14.01 Principles of Microeconomics, Fall 2007
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Lecture 12

Production Functions and Cost of Production

Outline

1. Chap 6: *Returns to Scale*
2. Chap 6: *Production Function Derivation*
3. Chap 7: *Cost of Production*

1 Returns to Scale

Increasing Returns to Scale

(Lecture 11)

Constant Returns to Scale

- Doubling the inputs leads to double the output:

$$Q(2K, 2L) = 2Q(K, L).$$

- One big firm is as good as many small firms.
- Isoquants are equally distant apart (see Figure 1).

Decreasing Returns to Scale

- Doubling the inputs leads to an output less than twice the original output:

$$Q(2K, 2L) < 2Q(K, L).$$

- Small firms are more efficient.
- Isoquants become further apart (see Figure 2).

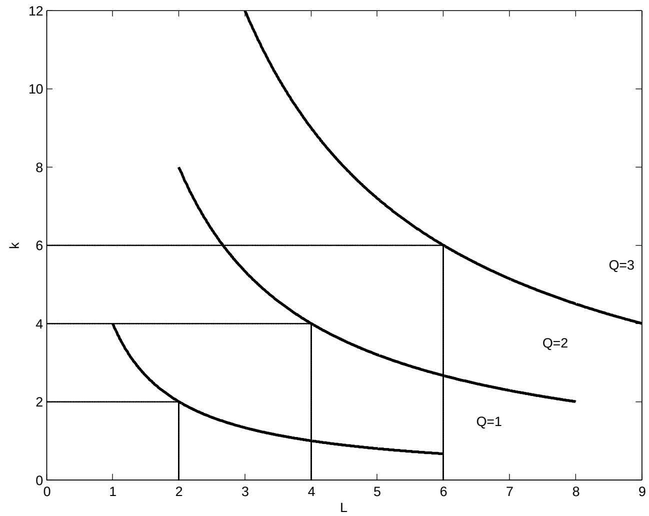

Figure 1: Isoquant Curves, Constant Returns to Scale.

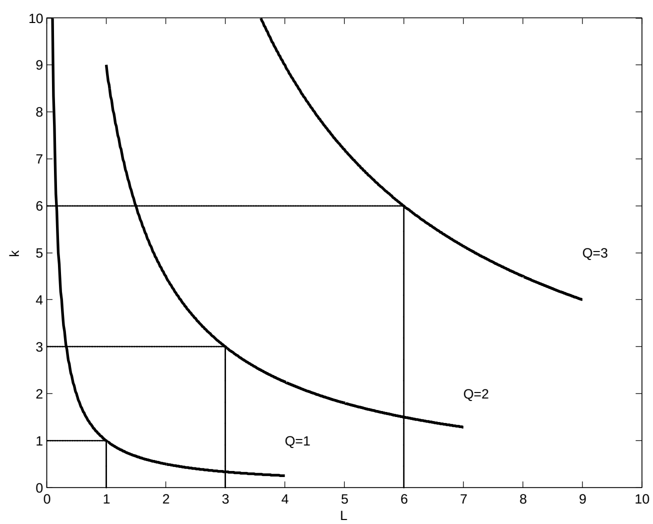

Figure 2: Isoquant Curves, Decreasing Returns to Scale.

Example (Cobb-Douglas Production Function.).

$$Q(K, L) = AL^{\alpha}K^{\beta}.$$

We double both inputs to see what type of returns to scale the production function has.

$$Q(2K, 2L) = A(2L)^{\alpha}(2K)^{\beta} = 2^{\alpha+\beta}AL^{\alpha}K^{\beta} = 2^{\alpha+\beta}Q(K, L).$$

1. If
$$\alpha + \beta > 1,$$
returns to scale is increasing.
2. If
$$\alpha + \beta = 1,$$
returns to scale is constant.
3. If
$$\alpha + \beta < 1,$$
returns to scale is decreasing.

2 Production Function Derivation

Assume that the firm has two technologies A and B, and the corresponding outputs are

$$q_A = \min\{\frac{x}{2}, \frac{y}{1}\},$$
$$q_B = \min\{\frac{x}{1}, \frac{y}{2}\},$$

where the inputs x and y are perfect complements (see Figure 3).
To derive production function, we must know which technology the firm chooses. If the firm choose either A or B, but not both, the isoquant curve for the production function is the black line (see Figure 3). This isoquant curve is not convex. However, the firm can adopt technologies at the same time, and this makes the isoquants convex (see Figure 4).

