Composites Cost Modeling: Complexity

by

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and

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at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May, 1993

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ABSTRACT

The reduced funding of defense-related projects since the end of the Cold War has caused a shift in the field of composites to cost orientation, as applications in the commercial sector have become the leading technology drivers. This shift in orientation has prompted the exploration of ways to make advanced composites more cost effective. Cost effectiveness is being accomplished by automated manufacturing technologies, improved material systems, and concurrent design methodologies. The motivation of this research is the latter of these, specifically the incorporation of cost as a variable in the design of composite structures. This has led to the development of a theoretical framework for a designer's cost model based on complexity.

Various measures of complexity are explored, and metrics based on in-plane shear and ply stacking information are proposed. A cost modeling methodology based on a first order model approximation for manufacturing processes is introduced, and a case study is presented to illustrate its use. Sources of variation in cost data and their effects on the reliability of the cost model are discussed.

The impact of environmental factors on the use of advanced composite materials is discussed in the context of policy-making and materials selection. The effects of environmental legislation on the acceptance of composites are examined, and an energy-based methodology for making materials choice decisions is proposed.

Thesis Supervisor: Dr. Timothy G. Gutowski
Professor, Department of Mechanical Engineering
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As I sit here in the wee hours putting the final touches on this thesis, my state of consciousness allows me to think only of one thing: how grateful I am for the existence of computers and coffee, without which this whole endeavor would have been orders of magnitude more difficult and laborious, if not impossible. Ah, the wonders of modern technology and caffeine.

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Chapter 1

Introduction: Composites and Cost

1.1 Background

The end of the Cold War has brought about major changes in the driving forces behind the development and implementation of advanced composite material technologies in the United States. In the 1980's, the Department of Defense was a major contributor to the growth of the advanced composite sector by way of various military programs. Programs such as the B-2 bomber and the F-117 stealth fighter were prime examples of the ever-increasing performance requirements of combat aircraft for which advanced composites are ideally suited. These programs had a heavy emphasis on performance, and a 'cost-is-no-object' philosophy was generally the rule. With the dissolution of the Communist block in the early 1990's came also a shift in U.S. thinking on defense-related issues, which resulted in major cutbacks of defense projects. These cutbacks have led to a paradigm shift in the approaches to the development of advanced composites. As commercial enterprises are becoming the major innovators and pioneers in the use of advanced composites, greater emphasis is now being placed on cost-effectiveness.
In an effort to reduce the cost of manufacturing composites parts and structures, it is necessary to look at how opportunities for cost reduction can be identified, and at what stages of the product development process these opportunities arise. Figure 1.1 shows the magnitude of cost savings that can be achieved throughout the product cycle up to production. It is important to notice that although only a small fraction of program costs have been deployed at the early design stages, the decisions made then could influence up to 95% of the total cost.¹ This leads to the notion that some of the best

![Figure 1.1: Decreasing opportunity for cost savings in program cycle. (source: Noton, p420)](image)

opportunities for cost reduction arise at the design stage of product development.

Cost considerations can be taken into account at the design stage by incorporating cost as a variable into design methodologies. This concept is referred to as a Designer's Cost Model, wherein a design engineer can obtain quick cost estimates of alternate design in order to make trade-offs based on cost.

1.2 Designer's cost model

For a cost model to be useful to the design engineer, it should ideally incorporate three elements:

- physical significance
- ease of use
- accuracy

The cost model should first be based on principles which make physical sense. All too often cost models are based on statistical fits to parameters over which a designer has no control. Such models are of little use, since they provide no direct feedback as to what are the cost drivers that the designer can vary.

Secondly, the cost model should be simple to use, in order to provide the designer an incentive to use it. It should require minimal involvement by the designer, and should provide results in a timely fashion. Many cost models have a cumbersome number of steps and a large number of parameters with which the designer must deal; often, these models are not used at the design stage due to the large time commitment they require.
Lastly, the cost model must be accurate, and provide the designer with some notion of the reliability of an estimate. No cost model can claim to be perfect, but if the degree of uncertainty with which a model predicts cost is known, the designer can make better informed trade-off decisions by placing appropriate levels of reliance on particular estimates.

1.3 Complexity

It has been established that the complexity of a part is a significant cost driver in parts manufacture. A cost model for composites manufacture should therefore incorporate measures of complexity that a designer can easily abstract from a design with readily available design tools. The complexity metrics used should also conform to the first guideline proposed in the previous section, and should therefore have some physical significance.

1.4 Scope of thesis

The need for a designer's cost model in the design and engineering of composites structures has become more and more apparent over the past several years, as composites have gained ground in the commercial aerospace sector. New programs which promote the use of advanced composites are typically evaluated using traditional costing methods, which require prolonged periods of time (months, sometimes up to one year) and tremendous effort (companies have staffed costing departments which devote themselves fully to this task). In an effort to reduce development times and manufacturing costs before they are incurred, the NASA Advanced
Composites Technologies (ACT) program has set out to develop a theoretical framework for a designer's cost model. This thesis is a result of the ongoing research at M.I.T. sponsored under the ACT program.

1.5 Overview of thesis

Chapter 2 takes a look at existing methodologies for predicting the cost of composite structures. Existing composites cost models are presented in the context of two classical approaches to cost modeling – bottom-up and top-down methodologies. The models are briefly described and discussed.

Chapter 3 first reviews previous work on quantifying complexity. Existing complexity measures, as well as some of their applications are described. Two new complexity measures are then introduced and explained: shear and ply stacking information. The determination of shear for two part geometries, curved contours and beads, is described.

Chapter 4 presents the framework for a designer's cost model based on complexity measures. The First Order Model is described as background, and a simplified version thereof, The Basic Form, is proposed. Cost data is discussed in the context of the Basic Form, upon which the data is dissected into three regimes – complex, extensive, and mixed regimes. The complex regime is discussed. A cost data reduction methodology is then proposed, and a case study based on actual composites production data is presented.

Chapter 5 is a brief discussion of sources of variation in cost data and the effects thereof on the reliability of a cost model. Lessons learned from the case study are then discussed.
The last chapter is a policy discussion of composites in the context of growing environmental awareness. The environmental pros and cons of advanced composites are discussed, and the influences of these factors on composites usage are examined. The effect of environmental legislation on the acceptance of composites is discussed, and an energy-methodology for making sounder materials choice decisions is proposed.
Chapter 2

Composites Cost Models

2.1 Overview

In formulating new approaches to classical problems, it is important to examine and understand their history, so as to build upon the experiences of others, rather than try to reinvent the wheel. As far as advanced composites are concerned, efforts to develop tools for predicting costs has a limited history. This chapter will examine some of the approaches taken over the past two decades to predict the costs of manufacturing advanced composite structures. The last section of this chapter will present the basis of the work presented in this thesis.

Cost prediction methodologies have typically taken on one of two forms - the bottom-up approach and the top-down approach. In the bottom-up approach, the processes used to fabricate parts are broken down to the detail stage, and every physical step is accounted for. The top-down approach usually consists of evaluating costs on a systems basis, bringing into play characteristics and uses of the final product.
2.2 Bottom-up approaches

2.2.1 ACCEM

One of the earliest attempts to predict the costs associated with composites manufacture is embodied in the Advanced Composite Cost Estimating Manual Program (ACCEM).\textsuperscript{1} In 1975, Northrop Corporation's Aircraft Division was contracted by the U.S. Air Force to develop a computerized methodology to estimate the fabrications costs of composite components. The result of this study was a model based on Industrial Engineering Standards (IES) relationships, obtained by the reduction of time-motion data which was produced in-house at Northrop. An example of these equations, which are mostly power law curve fits, is shown in Table 2.1, which shows the level of detail involved in the IES type of approach. The ACCEM model does make some provisions for part complexity by the use of complexity adders for parts with curved flanges. This aspect of the model will be examined in Chapter 3.

A typical cost analysis using ACCEM necessitates 60 to 70 relations of the type shown in Table 2.1 to account for all the steps involved in making a composite part, and requires a ply-by-ply, strip-by-strip analysis of the part. Due to the purely empirical basis for the cost estimating relationships (CER), the resulting power law equations and constants lack physical significance, and are thus of little use in helping a designer understand the cost drivers of a particular design or process. Furthermore, the magnitude of an ACCEM cost analysis make it undesirable to use for the quick type of cost estimation which designers require in order to make trade-offs based on cost.

<table>
<thead>
<tr>
<th>Detail Elements</th>
<th>Setup</th>
<th>Runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean lay-up tool surface</td>
<td></td>
<td>0.0000006A</td>
</tr>
<tr>
<td>Apply release agent to lay-up tool surface</td>
<td></td>
<td>0.0000009A</td>
</tr>
<tr>
<td>Position template (mylar) on table and tape down</td>
<td></td>
<td>0.000107A^{0.77006}</td>
</tr>
<tr>
<td>Ply deposition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manual</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- 3&quot; tape</td>
<td>0.05</td>
<td>0.00140L^{0.6018}</td>
</tr>
<tr>
<td>- 12&quot; tape</td>
<td>0.05</td>
<td>0.001454L^{0.8245}</td>
</tr>
<tr>
<td>- Woven material</td>
<td>0.05</td>
<td>0.000751A^{0.6295}</td>
</tr>
<tr>
<td>Hand-Assist</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- 3&quot; tape</td>
<td>0.10</td>
<td>0.000368L^{0.8446}</td>
</tr>
<tr>
<td>- 12&quot; tape</td>
<td>0.10</td>
<td>0.001585L^{0.5580}</td>
</tr>
<tr>
<td>CONRAC Auto</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(720 ipm)</td>
<td>0.15</td>
<td>0.000663L^{0.4942}</td>
</tr>
<tr>
<td>(360 ipm)</td>
<td>0.15</td>
<td>0.00058L^{0.5716}</td>
</tr>
<tr>
<td>Transfer ply from template to stack or lay-up tool</td>
<td></td>
<td>0.000145A^{0.6711}</td>
</tr>
<tr>
<td>Transfer stack to lay-up tool</td>
<td></td>
<td>0.000145A^{0.6711}</td>
</tr>
<tr>
<td>Clean curing tool surface</td>
<td></td>
<td>0.0000006A</td>
</tr>
<tr>
<td>Apply release agent to curing tool surface</td>
<td></td>
<td>0.0000009A</td>
</tr>
<tr>
<td>Transfer lay-up to curing tool</td>
<td></td>
<td>0.000146A^{0.6711}</td>
</tr>
</tbody>
</table>

Where:  
A = Area of ply, or greatest ply area of stack or lay-up, in square inches  
L = Length of ply strip, in inches

The ACCEM model, though, has gained a reputation among composites manufacturers of being fairly reliable, although it is somewhat dated. On this basis, results from ACCEM analyses have been used to substantiate the First Order Model, presented in Chapter 4. The power law relationships, such as shown in Figure 2.1, can be described by first order systems, resulting in
Figure 2.1: ACCEM power law fits for unidirectional tape lay-up.

extremely good curve fits. This leads to the notion that most physical processes can be modeled as first order systems. This will be explained in more detail in Chapter 4.

