THEORY OF FROST FORMATION

by

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ABSTRACT

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Submitted to the Department of Chemical Engineering on December 30, 1968, in partial fulfillment of the requirements for the degree of Doctor of Science.

The purpose of the present research was two fold. First, the physical and thermal properties of water frost on a liquid nitrogen-cooled surface were to be measured over a wide range of deposition conditions. Secondly, the experimental results were to be analyzed and interpreted with several analytical models of varying degree of sophistication which considered processes occurring both in the gas phase and in the frost layer.

The time variation in frost density, thickness and thermal conductivity and the heat and mass fluxes were measured for the water frost deposition from a humidified stream of air onto a short copper plate, maintained at about \(-315^\circ F\). The plate formed part of the underside of a long rectangular duct and was mounted a distance of 72 equivalent duct diameters from the entrance. The duct aspect ratio was 25:1.

The experimental results were obtained for the values of Reynolds numbers between 3,770 and 15,800, the gas humidities between dew point at \(14.3^\circ F\) and \(58^\circ F\), the gas temperatures between \(34^\circ F\) and \(93^\circ F\) and the time elapsed from the start of the frosting between 20 and 180 minutes. In all the experiments, the gas pressure over the plate was essentially atmospheric.

The experimental results for the time variation in average frost density, frost thickness and the heat and mass fluxes were successfully correlated by a simple theoretical model. This model was based on the heat and mass balances on a differential element at the frost surface. Water vapor was assumed to diffuse in the frost interior by the vapor pressure gradient at the frost surface. No consideration, however, was given to the mechanism of water transport at every layer in the frost interior. The frost density was assumed to be uniform.
with depth at all times.

In order to explain a complete physical picture of the frosting process, an improved model was examined in which the heat and mass balances were taken on a differential element within the frost. The equilibrium conditions between the ice and the interstitial gas were assumed everywhere in the frost interior. This model indicated that if the equilibrium conditions exist everywhere within the frost, then the frost layer will densify non-uniformly across its depth as it accumulates on the cold surface.

For a typical set of experimental conditions, the density distributions within the frost were measured by slicing the frost layers. These measurements indicated that the density of the frost layer deposited on a liquid nitrogen-cooled surface always stays uniform with depth. Thus, the equilibrium conditions could not exist everywhere in the frost interior.

The critical evaluation of the experimental results on the density distribution within the frost indicated that the measured water flux within the frost cannot be obtained by molecular diffusion under the non-equilibrium partial pressure gradients without causing nucleation. Based on this conclusion, another 'non-equilibrium model' was postulated to explain the phenomenon of uniform densification of the frost. This model assumed that water transport in the frost interior occurs by both molecular diffusion of water vapor under a non-equilibrium partial pressure gradient and the thermal diffusion of ice nuclei. The model has been examined only qualitatively in the present research.

Thesis Supervisor: Robert C. Reid

Title: Professor of Chemical Engineering

Thesis Supervisor: P. L. Thibaut Brian

Title: Professor of Chemical Engineering
December 30, 1968

Professor E. Neal Hartley
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

Dear Professor Hartley:

In accordance with the regulations of the Faculty, I herewith submit a thesis, entitled "Theory of Frost Formation," in partial fulfillment of the requirements for the degree of Doctor of Science in Chemical Engineering at the Massachusetts Institute of Technology.

Respectfully submitted,

Yatish T. Shah
Acknowledgement

Some of the original ideas of this thesis were first suggested by the supervisors, Prof. P.L.T. Brian and Prof. R. C. Reid. The author wishes to acknowledge with appreciation these suggestions along with their patient and perspicacious direction during the course of the thesis. The author is particularly grateful to his supervisors for their suggestions in the theoretical aspects of this thesis.

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SUMMARY

The formation of water frost on a liquid nitrogen-cooled surface from a flowing humid gas is a complex phenomenon but one which occurs with sufficient frequency in industrial practice to warrant both theoretical and experimental studies. Although some research studies have been reported on the water frosting process (3, 6, 27, 52, 89, 92, 114), only one successful preliminary attempt (14) has been made to explain the various aspects of the frosting process by means of a quantitative theory. The present research was undertaken to explore this theory in greater detail.

The present thesis first presents the results of an experimental study wherein the physical and thermal properties of water frost were measured over a wide range of deposition conditions. These results were then analysed and interpreted with various analytical models which considered processes occurring both in the gas phase and in the frost layer.

Similar Previous Studies

A. Experimental

In a similar study Brazinsky (14) obtained experimental data on the thickness, density and thermal conductivity of water frost deposited from flowing humid air onto a liquid nitrogen-cooled copper plate. The plate was a part of the under-side of a rectangular duct. The independent system parameters studied were the gas stream Reynolds number, the
humidity, and the time elapsed since the start of frosting. The data were taken for a Reynolds number range of 5,600 to 14,500, a humidity range corresponding to gas dew points of 29°F to 40°F, and for the frosting times as high as 120 minutes. The gas temperature was always maintained at about 75°F.

B. Theoretical ('Simple Model')

Brazinsky's experimental data were generally well correlated by a rather simple theoretical model which was based on the following assumptions:

1. After a smooth layer of frost is deposited on the liquid nitrogen cooled surface, the transport of heat and mass to the frost surface can be analysed using ordinary gas phase correlations (i.e., no fogging is assumed to occur in the gas phase).

2. Water vapor is transferred from the frost surface to the interior of the frost by a diffusional mechanism resulting from a water vapor pressure gradient. The rate of transfer is proportional to the gradient at the surface. At no time, however, is any consideration given to the mechanism of water transport at every layer in the frost interior.

3. The density of the frost stays uniform with depth at all times.

The mathematical equations applicable for this model were obtained from heat and mass balances on a differential layer at the frost surface. The heat flux within the frost
was assumed to be uni-directional. The resulting equations were solved numerically to obtain the time variation in average frost density, frost thickness and heat and mass transfer characteristics of the frosting process.

Experimental Apparatus of the Present Research

A schematic of the frosting system of the present research is shown in Figure 1. This system was very similar to one used by Brazinsky. There were, however, two improvements in the present system. The gas temperature was varied by means of a heat exchanger, and the frost surface was maintained flush with the bottom of the duct during frosting. To accomplish the latter, the copper plate and calorimeter assembly, onto which the frost deposited, was rested on a jack. Thus, the plate could be lowered during an experiment to keep the frost surface and the bottom of the rectangular duct flush at all times.

Experimental and Theoretical ('Simple Model') Results and Discussion

In the present research, the properties of water frost and the heat and mass transfer characteristics of the frosting process were studied under a considerably wider range of system variables than those investigated by Brazinsky (14). The range of the independent system variables examined in the present thesis are shown in the following table.
Table 1

Ranges of System Variables

<table>
<thead>
<tr>
<th>System Variables</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reynolds number</td>
<td>3,770 - 15,800</td>
</tr>
<tr>
<td>Gas Humidity</td>
<td>dew point at 14.3 to 50°F</td>
</tr>
<tr>
<td>Gas Temperature</td>
<td>34°F - 93°F</td>
</tr>
<tr>
<td>Frosting Time</td>
<td>20 minutes to 180 minutes</td>
</tr>
</tbody>
</table>

The measurements carried out in each frosting experiment were identical to ones obtained by Brazinsky, with the exception of heat flux measurements. In the present research, the latter were determined in the transient as well as in the quasi-steady state of the frosting process.

The experimental results on the time variation in average frost density, frost thickness and heat and mass transfer rates were found to correlate well with those predicted from simple theoretical model over the entire range of system variables stated in Table 1.

The effects of Reynolds number and gas humidity on the frost properties and the rates of heat and mass transfer obtained in the present research were almost identical to ones obtained by Brazinsky (14). The Reynolds number was found to have a significant effect on the thermal resistance of the frost. The gas humidity was found to affect the mass deposition significantly.

Figures 2 and 3 show the time variation of average frost density, frost thickness and the rates of heat and mass transfer during the frosting process at various gas
temperatures (but at constant-Reynolds number and gas humidity). These data indicate that a higher gas temperature produces a frost of lower thermal resistance. As shown in the figures, the predictions of the 'simple model' were found to be in good agreement with these measured experimental facts.

If the basic assumption of the 'simple model' regarding the heat and mass transfer processes occurring in the gas during the frosting process is true, then experimentally measured frosting heat and mass transfer coefficients should agree well with ones predicted from a heat and mass transfer analysis of the frosting system. This is verified in the present research as shown in Figures 4 and 5.

The experimental heat and mass transfer coefficients were obtained from the measurements of the gas temperature, humidity, the heat and mass fluxes and the frost surface temperature. The theoretical heat transfer coefficients were obtained from Sleicher and Tribus's (105) theoretical Equation 1 shown below.

\[
Nu(x) = \frac{\dot{h}(x)D_w}{k_g} = \frac{\sum \alpha_n \exp(-\lambda_n^2 X_s)}{2 \sum (\alpha_n/\lambda_n^2) \exp(-\lambda_n^2 X_s)}
\]

(1)

where the constants \( \alpha_n \) and \( \lambda_n \) are functions of Reynolds and Prandtl numbers and can be obtained from Reference 105.

The theoretical mass transfer coefficients were obtained from the Lewis analogy (39) for simultaneous heat and mass transfer, which can be mathematically described by the following Equation 2.
As shown in Figures 4 and 5, the arithmetic mean of the percentage deviations (percentage deviation = absolute value of \(( \text{Experimental Value} - \text{Theoretical Value} ) \times 100 \) between the experimental and theoretical values of both heat and mass transfer coefficients under the frosting conditions was found to be approximately 10\%. Heat transfer coefficients were also measured under non-frosting conditions. They agreed within 3\% of theoretical values. It is believed that experimental errors in the frost surface temperature measurement, errors resulting from uneven transition between the duct floor and the frost surface, and errors due to the slight roughness of the frost surface were the major causes for the observed deviation between experimental and theoretical values of the heat and mass transfer coefficients. These results prove, however, that the first assumption of the simple model, as mentioned above is reasonably valid.

The mathematical solution of the equations for the simple model requires the knowledge of initial values of average frost density and frost thickness. These values may not be always available. Hence, in order to obtain these values for an arbitrary set of system conditions, the following empirical correlations were developed from the experimental data of the present research at the time of forty minutes from the start of the frosting process.

\[
\frac{\kappa}{K_g} = \frac{\zeta g c_p (Le)^{3/3}}{\zeta g c_p \left( \frac{Sc}{Pr} \right)^{2/3}}
\] (2)
The above correlations were obtained at the time of forty minutes because it was the earliest time common to all the experimentally studied system conditions for which the measurements of average frost density and frost thickness were carried out. It should be noted that Equations 3 and 4 are applicable only to the frost deposited on a liquid nitrogen-cooled metal surface from flowing humid air. The accuracy of the predictions of average frost density and frost thickness from Equations 3 and 4 is estimated to be within ± 8%.

Improved Model' of the Frosting Process

Even though the simple model correlates well the experimentally measured effects of the environmental conditions on the frost properties and the heat and mass fluxes, it does not give the complete picture of the frosting process. Specifically, it neglects the mechanism by which every layer in the frost interior densifies with time. Hence a further theoretical study was carried out to develop a more improved model which could take into account the physical processes occurring in the frost interior along with those occurring in the gas phase.
The 'simple model' assumed that water vapor is transported into the frost interior due to a vapor pressure gradient at the frost surface. This vapor pressure gradient is caused by the temperature gradient. Since this assumption leads to successful predictions of the water transport past the frost surface, it appears logical to assume that the same mechanism would cause the water transport at every layer in the frost interior. Thus, the first attempt towards the attainment of an improved model was carried out with the help of this assumption. The mathematical equations were obtained by taking the heat and mass balances on a differential element in the frost interior. The transport processes occurring at the frost surface were one set of boundary conditions for the theoretical equations. The other set of boundary conditions were taken at the cold wall. The final mathematical equations with appropriate boundary conditions can be summarized as follows. The schematic of the frosting process as visualized by this model is shown in Figure 5.

**Mass Balance**

\[
\frac{\partial}{\partial x} \left( \frac{D_p (1 - \xi_f/\xi_{ice})}{RT} \frac{\partial P_p}{\partial x} \right) = \frac{\partial \rho_f}{\partial \Theta} \tag{5}
\]

also,

\[
\frac{\partial P_p}{\partial x} = \frac{\partial P_p}{\partial T} \frac{\partial T}{\partial x} \tag{6}
\]

from Clausius-Clapeyron equation (80)

\[
\frac{\partial P_p}{\partial T} = \frac{M \Delta H P_p}{RT^2} \tag{7}
\]

hence,

\[
\frac{\partial}{\partial x} \left( \frac{D_p (1 - \xi_f/\xi_{ice})}{RT} \frac{M \Delta H P_p (\frac{dT}{dx})}{RT^4} \right) = \frac{\partial \rho_f}{\partial \Theta} \tag{8}
\]
Heat Balance

\[ \dot{Q} = k_f \frac{dT}{dx} \]  \hspace{1cm} (9)

Boundary Conditions

At Cold Wall

\[ T(0) = T_w \]  \hspace{1cm} (10)
\[ j_0 = 0 \]  \hspace{1cm} (11)

At the Frost Surface

\[ h_f (T_s - T_0) + k_f (p_g - p_s) \Delta H = k_f \frac{dT}{dx} \bigg|_{x=d} \]  \hspace{1cm} (12)
(neglecting sensible cooling of condensible component)

\[ \frac{k_f (p_g - p_s)}{\Delta H} = \frac{\rho_f s}{\rho_f} \frac{d\delta}{d\theta} + \frac{D_g (1 - \rho_f s / \rho_f \delta)}{\Delta H} \frac{dP_{wp}}{dx} \bigg|_{x=d} \]  \hspace{1cm} (13)

With the knowledge of independent system parameters, the initial values for \( q_f(x,0) \) and \( \delta(0) \) and the correlation for the frost thermal conductivity (which was obtained experimentally as shown in Figure 7), the above equations were solved numerically to obtain \( q_f(x,\theta) \), \( T(x,\theta) \) and \( \delta(\theta) \).

The predictions of the above equations for \( q_f(x,\theta) \) and \( \delta(\theta) \) under a typical set of system conditions of \( Re = 5,600 \), humidity = gas dew point at 29.2 \(^\circ\)F and \( T_s = 70 \, ^\circ\)F are shown in Figure 8. The initial values of density were assumed to be constant everywhere in the frost interior. As shown in the figure, the model predicts that if water transport at every layer in the frost interior is caused by the prevailing vapor pressure gradient, then the entire frost layer will densify non-uniformly. The layer at the frost surface (where the ice vapor pressure is high) densifies at a considerably higher rate than one near the
cold wall. As a matter of fact, in eighteen minutes of the frosting process the density of a thin layer at the frost surface attains a value three times larger than one near the cold wall.

**Experimental Measurement of a Density Gradient in the Frost Interior**

The validity of the predictions of density distribution within the frost by the above described improved theoretical analysis was investigated by means of experimental measurements. These measurements were made on frost deposited at Re=15,800, a humidity corresponding to a dew point at 30°F, and with T_g = 71°F.

The frost was sampled with a polystyrene tube which was made by glueing 0.0625 inch thick equal diameter rings. Frost samples were then cut with a cold razor. Several slices could be obtained. The slicing process was carried out in a cold room (temperature = -20°F). The density of each slice was then calculated from the measurements of mass and volume.

Figure 9 shows the results of these measurements for the various frosting experiments. These results indicate that the density of the bottom 0.0625 inch thick layer was at all times almost the same as that of the entire frost layer, whose thickness varied from 0.091 inch at 26 minutes to 0.256 inch at 144 minutes. The density values shown in Figure 9 are estimated to be accurate within ±12%. Thus, it was concluded from all these results that, unlike
the predictions of the improved theoretical analysis, the density of the frost deposited on a liquid nitrogen-cooled surface always stays uniform with depth.

Discussion on the Above Described Experimental and Theoretical Results

The experimental fact that the density of the frost is always uniform with depth indicates that the assumption of equilibrium conditions between the interstitial gas and ice everywhere in the frost interior is not valid.

Further proof for the invalidity of the existence of equilibrium conditions in the frost interior can be shown by comparing the predictions of the improved and simple models for the time variation in average frost density, frost thickness and heat flux. This is shown in Figure 10. For a short duration of the frosting process, the agreement between their predictions is found to be fairly good. However, as shown in the figure, the deviation between the predictions increases with time. This should be expected because as the improved model predicts a higher and higher density gradient, the temperature distribution in the frost interior predicted by both the models would differ more and more (see Figure 11). Hence the predictions of the improved model will not agree with those of the simple model over a long period. Since the predictions of the simple model agree well with the experimental results, the improved theoretical analysis will fail to explain any observed effect in the frosting
process for a long period.

It should be noted that the predictions of the improved and the simple models also indicate that even though the experimentally observed rate of mass transport in the frost interior cannot be explained with the assumption that at every layer water vapor is in equilibrium with ice at the prevailing temperature, there could be only a very slight deviation from equilibrium between the gas and ice at the frost surface. Otherwise, the non-equilibrium partial pressure gradient and the vapor pressure gradient would just have to coincide with each other at the frost surface under all diverse gas phase conditions.

In the absence of equilibrium conditions within the frost, the true mechanism of water transport at every layer in the frost interior may be molecular diffusion of water vapor under a non-equilibrium partial pressure gradient. The validity of this hypothesis can be examined from an analysis of the experimental results of Figure 9. This analysis is briefly described below.

If the water transport at every layer in the frost interior occurs by the diffusional mechanism under a non-equilibrium partial pressure gradient, one can write

\[
\frac{\partial}{\partial x} \left( \frac{D_v (1 - \frac{v}{v_{ice}})}{RT} \frac{\partial p}{\partial x} \right) = \frac{\partial q_v}{\partial \Theta} \tag{14}
\]

Integrating the above equation for the uniform (with depth) rate of frost densification and substituting the boundary condition that at \( x=0, \frac{\partial}{\partial x} = 0 \), one obtains
The above Equation 15 can be integrated to obtain the partial pressure at \(x=0.0625\) inch by either setting the boundary condition for \(p\) at \(x=0\) (at the cold wall) or at \(x=\delta\) (at the cold surface). Both of these cases can be examined as follows.

If the boundary condition for partial pressure is taken at \(x=0\), one can write:

\[
\frac{\partial p}{\partial x} = \frac{d}{dx} \int_{x=a}^{x=0} \frac{\tau R T}{(\gamma f / \gamma ice) D_B} \, dx
\]

Equations 18 and 19. can be examined as follows.

Assuming \(p_{x=0} = p_{vp} x=0\) (vapor pressure of ice at \(-320^\circ F\)), using the data of Figures 9 and 12 at a time of 79 mins. and taking the values of \(\gamma\) and \(D_B\) from the following. Equations 18 and 19.

\[
\gamma = 1.10 \quad (18)
\]

\[
D_B = 3.3 \times 10^{-6} T^2.0087 \quad (19)
\]

where \(D_B\) is in \(\text{ft}^2/\text{hr}\) and \(T\) is in \(^\circ R\), the integral on the right hand side of Equation 17 for the bottom 0.0625 inch thick frost layer can be calculated to be 1.485 mm. of Hg. Hence, \(p_{x=0.0625''} = 1.485 \text{ mm. of Hg}\).

It should be noted that the assumption of \(p_{x=0} = 0\) makes the above calculated value of \(p_{x=0.0625''}\) the most conservative one. Also, at time of 79 mins. \(p_{vp,x=0.0625''}\)
= 5.2 x 10^{-4} \text{ mm. of Hg.} \quad \text{Hence, the required partial pressure at } \text{x=0.0625 inch indicate the existence of supersaturation ratio of } \frac{1.485}{5.2 \times 10^{-4}} = 2.86 \times 10^3. \quad \text{Such a high value of supersaturation ratio, of course, indicates a strong possibility for occurrence of nucleation.}

The strong possibility for nucleation indicates that Equation 14 may not be a valid representation of the true physical processes occurring in the frost interior. This indication can be verified by examining the realistic nature of the partial pressure value at the frost surface calculated from Equation 16. By setting the upper limits on the integrals of Equation 16 at \text{x=6}, and using the data of Figures 9 and 12 and Equations 18 and 19, the partial pressure at the frost surface can be calculated to be 16.86 mm. of Hg. Since the experimental results of Figures 9 and 12 were obtained with the partial pressure of water vapor in the bulk gas at the value of 4.14 mm. of Hg., the above calculated value of partial pressure at the frost surface is very unrealistic. Hence, Equation 14 could not be the true representation of the frost densification process.

It should be obvious to note that if the boundary condition for \text{p} in Equation 15 is set at the frost surface, then the partial pressure at \text{x=0.0625 inch} can be calculated from the following Equation 20

\[
 p_{x=6} \quad \text{such that} \quad p_{x=0.0625} = \int_{x=0.0625}^{x=6} \frac{\gamma RT}{(1-\gamma P_{\text{sat}}) D_{\text{B}}} \, dx \quad (20)
\]

Assuming \[\text{p}_{x=6} = p_{\text{sat}}|_{x=6} = 1.632 \text{ mm. of Hg.} \quad \text{(note that this}}\]
assumption is quite realistic as indicated by simple model) and calculating the right hand side of above Equation 20 from the data of Figures 9 and 12 and Equation 18 and 19 \( p |_{x=0.0625"} \) was found to be -13.65 mm. of Hg. This absurd value of partial pressure at \( x=0.0625 \) inch indicates, once again, that Equation 14 does not represent the true physics of the frost densification process.

Densification of Frost under Non-Equilibrium Conditions
(Non-Equilibrium Model)

Based on the results described above, a new model was postulated to represent the true mechanism of water transport in the frost interior. This model has only been examined qualitatively in the present research. It can be briefly described as follows.

The model postulates that the frost interior can be divided into three sections on the basis of state of water existing in the bulk interstitial gas. Near the frost surface (region 1), where the vapor pressures of ice are high, all the vapor exists in the vapor form. Near the cold wall (region 2), where the vapor pressures of ice are extremely small, all the water exists in the form of ice nuclei. In between these two regions, there is a so-called transition region (region 2), where the water exists both in the form of ice nuclei as well as vapor.

As water vapor reaches the frost surface from the bulk gas by diffusional mechanisms, it penetrates in the frost
interior due to a partial pressure gradient caused by a temperature gradient at the frost surface. Since the 'simple model' has been found to predict the water vapor transport past the frost surface fairly successfully, it can be assumed that at the frost surface water vapor is almost in equilibrium with ice at the existing temperature. The postulation regarding the state of water implies that the supersaturation of water vapor (p/\text{P}_{\text{vp}}) in the interstitial bulk gas should be considerably high at the interface of regions 1 and 2, so that nucleation can occur. Thus, as water vapor diffuses from the frost surface to the interior, the supersaturation ratio increases from the value of approximately 1 at the frost surface to a considerably high value at the interface of regions 1 and 2.

The ice crystals in region 1 grow by the condensation of water vapor. Hence, the crystal growth in this region is controlled by the rate at which water vapor arrives at the crystal surface from the bulk interstitial gas. The present model visualizes the uniform growth of the ice crystals over the entire region 1 as a result of counter-balancing effects of increase in driving force (p-\text{P}_{\text{vp}}) for the mass transfer between interstitial gas and the crystal surface and the decrease in the molecular diffusivity of water vapor in air (due to decrease in temperature) with the increase in depth from the frost surface.

As water continues to move deeper in the region 2,
water vapor is now not only used up due to the growth of ice crystals by vapor condensation but also by the generation of ice nuclei. At every layer, the generated ice nuclei are kicked towards the cold wall by a thermal force caused by the existing temperature gradient. While moving, some of these nuclei are impinged (or trapped) on the existing crystals. Thus, the crystals in this region grow by deposition of ice nuclei on the existing crystals as well as by vapor condensation. The model visualizes the uniform growth of ice crystals in region 2 as a result of a complex combination of these two separate mechanisms of the growth.

The continuous depletion of the water vapor will result in the existence of water only in the form of ice nuclei at the interface of regions 2 and 3. Thus, water is transported in region 3 only in the form of ice nuclei. Since the temperature decreases with depth in this region, nuclei will remain in the solid form as they move towards the cold wall.

The ice crystals in region 3 can be expected to grow uniformly due to a constant rate of trapping of the nuclei on the existing crystals at every layer as the nuclei move towards the cold wall. This is because for a given frost structure the probability for the rate of nuclei impingement can be visualized as a direct function of nuclei concentration and the momentum of each nuclei. Brock's equation (15) for thermal diffusive velocity indicate that
this velocity for the ice nuclei of a given size in the frost interior would vary approximately linearly with temperature gradient. Since the temperature gradient increases with depth (see Figure 12), the thermal diffusive velocity would also increase with depth. Thus, the constant rate of trapping at every layer in region 3 could be caused by the counterbalancing effect of increase in velocity of the nuclei but decrease in nuclei concentration with increase in depth from the frost surface.

The mathematical formulation of this 'non-equilibrium' model is possible. However, due to extreme difficulty involved in the accurate predictions of the nucleation rate and the trapping rate of ice nuclei, the solution of the mathematical equations is, at present, not possible.

Conclusions

From the results of the present research the following conclusions are made regarding the deposition of water frost on a cryogenically cooled surface.
1. The 'simple model' can be successfully used for the design of cryogenically cooled equipment under a wide range of frosting conditions.
2. At every layer in the frost interior, the bulk interstitial gas and the ice crystals do not exist under equilibrium conditions at the prevailing temperatures. Thus, even though the water transport past the frost-gas interface can be successfully predicted with the assumption of an equilibrium between ice and gas at the frost surface,
the complete picture of the frosting process can only be successfully modelled with the assumption that water transport at every layer in the frost interior occurs by diffusional mechanism under a non-equilibrium partial pressure gradient as well as by the transfer of ice nuclei. The density of the frost deposited on a liquid nitrogen-cooled surface stays always uniform with depth. The 'non-equilibrium model' appears to explain the true mechanism for uniform densification of the frost.
Figure 1
Flowsheet of Equipment
Figure 2 Experimental Facts and Theoretical Predictions on Effect of Gas Temperature on the Time Variation of Frost Properties and Heat and Mass Fluxes
Experimental Data

- $Re=8,430$; Humidity=Gas dew point at $29.8^\circ F$; $T = 52^\circ F$
- $Re=8,000$; Humidity=Gas dew point at $30.4^\circ F$; $T_g = 76^\circ F$

--- Theoretical Predictions of 'Simple Model'

**Figure 3** Experimental Facts and Theoretical Predictions on Effect of Gas Temperature (at Constant Humidity and Reynolds Number) on Variation of the Frost Properties and Heat and Mass Fluxes with Time


### Experimental Data:

<table>
<thead>
<tr>
<th>Gas Temperature</th>
<th>Cold Surface Temperature</th>
<th>System Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>93°F</td>
<td>15°F</td>
<td>Frosting</td>
</tr>
<tr>
<td>75°F</td>
<td>8°F</td>
<td>Frosting</td>
</tr>
<tr>
<td>50°F</td>
<td>5°F</td>
<td>Frosting</td>
</tr>
<tr>
<td>34°F</td>
<td>2°F</td>
<td>Frosting</td>
</tr>
<tr>
<td>82°F</td>
<td>-11.5°F</td>
<td>Non-Frosting</td>
</tr>
</tbody>
</table>

#### Theoretical Predictions under Non-Frosting Conditions (105)

#### Theoretical Predictions from Lewis Analogy (104)

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**Figure 4**

Heat Transfer Coefficient versus Reynolds Number (at Various Gas Temperatures) under Frosting and Non-Frosting Conditions

- 1. $T_e = 93°F$, $T_s = 15°F$
- 2. $T_e = 75°F$, $T_s = 8°F$
- 3. $T_e = 50°F$, $T_s = 5°F$
- 4. $T_e = 34°F$, $T_s = 2°F$

---

**Figure 5**

Mass Transfer Coefficient versus Reynolds Number (at Various Gas Temperatures) under Frosting Conditions

- 1. $T_e = 93°F$, $T_s = 15°F$
- 2. $T_e = 75°F$, $T_s = 8°F$
- 3. $T_e = 50°F$, $T_s = 5°F$
- 4. $T_e = 34°F$, $T_s = 2°F$
Reynolds Number = Re
Gas Temperature = T
Humidity = p

Heat Transfer Coefficient = h
Mass Transfer Coefficient = K

Direction of Heat and Mass Transfer

Heat Flux = h (T₆ - Tₛ) + K₆ (p - pₛ) ΔH
Mass Flux = K (pₒ - pₛ)

Frost Surface
(Temperature = Tₛ)

Cold Wall
(Temperature = T_w)

Figure 6  Schematic of Frosting Process as Visualized by Improved Model.
Figure 7  Point Thermal Conductivity as a Function of Density with Temperature as a Parameter
System Conditions
Re = 5,600
Humidity = Gas dew point at 29.2°F
Gas Temperature = T
ρ,initial = 0.026 gms./cc.
δ,initial = 0.023 inches
θ,initial = 2 mins.

Experimental Conditions
Re = 15,800
Humidity = Gas dew point at 29.8°F
T = 71°F
δ = 0.0625
δ = 0.125

Figure 8  Predictions of 'Improved Model' for Time Variation in Frost Density and Thickness
Figure 9  Density Distribution within the Frost as a Function of Time
Predictions at the Variation in Temperature Distribution within the Frost by 'Simple' and 'Improved' Models

Figure 10 Comparison between Predictions of 'Simple' and 'Improved' Models

Figure 11 Predictions of Time Variation in Temperature Distribution within the Frost by 'Simple' and 'Improved' Models
Experimental Conditions
Re = 15,000; Suction of 0.20; Gas low point at 30°F; T = 73°F

Distance from Coll Wall (inches)

Figure 12 End of Run Temperature Profile within the Frost
SECTION I

INTRODUCTION

Summary

Although there are some data reported in the literature on the structure and properties of water frost as well as for the heat transfer under frosting conditions, there is as yet no completely satisfactory quantitative theory for the frosting process.

The frosting process with both small and large wall-gas temperature differences have been investigated. The frosting heat transfer studies have dealt with both natural and forced convection. In all the studies heat flux through the frost has been found to reach a nearly steady value after a short duration of frosting.

For small wall-gas temperature differences diffusional mechanisms control the transport of water vapor from gas to cold surface. For the frost deposition on a surface at cryogenic temperatures, diffusional mechanisms control the water vapor transport after a smooth layer of frost is formed.

The formation of a smooth frost layer on a surface cooled by liquid nitrogen appears to occur in a very complex manner. It seems that at first, frost deposits are formed in the forms of clumps and then very soon these clumps are "filled in". Due to the fluffy nature of the frost, a small deposit of it creates significant insulation on the cold surface; and hence a smooth thin layer of this frost raises the surface
temperature to the extent that fogging in the gas phase boundary layer becomes highly improbable. The time for the formation of a smooth frost layer has been found to depend upon the gas phase turbulence.

The quasi-steady state heat flux during the frosting process is achieved by simultaneous increase in frost conductivity and thickness such that the thermal resistance of the frost remains almost constant. The frost conductivity is increased by increase in density. The density of the frost increases by the diffusion of water vapor in the frost interior which is caused by the vapor pressure gradient at the frost surface.

It is the objective of this research to obtain a quantitative model of the frosting process which could enable one to explain the experimentally observed effects of the various system variables on the frost properties. Both experimental as well as theoretical studies are planned. Some light will also be shed upon the physical processes occurring in the frost interior.

I-A Practical Application of the Frosting Phenomenon

Process in which heat is transferred to a refrigerated surface with the simultaneous deposition of a frost layer are important in gas coolers, refrigeration, regenerators, freeze-out purification of gases, cryopumping and the storage of cryogenic liquids. In these processes, frost deposition may be required for successful operation of a process or it may be an undesirable side effect.
The practical application of the frosting process can be well illustrated by its effect on the aerospace industry. In this industry frost deposits are found to be of mixed blessings. Frost accumulates on the walls of uninsulated missile fuel tanks. These frost layers partially insulate the Oxygen supply from external heat leak, thus reducing the amount of liquid required for 'topping' before firing. However, the accumulated frost adds an undesirable (and unknown) weight to the missile. These effects of frost depositions on the aerospace industry are typical examples of the industrial occurrences of the frosting phenomenon.

Even though there has been research work in each of the above mentioned industries, the data, when available, are widely scattered throughout the literature. Most of the published work is directed toward refrigeration and freeze-out purification. Frosting process under natural convection has been studied somewhat extensively by missile industries. Forced convection frosting has also been reported recently by several investigators (7,14,23,24,26,50,51,58,86,89,90,114). The final results of all these investigations are, however, correlated empirically and are applied specifically to the system conditions taken under examination. For these reasons, a generalized theory for the frosting process is of significant practical interest.

I-B Description of the Frosting Process

Frost deposits form when a gas mixture comes in contact
with a surface whose temperature is below the dew point of the gas and the freezing point of the vapor. As the mixture flows over the cold surface a concentration gradient is established normal to the surface, resulting in a mass flux towards the cold wall. Simultaneously, a temperature gradient is established causing a heat flux which cools the gas.

For small temperature differences between the bulk gas and the cold surface, the condensible component in the gas phase will move towards the wall by molecular and turbulent diffusional processes. If these temperatures are known, the heat and mass fluxes can be predicted with the usual heat and mass transfer correlations.

As the cold surface temperature is decreased and the temperature difference between the surface and the gas is increased, the diffusion mass flux at first increases rapidly and then levels off because the equilibrium concentration of the condensible component at the surface is approximately an exponential function of the surface temperature as shown in Equation I-B1.

\[ \ln p_w = \frac{a}{T_s} + b \quad (I-B1) \]

The heat flux, however, continues to increase almost linearly according to Equation I-B2

\[ q = h \left( T_g - T_s \right) \quad (I-B2) \]

Thus, the gas stream may be cooled below the dew point for low surface temperatures. This may cause sufficient
supersaturation in the gas phase thermal boundary layer to form fog particles or ice nuclei. Particle impingement or trapping of the particles on the surface then becomes the dominant mechanism of frost deposition. Therefore, the difference in deposition mechanism for large and small temperature differences depends on the formation of fog in the gas stream.

The limits for fog formation within the gas film adjacent to a cold surface have been derived by Johnstone and co-workers (57). The derivation assumed that a gas vapor mixture flowed in fully developed pipe flow. It was also assumed that the film theory adequately described the flow (i.e. a turbulent core without concentration and temperature variation normal to the surface, and a stagnant film where heat and mass transfer resistances are concentrated). Bulk temperatures and humidities were found for which the film remained everywhere unsaturated. A plot of fog formation limits for the water-air system is shown in Figure I-B1. The plot shows water vapor concentration as a function of gas phase dry bulb temperature at various surface temperatures. For any surface isotherm, operating points below this curve show regions where no fogging can occur anywhere in the gas phase. Fog may form for all bulk gas conditions lying above the curve for a particular surface temperature. The plot shows that, for a given bulk gas temperature, the maximum humidity without fog formation is a strong function of the surface temperature. Thus, this plot indicates approximately the
regions of high and low temperature differences. The theories for fog formation limits in various other systems are described by Amelin (2).

The deposition of the frost on a very cold surface (about \(-315^\circ F\)) with a large temperature difference between the cold surface and the gas appears to be a very complex process. Initially, fog or ice nuclei are formed in the gas phase (92, 116). These nuclei probably grow by diffusion of water vapor and by collisional processes. The nucleated particles are quite small and most likely follow the motion of the fluctuating eddies in the turbulent gas stream. Some particles deposit on the cold wall; others are apparently swept away from the cold surface and sublime. The initial deposit is an irregular patchy frost and could be resulted from the formation of a few micro-islands of frost which acted as traps for additional ice particles. A good discussion of phenomenon of particle transfer from the gas phase is given by Weber (114). During this initial phase, temperatures in the frost are probably not significantly different from the cold surface temperature. However, as the clumps of frost grow and the space between these clumps is filled in, a layer of frost, which now has significant insulative effect, builds up. This type of deposition is usually of short duration, and the time depends upon the conditions of the system variables such as Reynolds number, humidity and gas temperature (14). Thus, after a short while, a smooth layer of frost insulates the cold surface and, in effect, imposes a thermal resistance which
results in a sufficient increase in the frost surface
temperature so that diffusional mechanisms of frost
formation predominates.

Since the formation of fog in the gas boundary layer
is usually of short duration for the frost deposition by
forced convection, these mechanisms are neglected in the
theoretical study of the present research.

I-C Previous Studies

The key original ideas for the development of those
theoretical models studied in the present research evolved
from the logical explanation of the experimentally observed
phenomena by previous investigators. Hence, in this section
a brief survey of the important previous studies in the
present research area is made.

A. NATURAL CONVECTION:

a. Frosting with Small Temperature Differences:

Experimental data for natural convection on horizontal
cylinders with a small temperature difference between the
gas and the cold surface (18, 41, 102) show that the total
heat flux through the frost at first increased, then decreased
to a steady value, smaller than the initial value. This
increase of about 20% is believed to be caused by the
increased roughness of the frost deposit. Alternatively,
the increase may be viewed as due to the fin effect of the
individual frost elements. The true reason, however, is
not known as yet.
Frost formation in natural convection over a vertical plate two feet high has been investigated by Whitehurst (115, 117). His measurements showed that the total heat flux for the plate decreased fairly rapidly at first and then reached a steady value. Higher gas humidities gave a higher initial heat flux and a lower steady state heat flux than the lower humidity. The initial heat flux value was found to be higher at lower plate temperatures but the steady state value of heat flux was found to be independent of plate temperature.

Whitehurst (115, 117) as well as Stoecker (109) found that the frost thickness increased even after the asymptotic heat flux had been reached. Since the thermal resistance of the frost layer must remain constant after the steady state heat flux has been reached, they concluded that the frost thermal conductivity increased to compensate for the increased thickness. Schropp (102) found that the average frost conductivity increased right from the beginning of frost accumulation. He found that both frost density and thermal conductivity increased as frosting proceeded. This simultaneous increase in frost density and thermal conductivity agrees with the work of other investigators who have found that frost or snow thermal conductivity increases with density (25, 38, 120). Schropp (102) and Stoecker (109) found that for natural convection with small temperature differences frost density values ranged from
about 0.091 gms./cc. to nearly 0.917 gms./cc. depending upon the system conditions.

b. **Frosting with Large Temperature Differences:**

Natural convection frosting with temperature differences greater than 100°F have been studied for various geometries such as vertical cylinders (92), vertical plates (117) and spheres (52). In these studies, a constant heat flux through the frost was reached after a short time of frosting. The rate of frost accumulation on the cold surface, at first increased slightly, then decreased and levelled off, while the density increased from a very small value to within 5 per cent of that of solid ice. The mass of frost adhering to the cold surface was only 10 to 30 per cent of that predicted by the Lewis relation or the Chilton and Colburn analogy. Ruccia and Mohr (92) obtained the frosting heat flux correlation based on the data for a cylindrical liquid oxygen tank 18 inches in diameter. This correlation predicts that higher humidities yield lower steady state over-all heat transfer coefficients. The data of Holten (52), however, show no humidity effect. The reason for this discrepancy is not known.

**B. FORCED CONVECTION**

a. **Small Temperature Differences:**

Forced convection has been investigated in a number of geometries. Frosts were deposited inside tubes (23, 51, 90) and on the inner surface of an annulus in a double pipe...
exchanger (59) for counter current flow of coolant and air. Frosts were also formed on a flat plate in a wind tunnel (26) and on the inner surface of an annulus (7) with constant surface temperature. Single cylinders in cross flow (24) as well as staggered (50, 86) and in-line (50) tube banks have been used. In all the cases the total heat flux and the convective heat flux for the frosted surface attained a steady state value lower than the unfrosted flux. For internal frosting (i.e. inside tubes and annular passages) the over-all heat transfer coefficient increased at first and then decreased, as in natural convection.

The effect of frost roughness was investigated by Coles and Ruggieri (26). Their frost layers were formed by diffusion and convection from a high velocity gas stream.

Chen (23) was the first to propose a mechanism for frost growth. He postulated a two step mechanism; the layer first grows towards a stable roughness height and then, as deposition proceeds, it remains at the stable height. The stable roughness height is reached when the rate of mass transfer to the top of the element equals that to the "valleys" between rough height decreases as Reynolds number increases and that, as deposition proceeds, the stable roughness height decreases due to area blockage. The net effect of these two steps is to raise the gas side coefficients (78) and thus the over-all coefficient. However, as frost continues to accumulate, the insulation effect takes over and the heat flux decreases. The pressure drop increases rapidly as
the roughness builds up to its stable value, then increases more slowly as frost accumulation blocks the flow area. Fair agreement was obtained between Chen's data and the prediction of this theory. However, he did not account for the increased temperature difference in the unfrosted portions of the exchanger, and frost thermal conductivity and density were taken as adjustable constants.

Chung and Algren (24), Hardy, et al. (50) and Prins (86) found that frost continued to accumulate after the steady state heat flux had been reached. Rische (90) measured the density of benzene frosts formed inside a tube over a range of Reynolds number. His data show that the density increased with time and approached that of solid benzene for long times and high gas velocities (Reynolds numbers). Higher velocities yielded higher density at any given time. Extrapolation of his density time curve back to zero time indicated that higher velocity formed denser layers initially.

b. Large Temperature Differences:

Very few investigations have been done on the formation of the frost on liquid nitrogen cooled surfaces in forced convection. Richards et al. (89) studied the frost deposited outside of the liquid nitrogen cooled cylinder. Liquid nitrogen was continually circulated through the cylinder by natural convection due to the heat flow to the cylinder. The vapor was separated from the liquid and measured to determine the total heat flux. In these tests the heat
flux attained a steady state value which was a function of the velocity and humidity of the air stream. For water contents greater than about one percent by volume the steady state heat flux was greater than the initial flux. For humidities about 2 mole per cent water the heat flux fluctuated around the steady value. This fluctuation was believed to be due to the condensation of liquid water on the frost surface. This water soaked into the frost causing it to shrink and pack together forming a layer of ice at the frost surface. Snow-like frost was then deposited on the layer of ice until the surface again reached 32°F and water droplets formed. The cycle was then repeated. The heat flux decreased during frost deposition but increased when the layer froze.

Figure I-C1 shows typical values of the heat flux from Reference 89 for a Reynolds Number of 132,000 (velocity of 80 ft/sec.), gas temperature 100°F and three humidities. No fluctuations appeared for the lowest humidities, but they were very pronounced for the highest water vapor content. Frost layer thermal conductivities also showed fluctuations as shown in Figure I-C2.

Weber (114) studied the heat transfer and the frost accumulation rates for flow through a tube when the tube was cooled externally by liquid nitrogen. He found that as frosting began the frictional pressure drop increased and the overall heat transfer coefficient decreased. After
some time, however, the heat flux reached a constant value even though frost thickness continued to increase. As the inlet humidity increased (at a given flow rate) these changes occurred more rapidly and the steady state overall heat transfer coefficient decreased.

A typical plot of the apparent overall heat transfer coefficient versus time for a given Reynolds number at three different humidities as obtained by Weber (114) is shown in Figure I-C3. The effect of Reynolds number at a given humidity on the apparent overall heat transfer coefficient versus time profile is shown in Figure I-C4. The apparent overall heat transfer coefficient was defined by the following Equation I-C3.

\[
U_a = \frac{Wc_p}{D_o L} \ln \left( \frac{T_1 - T_w}{T_2 - T_w} \right) \quad \text{(I-C3)}
\]

Weber explained his results by saying that during the steady state heat transfer both frost density and thermal conductivity increased when more frost accumulated to yield the constant overall heat transfer coefficients. He postulated that the increase in frost density was caused by the diffusion of water vapor towards the cold wall due to temperature gradient across the frost layer. The increased frost density gave an increased frost thermal conductivity. The steady state heat transfer coefficients were lower for the higher inlet humidities because less time was available
for frost densification by diffusion. Therefore, at the heat transfer steady state the higher humidities gave frosts with lower densities and thus lower thermal conductivities. Weber's postulation for the internal diffusion is the basis for the first two theoretical models studied in the present thesis. It is described in more detail in Section I-D.

Initially Weber (114), and later Brazinsky (14), determined the thermal conductivity of the frost deposited on a liquid nitrogen cooled surface. Frosts were formed on a liquid nitrogen cooled copper plate which was part of the underside of a rectangular duct. Their system was very similar to the one used in the present thesis. Their results showed that the thermal conductivity of the low temperature frosts increased with density. However, for a given frost density the thermal conductivity was lower for the lower frost surface temperature.

Brazinsky (14) also found that frost density increased with time and at any time density increased with the Reynolds number. The steady state value of heat flux was found to be increased with Reynolds number and decreased with increase in humidity. These results are shown in Figure I-C5. In quasi-steady state heat transfer region, the frost surface temperature was found to remain constant and almost independent of system conditions. The values of the frost surface temperatures for typical experimental conditions
are shown in Table I-Cl. Brazinsky (14) also obtained thermal conductivity of frost as a function of density and temperature from the measurement of temperature distribution in the frost interior. A typical temperature profile in the frost interior is shown in Figure I-C6.

C. Heat and Mass Transfer Analogies for Frosting

The applicability of the heat and mass transfer analogies to the process of frost formation has been tested experimentally by Chung and Algren (24) and Coles and Ruggieri (26). Two simple analogies are the Chilton and Colburn analogy and the Lewis analogy. The Chilton-Colburn analogy is

\[ J_H = J_D \quad (I-C4) \]

For small vapor concentration the analogy becomes

\[ \frac{h}{c_p G} \left( Pr \right)^{2/3} = \frac{K_a Q_a}{c_p} \left( Sc \right)^{2/3} \quad (I-C5) \]

The Lewis analogy for small vapor concentration may be written as (104)

\[ \frac{h}{K_a Q_a c_p} = \left( Le \right)^{2/3} \quad (I-C6) \]

Hence, from definition of the \( j \)-factors, the Lewis analogy in terms of \( j \)-factors is

\[ \frac{J_H}{J_D} = \left( Pr/Sc \right)^{2/3} = \left( Le \right)^{2/3} \quad (I-C7) \]

The \( j \)-factors for frosted surface from the experimental data of References 24 and 26 were calculated. The results
are summarized in the following Table I-C2.

<table>
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<th>I-C2</th>
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<td><strong>Verification of Heat and Mass Transfer Analogies</strong></td>
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<td>plate</td>
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<tr>
<td>Reynolds number</td>
<td>sublimation</td>
</tr>
<tr>
<td>$j / j$</td>
<td>0.8-4 $10^6$</td>
</tr>
<tr>
<td>$H / D$</td>
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The data of Table I-C2 are within 12% of the simple Lewis relation (Equation I-C7). The Chilton and Colburn analogy was well verified when the frost surface temperature reached 32°F and liquid water condensed (59).

D. Properties of Cryogenic Frosts

The density of the cryogenic frost has been found to be much lower than those of the frost deposits formed at high temperatures (14). Until now, the variables which significantly affect the frost density are found to be Reynolds number (or gas velocity), humidity and time elapsed since the start of frosting (14).

The thermal conductivity of the frost deposited on a cryogenically cooled surface has been studied by few investigators (6, 14, 52, 114, 115). Holten (52) obtained the average thermal conductivity from the calculated temperature differences across the frost, and measured
values of the heat flux and frost thickness. He observed that average frost layer thermal conductivity was a strong function of the average frost temperature. Barron (6) and Whitehurst (115) also calculated thermal conductivity of the frost from the measurements of heat flux, frost thickness and temperature difference across the frost. Their frosting system was a vertical flat plate. Although, their conductivity values show considerable scatter, the trend of their data showed that the frost thermal conductivity increased with increasing density and temperature. All these investigators have indicated that there is a large temperature gradient existing in the frost interior (10^4 to 10^5 °F/ft.).

The measurements of point thermal conductivity as a function of density and the temperature in the frost interior were first reported by Weber and Brazinsky (114). They measured the temperature distribution in the frost interior along with heat flux, density and thickness measurements. A correlation between point thermal conductivity, density and temperature obtained by Brazinsky (14) is shown in Figure I-C7. Both Weber and Brazinsky concluded from their measurements that point thermal conductivity of the frost increases with density and temperature.

As yet, the microstructure of the cryogenic frosts has not been studied extensively. Weber (114) and Brazinsky (14) correlated their point thermal conductivity data by modelling the frost microstructure. Their models
predicted the variation of thermal conductivity with density and temperature in fair agreement with the data. Brazinsky (14) has also reported some data on the microscopic observations of the frost structure.

I-D Background of this Thesis

The original physical ideas involved in the development of the theoretical models of the present thesis were first suggested by Prof. P.L.T. Brian and Prof. R.C. Reid. These ideas were logically deduced from the Weber's (114) postulation for the explanation of the quasi-steady state heat transfer condition in the frosting process. In this section, they are briefly described.

As described previously, a smooth layer of frost formed on a liquid nitrogen cooled surface imposes a thermal resistance which results in a marked increase in the frost surface temperature. Hence, after a smooth layer of frost is formed, the rate of frost deposition is governed by the diffusional mechanisms in the gas phase thermal boundary layer.

As additional frost deposits upon this cold surface, the rate of heat transfer from the warm gas to the cold surface decreases. However, as described in Section I-C, numerous investigators observed (3, 6, 14, 23, 52, 76, 77, 89, 114, 115) that after an initial transient period (in which heat flux decreases), the heat flux levels off and becomes essentially constant with time.
Weber (114) explained this "steady state" heat transfer rate by postulating that the frost layer upon the cold surface densifies with time. During densification, the frost thermal conductivity increases very substantially, presumably because of greater inter-particle contact. This density increase was visualized as occurring by the diffusion of water vapor within the frost layer. This diffusion is produced by the ice vapor pressure gradient perpendicular to the cold surface. Mathematically, the rate of diffusion within the frost may be written as,

$$J = \frac{D_B c_T}{(1-Y)} \frac{(1- \frac{\rho_f}{\rho_{ice}})}{\gamma} \frac{dY}{dx}$$  \quad (I-D1)$$

where the term $1- \frac{\rho_f}{\rho_{ice}}$ is the void fraction in the frost interior and $\gamma$ is the tortuosity. The partial pressure gradient is produced in the frost when heat is transferred across the layer. Bader et al. (5) have found that a snow layer is saturated with water at local temperature. Assuming this would be also the case for cryogenic frost, the temperature gradient through the frost would produce an ice vapor pressure gradient. Therefore, the concentration gradient in Equation I-D1 can be replaced by

$$\frac{dY}{dx} = \frac{dY^S}{dx} = \frac{dY^S}{dT} \frac{dT}{dx} = -\frac{1}{P} \frac{dP_{vp}}{dT} \frac{dT}{dx}$$  \quad (I-D2)$$

where the superscript $s$ denotes a saturation value.

Therefore, as heat is transferred to the wall, water vapor
diffuses in the same direction, increasing the density of the frost layer.

From Equations I-D1 and I-D2 and using the ideal gas law and the Clausius-Clapeyron equation (80) gives

\[ J_{M_v} = -D_{v} M_{v} c_{T} \frac{(1 - \frac{\varrho}{\varrho_{ice}})}{\tau} \frac{Y^{S}}{1 - Y^{S}} \frac{M_{v} \Delta H}{RT} \frac{dT}{dx} \] (I-D3)

Examination of Equation I-D3 shows that the rate of vapor diffusion within the frost is a function of the local temperature gradient in the frost. Therefore, the amount of water vapor that has diffused and thus the "ageing" of the frost layer depends on the time lapse since the start of deposition, the temperature level within the frost and the temperature gradient (or heat flux).

The above described mathematical explanation for the frost densification is used in the mathematical formulation of the several models for the frosting process studied in the present thesis.

After postulating this mechanism for the frost densification, Weber (114) concluded that this frost densification resulted in an increase in thermal conductivity of the frost layer which just balanced the increasing mass of frost layer as frost continued to deposit and thus stabilized the heat transfer rate at a steady value.

Compensating effects such as this are often observed in nature, but the existence of a steady state heat transfer rate even while frost continues to deposit on a cold surface
is a wide-spread observation common to all investigations of the subject. It seems highly improbable, considering such a diversity of experimental situations, that the increase in frost thermal conductivity due to frost densification would just happen by chance to counterbalance the increasing frost accumulation and keep the heat transfer rate constant.

Brazinsky (14) shed considerable light upon this phenomenon. He conducted frosting heat transfer studies in a wind tunnel with a cryogenically cooled copper plate. Along with the heat transfer rates, he also measured the temperature profiles within the depositing frost layer and thus the thermal conductivity of the frost deposit and the frost surface temperature were monitored throughout the run.

Furthermore the increase of frost layer density with time was measured. These results confirmed the postulate that the frost layer does densify and that this densification does indeed result in an increase in the thermal conductivity of the frost layer which just compensates for the increased mass of frost to keep the heat transfer rate steady. The results also show, as would be expected, that the frost surface temperature is essentially constant during this steady state heat transfer phase. Furthermore, the frost surface temperature is essentially independent of gas phase Reynolds number and the humidity of the bulk
gas stream.

The fact that the frost surface temperature varied only slightly over a wide range of experimental conditions suggested the presence of a high-gain feedback effect tending to keep it constant and confirms the suspicion that the compensating effects which result in a steady state heat transfer rate are by no means compensating simply by chance. The rate of internal diffusion of water vapor within the frost layer is a strong function of frost temperature. The partial gradient which forces the water vapor diffusion can be thought of as the product of a temperature gradient and the slope of the curve of water vapor pressure versus temperature. This latter quantity increases exponentially as the frost temperature increases, and thus the importance of water vapor diffusion in the frost layer is strongly dependent upon temperature level.

As the insulating frost layer builds up on the cold plate, the heat flux decreases and the frost surface temperature rises. Finally, however, the frost surface temperature reaches a value at which internal water vapor diffusion within the frost layer becomes very substantial, and the frost layer begins to densify. As more frost accumulates and the frost layer gets thicker, there is a tendency for the frost layer to insulate the surface even more, causing a further increase in the frost surface temperature. But because of the very high sensitivity
of internal diffusion to the frost surface temperature, a very small increase in the surface temperature results in a sufficiently great increase in the rate of diffusion of water vapor that the thermal conductivity and density of the frost layer are increased to the point where the heat flux falls off only very slightly and the surface temperature rises only very slightly as additional frost is deposited. Thus the frost surface temperature and the heat transfer rate are stabilized by a very high-gain negative feed-back effect of frost surface temperature upon the rate of densification of the frost layer. Since the frost surface temperature will vary but little, the thermal resistance of the frost layer is automatically adjusted to be in a given ratio with the thermal resistance of the gas phase boundary layer. For example, increasing the gas phase Reynolds number will decrease the thermal resistance of the gas phase and thus will force the frost to become denser in order that it too will have a decreased thermal resistance. This is exactly the effect of gas phase Reynolds number upon frost density found by Brazinsky (14).

