THE INTERACTION BETWEEN FOREIGN EXCHANGE VOLATILITY AND PRICE
by
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Laurea degree in Mechanical Engineering
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May 15, 1993

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ABSTRACT

This research examines the relationship between daily volatility and daily prices in the foreign exchange market. The study includes empirical tests on the foreign exchange market of models and findings reported in other authors' studies as well as new tests leading to original findings. For each empirical test the underlying economic theory is discussed in its application to the foreign exchange market.

The empirical results of the study indicate, consistently with the findings presented in previous studies, an overall homogeneous structure about the relationships among volatility and price. The model developed by J. Campbell, S. J. Grossman, J. Wang [2] on the behavior of the stock index has been tested on the foreign exchange market. The test results indicate that their model applies also in the foreign exchange market.

The interaction between volatility and foreign exchange prices has also been explored. The study reveals two main findings. First, heteroskedasticity is higher when the price follows a trend, while it is lower in those periods of no trend. Second, the sign of the correlation between volatility and price is dependent on the trend direction of the exchange rate market. Correlation is positive during periods of "up trend" and negative during period of "down trend".

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SECTION I - Introduction

My motivation in exploring the relationship between foreign exchange daily volatility and daily price is twofold. First, the analysis of the incidence and behavior of daily volatility in the foreign exchange market. Second, the investigation and interpretation of the possible relationship between daily volatility and prices.

As to the first analysis, the research on daily volatility in the foreign exchange market seems to be particularly crucial, given the always increasing use of derivative instruments in investment strategies. As to the second part of the research, to my knowledge, no one has ever performed the same basic statistical analysis on the relationship between daily volatility and prices in the foreign exchange market, presenting similar results and interpretation. The aim of this exploratory examination is to further analyze the incidence of previous price movements and volatility on the behavior and the pattern of foreign exchange prices. I hope that the first findings presented in this paper will indicate interesting directions for further research and analysis.

The structure of this paper is organized as follows. Section II provides a comprehensive description of the previous studies relevant to this research. Section III describes the data sources, characteristics, and basic statistics. Section IV presents the analysis and the results of two empirical tests. The two tests are run to verify the application to the foreign exchange market of a model developed by Campbell, Grossman, and Wang [2] concerning the relationship between stock market trading volume and the autocorrelations of daily stock index returns. Section V presents the results and the interpretation of the analysis performed on the interaction between daily volatility and prices in the foreign exchange market. Section VI summarizes the study, draws the conclusions and provides some directions for further analysis and research.
In this research I will also take into account, in interpreting the results, previous studies about the price-volume relation in financial markets. There are at least three main reasons why the price-volume relation is of fundamental importance. First, analyzing and understanding the dynamic inter temporal relationships among prices, volatility and volume better characterizes and describes the economics of the events occurred in the market providing more complete and comprehensive explanation.

Second, volume analysis provides useful insight into the structure of financial market. Third, in event studies the price-volume relation can provide useful information from which to draw interesting inferences. Thus when contemporaneous price changes and volume are available, incorporating their mutual relation increases the power of the model and tests used to perform the analysis. In fact in some studies, as we shall see, the price changes indicate the market evaluation of new information, while the corresponding volume is interpreted as the degree of investors' disagreement about the meaning of the information.

In the foreign exchange market data on trading volume are not available, therefore volatility will be used as best indicator for trading volume to draw interesting inferences from the actual results obtained. Previous studies support the idea of a direct relationship between volatility and trading volume. Thus I will constantly refer to previous empirical and theoretical research into the price-volume relation to support my analysis.

SECTION II - Early Studies

Previous studies on trading volume are numerous, but mostly focused on the relationship between volume and the volatility of stock return. They consistently demonstrate and document that high stock market volume is associated with volatile returns.
Campbell, Grossman, and Wang [1] developed a model to study the relations between volume and price in the stock market. The model states that a stock price decline on a high volume day is more likely to be associated with a higher expected stock return than a stock price decline on a low volume day. Their underlying economic explanation goes as follows. There may be two different alternative explanations for a decline in stock prices. First, a public information is released and causes traders to decrease their valuation of the stock. In this case it is clear that there would be no reason for the expected return on the stock market to change. Second, "Non-informational" or "liquidity" traders can change their attitude towards risk. If they become, due to exogenous reasons, more risk adverse they can exert a selling pressure on the market. In this case market makers buying stock to accommodate this selling pressure will ask for higher expected return. Thus the price in the following days should be expected to increase. The way of recognizing the two different events occurring in the market, according to the authors, is to expect a unusually high trading volume in the second case, and no change in trading volume in the first case.

In conclusion, Campbell, Grossman, and Wang [2] demonstrate and find that stock return autocorrelations tend to decline with trading volume. In order to perform the test they regressed the one-day-ahead stock return on the current stock return, the current return interacted with volume, the current return interacted with volume squared, and the current return interacted with conditional variance:

\[ r_{i+1} = \alpha + (\beta_0 + \beta_1 V_i + \beta_2 (V_i)^2 + \beta_3 (\sigma_i)^2 ) r_i \]  \hspace{1cm} (1)

where

\[ r_{i+1} \] = one-day-ahead stock return

\[ V_i \] = current volume

\[ (\sigma_i)^2 \] = conditional variance

\[ r_i \] = current stock return
\[ \alpha \] = intercept coefficient

\[ \beta_0, \beta_1, \beta_2, \beta_3 \] = regression coefficients

Table 1 and Table 2 present the results of the regression based on the sample period 1962-87 and on the sample period 1975-87 respectively.

