MINIMIZING DISTORTION AND CROSSTALK
IN ASYMMETRIC CHANNELING

by

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SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
June, 1953

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ABSTRACT

The analysis of asymmetric filters is carried out on the basis that general modulation can be construed as amplitude modulation of two carriers in quadrature. Since the response of an asymmetric filter to an amplitude-modulated carrier is a generally modulated carrier, it is possible to analyze asymmetric filters in terms of the direct and quadrature response characteristics.

The problem of minimizing distortion and crosstalk becomes one of idealizing the direct response characteristics as much as possible and of minimizing the amplitude response of the quadrature response characteristics.

In Part 1 of this thesis, techniques for finding the direct and quadrature poles and zeros from the poles and zeros of the asymmetric filter are developed, and it is found that the process can be made one of expanding the transfer function for the asymmetric filter into real and imaginary functions of the complex frequency variable, s, by properly interpreting general modulation as complex modulation.

Part 2 is concerned with the analysis of a typical asymmetric filter problem presently of importance. The NTSC chroma channel is necessarily asymmetric yet is required to pass general modulation with a minimum of distortion and crosstalk. The problem is explored in terms of the methods developed in Part 1 and a definite contribution is made toward clarifying the issues of importance. Unfortunately, the problem is still quite flexible and is of such great scope that it is impossible to attempt anything but a design suggestion.

Thesis Supervisor: Ronald E. Scott
Title: Assistant Professor of Electrical Engineering
PREFACE

The greatest use for the asymmetric channeling of information has perhaps been brought about by television, where the use of vestigial channeling results in an effective doubling of the available spectrum. In this paper, we shall attempt a study of the effects of asymmetric filtering and the possibilities of minimizing the undesirable distortions introduced thereby.

Perhaps the major difficulty in the treatment of asymmetric filters is the lack of adequate information on the subject. Although various authors have treated the asymmetric filter, they have restricted their analyses to applications involving amplitude modulation only and have attacked the problem too directly. The direct approach, in this case, leads almost inevitably to complications so that the value of such analyses to a practicing engineer becomes questionable.

The primary purpose of this thesis is to place the theory and analysis of asymmetric filters on the more stable and workable Laplace and Fourier methods. Part 1 of this thesis is devoted to that purpose. Other useful techniques are developed so that insight may be gained into the limitations imposed by given restrictions on a given filter.

We shall also be concerned, in Part 2, with the presentation, analysis, and formulation of conclusions regarding a typical asymmetric filter problem - the NTSC chroma channel.

Sergio F. Valdes

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ACKNOWLEDGMENT

This thesis was prepared and written in the Philco Research Laboratories under a co-operative agreement between Philco Corporation and the Massachusetts Institute of Technology.

I am indebted to Mr. Monte I. Burgett, Project Engineer of the Philco Corporation, for his suggestion of this thesis topic and for his interest and guidance throughout the work involved and to Mr. Glenn W. Preston, Consulting Engineer of the Philco Corporation, for his many valuable suggestions and for his help in simplifying the proofs of some necessary relations.

It is with pleasure that I extend my gratitude to my supervisors, Mr. Joseph Tellier, Section Engineer of the Philco Corporation, and Professor Ronald E. Scott of the Massachusetts Institute of Technology, for their assistance and encouragement, without which this thesis could hardly have been done.

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EXPLANATORY NOTE

In the text, transfer function is used freely to denote transfer impedance, transfer admittance, transfer voltage ratio, or transfer current ratio. The exact type of function is immaterial to the analysis. In problems of synthesis where the exact meaning of the term is of importance, transfer impedance has been used.

The complex frequency variable $s = \sigma + j\omega$ is used throughout the text in agreement with the convention used by Gardner and Barnes.

In general, all forms of the letter $h$ refer to the transfer function, $f$ to the input, and $g$ to the output. The subscripts 1 and 2 are used to denote "direct" and "quadrature" respectively.
PART 1

ASYMMETRIC FILTER ANALYSIS
I. GENERAL SURVEY

The main objective of this thesis is to discuss the minimization of distortion and crosstalk in asymmetric filters and channels. The material presented in Section I is a review of the fundamental aspects of modulation and filtering as applied to asymmetric filters. More complete presentations of these fundamentals have been made*, but this section serves to unify and complete the contents of this thesis.

A. TYPES OF MODULATION INTERPRETATION

A filter can only be asymmetric to modulation. Any physical filter is necessarily symmetric** about the frequency axis origin because only real numbers describe the values of physical elements. The coefficients of the integro-differential equations which describe the transfer impedance, or transfer function, of a physical network are therefore real, and this results in a symmetric filter about the frequency scale origin.

A modulation signal, on the other hand, can be centered about any frequency, as can the pass band of a bandpass filter. If we wish only to examine the modulation response of a bandpass filter, the modulation carrier frequency can be considered the origin of a new frequency scale. With respect to such a scale, the pass band of the filter becomes an equivalent low-pass filter, although in general it will be asymmetric. The amplitude need no longer be an even function of frequency, and the phase need no longer be an odd function of frequency.


** By symmetric we mean that the amplitude is an even function of frequency and that phase is an odd function.
Unless otherwise indicated, all references to frequency in this thesis shall be understood to be with respect to the carrier radian frequency (hereafter $\omega_c$) as zero. In all vector diagrams, the coordinates will implicitly be rotating at a constant carrier reference frequency.

**Figure I-1**

**Figure I-2**

Figure I-1 shows the usual type of modulation imposed on a carrier. Each point in the complex plane is described by two coordinates, magnitude ($M$) and phase ($\phi$). The use of amplitude and phase modulation of the vector $\mathbf{C}$ to produce the vector $\mathbf{P}$ serves to convey the position and time sequence of a set of points anywhere on the plane.

The use of quadrature carrier vectors can also be made to convey the same information. Figure I-2 shows the vector diagram of two carriers of the same frequency but $90^\circ$ out of phase. Amplitude modulation of the two carriers then conveys the rectangular coordinates of the set of points.

The systems, of course, are equivalent; their difference lies only in the type of interpretation to be used at the receiver. It is easier to analyze asymmetric filters in terms of quadrature vector modulation, although the usual practice is to refer to modulation in terms of amplitude and phase.

Conversion between the two systems is accomplished by the use of the familiar polar:rectangular conversion equations.
The amplitude and phase modulated signal is of the form

\[(I-1) \quad M(t) \cos \left(\omega_c t + \Phi(t) \right) = S(t),\]

and the quadrature carrier signal form is

\[(I-2) \quad F(t) \cos \omega_c t + Q(t) \sin \omega_c t = S(t)\]

where a constant phase has been ignored. Equations (I-3) and (I-4) are then the conversion relations.

\[(I-3) \quad M(t) = \sqrt{F^2(t) + Q^2(t)} \quad \text{and} \quad \Phi(t) = \tan^{-1} \frac{Q(t)}{F(t)}\]

\[(I-4) \quad F(t) = M(t) \cos \Phi(t) \quad \text{and} \quad Q(t) = M(t) \sin \Phi(t)\]

Modulation of the form (I-2) shall be used henceforth in this report for reasons that will become progressively obvious.

**B. EFFECTS OF ASYMMETRY**

In the following discussion, the input signal to the filters will be \(f_1(t) \cos \omega_c t\) where, for simplicity, \(f_1(t) = 2a \cos \omega_m t\). The input-signal vector diagram is shown in Figure I-3.

![Input Signal Diagram](image)

**INPUT SIGNAL**
**FIGURE I-3**

![Symmetric-Filter Characteristics Diagram](image)

**SYMMETRIC-FILTER CHARACTERISTICS**
**FIGURE I-4**

![Output Signal Diagram](image)

**OUTPUT SIGNAL**
**FIGURE I-5**

Figure I-4 shows the amplitude and phase characteristics of a symmetrical filter with the modulation input spectrum superposed. The symmetric filter affects the vectors in the manner shown in Figure I-5. The resultant vector differs from the input because of amplitude and phase distortion. The resultant, however, remains on the real axis and no quadrature component results. The output remains as modulation on a \(\cos \omega_c t\) carrier.
1. Amplitude Asymmetry

The filter spectrum of Figure I-6 has a phase characteristic that is zero everywhere, and the amplitude characteristic is not an even function of frequency about the origin. Superposed on the filter spectrum is the line spectrum of the input signal.

The result of passing the signal through the filter is shown in Figure I-7. The side-band vectors are of different lengths, and their resultant is a vector which traces out an ellipse during each period as shown.

If we express the output signal of the filter in terms of \( A_u \) and \( A_l \), the upper and lower amplitude characteristics respectively, we obtain;

\[
(I-5) \quad \text{Output} = g(t) = g_1(t) \cos \omega_c t + g_2(t) \sin \omega_c t,
\]

where

\[
(I-6) \quad g_1(t) = (A_u + A_l) a \cos \omega_m t \quad \text{and} \quad \]
\[
(I-7) \quad g_2(t) = (A_u - A_l) a \sin \omega_m t.
\]

The asymmetry of the amplitude characteristic has introduced two effects. The direct output, \( g_1(t) \), has amplitude distortion if \( A_u + A_l \) varies as a function of frequency, and where the coefficient of \( \sin \omega_c t \) is zero at the input, it is a function of \( f_1(t) \) at the output. The latter effect is crosstalk.

2. Phase Asymmetry

Consider now the filter of Figure I-8. The amplitude characteristic
is everywhere equal to unity, but the phase is as shown. Figure I-9 shows the effect produced by the lack of skew-symmetry in the phase characteristic to the input signal $2a \cos \omega_m t \cos \omega_c t$.

\[ \begin{align*}
\phi_u & \quad \phi_l \\
-\omega_m & \quad \omega_m
\end{align*} \]

**ASYMMETRIC PHASE FILTER**

**VECTOR RESPONSE**

**FIGURE I-8**

**FIGURE I-9**

The effect of the phase asymmetry is that the resultant of the two sideband vectors has been rotated by an angle $(\phi_u + \phi_1)/2$; where the resultant represents $2a \cos \omega_m t$ at the input, it is $2a \cos(\omega_m t + \frac{\phi_u - \phi_1}{2})$ at the output.

The output is:

(I-9) $g_1(t) \cos \omega_c t + g_2(t) \sin \omega_c t$, where

(I-9) $g_1(t) = 2a \cos \left[\frac{(\phi_u + \phi_1)}{2}\right] \cos \left[\omega_m t + \frac{(\phi_u - \phi_1)}{2}\right]$ and

(I-10) $g_2(t) = 2a \sin \left[\frac{(\phi_u + \phi_1)}{2}\right] \cos \left[\omega_m t + \frac{(\phi_u - \phi_1)}{2}\right]$

The subscripts $u$ and $l$ again refer to the upper and lower characteristics respectively.

