Modeling, Path-Planning and Control of Space Manipulators: The Coupling Map Concept

by

Miguel Angel Torres

Submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Mechanical Engineering

at the

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Abstract

Addressing the need of space scientists and engineers to better understand the fundamental dynamic characteristics of space manipulator systems, this thesis proposes the Coupling Map as an analytical tool for graphically describing the dynamic interaction between a manipulator and its base.

The Coupling Map is used to develop three motion-planning algorithms that exploit the nonlinear characteristics of a space manipulator system to regulate the amount of energy transferred between the manipulator and its base. These are the Base Relocation algorithm, the Hot Spot method, and the Redundancy Resolver algorithm. For free-flying systems, these algorithms are shown to generate paths that, when followed, result in the reduction of fuel required to keep the spacecraft stationary during a manipulator maneuver. For elastically constrained systems, the algorithms succeed in generating paths that, when followed, result in a reduction of residual vibration to the system base. The Coupling Map can also be used in conjunction with pre-filtering techniques to improve their performance.

This thesis also introduces the Pseudo-Passive Energy Dissipation (P-PED) method. Simulation and experimental evaluation show the P-PED method to be an effective closed-loop control scheme for increasing the overall damping of elastically constrained systems and reducing unwanted base vibrations.

A 19-DOF prototype system, representing currently proposed space manipulator systems, is designed and used for simulation and testing of the path-planning and control algorithms proposed in this thesis.

Thesis Supervisor: Steven Dubowsky
Title: Professor of Mechanical Engineering
To my parents Alba, Miguel, Carmen and Pablo for providing encourangement,
to Andrea for providing support and love,
to Miguel and Alexa for providing distraction and giving me the opportunity to see
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Chapter 1

Introduction

This chapter will explain the motivation for this research, review previous work in this area, including a literature survey, highlight the contributions to the field made by this thesis, and, finally, outline how the thesis is organized.

1.1 Motivation

Many future space missions will be aimed at solving fundamental problems vital to the future of mankind. These problems include acquiring greater knowledge of our environment, searching for resources of raw materials and energy, and improving information support, to name just a few [6]. In order to meet these challenges, a number of scientific and technical problems must be solved, including those associated with the construction, inspection and servicing of space systems in orbit. Some of these tasks are currently performed by astronauts in what are known as Extra Vehicular Activities or EVAs. EVAs are dangerous and costly operations and have proven inadequate for some tasks, as evidenced by a recently publicized NASA mission, which required several attempts and, ultimately, three astronauts to retrieve a satellite [16].

Space manipulators have been proposed to assist astronauts in performing some of these tasks. Due to the variety of operations and the unstructured environments in which these operations will be performed, space manipulators are required to be mobile, versatile, dextrous and autonomous. Current proposed concepts of space ma-
nipulators [32, 14, 19], are designed to be either free-floating, free-flying or elastically constrained, that is, mounted a long flexible arm, such as the Space Shuttle's Remote Manipulator System. These arms are designed to be autonomous or teleoperated by an astronaut.

These proposed concepts of space manipulator systems present a number of dynamic and control problems due to the dynamic interaction between the manipulator and its mounting base or space structure. In free-floating systems, the motion of the manipulator will disturb the position and orientation of its uncontrolled base which can reduce the accuracy and compromise the safety of the maneuver. In a free-flying system, the base is controlled by mean of reaction jets, reaction wheels or both. However, the motion of the manipulator can saturate the reaction jets/wheels of its base [24, 25] and degrade system performance. Moreover, the control of the attitude and position of the base of these systems during a manipulator maneuver could require large amounts of fuel which is a limited resource and restricts the life of the system in space. In an elastically constrained system, in which the space manipulator is mounted on a flexible structure, motion of the manipulator can excite low-frequency modes of vibration in its supporting structure. This can seriously degrade the performance of the system and can limit the types of operations that such systems can perform in space.

Clearly, before space manipulator systems are deployed, we must develop a better understanding of their dynamic behavior and, from that understanding, ways to predict, control and plan the motion of these systems. This is the main focus and motivation of this thesis.

1.2 Background and Literature Review

Several studies conclude that space manipulators will become an integral part of the technology necessary for future space missions [2, 5, 92]. Robotic systems can increase productivity, reduce system complexity, lessen the danger to astronauts and cut costs of space missions. The complex life-support system, necessary to sustain life in the
hazardous environment of outer space would not be necessary if robotic manipulators were used in tasks currently conducted by astronauts in EVAs.

Studies have identified a number potential operations to be performed by space manipulators [2, 12, 14, 32, 64]. These include inspection of remote sites, assembly of space structures, servicing of spacecrafts and satellites, refueling, retrieval of tumbling objects in space and contingency operations [61].

In this study the concept of a space manipulator refers to a manipulator that will conduct its operations in outer space and has one or more robotic arms. The concept does not includes robots used in planetary exploration or space probes with no robotic arms.

Several concepts for space manipulator systems that would perform a variety of tasks have been proposed by space scientists and engineers, see Chapter 2. Based on their configurations, these systems can be classified in three categories: free-floating, free-flying and elastically constrained. All of these systems include one or more rigid dextrous robotic arms mounted on a spacecraft or base\(^{1}\). In free-flying systems, the spacecraft carrying the manipulator is controlled by reaction jets, reaction wheels or both. In this way, the entire spacecraft/manipulator system can be arbitrarily translated and/or rotated to any location or orientation in inertial space. In free-floating systems, the manipulator’s spacecraft is uncontrolled so that it is free to move in reaction to forces and torques produced by the manipulator motion. In elastically constrained systems, the manipulator’s base is mounted to a flexible structure, such as a long-reach manipulator system or a flexible space structure.

Much attention has been given to the study of space manipulator systems. Significant contributions have been made in several areas including those related to the dynamic and control, which are the main focus of this thesis.

\(^{1}\)In this thesis, the words base and spacecraft are used to represent the body to which the manipulator is mounted. The word spacecraft applies to free-floating and free-flying systems, while the word base applies to elastically constrained systems.
1.2.1 Modeling Rigid Multibody Systems

The first step towards controlling and planning the motion of space manipulator systems is developing the tools to generate good models. Free-flying and free-floating systems can be modeled as rigid multibody systems. Various formulations for the modeling of multibody systems have been developed in recent years [36, 43, 4]. The robotic community has focused on the Newton-Euler formulation [47], the Lagrangian formulation [33] and recently on Kane's method [39]. All of these formulations result in the same dynamic equations, which describe the motion of a free-floating or free-flying system. However, the equations of motion generated by these formulations are, in general, very complex and do not provide a clear picture of the system's dynamic and kinematic structure, which is fundamental to understanding, controlling and predicting these systems.

In an effort to simplify the equations of motion for free-floating systems, and to get an insight into the dynamic behavior of such systems, Vafa [88, 90] proposed the Virtual Manipulator (VM). The VM takes into account conservation of angular momentum to develop a dynamically and kinematically equivalent system whose base is fixed in inertial space, thus reducing the complexity of the system's dynamic and kinematic equations [89, 91, 90].

Papadopoulos [61, 58] proposed a formulation based on barycenters [34], which takes into account conservation of both linear and angular momentum and further reduced the complexity of the equation of motion for free-floating systems. Papadopoulos' formulation also showed the existence of dynamic singularities [61, 59]. Dynamic singularities are important in the study of all free-floating space manipulators, especially when inverse Jacobian-based algorithms [86] are used to control their motion [61, 60].

Both Vafa and Papadopoulos based their work on the idea of free-floating and free-flying systems in which the spacecraft position and/or orientation may or may not be controlled. Elastically constrained space manipulator systems are a relatively new concept and have not received much attention by the research community. However, a significant amount of work has been done in the modeling and control of elastic
systems in general.

1.2.2 Modeling Flexible Structures and Manipulators

The fundamental principles of elasticity are well known. Closed form solutions to the dynamic equations for relatively simple structures have been thoroughly studied [81, 18]. However, the behavior of machines that exhibit flexibility tend to be highly complex. In general, machines that are modeled by continuous structural elements that move through space lead to highly coupled, nonlinear equations.

Many papers have been published on the subject of modeling and control of flexible systems. Balas [8] gives a good survey of this literature. Also, in aerospace literature, Nurre [57] surveys the areas of dynamics and control of large space structures.

Extensive work has been done in modeling mechanical systems that exhibit flexibility, with the goal of controlling such systems. Some examples are [1, 9, 10, 13, 79, 38, 41, 40, 42, 46, 54, 56, 62, 63, 20, 97]. A series of formulations for simplifying the generation of flexible models has been suggested by Book [11, 15]. Investigators have also looked at computer methods for generating symbolic equations for flexible systems [29, 15]. For complex systems, such as spatial mechanisms, finite element analysis is the preferred tool of many researchers. A short review of the main finite element approaches is given by Thompson [80].

Finite element analysis has been used to model flexible spatial mechanisms, such as flexible-link manipulators [21, 78, 79], and has led to the development of extensive but generalized computer programs for the analysis and simulation of such systems. However, finite element models of flexible spatial mechanisms are very complex and extensive computational effort is required to simulate these systems [21]. Moreover, they do not provide an insight into the dynamic structure of the system and cannot be used in path-planning or control strategies.

In the analysis of flexible-link manipulators, researchers often focus on vibratory behavior, and linear theory is introduced to simplify the analysis [7, 69]. Linear theory is well suited to the study of flexible manipulators at a given configuration but cannot be applied to the analysis of the gross motion of the system.
The number of studies devoted to the modeling and control of compliant base manipulator systems is limited. The literature either focuses on the modeling and control of rigid multibody systems or attempts to apply linear theory to cases in which flexibility cannot be ignored.

1.2.3 Path-Planning and Motion Control

Motion control in space manipulator systems involves the control of both the space manipulator end-effector and its base\(^2\). Path-planning refers to the high level decision process that determines the desired path taken by the manipulator and its base during a maneuver.

Control and path-planning for space manipulator systems is difficult due to the dynamic interaction between the manipulator and its base. Motion of the manipulator will affect the position and orientation of its base [87]. The amount of base disturbance produced by the manipulator motion is a strong function of the path taken by the manipulator in the maneuver [82].

A number of path-planning and control techniques for free-flying/free-floating systems have been proposed. Papadopoulos [61] classifies them in three categories. The first category refers to systems in which the position and orientation of the system's base or spacecraft is controlled by the use of reaction jets, reaction wheels or both. Forces and torques generated by the motion of the manipulator are counteracted by the spacecraft control system. For this system, control laws for earth-bound manipulators can be used. However, the motion of the manipulator can saturate the reaction jet system [24, 25] and can consume large amounts of position and attitude control fuel, limiting its useful life [82].

The second category of control and path-planning techniques includes systems in which the spacecraft's attitude is controlled while its position remains uncontrolled. Several investigators have proposed control algorithms that estimate the required

\(^{2}\)In most cases the system's base is keep stationary in inertial space while the manipulator performs a maneuver but in general the system's base is under control or its motion must be taken into consideration for path-planning.
torque to keep the system's spacecraft orientation fixed using reaction wheels [45]. The VM technique can be used in the modeling, path-planning and workspace analysis of these systems [91].

The third category of control and path-planning techniques involves free-floating manipulators in which both the spacecraft's position and orientation are uncontrolled. Some examples of path-planning and motion control studies in this area [3, 44, 49, 86, 91]. The modeling and analysis of these systems can also be simplified using the VM.

End-point control has also received some attention as a way of dealing with flexible base systems [35]. Alexander and Cannon proposed a control algorithm based on resolved acceleration and successfully implemented it on an experimental two-DOF free-floating manipulator [3]. Their algorithm uses end-point sensing provided by a vision system mounted on the spacecraft.

End-point control depends on the use of end-point sensing which becomes very difficult in an unstructured environment such as space.

Umetani and Yoshida developed a generalized Jacobian-based algorithm for free-floating systems and proposed a control scheme based on resolved rate control [86]. A transposed Jacobian has also been proposed by Masutani et al using a Jacobian derived from the fixed-base system [49]. This algorithm provides good end-point control as long as the system's base mass and inertia are large. For systems with small spacecraft mass and inertia, stability problems are encountered [61].

Spofford and Akin proposed a system that alternates between free-floating and free-flying conditions[73] to reduce fuel consumption while maintaining a large workspace. This work is also motivated by man-machine considerations.

Nakamura and Mukherjee explored the non-holonomic nature of free-floating systems and proposed a path-planning algorithm based on a Lyapunov function [55]. Vafa also proposed a Virtual Manipulator (VM) based planning scheme that uses the redundant nature of the free-floating system to simultaneously control the system's manipulator and its base [91].

The VM technique led to the development of the Disturbance Map (DM) [87]. The
DM showed directions of motion of the manipulator which result in maximum and minimum dynamic disturbances to the spacecraft for a given manipulator configuration. The DM as proposed by Vafa, although insightful and simple, did not provide a convenient graphical representation for the visualization of manipulator-spacecraft dynamic interaction. It also proved inadequate as a tool for path-planning to regulate the amount of disturbance produced by the manipulator motion. Torres [82] recast the DM into a new graphical representation called the Enhanced Disturbance Map (EDM) and used it to develop a number of path-planning algorithms for reducing the amount of fuel spent by the reaction jet system in preventing the manipulator base from moving while the manipulator performs a maneuver [22, 23, 84]. The EDM also aided in the understanding of the dynamic coupling between the manipulator and its base, and served as the foundation of the Coupling Map, which one of the main focus of this thesis.

A significant amount of work has been done in the area of control of mechanical systems that exhibit flexibility [51, 50, 71, 70, 72, 69, 65]. Singer [69] and Meckl [51] proposed a command-shaping technique to reduce the residual vibration in computer controlled flexible systems. This method is based on linear theory but has been applied with some success to a variety of nonlinear systems, including manipulators with flexible links [50]. However, this technique has not been applied to elastically supported systems.

As we conclude this review, we must point out that past work on modeling and control of multibody and flexible systems has generally proposed particular algorithms and shown their validity on a case by case basis. According to our literature review, none of the previously developed algorithms has been evaluated on elastically constrained space manipulator systems. Furthermore, most algorithms developed for the control of flexible systems are based on linear theory. With the exception of the Enhanced Disturbance Map, no method exists that exploits the nonlinear characteristics of moving-base space manipulator for the purpose of regulating the dynamic interaction between the manipulator and its base.
1.3 Contributions of this Thesis

This thesis focuses on the fundamental dynamic characteristics of moving-base space manipulators, with a special interest in the development of new control and path-planning algorithms to regulate the amount of dynamic interaction between the manipulator and its moving base. The word regulate indicates that the path-planning and control methodology developed in this thesis can be used to increase as well as reduce the amount of dynamic interaction between the space manipulator and its moving base. In most cases it is desired to minimize this interaction. However, as will be presented later in this thesis (Chapter 6), for some applications it is desirable to maximize the dynamic interaction between the manipulator and its moving base.

Two types of space manipulator concepts are considered in this investigation. These are free-flying and elastically constrained systems. The dynamic interaction between the manipulator and its moving base is a problem that affects the planning and control of both of these systems. Elastically constrained space manipulator systems is the result of an evolution in the concept of space manipulator systems. It is expected that the first system to be flown will be a rigid manipulator mounted to a long flexible structure and therefore this thesis focuses largely on these type of systems. However, as we will show, the path-planning and control algorithms developed for elastically constrained systems apply directly to free-flying systems as well.

This thesis proposes the Coupling Map as an analytical tool to graphically describe the dynamic interaction between a manipulator and its moving base. Several control and path-planning algorithms are also proposed for reducing the amount of dynamic interaction between the manipulator and its moving base.

The Coupling Map is the result of extending the concept of the Enhanced Disturbance Map [82] to elastically constrained systems. It takes into consideration the mass properties of the system as well as the elastic properties of its base. The Coupling Map graphically describes the sensitivity of the system to the transfer of energy between the manipulator and its base. This energy transfer is fundamental to the behavior of moving-base space manipulators in general. In free-flying systems, this
energy is related to the amount of fuel required to keep the spacecraft stationary during a manipulator maneuver. In elastically constrained systems, this transfer of energy is related to the amount of residual vibrations generated by the motion of the manipulator as it performs its maneuver. Motion of the manipulator arm using Coupling-Map-selected paths results in reduced motion of the system base in free-floating systems, low fuel consumption in free-flying systems and low structural vibration amplitude in compliant base systems. Low disturbance paths generated by the Coupling Map can be used directly or as "good" starting solutions for more computationally intensive numerical optimization methods. In addition, these low disturbance paths can be used to improve the performance of time-domain motion-planning techniques, such as input pre-shaping [51, 71].

Three path-planning algorithms are proposed in this thesis to reduce the amount of energy transferred into the base of a general space manipulator system. These are the Base Relocation algorithm, the Hot Spot method, and the Redundancy Resolver algorithm. All of these use the Coupling Map to exploit the nonlinear characteristics of the system for reducing the amount of energy transferred into the system's base.

Another major contribution of this thesis is the Pseudo-Passive Energy Dissipation (P-PED) method. According to the literature survey conducted for this research, a method such as this has never before been presented. The P-PED method is proposed as an effective control scheme for increasing the overall damping of the system and reducing unwanted base vibrations resulting from either external disturbances or motion of the manipulator, thereby dramatically decreasing the time necessary to perform space manipulator maneuvers. In addition, when used in conjunction with Coupling-Map-selected paths, P-PED is shown to further improve space manipulator performance.

The development of the P-PED also includes a methodology for computing the optimal P-PED control gains. The method uses the Virtual Manipulator to simplify the modeling of the system and the Coupling Map to determine the most desirable configurations of the manipulator to increase the dynamic coupling between the manipulator and it base.
As an example of a general space manipulator system, a 19-DOF prototype system is designed in some detail. This system is representative of currently proposed systems which might be used in future space missions and provides a baseline for simulation and testing of the different path-planning and control algorithms developed in this thesis.

All of the path-planning algorithms proposed in this thesis are evaluated in simulation using several space manipulator models, including the prototype system. The simulations show favorable results. For elastically constrained systems, the simulations show from 14 to 60 percent reduction in the amplitude of the residual vibration for various systems using the Coupling-Map-based path-planning algorithms against conventional paths. In this thesis, the Coupling Map techniques are shown to be applicable to systems with more than three links as long as the planned maneuvers use only two or three links at a time; the rest of the links are not used in the maneuver and do not move.

The P-PED control scheme is also evaluated in simulation in a variety of conditions and is shown to be an effective and robust method for damping out residual vibrations in elastically constrained systems.

This thesis also presents experimental verification of some of the path-planning methods and the P-PED control scheme. These experiments were conducted on the Martin Marietta Harmonic Drive Manipulator Testbed. The results show the proposed methodologies to be viable for practical implementation on space manipulator systems.

Also, as part of this research effort, several special-case path-planning and control techniques were developed. These techniques emerged from the study of special configurations of elastically constrained space manipulator systems and have limited use since they can only be effective in unrealistic systems with very particular mass and elastic properties. However, they provide significant insight into the dynamics of elastically constrained space manipulator systems.
1.4 Thesis Organization

This thesis is organized in eight chapters and eight appendices. In Chapter 2, we present a formal statement of the problem addressed in this thesis. To provide a context for understanding this problem, we then discuss several examples of proposed space manipulator systems, focusing on the concept evolution and some of the problems pertaining to these manipulators. Finally, we present a more detailed description of the problem of manipulator-base dynamic interaction, including a case study that illustrates the importance of solving these problems for future space missions.

Chapter 3 develops the modeling methodology used throughout this thesis. This methodology includes equations of motion for a general free-flying and elastically constrained system. Also developed is the momentum equation, which is fundamental to the development of the Coupling Map. This modeling methodology also forms the basis for the simulation program used to evaluate the different control and path-planning techniques developed in this thesis.

Chapter 4 uses the methodology presented in Chapter 3 to develop the fundamentals of the Coupling Map. The Coupling Map is presented as an evolution of the Disturbance Map and the Enhanced Disturbance Map.

Chapter 5 shows how the Coupling Map can be used as an effective tool for planning manipulator motion that results in low energy transfer between the manipulator and its base. Three path-planning algorithms that can be used in either free-flying or elastically constrained systems are presented. These algorithms are called the Base Relocation algorithm, the Hot Spot method, and the Redundancy Resolver algorithm. This chapter also presents a case study that shows how the Coupling map can be used with complex systems such as a 19-DOF prototype system designed as part of this research effort. Appendix A describes the design of the prototype system and shows a complete detail of the system including its mass properties, actuator characteristics, etc.