**Table 1 - Volume, Volatility, and the First Autocorrelation**

<table>
<thead>
<tr>
<th>Specification</th>
<th>( \beta_0 ) (t-ratio)</th>
<th>( \beta_1 ) (t-ratio)</th>
<th>( \beta_2 ) (t-ratio)</th>
<th>( \beta_3 ) (t-ratio)</th>
<th>R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.219 (17.9)</td>
<td></td>
<td></td>
<td></td>
<td>0.048</td>
</tr>
<tr>
<td>Volume</td>
<td>0.256 (19.1)</td>
<td>-0.303 (6.72)</td>
<td></td>
<td></td>
<td>0.055</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.221 (11.1)</td>
<td></td>
<td>-18.1 (0.125)</td>
<td></td>
<td>0.048</td>
</tr>
<tr>
<td>Volume and Volatility</td>
<td>0.237 (11.4)</td>
<td>-0.388 (6.37)</td>
<td>0.226 (2.01)</td>
<td>83.1 (0.570)</td>
<td>0.055</td>
</tr>
</tbody>
</table>

From the coefficient shown on Table 1 we can infer that strong results on volume data are not matched by any other significant evidence of relationship with volatility measure. In fact from the second row of Table 1 we can see that R² statistic can be increased by 0.7 percentage points (or 15% of its initial value) by interacting the regressor with the trading volume. On the other hand, when volume is excluded from the regression, volatility enters negatively, but it is statistically less significant. Thus Table 1 indicates a strongly significant relationship between one-day-ahead stock return and the linear volume. However, in Table 2, we can notice that the first order autocorrelation is lower on average, and that volume contributes less to regression forecasts of the return, even if still statistically significant. In this sample period there is evidence of a strong negative relationship between volatility and autocorrelation, even stronger than the relationship between volume and autocorrelation.
Table 2 - Volume, Volatility and the First Autocorrelation

<table>
<thead>
<tr>
<th>Specification</th>
<th>$\beta_0$ (t-ratio)</th>
<th>$\beta_1$ (t-ratio)</th>
<th>$\beta_2$ (t-ratio)</th>
<th>$\beta_3$ (t-ratio)</th>
<th>R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.166 (9.65)</td>
<td></td>
<td></td>
<td></td>
<td>0.028</td>
</tr>
<tr>
<td>Volume</td>
<td>0.197 (9.66)</td>
<td>-0.178 (2.88)</td>
<td></td>
<td></td>
<td>0.030</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.248 (7.17)</td>
<td></td>
<td>-0.935 (0.275)</td>
<td></td>
<td>0.030</td>
</tr>
<tr>
<td>Volume and Volatility</td>
<td>0.258 (7.39)</td>
<td>-0.171 (1.78)</td>
<td>0.051 (0.328)</td>
<td>-0.771 (2.18)</td>
<td>0.031</td>
</tr>
</tbody>
</table>

I did not report the results of the regression performed on the first period (1962-1974), since these results are similar to those of the entire period 1962-87.

In addition to Campbell, Grossman, and Wang, I would add another comment in their second explanation provided for a price change. The explanation holds also in the case "liquidity" traders become less risk-adverse, exerting a buying pressure on the market. In fact market makers selling stocks to accommodate this shift in risk preference would allow buyers to earn a lower expected return on the same stock requiring a higher actual price to execute the transaction. I believe that in the case of "liquidity" traders exerting any form of pressure on the market there may be an alternative possible explanation that leads to the same conclusions. Market makers accommodating the shift in preference may fear a case of information asymmetry with the "liquidity" traders and for this reason require a higher expected return in order to buy or allow the buyers a lower expected return in order to sell. Therefore daily trading volume is an indicator of high-frequency shift in investors' risk preferences.

One obstacle to study the relation between volume and price in the foreign exchange market is that trading volume of foreign exchange is unknown to the public. However numerous studies [4,5,6,7,8,9] have indicated that volume and volatility are positively correlated in the stock market.
Clark [3], Epps and Epps [6] and other authors [5,13,14,15] who developed empirical studies of both futures and equity markets, consistently showed a positive correlation between price variability and the trading volume. Two possible different explanations for the relationship are followed and considered fundamental by most of the authors.

Clark's explanation [3] is based on a model where the daily price change is the sum of a random number of intra-day price changes. Thus the variance of the daily price change is a random variable with the mean proportional to the mean number of daily transaction. Furthermore Clark [3] claims that the trading volume is positively correlated to the number of intra-day transactions. Therefore the trading volume is positively correlated to the variance of the price change.