The effect of phase asymmetry is similar to the effect amplitude asymmetry in that direct distortion and quadrature crosstalk are introduced.

C. **DIRECT AND QUADRATURE EQUIVALENT TRANSFER FUNCTIONS**

If we interpret the quadrature carriers as two available separate lines for the transmission of two independent pieces of information, then Figures I-10 and I-11 can be used to represent the respective analogies of symmetric and asymmetric filtering.
In Figure I-10, the information on each line remains independent of whatever information may be contained in the other, and both are affected by the transfer function, \( H \), of the symmetric filter. Figure I-11, on the other hand, shows that there is mutual coupling existing between the two lines so that the input \( f_1(t) \) produces an output at each of the two lines. With an input \( F_1(t) \), \( g_1(t) \) is then the output of \( H_1 \), and \( g_2(t) \) is the output of \( H_2 \) to the input \( f_1(t) \).

On the basis of the previous arguments contained in sections B.1 and B.2, we see that it is possible to obtain the equivalent direct and quadrature transfer functions, \( H_1 \) and \( H_2 \) respectively, for any given filter in terms of its upper and lower amplitude and phase characteristics.

We shall not attempt here to derive the equivalent transfer functions in terms of the amplitude and phase asymmetry because this has been adequately done in the previously cited references. Suffice it to say that the expressions for \( H_1 \) and \( H_2 \) obtained in terms of the upper and lower amplitude and phase characteristics are rather involved and fail to take advantage of the fact that the amplitude and phase are fundamentally related. It seems reasonable to assume that any means of working with the amplitude and phase characteristics as separate and independent functions must involve some degree of redundancy. There are also
operational difficulties encountered because it is necessary to know the phase corresponding to given amplitude characteristics.

A possible asymmetric filter design technique, developed on the basis of our present knowledge, would then be along the following lines.

D. POSSIBLE DESIGN APPROACH

Suppose that the boundary conditions of an asymmetric filter are given in terms of the amplitude response. As an example, assume that the filter is to have little or no response for all radian frequencies $\omega_{uc}$ above the carrier and $\omega_{1c}$ below the carrier. Now assume that the equivalent quadrature transfer function, $H_2$, is to be minimized in a region $\pm \omega_q$ about the carrier.

Reference to sections B.1. and B.2. shows that both the amplitude and phase asymmetry must be minimized over any region where $|H_2|$ is to have minimum response. Figures I-7 and I-9 bear out the fact that one type of asymmetry cannot be made to cancel out the other because of their fundamentally different natures. Amplitude asymmetry produces a rotating resultant, whereas phase asymmetry shifts the resultant by a fixed angle.

Bode* gives the phase resulting from straight-line approximations of the amplitude characteristics. It is then possible to attack the above problem by using Bode's method of approximating the phase corresponding to given amplitude with straight-line approximations in an effort to minimize both the amplitude and phase asymmetry or to strike a balance between the amplitude and phase asymmetry so that $|H_2|$ will be small over the region $\pm \omega_q$.

Such an attack is based on trial and error, and if a satisfactory solution is found, it exists only in terms of the desired amplitude and phase characteristics so that the asymmetric filter can be realized only after resorting to the approximation problem of network synthesis.

II. **ASYMMETRIC FILTER ANALYSIS**

The purpose of the rest of this report is to develop better tools with which one may approach asymmetric filter problems. The obvious limitations of the method contained in section I.D. make it imperative that a more workable approach be developed if asymmetric filter problems are to be handled properly.

A little thought leads to the conclusion that, if the asymmetric filter be handled in terms of pole and zero theory,* great advantages would result. The singularities of a filter offer a means of lumping the phase and amplitude characteristics, thereby eliminating the necessity for relating the amplitude and phase. There is also the advantage that the results would be in a form that could be readily synthesized. There are many other important advantages, but these alone would suffice to make us seek tools that make use of filter singularities instead of the more direct but cumbersome amplitude and phase characteristics.

A. **REINTERPRETATION OF THE PROBLEM**

Consider a generally modulated carrier as given in equation (I-2),

\[ S(t) = F(t) \cos \omega_c t + Q(t) \sin \omega_c t. \]

If we use complex notation, it is,

\[ (II-1) \quad S(t) = \text{Re} \left[ F(t) - j Q(t) \right] e^{j\omega_c t}. \]

Figure II-1 shows the equivalent vector representation of \( S(t) \), where the real part is to be taken.

---

* References (a), (c), (d), (e), (h), and (k) on "List of References".

-9-
If we wished to detect the modulation, we could use a pair of synchronous
demodulators. One demodulator detects the amplitude modulation on the \( \cos \omega_c t \)
carrier and the other detects the amplitude modulation on the \( \sin \omega_c t \) carrier.

Nothing changes basically, however, if the signal \( S(t) \) is taken as complex
and the demodulators are complex demodulators. If we use complex demodulators,
then there is no need to carry through the "real part of" notation, \( \text{Re} \). One
demodulator detects the coefficient of \( e^{j\omega_c t} \), and the other takes the
coefficient of \( je^{j\omega_c t} = e^{j\omega_c t + \pi/2} \).

We can now normalize the problem so that the carrier frequency \( \omega_c \) is
considered zero reference frequency. The signal is then, (II-2) \( S(t) = F(t) \)
\(- J Q(t) \), and one demodulator detects the real part and the other the
imaginary part of the signal.

In terms of this new system, then, the effect of an asymmetric filter is
that a complex output results from a real input. If the input to the
asymmetric filter is \( f_1(t) \), the output is \( g_1(t) + J g_2(t) \), where we are using
the same notation defined in Section I.

The indication is that an asymmetric filter can be split up into two
filters as in equation (II-3), where \( H \) is an asymmetric filter transfer function.

\[
\text{(II-3)} \quad H = H_1 + JH_2
\]

Clearly, \( g_1(t) \) is the response of \( H_1 \) to \( f_1(t) \) and \( g_2(t) \) is the response
of \( H_2 \) to the input \( f_1(t) \). \( H_1 \) and \( H_2 \) are referred to as the equivalent direct
and quadrature transfer functions respectively.

B. \text{OBTAINING THE EQUIVALENT TRANSFER FUNCTIONS}

Consider some function of time \( K(t) \) which has a transform \( K(\omega) \); that is,

\[
\text{(II-4)} \quad K(t) \longleftrightarrow K(\omega)
\]

The transform of \( K^*(-\omega) \) is then \( K^*(t) \), where \( (\ast) \)
indicates the conjugate function.

-10-
If \( K(\omega) \) is symmetrical about its origin, then \( K(\omega) = K^*(-\omega) \).

(II-5) \( K(\omega) = \int_0^\infty K(t) e^{-j\omega t} dt \), but

(II-6) \( K^*(-\omega) = \int_0^\infty K^*(t) e^{j\omega t} dt = \int_0^\infty K^*(t) e^{-j\omega t} dt \).

If we let \( K(\omega) = K^*(-\omega) \) as required for symmetry, we obtain,

(II-7) \( K(t) = K^*(t) \). Clearly, \( K(t) \) must be real in order that it be equal to its own conjugate.

We can now draw the conclusion that, if the transform of a time function is symmetric, the time function is real; conversely, a real time function gives rise to a symmetric transform.

Now, if \( K(\omega) \) is skew-symmetric, then

(II-8) \( K(\omega) = -K^*(-\omega) \). Setting the integral expressions for these two transforms equal to each other then yields the result that (II-9)

\( K(t) = -K^*(t) \). Equation (II-9) can only be true if \( K(t) \) is imaginary.

We can now state that a skew-symmetric transform results from an imaginary time function and conversely that an imaginary time function gives rise to a skew-symmetric transform. The foregoing arguments are independent of the differences that exist between Fourier and Laplace transforms so that they are applicable to both.

Now consider the application of our results to a filter. If we express a symmetric filter \( H(s) \) in terms of its memory function \( H(t) \), then \( H(t) \) is real by the previous arguments.

(II-10) \( H(s) = \int_0^\infty H(t) e^{-st} dt \)

Now if we shift the symmetric filter along the frequency scale by an amount \( \omega_0 \), then the filter is asymmetric and it can be expressed as

-11-
(II-11) \[ H(s - j\omega_0) = \int_0^\infty H(t) e^{-(s - j\omega_0)t} \, dt \]

\[ = \int_0^\infty \left[ H(t) \cos \omega_0 t + j H(t) \sin \omega_0 t \right] e^{-st} \, dt \]

\[ = \int_0^\infty H_1(t) e^{-st} \, dt + j \int_0^\infty H_2(t) e^{-st} \, dt \]

where

(II-12) \[ H_1(t) = H(t) \cos \omega_0 t \quad \text{and} \quad H_2(t) = H(t) \sin \omega_0 t. \]

The functions \( H_1(t) \) and \( H_2(t) \) are both real. Therefore, \( H(s - j\omega_0) \) has been separated into symmetric and skew-symmetric parts.

(II-13) \[ H_1(s) = \int_0^\infty H_1(t) e^{-st} \, dt \quad \text{and} \]

(II-14) \[ H_2(s) = \int_0^\infty H_2(t) e^{-st} \, dt, \] where,

(II-15) \[ H(s - j\omega_0) = H_1(s) + j H_2(s). \]

Now we can draw the conclusion that any expansion of an asymmetric filter expression into the form \( H_1(s) + j H_2(s) \), such that \( H_1(s) \) and \( H_2(s) \) are symmetric, yields the equivalent direct and quadrature transfer functions respectively for the asymmetric filter.

If we have a filter with a symmetric transfer function \( H(s) \), then the equivalent direct and quadrature transfer functions of the same filter shifted by \( \omega_0 \) are obtained by expanding \( H(s - j\omega_0) \) into real and imaginary functions of \( s \).

1. **Example**

As a simple example, let us find the direct and quadrature transfer functions of a simple one-pole filter.
\[ H(s) = 1/(s + a) \]

\[ H(s - j\omega_0) = \frac{1}{(s + a) - j\omega_0} \]

\[ H(s - j\omega_0) = \frac{s + a}{(s + a)^2 + \omega_0^2} + j\frac{\omega_0}{(s + a)^2 + \omega_0^2} \]

Therefore, we have found,

(II-16) \[ H_1(s) = \frac{s + a}{(s + a)^2 + \omega_0^2} \], and

(II-17) \[ H_2(s) = \frac{\omega_0}{(s + a)^2 + \omega_0^2} \]

Figures II-3, II-4, and II-5 show the pole and zero configurations for the asymmetric filter and the equivalent direct and quadrature transfer functions.

2. Extension

Now it is a simple matter to generalize the procedure, which has so far been restricted to a filter symmetric about some radian frequency \(\omega_0\), to apply to a generally asymmetric filter.