Chapter 6 presents the development of the Pseudo-Passive Energy Dissipation Method. The effectiveness of the P-PED method is experimentally demonstrated on
a simple planar two-link elastically constrained system. This chapter also shows a method for computing the optimal P-PED gains for a two-link manipulator.

In Chapter 7, simulations and experimental evaluations of some of the techniques presented in Chapter 5 and Chapter 6 are presented. The simulations were conducted based on the dynamic characteristics the Martin Marietta Harmonic Drive Manipulator (HDM) and the experiments were carried out on the HDM Testbed in Denver, Colorado. (A detailed description of the HDM is presented in Appendix H.) Chapter 7 shows how the P-PED method can be used in conjunction with the Coupling Map to aid in reducing the amount of vibration produced as a result of a manipulator maneuver. Experimental results also show that the Coupling Map can be used to improve the performance of pre-filtering techniques.

Finally, Chapter 8 summarizes this thesis and suggests some directions for future research.
Chapter 2

Space Manipulator Systems: Concept Evolution and Problem Description

2.1 Introduction

This chapter begins by presenting a formal statement of the problems addressed in this thesis (Section 2.2). In order to provide a better understanding of the context and significance of those problems, Section 2.3 briefly reviews several examples of proposed space manipulator systems, focusing on their concept evolution and some of the issues related to their control. Finally, Section 2.4 presents an expanded problem description, including a case study that illustrates the importance of solving these problems for future space missions.

2.2 Problem Statement

This thesis solves the problem of regulating, through motion planning and control, the dynamic interaction between a space manipulator and its moving base. This problem is considered for both free-flying and elastically constrained systems. In free-flying systems, the problem is manifested as one of fuel minimization, while in elastically
constrained systems, the problem becomes one of vibration control.

In order to solve this problem, this thesis also addresses the need for new analytical tool for graphically describing the complex dynamic behavior of space manipulator systems. Such an analytical tool is critical for the understanding, control and design of future systems.

2.3 Space Manipulators: Concept Evolution

Inspired by recent advancements in computer and robotic technology and motivated by future objectives of space missions, such as inspection, assembly of space structures, servicing of spacecrafts and satellites, and refueling operations [61], the concept of a space manipulator has been suggested to perform many of the tasks currently carried out by astronauts.

One proposed space manipulator concept is based on a long-reach mechanical arm carried by a spacecraft. An example of this concept is the Space Shuttle's Remote Manipulator System (RMS), shown Figure 2-1. This system can operate in either a free-floating or free-flying configuration. In its free-floating configuration, the Space Shuttle acts as the system's moving base and is free to move in reaction to the RMS motion. The shuttle's motion is most significant when the RMS grasps a large object, such as a large satellite. In a free-flying configuration, the Space Shuttle's reaction jets, reaction wheels or both are used to counteract the effect of the manipulator motion.

The RMS provides a large workspace and has allowed tasks such as satellite retrieval to be carried out. However, this type of space manipulator system is limited in the kinds of operations that it can perform, since it must be operated slowly in order to avoid dangerous vibrations. An astronaut aboard the Space Shuttle must wait between 20 and 40 seconds for the manipulator to settle after a move [69].

Unlike the RMS, future space manipulator systems must be mobile, versatile, dextrous and autonomous. These requirements have led to new concepts in space manipulator systems. For example, the EVA Retriever [14], shown in Figure 2-2, was
Figure 2-1: The Space Shuttle and its Remote Manipulator System. (Illustration taken from the cover of the proceedings of the ASME Annual Winter Meeting, 1983, See Reference [95])
conceived as an autonomous free-flying system used in satellite retrieval, remote site inspection and contingency operations. This manipulator is equipped with reaction jets which allow it to free-fly and cover a large workspace. Since motion of the manipulator can affect the position and orientation of the system’s base, the reaction jet system is also used to keep the system’s base stationary with respect to its orbit while its robotic arms perform tasks. Fuel consumption is a concern in this type of operation since fuel is a limited resource which determines the useful life of the system in space. Fuel consumption during a manipulator maneuver is a function of the amount of dynamic interaction between the manipulator and its base [myMSthesis].

As the concepts of free-floating and free-flying manipulators continued to develop, issues of performance led to several studies of the systems' complex dynamic behavior. Major contributions were made which helped researchers better understand the complexities of these systems, see Section 1.2. Because their characteristics free-flying systems and long-reach, large manipulators are inadequate for operations that require dexterity and precise motion. Free-flying systems can spent large amounts of
attitude and position control while the manipulator performs a maneuver, and long-reach, large manipulators must be operate slowly to avoid excite large vibrations. To solve these problems, it was suggested that a manipulator free-fly to within reach of its target and then use one of its arms to attach itself to a space structure near its target, and perform its task. Assuming there is a place for the manipulator to attach itself, this strategy eliminates the need for reaction jets to keep the system's base stationary while the manipulator performs its maneuvers. The Flight Tele-robotic Servicer (FTS)[48, 32], shown in Figure 2-3, was designed with this strategy in mind. The FTS is a variation of the EVA Retriever, designed to operate either in a free-flying mode or attached to a space structure by means of its third arm, see Figure 2-3.

With the introduction of this new configuration, new issues of dynamic behavior arise. Since space structures are inherently flexible [57], the motion of an attached manipulator arm can cause low-frequency vibrations in the supporting structure. These vibrations can result in damage to the supporting structure, increase to time to perform a task, lessen the safety of the maneuver, and make control of the system difficult. This type of system can be modelled as a rigid-link manipulator coupled to an inertial ground by an elastic member, and, thus, is called in this thesis an **elastically constrained space manipulator system**.

Another example of an elastically constrained system is found on the proposed NASA Space Station Freedom. Freedom is intended to be a manned facility designed to serve as a permanent multipurpose research laboratory, servicing, storage and staging base [96]. In order to meet the objectives of this facility, the Space Station Remote Manipulator System (SSRMS) and the Special Purpose Dextrous Manipulator (SPDM) [19] have been proposed, see Figure 2-4.

Figure 2-4 suggests the use of the SPDM at the end of the SSRMS, thus creating an elastically constrained system. In this configuration, the low-bandwidth, less accurate SSRMS provides a large working envelope, while the smaller, more accurate SPDM provides fast and precise motion. The SPDM has also been designed to attach itself to predetermined points along the outer structure of the space station to perform
Figure 2-3: The Flight Telerobotic Servicer (FTS). (Illustration taken from [32])

various tasks.

The Japanese Experimental Module (JEM)\[94\] is another example of small/large manipulator design. It consists of a large Remote Manipulator System (RMS) and a Small Fine Arm (SFA) configuration for precise manipulation.

Systems like the FTS, the SPDM and the JEM-RMS/SFA are at the forefront of today's space manipulator technology. Their design meets many of the requirements of future space missions, which will involve assembly, inspection, and other extra vehicular activities. However, the dynamic interaction between the manipulator and its base is a problem that affects the performance of both free-flying and elastically constrained systems. Before these systems are deployed, new control laws and path-planning algorithms must be developed.

The following section discusses in greater detail the problem of dynamic interaction between the manipulator and its moving base and a case study is presented to help better understand the importance of motion-planning in space manipulator systems.
Figure 2-4: The Proposed Space Station Remote Manipulator System (SSRMS) and Special Purpose Dextrous Manipulator (SPDM). (Illustration provided by NASA Langley Automation Technology Branch)
2.4 Problem Description

The dynamic interaction between the manipulator and its spacecraft, base or supporting structure is a fundamental problem exhibited by all the systems mentioned earlier in this chapter. Motion of the manipulator will disturb the position and orientation of its base. In free-flying manipulators, such as the EVA Retriever and FTS, the reaction jet system can compensate for these disturbances [82]. However, fuel is a limited resource and can limit the life of the system in orbit. Moreover, fast manipulator motion can exceed the saturation levels of the reaction jets, causing excessive spacecraft motion [25].

In elastically supported systems, such as the FTS attached to a flexible space structure or the SPDM mounted at the end of the SSRMS, motion of the manipulator can excite low-frequency vibrations in its supporting structure. This can greatly limit the kinds of operations that this system can carry out and can increase the time required to perform such tasks.

The amount of dynamic interaction between the manipulator and its moving base is a complex phenomenon. In general, it is a function of the instantaneous direction of motion of the manipulator, the velocity of the maneuver, the mass properties of the system, including its base and payload, the stiffness of the compliant structure (for elastically constrained systems\(^1\)), and the configuration of the system. In addition, due to the non-holonomic nature of space manipulators [55, 87], the total amount of dynamic disturbance generated by the manipulator motion depends on the path taken by the manipulator in joint space. As mentioned above, this disturbance is fundamental to the problem of fuel consumption in free-flying systems and to the problem of vibration control in elastically constrained systems. In order to appreciate the importance of the path dependency phenomenon in space manipulator systems and its effect on fuel consumption in free-flying systems, let us look at the following simple example.

\(^1\)In free-floating systems this stiffness is zero. For free-flying systems this stiffness is associated with the close loop gains of the spacecraft position and attitude control system. See Chapter 3 for a complete description of the modeling methodology used in this thesis.
Table 2.1: Fuel Required to Keep the Spacecraft Stationary for the Three Maneuvers.

<table>
<thead>
<tr>
<th></th>
<th>Mass (kg)</th>
<th>cm (m)</th>
<th>Inertia (kg.m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>1.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Link 1</td>
<td>1.0</td>
<td>-3.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Link 2</td>
<td>1.0</td>
<td>-5.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

![Diagram of a two-link planar free-flying manipulator]

Figure 2-5: A Two-Link Planar Free-Flying Manipulator.

2.4.1 Problem Description: A Simple Example

Figure 2-5 shows a two-link Planar free-flying manipulator. Its mass properties are shown in Table 2.4.1. Three different paths were computed that take the manipulator’s end-effector from an initial to a final configuration in inertial space. Figure 2-6 shows the three paths in both inertial and joint space. The position and orientation of the spacecraft in this system is controlled by reaction jets under PD control. The task is to move the manipulator’s end-effector from an initial position, point A, to a final position, point B (see Figure 2-6) while keeping the base stationary. In this example, the maneuvers were designed to be completed in the same amount of time (2 seconds).

The total amount of fuel required to keep the spacecraft stationary for each of the
Figure 2-6: Three Arbitrary Paths

Table 2.2: Fuel Required to Keep the Spacecraft Stationary for the Three Maneuvers.

<table>
<thead>
<tr>
<th>Path</th>
<th>Fuel (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path 1</td>
<td>50.3</td>
</tr>
<tr>
<td>Path 2</td>
<td>15.5</td>
</tr>
<tr>
<td>Path 3</td>
<td>210.2</td>
</tr>
</tbody>
</table>
three cases was computed using RiBS\textsuperscript{2} [83] and the results are shown in Table 2.4.1.

Notice that for this example, there is a considerable difference in fuel usage between paths for the same initial and final manipulator configuration and it is not obvious which path will require less fuel. This example demonstrates the strong relationship between the path taken by the manipulator and the amount of fuel spent by the position-attitude control system in keeping the spacecraft stationary during a manipulator maneuver.

Clearly, a methodology for computing low-fuel paths is critical to the operation of future free-flying space manipulator systems in order to maximize the useful life of the system and reduce the cost of transportation and storage of reaction jet fuel. Moreover, since consumption of fuel can determine the feasibility of a space mission, this kind of methodology could allow missions that would otherwise be impossible.

If the system shown in the case study were attached to a compliant structure, these paths would have produced different levels of residual vibrations to the supporting structure depending on the compliant characteristics of the structure. Specific examples of elastically constrained systems are considered throughout the rest of this thesis and, therefore are not discuss in detail at this time. However, it should be noted here that, in elastically constrained systems, a methodology for vibration control through motion-planning is necessary in order avoid damaging the system’s supporting structure, increase the accuracy and safety of the maneuver, and reduce the time to perform a task.

2.5 Understanding Dynamic Behavior

Clearly, before a solution to the problem of dynamic interaction in space manipulators is found, an analytical tool must be developed to aid in the understanding, prediction and control of this complex phenomenon. This thesis addresses the problem of developing such a tool. Since visual thinking is an integral part of scientific and engineering communication, and often presents some advantage over conventional mathematical

\textsuperscript{2}The RiBS Library is a dynamic simulation package developed as part of this research.
formulations, a graphical representation is sought as a powerful, efficient and intuitive tool [27].

2.6 Summary

In summary, this thesis focuses on the problem of motion-planning in space manipulator systems to reduce the amount of dynamic interaction between a space manipulator and its mobile base. The problem is dealt with for two types of space manipulator systems: free-flying systems and elastically constrained systems.

For free-flying systems, this thesis addresses the problem of motion-planning to reduce the amount fuel spent by the system's position and attitude control systems in keeping its base stationary while the manipulator performs a maneuver.

For elastically constrained systems, this thesis addresses the problem of motion planning to reduce the amount of residual vibration produced by movements of the manipulator on its supporting structure.

An analytical tool is sought to aid in the understanding, prediction and motion-planning of free-flying and elastically constrained space manipulator systems.
Chapter 3

Modeling of Space Manipulator Systems

3.1 Introduction

In this chapter a model of a space manipulator system is presented. This dynamic model forms the basis for a dynamic simulation package developed as part of this research effort and is used throughout this thesis in the analysis and evaluation of the different path-planning and control algorithms proposed in this thesis. This dynamic model is based on a general $n$ DOF rigid-link manipulator mounted on a 6 DOF base that can be either elastically constrained or controlled by reaction jets.

3.2 System Dynamic Model

Consider the $n$ DOF rigid-link space manipulators depicted in Figure 3-2 and Figure 3-1. The types of systems we are interested in can be either free-flying or elastically constrained. Although these two systems have fundamentally different dynamic behavior, this thesis proposes a simplified model that can be used to describe the dynamic behavior of both systems, depending on the physical interpretation of the model parameters. As with any modeling methodology, the development of this dynamic model is based on a number of assumptions. These assumptions are discussed
3.2.1 Assumptions

The first assumption relates to the environment in which these types of manipulators will operate. Space manipulators are designed to operate in the microgravity environment of outer space. Thus, no gravity terms are included in the formulation of this model.

The manipulator arm is modelled as an n DOF open kinematic chain. The manipulator links are assumed to be rigid with no compliance in the powertrain of their
Elastically Constrained Systems

Typically, the space structure or large flexible manipulator to which our rigid space manipulator would be attached would be made of very light and strong materials, such as exotic alloys and composite materials.

It is well documented that the Space Shuttle Remote Manipulator System exhibits low-frequency vibration characteristics which range from 0.35 to 3.2 Hz, depending on its configuration [1, 31, 85]. These numbers are based on a no-payload configuration. When the RMS grabs an object, say the Flight Telerobotic Servicer, this frequency is expected to be even lower for some vibratory modes. Based on this scenario, the vibration characteristics of the elastically constrained systems considered in this study will be assumed to range from 0.5 to 1.0 Hz.

It is assumed that the dynamic effect of the distributed mass of the system's supporting structure is small compared to the dynamic effect of the mass of the manipulator's base at these frequencies. Based on this assumption, the space manipulator's base is modelled as a rigid body that possesses a given mass and inertia and is connected to an inertial ground through a massless 6 DOF spring/damper system.

The elasticity of the system's supporting structure enters the formulation of this model as a symmetric positive-definite stiffness matrix $K_b \in \mathbb{R}^{6 \times 6}$.

It is also expected that space structures would exhibit low damping characteristics, which enter the model formulation as a $6 \times 6$ symmetric positive-definite damping matrix $D \in \mathbb{R}^{6 \times 6}$. For the systems considered here, the damping ratio is assumed to range from 0.01 to 0.05.

Finally, it is assumed that the manipulator arm exhibit faster dynamic characteristics than that of its supporting base.

Free-Flying Systems

In a free-flying system, the base position and orientation are controlled by reaction jets, reaction wheels or both. In this study, it is assumed that the space manipulator
will control its base position and attitude using reaction jets only. The problem of fuel consumption is must important for systems with reaction jets since they depend on unrenewable fuel resources.

Due to the way in which reaction jet systems are operated, typically in an on/off mode, the modeling of the control of the attitude and position of the manipulator's base is difficult. In order to simplify the modeling of this system, it is assumed that the reaction jets will be pulse-width modulated and that the modulating frequency will be high enough to be unnoticied by the relatively low bandwidth of the system. Based on this assumption, the reaction jet system is composed of linear actuators in a simple proportional-derivative (PD) control loop. For the case in which the reaction jet system is used to counteract the effect of manipulator motion, the closed loop PD gains make the system behave as if the base of the manipulator were sitting on a rather stiff 6 DOF spring/damper system.

We can conclude that, given the assumptions made above, the dynamic behavior of the base in both free-flying and elastically constrained systems can be modelled using the same formulation, namely, a $6 \times 6$ stiffness and damping matrix. For elastically constrained systems, $K_b$ and $D_b$ represent the actual stiffness and damping characteristics of the system's base, respectively. In free-flying systems, the matrices $K_b$ and $D_b$ contain the PD gains of the position-attitude control system. The magnitudes of $K_b$ and $D_b$ are typically less in elastically constrained systems than in free-flying systems. Thus, free-flying systems are modelled here as stiff, well-damped elastically constrained systems.

This relationship between free-flying and elastically constrained systems is seen again in Chapter 4 as we explore the properties of the Coupling Map.

3.2.2 Equations of Motion

We begin writing the equations of motion for the free-flying and elastically constrained systems depicted in Figures 3-2 and 3-1 by defining a convenient set of generalized coordinates. For this model, we choose the following coordinates:
$X \in \mathbb{R}^3$ representing the position of the center of mass of the system's base measured with respect to a Newtonian reference frame,

$\Theta$ representing the orientation of the system's base given by either Euler angles or Roll, Pitch, Yaw angles and

$q \in \mathbb{R}^n$ representing the displacement of the manipulator joints.

Based on this choice of generalized coordinates, a generalized coordinate vector can be defined as:

$$\xi = [X, \Theta, q]^T$$  \hspace{1cm} (3.1)

A corresponding generalized force/torque vector is defined as $\Xi = [\tau_X, \tau_\phi, \tau_q]^T$. The system's equations of motion can be written as [18]:

$$H(\xi)\ddot{\xi} + C(\xi, \dot{\xi})\dot{\xi} + D\dot{\xi} + K\xi = \Xi$$  \hspace{1cm} (3.2)

where $H(\xi) \in \mathbb{R}^{n+6 \times n+6}$ is a symmetric, positive-definite inertia matrix, $C(\xi, \dot{\xi})\dot{\xi} \in \mathbb{R}^{n+6}$ is the force/torque vector accounting for centrifugal and Coriolis effects,

$$D = \begin{bmatrix} D_b & 0 \\ 0 & D_q \end{bmatrix} \in \mathbb{R}^{n+6 \times n+6}$$  \hspace{1cm} (3.3)

is a symmetric positive-definite matrix representing the overall damping characteristics of the system, and

$$K = \begin{bmatrix} K_b & 0 \\ 0 & K_q \end{bmatrix} \in \mathbb{R}^{n+6 \times n+6}$$  \hspace{1cm} (3.4)

is a symmetric, positive-definite matrix representing the overall stiffness characteristics of the system. Notice that the base stiffness matrix, $K_b$, and the base damping matrix, $D_b$, come into the formulation as part of the overall stiffness and damping characteristics of the system.

The matrices $D_q \in \mathbb{R}^{n \times n}$ and $K_q \in \mathbb{R}^{n \times n}$ represent the damping and stiffness characteristics of the manipulator joints, respectively. When the manipulator is controlled
using localized PD control, these matrices become diagonal matrices representing the derivative and proportional joint gains.

The generalized force/torque vector \( \Xi \) contains the control forces/torques for the manipulator base and joints. In elastically constrained systems, \( \Xi \) has the following form:

\[
\Xi = \begin{bmatrix}
0 \\
0 \\
\tau_q
\end{bmatrix}
\]  \hspace{1cm} (3.5)

The zeros in Equation (3.5) represent the 6 uncontrolled DOF of the base and \( \tau_q \) the \( n \) DOF joint force/torque vector.