Epps and Epps [6] provide the second explanation. Their model explores and analyses the structure and the functioning of intra-day trading. The market price change occurring on each intra-day transaction is the average of the changes in all the traders' reservation prices. Epps and Epps' assumption is that the degree of disagreement among traders when they modify their reservation prices is positively correlated to the total absolute value of the change in the market price. This means that if the disagreement among traders increases, the absolute value of the price change will be larger. Since we know that the trading volume is positively correlated to the extent to which traders disagree in revising their reservation prices, price variability is positively correlated to trading volume.

T. Copeland [4] developed a model of asset trading under the assumption of sequential information arrival that predicts a positive correlation between the absolute value of price changes and the volume traded, consistently with the other theories. Copeland assumes that there exists an asset market where individuals receive information sequentially and in random order. At the initial equilibrium, all the individuals possess an identical set of information. Then when a single information is
released, each individual reacts by shifting his or her demand curve. Finally a new equilibrium will be reached when all individuals have received the news, and possess again an identical set of information. In this world the price change between the initial and final equilibrium is known with certainty. However Copeland [4] shows that the price adjustment paths and as well as the total volume of trading are random variables. More specifically the model developed uses probability theory to obtain the expected number of trades generated by a given piece of new information. The author shows that the expected number of trades depends on the number of individuals in the market, the number of shares of the asset, the strength of the new information, and the percentage of individuals who react by shifting their demand curves upward. According to the same model, the expected number of trades is also shown to be positively related to the absolute value of price changes.

A. R. Gallant, P. E. Rossi, and G. Tauchen [9] in a study on stock prices and volume undertook a comprehensive investigation of price and volume co-movement using daily NYSE data from 1928 to 1987. Among others, two interesting empirical regularities were found. First, positive correlation between conditional volatility and volume. Second, large price movements are followed by high volume. As to the contemporaneous volume-volatility relationship the study indicates that the daily trading volume is positively and non linearly related to the magnitude of the daily price change. This relation is a characteristic of both the unconditional distribution of price changes and volume and the conditional distribution given past price changes and volume constant. As to the dynamic price-volume relationship, the research suggests that large price changes lead to increases in both the mean and variability of the volume. Furthermore the study indicates that market declines have the same effect on subsequent volume as market increases.

Blake LeBaron in an unpublished paper [11], demonstrates some interesting connections between serial correlations and volatility in stock return indices. These
patterns were observed in both daily and weekly returns for the S&P index. As I also mentioned before, a last well known result on stock index returns is the daily serial correlation. At the daily frequency there is a significant positive serial correlation.

Fisher [7] provided a first explanation based on the concept of non-synchronous trading. The theory can be described easily with a basic application. A new piece of information released in the market will affect first those stocks that are more frequently traded and then the effect will be delayed on those stocks less frequently traded to the next trade. If this delay will go over the end of day close into the next trading day, a stock index with a broad base formed form both frequently traded and infrequently traded stocks, will show positive serial correlation. More specifically, Blake LeBaron [11] shows (consistently with the results reported by J. Y. Campbell, S. J. Grossman, and J. Wang [2]) that daily and weekly serial correlations will be shown to be inversely related to the conditional volatility of the indices under study.

The sequential information arrival model presented by Copeland [4] showed a positive relation between volatility and volume in a very stilished and simple framework. The same model has led to some interesting empirical tests. Morse [12], applying the Copeland model [4], performed tests very similar to those presented by LeBaron [11], with opposite results. Morse [12] has analyzed the serial correlations for individual stocks conditional to the level of trading volume. He has found positive relation between volume and serial correlation. His explanation relies on an asymmetric information theory. Periods with high trading volume, are those when adjustment to new information is occurring in the market, causing positive serial correlation.

Conversely B. LeBaron [11], as I mentioned before, presented opposite results in his paper if we assume that volume and volatility are positively related. This can be justified and explained if we assume a different behavior of stocks index and individual stocks to the extent of the present research. B. LeBaron [11] has not provided any explanation for the results presented that clearly run against the asymmetric information
ideas. However, as I said before, J. Y. Campbell, S. J. Grossman, and J. Wang [2] have provided in a very recent paper a careful and comprehensive economic explanation of the same phenomena, with their theory about "market makers" and "liquidity" or "non-informational" traders.

Jonathan M. Karpoff [10] in his study on the relation between price changes and trading volume reviews previous and current research on the relation between price changes and trading volume in various financial markets. Karpoff [10] confirms with his empirical tests the existence of a positive correlation between absolute price changes and trading volume in equity markets. However, he also highlights that in equity markets the price-volume relation is asymmetric; trading volume is higher when prices increase than when prices decrease. To this extent Karpoff [10] provides an interesting possible explanation. The relatively higher cost of taking a short position, in equity markets, explains why the volume associated with a price increase generally exceeds that with an equal price decrease, since costly short sales restrict some investors' abilities to trade on new information.

