C is within $FEU_b$, country B gains even with retaliation, and if C is within $G FU_a$, country A gains even with retaliation. In a similar way, mutatis mutandis, the Stackleberg points can be ascertained, and their effect on welfare determined.
IV. COMMODITY MOVEMENTS AND FACTOR MOBILITY
Commodity movements are, at least to some extent, a substitute for factor movements.\textsuperscript{1} The absence of trade impediments implies commodity price equalization and, even when factors are immobile, a tendency toward factor price equalization. It is equally true that perfect mobility of factors results in factor price equalization and, even when commodity movements cannot take place, in a tendency toward commodity price equalization.

There are two extreme cases between which conditions in the real world approximate; there may be perfect factor mobility but no trade or factor immobility with unrestricted trade. The classical economists generally chose the special case where factors of production were internationally immobile.

This chapter will show some of the effects of relaxing this assumption and allowing not only commodity movements but also some degree of factor mobility. More specifically, it will show that an increase in trade impediments stimulates factor movements and that restrictions to factor movements encourage trade.\textsuperscript{2} It will also make more specific an old argument for protection.

\textsuperscript{1}I am much indebted for helpful criticism and valuable advice from Professor J. E. Meade, Professor H. G. Johnson, Dr. S. A. Ozga and Dr. M. Corden. I am also grateful to Mr. R. Lipsey and Mr. T. Rybczynski for helpful comment. Any mistakes are of course my own creation.

\textsuperscript{2}This proposition is implied in Bertil Ohlin's \textit{Interregional and International Trade} (Cambridge, Harvard University Press, 1935). See Chapter IX for the relevant discussion. Also Professor Meade discusses the problem in the second volume of his \textit{Theory of Internation Economic Policy}. See Meade, J. E., \textit{Trade and Welfare} (New York, Oxford University Press, 1955), Chapter XXI.
I

Under certain rigorous assumptions the substitution of commodity
for factor movements will be complete. In a two-country, two-commodity,
two-factor model, if:

(a) production functions are homogeneous of the first
degree (i.e., marginal productivities, relatively
and absolutely, depend only on the proportions in
which factors are combined) and identical in both
countries,

(b) one commodity requires a greater proportion of
one factor than the other commodity at any factor
prices at all points on the production function,
and

(c) factor endowments are such as to exclude speciali-

zation,
then commodity price equalization is sufficient to ensure factor price
equalization and factor price equalization is sufficient to ensure com-
modity price equalization.\(^1\)

These assumptions, inasmuch as they are not always satisfied in
the real world, limit somewhat the usefulness of a model employing them.
But they do isolate some important influences determining the pattern of
international trade, and for present purposes will be adhered to. It
will become clear later that the consequences of relaxing these assump-
tions do not seriously affect the conclusions of the paper.

Assuming the production conditions (a, b, and c above) are satis-
fied, when factors are immobile, the condition necessary for commodity
price equalization is the absence of trade barriers; when commodities

\(^1\) For the necessity of these assumptions and also a complete list of
references to the literature on factor price equalization, see Samuelson,
P. A., "Prices of Factors and Goods in General Equilibrium," The Review
are immobile, the condition necessary for factor price equalization is that at least one of the two factors is perfectly mobile. The production conditions should be kept analytically separate from the conditions required for free commodity and factor movements. Throughout this chapter we shall assume that the production conditions are satisfied and concentrate on the effects of changing the assumptions regarding commodity and factor mobility.

Neither commodity nor factor prices are equalized in the real world because, inter alia, various impediments inhibit the free movement of commodities and factors. Economic impediments can be conveniently classed for our purposes under transport costs, taxes and quotas; this classification refers to impediments to factor as well as commodity movements. For example, labor migration is more difficult because of such impediments as travelling costs, immigration taxes and immigration quotas.

Immobility exists when transport costs are infinite, tariffs prohibitive or quotas zero. When transport costs are less than infinite, immobility must be defined with reference to a given equilibrium; this applies also to a tariff which may be prohibitive at one set of prices but not at another. Some of the problems resulting from this will be discussed below.
II

First, we shall show that an increase in trade impediments encourages factor movements. In the simplified model assumed, there are two countries, A and B, two commodities, cotton and machinery, and two factors, labor and capital.\(^1\) Country A is well endowed with labor but poorly endowed with capital relative to country B; cotton is labor-intensive relative to machinery. Also we assume that tastes can be represented by a set of community indifference curves (this is for expository convenience only and is not necessary for the conclusions reached).

For the moment we shall consider country B the rest of the world and country A so small that its production conditions or factor endowments can have no effect on factor prices in B.\(^2\)

We begin with a situation where factors are immobile between A and B but where impediments to trade are absent so commodity and hence factor prices are equalized. A exports its labor-intensive product, cotton, in exchange for capital-intensive machinery. In Figure I, TT is A's transformation function (production possibility curve); production is at P and consumption is at S. A is exporting \(PR\) of cotton and importing \(RS\) of machinery. A's income in terms of machinery is \(OY\).

\(^1\)Factors are given names for verisimilitude. All the problems of capital requiring special treatment as a factor are ignored. It is here considered a physical, homogeneous factor which, when it moves internationally does not create any balance of payments problems. It is further assumed that capitalists qua consuming units do not move with their capital so national taste patterns are unaltered. For a discussion of these and other problems concerning international factor movements see Professor Johnson's lectures at Manchester, especially Lecture III. Also Meade, loc. cit.

\(^2\)It will become evident in Section III that the terms of trade and factor prices will not change, even if B is fairly small.
Suppose that some exogenous factor removes all impediments to the movement of capital. Clearly, since its marginal product is the same in both A and B, no capital movement will take place, and equilibrium will remain where it is.

But now assume that A imposes a tariff on machinery, and for simplicity make it prohibitive. Initially the price of machinery will rise in A and both production and consumption will move to Q, the autarky (economic self-sufficiency) point. Factors will move out of the cotton into the machinery industry; but, since cotton is labor-intensive and machinery is capital-intensive, at constant factor prices there would be an excess supply of labor and an excess demand for capital; consequently, the marginal product of labor must fall and the marginal product of capital must rise. This is the familiar Stolper-Samuelson tariff argument.

Now, since capital is mobile, its higher marginal product in A causes a capital inflow into A from B, changing factor endowments in such a way as to make A more capital-abundant. With more capital, A's transformation curve expands until a new equilibrium is reached. But where will this new equilibrium be?

Some help in answering this question is provided by the box diagram in Figure II. A has initially OC of capital and OL of labor; O

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1 Actually, any tariff is prohibitive, as will become evident later in the argument.

Figure II
is the efficiency locus (contract curve) along which marginal products of labor and capital are equalized in machinery and cotton. Equilibrium is initially at P, which corresponds to P on the production block in Figure I. Factor proportions in machinery and cotton are given by the slopes of OP and OP', respectively.

After the tariff is imposed, production moves along the efficiency locus to Q, corresponding to the autarky point Q in Figure I. The slopes of OQ and OP indicate that the ratios of labor to capital in both cotton and machinery have risen — i.e., the marginal product of capital has risen and the marginal product of labor has fallen. Capital flows in and the cotton origin O shifts to the right.

