THE DESIGN OF A LIQUID FLUIDIC REACTION JET SYSTEM

by

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B.S., University of Maryland (1966)

Submitted in Partial Fulfillment
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ABSTRACT

An analytical and experimental investigation has been performed to determine the feasibility of a liquid fluidic reaction jet system. A rational basis for the prediction of static and dynamic performance of a bi-stable fluid amplifier with reaction jet nozzles as a load has also been established. A fluidic reaction jet system was built, and, when tested, compared favorably with an existing system controlled by an electromagnetic shuttle valve with respect to efficiency and speed of response.

As part of the detailed design study, a method is developed to predict the time required to switch the bi-stable amplifier and the dynamic effect of the receiver inertance on the instantaneous thrust is analyzed. Experimental results supported the analytical conclusions developed in the design study.

Thesis Supervisor:  David N. Wormley
Title:  Assistant Professor of Mechanical Engineering
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Nomenclature

A - Area (in²)
b - Supply nozzle area (in)
d - Wall setback (in)
J - Momentum flux (lb)
K - Power (watt)
m - Mass (lb·sec²/in)
P - Pressure (psig)
Q - Flow (in³/sec) (except where specified gpm)
Sₜ - Empirical transport time constant
t - Time (sec)
T - Thrust (lb)
t' - Streamline coefficient
V - Fluid Velocity (in/sec)
v₁ - Empirical volume constant
v - Volume (in³)
w - Receiver width (in)
x - Axial distance from supply nozzle (in)
y - Lateral distance from centerline (in)
z - Height of amplifier (in)
α - Wall angle (radians)
β - Fluid bulk modulus (psi)
θ - Jet attachment angle (radians)
\[ \eta_p \] - Receiver diffuser efficiency
\[ \rho \] - Fluid density (lb\-sec\(^2\)/in\(^4\))
\[ \sigma \] - Goertler jet entrainment constant

Subscripts

\( c \) - Control port
\( co \) - Control to supply ratio
\( e \) - Receiver exit or Entrainment flow
\( h \) - Section at horizontal jet streamline
\( o \) - Supply port
\( r \) - Point of reattachment
\( s \) - Splitter
\( sr \) - Splitter to reattachment ratio
\( t \) - Total
\( 1, 2, 3 \) - Sections of receiver
\( d \) - Centerline
CHAPTER 1
THE PROBLEM

1.1 Introduction

The purpose of this thesis is to investigate the feasibility of a fluidic reaction jet system. To accomplish this goal, a secondary objective has been to establish an analytical design procedure for a bi-stable fluid amplifier.

In the reaction jet systems there is a need for a valving mechanism to modulate the net thrust of two opposing reaction jet nozzles (see Figure 1.1). In the application being considered in this investigation, the modulation is by an electromagnetic shuttle valve. In a search for better performance, smaller size and longer lifetime, the pure fluid amplifier has been considered as a replacement for this shuttle valve. The object of this study is to determine if the fluidic reaction jet system is indeed a desirable substitute. However, the method of obtaining the electric to fluid drive signal is not considered.

The field of fluidics is advancing rapidly as the merits of pure fluid control and computational devices are discovered and applied. A wide variety of these devices have been conceived and are now commercially available. Fluidic devices are very attractive because of their simplicity, reliability, and their ability to operate in hostile environments. These desirable characteristics are derived from the fact that fluidic devices have no moving parts.
Even though fluidic devices have developed considerably in the last decade, most designs have evolved through successive trial and error and are heavily dependent on experimentally derived performance characteristics. When a new application of a fluidic component is desired, considerable experimental effort must usually be expended. The need exists, then, for the establishment of an analytical basis for predicting the static and dynamic performance characteristics of fluidic devices. A considerable amount of analytical work has been done to date on the separate parts of the devices considering their basic fluid mechanics. This work is summarized in the NASA\textsuperscript{(1)} report on the state of the art of fluid amplifiers. However, additional study is needed to form these separate works into a rational design procedure for a complete system.

Reaction jet systems are usually the actuating component of a larger control system. The reaction jet transforms fluid power to a change in momentum of the fluid and uses the resultant reaction force to accomplish control. If directional control is desired, an active device is usually required to modulate the fluid power. Reaction jet systems have many applications in the control of rockets, missiles, and water borne vehicles which can utilize the merits of fluid amplifiers with regard to reliability and maintenance in the hostile environments in which they operate. It is in regard to this interest that the author has undertaken this study.
1.2 Specific Application

In the reaction jet system described in this study, the valving mechanism currently in use is an electromagnetic shuttle valve. The particular fluid amplifier chosen to be studied is of the same type as the electromagnetic valve to facilitate comparison. It is a bi-stable device and is operated in a pulse-ratio modulated bang-bang mode. The principal design constraints listed in the order of their importance are:

a. Operation with a dense liquid
b. Size
c. The time required to switch
d. The hydraulic power necessary to achieve the desired amount of reaction thrust
e. Operation in a pulse-ratio modulated bang-bang mode

A sketch of the present shuttle valve used in the system is shown in Figure 1.2. Table 1.1 lists the specifications of this electromagnetic design. These will be used as a base line for comparing performance.

A bi-stable fluid amplifier has several inherent characteristics which are significant with respect to the goal of meeting the two critical constraints of switching time and thrust per unit power. In most conventional valving mechanisms a metering orifice is used to control the flow. These generally have the characteristics of a "quadratic"
Figure 1.2 Schematic of Electromagnetic Shuttle Valve (scale: 5X)
<table>
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<td>16.5 Hydraulic Watts</td>
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<tr>
<td>Control Power</td>
<td>4.0 Watts</td>
</tr>
<tr>
<td>Supply Pressure</td>
<td>26 psi</td>
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<tr>
<td>Supply Flow</td>
<td>1.45 gpm</td>
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<td>Maximum Output Thrust</td>
<td>.42 lbs.</td>
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<td>Switching Time</td>
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<td>Fluid Specific Gravity</td>
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<td>Fluid Bulk Modulus</td>
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<tr>
<td>Overall Size</td>
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orifice in which the pressure drop across the orifice is proportional to the square of the flow through it. Usually no means is provided to recover the kinetic energy developed by accelerating the flow to high velocity at the exit of the orifice. Hence, the maximum ideal efficiency of such a device, power out divided by power in, is limited to 66 2/3 percent (see Reference 3). In the reaction jet application, the use of a fluid amplifier allows recovery of the velocity head since the thrust is related to the momentum of the fluid leaving the nozzles.

In the present application, both the thrust per unit power and the response are of particular interest. It should be noted, however, that having high momentum recovery is not the same as having high thrust per unit power. Output power in this case is the product of thrust and flow velocity. Therefore, a device which operates at low fluid velocities can develop its thrust with less power consumption (see Appendix A). However, response time generally has an inverse relationship to fluid velocity. This is due to the transport time in fluidic devices and the size of the metering orifice in electromagnetic devices. Hence, depending on the velocity needed to meet the response requirement, a valve with lower momentum recovery could have a higher thrust per unit power than a different type of valve of high momentum recovery. The significance of these factors is treated analytically in the sections that follow.
It is significant to note that in the present application, the fluid amplifier load is constant. Since there is only one load condition, the design may be optimized for that case. It is also noted that the system is operated in a pressurized environment to minimize the effects of cavitation.
CHAPTER 2
ANALYTICAL DESIGN

2.1 Introduction

The object of this chapter is to develop an analytical design procedure for a bi-stable fluid amplifier with reaction jet nozzles as a load. The design procedure will be developed by adapting the studies of the separate phenomenon which occur in a fluid amplifier to the specific application previously listed. These phenomenon are:

a. The free submerged jet
b. The attachment of a submerged jet to an adjacent wall
c. The interaction of a submerged jet with a receiver load
d. Interaction of a jet with control flow
e. The time required for a jet to switch from one wall and reattach to the other.

The analysis of each area provides design parameter values which must be met to accomplish the overall design objectives. As noted, previous work has been done in each of these areas and will be utilized where appropriate.

A description of a liquid fluidic reaction jet system is shown in Figure 2.1. It consists of a bi-stable fluid amplifier where the output ports function as two opposing reaction jet nozzles. Supply flow is
Figure 2.1  Principal Elements of a Bistable Fluid Amplifier - Reaction Jet System
emitted as a submerged jet into the interaction region from the supply nozzle. The jet then attaches to one of the walls and the flow goes into the receiver and out the nozzle. In order to switch the jet to the opposite side, control flow is introduced on the attached side.

