The Design of a Closed Loop Linear Motor System

by

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Submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Abstract

This thesis demonstrates the design of an air cooled, linear servo motor system. The
design is based upon linear stepping motor (or Sawyer linear motor) technology. A
linear stepping motor consists of a magnetic forcer assembly which is moved (stepped)
electromagnetically along a platen, or work surface using variable frequency excitation
currents. Since linear motors are stepping motors, they perform very well using
open-loop control. They are however, subject to loss of synchronicity and are prone
to skewing, a misalignment of forcer and platen teeth, which both effectively halt
motor function. Two dimensional linear stepping motors almost exclusively utilize
an air bearing to maintain separation between the moving motor forcer and the fixed
platen. Air bearings have extremely low friction and long life characteristics, but,
they consequently have very low damping due to the low viscosity of the pressurized
air. This low damping can cause resonant behavior, particularly for fine positioning
requirements. These open loop operational drawbacks can be a deterrent from using
linear stepping motors for automated applications requiring continuous operation
with little or no operator monitoring.

The first step towards designing a closed loop linear motor system was to under-
stand linear motor operating principles of operation using dynamic modeling tech-
niques. A simple linear mass-spring model, and a more advanced state-space model
were developed. The thermal aspects of the motor were also considered. These
models were then applied to a commercially available, 2-axis linear stepping motor
manufactured by Northern Magnetics, Inc.

Next, several sensor system options were considered for providing feedback signals.
Finally, a closed loop motor system was designed based on the results of system
simulation. It is intended to use such a motor in a flexible, high-speed material
handling application. The proposed machine would operate continuously with very
fast cycle times.

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Chapter 1

Introduction to Linear Motor Systems

1.1 Introduction

A linear motor, like a rotating motor, generates mechanical force by the interaction of magnetic fluxes due to currents through conducting coils and magnetic fluxes provided by permanent magnets; while an aerostatic or mechanical bearing mechanism maintains the required air gap between parts which move relative one to another. In many ways, a linear stepping motor resembles a hybrid Permanent Magnet (PM) rotary stepper motor whose stator has been unrolled and extended over the length of desired motion. However, a rotary motor requires a mechanical interface to transform the rotary motion into linear. Any mechanical interface (lead screw, etc.) has added costs, both in product cost, hardware space requirements and system losses. Therefore, a linear motor has an inherent advantage over a rotary motor in that it precludes the need for any intermediate interface between the motor and its application.

This first chapter provides a preliminary background on linear stepping motors; including a description of their advantages and disadvantages. Next, an overview of the thesis is provided, including what issues are to be addressed, as well as an outline of the approach towards dealing with these issues.
1.2 Background on Linear Stepping Motors

Invented in 1969, the Sawyer linear stepping motor has the capacity for long, nearly frictionless motion with precise, open-loop positioning. In fact, what makes Sawyer motors unique is that when they are commanded to move to a position, they move with relatively good precision without the use of any type of feedback device. It is an ideal motor for automated systems where precise positioning and long life are required.

One of the first companies to apply linear stepping motor technology was Xynetics, Inc. This company developed several products, including an ‘Automated Drafting System’, in which a drafting head was fixed to a platform which was suspended under a 2-dimensional X-Y platen; and a laser head fabric cutting system, used in apparel manufacturing. The laser optics were fixed to a positioning head which rode on a Y-direction beam. The Y-beam in turn was driven over an X-direction beam. Another Xynetics product was a 3-dimensional, semiconductor wafer micro-positioning system. An X-Y stepping motor carried a Z-stage, which in turn carried a semiconductor wafer on a vacuum chuck. All of these systems would operate at relatively impressive speeds and accuracy’s.

Currently, there is one US company, Northern Magnetics, Inc., which produces 2-dimensional linear stepping motors, and two primary companies, Megamation Inc. and AT&T, which integrate these motors into [open loop] micro-electronic assembly robots.

The application of linear stepping motor systems has not been terribly widespread. The reasons for this are: patent control, which only ended in the late 1980’s; subsequent high hardware cost due to limited availability; and few technical advances since their introduction. Linear step motors are not particularly more cost effective than most other comparable motion control technologies for typical applications. The advantage of 2-axis linear stepper motors, however, is their functional flexibility. Not only do they preclude the need for a mechanical interface to convert from rotary to linear motion, but their cycle path configuration and size are limited only by platen
size.

Single axis linear motors are also available, both form Northern Magnetics, Inc. and Compumoter, another US firm. These motors may use mechanical or aerostatic bearings to maintain the necessary air gap. Industrial applications of single axis motors have included fiber optics manufacturing, print head, water jet cutting, packaging, gauging, inspection, light assembly, pick and place, and parts transfer machinery [4] [16].

1.3 System Definition

Linear stepping motors, like rotary stepping motors, are generally run open loop. This is where an important problem arises: loss of synchronicity or position due to commanded acceleration exceeding the maximum motor acceleration capability, and thus exceeding the characteristic motor pullout force. This is unacceptable if continuous operation is desired with little or no operator intercession.

A second problem is also due to loss of position, but has a rotary rather than a linear effect. This effect is due to the configuration of a typical two-dimensional X-Y motor. Multiple forcer assemblies are arranged about the center of mass of the motor, as shown in Fig. 1-1. If one forcer in an axial pair loses or gains position relative to the other, the motor will rotate or 'skew', causing misalignment of the forcer and platen teeth, and the force generation capability of the motor will drop to zero. The only way to resume operation of a skewed motor is by manual realignment of the teeth, followed by system initialization/origin location.

Like rotary stepping motors, linear stepping motors are prone to oscillations around detent positions. That is, the motor is subject to 'ringing' when a virtually frictionless aerostatic bearing separates a 2-dimensional forcer from a 2-dimensional platen. These oscillations result primarily from the interaction of the forcer mass and the capacitive transformation between magnetic and mechanical energy in the air gap between the forcer and platen.

Also like rotary stepping motors, linear stepping motors can be micro stepped
by proportioning current to the phases of the forcer. The benefits of micro stepping include:

- higher resolution for positioning accuracy,
- smoothness at low speeds,
- wide speed range,
- minimal force loss at resonant points.

The ultimate purpose of this thesis is to design a linear motor system for use in a high-speed, flexible packaging system. In order to do this, a linear motor must meet the following requirements:

- The system must have the ability to move a specified payload a certain distance within a specified time.
- The system must operate continuously and reliably in a typical industrial environment.
To meet these requirements, some modification of current motor characteristics will be required.

1.4 Thesis Content

This thesis will address the issues involved in designing a linear stepping motor system which will operate continuously and reliably, without any loss of position, without any 'skewing,' and within time/position requirements.

First a mathematical model will be developed and validated for an open loop linear stepping motor. Next, various closed loop configurations will be considered. Then a suitable sensor system will be selected for a closed loop system, followed by a duty cycle description for a proposed application. For this application, an 'off-the-shelf' Northern Magnetics, Inc. 2-axis, open-loop, linear stepping motor system will be analyzed, studied, and reproduced. From this experience, a new motor will be designed.

1.5 Summary

This chapter reviewed briefly the origin and application of linear stepping motors. It then touched on the problems inherent to linear stepping motors in their operation. Finally, it was outlined how these problems are to be analyzed, investigated and addressed.
Chapter 2

Linear Motor Theory

2.1 Introduction

This chapter reviews the basic theory behind linear stepping motors. By reviewing the system hardware and system operating principles, several key parameters for effective linear motor design and implementation will become apparent.

2.2 Linear Motor Operation

A linear motor, like a rotating motor, generates mechanical force by the interaction of magnetic fields due to currents through conductors and fields provided by permanent magnets. As previously mentioned, a linear stepping motor is essentially a rotary hybrid stepping motor whose rotor and stator have been cut axially, unrolled and flattened. A simplified version of a linear stepping motor system consists of a motor 'forcer' which moves about on a fixed surface, or platen, as shown in Fig. 2-1.

A motor forcer mechanism consists of a permanent magnet bridging two electromagnets with toothed pole faces. The teeth on the pole faces correspond to the platen teeth, as shown in Fig. 2-2. The flux lines from the permanent magnet flow into the core of one of the electromagnets, divide evenly through each leg of the electromagnet, cross the air gap between the electromagnet and the platen, and then flow into the ferromagnetic platen. The flux lines continue through the platen volume under
Figure 2-1: Simplified Two Phase Linear Stepper Motor

Figure 2-2: Alignment of Motor Forcer Teeth With Platen Teeth
the forcer, cross the other air gap and traverse equally through the legs of the second electromagnet, thereby completing the magnetic circuit at the opposite end of the permanent magnet. A complete magnetic circuit is illustrated in Fig. 2-1.

The platen is fabricated from a thin sheet of high permeability ferromagnetic material, such as 1010 steel. The surface of the platen has teeth etched into its surface, and resembles the surface of a waffle iron. The tooth pitch on a typical platen surface is 1.00 mm. This toothed tile is then bonded to a honeycomb material for structural integrity, as shown in Fig. 2-3. In order to use an air bearing, the valleys between the teeth must be filled, which can be done using an epoxy resin.

Considering again Fig. 2-1, and assuming a two-phase motor, the permanent magnet flux will pass through any two adjacent poles in the same direction. Each electromagnet has an associated coil. Full coil current excitation in one coil (coil A, for example,) by superposition, commutes the flux entirely into one leg of the electromagnet (pole 1) and diminishes or cancels it at the other adjacent pole (pole 2), as shown in Fig. 2-4. The current required to completely commutate the permanent
magnet flux to one pole face represents the peak amplitude of the driving current
wave form. Note also that peak current in one phase corresponds to zero current in
the other phase.

The commutation of the coil flux, along with the misalignment of an electromag-
net pole face and platen tooth, produces a tangential force which drives the forcer
mechanism parallel to the platen in such a manner as to minimize the length of, and
therefore the energy in the air gap, by attempting to align the teeth of a pole face
with platen teeth, thus reducing the reluctance of the magnetic circuit. Looking at
Fig. 2-4, pole 1 is shown just prior to alignment with the closest platen tooth. Upon
tooth alignment, a forcer pole and platen tooth would align as shown in Fig. 2-2. By
varying the frequency of the currents through the two electromagnet windings, the
forcer assumes a series of different incremental positions along the platen in such a
way as to effect continuous linear motion. The actual driving currents required to
command smooth motion for a two phase motor are two sinusoidal wave forms which
are 90 [electrical] degrees out of phase. Each cyclic supply to the coils produces a
step-by-step linear displacement of \( p/4 \), where \( p \) is the tooth pitch of the forcer and the platen. The net result is one tooth pitch of forcer motion per cycle of input current, as shown in Fig. 2-5.

The alignment of the forcer and platen can be thought of as similar to a mass-spring system. To understand this statement, it is necessary to understand the motor force displacement characteristic curve, shown in Fig. 2-6. The forcer is 'held' at a stable equilibrium point (point P1 on Fig. 2-6, which corresponds to the first cardinal position shown in Fig. 2-5) when the magnetic flux follows a path of least reluctance. The forcer can be displaced from this equilibrium point by applying an external force, which results in the production of a restoring force by the motor. The further the forcer is displaced from point P1, the greater the restoring force exerted by the motor. If the displacement is less than one quarter of a tooth pitch (between points C and D in Fig. 2-6); or that is, the external force imposed on the motor is less than the peak characteristic motor force (at points C and D), the forcer will return to the stable position. If the displacement is more than a quarter tooth pitch, the forcer will move to another stable position point (point P2 on Fig. 2-6) when released. The points U1 and U2 are actually unstable equilibrium positions for the motor. They can be thought of as an alignment of a forcer tooth with a platen valley (see the first figure of Fig. 2-5, and imagine pole 1 shifted 1/2 tooth pitch to the left or right) the slightest 'push' to the forcer will cause it to jump and align itself with a platen tooth, since the magnetic reluctance is so much less in this latter position. During operation, as the forcer is subjected to inertial and drag forces, it is actually displaced from a stable equilibrium position. If the drag forces exceed the maximum characteristic force, the forcer will slip, thereby losing synchronicity between commanded and actual position. The current input wave forms are typically supplied through power transistors switched by digital controllers. The motor can also be micro-stepped through small fractions of a complete current cycle to achieve much finer resolution than a tooth pitch. Further, a single motor can combine two forcers at right angles to each other, and then be moved over a large 2-D platen surface following any desired trajectory. The platen in this case would require teeth which are cut in two directions, and would
Figure 2-5: Flux Path of Four Cardinal Step Positions for Two Phase Stepping Motor
consist of an array of square plateaus standing above square valleys. This is generally referred to as a 'waffle platen' because its surface looks like a waffle iron.

2.3 Saturation Effects

The discussion so far has ignored system losses. High forceer accelerations require high magnetic flux densities. For the flux density to increase, the amplitude of the coil currents must increase and the reluctance of the iron path will no longer be negligible with respect to the air gap reluctance.

Saturation of the iron path can be considered in two ways. In the definitive sense, it is the point at which the iron core ceases to carry any increase in flux density. In a more practical view, saturation is considered as the point where the iron actually begins to saturate, because after that point, the law of diminishing returns takes over and there is little flux carrying capacity left in the iron path. Applied to a linear motor, the saturation effect introduces distortion in magnetic flux wave
shapes. For any given core material there is an almost fixed relationship between the cross sectional area of the flux carrying electromagnet cores, and the motor force. Since the motors operate by commutating flux from one pole set to another, the force available is controlled by the flux, which is supported by a pole piece. The flux is determined by the properties of the material and by how far the material is run into the saturation region. Increasing saturation may allow small increases in force, but the trade-off is increased cyclic error and hysteresis losses (see the following section).

Before the force iron reaches saturation, both phases of a two phase motor contribute equally to force generation via sinusoidal flux variations. Flux densities are directly proportional to coil current. As saturation appears, the flux and current do not coincide. The flux wave is distorted, and a harmonic position error is introduced into the system. This is an error which repeats with each current cycle and therefore with each pole pitch of motion; and is referred to as cyclic error. In a two-phase motor, most of this cyclic error occurs with a period of one-fourth of a pole pitch (each cardinal step,) hence, fourth harmonic pitch error. Fourth harmonic pitch can be dramatically reduced by combining two forcers displaced by one-eighth of a pole pitch from each other, and driving the two forcers with four signals 45 degrees out of phase. This translates to one-eighth of a tooth pitch of forcer advancement with each current peak. This so-called four-phase motor has smaller cyclical errors which are easier to compensate, because most of the remaining error is at a fundamental frequency.

2.4 Eddy Currents and Hysteresis

There are in fact, drag effects which limit operation. As the velocity of the motors increases, the frequency of the current sine waves through the motor windings increase, as well as the magnetic flux throughout the structure. Hence, losses in the motor forcer and the platen material go up as the motor velocity increases. Eddy current losses are a major drag force and potential error source. The eddy currents are the result of relative motion between a magnetic field and a conductor (Lenz'
Law). It is interesting to note here that any conductive material, not just ferromagnetic materials can contribute to eddy current losses. The hysteresis loss is the result of successive magnetization’s (and the energy required for such magnetization’s) of different portions of the sheet as the magnet traverses its surface. Actually, for soft magnetic materials, the $B$ versus $H$ curve hysteresis loop, and thus the energy loss per unit volume are generally quite narrow. Since this is the case, hysteresis losses are assumed to play a much less significant role in system losses than eddy currents.

To understand this phenomena, consider a horse shoe magnet being slid across the surface of a steel sheet, with the poles of the horse shoe a fixed distance from the surface but not actually touching it. As long as the sheet is continuous, that is, always fully spanning the poles, and the air gap is not changing; then the frictional forces will be practically nonexistent, and there will be no resistance to the magnet’s motion, nor any type of retarding force. However, if the magnet tries to move off of the edge of the steel sheet, there will no longer be a constant permanent magnet cross section spanning the sheet. The cross sectional area through which the flux travels is changed. The operating point of the magnet changes, and the magnet will, in effect, be operating under dynamic conditions. Since flux travels through steel with a lower reluctance than through air, as any part of the magnet’s pole moves off of the sheet, a force will be produced that will oppose the original sliding force. This opposing force will try to keep both magnet poles in contact with or directly over the sheet. This effect is also evident if the surface of the steel sheet is irregular, for example, a toothed surface. The speed of sliding, and the flux density in the material are also important. If either sliding rate or flux density are high, significant drag may be produced by eddy current and hysteresis losses in the sheet.

When a linear step motor is accelerated to high velocities, eddy current and hysteresis losses act like a drag force which opposes motor motion. Since this drag force is caused by the force mechanism ‘sweeping’ over the platen, it is proportional to velocity. If the platen were to be constructed from a solid, thick block of material, the eddy current loss would dominate the performance in the high speed area. To control these losses, laminated structures are used, with subsequent increased fabrication
Figure 2-7: Characteristic Fringing and Leakage Losses

cost.

There are also other magnetic losses due to core geometry and the desire to follow a path of least reluctance by the magnetic flux. The flux in the vicinity of the air gap that does not pass directly across the air gap but runs parallel to it is called the fringing flux. The flux that radiates between the pole legs or across the back of all parts of a magnetic circuit is normally referred to as the leakage flux. These two loss modes are illustrated in Fig. 2-7. Serious consideration must be given to these losses, because their net effect is to increase magnet area requirements to achieve the necessary flux densities.

2.5 Maintaining Air Gap Distance

In addition to a tangential force, there is a larger normal force attracting the motor forcer to the platen. This force must be offset by a bearing mechanism to maintain clearance between the two parts. The most typical bearing mechanisms used in
current applications are: a ball-bearing interface, and a single pad gas/air bearing. Of the two, the air bearing introduces less friction. Ball-bearing separation mechanisms are used mostly for one-axis linear stepper motors, while air-bearing separation mechanisms are used with both one and two-axis linear stepper motors.

There are several inherent advantages to using air or ‘aerostatic’ bearings. Typically the single pad air bearing is an air cushion formed by injecting compressed air through pin holes in the underside of the forcer. Such a bearing, for all intensive purposes, does not wear mechanically, and precludes hysteresis, friction (at lower speeds,) and backlash. Air bearings can also support large loads, because the load is distributed over a large area. The effective bearing area is approximately equal to the area contained between the air inlet orifices plus one-half of the area between the orifices and the outer perimeter of the bearing. Or, in other words, the bearing area is a function of the distribution of air ‘outlets’ around a particular bearing face; for example, the face of a linear motor forcer.

Another advantage of an air bearing is its ability to ‘smooth out’ surface errors (these surface errors, however, must be fairly small: on the order of one-fourth the total gap distance.) Also, since there is no contact between the pad bearing and the platen, there should be no ‘break-in’ period required, and no wear on components, provided proper equipment cleanliness and gas supply criterion are met (air is always flowing out of the bearing, so aerostatic bearings are self cleaning). Also, since there is no contact, shock and vibration resistance is better than for contact type bearings.

Probably the biggest draw back of air bearings are the plumbing requirements. The bearing portion itself takes up very little volume, but not so for the plumbing. The connection to an air supply system is usually by means of only one hose; but this implies significant drilling to the motor forcer, in order to ‘manifold’ the gas to all of the exit orifices, with subsequent impact to the electromagnetic characteristics of the forcer. Obviously for any linear motor application, the supply pressure, and thus the supply plumbing could be reduced if the forcer mechanism is suspended from the platen, rather than vice versa. This way, the gas bearing generated separation force works in the same direction as gravitational force, rather than against it, and thus
must only counter-act the magnetic normal attraction force.

Because air has a very low viscosity, the bearing gap must be very small, ideally on the order of 1 to 10 microns, though this may be difficult and expensive to manufacture. The only viscous friction forces associated with a gas bearing is due to the shear of the gas layer during motion of the bearing (obviously static friction is zero). Therefore, for high velocity applications the gap distance should be increased slightly. This is to ensure that the friction power in the gap is at most one-half of the gas pumping power; thereby offsetting temperature rise due to friction by expansion effects of the gas film in the gap. Because a gas has such low viscosity, the damping capabilities of the bearing are low. As mentioned, the gas delivery system must be kept extremely clean - usually the air is filtered and dried (to minimize condensation upon gas expansion). Another drawback of an aerostatic bearing is that the air film collapses very quickly in the event of an interruption to gas supply. If the gas supply system has a large backup reservoir and a control 'kill' interlock to the motor for pump failure, serious damage to the motor (and observers) can be avoided. Touch down of the forcer at high acceleration/velocity could be catastrophic.