The concept of the high-gain feed-back effect regulating the frost density leads to the conclusion that the frost density would vary substantially with the bulk gas phase temperature. For example, if the gas phase temperature were increased the rate of heat transfer through the gas
phase boundary layer to the frost could be increased. Therefore, either the frost surface temperature would rise substantially or the thermal resistance of the frost layer would be decreased. The above physical ideas indicate that a rather small increase in frost surface temperature would result and that this would be sufficient to increase greatly the rate of internal water vapor transfer so as to increase the frost density and decrease the thermal resistance of the frost layer sufficiently. Therefore the concept in these physical ideas is that of a frost layer responding to thermal forces upon it and to the high-gain feed-back effect of internal diffusion so that its density and, therefore, its thermal conductivity adjust themselves and keep the frost surface temperature almost invariant.

The physical ideas described above are used to obtain quantitative theoretical models of the frosting process as described in Sections II-A and II-B.

I-E Scope of this Thesis

It is the purpose of this thesis to obtain a theoretical model or models to explain quantitatively the effect of various system variables on the frost properties such as density and thermal conductivity and on the heat and mass transfer rates in the frosting process. The experimental, as well as theoretical studies, will be carried out. The water frost will be deposited by forced convection over a
liquid nitrogen cooled surface. Some specific comments and recommendations will also be made for the application of the results of the present thesis to other system geometries and the formation of other kinds of frost.

Experimental work is undertaken for a system in which humid air is flown through infinitely parallel plates. The frost will be deposited on a liquid nitrogen cooled copper section which will form part of the lower plate. The following significant studies will be carried out.

A. The frost density, thickness, heat flux through the frost and the temperature distribution in the entire frost layer as a function of time will be measured for the various ranges of the system variables. The system variables that will be considered in the present research are Reynolds number, gas temperature and humidity.

B. The density gradient in the frost interior is to be measured for several frosting experiments.

Theoretical models will be developed to correlate all the experimental facts and to understand the mechanisms of the frosting process.

Two other short experimental programs will also be undertaken. The first one is undertaken to verify the validity of the postulated mechanism of internal diffusion as described in Section I-E.

The second one involves the experimental measurements of heat transfer coefficient in non-frosting conditions when the cold surface temperature is kept at about -15°F.
These data will be used to evaluate the theoretical models. The details of these experimental programs are shown in Section III-B and III-D.

I-F Possible Contribution of this Thesis

It is hoped to make the following contributions with this thesis.

I. Obtain a theoretical model for the frosting process which could be used for the quantitative prediction of the frost properties and the heat and mass transfer characteristics of the water frosting process under a given set of environmental conditions.

II. Some qualitative or quantitative, if possible, evidences showing the applicability of the theoretical results of the present thesis to a wide variety of system geometries and to other kinds of frost (e.g. carbon dioxide frost).

III. Discuss microscopic process occurring inside the frost to account for densification.
Table I-C1

Experimentally Measured Surface Temperatures in Quasi-steady State Heat Transfer Region

As a Function of Reynolds Number and Humidity (°F)

<table>
<thead>
<tr>
<th>Reynolds Number</th>
<th>Humidity (dew point °F)</th>
<th>Steady Value of Frost Surface Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14,500</td>
<td>40.0</td>
<td>+24</td>
</tr>
<tr>
<td>9,300</td>
<td>40.1</td>
<td>+25</td>
</tr>
<tr>
<td>5,600</td>
<td>39.9</td>
<td>+20</td>
</tr>
<tr>
<td>9,300</td>
<td>28.7</td>
<td>+12</td>
</tr>
<tr>
<td>5,600</td>
<td>29.1</td>
<td>+12</td>
</tr>
</tbody>
</table>
Figure I-B1 Fog Formation Limits at One Atmosphere
Development of Theoretical Models for the Frosting Process

Several models, of varying sophistication, are examined critically in this section to interpret the complex phenomena of frosting.

II-A Simple Model of the Frosting Process

Postulations and Assumptions of Simple Model

1. After a smooth layer of frost is formed, the transport of heat and mass to the frost surface can be analyzed using ordinary gas phase correlations. That is, no nucleation of water vapor occurs anywhere in the gas phase.

2. Water vapor is transferred from the frost surface to the interior by a diffusional mechanism resulting from a vapor pressure gradient. The rate of transfer is proportional to the gradient at the surface. No consideration is given to the water transport within the frost except to assume that the entire frost layer densifies uniformly...... there is no density gradient present in the frost.

3. Heat is transferred through the frost by a combined mechanism of conduction in the solid and gas phase and by the enthalpy transport of water vapor. This heat transfer in the frost interior is assumed to be one dimensional only.

4. The point thermal conductivity of the frost assumed
to be a function of temperature and frost density. The average frost layer thermal conductivity, on the other hand, assumed to be only a function of average frost density.

Qualitative Description of the Model

As postulated before, for the frost deposition on a liquid nitrogen cooled surface, this model is applied only after a smooth layer of frost is formed. The schematic of the frosting process as visualized by this model is shown in Figure II-A0.

Qualitatively, according to this model, one visualizes the frosting process as one in which water vapor from the gas phase is transferred to the frost surface by a simultaneous heat and mass transfer processes. Once water vapor deposits on the existing frost, part of it resublimes and diffuses in the frost interior and the rest of it increases the frost thickness. Thus both frost density and thickness increase with time. The model assumes that the internal diffusion of water vapor is proportional to the vapor pressure gradient existing at the frost surface and hence it is a very strong function of frost surface temperature.

Although this model postulates the internal diffusion as the cause for frost densification, it is important to note that it ignores the explanation for the mechanism by which diffused water vapor spreads itself in the frost interior, after it leaves the frost surface. Thus it
considers only a differential element at the frost surface for explanation of the effects of various independent system parameters on the frosting process and on the variation of the frost properties with time.

Previous experimental studies (14, 23, 25, 114, 116) have indicated that after an initial short duration of the frosting process, the rate of heat transfer through the frost attains a quasi-steady state, even though the frost continues to accumulate on the cold surface. The simple model explains this phenomenon on the basis of a high gain negative feed-back effect of the frost surface temperature on the internal diffusion.

In the quasi-steady state heat transfer condition, as frost accumulates and as frost layer gets thicker, there is a tendency for the frost layer to insulate the surface, causing an increase in the frost surface temperature. But, because of the very high sensitivity of internal diffusion to the frost surface temperature, a very small increase in the surface temperature results in a sufficiently great increase in the rate of diffusion of water vapor that the thermal conductivity and density of the frost layer are increased to the point where the heat flux falls off only very slightly as additional frost is deposited. Thus this high gain negative feed back effect of frost surface temperature on internal diffusion causes the frost surface temperature and hence the heat transfer through the frost to remain almost constant. One can also deduce from the
above discussion that the model predicts that frost responses to the thermal forces acting upon it, because at constant frost surface temperature, the thermal resistance of the frost is adjusted to be in a given ratio with the thermal resistance of the gas phase boundary layer.

The basic mathematical equations of this model predict the variation of frost density, thickness, mass deposition and heat transfer rate through the frost with them. The knowledge of the independent system variables such as Reynolds number, humidity, gas temperature and cold wall temperature are required to obtain these predictions.

For a well defined system (i.e., one in which heat and mass transfer coefficients can be characterized), if one knows thermal conductivity of frost as a function of density and temperature (note that this can be obtained experimentally or can be obtained from a frost structure model as shown in Appendix F), one can predict the frost surface temperature from the mathematical formulation of the model. Knowledge of frost surface temperature then allows one to predict the variation in frost density, thickness, rate of frost formation and heat transfer through the frost as it deposits on the cold surface.

Mathematical Formulation of Simple Model

The average frost density, thickness, surface temperature of the frost, and heat transfer through the frost are obtained from heat and mass balance at the frost surface.
The principal equations resulting from the heat and mass balances at the frost surface can be obtained as follows.

**Heat Transfer from the Gas to Frost Surface**

Heat is transferred from the gas to the frost both by convection and by material transport of the condensible component. This rate of heat transfer may be written as,

$$q = h(T_g - T_s) + N_s \Delta H + N_s c_p(T_g - T_s)$$  \hspace{1cm} (II-A1)

The sensible cooling of the condensible component in Equation II-A1 is usually small and can be safely neglected. Equation II-A1, hence, can be rewritten as,

$$q = h(T_g - T_s) + N_s \Delta H$$ \hspace{1cm} (II-A2)

At the frost surface, there is energy transfer to the frost layer by two mechanisms. First there is the usual conduction process. Secondly, there is an internal mass flux caused by the temperature gradient in the frost layer. Both are proportional to the surface temperature gradient. Thus, one may write,

$$q = k_{zs} \left( \frac{dT}{dx} \right)_s = (k_{f1} + k_{\text{excess}}) \left( \frac{dT}{dx} \right)_s$$ \hspace{1cm} (II-A3)

where $k_{zs}$ is an effective conductivity including both the intrinsic conductivity, $k_{f1}$, and the excess conductivity due to evaporative mass transport, $k_{\text{excess}}$, is evaluated at the frost surface temperature and density. The detailed description of $k_{\text{excess}}$ is given in Appendix F.

**Mass Balance over Frost Layer**

The total mass of frost at some specified time is,
The frost thickness is time dependent. The variation of \( m_f \) with time is obtained by differentiating Equation II-A4, i.e.,

\[
N_s = \frac{d m_f}{d \theta} = \frac{d}{d \theta} \left( \int_0^\delta \varphi_f \, dx \right) = \int_0^\delta \frac{d \varphi_f}{d \theta} \, dx + \varphi_f, s \frac{d \delta}{d \theta}
\]  

(II-A5)

Also the mass flux, \( N_s \), can be expressed by a gas phase mass transfer coefficient and partial pressure driving force as,

\[
N_s = K_g \left( p_g - p_s \right)
\]  

(II-A6)

where \( K_g \) is presumable determined by the gas phase fluid dynamics, \( p_g \) by the concentration of condensible component in the gas phase and \( p_s \) by the frost surface temperature.

Returning to Equation II-A5, the rate of increase in frost mass has been broken down into two parts. The first represents the rate of densification of the existing layer while the second allows for an increase in frost thickness with time. The densification results from internal diffusion of the condensible as there is a temperature gradient within the frost layer. The term

\[
\frac{d P_f}{d \theta} = \frac{d N_s}{d x} = N_{s_i}
\]  

(II-A7)

is equal to the mass flux from the frost surface, \( N_{s_i} \). It should be noted that in Equation II-A7, \( \frac{d N_s}{d x} \) is not a function of \( x \).

\( N_{s_i} \) is assumed to be limited by ordinary diffusional
processes.

\[ N_{si} = -M_v D'_{\text{eff}} \left( \frac{dP_{vp}}{dx} \right) \]  

(II-A8)

\[ D'_{\text{eff}} \text{ is related to the usual diffusion coefficient as}, \]

\[ D'_{\text{eff}} = \frac{D_0 \varepsilon}{\mathcal{L}} \left( \frac{P}{P - P_{vp}} \right) \left( \frac{1}{RT} \right) \]  

(II-A9)

in which \( \mathcal{L} \) is the tortuosity and \( \varepsilon \) is the void fraction

i.e., \( \varepsilon = (1 - \frac{\rho_f}{\rho_{ice}}) \)

The vapor pressure gradient can also be written as

\[ \left( \frac{dP_{vp}}{dx} \right) = \left( \frac{dP_{vp}}{dT} \right) \left( \frac{dT}{dx} \right) \]  

(II-A10)

Using Clausius-Clapeyron equation (80) one can write,

\[ \frac{dP_{vp}}{dx} = \left( \frac{M_v \Delta H P_{vp}}{RT^2} \right) \left( \frac{dT}{dx} \right) \]  

(II-A11)

Combining Equations II-A9 and II-A11, and assuming \( P_{vp} \) to be small so that \( (P - P_{vp}) \) is approximately equal to the system pressure \( P \), then Equation II-A9 becomes

\[ N_{si} = -\beta(T_s) \left( \frac{\varepsilon}{\tau} \right) \left( \frac{dT}{dx} \right) \]  

(II-A12)

where,

\[ \beta(T_s) = D_0 M_v \Delta H P_{vp} \rho_{T_s}^2 \]  

(II-A13)

Hence,

\[ \frac{\partial q_f}{\partial \theta} = -\beta(T_s) \left( 1 - \frac{\rho_f}{\rho_{ice}} \right) \left( \frac{dT}{dx} \right) \]  

(II-A14)

The function \( \beta(T_s) \) has the units of mass/length-time

temperature and is a strong function of \( T_s \).

From Equations II-A5, II-A6, II-A8 and II-A11

\[ N_s = K_q (p_g - p_s) = q_f \frac{d\theta}{d\theta} - \beta(T_s) \left( \frac{\varepsilon}{\tau} \right) \left( \frac{dT}{dx} \right) \]  

(II-A15)

and heat balance equation can be written as

\[ q = h(T_g - T_s) + K_g (p_g - p_s) \cdot \Delta H = R_{f,s} \left( \frac{dT}{dx} \right) \]  

\[ = K_{f,x} \left( \frac{dT}{dx} \right)_{f,x} = \frac{\int_{r} k_f d\tau}{\delta} \]  

(II-A16)
Computer Solution of Mathematical Equations

The equations derived above were solved by numerical methods. A computer program was written to obtain frost density, thickness, and heat transfer through the frost versus time profiles from numerical solution of the equations. The functional relation \( k_f = f(\varrho_f, T) \) was obtained by an empirical correlation of the experimental data of Brazinsky (14). The details of the computer program is described in Appendix D.

Computer Results

The computer solution was used to study the following aspects of the simple model.

1. Comparison between the predictions of the model and experimental results for time variation of the frost density and thickness as well as the rate of heat transfer through the frost.

2. The effect of the initial values (of frost density and thickness) on the model's predictions for the frost density and thickness as a function of time.

3. Predictions of the model for the effect of step changes in the environmental conditions on the frost density, thickness and heat transfer through the frost.

Figures II-Al to IIA10 compare the predictions of the simple model for the frost density, thickness, heat transfer through the frost and mass deposition of the frost versus time profiles for ten different experimental conditions studied in the present research. In all the
cases, the first experimental data points of the frost density and thickness were taken as the starting values for the theoretical predictions. The heat transfer coefficients used in the model were obtained from the Sleicher and Tribus's (105) theoretical heat transfer analysis (see Appendix L) of the present system under non frosting conditions.

The results of the Figures II-A1 to II-A10 show surprisingly good agreement between the experimental facts and the predictions of the model. The maximum deviation between the theoretical and experimental frost density values was about 8% and that for the frost thickness values was approximately within 10%. The mean average deviations for all the results was less than 3% both for frost density and thickness values. The slightly poor agreement between the experimental and theoretical heat fluxes in some of the cases (Figure II-A8, II-A9, and II-A10) was due to the difference in the values of the theoretical and experimental heat transfer coefficients.

Figures II-A11 and II-A12 show the comparison between the predictions of the simple model and the experimental data of Brazinsky (14). Although the starting values were once again chosen to be the first experimental data points of the frost density and thickness, the theoretical calculations were carried out using the experimentally determined heat transfer coefficients. It can be seen from these figures that the agreement between the predictions
of the model and the experimental facts was quite good. In order to obtain frost density and thickness versus time profiles from the solution of the mathematical equations of the simple model, initial frost density and thickness values have to be known. In most real situations, these values at any arbitrary time may not be known. In order to increase the range of application of the theoretical model, it is important to study the effect of the starting values on the predictions of the model for the time variation of the frost density and thickness.

Figure II-A13 shows the theoretical results (for a set of experimental conditions) of the time variation of the frost density and thickness obtained from the various assumed starting values. Two different starting values of the thickness (0.0015 inch and 0.015 inch) and three different starting frost density values (0.026 gms./cc., 0.035 gms./cc., 0.046 gms./cc.) were examined. The starting time was assumed to be one minute (from the start of frosting) for the lower initial thickness and two minutes for the higher initial thickness. The results indicate that the time variation of the frost density in the quasi-steady state region is slightly dependent on the initial values. The time variation of the frost thickness in the quasi-steady state heat transfer region is, however, almost independent of the initial values. It should be noted, that the theoretical predictions for the comparison of the Figures II-A1 to II-A12
were carried out with the first experimental data points of the frost density and thickness as the starting values because erroneously assumed initial values will introduce additional deviation between the experimental facts and the theoretical predictions.

It can also be observed from the results of the Figures II-A13 and II-A14 that assumed initial values of the frost density and thickness affect the frost densification rate only in the transient heat transfer state.

As postulated before, the simple model can be applied to the process of frost formation on a liquid nitrogen cooled surface only after a smooth layer of frost is formed (i.e. when no fogging occurs). It has been experimentally found in the present research as well as in the investigation of Brazinsky (14) that time for the formation of a smooth layer of frost is a strong function of Reynolds' number (i.e. heat and mass transfer coefficients) and a weak function of gas temperature and humidity. Table II-A1 shows the experimentally observed relationship between the time for the formation of a smooth layer of frost and the heat transfer coefficient.

The results of Figures II-A13 and II-A14 and of Table II-A1 indicate that if the initial values for the frost density and thickness are taken at the time for the formation of a smooth layer of frost, any reasonable values of the frost density and thickness (see discussion of the results in this section) will be
sufficient for the good predictions of the time variation of the frost density and thickness in a quasi-steady state region.

Since in the real frosting process the chances for a step change in the environmental conditions are quite high, the predictions of the simple model for the effects of the step changes in the environmental conditions on the frosting process and frost properties are of practical significance.

For a set of experimental conditions (Re=5,600, humidity = dew point at 29.2°F, $T_e=530^\circ R$ and $T_w=144^\circ R$), at time of seventy five minutes from the start of frosting, when the cold wall temperature increased from about $144^\circ R$ to $350^\circ R$ (keeping all other system conditions unchanged), the model predicted that the frost surface temperature would increase from about 9°F to about 32°F. Hence, it predicted erroneous results for the time variation of the frost density and the thickness at $T_w=350^\circ R$. The results of this test is shown in Table II-A2.

One can deduce from the above-mentioned results that in general, the model predicts increase in frost surface temperature with the increase in the cold wall temperature. The frost densification rate will be increased and the heat and mass fluxes and the time variation of the frost thickness will be decreased by this increase in the frost surface temperature.

Figure II-A15 shows the model's predictions for a
step change in Reynolds number on the frost properties and heat transfer through the frost. The step variation in Reynolds number was carried out at eighty five minutes from the start of frosting (all other system conditions were kept unchanged). The model predicted that the step increase in Reynolds number (from 8,000 to 15,800), would cause faster rate of change of density, slower rate of change of thickness and faster rate of mass deposition with time. The step increase in Reynolds number would cause the heat flux and the frost surface temperature to be increased initially sharply, but heat flux levels out, later, at another (but higher) and the frost surface temperature at another (but lower) quasi-steady state values. The step decrease in Reynolds number (from 8,000 to 5,600) would have an opposite effect on the frost properties, surface temperature and the heat transfer through the frost.

The predictions of the model for the effects of a step change of gas temperature on the frost properties and heat transfer through the frost are described in Figure II-A16. The step increase (from 75°F to 110°F) and step decrease (from 75°F to 30°F) in gas temperature (at all other system conditions unchanged) would have the similar effects of the frost density, thickness, surface temperature and heat transfer through the frost as those obtained for the step variation in Reynolds number.

Figure II-A17 shows the predictions of the simple
model regarding the effects of a step change in humidity on the frost properties and heat transfer through the frost (all other system conditions were kept unchanged). The step variation in humidity (from dew point of 33.2°F to dew point of 40.3°F and to dew point of 14.3°F) will change the rate of frost densification only slightly but significantly change the frost surface temperature and the time variation of frost thickness. Both frost thickness and the surface temperature will increase with a step increase in humidity. The heat transfer through the frost will be only slightly affected by a step variation in humidity.

Discussion of Computer Results

The results of Figures II-A1 to II-A12 reveals the surprising success of the simple model in representing the complex frosting process. Even though the simple model ignores the explanation for a mechanism by which water vapor penetrates the interior of the frost, its significant contribution lies in a simple quantitative explanation for the time variation of the frost density, thickness and heat transfer through the frost for the wide variety of the system conditions.

In general, all the experimental data on the frost density, thickness and heat transfer through the frost agree well with the predictions of the simple model. The large deviation between the experimental and the predicted values for the heat transfer through the frost for
certain experimental conditions (Figure II-A8, II-A10) are mainly due to the large difference in the values of the experimental and the theoretical heat transfer coefficients. Some deviation between the experimental and the theoretical values of the frost density in Figure II-A7 and of the frost thickness in Figure II-A9 can be accounted by the difference in the theoretical and experimental values of the mass deposition at a later time.

Qualitative explanation for the occurrences of the experimental facts and the effects of various independent system parameters such as Reynolds number, humidity and gas temperature on the frost properties and the frosting process predicted by the model can be given as follows.

Physically, the process of frost formation essentially involves the transfer of heat and mass of a condensible component from the gas to the cold surface. This occurs because as soon as warm gas comes into contact with the cold surface, temperature and the concentration gradients are established in the gas boundary layer. Thus, the condensible component is transported towards the cold surface by the concentration driving force, and similarly heat is transferred from gas to the cold surface by the temperature driving force. The intrinsic constants which control the rate of mass and heat transfer at the given driving forces are, of course, the usual mass and heat transfer coefficients respectively.

If the cold surface temperature is very low (for
example, liquid nitrogen temperature), then as seen previously (see Section I), the above described mechanisms control the frosting process only after a smooth layer of frost is just formed on the cold surface.

It should be noted that the various independent system parameters involve in the frosting process should affect either the various driving forces or the intrinsic constants described previously. In fact, bulk gas humidity and temperature will affect the driving forces for the mass and heat transfer (i.e., $\Delta p = p_g - p_s$ and $\Delta T = T_g - T_s$ ) respectively and Reynolds number will affect the intrinsic constants for the heat and mass transfer (i.e., $h$ and $K_g$).

If Reynolds number (or gas velocity) is increased, the gas phase turbulence will be increased. This will increase the thermal and mass momentum in the gas particles and as a result the heat and mass transfer coefficients will be increased. The physics of the frosting process indicate that an increase in Reynolds number (at constant gas temperature and humidity) will increase the heat and mass transfer rates due to increase in heat and mass transfer coefficients. Also, a higher thermal momentum will cause the ice particles to pack more tightly so that the frost of a higher density and hence of a higher thermal conductivity will be formed.
Thus, the key concepts of the simple model that the frost responses to the thermal forces acting upon it and that the effect of Reynolds number is to increase the thermal and mass momentum are in agreement with the physics of the frosting process. The results shown in the Figures II-A, II-A7, and II-A10 substantiate the above discussion quantitatively with the help of experimental facts and the theoretical predictions.

It should be noted that in several investigations (89, 114), in which the fog particles were formed in the gas phase, the density variation with Reynolds number was explained by saying that since the fluctuating velocity component normal to the main gas flow increases with Reynolds number, the impact velocity of the fog particles with the frost layer is greater at the higher Reynolds number. Greater impact velocities cause the particles to pack more tightly and hence a frost layer of higher density results.

The above explanation is erroneous for three reasons. First, the statement that higher particle impact velocities lead to more tightly packed arrays has not been proven so far. Secondly, the explanation for the frost density behavior based on the particle impact velocities is not applicable for the cases where fog particles do not form in the gas phase (i.e. frost formation at high temperatures). Lastly, it does not explain the experimental facts regarding the effects of Reynolds number on the heat and mass transfer.
processes during the frost deposition.

The increase in humidity will, of course, increase the mass transfer rate from gas to the cold surface due to an increase in the concentration driving force ($\Delta p = p_g - p_s$). Since the thermal forces imposed on the frost are not changed by varying the humidity, the density of the frost will not be significantly affected by the variation in humidity. Frost thickness, however, will increase significantly with increase in humidity because of increase in mass flux. Thicker frost at a higher humidity will insulate the cold surface better. Hence, the frost surface temperature will be a little higher and the quasi-steady state heat flux from gas to the frost surface will be lower at a higher humidity. All these effects of the variation in humidity, as dictated from the physics of the frosting process, are in agreement with the experimental facts and the theoretical predictions of the Figures II-A2, II-A8 and II-A1,II-A3.

The increase in gas temperature will increase the driving force for the heat transfer ($\Delta T = T_g - T_s$). This will, of course increase the heat flux from gas to the frost surface. In order to achieve the thermal balance on the system, increase in the gas temperature should also increase the heat flux through the frost. This can occur either by increase in the frost surface temperature or by increase in the frost density. Increase in the frost surface temperature, however, will decrease the mass
flux. Hence, the tendency of the frost layer will be to decrease its thermal resistance such that the thermal balance is regained. The experimental facts and the theoretical predictions of the Figures II-A5, II-A6 and II-A7, II-A9 are in agreement with these physical expectations.

The initial values of the frost density and thickness for the theoretical calculations of the Figures II-A1 to II-A12 were taken to be the first experimental data points. In order to expand the range of application of the model and to eliminate the need for the experimental data points for the theoretical calculations, a study was made to determine a judicious choice of the initial values for a wide variety of the system conditions. The important results of this study are shown in Figures II-A13, II-A14 and in Table II-A1.

There are three parameters to be fixed at the initial conditions: frost density, thickness and the starting time. Since the model applies only after a smooth layer of frost is formed, the time for formation of a smooth layer of frost should be taken as the starting time. The approximate value of this for given system conditions can be obtained from the data of Table II-A1.

It has been found experimentally in the present research as well as in the study of Brazinsky (14) that the density of the frost clumps formed in the initial phase of the frosting process at the low temperature lies between 0.02 gms./cc. to 0.03 gms./cc. for all experimental
conditions. Hence, 0.025 gms/cc. would be a reasonable value for the frost density at the time when smooth layer of frost is just formed.

The results of the Figure II-A13 indicate that the time variation of the frost thickness in the quasi-steady state heat transfer region is very weakly dependent upon its initial values. However, since thickness is a continuously increasing function of time, the initial value of the frost thickness should be lower than a true value at a little later time. A very approximate but reasonable guess for the frost thickness value at the time when a smooth layer of frost is just formed can be obtained as follows.

A very approximate value for the mass deposition on the cold surface at the time for the formation of a smooth thin frost layer can be given by the following Equation II-A17.

\[ \phi_f \cdot \delta = \int_{0}^{t} K_{g_i} (p_g - p_s) d\theta \]  

(II-A17)

Neglecting \( p_s \) (since the frost surface temperature is extremely low during this time), one can write

\[ \delta = \frac{K_{g_i} p_g t}{\rho_f} \]  

(II-A18)

In Equation II-A18, \( K_{g_i} \) and \( p_g \) are the known experimental variables. An approximate value of 't' can be obtained from the data of Table II-A1. Finally, \( \rho_f \) can be obtained from the following experimentally measured correlation between average frost density and the heat transfer coefficient.
Thus, an approximate value of the frost thickness at the time when a smooth layer of frost is just formed can be calculated from Equation II-A18.

Just to demonstrate the usefulness of the above-mentioned procedure, theoretical predictions for the system conditions of the Figure II-A1 will be obtained with the starting values dictated by it. Thus, the starting time will be 12.5 minutes (from the data of Table II-A1) and the initial frost density will be 0.025 gms./cc. The initial value of the frost thickness can be calculated using Equation (II-A18) as follows.

\[ \phi_t = 0.025 \left(\frac{h}{2.5}\right)^{-2} \]  

(II-A19)

\[ K_{eq} = 0.016 \text{ lb}_m/\text{hr ft}^2 \text{ mm.Hg.} \]
\[ p_g = 6.34 \text{ mm.Hg} \]
\[ t = 12.5 \text{ minutes (from Table II-A1)} \]
\[ \phi_t = 0.090 \text{ gms./cc.} \]
\[ \delta = \frac{0.016 \times 6.34 \times 12 \times 12.5}{0.09 \times 62.5 \times 60} \]
\[ = 0.044 \text{ inches} \]

The theoretical predictions for the time variation of the frost density and thickness with the starting values of \( \phi_t = 0.025 \text{ gms./cc.} \) and \( \delta = 0.044 \text{ inches} \) at the time of 12.5 minutes, are shown in the Figure II-A13. It can be seen that the theoretical predictions with these starting values agree well with the experimental results.

There are two other attempts made to obtain a judicious choice for the initial values at an arbitrary time.
In the first attempt, available frost density and thickness versus time profiles in the quasi-steady state heat transfer region are extrapolated backwards by using the computer solution with negative time increment. The results of this attempt is shown in Table II-A2. The results show that this method for obtaining the frost density and thickness at an earlier time is not successful because of the erroneous calculation of the frost surface temperature during backward extrapolation. The frost surface temperature is found to be increased as time is moved backwards.

It appears, from the results, that a slight error in the calculation of the frost surface temperature (by Newton Raphson's method) at the starting time is propagated and expanded at a subsequent time. If, however, the transcendental equation for the surface temperature is solved by an analytical method, it will be possible to avoid this error in the calculation of the frost surface temperature. Unfortunately, a closed form analytical solution of the transcendental equation is not possible because it contains fifth and the higher order terms in unknown $T_s$ (according to Abbel's law). Further efforts to obtain a correct solution for the frost surface temperature are not carried out.

In the second attempt to obtain a judicious choice for the initial values, a correlation between the frost density, thickness and the system variables is obtained.
The correlation is obtained forty minutes from the start of frosting. The experimental data of the present research are used for this purpose. The following results are obtained.

\[ \Phi_f = 1.930 \times 10^{-3} h^{0.933} T_g^{0.2062} P_g^{0.1511} \quad (II-A20) \]

and,

\[ \delta = 6.21 \times 10^5 h^{-0.1231} T_g^{-2.878} P_g^{0.779} \quad (II-A21) \]

All the nomenclatures of Equations II-A20 and II-A21 are described in Appendix R.

The details of the correlation procedure is described in Appendix G.

In the real frosting process, constant environmental conditions are rare. Hence the effect of a step change in environmental conditions on the frosting process is of considerable practical significance. The predictions of the model in this regard are described in Figures II-A15 to II-A17. The results are in qualitative agreement with the physics of the frosting process.

If the cold wall temperature is suddenly increased then \( \Delta T \) across the frost is decreased and the heat flux through the frost will be decreased. In order to achieve a thermal balance on the system, heat flux from the gas to the frost surface should also go down and hence the frost surface temperature will increase up to the point that the heat transfer through the frost is equalled to
that from gas to the frost surface. Thus physics of the frosting process dictates the increase in the frost surface temperature with a step increase in the cold wall temperature.

Figure II-A15 shows that the step increase in Reynolds number will cause an initial sharp increase in the heat flux through the frost followed by the frost growth under the transient heat transfer situation and finally attainment of a new quasi-steady state heat transfer condition. The results of the Figure II-A15 also indicate that the decrease in Reynolds number will have the converse effects on the frost properties. Since, physics of the frosting process indicates that the frost will respond to the thermal forces acting upon it, the behavior of the frosting process under a step change in the Reynolds number will be in qualitative agreement with the results of Figure II-A15.

A step variation in the gas temperature also changes the thermal force acting upon the frost. Hence, it will have a similar effect on the heat flux and frost thermal resistance as obtained for a step change in Reynolds number. The mass flux will also be affected by a step variation in the gas temperature because of the variation in the frost surface temperature. These deductions which are based on the physics of the frosting process are in qualitative agreement with the results of Figure II-A16.
It has been found experimentally that a step variation in humidity affects the frost thickness much more significantly than the frost density and the heat transfer through the frost. It is expected that a step variation in humidity will also have a similar effect on the frosting process. The results of Figure II-A17 are in qualitative agreement with these physical expectations.

Conclusions

As a result of the theoretical analysis of the simple model, the following conclusions can be made.

1. The model predicts frost density, thickness and heat transfer through the frost versus time profiles which are in agreement with the experimental facts for a wide range of experimental conditions.

2. The theoretical calculations for time variation of the frost density and thickness can be started at the time when a smooth layer of frost is just formed. The frost density of 0.025 gms./cc. and the thickness calculated from the Equation II-A18 can be used as the initial values for the calculations. The initial values calculated in this manner can be used only when the frost is deposited by forced convection on a cryogenically cooled surface.

3. The values of the frost density and thickness at forty minutes from the start of frosting on a liquid nitrogen cooled surface by forced convection can be obtained from the Equations II-A20 and II-A21.
4. The model predicts the effects of a step variation in environmental conditions on frost properties in agreement with the physics of the frosting process.

II-B Speculations of Processes Occurring within the Frost

As seen in Section II-A, the simple model correlates the effects of various independent system parameters on the frost properties and the frosting process. This correlation is obtained by considering the energy and mass balances on a differential element at the frost surface. The simple model, however, ignores the explanation for the mechanisms by which water vapor spreads itself in the frost interior once it leaves the frost surface. In order to understand the mechanism by which every layer of the frost interior densifies with time, an 'improved model' (or 'sophisticated model') of the frosting process was made. This section describes in detail this first attempt to understand the microscopic phenomena in the frost interior.

The basic postulations and the assumptions of the new model considered here are as follows:

1. The partial pressure of water vapor at every point in the frost interior is assumed to be the equilibrium vapor pressure at existing temperatures.

2. Every layer of the frost interior densifies with time because of the water vapor transport in the frost interior. The vapor pressure gradient due to temperature gradient causes this transport of water vapor.
3. The thermal conductivity of the frost can be represented as a function of the frost density, temperature and frost structure.

4. Heat conduction in the frost interior is one dimensional only.

5. On the gas side of the gas-frost interface, the heat and mass transfer characteristics of the boundary layer control the frosting process.

Qualitative Description of the "Sophisticated" Model

Like the "simple model" this model is applied only after a smooth layer of frost is formed. The schematic of the frosting process as visualized by this model is shown in Figure II-B0.

Qualitatively, this model is based on the same physical ideas as those used in the simple model. However, some of the physical ideas are extended to explain the microscopic occurrences in the frost interior.

It was assumed in the simple model that water vapor diffuses in the frost interior by vapor pressure gradient existing at the frost surface. The "sophisticated" model extends this physical idea to explain the processes occurring in the frost interior. It assumes that at every layer in the frost interior the bulk gas is in equilibrium with the ice at the prevailing temperature and thus there is a vapor pressure gradient exist at every point. During a frosting process, each layer of the frost interior densifies under the prevailing equilibrium conditions.
The mathematical formulation of this model is obtained by writing the heat and mass balances on a differential element of the frost. The frost surface conditions, which were the basis for the complete mathematical formulation of the 'simple model', are taken as only one set of boundary conditions. The other set of boundary conditions is taken at the cold wall. This model describes the development of the density distribution within the frost on the basis of the mechanism of water transport in the frost interior postulated above. Thus, it attempts to seek an explanation for the mechanism by which water vapor can diffuse through every layer of the frost interior. The principal mathematical equations of this model can be obtained as follows.

**Mass Balance**

Mass balance on a differential element of the frost can be written as

\[
A J \bigg|_{x+\Delta x} - A J \bigg|_x = \Delta x \frac{\partial \rho_f}{\partial \theta} \tag{II-B1}
\]

The mass flux, \( J \), by the vapor pressure gradient can be written as

\[
J = \frac{D_{\text{eff}}}{RT} \left( \frac{1}{1-P_{vp}/P} \right) \frac{\partial P_{vp}}{\partial x} \tag{II-B2}
\]

where,

\[
D_{\text{eff}} = \frac{D_B (1-\eta_f/\eta_{ice})}{\tau}
\]

All the nomenclatures of Equations II-B2 and II-B3 are described in Appendix R.

Substituting Equations II-B2 and II-B3 into II-B1, one
obtains,
\[
\frac{\frac{D_B (1 - \phi_f/\phi_{ice})}{RT \tau (1 - P_{rP}/P)}}{\Delta x} \left[ \frac{\partial \rho_f}{\partial T} \right]_{x+\Delta x} - \frac{\frac{D_B (1 - \phi_f/\phi_{ice})}{RT \tau (1 - P_{rP}/P)}}{\Delta x} \left[ \frac{\partial \rho_f}{\partial T} \right]_x = \frac{\partial \rho_f}{\partial \theta} \tag{II-B4}
\]

or,
\[
\frac{\partial}{\partial x} \left[ \frac{D_B (1 - \phi_f/\phi_{ice})}{RT \tau (1 - P_{rP}/P)} \left( \frac{\partial \rho_f}{\partial x} \right) \right] = \frac{\partial \rho_f}{\partial \theta} \tag{II-B5}
\]

Also,
\[
\frac{\partial \rho_f}{\partial x} = \left( \frac{\partial \rho_f}{\partial T} \right) \left( \frac{\partial T}{\partial x} \right) \tag{II-B6}
\]

Using Clausius-Clapeyron equation (80), one can write
\[
\frac{\partial \rho_f}{\partial x} = \left( \frac{M \Delta H_{vp}}{RT^2} \right) \left( \frac{\partial T}{\partial x} \right) \tag{II-B7}
\]

Combining Equations II-B5 and II-B7, one can write
\[
\frac{\partial}{\partial x} \left[ \frac{D_B (1 - \phi_f/\phi_{ice})}{RT \tau (1 - P_{rP}/P)} \left( \frac{M \Delta H_{vp}}{RT^2} \right) \left( \frac{\partial T}{\partial x} \right) \right] = \frac{\partial \rho_f}{\partial \theta} \tag{II-B8}
\]

Heat Balance

One dimensional steady state heat conduction in the frost interior can be written as,
\[
q = k_f \frac{dT}{dx} \tag{II-B9}
\]

where,
\[
k_f = k_f + k_{\text{excess}} \tag{II-B10}
\]

The excess thermal conductivity, \( k_{\text{excess}} \), can be represented as (see Appendix F)
\[
k_{\text{excess}} = \frac{D_B \Delta H^2 p_{VP} M^2}{R^2 T^3} \tag{II-B11}
\]

Physically, \( k_{\text{excess}} \) is the additional component of
the conductivity caused by evaporative mass transport in the frost interior.

Boundary Conditions

1. At Cold Wall

The schematic of the boundary conditions at the cold wall is shown in Figure II-B1.

\[ T \bigg|_{x=0} = T_w \quad \text{cold wall} \]

\[ J = 0 \]

\[ T \bigg|_{x=0} = T_w \quad \text{at all time} \quad (II-B12) \]

A. Heat Boundary Condition

Temperature at the cold wall remains constant at all time. Hence,

\[ T \bigg|_{x=0} = T_w \quad \text{at all time} \quad (II-B12) \]

B. Mass Boundary Condition

There is no mass flux past the cold wall. Hence,

\[ J \bigg|_{x=0} = \frac{D_k \left(1 - \frac{P_v}{P_{ice}}\right)}{RT \left(1 - \frac{P_v}{P}\right)} \frac{\partial P_r}{\partial x} \bigg|_{x=0} = 0 \quad (II-B13) \]

2. At the Frost Surface

The schematic of the boundary conditions at the frost surface is shown in Figure II-B2.

Gas Flow

\[ \text{Heat Flux} = h(T_s - T_f) + K_g \left(\frac{P_g - P_s}{P_{ice}}\right) \cdot \Delta H \]

Mass allocated to increase the frost thickness

\[ = \rho_s \frac{d\delta}{d\theta} \]

\[ \text{Heat Flux} = \frac{dF}{dt} \]

\[ \text{Mass Flux} = \frac{dM}{dt} = \frac{\partial P_{r_f}}{RT \left(1 - \frac{P_{r_f}}{P}\right) \partial x} \bigg|_{x=\delta} \]

\[ \text{Figure II-B2} \quad \text{Schematic of the conditions at the frost surface} \]
A. Heat Boundary Condition

Heat is transferred from the gas to the frost both by convection and by material transport of the condensable component. This rate of heat transfer can be written as

\[ q = h (T_g - T_s) + \frac{K}{g} (p_g - p_s) \Delta H \]  \hspace{1cm} (II-B14)

It should be noted that in above equation, the heat required for the sensible cooling of the condensable component is neglected.

On the frost side (at the frost surface), there is energy transfer through the frost layer by two mechanisms. First, there is the usual conduction process. Secondly, there is an internal mass flux caused by the temperature gradient. Both are proportional to the surface temperature gradient. Thus, one may write

\[ q = k_{f,s} \left( \frac{dT}{dx} \right)_s = (k_{f1} + k_{excess})_s \left( \frac{dT}{dx} \right)_s \]  \hspace{1cm} (II-B15)

Thus, the heat balance at the frost surface can be written as

\[ h (T_g - T_s) + \frac{K}{g} (p_g - p_s) \Delta H = k_{f,s} \left( \frac{dT}{dx} \right)_s \]  \hspace{1cm} (II-B16)

B. Mass Boundary Condition

The total mass that comes to the frost surface is given by Equation II-B17.

\[ J_s = \frac{K}{g} (p_g - p_s) \]  \hspace{1cm} (II-B17)

Part of this mass diffuses in the frost interior due
to vapor pressure gradient existing at the frost surface. The rest of it increases the frost thickness.

The mass flux which diffuses in the frost interior can be written as

\[
J_{s1} = \frac{D_B}{RT \gamma} \frac{(1 - \frac{\phi_f}{\phi_{ice}})}{(1 - \frac{P_{vp}}{P})} \frac{\partial P_{vp}}{\partial x} \bigg|_{x=\delta}
\]

and the mass allocated to increase the frost thickness can be written as

\[
J_{so} = \rho_{fs} \frac{d\delta}{d\Theta}
\]

Combining Equations II-B17, II-B18 and II-B19, the mass boundary condition at the frost surface can be written as

\[
J_s = J_{s1} + J_{so}
\]

or,

\[
K_s (P_j - P_s) = \rho_{fs} \frac{d\delta}{d\Theta} + \frac{D_B}{RT \gamma} \frac{(1 - \frac{\phi_f}{\phi_{ice}})}{(1 - \frac{P_{vp}}{P})} \frac{\partial P_{vp}}{\partial x} \bigg|_{x=\delta}
\]

or,

\[
K_s (P_j - P_s) = \rho_{fs} \frac{d\delta}{d\Theta} + \frac{D_B}{RT \gamma} \frac{(1 - \frac{\phi_f}{\phi_{ice}})}{(1 - \frac{P_{vp}}{P})} \left( \frac{M \Delta H F_{vp}}{RT^2} \right) \frac{\partial T}{\partial x} \bigg|_{x=\delta}
\]

**Dimensionless Form of the Equations**

The equations described above can be dedimensionalized as follows.

Let \( z = x/\delta \) \hspace{1cm} (II-B23)

then,

\( \phi_f = \phi_f(z, \Theta) \)
Hence, Equation II-B8 can be written in the dimensionless form as

\[
\frac{1}{\delta} \frac{\partial}{\partial z} \left[ D_f \frac{(1 - \rho_f \rho_{\text{ice}})}{V \Delta H \rho_f} \frac{I}{\delta} \frac{\partial T}{\partial z} \right] = \frac{\partial T}{\partial z} \bigg|_{z = 0} - \frac{\partial T}{\partial z} \bigg|_{z = 0} \theta
\]  

Similarly, Equation II-B9 can be written in the dimensionless form as

\[
\delta = \frac{k_f}{\delta} \frac{\partial T}{\partial z}
\]  

Dimensionless Form of the Boundary Conditions

The boundary conditions can take the following dimensionless form

1. At Cold Wall
   A. Heat Boundary Condition

   \[ T \bigg|_{z=0} = T_w \]  

   at all time  

   (II-B31)

   B. Mass Boundary Condition

   \[ J \bigg|_{z=0} = \frac{D_f (1 - \rho_f \rho_{\text{ice}})}{RT \tau (1 - \rho_{\text{rP}} \rho)} \delta \frac{\partial P_{\text{rP}}}{\partial z} \bigg|_{z=0} = 0 \]  

   (II-B32)
2. At the Frost Surface

A. Heat Boundary Condition

\[ q \bigg|_{z=1} = h \left( T - T_{\text{g}} \right)_{z=1} + K \frac{g - g_{\text{f}}}{g_{\text{f}}} \Delta H = \frac{k_f}{\delta} \frac{\partial T}{\partial z} \bigg|_{z=1} \quad (\text{II-B33}) \]

B. Mass Boundary Condition

\[ J_s \bigg|_{z=1} = K \frac{g - g_{\text{f}}}{g_{\text{f}}} = \rho_{\text{f}} d\delta + \frac{D_g (1 - P_{\text{f}}/P_{\text{ice}})}{RT \gamma (1 - P_{\text{f}}/P)} \frac{\partial P}{\partial z} \bigg|_{z=1} \quad (\text{II-B34}) \]

Computer Solution of the Equations

The finite difference solution of the above equations was obtained with the help of a computer. The solution involves the following steps.

1. Starting with an initial value of uniform (with depth) frost density and thickness, Equation II-B30 along with boundary conditions Equations II-B31 and II-B33 are used to obtain the temperature distribution in the frost interior at the starting time. The fourth order Runge-Kutta method is used to solve Equation II-B30.

2. Equations II-B29 along with the boundary conditions Equations II-B32 and II-B34 are then solved by a six point finite difference technique to obtain the density distribution within the frost and the frost thickness at new time.

3. Step 1 is repeated to obtain the temperature distribution within the frost at new time, and thus the calculation process is iterated.

The details of the computer solution is described in Appendix E.

Computer Results

The computer solution of the mathematical equations
of the 'sophisticated model' gives a density and a
temperature distribution within the frost and the frost
thickness as a function of time. Figures II-B3 to II-B6
show the results of this solution for two typical sets of
system conditions. Figure II-B3 describes the predictions
of the density distribution in the frost interior when it
is deposited under the system conditions of Reynolds number
= 5,600, humidity = gas dew point at 29.2°F and gas
temperature = 70°F. The predictions for the variation in
temperature distribution within the frost with time for
the same experimental conditions (in the transient state
of heat transfer) is shown in Figure II-B4. The theoretical
curves shown in Figures II-B3 and II-B4 are obtained with
an assumed starting value of frost density = 0.026 gms./cc.,
uniform with depth and a thickness of 0.023 inches. The
initial values are taken at the time of 2 minutes from the
start of the frosting process.

Figure II-B5 describes the similar results for another
set of experimental conditions (Re=14,700, humidity= dew
point at 40°F, T_g = 70°F) in which the starting values are
taken in the quasi-steady state heat transfer region (at
the time of 30 minutes from the start of the frosting
process). The values of the initial uniform (with depth)
frost density and frost thickness in this case are 0.076
gms./cc. and 0.144 inches respectively. The predictions
for the variation in temperature distribution within the
frost with time in this quasi-steady state heat transfer
region is shown in Figure II-B6.

The results shown in Figures II-B3 and II-B4 indicate that the model predicts a large difference in the rate of frost densification at various points in the frost interior. The rate being the highest at the frost surface (where the vapor pressures of ice are the highest). Figure II-B4 shows that the temperature distribution in the frost interior changes significantly in the transient heat transfer state of the frosting process. On the other hand, as shown in Figure II-B6, the variation in the temperature distribution within the frost with time is very slow in the quasi-steady state of heat transfer in the frosting process.

Since mathematical formulations of both 'simple' and 'sophisticated' models are based on the very similar physical ideas, one would expect a good agreement between the values of the average frost density (average frost density = $\frac{\int \rho T \, d\theta}{\int d\theta}$), frost thickness and heat transfer through the frost as a function of time predicted by the them for the identical system conditions. The comparison between the average frost density, frost thickness and heat transfer through the frost predicted by the models at various times is described in Figures II-B7 and II-B8 for two different sets of system conditions. Figure II-B7 shows the comparison for a case in which the initial values of frost density and frost thickness are taken in the transient state of heat transfer. Figure II-B8, on the other hand, shows the comparison for a case in which the starting values of frost density and
and thickness are taken in the quasi-steady state of heat transfer. The results of Figures II-B7 and II-B8 indicate that even though there is reasonably good agreement between the results predicted by both the models, the deviation between their predictions increases with time. Since, as seen in Section II-A, the predictions of 'simple model' agree well with the experimental results, the above described results also indicate that the predictions of 'sophisticated model' (for the time variation in average frost density, frost thickness and heat transfer through the frost) will not agree with the experimental results for a long period of the frosting process.

The predictions of the 'sophisticated model' for the density distribution in the frost interior are not agreement with the experimental results shown in Figure II-B9. Figure II-B9 shows the experimental data for the density of the various frost slices at different depths and at different times deposited under a typical set of system conditions. One can conclude from these results that the frost density remains essentially uniform (with depth) at all time. The theoretical predictions shown in Figures II-B3 and II-B5, on the other hand, indicates the existence of a large density gradient in the frost interior after a short duration from the start of frosting. Thus, it can be concluded that the 'sophisticated model' does not speculate well, the microscopic occurrences in the frost interior.

Discussion of Computer Results
The computer results of Figures II-B3 and II-B5 indicate that the sophisticated model shows that the assumption of equilibrium conditions between interstitial gas and ice within the frost is not valid.

The predicted density gradient can be explained on the basis of the driving force which causes it. The development of a large density gradient indicates that the driving force for the rate of frost densification, namely \( \frac{\partial}{\partial x} \left( \frac{D_B^2}{T} \frac{\partial P}{\partial x} \right) \) varies significantly across the frost layer. Since an empirical correlation between molecular diffusivity and temperature (see Figure II-B10) shows that \( D_B/T \) is approximately proportional to \( T \), it is in fact \( \frac{\partial}{\partial x} \left( T \frac{\partial P}{\partial x} \right) \) that varies significantly across the frost layer.

The reason for a significant variation of \( \frac{\partial}{\partial x} \left( T \frac{\partial P}{\partial x} \right) \) in the frost interior is the extremely sharp drop of vapor pressure of ice at low temperatures. The vapor pressure profile in the frost interior for a typical set of experimental conditions is shown in Figure II-C3.

The deviation between the predictions of the simple and sophisticated models for the time variation in average frost density, frost thickness and heat transfer rate (see Figures II-B7 and II-B8) should be expected to increase with time elapsed from the start of the calculations. This is because, as sophisticated model predicts higher and higher density gradient in the frost interior, the temperature distribution within the frost predicted by both the models would differ more and more. This is shown in Figures II-B4.
and II-B6. This increasing deviation between the predicted
temperature distributions would obviously cause the other
predictions (see Figures II-B7 and II-B8) of the models
to deviate more and more. This, of course, means that the
predictions of the sophisticated model for the time variation
in average frost density, frost thickness and the rate of
heat and mass transfer would not agree with the experimental
results for a long period.

Conclusions

Two important conclusions can be drawn from the
analysis of the sophisticated theoretical model.
1. The sophisticated model will not predict the time
variation in average frost density, frost thickness and
heat transfer through the frost well in agreement with
the experimental results over a long period.
2. The driving force for the rate of frost densification
assumed in the sophisticated model, namely \( \frac{\partial}{\partial x} \left( \frac{D_{eff} \partial r_v}{\partial T} \right) \)
varies significantly across the frost layer and this causes
a significant build-up of density gradient within the frost
as it accumulates on the cold surface. Since the experimental
facts indicate that the density of the frost stays uniform
(with depth) at all time, the sophisticated model does not
speculate the true physical processes occurring in the
frost interior.

II-C Critical Evaluation of the Speculated Physical Processes
(by "Sophisticated" Model) Occurring in the Frost Interior

The results of the sophisticated model indicated that if
at every layer in the frost interior, an equilibrium between the gas and ice exists then the frost densifies non uniformly (with depth) with time. The non uniform frost densification is caused by the significant variation in the driving force $\frac{\partial}{\partial x} \left( \frac{D_{\text{eff}}}{T} \frac{\partial p}{\partial x} \right)$ for it. The experimental data in Figure II-B9, on the other hand, indicate that the frost density stays almost uniform (with depth) at all time. This experimental fact demands a critical evaluation of the speculated physical processes occurring in the frost interior.

The sophisticated model assumed that the partial pressure of water vapor at every point in the frost interior was the vapor pressure at that point i.e. at the temperature prevailing there. Since this assumption governs the mechanism under which every layer in the frost interior densifies with time, it is examined critically, in this section, with the help of some experimental data as follows. **Quantitative Verification of Equilibrium Situation in the Frost Interior**

If every layer of the frost interior densifies uniformly (i.e. frost density remains uniform with depth at all time), the mass balance Equation II-B4 on the differential element of the frost can be written as

$$\frac{\partial}{\partial x} \left[ \frac{D_{\text{eff}}}{RTC} \frac{(1-p/P)}{(1-p/P)} \frac{\partial p}{\partial x} \right] = \frac{d\rho}{d\theta}$$  \hspace{1cm} (II-C1)

Assuming $(1-p/P) \approx 1.0$, Equation II-C1 can be written as
\[
\frac{\partial}{\partial x} \left[ \frac{D_b \left(1 - \frac{\eta_f}{\eta_{ice}}\right)}{RT} \frac{\partial \eta}{\partial x} \right] = \frac{d\eta_f}{d\theta} \tag{II-C2}
\]

Integrating Equation II-C2, one obtains

\[
N_x = \frac{D_b \left(1 - \frac{\eta_f}{\eta_{ice}}\right)}{RT} \frac{\partial \eta}{\partial x} = x \frac{d\eta_f}{d\theta} + c_1 \tag{II-C3}
\]

Substituting Boundary Condition at \( x = 0, N_x = 0 \) in Equation II-C3, one obtains that \( c_1 = 0 \) hence Equation II-C3 becomes

\[
\frac{D_b \partial \eta}{T \partial x} = x \frac{d\eta_f}{d\theta} \frac{R \varpi}{(1 - \frac{\eta_f}{\eta_{ice}})} \tag{II-C4}
\]

A typical experimental condition of Brazinsky (14) is used to obtain \( p(x) \) curve in the frost interior from Equation II-C4. It should be noted that similar results can be obtained from the experimental data of the present research.

From Figure II-C1 of \( \eta_f \) versus time profile at \( \eta_f = 0.111 \) gms./cc.

\[
1 - \frac{\eta_f}{\eta_{ice}} = 1 - (0.111/0.917) = 0.879 \tag{II-C5}
\]

and

\[
\left. \frac{d\eta_f}{d\theta} \right|_{\eta_f = 0.111} = 2.925 \text{ lbm./ft}^3 \text{ hr.} \tag{II-C6}
\]

Substituting the values of the Equation II-C5 and II-C6 into the Equation II-C4, one obtains

\[
\frac{\partial \eta}{\partial x} = \frac{T}{D_b} \times \frac{(2.925) \times 85.85 \times 1/10 \times 0.879}{0.879} = 315 \frac{T \eta}{D_b} \tag{II-C7}
\]

The molecular diffusivity of water vapor in nitrogen can be correlated with temperature by the following correlation (94).
\[ D_B = 3.30 \times 10^{-6} T^{2.0087} \]  

where \( T \) is in °R and \( D_B \) is in ft²/hr

The details of this correlation are shown in Figure II-B10.

Substituting the Equation II-C8 into II-C7 and integrating,

\[ \int_{x=0}^{x=x} dp = \int_{x=0}^{x=x} \frac{9.54 \times 10^{-7}}{T^{1.007}} \, dx \]  

\[ \Rightarrow \quad p \bigg|_{x=x} = p \bigg|_{x=0} + 9.54 \times 10^{-7} \int_{x=0}^{x=x} \frac{x}{T^{1.007}} \, dx \]  

where \( p \) is in lb_f/ft²

or

\[ p \bigg|_{x=x} = p \bigg|_{x=0} + 3.41 \times 10^{-7} \int_{x=0}^{x=x} \frac{x}{T} \, dx \]  

where \( p \) is in mm. Hg.