### 3.2.3 Momentum Equation

Based on the selection of generalized coordinates, see Equation (3.1), a generalized momentum vector \( \pi \) can be written as:

\[
H(\xi)\dot{\xi} = \pi
\]  \hspace{1cm} (3.6)

where the elements of \( \pi = [\pi_X, \pi_\theta, \pi_q]^T \) correspond to the components of the generalized momentum in the direction of the generalized coordinates. Equation (3.6) is used in Chapter 4 as part of the development of the Coupling Map. Equation (3.2) is the basis of a simulation program called RiBS [83] and is used throughout this thesis in the analysis and evaluation of the proposed path-planning and control algorithms.

### 3.3 Summary

This chapter has introduced the dynamic model to be used as the bases for simulation and analysis and evaluation of the different path-planning and control algorithms proposed in this thesis. This chapter has also developed the momentum equation, which is fundamental to the development of the Coupling Map in Chapter 4. This modeling methodology forms the basis for the simulation program called RiBS, [83]
used to evaluate the different control and path-planning techniques developed in this thesis.
Chapter 4

The Coupling Map

4.1 Introduction

This chapter presents the theoretical foundation of the Coupling Map. The Coupling Map is an extension of the Disturbance Map (DM) and the Enhanced Disturbance Map (EDM). It is conceived as an analytical tool to graphically describe the nonlinear dynamic interaction between a space manipulator and its moving base. While the DM and EDM are applicable to only specific types of systems, namely, free-floating and free-flying systems, the Coupling Map can describe a broader range of systems, including free-floating, free-flying and elastically constrained systems.

4.2 Foundation of the Coupling Map

4.2.1 The Disturbance Map

Vafa [87] introduced the concept of the Disturbance Map (DM) as a method for describing the nonlinear dynamic behavior of a space manipulator and its free-floating base. This concept, inspired by Shiller's acceleration lines for manipulator time-optimization [68, 67], was designed to graphically depict the directions in joint space of minimum and maximum disturbance of a spacecraft’s attitude due to manipulator movement. Vafa's work dealt only with the relationship between the manipulator...
Figure 4-1: Example of a Disturbance Map for a Two-Link Manipulator

motion and the spacecraft’s attitude; the relationship between the absolute position of the spacecraft and the movement of the manipulator was not considered. An example of the Disturbance Map as conceived by Vafa is shown in Figure 4-1.

The Disturbance Map is constructed based on conservation of angular momentum. Based on the equations of motion derived for the Virtual Manipulator a relationship between infinitesimal manipulator joint motion, $\delta q$, and infinitesimal base angular displacement, $\delta \theta$ is written as

$$\delta \theta = F \delta q$$  \hspace{1cm} (4.1)

were the matrix $F$ is called the Angular Disturbance Matrix [87]. A singular value decomposition [77] of matrix $F$ yield directions in joint space of maximum and minimum angular disturbance. These direction are represented in Figure 4-1 as arrows which indicate directions and magnitudes of spacecraft attitude disturbances. The
larger the disturbance, the longer the vector, which shows in the direction of movement. For the two-link planar case the magnitude of the minimum disturbances is always zero.

Vafa suggested the idea that the DM could be used to select paths of low angular disturbance, which might then result in low energy expenditure by the spacecraft’s attitude control system. Specifically, he speculated that, by reducing the amount of angular disturbance to the spacecraft, the reaction wheels system would consume less photo-voltaic electricity, allowing the space manipulator to perform more tasks [87, page 98].

The DM served as an important step towards understanding, through a graphical representation, the complex dynamic interaction between a space manipulator and its base. However, its format made it difficult to visualize and understand the valuable information it provided, and was not adequate for effective path-planning.

4.2.2 The Enhanced Disturbance Map

In an effort to permit a better understanding of the dynamic interaction between a space manipulator and its base, Torres [82, 22, 23, 84] recast the DM into the Enhanced Disturbance Map (EDM).

In this new format, as shown in Figure 4-2, directions of minimum disturbance are represented by continuous lines, which are called Minimum Disturbance Lines. Motion of the manipulator along the Minimum Disturbance Lines results in minimum angular disturbance to the spacecraft, while motion perpendicular to the Minimum Disturbance Lines results in maximum angular disturbance. The magnitudes of the maximum disturbances are shown by shading the lines proportional to the maximum disturbance magnitude at any given point. This shading makes obvious high and low maximum disturbance areas, called hot spots and cool spots, respectively.

The EDM shown in Figure 4-2 is for a two-link planar manipulator similar to the one used to compute the DM shown in Figure 4-1 and therefore the magnitude of the disturbance for motions along the minimum disturbance lines is zero. The relative magnitude of the disturbance for motion of the manipulator in any direction

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Figure 4-2: Example of an Enhanced Disturbance Map for a Two-Link Manipulator is illustrated in Figure 4-3.

The EDM offered a number of advantages over the DM. One of the most significant was the convenient and clear way in which the information was conveyed. The use of continuous lines rather than arrows to indicate directions of minimum-disturbance motion, and shading to indicate the magnitudes of maximum disturbances, allowed for a more comprehensive and intuitive understanding of the dynamic interaction between a space manipulator and its base.

In addition, the continuous lines and hot/cool regions helped researchers visualize low-disturbance paths and led to the development of several heuristic path-planning algorithms for reducing the amount of angular disturbance to the base produced by manipulator motion. These algorithms were used successfully to generate paths that resulted in low fuel usage when reaction jets were commanded to keep the base stationary during manipulator maneuvers [82].

The DM and the EDM are two distinct representations of the same information. This information is given in a mathematical way by what is known as the Angular Disturbance Matrix, see Equation (4.1). The research that led to the development of the EDM also led to the development of a more efficient way of computing the Angular Disturbance Matrix. This computational method played an important role in the development of the Coupling Map, as will be seen later in this chapter.
Figure 4-3: The Locus of the Magnitude of the Disturbance for and Arbitrary Point in the Enhanced Disturbance Map. (For a Two-Link Planar Manipulator)

The EMD succeeded in providing a better understanding of the complex dynamic behavior of free-floating and free-flying space manipulator systems [82, 22, 23, 84], and pointed to new directions for research. Most significantly, it led to the consideration of the problem of vibration in elastically constrained space manipulator systems. The strong relationship between dynamic disturbances and fuel consumption that was demonstrated through the EDM study [82, 22, 23, 84] suggested that, similarly, there might be a relationship between dynamic disturbances and residual vibrations in elastically constrained space manipulator systems. This realization led to the development of the Coupling Map and related motion-planning algorithms.

4.2.3 The Coupling Map

The Coupling Map is formulated to deal with a broader range of systems than the EDM, namely, free-floating, free-flying, and elastically constrained systems. As mentioned earlier, the EDM was formulated to deal with angular disturbances, but failed to address disturbances of a translational nature. This meant that it was useful as an aid in understanding of the relationship between the manipulator motion and the spacecraft's attitude, but did not illustrate the relationship between the absolute position of the spacecraft, or base, and the movement of the manipulator.
In elastically constrained systems, residual vibration is a function of both angular and translational disturbances. Thus, a new formulation is needed to deal with these more general types of systems. In this study this new formulation is called the Coupling Map. The term coupling is introduced to refer to the property of two dynamic systems, namely the manipulator arm and its mounting base, to mutually influence one another, in this case by transferring energy from one to the other.

Before discussing the development of the Coupling Map it should be recognized that this formulation is largely based on fundamental mechanics. As part of this development a number of assumptions were made. At some point throughout the development some of the assumptions will be violated but as it will be shown later in this thesis, simulations as well as experimentation yield good results.

The development of the Coupling Map begins by recalling the system's momentum equation (Equation (3.6)), derived in Chapter 3, and assuming that the system is initially at rest, with zero initial deflection in the elastic supporting structure. Also, it is assumed that no external forces/torques are acting on the manipulator. With these assumptions made, Equation (3.6) becomes

\[ H(\xi)\dot{\xi} = \pi = 0 \]  

(4.2)

Based on the choice of generalized coordinates (see Equation (3.1), page 46), the inertia tensor \( H(\xi) \) can be written as

\[ H(\xi) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \]  

(4.3)

where \( A \in \mathbb{R}^{3\times3} \) is a symmetric positive-definite submatrix relating the linear and angular velocity vectors of the system's base to its linear and angular momenta. \( B \in \mathbb{R}^{3\times3} \) is a mass submatrix relating the motion of the manipulator joints to the system's base linear and angular momenta. Submatrix \( C = B^T \) and submatrix \( D \in \mathbb{R}^{n\times n} \) relates the manipulator joint velocities to their momenta.

To simplify this development, let us define
\[ \Phi = [X, \Theta]^T \]  
(4.4)

Using Equation (4.3) and recalling that \( \dot{\Phi} = [\dot{\Phi}, \dot{\Theta}]^T \), Equation (4.2) is solved for \( \dot{\Phi} \), to yield

\[ \dot{\Phi} = -A^{-1}Bq \]  
(4.5)

By replacing the derivative operation with a variation operator and letting \( G = -A^{-1}B \), Equation (4.5) becomes

\[ \delta\Phi = G\delta q \]  
(4.6)

The matrix \( G \) is called the system’s *Disturbance Matrix* and relates infinitesimal manipulator motion, \( \delta q \), to infinitesimal base motion \( \delta\Phi \) for a free-floating system [87]. Notice that the Angular Disturbance matrix, matrix \( F \) in Equation (4.1) is a contained in matrix \( G \). For elastically constrained and free-flying systems, this physical interpretation does not apply, since Equation (4.6) is based on the assumption of zero external forces/torques acting on the system. For the type of elastically constrained system depicted in Figure 3-2 (page 43), the quantity \( \delta\Phi \) represents the measure of the dynamic coupling between the manipulator and its supporting structure, and is called the *Instantaneous Base Dynamic Disturbance Vector*. By recalling Equation (4.4), we can see that the Instantaneous Base Dynamic Disturbance Vector, \( \delta\Phi \) is composed of the *Instantaneous Translational Disturbance Vector*, \( \delta X \) and *Instantaneous Angular Disturbance Vector*, \( \delta\Theta \).

For systems which consist of a spacecraft-borne manipulator in which the spacecraft is kept stationary in inertial space by spacecraft attitude control reaction jets, see Figure 3-1, the vector \( \delta\Theta \) has been shown to be proportional to the reaction torques required to keep the spacecraft stationary [87]. Also, the integral over time of \( \delta\Theta \) has been shown to be proportional to the amount of fuel required by the reaction jets in keeping the spacecraft stationary during the maneuver [82].

For elastically constrained systems, the force/torque vector, \( \tau_\bullet \), exerted by the
supporting structure to restrain the manipulator base is also proportional to the
dynamic disturbance $\delta \Phi$ or

$$
\tau_\phi \sim \delta \Phi
$$

These forces/torques are equal in magnitude and opposite in direction to the forces/torques
acting on the structure. Over a given manipulator motion, these forces/torques result
in energy being transferred into the supporting structure. The residual vibration ex-
cited by the manipulator motion is directly related to these forces/torques. So, for a
system with given stiffness characteristics and a given speed of maneuver, manipulator
paths that produce low instantaneous disturbances, $\delta \Phi$, will produce relatively small
forces/torques acting on the supporting structure. Hence, relatively small amounts
of energy will be transferred to the structure and low residual vibrations will be ex-
pected. While it has not been possible to rigorously prove these points, they are
consistent with the physics of the system and as will be shown later in this thesis,

For a given set of disturbance $\delta \Phi$, the energy transferred to the supporting struc-
ture depends on the stiffness characteristics of structure. It can be shown, see Ap-
pendix F, that a softer spring will store more energy per unit force than a stiffer one.
Since $\tau_\phi \sim \delta \Phi$, a softer structure should absorb more energy for a given $\delta \Phi$ than a
stiffer structure.

The amount of energy that a given $\delta \Phi$ can transfer to the system's supporting
structure depends on the direction of the disturbance for a non-homogeneous structure.

The strain coenergy $V^*$ [18], stored in the system's compliant structure can be
written in terms of its stiffness matrix, $K_b$, and the forces/torques exerted on this
structure by the manipulator's base as:

$$
V^* = \frac{1}{2} \tau_\phi^T K_b^{-1} \tau_\phi
$$

Using the relation given in Equation (4.7), the strain coenergy introduced into the
structure by a given disturbance $\delta \Phi$ becomes

$$
V^* \sim \delta \Phi^T K_b^{-1} \delta \Phi
$$
Figure 4-4: A Typical Coupling Map for a Two-Link Manipulator Mounting on a Compliant Base

By recalling Equation (4.6), Equation (4.9) can be written as

\[ V^* \sim \delta q^T G^T K_b^{-1} G \delta q \]  

(4.10)

If we define a matrix \( Q = G^T K_b^{-1} G \), Equation (4.10) can be written as

\[ V^* \sim \delta q^T Q \delta q \]  

(4.11)

The matrix \( Q \) is called the Coupling Matrix and is a function of the manipulator's configuration and the stiffness characteristics of the base. The Coupling Matrix is a measure of the sensitivity of the transfer of strain energy to the system's supporting structure as a function of the manipulator motion. The structure's stiffness matrix, \( K_b \), enters into the formulation in a way such that directions of base motion with low stiffness carry a higher weight than those of high stiffness.

A singular value decomposition [77] of the Coupling Matrix \( Q \) yields directions and magnitudes of maximum and minimum energy coupling in configuration space defined by the manipulator joint variables \( q \). The plotting of lines representing the directions
of minimum energy in joint space is what is called the Coupling Map. Figure 4-4 shows the Coupling Map for a two-link planar space manipulator mounted at the end of the Space Shuttle’s Remote Manipulator system. See Appendix G and Appendix B for the mass properties and stiffness characteristics of this system, respectively.

### 4.3 Properties of the Coupling Map

The shaded lines in the Coupling Map (see Figure 4-4) are called Minimum Coupling Lines. Their slopes represent local directions in joint space for which motion of the manipulator tangent to these directions will result in relatively low energy transfer to the system’s supporting structure. Moving perpendicular to the minimum coupling lines result in relatively high energy transferred between the manipulator and its supporting structure. The relative strength of the coupling for motions perpendicular to the minimum coupling lines depends on the configuration of the manipulator, or its location in joint space. This information is represented in the form of shaded regions. Regions with dark shading, called hot spots, represent areas in which moving perpendicular to the minimum coupling lines results in relatively high coupling compared movement in lighter or cool spots regions.

The hot spots of the Coupling Map show the relative sensitivity of the system to the transfer of energy between the manipulator and its supporting structure as a function of the configuration of the manipulator. It is important to notice that energy can flow to or from the system’s supporting structure. For path-planning, it is desirable to minimize the coupling between the manipulator and its supporting structure, thus reducing the capacity of the system to transfer energy from the manipulator to its base. However, if the structure is vibrating and the manipulator is used to actively dissipate this vibration, it is desirable to maximize the coupling between the manipulator and its supporting structure. The Pseudo-Passive Energy Dissipation Method, introduced in Chapter 6, can be used in combination with the Coupling Map to maximize the performance of the manipulator as an active damper.
4.3.1 The Coupling Map in Higher Dimensional Spaces

The Coupling Map as a graphical tool cannot be easily used to represent, on paper, dynamic interaction for systems with more than three links. However, the concept can be extended to higher dimensional spaces since there are no constraints on the size of the Coupling Matrix $Q$. For the two dimensional case shown in Figure 4-4, the lines of minimum coupling represent one-dimensional manifolds in a two-dimensional space [77]. For higher dimensional spaces, these minimum coupling manifolds can have more than one dimension, such as is the case of the Coupling Map for a three-link planar redundant manipulator. For this case, discussed later in this thesis, if the manipulator motion is constrained to a given two-dimensional manifold or surface, low energy coupling between the manipulator and its supporting structure will result.

A two- or three-dimensional Coupling Map can be used to graphically describe systems with more than three links. This is possible if the manipulator's maneuvers are limited to the motion of two or three links at a time or if some of the links are relatively small and possess low mass so that their effect on the base is negligible (an example of this are the wrist links of most manipulators [75]). This will be seen again in Chapter 5.3 to plan low vibration path in a 19 DOF prototype system.

Algorithms for higher dimensional spaces can be developed based on the understanding of the structure of the Coupling Map in higher dimensional spaces.

4.3.2 The Coupling Map and Free-Flying Systems

In order to apply the Coupling Map to free-flying space manipulator systems, we must go back to the dynamic model developed in Chapter 3. Recall that this dynamic model is proposed to describe the behavior of both free-flying and elastically constrained systems, depending on the interpretation of the model parameters, or, more specifically, the interpretation of the $K_b$ and the $D_b$ matrices.

Notice in Equation (4.10) that the matrix $K_b$ enters into the formulation of the Coupling Matrix as a weighting matrix. For elastically constrained systems, this weighting matrix is necessary to account for the fact that motions of the base that

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The Coupling Map describes the sensitivity of manipulator motion to the transfer of energy into the system's supporting structure or spacecraft. This transfer of energy is a complex phenomenon and, in order to simplify its analysis, several assumptions have been made during the development of the Coupling Map. In general, these assumptions will not rigorously hold for a general situation.

One important assumption is that there is conservation of generalized momentum, see Equation (4.2). This assumption would not be valid for manipulator motions that result in large forces/torques applied to its supporting structure. However, if a manipulator motion results in low base reaction forces/torques, then these forces/torques will result in a small structural deflection and as a consequence the magnitude system's generalized momentum will remain essentially small. In this case, the Coupling Map would provide a good approximation of the actual behavior of the system. The same principle applies to large base velocities. Since it is assumed that the system has zero momentum, the Coupling Map is not expected to faithfully describe the dynamic behavior of the system under large velocities. However, in the formulation of the Coupling Map, it is assumed that: 1) the manipulator base is mounted on a relatively soft elastic structure, and/or 2) is held in place by a dynamic system that emulates a very stiff but well-damped structure, such as the reaction jet system on a free-flying configuration. With these assumptions made, a high structural vibration frequencies are not expected.

One way of looking at the Coupling Map is as a quasi-static approximation of the real, constantly changing coupling phenomenon. Given the shape of the Coupling Map depicted in Figure 4-4, it is possible to envision an exact or constantly changing Coupling Map, which, for the purpose of this argument, we call the Constantly Changing Coupling Map. Such a map would take into consideration not only the direction of motion of the manipulator, but also the instantaneous system velocities and base reaction forces/torques. This constantly changing map would provide an exact picture of the dynamic coupling phenomenon of the system as it occurred. As the forces/torques and velocities converged to zero, the shape of this Constantly Changing Coupling Map would converge back to the shape of the quasi-static Coupling Map
computed using conservation of generalized momentum. Therefore, it is reasonable to assume that if the manipulator motions are successfully planned, say, using the Coupling Map, the forces/torques acting on the supporting structure will be small and the shape of the Coupling Map will be a good approximation to the Constantly Changing Coupling Map.

4.4 Summary

This chapter discussed the development of the Coupling Map. It showed how the Coupling Map evolved from the EDM out of the need to describe a broader range of systems, including free-floating, free-flying and elastically constrained systems. Unlike the EDM, the Coupling Map meets this need by dealing with translational as well as angular disturbances. Furthermore, the formulation of the Coupling Map takes into consideration the mass properties of the space manipulator as well as the stiffness characteristics of its supporting base.

The Coupling Map aids in the understanding of the phenomenon of energy transfer between a manipulator and its supporting structure. This is done by providing a direct and intuitive way of visualizing the degree of dynamic coupling between the manipulator and its moving base as a function of manipulator motion. The Coupling Map also provides a useful graphical representation which can be used for path-planning to reduce the amount of energy transferred between the manipulator and its supporting structure, and to reduce the amount of vibration produced by such maneuvers.

For free-flying systems, the formulation of the Coupling Map introduces a weighting matrix that can be used to account for the physical characteristics of the base actuators, such as jet size, jet location, whether reaction wheels are used, and so forth. This weighting matrix allows for better modeling of the base control system, resulting in a better representation of the real system, which can help in improved path-planning for reducing the amount of fuel/energy consumed by the attitude/position control system.
degrees of freedom of a redundant manipulator to plan low energy paths.

These path-planning algorithms are based on algorithms that were developed to be used with the Enhanced Disturbance Map [82]. In this thesis, they have been updated to apply to a broader range of systems and to be used with the Coupling Map.

For each algorithm, simulation results are presented, and, for the Hot Spot method, a case study based on a prototype system demonstrates the validity of the algorithm for systems of more than two-links.