Any filter can be made up of a product of functions such that each
function is symmetric about some radian frequency. Thus, \( H = H_a H_b H_c \ldots H_n \) is a generally asymmetric filter such that each \( H_i \) is symmetric about a radian frequency \( \omega_i \). We can, therefore, obtain \( H_{11} + j H_{21} \) for each \( H_i \) and then obtain \( H_1(s) + j H_2(s) \) for the total filter function by finding the real and imaginary parts of the product \( \prod_{i=2}^{n} (H_{11} + j H_{21}) \).

Since a single pole or a single zero is symmetrical about some radian frequency, we see that we could ultimately treat each pole and zero separately to cover any possible asymmetric filter function.

It is worth pointing out here that a single pole (or zero) can be considered to be perfectly symmetrical about its center frequency, although in reality its amplitude characteristic is not symmetric because of the action of the conjugate pole (or zero) on the opposite side of the true frequency origin. It can be considered symmetrical because the modulating signals are affected in the same manner as are the singularities of the filter. This is quite obvious in the case of an infinite period modulating function which is transformable and yields a set of poles and zeros. The poles and zeros of the transformed function also have conjugate singularities on the opposite side of the true frequency origin, and the amplitude characteristics are therefore not truly symmetrical, although they must be symmetrical for our purposes if the modulating function is real.

If the amplitude and phase characteristics of a single pole exactly centered at the carrier frequency were analyzed by the methods of Section I, a quadrature response would result because of the slight asymmetry of the amplitude characteristic. There is no quadrature response, however, because the transformed signal input, although real, is not exactly symmetrical either. The output is also slightly asymmetrical by the reasoning of Section I, yet it
must be the transform of a real signal by the reasoning of the present section. Now we see that the method thus far developed is superior to methods which use the upper and lower amplitude and phase characteristics because we are not led astray by asymmetry introduced by the proximity of the conjugate singularities. Such asymmetry is not true asymmetry in the sense that it does not produce a quadrature response function.

C. PHASE NORMALIZATION

Up to this point, we have considered the direct and quadrature response characteristics for unphased demodulators. We should, however, take into account the fact that the demodulators may be "rotated" or phased. Phasing the demodulators by an angle $\phi$ is equivalent to rotating the reference real and imaginary axes by an angle $\phi$. That is, instead of taking the coefficients of $e^{j\omega ct}$ and $e^{j(\omega ct + \pi/2)}$ in the process of demodulation, we could take the coefficients of $e^{j(\omega ct + \phi)}$ and $e^{j(\omega ct + \pi/2 + \phi)}$.

Although the phase of the demodulators does not change the properties of the filter, it is often desirable to lump the phase of the demodulators with the filter. The expressions that result for the direct and quadrature transfer functions are not the equivalent functions for the filter, but rather composite functions that can be interpreted as equivalent transfer functions to the output of the demodulators. Usually, it is desirable to correct the phase of the demodulators so that one of the equivalent transfer functions has zero dc response (a zero at the origin).

If we multiply the filter function by $e^{j\phi}$, then the equivalent direct and quadrature transfer functions obtained in the expansion correspond to the transfer functions of the demodulators when they have been phased by $-\phi$. In multiplying the filter transfer function by $e^{j\phi}$, we are recognizing the fact that the direct equivalent transfer function of the filter as interpreted by
the demodulators is no longer a transfer function that produces a response along the real axis, but rather one that yields a response at an angle $-\phi$ from the real axis. The same interpretation applies to the quadrature response and the imaginary axis.

If we define $H'_1(s)$ as the equivalent direct transfer function as interpreted by the phased demodulators and $H'_2(s)$ as the equivalent quadrature transfer function, then

$$(II-18) \quad H'_1(s) + jH'_2(s) = \left[H_1(s) + jH_2(s)\right] \left(\cos \Phi_o + j\sin \Phi_o\right)$$

$$(II-19) \quad \begin{cases} H'_1(s) = H_1(s) \cos \Phi_o - H_2(s) \sin \Phi_o \\ H'_2(s) = H_1(s) \sin \Phi_o + H_2(s) \cos \Phi_o \end{cases}$$

In the usual case, the demodulators are phased so that the quadrature response for dc input is zero.

$H'_2(0) = 0 = H_2(0) \cos \Phi_o + H_1(0) \sin \Phi_o$ and therefore,

$$\tan \Phi_o = -\frac{H_2(0)}{H_1(0)}$$

$$\left(H'_1(s) = \frac{1}{D(0)} \left[H_1(s) H_1(0) + H_2(s) H_2(0)\right]\right)$$

$$(II-20) \quad \left.H'_2(s) = \frac{1}{D(0)} \left[H_2(s) H_1(0) - H_1(s) H_2(0)\right]\right)$$

where $D(0) = \sqrt{H_1^2(0) + H_2^2(0)}$.

as the equivalent direct and quadrature transfer functions when the demodulators are phased to produce a zero at the origin of the quadrature transfer function.

If we now normalize $H'_1(s)$ and $H'_2(s)$ to $H'_1(0)$, we can define,

$$(II-21) \quad h_1(s) = \frac{H'_1(s)}{H'_1(0)} \quad \text{and} \quad h_2(s) = \frac{H'_2(s)}{H'_1(0)}$$

and obtain,

$$(II-22) \quad \begin{cases} h_1(s) = \frac{H_1(s)H_1(0) + H_2(s)H_2(0)}{H_1^2(0) + H_2^2(0)} \\ h_2(s) = \frac{H_2(s)H_1(0) - H_1(s)H_2(0)}{H_1^2(0) + H_2^2(0)} \end{cases}$$
The equivalent direct and quadrature transfer functions are $h_1$ and $h_2$ respectively, normalized with respect to amplitude and phase such that $h_1(0) = 1$ and $h_2(0) = 0$. They are derivable in terms of $H_1(s)$ and $H_2(s)$ by equation (II-22) or they can be obtained by finding the real and imaginary functions of $s$ in an expansion of

$$H/H(0) = h_1(s) + j h_2(s) = h \equiv H/H(0)$$

Equation (II-23) could have been presented at the outset of this section, but it does not lend itself very easily to the phased demodulator interpretation which makes it valid. The arduous route taken to obtain equation (II-22) is justified inasmuch as the physical meaning of equation (II-23) is clarified. In the problem presented in Part 2 of this report, quadrature demodulators are used; it is a physical property of the demodulators that they can be phased. Furthermore, Cherry* effectively finds the equivalent direct and quadrature amplitude and phase characteristics of $h$ rather than $H$, and we are now in a position to check our results with those obtained by Cherry on the following simple example:

Cherry gives a set of direct and quadrature response curves obtained by detuning a single pole by various amounts. We shall apply equation (II-22) to Cherry's example and thereby check our results.

Cherry uses the independent variable $\alpha \equiv j\omega / \omega_0 = j\omega / 2a$; where $Q = \omega / 2a$ is the well-known figure of merit of a pole; $\omega_0$ is the tuned frequency of the pole, or more appropriately the radian frequency by which the pole has been detuned; and $a$ is the imaginary frequency coordinate of the pole (negative real coordinate of the pole on the complex s-plane).

$H_1(s)$ and $H_2(s)$ for this example have already been derived in equations (II-16) and (II-17) of Section II.B.1.

\( H_1(s) = \frac{s + a}{(s + a)^2 + \omega_0^2} \)

\( H_2(s) = \frac{\omega_0}{(s + a)^2 + \omega_0^2} \)

Therefore, \( H_1(0) = a/(a^2 + \omega_0^2) \) and \( H_2(0) = \omega_0/(a^2 + \omega_0^2) \)

Using \( \alpha = j\omega/2a = s/2a \) and \( \alpha_o = \omega_0/2a \), we then obtain the following expressions for \( h_1(\alpha) \) and \( h_2(\alpha) \).

\[
\begin{align*}
\left\{ \begin{array}{l}
 h_1(\alpha) = \frac{\alpha + (\alpha_o + \frac{1}{2})}{2 \left[ (\alpha + \frac{1}{2})^2 + \alpha_o^2 \right]} \\
 h_2(\alpha) = \frac{\alpha_o \alpha}{\left[ (\alpha + \frac{1}{2})^2 + \alpha_o^2 \right]}
\end{array} \right.
\]

Figures II-7 and II-8 show the pole-zero patterns of \( h_1 \) and \( h_2 \) respectively for the detuned single pole of Figure II-6.

The magnitude and phase of \( h_1 \) and \( h_2 \) resulting from the given patterns of Figures II-7 and II-8 are in perfect agreement with the curves given by Cherry.

D. TRANSIENT RESPONSE

Frequently, a knowledge of the transient response or the step-function response of a filter is of greater importance than a knowledge of the
amplitude and phase characteristics. This is particularly true for applications concerning television. The intended application of this first part of the thesis to television filters therefore makes it important that we consider the transient response of asymmetric filters.

1. Direct and Quadrature Transient Responses

It is possible, of course, to obtain the direct and quadrature transient and step-function responses from the equivalent transfer functions $h_1$ and $h_2$ as derived in the previous sections, but the intermediate step of finding these equivalent transfer functions from the asymmetric filter transfer function, $H$, is quite unnecessary. We can simply separate the time function response of the asymmetric filter itself into real and imaginary parts to get the direct and quadrature time function responses respectively. Such a procedure is valid because the equivalent transfer functions are symmetric and produce real time functions.

Therefore, (using the notation $(\mathcal{L})$ for Laplace transform operator)

$$\mathcal{L}^{-1}(h) = \mathcal{L}^{-1}(h_1 + j h_2) = g_1(t) + j g_2(t)$$

and $g_1$ and $g_2$ must be the transient responses of $h_1(s)$ and $h_2(s)$ respectively.

The direct and quadrature step-function responses can be found by using a real step-function input. The transform of a real step input to the filter $h$ is $1/s$ and therefore,

$$\mathcal{L}^{-1}(h/s) = \mathcal{L}^{-1}(h_1/s + j h_2/s) = g_1(t)_{\text{step}} + j g_2(t)_{\text{step}}$$

The requirement $h_1(0) = 1$ and $h_2(0) = 0$ simply means that the dc response of the asymmetric filter to a step input should be real and equal to unity. It is therefore possible to find the step-function response of the uncorrected filter $H$ and then normalize the time function output so that the dc term is $1 + j0$. 

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2. Example

As an example, we shall use the asymmetric-single-pole case previously considered.