Assuming partial pressure of the water vapor at the cold wall is an equilibrium vapor pressure at the cold wall temperature and using the experimental data for the temperature versus \( x \) profile in the frost interior the results of Table II-C1 can be obtained. The deviation from the equilibrium at every point in the frost interior can be seen from the listed values of vapor pressure and the supersaturation ratio in Table II-C1.

The results of Table II-C1 are plotted in Figures II-C3 and II-C4.
Discussion of Results

The results of the above analysis, as described in the Figures II-C3 and II-C4, show that there is a large deviation from the equilibrium at every point in the frost interior. If no nucleation is assumed then the true water vapor pressure exceeds the local vapor pressure by a large amount. Hence, a realistic model of the physical processes occurring in the frost interior should take into account a possibility of nucleation and should consider the transport of water vapor under a true partial pressure gradient.

It should be noted that since the theoretical results of the sophisticated model Section II-B indicate that ice particles can not grow under their own vapor pressure at cryogenic temperatures, the results of Figure II-B9, for the uniform densification of the frost can be explained only if the ice particles near the cold wall (at liquid nitrogen temperature) grow in non-equilibrium situation. The strong possibility of nucleation in the frost interior as shown by the theoretical analysis of this section indicate that ice at that temperature will grow by both impingement of ice nuclei on the existing crystals and by the molecular diffusional mechanisms under non-equilibrium situation.

Conclusions

The principal conclusions of the above calculations can be stated as follows.
1. If no nucleation is assumed, the calculated water vapor pressure exceeds the local vapor pressure by a large amount.

2. In the colder region of the frost, ice crystals will grow by the impingement of the ice nuclei on the already existing crystal as well as by the molecular diffusional mechanisms under non-equilibrium situation.

II-D  Summary of the Available Literature Information on the Possible Physical Processes Occurring in the Frost Interior

The results described in Sections II-B and II-C indicate that in order to obtain a model which can explain the uniform densification of the entire frost layer, a critical evaluation of the possible physical processes occurring in the frost interior is needed. This section briefly summarizes the significant and useful available literature information on the various possible physical processes which might be taking place within the frost.

Numerous physical processes can take place in the heterogeneous structure of the frost interior. In the 'sophisticated model' it was assumed that at every layer in the frost interior, the interstitial gas is in equilibrium with ice at the prevailing temperature. However, the analysis described in Section II-C indicate that in actual fact there will be a large deviation from equilibrium at every layer in the frost interior. Under the non-equilibrium conditions, the significant physical processes which could occur within
the frost are as follows.

1. Nucleation of liquid or solid particles of water from the vapor phase.
2. Non-equilibrium growth of ice crystals from the vapor phase.
3. Thermal diffusion of fog or ice nuclei in the interstitial gas.

Several effects or occurrences which assumed to have negligible effect on the densification of the frost are stated below.

1. Gravitational effect.
2. Electrostatic forces on ice nuclei.
3. Thermal diffusion of water vapor in air.

All the above stated occurrences are neglected for the following reasons.

1. **Gravitational Effect**

   The bottom layer of the frost could densify by the gravitational force exerted by the top layer. This gravitational force, however, could not be important when the frost is deposited in a vertical tube. Also, it varies from layer to layer. Thus, the significance of gravitational force for the explanation of the uniform densification of the frost (with depth) deposited on a surface with an arbitrary geometrical shape does not appear very promising. For the present purpose, therefore, this effect is neglected.

2. **Electrostatic Forces on Ice Nuclei**

   If nucleation occurs predominantly in the frost interior,
then the possible electrostatic charges on these nuclei could cause the transfer of these particles from layer to layer. At present, however, it is difficult to conceive the cause for the development of the possible electrostatic charges on the nuclei. Hence, this effect is neglected in the present thesis.

3. Thermal Diffusion of Water Vapor in Air

Since there is a large temperature gradient existing within the frost (see Section IV-A), one would expect that the diffusion of water vapor by the thermal gradient could be equally important as one due to the concentration (or vapor pressure) gradient. Contrary to the expectations, however, the following calculations indicate that the thermal diffusion of water vapor in air within the frost is negligible compared to the diffusion of water vapor under concentration gradient.

Comparison of Mass Fluxes under Concentration (Vapor Pressure) and Temperature Gradient

The total flux of water vapor within the frost due to both thermal and concentration gradient can be given by the following Equation II-D1.

\[ N_w = \text{total flux of water vapor} = \frac{D_w-A}{c_w c_A} \left( \frac{dc_w}{dx} + \frac{k_T}{T} \frac{dT}{dx} \right) \] (II-D1)

where, the term \( \frac{D_w-A}{c_w c_A} \frac{dc_w}{dx} \) represents the mass flux of water vapor due to ordinary concentration gradient and the term \( \frac{D_w-A}{c_w c_A} \frac{k_T}{T} \frac{dT}{dx} \) represents the water flux due to
Grew and Ibbbs (49) define $k_T$, the thermal diffusion coefficient, to be equal to $\alpha n_1 n_2$, where $\alpha$ denotes the thermal diffusion factor. Various methods for evaluating $\alpha$ are presented in Reference 49. In the present calculation maximum value of $\alpha$ is chosen, in order to obtain the value for maximum mass flux by thermal diffusion. From the graph on page 29 of Reference 49, it can be found that $\alpha = 0.180$ for very low water vapor concentrations of air-water vapor mixture. This value is assumed to be applicable at all positions in the frost interior.

From Equation II-D1, it is obvious, that contribution of thermal diffusion to the total mass flux can be evaluated by comparing the values of $\frac{dc_w}{dx}$ and $\frac{k_T}{T} \frac{dT}{dx}$ at various points in the frost interior.

Also, $\frac{dc_w}{dx} = \frac{dc_w}{dT} \frac{dT}{dx}$ and so a comparison between $\frac{dc_w}{dT}$ and $\frac{k_T}{T}$ or between $\frac{dc_w}{dT}$ and $\frac{dc_w c_A}{T}$ should be made.

Values of $\frac{dc_w}{dT}$ and 0.18 $c_w c_A / T$ at different temperatures are shown in Table II-D1. At any given temperature $\frac{dc_w}{dT}$ can be obtained from the slope of the vapor pressure curve. At all temperatures between -300°F and +20°F the value of 0.18 $c_w c_A / T$ found to be less than 1.32% of $\frac{dc_w}{dT}$. It can be therefore concluded that at all positions within the frost the mass flux due to thermal diffusion is small compared with the flux due to concentration gradient. Hence, the effects of thermal diffusion on the
frost densification can be ignored for the present purpose.

Available Literature Information on the Physical Processes Occurring within the Frost

1. **Nucleation from the Vapor Phase**

   There are three kinds of nucleation that can occur in the frost interior: homogeneous, heterogeneous and secondary.

   The secondary nucleation is, at present, a very poorly understood phenomenon. It can either occur by shearing off the dendrites of the existing crystal by thermal forces or due to mere presence of ice crystals. Both of these are possible to occur in the frost interior. The occurrence of this phenomenon may not required a considerably high supersaturation. There are, as yet, no significant experimental as well as theoretical studies done in this area.

   The heterogeneous nucleation is a very complex process because of its diverse relationships to the amount, size and nature of the impurities present in the gas. For the purpose of finding an explanation for the uniform densification of the frost under an arbitrary set of system conditions, however, this phenomenon could not play a significant role for the following reason.

   It appears highly probable that uniform densification of the entire frost layer will be observed if frost is deposited from a mixture of ultra pure air and water vapor. Both Brazinsky (14) and Weber (114) used purified Nitrogen from Nitrogen Bottle (purity of over 99%) as a carrier
gas for water vapor in the frosting experiments, and they still observed the same amount of frost densification as observed in the present research. In the present studies ordinary air is used as a carrier gas which is expected to have a little higher amount of impurity than nitrogen from nitrogen bottle. These results indicate that heterogeneous nucleation can not be used to explain the uniform densification of the frost under an arbitrary set of system conditions. Hence, it is neglected for the present purpose.

The homogeneous nucleation has been studied somewhat extensively both experimentally as well as theoretically. The detailed survey of the available literature for this phenomenon is described in Appendix H. The occurrence of homogeneous nucleation requires a significant amount of supersaturation of water vapor in the gas. If secondary nucleation is not a predominant mechanism, then it is possible, as seen in Section II-C, to achieve a sufficiently high supersaturation in the gas for homogeneous nucleation. The important studies done for this process can be summarized as follows.

1. The experimental data on homogeneous nucleation rate are obtained only for temperatures above $-73^\circ C$. These data are obtained in Wilson Chamber or in sonic nozzles. The data agree well with the theoretical relationship of Becker and Döring (8).
2. Volmer (112), Becker and Döring (8) and Frankel (9, 40, 42) equations are the three most significant
theoretical relationships for the homogeneous nucleation rate. The predictions of nucleation rate by Becker and Döring (8) and by Frankel (9,40,42) equations have been found to agree well with the experimental data (12,43,111).

2. Non-Equilibrium Growth of Ice Crystals from the Vapor Phase

The frost densification is essentially a result of growth of ice crystals network. The ice crystals can grow either by particle impingement of ice nuclei which are formed either in bulk interstitial gas or near the crystal surfaces, or by condensation of the water vapor which is transported from the bulk gas to the crystal surface. The detailed survey of available literature for the latter kind of growth is described in Appendix J.

The study of crystal growth by particle impingement is an extremely difficult task, particularly for the ice network in the frost interior. It requires the knowledge of frost structure, the position of the particles in the bulk interstitial gas and the possible forces acting upon the particles. There is, as yet, no significant study done to be able to predict all these requirements.

Usually, the phenomenon of particle impingement can be handled by assuming a value for so called 'sticking coefficient'. If no ice particles stick to the existing crystal then sticking coefficient has a zero value. On the other hand, if all the ice nuclei formed in the gas stick to the existing crystal, then sticking coefficient
will have a value of unity. There is, as yet, no systematic guidelines developed to obtain a value of sticking coefficient under a given set of system conditions.

The research work that has been done to study the crystal growth from vapor phase by diffusional mechanisms can be summarized as follows.

A. The experimental data for the growth of ice crystals from vapor phase have been obtained only at temperatures above -30°C. In initial part of the growth these data found to agree well with a model (68) which assumed that growth process is controlled by simultaneous heat and mass transfer from bulk to the crystal surface. After an initial short duration, the growth process has been found to occur in the dendritic manner.

B. The growth studies have been made with the crystals of various shapes.

C. So far, no significant theoretical as well as experimental studies of dendritic crystal growth process have been carried out.

3. Thermal Diffusion of Aerosols in Air

The existence of a large temperature gradient (see Section IV-A) and the possibility for an existence of a large number of water or ice nuclei indicate the probability for a significant flux of water or ice by thermal diffusion of these aerosols in the colder region of the frost. There has been a significant amount of theoretical as well as experimental work done in this area. None of these studies
however, have been dealt with ice particles or with low temperature conditions (below room temperature). The detailed available literature information for this phenomenon is described in Appendix K. The important information can be summarized as follows.

A. The thermal force on a solid particle of size greater than 4 microns is usually very small in magnitude.

B. The guidelines for the development of theoretical equations for this process depend upon the flow regimes (i.e. on the value of ratio \( \lambda/r_p \); where \( \lambda \) is mean free path of molecules and \( r_p \) is the radius of liquid or solid particles). The theoretical equations for both \( \lambda/r_p \ll 1.0 \) and \( \lambda/r_p \gg 1.0 \) have been reported extensively by several investigators (15,16,22,35,95,113). The theoretical analysis of the process for \( \lambda/r_p \approx 1.0 \) (transition regime) is extremely difficult task and no quantitative theoretical relationships have been obtained so far. For the present purpose, Brock's equation for the thermal force (15) is used for the calculation of thermal diffusive velocity of ice or water nuclei in the interstitial gas within the frost (see Section II-E). The details of Brock's equation as well as the brief descriptions of all other theoretical relationships are shown in Appendix K.

C. The experimental measurements of the thermal force on the aerosol particles of wide range of thermal conductivity (\( 10^{-4} \) cal./cm. sec. \( ^\circ \)C to 0.1 cal./sec. cm. \( ^\circ \)C) have been studied. The experimental data have been found to be in
fair agreement with the theoretical predictions. As yet, no experimental data for thermal force on aerosols have been taken at low temperatures (i.e. below room temperature).

II-E Model for Frost Densification ('Non-Equilibrium Model')

As seen in Section II-B, the 'improved model' (or 'sophisticated model') for the frosting process predicts that if every layer of the frost densifies under the prevailing equilibrium conditions then the frost will densify non-uniformly across its depth. The experimental measurements on the density gradient within the frost, on the other hand, indicated that the frost density remains uniform (with depth) at all times.

The discrepancy between the experimental facts and the theoretical predictions for the uniformity of the frost density was examined theoretically in Section II-C. It was concluded that the uniform densification of the frost could be obtained if the ice crystals grew in a non-equilibrium situation. The calculations also indicated that in the colder region of the frost (near the cold wall) a significant amount of water could exist only in the form of ice nuclei.

With the results of Section II-B and II-C along with the available literature information on the possible phenomena occurring in the frost interior (as described in Section II-D), a new model for the frost densification was developed. In this section, the new model is described in detail and discussed qualitatively as an explanation.
for the uniformity of the frost densification.

The basic postulations and the assumptions of the new model are as follows.

1. At the frost surface, water vapor is in equilibrium with ice at the prevailing temperature. The partial pressure gradient due to the temperature gradient at the frost surface causes the transport of water vapor within the frost. (It should be noted that the 'simple model' was also based on this assumption).

2. The interior of the frost is divided into three regions for convenience of discussion and calculation. Near the frost surface, where the vapor pressure of ice is large (probably above 
\(-40^\circ F\)), all the water is transported as vapor. Near the cold wall, on the other hand, where the vapor pressure of water is extremely small, water is transported through the frost only in the form of ice nuclei. In between these two regions, there is a so called 'transition' region in which water is transported in the form of both ice nuclei and vapor.

3. The transport of water near the frost-gas interface (region 1) is believed to be caused by the partial pressure gradient due to a temperature gradient. Near the cold wall (region 3) water is transported through every layer by thermal diffusion of ice nuclei. Both thermal diffusion of ice nuclei and molecular diffusion of water vapor contribute to the water transport in the 'transition' region (region 2).
4. Thermal diffusion of water vapor is assumed to be negligible compared to diffusion by concentration gradient.

5. At each layer in regions 2 and 3 the mass of the ice nuclei present is small compared to the mass of the existing crystalline phase.

6. The nuclei concentration at each layer in the regions 2 and 3 remains constant with time.

7. In regions 2 and 3 the thermal diffusive velocity of an ice nuclei is sufficiently high so that the nuclei will impinge (or be trapped) on the existing crystals before they can grow appreciably.

8. In region 1, ice crystals grow by condensation of water vapor. In region 3, ice crystals grow by the impingement (or trapping) of ice nuclei as they are driven towards the cold wall by a thermal force. Both particle trapping and vapor condensation contribute to the ice crystal growth in region 2.

Qualitative Description of the Model

As water reaches the frost surface from the bulk gas, it diffuses into the frost interior due to partial pressure gradient caused by the temperature gradient. Since the simple model predicts the water transport past the frost surface well in agreement with the experimental facts, it can be assumed that at the frost surface, ice and the gas are in equilibrium with each other.

The postulations regarding the state of water imply that the supersaturation of water vapor \( \frac{p}{P_{vp}} \) in the
interstitial bulk gas should be high at the interface of regions 1 and 2 so that nucleation can occur. Thus, as water vapor moves from the frost surface to the interior, the supersaturation ratio increases from the value of about 1 at the frost surface to a considerably higher value at the interface of region 1 and 2.

The ice crystals in region 1 grow by the condensation of water vapor. Hence, the crystal growth in this region is controlled by the rate at which water vapor arrives at the crystal surface from the bulk interstitial gas. The present model visualizes a uniform growth of an ice crystal over the entire region as the result of counter-balancing effects of an increase in driving force for the mass transfer \( \left( \frac{P}{P_{\text{vp}}} - 1 \right) \) and the decrease in molecular diffusivity of water vapor in air with increase in depth from the frost surface.

As water penetrates deeper into region 2 water vapor is now not only used up due to the growth of ice crystals by diffusional mechanisms but also by the generation of ice nuclei. The crystal growth by vapor condensation in this region will be controlled by the same mechanisms as described above.

The ice nuclei generated at every layer in the region 2 are directed towards the cold wall by a thermal force. The force results from the temperature gradient. While moving, some of these nuclei are impinged (or trapped) on the existing crystals. Thus, the crystals in this region also
grow by deposition of ice nuclei on the existing crystals.

The model visualizes the uniform growth of ice crystals in region 2 as a result of the complex combination of the two separate mechanisms of growth (growth by vapor condensation and the growth by particle impingement). As seen previously, growth by vapor condensation is controlled by $p(x)$ and $T(x)$ distributions. It can also be expected that the crystal growth by impingement or trapping would be governed by the concentration of nuclei and the velocity at which each nuclei is moving towards the cold wall. This is because the probability for the rate of nuclei impingement can be visualized as a direct function of number of nuclei and the momentum of each nuclei. As shown in Section II-E-1, it has been found that the thermal diffusive velocity in the frost interior varies approximately linearly with temperature gradient. Thus, just like crystal growth by vapor condensation, the frost densification by impingement of nuclei is also expected to be governed by water content (i.e. nuclei concentration) and temperature distribution.

The continuous depletion of water vapor will result in the existence of water only in the form of ice nuclei at the interface of region 2 and 3. Thus, water is transported in the region 3 only in the form of ice nuclei. Since the temperature decreases with depth in this region, nuclei will remain in the solid form as they move towards the cold wall.

The ice crystals in region 3 will grow uniformly by impingement of the nuclei on the existing crystals at a
constant rate as the nuclei move towards the cold wall. As discussed above, this growth will be controlled by the water content and the temperature distribution at each layer in the region.

**Mathematical Formulation of the Model**

**Region 1**

The mass balance on a differential element of frost in region 1 can be given by the following equation.

\[
\frac{\partial}{\partial x} \left( \frac{D_B}{RT} \left( 1 - \frac{q_f}{q_{ic}} \right) \frac{\partial p}{\partial x} \right) = \frac{\partial \rho_f}{\partial \theta} \tag{II-E1}
\]

All the terms in Equation II-E1 are defined in Appendix R.

It should be noted that \( D_B \) decreases as \( x \) decreases, i.e., the depth from the frost surface increases. If \( N \) is the number of large ice crystals in each differential element of frost, then rate of frost densification can also be written by Equation II-E2

\[
\frac{\partial \rho_f}{\partial \theta} = N \frac{d m}{d \theta} \tag{II-E2}
\]

where \( \frac{d m}{d \theta} = \) growth rate of a single ice crystal

**Mass Boundary Conditions at the Frost Surface**

\[
N \bigg|_{x=s} = \frac{D_B}{RT} \left( 1 - \frac{q_f}{q_{ic}} \right) \frac{\partial p}{\partial x} \bigg|_{x=s} \approx \frac{D_B}{RT} \left( 1 - \frac{q_f}{q_{ic}} \right) \frac{\partial \rho_f}{\partial x} \bigg|_{x=s} \tag{II-E3}
\]

and,

\[
K_g (p_y - p_s) = \rho_f \frac{d \rho}{d \theta} + N \bigg|_{x=s} \tag{II-E4}
\]

**Region 2**

The mass balance on a differential element of the
frost in this region can be given by the following equation.

\[ \frac{\partial}{\partial x} \left[ \frac{D_0}{RT} \left( 1 - \frac{\rho_f}{\rho_{ice}} \right) \frac{\partial p}{\partial x} + \rho \right] = \frac{\partial \rho_f}{\partial \theta} \]  

(II-E5)

**Nuclei Balance**

If nuclei concentration remains constant with time, the nuclei balance on a differential element of the frost can be given by the following equation.

\[ \frac{\partial}{\partial x} (\rho \nu_f) + \dot{N}_r - \dot{N}_d = 0 \]  

(II-E6)

where \( \dot{N}_r \) is the rate of formation of nuclei and \( \dot{N}_d \) is the depletion rate of nuclei from the interstitial gas. All other terms in Equations II-E5 and II-E6 are defined in Appendix R.

It is easy to see that Equation II-E5 can be divided into two parts as follows.

The frost densification due to diffusional mechanism can be written as

\[ \frac{\partial \rho_f'}{\partial \theta} = \frac{\partial}{\partial x} \left[ \frac{D_0}{RT} \left( 1 - \frac{\rho_f}{\rho_{ice}} \right) \frac{\partial p}{\partial x} \right] \]  

(II-E7)

On the other hand, the frost densification by the particle impingement can be written as

\[ \frac{\partial \rho_f''}{\partial \theta} = \frac{\partial}{\partial x} (\rho \nu_f) = f (\rho, \nu_f) \]  

(II-E8)

Also, from Equation II-E5

\[ \frac{\partial \rho_f}{\partial \theta} = \frac{\partial \rho_f'}{\partial \theta} + \frac{\partial \rho_f''}{\partial \theta} \]  

(II-E9)

**Region 3**

In this region, water transport occurs solely by
thermal diffusion of ice nuclei. Thus,

$$\frac{\partial^2 n}{\partial x^2} (n V) = N_d = f(n, V)$$  \hspace{1cm} (II-E10)

The nuclei balance can be written as

$$\frac{\partial}{\partial x} (n V) - N_d = 0$$  \hspace{1cm} (II-E11)

**Mass Boundary Condition at the Cold Wall**

Since no mass passes the cold wall,

$$N_x \bigg|_{x=0} = n V_t \bigg|_{x=0} = 0$$  \hspace{1cm} (II-E12)

**Heat Balance**

An energy balance in all three regions at the quasi-steady state of heat transfer can be written as

$$Q = k_f \frac{dT}{dx} = (k'_f + k_{\text{excess}}) \frac{dT}{dx}$$  \hspace{1cm} (II-E13)

where, $k'_f = \text{intrinsic thermal conductivity of frost}$

$k_{\text{excess}} = \text{excess thermal conductivity due to water transport.}$

**Heat Boundary Conditions**

1. At the frost surface

$$k_f (T_g - T_s) + k_g (P_g - P_s) \Delta H = k_f \frac{dT}{dx}$$  \hspace{1cm} (II-E14)

(note that this is the same as one used in the 'sophisticated' model)

2. At the cold wall

$$T \bigg|_{0} = T_w$$  \hspace{1cm} (II-E15)

**Qualitative Discussion on the Model**
Due to the extreme difficulties involved in obtaining the values of \( \dot{N}_r \) and \( \dot{N}_d \), the mathematical equations derived above are not solved quantitatively in the present thesis. However, in the discussion that follows the new model is evaluated qualitatively to examine whether it can explain the phenomenon of uniform densification of the frost. For the sake of clarity, each region (as shown in Figure II-E1) will be treated separately.

Since the simple model of the frosting process (see Section II-A) has been found to predict the effects of the independent system parameters on the frost properties fairly successfully, the heat and mass boundary conditions at the frost surface in this model are taken to be identical to those in the 'simple model'.

The mathematical formulation of the 'simple model' indicated that the amount of water vapor that diffuses into the frost interior from the frost-gas interface can be well predicted from the following equation.

\[
\left. N_{x=\delta} = \frac{D_\beta}{RT} \frac{(1 - \varphi_f \varphi_{ice})}{\gamma} \frac{\partial P_r}{\partial x} \right|_{x=\delta} \quad \text{(II-E16)}
\]

It should be noted that, since the equilibrium conditions can not exist within the frost, the true mechanism of water transport past the frost surface could be molecular diffusion under a non-equilibrium partial pressure gradient. Thus, a general equation for the internal diffusion of water vapor at the frost surface can be written as,
Since nucleation is assumed to be negligible in region 1 (note that this assumption is based on the literature information on nucleation phenomenon as described in Section II-D and Appendix H), the ice crystals in this region will grow by the condensation of water vapor which is transported from the bulk interstitial gas to the crystal surface. At the frost-gas interface equilibrium is assumed to exist between ice and gas. At the interface of regions 1 and 2, on the other hand, nucleation first occurs. Thus, it is obvious that the postulations of the model imply that the deviations from equilibrium would be increased with depth (or distance from the frost surface) in this region. The ice crystals in this region will, thus, grow under non-equilibrium conditions.

The non-equilibrium heat and mass transfer controlled growth of the crystals in the region 1 can be mathematically correlated from the following equation. The details for the derivation of this equation is shown in Appendix J.

\[
\frac{dm}{d\theta} = \frac{4\pi C \left( p/P_p - 1 \right)}{\frac{\Delta H^2M}{k_qRT^2} + \frac{RT}{D_g MP_v p}} \quad (\text{II-E18})
\]

\[
= \frac{4\pi C \left( p/P_v p - 1 \right)}{f(T)} \quad (\text{II-E19})
\]
All the nomenclatures of Equations II-E18 and II-E19 are described in Appendix R.

Since $k_g$ and $D_B$ decreases with temperature, $f(T)$, in Equation II-E19, would decrease with increase in temperature. The relation between $f(T)$ and temperature is shown in Table II-El.

**Table II-El**

<table>
<thead>
<tr>
<th>$T$ ($^\circ$R)</th>
<th>$f(T)$ ($\frac{ft\cdot hr}{lbm}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>460</td>
<td>$2.474 \times 10^4$</td>
</tr>
<tr>
<td>450</td>
<td>$4.114 \times 10^4$</td>
</tr>
<tr>
<td>440</td>
<td>$7.069 \times 10^4$</td>
</tr>
<tr>
<td>430</td>
<td>$1.275 \times 10^5$</td>
</tr>
<tr>
<td>420</td>
<td>$2.274 \times 10^5$</td>
</tr>
<tr>
<td>400</td>
<td>$9.066 \times 10^5$</td>
</tr>
<tr>
<td>360</td>
<td>$2.165 \times 10^7$</td>
</tr>
</tbody>
</table>

Since, both $(p/P_{vp}-1)$ and $f(T)$ increases with depth from the frost surface, the model visualizes that uniform densification of the frost in region 1 could be achieved by the proper adjustment between water vapor content (partial pressure) and the temperature at every layer such that the ratio of $(p/P_{vp}-1)$ and $f(T)$ remains constant over the entire region.

In order to illustrate the above discussed point more
clearly, a sample calculation was carried out. For a typical set of experimental conditions (Run number 29), a p(x) distribution in region 1 was obtained with the assumption that \((p/P_{vp}-1)/f(T)\) remains constant at all x's (note that \(f(T)\)'s at various x's can be calculated from experimentally measured T(x) profile). Table II-E2 shows the calculated p(x) distribution.

**Table II-E2**

<table>
<thead>
<tr>
<th>T(°R)</th>
<th>(f(T)) (ft-hr/1bm)</th>
<th>Calculated p(x) mm. of Hg.</th>
<th>x (inches)</th>
<th>P_{vp} (x) mm. of Hg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>462</td>
<td>2.12x10^4</td>
<td>1.080</td>
<td>0.202</td>
<td>1.080</td>
</tr>
<tr>
<td>460</td>
<td>2.474x10^4</td>
<td>1.048</td>
<td>0.200</td>
<td>0.984</td>
</tr>
<tr>
<td>450</td>
<td>4.114x10^4</td>
<td>0.610</td>
<td>0.184</td>
<td>0.569</td>
</tr>
<tr>
<td>440</td>
<td>7.0695x10^4</td>
<td>0.358</td>
<td>0.173</td>
<td>0.330</td>
</tr>
<tr>
<td>430</td>
<td>1.275x10^5</td>
<td>0.2115</td>
<td>0.163</td>
<td>0.174</td>
</tr>
<tr>
<td>420</td>
<td>2.274x10^5</td>
<td>0.134</td>
<td>0.153</td>
<td>0.100</td>
</tr>
</tbody>
</table>

From the values of p(x) distribution shown in Table II-E2, \(dP/d\theta\) of the frost slice between x=0.158 inch and x=0.2010 inch is calculated using Equation II-E1. Thus,

\[
\frac{dP}{d\theta} = \frac{DB (1-\frac{q_f}{q_{ice}}) \frac{\partial P}{\partial x}}{\frac{RT}{\gamma} \frac{1}{x}} \bigg|_{x=0.2010} - \frac{DB (1-\frac{q_f}{q_{ice}})}{\frac{RT}{\gamma} \frac{1}{x}} \bigg|_{x=0.158}^{0.043/12}
\]

\[
= \frac{0.0629-0.02995}{0.043/12}
\]

\[
= 0.897 \text{ lbm/hr ft}^3
\]
The experimentally measured \( \frac{d\rho}{dT} \) under the same conditions of the system has been found to be 0.95 \( \frac{lbm}{hr \cdot ft^3} \) (see Figure IV-A1). It should be noted that the data shown in Table II-E2 indicate that the value of C in Equation II-E26 for the investigated experimental conditions would be 17 microns. This value is comparable with 20 microns which is calculated from the assumed dimensions of an ice needle.

The above calculations indicate that there exists a \( p(x) \) distribution (for a typical set of experimental conditions) which could cause the experimentally measured densification of the frost at every layer in region 1 by the heat and mass transfer controlled growth process. The calculated \( p(x) \) distribution indicate that supersaturation ratio \( (p/P_{vp}) \) increases as \( x \) decreases.

In summary, it can be said that the present model for the frost densification suggests that the uniform densification of the frost in region 1 would be controlled by \( T(x) \) and \( p(x) \) distributions.

As postulated previously, the transfer of water in region 3 occurs due to thermal diffusion of ice nuclei. The validity of this postulation can be examined as follows.

For typical experimental mass flux of 0.05 \( lbm/hr \cdot ft^2 \), assuming nucleus diameter of 500A (see Appendix H) and taking the value of thermal diffusive velocity of 93.10 \( ft/hr \) from Brook's equation (see Section II-E-1) at temperature of \(-135^\circ F\), the nuclei concentration required for the mass flux of 0.05 \( lbm/hr \cdot ft^2 \) would be,
\[ n' = \frac{N_x}{Vx} \times \frac{1}{\text{ice}} = \frac{0.05 \times 4.54}{93.10 \times 6 \times 10^{-20}} = 4.07 \times 10^{18} \text{ molecules cu. ft.} \]

\[ = 1.432 \times 10^{14} \text{ Molecules cc} \]

If it is assumed that the residence time for these nuclei is \(10^{-3}\) secs. (note that there is no basic ground for this assumption except that it might give a conservative value for nucleation rate), then \(N_r\) required for this much nuclei concentration at all time would be \(1.432 \times 10^{17}\) molecules/cc. sec.

If homogeneous nucleation is a predominant mechanism for nuclei generation (it should be noted that this is the worst kind of nucleation mechanism), then partial pressure required for the \(N_r = 1.432 \times 10^{17}\) molecules at temperature \(-135^\circ F\) would be 250x.00001=.0025 mm. Hg. (from Figure H-4).

The partial pressure at the frost surface typically varies between 1 to 3 mm. of Hg. Thus, the magnitude of partial pressure at the temperature level of \(-135^\circ F\) compared to one at the frost surface indicate that the postulation that mass flux in region 1 occurs by thermal diffusion of ice nuclei appears physically realistic.

At every layer in region 3 some nuclei are expected to be impinged (or trapped) in the existing structure. Thus, mathematically, frost densification process can be defined by the following equation.

\[ \frac{d\phi_x}{d\theta} = \frac{\partial}{\partial x} (n_4) \quad (\text{II-120}) \]
There are three possible forces that could cause the movement of ice nuclei in the colder region. They are thermal, gravitational and the force due to electrostatic charges on the ice nuclei. The reasons for possible development of electrostatic charges on the ice nuclei are, at present, unknown and hence in the present thesis possible force due to electrostatic charge is neglected. The preliminary calculations indicated that thermal force is considerably higher than gravitational force on the ice nuclei. Thus, the movement of ice nuclei mainly occurs by thermal force. As shown in Section II-E-1, the thermal diffusive velocity has been found to be an approximate function of only the temperature gradient in the frost interior. In fact, the following approximate correlation for thermal diffusive velocity in the frost interior was obtained from Brock's equation (see Section II-E-1)

\[ V_t = \frac{d\alpha}{dx} \]  

(II-E21)

where \( \alpha \) is constant.

Since it has been found experimentally that \( \frac{d\alpha}{dx} \) increases with depth from the frost surface, \( V_t \) would be increased with depth. Since the ice crystals in region 3 grow by impingement (or trapping) of ice nucleation the existing crystals, nuclei concentration or water content has to decrease with increase in depth from the frost surface. Thus, \( n \) decreases and \( V_t \) increases with depth from the frost surface.
The ice crystals in region 3 grow by the impingement (or trapping) of nuclei on the crystal network as they move towards the cold wall. This is an analogous situation to one in which a bunch of ping pong balls are thrown through a network. At every point (along its depth), the ping pong balls may or may not collide or get trapped in the network. It can be easily visualized that the probability for the ping pong balls to collide on solid wall in the network would be higher at higher concentration of balls and higher velocity. Similarly, the rate of collision (or trapping) of ice nuclei on the ice crystals is expected to be higher at higher nuclei concentration and the thermal diffusive velocity. Thus, in general, the rate of depletion of ice nuclei from the bulk interstitial gas would be proportional to $n$, $V_t$ and $'a'$. Where $'a'$ is the effective surface area to volume ratio for the collision. For constant $'a'$, $\dot{N}_d$ would be an increasing function of $n$ and $V_t$.

It is an extremely difficult task to obtain an accurate quantitative relation between $\dot{N}_d$, $n$ and $V_t$. However, from the qualitative description shown above, it can be predicted that the crystal growth by nuclei impingement in region 3 would be constant at each layer because continuous increase in $V_t$ but decrease in $n$ with increase in depth.

In summary, it can be said that the model visualizes that water content and temperature distribution would govern the uniform densification of the frost in region 3.
The evaluation of frost densification process in region 2 is quite difficult. The frost in this region will densify by both vapor condensation as well as by impingement of ice nuclei on the existing crystal. Since, region 2 is the transition between regions 1 and 3 and since regions 1 and 3 are expected to densify uniformly, it is more than likely that region 2 would also densify uniformly. It should be noted that physically, region 2 is one in which particle impingement takes over the control for frost densification from the process of vapor condensation.

II-E-1 Determination of a Correlation for Thermal Diffusive Velocity in the Frost Interior

As discussed in Appendix K, there are two significant correlations for evaluation of thermal force exerted on an aerosol particle in slip region ($\lambda/r_p < 1.0$, where $\lambda$, mean free path and $r_p$, radius of the particle). Epstein (95) initially derived a correlation for thermal force by a simple treatment of the process of thermal diffusion. The predictions of the thermal force from Epstein equation, however, found to agree well with the experimental facts only for solid particles with low thermal conductivity ($10^{-4}$ cal/gm. cm. °F). For the particle of large thermal conductivity, Epstein equation has been found to predict much lower value of thermal force than the actual (experimental) one. This is because the thermal conductivity of solid has been believed to give undue importance in the correlation
for thermal force. It should be noted that the thermal conductivity of ice is approximately $10^{-2}$ cal/gm·cm°C at low temperatures. Also thermal conductivity of ice increases with decrease in temperature. Thus, it is expected that Epstein equation would predict low thermal force on ice nuclei than the actual ones.

Realizing the shortcomings of the Epstein equation, Brock (15) carried out more rigorous first order slip flow continuum analysis of the thermal diffusion process. Brock's correlation for the thermal force has been found to predict the values of thermal forces well in agreement with the experimental facts for all the values of thermal conductivity of solid particles. The value of $\lambda/r_p$, however, has been found to affect the validity of Brock's equation. It is found to be accurate for $\lambda/r_p < 0.25$, but only approximately valid for $\lambda/r_p > 0.25$. For $\lambda/r_p > 0.25$, Brock's equation has been found to predict thermal force as low as 50 to 100% of experimental ones. For the present purpose, since $\lambda/r_p$ is as high as 0.50 at high temperature, it is expected that the values of thermal force at high temperatures calculated by Brock's equation could be as low as 30 to 50% of experimental ones.

In this short section, the thermal diffusive force and the valocity for an ice nuclei (of 25Å radius) in the frost interior are calculated using both Epstein's and Brock's theoretical analysis. The required temperature gradients at various points were obtained from a typical
T(x) (experimental) profile in the frost interior. The results of these calculations are shown in Table II-E3. The equations used for the calculations are shown below.

**Epstein equation** (95)

**A. Thermal Force**

\[
F_t = -9\pi r_p \left( \frac{\eta}{\varphi} \right)^2 \left( \frac{K_f}{2K_i + K_{ice}} \right) \left( \frac{dT}{dx} \right) \tag{II-E22}
\]

**B. Thermal Diffusive Velocity**

\[
V_t = \frac{3}{2} \left( 1 + \frac{0.82 \times 10^{-5}}{r_p} \right) \left( \frac{K_f}{2K_i + K_{ice}} \right) \left( \frac{\eta}{\varphi} \right)^2 \left( \frac{dT}{dx} \right) \tag{II-E23}
\]

**Brock equation** (15)

**A. Thermal Force**

\[
F_t = 4\pi \gamma^2 \left( \frac{1}{1 + 6\lambda r_p} \right) \left( \frac{R_i + 15\lambda}{5\varphi} \right) \left( \frac{dT}{dx} \right) \tag{II-E24}
\]

**B. Thermal Diffusive Velocity**

\[
V_t = \frac{3}{2} \left( \frac{\gamma}{\varphi} \right) \left( \frac{1}{1 + 6\lambda r_p} \right) \left( \frac{R_i + 15\lambda}{5\varphi} \right) \left( \frac{dT}{dx} \right) \tag{II-E25}
\]

where,

\[
\gamma = 0.499 \sqrt{\pi} \lambda \tag{II-E26}
\]

All the nomenclature in Equation II-E22 to II-E26 are described in Appendix R.

From the results of Table II-E3, it can be seen that Epstein equation predicts that velocity due to thermal force would decrease (more sharply near the cold wall) with decrease in the temperature in the frost interior. As seen previously, however, these predictions may be erroneous due
to high thermal conductivity values of ice at low temperatures. Brock's equation, on the other hand, predicted that thermal diffusive velocity would be increased with depth in the frost interior and would be approximately proportional to the prevailing temperature gradient. Since Brock's equation has been found to predict thermal force and thermal diffusive velocity well in agreement with the experimental results of many other materials (whose thermal conductivities are in the range of those of ice), its predictions are taken to be more valid than the predictions of Epstein's equations, for the present purpose. Thus, in summary, it is concluded that the thermal diffusive velocity in the frost interior would be increased with depth from the frost surface and would approximately vary as dT/dx.
Table II-A1

Correlation between Heat Transfer Coefficient and Time for the Formation of a Smooth Layer of Frost

<table>
<thead>
<tr>
<th>Heat Transfer Coefficient (Btu/hr ft² °F)</th>
<th>Time to Form Smooth Layer of Frost (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.62</td>
<td>39</td>
</tr>
<tr>
<td>7.25</td>
<td>28</td>
</tr>
<tr>
<td>11.08</td>
<td>13.1</td>
</tr>
</tbody>
</table>

The above data fits the equation:

\[ t = \frac{(13 - h)}{0.19} \]

for \( h \geq 13 \) Btu/hr ft² °F

and:

\[ t = 0 \]

for \( h < 13 \) Btu/hr ft² °F

Table II-A2

Effect of Negative Time Increment on the Computer Solution of 'Simple Model'

System Conditions:
- Reynolds number = 9,300
- Humidity = dew point at 29.3°F
- Gas temperature = 70°F
- Wall temperature = -315°F

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Frost Surface Temperature (°F)</th>
<th>Frost Density (gms./cc.)</th>
<th>Frost Thickness (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>13.0940</td>
<td>0.0685</td>
<td>0.176040</td>
</tr>
<tr>
<td>71</td>
<td>13.1774</td>
<td>0.066647</td>
<td>0.174002</td>
</tr>
<tr>
<td>69</td>
<td>13.2263</td>
<td>0.065462</td>
<td>0.172198</td>
</tr>
<tr>
<td>67</td>
<td>13.2585</td>
<td>0.0636945</td>
<td>0.170376</td>
</tr>
<tr>
<td>65</td>
<td>13.2683</td>
<td>0.0619888</td>
<td>0.168644</td>
</tr>
<tr>
<td>63</td>
<td>13.2563</td>
<td>0.0602259</td>
<td>0.167026</td>
</tr>
<tr>
<td>61</td>
<td>14.1311</td>
<td>0.0584016</td>
<td>0.165539</td>
</tr>
<tr>
<td>59</td>
<td>14.4781</td>
<td>0.0565129</td>
<td>0.164264</td>
</tr>
<tr>
<td>57</td>
<td>14.9780</td>
<td>0.0545561</td>
<td>0.163020</td>
</tr>
<tr>
<td>55</td>
<td>15.4791</td>
<td>0.0525278</td>
<td>0.162438</td>
</tr>
<tr>
<td>53</td>
<td>16.0664</td>
<td>0.0504250</td>
<td>0.161204</td>
</tr>
<tr>
<td>51</td>
<td>16.8213</td>
<td>0.0482455</td>
<td>0.162112</td>
</tr>
<tr>
<td>49</td>
<td>17.7209</td>
<td>0.0459885</td>
<td>0.162763</td>
</tr>
<tr>
<td>47</td>
<td>18.7844</td>
<td>0.0436556</td>
<td>0.164221</td>
</tr>
<tr>
<td>45</td>
<td>20.0300</td>
<td>0.0412520</td>
<td>0.166645</td>
</tr>
<tr>
<td>43</td>
<td>21.4702</td>
<td>0.0387878</td>
<td>0.170335</td>
</tr>
<tr>
<td>41</td>
<td>23.1089</td>
<td>0.0362807</td>
<td>0.175652</td>
</tr>
<tr>
<td>39</td>
<td>26.9333</td>
<td>0.0337574</td>
<td>0.183044</td>
</tr>
<tr>
<td>37</td>
<td>29.9849</td>
<td>0.0312549</td>
<td>0.193055</td>
</tr>
<tr>
<td>35</td>
<td>31.8066</td>
<td>0.0288208</td>
<td>0.206300</td>
</tr>
<tr>
<td>33</td>
<td>33.1125</td>
<td>0.0265090</td>
<td>0.223467</td>
</tr>
<tr>
<td>31</td>
<td>35.0044</td>
<td>0.0247264</td>
<td>0.245158</td>
</tr>
<tr>
<td>29</td>
<td>36.7048</td>
<td>0.0207978</td>
<td>0.271388</td>
</tr>
<tr>
<td>27</td>
<td>38.1838</td>
<td>0.0193517</td>
<td>0.303928</td>
</tr>
</tbody>
</table>
### Table II-C1

**Calculated Distribution of Supersaturation Ratio within the Frost**

<table>
<thead>
<tr>
<th>X (inches) distance from cold wall</th>
<th>P (mm Hg) partial pressure of water vapor</th>
<th>( P_{vp} ) (mm Hg) Vapor pressure of water vapor</th>
<th>( P/P_{vp} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.0711</td>
<td>3.59 x 10^{-35}</td>
</tr>
<tr>
<td>0.01</td>
<td>0.2630</td>
<td>3.98 x 10^{-25}</td>
<td>6.61 x 10^{23}</td>
</tr>
<tr>
<td>0.02</td>
<td>0.8275</td>
<td>7.14 x 10^{-7}</td>
<td>1.11 x 10^{6}</td>
</tr>
<tr>
<td>0.04</td>
<td>1.558</td>
<td>0.0001</td>
<td>15.58</td>
</tr>
<tr>
<td>0.06</td>
<td>2.515</td>
<td>0.00134</td>
<td>1.872</td>
</tr>
<tr>
<td>0.08</td>
<td>3.670</td>
<td>0.00350</td>
<td>1.045</td>
</tr>
<tr>
<td>0.10</td>
<td>5.000</td>
<td>0.043</td>
<td>116.20</td>
</tr>
<tr>
<td>0.12</td>
<td>6.500</td>
<td>0.130</td>
<td>50.0</td>
</tr>
<tr>
<td>0.14</td>
<td>8.1400</td>
<td>0.364</td>
<td>22.35</td>
</tr>
<tr>
<td>0.16</td>
<td>9.930</td>
<td>0.870</td>
<td>11.40</td>
</tr>
<tr>
<td>0.20</td>
<td>11.850</td>
<td>1.750</td>
<td>6.775</td>
</tr>
<tr>
<td>0.22</td>
<td>13.900</td>
<td>3.10</td>
<td>4.40</td>
</tr>
</tbody>
</table>

### Table II-B1

**Comparison Between Molecular and Thermal Diffusion in the Frost Interior**

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>( \frac{dc_a}{dT} )</th>
<th>( 0.180 \frac{c_w c_A}{T} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>+20</td>
<td>1.70 x 10^{-4}</td>
<td>0.0131 x 10^{-4}</td>
</tr>
<tr>
<td>-30</td>
<td>7.65 x 10^{-7}</td>
<td>0.0352 x 10^{-7}</td>
</tr>
<tr>
<td>-145</td>
<td>2.27 x 10^{-9}</td>
<td>0.0138 x 10^{-9}</td>
</tr>
<tr>
<td>-215</td>
<td>2.61 x 10^{-12}</td>
<td>0.0199 x 10^{-12}</td>
</tr>
<tr>
<td>-300</td>
<td>6.98 x 10^{-16}</td>
<td>0.0742 x 10^{-16}</td>
</tr>
</tbody>
</table>

\( \frac{dc_a}{dT} \) = Term proportional to the driving force gradient for ordinary molecular diffusion

\( 0.180 \frac{c_w c_A}{T} \) = Term proportional to the driving force gradient for thermal diffusion.
## Table II-E3
### Thermal Force and Velocity at various Positions in the Frost Interior Calculated from Epstein(95) and Brock(15) Equations

<table>
<thead>
<tr>
<th>Temp. (°F)</th>
<th>Position from Cold Wall (inches)</th>
<th>( \frac{(F_t)\text{E}}{\text{dx}} )</th>
<th>( \frac{(F_t)\text{B}}{\text{dx}} )</th>
<th>( \frac{(dT)}{\text{dx}} ) (Ft)</th>
<th>( \frac{(V_t)\text{E}}{\text{hr}} ) (ft^2-oR hr)</th>
<th>( \frac{(V_t)\text{B}}{\text{hr}} ) (ft^2-oR hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-270</td>
<td>0.01</td>
<td>-14</td>
<td>-11</td>
<td>4</td>
<td>-5</td>
<td>-3</td>
</tr>
<tr>
<td>-214</td>
<td>0.025</td>
<td>-13</td>
<td>-11</td>
<td>4</td>
<td>-5</td>
<td>-3</td>
</tr>
<tr>
<td>-135</td>
<td>0.055</td>
<td>-12</td>
<td>-11</td>
<td>4</td>
<td>-4</td>
<td>-3</td>
</tr>
<tr>
<td>-80</td>
<td>0.095</td>
<td>-12</td>
<td>-11</td>
<td>4</td>
<td>-4</td>
<td>-3</td>
</tr>
<tr>
<td>-30</td>
<td>0.145</td>
<td>-12</td>
<td>-11</td>
<td>4</td>
<td>-4</td>
<td>-3</td>
</tr>
</tbody>
</table>
Heat Transfer Coefficient = $h$
Mass Transfer Coefficient = $K_g$

Direction of Heat and Mass Transfer

Reynolds Number = $Re$
Gas Temperature = $T_g$
Humidity = $p_g$

Figure II-A6 Schematic of Frosting Process as Visualized by 'Simple Model'
Figure II-A1  Comparison between Experimental Facts and Theoretical Predictions of 'Simple Model'
Figure II-A2 Comparison between Experimental Facts and Theoretical Predictions of 'Simple Model'
Comparison between Experimental Data and Theoretical Predictions of 'Simple Model'

**System Conditions**
- Re = 5,530
- Humidity = Gas dew point at 33.2°F
- T = 48°F

Figure II-A3  Comparison between Experimental Facts and Theoretical Predictions of 'Simple Model'
Figure II-A4 Comparison between Experimental Facts and Theoretical Predictions of 'Simple Model'
Figure II-A5 Comparison between Experimental Facts and Theoretical Predictions of 'Simple Model'
Figure II-A6 Comparison between Experimental Facts and Theoretical Predictions of 'Simple Model'
Figure II-A7 Comparison between Experimental Facts and Theoretical Predictions of 'Simple Model'
Figure II-A8  Comparison between Experimental Facts and Theoretical Predictions of 'Simple Model'
Figure II-A9 Comparison between Experimental Facts and Theoretical Predictions of 'Simple Model'
Figure II-A10 Comparison between Experimental Facts and Theoretical Predictions of 'Simple Model'
Experimental Conditions

- Re=18,500; Humidity-Gas dew point at 29°F; T = 70°F
- Re=5,600; Humidity-Gas dew point at 29.2°F; T = 70°F
- Re=9,300; Humidity-Gas dew point at 40°F; T = 70°F
- Re=5,600; Humidity-Gas dew point at 40°F; T = 70°F
- Re=18,500; Humidity-Gas dew point at 40°F; T = 70°F

--- Computer Results of 'Simple Model'

--- Predictions of 'simple model'

Experimental Data

- Re=18,500; Humidity-Gas dew point at 50°F; T = 70°F
- Re=5,600; Humidity-Gas dew point at 29.2°F; T = 70°F
- Re=9,300; Humidity-Gas dew point at 29.2°F; T = 70°F

--- Predictions of 'simple model' and Experimental Data of Brazinsky [1b]
Figure II-A13
Effect of Initial Values of Average Frost Density and Frost Thickness on the Predictions of 'Simple Model'

Figure II-A14
Effect of Starting Values on the 'Simple Model' Predictions for Time Variation in Average Frost Density and Frost Thickness

Experimental Conditions
△ R = 9,300; Humidity = Gas dew point at 294°F, T = 70°F

- - - 'Simple Model' Predictions with initial \( \kappa = 0.025 \)
- - - 'Simple Model' Predictions with initial \( \kappa = 0.032 \)
- - - 'Simple Model' Predictions with initial \( \kappa = 0.068 \)

Curve obtained with the initial values taken at the time for formation of a smooth frost layer.

Curve obtained with the initial values taken at the time for formation of a smooth frost layer.
System Conditions

- \( R_e = 5,600 \); Humidity = Gas dew point at \( 14.3 \) °F; \( T_g = 74 \) °F
- \( R_e = 8,000 \); Humidity = Gas dew point at \( 14.3 \) °F; \( T_g = 74 \) °F
- \( R_e = 15,800 \); Humidity = Gas dew point at \( 14.3 \) °F; \( T_g = 74 \) °F

Figure II-A15 'Simple Model' Predictions for Effect of a Step Change in Reynolds Number on Frost Properties
Figure II-A16  'Simple Model' Predictions for Effect of a Step Change in Gas Temperature on Frost Properties
Figure 11-A17 "Simple Model" Predictions for Effect of a Step change in Humidity on Frost Properties.
Heat Transfer Coefficient = $h$
Mass Transfer Coefficient = $K_g$

Reynolds Number = $Re$
Gas Temperature = $T_g$
Humidity = $p_g$

Direction of Heat and Mass Transfer

Gas Flow

Heat Flux = $h(T_g - T_s) + K_g(p - p_s) \Delta H$
Mass Flux = $K_g(p - p_s) g$

Thermal Boundary Layer

Frost Surface
(Temperature = $T_s$)

Frost Layer

Temperature Profile
(Temperature = $T_w$)

Figure II-B0  Schematic of Frosting Process as Visualized by 'Improved Model'
System Conditions

Re = 5,600
Humidity = Gas dew point at 29.2°F
Gas Temperature = T

θ_f, initial = 0.026 gms./cc.
δ_{initial} = 0.023 inches
θ_{initial} = 2 mins.

---

Predictions of 'Sophisticated Model'
---
Predictions of 'Simple Model'

System Conditions

Re = 5,600; Humidity = Gas dew point at 29.2°F
T = 70°F

Starting Time for Theoretical Predictions = 2 mins.

θ = 7 mins.
θ = 20 mins.
θ = 2 mins.
θ = 4 mins.
θ = 17 mins.
θ = 12 mins.
θ = 6 mins.
θ = 2 mins.

Figure II-B3 Predictions of 'Sophisticated Model' for Time Variation in Frost Density and Thickness

Figure II-B4 Predictions of Time Variation in Temperature Distribution within the Frost by 'Simple' and 'Sophisticated' Models
System Conditions
Re = 14,700
Humidity = Gas dew point at 29.2°F
Tg = 70°F

θ_{initial} = 0.076 gms./cc.
δ_{initial} = 0.144 inches
θ_{initial} = 30 mins.

Predictions of the Variation in Temperature Distribution within the Frost by 'Simple and 'Sophisticated' Models

Figure II-B5 Predictions of 'Sophisticated Model' for Time Variation in Frost Density and Thickness

Figure II-B6 Predictions of Time Variation in Temperature Distribution within the Frost by 'Simple and 'Sophisticated' Models

Starting Time for Theoretical Predictions = 30 mins.
Figure II-B7: Comparison between Predictions of 'Simple' and 'Sophisticated' Models

System Conditions
Re = 5,600
Humidity: Gas dew point at 29.2°F
Tg = 70°F

Figure II-B8: Comparison between Theoretical Predictions of 'Simple' and 'Sophisticated' Models
Figure II-B9  Density Distribution within the Frost as a Function of Time

\[ D_{1-2} = \frac{0.001858 \times T^{1.5}}{C_{D} F \Pi \rho} \]

\[ \sigma_{d} = 3.165 \text{ (from Reference 94)} \]

\[ N_{1} = 18 \]

\[ N_{2} = 29 \]

\[ \Omega_{a} = 1.449 \left( \frac{W}{L} \right)^{0.50877} \text{ (from this Figure)} \]

\[ \frac{E_{\Pi}}{R} = 180.5 \text{ (94)} \]

\[ D_{1-2} = 2.6795 \times 10^{-6} \times T^{2.00877} \]

where, \( T \) is in \( {\text{deg}} \) and \( D_{1-2} \) is in \( \text{cm}^{2} / \text{sec} \),

\[ \Omega_{a} = 1.449 \left( \frac{W}{L} \right)^{0.50877} \]

Figure II-B10  Correlation for Diffusion Coefficient of Water Vapor in Air
Figure II-C1 Experimental Data of Brazinsky (1b) for Average Frost Density as a Function of Time

Figure II-C2 End of Run Temperature Profile within the Frost Measured by Brazinsky (1b)
Figure II-C3 Partial Pressure and Vapor Pressure as a Function of Position in the Frost Interior

Figure II-C4 Supersaturation Ratio ($p/P_{\text{vP}}$) as a Function of Position in the Frost Interior

System Conditions

Re = 14,700
Humidity-Gas dew point at 40°F
$T_g = 70°F$
Gas Flow
Reynolds Number = Re
Humidity = \( p_g \)
Gas Temperature = \( T_g \)

Water Vapor Transport
\[
\frac{D_g}{RT} \frac{(1 - \theta_{ice}) \partial \theta}{\partial x} \quad x = \delta
\]

\[
= \frac{D_g}{RT} \frac{\partial \theta_{ice} \partial \theta_{cf}}{\partial x} \quad x = \delta
\]
Gas Flow

Figure II-E1 Schematic of Frosting Process as Visualized by 'Non-Equilibrium Model'
SECTION III

APPARATUS AND PROCEDURE


I. Apparatus

1. Flow Sheet

A diagram of flow system is shown in Figure III-A1. The most important feature of the equipment was the cold plate onto which the frost was deposited. As shown, it is a small circular copper plate, which formed part of the underside of a long rectangular duct and was mounted at a distance of 72 equivalent duct diameters from the duct entrance. The air from a compressor was filtered and divided into two streams. One was dried with 4-A molecular sieve while the other was saturated with water vapor by passing it counter-current to water in a humidifier packed with one-inch raschig rings. The two streams when remixed produced a gas with the desired humidity. The humidified air was then metered with an orifice passed through a heat exchanger to vary the gas temperature and then through a mixer to obtain a well-mixed stream.