With perfect mobility of capital the marginal products of both labor and capital must be equalized in A and B. This follows from the assumption that the production functions are linear, homogeneous and identical in each country. Since marginal products in the rest of the world can be considered constant, the wage and interest rate in A will not change. Factor proportions in both machinery and cotton in A then must be the same as before the tariff was imposed — that is, equilibrium must lie along OP extended at the point where it is cut by a line OP' parallel to OP, where O' is the new cotton origin. But this is not yet sufficient to tell us exactly where along OP extended the point P' will be.

Since marginal products in the new equilibrium are the same as before the tariff, commodity prices in A will not have changed. If
commodity prices have not changed, it is clear that no trade can take place at the new equilibrium (otherwise prices would differ by the amount of the tariff).

With marginal products the same as before, income earned by owners of factors previously in A must be the same as before. Hence, consumption will remain at S. But, since prices are also unchanged, production clearly must be greater than S because interest payments must be made to B equal in value to the marginal product of the capital inflow. In Figure I, then, equilibrium must be at some point above or to the northeast of S.

To find the exact point, we must show the effects of a change in capital endowments on the production block. Because machinery is capital-intensive, we should expect the production block after the capital movement has taken place to be biased in favor of machinery. That this is so has been proved by T. Rybczynski.¹

If prices remain unchanged as capital flows in, the new equilibrium must be at a point where (absolutely) less cotton and (absolutely) more machinery is produced. If the same price ratio as at P prevailed, the locus of all tangents to larger and larger production blocks must have a negative slope. Such a line, the "R-line," is drawn in Figure I. As argued above, prices in fact remain unchanged so the new equilibrium must lie along this line.

¹Rybczynski, T. M., "Factor Endowment and Relative Commodity Prices," Economica, November, 1955, pp. 336-341. The proof is easily demonstrated by reference to Figure II. At unchanged prices, factor proportions must be the same as before the capital movement; equilibrium, then, must lie along OP extended. With the larger endowment of capital, OP' must be shorter than OP. Since they have the same slope and production functions are homogeneous of the first degree, output of cotton at P' must be less than at P. An as yet unpublished paper by R. Jones written at M.I.T. in the spring of 1955 contained a similar proof.
Capital will flow in until its marginal product is equalized in A and B which will be at the point where A can produce enough machinery and cotton for consumption equilibrium S without trade, and at the same time make an interest payment abroad to pay the required return on the foreign capital. This point is clearly reached at P' directly above S. Any point along the "R-line" to the left of P' would mean that A would have to import machinery in order to consume at S --- i.e., demand conditions could not be satisfied in A to the left of P'. At P' demand conditions in A are satisfied and the interest payment can be made abroad at the same price ratio as before the tariff was levied so the capital movement need not continue past this point, although any point to the right of P' would be consistent with equilibrium.

Production is at P', consumption is at S and the transfer of interest payments (a flow) is the excess of production over consumption in A, SP' of cotton. The value of A's production has increased from OY to OY', in terms of machinery, but YY' (equals SP' in terms of cotton) must be transferred abroad, so income is unchanged.¹

The tariff is now no longer necessary! Since marginal products and prices are again equalized, the tariff can be removed and no movement of either commodities or factors will take place. As a result of

¹SP' must equal in value the marginal product of the capital inflow at constant prices; this is shown by referring to Figure I. PP' is the change in output associated with the increase in capital; machinery output increases by RS, but cotton output decreases by PT. The marginal product of the capital inflow is then the value of RS minus the value of PT, which in terms of cotton is P'S.
the tariff, factor and commodity prices are unchanged so neither country nor the world as a whole has suffered from any misallocation of resources. Real incomes are the same as before the tariff so there has been no change in the distribution of income. No loss to either country has occurred.

III

But are the conclusions of this model valid if A is of sufficient size in relation to B to change significantly B's factor endowments? We shall discuss this case in some detail because it will further clarify the determination of the new equilibrium point in the previous section. It turns out that the relative sizes of the two countries do not affect the conclusions of the previous section (barring complete specialization in one product).

Suppose, as before, that A is exporting cotton in exchange for machinery. There are no impediments to trade and capital is mobile. But we no longer assume that A is very small relative to B.

Now A imposes a tariff on machinery, raising the internal price of machinery in relation to cotton, shifting resources out of cotton into machinery, raising the marginal product of labor and lowering the marginal product of capital. A's demand for imports and her supply of exports fall -- that is, there is a decrease in demand for B's machinery exports and a decrease in supply of B's cotton imports -- so the price of cotton relative to machinery rises in B. Labor and capital
in B shift out of machinery into cotton, raising the marginal product of labor and lowering the marginal product of capital. The effects in B then reinforce the price changes in A in stimulating a capital movement from B to A. Because of this capital movement, B's production block contracts and A's production block expands.

The assumption that capital is perfectly mobile means that factor and commodity prices must be equalized after the tariff. It is necessary to show now that they will not only be equalized but also unchanged.

The prices of cotton and machinery are determined by world demand and supply curves. To prove that prices remain unchanged it is sufficient to show that these demand and supply curves are unchanged -- or that at the pre-tariff price ratio, demand equals supply.

But we know immediately that at the old price ratio, marginal products and hence incomes are unchanged -- hence demand is unchanged. All that remains, then, is to show that at constant prices production increases in one country cancel out production decreases in the other country.

The problem is illustrated in Figures IIIa and IIIb. In Figure IIIa, $T_aT_a'$ is A's transformation curve before the tariff. $T_a' T_a'$ is the transformation curve after the tariff and capital movement have taken place. At constant prices, equilibrium moves along A's R-line from $P_a$ to $P_a'$: the production changes are an increase of $R_{P_a}'$ of machinery and a decrease of $R_{P_a}$ of cotton. In Figure IIIb $T_bT_b'$ is B's initial transformation curve; $T_b' T_b$ is the transformation curve after
the capital has left B. At constant prices, production moves along B's R-line to \( P'_b \): machinery production decreases by \( SP_b \) and cotton production increases by \( SP'_b \).

To demonstrate the proposition that world supply curves are unchanged, it is necessary to prove that the increase in machinery production in A equals the decrease in machinery production in B; and that the decrease in cotton production in A equals the increase in cotton production in B -- i.e., that \( RP'_a \) equals \( SP_b \) and \( RP_a \) equals \( SP'_b \).

The proof is given in Figure IV. \( OL_a \) and \( OC_a \) are A's initial endowments of labor and capital, respectively; \( OL_b \) and \( OC_b \) are the endowments of B; \( OO_a \) and \( OO_b \) are the contract curves of A and B. Production takes place along these contract curves at \( P_a \) and \( P_b \).

Now when A imposes a tariff on machinery, suppose that \( C_b C'_b \) of capital leaves B. This shifts B's cotton origin from \( O_b \) to \( O'_b \). At constant prices labor-capital ratios in each industry must be the same as before -- that is, the new equilibrium must be along \( OP_b \) at a point where \( OP_b \) is intersected by a line from the new cotton origin parallel to \( O'_b P_b \). This point is \( P'_b \) which corresponds to \( P'_b \) in Figure IIIb.