2.2.2 FACET

In the early 1980's, Lockheed Georgia was contracted jointly by the Department of Defense and NASA to work on an extension of the ACCEM program, which resulted in the Fabrication Cost Estimating Technique
(FACET).\textsuperscript{2} The Lockheed study is based on an extensive data base of composites manufacture. The FACET model, similar to the ACCEM model, uses a bottom-up type of approach to cost prediction, and thus suffers from the same type of setbacks when considered from a designer's point of view.

\section*{2.3 Top-down approaches}

\subsection*{2.3.1 RAND}

The RAND Corporation was contracted in the late 1980's by the U.S. Air Force to study the impact of advanced materials usage on airframe structure costs. The results of this study was the report "Advanced Airframe Structural Materials: A Primer and Cost Estimating Methodology."\textsuperscript{3} The report includes a methodology for assessing airframe costs using a top-down approach. This study is based on an extensive database of military aircraft manufacturing data, from which cost estimating relationships (CER) were developed for the major cost elements. The cost elements are the following

- Nonrecurring engineering
- Nonrecurring tooling
- Development support
- Flight test
- Recurring engineering
- Recurring tooling
- Recurring tooling
- Recurring manufacturing material
- Recurring quality assurance


The inputs to the CER's are the following

- Aircraft empty weight (lb)
- Maximum speed of craft (kn)
- Number of flight test aircraft
- Type of aircraft (cargo or noncargo)
- Total structure weight by material type
- Percentage of functional cost elements attributable to structure

To obtain an estimate for a particular aircraft, it is necessary to approximate these factors, as well the proportion of composites usage.

This type of rough program cost estimation is extremely handy for deciding procurement issues, but from the designer's point of view, the resulting figures would be of little use in making design trade-offs between different features and different material types.

### 2.3.2 CAIG

In an initiative taken by the Cost Analysis Improvement Group (CAIG) in the Office of the Secretary of Defense, two cost estimating relationships were developed in 1991, one for estimating overall airframe costs, and the other for estimating the cost of individual parts and assemblies. This study is based on an extensive data base including data from the FACET and ACAP programs, as well as production data from the AV-8B (Harrier) program and from the Design and Manufacturing of Advanced Thermoplastic Structures (DMATS) program.

The first relation is for airframe 100th-unit cost \( AF_{100} \), and depends on three parameters: airframe unit weight (AUW), composite materials

---

percentage (ADVMAT), and a term called the weight complexity factor (WCF), which is simply the ratio of the aircraft empty weight to the structural weight of the craft.

\[ AF_{UC^{1st}} = 0.00286 (AUW)^{0.89} (ADVMAT)^{0.16} (WCF)^{0.94} \] (2.1)

This relation is based on a sample size of 16, and an R\(^2\) value of 0.83 is claimed. Even though the fit seems to be very good, due to the magnitude of the values of \( AF_{UC^{1st}} \), a prediction using this relation could be off by several million dollars on aircraft ranging from $5 million to $40 million.

The second relation is for first-unit part cost (1\(^{st}\)UC) given in labor hours, and accounts for ply cutting, material transfer, lay-up and preform placement, demold and cure preparation, finishing operations, and quality control. It is based on the weight of the finished part (W), three binary parameters describing the use of the part, and one binary parameter to account for differences in manufacturers.

\[ 1^{st} UC = 36.8(W)^{0.67} 1.87^{(F)} 0.68^{(F)} 1.25^{(FA)} 4.8^{(LTV)} \] (2.2)

The parameters are defined as:

- **P** = indicator variable; 1 if the part is a primary structural component, 0 if it is a secondary structural component
- **F** = indicator variable; 1 if the part is a fuselage component, 0 if it is a wing or empennage component
- **FA** = indicator variable; 1 if the part is associated with a fighter or attack aircraft, 0 if the part is associated with a transport/passenger aircraft or helicopter
- **LTV** = indicator variable; 1 if the part is manufactured by LTV Aerospace, 0 otherwise
The sample size is 217 parts, and an $R^2$ value of 0.82 is claimed. As can be seen by the definition of the parameters, this equation is probably of little use to a designer, although the parameters $P$ and $F$ might be seen as a general indicator of the part geometry. Furthermore, due to the large range in part weights, the $R^2$ value tends to discount the variance in favor of definite trends. This can be seen in Figure 2.2.

![Figure 2.2: Actual versus predicted first-unit cost for airframe parts.](source: Harmon & Arnold, p.652)

Due to the lack of physical significance of this equation and its relative inaccuracy, it can only be used to demonstrate overall trends in advanced composite structure fabrication – one of which is a definite weight dependence, and another a general trend of dependence on geometric factors associated with use and location of a specific part on an aircraft.
2.4 ACT designer's cost model

To address the inadequacies of existing costing methodologies and models for use by designers, NASA incorporated the development of a designer's cost model into its Advanced Composite Technologies (ACT) program in an effort to develop technologies which will be of aid to American companies facing international competition. The aim of this study is to develop a framework for incorporating cost-as a design variable, as an additional DFX factor, in an effort to reduce costs and to lead toward more optimally designed advanced composite structures. This thesis represents a portion of the efforts undertaken at M.I.T. under this program, and presents a suggested approach to the problem of estimating manufacturing costs of composites.

This approach will try to alleviate some of the deficiencies encountered in existing methodologies, namely a lack of physical significance, difficulty of use, and uncertain levels of accuracy.

\(^5\text{DFX is a term used to describe the various 'design-for' trends and methodologies such as design-for-assembly, design-for-manufacture, and more recently design-for-disassembly and design-for-environment.}\)
Chapter 3

Complexity measures

3.1 Introduction

A major cost driver in composites manufacture is the complexity of the parts being made. Burnet suggests the need to develop complexity measures which can be used as bases and guidelines for cost estimating methodologies.\(^1\) He argues that efforts to quantify complexity in the past have been too subjective and have therefore lacked reproducibility.

Burnet stipulated that part complexity can be categorized into three different types:

- Complexity of concentration
- Complexity of distribution
- Complexity of state

The complexity of concentration is an indicator of the concentration of parts or features within a certain space. Conversely, the complexity of distribution relates to the intricacies encountered with widely distributed systems. The complexity of state is associated with the difficulties that arise with specific materials or manufacturing processes and environments.

Deshmukh cites two categories for classification of system complexities in the context of manufacturing: static and dynamic complexity\(^2\). Static complexity, also referred to as part mix complexity, is a measure of the diversity of components in a system and the interaction between them; it is a function not only of the number of parts and operations in a particular process, but also of the connectivity between parts and processes. A static complexity metric is thus a good indicator of the difficulty associated with scheduling and planning decisions with multiple parts and part types. On the other hand, dynamic complexity is a measure of the unpredictability of outputs of a system. A dynamic complexity metric is a good indicator of the predictability of the parts produced by a manufacturing system.

In order to gain a better understanding of the notion of complexity, the first part of this chapter will review work previously done on relating complexity metrics to manufacturing costs. The second part of this chapter will then present complexity metrics proposed by the author. The work in this thesis will concentrate on geometric complexity metrics, which are of importance in a designer's cost model. Geometric complexity metrics are methods for quantifying complexity based on geometric features, and they thus indicate the relative difficulty of manufacturing a part.

3.2 Existing measures

3.2.1 ACCEM

The ACCEM cost model discussed in Chapter 2 does account for certain aspects of part complexity. The model relies on 'layup complexity increments' which consist of a set of equations, determined from parametric fits to data, that impose additional time requirements for making parts with straight, radial and curved bends. Table 3.1 gives these relations and Figure 3.1 illustrates the parameters used for curved flanges. These complexity factors are easily computable with information that is readily available to the design engineer.

Table 3.1: ACCEM layup complexity increments

<table>
<thead>
<tr>
<th>Bend Type</th>
<th>Bend Factors</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight bends</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sharp, male</td>
<td>0.00007Lb</td>
<td></td>
</tr>
<tr>
<td>Sharp, female</td>
<td>0.00016Lb</td>
<td></td>
</tr>
<tr>
<td>Radial, male</td>
<td>0.00007Lb</td>
<td>when R ≤ 2&quot;</td>
</tr>
<tr>
<td></td>
<td>no factor applied</td>
<td>when R &gt; 2&quot;</td>
</tr>
<tr>
<td>Radial, female</td>
<td>0.00016Lb</td>
<td>when R ≤ 2&quot;</td>
</tr>
<tr>
<td></td>
<td>[0.00047R^{−1.3565}]Lb</td>
<td>when R &gt; 2&quot;</td>
</tr>
<tr>
<td>Curved bends</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tape</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stretch flange</td>
<td>[0.015R^{−0.5532}F^{0.7456}]Lb</td>
<td></td>
</tr>
<tr>
<td>Shrink flange</td>
<td>[0.0064R^{−0.5379}F^{0.5178}]Lb</td>
<td></td>
</tr>
<tr>
<td>Woven</td>
<td>[0.00444R^{−0.55958} + 0.0007]Lb</td>
<td></td>
</tr>
</tbody>
</table>

Where:
- \( L_b \) = Length of bendline
- \( R \) = Radius of curvature
- \( F \) = Flange width
3.2.2 Pugh

Pugh propounds that complexity is a driver of the cost of assemblies, and presents a complexity factor which he suggests be used by designers as a guideline for a design-for-simplicity approach, wherein the designer would try to minimize overall complexity. He argues that simplicity not only decreases overall cost, but also increases reliability and enhances quality.