Analysis of the free jet produces a velocity profile as a function of nozzle dimension, b, (see Figure 2.2), and distance from the nozzle, x. This information is used in designing the receiver and determining the distance required for the jet to attach to the wall.

The work done on wall attachment relates the non-dimensional attachment distance, $x_r/b$, with wall angle, $\alpha$, and wall offset, $d/b$. This determines the geometrical shape of the amplifier and is affected by the relative size of the receiver.

The interaction of the submerged jet and the receiver is quite frequently neglected due to its complexity. However, it is very important to a complete design. The receiver design produces a relation for momentum recovery, $J/J_o$, as a function of receiver to supply area ratio, $A/A_o$ and splitter distance, $x_s/b$.

Switching time is a function of the absolute size of the amplifier, the supply velocity and the control flow. Given the control to supply flow ratio, $Q_c/Q_o$, along with response and thrust requirements, the absolute size and power consumption is determined. To summarize, the wall attachment and receiver analysis determine the non-dimensional geometry consistent with the requirement for high efficiency while the response and output requirements determine the size and power level.
Figure 2.2 Schematic Showing Parameters
2.2 The Interaction Region

The first area one must study when analyzing a fluid amplifier is the free jet and its attachment to an adjacent wall. From this study the basic geometrical relation between wall offset, \( d/b \), wall angle, \( \alpha \), and attachment distance, \( x_r/b \), is obtained.

Wall attachment is the basic phenomenon of the bi-stable fluid amplifier. It is the effect caused by sidewalls adjacent to a submerged jet and is called the "Coanda-effect." The phenomenon occurs because of the viscous shear forces present when a submerged jet exhausts from a sharp edged orifice (see Figure 2.3). The velocity difference across the boundary of the jet develops a shear force which entrains the surrounding fluid into the jet flow. Additional fluid moves in to replenish the fluid being entrained. The presence of a wall restricts the path of the entrainment flow so that the flow increases in velocity. If the jet is closer to one wall than the other, the static pressure will be lower on that side. Thus there is a pressure gradient across the jet which forces the jet even closer. The jet will reach a stable position when it is attached to the wall and a vortex is developed by entrainment and return flow since no net flow can leave this "bubble" region. If a fluid amplifier is designed properly, a small amount of flow may be induced into the bubble region to allow the pressure to increase, causing the jet to leave that wall and attach to the other.

A lot of attention has been given to analytical descriptions of the free jet. One of the first was by Albertson\(^{(2)}\). Albertson demon-
Fig 2.3 Schematic Representation of Jet Diffusion (ref. 2)
strated that all turbulent free jets are nearly identical when described by non-dimensional parameters regardless of the fluid or operating conditions. Another description of the free jet is given by Goertler\(^4\), whose solution is different in form from that of Albertson, but gives identical results. Goertler's model for the velocity profile, \( V \), is the following:

\[
V = \left[ \frac{3 J \sigma}{4 \rho (x + x_0)} \right] \cdot \text{sech}^2 \left( \frac{\sigma y}{x + x_0} \right)
\]  

(1)

where \( x \) = axial distance from slot
\( y \) = perpendicular distance from the axis of the jet
\( \sigma \) = a floating constant entrainment parameter, which permits some variation in the model.

This equation is simply a two dimensional description of the velocity profile as a function of lateral and longitudinal distances from a two-dimensional nozzle. \( \sigma \) is left as a floating constant to allow variation of conditions. However, Goertler claims the value to 7.69 for a completely free turbulent jet.

The first study of wall attachment effects was performed by Bourgue and Newman\(^5\). They predicted attachment distances for the two cases of a parallel wall set back from the nozzle and an inclined wall with no setback. Levin and Manion\(^6\) have extended this study to include the possibility of both wall offset and angle.
In both of these papers, the attachment distances for several values of the entrainment parameter, $\sigma$, and various conditions such as different Reynold's numbers and different fluids are predicted. The authors then compare their predictions to their measured values. The data from Levin and Manion is presented in Figures 2.4 and 2.5. The data demonstrates that, except for extreme conditions, the value of $\sigma$ is very close to that suggested by Goertler, even though the jet is considerably affected by the pressure of an adjacent wall. This is an important point, because the value of $\sigma$ is a measure of the amount of entrainment. Even though the jet is considerably affected by the presence of adjacent walls, it still diffuses and entrains the same amount of flow.

Levin and Manion's work is very conclusive in demonstrating the validity of their analytical derivations. Their control-volume model shows especially good correlation with their liquid jet at $\sigma = 7.69$ and therefore will be used here.

The equations of Levin and Manion are as follows:

$$\frac{x_r}{b} = \frac{\sigma}{3(\theta + \alpha)} \left( \frac{1}{t' \omega} - 1 \right) \left( \sin \alpha + \sin \theta \right) \left( \frac{\tanh^{-1} t'}{3 t' \omega \sin \theta} \right)$$

$$- \left( \frac{d}{b} + \frac{1}{2} \right) \sin \alpha$$  \hspace{1cm} (2)
Fig 2.4  Comparison of Observed Attachment Distance with Theoretical Computations Based on Control Volume Model, Reference (6)
Fig. 2.5 Comparison of Observed Attachment Distance with Theoretical Computations Based on Control Volume Model, Reference (6)
\[
\frac{d}{b} = \frac{\sigma}{3(\theta + \alpha)} \left( \frac{1}{t'^2} - 1 \right) \left( 1 - \frac{\cos \theta}{\cos \alpha} \right) - \frac{1}{2}
\]  

(3)

\[\cos \theta = 0.5 + 0.75 t' - 0.25 t'^3\]

where \(\sigma = 7.69\)

\(t'\) = streamline coefficient

\(d\) = wall setback

\(b\) = nozzle width

\(\alpha\) = wall angle

\(\theta\) = streamline attachment angle

\(x_r\) = attachment distance

Again, these equations make a major contribution toward the design by fixing the relationship between non-dimensional wall setback, \(d/b\), wall angle, \(\alpha\), and attachment distances, \(x_r/b\).
2.3 Receiver Design

Another major contribution to the design of fluid amplifiers is the study of receiver interaction by Reid \(^{(7)}\). From his study it is possible to determine the relationship between the receiver area ratios and the receiver to nozzle area ratio to optimize the receiver with respect to momentum recovery (see Figure 2.6). He provides three different methods of varying complexity for determining receiver flows as a function of distance from the nozzle, \(x_s/b\), area ratio, \(A/A_0\), diffuser efficiency, \(\eta_p\), load condition, \(P_4/P_0\), and free jet parameters, \(V_h/V_o\). Reid's work allows inclusion of the effect of receiver interaction in the design.

Reid's study was for an axisymmetric jet with the interaction region completely open to the atmosphere. However, since his study is the only one available, it will be used to approximate the momentum recovery of the wall attachment device. The validity of this approximation will be determined by experiment. It is significant to note that Reid allowed a variation of Albertson's free jet profile to include possible affects of the presence of a receiver, but finally used a constant very close to that which Albertson suggested.

Since reaction jets operate as a constant load, it was decided that Reid's third theory, which neglects viscous losses, would suffice. It would be very beneficial, in a more exhaustive study, to use his more complex theory so as to take into account affects of receiver
viscous losses and velocity profile non-uniformity. However, he points out that a receiver less than five diameters in length has negligible viscous losses. Reid's theory gives the following relation:

\[
\frac{A_o}{A_1} - \frac{1}{2} \eta_p \left( \frac{A_o}{A_1} \right) \left( 1 - \frac{A_1^2}{A_o^2} \right) \left( \frac{Q}{Q_o} \right)^2 \\
+ \left[ \frac{1}{2} \frac{A_1}{A_o} f_1 \left( \frac{x}{D_o} \right) - C \right] \left( \frac{Q}{Q_o} \right) \\
+ \frac{1}{2} \frac{A_1}{A_o} \left[ \frac{P_4}{P_o} - f_1 \left( \frac{x}{D_o} \right) \right] = 0
\]

(5)

where, \(Q\) = flow rate (in\(^3\)/sec)

\(A\) = area (in\(^2\))

\(P\) = pressure (psi)

\(\eta\) = diffuser efficiency

\(f_1 \left( \frac{x}{D_o} \right) = C^2 = \left( \frac{V_L}{V_o} \right)_{\text{free jet}}\)