\( H = \frac{1}{s - s_1} \) where \( s_1 = -a_1 + j\omega_1 \)

\( \mathcal{L}^{-1}(H/s) = \mathcal{L}^{-1}(1/s(s - s_1)) = g(t) \)

from which we obtain,

\( g(t) = -\frac{1}{s_1}e^{s_1t}/s_1 = -\frac{1}{s_1}(1 - e^{s_1t}) \)

After normalizing and separating the real and imaginary parts, we obtain,

\[
\begin{align*}
\frac{g_1(t)}{g_2(t)} &= 1 - e^{-\alpha_1 t} \cos \omega_1 t \\
\frac{g_2(t)}{g_1(t)} &= e^{-\alpha_1 t} \sin \omega_1 t,
\end{align*}
\]

In general, the step-function response in an asymmetric case will be,

\[
1 + \sum_{i=1}^{n} K_i e^{j\omega_i t} = 1 + \sum_{i=1}^{n} K_i e^{-\alpha_i t} \cos(\omega_i t + \phi_i)
\]

\[
+ j \sum_{i=1}^{n} K_i e^{-\alpha_i t} \sin(\omega_i t + \phi_i)
\]
III. DESIGN TECHNIQUES

The purpose of this section is to correlate the knowledge presented in Section II and to develop proper design techniques and insight into the problem of minimizing distortion and crosstalk. In their present form, the methods of Section II are primarily analysis tools and we must now apply them to the inverse problem of design.

A. THE MUTUAL TRANSFER FUNCTION

It is cumbersome to work with the two equivalent transfer functions $H_1$ and $H_2$ separately in efforts to simultaneously satisfy given conditions to both. For this reason we shall define,

(III-1) $M(s) = j \frac{H_2}{H_1}$ and

(III-2) $m(s) = j \frac{h_2/h_1}{H_1}$. For want of better names we shall refer to $M(s)$ as the mutual transfer function and $m(s)$ as the normalized mutual transfer function.

$M(s)$ and $m(s)$ actually represent the equivalent quadrature transfer functions of an asymmetric filter which has a perfect direct transfer function. If we multiply the asymmetric filter by the symmetric function $1/H_1$,

(III-3) $H/H_1 = 1 + j \frac{H_2}{H_1}$, all the ill-effects caused by the asymmetry of $H$ are lumped into a single crosstalk-producing term. It is desirable, therefore, to work with the functions $M(s)$ or $m(s)$ because they tell us all about the asymmetry, distortion, and crosstalk of a filter.

If, after analyzing the filter $H/H_1$ or $h/h_1$ in terms of the mutual transfer function, we know the allowable direct distortion, then it will be a simple matter to multiply the ideal direct transfer function by an appropriate transfer function. The mutual transfer function is also multiplied by the desired direct transfer function to yield the equivalent quadrature transfer function.

Thus, if we wish a Butterworth distribution of poles as the direct transfer function, we merely add a Butterworth distribution of poles to the
pole-zero pattern of the mutual transfer function to obtain the quadrature transfer function.

1. Analysis of the Mutual Transfer Function

If we define $H^*$ as an asymmetric filter transfer function having the conjugate singularities of $H$, then,

$$III-4 \quad H^* = H_1 - jH_2 \quad \text{where} \quad H = H_1 + jH_2.$$ 

Now we define $W$ as the ratio of $H$ to $H^*$,

$$III-5 \quad W = H/H^* = (H_1 + jH_2)/(H_1 - jH_2) \quad \text{and therefore,}$$

$$III-6 \quad W = (1 + M)/(1 - M).$$

Solving for $M$ in terms of $W$ we obtain,

$$III-7 \quad M = (W - 1)/(W + 1).$$

We have now succeeded in relating $M$ simply in terms of a function $(W)$ that is immediately known from the singularities of $H$ itself. $W$ is obtained from $H$ by adding opposite logarithmic singularities at the conjugate positions of the singularities of $H$.

Equation (III-7) shows that $M$ can have poles and zeros only where $W$ has a magnitude of 1. A determination of the curves on the $W$ $s$-plane where the magnitude of $W$ is 1 immediately serves to inform us where $M$ can have poles and zeros. In particular, at all points of such curves where the phase is $2K\pi$ ($K = 0, 1, 2...$ etc) $M$ has zeros and at all points where the phase is $\pi + 2K\pi$ $M$ has poles.

The best way to clarify the above is by the use of a simple example. We shall use the single asymmetric pole used previously.

Let $H = 1/(s + a - j\omega_o)$ and therefore,

$$W = H/H^* = (s + a + j\omega_o)/(s + a - j\omega_o).$$
Figures III-1 (a) and (b) show the pole-zero diagrams of $H$ and $W$ respectively. Figure III-1(b) shows clearly that the magnitude of $W$ is 1 only along the real axis. Within the finite real axis, the phase determines a single pole for $M$ as shown in Figure III-1(c). This is clear because the phase of $W$ along the real axis becomes $\pi$ at a point on the real axis directly between the two singularities as shown in Figure III-1(b).

Furthermore, equation (III-7) also shows that $M$ becomes unity where $W$ is infinite. This helps to determine the constant factor in $M$.

Since we have

(III-8) $M(s) = A/(s + a)$ and we demand $[M(s)]_s = -a + j\omega_o = 1$,

then we find $A = j\omega_o$ so that,

(III-9) $M(s) = j\omega_o/(s + a)$.

To find $m(s)$, we recall that $h = H/H(0)$, equation (II-23), requires that $h_2(0) = 0$. Let $W' = h/h^* = [H/H^*]e^{j\phi_o}2$ where $-2\phi_o$ is twice the argument of $H(0)$. Since

(III-9) $m(s) = (W'-1)/(W'+1)$ but $W'$ differs from $W$ only by an angle necessary to have the phase of $W'(0)$ be zero, $m(s)$ can be found from the $W$ s-plane in the same manner as $M(s)$ except that all phases are with reference to the origin as zero phase.

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In our example, \( m(s) \) has a zero at the origin and a pole on the real axis where the phase differs from that at the origin by \( \pi \) radians. Thus, the \( W \) s-plane of Figure III-2 shows that \( m(s) \) will have a pole at \( s = -(a^2 + \omega_0^2)/a \) as shown in the \( m(s) \) pole-zero diagram of Figure III-3.

![Diagram](image)

**Figure III-2**

**Figure III-3**

We now have,

(III-10) \( m(s) = A' \frac{s}{(s + \frac{a^2 + \omega_0^2}{a})} \) and we again require that \( m(s) = 1 \) at a pole of \( W \), from which condition we obtain \( A' = j\omega_0/a \).

(III-11) \( m(s) = \frac{j(\omega_0/a)}{s/(s + \frac{a^2 + \omega_0^2}{a})} \).

\( M(s) \) and \( m(s) \), of course, could be solved for algebraically, but the use of the transformation \( W \) enables one to see how variations in the placement of the \( H \)-filter singularities will affect \( M(s) \) or \( m(s) \).

2. **Another Approach**

Another tool which makes use of the function \( W \) can be obtained in the following way.

(III-12) \( j \frac{H_2}{H_1} = j \tan \left[ \tan^{-1} \left( \frac{H_2}{H_1} \right) \right] \) is obviously an identity.

But \( \tan^{-1}Z = (1/2j) \log_e \left[ (1 + jZ)/(1-jZ) \right] \). Therefore,

(III-13) \( M(s) = j \frac{H_2}{H_1} = j \tan \left[ (1/2j) \log_e(W) \right] \). Let \( \log_e(W(j\omega)) = \log_e|W| + j\theta \)

(III-14) \( M(s) = j \frac{\tan(\theta/2) + j \tanh \left[ \frac{1}{2} \log_e|W| \right]}{1 - j \tan(\theta/2) \tanh \left[ \frac{1}{2} \log_e|W| \right]} \)
Thus, the amplitude and phase response of \( W(j\omega) \) yields \( M(j\omega) \).

In particular, if \( \log_e|W| \) and \( \theta \) are small as in cases where we wish to minimize the amplitude response of \( M(s) \) or \( m(s) \), then we may approximate,

\[
(III-15) \quad M(s) \approx j \frac{\theta/2 - j/2 \log_e|W|}{1 + j(\theta/2)^{1/2} \log_e|W|}
\]

and,

\[
(III-16) \quad |M(j\omega)| \approx 4 \frac{\theta^2 + \left[\log_e|W(j\omega)|\right]^2}{16 + \theta^2\left[\log_e|W(j\omega)|\right]^2}
\]

gives the approximate amplitude response of \( M \) along the \( j\omega \) axis in small response regions. All the above relations for \( M \) hold for \( m \) except that a constant phase is added to \( \theta \) so the total is zero at the origin.

Equation (III-16) shows clearly that both the amplitude and phase response of \( W \) must be minimized along any region where we wish to minimize the quadrature response. The purpose of equation (III-16) is not primarily operational, but rather it is intended to provide a means of gauging the effects on the magnitude of \( M \) or \( m \) as the singularities of \( H \) are changed or varied in position.
PART 2

A TYPICAL APPLICATION
IV. THE NTSC CHROMA CHANNEL

In order to transmit color, three independent pieces of information are necessary. These independent variables are usually brightness, hue, and saturation. The compatible National Television Systems Committee (NTSC) color television system utilizes the available television channel to transmit color in the following manner.

![Television Channel Diagram](image)

**FIGURE IV-1**

Figure IV-1 shows the placing of critical frequencies within the 6-megacycle television channel spectrum. For reasons of compatibility, a carrier at B (picture carrier) must be amplitude modulated with the brightness information, and similarly the sound information must be contained on a carrier at position S.

The hue and saturation information must now be added in the spectrum so that a minimum of interference is introduced to present black-and-white sets. Most monochrome sets have a video pass band that has poor response in the region over 3 megacycles above the picture carrier. The dotted curve in Figure IV-1 is indicative of a typical monochrome television video response.

It has been empirically determined* that a chroma subcarrier can be positioned as shown at C in Figure IV-1, although there is presently some question as to whether or not it can be lowered to C'.

---

The band available for the transmission of the hue and saturation information is shown as a solid curve in Figure IV-1. The presence of the sound at approximately 0.6 megacycle above the chroma carrier makes for an asymmetric chroma channel.

The modulated chroma carrier takes the form,

\[ F(t) \cos \omega_c t + Q(t) \sin \omega_c t \]

because it is generally modulated, and \( F(t) \) and \( Q(t) \) convey the hue and saturation as functions of time.

The question now arises as to the allowable bandwidth of \( F(s) \) and \( Q(s) \), the respective transforms of \( F(t) \) and \( Q(t) \). In this connection there are two systems, the CPS and the OCW, which we shall discuss below.

A. LIMITATIONS ON THE F AND Q BANDWIDTHS

Figure IV-1 shows that about two-thirds of the total chroma channel, or about 1.2 megacycles, lies below the chroma carrier. The rest of the channel, about 0.6 megacycle, lies above the chroma carrier.

For frequencies in the modulation exceeding 0.6 megacycle,

\[ H = 0 = H_1 + j H_2 \] \label{eq:iv-2}

and therefore,

\[ H_2 = j H_1 \] \label{eq:iv-3}

at all frequencies above 0.6 megacycle.