This notion of complexity is rooted in the fact that discontinuities in a product add to cost, and therefore complexity should reflect the magnitude of discontinuity within a design. The level of discontinuity involves three parameters:

- The number of parts, $N_p$
- The number of types of parts, $N_t$
- The number of interconnections and interfaces, $N_i$

---

An additional parameter \( f \) denoting the number of functions that the product is expected to perform, and a constant of convenience \( K \) are also included. The expression for the complexity factor is then

\[
C_i = \frac{K}{f \sqrt{N_1 N_2 N_3}} \tag{3.1}
\]

This approach to design simplification has had some success in the electronic equipment industry, where the possibilities for reductions in part complexity as defined in Equation 3.1 abound.

### 3.2.3 Pearce

Pearce has proposed a complexity index for estimating the costs of making injection-molding molds based on a statistical and knowledge-based approach.\(^4\) Through the use of an industry survey and statistical data reduction, he determined that moldmaking costs were driven by six parameters:

- Number of dimensions on the print
- Number of different surface finishes required
- Length of part
- Depth of part
- Tightest tolerance
- Number of cavities

He concluded that the first parameter, the number of dimensions on a detail drawing, was a good indicator of the complexity of a mold. This is the same conclusion reached by Muter for machined parts\(^5\), as shown in Section 3.2.6.

---


Pearce claims a 85% to 90% accuracy in predicting mold cost using this approach.

### 3.2.4 Boothroyd & Dewhurst

Boothroyd & Dewhurst have done a great deal of work in the area of design-for-assembly, and have been able to identify a complexity metric based on part symmetry.\(^6\) They have related the degree of part symmetry to the handling time required to add a part to a built up assembly. The part symmetry is simply the sum of two angles \(\alpha\) and \(\beta\), where \(\alpha\) is the rotational symmetry about an axis perpendicular to the axis of insertion, and \(\beta\) is the rotational symmetry about the axis of insertion. Figure 3.2 gives examples of part symmetry.

![Diagram of part symmetry examples](image)

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>0</th>
<th>180</th>
<th>180</th>
<th>90</th>
<th>360</th>
<th>360</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>0</td>
<td>0</td>
<td>90</td>
<td>180</td>
<td>0</td>
<td>360</td>
</tr>
</tbody>
</table>

**Figure 3.2:** Examples of part symmetry.  
(source: Boothroyd & Dewhurst)

It was determined through experiments that the handling time of a part varies in an approximately linear fashion with the total angle of symmetry.

---

3.2.5 Information Theory

In the 1940's, Shannon presented a concept called Information Theory, which models communication systems as shown in Figure 3.3. A message is emanated from an information source, after which a transmitter produces a signal from the message, and sends it over a channel. A receiver then decodes the received channel, which is the original signal plus noise produced over the channel, and sends it to a destination.

A mathematical model for the transmission of a message over such a system can be formulated based on the probability that a particular message will be conveyed. Equation 3.2 describes the information content of a message in terms of the probability $p = \frac{1}{N}$ that this message will assume a particular value among a group of $N$ possible and equiprobable values.

$$I = \log_2 \left( \frac{1}{p} \right)$$

(3.2)

The logarithm to the base 2 is used so that the resulting information measure is in bits, although the natural log or logarithm to the base 10 could equally be
used. An adapted version of this equation has been proposed to account for the information associated with particular physical tasks and design features.\textsuperscript{7}

\[ I = \log_2 \left( \frac{\text{range}}{\text{tolerance}} \right) \]  \hspace{1cm} (3.3)

The range is the domain of possible values the particular task or feature could assume, and the tolerance is the degree of accuracy with which one wishes to measure this value. The quantity range/tolerance is in fact the probability that the measured quantity will fall within one unit of the tolerance in the given range.

If one applies Equation 3.3 to the simple task of having a person position a coin on a plate which is sitting on a table one arrives at the following equation.

\[ I_{\text{position}} = \log_2 \left( \frac{\text{range of motion}}{\text{size of target}} \right) \]  \hspace{1cm} (3.4)

In this case, the range is the extent of the person's reach onto the table and the tolerance is the size of the plate. If one uses smaller and smaller plates, it will take the person longer and longer to accomplish this task; likewise, the information content of the task will also grow. Fitts studied this phenomenon in the 1950's and 1960's, and demonstrated that positioning time is proportional to information content, which he called the index of difficulty.\textsuperscript{8}

Information theory has been applied to several areas of cost modeling, as will be discussed in the following sub-sections, and it is also the basis for a proposed complexity measure presented in section 3.3.2.

3.2.6 Fiber bending information

Tse has applied Information Theory to the bending of fibers in composite fiber-matrix systems.\(^9\) A fiber can be modeled as an information storage device, which, when bent into various configuration while being processed, stores a certain amount of information. In order for a fiber to assume a specific shape, it must thus be 'given' information by a process, be it through a tool, an operator, or a machine.

If one discretizes the curved portion of a single fiber into equal length straight segments, as shown in Figure 3.4, and one assumes a constant radius of curvature \(\rho\) for that portion of the bend, the linear deflection \(\delta\) of the fiber from one segment to the next is

\[ \delta = \rho \frac{\zeta^2}{2} \]  \hspace{1cm} (3.5)

\[\text{Figure 3.4: Discretization of the curved portion of a fiber.}\]

where $\zeta$ is the angular deflection which corresponds to $\delta$. Assuming that linear deflections which are less than $\delta$ cannot be measured, the information required to deflect an fiber segment by an angle $\zeta$ is

$$ I = \log_2 \left( \frac{1}{p} \right) = \log_2 \left( \frac{\theta_{\text{max}}}{\zeta} \right) \quad (3.6) $$

where $\theta_{\text{max}}$ is the greatest possible deflection a fiber could undergo. If one divides the curved portion of a fiber with enclosed angle $\theta$ into $N$ equal length segments with angular deflection of $\zeta$ relative to each other, we note that

$$ N = \frac{\theta}{\zeta} = \theta \sqrt{\frac{p}{2\delta}} \quad (3.7) $$

Summing up the information over the entire curved portion of a fiber then yields

$$ I_{\text{bend}} = N \log_2 \left( \frac{\zeta}{\theta_{\text{max}}} \right) = \theta \sqrt{\frac{p}{2\delta}} \log_2 \left( \frac{2\delta}{\theta_{\text{max}}} \right) \quad (3.8) $$

Equation 3.8 demonstrates that for a fiber bend of constant radius of curvature, the information stored in the bend is directly proportional to its enclosed angle. Thus we have

$$ I_{\text{bend}} \propto \theta \quad (3.9) $$

In order to obtain the information content of an entire fiber, one would thus add up all the $\theta$'s the fiber undergoes along its length.
Fiber bending information has been applied to computing the costs of stringer type shapes with a degree of success.\textsuperscript{10} When correlated to ACCEM cost estimates, the information content of various stringer shapes yields a linear relationship with good correlation coefficient, as shown in Figure 3.5.

![Graph showing linear relationship between ACCEM layup time (hours) and information content (MB)].

**Figure 3.5**: Fiber bending information to cost correlation for stringers.

### 3.2.7 Dimension information

Muter proposes an approach to defining part complexity based on the idea that the information required to make a part is embodied in the detail part drawings.\textsuperscript{11} An Information Theory approach is taken to quantify the complexity associated with each nonredundant dimension on a part drawing.

\textsuperscript{10}Tse.
\textsuperscript{11}Muter.
The information content of a part is then the sum of the individual dimension information terms, given by Equation 3.10.

\[ I = \sum_{i=1}^{N} \log_2 \left( \frac{\text{dimension}_i}{\text{tolerance}_i} \right) \]  

(3.10)

Both the individual dimensions and the associated tolerances are typically indicated on detail part drawings.

A good correlation was found between dimension information and machining cost data when fit with a second order equation. Cost estimates produced using this equation were found to exhibit a similar degree of accuracy when compared to estimates performed by an experienced estimator.

Due to the logarithm operator and the nature of the ratio dimension:tolerance for machined components, Equation 3.10 can in most cases be reduced to

\[ I \propto (\# \text{ of dimensions}) \]  

(3.11)

which agrees with Pearce's concept of a complexity index for moldmaking.

### 3.3 Proposed measures

#### 3.3.1 Shear

It has been determined that for certain composite manufacturing processes, such as drape forming and diaphragm forming, in which in-plane shear is a significant deformation mode, the amount of shear induced in the laminate to generate the final part geometry is a good indicator of the difficulty of
producing the part. It is therefore logical to examine shear as a possible candidate for a complexity metric in the cost modeling of composite materials.

Using a CAD fiber mapping method developed by Gonzalez-Zugasti, ideal fiber mappings were obtained for various typical aerospace composite part geometries. Features that increase the difficulty in making a part were chosen for analysis: curved contours (shrink and stretch flanges) and beads. Ideal mappings for a curved C-channel and for beads of various aspects are shown in Figure 3.6 and Figure 3.7, respectively.

![Figure 3.6: Ideal mapping of a curved C-channel.](image)

---


3.3.1.1  Methodology for obtaining shear

Shear is measured from a fiber mapping using a method based on local incremental shear, which is defined as

\[ d\Gamma_{12} \equiv \frac{ds_1 - ds_n}{du} \]  (3.12)
where $\text{d} s_n$ and $\text{d} s_1$ are incremental lengths along two adjacent fibers, and $\text{d} u$ is the inter-fiber spacing, as shown in Figure 3.8. The net shear between fibers on a mapping is obtained by calculating the difference in orthogonal lengths of adjacent fibers and dividing by the spacing between the two fibers.