The above relation arises from an application of the "cowl" theory used in predicting flow rates of air breathing engines. The flow conditions of the fluid entering the receiver are matched to the conditions in the free jet at some arbitrary point, \(h\), where the streamlines are parallel. The theory requires the average value of the velocity at the point, \(h\).
Since \( V_h \) is not known, Reid suggests as an approximation, the center-line velocity of supply. However, since \( V_h \) is a function of axial distance, \( x_s \), the author will consider \( V_h \) as a variable and attempt to evaluate it experimentally. Hence,

\[
f_1 \left( \frac{x}{D_o} \right) = \left( \frac{V_h}{V_o} \right)^2 = C^2
\]  

(6)

The load on the receiver is the reaction jets. This load can be approximated by a quadratic orifice open to a reservoir at ambient pressure. Therefore,

\[
\frac{P_4}{P_o} = \left( \frac{Q}{Q_o} \right)^2 \left( \frac{A_o}{A_e} \right)^2 \left[ 1 - \left( \frac{A_e}{A_3} \right)^2 \right]
\]

(7)

Substituting (6) and (7) into Equation (5),

\[
\left( \frac{Q}{Q_o} \right)^2 = \frac{\frac{A_1}{A_o} \frac{V_h}{V_o} \left( \frac{1}{2} \frac{A_1}{A_3} \frac{V_h}{V_o} - 1 \right) Q}{\left[ 1 - \frac{\eta_p}{2} \left( 1 - \frac{A_1^2}{A_3^2} \right) + \frac{1}{2} \left( \frac{A_1}{A_e} \right)^2 \left( 1 - \frac{A_e^2}{A_3^2} \right) \right]}
\]

(8)
The performance criterion to which the above relation must be optimized is the thrust per unit power. As shown in Appendix A, the thrust per unit power is simply a function of the velocity of the exhausting fluid.

\[
\frac{J}{K} = \text{Const.} \frac{V}{V}
\]

where K is power in watts \hspace{1cm} (9)

However, since the receivers are independent of the supply, except through the momentum of the fluid, the maximum momentum which the receivers can exchange with the environment to obtain thrust is that which it recovers from the supply jet. The momentum of the supply jet is described by Equation (9). Hence, to optimize the receivers, the momentum recovery should be optimized.

The momentum recovery can be characterized by the ratio,

\[
\frac{J}{J_0} = \frac{Q/Q_0}{A_e/A_0}
\]

in which the flow ratio is determined by Equation (8). Equation (10) assumes a constant velocity profile which is usually the case with a converging nozzle or turbulent flow.
Before attempting to optimize the complete expression, a few observations will make the task a little simpler. First, Equation (8) is of the form

$$Q^2 + \frac{b}{2a} Q - \frac{c}{a} = 0$$

(11)

The solution for $Q$ is,

$$Q = -\frac{b}{2a} + \left[\left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right]^{1/2}$$

(12)

For maximum momentum recovery, $J/J_o$, the value of "a" should be minimized.

The expression of interest is,

$$a = 1 - \left[\frac{\eta_P}{2} 1 - \left(\frac{A_1}{A_3}\right)^2\right] + \frac{1}{2} \left(\frac{A_1}{A_e}\right)^2 \left[1 - \left(\frac{A_e}{A_3}\right)^2\right]$$

(13)

This factor contains the terms associated with internal diffuser losses and area changes. It should be mentioned that this theory does not take into account viscous losses. For simplicity, viscous losses in the receiver will be assumed to contribute less than 10 percent loss in momentum since the receivers will be kept short as advised by Reid.
For the simple case of constant area diffusion with no losses, the value of the above expression is unity. Unity is approximately the optimum value even though it may be made smaller by letting

\[ A_e = A_3 < A_1 \]

However, any increase in \( A_e \) adversely affects the momentum at the exit due to the area term in Equation (10). The value of unity will be used for this analysis.

Writing Equations (8) and (10) with non-dimensional variables,

\[ Q^2 + AV \left( \frac{1}{2} AV - 1 \right) Q - \frac{1}{2} (AV)^2 = 0 \]  \hspace{1cm} (14)

\[ J = \frac{Q^2}{A} \]  \hspace{1cm} (15)

where \( Q = \frac{Q}{Q_o} \)

\[ A = \frac{A_e}{A_o} = \frac{A_1}{A_o} \]

\[ V = \frac{V_h}{V_o} \]
Solving for $Q$,\
\[ Q = \frac{1}{2} \ AV \left\{ - \left( \frac{1}{2} \ AV - 1 \right) + \sqrt{\left( \frac{1}{2} \ AV - 1 \right)^2 + 2} \right\}^{1/2} \] \hspace{1cm} (16)

Squaring and substituting into Equation (10),\
\[ J = \frac{1}{2} \ AV^2 \left\{ \left( \frac{1}{2} \ AV - 1 \right)^2 + 1 \right. \]
\[ + \left. \left( \frac{1}{2} \ AV - 1 \right) \sqrt{\left( \frac{1}{2} \ AV - 1 \right)^2 + 2} \right\} \] \hspace{1cm} (17)

It is now noted that if the positive sign is used, the momentum will go to infinity if the area ratio is increased to infinity. This solution is physically unrealistic so only the negative sign will be used.

The above equations are plotted in Figures 2.7 and 2.8. Eq. (17) indicates it is possible to obtain full momentum recovery with the proper area and the receiver setback sufficiently small to obtain a high average velocity, $V_h$.

To optimize the momentum recovery analytically, differentiate Equation (17) with respect to area and set equal to zero.
Fig 2.8  Receiver Momentum Recovery, Equation (17)

MOMENTUM RATIO

RECEIVER AREA RATIO, A/A₀

V₀/V₀ = 0.50
\[
\frac{dJ}{dA} = \frac{V^2}{2} \left\{ 1 + \left( \frac{1}{2} AV - 1 \right)^2 \left( \frac{3}{2} AV - 1 \right) \right\} = 0
\]

This equation is satisfied if \( AV = 1.44 \). The solution is verified graphically in Figure 2.8. Hence, there is now a geometric relation which may be used in the design of the amplifier. For purposes of investigation let

\[
\frac{V_h}{V_o} = 1.0
\]

\[
\frac{A_1}{A_o} = \frac{w}{b} = 1.44
\]

2.4 Switching Time

The determination of switching time is the most complex study of the fluid amplifier since it deals with a dynamic situation and is the least understood area. The study provides a relationship between jet velocity, \( V_o \), control to supply flow ratio, \( Q_{co} \), switching time, \( t \), and
actual size of the amplifier, b. If the control flow ratio, response requirements and the output thrust are given the actual size and power level are determined.

The time response requirement in the case studied here is sufficiently stringent to necessitate examination of the dynamic behavior of the fluid being used. As noted earlier, the fluid is a very dense liquid. In Appendix B the compressibility and inertance effects of the receivers are analyzed. It is demonstrated that the compressibility effects are negligible and the effect of the fluid inertance is very considerable. However, it is also shown that, although the inertance of the fluid causes a large delay in obtaining fluid motion, thrust can be obtained at the instant the jet switches due to acceleration of the fluid in the receiver. Hence, in this study, only the time necessary to switch the free jet in the interaction region will be considered.

After studying the switching it was found that one link in the chain of information is missing. To the author's knowledge there is no information published which is sufficiently general to predict the switching time of a bi-stable wall attachment device. There are, however, several papers which point out the direction one must take to find this relationship. The most enlightening of these is the thesis by Johnston\(^{(8)}\) which predicts switching times and describes the mechanism which occurs.
The basic mechanism described by Johnston for pressure controlled amplifiers is the following: once the required entrainment flow is supplied, the jet will consistently move to a certain point before it will separate from one side and go to the other and this point is constant for any particular design. This suggests that the separation bubble, the volume between the jet and the wall, must reach a specific size before separation occurs. Hence, a critical amount of mass must be introduced into the bubble to increase it to that size. If one knew that critical volume and then set the control flow rate, he could quickly calculate the time to obtain the required amount of mass.

This mechanism is also suggested by Katz and Brown. However, the problem arises in determining that critical position of the jet or, similarly, the critical mass. Johnston's model was particularly easy to predict because receiver interaction was eliminated in his analysis and he used a terminated wall device in which the critical position was simply the point at which the jet left the wall. However, Johnston's means of determining his result could help considerably, in experimentally analyzing more complicated situations. Manion has done some work in predicting dynamic response of proportional amplifiers in which he uses a control-volume method which he feels could be adapted to analysis of bi-stable devices.