5.2 Base Relocation: Path-Planning for Repetitive Maneuvers

Space manipulator systems are expected to perform a variety of activities, many of which require repetitive tasks. For example, activities such as loading space equipment, assembling space structures or repairing a satellite in orbit can require a manipulator to move in a nearly repetitive motion between two points in inertial space. In elastically constrained systems, activities that require repetitive tasks are more likely to take longer to execute because the manipulator must wait for the vibrations to settle before tasks. In free-flying systems, this transfer of energy will result in excessive fuel consumption, and in elastically constrained systems, will result in longer time to accomplish a given activity since the manipulator may have to wait a relatively long period of time to allow the base to settle between tasks.

In many cases, the location of the manipulator base can be arbitrarily placed for the manipulator to perform its assigned activities. However, not every arbitrary base location will allow the manipulator to follow a low energy path for a given task. The Coupling Map can be used to find a good base location from which the manipulator can perform a task by following a low energy path, and can also identify such a path.

In order to illustrate how the Base Relocation algorithm works, let us look at the following simple free-flying system:

The dark-line drawing of Figure 5-1 depicts a two-link free-flying space manipula-
tor. The task is to move the end-effector from a given initial point I to a final point F in inertial space, see Figure 5-1. Notice that, given the base location number 1 (the dark-line drawing) in Figure 5-1, there exists a corresponding final configuration that places the end-effector at final point F, see the dark-line drawing Figure 5-2, assuming the system is held stationary. This final configuration is represented by a point in joint space and is labeled 1, see Figure 5-3. Now let us consider a set of candidate base positions and orientation such as the one depicted in Figure 5-1 (light-colored lines). These candidate base positions and orientation are generated by rotating the entire system around the point I in inertial space, as seen in Figure 5-1. For each base location there is a different configuration that results in the end-effector reaching the final point F, see Figure 5-2. These configurations are represented by points in joint space, numbered accordingly, in Figure 5-3. The set of all the points corresponding to the configurations that reach point F in inertial space as we rotate the system around point I is called the Locus of F. Notice that as the base position moves around the initial point I, the locus of F forms a closed path in joint space enclosing the initial configuration point, or point I. If we now superimpose the minimum coupling line passing through configuration point I, we can see that it crosses the locus of F at two points, see Figure 5-3. The two points of intersection of the minimum coupling line and locus of F, called here $F_{opt1}$ and $F_{opt1}$, represent the two optimal manipulator configurations. The base locations corresponding to these two arm configurations are the optimal base locations. For each of these optimal base locations, a low energy path exists, one represented by the segment of the minimum coupling line between point I and point $F_{opt1}$ and the other represented by the segment of the minimum coupling line between I and $F_{opt2}$.

This approach, explained here graphically, has been implemented numerically for a simple two-link planar system for use with the Enhanced Disturbance Map, for which it was originally developed [82]. A flowchart of the algorithm is shown in Figure 5-5 [82]. This algorithm has been extended to be used with the Coupling Map. In order to illustrate the effectiveness of the Base Relocation algorithm, let us look at the following example, which implements the algorithm on an elastically constrained
Figure 5-1: A Two-Link Free-Flying Manipulator: Rotation of the System Around the End-Effector Initial Location

Figure 5-2: Manipulator Configurations that Place the End-Effector at the Final Configuration Given the New Base Locations
Figure 5-3: Representation of the Various Final Configurations in Joint Space

Figure 5-4: Optimal Manipulator Configurations
Input: Initial Position and Orientation of the base.
The task: Initial point, I
Final point, F

Compute minimum coupling line passing through qi. This line is defined numerically by a set of discrete points called the λ Points.

Compute the distances from each λ point to the final configuration, qf for the given base position.

Find the shortest of these distances (called dmin)

Calculate new base location by rotating the system base around point I in inertial space an amount Δφ. The base rotation, Δφ, is proportional to the magnitude of the dmin.

Given Δφ, calculate a new position and orientation of the system base

Diamond: does dmin = 0?

Output: Path, New Position and Orientation of the System Base

Figure 5-5: Flowchart of the Base Relocation Algorithm
Initial Base Location

Figure 5-6: Harmonic Drive Manipulator at Initial Configuration

Figure 5-7: System's Coupling Map and a Straight-Line Path (*path1*). The Circles Represent Control Points That Define The Path As A Four-Point Spline.

system using the Coupling Map.

### 5.2.1 Base Relocation: A Case Study

Figure 5-6 shows a two-link planar space manipulator attached to cantilever beam. The mass properties of this system, which are described in Appendix H, are based on the Martin Marietta Harmonic Drive Manipulator. In this model the cantilever beam can represent a large flexible manipulator designed to move the small manipulator base to a given location in inertial space.

The system is required to move *s* end-effector from a given initial point I to a
Figure 5-8: Simulation Results for the HDM Performing path1 in 1.6 Seconds. Base Acceleration in the Direction Perpendicular to the Beam Axis.

given final point F and after some period of time move it back to point I. This task will be repeated a number of times. Figure 5-7 shows the Coupling Map for this system and a straight line path, called path1, that can take the manipulator from point I to point F and back. Given the location of the base, the initial and final points I and F in inertial space are represented on the Coupling Map by the points I and F. Notice that, given the task and the location of the base, the selected path lies perpendicular to the lines of minimum coupling. Therefore, it is expected that this path will generate relatively large residual vibrations on the cantilever beam. This behavior is confirmed in simulation, the results of which are shown in Figure 5-8.

The mapping of the inertial points I and F into joint space, namely, mapping I and F, is a function of the base location. Changing the location of the base in inertial space changes the position of points I and F in the Coupling Map.

For this example, the base was relocated as shown in Figure 5-9. Figure 5-10 shows the new path, called path2, for the new base location. Notice that like path1, path2 takes the manipulator’s end-effector from point I to point F in inertial space, but the maneuver is now along a minimum coupling line. Therefore, it is expected that this maneuver will result in relatively low residual vibrations compared to the maneuver using path1. This maneuver was also simulated and the results are shown.
in Figure 5-11.

Notice that the use of path 2 resulted in a 75% reduction in the magnitude of the residual vibration compared to the residual vibration produced by the use of path 1. These results agree with the information provided by the Coupling Map. However, in order to test the performance of the algorithm in a real system, an experimental evaluation of these maneuvers was conducted on the actual Martin Marietta Harmonic Drive Manipulator and is presented in Chapter 7.

The new location of the system's base was found using a modified version of a base relocation algorithm developed in [82]. The original algorithm finds the new base relocation by rotating the manipulator's base around the manipulator's initial end-effector location in inertial space. This rotation keeps mapping of point I into joint space constant but moves the mapping of point F in a closed loop path in joint space. In the modified version presented here, a new manipulator initial configuration is chosen prior to the base rotation procedure in order increase the chances that point F will be within reach of the new computed base location.

Since this path planning method requires the relocation of the system's base, the
Figure 5-10: path2 and its Relation to The Minimum Coupling Lines. The Circles Represent Control Points That Define The Path As A Four-Point Spline.

Figure 5-11: Simulation Results for the HDM Performing path2 in 1.6 Seconds. Base Acceleration in the Direction Perpendicular to the Beam Axis.
method is only effective in cases in which the spending of the energy and time in moving the manipulator's base can be justified, such as in repetitive maneuvers. For cases in which relocating the base is not possible, path-planning schemes such as the Hot Spot method or the Redundancy Resolver algorithm can be more effective.

5.3 The Hot Spot Method

The Hot Spot method is a heuristic graphical method for path-planning that uses the information provided by the Coupling Map to find paths in joint space that avoid moving perpendicular to the minimum coupling lines when the manipulator configuration lies on a hot spot. In order to illustrate this method, consider the system in Figure 5-12 and its Coupling Map, depicted in Figure 5-13. The dynamic characteristics of this system have been chosen to be similar to those of an experimental manipulator called Advanced Robotic Manipulator II or ARM II, currently located at NASA's Langley Research Center. See Appendix G for a complete description of the dynamic properties of this system. The system shown in Figure 5-12 is mounted at the end of a flexible structure that possesses elastic properties similar to those of the Space Shuttle's Remote Manipulator System, or RMS, see Appendix B. The system's first vibration mode frequency is between 0.4 Hz and 0.7 Hz, depending on the ARM II's configuration. It is assumed to have a structural damping ratio of 0.05.

The task for the system is to move the manipulator's end-effector from an initial point $I$ to a final point $F$ in inertial space. The initial and final configurations on the Coupling Map corresponding to these points are shown in Figure 5-13. As described in Section 4.3, the dark shaded areas, of the Coupling Map represent hot spots, or areas in which moving perpendicular to the minimum coupling lines would produce relatively large transfer of energy into the manipulator's supporting structure. Notice that the final configuration, point $F$ in this example, lies in a hot spot. Motions of the manipulator joints perpendicular to the minimum coupling lines in this region would produce a relative large transfer of energy to the base's softer modes. Thus the Hot Spot method is based on the following heuristic rule:
Figure 5-12: Elastically Constrained System. Dynamic Characteristics Based on the NASA’s Advanced Robotic Manipulator II (ARM II). Mounted on a Flexible Structure with Elastic Characteristics Similar to those of the Remote Manipulator System (RMS).

*If a path must go through Coupling Map regions of large energy coupling, then it should follow minimum coupling lines as closely as possible. When coupling is low, at a cool spot, the path may move across the minimum coupling lines.*

### 5.3.1 Hot Spot Method: Case Study I

To demonstrate this approach, three paths are considered. These are shown in Figure 5-13. The first path, \textit{path1}, is a straight line from point \textit{I} to point \textit{F}. The second path, \textit{path2}, was chosen using the Hot Spot Method. Finally, \textit{path3}, a seemingly appropriate path, but one which moves perpendicular to minimum coupling lines in the hot spot, was studied. The three maneuvers were designed to take approximately 2 seconds.

The dynamic response of the system’s supporting structure for these three paths, was computed using the \textit{RiBS} library [83]. The time history of the vertical motion of the system’s base (z-direction in Figure 5-12) for the three maneuvers is shown
Figure 5-13: Coupling Map for Links 2 and 3 of the NASA ARM II Mounted at the End of RMS.

Figure 5-14: Residual Vibrations in the Z-Direction for path1, path2, and path3
in Figure 5-14. Notice that \textit{path2}, the path computed using the Hot Spot Method, yielded the lowest vibration amplitude. The path called \textit{path3} produced the highest residual vibration agreeing with the predictions made by the Coupling Map since it crosses the minimum coupling lines in a hot spot. Figure 5-14 also shows that, given a base motion tolerance of $\pm 0.02 \text{m}$, \textit{path2} resulted in the lowest settling time of 10.5 seconds. The straight line path, \textit{path1}, produced a base vibration that took more than twice as long to settle. This increased settling time can dramatically decrease system performance and increase the cost of the space mission. For example, as of 1989, the Space Shuttle cost approximately $20,000 per minute to operate [69]. If these figures are applied to the example above, this one simple maneuver using \textit{path2} would have saved approximately $3,300 over the more conventional straight line path.

In the next case study the Hot Spot method is applied to a more complex system and techniques such as projection and path-planning in a three dimensional space are discussed.

\subsection*{5.3.2 The Hot Spot Method: Case Study II}

In order to test the viability of the Coupling Map and the Hot Spot method on a more realistic systems, a prototype of an elastically constrained space manipulator system was designed in some detail. This prototype, called the Prototype Dextrous Manipulator (PDM), see Figure 5-15, is based on a 19-DOF manipulator mounted at the end of a 50 \text{ft.} long cantilever beam with stiffness characteristics of those of the Remote Manipulator System (RMS). Appendix A describes the design of the prototype system and shows a complete detail of the system including its mass properties, actuator characteristics, etc. The PDM has two seven-DOF rigid-link arms attached to a five-DOF body.

A two-parts task was chosen for the PDM which consist of moving the PDM's five-DOF body to a given configuration so as to bring its two dextrous arms and sensor equipment close to a working station, and then move its right arm from an initial to a final configuration so as to bring the right arm end-effector closer to an object within the working station. In the first part of the maneuver, residual base
vibrations are expected. Only when these vibrations have die out, the PDM's right arm will then be commanded to perform its maneuver.

The fist step requires moving three of the system's five body joints from an initial to a final configuration while keeping the two seven-DOF arms stationary. The initial and final configuration of the prototype body is shown in Figure 5-16. This three joint system requires a three dimensional Coupling Map. However, a full graphical representation of the Coupling Map is not necessary to compute the hot-spot based path. For this example only the minimum coupling lines passing through the initial and final configuration, point \( I \) and \( F \), respectively, are computed, see Figure 5-16. Figure 5-16 shows that point \( F \) lies on a "hotter" region than point \( I \). This suggests that a low coupling path can be found by starting the path perpendicular to the minimum coupling line at point \( I \) and approaching point \( F \) parallel to the minimum coupling line at \( F \). This path is shown is shown as \( path2 \) in Figure 5-16 and was designed to take 3 seconds to complete. The simulation results are shown in Figure 5-17. A straight line path, called \( path1 \), see Figure 5-16, was used for comparison. Figure 5-17 shows that \( path2 \) resulted in a 14.3 percent reduction in the
Figure 5-16: Initial and Final PDM's Body Configuration and the Two Selected Paths.

Figure 5-17: Simulation Results for path1 and path2.
Figure 5-18: Coupling Map for Joints 1 and 4 of PDM Right Arm and Computed Paths.

magnitude of the residual vibration based on the system's base absolute motion when compared to path1 for the same maneuver time.

The second step of the task required two of the seven joints of the system right arm to move from an initial to a final configuration while keeping the body stationary at the final configuration described above. The motion was planned to take 2.0 seconds. A Coupling map was computed and two paths were generated and simulated. Figure 5-18 shows the Coupling Map for joints 1 and 4 of the PDM's right arm, assuming that the rest of the system joints are kept stationary. The two paths were simulated and the base motion recorded and is shown in Figure 5-19. The simulation showed that the low coupling path, path2, resulted in a 25.33 percent reduction in the amplitude of the residual vibration of the base when compared to the amplitude of the residual vibrations produced by the straight line path or path1.

This case study showed that the Coupling Map can be used with complex sys-
Figure 5-19: Base Vibration: Simulation Results for Low Coupling Path and Straight Line Path.

tem such as the 19-DOF PDM. It also shows how the Hot Spot method is used for computing path in two- and three-dimensional joint spaces. Notice that for this cases study some of the system joints were kept stationary during the maneuvers. Although joints that remain stationary during the maneuver need to be included in the formulation of the Coupling Map, they do not contribute to the dimension of the Map or to the path-planing algorithm since they do not generate any disturbance to the system supporting structure. The construction of a Coupling Map using a subset of the total system DOF is called in this study projection since the resulting Coupling Map is a projection of the actual full n DOF Coupling Map into a two or three dimensional space. This technique can save computational effort in generating the Coupling Map and in path-planning when some of the system’s DOF can be neglected due to its relative inability to generate disturbances to the system’s base during a given maneuver. For example, the joints near the end-effector or wrist joints, in some manipulators (with not payload) have a low mass and it kinematic structure is such that motion

\footnote{In the case of the spherical wrist joint three revolute joints always intersect at one point so the}
of these joints do not produce large disturbances to the system supporting structure. Notice however, that a projected Coupling Map is only valid for the given plane of projection. In other words, a new projection or projected Coupling Map is needed if the configuration of the omitted DOF changes significantly.

5.4 The Redundancy Resolver Algorithm

If the number of degrees of freedom in a manipulator are greater than the number required to perform a task, the manipulator is said to be redundant [17]. In the general spatial case, a manipulator with six degrees of freedom is required to position and orient its end-effector in an arbitrary position within its working space. However, not all tasks require a full six degree of freedoms. For example, if the end-effector has one or more axes of symmetry, such as in the case of a 6 DOF manipulator pointing a beam of light in a given direction for inspection purposes, there will be an infinite number of configurations that satisfy this task. In this case, the manipulator is said to be redundant for the given task.

Current designs of space manipulator systems propose the use highly redundant manipulators, such as the proposed Special Purpose Dextrous Manipulator or SPDM [19]. See Figure 2-4, page 36. In this case the SPDM has been designed with 19 degrees of freedom.

The kinematic characteristics of these systems suggest the use of the extra degrees of freedom for a variety of tasks including vibration control and fuel minimization. In this section, the Coupling Map is proposed as an analytical tool to aid in exploiting the potential of a redundant manipulator in minimizing the transfer of energy to its supporting structure during a maneuver.

To demonstrate this algorithm, let us consider the three-link planar free-flying manipulator depicted in Figure 5-20. The task is to move the manipulator’s end-effector from point I to point F in inertial space, while the base is kept stationary using reaction jets. Since fuel will be used in keeping the base stationary, it is the goal

mass of these joints generally do not deviate from this point during a maneuver. See Ref [75]
of the path-planning algorithm to find a path that results in low fuel consumption.

The form of the Coupling Map for such three-DOF systems is shown in Figure 5-21. Notice that a three-dimensional space is necessary to graphically describe the Coupling Map for a three-link manipulator. The shape of this map is based on the mass properties of the system and on the selection of a weighting matrix, see Section 4.3.2 on page 59, that describes the fuel consumption characteristics of the reaction jet system. For this hypothetical system, we have chosen a weighting matrix that assigns a higher cost to attitude control fuel than to position control fuel, see Equation (5.1).
Figure 5-21: The Low-Coupling Manifold Passing Through the Initial Configuration

\[
K^* = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 10 & 0 & 0 \\
0 & 0 & 0 & 0 & 10 & 0 \\
0 & 0 & 0 & 0 & 0 & 10 \\
\end{bmatrix}
\]  \hspace{1cm} (5.1)

For this system, the attitude control system consumes 10 times more fuel than the position control system. For this redundant manipulator there are two directions of low coupling and one of high coupling. The two directions of low coupling are represented by a family of two-dimensional manifolds or surfaces in joint space.

The path-planning algorithm is implemented in the following way: Given the initial position \( I \) of the end-effector and the corresponding manipulator configuration \( I \), a low coupling surface passing through \( I \) can be computed by Equation (4.13), see page 60.

Next, it is noted that, because the manipulator is redundant, the set of all configurations that satisfy the placement of the end-effector at location \( F \) in inertial space
Figure 5-22: A Method for Computing $S(q)$

is called the *Locus of* $F$, which is described by a curved line in joint space, as shown in Figure 5-21. The optimal final configuration, called $F$, is is found at the intersection of the low coupling surface and the locus of $F$. Now, the low coupling path, called $S(q)$, can be identified; it is represented by a curve that runs from $I$ to $F$ with the constrained to the low coupling surface, see Figure 5-21.

To compute $S(q)$, one starts by computing a unit vector $\hat{u}$ from point $I$ in the direction of the vector $F - I$. The projection of $\hat{u}$ onto the plane tangent to the minimum coupling surface at point $I$ defines a direct path toward the desired final point $F$, see Figure 5-22. The surface at $I$ is found by using the two perpendicular low coupling direction vectors resulting from a singular value decomposition of the system coupling matrix $Q$, see Equation (4.13), page 60. The path $S(q)$ is then determined by multiplying the $\hat{u}$ by a small quantity $\Delta S$ to find a new point $I'$ on the low coupling surface. The process is repeated until the vector $F - I'$ equals zero. It should be noted that this algorithm was developed to be used with the Enhanced Disturbance Map [82] and it is directly applicable to the Coupling Map. The following is an example of the results obtained using the Redundancy Resolver Algorithm.

Figure 5-23 shows the manipulator introduced in Figure 5-20 following two paths, an arbitrary path and a low coupling path computed using the Redundancy Resolver
Figure 5-23: Two Paths with the Same Initial and Final End-Effector Position.

algorithm. Figure 5-24 shows the low coupling surface passing through \( I \) and the arbitrary path in configuration space. Notice that an arbitrary final configuration was chosen to lie outside the low coupling surface. Figure 5-25 shows the low coupling surface, the computed optimal final configuration and the computed low coupling path. Both the optimal configuration and the low coupling path lie on the low coupling surface.

Motion along both the arbitrary path and the low coupling path were simulated using \textit{RiBS} [83]. The effect of reaction jets to compensate for any dynamic disturbance was also simulated, and the resulting fuel usage was recorded. The velocity profile along the two paths was chosen to be a half sinusoidal wave with initial and final velocities equal to zero. The highest velocity for each maneuver was computed such that each path would take two seconds to complete. Figure 5-26 shows the rate of fuel consumption required to correct for base disturbances for each path. For the arbitrary path, the total amount of fuel consumed at the end of the maneuver was 0.107 units of fuel. By contrast, the low coupling path required only 0.075 units, a 30 percent reduction.
Figure 5-24: Low Coupling Surface and the Arbitrary Path

Figure 5-25: Low Coupling Surface and the Low Coupling Path
Figure 5-26: Simulation Results: Rate of Fuel Consumption for Arbitrary Path and Low Coupling Path.