In a study on stock return variances "The arrival of information and the reaction of traders", K. R. French and R. Roll [8] examine the possible explanations for higher volatility of asset prices during exchange trading hours than during non-trading hours. The authors conclude stating that even though a significant part of the daily variance is caused by mispricing, the pattern of asset returns around exchange holidays suggests that high volatility is due to the private information flow. This flow influences prices when informed investors trade on the basis of their own interpretation of the private information then available. This view is entirely consistent with the explanation provided by Epps and Epps [6] for the price variability-volume relationship when they say that the trading volume is positively related to the extent to which traders disagree when they revise their reservation prices. This conceptual similarity is another confirmation of the association between price variability and trading volume.
A. Admati and P. Pfleiderer developed in their study [1] a theory of intraday patterns for trading volume and price variability. Concentrated trading patterns are found to be the results of the strategy of liquidity traders and informed traders. Analyzing the trading strategy of both liquidity and informed traders and allowing for a reasonable possibility for liquidity traders to execute trades with a certain discretionary timing ("discretionary liquidity traders), the authors build a new model for intra-day trading patterns that lead to very interesting results. First, trading volume seems to be concentrated in particular time periods within the trading day. Second, returns variability is definitely higher in specific time periods than in others within the trading day. Third, periods of higher trading volume seem to be also the periods of higher returns variability within the trading day.

In conclusion the theory of intra-day trading patterns developed by A. Admati and P. Pfleiderer [1], confirms that high trading volume and high returns variability tend to occur simultaneously even within a trading day, which is clearly consistent with was found by other authors considering longer time horizons.

I believe that in the foreign exchange market, Campbell, Grossman, and Wang [2] model should apply rather than Morse's model [12] (based on the behavior of individual stocks), given the quick, symmetric, and contemporary flow of information and consequent adjustments, as well as the large blocks usually traded in the foreign exchange market. My point is that the asymmetric information explanation should not hold in the foreign exchange market that, intuitively, for structural characteristics, seems to be more similar to the behavior of the stock index, than to the behavior of individual stocks that have very different trading volume and intensity. Therefore I would expect from my analysis results consistent with those presented by Blake LeBaron [11] and Campbell, Grossman, and Wang [1].
SECTION III - Data sources

In this study Deutsche mark exchange rates data from the entire year 1990 and 1991 are used. The tick-by-tick data, collected and recorded from the market news services, were provided by JP Morgan. They include spot rate quotes from Reuters and Telerate. The Deutsche mark has been chosen over other currencies because the Deutsche mark is the most active and traded currency against the US dollar.

Daily prices and daily volatility were extracted from tick-by-tick data. In order to obtain daily data a precise definition of the trading day is required since the foreign exchange market is twenty-four hour worldwide market. Twenty-four hour period from 0:00 Greenwich Mean Time (GMT) has been chosen as a day, because 0:00 GMT is 9:00 am Tokyo time and 24:00 GMT is 7:00pm New York time. This twenty-four hour period seems to include most activities in the worldwide foreign exchange market. Thus daily prices are the closing prices of the defined trading day, while daily volatility data are calculated on the basis of the defined trading day. Daily volatility has been calculated according to the estimator suggested by Bin Zhou [16]. Saturdays and Sundays data show, generally, very low volatility figures since the activity in the market tends to be smaller. Therefore Saturdays, Sundays were excluded from the analysis in order to avoid additional noise. As I found in many other studies, I will use the compound returns (difference in the logarithmic value of the prices), rather than the simple prices in my research. Table 3 shows the basic statistics of the daily returns in the year 1990 and 1991.

<table>
<thead>
<tr>
<th>Specification</th>
<th>1990 data</th>
<th>1991 data</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum</td>
<td>-3.158E-02</td>
<td>-2.963E-02</td>
</tr>
<tr>
<td>maximum</td>
<td>2.385E-02</td>
<td>3.399E-02</td>
</tr>
<tr>
<td>mean</td>
<td>-4.773E-04</td>
<td>2.556E-04</td>
</tr>
<tr>
<td>standard deviation</td>
<td>6.751E-03</td>
<td>8.694E-03</td>
</tr>
<tr>
<td>skewness</td>
<td>-3.739E-01</td>
<td>-1.241E-01</td>
</tr>
<tr>
<td>kurtosis</td>
<td>5.004E+00</td>
<td>4.458E+00</td>
</tr>
</tbody>
</table>
The average daily return is small compared to the standard deviation in both years. However, the average return and the standard deviation of returns are comparable in both years. The returns are significantly skewed to the left in both years. The kurtosis is higher than 3, the kurtosis of the normal distribution. Bin Zhou showed in his study [16] the empirical evidence that kurtosis increases when periodicity becomes shorter and frequency becomes higher. Thus we should expect a higher kurtosis for hourly and minute-by-minute data and a lower kurtosis for weekly and monthly data.

Daily volatility estimates have been obtained on the basis of tick-by-tick data according to the procedures and recommendations suggested by Bin Zhou [16]. An estimate based on high frequency observations leads to an assessment of the volatility closer to the true value. In fact, for any given sample size, the higher is the data frequency, the better is the estimate of the volatility. The basic statistics for the daily volatility data are shown below in Table 4.