![Diagram](image)

**Figure 3.8:** In-plane local incremental shear.

It can be shown that the in-plane shear deformation can also be related to the geodesic curvature of a fiber as

$$\Gamma_{12} = \int_0^L \kappa_k(s) \text{d}s$$

(3.13)

By application of the Gauss-Bonnet theorem to an enclosed region bounded by two adjacent fibers, denoted by the paths $C_1$ and $C_1$, as shown in Figure 3.9, one obtains the relation

$$\int_{C_1} \kappa_k \text{d}s + \iint_{\mathcal{D}_k} K \text{d}A = 2\pi - \sum_{i=1}^{\infty} \theta_i$$

(3.19)
Figure 3.9: The enclosed region $\mathcal{R}$ bounded by $C_1$, smooth curves.

where $K$ is the gaussian curvature of the region. If the path $C_3$ is that of the fiber for which shear is to be determined relative to the fiber represented by the path $C_1$, and the assumptions are made that paths $C_1$, $C_2$, and $C_4$ are along geodesic paths ($\kappa_s = 0$) and that the $\theta_i$'s sum up to $2\pi$, the relation for shear is then

$$\Gamma_{12} = \int_{C_1} \kappa_s ds = -\iint_{\mathcal{R}} K da = -K_T(\mathcal{R})$$

(3.15)

$K_T$ is the total curvature of the region. A more detailed derivation of this formulation can be found in [Tam].
3.3.1.2 Curved contour

Due to the multitudinous possible mappings on a curved C-channel depending on the initial fiber orientation, a singular mapping was chosen in which the initial fiber is along an axis of symmetry. Figure 3.10 shows the mapping of a fiber on a curved flange with the initial fiber, shown as a thick line on the edge of the contour, on an axis passing through the center of curvature of the part. A fiber mapped onto the top flange, such as the one shown at an arc length corresponding to an included angle \( \phi \), has the characteristic of behaving as an involute. The involute is perpendicular to the tangent of the contour where the fiber crosses over the edge of the contour. This leads to the results that the shear in a fiber crossing a curved contour of constant radius is equal to the angular distance between this fiber and the initial fiber, assuming the initial fiber is on a geodesic path (that it undergoes no in-plane deformation).

Figure 3.10: Singular mapping resulting in involute path.
By this reasoning, it can be determined that the maximum attainable shear for a curved flange is directly proportional to the enclosed angle $\alpha$ as shown in Figure 3.11, giving

$$\Gamma_{\text{max}} \propto \alpha \quad (3.16)$$

A more detailed description of shear for a curved C-channel can be found in Appendix A.

![Diagram of the enclosed angle of a curved flange](image)

**Figure 3.11:** The enclosed angle of a curved flange.

### 3.3.1.3 Beads

In order to facilitate the analysis of beads, they are approximated as tapered boxes, as shown in Figure 3.12. The shear of a fiber on the bead is obtained by measuring the enclosed angle the fiber makes when it is developed in the flat (when the surfaces are unfolded so that they all lie in one plane). Figure 3.13 shows part of the developed surface of a bead with a fiber displaying in-plane
Figure 3.12: Approximation of a bead showing mapped fiber.

Figure 3.13: Developed surface of bead showing shear in fiber.
shear. The total cumulative shear over this fiber is $4\beta - \pi$, which is also the maximum shear a fiber can attain on a bead with surface edge angle $\beta$. Thus in the case of a bead, the maximum attainable shear is proportional to the surface edge angle $\beta$, or

$$\Gamma_{\text{max}} \propto \beta$$

(3.17)

3.3.2 Ply stacking information

An inherent complexity that arises in the lay-up of composite laminates is that associated with the location and placement of successive plies on top of one another. In order to quantify this type of complexity, an Information Theory approach is taken. If the ply configuration of a particular part is examined, the lay-up of plies can be viewed as a stack of $N$ elements, where each element is a ply. If all $N$ elements are identical, only one configuration is possible, as shown in Figure 3.14a, and thus the probability of this configuration is 1.

When number of type of elements, denoted by $m$, changes, indicating a different ply size or shape, the number of possible configurations also changes, leading to a different probability of obtaining a particular configuration. Figure 3.14b shows two different configurations for the same $N$ and $m$. Equation 3.17 defines the probability of a particular stacking configuration of $N$ plies with $m$ types of plies (assuming a large inventory).

$$p = \frac{1}{m^N}$$

(3.17)
Figure 3.14: Various ply stack-ups.

From this one can obtain the information content of a ply stack-up:

\[ I = \log_2 \left( \frac{1}{p} \right) = \log_2 (m^n) \]  \hspace{1cm} (3.18a)
which can be rewritten as

\[ I = N \log_2 m \]  

(3.18b)

From a practical viewpoint, the \( m \) types of plies can be abstracted from a design as the number of ply drop-offs plus one. Equation 3.18b can be viewed as

\[ I = (\text{# of plies}) \times \log(\text{# of ply dropoffs } + 1) \]  

(3.19)

This complexity metric seems to be a significant cost driver in the hand lay-up process, as will be shown in the case study in Section 4.4.2.
Chapter 4

The Basic Form

4.1 Overview

This chapter will examine a theoretical framework for manufacturing cost analysis. The general framework of the First Order Model will be presented as an introduction and basis for the Basic Form. Aspects of the Basic Form will then be examined, and a generalized methodology for cost data reduction will be presented. The results of such an analysis will be presented and explained.

4.2 The First Order Model

Upon the examination of physical tasks and activities, it can be observed that these behave as first order systems.1 Such systems are of the form shown in Figure 4.1, which models a physical system as a signal with gain G and feedback H. The gain can be thought of the physical part of the process, such as moving an object through a certain distance, while the feedback, or delay, can be thought of as the cognitive part of the process, such as positioning an object at a specific location. In the context of a first order system, these two

---

parts of the process are respectively characterized by a steady state rate $v_n$ and a time constant $\tau$, as shown in Equation 4.1 for the processing rate $v$.

$$v = v_0(1-e^{-\frac{t}{\tau}}) \quad (4.1)$$

In terms of a one-dimensional process, such as measuring a length, this equation can be integrated, resulting in Equation 4.2 for the length $x$.

$$x = v_0 \left[ t - \tau \left(1-e^{-\frac{t}{\tau}}\right) \right] \quad (4.2)$$

A characteristic plot of time $t$ versus distance $x$ is shown in Figure 4.2. In order to obtain an explicit relationship for time in terms of distance, it is necessary to make some approximations, since Equation 4.2 cannot be reduced to a closed form solution for $t$. For $t < \tau$, a Taylor series approximation yields Equation 4.3, and for $t > \tau$, the expression is reduced to Equation 4.4.

$$t = \frac{2\tau}{v_0} \sqrt{x} \quad (4.3)$$

$$t \approx \tau + \frac{x}{v_0} \quad (4.4)$$
The majority of manufacturing processes fall into the latter category, and thus Equation 4.4 can be used as a realistic approximation for modeling manufacturing processes as first order systems.

For additive processes such as tape lay-up, it is necessary to project the first order approximation into three dimensions. This is done by first summing up times for n strips of width w and length x, for a ply, resulting in Equation 4.5b which gives time in terms of area A,

$$T = \sum t = \sum_{i=1}^{n} \left( \tau + \frac{x_i}{v_0} \right) = n\tau + \frac{1}{v_0} \sum_{i=1}^{n} (x_i)$$ \hspace{1cm} (4.5a)

$$T = n\tau + \frac{A}{v_0w}$$ \hspace{1cm} (4.5b)
and then adding up the times for N plies of thickness h through the part resulting in Equation 4.6, giving time in terms of part volume V.

\[
T = \sum_{i}^{N} \sum_{l}^{n} t = nN\tau + \frac{V}{v_{n}wh} \tag{4.6}
\]

Equation 4.6 can further be reduced to a time-weight relation by introducing material density, as shown in Equation 4.7.

\[
T = nN\tau + \frac{W}{\rho v_{n}wh} \tag{4.7}
\]

The ensemble of Equations 4.4, 4.5b, 4.6 and 4.7 form the basis of the First Order Model, first presented by Tse.\textsuperscript{2} Depending on the nature of a particular part or process, either of these equations can be applied.

Upon examination, Equations 4.4, 4.5b, 4.6 and 4.7 can be split up into two terms, the first term which contains \(\tau\), and the second which contains \(v_{n}\). The \(\tau\) term scales mainly with the complexity of the part being made, and the \(v_{n}\) term scales with extensive physical attributes of the part, such as length, area, volume, or weight. This aspect of these equations will be dealt with in the following sections.

### 4.3 The Basic Form

Upon analysis of actual composites manufacturing cost data, one can derive a simplified time-weight correlation, which is congruous with and analogous to the First Order Model. Figure 4.3 shows a typical plot of the weight rate of time versus weight for composite parts, from which we derive the relation
Figure 4.3: Typical plot of weight rate of time vs weight.

\[
\frac{T}{W} = \frac{\overline{A}}{W} + \overline{B} \tag{4.8}
\]

This can also be written

\[T = \overline{A} + \overline{B}W \tag{4.9}\]

which is directly analogous to Equation 4.7. In this case, \(\overline{A}\) is the complexity term and \(\overline{B}\) is the extensivity term. Equation 4.9 is what is termed the Basic Form, as manufacturing time data for a variety of processes take on this format.


4.3.1 The three regimes

Upon further examination, the data can be subdivided into three regimes - a complexity-driven zone, an extensivity-driven zone, and a zone in between in which interaction of complexity and extensivity occurs. These zones are identified on Figure 4.4.