Both \( H_1 \) and \( H_2 \) are nonzero in the frequency range 0.6 to 1.2 megacycles because \( H \) has response in that region below the carrier. Above 1.2 megacycles, both \( H_1 \) and \( H_2 \) become zero.

From equation (IV-3), we see that frequency components in \( F \) and \( Q \) above 0.6 megacycle will be subject to complete quadrature crosstalk.

1. The CPA System

The color phase alternation system (CPA) utilizes the chroma channel in the following manner:
The frequency ranges of both \( F(s) \) and \( Q(s) \) are allowed to extend up to 1.2 megacycles.* The crosstalk introduced by the asymmetric channel is then eliminated by changing the phase of \( F(s) \) and of one demodulator by 180° on each successive picture frame.

The transforms of the output of the demodulators on successive picture frames then become;

\[
\begin{align*}
(IV-4) \quad F(s) H_1 - Q(s) H_2 & \longrightarrow -[-F(s) H_1 - Q(s) H_2] \\
(IV-5) \quad Q(s) H_1 + F(s) H_2 & \longrightarrow Q(s) H_1 - F(s) H_2
\end{align*}
\]

The average output of the demodulators for an even number of frames is then appropriately,

\[
(IV-6) \quad F(s) H_1 \text{ and } Q(s) H_1 \text{ respectively.}
\]

The response of both demodulators to \( H_2 \) has been removed, but at the expense of cancellation on successive frames. The effect of \( H_2 \), therefore, is to introduce color flicker at a 30-cycle rate in color areas where the component frequencies fall in good response regions of \( H_2 \).

The object is then to minimize the response of \( H_2 \) over as much of the low-frequency range as possible so that the flicker will not occur for appreciably large color areas. Now we see that the purpose of phasing the demodulators as explained in section II.B. in order that the direct and quadrature transfer functions become \( h_1 \) and \( h_2 \) where \( h_2(0) = 0 \) is simply to eliminate the very large area flicker.

2. The OCW System

The other chroma transmission system has only recently been introduced and is still under consideration. This system is called the orange-cyan wideband (OCW) system.

* \( F \) and \( Q \) are here used for \( B-Y \) and \( R-Y \) of the literature on the CPA system.
In the OCW system, $F(s)$ is allowed the same range of frequencies as in
the CPA system (up to 1.2 megacycles), but $Q(s)$ is limited in bandwidth to
approximately 0.35 megacycle.* The high-frequency color regions are produced
only from $F(s)$ and the $F(s)$ signal is used to transmit the orange-cyan axis
information. Thus, the large color areas are made up of a three-color system,
but the smaller color areas are in a two-color system.

$F$ and $Q$ in the OCW system can theoretically be kept free of crosstalk
to the extent that $H_2$ (or $h_2$) has negligible response below 0.35 megacycle.

The crosstalk term in the $F$ channel is $QH_2$, but if the range of frequencies
in $Q$ is small and over a region where $H_2$ has poor response, then the term $QH_2$
will be of little consequence.

The crosstalk term in the $Q$ channel is $FH_2$, but this term is of consequence
for frequencies above the range encompassed by $Q$ because of the poor amplitude
response of $H_2$ in the $Q$ region of frequencies. A low-pass filter which
attenuates frequencies above the $Q$ range of frequencies in the $Q$ channel will
therefore eliminate most of the crosstalk in the $Q$ channel.

More precisely, if $H_3$ is the low-pass filter in the $Q$ channel, the two
outputs are;

$$ FH_1 - QH_2 \approx FH_1 $$
$$ QH_1 H_3 + FH_2 H_3 \approx QH_1 H_3 $$

if $|QH_2|$ and $|H_2 H_3|$ are small for all frequencies, which they will be if $H_2$ is
high-pass complementary to $H_3$ and $Q$, as shown in the figure below.

* $F$ and $Q$ here are used for the $I$ and $Q$ color signals respectively in OCW
literature.
B. THE PROBLEM

The chroma detection system is shown in Figure IV-3. The chroma filter shown has the primary purpose of attenuating the sound which is present in a 25-kilocycle region about the sound carrier at \( \omega_q \). The synchronous demodulators can be arbitrarily phased so that the direct and quadrature equivalent transfer functions are \( h_1 \) and \( h_2 \) as defined by equation (II-22).

The objective of the system is to separate the F and Q signals as completely as possible while introducing a minimum of distortion.

For purposes of analysis, we shall lump all the various filters into a single chroma filter. The low-pass filters after the detection stages are merely symmetrical filters centered at the chroma carrier frequency before demodulation. The RF and IF filters are asymmetric with respect to the chroma carrier frequency.
V. ANALYSIS OF THE PROBLEM

In both the CPA and the OCW systems, it is desirable that the quadrature transfer functions have the minimum possible amplitude characteristic over the largest possible low-frequency range. At the same time, we must be careful to insure that the direct transfer function does not introduce undue distortion.

In this section, we shall discuss the possibilities of minimizing distortion and crosstalk introduced by the NTSC chroma channel.

A. SYSTEM DESIGN

Before attempting an analysis of the chroma filter itself, let us examine the possibilities of removing the crosstalk after it has been introduced by the filter. The possible use of systems to remove crosstalk has received much attention. As we shall see, however, such systems are not only impractical and critical, but where they appear to remove crosstalk, they do so at the expense of undoing the job intended for the asymmetric filter.

The filter is asymmetric because it must remove or filter out an asymmetric signal. In particular, assume that some signal that is present at $\omega_1$ radians/sec above the carrier is to be removed or greatly attenuated by the asymmetric filter.

We can write the undesirable signal as,

$$a e^{j\omega t} = a \cos \omega_1 t + j a \sin \omega_1 t.$$  \hspace{1cm} (V-1)

We shall now show that any linear system which is to eliminate such a signal must introduce complete quadrature crosstalk at the frequency $\omega_1$. If it does not introduce crosstalk, then the system, whatever it is composed of, will not eliminate the asymmetric signal.
SYSTEM ANALYSIS

FIGURE V-1

Consider the X system of Figure V-1. Regardless of what composes the X system, it must have provision for real and imaginary inputs and outputs. Let the subscripts 1 and 2 refer to the real and imaginary terminals of the X system and then define,

\[
\begin{align*}
G_{11} & = \text{real input to real output transfer function.} \\
G_{22} & = \text{imaginary input to imaginary output transfer function.} \\
G_{12} & = \text{real input to imaginary output transfer function.} \\
G_{21} & = \text{imaginary input to real output transfer function.}
\end{align*}
\]

(\text{V-2})

Now let \(I_1(s)\) be the transform of a \(\cos \omega_1 t\) and \(I_2(s)\) be the transform of a \(\sin \omega_1 t\).

Figure V-1(a) shows the output of the system for the real input \(I_1(s)\) and Figure V-1(b) shows the output of the system for the imaginary input \(I_2(s)\). Denoting the total real output by \(E_1(s)\) and the total imaginary output \(E_2(s)\), we have,

\[
\begin{align*}
(V-3) & \quad E_1(s) = G_{11}I_1 + G_{21}I_2 \\
(V-4) & \quad E_2(s) = G_{12}I_2 + G_{22}I_2
\end{align*}
\]

Since \(I_1\) and \(I_2\) are not zero and we require \(E_1\) and \(E_2\) to be zero, \(G_{12}\) and \(G_{21}\) cannot be zero unless \(G_{11}\) and \(G_{22}\) are also zero. Requiring \(G_{11}\) and \(G_{22}\) to be zero at \(s = j\omega_1\), however, is the trivial solution which merely implies that
the system is symmetrical, attenuating signals at a frequency \( \omega_1 \) above and below the carrier reference.

In any case, we see that the system must rely on \( G_{12} \) and \( G_{21} \), the crosstalk-producing transfer functions, to eliminate an asymmetric signal and, in fact, \( |G_{21}| = |G_{11}| \) and \( |G_{12}| = |G_{22}| \) at the frequency \( \omega_1 \). The above argument is perhaps obvious and trivial, but unfortunately it is rather easy to fall into the trap of trying to eliminate crosstalk with the use of elaborate active networks.

Since there is nothing (or little) to gain by using systems, we shall confine ourselves to asymmetric filter design and analysis in the investigation of the NTSC chroma channel.

**B. CHROMA FILTER REQUIREMENTS**

![Figure V-2](image1)

![Figure V-3](image2)

**THEORETICAL CHROMA FILTER RESPONSE**

The theoretical limitations of the chroma channel would indicate that the chroma filter response would have to be somewhat as shown in Figure V-2, where the dotted line indicates the sound position at 0.6 megacycle and the solid line indicates the television channel limit.

In practice, however, it is possible to extend the chroma filter response up to about 0.5 megacycle above the channel limit because of a guard band provided in that region. Figure V-3 shows that the chroma channel can be made symmetrical except for the attenuation of the sound signal at 0.6 megacycle.
It is required that the sound be attenuated by 30 to 40 db.

The chroma filter can now be split up into two cascaded sections such that one section is perfectly symmetrical as in Figure V-4(a) and the other section provides for the sound attenuation and is approximately as shown in Figure V-4(b).

![Symmetric and Asymmetric Chroma Filter](image)

**Figure V-4**

The symmetric part of the filter, of course, takes into account the correction required for any asymmetry caused by the RF and IF sections so that the input into the sound attenuating section is symmetric and does not contain crosstalk.

The analysis of the chroma filter has now been reduced, without oversimplification, to the analysis of a simple asymmetric filter with the sole purpose of attenuating a signal at 0.6 megacycle above the carrier.

C. **Basis of Analysis**

If we consider the step-function response of the symmetrical part of the chroma filter, Figure V-4(a), to be very nearly as shown by the straight-line approximation of Figure V-5, then the step-function response of the total chroma filter (including the asymmetric sound attenuation filter) can be obtained by considering the time function of Figure V-5 as the input to the asymmetric part of the filter and finding the response. $T_0$ of Figure V-5 is very nearly one-half the period of the half-power frequency of the symmetric.
part of the chroma filter. Figure V-4 shows that $T_0 \approx \pi/2 \omega_s$ where $\omega_s$

![Approximate step response of symmetric part of chroma filter](image)

**Figure V-5**

is the sound radian frequency.

We should like to consider the direct and quadrature step-function response of the chroma filter because it is of particular importance in the CPA system; the CPA system produces flicker where there is a quadrature signal. The CPA system, therefore, should logically be considered in terms of the quadrature step-function response of the chroma filter. In consideration of the OCW system, the quadrature transfer function amplitude response in the frequency region of $\pm 0.35$ megacycle must be known. This is the logical way to consider the OCW system because one of the quadrature signals is allowed a bandwidth of only 0.35 megacycle.