Obviously, the capital outflow from B must equal the capital inflow to A. A's cotton origin moves to the right by just the same amount as B's cotton origin moved to the left -- i.e., to \( O'_a \). A's production equilibrium at constant prices then moves from \( P_a \) to \( P'_a \).

The proof that world supply at constant prices is unchanged is now very simple. Since \( O_a O'_a \) equals \( O'_b O_b \), then \( P'_a P_a \) equals \( P'_b P_b \). But \( P'_a P_a \) represents the increase in machinery output in A and \( P'_b P_b \)
Figure IV
represents the decrease in machinery output in B -- so, because factor proportions are unchanged and constant returns to scale apply, these must be equal. Similarly, $J\tilde{P}_a$, representing the decrease in output of cotton in A, must equal $K\tilde{P}_b'$, representing the increase in output of cotton in B. Q.E.D.

This relationship holds at all combinations of commodity and factor prices, provided some of both goods is produced in both countries. It means that world supply functions are independent of the distribution of factor endowments. More simply, it means that it makes no difference to world supply where goods are produced if commodity and factor prices are equalized -- a conclusion which may be intuitively obvious to some.

The capital movement, then, does not change world production at the same price ratio. Marginal products are the same; and, since the marginal product of the capital outflow from B is returned to capitalists in B, world demand conditions are the same as before. The new price equilibrium is unchanged. Thus the assumption made in the previous section that A is very small in relation to B is an unnecessary one because no change in the terms of trade is possible.\(^1\)

\(^1\)One qualification to the argument must be noted which is not necessary when the rest of the world is large. A condition for the marginal product of capital in A to rise as a result of the tariff is that the price of machinery rise in relation to the price of cotton. It is possible, if the foreign offer curve is very inelastic, that the improvement in A's terms of trade in raising the relative price of exports (cotton) will more than offset the effect of the tariff in raising the relative price of imports. The condition that the "normal" case is satisfied requires that the sum of the foreign elasticity of demand and the domestic marginal propensity to import (this is relevant because the improved terms of trade increase income) be greater than unity. This is Professor Metzler's qualification to the Stolper-Samuelson tariff argument. See Metzler, L., "Tariffs, the Terms of Trade and the Distribution of the National income," Journal of Political
The general conclusion of sections II and III is that tariffs will stimulate factor movements. When only capital is mobile, a labor-abundant country can attract capital by levying tariffs on its imports; a capital-abundant country can encourage foreign investment by protecting its import-competing industry. It is also true, because of the symmetry of export and import taxes, that an export tax imposed by a labor-abundant country will attract capital, and that an export tax imposed by a capital-abundant country will encourage foreign investment.

The analysis is not restricted to tariffs; it can apply as well to changes in transport costs. An increase in transport costs would increase domestic production of imports, raise the real return of, and thus attract, the scarce factor; similarly, a reduction in costs of transport would attract the abundant factor.\(^1\)

(Footnote 1 continued)

\textit{Economy, February, 1949.}

If this criterion is less than unity in our model, a tariff imposed by a labor-abundant country would stimulate foreign investment rather than attract capital. This conclusion, however, is based on the assumptions of a purely static model; if dynamic elements are involved, the direction of the capital movement would depend on whether the effects of the tariff on production preceded or followed the effects on the terms of trade.

\(^1\)It will be useful to note a few characteristics relating to the production blocks in Figures III and IV: (a) The R-line must be straight. This follows from the proof that an increase in production resulting from any given increment of capital at constant prices must be equal to the decrease in production resulting from any given decrement of capital. (b) The slope of the R-line depends on the units of machinery and cotton chosen; but the proportional increment of cotton divided by the proportional increment of machinery (one is negative) is determined by the production conditions in the box diagram. (c) In Figures IIIa and IIIb the R-lines must be parallel.
IV

This section will conclude that an increase in impediments to factor movements stimulates trade. To show this we shall assume that some capital is foreign-owned and illustrate the effects on trade of taxing this capital. The conclusion is a corollary of that of the previous sections.

We shall use Figures I and II. Begin with equilibrium initially at P' in Figure I. There are no impediments to trade, but since factor and commodity prices are already equalized, no trade takes place. CC' of capital (Figure II) is foreign-owned, so a transfer equal in value to YY' (Figure I) is made permitting consumption equilibrium at S.

If a tax is now levied on all foreign capital, its net return will be decreased, and since factor prices must be equalized, all of it (CC') will leave A. As capital leaves A, its production block contracts, this contraction being biased in favor of cotton against machinery. The price of machinery relative to cotton tends to rise, but since impediments to trade are absent, is prevented from doing so by machinery imports and cotton exports.

Since all foreign capital leaves A, the final size of A's production block, then, is TT, that consistent with domestically-owned capital. Production equilibrium moves from P' to P, consumption remains at S; no interest payment now is made abroad.
The effect of the tax has been to repatriate foreign capital and increase trade. By similar reasoning it could be shown that a subsidy will attract capital and decrease trade. In the case of the subsidy, however, the capital movement will stop only when factor prices change — i.e., specialization takes place.

V

In order to achieve efficiency in world production it is unnecessary that both commodities and factors move freely. As long as the production conditions are satisfied, it is sufficient that either commodities or factors move freely.

But if some restrictions, however small, exist to both commodity and factor movements, factor and commodity price equalization cannot take place unless the pre-trade prices are equal. This principle applies of course only to those restrictions which are operative; it does not apply to import taxes on goods which are exported, transport costs of factors which are immobile anyway or quotas larger than those required for equalization to take place.

Linear homogeneous production functions have been assumed throughout this chapter. But the conclusions reached here are not entirely dependent on that restricted type of production function. For example, factor prices do not have to be equal for a tariff to encourage capital imports — it is enough that the tariff raises the marginal product of
capital above the difference in marginal products at which foreign capitalists are indifferent between foreign and domestic investment. The argument does, however, require that each country export its abundant factor-intensive commodity.

It would be interesting to see what help this model is in explaining the considerable factor movements of the late nineteenth and early twentieth centuries. Did these factor movements, after a reasonable time-lag, coincide with the growth of protection? Is there any significant historical correlation between decreased factor mobility and increased trade in this country?

These are questions which cannot be answered here.
V. CAPITAL MOVEMENTS
This chapter will consider the effects of capital movements on the terms of trade and income when the analysis of capital movements distinguishes between gifts and loans. We shall first develop criteria for changes in the terms of trade in the absence of impediments, and under the assumption that production functions are linear, homogeneous and identical in each country. Later impediments to both factor and commodity movements will be introduced, and the direction of bias they exert on the terms of trade will be ascertained.

We shall consider loans and gifts from A to B: the former country is well endowed with capital, but poorly endowed with labor relative to B; country A initially exports machinery, which is always relatively capital-intensive, and imports cotton, which is always relatively labor-intensive.

Assume first that the capital movement is in the form of a loan: we assume also that there are no impediments to trade, so factor and commodity prices are equalized.

A. No Impediments

1. Loan

Consider first a capital loan from A to B. There are no impediments to trade, so factor and commodity prices are equalized. Assume that country A is well endowed with capital but poorly endowed with labor relative to B, and that cotton is labor-intensive relative to
machinery. A is exporting machinery in exchange for cotton. Commodity and hence factor prices are equalized in A and B.