![Graph showing three regimes of composites manufacture](image)

**Figure 4.4**: The three regimes of composites manufacture.

Parts weighing less than $W^*$ are considered to be in the complexity-driven zone, in which manufacturing time is mainly a function of part complexity. In the zone above $W^{**}$, time is considered extensivity-driven, while in the zone between $W^*$ and $W^{**}$, time is a function of both complexity and extensivity. This becomes more clear upon the examination of the data in a log-log plot, as shown in Figure 4.5. The data in the complex region conforms to a line with a slope of -1, indicating the absence of weight dependence in that particular region. In the extensive region, the data can be thought of as
conforming to a horizontal line, indicating strong weight dependence. In the mixed region, the slope is between -1 and 0, which suggest a blend of the two behaviors.

\[ W^* = \frac{A}{B} \]  

(4.10)

We also define

\[ W^{**} = \frac{\sigma_A}{\sigma_B} \]  

(4.11)

where \( \sigma_A \) and \( \sigma_B \) are the standard deviation for \( A \) and \( B \) respectively.
Since $\bar{B}$ scales with extensive parameters, such as weight in this case, its determination and interpretation are fairly simple. $\bar{B}$ is simply the $v_0$ term of the First Order Model, and is the inverse of either a distance, area, volume, or weight rate of material deposition for composite materials. With $\bar{B}$ defined, the extensive regime is accounted for.

### 4.3.2 The complex regime

As in the extensive regime, the complex regime's $\bar{A}$ is analogous to the $\tau$ term of the First Order Model. $\bar{A}$ is a function of part complexity as well as being a function of some form of extensive measure, and in itself also takes on the form of the Basic Form:

$$\bar{A} = \bar{A}_c + \bar{B}_c(x_c)$$  \hspace{1cm} (4.12)

The subscript $C$ denotes the complex regime. The $\bar{A}_c$ term is the complexity term, and the $\bar{B}_c(x_c)$ term accounts for the extensivity dependence that exists in the complex regime. The extensivity measure for $\bar{B}_c$ is not necessarily the same as for the $\bar{B}$ term. For example, the $\bar{B}$ discussed in the previous section is related to the part weight whereas $\bar{B}_c$ could be related to some other measure of extensivity such as the part perimeter or the square root of the part area.

The $\bar{A}_c$ term is comprised of different complexity parameters, which could vary from process to process, or even from manufacturer to manufacturer. Many candidates for part complexity exist, as delineated in Chapter 3; choosing the appropriate measures, the ones that drive cost, can be achieved by the method explained in the next section. Generally, $\bar{A}_c$ will take on the following form
\[ \overline{A}_c = a_1C_1 + a_2C_2 + a_3C_3 + \cdots \] (4.13)

where the \( a_i \)'s are relation coefficients, and the \( C_i \)'s are the different complexity metrics.

### 4.4 Cost data reduction

In order to make the Basic Form usable to predict manufacturing times, it is necessary to determine all the constants which it comprises. This is done by analyzing manufacturing data for a given process, and therefore requires that a process be established and have a recorded history. A data set which includes manufacturing times, part weights, and detailed part drawings, as well as material type and process information, is required. Information on fabrication sequence is also desirable. A spreadsheet application such as Microsoft Excel or Lotus 1-2-3 is desirable so as to facilitate and expedite the analysis.

#### 4.4.1 General methodology

A logical starting point for this analysis is to plot the data in the form of Figure 4.3, plotting time/weight versus part weight. In doing so, the familiar L shape will become apparent. It is relatively easy to visually estimate \( W^* \) and \( W^{**} \), as the former is near the vertex of the L, and the latter occurs at a point where values seem to reach a steady level. To the left of \( W^* \) is the reduced data set that is used to determine the overall \( \overline{A} \), the \( \overline{B}_c \), and the appropriate \( C_i \)'s and corresponding \( a_i \)'s. The overall \( \overline{B} \) is determined by
taking the mean value of the points to the right of $W^{**}$, as shown in Figure 4.6.

![Figure 4.6: Determining $\overline{B}$](image)

In order to determine the overall $\overline{A}$, it is necessary to plot the reduced data set with time/weight as a function of 1/weight. The slope of the line formed by the data points, which can be determined either graphically or with a linear curve fit, is the overall $\overline{A}$ of the Basic Form. This is shown in Figure 4.7. Once the $\overline{A}$ and $\overline{B}$ have been determined, their standard deviations $\sigma_A$ and $\sigma_B$ can also be calculated, and the actual values of $W^*$ and $W^{**}$ can then be established by Equations 4.10 and 4.11.
As can be noticed in Figure 4.6, there exists a large amount of scatter in the complex regime. The $\overline{B}_c$, the $C_i$'s and the corresponding $a_i$'s are used to account for a portion of this scatter. The remainder of the scatter can be accounted for other by factors, which will be addressed in the following chapter.

To determine what $\overline{B}_c$ and $C_i$'s are appropriate for a given process, it is necessary to analyze the part drawings, from which the relevant information can be extracted. Candidates for $\overline{B}_c$ are extensivity measures such as length, tool area, the square root of tool area, part perimeter, part weight, projected part area, and so on. Candidates for the $C_i$'s are ply stacking information, cumulative in-plane shear, fiber information, and so on, and simplifications thereof, such as the number of interacting features and the number of sharp compound curvature bends. A tally of these quantities must be made from a representative set of parts. When such a study has been completed, a
multivariable linear regression analysis (MLRA) must be performed. For the purpose of a design cost model, it is desirable to minimize the number of input parameters, and it is thus recommended to perform the MLRA with at most four or five parameters simultaneously. A combination of parameters which results in a high coefficient of determination ($r^2$) denotes a good fit. Care must be taken to insure that each parameter used has a high marginal contribution to the $r^2$, in order to eliminate terms which are not significant. An $r^2$ value of 0.7 or higher indicates a good correlation.

Upon the combination of $\overline{B}$, $W$, $\overline{B}_c$, $x_c$, the $C_i$'s and the $a_i$'s, the Basic Form for a particular process can be assembled, resulting in

$$T = a_1C_1 + a_2C_2 + a_3C_3 + \cdots + \overline{B}_c(x_c) + \overline{B}W$$ (4.14)

Assuming the use of four complexity metrics, this equation then simply requires a total of six parameters, and only one step, to estimate manufacturing times for specific parts utilizing a chosen manufacturing process.

The analysis of actual cost data presented in the following section, though, suggests the total absence of dependence on the $\overline{B}$ extensity term in the complex regime, and the reduced influence of the complexity term in the mixed and extensive regimes. The nature of this behavior has not been determined, but it could take on one of many different forms, be it linear or nonlinear. Figure 4.8 describes this phenomenon in terms of a linear variation of the extensity and complexity terms, although, as stated, this behavior could well be nonlinear.
Figure 4.8: The relative influence of the Basic Form terms on cost.

This departure from the First Order Model theory might be explained by several reasons. First of all, the first order approximation of Equation 4.4 yields errors for parts that are within the complex regime, and thus have large time constants $\tau$. This error is represented by the shaded area in Figure 4.9.

Secondly, the First Order Model does not account for system complexities that arise in the actual manufacture of parts—issues such as queuing, system delays and shop floor dynamics are not accounted for. The estimate produced acts as a lower bound estimate for part production time, resulting in the lowest possible time in which a part can be produced.
4.4.2 Case study and results

In order to validate the previous analysis, a case study based on production data from Sikorsky Aircraft is presented. This data is a result of the Army's Advanced Composite Airframe Program (ACAP) carried out in the 1980's as a technology demonstration program, under which Sikorsky Aircraft designed and built prototype helicopters with all-composite airframes.³

The data set consisted of 177 data points for which both production time and weight were available. These parts were all fabricated using the hand lay-up process, using mainly woven prepreg broadgoods and some unidirectional tape. A plot such as Figure 4.3 was produced, and from the methodology described in Section 4.4.1, a $W^*$ of approximately 0.5 pounds and a $W^{**}$ of

approximately 7.5 pounds were calculated. Drawings were obtained for 61 parts, and the analysis for the complex regime was based on 29 of these parts, all of which weighed less than 0.5 pounds. Candidates for the complexity term parameters were extracted from the part drawings and specifications, after which an MLRA was performed. The analysis resulted in three $C_i$'s.

Table 4.1: $C_i$'s and $a_i$'s for Sikorsky data.

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<thead>
<tr>
<th>$C_i$</th>
<th>$a_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ply stacking information</td>
<td>0.297</td>
</tr>
<tr>
<td># of sharp compound curvature bends</td>
<td>0.543</td>
</tr>
<tr>
<td># of flanges with beads in web</td>
<td>1.10</td>
</tr>
</tbody>
</table>

The $\bar{B}_c$ had a value of 0.247 and was related to an $x_c$ which was the square root of the tool area. The resulting relationship for the complex regime yields an $r^2$ value of 0.774, which indicates a good relationship. Definitions for the terms in Table 4.1 can be found in Appendix B. Figure 4.10 shows estimated times using these factors plotted against the actual production times for parts in the complex regime. The diagonal line represents a perfect correlation. Points above the diagonal overestimate the production times and points under the diagonal are underestimated. The standard error in the estimated time is 0.88 hours, and as can be seen in Figure 4.10, about two out of three points fall within a 0.88 hour range of a perfect correlation. It is interesting to note that the absolute error in the estimate does not seem to be a function of scale, and as a result, the relative error decreases with increased time requirements. Estimates for parts that take longer to make would thus seem to be more accurate than estimates for smaller, less complicated parts.
Figure 4.10: Estimated versus actual production times in complex regime.
Chapter 5

Variation and reliability

5.1 Overview

A cost model can only be as good as the data from which it is derived. As indicated in the previous chapter, variation in data, or scatter, can be accounted for in part by the application of complexity metrics. However, manufacturing processes will inherently have a certain quantity of residual variation, or noise, which cannot be described explicitly in a model. In light of this, it is helpful to know where this residual scatter originates, and how the noise in a process affects the reliability of data produced. This chapter will take a brief look at some known sources of variation, and will abstract therefrom some lessons that were learned in the process of analyzing data with large amounts of scatter which cannot be described consistently.