One can, however, assume that the mechanism of switching is of the type mentioned above. Even though the size of the critical bubble volume is unknown, one could assume a reasonable size, subject to al-
teration by experimental investigation. The assumption will be, then, that some critical bubble volume is reached before separation occurs and then the jet will reattach to the opposite wall in some time proportional to the transport time.

This assumption takes the following form:

\[ t_{\text{switch}} = t_{\text{separation}} + t_{\text{reattachment}} \]

\[ = \frac{v_x}{Q_c} + S_t \frac{x_s}{V_o} \]  

(19)  

(20)

where \( v_x \) = the critical volume (in \(^3\))  
\( Q_c \) = control flow (in \(^3\)/sec)  
\( x_s \) = splitter distance (in)  
\( V_o \) = supply velocity (in/sec)  
\( S_t \) = transport time constant

Letting,

\[ v_x = v_1 x_s^2 z, \]

\[ t = v_1 \frac{x_s^2 z}{Q_c} + S_t \frac{x_s}{V_o} \]  

(21)  

(22)
Substituting,

$$Q_c = \frac{Q_c}{Q_o} Q_o = Q_{co} z b V_o$$  \hspace{1cm} (23)$$

one obtains the following;

$$t = \left[ \frac{v_1}{Q_{co}} \left( \frac{x_s}{b} \right) + S_t \right] \frac{x_s}{V_o}$$  \hspace{1cm} (24)$$

where \( v_1 \) = volume constant
\( Q_{co} \) = control flow ratio

This relation is plotted in Figure 2.9. However,

$$V_o = \left[ \frac{J}{\rho z b} \right]^{1/2}$$  \hspace{1cm} (25)$$

Substituting (25) into (24) and rearranging, an expression for the nozzle width is obtained.

$$b = \left[ \frac{x_s}{b} \left( \frac{x_s}{b} \frac{v_1}{Q_{co}} + S_t \right) \right]^{2/3} \left[ \frac{J}{\rho z} \right]^{1/3}$$  \hspace{1cm} (26)$$
Fig. 2.9 Switching Time versus Control Flow Ratio and Supply Velocity Equation (24)
where \( v_1 \) = critical volume constant
\( S_t \) = transport time constant

A rational choice of these two constants must now be made. By assuming the Strouhal number (total switching time divided by transport time), approximate splitter distance, approximate flow gain (inverse of control flow ratio) and dividing the time equally between separation and reattachment, the value of the constants can be obtained. For purposes of investigation, let,

\[
\text{Strouhal No.} = \frac{t}{x_s/V_o} = 2.0
\]

\[
\frac{x_s}{b} = 6.0, \quad Q_{co} = 0.3
\]

Substituting into Equation (24),

\[
\frac{v_1}{Q_{co}} \left( \frac{x_s}{b} \right) + S_t = 2.0
\]

Dividing the time equally, \( S_t = 1.0 \), hence,

\[
v_1 = \frac{Q_{co}}{x_s/b} = .05
\]
In the above example a relatively large value was assumed for the control flow ratio, $Q_{co'}$, because the control flow required for a dynamic switch will necessarily be greater than that required to switch in a steady state sense. The control flow required for steady state switching will be determined by experiment.

2.5 Simultaneous Solution

There is now enough basic information to make a complete design. The equations must now be put together to obtain a simultaneous solution. The equations are:

$$\cos \theta = 0.5 + 0.75 t' - 0.25 t'^3$$  \hspace{1cm} (4)

$$\frac{d}{b} = \frac{2.55}{\theta + \alpha} \left( \frac{1}{t''^2} - 1 \right) \left( 1 - \frac{\cos \theta}{\cos \alpha} \right) - \frac{1}{2}$$ \hspace{1cm} (3)

$$\frac{x_r}{b} = \frac{2.55}{\theta + \alpha} \left( \frac{1}{t''^2} - 1 \right) \left( \sin \theta + \sin \alpha \right) \left[ \frac{1}{2} \ln \frac{1 + t'}{1 - t'} \right]$$

$$\left[ \frac{3}{2} \frac{t'}{\sin \theta} \right]$$

$$- \left( \frac{d}{b} + \frac{1}{2} \right) \sin \alpha$$ \hspace{1cm} (2)

$$b = \left[ \frac{t}{x_s} \left( \frac{x_s}{b} \frac{v_1}{Q_{co}} + S_t \right) \right]^{2/3} \left[ \frac{J}{\rho z} \right]^{1/3}$$ \hspace{1cm} (26)
Due simply to geometrical constraints the following relation is obtained.

\[
\frac{x_s}{b} = \frac{1}{\tan \alpha} \left[ \frac{w/b}{\cos \alpha} - \left( \frac{d}{b} + \frac{1}{2} \right) \right]
\]  

(27)

In order to link the wall attachment study to the geometry of the amplifier, a relationship is needed between the attachment distance and the receiver distance. The receiver study showed that momentum recovery was highest for the shortest possible receiver distance. For purposes of this study, it will be assumed that the attachment distance equals the receiver distance. Therefore, let,

\[
\frac{x_s}{x_r} = 1.0
\]

and Equation (2) becomes

\[
\frac{x_s}{x_r} = \frac{2.55}{\theta + \alpha} \left( \frac{1}{t'^2} - 1 \right) (\sin \theta + \sin \alpha) \left\{ \frac{1}{3} \ln \frac{1 + t'}{1 - t'} \right\}
\]

(28)
These equations are complicated by the implicit streamline function, $t'$. In order to solve them simultaneously an iteration procedure is used. The format is to assume a value for $t'$ and use the Newton-Raphson iteration procedure to find the solution to Equation (28). The solution accomplished by the computer program in Appendix C is as follows:

Assume $t'_{_0}$, $t'_{_1}$

\[ \theta = \cos^{-1}(0.5 + 0.75 t' - 0.25 t'^3) \]  \hspace{1cm} (4)

\[ \frac{d}{b} = \frac{2.55}{\theta + \alpha} \left( \frac{1}{t'^2} - 1 \right) \left( 1 - \frac{\cos \theta}{\cos \alpha} \right) - \frac{1}{2} \]  \hspace{1cm} (3)

\[ \frac{x_s}{b} = \frac{1}{\tan \alpha} \left[ \frac{w/b}{\cos \alpha} - \left( \frac{d}{b} + \frac{1}{2} \right) \right] \]  \hspace{1cm} (27)

\[ G(t') = \frac{x_s}{x_r} \left\{ \frac{2.55}{\theta + \alpha} \left( \frac{1}{t'^2} - 1 \right) \sin \theta + \sin \alpha \right\} \]  \hspace{1cm} (28)

\[ \frac{d}{dt'} \frac{G}{t'_{_1} - t'_{_0}} \]  \hspace{1cm} (29)
\[ t'_2 = t'_1 - \frac{G(t'_1)}{G'(t'_1)} \]  

When the value of \( G(t') \) is reduced to zero, the values of the following parameters are found:

\[ \theta, \frac{d}{b}, \frac{x_s}{b}, t' \]

The values of these parameters are used in the following relations to find the additional parameters pertinent to the design.

\[ b = \left[ \frac{t}{\frac{x_s}{b} \left( \frac{x_s}{b} \frac{V_1}{Q_{co}} + s_t \right)} \right]^{2/3} \left[ \frac{J}{\rho z} \right]^{1/2} \]  

(26)

\[ V_o = \left( \frac{J}{\rho z b} \right)^{1/2} \]  

(31)

\[ Q_o = V_o b z \]  

(32)

\[ Q_c = Q_{co} Q_o \]  

(33)

\[ P_o = \frac{J}{\rho z b} \]  

(34)
\[ c = \frac{Q_C}{z} \left( \frac{\rho}{2 \ P_C} \right)^{1/2} \]  

\[ V_L = \frac{2.28}{(x_s/b)^{1/2}} \text{ (from Albertson's velocity profile)} \]  

\[ w = \frac{w}{b} \]  

\[ K_O = 0.435 \ P_O \ Q_O \]  

\[ K_C = 0.435 \ P_C \ Q_C \]  

\[ K_T = K_C + K_O \]  

\[ J_C = 2 \ c \ z \ P_C \]  

For the test amplifier design the following parameters were assumed.