2.6 Platen Considerations

The characteristics of the platen are equally as important as the characteristics of the motor forcer. In fact, the platen is more difficult to manufacture than a forcer. The platen must be at least as long as the motion of the system. Over its X and Y lengths, geometrical errors can be distributed in every way imaginable. Compensation for these errors may require detailed mapping of the errors over the entire platen area. A platen is produced either by photochemical etching or by stamping. A stamped platen can be made in almost any length, limited only by the size of the assembly tool. Stamped platens are, however, less accurate and have larger periodic components in their errors than do etched platens. Etched platens, which require master artwork, are limited in size by the ruling machines on which the artwork is generated, and the size of the etching machine. Splicing platen sections is difficult, due to the size and
tolerance requirements on platen teeth.

Obviously the tooth dimensions on a platen can be varied. Using a large tooth pitch makes the platen and forcer easier to fabricate by making the tolerances required somewhat larger. An effect of this, however, is that the motor loses force and stiffness characteristics as the pitch increases. Referencing again Fig. 2-6, an increased pole pitch would ‘stretch out’ the sinusoidal characteristic curve, and so, for the same external force applied to the motor forcer, the forcer would be displaced further from the stable equilibrium point. Making the pole pitch smaller does just the opposite, and produces a stiffer motor, but also makes the motor more sensitive to dimensional error. Also, as the periodic pitch decreases, the period of cyclic errors is reduced, as well as the amplitude of these errors.

2.7 Summary

From the above theoretical review, several key parameters of linear stepper motor performance are identified:

- Motor force production

  - mass
  - pole size
  - tooth pitch
  - air gap
  - hysteresis loss
  - saturation limit
  - permanent magnet strength
  - current into coils
  - max. coil operating temperature
  - max. flux density in air gap
• Separation of forcer and platen/Air bearing considerations
  
  – pressure/force
  
  – stiffness
  
  – air gap height

• Platen parameters
  
  – thickness and flatness
  
  – tooth pitch
  
  – eddy current and hysteresis loss
  
  – saturation limit
Chapter 3

Open Loop Linear Motor Modeling

3.1 Introduction

This chapter develops a model of an open loop, two phase linear stepping motor system. A block diagram of this prototype system is shown in Fig. 3-1. The motor elements to be characterized are a pair of (two-phase) motor forcers, an aerostatic bearing, and the controller and driver electronics. In addition, it is necessary to characterize the relevant system losses. To accomplish this, the chapter is organized as follows: section 2 describes the force generation in the motor. Section 3 describes the losses in the motor system, and section 4 examines the controller and drive electronics portions of the system. Section 5 examines air bearing considerations. Section 6 develops a fairly simple linear dynamic model, while section 7 considers a more advanced model using bond graph technique. Section 8 review thermal considerations for the motor.

3.2 Linear Stepping Motor Force Generation

In order to effectively design a linear motor system, it is first necessary to understand how a typical system behaves both statically and dynamically. A typical forcer mech-
Figure 3-1: Block Diagram of a Two-phase Open Loop Linear Stepping Motor System

The mechanism of a multi-phase linear motor consists of a permanent magnet which bridges two electromagnets. Each electromagnet has toothed pole faces. Full coil current excitation in an electromagnet commutes the flux entirely into one leg of the electromagnet and diminishes or cancels it at the other, as described in Chapter 2. The current required to completely commutate the permanent magnet flux to one pole face or the other represents the peak amplitude of the driving current waveform. The force generated by the two forcers of a two phase motor depends upon: the cross sectional area of the electromagnet poles, the air gap distance between the electromagnet pole tooth faces and the platen tooth faces, the strength of the bridging permanent magnets, and the amplitude and direction of the currents into the electromagnet coils.

In order to model a linear motor forcer assembly, an electromagnet pole with a given magnetic flux through it will first be considered. For the sake of simplicity, this pole will be considered to have a one-half tooth pitch width, corresponding to one half of a platen tooth pitch. It will be initially assumed that the permeability of the ferromagnetic electromagnet core material is large enough to ignore the magnetic...
reluctance of the material path. It will also be initially assumed that the leakage flux between the forcer assembly and platen is negligible. The flux flows through the pole, through the air gap, and into the platen, as shown in Fig. 3-2. In order to characterize a magnetic circuit, it is necessary to understand a few magnetic relationships. The driving force which tends to setup the magnetic flux $\phi$ in an electromagnetic circuit is the magneto-motive force $M$, or $mmf$. In the ideal case, $M$ and $\phi$ for a typical soft ferromagnetic material are related as shown in Fig. 3-3. There is a typical saturation phenomenon, where there is a diminishing amount of increase in $\phi$ as $M$ increases further. In the case of the linear motor, the saturation limit of the electromagnet cores is the limiting case for the amount of flux in the air gap between the motor and platen. If it is assumed that the $mmf$ in the air gap is linearly proportional to the flux in the air gap:

$$M_g = R g \phi_g,$$  \hspace{1cm} (3.1)
Figure 3-3: a) Magneto-motive Force versus Flux (b) Field Strength versus Flux Density [9]

where,

\[ M_g = \text{magneto-motive force (mmf) in the air gap [Amp-turns]} \]
\[ R_g = \text{air gap reluctance [1/H]} \]
\[ \phi_g = \text{air gap flux [Wb]} \]

The reluctance of the air gap, or the inverse of the permeance \( P \) (\( P \) is the 'relative ease' with which flux flows through a magnetic circuit element) is described by:

\[ R = \frac{1}{P} = \frac{l}{\mu A}. \]  \hspace{1cm} (3.2)

The \( l \) term is the length and the \( A \) term is the cross-sectional area of the magnetic circuit element being described, which in this case is the air gap. The \( \mu \) term is the permeability constant for a [magnetically] isotropic material. In this case, since
air is the material, the permeability constant is very close to $\mu_0$, the permeability of free space. The potential energy stored in the air gap is the integral of the above relationship, or using the relationship between the flux $\phi$ and the flux density $B$, can be described by:

$$U_{\text{gap}} = \frac{1}{2} \frac{V_g}{\mu_0} B_g^2 = \frac{1}{2} \frac{A_g z_0}{\mu_0} B_g^2$$

(3.3)

where:

- $U_{\text{gap}}$ = potential energy in the air gap [J]
- $B_g$ = air gap flux density [T]
- $V_g$ = air gap volume [$m^3$]
- $\mu_0$ = permeability of free space/vacuum [$4\pi \times 10^{-7} \, H/m$]
- $A_g$ = air gap area [$m^2$]
- $z_0$ = air gap height [$m$].

This assumption means that the flux in the air gap is the same as the flux in the electromagnet core. And:

$$V_g = A_g z_0 = r h z_0$$

where

- $r$ = pole/platen tooth engagement length [$m$]
- $h$ = electromagnet pole width (into the page in Fig. 3-2) [$m$].

The tooth engagement, $r$, can be thought of as a ‘local’ position variable over one electromagnet/platen tooth pitch, as opposed to $x$, the position variable over the
entire length of the platen. Substituting the above into the original equation:

\[ U_{\text{gap}} = \frac{1}{2} \frac{r \ h \ z_0}{\mu_0} B_g^2. \]  \hspace{1cm} (3.4)

### 3.2.1 Horizontal Force Generation

A horizontal force, \( F_x \), results when stored magnetic energy is transformed into mechanical work by a relative displacement between the electromagnet pole and platen teeth:

\[ Work = F_x \, dr \]

The force generated in the air gap can be approximated using the method of virtual work:

\[ \vec{F}_{\text{gap}} = -\nabla U_{\text{gap}}. \]  \hspace{1cm} (3.5)

A positive force works in the opposite sense of an increase in tooth engagement length, or, in other words, a horizontal force seeks to align the teeth to achieve a magnetic path of minimum reluctance. So:

\[ F_x = -\frac{\partial U_{\text{gap}}}{\partial r} = -\frac{B_g^2 \ h \ z_0}{2\mu_0} \]  \hspace{1cm} (3.6)

It is assumed that \( h \), \( z_0 \) and \( \mu_0 \) are constant, and the air gap flux density, \( B_g \), is a function of the tooth engagement, \( r \). It is assumed then, as stated before, that all of the magnetizing energy in the electromagnet core is also in the air gap between the core and platen teeth; and this flux density is the superposition (or the sum) of the flux due to the permanent magnet and the flux due to the current in the electromagnet coil.

To find the air gap flux density in terms of the permanent magnet flux and the current into the electromagnet coil, it is assumed that the magnetic energy in the air gap is equal to the flux density of the air gap multiplied by the air gap reluctance.

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Figure 3-4: Generic Permanent Magnet Demagnetization Curve

Utilizing equation 3.4, the magnetizing force in the air gap can be approximated by:

\[ H_g = \frac{B_g}{\mu_0}. \]  

(3.7)

where \( H_g \) is the magnetic field strength in the air gap in [A/m] and is defined as the magnitude of force required to produce a particular magnetic field.

Permanent Magnet Strength

When using a permanent magnet in a design, the primary design requirement is to establish a permanent magnet volume and configuration that most efficiently produces the required magnetic field. In order to make this selection, it is necessary to become familiar with the format of typical demagnetization (\( B_d \) vs. \( H_d \)) curves published by Permanent Magnet vendors [see Appendix A]. A demagnetization curve, as shown in Fig. 3-4, is actually the second quadrant portion of the characteristic, saturated condition magnetic hysteresis loop for a magnetic material. These curves include the energy product (\( B_d H_d \)) scale and the \( B_d/H_d \) ratio. There are a few points
concerning demagnetization curves which must be understood. First, for any particular material, the point at which the demagnetization curve meets the $B$-axis is the residual induction, $(B_r)$, which is the maximum flux density that can occur in a fully closed magnetic circuit (no air gap.) Any air gap in a magnetic circuit has a ‘self-demagnetizing’ effect that acts to reduce the flux density in the magnet. So the flux density in a permanent magnet $(B_{PM})$ can never equal $B_r$. Second, the point at which a demagnetization curve meets the $H$-axis is the coercive force $(H_c)$; or the demagnetizing force required to reduce the flux density in the permanent magnet to zero.

A common criteria for permanent magnet selection is to maximize the total energy in the circuit air gaps. Rearranging equation 3.3 using equation 3.1 yields:

$$U_{gap} = \frac{M_g \phi_g}{2} = \frac{R_g \phi_g^2}{2}. \quad (3.8)$$

The pole material reluctance is ignored, and the permanent flux is assumed equal to the air gap flux. Then the $mmf$ of the permanent magnet is the negative of the $mmf$ required to establish the air gap flux:

$$M_{PM} = -M_g, \quad (3.9)$$

and so:

$$\phi_{PM} = \phi_g, \quad (3.10)$$

Then if a uniform flux density $B$ and field strength $H$ are assumed, the $mmf$ and flux can be related the $BH$ operating point of the permanent magnet:

$$|M_g \phi_g| = |M_{PM} \phi_{PM}|$$
$$= (A_{PM} B)(l_{PM} H)$$
$$= (A_{PM} l_{PM})(B H), \quad (3.11)$$

where $l_{PM}$ is the length of the permanent magnet from pole to pole. The gap energy
in equation 3.8 will be maximized when the \( BH \) product (which corresponds to the permanent magnet operating point) is maximized. If \( B_{PM} \) and \( H_{PM} \) are defined as this operating point on the \( BH \) curve, then the following equations will apply, since permanent magnet and air gap fluxes are assumed equal:

\[
B_{PM} A_{PM} = B_g A_g, \quad (3.12)
\]

\[
H_{PM} l_{PM} = H_g l_g, \quad (3.13)
\]

\[
B_g = \mu_0 H_g. \quad (3.14)
\]

Eliminating \( B_g \) and \( H_g \) results in the equation:

\[
\frac{A_g l_{PM}}{A_{PM} l_g} = \frac{B_{PM}}{\mu_0 H_{PM}}. \quad (3.15)
\]

It is interesting to point out that the \( B \)-value in the air gap can be greater than that in the magnet by simply adjusting the cross-sectional area ratio.

To select a suitable permanent magnet material, a trial-and-error 'filtering approach' can be used. For a particular permanent magnet material, a trial magnetic circuit calculation can be made for the material in the proposed application, which will determine the suitability of that material. The selection is then reviewed, recalculated, and verified. Several iterations may be necessary.

As a general note on characterizing hard/permanent magnet materials: the energy product scale is developed by multiplying specific values of \( B_d \) by the corresponding value of \( H_d \) that occurs on the \( H \)-axis of the characteristic \( BH \) curve. With this energy product scale added to the demagnetization curve, the maximum energy product, \((B_dH_d)_{max}\), can be obtained by noting where the highest point (the 'knee') of the material characteristic curve falls in relation to the energy product scalar plot. Operation above or below the 'knee' is related to external demagnetization resistance. Operating above the knee infers higher resistance to external demagnetizing fields due to less change in \( B_{PM} \) as \( H_{external} \) is applied. If the demagnetization curve of a permanent magnet material is flat (with no knee), the self-demagnetization effect of a changing
air gap is minimized, since reducing the air gap will result in $B_{PM}$ approaching $B_r$. In this case also, the magnet should be operated as high up on the curve as possible. In the case of a linear stepping motor, both of these considerations are important, and a ‘best compromise’ material must be selected.

The traditional approach towards calculating the relative relationships between magnetic circuit flux paths is by means of ‘permeance’ calculations [12]. Permeance, $P$, with units of [Henry], is the inverse of reluctance in magnetic circuits, and is treated as analogous to conductance in electrical circuits. The permeance of an air gap can be approximated by:

$$P_g = \frac{\mu_0 A_g}{L_g}$$  \hspace{1cm} (3.16)

Equation 3.16 describes the permeance, or relative ease, with which magnetic flux will pass through a given material or space. Just about any static magnetic circuit can be reduced to a series of spatial configurations of component permeances, based on the three dimensional geometry. These component permeances can then be assembled into an analogous electrical circuit diagram, to which Kirchhoff’s electrical laws can be applied for a complete circuit analysis. An alternative approach, bond graph modeling will be considered shortly.

**Electromagnet Strength**

Ampere’s Law [7] says:

$$\oint B \cdot dl = \mu_0 i$$

This relationship is to be applied to an arbitrary hypothetical closed loop drawn in a region where, in general, magnetic fields and current distributions exist, as shown in Fig. 3-5. The quantity $i$ on the right of the above equation is the net current that pierces the closed loop. The quantity $B \cdot dl$ is a scalar product. The magnetic field set up by a solenoid is the vector sum of the fields set up by all of the turns that make up the solenoid. In the limiting case of adjacent square, tightly packed wires, the solenoid becomes essentially a cylindrical current sheet and the requirements of symmetry then suggest that within the solenoid, $B$ is parallel to the solenoid axis. In
Figure 3-5: Ampere's Law: Long Straight Wire Carrying Current with Circular 'Ampere’s Loop'

In fact, it can be shown that a very good approximation for $B$ inside a solenoid is given by:

$$B = \mu_0 i n$$

where $n$ is the number of turns per unit length [7].

Since the reluctance of the electromagnet core material is assumed negligible, then the value of $B$ is constant throughout the core material, as well as within the air gap between the core pole face and platen surface. Focusing on the flux density inside the electromagnet coil, $B_{EM}$, and using Ampere's Law yields $B_{EM}$ in the air gap to be:

$$\oint B_{EM} \cdot dl = \mu_0 i N$$

where:

$N = \text{effective number of coils}$

$i = \text{coil current, [Amps]}$
Solving the above integral for the air gap perimeter yields:

\[ B_{EM} z_0 = N i \mu_0 \]
\[ = \frac{N i \mu_0}{z_0} \]

(3.17)

where \( z_0 \) is the length of the air gap.

Finally, using the result just found, and substituting into equation 3.6 yields the equation for tangential force in terms of coil current and permanent magnet flux density:

\[ F_x = -\frac{h \cdot z_0}{2 \mu_0} \left( B_{PM} + \frac{N i \mu_0}{z_0} \right)^2 \]

(3.18)

Conspicuous in its absence from equation 3.18 is the variable \( r \), the tooth engagement or overlap. The amount of tooth overlap is very important to tangential force generation, since the tooth overlap determines the air gap reluctance. In actuality, equation 3.18 is a static force equation, for a fixed cross sectional area and a fixed air gap, and thus for a fixed reluctance. As will become obvious in future sections, the horizontal force production changes rapidly as the air gap height, \( z \) changes. It will be seen that this is directly proportional to the amount of change in the reluctance.

### 3.2.2 Attractive Force Generation

Once again, returning to equation 3.6:

\[ \vec{F}_{gap} = -\nabla U_{gap}, \]
\[ = -\nabla \left( \frac{B_g^2 A_g z}{2 \mu_0} \right) \]

the attractive force produced can be described by:

\[ F_x = -\frac{\partial U_{gap}}{\partial z} \]
\[ = -\frac{B_g^2 A_g}{2 \mu_0}. \]

(3.19)
This describes the total magnetic attractive force for a pole. Notice that this expression is for a fixed air gap length \( z_0 \) and thus only includes the attraction between the pole and platen tooth faces, and ignores the fringing flux lines emanating from tooth sides, between teeth, etc. This is because the gap between tooth faces are usually very small, around 0.003 \( mm \), while the gap between tooth valleys are on the order of 0.5 \( mm \); or 100 times more. The corresponding reluctance of these longer flux paths is much greater.

### 3.3 System Losses [5]

#### 3.3.1 Eddy Currents and Hysteresis

Every real system has losses and non-linearities, and of course, a linear stepping motor is no exception. Once the magnetic forcer assemblies have been designed and assembled, the performance losses will increase as operation velocities increase. As the motor forcer moves, the forcer magnetic field sweeps over the surface of the platen. Since the platen is by necessity made from a ferromagnetic conductive material, each platen tooth element is the seat of eddy current and hysteresis losses. The characteristics of eddy current and hysteresis losses are discussed in Chapter 2, and as noted, hysteresis losses will be assumed to have a negligible loss effect.

The eddy current platen losses translate into a drag force opposing the motor motion, and are directly proportional to the forcer velocity. The drag power per unit volume can be approximated as follows [5]:

\[
P_V = \frac{\pi^2 B_g^2 f^2 t^2 V}{6 \rho}
\]

where

\[
P_V = \text{power loss per unit volume [W/m}^3],
\]

\[
B_g = \text{peak air gap flux density, [T]}
\]

\[
f = \text{electromagnet coil current frequency, [Hz]}
\]
\[ t = \text{tooth thickness, [m]} \]
\[ V = \text{ferromagnetic material volume, [m}^3]\]
\[ \rho = \text{resistivity, [V m/A]}. \]

Then the drag power for the motor is the product of \( P_V \) and the volume. If this is so, then the drag force is found by dividing the power loss by the force velocity:

\[ F_d = \frac{P_V V}{\dot{x}}. \] (3.21)

The total motor tangential force available for acceleration at any given velocity is now given by:

\[ F_{\text{accel}} = F_{\text{z_total}} - F_d, \] (3.22)

where \( F_d \) is the drag force at that particular velocity, and \( F_{\text{z_total}} \) is the force production to an entire motor assembly (versus a single pole).

As has been previously noted, the air bearing in a linear motor system is virtually frictionless. This being the case, the eddy current loss factor, since it is proportional to velocity, is solely responsible for any damping in the system (for a normal sinusoidal current input). So, from the above equations, the motor damping factor, \( K_{ee} \) is given by:

\[ K_{ee} = \frac{F_d}{\dot{x}} = \frac{P_V V}{\dot{x}^2}. \] (3.23)

The significance of this damping factor will become clear in future sections.

### 3.3.2 Saturation

Saturation of the electromagnet core material is not so much a system loss as it is an upper limit on system performance. For linear systems, the energy and the co-energy are equivalent quantities as shown in part a of Fig. 3-6. For nonlinear systems, this is not the case (part b of Fig. 3-6.) This is also obvious in Fig. 3-3. The net result is that there are diminishing increases in force generation for increased current through the motor coils. It is the characteristic material properties
of magnetic circuit elements which determine how far into the saturation zone an assembly can be driven. Appendix A shows the characteristic $B - H$ curves for several ferromagnetic materials. There are various methods of modeling saturation, including look-up tables, or incorporating non-linear constitutive laws, etc. Exact methodology will be addressed in future sections.