Before entering the duct, the air was passed through a diverging nozzle, an aluminum box containing 2 inch thick honeycomb and two 16 mesh screens and finally through a converging nozzle. The purpose of this inlet section of the duct was to obtain a uniform velocity profile in the flowing gas. The schematic of this section is shown in Figure III-A3.

The gas which was now—at the desired temperature and
humidity, flowed through five feet of duct (see Figure III-A3) before it passed over a cold circular copper plate. The purpose of this five feet duct length was to obtain fully developed velocity profile in the gas before it passed over the cold plate. The air leaving the duct was purged out to the atmosphere. The pipings of the system were made of one inch diameter or smaller polyvinyl chloride, metal and rubber tubings. The instrumentation used in this system is described in Appendix A.

The three independent system parameters studied in the present research work were Reynolds number (based on the hydraulic diameter of the duct), gas temperature and humidity. The Reynolds number was varied by changing the flowrate of air. The gas temperature was varied by submerging the heat exchanger in an ice and brine cold bath for low gas temperature experiments and by heating the gas by means of an electrical heating tape would around the heat exchanger for high gas temperature experiments. The gas humidity was varied by adjusting the air flow rate through the drier and the humidifier.

2. Copper Plate and Associated Boil-off Calorimeter.

The copper plate onto which the frost deposited was a circular plate of 5.54 inch diameter and $\frac{1}{4}$ of an inch thick. The plate actually consisted of two sections; an inner section which served as the heat transfer test section and an outer annular "ring" which served as a guard section. The inner section was 2.235 inches in diameter.

A 0.02 inch thick epoxy ring separated the two sections of
the plate. The primary purpose of the epoxy ring was to reduce heat conduction from the outer annular section into the inner section. This ring also bonded the two sections of the plate together and supported the weight of the stainless steel liquid nitrogen calorimeter vessels, the inner section of copper plate and the 17 copper equilibrator rods silver soldered to the bottom of this inner section. A detailed sketch of the plate arrangement is shown in Figure III-A2. Ninety-one copper equilibrator rods were silver soldered to the bottom of the outer annular section. (Before soldering the rods to the plate, each rod was inserted in a hole, 1/8 of an inch in diameter by 1/8 inch deep). The rods provided a low thermal resistance path between the copper plate and the liquid nitrogen in the calorimeter, thus maintaining the plate at a temperature within several degrees of the normal boiling point of liquid nitrogen.

An outer brass cylinder filled with liquid nitrogen served as a heat transfer guard chamber for the inner stainless steel test vessel. The outer annular section of the copper plate was fitted into the brass chamber by means of a 1/8 inch deep groove, machined in the outer copper plate. The connection between the copper plate and the brass chamber was sealed by epoxy resin. All the 91 rods were contained within the brass chamber (see Figure III-A2). The entire calorimeter assembly could be jacked up-and-down whenever desired.
The copper plate was surrounded by ½ inch thick styroform plate. This assembly is shown in Figure III-A4. The two ends of the styroform plate were flush with the bottom of the duct. The purpose of the styroform plate was to insulate the copper plate from the bottom of the aluminum duct. The frost growth during an experiment was viewed through the Lucite "windows" which formed part of the sides of the duct.

The brass chamber was insulated with four layers of fiberglass aircraft insulation. Each layer was separated by aluminum foil to reduce radiation to the chamber.

Four lines were silver soldered to the brass cylinder (see Figure III-A2). Line I provided continuous ventilation of the liquid nitrogen boil-off. Thermocouples measuring the temperature of the outer plate and temperature of the vapor space in the outer cylinder were inserted through Line II. Line III served as the liquid inlet, and Line IV was the liquid nitrogen overflow line. After each run the liquid nitrogen was drained from the calorimeter through Line III.

3. Rectangular Duct.

The rectangular duct was 7 feet long, 10.25 inches wide and 0.4 inch high. Five feet of warm duct preceded the cold copper plate and surrounding styroform plate assembly to ensure that the turbulent velocity profile in the gas passing over the cold plate was fully developed.

A large aspect ratio (width/height = 25) was used in order to simulate the case of flow between infinitely wide parallel
plates. It should be noted that for this geometry, the clearance between the parallel plates does not affect the Reynolds number (since \( Re = \frac{2g}{L^2} \)).

The top of the duct above the plate was made of 0.25 inch thick Lucite. This Lucite section was easily removed, allowing the frost to be sampled at the end of each run. The movable thermocouple probe, which was used to measure the frost surface temperature, was mounted in this Lucite section (see Figure III-A2). The entire inside of the duct was lined with aluminum foil in order to minimize radiant heat transfer to the frost surface.

The duct was made of No. 14 gauge aluminum. Before each run, the copper plate was jacked up flush with the bottom of the duct. During a run, as frost deposited on the copper plate, the calorimeter assembly was lowered almost continuously such that the frost surface always remained flushed with the floor of the duct.

4. **Humidity Control System.**

A drier with 4-A molecular sieves (see Section III-A5) and a humidifier packed with one-inch Raschig rings (see Section III-A6) arranged in parallel were used to control the gas stream humidity (Figure III-A1). The desired humidity was obtained by controlling gas flow through both the humidifier and drier, and subsequently remixing these separate streams. The gas flowing through the humidifier could be saturated at any temperature between 40°F and 75°F depending upon the temperature of the water circulating through the tower. The
drier was designed such that the completely dried and fresh 4-A molecular sieve would lower the dew point of flowing gas up to approximately -90°F. Thus, with this arrangement, a very wide range of humidities could be obtained. In the present thesis, however, the humidity was varied from a dew point of about 14°F to a dew point of about 33°F except for a few experiments in which flowing air was at dew point of approximately 58°F. The water for the humidifier and cooling coils in the drier was supplied from the building high pressure main.

5. Drier.

The drier containing about 50lbs. of 4-A molecular sieve was made of No. 14 guage aluminum sheet. It was a cylindrical tank 3 feet high and 14 inches in outside diameter. The molecular sieve was supported by a perforated plate fastened to the walls. A filter cloth was laid on top of the plate to filter the gas leaving the drier and a thin layer of glass wool was placed on the top of the molecular sieve to filter the gas before it came into contact with the sieve.

The top of the drier was equipped with two circular Eisenglass windows. If silica-gel were to be used for drying water vapor, its color change due to adsorption of water could be visualized from these windows. Moist gas entered the top and exited at the bottom of the drier. For safety reasons, the drier was equipped with a safety valve.
Heat is evolved when water vapor is adsorbed by molecular sieve. In fact, if the temperature of the bed is allowed to rise significantly, water vapor adsorption capacity is drastically reduced. The bed of molecular sieve was therefore cooled with water flowing through a coil imbedded in the sieve. Four thermocouples were inserted to measure the temperatures of the bed (see Figure III-A5). The bed was kept essentially at room temperature.

When the dew point of the air coming out of the drier exceeded 100°F, the molecular sieve was regenerated. The regeneration procedure consisted essentially of maintaining the sieve at a temperature of about 350°F for approximately 24 hours while dry nitrogen gas was being bled through the bed. The sieve was maintained at this temperature by high pressure steam which was passed through the imbedded stainless steel coil and by an electrical heater wrapped around the outside of the drier. The drier was insulated from its surroundings only during the regeneration period.

The insulation was removed before each experiment. Figure III-A5 shows the schematic of the regeneration process.

6. **Humidifier.**

The humidifier was a cylindrical tank made of No. 14 gauge aluminum. It was 3½ feet high and 14 inches in outside diameter. It was packed with 1 inch Raschig rings which were supported by a ½ inch mesh aluminum screen attached to the wall of the tank. Water was sprayed onto the packing
from an aluminum coil whose end was closed off and which contained holes on its underside. Water was supplied to it from a high pressure main. The schematic of the humidifier is shown in Figure III-A6.

As shown in the Figure III-A6, the gas entered at the bottom and exited at the top of the humidifier. During operation of the tower, a constant liquid seal of about 3½ inches was maintained over the liquid outlet in order to prevent gas from escaping by this route. The water level was viewed through a sight glass.

At the gas exit end, a baffle was placed 1 inch in front of the outlet to prevent the gas from jetting into the outlet tube. Glass wool and a filter cloth trapped entrained water droplets and large dust particles which would have been carried along with the exiting gas stream.

The humidifier was designed such that for a sufficiently large ratio of liquid to gas flow rate, the gas stream became saturated at all gas flow rates. For safety reasons, the packed tower was equipped with a safety valve.


The mixed gas stream from the humidifier-drier parallel combination was heated (cooled) in a heat exchanger. The heat exchanger was a 0.5 inch i.d. coiled copper tubing with an equivalent straight length of about 20 feet. The coiled heat exchanger had ten turns.

For experiments in which the gas temperature was below room temperature, the copper coil heat exchanger was
immers ed in an ice or ice and brine bath. The gas temperature above room temperature was obtained by winding an electrical heating tape around the heat exchanger. The gas temperature for these experiments was controlled by adjusting the electrical power input to the heater. The heated section in these experiments was surrounded by fiberglass insulation in order to prevent significant heat loss to the surroundings.

The gas temperature was measured at three locations in the system, at the entrance of the duct, just above the copper plate, and at the downstream end of the duct.


The gas flow rate was controlled by the needle valves placed at various locations in the system as shown in Figure III-A1.

The gas flow rate was measured with an orifice meter. Details of the calibration and construction are presented in Appendices B and A respectively.

9. Duct Inlet and Outlet Chambers.

Before the gas entered the duct, it flowed through a diverging nozzle, a rectangular box containing a 2 inches thick honeycomb and two 16 mesh aluminum screens, and finally through a converging nozzle. All these sections were made of No. 14 gauge aluminum sheet and were connected by No.8 gauge aluminum flanges. The dimensions of each piece are shown in Figure III-A3.
The purpose of the nozzles, honeycomb and the screens was to provide the gas with a uniform velocity profile at the entrance to the duct. The five feet of warm duct preceding the copper plate was used \((39, 72)\) to obtain the fully developed velocity profile gas flow.

After leaving the cold plate and surrounding styrofoam assembly, the gas passed through a ten inch long duct and then through a converging nozzle before it was purged out to the atmosphere. The outlet end of the duct was made of No. 14 gauge aluminum sheet. The detailed dimensions of which are shown in Figure III-A.

II. Experimental Procedure

1. Preliminaries Before a Run

Before each run was started, it was necessary to determine the appropriate settings for the flow control valves in order that the gas entering the duct would have the desired humidity, flow rate and temperature. The pre-run procedure for finding these appropriate valve settings is described below.

The knowledge of the humidities of the gas which would leave the drier and the humidifier was obtained by making two experiments. In the first experiment, no gas was flown through the humidifier and the humidity of the gas leaving the drier was measured by the method described in Appendix A. In the second experiment, no gas was flown through the drier and the humidity of the gas leaving the humidifier was measured by the same method. In the second experiment, the
maximum attainable humidity of the exit gas was obtained by measuring the temperature of the water circulating through the tower. From these information, the necessary flow rates through the drier and the humidifier for the required gas humidity were calculated. A non-frosting run at the desired gas flow rate was then made to adjust the valve settings for the required humidity. Once the settings of the flow control valves were determined during the preliminaries, it was generally not necessary to change them during the actual run.

A second step in the preliminary procedure involved adjusting the temperature of the gas entering the duct. For low gas temperature experiments, this was accomplished by adjusting the temperature of the ice and brine bath and the length of the heat exchanger immersed in the bath. For high gas temperature experiments, the required temperature was accomplished by changing the electrical power input to the heater. The variac setting determined during the preliminary procedure was then used for the actual run.

2. Start-up

Once the preliminaries described above were completed, the over-all system was purged with dry nitrogen gas. After about 5 to 10 minutes of purging, liquid nitrogen was supplied to the calorimeter from a 25 litre dewar. The liquid filled the inner and outer cylinders of the calorimeter and then flowed out through the overflow line (see Figure III-A2). This overflow was collected in an empty dewar. (During the
run, the liquid overflow rate was maintained as constant as possible. The frosting experiment was not started, until the temperature of the inner and outer sections of the copper plate became constant at approximately \(-320^\circ F\).

Before the run actually started, (i.e. before the gas was allowed to flow over the cold copper plate), the variac in the heater circuit was set to the desired values for high temperature experiments and for low temperature experiments the known heat transfer area of the heat exchanger was immersed in the prepared ice or ice and brine bath. The thermocouples used to measure the temperature distribution within the frost were fastened to the copper plate by two metal poles 0.003 inches thick and about 0.3 inches high. The distances between the thermocouples and the copper plate were measured by a depth micrometer. The calorimeter was rested on a jack, such that during a run it could be lowered. The copper plate was flushed with the bottom of the duct and the removable portion of the duct right above the plate was sealed with the taps.

Finally the water was circulated in the humidifier and the gas flow was started using the regulating value on the compressor. The opening of the regulating value on the compressor corresponded to the actual start of the run and hence to time zero.

3. **Procedure During the Actual Experiment.**

During an experiment the following measurements were carried out at various time intervals.
A. The liquid nitrogen boil-off rate from the inner cylinder was measured with a calibrated rotameter. In some experiments this reading was checked with a wet test meter connected in series with the rotameter. The boil-off measurement was used to calculate the heat flux into the heat transfer test section (inner cylinder of the calorimeter; see Appendix H). The boiled-off vapor was superheated as it flowed from the calorimeter to the measuring rotameter.

B. The pressures in the inner and outer chamber of the calorimeter and on the orifice meters were measured with the water manometers.

C. The various thermocouple readings were measured periodically. Thermocouple locations in the system were as follows:
   a. Under the inner and outer copper plates.
   b. In the inner and outer chamber of the calorimeter.
   c. In the frost interior.
   d. In the gas flow passage, as shown in Figure III-A3.

D. As frost was deposited, the calorimeter assembly was slowly but almost continuously lowered so as to keep the frost surface flushed with the bottom of the duct. This was necessary for maintaining the velocity profile in the gas flowing over the cold plate well developed and to keep heat and mass transfer coefficients between gas and frost surface constant during the run. The smooth transition between the floor of the duct and the frost surface was verified by visual examination through the Lucite "windows". By this method, the smooth transition could be obtained only within
E. In most of the experiments, it was necessary to switch liquid nitrogen supply dewars as the first dewar was depleted. The switch was accomplished by shutting the valve on the liquid nitrogen inlet line, lifting the liquid transfer tube out of the first dewar by means of a hot blower and inserting it into the second one. After the new connection was made leak tight, the valve on the liquid line was re-opened, allowing liquid nitrogen to flow again from the supply dewar to the calorimeter.

F. Towards the end of an experiment, the frost surface temperature was measured with a "movable" thermocouple probe. In making the measurement the probe assembly was lowered towards the frost surface. The millivolt output of this thermocouple was read on a potentiometer. The output was read and plotted as a function of distance from the centre of the duct. Figure III-A7 shows a typical plot of this output. The sharp change of the slope of the above plot determines the position of the frost surface. The location of the frost surface by this method can be obtained only within 0.02 inches. The visual observation of the movement of the thermocouple was done through the Lucite windows.

In general, care had to be exercised to prevent the thermocouple wire from causing any disturbances to the frost surface when it was moved in and out of the frost. If large chunks were torn out of the frost layer while retracting the thermocouple from the frost interior, the experiment was
discontinued. This type of accident was generally prevented by lowering the thermocouple wire slowly towards the frost surface. The wire would therefore cool significantly before contact. When a warm wire contacts the frost, localized melting results. The molten frost subsequently refreezes and bonds other parts of the frost to the wire. Retraction of the wire, of course, results in large chunks of frost being ripped out.

4. **Shutdown.**

At a predetermined time, the gas flow was stopped. The following operations were performed just after the shutdown in the chronological order:

A. The water flow rate through the humidifier was turned off.
B. The Lucite section of the duct (i.e., the short section just above the copper plate) was removed. Several frost samples at various locations on the copper plate were taken with the glass vials of known cross section.
C. The frost thickness was measured with a depth micrometer at various locations on the copper plate.
D. In some experiments, several samples of the frost were taken with the polystyrene tubes (see Section III-C) to determine the density gradient within the frost. These samples were immediately stored in a small closed container which was kept in a liquid nitrogen bath.
E. The frost was cleared away from the areas adjacent to the thermocouples without disturbing the probes. The heights of these thermocouples above the plate were then
measured with a depth micrometer.

F. If step (D) had been carried out then the samples of the various frost slices were obtained by the method described in Section III-C.

G. The glass vials containing the frost collected by step (B) and (F) were weighed. (By the time the vials were weighed, the frost had melted to liquid water). The actual frost weight was determined by subtracting the weight of the dry vials (in case of step (B)) or dry vial and polystyrene ring (in case of step (F)) from the tare weight. Densities were calculated from the measured frost weights and the volumes of the collected frost samples.

III-B Experimental Measurement of the Effect of a Temperature Gradient across the Frost on its Density.

1. Apparatus

To determine the effect of a temperature gradient on the frost density, an apparatus was constructed as shown in Figure III-B1.

Two hollow cylinders were mounted vertically end-to-end and the frost under investigation was placed in the measured space between the ends (see Figure III-B1). The sandwiched frost was divided into several slices by inserting 200 mesh, 0.002 inch thick phos-bronze screens. The frost was insulated from the surroundings by two inch thick fibre glass insulation. During a run, the two ends of the cylinders were maintained at different but constant temperatures. Experiments were made with both large and small temperature
differences across the frost. The complete experimental set-up was placed in a cold room whose temperature can be varied between +20°F to -20°F.

In the experiments with a large temperature difference across the frost, the copper plate of the lower hollow cylinder was maintained at liquid nitrogen temperature (-310°F). In order to reduce the heat loss, this cylinder was insulated with two inch thick asbestos. The top hollow cylinder, in these runs, was filled with Nujol (oil) and an electrical heater was inserted in this oil bath. With the help of this heat reservoir of oil bath, the copper plate of the top cylinder was maintained at approximately 15°F. The temperatures of both the lower and the upper copper plates were measured by copper-constantan thermo-couples. The experimental set-up for these experiments is shown in Figure III-B1.

In the experiments with a small temperature difference across the frost, the upper copper plate was kept once again at approximately 15°F. The lower copper plate, however, was kept at approximately -4°F by blowing cold air under the plate. The temperature of the cold room was maintained at approximately -4°F. The rest of the set-up was identical to the previous one. Figure III-B2 shows the set-up for these experiments.

2. Procedure.

The frost for these experiments was obtained from natural snow. The frost was sprinkled over the lower
copper plate. During this process of sprinkling, the phos bronze screens were inserted in the frost as shown in Figure III-B1. In order to determine the uniformity of the laid frost, the thickness and density of each slice of the frost were measured by the methods described in Section III-A. The top cylinder was then adjusted such that the top copper plate just touched the frost surface (see Figure III-B1). Thus, the frost was sandwiched between the plates. The sandwiched frost was then insulated on its sides by fibre glass insulation.

During all the experimental runs the temperature of both the copper plates was measured at intervals of ten minutes and a good contact between the top copper plate and the frost surface was assured by visual observation.

At the end of each experiment, density and thickness of each slice of the frost was again measured, taking a few samples on each screen and on the lower copper plate.

Experiment number R-7 was carried out to measure the density gradient in the frost deposited by natural convection on liquid nitrogen cooled copper plate. In this experiment liquid nitrogen was circulated in the lower cylinder. As soon as a certain thickness of frost was deposited, a phos bronze screen was placed over it. This phos bronze screen was supported by four small wooden supports such that the screen touched the frost without crushing it down. More frost was then allowed to accumulate on this screen and then the above process was repeated. Thus, several slices of
the frost were obtained. The density and thickness of each slice was then measured by the method described previously.

**III-C Experimental Measurement of Density Gradient Within the Frost.**

**Apparatus and Procedure.**

To determine if the frost deposited early in a run was of a different density from that deposited at a later stage, some frost cores were sliced horizontally to separate discrete portions identified by being formed at different times. This slicing was not easily done. After numerous attempts, a simple technique was developed using a small polystyrene tube. The tube was itself sliced into several rings 0.0625 inch thick. These rings were glued together to form the original tube. However, they could be easily separated with a knife. The tube was cooled in liquid nitrogen. The frost at the end of an experiment was then sampled with the cold polystyrene tube and placed into a cold box at liquid nitrogen temperature. The whole assembly was then carried to the -20°F cold room. The tube made of rings was cut with a cold razor. The density of the frost contained in each ring was measured by the method described previously.

**III-D Measurement of Heat Transfer Coefficient in Non-Frosting Conditions**

As seen in Section II, a theoretical evaluation of the frostling process requires an accurate knowledge of heat transfer coefficient. Hence, the heat transfer coefficients
under non-frosting conditions were measured in the experimental set-up of Figure III-A1.

In these experiments, the copper plate was maintained at a temperature of approximately -12°C by circulating liquid Freon -12 through the calorimeter. A corrugated aluminum foil was cemented over the copper plate in order to obtain frost like surface conditions. The measurements of heat flux were carried out in exactly the same manner as done in the frosting experiments (see Section III-A). The plate temperature was measured by imbedding a thermocouple in the cement.

The dried air was obtained by allowing all the air to flow through a drier containing 4-A molecular sieve. The dew point of the gas coming out of the drier was sufficiently low so that no frost was deposited on the cold plate during an experiment.

The heat transfer coefficients at various Reynolds numbers were obtained in this manner. In all the experiments the temperature of air was maintained at room temperature.
Figure III-A1 Flowsheet of Equipment
Figure III-A2  Copper Plate and Associated Boil-off Calorimeter
Figure III-A4 Schematic of Styroform Assembly Surrounding Copper Plate
Figure III-A5  Schematic of Drier Unit
Figure III-A7  Millivoltage Output versus Distance to Determine the Frost Surface Temperature
Figure III-81 Experimental Set-up for Measurement of Effect of a Large Temperature Difference on the Frost Density

Figure III-82 Experimental Set-up for Measurement of a Temperature Difference (Small) on the Frost Density
SECTION IV
EXPERIMENTAL RESULTS

The experimental results of this thesis are separated into four categories as follows.
A. The effect of the independent system parameters, Reynolds number, humidity and gas temperature on the average frost density, frost thickness, thermal conductivity, and rate of heat transfer through the frost.
B. The effect of a temperature gradient in the frost on its density.
C. The measurement of density gradients in the bulk of the frost.
D. The measurement of heat transfer coefficients for forced flow through a rectangular duct under the non-frosting conditions.

IV-A Effects of Various Independent System Parameters on Frost Properties

Summary

Average frost densities (densities of the entire frost layer), frost thicknesses and mass deposition rates were measured only after a smooth layer of frost was formed on the cold plate. The heat flux measurements were made over most of the frosting time (except for about the first ten minutes).

The range of independent system variables studied were: Reynolds number from 3,770 to 15,800, humidity
from a dew point at 14.3°F to a dew point at 58°F: and
gas temperature from 34°F to 93°F.

The most important measurements made in all the
frosting experiments are shown in Tables IV-A1 to IV-A5.
Average frost densities ranged from 0.053 gms./cc
to 0.163 gms./cc. Frost thickness varied from 0.08
inches to 0.40 inches. The total elapsed time for an
experiment was from 20 minutes to 180 minutes. The
quasi-steady state value of heat fluxes ran from
338 Btu/hr ft² to 830 Btu/hr ft².

The calculations of average thermal conductivity
of the frost (defined by Equation IV-A1) indicate that
over the average frost density range of 0.055 gms/cc to
0.163 gms./cc, the conductivity value varied from
0.0190 Btu/hr ft°F to 0.0534 Btu/hr ft°F. The point
thermal conductivity values of the frost (defined by
Equation IV-A2) increased both with density and temperature
and it ranged from approximately 0.01 Btu/hr ft°F to
0.1257 Btu/hr ft°F over the density range from 0.055
gms./cc to 0.163 gms./cc and temperature range from
-215°F to +100°F.

1. **Initial Phases of a Frosting Experiment**

In the initial stage of the frosting experiments,
before a significant amount of frost was deposited, a
fog was observed in the gas space immediately adjacent
to the plate. The amount of fog increased with humidity
and gas temperature. Shortly thereafter, islands of
frost began to form. Initially, the heights of the individual islands increased. Before long, however, the spaces between them began to fill in and very soon a smooth, thin ( ≈ 0.05 inch) layer of frost was formed. The formation of a smooth layer of frost occurred first at the upstream part of the test plate. Once a smooth layer of frost was formed, the gas over the frost appeared to be clear of fog particles.

The time required for the frost on the entire cold plate to become macroscopically smooth decreased with an increase in Reynolds number, but it was almost independent of gas phase humidity and gas temperature. The relationship between time required and the Reynolds number is shown in Table IV-A1. This relationship is very similar to one obtained by Brazinsky (14).

2. Frost Behavior as a Function of Time

The average frost density, frost thickness and the total mass deposition of the frost are continuously increasing functions of time as shown for a typical set of experimental conditions in Figures IV-A1 and IV-A2. The time dependence of these parameters were, however, found to be strong functions of the system conditions as described in Sections IV-A3 to IV-A5.

The heat flux through the frost decreased sharply in the initial phase of an experiment, but later levelled out to a quasi-steady state value. The time dependence of heat flux through the frost for a typical set of experimental
conditions is shown in Figure IV-A3. The frost surface temperature increased with time in the initial transient (heat transfer) state of the frosting process, but once a quasi-steady state of heat transfer was reached, it attained almost a constant value.

The average thermal conductivity of the frost increased with time, particularly in the quasi-steady state of heat transfer. The temperature of any differential layer of frost decreased with time, as the additional frost continued to deposit on the cold plate. This can be seen from the data of Table IV-A3.

3. Effect of Reynolds Number on Average Frost Density, Frost Thickness, Mass Deposition and Heat Transfer through the Frost

In the present study, three gas phase Reynolds numbers (based on hydraulic diameter), namely, 5,580, 8,000 and 15,800 were used. At a constant gas temperature of approximately 73°F and a constant humidity of dew point at 29.8°F, the effects of these three levels of Reynolds numbers on average frost density (density of the entire frost layer), frost thickness and on the mass deposition of the frost are described in Figures IV-A1 and IV-A2.

It should be noted that each point in the figures represent the measurement of a separate run.

As shown in Figure IV-A1, the increase in Reynolds number from 5,580 to 15,800 increased significantly both the average frost density and the rate of frost densification.
Increases in Reynolds number however, decreased the frost thickness slightly at a given time. The total mass deposition, as shown in Figure IV-A2, increased considerably with the increase in Reynolds number.

The heat flux through the frost at various Reynolds numbers are shown in Figures IV-A3 to IV-A5. The results of these figures indicate that at all Reynolds numbers, the heat flux through the frost decreased sharply initially but levelled out at a later time to (different) quasi-steady state values. The magnitude of quasi-steady state heat flux value increased considerably with an increase in Reynolds number (from 510 Btu/hr ft$^2$ at Reynolds number = 5,580, to 830 Btu/hr ft$^2$ at Reynolds number = 15,800). These results are in qualitative agreement with the observation that the frost of a lower thermal resistance was deposited with increase in Reynolds number, as shown in Figure IV-A1.

4. Effect of Humidity on Average Frost Density, Frost Thickness, Mass Deposition and Heat Transfer through the Frost

The effects of varying the gas phase humidity on the average frost density, frost thickness, and mass deposition at two different sets of Reynolds number and gas temperature, two levels of humidities were studied.

Figure IV-A6 shows the effect of variation in humidity from gas dew point at 18°F to dew point at 33.2°F (at Reynolds number of approximately 5,800 and gas temperature
of 48°F) on average frost density and thickness. For this humidity range, it was found that at an equivalent time, an increase in humidity increased the average frost density slightly but frost thickness considerably. This, of course, implies that the total mass deposition of the frost would be significantly increased with humidity. This is shown in Figure IV-A7. The slight effect of humidity on the heat flux through the frost is demonstrated in Figures IV-A10 and IV-A11.

Figures IV-A8, IV-A9, IV-A12 and IV-A4 describe the results similar to the ones described above for two other humidities and at Reynolds number of approximately 8,050 and at gas temperature of 75°F. Once again, as shown in Figures IV-A8 and IV-A9, a slight increase in average frost density and a considerable infrost thickness and mass deposition rates were observed with an increase in humidity. The slight decrease in the quasi-steady state value of heat flux with increase in humidity for the same experimental conditions can be seen from the results of Figures IV-A12 and IV-A4.

In summary, even though the effects of humidity on the time variation in frost thickness, mass deposition and heat flux found in the present thesis were very similar to ones obtained by Brazinsky (14), the present research indicate that the average frost density would be increased slightly with humidity for the range of gas dew point at 14.3°F to dew point at 33.2°F.
5. Effect of Gas Temperature on Average Frost Densities, Frost Thicknesses, Mass Deposition Rates and Heat Fluxes through the Frost

The effects of gas temperature on the frost properties were examined for two different sets of Reynolds numbers and humidities. In both the cases, two gas temperatures were studied.

As shown in Figure IV-A13, at a Reynolds number of approximately 5,500 and a gas dew point at 27.2°F, when the gas temperature was increased from 3°F to 93°F, the average frost density increased but frost thickness decreased so that the total mass deposition of the frost at any time was found to be almost unaffected (see Figure IV-A14). Thus, a frost of a considerably lower thermal resistance deposited at a higher gas temperature. It should be noted, however, that the gas temperature affects the frost properties significantly only in the quasi-steady state heat flux region. The density and the thickness of the frost at a very earlier time of the frosting process (in the transient heat transfer state) are nearly independent of the gas temperature as shown in Figure IV-A13.

Figures IV-A15 and IV-A16 show similar results for another set of gas temperatures. The results of these figures were obtained at a Reynolds number of approximately 8,200, a gas dew point of about 30°F and at the gas temperatures of 52°F and 76°F.

As shown in Figures IV-A17 and IV-A18, for Reynolds
number of approximately 5,500 and a gas dew point at 27.2°F, when the gas temperature was increased from 34°F to 93°F, the quasi-steady state heat flux increased from 334 Btu/hr ft\(^2\) to 535 Btu/hr ft\(^2\). Similarly, the heat flux through the frost versus time data for Reynolds number = 8,200, humidity = dew point at 29.8°F and for the two values of gas temperatures (52°F and 76°F) are described in Figures IV-A19 and IV-A4 respectively. As shown in the figures, the increase in gas temperature (at constant Reynolds number and humidity) from 52°F to 76°F increased the quasi-steady state heat flux from 460 Btu/hr ft\(^2\) to 560 Btu/hr ft\(^2\).

6. Frost Thermal Conductivity

a. Average Conductivity of the Frost Layer

The average conductivity of the frost layer was calculated from Equation IV-A1 below, using measured values of the heat flux, the frost surface temperature thickness and the temperature of the copper plate. All the thermal conductivity values were measured in the quasi-steady state of heat transfer region.

\[
q = \frac{k_{\text{avg}}}{\delta} (T_s - T_w) \quad (\text{IV-A1})
\]

Conductivity values calculated from Equation IV-A1 are tabulated in Table IV-A2. In Figure IV-A20, the average frost conductivity is plotted as a function of frost density. (The straight line shown in this diagram was obtained such that the total sum of the square of the deviation between the experimental values and the values
Figure IV-A20 also shows the experimental data of average frost thermal conductivity as a function of density obtained by Brazinsky(14). It can be concluded from these results that there is a close agreement between the average thermal conductivity data of the present research and those obtained by Brazinsky (14). The data of both the research works indicate that over the density range of 0.055 gms./cc. to 0.163 gms/cc. the average thermal conductivity increases about linearly with average frost density.

The various points in Figure IV-A20 correspond to frosts which were formed at different Reynolds numbers, humidities and gas temperatures. The measured surface temperatures of these frosts ranged from -20°F to +18°F. In plotting thermal conductivity as a function of density only, it was implicitly assumed that a given value of density implied a given frost structure, and therefore that the average thermal conductivity of the frost layer was a unique function of density. This is equivalent to assuming that the frosts of the same density formed at different Reynolds numbers, humidities and gas temperatures have the same structure. Actually, some of the scatter in the data, shown in Figure IV-A20, may indeed be caused by structural differences of the frosts of the same density. No definite trend, however, was observed. (it should be noted that the scatter may also be partly due to the different surface temperatures).
The measurements of average thermal conductivity by this method are accurate approximately within ± 7%. The detailed error analysis is described in Appendix N.

It should be noted that Equation IV-A1 is rigorously valid only for a true steady state situation. So, the results of Figure IV-A20 can be used to predict thermal conductivity of the frost only in the quasi-steady state heat transfer period.

b. Local or Point Thermal Conductivity

In order to obtain frost thermal conductivity as a function of density over a wide temperature range, temperatures within the frost as well as at the frost interfaces were measured. More specifically, temperatures within the frost were measured at two different vertical positions (in most of the experiments) with 0.003 inch diameter copper-constantan thermocouples. The measuring junctions of these probes were located at known heights from the plate. The stationary probe readings and the measured frost interface temperatures (i.e., frost-gas and frost-plate interfaces) were used to obtain a temperature profile at the end of each experiment. A typical temperature profile is shown in Figure IV-A21. In particular, Figure IV-A21 shows the temperature distribution within the frost at time of 79 minutes from the start of frosting process, (Run number 35) deposited at a Reynolds number of 15, 800, humidity of dew point at approximately 30°F.
and gas temperature of 73°F. The data on temperature distribution within the frost for the frosting experiments are listed in Table IV-A3.

The temperature gradient, \( \frac{dT}{dx} \), was obtained at any particular location by graphical differentiation (i.e., by eye) of the frost temperature profile. These values of the gradient and the measured heat fluxes were used to calculate the local (or point) thermal conductivity from the following Equation IV-A2.

\[
q = k_f \left( \frac{dT}{dx} \right)
\]  

(IV-A2)

Just as Equation IV-A1, above equation can be used only during the quasi-steady state period.

The frost thermal conductivities were generated over a temperature range of -215°F to +10°F and a density range of 0.055 gms./cc. to 0.163 gms./cc. Their values, as shown in Figure IV-A22, lie between 0.01 Btu/hr ft °F to 0.126 Btu/hr ft °F. The straight lines through the data were calculated by the same method as used for the achievement of the line shown in Figure IV-A20 (by the method of least squares). At any given density, the thermal conductivity showed a marked increase with temperature. For a given temperature, the thermal conductivity increased with density; the rate of this increase being greater at higher temperatures. In calculating point thermal conductivity data from Equation IV-A2, once again, it was implicitly assumed that at any given temperature a particular frost structure was implied.
by a given value of frost density.

Figure IV-A22 shows the experimental data of Brazinsky (14) for thermal conductivities. It can be seen from the figure that the data of Brazinsky (14) as well as of the present research gave an almost identical correlation for thermal conductivity as a function of density and temperature.

The model of the frost structure as one in which a symmetrical matrix of ice cubes are interconnected by ice rods (14) is found to predict thermal conductivity correlation well in agreement with one obtained from experimental data. The dotted lines in Figure IV-A22 show these correlations for various temperatures. It can be seen from the results of this figure that the theoretical predictions agree quite well with the experimental results. The details of the theoretical model is described in Section V-A.

Table IV-A4 lists the values of point thermal conductivity as a function of density at five different temperatures (-215°F, -145°F, -80°F, 0°F, and +10°F), as described in Figure IV-A22. These values are estimated to be accurate within approximately ±10%. The details of the error analysis is shown in Appendix N.

IV-B Experimental Measurements of the Effects of a Temperature Gradient Across the Frost on its Density

As shown in Section IV-A, it was found experimentally
that there existed a large temperature gradient in the frost interior. In order to evaluate the effect of this temperature gradient on the frost density, an experiment program was undertaken. Since, the experimental study was essentially divided into two parts based on the magnitude of the temperature difference across the frost, the results are also separated on the same basis as described below.

1. Effect of a Large Temperature Difference across the Frost on its Density

In this study, a frost layer was exposed to a large temperature gradient which was very similar to one obtained in the frosting experiments of Section IV-A (i.e. bottom end at -315°F and top surface at about 15°F).

Several experiments were made for various times, densities and thicknesses of the frost. The temperature difference across the frost was almost identical in all the experiments. The ranges of the independent system variables considered were as follows:

a. The length of time for an experiment varying from 2.5 hours to 7.0 hours.
b. The initial average frost density varying from 0.148 gms./cc. to 0.314 gms./cc.
c. The initial frost thickness varying from 0.382 inch to 0.582 inch.

In three tests, the frost layer was divided into two slices and in the remaining one it was divided into three slices (see Section III-B). The results of these experiments
are described in Table IV-B1.

The uniformity of the laid frost in each experiment was determined by measuring the densities at various positions on the copper plate. A typical set of results for this kind of measurements are shown in Table IV-B2. From these results it can be concluded that the average frost density values at various positions on the plate were approximately within 5% of each other.

The density values reported in Table IV-B1 are accurate within ± 8.5%. The results of Table IV-B1 indicate that the density values of the frost slices are changed by the temperature gradient only within the accuracy of their measurements. The details of the error analysis of the density measurements is shown in Appendix N.

The experimental results indicate that there was no significant effect of the length of an experiment and of the initial values of the frost density and the thickness on the density of the frost slices. An average of the differences in the initial and the final values of all the frost slices was 0.008 gms./cc. or less than 4% of the average initial frost density values for all the experiments. Thus, it can be concluded from these results that within the accuracy of this kind of measurements, a large temperature difference across the frost layer did not change the density significantly.

2. Effect of a Small Temperature Difference across the Frost on its Density
Two experiments were performed with a small temperature difference across a frost (bottom surface at -4°F and top surface at +23°F; see Section IV-B). The lengths of time for the two experiments were six and twelve hours. In each experiment, the frost under investigation was divided into two slices of different thickness. In both the experiments, the total initial thickness was approximately 0.37 inch and the initial average frost density was approximately 0.15 gms./cc. The results of this study are described in details in Table IV-B3.

The results of Table IV-B3 indicate that there is a significant mass transfer (mass transfer = 9.50x10^-4 gms./cm²/hr) from the top slice to the bottom slice of the frost at the temperature level considered in this study. The magnitude of mass transfer also increased with time. Thus, it can be concluded from these results that there will be a mass transfer due to temperature gradient at the temperature level of about 15°F.

The uniformity of the laid frost in this study was identical to one described in Table IV-B2.


As described in Section III-B, one experiment (Run number R-7) was performed to determine the density gradient in the frost deposited by natural convection on a liquid nitrogen cooled plate. The results of this experiment
are shown in Table IV-B4. In this experiment the deposited frost was divided into three slices by the method described in Section III-B. The measured density values of each slice of the frost (of varying thickness) after 6.5 hours of the experiment was found to be identical with a deviation of approximately ±4% from the mean value. The average density value was approximately 0.035 gms./cc. The surrounding temperature in this experiment was 15°F.

Thus it can be concluded from the results of Table IV-B4 that there is no significant density gradient in the frost deposited by natural convection on a surface at cryogenic temperature.

The measurements of density in all the experiments described above were, once again accurate within ±8% of its value. The details of error analysis is shown in Appendix N.

IV-C Measurement of Density Gradient in the Frost Deposited by Forced Convection on a Liquid Nitrogen Cooled Plate.

For a typical set of experimental conditions, (Re = 15,800; E = dew point at 30°F; Tg = 73°F), the density gradient within the frost was measured by the method described in Section III-C. The results of these measurements are described in Figure IV-C1. As shown in the figure, the density of the bottom 0.0625 inch thick frost was found to increase almost identically as the density of the entire frost layer between the time of 26 min to 1 hour 44 min from the start of the frosting process.
In one experiment, (at time = 106.9 min from the start of frosting) the density of the bottom 0.1250 inch frost was measured and was found to be identical to the density of the entire frost layer. The thickness of the frost during this density gradient measurement was found to increase from 0.091 inch to 0.256 inch.

The variation in temperature distribution with the frost with time during this period of density gradient measurement is shown in Figure IV-C2. As expected, the temperature of every existing frost layer decreased as more frost was deposited.

The results of Figure IV-C1 and IV-C2 indicate that in order for a densification of the bottom 0.0625 inch thick frost layer from 0.085 gms./cc. to 0.161 gms./cc., a significant amount of water had to pass the temperature level of the frost layer located at 0.0625 inch away from the cold wall (which varied from -44°F to -173°F)

A possible mechanism by which the required amount of water can be transported at that temperature level (average of -108°F) is discussed in Section V-C.

The measurement of the density of each slice of the frost by this method is accurate within approximately ±12%. The detailed error analysis for this kind of measurement is shown in Appendix N.

**IV-D Measurement of Heat Transfer Coefficients under the Non-frosting Conditions**

In order to evaluate heat and mass transfer characteristics
of the frosting process, an experimental program was carried out to measure the heat transfer coefficients under non-frosting conditions in the system of Figure III-A1. Dry air was blown over the copper plate which was covered with a slightly corrugated aluminum foil, in order to obtain a frost like surface over the copper plate. The temperature of the aluminum foil was maintained at approximately -12°F by circulating liquid Freon-12 in the calorimeter.

The results of this experimental program are shown in Figure IV-D1. The figure shows the heat transfer coefficient as a function of Reynolds number for the gas temperature of approximately 82°F. The experimental data were obtained with the Reynolds number range from 3,380 to 15,720. The heat transfer coefficient varied from 3.12 Btu/hr ft²°F to 11.08 Btu/hr ft²°F over this range of Reynolds number.

A solid line in Figure IV-D1 shows the correlation between heat transfer coefficient and Reynolds number as obtained from the Sleicher and Tribus's theoretical analysis at the conditions of the experiments (i.e. $T_g = 82°F, T_w = 12°F$). One can see from the figure that the theoretical correlation agree well with the experimental results. The maximum absolute deviation between experimental and theoretical values of heat transfer coefficient is found to be within 5%. The experimental data are estimated to be accurate within ±4%. The
details of the error analysis of the experimental data is shown in Appendix N.

The experimental data shown in Figure IV-D1 are also listed in Table IV-D1.
Table IV-A1

Effect of Reynolds Number on Length of Time Required to Form a Smooth Frost Layer

<table>
<thead>
<tr>
<th>Reynolds Number</th>
<th>Time to Form Smooth Frost (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,500</td>
<td>~39</td>
</tr>
<tr>
<td>8,000</td>
<td>~28</td>
</tr>
<tr>
<td>15,800</td>
<td>~10</td>
</tr>
<tr>
<td>Run No.</td>
<td>Time Start (min.)</td>
</tr>
<tr>
<td>---------</td>
<td>------------------</td>
</tr>
<tr>
<td>2</td>
<td>69.0</td>
</tr>
<tr>
<td>3</td>
<td>36.5</td>
</tr>
<tr>
<td>4</td>
<td>20.0</td>
</tr>
<tr>
<td>5</td>
<td>66.7</td>
</tr>
<tr>
<td>6</td>
<td>98.0</td>
</tr>
<tr>
<td>7</td>
<td>141.0</td>
</tr>
<tr>
<td>8</td>
<td>73.0</td>
</tr>
<tr>
<td>9</td>
<td>68.0</td>
</tr>
<tr>
<td>10</td>
<td>43.0</td>
</tr>
<tr>
<td>11</td>
<td>69.0</td>
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<tr>
<td>12</td>
<td>61.0</td>
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<tr>
<td>13</td>
<td>127.0</td>
</tr>
<tr>
<td>14</td>
<td>95.0</td>
</tr>
<tr>
<td>15</td>
<td>57.0</td>
</tr>
<tr>
<td>16</td>
<td>132.5</td>
</tr>
<tr>
<td>17</td>
<td>61.0</td>
</tr>
<tr>
<td>18</td>
<td>97.5</td>
</tr>
<tr>
<td>20</td>
<td>61.5</td>
</tr>
<tr>
<td>21</td>
<td>99.9</td>
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<tr>
<td>23</td>
<td>72.4</td>
</tr>
<tr>
<td>24</td>
<td>62.6</td>
</tr>
<tr>
<td>25</td>
<td>98.2</td>
</tr>
<tr>
<td>26</td>
<td>36.2</td>
</tr>
<tr>
<td>27</td>
<td>122.5</td>
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<tr>
<td>28</td>
<td>60.0</td>
</tr>
<tr>
<td>29</td>
<td>76.3</td>
</tr>
<tr>
<td>30</td>
<td>104.4</td>
</tr>
<tr>
<td>31</td>
<td>45.1</td>
</tr>
<tr>
<td>32</td>
<td>68.2</td>
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<tr>
<td>33</td>
<td>180.2</td>
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<tr>
<td>34</td>
<td>182.5</td>
</tr>
<tr>
<td>35</td>
<td>79.1</td>
</tr>
<tr>
<td>36</td>
<td>26.5</td>
</tr>
<tr>
<td>37</td>
<td>58.2</td>
</tr>
<tr>
<td>38</td>
<td>144.8</td>
</tr>
<tr>
<td>39</td>
<td>106.9</td>
</tr>
</tbody>
</table>
Table IV-A2

Temperature as a Function of Position in Frost

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Position in Frost, i.e., Distance from Copper Plate (inches)</th>
<th>Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.086</td>
<td>-80</td>
</tr>
<tr>
<td>11</td>
<td>0.046 0.073</td>
<td>-131 -53</td>
</tr>
<tr>
<td>13</td>
<td>0.023 0.077</td>
<td>-253 -120</td>
</tr>
<tr>
<td>14</td>
<td>0.078</td>
<td>-116.6</td>
</tr>
<tr>
<td>15</td>
<td>0.023</td>
<td>-225</td>
</tr>
<tr>
<td>16</td>
<td>0.062 0.125</td>
<td>-166 -60</td>
</tr>
<tr>
<td>17</td>
<td>0.031</td>
<td>-181.5</td>
</tr>
<tr>
<td>18</td>
<td>0.054 0.085</td>
<td>-161 -97</td>
</tr>
<tr>
<td>20</td>
<td>0.062 0.090</td>
<td>-119 -56</td>
</tr>
<tr>
<td>21</td>
<td>0.040 0.062</td>
<td>-109 -65</td>
</tr>
<tr>
<td>22</td>
<td>0.078</td>
<td>-76</td>
</tr>
<tr>
<td>23</td>
<td>0.034</td>
<td>-131</td>
</tr>
<tr>
<td>24</td>
<td>0.069 0.125</td>
<td>-151 -52</td>
</tr>
<tr>
<td>25</td>
<td>0.085 0.164</td>
<td>-157 -53</td>
</tr>
<tr>
<td>27</td>
<td>0.023 0.102</td>
<td>-203 -69</td>
</tr>
<tr>
<td>28</td>
<td>0.023 0.093</td>
<td>-221 -37</td>
</tr>
<tr>
<td>29</td>
<td>0.040 0.109</td>
<td>-236 -100</td>
</tr>
<tr>
<td>30</td>
<td>0.032 0.071</td>
<td>-236 -160</td>
</tr>
<tr>
<td>33</td>
<td>0.074</td>
<td>-214</td>
</tr>
<tr>
<td>34</td>
<td>0.101</td>
<td>-163.5</td>
</tr>
<tr>
<td>35</td>
<td>0.057 0.103</td>
<td>-117 -33</td>
</tr>
<tr>
<td>36</td>
<td>0.046</td>
<td>-76</td>
</tr>
<tr>
<td>37</td>
<td>0.056</td>
<td>-77</td>
</tr>
<tr>
<td>38</td>
<td>0.044 0.070</td>
<td>-205 -159</td>
</tr>
<tr>
<td>39</td>
<td>0.069</td>
<td>-143</td>
</tr>
</tbody>
</table>
## Table IV-A4

### Calculated Point Conductivities as a Function of Temperature and Density

#### Temperature = -215°F

<table>
<thead>
<tr>
<th>Run number</th>
<th>( \phi_f ) (gms./cc.)</th>
<th>( k_f ) (Btu/hr ft(^2)°F/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.058</td>
<td>0.0140</td>
</tr>
<tr>
<td>11</td>
<td>0.064</td>
<td>0.0122</td>
</tr>
<tr>
<td>13</td>
<td>0.066</td>
<td>0.0105</td>
</tr>
<tr>
<td>14</td>
<td>0.062</td>
<td>0.0107</td>
</tr>
<tr>
<td>15</td>
<td>0.059</td>
<td>0.0128</td>
</tr>
<tr>
<td>16</td>
<td>0.078</td>
<td>0.0185</td>
</tr>
<tr>
<td>17</td>
<td>0.058</td>
<td>0.0133</td>
</tr>
<tr>
<td>18</td>
<td>0.074</td>
<td>0.0166</td>
</tr>
<tr>
<td>20</td>
<td>0.060</td>
<td>0.0150</td>
</tr>
<tr>
<td>21</td>
<td>0.077</td>
<td>0.0111</td>
</tr>
<tr>
<td>22</td>
<td>0.090</td>
<td>0.0151</td>
</tr>
<tr>
<td>23</td>
<td>0.066</td>
<td>0.0134</td>
</tr>
<tr>
<td>24</td>
<td>0.068</td>
<td>0.0164</td>
</tr>
<tr>
<td>25</td>
<td>0.074</td>
<td>0.0198</td>
</tr>
<tr>
<td>27</td>
<td>0.089</td>
<td>0.0155</td>
</tr>
<tr>
<td>28</td>
<td>0.070</td>
<td>0.0146</td>
</tr>
<tr>
<td>29</td>
<td>0.064</td>
<td>0.0184</td>
</tr>
<tr>
<td>30</td>
<td>0.075</td>
<td>0.0197</td>
</tr>
<tr>
<td>33</td>
<td>0.095</td>
<td>0.0222</td>
</tr>
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<tr>
<td>35</td>
<td>0.126</td>
<td>0.0190</td>
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<tr>
<td>36</td>
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<td>0.0164</td>
</tr>
<tr>
<td>37</td>
<td>0.124</td>
<td>0.0152</td>
</tr>
<tr>
<td>38</td>
<td>0.163</td>
<td>0.0321</td>
</tr>
<tr>
<td>39</td>
<td>0.136</td>
<td>0.0278</td>
</tr>
</tbody>
</table>

#### Temperature = -145°F

<table>
<thead>
<tr>
<th>Run number</th>
<th>( \phi_f ) (gms./cc.)</th>
<th>( k_f ) (Btu/hr ft(^2)°F/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.058</td>
<td>0.0175</td>
</tr>
<tr>
<td>13</td>
<td>0.066</td>
<td>0.0139</td>
</tr>
<tr>
<td>14</td>
<td>0.062</td>
<td>0.0155</td>
</tr>
<tr>
<td>16</td>
<td>0.078</td>
<td>0.0242</td>
</tr>
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<td>18</td>
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</tr>
<tr>
<td>20</td>
<td>0.060</td>
<td>0.0228</td>
</tr>
<tr>
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</tr>
<tr>
<td>22</td>
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<td>0.0224</td>
</tr>
<tr>
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<td>0.0168</td>
</tr>
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</table>
Table IV-A4 (contd.)

<table>
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<tr>
<th>Run number</th>
<th>$\phi_f$ (gms./cc.)</th>
<th>$k_f$ (Btu/hr ft°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>0.068</td>
<td>0.0214</td>
</tr>
<tr>
<td>25</td>
<td>0.074</td>
<td>0.0255</td>
</tr>
<tr>
<td>27</td>
<td>0.089</td>
<td>0.0263</td>
</tr>
<tr>
<td>28</td>
<td>0.070</td>
<td>0.0188</td>
</tr>
<tr>
<td>29</td>
<td>0.064</td>
<td>0.0236</td>
</tr>
<tr>
<td>30</td>
<td>0.075</td>
<td>0.0283</td>
</tr>
<tr>
<td>33</td>
<td>0.095</td>
<td>0.0330</td>
</tr>
<tr>
<td>34</td>
<td>0.083</td>
<td>0.0278</td>
</tr>
<tr>
<td>35</td>
<td>0.126</td>
<td>0.0302</td>
</tr>
<tr>
<td>36</td>
<td>0.090</td>
<td>0.0204</td>
</tr>
<tr>
<td>37</td>
<td>0.124</td>
<td>0.0216</td>
</tr>
<tr>
<td>38</td>
<td>0.163</td>
<td>0.0451</td>
</tr>
<tr>
<td>39</td>
<td>0.136</td>
<td>0.0354</td>
</tr>
</tbody>
</table>

Temperature = -80°F

<table>
<thead>
<tr>
<th>Run number</th>
<th>$\phi_f$ (gms./cc.)</th>
<th>$k_f$ (Btu/hr ft°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.058</td>
<td>0.0251</td>
</tr>
<tr>
<td>13</td>
<td>0.066</td>
<td>0.0232</td>
</tr>
<tr>
<td>14</td>
<td>0.062</td>
<td>0.0278</td>
</tr>
<tr>
<td>16</td>
<td>0.078</td>
<td>0.0299</td>
</tr>
<tr>
<td>18</td>
<td>0.074</td>
<td>0.0321</td>
</tr>
<tr>
<td>20</td>
<td>0.060</td>
<td>0.0317</td>
</tr>
<tr>
<td>21</td>
<td>0.077</td>
<td>0.0267</td>
</tr>
<tr>
<td>22</td>
<td>0.090</td>
<td>0.0336</td>
</tr>
<tr>
<td>23</td>
<td>0.066</td>
<td>0.0287</td>
</tr>
<tr>
<td>24</td>
<td>0.068</td>
<td>0.0294</td>
</tr>
<tr>
<td>25</td>
<td>0.074</td>
<td>0.0317</td>
</tr>
<tr>
<td>27</td>
<td>0.089</td>
<td>0.0343</td>
</tr>
<tr>
<td>28</td>
<td>0.070</td>
<td>0.0258</td>
</tr>
<tr>
<td>29</td>
<td>0.0641</td>
<td>0.0303</td>
</tr>
<tr>
<td>30</td>
<td>0.075</td>
<td>0.0354</td>
</tr>
<tr>
<td>33</td>
<td>0.095</td>
<td>0.0495</td>
</tr>
<tr>
<td>34</td>
<td>0.083</td>
<td>0.0366</td>
</tr>
<tr>
<td>35</td>
<td>0.126</td>
<td>0.0384</td>
</tr>
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<td>36</td>
<td>0.090</td>
<td>0.0444</td>
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<tr>
<td>37</td>
<td>0.124</td>
<td>0.0459</td>
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<tr>
<td>38</td>
<td>0.163</td>
<td>0.0629</td>
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<tr>
<td>39</td>
<td>0.136</td>
<td>0.0494</td>
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</table>
### Table IV-A4 (contd.)