5.5 Summary

This chapter showed how the Coupling Map can be an effective tool for planning manipulator motion that results in low energy transfer between the manipulator and its base. Three path-planning algorithms are presented which can be used in either free-flying or elastically constrained systems. Simulation showed that each algorithm was capable of generating paths of low energy coupling (resulting in low residual vibrations or low fuel usage) and that the Coupling Map was capable of predicting such low energy paths. Also, a case study based on the 19-DOF prototype system demonstrated the validity of the Hot Spot method of more complex systems.
Chapter 6

The Pseudo-Passive Energy Dissipation Method

6.1 Introduction

Unlike the open-loop path-planning algorithms developed in Chapter 5, the Pseudo-Passive Energy Dissipation (P-PED) Method is a technique designed to change the dynamic behavior of a space manipulator system by means of tuning the manipulator’s closed loop gains so as to increase the transfer of energy from the manipulator’s flexible, (very lightly damped) supporting structure to its actuators. This energy is then dissipated by the actuators since they are commanded to behave as passive linear springs and dampers.

The word pseudo-passive indicates that passive elements such as springs and dampers are used to damp out the vibration, but they are implemented using active elements, such as electromechanical actuators.

The P-PED method is not a path-planning algorithm. It is a closed loop control scheme designed to help damp out vibrations to the manipulator’s base which result from either external disturbances to the system, motions of the manipulator itself or motion of the base as the system is relocated.

The P-PED methodology derives from the results obtained from the Lyapunov stability analysis presented in Appendix E. This analysis yielded Equation (E.5),
which shows that, due to the structure of the $\Xi$ vector for elastically constrained systems, see page 47, the only way to remove energy from the system (i.e., $\dot{v} < 0$) is if the torque applied to the joints is opposite in direction to the joint velocity vector. This behavior is, in fact, the behavior of a linear dashpot whose constitutive equation is

$$f_d = -K_d \dot{x}_d \tag{6.1}$$

where $K_d$ is called the damping coefficient [66]. Making the manipulator joint actuators behave as linear dashpots is what is known in classical control theory as derivative control and is a common technique to increase the stability margin of a given dynamic system [28]. However, in order for the energy stored in the vibratory modes of the base to be most effectively transferred to the manipulator actuators, the derivative as well as the proportional closed loop gains of the manipulator joints must be carefully selected. For instance, if the proportional-derivative, or PD, gains of the system are selected to satisfy a given high-bandwidth, high performance specification, motion of the manipulator will generate large vibrations to its supporting structure but the manipulator will be unable to damp out these vibrations. This is because, once the maneuver is completed, the high bandwidth characteristics of the manipulator makes it look like a rigid body to the slowly vibrating base.

Tuning down the PD gains can increase the dynamic interaction between the manipulator and its compliant base as so increasing the effective damping of the base vibratory mode. However, as shown in Chapter 4, this interaction also depends on the configuration of the system. In general, the configuration of the manipulator cannot be arbitrarily selected for P-PED control since the manipulator's end-effector might be required to be at a given location in inertial space. However, in cases in which the manipulator will be maintained in an idle position while the base is relocated, or when external disturbances are expected, instead of locking the manipulator in its, say, home position, it is preferable to position it in a Coupling May's Hot Spot area, see Chapter 4, and set the system to P-PED control. This way, the manipulator can act as an active damping system and help the system damp out base vibrations faster.
6.2 Computing the Optimal Gains for a P-PED Control

In order to better illustrate the technique of computing the optimal P-PED gains for a general n-DOF manipulator, and to get an insight into the dynamic behavior of a system under P-PED control, a two-link manipulator is used in the development of the equations used for computing the P-PED gains. These equations are then generalized for an arbitrary n-DOF system.

Let us look at the hypothetical case of a two-link flexible base manipulator in which its end-effector is attached to an inertial ground, see Figure 6-1 System A, and the manipulator joints are commanded to behave as pure dampers. In this configuration any motion or vibration of the base would impose a motion to the manipulator joint and energy would be dissipated as the joint damping action resisted this motion.

Now let us look at another hypothetical case of the same two-link flexible base manipulator, which is this time holding a large object. See Figure 6-1 System B. In this case, if the mass of this object were infinite, System A would be exactly the same as the System B. However, if the mass of this large object has a finite magnitude, say, of the same order of magnitude as the manipulator's base mass, then the dynamics of the system become more complex. For instance, vibratory motions of the base can induce motion to the large object, depending on how much damping is programmed into the joint control loops. Large PD gains mean the manipulator will not move and the base, the manipulator and it payload will vibrate together. Gains equal to zero indicate complete decoupling of the base and the manipulator payload.

If the correct set of gains were found, the manipulator, in combination with the its payload, would act as an active damper absorbing and dissipating the energy contained in the vibratory modes of the base. However, if this were all we cared about,

\(^1\)In fact, if the joints were locked in position, no motion could occur at the base, which is a form of bracing [93]. But this case is not relevant to this discussion.
Figure 6-1: Two Elastically Constrained Systems: System A End-Effector is Connected to an Inertial Ground. System B End-Effector is Connected to Large Mass Object.
commanding the manipulator actuators to behave as pure dampers with the proper gains would be enough. Unfortunately, this setup does not guarantee that once the vibrations are damped out, the manipulator will return to its original configuration. In order to assure that once the vibrations have stopped, the manipulator returns to its nominal configuration, and to prevent the manipulator and the payload from wandering too much while the base is vibrating, a proportional gain must be added to the control equation. This proportional gain should not be too high or the manipulator will become too stiff and will lose its ability to absorb energy from the base.

Said another way, the proportional and derivative gains of the system must be set in such a way that the impedance of the manipulator-payload system matches the impedance of the base-compliant structure system [66].

### 6.2.1 Root Locus Analysis

Let us look at a simple two-DOF mass-spring-damper system, see Figure 6-2, with homogeneous differential equation

\[ \dot{x} = Ax \]  

(6.2)

where \( x = [y_b, y_r, \dot{y}_b, \dot{y}_r]^T \) and \( A \) is the system matrix given by

\[
A = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-\frac{k}{m_b} & \frac{k_p}{m_b} & -\frac{b}{m_b} & \frac{k_d}{m_b} \\
\frac{k}{m_b} & -(\frac{k}{m_b} + \frac{k_p}{m_b}) & \frac{b}{m_b} & -(\frac{k_d}{m_b} + \frac{k_d}{m_b})
\end{bmatrix}
\]  

(6.3)

The mass \( m_r \) in this system can be seen as the total mass of the robot and \( m_b \) as the mass of the system's base. This model will be developed into a general n-DOF system as the analysis progresses, so it is convenient at this early stage of the analysis to think of \( m_r \) as the total mass of the manipulator. In a similar fashion, \( k_p \) and \( k_d \) are the proportional and derivative P-PED gains of the system, respectively.

For this system, we are interested in finding the optimal values of \( k_p \) and \( k_d \) that maximize the overall damping characteristics of the system. Figure 6-3 shows the
realationship bewteen the damping ratio $\zeta$, of the most dominant poles of the systems and the values of $k_p$ and $k_d$. Notice that for this system there is a set of values for $k_p$ and $k_d$ that maximizes the damping of the system dominant poles. In order to find this set of values an iterative procedure is followed. First, set $k_P = 0$ in Equation (6.3) and use the root locus methodology to graphically determine a sub-optimal value for $k_d$. This sub-optimal value corresponds to the highest value of $k_d$ that gives the highest damping to the system. Second, fix $k_d$ at this sub-optimal value and use a root locus plot again, this time varying $k_p$ until a sub-optimal value for $k_p$ is obtained. Repeat the process always increasing the values of $k_d$ and $k_p$ in each step until they can no longer be increased and improve system stability. In order to understand this procedure let us look at the following example.

To choose the optimal gain values of $k_p$ and $k_d$ from a root locus plot, we must first understand the behavior of the poles as these gains are varied. For the system depicted in Figure 6-2, setting $k_p = 0$ and varying $k_d$ will produce a root locus of the shape shown in Figure 6-4. Notice that when $k_p$ and $k_d$ are equal to zero, the root locus starts out with two poles at zero, which represent the free floating mass $m_r$, and a pair of complex poles, which represent the mass-spring system $k, m_b$. As $k_d$ increases, one of the pairs of poles at the origin moves to the left of the complex plane.
Figure 6-3: Damping of the Most Dominant Poles of the System in Figure 6-2 vs. $k_p$ and $k_d$

along the negative real axis, while the other remains at the origin. As for the pair of complex poles, as $k_d$ increases, they migrate to the left side of the complex plane, making the system more stable. However, as $k_d$ continues to increases, the complex pair of poles moves to the complex plane and then begins to move back towards the imaginary axis. As $k_d$ approaches $\infty$, the two pairs of complex poles return to the imaginary axis, this time at a lower frequency, given by $\omega = \sqrt{k/(m_b + m_r)}$. The optimal value for $k_d$ is, therefore, the value that maximizes angle $\phi$, which is the angle between the imaginary axis and a line between the origin and the location of the pole. The $\sin \phi = \zeta$, $\zeta$ being the damping ratio of the system. The other two poles will always lie on the real axis, so they do not contribute to vibration. Therefore, by knowing of the shape of the root locus as a function of $k_d$, it is possible to look at the root locus and choose the optimal value for $k_d$ given $k_p = 0$.

Now a similar procedure is conducted to determine the optimal value for $k_p$. However, this time, varying $k_p$ produces a completely different root locus graph. Given the optimal value of $k_d$ for $k_p = 0$ obtained before, increasing the value of $k_p$ will generate a root locus like the one depicted in Figure 6-5. Notice that as $k_p$
Figure 6-4: Root Locus Resulting from Setting $k_p = 0$ and Varying $k_d$
Figure 6-5: Root Locus Resulting from Setting $k_d = k_d$ sub-optimal and Varying $k_p$
increases, the two poles on the real axis begin to move toward each other but remain in the negative real axis of the complex plane. The two pairs of complex poles begin to migrate further to the left, which makes the system even more stable. However, at some value of \( k_p \), the two pairs of real poles meet and start to move vertically in the complex plane and, thus, become complex poles. The two pairs of complex poles we began with at some point no longer migrate to the left and begin to move vertically. If we increase \( k_p \) further, the pair of poles initially on the negative real axis become complex and the other pair of complex poles continues to move vertically. The criteria for selecting the optimal value of \( k_p \) is similar to the one used for selecting \( k_d \), but this time two pairs of complex poles must be taken into consideration. So, to obtain the optimal value for \( k_p \), select the value of \( k_p \) that maximizes the angles between the imaginary axis and the lines connecting the two complex poles and the origin.

Once this value of \( k_p \) is obtained we then fix \( k_p \) at this value and start increasing \( k_d \) again following the pole behavior depicted in Figure 6-4. The iterative process is then repeated until the values of \( k_p \) and \( k_d \) can no longer be increased to improve stability. For the cases looked in this study we found that one or two iterations were enough.

For some applications, \( k_p \) must be selected first, to satisfy a motion constraint. For instance, if the manipulator is near an obstacle, it is important to select the value of \( k_p \) so that when the manipulator is under P-PED control, the base-induced manipulator maneuvers will not interfere with the surroundings. In such cases, the value for \( k_p \) must be selected first and then a value of \( k_d \) can be obtained using the root locus methodology described above.

### 6.2.2 Kinematic Analysis

The system depicted in Figure 6-2 represents a simplified dynamic model of a flexible base manipulator system, where \( m_r \) represents the total mass of the manipulator and its payload, and \( m_b \) represents the mass of the system's base. Motion of \( m_r \) with respect to \( m_b \) in Figure 6-2 represents the motion of the manipulator's center of mass with respect to its base. The spring and damper between the two masses in the system
in Figure 6-2 represent the effective spring-damping characteristics present where the joint PD gains are active. In order to compute the manipulator joint P-PED gains that result in the effective dynamic behavior exhibited by $k_p$ and $k_d$ in the model in Figure 6-2, the values of $k_p$ and $k_d$ need to be translated into the manipulator joints. This is more effectively done by combining the Virtual Manipulator concept with the notion of impedance control [87, 4].

First, the virtual manipulator is constructed for the system as shown in Figure 6-6. This virtual manipulator kinematic chain starts at the system base and its end-effector corresponds to the real manipulator's center of mass. The virtual manipulator is a massless kinematic chain whose joints axes are always parallel to the real system's joint axes and whose end-effector always coincides with the real manipulator's center of mass [87]. The construction of the virtual manipulator simplifies the analysis.
considerably, since now we can look at the virtual system as a massless manipulator holding a payload of a mass equal to the total mass of the real manipulator. Notice that this model is similar to the one depicted in Figure 6-1, 91, discussed earlier.

For the system shown in Figure 6-6, the dimensions of the links of the virtual manipulator are given by

\[
\begin{align*}
    a &= \frac{m_1(l_2-s_2)+m_2l_1}{m_1+m_2} \\
    b &= \frac{m_2(l_2-s_2)}{m_1+m_2}
\end{align*}
\]  

(6.4)

The expressions for \( a \) and \( b \) in Equation 6.4 are obtained by using the Virtual Manipulator technique [87]. The joints of the virtual manipulator are always parallel to the real system's joint axes, the expression for the virtual manipulator Jacobian, or \( J_{cm} \), is similar to the real system Jacobian, \( J \). The only difference is the length of the links. In this case, the Jacobian for the real system is given by

\[
J = \begin{bmatrix}
-l_1 \sin(q_1) - l_2 \sin(q_1 + q_2) & -l_2 \sin(q_1 + q_2) \\
l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2)
\end{bmatrix}
\]  

(6.5)

and the Jacobian for the virtual manipulator is given by

\[
J_{cm} = \begin{bmatrix}
-a \sin(q_1) - b \sin(q_1 + q_2) & -b \sin(q_1 + q_2) \\
 a \cos(q_1) + b \cos(q_1 + q_2) & b \cos(q_1 + q_2)
\end{bmatrix}
\]  

(6.6)

The subscript in \( J_{cm} \) indicates that this expression relates motion of the manipulator joints to motion of the manipulator's center of mass.

\[
\dot{X}_{cm} = J_{cm} \dot{q}
\]  

(6.7)

In general, the manipulator Jacobian also relates force and torque acting on the manipulator's end-effector to actuator torques [4]. This is also true for the \( J_{cm} \); however, in this case it relates forces and torques acting between the manipulator base and its center of mass, \( F_{cm} \), to the manipulator actuator torques, \( \tau \). This relation is given by

\[
\tau = J_{cm}^T F_{cm}
\]  

(6.8)
Now, if the concept of $k_p$ and $k_d$, as discussed in Section 6.2.1, is generalized to a 3 DOF motion, an expression for $F_{cm}$ can be written as

$$F_{cm} = K_p X_{cm} - K_d \dot{X}_{cm} \quad (6.9)$$

where $K_p \in \mathbb{R}^{3 \times 3}$ and $K_d \in \mathbb{R}^{3 \times 3}$ are diagonal positive-definite matrices describing the desired stiffness and damping characteristics, respectively, between the manipulator center of mass and its base. The diagonal elements of the matrices $K_p$ and $K_d$ are computed using the root locus iterative procedure discussed in the previous section.

Combining Equation (6.8) and Equation (6.9) yields

$$\tau = J_{cm}^T (K_p X_{cm} - K_d \dot{X}_{cm}) \quad (6.10)$$

which, combined with Equation (6.7), yields

$$\tau = J_{cm}^T K_p J_{cm} q_{err} - J_{cm}^T K_d J_{cm} \dot{q} \quad (6.11)$$

where $q_{err}$ is the joint error vector, or deviation from the nominal configuration.

Equation (6.11) leads to the expressions for the manipulator’s proportional and derivative joint gains as a function of the previously computed $K_p$ and $K_d$ matrices. These expressions are

$$K_{P-PED} = J_{cm}^T K_p J_{cm} \quad (6.12)$$

and

$$D_{P-PED} = J_{cm}^T K_d J_{cm} \quad (6.13)$$

where $K_{P-PED}$ and $D_{P-PED}$ are the manipulator P-PED control gain matrices. Notice Equations (6.12) and (6.13) provides the expressions for computing $K_{P-PED}$ and $D_{P-PED}$, respectively, for a general $n$-DOF manipulator. In this simple example the matrix $J_{cm}$ is $\in \mathbb{R}^{2 \times 2}$. For a general spatial $n$-DOF manipulator the matrix $J_{cm}$ is $\in \mathbb{R}^{3 \times n}$.

Notice that $K_{P-PED}$ and $D_{P-PED}$ are, in general, full matrices. In order to obtain the correct system dynamic response, as expected by the root locus analysis, the full
\(K_{P-PED}\) and \(D_{P-PED}\) matrices must be used in the implementation of the control algorithm. It is evident that this control scheme cannot be fully implemented using conventional local PD control. A more comprehensive control architecture is required, in which the position and velocity of the manipulator joints are globally available.

For a manipulator with local PD control, only the diagonal values of the matrices \(K_{P-PED}\) and \(D_{P-PED}\) can be used in the implementation of the algorithm. The experimental results shown in Figure 6-9, page so, were obtained with a local P-PED implementation, which shows that, at least for this system, a local scheme implementation can significantly increase the system’s ability to damp out base vibrations.

\(K_{P-PED}\) and \(D_{P-PED}\) depend on \(J_{cm}\), which, in turn, is a function of the system configuration. For those configurations in which the matrix \(J_{cm}\) becomes singular, values for \(K_{P-PED}\) and \(D_{P-PED}\) that satisfy the required dynamic behavior, will not be found. For cases in which the configuration of the manipulator can be arbitrarily selected, the Coupling Map can be used in selecting such configurations. As discussed in Chapter 4, the hot spot areas in the Coupling Map represent configurations of the manipulator that exhibit high coupling between the manipulator and its base. These high coupling configurations also coincide with configurations for which \(J_{cm}\) will not become singular, as suggested by Equation (6.7).

The following is an experimental demonstration of the P-PED method using the Martin Marietta Harmonic Drive Manipulator.

### 6.3 Experimental Demonstration

In order to illustrate this method, let us look at the following experiment conducted on the Martin Marietta Harmonic Drive Manipulator Testbed, shown in Figure 6-7. The mass properties of this system are described in Appendix H. In this experiment the manipulator will be kept at the configuration shown in Figure 6-7 while a disturbance is applied to its base.

For the first part of the experiment, the manipulator PD gains were tuned to its normal high bandwidth values, see Table 6.3. The system was commanded to main-
Figure 6-7: The Martin Marietta Harmonic Drive Planar Arm Testbed.

Table 6.1: Manipulator Gains for Conventional PD and P-PED Control

<table>
<thead>
<tr>
<th>Joint</th>
<th>PD Gains</th>
<th>P-PED Gains</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kp</td>
<td>Kd</td>
</tr>
<tr>
<td>Joint 1</td>
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</tr>
<tr>
<td>Joint 2</td>
<td>200.00</td>
<td>7.0</td>
</tr>
</tbody>
</table>
Figure 6-8: Base Response to an External Disturbance: Conventional PD Control.

Figure 6-9: Base Response to an External Disturbance: Under P-PED Control.

tain its configuration while a disturbance was applied to the base. The acceleration of the base was recorded and is shown in Figure 6-8.

For the second part of the experiment, the manipulator P-PED gains were computed for this configuration based on the approach described above and the manipulator was commanded to keep its configuration under P-PED control. The value of the gains are shown in Table 6.3. A similar disturbance was applied to the base and the base acceleration was recorded. The results are shown in Figure 6-9.

Notice that under P-PED control the system was able dissipate the disturbance produced base vibration in only 3 seconds, which was significantly faster than when
the system was commanded to keep its position using conventionally tuned PD control, which required more than 20 seconds.

The effectiveness of P-PED control is further evaluated in Chapter 7, where the method is applied to damping residual vibrations produced by a manipulator maneuver.