### 3.3.3 Electromagnet Coil Inductance

A characteristic of motor windings is inductance, $L$, due to the length of the copper path. As a result of this inductance, the current in the motor windings does not rise instantly to its expected value of Volts/Ohms with typical voltage control techniques. Instead, it follows the formula:

$$ I(t) = \frac{V}{R(1 - e^{-R/Lt})} $$

(3.24)
where

\[ I(t) = \text{current in winding, [Amps]} \]
\[ V = \text{supply voltage, [Volts]} \]
\[ R = \text{winding resistance, [Ohms]} \]
\[ L = \text{winding inductance, [Henry]} \]
\[ t = \text{time, [sec]} \]

which starts out linearly and then asymptotically approaches the level expected from the winding resistance and the applied voltage values. At low step rates, the winding current has sufficient time to reach its full value, providing rated force. As the step rate increases, however, the winding current can only build to a fraction of its full force-producing value before it is switched off. As a result, motor force falls with increasing step frequency; until eventually, there is insufficient force to drive the motor and its load, and the motor stalls. In the case of the system being studied, it will be assumed that a high-voltage motor driver will be used, which effectively means that the driver acts as an ideal current source for the operational mode(s) being considered.

### 3.3.4 Cyclic Force and Velocity Variation

To understand force ripple, it is necessary to consider the force generation. Each microstep input to the forcer tends to displace the motor equilibrium position from the actual position, and the motor 'readjusts' its position to match that of equilibrium. This produces a force which tends to move the forcer in the direction the motor drive phase is moving. This force increment due to the application of a microstep can be approximated by:

\[ \Delta F = \frac{2\pi}{p} F_{\text{max}} S. \]  

(3.25)
Figure 3-7: Velocity Ripple due to Microstepping

where:

\[ \Delta F \] = increment of force corresponding to an input of one microstep,

\[ F_{max} = \] motor pullout force

\[ S = \] p/ no. of microsteps per full step.

Since the motor is oscillatory, due to the air bearing, the effect of a driving force increment is to produce steps in acceleration (due to the motor settling time) which eventually lead to variations in velocity, as shown on Fig. 3-7.

Using Fig. 3-7, the maximum value of the velocity ripple can be found by:

\[ V_{r_{max}} = \int_0^L \frac{F(t)}{m} \, dt \]

or substituting in the force variation due to microstepping:

\[ V_r = \frac{\Delta F \, t}{2m} \]
\[ \Delta F t = \frac{\Delta F t}{4m} \]

where \( t \) is the time duration of a microstep. The frequency of the velocity ripple is given by \( 1/t \).

### 3.4 Controller and Driver Electronics

To achieve controlled motion of a two phase linear motor, two sinusoidal currents (which are out of phase by 90 degrees) must be supplied to the two electromagnet coils of a forcer assembly. The frequency of the excitation current waves, \( f \), determines the commutation rate of the magnetic flux, and hence the velocity of the motor forcer:

\[ v = pf \]

where

\[ v = \text{velocity, \[m/sec\]} \]
\[ p = \text{tooth pitch, \[m\]} \]
\[ f = \text{excitation frequency, \[Hz\]} \]

Also, the motor must be accelerated to and decelerated from a desired velocity, and at such a rate so as not to exceed the maximum pullout force of the motor. The equipment required for this controlled motion is a controller and a driver (see Fig. 3-1.) The controller is the interface between the user and the motor system, while the driver electronics is actually part of the motor system, and can be assumed to be an ideal current supply for the motor electromagnet coils.

Use of microstepping improves the performance of standard step motors by reducing resonance, thus improving low speed motion and increasing the positioning resolution; without a sacrifice in top speed capability. Conventional bipolar motor drives alternate the current direction in one coil at every step - that is, the driving
current wave resembles a square wave as opposed to a sine wave. In microstepping, the coil current is changed in much smaller increments, and for a two phase motor, the current is increased in one coil as it is decreased in the other. Since stepping motors by definition, move in discreet increments, operation at low step rates (near the fundamental frequency of the motor) generates noise and vibration. Microstepping decreases the size of the increments, and increases their frequency for a given velocity. The result is significantly smoother low speed operation.

Utilization of current control of the motor allows higher speeds versus typical voltage control techniques. The challenge in driving stepper motors is get the current to rise as quickly as possible, thereby providing full coil current instantaneously. This provides the means to achieve to full force production, as previously described. Current control is achieved using a high driving voltage, which essentially overcomes the characteristic $L/R$ ratio of a particular motor coil. Obviously, the feasibility of this approach decreases as motor size (and cost) increases.

3.4.1 Controller Board

The controller board acts as a motion processor. The controller card used for this research project, the MoProIIPC built by Motion Science, Inc., communicates with a host PC through an I/O bus. Communication between the controller card and the host PC is via English-like commands, and takes place through Read/Write and status registers in the controller. The MoProII controller produces control signals to achieve three types of motion: point-to-point positioning (most commonly used); continuous motion, with constant or varying velocity; and location of a physical reference position (via limit switch signals.) Axis motions can be performed independently (of other axis movements) or jointly. These motion algorithms are typical of most commercially available controllers. For simplicity sake, motion in one independent axis will be considered for point-to-point positioning.

Point-to-point positioning allows movement of a specific distance from a current position. A move may have either a trapezoidal (default) or a parabolic velocity profile. For a typical move, the user must specify maximum velocity and acceleration, as
Figure 3-8: Trapezoidal Velocity Profile (With Corresponding Acceleration and Move Distance Profiles)

well as a move distance (in steps or inches.) The controller determines the appropriate motion profile, and hence determines the number and width of steps to perform the move. For a trapezoidal velocity profile, as shown in Fig. 3-8, the acceleration is constant, and:

$$ t_a = \frac{v_{\text{max}}}{a} \quad (3.26) $$

where

$$ v = \text{specified max. velocity, } [m/sec] $$

$$ a = \text{specified max. acceleration, } [m/sec^2] $$

$$ t_a = \text{acceleration/deceleration time } [sec]. $$

The distance the forcer travels in the acceleration and deceleration time periods, $t_a$, is:

$$ d_a = \frac{1}{2} a_{\text{max}} t_a^2, \quad (3.27) $$
with the same result for deceleration distance. Once the motor reaches the specified maximum velocity, the distance traveled during the coasting period (or period of zero acceleration,) \( t_c \), is given by:

\[
d_c = v_{\text{max}} t_c.
\]  

(3.28)

Thus for a given move, the total distance traveled is:

\[
d_{\text{total}} = \frac{1}{2} a_{\text{max}} t_a^2 + v_{\text{max}} t_c + \frac{1}{2} a_{\text{max}} t_c^2.
\]  

(3.29)

For a given commanded move, the motion processor determines the distance required to accelerate to peak velocity as well as decelerate to zero, plus the ‘cruise time,’ the time of motion required at peak velocity to achieve the total move distance. If, however, the commanded move is shorter than the distance required to accelerate and decelerate from the maximum velocity, the motion will have a triangular velocity profile, as shown in Fig. 3-9. For the first half of the move, the force will accelerate at the specified maximum rate; and decelerate at the same rate for
the second half. The specified maximum velocity will never be reached. The total distance traveled for a triangular velocity profile is:

$$d_{\Delta total} = \frac{1}{2} a_{max} t_{\Delta a}^2 + \frac{1}{2} a_{max} t_{\Delta a}^2.$$  \hspace{0.2cm} \text{(3.30)}$$

After determining the time periods required for a specified move, the controller must then convert this information to signals suitable for translation and conversion by the driver electronics. This is usually done using very short duration pulses, which are generated at the correct frequency.

3.4.2 Driver Electronics

The driver electronics must translate the incoming timing pulses from the controller, translate them to corresponding sinusoidal currents of appropriate frequencies, and deliver these current waves to the motor coils at the correct amplitude, as shown in Fig. 3-10.

3.5 Air Bearing

The magnetic attractive force between a linear motor forcer and platen must be counteracted, and for a two dimensional linear motor system, the most effective method is to use an air bearing. The only friction associated with air/aerostatic bearings is the shearing of the air film layer during motion of the bearing components.

A rectangular, flat pad aerostatic bearing is difficult to design efficiently because of the complex flow paths from the inlet orifices to the bearing outlet. Subsequently, it has been realized that simple approximate models or simple analytical solutions often times yield unsatisfactory solutions. However, to understand the behavior of a rectangular pad bearing, an estimate for the load capacity and stiffness can be found by multiplying the effective bearing area by the entry pressure of the gas as it enters the bearing clearance as shown in Fig. 3-11. The effective rectangular bearing pad area can be approximated as the area contained between the inlet orifices plus
Figure 3-10: Driver Current Output Corresponding to Controller Input Microstep Pulses (16 microsteps/pitch)

one-half of the area between the orifices and the edge of the bearing pad, or, using Fig. 3-11(a) [24]:

\[
A_{eff} = (l - 2a)(L - 2a) + \frac{1}{2}[l \cdot L - (l - 2a)(L - 2a)]
= (l - a)(L - a) + a^2.
\]

From which the effective force can be estimated:

\[
F_{pad} = P_e \cdot A_{eff}
= P_e \cdot [(l - a)(L - a) + a^2],
\]

where \(l\), \(L\), and \(a\) are as described in the figure. For a double row pad bearing, the entry pressure, \(P_e\), is approximated as one-half of the supply pressure into the bearing.

The stiffness for an opposed pad rectangular bearing can then be estimated by
Figure 3-11: Estimating Opposed Pad Rectangular Air Bearing Load and Stiffness.
b) Inherent Versus Pocket Compensated Restrictions.
dividing the effective force by one-half of the no-load bearing gap:

\[ K_{pad} = \frac{F_{pad}}{z_0/2} = \frac{2 P_e}{z_0} \cdot [(I - a)(L - a) + a^2] \]

where \( z_0 \) is the height of the bearing gap. Since the load is distributed over a relatively large bearing area, air bearings can typically support large loads, and with a very high stiffness across the air gap. For a linear motor with a properly designed air bearing, it is often times safe to assume that the bearing air gap distance is constant.

To gain a clearer understanding of why air bearings work, Fig. 3-11(b) shows gas at supply pressure \( P_o \) being admitted into the bearing clearance by a restricting device which reduces the gas pressure from \( P_o \) to \( P_d \) (the orifice downstream pressure). Some resistance must exist at the inlet orifice between the pressure source and the bearing pads in order to allow a pressure differential to form between opposed pads. The type of orifice design used in a bearing determines the bearing stiffness. Orifices, or restrictors as they are also called, made by drilling a hole through the surface of one bearing pad into an air supply reservoir are called ‘inherently compensated restrictions.’ Restrictors made with recessed pockets around the supply orifice are referred to as ‘pocketed restrictors.’ Pocketed restrictors provide a preliminary area for gas expansion before expanding further in the bearing gap. This preliminary expansion allows the gas to maintain more of its dynamic pressure than an inherently compensated restriction, thereby increasing the load capacity of the bearing. It has been shown [24] that bearings with pocketed orifices yield greater stiffness (up to 1.5 times stiffer) than those with inherently compensated orifices.

Returning again to the pocketed restrictor of part b., Fig. 3-11, the gas initially flows through the orifice restrictor with diameter \( d_o \) and area \( \pi d_o^2/4 \), and attains a pocket pressure, \( P_p \). At the edge of the pocket, the gas further expands through a secondary restrictor with a curtain area of \( \pi d_{Rz_0} \), where a ‘vena contracta’ occurs as the gaseous air enters the bearing air film in the bearing gap. Downstream of the pocket, the gas flows into the bearing clearance and recovers some of its dynamic
pressure as the air velocity reduces. Eventually, viscous losses prevail in the bearing clearance, until finally the gas pressure is further reduced to atmospheric pressure, $P_a$ as it exits the bearing gap. Changes in bearing gap clearance modify the resistance over the bearing lands, and affects the orifice downstream pressure, $P_d$, which in turn, affects load capacity. A smaller gap clearance leads to a higher $P_d$ and thus a higher load capacity. Increasing the film clearance has the opposite effect, that is, reducing bearing load capacity. A major air bearing design objective, therefore, is to choose the orifice dimensions in conjunction with film clearance to maximize stiffness.

3.6 System Dynamics

It will now be attempted to incorporate all of the material presented thus far into a useful and comprehensive dynamic model to describe the behavior of a simple linear motor. This will be accomplished by beginning with a simple linear spring-mass model, followed by a more sophisticated model using a bond graph development method.

3.6.1 Linear Mass-Spring System Characteristics

Ideally, when a single pulse is applied, a linear motor should instantaneously advance through one-quarter of a pitch distance and stop at the commanded detent position. The actual response for an open loop motor is not ideal. Recalling now the force vs. displacement curve for a linear stepper motor (Chapter 2) it is observed that at rest (i.e. currents are fixed, not cycling,) the forcer develops a restoring or holding force which opposes any attempt at displacement from equilibrium. As the resting motor is displaced, the restoring force increases, until the displacement reaches one quarter of a tooth pitch. Beyond this point, the point of peak holding or breakaway force, the restoring force drops and the motor slips or jumps and comes to rest at an integral number of tooth intervals away from the original location. If this occurs while the forcer is traveling along the platen, it is referred to as a loss of synchronicity.

For DC coil currents, the force versus displacement curve is a motionless wave
which extends over the length of the platen. When the currents begin cycling, the force versus displacement curve becomes a traveling curve which moves with the same velocity as the coil excitation frequency. Note that this curve travel is related to the winding currents, and not to the forcer position. To summarize, a horizontal force is exerted on the forcer only when the motor position does not coincide with a stable equilibrium point on the characteristic force-displacement curve for the motor system. So, in order to overcome any inertia or drag forces, the actual motor position must lag or lead the commanded position. (This lag or lead must generally be less than one-eighth of the tooth pitch for a reasonable open loop margin so as not to exceed the pullout force.)

If the platen is considered to have infinite mass compared to the motor forcer, then the platen can be considered as ‘ground.’ Then summing the forces on the motor for a commanded input position, \( c(t) \) and actual position \( x(t) \) yields:

\[
F_{\text{max}} \cdot \sin \left( \frac{2\pi (c(t) - x(t))}{p} \right) - K_{\text{ec}} \frac{dx}{dt} - m \frac{d^2x}{dt^2} = 0.
\]

(3.31)

where the second term approximates the eddy current/hysteresis drag forces (Section 3.3). This non-linear, second order differential equation would be easier to use as a linear model.

Towards this end, it is observed that a consequence of the force vs. position characteristic curve for a linear stepper motor is that the linear motor exhibits behavior similar to a spring-mass resonant system as shown in Fig. 3-12. If a mass hanging from a spring is displaced from its point of equilibrium, a ‘restoring force,’ opposite in direction to the motion of the mass will be generated by the spring. The magnitude of the restoring force will be proportional to the stiffness of the spring and to the displacement of the mass. The mass will oscillate back and forth at a frequency determined by the mass and the spring stiffness. When the stepping motor performs a single step, the nature of the response is also oscillatory, and can be thought of as a mass hanging from a ‘magnetic spring’ which generates a restoring force, also proportional to its ‘stiffness.’ The dynamic behavior resembles a mass-spring system.
In other words, the mass of the forcer, floating on a layer of air, and the magnetic 'springy' restoring force form an under-damped resonant system which rings in response to acceleration transients. These acceleration transients occur whenever there is a change in velocity at the beginning, end, or during a move. The time constant of these decaying oscillations is long enough to create an accumulation of resonant activity from one transient to the next and the displacements can become large enough for the forcer to stall or lose synchronicity. There are some simplifying assumptions which are implicit in this modeling technique. First, as stated before, the platen is assumed to have infinite mass. Second, the 'magnetic spring' is assumed to be have a linear spring constant which is equal across the face of the motor forcer, that is, at the face of each electromagnet pole of the forcer. Finally, the drag force is assumed to be a linear function of the velocity, with coefficient $K_{ec}$.

If the 'spring constant,' $K_m$, of the motor is considered linear with displacement
(which is approximately true for $\pm p/4$), then the above equation becomes:

$$K_m [c(t) - x(t)] - K_{ee} \frac{dx}{dt} - m \frac{d^2x}{dt} = 0. \quad (3.32)$$

Assuming zero initial conditions, and taking the Laplace transform of the above:

$$K_m [C(s) - X(s)] - sK_{ee}X(s) - s^2mX(s) = 0. \quad (3.33)$$

Putting this in standard form (output over input:)

$$\frac{X(s)}{C(s)} = \frac{K_m/m}{s^2 + sK_{ee}/m + K_m/m}, \quad (3.34)$$

which is the typical response for a mass/spring system with one degree-of-freedom:

$$\frac{X(s)}{C(s)} = \frac{\omega_n^2}{s^2 + s\zeta\omega_n + \omega_n^2},$$

Fig. 3-13 shows the development of a transfer function from $C(s)$ to $X(s)$.

The motor system damping is given by:

$$\zeta = \sqrt{\frac{K_{ee}^2}{4K_mm}}. \quad (3.35)$$

At certain speeds the cyclic force ripple of the motor will match the resonant frequency and cause an inordinately large velocity ripple. The resonant frequency is approximated by:

$$F_0 = \frac{1}{2\pi} \sqrt{\frac{K_m}{m}}, \quad (3.36)$$

where

$$K_m = \text{spring constant for the motor [N/m]}$$

$$m = \text{mass of forcer (and load.)}$$
3.6.2 Bond Graph Model

Bond graphs are an excellent technique to represent dynamic systems in a compact and intuitively understandable form. This technique utilizes a symbolic notation for energy storage and flow through a system model. The derivation of state equations is greatly simplified, and allows inclusion of elements from different energy domains such as electrical, magnetic and mechanical. For design purposes, bond graphs also provide significant insight into the effects of different physical parameters. For a description of how bond graphs are developed, the reader is referred to other works [9] for a comprehensive background.

Bond graph modeling allows incorporation of all of the previous motor system assumptions into a single, cohesive model. A suitable starting point is to consider the potential energy in the air gap. As previously discussed, the energy in the air gap between the motor fade and platen is a function of the magnetic flux, and the cross sectional area through which the flux lines travel. This magnetic energy is translated
into mechanical energy, thus causing the motor to move. This relationship can be described in bond graph technique using a multi-port C-field; that is, a three port capacitance. This C-field relates the mechanical (X or Y, and Z) effort (force) and flow (displacement) to the magnetic effort (mmf) and flow (flux rate). The electromagnets convert energy from the electrical domain to the magnetic domain, in proportion to the number of electromagnet coils. This implies that the electrical/magnetic domains can be modeled using gyrator elements with the number of coils, N, as the modulus. On the electrical side, the driver can be considered an ideal current source. The coils themselves have both resistance and inductance.

On the magnetic side, the flux in the forcer poles is of course due to the permanent magnets and the electromagnets. In the electrical analogy (permeance analysis method) for a permanent magnet, the permanent magnet is represented as an effort source, like a battery. However, no net power is actually provided by the permanent magnet. For modeling purposes it will be assumed that the permanent magnet operates with a constant reluctance. This is a valid assumption since the permanent magnets being considered (Somarium Cobalt and NdFeBo, see Appendix A) have fairly linear operating curves, and are extremely resistant to external demagnetizing fields. The system mechanical losses, as reviewed earlier, are essentially proportional to the velocity of the forcer mechanism. This translates to a resistive element in a bond graph.

It is obvious from an earlier section that an air bearing system is a fairly complex dynamic system. It was also shown that air bearings maintain a substantially stiff separation between motor and platen. The simplest way, then, to model an air bearing for its bandwidth of operation in a motor application would be as very stiff preloaded linear springs, similar to the premise of the previous linear spring-mass model.

The simple motor forcer itself will be considered as a lumped mass, m with 3 degrees-of-freedom: X or Y (depending upon the axes of tooth direction), Z, and rotation about the X or Y axes, or θ_{cg}, where cg refers to center of gravity. The poles will also be considered as being a fractional distance of L, the length of the motor forcer, from the center of gravity. These dimensions and axes are labeled in Fig. 3-14.
Figure 3-14: Dimensions and degrees-of-Freedom for Simple Linear Motor Bond Graph Model

The bond graph for the motor as shown in Fig. 3-14 is shown in Fig. 3-15 [1].

Several interesting things become obvious when studying the bond graph model. First, since the driver is assumed to be an ideal current source as opposed to a voltage source, the inductance of the coils do not have integral causality, and therefore are not significant. Rather, they are redundant, since the inductance in the system is due to the capacitance elements in the magnetic domain which are reflected through the gyrator elements. Also, the reluctance/capacitance of the permanent magnet does not have integral causality and therefore does not contribute a state variable. The air bearing 'spring constants,' $K_{AH}$, have integral causality, but the state variables, $z_i$, are the same flow variables which contribute to the air gap C-fields, so the effect is 'combined' into the momentum equations, as will be shown.