**Temperature = 0°F**

<table>
<thead>
<tr>
<th>Run number</th>
<th>( q_f ) (gms./cc.)</th>
<th>( k_f ) (Btu/hr ft°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>0.066</td>
<td>0.0557</td>
</tr>
<tr>
<td>14</td>
<td>0.062</td>
<td>0.0507</td>
</tr>
<tr>
<td>16</td>
<td>0.078</td>
<td>0.0531</td>
</tr>
<tr>
<td>18</td>
<td>0.074</td>
<td>0.0535</td>
</tr>
<tr>
<td>21</td>
<td>0.077</td>
<td>0.0594</td>
</tr>
<tr>
<td>22</td>
<td>0.090</td>
<td>0.0672</td>
</tr>
<tr>
<td>23</td>
<td>0.066</td>
<td>0.0577</td>
</tr>
<tr>
<td>25</td>
<td>0.074</td>
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<td>0.070</td>
<td>0.0652</td>
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<tr>
<td>29</td>
<td>0.0641</td>
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<td>0.075</td>
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<td>0.126</td>
<td>0.0980</td>
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<tr>
<td>37</td>
<td>0.124</td>
<td>0.0865</td>
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<tr>
<td>38</td>
<td>0.163</td>
<td>0.1152</td>
</tr>
<tr>
<td>39</td>
<td>0.136</td>
<td>0.0988</td>
</tr>
</tbody>
</table>

**Temperature = 10°F**

<table>
<thead>
<tr>
<th>Run number</th>
<th>( q_f ) (gms./cc.)</th>
<th>( k_f ) (Btu/hr ft°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>0.078</td>
<td>0.0749</td>
</tr>
<tr>
<td>18</td>
<td>0.074</td>
<td>0.0720</td>
</tr>
<tr>
<td>38</td>
<td>0.163</td>
<td>0.1257</td>
</tr>
<tr>
<td>39</td>
<td>0.136</td>
<td>0.1152</td>
</tr>
</tbody>
</table>
Table IV-B1

Measurements for the Effects of a Large Temperature Difference across the Frost on its Density

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Length of Time for run (hours)</th>
<th>Total Thickness of Frost (inches)</th>
<th>Initial Density of Frost (gms./cc.)</th>
<th>Final Density of Frost (gms./cc.)</th>
<th>Initial Thickness of Frost (inches)</th>
<th>Frost Thickness of Frost (inches)</th>
<th>Surface Temp of Frost (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-3</td>
<td>2.5</td>
<td>0.534 - 0.314 - 0.277</td>
<td>0.327 - 0.267</td>
<td>0.187 - 0.347</td>
<td>0.187 - 0.348</td>
<td>15 - 305</td>
<td></td>
</tr>
<tr>
<td>H-4</td>
<td>7.0</td>
<td>0.566 - 0.243 - 0.243</td>
<td>0.237 - 0.268</td>
<td>0.219 - 0.367</td>
<td>0.219 - 0.367</td>
<td>13 - 305</td>
<td></td>
</tr>
<tr>
<td>H-5</td>
<td>6.0</td>
<td>0.500 - 0.217 - 0.238</td>
<td>0.216 - 0.234</td>
<td>0.187 - 0.125</td>
<td>0.187 - 0.125</td>
<td>15 - 305</td>
<td></td>
</tr>
<tr>
<td>H-6</td>
<td>3.5</td>
<td>0.382 - 0.142 - 0.145</td>
<td>0.138 - 0.155</td>
<td>0.195 - 0.187</td>
<td>0.187 - 0.187</td>
<td>13 - 305</td>
<td></td>
</tr>
</tbody>
</table>

Table IV-B2

Determination of Uniformity of the Laid Frost in Density Measurements of the Experiments Described in Section IV-B

<table>
<thead>
<tr>
<th>Density at Position 1 on the Plate</th>
<th>Density at Position 2 on the Plate</th>
<th>Density at Position 3 on the Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.441 gms./cc.</td>
<td>0.448 gms./cc.</td>
<td>0.469 gms./cc.</td>
</tr>
</tbody>
</table>

Positions 1, 2 and 3 are random positions on the Copper Plate.
**Table IV-83**

**Measurements of effects of a Small Temperature Difference across the Frost on its Density.**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>R-8</td>
<td>6.0</td>
<td>0.1326 Top, 0.25 Bottom</td>
<td>0.064</td>
<td>0.0909</td>
<td>0.0377 Top, 0.0978 Bottom</td>
<td>0.1305 Top, 0.141 Bottom</td>
<td>0.111 Top, 0.150 Bottom</td>
<td>-23</td>
</tr>
<tr>
<td>R-9</td>
<td>12.0</td>
<td>0.1641 Top, 0.2031 Bottom</td>
<td>0.0664</td>
<td>0.0893</td>
<td>0.0560 Top, 0.0986 Bottom</td>
<td>0.1592 Top, 0.173 Bottom</td>
<td>0.136 Top, 0.192 Bottom</td>
<td>-23</td>
</tr>
</tbody>
</table>

**Table IV-84**

**Density Gradient Measurements in Frost Deposited by Natural Convection on a Liquid-Nitrogen Cooled Surface**

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Length of Run (hours)</th>
<th>Total Thickness of Frost (inches)</th>
<th>Final Thickness of Frost (inches)</th>
<th>Final Density of Frost (g/cc)</th>
<th>Cold Plate Temp. °F</th>
<th>Surrounding Temp. °F</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-7</td>
<td>6.5</td>
<td>0.414</td>
<td>0.226</td>
<td>0.038 Top, 0.034 Middle, 0.036 Bottom</td>
<td>-305</td>
<td>20</td>
</tr>
<tr>
<td>Run Number</td>
<td>Gas Temperature (°F)</td>
<td>Plate Temperature (°F)</td>
<td>Reynolds Number</td>
<td>Heat Transfer Rate (Btu/hr ft²)</td>
<td>Heat Transfer Coefficient (Btu/hr ft²°F)</td>
<td></td>
</tr>
<tr>
<td>------------</td>
<td>----------------------</td>
<td>------------------------</td>
<td>----------------</td>
<td>-------------------------------</td>
<td>------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>H-1</td>
<td>82.6</td>
<td>-11.5</td>
<td>12,280</td>
<td>870</td>
<td>9.25</td>
<td></td>
</tr>
<tr>
<td>H-2</td>
<td>82.6</td>
<td>-11.5</td>
<td>15,720</td>
<td>1,087</td>
<td>11.08</td>
<td></td>
</tr>
<tr>
<td>H-3</td>
<td>82.6</td>
<td>-11.5</td>
<td>13,650</td>
<td>931</td>
<td>9.90</td>
<td></td>
</tr>
<tr>
<td>H-4</td>
<td>82.6</td>
<td>-11.3</td>
<td>12,300</td>
<td>886</td>
<td>9.45</td>
<td></td>
</tr>
<tr>
<td>H-5</td>
<td>82.6</td>
<td>-11.3</td>
<td>11,080</td>
<td>779</td>
<td>8.30</td>
<td></td>
</tr>
<tr>
<td>H-6</td>
<td>82.6</td>
<td>-15.3</td>
<td>8,110</td>
<td>698</td>
<td>7.12</td>
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</tr>
<tr>
<td>H-7</td>
<td>82.6</td>
<td>-11.5</td>
<td>13,410</td>
<td>900</td>
<td>9.55</td>
<td></td>
</tr>
<tr>
<td>H-8</td>
<td>82.6</td>
<td>-11.5</td>
<td>15,280</td>
<td>1,087</td>
<td>11.08</td>
<td></td>
</tr>
<tr>
<td>H-9</td>
<td>82.6</td>
<td>-11.85</td>
<td>3,380</td>
<td>296</td>
<td>3.14</td>
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</tr>
</tbody>
</table>
Figure IV-A1  Effect of Reynolds Number on Average Frost Density and Frost Thickness

Figure IV-A2  Effect of Reynolds Number on Mass Deposition
Experimental Conditions

- **Figure IV-A6** Effect of Humidity on Average Frost Density and Frost Thickness

- **Figure IV-A7** Effect of Humidity on Mass Deposition

- **Experimental Conditions**
  - Re=5,530; Humidity= Gas dew point at 33.2°F; T = 48°F
  - ○ Re=6,100; Humidity= Gas dew point at 16°F; T = 48°F
Figure IV-A8 Effect of Humidity on Average Frost Density and Frost Thickness

Figure IV-A9 Effect of Humidity on Mass Deposition
Figure IV-410  Heat Flux versus Time

Figure IV-411  Heat Flux versus Time

Figure IV-412  Heat Flux versus Time
Figure IV-A13 Effect of Gas Temperature on Average Frost Density and Frost Thickness

Figure IV-A14 Effect of Gas Temperature on Mass Deposition
Experimental Conditions

- Re= 8,000; Humidity= Gas dew point at 30.4°F, T_g = 76°F
- Re= 8,430; Humidity= Gas dew point at 29.8°F, T_g = 52°F

Figure IV-A15 Effect of Gas Temperature on Average Frost Density and Frost Thickness

Figure IV-A16 Effect of Gas Temperature on Mass Deposition
Experimental Conditions

- Run Number 11
- Run Number 12
- Run Number 13
- Run Number 14

- Run Number 25
- Run Number 26
- Run Number 27

- Run Number 35
Experimental Data of Present Work

- Experimental Data of Brazinsky (14)

Line Drawn by Least Square Fitting of the Experimental Data of Present Work

Figure IV-A20  Average Thermal Conductivity of the Frost Layer as a Function of Average Frost Density
Experimental Conditions
Re = 15,800; Humidity = Gas dew point at 30°F; T_c = 73°F

Distance from Cold Wall (inches)

Temperature (°F)

Figure IV-A21  End of Run Temperature Profile within the Frost
Experimental Results:
- Shaded data = present research
- Unshaded data = study of Brazinsky (14)

- $T = -215^\circ$F
- $T = -145^\circ$F
- $T = -90^\circ$F
- $T = 0^\circ$F
- $T = +10^\circ$F

- Least square line through data
- Least square line extrapolated to zero density
- Theoretical Lines

Figure IV-A22: Point Thermal Conductivity as a Function of Density with Temperature as a Parameter
Figure IV-C1  Density Distribution within the Frost as a Function of Time

Figure IV-C2  Temperature Distribution within the Frost as a Function of Time
Experimental data at $T_g = 82^\circ F; T_v = -12^\circ F$

Theoretical predictions of Sleicher and Tribus (105)

Figure IV-D1 Heat Transfer Coefficient versus Reynolds Number under Non-Frosting Conditions
SECTION V

DISCUSSION OF RESULTS

The discussion of results is divided into four main categories as follows:
A. The effect of various independent system parameters on frost properties.
B. The results of a temperature gradient across the frost layer on its density.
C. The discussion on experimentally measured density gradient across the frost layer.
D. The experimentally measured heat transfer coefficient under the non-frosting conditions.

V-A Measured Effects of Various Independent System Parameters on Frost Properties

1. Initial Phases of Frost Formation

No data were taken during the very initial stages of frost formation other than the heat flux (except in a few cases, an approximate value of the density was determined). However, this period prior to the time when a smooth frost was attained was observed visually. At the very start of a run, the heat flux was quite high and a snowstorm of small ice particles was observed in the gas phase. This snowstorm disappeared entirely when the plate was covered by an even frost layer.

Fogging is a common occurrence in condensers in those
cases where an appreciable fraction of the gas is cooled below the saturation temperature. In the experiments carried out here, the small cold surface affected the bulk gas temperature but only slightly and one hypothesizes that the formation of the snow in the vapor was due to rapid cooling of the boundary layer. This hypothesis may be shown reasonable from the results shown in Figure I-B1. In this figure, the limits of fog formation in the gas phase boundary layer in the system similar to one used in the present study are shown as a function of bulk gas humidity, temperature and the cold wall temperature. The results of this figure indicate that for the cold wall (or the frost surface) temperatures below -30°F, the fog formation in the gas phase boundary layer is quite possible for all the experimentally studied conditions of bulk gas humidity and temperature. Thus in the initial phases of an experiment, the formation if fog in the gas phase boundary layer should be expected.

Once a smooth thin layer of frost is deposited on the cold surface, due to highly insulative nature of the frost, its surface temperature rises significantly. Hence, the probability of the formation of fog diminishes significantly. The criteria of Johnstone and co-workers (57), as shown in Figure I-B1, indicate that between the experimental humidity range of dew point at 14.3°F to 33.2°F and bulk gas temperature range of 34°F to 93°F, the range of the frost surface temperature for which no fog is formed in the
gas phase boundary layer is $-5^\circ F$ to $+5^\circ F$. The required frost surface temperature for no fog formation in the gas phase boundary layer increases with humidity and the gas temperature. In the present study the values of the frost surface temperature in the quasi-steady state of heat transfer were found to be between $-2^\circ F$ to $+7^\circ F$. Since quasi-steady state of heat transfer is found to be attained a very short time after the formation of a smooth frost layer it could be expected that the frost surface temperature at this time in all the experiments would be very close to the above stated range of $-2^\circ F$ to $+7^\circ F$. Thus, the results of Figure I-Bl indicate that almost no fog should be observed in the experiments of the present thesis after a smooth layer of frost is formed. This is in agreement with the experimental observation.

The fact that the initial portion of a run resulted in the formation of ice particles in the gas phase rather than ice on the cold wall introduces the question - how did the initial frost layer form on the wall? Several ideas have been advanced, e.g. transfer of fog particles by gravity, thermal or electrostatic force etc., but the most reasonable one appears to be that turbulent fluctuating eddy components normal to the cold wall actually drive ice particles against the surface. On the basis of this idea the higher intensity of turbulence will cause the faster rate of mass transfer in the initial phase of a run. This
is in agreement with the experimental observation. The
time for the formation of a smooth frost layer has been
found to depend significantly on the gas Reynolds number
(gas phase turbulence). The brief evaluation of the
various mechanisms which might be causing the transfer of
fog particles in the early phases of a run is shown in
Appendix 0.

The time for the formation of a smooth frost layer has
been found to be a very weak function of gas temperature
and humidity. This is because, these independent system
parameters affect only the rate of formation of mass of
fog particles. On the other hand, the time for formation
of a smooth layer should be expected to depend upon the
rate at which these particles are transferred towards the
cold wall.

When ice particles are transferred towards the cold
wall, whether they stick or not is an interesting question.
On very smooth walls, sticking may be difficult; microscopic
projections on the surface act as traps for the first frost
clumps. If the surface is covered with silicon oil, the
frost particles may never be trapped by the cold wall.
Thus, one should expect that the time for the formation
of the first few clumps of frost (and hence for the formation
of a smooth frost layer) depends significantly upon the
surface roughness.

After a few clumps are formed on the surface, each
clump acts as a trap for additional frost particles.
Initially, the heights of the individual clumps increase as more particles are thrown onto them. Before long, however, the valleys between the clumps fill in and a smooth layer of frost is formed. Even though, at present some research work is being carried out at Lehigh University (New York) to understand the mechanisms for clump formation and valley 'filling' process, as yet no conclusive results have been reported.

For certain experimental conditions, the density of the clumps were measured. Even though, these experiments are not reported in this thesis, density values in most cases were observed to lie between 0.02 to 0.03 gms./cc. It should be noted that from the knowledge of time for the formation of a smooth frost layer, an approximate value of frost density at that time can be obtained from an extrapolation back in time of the measured average frost density.

2. Frost Behavior as a Function of Time

As shown in Section IV, both the average frost density (density of the entire frost layer) and the frost thickness have been found to be a continuously increasing function of time. In all the cases mass deposited continuously onto the cold plate throughout the experiment.

Based solely on the experimental fact that frost densifies continuously through out an experiment, it can be concluded that a concentration driving force between the gas
phase and the frost surface was always present. Thus, the frost surface temperature always had to be lower than the gas phase dew point. This was verified experimentally. When the gas phase dew point was varied from 14.3 to 33.2 \(^{\circ}\)F, the measured values of the surface temperature during the quasi-steady state period of heat transfer varied from -2 to +7\(^{\circ}\)F respectively. As discussed in Section II, it is the mechanism of internal diffusion which causes the frost surface temperature to remain constant during quasi-steady state period of heat transfer and which prevents it from ever reaching either 32\(^{\circ}\)F or the gas phase dew point, whichever is lower.

The internal diffusion at the frost surface causes the frost density to increase continuously with time. This internal diffusion also by its very high gain negative feedback relation with the surface temperature causes the frost surface temperature to be constant in the quasi-steady state heat transfer period. The details of this feed-back effect is described in Section I-D and II-A.

The heat flux through the frost have been found to decrease and the frost surface temperature have been found to increase with time in initial phases of the frosting experiment. As more and more frost deposits, the cold plate is insulated and the heat flux decreases. Very shortly, however, frost surface temperature reaches a value at which internal water vapor diffusion becomes quite significant; with the result that frost density increases significantly.
Frost thickness still continues to increase during this period because of the continuous deposition of mass.

A very high sensitivity of internal diffusion to the frost surface temperature causes the thermal resistance, frost surface temperature and the heat flux to attain a constant value. A very small increase in the surface temperature results in a large increase in the internal water vapor diffusion rate. The density (and hence the thermal conductivity of the frost), therefore, increases to a point where the heat flux falls off only very slightly, and the surface temperature rises only very slightly as additional frost is deposited.

During the constant heat transfer period, the frost thermal resistance remains essentially constant as the increasing thickness is balanced by the increasing thermal conductivity. It is highly unlikely that these two effects (i.e. increase in density and thickness) balanced each other merely by chance. It is believed that the heat flux and the frost surface temperature are stabilized by a very high-gain negative feed-back effect. The details of this mechanism are described in Section I-D and II-A.

3. Effect of Reynolds Number on the Average Frost Density, Frost Thickness, Mass Deposition and Heat Transfer through the Frost

The results of Figure IV-A1, indicate that an increase in Reynolds number (at constant humidity and gas temperature) causes the formation of the frost with higher density at
the time when a smooth layer of frost is just formed. This is to be expected, because both the thermal momentum (i.e. momentum imparted on each molecule for the transfer of the thermal energy) as well as the velocity component normal to the cold plate are higher at higher Reynolds number. It is expected that the higher velocity component of the fog particles normal to the cold plate will lead to higher rates of mass transfer. In order to achieve the heat balance on the frosting process, the frost of lower thermal resistance (and hence of higher density) should be produced at the higher thermal forces existing in the gas phase boundary layer.

As seen before (see Sections I-B and II-A), once a smooth layer of frost is formed, diffusional mechanism control the transfer of water vapor from gas to the frost surface. The results shown in Figure IV-A2 indicate that an increase in Reynolds number causes the higher rate of mass transfer. This is in agreement with the theoretical prediction that an increase in Reynolds number will increase the mass transfer coefficient. Quantitatively, it has been reported by Sherwood and Pigford (104) that mass transfer coefficients for flow through a rectangular duct vary as approximately 0.8 power to the Reynolds number. Thus,

\[ K_g \propto \text{Re}^{0.8} \]  

(V-A1)

Also, the mass flux \( N_s = K_g (p_g - p_s) \)  

(V-A2)

hence,

\[ K_g = \frac{N_s}{(p_g - p_s)} \]  

(V-A3)
The mass transfer coefficients calculated from above equations for the three different system conditions, as shown in Figure IV-A2, are listed in Table V-Al.

**Table V-Al**

Experimental Mass Transfer Coefficients as a Function of Reynolds Number

Experimental Conditions: Humidity=Gaa dew point at 29.9°F,

\[ T_g = 73°F \]

<table>
<thead>
<tr>
<th>Reynolds Number</th>
<th>Mass Transfer Coefficient (ft/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,580</td>
<td>332</td>
</tr>
<tr>
<td>8,000</td>
<td>402</td>
</tr>
<tr>
<td>15,800</td>
<td>762</td>
</tr>
</tbody>
</table>

From the data of Table V-Al, comparing the values of \( K_g \) at Reynolds number of 5,580 and 15,800, it can be seen that \( K_g \) varies as 0.796 power of Reynolds number between this range of Reynolds number. Thus, the experimentally measured dependence of the mass transfer coefficient on the Reynolds number is in excellent agreement with the reported correlation by Sherwood and Pigford (104).

The increase in Reynolds number also increases the heat transfer coefficient between the gas and the frost surface. The experimentally measured correlation between heat transfer coefficient and the Reynolds number is shown
The increased heat transfer rate would tend to overheat the surface because of an increase in the heat transfer coefficient. In the frosting process, the surface temperature of the frost varies with time. An increase in Reynolds number would increase the heat flux from the gas to the frost surface because of an increase in the heat transfer coefficient. The increased heat transfer rate would tend to overheat the frost surface. However, as discussed in Section I-C, the higher surface temperature would result in a greater densification rate because of the very high-gain negative feed-back effect of the frost surface temperature on internal diffusion. Thus, the frost deposited at higher Reynolds number would be of greater density and hence increased conductivity. The increased conductivity will maintain the surface temperature nearly constant at various Reynolds numbers. The experimental results of the present studies (see Table IV-A2) borne out these expectations.

As discussed above, the experimental data of Figure IV-A1 indicate that the frost of lower thermal resistance is deposited at higher Reynolds number. At comparable times,
the average frost density increases and the frost thickness decreases with increase in Reynolds number. The frost densification is, obviously, expected to be higher at higher Reynolds number. This expectation is in agreement with the results of Figure V-\textbf{A}.

It should be noted that, once a smooth layer of frost is formed, the behavior of frost with respect to Reynolds number can not be explained in terms of fluid mechanics, because at that stage the transfer of fog particles is not a significant mechanism of frost deposition. A smooth layer of frost insulates the cold surface sufficiently so that the frost surface temperature is increased to the point where the frost deposition occurs only by diffusional mechanisms. Thus, the intrinsic constants for heat and mass transfer ($h$ and $K_g$) control the frost behavior.

The experimentally measured effects of Reynolds number on the average frost density, frost thickness, mass deposition and heat transfer through the frost are in agreement with the reported data of the previous investigators (3, 14, 25, 89, 114). Cole (25) obtained the frosts of higher densities at higher Reynolds number. Richards et al. (89) also obtained low thermal resistance frosts at higher Reynolds number. Bothe Richards (89) and Kamei (58) observed that the thickness of the frost increased with decrease in Reynolds number. They explained this phenomenon by saying that a greater shearing action at the higher velocity streams causes the higher Reynolds number frosts.
to have smaller thicknesses. While it is true that the higher density frosts, which are formed at higher Reynolds numbers, have greater shear strengths than lower density frosts, it is implied in Reference 80 that the greater shearing action of the higher velocity gas stream overrides this consideration. Brazinsky (14) obtained the similar qualitative effects of Reynolds number on average frost density, frost thickness, mass deposition and heat transfer through the frost as obtained in the present study. Based on the same preliminary calculations, he concluded, however, that the lower thickness of the frost at higher Reynolds number cannot be explained on the basis of shearing action as described above. However in his system, as frost deposited, it created a hump in the flow path of air. Hence, due to this complex geometry for heat transfer, he could not correlate his experimentally measured heat transfer coefficient with Reynolds number by means of theoretical analysis.

It should be noted that in the present studies the maximum shear strength exerted on the frost (at Reynolds number of 15,800) was approximately $6 \times 10^{-5}$ psi. (Note that this can be easily calculated from pressure drop estimates over the frosted surface. See page 5-19 of Reference 80). Based on the data presented in References 5 and 20, it was estimated that the frost with a surface temperature of $10^\circ$F has a shear strength of approximately $9 \times 10^{-3}$ psi. Thus, the magnitude of above two shear strengths
indicate that the shearing action on the frost would be negligible in the present experimental conditions.

It can be expected that in earlier phases of a frosting process when the frost deposits in the clumps, some frost is sheared off at higher Reynolds number because in this situation, the surface temperature of the frost is quite low and the corresponding shear strength of the frost would also be small. The experimental values of frost thickness at time of 26 minutes, as shown in Figure IV-A1, however, indicate that this is really not the case and frosts of almost identical thicknesses are deposited during this stage of the frosting process. Thus, it is believed that after a smooth layer of frost is formed a frost of lower thickness is deposited at higher Reynolds numbers in response to the thermal force acting upon the surface.

4. Effect of Humidity on Average Frost Density, Frost Thickness, Mass Deposition and Heat Transfer through the Frost

The results of Figures IV-A6 and IV-A8 indicate that for the humidity range of dew points at 14.3°F to 33.2°F both the average frost density and frost thickness increase with humidity (at constant Reynolds number and gas temperature) at equivalent time. Thus, the increase in humidity causes the higher rate of mass transfer as shown in Figures IV-A7 and IV-A9.

The increase in mass flux with humidity is caused by the increase in driving force between the gas and the cold surface of the frost. The driving force for the mass transfer rate is $p_g - p_s$, where $p_g$ is the partial pressure
of water vapor in the bulk gas phase and $p_s$ is the equilibrium vapor pressure of ice at the frost surface. It is observed experimentally that, even though the frost surface temperature with $p_g$ (see Table IV-A2), the difference between $p_g$ and $p_s$ continues to increase with $p_g$.

The mass flux between the gas and the frost surface can be mathematically obtained from Equation V-A2.

From the results of Figure IV-A7, the validity of the representation of Equation V-A2 can be examined as follows.

For Reynolds number = 5,800 and gas temperature 48°F, at dew point of 33.25°F, mass flux = 0.0608 lbm/hr ft$^2$ and, at dew point of 18.02°F, mass flux = 0.0354 lbm/hr ft$^2$.

The concentration driving force ($p_g - p_s$) for the two experimental conditions are as follows, at a humidity = gas dew point at $33^\circ$F, $p_g - p_s$ = 1.728x10$^{-4}$ lbm/cu ft. 

(V-A5)

at a humidity = gas dew point at $18^\circ$F, $p_g - p_s$ = 0.904x10$^{-4}$ lbm/cu ft. 

(V-A6)

It should be noted that in the above calculations $p_s$'s are obtained as equilibrium vapor pressures at the measured frost surface temperatures, whose values are listed in Table IV-A2.

Thus, the mass transfer coefficients for the two sets of experimental conditions are

at a dew point of 33.2°F, $K_g = N_s/(p_g - p_s)$ = 352 ft/hr

similarly, at a dew point of 18°F, $K_g$ = 392 ft/hr.

Thus, the values of $K_g$ at two different humidities agree with each other within approximately 10%. Considering
the experimental errors involved, particularly in the measurement of $T_s$ (see Appendix A), it can be said that Equation V-A2 is quite good mathematical representation of the mass flux from gas to the frost surface.

As shown in Table IV-A2, an increase in humidity increases slightly the quasi-steady state value of frost surface temperature. On the basis of theoretical models examined in Section II-A, the mass of water vapor diffusing in the frost interior increases with an increase in the frost surface temperature. The increase in humidity, however, also increases the frost thickness (at equivalent time, Reynolds number and gas temperature). So, the increase in humidity may or may not give the higher average frost density. For the humidity range considered in the present thesis, the results of Figure IV-A6 and IV-A8 show that at equivalent times, the average frost density is slightly larger at a higher humidity.

It should be noted, that in the initial phases of the frosting process when the particle transfer is a significant mechanism for the frost densification, the humidity would have a very little effect of the frost densification because the frost density responses only to the thermal force exerted by the gas phase boundary layer. The extrapolation of the average frost density versus time data of Figure IV-A6 and IV-A8 to a very low time indicate this to be the case.

As seen above, the slight effect of humidity on the
average frost density, as described in Figure IV-A6 and IV-A8 may not hold true outside the humidity range considered in the present study. Actually, Brazinsky (14) found that for the humidity range of dew point at 29°F to dew point at 40°F, the average frost density is essentially independent of humidity. Richards et al. (89), on the other hand, obtained the frosts consisting of alternate layers of ice and porous frosts for humidity higher than that corresponding to a dew point of 69°F. The surface temperature in this kind of frost exceeded 32°F (the melting point of ice), which caused the melting at the frost surface. Such frosts have much higher densities than frosts formed at the lower gas phase humidities and equivalent Reynolds number and gas temperature.

As mentioned above, the lower humidities result in lower frost surface temperatures. In most cases, the energy transfer due to water deposition contributes only a very small portion of the total heat flux from gas to the frost surface. Since, the heat transfer coefficient is not affected by the gas phase humidity, it would be expected that the quasi-steady state heat flux would increase as the humidity is lowered (at constant gas temperature and Reynolds number). This is indeed found to be the case as seen in Figures IV-A10, IV-A11, IV-A12 and IV-A4.

Alternatively, the variation of heat transfer rates with gas phase humidities may be explained by focusing
attention on the frost. For the humidity range considered in this study, the frost surface temperature was changed approximately within 7°F (increases in humidity increase frost surface temperatures). This increase in the frost surface temperature increased the ΔT across the frost only by 2%. However, at an equivalent time, the thickness of the frost increased as much as 50% and the average frost density increased as much as 15% with increase in humidity. It is expected that due to the significant increase of frost thickness compared to the increase in the average frost density, the frost thermal resistance will increase with increase in humidity. Thus, despite the slightly lower temperature driving force across the lower humidity frost, the smaller thermal resistance of the frost formed at lower humidity resulted in higher heat fluxes.

5. Effect of Gas Temperature on the Average Frost Density, Frost Thickness and Rates of Mass and Heat Transfer

The variation in gas temperature changes the driving force for the heat transfer between gas and the frost surface. This would either tend to heat the frost surface or frost of lower thermal resistance will be deposited to achieve the heat balance on the frosting process.

The experimental results shown in Figures IV-A13 and IV-A15 indicate that at equivalent times (and at constant Reynolds number and humidity) an increase in gas temperature results in frost of higher density and lower thickness. Thus, the frost responses to the thermal forces acting upon
its surface. The data of Table IV-A2 indicate that the frost surface temperature increases only by few degrees with increase in gas temperature of as much as 50°F.

It is expected that the gas temperature would have a significant effect on the average frost density and the frost thickness only after a smooth layer of frost is formed. In initial phases of the frosting experiment, when the transfer of fog particles is a dominant mechanism for the frost deposition, the gas temperature should have a very little effect on the average frost density, frost thickness and the rate of mass transfer. The experimental data shown in Figures IV-A13 and IV-A15 bear out these expectations. As shown in these figures, both the average frost density and frost thickness at different gas temperatures have almost identical values at the time of approximately 40 minutes from the start of the frosting process.

Since the frost surface temperature remains essentially independent of the gas temperature, it is expected that the mass flux from gas to the cold surface remains essentially unchanged with variation in gas temperature. The gas temperature affects the mass transfer coefficient only very slightly because of its effect on the gas properties. These expectations are borne out by the results shown in Figures IV-A14 and IV-A16. As shown in the figures, the mass flux from gas to the frost surface is essentially independent of variation in gas temperature (at constant humidity and Reynolds number).
Since, the thermal resistance of the frost decreases and $\Delta T$ across the frost increases slightly with an increase in gas temperature, the quasi-steady state heat flux would increase with increase in the gas temperature. The results shown in Figures IV-A17, IV-A18, IV-A19 and IV-A4 are in agreement with these expectations.

It should be noted that the concept of the frost responding to the thermal forces acting upon it is very significant in explaining the frosting process. The theoretical models studied in the present thesis which are based on this physical idea are described in Sections I-D and II.

6. Thermal Conductivity of Frost

Frost is a heterogeneous material. It consists of a matrix of interconnected ice particles with interstitial gas. It is believed that the frost thermal conductivity is dependent upon the local frost density, temperature and the frost structure. Since frost density does not necessarily imply a particular structure, frosts at the same temperature and density can have significantly different values of thermal conductivity. For the purpose of the present thesis, however, it was assumed, despite any shortcomings of this assumption, that a given value of frost density implied a particular structure regardless of the conditions under which the frost was formed. This is tantamount to assuming that a property such as thermal conductivity of frost depends solely on temperature and density. Experimental
values of point or local frost thermal conductivity were therefore plotted as a function of local frost density with temperature as a parameter (see Figure IV-A22). The scatter in data may possibly due to small structural differences of frosts of the same density.

The average thermal conductivities of the frost layer (Figure IV-A20) were obtained using the measured temperature differences across the layer (see Equation IV-A1). The temperature differences across the frost samples, whose conductivities are shown in Figure IV-A20, are substantially the same. However, it should be noted that, as previously indicated, that the measured surface temperatures of the frost varies from $-1^\circ F$ to $+18^\circ F$. The scatter in the data may be partly due to the different surface temperatures. However, the more plausible explanation is that structural differences of frosts having equivalent densities are primarily responsible for the scatter.

It should be noted that heat also flows through the frost by mechanisms other than conduction through the solid-gas network. Thus, the values of thermal conductivity shown in Figures IV-A20 and IV-A22 include contribution made by other mechanisms. These mechanisms are:

A. Radiation between the ice particles in the frost.
B. Enthalpy transport by the diffusion of water through the frost. This diffusion may be due either to the concentration gradient, resulting from the temperature gradient, or to the thermal diffusion of water vapor or of nucleated
ice particles.

A. Effect of Radiation

Radiation from the warmer frost particles to the colder ones constitutes a component of the total or overall frost thermal conductivity, the contribution made by interparticle radiation was calculated using Equation V-A7, which was presented by Damkohler (30) (see Appendix P)

$$k_r = \frac{0.685}{100} \varepsilon D_p \left(\frac{T}{100}\right) e_H$$  \hspace{1cm} (V-A7)

(In experiments involving heat transfer through a packed bed, Polack (82) found that contribution of radiation to the bed conductivity was in fair agreement with the predictions of Damkohler equation).

For the experiments of present thesis, it was calculated from the Damkohler equation that the component of thermal conductivity due to radiation was less than 1.3% of the lowest measured value of the point thermal conductivity (see Appendix P). It should be noted that values substituted into Equation V-A7 made the computed values of the contribution to the thermal conductivity by inter-particle radiation a maximum. Since this maximum contribution to the overall thermal conductivity was small, it was ignored. It should be noted that using reasonable values of $T$, $\varepsilon$, $e_H$ and $D_p$ in Equation V-A7, the contribution of radiation can be reduced to well under 1%.

The above described results regarding the internal radiant heat transfer is in agreement with the conclusion of Van der Held, who demonstrated that for particulate
insulators the radiation heat transfer under atmospheric conditions becomes significant only at temperatures above 212°F and is negligible below 32°F. Brazinsky (14) arrived at the same conclusions for his experimental data from the calculations similar to one described above.

B. Diffusion of Water through the Frost

As discussed in Section II, there are several mechanisms by which water can diffuse in the frost interior. These can be stated as follows.

1. Diffusion of water vapor by the vapor pressure gradient.
2. Thermal diffusion of water vapor.
3. Thermal diffusion of ice nuclei.

1. Diffusion of water vapor by the vapor pressure gradient

The diffusion of water vapor under vapor pressure gradient would add the additional component of thermal conductivity added to the ordinary thermal conductivity of air. This additional component can be designated as $k_{\text{excess}}$.

Thus, the total or effective value of gas thermal conductivity is equal to the sum of $k_{\text{excess}}$ and $k_g$, where $k_g$ is the gas thermal conductivity. Thus, in calculating the thermal conductivity of frost from different assumed structural models, the total thermal conductivity of the gas phase ($k_g + k_{\text{excess}}$) should be used.

An expression for $k_{\text{excess}}$ can be obtained by considering the total heat flux through the pore phase at some point in the frost. Thus,
q/A = k_g,e \frac{dT}{dx} = k_g \frac{dT}{dx} + \frac{D_B}{RT} M_v \Delta H \frac{dP_v p}{dx} \quad (V-A8)

A more detailed explanation of Equation V-A8 is presented in Appendix F. From Equation V-A8 and the definition of \( k_{\text{excess}} \),

\[ k_{\text{excess}} = k_g,e - k_g \quad (V-A9) \]

Equation V-A10 can be derived as,

\[ k_{\text{excess}} = \frac{D_B}{RT} \left( \frac{dP_v p}{dT} \right) M_v \Delta H \quad (V-A10) \]

Combining Equation V-A10 with Clausius-Clapeyron equation (80)

\[ \frac{dP_v p}{dT} = \frac{M_v \Delta H}{RT^2} P_v p \quad (V-A11) \]

the final expression for \( k_{\text{excess}} \) can be obtained

\[ k_{\text{excess}} = (D_B \Delta H^2 P_v p) \frac{M_v^2}{R^2 T^3} \quad (V-A12) \]

The computed values of \( k_{\text{excess}} \) as a function of temperature are shown in Table V-A2. At positions within the frost, where the temperature is lower than -10°F, \( k_{\text{excess}} \) makes a small contribution to the total gas phase thermal conductivity and hence the point thermal conductivity of the frost. However, \( k_{\text{excess}} \) becomes significant at higher temperatures. For example, at 20°F, the contribution is about 37% of the total gas phase conductivity. Obviously, \( k_{\text{excess}} \) increases the over-all conductivity of the frost since it increases the total gas phase conductivity. It can
be easily seen that the exact contribution made by excess thermal conductivity to the over-all thermal conductivity of the frost is dependent on the real frost structure. This contribution cannot be calculated exactly as the true frost structure generally cannot be mathematically characterized.

Thus, in order to calculate the fractional contribution made by $k_{excess}$, a particular structure has to be assumed. As an illustration, suppose the frost consists of a series of parallel slabs of ice with heat flow parallel to these slabs. Then for a $20^\circ F$ frost having a density of 0.09 gms./cc. thermal conductivity with $k_{excess}$ is equal to 0.145 Btu/hr ft $^0F$ (14), whereas without $k_{excess}$ the conductivity is 0.138 Btu/hr ft $^0F$. Thus, for this particular model, $k_{excess}$ adds 5.1% to the point conductivity. However, if the frost structure is idealized to consist of parallel array of slabs with heat flow normal to the array, then the conductivity including $k_{excess}$ is 0.024 Btu/hr ft $^0F$ (14). Without $k_{excess}$ the thermal conductivity is 0.015 Btu/hr ft $^0F$. Thus, in this case $k_{excess}$ increases point conductivity by 60%.

As described above, since the contribution of $k_{excess}$ is significant only for the temperature above $-10^0F$, it is expected that the average thermal conductivity of the frost will not be significantly affected by $k_{excess}$. This can be seen more clearly from the results shown in Figure V-A1. In this figure, the thermal conductivity of the frost (based on the Woodside model of the frost structure, which
is a model, as shown in Appendix F, consisting of a cubic array of ice sphere) is plotted as a function of temperature for the density of 0.09 gms./cc. The curve ABC shows the conductivity correlation which takes into account the excess thermal conductivity of the frost. The curve ABC', on the other hand, does not take into account the excess thermal conductivity of the frost. The average thermal conductivity of the frost can be obtained from these correlations using Equation V-13

\[ k'_{\text{avg,}f} = \frac{\int_T k_f \, dT}{T_W} \left( \frac{T_w}{T_s - T_W} \right) \] (V-13)

Thus, the average conductivity calculated from curve ABC was found to be equal to 0.0214 Btu/hr ft °F, whereas average thermal conductivity calculated from curve ABC' was found to be 0.0209 Btu/hr ft °F. The value of the average conductivity of a frost layer having a surface temperature of 20°F, was thus affected by 4.42% due to excess thermal conductivity.

B. **Thermal Diffusion of Water Vapor**

The existence of a large temperature gradient (see Section IV) in the frost interior indicate that there could be a significant enthalpy transport due to thermal diffusion of water vapor. This, however, is not the case. As shown in Section II-D, it has been found that the water vapor flux caused by thermal diffusion is negligible compared with the flux due to concentration gradient (vapor pressure gradient) at all levels within the frost. The enthalpy transport due
to thermal diffusion of water vapor is therefore ignored in the present thesis.

C. Thermal Diffusion of Ice Nuclei

As discussed in Section II, it is believed that thermal diffusion of ice nuclei plays a significant role in the densification of the frost, particularly in the colder region of the frost. This transport of ice nuclei will also cause the enthalpy transport. Mathematically, this kind of enthalpy transport can be written as,

\[ q_h = n v_t c_p \Delta T = k'_{\text{excess}} \frac{dT}{dx} \quad (V-A14) \]

From the above equation, it can be seen that \( k'_{\text{excess}} \) varies with the position in the frost interior. For a typical set of experimental conditions (Re=15,800, Humidity= Gas dew point at 30°F, \( T_g = 70°F \)) it was estimated that at \( Q_f = 0.133 \) gms./cc., and \( T = -133°F \), (see Section V-C)

\[ \frac{dT}{dx} = 2.52 \times 10^{-6} F/ft, \quad n v_t = 0.0125 \text{ lbm/hr \cdot ft}^2 \]

and assuming \( \Delta T = 100°F \) and \( c_p = 1.008 \text{ Btu/lbm °F} \) (80)

the value of \( k'_{\text{excess}} = (0.0125 \times 4.032 \times 25 \times 10^{-4}) / 252 \)

\[ = 4.8 \times 10^{-7} \text{ Btu/hr ft °F} \]

From Figure IV-A22, \( k_f \) at \( Q_f = 0.133 \) gms./cc. and \( T = -133°F \) has the value of 0.037 Btu/hr ft °F.

Thus, for the chosen experimental conditions \( k'_{\text{excess}} \) due to thermal diffusion of ice nuclei is negligible compared to the total thermal conductivity.

Since, the experimental conditions chosen in the above
calculations were such that the densification of the frost and hence the mass flux in the frost interior were the maximum for the present study, it was concluded that the enthalpy contribution due to mass transfer was negligible for the present purpose.

7. Experimental Error in Thermal Conductivity Measurements

Due to experimental errors involved in the measurements of various quantities, it is believed that the average thermal conductivity values reported in this thesis are accurate within approximately ± 7%. The details of the error analysis is shown in Appendix N.

The calculation of point thermal conductivity from Equation IV-A2 could be inaccurate due to errors in the measurements of slope of a $T(x)$ curve or in heat flux measurements.

The slope of the temperature versus $x$ profile at various points were measured by graphical differentiation with the help of a mirror. For a typical $T(x)$ curve, value of the slope obtained by this method was compared with one obtained by analytical differentiation. As shown in Appendix N, the values obtained by these two different methods agreed quite well. It was estimated that the error in temperature gradient measurement was approximately within ± 4%. The error in heat flux measurement was within approximately ±2%. Thus, the point thermal conductivity values were estimated to be accurate within ± 10%. The details of the error analysis is shown in Appendix N.
8. Variation of Conductivity with Density

A. Average Frost Layer Conductivity

The average frost layer conductivity is plotted as a function of density is shown in Figure IV-A20. Since it has been shown that radiation and excess thermal conductivities do not affect the average thermal conductivity of the frost layer significantly, it can be concluded that the major contributor to the average frost thermal conductivity is conduction through the solid gas matrix.

The thermal conductivity increases with increasing frost density because at the higher densities larger areas of low thermal resistance paths are present. The larger contact areas are a consequence of the increased solid fraction. In addition, higher density frosts may have structures which are significantly different than lower density frosts. These different structural characteristics may also be partly responsible for the increased conductivities at the higher densities.

B. Point Thermal Conductivity

Point thermal conductivity at different density value and with temperature as a parameter are plotted in Figure IV-A22. As was true for average frost layer conductivities, higher densities result in larger values of point conductivities because of the larger areas of low thermal resistance paths. These large areas are a consequence of the increased percentages of solid present at the higher frost densities. In addition, possible structural differences implied by the higher densities may be partly responsible for the increased
conductivities.

9. Variation of Thermal Conductivity with Temperature

As shown in Figure IV-A22, at equivalent densities, higher frost temperatures lead to increased thermal conductivity values for three reasons. First, the contribution of $k_{\text{excess}}$ to the overall thermal conductivity increases with temperature. Secondly, it is believed that the contact areas between the various ice particles increase with increasing temperature. At the higher temperatures the processes of constructive and destructive metamorphism (34, 62) are more active, and it seems likely that additional water vapor is transferred to the contact areas between the particles. Thirdly, the conductivity of the gas phase increases with increasing temperature and, at the high frost porosities encountered in this thesis, the gas conductivity is very important in determining the overall conductivity of the frost. Thus, despite the fact that ice thermal conductivity decreases with increasing temperature, the intrinsic conductivity of the ice-gas matrix increases with temperatures.

10. Comparison of Experimental Data of Thermal Conductivity with those Predicted from a Frost Structure Model

Frost is a heterogeneous structure consisting of an array of irregular particles. In order to obtain a mathematical relation between frost thermal conductivity, density and temperature, it is important to model the frost structure. It is obvious, considering the irregularities involved
in the frost structure, that a unique mathematical representation of frost structures is almost impossible. Usually the frost structure is visualized by a simple model which would not only fairly well represent the true structure but which can be mathematically defined such that a correlation between thermal conductivity, density and temperature can be obtained.

The previous investigators \( (3, 14, 25, 47) \) of this area have analyzed various idealized models of the frost structure. The detailed descriptions of these models are given in References 14 and 47.

As shown in Figure IV-A22, the thermal conductivity data of the present thesis are in quite good agreement with those obtained by Brazinsky \( (14) \). A model which was found to be successful in correlating the experimental data of Brazinsky \( (14) \) for thermal conductivity is, therefore, also used in the present study.

**A. Model of the Frost Structure: Cubes of Ice Connected by Ice Rods**

The assumed model is a symmetrical matrix of ice cubes interconnected by ice rods. The rods are square prisms (see Figure V-A2). As is the case for other structural models, this structure only very approximately models the true physical situation. However, a desirable characteristic of this model is that the dimensions of the rods may be taken as a measure of the actual contact between the ice particles.

An equation for calculating frost thermal conductivity
is derived by considering a typical unit cell of heterogeneous structure (see Figure V-A3). \( x \) is the linear dimension of the ice cube, and \( z \) is the width of the square ice rod. Actually \( x \) and \( z \) are dimensionless as they represent, the ratio of the ice cube length to unity and the ratio of rod width to unity, respectively. It should be noted that the actual size of the unit cell does not affect the computed value of the **frost thermal conductivity**.

From Figure V-A3 it can be seen that \((1-\epsilon)\), the solid fraction, is given by

\[
1 - \epsilon = \frac{\text{volume of solid in a unit cell}}{\text{volume of a unit cell}}
\]

or

\[
1 - \epsilon = x^3 + 6 \left( \frac{1-x}{2} \right) z^2
\]

\((V-A16)\)

or,

\[
1 - \epsilon = x^3 + 3 \: z^2 \: (1-x)
\]

\((V-A17)\)

\((1-\epsilon)\) is related to the densities of ice, gas and frost by

\[
1 - \epsilon = \frac{\rho_i - \rho_g}{\rho_{\text{ice}} - \rho_g}
\]

\((V-A18)\)

**B. Equation for Frost Thermal Conductivity**

The three dimensional and planar views of the frost structure are shown in Figures V-A3 and V-A4 respectively. A representative unit cell of the homogeneous structure is shown in Figure V-A4. Only the case of infinite sidewise thermal conductivity will be considered. Thus the equivalent
thermal resistance of the unit cell is obtained by considering the resistances of zones 1, 2, 3, 4 and 5 to be in series.

The length of the ice cube is \( x \). The rods are square prisms with cross sectional area equal to \( z^2 \). It should be noted that the resistances of zones 1 and 5 are identical and so are the resistances of zones 2 and 4. The combined thermal resistance of zones 1 and 5 is

\[
R_{1-5} = \frac{1 - x}{k_{g,e}(1-z^2) + k_1 z^2} \tag{V-A19}
\]

The combined thermal resistance of zones 2 and 4 is

\[
R_{2-4} = \frac{(x - z)}{k_{g,e}(1-x^2) + k_1 x^2} \tag{V-A20}
\]

For zone 3 the thermal resistance is

\[
R_3 = \frac{z}{2k_1 z (1-x) + k_1 x^2 + k_{g,e}(1-x^2-2z(1-x))} \tag{V-A21}
\]

The combined thermal resistance is obtained by adding Equations V-A19, V-A20 and V-A21 as these resistances are in series.

Total resistance of unit cell

\[
\frac{1}{k_f} = \frac{1}{k_{g,e}(1-z^2)+k_1 z^2} + \frac{z}{2k_1 z (1-x) + k_1 x^2 + k_{g,e}(1-x^2-2z(1-x))} \tag{V-A22}
\]

Thus,

\[
\frac{1}{k_f} = \frac{(1-x)}{k_{g,e}(1-z^2)+k_1 z^2} + \frac{z}{2k_1 z (1-x) + k_1 x^2 + k_{g,e}(1-x^2-2z(1-x))}
\]
Equation V-A23 can be used to calculate frost thermal conductivities with the knowledge of adjustable constants, namely the ratio of cube length to the width of the ice rod. This is because there are two equations (V-A17 and V-A23) and three unknowns \(k_f, x \text{ and } z\). The \(x/z\) ratio is a function of temperature. In the present thesis a correlation (14) between \(x/z\) and temperature described in Figure V-A5 is used to obtain the point thermal conductivity values shown in Figure IV-A22. In general, however, a particular value of adjustable constant can be obtained by matching the thermal conductivity calculated from Equation V-A23 with a value on experimental curve. The specific procedure is described in Reference 14.

The results shown in Figure IV-A22 indicate that at all temperatures there is a fair agreement between theoretical values and the experimental data. This agreement tends to indicate that over the density range involved, the contacts between the ice particles in the frost are reasonably well represented by the dimensions of the interconnecting ice rods. In addition, it should be noted that the ratio \((x/z)\) increases with increasing temperature, thus indicating that contact areas between particles are greater in the higher temperature sections of the frost than in the lower temperature sections. This conclusion seems reasonable.
since higher temperature results in increased rates of the metamorphic processes (24, 62). These increased rates cause additional mass to be deposited in the particle contact areas hence results in a greater degree of bonding. The various types of metamorphic processes are discussed in Reference 14.

11. Applicability of an Analogy for Simultaneous Heat and Mass Transfer for the Frosting Process

As discussed in Section II-A, the formation of frost on a liquid nitrogen cooled surface is governed by the simultaneous heat and mass transfer from gas to the cold surface, once a smooth layer of frost is deposited. It is expected, therefore, that an analogy for simultaneous heat and mass transfer can be applied to the frosting process.

For the purpose of evaluating the applicability of such an analogy, the experimentally measured coefficients of heat and mass transfer are compared with ones obtained from the theoretical correlations. The experimental heat transfer coefficients are calculated from the following heat balance Equation V-A24 on the frosting process.

\[ q = h (T_g - T_s) + N_s \Delta H \]  \hspace{1cm} (V-A24)

With the knowledge of experimentally measured quantities \( q, T_g, T_s \) and literature value of \( \Delta H \) (s/1), above equation can be used to calculate heat transfer coefficients for various system conditions.

The theoretical heat transfer coefficients are calculated using the theoretical analysis of Sleicher and
A comparison between the experimental and theoretical correlation between heat transfer coefficient and Reynolds number at various gas temperatures is shown in Figure V-A6. The details of the theoretical heat transfer analysis are shown in Appendix L.

From the results of Figure V-A6, it can be seen that for all the experimental conditions studied in the present research there is a good agreement between the values of theoretical and experimental heat transfer coefficients. The maximum percentage deviation between the experimental and the theoretical values (\( \% \) deviation = absolute value of \( \left( \frac{h_{\text{exp}} - h_{\text{theo}}}{h_{\text{exp}}} \right) \times 100 \)) is approximately within 17% and the average of the deviations for all the experimental conditions is approximately within 10%. The experimental heat transfer coefficients are estimated to be accurate within approximately ± 12.7%. The details of the error analysis is shown in Appendix N.

As discussed previously, the experimental mass transfer coefficients can be calculated from the mass balance Equation V-A2 for the frosting process. The theoretical mass transfer coefficients are calculated using the theoretical heat transfer coefficients and Lewis analogy (39, 72) for the simultaneous heat and mass transfer. For small vapor concentration, the Lewis analogy can be written as,

\[
\frac{h}{K_g} = \frac{q}{\varphi_g c_p} L e^{2/3}
\]  

(V-A25)

The details of Lewis analogy are shown in Appendix L.
A comparison between experimental and theoretical mass transfer coefficients for the various experimental conditions is shown in Figure V-A7. From the figure it can be seen that the maximum percentage deviation between the experimental and theoretical values of mass transfer coefficient ($\% \text{ deviation} = \text{absolute value of } \left( \frac{K_{\text{exp}} - K_{\text{theo}}}{K_{\text{exp}}} \right) \times 100$) is approximately within 17% and the average of deviations for all the experimental conditions is approximately within 11%. As shown in Appendix N, the experimental mass transfer coefficients are accurate within approximately ±7%. Thus, it can be said that the theoretical and the experimental values of mass transfer coefficients agree well for all the experimental conditions studied in the present thesis.

Possible Reasons for Discrepancy between Experimental and Theoretical Values of Heat and Mass Transfer Coefficients

There are several possible reasons for slight discrepancy between the experimental and theoretical heat and mass transfer coefficients. The errors lie both in experimental measurements as well as in theoretical predictions.

There are two main possible errors involve in experimental measurements. First, there could be an error involved in the measurement of frost surface temperature. This is because a significant difficulty involved in locating the exact position of the frost surface (see Section III). The error in this measurement causes the inaccuracy in the calculations of heat and mass transfer coeff-
Secondly, there is a possibility of an error in the determination of the exact value of \( D_h \) (hydraulic diameter of the duct). As frost deposits on the cold surface during a frosting experiment, the effective clearance of the duct for the flow decreases. In present research, the duct clearance is kept almost constant during an experiment by lowering the cold plate as frost deposits upon it such that the frost surface and bottom of the duct are always flushed. This is done, however, by visual examination through lucite side windows and the accuracy with which the frost surface and bottom of the duct can be flushed together by this method is approximately within 0.03 to 0.04 inch.

The clearance of the duct in the present frosting system is 0.4 inch. The reduction of this clearance by 0.04 inch causes 10% reduction in clearance and hence at the same Nusselt number the heat transfer coefficient increases by the same amount. Thus, the reduction in clearance of the duct gives higher value of experimental heat transfer coefficient than the expected one. The same statement can be made for the mass transfer coefficient.

Levy (65) states that heat transfer characteristics of turbulent flow in a rectangular duct varies significantly with the nature of the boundary conditions at the two walls of the duct. This is particularly true for low Prandtl number (\(< 1.0\)) when thermal boundary layer thickness can exceed velocity boundary layer thickness. Since Prandtl
number of air is 0.73 and the nature of the boundary conditions in the theoretical and experimental heat and mass transfer analysis are different, one should expect the slight difference in the values of theoretical and experimental heat and mass transfer coefficients.

Finally, the roughness of the frost surface causes the experimental heat and mass transfer coefficients to be higher than ones calculated from the theoretical analysis. This is because the theoretical analysis assumes the duct wall to be completely smooth (i.e. no loss due to friction).