6.4 Summary

This chapter presented the development of the Pseudo-Passive Energy Dissipation Method. The effectiveness of the P-PED method has been experimentally demonstrated on a simple planar two-link elastically constrained system. This chapter has also shown a method for computing the optimal P-PED gains for a general n-link manipulator.
Chapter 7

Experimental Evaluation of the Coupling Map and the P-PED Method

7.1 Introduction

In this Chapter, the Coupling Map, the Base Relocation algorithm and the Pseudo-Passive Energy Dissipation Method are experimentally evaluated on a two-link planar manipulator. The experimental evaluation is conducted on the Martin Marietta Harmonic Drive Manipulator Testbed. For this evaluation, the Harmonic Drive Manipulator is mounted to the free end of a flexible cantilever beam. The first part of this experiment demonstrates how the Coupling Map, using the Base Relocation algorithm, generates a path of low energy transfer that, when followed, produces significantly reduced residual vibration and, therefore, reduced base settling time. The second part of this experiment demonstrates how, by planning a low-coupling path that produces low residual vibration, the Coupling Map can be used effectively to improve the performance of pre-filtering techniques [72, 70, 71, 51, 50]. This experiment also demonstrates how the P-PED method can be used in conjunction with the Coupling Map to reduce magnitudes of residual vibration and the amount of time necessary to cancel out of that vibration, thus improving the overall performance of
the system.

### 7.2 System Characteristics and Analytical Model

The Harmonic Drive Manipulator, or HDM, is a three-link rigid body planar manipulator designed to operate on an air-bearing table for frictionless motion. See Figure 7-1 and Appendix H. This configuration provides a convenient and cost-effective way of emulating a microgravity environment and space-like motions. Each actuator is a harmonic drive transmission with an 80:1 gear reduction and is driven by a brush DC motor. These actuators are driven by current-controlled, pulse-width modulated amplifiers. Local joint torque feedback loops allow the arm to behave as a frictionless direct-drive system.

Each joint is instrumented with a motor-side tachometer, an output torque transducer and a resolver to measure the joint angle. An accelerometer is mounted on the system's base in order to record its vibrations.

For the experiment, the HDM is attached to a 886 mm × 89.0 mm × 13.0 mm aluminum beam and floats across a 2.565 m × 1.295 m glass table. For modeling purposes, the aluminum beam is assumed to be massless with compliance only in one direction, i.e., $y$ direction, see Figure 7-1. All other directions are assumed to be rigid, and higher order modes are neglected.
Figure 7-2: The Coupling Map for the Martin Marietta Harmonic Drive Planar Arm Manipulator and the Admissible Configurations Due to the Glass Table Constraints.

In this experimental evaluation, only the HDM's first and second links, called shoulder and elbow, respectively, are used. The third link, the wrist, remains locked in the configuration shown in Figure 7-1 during the evaluation.

The mass properties, including a 9.1 kg (20 lb) payload, are known to within a 10 percent margin of error and are shown in Appendix H. These mass properties are used to predict the system's behavior in simulation and to construct the Coupling Map for the system shown in Figure 7-2. They are also used to compute optimal P-PED gains for the system.

Due to the geometric constraints given by the area of the glass table on which the HDM operates, not all of the manipulator's joint space is available for maneuvering. Figure 7-2 shows the area in joint space of admissible configurations given the geometric constraints of the table. As shown in Figure 7-2 the area of admissible configurations does not contain hot spots and so the Hot Spot method could not be tested in this experiment. However, Base Relocation Algorithm was evaluated and
Figure 7-3: Initial and Final Configuration of the HDM for path1 in Inertial Space.

the results shown in the next section.

7.3 Path-Planning Using the Coupling Map's Base Relocation Algorithm

In this experiment, the effectiveness of the Coupling Map as a tool for path-planning to reduce the dynamic interaction between the manipulator and its compliant base is evaluated. For the first part of the experiment, a path was chosen in joint space to take the manipulator end-effector from point I to point F in inertial space. See Figure 7-3. This path, called path1, was purposely chosen to lie perpendicular to the lines of minimum coupling in the Coupling Map. See Figure 7-4\(^1\).

\(^1\)The circles represent the control points of the spline that define the path.
Figure 7-4: The path1 and its Relation to the Minimum Coupling Lines in the Coupling Map.

The dynamic characteristics of the HDM (see Appendix H) were supplied to RiBS\textsuperscript{2} and path1 was simulated. For the simulation, a velocity profile along the path was chosen that resulted in a final execution time of 1.6 seconds; a very fast maneuver. The simulation results of the base acceleration in the direction perpendicular to the beam axis is shown in Figure 7-5. The same maneuver was conducted on the real system and the base acceleration results are shown in Figure 7-6. Notice that the simulation and the experiment agree with the behavior of the system. For this case notice that the model used in the simulation is slightly more damped than the actual system but in general the base has a 0.5Hz damped frequency and took more than 25 seconds for its vibrations to die out.

For the second part of the experiment, the base relocation algorithm discussed in Section 5.2 was used to find a more suitable base location from which to perform the same task discussed above, and a new path, called path2, was computed. The new base location in relation to the old one is shown in Figure 7-7. A similar velocity profile was chosen for a 1.6 second maneuver. Since this path, shown in Figure 7-8, lies along a minimum coupling line, it is expected that this maneuver will result in dynamic

\textsuperscript{2}The RiBS library is a dynamic simulation package developed as part of this research effort. For more information, see Reference [83].
Figure 7-5: Simulation Results for the HDM Performing path1 in 1.6 seconds.

Figure 7-6: Experimental Results of the HDM Performing path1 in 1.6 Seconds.
interactions between the manipulator and its base that produce lower energy transfer than those generated by the maneuver using \textit{path1}. This was verified in simulation, as is shown in Figure 7-9 and experimentally demonstrated, as shown in Figure 7-10. As expected from the simulation results, \textit{path2} yielded a 75 percent reduction in the amplitude of the residual vibration when compared to \textit{path1}. This translates into a smaller base settling time, which increases the overall system performance. This behavior confirms the predictions made by the Coupling Map, since \textit{path1} moves perpendicular to the minimum coupling lines while \textit{path2} moves in the same direction as these lines.

As mentioned in Section 5.2, the base relocation algorithm can be effective when repetitive maneuvers of the manipulator from point \textit{I} to point \textit{F} are expected. This experiment demonstrates that by moving the base to the new location, we can improve the performance of the system by taking advantage of the system’s stiffness and dynamic characteristics.

In the next set of experiments, the same two paths as were used here will be used to evaluate other vibration control methods and to show how the Coupling Map can be
Figure 7-8: The path2 and its Relation to the Minimum Energy Lines.

Figure 7-9: Simulation Results for the HDM Performing path2 in 1.6 seconds.
Figure 7-10: Experimental Results of the HDM Performing path2 in 1.6 Seconds. The initial pick acceleration is due to the switching on of the closed-loop control system. 

used in combination with these other methods to help improve system performance.

7.4 P-PED in Fast Maneuvers

In Section 6.3, the effectiveness of the P-PED method in damping out externally-caused base vibration was demonstrated on the HDM. In this section, the P-PED algorithm is used in a path-planning and gain-scheduling control scheme to generate fast HDM maneuvers that result in low base settling times. This is done by combining the Coupling Map and the P-PED method into one path-planning and control scheme. The basic idea is to compute a low coupling path using the Coupling Map, execute the command under a normally tuned PD, high-bandwidth control, and then switch the system to P-PED control in real time at the end of the maneuver to damp out the residual vibrations produced by the maneuver.

In order to evaluate the performance of the HDM under this new proposed scheme, an input pre-filtering technique will also be demonstrated and the results will be used for comparative purposes. Input pre-filtering techniques have been proposed in recent years to deal with the problem of residual vibrations in systems that exhibit dynamic
behavior similar to the behavior seen in the system studied here [72, 70, 71, 51, 50]. Also, it is of some interest to observe how the Coupling Map can be used to help improve the performance of the pre-filtering scheme.

### 7.4.1 Implementation of P-PED on the HDM

In the experiments described above, the P-PED method was implemented using a local scheme. Recall from Section 6.2.2 that the matrices $K_{qp}$ and $K_{qp}$ in Equations (6.12) and (6.13), see page 100, are, in general, full matrices, and that, in order to obtain the correct dynamic response from the system, $K_{qp}$ and $K_{qp}$ must be used in full. Since the control architecture used in the HDM for this experiment is based on a local PD scheme, only the diagonal elements of the matrices $K_{qp}$ and $K_{qp}$ could be used in the implementation of the method. The results of the experiment show that, at least for this system, a full implementation of the P-PED control algorithm is not necessary. A simple local P-PED implementation of the algorithm significantly increases the system's ability to damp out base vibrations and improve overall system performance.

In order to implement P-PED control in fast maneuvers, the system must have the ability to switch gains in real time. Due to hardware constraints, this operation cannot be fully implemented on the HDM system. In this system, since the proportional gains are set in software, it is a simple procedure to change them in real time once the control software has detected that the arm has reached its final configuration. However, since the derivative gains for the HDM system are implemented in the analog domain and can only be tuned through potentiometers located in front of an electronic rack, it becomes physically impossible to tune the derivative gains in real time. Therefore, for this experiment, the derivative gains on the potentiometers were set at the beginning of the maneuver to the values computed by the P-PED method. The proportional gains were changed in real time once the manipulator reached its final configuration.
Figure 7-11: Experimental Results: Base Acceleration for path1 with Pre-Filtering. The initial pick acceleration is due to the switching on of the closed-loop control system. Notice that it took 4 seconds for the maneuver to complete.

7.4.2 Experimental Results

As mentioned above, the two paths, path1 and path2, used in the experiment described in Section 7.3 were used here for the evaluation of the P-PED in a fast maneuver scenario. For each path, two maneuvers were compared. We began with path1. In the first maneuver, the HDM was commanded to move along path1 using a three impulse pre-filter [69, 72, 70, 71]. In the second maneuver, the HDM was again commanded to move along path1, this time without pre-filtering but using conventional PD control at the beginning of the maneuver and P-PED control at the end of the maneuver. In a similar fashion, the HDM was then commanded to move along path2 using pre-filtering in the first maneuver, and then, in the second maneuver, to move along path2 using conventional PD control followed by P-PED control at the end of the maneuver. The results of these experiments are shown in Figures 7-11, 7-12, 7-13 and 7-14 respectively.

Figure 7-11 shows the HDM base acceleration history for the maneuver of path1 with a three-impulse pre-filter [69]. Notice that there was a significant reduction in
Figure 7-12: Experimental Results: Base Acceleration for path1 with P-PED. The initial pick acceleration is due to the switching on of the closed-loop control system. The high-frequency (50 Hz) vibration or humming at the end of the maneuver is not a base vibration. It is the effect of the system joint torque sensors as they sense bending in the joint shaft at such configuration.

Figure 7-13: Experimental Results: Base Acceleration for path2 with Pre-Filtering.
the residual vibration produced when pre-filtering was used, as compared with the amount produced in the maneuver through path1 that used only conventional PD control (Figure 7-6). However, due to the pre-filtering effect, a 1.6 second maneuver resulted in a 4.0 second maneuver time. Notice also in Figure 7-11 that there was some residual vibration due to the nonlinear behavior of the system; that is, the natural frequency of the base changed as the configuration changed. This filter was designed to cancel out a 0.5 Hz frequency. The frequency of the base residual vibration observed in Figure was 0.45 Hz. In order to increase the robustness of the pre-filtering technique, a greater number of impulses would have been needed. However, this would have had the negative effect of slowing down the manipulator motion during the maneuver and, thus, increasing the overall maneuver time.

Figure 7-13 shows the HDM base acceleration history for the maneuver along path2, which used the same three-impulse filter used in the path1 maneuvers described above. Notice that the filter used for path2 yielded a better performance than it did when used for path1, despite the system nonlinearities. This is because path1 was a high-coupling path, whereas path2 was a low-coupling path, as demonstrated by the Coupling Map. Because the amplitudes of the manipulator-caused base disturbances were lower for path2, the pre-filter did not have to "work so hard". From this we can
conclude that the Coupling Map can be used in conjunction with any pre-filtering technique to improve its robustness and to improve overall system performance.

Figure 7-12 shows the HDM base acceleration history for the maneuver along path1 using P-PED control at the end of the path. Notice that since no pre-filtering was introduced in this maneuver, the maneuver took 1.6 seconds as commanded. When the manipulator reached the end of the path, the P-PED control was turned on in order to damp out the large residual vibrations produced by this maneuver. Approximately 2.0 seconds later, all the base residual vibrations had been dissipated. This resulted in a 3.6 seconds total maneuver time. This maneuver took approximately the same amount of time as did the maneuver along path1 using pre-filter, but this time no residual vibrations were observed. This was because P-PED actively damped out any residual vibration in a closed loop fashion whereas the pre-filtering technique was an open loop technique. The P-PED technique was put to work at the moment the system began vibrating, whereas the pre-filtering technique worked before the maneuver began. After a maneuver is completed, the pre-filtering technique is unable to affect the system and, if the filter parameters have not been computed correctly or if nonlinearities are present, the performance of the system will degrade. The P-PED technique is more robust than the pre-filtering technique since Equation (E.5) guarantees that the system under P-PED is asymptotically stable and that the base vibrations will die out in a finite amount of time.

Figure 7-14 shows the HDM base acceleration history for the maneuver the path2 with P-PED control at the end of the path. This maneuver took about 4.0 seconds, which is approximately the same time taken by path2/pre-filtering maneuver. It is important to notice, however that just as with path1/P-PED maneuver, path2/P-PED maneuver resulted in zero residual vibrations, suggesting that path2/P-PED is more robust than the path2/pre-filtering maneuver. Thus, just as the Coupling Map can be used to improve the performance of a pre-filtering technique, it can also be used in conjunction with the P-PED method to improve overall system performance.
7.5 Summary

In this Chapter, the Coupling Map and the Pseudo-Passive Energy Dissipation Method were experimentally evaluated on a two-link planar manipulator. The experimental evaluation was conducted on the Martin Marietta Harmonic Drive Manipulator Testbed. For this evaluation, the Harmonic Drive Manipulator was mounted to the free end of a flexible cantilever beam. The first part of this experiment demonstrated that the Coupling Map, using the Base Relocation algorithm, will generate a path of low energy transfer that, when followed, produces significantly reduced residual vibration and, therefore, reduced base settling time. The second part of this experiment demonstrated that, by planning a low-coupling path that will produce low residual vibration, the Coupling Map can be used effectively to improve the performance of pre-filtering techniques. This experiment also demonstrated that the use of the P-PED method in conjunction with the Coupling Map will result in lower magnitudes of residual vibration and a faster cancelling out of that vibration, thus improving the overall performance of the system.

The Coupling Map and the P-PED method are currently been experimentally evaluated for more complex systems using the MIT Vehicle Emulation System II or VES II [37, 53, 26]. Also the Coupling Map and the P-PED method are been proposed to for experimental flight evaluation aboard the NASA Space Shuttle as part of a NASA In-Step proposal [74]. This work is being proposed by Martin Marietta Astronautics Group, NASA Langley Research Automation Technology, MIT and the University of Puerto Rico.
Chapter 8

Conclusions

8.1 Contributions of this Research

This thesis was motivated by the need to better understand the fundamental dynamic characteristics of space manipulator systems, with a special interest in developing new control and path-planning algorithms for regulating the dynamic interaction between a space manipulator and its base.

Two types of space manipulator concepts were considered in this investigation: free-flying systems and elastically constrained systems focusing largely on elastically constrained systems since it is expected that the first systems to be flown by NASA will be a system of this kind.

This thesis proposed the Coupling Map as an analytical tool to graphically describe the dynamic interaction between a manipulator and its moving base. The Coupling Map was used to develop three motion-planning algorithms designed to reduce the amount of energy transferred between the manipulator and its base. These are the Base Relocation algorithm, the Hot Spot method, and the Redundancy Resolver algorithm. All of these algorithms use the Coupling Map to exploit the nonlinear characteristics of the system for reducing the amount of energy transferred into the system’s base. For free-flying systems, these algorithms were shown to generate paths that, when followed, resulted in the reduction of fuel required to keep the spacecraft stationary during a manipulator maneuver. For elastically constrained systems, the
algorithms succeeded in generating paths that, when followed, resulted in a reduction of residual vibrations to the system base. Simulations showed from 14 to 75 percent reduction in the overall dynamic interaction between the manipulator and its base for both free-flying and elastically constrained systems. In general, these paths can be used as "good" starting points for more computationally intensive numerical optimization methods.

This thesis also introduced the *Pseudo-Passive Energy Dissipation* (P-PED) method as an effective control scheme for increasing the overall damping of elastically constrained systems and reducing unwanted base vibrations. Simulation as well as experimental evaluation of the P-PED method showed that this was an effective and viable tool for vibration control in elastically constrained systems. In addition, when used in conjunction with Coupling-Map-selected paths, P-PED was shown to further improve space manipulator performance. The development of the P-PED method also included a formulation for computing the optimal P-PED control gains.

A 19-DOF prototype system was designed in some detail to represent currently proposed space manipulator systems which might be used in future space missions. This prototype was used for simulation and testing of the different path-planning and control algorithms developed in this thesis.

Several special-case path-planning and control techniques were also developed which, although limited in use, provided significant insight into the dynamics of elastically constrained space manipulator systems.

This thesis also presented experimental verification of the Base Relocation path-planning method and the P-PED control scheme. It was also experimentally demonstrated that, by planning a low-coupling path that will produce low residual vibration, the Coupling Map can be used effectively to improve the performance of pre-filtering techniques. These experiments were conducted on the Martin Marietta Harmonic Drive Manipulator Testbed. The results showed the proposed methodologies to be viable for practical implementation on space manipulator systems.
8.2 Future Work

The Coupling Map has proven to be a successful tool for graphically describing the complex dynamic interaction between a manipulator and its moving base. However, as a graphical tool, it cannot be used to describe systems that use more than three-links for their maneuvers. Although techniques like projection, demonstrated in Section 5.3, have been used in cases in which only two or three links are used at a time for a given maneuver, more research is necessary in order to extend some of the algorithms to multidimensional spaces.

The concept of the Coupling Map and the motion-planning algorithms developed in this thesis have been successfully applied in simulation to relatively complex systems and experimentally evaluated on a simple two-link planar system. Clearly more experimental evaluation of this analytical tool is desirable in order to further examine the full capability of this technology.

Future space manipulators are expected to work with flexible payloads which must be handled carefully in order to avoid exciting dangerous vibrations which can cause damage to the payload itself and/or jeopardize the safety of the operation. Manipulators handling flexible payloads appear to have dynamic characteristics similar to those of elastically constrained systems, which suggests that the Coupling Map concept can be applied to the problem of motion-planning to reduce dynamic interaction between the manipulator and its flexible payload. This is promising area of research which could lead to the better understanding, prediction and control of these types of systems.

The success demonstrated in this thesis using the P-PED method for damping out unwanted vibrations in elastically constrained systems suggests that the P-PED method may be a suitable control method for damping out unwanted vibrations in more general flexible systems, such as compliant-link mechanisms and manipulator handling flexible payloads. This appears to be an area of research which can lead to better control of these systems.

More research is needed in order to effectively utilize the information provided
by the Coupling Map for motion-planning. Path-planning algorithms need to be developed to navigate through multidimensional spaces. Also, methods for efficiently computing and storing the Coupling Map are necessary. When a space manipulator grabs an object, its dynamic characteristics change and so does the shape of the Coupling Map. Practical implementation of the Coupling Map methodology for real-time path-planning requires that the Coupling Map be computed quickly and efficiently. Further research is suggested in the areas of parallel computation of the Coupling Map, neural-network implementation of the Coupling Map, and parameter identification for generation the Coupling Map.
Bibliography


[91] Z. Vafa and S. Dubowsky. On the dynamics of space manipulators using the virtual manipulator, with applications to path planning. *Journal of Astronauti-


Appendix A

Design of a Prototype Space Manipulator System

A.1 Introduction

A prototype of a space manipulator has been designed in some detail to aid in the study and evaluation of the problem of controlling base vibrations in such systems. The prototype design presented here is an attempt to represent the basic dynamic characteristics of a relatively complex space manipulator system and serves as the basis for the simulations and evaluations of the various path-planning and control algorithms proposed in this thesis.

A.2 Prototype Design: General Characteristics

The prototype design consists of a dextrous two-arm manipulator attached to a long flexible arm similar to the Remote Manipulator System (RMS). The design of the dextrous two-arm system is based on the preliminary design of the Canadian Special Purpose Dextrous Manipulator System, or SPDM, proposed to NASA for use on NASA’s future Space Station[96]. Since this system is currently under development, its actual dimensions and mass properties are unknown. The Martin Marietta Flight Telerobotic Servicer (FTS)[32] is a similar concept and provided an estimate of the
dimensions and mass properties for the SPDM system.