From the bond graph, the state variables for the simple linear motor system are $\phi_1$ and $\phi_4$, the magnetic fluxes in poles 1 and 4 respectively, $p_{\theta_{cg}}$, the angular momentum, and $p_x$ and $p_z$, the momentums in the horizontal and vertical directions.
Figure 3-15: Simple Linear Motor Bond Graph Model
When the bond graph is solved for the state variables using normal bond graph reduction technique, the system state equations are:

\[
\dot{\phi}_1 = \frac{d}{dt} \left( \frac{N_i A + N_i B + H_{CLPM} - R_1 \phi_1 - R_4 \phi_4}{R_{PM}} \right) + \frac{d}{dt} \left( \frac{N_i A - R_1 \phi_1}{R_2} \right) \quad (3.37)
\]

\[
\dot{\phi}_4 = \frac{d}{dt} \left( \frac{N_i A + N_i B + H_{CLPM} - R_1 \phi_1 - R_4 \phi_4}{R_{PM}} \right) + \frac{d}{dt} \left( \frac{N_i B - R_4 \phi_4}{R_3} \right) \quad (3.38)
\]

\[
\dot{p}_x = -\frac{1}{2} \left[ \phi_1^2 \frac{\partial R_1}{\partial x} + \phi_4^2 \frac{\partial R_4}{\partial x} + \left( \frac{N_i A - R_1 \phi_1}{R_2} \right)^2 \frac{\partial R_2}{\partial x} + \left( \frac{R_4 \phi_4 - N_i B}{R_3} \right)^2 \frac{\partial R_3}{\partial x} \right] - R_L \dot{x} \quad (3.39)
\]

\[
\dot{p}_{cg} = -\frac{1}{2} \left[ \phi_1^2 \frac{\partial R_1}{\partial z} + \phi_4^2 \frac{\partial R_4}{\partial z} + \left( \frac{N_i A - R_1 \phi_1}{R_2} \right)^2 \frac{\partial R_2}{\partial z} + \left( \frac{R_4 \phi_4 - N_i B}{R_3} \right)^2 \frac{\partial R_3}{\partial z} \right] + F_{AB} \quad (3.40)
\]

\[
\dot{\theta}_{cg} = \frac{-1}{2} \left[ \frac{L}{2^2} \left( \phi_1^2 \frac{\partial R_1}{\partial z} + \frac{F_{AB}}{4} + K_{AB}(z_1 - z_{cg}) \right) - \phi_4^2 \frac{\partial R_4}{\partial z} - \frac{F_{AB}}{4} + K_{AB}(z_4 - z_{cg}) \right] + \frac{L}{4} \left( \left( \frac{N_i A - R_1 \phi_1}{R_2} \right)^2 \frac{\partial R_2}{\partial z} + \frac{F_{AB}}{4} + K_{AB}(z_2 - z_{cg}) \right)
\]

\[
- \frac{L}{4} \left( \left( \frac{R_4 \phi_4 - N_i B}{R_3} \right)^2 \frac{\partial R_3}{\partial z} + \frac{F_{AB}}{4} + K_{AB}(z_3 - z_{cg}) \right) \quad (3.42)
\]

where \( R_i \) refers to the reluctance of the air gap at a particular pole. Some constitutive and kinematic relationships are related to these equations:

\[
\dot{x} = \frac{1}{m} p_x \quad (3.43)
\]

\[
\dot{z}_{cg} = \frac{1}{m} p_x \quad (3.44)
\]

\[
\dot{\theta}_{cg} = \frac{1}{J} p_{\theta_{cg}} \quad (3.45)
\]

\[
(3.46)
\]
and for small $\theta_{cg}$:

\[
\begin{align*}
    z_1 &= z_{cg} + \frac{L}{2} \theta_{cg} \\
    z_2 &= z_{cg} + \frac{L}{4} \theta_{cg} \\
    z_3 &= z_{cg} - \frac{L}{2} \theta_{cg} \\
    z_4 &= z_{cg} - \frac{L}{4} \theta_{cg}.
\end{align*}
\]

(3.47) (3.48) (3.49) (3.50)

It is probably obvious that the force expressions in the bond graph state equations do not exactly resemble the force equations derived in earlier sections. If equation 3.6 is revisited:

\[
F_x = -\frac{1}{2} \frac{B^2_g h z_0}{\mu_0}.
\]

The constitutive relationship between flux density and flux is:

\[
B = \frac{\phi}{A}
\]

or in this case:

\[
B_g = \frac{\phi_g}{h x}.
\]

Now, slightly modifying equation 3.6:

\[
F_x = -\frac{1}{2} \frac{B^2_g h z_0 (h x^2)}{\mu_0 (h x^2)} = -\frac{1}{2} \frac{\phi^2_g z_0}{\mu_0 (h x^2)}.
\]

The air gap reluctance was also previously defined:

\[
R = \frac{l}{\mu_0 A} = \frac{l}{\mu_0 h x}
\]

Then:

\[
\frac{\partial R}{\partial x} = -\frac{l}{\mu_0 h x^2},
\]
and the above force equation for a pole becomes:

$$F_x = -\frac{1}{2} \phi^2 \frac{\partial R}{\partial x},$$

which looks very familiar.

If the state equations for the simple motor are converted to a block diagram, suitable for analysis using MATLAB/SIMULINK, the result is as shown in Fig. 3-16. A significant benefit of using SIMULINK is the ability to incorporate saturation into the analysis.

A topic which requires further discussion is the reluctance function for the air gaps. Depending upon the alignment of the teeth of a motor pole with the platen teeth, the air gap reluctance will have a minimum and a maximum value. So, as the motor sweeps over the platen, the reluctance in the pole air gaps will vary as a function of position (tooth alignment), as shown in Fig. 3-17. The Fourier series expansion for a periodic triangular wave form is:

$$f(x) = \frac{1}{2} - \frac{4}{\pi^2} \sum_{n=1,3,5,...} \frac{1}{n^2} \cos \left(\frac{n\pi x}{L}\right)$$

where $L$ is one-half of a wave form period. In this case, for the reluctance function:

$$R_i(x, z) = \frac{1}{2}(R_{\text{max}} + R_{\text{min}}) - \frac{4(R_{\text{max}} - R_{\text{min}})}{\pi^2} \sum_{n=1,3,5,...} \frac{1}{n^2} \cos \frac{n\pi x}{p/2}$$

(3.51)

It is important to note that the values for $R_{\text{max}}$ and $R_{\text{min}}$ are both dependent upon $z$, the air gap height. Then the partial derivatives of $R(x, z)$ are:

$$\frac{\partial R}{\partial x} = \frac{4(R_{\text{max}} - R_{\text{min}})}{\pi(p/2)} \sum_{n=1,3,5,...} \frac{1}{n} \sin \frac{n\pi x}{p/2}$$

(3.52)

$$\frac{\partial R}{\partial z} = \left(\frac{1}{2} - \frac{4}{\pi^2} \sum_{n=1,3,5,...} \frac{1}{n^2} \cos \frac{n\pi x}{p/2}\right) \frac{\partial R_{\text{max}}}{\partial z} + \left(\frac{1}{2} + \frac{4}{\pi^2} \sum_{n=1,3,5,...} \frac{1}{n^2} \cos \frac{n\pi x}{p/2}\right) \frac{\partial R_{\text{min}}}{\partial z}$$

(3.53)
Figure 3.16: Simple Linear Motor Bond Graph Simulink Block Diagram
and its derivative with respect to time, using the chain rule, is:

\[
\frac{dR}{dt} = \frac{\partial R}{\partial x} \frac{dx}{dt} + \frac{\partial R}{\partial z} \frac{dz}{dt}.
\]  \hspace{1cm} (3.54)

3.7 Thermal Considerations

[10]

All systems must obey the fundamental law of thermodynamics; that is, that the amount of energy flowing into a system must equal the total energy out. The most common form of energy dissipation in an electro-mechanical system is in the form of heat. Thermal systems have traditionally been represented as electrical circuits, with temperature considered as voltage (effort) and heat flow as current (flow). There is a problem with this, however, when trying to convert to bond graph techniques. This is because the product of the relevant effort and flow variables, temperature and heat flow, is not power. Heat flow itself has the dimensions of power, and so,
the fundamental premise of bond graph modeling is violated. It is possible to use so-called pseudo bond graphs, but for purposes of this thesis, the traditional analysis methods will be used.

Whenever a temperature difference exists, heat flows from a region of higher heat concentration to a region of lower concentration. If heat is being generated by a source at some constant rate, the temperature of the source will rise until its temperature difference from the surrounding environment is sufficiently high such that some heat transfer mechanism will remove heat at the same rate as generation. There are three basic mechanisms for heat transfer: conduction, convection, and radiation (transfer of photons). That is, heat energy can be transferred by the coupling of motion from one mass to another, or the transmission of a photon from a particle to free space or to another particle.

In real systems, heat is usually transferred in steps through a number of different series connected sections, and frequently occurs by two mechanisms in parallel. For the present system under consideration, heat generation and transmission should be considered for at least two reasons. The first is that the insulation on the electromagnetic coils does have temperature limit, which if exceeded will cause the insulation to melt, thus shorting the coil. Also, increases in system temperature means thermal expansion, and thus increased geometric misalignment between motor and platen teeth. So it is obvious that thermal characteristics must be considered. For modeling purposes, it will be assumed that the source of heat generation in a linear motor is due to current flow through the resistances of the electromagnet coil windings. Further, it will be assumed that conduction and convection are the primary modes of heat dissipation. Any additional heat transfer due to a radiation mode can only improve the expected dissipation.

3.7.1 Conduction Heat Transfer

The only heat transfer mode in opaque solid media is via conduction. The rate at which heat is transferred by conduction, $q_k$, is proportional to the temperature gradient times the area through which heat is transferred. The actual rate of the
heat flow depends upon the thermal conductivity, $k$ [in $W/mK$], of the medium. The rate equation can thus be written:

$$q_k = -k A \frac{dT}{dx} \tag{3.55}$$

The minus sign satisfies the second law of thermodynamics which requires that heat must flow in the direction of lower temperature. The above equation is referred to as Fourier's law of heat conduction. For the purposes of linear motor modeling, conduction will be assumed the mode of heat transfer from the electromagnetic core to the exterior surfaces of the motor forcer.

**Convection Heat Transfer**

Convection heat transfer results from a fluid contacting a solid surface at a different temperature. There are two kinds of convection processes: natural (or free) convection, and forced convection. In free convection, the motive force for the fluid comes from density differences in the fluid. The density differences result from fluid contact with the surface at a different temperature, thus causing density differences in the fluid, which gives rise to buoyant forces in the gas. This effect could not take place in the absence of an acceleration field such as gravity.

Forced convection occurs when an outside motive force moves a fluid past a surface at a higher or lower temperature than the fluid. Since the fluid velocity in forced convection is larger than in free convection, more heat can be transferred at a given temperature difference; with a subsequent increase in the work required to move the fluid past the surface.

The rate of convective heat transfer, $q_c$ can be written in the form of Newton's cooling law:

$$q_c = \bar{h}_c A (T_s - T_{f,\infty}) \tag{3.56}$$

where

\[
\bar{h}_c = \text{average convection heat-transfer coefficient at fluid to solid interface, } [W/m^2K]
\]
\[ A = \text{surface area in contact with fluid, [m}^2] \]
\[ T_S = \text{surface temperature, [K]} \]
\[ T_{I,\infty} = \text{undisturbed fluid temperature at infinite distance from heat transfer surface, [K].} \]

Natural convective heat transfer will be assumed as the mechanism for heat removal from the exterior surfaces of the motor forcer to the surrounding ambient environment.

### 3.7.2 Heat Generation and Combined Heat Transfer Modes

To approach a problem involving internal heat generation, the appropriate form of the energy equation can be solved for the temperature distribution in the material:

\[ \nabla^2 T + \frac{q_G''}{k} = 0. \quad (3.57) \]

The generation per unit volume is denoted by the symbol \( q_G'' \). In this case, heat is generated by sinusoidal electric currents passing through resistances in copper wire coils. The solution to the gradient equation will result in two constants of integration that can be determined using two boundary conditions. Next, the Fourier law (equation 3.55) is used to determine the heat flux through the solid.

A heat transfer problem is considered transient whenever the temperature within the system being evaluated changes with respect to time. In general, the effort required to solve a transient problem is greater than that to solve a similar steady state problem. To determine the transient temperature distribution and ultimately the heat-transfer rate, it is necessary to include an energy storage term in the general conduction equation:

\[ \nabla^2 T + \frac{q_G''}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (3.58) \]

where

\[ q_G'' = \text{heat generation per unit volume, [W/m}^3] \]
\[ \alpha = \text{thermal diffusivity, [m}^2/sec] \]
\[ \frac{k}{\rho c} \]
\[ \rho = \text{density, } [kg/m^3] \]
\[ c = \text{specific heat capacity, } [J/kg K]. \]

A simplified approach is to consider the temperature in a solid body as varying with time but not position; that is, the temperature at all locations inside the solid varies uniformly with time. If it is further assumed that the energy transferred from the solid is removed by convection to a fluid, then the condition for a uniformly varying temperature within the solid would be satisfied when resistance to conduction is much less than the resistance to convection from the surface. Such a condition is referred to as a system with negligible internal resistance, or lumped capacitance. When a body has negligible internal resistance, the temperature gradients inside the body are much less than those occurring in the surrounding fluid. To determine if a body surrounded by a fluid has negligible internal resistance, the magnitudes of the two resistances must be evaluated and compared. This can be done using the Biot number, \( Bi \), a dimensionless group defined as the ratio of conductive resistance to convective resistance. Then, if:

\[ Bi = \frac{\bar{h}_c L}{k} \ll 1.0 \]  \hspace{1cm} (3.59)

is true, the condition for negligible internal resistance is met. The symbol \( L \) represents a characteristic length, and is frequently defined as the volume of a body divided by its surface area. If \( Bi \) is less than 0.10, the error in the temperature history given in the above equation can be shown to be less than 5 percent [10]. As the Biot number decreases further, the accuracy increases. Then, considering a solid of arbitrary shape, applying an energy balance indicates that the decrease in stored energy in the solid must be equal to the heat-transfer rate from the surface by the convective mode:

\[ -\rho V_c \frac{dT(t)}{dt} = \bar{h}_c A_s [T(t) - T_\infty]. \]
where:

\[ V = \text{solid volume,} \ [m^3] \]
\[ A_s = \text{solid surface area,} \ [m^2] \]

Assuming negligible internal resistance, \( T(t) \) will specify the temperature for all points within the solid, including the surface. The above equation can be simplified still further by defining a new dependent variable as:

\[ \theta(t) = T(t) - T_\infty \]

Then, to solve the previous equation, the temperature of the body must be specified at a particular time. If the temperature of the body is assumed to be \( T_0 \) at \( t = 0 \), then:

\[ \theta(0) = T_0 - T_\infty \]

and:

\[ \frac{\theta(t)}{\theta(0)} = e^{-(\frac{h_c A_s}{\rho V c})t} \]

which can actually be rewritten as:

\[ \frac{\theta(t)}{\theta(0)} = e^{-(Bi)(Fo)}. \]

\( Fo \) is the Fourier number:

\[ Fo = \frac{\alpha t}{L^2}. \]

Once the time history of the solid is known, the total heat transfer and instantaneous heat-transfer rates from the surface of the solid can be calculated by determining the amount of heat that leaves the surface. The instantaneous heat-transfer rate at a particular time \( t \) would be:

\[ q(t) = h_c A_s [T(t) - T_\infty]. \quad (3.60) \]

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Substituting the instantaneous temperature, the instantaneous heat transfer rate can be written in non-dimensional form as:

\[
\frac{q(t)}{h_c A_s [T_0 - T_\infty]} = e^{-(Bi)(Fo)}.
\]

The total amount of heat transferred from the solid between time \( t = 0 \) and time \( t \) can be determined by integrating the above equation between the time limits:

\[
Q(t) = \int_{t=0}^{t} q(t) dt = \frac{h_c A_s (T_0 - T_\infty)}{1} \int_{t=0}^{t} e^{-(Bi)(Fo)} dt.
\]

or, in dimensionless form:

\[
\frac{Q(t)}{h_c A_s [T_0 - T_\infty]} = \left[ 1 - e^{-(Bi)(Fo)} \right] \frac{1}{(Bi)(Fo)}.
\]

The instantaneous heat transfer rate \( q(t) \) is expressed in \( \text{watts} \), and the total heat transfer \( Q(t) \) is in \( \text{watt} \text{-sec} \) or \( \text{joules} \).

These thermal modeling techniques will be applied to linear stepping motors, in order to understand how heat energy is dissipated in the motor.
Chapter 4

Evaluation and Redesign of a ‘Stock’ Linear Motor

4.1 Introduction

This chapter applies the previously developed modeling techniques to characterize the largest, commercially available, two-phase linear stepping motor system manufactured by Northern Magnetics Inc. (or ‘Normag;’) model no. 4XY-...304-2-S05. The manufacturers specifications for this motor are presented first, followed by an analysis of the system performance.

In parallel with the Normag motor evaluation, a new motor will be designed, utilizing and improving on linear stepping motor technology. To begin, the motor design parameters are reviewed, as shown in Fig. 4-1.

4.2 Specifications for a Normag Motor

As shown in the Fig. 1.1 in Chapter 1, and in Appendix B, there are multiple magnetic forcer assemblies for each of the two axes of motion. In fact, for this particular Normag model, there are four such assemblies for each of the X and Y axes. A cross-section through a typical forcer assembly is as shown in Fig. 4-2.

The characteristics for the motor under consideration are given in the following
<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Defining Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Force</td>
<td>flux  \common{reluctance} \common{bearing stiffness} \common{mass}</td>
</tr>
<tr>
<td>Flux</td>
<td>reluctance \common{core material properties} \common{Permanent Magnet material properties} \common{Permanent Magnet cross sectional area} \common{core cross sectional area} \common{No. of electromagnet coil turns} \common{electromagnet peak current amplitude}</td>
</tr>
<tr>
<td>Reluctance</td>
<td>air gap height \common{pole/tooth engagement cross sectional area}</td>
</tr>
<tr>
<td>Drag Force</td>
<td>total pole area \common{velocity}</td>
</tr>
<tr>
<td>Velocity</td>
<td>excitation current frequency \common{pole/platen tooth pitch} \common{electromagnet coil inductance} \common{drag forces}</td>
</tr>
</tbody>
</table>

Figure 4-1: Trade-off Summary of Linear Motor Design Parameters

Figure 4-2: Cross Section View, Single Forcer Assembly, Normag Model No. 4XY-2504-2-S05
table, showing data published by Normag. Dimensioned drawings of the motor are given in Appendix B. It should be noted that the performance figures for the motor are for two axis operation, not single axis. This implies a $1/\sqrt{2}$ factor for single axis performance.

4.3 Force Production

The primary function of the magnetic forcer assembly/assemblies in a linear motor system is to produce a controllable, horizontal force capable of moving the motor forcer about the surface of the platen. To do this, magnetic flux must be directed or 'manifolded,' if you will, via ferromagnetic poles. The Normag motor forcer assemblies are very efficient for doing this. By 'ganging' several smaller forcer assemblies to produce force, the overall core volume (and therefore) mass, is reduced. For a two dimensional linear motor, the distribution of the several forcer assemblies about the center of mass of the motor is important. Bond graph modeling, which was reviewed in the previous chapter gives perhaps the most insight into the energy transformation and flow required to produce the desired mechanical force.