Considering all the possible sources of errors described above it can be said that there is good agreement between experimental and theoretical heat and mass transfer coefficients.

The applicability of the Lewis analogy for the frosting process has been studied by the various previous investigators (6, 14, 52, 89, 92, 114). In low temperature natural convection studies (6, 52, 92), it has been found that the Lewis analogy was not successful in predicting the experimental mass transfer rates. Since Lewis analogy is applicable to the case when heat and mass transfer rates are controlled by diffusional mechanisms in the gas phase thermal boundary layer, this discrepancy has been generally attributed to the formation of ice particles in the boundary layer.

In the forced convection low temperature studies the experimental results of Richards et al. (89) and of
Brazinsky (14) for heat and mass transfer rates did not agree well with ones predicted from Lewis relation. In early phases of the experiments Brazinsky's mass transfer data (from mass versus time curve) were only 50% of those predicted from Lewis relation. These results can be explained on the basis of formation of ice particles in the gas phase boundary layer. Both Richards (89) and Brazinsky (14) could not correlate their data with Lewis relation even in the later phases of the frosting experiments. It is believed that the reason for discrepancy was a complicated geometry of the frosting system which failed them to carry out accurate theoretical heat and mass transfer analysis.

In the high temperature, forced convection studies, on the other hand, Chung and Algren (24) evaluated theoretical mass transfer rates using a correlation for mass transfer coefficient applicable to flow past single cylinder. These investigators found that the mass transfer rates predicted from Lewis analogy (39, 72) showed good agreement with their experimental values of frost accumulation rates. This would be expected as the studies were conducted at relatively high temperature thus ensuring that deposition occurred solely by the molecular and turbulent diffusion of water vapor from the gas phase to the frost surface. It is believed that the better agreement between the experimental and the theoretical mass transfer rates in the present study as well as in the study of Chung and Algren (24) are due to the better defined systems used.
in the studies of Richards et al. (89) and Brazinsky (14).

In summary, the results shown in Figures V-A6 and V-A7 indicate that if no fogging occurs in the gas phase, then the Lewis analogy can be used to study the simultaneous heat and mass transfer phenomena during the frosting process.

V-B Effect of a Temperature Gradient on the Frost Density

Frost is a heterogeneous material consisting of mainly a solid and a gas phase. Its density varies from 0.02 gms./cc. (for a snow formed at cryogenic temperatures) to almost that of solid ice (0.917 gms./cc.) for the high temperature frosts deposited from humid air flowing at extremely high velocity.

The frost formed on a liquid nitrogen cooled surface is very fluffy and hence has a very low density. Due to the fluffy structure, this frost has a very low thermal conductivity. Thus, a thin layer of this kind of frost creates a considerable insulation on the cold surface.

As seen in Section IV-A, there exists a large temperature gradient in the frost interior. It has also been found that the density of the frost increases with time. Thus, this experimental program was undertaken to evaluate a relation between above two experimentally observed occurrences.

A uniform layer of the frost in all the experiments was obtained by sprinkling the snow on the cold plate. The
data of Table IV-B2 indicate that a uniform frost density over the entire plate within an accuracy of 4% of its value can be obtained by this method. It should be noted that an achievement of a uniform layer of frost is important because a representative sample for the frost density can then be obtained from any position on the plate.

The data of Table IV-B1 indicate that in general, there was no significant mass transfer between two adjacent slices of the frost when the entire frost layer was exposed to a large temperature difference in the manner described in Section III-B. This was found to be the case for various values of frost densities. It was, however, qualitatively observed in all the experiments that the surface of the frost at the end of an experiment was quite hard. Thus, it is possible that even though there was no significant mass transfer between two adjacent slices of the frost, there was a significant mass flux between the adjacent layers of the top slice in each experiment and the density of a very thin layer at the frost surface was increased significantly by a temperature gradient.

In order to examine more critically the effect of a temperature gradient on the top slices of the frost layers in the above discussed experiments, a few more experiments were performed in which a frost layer was exposed to a small temperature difference at the high temperature level (the average temperature of the frost layer being approximately 15°F). The results of these experiments are shown in Table
As shown in the table, when a frost layer of density approximately 0.15 gms./cc. and thickness of 0.317 inch was exposed to a temperature difference of about 27°F, there was a noticeable amount of mass transfer from the top to the bottom slice of the frost. This mass transfer also increased with time. Thus, qualitatively it can be said that the mass transfer of water vapor due to a temperature gradient does take place at the temperature level of approximately 15°F. The quantitative explanation of the results of Table IV-B3 can be given as follows.

If it is assumed that at every layer in the frost interior, there was an equilibrium between the ice crystals and the interstitial bulk gas at the prevailing temperature, then the temperature gradient causes a vapor pressure gradient at every layer. The amount of water vapor that can be transferred from the top to the bottom slice under this situation can be approximately calculated as follows (see Figure V-B1).

\[
N = \frac{D_B}{RT} \frac{1 - \frac{\xi}{\xi_{ic}}} \frac{dP_{vp}}{dx} \bigg|_{x=5/2} = \frac{D_B}{RT} \frac{1 - \frac{\xi}{\xi_{ic}}} \frac{\Delta P_{vp}}{\delta}
\]

(V-B1)

It should be noted that above calculation for the mass transfer rate is very approximate because vapor pressure does not vary linearly with distance along the depth (x). However, it is believed that the vapor pressure gradient at x = 5/2, would be reasonably close to \( \Delta P_{vp}/\delta \) (see Figure...
For,
\[
\tau = 1.10 \quad (94)
\]
\[
1 - \frac{P_i}{P_{\text{ice}}} = 1 - 0.15/0.917 = 0.8365
\]
\[
\Delta P_{\text{vp}}/\delta = (3.013 - 0.776) \times 12/0.37 = 7.25 \text{ mm. of Hg. ft.}
\]
\[
T = 470^\circ R \text{ (average)}
\]
\[
D_B = 0.1965 \text{ cm}^2/\text{sec} \quad (94)
\]
\[
R = 555.0 \text{ (mm. of Hg. cu. ft.)/(}^\circ R \text{ lb mole)}
\]

from Equation V-B1
\[
N = \frac{0.1965 \times 0.8365 \times 7.25 \times 3.600 \times 18 \times 454}{1.10 \times 555.0 \times 470 \times (30.47)^4}
\]
\[
= 1.51 \times 10^{-3} \text{ gms./cm}^2 \text{ hr.}
\]

The data of Table IV-B3 indicate that experimental mass flux from top to bottom slice was \(9.50 \times 10^{-4} \text{ gms./cm}^2 \text{ hr.}

The diagram shows the pressure and temperature conditions at top and bottom slices.

**Figure V-B1** Schematic of Physical Process Visualized by Equation V-B1.

From the above described calculation, it can be seen that the experimentally measured mass flux agrees well with one calculated by Equation V-B1. Thus, if the magnitudes of the vapor pressures are high, a temperature gradient does cause the significant transfer of mass due to vapor pressure.
driving force.

The results of both large and small temperature differences across the frost indicate that in the static system conditions, as shown in Figures IV-B1 and IV-B2, the sandwiched frost layer probably acquires the equilibrium conditions in its interior. This means that under such system conditions, at every layer of the frost interior, ice crystals and the interstitial gas are in equilibrium at the prevailing temperature.

Under the equilibrium conditions, the mass transfer of water vapor at every point in the frost interior will occur by the prevailing vapor pressure gradient. Since the vapor pressure of ice is appreciable only for temperatures above -10°C, the mass transfer by the physical mechanism described above will take place only for a temperature level above -10°C. The experimental data with a large and a small temperature difference across the frost indicate this to be the case.

V-B1 Density Distribution within the Frost Deposited by Natural Convection

It is believed that, even though the theoretical models of the present thesis are verified only for the occurrence of the frosting process by forced convection, they can still be applied to the frosting process by natural convection. So, in order to examine the density gradient in the frost deposited by natural convection experiment number R-7 (see Table IV-B4) was carried out.
The results of this experiment indicate that there was no significant density gradient in the frost deposited by natural convection on a liquid nitrogen cooled surface. Since, the density measurements in this experiment were carried out at time of 6.5 hours from the start of the frosting process, one can expect the similar kind of density distribution in the frost interior at any earlier time. Thus, just like density of the frost deposited by forced convection, the natural convection frost is also of uniform density (with depth) at all time. The possible explanation for the uniformity of the density is described in Section II-E.

V-C Discussion on Results of Measurements on Density Gradient within the Frost

The results of Figure IV-Cl indicate that for a typical set of experimental conditions (Re=15,800, Humidity= Gas dew point at 30°F, T_g=71°F) the density of the bottom 0.0625 inch thick layer of cryogenic frost increases essentially at the same rate as the average density of the entire frost over the period of 26 to 144 minutes from the start of the frosting process. The thickness of the frost for the same experimental conditions increased from 0.093 to 0.256 inch over the same period of time. The density gradient measurements were carried out at various positions on the heat transfer section of the copper plate. The results similar to ones shown in Figure IV-Cl were obtained.

The maximum possible error in the density values of
Figure IV-C1 is within ±12%. The major errors were caused in slicing and in thickness measurements. The details of the error analysis is shown in Appendix N.

Figure IV-C2 shows the variation in temperature distribution within the frost with time for the same experimental conditions as used for the results of Figure IV-C1. The results of this figure show that the temperature of the frost layer, 0.0625 inch away from the cold wall varied from -44°F to -173°F over the period of 26 to 144 minutes. Thus, the average temperature of that layer during the frost densification process was -108°F.

The densification of the bottom 0.0625 inch thick frost from 0.085 gms./cc. to 0.161 gms./cc. over a period of 118 minutes required the transport of 0.0125 lbm/hr ft² of water in that region, i.e., this amount of water has to pass the frost layer at x=0.0625 inch from the cold wall. In the discussion that follows, a feasible mechanism for this transport of water will be developed.

If, it is assumed that there exists an equilibrium between bulk gas and ice crystals at every point in the frost interior then the prevailing temperature gradient will establish a vapor pressure gradient at every point. Under these circumstances, molecular diffusion of water vapor by vapor pressure gradient will transport the water at every point in the frost interior. Mathematically, the water flux by this mechanism can be written as,
\[ N = \frac{D_B}{RT} \left( 1 - \frac{q_f}{q_{ice}} \right) \frac{\partial P_{vp}}{\partial x} \quad (V-\text{Cl}) \]

If Equation \( V-\text{Cl} \) represents truly the mechanism of water transport in the frost interior, then it should satisfy the experimental results for mass flux at \( x=0.0625 \) inch. From the results of Figure IV-C2 and with the help of Equation \( V-\text{Cl} \), the water vapor flux at \( x=0.0625 \) inch can be calculated as follows:

At \( x = 0.0625 \) inch, and at time = 85 mins. (Arithmetic mean time),

\[ T = 342^\circ R = 190^\circ K \]

\[ \frac{\partial P_{vp}}{\partial x} = 6.25 \frac{\text{lb}_f}{\text{cu ft.}} \quad \text{ (from Figure IV-C2)} \]

\[ D_B = 2.6795 \times 10^{-6} T^{2.0087} = 0.0965 \text{ cm./sec.} \quad \text{ (from Figure II-B10)} \]

\[ = 0.133 \text{ gms./cc.} \]

\[ N = \frac{0.0965 \times 118 \times 18 \times (1-0.133/0.917) \times 6.25}{342 \times 1.544 \times 10^{-3} \times 1.10} \]

\[ = 0.00189 \text{ lbm/hr ft}^2 \]

Comparing the above value of mass flux with the experimental one, it can be seen that the experimental mass flux is considerably higher than one calculated from Equation \( V-\text{Cl} \). Thus, Equation \( V-\text{Cl} \) can not represent the true process for water transport in the frost interior.

If water vapor transport occurs by non-equilibrium
partial pressure gradient, then as shown in Section II-C, partial pressure at \( x=0.0625 \) inch can be calculated from the following Equation V-C2

\[
P_x = 0.0625 = \int_0^{x} \frac{RT}{D_B (1 - \frac{\varphi}{\varphi_{ice}})} \, dx
\]  

(V-C2)

Substituting \( \varphi = 1.10 \) (\text{ft}^2/\text{hr}) and \( D_B = 3.3 \times 10^{-6} \) \( \text{ft}^2/\text{hr} \) and using the data of Figures IV-C1 and IV-C2 at time of 85 minutes, \( p_x = 0.0625 \) can be calculated to be 1.485 mm. of Hg. Hence, this calculated partial pressure indicates that at \( x=0.0625 \), supersaturation ratio \( \frac{p}{p_{vp}} = \frac{1.485}{5.2 \times 10^{-4}} = 2.86 \times 10^3 \).

It should be noted that in the above calculation \( p_x = 0 \) was assumed to be zero. This assumption makes the above calculated value of \( p_x = 0.0625 \) the most conservative one.

The strong possibility for nucleation indicate that water transport at every layer could not occur by the diffusional mechanism under the non-equilibrium partial pressure gradient. This indication can be verified by calculating the partial pressure at the frost surface from Equation V-C2 by setting upper limits on the integrals as \( x = 6 \). For the present experimental conditions (see Figures IV-C1 and IV-C2), this value is found to be 16.865 mm. of Hg which is higher than even the bulk gas humidity (4.14 mm. of Hg.). Thus, this result shows that Equation II-C1, does not represent the true mechanism of the frost.
densification process.

It should be obvious to note that if the boundary condition for Equation II-C4 is set at the frost surface, then the partial pressure at \( x = 0.0625 \) inch can be calculated from the following Equation V-C3.

\[
\left. p \right|_{x=\xi} - \left. p \right|_{x=0.0625} = \int_{x=0.0625}^{x=\xi} \frac{\gamma RT}{(1 - \frac{\xi_i}{\xi_{ix}}) D_\phi} \ dx
\]  

(V-C3)

Assuming \( \left. p \right|_{x=\xi} = \left. p \right|_{x=0.0625} = 1.632 \) mm. of Hg. (Note that this assumption is quite realistic as indicated by simple model) and calculating the right hand side of the above Equation V-C3 from the data of Figures IV-C1 and IV-C2, \( p_{x=0.0625} \) can be found to be -13.648 mm. of Hg. This absurd value for partial pressure, once again, indicates that Equation II-C1 does not represent the true mechanism of the frost densification process. The water transport past every layer in the frost interior should occur by a mechanism other than only molecular diffusion of water vapor as assumed in Equation II-C1.

So far, the critical evaluation of the experimental results of Figures IV-C1 and IV-C2 have indicated that in the colder region of the frost, most of the water should exist in the form of ice nuclei. These nuclei could be formed by any of the three mechanisms of nucleation described in Appendix H. The existence of a large temperature
driving force as shown in Figure IV-C2, suggests that the required water flux at $x=0.0625$ inch can be obtained by thermal diffusion of ice nuclei. This hypothesis can be verified as follows.

The phenomenon of thermal diffusion of solid or liquid particles in air is discussed in detail in Appendix K. A literature survey has indicated that a theoretical correlation of Brock (15) is the best known, as yet, for the calculation of the force acting on a solid particle in a thermal unequilibrium state. This correlation along with Stoke-Cunningham equation (26) can be used to calculate a diffusive velocity of a particle under a thermal force.

Assuming that ice nuclei are $50 \, \text{Å}$ in diameter (See Figure H-1) and calculating thermal diffusive velocity to be $82 \, \text{ft/hr}$ from Equation II-E32, the nuclei concentration needed for the required water flux will be

$$n' = \frac{0.0125 \times 454}{82.0 \times 2.81 \times 10^4 \times 6 \times 10^{-20}} = 4 \times 10^{13} \text{ molecules/ cc.}$$

If it is assumed that the above calculated nuclei are formed in $10^{-5}$ sec (note that there is no basic ground for this assumed time except that it may give a maximum value of nucleation rate), then required nucleation rate at $x=0.0625$ inch will be $4 \times 10^{18}$ nuclei/cm$^3$ sec.

Even in the worst case of nucleation, when homogeneous nucleation is a predominant mechanism, the partial pressure required to form above calculated nuclei concentration rate can be obtained from the theoretical relationship of Becker
and Doring [8] as shown in Figure H-4.

From Figure H-4, at $T=190^\circ K$ and for nucleation rate of $4 \times 10^{18}$ nuclei/cm$^3$ sec, $p/P_{vp} = 282$

Since, at $T=190^\circ K$ $P_{vp} = 2.45 \times 10^{-4}$ mm. of Hg., partial pressure at $x = 0.0625$ inch, necessary to form a sufficient number of nuclei for the required water flux, is .069 mm. Hg.

The magnitude of this partial pressure and its comparison with the partial pressure calculated from Equation V-C2 indicate that the required water flux by the mechanism of thermal diffusion of ice nuclei appears quite feasible.

Thus, the above described discussion shows that the quantitative explanation of the results of Figures IV-C1 and IV-C2 demands a feasible explanation for the transport of water in the colder region of the frost. The most conceivable physical process that could explain this transport is the thermal diffusion of ice or water nuclei. The detailed theoretical discussion for this experimentally observed phenomenon of uniform densification of the frost is described in Section II-B to II-E.

In order to examine the phenomenon of uniform densification more critically, an experiment was performed in which the transfer of water in the colder region was examined qualitatively. The set-up for this experiment was as follows.

A stainless-steel screen was sandwiched between two
glass wool plugs of total thickness of approximately 0.05 inches. The glass wool screens were so designed that their porosities were almost the same as that of cryogenic frost. The total thickness of this glass wool-screen assembly was approximately 0.0625 inch. The assembly was fastened on to the copper plate and the surface of the top glass wool was flushed with the bottom of the duct.

A regular frosting experiment was then performed with this kind of experimental set-up. The experiment lasted for about an hour. The frost deposition on the top glass wool from flowing humid air was observed during this time.

After the experiment, the top glass wool screen and the deposited frost were removed gently. Both the stainless steel screen and the glass wool screen near the cold wall were examined visually for the presence of frost particles. It was found that both screen and lower glass wool were filled with scattered collections of tiny frost particles. (size < 0.05 cm.).

Comparing this experiment with the usual frosting experiment, it can be seen that is this experiment the bottom 0.0625 inch thick glass wool-stainless steel screen assembly was used as the replacement for the frost layer. Thus, if the results of Figure IV-C1 are actual facts then there should be water flux into the glass wool screens in the present experiment. As described above, the results of the experiment indicate qualitatively
that there is indeed transfer of water flux in the colder region of the glass wool screens. Thus, this additional experimental fact supports the previous conclusion that there is a transport of water in the colder region of the frost.

As discussed previously, the thermal diffusion of ice nuclei appears to be, at present, the most feasible explanation for the water flux in the colder region of the frost.

V-D Discussion of Results of Heat Transfer Coefficients in Non-frosting Conditions

The data of Figure IV-D1 indicate that heat transfer coefficients at various Reynolds numbers under non-frosting conditions agree quite well with ones predicted by Sleicher and Tribus's theoretical analysis (105). The cold wall temperature in these experiments were maintained at the same temperature level as that of frost surface temperature obtained (in the quasi-steady state of heat flux) during the frosting experiments. Thus, it is expected that the heat transfer characteristics of the non-frosting and the frosting experiments are almost identical.

While comparing the experimental data with the theoretical analysis of Sleicher and Tribus (105), it should be noted that the measurements of heat transfer coefficients were carried out for gas flow over a rough surface (i.e., surface of corrugated aluminum foil). The
Sleicher and Tribus's theoretical analysis, on the other hand, assumed that there was no loss of energy due to frictional force between the flowing gas and the rough cold surface. One would expect, however, that for low viscous fluid like air the dissipation of energy due to friction is negligible. The following calculations evaluate the importance of frictional force in the present experimental conditions.

The Calculation of Energy Dissipation due to Frictional Force in the non-frosting Experiments:

The energy loss due to frictional force between the flowing gas and the rough surface can be calculated from Equation V-D1 shown below (page 5-19 of Reference 80)

\[ F = \frac{32 \cdot f \cdot L \cdot q^2}{2 \cdot g \cdot D_h^5} \quad \text{(V-D1)} \]

At Reynolds number=10,000, \( f = 0.01 \) (for the surface roughness like corrugated aluminum from Figure 5-25 in Reference 80).

\[ L = 0.23 \text{ ft. (radius of the copper plate)} \]
\[ q^2 = (42/60)^2 = 0.49 \text{ ft}^2/\text{sec}^2 \]
\[ D_h^5 = (0.8/12)^2 \text{ ft}^2 \]

Substituting all the values in Equation V-D1, one obtains

\[ F = \frac{32 \times 0.01 \times 0.23 \times 0.49 \times (1.2)^5 \times 10^5}{9.87 \times 32.17 \times 327} \]

\[ = 35.2 \text{ ft lb force/lb of gas flowing} \]

Thus, the rate of frictional loss of energy
\[ = (35.2) \times (42 \times 0.0734) \times (32.2 \times 3.995 \times 10^{-5} \times 60) \]
\[ = 8.37 \text{ Btu/hr} \]

The heat flux from gas to cold wall as measured experimentally for Reynolds number of 10,000 is 784 Btu/hr ft². If it is assumed that a constant heat transfer over the entire plate, then the total heat transfer rate over the entire rough surface

\[ Q = 784 \times \pi \times (5.54 / 12)^2 = 131.5 \text{ Btu/hr} \]

percentage contribution of the frictional energy to the heat energy transport is \((8.37 \times 100) / 131.5 = 6.36\%\)

It should be noted that in actual case, the heat transfer rate over the entire plate would be considerably higher than the one calculated above because the heat transfer coefficient in the thermal entrance region would be much higher than one used above. Thus, in reality, the percentage contribution of frictional energy to the heat energy would be considerably lower than one computed above. The application of Equation V-D1 for the present calculation is only approximately correct because it is derived only for isothermal conditions of the system.

The above described calculations indicate that frictional energy loss is insignificant and can be safely neglected in the evaluation of the heat transfer coefficient measurements under the non-frosting conditions. Thus, a good agreement between the experimental and the theoretical values of heat transfer coefficient as shown in
Figure IV-D1, should be expected.

It is assumed that Sleicher and Tribus's theoretical correlations between the heat transfer coefficients and Reynolds number at other gas temperatures will be equally close to the truth as it is for the gas temperature studied in the present experimental studies.
### Table V-A2

**Excess Thermal Conductivity as a Function of Temperature**

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>$K_{\text{excess}}$ Btu/hr ft² °F/ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>-215</td>
<td>negligible</td>
</tr>
<tr>
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Figure V-A1  Point Thermal Conductivity as a Function of Temperature for a Frost Density of 0.09 gm/cc
Note: Though it is not shown in this diagram, rods extend from all six faces of each cube.

Figure V-A2 Assumed Frost Structure; Cube and Rod Model

Figure V-A3 Planar View of a Representative Unit Cell

Figure V-A4 Representative Unit Cell for Cube and Rod Model
Figure V-A5 $x/z$ as a Function of Temperature for Cube and Rod Model
**Experimental Data:**

<table>
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<th>System Conditions</th>
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</tr>
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<td>$-11.5^\circ F$</td>
<td>Non-Frosting</td>
</tr>
</tbody>
</table>

---

**Theoretical Predictions**

- Non-Frosting Conditions
- Lewis Analogy (105)

---

**Figure V-A6** Heat Transfer Coefficient versus Reynolds Number (at Various Gas Temperatures) under Frosting and Non-Frosting Conditions

**Figure V-A7** Mass Transfer Coefficient versus Reynolds Number (at Various Gas Temperatures) under Frosting Conditions
SECT ION VI

CONCLUSIONS

1. After a smooth layer of frost is formed on a liquid nitrogen cooled surface, heat and mass transfer during the frosting process are governed by diffusional mechanisms in the gas phase boundary layer (i.e. no fogging occurs).

2. For all the system conditions,
   A. Both frost density and thickness increase continuously with time.
   B. The heat flux through the frost decreases at first, but levels off and attains a constant quasi-steady state value.
   C. The frost surface temperature increases rapidly at first, but attains a constant value at a later time.

3. In the quasi-steady state of heat flux, both surface temperature and the thermal resistance of the frost remain constant due to a high gain negative feed-back effect of internal diffusion of water vapor to the frost surface temperature.

4. The frost thermal conductivity increases with frost density and temperature.

5. Increase in Reynolds number (at constant humidity and gas temperature),
   A. Increases the frost density at equivalent time.
   B. Decreases the frost thickness at equivalent time.
   C. Increases the quasi-steady state heat flux.
D. Slightly increases the frost surface temperature in the quasi-steady state of heat flux.

6. Increase in gas temperature (at constant humidity and Reynolds number):
   A. Increases the frost density at equivalent time.
   B. Decreases the frost thickness at equivalent time.
   C. Increases the quasi-steady state of heat flux.
   D. Slightly increases the frost surface temperature in the quasi-steady state of heat flux.

7. Increase in humidity (at constant gas temperature and Reynolds number):
   A. Slightly increases the frost density at equivalent time for the humidity range of gas dew point at 14.3°F to gas dew point at 33.2°F.
   B. Increases the frost thickness at equivalent time.
   C. Slightly decreases the quasi-steady state heat flux.
   D. Increases the frost surface temperature in the quasi-steady state of heat flux.

8. With the help of accurate knowledge of heat and mass transfer coefficients, the computer program of Appendix D will successfully predict the time variation of average frost density, frost thickness, mass deposition and heat flux through the frost. The predictions can be made for all the practical range of system conditions, once a smooth layer of frost is deposited from a flowing gas on the liquid nitrogen cooled surface.
9. If an accurate correlation between the frost thermal conductivity, temperature and the frost density is known for the range of interest of density and temperature, then a computer program similar to one described in Appendix D can be used to predict the frost behavior under an arbitrary set of system conditions. Thus, 'simple model' can be used for design of cryogenically cooled equipment under frosting conditions.

10. A temperature difference imposed across the frost in the static conditions (no transfer of heat and mass from outside) causes the mass transfer solely due to temperature gradient only at high temperature levels (where the vapor pressures of ice are of significant magnitude) within the frost.

11. Bleicher and Tribus's theoretical analysis can be well used for the heat transfer calculations for the frosting process in the systems similar to one used in the present thesis.

12. The densification of the frost by the transport of water vapor under the driving force of vapor pressure gradient (caused by a temperature gradient) prevailing at every layer in the frost interior will cause a significant build-up of density gradient in the frost interior.

13. The density of the frost deposited on a liquid nitrogen cooled plate stays always uniform with depth.

14. Even though the water transport past the frost-gas interface can be successfully predicted with the assumption of an equilibrium between ice and gas at the frost surface,
the complete picture of the frosting process can only be successfully modelled with the assumption that water transport at every layer in the frost interior occurs under non-equilibrium conditions. The uniform densification of the frost appears to be explained by 'Non-Equilibrium Model'.
SECTION VII

RECOMMENDATIONS

1. In order to expand the range of application of the successful theoretical model ('simple model') of the present thesis, the experimental studies of carbon dioxide frosting process should be carried out in the system similar to one used in the present study.

2. As seen in Section II, the quantitative theoretical explanation for the uniform (with depth) densification of the frost demands the following investigations:

   A. The experimental study of phenomenon of secondary nucleation from vapor phase at low temperatures.
   B. The experimental study of homogeneous nucleation from vapor phase at temperatures below 200°K.
   C. The experimental study of thermal diffusion of aerosols in air at low temperatures (at all temperatures below 32°F).
   D. The experimental study of crystal growth from vapor phase at the temperatures below -30°C.
   E. The theoretical study of dendritic crystal growth from vapor phase.
APPENDIX  A

Instrumentation for the Apparatus of Figure III-Al

1. Temperature Measurements

Temperature measurements were made with both chromel-constantan and copper-constantan thermocouples. The various temperature measurements carried out in the system of Figure III-Al are described below.

A. Gas Temperatures

Gas temperatures were measured at the inlet and the outlet of the duct and right above the copper plate. The gas temperature above the copper plate was measured with a movable thermocouple probe—-the same probe used to measure the temperature of the frost surface. The gas temperatures at the duct inlet and outlet were measured with 72 inches long No. 24 gauge chromel-constantan thermocouples fixed in the gas phase. (Thermocouples were made by Nonmac Corporation, 140, Crescent Road, Needham, Massachusetts; according to their catalogue No. 40-E specification). The long and thin thermocouple wires were used to reduce the error in the measurement due to heat conduction along the wires to the measuring junction.

B. Temperature Measurements on Copper Plate

The thermocouples located on the copper plate were made of No. 24 gauge copper-constantan wire. Two thermocouples were silver soldered to the underside of the inner and the outer section of the plate as shown in Figure A-l. One thermocouple was used to check the reading of the other.
C. Calorimeter Temperature

The temperature of the gas space in the outer chamber was measured with No. 24 gauge, copper-constantan thermocouple. During normal operation of the calorimeter, this temperature was usually within few degrees of the liquid temperature (The liquid nitrogen in the guard chamber was essentially at the normal boiling point ). The location of this thermocouple in the calorimeter is shown in Figure A-1.

D. Temperature Measurements in the Frost Interior

Temperatures in the frost interior were measured with No. 30 gauge (wire diameter = 0.005 inch ) thermocouples located at the fixed heights over the cold plate. The exact distances between the copper plate and the centre of thermocouple beads were determined at the beginning and the end of each experiment. The wires were kept taut and parallel to the plate as shown in Figure A-2. The gas flowed in a direction parallel to the length of the wire. The distance between the two closest thermocouples and between the lowest thermocouple and the plate was about 0.05 inch to 0.06 inch in most of the experiments. The maximum thickness of the thermocouple bead was about 0.02 inches. Since there is a large temperature gradient existing in the frost interior, there could be significant error in the temperature versus distance profile due to inaccuracy involved in the determination of the exact location for the thermocouple joint.

E. Movable Probe for Measurement of Frost Surface Temperature

A movable thermocouple probe was used to measure the
surface temperature of the frost. This probe, which could move up and down in a vertical plane, was a loop type thermocouple as shown in Figure III-A2.

The maximum thickness of the measuring junction was about 0.02 inches. With the help of this thermocouple, the frost surface temperature was measured by the method described in Section III-A. The thermocouple was made of No. 30 gauge copper-constantan wires.

F. Errors in the Surface Temperature Measurement

A possible source of error in the measurement of the frost surface temperature was conduction to the junction along the thermocouple wires. This error was made negligible by taking two precautions: first, thin thermocouple wire was used; secondly, when a surface temperature measurement was being made, a copper and a constantan wire, each two inches long, extended from the junction and contacted the frost surface. This ensured long isothermal paths on either side of the junction and hence conduction to the bead was minimized.

Another possible source of error was radiant heat transfer from the warm duct to the bead in contact with the surface. The magnitude of this error could not be estimated accurately as the contact thermal resistance between the thermocouple junction and the frost itself was unknown. The error was reduced by taking two precautions. First, when making a surface temperature measurement, the thermocouple junction was allowed to remain contact with the frost surface for a
relatively long length of time (~about one minute~). Thus, presumably, a substantial portion of bead was covered by the growing frost for a long time to minimize radiation effect. Secondly, radiant heat transfer from the warm inside of the duct to the colder thermocouple junction was minimized by lining the inside of the Lucite section right above the copper plate with aluminum foil.

The magnitude of the error caused by gas flow over the thermocouple bead in contact with the frost surface also cannot be estimated accurately, since the thermal contact resistance between bead and frost is unknown. Again, this error was reduced by allowing a significant portion of the thermocouple bead to become covered with the frost when a surface temperature measurement was being made. Errors caused by both radiation and gas flow over the thermocouple bead would result in thermocouple readings which are too high.

As mentioned earlier, the movable thermocouple probe also measured the temperature of the gas stream over the copper plate. The gas flow past the probe did not cause a significant error in the gas stream temperature measurement.

2. Pressure Measurements

Pressure measurements for the orifice meter were carried out with water-filled manometers. In the initial runs, pressures at the inlet and the outlet of the duct were measured with low pressure gauges. These pressure values are not reported in this thesis. It should be noted, though,
that the pressure in the gas stream over the copper plate was essentially atmospheric for all the flow rates studied in this thesis. Pressures in the inner and outer cylinders of the calorimeter were also measured with water-filled manometers.

3. Flow Measurement

As mentioned earlier the flow rate was measured with an orifice meter. Three different orifice meters with the hole diameters of 0.346", 0.535" and 0.80" were used. The meter was constructed according to the ASME code described in Reference 4. The centres of the upstream and the downstream tap holes were located exactly at distances of one tube diameter from the upstream and the downstream faces of the orifice plate respectively. The tap lines were ¼ inch copper tubes soldered into the ¼ inch tap holes.

For maximum accuracy the orifice hole was positioned so that it was concentric with the tube openings. To further ensure accurate flow rate determination the lengths of straight pipe upstream and downstream of the orifice plate were made longer than the recommended minimum requirements (80).

Despite all the precautions taken in constructing the orifice meter, it was still decided to calibrate (see Appendix B for details of the calibration procedure).

4. Measurement of Humidity

The humidity of the flowing gas was measured at a
short distance upstream of the duct by analyzing a sample drawn from the flowing stream. To be more specific, the humidity was measured by determining the quantity of water vapor contained in a measured volume of gas sample. A schematic of the humidity measuring equipment is shown in Figure A-3. The gas sample was flown through a tube containing molecular sieve 10X and then through a wet test meter. The flow rate of the gas sample was insignificant compared with the flow rate of the flowing gas in the system.

The volume of gas bled off for analysis was measured with a wet test meter. The weight of water vapor contained in this gas sample was determined by weighing the tube containing molecular sieve before and after the sample was passed through it. This method of measuring gas stream humidity was found to be quite reliable as the tube was designed long enough so that substantially all the moisture in the gas sample was adsorbed by molecular sieve. Complete drying of the air was also verified by changing the flow rate of the gas. The reliability of the method was checked by measuring the humidity of a gas stream which had been saturated with water vapor at a known temperature. "Checks" were performed at several humidities. In all the cases the humidity measured by the method described above was within 3% of the true humidity.

During a frosting run, the humidity of the flowing gas was measured several times. Frequencies of the measurement depended upon the length of the experiment. The measured
humidity variation during a run was generally within \( \pm 2\% \) of the values given in column 4 of Table IV-A2. The humidity variation was usually random with time; i.e., it did not show a consistent trend during the course of a run.

It should be noted that the observed humidity variation and the error in the humidity measurement were approximately equal. Thus, it is probable that the observed random humidity variation was due to errors in the measurement of the humidity rather than to variation in the performance of the drier and the humidifier system.

5. Pipings for the System

The majority of the pipings for the system consisted of polyvinyl chloride tubing and the rest was made of metal and rubber tubing. The majority of tubings were one inch in diameter. Smaller diameter tubings, however, were employed for some of the parallel and by-pass gas flow lines. The section of the system after the heat exchanger was insulated with fibreglass aircraft insulation in order to control the gas temperature.
**Figure A-1** Schematic Diagram Showing the Temperature Measurement Locations in the Calorimeter Assembly

**Figure A-2** Schematic Diagram Showing the Fixed and Movable Thermocouples over the Copper Plate
Schematic Diagram for Gas Sampling
for Humidity Analysis
APPENDIX  B

Calibration of Orifice Meters

The two orifice meters (0.346 inches and 0.535 inches in inside diameters) were calibrated over the entire range of pressure measurements. Whereas 0.80 inches diameter orifice meter was calibrated only for its lower 25% range of the pressure measurements. These calibrations covered the entire range of their application.

The calibrating device was a large gasometer. Constant flow rates were produced by the displacement of gas from an inverted vessel which fell with a constant velocity through a tank containing oil. Gas flows up to about 60 std. ft³/minute were produced. The schematic of the set-up for the calibration is shown in Figure B-1.

Agreement to within 2% was obtained between the actual flow rates determined by measuring the displacement of gasometer and the calculated rates. "Calculated" rates were obtained using standard sharp edged orifice meter equation (4). The error values at the various flow rates did not exhibit any trend with flow rate.
Flow Regulator

Air Flow

Manometer

Orifice Meter

Flow Regulator

Air Line Connecting to Drum

Air From Building Main Line

Oil Bath

Figure B-1 Schematic Diagram of Set-up for the Calibration of Orifice Meter
Calibration of Calorimeter

The details of the calorimeter construction and operation have been presented in Section III-A. When heat flux is measured by the method described in Section III-A, errors can arise from various sources. For example, one possible source of error is the entrainment of liquid droplets in the calorimeter boil-off and the subsequent vaporization of these droplets in the inner cylinder outlet line (see Figure III-A2). In addition, another source of error is the superheating of the boil-off nitrogen vapor in the inner stainless steel calorimeter vessel. Thus in order to have confidence in the heat flux measurements the calorimeter had to be calibrated.

The source of heat for the calibration was an electrical heater, which was well insulated. The electrical energy output of the heater could be varied by changing the voltage. This electrical heater was sandwiched between two different slabs of cork. The top slab was approximately 3 inches thick. The thickness of the lower slab varied from 2 to 3 inches. Some experiments were also made without the lower slab (i.e., by direct contact of heater with copper plate). The sides of the heater were insulated by 6 inches thick fibre glass insulation. The complete heater assembly was placed on the copper plate. The details of the experimental "set-up" are shown in Figure C-1.

With the calibration set-up described above, substantially all the electrical energy produced by the heater was passed
into the copper plate through the lower slab of cork. The following measurements were carried out during a calibration experiment.

1. The current and the voltage across the heater were measured to calculate energy output from the heater. (Direct current was employed for all the calibration experiments).

2. Temperature differences across both lower and upper cork slabs were measured by the thermocouples. The thermal conductivities of the cork slabs were obtained from the literature (48, 54, 118).

3. The boil-off of liquid nitrogen was measured by a calibrated rotameter.

The comparison between the heat fluxes calculated from the liquid nitrogen boil-off measurement, electrical heat input to the heater and the calculated heat flux through the bottom cork slab for three different calibration experiments is shown in Table C-1. The close agreement among all of them indicate that the measurement of the liquid nitrogen boil-off is a reliable method for obtaining the true heat flux into the calorimeter. In the worst case, the heat flux values disagreed by 3.90%.

No calibration curve for the calorimeter was established as the agreement between known heat fluxes and the values determined from the boil-off measurements was quite good. The reported heat flux values for the various frosting runs were, therefore, determined solely from the liquid nitrogen boil-off measurement. The brief descriptions
of the electrical heater and the available literature information on the cork thermal conductivities are given below.

A. Electrical Heater

The heating element was Nichron replacement electrical heating wire (maximum output 600 watts). It was wound in a number of horizontal loops. These loops were all located in the same plane and were sandwiched between two 1/8 inch stainless steel plates. The stainless steel plates were cemented to each other and the space between the two plates was filled with glass wool to minimize heat loss.

The measured electrical resistance of the heater was 19.437 ohms. This value was essentially independent of wire temperature or applied voltage over the range of the conditions used in the calibration runs.

B. Cork Thermal Conductivities

In order to calculate the heat fluxes through the cork slabs, it was important to know the accurate values of the cork thermal conductivities. Cork conductivity data were obtained from different sources (48,54,118). These references furnished the data over a wide range of temperature and cork density. These data indicated that conductivity is a strong function of density. The densities of the various pieces of cork employed in the calibration experiments were therefore measured and these values were used to obtain the appropriate conductivities.
C. Sample Calculation for Calibration Experiment

Case 1 in Table C-1

1. Calculation of Electrical Heat Output

 resistence of electric heater = 19.437

Voltage = 10 volts

Electrical heat input = \( \frac{V^2}{R} = \frac{100}{19.437} \approx 5.14 \) watts

Area of heater = \( \frac{6'' \times 6''}{144} = 0.25 \) ft

heat input/area = \( 4.87 \times 3.6 \times 10^3 \times \frac{144}{36} \times 10^{-3} \) = 70.2 Btu/hr ft


The cork slab density = 0.1144 gms/cc

\( \Delta T \) across the cork slab = 80 - 72 = 8°F

\( k_{avg} \), at 76°F and density of 0.1144 gms/cc = 0.28 Btu/hr ft²°F/inch (54)

Thickness of cork slab = 3.08 inch

heat flux through the upper cork slab = \( \frac{k_{avg}}{8} \Delta T = \frac{0.28 \times 8}{3.08} \) = 0.736 Btu/hr ft²

3. Calculation of Heat Flux Through Lower Cork Slab

The cork slab density = 0.1144 gms/cc.

\( \Delta T \) across the cork slab = 315 + 70 = 385°F

Thickness of cork slab = 1.05 inches

\( k_{avg} \) at density of 0.1144 gms/cc and temperature of -123°F = 0.193 Btu/hr ft²°F/inch (54)
heat flux through the lower cork slab

\[ k_{\text{avg}} \frac{\Delta T}{\delta} = 0.193 \times \frac{385}{1.05} = 70.7 \text{ Btu/hr ft}^2 \]

4. Calculation of Heat Flux Through Calorimeter

Liquid nitrogen boil-off as measured by calibrated rotameter = 0.0053 ft$^3$/minute

heat flux = \[ \frac{0.0053 \times 2405 \times 0.07245}{0.0277 \times 28} = 71.1 \text{ Btu/hr ft}^2 \]
| Run | Electrical Heat Output (Btu/hr ft²) | Heat Flow Through Lower Cork (Btu/hr ft²) | Heat Flow Through Upper Cork (Btu/hr ft²) | HeatMeasured By LN₂-Boil off | % Error (Heat by LN₂ Boil off + Heat Flow Through upper Cork-Electrical Heat x100) Electrical Heat |
|-----|----------------------------------|----------------------------------------|----------------------------------------|----------------------------|-------------------------------------------------------------------------------------------------
| 1   | 70.2                             | 70.7                                   | 0.736                                  | 71.1                       | +2.33%                                                                                             |
| 2   | 280.8                            | -                                      | 2.19                                   | 279.0                      | +0.1385%                                                                                           |
| 3   | 631.8                            | -                                      | 3.19                                   | 604.0                      | -3.90%                                                                                             |
Figure C-1  Arrangement for Calibration of Calorimeter
APPENDIX D

Computer Program For the "Simple Model" of the Frosting Process

The three basic equations of the "simple model" which are solved on the computer can be summarized as follows.

Heat Balance

\[ q = h (T_g - T_s) + K_g (p_g - p_s) \Delta H = k_f (\frac{dT}{dx})_s = k_f x (\frac{dT}{dx})_x \]

\[ = \left( \int k_f \, dT \right)/\delta \]  \hspace{1cm} (D-1)

Mass Balance

\[ K_g (p_g - p_s) = \rho_f \frac{d\rho_f}{d\theta} + \delta \frac{d\rho_f}{d\theta} \]  \hspace{1cm} (D-2)

and Internal Diffusion Equation,

\[ \frac{d\rho_f}{d\theta} = - \beta(T_s) \left( \frac{1 - \rho_f/\rho_{in}}{\rho_f} \right) \left( \frac{dT}{dx} \right)_s \]  \hspace{1cm} (D-3)

where, \( \beta(T_s) = \frac{D_B M \Delta H P_{vp}}{R^2 T_s^3} \)

For simplicity in programming, \( \beta(T_s) = f(T_s) \) and \( k_f = f(T, \rho_f) \) were obtained in the form of empirical equations.

The correlation among the frost thermal conductivity, density and temperature was obtained from the experimental data of Brazinsky (14). The following Equations D-4 and D-5 show these correlations.

For \( \rho_f > 0.025 \text{ gms./cc. and } T < 460^\circ R \)

\[ k_f = 3.875 \times 10^{-6} T^{1.441} + 4.08 \times 10^{-9} x (\rho_f - 0.025) x T^{3.055} \]  \hspace{1cm} (D-4)

For \( \rho_f > 0.025 \text{ gms./cc. and } T > 460^\circ R \)

\[ k_f = 8.49 \times 10^{-17} T^{5.44} + 6.86 \times 10^{-14} x (\rho_f - 0.025) x T^{4.84} \]  \hspace{1cm} (D-5)

where, \( T \) is in \( ^\circ R \), \( \rho_f \) is in gms/cc. and \( k_f \) is in Btu/hr ft \( ^\circ F \)

The following correlation for \( \beta(T_s) = f(T_s) \) was obtained:
\[ \beta(T_s) = 3.42 \exp\left(-\frac{4460.0}{(T_s-140.0)}\right) \]  
\[ \text{(D-6)} \]
where, \( T_s \) is in °R and units of \( \beta(T_s) \) are \( \text{lbm/ft}^3 \text{hr} \)

It should be noted that since there is no thermal conductivity correlation for the frost density below 0.025 gms./cc. included in the computer program, theoretical calculations for \( \rho_f < 0.025 \text{ gms./cc.} \) can not be carried out with this program. However, the following correlation for the thermal conductivity of water frost can be used when \( \rho_f < 0.025 \text{ gms./cc.} \).

For \( \rho_f < 0.025 \text{ gms./cc.}, \ T < 460^\circ R \)
\[ k_f = 6.58 \times 10^{-6} T^{1.272} + 8.17 \times 10^{-6} \rho_f T^{1.74} \]  
\[ \text{(D-7)} \]
where once again, \( T, \rho_f \) and \( k_f \) are in the same units as those used in Equations D-4 and D-5.

**Method of Solution**

At starting time, knowledge of \( \delta \) and \( \rho_f \) is required. Starting with these initial values of \( \delta \) and \( \rho_f \), transcendental Equation D-1 was solved for \( T_s \) at starting time using Newton-Raphson method. Equations D-2 and D-3 were then used to calculate \( \delta \) and \( \rho_f \) at new time. The following finite difference analogues of Equations D-2 and D-3 were used for this purpose in the listed sequence. The finite difference equations were solved by a simple Euler's method.

\[ \frac{\rho_{f,n} - \rho_{f,0}}{\Delta \theta} = \frac{\left(1 - \rho_{f,0}/0.917\right) \beta(T_{50}) K_{ar}(\rho_{f,0}, T_{50}) \left(T_{50} - T_w\right)}{(4.41) (\tau) (\delta_{so})} \]  
\[ \text{(D-8)} \]
and
\[ \frac{\delta_n - \delta_0}{\Delta \theta} = \frac{\left[K_{fg} (\rho_g - \rho_{g,0} (T_{50})) - \delta_0 \left(\right)\right]}{\left((\rho_{f,N} + \rho_{f,0})/2.0\right)} \]  
\[ \text{(D-9)} \]
where, $\tau$, tortuosity was assumed to be 1.10 (94)

The time variations of the average frost density, frost thickness and surface temperature were obtained by iterating the above described calculations.

**Convergence Test for the Computer Results**

In order to verify the convergence of the computer solution for the frost density, thickness and surface temperature, the final results were obtained for various time increments. These results are shown in Table D-1.

The computer results shown in Figures II-A1 to II-A12 and in Figures D-1 to D-3 were obtained with time increment of 1 minute. From the results shown in Table D-1, therefore, one can conclude that all the computer results of the "simple model" presented in this thesis are convergent.

**Additional Computer Results of the "simple model"**

Figures D-1 to D-3 show the predictions of the "simple model" regarding the effects of gas temperature on the frost density and thickness versus time profiles for the typical sets of experimental conditions. The results shown in Figures D-1 and D-2 indicate that at constant Reynolds number and humidity, the frost densifies at a faster rate and the thickness of the frost increases at a slower rate as gas temperature is increased. Thus, the thermal resistance of the deposited frost is higher at a higher gas temperature. Figure D-3 shows the effect of a step change in gas temperature on the average frost density and frost thickness. As expected, the model predicts that a step decrease in gas temperature
at constant Reynolds number and humidity would decrease the rate of frost densification and increase the rate of change of frost thickness. Thus, the conclusions regarding the effects of gas temperature on the frost density and thickness versus time profiles one can draw from the Figures D-1 to D-3 are the same as those stated in Section II-A.

A listing of the computer program written in Fortran 4 language is included in this appendix.

Application of the "simple model" to the Carbon Dioxide Frosting Process

In order to apply the "simple model" to the carbon dioxide frosting process, another similar computer program was written. The functional correlation $k_f(\rho_f, T)$ for this program was obtained from Riemann's model for the frost structure (see Appendix F), and hence the program can be used only for the carbon dioxide frost density greater than 0.85 gms./cc. Since this $k_f(\rho_f, T)$ correlation needs experimental verification, the computer program has not been used to obtain theoretical predictions. However, once an experimentally verified thermal conductivity correlation is obtained, a similar computer solution can be used to apply the "simple model" to carbon dioxide frosting process. A listing of this program written in Fortran 2 language is included in this appendix.
Table D-1

Convergence Tests for the Computer Solution of Mathematical Equations of the "Simple Model"

A. Experimental Conditions

Re = 5,600; Humidity = dew point at 29.2\(^o\)F; \(T_g = 70\(^o\)F

<table>
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<th>(T_s) ((^o)F)</th>
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<tr>
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<td>0.0589220</td>
<td>0.190156</td>
<td>13.5598</td>
</tr>
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</table>

B. Experimental Conditions

Re = 14,500; Humidity = dew point at 40\(^o\)F; \(T_g = 75\(^o\)F

<table>
<thead>
<tr>
<th>(\theta) (mins.)</th>
<th>(\Delta \theta) (mins.)</th>
<th>(Q_f) ((\text{gms.}/\text{cc.}))</th>
<th>(\delta) (inches)</th>
<th>(T_s) ((^o)F)</th>
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<td>0.0877733</td>
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<td>$\Delta \theta$ (mins.)</td>
<td>$q'(\text{Kms}^2/\text{cc})$</td>
<td>$\delta$ (inches)</td>
<td>$T_s$ (°F)</td>
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<td>5</td>
<td>0.147114</td>
<td>0.230220</td>
<td>19.4170</td>
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</tbody>
</table>
Experimental Conditions

1. Re = 14,500; Humidity = Gas dew point at 40°F; T_g = 70°F
2. Re = 9,300; Humidity = Gas dew point at 29.2°F; T_g = 70°F
3. Re = 5,600; Humidity = Gas dew point at 29.2°F; T_g = 70°F
4. Re = 14,500; Humidity = Gas dew point at 40°F; T = 30°F
5. Re = 9,300; Humidity = Gas dew point at 29.2°F; T = 30°F
6. Re = 5,600; Humidity = Gas dew point at 29.2°F; T_g = 30°F

Figure D-1 'Simple Model' Predictions Regarding Effect of Gas Temperature on Average Frost Density

Figure D-2 'Simple Model' Predictions Regarding Effect of Gas Temperature on Frost Thickness
Figure D-3 Predictions of Simple Model for the Effects of Gas Temperature on Average Frost Density and Frost Thickness
Computer Program for 'Simple Model' (Water Frosting Process)

DIMENSION FLUX(4001), FLUXP(4001)
READ (5, 100) H, RHOF1, TG, P, FLUX1, F1
FORMAT (6F10.5)
READ (5, 101) TW, DELX, DELT, TZ1, N
FORMAT (6F10.5, 13)
RHOF0=RHOF1
DELO=DELI
TS=460.0
IJ=0
TZ=TZ1
DO 10 I=1, N
IF(I-1) 487, 483, 487
483 FLUX(I)=FLUX1
487 IF(TZ<46.0) 482, 482, 28
28 DO 493 J=1, 25
TZ=TZ1.80
P1=8.2312*S(4.434*ALOG(TZ))
PVP=EXP((-2445.5646/TZ)1.31-(0.3157/TZ))&11.2514*(1.0*(1-0.0)
1)*TZ1.7571691/(4.3431)
TZ=TZ1.90
IF(I-1) 511, 510, 511
510 P0=1(1.2445.56461.81)/TZ&1(I+4.3438*2.712/17.01.47706/
1.801+7.41028*(10.**(-0.0)*TZ1)/3.2501)/10.4343)
PVP=PVP*1.0
IF(I-1) 33, 34, 33
34 F=1
33 FLUX(I)=(-F*PVP)
C1=
11*(TS**1.544)*[1.175*(RHOF0-.025)*([0.**(-14.0)]*TS**(15.84)]
25*(1.587*([10.**(-6.0)]*TS**(2.441)))&11.058*([10.**(-3.0)]*TS**(4.055))
1+(RHOF0-.025)*([TW**(-4.55)]
FTZ=1.319*([10.**(-17.0)]*TS**(-6.441))&11.175*(RHOF0-.025)*
1)*TZ**(-14.01)*TS**(-5.341)*TL*DEL0*TSI-FLUX1*DEL0,(TZ)**TS**(-2.441)
25*(1.058*([10.**(-9.0)]*RHOF0-.025)*([TS**(-4.55)]
PFTZ=(6.440*1.3191.0)**(TS**(-5.341))6.4)*RHOF0-.025
1)*1.175*(10.0)**(-14.01)*12*(TS**(-4.841))*(DEL0)*FLUX1*DEL0
34 FLUX(I)=(-F*PVP)
C1=
11*(TS**1.544))&11.175*(RHOF0-.025)*([0.**(-14.0)]*TS**(15.84))&11.058*([10.**(-6.0)]*TS**(2.441))
25*(1.587*([10.**(-4.0)]*TS**(4.055))
1)*TZ1+7571691/(4.3431)
10*FUT=(1.0, 17, 12)*TS*-(0.82811) (-DEL0)*1.22.5*1.0*(1.71721)
34 FLUX(I)=(-F*PVP)
C1=
11*(TS**1.544))&11.175*(RHOF0-.025)*([0.**(-14.0)]*TS**(15.84))&11.058*([10.**(-6.0)]*TS**(2.441))
25*(1.587*([10.**(-4.0)]*TS**(4.055))
1)*TZ1+7571691/(4.3431)
10*FUT=(1.0, 17, 12)*TS*-(0.82811) (-DEL0)*1.22.5*1.0*(1.71721) IF(ABS(EP)=(10.0)**(-4.0)) 491, 28, 28
493 CONTINUE
GO TO 491
482 DO 490 K=1, 25
TZ=TZ1.80
P1=8.2312*4.434*ALOG(TZ)
PVP=EXP((-2445.5646/TZ)1.31-(0.3157/TZ))&11.2514*(10.**(-0.0)
1)*TZ1.7571691/(4.3431)
TZ=TZ1.90
IF(I-1) 16, 507, 16
16 FLUX(I)=F*(P-PVP)
Computer Program for 'Simple Model'

(Carbon Dioxide Frosting Process)

```
DIMENSION FLUX(400),FLUXP(400)
L=1
GO TO *(4,1)+*L
4 READ (2,1CC) 4,*HOF,TG,F,F,KG
10 FORMAT(6F10.4)
READ (2,1LC) *DEL,TZI,N
11 FORMAT(4FLUX*13)
HOF=HOF
DEL=DEL
TZ=TZI
DO 16 I=1,N
DO 493 J=1,25
T7=TZ+1.80
IF (TZ=138.0) 41,41,42
41 PVV=EXP((-1.275*E+0/TZ)+(U.09683*TZ)+8.3011/(U.4343))
TZ=TZ+1.80
PV=(1.275*E+0/1.80)+(1.1469)/(U.4343)
GO TO 815
42 PVV=EXP((-1.3673448/TZ)+9.9082/(U.4343))
TZ=TZ+1.80
PV=(1.36734481.80)+(1.1469)/(U.4343)
815 PVVP=PV*PP
FLUX(1)=FL(D-PP)
FLUXP=FL(I*PPP)
IF (HOF<=1.0) 11,11,12
11 CI=(HDELO+T7)+(3.558*(T7**(-0.584)))*KHFUS*(23.920*(T7**(-1.0239)))*TW1
FT2=(3.558*(T7**(-0.584)))*(HOFU**((23.920*(T7**(-1.0239)))))*T2
1+(HDELO*T2)-CI=(FLUX1)*DELU*(1.58)304*(T7**(-0.1469)))
FTL=(1.358*(T7**(-0.584)))*(KHFUS*(23.920*(T7**(-1.0239))))
1+(HDELO*T2)-CI=(FLUX2)*DELU*(1.58)304*(T7**(-0.1469)))
FT2=(3.558*(T7**(-0.584)))*(HOFU**((23.920*(T7**(-1.0239)))))*T2
1+(HDELO*T2)-CI=(FLUX1)*DELU*(1.58)304*(T7**(-0.1469)))
FTL=(1.358*(T7**(-0.584)))*(KHFUS*(23.920*(T7**(-1.0239))))
1+(HDELO*T2)-CI=(FLUX2)*DELU*(1.58)304*(T7**(-0.1469)))
GO TO 13
12 CI=(HDELO+T7)+(3.558*(T7**(-0.584)))*(HOFU**((23.920*(T7**(-1.0239)))))*TW1
FT2=(3.558*(T7**(-0.584)))*(HOFU**((23.920*(T7**(-1.0239)))))*T2
1+(HDELO*T2)-CI=(FLUX1)*DELU*(5.83504)*(T7**(-0.1469)))
FTL=(1.358*(T7**(-0.584)))*(KHFUS*(23.920*(T7**(-1.0239))))
1+(HDELO*T2)-CI=(FLUX2)*DELU*(5.83504)*(T7**(-0.1469)))
1+(HDELO*T2)-CI=(FLUX1)*DELU*(5.83504)*(T7**(-0.1469)))
FLUX(1)=F1(F-PP)
IF (HOF<1.0) 61,61,62
61 CAVG=(3.558*(T7**(-0.584)))*(HOFU**((23.920*(T7**(-1.0239)))))
FU=F(CAVG*(T7-TZ))/DELU
```

```f
FUP = (H*(TG-TZ)) + (FLUX(1)*(283.5U)*(TZ**(-0.1466)))
EP = (FUP/FUP) =1.0
GOTO 14

62 CAVG = (3.5E5*(TZ**(-0.564))*(RHOF0)**(2.27G*(TZ**(-0.0345))))
FUT = (CAVG*(TZ-TA))/DELO
FUP = (H*(TG-TZ)) + (FLUX(1)*(583.5U)*(TZ**(-0.1466)))

14 IF (ABS(EP) - (1.0)*(4.0)) 481, 481, 493
493 CONTINUE
461 WRITE (6,192) EP,J
192 FORMAT (E15.6,I3)
14 IF (RHOF0-1.0) 54, 64, 65
64 IF (TZ-265.0) 66, 66, 67
66 CAS = (4.150*(TZ**(-1.081))*(RHOF0)**(9.50*(TZ**(-0.5072))))
GOTO 918
67 CAS =(0.100*(RHOF0)**(8.060)*(TZ**(-3.380))*(10.0)**(8.0)))
GOTO 918
65 IF (TZ-265.0) 68, 68, 69
68 CAS = (4.150*(TZ**(-1.081))*(RHOF0)**(2.96*(TZ**(-0.0862))))
GOTO 918
69 CAS = (0.100*(RHOF0)**(1.175)*(10.0**(-1.5750))*(10.0)**(4.0))))
818 DELC = (CAVG*(TZ-TW))/CEL
TZ = TZ+1.0 B
DIF = (2.380)**(10.0)**(6.0)**(10.0)**(7.0)**(2.0)**(3.377))
TZ = TZ+1.80
DEL2 = DELO*DEL3
RHOF1 = (1.00*(RHOF0/1.60))*DELC*DIF*PVPP/(1.10**62.4C*DEL2**6*TZ)
DELO = DELO/12.00
TZ = TZ-459.67
WRITE (3,108) RHOF0, DELO, TZ
108 FORMAT (3E16.6)
TZ = TZ+459.67
DELO = DELO/12.00
RHOFN = RHOF0 + DELX*RHOF;
RHOF = RHOF0 + DELX*RHOF;
RHOF = (1/(RHOF0+RHOFN)/2.0)*62.40
DELN = DELO*(DELC*(FLUX(1)-DELU*RHOF))/2.0
RHOF = RHOF0
10 DELO = DELN
GOTO 5
2 CALL EXIT
END
```
APPENDIX  E

Computer Solution of the Mathematical Equations of the 'Sophisticated Model'

The basic equations of the 'sophisticated model' in the dimensionless form can be summarized as follows.