\[ J = 0.4 \ lb \]  

\[ t = 0.002 \ sec \]  

\[ \rho = 1.655 \ (10)^{-4} \ lb\text{-sec}^2/\text{in}^4 \]
\[ z = 0.200 \text{ inches} \]

\[ \frac{x_s}{x_r} = 1.0 \]

\[ \frac{Q_c}{Q_o} = 0.3 \]

\[ \frac{P_c}{P_o} = 1.0 \]

\[ \frac{w}{b} = 1.44 \]

\[ S_t = 1.0 \]

\[ \frac{v_1}{V_o} = 0.05 \]

\[ \frac{V_h}{V_o} = 1.0 \]

When these values were used the following design parameters were obtained.
\[ b = 0.071 \text{ in.} \quad t' = 0.616 \]

\[ \frac{x_s}{b} = 5.9 \quad \theta = 28^\circ \]

\[ \frac{d}{b} = 0.126 \quad w = 0.102 \text{ in.} \]

\[ P_o = 14.1 \text{ psi} \quad c = 0.021 \text{ in.} \]

\[ P_c = 14.1 \text{ psi} \quad V_d = 0.939 \]

\[ Q_o = 1.51 \text{ gpm} \quad K_o = 9.26 \text{ Watt} \]

\[ Q_c = 0.452 \text{ gpm} \quad K_c = 2.78 \text{ Watt} \]

\[ J_c = 0.12 \text{ lb.} \quad K_t = 12.04 \text{ Watt} \]

These parameters characterize performance better than that which the specifications require. However, the validity of this analytical work must be determined. Also the following assumed parameters must be investigated to find their correct values.

\[ \frac{V_h}{V_o}, \frac{Q_c}{Q_o}, \frac{P_c}{P_o}, \frac{x_s}{x_r}, S_t, V_1 \]

This is accomplished in the following chapter.
CHAPTER 3
Experimental Results

3.1 Objectives

The previous chapter provides an analytical design of a fluid amplifier with several experimentally determined parameters. It is now necessary to check the parameter values and to determine the validity of the analytical results. The investigation is accomplished in three basic experiments.

a. A test is performed to determine the static control flow required to switch. Receiver flow ratio, \( Q/Q_0 \), is plotted versus control flow ratio, \( Q_c/Q_0 \), to determine a method of predicting the maximum control flow.

b. The receiver area ratio, \( A/A_0 \), and receiver distance, \( x_s/b \), are varied and receiver flow ratio, \( Q/Q_0 \) measured. This is compared with the analytical results with different values of \( V_h/V_o \).

c. The control flow, \( Q_c \), and supply velocity, \( V_o \), is varied as the control ports are being supplied a square wave of pressure. The switching constants are determined and the switching curves are compared with analytical results.
3.2 Apparatus

The fluid amplifier used for testing was made to allow as much variation of parameters as possible. It consists of separate flat pieces which make up the supply nozzle, the sidewalls and the splitter (see Figures 3.1 through 3.5). The pieces are then sandwiched between two plates. This arrangement allows movement of the pieces and separate removal or replacement. The pieces were ground to very close tolerances to prevent leakage and allow setting dimensions accurately with thickness gauges. The apparatus was very successful in this respect.

The amplifier was enclosed in a pressure vessel in which it could be submerged and pressurized to prevent cavitation. The hydraulic power supply consists of a centrifugal pump, five flowmeters and a reservoir with temperature control included.

For experiments in which steady state control flow was desired, each control port was fed separately through two flowmeters of different range for accurate measurement. When switching was desired, an electromagnetic valve was installed as a first stage. This first stage supplied a pressure square wave which was measured by high frequency pressure transducers in the control ports. The jet switching was measured by pressure transducers in the receivers.

The pressure signals were measured by an accurate voltmeter for steady state signals and an oscilloscope with camera for transient signals.
Fig 3.1  Schematic of the First Test Amplifier
Fig. 3.3  Schematic of the Second Test Amplifier
Figure 3.5 Second Test Amplifier with Instrumentation
The adjustable amplifiers were made in two configurations. The first was made with large inefficient receivers to allow variation of the interaction region without varying the receiver performance. The second amplifier was made as close as possible to the final design to check the results.

3.3 Steady State Switching

This test was performed to demonstrate the degree of asymmetry in the amplifier and to provide some insight into the mechanism of switching. The supply flow was 1.4 gpm and the control flow was fed through a flowmeter with a full scale range of 0.1 gpm. The procedure was to increase control flow on the attached side until switching occurred. Three different values of control flow on the opposite side were used to give a curve of normalized receiver flow versus normalized control flow difference as shown in Figure 3.6.

Several conclusions can be drawn from Figure 3.6. First, the amplifier is quite bi-stable. Stability is demonstrated by the fact that entrainment flow is high and is affected little by positive control flow. Entrainment is the amount of flow in the receiver above that of the supply and positive control flow is that which is supplied to the unattached side. These are characteristics of a very low receiver impedance.
Fig. 3.6  Steady State Switching Curves
Secondly, the amplifier is quite sensitive to control flow. The figure shows a flow gain of approximately twenty; flow gain is defined as the supply flow divided by the control flow required to switch. High flow gain is an indication of conditions approaching unstable oscillations. Unstable attachment is caused by short splitter distance, $x_s$, and little wall setback, $d$. These conditions require little jet curvature and hence, a small pressure gradient across the jet for attachment. This pressure gradient could easily be overcome by receiver impedance. This conclusion would seem to contradict the previous one, but the fact is that the very low receiver impedance makes a geometry very stable which would otherwise be unstable. This condition will prevail in all designs for the reaction jet system due to the requirement that receiver flow be greater than supply flow for high momentum recovery.

The flow recovery predicted by Reid, as in Figure 2.7, can be greater than unity because of the internal diffusion process which occurs in the receiver. Constant area diffusion occurs when the parabolic velocity profile is transformed to the profile shape of turbulent pipe flow. This diffusion reduces the pressure at the entrance of the receiver which entrains flow from the surroundings.

Finally, the data shows considerable asymmetry in the control flow required to switch. This asymmetry was caused by what was later found to be an error in the center line position of the supply nozzle of .001 inch, causing a difference in wall setback of .002 from one side to the other. This difference is ten percent of the wall setback of .021.
It should be noted that this data was taken on the first apparatus. As was mentioned earlier, the receiver entrance area ratio was not of optimum value and the receiver provided little momentum recovery because of the need to accommodate adjustment. However, the receiver exit to entrance area ratio was increased to reduce the receiver impedance and obtain the desired high entrainment flow. This is the reason for low momentum recovery.

In Figures 3.7 and 3.8, the data is plotted in such a manner as to provide some insight into the switching mechanism. The control flow is subtracted from the receiver flow and then plotted versus the difference in control flow. The horizontal line at unity depicts the supply flow. The difference between the curve and the horizontal line is the amount of flow being drawn into the opposite receiver. The conclusion which can be drawn from these figures is that the jet switches when the difference in control flow is approximately one half the original entrainment. In this condition, a small portion of the supply flow goes out the opposite receiver.

3.4 Examining Momentum Recovery

The objectives of the experiment described below were to determine whether the equations derived from Reid's study were valid for a wall attachment device and to determine the relationship between \( V_h/V_0 \) and splitter distance, \( x_s/b \).
Fig 3.7  Steady State Switching
Fig 3.8  Steady State Switching Showing Asymmetry and Reverse Receiver Flow
To accomplish the first objective, the flow recovery $Q/Q_o$ was measured as the receiver area ratio, $A/A_o$, was varied with the splitter distance held constant. The second apparatus was made for this test with receivers of uniform area. In order to vary the receiver area without changing the attachment distance, the wall angle and offset also had to be adjusted. Hence, separate sidewalls had to be made for each area ratio.

In order to perform these tests, the control port on the side opposite the attached jet had to be vented or provided with sufficient control flow to raise the interaction region to ambient pressure. It is suggested by the author that the entrainment flow was prevented from coming in the opposite receiver by the small wall angle and offset on this, the second apparatus. The small wall angle and offset were required by the geometrical constraint of the small receiver area ratio. A blunt or cusp shaped splitter is considered an improvement over the conventional pointed splitter by some (see Reference 11) and would provide some geometrical flexibility.