4.3.1 Normag Motor Bond Graph

Besides the number of permanent magnet/electromagnet assemblies per axis, there is another subtle difference between the actual configuration of the Normag motor and the configuration assumed for the simple motor bond graph of Chapter 3. The Normag motor actually has a permanent magnet bridging two electromagnet cores for each phase (see Fig. 4-2), whereas the simple motor of Chapter 3 only had one permanent magnet bridging the two electromagnet phases of the motor (Fig. 3-14). The effect is to expand the bond graph to look like Fig. 4-4. This bond graph includes only the horizontal force production. The vertical force production will be addressed shortly. The corresponding state equations for the pole fluxes are then:

$$\dot{\phi}_1 = \frac{d}{dt} \left( \frac{N_i A - R_1 \phi_1 + H_{c1} p_m - R_3 \phi_3}{R_{PM}} \right) + \frac{d}{dt} \left( \frac{N_i A - R_1 \phi_1}{R_2} \right) \quad (4.1)$$

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### Published Normag Motor Specifications:

<table>
<thead>
<tr>
<th></th>
<th>4XY2504-2-0</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Axes</strong></td>
<td>-</td>
</tr>
<tr>
<td><strong>Number of Sets/Axis</strong></td>
<td>-</td>
</tr>
<tr>
<td><strong>Number of Phases</strong></td>
<td>-</td>
</tr>
<tr>
<td><strong>Static Force</strong></td>
<td>-</td>
</tr>
<tr>
<td><strong>Force @ 40 in/sec</strong></td>
<td>lbs</td>
</tr>
<tr>
<td><strong>Resistance/Phase/Set</strong></td>
<td>ohms</td>
</tr>
<tr>
<td><strong>Inductance/Phase/Set</strong></td>
<td>mh</td>
</tr>
<tr>
<td><strong>Amps/Phase/Set</strong></td>
<td>-</td>
</tr>
<tr>
<td><strong>Maximum forcer temp</strong></td>
<td>C</td>
</tr>
<tr>
<td><strong>Weight</strong></td>
<td>-</td>
</tr>
<tr>
<td><strong>Repeatability</strong></td>
<td>-</td>
</tr>
<tr>
<td><strong>Resolution</strong></td>
<td>-</td>
</tr>
<tr>
<td><strong>Bearing Type</strong></td>
<td>-</td>
</tr>
<tr>
<td><strong>Airgap</strong></td>
<td>-</td>
</tr>
<tr>
<td><strong>Air Pressure</strong></td>
<td>-</td>
</tr>
<tr>
<td><strong>Airflow</strong></td>
<td>-</td>
</tr>
</tbody>
</table>

*dependent on drive electronics and system implementation

**four phase is available

### Platen Specification:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Length</strong></td>
<td>inch</td>
</tr>
<tr>
<td><strong>Width</strong></td>
<td>inch</td>
</tr>
<tr>
<td><strong>Tooth Pitch</strong></td>
<td>inch</td>
</tr>
</tbody>
</table>

Figure 4-3: Published Specifications, Normag Model No. 4XY-2504-2-S05
Figure 4-4: Normag Motor Horizontal Force Production Bond Graph - Single Forcer
\[ \dot{\phi}_3 = \frac{d}{dt} \left( \frac{Ni_A - R_1\phi_1 + H_e l_{PM} - R_3\phi_3}{R_{PM}} \right) - \frac{d}{dt} \left( \frac{Ni_A + R_3\phi_3}{R_4} \right) \quad (4.2) \]

\[ \dot{\phi}_6 = \frac{d}{dt} \left( \frac{Ni_B - R_6\phi_6 + H_e l_{PM} - R_6\phi_6}{R_{PM}} \right) - \frac{d}{dt} \left( \frac{Ni_B + R_6\phi_6}{R_5} \right) \quad (4.3) \]

\[ \dot{\phi}_8 = \frac{d}{dt} \left( \frac{Ni_B - R_8\phi_8 + H_e l_{PM} - R_6\phi_6}{R_{PM}} \right) - \frac{d}{dt} \left( \frac{Ni_B - R_8\phi_8}{R_7} \right) \quad (4.4) \]

\[ \dot{p}_z = \frac{1}{2} \sum_{i=1}^{8} \phi_i \frac{\partial R_i}{\partial x} - R_L \dot{x} \quad (4.5) \]

where \( i \) corresponds to the appropriate pole number. The corresponding constitutive relationships are:

\[ \dot{x} = \frac{1}{m} p_z \quad (4.6) \]

\[ \phi_2 = \frac{1}{R_2} (Ni_A - R_1\phi_1) \quad (4.7) \]

\[ \phi_4 = \frac{1}{R_4} (Ni_A + R_3\phi_3) \quad (4.8) \]

\[ \phi_5 = \frac{1}{R_5} (Ni_B + R_6\phi_6) \quad (4.9) \]

\[ \phi_7 = \frac{1}{R_7} (Ni_B - R_8\phi_8). \quad (4.10) \]

The above equations are for a single force assembly, when in fact there are four such force assemblies per axis. The force assemblies are arranged symmetrically about the force \( cg \), and their force production can be summed for modeling purposes. It is possible, then to concentrate on the force produced by a single pole of an electromagnet core.

**Reluctance**

From the previous chapter, the reluctances in the air gap can be estimated as a function of the air gap height, the cross sectional area of the overlapping pole/tooth faces, and the permeability constant of air, \( \mu_0 \) as described in Chapter 3. \( R_{max} \) and \( R_{min} \) correspond to complete alignment of pole teeth with platen teeth, and complete misalignment of pole teeth with platen teeth, respectively. The maximum
and minimum air gaps are:

\[ z_{\text{max}} = 0.0408 \text{ in} = 1.03632 \text{ mm} \]
\[ z_{\text{min}} = 0.0008 \text{ in} = 0.02032 \text{ mm}. \]

So, treating the individual pole tooth and valley face reluctances as conductances in parallel yields:

\[ R_{\text{max}} = 679,000 \frac{A}{V_{\text{sec}}} \]
\[ R_{\text{min}} = 232,000 \frac{A}{V_{\text{sec}}}. \]

These values are for a single pole of a forcer assembly. It should be noted that the actual pole tooth width on the Normag motor is slightly larger than the platen tooth width, 0.025 versus 0.020. This 'bit' of overlap has a significant effect on \( R_{\text{max}} \): 679,000 \( \frac{A}{V_{\text{sec}}} \) versus 2,078,000 \( \frac{A}{V_{\text{sec}}} \) if the teeth were of the same width.

The reluctance functions for the four poles of a single phase of a forcer assembly are approximated thus:

\[ R_1 = R_3 = 455,500 - 181,162 \cos(1,969\pi x) \quad (4.11) \]
\[ R_2 = R_4 = 455,500 + 181,162 \cos(1,969\pi x), \quad (4.12) \]

to first order expansion. The value of expanding \( R \) beyond first order drops off rapidly, as shown by the Fourier series approximation for reluctance in Chapter 3. Continuing, the partial derivatives of the reluctance are:

\[ \frac{\partial R_1}{\partial x} = 3.56 \times 10^5 \sin(1,969\pi x) \quad (4.13) \]
\[ \frac{\partial R_2}{\partial x} = -3.56 \times 10^5 \sin(1,969\pi x). \quad (4.14) \]

And, using the chain rule:

\[ \frac{dR_i}{dt} = \frac{\partial R_i}{\partial x} \frac{dx}{dt}. \quad (4.15) \]
Permanent Magnet Properties

For an NeFeBo-27 permanent magnet material [Appendix A], the properties and characteristics which are relevant to the Normag motor are:

\[ H_c = 763,940 \, A/m \]
\[ B_r = 1.08 \, T \]
\[ l_{PM} = 0.075 \, \text{in} = 1.905 \, \text{mm} \]
\[ A_{PM} = 0.375 \, \text{in}^2 = 2.42 \times 10^{-4} \, \text{m}^2 \]
\[ R_{PM} = \frac{H_c l_{PM}}{B_r A_{PM}} = 5.568 \times 10^6 \, A/Wb. \]

If the above parameters are substituted into the original state equation for \( \phi_1 \), and a constant peak current of 2 Amps is assumed, and the motor forcer is constrained from horizontal motion with an assumed misalignment of pole/platen teeth of \( p/4 \), then the peak flux through pole 1 will be:

\[ \phi_{1_{max}} = 8.5 \times 10^{-3} \, Wb. \]

The corresponding predicted peak horizontal force production for the pole is:

\[ F_{1_{max}} = 12.9 \, N \, (2.9 \, lbs). \]

The predicted peak motor force would be the sum of the 8 ‘in phase’ poles for the axial direction, or:

\[ F_{max} = 103.2 \, N \, (23.2 \, lbs). \]

The electromagnet core material is assumed to be silicon steel, the \( BH \) curve for which is shown in Appendix A. Using the approximation:

\[ H = \frac{M}{l}, \]
where $l$ is the path length through the core, the magnetizing force in the core is approximately 263,000 $\text{A/m}$ or 3300 $\text{Oersted}$. Per Appendix A, the material is operating well into the saturation region, at its highest flux (per unit area) carrying capacity.

Certain conclusions can be drawn from these calculations, and Fig. 4-1: first, the primary limiting factor to increasing force production per unit of cross sectional area of core material is the flux saturation limit of the material itself. Therefore, the next most obvious way to increase the horizontal force capacity of a motor system is to decrease the system mass. Therefore, any motor redesign should carefully address optimization of structure and careful component selection (this also relates directly to system cost.) Secondly, since the flux carrying capacity of an electromagnet core cannot be increased beyond its characteristic saturation level, to carry more flux requires an increase in the core cross sectional area, which implies increased core volume and mass.

4.3.2 Modular Structure

The previously discussed Normag motor utilizes a multi-functional, machined aluminum housing. This housing functions structurally; as an air bearing manifold; and as a heat sink and heat path for heat transmission from the electromagnet core to the ambient environment.

The basic structural/chassis component for a new motor could be limited in function to a structural role. Essentially, all motor subassemblies/components would be attached to this basic component. It is proposed that this component be a simple flat plate, as shown in Fig. 4-5.

This plate would have four identical and symmetrical machined patterns for mounting of magnetic forcer assemblies, sensors, and air bearing components. It is further proposed that these other subassemblies and components be attached using screw-type hardware. This lends itself more easily to assembly/disassembly, servicing, component repair/replacement, and general system adjustments. Also, since the role of the motor plate is structural only, the material selected does not necessarily
Figure 4-f: Motor Plate

have to be metallic, but could instead be a more weight efficient composite materials.

An increased force motor could be accomplished using a similar configuration to the Normag motor, utilizing longer electromagnet cores for increased flux carrying capability. These electromagnets would also have more coil turns, and would be actively cooled, as described in a future section.

4.4 Air Bearing Characterization

The air bearing of the Normag motor is configured as shown in Fig. 4-6. The air supply is divided into four separate 'branches'. Each of the four branches is then bisected into two more branches, one branch supplying two orifices, and the other supplying one orifice. Although it is beyond the scope of this thesis to accurately model and analyze the complete air-bearing design, it is possible to estimate the air bearing performance characteristics. Because of the distribution of the orifices, it will be assumed that the air bearing pressure distribution is uniform over the face of the
motor forcer. That is, flow path losses are ignored and it is assumed that each bearing orifice has equal effort and flow values. If this is the case, the estimation guidelines from Chapter 3 can be used to estimate the total air bearing force and stiffness:

\[ P_0 = \frac{80}{2} \text{ psi} \]
\[ A_{eff} = 49 \text{ in}^2. \]

Then:

\[ F_{AB} = 1960 \text{ lb} \]
\[ K_{AB} = 4.0 \times 10^6 \text{ lb/in}. \]

This high stiffness approximation is not that unreasonable [24], since air bearings are known for having very high stiffness with very small air gaps. Thus, for a constant air bearing supply pressure, and uniform bearing surfaces geometry, it can be assumed that the air gap between the forcer and platen is essentially constant.
Figure 4-7: Air Bearing Configuration in Linear Motor

This assumption greatly simplifies the system bond graph: as shown in Fig. 4-4. The change in air gap height no longer affects the horizontal force generation, since it is now assumed that there is no change in the air gap.

4.4.1 Air Bearing Implementation, New Motor

Instead of using the motor housing as an air bearing manifold as in the Normag motor, it would be simpler to use a pneumatic manifold which distributes air to every bearing orifice individually, as shown in Fig. 4-7. This has a threefold benefit:

- simplifies motor structure,
- each orifice is delivered nearly equal air pressure and mass flow,
- air bearing losses are reduced due to simplified distribution.

As stated before, air bearing design is a complex process, and considered beyond the scope of this thesis. However, a simplified ‘cook-book’ methodology for a double-row, opposed pad air bearing [24] is presented in Appendix E. The design is based
on use of pocketed, inherently compensated orifices, using three such orifices in each row, or six total. It will also be assumed that the air gap separation between motor and platen should be the same as that for the Normag motor (0.0008 in).

4.4.2 Electrical and Structural Connections

In the Normag motor, each of the motor quadrants consists of two magnetic forcer assemblies, with the electromagnetic coils for each phase connected in parallel, as opposed to series connection. This parallel connection lends itself to better high speed performance, due to lower inductance (versus series connection). As previously mentioned, the inductance of a coil is related to the rise time of a current into a coil.

For the new motor, the aluminum cooling fin portion of the forcer assembly (Fig. 4-10) could also serve as the structural tie to the motor plate. As shown in Fig. 4-8, long, T-shaped 'rails' could run the length of each side of the forcer assembly. These rails are attached to the motor plate via flat-head screws, and provide stiffening for the motor plate, vertical constraint for the forcer assembly, and a horizontal 'squeezing' force on the electromagnetic cores due to the chamfered edge interface.

4.5 Thermal Effects

The primary thermal concern to motor operation is the magnet wire insulation. For typical magnet wire, the rated temperature limit is 130 $^\circ$C. If this temperature is exceeded, the insulation will melt, causing coil shorting and motor failure. Temperatures which would affect the strength of the permanent magnet are significantly higher than the coil wire insulation limit, and so are not considered relevant.

The motor will be considered as a lumped mass. Of the total mass of the motor, 1.8 kg, or approximately 45 percent is the aluminum structure. Using the density of aluminum, this implies an aluminum volume of $2.95 \times 10^{-3}$ m$^3$. The surface area of the motor exposed to free convection includes the four shorter sides and one of the large faces, for a total of 0.050 m$^2$. The equivalent length of the motor is then, from Chapter 3, $V/A = .0059$ m. For free convection in air, the value for $\bar{h}_c$ ranges from 5
Figure 4-8: One Quadrant Views of Installed Magnetic Forcer Assemblies (Top and End Views)
to 25 $W/m^2K$, and the value of $k$ for aluminum ranges from 171 to 216 $W/mK$. The Biot number for the motor (equation 3.59) is $1.5 \times 10^{-4}$. Since this is much less than 1, then the system does indeed satisfy the condition for negligible internal resistance, and can be modeled as a lumped parameter.

Applying an energy balance to the motor (assuming the motor meets the requirement for negligible internal resistance) yields:

$$q''_G = q_k + q_e + \rho V c \frac{dT(t)}{dt}.$$

But $q_k = 0$, and expanding the expression, it can be written:

$$(\bar{h}cA_S)T(t) + (\rho c) \frac{dT(t)}{dt} - (\bar{h}cA_S)T_{f,\infty} = I^2 R.$$

For a constant current input, the motor temperature would have a first order system response, as demonstrated in [3].

A secondary effect of motor heat generation is dependent on the path of heat dissipation. The result can be thermal growth of motor teeth, both on the platen and the forcer, with obvious resultant geometric errors. This is also discussed in [3], but warrants some further discussion here. The thermal dissipation paths for a Normag forcer assembly are shown in Fig. 4-9, and includes the corresponding thermal circuit diagram. The heat is generated in the copper coils, and travels outward through the steel core. The heat then is transferred to the ambient air via the forced convection due to the air bearing, or is transferred into the aluminum housing, and then to the ambient air via free convection from the housing surface. As can be seen, the path of least resistance for heat transfer is actually via the core, into the aluminum housing, and then into the ambient environment via the free convection mechanism. The conclusions that can be drawn from this thermal analysis is that the method of thermal dissipation in the Normag motor performs well. The heat is transported to the ambient atmosphere via the aluminum structure, and is not stored in the coil windings (the insulation will not melt), and does not use the electromagnet cores as primary heat paths (the pole teeth will not ‘grow’).
Figure 4-9: a. Normag Motor Heat Dissipation Path b. Equivalent Heat Dissipation Circuit
4.5.1 Active Thermal Dissipation

Since the primary heat generation in the motor is due to the electromagnet, it seems to make sense to remove the heat from the source as directly as possible. An idea for the new motor design is to cool each magnetic forcer assembly by attaching a heat sink to each forcer assembly, as shown in Fig. 4-10.

In addition, an extra pneumatic line could be attached to the forcer via the umbilical. The sole purpose of this air line would be to provide forced air convection cooling of the forcer fins. The net effect would be to increase the combined heat transfer coefficient, $h_c$. The air would be distributed in all directions from the center of the motor and travel over the cooling fins, as shown in Fig. 4-11.

The sizing of cooling fins for such an application is a fairly straightforward process, and is presented in Appendix D, while the resulting fin design is shown in Appendix F.
4.6 Normag Motor Characteristic Parameters

This section develops the parameters required to build a second order, linear mass-spring model of the Normag motor, in order to easily predict the dynamic system behavior.

4.6.1 System Loss Characteristics

The eddy current drag force information from Chapter 3 can be directly applied to the Normag motor. The first step is to determine the eddy current power loss per unit volume:

\[ P_V = \frac{\pi^2 B_g^2 f^2 t^2 V}{6 \rho} \]

where

\[ P_V = 34,565 \, [W/m^3], \]
\[ B_g = 4.3 \, [T] \]
\[ f = 1000 \, [Hz] \]
\[ t = 5 \times 10^{-4} \, [m] \]
\[ V = 1.35 \times 10^{-3} \, [m^3] \]
\[ \rho = 1.8 \times 10^{-6} \, [V \, m/A]. \]

If this value for \( P_V \) is multiplied by the volume \( (1.35 \times 10^{-3} \, [m^3]) \), and divided by the square of the velocity corresponding to the 1000 \( Hz \) frequency \( (1 \, m^2/\text{sec}^2) \), the value for the loss factor is calculated to be:

\[
K_{ec} = 46.7 \, N/m/sec
\]
\[
= 0.27 \, lb/in/sec.
\]

The Normag motor was tested under various loading conditions in order to determine the maximum achievable acceleration to reach various motor velocities.

The motor was tested with loads of 3 \( lbs \), 5 \( lbs \) and 7 \( lbs \). For each of these loads, the motor was accelerated (from zero velocity) to 5, 10, 20, 30, 40 and 50 \( m/sec \) in one axis, following a trapezoidal (constant acceleration) velocity profile. For each trial at a particular load and test velocity, the acceleration was increased until the motor failed to achieve the test speed. Failure was defined as the point where the motor stalled and failed to achieve the commanded velocity. The results of this testing were as shown in Fig. 4-12. The data points were then analyzed using a MATLAB curve-fit file (Appendix C) for the best fit line: The best fit line is as shown in Fig. 4-13. The slope of this line yields the actual motor loss coefficient. This loss coefficient was found to be \(-0.26 \, lb/in/sec\). This is approximately equal to the expected axial loss coefficient from the motor specification (using manufacturer specified average values):

\[
K_{ec} = \left( \frac{19.5 - 28}{40} \right) \, \frac{[lb]}{[in/sec]}
\]
\[
= -0.21 \, lb/in/sec.
\]
Figure 4-12: Data Results From Normag Motor Maximum Acceleration Testing

Figure 4-13: 'Best Fit' Line for Data Points From Motor Maximum Acceleration Testing
These values also agree well with the predicted drag coefficient.

### 4.6.2 Controller and Driver Simulation

The controller board and driver electronics modeling are based on the MoPro IIPC and MS Series Micro Step Motor Drive, respectively; both from Motion Science Inc., San Jose, CA. (See Appendix C)

After determining the time periods required for a specified move, the controller must then convert this information to signals suitable for translation and conversion by the driver electronics. This is done using 470 nanosecond duration pulses, which are generated at the correct frequency. The Motion Science Inc. MS series drivers utilize MOSFET power stages operating in a pulse width modulating configuration. An MS series driver produces from 4 to 125 microsteps per full motor step (one tooth pitch.) A functional block diagram of a typical driver board is shown in Fig. 4-14. The ‘Step’, ‘Direction’, ‘Shutdown In’ and ‘Low Power In’ are the inputs to the driver from the controller. A Step pulse will advance the motor one microstep. The Direction
input logic level (high or low) determines the motor direction. Shutdown In is used to disable power to the motor, and Low Power In is used to reduce current amplitude to the motor. The step and direction inputs are the minimum control signals required for driver operation. Stepping will occur on the falling edge of a step input. Step and direction timing are as shown in Fig. 4-15.