In the following discussions, therefore, both the step-function response and the amplitude of the quadrature transfer function will be considered.

D. **REALIZABILITY RESTRICTIONS**

Before we proceed with an analysis of the sound attenuation filter, we shall have to determine how near the $j\omega$ axis it is possible to place the singularities that may be required.

If we assume that a $Q$ (figure of merit of a tuned circuit $\approx \omega_0/2\alpha$ where $\omega_0$ is the tuned frequency and $\alpha$ is the real part of the pole or zero) of 100 is obtainable, then in the vicinity of the sound signal and with reference to the
carrier as zero the equivalent Q becomes about 13. This follows because
the realizable Q of 100 must be with respect to the true origin at the output
of the second detector, on which scale the sound signal is at 4.5 megacycles.
If we then refer to the carrier, on which scale the sound is at 0.6 megacycle,
the relative Q becomes \(100 \times 0.6 / 4.5 = 13.3\). Looking at it another way, the
minimum \(Q\) obtainable for singularities in the vicinity of the sound is about
\((0.02)2\pi \times 10^6\).

The above information is necessary in order that the sound attenuation
requirements be met with easily realizable circuits.

E. POSSIBLE ATTENUATION FILTERS.

The usual practice has been to eliminate, or attenuate, the sound by the
use of a single zero, referred to as the sound trap. For this simple case,
shown in Figure V-6(a), we obtain a simple differentiating transfer function
for \(m(s)\) as shown in Figure V-6(b).

\[
\begin{align*}
\text{SOUND-TRAP FILTER} & \quad m(s) \quad S-PLANE \\
\text{a.} & \quad \text{b.}
\end{align*}
\]

\textit{FIGURE V-6}

Figure V-7 shows the step-function response, direct and quadrature, for a
chroma filter using a simple sound trap at the sound frequency. Figure V-7(c)
shows the amplitude of the quadrature response. For this particular case, \(m(s)\)
and \(h_2\) are very nearly the same. There is a slight discrepancy, which has been
ignored, introduced by the fact that the zero must have a small real part as
discussed in section V.D. In reality, therefore, \(m(s)\) should have a pole as
shown by the example considered on page 24. The pole is too far removed to be
of any consequence, however, and for this reason it can be neglected.
The results obtained for this simple case are borne out by experimental evidence* which indicates that the quadrature step-function response is the derivative of the direct step-function response. Figure V-7(a) and (b) show that CPA will introduce considerable flicker under the circumstances. Figure V-7(c) shows that there is room for improvement under the OCW system also. The quadrature response at 0.35 megacycle is over half the direct response. If possible, this should be reduced.

1. Addition of a Pole

Now let us consider what may be gained by adding a pole to the asymmetric filter. In adding a pole to the sound attenuation filter, we must first determine the restrictions that apply.

In section V.B., one of the requirements set on the chroma filter is that it shall attenuate the sound by 30 to 40 decibels. In section V.D., it is determined that the closest a zero can be to the $j\omega$ axis, if we restrict ourselves to circuits having $Q$'s within 100, is $(0.02)2\pi \times 10^6$ at $\omega_g = (0.6)2\pi \times 10^6$. The attenuation requirements, coupled with the maximum available $Q$, determine a circle centered at $j\omega_g$ on the s-plane within which the pole cannot lie. For an attenuation of 40 db, the pole must be 100 times farther away from the point $j\omega_g$ than the zero. The radius of such a circle on the s-plane is then about 2 megacycles, which for practical purposes indicates that nothing will be gained by the addition of a pole at such a great distance. (See Figure V-8.)

* D. H. Pritchard, "Investigation of Quadrature and In-Phase Components in the NTSC Color Television System," David Sarnoff Research Center, RCA Laboratory Division, Princeton, N.J.
\[
\frac{H}{H(s)} = h = (s - j\omega_s)\left(\frac{1}{j\omega_s}\right) = 1 + j\frac{s}{\omega_s} = h_1 + jh_2
\]

\[
m(s) = jh_2 = j\frac{s}{\omega_s}
\]
2. The Double-Zero Case

A very important limitation imposed on \( m(s) \) is brought out by consideration of a double-zero sound trap.

If we expand,

\[
H = (s - jw_s)^2 = (s^2 - \omega_s^2) - j(2 \omega_s s)
\]

we note that \( H_1 = (s - \omega_s)(s + \omega_s) \) has a zero in the positive real half of the complex plane. Thus, \( M(s) = m(s) \) in this case cannot be realized as the quadrature transfer function because to do so would require the use of a filter containing a pole in the positive real half of its complex \( s \)-plane.

Since \( m(s) \) cannot be realized, the direct transfer function cannot be made ideal. The best that could probably be done for the direct transfer function might be to make it an "all-pass" transfer function. We would then multiply \( H \) by \( 1/(s + \omega_s)^2 \) to obtain,

\[
\text{H}_1(s) = (s - \omega_s)/(s + \omega_s) \quad \text{and} \quad \text{H}_2(s) = 2\omega_s/(s + \omega_s)^2
\]

The direct step-function response is easily found to be,

\[
-1 + 2e^{-\omega_st} \quad \text{and the quadrature step response is,}
\]

\[
\frac{1}{(2-1)}, \frac{d}{ds} \left[ (s + \omega_s)^2 \frac{2 \omega_s e^{st}}{(s + \omega_s)^2} \right]_{s = \omega_s} = -40
\]
Figures V-9(a) and (b) show approximately the plots of the direct and quadrature step-function responses respectively. The ideal direct step-function response of the filter, as shown in Figure V-5, is superposed in Figure V-9(a) for comparison purposes.

The sharp negative spike of Figure V-9(a) will, of course, be removed by the symmetrical part of the chroma filter. Removal of the shaded region leaves a step-function response that is noticeably delayed.

We can now sum up some important points that have been highlighted in the present discussion. The first of these is that poles of \( m(s) \) in the positive real half of the complex plane imply that the direct transfer function cannot be idealized. The direct transfer function can, however, be made into an all-pass filter, and phase corrective networks can be used to ensure that the direct transfer function only introduces delay.

A note of caution must be introduced at this point pertaining to delay introduced by the chroma filter. The chroma channel signal will have to be superposed on the monochrome signal at the picture tube, and therefore any delay introduced by the chroma filter which is not also introduced into the monochrome signal will cause the color picture at the picture tube to be shifted.
with respect to the black-and-white picture. It is therefore necessary to keep in mind that excessive chroma delay will require the introduction of delay in the monochrome channel.

The double-zero sound-trap case under consideration is summarized in Figure V-10. Figure V-10(b) shows the pole-zero diagram for \( H_1(s) \), clearly indicating the phase characteristic which introduces delay over the low-frequency range.

Figure V-10 shows that the quadrature amplitude response for the double-zero case as developed to this point is inferior to the single-zero case previously considered. The amplitude response of \( H_2 \) in the \( Q(s) \) range of frequencies has been increased. We are able to improve the situation, however, because the asymmetric filter of Figure V-10(a) is attenuating the sound much more than the 40 db called for. Recall that in the single-zero case a pole could not be placed within a circle having a 2-megacycle radius from the zero. For the double-zero case then, a double pole can be placed anywhere outside a circle having a 0.2-megacycle radius from the double zero, and the attenuation of the sound will exceed the 40 db called for (assuming that the zeros have \( Q \)'s of 100). The region within the dashed circle in Figure V-10(a) cannot contain the two poles if the frequency \( \omega_s \) is to be attenuated by 40 db.

3. The Double-Zero, Double-Pole Case

In the previous section, a symmetrical double pole (on the real axis) was added in order to minimize the direct distortion to the point where the direct distortion was in the form of delay. The quadrature transfer function, however, is less satisfactory than the more simple sound trap considered in section V.E. Improvement is possible because the sound need be attenuated by only 40 db.
DOUBLE-ZERO TRAP ANALYSIS

Fig. V-10
whereas the filter of Figure V-10(a) provides an attenuation of almost 60 db.

In this section, therefore, we shall study the more general case where the poles are placed asymmetrically also.

Before proceeding with the analysis of the best placement of the poles, we should recognize that although complexity may produce better performance it must be paid for. By leaving the simple single-zero case and examining the present four-singularity case, we have effectively said, "The consequences of using a single sound trap are intolerable to the extent that we will gladly quadruple the price of the sound attenuation filter to obtain better performance."

Assuming for the present that such is the case, we again reach a new economic dilemma that is relatively clear cut. We have set ourselves the task of finding the best placement of two poles on the arc shown dotted in Figure V-11(a) such that the quadrature distortion and crosstalk shall be minimized. Figure V-11(b), however, shows a particular placement for the poles along an approximately straight line in the real direction and crossing the \( \omega \) axis at the sound frequency. (The locus is really a circle having a 4.5-megacycle radius centered at the true frequency origin.) In a later section, we shall show that the poles, if placed on the locus shown in Figure V-11(b), can be realized at less cost than if they are not.

\[\begin{array}{c}
\begin{array}{c}
\text{LOCI FOR ADDING POLES} \\
\text{FIGURE V-11}
\end{array}
\end{array}\]
We should, therefore, investigate the possible advantages that might accrue to compensate the use of some locus other than that of Figure V-11(b). If no positive advantages result or if any such advantages are incommensurate with the added cost that would result, then we should accept the locus of Figure V-11(b).

Consider the pole-zero configuration of Figure V-12.

DOUBLE-ZERO, DOUBLE-POLE SOUND TRAP FILTER

FIGURE V-12

The expressions for \( h_1(s) \), \( h_2(s) \), and \( m(s) \) for the filter having the singularities of Figure V-12 are derived in Appendix A and are presented below.

\[
(V-9) \quad h_1(s) = \left( \frac{8}{7} \right) \frac{\left[ (s + \frac{\omega_s}{6})^2 + \omega_1^2 \right] \left[ (s + \frac{\omega_s}{6})^2 + \omega_2^2 \right]}{\left[ (s + \frac{\omega_s}{3})^2 + \omega_s^2 \right]^2}
\]

\[
(V-10) \quad h_2(s) = \left( \frac{2}{3} \right) \frac{s(s + \frac{\omega_s}{3}) \left[ (s + \frac{\omega_s}{6})^2 + \omega_3^2 \right]}{\left[ (s + \frac{\omega_s}{3})^2 + \omega_s^2 \right]^2}
\]

\[
(V-11) \quad m(s) = j \left( \frac{3}{4} \right) \frac{s(s + \frac{\omega_s}{6}) \left[ (s + \frac{\omega_s}{6})^2 + \omega_3^2 \right]}{\left[ (s + \frac{\omega_s}{6})^2 + \omega_1^2 \right] \left[ (s + \frac{\omega_s}{6})^2 + \omega_2^2 \right]}
\]
\[
\begin{align*}
\omega_1 &= 1.28 \omega_b \\
\omega_2 &= 0.9 \omega_b \\
\omega_3 &= 1.04 \omega_b
\end{align*}
\]

Figure V-13 shows the amplitude response of \(m(\omega)\) for the filter of Figure V-12. The response of \(m(s)\) to the input depicted in Figure V-5 is shown in Figure V-15 and is derived in Appendix B.