The effect mentioned above may also have been caused by the very short splitter distance. The splitter to attachment distance ratio was kept at unity for both the first and second apparatus. The first apparatus had an attachment distance of 7.72 nozzle widths compared to 5.9 for the second apparatus. Although the effect of the splitter to attachment distance ratio was not studied, it may be that a value of unity is insufficient for short attachment distances.
The results, shown in Figures 3.9 and 3.10, match the theoretical prediction very well. The error at the low area ratio was caused by viscous losses. All of the receivers used were of the same length so that the length to diameter ratio became very high for the small receiver. This allowed boundary layers to form and hence, viscous forces to take affect.

The data approximates the curve corresponding to a value of 0.75 for \( V_h/V_o \). The maximum momentum recovery is approximately 78 percent at the receiver area ratio of 1.9. The splitter distance being held constant at 6.7 nozzle widths.

To find the value of \( V_h/V_o \) at other splitter distances, the momentum recovery was measured as the splitter distance was changed. The area ratio was kept constant at 1.44. To prevent large changes in the wall setback as the splitter was moved, sidewalls were used with zero wall angle.

The results of this test are plotted in Figure 3.11 to compare the effects of splitter distance on momentum recovery with the theoretical effect of \( V_h/V_o \). It is significant to note that both \( V_h/V_o \) and \( x_s/b \) are linearly related to momentum recovery and hence linearly related to each other. From Figure 3.11,

\[
\frac{J}{J_o} = 1.3 \frac{V_h}{V_o} - 0.21
\]  

(3.1)
Fig 3.9  Flow Recovery versus Area Ratio
Fig 3.10  Momentum Recovery versus Area Ratio
\[
\frac{J}{J_0} = 1.345 - 0.0085 \frac{x_s}{b} \tag{3.2}
\]

\[
\frac{V_h}{V_o} = 1.189 - 0.0681 \frac{x_s}{b} \tag{3.3}
\]

The relationship between splitter distance and \( \frac{V_h}{V_o} \) can be included in the amplifier design in place of the assumption that \( \frac{V_h}{V_o} \) is equal to unity. The values of momentum recovery and \( \frac{V_h}{V_o} \) for a splitter distance of 5.9 are respectively 0.823 and 0.787. According to the solution of Equation 19 and Figure 2, the area ratio should be adjusted for maximum momentum recovery.

\[
\frac{A}{A_o} \frac{V_h}{V_o} = 1.44
\]

\[
\frac{A}{A_o} = 1.83
\]

The momentum recovery will then be 85 percent if the splitter will remain at 5.9 nozzle widths when the area is increased to this value. However, due to geometrical constraints the closest possible splitter distance for this area ratio is 6.2. Then the momentum recovery would be 80 percent. But an area ratio of 1.6 can be attained with a splitter
distance of 5.9, which gives a momentum recovery of 84 percent. This appears to be the optimum value for a straight walled, pointed splitter geometry.

The author feels that other geometries would give higher momentum recovery. The angle at which the jet attaches to the wall is nearly 30° which suggests a receiver at that angle to the attachment wall to avoid distortion of the velocity profile. Sarpkaya\textsuperscript{11} has experimented with convex walled amplifiers and found a considerable increase in performance. A detailed study similar to that completed by Levin and Manion is needed to predict attachment distances for curved wall devices.

It can be concluded that Reid's study gives a good approximation of the flow and momentum recovery in a wall attachment device.

3.5 Measurement of Switching Time

The object of the experiment described below was to determine the validity of Equation (22) for predicting switching times and to evaluate the constants in the equation. The switching time was measured as the control flow and supply velocity were varied.

The switching time was determined by measuring the delay between a pressure pulse in the control port and a pressure pulse in the receiver entrance. Advantage was taken of the fact that the receiver inertia causes a momentary pressure gradient to develop through the
Figure 3.12A Switching Time versus Control Flow and Supply Velocity

-73-

a) $V_1 = 0.7$, $S_t = 3.0$

b) $V_1 = 0.8$, $S_t = 2.0$

c) $V_1 = 0.9$, $S_t = 1.0$

$V_o = 40-45$ ft/sec
$x_s = 0.463$
$z = 0.200$

SWITCHING TIME (msec)

CONTROL FLOW RATIO, $Q_c / Q_o$
Figure 3.12B Switching Time versus Control Flow and Supply Velocity

-74-
receiver. Hence, an impulse is seen at the entrance of the receiver as the jet switches. The control pulse was a square wave provided by an electromagnetic shuttle valve.

The data is plotted in Figures 3.12 A and 3.12 B compared with theoretical approximations. In Figure 3.12 A the theory matches very well with switching constants of the following value:

\[ V_1 = 0.08 \]

\[ S_t = 2.0 \]

In Figure 3.12 B, considerable variation of the switching constants can match the experimental data. This data was inaccurate due to corruption of the control signal by vibrations from the shuttle of the driver valve hitting the stops. The vibrations can be seen clearly in Figure 3.13. The pressure waves were removed when obtaining the data of Figure 3.12 A by including a small air pocket in the control port which provided damping.

The control port size was kept constant as the control flow was increased. This caused a large change in the control flow velocity. The momentum of the control flow restricted the path of the supply jet. To maintain a constant supply flow, the supply pressure and hence, the supply velocity were increased. The supply velocity was evaluated by measuring the pressure difference across the supply nozzle.
Figure 3.13 Switching Time Measurement

Figure 3.14 High Frequency Operation

\[ Q_{co} = 0.2 \]
\[ V_o = 40 \text{ ft/sec} \]
\[ f = 20 \text{ cps} \]

\[ Q_{co} = 0.3 \]
\[ V_o = 35 \text{ ft/sec} \]
\[ f = 200 \text{ cps} \]
It is concluded that Equation (22) gives a good approximation of
the switching time if the constants are experimentally evaluated for
each design. However, one can estimate those constants closely after
some experience.

It should be noted that these amplifiers exhibited satisfactory
performance at higher frequency than the measured switching times
warrant. Performance at 200 cps is pictured in Figure 2.14. The
amplifier performance deteriorates after 200 cps due to deterioration
of the driving signal. The switching time measured at the control flow
and supply velocity of Figure 3.14 is 5 milliseconds. This corre-
ponds to a maximum cycling frequency of 100 cps. The reason that
higher frequencies are attainable is that the jet need not completely
attach to the wall to deliver a pressure pulse, but simply pass the
splitter. When driven at high frequency, the jet oscillates from side
to side with an amplitude inversely proportional to the mass of the jet
and the driving frequency.

It is also significant that the switching time approaches an
asymptote at approximately the same control flow required for steady
state switching. Also, the switching time curve levels off at control
flows greater than five times that value. This information would im-
ply that steady state switching control flow should be minimized to
reduce the control flow necessary to attain desired switching times.

3.6 Conclusions

A design procedure has been established in Chapter 2 and the
previous sections of Chapter 3 have established its validity. The fol-
lowing is a summary of other pertinent conclusions reached.
a. The control flow required for steady state switching is approximately one half the jet entrainment flow and dynamic switching requires three to five times that much flow.

b. The momentum recovery and $V_h/V_o$ are linear functions of splitter distance for a constant area ratio. For $A/A_o = 1.44$,

\[
\frac{J}{J_o} = 1.345 - 0.0885 \frac{x_s}{b}
\]  \hspace{1cm} (3.1)

\[
\frac{V_h}{V_o} = 1.189 - 0.0681 \frac{x_s}{b}
\]  \hspace{1cm} (3.2)

Using this result, and optimizing the splitter distance and area ratio, optimum momentum recovery is obtained:

\[
\frac{J}{J_o} = 0.84 \quad \frac{Q}{Q_o} = 1.17
\]

\[
\frac{x_s}{b} = 5.9 \quad \frac{A}{A_o} = 1.6
\]
c. The switching times measured were about double the times assumed initially, but the amplifier was found to perform satisfactorily at higher cycling frequencies than the switching times warrant due to the fact that the jet need not completely attach to the wall to deliver a pressure pulse of full amplitude. Hence, longer switching times may possibly be sufficient to meet the cycling frequency requirement. However, the switching constants were found to be:

\[ V_1 = 0.08 \]

\[ S_t = 2.0 \]