<table>
<thead>
<tr>
<th>Specification</th>
<th>1990 data</th>
<th>1991 data</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum</td>
<td>2.283E-06</td>
<td>6.505E-08</td>
</tr>
<tr>
<td>maximum</td>
<td>2.293E-04</td>
<td>4.545E-04</td>
</tr>
<tr>
<td>mean</td>
<td>3.887E-05</td>
<td>5.636E-05</td>
</tr>
<tr>
<td>standard deviation</td>
<td>2.584E-05</td>
<td>4.538E-05</td>
</tr>
<tr>
<td>skewness</td>
<td>2.828E-00</td>
<td>4.006E-00</td>
</tr>
<tr>
<td>kurtosis</td>
<td>16.711E-00</td>
<td>28.581E-00</td>
</tr>
</tbody>
</table>

SECTION IV - An Empirical Test

In this section, I will present the results of a series of empirical tests performed in order to analyze the application to the foreign exchange market of the economic theory presented by John Y. Campbell, Sanford J. Grossman, and Jiang Wang [2].
The results obtained will also be compared with those presented by Blake LeBaron [11]. The analysis will be performed on the data described in section III, daily foreign exchange prices and daily volatility from the year 1990. Two different tests have been performed. The model shown by Campbell, Grossman and Wang [2], implies that a stock price decline (increase) on a high volume day is more likely than a stock price decline (increase) on a low volume day to be associated with an increase (decline) in the expected stock return. In order to test this statement, daily volatility has been used as an estimate for trading volume, given the obvious difficulty of finding volume data in the foreign exchange markets. The close direct relationship between volume and volatility proved by the many studies presented in section II, improve the reliability and the significance of this type of test. Daily prices and volatility from year 1990 with Saturdays and Sundays data excluded, are shown in Exhibit 1, and 2. To perform the test on the Hypothesis, the following procedure has been followed:

i) transform exchange rate prices in log-returns (See Exhibit 3)

ii) sort the whole set of data, including volatilities and prices, according on an increasing order of volatility magnitude (from lowest to highest volatility). divide the sorted data has been into five different groups on the basis of relative volatility magnitude, bottom 50%, 25%, 10%, 10%, top 5%.

iii) count the number of days with absolute return greater than 0.006 in each group.

iv) count the number of days with absolute return greater than 0.006 and whose return in the following day changes sign.

v) calculate the percentage of the numbers in iv) over the number in v). This percentage is used to verify the hypothesis.

The results for year 1990 are shown on Table 5.

According to the theory by Campbell, Grossman, and Wang [2], we should expect an increasing percentage from the group characterized by low volatility to the
group characterized by high volatility. This is exactly the pattern revealed by table 1.

Thus the empirical test indicates that the theory may hold and apply also in the foreign exchange market.

Table 5 - Test of hypothesis on 1990 data

<table>
<thead>
<tr>
<th></th>
<th>Days with high absolute return</th>
<th>Days with high absolute return followed by a reverse trend</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>low volatility</td>
<td>26</td>
<td>15</td>
<td>57.69%</td>
</tr>
<tr>
<td></td>
<td>23</td>
<td>12</td>
<td>52.17%</td>
</tr>
<tr>
<td>med. volatility</td>
<td>10</td>
<td>8</td>
<td>80.00%</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>4</td>
<td>30.77%</td>
</tr>
<tr>
<td>high volatility</td>
<td>8</td>
<td>7</td>
<td>87.50%</td>
</tr>
</tbody>
</table>

I further examined the application of the same model to the foreign exchange market exploring and testing the relationship between price return serial autocorrelation and volatility, taken again as an indicator of trading volume (given the direct relation).

In order to perform the analysis, I regressed the one day ahead price return on the current price return, and on the current return interacted with volatility:

\[ r_{t+1} = \alpha + \beta_1 r_t + \beta_2 \{ \text{Vol}_t \times r_t \} \]  

(2)

where

\[ r_{t+1} \] = one-day-ahead return
\[ r_t \] = current return
\[ \text{Vol}_t \] = current volatility
\[ \alpha \] = intercept coefficient
\[ \beta_1, \beta_2 \] = regression coefficients

The results of the regression are shown below in Table 6:
Table 6  Coefficients and t-ratios of regression (2)

<table>
<thead>
<tr>
<th>Specification</th>
<th>$\beta_1$ (t-ratio)</th>
<th>$\beta_2$ (t-ratio)</th>
<th>R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1271 (1.1568)</td>
<td>-3285.8522 (-2.1315)</td>
<td>0.0223</td>
</tr>
</tbody>
</table>

I also regressed the one day ahead price return on the current price return, and the current price return interacted with the square root of volatility:

$$r_{t+1} = \alpha + \beta_1 r_t + \beta_2 \{ \sqrt{(Vol)} \times r \}$$  \hspace{1cm} (3)

The results of the regression are shown below in Table 7:

Table 7 - Coefficients and t-ratios of regression (3)

<table>
<thead>
<tr>
<th>Specification</th>
<th>$\beta_1$ (t-ratio)</th>
<th>$\beta_2$ (t-ratio)</th>
<th>R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.3486 (1.7088)</td>
<td>-56.6419 (-2.1387)</td>
<td>0.0225</td>
</tr>
</tbody>
</table>

Both the regressions seem to confirm the empirical findings of the first test and the validity in the foreign exchange market of the model discussed before. In fact in the first regression shown on Table 4 the coefficient of the factor "current price return interacted with volatility" is negative and, as it can be seen from the t-value, statistically significant at a 5% level. Furthermore the second regression shows very similar results (Table 5) with a negative coefficient of the factor "current price interacted with square root of volatility" statistically significant at 5% level.