Thus the production function is:

$$q(x, y) = \begin{cases} \min\{\frac{x}{2}, \frac{y}{1}\}, \text{when } x > 2y. \\ \frac{x+y}{3}, \text{when } \frac{1}{2}y \leqslant x \leqslant 2y. \\ \min\{\frac{x}{1}, \frac{y}{2}\}, \text{when } x < \frac{1}{2}y. \end{cases}$$

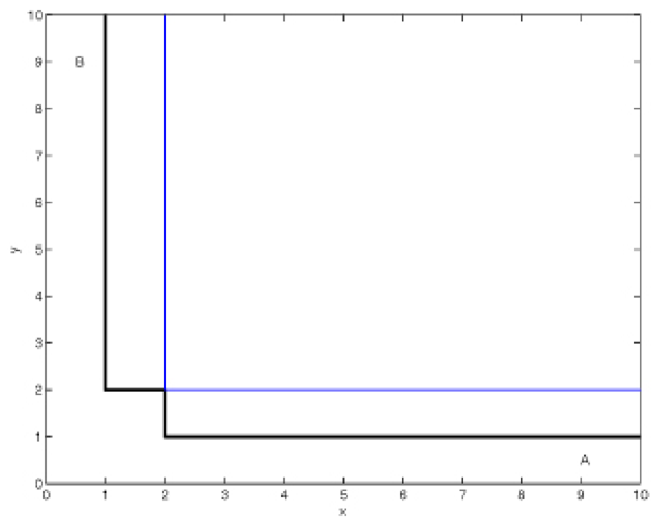

Figure 3: Deriving Production Function, Using Technology A or B.

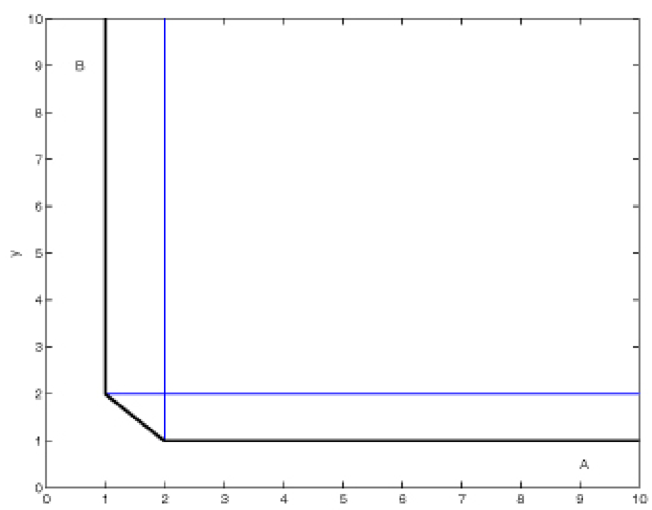

Figure 4: Deriving Production Function, Using Technology A and B.

3 Cost of Production

Cost comes from factor price and how many units are used.

Accounting Cost. Actual expenses plus depreciation.

Economic Cost. Cost to a firm of using resources in production. Also called opportunity cost, the most valuable forgone alternative.

	Wage	Transportation Cost	Accounting Cost	Opportunity Cost
Job 1	150	0	0	180
Job 2	200	20	20	150

Table 1: Accounting Cost and Opportunity Cost.

Example (Two job opportunities (see Table 1)). If the person accepts Job 2, the most valuable forgone opportunity is Job 1.
Opportunity cost does not really happen but must be considered.

Sunk Cost. Expenditure that has been made and cannot be recovered.

Example (Two building choices). A firm has two building choices. For Building 1, they have paid 500,000, and will pay 5,000,000 in the future; for Building 2, they have not paid anything, and will pay 5,300,000 in the future. Although Building 2 is cheaper than Building 1, the firm will choose Building 1 because the 500,000 is sunk.

Total Cost.

$$\text{Total Cost} = \text{Variable Cost} + \text{Fixed Cost}.$$

Fixed Cost. A cost that is actually incurred, but independent of the level of output.

Variable Cost. A cost that is actually incurred, and dependent of the level of output.

Example (Short Run). Capital K is fixed, and Labor L is variable; hence, the cost of K is a fixed cost, and the cost of L is a variable cost.

Here is another definition of sunk cost.

Sunk Cost. A fixed cost which is also independent of output, but whose cost is not incurred, because of no cash outlay and no opportunity cost.
Usually fixed costs are considered sunk costs because they happen before production begins.