Our Prototype Dextrous Manipulator (PDM) consists of a 19-degree-of-freedom (DOF) system: two seven-DOF rigid arms connected to a five-DOF rigid body. This system has a total mass of 455 kg (1000 lbm). A conceptual design is shown in Figure A-1, A-2, A-3.

A.3 Structural Resonance Frequency Analysis and Link Design

Motion of the existing RMS is quite slow; on the Space Shuttle, an astronaut must wait between 20 and 40 seconds for the RMS to settle after a move [69]. However, since systems like the SPDM are designed to be teleoperated, their bandwidth is assumed to be of the same order of magnitude as that of a human operator working within an envelope of a 1.5 meter radius. With this in mind, a typical maneuver time of 3 seconds was chosen; i.e., the manipulator would take up to 3 seconds to move between two arbitrary end-effector positions with zero initial and final end-effector
Figure A-2: Prototype Dextrous Manipulator: Dimensions and Kinematic Structure of the Five-DOF Body
Figure A-3: Prototype Dextrous Manipulator: Dimensions and Kinematic Structure of the Right Arm
velocity. Based on this estimate, the control bandwidth of the two seven-DOF arms should be at least 1.5 Hz and the bandwidth of the five-DOF body at least 1.0 Hz. This system is assumed to be rigid; therefore, for a real system, the first structural resonance frequency would have to be above 15 Hz for the seven-DOF arm and above 10 Hz for the five-DOF body.

A.3.1 Seven-DOF Dextrous Arms

In order for the first structural resonance frequency to be above 15 Hz, the seven-DOF arms were chosen to be 1.5 meters long, see Figure A-1. The two longest links; links 3 and 4, are made of thin-walled composite tubes with an outer radius of 8.25 cm (3.25 in) and wall thickness of 3.2 mm (1/8 in). This model has a first structural resonance frequency of 27.8 Hz when a payload of 50 kg is attached to its end-effector. Without a payload, the first structural resonance frequency is 40.3 Hz. These frequencies were calculated using elementary structural mechanics and modeling the manipulator as a Bernoulli-Euler beam [81]. The mass and inertia of the actuators were included. These theoretical resonance frequencies would probably be lower in a real system due to the effects of transmission compliance.

A.3.2 Five-DOF Body Structure

For the five-DOF body similar analysis was carried out. The two most critical links in this mechanism are links 3 and 4. These are designed to be rather wide structures with interior room for sensors and a variety of tools used by the two seven-DOF arms, see Figure 4. Each of the links 3 and 4 is a 1.1 x 0.2 x 0.6 m rectangular structure. A simplified model, based on a thin-walled 2.3 m rectangular cantilever beam with a mass at its end, was used to calculate the first structural resonance frequency of this system. The beam is made of steel with a cross-section similar to the cross-section of links 3 and 4, 20x60 cm, and a wall thickness of 10 mm (3/8in). A 300 kg mass at its end accounts for the mass of the two seven-DOF arms, its payload, cameras and sensors. Based on this model, the first structural resonance frequency is 20.9
Table A.1: Seven-DOF Joint Actuator Maximum Torque Capacities

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Hz with a payload and 23.5 Hz without a payload. As with the structural analysis of the seven-DOF arms, these higher theoretical resonance frequencies were chosen to compensate for the fact our simplified model does not take into account joint servo compliances. Also, this simple model neglects the inertial effect of the two seven-DOF arms.

A.4 Actuator Selection

A.4.1 Seven-DOF Dextrous Arms

Rotary electrical actuators, such as the ones used in the FTS concept, were chosen for this prototype model. This system is intended for operation in a microgravity environment, so the actuators do not need to be designed to support the weight of the arms themselves. Instead, maximum joint acceleration is the important factor in designing actuator maximum torque capacity. Table A.4.1 shows the designed peak torques for the seven actuators in the seven-DOF arm.

The numbers in Table A.4.1 are based on the maximum acceleration required to follow the trajectory described by Equation A.1. Figure A-4 shows a graphical
Figure A-4: Graphical Description of the Trajectory Used to Determine Maximum Joint Torque Capacities

description of this trajectory.

\[ y(t) = \pi(1 - \cos\left(\frac{\pi}{3}t\right) \quad (A.1) \]

\[ A.4.2 \quad \text{Five-DOF Body Structure} \]

The maximum torque capacities for the five-DOF body structure were designed in a similar way. Table A.4.2 shows the designed peak torques for the five-DOF body system.

\[ A.5 \quad \text{Summary of Prototype Design Parameters} \]

Tables A.5 and A.5 represent a summary of the kinematic and dynamic properties of the PDM. These values were used for simulation using the RiBS [83] simulation package.

Note that the total mass includes the mass of actuators, sensors, etc. The center of mass of each link is measured with respect to a local coordinate system defined by the Denavit-Hartenberg parameters [4]. Moment of inertia is measured about the geometric center of each link. The product of inertia for each link is assumed
Table A.2: Five-DOF Joint Actuator Maximum Torque Capacities

<table>
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<th>Joint</th>
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to be equal to zero. The Denavit-Hartenberg parameters are used to described the kinematic relationship between links. See Reference [4] for a detailed description of the variables.
<table>
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<td>0.0</td>
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<td>0.0</td>
</tr>
<tr>
<td>Center of Mass Z (m)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.75</td>
<td>0.0</td>
</tr>
<tr>
<td>Inertia $I_{xx} (kg - m^2)$</td>
<td>3.0</td>
<td>2.0</td>
<td>1.0</td>
<td>3.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Inertia $I_{yy} (kg - m^2)$</td>
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<td>0.1</td>
<td>3.0</td>
<td>2.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Inertia $I_{zz} (kg - m^2)$</td>
<td>3.0</td>
<td>2.0</td>
<td>2.5</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Denavit-Hartenberg</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height (m)</td>
<td>0.5</td>
<td>0.3</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Theta (deg)</td>
<td>$\theta_1$</td>
<td>$\theta_2$</td>
<td>$\theta_3$</td>
<td>$\theta_4$</td>
<td>$\theta_5$</td>
</tr>
<tr>
<td>Length (m)</td>
<td>0.0</td>
<td>0.0</td>
<td>1.5</td>
<td>0.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Alpha (deg)</td>
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<td>90.0</td>
<td>0.0</td>
<td>90.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Table A.4: Mass and Kinematic Parameters for the Seven-DOF Arm

<table>
<thead>
<tr>
<th>Five-DOF Body Subsystem</th>
<th>Link1</th>
<th>Link2</th>
<th>Link 3</th>
<th>Link 4</th>
<th>Link 5</th>
<th>Link 6</th>
<th>Link 7</th>
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<tr>
<td>Mass (kg)</td>
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<td>10.0</td>
<td>30.0</td>
<td>30.0</td>
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<td>Center of Mass X (m)</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.75</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Center of Mass Y (m)</td>
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<td>0.75</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Center of Mass Z (m)</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Inertia $I_{xx} (kg \cdot m^2)$</td>
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<td>1.5</td>
<td>50.0</td>
<td>1.0</td>
<td>1.5</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Inertia $I_{yy} (kg \cdot m^2)$</td>
<td>0.5</td>
<td>0.1</td>
<td>1.0</td>
<td>50.0</td>
<td>1.5</td>
<td>1.0</td>
<td>1.0</td>
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<td>50.0</td>
<td>50.0</td>
<td>1.5</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Denavit-Hartenberg</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.75</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Theta (deg)</td>
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<td>$\theta_2$</td>
<td>$\theta_3$</td>
<td>$\theta_4$</td>
<td>$\theta_5$</td>
<td>$\theta_6$</td>
<td>$\theta_7$</td>
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<td>0.0</td>
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<tr>
<td>Alpha (deg)</td>
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</tr>
</tbody>
</table>
Appendix B

Prototype Model of a Long Flexible Arm

B.1 Introduction

This appendix presents a description of the elastic characteristics of a Prototype Long Flexible Arm (PLFA). This structure is based on the elastic characteristics of the Space Shuttle's Remote Manipulator System. This Prototype Elastic Structure is part of the Prototype Dextrous Manipulator (PDM) designed as part of this research, see Appendix A, and is aimed at the study and evaluation of the problem of controlling base vibrations in elastically constrained systems. The PLFA and the PDM are an attempt to represent the basic dynamic characteristics of a relatively complex space manipulator system and serves as the basis for the simulations and evaluations of the various path-planning and control algorithms proposed in this thesis.

B.2 The Flexible Arm

Figure B-1 shows a graphical representation of the Prototype Long Flexible Arm which dimension and mass properties are based on the Space Shuttle's Remote Manipulator System. Table B.2 shows the Denavit and Hartenberg parameters for the PLFA and Table B.2 shows its mass properties. The PLFA is designed to be a 50 ft
Table B.1: The Prototype Long Flexible Arm Denavit and Hartenberg Parameters

<table>
<thead>
<tr>
<th>Link</th>
<th>height(m)</th>
<th>theta(deg)</th>
<th>length(m)</th>
<th>alpha(deg)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.00</td>
<td>-90.00</td>
</tr>
<tr>
<td>2</td>
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<td>90.00</td>
<td>-0.31</td>
<td>90.00</td>
</tr>
<tr>
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<td>90.00</td>
<td>6.38</td>
<td>0.00</td>
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<tr>
<td>4</td>
<td>0.00</td>
<td>0.00</td>
<td>7.06</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>0.00</td>
<td>0.00</td>
<td>0.46</td>
<td>90.00</td>
</tr>
<tr>
<td>6</td>
<td>0.00</td>
<td>-90.00</td>
<td>0.00</td>
<td>-90.00</td>
</tr>
<tr>
<td>7</td>
<td>1.43</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

long, seven-DOF arm with links 3 and 4 modelled as flexible links made out of thin-wall composite tubing. Links 1, 2, 5, 6, and 7 are assumed to be rigid to simplify the analysis. Also, since the manipulator is intended to serve as a flexible structure to support the Prototype Dextrous Manipulator (PDM), its joints will be assumed to be locked in the configuration shown in Figure B-1.

The composite material from which links 3 and 4 are made of are assumed to have a modulus of elasticity and shear modulus of that of steel. Figure B-1 shows the dimensions of the tubing cross section form which the stiffness characteristics of the arm were calculated. A coordinate system is assigned to link 7, see Figure B-1 and the stiffness characteristics are derived for that point in the structure. The following is the stiffness matrix at such point:

\[
K_b = \begin{bmatrix}
9798.72 & 0.00 & 0.00 & 0.00 & 0.00 & 76430.0 \\
0.00 & 305128.0 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 9798.72 & -76430.0 & 0.00 & 0.00 \\
0.00 & 0.00 & -76430.0 & 794872.0 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 153846.0 & 0.00 \\
76430.0 & 0.00 & 0.00 & 0.00 & 0.00 & 794872.0
\end{bmatrix}
\]  

(B.1)

A 5 percent structural damping is assumed which yield the following damping
Figure B-1: Prototype Model of Long Flexible Arm (PFLA): Conceptual Drawing
Table B.2: The Prototype Long Flexible Arm Mass Properties

<table>
<thead>
<tr>
<th>Link</th>
<th>Mass (kg)</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>$I_{xx}$</th>
<th>$I_{yy}$</th>
<th>$I_{zz}$</th>
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<tbody>
<tr>
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<td>0.00</td>
<td>3.67</td>
<td>0.69</td>
<td>3.80</td>
</tr>
<tr>
<td>2</td>
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<td>0.00</td>
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<td>1.12</td>
</tr>
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<td>997.17</td>
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<tr>
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<td>-0.34</td>
<td>2.83</td>
<td>3.82</td>
<td>1.90</td>
</tr>
</tbody>
</table>

matrix:

$$D_b = \begin{Bmatrix} 109.26 & 0.00 & 0.00 & 0.00 & 17.68 & 136.28 \\ 0.00 & 1073.17 & 11.97 & -139.91 & 0.00 & 0.00 \\ 0.00 & 11.97 & 105.26 & -92.81 & 0.00 & 0.00 \\ 0.00 & -139.91 & -92.81 & 873.93 & 0.00 & 0.00 \\ 17.68 & 0.00 & 0.00 & 0.00 & 549.76 & 32.37 \\ 136.28 & 0.00 & 0.00 & 0.00 & 32.37 & 1360.15 \end{Bmatrix} \quad (B.2)$$

B.3 Summary

This appendix presented a description of the Prototype Long Flexible Arm. This structure is based on the elastic characteristics of the Space Shuttle’s Remote Manipulator System.
Appendix C

Case Studies: Translational Compliant Systems

C.1 Introduction

This Appendix presents a study of several special configurations of elastically constrained system. These configurations, while based on unrealistic systems with very particular mass and elastic properties, provide significant insight into the dynamics of elastically constrained space manipulator systems.

The discussion starts by dividing the set of all dynamic systems described by Equation (3.2), page 46, into three distinct groups, see Figure C-1. These are: Translational Compliant Systems (TCS), Rotational Compliant Systems (RCS) and Translational plus Rotational Compliant Systems (TRCS). As the names imply, TCS are systems which base vibrations are translational in nature with very high torsional stiffness at the base. High stiffness implies zero motion in such direction. RCS, opposite to TCS, are systems which base vibrations are rotational in nature with very high translational base stiffness. Finally, TRCS, are systems which translational and rotational base stiffness are of the same magnitude and must be accounted for in the model.

In this appendix we analyze the TCS model and propose several techniques for minimizing the amount of residual base vibration produced by the manipulator mo-
Figure C-1: Translational Compliant Systems, Rotational Compliant Systems and Translational plus Rotational Compliant Systems.

C.2 Translational Compliant Systems

The Translational Compliant System, is a special class of flexible base space manipulator systems which flexibility is largely translational in nature with little or no rotational motion. Figure C-2 shows an example of a TCS.

If no disturbances are present, vibrations in these systems are due to the manipulator-caused base forces. These forces are produced as a consequence of accelerating the center of mass of the manipulator during a maneuver.

Three TCS systems are presented that reduce the amount of residual vibrations to the flexible base. The first two are designed to reduce the amount of acceleration to the center of mass of the manipulator. The last system uses pre-filtering techniques to plan the path of the manipulator's center of mass.
Figure C-2: An Example of a Translational Compliant System.

C.2.1 Configuration-Independent TCS Systems

The Configuration-Independent System is a special type of TCS system in which the center of mass does not change with configuration. This is due to the special mass distribution of its links. Maneuvers produced by this type of manipulator will generate no acceleration to the manipulator’s center of mass and therefore no force will be experienced by its base. Figure C-3 shows an example of this type of system.

C.2.2 Redundant TCS Systems

Another way to eliminate accelerating the manipulator's center of mass during a maneuver is to use redundancy. Figure C-4 shows a three-link planar TCS system. Due to the redundant nature of this system, paths can be found that produce no translation of the center of mass resulting in zero base disturbances and no vibrations. Figure C-5 shows a closed-loop path that results in a zero motion of the manipulator’s center of mass.

Configuration independent systems and redundant manipulators will generate a vibration free maneuver on translational compliant base manipulator systems. How-
Figure C-3: Configuration-Independent Translational Compliant System.

Figure C-4: Redundant Translational Compliant System.
Figure C-5: Closed-Loop Path that Results in no Motion to the Manipulator's Center of Mass.

However, it is important to notice that these systems will not produce torque-free maneuvers which will cause vibrations in systems that show rotational compliance such as RCS and TRCS.

C.2.3 Pre-filtering to the Manipulator Center of Mass

Several input shaping techniques has been applied to a number of flexible systems including flexible-link manipulators [72, 70, 71, 51, 50]. However, this methodology has not been applied to compliant base manipulators. Here a path-planning algorithms based on pre-filtering of the manipulator's center of mass is proposed as a way to minimize manipulator-caused residual vibrations in TCS systems.

In TCS systems, once the base is vibrating, manipulator-caused base forces will increase or decrease the magnitude of this vibration depending on the time of application of such forces. For example, if the maneuver produces an instantaneous base force in the same direction as the base velocity, then this force will result in an increase in the magnitude of the vibration. However, if the instantaneous manipulator-caused base force is opposite in direction to the base velocity, then the result will be a de-
crease in the magnitude of the base vibration. This is true for any dynamic system described by Equation (3.2) as concluded in Appendix E.

Notice also that due to orthogonality, the motion of the manipulator's center of mass can be prescribed in terms of the three directions, \( x \), \( y \) or \( z \), corresponding to the vibration directions of the base. This means that, given a proper manipulator motion, the corresponding motion of the manipulator's center of mass can be decoupled in its orthogonal directions in inertial space.

Finally, notice that the base's natural frequency for TCS is configuration independent. This means that we can treat the base of a TCS as a linear system.

These observations suggest the use of techniques based on linear theory to deal with the problem of manipulator-caused base vibrations in TCS systems and have lead to the development of a path-planning algorithm based on pre-filtering techniques. This algorithm is as follows:

Let us consider a two-link planar system mounted on a two-DOF translational compliant base such as the one in Figure C-6. Our goal is to move the manipulator's end-effector from point \( I \) to point \( F \) in inertial space (see Figure C-6) both in the minimum amount of time and with the minimum amount of residual vibrations to the base. Notice that moving from configuration \( I \) to \( F \) requires the translation of the manipulator's center of mass from point \( cm_1 \) to point \( cm_2 \). It is also required that the initial and final velocities of the maneuver be zero, which implies that the center of mass of the manipulator will experience acceleration and deceleration during this maneuver.

One way to minimize the residual vibrations of this maneuver is to compute a path between configuration \( I \) and \( F \) that takes into consideration the acceleration history of the manipulator's center of mass.

This can be done by independently pre-shaping the \( x \), \( y \) and \( z \) components of an arbitrary path followed by the manipulator's center of mass.

The following steps outline the algorithm.

1. We start out with the knowledge of the initial configuration and the desired final end-effector position. It is assumed that the dimensions as well as the mass
Figure C-6: Left: A Two-Link Planar Manipulator Mounted on a Two-DOF Translational Compliant Base. Right: Computed Path Using Pre-Filtering to the Manipulator Center of Mass and Corresponding CM trajectory.
properties of the manipulator and its base are known. Also knowledge of the natural frequency of the base is available.

2. With knowledge of the manipulator's initial configuration, the initial position of the manipulator's center of mass can be computed. This can be done through forward kinematics [4] on the system's virtual manipulator[87].

3. Given the desired final end-effector position, the equations of inverse kinematics [4] can be used to compute the final configuration F and the final manipulator's center of mass cm2.

4. Moving the manipulator's end-effector from point I to point F (Figure C-6) implies moving the manipulator's center of mass from cm1 to cm2. The path is then obtained by convolving a step function and a sequence of pulses designed to eliminate vibrations in the x, y and z directions. Since in general the system base can exhibit different stiffness characteristics in each of its x, y and z direction, a set of impulse sequences must be computed for every direction of motion. Figure C-7 summaries this technique.

The right side of Figure C-6 shows the computed path using the method proposed above. Notice the trajectory of the manipulator's center of mass. This path was simulate and showed no residual vibrations in any of the two vibratory axes.

This technique has potential applications in systems in which torsional base stiffness is high and rotational base motions can be neglected. An example of such a system is a manipulator sitting at the end of a long flexible cantilever beam such as the one depicted in Figure 5-12, 74. For this system, rotational motion do occurs but can be considered negligible when compared to the magnitude of the translational motion for a given disturbance.

Path planning using pre-filtering of the manipulator's center of mass present some limitations. The system can only be applied to TCS systems. In RCS or TRCS systems the base does not behave in a linear fashion so liner superposition of input commands does no hold. Also, just as with any input shaping technique, knowledge of the system dynamic properties is required. A discussion on robustness of pre-filtering techniques can be found in [69].
Figure C-7: Summary of the Pre-Filtering on the Center-of-Mass Technique.
C.3 Summary

This appendix presented a study on translational Compliant Systems. It showed that vibrations in TCS systems are a consequence of accelerating the manipulator's center of mass during a manipulator maneuver. Two special configurations were proposed to avoid accelerating the manipulator's center of mass during a maneuver. These were *Configuration-Independent TCS Systems* and *Redundant TCS Systems*.

Also, a pre-filtering technique was presented to plan the acceleration history of the manipulator's center of mass for a given initial and final configuration. This motion-planning technique can have potential applications in systems high torsional stiffness characteristics. However, as with any pre-shaping technique robustness is an issue that must be dealt with in a case by case basis.

TCS systems are in general unrealistic systems but they provide significant insight into the dynamics of elastically constrained space manipulator systems.
Appendix D

Case Study: Rotational Compliant Systems

D.1 Introduction

The Rotational Compliant System, is a special class of flexible base space manipulator system in which flexibility is largely rotational in nature with little or no translational motion. See Figure C-1.