In Figure 1a, production equilibrium in A is initially at $P_a$ on A's trade formation curve, $T_a^a$. A exports $P_a$ of machinery and imports $R_a$ of cotton, consuming at $C_a$. In Figure 1b, production equilibrium in B is at $P_b$, consumption equilibrium is at $C_b$; B exports $S_b$ of cotton and exports $S_C_b$ of machinery.

With the loan, capital moves out of A and into B, contracting A's and expanding B's, production block. Since A before the loan was importing capital-intensive machinery, the loan will decrease exports and imports. But what happens to the terms of trade?

To show the direction in which the terms of trade change, it is convenient to consider the changes in world demand and supply resulting from the capital movement at constant prices. If at constant prices there is an excess demand for one commodity (implying an excess supply of the other), the relative price of that commodity must rise. We shall, then, consider demand changes and supply changes at constant prices.

From the Rybczynski proof we know that the output of machinery will decrease in A and increase in B, and that the output of cotton will increase in A and decrease in B. From the previous chapter, since factor and commodity prices are equal, we know that the production changes in A exactly cancel out the production changes in B. At constant prices, then, world supply does not change.

1See the previous chapter.
But at constant prices marginal products do not change; and, since the capital movement is in the form of a loan, interest payments equal to the marginal product of capital inflow (to B) must be paid to A. The capital that has moved from A to B is earning the same amount, at constant prices, in B as it formerly earned in A; and this amount (its marginal product) is returned to capitalists in A. Incomes of the factors in A and B then have not changed at constant prices -- therefore at constant prices demand has not changed. Neither demand nor supply changes at constant prices; therefore the terms of trade do not change.

In Figure 1a, output at constant prices moves from \( P_a \) to \( P_a' \) along A's R-line; and in Figure 1b, output moves from \( P_b \) to \( P_b' \) along B's R-line. From the proof in the previous chapter, we know that the two R-lines must be parallel and that, following a reduction in capital in one country equal to the augmentation in the other, the distance \( P_bP_b' \) to the right along B's R-line equals the distance to the left along A's R-line; world supply is unchanged, then, at \( P_a' \) and \( P_b' \).

Consumption remains at \( C_a \) in A and \( C_b \) in B; the difference between production and consumption in A is cancelled out by the transfer of income \( VC_a \) of cotton and the smaller volume of trade. A now exports \( QP_a' \) of machinery and imports \( QV \) of cotton.

We may conclude therefore that, under the conditions when factor and commodity prices are equalized, a loan from one country to another will not change the terms of trade.
2. Gift or Tribute

In the theoretical and empirical work on the Transfer Problem, it is convention to treat reparations payments in terms of income transfers, even though actual reparations payments have often been fixed in other terms. Historically, territorial reparations, conscription of slave labor and capital endowment transfers have played a not unimportant role as well as the more usually analyzed tribute (income transfer). Now little can be gained by disputing the definition of the Transfer Problem; classically it has referred to the problem of effecting an excess of exports over imports and the burdens involved in relative price changes; it can be distinguished from capital transfers in that it relates to a flow rather than a stock. In a dynamic model we should expect the two types of transfers to be less distinguishable——income transfers affecting stocks through the rate of saving.

However, there is some value in showing, in a static model, the relationship which exists between capital transfers and income transfers; it can be shown that in some models, with conventional even if rigorous assumptions, the criteria for changes in the terms of trade are the same in both cases. In some more complicated models involving transport costs it can be shown how capital transfers can be used to effect income transfers, and also the situations in which a better allocation of resources will result through one rather than the other method.

It will be assumed, as before, that commodity and factor prices are equalized and that A makes a capital transfer to B. The method of
determining whether or not the terms of trade change is the same as that used in the analysis of a capital loan from A to B, except that, in the case of the capital transfer, the marginal product of the capital which moves from A to B will not be returned to A. Because of this, income in A will fall and income in B will rise.

In order to use unchanged indifference maps in A and B, we assume that A's government raises the cost of the capital transfer by a capital levy, and taxes and subsidizes incomes to distribute equally the burden of the capital levy and make the reduced incomes in A consistent with the indifference map as drawn. Similarly, B's government receives the capital, lends it and redistributes the marginal product.¹

¹The author possesses few illusions about the verisimilitude of this way of taxing and redistributing incomes; it is simply a convenient way of begging a question he does not wish to consider here. A more appropriate assumption would be not that a levy is made on what may be sunk capital, but that the transfer affects the capital stock through the rate of saving. It could be assumed that an income tax is levied; that this income tax reduces consumption and saving in a proportion depending on the marginal propensity to save; and that the decrease in saving reduces the rate of growth of the capital stock. When the income is transferred to country B and redistributed, consumption and savings increase in a proportion depending on the marginal propensity to save in B; and that new saving enhances the rate of growth of the capital stock in B.

If these assumptions of saving and investment were introduced here, an explanation consistent with the example in the text could be found along the following lines: before the income tax is levied in A, savings and investment are zero; the income tax comes entirely out of saving -- that is, dissaving occurs by the amount of the tax. In B, if savings and investment are zero before the transfer, the effect of the latter must be to increase saving by the amount of the transfer. The result is that the capital stock increases in B and decreases in A by the amount of capital which can be created by the income transfer.

This assumption supposes a very unique savings-income function, but it is easy to see how it could be modified into something more realistic. If part of the transfer comes out of consumption in A and goes into consumption in B, the result is a mixed income-capital transfer. If the marginal propensities to save are identical in each country, no change in the world endowment of capital takes place: but the world stock of capital will increase or decrease depending on whether the marginal propensity to save in B is greater or smaller than the marginal propensity to save in A, providing the effects of any possible change in the terms of trade on saving are negligible.
We analyze the case of capital transfers, then show the supply and demand changes at constant prices. But we know that, at constant prices, production changes in A and B exactly cancel out, so any changes in the terms of trade can be deduced from demand changes alone. Consider Figures IIa and IIb. Production in A and B moves along the R-lines from \( P_a \) to \( P'_a \) and from \( P_b \) to \( P'_b \), respectively. Consumption points in A and B are initially at \( C_a \) and \( C_b \); as income decreases in A and increases in B, consumers at constant prices move along their Engel's curves. In the absence of inferior goods, these Engel's curves must slope northeast.

Because production changes cancel out, the income in B at constant prices is equal to the decrease in income in A. Demand at constant prices is thus given by the intersection of the Engel's curves of A and B with the income lines \( Y_{a'} \) and \( Y_{b'} \), of A and B, respectively.

It should now be clear that the criterion for changes in the terms of trade depends on the slopes of the Engel's curves as in the usual analysis of the transfer problem. If the Engel's curves are parallel, the increase in demand for cotton and machinery in B is just offset by the decrease in demand for cotton and machinery in A, so the terms of trade do not change; if the slope of B's Engel's curve is steeper than the slope of A's Engel's curve, there is an increase in the world demand for cotton and a decrease in the world demand for machinery, so B's terms of trade must improve; and if the slope of B's Engel's curve is flatter than that of A's, as in Figures IIa and IIb, A's terms of trade must improve.