Motor current is set using a Current Level Potentiometer and a current level test point. The voltage level at the current level test point is scaled to 1 volt per motor ampere. Step resolution is set in terms of microsteps per full step and is set with 4 DIP switches.

For the purposes of this thesis, the controller card and driver electronics are simulated using an m-file for input to the MATLAB/SIMULINK environment. The m-file uses specified values for the maximum velocity and acceleration, and the number of microsteps/step. The input to the model is the desired move distance. To determine a move profile, the model first determines whether the move distance is far enough to accelerate to maximum velocity using a trapezoidal velocity profile. If not, a trian-
regular profile is used. The outputs of the model are a monotonically increasing time vector, \( T \), with a corresponding move distance vector, \( U \). The resultant model output vectors are the inputs required for the mass-spring dynamic model. A listing of the controller/driver model is shown in Appendix C.

4.6.3 Normag Linear Spring-Mass Dynamic Model

There are three primary pieces of information required to apply the previously developed linear spring-mass model to the Normag motor: the motor ‘spring constant,’ \( K_m \), the motor loss coefficient, \( K_{ec} \), and the motor mass. The values for these are:

\[
K_m = 3455 \, \text{lbs/in or 605,033 N/m},
\]
\[
K_{ec} = 0.26 \, \text{lbs/in/sec or 45.5 N/m/sec},
\]
\[
m = 4.5 \, \text{lbs or 2.05 kg}.
\]

The motor ‘spring constant,’ \( K_m \), was found by plotting the characteristic motor force curve, and then approximating the slope of the curve about a stable equilibrium point as shown in Fig. 4-16. It is then a simple matter using MATLAB/SIMULINK to generate system simulations, such as the motor step response of Fig. 4-17. The step input can be considered as a current step or microstep from the driver. As expected from modeling assumptions, the motor has a highly underdamped response, with a damping coefficient and natural frequency of:

\[
\zeta = .02
\]
\[
\omega_n = 543.2 \, \text{rad/sec}.
\]

The corresponding frequency response of the motor per this model output is as shown in Fig. 4-18.

If these results are compared to the previous results, and from previous analysis [1], it is seen that the model outputs are display similar motor behavior. Since this is the case, the linear mass-spring motor model will be used for subsequent dynamic
Figure 4-16: Normag Motor Characteristic Force Curve

Figure 4-17: Normag Motor Linear Spring-Mass Model Step Response - Single Axis
response modeling, since this is an easier model to implement, both in terms of computer implementation, and in terms of easier interpretation of results for design purposes.

4.7 Redesign Summary

A repeatable, producible, high-performance linear motor system should be designed with a modular approach - that is, assembled from lower subassemblies which are assembled to an overall system structure or chassis. Such an approach will facilitate system fabrication and assembly, as well as more cost effective and flexible perhaps, than currently available motor systems. A high performance motor will require a weight efficient motor structure/housing, larger, actively (forced air) cooled magnetic forcer assemblies, a redesigned air bearing configuration, and potential for incorporation of feedback sensors for closed loop control.
Chapter 5

Linear Motor Application

5.1 Introduction to Proposed Application

It has been proposed that a linear motor system be incorporated into a high speed collating and packaging system. The requirements for such a system are briefly defined as follows:

1. The system should be able to move a payload (or payloads) of variable mass (0 to 2 kg) from an initial position to a new position up to 250 mm away.
2. The system should have the capability of moving 1000 payloads/minute.
3. The system should have the capacity to select different payload source locations from the supply system, and be able to deliver the payloads to any location within a specified area, within an accuracy zone of ±2 mm.
4. The system must operate continuously [reliably] in a typical industrial environment with minimal operator intervention.

The key requirement of this system which points to use of a linear motor system is the need for flexibility: flexibility in potential pickup and drop off points, and in potential load requirements. This chapter will address moving a payload a distance of 250 mm reliably and within specified time restrictions. An appropriate end effector design will be implied in this analysis, but not specified.

This chapter examines applying open and closed loop linear motors by first reviewing, current open and closed loop linear motor techniques, and then applying
control techniques to the Normag and newly designed motor systems which were characterized in the previous chapter.

5.2 Application Duty Cycle Description

It will be initially assumed that one payload is being moved at a time, with a quantity of 1000 payloads being moved per minute. If each payload is moved sequentially; that is, one forcer moves 1000 payloads per minute, one at a time; the required operating frequency is 16.7 cycles per second, or .060 sec per payload. In actuality, the time required to move each payload is about half of this time, or .03 sec, since if each payload is being moved from point A to point B, the motor forcer must carry each payload and return to point A in order to obtain another payload. Hence one cycle is defined to be the movement of the forcer from point A to point B, 250 mm away, with a payload, and then from point B back to point A, without a payload. As stated, the means by which a payload is picked up and dropped off, and held while being transported, will not be addressed in this thesis. Also, it will be assumed that any load/grip/unload mechanization does not affect the dynamic motion of the forcer.

In order to apply linear motor technology to the desired material handling application, it is first necessary to develop a duty cycle description of the application.

1) Payload Parameters:

- Mass of Load: 2 kg (maximum)
- Mass of drive-train components (forcer): \( m_{dt} \)
- \( \text{Mass}_{\text{Total}}: 2 \text{ kg} + m_{dt} \)

2) Move Profile Parameters:

There must be some estimate of the velocity profile as a function of time in order to understand system operation. In this particular application, it is desired to move a specified load \( m_{DT} \) a specified distance and within a specified time.

- Move Time: 1.060 seconds (total)
Initially, a trapezoidal profile will be assumed. A triangular profile could be assumed, which would result in a lower maximum acceleration requirement, but would also require a higher maximum velocity. Since it has been shown that motor losses for a linear motor system are directly proportional to velocity, it seems to make more sense to try to reduce the maximum velocity required.

The total cycle time is .060 sec, and as stated before, it will initially be assumed that the actual time for the motor to move the payload is one-half of the total cycle time, or .030 sec. A full cycle for the linear motor consists of carrying a payload from point A to point B (a distance of 250 mm) and then returning to point A in order to pick up the next payload.

The trapezoidal velocity profile will look like Fig. 3-8 of Chapter 3.

The cycle parameters can be found using the equations:

\[
d_{total} = \frac{a_{max}}{2} \left( \frac{t_t}{3} \right)^2 + a_{max} \left( \frac{t_t}{3} \right) \left( \frac{2t_t}{3} \right) - \frac{a_{max}}{2} \left( \frac{t_t}{3} \right)^2
\]

where:

\[
t_t = t_a + t_c + t_d
\]

\[
= .030 \text{ sec.}
\]

Solving for \( a_{max} \):

\[
a_{max} = \frac{9d_{total}}{2t_t^2}
\]

and then:

\[
v_{max} = \frac{3d_{total}}{2t_t}
\]

Using the known values for \( t_t \) (0.03 sec) and \( d_{total} \) (250 mm), the above equations produce:

\[
a_{max} = 1,250 \frac{m}{s^2} \quad [= 128g's]
\]
\[ v_{\text{max}} = 12.5 \frac{m}{s} \]
\[ t_a = 0.01 \text{ sec} \]

These requirements for \( a_{\text{max}} \) and \( v_{\text{max}} \) seem to be exceedingly large.

A reasonable 'reality check' is to determine the power rate for the motor and load. This is essentially a measure of the efficiency of electrical to mechanical power conversion which will be required to meet duty cycle requirements. For the application being studied, motor and the payload are assumed integral. For the motor to accelerate the payload over a time period of \( t_a \), the power rate is defined as:

\[ P_{\text{load}} = (m_{\text{load}} a_{\text{max}} + F_{\text{friction}}) a_{\text{max}} \]
\[ = 3.125 \text{ MW/sec} \]

In the case of a linear motor, \( a_{\text{max}} \) is the same as \( a_{\text{motor}} \) and the friction force is assumed negligible. The load acceleration power requirement is:

\[ P_{\text{load}} = P_{\text{Rload}} t_a \]
\[ = 31.25 \text{ kW} \]

This means that the work required to accelerate the payload from zero to maximum velocity is:

\[ W = P t \]
\[ = 312.5 \text{ J} \]

The required tangential force for the load is then given by:

\[ F_x = m_{\text{load}} a_{\text{max}} \]
\[ = 2500 \text{ N} \]

Once again, these requirements are exceedingly high to be achieved by a single forcer.
assembly, perhaps more suited for a military application rather than an industrial application.

Another approach towards meeting the duty cycle requirements must be developed. Multiple forcers can be used to meet the required payload throughput rate. If it is assumed that multiple forcers handle 1000 payloads per minute, and using the relationship:

\[ a_{\text{max}} = \frac{9d_{\text{total}}}{2t^2} \]

The results are presented in a trade-off chart (Fig. 5-1). The chart indicates that as the number of motor forcers increases, the operational characteristics become much more "feasible".

Another item to note is that the duty cycle need not be divided equally. That is, the forcer, when unloaded, can accelerate/decelerate faster than when loaded. This results in an "uneven" velocity profile, as shown in Fig. 5-2.

### 5.3 Predicted Performance of an Open-Loop Normag Stepping Motor

As a starting point, the Normag motor previously characterized in Chapter 4 will be analyzed for operation at its maximum capacity towards meeting this particular
application. It will be assumed that the motor will be driven simultaneously in both axes at maximum capacity. This being the case, the parameters are:

\[
\begin{align*}
m_{\text{loaded}} &= 8.9 \text{ lbs} \\
m_{\text{unloaded}} &= 4.5 \text{ lbs} \\
F_{\text{max-loaded}} &= 12.5 \text{ lbs} \\
F_{\text{max-unloaded}} &= 10.8 \text{ lbs} \\
v_{\text{max-loaded}} &= 40.5 \text{ in/sec} \\
v_{\text{max-unloaded}} &= 46.9 \text{ in/sec}
\end{align*}
\]

Using the system loss data curves from Chapter 4, the theoretical cycle time for the Normag motor operating at peak capacity is 0.70 sec. Therefore, the maximum through-put rate for a single Normag motor is 85 payloads/minute. A total of 12 Normag motor forcers would therefore need to operate in parallel to achieve the desired 1000 payloads/minute desired rate. At approximately 8,500 dollars per
forcer/step-driver combination, the cost would exceed 100,000 dollars just for the required number of Normag motor forcers.

Analysis of a stock Normag linear stepper motor has indicated that it does not have a reasonable operational capacity for an as defined continuous, flexible packaging system. Instead, a new motor configuration can be designed to better suit the system requirements.

5.4 Open Loop Versus Closed Loop Motor Systems

The linear stepper motor can be operated without feedback (open-loop) as long as the excitation energy is limited in shape and amplitude to avoid loss of synchronism. It is possible to operate motors beyond their capabilities so that they pull out of registration, but this is no different from rotary stepper motor systems. For a typical application, open loop stepping motors are generally sized to operate at about 50 percent of their rated peak torque or force.

In the normal open loop operating mode, the linear stepper motor motion is synchronized with a command pulse train. As a result of stepwise operation, the position error is generally non-cumulative. This is usually adequate for many applications, especially at low speeds and steady-state operation.

The big disadvantage of open loop operation, however, is that the actual response of the motor is not being measured. There is no way of knowing whether or not significant error is present due to missed pulses. If the motor is operated near its rated force, 'pulse missing' is a potential problem.

There are two main reasons for pulse missing. The first can be due to the phase translator in the driver. If pulses are being received at a high stepping rate, the phase translator may not respond to a particular pulse. The corresponding phase would then not be energized before the next pulse arrives. In this case, the next pulse reaching the translator will not energize the phase corresponding to the missed pulse but will energize its own phase.
The second error source can be due to the motion processor - a pulse may not actually be generated, or extra pulse(s) can also be generated. In this case, the logic sequence within the translator will be interpreted as the lost pulse and will energize the phase corresponding to the lost pulse. This introduces a time delay into the command pulse sequence. In both cases of pulse missing, the net result is a deceleration of the motor because of a negative force due to a phase that is not switched off. Depending on the timing of subsequent pulses, a negative force can continue to exist in the motor. The net force can be much smaller than what is required to maintain the normal operating speed, and the motor might stall. To avoid this situation, pulse missing should be detected by response measurement (using a feedback sensor), and corrective action should be applied to modify the future pulse sequence in order to accelerate the motor back into the desired trajectory. Feedback control can be used to compensate for motion errors in a stepper motor.

In this feedback control scheme, the actual response of the stepper motor is measured and compared with the desired response. If an error is detected, the pulse train to the drive system is modified to reduce the error. A closed-loop system can operate near its rated capacity, without introducing excessive error and stability problems.

Another potential error source is particular to linear motors; linear motors with multiple forcer assemblies which are not necessarily coaxial - such as the Normag motor addressed in the previous chapter. This motor can be subject to a 'skewing error,' where the forcer assemblies do not provide balanced horizontal force production, thereby 'skewing' or slightly rotating the motor about its $Z$-axis. Since the alignment of forcer teeth with platen teeth is critical to system functionality, the net result of motor skewing is a stall condition, requiring external corrective measures (i.e., realigning the forcer on the platen.) By including position feedback sensors for for either edge of a multiple forcer motor, the signals can be compared to prevent any skewing errors from occurring.
5.5 Control of Linear Motor Systems

There seem to be two major characteristics of linear motors which implicate their need for 'special' control techniques. The forcer, floating on a layer of air, is highly resonant. The damping in the system is therefore quite low. The result is an excessive response to excitations near its natural resonance frequency, and prolonged 'ringing' from disturbance forces, and starting and stopping forces as described in Chapters 3 and 4. To review this phenomenon, it is necessary to look at the motor force displacement characteristic curve, as shown in Chapters 2 and 4.

The motor is 'held' at a stable equilibrium point. The motor can be displaced from the stable equilibrium point and will, when released, return to the stable point provided that the displacement is less than ± one-quarter of a tooth pitch. If the displacement is more than this, the motor will move to a different stable point at another location when released. During operation, as the motor is subjected to inertial and drag forces, it experiences setback that displaces it from the stable point. If the setback reaches the limit (the maximum force at acceleration, or minimum force for deceleration), the motor will slip, thereby losing synchronism between commanded position and actual position.

5.5.1 Open Loop Linear Motor Systems: Acceleration Roll-off and Burst

Consider now moving the motor from one point to another, with the initial and final velocity being zero. The simplest approach would be to apply a constant acceleration, until the midpoint of the move is reached, and then an equal amount of deceleration for the second half of the move (a triangular velocity profile). The driving function (position) would have the form:

\[ c(t) = \frac{1}{2} At^2 \]

where A is the constant acceleration. The second derivative of this driving function is a step function, and the open loop response of the second order (under damped)
system would be oscillatory, as has been shown. Also, as the velocity increases, the
drag forces will increase, meaning increased risk of slipping; and will most likely
occur, especially at the large deceleration step at the midpoint of the move. To
counteract this, one strategy is to use acceleration roll-off. This requires adjusting
the acceleration/deceleration command pulse-train such that the sum of the inertial
and drag forces are a constant. This strategy has limited success in actual practice,
and is valid only for low levels of acceleration.

What is required is a control strategy which will allow the motor to lag the com-
manded position by a constant, acceptable error without allowing slip:

\[ x(t) - c(t) = \text{constant} , \]

and, taking the Laplace transform:

\[ X(s) - C(s) = \frac{-E}{s} \]

where \( E/s \) is an error step of magnitude \( E \). The negative sign indicates a lag error. If
\( X(s) \) is eliminated from the above equation (and using the linear mass-spring model):

\[
C(s) \left[ \frac{K_m/m}{s^2 + sK_{ec}/m + K_m/m} \right] - C(s) = \frac{-E}{s}
\]

or, solving for \( C(s) \):

\[
C(s) = \frac{E}{s^2} \left( \frac{s^2 + sK_{ec}/m + K_m/m}{s + K_{ec}/m} \right)
\]

or, substituting \( \omega_n \) and \( \zeta \):

\[
C(s) = \frac{E}{s^2} \left( \frac{s^2 + 2\zeta \omega_n s + \omega_n^2}{s + 2\zeta \omega_n} \right).
\]

This is a driving function that the motor will [theoretically] follow with a constant
error.
Looking at the input function in the time domain yields:

\[ c(t) = E \left( 1 - \frac{1}{4\zeta^2} + \frac{1}{4\zeta^2}e^{-2\zeta\omega_n t} + \frac{\omega_n}{2\zeta}t \right) \]

The derivatives of the input function are:

\[ \frac{dc(t)}{dt} = E \left( \frac{\omega_n}{2\zeta} - \frac{\omega_n}{2\zeta}e^{-2\zeta\omega_n t} \right) \]

and,

\[ \frac{d^2c(t)}{dt^2} = E\omega_n^2e^{-2\zeta\omega_n t} \]

Notice that the acceleration is rolled off exponentially from an initial value of \((E\omega_n^2)\), and that the end velocity is \((E\omega_n/(2\zeta))\). This is the same as acceleration roll-off except it has the error value added to the basic rolled-off function. This initial step is referred to as 'burst'. Burst is a function of acceleration and velocity, and is a control technique which supplies an offset that compensates for lag or lead in a motor, thereby bringing the motor into alignment with the intended position. The higher the velocity or acceleration, the higher the burst. Burst can be applied whenever there is a transition in motor movement, for example, at the end points or midpoint of a move, or when the velocity is limited in value to less than \((E\omega_n/(2\zeta))\). Acceleration roll-off and burst are actually a form of cascade compensation, and work well if the system is modeled accurately and is reasonably linear. The control action consists of adding zeros exactly at the anticipated position of the transfer function conjugate poles. Fig. 5-3 shows how burst and acceleration roll-off combine.

Burst is added at point A in the figure. This burst 'level' will remain constant if corresponding acceleration roll-off is used. At point B, where the motor reaches constant velocity, the acceleration dependent burst is removed. For constant velocity, only velocity dependent burst is required (notice that it is leading throughout the acceleration and constant velocity phases). When the motor decelerates (starting at point C), the velocity dependent burst must be changed from leading to lagging, and in addition, deceleration dependent burst is also subtracted from the input (this is
also constant if proper deceleration roll-off is used.) Burst can be implemented in hardware (analog) or software (digital). Analog burst generation is more effective than digital since digital generation is resolution limited. Digital burst generation does have a cost advantage, however.

The use of burst and acceleration roll-off improves the open loop performance of a linear motor, and permits greater and more efficient force utilization. But, there is still no system verification that motor actually follows the input commands.

### 5.5.2 Commercially Available Closed Loop Linear Motor Systems

The control technique outlined previously does not provide for the effect of outside disturbances. If the mass carried by the motor changes, the amount of burst required also changes. Also, the actual force versus displacement curve is not a perfect sinusoid, and the amplitude varies over a complete cycle. Frictional forces have some responsibility for this, as well as geometrical errors and numerous other small pertur-
Feedback control has been added to linear motor systems. One simple, fairly inexpensive method is to mount an accelerometer onto the 2-axis forcer mechanism. To use this signal, the accelerometer output would be integrated through a low pass filter to obtain a velocity signal. This signal can then be subtracted from the input function, as shown in Fig. 5-4.

The closed loop transfer function is given by:

\[
\frac{X(s)}{C(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{\omega_n^2}{s^2 + (2\zeta + K)\omega_n s + \omega_n^2}
\]

By adjusting the proportional gain, \(K\), the closed loop system can be given any amount of damping. There is, however, a trade-off involved. If \(K\) is selected to achieve a critical damping ratio \((\zeta' = .707)\), the following error will increase, by a factor of about fifty percent. The result is sluggish system performance. If the input function
to the system is as close to the ideal model as possible, the activity in the feedback loop will be minimized. The activity in the loop removes resonant energy from the system, but at the expense of instantaneous position accuracy. Noise is a serious consideration also in this scheme. Noise can arise from various sources. For example, a velocity ramp during acceleration/deceleration will probably consist of a staircase of velocity steps. A velocity step requires action in the feedback loop as the motor settles to a new equilibrium; therefore, velocity steps should be as small as practical. Other noise may arise from a digital controller. If the sampling frequency is within the operational bandwidth, a bias frequency can be present in the input function, causing slight alterations to the trajectory. This can be especially significant in X-Y motion, particularly if the two axes have different bandwidths. An unexpected latency in a digital controller (interrupt processing, etc.) can introduce significant error also, especially at high speeds. Yet another consideration are additional degrees of freedom due to structural modes of the platen (which can be excited by forcer motions or the platen support base).