These systems are more complex than TCS systems since base vibrations depend on both forces and torques produced by manipulator motions. Moreover, the natural frequency of the base is configuration dependant so the system is in general nonlinear.

In this appendix a method for "pumping" energy out of a given rotational mode of vibration is developed. This method explores the phenomenon known as parametric excitation [76].

D.2 Parametric Excitation

The nonlinearities of Rotational Compliant Systems (RCS) are only due to the dependency of the system’s mass matrix on the configuration of the system. The base stiffness is assumed linear in this study, see Chapter 3. In order to illustrate the parametric excitation phenomenon in RCS systems, we are going to select a spe-
cial type of RCS system with the property that motion of the manipulator does not produces forces to the base. Such system is depicted in Figure D-1 and is named Symmetric-RCS due to the symmetry of the arms about the base center of mass.

Normally, the system depicted in Figure D-1 can move its arms independently, however in this study we are are going to constrain the motion of the two arms. In Symmetric-RCS, the left arm configuration is a mirror image of the right arm at all time. This way the torque produced by one arm cancels out the torque produced by the other. With this motion constrain, the arms cannot produce any net force or torque to the system base regardless of their motion.

It would appear that the arms and the base of a Symmetric-RCS system are dynamically decoupled from one another, however, as shown below this is not the case.

If the initial base strain energy is zero, namely the initial deflection of the base elastic member is zero, then the base will remain undisturbed independent of the motion of the manipulator. However, if there is an initial deflection of the base elastic structure, cyclical motions of the manipulator arms can pump energy in or out a given base mode of vibration depending on the time history of this motion. This phenomenon is known as parametric excitation. Dynamical systems of this type are
Figure D-2: Description of the Cyclical Path Designed to Pump Energy Out of a Rotational Mode of Vibration.

referred as linear system with periodic coefficients and the theory of such systems of equations is frequently referred as Floquet's theory [76].

A general review of Floquet's theory of beyond the scope of this thesis (for a detailed description of Floquet's theory see references [76, 52]), however, in this case study, a controlled cyclical motion is proposed based on the phenomenon of parametric excitation to help pump energy out of a given base mode of vibration.

For system described above the following cyclical path is proposed:

\[
\begin{align*}
\text{if } ||\theta|| \leq \epsilon & \text{ then } q_1 = -q_2 = 0.0 \\
\text{if } ||\dot{\theta}|| \leq \zeta & \text{ then } q_1 = -q_2 = 0.4\pi
\end{align*}
\]

(D.1)

where \(\epsilon\) and \(\zeta\) are two arbitrary small values and \(q_1\) and \(q_2\) refers to the position of joint 1 and 2 respectively for both right and left arms. Equation (D.1) said that when the base deflection, \(\theta\) is approximately zero, the system must extend or open it arms as far away from the base center of mass as possible. This configuration is to be maintained until the system base has reached its maximum deflection, i.e., \(\theta = \theta_{max}\) or \(\dot{\theta}\) is almost zero, the system must close or fold its arms as close as possible to the base (for this example 0.4\(\pi\) or 72 deg). This configuration is then held until \(\theta\) is almost zero and the process is repeated until all the energy of the system has been dissipated. Figure D-2 presents a graphical description of this cyclical path.

The proposed cyclical motion described by Equation (D.1) was evaluated in sim-
Figure D-3: Phase Plane Showing the Time History of the Base Angular Motion as Cyclical Motion Takes Place.

ulation and the results are shown in Figure D-3. An initial deflection of the base of $\theta_0 = \pi$ was set as initial conditions and the arms started in it folded position.

Figure D-3 shows the time history of the base motion in the phase plane. The abscissa represents the base angular position $\theta$, and the ordinate represents the base angular velocity $\dot{\theta}$. Notice in Figure D-3 that the system starts with an initial deflection of 0.41 rads or 23.5 degrees. The arrow on the curve indicates the direction of time as the system evolves under parametric excitation. The distance from the origin in the phase plane is related to the system’s total energy. Notice that as time passes the total energy stored in the base in the form of potential and kinetic energy is *cyclically* dissipated as oppose to *asymptotically* dissipated as in the case of a linear damper. In other words, the energy stored in the base is *pumped* out of the system. This phenomenon is explained below.

The Symmetric-RCS system moving under the cyclical motion described by Equation (D.1) can be represented by a differential equation known as a *Meissner equation* which stability and form of the general solution has been studied in great detail in
[76]. In this study an different approach is introduced to the analysis of this system which provides a better insight into the dynamics of the problems and help define an upper bound on the maximum amount of energy it can be pump in or out of the base using this method.

Let us start by looking at the total energy of the system at the beginning of the simulation. Since the initial velocity of the system is zero, the total energy of the system, \( e \), is equal to the potential energy stored in the elastic structure of the base, \( V \), and given by the following expression:

\[
e = V = \frac{1}{2} k \theta_0^2 \tag{D.2}
\]

where \( k \) is the rotational stiffness of the base and \( \theta_0 \) is the initial angular deflection of the base.

As the system starts moving, the potential energy \( v \), gets transfer into kinetic energy \( T \). When the all the potential energy is transferred into kinetic energy ( \( \theta = 0 \) ) the total energy in the system is given by

\[
e = T = \frac{1}{2} I_c \dot{\theta}^2 \tag{D.3}
\]

where \( I_c \) is the inertia of the system along the axis of vibration for the arms closed. So far no motion of the arms has occurred so the total energy of the system is conserved.

At this point, based on Equation (D.1), the system arms are open very fast relative to the motion of the base\(^1\). During this process the base angular deflection is approximately zero. Therefore it is reasonable to assume that no torques act on the base during this short period of time. Since the base torque equals zero, the angular momentum around the axis vibration is conserved during the opening of the arms and we write

\[
p_c = I_c \dot{\theta}_c = I_o \dot{\theta}_o = p_o \tag{D.4}
\]

where \( p_c \) is the angular momentum of the base for the arms fully closed, \( p_o \) is the angular momentum of the base for the arms fully extended or open, \( \dot{\theta}_c \) is the base

\(^1\)Fast relative to the motion of the base. This is necessary in order for this quasi-static analysis to be valid.
angular velocity for the arms closed, \( \dot{\theta}_o \) is the base angular velocity for the arms open and \( I_o \) is the moment of inertia of the base about the axis of vibration when the arms are fully open. For the system under study

\[
I_o > I_c
\]  
(D.5)

since the opening of the arms increases the inertia of the system about the axis of vibration. The kinetic energy of the system for the system after the arms are open is written as

\[
T_o = \frac{1}{2} I_o \dot{\theta}_o^2
\]  
(D.6)

Notice that the kinetic energy of the system is not conserved during the opening of the arms. From Equation (D.4) an expression for the angular velocity of the base after the arms are open is obtained.

\[
\dot{\theta}_o = \frac{I_c}{I_o} \dot{\theta}_c
\]  
(D.7)

Combining Equation (D.6) and Equation (D.7) we have

\[
T_o = \left( \frac{1}{2} I_c \dot{\theta}_c^2 \right) \frac{I_c}{I_o}
\]  
(D.8)

The expression in parenthesis in Equation (D.8) is the kinetic energy of the system just prior to the opening of the arm, \( T_c \), therefore

\[
T_o = T_c \frac{I_c}{I_o}
\]  
(D.9)

Combining Equation (D.5) and Equation (D.9) we can show that the kinetic energy of the base just after the opening of the arms, \( T_o \), is less than the kinetic energy of the base prior to the opening of the arms, \( T_c \), by a factor equal to the ration \( \frac{I_c}{I_o} \).

Now where does this energy goes? The fraction of the energy \( T_c - T_o \), is transferred into the arms and is dissipated by the manipulator actuators. Notice that in opening the arms the manipulator actuators do not require as much energy since the centrifugal force produced by the angular velocity of the base helps the manipulator actuators to open the arms. This is how the energy is transferred into the arms. Now
this energy is dissipated by the manipulator actuator as they decelerate the arm into its final open arm position.

After the arms are open the magnitude of the angular velocity of the base is smaller and so does the total kinetic energy of the system. Now the base continues its motion with the arms fully extended and the remainder of the kinetic energy is then transfer into the base compliant structure. Since a fraction of the total energy has been dissipated during the maneuver, the final deflection of the base will be less than the initial deflection.

When the system transfers all its kinetic energy into potential energy, at $\dot{\theta} = 0$, Equation (D.1) tells the system to close the arms. Since $\dot{\theta} = 0$ the total kinetic energy of the system is zero regardless of the change in inertia. At this point the system starts gaining momentum again and the process is repeated until all the energy of the system is pumped out of the system.

The analysis described above is based on a quasi-static model of the actual dynamic system designed to aid in the understanding of the complex phenomenon of parametric excitation. It does not describe the exact dynamics of the system since it assumes that the arms will open and close almost instantaneously. However, for symmetric-RCS system with very low base vibration frequency and a fast controller this model provides a good approximation. For slower arm systems the quantity $T_0$ will be higher that the one predicted by Equation (D.9) and the efficiency of the system will be lower. The faster the maneuver the more energy can be pumped out of the system with an upper bound given by Equation (D.9) since this equation is derived for instantaneous maneuvers.

The cyclical motion scheme described above has potential applications in vibration control and can be easily applied in real systems. Any two-arm system such as the SPDM shown in Figure 2-4 page 36, or the FTS shown in Figure 2-3 page 35, is a potential candidate for implementation of this type of scheme. The method can be implemented in a closed loop nonlinear controller scheme in which the base angular position and velocity are sensed, say using strain gages attached to the structure and accelerometers, and a command send to the arms to open and close based on
Equation (D.1). The system has limited application on system with translational compliant behavior since this method will not pump energy out of a translational mode of vibration.
Appendix E

Lyapunov Stability Analysis

This appendix presents a stability analysis of the dynamic system described by Equation (3.2). In order to be able to conclude the conditions of stability for such system without having to solve for the solution of the differential equation a Lyapunov stability analysis is conducted [17]. For this analysis we proposed the expression for the total energy of the system as candidate Lyapunov function which can be written as

\[ v = \frac{1}{2} \dot{\xi}^T H(\xi) \dot{\xi} + \frac{1}{2} \xi^T K \xi \]  

(E.1)

The function described by Equation (E.1) is always positive or zero because the manipulator inertia matrix \( H(\xi) \) and the stiffness matrix \( K \) are positive-definite matrices. Differentiation of Equation (E.1) yields

\[ \dot{v} = \dot{\xi}^T H(\xi) \ddot{\xi} + \frac{1}{2} \dot{\xi}^T \dot{H}(\xi) \dot{\xi} + \dot{\xi}^T K \xi \]  

(E.2)

\[ \dot{v} = \ddot{\xi}^T [H(\xi) \ddot{\xi} + \frac{1}{2} \dot{H}(\xi) \dot{\xi} + K \xi] \]  

(E.3)

By careful examination of the structure of the Lagrange's dynamics it can be shown that \( \dot{H} - 2C(\xi, \dot{\xi}) \) is antisymmetric [4] so Equation (E.3) reduces to

\[ \dot{v} = \ddot{\xi}^T \Xi \]  

(E.4)
Due to the structure of the $\Xi$ vector described in Equation (3.5), Equation (E.4) can be written in terms of the manipulator joint variables as

$$\dot{v} = q^T \tau$$  \hspace{1cm} (E.5)

Equation (E.5) shows that when the manipulator joint torques are in the same direction of the joint velocity, energy is put into the system; i.e. $\dot{v} > 0$. It also shows that, for the system described by Equation (3.2), the only way to remove energy out of the system; i.e. $\dot{v} < 0$, is if the torque applied to the joints is always opposite in direction to the velocity of the joints. This is an important conclusion which becomes fundamental in the development of the Pseudo-passive Energy Dissipation Method described in Chapter 6.
Appendix F

Spring Energy Analysis

This Appendix compares the energy stored in two spring with different stiffness characteristics. It concludes that a softer spring has more capacity to store energy that a stiffer one for the same applied force. This conclusion is fundamental for the formulation of the Coupling Map developed in Chapter 4.

Let's take two springs with stiffness constants $k_1$ and $k_2$. Let's assume that the stiffness of the first spring, $k_1$ is greater than the stiffness of the second or

$$k_1 > k_2 \quad \text{(F.1)}$$

The constitutive equations of the two springs are

$$f_1 = k_1 x_1 \quad \text{(F.2)}$$

$$f_2 = k_2 x_2 \quad \text{(F.3)}$$

where $x_i$ is the deflection of of spring $i$ for a given force $f_i$. If both springs resist the same force then

$$f_1 = f_2 \quad \text{(F.4)}$$

Since the force rather than the spring deflection has been chosen as the independent variable, the amount of energy stored in the springs is given by the potential
coenergy (see reference [18]), $V^*$, expressed as

$$V_1^* = \frac{1}{2} \frac{1}{k_1} f_1^2$$  \hspace{1cm} (F.5)

$$V_2^* = \frac{1}{2} \frac{1}{k_2} f_2^2$$  \hspace{1cm} (F.6)

Now combining Equation (F.4), Equation (F.5) and Equation (F.6), it can be shown that

$$\frac{V_1^*}{V_2^*} = \frac{k_2}{k_1}$$  \hspace{1cm} (F.7)

or

$$V_1^* = V_2^* \frac{k_2}{k_1}$$  \hspace{1cm} (F.8)

Since $k_1 > k_2$ then $\frac{k_2}{k_1} < 1$ and

$$V_2^* > V_1^*$$  \hspace{1cm} (F.9)

Equation (F.9) shows that more energy is stored in a softer spring than in a stiffer one for the same applied force. This is an important observation which will become fundamental in the development of the Coupling Map described below.
Appendix G

The Advanced Research Manipulator II

G.1 Introduction

This appendix presents some of the features and mass properties of the NASA Advanced Research Manipulator II (ARM II). This model is used to evaluate in simulation the different path-planning and control algorithms presented in this thesis.

G.2 The ARM II properties

The ARM II has been designed as research manipulator incorporating many features not found in industrial robots. Some of these features are [30]:

- 1:3.6 payload to weight ratio.
- A 40 lb payload at 60-in.
- Operation form JPL Universal Motor Controller.
- A reconfigurable modular joint design.
- 8-DOF revolute joints.
- Graphite/epoxy links.
Figure G-1: The Advanced Research Manipulator II (ARM II): Conceptual Drawing

- Infinite roll of joint 8.
- Input and output joint position sensing.
- Extensive use of space flight qualifiable technology.
- 4 axis gimbaled wrist.

Figure G-1 shows a simpler conceptual drawing of this system. Table G.2 show the Denavit and Hartenberg parameters for this 8-DOF manipulator. The mass properties of this system were not known at the time this research was conducted and were estimated. These estimated values are shown in Table G.2.

G.3 Summary

This appendix presented a model of the NASA Advanced Research Manipulator II (ARM II). This model is used to evaluate in simulation the different path-planning and and control algorithms presented in this thesis.
Table G.1: The Advanced Research Manipulator II Denavit and Hartenberg Parameters

<table>
<thead>
<tr>
<th>Link</th>
<th>height(m)</th>
<th>theta(deg)</th>
<th>length(m)</th>
<th>alpha(deg)</th>
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<td>90.00</td>
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<td>0.00</td>
<td>90.00</td>
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<td>0.00</td>
<td>90.00</td>
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<td>0.00</td>
<td>90.00</td>
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<td>0.00</td>
<td>0.47</td>
<td>0.00</td>
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</table>

Table G.2: The Prototype Long Flexible Arm Mass properties

<table>
<thead>
<tr>
<th>Link</th>
<th>Mass(kg)</th>
<th>Center of Mass(m)</th>
<th>Inertia(kg – m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>x</td>
<td>y</td>
</tr>
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<td>0.00</td>
<td>-0.08</td>
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<tr>
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<tr>
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<td>5.20</td>
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<td>0.00</td>
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</tbody>
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Appendix H

Characteristics of the Martin Marietta Harmonic Drive Planar Arm System

H.1 Introduction

This appendix presents a description of the mass properties and general hardware characteristics of the Martin Marietta Harmonic Drive Manipulator. This research testbed was used to experimentally demonstrated the capability of the Coupling Map and the P-PED method as viable tools for regulating the amount of vibration produced by the manipulator motion to its supporting structure. The results of these experiments are shown in Chapter 7. The dynamic properties of the system as presented in this appendix were also used to simulate the behavior of the HDM in order to predict the outcome of the experiments before they were carried out.

H.2 General Hardware Characteristics

The Martin Marietta Harmonic Drive Manipulator (HDM), see Figure H-2, was designed as a research testbed to study a number of fundamental problems related to the control of space manipulator systems. The HDM is a three-link planar manipulator
Figure H-1: Martin Marietta Harmonic Drive Planar Arm System: Conceptual Drawing

supported by air bearings on a glass table top for frictionless motion. The actuators are brush DC motors driving an 80:1 harmonic drive transmission. Local joint torque feedback loops allows the arm to behave as a frictionless direct-drive system. Each joint has a motor-side tachometer, an output torque transducer, and a resolver to measure joint angle.

For this experiment the HDM was mounted to a 0.886m aluminum beam which acted as a flexible structure and an accelerometer was mounted at the outer casing of the shoulder joint to monitor base vibrations. Figure H-1 shows the overall dimensions of the system including the dimensions of the glass table top and location of the base accelerometer. A 20 lb (9.1 kg) payload was attached to the manipulator wrist joint. For this experiment only the shoulder and the elbow joints were used in motion planning. The wrist joint was keep locked at the configuration shown in Figure H-1. For this configuration the HDM base vibration was between 0.45 to 0.5 Hz depending on the manipulator configuration.

H.3 Mass Properties

The mass properties of the HDM are shown in Table H.3.
Table H.1: Mass and Kinematic Parameters for the Harmonic Drive Planar Arm Manipulator

<table>
<thead>
<tr>
<th>HDM</th>
<th>base</th>
<th>Shoulder</th>
<th>Elbow</th>
<th>Wrist</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
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<td>13.80</td>
<td>12.12</td>
<td>13.70</td>
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<tr>
<td>Center of Mass X (m)</td>
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<td>-0.19</td>
<td>-0.08</td>
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<td>Center of Mass Y (m)</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Center of Mass Z (m)</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Inertia $I_{zz} (kg \cdot m^2)$</td>
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<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Inertia $I_{yy} (kg \cdot m^2)$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Inertia $I_{yy} (kg \cdot m^2)$</td>
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<td>0.73</td>
<td>0.11</td>
</tr>
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<td>Denavit-Hartenberg</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height (m)</td>
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<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Theta (deg)</td>
<td>$\theta_1$</td>
<td>$\theta_2$</td>
<td>$\theta_3$</td>
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</tr>
<tr>
<td>Length (m)</td>
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<td>0.56</td>
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<tr>
<td>Alpha (deg)</td>
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<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>
H.4 Base Damping and Stiffness Characteristics

Based on the geometric and materials properties of the aluminum beam the stiffness and damping matrices for the HDM supporting structure were calculated. These are

\[
K_b = \begin{bmatrix}
488775.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 10020.8 & 0.00 & 0.00 & 0.00 & -4444.05 \\
0.00 & 0.00 & 360748.00 & 0.00 & 159986.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 14469.30 & 0.00 & 0.00 \\
0.00 & 0.00 & 159986.00 & 0.00 & 94601.50 & 0.00 \\
0.00 & -4444.05 & 0.00 & 0.00 & 0.00 & 2627.82 \\
\end{bmatrix}
\]

(H.1)

and

\[
D_b = \begin{bmatrix}
1831.19 & -3.65 & 0.00 & 0.00 & 0.00 & 2.29 \\
-3.65 & 15.28 & 0.00 & 0.00 & 0.00 & -1.20 \\
0.00 & 0.00 & 90.59 & -0.48 & 6.53 & 0.00 \\
0.00 & 0.00 & -0.49 & 11.90 & 3.23 & 0.00 \\
0.00 & 0.00 & 6.53 & 3.23 & 30.96 & 0.00 \\
2.29 & -1.20 & 0.00 & 0.00 & 0.00 & 5.27 \\
\end{bmatrix}
\]

(H.2)
Figure H-2: Photograph of the Martin Marietta Harmonic Drive Planar Arm System
I might be a Captain by rank,
but all I ever wanted to be was an Engineer.

Scotty, "Star Trek: The Next Generation"