Using the above conclusions the final design was produced. Assuming the following parameters:

\[ J_0 = 0.475 \text{ lbs} \]

\[ \frac{w}{b} = 1.6 \]

\[ t = 0.002 \text{ sec} \]

\[ \frac{V_n}{V_o} = 0.787 \]

\[ \rho = 1.65 \times 10^{-4} \text{ lb.-sec}^2/\text{in}^4 \]

\[ V_1 = 0.08 \]
\[ \alpha = 10.3^\circ = 0.18 \text{ rad.} \] \[ S_t = 2.0 \]

\[ \frac{x_s}{x_r} = 1.0 \]

\[ \frac{Q_c}{Q_o} = 0.3 \]

\[ \frac{P_c}{P_s} = 0.333 \]

\[ z = 0.200 \]

The following design parameters were obtained:

\[ b = 0.051 \]

\[ t' = 0.615 \]

\[ \frac{x_s}{b} = 5.91 \]

\[ \theta = 25^\circ \]

\[ d = 0.050 \]

\[ V_o = 44.4 \text{ ft/sec} \]

\[ w = 0.081 \text{ in.} \]

\[ c = 0.026 \text{ in.} \]

\[ Q_o = 1.39 \text{ gpm} \]

\[ Q_c = 0.417 \text{ gpm} \]

\[ P_o = 23.4 \text{ psi} \]

\[ P_c = 7.8 \text{ psi} \]
$K_o = 14.2$ watt \hspace{1cm} $K_c = 1.42$ watt

$K_t = 15.6$ watt \hspace{1cm} $J = 0.4$ lbs

The performance parameters listed above are slightly better than the those of the electromagnetic reaction jet system listed in Table 1.1. It is concluded, then, that the liquid fluidic reaction jet system is indeed a feasible substitute.
APPENDIX A

Relating Steady State Thrust and Power for Nozzle Flow

A fundamental form of Newton's law is the momentum theorem for a control volume. The theorem states that the time rate of change of momentum of the mass in the control volume plus the summation of momentum crossing the control surface equals the sum of all external forces. For the one-dimensional problem of Figure A.1,

$$\sum F_x = T = \frac{\partial}{\partial t} \int \int \int \rho \ V_x \ dv + \int \int \rho \ V_x \ (\bar{V} \cdot d\bar{A})$$

where $F_x$ = external forces on the control volume in the x-direction (lbf)

$T$ = external force which equals reaction force in x-direction (lbf)

$t$ = time (sec)

$\rho$ = density (lb-sec$^2$/in$^4$)

$V$ = volume (in$^3$)

$A$ = area (in$^2$)

$C_v$ = control volume

$C_s$ = control surface

(A-1)
Since a steady state condition is being examined, the time rate of change of momentum is zero. Hence,

\[ T = \int \int \frac{\rho V_x}{C_s} (V_x \, \text{d}y \, \text{d}z) = \rho V^2 A = J \]  

This equation says that the thrust in the x-direction is equal to the momentum flux in the x-direction across the boundary, \( J \). It is assumed that the velocity is constant over the cross section of the exit.

The fluid power is the product of the total pressure and the flow rate.

\[ K = P Q \]  

If the nozzle is short and smooth, the Bernoulli equation holds

\[ P_{\text{total}} = P_{\text{static}} + \frac{1}{2} \gamma V^2 \]  

In the case considered the static pressure is zero at the exit of the nozzle. Also, since the velocity is assumed to be nearly constant over the cross section of the exit,

\[ Q = A V \]
Combining these three equations,

$$K = \frac{1}{2} \rho A V^3 \quad (A-6)$$

And now combining this with Equation (A-2)

$$K = \frac{1}{2} J V \quad (A-7)$$

Hence, the power is simply a product of the thrust and the exhaust velocity if there are no losses. Rearranging, one can see that the ideal thrust per unit power is simply a function of the velocity.

$$\frac{J}{K} = \frac{4.6}{V} \quad (A-8)$$

with $K$ in watts. This relation is plotted in Figure A.2.

The conclusion from this result is that the lower the velocity of the operating point, the higher the thrust per unit power.
Fig A-2  Thrust, Velocity and Power Relationship for Nozzles
APPENDIX B

Dynamic Affects of the Receivers

Since the receivers of a fluid amplifier represent a mass of fluid between the switching jet and the output port, they must be examined to determine how the fluid inertance and compressibility affect the output. It shall be determined how the reaction thrust varies in response to a step input of momentum flux impinging on the receiver. Although it requires a finite time for the supply jet to move across the splitter, the simpler case of the step input will be discussed here.

Consider the control volume of Figure (B-1) and write the momentum theorem as in Appendix A.

\[
\sum F_x = \frac{\partial}{\partial t} \iiint \rho \, V_x \, d \, v + \iiint \rho \, V_x \, (V \cdot d \, A) \tag{A-1}
\]

This equation will now be evaluated term by term. Where,

\[
\sum F_x = T(t) \tag{B-1}
\]

Since the velocity is a function of time, the first term on the right becomes,
Figure B.1: Dynamic Model of Receiver
\[
\frac{\partial}{\partial t} \int_{V} \rho \mathbf{V}_x (t) \, d\mathbf{v} = \int_{V} \frac{\partial}{\partial t} \left[ \rho \mathbf{V}_x (t) \right] \, d\mathbf{v} \quad (B-2)
\]

The continuity equation will be needed to evaluate the portion within the integral. It states that the time rate of change of the mass of the system is equal to the rate of change of the density of the control volume plus the net flow across the control surface.

\[
\frac{\partial m}{\partial t} = \frac{\partial}{\partial t} \int_{V} \rho \, d\mathbf{v} + \int_{s} \rho \mathbf{V} \cdot d\mathbf{A} \quad (B-3)
\]

Since the fluid density is uniform throughout the volume and the velocity is uniform over the entrance and exit areas,

\[
\frac{\partial m}{\partial t} = \frac{\partial}{\partial t} (\rho \mathbf{v}) + \rho (\mathbf{V} \cdot \mathbf{A})_{\text{in}} - \rho (\mathbf{V} \cdot \mathbf{A})_{\text{out}} \quad (B-4)
\]

\[
= \mathbf{v} \frac{\partial \rho}{\partial t} + \rho \frac{\partial \mathbf{v}}{\partial t} + \rho (Q_{\text{in}} - Q_{\text{out}}) \quad (B-5)
\]

Since the geometry is fixed, the volume cannot change with time. It must now be determined whether the density changes significantly. It is known (Reference 3) that if the time required for a sound wave to
traverse the length of the passage is small compared to the characteristic time of the dynamics of interest, then the fluid nozzle may be considered incompressible. To determine that time, consider

$$T_c = L \left( \frac{\rho}{\beta} \right)^{1/2} = .04 \text{ millisec.} \quad (B-6)$$

where

$$L = 1 \text{ in (approximate length of passage)}$$
$$\rho = 1.655 \times 10^{-4} \text{ lb-sec}^2/\text{in}^4$$
$$\beta = 9.5 \times 10^4 \text{ psi}$$
$$\left( \frac{\beta}{\rho} \right)^{1/2} = \text{speed of sound (in/sec)}$$

Since the jet will require at least two milliseconds to switch, the time for a sound wave to reach the outlet is only about two percent of this time. Hence, the fluid will be assumed incompressible. Using this result in Equation (B-5) the continuity equation simply states that the mass rate of flow is constant throughout the passage.

$$\rho A V_{in} = \rho A V_{out} \quad (B-7)$$
Returning now to Equation B-2

\[
\int_v \frac{\partial}{\partial t} \left[ \rho \mathbf{V}_x (t) \right] \, dv = \int_v \rho \frac{\partial \mathbf{V}_x}{\partial t} \, dv \quad (B-8)
\]

This integral represents the force caused by the sum of each element of mass multiplied by its x-component of velocity. An example of its evaluation can be demonstrated with the receiver model of Figure B-1.

\[
\text{total volume, } v = A (r \theta + 1) = A \left( \frac{\pi}{2} r + 1 \right) \quad (B-9)
\]

\[
dv = A (rd\theta + dl) \quad (B-10)
\]

\[
\frac{\partial \mathbf{V}_x}{\partial t} = \frac{\partial \mathbf{V}}{\partial t} \cos \theta \quad (B-11)
\]

\[
\int_v \rho \frac{\partial \mathbf{V}_x}{\partial t} \, dv = \rho A \frac{\partial \mathbf{V}}{\partial t} \int_0^{\pi/2} r \cos \theta \, d\theta + (1) \, dl \quad (B-12)
\]

\[
= \rho A r \frac{\partial \mathbf{V}}{\partial t} \int_0^{\pi/2} \cos \theta \, d\theta \quad (B-13)
\]
The value of this term is simply the portion of the total mass which moves in the x-direction times the total acceleration. That portion will be called, \( m_x \). For this case,

\[
\frac{m_x}{m_t} = \frac{Ar}{(A \frac{\pi}{2} r + 1)} = \frac{1}{\frac{\pi}{2} + \frac{1}{r}} \tag{B-14}
\]