The meaning of a negative coefficient for the current volatility interacted with the current price return is that on any given day characterized by a positive (negative) price return the higher is the volatility the more likely seems to be a decrease (increase) in the expected price return.
Thus we can conclude that the results of both the tests performed are consistent with the model by Campbell, Grossman, and Wang [1] presented in section II. Therefore their theoretical explanation may also hold in the foreign exchange market.

SECTION V - The Interaction between Volatility and Price

In this section, an exploratory study on the relationship between volatility and foreign exchange prices is presented. Original results are found and shown and a first tentative economic interpretation of these results is provided. Daily volatility and daily prices from the year 1990 and 1991 separately are examined. The data under analysis do not include Saturday and Sunday data.

Daily price, return, and volatility for the year 1990 and 1991 are shown in Exhibit 1, 2, 3, 4, 5, and 6. The first step in the analysis is to examine the behavior of volatility and prices in each individual period when daily price show a constant and uniform pattern of returns (i.e.: "up trend", "down trend", "no trend"). Thus, in order to start the analysis the whole set of data has been divided in a number of subsets according to the shown pattern of returns. These subsets for year 1990 and 1991 are shown respectively in Exhibit 7 and 8.

For both year 1990 and year 1991, I found that there are some correlations between daily price and daily volatility, mean of volatility, median volatility, variance and standard deviation of volatility. The results of these analyses are reported in table 8 and 9.

Table 8 - Statistics for each individual subset on 1990 data

<table>
<thead>
<tr>
<th>Year 1990</th>
<th>correlation</th>
<th>mean</th>
<th>median</th>
<th>st. dev.</th>
<th>variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>subset 1</td>
<td>1.8271E-01</td>
<td>3.3849E-05</td>
<td>3.1159E-05</td>
<td>1.7740E-05</td>
<td>3.1471E-10</td>
</tr>
<tr>
<td>subset 2</td>
<td>-2.0519E-01</td>
<td>2.5465E-05</td>
<td>2.5341E-05</td>
<td>1.0551E-05</td>
<td>1.1133E-10</td>
</tr>
<tr>
<td>subset 3</td>
<td>-5.3030E-01</td>
<td>4.0567E-05</td>
<td>3.0347E-05</td>
<td>3.3787E-05</td>
<td>1.1415E-09</td>
</tr>
<tr>
<td>subset 4</td>
<td>-4.1253E-01</td>
<td>4.4757E-05</td>
<td>4.3961E-05</td>
<td>1.1307E-05</td>
<td>1.2785E-10</td>
</tr>
<tr>
<td>subset 5</td>
<td>-3.1490E-01</td>
<td>4.9668E-05</td>
<td>4.6180E-05</td>
<td>1.7634E-05</td>
<td>3.1095E-10</td>
</tr>
</tbody>
</table>
Table 9 - Statistics for each individual subset on 1991 data

<table>
<thead>
<tr>
<th>Year 1991</th>
<th>correlation</th>
<th>mean</th>
<th>median</th>
<th>s' dev.</th>
<th>variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>subset 1</td>
<td>6.5655E-01</td>
<td>5.7975E-05</td>
<td>4.3188E-05</td>
<td>4.3504E-05</td>
<td>1.8926E-09</td>
</tr>
<tr>
<td>subset 2</td>
<td>-5.9650E-02</td>
<td>8.0827E-05</td>
<td>7.2159E-05</td>
<td>3.0196E-05</td>
<td>9.1181E-10</td>
</tr>
<tr>
<td>subset 3</td>
<td>3.9338E-01</td>
<td>7.0960E-05</td>
<td>6.0980E-05</td>
<td>3.7421E-05</td>
<td>1.4003E-09</td>
</tr>
<tr>
<td>subset 4</td>
<td>-4.0216E-01</td>
<td>4.0605E-05</td>
<td>4.2481E-05</td>
<td>1.2439E-05</td>
<td>1.5472E-10</td>
</tr>
<tr>
<td>subset 5</td>
<td>-4.9282E-02</td>
<td>4.7894E-05</td>
<td>4.4457E-05</td>
<td>1.8317E-05</td>
<td>3.3553E-10</td>
</tr>
<tr>
<td>subset 6</td>
<td>8.8408E-01</td>
<td>8.9279E-05</td>
<td>3.8496E-05</td>
<td>1.3907E-04</td>
<td>1.9339E-08</td>
</tr>
<tr>
<td>subset 7</td>
<td>-1.2613E-01</td>
<td>3.7238E-05</td>
<td>3.7364E-05</td>
<td>1.5650E-05</td>
<td>2.4492E-10</td>
</tr>
<tr>
<td>subset 8</td>
<td>-1.8184E-01</td>
<td>5.4957E-05</td>
<td>4.9740E-05</td>
<td>3.5532E-05</td>
<td>1.2626E-09</td>
</tr>
</tbody>
</table>

Looking at Exhibit 7 and 8 and at the corresponding results shown on Table 8 and Table 9, two interesting findings are revealed by the analysis. First, heteroskedasticity is higher when the price follows an obvious "up or down trend", while it is lower in those periods of "no trend" characterized by a flat unclear pattern. Second, the correlation between volatility and price tends to change sign when the pattern of the exchange rate inverts direction. Correlation is positive during periods of "up trend" and negative during period of "down trend".