The filter of Figure V-12 can be realized by cascading two sections as shown in Figure V-14.

---

To realize a filter in which the poles are shifted to some other position will, in general, require more complex circuits and more difficult tuning procedures. The question of whether it will be worth the added complexity can, in most cases, be easily answered by use of the methods of section III.A.2.

If we vary the position of the double pole, the amplitude and phase response of \(W(\omega)\) will give an indication of any improvements that may result. One quickly discovers, upon such an investigation, that moving the poles from the position shown in Figure V-12 will not bring about any radical improvement in the amplitude response of \(m(\omega)\). Where the amplitude response of \(W(\omega)\) is improved, the phase response deteriorates, and vice versa, in the frequency range of interest.
INPUT SIGNAL AND DIRECT RESPONSE

QUADRATURE

RESPONSE OF m(s) TO INPUT SHOWN IN FIGURE V-5
4. **Nonminimum Phase Case**

At this point, we might notice that if we do not restrict ourselves to minimum phase networks, a means of introducing further improvement is possible. We can eliminate the phase of $W(j\omega)$ entirely by placing the singularities of $W(s)$ symmetrically about the $j\omega$ axis. The fact that poles will be necessary in the positive real half of the complex $s$-plane should not disturb us because each pole of $W$ has an associated complex conjugate zero which we may attribute to the asymmetric filter $H(s)$. Thus, the poles of $W$ in the positive real half of the complex $s$-plane are produced when $H$ is nonminimum phase.

For example, consider the pole-zero diagram of the $W$ function shown in Figure V-16. The phase is zero all along the $j\omega$ axis. By proper disposition of the singularities to positions approximately as shown, it is possible to obtain an almost ideal amplitude response for $m(j\omega)$. Unfortunately, however, $m(s)$ will have poles in the positive real half of the complex plane as can be shown by the principles put forth in section III.A.1. By the reasoning used in section V.E.2., this will result in an uncorrectable direct transfer function which, in fact, will introduce delay in the chroma signal.

![Nonminimum Phase W(s) S-Plane Diagram](image)
Figure V-17 shows the filter that is associated with the $W(s)$ function shown in Figure V-16.

The tremendous increase in complexity that has resulted can hardly be compensated for by any advantages that may accrue in minimizing distortion and crosstalk. Six singularities must be used in the sound-eliminating filter and two zeros must be realized in the positive real half of the complex plane. In addition, we are assured that direct distortion to the direct signal can not be entirely eliminated, and indeed it may be necessary to add delay to the monochrome channel to balance the delay introduced into the chroma channel.

Without having gone through lengthy calculations, but merely by applying the techniques of sections III.A.1, III.A.2, and V.E.3. we are encouraged to discard further investigation along these lines.

F. CONCLUSIONS

The analysis which has been carried out is far from exhaustive. The attempt has not been made to solve the chroma filter problem because no clear-cut solution exists. Any solution must always be dependent on factors that cannot be taken into account in a paper of this scope and which, in many cases, are in a state of flux. Wherever possible, for example, we have kept to the use of $\omega_s$ to denote the sound frequency rather than a particular value simply
because the chroma carrier frequency has not as yet been definitely fixed. The value of $\omega_8$, therefore, is subject to change.

The greatest value of the analysis is the insight that it will give toward the problem as a whole rather than any passing benefit resulting from particular solutions to be found here.

A comparison of Figures V-7(b) and V-15 shows that, insofar as the CPA system is concerned, there is little to gain by the use of elaborate sound traps. Only by changing $\omega_8$ or the attenuation requirements can we materially reduce the crosstalk.

Figure V-13 definitely shows that improvement is possible in the OCW system and, in fact, that the solution obtained can hardly be improved upon. For most of the range of frequencies in the $Q(s)$ signal there will be less than 20 percent crosstalk. Here again, increasing $\omega_8$ results in improvement because the range of frequencies in $Q(s)$ can be proportionately increased for the same amount of crosstalk. The attenuation requirements also affect the crosstalk in the OCW system directly, as can be seen by a comparison of Figures V-10(c) and V-13, in which the attenuation was changed from 60 to 40 db respectively.

The effects of the attenuation requirements can best be illustrated by the examination of a simple case. Consider the pole-zero configuration of Figure V-18. $s_1$ is the zero and $s_2$ is the pole.

![Diagram](image)

**FILTER USED TO ILLUSTRATE IMPORTANCE OF REQUIREMENTS**

**FIGURE V-18**
\[(V-13) \quad H(s) = \frac{(s-s_1)}{(s-s_2)}\]

The response of the filter to a step is;

\[(V-14) \quad g(t) = \frac{s_1}{s_2} \left[ 1 - \left( \frac{s_2-s_1}{s_1} \right) e^{s_2t} \right]\]

from which we obtain, after normalizing so that \(s_1/s_2\) disappears,

\[(V-15) \quad g_1(t) = 1 - \left( \frac{s_2-s_1}{s_1} \right) e^{-\alpha_2 t} \cos (\omega_2 t + \phi)\]

\[(V-16) \quad g_2(t) = -\left( \frac{s_2-s_1}{s_1} \right) e^{-\alpha_2 t} \sin(\omega_2 t + \phi).\]

If we are interested in the attenuation at some frequency \(s_o\), where \(s_1\) is very nearly equal to \(s_o\), the vector magnitude \(|s_2-s_o| \approx |s_2-s_1|\) is one of the determining factors. The other factor of importance is the real part of the zero.

\[(V-17) \quad \text{Attenuation} \approx 20 \log \frac{|s_2-s_1|}{\alpha_1} = A\]

We can now substitute \(A\) into the expressions (V-15) and (V-16).

\[(V-18) \quad g_1(t) = 1 - \left[ \frac{10(A/20)}{|s_1|} \right] e^{-\alpha_2 t} \cos(\omega_2 t + \phi)\]

\[(V-19) \quad g_2(t) = -\left[ \frac{10(A/20)}{|s_1|} \right] e^{-\alpha_2 t} \sin(\omega_2 t + \phi)\]

Since \(|s_1| \approx \omega_s\), the magnitude of the crosstalk and distortion terms becomes very nearly,

\[(V-20) \quad (\alpha_1/\omega_s) 10^{(A/20)}\]

Equation (V-20) gives a clear indication of the importance of the constants that apply to the sound-attenuating filter. It is easily seen that, if the attenuation to be had and the frequency of the chroma carrier are fixed, the only parameter that can be varied to reduce distortion and crosstalk is \(\alpha_1\), which is
limited by the available Q's of the tuned circuits to be used. The higher
the Q of the zero, the smaller the value of $\alpha_1$ and therefore the smaller
the undesired terms become.

$\alpha_1$ cannot be reduced indiscriminately, however, because the
attenuation bandwidth is proportional to $\alpha_1$. The job of the filter is really
to attenuate a band of frequencies and not a single frequency.

The simple example considered above serves to point out the validity
of the assertions made relative to the attenuation requirements and the chroma
carrier frequency. The attenuation requirements have not as yet been rigidly
fixed nor, in fact, thoroughly investigated. When such investigations are
carried out, it would be well to keep in mind the problem of crosstalk, as well
as the other factors that may enter in to the considerations.

Perhaps the most important factor in design problems is that of cost.
Wherever possible, we have tried to determine and discard designs which are
economically unsound. This is not to say, however, that the remaining designs
are economically sound. Indeed, economy is based on factors too remote and
widespread for proper consideration here.

For example, throughout the preceding analysis we have assumed that
only the sound-eliminating filter will be the asymmetric part of the system.
Actually, such an assumption is valid only to the extent that the asymmetry
due to the RF and IF filters is properly corrected by the chroma filter. Such
correction may or may not be realistic, depending on the complexity of the RF
and IF filters.

We should also note that the possibility of using crystals in the
sound-attenuating filter has not been considered. At present, there is a
question about their cost, although it may prove groundless if a proper study
of the subject shows the advantages to be gained by their use outweighs the
cost of crystals in quantity. Certainly, crystals afford a means of producing
zeros with Q's in the order of thousands (compared to the Q of 100 used in all
of the foregoing analysis).

In conclusion, we should point out that this thesis has been
primarily concerned with the analytic approach to the problem of asymmetric
filters and channels. The experimental approach will have to provide the
concrete results to specific problems such as the NTSC chroma filter. The
deliberations contained herein, however, may be of great value, and it is to
be hoped that they represent a substantial contribution to the art.
LIST OF REFERENCES

Books.


Articles.


(h) Lecture Notes on Advanced Network Theory, for course 6.562 at M.I.T.


APPENDIX A

DETERMINATION OF $h_1(s)$, $h_2(s)$, AND $m(s)$

\textbf{Given:}\quad H(s) = \frac{(s - j\omega_s)^2}{(s + \alpha - j\omega_s)^2} \quad \text{where} \quad \alpha = \frac{\omega_s}{3}

\textbf{To Find:}\quad h_1(s), h_2(s) \text{ and } m(s)

where \( h(s) = \frac{H(s)}{H(0)} = h_1(s) + jh_2(s) \)

and \( m(s) = j \frac{h_2(s)}{h_1(s)} \)

\( H'(s) = \frac{s - j\omega_s}{s + \alpha - j\omega_s} \)

\( h(s) = \frac{H(s)}{H(0)} = \left[ \frac{H'(s)}{H'(0)} \right]^2 = \left[ h_1'(s) + jh_2'(s) \right]^2 \)

therefore:

\( h_1(s) = \left[ h_1'(s) \right]^2 - \left[ h_2'(s) \right]^2 \)

and

\( h_2(s) = 2h_1(s)h_2'(s) \)

We first solve for $h_1'(s)$ and $h_2'(s)$ and then use (3) and (4) to find $h_1(s)$ and $h_2(s)$.