(Footnote 1 continued)

There is a difference still in the "capital transfer" caused by the income transfer and the usual income transfer: the latter usually refers to a regular flow of payments, while the former, as discussed above, refers to a once-for-all payment.
Figure II-a

Figure II-b
Because of the identity of income and expenditure in this classical model, the criterion in terms of the slopes of the Engel's curves is equivalent to a criterion in terms of the demand for any one commodity. For example, A's terms of trade will improve or worsen depending on whether A's marginal propensity to consume machinery (MPC_{ma}) is smaller or larger than B's marginal propensity to consume machinery (MPC_{mb}); or, in terms of cotton, A's terms of trade will improve or worsen depending on whether A's marginal propensity to consume cotton (MPC_{ca}) is greater or less than B's marginal propensity to consume cotton. This is equivalent to the familiar criterion in terms of import propensities; A's terms of trade will improve or worsen depending on whether the sum of the marginal propensities to import is greater or less than unity.

The relationship between capital transfers and income transfers is more easily seen in Figure III. O_a and O'_b are the pre-transfer offer curves of A and B intersecting at F; the terms of trade are given by a. Now OO' represents the changes in production in A and B at constant prices, and has the same length and slope (inverted) as the R-lines in Figures IIa and IIb. The value of the capital transfer, at constant prices, in terms of annual income is OM in terms of machinery, or OC in terms of cotton. (It will be remembered that machinery output increases in B but cotton output decreases at constant prices, so the transfer OM is actually a transfer from A to B of OA of machinery minus O'A of cotton.) At constant prices, O' represents the new endowment positions of A and B, from which the new offer curves originate.
Now at constant prices, B's expenditure will move to the right (increase) along \( PE_b \), while A's expenditure will move to the right (decrease) along \( PE_a \). The contract curve KK will separate these two expenditure lines. If \( PE_b \) lies above \( PE_a \) below the contract curve, as in Figure III, then

\[
\frac{MPC_{mb}}{MPC_{cb}} > \frac{MPC_{mb}}{MPC_{cb}}
\]

i.e., there is at constant prices an excess demand for machinery and an excess supply of cotton, so A's terms of trade will improve. If \( E_b^P \) were below \( PE_a \), A's terms of trade would fall, and if \( E_b^P \) coincided with \( E_a^P \), there would be no change.

This demonstrates that a capital transfer can be treated in the same way as an income transfer in deriving criteria for changes in the terms of trade, under the assumption that factor and commodity prices are equalized. However, the quantitative changes in the terms of trade will not be the same except in the special and trivial case when no change takes place in the terms of trade. Consider again Figure III and the set of offer curves determining equilibrium. These offer curves must be derived from trade indifference curves rather than consumption indifference curves, since production was assumed to be variable. Now if a transfer of OA of machinery is made from A to B and a transfer of \( O'A \) of cotton is made from B to A, it is possible to derive offer curves from the point \( O' \); and, since trade rather than consumption indifference curves are used, the degree to which the Engel's curves diverge reflects not only the qualitative direction of change of the terms of trade but also the quantitative intensity of that change. So much for income transfers.
With a capital transfer it is permissible to take $O'$ as the new origin only if the terms of trade do not change. Since world supply is unchanged at constant prices, it is easy to see that the terms of trade will be changed if demand changes; and, since supply will always move at increasing cost in the direction in which demand changes, demand criteria alone are sufficient to establish the direction of change in the terms of trade. However, when the terms of trade actually change, demand conditions will not reflect the quantitative intensity of that change because of substitution effects in production.

And if production does change because of relative price changes, then OM can no longer reflect the income transfer implicit in the capital transfer. These effects cannot be grouped in the same diagram with income transfers and be expected to reflect the degree to which production and prices change because the trade indifference curves drawn up from the different production blocks will be different.

It remains true, however, that the criterion for the direction of change is the same for an income transfer as for a capital transfer. Figure III can be used effectively to illustrate the effects of a capital loan, analyzed in Section I. The effect of the capital loan is to increase income in B by OC; but this is the marginal product of the capital which must be returned to capitalists in A. B therefore, after receiving the loan, must make a transfer to A equal in value to OC of cotton, so equilibrium must remain at $P$. 
B. Capital Transfer

1. Impediments

We shall now consider the effects of introducing impediments to the transfer of capital from A to B. Although the explicit impediment will be treated as a cost of transport, it could apply as well to banking charges, etc., or to a difference in marginal productivities between industries in A and B due to, say atmospheric conditions.

We shall assume that the cost of transport involves a proportion of the capital which leaves A being used up en route.\(^1\) Let us call \(a_x\) the proportion of the capital leaving A which arrives in B. Under the same assumptions where A exports machinery and B exports cotton before and after the transfer, B's income will increase and A's income will decrease as before; but now, because of the cost of transport, the increase in income in B will be less than the decrease in income in A at constant prices -- the difference will be the marginal product of the capital which is used up as transport cost. In order to show the effects of the terms of trade, then, considering demand and supply changes in each country at constant prices, it will be necessary to consider absolute as well as relative changes in demand.

In order to absorb new capital at constant prices and constant factor proportions, the output of (capital-intensive) machinery must increase; in order for machinery output to increase, some labor (hence some capital) must be released from cotton production. The net result is that machinery output increases and cotton output decreases. The

\(^1\)The author, of course, does not treat this assumption too seriously, but it does hint at some important problems concerning costs of transporting capital. Its obvious limitation is that it considers the transport industry as completely capital-intensive -- hence the effect is to tend to increase the marginal product of capital and thus the terms of trade of the country exporting capital-intensive products.
opposite happens in A: cotton production increases in order to absorb the labor left idle (at constant factor prices) by the exodus of capital from machinery production. Now we know that if the increase in capital in B is equal to the decrease in capital in A, output changes in each country will cancel out. But our assumption that only a proportion \( a_x \) of the capital leaving A reaches B means that, at constant prices, the increase in output of machinery in B will only be \( a_x \) times the decrease in output of machinery in A; and the decrease in output of cotton in B will be only a proportion, \( a_x \), of the increase in output of cotton in A. We know therefore that the world output of cotton must rise and the world output of machinery must fall absolutely at constant prices. The presumption certainly exists then that, in this example, the world price of cotton will fall in relation to the world price of machinery; although strongly biassed demand conditions could offset the supply bias.