5.2 Sources of variation

As can be seen in Figure 5.1, which shows the direct labor hours for an aerospace composite part over a 230 unit production period, tremendous variation exists in producing one specific part. Although this large a variation is not typical, it is representative of what can occur in composite
Figure 5.1: Hours v. unit number plot for a composite part.

manufacturing processes. Several factors can be cited as significant causes of some of this variation:

- differences in operators
- use of manual rather than automated process
- lack of process structure and task characteristics
- multi-step processes
- design or procedure changes during production
- material changes during production

Firstly, differences between operators in performing cutting, layup and bagging operations can result in 75% to 80% variation for the same part.¹

¹From personal communication with Christos Kassapoglou at Sikorsky Aircraft, March 1993.
Secondly, as demonstrated by the first factor, the use of manual processes such as hand layup inherently results in larger scatter than the use of automated fabrication processes, such as automated tape layup. Thirdly, the lack of process structure and task characteristics for parts manually produced often gives the shift supervisor discretionary power in allocating human resources to different tasks and parts, and thus a part that was made by one person on a particular day could possibly be made by two on another day. Fourthly, the more steps that are involved in a process, the more opportunity there is for noise to accumulate in the total fabrication time. With multiple steps in a process, buffers are created to accommodate for in-process variation of the individual steps, and thus a greater potential for variation exists. For example, an operator will deliberately take longer to complete her/his task if she/he observes a large number of parts in the buffer before the next process. Fifthly, design or procedure changes often occur during production as a result of high reject and rework rates. These changes can drastically affect production times for specific parts, and if not documented in a cost data set, could lead to unaccounted erratic behavior in cost trends. Lastly, material changes during production, resulting from either a change in raw materials supplier or changes in design specifications can also lead to variation in production times, as some materials are easier to handle than others, due to differences in characteristics such as tack and drapeability in the case of composite pre-pregs.

Additional factors such as learning effects can also contribute significantly to part to part variation. Part to part learning, among different parts, can lead to reduced fabrication times. For example, an operator could take significantly less time to make a part if she/he has already made other similar parts.
Data acquisition and cost accounting methods can also influence the reliability of cost data. The most reliable method for acquiring data is that of time-motion studies, where a person observes a manufacturing process with stopwatch in hand, and times the various steps involved in the fabrication of a part. The type of data typically available in most cases, though, is based on accounting data; labor hours are appropriated to specific tasks or parts to account for an operator's scheduled work time. This distorts the cost data, as an operator's non-productive time is also appropriated, resulting in increased variation in the data.

5.3 Lessons learned

Care has to be taken when using cost models formulated from production cost data so as to avoid common pitfalls. As stated, a model can only be as accurate as the data it is based on, and if large amounts of scatter exist that cannot be accounted for by the use of complexity factors, the degree of this scatter should be taken into consideration when making cost predictions. Thus a cost prediction should be accompanied by a standard error term, indicating a range of possible costs, rather than one single dollar figure.

As stated in the previous chapter, it would seem that cost predictions tend to be more accurate for larger, more complicated parts, as relative error tends to decrease with scale. Less reliance on cost estimates should thus be placed on parts whose predicted value is close to the standard error. A complexity-based model, though, should be able to rank parts correctly, and could therefore still be useful in that particular region for the purpose of making trade-offs between similar designs.
Chapter 6

Environmental considerations to materials selection and policy-making in the advanced composites sector

6.1 Overview

Advanced composite materials such as carbon-fiber reinforced plastics have been in existence for several decades, but it has not been until recently that these materials have seen more widespread commercial applications. The inherent qualities of composites – high strength and stiffness to weight ratios – have made them ideal for demanding structural applications where in the past only metals, namely steel and aluminum, had been used. Additionally, qualities such as corrosion and fatigue resistance give these materials an additional edge over their metal counterparts.

In the '70s and '80s, the adoption of composite materials by industry was a relatively slow process, mainly due to the prohibitive costs of the raw materials and the high labor content of composite manufacturing processes. The main area of development in composites during that time was in military applications, where the focus was on performance, and cost was no object. With the end the Cold War several years ago, defense budgets
diminished, and so did support for research and development in composite technologies. This led to a shift in innovative forces in this field from the military to the private sector.

The private sector, though, has a strong cost sensitivity which is partly due to risk-aversion, and in order to make composites economically viable alternatives to metals, the costs of composites must also be competitive with metals costs. This competitiveness is being achieved through trends in the automation of manufacturing processes, alternate design methodologies which take advantage of the ability to integrate many parts into one (so as to diminish assembly operations), and decreasing raw materials costs. On a per weight basis, composites are still much more expensive than metals. But if a comparison is made on a functional basis, wherein a composite assembly is designed to take advantage of the material's directional properties, a weight savings of 20-40% over aluminum can usually be achieved, and the part count can be reduced by an order of magnitude, the composite assembly may then become cost competitive with its aluminum or steel counterpart, assuming optimized structural design and integration of parts.

What then, is the decisive factor in selecting composites over metals for particular applications? Traditionally, weight savings has been composite's strongest selling point in transportation applications, such as the automobile and the commercial aerospace industries. A growing concern for the environment, though has caused a shift in focus to disposal issues of materials at the end of a product's life cycle. Legislation such as Germany's recycled content law has caused manufacturers to rethink their strategies on materials selection; Opel, for instance, has opted not to use sheet molding compound, a tough to recycle composite, in their new model lines for that
particular reason.\textsuperscript{1} Is this type of legislation justified, and is it leading to environmentally and economically sound decisions? This chapter will address these questions by first looking at the pros and cons of composites in an environmental context, as compared to metals; secondly, the impact of environmental factors on the acceptance of composites will be examined; thirdly, the effect of various legislations on materials usage will be examined; and lastly, a method for better informed decision and policy-making with regards to materials selection and the environment will be proposed.

\section*{6.2 Pros and cons of composites}

Advanced composite materials have been the center of much environmental debate as they have become more and more widespread in higher volume application such as in automobiles. In order to better understand the environmental issues, it is first necessary to look at the environmental benefits and detriments of composite materials as compared to the materials they replace - steel and aluminum.

\subsection*{6.2.1 How composites are good for the environment}

Composite materials exhibit three distinct environmental advantages over metals:

- Lower weight
- Longer life
- Lower energy requirement

\textsuperscript{1}Smock, Doug, "Plastics in new Opels will be 'easily' recyclable," Plastics World, April 1992, p12.
The lower weight of composites in transportation applications can be translated into higher fuel economies, and thus less use of depletable natural resources as well as reduced production of greenhouse gases. Weight reduction also leads to a secondary benefit of downsizing of airplane and automobile powerplants, which also leads to increased fuel economy. It has been estimated that in automobile applications, each kilogram of weight reduction results in an economy of 9 liters of gasoline over a 150,000 km life cycle.\textsuperscript{2} Due to the fatigue and corrosion resistance characteristics of composites, longer product lives can be expected. Analysts have estimated that use of composites in automobiles could more than double the average vehicle lifetime.\textsuperscript{3} This could result in a drastic reduction in the automobile scrappage rate, and in turn, a reduction in solid waste production. A little known environmental advantage of composite materials, but one that could have the most impact on the usage of these materials, is the lower energy requirements for producing composites. On a volume basis, composites require one third to one fourth the energy to produce compared to steel, and approximately one sixth the energy compared to aluminum.\textsuperscript{4} Lower energy requirements translate into reduction in use of nonrenewable resources, and the reduction in atmospheric emissions produced by the combustion of fuels.

\subsection{6.2.2 How composites are bad for the environment}

Composite materials exhibit three environmental disadvantages when compared to metals:

\textsuperscript{3}Office of Technology Assessment (OTA), \textit{Advanced Materials by Design}, 1988, p 86.
\textsuperscript{4}Fussler, p123.
• Hard to recycle
• Bulky - use up more space in landfill
• Emit toxins when incinerated

These three arguments all stem from the issue of disposal after use, and deal with waste disposal options. Due to the heterogeneous nature of composites, they are difficult to recycle. Although recent developments have been made in the recycling of sheet moulding compound, making it economically feasible\textsuperscript{5}, the technologies generally available lag drastically behind those in the well-established metals recycling industry. Composites, although chemically inert once fully cured, do pose a disposal problem when landfilled in that they are bulky due to their relatively low density, and tend to occupy substantial volumes. The limited use of composites up to now has made this a non-issue in the past, but with increased usage in high volume applications, composites may become an solid waste disposal problem. The last option for disposal is incineration with energy recovery, which has been the subject of much controversy. Due to the long molecular chain structure of polymers, their calorific content is close to that of fuel oil, making them good candidates for energy sources in incineration with electric power production. But incineration also produces dioxins and other toxic emissions, as well as contributing to the emission of greenhouse gases.

6.3 Impact of environmental factors on composites

Product 'Green-ness', or environmental-friendliness, has become a growing concern among increasingly environmentally conscious consumers. 'Green-ness' is perceived by the layperson as ease of recyclability, and amount of recycled content. This perception, however, contains a serious flaw in that it takes into account only the disposal aspect of a product's life-cycle. This in turn has led to a negative image of materials which are hard to recycle, such as advanced composites. In response to these concerns, auto manufacturers are placing heavier weight on recyclability as a factor for materials selection. For instance, the choice of material for all exterior panels of Saturn automobiles was heavily influenced by its recyclability.\(^6\) In addition to consumer attitudes, rising landfill costs and the threat of recycled content laws have also accelerated this trend. Moreover, the issue of recyclability has become a competitiveness argument for U.S. auto manufacturers, as European car makers have established a clear lead in this domain, spurred by German legislation aiming for 25% recycled content by 1994.\(^7\) In an effort to gain competitiveness in the recycling of composites, General Motors, Ford and Chrysler (the Big Three) have undertaken joint projects in recycling technologies.\(^8\)

The push to increase the recyclability of automobiles has brought about the shift in design paradigms to Design for Disassembly (DFD), to ease the retrieval of components for recycling. The concept of DFD in many ways

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\(^7\)Leaversuch, R D, "Auto-part recycling is a top industry priority," Modern Plastics, Mar 1992, p 37.
\(^8\)Leaversuch, R D, "Alliance will intensify auto recycling effort," Modern Plastics, Feb 1993, p 32.
conflicts with the trends for weight reduction.\textsuperscript{9} Highly integrated lightweight structures which are made possible by the use of advanced composites, such as sandwich panels, do not allow for easy dismantling, and are therefore difficult to recycle. This has led some designers to rule out such structures right from the start, without giving consideration to the benefits to be gained by the use of composites. DFD has in fact become a barrier against the more widespread implementation of lightweight composites structures.