Heat Balance
\[ q = \frac{k_f}{\delta} \frac{\partial T}{\partial z} \] (E-1)

Mass Balance
\[ \left( \frac{\partial^2 \dot{q}}{\partial z^2} \right) - \frac{z}{\delta} \left( \frac{\partial \dot{q}}{\partial z} \right) = \frac{1}{z} \left[ \frac{\left( l - \frac{\eta / \eta_{\text{ref}}}{\eta} \right) \partial \alpha}{RT \tau (1 - \phi / \phi_{\text{ref}})} \left( \frac{M \Delta H}{P_{\text{vp}}} \right) \frac{1}{\delta} \left( \frac{\partial T}{\partial z} \right) \right] \] (E-2)

Boundary Conditions

1. At Cold Wall
   A. Heat Boundary Condition
   \[ T \bigg| _{z=0} = T_w \text{ at all time} \] (E-3)
   B. Mass Boundary Condition
   \[ J_s \bigg| _{z=0} = \frac{D_B}{RT \tau (1 - P_{\text{vp}} / P)} \left( \frac{M \Delta H}{P_{\text{vp}}} \right) \frac{1}{\delta} \frac{\partial T}{\partial z} \bigg| _{z=0} = 0 \] (E-4)

2. At Frost Surface
   A. Heat Boundary Condition
   \[ q \bigg| _{z=1} = h \left( T_g - T_{z=1} \right) + k_g \left( p_{g} - p_{z=1} \right) \Delta H = \frac{k_f}{\delta} \frac{\partial T}{\partial z} \bigg| _{z=1} \] (E-5)
   B. Mass Boundary Condition
   \[ J_s \bigg| _{z=1} = \frac{D_B}{RT \tau (1 - P_{\text{vp}} / P)} \left( \frac{M \Delta H}{P_{\text{vp}}} \right) \frac{1}{\delta} \frac{\partial T}{\partial z} \bigg| _{z=1} \] (E-6)

Finite difference analogues of Equations E-2, E-4 and E-6 used in the computer solution can be written as,

...
$$D_B = 2.67953 \times 10^{-6} \times T^{2.00877} \quad (E-7)$$

where, $D_B$ is in cm$^2$/sec. and $T$ is in K.

The brief description of the numerical method used in this program to obtain $T(z)$, $\varphi(z, \theta)$ and $\delta(\theta)$ profiles is shown below.

At starting time, $\varphi(z, \theta)$ and $\delta(\theta)$ values were assumed. In all the cases studied in the present thesis, initial values of $\varphi(z, \theta)$ were assumed to be constant across the frost layer. Equations E-1, E-3 and E-5 were then used to solve the temperature distribution in the frost interior at the starting time. The fourth order Runge-Kutta method was used for this purpose. A thorough description of the fourth order Runge-Kutta method is given by Mickley et al. (74).

Knowing $T(z)$, $\varphi(z, \theta)$ and $\delta(\theta)$ at the starting time, Equations E-2, E-4 and E-6 were then used to obtain $\varphi(z, \theta)$ and $\delta(\theta)$ at new time. The schematic of a six point finite difference scheme used to solve these equations is shown in Figure E-0. As shown in the figure, in the finite difference scheme, two levels in time and three levels in distance (z-direction) were used. The weighing coefficients were given equal values ($a=b=c=d=1/2$).

![Figure E-0: Schematic of Six Points Finite Difference Method](image-url)
The boundary condition at the cold wall will be,

$$\frac{\phi_j - \phi_{j-1}}{\Delta \theta} = \frac{1}{\delta_{i,v_{i,j}}} \left[ \frac{\phi_j - \phi_{j+1}}{\Delta z} \right] + \frac{1}{\delta_{i,v_{i,j}}} \frac{1}{\Delta z}$$

and the boundary condition at the frost surface will be,

$$\frac{\phi_j - \phi_{j+1}}{\Delta \theta} = \frac{1}{\delta_{i,v_{i,j}}} \left[ \frac{(1-\phi_l/\phi_{ce}) S_{B} \left( \frac{M \cdot \Delta H \cdot P_r}{RT^2} \right) \phi_j}{RT \cdot (1-P_r/p)} \right] \frac{1}{\Delta z}$$

Method of Solution

Above described equations were solved by finite difference techniques on computer. A listing of the computer program written for this purpose is shown at the end of this appendix. This program is written in Fortran 4 language.

For simplicity in programming, the empirical correlation among the frost thermal conductivity, density and temperature, described in the Appendix D was used in this program. Similarly, an empirical correlation between molecular diffusivity and the temperature was used in the program. This correlation is shown in Equation E-7 and described in Figure II-B10.
Thus, the method of solution of mathematical equations essentially involve the iterative procedure for the solution of the Equations E-1, E-3 and E-5 by Runge-Kutta method and the solution of the finite difference Equations E-7 to E-9.

Stability and Convergence Tests for the Finite Difference Solution:

The stability and convergence of the computer results were examined by varying $\Delta z$ and $\Delta \theta$ values respectively. Figures E-1 to E-4 show the effects of variation in $\Delta z$ and $\Delta \theta$ values on the frost density at various points in the frost interior. The results of the figures indicate that the finite difference method used in the computer solution gave stable and convergent results.

Additional Results of the 'Sophisticated Model'

Figure E-5 shows the additional results for the time variation of the density distribution within the frost and the frost thickness for a set of system conditions ($Re=5,600$, humidity= dew point at $29.2^\circ F$ and $T = 70^\circ F$). These system conditions were studied experimentally by Brazinsky (14). The initial values for the theoretical calculations were taken as uniform frost density (with depth) of 0.0575 gms./cc. and the frost thickness of 0.189 inches. Brazinsky(14) showed experimentally that these values of frost density and thickness are obtained in the quasi-steady state of heat transfer. The results of Figure E-5 convey the same meaning as those obtained from the results of Figures II-B1 and II-B3.
Modification of the 'Sophisticated Model'

As an additional thought to the mathematical formulation of the frosting process, a slight modification of the 'sophisticated model' was examined. The mathematical equations of this modified 'sophisticated model' were based on the postulation that the driving force for the frost densification is 

$$\frac{\partial}{\partial x} \left[ \frac{D_B (1-P_v/P_{ice})}{\tau (1-P_v/P)} \frac{\partial P_{vp}}{\partial x} \right]$$

instead of

$$\frac{\partial}{\partial x} \left[ \frac{D_B (1-P_v/P_{ice})}{RT \tau (1-P_v/P)} \frac{\partial P_{vp}}{\partial x} \right]$$

as taken in the 'sophisticated model'. All other features of the modified 'sophisticated model' were identical to those of 'sophisticated model'.

The computer solution of the mathematical equations of the modified 'sophisticated model' by the same numerical technique as described before was found to be stable and convergent.

Figure E-6 shows the comparison between the predictions of the 'sophisticated' and the 'modified sophisticated' models for the time variation of the density distribution in the frost interior and the frost thickness. It can be seen that both the models predict almost identical results for a set of experimental system conditions. The modified 'sophisticated' model, however, predict a slightly lower densification rate and a slightly higher rate of change of thickness than those predicted by the 'sophisticated' model.

The results of Figure E-6 also indicates that the
simple model will also predict a little lower rate of the frost densification and a little higher rate of change of frost thickness, if the driving force for the densification rate is taken as \( \frac{1}{3x} \left( \frac{\rho_{ep}}{RT} \right) \) instead of \( \frac{1}{RT} \frac{\partial P_v}{\partial x} \) at the frost surface. This can be seen mathematically by realizing that the driving force for the former case is \( \left( \frac{1}{RT} \frac{\partial P_v}{\partial x} - \frac{\rho_{ep}}{RT^2} \frac{\partial T}{\partial x} \right) \), which is smaller than the driving force for the latter case by the quantity \( \frac{n_b}{RT^2} \frac{\partial T}{\partial x} \). Since this difference is quite small at the frost surface, the modification of the simple model was not examined.

It should be noted, however, that the results of Figures II-A1 to II-A12 indicate that the above-mentioned modification of the simple model will give the results in better agreement with the experimental facts.
Figure E-1 Stability and Convergence Test for Finite Difference Technique Used for Solution of Equations of 'Sophisticated Model'

Figure E-2 Stability and Convergence Tests for Finite Difference Technique used for Solution of Equations of 'Sophisticated Model'
Figure E-3 Stability and Convergence Tests for Finite Difference Technique Used for Solution of Equations of 'Sophisticated Model'.

Two values (at different \( \Delta z \)) overlapping

- Shaded Points - Values obtained with extended precision
- Unshaded Points - Values obtained with single precision

\( \Delta z = 0.04 \)
\( \Delta z = 0.025 \)
\( \Delta z = 0.02 \)

\( \theta = 8 \text{ mins.} \)
\( \theta = 9 \text{ mins.} \)
\( \theta = 12 \text{ mins.} \)

Figure E-4 Stability and Convergence Tests for Finite Difference Solution of 'Sophisticated Model'.

Three values (at different \( \Delta z \)) overlapping

- Shaded Points - Values obtained with extended precision
- Unshaded Points - Values obtained with single precision

\( \Delta z = 0.04 \)
\( \Delta z = 0.025 \)
\( \Delta z = 0.02 \)

\( \theta = 6 \text{ mins.} \)
\( \theta = 9 \text{ mins.} \)
\( \theta = 8 \text{ mins.} \)
Figure E-5 Predictions of 'Sophisticated Model' for the Time Variation in Density Distribution within the Frost and the Frost Thickness

Figure E-6 Comparison between Predictions of 'Sophisticated Model' and 'Modified Sophisticated Model'
Computer Program for 'Sophisticated Model'

```
DIMENSION FLUX1(1), RHOF(1), T(1), RHOF2(1), F(U(1)),
11FU(100), RHOF1(100), FLUX1(100), TP(100)
1
50 READ (2,10) H, T, P, FLUX1, KG, PP,
100 READ (2,100) T, DELX, DELZ, DELE, TZ, THETA, R
101 READ (2,102) (RHOF(I), I=1, N)
102 READ (2,105) (TP(I), I=1, N)
105 READ (10,5)
DO 112 I=1, N
112 RHOF(I)=RHOF(I-1)
RHOF2=RHOF(1)
KRHOFS=KRHOF(N)
FLUX(I)=FLUX1
DELE=DELE
TZ=TZ+DELX
DO 31 I=1, N
IF(I-1) 53, 54, 53
53 THETA=THETA+DELX
54 M=0
55 IF(I-1) 36, 126, 37
126 IO=I-1
10 IF(I-1) 42, 43, 42
T2=TP(I)
IF(I-1) 42, 43, 42
P1=6.23126*4.543*ALOG(TZ)
T2=-(244.6*3.546/TZ)-1.01677066*TZ-1.20514*(10.**(-5.0))
T3=7.57692*(10.**(-4.8))
T2=1.40
50 FLUX(I)=FLUX(I)*(P-PVP)
51 HEAT=HEAT+FLUX(I)*(422.5*(TZ**0.1712))
DO 10 K=1, T(I)
10 IF(K-460.0) 1,1, 11, 181, 182
101 IF(T(K)-460.0) 1,1, 11, 181, 182
52 IF(T(K)-460.0) 1,1, 11, 181, 182
IF(K=460.0) 1,1, 11, 181, 182
T2=-(14.0)*T**4.84*(RHOF(K)-0.025))
T2=T(K)+DELG*2)/2.0
56 G3=DELO*HEAT/(1.0,*(17.0)*T**5.44*(6.636*10.
58 G4=DELO*HEAT/(1.0,*(17.0)*T**5.44*(6.636*10.
194*(T(K)**4.84*(RHOF(K)-0.025))
GO TO 18
181 G1=DELO*HEAT/((3.875*(10.**(-6.0))*(T(K)**1.441))+(4.080*10.
IF(T(K)-460.0) 51, 51, 15, 152
51 G2=DELO*HEAT/((3.875*(10.**(-6.0))*(T(K)**1.441))+(4.080*10.
IF(T(K)-460.0) 55, 55, 55
55 G3=DELO*HEAT/((3.875*(10.**(-6.0))*(T(K)**1.441))+(4.080*10.
194*(T(K)**4.84*(RHOF(K)-0.025))
T3=T(K)+(DELG*2)/2.0
IF(TZ-460.0) 55, 55, 55
55 G4=DELO*HEAT/((3.875*(10.**(-6.0))*(T(K)**1.441))+(4.080*10.
```

18  
T(K+1) = T(K) + DELX * (G1*2.0*G2 + 2.0*G3 + G4) / 6.0

10  
CONTINUE

62  
IF(1 < T) 36,32,36

32  
P1 = B2312 * 0.4343 * ALOG(TZ)

PV = P1 * (2445.5646/TZ) + P2 - (0.01677006*TZ) + (1.20514*(10.**(-5.0))

14*T+T—0.757169) / (0.4343)

TZ=TZ*1.80

F=FLUX1/(P-PV)

GO TO 126

36  
IF(TZ<6.0) 39,39,38

39  
IF(MM<119,119,120

119  
FLUX1=F/(P-PV)

120  
YH1 = 1.0-(PVP1/P)

PV1 = ((2445.5646*1.80) / (T(J+1)) + (0.4343*8.2312 / T(Z-0.01677006 /

11.80) + (2.41028*10.**(-5.0)) / T(Z—3.240) / (0.4343)

PVPP1 = (PV1*PP1)

DIP = (FLUX1*IF*(1.0—RHOFSP/0.4343*PVPP1*DEL)/T*DELO)) /(RHOFSP/62.46)

119  
IF(MM=119,119,120

120  
DIF1 = (2.760*(10.**(-6.0))* (T(Z—2.009))

P2 = B2312 * 0.4343 * ALOG(TZ)

PV2 = EXP(((—2445.5646/TZ) + P2 — (0.01677006*TZ) + (1.20514*(10.**(-5.0))

14*T+T—0.757169)/ (0.4343)

YH2 = 1.0-(PVP2/PP)

PV3 = (2445.5646*1.80) / (T(J+1)) + (0.4343*8.2312 / T(J+1))

P4 = B2312 * 0.4343 * ALOG(T(J))

PV4 = EXP((—2445.5646/T(J)) + P4 — (0.01677006*T(J)) + (1.20514*(10.**(-5.0))

14*T+T—0.757169) / (0.4343)

YH2 = 1.0-(PVP2/PP)

PV5 = (2445.5646*1.80) / (T(J+1)) + (0.4343*8.2312 / T(J)) — (0.01677006*T(J)) + (1.20514*(10.**(-5.0))

14*T+T—0.757169)/ (0.4343)
3!-:33

I

45

Y

\text{RHOF} = \text{RHOF} (J+1) / \text{DEL}O

1.5

DELS=DELO

\text{THETA=THETA/60.0}

\text{DO 113 J=1,M}

\text{WRITE (3,108) (T(J),J=1,M)}

\text{DO 114 J=1,M}

\text{RHOF=RHOF(M)}

\text{DELO=DELO/12.0}

\text{THETA=THETA/60.0}

\text{DELS=DELO}

CONTINUE

\text{GO TO 50}

2 CALL EXIT
Appendix F

Empirical Correlation between Carbon Dioxide Frost Thermal Conductivity, Density and Temperature from the Frost Structure Models.

As demonstrated in Section II of the thesis an application of any theoretical model to the frosting process requires an accurate knowledge of thermal conductivity of frost. The thermal conductivity of the frost is usually assumed to be the function of frost density, temperature and frost structure. For the purpose of theoretical development of models for water frost formation process this correlation was obtained from an empirical curve fitting of the experimental data of Brazinsky (14) and of the present work. In order to use the successful models of the present thesis to explain the frosting process of other materials (e.g. carbon dioxide), a similar thermal conductivity correlation is needed.

At present there are no experimental data available on properties of carbon dioxide frost. Hence, in the absence of experimental data, thermal conductivity correlation for carbon dioxide frost can only be obtained by modelling of the frost structure.

The heterogeneous frost structure can be usually classified into three categories: (1) dispersions, (2) packed beds and (3) continuous pairs.

Dispersions are defined as a discontinuous phase
dispersed in a continuous phase. Continuous pairs refers to materials where both phases are significantly continuous. Packed beds are limiting cases between dispersions and continuous pairs, e.g. dispersions with point contacts.

Usually low density structure is expected to be of dispersion model and high density structure is expected to be like the continuous pairs. The dispersion frost structure have been often successfully characterized by Woodside model (121). The continuous pairs structure, on the other hand, have been well characterized by Riemann's model (47). Hence in this appendix, the carbon dioxide frost thermal conductivity at various density and temperature are obtained with the help of these two frost structure models. For low density frost \( \rho_f < 0.85 \), Woodside model is used for the thermal conductivity correlation and Riemann's model is assumed to represent high density \( \rho_f > 0.85 \) frost structure.

The thermal conductivity values reported in this appendix include the excess thermal conductivity component of carbon dioxide. In order to study the theoretical models for the carbon dioxide frosting process, empirical equations of this appendix could be used for thermal conductivity calculations. The results, however, should be checked with the help of experimental data in order to verify the validity of each model.
Empirical Correlation for low Density Frost

The low density carbon dioxide frost ($\varphi_f < 0.85\text{gms/cc}$) structure is modelled by Woodside model (121). This model assumes that the heterogeneous structure of frost consists of a simple cubic array of ice spheres and that conductivity is infinite in a sidewise direction (see Figure F-1 for a three dimensional view of the structure). This is tantamount to assuming that the isotherms are parallel planes. The final expression $k_f / k_{g,e}$ was derived by considering the resistance of Zone I and II (see Figure F-2) to be in series. The resistance of Zone I was a parallel combination of the resistances of the solid and gas in that zone. The derivation of an expression for calculating the thermal conductivity of this array is presented in Reference 90. The final results are

$$\frac{k_f}{k_{g,e}} = \left\{1 - \left( \frac{6(1-\varphi)}{\pi} \right)^{1/3} \left[1 - \frac{(a^2-1)}{2a} \ln(a+1) \right] \right\}^{-1}$$  \hspace{1cm} (F-1)

where

$$a = 1 + \sqrt[3]{\frac{6(1-\varphi)}{\pi}}$$  \hspace{1cm} (F-2)

$$\varphi = \frac{k_f - k_{g,e}}{k_{g,e}} \quad \text{and} \quad (1-\varphi) = \left( \frac{\varphi_f - \varphi_g}{\varphi_s - \varphi_g} \right)$$  \hspace{1cm} (F-3)

Here $k_{g,e}$ is the total conductivity of the gas phase which includes $k_{g}$, the intrinsic gas conductivity and $k_{\text{excess}}$, the additional component of conductivity due to internal mass transport.

An expression for $k_{\text{excess}}$ can be obtained by considering the total heat flux through the pore phase at some point
in the frost.

\[ q = \frac{k_{g,e}}{A} \frac{dT}{dx} = k_g \frac{dT}{dx} + \frac{D_v}{RT} \frac{M_v}{dx} \frac{dP_v}{dx} \Delta H \]  

(F-4)

where the second term in the right hand side of Equation F-4 is enthalpy transport due to mass transport.

From Equation F-4 and the definition of \( k_{excess} \)

\[ k_{excess} = k_{g,e} - k_g \]  

(F-5)

Equation F-6 can be derived as

\[ k_{excess} = \frac{D_v}{RT} (\frac{dP_v}{dT}) \Delta H M_v \]  

(F-6)

Combining Equation F-6 with Clausius-Clapyson Equation (80)

\[ \frac{dP_v}{dT} = \frac{M_v \Delta H P_v}{RT^2} \]  

(F-7)

The final expression for \( k_{excess} \) is obtained as

\[ k_{excess} = \frac{D_v \Delta H^2 P_v}{RT^2} (M_v)^2 \]  

(F-8)

**Computer Results**

The computer results of solution of Equations F-1 to F-3 are shown in Table F-1. The following empirical correlations for various properties of carbon dioxide and air were used in the computer program

\[ \Delta H = 583.0 \times T^{-0.1469} \]  

(F-9)

where \( T \) is in \( ^\circ\text{R} \) and \( \Delta H \) is in Btu/lb

\[ K_s = 325/T^{1.22} \]  

(F-10)

where \( T \) is in \( ^\circ\text{R} \) \( K_s \) is in Btu/hr ft\(^2\) \( \text{OR/ft} \)  

\[ k_g = 57.86 \times 10^{-6} \left( \frac{T}{273.16} \right) 0.773 - 0.57 \times 10^{-4} t 0.4 \times 10^{-7} t^2 \]  

(F-11)

where \( t = ^\circ\text{C} \) \( T = ^\circ\text{K} \) and \( k_g = \frac{\text{Cal}}{\text{cm sec deg C}} \)
Also, \( \dot{g}_g = \frac{38.3}{T} \)

where \( T \) is in \( ^\circ R \) and \( \dot{g}_g \) is in \( \frac{lbm}{cu \ ft} \) (F-12)

For \( T < 270^\circ R \)

\( \dot{g}_s = 1.7304 - 4.01 \times 10^{-4} T \) (F-13)

and for \( T > 270^\circ R \)

\( \dot{g}_s = 1.8318 - 7.76 \times 10^{-4} T \) (F-14)

where \( T = ^\circ R \)

\( \dot{g}_s = \text{gms.}/\text{cc.} \)

\( D_{co2-Air} = 1.765 \times 10^{-6} T^2 \cdot 2.009 \) (F-15)

\( D_{co2-N_2} = 2.38 \times 10^{-6} T^2 \cdot 0.087 \) (F-16)

Details of the above Equations are shown in Figure II-B10 and Reference 94.

where \( T = ^\circ k \)

\( D_{1-2} = \text{cm}^2/\text{sec.} \)

\( k_g = 0.0089 \left( \frac{T}{360} \right) \cdot 95 \) (F-17)

where \( T = ^\circ R \)

\( k_g = \text{Btu/hr ft}^\circ R \) (F-18)

for \( 138^\circ k < T < 216.3^\circ k \)

\( \log_{10} P_{vp} \ (\text{mm. Hg.}) = \frac{-1367.3448 + 9.9082}{T} \) (F-19)

for \( 90^\circ k < T < 138^\circ k \)

\( \log_{10} P_{vp} = \frac{-1275.62 + 0.006833T + 8.3071}{T} \) (F-20)

**Empirical Correlation for High Density Frost.**

The high density carbon dioxide frost structure is
represented by a truncated-sphere model by Riemann (47). This model is tested extensively by Gorring and Churchill (47). In this model the solid phase is assumed to be composed of spheres of diameter $2R$ with common intersecting circles of diameter $2r$ (see Figure F-3). The derivation of the mathematical equation are summarized in Reference 47. The final results are as follows:

$$\frac{k_{f1}}{k_{g,e}} = \left( \frac{R}{r} + \frac{1}{n} \ln \left( \frac{2R}{r} \right) \right)^{-1}$$  \hspace{1cm} (F-21)

$k_{f1}$ is the contribution to the frost conductivity due to the solid sphere.

The ratio $(R/r)$ is related to density parameter

$$S = \frac{(\phi_f - \phi_g)}{(\phi_s - \phi_g)}$$

by a material balance

$$S = \frac{\pi}{6} \left[ 1 - \left( \frac{r}{R} \right)^2 \right]^{3/2} \left\{ 1 - \frac{3}{2} \left[ \left( \frac{r}{R} \right)^2 + 1 - \left[ 1 - \left( \frac{r}{R} \right)^2 \right]^{3/2} \right] \right\}$$  \hspace{1cm} (F-22)

or

$$\epsilon = \text{void fraction} = \frac{1}{6} \left[ 1 - \left( \frac{r}{R} \right)^2 \right]^{2/3} \times \left[ \left( \frac{r}{R} \right)^2 - (2 - \left[ 1 - \left( \frac{r}{R} \right)^2 \right]^{3/2}) \right]$$  \hspace{1cm} (F-23)

Equation F-17 is not applicable to dispersion models i.e. \( \psi \).

Solving Equations F-21 and F-22 simultaneously $k_{f1}/k_{g,e}$ as a function of $S$ can be obtained.

Equation F-21 allows only for the contribution due to the solid phase. Some heat flow will be due to the interstitial gas (air) and this contribution $k_{f2}$ is estimated approximately as (47)

$$\frac{k_{f2}}{k_{g,e}} = 2(1-S)/(2+S)$$  \hspace{1cm} (F-24)
where \( k_{e} = k_{g} + k_{\text{excess}} \) \hspace{1cm} (F-25)

\( k_{g} \) = dry gas conductivity

\( k_{\text{excess}} \) = excess thermal conductivity due to excess evaporative transport as defined previously.

The total frost conductivity is defined as

\[ k_{f} = k_{f1} + k_{f2} \] \hspace{1cm} (F-26)

This model is applicable for high density frost where \( S > 0.524 \) but less than about 0.9.

The results of the computer solution of the above described equations are shown in Table F-2.
<table>
<thead>
<tr>
<th>T (g/m³ cc)</th>
<th>α (K)</th>
<th>k (Btu/ft²h°F)</th>
<th>kV (BTU/ft²h°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.100000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>0.150000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>0.200000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>0.250000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>0.300000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>0.350000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>0.400000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>0.450000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>0.500000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>0.550000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>0.600000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>0.650000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>0.700000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>0.750000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>0.800000</td>
<td>0.200000</td>
<td>0.150000</td>
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</tr>
<tr>
<td>0.850000</td>
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<tr>
<td>0.900000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>0.950000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
<tr>
<td>1.000000</td>
<td>0.200000</td>
<td>0.150000</td>
<td>0.127334</td>
</tr>
</tbody>
</table>

Table F-1
Thermal Conductivity as a Function of Density and Temperature from Woodside Model (121)

\[ \alpha (K) = \frac{k}{\rho c_p} \]

\[ k = \frac{kV}{T} \]

\[ kV = \frac{k}{T} \]

\[ T = -355°C \]
Table F-1 (contd.)

<table>
<thead>
<tr>
<th>Value</th>
<th>Expression</th>
<th>Value</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000000E+00</td>
<td>0.111621E9</td>
<td>0.000000E+00</td>
<td>0.349972E2</td>
</tr>
<tr>
<td>0.100000E+00</td>
<td>0.111621E9</td>
<td>0.200000E+00</td>
<td>0.349972E2</td>
</tr>
<tr>
<td>0.200000E+00</td>
<td>0.111621E9</td>
<td>0.300000E+00</td>
<td>0.349972E2</td>
</tr>
<tr>
<td>0.400000E+00</td>
<td>0.111621E9</td>
<td>0.600000E+00</td>
<td>0.349972E2</td>
</tr>
<tr>
<td>0.800000E+00</td>
<td>0.111621E9</td>
<td>1.200000E+00</td>
<td>0.349972E2</td>
</tr>
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<td>0.349972E2</td>
</tr>
<tr>
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<td>0.111621E9</td>
<td>3.000000E+01</td>
<td>0.349972E2</td>
</tr>
<tr>
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<td>0.111621E9</td>
<td>6.000000E+01</td>
<td>0.349972E2</td>
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<td>2.000000E+02</td>
<td>0.349972E2</td>
</tr>
<tr>
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<td>0.111621E9</td>
<td>3.000000E+02</td>
<td>0.349972E2</td>
</tr>
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<td>0.400000E+02</td>
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<td>0.349972E2</td>
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<td>0.349972E2</td>
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<tr>
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<td>0.111621E9</td>
<td>3.000000E+03</td>
<td>0.349972E2</td>
</tr>
<tr>
<td>0.400000E+03</td>
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<tr>
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<td>0.349972E2</td>
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<td>3.000000E+04</td>
<td>0.349972E2</td>
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Note: The table continues with similar entries for various values and expressions.
<table>
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<tr>
<th>Table F-1 (cont.)</th>
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<tbody>
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<td>( e \times 10^6 )</td>
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</tr>
<tr>
<td>0.100000</td>
</tr>
<tr>
<td>0.300000</td>
</tr>
<tr>
<td>0.400000</td>
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<td>0.500000</td>
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</tr>
</tbody>
</table>

**Notes:**
- All values are in \( e \times 10^6 \) format.
- Table continues with similar entries.
Table F-2
Thermal Conductivity as a Function of Density and Temperature function of Riemann’s Model \( f(T_w) \)

<table>
<thead>
<tr>
<th>( \rho ) (gms./cc.)</th>
<th>( T_w = -315^\circ F )</th>
<th>( (8 \text{ Btu/hr ft}^\circ \text{R}) )</th>
<th>( (5 \text{ Btu/hr ft}^\circ \text{R}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0249999E 00</td>
<td>0.160000E 03</td>
<td>0.127239E 00</td>
<td>0.032188E 01</td>
</tr>
<tr>
<td>0.0249999E 00</td>
<td>0.160000E 03</td>
<td>0.127239E 00</td>
<td>0.032188E 01</td>
</tr>
<tr>
<td>0.999999E 00</td>
<td>0.160000E 03</td>
<td>0.127239E 00</td>
<td>0.032188E 01</td>
</tr>
<tr>
<td>0.104999E 01</td>
<td>0.160000E 03</td>
<td>0.126909E 00</td>
<td>0.031843E 01</td>
</tr>
<tr>
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<td>0.160000E 03</td>
<td>0.126909E 00</td>
<td>0.031843E 01</td>
</tr>
<tr>
<td>0.114999E 01</td>
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<td>0.126909E 00</td>
<td>0.031843E 01</td>
</tr>
<tr>
<td>0.119999E 01</td>
<td>0.160000E 03</td>
<td>0.126909E 00</td>
<td>0.031843E 01</td>
</tr>
<tr>
<td>0.124999E 01</td>
<td>0.160000E 03</td>
<td>0.126909E 00</td>
<td>0.031843E 01</td>
</tr>
<tr>
<td>0.899999E 00</td>
<td>0.160000E 03</td>
<td>0.126909E 00</td>
<td>0.031843E 01</td>
</tr>
<tr>
<td>0.949999E 00</td>
<td>0.160000E 03</td>
<td>0.126909E 00</td>
<td>0.031843E 01</td>
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Figure F-1  Three Dimensional View of Structure Consisting of a Simple Cubic Array of Non-Touching Ice Spheres

Figure F-2  Unit Cube of Heterogeneous Structure Consisting of a Simple Cubic Array of Non-Touching Ice Spheres

Figure F-3  Model for Riemann Equation
Appendix G

Empirical Correlations for 'Starting Values' of Frost Density and Thickness

As discussed in Section II-A, the success of the predictions of the simple model depends on judicious choice of starting or initial values of frost density and thickness. One of the judicious choices for the starting values can be obtained from an empirical correlation between the experimentally measured average frost density, thickness and the independent system parameters at a certain time. In this appendix such a correlation is obtained from the experimental data of the present thesis at the time of forty minutes from the start of the frosting process.

The three important parameters which affect the starting values of frost density and thickness are believed to be humidity, gas temperature and heat (hence mass) transfer coefficient. Hence empirical correlations between starting values of frost density and frost thickness and these three parameters are obtained.

Since simple model applies only after a smooth layer of frost is formed (see Section II-A), it is difficult to pick a starting time which could be used for the wide range of experimental conditions. This is because the time for a smooth layer of frost is found to be dependent upon independent system parameters. For these reasons, in order to apply the empirical correlation the complete
range of experimental conditions studied in the present thesis, it was obtained at forty minutes from the start of frosting runs.

Method of Attack

The empirical correlations were obtained by assuming polynomial relationships between frost density and frost thickness and three independent system variables. Thus it was assumed that

\[ \phi_f = a h^\beta T_g^\gamma p_g^\delta \]  \hspace{1cm} (G-1)

and  \[ \delta = b h^\gamma T_g^\phi p_g^\gamma \]  \hspace{1cm} (G-2)

where \( \phi_f \) = Frost density (gms/cc)

\( h \) = Heat transfer coefficient (Btu/hr ft²oF)

\( T_g \) = Gas temperature (oR)

\( p_g \) = Dew point of gas (oF)

\( \delta \) = Frost thickness (inches)

The constants in Equation G-1 and G-2 were obtained from the experimental data as follows:

The polynomials \( \phi, \beta, \gamma \) and \( \delta \) were obtained from the log-log plots of frost density versus heat transfer coefficient (at constant \( T_g \) and \( p_g \)), versus gas temperature (at constant \( h \) and \( p_g \)) and versus dew point of gas (at constant \( h \) and \( T_g \)) respectively. The polynomials of Equation G-2 \( \gamma, \phi \) and \( \delta \) were obtained similarly. Once all the polynomials were calculated, the constants \( a \) and \( b \) were obtained from the experimental data point of frost density and thickness at known heat transfer coefficient, gas temperature and humidity. The experimental results
used for the correlation are described in Tables G-1 to G-4. The following correlations were obtained by this method.

\[ \theta_f = 1.930 \times 10^{-3} \cdot h^{0.933} \cdot T_g^{0.2062} \cdot p_g^{0.1511} \]  \hspace{1cm} (G-3)

and,

\[ \delta = 6.21 \times 10^{-5} \cdot h^{-0.1231} \cdot T_g^{-2.878} \cdot x_{pg}^{0.779} \]  \hspace{1cm} (G-4)

As shown in Tables G-1 to G-4, the ranges of independent variables used to obtain above mentioned were \( h \) varying from 5.60 Btu/hr ft\(^2\)°F to 10.8 Btu/hr ft\(^2\)°F, \( T_g \) varying from 494°R to 553°R and \( p_g \) varying from dew point of 14.3°F to dew point of 33.25°F.

It is the author's belief that the empirical correlations G-3 and G-4 should be applicable in predicting frost density and thickness values at forty minutes from the beginning of the frosting process for the system conditions outside the ranges considered above, if the criterion of smooth layer of frost is satisfied. It should also be noted that these correlations are independent of system's geometries as long as it is well defined such that heat and mass transfer coefficients can be obtained accurately.

A comparison between the experimental values and the values predicted from Equation G-3 and G-4 for various system conditions are shown in Table G-5.
### Table G-1

Experimental data for Calculation of $\alpha$ and $\gamma$

<table>
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<tr>
<th>$h$ (Btu/hr ft$^2$O$^\circ$F)</th>
<th>$\varepsilon_f$ (gms./cc.)</th>
<th>$\delta$ (inches)</th>
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### Table G-2

Experimental data for Calculation of $\beta$ and $\psi$

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### Table G-3

Experimental data for Calculation of $\phi$ and $\psi$

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Table G-3 (Contd.)

\[ \begin{array}{ccc}
\text{pg (dew pt°F)} & \text{\(q_f\) (gms.)} & \text{\(\delta\) (inches)} \\
30.4°F & 0.0620 & 0.1310 \\
14.3°F & 0.0535 & 0.0750 \\
\end{array} \]

Table G-4

Experimental data for Calculation of \(a\) and \(b\)

\[ \begin{array}{cccc}
\text{h (Btu/hr ft}^2\text{°F)} & \text{Tg (°R)} & \text{pg (dew pt°F)} & \text{\(q_f\) (gms.)} & \text{\(\delta\) (inches)} \\
10.80 & 530°R & 29.84°F & 0.1030 & 0.125 \\
\end{array} \]

Table G-5

Experimental Data versus Values predicted from Equations (G-3) and (G-4) for various System Conditions

\[ \begin{array}{ccccc}
\text{Re} & \text{Tg} & \text{Humidity (dew point) } & \text{\(q_f \text{ exp (gms.)}\)} & \text{\(q_f \text{ exp (gms.) from Eq(G-3)\)} } & \text{\(\delta \text{ exp (inch)}\)} & \text{\(\delta \text{ (inch) from Eq(G-4)}\)} \\
5,100 & 93°F & 27.16 & 0.0620 & 0.0612 & 0.058 & 0.060 \\
8,000 & 73°F & 30.0 & 0.0620 & 0.0625 & 0.131 & 0.129 \\
6,100 & 48°F & 18.02 & 0.0523 & 0.0575 & 0.0825 & 0.0820 \\
\end{array} \]
Appendix H

Literature Survey on Homogeneous Nucleation from Vapor Phase

As seen in Section II, the transport of water in the colder region of the frost is possible only by the thermal diffusion of water or ice nuclei. Since homogeneous nucleation is expected to occur in the frost interior, the available literature on this phenomenon is reviewed in this appendix.

Numerous literature is available on the subject of homogeneous nucleation from vapor phase. The critical reviews of all the existing theories and the experimental facts about this process are given by Amelin (2) and Byers (21). Sander and Damkohler (93) and Hinckley et al. (3) have also given some discussion on the mathematical aspects of this process.

For the present purpose, the knowledge of nucleation rate and the critical size of a nuclei under a set of environmental conditions are of considerable significance. The critical nucleus size can be calculated from Kelvin's equation (60) shown below.

\[
\ln\left(\frac{P}{P_{vp}}\right) = \frac{2\sigma M}{RT\xi r} \quad (H-1)
\]

All the nomenclatures of Equation H-1 are described in Appendix R.

Schaefer (100) calculated critical size of a nuclei from his experimental data and found that it was in good agreement with one computed from Equation H-1. These results are shown in Figure H-1.
The three significant theoretical relationships for the calculation of the homogeneous nucleation rate are those of Volmer's (112), Becker and Doring's (8) and Frankel's (9, 40, 42). All these relationships were obtained from Gibbs stability criteria for formation of a stable aggregate of nuclei (embryo). The mathematical formulations of all the three theoretical models were based on the same assumptions. The final expressions, however, were arrived at with increasing degree of sophistication in the mathematical derivation, in the order of the investigation listed above.

The experimental results on nucleation rates (12, 43) have been found to be in better agreement with Becker and Doring equation (8) and Frankel equation (42) than with Volmer equation (112). A comparison between the predictions of nucleation rate as a function of supersaturation ratio and temperature obtained from Volmer and Becker-Doring equations is shown in Figure H-4.

Since Becker-Doring equation has been proven to be good with the help of experimental results (12, 43), it is used for the preliminary calculations described in Section II-E. This equation is shown below.

\[
\dot{N}_t = 10^{26} \frac{\alpha}{\beta} \left( \frac{p}{T} \right)^2 \left( \frac{M}{\sigma} \right)^{1/2} e^{-A(1-3g^{2/3}+2g^{-1})}
\]

where,

\[
g = \frac{35.2}{\ln^3 S} \left( \frac{M}{\sigma} \right)^2 \left( \frac{\sigma}{T} \right)^3
\]

and

\[
A = \frac{17.6}{\ln^2 S} \left( \frac{M}{\sigma} \right)^2 \left( \frac{\sigma}{T} \right)^3
\]
All the nomenclatures of above equation are described in Appendix R.

The experimental measurements of the homogeneous nucleation rate have been found to be an extremely difficult task. The major problems involved in the measurement are the inaccuracy involved in the counting of the nuclei and the difficulties in complete removal of dust or other foreign particles from air. Most of the experimental studies are carried out in Wilson's chamber \((44, 85, 99, 119)\) and in sonic nozzles \((73)\).

Wilson \((119)\) and Powell \((85)\) carried out some original experimental investigations of the process of homogeneous nucleation. Their results, however, were qualitative in nature. Their experimental technique was elaborated and used for achievement of quantitative data by the later investigators. Volmer and Flood \((111)\) checked Equation H-2 by determining experimentally the value of \(p/p_{vp}\) at which the rate of nucleation, \(\dot{N}_v\), attained a value of 1 nuclei/cm\(^3\) sec, and obtained very good agreement between the theory and the observation. The lowest system temperature they considered was 261 K, and the nucleation rate at that temperature was observed to be increased rapidly at supersaturation ratio greater than 5. Schaefer \((100)\) and Maybank and Mason \((71)\) later on carried out the measurements of homogeneous nucleation rate at the system temperatures slightly lower than that studied by Volmer and Flood \((111)\). Both of them arrived at the same conclusion that in clean air, crystals
formed by spontaneous condensation followed by homogeneous crystallization, appeared only when the supersaturation exceed 400% and the terminal temperature fell below -40°C. There was discrepancy, however, in crystal concentration formed by both the investigators under these conditions. Maybank and Mason (71) reported nuclei concentration of $4.0 \times 10^{11} / \text{cm}^3$ as opposed to $10^{10} / \text{cm}^3$ found by Schaefer (100). They claimed that Schaefer’s results were erroneous because of maltechnique involved in their experimental measurements. The final results of these investigators are shown in Figures H-2 and H-3.

Sander and Damkohler (93), Rau (87) and Pound et al. (84) have also obtained quantitative measurements of nucleation rate at the system temperature as low as -73°C. Pound et al. (84) suggested that because homogeneous nucleation of liquid water droplet at -40°C had been well established, the particles appearing between this temperature and -65°C were frozen spheres which had not developed crystalline faces. Sander and Damkohler (93) correlated their data for nucleation rate of 1 nuclei/cm$^3$ sec in a following way.

above -62°C $\ln \frac{P}{P_{vp}} = \frac{780}{T} - 1.521$ (with ions) (H-3)

above -62°C $\ln \frac{P}{P_{vp}} = \frac{765}{T} - 1.537$ (without ions) (H-4)

below -62°C $\ln \frac{P}{P_{vp}} = \frac{1,373}{T} - 3.748$ (with or without ions) (H-5)
Above empirical correlations are found to be in well agreement with Becker-Döring Equation H-2 for predicting supersaturation ratios \( \frac{p}{p_{vp}} \) and temperatures for nucleation rate of 1 nuclei/cm\(^3\) sec.

At present, there are no quantitative experimental data available for nucleation rate at the temperatures below \(-73^\circ C\). Some observations, however, have been made regarding the nature of ice crystals formed at temperatures below \(-73^\circ C\). Mason (67) observed the mother of pearl clouds at temperature of about \(-80^\circ C\) in the stratosphere. Blackman and Lisgarten (10) have described the structure of ice formed at temperatures below \(-73^\circ C\). Cwilong (29) has given a good qualitative description of the nature of ice cloud formed in the Wilson chamber at the temperatures as low as \(-120^\circ C\). He also concluded that at that low temperature, the nature of cloud formed would make the quantitative measurement of nucleation rate extremely difficult.
The Experimental Value of Schaefer (108) and Values Calculated from Kelvin Equation H-1 are shown in Figure H-1. The number of crystals predicted as a function of the final temperature is shown by the equation (107) in Figure H-1.

Figure H-4 demonstrates the Homogeneous Nucleation Rate as a function of Supersaturation Ratio and Temperature.
APPENDIX J

Literature Survey on Non-Equilibrium Growth of an Ice Crystal from Vapor Phase

Since frost densification is essentially a result of growth of ice crystal network, in this appendix, a very brief description of the available literature on the non-equilibrium growth of an ice crystal is presented.

In the usual case, diffusion, degree of supersaturation, kinetics of bonding and presence of impurities are the factors which modify the crystal form and habit. Hence, in order to obtain a theoretical model for the crystal growth process, it is important to determine the factors which control the growth under a given set of environmental conditions.

The probability for presence of impurities in the ice crystals of frost is highly remote. The ice crystal growth from a stagnant vapor phase is expected to be controlled by the rate of diffusion and heat conduction since molecular diffusional processes in this environmental condition are usually small in magnitudes. The calculations shown in Section II-E describe one of the ways for determining the controlling factors for crystal growth. The results of these calculations indicate that most likely, the growth of ice crystals from vapor phase in the present case would be controlled by molecular diffusional mechanisms.

The first theoretical equation for the diffusion controlled growth of a single crystal, which is at rest
relative to air and that the diffusion field around the
crystal has achieved a steady state, was given by Jeffrey
(56) as follows,

\[ \frac{dm}{d\theta} = 4\pi C D_B (\rho - \rho_s) \]  

(J-1)

Equation J-1 was later used by Houghton (53) to calculate
the growth rate of crystal by a trial and error method
based on a table of values of \( \rho_s \) against crystal surface
temperature.

Mason (68) carried out first systematic mathematical
approach to derive an analytical expression for \( \frac{dm}{d\theta} \) in terms
of parameters which, for a water saturated environment, are
determined only by the temperature and by the geometry of
the crystal. This significant contribution to the present
subject can be briefly summarized as follows:

If it is assumed that the vapor pressure \( p_s \) in the
viscosity of the crystal surface is the equilibrium vapor
pressure with respect to ice at the surface temperature \( T_s \)
of the crystal, i.e. \( p_s = p_{e}(T_s) \), Equation J-1 can be written
as,

\[ \frac{p - p_{e}(T_s)}{p_{e}(T)} = \frac{RT}{4\pi D_B C_{p} e(T) M} \frac{dm}{d\theta} \]  

(J-2)

where, \( p_{e}(T) \) is the saturation vapor pressure over a plane
ice surface at the ambient temperature \( T \), \( p \) is the ambient
vapor pressure and \( M \) the molecular weight of ice.

While the crystal is growing, the surface temperature \( T_s \)
will be higher than that of the surrounding air. If it is assumed that latent heat of sublimation $\Delta H$ is dispersed solely by conduction through air, we have

$$\frac{\Delta H}{\theta} = 4\pi C k_{a}(T_{s} - T) \quad (J-3)$$

where, $k_{a}$ is the thermal conductivity of air.

Now,

$$\frac{1}{p_{e}} \frac{d p_{e}}{dT} = \frac{\Delta H M}{RT^2} \quad \text{(Clausius-Clapeyron)} \quad (J-4)$$

and integrating this equation from the crystal surface to a boundary in the environment remote from the crystal gives

$$\ln \frac{p_{e}(T_{s})}{p_{e}(T)} = \frac{\Delta H M (T_{s} - T)}{RT} = \frac{\Delta H M}{RT} \quad (J-5)$$

From Equation $J-3$

$$\frac{p_{e}(T_{s}) - p_{e}(T)}{p_{e}(T)} = \exp \left( \frac{\Delta H^2 M}{4\pi C k_{a} T^2} \frac{dm}{d\theta} \right) - 1 \quad (J-6)$$

Adding Equations $J-2$ and $J-6$

$$\frac{p}{p_{e}(T)} = \exp \left( \frac{\Delta H^2 M}{4\pi C k_{a} T^2} \frac{dm}{d\theta} \right) + \frac{RT}{4\pi C D B p_{e}(T) M} \frac{dm}{d\theta} \quad (J-7)$$

When the exponent in Equation $J-7$ is small compared with unity, one may write

$$\frac{dm}{d\theta} = 4\pi C \sigma / \left( \frac{\Delta H^2 M}{k_{a} T^2} + \frac{RT}{D_{p} M p_{e}(T)} \right) = 4\pi C \sigma / f(T) \quad (J-8)$$

where, $\sigma = (p/p_{e}(T)) - 1$ is the supersaturation of the environment relative to ice and the term in the brackets
is a function of temperature only at constant air pressure.

Electrostatic capacity of crystal \( C \), was calculated by Houghton (53), for several geometries. He treated a then hexagonal prism as a prolate spheroid of revolution of large accentricity and obtained (53) the following area terms.

For sphere \( C = r \) \hspace{1cm} (J-9)

For circular disc \( C = 2r/\pi \) \hspace{1cm} (J-10)

and for prolate spheroid \( C = \frac{2ae}{\ln(1+e)/(1-e)} \) \hspace{1cm} (J-11)

Where, accentricity \( e = \frac{(1- \frac{b^2}{a^2})^{\frac{3}{2}}}{a^2} \) and \( a \) and \( b \) are major and minor semi-axes respectively.

The Equation J-8 can be used to calculate growth rate of crystal as long as crystal retains its geometrical shape. In reality, however, it has been found that after the early stages of growth, the crystals generally lose their simple geometrical shapes and develop into dendritic structures. Equation J-8 could not be applied for the dendritic crystal growth.

The growth rate Equation J-7 has been found to agree with experimental results of Mason (68) and Houghton (53) only in the early stages of growth. In both the studies, the crystal developed into dendritic structure after a short duration. The criterion for this transition from polyhedral to dendritic growth has been derived by Mason (68).

The experimental studies of Mason (68) and Houghton (53) were carried out in the temperature range between
0°C to -30°C. Mason (67) also reported experimental as well as theoretical studies on growth rate of a spherical droplet. Schaefer (98) and Bradley (13) have also reported some data on growth rate of a single ice crystal. At present, however, no experimental data available for growth of an ice crystal from vapor phase below -30°C.

As yet, no systematic theoretical or experimental studies of dendritic crystal growth have been carried out. This phenomenon was, however, correctly interpreted by Kossel (19,63) as to be due to the fact that the corners of a growing crystal have a greater surface energy (because of higher area to volume ratio) than the edges which, in turn, have a greater energy than the faces of the crystal. The greater area to volume ratio provides more heat conduction to the surrounding vapor, and makes the dissipation of the heat of crystallization take place at a smaller difference in temperature between the crystal surface and the vapor phase, giving the corners and edges a higher effective supersaturation than the faces of the crystal. The higher effective supersaturation brings on an increase in the growth rate at these locations, thus perpetrating and intensifying the effect. The mathematical study of this perpetrating and intensifying effect is an extremely difficult task.
APPENDIX K

Literature Survey on Thermal Diffusion of Solid Particles in Stagnant Gas

A radiometric or thermal force is defined as a force on a body suspended in gas which is not in thermal equilibrium. The movement or diffusion of this suspended body by thermal force is called the process of thermal diffusion.

The strong possibility for an existence of fog or ice nuclei (see Section II-C) along with the existence of a large temperature gradient (see Section IV) in the frost interior suggests the occurrence of process of thermal diffusion of aerosols in gas within the frost. For the same thermal force, the mass transport of a material by this process, as reported by Mason (69), could be as high as $10^6$ times that due to thermal diffusion of the same material in the gaseous phase.

In trying to seek an explanation for the phenomenon of uniform densification of frost, one must find a mechanism by which frost at extremely low temperature densifies with time. Since at that low temperature one would expect that appreciable water vapor could not exist before nucleation takes place, the crystal growth from the vapor phase may not be a sufficiently rapid process. Under this situation, in the colder region of the frost, densification could occur only if water vapor is transported there in an other phase. Since both nucleation and thermal diffusion of fog or ice nuclei in
gas are highly conceivable processes occurring in the frost interior, it is possible that thermal diffusion of nucleated fog or ice particles could transport water vapor in the colder region of the frost. It is for these reasons, that thermal diffusion of aerosols in gas may be a very significant phenomenon in interpreting and clarifying results. The available literature on both theories as well as some pertinent experimental facts are outlined below.