A criterion can easily be developed to show the conditions under which the price of cotton falls relative to machinery. This is done in the following way:¹

Let \( R_a \) = the value of the marginal product of capital in A,

\[ R_b \] = the value of the marginal product of capital in B,

\( Ca \) = the proportion of capital returns to total earnings in cotton in A,

\( Ma \) = the proportion of capital returns to total earnings in machinery in A,

¹The following technique is used by Professor Meade in his Trade and Welfare, Appendix 9, and in his Trade and Welfare Mathematical Supplement, Section XIX. Meade does not consider impediments to capital movements. His analysis is limited, while mine is not, to the assumption that import propensities are the same in both countries.
\[ \delta_b = \text{the proportion of capital returns to total earnings in cotton in } B, \]

\[ \delta_b = \text{the proportion of capital returns to total earnings in machinery in } B, \]

\[ P_{ca} \cdot dC_a = \text{the increased value of cotton output in } A, \]

\[ P_{cb} \cdot dC_b = \text{the increased value of cotton output in } B, \]

then \[ R_a \cdot dC_P = \text{the increased value of machinery output in } A, \]

and \[ R_b \cdot dC_P = \text{the increased value of machinery output in } B. \]

Also let \[ \frac{dk_b}{a_x} = \text{the capital which leaves } A, \]

\[ a_x = \frac{dk_b}{dka}. \]

Now the total increase in capital returns in A is equal to the sum of changes in returns to capital in cotton and machinery, i.e.,

\[ R_a \cdot \frac{dk_b}{a_x} = \Gamma_a \cdot P_{ca} \cdot dC_a + \Gamma_{ma} \left( R_a \cdot \frac{dk_b}{a_x} - P_{ca} \cdot dC_a \right) \]

so \[ P_{ca} \cdot dC_a = -R_a \cdot \frac{dk_b}{a_x} \left( \frac{1 - \Gamma_{ma}}{\Gamma_a - \Gamma_{ma}} \right). \]

Similarly in B,

\[ R_b \cdot dk_b = \Gamma_b \cdot P_{cb} \cdot dC_b + \Gamma_{mb} \left( R_b \cdot dk_b - P_{cb} \cdot dC_b \right) \]

\[ P_{cb} \cdot dC_b = R_b \cdot dk_b \cdot \frac{1 - \Gamma_{mb}}{\Gamma_b - \Gamma_{mb}}. \]

Now, because factor and commodity prices are equalized before the capital transfer, and we are considering changes at constant prices, then
\[ \frac{1 - \Gamma_{ma}}{\Gamma_{ca} - \Gamma_{ma}} = \frac{1 - \Gamma_{mb}}{\Gamma_{cb} - \Gamma_{mb}} = \Delta. \]

\( \Delta \) is negative because \( \Gamma_{ca} (= \Gamma_{cb}) < \Gamma_{ma} (= \Gamma_{mb}) \)
(i.e., cotton production is less capital-intensive than machinery production);

and \( R_b = R_a = R \) and \( p_{ca} = p_{cb} = p_c \)

The change in world value of cotton production is

\[ p_c \cdot dC = p_c (dC_a - dC_b) = R_a \cdot \frac{dK_b}{a_x} \cdot \frac{1 - \Gamma_{ma}}{\Gamma_{ca} - \Gamma_{ma}} - R_b \cdot dK_b \cdot \frac{1 - \Gamma_{mb}}{\Gamma_{cb} - \Gamma_{mb}}. \]

The change in world machinery production value is:

\[ p_m \cdot dM = \frac{dK_b}{a_x} \cdot R_a - p_c \cdot dC_a - dK_b \cdot R_b + p_c \cdot dC_b \]

\[ = \frac{dK_b}{a_x} \cdot R_a - dK_b \cdot R_b + R_b \cdot dK_b \cdot \Delta(1 - \frac{1}{a_x}) \]

Adding the change in world cotton production value and the change in world machinery production value, we get the change in world income at constant prices.

\[ p_c \cdot dC + p_m \cdot dM = - R \cdot dK_b \cdot \Delta(1 - \frac{1}{a_x}) + R \cdot dK_b \cdot \Delta(1 - \frac{1}{a_x}) \]

\[ + R_a \cdot \frac{dK_b}{a_x} - dK_b \cdot R_b. \]

\[ = R \cdot dK_b \cdot (1 - \frac{1}{a_x}). \]

of which \( dK_b \cdot R \) is B's increase in income and \( \frac{R}{a_x} \cdot dK_b \) is A's increase in income.
If $\Pi_{ca}$ is A's marginal propensity to consume cotton, and $\Pi_{cb}$ is B's marginal propensity to consume cotton, then the increase in world demand for cotton is:

$$\Pi_{cb} \cdot R \cdot d K_b - \Pi_{ca} \cdot \frac{d K_b}{a_x} \cdot R_a.$$ 

The increase in world supply of cotton at constant prices is:

$$- \Delta \cdot R \cdot d K_b \left(1 - \frac{1}{a_x}\right).$$

The excess demand for cotton is therefore

$$R \cdot d K_b \left(\Pi_{cb} - \frac{\Pi_{ca}}{a_x}\right) - R \cdot d K_b \cdot \Delta \left(1 - \frac{1}{a_x}\right).$$

The price of cotton will rise or fall depending on whether

$$R \cdot d K_b \left[\Pi_{cb} - \frac{\Pi_{ca}}{a_x} - \Delta \left(1 - \frac{1}{a_x}\right)\right] \geq 0.$$ 

\[ \text{i.e., whether} \]

$$\left[\Pi_{cb} - \frac{\Pi_{ca}}{a_x} - \Delta + \frac{\Delta}{a_x}\right] \geq 0. \quad \text{or,}$$

$$\Pi_{cb} - \frac{\Pi_{ca}}{a_x} \geq \Delta - \frac{\Delta}{a_x}.$$

Now $a_x$ is less than unity, so $\frac{\Pi_{ca}}{a_x} > \Pi_{ca}$ and $\Delta < \frac{\Delta}{a_x}$

but since $\Delta$ is negative, the right-hand argument is positive.
Therefore $\frac{\Pi_b - \frac{\Pi c_a}{a_x}}{\Pi}$ must be at least positive for the price of cotton to rise. The factor $a_x$ on the demand side, less than unity, is a factor tending to make the price of cotton fall, as is $a_x$ on the supply side. Consider a special case where $\Pi c_b = \Pi c_a = \Pi_j$; then the criterion becomes whether
\[\Pi (1 - \frac{1}{a_x}) \geq (1 - \frac{1}{a_x}) \Delta\]
(i.e., whether $\Pi \geq \Delta$)
but $\Pi$ is positive in the absence of inferior goods and $\Delta$ is negative so the price of cotton must fall.

From the definition of $\Delta$ it is obvious that the smaller is $\Gamma_m$ (the less capital-intensive is machinery and the more capital-intensive is cotton, the greater the tendency for the price of cotton to fall). The more that output of machinery must change in A and B in order to absorb in constant factor proportions the new amounts of capital displaced.

2. Impediments to Commodity Movements

This section will be concerned with capital transfers and the terms of trade when differences in commodity prices exist. These price differences may be due to tariffs, transport costs or some other impediment; but in this section we shall be concerned exclusively with transport costs, an impediment which any welfare policy will treat as data.

With transport costs, prices will differ by a constant factor. As in our analysis of the Transfer Problem, we shall assume that part
of the export good is used up as costs of transport. When \( k_c \) and \( k_m \) are the proportions of cotton and machinery used up, price ratios differ by the constant factor \( k_c k_m \).

Equilibrium is illustrated in Figure IV. \( Q_a \) and \( Q_b \) are the offer curves of A and B; the transport-modified offer curves (not drawn) intersect at P, so \( Q_a \) and \( Q_b \) are the equilibrium trading points, permitting consumption points at \( C_a \) in the northwest and \( C_b \) in the southeast quadrants for A and B, respectively. Price ratios are \( a' \) and \( b' \) parallel to \( a \) and \( b \) in the trading quadrant. The rectangle \( PQ_a^*a'Q_b \) (shaded) is used up as transport costs.