Persistently low fuel prices over the past five years has also been a factor affecting the commercialization of composite technologies. Lower fuel prices have resulted in decreased savings from weight reduction in transportation applications, and thus manufacturers have been less eager to switch to composites.\textsuperscript{10} A problem with basing monetary economies on fuel prices is that these prices do not account for environmental externalities such as the creation of greenhouse gases and the depletion of non-renewable resources.

In an effort to minimize waste, which has obvious environmental benefits, much research work has been dedicated to making composite processes more efficient. Reduction in scrap rates and the advent of more efficient materials such as Net Resin Systems, which minimize the waste created by excess resin, have reduced the amount of waste produced in the manufacturing process, and in turn have reduced costs so as to make composites more cost competitive with other materials.

\textsuperscript{10}OTA, p 17.
6.4 Effect of environmental legislation on composites usage and industry trends

This section will examine two different approaches taken by policy makers to mitigate adverse environmental trends, initiatives which also have had effects on materials selection and usage.

6.4.1 German Recycled Content Law for automobiles

Spurred by an ever-increasing solid waste disposal problem, the German government proposed "The Federal Government's Policy on the Reduction, Minimization, or Utilization of Scrapped Vehicle Wastes" in August of 1990. Along with requiring the collection and dismantling of used vehicles, the policy mandates that recycling and reuse be integral concerns in the initial design of vehicles. A later addition to the policy mandates that a 25% recycled content goal be met by 1994.\textsuperscript{11} The policy attempts to mitigate the German solid waste disposal problem, as well as to promote the conservation of material and energy resources. This legislation heavily promotes the Design for Disassembly trend which was discussed earlier, which has had an adverse effect on the usage of composites in the auto industry. Furthermore, the policy, by concentrating on the disposal aspect of a vehicle's life-cycle, does not take into account the environmental impacts of the automobile at other stages of its life-cycle, such as vehicle emissions and energy consumption.

The prospect of such legislation being passed in the United States on the Federal level is somewhat remote, since solid waste disposal policy has

\textsuperscript{11}Leaversuch, Mar 1992, p 37.
traditionally been made at the state level. Nevertheless, a concern that the U.S. auto industry is losing competitive ground and restricting its access to European markets has prompted the Big Three to examine their recycling strategies. The competitiveness argument combined with increasing public pressure will eventually force U.S. carmakers to develop practical recycling programs even without legislation, says Alexander Trotman, president and chief operating officer of Ford Automotive Group. This is made apparent by joint efforts by the Big Three and composites manufacturers to demonstrate economical composites recycling programs.

### 6.4.2 Energy Policy and Conservation Act - Corporate Average Fleet Economy (CAFE) requirements

In the United States, the Energy Policy and Conservation Act of 1975 has required yearly increases in the average fuel efficiency of a manufacturer's fleet of vehicles, hence the name Corporate Average Fleet Economy, or CAFE. The main goal of this policy was, as its name implies, to promote the conservation of nonrenewable resources, and reduce dependence on foreign energy sources. As a by-product of this policy, a reduction in emissions due to greater fuel efficiency is also achieved. In an effort to comply with CAFE requirements, automobile manufacturers have exercised several options - they have downsized their car fleet, they have striven to increase the powertrain efficiency through R & D in engine technologies, and they have reduced vehicle weights. The latter of these has been a boost to the use of lighter weight materials, such as plastics and advanced composites. It has

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been predicted that an increase in CAFE standards would accelerate the development and increase the use of advanced composites.\textsuperscript{13}

6.5 Making trade-offs: the ecobalance approach to materials selection and policy-making

As the costs of composites manufacture is being brought down due to advances in materials processing technologies, automated manufacturing processes, and improved design practices, major hurdles to the commercialization of composite technologies still exist. Among the newest and most visible of these has been the relation between the manufacture and the application of composites and the environment. In order to make well-informed decisions on materials selection and on environmental policy which can influence the future of the composites industry, it is necessary to take an all-encompassing approach to assessing environmental costs and benefits by using the appropriate policy tools.

Classical cost benefit analysis in this case tends to lose its validity, as environmental externalities cannot be captured, or are weighed in a subjective fashion, reflecting the values of the person performing the analysis. It is therefore suggested that energy-based life cycle analyses, or ecobalance, be performed. The rationale behind using energy rather than dollar costs for performing this analysis is that energy usage is a better indicator of environmental impact. Fussler and Krummenacher have performed such an analysis for a typical automotive fender, comparing the life cycle energy requirements for steel, aluminum, and Reaction Injection

\textsuperscript{13}OTA, p182.
Molded (RIM) polyurethane (PU), a glass fiber composite. The results of this analysis are shown in table 1.

**Table 6.1: Total energy balance of fender materials.**
*source: Fussler and Krummenacher*

<table>
<thead>
<tr>
<th></th>
<th>Steel</th>
<th>Aluminum</th>
<th>PU RIM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(MJ)</td>
<td>(%)</td>
<td>(MJ)</td>
</tr>
<tr>
<td>Energy invested in part</td>
<td>285</td>
<td>16</td>
<td>850</td>
</tr>
<tr>
<td>Energy for operation</td>
<td>1521</td>
<td>84</td>
<td>832</td>
</tr>
<tr>
<td>Energy total in use</td>
<td>1806</td>
<td>100</td>
<td>1682</td>
</tr>
<tr>
<td>Energy value of recycling</td>
<td>(73)</td>
<td>(4)</td>
<td>(548)</td>
</tr>
<tr>
<td>Energy total in life cycle</td>
<td>1733</td>
<td>1134</td>
<td>910</td>
</tr>
<tr>
<td>Life cycle energy saving of PU RIM versus steel</td>
<td>599</td>
<td>35</td>
<td>823</td>
</tr>
<tr>
<td>Life cycle energy saving of PU RIM versus aluminum</td>
<td>224</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

( ) denotes recovery

From this table, we can see that the benefit of recycling this particular composite does not offer a significant energy savings over using virgin raw materials. Furthermore, if we eliminate the recycling step from the PU RIM analysis, a savings of close to 200 MJ over aluminum is still accomplished.

In order to weigh this quantity against options such as landfill disposal costs, it is necessary to translate the 197 MJ of energy savings into monetary terms. If standard industrial electric rates are used, a savings of 5¢ per kilowatt-hour of saved energy can be achieved. With the fender used in the
analysis, this translates into $2.74 of energy savings per fender. At 2.5 kilograms per part, the resulting energy savings are $1094.44 per ton of material. When weighed against typical landfill tipping fees of $15 to $100 per ton, it is clear that the life-cycle energy savings of using composites instead of aluminum or steel far outweigh the additional disposal costs incurred at the disposal stage of the life cycle.

6.6 Conclusions

Weber argues that "a reliable ecological balance ought to be struck between the energy that can be saved by weight-saving and that which can be achieved by recycling the materials."14 This kind of analysis, although still deficient in some aspects, offers policy makers and managers alike, a way of making these kinds of trade-offs with a minimization of distortion compared to other methodologies. If adopted on a larger scale, ecobalances could ultimately lead to the wider acceptance and commercialization of the form of materials known as advanced composites.

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14 Weber, p 207.
Bibliography


Appendix A

Shear formulation for C-channel

Given a curved C-channel of inner radius $R_1$ and outer radius $R_2$, and enclosed angle $\alpha$, we obtain the unique mapping, as shown, when the initial fiber is placed along a geodesic path on the radial cross-section of the part (shown in bold in Figure A.1).
The shape the fibers assume on the top face is that of an involute, which has the property of being perpendicular to the inner contour at the point where the fiber meets the edge, and also the property of being perpendicular to line segments which are tangent to the inner circular edge.

We adopt as our coordinate system the parameter $S_n$, which is the offset distance from the initial fiber. In Figure A.2, the three arrows all represent the same $S_n$.

![Figure A.2](image)

We define $S_{nc}$, the critical offset distance, as the length $S_n$ at which the line segment tangent to the inner contour at $S_n=0$ intersects the outer edge (Figure A.3). From the resulting right triangle with leg of length $R_1$ and hypotenuse of length $R_2$, we find
\[ S_{nc} = (R_2^2 - R_1^2)^{1/2} \]

Similarly, we can determine the critical angle
\[ \theta_c = \cos^{-1}\left( \frac{R_1}{R_2} \right) \]

![Diagram](image-url)
From this, we can foresee three distinct cases (Figure A.4). In each case, the shear in a fiber at a distance $S_n$ from the initial fiber is due to in-plane deformation caused by the fiber crossing the inner ($\Gamma_{R_1}$) and outer ($\Gamma_{R_2}$) contours. The maximum shear for a fiber is $\Gamma_{\text{max}} = \Gamma_{R_1} + \Gamma_{R_2}$.

Case I

$$R_2 > \frac{R_1 \alpha}{\sin \theta_i}$$

Case II

$$R_2 = \frac{R_1 \alpha}{\sin \theta_i}$$

Case III

$$R_2 < \frac{R_1 \alpha}{\sin \theta_i}$$

Figure A.4
Case I: \( R_2 > \frac{R \alpha}{\sin \theta_1} \)

Region 1: \( 0 \leq S_n \leq \alpha R_1 \) (Figure A.5)

The shear contribution from the inner contour is

\[ \Gamma_{R_1} = \theta_1 = \frac{S_n}{R_1} \]

The shear contribution from the outer contour is

\[ \Gamma_{R_1} = \theta_2 = \sin^{-1} \left( \frac{S_n}{R_2} \right) \]

![Figure A.5](image)

Region 2: \( \alpha R_1 < S_n \leq S_{nc} \) (Figure A.6)

The shear contribution from the inner contour in this region is limited by the end of the C-channel, dictated by the enclosed angle \( \alpha \) of the contour. The
maximum shear is the angle $\theta_2$ at which the fiber crosses the end of the C-channel.

$$S_n = \overrightarrow{ab} + \overrightarrow{bc}$$
$$\overrightarrow{ab} = R_1 \theta_2$$
$$\overrightarrow{bc} = \frac{R_1}{\tan \theta_2}$$

$$\theta_2 = \frac{S_n}{R_1} - \frac{1}{\tan \theta_2}$$

Figure A.6

From the geometry of the C-channel and the involute, we derive an implicit expression for the maximum shear on the top face.

$$\Gamma_{R_1} = \theta_2$$

The shear contribution from the outer contour follows the same relation as in region 1:

$$\Gamma_{R_2} = \sin^{-1}\left(\frac{S_n}{R_2}\right)$$

Region 3: $S_{nr} < S_n < \left[S_{nr} + (\alpha - \theta_1)R_1\right]$ (Figure A.7)

In this region, the shear in the top face is limited by the outer contour on one side, and by the end of the C-channel on the other. The maximum shear in
this section of the fiber is the enclosed angle of the curving fiber, which is the difference of the angles $\theta_1$, where the fiber meets the outer contour, and $\theta_2$, where the fiber meets the end of the C-channel.