Poppoff (83) showed that the thermal force is not appreciable for particle greater than 10 microns in diameter. In fact, the efficiency with which particles can be separated from a gas by thermal diffusion have been found to decrease rapidly when particle radius exceeds 2 microns (46). Hence this literature on thermal diffusion can be meaningfully applied only to particles with a diameter of less than 4 microns. Since the size of ice nuclei is always smaller than this, the literature information would be meaningfully applied to the present purpose.

Theoretical treatments to the subject have been carried out by several investigators. Since the guide lines for the development of the theoretical equations depended upon the flow regimes (i.e. on the value of ratio $\lambda/r_p$; where $\lambda$ is mean free bath and $r_p$ is particle radius) the discussion of the process will be separated on that basis.

Although Tyndall(110), Rayleigh(88), Aitken (1) and Lodge and Clark (66) first conceived and performed some early pioneering experiments for the understanding of
movement of aerosols under the effect of thermal force. The first significant theoretical treatment to the thermal diffusion phenomenon in the free molecular regime (\(\frac{\lambda}{r_p} \gg 1.0\)) was given by Cawood (22).

Considering a cylinder of gas, of length about twice the mean free path of the gas, whose axis is perpendicular to a hot surface and radius of the magnitude of the mean free path of the gas, being situated in a temperature gradient, \(\frac{dT}{dx}\), Cawood concluded that a particle inside this cylinder would experience bombardment due to the motion of gas molecules parallel to the axis of the cylinder but would experience no net lateral bombardment.

The net energy flow through the cylinder is given by

\[
E = \frac{1}{6} \left( N_2 V_{g2} \right) \left( \pi r_p^2 \right) \left( \frac{1}{2} m V_{g2}^2 \right) - \frac{1}{6} \left( N_1 V_{g1} \right) \left( \pi r_p^2 \right) \left( \frac{1}{2} m V_{g1}^2 \right)
\]

Where subscripts 1 and 2 refer to temperature conditions \(T_1\) and \(T_2\), \(T_2 > T_1\). The flux of molecules in one direction is equal to that in the opposite direction so that \(N_1 V_{g1} = N_2 V_{g2}\). Replacing \(\frac{1}{2} m V_{g1}^2\) with \(3/2 \left( \frac{RT_1}{N_0} \right)\) Equation K-1 simplifies to

\[
E = -\frac{1}{2} \left( \frac{N_1 R}{N_0} \right) \left( \pi r_p^2 V_{g2} \right) \left( \lambda \frac{dT}{dx} \right)
\]

The net force acting on a particle can now be found by dividing the net energy flow per unit time by the average velocity of the gas molecules whereby

\[
F_t = \frac{E}{V_g} = -\frac{1}{2} \left( \frac{N_1 R}{N_0} \right) \left( \pi r_p^2 \right) \left( \lambda \frac{dT}{dx} \right)
\]

It is assumed here that \(\frac{V_g}{2} = \frac{V_{g1} + V_{g2}}{2}\) is very nearly
Replacing $N_R/N_0$ by $p/T$, Equation (K-3) then becomes

$$F_t = -\frac{1}{2} \left( \frac{\pi R^2}{p} \right) (p \lambda /T) \frac{dT}{dx} \quad (K-4)$$

which is Cawood's equation in the final form.

The values of thermal forces given by Cawood's equation were within an order of magnitude of those found by experiments.

The elementary treatment of Cawood (22) was later improved by Derjaguin, Bakanov (35) and by Waldmenn (113). The improved theory starts from the basic statement that the velocity distribution of incoming gas molecules, as they approach a suspended particle, is not appreciably changed by the presence of the particle. The Chapman-Enskog velocity distribution function for a gas with an imposed temperature gradient is used to describe the incoming gas molecules. The rebounding molecules are considered to have a velocity distribution made up in part by a fraction of molecules specularly reflected and in part by those molecules which leave the particle surface after having reached equilibrium with it. The expression for thermal force in this case is given by:

$$F_t = -\frac{32 \pi p^2 K}{15 V_g} \frac{dT}{dx} \quad (K-5)$$

This equation can be shown to have the same form as Cawood's equation but to contain a different numerical coefficient by making the following substitutions for the monoatomic gases.
Thus, assuming ideal gas behavior, equation K-6 becomes

\[ F_t = \frac{-32 \pi^2 k}{15 \nu g} \frac{dT}{dx} = -\frac{4 \pi^2}{15 \nu g} (p \lambda / T) \frac{dT}{dx} \]  

(K-7)

The thermophoretic velocity in this regime is given by,

\[ V_t = \frac{15 \pi}{14 \psi T} (v_g \lambda) \frac{dT}{dx} \]  

(K-8)

where \( \psi \) is dependent upon the nature of the gas particle interaction. The velocity, \( V_t \), does not depend on the size of the particle at all, and only to a small degree on their nature through \( \psi \).

For slip region ( \( \lambda / r_p \ll 1.0 \)), when the size of a particle becomes large compared to the mean free path, the molecule particle interaction changes; the molecule no longer sees the particle as another molecule, but sees the particle as a solid surface. Hence a different approach must be taken to define the thermal force. Epstein (95) was the first to solve this problem analytically. He considered both the kinetic and hydrodynamic aspects of gas flow around a spherical particle and took into account the fact that the behavior of the particle in the gas would depend on the ratio of heat transported through the interior of the particle to the heat received on the surface from molecular impacts. Therefore, the thermal conductivities of the gas and the immersed material should come into the derivation of the thermal stress. Three basic assumptions were made:

1. \( \lambda / r_p \ll 1.0 \)

2. A uniform temperature distribution exists in the gas
at a great distance from the particle.

3. Convective heat transfer to the particle is negligible compared to heat transfer by conduction.

He then broke down the problem into two phases of hydrodynamics and conduction which were treated independently initially and later combined through the concept of the thermal creep. Epstein's procedure may be summarized as follows:

Heat conduction problem was solved to obtain the temperature gradient over the surface of the sphere. Using the thermal creep velocity, defined by Maxwell (70), as a boundary condition, the Navier-Stokes equations were solved. By integrating over the entire surface, the total force on the particle was obtained. The thermal force was then calculated as a component of the total force parallel to the direction of the temperature gradient. Epstein's final equation for the thermal force is given by

\[ F_t = (-9 \pi) (r_p) (n^2/k) \left( \frac{k_g}{2k_g + k_p} \right) \frac{dT}{dx} \]  \hspace{1cm} (K-9)

Numerous experimental investigations have been carried out which can test Epstein's equation. The data of Rosenblatt and Lamer (91) showed satisfactory agreement with the theory considering the assumptions used in its development. Additional support for the theory was given by the work of Saxon and Rans (95). However, results obtained by Schadt and Cadle (96, 97) cast some doubt on the Epstein theory particularly for aerosol particles.
of high thermal conductivity. Equation K-9 shows, when \( k_p \) becomes large the thermal force tends to zero. Yet Schadt and Cadle found no such tendency in their experimental observations while using particles where thermal conductivity ranged from \( 1.45 \times 10^{-2} \text{ Btu/hr ft^0F} \) to \( 39.4 \text{ Btu/hr ft^0R} \). In fact, the thermal force on iron and sodium chloride particles was from twenty to thirty times greater than those predicted by Equation K-9.

Brock (15,16), has been able to account for these differences by resolving the steady state Navier-Stokes equation subject to two corrections of the Epstein theory. 1. Boundary conditions more suitable for the step flow regime were used and 2. the continuous energy equation has been solved taking into account convective terms.

Brock's equation which is applicable into the slip flow regime \( (\lambda/r_p \ll 1.0) \) for thermal diffusive force of particles in air has the form

\[
F_t = (-9\pi)(r_p)(\eta^2) \left( \frac{1}{1+6\lambda/r_p} \right) \frac{k_n}{k_p} \left( \frac{1+2k_n+15\lambda}{r_p} \right) dT
\]

(K-10)

Where \( \lambda \) is calculated from Equation D-24 as follows,

\[
\eta = 0.499 \sqrt[3]{\lambda}
\]

(K-11)

All the nomenclatures in Equations K-10 and K-11 are described in Appendix R.

On the basis of coefficients obtained by these equations, the thermal force as calculated by Equation K-10 has been shown in good agreement with the thermal force found for liquid aerosols by Jacobsen and Brock (55).
A completely different approach to defining thermophoresis of large particles has been taken by Derjaguin and Bakanov (35). Their calculations give a thermal slip coefficient which is 35 times smaller than that obtained by Maxwell (70), the latter being the basis of boundary conditions used by Epstein and Brock. Derjaguin and Bakanov use the Chapman-Enskog method of solving the kinetic equation which make possible the calculation of the isothermal heat transfer in a gas stream when the space derivatives of the velocity gradient are not equal to zero. This approach introduces a new turn into the boundary condition for the component of the heat flux normal to the surface. Their result for the thermophoretic force is,

\[ F_t = (-) \frac{3m \eta^2 r_p}{q_T} \frac{(8k_g+k_p)}{(2k_g+k_p)} \frac{dT}{dx} \quad (K-12) \]

More recently Derjaguin and Yalamov (37) have taken the temperature jump into account with the result

\[ F_t = (-) \frac{3m \eta^2 r_p}{q_T} \frac{(8k_g+k_p+2C_t^2r_p k_p)}{(2k_g+k_p+2C_t^2 r_p k_p)} \frac{dT}{dx} \quad (K-13) \]

This equation, valid only for large particles, gives results to within 20% of experimental (36). But the theory infers that the thermal velocity is size independent which does not agree with the data of Rosenblatt and Lamer (91) or the results of this work.

Due to the complexity of the problem, no work has been done to define the thermal force when the size of the...
particle becomes comparable to the mean free path ($\lambda = 1.0$, $R_T$ the transition region) until recently. Brock (17) is the first to give a description of the mechanism in this regime. In principle, the approach is similar to that of the free molecular regime, that is, the calculation is based on momentum transfer between the gas molecule and the suspended particle. The important difference has to do with velocity distribution of the incoming and outgoing gas molecules. The qualitative description of Brock's first attempt is given in Reference 17.
APPENDIX L

Heat and Mass Transfer Analysis of the Forced Flow in a Smooth Pipe

As seen in Section II, a quantitative theoretical analysis of the frosting process requires an accurate knowledge of heat and mass transfer characteristics of the system. This appendix briefly describes the applicability of various theoretical analysis for the evaluation of the heat and mass transfer characteristics of the frosting system of the present research. The theoretical studies are carried out both in laminar and turbulent flow regimes.

A. Heat Transfer Characteristics of the Forced Flow in a Smooth Pipe

In the frosting system of the present research, a long calming section was used to obtain a fully developed velocity profile in gas before it passed over the heat transfer section of the duct. Also, the measurements of heat flux through the frost were carried out in the thermal entrance region (where thermal boundary layer is being developed). Hence, for the present purpose, heat transfer characteristics of a fully developed flow through a duct (or pipe) in thermal entrance region are of significance. The boundary condition of constant wall temperature is assumed to be valid one for theoretical predictions. Since the flow is usually divided into the turbulent, laminar and transition regimes, the following discussion of forced convection heat transfer is also separated on that basis.
1. **Turbulent Heat Transfer**

The turbulent heat transfer analysis for a fully developed flow through a smooth pipe (see Figure L-1) can be theoretically studied with the help of the basic energy Equation L-1 and boundary conditions Equations L-2 to L-4 as described below.

\[
\frac{u}{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial y}\left[\gamma \left(\frac{\partial}{\partial r} + \epsilon_H \frac{\partial}{\partial T}\right)\right] \quad \text{(L-1)}
\]

![Figure L-1 Schematic of Heat Transfer Process for a Fully Developed Forced Flow through a Smooth Pipe](image)

**Boundary Conditions**

\[
T(x, r) = T_o \quad ; \quad x < 0 \quad \text{(L-2)}
\]

\[
T(x, r_o) = T_w \quad ; \quad x > 0 \quad \text{(L-3)}
\]

\[
T(0, r_o) = T_0 \quad ; \quad r \neq r_o \quad \text{(L-4)}
\]

All the nomenclatures of Equations L-1 to L-4 are described in Appendix R.

The system satisfying the Equation L-1 is subjected to the following restrictions.

1. Fluid properties are constant.
2. Mean velocity in axial direction is independent of angular position.
3. Mean radial velocity is zero.
4. Mean temperature at any radius does not vary with time.
5. Frictional dissipation of energy is negligible.

6. Axial diffusion is negligible with respect to bulk transport in the x-direction.

Several attempts have been made to obtain a mathematical solution of Equation L-1 with varying degree of sophistication. First, Latzko (64) obtained an approximate solution of Equation L-1 for Pr = 1.0. His final result for heat transfer coefficient is as follows:

\[ h(x) = h_\infty \left( 1 + 0.1 e^{\frac{-2.7x}{\text{Re}^{0.25} DH}} + 0.90 e^{\frac{-29.27x}{\text{Re}^{0.25} DH}} - 0.023 e^{\frac{-31.96x}{\text{Re}^{0.25} DH}} \right) \]  

(L-5)

where,

\[ h_\infty = \frac{0.0384 G c_p}{\text{Re}^{0.25}} \]  

(L-6)

All the nomenclatures of Equations L-5 and L-6 are described in Appendix R.

Deissler (31,32,33) and Sleicher and Tribus (105,106), later on, carried out sophisticated mathematical analysis of Equation L-1. The main difference in their solutions lies in the estimation of \( \epsilon_H \).

Deissler (31,32) used experimentally determined velocity and concentration distribution to obtain the following correlations for \( \epsilon_M \). He, then, assumed \( \epsilon_H = \epsilon_M \).

Near cold wall,

for \( y^+ < 26 \)
\[ \varepsilon_m = n^2uy (1 - e^{-(n^2uy/\nu)}) \]  
where, \[ y^+ = y \left( \frac{g_c \zeta_c}{\nu} \right)^{1/2} \]  
for \( y^+ > 26 \)  
\[ \varepsilon_m = k^2 \frac{(dy/du)^3}{(du/du^2)^2} \]  

The values of the constants \( n=0.124 \) and \( k=0.36 \) in above equations were obtained from the mass transfer experiments in turbulent flow through pipe.

Sleicher and Tribus (105,106) measured not only the velocity distribution but also the temperature distribution in pipe to calculate \( \varepsilon_m \). They found that the ratio of \( \varepsilon_m \) and \( \varepsilon_m \) was 1.4 instead of unity as assumed by Deissler (31,32). The results of Sleicher and Tribus (105,106) for agree well with ones obtained by Corcoran et al. (27). Their final result for heat transfer coefficient in thermal entrance region can be given by the following mathematical equation (105,106).

\[ \text{Nu}(x) = \frac{h(x) D_H}{k} = \frac{\sum_{n=0}^{\infty} A_n \exp(-\lambda_n^2 X_*)}{2 \sum_{n=0}^{\infty} (A_n/\lambda_n) \exp(-\lambda_n^2 X_*)} \]  

where,  
\[ X_* = 2 x / \text{Re} \text{ Pr} D_H \]

The constants \( A_n \) and \( \lambda_n \) are dependent on the Reynolds and Prandtl numbers. Their values are reported in Reference 105. The heat transfer coefficients calculated from Equation L-9 have been found to agree with the experimental data of Boelter (11).
In order to evaluate the applicability of each of the above described method for the present purpose, the heat transfer coefficient at the centre of the test section under a set of system conditions are calculated using Equations L-5 and L-9 and Figures 6 and 8 of Reference 32. These values are then compared with the experimental one. The final results of these calculations are summarized below.

For **system conditions**:

Reynolds number = 6,100
Humidity = dew point at 14.3°F
Gas Temperature = 48°F
x = 2.77 inches
D = 0.8 inch

<table>
<thead>
<tr>
<th>Method</th>
<th>h (Btu/hr ft² °F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latzko</td>
<td>4.24</td>
</tr>
<tr>
<td>Deissler</td>
<td>5.61</td>
</tr>
<tr>
<td>Sleicher-Tribus</td>
<td>5.95</td>
</tr>
<tr>
<td>Experimental(frosting)</td>
<td>6.92</td>
</tr>
</tbody>
</table>

From the above described results it can be concluded that, as expected, Latzko method (64) predicts too low heat transfer coefficient compare to the experimental value. Both Sleicher-Tribus method (105) and Deissler method (32) predict almost identical values of heat transfer coefficient. These values are also in good agreement with the experimental one. Since Sleicher-Tribus (105) uses the experimentally measured values of $\epsilon_m$ in solution of Equation L-1, their
theoretical analysis is believed to be more realistic than that of Deissler (31, 32). Hence, it is used for the purpose of a comparison between theoretical and experimental values of the heat transfer coefficients in the frosting experiments.

It should be noted that in the present research, the heat transfer coefficients were also measured under non-frosting conditions (see Section IV-D). These results were found to be in very good agreement with the predictions of Sleicher-Tribus Equation L-9.

2. Laminar Heat Transfer

Although, no frosting experiments were carried out to obtain the properties of frost deposited from the gas in laminar flow regime, it is believed that the theoretical models for the frosting process are valid in this flow regime. Thus, the application of the theoretical models in the laminar flow regime also demands the accurate knowledge of heat transfer coefficient.

Heat transfer characteristics of a laminar flow through a pipe or a duct are well studied by Graetz (61). Graetz solution for the heat transfer coefficient, however, can not be used when Reynolds number is less than about 130 (101) because axial conduction then becomes significantly important. Graetz solution, of course, does not take into account the effect of axial (or back) conduction on the heat transfer coefficient. Schneider (101) has solved the problem of heat transfer in laminar flow (with flat velocity profile) when back conduction is important. The problem of heat
transfer in laminar flow, both with or without back conduction and for both flat and parabolic velocity profiles is solved by the author and co-workers (103). The effects of back conduction and of velocity profile on the laminar heat transfer characteristics in a rectangular duct are described in Figures L-2 and L-3 respectively for two different Reynolds numbers of air. As shown in Figure L-2 the back conduction at very low Reynolds number causes the heat transfer coefficient to attain a finite value at the entrance of the heat transfer regime. It should be noted that in the absence of back conduction heat transfer coefficient is infinity at the entrance of the heat transfer regime. The thermal boundary layer thickness (at any downstream distance) is higher hence heat transfer coefficient is lower in case of laminar flow with parabolic velocity profile than ones in case of flat velocity profile laminar flow as shown in Figure L-3.

3. Heat Transfer in Transition Flow

The theoretical analysis of heat transfer in transition flow regime is quite difficult. Usually, the correlation for heat transfer coefficient in transition flow regime is obtained by interpolating between the correlations for laminar and turbulent flow regimes.

B. Mass Transfer Analysis of the Forced Flow in a Rectangular Duct

The process of frosting occurs with simultaneous transfer of heat and mass. Hence, the mass transfer
characteristics of the frosting process can be studied with the help of heat transfer analysis of the process and using an analogy between heat and mass transfer.

Two simple analogies between heat and mass transfer are those of Lewis (39) and of Chilton and Colburn (104). The experimental data of Coles (26) and Chung (24) for the studies of heat and mass transfer analogies have been found to agree well with Lewis relation (39, 72). Hence, for the present purpose of evaluating the mass transfer characteristics of the frosting process, Lewis relation is used.

**Lewis Relation for Simultaneous Heat and Mass Transfer**

The Lewis relation for simultaneous heat and mass transfer can be used in all the flow regimes (turbulent, transition and laminar). For small vapor concentration it can be written as (39)

\[ \frac{j_D}{j_H} = \left( \frac{Sc}{Pr} \right)^{2/3} \quad (L-10) \]

or,

\[ \frac{h}{kg} = \frac{\rho c_p \left( \frac{Sc}{Pr} \right)^{2/3}}{Le^{2/3}} \quad (L-12) \]

Thus, the mass transfer coefficient in the frosting process can be calculated from Equation L-12 with the knowledge of \( h \) (heat transfer coefficient) and gas properties.
Constant Wall Temperature

Reynolds Number

Plug Velocity Profile

Parabolic Velocity Profile

System Conditions
Reynolds Number = 274
Constant Wall Temperature

With Back Conduction
Without Back Conduction

Region of Back Conduction

Figure L-2 Effect of Back Conduction on Heat Transfer Coefficient for Plug Flow of Air in a Rectangular Duct (103)

Step Change in Wall Temperature

Distance from Entrance (X/Dh)

Figure L-3 Effect of Velocity Distribution on Convection Heat Transfer Coefficient for Laminar Flow of Air in a Duct (103)
APPENDIX N

Sample Calculations and Error Analysis

1. **Average Frost Density**

   All the frost density values reported in this thesis are actually indirectly measured quantities, since they were calculated from the measured weight of the collected frost samples and the correspondingly measured sample thicknesses. A sample calculation for the frost density is described below.

   For Run number 30, at time of 104.36 minutes from the start.

   Mass of the frost collected by 0.866 cm² cross section (inside), cylindrical, hollow glass vial = .0372 gms.

   Thickness of the frost = 0.225 inch (as measured by depth micrometer)

   Density of the frost = \[ \frac{0.0372}{0.866 \times 2.54 \times 0.225} = 0.0751 \text{ gms./cc.} \]

   The error in the density measurement is related to the errors involved in weighing the frost sample and in measuring the frost thickness. The error in the density measurement, \( \Delta \rho_f \), can be estimated from Equation N-1 (74) shown below.

   \[
   \Delta \rho_f = \frac{\partial \rho_f}{\partial m_f,v} \Delta m_f,v + \frac{\partial \rho_f}{\partial A_v} \Delta A_v + \frac{\partial \rho_f}{\partial \delta} \Delta \delta \quad (N-1)
   \]

   since \( \rho_f = \frac{m_f,v}{A_v} \), Equation N-1 reduces to

   \[
   \Delta \rho_f = \frac{1}{A_v \delta} \Delta m_f,v - \frac{m_f,v}{8 A_v^2} \frac{1}{A_v} \Delta A_v - \frac{m_f,v}{A_v \delta^2} \Delta \delta \quad (N-3)
   \]
It was estimated that in general the weight of the collected frost sample could be determined to the nearest 0.002 gms. This limitation on the accuracy was due to loss of material during the collection process. In the measurements of density gradient in the frost interior (the results described in Section IV-C), the weight of the frost slice could be determined to the nearest 0.003 gms. because of additional error involved during the slicing. It was also estimated that the frost thickness could be measured accurately to the nearest 0.005 inch (or 0.0127 cm).

The primary cause of error in this measurement is the difficulty involved in determining the actual location of the frost surface.

The diameter of the vial used to collect the frost sample could be estimated to the nearest 0.002 inches (0.00508 cm). From Equation N-4 shown below

\[ \Delta A_v = \frac{D_v \pi}{2} \Delta D_v \]  

\( (N-4) \)

it was calculated that this uncertainty in the diameter implied an error of 0.00836 cm\(^2\) in the vial cross sectional area. Thus, from the preceding discussion, the values of \( \Delta A_v, \Delta m_{f,v} \) and \( \Delta \delta \) chosen for this calculation were

\[ \Delta A_v = 0.00836 \text{ cm}^2 \]

\[ \Delta m_{f,v} = 0.002 \text{ gms.} \]

and \( \Delta \delta = 0.0127 \text{ cm.} \)

It should be noted that these error values are applicable to all the frosting experiments.
Substituting in Equation N-3 the values of $\Delta A_v$, $\Delta m_f$, and $\Delta \delta$ listed above, and the values of $A_v$, $m_f$, and $\delta$ shown earlier, $\Delta A_f$ was found to be 0.00644. Since $\rho_f$ for Run 30 is equal to 0.0751 gms./cc., the maximum percentage error in density measurement is $\frac{0.00644 \times 100}{0.0751}$ or $\pm 8.57\%$.

In the density gradient measurement, where $\Delta m_f$, $\delta = 0.003 gms.$, the maximum percentage error for a typical experiment can be calculated as follows.

For Run number 37, the mass of the bottom 0.0625 inch thick frost = 0.01468 $gms.(=m_f,v)$, $A_v = 0.714 \text{ cm}^2$.

Thus, $\rho_f = \frac{0.01468}{2.54 \times 0.0625 \times 0.714} = 0.1295 \text{ gms./cc.}$

Since, once again,

$\Delta A_v = 0.00836 \text{ cm}^2$

$\Delta m_f,v = 0.001 gms.(\text{since } 0.003 gms \text{ for three slices})$

and $\Delta \delta = 0.0042 \text{ cm}$.

Thus, the maximum percentage error in this kind of measurement is calculated as follow,

$\Delta \rho_f = 0.01582$

maximum % error = $\frac{0.01582 \times 100}{0.1295}$ or $\pm 12.22\%$

It should be noted that the calculation of error by Equation N-3 does not allow for compensating errors. The true magnitude of the errors (maximum) are most certainly lower than the ones calculated above.

2. Thermal Conductivities

A. Average Frost Layer Conductivity
The average frost layer thermal conductivity was calculated from Equation N-5 shown below.

\[ q = \frac{k_{avg}}{6} (T_s - T_w) \quad (N-5) \]

From the experimental measurements of \( q, T_s, T_w \) and \( k_{avg} \) for a typical set of experimental conditions can be calculated as follows.

For Run number 16,
\( q = 539 \) Btu/hr ft\(^2\) (quasi-steady state value)
\( T_s = 15^\circ\text{F} \) (value in the quasi-steady state of heat flux)
\( T_w = -315^\circ\text{F} \)
\( \delta = 0.1875 \) inch

hence,
\[ k_{avg} = \frac{539 \times 0.1875 \times 1}{330 \times 12} = 0.02784 \text{ Btu/hr ft}^\circ\text{F} \]

The method if determining the error in the conductivity measurement is essentially the same as that used for determining \( \Delta q_T \).

The error in the conductivity measurement is given by

\[ \Delta k_{avg} = \frac{\partial k_{avg}}{\partial \delta} \Delta \delta + \frac{\partial k_{avg}}{\partial Q} \Delta Q + \frac{\partial k_{avg}}{\partial A} \Delta A + \frac{\partial k_{avg}}{\partial T_s} \Delta T_s + \frac{\partial k_{avg}}{\partial T_w} \Delta T_w \quad (N-6) \]

where, \( Q/A = q \)

The combination of Equations N-5 and N-6 yields Equation N-7 shown below.

\[ \Delta k_{avg} = \frac{Q \Delta \delta}{A(T_s - T_w)} + \frac{\delta}{A(T_s - T_w)} \Delta Q + \frac{\delta Q}{A^2(T_s - T_w)} \Delta A + \frac{\delta Q}{A(T_s - T_w)} \Delta T_s + \frac{\delta Q}{A(T_s - T_w)^2} \Delta T_w \quad (N-7) \]
The signs of all terms have been taken as positive as was done in estimating $\Delta P_f$.

Due to thermocouple error, the plate temperature could only be measured to about the nearest 2°C. The measurement of surface temperature, on the other hand, may involve an error as large as 5°C. The reasons for an error of such magnitude have been previously discussed in section II-A and in Appendix A.

Based on the calorimeter calibration results, it was estimated that the maximum error in the heat flux measurement was 0.4 Btu/hr. In addition, the uncertainty with which the area of the heat transfer test section is known is about 0.0001228 ft$^2$. This value of $\Delta A$ was obtained from Equation N-8 using a value of $\Delta D$ equal to 0.000416 ft (i.e., 0.005 inches).

$$\Delta A = \frac{D\pi}{2} \Delta D \quad \text{(N-8)}$$

The diameter of the inner copper plate was actually known more accurately than to the nearest five thousandths of an inch. However, the heat transfer test section area also included one-half the area of the epoxy ring, as it was assumed that the heat transferred into the inner half of the epoxy ring boiled off liquid nitrogen in the inner stainless steel chamber, whereas the heat transferred into the outer half of the epoxy ring boiled off liquid nitrogen in the outer chamber. It is the difficulty involved in determining half the thickness of the epoxy
ring which leads to an error estimate as large as 0.005 inches. The diameter of the actual copper inner plate could be determined to the nearest 0.002 inches.

As mentioned earlier, the error in the frost thickness measurement is five-thousandths of an inch or 0.000416 ft. Based on the above discussion, the errors involved in the various measured quantities were taken as follows:

\[
\begin{align*}
\Delta Q &= 0.4 \text{ Btu/hr} \\
\Delta T_w &= 2^\circ F \\
\Delta T_s &= 5^\circ F \\
\Delta \delta &= 0.000416 \text{ ft} \\
\text{and, } & \Delta A = 0.0001228 \text{ ft}^2
\end{align*}
\]

Using the data of Run number 16 for the calculation one can write

\[
\begin{align*}
Q &= 14.92 \text{ Btu/hr} \\
A &= 0.0277 \text{ ft}^2 \\
\delta &= 0.01561 \text{ ft} \\
T_s &= 15^\circ F \\
T_w &= -315^\circ F
\end{align*}
\]

Substituting these values into Equation N-7, \( \Delta k_{\text{avg}} \) was calculated to be 0.002019. The measured \( k_{\text{avg}} \) for Run 16 is 0.02784 and, therefore, the estimated error is

\[
\frac{0.002019 \times 100}{0.02784} \% = \pm 7.24\%
\]

Once again it should be noted that since Equation N-7 does not allow for compensating errors, the true magnitude of the maximum error is most likely to be lower than one
calculated above.

3. Heat and Mass Transfer Coefficients in the Frosting Experiments

A. Heat Transfer Coefficient

The heat transfer coefficients in the frosting experiments for various system conditions were calculated from Equation N-9 shown below.

\[ h = \frac{q - N_s \Delta H}{(T_g - T_s)} \] (N-9)

For a typical set of experimental conditions, the heat transfer coefficient can be calculated as follows.

For Run number 38, \( q = 832 \) Btu/hr ft²
\( N_s = 0.1125 \) lbm/hr ft² (from mass deposition versus time curve)
\( \Delta H = 1,210 \) Btu/lbm \( \) (54)
\( T_g = 73^\circ F \)
\( T_s = 8^\circ F \)

hence, from Equation N-9

\[ h = \frac{832 - 0.1125 \times 1210}{65} = 10.71 \text{ Btu/hr ft}^2\text{OF} \]

The errors involved in the calculation of heat transfer coefficient by the above method can be obtained as follows.

From Equation N-9

\[ N = \frac{Q/A - n/A_v (\Delta H)}{(T_g - T_s)} \] (N-10)

where, \( Q/A = q \) and \( n/A_v = N_s \)

The error (maximum) in heat transfer coefficient is given by
Combining Equations N-10 and N-11, following relation for \( \Delta h \) can be obtained,

\[
\Delta h = \frac{\Delta Q}{A(T_g - T_s)} + \frac{\Delta H}{A_v(T_g - T_s)} + \frac{n/A_v \Delta(\Delta H)}{(T_g - T_s)^2} + \frac{Q}{A^2(T_g - T_s)} \Delta A + \frac{n \Delta A_v}{A_v^2 (T_g - T_s)}
\]

The errors involved in the various measured quantities were taken as follows.

\[
\begin{align*}
\Delta Q &= 0.4 \text{ Btu/hr} \\
\Delta A &= 0.0001228 \text{ ft}^2 \\
\Delta n &= 4.41 \times 10^{-6} \text{ lbm/ft} \\
\Delta A_v &= 9.0 \times 10^{-6} \text{ ft}^2 \\
\Delta(\Delta H) &= 2.5 \text{ Btu/lb} \\
\Delta T_g &= 1^\circ \text{F} \\
\text{and} \quad \Delta T_s &= 5^\circ \text{F}
\end{align*}
\]

Again Run 38 is used to illustrate the following calculation of \( h \). For this run

\[
\dot{Q} = 23.2 \text{ Btu/hr} \\
A = 0.0277 \text{ ft}^2 \\
n = 1.049 \times 10^{-4} \text{ lbm/hr} \\
\Delta H = 1210 \text{ Btu/lb} \\
A_v = 9.32 \times 10^{-4} \text{ ft}^2
\]
\[ T_g = 73^\circ F \]
\[ T_s = 8^\circ F \]

Substituting all the above values into Equation N-12, \( \Delta h \) was calculated to be 1.36. The value of \( h \) for this run is 10.71 Btu/hr ft\(^2\) \(^\circ\)F (as calculated before). Hence, the estimated error is \( \frac{1.36}{10.71} \times 100 \approx 12.69\% \).

The true error in the measurement of heat transfer coefficient will be most likely less than one calculated above because of presence of compensating errors.

B. Mass Transfer Coefficient

The mass transfer coefficients for various system conditions studied in the present thesis were calculated from Equation N-13 shown below.

\[ K = \frac{N_s}{g \left( p - p_s \right)} \]  
(N-13)

For a typical set of experimental conditions, mass transfer coefficient can be calculated as follows.

For Run number 38, \( N_s = 0.1125 \) lbm/hr ft\(^2\) (from Figure IV-A2)

\[ p_g = 2.542 \times 10^{-4} \] lbm/ft\(^3\)
\[ T_s = 8^\circ F, \text{ hence } p_s = 1.03 \times 10^{-4} \] lbm/ft\(^3\)

Thus, from Equation N-13

\[ K = \frac{0.1125 \times 10^{-4}}{1.512} = 744 \text{ ft/hr}. \]

The method for determining the error in the mass
transfer coefficient measurement is essentially the same as that used for $\Delta k_{\text{avg}}$.

The mass transfer coefficient is given by

$$K = \frac{n}{g A_v (p_g - p_s)} \quad (N-14)$$

Hence, the error in the mass transfer coefficient measurement is given by

$$\Delta K_g = \frac{\Delta K g}{\Delta n} \Delta n + \frac{\Delta K g}{\Delta A_v} \Delta A_v + \frac{\Delta K g}{\Delta p_g} \Delta p_g + \frac{\Delta K g}{\Delta p_s} \Delta p_s \quad (N-15)$$

The combination of Equation N-14 and N-15 yields

Equation N-16 shown below

$$K_g = \frac{n}{A_v (p_g - p_s)} + \frac{\Delta A_v n}{A_v^2 (p_g - p_s)} + (\Delta p_g + \Delta p_s) \left( \frac{n}{A_v (p_g - p_s)^2} \right) \quad (N-16)$$

The signs of all the terms have been taken as positive in order to obtain the maximum value of $\Delta K_g$.

The errors involved in the measured quantities were taken as follows.

$$\Delta n = 0.002 \text{ gms./hr or } 4.41 \times 10^{-6} \text{ lbm/hr}$$
$$\Delta A_v = 9.0 \times 10^{-6} \text{ ft}^2$$
$$\Delta p_g = 0.1007 \times 10^{-4} \text{ lbm/ft}^3$$
$$\Delta p_s = 0.264 \times 10^{-4} \text{ lbm/ft}^3 \quad \text{(since } T_s = 5^\circ \text{F, as discussed before)}$$

Again, Run number 38 will be used to illustrate the calculation of $\Delta K_g$. For this run

$$n = N_S A_v = 1.049 \times 10^{-4} \text{ lbm/hr}$$
$$A_v = 9.32 \times 10^{-4} \text{ ft}^2$$
\[ p_g = 2.542 \times 10^{-4} \text{ lbm/ft}^3 \]
\[ p_s = 1.03 \times 10^{-4} \text{ lbm/ft}^3 \]

Substituting all these values in Equation N-16, \( \Delta K_g \) was calculated to be 49.81. The measurement of \( K_g \) for this run is 744 (as calculated before), and therefore, the estimated maximum error is \( \frac{49.81 \times 100}{744} \% \) or \( \pm 6.7\% \).

The true error will be most likely less than one calculated above because of the presence of the compensating errors.

4. Heat Transfer Coefficient in Non-frosting Experiment

The heat transfer coefficient in the non-frosting experiment can be calculated from Equation N-17 shown below.

\[ h = \frac{q}{(T_g - T_s)} = \frac{Q}{A \ (T_g - T_s)} \quad (N-17) \]

where all the nomenclatures are the same as the ones described before.

For a typical set of experimental conditions, heat transfer coefficient in the non-frosting conditions can be calculated as follows.

For Run number \( \text{H-2} \), \( q = 1,087 \text{ Btu/hr ft}^2 \)

\[ T_g = 82.6^\circ \text{F} \]
\[ T_s = -11.5^\circ \text{F} \]

hence, from Equation N-17

\[ h = \frac{1,087}{(94.10)} = 11.08 \text{ Btu/hr ft}^2^\circ \text{F} \]

The errors involved in the calculation of heat transfer coefficient can be obtained as follows.

From Equation N-17

\[ h = \frac{Q}{A \ (T_g - T_s)} \quad (N-18) \]
The maximum error in heat transfer coefficient is given by
\[
\Delta h = \left( \frac{\partial h}{\partial Q} \right) \Delta Q + \left( \frac{\partial h}{\partial A} \right) \Delta A + \left( \frac{\partial h}{\partial T_g} \right) \Delta T_g + \left( \frac{\partial h}{\partial T_s} \right) \Delta T_s \quad (N-19)
\]

Combining Equation N-18 and N-19, Equation N-20 can be obtained as follows.
\[
\Delta h = \frac{\Delta Q}{A(T_g-T_s)} + \frac{\Delta A}{A^2(T_g-T_s)} + \frac{(\Delta T_g + \Delta T_s) \times Q}{A(T_g-T_s)^2} \quad (N-20)
\]

The errors involved in the various measured quantities were taken as follows:

- \( \Delta Q = 0.30 \text{ Btu/hr.} \)
- \( \Delta A = 0.0001228 \text{ ft}^2 \)
- \( \Delta T_g = 1^\circ F \)
- \( \Delta T_s = 1^\circ F \)

Again, Run number H-2 is used to illustrate the following calculation of \( \Delta h \). For this run:

- \( Q = 30.15 \text{ Btu/hr.} \)
- \( A = 0.0277 \text{ ft}^2 \)
- \( T_g = 82.6^\circ F \)
- \( T_s = -11.5^\circ F \)

Thus, substituting all these values into Equation N-20, \( \Delta h \) was calculated to be 0.4124. The measured \( h \) for this Run is 11.08. Hence, the maximum % error in the calculation of heat transfer coefficient would be \( \frac{0.4124 \times 100}{11.08} \pm 3.72\% \)

Once again, it should be noted that compensating errors will most likely cause the true error to be less than the one calculated above.
5. **Point Thermal Conductivities**

The point thermal conductivity values shown in Table IV-A5 and in Figure IV-A22 were obtained from the heat flux measurements and the graphical differentiation (with the help of a mirror) of the measured temperature distribution within the frost. A sample calculation for the point thermal conductivity is described below.

For Run number 35, at time of 79 minutes from the start of frosting the heat flux was found to be 871 Btu/hr ft\(^2\) and average frost density was found to be 0.1258 gms./cc. From Figure IV-A21, the following values of \(\frac{dT}{dx}\) at various temperatures were obtained.

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>(\frac{dT}{dx}) (°F/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-215</td>
<td>4.57 \times 10^4</td>
</tr>
<tr>
<td>-145</td>
<td>2.883 \times 10^4</td>
</tr>
<tr>
<td>-80</td>
<td>2.267 \times 10^4</td>
</tr>
<tr>
<td>0</td>
<td>0.889 \times 10^4</td>
</tr>
</tbody>
</table>

Since, \(k_f = \frac{q}{\frac{dT}{dx}}\), the following values of point thermal conductivities at various temperatures can be obtained.

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>(k_f) (Btu/hr ft °F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-215</td>
<td>0.01905</td>
</tr>
<tr>
<td>-145</td>
<td>0.03020</td>
</tr>
<tr>
<td>-80</td>
<td>0.03840</td>
</tr>
<tr>
<td>0</td>
<td>0.09800</td>
</tr>
</tbody>
</table>
Error Analysis

The error in point thermal conductivity measurements can be evaluated in the manner analogous to one described previously. Thus,

\[
\Delta k_f = \frac{\Delta Q}{A \left(\frac{dT}{dx}\right)} + \frac{Q \Delta A}{A^2 \left(\frac{dT}{dx}\right)} + \frac{Q \Delta \left(\frac{dT}{dx}\right)}{A \left(\frac{dT}{dx}\right)^2}
\]  \hspace{1cm} (N-21)

For a typical set of experimental conditions (Run number 35), \(\Delta k_f\) can be evaluated as follows,

for, \(T = 0^\circ F\)

- \(Q = 24.13 \text{ Btu/hr}\)
- \(A = 0.0277 \text{ ft}^2\)
- \((dT/dx) = 0.889 \times 10^4 \text{ °R/ft}\)
- \(\Delta Q = 0.40 \text{ Btu/hr}\)
- \(\Delta A = 0.000128 \text{ ft}^2\)
- \(\Delta (dT/dx) = 7.0 \times 10^2 \text{ °F/ft}\)

Thus, substituting all these values in Equation N-21, one obtains,

\[
\Delta k_f = 9.70 \times 10^{-3} \text{ Btu/hr ft}^2 \text{ °F/ft}
\]

or, the maximum % error = 0.978/0.0980 = \(\pm 9.95\%\)

The actual error is likely to be lower than one calculated above due to presence of compensating errors.

6. Temperature Gradient Measurements

In order to calculate point thermal conductivity accurately, the precise determinations of temperature gradients are required. In the present thesis, the temperature gradients were measured with the help of a mirror. However, an accuracy of this differentiation procedure was tested
for a typical run by formulating a cubic equation which passed through the measured temperatures in the frost interior and at the interface of gas and frost. Temperature gradients were obtained by analytically differentiating this equation.

Run number 35 was chosen as a typical run. The measured temperature profile in the frost interior for this run is shown in Figure IV-A21. Temperature gradients calculated by analytical and graphical differentiations at temperatures of -215°F, -145°F, -80°F and 0°F are compared in Table N-1. The agreement was found to be quite good. From these results, it was concluded that the temperature gradient by the present method can be measured with an accuracy of approximately within ± 4%.
<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>( \frac{dT}{dx} ) (°F/ft) Obtained by Graphical Differentiation</th>
<th>( \frac{dT}{dx} ) (°F/ft) Obtained by Analytical Differentiation</th>
<th>( % ) Error: ( \frac{(\frac{dT}{dx})_G - (\frac{dT}{dx})_A}{(\frac{dT}{dx})_G} \times 100 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-215</td>
<td>( 4.57 \times 10^{-4} )</td>
<td>( 4.635 \times 10^{-4} )</td>
<td>-1.402%</td>
</tr>
<tr>
<td>-145</td>
<td>( 2.883 \times 10^{-4} )</td>
<td>( 2.912 \times 10^{-4} )</td>
<td>-0.996%</td>
</tr>
<tr>
<td>-80</td>
<td>( 2.267 \times 10^{-4} )</td>
<td>( 2.187 \times 10^{-4} )</td>
<td>+3.655%</td>
</tr>
<tr>
<td>0</td>
<td>( 0.889 \times 10^{-4} )</td>
<td>( 0.895 \times 10^{-4} )</td>
<td>-0.675%</td>
</tr>
</tbody>
</table>
APPENDIX 0

Evaluation of the Possible Mechanisms for the Transfer of Fog Particles from a Gas Stream to a Solid Surface in Early Phases of a Frosting Experiment

As seen in Section V, in early phases of a frosting experiment, transfer of fog particles control the frost deposition. The purpose of this appendix is to examine the various forces acting on a particle suspended in a gas stream, and to determine which mechanisms are important in causing the particles to be deposited. The concepts regarding particle deposition mechanisms developed by Owen (79), and by Friedlander and Johnstone (45), will be utilized in examining the deposition of ice particles from a gas stream onto a frost layer. These concepts are reviewed below.

1. Previous Study

The fog particles formed in a gas stream can be transferred to a wall by one or more of the following mechanisms.

A. turbulence in the gas stream (45, 79)
B. gravity, if deposition is from above
C. force due to temperature gradient in the gas stream (95)
D. electrostatic forces (45)
E. fluid shear (79)

In their investigation Friedler and Johnstone (45) as well as Brazinsky (14) concluded that it is gas phase turbulence which is primarily responsible for transporting
the particles to the wall."

To corroborate their thesis that the turbulent eddies were responsible for transporting the particles to the wall, Friedlander and Jonstone noted that no particle deposition occurred in the inlet section of pipe where a relatively thick boundary layer existed adjacent to the wall. However, after a length of pipe corresponding to $Re_x = 100,000$, the position at which the laminar boundary layer is expected to go turbulent (72), deposition of particles onto the pipe wall was clearly observed.

Brazinsky (14) also made two experiments (at Reynolds numbers of 600 and 1,800) in the laminar flow regime to evaluate the effect of turbulence on the particle transfer. He observed that even though there was a considerable amount of fog present in the gas phase, substantially no fog was deposited on the cold wall for at least 37 minutes of the frostig process. He concluded from these results that turbulence in the gas stream plays significant role in the transfer of fog particles from gas to frost surface.

The process of transferring particles by turbulence in a direction transverse to the direction of the gas stream is not completely understood. The investigations of References 45 and 79 have, however, provided some insight into this phenomenon. The details of the mechanism for transporting solid particles through a turbulent gas stream, when molecular transport in the laminar boundary is not important, is also shown in the same references.
Since the experimental set-up for the present thesis is very similar to one used by Brazinsky (14), the calculations similar to one described in References 14 indicate that, in the present study, turbulence in the gas plays a significant role in the transfer of fog particles from gas to frost surface. The details of the calculations is shown in Reference 14.
APPENDIX - P

Contribution of Radiation to the Frost Thermal Conductivity

As mentioned in Section V-A, heat is not only transferred through the frost by conduction through the solid gas network, but also by radiation between ice particles and by water flux through the interior of the frost. In this appendix, the contribution of radiation to the over-all frost thermal conductivity is briefly evaluated.

The warmer ice particles, located at the top of the frost layer, radiate heat to the colder particles located below them. Each layer, in this way, absorbs part of the radiation striking it, and then re-emit radiant heat to the next lower layer. The experimentally measured thermal conductivity, includes this radiation component of thermal conductivity. The value of this radiative component was calculated from the Damkohler equation presented below (30). This equation was obtained from the radiation study in a packed bed onto which a large temperature gradient was imposed.

\[ k_r = \frac{0.684}{100} \varepsilon D_p \left( \frac{T}{100} \right) e_H \]  

(P-1)

where, \( D_p \) is in ft., \( T \) is in °R and \( k_r \) is in BTU/hr ft °F.

In Equation P-1, \( k_r \) is the contribution to the over-all frost conductivity made by radiation between the particles. The values chosen for substitution into Equation P-1 were those which would yield the largest possible contribution radiation could make to the frost thermal
conductivity.

The size of ice particles was chosen to be 100 microns \( (14) \). Thus, \( D_p = 100 \) micron was substituted into Equation P-1. The void fraction \( \varepsilon \) chosen for calculating \( k_r \) was taken as 0.94, the largest frost void fraction encountered in the experiments of present thesis. \( 32^{\circ}\text{F} \) was the temperature chosen for substitution into Equation P-1 as this is the highest possible temperature that can exist within the frost layer (In the experiments of the present thesis, the frost surface temperature never reached \( 32^{\circ}\text{F} \)).

Several values of frost emissivity are reported in the literature \( (75, 82) \). Reported values generally vary between 0.082 and 0.098. 0.098 was chosen, as this value represented the upper limit of the reported emissivities. Using the values of \( \varepsilon, D_p, T \) and \( e_H \) presented above, the value of \( k_r \) was calculated to be 0.000122 Btu/hr ft \( ^{\circ}\text{F} \). Assuming Damkohler’s equation and various assumed values (particularly \( D_p \)) are valid, this value of \( k_r \) is the highest possible contribution that radiation can make to the frost thermal conductivity. Since 0.000122 Btu/hr ft \( ^{\circ}\text{F} \) is negligible compared with 0.01 Btu/hr ft \( ^{\circ}\text{F} \), the lowest measured thermal conductivity, it is concluded that radiation makes a small contribution to the frost conductivities. Radiant heat transfer is, therefore, neglected in calculating theoretical values of conductivity.
**APPENDIX R**

**Table of Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Constant in Equation I-B1</td>
</tr>
<tr>
<td>a</td>
<td>Constant in Equation G-1</td>
</tr>
<tr>
<td>A</td>
<td>Cross sectional area for heat and mass transfer</td>
</tr>
<tr>
<td>A_v</td>
<td>Cross sectional area of glass vial</td>
</tr>
<tr>
<td>A_n</td>
<td>Constants in Equation L-9</td>
</tr>
<tr>
<td>b</td>
<td>Constant in Equation G-2</td>
</tr>
<tr>
<td>b</td>
<td>Constant in Equation I-B1</td>
</tr>
<tr>
<td>c_1</td>
<td>Constant in Equation II-C3</td>
</tr>
<tr>
<td>c_A</td>
<td>Concentration of air</td>
</tr>
<tr>
<td>c_p</td>
<td>Specific heat of gas</td>
</tr>
<tr>
<td>c_w</td>
<td>Concentration of water vapor</td>
</tr>
<tr>
<td>c_T</td>
<td>Total concentration of gas</td>
</tr>
<tr>
<td>C</td>
<td>Capacity factor as defined in Equation J-1</td>
</tr>
<tr>
<td>C_t</td>
<td>Constant in Equation K-13</td>
</tr>
<tr>
<td>D_B</td>
<td>Molecular diffusivity</td>
</tr>
<tr>
<td>D'_B</td>
<td>Molecular diffusivity in the free convection boundary layers within the frost</td>
</tr>
<tr>
<td>D_{eff}</td>
<td>Effective diffusivity</td>
</tr>
<tr>
<td>D_H</td>
<td>Hydraulic diameter</td>
</tr>
<tr>
<td>D_o</td>
<td>Pipe diameter</td>
</tr>
<tr>
<td>D_p</td>
<td>Particle diameter</td>
</tr>
<tr>
<td>D_{w-A}</td>
<td>Molecular diffusivity of water vapor in air</td>
</tr>
<tr>
<td>D_{1-2}</td>
<td>Molecular diffusivity of component 1 into 2</td>
</tr>
<tr>
<td>e_H</td>
<td>Emissivity</td>
</tr>
<tr>
<td>E</td>
<td>Energy</td>
</tr>
</tbody>
</table>
\( f \) Friction factor

\( f(\beta) \) Functionality relationship for \( \beta \)

\( F \) Frictional dissipation of energy

\( F_t \) Thermal force

\( g_c \) Gravitational constant

\( h \) Heat transfer coefficient

\( i \) Grid number in direction as shown in Figure E-0

\( j \) Grid number in z direction as shown in Figure E-0

\( j_D \) \( j \)-factor for mass transfer as shown in Equation I-C4

\( j_H \) \( j \)-factor for heat transfer as shown in Equation I-C4

\( J \) Grid number corresponding to the frost surface

\( J_s \) Mass flux from bulk gas to the frost surface

\( J_{si} \) Mass flux which diffuses in the frost interior

\( J_{so} \) Mass flux allocated to increase the frost thickness

\( k \) Thermal conductivity

\( k_{\text{excess}} \) Excess thermal conductivity

\( k_{f1} \) Intrinsic thermal conductivity of the frost

\( k'_{g} \) Thermal conductivity of air in the free convection boundary layers within the frost

\( k_{g,e} \) Total gas thermal conductivity

\( k_i \) Thermal conductivity of ice

\( k_{\text{ice}} \) Thermal conductivity of ice

\( k_r \) Radiative component of thermal conductivity of the frost

\( k_T \) Thermal diffusion coefficient

\( k'_{\text{excess}} \) Excess thermal conductivity of frost due to thermal diffusion of ice nuclei
\( K_g \) Mass transfer coefficient between bulk gas and the frost surface

\( K_{gi} \) Mass transfer coefficient at the start of the frosting process

\( K_s \) Ice thermal conductivity

\( L \) Length of a tube (also width of a duct)

\( Le \) Lewis Number

\( m \) Mass of a particle

\( m \) Mass of a single ice crystal

\( M \) Molecular weight

\( n \) Mass rate of water vapor

\( n \) Mass concentration of ice nuclei

\( n' \) Nuclei concentration per unit volume

\( N \) Number of ice crystals per unit volume

\( N_d \) Depletion rate of ice nuclei due to impingement (or trapping)

\( N_r \) Nucleation rate

\( N_s \) Mass flux of water vapor from bulk gas to the frost surface

\( N \) Concentration of molecules

\( p \) Pressure

\( p_w \) Partial pressure at the cold wall (or cold surface)

\( P \) Total pressure

\( P_{vp} \) Vapor Pressure

\( Pr \) Prandtl Number

\( q \) Heat flux

\( q' \) Volumetric flow rate
- $Q$: Heat transfer rate
- $r$: Radius
- $R$: Thermal resistance of a zone in a unit cell of the frost structure model
- $R_e$: Reynolds Number
- $S$: Supersaturation ratio ($p/P_{vp}$)
- $S_c$: Schmidt Number
- $t$: Time for the formation of a smooth frost layer
- $T$: Temperature
- $T_1$: Temperature at the tube entrance
- $T_2$: Temperature at the tube outlet
- $u$: Velocity of the gas
- $U_a$: Apparent over-all heat transfer coefficient between bulk gas and the cold wall in the frosting conditions
- $\bar{v}$: Mean average random velocity of a molecule
- $v$: Volume
- $V$: Velocity
- $V_t$: Thermal diffusive velocity of a solid particle
- $w$: Mass flow rate
- $x$: Dimensionless linear dimension of an ice cube in the frost structure model
- $x$: Distance from the cold wall towards the bulk gas phase
- $x$: Distance from the thermal entrance (Appendix L)
- $X_*$: $2x/Re Pr D_H$
- $y$: Distance from pipe wall (Appendix L)
- $Y$: Mole fraction of water vapor
z  Dimensionless distance from the cold wall \( (x/\delta) \)

z  Dimensionless width of the square ice rod in the frost structure model

**Greek Letters**

\( \alpha, \beta, \gamma \)  Constants used in Equations G-1 and G-2

\( \gamma, \psi, \xi \)

\( \alpha \)  Constant in Equation H-2

\( \alpha \)  Thermal diffusion factor

\( \delta \)  Boundary layer thickness

\( \sigma \)  Surface tension

\( \sigma \)  \( p/P_{vp} - 1 \)

\( \varrho \)  Density

\( \varrho_{ice} \)  Ice density

\( \varepsilon \)  Void fraction

\( \Delta H \)  Heat of sublimation

\( \epsilon_H \)  Eddy diffusivity of heat

\( \epsilon_m \)  Eddy diffusivity of momentum

\( \kappa \)  Constant in Equation K-5

\( \psi \)  Constant in Equation K-8

\( \eta \)  Viscosity

\( \mu \)  Viscosity

\( \varepsilon \)  Constant in Equation K-6

\( \nu \)  Kinematic viscosity

\( \zeta_0 \)  Shear stress

\( \tau \)  Tortuosity
Δz  Grid size in z direction
Δθ  Grid size in θ direction
Δ(κ) Differential variation (or increment) in an arbitrary variable κ
Θ  Time
δ  Frost thickness
\dot{δ}  Rate of change of frost thickness
λ  Mean free path
λ  Constants in Equation L-9

Subscripts

g  Gas
s  Frost surface
w  Cold wall
f  Frost
v  Vapor
v  Glass vial
f,s  Frost surface
p  Particle
b  Bulk gas
avg  Average
o  Old value
N  New value
o  Total value
e  Value at the prevailing equilibrium condition
Appendix T

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