\( h_1'(s) = \text{Re} \left\{ \frac{s - j\omega_s}{s + \alpha - j\omega_s} \cdot \frac{\alpha - j\omega_s}{-j\omega_s} \right\} = \text{Re} \left\{ \frac{(s-j\omega_s)(s+\alpha+j\omega_s)(j\frac{\alpha}{\omega_s}+1)}{(s+\alpha)^2+\omega_s^2} \right\} \)

\( h_2'(s) = \frac{g_m \left[ (s-j\omega_s)(s+\alpha+j\omega_s)(j\frac{\alpha}{\omega_s}+1) \right]}{(s+\alpha)^2+\omega_s^2} \)

\( h_1'(s) = \frac{s(s+\alpha)+\alpha^2+\omega_s^2}{(s+\alpha)^2+\omega_s^2} \)
(8) \[ h'_2(s) = \frac{\alpha}{\omega_s} \frac{s(s+\alpha)}{(s+\alpha)^2 + \omega_s^2} \]

From (3):

(9) \[ h_1(s) = \frac{\left[s(s+\alpha) + \omega_s^2\right]^2 - \left[\frac{\alpha}{\omega_s}(s+\alpha)s\right]^2}{(s+\alpha)^2 + \omega_s^2} \]

but \( \frac{\alpha}{\omega_s} = \frac{1}{3} \)

(10) \[ \therefore h_1(s) = \frac{\frac{8}{9}s^2(s+\alpha)^2 + 2(\alpha^2 + \omega_s^2)s(s+\alpha) + (\alpha^2 + \omega_s^2)^2}{(s+\alpha)^2 + \omega_s^2} \]

By the reasoning of section III.A.1, we know that the real parts of the zeros will be \( \frac{\alpha}{3} \leq a \) and therefore

(11) \[ h_1(s) = \frac{\frac{8}{9}\left( (s+a)^4 - 2a^2(s+a)^2 + a^4 + \frac{2}{9}(4a + \omega_s^2)((s+a)^2 - a^2) + \frac{9}{8}(4a^2 + \omega_s^2)^2 \right)}{(s+2a)^2 + \omega_s^2} \]

(12) \[ h_1(s) = \frac{(s+a)^4 + (s+a)^2\left[ \frac{7a^2}{2} + \frac{1}{2}\omega_s^2 \right] + \left[ - \frac{8a^4}{9} - \frac{2}{1296}\omega_s^2 \frac{4a^2 + \omega_s^2}{3} \right]^2}{\frac{3}{8}\left[(s+2a)^2 + \omega_s^2\right]^2} \]

(13) \[ a = \frac{\alpha}{2} = \frac{\omega_s}{6} \]

(14) \[ h_1(s) = \frac{(s + \frac{\omega_s}{6})^4 + (s + \frac{\omega_s}{6})^2\left[ \frac{\omega_s^2}{2} (\frac{22}{9}) \right] + \left[ \frac{1711}{1296} \frac{\omega_s^4}{9} \right]}{\frac{9}{8}\left[(s + \frac{\omega_s}{3})^2 + \omega_s^2\right]^2} \]

(15) \[ h_1(s) = \frac{8}{9}\left[ (s + \frac{\omega_s}{6})^2 + \frac{\omega_1^2}{\omega_2^2} \right]\left[ (s + \frac{\omega_s}{6})^2 + \omega_2^2 \right]\left[ (s + \frac{\omega_s}{3})^2 + \omega_3^2 \right]^2 \]

(16) \[ \text{where} \begin{cases} \omega_1 \approx 1.28 \omega_s \\ \omega_2 \approx 0.9 \omega_s \end{cases} \]
From (4), (7), and (8) we get:

\begin{align*}
(17) \quad h_2(s) &= 2 \frac{\omega_s}{\omega_s} \left[ \frac{(s + \alpha)^2 + \omega_s^2}{(s + \alpha^2 + \omega_s^2)^2} \right] \frac{s(s + \alpha)}{(s + \alpha)^2 + \omega_s^2} \\
(18) \quad h_2(s) &= \frac{\omega_s}{2\alpha} \left[ \frac{(s + \alpha)^2 + \omega_s^2}{(s + \alpha^2 + \omega_s^2)^2} \right] (s + \alpha)(\alpha^2 + \omega_s^2) \\
(19) \quad h_2(s) &= \frac{s}{\omega_s \alpha} \left[ \frac{(s + \alpha)^2 + \omega_s^2}{(s + \alpha^2 + \omega_s^2)^2} \right] (s + \alpha)(\alpha^2 + \omega_s^2) \\
(20) \quad h_2(s) &= \frac{s(s + \alpha)}{\omega_s} \left[ \frac{(s + \alpha)^2 + \omega_s^2}{(s + \alpha^2 + \omega_s^2)^2} \right] (s + \alpha) + (\alpha^2 + \omega_s^2) \\
\end{align*}

Substituting \( a = \frac{\alpha}{2} = \frac{\omega_s}{c} \)

\begin{align*}
(21) \quad h_2(s) &= \frac{s(s+2a)}{\omega_s \alpha} \left[ \frac{(s+a)^2 + \omega_s^2}{(s+2a)^2 + \omega_s^2} \right] \\
(22) \quad h_2(s) &= \frac{2}{3} \frac{s(s + \omega_s^2)}{\omega_s} \left[ \frac{(s + \omega_s^2)^2 + \omega_s^2}{(s + \omega_s^2)^2 + \omega_s^2} \right] \\
(23) \quad \text{where } \omega_s &\approx 1.04 \omega_s \\
\end{align*}

Now we find

\begin{align*}
(24) \quad m(s) &= \frac{3}{4} \frac{s(s + \omega_s^2)(s + \omega_s^2)^2 + \omega_s^2}{\omega_s} \\
\end{align*}

Results:

\[ h_1(s) = \frac{8}{9} \frac{(s + \omega_s^2)^2 + \omega_1^2}{(s + \omega_s^2)^2 + \omega_2^2} \left[ \frac{(s + \omega_3^2)^2 + \omega_s^2}{(s + \omega_3^2)^2 + \omega_1^2} \right] \\
\]
\[ h_2(s) = \frac{2}{3} \frac{s(s + \omega_s)}{(s + \frac{\omega_s}{3})^2 + \omega_s^2} \]

\[ m(s) = \frac{3}{4} \frac{s(s + \omega_s)}{(s + \frac{\omega_s}{3})^2 + \omega_s^2} \]

where

\[
\begin{align*}
\omega_1 &= 1.28 \omega_s \\
\omega_2 &= 0.9 \omega_s \\
\omega_3 &= 1.04 \omega_s
\end{align*}
\]
APPENDIX B

DETERMINING THE QUADRATURE RESPONSE TO THE INPUT OF Figure V-5 FOR IDEALIZED DIRECT RESPONSE

We wish to find the quadrature response to the input of Figure V-5 (page 36) for the case where the direct response is idealized. That is, the quadrature transfer function is \( \frac{1}{j} m(s) \).

First we find the unit ramp (Laplace transform = \( \frac{1}{s^2} \)) response because:

![Diagram of unit ramp](image)

**Figure B-1**

The unit ramp response is:

\[
(1) \quad \mathcal{L}^{-1} \left[ \frac{(s+2a)[(s+a)^2 + \omega_3^2]}{s \left[ (s+a)^2 + \omega_1^2 \right] \left[ (s+a)^2 + \omega_2^2 \right]} \right] = f_r(t) \quad \text{where} \quad a = \frac{\omega_2}{\omega_3}
\]

and \( \omega_1, \omega_2, \omega_3 \) are given in Appendix A.

\[
(2) \quad f_r(t) = a_0 + a_1 e^{-at} \cos(\omega_1 t + \phi_1) + a_2 e^{-at} \cos(\omega_2 t + \phi_2)
\]

\[
(3) \quad a_0 = \left( \frac{3}{4} \right) \frac{(2a)(a^2 + \omega_3^2)}{(a^2 + \omega_1^2)(a^2 + \omega_2^2)}
\]

\[
= \frac{3}{2} \frac{\omega_3}{\omega_0} \frac{\left[ \frac{1}{36} + (1.04)^2 \right]}{\omega_0^2} \frac{\omega_3^2}{\omega_0^2} \left[ \frac{1}{36} + 0.81 \right] \left[ \frac{1}{36} + 1.28^2 \right]
\]

\[-60-\]
(4) \[ a_0 = \frac{0.79}{\omega_s} \]

(5) \[ a_1 = \frac{3}{2} \frac{\left(\omega_2^2 - \omega_1^2\right)}{\left(\omega_2^2 - \omega_1^2\right)(2\omega_1)} = \frac{3}{4} \frac{(1.08 - 1.64)}{(0.81 - 1.64)(1.28)} \]

(6) \[ a_1 = \frac{0.32}{\omega_s} \]

(7) \[ a_2 = \frac{3}{2} \frac{\left(\omega_3^2 - \omega_2^2\right)}{\left(\omega_2^2 - \omega_1^2\right)(2\omega_2)} = \frac{3}{4} \left[ \frac{(1.08 - 0.81)}{(1.64 - 0.81)(0.81)} \right] \]

(8) \[ a_2 = \frac{0.27}{\omega_s} \]

(9) \[ \phi_1 = -\frac{\pi}{2} + \tan^{-1} \frac{\omega_1}{a} - \tan^{-1} \left(\frac{\omega_1}{a}\right) \]

\[ = -\frac{\pi}{2} + \tan^{-1}(1.28)(6) - \left[ \pi - \tan^{-1}(1.28)(6) \right] \]

\[ \phi_1 = -270^\circ + 2 \times 82.5^\circ = -270^\circ + 165.2^\circ \]

(10) \[ \phi_1 \approx -105^\circ \]

(11) \[ \phi_2 = -\frac{\pi}{2} + \tan^{-1} \left(\frac{\omega_2}{a}\right) - \tan^{-1} \left(\frac{\omega_2}{a}\right) \]

\[ = -\frac{\pi}{2} + \tan^{-1}(0.9x6) - (\pi - \tan^{-1} 0.9x6) \]

\[ = -270^\circ + 2x79.5^\circ = -270^\circ + 159^\circ \]

(12) \[ \phi_2 = -111^\circ \]

(13) \[ f_r(t) = \frac{1}{\omega_s} \left[ 0.79 + 0.39 e^{-\omega_s t} \cos(1.28 \omega_s t - 105^\circ) \right. \]

\[ + 0.27 e^{-\omega_s t} \cos(0.9 \omega_s t - 111^\circ) \]

On the basis of the unit ramp response of equation (13), we are able to plot the response for the input shown in Figure V-5 by the superposition
technique shown in Figure B-1.

The input slope is \( \frac{1}{T_0} = \frac{2w_s}{\pi} \), so the output is obtained by multiplying equation (13) by \( \frac{2w_s}{\pi} \).

\[
(14) \quad f_c(t) = \frac{2}{\pi} \ 0.79 \ 1 + 0.5 \ e^{-\frac{\omega_s}{6} t} \ \cos(1.28 \ \omega_s t - 105^\circ) + \\
+ 0.34 \ e^{-\frac{\omega_s}{6} t} \ \cos (0.9 \ \omega_s t - 110^\circ)
\]

\( f_0(t) - f_0(t-T_0) \) is used to obtain the response curve shown in Figure V-15.