Now suppose that A makes a capital transfer to B. A's production block contracts and B's production block expands. As before, we can derive criteria for changes in the terms of trade by considering supply and demand changes at constant prices. Now, however, the analysis is made more difficult because we must consider expansion of production and consumption at different price ratios in A and B. Considering first supply changes, we must determine the direction and slope of the R-lines extending, in Figure IV from \( P_a \) and \( P_b \).

The problem of output changes at constant prices is reproduced in Figures Va and Vb. \( T_a'T_a \) is A's pre-transfer and \( T_a'T_a' \) is A's post-transfer production block; similarly, \( T_b'T_b \) is B's pre-transfer and \( T_b'T_b' \) is B's post-transfer production block. Equilibrium is at \( P_a \) and \( P_b \). The problem is to determine the shape of the new production blocks and, specifically, the slope of the R-line to the left of \( P_a \) and to the right of \( P_a \).
We know, of course, from the Rybczynski proof that more machinery and less cotton will be produced in B, and that less machinery and more cotton will be produced in A. But the price of machinery relative to cotton is higher in B than in A because of transport costs; if prices were the same, the output changes would exactly offset one another. The problem is therefore: will the increase in machinery output in B be greater or less than the decrease in machinery output in A; and will the decrease in cotton output in B be greater or less than the decrease in cotton output in A?

Turn now to Figure VI; it is similar to the figure used in proving that at constant prices, output changes would be offsetting. The difference is that now, since commodity prices differ in A and B, factor prices must differ. It will therefore be more economical within each country to use different factor proportions. Now the relative price of machinery, the capital-intensive good, is higher in B than in A, so the ratio of capital returns relative to wages must be higher in B than in A. In that case, the labor-capital ratios in B in both industries must be higher than the labor-capital ratios in A in both industries. In Figure VI B's pre-transfer equilibrium is at $P_b$ along $OO_b$, B's contract curve; A's pre-transfer equilibrium is at $P_a$ along $OO_a$, A's contract curve. The difference in slopes from $P_a$ and $P_b$ shows the difference in factor proportions in each industry.

Now A transfers $K_aK_a'$ of capital to B. $K_aK_a'$ equals $O_bO_b'$. Equilibrium moves to $P_a'$ in A where less machinery and more cotton
is produced; and equilibrium moves from $P_b$ to $P_b'$ in $B$ where more machinery and less cotton are produced. The difficulty that now presents itself is evident -- it is no longer meaningful to compare the lengths $P_aP_a'$ and $P_bP_b'$ because they represent equilibria at different points on the production functions, at different factor price ratios. Even if we could prove that $P_bP_b'$ is longer than $P_aP_a'$ it would not prove that output of machinery has increased. Nor is it meaningful to compare the ratios of 

$$\frac{P_aP_a'}{P_bP_b'} : \frac{\omega P_a'}{\nu P_b'}$$

(this must be greater than unity), which is not invariant under different commodity units chosen.

Nevertheless it is possible to derive the conditions under which world supply of machinery will increase and world supply of cotton will decrease. We shall argue the result verbally and then derive the criterion algebraically.

Capital is moving from a country of low marginal productivity to a country of high marginal productivity. The value of the marginal product of capital in $B$ must be higher than in $A$, then, so world incomes at constant prices must increase. The value of the marginal product that enters $B$ is equal to the value of the increase in machinery output less the value of the decrease in cotton output; and this must be greater than the increase in the value of cotton output less the value of the decrease in machinery output in $A$. 
Now in order to absorb more capital in B at constant factor proportions, labor and capital flow out of cotton into machinery; and in order to keep factor proportions constant in A, labor and capital must flow out of machinery into cotton. But because cotton and machinery are both produced in a more labor-intensive way in B than in A, the increase in machinery output in B is probably greater, because for every unit of capital absorbed into machinery, a larger amount of labor is required as its complement. This is a factor tending to make the increase in machinery output greater and also the decrease in cotton output greater in A than in B. Also, the marginal product of the capital used in B in increasing machinery output is greater than that used in A in decreasing machinery output, so this factor would also contribute in making the increase in machinery output in B greater than the decrease in machinery output in A. There is another effect that must be considered which acts in the opposite direction. As cotton output decreases in B, to release labor to combine with the new capital in the machinery industry, only a small amount of capital is released with it so the machinery industry will not have to expand, and the cotton industry will not have to contract, so much on this account.

The net effect of these changes, the most important of which is the higher marginal productivity of capital in B, should be to increase machinery output in B more than the decrease in A; and cotton output in B should decrease more than it increases in A. If this happens, the world output of machinery would increase and the world output of cotton
would decrease -- cet. par. then, the price of machinery should fall relative to the price of cotton.

If this conclusion is general, it means that a transfer of capital from a country of low productivity to a country of high marginal productivity will increase the world output of the capital-intensive good and decrease the world output of the labor-intensive good, tending to cause a deterioration of the terms of trade of the country exporting the capital-intensive good -- i.e., the capital-abundant country, A.

But to test the generality of this proposition we must derive the criterion for output changes after capital transfer. It is possible in this real model to test this proposition in terms of the excess demand for one commodity at constant prices. The analysis is similar to that of the previous case, although now we must distinguish between prices, factor proportions and marginal productivities in each country.

Using the same symbols as in the previous section, the value of the marginal product of the capital entering B is

$$R_b \cdot dK_b = \Gamma_{cb} \cdot P_{kb} \cdot dC_b + \Gamma_{mb} \left( R_b \cdot dK_b - P_{cb} \cdot dC_b \right)$$

so the value of the change in cotton output in B is

$$P_{cb} \cdot dC_b = R_b \cdot dK_b \cdot \frac{1 - \Gamma_{mb}}{\Gamma_{cb} - \Gamma_{mb}}$$

which is obviously negative, since \( \Gamma_{mb} > \Gamma_{ma} \).
Similarly, the value of the marginal product of the capital leaving A is

\[ R_a \cdot dK_a = \Gamma_{ca} \cdot p_{ca} \cdot dc_a + \Gamma_{ma} (R_a \cdot dK_b - p_{ca} \cdot dc_a) \]

so the value of the change in cotton output in A is

\[ p_{ca} \cdot dc_a = R_a \cdot dK_a \cdot \frac{1 - \Gamma_{ma}}{\Gamma_{ca} - \Gamma_{ma}}. \]

\[ \Gamma_{ma} > \Gamma_{ca} \text{ but } dK_a = -dK_b < 0 \]

so the increase in cotton output in A is positive.