Two main lines of reasoning may provide a first intuitive explanation for the findings presented. If volatility is considered as a good estimate of volume traded, I can say that higher heteroskedasticity means a period of more volatile trading volume, and, in general, a period of higher activity in the market. In fact when an obvious positive or negative trend is revealed in the market, large and small institutional and individual investors may take position in turn providing a higher volatility to the volume traded. On the other hand when the price pattern reveals a "no trend" period, volume traded should be generally stable unless specific exogenous causes occur. However one may argue that even when "no trend" is revealed by the market, the price can be very volatile (Exhibit 8 and 9) so that traders can apply so called "channel trading strategy", and both variability of trading volume and the volume will be high. My counter argument is that in periods of clearly revealed "up or down trend" small and large investors alternatively may join the trend, perceiving opportunities for a gain,
providing more variance to the volume traded; instead in periods of "no trend" only traders that can play with large volume may take advantage of the "channel strategy" (small frequent possible gains) overcoming the problem of transaction costs, so that the variance of the volume will be lower.

The underlying intuition of the second finding is consistent with the explanation provided for the first finding. When a pattern is very clearly revealed in the market (either up or down) volatility (volume traded) tends to increase because the clearer is the trend in the market, the larger the number of investors who will be active in the market perceiving opportunities for capital gain. Thus volatility (volume) shows a positive correlation with price in period of "up trend" (both increase) and a negative correlation in period of "down trend" (volume increases as price decreases). Further research and investigations are needed to verify these findings.

In general, the longer is the trend the more both the arguments should hold and apply. In the first case if the trend is lasting longer, the variability of volume would be even higher because discretionary liquidity traders or informational traders may decide to trade with higher flexibility and freedom and not all together given a fairly stable positive or negative trend (perceived to continue). In the second case the longer is the trend the higher is the probability that more and more investors will take larger position increasing the volume traded. However further investigation is needed to increase the reliability of these first findings and to draw any conclusions from the analysis. With this aim I computed the correlation coefficients and performed the following regressions:

**Regression equation** - Series of k-days absolute return against series of variances of volatility in the correspondent identical k days period.

\[
| \log( P_{t+k} ) - \log( P_t ) | = \alpha + \beta_1 [ \text{var} (\text{Vol}_t, \text{Vol}_{t+1}, \ldots, \text{Vol}_{t+k}) ]
\]  

(4)

where \( t = 1, 2, \ldots \)
Correlation coefficients were calculated in addition to the regressions.

The results of this test should provide the statistical significance of the empirical findings revealed by the first test. Table 10 and 11 are a summary of the correlation coefficients calculated for various values of the parameter k.

<table>
<thead>
<tr>
<th>Lag k</th>
<th>Correlation coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-0.0432</td>
</tr>
<tr>
<td>10</td>
<td>0.1528</td>
</tr>
<tr>
<td>15</td>
<td>0.2062</td>
</tr>
<tr>
<td>20</td>
<td>0.1518</td>
</tr>
<tr>
<td>25</td>
<td>0.1993</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lag k</th>
<th>Correlation coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.1553</td>
</tr>
<tr>
<td>35</td>
<td>0.2187</td>
</tr>
<tr>
<td>40</td>
<td>0.2870</td>
</tr>
<tr>
<td>45</td>
<td>0.3180</td>
</tr>
<tr>
<td>50</td>
<td>0.3495</td>
</tr>
</tbody>
</table>

The graph on Exhibit 9 clearly shows that correlation increases with time according with the explanation I provided before. The pattern of this curve is clearly dependent on the price behavior in the specific year considered. In fact the longer is the average trend in any given year the higher should be our correlation coefficient around that time lag represented by the value of k. The specific results for the regressions (4) are shown in Table 12 and the corresponding graphs are presented from Exhibit 10 to Exhibit 19.

Table 12 reveals that the relationship between absolute return and mean of volatility becomes statistically significant at 5% level from lag = 10 to all the longer lags, while it does not seem to be significant for lag = 5. The R-square shows that 2.34% of the absolute log-return can be explained by a regression of the variance of volatility at lag = 10. At lag = 50 The R-square shows that 12.21% of the absolute log-return can be explained by a regression of the mean of volatility. The explanatory power of the variance of volatility seems to be lower than that of the mean of
volatility. The longer is the time lag, the higher seems to be the explanatory power of the variance of volatility.