5.6 Stepper versus Servo

As seen, closed loop linear motors are commercially available. Parker/Compumotor supplies one-dimensional linear steppers with accelerometer feedback or linear encoder feedback [4]. Two-dimensional systems are available which incorporate accelerometers (Megamation, AT&T). A prototype system was developed for the US Navy using laser interferometers (Northern Magnetics, Inc.) But these systems are either too simple (only 1-D, or slow response), or are too costly for typical high performance industrial applications.

A closed loop system can be either a stepper system with a closed position loop, or a servo system with position/velocity feedback. A closed loop stepper system with encoder feedback to the motion controller is referred to as a position tracking system. The controller compares the number of pulses it receives from the encoder with the number of pulses that it sends to the drive. This comparison provides operator inde-
dependent verification of the motor's position, and allows stall detection or end-of-move position corrections. Position tracking step motor systems send position commands, and then 'look' at the encoder response to check for position error. Corrections are made after the initial move.

A servo system, on the other hand, constantly monitors the commanded versus actual position and velocity via feedback elements, and continuously makes dynamic adjustments to the system in order to drive the errors to zero. The system characteristics are thereby compensated for, allowing maximum use of the system capabilities. A further advantage to servo systems over stepper systems is the current supply at zero velocity. A stepper system supplies a DC current at zero velocity to maintain position. A servo, on the other hand, draws no current at a fixed position, and only requires current to move or to compensate for an error. Therefore, it can be concluded that the most useful linear motor system, for general industrial applications, would be a servo system. A hybrid rotary stepper motor can be thought of as a brushless servo motor with more poles than usual. Therefore, a hybrid stepper can be used as a brushless servo simply by fitting a feedback device to control the commutation. This is referred to as a 'hybrid servo.' A hybrid servo generates approximately the same force output as the equivalent step motor, assuming the same drive current and supply voltage. This thinking can be extended to a linear step motor.

5.6.1 Proportional Control

For proportional control the root locus for the Normag mass-spring motor model from Chapter 4 is shown in Fig. 5-5:

\[
\frac{X(s)}{C(s)} = \frac{K_p K_m/m}{s^2 + s K_{ec}/m + K_p K_m/m}
\]

The open loop poles are still complex - indicating an under damped time domain response. As the proportional control gain, \( K_p \), increases to greater than one, the poles migrate even further from the real axis, and the system oscillatory response only gets worse.
5.6.2 Proportional-Derivative Control

The addition of derivative control to a system has the effect of adding a finite zero whose position in the s-plane depends upon the derivative gain setting, $K_d$. Derivative control contributes to controller action in a manner which is proportional to the rate of change of the controller input (usually the error between actual and desired position). It is especially effective at the beginning of a transient response, but is not as useful when the input asymptotically nears the steady state, as the variation in input becomes very low. In fact, application of the final value theorem confirms that the derivative gain plays no role in the steady state value. Derivative control also tends to amplify high frequency noise within the system. The position of the added zero can be of critical importance, as seen in Fig. 5-6. If the zero were to be placed in the right-hand s-plane (positive feedback), the system poles would quickly migrate to the unstable right plane region for very low proportional gain values.
5.6.3 Proportional-Integral-Derivative Control

Qualitatively, integral control action is used to bring the steady state control error to zero. It acts proportionally to the integral of errors over time. This makes it useful for the late stages of the transient response when the error is small, but it also has the effect of increasing the overshoot of a response when the error is large.

Qualitatively, the addition of integral action to a controller has the effect of placing an extra pole on the system zero-pole plot. With integral control added to a P-D controller, the controller zeros can be coupled by the characteristic equation:

\[ K_d s^2 + K_p s + K_i = 0. \]

The pole is a pure integrator, fixed at the origin. Examination of the controller equation reveals that if the zeros are complex, the real portion of the zeros are determined by the ratio \( K_p / K_d \), while the distance the imaginary parts are from the real axis are determined by \( K_i \). If the zeros are strictly real, increasing the value of \( K_i \) will move
the zeros closer together up to the limit defined by $K_d/K_p$. This is useful information for designing a controller. It would be expected that there are many possible root loci for the Normag motor with a PID controller. Fig. 5-7 confirms this.

**PID Controller for the Normag Motor**

To design a reasonable PID controller for the Normag linear motor, it is best to decide what the desired characteristics are, and then work backwards. In this case, the desired motor behavior includes:

- better motor damping/less ringing step response (PID control),
- no penalty in following error or rise time (P control),
- stability in the face of disturbances/feedback noise (P-D control, low I-gain),
- zero steady state error (I control).

Using the logic from above, reasonable gain factors can be found for a PID controller, with much improved system transient response, as shown in Fig. 5-8. The stability
This is the most complete text of the thesis available. The following page(s) were not correctly copied in the copy of the thesis deposited in the Institute Archives by the author:
mechanically connected to, or in other mechanical contact with the point or object whose displacement is to be measured. For a two dimensional linear motor, any mechanical contact between the forcer and platen would introduce additional system dynamics, such as frictional and disturbance forces. This limits selection to non contacting displacement sensing devices in which an optical, inductive, or so on, coupling is used between the measured object and the sensing element.

An absolute sensor, one with an output signal which is referenced to a known fixed reference which is independent of the initial state of the sensor, is also desirable, especially if the system is to be powered down frequently. The alternative to this is an incremental sensor which produces a pulse for each change in a measured quantity. The output is therefore measured with respect to some initial state which is defined before measurements are taken. In this case, although an absolute sensor is preferable, an incremental sensor is much more achievable.

Some other considerations for any sensing application are accuracy, resolution, precision, and sensitivity requirements. Accuracy is defined as the deviation between the sensor output and the actual measured quantity. Resolution is the smallest detectable change of the sensor output signal in response to a change in the measured quantity. Repeatability, or precision, is the ability of the sensor to produce the same output in response to the same input. And finally, sensitivity is defined as the ratio of the change in output corresponding to a change in the measured input. Sensitivity is often times a directional property of a sensor, in other words a sensor is usually sensitive in a specific direction and not (or less) so in all other directions. A two-dimensional motor may require directional sensitivity if the platen is to be used as the measurand. The topological profile of the platen is identical in both the X and Y directions, since it is a 'waffle' platen.

Another sensor consideration is the slew rate: the maximum speed beyond which the sensor cannot keep up with the rate of the change of the quantity being measured. The bandwidth of the sensor is then defined as the frequency range in which the sensor can operate - a sensor with a high bandwidth has a fast speed of response and is therefore capable of operating at a high slew rate.
The frequency response of a sensor system is the system's ability to respond to changes in the measurand. Generally, the faster the process being measured, the less accurate the measurement. The bandwidth of a sensor is usually defined in terms of the frequency at which the sensor's output tends to decrease because it can no longer accurately detect changes in a rapidly changing measurand. If a sensor is used to detect motion of a part and the output from the sensor is used to control an axis to correct for error, the sensor should be operated well before its -3 db frequency response point. The phase angle portion of dynamic response also affects whether a sensor can be effectively used in a control system. If the response of the sensor lags behind the actual physical process too much, then it may not be possible for the mechanism to correct for errors sensed because the error may have already irreversibly affected the process.

A differential measuring system allows for the output of one sensor to be subtracted from the output of another which cancels errors. Assuming one sensor is looking at a fixed target or the opposite side of the other sensor's target, motion will be recorded and environmental errors will still be canceled. This requires the systems to be matched and hybrid electronic circuits to be used. Matching ensures that both sensors have similar characteristics and they will be affected equally by changes in the environment. Hybrid circuits ensure that all signal processing is done in a small enclosed area to ward off other environmentally induced errors.

In general, proximity sensors are entirely solid state electronic sensors which contain no moving parts to wear out. They require no physical contact for actuation, no cams or linkages, have no contacts to bounce or arc and are usually completely encapsulated, making them impervious to most liquids, chemicals and corrosive agents. They are characterized by rapid response and high switching rates. In this case it is important to differentiate between proximity sensors and proximity transducers. A proximity transducer includes the sensor supply electronics (usually a regulated voltage supply) and additional signal conditioning electronics to make the output usable for an application (usually a an amplifier stage, and perhaps a trigger/latch stage). Proximity transducers can be obtained with AC or DC output.
One final item of consideration is standoff distance - how far a sensor is nominally placed from the target. The actual sensing range is usually some fraction of the standoff distance. By reducing the noise level in a system, the resolution and the repeatability can often be increased.

5.7.2 Sensors Considered

Four categories of proximity sensors were considered for this application: capacitance, ultrasonic, optical, and magnetic. Of the four categories, active magnetic sensors were selected for two major reasons: cost, and the operational environment (platen size, materials, etc.)

In general, optical sensors are far more accurate and have greater frequency response than non-optical sensors. All sensor systems have to be traceable to a standard, and perhaps no reference can be made more stable than the wavelength of light. However, they are also generally quite costly, and the reflectivity of surfaces in a linear motor system would probably be a hindrance rather than an advantage.

Magnetic sensors include Hall effect, inductive and magnetostriction sensors. Magnetic sensors are non-contact transducers which convert mechanical motion into electrical control signals. The motion can be rotation, slide-by, or oscillation. The area to be measured must be of a ferromagnetic material. Because of their wide spread usage, magnetic sensors have become known by many names, including: magnetic pick-ups, speed sensors, motion sensors, variable reluctance sensors, and magnetic probes.

Magnetic sensors are divided into two classes: active and passive. Passive magnetic sensors do not require any external power supply, and have an analog output signal. The output waveform is a function of the actuator being measured (usually sinusoidal with gear actuators). The amplitude and the frequency are both proportional to the surface speed of the actuator as it passes the sensor's pole piece. Active magnetic sensors require an external power supply to operate. The output signal is usually a digital signal with a constant amplitude over the operating range for a fixed supply voltage level. Active magnetic sensors provide usable output signals at very
low actuator speeds and with relatively large air gaps between the sensor and the actuator. A magnetic sensor operates by directing a magnetic field through a coil via a pole piece. As a discontinuity (such as a tooth edge) approaches, the change in the magnetic field produces a voltage in the coil, exactly as occurs in a conventional electric generator. The voltage amplitude and frequency are directly proportional to the speed of the discontinuity. The variable reluctance of the magnetic flux path varies the quantity of magnetic flux in the path. This change in magnetic flux generates a voltage at the terminals of the coil that is exactly proportional to the rate of change of magnetic flux.

The sensor chosen as most suitable for feedback purposes was a Hi-resolution Digitrac\textsuperscript{R} from Transducer Systems, Inc. (TSI), and specifications are given in Appendix F. The Digitrac is an active, solid state magnetic sensor which initially produces an analog signal with the passage of a target. This analog signal can be fed through a comparator which converts the signal to a switched, TTL/CMOS-compatible digital output. The sensing portion consists of dual, coplanar, sensor lamina, which are essentially ferromagnetic pole pieces for two separate electromagnet coils. The analog signal is a comparison of ferromagnetic target contour immediately in front of each sensor lamina, as described in the previous paragraph. The poles are chisel shaped, and placed parallel to each other. The output then is actually the difference between what the two sensors are 'seeing.' The slope of this analog wave form at its zero crossing represents the transition of the differential between the target contours sensed by the two laminas, and is not speed influenced. Target rising and falling edges must be well-defined, as is the tooth pattern of a linear motor platen. The analog signal spans 180 electrical degrees with the passage of a target across its sensor face.

A supply voltage is used to excite the sensor to detect the presence or absence of ferromagnetic targets. This gives the Digitrac zero speed capability, which extends to a maximum frequency of 20 kHz (very low inductance). The magneto resistive technology of the sensor lamina provides a high-level signal to the signal conditioners. Since the sensor lamina form arms of a bridge configuration, the primary signal is
a differential signal and does not require amplification prior to signal conditioning, thus decreasing sensitivity to noise. The cost for a Digitrac sensor is 175 dollars at the time this thesis was written. A self-contained switching circuit can convert the initial analog signal generated by the passing of a ferro-magnetic material, into a digital pulse train. The frequency generated by the sensor can be converted directly to speed by means of a frequency counter or digital tachometer. Another method of speed measurement is to change the frequency into a proportional dc current with a frequency-to-dc converter driving a speed indicating meter. Fig. 5-9 shows a schematic for a Digitrac sensor.

The resolution of a sensor like the Digitrac is a function of the width of a sensor lamina face, and the distance between the two sensor lamina. By making the sensor lamina rectangular in shape, and offsetting the two lamina such that the center distance between them can be less than a lamina thickness (see Fig. 5-10) causes unidirectional sensing with very high resolution. It is very feasible to produce a uni-directional sensor with the resolution required for a linear motor system with a 0.040
in pitch platen.

5.7.3 Sensor Incorporation

The Digitrac sensors can be mounted symmetrically at each corner of the motor as shown in Fig. 5-11. Two uni-directional sensors would be used for each axis of motion. Each sensor provides unidirectional position feedback. Each axial sensor pair would provide position error relative to each other, and could therefore be used to, to prevent ‘skewing’ of the motor.

5.7.4 Hardware Implementation - Digital Servo

The most cost effective way to implement a closed loop 2-D linear servo motor is by using a single loop servo control system, as shown in Fig. 5-12. All control feedback signals and control commands are processed by the (programmable) controller algorithm. This provides a very flexible solution to control under varying conditions such as multiple applications, multiple motors, etc. Performance is only limited by
the selected digital platform. The servo amplifier(s) simply produces a current signal proportional to the controller commands. By using the microprocessor controlled servo gains in the digital controller, there can be multiple gain settings in memory and selection of different gain sets for different situations: lower gain set for lower mass and higher gain set for higher mass or as the system starts each move.
Figure 5-12: Single Loop Servo Control
Appendix A: Material Magnetic Properties and Magnet Vendors

**Permanent Magnet Vendors**

Dexter Magnetic Materials  
Billerica, MA  
Tel: 508-663-7500

Tridus International  
500 Carson Plaza Drive  
Carson City, CA 90746  
Tel: 310-329-7777

Magnet Sales & Mfg. Co.  
Tel: 1-800-421-6692

Arnold Engineering Co.  
Tel: 815-568-2000

The Magnet Source  
Tel: 1-800-525-3536

Electro-Core Inc.  
Tel: 314-239-2703
Appendix B: Normag Model 4XY-2504-2 Specifications, Motion Science Information
DUAL AXIS HIGH PERFORMANCE LINEAR STEPPER MOTORS

GENERAL DESCRIPTION:

NORMAG high performance dual axis, single plane, linear stepper motors are capable of high position accuracies without a position feedback loop.

The moving electromagnetic primary or "forcer" which consists of two sets of single axis forcers mounted at 90 degrees to each other, travels over a grooved stationary secondary or "waffle platen" that provides X-Y motion in a single plane.

The airgap between these parts is maintained by integral airbearings and as a result there is no mechanical contact between the forcer and the platen.

Motion is achieved by powering the forcer with a two or four phase, two axis, micro stepping motor driver. Each pulse causes the forcer to move one microstep and the frequency at which these microsteps are generated determines the velocity of the forcer.

The platens are available in different sizes and mounting arrangements to meet customer requirements.

ADVANTAGES:

- High reliability, long life.
- No mechanical linkages \(\Rightarrow\) no backlash.
- High acceleration.
- Smooth and reversible travel.
- Two dimensional travel can be very large without performance reduction.
- Compact, low profile design.
- Higher accuracies possible, using a position feedback loop.
- Velocities up to 100 in/sec.
- Can have multiple forcers on a single platen.

OPTIONS:

- Two or four phase operation.
- Any customer required modifications.
- Custom platens to meet special requirements.

APPLICATIONS:

- High speed linear positioning and velocity control.
- Pick and Place equipment.
- PCB assembly and inspection system.
- Automated inspection system.
- Robotics.
- Parts transfer systems.
- Many others.
### MOTOR SPECIFICATIONS:

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<th>4XY2002-2-0</th>
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*dependent on drive electronics and system implementation.

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SPECs: NORTHERN MAGNETICS, INC.  
1624 WYANDOTTE STREET  
VAN NUYS, CA 91406 USA |
| 2       | MOTOR HOUSING, NORMAG CLONE | 1        | DRA. NO.: MIT-0001  
DATA BASE: MOTORDWG |
| 3       | COVER, MOTOR, NORMAG CLONE | 1        | DRA. NO.: MIT-0002  
DATA BASE: COVERDWG |
| 4       | CORE, ELECTROMAGNET, NORMAG CLONE | 15       | DRA. NO.: MIT-0003  
DATA BASE: COREDWG |
| 5       | ENVELOPE, PERMANENT MAGNET, NORMAG CLONE | 15       | DRA. NO.: MIT-0004  
DATA BASE: PMDWG |
| 6       | SCHEMATIC, NORMAG CLONE | -        | DRA. NO.: MIT-0005  
DATA BASE: SCHEMDWG |
| 7       | WIRE, MAGNET, 26 AWG. | AR       | SOURCE: MWS WIRE INDUSTRIES  
3200 CEDAR VALLEY DRIVE  
WESTLAKE, CA 91362 USA  
PART NO: 265PN |
| 8       | PLATE, PROTECTIVE  
COVER PLATE, NORMAG CLONE | 8        | DRA. NO.: MIT-0006  
DATA BASE:  
PLATEDWG |
| 9       | CONNECTOR, 25 PIN, MALE AND FEMALE | 1        | ITT CANNON  
PART NOS. DB-25P  
DB-25S |
| 10      | 0.020" DIA. ORIFICE, POCKETED, AIR BEARING | 12       | NORTHERN MAGNETICS, INC  
SEE ITEM 11 |
| 11      | QUICK DISCONNECT FITTING, AR, MALE ELBOW | 1        | SMC PNEUMATICS INC.  
PART NO. K007-32 |
| 12      | 4-40UNC-2A x 0.25 SLOT HD FLAT PT SET SCREW | 10       | NORTHERN LABS  
41 CHESTNUT STREET  
GREENWICH, CT 06830  
PART NO. NORCAST 3259D |
| 13      | EPOXY CASTING SYSTEM | AR       | DRAWING NO: MIT-0000  
REV:  
6/30/92  
SH. 2 OF 2 |
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Units: INCHES
Title: NORMAG LINEAR MOTOR HOUSING

Drawing No: MT-0001

Drawn by: P. GJELTEMA

Date: JUNE 1992
Rev: 6/30/92
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
HIGH-SPEED FLEXIBLE AUTOMATION PROJECT
PROF. KAMAL YOUSEF-TOUMI

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<td>JUNE 1992</td>
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COVER.DWG
0.188" DIA. THRU CBORE 0.280" x 0.750" DP.
8 HOLES

2.850'

6.750'

3.900'

7.000'

3.900'

6.750'

7.000'
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CORE.DWG
**Material:**
- AS LISTED

**Units:**
- INCHES

**Tolerances:**
- \( x: \pm \)
- \( xx: \pm \)
- \( xxx: +/- \)
- \( angle: +/- \)

**Title:**
- WIRING SCHEMATIC
- NORMAG CLONE

**Drawing No.:**
- MIT-0005

**Drawn by:**
- P. GJELTEMA

**Date:**
- JUNE 1992

**Rev.:**
- 6/30/92

**Scheme.Dwg**
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PLATE.DWG
Appendix C: Controller/Driver Simulation and Motor Loss Analysis M-Files
condrive2.m
a matlab m-file to simulate controller/driver position output

Output vectors:

[T] time
[U] position

Trapezoidal Velocity Profile is default

clear
pack

"PARAMETERS"

amax = 100;
vmax = 10;
ta = vmax/amax;
da = 0.5*amax*ta^2;

pitch = 0.040
stepstep = 4 % no. steps to move one pitch distance
stepin = stepstep/pitch % steps per inch
ddel = 1/stepin % inches per step
tdel = ddel/vmax % min. step time to move ddel

"COMMANDED MOVE DISTANCE"

c = 1.1;

"GENERATE STEP TIMING"

"TRAPEZOIDAL PROFILE"

if c/2 > da,

"ACCELERATION"