Hence,

\[
\frac{\partial}{\partial t} \int \rho \, v_x \, d \, v = m_x \frac{\partial v}{\partial t} \tag{B-15}
\]

The second term of Equation (A-1) is the same as in Appendix A except that the velocity is a function of time.

\[
\int \rho \, v_x \, \overline{v} \cdot d \, \overline{A} = \rho \, A \, v^2(t) \quad \text{c.s.} \tag{B-16}
\]

Combining these results, (A-1) becomes

\[
T = m_x \frac{\partial v}{\partial t} + \rho \, A \, v^2(t) \tag{B-17}
\]
To evaluate $\partial V / \partial t$, sum the forces on the fluid at the entrance of the receiver. Since the area is constant the acceleration is the same on all the fluid particles; therefore,

$$
\sum F = J_0 - J(t) = m_t \frac{dV}{dt}
$$

(B-18)

Where,

$$
J(t) = \rho A V^2(t) \quad \text{and} \quad J_0 = \rho A V_0^2
$$

(B-19)

Substituting (B-19) into (B-18),

$$
\frac{m_t}{\rho A} \frac{dV}{dt} = V_0^2 - V^2(t)
$$

(B-20)

Rearranging and integrating,

$$
\int_{V_0}^{V} \frac{dV}{V_0^2 - V^2} = \frac{\rho A}{m_t} \int_{0}^{t} dt
$$

(B-21)

performing the integration one obtains,
\[ \frac{1}{2 V_o} \log \frac{V_o + V}{V_o - V} = \frac{\rho A}{m_t} t \]  \hspace{1cm} (B-22)

Again, rearranging, an expression for the velocity is obtained as a function of time.

\[ V(t) = V_o \left[ \frac{e^{t/\tau} - 1}{e^{t/\tau} + 1} \right] \]  \hspace{1cm} (B-23)

where

\[ \tau = \frac{m_t}{2 \rho A V_o} = \frac{1}{2} \frac{L}{V_o} \]

This equation is plotted in Figure B. 2.

Returning now to the thrust, combine Equations (B-17), (B-18), and (B-19).

\[ T(t) = \frac{m_x}{m_t} \left[ J_o - J(t) \right] + J(t) \]  \hspace{1cm} (B-24)

or

\[ \frac{T(t)}{J_o} = \frac{m_x}{m_t} \left[ 1 - \frac{m_x}{m_t} \right] \frac{V(t)^2}{V_o} \]  \hspace{1cm} (B-25)
\[ \tau = 1.19 \text{ millisecond} \]
for \( l = 1.0 \text{ inch} \)
\[ V_0 = 35 \text{ ft/sec} \]

Receiver Velocity Ratio, \( \frac{V}{V_0} \)

Fig B-2  Receiver Velocity Response to Jet Switch

ELAPSED TIME, \( \frac{1}{\tau} \)

1.0  2.0  3.0  4.0  5.0  6.0
The conclusion to be drawn from this result is that the thrust is obtained as the jet is switched even though the velocity may be initially zero (see Figure B.3.). However, the magnitude is reduced by the factor $m_x/m_t$, which is a function of the geometry. Assuming that the geometry could be adjusted so that this factor was near unity, the above equation would imply that the thrust could be changed as fast as the jet could be switched. This result would also apply to a step decrease in the momentum applied to the receiver because the established flow would have to decelerate, causing a cancelation to the thrust of that flow. In summary, the acceleration of the flow should be considered in reaction jet thrust and not simply the velocity of the flow.
Fig B-3  Reaction Force Response to Jet Switch

\[ \frac{m_x}{m_T} = 0.7 \]

- ELAPSED TIME, t/τ -

- NET THRUST -

- MOMENTUM FLUX -

- FLUID ACCELERATION -

REACTION FORCE F\textsubscript{R0}
APPENDIX C
Computer Programs and Additional Photographs

Program for Receiver Study

A=1.44
V=0.9346
A13=1.0
A1E=1.0
NPM=1.0
DLA=.1
DLV=0.
DLA13=0.
DLA1E=0.
TN=0
AM=3.0

BEGIN
T='H REPEAT, FOR NP=0,.2,NP.G,NPM
T=1.-.5*A*V
Q=(A*V/(2.*DE))*(T+(T.P.2.+2.*DE).P.5)
J=0.P.2.*A1E/A
PRINT ONLINE FORMAT A01,J,Q,DE,A,V,A13,A1E,NP
V'S A01=$8F10.4*$

REPEAT
CONTINUE
A=A+DLA
V=V+DLV
A13=A13+DLA13
A1E=A1E+DLA1E
PRINT COMMENT "$ W'R A.L.E.AM, TRANSFER TO BEGIN
W'R TN.E.O,TRANSFER TO START
INTEGER TN
EXECUTE EXIT.
END OF PROGRAM
Program for Amplifier Design

PRINT COMMENT $DESIGN OF A FLUID AMPLIFIER$
J=.475
T=.002
RO=1.65E-4
A=.18
Z=.2
XSR=1.
QCO=.3
VL=0.08
PCS=.333
ST=2.0
ERRF=1.0E-5
N=100
WB=1.6
BEGIN
PRINT COMMENT $READ DATA$
READ DATA
T1=.59
T2=.595
TN=0
INTERNAL FUNCTION CF.(T1)=XSR*(((2.55/(A+A1))
1*(1./(T1)*P.2)-1.)*(SIN.(A)*SIN.(A1))*(.5*LOG.((1.+T1)/
2*(1.-T1))/((.3*SIN.(A1)*(T1.*P.2)))-(DB+.5)*SIN.(A))-XS1
INTERNAL FUNCTION AIF.(X)=ACOS.((.5+.75*X-.25*X.*P.3.0)
INTERNAL FUNCTION DBF.(X)=(2.55/(A+A1))*(X.*P.-2.-1.)*
1*(1.-COS.(A1)/COS.(A))-DB+.5
INTERNAL FUNCTION XE.(DB)=(1./TAN.(A))*((A1+2)/(A1+COS.(A)))-DB+.5
A1=AIF.(T1)
DB=DBF.(T1)
XS1=CF.(DB)
G1=GF.(T1)
THROUGH REPEAT, FOR K=1,1(.ABS.(T2-T1),L.ERRF).OR.K.GE.N
A1=AIF.(T2)
DB=DBF.(T2)
XS1=CF.(DB)
G2=GF.(T2)
L=(T2-T1)/(G2-G1)
G1=G2
T1=T2
REPEAT
T2=T1-L*G1
WHENEVER K.GE.N, PRINT COMMENT $NO CONVERGENCE$
V=(T/((ST+XS1*VL/QCO)*XS1)).P..666*(J/(RO+Z)),P..333
P'S B, XS1, DB, T1
V0=(((J/(RO+Z*B)).P.+5)/12.0
Q0=((J*B+Z)/RO).P.+5)/3.886
\[ QO = \frac{(J \cdot B \cdot Z) / RO \cdot P \cdot +.5)}{3.886} \]
\[ QC = QC0 \cdot QO \]
\[ PS = \frac{(2 \cdot Z \cdot B)}{PC} \]
\[ PC = PCS \cdot PS \]
\[ C = \frac{(QC \cdot 3.886) / Z) \cdot (RO / (2 \cdot PC) \cdot P \cdot +.5)}{VCLO = 2.28 / XS1 \cdot P \cdot +.5} \]
\[ W = \sqrt{B} \]
\[ KS = 4355 \cdot PS \cdot QC \]
\[ KC = 4355 \cdot PC \cdot QC \]
\[ KT = KS + KC \]
\[ JC = 2 \cdot C \cdot Z \cdot PC \]
\[ P1S W, C, VCLO, VO \]
\[ P1S PS, PC, QC, QO, QC \]
\[ P1S KS, KC, KT, JC \]
\[ P1S A1 \]
PRINT COMMENT $IF RESULTS ARE UNSATISFACTORY, SET TN=1$
READ DATA
WHENEVER TN.E.1, TRANSFER TO BEGIN
INTEGER K, N, TN
END OF PROGRAM
Figure C-4  Parts for Test Amplifiers