Table 12 - Coefficients, t.statistic, R-square for regression (5) from lag=5 to lag=50

<table>
<thead>
<tr>
<th>Regression type 2</th>
<th>coefficient</th>
<th>t.statistic</th>
<th>R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>lag = 5</td>
<td>5.77E-09</td>
<td>-0.6830</td>
<td>0.0019</td>
</tr>
<tr>
<td>lag = 10</td>
<td>1.23E-08</td>
<td>2.4158</td>
<td>0.0234</td>
</tr>
<tr>
<td>lag = 15</td>
<td>1.13E-08</td>
<td>3.2582</td>
<td>0.0425</td>
</tr>
<tr>
<td>lag = 20</td>
<td>6.63E-09</td>
<td>2.3497</td>
<td>0.0231</td>
</tr>
<tr>
<td>lag = 25</td>
<td>6.62E-09</td>
<td>3.0783</td>
<td>0.0397</td>
</tr>
<tr>
<td>lag = 30</td>
<td>4.06E-09</td>
<td>2.3527</td>
<td>0.0241</td>
</tr>
<tr>
<td>lag = 35</td>
<td>4.56E-09</td>
<td>3.3159</td>
<td>0.0478</td>
</tr>
<tr>
<td>lag = 40</td>
<td>4.95E-09</td>
<td>4.3822</td>
<td>0.0823</td>
</tr>
<tr>
<td>lag = 45</td>
<td>4.61E-09</td>
<td>4.8482</td>
<td>0.1011</td>
</tr>
<tr>
<td>lag = 50</td>
<td>4.41E-09</td>
<td>5.3277</td>
<td>0.1221</td>
</tr>
</tbody>
</table>

To conclude, this last test seems to confirm the findings and the explanation provided about the interaction between volatility and foreign exchange prices. These are only first findings and intuitive explanation on the relationship between volatility and price in the foreign exchange market as well as on the functioning of the same market. I hope some other inferences and directions can be drawn from these results for further research.

SECTION VI - Conclusions

The study has analyzed the interaction among volatility, price, and trading volume in the foreign exchange market. Previous studies, models of capital market functioning, and empirical tests were presented and the main findings utilized in the research that has been performed. The actual results and the conclusions of these studies seem to consistently indicate an overall homogeneous structure about the relationship among price, volatility and trading volume. Volatility presents a direct relation with trading volume, as it was observed with different time horizons, even in
studies analyzing intra-day patterns. For this reason, as it was done in other studies, I used volatility as an indicator of trading volume in my interpretations of the results obtained. In fact actual data on foreign exchange trading volume are obviously very difficult to find.

The empirical test in the foreign exchange market of the model developed J. Campbell, S. J. Grossman, J. Wang [2] indicates that the model presented holds and applies also in the foreign exchange market. Thus, in the foreign exchange market, a price decline (increase) on a high volatility day seems to be more likely than a price decline (increase) on a low volatility day to be associated with a higher (lower) expected price return.

The studies on the interaction between volatility and foreign exchange prices with both a static and a dynamic approach revealed two main findings. First, heteroskedasticity is higher when the price follows an obvious "up or down trend", while it is lower in those periods of "no trend" characterized by a flat unclear pattern. Second, the correlation between volatility and price tends to change sign when the pattern of the exchange rate inverts direction. Correlation is positive during periods of "up trend" and negative during period of "down trend".

In order to further test the statistic significance of these results with a dynamic analysis, a set of regressions was run. All the regressions were found to be statistically significant at 5% level. Therefore the results of the first have been empirically confirmed. Tentative economic explanations of these two findings are, that assuming volatility as a good indicator for trading volume,

i) higher heteroskedasticity means a period of more volatile volume and, in a sense, of higher activity in the market. A period of clearly defined positive or negative trend, tends to invite a higher variety of smaller and larger investors to take positions and execute trades in turn, providing a higher variance to the volume traded. Conversely there does not seem to be any endogenous economic
reason why the volume should not be fundamentally stable in period characterized by a flat, stable trend.

ii) a period of clearly defined positive or negative trend tends to convince a larger number of investors and, more generally, discretionary traders to take positions and trade, providing a higher volume to the market. Again there does not seem to be any endogenous economic reason why the volume should not be fundamentally stable in period characterized by a flat, stable trend.

I hope that these findings will lead to further investigations and research. One possibility could be to analyze the same series of data with additional regression analysis and see if the results still hold and if any other useful qualifications may be added to my tentative explanation.
1990 US Dollar/Deutsche Mark Daily Price

Exhibit 1
1990 US Dollar/Deutsche Mark Daily Return

Exhibit 2
1990 US Dollar/Deutsche Mark Daily Volatility

Exhibit 3
1991 US Dollar/Deutsche Mark Daily Price

Exhibit 4
1991 US Dollar/Deutsche Mark Daily Return

Exhibit 5
1991 US Dollar/Deutsche Mark Daily Volatility

Exhibit 6
Subsets of 1990 US Dollar/Deutsche Mark Daily Price

Exhibit 7
Subsets of 1991 US Dollar/Deutsche Mark Daily Price

Exhibit 8
Correlation Coefficients at Different Lags

Exhibit 9
Regression (4) at Lag = 5

Exhibit 10
Regression (4) at Lag = 10

Exhibit 11
Regression (4) at Lag = 15

Exhibit 12
Regression (4) at Lag = 20

Exhibit 13
Regression (4) at Lag = 25

Exhibit 14
Regression (4) at Lag = 30
Regression (4) at Lag = 35

Exhibit 16
Regression (4) at Lag = 40

Exhibit 17
Regression (4) at Lag = 45

Exhibit 18
Regression (4) at Lag = 50
References