T(1) = 0;
U(1) = 0;
for i = 1:ta/tdel,
    T(i+1) = i*tdel;
    tee = T(i+1);
\[ u = \frac{1}{2} a_{\text{max}} T(i+1)^2; \]
if \( u - U(i) \geq \delta_{\text{del}} \),
\[ U(i+1) = u; \]
else,
\[ U(i+1) = U(i); \]
end
\[ u_{\text{ee}} = U(i+1); \]
end

\[ d_{\text{cru}} = c - 2d_{\text{a}}; \]
for \( i = 1:d_{\text{cru}}/d_{\text{del}} \),
\[ T(i+\text{ta}/t_{\text{del}}+1) = t_{\text{ee}} + i*t_{\text{del}}; \]
\[ U(i+\text{ta}/t_{\text{del}}+1) = u_{\text{ee}} + i*d_{\text{del}}; \]
\[ g_{\text{ee}} = T(i+\text{ta}/t_{\text{del}}+1); \]
\[ w_{\text{ee}} = U(i+\text{ta}/t_{\text{del}}+1); \]
\[ e_{\text{ee}} = i*\text{ta}/t_{\text{del}}+1; \]
end

\[ d_{\text{cru}} = c - 2d_{\text{a}}; \]
for \( i = 1:ta/t_{\text{del}} \),
\[ T(i+\text{eye}) = g_{\text{ee}} + i*t_{\text{del}}; \]
\[ u = u + v_{\text{max}} i*t_{\text{del}} - 1/2 a_{\text{max}} (i*t_{\text{del}})^2; \]
if \( u \geq \delta_{\text{del}} \),
\[ U(i+\text{eye}) = w_{\text{ee}} + u; \]
\[ u = 0; \]
else,
\[ U(i+\text{eye}) = U(i+\text{eye}-1); \]
end
end
lossreal.m

this program converts data from drag force lab testing
to force vs. velocity curves

motor weight
[w] lbs
w = 4.0;

velocity vector (velocities tested)
[v] [in/sec]
v = [10
20
30
40
50];

motor force limit
[f] lbs
flimit = [21.2
15.6];
vlimit = [0
40];

Graph Set-up
hold off
axis([lo x, hi x, lo y, hi y])
axis([0,60,0,25]);
plot(vlimit,flimit,'--');
title('Motor vs Velocity - One Axis');
ylabel('Velocity [in/sec]');
xlabel('Force [lbs]');
grid;
hold on

0.5 lb acc ramp
ahalf = [2.64286
2.39286
2.14286]
% 1.91071
% 1.66071]
%fhalf = (w+0.5) * ahalf;
%dhalf = 8 - fhalf;
%plot(v,fhalf,'*');
%plot(v,fhalf,'-.',v,dfhalf);
%
% 3 lb acc ramp
%
a3 = [1.0808/.386
 .9264/.386
 .772/.386
 .6562/.386
 .4632/.386];
f3 = (w+3) * a3;
df3 = 30 - f3;
%plot(v,f3,'.',v,df3);
plot(v,f3,'.',v,f3,'+');
%
% 5 lb acc ramp
%
a5 = [.8106/.386
 .7334/.386
 .6176/.386
 .5018/.386
 .386/.386];
f5 = (w+5) * a5;
df5 = 30 - f5;
%plot(v,f5,'-',v,f5);
plot(v,f5,'-',v,f5,'*');
%
% 7 lb acc ramp
%
a7 = [.6562/.386
 .6176/.386
 .4632/.386
 .4246/.386
 .3088/.386];
f7 = (w+7) * a7;
df7 = 30 - f7;
%plot(v,f7,'--',v,df7);
plot(v,f7,'--',v,f7,'o');
%
% 10.6 lb acc ramp
%

a10 = [.4246/.386
.386/.386
.3497/.386
.1158/.386
.0386/.386];
f10 = (v+10.6) * a10
df10 = 30 - f10;
% plot(v,f10,v,df10)
% plot(v,f10,'x');
% pause
%
% CURVE FITTING
%
hold off
spread = 0:2:100;
%chalff = polyfit(v,dfhalf,3)
%fithalf = polyval(chalff,spread);
c3 = polyfit(v,f3,1)
fit3 = polyval(c3,spread);
c5 = polyfit(v,f5,1)
fit5 = polyval(c5,spread);
c7 = polyfit(v,f7,1)
fit7 = polyval(c7,spread);
c10 = polyfit(v,f10,1)
fit10 = polyval(c10,spread);
x = [c3
c5
c7]
a = [x(:,1), x(:,2)]
coef = [sum(a(:,1))/3 , sum(a(:,2))/3]
fitco = polyval(coef,spread);
axis([0,100,0,25]);
plot(spread,fit3,'i',v,f3,'+');
hold on
plot(spread,fit5,'i',v,f5,'*');
hold on
plot(spread,fit7,'i',v,f7,'o');
hold on
%plot(spread,fit10,v,f10,'x');
plot(spread,fitco)
hold on
title('Characteristic NORMAG Motor Force vs Velocity - One Axis');
xlabel('Velocity [in/sec]');
ylabel('Force [lbs]');
grid:
grid
hold off
Appendix D: Cooling Fin Design Procedure
If convection is the primary mode of heat dissipation from a heat sink, and since the convection rate is proportional to surface area, the heat dissipated can be increased by adding fins. The heat is conducted through the solid material of the fin and removed from the surface to the surrounding fluid by convection. The temperature of the ambient fluid, \( T_\infty \) and the combined heat transfer coefficient, \( \tilde{h}_c \) are considered constant. To determine the temperature distribution in the fin, and eventually the heat transfer rate from the surface, it is necessary to perform an energy balance on a differential volume of fin material. Applying the Fourier law for two conduction terms and Newton’s law of cooling leads to a second order differential equation for the temperature distribution:

\[
\frac{d^2 T}{dx^2} - \frac{\tilde{h}_c P}{kA} [T(x) - T_\infty] = 0
\]

where:

\[
x = \text{distance along the fin, [m]}
\]

\[
P = \text{fin perimeter, [m]}
\]

\[
A = \text{fin cross-sectional area, [m}^2]\]

This equation can be nondimensionalized by defining a dimensionless temperature and coordinate as:

\[
\theta(x) = \frac{T(x) - T_\infty}{T_b - T_\infty}
\]

where \( T_b \) is the fin base \((x = 0)\) temperature, and:

\[
\zeta = \frac{x}{L}
\]

The temperature distribution equation then becomes:

\[
\frac{d^2 \theta}{d\zeta^2} - \frac{\tilde{h}_c P L^2}{kA} \theta = 0.
\]

The perimeter of the fin times the length \( L \) of the fin is equal to the total surface
area, $A_s$. Then:
\[
\frac{PL^2}{A} = \frac{A_sL}{A}.
\]

which can be defined as $l$, the characteristic fin length. Again substituting into the temperature distribution equation:
\[
\frac{d^2\theta}{d\zeta^2} - \frac{\bar{h}_c l}{k} \theta = 0
\]
or, noting the similarity to the Biot number used in Chapter 3:
\[
\frac{d^2\theta}{d\zeta^2} - (Bi)\theta = 0.
\]
The solution of this equation is:
\[
\theta(\zeta) = C_1 e^{-\left(Bi\right)^{1/2} \zeta} + C_2 e^{\left(Bi\right)^{1/2} \zeta}
\]
The values for the two constants of integration can be calculated once the boundary conditions are specified. The most commonly known is the base temperature, $T_0 = T_b$. The second boundary condition for a fin with convective heat loss from the tip surface area becomes:
\[
-kA \left. \frac{dT}{dx} \right|_L = \bar{h}_c [T(L) - T_\infty]
\]
or
\[
-\left. \frac{d\theta}{d\zeta} \right|_{1,0} = \frac{\bar{h}_c L}{k} \theta(1).
\]
Once the temperature distribution of the fin is known, the heat dissipation from the fin can be determined. The easiest method of evaluating the heat transfer from the fin involves determining the amount of heat conducted through the base of the fin:
\[
q_f = -kA \left. \frac{dT}{dx} \right|_{x=0} = \left. -\frac{kA}{L} (T_b - T_\infty) \frac{d\theta}{d\zeta} \right|_{\zeta=0}
\]
The temperature distribution and heat-transfer rates from the fins that satisfy the boundary conditions are:

\[ \theta(\zeta) = \frac{T(\zeta) - T_\infty}{T_b - T_\infty} = \frac{\cosh \left( (Bi)^{1/2}(1 - \zeta) \right) + (Bi)^{1/2}(A/PL) \sinh \left( (Bi)^{1/2}(1 - \zeta) \right)}{\cosh (Bi)^{1/2} + (Bi)^{1/2}(A/PL) \sinh (Bi)^{1/2}} \]

and

\[ q_f = (Bi)^{1/2} \frac{kA}{L} (T_b - T_\infty) \left[ \frac{\sinh (Bi)^{1/2} + (Bi)^{1/2}(A/PL) \cosh (Bi)^{1/2}}{\cosh (Bi)^{1/2} + (Bi)^{1/2}(A/PL) \sinh (Bi)^{1/2}} \right]. \]
Appendix E: Air Bearing Design
When a motor force is moving at high speeds relative to a platen, the bearing gap between them must be large enough to ensure that the friction power in the gap is less than twice the pumping power of air into the bearing. In such situations, the temperature rise due to friction within the bearing gap is offset by the refrigeration effects of the gas film as it expands after leaving the orifice and enters the gap.

First, a double row bearing system for a motor will be designed using the guidelines developed in [24]. The parameters for the motor are:

\[ d_R = \text{feedline diameter, } [5.08 \times 10^{-4} \, \text{m}], \]
\[ P_o = \text{supply pressure, } [652, 910 \, \text{Pa}], \]
\[ P_a = \text{atmospheric pressure, } [101, 325 \, \text{Pa}], \]
\[ L = \text{bearing width, } [0.178 \, \text{m}], \]
\[ B = \text{bearing length, } [0.178 \, \text{m}]. \]

In addition, the following parameters will be selected, based on the motor configuration:

\[ n = \text{number of orifices per supply line, } [3], \]
\[ N = \text{total number of orifices, } [6]. \]
\[ z_o = \text{air gap distance } [0.0008 \, \text{in}] \]
\[ a = \text{orifice edge distance, } [0.045 \, \text{m}]. \]

First, the shape factor \( \xi \) is determined using the equation

\[ \xi = \frac{\pi 2a}{L} \]

and found to be 1.57. Next, the equation:

\[ \lambda \xi = \frac{7.89 \times 10^3 nd_o^2 \pi B}{P_o z_o^3 L} \]
is solved for the orifice diameter $d_0$, with the assumption that the feeding parameter $\lambda \xi$ has its optimum value of 0.55. If this is the case, $d_0$ is 0.174 $mm$. At this point, the value for the line feed correction factor $1/\lambda$ can be estimated using Fig. 9.3.9 from [24], and is found to be approximately 0.60.

The values for $1/\lambda$ and $\lambda \xi$ are used with Fig. 9.5.3 from [24] to determine the bearing stiffness factor, $\bar{K}$. Using the equation:

$$ K = \frac{\bar{K}LB(P_o - P_a)}{z_0} $$

yields a bearing stiffness of 463.7 $N/\mu m$. Similarly, using Fig. 9.5.4 from [24] and the equation:

$$ W = LB(P_o - P_a)\bar{W} $$

the bearing load is calculated as 6107.6 $N$, which is just about equal to the attraction force calculated previously, as it should be.

A final note on air bearing design is the subject of pneumatic hammer instability, which is most often found associated with pocketed restrictions. This cyclical phenomena is due to a large pocket volume compared to the volume of air between the lands. The pocket pressure then rises, increasing the air gap between the bearing faces, which in turn, reduces the pocket pressure, thereby reducing the air gap clearance, etc., until over compensation or 'pneumatic hammer' vibration may result. There are two ways to overcome this problem; the first is to use inherently compensated orifices, thereby reducing the load capacity; and, secondly, to reduce the pocket volume. The general rule for pocket depth $p$ in $mm$ is:

$$ p \leq \frac{0.2LBz_0}{\pi d_R^2N \times 10^3} $$

where $z_0$ is expressed in microns, and all other dimensions are in $mm$. 

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Figure 9.3.9 Determination of $\psi_\lambda$.

Figure 9.5.3 Stiffness parameter for rectangular double entry thrust bearings with pocketed orifices where $a/B = 0.25$, $P_0/P_\alpha = 5$, $C_d = 0.8$, and $\gamma = 1.4$. For inherently compensated orifices, multiply the ordinate by 0.67. For single entry bearings, multiply the ordinate by 0.75.

Figure 9.5.4 Load parameter for rectangular double entry thrust bearings with pocketed orifices where $\epsilon = 0$, $a/B = 0.25$, $P_0/P_\alpha = 5$, $C_d = 0.8$, and $\gamma = 1.4$. For single entry bearings, multiply the ordinate by 0.75.
Appendix F: 40-lb Linear Motor Specifications and Digitrac Sensor Specification
Digitrac®

A solid-state magnetic sensor capable of slow-to-zero speed as well as high-speed sensing, and critical position control. Digitrac reduces motion to increments of position change, accurately and repeatably. Digitrac also senses a wide variety of target contours. Applications include:

- Sensing of band-printer timing marks
- Head positioning and needle-point registration in programmable looms
- Control valve angular position sensing
- Fluid-level readout in pharmaceutical liquid-fill equipment
- Ejection and retraction sensing in injection molding
- Electronic tachometers, speedometers, totalizers, ratio indicators
- Proximity sensing at very slow repetition rates as well as those to 20 kHz

Transducer as sensors for speed and position control.

Cams, gears, shafts with keyways, slotted computer memory discs, pulleys with notches or cutouts—in operation, all these items rotate, spin, or otherwise move in regular periodic motion. Their indentations or protrusions can serve as actuators for electromagnetic transducers (figure 1).

![Ferromagnetic actuators for electromagnetic transducers](image)

Passive transducers.

Passive transducers, such as variable-reluctance magnetic pickups, produce an output that is analogous in waveform to both the shape and the motion of the ferromagnetic actuating mechanisms they sense (TSI bulletin 1101).

Digital-output magnetic pickups condition this analog signal and produce TTL/CMOS-compatible output (TSI bulletin 1024).

TSI manufactures hundreds of models of analog-output magnetic pickups, as well as the most popular digital-output magnetic pickup, Digitrac, in use today.

These components offer many advantages for a broad range of applications. Passive transducers, they are primarily speed-sensing devices, requiring actuator surface speeds of generally greater than 50 ips to produce outputs of useful level. They serve well as position-control devices, provided the actuators maintain these minimum surface speeds. When using passive transducers, position control is a function of speed, and thus subject to this minimum-speed limitation.

Digitrac® output—useful and informative, independent of target speed.

Digitrac is an active (vs. passive) transducer. Unlike digital-output magnetic pickups which employ a supply voltage only for signal conditioning, Digitrac uses its supply voltage also for exciting the sensor to detect the presence or absence of ferromagnetic targets. The result—output which tracks the motion of slow-moving targets, and indicates the position of stationary targets relative to the sensor head—gives Digitrac an important advantage for position control, zero-speed output capability.

One target, one maximum air gap.

Digitrac senses the shaped contour of ferromagnetic target/actuators. The larger the target and the greater the contour variations, the larger Digitrac's maximum operational air gap. This effect is illustrated (figure 2) using involute gears. Common actuators for Digitrac are design simplicity—no 163 operational air gap does not vary with changes in target speed.

Differential sensors—digital output.

Many active transducers, proximity switches for example, signal the presence or absence of moving or stationary targets. Digitrac does more. Its dual-sensor design produces an analog signal initially, which is actually a comparison of ferromagnetic target contour immediately in front of each sensor lamina. Provided the slope of this analog waveform at its zero crossing is sufficient, Digitrac's TTL/CMOS-compatible output identifies not only presence or absence, but also the very center of the target—accurately and repeatably. This is an outstanding capability for very slow-speed control and positioning tasks (figure 3).

![Relative position of sensor center to target](image)

Relative position sensing via analog output.

Although Digitrac is primarily a digital-output device, its initial analog signal can be tapped as a separate signal to accompany its digital output. Digitrac can also be supplied as a strictly analog-output sensor. Digitrac's analog signal spans 180 electrical degrees with the passage of a target (gear tooth for example) across its sensor face. Zero crossing occurs as the sensor faces the target center. Subject to limitations (e.g., temperature fluctuations, etc.), this analog signal can identify the relative position of the

*The slope of this analog signal at zero crossing represents the transition of the differential between the target contours sensed by the two laminae, and is not speed-influenced. For best results, target width should approximate the center-to-center distance between Digitrac sensor laminae. Figure 5, cutout D, This distance is 0.056", the distance across the face of a 64-pitch gear tooth for the high-resolution Digitrac and 0.067", the distance across the face of a 24-pitch gear tooth for the standard-resolution Digitrac. Consult TSI for minimum and maximum target widths. Target sizing and tooth margins may be pre-determined.
target tooth at all points in its path across the sensor face. This applies to target indentations as well.

**High-speed sensing.**

For fast-moving/rapidly recurring targets, Digitrac output is compatible with TTL and CMOS circuits to a maximum frequency of 20 kHz, twice the frequency of functionally similar Hall-effect devices.

**Magnetoresistive rather than Hall-effect sensing eliminates noise amplification.**

Digitrac's unique magnetoresistive technology (figure 4) provides a high-level signal from the sensor head to its signal conditioners. There is no need to amplify this primary signal, and the noise that can accompany it, prior to conditioning. An initially high signal-to-noise ratio contributes to improved output clarity, and broader product application in areas with significant EMI. Digitrac's digital output can feed up to 20 TTL circuits without amplification.

(Figure 4) Digitrac Block Diagram

Digitrac** is easy to design in, easy to install.

Compared with other technologies offering similar sensing capability (optics, encoders, etc.), Digitrac is simple to work with—both in design and in installation. Digitrac comes in two resolution packages, and offers a choice of supply voltages. Installing Digitrac requires only simple mechanical alignment within generous tolerances (consult TSI). And Digitrac's low cost is competitive with less-capable sensors—another advantage to figure into your plans.

**Specifications**

<table>
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<tr>
<th></th>
<th>Standard-resolution Digitrac</th>
<th>High-resolution Digitrac</th>
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</thead>
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<tr>
<td>Supply voltage, $V_s$</td>
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</tr>
<tr>
<td>5V unit</td>
<td>5 Vdc ±10%</td>
<td>5 Vdc ±10%</td>
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<tr>
<td>12V unit</td>
<td>7-15 Vdc</td>
<td>7-15 Vdc</td>
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<td>Supply current*</td>
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<td>put (PNP collector with</td>
<td>High ($V_{Qh}$), 5V** and</td>
<td>High ($V_{Qh}$), 5V** and</td>
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<td>internal pull-up resistor)</td>
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<td>12V** units</td>
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<td>0.95 X $V_s$ minimum</td>
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<td>5V and 12V units</td>
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<td>2 μs typical</td>
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<td>5 μs maximum</td>
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<td>Output fall time (90-10%)</td>
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<tr>
<td></td>
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<tr>
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<tr>
<td>12V unit</td>
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<td>5.0 kΩ typical</td>
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<td></td>
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<td>see figure 5</td>
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<td>Case material</td>
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<td>6061 aluminum, black anodized</td>
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<tr>
<td>Other</td>
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<td>available with analog output</td>
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</table>

*dependent upon load resistor

**TTL- and CMOS-compatible

***CMOS-compatible, available as TTL-compatible

**Table of lamina dimensions from figure 5**

<table>
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<tr>
<th>CODE</th>
<th>STANDARD- RESOLUTION MODEL</th>
<th>HIGH- RESOLUTION MODEL</th>
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<td>(1.05 mm)</td>
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<tr>
<td>B</td>
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<td>0.0256</td>
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<tr>
<td></td>
<td>(1.7 mm)</td>
<td>(0.65 mm)</td>
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<tr>
<td>C</td>
<td>0.039</td>
<td>0.0157</td>
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<tr>
<td></td>
<td>(1.0 mm)</td>
<td>(0.40 mm)</td>
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<td>D</td>
<td>0.028</td>
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<td>(0.7 mm)</td>
<td>(0.80 mm)</td>
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<tr>
<td>E</td>
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<td>0.051</td>
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<tr>
<td></td>
<td>(1.3 mm)</td>
<td>(1.3 mm)</td>
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<tr>
<td>F</td>
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<td>0.0825</td>
</tr>
<tr>
<td></td>
<td>(2.0 mm)</td>
<td>(2.10 mm)</td>
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</table>

**Dimensions (figure 5)**

- The plane formed by the orientation groove and the axis of the sensor must intersect orthogonally. Contact TSI for tolerances.
Top View (Air Bearing and Cooling Manifolds/Hardware not Shown)
Cooling Fins
Forcer Assembly Mounting Rib
Core, X-section (Length : 4.650 in)
Bibliography