![Diagram](image)

$S_n = \overline{ab} + \overline{bc}$

$bc = S_{nc}$

$\overline{ab} = S_n - S_{nc}$

$\theta_1 = \frac{S_n - S_{nc}}{R_1}$

Figure A.7

From these relations we obtain the shear in the top face

$\Gamma_{R_1} = \theta_2 - \theta_1$

The shear contribution from the outer contour in region 3 is constant:

$\Gamma_{R_2} = \theta_c$

**Case II:**

$R_2 = \frac{R_c \alpha}{\sin \theta_c}$

**Region 1:** $0 \leq S_n \leq S_{nc}$

The same relations as in region 1 of Case I apply here:
\[ \Gamma_{R_1} = \frac{S_n}{R_1} \]

\[ \Gamma_{R_2} = \sin^{-1} \left( \frac{S_n}{R_2} \right) \]

**Region 2:** \( S_{nc} < S_n < S_{nc} + (\alpha - \theta_c)R_1 \)

The same relations as in region 3 of Case I apply here:

\[ \Gamma_{R_1} = \theta_2 - \theta_1 \]

\[ \Gamma_{R_2} = \theta_c \]

**Case III:** \( R_2 < \frac{R_1 \alpha}{\sin \theta_c} \)

**Region 1:** \( 0 \leq S_n \leq S_{nc} \)

The same relations as in region 1 of Case I apply here:

\[ \Gamma_{R_1} = \frac{S_n}{R_1} \]

\[ \Gamma_{R_2} = \sin^{-1} \left( \frac{S_n}{R_2} \right) \]

**Region 2:** \( S_{nc} < S_n < \alpha R_1 \)

In this region, the shear in the top face is limited by the outer contour. The maximum shear in the top face is

\[ \Gamma_{R_1} = \frac{S_n}{R_1} - \theta_1 \]

The shear contribution from the outer contour in region 2 is constant:
$\Gamma_{R_2} = \theta_c$

Region 3: $\alpha R_1 < S_n < S_{nw} + (\alpha - \theta_c)R_1$

The same relations as in region 3 of Case I apply here:

$\Gamma_{R_1} = \theta_2 - \theta_1$

$\Gamma_{R_2} = \theta_c$
Appendix B

Definition of parameters

Parameters used in analysis of Sikorsky data

Ply stacking information

This complexity metric is explained in section 3.3.2.

Number of sharp compound curvature bends

For the analysis of the Sikorsky data, it was determined that curved flanges did not contribute significantly to cost. A reason for this is that most parts were made of woven broadgoods, which are easier to drape over gentle contours than unidirectional prepregs. Another reason is that most curved flanges in the parts analyzed were relatively shallow – most were under one inch deep. The combination of these two factors, woven material and shallow flanges, made the shear in curved contours an insignificant measure of complexity for these parts.

The importance of shear, though, was manifested in the significance of high shear areas denoted by the sharp compound curvature metric used for this analysis. This parameter is defined as a part feature wherein at least three
surfaces meet at angles close to 90°. The corner of a box would qualify as such a feature.

**Number of flanges with beads in web**

Beads are added to structures to improve stiffness. Composite manufacturers typically standardize beads so as to simplify tooling and manufacturing requirements. With standardized beads, the bead shear complexity measure would simply reduce to the number of beads in a particular structure, as the shear for all beads would be identical. In the analysis of the Sikorsky data, it was found that the number of beads alone was a poor cost correlator. However, an interaction term, defined as the existence of both a flange and a bead in one part direction, was found to be significant. Examples of this are given in Figure B.1.
Figure B.1: Examples of the # of flanges with beads in web measure.
Appendix C

Analysis of Sikorsky data

Results of multivariable regression analysis

Table C1
Regression Statistics

<p>| | |</p>
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<tr>
<td>Adjusted R Square</td>
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<td>Standard Error</td>
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Table C2
Analysis of Variance

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<th>df</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance F</th>
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<td>64.0264</td>
<td>16.0066</td>
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<td>Residual</td>
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<td>18.6715</td>
<td>0.7780</td>
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<tr>
<td>Total</td>
<td>28</td>
<td>82.6979</td>
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Table C3

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t-Statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.0426</td>
<td>0.4490</td>
<td>-0.0949</td>
<td>0.9251</td>
</tr>
<tr>
<td>Bead +flange</td>
<td>1.0987</td>
<td>0.2385</td>
<td>4.6059</td>
<td>0.0001</td>
</tr>
<tr>
<td>Sharp bends</td>
<td>0.5484</td>
<td>0.2085</td>
<td>2.6305</td>
<td>0.0137</td>
</tr>
<tr>
<td>M^*ln (m + 1)</td>
<td>0.2985</td>
<td>0.0626</td>
<td>4.7675</td>
<td>0.0001</td>
</tr>
<tr>
<td>SORT(AREA)</td>
<td>0.2512</td>
<td>0.0471</td>
<td>5.3274</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
The preceding regression statistics and analysis of variance data can allow us to make useful observations on the quality of the multivariable fit generated to explain the dependence of labor hours on the selected parameters. The estimated values for labor hours are referred to as the response variable, and the parameters used to fit the data, (Bead + flange, # Sharp bends, $M^*\ln(m+1)$, and the SQRT(AREA)) are referred to as the explanatory variables. Table C1 summarizes the regression statistics for the fit. The first useful indicator is the $R^2$ value, (which is simply the square of the correlation coefficient). The value of 0.7742 indicates that 77.42% of the variation in the data can be explained by the fit to the data achieved with the optimized parameters presented. Conversely it implies that 22.58% of the variation remains unexplained. Note that $R^2$ is the regression sum of the squares (SSR) divided by the total sum of the squares (SSTO). Therefore SSR is the variation in the data that can be explained by the fit, and the residual, or error sum of the squares (SSE), is the unexplained error.

The regression coefficients explained in Appendix B, and their associated errors are themselves random variables, each with a sampling distribution and a population variance. The $SSE/\sigma^2$ is described by a chi-squared distribution such that the mean squared error (MSE) is equal to the SSE divided by the number of degrees of freedom of the descriptive (chi squared) distribution. A similar statement is true for the mean squared regression (MSR). For a multivariable fit the degrees of freedom associated with the SSE is $(n-p-1)$, where $n$ is the number of observations, and $p$ is the number of explanatory variables. Practically the MSR and MSE are estimators for the variance of the regression sum of the squares and the error sum of the squares respectively. The parameter $F$ is a useful measure of the overall quality of the fit and is the distribution created by the ratio MSR/MSE. It is
described by an F - distribution \( F(\alpha; p, n-p-1) \) and is used to comment on the hypothesis that none of the chosen parameters have an influence on the resulting time values. In other words, it tells us if we are testing entirely the wrong set of parameters. It is evaluated at some significance level (usually 0.01) which can be read from statistical tables for the relevant degrees of freedom. The relevant value for the case presented here is \( F(0.01, 4, 24) = 4.22 \). This is compared to the F value calculated conveniently from our solution, by the expression:

\[
F = \frac{R^2}{1-R^2} \left(\frac{n-p-1}{p}\right)
\]

The F value for our solution is given in table C2 as 20.57. This is much larger than the value read from the tables and therefore the parameters chosen are a reasonable representation of the data. In practical terms this simply tells us that at least one of the chosen parameters has a significant influence on the result.

For a statement on the relevance of each of the individual parameters we refer to Table C3. As stated above the coefficients for each of the parameters are themselves random variables. The values given are unbiased estimators for the parameters. The standard errors can be thought of as estimators for the variances associated with each. With this standard error, the sampling distribution;

\[
\frac{\text{Random value for the parameter} - \text{the determined value}}{\text{Standard error}}
\]

is described by a \( t \)-distribution with \((n-p-1)\) degrees of freedom. With this sampling distribution we can establish confidence intervals for the estimates of the coefficients. More importantly we can use the \( t \)-statistic to test
the hypotheses that each parameter is of no consequence in determining the times. Of course the desired result is the null hypotheses, i.e. that all parameters are significant. We simply compare the t-value read from tables at some significance level, say 5 % to the value determined from our data. From the t-distribution table the critical value is 2.064. From table C3 all of the calculated values are much greater than this critical value, therefore each of the parameters can be thought of as contributing to the overall quality of fit. To determine the individual contributions to the overall scatter of the fit we would perform a sequential regression analysis where we remove each of the parameters from the fit in turn and determine the new regression constants and ANOVA statistics. In this way we can establish the relative effects that each of the parameters have on the overall quality of fit. Two commonly used methods for doing this are known as backward elimination and forward selection, and should yield the same result provided the parameters are not correlated. A logical sequence for the backward elimination technique would be in ascending values of the t-value. That is, we would eliminate the sharp bends parameter first, the bead and flange next and so on. Our expectation is that the worst correlators are the greatest contributors to the scatter. Therefore we might expect the relative contributions to scatter to rank, in increasing order (SQRT Area, M*ln(m+1), Bead and flange, Sharp bends). Eliminating each of the parameters from the fit, and observing the $R^2$ value obtained from the three remaining parameters we observe exactly the same ranking as above.