At constant prices the change in value of world cotton output is

\[ p_{ca} \cdot dc_a + p_{cb} \cdot dc_b = R_a \cdot dK_a \cdot \frac{1 - \Gamma_{ma}}{\Gamma_{ca} - \Gamma_{ma}} + R_b \cdot dK_b \cdot \frac{1 - \Gamma_{mb}}{\Gamma_{cb} - \Gamma_{mb}} \]

\[ = [R_b \cdot \frac{1 - \Gamma_{mb}}{\Gamma_{cb} - \Gamma_{mb}} - R_a \cdot \frac{1 - \Gamma_{ma}}{\Gamma_{ca} - \Gamma_{ma}}] \cdot dK_b. \]

The value of world cotton output will decrease if

\[ R_a \cdot \frac{1 - \Gamma_{ma}}{\Gamma_{ca} - \Gamma_{ma}} > R_b \cdot \frac{1 - \Gamma_{mb}}{\Gamma_{cb} - \Gamma_{ma}} \]

i.e., if

\[ R_b \cdot \frac{1 - \Gamma_{mb}}{\Gamma_{mb} - \Gamma_{cb}} > R_a \cdot \frac{1 - \Gamma_{ma}}{\Gamma_{ma} - \Gamma_{ca}} \quad (\text{since } \Gamma_{cb} < \Gamma_{mb}, \text{ etc}) \]

Nothing a priori, as far as I know, can be said about whether

\[ \Gamma_{mb} - \Gamma_{cb} \geq \Gamma_{ma} - \Gamma_{ca}. \]

(the difference between the proportions of capital returns in machinery and cotton in A and B).
But if this difference is "neutral," the value of world cotton output would decrease, since \( R_b > R_a \) and \( 1 - \Gamma_{ma} > 1 - \Gamma_{mb} \). We should expect, then, that the world output of cotton would decrease.

The exact condition for it to decrease is that

\[
\frac{R_b}{R_a} \cdot \frac{1 - \Gamma_{mb}}{1 - \Gamma_{ma}} > \frac{\Gamma_{mb} - \Gamma_{cb}}{\Gamma_{ma} - \Gamma_{ca}}.
\]

Now the change in world income must be \( dK_a \cdot R_a + R_b \cdot dK_b \).

The increase in demand for cotton in A is therefore, at constant prices,

\[
K_c \cdot \Pi_a \cdot dK_a \cdot R_a + \Pi_b \cdot dK_b \cdot R_b.
\]

(it will be remembered that \( k_c \) is the proportion of cotton leaving B which reaches A).

The price of cotton will rise if there is an excess demand for cotton after the capital movement -- i.e., if

\[
d K_b \left( \Pi_b \cdot R_b - K_c \cdot \Pi_a \cdot R_a \right) = dK_b \left[ \frac{R_b}{\Gamma_{cb} - \Gamma_{ma}} - \frac{R_a}{\Gamma_{ca} - \Gamma_{ma}} \right]
\]

i.e., if

\[
\Pi_b \cdot R_b - \Pi_a \cdot K_c \cdot R_a > R_b \cdot \frac{1 - \Gamma_{mb}}{\Gamma_{cb} - \Gamma_{ma}} - R_a \cdot \frac{1 - \Gamma_{ma}}{\Gamma_{ca} - \Gamma_{ma}}.
\]

The larger is \( \Pi_b \) and the smaller is \( K_c \cdot \Pi_a \), the greater the tendency for the price of cotton to rise. The larger is \( k_c \), the cost of transporting cotton, the greater the tendency for the price of cotton to fall.
Now it cannot be concluded, as suggested above, that the world supply of cotton will definitely decrease, even if a presumption exists for a change in that direction. But it can be concluded that, if the world supply of cotton does decrease, then the world supply of machinery must increase. This follows from the identity of income and expenditure; since world income increases by $dK_b(R_b - R_a)$, which is positive, then the excess value of increased machinery production over the value of decreased cotton production must equal the increase in world income; so at constant prices, if cotton production declines, machinery production must increase.

**Fixed Coefficients**

Consider as a special case, in order to eliminate production changes and thus isolate the effects of transport costs, the case where production coefficients are fixed. This results in a kinked contract curve as in Figure VII, which permits differences in marginal value products without differences in factor proportions. We assume that there is no unemployment of either factor, so production in each country must take place at the kinked part of the contract curves, $P_a$ and $P_b$. Now a capital transfer is made and equilibrium moves to $P_a'$ and $P_b'$. Clearly no change in world output takes place (this is the same result as when substitution is allowed but output changes are made at constant prices). Now the slopes at $P_a$, $P_a'$, $P_b$, $P_b'$ indicate the marginal value product ratios of labor and capital. We assume that commodity prices differ because of transport costs; marginal physical products of machinery and
Figure VII
cotton are the same in A and B, so marginal value products differ as shown by the slopes $b, b', a', a$.

Since world outputs do not change, the criterion for changes in the terms of trade can be derived from demand conditions alone. Excess demand for cotton is

$$\Pi_{cb} R_b - k_c \Pi_{ca} R_a > \frac{1 - \Gamma_m}{\Gamma_c - \Gamma_m} R_b - \frac{1 - \Gamma_m}{\Gamma_c - \Gamma_m} R_a.$$

Since factor proportions are the same in A and B, this is equivalent to whether

$$\Pi_{cb} R_b - k_c \Pi_{ca} R_a > \frac{1 - \Gamma_m}{\Gamma_c - \Gamma_m} (R_b - R_a).$$

Since $R_a = \frac{2c_a}{2k} P_{ca}$ and $R_b = \frac{2c_b}{2k} P_{cb}$ and $\frac{2c_a}{2k} = \frac{2c_b}{2k} = \frac{2C}{2k}$

then $\frac{2C}{2k} (P_{cb} \Pi_{cb} - k_c \Pi_{ca} P_{ca}) > \frac{2C}{2k} (P_{cb} - P_{ca}) \frac{1 - \Gamma_m}{\Gamma_c - \Gamma_m}$.

Since $P_{cb} = k_c P_{ca}$ then this is equivalent to whether

$$P_{cb} (\Pi_{cb} - \Pi_{ca}) > \left(\frac{P_{cb} - P_{cb}}{k_c}\right) \frac{1 - \Gamma_m}{\Gamma_c - \Gamma_m}$$

or $\Pi_{cb} - \Pi_{ca} > \left(1 - \frac{1}{k_c}\right) \frac{1 - \Gamma_m}{\Gamma_c - \Gamma_m}$

$$\Pi_{cb} - \Pi_{ca} > \left(\frac{1}{k_c} - 1\right) \frac{1 - \Gamma_m}{\Gamma_m - \Gamma_c}.$$. 
The right-hand side is positive; $K_c$ operates to increase the right-hand side, so it is a factor tending to depress the relative price of cotton in both $A$ and $B$. This conforms to commonsense, since cotton resources are saved through a reduction in transport costs.
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Biographical Note

The author, Robert Alexander Mundell, was born on October 24, 1932, in Kingston, Ontario; attended a series of public schools in Kingston and environs; enrolled in the Kingston Collegiate-Vocational Institute before moving to British Columbia, where he graduated from the Maple Ridge High School in June, 1949.

He majored in Economics and Slavonic Studies at the University of British Columbia, graduated with the degree of Bachelor of Arts in May, 1953. He studied at the University of Washington as a teaching fellow in 1953-54, before enrolling at M.I.T. the next year; at M.I.T. the author completed general examinations in May, 1955. Under a Mackenzie King Travelling Scholarship, he studied at the London School of Economics the next year, returning to M.I.T. in the summer of 1956.

The author was awarded The Post-Doctoral Fellowship in Political Economy at the University of Chicago for the year 1956-57.